

## A fuzzy logic approach to the selection of cranes

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### Abstract

This paper presents a fuzzy logic approach to select the best crane type in a construction project from the main crane types, namely, mobile, tower and derrick cranes. Each factor of the project is classified as being *dynamic or static* according to whether the factor does or does not depend on the particular project. Linguistic information about the suitability of each crane type with respect to each factor of the project is translated into either fuzzy sets (for static factors) or fuzzy if–then rules (for dynamic factors). The fuzzy rules are then aggregated into a fuzzy relation between the space of factor property and the space of crane efficiency. In a particular project the experts describe the property as well as the relative importance of each factor. The rules are then fired using the max–min extension principle, and the resulting efficiencies are aggregated with their importance weights. The process identifies the best crane as the one with the highest expected overall efficiency. © 1999 Elsevier Science B.V. All rights reserved.

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### 1. Introduction

Cranes are the most useful and versatile piece of equipment on a vast majority of construction projects. They vary widely in configuration, capacity, mode of operation, intensity of utilization and cost. On a large project, a contractor may have an assortment of cranes for different purposes. Small mobile hydraulic cranes may be used for unloading materials from trucks and for small concrete placement operations, while larger crawler and tower cranes

may be used for the erection and removal of forms, the installation of steel reinforcement, the placement of concrete, and the erection of structural steel and precast concrete beams.

The selection of cranes is a central element of the life cycle of the project. Cranes must be selected to satisfy the requirements of the job. An appropriately selected crane contributes to the efficiency, timeliness, and profitability of the project. If the correct crane selection and configuration is not made, cost and safety implications might be created (Hanna, 1994). Decision to select a particular crane depends on many input parameters such as site conditions, cost, safety, and their variability. Many of these parameters are qualitative, and subjective judgements

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implicit in these terms cannot be directly incorporated into the classical decision making process. One way of selecting crane is achieved using fuzzy logic approach, since it converts the qualitative criteria into numerical measures for the optimum selection of a crane to aid the contractors.

## 2. Literature review

The literature review revealed some work regarding crane selection. Most of the work has been conducted using knowledge-based systems to select the most appropriate crane for building. Three existing applications of crane selection using a knowledge-based system have been published, namely CRANES, LOCRANE, and SELECTCRANE.

CRANES is a knowledge-based system designed to configure cranes based on site conditions (Gray and Little, 1985). It allows graphical location of multiple tower cranes on the site so as to optimize coverage and minimize cost. CRANES outputs the best technically sound crane and evaluates the least cost option to be selected. CRANES restricted its solution to large buildings which require one or more cranes (Warszawski, 1990).

LOCRANE is another knowledge-based system developed to assist the construction planner in selecting and locating a crane for a construction site (Warszawski, 1990). This system was developed using a commercially available shell (SAVOIR). The system asks the user to input all information related to building geometry and possible application for the proposed crane. The system outputs the most appropriate alternative from a set of cranes available.

SELECTCRANE (Hanna, 1994) is a knowledge-based expert system developed using EXSYS PROFESSIONAL shell, and can assist the contractor in selecting the type and configuration of cranes. The user provides the system with the expected weights, dimensions and lift radii of heaviest loads, wind speed, the rental charges and other project information. SELECTCRANE will then provide the user, the recommended type of crane. The system will facilitate the decision making process and serve as an advisor for field engineers.

Shapira and Glascock (1996) have described the culture of using mobile and tower cranes for building

construction. They demonstrated project characteristics and compared tower cranes and mobiles cranes to select the favored alternative.

The problem with the current literature of crane selection is that it does not address how multiple qualitative attributes are transformed into successful selection of a crane.

For example, to select the appropriate cranes, the two systems CRANES and LOCRANE used only quantitative factors such as site geometry and load characteristics.

## 3. General cranes classification

A crane is defined as a mechanism for lifting and lowering loads with a hoisting mechanism (Shapiro and Shapiro, 1991). On many construction sites a crane is needed to lift loads such as concrete skips, reinforcement, and formwork. As the lifting needs of the construction industry have increased and diversified, a large number of general and special purpose cranes have been designed and manufactured. These cranes fall into two categories, those employed in industry and those employed in construction. The most common types of cranes used in construction are mobile, tower, and derrick cranes.

### 3.1. Mobile cranes

A mobile crane is a crane capable of moving under its own power without being restricted to predetermined travel. Mobility is provided by mounting or integrating the crane with trucks or all terrain carriers or rough terrain carriers or by providing crawlers. Truck-mounted cranes have the advantage of being able to move under their own power to the construction site. Additionally, mobile cranes can move about the site, and are often able to do the work of several stationary units.

