Bayesian Analysis in R: brms vs rstanarm

Alice Milivinti
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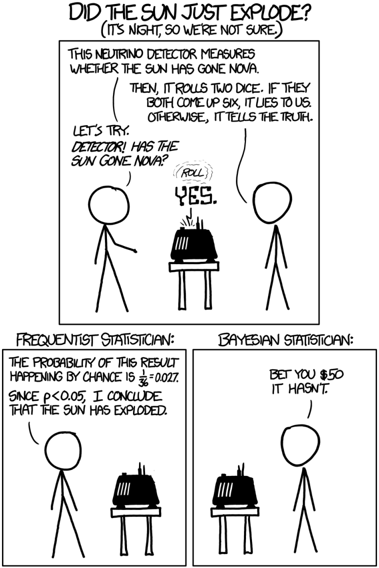
12 april 2018

Bayesians vs Frequentist



Probability is...

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Frequentists

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Fundamentally related to the frequencies of repeated events.

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$Pr(A \mid B)$: Posterior;
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Given the observed data, what is the best estimate of the true value?

Maximum Likelihood

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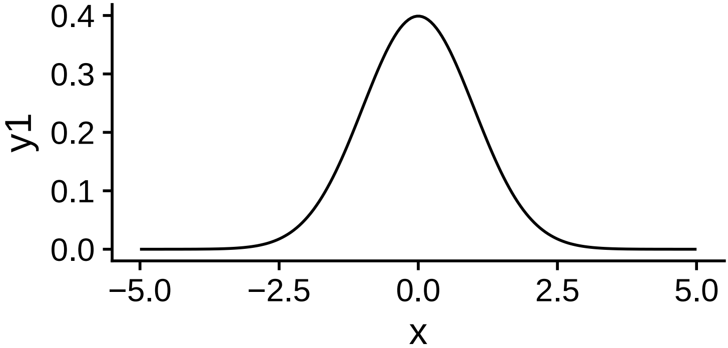
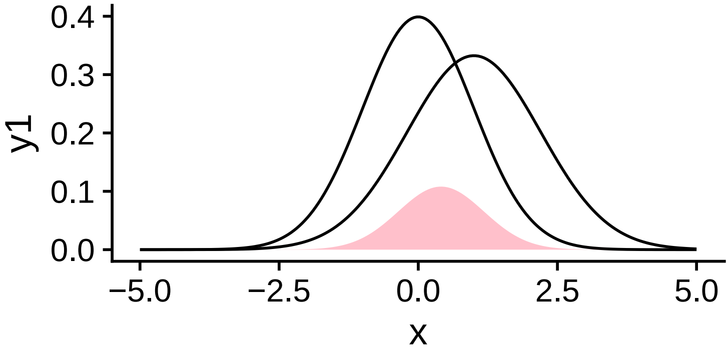
Model: each observation A_i drawn from a Gaussian of width e_i

$$P(B_i \mid A_{true}) = \frac{1}{\sqrt{2\pi e_i^2}} \exp\left[-\frac{(A_i - A_{true})^2}{2e_i}\right]$$



Buldind the Maximum Likelihood

$$\mathcal{L}(B \mid A_{true}) = \prod_{i=1}^{N} P(B_i \mid A_{true})$$



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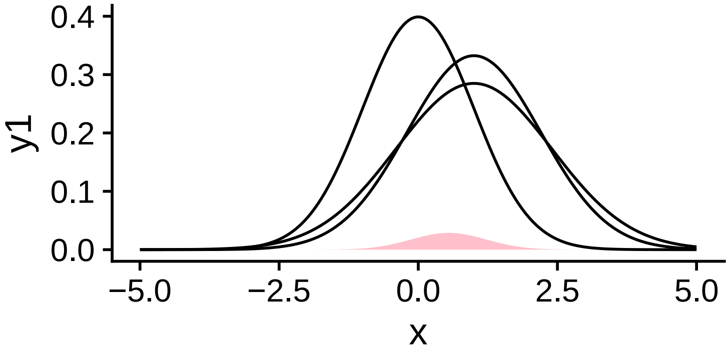

