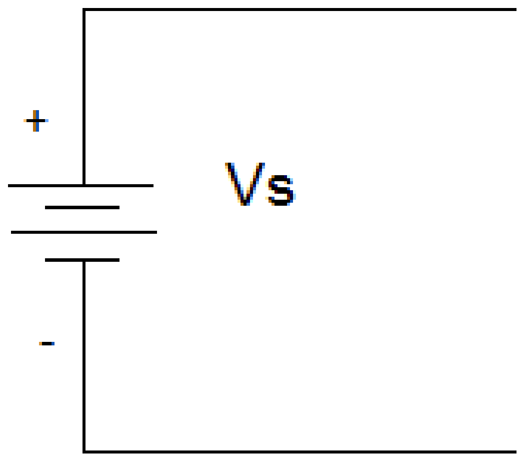
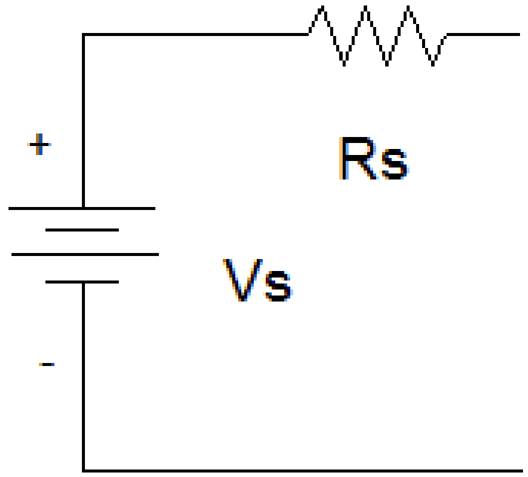
Network Theorems

Voltage Sources

- Ideal voltage sources can produce as much current as is needed to provide power to the rest of the circuit.
- An ideal voltage source has zero internal resistance.
- A real voltage source is modeled as an ideal voltage source in series with a resistor.
- There are limits to the current and output voltage from the source.



Ideal Voltage Source



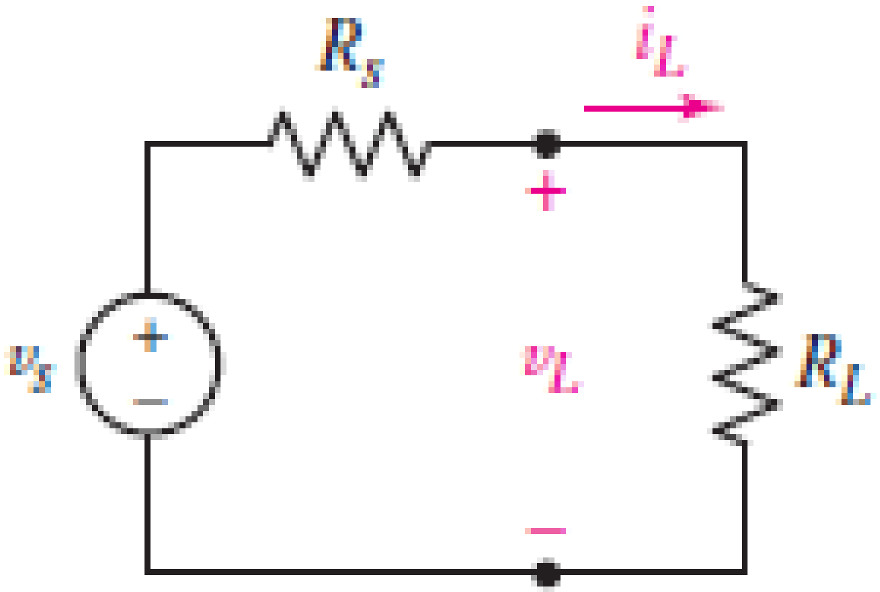
Real Voltage Source

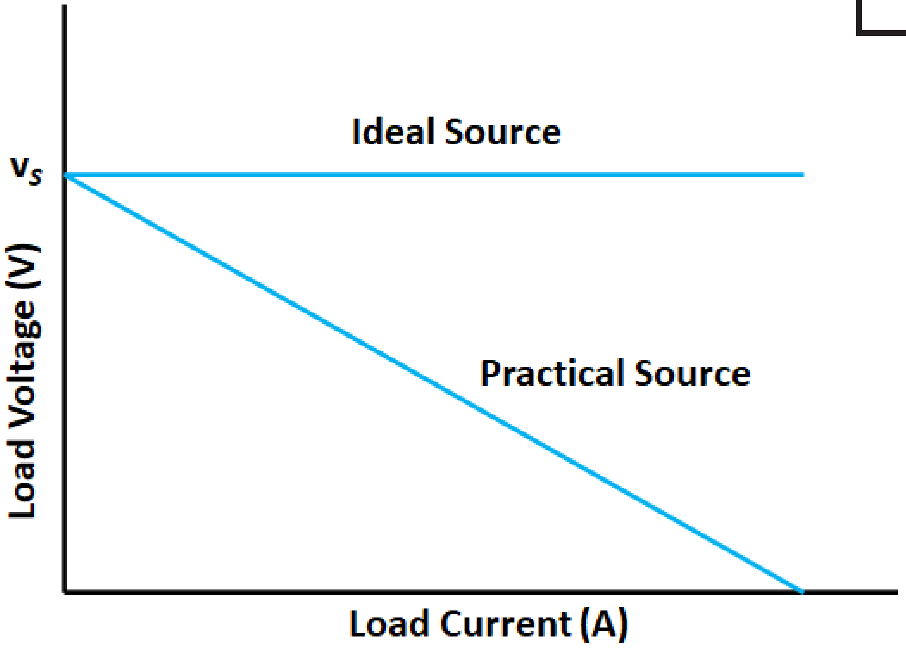
Voltage Sources….continued

- When R_L = ∞ no current flows through the load, the practical source is open-circuited and the terminal voltage, or open-circuit voltage, is $V_{OC}= v_S$.