### 3.2. Tower cranes

The tower crane is a crane with a fixed vertical mast that is topped by a rotating boom and equipped

with a winch for hoisting and lowering loads (Dickie, 1990). Tower cranes are designed for situations which require operation in congested areas. Congestion may arise from the nature of the site or from the nature of the construction project. There is no limitation to the height of a high-rise building that can be constructed with a tower crane. The very high line speeds, up to 304.8 m/min, available with some models yield good production rates at any height. They provide a considerable horizontal working radius, yet require a small work space on the ground (Chalabi, 1989). Some machines can also operate in winds of up to 72.4 km/h, which is far above mobile crane wind limits.

The tower cranes are more economical only for longer term construction operations and higher lifting frequencies. This is because of the fairly extensive planning needed for installation, together with the transportation, erection and dismantling costs.

### 3.3. Derrick cranes

A derrick is a device for raising, lowering, and/or moving loads laterally. The simplest form of the derrick is called a Chicago boom and is usually installed by being mounted to building columns or frames during or after construction (Shapiro and Shapiro, 1991). This derrick arrangement (i.e., Chicago boom) becomes a guy derrick when it is mounted to a mast and a stiffleg derrick when it is fixed to a frame.

## 4. Fuzzy set concepts

### 4.1. Fuzzy sets

Since Zadeh (1965) introduced the concept of a fuzzy set, it has been employed in numerous areas. The concept is founded on the fact that some notions, though meaningful, may not be clearly defined. For example, the question whether a man is tall or not may not easily be answered by 'yes' or 'no'. Thus, the set of tall men is not clearly defined, and if we try to define it by arbitrarily setting a threshold height, above which a man is considered to be tall, we will end up with an artificial set that may

contain a man but does not contain a slightly shorter one.

This problem can be solved by using a fuzzy set to describe the notion of tall men. While for a classical set, any element either belongs to the set or does not belong to it, for a fuzzy set different elements belong to it with various strengths ranging from 0 to 1, where 0 means no membership and 1 means full membership.

In other words, while a classical set  $A$  living in a universe of discourse  $X$  (i.e.,  $A \subseteq X$ ) can be characterized by its characteristic function  $\chi_A: X \rightarrow \{0, 1\}$ , where for  $x \in X$ ,

$$\chi_A(x) = 1 \text{ iff } x \in A,$$

in fuzzy set theory,  $\chi_A$  is allowed to range over the real interval  $[0, 1]$ , with  $\chi_A(x)$  describing the strength of membership of the element  $x$  in the (fuzzy) set  $A$ .

In the literature of fuzzy logic, the characteristic function is usually denoted by  $\mu_A$  instead of  $\chi_A$ . However, in this paper we will just use the very name  $A$  of the fuzzy set to describe its characteristic function. After all, apart from its characteristic function we do not quite know what the fuzzy set really is.

Since a fuzzy set  $A$  is just a function, we can describe it by the set of ordered pairs  $\{(x, A(x)): x \in X\}$  (i.e., by its graph). If the universe  $X$  is a finite subset of the real line, and thus inherits its natural order, say  $X = \{x_1, x_2, \dots, x_n\}$ , where  $x_1 < x_2 < \dots < x_n$ , the fuzzy set  $A$  can be described by an ordered list of its images, i.e.

$$A = \langle A(x_1), A(x_2), \dots, A(x_n) \rangle,$$

or simply as a vector:

$$A = [A(x_1) \ A(x_2) \ \dots \ A(x_n)]$$

Likewise, if  $R$  is a fuzzy subset of the classical Cartesian product  $X \times Y$ , where both  $X$  and  $Y$  are finite subsets of the reals, say  $X = \{x_1 < x_2 < \dots < x_n\}$  and  $Y = \{y_1 < y_2 < \dots < y_m\}$ , then  $R$  may be described by a matrix:

$$R = \begin{bmatrix} R(x_1, y_1) & R(x_1, y_2) & \cdots & R(x_1, y_m) \\ R(x_2, y_1) & R(x_2, y_2) & \cdots & R(x_2, y_m) \\ \vdots & \vdots & \ddots & \vdots \\ R(x_n, y_1) & R(x_n, y_2) & \cdots & R(x_n, y_m) \end{bmatrix}$$

$R$  is then called a fuzzy relation between  $X$  and  $Y$ .

#### 4.2. Operations on fuzzy sets

Now let us consider two fuzzy sets  $A$  and  $B$  living in the same universe  $X$ . The following operations on them are defined.

##### Intersection:

$$(A \cap B)(x) = \min\{A(x), B(x)\}, \quad \text{for all } x \in X$$

##### Union:

$$(A \cup B)(x) = \max\{A(x), B(x)\}, \quad \text{for all } x \in X$$

##### Complement:

$$(A^c)(x) = 1 - A(x), \quad \text{for all } x \in X$$

Here the symbols  $\cap$ ,  $\cup$ , and  $c$  will stand for ‘and’, ‘or’, and ‘not’, respectively.