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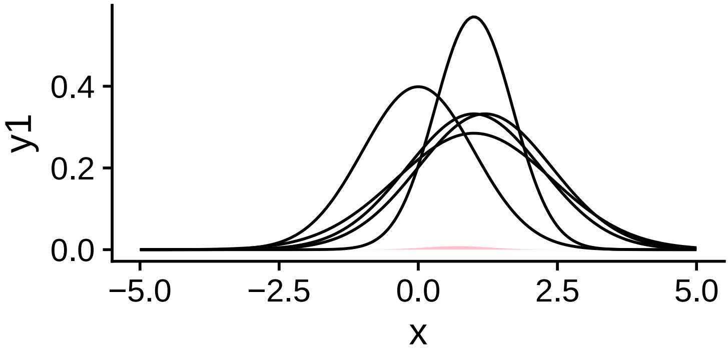

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Fundamentally related to OUR OWN certainty or uncertainty of events.

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$Pr(A \mid B)$: Posterior;
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$Pr(A)$: Prior = Our Knowledge.

Prior, Likeloihood and Posterior

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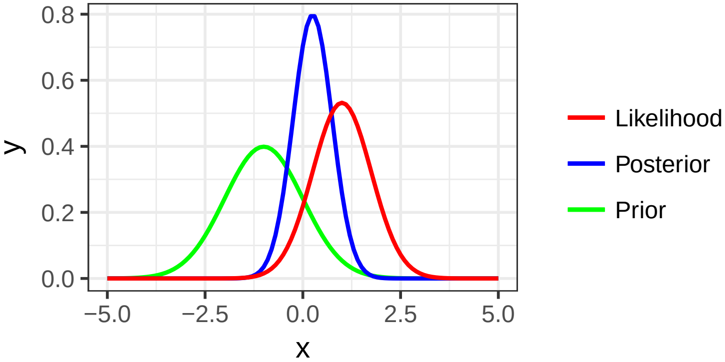
$$\mathbf{Pr(A \mid B)} \propto \mathbf{Pr(B \mid A)}\mathbf{Pr(A)}$$



A word on the priors...

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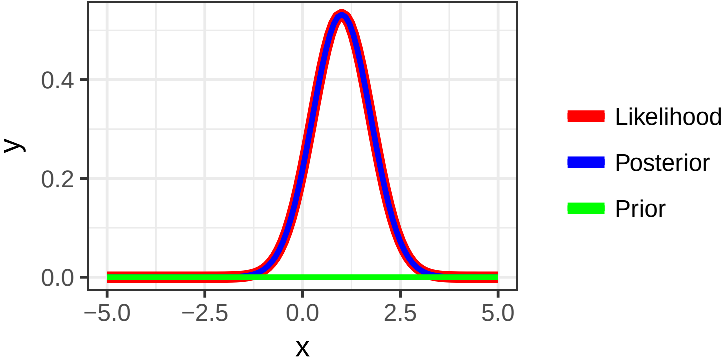
will give the same result as a frequentist analysis.



Sample the Posterior

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Sample the Posterior

So far we have investigated how to use Bayesian techniques to determine posterior probability distribution for a set of parameters in light of some data.
However, our parameter set may be highly-dimensional, and we may only be interested in a sub-set of (marginalized) parameters.
Markov Chain Monte Carlo (MCMC) is an efficient approach to this problem.

Markov Chain Monte Carlo

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MCMC Algorithms: ex. Metropolis-Hasting, Gibbs, etc.

Frequestist Approach

Frequentism is a probabilistic statement about a recipe for generating confidence intervals given a fixed model parameter

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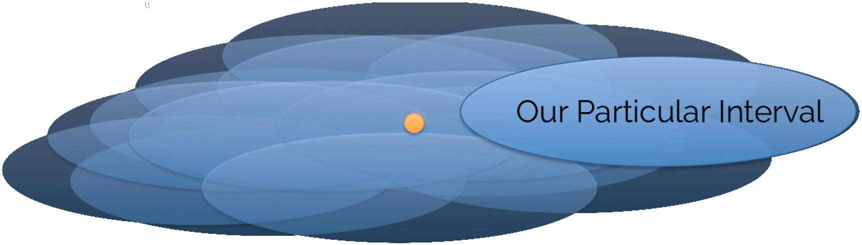

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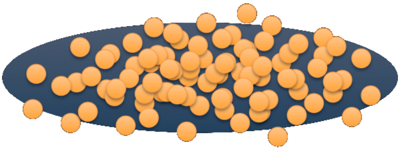

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A frequentist 95% confidence interval is NOT 95% likely to contain the true value!