- When R_L = 0, short-circuiting the load terminals, then a load current or short-circuit current, I_{Lsc} = ∞, would flow.

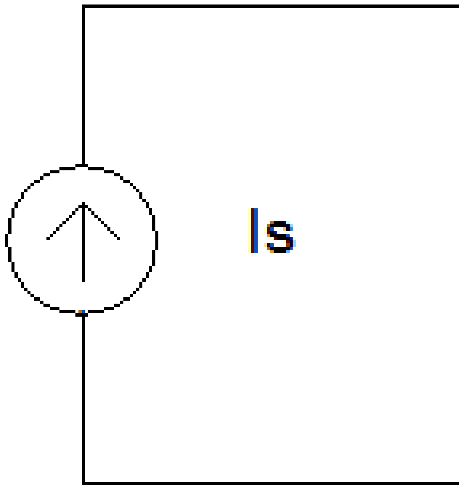
$$\boxed{v_L = v_S - i_L R_S}$$



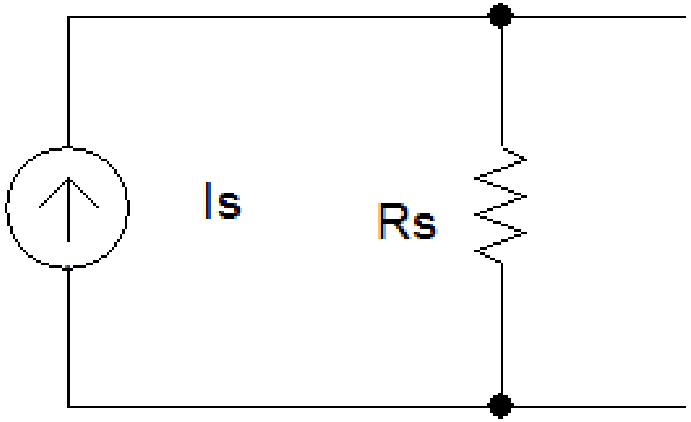


Current Sources

- An ideal current source can produce as much voltage as is needed to provide power to the rest of circuit
- An ideal current source has infinite internal resistance connected in parallel.
- A real current source is modeled as an ideal current source in parallel with a resistor.
- Limitations on the maximum voltage and current.



Ideal Current Source



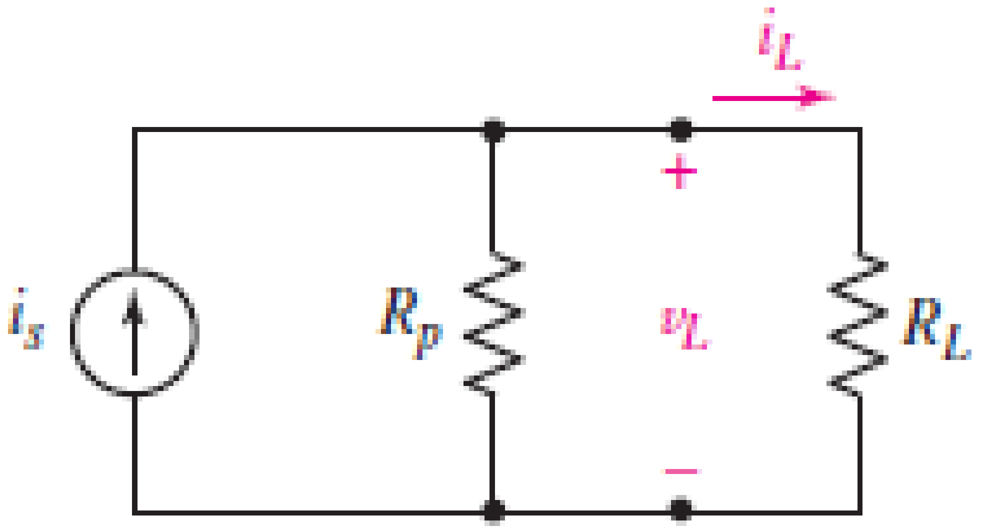
Real Current Source

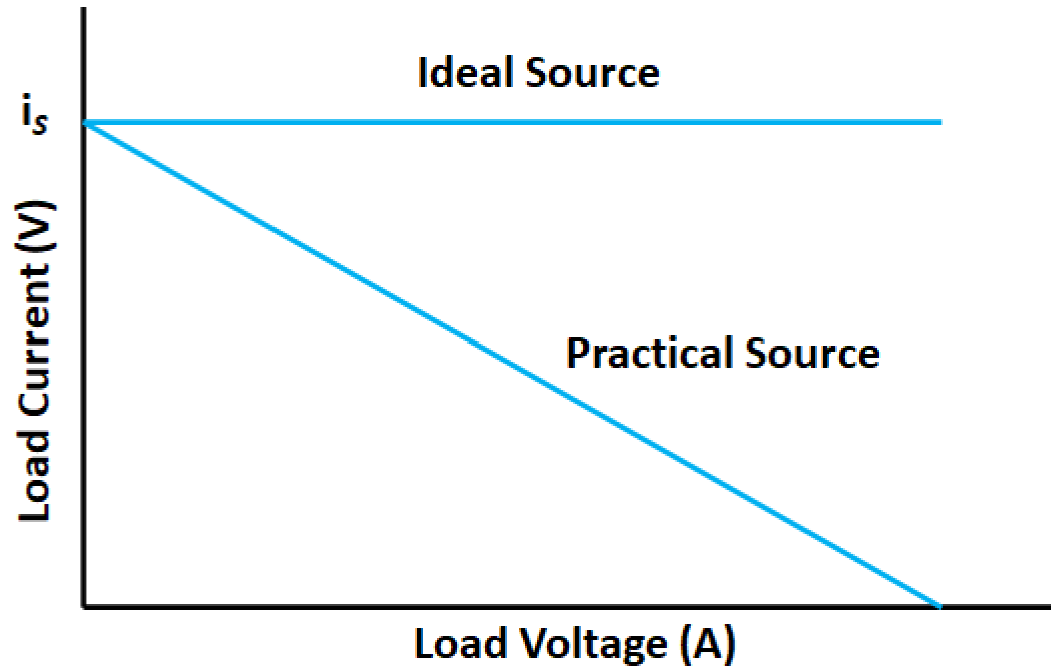
Current Sources….continued

- When R_L = ∞ no current flows through the load, the practical source is open-circuited and the open circuit current is zero.

