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A random-fuzzy analysis of existing structures

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Abstract: Two approaches are proposed to estimate the reliability of existing structures by considering both the randomness in some of the design parameters and the fuzzy imprecision in some other parameters representing the in-place condition of the aged structures. In the first approach, the fuzzy imprecision is transformed into random uncertainty using the entropy concept, and the reliability of existing structures is estimated using well-established theories of probability. In the second approach, a hybrid approach in the random-fuzzy domain is used to evaluate the reliability using an α-level concept. The multiple fuzzy variables case is also considered. Both discrete and continuous fuzzy variables are considered. The results obtained from the proposed approaches are compared with other techniques available in the literature whenever possible. Both methods are applied to civil structural engineering problems in this paper. The results obtained are very encouraging and demonstrate the applicability and robustness of the algorithms.

Keywords: Measure of fuzziness; linguistic modeling; probability theory and statistics; analysis; engineering.

1. Introduction

In conventional civil engineering practice, the designing of new structures got all the attention in the past. However, very recently the profession has realized its responsibility to assure the safety of an existing structure. This has caused a sense of urgency in the profession. According to the 1988 National Research Council Report, the estimated value of public infrastructure in the U.S. amounted to $49 trillion in 1984, and is growing rapidly. This clearly indicates that the amount of money already invested far exceeds new construction, and the protection of this investment is in the national interest.

To protect an existing structure, it is of utmost importance to understand its behavior. However, this is a challenging job. A structure deteriorates with time due to aging, excessive use, overloading, exposure to climatic variations, lack of sufficient maintenance, difficulties in carrying out some inspections, lack of methodologies for interpreting the results, and lack of understanding in developing remedial action when necessary. Engineering calculations and drawings of an existing structure when initially built are readily available from several sources. This information could be extremely valuable in evaluating an existing structure, but could be useless in some cases, because it fails to consider the in-place structural parameters. It fails to consider the structure's deterioration and degradation with time. For successful evaluation of an existing structure, the amount of degradation must be quantified and the in-place structural properties must be used to predict the structural behavior. Any mathematical model used must consider information from many different sources, including information from visual inspection or any other type of nondestructive evaluation procedure, as well as subjective assessment of the structure. In many cases, the subjective information could consist of the verbal assessment of experts. The mathematical representation of the linguistic variables can be achieved using fuzzy sets and systems theory.

It is also expected that the safety of an existing structure is evaluated in terms of risk or reliability. The state of the art in the evaluation of the reliability of engineering structures has advanced greatly in the last two decades. In a classical reliability analysis, the uncertain loads and resistance-related parameters are modeled as random variables, and the corresponding reliability or risk of a structural system is evaluated using well-established theories of probability [1, 2, 11]. In the safety evaluation of an existing structure, some of the variables are considered to be random (most of the load-related parameters, and some of the resistance-related parameters), some are considered to be deterministic (geometrical sizes of the structures), and some are neither deterministic nor random, but fuzzy (linguistic description of the structure reflecting the amount of degradation it has undergone). The class of problems where both random and fuzzy variables are present has received very little attention from the profession so far. The objective of this paper is the incorporation of fuzzy information to subjectively consider the present condition of the structure and the evaluation of the corresponding reliability of existing structures considering both the random and fuzzy variables.

2. Problem description

Evaluation of an existing structure is quite different than the process used when it was initially analyzed, designed and built. The mathematical model initally used to represent the structure may not be applicable to the aged structure. The exposure of the structure to the real environment may have altered many of the initial assumptions and structural parameters. From a probabilistic point of view, it can be viewed as introducing another source of uncertainty into the problem and must be accounted for accordingly. The information on physical condition, e.g., the presence of an excessive number of cracks or loss of material due to corrosion, may originate from visual inspection of the structure by experts. The experts' opinion could be linguistic in nature and must be incorporated in any risk evaluation procedure. Here, the parameter values obtained from a non-destructive evaluation procedure are fuzzified with the subjective information on the condition of the structure using expert opinions. The parameters thus rendered fuzzy are used to estimate the safety of the structure. The discussion of non-destructive evaluation schemes is beyond the scope of this paper.

