Application of Fuzzy Modeling in Civil Engineering

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Abstract

Fuzzy modeling in civil engineering comprises fuzzification, fuzzy analysis, defuzzification and safety assessment. It offers a tremendous potential for analyzing structures with non-stochastic imprecise input information. Non-stochastic input information arises in both existing and new structures. The imprecise results of fuzzy modeling permit an improved assessment of structural behaviour and provide a starting point for a new type of safety assessment based on possibility theory. We shall discuss all four steps of fuzzy modeling with special emphasis on the problems of fuzzy analysis. It will be shown that α-discretization may be advantageously applied in structural analysis. Taking into account the nonlinearity and nonmonotonicity of the deterministic computations, the ensuing fuzzy problems are solved using optimization algorithms. Two examples will be discussed which include the solution of a transcendental eigenvalue problem and a linear system of equations. The results are described and a structural safety assessment is carried out.

1 Fuzzification

Typical imprecise input information to structures include loads (e.g. earthquake loads), material properties (e.g. damping parameters) or geometrical data. The fuzzification of a material value is shown in Fig. 1. For a historical building in Dresden the compression strength of sandstone masonry was to be determined. As only a limited number of compression tests could be carried out, it was not possible to state a definite statistical distribution. Therefore the compression strength was consequently modeled as a fuzzy value. Taking account of heuristical engineering knowledge, two proposals were put forward for membership functions.

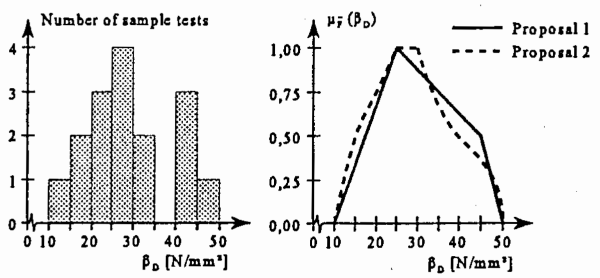

Fig. 1 Compression strength of sandstone samples and fuzzification

2 Fuzzy Analysis

The mapping of imprecise input information (taken to be convex in the following) onto an imprecise result space is called fuzzy analysis. This is based on the extension principle in conjunction with the cartesian product between fuzzy sets (see e.g. /1/, /2/).

Given are

- n+1 fundamental sets $X_1, \ldots, X_i, \ldots, X_n$, Y,
- n fuzzy sets $\tilde{A}_i$ on X_i with the membership functions $\mu_i(x_i)$, $i=1,\ldots,n$
- a mapping f: $X_1 \times \ldots \times X_n \rightarrow Y$ with $y=f(x_1,\ldots,x_n)$ and $y \in Y$.

The mapping $f(\tilde{A}_1, \ldots, \tilde{A}_n)$ leads to a fuzzy set $\tilde{B}$ on Y with

$$\tilde{B}=\{(y, \mu_{\tilde{B}}(y)) \mid y=f(x_1,\ldots,x_n), (x_1,\ldots,x_n) \in X_1 \times \ldots \times X_n\},$$

by where the membership function $\mu_{\tilde{B}}(y)$ is given by

$$\mu_{\tilde{B}}(y)=\begin{cases} \sup\limits_{y=f(x_1,\ldots,x_n)} \min\,[\mu_1(x_1),\ldots,\mu_n(x_n)], & \text{for the case} \exists\ y=f(x_1,\ldots,x_n) \\ 0 & \text{otherwise.} \end{cases}$$

The imprecise input values must be connectable through the cartesian product. The extension principle includes two operators, namely, the mapping operator f for generating the resulting value y and the min-max-operator for generating the membership function $\mu_{\tilde{B}}(y)$.

