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River assimilative capacity analysis via fuzzy linear programming☆

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Abstract

The assimilative capacity of a river, that is the so-called allowable pollution loading in a river basin, is a problem of water quality management involving optimization subject to constraints of standards of water quality and their models. This study is an up-to-date evaluation of assimilative capacity in a river basin with consideration of the imprecision of the water environment, i.e. water quality and river flowrate. The purposes of this study, in comparison with crisp linear programming (LP), is to select and evaluate four well-known fuzzy linear programming (FLP) approaches and to apply them to the assimilative capacity of a river basin, Tou-Chen River Basin in Taiwan. The results show the models of assimilative capacity of FLP are better than those of a crisp one, which reflect the flexible characteristics of the former approaches. The emphases in this paper are on how to formulate problems of assimilative capacity of a river according to FLP models and on how to solve them according to algorithms of crisp LP. Furthermore, the results of river analysis serve as criteria for regulatory agencies to implement the control of pollution sources and to accomplish finally the sustainable use of water resources.

Keywords: Fuzzy linear programming; Fuzzy numbers; Water quality management; Assimilative capacity

1. Introduction

Fuzzy set theory has extensively developed since Zadeh's pioneer work in 1965 [36]. The related areas of application include logic, control, expert systems, decision making, mathematical programming, pattern recognition, cluster analysis, and others [4, 9, 14, 15, 39–41] that are generally studied and that demonstrate satisfactory achievements of fuzzy set theory.

In modeling actual problems, the decision maker generally describes the problem in precise mathematical terms rather than in terms of imprecision according to natural and human aspects [37]. Until recently, in problems of mathematical programming, one invariably treated imprecision by means of probability theory. However, the main source of imprecision in realistic problems involving large-scale systems that contain uncertainty of many types should be appropriately stated as "fuzziness" rather than "randomness" such as economic systems, social systems and water resources systems [13,32]. Ordinarily, coefficients of the objective and constraints in mathematical programming are supposed to be precise numbers. This condition is

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not rational in many practical situations, e.g. programming involves demands and available capacities of renewable and nonrenewable resources, cost requirement, technological coefficients, etc. [18].

Up to now, fuzzy set theory has played an important role in treating imprecision, by providing membership functions to indicate explicitly the degree of belonging. Fuzzy mathematical programming self-evidently applies fuzzy set theory and mathematical programming approaches (e.g. linear programming, nonlinear programming, dynamic programming, goal programming, etc.) and is to optimize the alternatives of which constraints have fuzzy coefficients, fuzzy variables or fuzzy inequalities, the objective function might also have fuzzy coefficients. Fuzzy mathematical programming has the significance of solving optimization problems in an imprecise environment, which has been formulated into some models to allow flexibility of constraints and fuzziness in the objective function completed by Bellman and Zadeh [3], Negoita et al. [20], Tanaka et al. [32], and Zimmermann [38]. Recently, fuzzy mathematical programming evokes increasing interest and is developed in connection with the necessity to meet requirements of decision making as it can in many ways be well adapted to various decision problems [18].

From a practical point of view, a powerful and attractive fuzzy mathematical approach is fuzzy linear programming (FLP), with the assumption of linear addition of objective function and constraints, which typically converts to conventional linear programming (LP) and utilizes its ordinary algorithms to obtain the optimal solution [13,21,30]. The solution of FLP depends strongly upon the fuzziness of coefficients as the smaller the fuzziness of coefficients becomes, the more satisfactory is a solution obtained [31]. After foremost research in FLP by Tanaka et al. [32] and Zimmermann [37], problems of several kinds of this approach have appeared in the literature and varied approaches of resolution are proposed [8,19,24,29-31,33,38]. Verdegay [33] and Delgado et al. [8] formed general models of FLP problems, the former emphasizes the constraint set, both fuzzy numbers and fuzzy constraints; the latter includes the approaches of Tanaka et al. [32], Hamacher et al. [11] (fuzzy inequalities), and Zimmermann [37] (fuzzy objective and fuzzy inequalities). However, in this study, we induce a practical case according to the FLP approaches instead of numerical examples already presented [6,21,24,29,30,32].