##### Product:

Consider two fuzzy sets  $A$  and  $B$  living in two different universes  $X$  and  $Y$ . The Cartesian product  $A \times B$ , living in the classical Cartesian product  $X \times Y$ , is defined by:

$$(A \times B)(x, y) = \min\{A(x), B(y)\},$$

for all  $(x, y) \in X \times Y$

In other words,  $A \times B$  is nothing but the intersection of the cylindrical extensions of  $A$  and  $B$  to the product universe  $X \times Y$ . As we will see, the Cartesian product  $A \times B$  will play an important role in interpreting the fuzzy if–then rules.

##### Max–min–composition:

Consider a fuzzy set  $A$  in a universe  $X$ , and a fuzzy relation  $R$  between  $X$  and  $Y$  (i.e., a fuzzy subset of the classical product  $X \times Y$ ). The max–min–composition (or simply composition)  $A \cdot R$  of  $A$  and  $R$  is a fuzzy subset of  $Y$  defined by:

$$(A \cdot R)(y) = \max_{x \in X} \min\{A(x), R(x, y)\},$$

$$\text{for all } y \in Y$$

Thus, using the vector and matrix representation of  $A$  and  $B$ , respectively, we can view the max–min–composition as a vector–matrix multiplication with the number addition and multiplication being replaced by the max and min operations, respectively.

We call a fuzzy set  $A$  *normal* if it attains the maximum value 1 at some  $x_0 \in X$ , i.e., if  $\max_{x \in X} A(x) = 1$ . If  $A$  is a normal fuzzy set, we can use the previous definitions to verify the following identity:

$$A \cdot (A \times B) = B$$

Starting with the left hand side we have:

$$\begin{aligned} A \cdot (A \times B)(y) &= \max_{x \in X} \min\{A(x), \min\{A(x), B(y)\}\} \\ &= \max_{x \in X} \min\{A(x), B(y)\} \\ &= \min\left\{\max_{x \in X} A(x), B(y)\right\} \\ &= \min\{1, B(y)\} = B(y), \end{aligned}$$

where we used the normality of  $A$  in the last step.

#### 4.3. Linguistic variables

The concept of a linguistic variable, introduced by Zadeh (1975a,b), is a mean to capture natural language expressions like ‘The weather is good’. In this example, ‘The weather’ is a name of a linguistic variable, which can take the values ‘good’, ‘bad’, ‘moderate’, ‘very bad’, ‘more or less good’, ‘not good and not bad’, etc.

Here, the values ‘good’, ‘bad’ and ‘moderate’ are called the basic values. The other nonbasic values can be generated from the basic ones by a context-free grammar. They may include the modulators ‘not’ for negation, ‘very’ as an intensifier and ‘more or less’ as a diluter. Also, they may include the connectives ‘and’ and ‘or’ with its logical meaning.

Zadeh used fuzzy sets to interpret those linguistic values. To make things discrete, we assume that each weather is rated by a number in the set  $X = \{0, 0.1, 0.2, \dots, 1\}$ . The linguistic value ‘good’ is then defined to be the fuzzy set of (good) rates living in the universe  $X$ . In this paper we will define the value ‘good’ as the fuzzy set:

$$\text{‘good’} = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0.2 \ 0.4 \ 0.6 \ 0.8 \ 1]$$

As an opposite of ‘good’, the value ‘bad’ is defined by:

$$\text{‘bad’} = [1 \ 0.8 \ 0.6 \ 0.4 \ 0.2 \ 0 \ 0 \ 0 \ 0 \ 0]$$

Also, we define the value ‘moderate’ by:

$$\text{‘moderate’} = [0 \ 0.2 \ 0.4 \ 0.6 \ 0.8 \ 1 \ 0.8 \ 0.6 \ 0.4 \ 0.2 \ 0]$$

Note that the three basic values, as defined above, have the following features:

1. Normality: (defined in Section 4.2) as each attains the maximum value 1 at some  $x_0 \in X$ .
2. Unimodality: each is nondecreasing for  $x \leq x_0$  and nonincreasing for  $x \geq x_0$ .
3. Complementarity: as functions they add up to the constant function 1.

The other (potentially) infinite nonbasic values are defined by giving a suitable semantics to the modulators and the connectives. In particular, the connectives ‘and’, ‘or’ and the modulator ‘not’ are given the semantics of the intersection, union, and complementation, respectively, of fuzzy sets. The reader can now verify that the linguistic value ‘not good and not bad’ is the same as the value ‘moderate’. Thus, in theory, ‘moderate’ is a redundant value, but including it will be convenient for the experts to express their ratings.

