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AN ALGORITHM FOR SOLVING EXPECTED POSSIBILITY AND ITS APPLICATION IN CONSTRUCTION RESOURCE ALLOCATION

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Abstract: Allocation of labor and other resources to individual construction activities is at best a non-exact science even though there have been many algorithms developed to add rigor to the process. These algorithms assume known resource pool levels when experience indicates daily variations in the 'show up' ratio; they also assume the external factors of weather, economy, and so forth can be ignored or their impact directly factored into the network logic. This paper addresses the question of introducing this type of uncertainty into the resource assignment algorithm rather than the logic. First, an algorithm to find the degree of possibility for a body of fuzzy evidence regarding the external factors is introduced. This algorithm is then used to measure the weights of criteria that are required in the resource allocation process. Finally, these weights are utilized with susceptibility data to create the priority measures to be employed by the resource allocation algorithm.

Keywords: Fuzzy sets; membership functions; approximate reasoning; possibility; construction management; resource allocation.

1. Introduction

Allocation of labor and other resources to individual construction activities is at best a non-exact science even though there have been many algorithms developed to add rigor to the process. These algorithms assume known resource pool levels when experience indicates daily variations in the 'show up' ratio; they also assume the external factors of weather, economy, and so forth can be ignored or their impact directly factored into the network logic. This paper addresses the question of introducing this type of uncertainty into the resource assignment algorithm rather than the logic. (This eliminates the need to modify logic as project slippage or other delays change the impact of these external factors.)

This paper first reviews the underlying fuzzy logic theory that is applicable and then demonstrates its use in the construction resource allocation problem. Finally the conclusion allows extension to other related problems.

In *possibility theory* [10], a body of evidence called *Type II Evidence* is defined as a collection of fuzzy propositions, i.e.,

$$E = \{g_1, g_2, \ldots, g_n, g_1', g_2', \ldots, g_n'\} \tag{1.1}$$

where g_i and g_i' $(i = 1, 2, \ldots, n)$ are referred to as *granules* [10] and are defined as:

$$g_i = \text{if } X \text{ is } F_i \text{ then } Y \text{ is } G_i, \tag{1.2}$$

$$g_i' = X \text{ is } F_i \text{ is } \rho_i, \tag{1.3}$$

where X and Y are linguistic variables, F_i and G_i are fuzzy subsets of the universes of discourses U and V, respectively, and ρ_i is a linguistic probability.

From E, the following can be inferred [10]:

$$\text{“}Y \text{ is } Q\text{” is } ?\alpha. \tag{1.4}$$

or equivalently,

What is the degree of possibility that Y is Q? (1.5)

What is the degree of certainty that Y is Q? (1.6)

where Q is a specified fuzzy subset of V, $?\alpha$ is a desired linguistic probability, and the degree of certainty is equal to 1 minus the degree of possibility of "Y is Q'" with Q' being the complement of Q.

For example, assume that a set of rules has been established as the following:

rule 1: IF rain is *heavy*, THEN it is *very strongly* recommended to consider rain as a criterion in a decision making process.

rule 2: IF rain is *moderate*, THEN it is *strongly* recommended to consider rain as a criterion in a decision making process.

rule 3: IF rain is *small*, THEN it is *normally* recommended to consider rain as a criterion in a decision making process.

Now, assume that the following propositions have been established, based on experience or weather forecast data, for a particular time frame:

1. In the specified period, rain is heavy is *likely*.
2. In the specified period, rain is moderate is *very likely*.
3. In the specified period, rain is small is *unlikely*.

Then, the following can be inferred:

What is the degree of possibility that the recommendation of using the severity of rain as a criterion is *very strong*? (1.7)

or

What is the degree of certainty that the recommendation of using the severity of rain as a criterion is *very strong*? (1.8)

In this example, Y = "the recommendation of using the severity of rain as a criterion" and Q = "*very strong*".

In this paper, an algorithm is developed for computing the degree of possibility and degree of certainty. It is also shown that the degree of possibility (or degree of certainty) can be appropriately used in the decision making process of resource allocation in engineering and construction projects.

2. Computation procedure for the weights of criteria

Zadeh [8, 10, 11] has generalized the concepts of upper and lower probabilities in the theories of Dempster [3, 4] and Shafer [6] to define the *expected possibility*, $E\Pi(Q)$, and *expected certainty*, $EC(Q)$. The $E\Pi(Q)$ and $EC(Q)$ have been then interpreted as the degree of possibility in (1.5) and the degree of certainty in (1.6), respectively.

