ARTICLE IN PRESS
AUTCON-00990; No of Pages 9
Automation in Construction xxx (2009) xxx–xxx
Contents lists available at ScienceDirect
Automation in Construction
journal homepage: www.elsevier.com/locate/autcon

Estimating bridge performance based on a matrix-driven fuzzy linear regression model

Nang-Fei Pan [a,*], Tzu-Chieh Lin [a], Nai-Hsin Pan [b]

[a] Department of Civil Engineering, National Cheng Kung University, No. 1 University Road, Tainan 70101,Taiwan, ROC
[b] Department of Construction Engineering, National Yunlin University of Science and Technology, Taiwan, ROC

ARTICLE INFO

Article history:
Accepted 8 December 2008
Available online xxxx

Keywords:
Maintenance decision-making
Bridge inspection
Bridge performance
Deterioration prediction
Fuzzy regression analysis

ABSTRACT

Determining a reliable bridge maintenance and rehabilitation strategy relies on accurate predictions of bridge conditions. Conventional regression cannot handle visual inspection results that are inherently non-crisp or linguistic. On the other hand, fuzzy regression provides an effective means for coping with such fuzzy data or linguistic variables. However, many of the existing fuzzy regression models require substantial computations due to complicated fuzzy arithmetic. This paper presents a multiple fuzzy linear regression using matrix algebra. The proposed model is capable of dealing with a mixture of fuzzy data and crisp data. Moreover, the approach is intuitive and easy to implement as compared to other related fuzzy regression models. A case study using bridge inspection data is presented to establish estimated fuzzy regression equations produced by the proposed approach and examine the factors contributing to overall bridge performance. The results demonstrate the capability of the approach, which can assist bridge managers to make better maintenance policies based on the future bridge conditions predicted by the model.

1. Introduction

Bridges are chief elements in the transportation system. Maintenance of highway bridges plays an important role to assure the desirable service and adequate reliability of highway networks. A primary goal of a Bridge Management System (BMS) is to assist bridge managers in determining the best bridge maintenance, repair and rehabilitation strategy with respect to current or future bridge conditions. In essence, the government funds available for maintaining existing bridges are usually limited. With restricted funds, maximizing the effect of the investment on the improvement of serviceability and safety of the existing highway system is the major challenge for highway agencies. For example, between 2003 and 2007, the percentage of the nation's 599,893 bridges rated functionally obsolete or structurally deficient decreased slightly from 27.1% to 25.59%. To eradicate all bridge deficiencies, it will cost 9.4 billion US dollars a year for 20 years; however, long-term budget is compounded by the shortage of the Highway Trust Fund [1].

Bridges are composed of several components including decks, girders, piers, cap beams, bearings, joints, abutments and foundation, etc. Typically, highway bridges are exposed to increasing traffic volumes and detrimental environmental conditions and have been built some decades ago. As a result, many highway bridges have been deteriorated or damaged. Bridge deteriorations may reduce functional performance such as loss of comfort for the road user, reduce structural reliability and require higher maintenance expenditure. A structurally deficient bridge may be closed or restricted to light vehicles because of its deteriorated structural components. Typical bridge defects include cracking, surface distortion, disintegration, rebar corrosion, expansion joint damage, waterproof deterioration, bearing damage, and foundation erosion, etc [2]. A major cause for bridge deficiencies is inadequate and ineffective maintenance activities.

In general, the bridge maintenance decision-making process comprises the steps of (1) assessing bridge condition, (2) forecasting bridge deterioration, (3) determining the most desirable maintenance strategy, (4) prioritizing maintenance actions, and (5) optimizing resource allocations [3,4]. Forecasting the future bridge conditions in advance is useful for taking required or urgent repair actions in order to avoid disastrous consequences. Accordingly, accurate predictions of future bridge conditions based on periodic bridge inspection results are essential to develop an optimal maintenance policy. To estimate a bridge's condition in future, approximately 60% of BMS depends on periodic bridge inspection results [5]. However, bridge inspection observations are innately imprecise or fuzzy because they are usually collected from the bridge inspector's visual and subjective assessments using linguistic descriptions such as “The condition of this pier is very good” or “The condition rating of this concrete deck is about 80”. On the other hand, data or variables affecting a bridge's condition including bridge age, traffic load, bridge geometry such as bridge span, and environmental condition (e.g., rainfall and temperature) are numerical or crisp. Consequently, information on current bridge condition is a mixture of crisp data and fuzzy data.

* Corresponding author. Tel./fax: +886 62758455.
E-mail address: nfpan@mail.ncku.edu.tw (N.-F. Pan).

0926-5805/$ – see front matter © 2009 Elsevier B.V. All rights reserved.
doi:10.1016/j.autcon.2008.12.005

Various techniques have been applied to predict bridge condition [6–9]. The Markov chain is one of the most widely used probabilistic state-of-the-art techniques for forecasting the performances of bridge infrastructures [10–12]. However, the main problem of this approach is the assumption that the future state of infrastructure components depends only on the present condition, which may affect its prediction accuracy. State-of-the-art infrastructure assessment also applies regression analysis [13]. Regression analysis is primarily used to find the best-fitted mathematical model, so that a dependent or response variable can be predicted from independent or predictor variable(s). Descriptions and controls of the cause-and-effect relationship between variables are also major purposes that regression analysis serves. However, conventional regression techniques are suitable for dealing with non-fuzzy data. To analyze data containing ambiguity and imprecision, fuzzy set theory is an effective approach [14].

Recently, numerous studies have been conducted using artificial intelligence techniques such as Artificial Neural Network (ANN) methods [15–17], Genetic Algorithms [18,19], and fuzzy techniques [20–23] to evaluate bridge conditions. Bridge conditions are usually difficult to precisely estimate because of uncertainties and vague information involving the process of inspection. Fuzzy regression analysis is suitable for coping with problems in which human experts rely on subjective judgment or rules-of-thumb. Tanaka et al. [24] proposed the first fuzzy linear regression analysis for crisp input and fuzz output data. Following their work, various developments of fuzzy regression techniques and applications have been accomplished [25–31]. Many of the existing fuzzy regression models require a great deal of computations because of difficult fuzzy arithmetic. The regression model proposed by Tanaka et al. is quite popular and useful; however, this model is restricted to symmetric triangular fuzzy numbers. To overcome this limitation, Chang and Lee developed a fuzzy least-squares regression model [28]. However, in their model, the regression coefficients are derived from a nonlinear programming problem that requires considerable computations.

This paper presents a matrix-driven multiple fuzzy linear regression model to overcome the difficulties arising from Chang and Lee's approach and other models that require complex fuzzy mathematics. The proposed approach can deal with asymmetric and symmetric triangular membership functions. Furthermore, by the use of matrix algebra, the model is simpler to follow and easier to apply. An illustration using this model for the estimation of overall bridge conditions is presented using actual bridge inspection data from Taiwan.

2. Estimation of bridge conditions

A degree-extent-relevancy-urgency evaluation method is implemented in the Taiwan Bridge Management System and is currently executed by the bridge administration units for rating bridge conditions. The performance assessment of each component of a concrete bridge is performed by bridge inspectors to assess the current degree of deterioration (*D*), extent of deteriorated area (*E*) relevancy to serviceability and safety (*R*), and the urgency of the action for repairing defects (*U*). Table 1 depicts the DER&U condition ratings [32–34]. As defined in the table, the deterioration conditions are described on a discrete and ordinal scale of 1, 2, 3, and 4 to represent the states of “Good”, “Fair”, “Bad”, and “Serious”, respectively. The condition indicator given by 1, 2, 3, or 4 is used to describe the extent of deteriorated area being less than or equal to 10%, 30%, 60%, and 100%, respectively. Correspondingly, the relevancy to safety and serviceability including “minor”, “restricted”, “major” and “considerable” is also represented by 1, 2, 3, and 4, respectively. Based on *D*, *E*, and *R* ratings, an appropriate bridge maintenance action can be determined amongst “routine maintenance”, “repair in 3 years”, “repair in 1 year”, and “repair immediately”.