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This very common mistake is a Bayesian interpretation of a frequentist construct.

Take Home Message

Frequentists A 95% of such Confidence Intervals in repeated experiments will contain the true value!

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Frequentists A 95% of such Confidence Intervals in repeated experiments will contain the true value!

Bayesians A 95% Credible Region is 95% likely to contain the true value!

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Stan language (Stan Development Team, 2015) which makes use of the Hamiltonian Monte-Carlo Sampler (Neal et al., 2011) of Hybrid Monte-Carlo Sampler (HMC) (Duane et al., 1987) and its extension No-U-Turn Sampler (NUTS) (Hoffman and Gelman, 2014).

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- Much higher effective sample size per iteration for complex posteriors.
- Overall, much higher number of effective samples per second.
- Does not require any special behaviour for conjugate priors, which much impact the priors' choice (Hoffman and Gelman, 2014).

STAN & R

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- `rstan`: R Interface to Stan C++ library for Bayesian estimation. Upload your stan code and run it through R.

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- `rstanarm` & `brms`: Estimate previously compiled regression models using `rstan`. Users specify models via the R syntax with a formula and data.frame plus some additional arguments for priors.

rstanarm & brms

`rstanarm`

- manual's pages: 121
- topics documented: 47
- authors & contributors: 17, Jonah Gabry, Imad Ali, Sam Brilleman, Jacqueline Buros Novik, AstraZeneca, Trustees of Columbia University, Simon Wood, R Core Deveopment Team, Douglas Bates, Martin Maechler, Ben Bolker, Steve Walker, Brian Ripley, William Venables, Ben Goodrich

`brms`

- manual's pages: 154
- topics documented: 144 (so detailed!)
- author: 1, Paul-Christian Bürkner

Syntax Comparison with mtcars

With default weakly informative priors:

`rstanarm`:

```
stan_glm(formula = mpg ~ wt + am + cyl, data = mtcars,
         prior = NULL, family = gaussian(), chains = 4,
         iter = 2000, warmup = 1000)
```

`brms`:

```
brm(formula = mpg ~ wt + am + cyl, data = mtcars,
    prior = NULL, family  = "gaussian", chains = 4,
    iter = 2000, warmup = 1000)
```

system.time() and Marvok Chains

`rstanarm`

system.time():

user system elapsed

0.924 0.000 0.918

1st Chain

0.107433 seconds (Warm-up)

0.09435 seconds (Sampling)

0.201783 seconds (Total)

`brms`

system.time():

Compiling the C++ model

user system elapsed

50.728 1.276 52.083

1st Chain

0.070122 seconds (Warm-up)

0.068211 seconds (Sampling)

0.138333 seconds (Total)

Random Effect Coefficients

Generalized linear models with group-specific terms:

`rstanarm`:

```
stan_glmer(formula = mpg ~ wt + am + (1|cyl), data = mtcars,
           prior = NULL, family = gaussian(), chains=4,
           iter=2000, warmup=1000)
```

`brms`:

```
brm(formula = mpg ~ wt + am + (1|cyl), data = mtcars,
    prior = NULL, family="gaussian", chains=4,
    iter=2000, warmup=1000)
```

Smooth Terms

The implementation is similar to that used in the gamm4 package:
rstanarm:

```
dat <- mgcv::gamSim(1, n = 200, scale = 2)

stan_gamm4(y ~ s(x0) + x1 + (1|x2) + s(x3), data = dat,
                   prior = NULL, family = gaussian(), chains=4,
                   iter=2000, warmup=1000)
```

brms:

```
brm(y ~ s(x0) + x1 + (1|x2) + s(x3), data = dat,
    prior = NULL, family="gaussian", chains=4,
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Priors' Specification: the Dirty Job

The packages offer all the priors' you would ever need: Student t family, Hierarchical shrinkage family, Laplace family, Product-normal family, Dirichlet family, etc.

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`rstanarm`:

```
stan_glmer(mpg ~ wt + am + (1|cyl), data = mtcars,
           prior = student_t(df=4, location=0, scale=2.5),
           prior_intercept = cauchy(location=0, scale=10))
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```

In `rstanarm` you cannot (to the best of my knowledge), choose different priors for different coefficeints since this would break vectorization. In `brms` cou can, but it may slow down the process.

