Interval and Fuzzy Finite Element Analysis of mechanical structures with uncertain parameters

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Abstract

This paper uses a non-probabilistic Fuzzy Finite Element Method (FFEM) for the analysis of the dynamic behaviour of structures with uncertain parameters. In two case studies, the concepts of the interval and the fuzzy finite element frequency response function analysis are illustrated for different types of uncertainties, for undamped as well as for damped structures. The comparison of the obtained results with the results of Monte Carlo Simulations proves that the FFEM gives a good and reliable approximation of the dynamic behaviour of uncertain structures. The method is capable of handling large uncertainty intervals, and does not need assumptions on the statistical distribution of the uncertain model parameters.

1 Introduction

Nowadays, the Finite Element Method (FEM) has become an indispensable tool for the numerical optimisation and validation of structural designs. This numerical method is capable of predicting the static and dynamic behaviour of a structure based on its geometry and material characteristics, the applied loads and constraints. The finite element method allows to adopt a *Virtual Prototyping* concept, as a reliable FE analysis can reduce the need for expensive physical prototype production and testing.

However, it is often very difficult to define a reliable FE model, especially when a number of physical properties are uncertain, eg. variable material characteristics, geometric uncertainties due to production tolerances, non-ideal boundary conditions, etc. In these cases, the validation of a structure with the FEM can only be reliable if all uncertainties are taken into account. A probabilistic Monte Carlo Simulation (MCS) is the best known procedure to predict the effect of uncertain model parameters. However, when the design model contains uncertain physical properties for which no objective statistical data is available, a probabilistic analysis leads to subjective and misleading conclusions, and it is therefore not reliable for objective design validation purposes [1].

An alternative method to describe uncertainties is provided by the concept of fuzzy numbers, which has led to the development of the Fuzzy Finite Element Method for dynamic analysis [2, 3, 4]. In section 2, a brief overview of the basic concepts of the method is given. The strength and applicability of the method are illustrated in two case studies: the aircraft model of the GARTEUR benchmark problem (section 3), and the cover of the COROT telescope (section 4).

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2 The Fuzzy Finite Element Method for FRF analysis

The fuzzy finite element method for the calculation of frequency response functions (FRFs), developed by Moens, combines the concept of fuzzy sets with the modal superposition principle. A short overview of the basic principles of the method is given in this section. The complete mathematical description of the method can be found in [2, 4].

2.1 Fuzzy sets & fuzzy arithmetic

The concept of fuzzy sets, introduced by Zadeh [5] in 1965, has gained an increasing popularity during the last two decades. Its most important property is that it is capable of describing linguistic and therefore incomplete information in a non-probabilistic manner. Whereas a classical set clearly distinguishes between members and non-members, a fuzzy set introduces a degree of membership, represented by the *membership function*. For a fuzzy set $\tilde{x}$, the membership function $\mu_{\tilde{x}}(x)$ describes the grade of membership to the fuzzy set for each element x in the domain X:

$$\tilde{x} = \{(x, \mu_{\tilde{x}}(x)) \mid (x \in X)(\mu_{\tilde{x}}(x) \in [0,1])\} \tag{1}$$

If $\mu_{\tilde{x}}(x) = 1$, x is definitely a member of the set $\tilde{x}$, whereas if $\mu_{\tilde{x}}(x) = 0$, x is definitely not a member of the set $\tilde{x}$. For all x with $0 < \mu_{\tilde{x}}(x) < 1$, the membership is not certain. The most frequently applied membership function shapes are the triangular and Gaussian shape.

The description of an uncertain parameter using a membership function can practically be implemented using the α-*level strategy*. This approach subdivides the membership function range into a number of α-levels. The intersection with the membership function of the input uncertainties at each α-level results in an interval $\mathbf{x}_{i,\alpha} = [\underline{x}_i, \overline{x}_i]_\alpha$. With these input intervals of the α-sublevel, an Interval Finite Element (IFE) analysis is performed, resulting in an interval for the analysis result at the considered α-level. Finally, the fuzzy solution is assembled from the resulting intervals at each sublevel. Figure 1 clarifies this procedure for a function of two triangular fuzzy parameters.

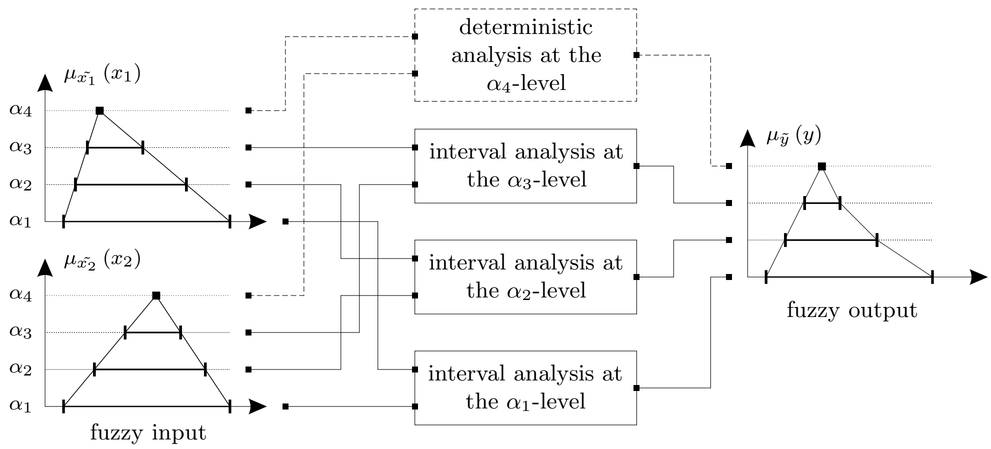


Figure 1: α-level strategy with 4 α-levels, for a function of two triangular fuzzy parameters

