

Matrix A

In[69]:= **A = {{1, 0}, {0, 1}, {1, 0}}**

Out[69]= {{1, 0}, {0, 1}, {1, 0}}

In[45]:= **x = {1, 0, 1}**

Out[45]= {1, 0, 1}

In[46]:= **y = {Norm[x], 0, 0}**

Out[46]=  $\{\sqrt{2}, 0, 0\}$

In[47]:= **u = {(x - y) / Norm[x - y]}**

Out[47]=  $\left\{ \left\{ \frac{1 - \sqrt{2}}{\sqrt{1 + (-1 + \sqrt{2})^2}}, 0, \frac{1}{\sqrt{1 + (-1 + \sqrt{2})^2}} \right\} \right\}$

In[49]:= **H = IdentityMatrix[3] - 2 \* Transpose[u].u**

Out[49]=  $\left\{ \left\{ 1 - \frac{2(1 - \sqrt{2})^2}{1 + (-1 + \sqrt{2})^2}, 0, -\frac{2(1 - \sqrt{2})}{1 + (-1 + \sqrt{2})^2} \right\}, \{0, 1, 0\}, \left\{ -\frac{2(1 - \sqrt{2})}{1 + (-1 + \sqrt{2})^2}, 0, 1 - \frac{2}{1 + (-1 + \sqrt{2})^2} \right\} \right\}$

In[60]:= **R = H.A // Simplify // MatrixForm**

Out[60]//MatrixForm=

$$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In[62]:= **Q = IdentityMatrix[3].H // Simplify // MatrixForm**

Out[62]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \end{pmatrix}$$

In[70]:= **{Q, R} = QRDecomposition[A]**

Out[70]=  $\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\}, \{0, 1, 0\} \right\}, \left\{ \left\{ \sqrt{2}, 0 \right\}, \{0, 1\} \right\} \right\}$

In[65]:= **Q // MatrixForm**

Out[65]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \end{pmatrix}$$

In[66]:= **R // MatrixForm**

Out[66]//MatrixForm=

$$\begin{pmatrix} \sqrt{2} & 0 \\ 0 & 1 \end{pmatrix}$$

In[74]:= **Transpose[R.Q] // MatrixForm**

Out[74]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

**Matrix B**

In[194]:= **B = {{1, 2}, {0, 1}, {1, 0}}**

Out[194]=

**{{1, 2}, {0, 1}, {1, 0}}**

**Stage 1**

In[195]:= **x = {1, 0, 1}**

Out[195]=

**{1, 0, 1}**

In[196]:= **y = {Norm[x], 0, 0}**

Out[196]=

**{ $\sqrt{2}$ , 0, 0}**

In[197]:= **u = {(x - y) / Norm[x - y]}**

Out[197]=

$$\left\{ \left\{ \frac{1 - \sqrt{2}}{\sqrt{1 + (-1 + \sqrt{2})^2}}, 0, \frac{1}{\sqrt{1 + (-1 + \sqrt{2})^2}} \right\} \right\}$$

In[198]:= **H = IdentityMatrix[3] - 2 \* Transpose[u].u**

Out[198]=

$$\left\{ \left\{ 1 - \frac{2(1 - \sqrt{2})^2}{1 + (-1 + \sqrt{2})^2}, 0, -\frac{2(1 - \sqrt{2})}{1 + (-1 + \sqrt{2})^2} \right\}, \{0, 1, 0\}, \left\{ -\frac{2(1 - \sqrt{2})}{1 + (-1 + \sqrt{2})^2}, 0, 1 - \frac{2}{1 + (-1 + \sqrt{2})^2} \right\} \right\}$$

In[199]:= **R = H.B // Simplify**

Out[199]=

**{{ $\sqrt{2}$ ,  $\sqrt{2}$ }, {0, 1}, {0,  $\sqrt{2}$ }}**

In[200]:= **Q = IdentityMatrix[3].H // Simplify**

Out[200]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\}, \{0, 1, 0\}, \left\{ \frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right\} \right\}$$