Table 1
DER&U rating criteria

Rating	Degree (*D*)	Extent (*E*)	Relevancy (*R*)	Urgency (*U*)
1	Good	<10%	Minor	Routine maintenance
2	Fair	10–30%	Restricted	Repair in 3 years
3	Bad	30–60%	Major	Repair in 1 year
4	Serious	60–100%	Considerable	Repair immediately

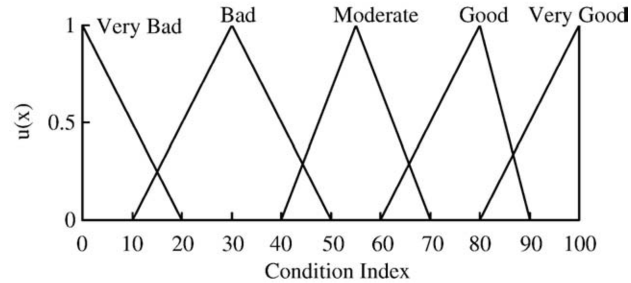

Fig. 1. Membership functions of bridge conditions.

Based on the inspection results, the condition of the component *i* can be estimated by using the performance index (*PI*). The calculation of *PI* is based on deterioration deduction points from the maximum score of 100 to the minimum score of 0, which is given by [32–34]

$$PI = 100 - 100 \times \frac{(D+E) \times R}{32}. \tag{1}$$

Concerning the assessment results for a given concrete deck: $D=2$, $E=2$, and $R=3$ as an example, the performance index of this concrete deck results in 62.5.

The overall bridge condition index (*CI*) can be defined as follows [32,33]:

$$CI = \frac{\sum_{i=1}^{n} PI_i \times w_i}{\sum_{i=1}^{n} w_i} \tag{2}$$

in which *n* is the number of the total components; w_i represents the relative importance or weight of each component from the inspector's point of view.

The relative weight in the above equation can be drawn from the use of Analytic Hierarchy Process (AHP). Since human inspector's assessment is naturally imprecise, the use of Eqs. (1)–(2) may not adequately represent overall bridge conditions. Furthermore, the DER&U ratings only define four levels of deterioration conditions, which overlook a necessary condition like very good or excellent. This type of condition is usually associated with newly constructed or repaired bridges. Hence, this paper defines five linguistic terms, Very Bad (VB), Bad (B), Moderate (M), Good (G), and Very Good (VG), corresponding to a nominal scale ranging from 0–100% to represent deterioration degrees. A questionnaire is used to direct seven experts to appraise these five linguistic terms. These experts are bridge inspectors who have more than ten-year working experience in bridge assessment. Typical questions are as follows:

> “On a scale rating from 0% to 100% condition index values with increments of 5%, please indicate (1) a likely range, and (2) a most-likely value that can best represent bridge conditions being *Very Good*?”

Similarly, the rest four linguistic terms can be measured through the survey. Based on the final agreement, the membership functions used to characterize the five levels of bridge conditions are constructed as shown in Fig. 1. It can be found in Fig. 1 that “Very Bad” and “Very Good” are represented by half triangular membership functions. “Good” is described by an asymmetric triangular membership function; whereas the remaining levels are characterized by symmetric triangular membership functions.

Each of the membership functions in Fig. 1 can also be mathematically expressed through Eqs. (3)–(7).

$$\mu_{\mathrm{VB}}(x) = \begin{cases} \dfrac{20-x}{20}, & 0 \le x \le 10 \\ 0, & \text{otherwise} \end{cases} \tag{3}$$

$$\mu_{\mathrm{B}}(x) = \begin{cases} \dfrac{x-10}{30-10}, & 10 \le x \le 30 \\ \dfrac{50-x}{50-30}, & 30 \le x \le 50 \\ 0, & \text{otherwise} \end{cases} \tag{4}$$

$$\mu_{\mathrm{M}}(x) = \begin{cases} \dfrac{x-40}{55-40}, & 40 \le x \le 55 \\ \dfrac{70-x}{70-55}, & 55 \le x \le 70 \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

$$\mu_{\mathrm{G}}(x) = \begin{cases} \dfrac{x-60}{80-60}, & 60 \le x \le 80 \\ \dfrac{90-x}{90-80}, & 80 \le x \le 90 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

$$\mu_{\mathrm{VG}}(x) = \begin{cases} \dfrac{x-80}{100-80}, & 80 \le x \le 100 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

3. Matrix-driven multivariate fuzzy linear regression model

Observations of bridge conditions obtained from human visual and subjective inspection cannot be fitted by using ordinary regression models. To tackle this problem, the proposed fuzzy linear regression model is introduced here.

Suppose that each fuzzy data point (fuzzy number) is represented by $\tilde{Y}_i=(Y_i, e_{i,L}, e_{i,R})$, where $i=1, 2,\ldots n$, and n is the sample size; Y_i is the fuzzy center; $e_{i,L}$ and $e_{i,R}$ is the fuzzy width and the fuzzy width of $\tilde{Y}_i$ at μ membership level, respectively. The values of the membership functions μ are measures of relative degree of belief. Fig. 2 exemplifies degrees of fitting $\hat{Y}_i$ to given fuzzy datum $\tilde{Y}_i$ concerning one crisp predictor and one fuzzy response variable [28]. In Fig. 2, $\mu_{\tilde{Y},L}$ and $\mu_{\tilde{Y},R}$ are the left bound and right bound of observed $\tilde{Y}_i$ at membership μ level; whereas $\mu_{\hat{Y},L}$ and $\mu_{\hat{Y},R}$ are the left bound and right bound of the predicted $\hat{Y}_i$ at membership μ value. Note that $a_0+a_1X_i$ represents the predicted $\hat{Y}_i$ at $\mu=1$ or the estimated conventional regression equation.

The use of μ membership level aims to adequately reflect the inspector's or the analyzer's degree of confidence in his or her assessment and to represent various degrees of uncertainty or fuzziness associated with the prediction environment. The value of μ is between 0 and 1. Larger μ values correspond to lower the fuzziness. $\mu=1$ indicates the highest confidence level (i.e., the minimum degree of fuzziness) or the forecasting condition has optimistic view; whereas $\mu=0$ represents the lowest degree of confidence or the forecasting condition has pessimistic view.

Fuzzy numbers in Fig. 2 are considered as symmetrical triangular membership functions in the following discussion. With k crisp independent variables and one fuzzy dependent variable, the estimated fuzzy linear regression can be expressed as

$$\hat{Y}_i = (a_0, c_{0,L}, c_{0,R}) + (a_1, c_{1,L}, c_{1,R})X_1 + (a_2, c_{2,L}, c_{2,R})X_2 + \cdots + (a_k, c_{k,L}, c_{k,R})X_k \tag{8}$$

where $(a_0,\ c_{0,L},\ c_{0,R})$ is the fuzzy intercept coefficient; $(a_1,\ c_{1,L},\ c_{1,R})$ is the fuzzy slope coefficient for X_1; $(a_2,\ c_{2,L},\ c_{2,R})$ is the fuzzy slope coefficient for X_2; $(a_k,\ c_{k,L},\ c_{k,R})$ is the kth fuzzy slope coefficient.