The aims of this study was analysis of the assimilative capacity, also called allowable pollution loading, of a river basin, by mathematical programming models taking into account the imprecision of water environment, i.e. water quality and river flowrate. The four models of FLP are approaches of Zimmermann [37], Tanaka and Asai [30], Chanas [6], and Julien [13] that can all treat fuzzy right-hand sides (RHS, in this study, water quality) and the latter approach can additionally solve fuzzy coefficients of constraints (in this study, river flowrate). Afterwards, we screen more suitable approaches for application in this analysis. Section 2 presents fundamentals and algorithms of approaches of FLP that we apply to the practical assimilative capacity of a river. Section 3 describes the environment of the research area, the Tou-Chen River Basin in Taiwan, and its models to manage water quality. Section 4 illustrates the results of optimization of these FLPs and indicate the assimilative capacity of a river basin. In Section 5, we show some remarks and conclusions for our study.

2. Fuzzy linear programming approaches

A classical model of LP, also called a crisp LP model, may have the following formulation:

$$\begin{aligned} &\text{Max} \quad Cx \\ &\text{s.t.} \quad A_i x \leqslant b_i, \quad i = 1, \dots, m, \end{aligned} \tag{1}$$

in which x is an $n \times 1$ alternative set, C is a $1 \times n$ coefficients of an objective function, A_i is an $m \times n$ matrix of coefficients of constraints and b_i is an $m \times 1$ RHS.

The traditional problems of LP are solved with a LINDO package (for the user's manual see [25] or equivalent software) and obtain the optimal solution in a precise way. If coefficients of constraints, objective function or the RHS are imprecise, in other words, being fuzzy numbers, traditional algorithms of LP are unsuitable to solve the fuzzy problem and to obtain the optimization.

However, in the real world, the coefficients are typically imprecise numbers because of insufficient information, for instance, demands, available capacities, cost requirement, technological coefficients. Many researchers formed FLP of various types, invented approaches to convert them into crisp LP, and finally solved the problems with available packages.

2.1. Zimmermann's approach

A LP with a fuzzy objective function and fuzzy inequalities first shown by Zimmermann [37] is indicated as follows:

$$\begin{aligned}&\widetilde{\text{Max}} \quad Cx \\ &\text{s.t.} \quad A_i x \lesssim b_i, \quad i = 1, \ldots, m,\end{aligned} \tag{2}$$

in which the meanings of the decision variables and all coefficients are the same as Eq. (1).

If we give an aspiration level b_0, then problem (2) becomes

$$Cx \gtrsim b_0, \tag{3a}$$

$$A_i x \lesssim b_i. \tag{3b}$$

Inequality (3) is a symmetrical model of which the objective function becomes one constraint. To write a general formulation, inequality (3) is converted to a matrix form as

$$Ax \leqslant b, \tag{4}$$

in which

$$A = \begin{bmatrix} -C \\ A_i \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix}.$$

The inequalities of constraint (4) signify "be as small as possible or equal" that can be allowed to violate the RHS b by extending some value. The degree of violation is represented by membership function as

$$\mu(Ax) = \begin{cases} 1 & \text{if } Ax \leqslant b, \\ 1 - (Ax - b)/d & \text{if } b \leqslant Ax \leqslant b + d, \\ 0 & \text{if } b + d \leqslant Ax, \end{cases} \tag{5}$$

in which d is a matrix of admissible violation.

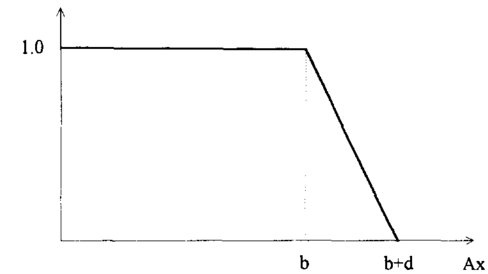

Fig. 1. Membership function of Ax in Eq. (5).

The membership function can also be shown as in Fig. 1; when the inequalities are rigidly established, the membership grade is 1. The more the inequality is violated, the less the membership grade, until 0 when $Ax \geqslant b + d$ is found rigorously.

The optimization of FLP then becomes to find the maximum value of the membership grade of Eq. (5). According to Bellman and Zadeh's "fuzzy decision" [3], max-min operation, the optimal solution of constraint (4) becomes

$$\begin{aligned}&\text{Max} \quad \lambda \\ &\text{s.t.} \quad \lambda \leqslant 1 - \frac{Ax - b}{d}, \quad \lambda \leqslant 1.\end{aligned} \tag{6}$$

Problem (6) can be solved as a conventional LP model with the LINDO package to obtain an optimal value of λ and its original fuzzy objective function can also be found jointly.