To compute $E\Pi(Q)$ as a fuzzy set from the Type II Evidence described in (1.1), (1.2) and (1.3), Zadeh [8, 9] claimed that the solution for the variational problem shown in (2.2), (2.3) and (2.4) was the appropriate answer, i.e.,

$$E\Pi(Q) = \int_{z \in [0,1]} \mu(z)/z \tag{2.1}$$

where

$$\mu(z) = \max_{v_1, v_2, \ldots, v_n} \{\min[\mu_{\rho_1}(v_1), \mu_{\rho_2}(v_2), \ldots, \mu_{\rho_n}(v_n)]\} \tag{2.2}$$

subject to

$$\sum_{i=1}^{n} v_i = 1 \tag{2.3}$$

and

$$z = \sum_{i=1}^{n} [\sup(G_i \cap Q)] v_i. \tag{2.4}$$

Note that the ρ_i $(i = 1, 2, \ldots, n)$ in (2.2) are the linguistic probabilities in (1.3), the G_i and Q in (2.4) are the fuzzy sets defined in (1.2), (1.5) and (1.6), and the $\sup(G_i \cap Q)$ in (2.4) is defined as follows:

$$G_i \cap Q = \int_{u \in [0,1]} \mu_{G_i}(u) \wedge \mu_Q(u) \tag{2.5}$$

or

$$G_i \cap Q = \sum_{j=1}^{n} \mu_{G_i}(u_j) \wedge \mu_Q(u_j) \tag{2.6}$$

where $u_j \in U = 0 + 0.1 + 0.2 + \cdots + 0.9 + 1.0$. (Note: the $+$ here represents union.) and,

$$\begin{aligned}
\sup(G_i \cap Q) &= \sup\left[\sum_{j=1}^{n} \mu_{G_i}(u_j) \wedge \mu_Q(u_j)\right] \\
&= \sup[\mu_{G_i}(0) \wedge \mu_Q(0) + \mu_{G_i}(0.1) \wedge \mu_Q(0.1) \\
&\qquad + \cdots + \mu_{G_i}(1.0) \wedge \mu_Q(1.0)] \\
&= \max\{\min[\mu_{G_i}(0), \mu_Q(0)], \min[\mu_{G_i}(0.1), \mu_Q(0.1)], \ldots, \\
&\qquad \min[\mu_{G_i}(1.0), \mu_Q(1.0)]\}
\end{aligned} \tag{2.7}$$

where the $\wedge$ is the min operator, i.e., for example, $0.5 \wedge 0.7 = 0.5$, and the $\sum_{j=1}^{n}$ is defined as the union operator.

To compute $EC(Q)$, we can use the following identity [8, 10]:

$$EC(Q) = 1 - E\Pi(Q') \tag{2.8}$$

where Q' is the complement of the fuzzy set Q.

Therefore, to find the expected possibility, it is necessary to solve the equation (2.2) subject to the two constraints in (2.3) and (2.4). The steps can be summarized as follows:

(1) Find $\alpha_i = \sup(G_i \cap Q)$ for $i = 1, 2, \ldots, n$ from (2.7).

(2) Find an array of $(v_1, v_2, \ldots, v_n)$ from (2.3).

(3) Determine one z value by (2.4), i.e., $z = \sum_{i=1}^{n} \alpha_i v_i$.

(4) find $\mu_{\rho_i}(v_i)$ for $i = 1, 2, \ldots, n$ from the definitions of membership functions for the linguistic probabilities of ρ_i's.

(5) Determine the $\mu(z)$ by (2.2). The max operator in (2.2) is used to resolve the situation when different arrays $(v_1, v_2, \ldots, v_n)$ result in the same z values in (2.4).

(6) Repeat (2) to (5) until all possible arrays $(v_1, v_2, \ldots, v_n)$ are found.

It can be seen that step (5) and (6) create difficult evaluation problems. First, in step (6), there are infinitive possible arrays of $(v_1, v_2, \ldots, v_n)$ that satisfy the constraint of (2.3) if the definitions of ρ_i are continuous membership functions such as the *likely*, *very likely* and *unlikely* shown in Figure 1.

To simplify the continuous formulation of ρ_i, use is made of the discrete membership functions. For example, the following definitions for the *likely*, *unlikely* and *very likely* will be used to represent the curves in Figure 1:

$$\begin{aligned}\rho_1 &= \textit{likely} \\ &= 0/0 + 0/0.1 + \cdots + 0/0.5 + 0.05/0.6 + 0.3/0.7 \\ &\quad + 0.8/0.8 + 0.95/0.9 + 1.0/1.0,\end{aligned} \tag{2.9}$$

$$\begin{aligned}\rho_2 &= \textit{very likely} = (\rho_1)^{1/2} \\ &= 0/0 + 0/0.1 + \cdots + 0/0.5 + 0/0.6 + 0.09/0.7 \\ &\quad + 0.64/0.8 + 0.9/0.9 + 1.0/1.0,\end{aligned} \tag{2.10}$$

$$\begin{aligned}\rho_3 &= \textit{unlikely} \\ &= 1.0/0 + 0.95/0.1 + 0.08/0.2 + 0.3/0.3 + 0.05/0.04 \\ &\quad + 0/0.5 + \cdots + 0/1.0.\end{aligned} \tag{2.11}$$

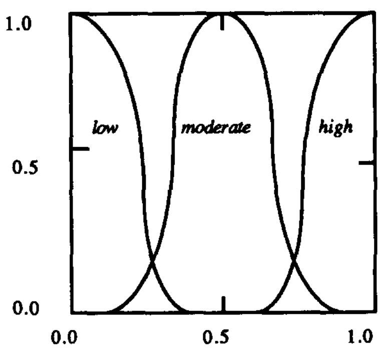

Fig. 1. Continuous membership functions for *likely*, *unlikely* and *very likely*.

