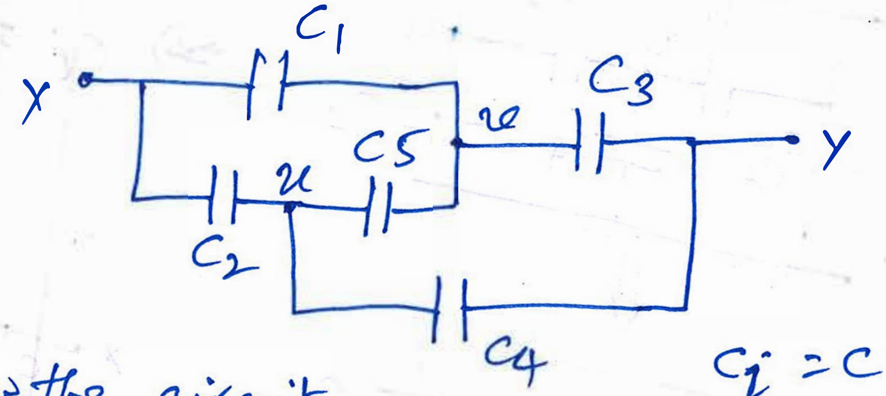
EECE105L Solutions to Tutorial Sheet 10

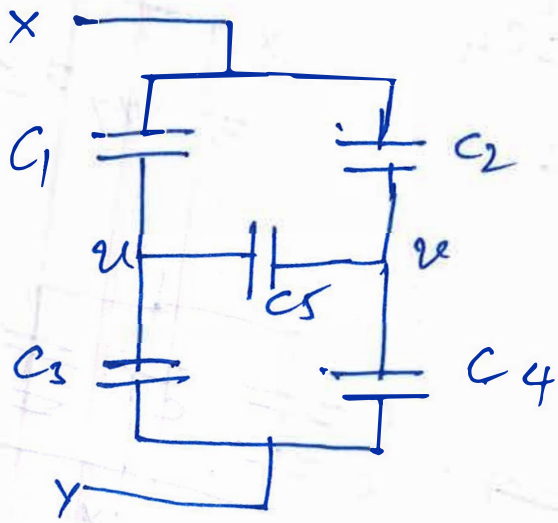
1)

Fig. 4

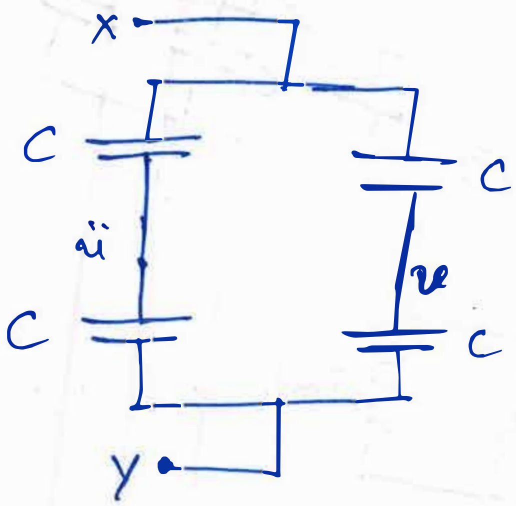


$C_i = C$

redraw the circuit

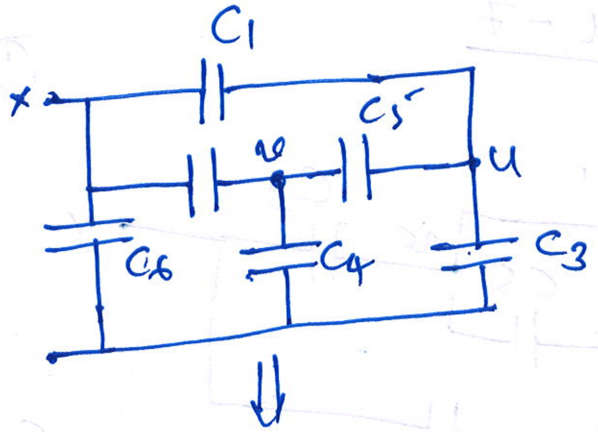


$\because C_i = C,$
Capacitor between u and v acts like a open. Then, re-drawing the circuit