Zimmermann's approach is applied successfully to solve practical problems, for example, water quality management problem [16], inventory planning [23], air pollution regulation problem [27] and media selection [35].

2.2. The approach of Tanaka and Asai

Tanaka and Asai [30] provide a theorem to deal with fuzzy symmetric LP that is represented as

$$\begin{aligned}
Y_1 &= a_{11}X_1 + \cdots + a_{1j}X_j + \cdots + a_{1n}X_n + B_1 \gtrsim 0, \\
Y_2 &= a_{21}X_1 + \cdots + a_{2j}X_j + \cdots + a_{2n}X_n + B_2 \gtrsim 0, \\
&\vdots \\
Y_i &= a_{i1}X_1 + \cdots + a_{ij}X_j + \cdots + a_{in}X_n + B_i \gtrsim 0, \\
&\vdots \\
Y_m &= a_{m1}X_1 + \cdots + a_{mj}X_j + \cdots + a_{mn}X_n + B_m \gtrsim 0,
\end{aligned} \tag{7}$$

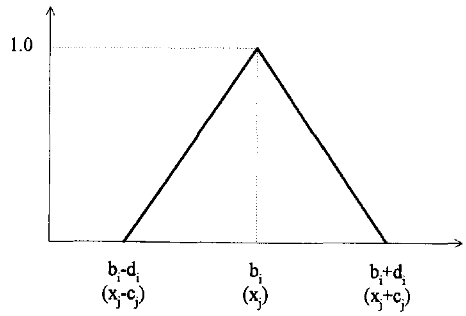

Fig. 2. Fuzzy numbers of b_i and x_i in inequality (7).

in which $B_i = (b_i, d_i)$ and $X_j = (x_j, c_j)$ are both triangular fuzzy numbers, with centers b_i and x_j and with extensions d_i and c_j, respectively. The fuzzy numbers can be shown as in Fig. 2. The membership function of inequality (7) is expressed as below according to a theorem proposed in [30]:

$$\mu_{Y_i}(y) = 1 - \frac{|y - \sum_{j=1}^{n} a_{ij} - b_i|}{\sum_{j=1}^{n} |a_{ij}| c_j + d_i}. \tag{8}$$

They further propose a definition to describe the fuzzy function and its membership function as

$$Y_i \gtrsim 0 \Leftrightarrow \mu_{Y_i}(0) \leqslant 1 - h_i, \tag{9}$$

in which h_i indicates the degree of $Y_i \geqslant 0$, that is, the larger is h_i, the more rigid is the meaning of "likely positive".

Consequently, inequality (9) becomes rewritten as

$$\sum_{j=1}^{n} (a_{ij} - h_i|a_{ij}|c_j) + b_i - h_i d_i \geqslant 0, \tag{10}$$

then on applying condition (10) to constraint (4) and maximizing the extension c_i, the optimization problem becomes represented as below:

$$\begin{aligned} \text{Max} \quad & \sum_{i=1}^{m} k_i c_i \\ \text{s.t.} \quad & \sum_{j=1}^{n} (a_{ij} - h_i|a_{ij}|c_j) + b_i - h_i d_i \geqslant 0, \qquad (11) \\ & 0 \leqslant h_i \leqslant 1, \end{aligned}$$

in which k_i is the weight parameter.

To set some h_i and k_i values, we can solve problem (11) to obtain the solution of the FLP.

2.3. Chanas's approach

To remedy a certain disadvantage of FLP approaches that obtain only the maximization alternative and lose information about a completely fuzzy decision, Chanas [6] proposed parametric programming as an alternative of FLP, by applying a parameter θ in inequality (3), which becomes a fuzzy parametric programming model as shown in inequality (12).

$$Cx \geqslant b_0 - \theta d_0, \tag{12a}$$

$$Ax \leqslant b_i + \theta d_i, \tag{12b}$$

in which θ is a number in the range [0, 1].

Also applying the Bellman and Zadeh's max-min operation [3], optimization of the membership grade of constraint (12b) denotes the following formulation:

$$\mu_c[A_i x(\theta)] = \min\{\mu_i[A_i x(\theta)]\} = \alpha = 1 - \theta, \tag{13}$$

in which α is the minimum membership grade of all constraints; the relation of α and θ can be simply used as a linear complementary form, also shown in Eq. (13), because the greater is the fuzziness of constraints, the larger is the value of θ and the smaller is the value of α.

