Computer-Aided Civil and Infrastructure Engineering **15** (2000) 134–146

Earthquake Response Spectra Models Incorporating Fuzzy Logic with Statistics

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Abstract: *It is well accepted that the ground motion at a site depends on the rupture mechanism, source-to-site distance, local geologic conditions, and energy released by the earthquake. However, design spectra represent expected responses that do not explicitly include the influence of the uncertainties associated with these fundamental features. The objective of this article is to present a viable methodology that can be used to develop a response spectra using fuzzy logic and statistical analysis and to demonstrate how fuzzy-statistical response spectra can be used to evaluate potential structural response.*

Site-specific response spectra from the Northridge earthquake are used to develop response spectra models that quantify uncertainties inherent to the ground motion. The uncertainty in these computational models is quantified using fuzzy-set logic, statistics, and random vibrations. The local geologic conditions are characterized as rock or alluvium, and fuzzy sets are used to represent near, intermediate, and far epicentral distances. Proposed ground-motion models are used to define uncertain input motion for use in dynamic analyses of an example building. The resulting structural responses are compared with those obtained from time-dependent accelerations. Comparisons are made with the current design codes, and suggested implementation strategies for the proposed models are discussed.

1 INTRODUCTION

The massive damage caused by recent earthquakes in urban settings has motivated the reevaluation of earthquake-prediction methodologies and design load requirements, especially for near sites. Current needs to reduce earthquake damage include an improved understanding of the uncertainties related to the characteristics of ground motion and refinement of design codes. Previous earthquakes, in particular, Mexico City (1985), Loma Prieta (1989), and Northridge (1994), have highlighted the influence of site characteristics, distance from the source, and nature or the intensity of ground shaking.[2,3,6,7,11]

The response spectrum gives the maximum response of a single-degree-of-freedom (SDOF) mass oscillator subjected to an earthquake ground motion. An advantage is that the spectrum gives responses for a range of frequencies and different levels of damping, making it adaptable to structural design. Given a frequency and damping coefficient, it is possible to obtain spectral ordinates for use in modal superposition. Design spectra selection is based on local soil conditions.[22] However, once the design spectrum is selected, the remaining analysis is performed deterministically using expected spectral parameters obtained from the code. Although these spectra are derived from many earthquake time histories, the result is a single smoothed line that depicts expected ground motion at a prescribed level of damping with a single line without explicit consideration of the uncertainty inherent in the local site conditions. Specifically, the uncertainties due to local site conditions, distance from the site to the source, earthquake magnitude, focal distance, and soil-structure interaction are not considered explicitly in the development of the design spectrum. The spectral ordinates are linearly scaled by only the peak acceleration, and site effects are considered by assigning larger spectral accelerations at longer periods for softer and deeper soil profiles. The resulting ordinates are deterministic. Limited data available before the more recent earthquakes made it difficult to isolate the influencing factors. The Northridge earthquake with a moment magnitude of

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6.7 and thrust rupture mechanism has produced one of the largest known earthquake data sets from a single event, making it possible to consider the local site characteristics on the response spectra for a single earthquake mechanism.

The discussion on amplification and attenuation characteristics of seismic waves due to local site conditions began in the 1970s. Studies by Seed et al.[23] and Mohraz[20] showed that soil conditions have a significant effect on the shape of the response spectrum. It is widely recognized that soft soil sites have narrow-banded spectra, while rock and stiff soils transfer more frequencies and are broadband.[2,13,14] Borehole studies provide direct measurements of shear-wave velocity.[15] Different site characteristics influence the response-spectrum shape differently. However, their effect will vary depending on the distance of the recording from the source. This dependence is due to the different frequencies of vibration generated by the earthquakes of different magnitudes and from frequency-dependent attenuation. Motion in the near field of moderate and large earthquakes is poorly documented at present. There is no doubt that this motion is governed by physical laws different from those which govern intermediate and far-field regions that are better documented. Therefore, intermediate and far-field data should not be used for distances closer to the source than the source-site distances of the data for which the relationships are based. Near-field motions must be established using judgment and physical arguments rather than the data alone.[19]

Earthquake ground motion is characterized as a random vibration dependent on a number of factors including those listed previously and can be described by a transient stochastic function.[12] Simulated acceleration time histories can be generated based on mean and standard deviation values of random variables that describe the physical conditions listed above. Although these time histories consider the uncertainty of a number of factors influencing the ground-motion response, it is difficult to establish design loads based on time histories. As a result, response spectra have become the standard tool for the determination of seismic loads. Fuzzy sets are used to describe qualitative descriptions or variables defined by imprecise data. Fuzzy sets are not new to the field of earthquake engineering. For example, fuzzy sets have been used to evaluate seismic hazard,[18] to correlate earthquake load to damage states,[25] to prioritize bridge repair due to seismic damage,[10] and to quantify the uncertainties in a structural model and the subsequent response due to earthquake ground motions.[28] The use of natural language to describe earthquake magnitude, site conditions, and a site's proximity to a seismically active fault motivates the use of fuzzy logic to describe some of the uncertainties in the input motion.

The primary objective of this article is to propose a methodology to obtain ground-motion models that consider the uncertainty of local site geology and source-to-site distance in earthquake ground motion. Fuzzy logic is used with a statistical-based approach to rationally incorporate natural language in the computational modeling of earthquake ground motion. The proposed ground-motion models are used to investigate structural response considering the uncertainties associated with epicentral distance and local geologic site conditions. Ground-motion data from the 1994 Northridge earthquake is used to develop the proposed models. For the purposes of comparison, the proposed response spectra are compared with those suggested by the current design codes, and the structural response obtained from these proposed ground-motion models is compared with the response obtained by implementing step-by-step time integration and random vibration analyses.

2 USE OF FUZZY SETS IN QUANTIFYING UNCERTAINTY

The fuzzy set, first defined by Zadeh,[30] is used to give mathematical precision to processes that are imprecise and ambiguous. A fuzzy set supports data that do not clearly belong in a set but rather partially in a set. Examples of data and the fuzzy set to which they belong include shades of color that can be considered "blonde," ages that are "young," and distances that are "near." The membership that describes how well the data belong to the set is an essential parameter in the definition of a fuzzy set. As opposed to the traditional binary assignment of membership used in crisp-set theory, data can be assigned to a fuzzy set with variable membership given by

$$\mu_{\text{NEAR}}(X) \in [0, 1] \tag{1}$$

where $\mu_{\text{NEAR}}(X)$ is the degree of membership that X has in the fuzzy set of sites NEAR the earthquake source and X is distance between the site and the epicentral region, as shown in Figure 1.