- When R_L = 0, short-circuiting the load terminals, then a load current or short-circuit current, I_{Lsc} = ∞, would flow.

$$\boxed{i_L = i_S - \frac{v_L}{R_P}}$$





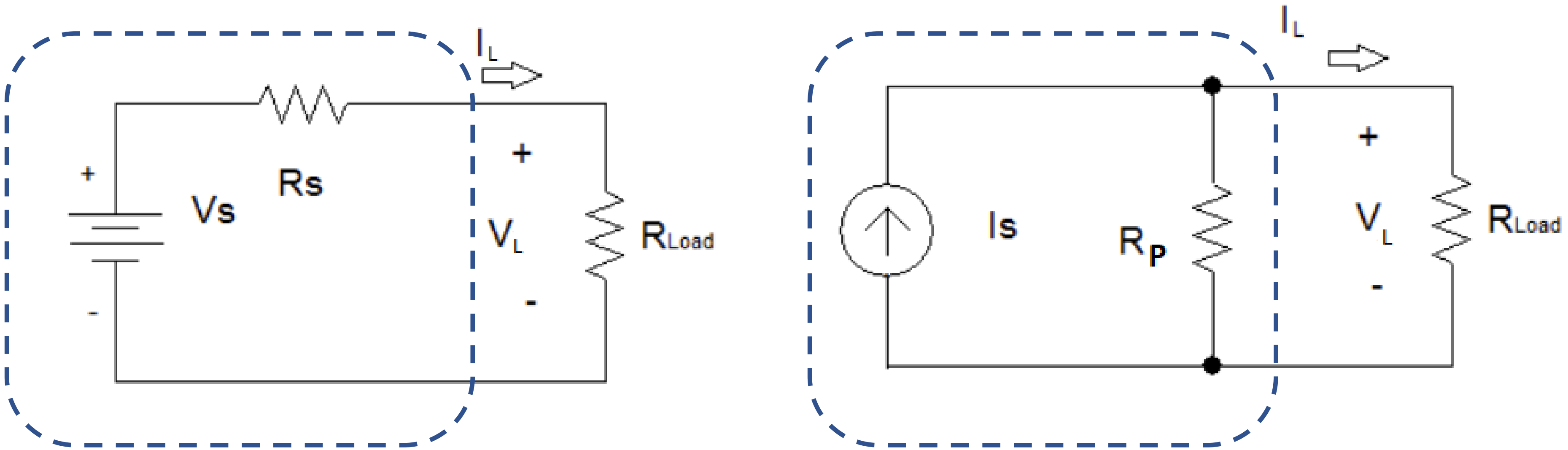
Source Transformations

- An equivalent circuit is one in which the i-v characteristics are identical to that of the original circuit.
- R_L in both circuits must be identical.

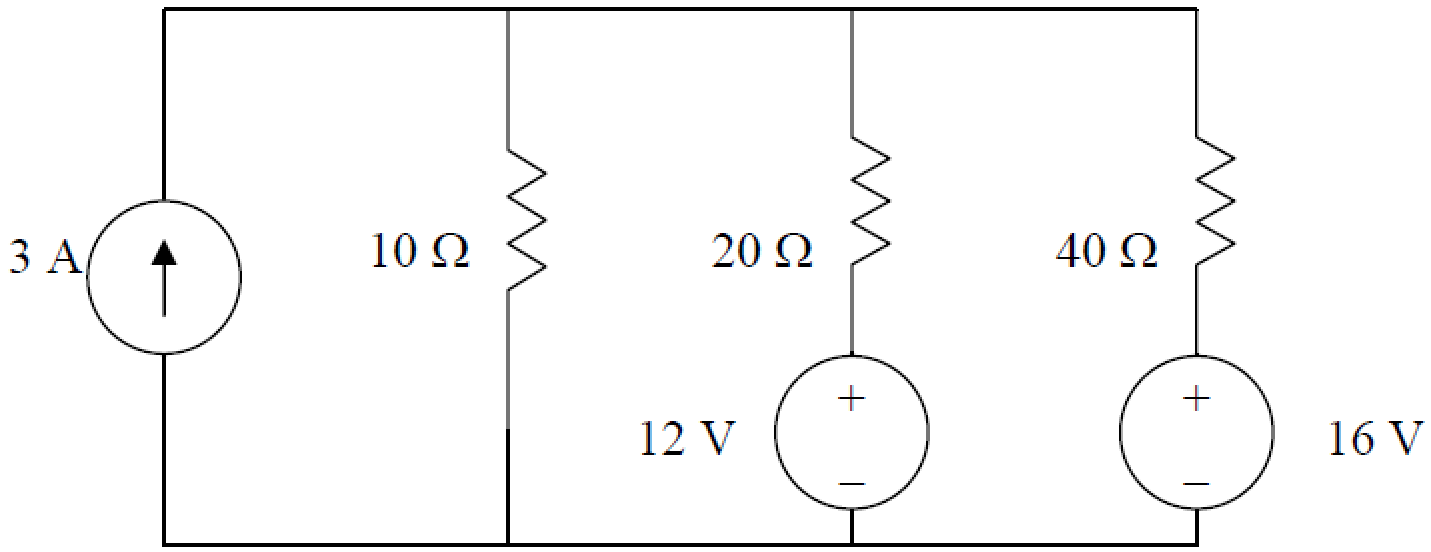
$$I_L = \frac{V_S}{R_S + R_{Load}} \qquad\qquad I_L = \frac{R_P}{R_P + R_{Load}} I_S$$

