

A fuzzy compromise approach to water resource systems planning under uncertainty

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Abstract

A fuzzy compromise approach to decision analysis is described within the context of water resource systems planning under uncertainty. The approach allows various sources of uncertainty and is intended to provide a flexible form of group decision support. The example compares the ELECTRE method with the fuzzy compromise approach. The comparison is intended to demonstrate the benefits of adopting a multicriteria decision analysis technique which presents subjectivity within its proper context while maintaining an intuitive and transparent technique for ranking alternatives. The fuzzy compromise approach allows a family of possible conditions to be reviewed, and supports group decisions through fuzzy sets designed to reflect collective opinions and conflicting judgements. Ranking of alternatives is accomplished with fuzzy ranking measures designed to illustrate the effect of risk tolerance differences among decision makers. Two distinct ranking measures are used – a centroid measure, and a fuzzy comparison measure based on a fuzzy goal. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

Multicriteria decision analysis (MCDA), has been moving from optimization methods to more interactive decision support tools. Some research areas of interest have been identified by Dyer et al. [12] as:

Sensitivity analysis and the incorporation of vague or imprecise judgements of preferences ... Development of improved interactive software for multicriterion decision support systems.

Uncertainty is a source of complexity in decision making which can be found in many forms. This includes uncertainty in model assumptions, and uncertainty

in data or parameter values. There may also be uncertainty in the interpretation of results. While some uncertainties can be modeled as stochastic variables in a Monte Carlo simulation for example, other forms of uncertainty may simply be vague or imprecise.

Traditional techniques for evaluating discrete alternatives such as ELECTRE [3], AHP [25], Compromise Programming [33, 34], and others normally do not consider subjective uncertainties when procuring criteria values. In fact, decision makers are often tempted to simplify the problem by discounting subjective decision criteria simply because they are subjective. Sensitivity analysis can be used to evaluate subjective uncertainty (such as uncertain preferences and ignorance), but this form of sensitivity analysis can be inadequate at expressing both

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the probabilistic and imprecise forms of uncertainty. There have been efforts to extend traditional techniques, such as PROTRADE [15], which could be described as a stochastic compromise programming technique. A remaining problem is that not all uncertainties easily fit the probabilistic classification.

Fuzzy decision analysis techniques have addressed some uncertainties, such as the vagueness and conflict of preferences common in group decisions [5,29,27,13, and others], and at least one effort has been made to combine decision problems with both stochastic and fuzzy components [22]. Application, however, demands some level of intuitiveness for decision makers, and encourages interaction or experimentation such as that found in Nishizaki and Seo [23]. Leung [20] and many others have explored fuzzy decision making environments. This is not always so intuitive to many people involved in practical decisions because the decision space may be some abstract measure of fuzziness, as opposed to a more tangible measure of alternative performance. The alternatives to be evaluated are rarely fuzzy. However, their perceived performance may be fuzzy.

An intuitive, and relatively interactive, decision tool for discrete alternative selection, under various forms of uncertainty, would be a valuable tool in decision analysis – especially for applications with groups of decision makers. This paper explores the application of fuzzy sets in conjunction with a standard MCDA technique, compromise programming.

1.1. Displaced ideals

Multicriteria decision analysis techniques can approach the analysis of multiobjective problems in a number of ways. They are generally based on: out-ranking relationships; distance metrics; and utility theory. The concept of the displaced ideal was used by Zeleny [33, 34] to form compromise programming, a multicriteria technique which resolves criteria into a commensurable, unitless, distance metric measured from an ideal point (for each alternative). The result is a direct ranking (strong ordering) of alternatives, valid for the selected weights and the chosen form of distance measurement. The following can be used to calculate a discrete compromise

programming distance metric (L), otherwise known as the Minkowski distance

$$L = \left[\sum_i w_i^p \left(\frac{f_i^* - f_i}{f_i^* - f_i^-} \right)^p \right]^{1/p}. \quad (1)$$

f_i is the value for criteria i ; f_i^* , f_i^- are the positive and negative ideal values for criteria i , respectively; w_i is a weight, indicating relative importance of a criteria; L is the distance from an ideal solution; and p is the distance metric exponent. It is assumed that $\sum w_i = 1$. Typically, the Euclidean distance ($p = 2$) is used to penalize large deviations from the ideal. However, the exponent can also carry an economic interpretation. The Hamming distance ($p = 1$) results in a case of perfect compensation between criteria. For the Chebychev distance ($p = \infty$), there is no compensation among criteria – the largest deviation from the ideal dominates the assessment.