The estimated $\hat{Y}_i$ at a particular μ value is given by

$$\begin{aligned} \mu_{\hat{Y},L} &= [a_0-(1-\mu)c_{0,L}] + [a_1-(1-\mu)c_{1,L}]X_1 + \ldots + [a_k-(1-\mu)c_{k,L}]X_k \\ &= (a_0 + a_1 + \cdots + a_k)-(1-\mu)(c_{0,L} + c_{1,L}X_1 + c_{2,L}X_2 + \cdots + c_{k,L}X_k) \end{aligned} \tag{9}$$

and

$$\mu_{\hat{Y},R} = (a_0 + a_1 + \cdots + a_k) + (1-\mu)(c_{0,R} + c_{1,R}X_1 + c_{2,R}X_{2i} + \cdots + c_{k,R}X_k) \tag{10}$$

in which $a_0, a_1,\ldots, a_k$ are the estimated coefficients of $\hat{Y}_i$ at $\mu=1$; $c_{0,L}+c_{1,L}X_1$ and $c_{0,R}+c_{1,R}X_1$ are the left fuzzy width and the right fuzzy width for X_1; $c_{0,L}+c_{2,L}X_2$ and $c_{0,R}+c_{2,R}X_2$ are the left fuzzy width and the right fuzzy width for X_2; $(1-\mu)(c_{0,L}+c_{1,L}X_1)$ and $(1-\mu)(c_{0,R}+c_{1,R}X_1)$ are the left fuzzy width and right fuzzy width for X_1 at a given μ value; $(1-\mu)(c_{0,L}+c_{2,L}X_2)$ and $(1-\mu)(c_{0,R}+c_{2,R}X_2)$ are the left fuzzy width and the right fuzzy width for X_2 at a given μ value.

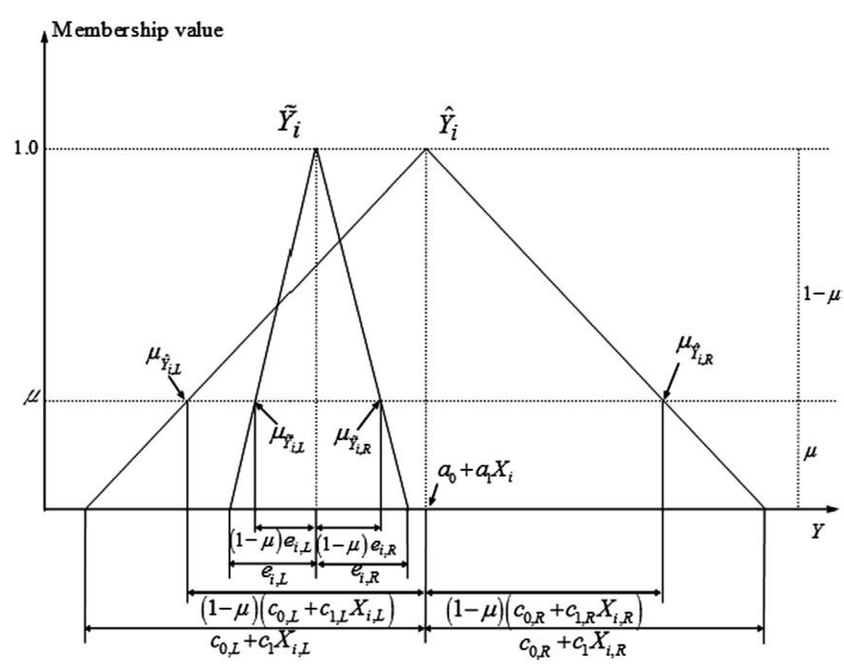

Fig. 2. Degrees of fitting $\hat{Y}_i$ to given fuzzy datum $\tilde{Y}_i$.

Observed data represented by asymmetrical triangular fuzzy numbers can be expressed as

$$\tilde{Y}_i = (y_i, (1-\mu)e_{i,L}, (1-\mu)e_{i,R}). \tag{11}$$

If $\tilde{Y}_i$ is a symmetric triangular fuzzy number, $e_{i,L}=e_{i,R}$. The method of least squares is used to find that particular regression line ($\hat{Y}_i$) where the sum of squared deviations of the data points ($\tilde{Y}_i$) above or below it is minimized. To facilitate the fuzzy regression analysis, matrix algebra is employed in this study. The general fuzzy linear model can be expressed in the following matrix form:

$$\tilde{\mathbf{Y}} = \mathbf{X}\hat{\tilde{\beta}} \tag{12}$$

where

$$\tilde{\mathbf{Y}} = \begin{bmatrix} (y_1, (1-\mu)e_{1,L}, (1-\mu)e_{1,R}) \\ (y_2, (1-\mu)e_{2,L}(1-\mu)e_{2,R}) \\ \vdots \\ (y_n, (1-\mu)e_{n,L}, (1-\mu)e_{n,R}) \end{bmatrix} \tag{13}$$

$$\mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{21} & \cdots & x_{k1} \\ 1 & x_{12} & x_{22} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{1n} & x_{2n} & \cdots & x_{kn} \end{bmatrix} \tag{14}$$

and

$$\hat{\tilde{\beta}} = \begin{bmatrix} \hat{\tilde{\beta}}_0 \\ \hat{\tilde{\beta}}_1 \\ \vdots \\ \hat{\tilde{\beta}}_k \end{bmatrix} = \begin{bmatrix} (a_0, (1-\mu)c_{0,L}, (1-\mu)c_{0,R}) \\ (a_1, (1-\mu)c_{1,L}, (1-\mu)c_{1,R}) \\ \vdots \\ (a_n, (1-\mu)c_{n,L}, (1-\mu)c_{n,R}) \end{bmatrix}. \tag{15}$$

In the above equations, matrices $\tilde{\mathbf{Y}}$ and $\mathbf{X}$ are the data matrices associated with response variable and predictor variables, respectively. Matrix $\tilde{\beta}$ contains the least squares estimates of the regression coefficients. To obtain the regression parameters, Eq. (12) can be transformed by

$$\left(\mathbf{X}'\mathbf{X}\right)\hat{\tilde{\beta}} = \mathbf{X}'\tilde{\mathbf{Y}} \tag{16}$$

where $\mathbf{X}'$ is the transpose matrix of $\mathbf{X}$.

By matrix operations, the regression coefficients can be derived as follows [35]:

$$\hat{\tilde{\beta}} = \left(\mathbf{X}'\mathbf{X}\right)^{-1}\mathbf{X}'\tilde{\mathbf{Y}} \tag{17}$$

where $(\mathbf{X}'\mathbf{X})^{-1}$ is the inverse matrix of $\mathbf{X}'\mathbf{X}$.

The fitted fuzzy regression equation can be developed based on the estimated regression coefficients. After establishing the regression equation, it is of interest to measure the quality or reliability of the fitted regression equation. The fuzzy coefficient of determination $(HR)^2$ is used to interpret the proportion of the total variation in Y explained by the regression line, which is defined by

$$(HR)^2 = \frac{\sum_{i=1}^{n}\left(\hat{Y}_i - \bar{\tilde{Y}}\right)^2}{\sum_{i=1}^{n}\left(\tilde{Y}_i - \bar{\tilde{Y}}\right)^2} \tag{18}$$

where $\bar{\tilde{Y}}$ is the mean of fuzzy data $\tilde{Y}$.