**Stage 2**

In[201]:= **x = {1, Sqrt[2]}**

Out[201]=

$$\{1, \sqrt{2}\}$$

In[202]:= **y = {Norm[x], 0}**

Out[202]=

$$\{\sqrt{3}, 0\}$$

In[203]:= **u = {(x - y) / Norm[x - y]}**

Out[203]=

$$\left\{ \left\{ \frac{1 - \sqrt{3}}{\sqrt{2 + (-1 + \sqrt{3})^2}}, \sqrt{\frac{2}{2 + (-1 + \sqrt{3})^2}} \right\} \right\}$$

In[204]:= **H = IdentityMatrix[2] - 2 \* Transpose[u].u**

Out[204]=

$$\left\{ \left\{ 1 - \frac{2(1 - \sqrt{3})^2}{2 + (-1 + \sqrt{3})^2}, -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2} \right\}, \left\{ -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2}, 1 - \frac{4}{2 + (-1 + \sqrt{3})^2} \right\} \right\}$$

In[205]:= **H2 = IdentityMatrix[3]**

Out[205]=

$$\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$$

In[206]:= **H2[[2 ;;, 2 ;;]] = H**

Out[206]=

$$\left\{ \left\{ 1 - \frac{2(1 - \sqrt{3})^2}{2 + (-1 + \sqrt{3})^2}, -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2} \right\}, \left\{ -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2}, 1 - \frac{4}{2 + (-1 + \sqrt{3})^2} \right\} \right\}$$

In[207]:= **H2 // MatrixForm**

Out[207]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 - \frac{2(1 - \sqrt{3})^2}{2 + (-1 + \sqrt{3})^2} & -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2} \\ 0 & -\frac{2\sqrt{2}(1 - \sqrt{3})}{2 + (-1 + \sqrt{3})^2} & 1 - \frac{4}{2 + (-1 + \sqrt{3})^2} \end{pmatrix}$$

In[208]:= **R = Dot[H2, R] // Simplify**

Out[208]=

$$\{\{\sqrt{2}, \sqrt{2}\}, \{0, \sqrt{3}\}, \{0, 0\}\}$$

In[209]:= **Q = Q.H2 // Simplify**

Out[209]=

$$\left\{ \left\{ \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{-1 + \sqrt{3}}{\sqrt{2}(-3 + \sqrt{3})} \right\}, \left\{ 0, \frac{1}{\sqrt{3}}, \frac{\sqrt{2} - \sqrt{6}}{-3 + \sqrt{3}} \right\}, \left\{ \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{3}}, -\frac{-1 + \sqrt{3}}{\sqrt{2}(-3 + \sqrt{3})} \right\} \right\}$$

In[211]:= **Q.R // MatrixForm**

Out[211]//MatrixForm=

$$\begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$$

In[193]:= **{Q, R} = QRDecomposition[B]**

Out[193]=

$$\left\{ \left\{ \left\{ \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} \right\}, \left\{ \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}} \right\} \right\}, \left\{ \left\{ \sqrt{2}, \sqrt{2} \right\}, \left\{ 0, \sqrt{3} \right\} \right\} \right\}$$

In[212]:= **Q // MatrixForm**

Out[212]//MatrixForm=

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{3}} & \frac{-1+\sqrt{3}}{\sqrt{2}(-3+\sqrt{3})} \\ 0 & \frac{1}{\sqrt{3}} & \frac{\sqrt{2}-\sqrt{6}}{-3+\sqrt{3}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{3}} & -\frac{-1+\sqrt{3}}{\sqrt{2}(-3+\sqrt{3})} \end{pmatrix}$$

In[213]:= **R // MatrixForm**

Out[213]//MatrixForm=

$$\begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{3} \\ 0 & 0 \end{pmatrix}$$