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Fuzzy timed Petri nets

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Abstract

The study is concerned with a new temporal version of fuzzy Petri nets—fuzzy timed Petri nets (ftPNs). These nets are augmented by temporal fuzzy sets that allow for the representation of timing effect (e.g., aging of information). We show how the time factor can be added as an integral part of the models of transitions and places. The formalism of the extended net is introduced and studied in depth. A series of illustrative examples are presented with intent of developing a better qualitative insight into the temporal behavior of the nets. A detailed learning scheme for the net is also given.
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1. Introductory comments

Fuzzy Petri nets have been an area of vigorous theoretical and experimental studies that resulted in a number of formal models and practical findings, cf. Fuzzy Petri Nets [2,8,9,11]. These models attempted to address an issue of partial firing of transitions, continuous marking of input and output places and relate such models to the reality of environments being inherently associated with factors of uncertainty.

The timed extensions of Petri Nets most frequently found in the literature associate time to transitions. Generally speaking, two classes can be identified. The first one, usually called Time Petri Nets, originated from the proposal found in [7]. It associates two integer numbers to each transition that represent the moment at which the transition can be fired, and the moment at which it must be

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fired at the latest, both with regard to the instant at which the transition is enabled. Furthermore, the firing of a transition takes no time. The second approach, usually called Timed Petri Nets, originated from [10] and is based on the association of an integer number to each transition. This number indicates the duration of the transition execution.

Petri Nets have been extended with time in several different ways, by considering time associated with tokens, transitions, arcs and places; refer to [1,3–6,12–14].

The objective of this work is to extend the Generalized Fuzzy Petri Net model defined in [8,9] by adding a time component, either associated with transitions or with places.

The application areas of Petri nets being vigorously investigated involve process control and diagnostics [8], knowledge representation and discovery [14,4], traffic control [3], to name a few highly representative domains. The research undertaken in this study directly augments the models of Petri nets developed and used therein by adding and quantifying the concept of information whose relevance (importance) is affected by the aging factor.

The material of this study is arranged into six sections. In Section 2, we review the existing models of Petri nets that were developed in the framework of fuzzy sets. Section 3 concentrates on the essence of the generalization proposed in this study: we start with a detailed motivation and then move on to the underlying formalism of the detailed model where we show how temporal aspects of information in the net can be associated with transitions and places. This section includes a number of illustrative numeric examples. Section 4 concentrates on the issue of learning in tfPN where we show how the nets can be adapted to some training data through some parametric optimization. We elaborate on the application facets of the timed fuzzy Petri nets in Section 5 by discussing selected categories of situations in which these models are natural reflections of the real world phenomena. Finally, conclusions are covered in Section 6.

2. Fuzzy Petri nets (fPN): a generic model in the setting of fuzzy sets

The generic model of the fuzzy Petri net (fPN) as originally introduced in [9] arises around fuzzy logic operators (t- and s-norms). The operations describing firing of transitions and ensuing migration of tokens at input and output places are deeply rooted in the language of logic (or being more specific fuzzy logic). The constructs introduced in this manner comply with the embedding principle meaning that when considering a Boolean variant of the net, it converts into its binary counterpart. To briefly summarize the functioning of the fPNs, refer to Fig. 1 that visualizes a collection of n-input places interacting with an output place through a single transition.

The transition $\mathscr{T}$ fires if the input places come with a nonzero marking of the tokens; the level of firing is inherently continuous. The level of firing (z) assuming values in the unit interval is governed by the expression

$$z = \operatorname*{T}_{i=1}^{n} ((r_i \rightarrow x_i) s w_i), \tag{1}$$

where $T(t)$ denotes a t-norm while "s" stands for any s-norm. More specifically, x_i denotes a level of marking of the ith place. The weight w_i is used to quantify an input coming from the ith place. The threshold r_i expresses an extent to which the corresponding place's marking contributes to the

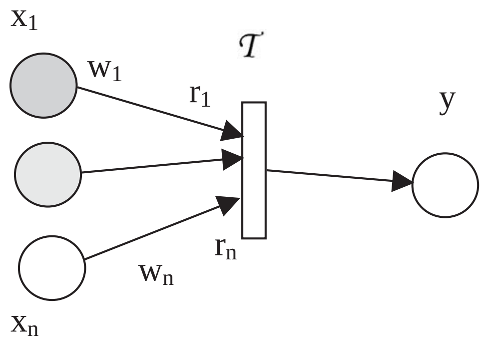


Fig. 1. An example of a fPN (a level of tokens is illustrated by using different shading intensity); see a detailed description in text.

firing of the transition. The implication operator ($\rightarrow$) expresses a requirement that a transition fires if the level of tokens exceeds a specific threshold (quantified here by r_i).

Once the transition has been fired, the input places involved in this firing modify their markings that is governed by the expression

$$x_i(\text{new}) = x_i t(1 - z), \quad i = 1, 2, \ldots, n. \tag{2}$$

(Note that the reduction in the level of marking depends upon the intensity of the firing of the corresponding transition, z.) Owing to the t-norm being used in the above expression, the marking of the input place gets lowered. The output place increases its level of tokens following the expression:

$$y(\text{new}) = ysz. \tag{3}$$

The s-norm is used to aggregate the level of firing of the transition with the actual level of tokens at this output place. This way of aggregation makes the marking of the output place increase.

The plots of the levels of firing of the transition and modifications to the level of tokens at input and output place are shown in Fig. 2.

The above fuzzy model directly generalizes the Boolean case of Petri nets. In other words, if x_i and w_i assume values in $\{0, 1\}$ then the rules governing the behavior of the net are the same as those encountered in the "standard" Petri nets.

Before moving on to the timed extension of the fPN, it is instructive to cast this fuzzy Petri net in some application setting to see how it captures its essence of this application. Let us note that a certain category of classification problems that can be expressed via a collection of rules

- if feature$_1$ is A and feature$_2$ is B then class ω.

The feature space has two variables (features), namely feature$_1$ and feature$_2$. A and B are fuzzy sets defined in the corresponding feature spaces. These could be linguistic terms (for instance, *small* and *large*). If the conditions of the rule are satisfied, then the rule "fires" and in this way identifies class ω.

The functioning of the rule is obvious. For a given pattern we want to classify, we end up with the "activation" levels of A and B caused by the actual values of this pattern, say $A(x_1)$ and $B(x_2)$

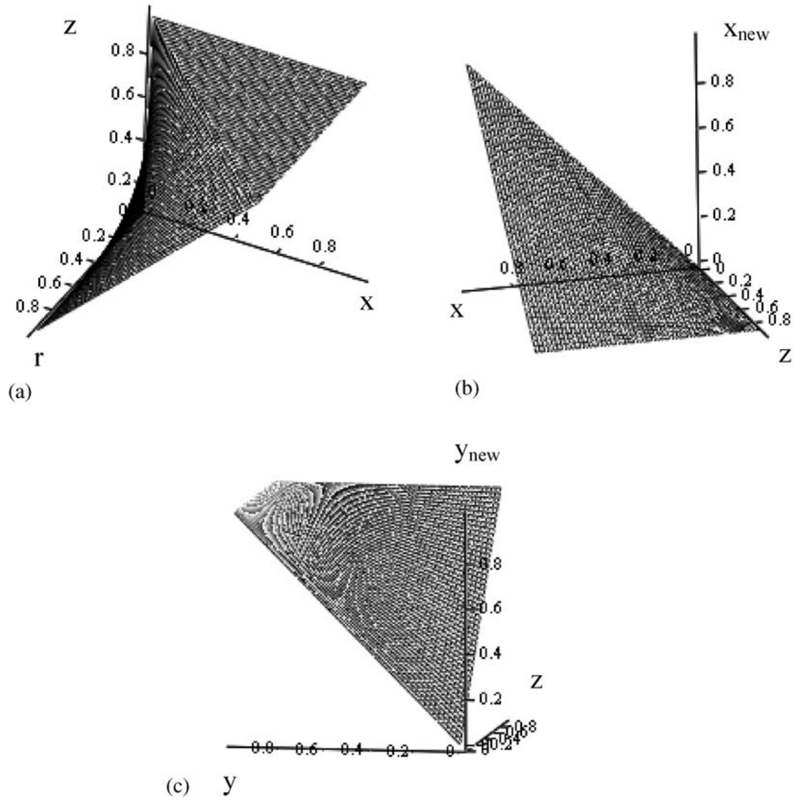


Fig. 2. Plots of levels of firing of transition (a) and changes to marking of the associated input (b) and output place (c). t-norm: product; s-norm: probabilistic sum; implication $a \rightarrow b = \min(1, b/a)$, $w = 0.1$.

considering that x_1 and x_2 are the numeric values of the features of the patterns. If these two are active (with nonzero membership grades), then the transition fires and this in turn translates into a new marking of the output place representing a degree of membership to the class. We may interpret this marking as a confidence level that the pattern belongs to class ω.

3. Timed fuzzy Petri nets

In this section, we introduce a class of timed fuzzy Petri nets (tfPNs), provide with some motivation and proceed with the pertinent formalism. Before embarking on the underlying formalism, it is instructive to elaborate on some areas of applications that motivate studies on this class of Petri Nets.

3.1. Motivation

The fPN summarized in the previous section has revealed how they can represent rules and what it means to carry out classification in the language of token passing across the net. Interestingly, the classification problem can be made more refined to reflect the character of such pattern recognition processes. This directly leads to the generalization of the net.

To emphasize the need for further augmentation of the generic form of the fPNs, let us consider two broad categories of situations

(a) The pertinent values of the features may be available in a certain time interval and do not appear in the same time instant. We would like to capture this effect and attach a time interval $\mathbf{T} = [t_1, t_2]$ to the transition of the respective places corresponding to this features. The time interval $\mathbf{T}$ means that the transition fires when all feature values have appeared and are available.
(b) To cope with the availability of the pertinent information about the features, we consider a time factor coming with each input place. In other words, the motivation is such that once we start observing the nonzero level of marking at the input place, this marking start " aging" with this aging effect being quantified by the time frame $\mathbf{T}_i$ (note that there could be different time frames $\mathbf{T}_i$ depending upon the input place).

Moving on with additional motivation, the importance of the timing factor can be emphasized in the following manner. In a recognition (classification) system we may have input signals (say, data coming from sensors) and on their basis we obtain the class membership grade. As the information may not be available from all sensors at the same time moment, the one that occurs earlier needs to be discounted over time as becoming less relevant. The timing function $\mathbf{T}$ associated with each sensor is used to quantify this effect. Some standard models of these temporal relationships include exponentially decaying functions, say $\mathbf{T}_i(t) = \exp(-\alpha t)$ where t is a time instance when the nonzero marking at this particular place happens. The parameter (α) controls the speed of deterioration of information. On the other hand, the timing effect associated with the transition rather than the individual input places emphasizes an importance of some temporal relationships between the presence of the nonzero tokens at the input places. In contrast to the time factor associated with places, the time factor associated to transitions does not produce an aging effect on each individual piece of information, but captures the requirement that they have to be available at the time specified by the time interval or they would not be relevant to that particular classification step.