The two practical sources are electrically equivalent if

$R_S = R_P$ and $V_S = R_P I_S$



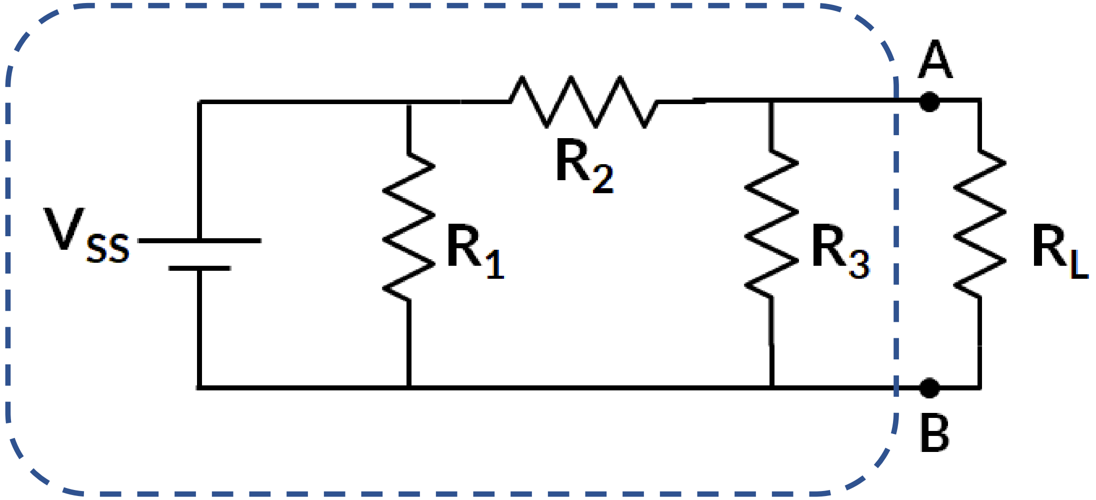
Problem1. Use source transformations to reduce the circuit to a single voltage source in series with a single resistor.

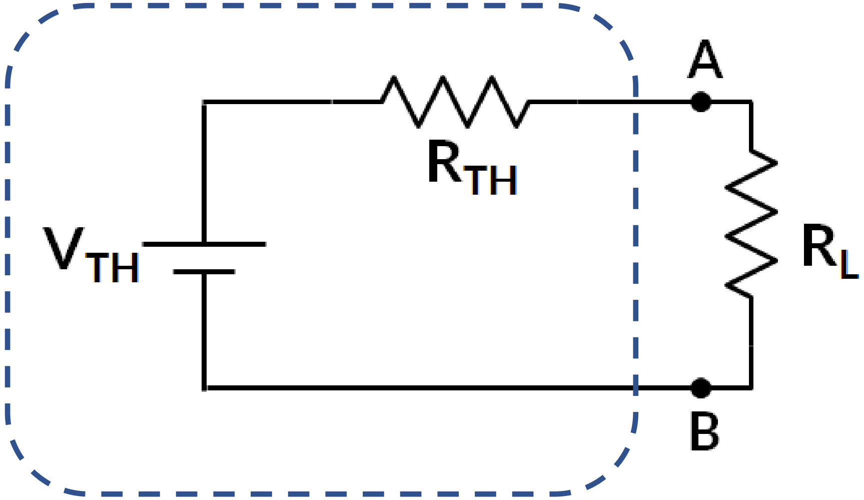


Hints: First convert voltage sources into current sources
Req=5.714Ω, Veq=22.856 V

Thevenin's Theorem

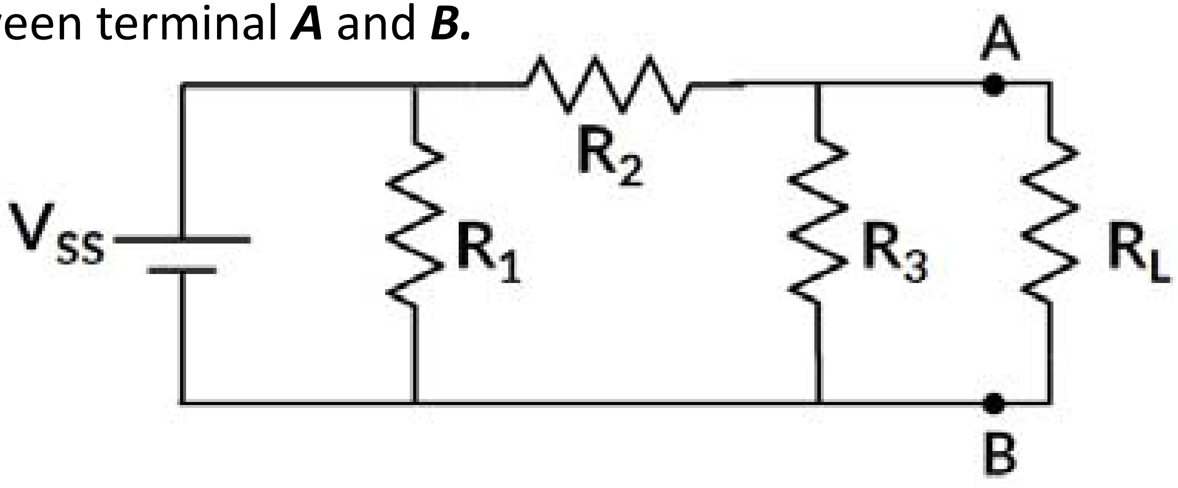
- Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor.
- Thevenin's theorem establishes an equivalence at the terminals.
- Internal construction and characteristics of the original network and the Thevenin equivalent are generally entirely different.