2.2 The deterministic modal superposition principle

For **undamped** structures, the deterministic modal superposition principle states that, considering the first n_{modes} modes, the frequency response function between degrees of freedom j and k equals:

$$FRF_{jk} = \sum_{i=1}^{n_{modes}} FRF_{jk}^{i} = \sum_{i=1}^{n_{modes}} \frac{\phi_{i_k}\phi_{i_j}}{\{\phi_i\}^T[K]\{\phi_i\} - \omega^2\{\phi_i\}^T[M]\{\phi_i\}} \tag{2}$$

with $\{\phi_i\}$ the i^{th} eigenvector of the system and ϕ_{i_j} the j^{th} component of the i^{th} eigenvector. Simplification of equation (2) yields

$$FRF_{jk} = \sum_{i=1}^{n_{modes}} \frac{1}{\hat{k}_i - \omega^2 \hat{m}_i} \tag{3}$$

with $\hat{k}_i$ and $\hat{m}_i$ the modal parameters defined as

$$\hat{k}_i = \frac{\{\phi_i\}^T [K] \{\phi_i\}}{\phi_{i_j} \phi_{i_k}} \quad , \quad \hat{m}_i = \frac{\{\phi_i\}^T [M] \{\phi_i\}}{\phi_{i_j} \phi_{i_k}} \ . \tag{4}$$

The function $\mathcal{D}(\omega) = (\hat{k}_i - \omega^2 \hat{m}_i)$ expresses the modal response denominator as a function of frequency.

For structures with **proportional damping**, the system damping matrix $[C]$ is written as

$$[C] = \alpha_K [K] + \alpha_M [M] \tag{5}$$

with α_K and α_M the proportional damping coefficients. In this case, the deterministic damped frequency response function yields:

$$FRF_{jk} = \sum_{i=1}^{n_{modes}} \frac{1}{(\hat{k}_i - \omega^2 \hat{m}_i) + \jmath\, \omega (\alpha_K \hat{k}_i + \alpha_M \hat{m}_i)} \tag{6}$$

with $\jmath$ the imaginary unit ($\jmath = \sqrt{-1}$).

2.3 Fuzzy Finite Element FRF analysis

The modal superposition principle for undamped and proportionally damped structures has been translated into an *interval finite element method* for FRF analysis. For the undamped case a graphical overview of the translation of the deterministic algorithm into an interval procedure is given in figure 2. In the remainder of this paper, the range of a function $f(x)$ taking into account all possible values of the function variable x inside an interval $\mathbf{x}$ is denoted by $\langle f \rangle_{\mathbf{x}}$.

The interval translation shows that the total envelope FRF can be calculated in three principal steps:

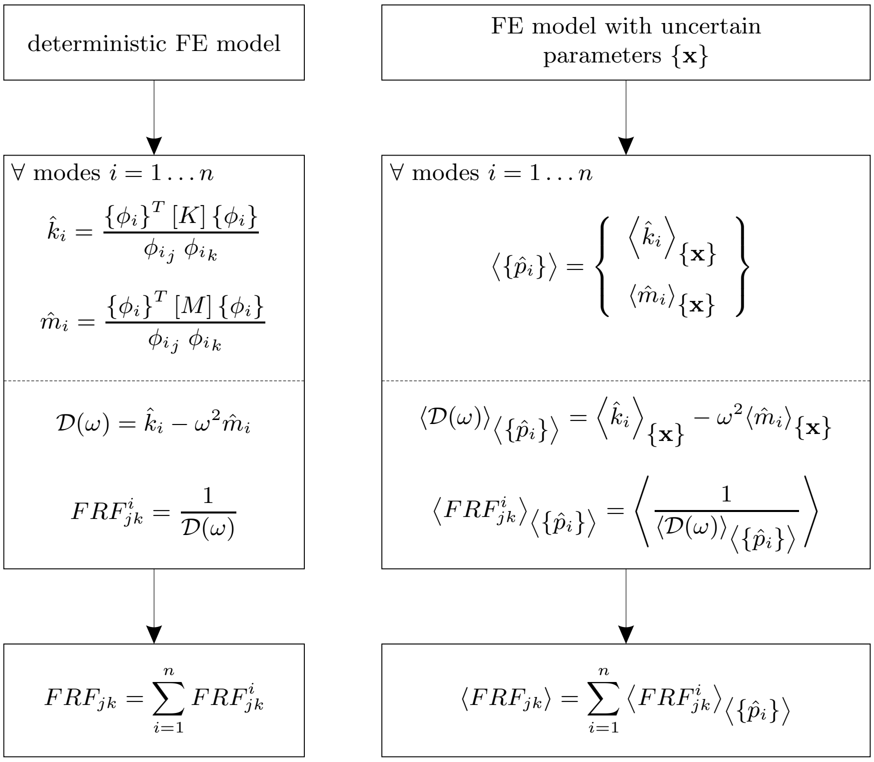
1. For all n_{modes} taken into account, the correct ranges of the modal parameters $\langle \hat{k}_i \rangle_{\{\mathbf{x}\}}$ and $\langle \hat{m}_i \rangle_{\{\mathbf{x}\}}$ corresponding to the vector $\{\mathbf{x}\}$ with the input uncertain parameters are determined using a global optimisation procedure. The modal parameter range vector $\langle \{\hat{p}_i\} \rangle$ combines the two modal parameter ranges in an independant way :

$$\langle \{\hat{p}_i\} \rangle = \left\{ \begin{array}{c} \langle \hat{k}_i \rangle_{\{\mathbf{x}\}} \\ \langle \hat{m}_i \rangle_{\{\mathbf{x}\}} \end{array} \right\} \ . \tag{7}$$

