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Systems framework for fuzzy sets in civil engineering

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Abstract: Civil engineering projects and designs are commonly developed in a systems framework that includes different types of uncertainty. In general, uncertainty can be of the ambiguity or vagueness type. The theory of probability and statistics has been extensively used in civil engineering to deal with the ambiguity type of uncertainty. The theory of fuzzy sets and systems have been used in civil engineering to model the vagueness type of uncertainty in many civil engineering applications. In this paper, the role of fuzzy sets in civil engineering systems is described using several example applications, e.g., quality assessment of wildlife habitat, construction engineering and management, structural reliability, and damage assessment of existing structures.

Keywords: Applications; civil; construction; engineering; environment; fuzzy; structures; wildlife.

1. Introduction

Uncertainties in civil engineering systems can be attributed to mainly ambiguity and vagueness in defining the parameters of the systems. The ambiguity component is due to (1) physical randomness; (2) statistical uncertainty due to the use of limited information to estimate the characteristics of these parameters; and (3) model uncertainties which are due to simplifying assumptions in analytical and prediction models, simplified methods, and idealized representations of real performances. The vagueness related uncertainty is common in (1) the definition of certain parameters, e.g., structural performance (failure or survival), quality, deterioration, skill and experience of construction workers and engineers, environmental impact of projects, conditions of existing structures; and (2) defining the inter-relationships among the parameters of the problems, especially for complex systems.

2. Systems framework

In general, civil engineers are concerned about investigating certain aspects and components of our environment in order to understand and predict their behavior for the purpose of using them to meet our needs. This process can be systematically performed within a systems framework. Generally, a civil engineering project can be modelled to include a segment of its environment that interacts significantly with it, defining a civil engineering system. The limits of the system are drawn based on the nature of the project, class of performances (including failures) under consideration and the objectives of the analysis. The first step in solving any civil engineering problem is to define the architecture of the system. The definition can be based on observations about the source and data elements, interaction among the elements and behavior of the system.

Each level of knowledge which is obtained about a civil engineering problem can be said to define a system on the problem. As additional levels of knowledge are added to previous ones, higher epistemological levels of system definition and description are generated which, taken together, form a hierarchy of such system descriptions. In this section, an epistemological hierarchy of systems suited to the representation of structural reliability problems with a generalized treatment of uncertainty is developed for illustration purposes. The system framework definition is based on the approach suggested by Klir [69] and Klir and Folger [70].

Any civil engineering system can be considered as a knowledge (epistemological) system. The hierarchy of the system can be divided into several levels. The *level* 0 *system,* which is called the *source system,* is defined within the perceived environment. A boundary is conceptually drawn which includes the project and a segment of its environment. The identification of an object in this manner represents the most primitive way in which a system can be epistemically defined. The source system is considered to be always in some state which is generally time-dependent. The space of possible states which the source system may possess is called the source state space. Generally, the source system can include several types of uncertainty in its definition. For example, a structure, its foundations, the soil that surrounds the foundations, and its loads can be considered as a source system. The uncertainties in this case include uncertainty in defining the soil boundaries of the system, load types, and the connections among system's components.

The *level* 1 (*parameter*) *system* is defined by establishing the parameters of the system. The parameters can be classified by partitioning them according to several criteria depending on the objectives of an analysis. For example, the set of parameters can be partitioned according to criteria such as: (1) operational procedures by which the parameters are estimated (e.g., observables versus perceptables, global versus local, material versus geometric), (2) structured model input/output partitioning induced by *level* 3 definition of the system (as described below), and (3) magnitude of the rate of change over time of the parameters (e.g., fast versus slow changing parameters). The parameter classification can be used to establish the uncertainty types to be considered in the analysis of a system. For example, observables commonly include probabilistic and statistical uncertainties; whereas perceptables include vagueness uncertainty. Generally, the system of parameters can include several sources of uncertainty in its definition.

The *level* 2 *system* called the *data system.* This system provides information about the actual values of the parameters that are defined in the level 1 parameter system. This information may be deterministic, or characterized by uncertainty expressed according to some appropriate measures that correspond to its different forms. To date, the type of uncertainty typically considered in structural reliability assessment has been characterized by the probability-based measures. A generalized treatment of uncertainty that considers both ambiguity and vagueness types in structural reliability was developed by Alvi and Ayyub [2].