It is important to note that the distances that belong to the set NEAR depend on how the fuzzy set will be used. For example, "near" a hospital and "near" an earthquake

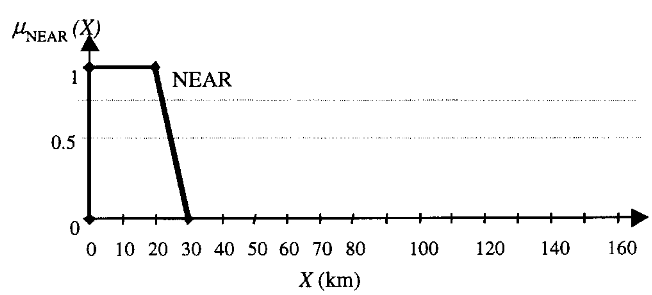


Fig. 1. Membership function for fuzzy set NEAR.

source imply different distances. The membership function indicates the membership grade of the elements in the set. The larger the likelihood that X will qualify as near, the higher is the degree of membership in the set. Membership can be assigned to fuzzy sets in a number of ways, including intuition (general intuitive understanding about a variable and its typical bounds), inference (knowledge about physical behavior), and rank ordering (achieved by polling populations). Fuzzy sets are used in this application to describe source-to-site distance and peak ground acceleration.

The following definitions[17,20] will be used in the development of the ground-motion models proposed in the next section. The *height* of the fuzzy set is the highest level of membership assigned to its members. *Normal* fuzzy sets have a height equal to 1, which is the maximum height permitted. Data with a membership of zero implies that the element is not a member of the fuzzy set, whereas membership greater than zero implies that the value does belong in the fuzzy set. The *alpha-cut* (α-cut) defines crisp-set bounds for the fuzzy parameter at a particular level of membership. The α-cut (N_α) and strong α-cut (N_{α^+}) for the fuzzy set NEAR (N) are defined by

$$N_\alpha = \{X | N(X) \geq \alpha\} \tag{2}$$

$$N_{\alpha^+} = \{X | N(X) > \alpha\} \tag{3}$$

The *support* of a fuzzy set is the crisp set that contains all the elements of the set with nonzero membership grades. A fuzzy set is *convex* if all the α-cuts are convex such that

$$\mu_N(y) \geq \min[\mu_N(x), \mu_N(z)] \qquad \text{where } x < y < z \tag{4}$$

Reworded, a fuzzy set is convex if there is only one crisp set contained by $N_{(\alpha=z)}$ and it is a subset of the crisp set contained by $N_{(\alpha=x)}$, where $x < z$. In Figure 1, an α-cut at 0.5 produces one crisp set from the fuzzy set NEAR. Thus NEAR is a convex fuzzy set. It is conceivable that an α-cut can produce two crisp sets, implying that the fuzzy set is nonconvex. However, most engineering contexts require convex fuzzy sets. The methodology presented in this article is restricted to convex fuzzy sets. The resolution principle can be used to define any fuzzy set with a family of its α-cuts. Two α-cuts are required to define a trapezoidal fuzzy set, with additional α-cuts leading to a more refined definition. Typical set operations such as intersection, union, and complement can be performed on fuzzy sets. Function mapping is performed on fuzzy sets using the extension principle[31] or approximated using the vertex method.[9] The vertex method, which is valid for convex and continuous fuzzy sets and implements interval analysis at the α-cut level, will be used here in conjunction with the proposed fuzzy models and modal superposition to estimate structural response.

Response spectra have a number of characteristics that can be considered vague or imprecise. While the data that are associated with a site-specific response spectra or an individual time history are quantitative and can be measured with a reasonable degree of accuracy, there are a number of design parameters that are ambiguous when the seismic load is unknown. These fuzzy parameters can be the shape of the smoothed response spectra, the soil type that corresponds to the design spectra, the magnitude of the earthquake that can be described as "severe," "moderate," or "mild," and the vicinity of the site to the epicentral region. Ambiguous seismic parameters can be expressed as fuzzy parameters.

In an earlier study, Wadia-Fascetti et al.[29] define the shape of the velocity response spectra with fuzzy sets. Since the response spectrum is the graphic representation of the maximum response of a series of SDOF systems as a function of period, the fuzzy response spectrum provides bounds for the maximum response of the structure under consideration. From a design point of view, the engineer is interested in the maximum response that a structure will experience in its lifetime. A fuzzy response spectrum provides a "ballpark" number for the parameter of interest, which can be more realistic, since the analyst is equipped with more information than what is obtained from current design approaches. In this study, fuzzy sets, which are used to define natural-language parameters, are used to construct response spectra that incorporate vagueness in distance and soil properties.

Local-site shear-wave velocity and source-to-site distance are the two fundamental fuzzy variables addressed in this article. Although shear-wave velocity of the soil profile is a representative parameter for the quantification of uncertainty, lack of detailed borehole measurements at all the recording sites makes this approach not feasible. In this study, sites are classified as rock and alluvium. Source-to-site distance is defined by fuzzy sets. Note that the shape (triangle, trapezoid, or parabolic) of a fuzzy set does not influence the final result (in this case, structural response) as much as the selection of variables and the amount of overlapping between the fuzzy sets.

Fuzzy sets are used to describe sites that are "near" (N), "intermediate" (I), and "far" (F) from the epicentral region. Most methods for constructing membership functions are almost invariably based on experts' judgment. Intuition and inference are used to define the fuzzy sets based on the meaning of the natural language in the context of earthquake engineering. The resolution principle is used to define normal trapezoid-shaped fuzzy sets with two α-cuts each at membership levels 1 and 0^+. The logic used to define the crisp sets at each α-cut level is as follows: The distances captured by α-cut 1 should be the distances that are most commonly associated with the linguistic description and have complete membership in the set. In a similar manner, the α-cut 0^+ bounds must contain the most extreme cases and include all values that may be considered part of the set. Most experts would agree that sites between 40 and 60 km of the epicentral region could be classified at an intermediate distance, while the most extreme case is to include sites between 20 and 70 km. The information gathered from the works of Chang et al.,[5] Somerville et al.,[24] Seed et al.,[23] Borcherdt and Glassmoyer,[3] and Campbell[4] is used to construct α-cut 0^+ and α-cut 1 bounds for each of the distance categories as shown in Figure 2.