The above expression can be represented by the following expression.

$$(HR)^2 = \frac{\sum_{i=1}^{n}(a_0 + a_1X_i - \bar{Y})^2 + (1-\mu)\sum_{i=1}^{n}(c_{0,L} + c_{1,L}X_i - \bar{e}_L)^2 + (1-\mu)\sum_{i=1}^{n}(c_{0,R} + c_{1,R}X_i - \bar{e}_R)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2 + (1-\mu)\sum_{i=1}^{n}(e_{i,L} - \bar{e}_L)^2 + (1-\mu)\sum_{i=1}^{n}(e_{i,R} - \bar{e}_R)^2} \tag{19}$$

in which $\bar{e}_L$ and $\bar{e}_R$ are the mean of left fuzzy width and mean of right fuzzy width, respectively.

Likewise, the fuzzy correlation coefficient (HR) is the root of $(HR)^2$, which can evaluate the strength of the linear relationship between predictor variables and response variables.

Table 2
Bridge inspection results

Sample no.	AAT ($\times 10^4$ pcu/yr), X_1	Girder length (m), X_2	Duration after the previous maintenance (yr), X_3	Overall bridge condition, $\tilde{Y}_i$	Y_i	e_L	e_R
1	2426	17.1	1	G	80	20	10
2	2426	20	1	G	80	20	10
3	2358	19.2	2	VG	100	20	0
4	2358	17.1	1	VG	100	20	0
5	2058	17.1	1	VG	100	20	0
6	2347	34.1	1	VG	100	20	0
7	2058	24.2	1	VG	100	20	0
8	2058	12.2	1	VG	100	20	0
9	2170	24.2	2	G	80	20	10
10	2170	12.2	1	VG	100	20	0
11	2170	16.6	1	VG	100	20	0
12	2070	18	1	VG	100	20	0
13	2070	16	3	G	80	20	10
14	3358	19.2	1	G	80	20	10
15	3358	58	3	M	55	15	15
16	3358	35	4	M	55	15	15
17	3588	17	1	VG	100	20	0
18	3588	25.7	5	M	55	15	15
19	3588	20	2	G	80	20	10
20	3588	29.1	4	M	55	15	15
21	1886	29.6	1	G	80	20	10
22	1886	46	2	G	80	20	10
23	1886	20	1	VG	100	20	0
24	1886	25	1	VG	100	20	0
25	1750	27.3	1	VG	100	20	0
26	1750	29.6	2	VG	100	20	0
27	1750	35	2	G	80	20	10
28	1750	27.3	1	VG	100	20	0
29	1613	29.1	1	VG	100	20	0
30	1613	11.4	1	VG	100	20	0

4. Case study

30 sample data obtained from the National Sun Yat-Sen Freeway in January, 2007, are used to evaluate the capability of the proposed model. These data were randomly selected from the concrete bridges on the middle section of this freeway that was built in 1975. Table 2 displays the inspection results [36]. The overall bridge conditions listed in Table 2 Column 5 are derived from the observed conditions of decks, girders, and piers using Eqs. (1) and (2). The reasons for considering these three components are because they are main structural supporting elements and their data are available. It can be discovered in Table 2 that they are a mixture of crisp data and non-crisp data. The overall bridge condition can be regarded as a fuzzy response variable ($\tilde{Y}$). The corresponding three non-fuzzy predictor variables are average annual traffic (AAT) volume (X_1), girder length (X_2) and duration after the previous maintenance or rehabilitation (X_3). Note that other variables including precipitation, temperature, deck area, and pier height had been considered and analyzed as a preliminary study. Nevertheless, adding these variables resulted in lessening of $(HR)^2$ indicating that their contributions are statistically insignificant [37]. As a result, only the three variables listed in Table 2 are chosen.

Since conventional regression analysis is incapable of coping with a mixture of fuzzy data and crisp data, the proposed model is applied

Please cite this article as: N.-F. Pan, et al., Estimating bridge performance based on a matrix-driven fuzzy linear regression model, Automation in Construction (2009), doi:10.1016/j.autcon.2008.12.005

here. First, the data matrices $\widetilde{\mathbf{Y}}$ and $\mathbf{X}$ deriving from Table 2 are given by

$$\mathbf{X}=\begin{bmatrix}1 & 2426 & 17.1 & 1\\ 1 & 2426 & 20 & 1\\ \vdots & \vdots & \vdots & \vdots\\ 1 & 1613 & 29.1 & 1\\ 1 & 1613 & 11.4 & 1\end{bmatrix} \tag{20}$$

$$\mathbf{Y}=\begin{bmatrix}(80,20,10)\\ (80,20,10)\\ \vdots\\ (100,20,0)\\ (100,20,0)\end{bmatrix}. \tag{21}$$

Next, the matrix of regression coefficients at μ=0 can be estimated by using Eq. (17) to produce the following

$$\widetilde{\beta}=\begin{bmatrix}(128.211,23.518,-8.829)\\ (-0.007,-0.007,0.002)\\ (-0.389,-0.036,0.140)\\ (-8.093,-0.956,2.612)\end{bmatrix} \tag{22}$$

where

$$\mathbf{X}'\mathbf{Y}=\begin{bmatrix}2640\\ 6037160\\ 62037\\ 3980\end{bmatrix} \tag{23}$$

and

$$\left(\mathbf{X}'\mathbf{X}\right)^{-1}=\begin{bmatrix}6.88\times10^{-1} & -2\times10^{-4} & -8.9\times10^{-3} & 6.32\times10^{-2}\\ -2\times10^{-4} & 1.16\times10^{-7} & 8.5\times10^{-7} & 4.42\times10^{-5}\\ -8.9\times10^{-3} & 8.5\times10^{-7} & 4\times10^{-4} & 1.8\times10^{-3}\\ 6.32\times10^{-2} & -4.42\times10^{-5} & 1.8\times10^{-3} & 5.12\times10^{-2}\end{bmatrix}. \tag{24}$$

Therefore, the estimated multiple fuzzy linear regression equation is obtained as follows:

$$\hat{\widetilde{Y}}=(128.211,23.518,-8.829)+(-0.007,-0.0007,0.002)X_1 + (-0.389,-0.036,0.140)X_2+(-8.093,-0.956,2.612)X_3. \tag{25}$$

Accordingly, the cause-and-effect relationship between the influencing factors and overall bridge condition can be examined. It can be discovered in the above equation that the slope of X_1 is (−0.007, −0.0007, 0.002), indicating that regardless of girder length (X_2) and duration of the previous maintenance (X_3), an estimated decrease of 0.007 in the condition index of the examined bridge ($\widetilde{Y}$), decrease of 0.0007 in the lower bound (left interval) and increase of 0.002 in the upper bound (right interval) occurs for each additional average annual traffic (X_1). Likewise, the estimated slope coefficient of X_2, (−0.389, −0.036, 0.140), signifies that regardless of X_1 and X_3 and for each additional X_2, $\widetilde{Y}$ decreases by 0.389, with decreases by 0.036 and increase by 0.140 in the lower bound and upper bound, respectively. Similarly, the estimated slope coefficient of X_3, (−8.093, −0.956, 2.612), denotes that regardless of X_1 and X_2 and for each additional X_3, an estimated decrease of 8.093 in $\widetilde{Y}$, with decrease of 0.956 in the lower bound and increase of 2.612 in the upper bound.