The *level* 3 (*model*) *system* accounts for various relations between parameters. The model system is usually constructed based on existing theoretical and empirical knowledge in a specific domain and is generally subject to uncertainties and bias due to approximations in representing the true relations between the parameters. In structural reliability theory, model systems are usually based on the principles of force equilibrium, geometric (strain and member connection) compatibility under deformations, and analytically as well as empirically based constitutive relations of materials. More specifically, model systems express relations which map a set of model input parameters to a set of model state parameters (outputs). Generally, it is assumed that the model system is time-invariant (static). Therefore, the model system provides estimate values of the output parameters without actually (or physically) measuring them. The differences between the values of output parameters predicted by the model system and those obtained in actual experiments which measure the same output parameters can be attributed partly to the fact that model systems generally represent approximations of actual performances of source systems. This level of approximation is additional to that which results in defining a level 1 parameter system. Therefore, this process results in uncertainties in the estimated output parameters.

The *level* 4 (*structure*) *system* generally represents a set of level 3 systems. It may include as well levels 0, 1, and 2 systems. The transitions and the interactions between these systems should be defined. The relationships among the systems represent coupling and are due to common variables among the lower-level systems, or they may interact in some other ways. All the previous systems are generally considered to be support-invariant, i.e., time-invariant, space-invariant, and/or population-invariant. Block diagrams can be used for the representation of level 4 systems. Thus, structure systems are essentially model systems at a higher hierarchic level. Examples include relating model systems of structural components to form structure systems of structural (engineering) systems, and relating local stiffness matrices within structural components to form global stiffness matrices which describe the entire structural system.

The next level is *level* 5 (*metasystem*) *systems* which are defined using lower-level systems. These systems change according to a support-invariant procedure. For example, systems defined on this level may include time-variant systems. These systems result from allowing any of the definitions of lower level systems to be dependent on the time parameter. More specifically, types of time-dependency which may exist for each system level include.

1. Time dependency of the boundary used to define and delineate the source system at level 0.

2. Time-dependency regarding which parameters are used to define the parameter system. For example, over time, various parameters may be added and deleted to the set used to define the parameter system.

3. Time-dependency of the data used in defining the data system. For example,

in structural reliability, values of the basic variables may be expressed in the form of stochastic processes. Similarly, if values of certain variables are assumed to be deterministic, they may be considered functions of time (e.g., increasing deterministic concrete strength).

4. Time-dependency of the domain-specific relations employed in forming the model system, i.e., a model structure which changes over time (e.g., changing material constitutive relations over the time span of an earthquake event).

5. Time-dependency of the relations among level 3 model systems (e.g., changes in component connections within a structural system during an earthquake event).

Metasystems that change according to a support-invariant metaprocedure are called *level* 6 (*meta metasystems*) *systems*. Similarly, higher order systems can be defined. In structural engineering, level 6 systems can be defined by considering the time-history response of time-dependent level 5 systems.

The theories of probability, statistics, fuzzy sets and possiblity can be used to deal with these types of uncertainty within a system framework. Many researchers considered the ambiguity type of uncertainty in modelling civil engineering systems. In this paper, the vagueness type of uncertainty in civil engineering systems is discussed along with applications of fuzzy set theory to such systems.

3. Civil engineering applications

Since the inception of fuzzy set theory (Zadeh [95]), it has received extraordinary recognition and consideration by scientists, researchers and engineers in many fields (Zadeh [95–99] and Zadeh et al. [100]). Additional information with example applications is provided by Kaufmann [67], Kaufmann and Gupta [68] and Negoita and Ralescu [75]. In civil engineering, the theory was proven to be a very useful tool in solving problems that involve the vagueness type of uncertainty. The realization of the research and engineering community to the suitability and potential of this tool started in the early 1970's (Brown [21] and Alley, et al. [1]). To date, many applications of the theory in civil engineering were developed. The theory has been successfully used in (1) quality of wildlife habitat; (2) analysis of construction failures, scheduling of construction activities, safety assessment of construction operations, and decisions during construction; (3) tender evaluation; (4) strength assessment of existing structures and other structural engineering applications; (5) risk analysis and assessment in engineering; (6) planning of river basins; and (7) other areas.