It can be stated at this stage that in the evaluation of existing structures, some of the uncertain parameters are random and others are fuzzy. Safety analysis in the fuzzy-random domain is expected to be complicated. At least three approaches can be used for this purpose. In the first approach, the problem can be considerably simplified if the fuzzy imprecision can be transformed to random uncertainty in a suitable format that will facilitate the safety evaluation process. The justification for this approach is the most reliability formats are probability-based, the area is well-developed and it is easier to interpret probabilistic results. In this study, it is proposed that the transformation of fuzzy imprecision to random uncertainty can be made using the concept of entropy.

Conceptually, random uncertainty can be transformed to fuzzy imprecision, and for discussion purposes can be called the second approach. However, as stated earlier, the area needs further development before it can be applied to practical problems. It will not be discussed further in this paper.

In the third approach, a hybrid model can be used to estimate reliability in the random-fuzzy domain. Both approaches are discussed in the following sections.

3. Proposed method – entropy approach

The success of this approach depends on the transformation of fuzzy imprecision to random uncertainty and then formulating the problem in a routine manner as is done in any conventional reliability model by treating all the variables as random [1, 2, 11]. Thus, it is essential in this approach to transform fuzzy imprecision to random uncertainty. The concept of entropy is used for this purpose.

Entropy is a measure of the uncertainty of a random variable. Entropy is also defined for fuzzy variables as a measure of imprecision. Entropy can be thus looked upon as a measure of uncertitude. The premise of this transformation is that this measure is invariant under transformation. It is therefore considered appropriate to work with entropies to establish the equivalence of fuzzy and random variables. This concept has been used before [4, 8, 9] to update the parameters of a random variable. Here an equivalent random variable is defined which has an appropriate measure of uncertainty equivalent to the level of imprecision of a fuzzy variable. According to Shannon [12], the probabilistic entropy can be defined as

$$H_x = -\int_x p(x) \ln p(x) \, dx \tag{1}$$

where H is the entropy of X and $p(x)$ is the probability density function of X.

The non-probabilistic entropy defined by DeLuca and Termini [5, 6] and used later by Brown et al. [4, 8, 9] can be represented as

$$G_x = -\int_x (f(x) \ln f(x) + (1 - f(x))\ln(1 - f(x))) \, dx \tag{2}$$

where G is the entropy of X and $f(x)$ is the membership function of X.

The second term in (2) corresponds to complementary events. Alternatively, entropy can be defined as [13]

$$G'(x) = -\int_x f'(x) \ln f'(x) \, dx \tag{3}$$

where

$$f'(x) = \frac{f(x)}{\int_x f(x) \, dx}. \tag{4}$$

The term corresponding to the complementary events is dropped from (3) as the membership function is standardized. No information is lost by standardizing the membership function as membership functions can be looked upon as the relative measure of belonging to a set [13].

The equivalent normal random variable is defined in this study such that is has the same entropy as the fuzzy variable. Thus, equating (1) and (3), one obtains

$$H_{x_{\mathrm{eq}}} = G'_x. \tag{5}$$

The mean of the equivalent normal random variable is assumed to be the same as the value obtained from the parameter identification procedures of nondestructive evaluation schemes. The error associated with any nondestructive evaluation scheme (NES) is problem-dependent, and without any additional information, it is quite adequate and reasonable to assume that the error is spread on both sides of the observed or estimated value, and the value obtained by the scheme is the mean value. This is a standard engineering practice, unless otherwise mentioned by the manufacturer of the instrument used. Of course, any nonsymmetric distribution of error can be taken into account accordingly. The choice of normal variable is purely for mathematical convenience. In general, any two-parameter random variable can be used for this purpose. To define the normal random variable uniquely, the information on standard deviation, σ_x, must be known at this stage, and can be obtained in the following way.