2. The modal envelope FRF is calculated by substituting the ranges of the modal parameters in the denominator function $\mathcal{D}(\omega)$, and subsequently inverting the resulting denominator function range. For the proportionally damped case, the real and imaginary part of the modal FRF contribution is determined.
   In the *Modal Rectangle* (MR) method the modal parameters $\hat{k}_i$ and $\hat{m}_i$ are assumed to be independent. This leads to a rectangle approximation of the modal domain $\langle \hat{k}_i, \hat{m}_i \rangle$. The approximation of the modal envelope FRF can be improved substantially by taking the exact eigenfrequency ranges $\langle f_i \rangle$ into account, resulting in the *Modal Rectangle method with Eigenvalue interval correction* (MRE) [2].

3. The total interval FRF is obtained by the summation of the contribution of each mode. For proportionally damped structures, this step also includes the calculation of the amplitude and phase of the total interval FRF based on the summation of the real and the imaginary parts.

On each α-level of a membership function, an IFE frequency response function analysis is performed, resulting in an upper and lower bound on the FRF. The results on each α-level are then combined to construct the fuzzy FRF.



(a) Deterministic algorithm (b) Set translated algorithm

Figure 2: Translation of the deterministic modal superposition algorithm to an equivalent IFE procedure

3 Case study: the Garteur benchmark problem

3.1 Problem definition

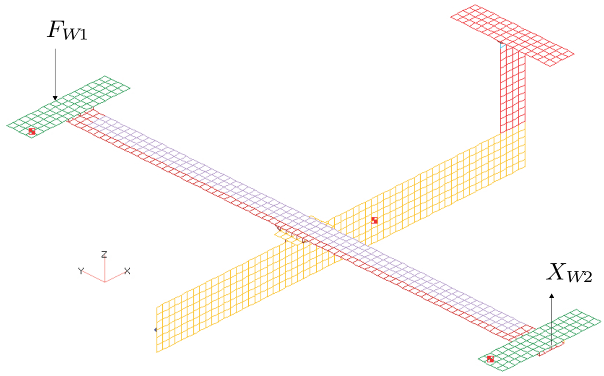
The simplified aircraft model of the Garteur benchmark problem is a common testbed designed and manufactured by the Garteur Action Group to evaluate the efficiency and reliability of different ground vibration test techniques [6, 7]. It has also been used extensively for testing model updating methods and model error localisation procedures.

The structure consists of a small-scale aluminium aircraft model with a length of 1.5 m, a wing span of 2 m and a mass of 44 kg. The fuselage of the aircraft consists of a rectangular plate with a thickness of 50 mm. The aluminium tail with a thickness of 10 mm is connected rigidly to the fuselage. The wings are connected to the fuselage through an intermediate steel plate. Wingtips are rigidly connected at both ends of the wings. Both the wings and wingtips are rectangular aluminium plates with a thickness of 10 mm. A visco-elastic layer is glued onto a part of the wings. Three concentrated masses are present : one on each wingtip and one on the fuselage.

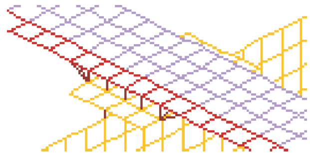
A finite element model has been created based on the description of the physical model, data of the FE model made by the Garteur Action Group and test data. The model, as illustrated in figure 3(a), is built with 1014 CQUAD8 elements, and it contains almost 20000 degrees of freedom [8].

The Garteur aircraft model contains some inherent uncertainties, due to a lack of knowledge on the physical model as well as due to uncertainty on the modelling level. Three uncertainties are considered during the dynamic analysis of the structure.

1. A first source of uncertainty is the visco-elastic layer, glued onto a part of the wings. The damping characteristics of this visco-elastic layer, as well as the quality of the glue connection are not exactly known. The combined structure of wing and visco-elastic layer connected by glue, is modelled using



(a) Finite element model of the aircraft



(b) Detail of the connection between fuselage and wings

Figure 3: Finite element model of the Garteur benchmark aircraft

a layered material. In the performed analyses, the uncertainty on the quality of the glue connection is represented by an uncertain thickness of the visco-elastic layer between 0.1 and 1.6 mm, with a nominal value of 1.1 mm.

2. A second source of uncertainty is the stiffness of a part of the connection between the wings and the fuselage - an inherent modelling uncertainty. In the assembled model, this connection is modelled with an interconnecting plate parallel to the wings, as shown in fig 3(b). The fuselage is rigidly connected to this plate. The degrees of freedom of the interconnecting plate are rigidly connected to the wings, except for the degrees of freedom on the edge of the plate. These DOFs are connected to the corresponding DOFs on the wings using linear springs [8]. The dimension and the stiffness of the connection between the wings and the fuselage can then be varied in a continuous way by changing the stiffness of these springs. In the performed analyses this stiffness ranges between 10 N/m and 10^{15} N/m with a nominal value of 10^8 N/m. The lower bound of this uncertainty interval simulates the situation in which the edge of the connection plate does not contribute to the connection between wings and fuselage. The upper bound simulates the situation in which the corresponding DOFs of the edge of the interconnecting plate and the wings are rigidly connected. An experienced engineer may have sufficient knowledge to specify a more narrow interval. The interval $[10; 10^{15}]$ N/m is taken artificially wide here, to demonstrate the capability of the developed methods to deal with highly uncertain parameters. Of course, care should be taken not to create near-singularities in the FE model.