In the following sections, selected applications are described in detail as example utilizations of fuzzy sets in civil engineering. Other significant contributions in the fuzzy set application area are available in the reference list.

3.1 *Quality of wildlife habitat*

Engineers are often criticized for not being sensitive to the effects of engineering projects on social values. The effect of engineering projects on the quality of wildlife habitat is one example where engineers have been criticized for their lack of awareness. One reason for the problem appears to lie not only in a lack of concern by engineers but also the lack of means to incorporate value goals into the sociotechnical decision framework used by administrators in making decisions about engineering projects. Too often, the quality of wildlife habitat is viewed as a nonquantifiable project goal and, therefore, is assigned to a second class of decision goals. This need not be the case, and the problem can be overcome by developing ways of assessing the quality of habitat and incorporating such assessments into the sociotechnical decision-making framework. But it is important to have methods of analysis that are capable of reflecting subjectivity in value goals. Ayyub and McCuen [12] developed a method for the impact assessment of engineering projects on the quality of wildlife habitat.

The requirements of wildlife for food and cover are widely documented [73, 88, 90]. In some areas, annual census evaluations of certain species are made. Trend analyses of the populations can be used to identify natural changes in species as well as the effects of both natural succession and engineering projects on the quantity and quality of a habitat. In order to take effective and realistic corrective actions, the habitat quality for a specific site needs to be assessed and compared to human demands. Only recently have there been attempts to develop systems for examining specific areas of land and providing a quantitative index of habitat quality for many forms of indigenous wildlife.

Federal agencies have always attempted to mitigate or minimize significant wildlife habitat losses connected with engineering projects. Until recent years, emphasis has been mainly in areas where the loss was obvious. The National Environmental Policy act of 1969 and growing public concern, however, have resulted in more consideration and efforts to study all environmental effects for every project. It is now necessary to evaluate accurately wildlife habitat in project areas, assess the changes that may occur, and identify mitigation and enhancement opportunities. The method suggested by Ayyub and McCuen [12] provides a means for quanitifying the existing habitat and displaying the net effects on the habitit of various project alternatives and proposed mitigating measures. Once the model has been calibrated to a specific area by experienced wildlife biologists, the methodology can be applied by field engineers and water resource technicians using aerial photographs and field evaluation of randomly located points.

The factors that determine the quality of wildlife habitat include the quantity and quality of land uses, the degree of interspersion of land uses, and the type of land use management. These factors cannot be precisely defined by various degrees. As a result, experience and judgment are used to supplement objective knowledge. The combination of objective information and value of subjective judgment can be performed methodically by the use of the theory of fuzzy sets and systems [95–100]. Additional information with example applications is provided by Kaufmann [67], Kaufmann and Gupta [68] and Negoita and Ralescu [75].

The major problem in quality assessment of wildlife habitat lies in the factors that are expressed in linguistic, rather than mathematical terms. For example, ideal quantity or inadequate quantity of land use fall into this category. Even the importance of each factor is usually expressed in linguistic terms. Therefore, two kinds of uncertainty are encountered in practice, ambiguity and vagueness. Ambiguity can be classified into random, statistical and model uncertainty. Random uncertainty arises from the randomness of precisely defined events or propositions and can be dealt with using the theory of probability. Statistical uncertainty arises from estimating some of the factors using limited data (or a finite sample size) and can be dealt with using the theory of statistics. Model uncertainty arises from using approximations and assumptions in our prediction models. Vagueness arises from a lack of precision or a lack of understanding of an event, a proposition, a value, or a system. The theory of fuzzy sets and systems has proved to be an effective tool in handling the vagueness uncertainty. Therefore, it is well suited to the problem of wildlife habitat quality assessment.

In this section, a method of assessment of the quality of wildlife habitat based on the judgment of experts is described. The method uses the concepts of fuzzy sets and systems. The method can be applied to describe the wildlife habitat in an area where planned project activities are expected affect vegetative patterns. It permits small portions of a project to be evaluated individually as well as the total project area to be evaluated. This method can also generate much of the other information required in preparing environmental impact statements.

