Passive Filters

Filter Applications

- Filters are specifically used to
  - remove unwanted frequency components from the signal
  - to enhance wanted ones
  - or both.

- Filters are essential building blocks in many systems,
  - Example: used in communication and instrumentation systems

- A common need for filter circuits is in high-performance stereo systems, where certain ranges of audio frequencies need to be amplified or suppressed for best sound quality and power efficiency

Filter Characteristics

- Filter is an electrical network that modifies the amplitude and phase characteristics of a signal with respect to frequency

- In electronic systems, filters are useful in emphasizing signals in certain frequency ranges and reject signals in other frequency ranges.

Different types of Filters

- **Low-pass filter**: low frequencies are passed; high frequencies are attenuated.
- **High-pass filter**: high frequencies are passed; low frequencies are attenuated.
- **Band-pass filter**: only frequencies in a frequency band are passed.
- **Band-stop filter or band-reject filter**: only frequencies in a frequency band are attenuated

Filter Characteristics

- Filter is characterized by two important observations

- **Transfer Function or Transfer Characteristics**: A mathematical function describing the output response of a filter system to the input or stimulus. Transfer function in filters is studied as a frequency response

- **Phase response**: How the phase of filter changes with frequency

- The order of the filter is decided by the order of the differential equation that need to be solved.

- 1st order differential equation- 1st order filter

- 2nd order differential equation- 2nd order filter

- Important properties of filter

- **3 dB Frequency or cut-off frequency**: The frequency at which the transfer function becomes half.

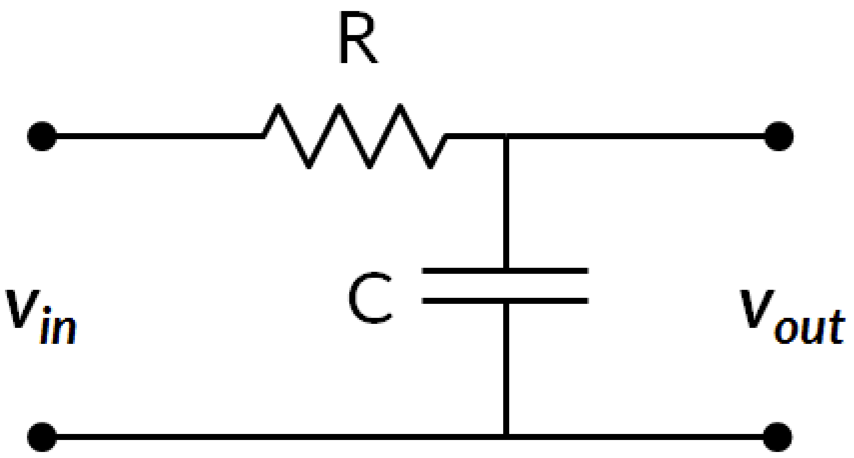
RC Low Pass Filters

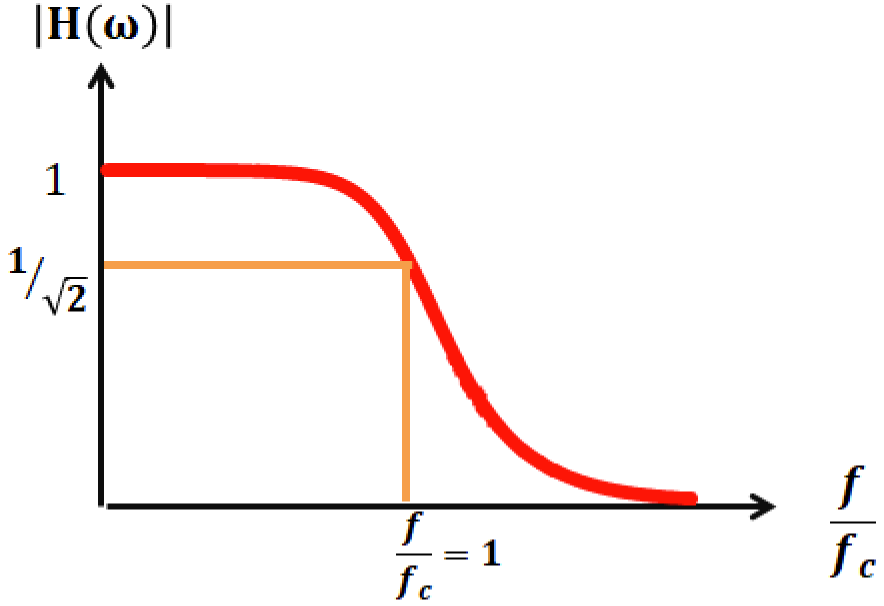
- Low pass filter passes low frequency signals and attenuates high frequency signals.
- Consider an input signal v_{in}. Output v_{out} is taken across capacitor.
- Transfer function H(ω):