Also, the modulators ‘very’ and ‘more or less’ are given the following semantics:

$$\text{‘very’ value}(x) = (\text{value}(x))^2,$$

$$\text{‘more or less’ value}(x) = (\text{value}(x))^{(1/2)},$$

#### 4.4. Fuzzy if–then rules and approximate reasoning

Zadeh (1979) introduced the theory of approximate reasoning. This theory provides a powerful framework for reasoning in the face of imprecise and uncertain information. Central to this theory is the concept of the fuzzy if–then rule, which is a mathematical interpretation of the linguistic if–then rule. A linguistic if–then rule is a linguistic sentence of the form:

If  $X$  is  $A$ , then  $Y$  is  $B$ .

Here  $X$  and  $Y$  are linguistic variables, and  $A$  and  $B$  are the corresponding linguistic values. We identify the names of the variables  $X$  and  $Y$  with the universes, in which the fuzzy values  $A$  and  $B$  live.

For example, consider the rule:

If ‘the weather’ is ‘good’,

then ‘the project duration’ is ‘short’.

Here ‘the weather’ is a linguistic variable, and ‘good’ is a fuzzy subset of the universe of weather ratings, which we also use ‘the weather’ for its name. Likewise, ‘the project duration’ is the name of both a linguistic variable and the universe of durations, of which ‘short’ is a fuzzy subset.

Mamdani and Assilian (1975) proposed that such fuzzy if–then rules can be mathematically represented by the Cartesian product  $R$  of the fuzzy sets  $A$  and  $B$ , i.e.,  $R = A \times B$ . The identity  $A \cdot (A \times B) = B$  can then be viewed as the verification of the following logical inference rule (called ‘modus ponens’):

Given:

(1) If  $X$  is  $A$ , then  $Y$  is  $B$ .

(2)  $X$  is  $A$ .

Infer:  $Y$  is  $B$ .

Now, what do we know about  $Y$  if  $X$  is not  $A$ ? In general,  $Y$  can be anything. However, assume we have several if–then rules  $R_1, R_2, \dots, R_k$  between  $X$  and  $Y$ , where for  $1 \leq i \leq k$ , each rule  $R_i$  is of the form:

If  $X$  is  $A_i$ , then  $Y$  is  $B_i$ .

I.e.,  $R_i = A_i \times B_i$ . Also assume that the fuzzy values  $A_i$  for  $i = 1, \dots, k$  are complementary, i.e., (as functions)  $\sum_{i=1}^k A_i = 1$ .

We can then aggregate all these rules into one rule  $R$  defined by their union, i.e.:

$$R = \bigcup_{i=1}^k R_i$$

Now if  $X$  takes the value  $A$  (which is in general different from all the  $A_i$ s), assuming some consis-

tency conditions we can infer that  $Y$  takes the value  $B$  defined by:

$$B = A \cdot R$$

## 5. Fuzzy theoretic methodology for crane selection

The goal of this paper is to use fuzzy theoretic concepts to choose the best possible crane type for a particular project. There are several factors that affect the proper selection of a crane. We call a factor *static* if it is constant and does not depend on a particular project, e.g., the productivity and the safety of the crane used. If, however, the factor depends on the particular project, e.g., the operating clearance and the building height, we then call it *dynamic*.

For each of the factors we translate the experts' vague information about the suitability of crane types either to fuzzy sets if the factor is static or to fuzzy if-then rules if the factor is dynamic.

### 5.1. Factors affecting crane selection

Hanna (1992) provided a tabular knowledge-based format for crane selection, shown in Table 1. This table shows the various qualitative and quantitative features of the cranes with respect to the following five factors, divided into subfactors.

(1) Site Conditions (F1): this qualitative factor is the most important factor affecting the selection of a crane. It generally deals with problems such as:

- (a) Soil Stability and Ground Conditions (F1a)
- (b) Access Road Requirement and Site Accessibility (F1b)
- (c) Operating Clearance (F1c)

(2) Building Design (F2): the nature and the size of each project usually dictate the type of crane to be used. For example, when tower cranes are used, they are usually attached to the building every two or three floors. This might mean that the structure requires some modification in terms of additional structural bracing and exterior fasteners to allow for attachment of the crane to the structure. The main factors here are:

- (a) Building Height (F2a)
- (b) Project Duration (F2b)

(3) Economy (F3): the choice of the crane should be such that the unit to be used on site is the most economical one. Crane cost includes initial cost, operation cost, transportation cost, and installation cost. Three factors contribute to the cost of the crane, namely:

- (a) Cost of moving in, setting up, and moving out the crane (F3a)
- (b) Cost of renting or leasing the crane (F3b)
- (c) Productivity (F3c)

For example, tower cranes are the most productive because of the larger area they can cover. Mobile cranes are less productive, and derricks are the least productive type.