The logic used in the definition of near, intermediate, and far sites requires that the fuzzy sets are normal and convex. The procedure presented in the next section involves statistical analyses on data sets defined by each α-cut. The number of sites that fall within α-cut 0^+ and α-cut 1 for each fuzzy set for alluvium and rock sites, respectively, are shown in Table 1. In total, there are 54 alluvium sites and 23 rock sites. It is worth noting that due to the overlapping nature of the fuzzy sets, some sites fall into two different fuzzy sets. Some boundaries were modified to ensure that sufficient data are available because the fuzzy sets prescribed here are used to define data sets for statistical analyses in the ground-motion model development. In particular, the near and far fuzzy sets were adjusted to account for limitations in the number of sites available in these regions. With additional data available, the α-cut 1 bounds would be narrowed for the near and far sites, creating a greater difference between the distances in membership levels 1 and 0. Regardless, the distance fuzzy sets make it possible to consider the potential effect of distance as a fuzzy variable in the ground motion.

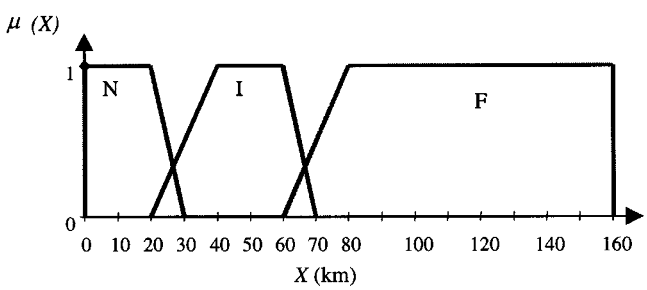


Fig. 2. Distance fuzzy sets.

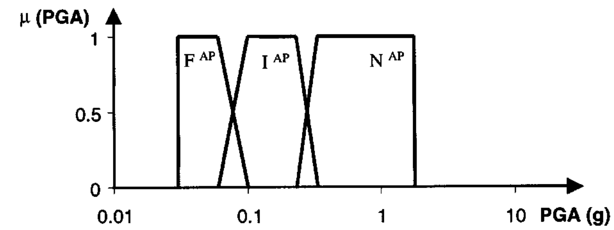
Peak ground acceleration (PGA) is a function of earthquake magnitude, distance, and local site conditions. The objective here is to represent the amplification and attenuation characteristics of PGA at sites that fall within the three distance categories defined in Figure 2. Fuzzy sets for near, intermediate, and far sites are defined in terms of the fuzzy parameter PGA for two soil types: rock and alluvium. The logic is that there are certain values of PGA that are associated with each site category and soil type for a given magnitude. For example, soft soil sites that are at far distances tend to amplify PGA in narrow frequency bands, while PGA is likely to attenuate more with distance for rock sites. The development presented in this article is based on data from a single event; thus the PGA fuzzy sets defined here are valid for a moment magnitude equal to 6.7.

While it is recognized that the soil sites can be distinguished further based on shear-wave velocity, the decision to discriminate between the two site types, alluvium and rock, produces sufficient data required for the statistical analyses presented in Section 4. This decision also coincides with the commonly used general site descriptors, soil and rock. Parameters from the Northridge earthquake are used to define the fuzzy sets using a rank-order approach. The PGA from each site is "polled" and ranked by distance. α-Cut 0^+ bounds are established so that the minimum and maximum PGA values of the response are captured by the distance categories shown in Figure 2. α-Cut 1 bounds represent the most frequent PGA values for the earthquake in that distance category. These two bounds define trapezoid-shaped membership functions for the peak ground-motion acceleration. The Tarzana record located 5 km on an alluvium site from the source and with an unexpectedly high PGA of $1.78g$ is still included in the fuzzy set as the upper bound of α-cut 0^+ because it represents an extreme case in a single event. Even though the Tarzana event is extreme, it is important to consider such an event when discriminating between distances. Somerville et al.[24] suggested that Northridge data are not anomalous but rather representative of the ground motions expected near thrust faults.

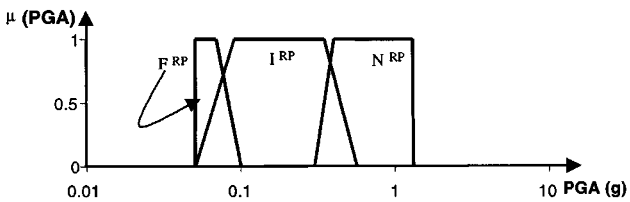
The fuzzy sets for PGA at alluvium and rock sites are shown in Figure 3. The notation used to distinguish between distance and soil category for the fuzzy sets is D^{SP}, where D is the distancecategory (N, I, or F), S is the

Table 1
Number of sites available in each distance category

	Near		*Intermediate*		*Far*	
Soil type	*α-cut* 0^+	*α-cut* 1	*α-cut* 0^+	*α-cut* 1	*α-cut* 0^+	*α-cut* 1
Alluvium	8	1	31	15	24	20
Rock	3	3	17	12	5	2



(a) Alluvium Sites



(b) Rock Sites

Fig. 3. Distance fuzzy sets for PGA.

soil type (A or R), and P denotes the parameter described, PGA. The PGA fuzzy sets are defined in accordance with the distance fuzzy sets and data from the Northridge earthquake. The polling procedure identified PGA values for sites that fall within the different distance categories at the two membership levels that define the PGA fuzzy sets. Using this procedure, the Northridge earthquake data suggest that the upper and lower bounds of the near and far PGA fuzzy sets are the same. For instance, the Tarzana site, which yielded the highest PGA value equal to $1.78g$ is at a distance that falls within membership values 1 and 0^+. Using the polling procedure described above, the $1.78g$ PGA should appear in both membership levels in the N^{AP} fuzzy set, which accounts for the vertical drop. Note that this also coincides with the vertical drops in the distance categories shown in Figure 2. Vertical drops are obtained in a similar manner for the far alluvium sites and the near and far rock sites.

While it is conceivable that PGA values can be larger than $1.78g$, larger values did not occur within this data set. It is difficult to justify bounds at an α-cut level without having the experience to validate the decision. One could argue that the PGA fuzzy sets should have a lower bound of zero as an extreme case. Since the PGA fuzzy sets are defined for an event with a 6.7 moment magnitude, it would not be logical to propose zero PGA values at sites that are expected by the earthquake. Finally, the approach used to define the PGA fuzzy sets in this study is based on observed data. It should be noted that this is not the only way PGA fuzzy sets could be defined, and the user's objectives may suggest a different approach such as mild, moderate, and severe PGA fuzzy sets.

3 GROUND-MOTION MODELS

Two common representations of ground motion are response spectra and power spectral density functions where response spectra are approximate, since they do not give a unique accelerogram. From an engineering standpoint, the response spectrum is the most convenient representation of ground motion because it characterizes the ground motion produced by earthquakes and their effect on elastic single-degree-of-freedom systems. In this study, ground motion is characterized in three ways using ground motion from the Northridge earthquake: (1) statistics of the response spectrum combined with fuzzy distances, (2) statistics of the normalized acceleration spectrum with fuzzy distances, and (3) the Kanai-Tajimi[16] spectral density function.