For solving fuzzy problems in civil engineering applications the α-discretization method offers more advantages than the extension principle. The problem of seeking the solution point in the y-$\mu_{\tilde{B}}(y)$ plane is thus inverted. For a given fixed value of $\alpha=\mu_{\tilde{B}}(y)$, the smallest and greatest value of y is searched for. The result of an analysis using α-discretization is always a convex fuzzy set. If a continuous mapping operator is presumed the same membership function is computed using the extension principle or α-discretization. In the case of a non-continuous mapping the extension principle can produce a non-convex (possibly not connected) imprecise result set. The application of α-discretization leads to membership values $\mu_{\tilde{B}}(y) \geq \alpha$ for $\min y_\alpha \leq y \leq \max y_\alpha$, corresponding to an envelope curve. An involved analysis of the mapping operator combined with an interpretation of all local minima and maxima of y within the solution interval can eliminate this shortcoming. The described effect is illustrated by the example of a single-span girder with a length l=5 m subjected to a single

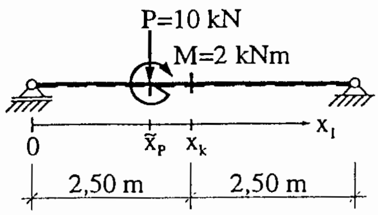

Fig. 2 Single-span girder with a single load at the imprecise point $\tilde{x}_P$

load and a single moment at the imprecise point $\tilde{x}_P$ along the bar axis (see Fig. 2). The mapping of the load point $\tilde{x}_P$ onto the bending moment $\tilde{M}_{xk}$ at the point $x_l{=}x_k$, $y{=}M_{xk}(x_P){=}f(x_P)$, is investigated. The functional equation for computing $M_{xk}(x_P)$ is obtained from Fig. 2 using the parameters indicated

$$M_{xk}(x_P)=\begin{cases} 1+5\cdot x_P & [\mathrm{kNm}] \mid x_P \le x_k \\ 24-5\cdot x_P & [\mathrm{kNm}] \mid x_P > x_k . \end{cases}$$

The procedures according to the extension principle and α-discretization are compared in Fig. 3. The α-discretization method is applicable when the errors introduced by disregarding local optima have no bearing on the interpretation of the imprecise results (a priori estimation), or when continuous mapping operators are used.

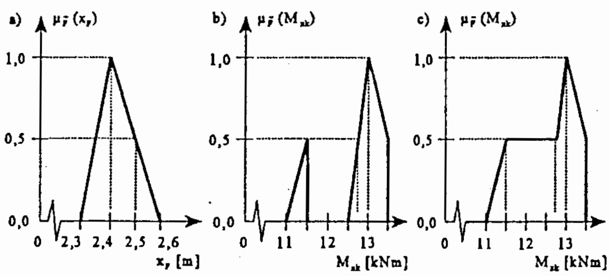

Fig. 3 a) imprecise load point $\tilde{x}_P$,
b) imprecise bending moment at $x_l{=}x_k$, computed by the extension principle,
c) imprecise bending moment at $x_l{=}x_k$, computed using α-discretization without accounting for local optima

In the α-discretization method the fuzzy input values to be coupled via the cartesian product are reduced to α-level sets, thus forming an imprecise input domain. This input domain describes an n-dimensional hyper-cuboid (n = number of fuzzy input values) in the input parameter space. The n-dimensional imprecise input domain is mapped onto an m-dimensional result domain, where m is the number of fuzzy result values. If the mapping operator additionally depends on crisp parameters (e.g. load factor ν, time parameter t), then the imprecise result domain develops by changing these values. Special cases result from the characteristics of the mapping operator and the dependency between the imprecise result domain and the crisp parameters. Such characteristics are

- Continuity,
- One to one correspondence,
- Relationship between n and m,
- Monotonicity,
- Linearity.

When searching for the solution point in the imprecise input domain or when following the development of the imprecise result domain with changing crisp parameters these characteristics may partly be used to simplify the computation. If monotonicity exists in all directions the solution points may, e.g., lie in the corners of the imprecise input domain. The above mentioned characteristics may be partially evaluated.