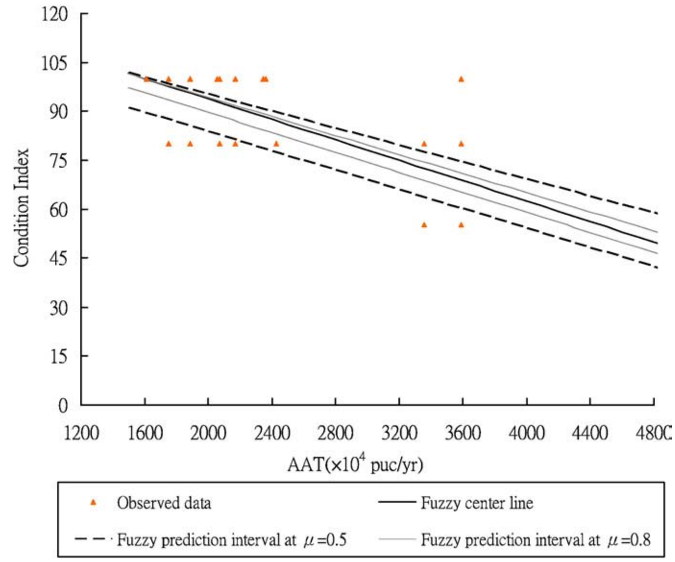

Fig. 3. Fuzzy regression models with X_1 at μ=0.5 and 0.8.

The result produced by the model indicate that duration of the previous maintenance is the most important variable for reducing bridge performance in this case study; whereas deterioration rates are higher in bridges with longer spans than those with higher traffic volume. Such information is useful for the maintenance unit to identify which variables should receive more attention to enhance bridge performance. Notably, the result may not be absolute because different sample data may yield unlike outcome.

Using Eq. (19), the fuzzy coefficient of determination is computed as

$$(HR)^2=\frac{5829.0087+69.7066+658.9797}{7380+86.6667+1050}=0.770. \tag{26}$$

The above result shows that 77% of the total variation in $\widetilde{Y}$ can be explained by the regression line relating to the inputs. The fuzzy correlation coefficient thus yields 0.877 ($=\sqrt{0.77}$), signifying a fairly strong positive linear correlation between the overall bridge condition and predictor variables. Accordingly, these three variables provide reliable predictions of bridge performance.

By the use of the foregoing procedure, the estimated regression equations based on μ=0, 0.05, 0.8 and 1 can be obtained and shown in Table 3. It can be found in this table that the width of the fuzzy regression decreases as μ increases. Besides, the HR values yield 0.877, 0.878, 0.883, 0.886, and 0.890 as μ=0, 0.05, 0.5, 0.8 and 1, respectively. The results confirm good quality (goodness-of-fit) of the estimated regression equations. Notice that the fuzzy regression equation fitted by μ=1 has zero fuzzy width, which is equivalent to the conventional regression model. The regression equations in Table 3 can be graphically displayed. For instance, Figs. 3–5 depict two-dimensional scatter diagrams with fuzzy regression lines at μ=0.5 and μ=0.8 for bridge condition plotting against X_1, X_2, and X_3, respectively. Figs. 3–5 illustrate that the prediction

Table 3
Fuzzy linear regressions with different μ values using the proposed approach

Membership value, μ	Estimated linear regression equation	$(HR)^2$	HR
μ=0	$\hat{\widetilde{Y}}=(128.211,23.518,-8.829)+(-0.007,-0.0007,0.002)X_1+(-0.389,-0.036,0.140)X_2+(-8.093,-0.956,2.611)X_3$	0.769	0.877
μ=0.1	$\hat{\widetilde{Y}}=(128.211,21.166,-7.946)+(-0.007,-0.0006,0.002)X_1+(-0.389,-0.032,0.126)X_2+(-8.093,-0.861,2.351)X_3$	0.771	0.878
μ=0.5	$\hat{\widetilde{Y}}=(128.211,11.759,-4.415)+(-0.007,-0.0003,0.001)X_1+(-0.389,-0.018,0.07)X_2+(-8.093,-0.478,1.306)X_3$	0.779	0.883
μ=0.8	$\hat{\widetilde{Y}}=(128.211,4.703,-1.766)+(-0.007,-0.0001,0.0005)X_1+(-0.389,-0.0007,0.028)X_2+(-8.093,-0.191,0.522)X_3$	0.785	0.886
μ=1	$\hat{\widetilde{Y}}=(128.211,0,0)+(-0.007,0,0)X_1+(-0.389,0,00)X_2+(-8.093,0,0)X_3$	0.790	0.890

Please cite this article as: N.-F. Pan, et al., Estimating bridge performance based on a matrix-driven fuzzy linear regression model, Automation in Construction (2009), doi:10.1016/j.autcon.2008.12.005

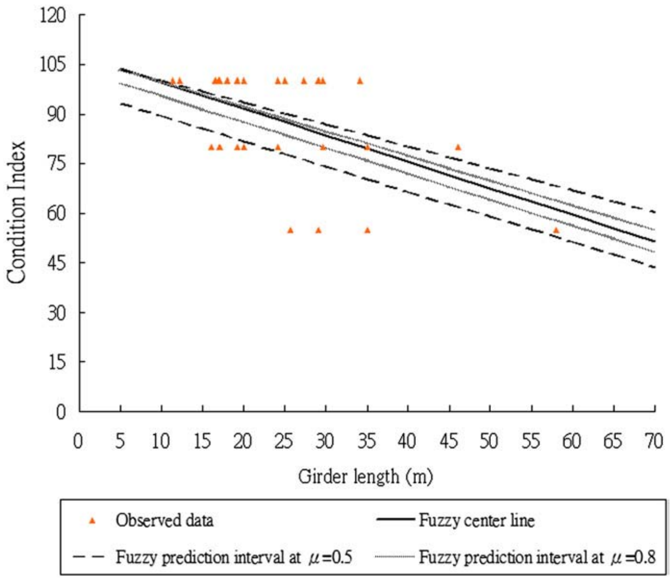

Fig. 4. Fuzzy regression models with X_2 at μ=0.5 and 0.8.

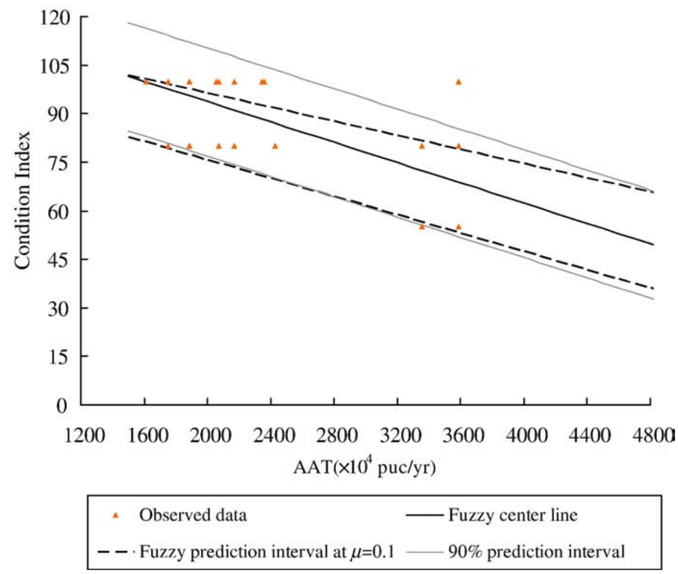

Fig. 6. Fuzzy prediction interval with X_1 at μ=0.1.

interval is wider at μ=0.5 than the one at μ=0.8 regarding each independent variable. Consequently, it is concluded that the outer bound of the fuzzy regression enlarges as μ values decreases. Notably, when μ=1, the center line of the fuzzy regression is equivalent to the classical regression. In other words, the predictions given by the two regression models are identical when all data are crisp.