The entropy of a normal variable with mean μ_x the standard deviation σ_x can be obtained by evaluating (1) as

$$H = \ln(\sqrt{2\pi e}\,\sigma_x). \tag{6}$$

From (5), the equivalent standard deviation, $\sigma_{x_{\mathrm{eq}}}$, can be shown to be

$$\sigma_{x_{\mathrm{eq}}} = \frac{1}{\sqrt{2\pi}} e^{G'_x - 0.5}. \tag{7}$$

The equivalent normal variable thus defined can be used with all other random variables already identified for the problem under consideration to estimate the reliability of the system.

In many practical problems, if the expert assessment is defined by a membership function, f_{xi}, based on rule-based inference schemes as in [14], the fuzzy variable x would be discrete. In this case, the procedure described above needs some modification as discussed below. The continuous equivalent normal variable needs to be discretized at x_i's by lumping the corresponding probability mass functions, p_i's. The entropy of a discrete variable is defined as

$$H = -\sum_{i=1}^{N} p_i \ln p_i. \tag{8}$$

For the problem under consideration, the p_i values can be calculated as

$$p_1 = \phi\left(x_1 + \frac{x_2 - x_1}{2}\right), \tag{9a}$$

$$p_j = \phi\left(x_j + \frac{x_{j+1} - x_j}{2}\right) - \phi\left(x_j - \frac{x_j - x_{j-1}}{2}\right), \quad j = 2, 3, \ldots, N-1, \tag{9b}$$

and

$$p_N = 1 - \phi\left(x_N - \frac{x_N - x_{N-1}}{2}\right).$$

$\Phi(x)$ is the distribution function of the normal variable and can be calculated as

$$\phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[-\frac{1}{2}\left(\frac{x-\mu_x}{\sigma_x}\right)^2\right] \mathrm{d}x. \quad (10)$$

The entropy of a discrete fuzzy variable is given by

$$G'_x = -\sum_{i=1}^{N} f'_x \ln f'_{xi} \quad (11)$$

where N is the number of data points and

$$f'_{xi} = \frac{f_{xi}}{\sum_{j=1}^{N} f_{xj}}. \quad (12)$$

As in the continuous variable case, (8) and (11) can be compared to calculate the equivalent normal standard deviation σ_{xeq}. However, since it is not possible to obtain a close-form expression for it in the discrete case, an iterative numerical scheme is necessary. It is necessary to assume an initial value for σ_{xeq}. Considering the mean value μ_{xeq} to be the same as that obtained from the parameter identification procedures or non-destructive evaluation schemes, the random entropy can be estimated from (8). Similarly, using (11), the fuzzy entropy can be calculated corresponding to the fuzzy membership functions. Comparing the two entropies thus obtained, an updated estimate of σ_{xeq} can be obtained. The procedure needs to be repeated until the old and new values of σ_{xeq} converge within a predetermined tolerance.

4. Random-fuzzy format

The error associated with the transformation from fuzzy to random uncertainty can be avoided by estimating risk or reliability by the direct hybrid approach in the random-fuzzy domain. However, the risk or reliability will be a fuzzy number with an associated membership function. A crisp measure of risk needs to be estimated according to some criterion as discussed below. The evaluation of risk in the random-fuzzy domain was proposed in the literature in the context of water supply systems [7]. This method is extended here to consider structural engineering problems and to consider more than one fuzzy variable using the α-level concept.

Case 1 – One fuzzy variable

For ease and clarity of discussion, a structural member is considered and is subjected to a random load Q. All structural parameters except one, i.e., the stress at failure σ_f, are considered to be deterministic. The stress at failure can not be accurately specified in this case, and is modeled as a fuzzy variable.