From Eqs. (2.9), (2.10) and (2.11), it can be seen that there are only five non-zero membership grades representing the *likely*, four non-zero membership grades representing the *very likely*, and five non-zero membership grades representing the *unlikely*. This simplification process (from the continuous curve to several discrete points) greatly reduces the evaluation problem and, more important, it still sufficiently defines the general concept of the fuzzy terms (such as *likely*, *very likely* and *unlikely*). Incidently, it can be seen that the discrete function could have as many points as desired depending on the needs of different applications. In general the 11 elements as shown in (2.9), (2.10) and (2.11) are recommended.

Using discrete membership functions, the combinations of $v_1, v_2, \ldots, v_n$ can be limited within a finite number. However, when the n in (2.2), (2.3) and (2.4) increases, the total number of the combinations increases exponentially. To see this, assume normalized members and memberships are used, i.e., let the member z and memberships $\mu(z)$ take their values in the unit interval [0, 1]. And, assume the total number of the members are limited to 11, i.e., let the 11 normalized members of 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1.0 represent the universe of discourse of the members in the fuzzy sets of ρ_i's. (The example membership functions defined in (2.9), (2.10) and (2.11) reflect the assumption of normalization and the limitation of 11 members.) Then, the combinations of $v_1, v_2, \ldots, v_n$ that satisfy (2.3) are as given in Table 1 (Appendix A describes the development of these numbers).

In fact, the n is controlled by the total number of rules in the Type II Evidence (1.1). For example, in the evidence regarding the severity of rain stated above, the n is equal to 3. For practical purposes, the number of rules (n) in a Type II Evidence would rarely exceed 10. For example, when considering the construction resource allocation problem, 3 to 5 rules (fuzzy production rules) are sufficient to cover the probabilistic phenomenon of the random variables that comprise the decision criteria.

To evaluate all combinations shown in Table 1, an efficient algorithm is needed. The flow chart in Figure 2 shows one possible algorithm. When implemented, the computer time required to find the 92 378 combinations was approximately 5 seconds (using a standard IBM AT personal computer and a program written in assembly code).

Step (5) represents the second evaluation problem and it involves (2.2). The problem is that the min operator in (2.2) always results in a zero membership grade (i.e., $\mu(z)$ is always 0) if there exists a ρ_i which 'contains' another $\rho_{i'}$. For

Table 1

n	combinations	n	combinations
1	1	6	3 003
2	11	7	8 008
3	66	8	19 448
4	286	9	43 758
5	1 001	10	92 378