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_C}{V_R + V_C}$$

$$H(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$





RC Low Pass Filters

Cut-off Frequency (f_c)

$$|H(\omega)| = \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{1 + (\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_C = \frac{1}{RC}$$

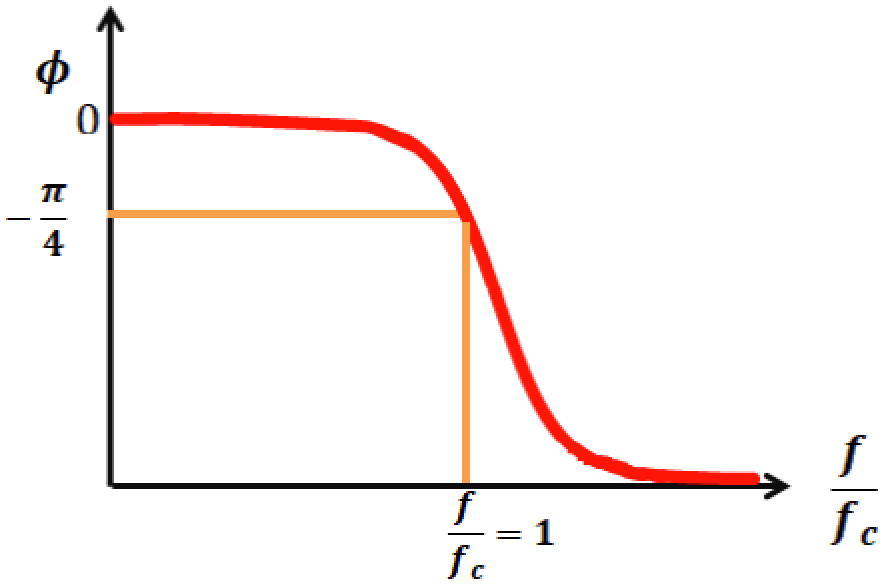
$$f_C = \frac{1}{2\pi RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega}{\omega_C}\right)^2}}$$

Phase Angle (ϕ)

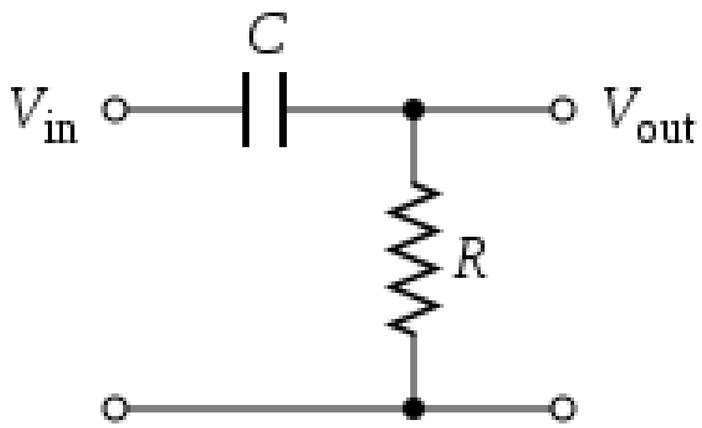
$$\phi = -\tan^{-1}(\omega RC)$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right) = -\tan^{-1}\left(\frac{f}{f_C}\right)$$



RC High Pass Filters

- High pass filter passes high frequency signals and attenuates low frequency signals