3.1 Unnormalized response spectrum for fuzzy distance groups

Fuzzy unnormalized response spectra are constructed for the three fuzzy distance groups. The model is obtained by calculating statistical properties of site-specific acceleration response spectra with 5 percent damping. For each α-cut, the mean $m_\alpha(T)$ and standard deviation $\sigma_\alpha(T)$ are calculated for each period T (the inverse of frequency) of the response spectra. The mean response spectrum is the plot of m_α versus T. The standard deviation quantifies the variance of the shape at each period and leads to confidence bounds for the mean shape. α-Cut bounds are defined by combining $m_\alpha(T)$ and $\sigma_\alpha(T)$ such that α-cut 1 is

$$Sa_{\alpha=1} = [m_{\alpha=1}, m_{\alpha=1} + \sigma_{\alpha=1}] \tag{5}$$

and α-cut 0^+ is

$$Sa_{\alpha=0^+} = [m_{\alpha=0^+}, m_{\alpha=0^+} + 2\sigma_{\alpha=0^+}] \tag{6}$$

where Sa is the fuzzy acceleration for a period T.

The α-cut 1 bounds contain the most possible values of Sa, and α-cut 0^+ bounds contain extreme cases of Sa. By definition, a response spectrum provides the maximum response for a system. Thus Equations (5) and (6) define upper and lower bounds for expected maximum response. Both bounds are given here for completeness; however, the upper bounds provide the most information in evaluating potential structural response. One-sided ranges are used in the α-cut definitions because the response spectra represent maximum response, and it would be counterintuitive to propose maximum responses for a lower bound that has a high probability of being exceeded. Since the distance bounds for α-cut 0^+ are larger than those for α-cut 1, the sites included in the statistical analysis will be different for each membership level. α-Cut 0^+ includes all the sites for a particular distance group. To consider extreme responses, the upper bound of α-cut 0^+ is defined as $m + 2\sigma$.

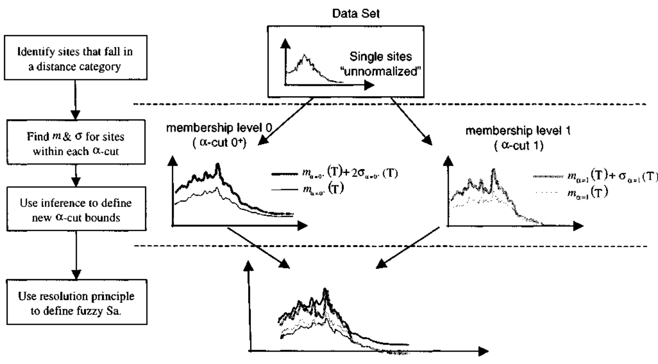


Fig. 4. Schematic describing construction of unnormalized fuzzy acceleration response spectra.

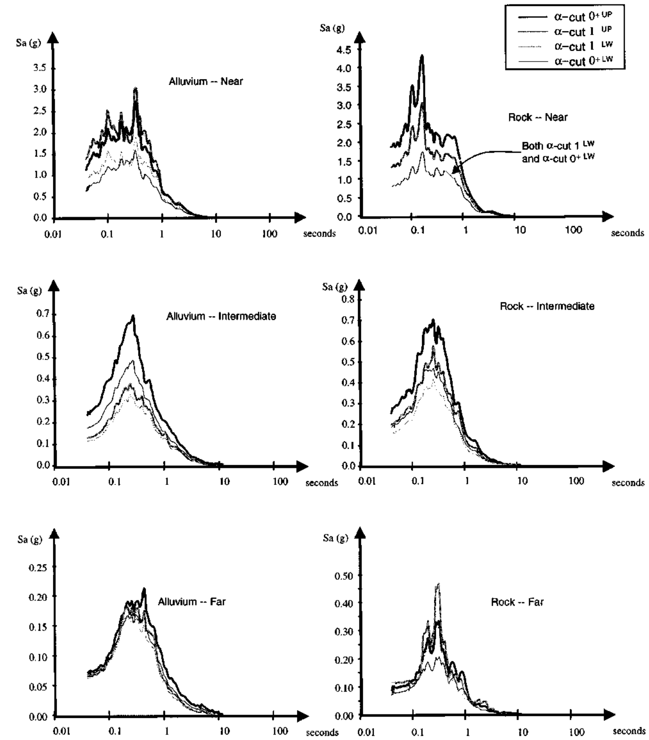


Fig. 5. Unnormalized fuzzy response spectra.

Figure 4 is a schematic of the process described in the preceding paragraph. The process begins with a data set comprised of individual site-specific acceleration response spectra from free-field sites from the California Strong Motion Instrumentation Program (CSMIP). Response spectra with 5 percent critical damping are selected for use in this article. Other structural damping levels can be incorporated into the model by modifying the spectra used in the original data set. The statistical properties are obtained for the six data sets defined by the three distance fuzzy sets (N, I, and F) at α-cuts 0^+ and 1 for each fuzzy set. Equations (5) and (6) define the bounds for the two α-cut levels, and finally, the resolution principle is used to define the final fuzzy response spectrum Sa. The fuzzy spectrum Sa has the traditional coordinates along the horizontal and vertical axis, while the membership scale is represented on an axis perpendicular to the plane of the paper. The resulting fuzzy response spectra for each of the distance categories for near, intermediate, and far alluvium and rock sites are shown in Figure 5 with membership bounds for α-cuts 0^+ and 1 overlaid on the traditional axes.

The bounds defined in Equations (5) and (6) are shown in each fuzzy response spectrum. The superscripts UP and LW denote the upper and lower bounds for each α-cut. It is not logical for any α-cut to result in two crisp sets; thus the resulting fuzzy response spectrum should be convex. The resulting fuzzy sets shown in Figure 5 must be modified to enforce convexity before proceeding with any analysis by comparing the upper bounds for α-cut 1 and α-cut 0^+. If the α-cut 1 bounds fall outside the α-cut 0^+ bounds, the α-cut 0^+ bounds should be redefined such that α-cut 0^+ = α-cut 1. This modification is performed when the ground-motion models are applied to the examples in Section 4.

The fuzzy response spectra characterize trends in ground-motion behavior. Frequency content narrows with distance for alluvium sites. Peak ground acceleration attenuates for all cases, with the greatest difference between near and intermediate sites. Since response spectra give maximum response values, the upper bounds are of most interest; both bounds are included here for completeness.