Requirements for food, cover, and water of individual species, especially game species, has been intensively studied since Leopold [71] outlined the basic principles of wildlife management. Species such as the turkey (*Meleagris gallopavo*) [61] and bobwhite quail (*Colinus virginianus*) [80] have had their habitat needs analyzed in every section of the country. The *Audubon Guides* [78] and other studies [13, 30, 79] describe the preferred habitat and food requirements of most bird species and outline techniques for providing food and nesting places.

Until recently, the evaluation of wildlife habitat to determine the effect of most engineering projects has been quite subjective. Such evaluations usually resulted in a habitat rating for each important species, generally with quantitative categories ranging from excellent to poor. Thus, the evaluation did not provide a quantitative indication of the effects of proposed alternative projects, and it could not express properly the effects of small or subtle changes that might be brought about by the large changes associated with development projects. It has long been recognized that the larger the diversity of vegetation, the more valuable the habitat will be for a variety of wildlife species. Leopold [71] in his 'law of interspersion' recognized that wildlife in general is a phenomenon of the edges. For such a complex system, three broad factors affecting the quality of wildlife habitat were considered in their model: the quantity of each land use, the degree of interspersion of land uses, and the management or vegetative condition of each land use. A fuzzy relation between the quality of wildlife habitat and each factor was developed. Then, the resulting fuzzy relations can be used to determine the impact of design alternatives on the quality of wildlife habitat. This process can be performed in the following four steps:

1. Due to the variety of species within any system and the variations in their

requirements of land use, the quantity of various land use classifications needs to be quantified. Therefore, a fuzzy relation between the quantity of each land use Q_{Ni} (i-th land use) and the quality of wildlife habitat Q_L was defined as

$$R_{1i} = \text{If } Q_{Ni} \text{ is } G_{Nj} \text{ then } Q_L \text{ is } G_{Lk}\text{; else if } Q_{Ni} \text{ is} \ldots \tag{1}$$

where G_{Nj} and G_{Lk} are the j-th grade of the quantity of the i-th land use and the k-th grade of the quality of wildlife habitat. Equation (1) can be expressed as

$$R_{1i} = (G_{N1} \times G_{Lk_1}^{\alpha_{1i}}) \cup (G_{N2} \times G_{Lk_2}^{\alpha_{1i}}) \cup \cdots \cup (G_{Nm} \times G_{Lk_m}^{\alpha_{1i}}) \tag{2}$$

where α_{1i} is the significance level of the first factor for the i-th land use.

2. In addition to the quantity of land use, the ability of the system to support a wide variety of wildlife depends on the spatial distribution of land use or its interspersion. A fuzzy relation R_{2j} between the average distance for the j-th combination of land uses, i.e., the degree of interspersion of land uses, and the quality of wildlife habitat can be developed with the help of experts. The fuzzy relation can take the following form:

$$R_{2j} = (G_{I1} \times G_{Lk_1}^{\alpha_{2j}}) \cup (G_{I2} \times G_{Lk_2}^{\alpha_{2j}}) \cup \cdots \cup (G_{Ip} \times G_{Lk_m}^{\alpha_{2j}})$$
$$\text{for } j = 1, 2, \ldots, n;\ n = \text{number of combinations of land use,} \tag{3}$$

where G_{Ii}, $i = 1, \ldots, p$, is the i-th grade of the degree of interspersion of land uses; G_{Lk} is the k-th grade of the quality of wildlife habitat, and α_{2j} is the significance level of judgement or the importance of the degree of the interspersion of land use.

3. The quality of wildlife habitat depends in part on land use management conditions that include the amount of food generally available, its nutritive value, its rate of decomposition, and the ease in which wild species can find and use it. Because of the subjectivity in assessing these factors, a fuzzy relation between the land use management of each vegetative type and the quality of wildlife habitat needs to be developed. For the i-th type $i = 1, \ldots, m$, this fuzzy relation can be expressed as

$$R_{3i} = (G_{M1} \times G_{Lk_1}^{\alpha_{3i}}) \cup (G_{M2} \times G_{Lk_2}^{\alpha_{3i}}) \cup \cdots \cup (G_{Mp} \times G_{Lk_m}^{\alpha_{3i}}) \tag{4}$$

where G_{Mj}, $j = 1, \ldots, r$, is the j-th grade of the i-th vegetative type used in the evaluation of the land use management; G_{Lk} is the k-th grade of the quality of wildlife habitat, and α_{3i} is the significance level of judgement or the importance of the i-th vegetative type.