Many of the traditional MCDA techniques, including compromise programming, attempt to preserve some level of transparency to problems. This is a valuable property in decision analysis tools. However, only a limited amount of information is typically utilized. Extensive sensitivity analysis is necessary to recommend a robust alternative. The marriage of a transparent technique such as compromise programming with fuzzy sets is an example of a hybrid decision making tool available to planners.

1.2. Existing applications using fuzzy ideals

The concept of a fuzzy displaced ideal was probably born with the comment by Carlsson [7]:

Zeleny's theory of the displaced ideal would ... be very useful in a fuzzy adaptation.

Lai [18] used distance metrics to reduce a multiobjective problem to a 2 objective problem. They are to (1) minimize the distance to an ideal solution, and (2) maximize the distance to the worst solution. Membership functions are assigned to the ideal and worst solutions to fuzzify the problem, weights are used to resolve the two remaining objectives. Decisions are reached by formulating the problem as a fuzzy linear programming problem, and solved using the standard Bellman and Zadeh [2] approach.

An example of decision analysis with fuzzy composite programming can be found in Bardossy and

Duckstein [1], where a MCDA problem is evaluated using a distance metric with one of the criteria being qualitative and subjective. A codebook, a set of membership functions used to describe categories of subjective information, is established which translates a cardinal scale selection of the subjective criteria into a fuzzy set. Application of the extension principle to combine the single fuzzy criteria with the other, quantitative, criteria is demonstrated graphically by Bardossy and Duckstein [1] and a similar paper by Lee et al. [19] who provide examples of using a fuzzy displaced ideal.

1.3. Fuzzy arithmetic operations

The theory of fuzzy sets, initiated by Zadeh [32], defines a fuzzy set, A , by degree of membership, $\mu_A(x)$, over a universe of discourse, X , as

$$\mu_A : X \rightarrow [0, 1]. \quad (2)$$

Many operations on fuzzy sets use connectives called triangular norms: t-norms; and s-norms. t models the intersection operator in set theory. Likewise, s models the union operator. The min and max operators are commonly used for t and s respectively, although the family of valid triangular norms is very large. Composition operators are also used to connect fuzzy sets. They include sup and inf. The sup operation is the supremum or maximum of its membership function over a universe of discourse. Likewise, inf refers to the minimum. The combination of composition operators and connectives produces a powerful framework for many operations. sup-t compositions (max–min), and inf-s compositions (min–max) are examples used in fuzzy operations. There are many texts on fuzzy sets, including Dubois and Prade [9], Zimmerman [35], Mares [21], Sakawa [26], and Pedrycz [24].

Fuzzy arithmetic is made possible by Zadeh's extension principle, which states that if $f : X \rightarrow Y$ is a function and A is a fuzzy set in X then $f(A)$ is defined as

$$\mu_{f(A)}(y) = \sup_{x \in X; f(x)=y} \mu_A(x), \quad (3)$$

where $f : X \rightarrow Y$, $y \in Y$.

From this extension principle, fuzzy arithmetic operations such as addition, subtraction, multiplication, division, and exponentiation can be described.

2. Fuzzy compromise approach

2.1. Fuzzy distance metrics

The transformation of a distance metric to a fuzzy set can be accomplished by changing all inputs from crisp to fuzzy and applying the fuzzy extension principle. Measurement of distance between an ideal solution and the perceived performance of an alternative can no longer be given a single value, because many distances are at least somewhat valid. Choosing the shortest distance to the ideal is no longer a straightforward ordering of distance metrics, because of overlaps and varying degrees of possibility. The resulting fuzzy distance metric, as the following approach will attempt to demonstrate, contains a great amount of additional information about the consequences of a decision and the effect of subjectivity.