(4) Capability (F4): this qualitative feature mainly reflects the capability of the crane. This characterization may include:

- (a) Power Supply (F4a)
- (b) Load Lifting Frequency (F4b)
- (c) Operators Visibility (F4c)

(5) Safety (F5): crane safety is an extremely important issue in construction. Crane accidents due to overturning result in severe damage to the crane and other equipments and can endanger human lives. This qualitative feature mainly emphasizes

- (a) Initial Planning and Engineering (F5a) necessary to overcome the problems likely to be encountered during the implementation of a project.
- (b) Safety (F5b)

Note that the factors F1a, F1b, F1c, F2a, F2b, F3b, F4b, and F5a are dynamic factors, while the other factors are static.

### 5.2. Efficiencies with respect to static factors

The efficiency of each crane type with respect to static factors is simply translated to a linguistic value in the efficiency space. Thus, if  $F$  is a static factor, and  $T$  is a crane type, then from Table 1 we can read a statement of the form:

With respect to the factor  $F$ ,

the crane type  $T$  is  $\text{eff}(F, T)$

where  $\text{eff}(F, T)$  takes a linguistic value over the basic values 'good', 'fair' and 'poor', the fuzzy interpretation of which are respectively the same as of the values 'good', 'moderate' and 'bad', defined in Section 4.3.

Table 1  
Factors affecting the crane selection

Factors	Mobile cranes	Tower cranes	Derrick cranes
<i>F1. Site Conditions</i>			
(a) Soil Stability and Ground Conditions	Can operate in muddy terrain but requires good ground conditions	Can operate where ground conditions are poor	Used when ground conditions does not allow the use of mobile or tower crane
(b) Access road requirement and site accessibility	Requires access to and from lifting position	Preferred when poor accessibility prevails since tower cranes are brought to site disassembled	Minimum site accessibility and access road conditions are adequate since derricks are always dismantled into smaller units for transit
(c) Operating Clearance	Needs adequate operating clearance	Used when site is constricted or congested	Used when clearance is inadequate for the other units and sufficient space is unavailable for the erection of a tower foundation or base
<i>F2. Building Design</i>			
(a) Building Height	Adequate for all types of structures (up to 107 m)	Preferable for high-rise (over 107 m)	Preferable for high-rise and apartment buildings
(b) Project Duration	Used for shorter projects duration (less than 4 months)	Used when crane requirement is for long term in a specific location	Can be used for both long term and short term projects
<i>F3. Economy</i>			
(a) Cost of move in, setup, and move out	Not expensive	Expensive to set up because it requires a foundation and possibly bracing to the structure being erected	Cheaper than mobile and tower cranes
(b) Cost for rent	Usually cheaper if required for projects of short duration (less than 4 months)	Usually costs less for the long term duration (greater than 4 months)	Cheaper to rent and cheaper to buy
(c) Productivity	Not very productive	Much more productive than mobile units	Least productive
<i>F4. Capability</i>			
(a) Power Supply	Usually powered by diesel engines	Usually electrically powered (requires power supply)	Usually powered by diesel engines
(b) Load lifting Frequency	Used when lift frequency is sporadic	Preferred when lift frequency is high	Used if lift frequency is not a major consideration or no other viable alternative crane type exists
(c) Operators Visibility	Usually not good and fair for smaller units	Better	Depends on the location
<i>F5. Safety</i>			
(a) Initial Planning and Engineering	Details are not very much needed, only job site has to be examined for adequate crane maneuverability	Extensive planning is needed to provide the crane with appropriate foundation	Not very detailed
(b) Safety	Not considered to be very safe due to lack of safety devices or limited switches to prevent overloading	Considered to be very safe due to the presence of limit switches	Not considered to be safe

Table 2  
Efficiencies of crane types with respect to static factors

Static factors	Mobile cranes	Tower cranes	Derrick cranes
F3a	not poor	poor	good
F3c	not very good	very good	fair
F4a	good	fair	good
F4c	not good	good	fair
F5b	not very good	very good	not good

For example, taking  $F = F3c$  (Productivity), and  $T$  the tower crane, we can let  $\text{eff}(F, T) = \text{'very good'}$ , expressing the fact that the tower crane is very productive. Thus, the above statement can be read as:

With respect to 'Productivity',

the 'tower' crane is 'very good'.

Table 2 shows the function  $\text{eff}(F, T)$ , where  $F$  ranges over static factors, and  $T$  ranges over the three crane types.