4. The impact of grades of the three factors, G_{Nj}, G_{Ij}, and G_{Mj}, on the quality of wildlife habitat can be determined using the developed three fuzzy relations (Eqs. (2), (3), and (4)) using the composition operator as follows:

$$\text{Quality of wildlife habitat}$$
$$= \left[\bigcup_{j=1}^{m} G_{Nj} \circ R_{1j}\right] \cap \left[\bigcup_{j=1}^{n} G_{Ij} \circ R_{2j}\right] \cap \left[\bigcup_{j=1}^{r} G_{Mj} \circ R_{3j}\right] \tag{5}$$

where R_{1j}, R_{2j}, and R_{3j} are the fuzzy relations as described in Eqs. (2), (3), and (4), respectively; and G_{Nj}, G_{Ij}, and G_{Mj} are the grades of the three factors.

The above method was used by Ayyub and McCuen [12] utilizing data for the Upper Choptank watershed for illustration purposes.

3.2. *Construction engineering and management*

Several applications were developed in the area of construction engineering and management. Blockley [15, 16] suggested methods for predicting the likelihood of structural accidents and the analysis of structural failures. The methods were based on fuzzy sets and approximate reasoning. These studies outlined approaches of predicting the likelihood of structural failures due to causes other than the stochastic variation in loads and strength. Ayyub and Haldar [7] developed a method for estimating the duration of construction activities based on fuzzy set models of the factors affecting activity durations. In other studies, Ayyub and Haldar [9], Ayyub and Eldukair [4, 5, 6], and Eldukair and Ayyub [40, 41] suggested decision methodologies for selecting and designing construction strategies using approximate reasoning. The decision analysis framework was developed based on considering information on relative risk, cost, benefit, and failure consequences of the construction strategies. Optimization methods were developed based on the fuzzy set theory. These studies are summarized in detail at the end of this section. Hadipriono [52, 53] used fuzzy set concepts in the assessment of falsework performance and the analysis of recent structural failures. Hadipriono and Toh [55] and Hadipriono and Wang [56] developed methods for the analysis of falsework failures in concrete structures and the consequences of failure. The assessment of the consequences of structural failure were based on classifying the inducing events into enabling and triggering events. Then, approximate reasoning based on linguistic variables were used to estimate the consequences of failure. Leung [72] used fuzzy set procedures for project selection with hierarchical objectives. Nguyen [76] developed methods for tender evaluation in construction engineering based on fuzzy sets. Juang et al. [65] used fuzzy system for bid proposal evaluation on microcomputers.

Two illustrative examples of the use of fuzzy sets in construction engineering and management are described in this section. In the first example, a method of estimation of construction activity duration is discussed (after Ayyub and Haldar [7]). It is common in construction management to divide construction projects into activities. The relationship and sequence of these activities are presented in the form of a network. Each activity requires a certain amount of resources which may include time, labor, material, and money. The objective of a construction manager is to find the combination of resources which will minimize the total cost of not only one activity but of all the activities involved in the project, and to finish the project on time. In order to estimate the completion time of a project, the time required to finish each activity (duration of an activity) needs to be estimated. The nominal duration, or the mean value and the standard deviation of the duration, or the probability distribution and its parameters of the duration of each activity need to be estimated, depending on which scheduling method is being used, i.e., critical path method (CPM), project evaluation technique (PERT), or simulation techniques. Obtaining reasonable activity duration estimates is important because all subsequent calculations and decisions are based on these estimates. There are many factors which affect the duration of an activity, e.g., weather, labor skill (which changes with time because of the learning effect), superintendent experience, type of equipment used, and level of operators' experience. The effect of these factors on the duration of an activity depends on the activity being considered. For example, pouring concrete in an open area is highly sensitive to weather conditions compared to other factors. The construction engineer or superintendent estimates the duration of the activities using experience and judgment. The level of experience and judgment affect the final outcome and result in uncertainties in the estimated durations. These uncertainties need to be modeled mathematically.