In all cases except for alluvium sites near and rock sites far from the epicentral region, the α-cut 0^+ upper bound captures the largest range of frequency. This can be explained by the different data sets used in the statistical analysis. Although the additional data included in the α-cut 0^+ bounds represent extreme cases in distance, the corresponding response spectra characteristics may not have been extreme, and the additional data available in α-cut 0^+ may pull the average response down. There is a greater difference between Sa at the α-cut 1 and α-cut 0^+ bounds for the near and intermediate categories and less at sites far from the source. This can be attributed to the fact that for farther distances, both magnitude and amplification decrease. The α-cut 0^+ bounds include more sites in the calculation of $m_{0^+}(T)$ and $\sigma_{0^+}(T)$, and the effect of the sites on the averages is insignificant. The definition of the far distance category was partially due to the limitations in the number of far rock sites. These distance categories, especially the bounds for far sites, can be refined with additional data. The additional data should be from an earthquake with a similar magnitude and rupture mechanism.

3.2 Normalized fuzzy response spectra

The second ground-motion model proposed is a fuzzy normalized acceleration response spectrum. In this case, the response spectra are normalized to peak ground acceleration. Site conditions are considered in the same manner as before, with the rock and alluvium data separately. The normalized fuzzy spectra are constructed using the same process except that $m_\alpha(T)$ and $\sigma_\alpha(T)$ are found using the normalized spectra. Use of normalized spectra has several advantages. First, the amplification characteristics for the sites are averaged together. Second, normalized response spectra can be combined with normalized spectra from other earthquakes with different magnitudes. Finally, removing the PGA from individual sites and then reapplying it as an uncertain parameter as part of the structural analysis makes the procedure applicable to earthquakes with different magnitudes. The resulting fuzzy response spectra for each of the distance categories for alluvium and rock sites are shown in Figure 6.

Normalization to PGA makes it possible to combine data from earthquakes with different magnitudes. Since ground-motion characteristics are dependent on focal depth, it is important to combine ground-motion data from similar rupture mechanisms. The response spectra shown in Figure 6 include the α-cut bounds and the design spectrum recommended by the code.[22] There is less uncertainty in these resulting spectra because PGA, a significant uncertain parameter in site response, has been removed. The design spectrum underestimates the maximum response for all cases; however, the importance of capturing each peak may not be significant because inelastic structural behavior will reduce the design loads. The shape of the design spectrum is a good match for all fuzzy response spectra. However, it is interesting to note that there is not much conservation, especially at the tails. There is difficulty in capturing the shape of the rock spectra at near sites, which is likely due to limited data. The best fit is for intermediate sites, since there was an insufficient amount of data available for near-source sites until the Northridge earthquake.

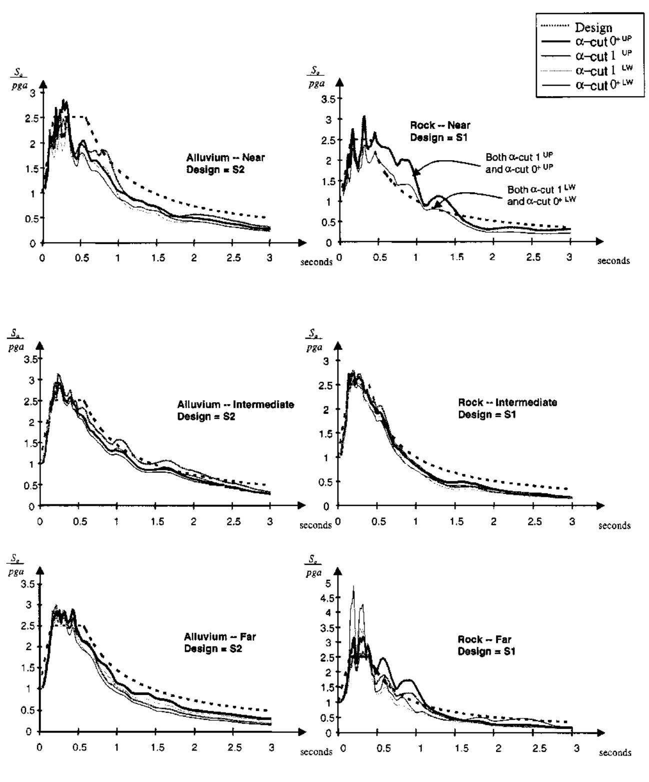


Fig. 6. Fuzzy normalized response spectra.

3.3 Probabilistic representation of ground motion: Spectral density functions

Random vibration theory is the third method used to represent uncertainties in ground motion in this article and is used as a comparison with the previous two models proposed. A brief description of random vibration theory is given here. The reader is referred to Vanmarke,[27] who gives an excellent review of random vibration theory, for an in-depth explanation of the concepts discussed in this subsection and subsection 4.3. The ground-motion is assumed to have a zero mean and a time-invariant spectral density function. Assuming that an earthquake has a limited duration and is of a stationary stochastic process makes it possible to represent the motion as a spectral density function (SDF) $G(\omega)$. The most commonly used $G(\omega)$ to describe ground motion is that of Kanai-Tajimi[16]:

$$G(\omega) = \frac{[1 + 4\zeta_g^2(\omega/\omega_g)^2]G_0}{[1-(\omega/\omega_g)^2]^2 + 4\zeta_g^2(\omega/\omega_g)^2} \qquad (7)$$

Equation (7) represents the stationary frequency content of the acceleration response for a single-degree-of-freedom (SDOF) system with natural frequency ω_g and viscous damping ζ_g when excited by a white-noise excitation intensity G_0. The spectral parameters that describe the shape of the Kanai-Tajimi SDF for a soil profile are location of the predominant frequency ω_g and its bandwidth ζ_g. Soft soil sites typically have a single predominate frequency and have narrow peaks (small ζ_g), while rock sites are capable of transferring wider-frequency bounds (large ζ_g), and the variability in relative frequency content is dependent on the variability in ω_g and ζ_g. Quantification of the variability in ω_g and ζ_g will describe the uncertainty in the local site conditions. The variability of the local site parameters include some of the ground-motion effects due to intensity and duration.

The spectral shape parameters (ω_g, ζ_g, and G_0) for the normalized Kanai-Tajimi $G(\omega)$ parameters (such that the area equals 1) for each site are used as the basis for comparison with the previous two models. Spectral shape parameters are selected such that the spectral moments of the Kanai-Tajimi $G(\omega)$ match the moments of the squared Fourier amplitudes $|f(\omega)|^2$ obtained from each of the strong-motion records.