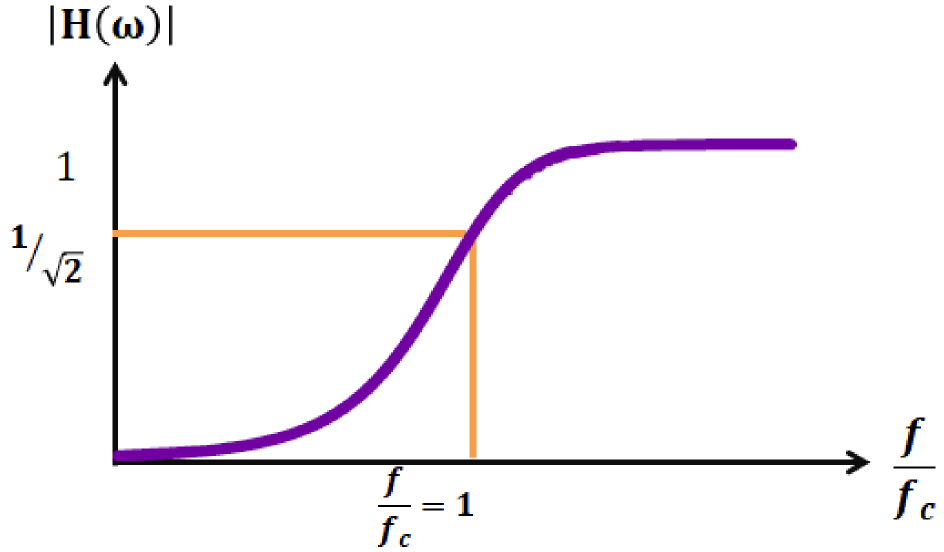
- Consider an input signal v_{in}. Output v_{out} is taken across resistor.

- Transfer function $H(\omega)$:

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_R}{V_R + V_C}$$

$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$|H(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$



RC High Pass Filters

Cut-off Frequency (f_c)

$$|H(\omega)| = \frac{1}{\sqrt{2}}$$

$$\frac{\omega RC}{\sqrt{1+(\omega RC)^2}} = \frac{1}{\sqrt{2}}$$

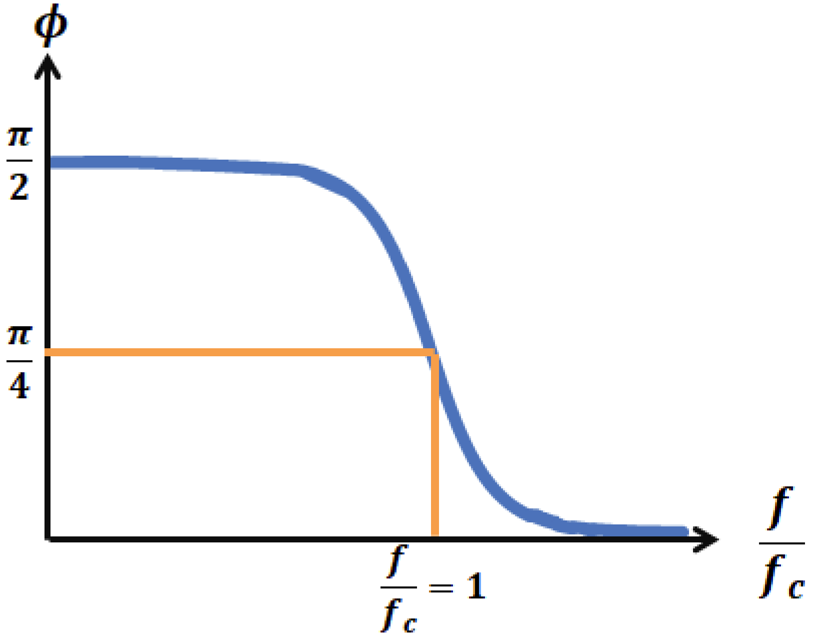
$$\omega_C = \frac{1}{RC}$$

$$f_C = \frac{1}{2\pi RC}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega_C}{\omega}\right)^2}}$$

Phase Angle (ϕ)

$$\begin{aligned}\phi &= \frac{\pi}{2} - tan^{-1}(\omega RC)\\ &= \frac{\pi}{2} - tan^{-1}\left(\frac{\omega}{\omega_C}\right)\\ &= \frac{\pi}{2} - tan^{-1}\left(\frac{f}{f_C}\right)\end{aligned}$$