Let us elaborate on a detailed application scheme in which this type of fuzzy Petri nets plays a pivotal role. In any task of decision-making, control, classification, etc. we encounter a variety of sources of information. These data records are collected from different parts of the process and they come at different time instants. For instance, in a fault monitoring process, we encounter data associated with different variables. To construct a model of fault-detection procedure, we consider that the measurement is assessed vis-à-vis some reference fuzzy set (specified for the purpose of the monitoring process), say elevated pressure, high temperature. The original numeric datum (x) available at the sensor is then cast in the setting of the reference set giving rise to the expression $A(x)$. Next this transformed measurement is used in the fault-detection model. Put it differently: $A(x)$ is a marking of a certain input place of the fuzzy Petri net being regarded as a fault-detection model (the output place of such net is associated with the level of alert produced by the net).

As the readings could come at different time instants and be collected at different sampling frequency, we encounter an inevitable aging of information collected by the sensor. It becomes apparent that its relevance is the highest at the time moment when the sensor captures it but then its relevance has to be discounted over the passage of time. This is an effect of aging that has to be viewed as an integral part of the model and it triggers interest in the class of the timed fuzzy Petri

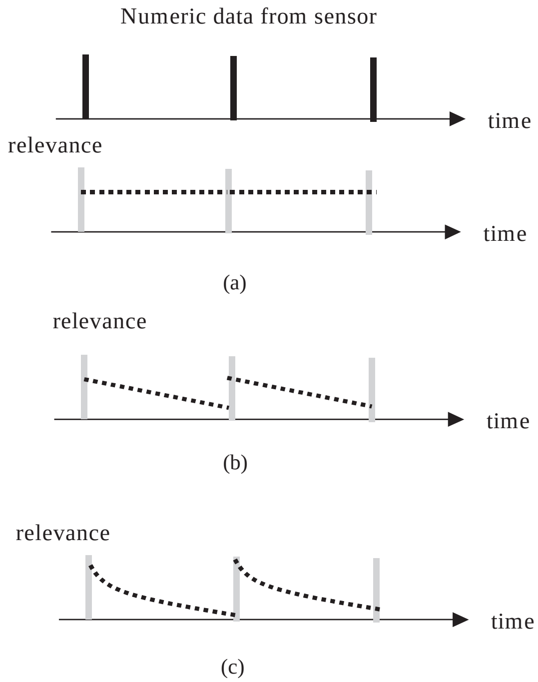


Fig. 3. Modeling relevance of information-various models of aging: no aging (a), linear effect of aging (b), and exponential model (c).

nets. A schematic view at the relevance of information is included in Fig. 3; note that the aging effect (reduced relevance) can be modeled in many different ways; for comparative purposes we also included a case in which no aging is considered.

Here we would like to stress that the effect of aging data is not "measured" by sensors but results from an assessment of the relevance of information that is processed. Intuitively, as usually encountered in signal processing we may anticipate an exponential model of relevance decay as shown in Fig. 4. This means that if a sensor under discussion provides us with its measurements obtained in discrete time instances, the relevance of information (related to its aging effect) decays exponentially with some positive intensity rate γ, say relevance$(t) = \exp(-\gamma t)$. Depending upon the value of the exponent, the decay may vary.

Taking into account the overall scheme of collecting data, their transformation through fuzzy sets and the related aging effect, we can build a general scheme outlined in Fig. 5.

The timing effect can be modeled by temporal fuzzy sets, viz. fuzzy sets whose membership functions are defined in time space. To underline this fact, we use a notation $\mathbf{T}$ (for the time attached to a certain transition) and $\mathbf{T}_i$ for the time associated with the individual places. The membership function of $\mathbf{T}$ could come in the form of a unimodal fuzzy set (usually a trapezoidal fuzzy set can capture a broad range of situations). To express the effect of aging occurring in the input places, one considers $\mathbf{T}_i$ to be monotonically decreasing functions of time.

The two categories of situations discussed in the previous section, motivate an emergence of a class of timed fuzzy Petri nets (tfPN). The factor of time affecting the relevance of marking of

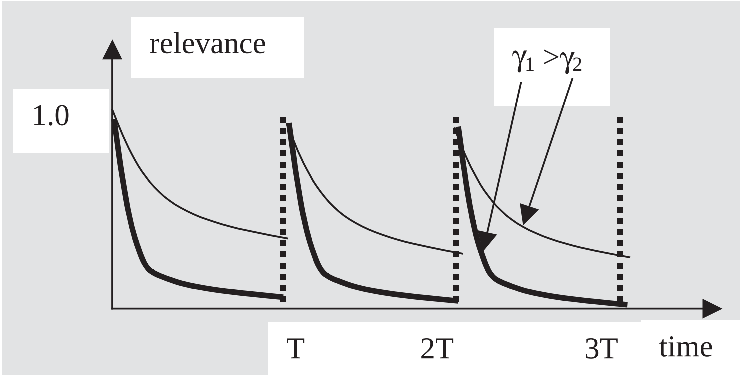


Fig. 4. An effect of information relevance related to its aging effect ($T, 2T, 3T$, etc. are the time instances when information is collected by the sensor).

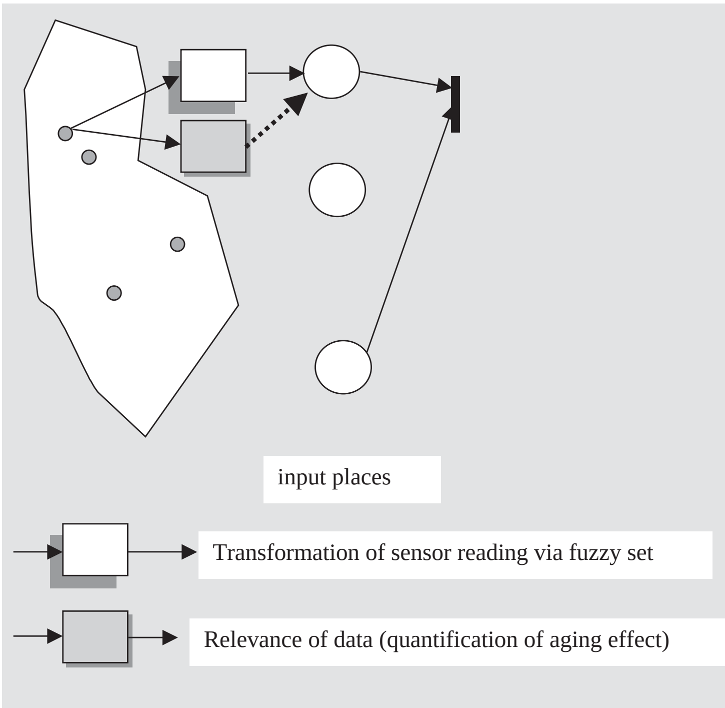


Fig. 5. A general framework of dealing with sensor information.

the input places has not been explicitly captured in the previous generic model of the fPN. In the classifier discussed so far we have assumed that we were provided with the relevant information over time and this effect manifests either by (a) or (b). Schematically, we visualize the timed facet of the fPN in Fig. 6.

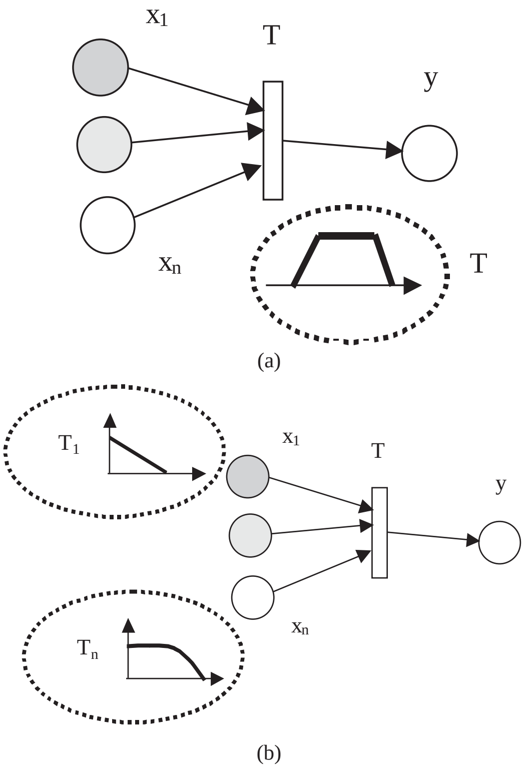


Fig. 6. A timed facet of fPNs; time associated with transition (a) and places (b). Note that the modeling of this effect is realized in terms of fuzzy sets; their membership functions are also indicated in the figure.

3.2. The formalism of the tfPN: a detailed model

The concept of the timing effect in the net needs to be converted into the formalism of the logic expressions governed a way in which the tokens flow across the net. The detailed formulas reflect a way in which the time component is inbuilt into the net. More precisely, it depends whether it associates with the transition or the individual input places.

Following the first way in which the transition comes with the time framework, we modify the expression for the firing of the transition by augmenting the previous expression (1) by

$$z = \left(\mathop{\mathrm{T}}_{i=1}^{n} ((r_i \to x_i) s w_i) \right) t \mathbf{T} \tag{4}$$

so as $\mathbf{T}$ changes over time, this impacts the level of firing "z". The formulas for the change in the marking of the input and output places remain the same as introduced before (Eqs. (2) and (3)).

Note that the time factor ($\mathbf{T}$), the degree of firing z and the markings x_i and y change continuously over time.

To elaborate on the temporal behavior of the net, suppose the current values of input and output markings are equal to $x_i(v')$ and $y(v')$, respectively and the last degree of firing is $z(v')$. Considering

"v" to be the time instant immediately following v', the resulting degree of firing is calculated as a function of time, through the evaluation of $\mathbf{T}(v)$, and considering the current value of the input marking $x_i(v')$. The detailed calculation can be completed by rewriting (4)

$$z(v) = \left(\mathop{\mathrm{T}}_{i=1}^{n} ((r_i \to x_i(v'))sw_i) \right) t\mathbf{T}(v), \tag{5}$$

After each firing, the input and output markings need to be updated. The next value of x_i is calculated from the last degree of firing found ($z(v)$), and the current value of x_i. Rewriting (2) in a more detailed way, the new input marking is calculated as

$$x_i(v) = x_i(v')t(1 - z(v)). \tag{6}$$

In a similar manner the next value of y, denoted by $y(v)$ is calculated from the last degree of firing found, and the current value of y. Referring to (3), we obtain

$$y(v) = y(v')sz(v). \tag{7}$$

At the next time instant, the degree of firing is calculated again as a function of time and of the recently updated values of x_i.

In the second approach where the time factor comes with the input places, we express it by considering an effective marking of the places augmented by the aging factor. Introduce the notation

$$x_i^{\sim} = x_i t\mathbf{T}_i. \tag{8}$$

Then the firing of the transition is governed by the expression

$$z = \left(\mathop{\mathrm{T}}_{i=1}^{n} ((r_i \to x_i^{\sim})sw_i) \right), \tag{9}$$

where $x_i^{\sim}$ becomes an effective level of firing of the ith input place.