3. A third uncertainty is introduced on the Young's modulus of the wing material, with a range of 67.5 GPa up to 68.5 GPa, with nominal value 68.0 GPa.

These uncertainties represent three different types of uncertainty : an early design uncertainty (thickness of the visco-elastic layer), a modelling uncertainty (the connection between wings and fuselage) and a global physical uncertain variability (Young's modulus).

The influence of the uncertain parameters on the frequency response function between the wingtips of the aircraft model is investigated. The input and output degrees of freedom W1 and W2 are indicated in figure 3(a). For the calculation of the interval FRF, 14 modes are taken into account, covering a frequency range up to 160 Hz.
In section 3.2 the influence of the thickness of the visco-elastic layer on the dynamic behaviour of the aircraft is investigated. The results for both the MR and the MRE method are presented. Section 3.3 studies the combined effect of the three considered uncertainties. The last section 3.4 presents the results of a fuzzy FRF analysis of the damped aircraft model including all three uncertain parameters.

3.2 Influence of the thickness of the visco-elastic layer

Figure 4 presents the results of the damped interval FRF analysis calculated with the MR and the MRE methods, compared with the results of a Monte Carlo Simulation with 100 uniformly distributed samples (green curves). The dashed line represents the envelope FRF calculated with the MR method, whereas the solid line represents the envelope FRF calculated with the MRE method. The values of the proportional damping coefficients α_K and α_M are $5 \cdot 10^{-5}$ and 2 respectively, resulting in damping ratios between 1.0% and 2.8% for all 14 considered modes. The figure clearly shows the improvement of the envelope FRFs achieved by the MRE method. This improvement is obtained by using the correct eigenfrequency intervals for the approximation of the modal domain (cfr. figure 5). The upper and lower bounds on the FRF, calculated with the MRE method, give a narrow circumscription of the MC results for the entire frequency range.

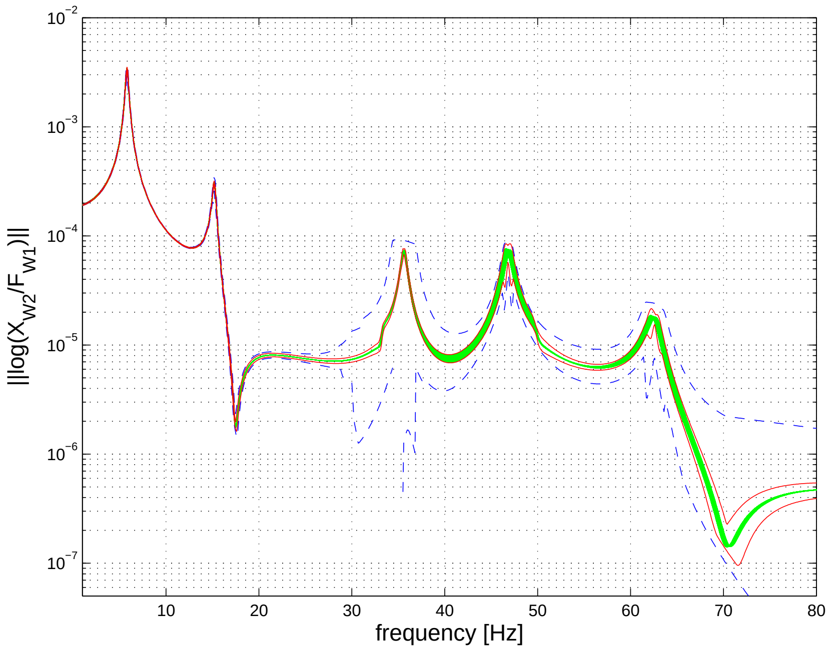
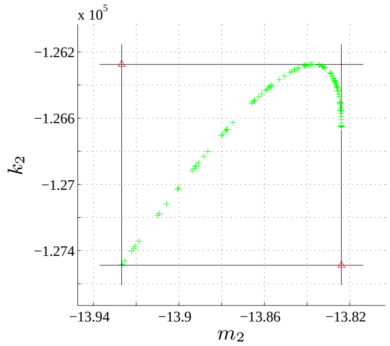


Figure 4: Damped interval FRF of the Garteur aircraft model with the thickness of the visco-elastic layer as uncertain parameter (MR: dashed, MRE: solid)

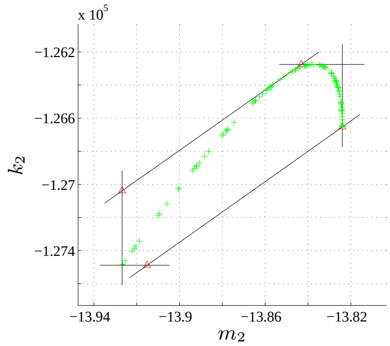
Table 1 lists the corresponding eigenfrequency intervals used in the MRE method. For several modes the eigenfrequency and the modal mass and stiffness parameters behave in a non-monotonic way with respect to the thickness of the visco-elastic layer, as illustrated in figure 6 for mode 2. A vertex method, which only considers the boundary values of the uncertainty intervals to calculate the boundary values of the modal parameters, does not correctly predict the actual boundaries on the modal parameters $\langle \hat{k}_i \rangle$, $\langle \hat{m}_i \rangle$ and $\langle f_i \rangle$. Therefore a global optimisation procedure for the calculation of the modal parameter intervals is required.