The stress, σ, due to the applied load can be expressed as

$$\sigma = f(Q, P) \quad (13)$$

where P is a structural parameter vector considered to be deterministic in this problem. The limit state equation for this structural element can be defined as

$$g(Q, \sigma_f, P) = \sigma_f - \sigma. \quad (14)$$

The structure fails when $\sigma \geqslant \sigma_f$ or when $g(\,) \leqslant \sigma$. The probability of failure is then

$$p_f = P[g(Q, \sigma_f, P) \leqslant 0.0]. \quad (15)$$

The limit state function in (14) is not just random but also a fuzzy number. Thus, (14) is no longer a crisp event but a fuzzy event. Equation (15) needs to be evaluated as a fuzzy number [3]. This can be done by estimating p_f conditioned on a specific value of σ_f and assigning the corresponding membership value to p_f. Experts opinions or information available in the literature can be used to assess the membership functions of σ_f. p_f thus obtained becomes a fuzzy number when evaluated over the whole range of σ_f.

Alternatively, fuzzy p_f can also be estimated from an α-level analysis as proposed here. Conceptually, this α-level method is similar to the method discussed above. However, it has some very desirable features. The α-level approach is very desirable for the case of multiple fuzzy variables as will be discussed later. It is especially convenient to compute the membership functions of a function of fuzzy variables.

A fuzzy number X, as described in [10] is an ordered pair defined as

$$x = (x, \mu_x(x)), \quad x \in R, \mu_x(x) \in [0, 1], \quad (16)$$

where $\mu_x(x)$ is the membership function of x. Alternatively, it can be represented as an

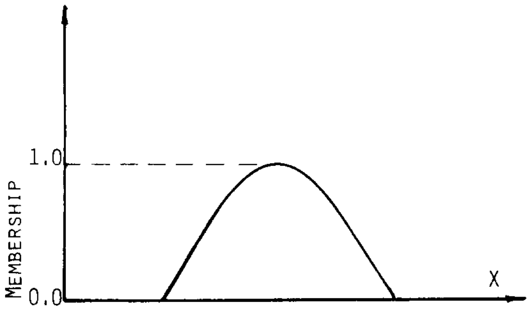

Fig. 1. Convex fuzzy membership function.

interval of confidence $x(\alpha)$ at any specified level of presumption α as

$$x(\alpha)=[x_1(\alpha), x_2(\alpha)], \quad \alpha \in [0, 1]. \tag{17}$$

A fuzzy number is convex and normal as shown in Figure 1 if

$$\alpha' > \alpha \Rightarrow X(\alpha') \subset X(\alpha),$$

$$\bigvee_x \mu_x(x)=1,$$

or

$$\alpha' > \alpha \Rightarrow [x_1(\alpha'), x_2(\alpha')] \subset [x_1(\alpha), x_2(\alpha)],$$

$$\bigvee_x \mu_x(x)=1.$$

Note that the membership function $\mu_x(x)$ in (16) is the inverse of $X(\alpha)$ in (17). Thus, σ_f considered here can be represented as

$$\sigma_f(\alpha)=[\sigma_{f1}(\alpha), \sigma_{f2}(\alpha)]. \tag{18}$$

Now varying α in [0, 1], $p_{f1}(\alpha)$ and $p_{f2}(\alpha)$ can be evaluated from (15), i.e.,

$$p_f(\alpha)=1-F_\sigma[\sigma_f(\alpha)] \tag{19}$$

where F_σ is the distribution function of σ derived from the density function of the load using (13). Equation (13) represents the stress at a particular point in the structure and is a function of the applied load and the geometric properties of the structure. Classical reliability approaches can be used to obtain the distribution function of σ from the statistical information of load and structural geometry-related parameters as discussed by Ayyub and Haldar [2] elsewhere.