In addition, the form of membership function of the objective function is the same as in Zimmermann's approach. The parameter θ and the optimal solution of the FLP also are obtained according to the fuzzy decision theory [3].

2.4. Julien's approach

Recently the corresponding possibilistic programming problem is viewed as an alternative to the stochastic in which the parameters are modeled as fuzzy variables instead of random variables. Julien [13] adapted Buckley's possibility programming [5] and combined an α-cut of fuzzy set theory. The model of FLP is shown as problem (14), indicating the fuzzy coefficients of a fuzzy objective and constraints, and a fuzzy RHS, can be formed as a coupled crisp LP shown in conditions (15) and (16).

$$\begin{aligned} &\text{Max} \quad \tilde{C}x \\ &\text{s.t.} \quad \tilde{A}x \leqslant \tilde{b}. \end{aligned} \tag{14}$$

$$\begin{aligned} &\text{Max} \quad C_{\mathrm{L}}^{\alpha}x \\ &\text{s.t.} \quad A_{\mathrm{U}}^{\alpha}x \leqslant b_{\mathrm{L}}^{\alpha}, \end{aligned} \tag{15}$$

$$\begin{aligned} &\text{Max} \quad C_{\mathrm{U}}^{\alpha}x \\ &\text{s.t.} \quad A_{\mathrm{L}}^{\alpha}x \leqslant b_{\mathrm{U}}^{\alpha}, \end{aligned} \tag{16}$$

in which the superscript α and subscripts L and U mean α-cut, lower and upper bounds of parameters, respectively. The optimal solution of conditions (15) and (16) can also be obtained by solution with algorithms of crisp LP; then the optimal solution of problem (14) can be represented as the range of those of conditions (15) and (16).

The fuzzy approaches play central roles as the basis applications in this study. Their algorithms of solution procedures are shown in Fig. 3. We take as an example analysis of assimilative capacity of a river basin shown in the next section.

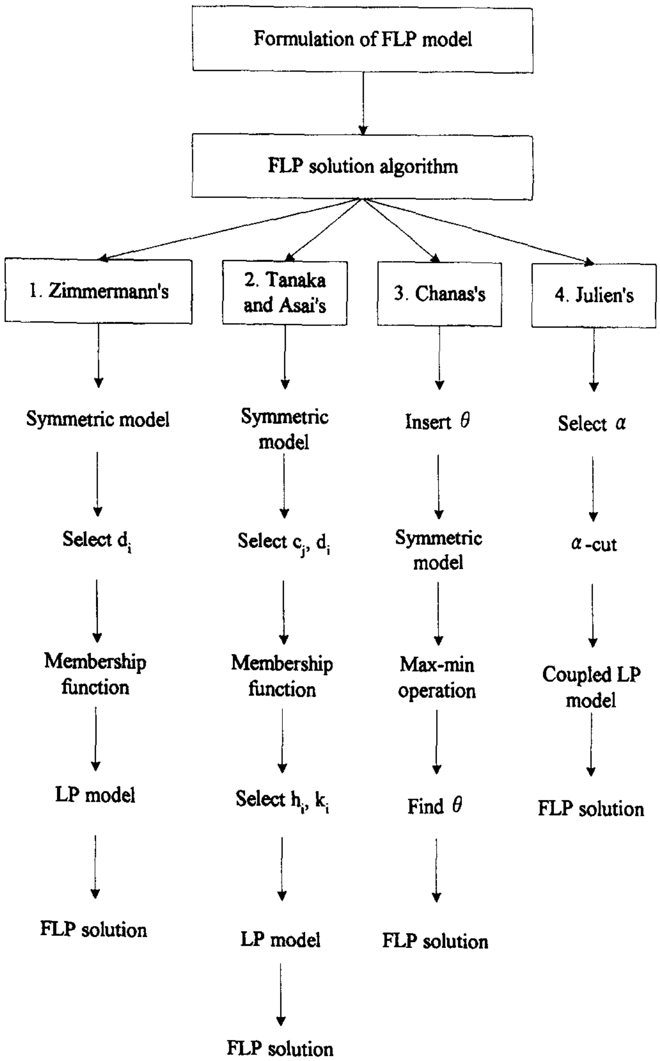

Fig. 3. Algorithms of the models of FLP selected in this study.