To Find R_{TH}

1. Find that portion of the network across which the Thevenin equivalent circuit needs to be found.
2. Load resistor R_L is temporarily removed from the network.
3. Mark the terminals of the remaining two-terminal network (say ***A*** and ***B***).
4. Identify all voltage and current sources and retain their internal resistances if any.
5. Replace the voltage sources by short circuits.
6. Replace the current sources by open circuits.
7. Find the resistance between terminal ***A*** and ***B.***



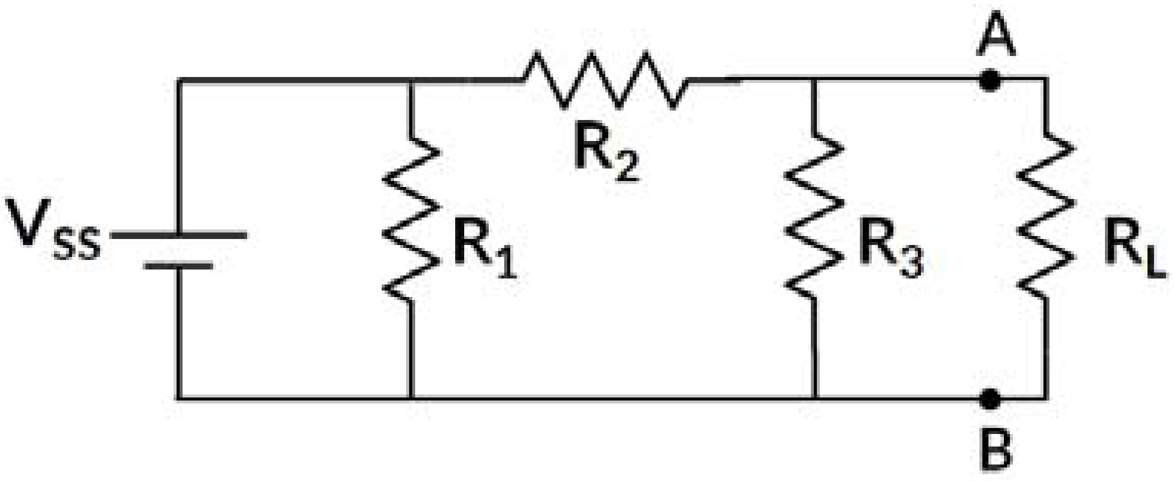
The resistance between **A** and **B** is called as Thevenin's resistance $\mathbf{R_{TH}}$

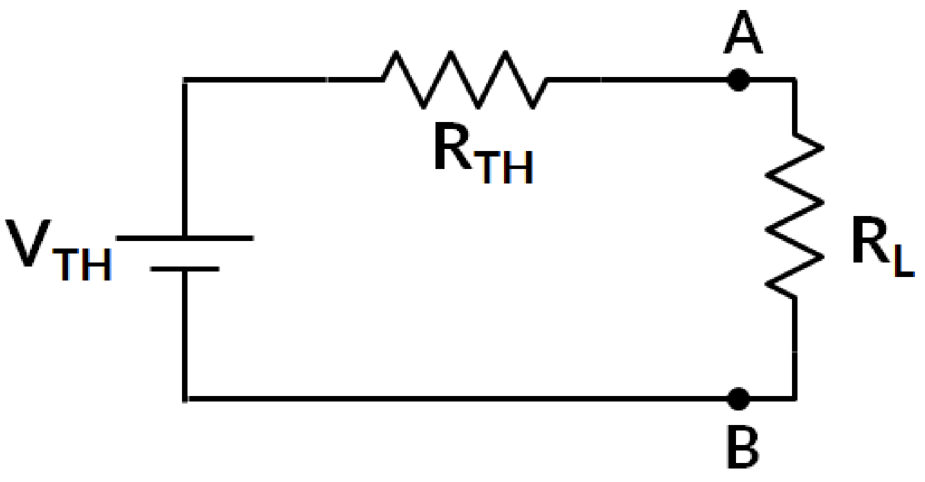
To Find V_{TH}

1. In the original circuit, remove the load resistor (R_L) connected between the marked terminals (***A*** and ***B***).

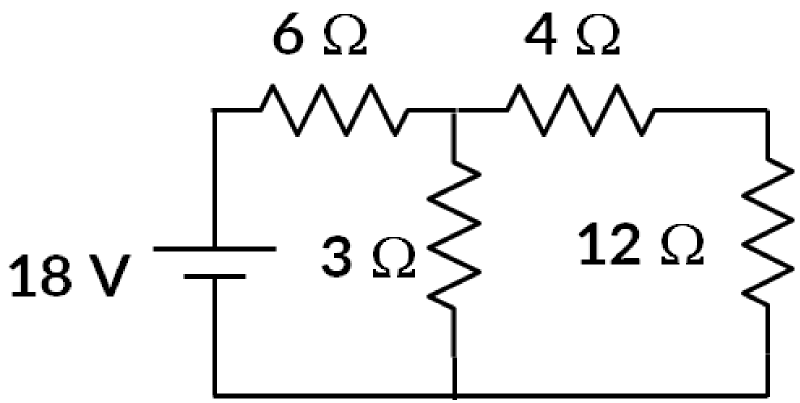
2. Find the open-circuit voltage (V_{TH}) between the marked terminals (***A*** and ***B***). V_{AB} is Thevenin voltage, denoted by symbol V_{TH}.

3. Draw the Thevenin equivalent circuit by keeping V_{TH}, R_{TH} and the load resistor (R_L) in series





Problem1 Using Thevenin's theorem, find the voltage across and current through the 12 Ω resistor.