Again we proceed with a detailed explanation of the behavior of the net, assuming that the current values of input and output marking are denoted by $x_i(v')$ and $y(v')$, respectively. The first step here is to evaluate the impact of the time factor on the input marking, and this leads to the following expression:

$$x_i^{\sim}(v) = x_i(v')t\mathbf{T}_i(v). \tag{10}$$

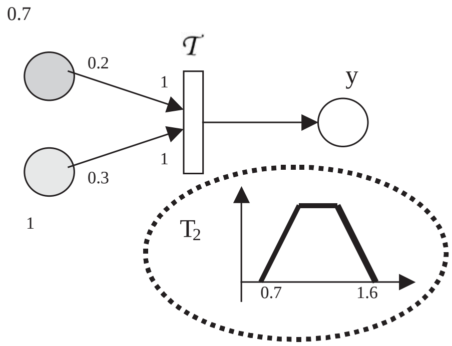
Subsequently, calculate the next degree of firing, through (9), and this can be rewritten in the form

$$z(v) = \left(\mathop{\mathrm{T}}_{i=1}^{n} ((r_i \to x_i^{\sim}(v))sw_i) \right). \tag{11}$$

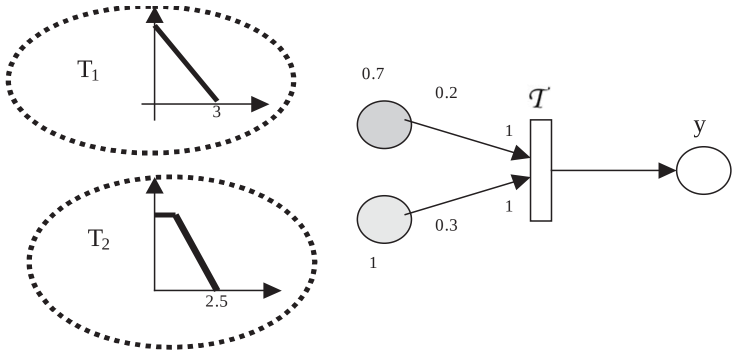
The update of the input and output markings is realized in the sequel, by applying (6) and (7).

The effects of timing can be included both at the level of the places and transitions by combining the two generic models studied so far.

In what follows, we come up with a series of illustrative examples that helps us gain a better qualitative insight into the temporal behavior of the nets. In a nutshell, we are interested in the



(a)



(b)

Fig. 7. Example of a input marking for a ftPN with two input places. Time associated with transition (a) and with places (b).

changes in the marking of the places and in the level of firing of the affected transitions. All changes are governed by the equations defined previously in this section. Let us consider, as an example, the particular net in Fig. 7; the values of the parameters (connections w_i and thresholds r_i) are shown in the same figure. The triangular t- and s-norm is implemented as a product and probabilistic sum. The time factor is represented via triangular or trapezoidal fuzzy sets. Starting with some initial marking of the net, also shown in Fig. 7 (the output place comes initially with a zero marking), we obtain the marking as in Fig. 8.

Lets now look at the changes over time for the degree of firing and markings for both cases in Fig. 7 that led to the final marking illustrated in Fig. 8. We consider first the ftPN with a timed transition (Fig. 7(a)). The temporal interval is modeled as a trapezoidal fuzzy set (see Fig. 9).

The changes over time in the input marking (Eq. (6)), output markings (Eq. (7)), as well as in the degree of firing (Eq. (5)) are illustrated in Fig. 10(a)–(c).

We consider now the ftPN with timed places (Fig. 7(b)). The respective time fuzzy sets are shown in Fig. 11.

The changes in the marking and the degree of firing are illustrated in Fig. 12. In order to observe how the time information attached to places affects each input value, the results for the input markings

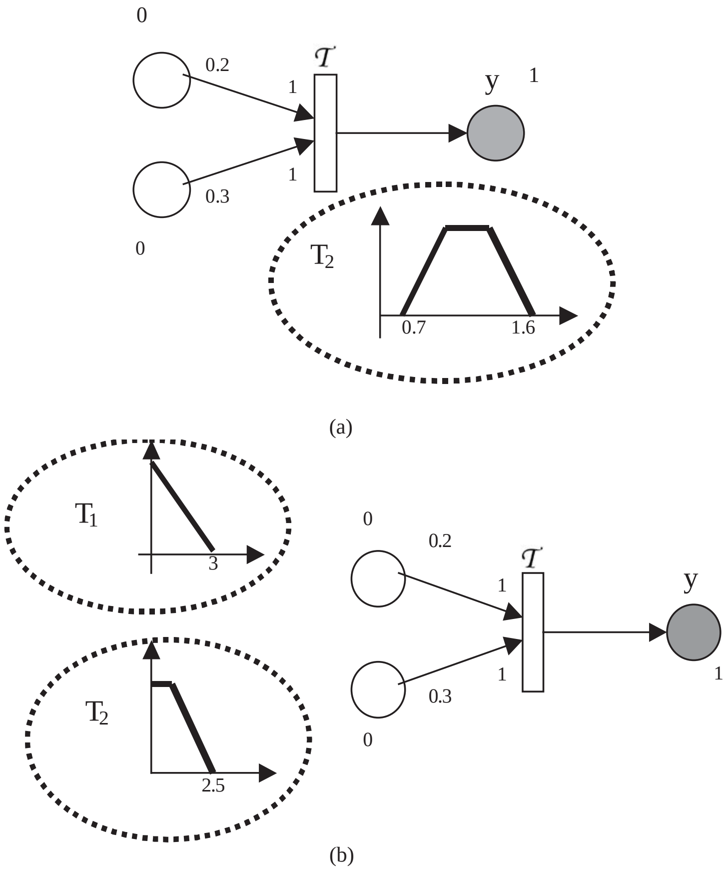


Fig. 8. Output marking obtained for the ftPN shown in Fig. 7. Time associated with transition (a) and with places (b).

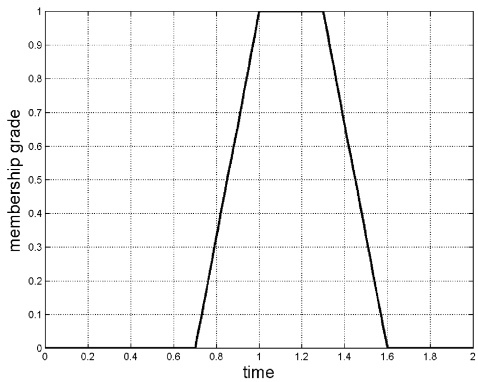


Fig. 9. A fuzzy set of time interval.

(obtained through Eq. (6)) are shown in Fig. 12(a). Likewise the results for the input marking with aging effect (obtained through Eq. (10)) are shown in Fig. 12(b). Note that in this particular situation, the aging effect was not strong enough to affect the result, and the final value of the output marking

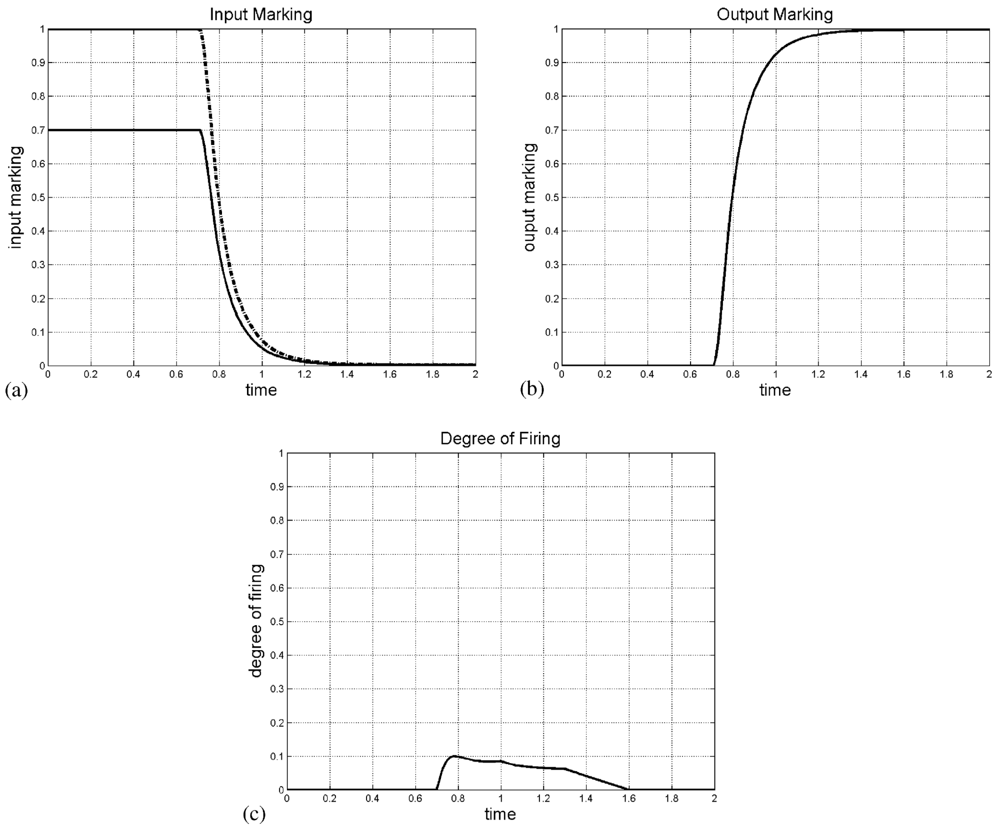


Fig. 10. Behavior of a ftPN with timed transition: changes in marking of input places (a), changes in marking of the output place (b) and temporal changes in the degree of firing z (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

is equal to one. In the setting of a classification problem as described in Section 2, this means that the confidence that the given pattern belongs the class is one.

Some interesting issues arise as a result of a comparative analysis of these nets. Let us first consider our original model as presented in Section 2, that is, the net without the factor of time explicitly affecting the degree of firing and marking updates. Since the transition firing is inherently continuous, the net behavior can be shown as a function of time. The changes in input marking, output marking and degree of firing along time are shown in Fig. 13. In this case the initial firing occurs immediately. Since there is no time restriction for the firing, there is a single spike of "strong firing" (high value of z, and significant initial changes in x and y) and the values change faster. In contrast, in the fuzzy timed Petri net, the firing levels are not so strong (Fig. 10(c)) and do not achieve a value equal to one; this is a manifestation of the existence of the fuzzy time.

The behavior of the net with fuzzy timed places is more similar to the one of the original net, assuming that all markings are available at initial time: there is a first "strong" firing (high degree of firing z, large decrease in the level of x_i, large increase in y at the first firing), except for the initial changes in z, y and x that are smoother.