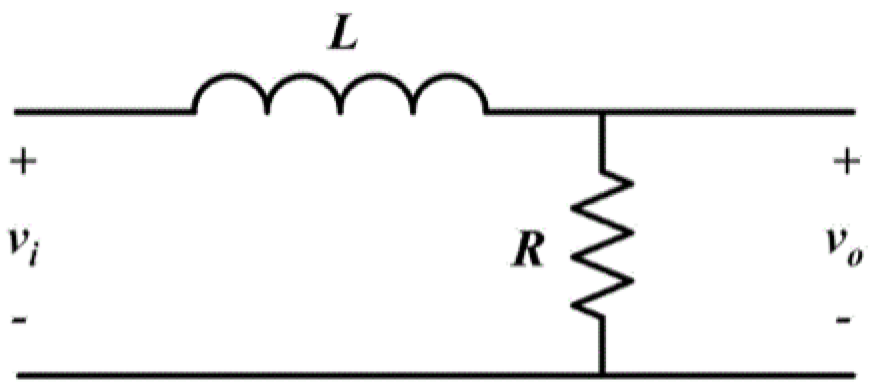
RL Low Pass Filters

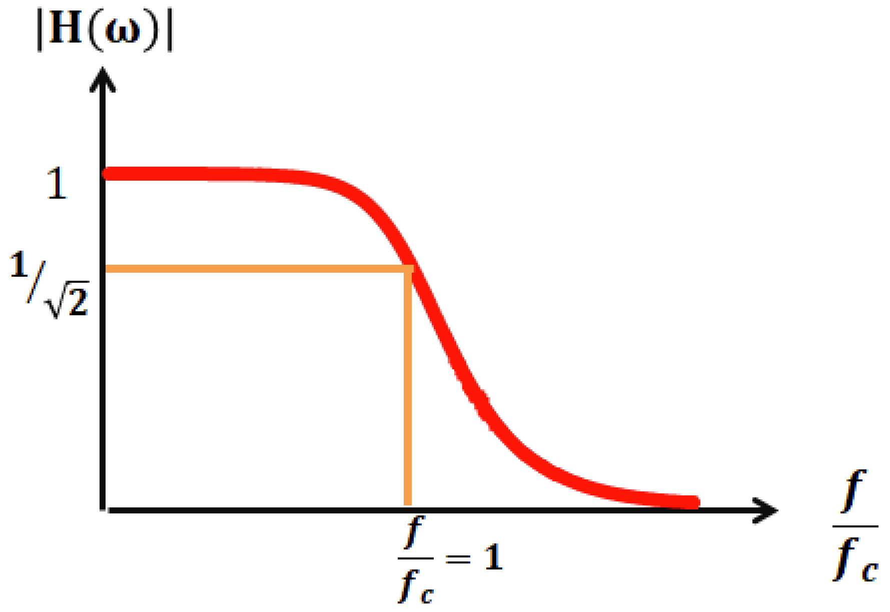
- Transfer function H(ω):

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_R}{V_R + V_L}$$

$$H(\omega) = \frac{R}{R + j\omega L} = \frac{1}{1 + j\omega \frac{L}{R}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega L}{R}\right)^2}}$$





RL Low Pass Filters

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$$|H(\omega)| = \frac{1}{\sqrt{2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega L}{R}\right)^2}} = \frac{1}{\sqrt{2}}$$