The process of generating input fuzzy sets is not trivial but there are many available techniques for encoding information and knowledge in a fuzzy set. The process of generating appropriate fuzzy sets, accommodating available data, heuristic knowledge, or conflicting opinions, should be capable of presenting information accurately. This topic is not addressed in this paper. Appropriate techniques for fuzzy set generation should be considered to be specific to the type of problem being addressed, the availability of different types of information, and the presence of different decision makers (remaining discussions on properties of fuzzy distance metrics are for maximization problems ... in other words, larger values for criteria are assumed to be better than smaller values, and the ideal solution tends to have larger values than the alternatives).

Fuzzification of criteria values is probably the most obvious use of fuzzy sets. There is a long history of published articles demonstrating decision problems with qualitative or subjective criteria. Fuzzy sets are able to capture many qualities of relative differences in perceived value of criteria among alternatives. Placement of modal values, along with curvature and skew of membership functions can allow decision makers

to retain what they consider degree of possibility for subjective criteria values.

Selection of criteria weights is an aspect which is typically subjective, usually with a rating on an interval scale. As a subjective value, criteria weights may be more accurately represented by fuzzy sets. Generating these fuzzy sets is also a subjective element. It may be difficult to get honest opinions about degree of fuzziness from a decision maker. It might actually be more straightforward to generate fuzzy sets for weights when multiple decision makers are involved! Then, at least, voting methods and other techniques are available for producing a composite, collective, opinion. Regardless, more information can be provided about valid weights from fuzzy sets than from crisp weights.

Ideal values for a criteria may also be very subjective. Certainly, the ideal solution may be significantly more subjective than the perceived performance of an alternative. For example, if profit is a criteria, what is the ideal amount of profit?

The distance metric exponent, p , is likely the most imprecise or vague element of distance metric calculation. There is no single acceptable value of p for every problem, and it can be easily misunderstood. Also, it is not related to problem information in any way except by providing parametric control over interpretation of distance. Fuzzification of the distance metric exponent, p , can take many forms but in a practical way it might be defined by a triangular fuzzy set with a mode of 2. Larger or smaller (fuzzy) values of p may also be valid but fuzzy exponential operations for large exponents results in difficult interpretation of the distance metric due to a large degree of fuzziness (range of possible values).

The benefits of adopting the general fuzzy approach to compromise programming are many. Probably the most obvious is the incorporation of subjective uncertainty. Expressing possibility values with fuzzy inputs allows experience to play a significant role in the expression of input information. The shape of a fuzzy set expresses the experience or the interpretation of a decision maker. Conflicting data or preferences can also be easily expressed using multimodal fuzzy sets, making the fuzzy compromise approach a candidate for application to group decision making.

Nonfuzzy distance-based techniques measure the distance from an ideal point, where the ideal alterna-

tive would result in a distance metric, $L : X \rightarrow \{0\}$. In a fuzzy compromise approach, the distance is fuzzy, such that it represents all of the possible valid evaluations, indicated by the degree of possibility or membership value. Alternatives which tend to be closer to the ideal may be selected. This fuzzified distance metric is analogous to a sensitivity analysis for the nonfuzzy case.

2.2. Selecting acceptable alternatives

The fuzzy compromise approach is further able to support decision analysis exercises by ranking the alternatives according to perceived performance.

As an attempt to standardize a procedure for judging which L is best among a set of alternatives, desirable properties can be defined. The most important properties are:

1. Possibility values tend to be close to the ideal, $x = 0$, distance.
2. Possibility values have a relatively small degree of fuzziness.

Some other performance indicators might favour modal values close to the ideal, or possibility values which tend to be far from poor solutions.