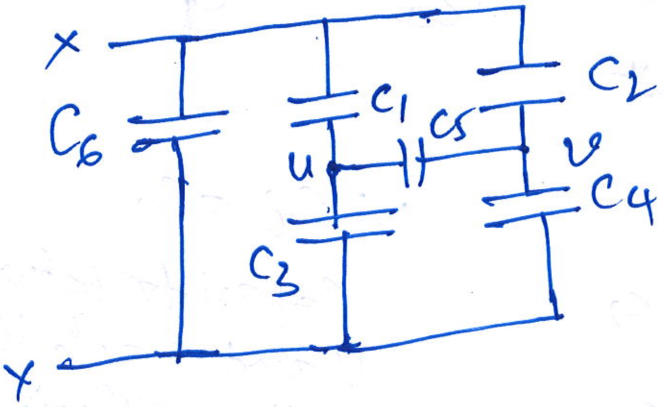


$\Rightarrow C_{xy} = C$

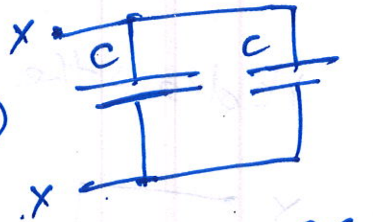
Note: Fig. 6 is similar except one more capacitor in parallel



$\Rightarrow C_i = C$

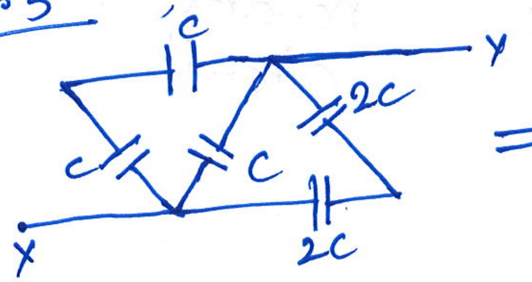


$\Rightarrow$

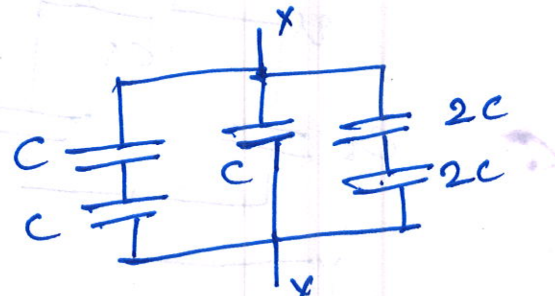


$\Rightarrow C_{xy} = \underline{2c}$

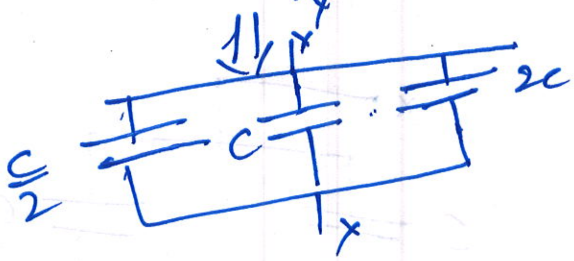
Fig 5



$\Longrightarrow$



$C_{eq} = 2.5c \Leftarrow$



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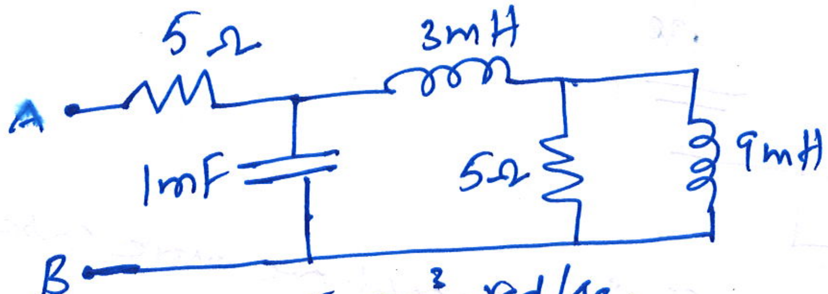


$\omega = 2.5 \times 10^3$ rad/sec.