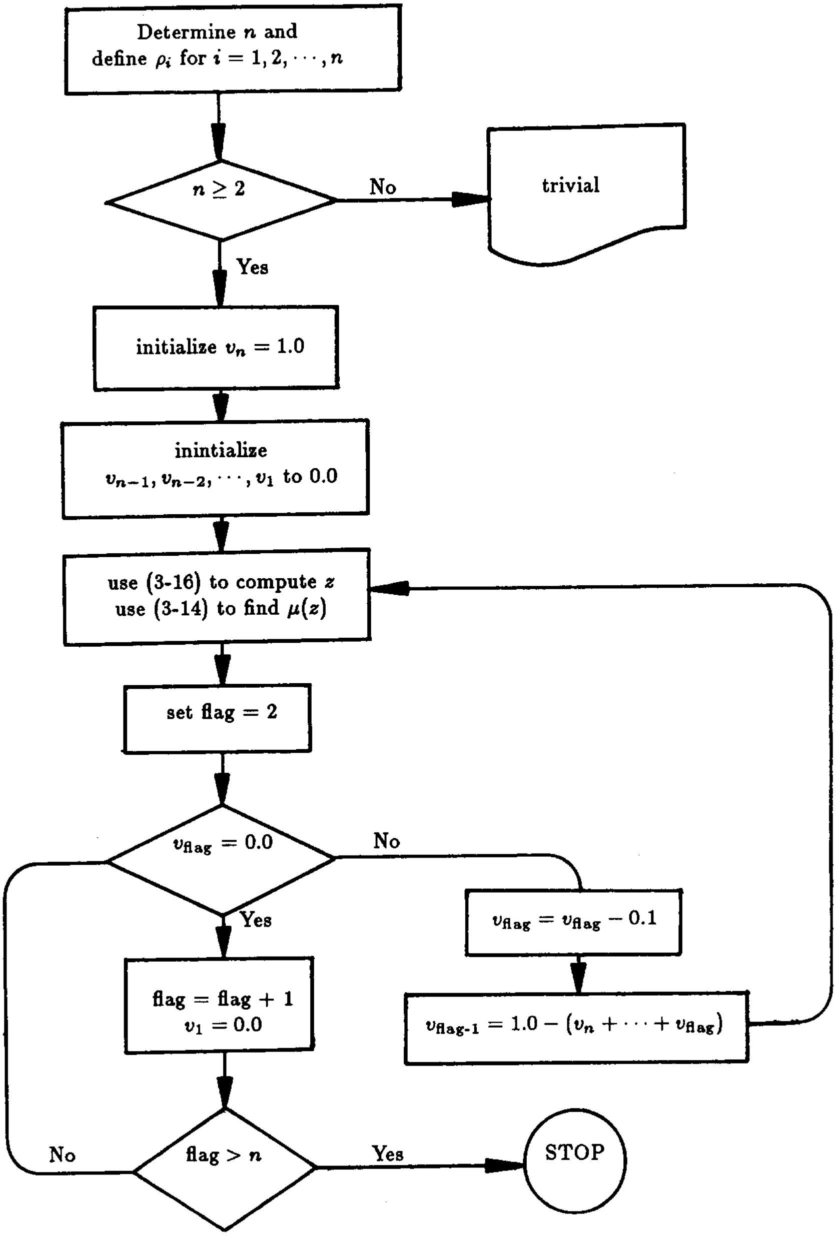

Fig. 2. Algorithm for evaluating the combinations of $v_1, v_2, \ldots, v_n$ such that $\sum_{i=1}^{n} v_i = 1.0$ and $v_i \in V = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$.

example, consider the Type II evidence regarding the severity of rain stated in section 1. Suppose the linguistic probabilities ρ_1 (*likely*), ρ_2 (*very likely*) and ρ_3 (*unlikely*) are defined in (2.9), (2.10) and (2.11). From these definitions, it can be seen that $\rho_2 = (\rho_1)^{1/2}$, i.e., ρ_1 (*likely*) 'contains' ρ_2 (*very likely*). Based on these definitions, the combination of $(v_1 = 0.6,\ v_2 = 0,\ v_3 = 0.4)$ satisfies (2.3). So, z

can be computed as follows:

$$z = 0.6\alpha_1 + 0\alpha_2 + 0.4\alpha_3 \tag{2.12}$$

where $\alpha_i = \sup(G_i \cap Q)$ for $i = 1, 2, 3$. (α_i's are constants as shown in (2.7).) And, the membership grade, $\mu(z)$, for the member z in (2.12) can be computed as follows:

$$\mu(z) = \min[\mu_{\rho_1}(0.6), \mu_{\rho_2}(0), \mu_{\rho_3}(0.4)] = \min[0.05, 0, 0.05] = 0. \tag{2.13}$$

By the same token,

$$\mu(0.7\alpha_1 \oplus 0\alpha_2 \oplus 0.3\alpha_3) = \min(0.3, 0, 0.3) = 0. \tag{2.14}$$

It can be shown that continued use of (2.12) and (2.13) to evaluate all possible $\mu(z)/z$, would eventually obtain a *zero* fuzzy set ($\mu(z) = 0$ for all z) for this example.

In fact the fuzzy set should be non-zero and it is the *min* operator (maximal restriction) in (2.2) will always cause this 'containment' to result in a *zero* fuzzy set; and does not reflect the meaning of the containment. In those situations when one fuzzy set is entirely contained in another, we suggest that use the *min* operator on all possible combinations of non-contained ρ_i's and then apply *max* operator as in (2.2). The numeric example in the next section shows this procedure.

3. Numeric example

Given the example of *Type II Evidence in* Section 1, the degree of possibility of (1.7) will be numerically demonstrated in this section. It is noted that the two linguistic variables X and Y for the given evidence are: X = *severity of rain*, and Y = the recommendation of using the severity of rain as a criterion. And, the fuzzy sets F_i, G_i, ρ_i and Q are summarized as below:

F_1 = *heavy*, F_2 = *moderate*, F_3 = *small*,

G_1 = *very strong*, G_2 = *strong*, G_3 = *normal*,

Q = *very strong*,

ρ_1 = *likely*, ρ_2 = *very likely*, ρ_3 = *unlikely*.

Table 2 shows the definitions of membership functions G_1, G_2, G_3 and Q; the parameters $\alpha_i = \sup(G_i \cap Q)$, $i = 1, 2, 3$, are also computed and listed in Table 2. It turns out that $\alpha_1 = 1.0$, $\alpha_2 = 1.0$ and $\alpha_3 = 0.3$.