An aspect of comparing fuzzy distance metrics is the possible occurrence of points of indifference between fuzzy sets. If the rising limb of a fuzzy distance metric (the arc which is closest to the ideal distance of zero) were to intersect the rising limb of another fuzzy distance metric (i.e. equal membership values for 2 or more alternatives at some distance from the ideal) – a point of indifference would exist. This concept of indifference may vary. Interpretation of “best” depends on which side of the indifference point is considered to be interesting in the evaluation of comparative best. In the special case where the modes are equal, while the rising and falling limbs vary drastically, selection of the mode as the point of interest in ranking the sets will result in equal ranking. Awareness of these indifference points may not be directly evident when ranking alternatives, but indifference points (depending on their location) cause ranking orders to change when different levels of risk tolerance are specified. The ability to express risk tolerance will be explored further.

Relative performance of alternatives may be visually intuitive when looking at the fuzzy distance

metrics but in cases where many alternatives display similar characteristics, it may be impractical or even undesirable to make a visual selection. A method for ranking alternatives can automate many of the visual interpretations – and create reproducible results. A ranking measure may also be useful in supplying additional insight into decision maker preferences, such as distinguishing relative risk tolerance levels.

Selection of a ranking method is subjective and specific to the form of problem and the fuzzy set characteristics which are desirable. A taxonomic examination of existing methods can be found in [6]. There exists an assortment of methods ranging from horizontal and vertical evaluation of fuzzy sets, to comparative methods. Some of these methods may independently evaluate fuzzy sets, while others use competition to choose among a selection list. Horizontal methods are related to the practice of defuzzifying a fuzzy set by testing for a range of validity at a threshold membership value. Vertical methods tend to use the area under a membership function as the basis for evaluation, such as center of gravity. The comparative methods introduce other artificial criteria for judging the performance of a fuzzy set, such as a fuzzy goal. Horizontal methods are not explored in this paper. The following selected methods are vertical and comparative, respectively. A discussion of their properties follows.

2.3. Weighted center of gravity measure

Given the desirable properties of a ranking method for the fuzzy compromise approach, one technique which may qualify as a candidate is the centroid method, as discussed by Yager [31] in terms of its ability to rank fuzzy sets on the range $[0, 1]$. The centroid method appears to be consistent in its ability to distinguish between most fuzzy sets. One weakness, however, is that the centroid method is unable to distinguish between fuzzy sets which may have the same centroid, but greatly differ in their degree of fuzziness. The weakness can be somewhat alleviated by the use of weighting. If high membership values are weighted higher than low membership values, there is some indication of degree of fuzziness when comparing rankings from different weighting

schemes. However, in the case of symmetrical fuzzy sets, weighting schemes will not distinguish relative fuzziness.

A weighted centroid ranking measure (WCoG) can be defined as follows:

$$\text{WCoG} = \frac{\int g(x)\mu(x)^q dx}{\int \mu(x)^q dx}, \quad (4)$$

where $g(x)$ is the horizontal component of the area under scrutiny, and $\mu(x)$ are membership function values. In practice, WCoG can be calculated in discrete intervals across the valid universe of discourse for L . WCoG, allows parametric control in the form of the exponent, q . This control mechanism allows ranking for cases ranging from the modal value $q = (\infty)$ – which is analogous to an expected case or most likely scenario, to the center of gravity ($q = 1$) – which signifies some concern over extreme cases. In this way, there exists a family of valid ranking values (which may or may not change too significantly). The final selection of appropriate rankings is dependent on the level of risk tolerance from the decision maker.

Ranking of fuzzy sets with WCoG is by ordering from the smallest to the largest value. The smaller the WCoG measure, the closer the center of gravity of the fuzzy set to the origin. As a vertical method of ranking, WCoG values act on the set of positive real numbers.

2.4. Fuzzy acceptability measure

Another ranking method which shows promise is a fuzzy acceptability measure, Acc, based on [17]. Kim and Park derive a comparative ranking measure, which builds on the method of Jain [16] using the possibility measure (Poss) to signify an optimistic perspective, and supplements it with a pessimistic view similar to the necessity measure (Nec).

The possibility measure, formally known as the *degree of overlap* between fuzzy sets, can be described as the possibility of something good happening, and can be stated mathematically as:

$$\text{Poss}(G, L) = \sup_{x \in R} T(\mu_G(x)\mu_L(x)), \quad (5)$$

where T is a t-norm, L is the fuzzy set defined by $L: X \rightarrow [0, 1]$ and G is a fuzzy goal, defined by $G: X \rightarrow [0, 1]$.