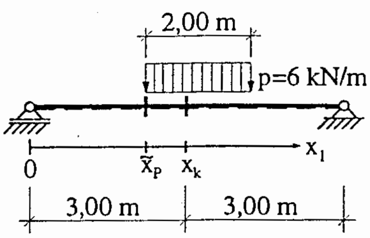

Fig. 4 Single-span girder with a load at the imprecise point $\tilde{x}_P$

If simplifications are not possible the entire imprecise input domain has to be searched for solution points. The mapping of the imprecise load point $\tilde{x}_P$ onto the bending moment $\tilde{M}_{xk}$ at the point x_k is now considered for a single-span girder stressed by a linear load (see Fig. 4). With the restriction $1\mathrm{m} \le x_P \le 3\mathrm{m}$ the continuous mapping operator f is given by

$$y(x_P) = M_{xk}(x_P) = -3\cdot x_P^{\,2} + 12\cdot x_P + 3 \ [\mathrm{kNm}].$$

This function attains its global maximum at the point $x_P{=}2{,}00$ m. If the fuzzy input value $\tilde{x}_P$ is prescribed in such a way that the value $x_P{=}2{,}00$ m is an element of at least one α-level set, when searching for the maximum, the solution point lies within the imprecise input domain. The fuzzy input value $\tilde{x}_P$ and the fuzzy result value $\tilde{M}_{xk}$ are plotted in Fig. 5.

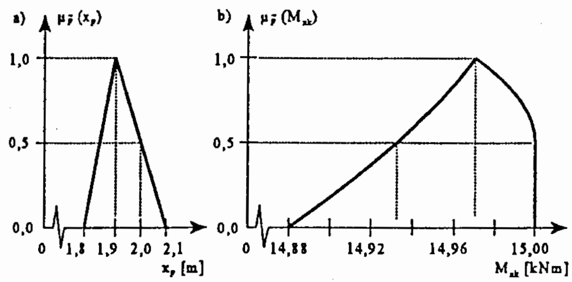

Fig. 5 a) imprecise load point $\tilde{x}_P$,
b) imprecise bending moment at $x_l{=}x_k$, computed by α-discretization

The vertical portion of the membership function curve $\mu_{\tilde{F}}(M_{xk})$ results from the described position of the solution point within several α-level sets yielding identical maximum solutions.

The search inside an imprecise input domain represents an optimization problem. The mapping operator f may be interpreted as an objective function. The (strictly speaking) vector optimization problem which arises (several result values = several objective functions) immediately disintegrates if the individual objective functions are considered successively and the remainder are weighted by 0. A true compromise solution has no relevance in this case. Different algorithms are available to solve the optimization problem. If the properties of the mapping operator permit an a priori estimation that the solution points only lie at the corners of the imprecise input domain, then a permutation search algorithm may be applied (see e.g. /3/). The input value vector is successively occupied by all possible interval-boundary combinations and the result is evaluated. If the solution point is situated on the surface, then a boundary search is practicable. A search throughout the entire input domain may be carried out by, e.g.

- Systematic networking,
- Monte-Carlo Method,
- Gradient method or
- Evolution strategy.

Application of the gradient method requires a knowledge of the directional derivatives of the objective function. This method may be advantageously applied for solving the systems of simultaneous first order ordinary differential equations. This procedure is described by Bontempi /4/.

In the case of nonlinear structural analysis there is generally very little or no information available concerning the behaviour of the objective function (of the mapping operator). Therefore the following examples were solved using both a multi-layer net search procedure and a modified evolution strategy. In applying net search procedures the computational effort increases exponentially with the number of fuzzy input values. The application of the evolution strategy can lead to non-detection of global optima. Both procedures yielded the same results within the limits of the prescribed termination criteria for the accaracy of the solution.

Example 1

A plane frame consisting of rolled steel joists (see Fig. 6) under the given vertical load P_V was investigated with regard to stability failure. The horizontal load P_H was 0. The column bases are rigidly connected to the square foundations. The stiffness of the semi-solid clay was taken to be E_S=5000...20000 kN/m². The foundations are idealized as linear elastic elements embedded in the foundation soil, thus yielding an imprecise rotational restraint of the column bases. The resulting fuzzy value for the rotational restraint is $\tilde{k}_{\varphi 1}=\tilde{k}_{\varphi 2}=\langle 6666;13333;26666\rangle$ kNm/rad. The end-plate shear connections at the corners of the frame were modeled as rotational springs and included in the analysis as fuzzy triangular values $\tilde{k}_{\varphi 3}=\tilde{k}_{\varphi 4}=\langle 20000;23400;25000\rangle$ kNm/rad. The horizontal translation spring at the right-hand corner of the frame was considered to be a crisp value, k_F=194 kN/m. This is determined by the rolled steel joist IPE 360 on the right, running orthogonal to the frame plane.