### 5.3. Efficiencies with respect to dynamic factors

To deal with dynamic factors we have to work with fuzzy rules rather than fuzzy sets. So, for each dynamic factor  $F$ , property value  $P$ , and crane type  $T$ , we read from Table 1 a rule  $R_p(F, T)$  of the form:

If the factor  $F$  has property  $P$ ,

then the crane type  $T$  is  $\text{eff}(F, P, T)$ ,

where  $P$  takes the basic linguistic values 'good', 'moderate' and 'bad'; or 'long', 'medium' and 'short'; or 'high', 'medium' and 'low', etc. depending on the factor  $F$ , and  $\text{eff}(F, P, T)$  again takes linguistic values over the basic ones 'good', 'fair' and 'poor', where as before all of the fuzzy interpretations of those triplets are respectively the same as of the values 'good', 'moderate' and 'bad', defined in Section 4.3.

For example, taking  $F = F2b$  (Project Duration),  $P$  the value 'short', and  $T$  the mobile crane, we can let  $\text{eff}(F, P, T) = \text{'good'}$ , expressing the fact that the

mobile crane is suitable for short term projects. Thus, the above statement can be read as:

If the 'Project Duration' is 'short',

then the 'mobile' crane is 'good'.

Table 3 shows the function  $\text{eff}(F, P, T)$ , where  $F$  ranges over dynamic factors,  $P$  ranges over their property values, and  $T$  ranges over the three crane types.

From Section 4.4 we know that the rules  $R_p(F, P)$  can be represented by the Cartesian product  $P \times \text{eff}(F, P, T)$ , where both of  $P$  and  $\text{eff}(F, P, T)$  are now considered as fuzzy sets living in different universes  $X$  and  $Y$ , respectively, i.e., we have:

$$R_p(F, T)(x, y) = (P \times \text{eff}(F, P, T))(x, y) \\ = \min\{P(x), \text{eff}(F, P, T)(y)\}.$$

For each dynamic factor  $F$  and crane type  $T$ , the rules  $R_p(F, T)$  are aggregated into one rule  $R(F, T)$  defined by:

$$R(F, T) = \bigcup_P R_p(F, T) = \bigcup_P (P \times \text{eff}(F, P, T))$$

To illustrate this equation, let us calculate the rule  $R(F3b, \text{'mobile'})$  mapping the property space of ( $F3b = \text{Cost of rent}$ ), which is the project duration, to the efficiency space of the (mobile) crane.

$$R(F3b, \text{'mobile'}) \\ = R_{\text{'long'}}(F3b, \text{'mobile'}) \\ \cup R_{\text{'mod'}}(F3b, \text{'mobile'}) \\ \cup R_{\text{'short'}}(F3b, \text{'mobile'}) \\ = (\text{'long'} \times \text{eff}(\text{'long'})) \\ \cup (\text{'mod'} \times \text{eff}(\text{'mod'})) \\ \cup (\text{'short'} \times \text{eff}(\text{'short'})) \\ = (\text{'long'} \times \text{'poor'}) \cup (\text{'mod'} \times \text{'fair'}) \\ \cup (\text{'short'} \times \text{'good'}).$$



Table 3  
Efficiencies of crane types with respect to dynamic factors

Dynamic factors	Qualities	Mobile cranes	Tower cranes	Derrick cranes
F1a	good	good	good	good
	moderate	poor	good	good
	bad	poor	fair	good
F1b	good	good	good	good
	moderate	poor	good	good
	bad	poor	good	fair
F1c	adequate	good	good	good
	moderate	fair	good	good
	inadequate	poor	fair	good
F2a	high	poor	good	good
	moderate	fair	good	good
	low	good	fair	fair
F2b	long	poor	good	good
	moderate	fair	fair	good
	short	good	poor	good
F3b	long	poor	good	good
	moderate	fair	fair	good
	short	good	poor	good
F4b	high	poor	good	fair
	moderate	fair	fair	fair
	low	good	poor	fair
F5a	detailed	good	good	good
	moderate	good	fair	good
	low	fair	poor	fair

Thus:

$$\begin{aligned}
 R(F3b, 'mobile')(x, y) \\
 &= ('long'(x) \wedge 'poor'(y)) \\
 &\vee ('mod'(x) \wedge 'fair'(y)) \\
 &\vee ('short'(x) \wedge 'good'(y)),
 \end{aligned}$$

where the symbols  $\vee$  and  $\wedge$  denote the max and min operations, respectively.