The necessity measure gives a pessimistic view, formally known as the *degree of containment*, described as the necessity for ensuring something bad does not happen. Nec can be expressed mathematically as

$$\text{Nec}(G, L) = \inf_{x \in R} [\mu_G(x) s \bar{\mu}_L(x)], \quad (6)$$

where $\bar{\mu}_L$ is the complement ($1 - \mu_L$) membership value.

These two measures, Poss and Nec, can be combined to form an acceptability measure (Acc):

$$\text{Acc} = \alpha \text{Poss}(G, L) + (1 - \alpha) \text{Nec}(G, L). \quad (7)$$

Parametric control with the acceptability measure (Acc) is accomplished with the α weight and the choice of the fuzzy goal, G . The α weight controls the degree of optimism and degree of pessimism, and indicates (an overall) level of risk tolerance. The choice of a fuzzy goal is not so intuitive. It should normally include the entire range of L , but it can be adjusted to a smaller range for the purpose of either exploring shape characteristics of L , or to provide an indication of necessary stringency. By decreasing the range of G , the decision maker becomes more stringent in that the method rewards higher membership values closer to the ideal. At the extreme degree of stringency, G becomes a nonfuzzy number requiring the alternatives be ideal. As a function, G may be linear, but can also adapt to place more emphasis or less emphasis near the best value ($x = 0$ for distance metrics).

Ranking of fuzzy sets using Acc is accomplished by ordering values from largest to smallest. That is, the fuzzy set with the greatest Acc is most acceptable. Acc values are restricted on the range $[0, 1]$ since both the Poss and Nec measures act on $[0, 1]$, and α reduces the range of possible values by a factor of 2.

2.5. Comparison of ranking methods

Comparison of ranking methods WCoG and Acc with those reviewed by Bortolan and Degani [6], suggested both to be superior to the methods given in the review, given the desirable properties of L . The problem with many available methods is that, although most are able to correctly identify the best fuzzy set, they may not be capable of distinguishing both degree of dominance and provide an ordinal ranking for more than 2 fuzzy sets. Many methods supplied rank-

ing values, for example, as $\{1, 0, 0\}$ for 3 fuzzy sets. Very little decision information is returned by those methods. Relative dominance among fuzzy sets is an important aspect for distinguishing between fuzzy distance metrics. Information of this type is provided by both WCoG and Acc.

WCoG is conceptually simple and visually intuitive. It's weakness in discerning between fuzzy sets with the same shape and modal value, yet with different degrees of fuzziness is offset, somewhat, by the unlikely event of having distance metrics with those properties. Fuzzy distance metrics may have very similar shapes considering that all alternatives are evaluated for the same fuzzy definition of p . They may also have similar modes, depending on criteria values. Degree of fuzziness, or at least some discrepancy in shape, provides the means by which the weighting parameter, q , is able to distinguish indifference points. In general, though, interpretation of indifference points is not usually very sensitive to the choice in q .

Acc provides more comprehensive, and possibly more relevant, parametric control over the interpretation of results. Acc is able to explore the "surface" of fuzzy distance metrics with a meaningful interpretation of the variables used for parametric control (α, G). However, the parameters for the Acc measure are difficult to justify if some combination is used to recommend an alternative. The appropriate use of Acc is strictly to determine sensitivity, if any, of alternative rankings to different attitudes displayed by a decision maker.

Regardless of the combination of characteristics for fuzzy distance metrics, both the WCoG and Acc methods produced similar results which corresponded with visual interpretation of fuzzy distance metrics. Both methods satisfy the desirable properties for ranking fuzzy distance metrics. Both may prove to be useful in a decision making problem with multiple alternatives. Choosing just one of these methods, or a completely different method (of which there are many), should be dependent on the desirable ranking properties of the given problem. In some cases, it may be advantageous to use more than one method as a form of verification.