3. Application – analysis of assimilative capacity of a river basin

In the problem of water quality management, the important works are to analyze cost-effectiveness, which includes to minimize the cost of wastewater treatment and to maximize the assimilative capacity of pollution loading in the whole river basin. Many researchers have applied their mathematical programming models with precise ways and most have devoted themselves to minimum cost problems [2, 7, 17, 26, 34]. Our study focuses on finding the maximization of assimilative capacity of a river basin, so as to form strategies to allocate the allowable pollution loading to each pollution source. Furthermore, regulatory agencies may request these pollution sources to take the best practical technology (BPT) or the best available technology (BAT) for the required removal of waste.

The assimilative capacity is a renewable resource that indicates the capacity of a river to "digest" the pollution loading by biological activities and physical self-purification in a river. Each river has its assimilative capacity that depends on the use of the water body and its standards of water quality. The analysis of this capacity is an optimization problem of operational research that has constraints of models and standards of water quality, and so on.

In fact mathematical programming models contain uncertainties that include model formulation, determination of model parameters, vagueness of planning objectives and constraints, and computational procedures. In particular, the parameters of models of water quality are not in general deterministic. The error rate of a model may approach 300% [1]. Besides, regulatory agencies define standards of water quality by assessing precise limits on the level of specific indicators of water quality. Standards are commonly set based on imprecise environment goals and their determinations require risk assessments that are influenced by subjective and imprecise value judgements [13].

Accordingly, it is not so objective to describe problems by conventional analysis of assimilative capacity. Among complementary approaches developed, fuzzy set theory is particularly relevant to problems of water quality management [12, 13, 16]. Our main work consists of applying FLP to find the assimilative capacity in each reach and to allocate allowable pollution loading to each source to plan the management of water quality. According to the fuzzy model, the crisp objective function is the summation of assimilative capacity of each reach, and in its constraint set, water quality and average river flowrate are fuzzy numbers. The research flowchart can be illustrated as shown in Fig. 4.

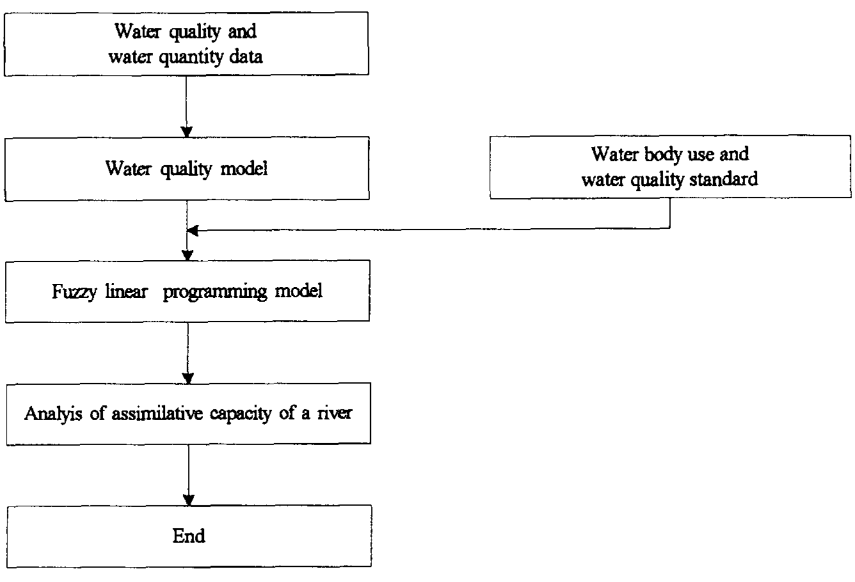

Fig. 4. Research flowchart of assimilative capacity of a river.