The probability of failure thus obtained is a fuzzy number and can be represented as

$$p_f(\alpha)=[p_{f1}(\alpha), p_{f2}(\alpha)]. \tag{20}$$

In a conventional engineering risk analysis, it is very common to express risk in terms of a crisp estimate of the probability of failure. A crisp measure of the probability of failure can be computed as the fuzzy mean [3] as

$$\bar{p}_f=\frac{\int p_f p_{f1}^{-1}(p_f)\,\mathrm{d}p_f+\int p_f p_{f2}^{-1}(p_f)\,\mathrm{d}p_f}{\int p_{f1}^{-1}(p_f)\,\mathrm{d}p_f+\int p_{f2}^{-1}(p_f)\,\mathrm{d}p_f}. \tag{21}$$

The procedure described above will be explained later with the help of an example.

Case 2 – Multiple fuzzy variables

The hybrid approach discussed in the previous section needs further generalization to consider multiple fuzzy variables. This is discussed in the following paragraphs.

In (13), P is considered to be a deterministic vector representing the structural parameters. In general, P would include the following structural parameters: E, the Young's modulus; L, the span; I, the moment of inertia; H, the depth; and B, the width of the cross section of a structural element. For the sake of discussion, all these parameters along with the failure stress σ_f can be considered to be fuzzy variables and can be represented using the α-level approach as

$$\sigma_f(\alpha)=[\sigma_{f1}(\alpha), \sigma_{f2}(\alpha)], \tag{22a}$$

$$E(\alpha)=[E_1(\alpha), E_2(\alpha)], \tag{22b}$$

$$L(\alpha)=[L_1(\alpha), L_2(\alpha)], \tag{22c}$$

$$I(\alpha)=[I_1(\alpha), I_2(\alpha)], \tag{22d}$$

$$H(\alpha)=[H_1(\alpha), H_2(\alpha)], \tag{22e}$$

$$B(\alpha)=[B_1(\alpha), B_2(\alpha)], \tag{22f}$$

for $\alpha \in [0, 1]$. The load Q is again considered to be a random variable.

For ease of discussion, a cantilever beam with a rectangular cross section and the properties given in (22) is considered. The cantilever beam assumption in no way reduces the generality of the proposed method. The maximum stress for the assumed beam can be expressed as

$$\sigma=\frac{6QL}{BH^2}. \tag{23}$$

This equation could be different for other types of structural elements. The probability of failure for the beam under consideration is then

$$p_f = P(\sigma \geq \sigma_f) = P\left(Q \geq \frac{\sigma_f BH^2}{6L}\right). \qquad (24)$$

Here, σ_f, B, H and L are all fuzzy variables. Defining Z as

$$Z = \frac{\sigma_f BH^2}{6L}, \qquad (25)$$

(24) can be rewritten as

$$p_f = P(Q \geq Z). \qquad (26)$$

Here Z is a fuzzy number and its membership function can be obtained from those of σ_f, B, H and L using Zadeh's extension principle [10]. The interval of confidence $Z(\alpha)$ can be expressed as

$$Z(\alpha) = [Z_1(\alpha), Z_2(\alpha)] \qquad (27)$$

where

$$Z_1(\alpha) = \frac{\sigma_{f1}(\alpha) B_1(\alpha) H_1^2(\alpha)}{6L_2(\alpha)} \qquad (28a)$$

and

$$Z_2(\alpha) = \frac{\sigma_{f2}(\alpha) B_2(\alpha) H_2^2(\alpha)}{6L_1(\alpha)}. \qquad (28b)$$

A close-form expression for the membership function of Z involves the inversion of (28), which is extremely complicated. However, it can be constructed numerically by varying α in [0, 1] for each one of the fuzzy variables; using (28), the corresponding α-level values for Z can be obtained. This procedure is valid since the inversion of $Z_1(\alpha)$ and $Z_2(\alpha)$ yields unique results in this problem. It can be easily proved that this procedure would be valid in all cases if all the fuzzy variables are convex [10]. Once the membership function of Z is available, the probability of failure can be computed as a fuzzy number by the α-level scheme described earlier for Case 1.