2.6. Water resource management

The following example is taken from the field of water resources planning. It is a multicriteria deci-

sion problem that has been addressed using standard MCDA techniques to select a most desirable water management system alternative, either as a best compromise or as a robust choice. The example redefines the problem in fuzzy terms to demonstrate the added value of adopting a fuzzy compromise approach.

The Tisza River basin in Hungary was studied by David and Duckstein [8] for the purpose of comparing alternative water resource systems for long-range goals. They attempt to follow a cost-effectiveness methodology to choose from five alternatives, but many of the 12 criteria are subjective. Eight criteria are subjective, and have linguistic evaluations assigned to them. Six of these subjective criteria are considered on a scale with five linguistic options {*excellent*, *very good*, *good*, *fair*, *bad*}. Two criteria are judged by different linguistic scales {*very easy*, *easy*, *fairly difficult*, *difficult*}, and {*very sensitive*, *sensitive*, *fairly sensitive*, *not sensitive*}. David and Duckstein [8] provide numeric differences along an interval scale are given so that a discordance index can be calculated for the ELECTRE method.

David and Duckstein [8] provide criteria weights to calculate the concordance index of ELECTRE. Weights were supplied from the set of {1, 2}. The technique used by ELECTRE somewhat alters the weighting issues in its use of a concordance index, and weights are not needed to calculate a discordance index, but it is not known what effect uncertainty in the weights has on assessing alternative tradeoffs.

As a conclusion, David and Duckstein [8] suggest that a mix of systems I and II would be appropriate – since they appear to somewhat dominate the other alternatives and show no overall domination over each other. Duckstein and Opricovic [11] reached similar conclusions for the same system, using a different artificial scaling for subjective criteria. A useful improvement to evaluating water resource systems such as the Tisza River may be to treat uncertainties as fuzzy.

Fuzzy definitions consist of a “codebook” of linguistic terms used to assess subjective criteria. Quantitative criteria can also be fuzzified, but are generally less fuzzy. Other fuzzy inputs include the expected ranges of criteria values, and the form of distance metric (degree of compensation) among criteria for different alternatives. Criteria weights are fuzzified on a range of [0, 1] by simple scaling of the weights as $\{1, 2\} \rightarrow \{0.3\tilde{3}, 0.6\tilde{6}\}$. All fuzzy inputs are treated in

Table 1

Tisza River alternative rankings from WCoG and Acc measures

Rank	Alt	WCoG ($p = 1$)	Acc ($G: [0, 8], \alpha = 0.5$)
1	1	1.49	0.81
2	2	1.59	0.80
3	4	2.38	0.75
4	3	2.83	7.72
5	5	2.85	0.71

a simple form, exclusively normal and unimodal, with either triangular or one-sided membership functions.

Assuming a fuzzy definition for the distance metric exponent (p), and knowing the form of criteria values and weights to be triangular, the resulting fuzzy distance metrics (L_i) possess the characteristic shape (Fig. 1) of near linearity below the mode, and a somewhat quadratic polynomial curvature above the mode. Although the degree of fuzziness is similar for all five alternatives, some of the alternatives are clearly inferior.

Rankings alternatives is reasonably straightforward because of the simplicity of the shapes, and similarity in degree of fuzziness. Both WCoG and Acc measures produced expected results (Table 1). Rankings are insensitive to changes in levels of risk aversion, as would be expected from visual inspection. The resulting ranks confirm the findings of David and Duckstein [8], that alternatives I and II are dominant.

In a live case study with multiple decision makers, there are opportunities for a group emphasis to collectively adjust the fuzzy inputs. The rankings may change considerably because the values defined for this experiment are predominantly simple triangular membership functions, given the form of nonfuzzy input data. Adjustments in relative fuzziness, and the presence of conflicting opinions will significantly alter the shape of the fuzzy distance metric, particularly within the vicinity of the modal value.