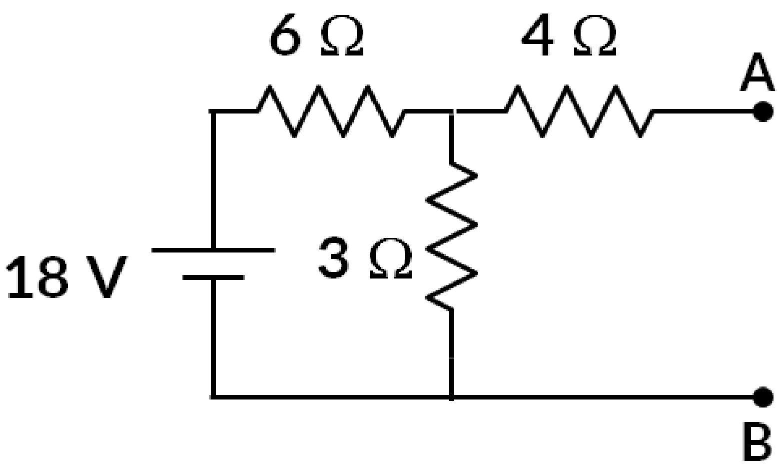


To Find R_{TH}

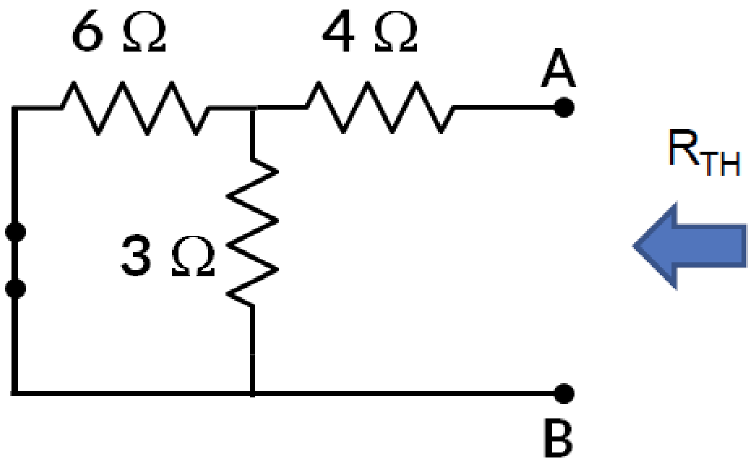
Step 1: Find that portion of the network across which the Thevenin equivalent circuit needs to be found. (Identifying the load as 12 Ω resistor, marking the nodes **A** and **B**)

Step 2: Load resistor RL is temporarily removed from the network.

Step 3: Mark the terminals of the remaining two-terminal network (say ***A*** and ***B***).



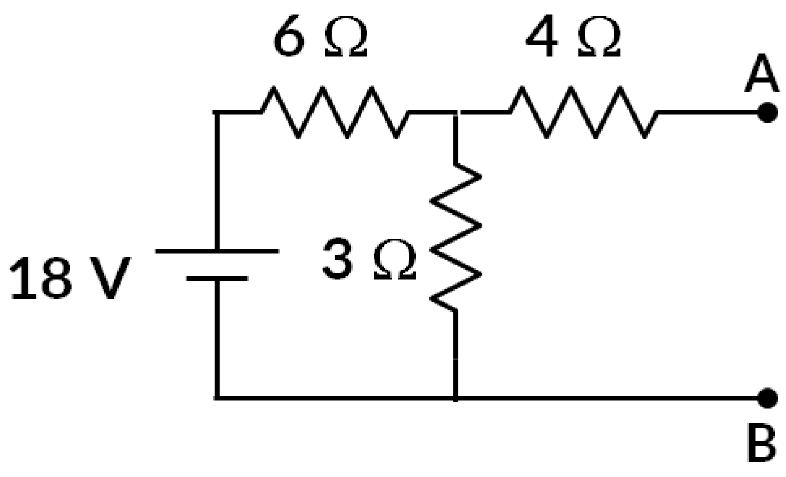
Step 4: Identify all voltage and current sources and retain their internal resistances if any. There is only one voltage source 18 V, with zero internal resistance
Step 5: Replace the voltage sources are replaced by short circuits (as there is only one voltage source in this example, replace it with a short circuit)
Step 6: Replace the current sources by open circuits (as there are no current sources, we won't act on this step)
Step 7: Find the resistance between terminal ***A*** and ***B***



R_{TH}=4+(6||3)=?

To Find V_{TH}

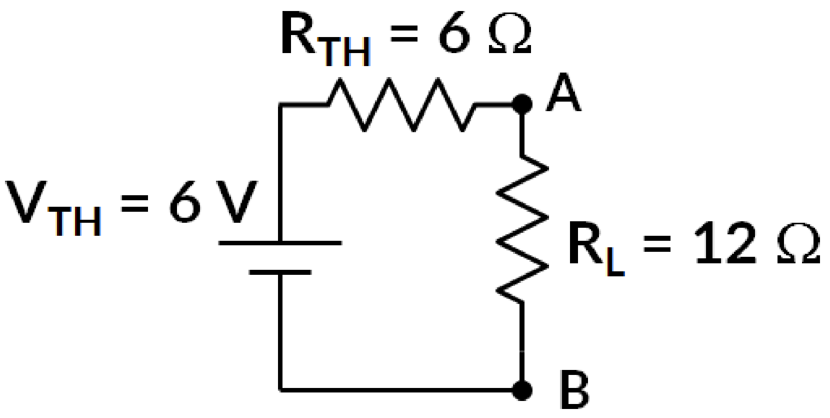
Step 1: In the original circuit, remove the load resistor (R_L) connected between the marked terminals (***A*** and ***B***).
Step 2: Find the open-circuit voltage (V_{TH}) between the marked terminals (***A*** and ***B***). V_{AB} is Thevenin voltage, denoted by symbol V_{TH}**.**