$X_{1mF} = -\frac{j}{\omega C} = -j0.4\,\Omega$

$X_{3mH} = j\omega L = j7.5\,\Omega$

$X_{9mH} = j\omega L = j22.5\,\Omega$

③

$$5 \parallel X_{9mH} = \frac{5 \times j22.5}{5 + j22.5} = \frac{112.5j\,(5 - j22.5)}{(5 + j22.5)(5 - j22.5)}$$

$$= \frac{562.5j + 2531.25}{(5)^2 + (22.5)^2} = 4.76 + 1.05j$$

$$(5 \parallel X_{9mH}) + X_{3mH} = 4.76 + 1.05j + j7.5$$

$$= 4.76 + j8.55$$

$$X_{1mF} \parallel \left[(5 \parallel X_{9mH}) + X_{3mH} \right] =$$

$$= \frac{(4.76 + j8.55)(-j0.4)}{(4.76 + j8.55 - j0.4)}$$

$$= \frac{3.42 - j1.904}{(4.76 + j8.15)}$$

writing in $|z| \angle \theta$ or $r \angle \theta$ form,

$\theta = \tan^{-1}(b/a)$

$r = \sqrt{a^2 + b^2}$

$$\underset{(a)}{3.42} - j\underset{(-b)}{1.904} = 3.91 \angle -29.1^\circ$$

$$4.76 + j8.15 = 9.438 \angle 59.71^\circ$$

$re^{j\theta}$

$\downarrow$

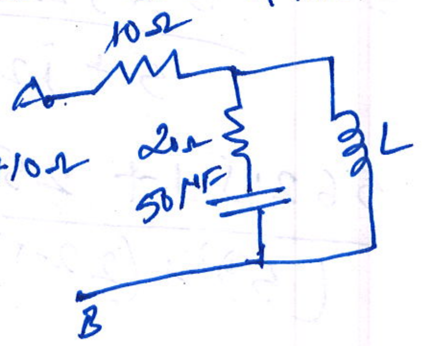
$a + jb$

$$X = \frac{3.91 \angle -29.1}{9.438 \angle 59.71^\circ} = 0.414 \angle -88.1^\circ = 0.414 + 0.014j$$

$$Z = 5\Omega + X = 4.586 + j0.014$$

4) given $Z_{AB} = 25 + j10\,\Omega$ $\omega = 4\times10^3$ rad/sec.

$Z_{AB} = \left[j\omega L \,//\, \left(20 - \frac{j}{\omega C}\right)\right] + 10\,\Omega$



$25 + j10 = 10 + j\omega L$ $\omega C =$

$20 - \frac{j}{\omega C} = 20 - j5$

$j\omega L \,//\, (20 - j5) = \frac{j\omega L(20 - j5)}{(20 - j5 + j\omega L)}$

$25 + j10 - 10 = \cancel{20j\omega} \frac{j20\omega L + 5\omega L}{(20 + j(\omega L - 5))}$

$(15 + j10)\left[20 + j(\omega L - 5)\right] = 5\omega L + j20\omega L$

Equating real parts

$300 - 10\omega L + 50 = 5\omega L \Rightarrow L = \underline{5.83\,mH}$

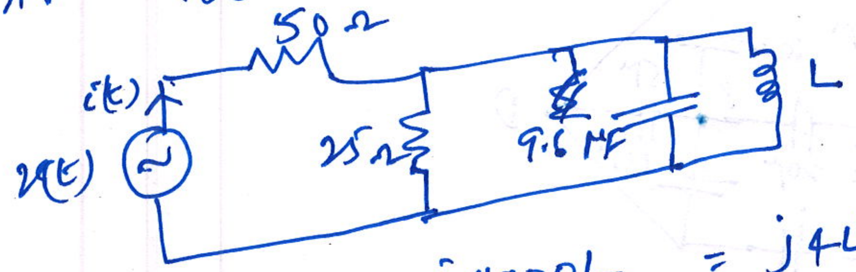
Equating imaginary parts,

$200j - 75j + j15\omega L = 20\omega L$

$125 = 5\omega L \Rightarrow \cancel{\omega} \underline{L = 6.25\,mH}$

So two inductance values are possible.