3.3 Interval analysis with three uncertain parameters

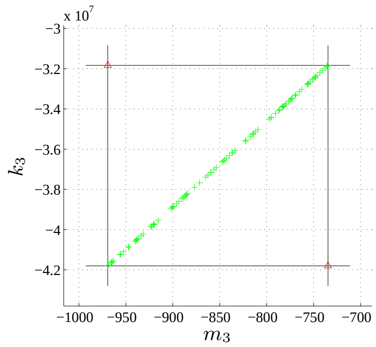
Undamped and damped IFE frequency response function analyses are performed, taking into account all three uncertain parameters : the thickness of the visco-elastic layer, the stiffness of the connection springs between the wings and the fuselage and the Young's modulus of the wing material (cfr. section 3.1). In figure 7 the **undamped** interval FRF, calculated with the MRE strategy, is compared with the results of a Monte Carlo Simulation with 50 samples. For all uncertain parameters uniform probability density functions over the interval are chosen. The figure reveals that the envelope FRF is conservative with respect to the samples and does not give a narrow circumscription of the Monte Carlo samples, especially in the frequency range between 44 and 48 Hz and between 59 and 76 Hz. However, a Monte Carlo Simulation with a uniform



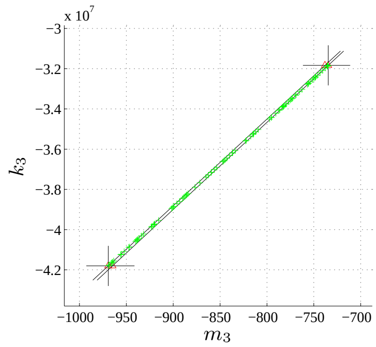
(a) $\langle \hat{k}_2, \hat{m}_2 \rangle$-domain approximation, MR method



(b) $\langle \hat{k}_2, \hat{m}_2 \rangle$-domain approximation, MRE method



(c) $\langle \hat{k}_3, \hat{m}_3 \rangle$-domain approximation, MR method



(d) $\langle \hat{k}_3, \hat{m}_3 \rangle$-domain approximation, MRE method

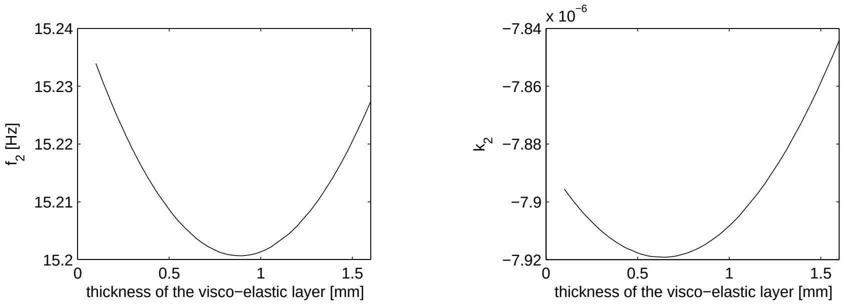
Figure 5: Comparison of the $\langle \hat{k}_i, \hat{m}_i \rangle$-domain approximations between the MR and the MRE method

Mode	Eigenfreq. interval [Hz]	Mode	Eigenfreq. interval [Hz]
1	[5.7989 ; 5.8324]	8	[54.003 ; 54.076]
2	[15.201 ; 15.234]	9	[62.290 ; 62.836]
3	[33.052 ; 33.130]	10	[67.594 ; 67.597]
4	[33.174 ; 33.262]	11	[100.22 ; 100.27]
5	[35.585 ; 35.660]	12	[128.73 ; 129.19]
6	[46.577 ; 47.078]	13	[137.10 ; 138.95]
7	[49.821 ; 49.825]	14	[150.70 ; 151.79]

Table 1: Eigenfrequency intervals of the first 14 modes of the Garteur aircraft model, for one uncertain parameter

distribution on the logarithm of the stiffness of the connection springs results in a much better correspondence in these frequency regions, as illustrated in figure 8. The samples of this Monte Carlo Simulation cover the actual stiffness interval more accurately, especially in the range of the low stiffness values. With a linear interval $k = [10; 10^{15}]$ low stiffness values have insufficient weight in MC sampling. With a logarithmic interval $\log k = [1; 15]$ a uniform distribution is closer to the actual range of the physical parameter dependency.

Figure 9 illustrates the amplitude of the **proportionally damped** envelope FRF, compared with the results of a combined Monte Carlo Simulation with 50 uniformly distributed and 50 logarithmically distributed samples. The proportional damping coefficients have the same values as in the previous damped analysis. The figure clearly shows that the damped IFE FRF analysis using the MRE strategy is capable of giving a very close approximation of the frequency response function range, for different types of uncertain parameters.