Getting into the Priors'

In brms only you can start with:

```
get_prior(formula = mpg ~ wt + am + (1|cyl),
          data = mtcars, family="gaussian")

##                   prior     class      coef group resp dpar
## 1                               b
## 2                               b        am
## 3                               b        wt
## 4 student_t(3, 19, 10) Intercept
## 5  student_t(3, 0, 10)        sd
## 6                              sd             cyl
## 7                              sd Intercept   cyl
## 8  student_t(3, 0, 10)     sigma
```

Priors' Specification: the Dirty Job

```
prior <- c(set_prior("normal(0,10)", class = "b"),
           set_prior("normal(1,2)", class = "b", coef = "wt"),
           # Sd of group-level (random) effects
           set_prior("cauchy(0,2)", class = "sd",
            group = "cyl", coef = "Intercept")),
           set_prior("student_t(3, 0, 10)", class = "sigma"))


brm(mpg ~ wt + am + (1|cyl), data = mtcars, prior = prior)
```

For gamm you can specify the standard deviation of the smooth terms: `class = sds`.

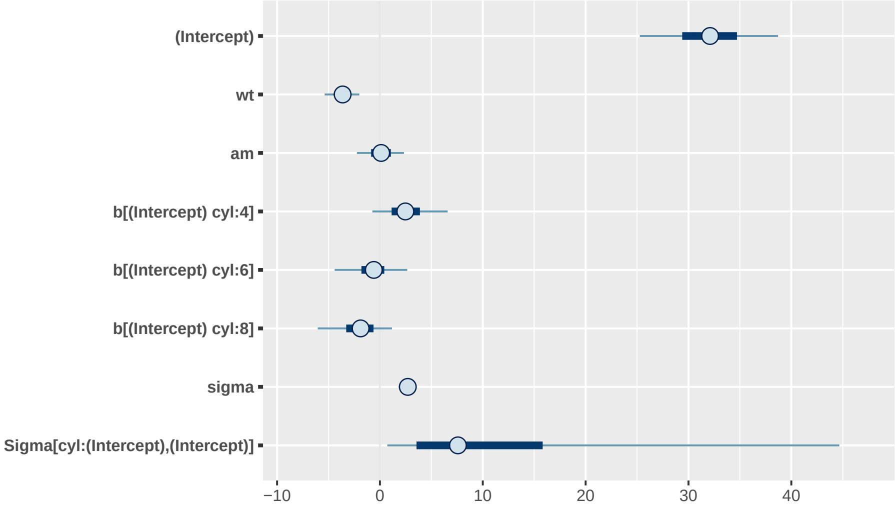
Parallelize the Chains

It is possible to parallelaze the Markov chains in both packages by using the argument `cores = ...` within the function.
Or by using more general parallel syntax:

```
options (mc.cores=parallel::detectCores ())
```

Plot the Results'

```r
plot(rstanarm_glmer)
```



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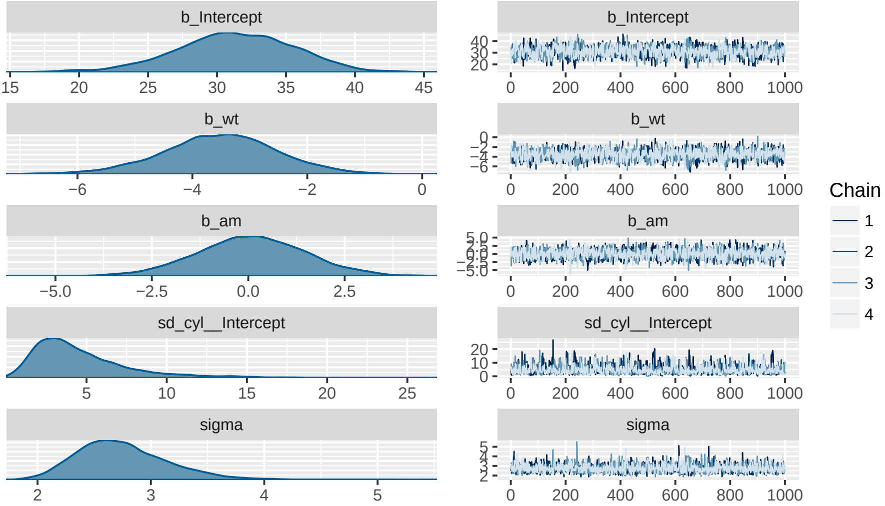
```r
plot(brm_re)
```