5) Given that $v(t)$ and $i(t)$ are in phase



$\omega = 4000 \text{ rad/sec.}$

$L' = 1000L$

$X_L = j\omega L = j4000L = j4L'$

$X_C = \frac{-j}{\omega C} = -26.04j$

$Z_{eq} = 50 + (X_L \parallel X_C \parallel 25)$

for phase to be zero, imaginary part needs to be zero.

$X_L \parallel X_C \parallel 25 = \frac{X_C * X_L * 25}{X_C X_L + X_C 25 + X_L 25}$

$\frac{2604L'}{104.16L' + j100L' - 651j} + 50$

$= \frac{2604L'(104.16L' - j(100L' - 651)j)}{(104.16L')^2 + (100L' - 651)^2} + 50$

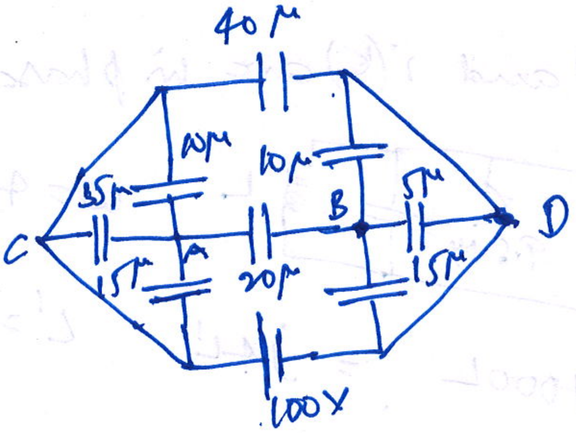
Equating imaginary part to zero,

$\cancel{104.16L'}\ 100L' - 651 = 0$

$L' = \frac{651}{100} = 6.51$

$L' = 1000L \therefore L = \underline{6.51 \text{ mH}}$

6)



(6)

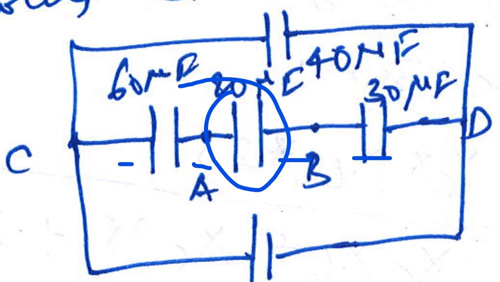
$10\mu \parallel 35\mu \parallel 15\mu = 60\mu F$