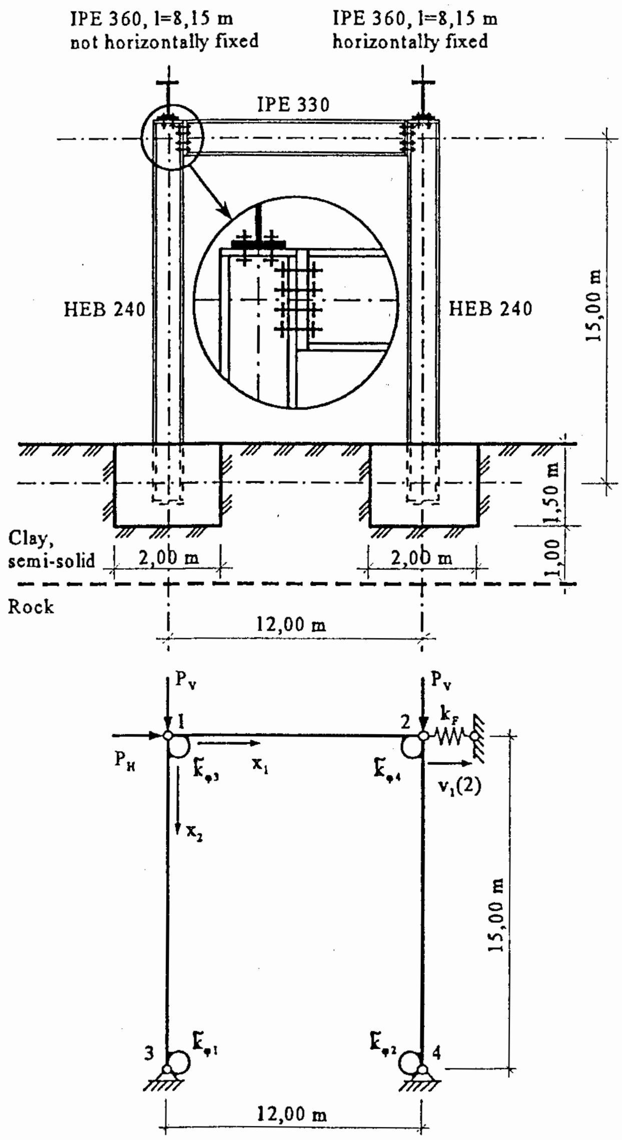

Fig. 6 Frame with imprecise rotational restraints, Construction sketch for modeling the static system and load

The four fuzzy input values (two rotational restraints at the column bases and two rotational restraints at the frame corners) are considered to be independent of each other and may thus be coupled via the cartesian product.

The linearized divergence load $\nu \cdot P_V$ is computed. The structure is stressed by an initial load P_V and the longitudinal force state S is computed by linear analysis. This longitudinal force state S is linearly increased by the factor ν until the coefficient matrix (total stiffeness matrix) of the equation system for computing the unknown node displacement components becomes singular, i.e. when the displacements approach infinity. The transcendental fuzzy eigenvalue problem to be solved is

$$\det(\tilde{K}(\nu \cdot S))=0$$

The nonlinear mapping operator f is given by

$$y=\nu=f(k_{\varphi 1},k_{\varphi 2},k_{\varphi 3},k_{\varphi 4}),$$

and is strictly monotonic in all directions, i.e. the solution points are situated at the corners of the four-dimensional imprecise input domain. The same combination of interval boundaries at all α-cuts forms the solution point. The nonlinearity is immediately recognizable, as all fuzzy input values are fuzzy triangular values (linear membership functions), whereas the fuzzy result value has a definite nonlinear membership function (see Fig. 7). The computation was performed at eleven α-cuts. In order to search for the solution points in the imprecise input domain the evolution strategy was adopted. Owing to the monotonicity of f, a permutation search algorithm also leads to the correct result.

