

Lecture 4: Model-Free Prediction

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Outline

- 1 Introduction
- 2 Monte-Carlo Learning
- 3 Temporal-Difference Learning
- 4 $TD(\lambda)$

Model-Free Reinforcement Learning

- Last lecture:
 - Planning by dynamic programming
 - Solve a *known* MDP
- This lecture:
 - Model-free prediction
 - Estimate the value function of an *unknown* MDP
- Next lecture:
 - Model-free control
 - Optimise the value function of an *unknown* MDP

Monte-Carlo Reinforcement Learning

- MC methods learn directly from episodes of experience
- MC is *model-free*: no knowledge of MDP transitions / rewards
- MC learns from *complete* episodes: no bootstrapping
- MC uses the simplest possible idea: value = mean return
- Caveat: can only apply MC to *episodic* MDPs
 - All episodes must terminate

Monte-Carlo Policy Evaluation

- Goal: learn v_π from episodes of experience under policy π

$$S_1, A_1, R_2, \dots, S_k \sim \pi$$

- Recall that the *return* is the total discounted reward:

$$G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T$$

- Recall that the value function is the expected return:

$$v_\pi(s) = \mathbb{E}_\pi[G_t \mid S_t = s]$$

- Monte-Carlo policy evaluation uses *empirical mean return* instead of *expected return*

First-Visit Monte-Carlo Policy Evaluation


- To evaluate state s
- The **first** time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- By law of large numbers, $V(s) \rightarrow v_\pi(s)$ as $N(s) \rightarrow \infty$



Every-Visit Monte-Carlo Policy Evaluation

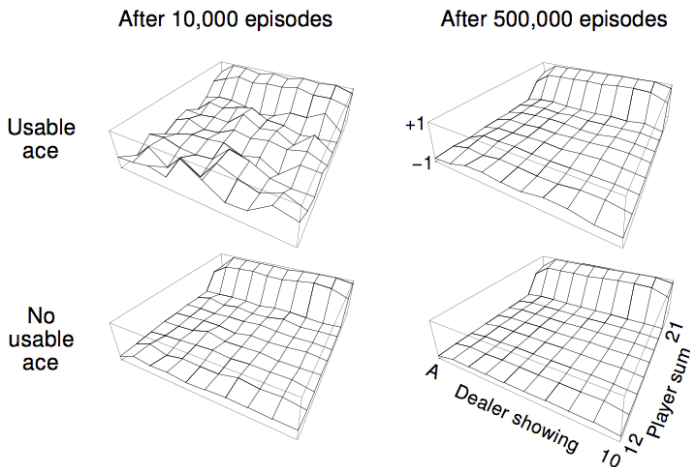
- To evaluate state s
- **Every** time-step t that state s is visited in an episode,
- Increment counter $N(s) \leftarrow N(s) + 1$
- Increment total return $S(s) \leftarrow S(s) + G_t$
- Value is estimated by mean return $V(s) = S(s)/N(s)$
- Again, $V(s) \rightarrow v_{\pi}(s)$ as $N(s) \rightarrow \infty$

Blackjack Example

- States (200 of them): 
 - Current sum (12-21)
 - Dealer's showing card (ace-10)
 - Do I have a "useable" ace? (yes-no)
- Action **stick**: Stop receiving cards (and terminate)
- Action **twist**: Take another card (no replacement)
- Reward for **stick**:
 - +1 if sum of cards $>$ sum of dealer cards
 - 0 if sum of cards = sum of dealer cards
 - -1 if sum of cards $<$ sum of dealer cards
- Reward for **twist**:
 - -1 if sum of cards $>$ 21 (and terminate)
 - 0 otherwise
- Transitions: automatically **twist** if sum of cards $<$ 12



Blackjack Value Function after Monte-Carlo Learning



Policy: **stick** if sum of cards ≥ 20 , otherwise **twist**

Incremental Mean

The mean μ_1, μ_2, \dots of a sequence x_1, x_2, \dots can be computed incrementally,

$$\begin{aligned}\mu_k &= \frac{1}{k} \sum_{j=1}^k x_j \\ &= \frac{1}{k} \left(x_k + \sum_{j=1}^{k-1} x_j \right) \\ &= \frac{1}{k} (x_k + (k-1)\mu_{k-1}) \\ &= \mu_{k-1} + \frac{1}{k} (x_k - \mu_{k-1})\end{aligned}$$



Incremental Monte-Carlo Updates

- Update $V(s)$ incrementally after episode $S_1, A_1, R_2, \dots, S_T$
- For each state S_t with return G_t

$$N(S_t) \leftarrow N(S_t) + 1$$

$$V(S_t) \leftarrow V(S_t) + \frac{1}{N(S_t)} (G_t - V(S_t))$$

- In **non-stationary** problems, it can be useful to **track a running mean**, i.e. forget old episodes.