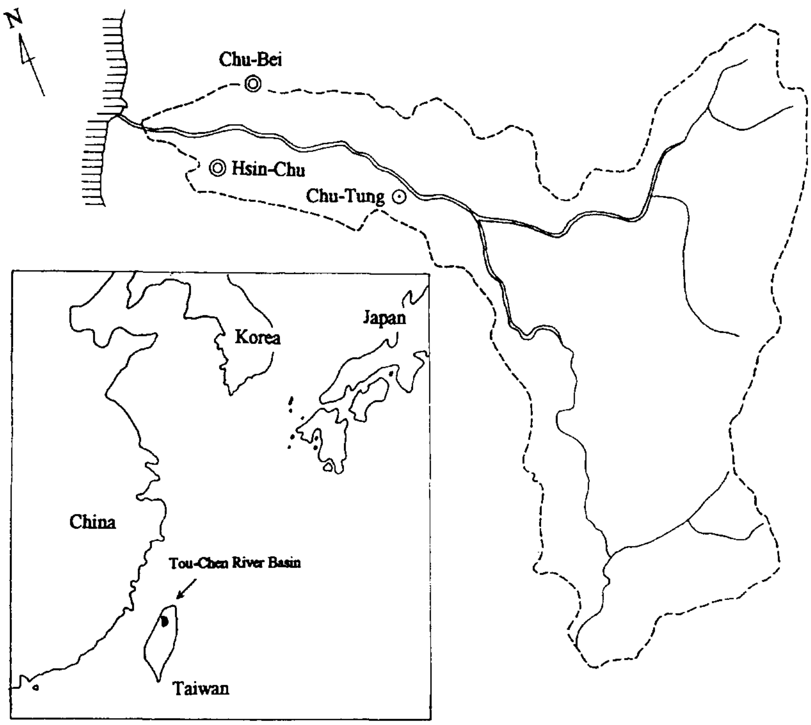

Fig. 5. Geographical location of Tou–Chen River Basin.

Table 1
Water quality models and standards of Tou-Chen River Basin

Reach	Water quality model	Water quality standard
1	$C = C_0 \times 10^{-0.016x}$	$BOD_5 \leqslant 1$ mg/l
2	$C = C_0 \times 10^{-0.017x}$	$BOD_5 \leqslant 1$ mg/l
3	$C = C_0 \times 10^{-0.156x}$	$BOD_5 \leqslant 2$ mg/l
4	$C = C_0 \times 10^{-0.133x}$	$BOD_5 \leqslant 2$ mg/l
5	$C = C_0 \times 10^{-0.067x}$	$BOD_5 \leqslant 2$ mg/l
6	$C = C_0 \times 10^{-0.003x}$	$BOD_5 \leqslant 2$ mg/l
7	$C = C_0 \times 10^{-0.044x}$	$BOD_5 \leqslant 3$ mg/l
8	$C = C_0 \times 10^{-0.034x}$	$BOD_5 \leqslant 3$ mg/l

X: distance from the starting point of each reach; C_0 and C: BOD_5 concentrations of the starting point and distance x of each reach.

The research area, the Tou-Chen River Basin, which is one of 21 main-streams, is located in the midnorth of Taiwan. The geographical location of the research area is shown in Fig. 5. The watershed area of Tou-Chen River Basin is 566 km^2, the stream length is 63 km, the population of the whole river basin is 314 700, its sources of wasterwater consist of municipal, animal, and industrial pollution in the ratio 89%, 5.5% and 5.5%, respectively (until 1992) [10]. The upper river area supports the use of drinking water in the region of Hsin-Chu. There are several cities in the river basin, the main ones being the Hsin-Chu, Chu-Tung, and Chu-Bei.

The standards of water quality are announced by the Environmental Protection Bureau of Taiwan and models of water quality employ the Streeter–Phelps formulation [28] adapted from National Central University, Taiwan [22], both as shown in Table 1. A programming model of assimilative capacity can be formulated as follows:

$$\text{Max} \quad Z = \sum_{i=1}^{8} A_i \tag{17}$$

subject to

(1) water quality model constraints (mass balance equation):

in the first reach

$$A_1 + Q_0 C_0 = Q_1 C_1,$$

in the following reaches

$$A_i + Q_{i-1} C_{i-1} \times 10^{-k^*_{i-1} x_{i-1}} = Q_i C_i, \quad i = 2, 3, \ldots, 8,$$