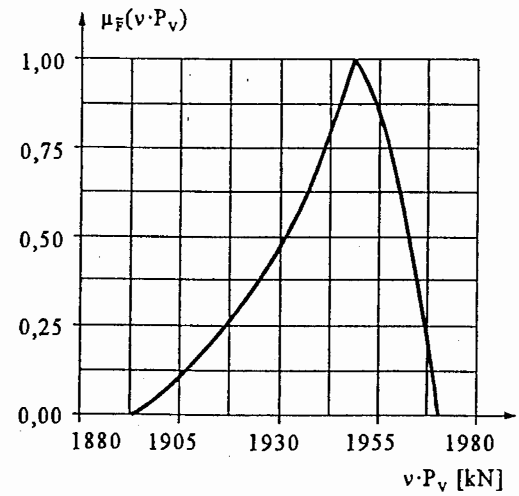

Fig. 7 Membership function for the linearized divergence load $\nu \cdot P_V$

Example 2

We shall now compute the load-bearing capacity based on second order elasticity theory. An initial horizontal displacement of the system is achieved by applying a small horizontal load P_H=1,00 kN. The vertical load P_V is increased in steps of 20 kN. At every load step the secant stiffeness, the internal forces and the displacements are computed iteratively. The displacement vector **v** for the system nodes is determined by solving a linear system of equations. The fuzzy system of equations to be solved is

$$\mathbf{P} - \tilde{\mathbf{K}}(\mathbf{S}) \cdot \tilde{\mathbf{v}}=\mathbf{0}.$$

The fuzzy result domain is chosen to be three-dimensional. The horizontal displacement $\tilde{v}_1(2)$ at the right-hand corner of the frame, the maximum stress $\tilde{\sigma}_{St}$ at the right-hand column cap and the maximum soil stress $\tilde{\sigma}_{Sohl}$ below the right-hand column foundation are considered. In order to compute the maximum soil stress a constant stress distribution is assumed as a first approximation below a rectangular replacement plane. The replacement plane is determined by the condition that the resultant load in the base joint is situated at the centre of this plane. The mapping operator f is given by

$$y=\begin{bmatrix} v_1(2) \\ \sigma_{St} \\ \sigma_{Sohl} \end{bmatrix} = f(k_{\varphi 1},k_{\varphi 2},k_{\varphi 3},k_{\varphi 4}).$$

Monotonicity can only be presumed in the direction of the result value $v_1(2)$, i.e. the mapping is nonlinear. The load-soil stress dependency is presented in Fig. 8 for α=0 and α=1. Fig. 9 shows the load-displacement dependency for the same α-cuts.

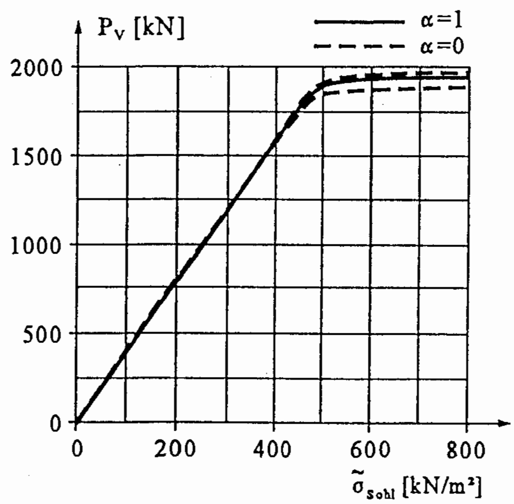

Fig. 8 Load-soil stress dependency

The membership function of the displacement $v_1(2)$ is shown in Fig. 10 for the load step P_V=1880 kN. The nonlinear relationships between the crisp load parameter P_V as well as the imprecise input values $\tilde{k}_\varphi$ and the result values are recognizable in the figures. The horizontal displacement $v_1(2)$ rises increasingly with linearly decreasing stiffeness k_φ. The linear increase of the crisp load parameter P_V produces a superlinear increase of the soil stress

σ_{SohI} and the displacement v_1 (2). The computation was performed using evolution strategy applied to eleven α-cuts. The application of a permutation or boundary search algorithm is only possible to a limited degree. A careful a priori appraisal of the mapping operator is necessary to avoid erroneous results. The expected computational effort is lower than that required by more general search algorithms.