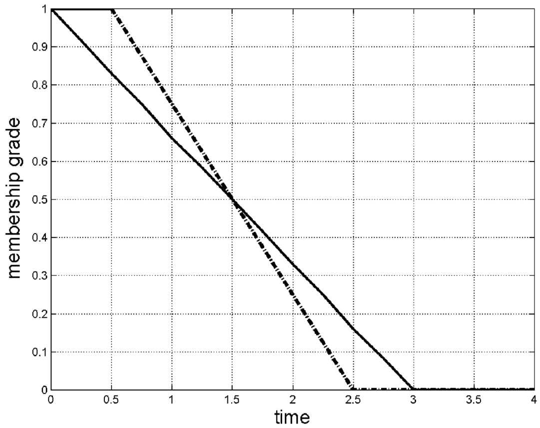


Fig. 11. Fuzzy time intervals used in the tfPN. The time interval associated with the first input place is denoted with a solid line while the time interval associated with the second place is denoted as a dash-dotted line.

Let us also consider, for means of comparisons between illustrative examples, a ftPN with timed transition like the one in Fig. 6(a), the time interval being a crisp interval, namely the one shown in Fig. 14. In this case, the changes in z, y and x, with respect to the same net with fuzzy timed transition, are faster as well, as can be seen in Fig. 15. It is also interesting to observe that in this net, there is a shift over time in the process of changing values with relation to the original net, that is, nothing changes until time gets into the specified time interval, and after that, the values change exactly like in the original net.

It is also of interest to consider some boundary cases as far as the marking levels are concerned, that is input marking equal to one or the values of the weights being equal to zero. Several examples were run in this case. We used the same four different types of nets considered so far, namely (a) original fPN without any explicitly factor of time, (b) fPN with time interval associated with transition, (c) fuzzy tPN with a fuzzy time interval associated with transition, (d) and fuzzy tPN with a fuzzy time interval attached to places. The time intervals considered in all these cases are the same as in the previous examples (shown in, Figs. 9, 11 and 14). First, we consider the following collection of initial markings and parameters of the net, $\mathbf{x} = [1, 1], \mathbf{r} = [1, 1], \mathbf{w} = [0, 0]$.

The temporal behavior of the original net coincides with the two-valued Petri net: there is just a single immediate firing where the degree of firing itself is equal to 1. Then the input markings go to zero while the degree of firing and the marking of the output place goes to one. In the net with the interval timed transition, there is a single firing as well. The input markings go to zero and output marking goes to one; however, the firing is not immediate and gets stretched throughout time. In the net with the fuzzy timed transition, the transition does not fire until time instants located in the allowed interval. The firing is continuous and the input and output values change gradually. Since the time interval is modeled as fuzzy, the degree of firing is not equal to one. The input and output marking may or may not reach 0 (input) and one (output), depending on how long is the time interval. The behavior of the net with the fuzzy timed places is exactly the same as the one for the original net: there is just one immediate firing, input markings go to zero and output marking goes to one.

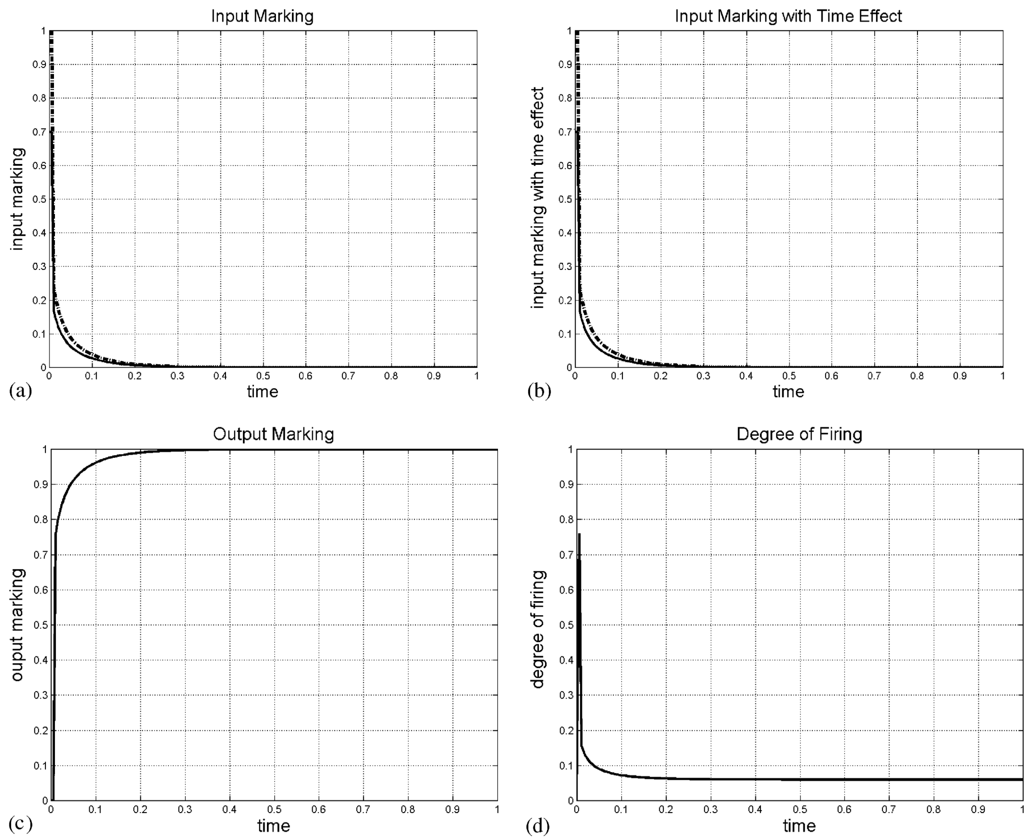


Fig. 12. Behavior of a ftPN with timed places: changes in input marking (a), changes in input marking affected by time (b), changes in output marking (c) and changes in the degree of firing z (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

Lets us now proceed with the discussion of the case where the arc weights are zero for more general situations than the previous one, like for instance when the initial markings are not all equal to 1, that is, $\mathbf{x}=[0.7,1]$, $\mathbf{r}=[1,1]$ and $\mathbf{w}=[0,0]$. This is exactly the example in Fig. 6, with zero weights.

In this situation, the original net behavior presents one first "strong firing", that is, a high value of z, resulting in large initial changes in x and y. The changes are softer than in the example with nonzero weight values that is the one in Fig. 6(a). In the crisp interval timed transition net, the addition of time information shifts the changes that would have occurred in the original net with the same parameters and input markings, so that they start at the beginning of the time interval, and stop at the end of the time interval. Since the weights make the changes faster, in both these zero weight nets the values change slower, compared to the same net with nonzero weight. As long as in the net with crisp interval timed transition, the marking changes are truncated when the time interval ends, the final marking may be far from 0 and 1.

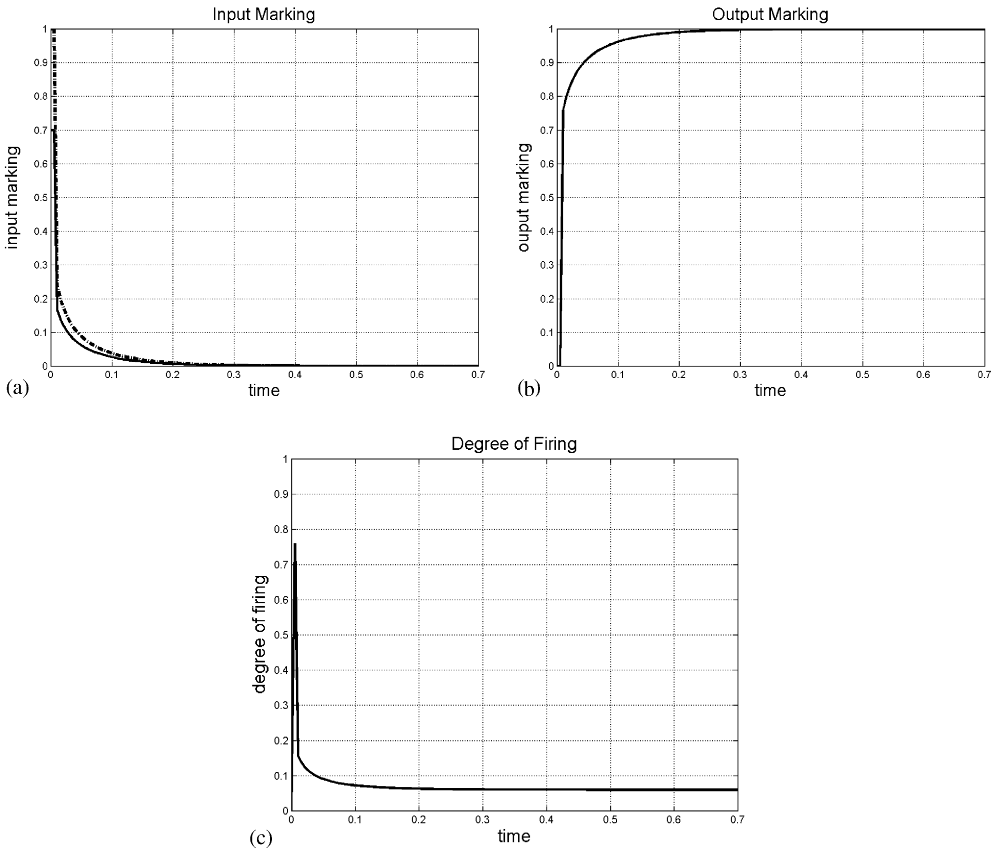


Fig. 13. Temporal behavior of the original net: input marking (a), output marking (b), and degree of firing (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

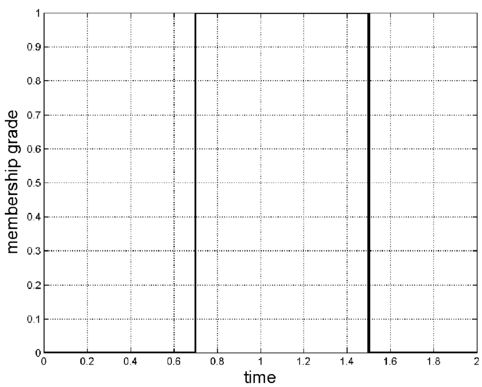


Fig. 14. Time interval represented as a set.