The method suggested for multiple fuzzy variables appears to be very simple. However, it must be emphasized that the proposed algorithm is applicable only when the limit state function is available in close form and it is possible to isolate and group all the fuzzy variables so that they can be represented by a single fuzzy variable. Further research is necessary for more general cases where it is not possible to separate fuzzy and random variables and the limit state function is not available in close form.

The method discussed here will now be elaborated further with the help of an example.

Numerical example

A cantilever beam shown in Figure 2 is considered here. All the geometrical structural parameters are shown in the figure. Initially, they are assumed to be deterministic. The stress at failure, σ_f, is considered to be fuzzy and is evaluated to be 50 ksi. Using the first approach, this value of σ_f is considered to be the mean, and the equivalent standard deviation needs to be calculated using the fuzzy information, as discussed earlier. The fuzzy information must be collected on a case-by-case basis. However, for illustrative purposes only, and to validate the proposed algorithm, several arbitrary fuzzy membership functions are assumed. Both discrete and continuous fuzzy variables are considered. Equivalent normal variable parameters are calculated for each case using the proposed algorithm. For comparison, Zadeh's [15] equivalent random variable statistics are also determined. Zadeh's mean is the geometric mean of the membership functions. The variance is the second moment of the membership functions about the mean.

At first, σ_f is considered to be a discrete fuzzy variable with the following membership functions.

Case 1:

$$20|0.3, 25|0.5, 35|0.6, 40|0.8, 45|0.9,$$
$$50|1.0, 55|0.9, 60|0.8, 65|0.6, 75|0.5, 80|0.3.$$

Case 2:

$$10|0.3, 20|0.4, 25|0.5, 30|0.7, 35|0.8,$$
$$40|0.9, 45|0.95, 50|1.0, 55|0.95, 60|0.9,$$
$$65|0.8, 70|0.7, 75|0.5, 80|0.4, 90|0.3.$$

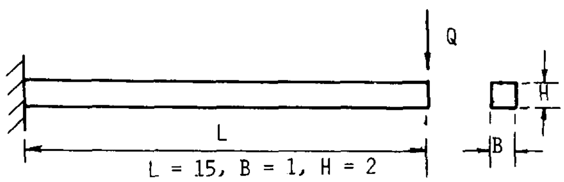

Fig. 2. A cantilever beam.

Case 3:

$30|0.3, 35|0.5, 40|0.8, 50|1.0, 60|0.9,$
$70|0.7, 80|0.4, 90|0.3, 100|0.10.$

Case 4:

$0|0.1, 10|0.3, 20|0.4, 30|0.7,$
$40|0.9, 50|1.0, 60|0.8, 70|0.3.$

Parameters for the equivalent normal variable according to the proposed algorithm as well as for the method suggested by Zadeh [15] are given in Table 1 for all four cases. The results will be discussed later.

Table 1. Equivalent normal statistics: Discrete case

	Proposed algorithm		Zadeh's statistics	
Case	Mean	Std. Dev.	Mean	Std. Dev.
1	50	15.77	50.0	15.09
2	50	19.27	50.0	18.49
3	50	24.60	56.1	17.81
4	50	24.60	41.6	17.25

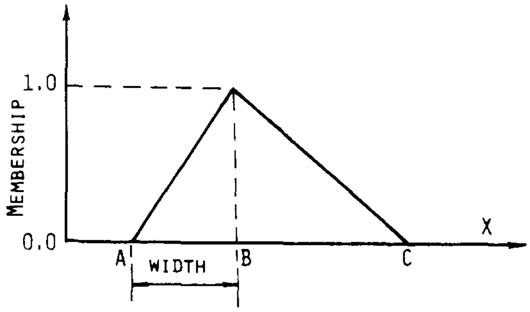

Fig. 3. Triangular fuzzy number (TFN).