Table 4  
Efficiency rule for the mobile crane with respect to cost of rent

Efficiency of the mobile crane											
Project duration	0	0	0	0	0	0	0.2	0.4	0.6	0.8	1
	0	0.2	0.2	0.2	0.2	0.2	0.2	0.4	0.6	0.8	0.8
	0	0.2	0.4	0.4	0.4	0.4	0.4	0.4	0.6	0.6	0.6
	0	0.2	0.4	0.6	0.6	0.6	0.6	0.6	0.4	0.4	0.4
	0	0.2	0.4	0.6	0.8	0.8	0.8	0.6	0.4	0.2	0.2
	0	0.2	0.4	0.6	0.8	1	0.8	0.6	0.4	0.2	0
	0.2	0.2	0.4	0.6	0.8	0.8	0.8	0.6	0.4	0.2	0
	0.4	0.4	0.4	0.6	0.6	0.6	0.6	0.6	0.4	0.2	0
	0.6	0.6	0.6	0.6	0.4	0.4	0.4	0.4	0.4	0.2	0
	0.8	0.8	0.6	0.4	0.2	0.2	0.2	0.2	0.2	0.2	0
	1	0.8	0.6	0.4	0.2	0	0	0	0	0	0

Table 4 shows the fuzzy rule  $R(F3b, 'mobile')$  in the matrix notation. For example, setting  $x = 0.8$ ,  $y = 0.1$ , we have by definition:

$$'long'(0.8) = 0.6, 'poor'(0.1) = 0.8,$$

$$'mod'(0.8) = 0.4, 'fair'(0.1) = 0.2,$$

and

$$'short'(0.8) = 'good'(0.1) = 0.0.$$

Thus we have:

$$R(F3b, 'mobile')(0.8, 0.1)$$

$$= (0.6 \wedge 0.8) \vee (0.4 \wedge 0.2) \vee (0.0 \wedge 0.0))$$

$$= 0.6 \vee 0.2 \vee 0.0 = 0.6,$$

which is the (0.8, 0.1) entry of Table 4.

Now, in a particular project, the property  $P(F)$  of each dynamic project factor  $F$  has to be described by an expert using linguistic values over the basic values. The efficiency  $\text{eff}(F, T)$  of each crane type  $T$  with respect to this dynamic factor is then computed by  $\text{eff}(F, T) = P(F) \cdot R(F, T)$ , i.e.

$$\text{eff}(F, T)(y) = P(F)(x) \cdot R(F, T)(x, y).$$

Of course, if  $F$  is static,  $\text{eff}(F, T)$  can be just read off Table 2.

#### 5.4. Aggregating efficiencies

For each crane type  $T$ , we need to aggregate the different efficiencies  $\text{eff}(F, T)$  into an overall efficiency  $\text{eff}(T)$ . To do this, we assign a weight  $w_F$  for each factor  $F$ , expressing the importance of factor  $F$  in our decision. Those weights may vary from some project to another and represent the balance between

Table 5

The important weights of factors

F1a	F1b	F1c	F2a	F2b	F3a	F3b	F3c	F4a	F4b	F4c	F5a	F5b
0.9	0.6	0.9	1.0	0.8	0.7	0.2	1.0	0.1	1.0	0.9	0.5	0.6

the cost and duration of the project and other factors like the quality and safety of the project.

The overall efficiency can then be defined as:

$$\text{eff}(T) = \bigcap_F ((w_F)^c \cup \text{eff}(F, T)),$$

i.e., as a function on the space  $\{0, 0.1, 0.2, \dots, 1\}$  we have:

$$\text{eff}(T)(x) = \min_F \max\{1 - w_F, \text{eff}(F, T)(x)\}.$$

Thus, if  $w_F$  is small,  $1 - w_F$  is large eliminating the effect of the fuzzy set  $\text{eff}(F, T)$  on the overall efficiency  $\text{eff}(T)$ . Note that the minimum operation indicates a worst case analysis.

Now to determine the best possible overall efficiency we use the Center-of-Gravity defuzzification method to calculate for each crane type  $T$  its efficiency center:

$$e_T = \frac{\sum_{x \in X} x \cdot \text{eff}(T)(x)}{\sum_{x \in X} \text{eff}(T)(x)}$$

Our final decision is then to choose the crane type with the highest efficiency center. This procedure is illustrated in the next section.

## 6. Case study

This section gives a case study as an example that we can apply our inference machinery on. The project is the construction of the Biochemistry building

at the University of Wisconsin-Madison. The major features of the project, viewed as describing the qualities of dynamic factors, were as follows:

(F1a) Ground conditions are ‘very good’.

(F1b) Accessibility to the site is ‘good’.

(F1c) Operating clearance is ‘adequate’.

(F2a) Building height is 24.18 m (‘low’).

(F2b/F3b) Project expected duration is 22 months (‘very long’).

(F4b) Lift frequency is ‘moderate’.

(F5a) Initial planning is ‘very detailed’.

The decision making had to be done to choose a suitable crane for the project out of the various alternative cranes available to the firm undertaking the project by considering the qualitative and quantitative factors of the present case study.