The major problem lies with the factors that are expressed in linguistic, rather than mathematical terms. Good or bad weather, long or short experience, etc., fall into this category. Even the sensitivity of the activity's duration to any of these factors is measured in linguistic terms, e.g., highly sensitive, strong influence, etc. Not only are future weather conditions uncertain at the present time, the definition of good or bad weather complicates the problem. Uncertainties in future weather conditions can be modeled using probability theory; however, additional sources of uncertainty due to the qualitative assessment of good or bad weather need to be considered. The linguistic variables can be translated into mathematical measures by fuzzy sets and systems theory. Conventional procedures like PERT can still be used if updated probabilistic input is used to obtain the required information.

In this application, the concept of fuzzy sets and systems was introduced and applied in construction project scheduling. The factors, weather conditions and labor skill, were considered by Ayyub and Haldar [7] to help explain the applicability of the fuzzy set concept in construction scheduling. However, any number of similar factors could have been modeled accordingly. The suggested method for systematically qualifying the linguistic factors is realistic and simple. It could easily be implemented in any available computer software or some project scheduling techniques, such as PERT.

The second example is based on the studies by Ayyub and Haldar [9], Ayyub and Eldukair [4, 5, 6] and Eldukair and Ayyub [40, 41] in which they suggested decision methodologies for selecting and designing construction strategies using approximate reasoning. The decision analysis framework was developed based on considering information on relative risk, cost, benefit, and failure consequences of the construction strategies. Optimization methods were developed based on the fuzzy set theory. Additional developments in this area were provided by Buckley [25], and Okuda et al. [77].

According to these studies, construction work is considered hazardous and subject to many accidents. Many factors affect the safety of construction operations and might lead to accidents, e.g., labor skill, supervisors' experience and attendance, condition of temporary structures, weather conditions, type of equipment, level of operators' experience, etc. The safety of the construction operations has different levels of sensitivity to each of these factors.

The potential losses due to the failure of structures during construction could be enormous. These losses may include property damage, human death and injury, construction delay, and discredit to the responsible contractor and/or engineer. Therefore, these potential losses need to be considered appropriately in the planning and scheduling stage of the construction of structures by means of a decision analysis of the possible construction strategies.

The possible construction strategies for a structure are numerous. For example, for reinforced concrete buildings, a construction strategy can be defined by the forming technique, construction cycle, number of shored and reshored floors in the construction cycle, level of experience of the personnel involved in the construction, level of supervision and quality control and type of construction equipment. These parameters defining construction strategies affect the total cost of the structure, the completion time of construction, and the safety at any stage of construction. The decision analysis methodology forms a logical basis for selecting the most desirable construction strategy for a given structure or class of structures, considering both the technical and economic aspects of the problem. The information on relative risk, along with the information on cost, benefits, and consequences of each construction strategy is valuable to engineers in selecting the optimum alternative. A framework of decision theory is an important tool to explore this area.

As stated earlier, the factors that can lead to accidents during construction and failure of construction strategies are numerous. Several researchers identified these factors. They cannot be avoided completely in a typical construction project. However, the responsibility of a construction engineer is to devise the most desirable construction strategy. The variables involved in the problem are usually qualified instead of quantified. That is, these variables are expressed in linguistic terms rather than mathematical measures. The workers' or the supervisors' experience, or the condition of falsework can be at best described as good or bad with no standard acceptable numerical value attached to these qualitative statements. If an engineer is faced with alternative construction strategies, e.g., (1) highly experienced workers, moderately experienced supervisors, and new falsework; or (2) moderately experienced workers, highly experienced supervisors, and new falsework; or (3) highly experienced workers and supervisors, but old falsework [used many times in the past and its reuse is questionable], he may not have any mathematical tools available to make a logical decision. He can only select an alternative using his intuition. To obtain the most desirable alternative using decision theory, the linguistic variables need to be translated into mathematical measures. The linguistic variables can be translated into mathematical measures by the theory of fuzzy sets and systems. Since in construction operations, linguistic variables are very common, they need to be considered appropriately in decision analysis problems. In these studies, a decision analysis framework was developed for construction operations where the variables are linguistic in nature.