The ith moment of any spectral density function is defined as

$$\lambda_i = \int_0^\infty \omega^i G(\omega) d\omega \qquad i = 0, 1, 2, \ldots \tag{8}$$

where $G(\omega)$ is replaced by $|f(\omega)|^2$ to obtain the moments for the actual ground motion. Frequency and bandwidth, the governing parameters for the normalized Kanai-Tajimi SDF, are related to the first three moments by

$$\omega_f = \sqrt{\frac{\lambda_2}{\lambda_0}} \tag{9}$$

and

$$\zeta_f = \sqrt{1 - \frac{\lambda_1^2}{\lambda_0 \lambda_2}} \tag{10}$$

where ω_f and ζ_f denote frequency and bandwidth parameters obtained directly from a site-specific digitized response spectrum. The moment λ_0 of the Kanai-Tajimi spectrum is assumed to equal that of the digitized response spectrum σ_f^2 and is matched to the variance of $G(\omega)$ by assuming $\sigma_0^2 = \sigma_f^2$. The intensity parameter G_0 can be computed analytically using the definition of moments and from the Kanai-Tajimi spectrum by evaluating the integral

$$\sigma_0^2 = \lambda_0 = \int_0^\infty G(\omega)\, d\omega \tag{11}$$

and solving for G_0,

$$G_0 = \frac{4\zeta_g \sigma_0^2}{\pi \omega_g (1 + 4\zeta_g^2)^{1/2}} \tag{12}$$

Spectral parameters that describe the shape of the normalized $G(\omega)$ are computed for each site using Equations (9), (10), and (12). The mean values for ω_g and ζ_g at the rock sites are 23.12 rad/s and 0.49, respectively. These parameters together with G_0 describe the uncertainty in the ground-motion for the random vibration analyses presented in Section 4.

4 STRUCTURAL RESPONSE

In this section the three models presented in Section 3 are used to calculate maximum structural responses. The structural responses obtained via these three models are compared with responses of the same buildings through time-history analysis and with responses from ground motions recorded during the San Fernando earthquake (1971). This analysis is performed for the following purposes:

- To demonstrate how the proposed fuzzy spectra can be used in a building analysis.
- To assess the error between the response obtained from the time histories and the response obtained from the proposed ground-motion models developed with the same earthquake records.
- To assess the integrity of the proposed ground-motion model by comparing it with records from a different earthquake that had a similar magnitude.
- To compare the results obtained from the proposed methodology presented in this article with the structural response obtained from the random vibration approach.

The governing equation of motion for a linear n degree of freedom (DOF) structural system is

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f}(t) \tag{13}$$

where $\mathbf{M}$, $\mathbf{C}$, and $\mathbf{K}$ are the system mass, damping, and stiffness matrices with dimensions $n \times n$. The response of the system is displacement $\mathbf{x}$ (with the first and second time derivatives denoting system velocity and acceleration), and the applied force as a function of time t is $\mathbf{f}$. Equation (13) can be solved using step-by-step integration in the time domain, modal superposition, or a frequency-domain response.

4.1 Structural response using fuzzy response spectra

System frequencies (ω) and mode shapes (ϕ) correspond to the undamped free-vibration eigenproblem

$$\omega_j^2 \mathbf{M}\phi_j = \mathbf{K}\phi_j \tag{14}$$

for mode j.

Solution of the eigenvalue problem is convenient, since the dynamics for each mode are equivalent to that of an SDOF problem. If a time history is available, the time-dependent responses for the modes are superimposed to obtain the complete system response. In design, a response spectrum is used to estimate the maximum response for each mode. The modes are superimposed using the complete quadratic combination rule (CQC) for modal combination.[8] In general, the maximum response can be approximated by

$$|P_{\max}| = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{m} L_i L_j \phi_i \phi_j S_p(i) S_p(j) \rho_{ij}} \tag{15}$$

where $P_{\max}$ is the maximum value of the parameter of interest, L is the participation factor for each mode such that $L_j = \phi_j \mathbf{M}\{1\}$, S_p is the maximum value of the parameter of interest (obtained from the response spectrum), and ρ_{ij} is a factor representing the correlation between modes i and j. The ground-motion models presented here give acceleration as the spectral parameter. This parameter can

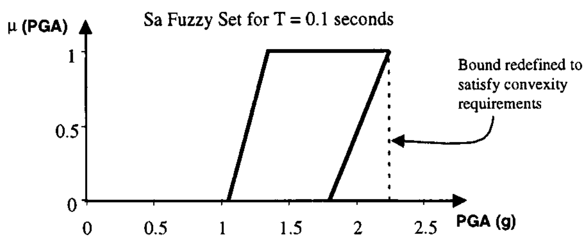


Fig. 7. *Sa* fuzzy set for near alluvium sites at $T = 0.1$ s.

be converted to any parameter of interest. For example, displacement is Sa/ω^2.

In this article, the structural system is assumed to be deterministic, and the only fuzzy parameter in the solution is Sa. Each mode corresponds to a fuzzy set for Sa from the fuzzy response spectrum. For example, Figure 7 shows the Sa fuzzy set for a structure on near-alluvium sites with a period T of 0.1 seconds ($T = 1/\omega$). The modal damping characteristics of the structure are included as Sa is selected based on the critical damping of the current mode. Consistent with the models developed in Section 3, 5 percent critical damping is assumed for each mode.

The vertex method[9] is implemented at each α-cut. Equation (15) is evaluated deterministically using the upper and lower bounds of each α-cut. The structural response becomes a fuzzy set with α-cuts defined by the resulting upper and lower bounds. The vertex method can support additional fuzzy sets, nonconvex fuzzy sets, and nonlinear mapping functions by adding permutations to the analysis.

4.2 Structural response using fuzzy normalized response spectra

Structural response is evaluated using the modal superposition approach described in subsection 4.1 with one exception. The spectral ordinates need to be unnormalized with the fuzzy sets defined for PGA. This fuzzy calculation reintroduces the amplification uncertainty into the ground-motion model. After scaling by PGA, the procedure continues by solving the free-vibration eigenproblem and solving Equation (15) in terms of the fuzzy variable Sa.