Using (2.9), (2.10) and (2.11), the ρ_1, ρ_2, ρ_3 and $\alpha_1 v_1$, $\alpha_2 v_2$, $\alpha_3 v_3$ can be tabulated as shown in Table 3. Then, the z and $\mu(z)$ can be computed as shown in Table 4.

Applying the maximum operation on the minimum values in the 4th and 8th columns of Table 4, the membership function μ can be obtained as shown below:

$$\mu = 0.05/0.72 + 0.30/0.79 + 0.80/0.86 + 0.95/0.93 + 1.0/1.0. \tag{3.1}$$

Table 2. Numeric example of computing the parameter $\alpha_i = \sup(G_i \cap Q)$, $i = 1, 2, 3$

	G_1	G_2	G_3	Q	$G_1 \cap Q$	$G_2 \cap Q$	$G_3 \cap Q$
0	0	0	0	0	0	0	0
0.1	0	0	0.05	0	0	0	0
0.2	0	0	0.30	0	0	0	0
0.3	0	0	0.80	0	0	0	0
0.4	0	0	0.95	0	0	0	0
0.5	0	0	1.0	0	0	0	0
0.6	0	0.05	0.95	0	0	0	0
0.7	0.09	0.30	0.80	0.09	0.09	0.09	0.09
0.8	0.64	0.80	0.30	0.64	0.64	0.64	0.30
0.9	0.90	0.95	0.05	0.90	0.90	0.90	0.05
1.0	1.0	1.0	0	1.0	1.0	1.0	0
					$\alpha_1 = 1.0$	$\alpha_2 = 1.0$	$\alpha_3 = 0.3$

Table 3. Example definitions of ρ_1, ρ_2 and ρ_3

v_1	$\mu_{\rho_1}(v_1)$	$\alpha_1 v_1$	v_2	$\mu_{\rho_2}(v_2)$	$\alpha_2 v_2$	v_3	$\mu_{\rho_3}(v_3)$	$\alpha_3 v_3$
0	0	0	0	0	0	0	1.0	0
0.1	0	0.1	0.1	0	0.1	0.1	0.95	0.03
0.2	0	0.2	0.2	0	0.2	0.2	0.80	0.06
0.3	0	0.3	0.3	0	0.3	0.3	0.30	0.09
0.4	0	0.4	0.4	0	0.4	0.4	0.05	0.12
0.5	0	0.5	0.5	0	0.5	0.5	0	0.15
0.6	0.05	0.6	0.6	0	0.6	0.6	0	0.18
0.7	0.30	0.7	0.7	0.09	0.7	0.7	0	0.21
0.8	0.80	0.8	0.8	0.64	0.8	0.8	0	0.24
0.9	0.95	0.9	0.9	0.90	0.9	0.9	0	0.27
1.0	1.0	1.0	1.0	1.0	1.0	1.0	0	0.30

Table 4. Computation of degree of possibility

v_1	v_3	$z = \alpha_1 v_1 + \alpha_3 v_3$	$\mu(z) = \min(\mu_{\rho_1}(v_1), \mu_{\rho_3}(v_3))$	v_2	v_3	$z = \alpha_2 v_2 + \alpha_3 v_3$	$\mu(z) = \min(\mu_{\rho_2}(v_2), \mu_{\rho_3}(v_3))$
0.6	0.4	0.72	0.05	0.7	0.3	0.79	0.09
0.7	0.3	0.79	0.30	0.8	0.2	0.86	0.64
0.8	0.2	0.86	0.80	0.9	0.1	0.93	0.90
0.9	0.1	0.93	0.95	1.0	0	1.0	1.0
1.0	0	1.0	1.0				

This result can be interpreted as '*very high*', i.e., the degree of possibility in (1.7) is *very high*. Using fuzzy average [2, 7], a plausible number can be computed to represent this fuzzy concept, i.e.,

$$\begin{aligned} &\text{degree of possibility} \\ &\quad = \frac{0.05 \times 0.72 + 0.3 \times 0.79 + .08 \times 0.86 + 0.95 \times 0.93 + 1.0 \times 1.0}{0.05 + 0.3 + 0.8 + 0.95 + 1.0} \\ &\quad = 0.92 \text{ (or } 92\%). \end{aligned} \tag{3.2}$$

4. Application in construction resource allocation

In the process of allocating limited resources to construction activities, it is necessary to establish a set of criteria for priority ranking [2]. The 'rain' example given in the previous section is but a possible criterion for this purpose. Usually, experienced engineers should be able to set up a list of criteria such as severity of rain, degree of temperature, resource availability, cash shortages, morale of labor, changes, and changed conditions that are significant to a specific construction project. This criteria are project related and are referred to as *external criteria* [1, 2].

