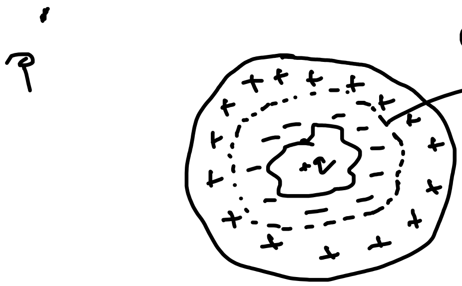
10.12.20

Conductors (contd.)

Gaussian surface

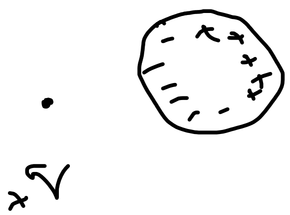
- Spherical conductor
- Cavity inside that holds a point charge 'q'



→ Inside the cavity, $\vec{E} \neq 0$

→ Outside the cavity but inside the conductor

$$\vec{E} = 0$$



Gauss' law : $\oint \vec{E} \cdot d\vec{a} = \frac{q_{enc.}}{\epsilon_0}$

If $q_{enc.} = 0 \Rightarrow \vec{E} = 0$

$q_{induced} = -q$

For points outside the conductor :

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

→ charge 'q' uniformly distributed over the outer surface.

⇒ Principle of Faraday cage (grounded)

Capacitor:

→ Two conductors with charges $+q$ & $-q$.

Both equipotential.

Potential difference:

$$V = V_+ - V_- = -\int \vec{E} \cdot d\vec{l}$$

↳ Evaluating $\vec{E}$ can be difficult if surface is not symmetric

(*) We know, $\vec{E} \propto q$

$$\Rightarrow V \propto q$$

$$\Rightarrow C = \frac{q}{V}$$

→ +ve number always.

↳ Proportionality constant.

→ Called the capacitance. (Measured in farads)

Depends on sizes, shapes and separation of the two conductors.

- For a single conductor: (with +ve charge q)
  ↳ we can think of the 'second conductor' with -ve charge & a radius which is infinity. (surrounds the conductor in question)

Ex: Parallel plate capacitor:

$\vec{E}$ (due to one of this plates) $= \frac{1}{\epsilon_0} \frac{Q}{A}$;

A → Area of the plate
Q → Total charge on the plate
d → Separation of the two plates.

Potential difference, $V = \frac{Q}{\epsilon_0 A} d$

Capacitance, $C = \frac{A \epsilon_0}{d}$

(*) Work done to charge up a capacitor to 'Q'

→ we are removing electrons from one plate and adding them to the other.

Say, at some point of time, we have on the positive plate a charge $= +q$

→ Potential difference (V) $= \frac{q}{C}$

In order to bring charge 'dq' from +ve to -ve plate: $\quad dW = V\,dq = \frac{q}{C}\,dq$

Hence, $\underset{\substack{\downarrow \\ \text{Total work done}}}{W} = \int_0^Q \frac{q}{C}\,dq = \frac{1}{2}\frac{Q^2}{C} \qquad | \; Q = CV$

$$= \frac{1}{2} CV^2$$