4.3 Structural response using random vibration theory

Dynamic analysis using random vibration theory is performed in the frequency domain and is governed by

$$G_Y(\omega) = |H^2(\omega)|G(\omega) \tag{16}$$

where $H(\omega)$ is the system transfer function, $G_Y(\omega)$ is the spectral density of the structural response, and $G(\omega)$ is the input motion presented in subsection 3.3. For a single-component input, the transfer function can be written to describe the structural system in terms of its stiffness, damping, and mass matrices and is given by

$$H(\omega) = \mathbf{a}^T[\mathbf{K} - \omega^2\mathbf{M} + \omega\mathbf{C}i]^{-1}\mathbf{Mr} \tag{17}$$

where $\mathbf{r}$ is an $n \times 1$ vector of values for each DOF when a unit displacement is applied at the ground, and $\mathbf{a}$ is an $n \times 1$ zero vector with a unit value at the DOF of interest.[1]

An approach presented by Vanmarke[26] is used to estimate the peak response due to $G(\omega)$. The moments λ_{0y}, λ_{1y}, and λ_{2y} are found by evaluating Equation (8) with the response spectral density function $G_Y(\omega)$, where the center frequency is defined as

$$\Omega_y = \frac{\lambda_{2y}}{\lambda_{0y}} \tag{18}$$

and a dimensionless measure of variability in the frequency content is defined as

$$\delta_y = \sqrt{1 - \frac{\lambda_{1y}^2}{\lambda_{0y}\lambda_{2y}}} \tag{19}$$

The maximum response at probability level p, duration s, and in terms of Ω_y and δ_y is approximated as

$$y_{\max} = \sigma_y\sqrt{2\ \ln\left[2n\left(1 - \frac{1}{e^{\alpha}}\right)\right]} \tag{20}$$

where $\sigma_y^2 = \lambda_{0y}$,

$$n = \frac{\Omega_y s}{2\pi(-\ln\ p)} \tag{21}$$

and

$$\alpha = \delta_y^{1,2}\sqrt{\pi\ \ln n} \tag{22}$$

The analysis approach presented here will be applied to the example in the next subsection as a comparison with the results found from the preceding two ground-motion models.

4.4 Illustrative examples

The three-story shear building shown in Figure 8 is used to demonstrate the use of the ground-motion models on structural response. The results from the three ground-motion models presented in Section 3 are also compared with those obtained using time-step integration.

Fuzzy sets for maximum roof displacements for alluvium and rock sites are shown in Figures 9 and 10. The solid gray and black lines denote the maximum roof displacements obtained from the unnormalized and normalized spectra, respectively. The dashed black line denotes the maximum response with the upper bounds of Equations (5)

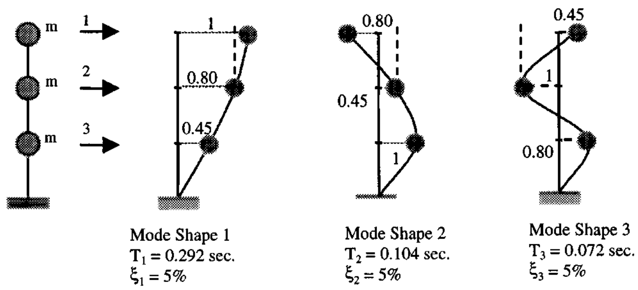


Fig. 8. Example three-story shear building.

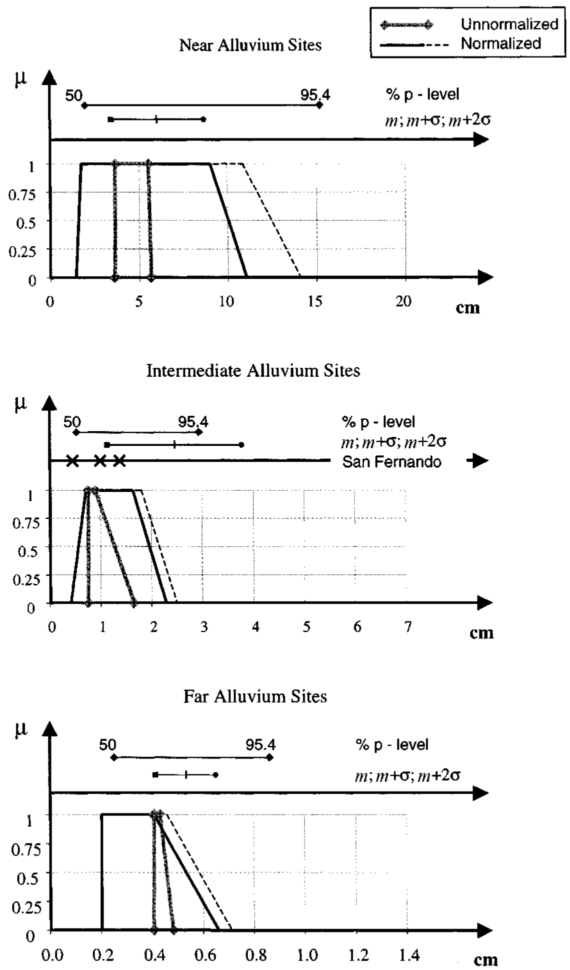


Fig. 9. Fuzzy sets for maximum roof displacements on alluvium sites.

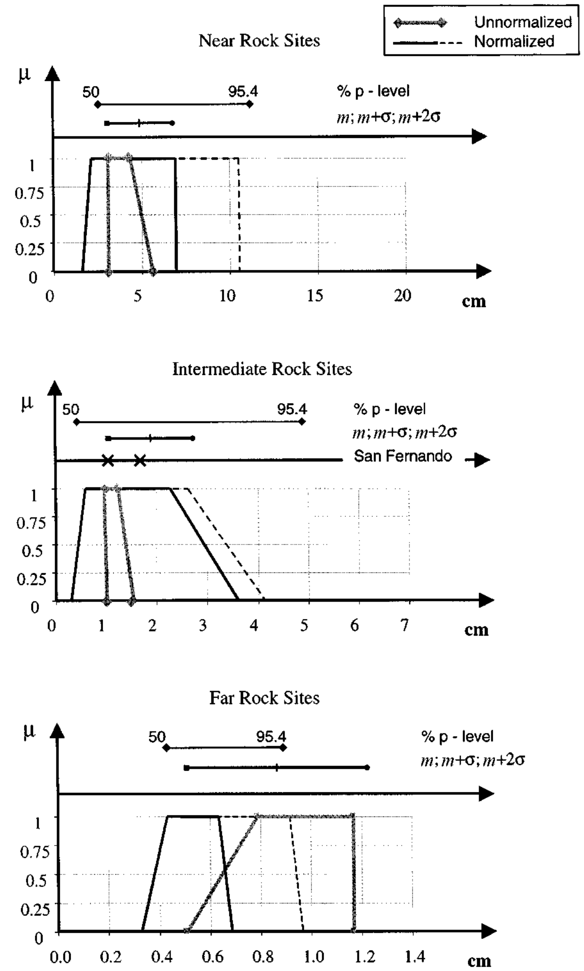


Fig. 10. Fuzzy sets for maximum roof displacements on rock sites.

and (6) modified to $m_{\alpha=0^+} + 2\sigma_{\alpha=0^+}$, and $m_{\alpha=0^+} + 3\sigma_{\alpha=0^+}$, respectively.