3. Conclusions

The fuzzy compromise approach may prove to be very useful to water resource systems planners. The idea of a displaced ideal is a relatively simple and

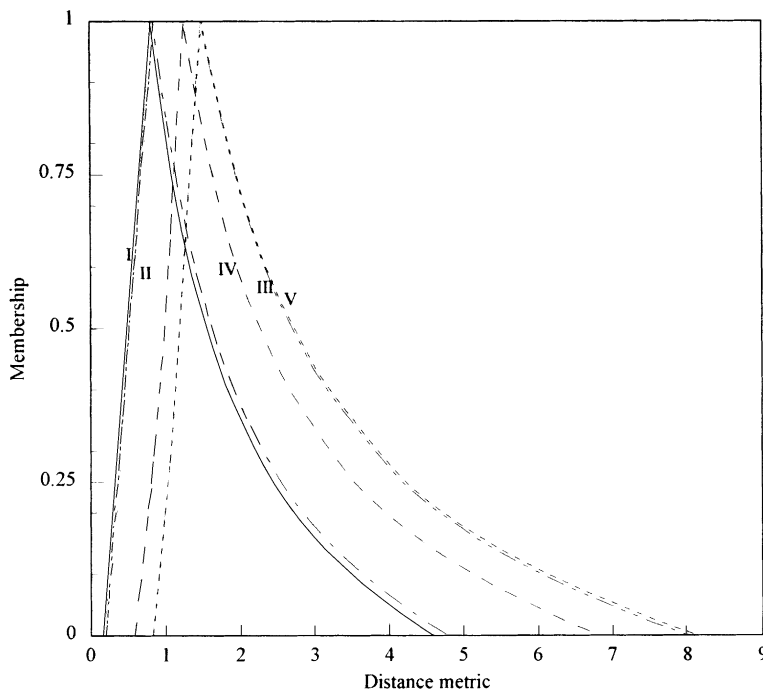


Fig. 1. Distance metrics for David and Duckstein [8]

intuitive concept used in MCDA, and it easily adapts to fuzzy inputs. Intuitiveness and simplicity are two properties that are seen as extremely important if a decision support tool is to be realized for multiple decision makers.

Fuzzy distance metrics can be inspected visually with a great degree of accuracy and minimal loss of information. This allows decision makers to make a qualitative evaluation of alternatives. Each decision maker may interpret the results differently, though, as a function of their relative degree of risk aversion. Subjective interpretation depending on risk tolerance can be modeled in a ranking measure. The centroid measure (WCoG) discussed in this paper is easily understood, but the acceptability measure (Acc) allows parametric control more specifically designed to model level of risk aversion from decision makers.

A fuzzy compromise approach has a number of comparative advantages over traditional (nonfuzzy) MCDA techniques. The most important is the direct, and often intuitive incorporation of vague and imprecise forms of uncertainty to the decision making

process. In real decisions, many of the criteria are subjective in nature. By their very nature, subjective criteria are fuzzy. By allowing a degree of fuzziness, more realism is added to the evaluation without compromising on the technique's ability to disseminate alternative preferences. Similar observations can be made about criteria weights and the decision maker's interpretation of degree of compensation between criteria (p). All of which possess sufficient vagueness and imprecision to warrant skepticism when using traditional MCDA techniques.

By applying a fuzzy compromise approach, MCDA feedback changes from "most robust to uncertainties in perception", to "most robust to uncertainties in perception and performance". The assessment of sensitivity to degree of risk aversion allows alternatives to be chosen which are robust to the context of a given problem, in addition to the more traditional robustness which is robustness to both criteria emphasis and criteria measurement.

The example demonstrates many characteristics of the fuzzy compromise approach as a multicriteria

decision analysis technique. The Tisza River example showed consistency of results, compared to the ELECTRE method, without a need for sensitivity analysis. The fuzzy compromise approach suggested a degree of dominance in the ordering, with both the WCoG and Acc ranking measures.

Group decision making is becoming more common. A fuzzy compromise approach facilitates more collaborative exploration of available alternatives and their associated risks. Collective opinions are incorporated by increasing (decreasing) the fuzziness of the inputs, and by locating ranges or multiple points of opinion. Fuzzy sets are able to process this kind of information, and are also able to present it effectively and intuitively.

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