The estimated regression equations can be used to forecast the future bridge condition with respect to particular input values. As an example, if a bridge manager is of interest to predict the bridge condition regarding X_1=3600×10^4 pcu/yr, X_2=35 m, X_3=5 years (i.e., 5 years after the previous repair), the estimated condition index at μ=0 is given by

$$\begin{aligned}\hat{\tilde{Y}} &= (128.211, 23.518, -8.829) + (-0.007, -0.0007, 0.002)\times 36000 \\ &+ (-0.389, -0.036, 0.140)\times 35 + (-8.093, -0.956, 2.612)\times 5 \\ &= (33.97, 48.93, 65.74)\end{aligned} \quad (27)$$

The above result indicates that the estimated minimum, most-likely, and maximum condition index of the bridges is 45.85, 62.47 and 74.48, respectively. This is valuable decision-making information for the bridge managers to determine the priorities of maintenance methods and resources. Moreover, discerning the future bridge conditions in advance is useful for taking necessary or urgent repair actions so as to avoid catastrophic bridge failures.

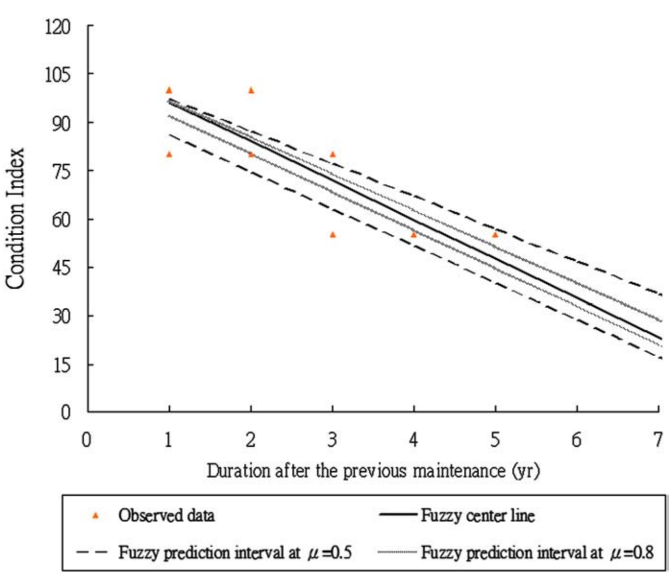

Fig. 5. Fuzzy regression models with X_3 at μ=0.5 and 0.8.

The quality of fuzzy predictions at a given membership level needs to examine. The quality of predictions depends on the precision and reliability of an estimate, where the reliability of an estimate is the probability that it is correct. It should be noted that fuzzy prediction interval at $\mu=\alpha$ is analogous to $100(1-\alpha)$% confidence prediction interval for individual Y in the conventional regression analysis. For instance, if a bridge manager wishes to forecast bridge condition with X_3=5 years, the 90% confidence prediction interval is estimated as follows [38]:

$$\begin{aligned}Y(X) &= \hat{Y}(X) \pm t_{\alpha/2} S_{Y\cdot X} \sqrt{\frac{\frac{1}{n} + (X-\bar{X})^2}{\sum X^2 - \frac{1}{n}(\sum X)^2} + 1} \\ &= 47.615 \pm 1.699\times 5.143\times 1.017 = (38.733, 56.5).\end{aligned} \quad (28)$$

The above result shows that 90% confidence prediction interval is (38.733, 56.5). Figs. 6–8 illustrate the resulting graphical displays of predicted $\hat{Y}$ at μ=0.1 given X_1, X_2, and X_3, respectively. It can be found in these figures that the outer bands of these fuzzy regressions increase as either X_1, X_2 or X_3 increases. In addition, all the outer bounds of the fuzzy regressions are narrower than 90% confidence prediction intervals of the ordinary regression. Since the interval of prediction represents the degree of precision, the result demonstrates that the proposed model is more accurate than the conventional regression model in this case problem.

5. Discussion

The degree of confidence in judgment by the inspector is represented by the μ value in the approach; thus, the user of the proposed model can choose an appropriate μ value corresponding to the level of uncertainty or fuzziness so that reliable predictions can be achieved. When μ=0 or all observed data are crisp, this model provides the same results as the conventional regression model. To further justify the effectiveness of this model, it is necessary to compare the results to other approaches. Since Chang and Lee's model is one of few existing fuzzy regression models that can handle

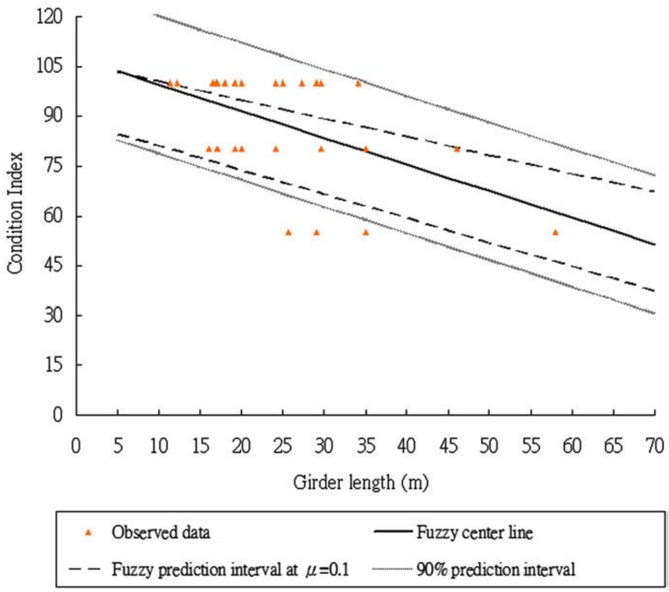

Fig. 7. Fuzzy prediction interval with X_2 at μ=0.1.

Table 4
Fuzzy linear regressions with different μ values using Chang and Lee's model

Membership value, μ	Estimated linear regression equation	$(HR)^2$	HR
μ=0	(137.0792, 19.8992, 0)+(−0.0111, 0.0049, 0.0050)X_1+(−0.6992, 0, 0.2308)X_2+(−5.9607, 0, 0)X_3	0.008	0.089
μ=0.1	(137.0796, 19.8985, 0)+(−0.0114, 0.0042, 0.0049)X_1+(−0.6719, 0, 0.2035)X_2+(−5.9607, 0, 0)X_3	0.161	0.401
μ=0.5	(137.0764, 19.9003, 0)+(−0.0111, 0.0020, 0.0046)X_1+(−0.5626, 0, 0.0943)X_2+(−5.9602, 0, 0)X_3	0.561	0.749
μ=0.8	(137.0766, 19.9002, 0)+(−0.0101, 0.0005, 0.0036)X_1+(−0.5684, 0, 0.1001)X_2+(−5.9602, 0, 0)X_3	0.640	0.800
μ=1	(137.0810, 8958, 0)+(−0.0100, 0.0004, 0.0035)X_1+(−0.5685, 0, 0.1001)X_2+(−5.9609, 0, 0)X_3	0.643	0.802

this type of case problem [30,31], it was applied for this purpose. Based on Table 2, the results derived by using Chang and Lee's model are listed in Table 4. It can be found in Table 4 that all the values of $(HR)^2$ are larger than those produced by the proposed approach. Moreover, Fig. 9 shows the graphical displays of predicted $\hat{Y}$ at μ=0.1 given X_1 through the model and Chang and Lee's method. As shown in the figure, the prediction interval produced by Chang and Lee's method is much wider than that of the approach. Accordingly, the results indicate that the proposed approach produces better reliability measure and prediction precision.