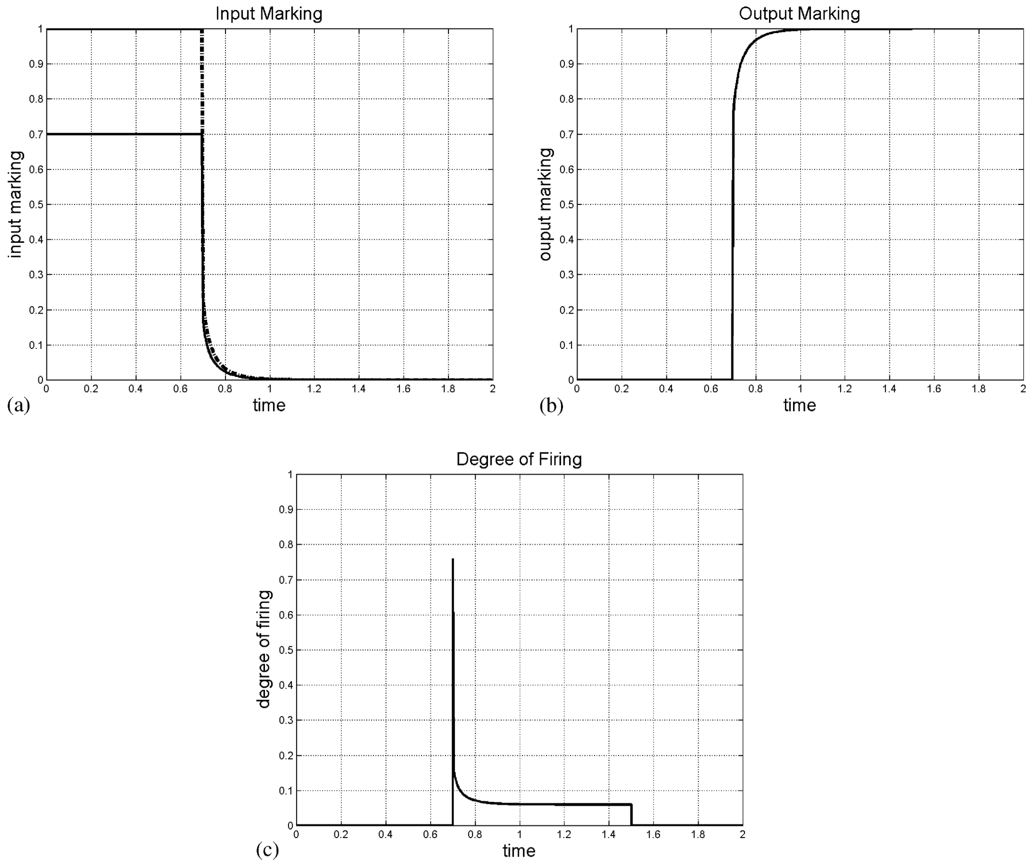


Fig. 15. Net with time interval: input marking (a), output marking (b), and degree of firing (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

In the net with fuzzy timed transition, shown in Fig. 16, changing in input and output markings are slower than in the net with nonzero weights. The initial values of z are smaller and decrease faster, as can be noticed by comparing Figs. 16(c) and 10(c). As a consequence, for the particular time intervals considered in the examples, input and output markings do not reach zero and one as final values.

The behavior of the fuzzy timed places net in the case where all arc weights are equal to zero is shown in Fig. 17. Noticeably the changes are smoother than in the net with nonzero weights.

It is important to note that the weights play a significant role when adding time information to the fPN. In general, for higher arc weights, the changes in the values of input and output markings are faster. One can expect that for the same time intervals nets with higher weights will behave similarly to the original net, in the sense of having the input and output values closer to zero and one, respectively, as the final marking.

In what follows we consider additional examples of ftPN with timed places to address the cases where the time intervals are very different for all inputs and where the markings are not available

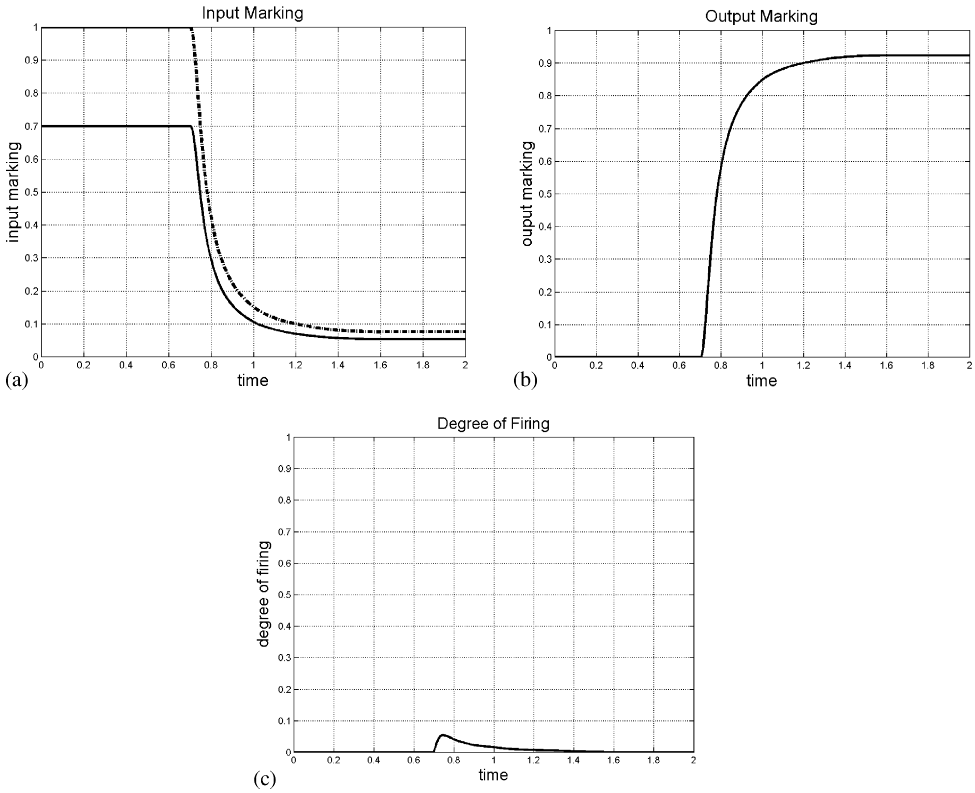


Fig. 16. Net with fuzzy timed transition and zero weights: input marking (a), output marking (b) and degree of firing (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

at the same time moment. When all input markings are available at the beginning the change in the values of z, y and x depend on the specific form of the time intervals (membership functions) associated with each place. If all membership functions of the time intervals are of triangular or trapezoidal shape with a small core, this implies that at some point, the values of z and y will stop changing (earlier than in the original net). The values of x continue to decrease, as it would be expected because of the aging effect in the timed places. One can then conclude that if the time intervals are long, the behavior of this net can be very similar to the one of the original net (see Fig. 13).

When the time intervals are quite different for all inputs, (such as in the case illustrated in Fig. 18), the differences in the net behavior with relation to the case presented before are very small. The net behavior in this specific case is visualized in Fig. 19. The main difference is that the input marking with the shortest aging time tends to decrease faster (as again can be seen in Fig. 19 (b)).

In the classification system, as already pointed out in the previous discussion, the information represented by tokens may not be available at the same time. The marking of input places can happen at any time and the aging process of the token starts from the point when its nonzero values appear at the input place. When the timed places nets come with markings that are not available

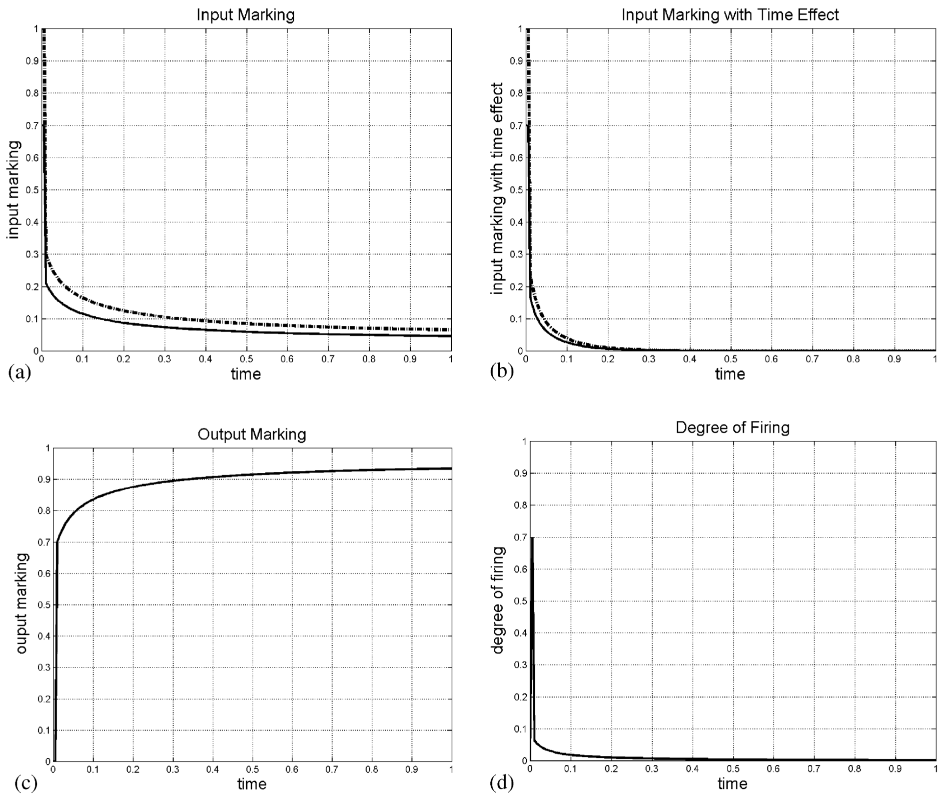


Fig. 17. Fuzzy timed places net with zero weights: input markings (a), input markings with time effect (b), output marking (c) and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

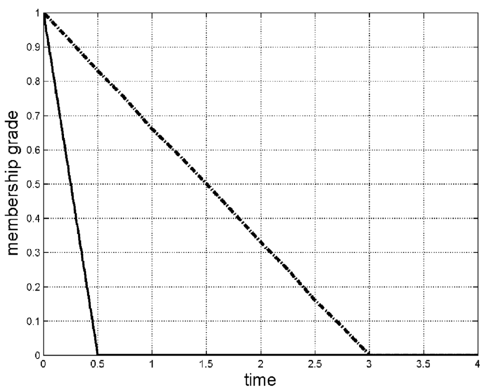


Fig. 18. Fuzzy time intervals with different supports of their membership functions.

at the same time moment, the transition is not allowed to fire until all input places have a nonzero marking. When the marking became available, the transition firing proceeds as usual. To visualize this situation, consider the fuzzy time intervals shown in Fig. 20.