Assume that there exist n external criteria. Let C_i $(i = 1, 2, \ldots, n)$ denote them. It can be shown that the importance of these criteria depends on time, project location, project characteristics, project manager's preferences, and other similar factors. The importance does not simply mean that C_i is either important or not important (crisp set concept). Rather, it can be better treated as a 'linguistic variable', or as the 'degree' of importance or the 'weight' of importance (fuzzy set concept). Symbolically, w_i will be used to denote the measure of the 'weight' for the importance of criterion C_i. For example, if there exists three criteria C_1, C_2 and C_3, they can be described such as criterion C_1 is *very* important (w_1 = very high), C_2 is *more or less* important (w_2 = more or less high), and C_3 is *less* important (w_3 = low). Using this concept it will be shown that the *degree of possibility* or *degree of certainty* discussed in the previous section can be used to establish an appropriate method to measure the weight (w_i).

In addition to the example Type II Evidence of rain given in Section 1, assume that the evidence for temperature and cash shortage have been established as follows:

Rules

Criterion 1 – *Severity of rain*

rule 1: IF rain is *heavy*, THEN it is *very strongly* recommended to consider rain as a criterion for resource allocation.

rule 2: IF rain is *moderate*, THEN it is *strongly* recommended to consider rain as a criterion for resource allocation.

rule 3: IF rain is *small*, THEN it is *normally* recommended to consider rain as a criterion for resource allocation.

rule 4: IF there is no rain at all, THEN it is not recommended to consider rain as a criterion for resource allocation.

Criterion 2 – *Degree of temperature*

rule 1: IF temperature is *high*, THEN it is *strongly* recommended to include temperature as a criterion for resource allocation.

rule 2: IF temperature is *moderate*, THEN it is *very very weakly* recommended to include temperature as a criterion for resource allocation.

rule 3: IF temperature is *low*, THEN it is *strongly* recommended to include temperature as a criterion for resource allocation.

Criterion 3 – *Cash shortage*

rule 1: If the degree of shortage in cash in your organization is *high* then the recommendation to use cash shortage as a criterion is *strong*.

rule 2: If the degree of shortage in cash in your organization is *moderate* then the recommendation to use cash shortage as a criterion is *average*.

rule 3: If the degree of shortage in cash in your organization is *low* then the recommendation to use cash shortage as a criterion is *weak*.

User input

Criterion 1 – *Severity of rain*

1. In the specified period, rain is heavy is *not likely*.
2. In the specified period, rain is moderate is *likely*.
3. In the specified period, rain is small is *very likely*.
4. In the specified period, there is no rain at all is *unlikely*.

Criterion 2 – *Degree of temperature*

1. In the specified period, temperature is high is *likely*.
2. In the specified period, temperature is moderate is *not likely*.
3. In the specified period, temperature is low is *very unlikely*.

Criterion 3 – *Cash shortage*

1. In the specified period, degree of cash shortage is high is *very likely*.
2. In the specified period, degree of cash shortage is moderate is *more or less likely*.
3. In the specified period, degree of cash shortage is low is *very unlikely*.

Now, according to the *Type II Evidence*, the following questions can be asked:

What is the degree of possibility that the recommendation of using the severity of rain as a criterion is *absolutely strong*? (4.1)

What is the degree of possibility that the recommendation of using the degree of temperature as a criterion is *absolutely strong*? (4.2)

What is the degree of possibility that the recommendation of using the cash shortage as a criterion is *absolutely strong*? (4.3)

where the fuzzy set for the linguistic term *absolutely strong* defined as:

$$0/0 + 0/0.1 + \cdots + 0/0.9 + 1.0/1.0 \tag{4.4}$$

i.e., the membership grade is 0 for every member that is less than 1.0; the membership grade is 1.0 when the member is 1.0.

The degree of possibility for (4.1), (4.2) and (4.3) can be now obtained by the algorithm described in the previous section. Note that in the above questions the Q (see (1.5)) is deliberately set to *absolutely strong* for all of the three criteria. Therefore the degrees of possibilities for (4.1), (4.2) and (4.3) become relative measures among the three criteria. For the purpose of solving the priority ranking problem for resource allocation [2], these relative measures can be interpreted as the *weights* of the criteria. In general, the weights of criteria can be defined as follows:

Definition. For a list of criteria C_i $(i = 1, 2, \ldots, n)$, the weight w_i is defined as the degree of possibility when the fuzzy set Q in (1.5) is set to an absolute level.

(Note that the term of 'degree of certainty' is dropped out from the definition above because the use of degree of certainty will result in a similar concept to the measurement of the weights as per (2.8).)