(a) Variation of the eigenfrequency of mode 2 with respect to the thickness of the visco-elastic layer

(b) Variation of the modal stiffness of mode 2 with respect to the thickness of the visco-elastic layer

Figure 6: Non-monotonous behaviour of modal parameters of mode 2 with respect to the thickness of the visco-elastic layer

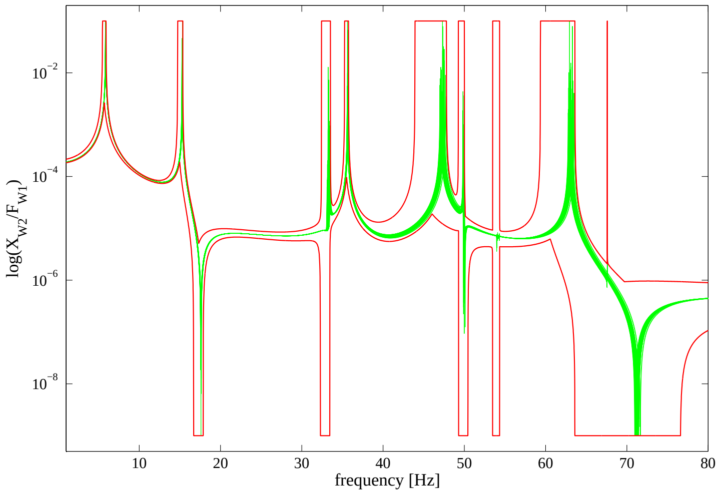


Figure 7: Undamped interval FRF (MRE) of the Garteur model, compared with a MCS with uniform sampling strategy

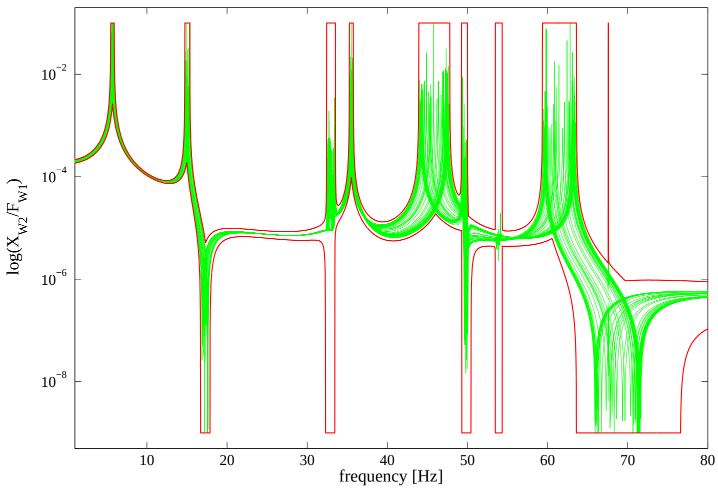


Figure 8: Undamped interval FRF (MRE) of the Garteur model, compared with a MCS with logarithmic sampling strategy

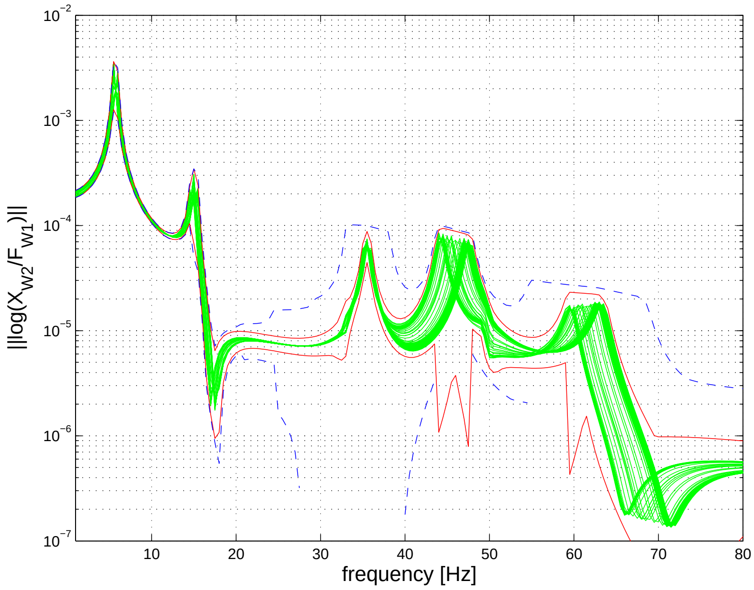
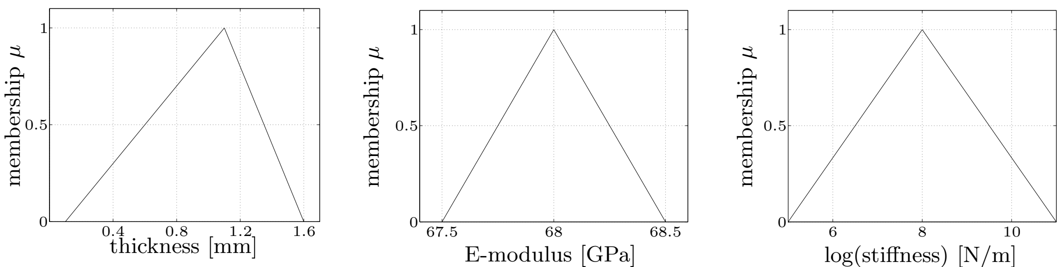


Figure 9: Damped interval FRF of the Garteur model: MR (dashed) versus MRE (solid) method

3.4 Fuzzy analysis of the Garteur aircraft

Finally, a fuzzy FRF analysis has been performed for the Garteur aircraft model. Figure 10 shows the triangular membership functions for the three uncertain parameters. For the thickness of the visco-elastic layer a non-symmetric membership function is used, while for the stiffness of the connection springs a triangular membership function of the logarithm of the stiffness is used. The support interval for this last uncertain parameter has been reduced, as a specific analysis with only the spring stiffness as uncertain parameter has revealed that the change of the stiffness outside the range of $10^6 - 10^{10}\ N/m$ has no influence on the modal parameter ranges $\langle \hat{k}_i \rangle$, $\langle \hat{m}_i \rangle$ and $\langle f_i \rangle$.