3.3. *Structural engineering*

One of the most significant contributions to the applications of fuzzy sets in structural engineering is in the area of damage assessment of existing structures

[62, 63, 86, 87, 92, 93]. In order to assess the damage condition of an existing structure (following, for example, the occurrence of a strong-motion earthquake) or the durability of a structure, or the structural integrity of a building, we need to deal with imprecise measures of the structural parameters and their effect on the damage condition, durability, or structural integrity, respectively. In such problems, the analyst is truly encountering a complex problem where a systems framework with a generalized uncertainty treatment is a viable solution procedure. According to these methods, the complex problem is divided into a series of detailed questions. Qualified persons (experts) are asked to provide judgments or answers to these questions with simple descriptive words (linguistic variables). The answers are provided in these terms because of the nature of the parameters being questioned. Then, these linguistic measures are translated into fuzzy sets with the help of a system analyst. Using the operations of fuzzy sets, an overall assessment of the condition of the structure, durability, or damage state can be obtained. Such methods, also, offer the possibility of combining the judgment of several experts about some of the questions. Weight factors can be used to combine the assessments of the experts that depend on experts as well as the analyst's assessments of the experts.

Another important area of application in structural engineering is in the field of structural reliability. Structural reliability, as is commonly known, emerged based on the objectives of ensuring the safety and performance of a structural engineering system for a given period of time and for specified loading conditions and functions. The absolute safety of a structural system cannot be guaranteed without an outlay of infinite resources. This is due to the uncertainties involved in structural parameters, modeling methods, prediction models, loading conditions, and structural behavior. However, through structural reliability methods, the risk of failure of the structural system and its components can be limited to acceptable levels. In order to have control on the risk of failure of structural systems and their components this risk needs first to be estimated and assessed. Reliability assessment of structures has received great attention by researchers since the early days of structural reliability. Since 1947, when Freudenthal published his first work on safety of structures, a great number of methods have been developed and suggested for reliability assessment [3, 8, 11, 14, 26, 32, 33, 43, 50, 57, 58, 60, 74, 83, 85, 89, 91].

These methods determine the probability of failure of a structural component or system according to a specified limit state equation. Consider the following performance function:

$$Z = g(X_1, X_2, \ldots, X_n) \tag{6}$$

where the X_i's are the basic random variables. Equation (6) defines the performance function, such that failure occurs where $g(\cdot) < 0$. The probability of failure can be determined by solving the following integral:

$$P_f = \int \cdots \int f_X(X_1, \ldots, X_n)\, \mathrm{d}x_1\, \mathrm{d}x_2 \cdots \mathrm{d}x_n \tag{7}$$

where f_X is the joint probability density function of $X = \{X_1, X_2, \ldots, X_n\}$ and the integration is performed over the region where $g(\cdot) < 0$.

Classical structural reliability assessment techniques, defined based on Eqs. (6) and (7), are based on precise and crisp (sharp) definitions of failure and non-failure (survival) of a structure in meeting a set of strength, function and serviceability criteria. These definitions are provided in the form of performance functions and limit state equations. Thus, the criteria provide a dichotomous definition of what real physical situations represent in the form abrupt change from structural survival to failure. However, based on observing the failure and survival of real structures according to the serviceability and strength criteria, the transition from a survival state to a failure state and from serviceability criteria to strength criteria are continuous and gradual rather than crisp and abrupt. That is, an entire *spectrum of damage* or failure levels (grades) is observed during the transition to total collapse. In the process, serviceability criteria are gradually violated with monotonically increasing level of violation, and progressively lead into the strength criteria violation.

Classical structural reliability methods correctly and adequately include the first three sources of uncertainty (physical randomness, statistical and modeling uncertainty) by varying amounts. However, they completely fail and are unable to incorporate the presence of a damage spectrum, and do not consider in their mathematical framework any sources of uncertainty due to vagueness and ambiguity. Vagueness can be attributed to sources of fuzziness, haziness, unclearness, indistinctiveness, sharplessness and grayness; whereas ambiguity can be attributed to nonspecificity, one-to-many relations, variety, generality, diversity and divergence [70]. Using the nomenclature of structural reliability, vagueness and ambiguity can be accounted for in the form of realistic delineation of structural damage based on subjective judgment of engineers. The inability of the classical structural reliability theory to incorporate these subjective elements is a significant deficiency. For situations that require decisions under uncertainty with cost/benefit objectives, the risk of failure should depend on the underlying level of damage and the uncertainties associated with its definition.