Results' Diagnostics: shinystan

shinystan workd both for stanreg and brmsfit objects:

```
m1 <- stan_glmer(formula = mpg ~ wt + am + (1|cyl),
                 data = mtcars,  prior = NULL,
                 family = gaussian())

launch_shinystan(m1)
```

Get the STAN code

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`rstanarm: shinystan`

Get the STAN code

`rstanarm: shinystan`
`brms: make_stancode!`

```
make_stancode(mpg ~ wt + am + cyl, data = mtcars,
    prior = NULL, family  = "gaussian")
```

model
vector[N] mu = Xc * b;
for (n in 1:N)
mu[n] = mu[n] + (r_1_1[J_1[n]]) * Z_1_1[n];
// priors including all constants
target += student_t_lpdf(temp_Intercept — 3, 0, 10);
target += student_t_lpdf(sd_1 — 3, 0, 10)

Some Differences

Some Differences

With `brms` you can also implement multinomial logistic regressions

```
brm(Species ~ Petal.Length + Petal.Width + Sepal.Length +
      Sepal.Width, data=iris, family="categorical",
    prior=c(set_prior("normal (0, 8)")))
```

Correlations

With `brms` you can also deal easily with different correlation structures by specifieng:

- **cor_arma**: autoregressive-moving average (ARMA) structure.
- **cor_arr**: response autoregressive (ARR) structure
- **cor_car**: Spatial conditional autoregressive (CAR) structure
- **cor_sar**: Spatial simultaneous autoregressive (SAR) structure
- **cor_bsts**: Bayesian structural time series (BSTS) structure
- **cor_fixed**: fixed user-defined covariance structure

```
brm(mpg ~ wt + am + (1|cyl), data = mtcars, prior = NULL,
    family="gaussian", cor_arma(formula = ~1, q = 1))
```

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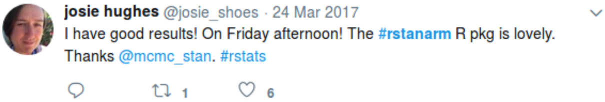
- `rstanarm` can be the easiest package to start with since precompiled, but it might be limiting (intentionally) for more advanced needs.
- `brms` is more flexible and customizable.
- `rstan` fully flexible, but you need to learn STAN programming (maybe with the help of `make_stancode`).

stanarm

stanarm


josie hughes @josie_shoes · 24 Mar 2017
I have good results! On Friday afternoon! The #rstanarm R pkg is lovely.
Thanks @mcmc_stan. #rstats
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Heidi Lorimor @heidi_lorimor · 28 Oct 2016

running my first bayesian models today! **#rstanarm** and #shinystan are amazing!

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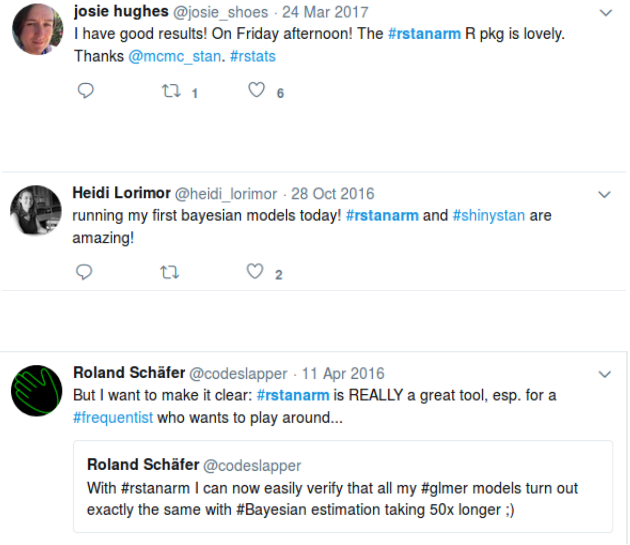
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Roland Schäfer @codeslapper · 11 Apr 2016
But I want to make it clear: #rstanarm is REALLY a great tool, esp. for a #frequentist who wants to play around...
Roland Schäfer @codeslapper
With #rstanarm I can now easily verify that all my #glmer models turn out exactly the same with #Bayesian estimation taking 50x longer ;)