(a) Membership function for the thickness of the visco-elastic layer

(b) Membership function for the E-modulus of the wing material

(c) Membership function for the stiffness of the connection springs

Figure 10: Membership functions for the three uncertain parameters of the Garteur aircraft model

Figure 11 illustrates the damped fuzzy FRF resulting from an IFE envelope FRF solution procedure at 11 α-levels. The proportional damping coefficients have the same values as in the previous damped analysis. The figure gives a clear indication of the sensitivity of both the upper and the lower damped FRF bound with respect to the input uncertainty level.

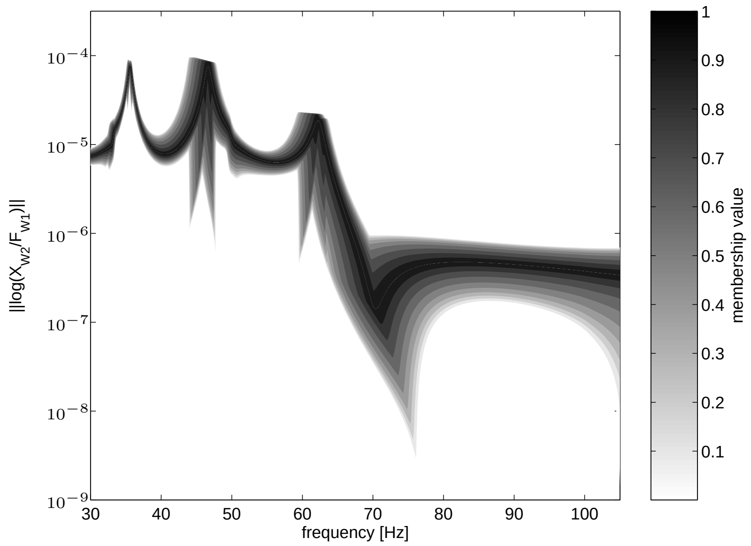


Figure 11: Fuzzy interval FRF of the Garteur model with three uncertain parameters

4 Case study: COROT Baffle Cover

4.1 Problem definition

In this section the influence of physical uncertainties on the dynamic behaviour of the telescope baffle cover (figure 12) for the COROT mission (Seismology of Stars: Convection and Rotation) is investigated. The cover prototype consists of a sandwich circular plate interfacing the baffle wall by means of two arms and compression springs [9]. The diameter of the cover is 832 mm, the thickness 22.7 mm. The FE model of this structure is built by Centre Spatial de Liège (B). It contains 964 nodes and 1042 elements. For the cover lid laminate plate elements are used. Both the hinge and the lock arm are modelled with beam elements and plate elements. Linear springs connect the nodes on the edge of the cover with nodes on the baffle which are clamped. In this way the springs model the boundary condition of simple support with no sliding assumed to the cover lid in contact with the baffle.

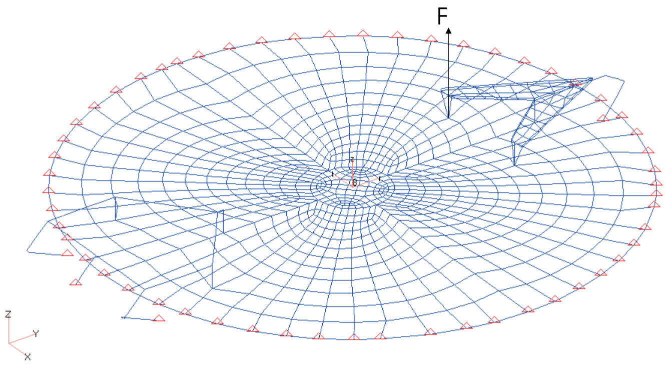


Figure 12: Finite element model of the cover with the hinge arm (left) and the lock arm (right) (model development by Centre Spatial de Liège)

Six uncertain parameters are identified, as listed in table 2. The first three uncertainties arise from an uncertainty of 5% on the stiffness of the compression springs. The uncertainty on the use of screws, washers and nuts is modelled by the range of the cover lid non-structural mass (NSM). Typical production tolerances are considered on the thickness of the sandwich skins and the honeycomb core.

uncertain parameter	nominal value	uncertainty interval
launch lock side arm preload stiffness	203 N/mm	[198 ; 208] N/mm
hinge side arm preload stiffness	99 N/mm	[96 ; 102] N/mm
hinge torsion stiffness	1000 Nmm	[975 ; 1025] Nmm
cover lid NSM	$3.7 \cdot 10^{-4}\ g/mm^2$	[2.96 ; 4.44]$\cdot 10^{-4}\ g/mm^2$
sandwich skin thickness	0.35 mm	[0.34 ; 0.36] mm
honeycomb thickness	22.0 mm	[21.8 ; 22.2] mm

Table 2: Uncertain parameters of the COROT baffle cover

4.2 Damped interval FRF analysis of the cover lid

The influence of the uncertain parameters on a damped direct FRF is investigated. Both the excitation and the response are considered in one of the two nodes of the cover lid that rigidly connect the cover lid with the lock arm, as indicated in figure 12. The first 20 modes are taken into account in the analysis, covering a frequency range up to 1300 Hz. The values of the proportional damping coefficients α_K and α_M are respectively $4 \cdot 10^{-6}$ and 25, resulting in damping ratios between 1.0% and 1.7% for all considered modes. Figure 13 illustrates the amplitude of the damped interval FRF, compared with 200 uniformly distributed Monte Carlo samples. The figure clearly proves the benefits of the MRE method with respect to the MR method, as the conservatism on the envelope FRF is substantially decreased for almost the entire frequency range. For the first 10 modes the correct eigenfrequency intervals used by the MRE method, are listed in table 3. Special attention was given to the first eigenfrequency, for which a strict design criterium of minimum 200 Hz is specified. Clearly, the first eigenfrequency interval is strongly affected by the defined uncertainties, and the design criterium is in all cases violated.