A mathematical model for structural reliability assessment that includes all sources of uncertainty was developed by Alvi and Ayyub [2]. The model result in the risk of failure over a damage spectrum that depends on the duration of exposure and interaction time of the structure with its environment. The resulting structural reliability estimates properly represent the continuous transition from serviceability to strength limit states over the ultimate time exposure of the structure. Researchers, e.g., Costa [29] used a non-fuzzy based method for determining the reliability of partly damaged structures. However, the fuzzy-based methods provide a complete and more effective treatment of both ambiguity and vagueness uncertainty types.

The theory of fuzzy sets was used in other example applications that include modeling the effect of workmanship on the strength of concrete [24]. Blockley [18] used fuzzy sets in the calculation of fatigue strength of structural details and the slenderness evaluation of steel columns. Hadipriono [54] used a repairability criterion for the assessment of structural damage due to for example explosive loading. The variables of repairability were determined as the amount of repair, repair time, repair cost, and resource availability. These variables were assessed based on expert judgement using qualitative subjective evaluation. Itoh and Itagaki [64] used fuzzy-Bayesian analysis in the reliability evaluation of structures. According to the suggested method, priori uncertain parameters can be estimated based on fuzzy information for structural reliability evaluation purposes. The fuzzy information was assembled based on decision analysis of inspection schedules. Dong et al. [35] used the notion failure possibility as a measure of structural reliability. This measure was based on fuzzy sets that were used to model the subjective assessments of experts about the condition of a structure, its environment and the criterion of failure. Furuta et al. [46–49] used fuzzy sets for the damage assessment of bridges and durability of structures in an expert system framework. Kaneyoshi et al. [66] used fuzzy regression analysis to optimize the cable tension in cable-stayed bridges. Elms [42] used fuzzy sets in developing code risk factors.

3.4. *Other application areas*

Several other civil engineering applications of fuzzy sets were reported in the literature. Alley et al. [1] used fuzzy set approaches in planning the Grand River basin in Ontario, Canada. The fuzzy set approaches were designed to study the results of questionnaires which were answered by local politicians and other interested parties. Non-quantitative and qualitative factors, and the viewpoints of interest groups, were incorporated in the decision making process. A dominance matrix was developed to rank, rate and compare the available alternatives. A comparison of the alternatives was made based on cost effectiveness.

In the area of geotechnical engineering, fuzzy sets were used to solve problems that involves vagueness uncertainty by Santamarina [81]. Gunaratne et al. [51] used fuzzy sets in the evaluation of pavements. Developments in membership functions were introduced for this purpose [82, 27, 28].

Several studies were performed in order to characterize the different sources of uncertainty in civil engineering [2, 17, 19–22, 36–38, 94]. Brown [22] separated uncertainty into objective and subjective, and then he suggested the merging of fuzzy subjective results with objective probabilities. Ayyub and Ibrahim [10] and Alvi and Ayyub [2] suggested the use of available uncertainty measures in structural engineering. These measures include Hartley measure [59], Shannon entropy [84], U-uncertainty, dissonance in evidence, confusion in evidence, and measure of fuzziness [31, 69, 70].

A workshop on civil engineering applications of fuzzy sets, that was sponsored by the National Science Foundation, was conducted at Purdue University in 1985 [23]. The participants identified the following potential and current applications: nuclear power plants; dam analysis for safety screening; safety during construction; quality of wildlife; construction management; environmental impact; performance of structures facilities and systems; damage analysis; fragility of nuclear power plants; contract bidding; codes and regulation; human error; human and social values; operation and control of water and reservoir systems; combining expert judgment with crisp information; stability of geotechnical structures; problems of seepage and grouting; offshore construction; and underground construction loads.

4. Summary and conclusions

Civil engineering projects and design are commonly developed in a systems framework that includes different types of uncertainty. In general, uncertainty can be of the ambiguity or vagueness type. The theory of probability and statistics has been extensively used in civil engineering to deal with the ambiguity type of uncertainty. The theory of fuzzy sets and systems have been used in civil engineering to model the vagueness type of uncertainty in many civil engineering applications. In this paper, the role of fuzzy sets in civil engineering systems is described using several example applications, e.g., quality assessment of wildlife habitat, construction engineering and management, structural reliability, and damage assessment of existing structures. It is evident from the examples presented in this paper that fuzzy sets and systems provide a complementary dimension to the classical set and probability theory in modeling uncertainty in civil engineering systems.

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