From the above features, we could read the property linguistic values for each of the dynamic factors. Using those values, the rules  $R(F, T)$  were fired to get the efficiencies of the three types of cranes with respect to the dynamic factors.

For example, the rule of Table 4 (for F3b and the mobile crane) was fired with the property value of:

‘very long’ = ‘long’,<sup>2</sup>

$$= [0 \ 0 \ 0 \ 0 \ 0 \ 0.2 \ 0.4 \ 0.6 \ 0.8 \ 1]^2$$

$$= [0 \ 0 \ 0 \ 0 \ 0 \ 0.04 \ 0.16 \ 0.36 \ 0.64 \ 1],$$

to get the fuzzy efficiency:

$\text{eff}(\text{F3b}, \text{‘mobile’})$

$$= [1 \ 0.8 \ 0.6 \ 0.4 \ 0.36 \ 0.36 \ 0.36 \ 0.36 \ 0.36 \ 0.2 \ 0]$$

Table 6

The aggregated overall efficiencies for the crane types

Mobile crane	0	0.2	0.36	0.36	0.2	0.1	0.2	0.36	0.36	0.2	0
Tower crane	0	0	0	0	0	0	0.04	0.16	0.3	0.3	0.3
Derrick crane	0	0.1	0.1	0.1	0.1	0.1	0.2	0.4	0.4	0.2	0

Table 7  
Efficiency centers for the crane types

Mobile crane	0.50
Tower crane	0.86
Derrick crane	0.62

To illustrate, let us calculate the 4th entry of this vector. Using the max–min composition we get:

$$\begin{aligned}
 \text{eff}(F3b, \text{'mobile'}) (0.3) &= (P(F3b) \cdot R(F3b, \text{'mobile'})) (0.3) \\
 &= \max_x \min \{ \text{'very long'}(x), \\
 &\quad R(F3b, \text{'mobile'})(x, 0.3) \} \\
 &= \max \{ 0, 0, 0, 0, 0, 0.04 \wedge 0.6, 0.16 \wedge 0.6, \\
 &\quad 0.36 \wedge 0.4, 0.64 \wedge 0.4, 1 \wedge 0.4 \} = 0.4
 \end{aligned}$$

Likewise, we got  $\text{eff}(F, T)$  for each dynamic factor  $F$  and each crane type  $T$ . Also, when the factor  $F$  is static, we just read  $\text{eff}(F, T)$  off Table 2. Since there are 13 project factors, this resulted into 13 efficiency vectors for each crane type.

To aggregate the different efficiencies for each crane type, we consulted the experts' opinion and got the factors' weights  $w_F$  shown in Table 5.

We then used the equation:

$$\text{eff}(T)(x) = \min_F \max \{ 1 - w_F, \text{eff}(F, T)(x) \},$$

to calculate the overall efficiency for each crane type (Table 6), where for each  $T$  we modified the efficiency vectors  $\text{eff}(F, T)$  by taking the maximum with  $(1 - \text{the factor weight } w_F)$  and took the minimum of all the 13 resulting vectors.

Finally, we calculated the efficiency center  $e_T$  for each  $T$ . For example:

$$\begin{aligned}
 e_{\text{'tower'}} &= \frac{\sum_{x \in X} x \cdot \text{eff}(\text{'tower'})(x)}{\sum_{x \in X} \text{eff}(\text{'tower'})(x)} \\
 &= \frac{0 + 0.6 + 0.04 + 0.7 \cdot 0.16 + 0.8 \cdot 0.3 + 0.9 \cdot 0.3 + 1 \cdot 0.3}{0 + 0.04 + 0.16 + 0.3 + 0.3 + 0.3} \\
 &= 0.86
 \end{aligned}$$

Table 7 shows the values of  $e_T$  for the three crane types, from which we can see that the best crane for the project is the tower crane with highest degree of

support of 0.86. Therefore, the tower crane should be selected for this project. The next best crane is the Derrick crane with a degree of support 0.62 and the worst crane is the mobile crane with a degree of support 0.50.

## 7. Conclusions

This paper presents an organized procedure for the selection of cranes based on a fuzzy logic approach. Using the concept of linguistic variables, the experts' vague knowledge of the suitability of crane types in various project conditions is translated into fuzzy sets and fuzzy rules.

In a particular project an expert has to describe the project conditions in words and a fuzzy inference engine can quantitatively identifies the best crane type for this project.

The inference engine can and should be carried out by a user-friendly computer software that asks the user some questions about the project considered and outputs the best decision about selecting the crane type.

The above application of fuzzy set theory is a step toward the elimination of bias or prejudice in the judgment of an expert, since the steps leading to the judgment are made explicit. This also helps to uncover any gap in the expert's thinking, for example in regard to qualitative factors which may not have been considered.

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