In order to evaluate the proposed algorithm for a continuous fuzzy variable, four sets of symmetric and four sets of nonsymmetric Triangular Fuzzy Numbers (TFN) are considered here representing various amounts and types of spread in the membership functions. The parameters A, B and C to define the TFNs are defined in Figure 3. The results are summarized in a format similar to Table 1.

It is important to discuss Tables 1 and 2 at this stage. In Table 1, four discrete cases are considered. The membership functions for the first two cases are symmetric and for the last two cases are nonsymmetric. Each case shows a different amount of spread or fuzziness in the membership functions. For the symmetric membership function cases, the normal statistics obtained from the proposed algorithm and the method suggested by Zadeh [15] are almost identical. For the nonsymmetric membership functions, the statistics are similar; however, the mean values shifted to some extent according to Zadeh's method, showing some reduction in the standard deviation estimation.

Similar observations can be made for the continuous fuzzy variable cases. However, for all continuous fuzzy variable cases, symmetric and nonsymmetric, the standard deviations obtained from the proposed method are smaller than Zadeh's. The mean value always remains the same in the proposed method, but takes

Table 2. Equivalent normal statistics: Continuous case

Case	TFN			Proposed algorithm		Zadeh's statistics	
	A	B	C	Mean	Std. Dev.	Mean	Std. Dev.
Symmetric							
1	0	50	100	50	19.95	50	20.41
2	10	50	90	50	15.96	50	16.33
3	20	50	80	50	11.97	50	12.25
4	30	50	70	50	7.98	50	8.17
5	40	50	60	50	3.99	50	4.08
Unsymmetric							
1	0	50	80	50	15.96	43.3	16.50
2	20	50	65	50	8.98	45.0	9.35
3	30	50	55	50	5.00	45.0	5.40
4	35	50	90	50	10.97	58.3	11.61
5	45	50	80	50	6.98	58.3	7.73

Table 3. Probabilities of failure for entropy-based and hybrid approaches

	TFN		Entropy method	Hybrid method
Case (1)	Width (2)	Std. Dev. (3)	p_f (4)	p_f (5)
1	50	19.9	0.893 E −01	0.444 E −00
2	40	15.96	0.486 E −01	0.399 E −00
3	30	11.97	0.157 E −01	0.220 E −00
4	20	7.98	0.134 E −02	0.127 E −01
5	10	3.99	0.240 E −05	0.129 E −04

different values for nonsymmetrical membership functions according to the method suggested by Zadeh. Since the mean values change according to Zadeh's method corresponding to the geometric mean of the TFN, at least for the nonsymmetrical cases, the corresponding standard deviations are expected to be smaller. However, that is not true in this case. This observation points out the desirability of the proposed algorithm, at least for continuous cases.

From this discussion, it is established that the entropy approach is a viable alternative, i.e., the fuzzy imprecision can be transformed to random uncertainty, and then the reliability of an existing structure can be estimated using the conventional probability approach.

The stage is now set to compare the two approaches proposed here, i.e., the entropy-based and hybrid, to estimate the reliability of an existing structure. The cantilever structure shown in Figure 2 is again considered here. All the geometrical structural properties are shown in the figure, and initially they are assumed to be deterministic, except that the stress at failure, σ_f, is considered to be a TFN. For the cantilever beam considered here, the maximum stress occurs at the fixed end and is given by

$$\sigma = \frac{6QL}{BH^2}.$$

The load is considered to be a random variable represented by a normal distribution with a mean of 1.0 and a standard deviation of 0.2. The probability of failure of the cantilever, i.e, $\sigma > \sigma_f$, is calculated using both methods for all 5