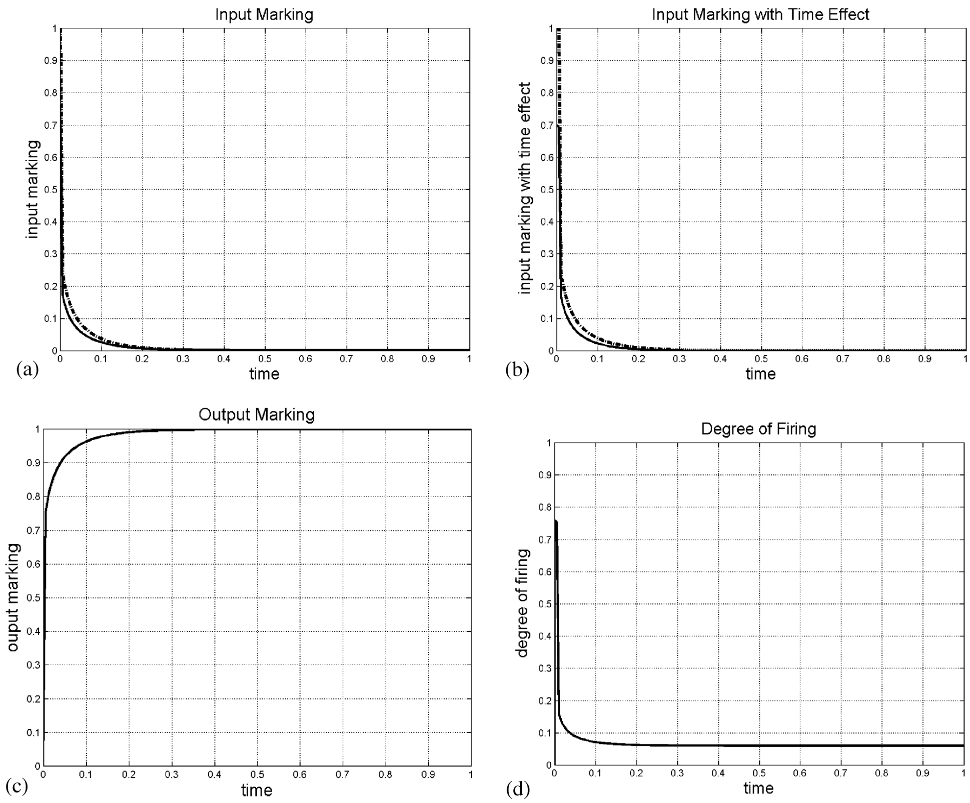


Fig. 19. Fuzzy timed places net with very different time intervals for each input: input markings (a), input markings with time effect (b), output marking (c), and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

The changes in markings considering the same parameters as the other examples, $\mathbf{r} = [1, 1], \mathbf{w} = [0.2, 0.3]$ with initial marking $\mathbf{x} = [0.7, 1]$, are visualized in Fig. 21.

In the next example, the weight vector is given as $\mathbf{w} = [0.8, 0.3]$. The changes in the input and output marking are a lot faster, as always happens for nets with higher weight values. Besides that the initial value for the degree of firing is very high because of the higher weight defined for the first place. The changes in input marking with and without aging effect, output marking and degree of firing are shown in Fig. 22.

Different realizations of t- and s-norms impact the behavior of the net. Figs. 23 and 24 show the markings and degree of firing for timed transition and timed places nets respectively, when min and max operators are used as t- and s-norm. In the timed transition net, input and output markings changes during the period of time when the fuzzy time interval membership is increasing and then stabilizes. In the timed places net the behavior is again very similar to the original net, and there is only one initial firing.

Lets also analyze the behavior of the net when the Lukasiewicz logic operators (Lukasiewicz product and sum are used are t-norm and s-norm operators in the net equations. In the tfPN with

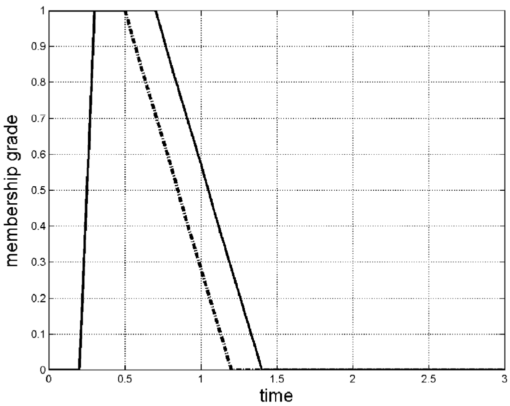


Fig. 20. Fuzzy time intervals that do not begin at the same time moment. A solid line denotes time for input x_1 and a dash-dotted line denotes time for input x_2. The initial information for place 1 is not available until the time moment is 0.3.

timed transitions, the values of input and output marking change gradually during the period of time where the fuzzy time values are increasing, and stops changing afterwards. The degree of firing z is always very small and after the input markings reach a certain value, it starts to bounce between two values (see Fig. 25).

In the ftPN with timed places, the result is the same as that in the original net, that is, there is only one initial firing, when the output marking reaches its final values and the degree of firing becomes zero. The input marking have some additional changes even if no more firings occur, because of the aging effect (see Fig. 26).

4. Learning in tfPNs

So far we have assumed that the parameters of the net are given (this concerns both the values of the connections weights as well as the fuzzy sets describing time). In general, however, these parameters may not be available and need to be determined (estimated). The estimation is realized on a basis of experimental data concerning marking of input and output places. The marking of the places is provided as a discrete time series. More specifically we consider that the marking of the output place(s) is treated as a collection of target values to be followed during the training process. As a matter of fact the learning is carried in a supervised mode. For illustrative purposes we discuss a net shown in Fig. 27 that involves several input places, a single transition followed by a single output place.

The connections of the net (namely weights w_i and thresholds r_i) as well as the time decay factors α_i are optimized (trained) so that a given performance index Q becomes minimized. The training data set consists of (a) initial marking of the input places $x_i(0), \ldots x_n(0)$ and (b) target values—markings of the output place that are given in a sequence of discrete time moments, that is $\text{target}(0), \text{target}(1), \ldots, \text{target}(K)$. The example of some training data of this format is shown in Fig. 28. Note that the marking of the input places can assume nonzero values at different time instances;

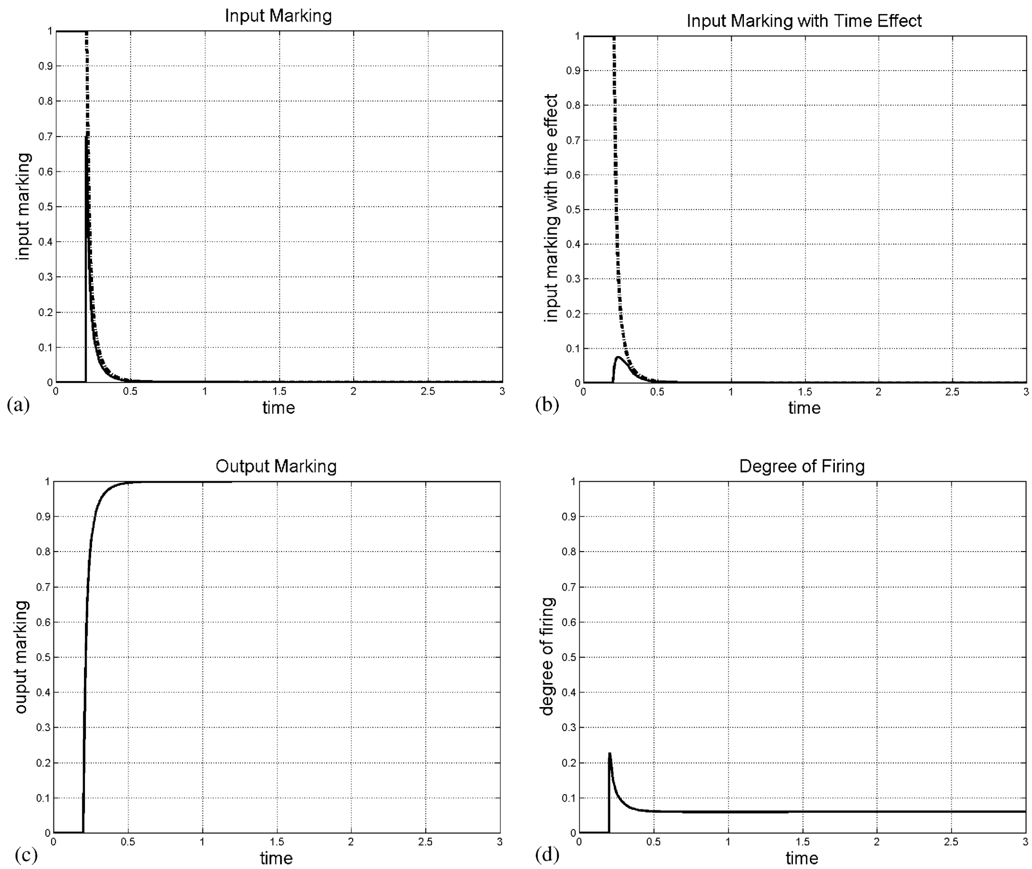


Fig. 21. Changes in timed places net with marking that are not available at the same time moment: input marking (a), input marking with time effect (b), output marking (c), and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

to make it clear we use the following notation:

$$x_i(k) = \beta\delta(k - k_i)$$

with β_i denoting the initial marking of the ith place. The δ function with a pertinent translation argument (k_i) captures the temporal shift in the initial marking (recall that $\delta(u)=1$ if $u=0$ and zero otherwise).

The crux of the training follows the general update formula being applied to the parameters

$$\mathbf{param}(\text{iter} + 1) = \mathbf{param}(\text{iter}) - \gamma\nabla_{\mathbf{param}}Q,$$

where $\gamma > 0$ is a learning rate and $\nabla_{\mathbf{param}}Q$ denotes a gradient of the performance index taken with respect to all parameters of the net (here we use a notation **param** to embrace all parameters of the net we indicated above).

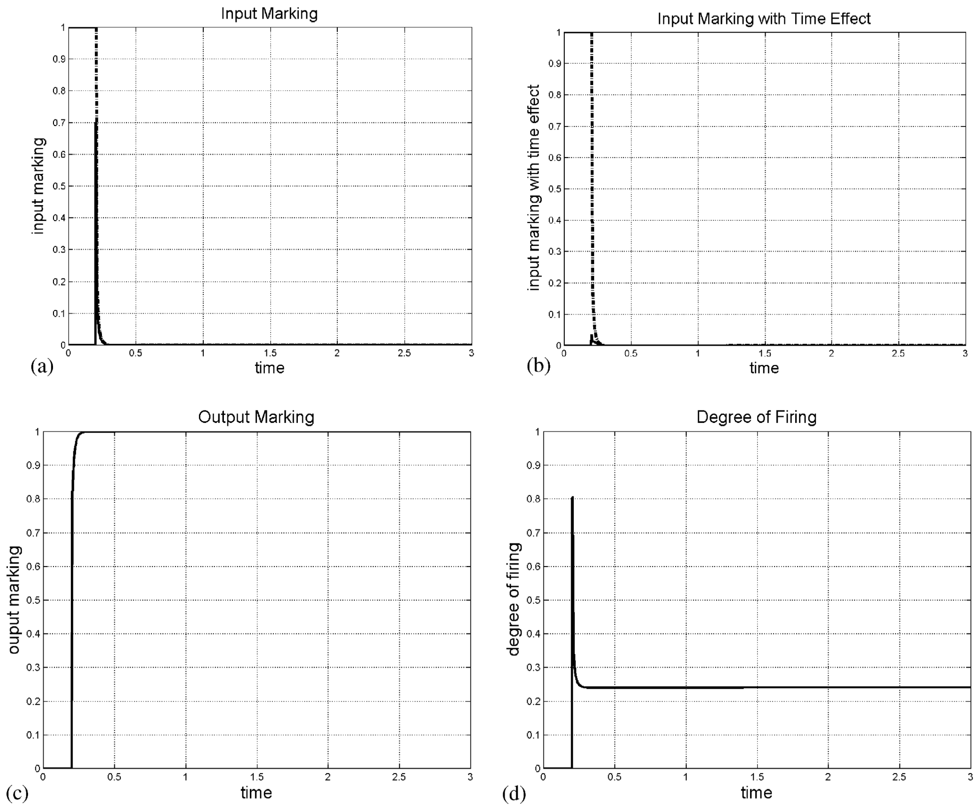


Fig. 22. Changes in timed places net with marking that are not available at the same time moment, with higher weights: input marking (a), input marking with time effect (b), output marking (c), and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