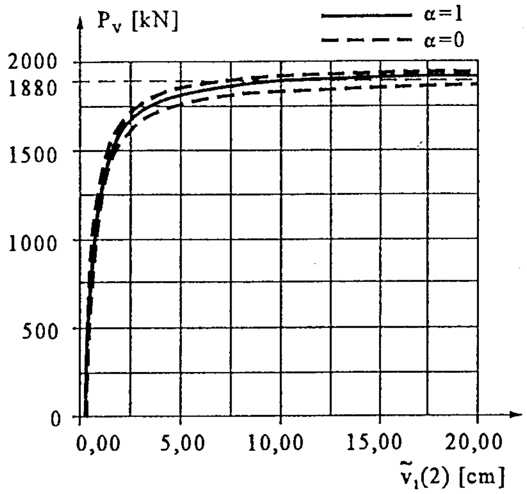

Fig. 9 Load-displacement dependency

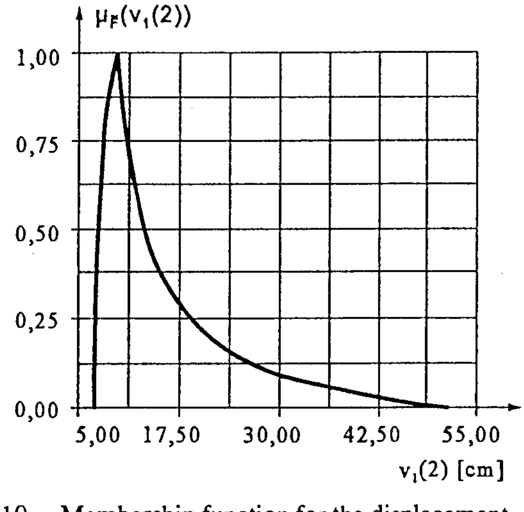

Fig. 10 Membership function for the displacement $v_1(2)$ at the load step P_V=1880 kN

3 Defuzzification, Safety Assessment

The imprecise results can be defuzzified or directly used for further evaluation (safety assessment, see e.g. /5/). Defuzzification represents a restricted subjective estimation; an imprecise item of information is mapped onto a crisp value by applying suitably chosen algorithms. The choice of the defuzzification algorithm depends on the required prognosis.

There are numerous methods available to achieve defuzzification. For example, the level-plane procedure by Rommelfanger /6/ can be used.

In order to arrive at a safety assessment it is necessary to formulate failure conditions, which may form either crisp or imprecise sets. Failure functions $\pi(x)$ are possibility functions. According to definition they are membership functions ($\pi(x)=\mu(x)$) which evaluate the possibility of failure. In the following example the yield stress f_y of a structural steel member, i.a., is taken to be the failure criterion. To generate the failure function $\pi(\sigma)$ the fuzzy value $\tilde{f}_y$ is used (see Fig. 11 above). The failure function is computed from (Fig. 11 below)

$$\pi(\sigma) = \bigvee_{f_y \le \sigma} \mu_{\tilde{F}}(f_y) = \sup_{f_y \le \sigma} \mu_{\tilde{F}}(f_y).$$