As a means for comparison to the fuzzy sets, two horizontal lines are also plotted on the same scale above each graph. The top line highlights maximum roof response obtained from the random vibration analyses using mean values for the Kanai-Tajimi spectral parameters. To ensure that the results are consistent with the original comparisons, the averages are recalculated for each soil and distance category, and response is evaluated at 50 and 95.4 percent probability levels. The second line is a statistical summary in terms of mean (m) and standard deviation (σ) of the maximum response from the time-history analyses. The markers denote, from left to right, m, $m + \sigma$, and $m + 2\sigma$ maximum roof displacement. The statistics performed for the random vibration analysis and time-history response used data defined by α-cut 0^+ for each distance fuzzy set.

The final comparison is to time-history response from the San Fernando earthquake, which had a similar rupture mechanism and magnitude ($M_w = 6.7$). Free-field data from the San Fernando earthquake are scarce, and only responses for a few intermediate sites were available. Individual maximum roof responses for ground motion

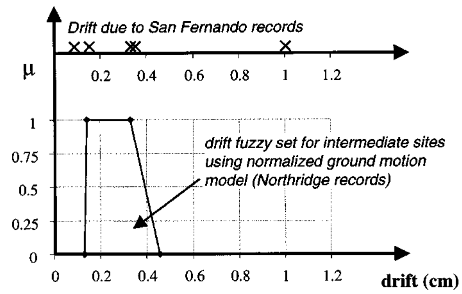


Fig. 11. Interstory drift fuzzy set compared with maximum drift values from the San Fernando records.

recorded from the San Fernando records are depicted on the top horizontal axis with "X" marks.

All fuzzy sets shown have been adjusted to satisfy convexity requirements. The primary difference between the responses obtained from the unnormalized spectra and the time-history analysis is due to the approximations inherent in modal superposition. The normalized spectra capture the maximum response in almost all cases, with α-cut 1 capturing the m and the $m + \sigma$ responses and α-cut 0^+ capturing extreme responses. The random vibration predictions did not capture the extreme responses for the far rock sites. The difference in the bounds between the unnormalized and normalized spectra is due to the use of the fuzzy PGA. The uncertainty in PGA, which was removed from the model construction, is reapplied as part of the structural analysis. This uncoupling of the uncertainty causes the bounds to broaden in the final analysis. Intermediate fuzzy sets capture the response from the San Fernando earthquake, although it is important to know that some extreme cases from this earthquake may not have been evaluated. The normalized fuzzy response spectrum combined with fuzzy PGA appears to be the most promising approach to quantify the effect of source-to-site distance and local site characteristics.

Interstory drift, the incremental lateral deflection between two stories, is a critical parameter for structural performance in the design of buildings subjected to lateral loads. The fuzzy set shown in Figure 11 is the maximum interstory drift between the middle floor and roof valid for intermediate alluvium sites. As a comparison, interstory drift is evaluated with San Fernando records from five intermediate sites. The fuzzy set captures the actual drift values reasonably well. The case that falls outside the fuzzy set corresponds to a site with an actual PGA that was not contained within the PGA fuzzy set used here for intermediate sites. This site should be analyzed at both near and intermediate distances. Thus evaluation for this site using parameters for near sources would have captured this response.

It should be emphasized that the responses shown in Figures 9 and 10 are predictions for maximum roof displacements. The fuzzy-set bounds are upper and lower estimates of maximum displacements. Thus the upper bounds contain the most significant information from a design point of view.

4.5 Discussion

The examples presented in this article demonstrate the use of the proposed fuzzy ground-motion models as an analysis tool with comparisons made to other commonly used tools. The scope of this work was to use data from a single earthquake to develop the methodology for fuzzy ground-motion models and to evaluate the potential usefulness of the proposed models. The merit of the proposed ground-motion models is demonstrated through the evaluation of structural response. The fuzzy normalized ground-motion model is the most promising from a design point of view. The normalized shape makes it possible to incorporate earthquakes of different magnitudes by using the PGA fuzzy sets. While not within the scope of this study, the fuzzy normalized ground-motion model should be improved through the integration of additional data sets, especially at near and far sites, before it can be used as a design tool. The additional sites will improve the model's ability to characterize ground motion, especially in regions with limited data such as the near and far sites. Additionally, the response spectra should be smoothed for use in design.

The fuzzy response spectra are selected based on the vicinity of the site to seismic fault areas, which is generally known and characterized as fuzzy—"near," "intermediate," or "far." To ensure that all possible ground-motion characteristics are considered, sites that fall within 30 km of a possible source should be evaluated for all three distances. Sites at distances greater than 30 km and less than 70 km should be evaluated for intermediate and far distances, and finally, sites greater than 70 km of a potential seismic source should be evaluated for far distances.

5 CONCLUSIONS

Ground-motion data recorded by CSMIP during the Northridge earthquake are used to develop fuzzy response spectra to quantify the uncertainties in ground motion due to local site conditions and source-to-site distance. Local sites are characterized as alluvium or rock, and epicentral distance is categorized as near, intermediate, and far, with fuzzy sets defined for distance and PGA. Epicentral distance and local site conditions are shown to affect the fuzzy response spectrum. The fuzzy bounds in the response spectra distinguish between frequency content for different site conditions and amplification magnitude for different distance groups. Spectral accelerations generally attenuate as the site distance increases, whereas high amplification is observed for far rock sites.

In general, the unnormalized fuzzy spectra capture the mean roof displacements, however, they are less effective in estimating the extreme responses. Displacement upper bounds estimated by the normalized fuzzy spectra envelop the time-history analysis results at near and intermediate distances for both site conditions. The bounds representing the results from the random vibration analyses envelop the time-history analysis in most cases. However, it should be noted that the random vibration approach is very sensitive to the specification of the parameters used.

The computational models for earthquake ground motion demonstrate the use of fuzzy logic in the quantification of uncertainties inherent in earthquake ground motion. The combination of fuzzy sets to describe natural language and statistics to quantify frequency in response lead to a rational design tool. The models presented here are attractive for design purposes because many designers already think in terms of bounding and enveloping response. This approach can be used to assess the potential variability in parameters that influence structural performance such as interstory drift. The proposed methodology makes it possible to estimate the uncertainty in structural responses due to ground motion at different source to site distances.

ACKNOWLEDGMENTS

The research presented in this article has been supported in part by the Research and Scholarship Development Fund at Northeastern University and was performed as part of the second author's graduate studies, which were supported by the Department of Civil and Environmental Engineering for 2 years. The random vibration analysis was performed using a Matlab-based code written by Professor Dionsio Bernal at Northeastern University. This support is gratefully appreciated.

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