(2) flowrate gauge station constraints (mass balance equation, to calibrate C_i^* and C_i^{**}):
in the third, fourth, and sixth reaches

$$Q_i C_i \times 10^{-k_i^* x_1} = Q_i^* C_i^*, \quad i = 3, 4, 6,$$

in the sixth reach

$$Q_i^* C_i^* \times 10^{-k_i^* x_i^*} = Q_i^{**} C_i^{**}, \quad i = 6,$$

(3) water quality standard constraints:
at water quality monitoring stations in the first, third, fourth, sixth, and eighth reaches

$$C_i \leqslant C_{i,s}, \quad i = 1, 8,$$

$$C_i^* \leqslant C_{i,s}, \quad i = 3, 4, 6,$$

$$C_i^{**} \leqslant C_{i,s}, \quad i = 6,$$

(4) non-negativity constraints:

$$A_i \geqslant 0, \quad i = 1, 2, \ldots, 8,$$

$$C_i \geqslant 0, \quad i = 1, 2, \ldots, 8,$$

$$C_i^* \geqslant 0, \quad i = 3, 4, 6,$$

$$C_i^{**} \geqslant 0, \quad i = 6,$$

in which

Z: objective function of assimilative capacity (total allowable pollution loading) of BOD_5 (biochemical oxygen demand for five days, kg BOD_5/day);
A_i: allowable pollution loading of BOD_5 for reach i, $i = 1, 2, \ldots, 8$ (kg BOD_5/day);
C_0: BOD_5 concentration above the first reach (mg/l), in this study, $C_0 = 0.5$;
C_i: BOD_5 concentration from the starting point of reach i, $i = 1, 2, \ldots, 8$ (mg/l);
C_i^*: BOD_5 concentration at the midway of reach i, $i = 3, 4, 6$ (mg/l);
C_i^{**}: BOD_5 concentration at the midway of reach i, $i = 6$ (mg/l);
$C_{i,s}$: BOD_5 standard for reach i (mg/l);
Q_0: river flowrate above the first reach ($10^3\,m^3$/day);
Q_i: river flowrate of reach i, $i = 1, 2, \ldots, 8$ ($10^3\,m^3$/day);
Q_i^*: river flowrate at the flowrate gauge station of reach i, $i = 3, 4, 6$ ($10^3\,m^3$/day);
Q_i^{**}: river flowrate at the second flowrate gauge station of reach i, $i = 6$ ($10^3\,m^3$/day);
k^*: BOD_5 degradation coefficient (km^{-1});
x_i: distance from the starting point of reach i (km);
x_i^*: distance from the midway of reach i (km).

Then a practical model of assimilative capacity can be represented in Table 2.

The crisp optimal solution of assimilative capacity is 7185 kg BOD_5/day. According to the four fuzzy approaches of Section 2, we extend the crisp LP to FLP, and obtain a fuzzy assimilative capacity on applying an aspiration level (b_0), an admissible violation (d_0 and d_i), and an h_i value in the approach of Tanaka and Asai.

Table 2
Programming model of Tou-Chen River Basin

Objective junction	Max ΣA_i, $i = 1, \ldots, 8$
Constraint set	
Water quality model	$-A_1 + 654.1C_1 = 327.1$
	$A_2 + 630.2C_1 - 1071.2C_2 = 0$
	$A_3 + 621.8C_2 - 973.6C_3 = 0$
	$A_4 + 317.3C_3^* - 820.8C_4 = 0$
	$A_5 + 646.8C_4^* - 820.8C_5 = 0$
	$A_6 + 700.2C_5 - 760.3C_6 = 0$
	$A_7 + 755.1C_6^{**} - 741.0C_7 = 0$
	$A_8 + 734.6C_7 - 736.6C_8 = 0$
Flowrate gauge station	$731.2C_3 - 973.6C_3^* = 0$
	$560.6C_4 - 820.8C_4^* = 0$
	$747.3C_6 - 760.6C_6^* = 0$
	$753.8C_6^* - 760.3C_6^{**} = 0$
Water quality standard	$C_1 \leqslant 1$
	$C_3^* \leqslant 2$
	$C_4^* \leqslant 2$
	$C_6^* \leqslant 2$
	$C_6^{**} \leqslant 2$
	$C_8 \leqslant 3$
Non-negativity	$A_i \geqslant 0$, $i = 1, \ldots, 8$
	$C_i \geqslant 0$, $i = 1, \ldots, 8$
	$C_i^* \geqslant 0$, $i = 3, 4, 6$
	$C_i^{**} \geqslant 0$, $i = 6$

4. Results

All models of FLP for assimilative capacity can be converted into a crisp LP model that is solved with conventional algorithms, in this study, with the LINDO package [25]. For the four fuzzy approaches, in addition to crisp LP, their optimal assimilative capacities are shown in Table 3. Evidently, FLP admits the mere violation of inequalities of constraints and the assimilative capacities solved in this way are larger than from crisp LP. The present fuzzy approaches are flexible programming of types and they draw nearer the phenomenon of water environment and human thinking than crisp LP.