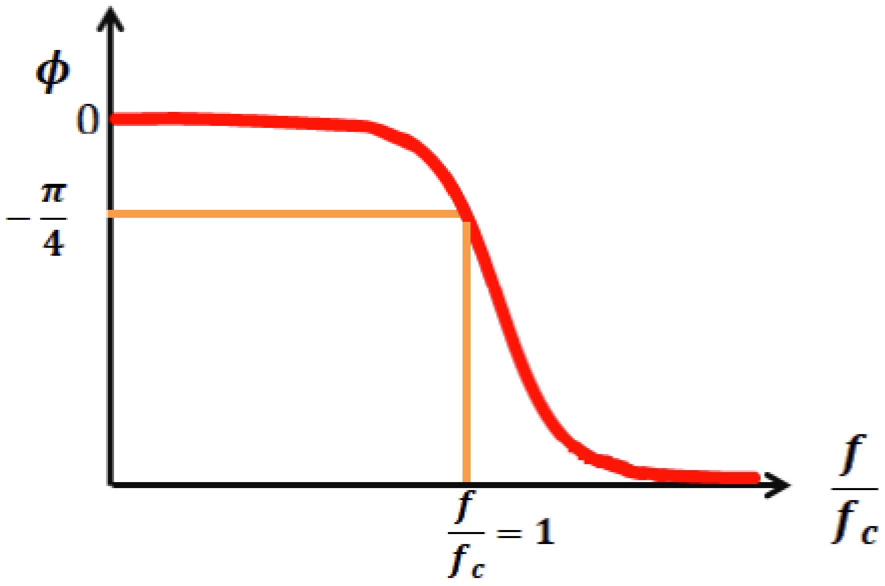
$$\omega_C = \frac{R}{L}$$

$$f_C = \frac{R}{2\pi L}$$

Phase Angle (ϕ)

$$\phi = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\phi = -\tan^{-1}\left(\frac{\omega}{\omega_C}\right) = -\tan^{-1}\left(\frac{f}{f_C}\right)$$



RL High Pass Filters

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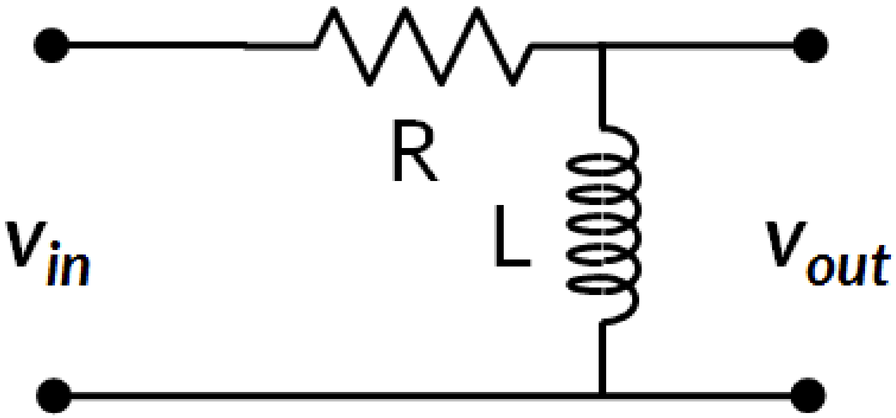
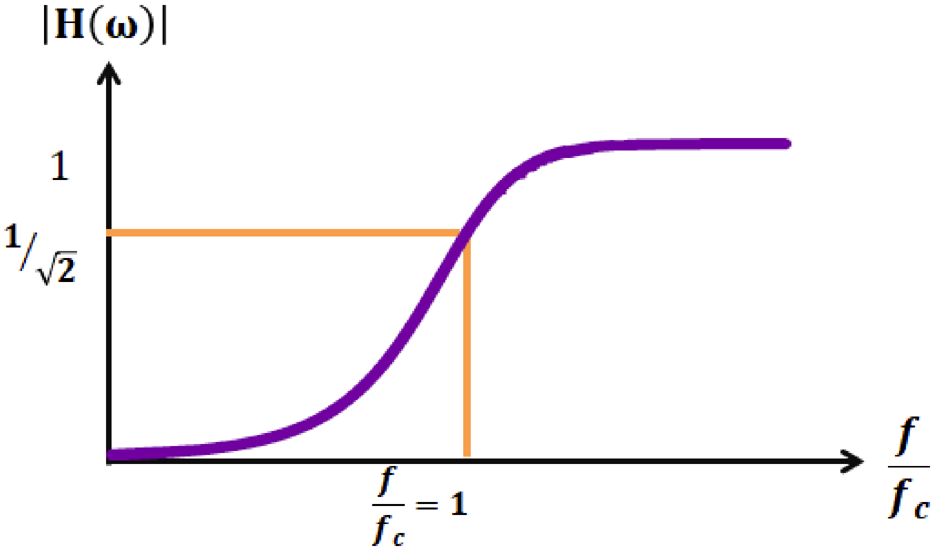
$$H(\omega) = \frac{j\omega L}{R + j\omega L} \qquad H(\omega) = \frac{1}{1 + \frac{R}{j\omega L}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}}$$





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Phase Angle (ϕ)

$$\phi = \frac{\pi}{2} - tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\phi = \frac{\pi}{2} - tan^{-1}\left(\frac{\omega}{\omega_C}\right)$$

$$= \frac{\pi}{2} - tan^{-1}\left(\frac{f}{f_C}\right)$$