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

Temporal-Difference Learning

- TD methods learn directly from episodes of experience
- TD is *model-free*: no knowledge of MDP transitions / rewards
- TD learns from *incomplete* episodes, by *bootstrapping*
- TD updates a guess towards a guess

MC and TD

- Goal: learn v_π online from experience under policy π
- Incremental every-visit Monte-Carlo
 - Update value $V(S_t)$ toward *actual* return G_t

$$V(S_t) \leftarrow V(S_t) + \alpha (G_t - V(S_t))$$

- Simplest temporal-difference learning algorithm: TD(0)
 - Update value $V(S_t)$ toward *estimated* return $R_{t+1} + \gamma V(S_{t+1})$

$$V(S_t) \leftarrow V(S_t) + \alpha (R_{t+1} + \gamma V(S_{t+1}) - V(S_t))$$

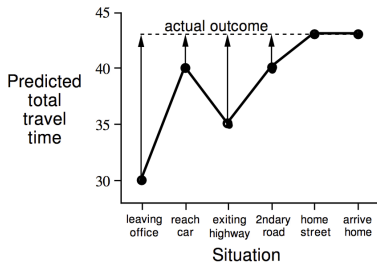
- $R_{t+1} + \gamma V(S_{t+1})$ is called the **TD target**
- $\delta_t = R_{t+1} + \gamma V(S_{t+1}) - V(S_t)$ is called the **TD error**

Driving Home Example

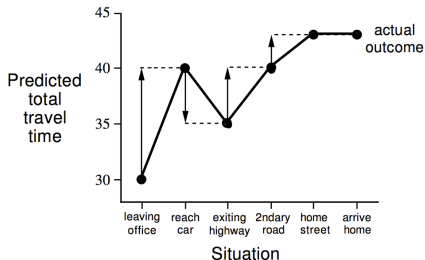
State	Elapsed Time (minutes)	Predicted Time to Go	Predicted Total Time
leaving office	0	30	30
reach car, raining	5	35	40
exit highway	20	15	35
behind truck	30	10	40
home street	40	3	43
arrive home	43	0	43

Driving Home Example: MC vs. TD

Changes recommended by
Monte Carlo methods ($\alpha=1$)



Changes recommended
by TD methods ($\alpha=1$)



Advantages and Disadvantages of MC vs. TD

- TD can learn *before* knowing the final outcome
 - TD can learn online after every step
 - MC must wait until end of episode before return is known
- TD can learn *without* the final outcome
 - TD can learn from incomplete sequences
 - MC can only learn from complete sequences
 - TD works in continuing (non-terminating) environments
 - MC only works for episodic (terminating) environments

Bias/Variance Trade-Off

- **Return** $G_t = R_{t+1} + \gamma R_{t+2} + \dots + \gamma^{T-1} R_T$ is *unbiased* estimate of $v_\pi(S_t)$
- **True TD target** $R_{t+1} + \gamma v_\pi(S_{t+1})$ is *unbiased* estimate of $v_\pi(S_t)$
- TD target $R_{t+1} + \gamma V(S_{t+1})$ is *biased* estimate of $v_\pi(S_t)$
- TD target is **much lower variance** than the return:
 - Return depends on **many** random actions, transitions, rewards
 - TD target depends on **one random action**, transition, reward

Advantages and Disadvantages of MC vs. TD (2)

- MC has high variance, zero bias
 - Good convergence properties
 - (even with function approximation)
 - Not very sensitive to initial value
 - Very simple to understand and use
- TD has low variance, some bias
 - Usually more efficient than MC
 - TD(0) converges to $v_{\pi}(s)$
 - (but not always with function approximation)
 - More sensitive to initial value