The overall organization of the learning scheme is arranged in the following pseudocode

Set up parameters to some random values in the unit interval

iter=1 //main iteration loop

start with $x_1(0), x_2(0), \ldots, x_n(0)$, $y(0) = 0$;
iterate over data points $k = 1, 2, \ldots, K$
for the kth data point target (k) update the parameters following the gradient-based scheme

$$\mathbf{param}(\text{iter}+1) = \mathbf{param}(\text{iter}) - \gamma \nabla_{\mathbf{param}} Q$$

for these values update marking of the input places

$$x_i^{\sim}(k) = x_i(0)\mathrm{T}_i(k)$$

compute a level of firing of the place

$$z = \left(\mathop{\mathrm{T}}_{i=1}^{n} ((r_i \rightarrow x_i^{\sim}) s w_i) \right)$$

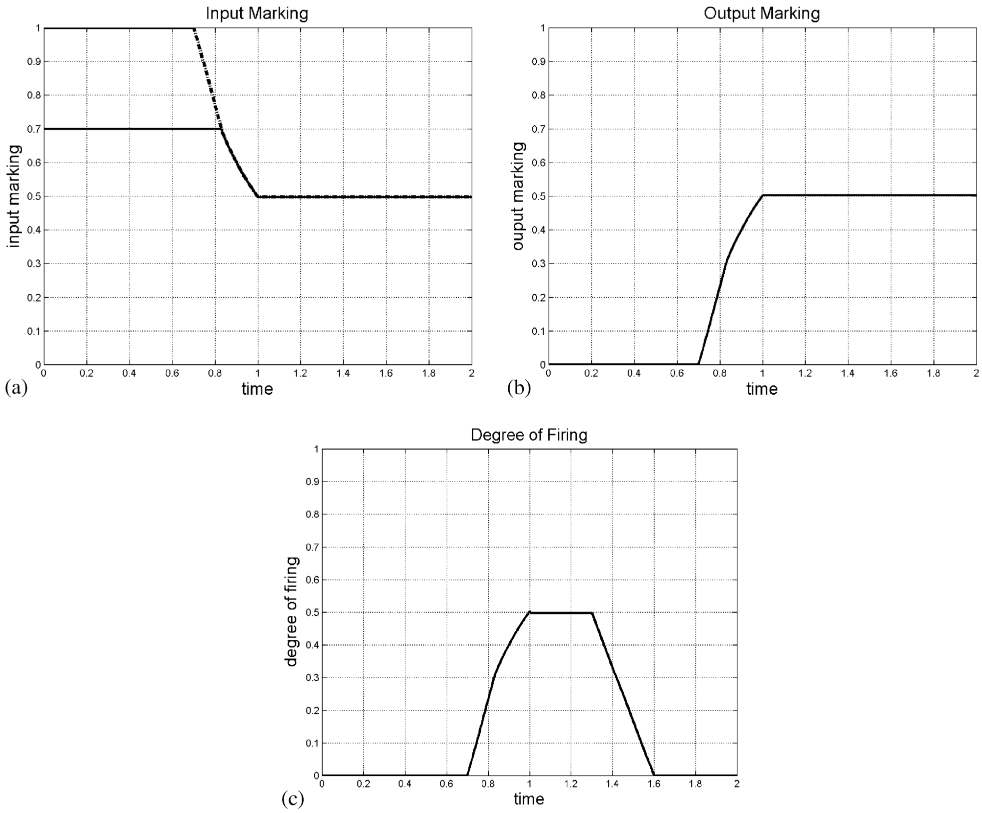


Fig. 23. Timed transition net with min and max: input marking (a), output marking (b) and degree of firing (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

and calculate the successive level of tokens at the output place and input places

$$y(k) = y(k-1)sz, \quad x_i(k) = x_i(k-1)t(1-z)$$

end (k)
end (iter)

To realize the computational details, we have to confine ourselves to specific t-norms, implication operation as well as admit a certain type of temporal decay of the marking of the places. In what follows, the temporal decay is modeled by an exponential function,

$$\mathrm{T}_i(k) = \begin{cases} \exp(-\alpha_i(k-k_i) & \text{if } k > k_i, \\ 0 & \text{otherwise} \end{cases}$$

and shown in Fig. 29.

We use the following realizations: product (t-norm) ab, probabilistic sum (s-norm) $a+b-ab$, Godel implication $a \rightarrow b = \min(1, b/a)$ where a, b are in the unit interval. The on-line optimization starts at $k=1$.

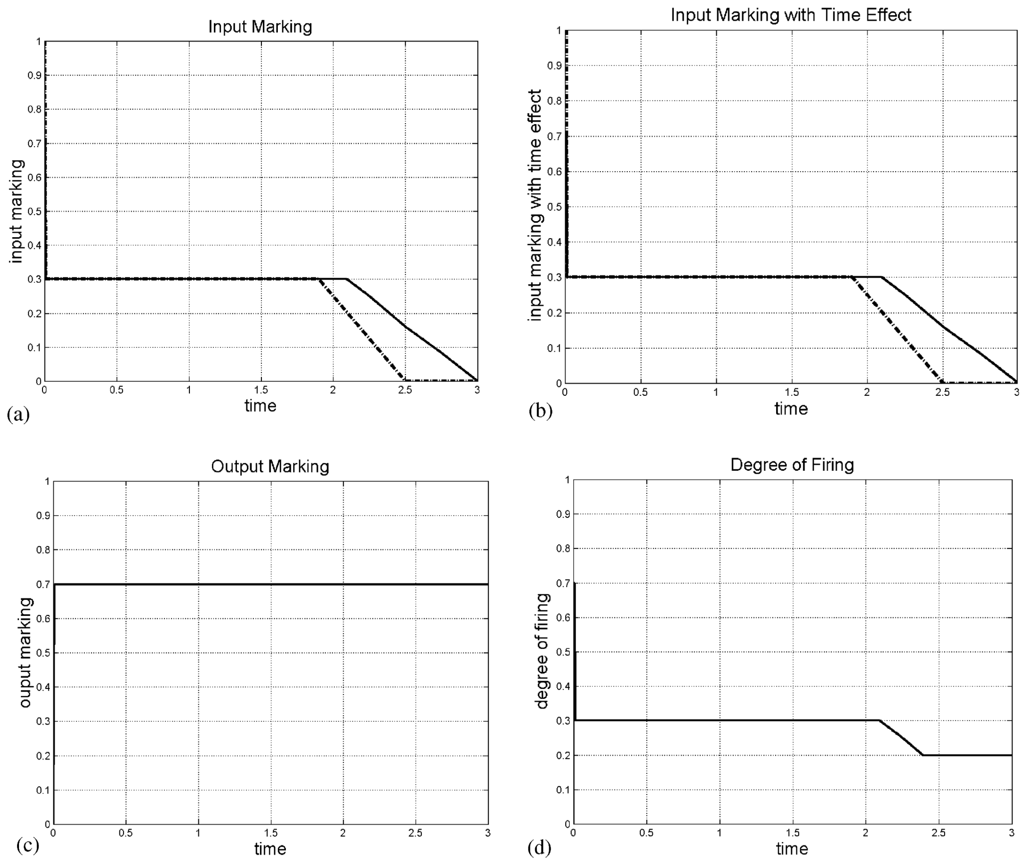


Fig. 24. Timed places net with min and max: input marking (a), input marking with time effect (b), output marking (c) and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

The performance index Q under discussion assumes the form of the following sum:

$$Q = \sum_{k=1}^{K} (\text{target}(k) - y(k))^2,$$

where the summation is taken over all time instants $(k = 1, 2, \ldots, K)$.

We assume that the initial marking of the output place $y(0)$ is equal to zero, $y(0) = 0$. The derivatives of the weights w_i are computed as follows:

$$\frac{\partial}{\partial w_i}(\text{target}(k) - y(k))^2 = -2(\text{target}(k) - y(k)\frac{\partial y(k)}{\partial w_i},$$

where $i = 1, 2, \ldots, n$. Note that $y(k+1) = y(k)sz(k)$. Specifically $y(1) = z(1)$. Then

$$\frac{\partial y(k)}{\partial w_i} = \frac{\partial}{\partial w_i}\left[\mathop{\mathrm{T}}_{j=1}^{n}((r_j \to x_j^{\sim}(k))sw_j)\right].$$

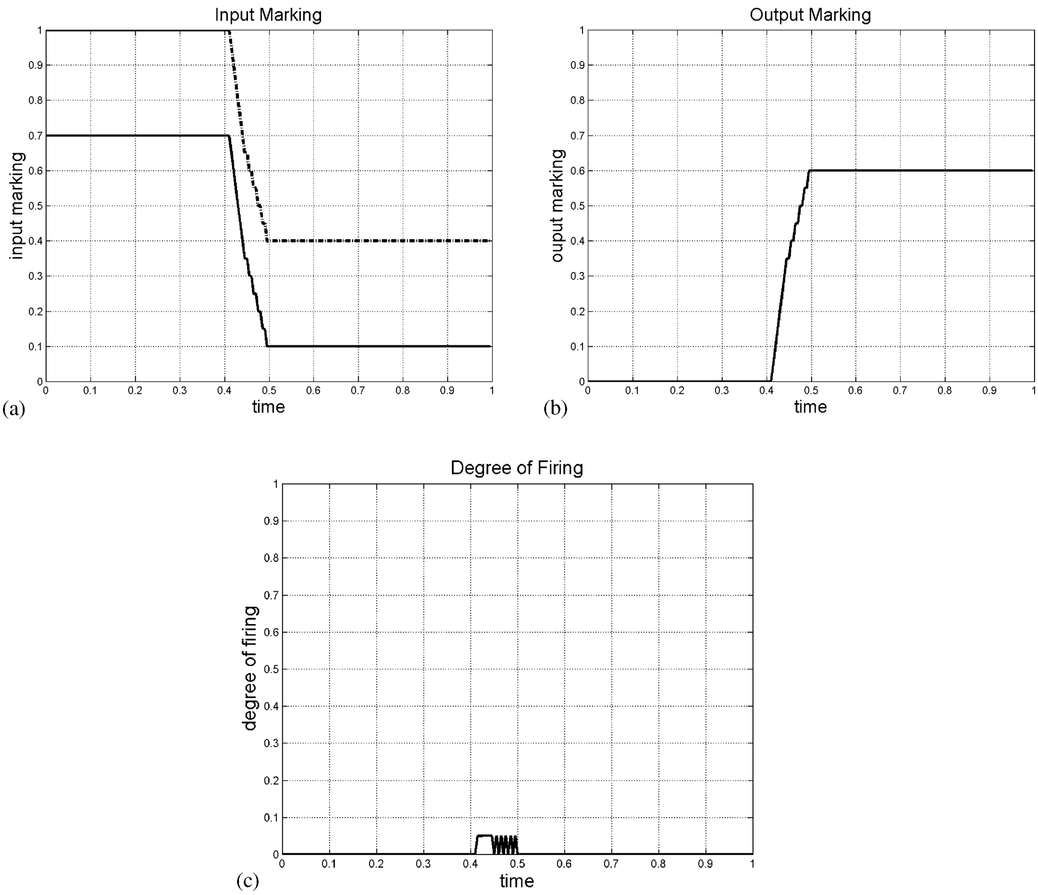


Fig. 25. Timed transition net with Lukasiewicz product and Lukasiewicz sum. Input marking (a), output marking (b) and degree of firing (c). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