brms

brms

Frank Harrell
@f2harrell

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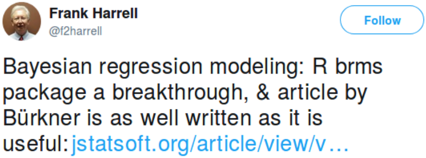
Bayesian regression modeling: R brms package a breakthrough, & article by Bürkner is as well written as it is useful: jstatsoft.org/article/view/v…

brms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Shravan Vasishth @vasishthlab · 22 Sep 2017
Replying to @f2harrell
Bürkner deserves a prize.

brms

Frank Harrell @f2harrell

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Shravan Vasishth @vasishthlab · 22 Sep 2017

Replying to @f2harrell

Bürkner deserves a prize.

Stephen Martin @smartin2018 · 23 Sep 2017

And brms is just crazy potent. Want a location-scale-shape crossed random effects mixture model? You can. Goodness.

1

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In forecasts since:

"...in terms of forecasting ability, ...a good Bayesian will beat a non-Bayesian, who will do better than a bad Bayesian."

C.W.J. Granger (1986, p. 16)

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In forecasts since:
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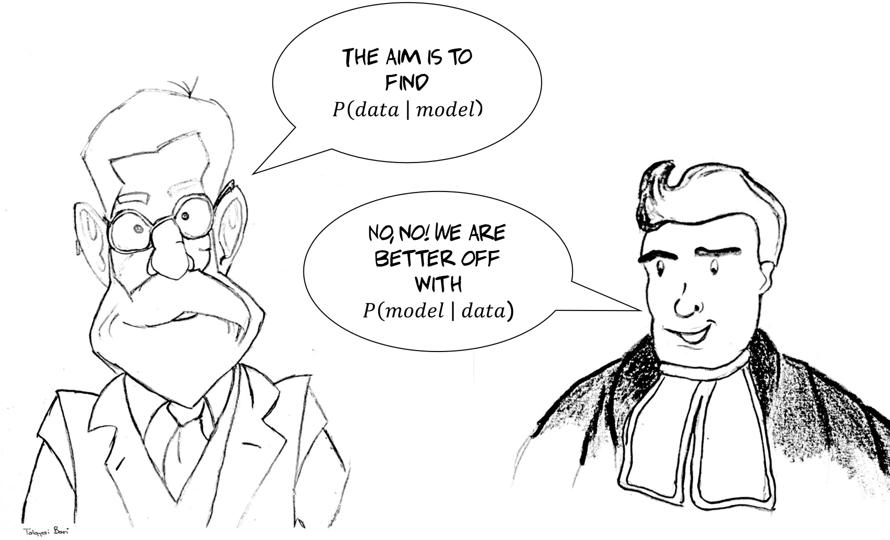
C.W.J. Granger (1986, p. 16)
At the end it is a real philosophical question about how you intend statistics.

The End


THE AIM IS TO FIND $P(data \mid model)$
NO, NO! WE ARE BETTER OFF WITH $P(model \mid data)$

References I

Duane, Simon et al. (1987). "Hybrid monte carlo". In: *Physics letters B* 195(2), pp. 216–222.

Hoffman, Matthew D and Andrew Gelman (2014). "The No-U-turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo." In: *Journal of Machine Learning Research* 15(1), pp. 1593–1623.

Neal, Radford M et al. (2011). "MCMC using Hamiltonian dynamics". In: *Handbook of Markov Chain Monte Carlo* 2, pp. 113–162.

Stan Development Team (2015). *Stan Modeling Language User's Guide and Reference Manual, Version 2.10.0*. URL: http://mc-stan.org/.