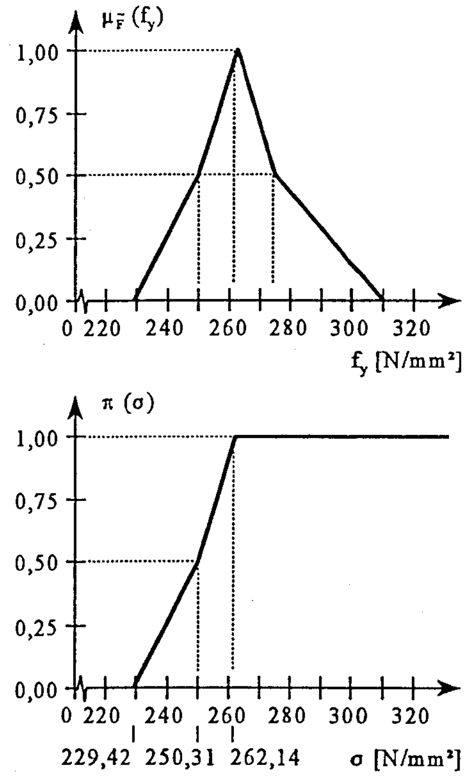

Fig. 11 Generation of imprecise failure functions; yield stress f_y of steel

The failure possibility $\Pi(\tilde{F})$ arising from a fuzzy event $\tilde{F}$ is computed from an analysis of the membership functions $\mu_{\tilde{F}}(x)$ in conjunction with the imprecise failure condition $\pi(x)$

$$\Pi(\tilde{F}) = \sup_{x \in X} \min(\, \mu_{\tilde{F}}(x),\, \pi(x)).$$

As a rule, several failure criteria have to be checked in statical structure analysis. Several values of failure possibility $\Pi(\tilde{F})$ exist at the same analysis time. The failure possibility V of a structure may thus be expressed by

$$V = \sup_{\tilde{F}} \Pi(\{\tilde{F}\}).$$

The failure possibility of the frame shown in Fig. 6 will now be computed. Failure is dependent on the fuzzy values $\tilde{v}_{\,l}(2)$, $\tilde{\sigma}_{\,St}$ and $\tilde{\sigma}_{\,Sohl}$. The three corresponding failure functions are plotted in Figs. 11 and 12.

For P_V=1880 kN the failure possibilities are computed to be

$$\Pi(\tilde{F}(\sigma_{St})) = \sup \min(\mu_{\tilde{F}}(\sigma_{St}), \pi(\sigma_{St})) = 0{,}096$$
$$\Pi(\tilde{F}(\sigma_{Sohl})) = \sup \min(\mu_{\tilde{F}}(\sigma_{Sohl}, \pi(\sigma_{Sohl})) = 0{,}491$$
$$\Pi(\tilde{F}(v_l(2))) = \sup \min(\mu_{\tilde{F}}(v_l(2)), \pi(v_l(2))) = 0{,}569.$$

The corresponding functions indicating the computed values are shown in Fig. 13.

An evaluation of all load steps leads to the failure possibility curve for the structure (see Fig. 14). A progressive increase in the failure possibility with increasing load P_V is evident. The rising increase is due to the nonlinear behaviour of the right-hand parts of the membership functions for the fuzzy result values (see Fig. 10). The kink in the failure functions at $\pi(x)$=0,5 is reflected in the failure possibility curve.

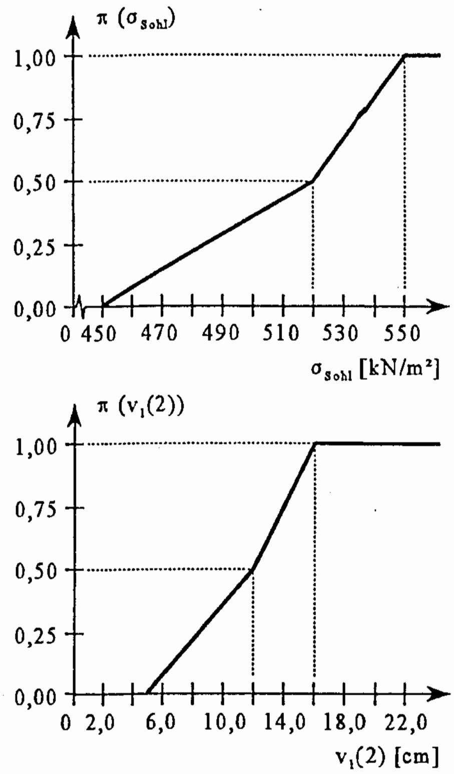

Fig. 12 Failure functions for the maximum soil stress σ_{Sohl} and the horizontal displacement $v_l(2)$