Let us introduce the abbreviated form

$$A_i = \mathop{\mathrm{T}}_{\substack{j=1\\ j\neq i}}^{n} ((r_j \rightarrow x_j^{\sim}(k))sw_j).$$

This allows us to compute the above derivative in the following manner:

$$\frac{\partial y(k)}{\partial w_i} = \frac{\partial}{\partial w_i}(A_i(r_i \rightarrow x_i^{\sim}(k))sw_i) = A_i\frac{\partial}{\partial w_i}(r_i \rightarrow x_i^{\sim}(k))sw_i).$$

Furthermore, we use the notation

$$u_i = r_i \rightarrow x_i^{\sim}(k)$$

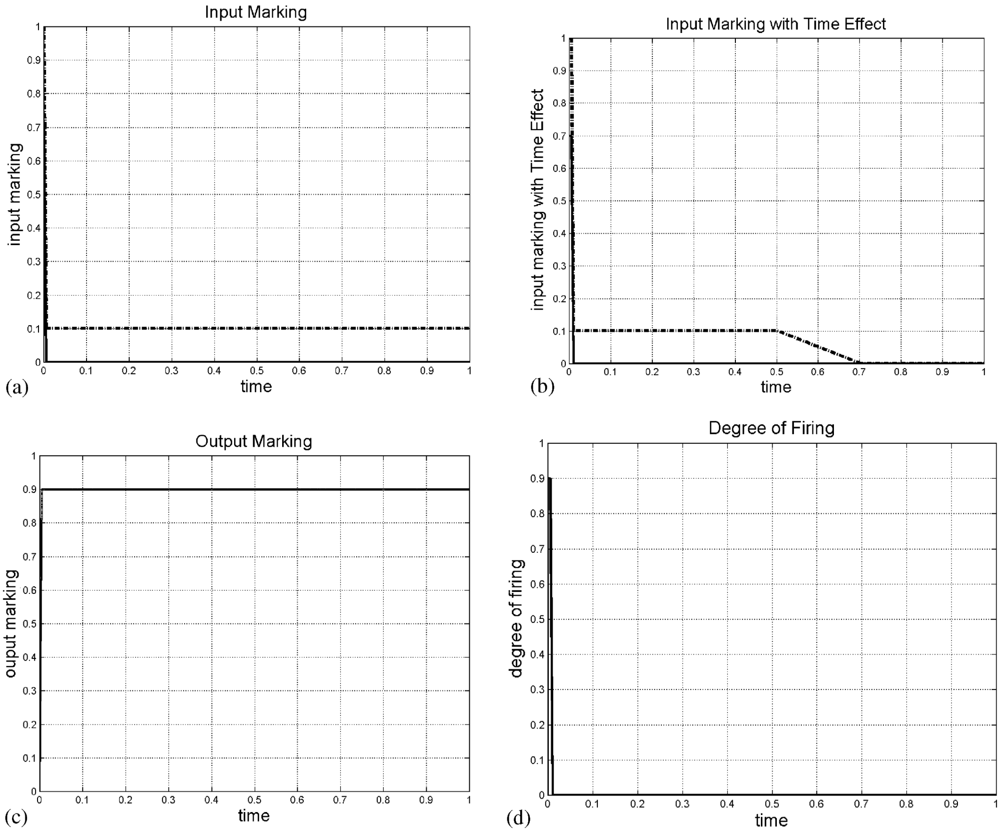


Fig. 26. Timed places net with Lukasiewicz product and Lukasiewicz sum: input marking (a), input marking with time effect (b), output marking (c) and degree of firing (d). Input marking at place 1 is denoted as a solid line and input marking at place 2 is denoted as a dash-dotted line.

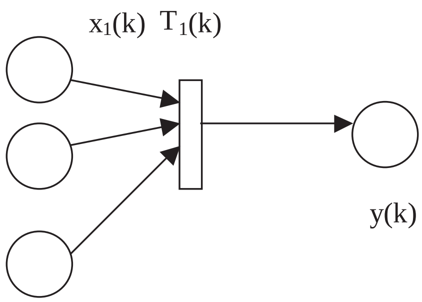


Fig. 27. An example of tfPN—training details.

that leads to the expression

$$\frac{\partial}{\partial w_i}(u_i s w_i) = \frac{\partial}{\partial w_i}(u_i + w_i - u_i w_i) = 1 - u_i.$$

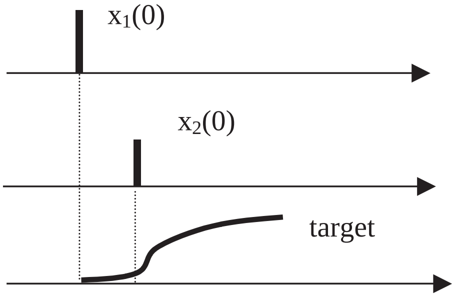


Fig. 28. Training data—marking of the input places in initial time instances and a series of marking levels of the output place (target).

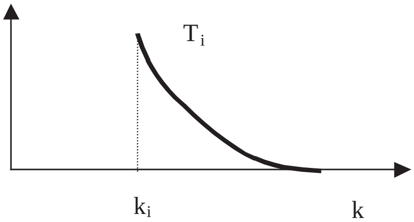


Fig. 29. The exponential model of temporal decay of marking of the *i*th input place; see details in the text.

The derivative with respect to r_i is determined using the same notation as before

$$\frac{\partial}{\partial r_i}(r_i \to x_i^{\sim}(k))sw_i) = \frac{\partial}{\partial r_i}((r_i \to x_i^{\sim}(k)) + w_i - (r_i \to x_i^{\sim}(k))w_i)$$

$$= (1 - w_i)\frac{\partial}{\partial r_i}((r_i \to x_i^{\sim}(k)).$$

The derivative in the above expression is computed taking into account the Godel implication. We have

$$\frac{\partial}{\partial r_i}(r_i \to x_i^{\sim}(k) = \frac{\partial}{\partial r_i}\begin{cases} 1 & \text{if } r_i \leqslant x_i^{\sim}(k), \\ x_i^{\sim}(k)/r_i & \text{if } r_i > x_i^{\sim}(k), \end{cases} = \begin{cases} 0 & \text{if } r_i \leqslant x_i^{\sim}(k), \\ -x_i^{\sim}(k)/r_i^2 & \text{if } r_i > x_i^{\sim}(k). \end{cases}$$

Finally, we compute the derivative of y with respect to α_i. In virtue of the detailed expression for the temporal decay of the marking, the derivative

$$\frac{\partial}{\partial \alpha_i}(r_i \to x_i^{\sim}(k))sw_i) = (1 - w_i)\frac{\partial}{\partial \alpha_i}((r_i \to x_i^{\sim}(k))$$

is given in the following fashion

- if $k < k_i$ the derivative returns zero

- if k is greater or equal to k_i we obtain

$$\frac{\partial}{\partial\alpha_i}(r_i \to x_i(k)\exp(-\alpha_i(k-k_i)))$$

$$= \frac{\partial}{\partial\alpha_i}\begin{cases} 1 & \text{if } r_i \leqslant x_i(k)\exp(-\alpha_i(k-k_i)) \\ \frac{x_i(k)\exp(-\alpha_i(k-k_i)}{r_i} & \text{otherwise.} \end{cases}$$

This results in the following expression:

$$\frac{\partial}{\partial\alpha_i}(r_i \to x_i(k)\exp(-\alpha_i(k-k_i))) = \begin{cases} 0 & \text{if } r_i \leqslant x_i(k)\exp(-\alpha_i(k-k_i)), \\ \frac{x_i(k)(k-k_i)\exp(-\alpha_i(k-k_i)}{r_i} & \text{otherwise.} \end{cases}$$

5. Application aspects of timed fuzzy Petri nets

Owing to the nature of the facet of time captured in this extension of the fuzzy Petri net, they become viable models in a wide range of engineering problem augmenting the already existing Petri nets, cf. [3,8]. Two main and commonly categories of models are worth elaborating here.

5.1. Classification and classifiers

The classification paradigm and subsequently a variety of classifiers are omnipresent in a number of areas. In a nutshell, in spite of the existing variety of models, a classifier realizes a mapping from the feature space (in which patterns are defined) to a collection of classes (where such classes could be discrete or continuous). The results of classification depend on the specific values of the features a pattern under discussion assumes and the form of the mapping. Petri nets come with a straightforward interpretation. Input places are concerned with the values of the features. For instance, the features are perceived through information granules (fuzzy sets or fuzzy relations) defined in the corresponding spaces or their Cartesian products. The markings of the input places are just the degrees of activation (matching) of the respective granules by the values of the given pattern. The marking of the output place is just a degree of class membership of this pattern. The transition and its parameters (weights and thresholds) describe the mapping from the feature space to the class space as we already alluded to. The time factor is evident in classification processes. The values of the features are usually acquired by sensors, polls, summarizations and these data acquisition processes can be realized asynchronously as pieces of information gathered over some time period (say in classifying customers, the results of a questionnaire could refer to a different time period in comparison to some other data being available in a database, etc.). This makes us think of expressing temporal relevance of individual values of the features vis-à-vis a level of membership of the classified pattern.

5.2. Models of dynamic systems

In models of dynamic systems as usually encountered in industrial practice, the readings of the sensors can be available at different sampling frequencies. For instance, in some dynamic readings of temperature could be less frequent than readings of velocity. This in turn, calls for adjustment of relevance of the information gathered at different time scales. The models of temporal information degradation (aging) become crucial to any tasks involving the usage of the models as it helps quantify the confidence of the inferred result (e.g., prediction made on a basis of the model).

6. Conclusions

We have introduced a new class of fuzzy Petri nets by incorporating a concept of time with the interval and fuzzy set-based models of temporal relationships. The factor of time is incorporated into the structure of the net in two different ways: we augment temporal relationships at the level of the transitions or/and places. We provided a number of illustrative examples to analyze an impact of the time factor on the performance of the net expressed in terms of firing of the transitions and the distribution of the level of marking of the input and output places. These examples shed light on the role of the parameters (weights and thresholds) of the net on its functioning. Additionally, we looked at the issue of implementing Petri nets with the aid of various t- and s-norms. Finally, we developed a detailed scheme of supervised learning in the tfPNs.

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