Partial computations of using Chang and Lee's method to establish the regression equation are illustrated in the Appendix A. It can be found that their approach resulted in a nonlinear programming problem that is every difficult to comprehend and construct the equations. Moreover, it requires considerable computations to solve the equations. The proposed model is relatively easy to implement and needs less computation time as compared to their model. Accordingly, it demonstrates the capability of the proposed method in terms of prediction accuracy and computational efficiency.

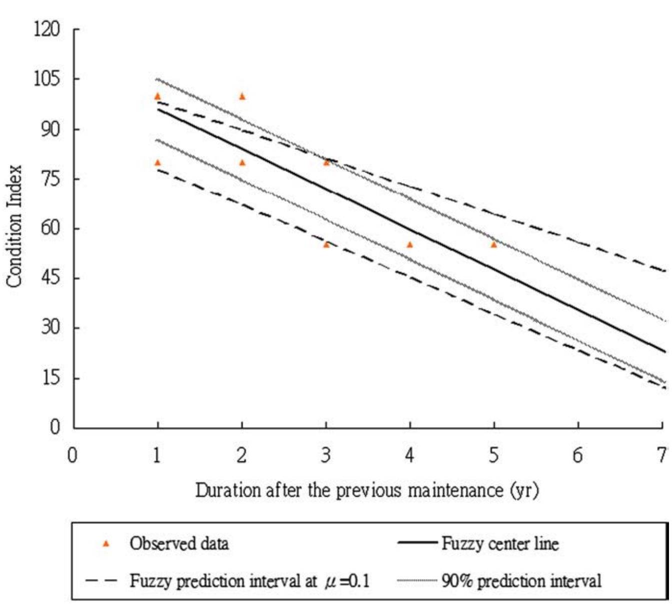

Fig. 8. Fuzzy prediction interval with X_3 at μ=0.1.

6. Concluding remarks

Bridge assessment results are usually qualitative; thus, ordinary regression cannot deal with such non-crisp data. This paper presents a matrix-driven multivariate fuzzy linear regression model which is capable of fitting a regression equation to fuzzy data represented by asymmetric and symmetric triangular fuzzy numbers, numerical data, and their combination. This approach also enables to adequately reflect the decision-makers' confidence in the collected data and in the established model by employing particular membership values. Moreover, this model is more intuitive and straightforward than other related models.

The main purpose of employing the proposed model is to establish fitted regression equations for estimating future conditions and investigating the cause-and-effect relationship between variables. Major advantages to using the method for bridge administration are as follows:

(1) Future bridge conditions estimated by the model are particularly useful for planning inspection, establishing rehabilitation, performing necessary repairs, and predicting the impact of undertaking maintenance policies in a bridge system.
(2) This model can assist bridge managers in prioritizing maintenance alternatives, determining the most appropriate maintenance strategy, and optimizing resource allocations.
(3) The prediction outcomes given by the approach can be used as a signal to warn the bridge maintenance units under the

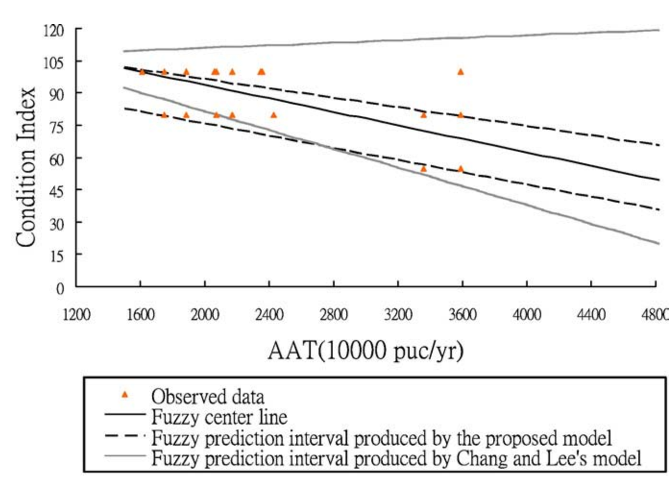

Fig. 9. Fuzzy prediction intervals with X_1 at μ=0.1 produced by the proposed model and Chang and Lee's approach.

situation when the estimated bridge condition is abnormally bad or close to the limit of allowance.

(4) This model can identify noteworthy variables affecting bridge performance and evaluate the magnitude of each variable, which can assist bridge engineers for designing bridges that will reduce the future bridges' maintenance cost.
(5) Bridge inspection is an expensive and time-consuming task. Using the predictions from the model, one may save considerably on inspection costs and find potential bridge damage in advance.
(6) The approach is practical in dealing with large-sized observations due to its computational efficiency and simplicity of use.

Therefore, this model can assist bridge engineers in making better decisions.

The consequence of the case study shows that this model has potential for use with similar types of prediction problems or in various areas of decision-making problems. The outcomes given by this model depend on the inspector's assessments; thus, sufficient personnel experience and training is essential. Basically, the use of the model is not restricted to the numbers of independent variables; however, computation time increases as the numbers of predictor variables and sample sizes increase. To facilitate matrix operations in this approach, the use of software such as MicroSoft™-Excel, Maple™ and MathWorks™-MATLAB is helpful. Notably, although the proposed method is suitable for fuzzy regression analysis, it can also be applied for a broad class of nonlinear functions. This can be accomplished by transforming variables to find a linear function involving the transformed variables.

This model is restricted to triangular membership functions; thus it can not handle other types of membership functions. In addition, the approach allows fitting a model to crisp-input and fuzzy-output data, but it is incapable of dealing with fuzzy independent variable(s). These are two main limitations of this approach, which are suggestions from the author for future studies.

Acknowledgments

The authors wish to express his gratitude to the National Science Council, Taiwan, for funding this research work through permit No. NSC96-2221-E-006-227.

Appendix A

Based on Table 2 and Chang and Lee's model, the following nonlinear programming problem can be derived [30,31]:

$$
\begin{aligned}
&+\{[(4\times 100-20+0)-4(a_0+1613a_1+11.4a_2+a_3)+(c_{0.L}+1613c_{1,L}+11.4c_{2,L}+c_{3,L})\\
&-(c_{0,R}+1613c_{1,R}+11.4c_{2,R}+c_{3,R})]^2+(20-c_{0,L}-1613c_{1,L}-11.4c_{2,L}-c_{3,L})^2\\
&+(10-c_{0,R}-1613c_{1,R}-11.4c_{2,R}-c_{3,R})^2\}
\end{aligned}
\tag{29a}
$$

subject to

$$
\begin{aligned}
&80-(1-\mu)\times 20\geq a_0+2426a_1+17.1a_2+a_3-c_{0,L}-2426c_{1,L}-17.1c_{2,L}-c_{3,L}\\
&80+(1-\mu)\times 10\geq a_0+2426a_1+17.1a_2+a_3+c_{0,R}+2426c_{1,R}+17.1c_{2,R}+c_{3,R}\\
&\qquad\vdots\\
&100-(1-\mu)\times 20\geq a_0+1613a_1+29.1a_2+a_3-c_{0,L}-1613c_{1,L}-29.1c_{2,L}-c_{3,L}\\
&100+(1-\mu)\times 0\geq a_0+1613a_1+29.1a_2+a_3-c_{0,R}+1613c_{1,R}+29.1c_{2,R}+c_{3,R}\\
&c_L,\, c_R\geq 0.
\end{aligned}
\tag{29b}
$$

Note that the number of constraints in Eq. (29b) are 60 in total (=2×30). Thus, solving the above problem is relatively complex.