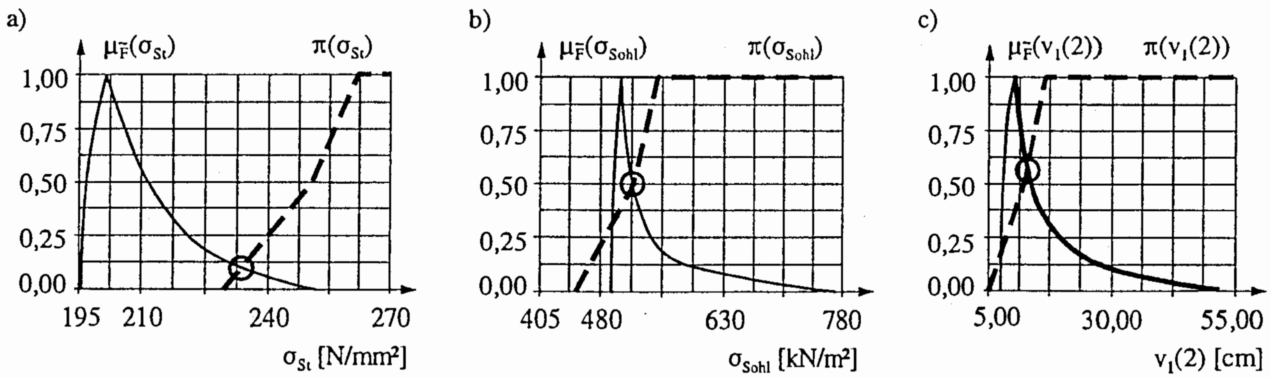

Fig. 13 Failure possibilities for the failure criterion with P_V= 1880 kN.
a) maximum stress $\tilde{\sigma}_{St}$ at the right-hand column cap,
b) maximum soil stress $\tilde{\sigma}_{Sohl}$ below the right-hand column foundation,
c) horizontal displacement $\tilde{v}_l(2)$ of the right-hand corner of the frame

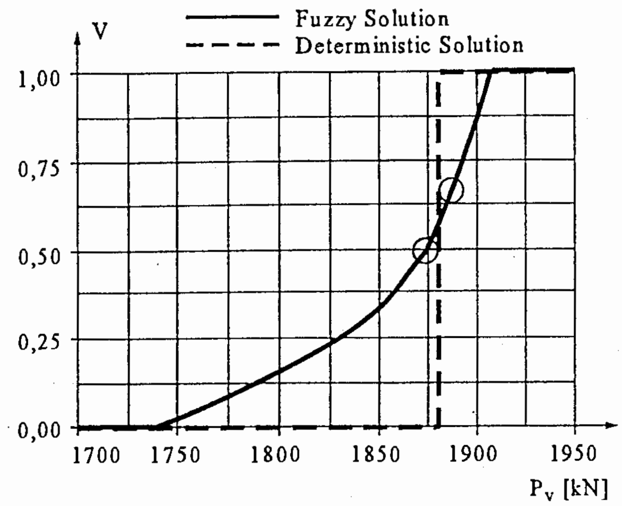

Fig. 14 Failure possibility function

A change in the active failure condition also produces a kink in the failure possibility function. Owing to the very slight increase in the gradient of the curve shown in Fig. 14 at P_V=1880 kN, the kinks (marked) are difficult to discern.

If the fuzzy input values of the structural analysis with α=1 are considered to be deterministic values and the following values are adopted for the crisp failure conditions

f_y = 240 N/mm²,
σ_{Sohl} = 500 kN/m²,
$v_1(2)$ = 10,0 cm,

then the deterministic failure load is computed to be

P_V= 1880 kN.

4 Conclusions

Fuzzy modeling represents a supplement to existing deterministic and stochastic models. The method is able to account for imprecise input values which cannot otherwise be specified in the form of stochastic distributions owing to insufficient information. Expert knowledge enters into the analyses by virtue of the subjective evaluation of the input values involved in fuzzification. By applying possibility theory to the computed fuzzy result values it is possible to realize a new type of safety assessment for structures. This permits a more realistic assessment of the behaviour of supporting structures on a case-by-case basis.

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