1. Voltage across 3 Ω resistance is equal to voltage between terminals **A** and **B**.
2. This is due to the fact that current through 4 Ω is zero. Hence voltage across is 4 Ω zero.
3. Use voltage divider formula to get voltage across 3 Ω resistor.

$$V_{3\Omega}=V\frac{R_1}{R_1+R_2}=18\frac{3\Omega}{3\Omega+6\Omega}=6\,V=V_{AB}=V_{TH}$$

Step 3: Draw the Thevenin equivalent circuit by keeping V_{TH}, R_{TH} and the load resistor (R_L) in series



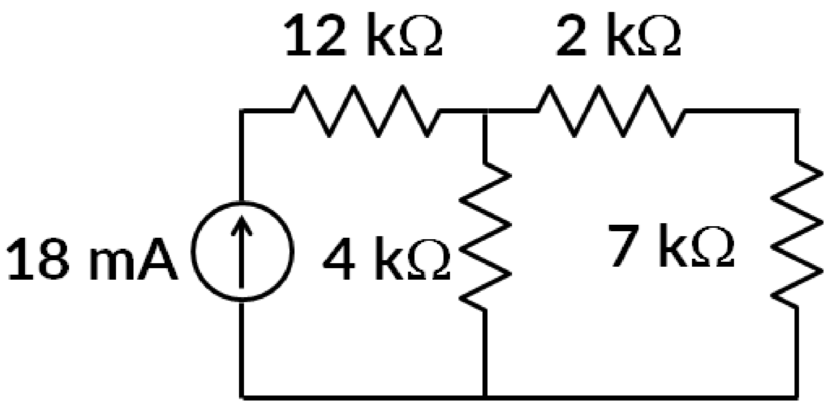
1. Use voltage divider formula to get voltage across 12 Ω resistor. (what was asked in the question)

$$V_{12\Omega} = V\frac{R_1}{R_1 + R_2} = 6\frac{12\Omega}{12\Omega + 6\Omega} = 4\ V$$

2. Current through the 12 Ω resistor. (what was asked in the question)

$$I_{12\Omega} = \frac{4\ V}{12\Omega} = 0.33\ A$$

Problem2 Using Thevenin's theorem, find the voltage across and current through the 7 kΩ resistor.

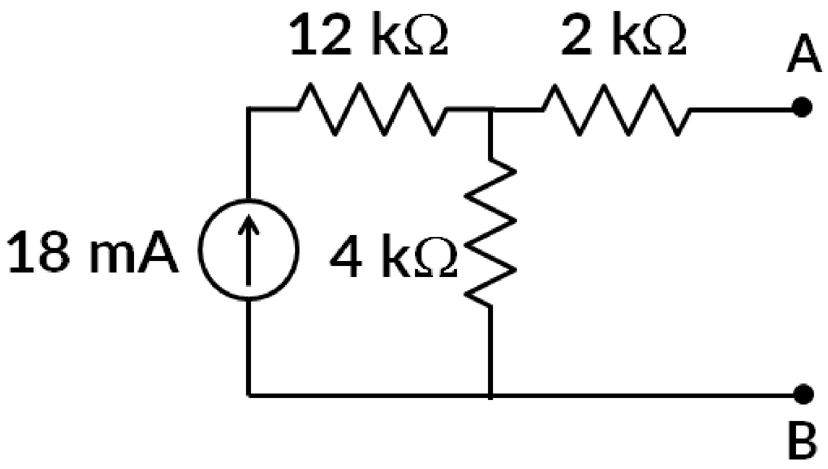


To Find R_{TH}

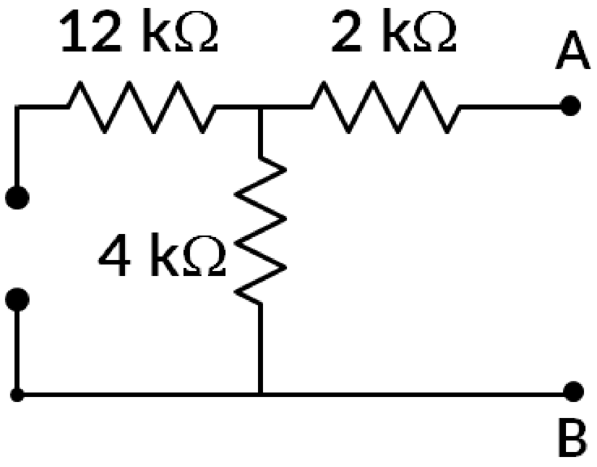
Step 1: Find that portion of the network across which the Thevenin equivalent circuit needs to be found. (Identifying the load as 7 kΩ resistor, marking the nodes **A** and **B**)

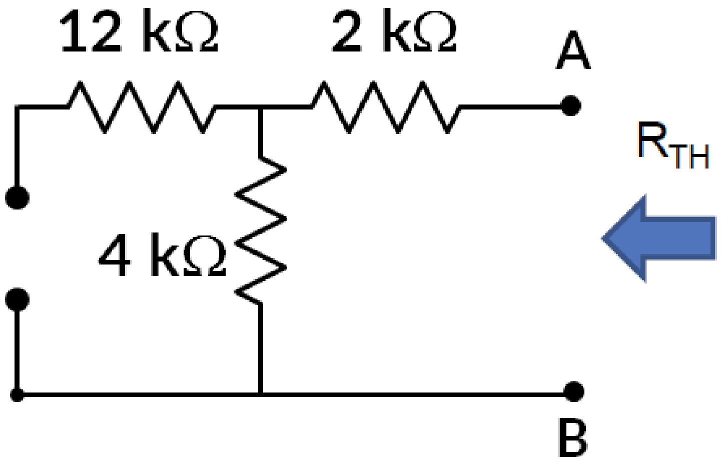
Step 2: Load resistor R_L is temporarily removed from the network.

