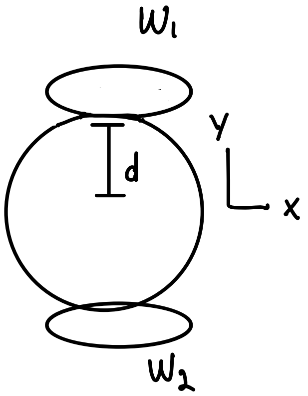
Kinematics for Differential Drive Robot



d = track length / 2

w = wheel

T_{b1} = transform from body {b} to wheel one {1}.

$T_{b1}(0, 0, D)$

$T_{b2}(0, 0, -D)$

$$A_{b1} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{b2} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Big\downarrow \text{ Inverse Adjoint}$$

$$A_{1b} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad A_{2b} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Body Twist $V_b = \begin{bmatrix} \dot{\theta} \\ x \\ y \end{bmatrix}$

Wheel 1: $V_1 = A_{1b} V_b$

$$\begin{bmatrix} \dot\theta \\ v_{x1} \\ v_{y1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ -d\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -d & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

$$\Rightarrow \quad r\dot\phi_1 = \begin{bmatrix} -d & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

$$\dot\phi_1 = \begin{bmatrix} -d/r & 1/r & 0 \end{bmatrix}$$

Wheel 2:

$$\begin{bmatrix} \dot\theta \\ v_{x2} \\ v_{y1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \dot\theta \\ d\dot\theta + v_x \\ v_y \end{bmatrix}$$

$$\begin{bmatrix} \dot\theta \\ r\dot\phi_2 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ d & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

$$r\dot\phi_2 = \begin{bmatrix} d & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot\theta \\ v_x \\ v_y \end{bmatrix}$$

$$\dot\phi_2 = \begin{bmatrix} d/r & 1/r & 0 \end{bmatrix}$$

Inverse Kinematics:

$$\begin{bmatrix} \dot{\Phi}_1 \\ \dot{\Phi}_2 \end{bmatrix} = \begin{bmatrix} -d/r & 1/r & 0 \\ d/r & 1/r & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}, \quad \dot{\Phi} = H V_b$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta} \\ v_x \\ v_y \end{bmatrix}, \quad 0 = Q V_b$$

$$\Rightarrow \quad \dot{\Phi}_1 = -d/r \, \dot{\theta} + 1/r \, v_x$$

$$\dot{\Phi}_2 = d/r \, \dot{\theta} + 1/r \, v_x$$

- Eq. (1-2): Inverse kinematics

New wheel position 1: $\dot{\Phi}_1 = -d/r \, \dot{\theta} + 1/r \, v_x$

New wheel position 2: $\dot{\Phi}_2 = d/r \, \dot{\theta} + 1/r \, v_x$

Forward Kinematics:

$u = H V_b \qquad H^{+} = H^{T}(HH^{T})^{-1}$

$V_b = H^{+} u$

$$V_b = \begin{bmatrix} r/2d & -r/2d \\ r/2 & r/2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

$$V_b = r/2 \begin{bmatrix} 1/d & -1/d \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\phi}_1 \\ \dot{\phi}_2 \end{bmatrix}$$

- Eq. 3: Body Twist V_b

$$V_b = r/2 \begin{bmatrix} \dot{\phi}_1/d - \dot{\phi}_2/d \\ \dot{\phi}_1 + \dot{\phi}_2 \\ 0 \quad 0 \end{bmatrix}$$

$$\mathcal{V}_b = T_{bb'} = T(\Delta\theta_b, \Delta x_b, \Delta y_b)$$

Δq_b

$$\Delta q = A(\theta, 0, 0)\,\Delta q_b$$

$$q' = q + \Delta q$$

$$\Delta q = A(\theta, 0, 0)\,\Delta q_b$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \Delta\theta_b \\ \Delta x_b \\ \Delta y_b \end{bmatrix}$$

- Eq. 4 : forward kinematics Δq

$$\Delta q = \begin{bmatrix} \Delta\theta_b \\ \cos\theta \cdot \Delta x_b - \sin\theta\,\Delta y_b \\ \sin\theta \cdot \Delta x_b + \cos\theta\,\Delta y_b \end{bmatrix}$$