The list of criteria C_i $(i = 1, 2, \ldots, n)$ and their related weight w_i will help to solve the priority ranking problem. To show this, assume that there is a pool of construction activities that have all logical precedence constraints met, and, therefore, are candidates for resource assignment and initiation of physical work. And, assume that the resources are limited (as true in most cases). Then the immediate problem is which candidate activity has the first priority. To resolve this, each candidate in the pool should be checked against the list of criteria C_i $(i = 1, 2, \ldots, n)$ to see the *susceptibility* of each candidate to each criterion. Let s_{ij} denote the *susceptibility* of an activity j in the pool to a criterion C_i of the list. And, let s_{ij} range between 0 and 1 for convenience (i.e., normalize s_{ij} to take values in the unit interval [0, 1]). Then, when $s_{ij} = 0$ it means that activity j is not susceptible to C_i. When $s_{ij} = 1$ it means that activity j is 'absolutely' susceptible to C_i. And, when s_{ij} = any other value in the range 0 to 1 it means that activity j has some 'degree' of susceptibility to C_i. Chang [2] has shown that the *susceptibility* (s_{ij}) can be determined using an approximate reasoning system based on the fuzzy set theory. Knowing w_i and s_{ij}, the priority rank (P_j) for activity j in the activity pool can be defined as follows:

$$P_j = \frac{\sum_{i=1}^{n} (1 - s_{ij} \cdot w_i)}{n} \tag{4.5}$$

where the term $1 - s_{ij} \cdot w_i$ can be interpreted as a measure of the priority for the activity j against the criterion C_i. (Note: a P_j which has a small value indicates that activity j is susceptible to the criterion and therefore has a low priority. Another way of expressing this is to recognize that an activity which is scheduled to start when it has a small valued P_j will most likely be adversely impacted by the controlling criteria during its duration.)

For example, assume that a criterion C_i called 'severity of rain' has been established, and an activity j called 'placing concrete' will be done in an exposed area where there is no protection from rain. Therefore, the activity is highly susceptible to rain (s_{ij} is large or close to 1). So, the priority to schedule the activity under the consideration of impact by rain should be low. The term $1 - s_{ij} \cdot w_i$ reflects this situation and it is defined as a relative value to measure the priority against each criterion. Note that each criterion has its own 'relative weight of importance' (w_i). So, the susceptibility (s_{ij}) would be diluted by its weight in relation to other criteria. The product of s_{ij} and w_i gives the measure of the dilution and represents the weighted susceptibility of activity j to criterion C_i. If there exist n criteria then the summation of all $1 - s_{ij} \cdot w_i$ for $i = 1, 2, \ldots, n$ yields a relative value for the priority rank of activity j when all available criteria are taken into account. The priority rank is then normalized by the total number of criteria (n) to yield a value that is in the range from 0 to 1.

With P_j determined by (4.5), each candidate in the activity pool can be assigned a priority value (a relative measure). Then, the limited resources can be allocated to the activities in the pool according to the priority rank. This will initiate the physical work of the construction activities.

5. Conclusions

The priority ranking problem in construction resource allocation involves the incorporation of the impacts of external forces such as weather conditions, resource availability, material delays, and other factors. hese factors are non-deterministic and are inherent with a degree of certainty (or degree of possibility) in their occurrences. It is recognized that the traditional analytic methods or mathematical models are difficult (although not impossible) to apply to measure the impacts of these factors. Thus, the impacts have most often been ignored in the resource allocation process, and therby in the project scheduling process, since the advent of the CPM/PERT [5] scheduling methods. With this omission, schedules produced by the basic CPM/PERT and resource allocation procedures ignore significant external factors that can adversely impact the project. Fortunately, this deficiency can be resolved with the help of the recently developed fuzzy logic system. This paper has clearly shown this possibility.

The reasoning process of 'Type II Evidence' syllogism provides the best way to resolve the difficulty in measuring the weights of a set of resource allocation criteria. This measurement is based on the concepts of *expected certainty* and *expected possibility*. In fact, this reasoning process is particularly appropriate for a rule system in which the antecedent part of a rule is intrinsic with uncertainty. Such a rule system is quite common; the rule system used to measure the criteria weights is but an example. (In general, the uncertainty is a mixture of probabilistic and possibilistic problems, and must be handled by a combination of probabilistic and possibilistic methods. The reasoning process of 'Type II Evidence' provides such an approach.)

Although the domain problem of this paper was the construction resource allocation area, the principles and theoretical background is not limited to this specific area. Actually, they could be applied to other disciplines of engineering and management as long as the domain problems have similar characteristics.

Appendix A: Evaluation of the combination of $v_1, v_2, \ldots, v_n$ such that $\sum_{i=1}^{n} v_i = 1.0$

Let $v_i \in V = \{0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0\}$. Then the combinations of $v_1, v_2, \ldots, v_n$ such that $\sum_{i=1}^{n} v_i = 1.0$ are as follows:

When $n = 1$, there is only one solution (trivial case).

When $n = 2$, there are 11 combinations, shown in Table A1.