Step 3: Mark the terminals of the remaining two-terminal network (say ***A*** and ***B***).



Step 4: Identify all voltage and current sources and retain their internal resistances if any. There is only one current source 18 mA, with zero internal resistance
Step 5: Replace the voltage sources are replaced by short circuits (as there are no voltage sources, we won’t act on this step)
Step 6: Replace the current sources by open circuits (as there is only one current source, in this example, replace it with a open circuit)
Step 7: Find the resistance between terminal ***A*** and ***B***

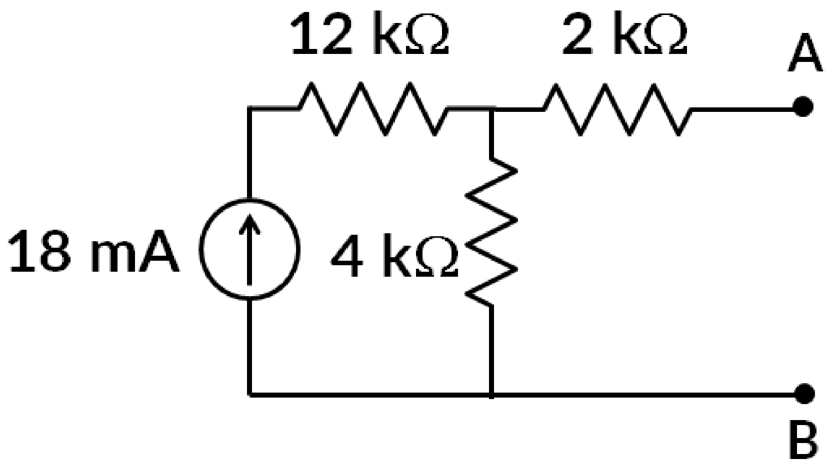




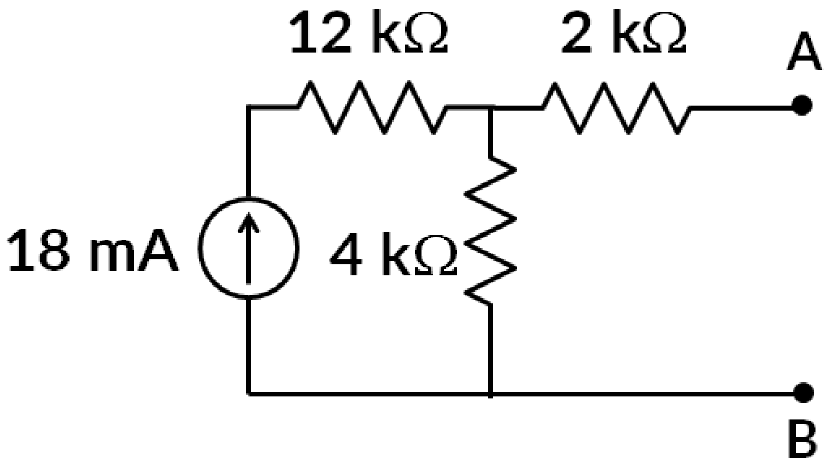
R_{TH}=2+4=6 kΩ

To Find V_{TH}

Step 1: In the original circuit, remove the load resistor (R_L) connected between the marked terminals (***A*** and ***B***).



Step 2: Find the open-circuit voltage (V_{TH}) between the marked terminals (***A*** and ***B***). V_{AB} is Thevenin voltage, denoted by symbol V_{TH}**.**



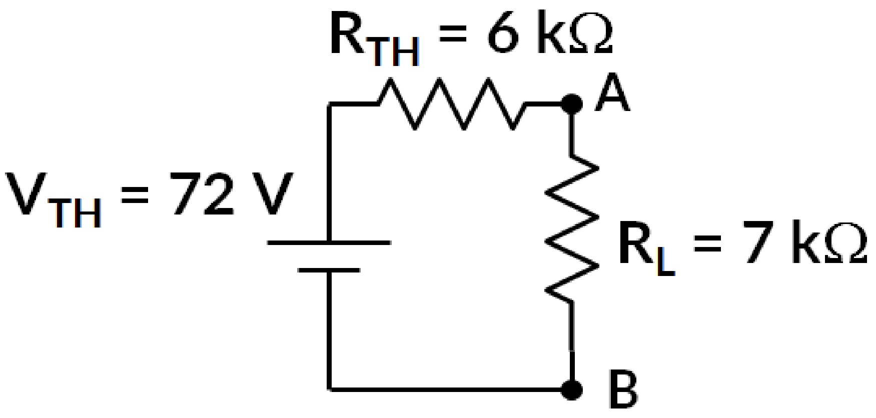
1. Voltage across 4 kΩ resistance is equal to voltage between terminals **A** and **B**.
2. This is due to the fact that current through 2 k Ω is zero. Hence voltage across 2 kΩ is zero.
3. Voltage across 12 k*Ω* and 4 k*Ω* is given by

$$V = IR = 18mA \times (4\ k\Omega + 12\ k\Omega) = 288V$$

4. Using voltage divider formula, voltage across 4 k*Ω* is given by

$$V_{4\,k\Omega} = 288 \frac{4\ k\Omega}{4\ k\Omega + 12\ k\Omega} = 72\ V = V_{AB} = V_{TH}$$

Step 3: Draw the Thevenin equivalent circuit by keeping V_{TH}, R_{TH} and the load resistor (R_L) in series



1. Use voltage divider formula to get voltage across 7 k resistor. (what was asked in the question)

$$V_{7\,k\Omega} = V\frac{R_1}{R_1+R_2} = 72\frac{7\,k\Omega}{7\,k\Omega+6\,k\Omega} = 38.77\,V$$

2. Current through the 7 kΩ resistor. (what was asked in the question)

$$I_{7k\Omega} = \frac{38.77}{7} = 5.54mA$$