According to Zimmermann's approach, we chose the aspiration level $b_0 = 7000$ (kg BOD_5/day), and an admissible violation $d_0 = 700$ (kg BOD_5/day); the optimal solution is 7000 and λ is 1.0. That choice of b_0 is poor because the aspiration level is less than the assimilative capacity of crisp LP, the value of λ has to be 1.0. A similar condition also emerges for the case of $b_0 = 9000$ and $d_0 = 900$ because the value of λ is only 0.1.

In comparison with the approaches of Zimmermann, Tanaka and Asai, and Chanas, where we chose the maximum possible capacity to be approximately 8000 kg BOD_5/day (in Table 3, 8000, 8004, and 8000, respectively), we found that those solutions have the characteristic of consistency, that is, the assimilative capacities are 7660, 7680, and 7690, respectively. Hence those approaches are suitable for analysis of the assimilative capacity of a river basin.

Table 3
Assimilative capacity of Tou-Chen River Basin

LP model	Assimilative capacity	Condition
Crisp LP	7185	–
Zimmermann	7000	$\lambda = 1$, $b_0 = 7000$, $d_0 = 700$
	7660	$\lambda = 0.57$, $b_0 = 8000$, $d_0 = 800$
	8190	$\lambda = 0.1$, $b_0 = 9000$, $d_0 = 900$
Tanaka and Asai	7683	$h_i = 0.5$, $b_0 = 7185$, $d_0 = 718.5$
Chanas	7687	$\theta = 0.38$, $b_0 = 8000$, $d_0 = 800$
Julien	[0, 27 620]	$\alpha = 0$
	[966, 20 495]	$\alpha = 0.25$
	[2660, 14 955]	$\alpha = 0.5$
	[4504, 10610]	$\alpha = 0.75$

Table 4
Allowable pollution loading of each reach in Tou-Chen River Basin

LP model	Assimilative capacity	Reach 1	2	3	4	5	6	7	8
Crisp LP	7185	327	3836	0	1769	521	0	732	0
Zimmermann	7660	355	4000	0	1946	565	0	794	0
Tanaka and Asai	7683	359	3954	0	1961	599	0	810	0
Chanas	7687	352	4046	0	1911	586	0	792	0
Julien[a]	4504	0	2934	0	1227	0	0	0	0
Julien[b]	10610	410	5388	0	2295	1152	0	1365	343

Unit: kg BOD_5/day.
[a] Upper bound at $\alpha = 0.75$.
[b] Lower bound at $\alpha = 0.75$.

According to Julien's approach, we set a variable α-cut and obtain a variable range of the assimilative capacity. The solutions indicate that the smaller is the value of α, the less narrow is the range of the assimilative capacity and the more sharply increasing is the upper bound of the range of the assimilative capacity.

In each reach, the allowable pollution loading that may be identified according to the four FLP approaches is presented in Table 4. The results form strategies for allocation of an allowable pollution loading to each pollution source. Regulatory agencies may request these pollution source to take the BPT or the BAT for the required removal of waste by treatment of the water and have a basis of total elimination of mass loading in a deteriorating water body.

From the results of these four fuzzy approaches, the decision maker can choose the better one to obtain a compromise assimilative capacity. From applicable point of view, Zimmermann's and Chanas's approaches are more suitable for this analysis because they have criteria of few assumptions and easy computation.

5. Conclusions

In this study, we applied four FLP approaches to analysis of the assimilative capacity of a river basin. In advance, by setting aspiration level, admissible violation and others, we obtained an optimal solution of this capacity. It is obvious that the computational effort to determine a fuzzy set decision is larger than that to compute the crisp maximizing decision [38], but in the application to systems of managing water quality, it is not tedious to run the LINDO package. The choice of aspiration levels remains subjective; that property must be improved for more popular applications. The results of these approaches have consistency, that is, they approach a solution at the same aspiration levels and admissible violations. The assimilative capacity according to FLP exceeds that of the crisp approach; hence the fuzzy analysis is suitable for water environment. The advantages of fuzzy approaches are overall consideration of the imprecision of water quality standards, model formulation, parameter selection, and planning objectives and constraints.

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