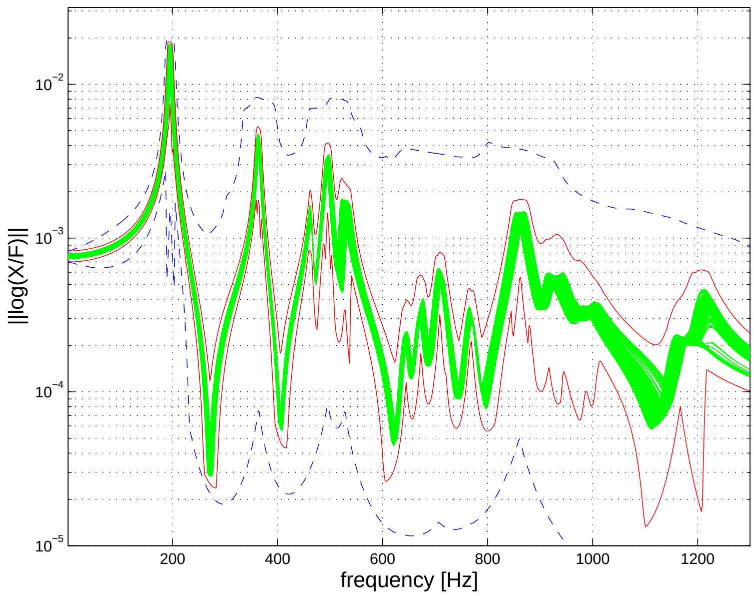


Figure 13: Amplitude of the damped interval direct FRF of the COROT baffle cover (MR: dashed, MRE: solid)

4.3 Fuzzy FRF analysis of the cover lid

A fuzzy FRF analysis of the COROT baffle cover is performed with enlarged uncertainty intervals. For all uncertain parameters, the uncertainty interval is taken twice as wide, except for the uncertainty interval on

Mode	Eigenfreq. interval [Hz]	Mode	Eigenfreq. interval [Hz]
1	[191.30 ; 199.66]	6	[490.59 ; 500.61]
2	[278.47 ; 282.71]	7	[517.89 ; 538.59]
3	[360.52 ; 367.48]	8	[638.71 ; 650.70]
4	[462.51 ; 462.59]	9	[667.19 ; 678.98]
5	[462.73 ; 465.25]	10	[702.54 ; 715.87]

Table 3: Eigenfrequency intervals of the first 10 modes of the COROT baffle cover

the non-structural mass of the cover lid that is 1.5 times the original interval. All membership functions are triangular and symmetric. Figure 14 presents the results of the fuzzy frequency response function analysis with 11 α-levels. The values of the proportional damping coefficients are the same as in the previous analysis. The figure gives a clear indication of the effect of the width of the input uncertainty intervals on the frequency response function envelopes.

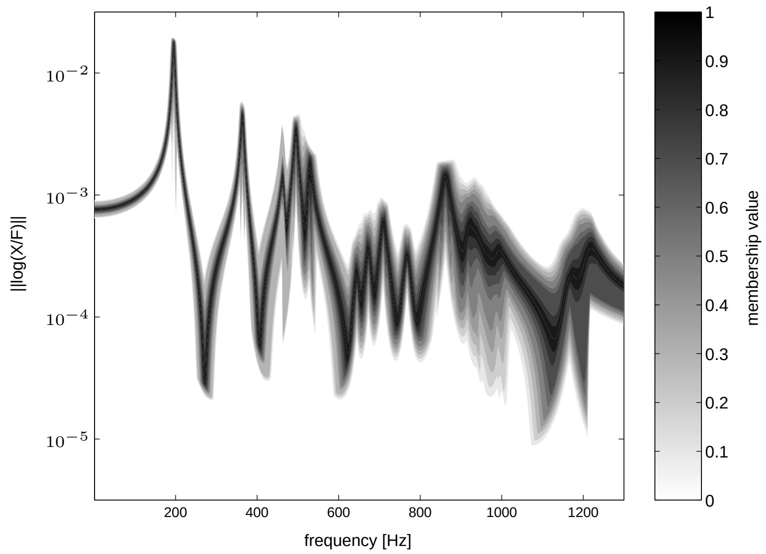


Figure 14: Fuzzy interval FRF of the cover lid with six uncertain parameters

5 Conclusions

This paper demonstrates the strength and applicability of the Interval and the Fuzzy Finite Element Method for the dynamic analysis of structures with uncertain parameters. Envelope and fuzzy FRFs are calculated for both undamped and damped structures and with different types of uncertainties.

The IFE method proves to be powerful and reliable: a conservative approximation of the upper and lower bound of an FRF is calculated in a single run. The method is capable of handling large uncertainty intervals and is less dependent on subjective input data. The IFE analysis of the Garteur benchmark problem clearly shows that a Monte Carlo Simulation can lead to false conclusions since the response data can be very dependent on the assumed probability density functions on the input uncertainty intervals.

The fuzzy FRF gives a clear indication of the variation of the worst-case response with the considered input uncertainty bounds, as it combines the results of several interval FRFs in one single graphical representation. The second example proves that industrial models can be run.

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