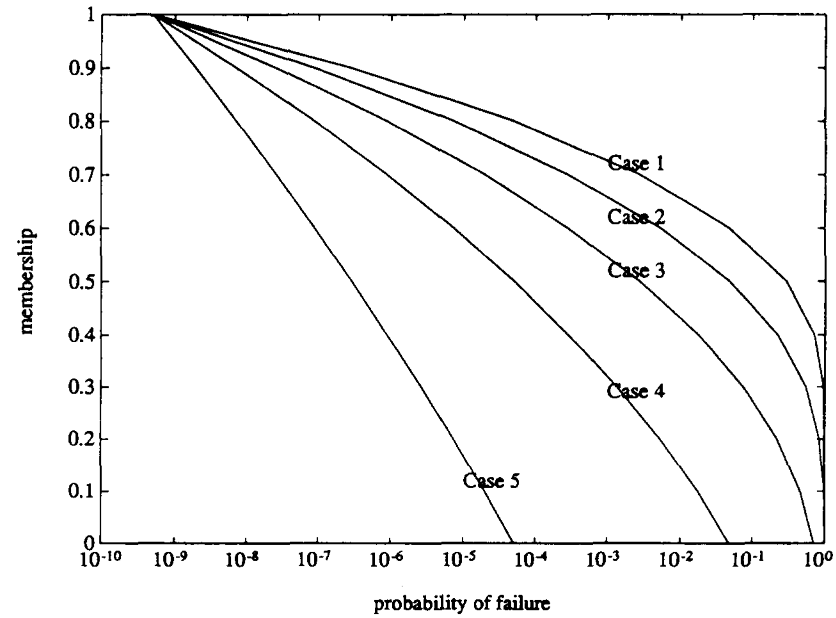

Fig. 4. Membership function of p_f.

Table 4. Probabilities of failure for the hybrid approach, multiple fuzzy variable case

	Yield stress TFN		Beam depth TFN		
Case	Mode	Width	Mode	Width	p_f
1	50	10	2.0	0.0	0.129 E −04
2	50	10	2.0	0.1	0.327 E −03
3	50	10	2.0	0.2	0.369 E −02
4	50	10	2.0	0.3	0.212 E −01
5	50	10	2.0	0.4	0.698 E −01

cases of Table 2 and the results are summarized in Table 3.

For the hybrid approach, the fuzzy probability of failure at various α-levels is calculated and is shown in Figure 4 for all five cases. The crisp measure of probability for each case is also estimated using Figure 4 and is shown in column 5 of Table 3.

Several important observations can be made from Table 3 and Figure 4. From Table 3, columns 2, 4 and 5, it can be seen that as the width of the TFN of the stress at failure increases, the corresponding probability of failure for both methods also increases. This is expected since the increase in the width of the TFN indicates an increase in the uncertainty level of the fuzzy variable. With increased uncertainty, a higher probability of failure is expected.

The probabilities of failure estimated by the two methods are shown in columns 4 and 5 of Table 3. They are not expected to be identical. The estimation of the crisp measure of probability for a fuzzy variable depends on the definition used. However, the similarities of these numbers are very encouraging and show that it is possible to reliably estimate risk in a random-fuzzy environment using either of the two methods suggested here. Further investigation is necessary in this area.

The hybrid approach is also applied to problems where multiple fuzzy variables are present. For illustrative purposes only, the cantilever beam discussed earlier can again be considered. However, in this case, in addition to the yield stress, the depth of the beam is also considered to be fuzzy. Both fuzzy variables are represented by symmetric TFN's with modal values of 50 and 2, respectively. The load is considered to be random as before and all other variables are considered to be deterministic. By changing the width of both the TFN's, the probabilities of failure are estimated from several cases using the proposed hybrid approach. The results are summarized in Table 4 for some representative cases.

When the width of the TFN representing the beam depth is zero, the problem reduces to a case with one fuzzy variable. The probabilities of failure are found to be identical to the values given in Table 3. However, for a given width of the TFN of the yield stress, the probability of failure increases with an increase in the width of the TFN representing the beam depth. The increase in the width of the TFN indicates larger uncertainty in the estimation of the beam depth. With larger uncertainty, the increase in the probability of failure is expected. This example demonstrates the applicability and robustness of the proposed algorithm.

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