References

[1] The American Society for Civil Engineers, Report card for America's infrastructure, 2008, available at http://www.asce.org/reportcard/2005/actionplan07.cfm, accessed 11/27/2008.
[2] A.P. Chassiakos, P. Vagitotas, D.D. Theodorakopoulos, A knowledge-based system for maintenance planning of highway concrete bridges, Advances in Engineering Software 36 (2005) 740–749.
[3] Y.-M. Cheng, S.-S. Leu, Constrain-based clustering model for determining contract packages of bridge maintenance inspection, Automation in Construction 17 (2008) 682–690.
[4] S. Yehia, O. Abudayyeh, I. Fazal, D. Randolph, A decision support system for concrete bridge deck maintenance, Advances in Engineering Software 39 (2008) 202–210.
[5] G. Heam, R.L. Purvis, P.D. Thompson, W.H. Bushman, K.K. McGhee, W.T. McKeel, Bridge maintenance and management: a look to the future, Proceedings of the TRB 81 st Annual Meeting: A3C06: Structures Maintenance and management, 2000, pp. 1–7.
[6] O.O. Ugwu, M.M. Kumaraswamy, F. Kung, S.T. Ng, Object-oriented framework for durability assessment and life cycle costing of highway bridges, Automation in Construction 14 (2005) 611–632.
[7] A. McCrea, D. Chamberlain, R. Navon, Automated inspection and restoration of steel bridges—a critical review of methods and enabling technologies, Automation in Construction 11 (2002) 351–373.
[8] D.M. Frangopol, J.S. Kong, E.S. Gharaibeh, Reliability-based life-cycle management of highway ridge, Journal of Computing in Civil Engineering, ASCE 15 (1) (2001) 147–157.
[9] S.J. Lorenc, B.E. Handlon, L.E. Bernold, Development of a robotic bridge maintenance system, Automation in Construction 9 (2000) 251–258.
[10] G. Morcous, Z. Lounis, S.M. Mirza, Identification of environmental categories for Markovian deterioration models of bridge decks, Journal of Bridge Engineering, ASCE 8 (6) (2003) 353–361.
[11] W.T. Scherer, D.M. Glagola, Markov models for bridge maintenance management, Journal of Transportation Engineering, ASCE 120 (1) (1994) 37–51.
[12] Y. Jiang, M. Saito, K.C. Sinha, Bridge performance prediction model using the Markov Chain, Transportation Research Record 1180 (1988) 25–32.
[13] S.M. Madanant, W.H. Wan, Ibrahim, Poisson and negative binomial regression models for the computation of infrastructure transition probabilities, Journal of Transportation Engineering, ASCE 121 (3) (1995) 267–272.
[14] J. Yen, R. Langari, Fuzzy Logic: Intelligence, Control, and Information, Prentice Hall, New Jersey, USA, 1999.
[15] J. Lee, K. Sanmugarasa, M. Blumestein, Y.-C. Loo, Improving the reliability of a bridge management system (BMS) using an ANN-based backward prediction model (BPM), Automation in Construction 17 (2008) 758–722.
[16] Y.H. Huang, T.M. Adams, J.A. Pincheira, Analysis of life-cycle maintenance strategies for concrete bridge decks, Journal of Bridge Engineering, ASCE 9 (3) (2004) 250–258.
[17] J. Cattan, J. Mohammadi, Analysis of bridge condition rating data using neural networks, Microcomputers in Civil Engineering 12 (1997) 419–429.
[18] G. Morcous, Z. Lounis, Maintenance optimization of infrastructure networks using genetic algorithms, Automation in Construction 14 (2005) 129–142.
[19] A. Miyamoto, K. Kawamura, H. Makamura, Bridge management system and maintenance optimization for existing bridges, Computer-Aided Civil and Infrastructure Engineering 15 (2000) 45–55.
[20] H. Furuta, J.H. He, A fuzzy expert system for damage assessment using generic algorithm and neural networks, Microcomputing in Civil Engineering 11 (1996) 37–54.
[21] N.-F. Pan, Performance prediction of highway bridges using fuzzy regression model, Proceedings of the Tenth International Conference on Civil, Structural and Environmental Engineering Computing, Civil-Comp Press, Edinburgh, UK, 2005, pp. 1–15, Paper 9.
[22] M.-T. Liang, T.-B. Chu, W.-H. Tsao, C.-J. Yeh, Determining the repair ranking of existing RC bridges using fuzzy synthetic evaluation method, Journal of Chinese Institute of Engineers, 29 (1) (2006) 37–50.
[23] S. Sasmal, K. Ramanjaneyulu, S. Gopalakrishnan, N. Lakshmanan, Fuzzy logic based bondition rating of existing reinforced concrete bridges, Journal of Performance of Constructed Facilities, ASCE 20 (3) (2006) 261–273.
[24] H. Tanaka, S. Uejima, K. Asai, Linear regression analysis with fuzzy model, IEEE Transactions on System, Man and Cybernet 2 (6) (1982) 903–907.
[25] D.A. Savic, Evaluation of fuzzy linear regression models, Fuzzy Sets and Systems 39 (1991) 51–63.
[26] P. Diamond, R. Korner, Extended fuzzy linear models and least-squares estimates, Computers and Mathematics with Applications 33 (1997) 15–32.
[27] B. Kim, R.R. Bishu, Evaluation of fuzzy linear regression models by comparing membership functions, Fuzzy Sets and Systems 110 (1998) 343–352.
[28] Y.-H.O. Chang, Hybrid fuzzy least-squares regression analysis and its reliability measures, Fuzzy Sets and Systems 119 (2001) 225–246.
[29] P. D'Urso, Linear regression analysis for fuzzy/crisp input and fuzzy/crisp output data, Computational Statistics and Data Analysis 42 (2003) 47–72.
[30] P.-T. Chang, E.S. Lee, Fuzzy least absolute deviations regression and the conflicting trends in fuzzy parameters, Computers & Mathematics with Applications 28 (5) (1994) 89–101.
[31] P.-T. Chang, Fuzzy regression analysis, Ph.D. Thesis, Industrial Engineering, Kansas State University, USA, 1994.
[32] N.-J. Yau, H.-K. Liao, An automated visual-aid system for inspecting concrete bridges, Proceedings of the 10th International Conference on Civil, Structural and Environmental Engineering Computing, Civil-Comp Press, Edinburgh, UK, 2005, pp. 1–14, Paper 3.
[33] Taiwan Area National Freeway Bureau, Visual inspection manual for freeway bridges (in Chinese), Taiwan Area National Freeway Bureau, 1995.

[34] E. de Fleuriot, P.A. Nordengen, Y.-F. Lee, A.-Y. Lin, A bridge management system for the Taiwan Area National Freeway Bureau, International Bridge Conference: Bridges into the 21st Century, Hong Kong, October 1995, pp. 611–618.
[35] J. Neter, W. Wasserman, M.H. Kutner, Applied Regression Models, 2nd ed. IRWIN, Inc., NY, 1996.
[36] Taiwan Area National Freeway Bureau, 2007, Bridge Management System, available at http://bms.iot.gov.tw, accessed 01/15/2007.
[37] N.F. Pan, Applying Fuzzy Regression to Forecast Bridge Conditions, Technical Report, the National Science Council, Taiwan, 2008.
[38] L. Lapin, Modern Engineering Statistics, Duxbury Press, NY, 1997.