$5\mu \parallel 15\mu \parallel 10\mu = 30\mu F$

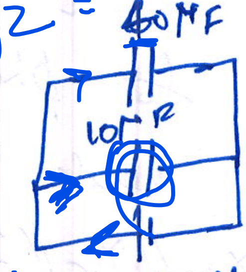
re-drawing circuit,

$\frac{Q^2}{2C}$ $\frac{1}{2}CV^2$

$\boxed{V = IR}$



$\Rightarrow$



$C = \frac{Q}{V}$

$Q = CV$

$\frac{10 \times 5000}{50}$

1000

As 60μF, 20μF, 30μF are in series,

$C_{eq} = 1/\left[\frac{1}{60} + \frac{1}{20} + \frac{1}{30}\right] = 10\mu F$

$10\mu F \parallel 40\mu F = 50\mu F$

$Q = C_{eq} V = 50 \times 10^{-6} \times 100 =$ CV

5000μC

Charge in 10μF capacitance is

$100 \times 10\mu F = 1000\mu C$

So charge on 20μF capacitance is

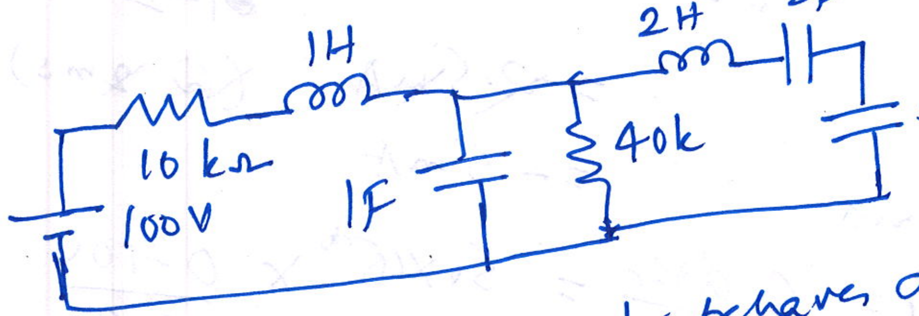
~~2000~~ 1000μC

So voltage across 20μF capacitance is

$V = \frac{Q}{C} = 50V$

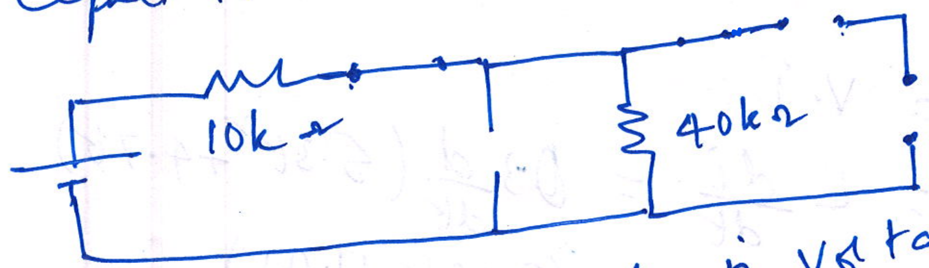
So energy stored in $\frac{1}{2}CV^2 = \underline{0.025 W}$

7)



$\frac{3}{5} \times 80$ $\boxed{1.2}$ $\frac{2}{5}\times 80$

In steady state, inductor behaves as a short, capacitor as an open. Redrawing the circuit



So voltage across 40kΩ is voltage across 2F, 3F capacitors combined.

$$V_{40k} = \frac{100 \times 40}{10+40} = \underline{80 V}$$

$V = \frac{Q}{C}$

$$V_{2F} = \boxed{\frac{80}{C_2} \Big/ \left(\frac{1}{C_2} + \frac{1}{C_3}\right)} = \frac{80 \times 3}{5} = \underline{\underline{48V}}$$

8) $V_L = L\frac{di}{dt}$ $\quad L = 2mH \quad i(t) = \sin(377t)$

$\therefore V_L = 2 \times 10^{-3} \times 377 \cdot \cos(377t)$

$= 0.754 \cos(60t) V \quad \omega = 377 \frac{rad}{sec}$

$E = \frac{1}{2}Li^2 = 10^{-3}\sin^2(377t)$

9) ~~8)~~ $i_C = C \frac{dv_C}{dt} = 50\times10^{-6} \times \frac{100-0}{1-2}$

$= \cancel{-2.5 mA} \quad \cancel{(@ 2ms)}$

$= -5 mA$

at 3ms

$i_C = C \frac{dv_C}{dt} = 50\times10^{-6} \times \frac{0-100}{2-3}$

$= \underline{5mA}$

~~9)~~ 10) $\underline{P = V.I}$

$\underline{V_L = L \frac{di}{dt}} = 0.3 \frac{d}{dt}(5.3t^2 + 4.7t)$

$(3.18t + 1.41)$

$P = (5.3t^2 + 4.7t)(3.18t + 1.41)$

$= (16.854t^2 + 22.419t + 6.627)t$

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(8)