Table A1

1	0
0.9	0.1
0.8	0.2
0.7	0.3
0.6	0.4
0.5	0.5
0.4	0.6
0.3	0.7
0.2	0.8
0.1	0.9
0	1.0

When $n = 3$, there are 66 combinations as shown in Table A2.

Note that the total combinations (66) are equal to $1+2+3+\cdots+10+11$ where 1 is the number of combination(s) starting with 1.0, i.e., the (1.0, 0, 0); 2 are the number of combinations starting with 0.9, i.e., the (0.9, 1, 0) and (0.9, 0, 0.1); 3 are the number of combinations starting with 0.8, i.e., the (0.8, 0.2, 0), (0.8, 0.1, 0.1) and (0.8, 0, 0.2); . . . ; 10 are the number of combinations starting with 0.1; 11 are the number of combinations starting with 0.

When $n = 4$, the same pattern of combinations can be obtained. Table A3 shows part of the total 286 combinations.

Note that the number of combination(s) starting with 1.0 is one, i.e., the (1.0, 0, 0, 0). The number of combinations starting with 0.9 are 3 (=1+2).

Table A2

1.0	0	0	0.4	0.6	0	0.2	0.8	0	0.1	0.9	0	0	1.0	0
0.9	0.1	0	0.4	0.5	0.1	0.2	0.7	0.1	0.1	0.8	0.1	0	0.9	0.1
0.9	0	0.1	0.4	0.4	0.2	0.2	0.6	0.2	0.1	0.7	0.2	0	0.8	0.2
0.8	0.2	0	0.4	0.3	0.3	0.2	0.5	0.3	0.1	0.6	0.3	0	0.7	0.3
0.8	0.1	0.1	0.4	0.2	0.4	0.2	0.4	0.4	0.1	0.5	0.4	0	0.6	0.4
0.8	0	0.2	0.4	0.1	0.5	0.2	0.3	0.5	0.1	0.4	0.5	0	0.5	0.5
0.7	0.3	0	0.4	0	0.6	0.2	0.2	0.6	0.1	0.3	0.6	0	0.4	0.6
0.7	0.2	0.1	0.3	0.7	0	0.2	0.1	0.7	0.1	0.2	0.7	0	0.3	0.7
0.7	0.1	0.2	0.3	0.6	0.1	0.2	0	0.8	0.1	0.1	0.8	0	0.2	0.8
0.7	0	0.3	0.3	0.5	0.2				0.1	0	0.9	0	0.1	0.9
0.6	0.4	0	0.3	0.4	0.3							0	0	1.0
0.6	0.3	0.1	0.3	0.2	0.5									
0.6	0.2	0.2	0.3	0.1	0.6									
0.6	0.1	0.3	0.3	0	0.7									
0.6	0	4												
0.5	0.5	0												
0.5	0.4	0.1												
0.5	0.3	0.2												
0.5	0.2	0.3												
0.5	0.1	0.4												
0.5	0	0.5												

Table A3

1.0	0	0	0
0.9	0.1	0	0
0.9	0	0.1	0
0.9	0	0	0.1
0.8	0.2	0	0
0.8	0.1	0.1	0
0.8	0.1	0	0.1
0.8	0	0.2	0
0.8	0	0.1	0.1
0.8	0	0	0.2
0.7	0.3	0	0
0.7	0.2	0.1	0
0.7	0.2	0	0.1
0.7	0.1	0.2	0
0.7	0.1	0.1	0.1
0.7	0.1	0	0.2
0.7	0	0.3	0
0.7	0	0.2	0.1
0.7	0	0.1	0.2
0.7	0	0	0.3

The number of combinations starting with 0.8 are 6 (=1+2+3). The number of combinations starting with 0.7 are 10 (=1+2+3+4). Same pattern can be applied to the combinations starting with 0.6, 0.5, ..., 0.1, and 0. And, the total number of combinations are

$$1+(1+2)+(1+2+3)+(1+2+3+4)$$
$$+(1+2+3+4+5)+\cdots+(1+2+\cdots+10+11)$$

or

$$\sum_{j=1}^{11}\left(\sum_{i=1}^{j} i\right)$$

and are equal to 286.

Using the same combination pattern, the total number of combinations for $n=5$ can be found as follows:

$$\sum_{k=1}^{11}\left(\sum_{j=1}^{k}\left(\sum_{i=1}^{j} i\right)\right)=1001.$$

In general, the total number of combinations can be expressed as the following equation:

$$f(n)=\sum_{i_{n-2}=1}^{11}\left(\sum_{i_{n-3}=1}^{i_{n-2}}\left(\sum_{i_{n-4}=1}^{i_{n-3}}\left(\cdots\left(\sum_{i_2=1}^{i_3}\left(\sum_{i_1=1}^{i_2} i_1\right)\right)\cdots\right)\right)\right)$$

where $f(n)$ = the total number of combinations. Note that there are $n-2$ levels of summations in this equation.

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