12.11.20

Curl of a gradient:

$$\vec{\nabla} \times (\vec{\nabla} T) = 0 \qquad T(x, y, z)$$

$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

$$\vec{\nabla} \times (\vec{\nabla} T) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{vmatrix}$$

$$\boxed{\vec{\nabla} \cdot (\vec{\nabla} T) = \nabla^2 T \equiv \text{Laplacian}}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}\left(\frac{\partial T}{\partial z}\right) - \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial y}\right)\right) + \cdots$$

$$= 0$$

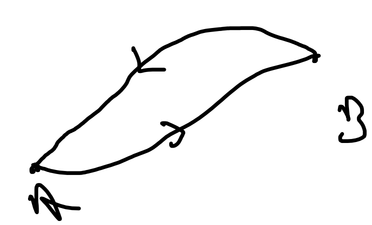
Line integral:

$$\int_a^b \vec{v} \cdot d\vec{l}$$

$d\vec{l}$ → displacement vector

$\vec{v}$ → arbitrary vector

→ if a close loop

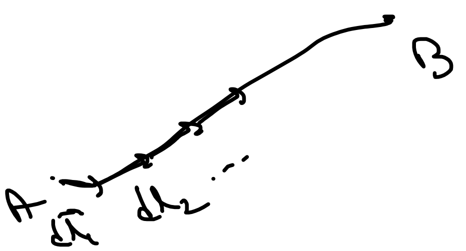


$$\oint \vec{v} \cdot d\vec{l} = \int_a^b \vec{v} \cdot d\vec{l} + \int_b^a \vec{v} \cdot d\vec{l}$$

(*) Usually $\int_a^b \vec{v} \cdot d\vec{l}$ depends on path taken.

Say, you apply force '$\vec{F}$' to displace an object by $d\vec{l}$. $\Rightarrow$ Work done $= \vec{F}\cdot d\vec{l}$

(*) If we take line integral with the gradient of a fn:



$$dT = \underline{\underline{\vec{\nabla} T}} \cdot d\vec{l}$$

$$d\vec{l} = (\hat{x}\,dx + \hat{y}\,dy + \hat{z}\,dz)$$

$$dT_i = \vec{\nabla} T \cdot d\vec{l}_i$$

[$\displaystyle\int_a^b \frac{dT}{dx}\,dx = T(b) - T(a)$ if $T \equiv T(x)$]

$$\int_a^b dT = \int_a^b (\vec{\nabla} T)\cdot d\vec{l} = \underbrace{T(b) - T(a)}_{\text{independent of path}}$$

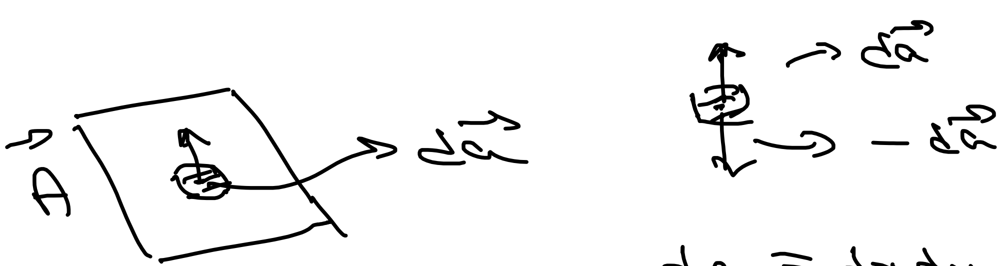
[$T \equiv T(x,y,z)$]

(*) $\displaystyle\int_a^b (\vec{\nabla} T)\cdot d\vec{l}$ is independent of path.

(*) $\displaystyle\oint (\vec{\nabla} T)\cdot d\vec{l} = 0$ since begining and end points are same.

<u>Surface integral:</u> $\displaystyle\int_S \vec{v}\cdot d\vec{a}$

$\rightarrow$ infinitesimal patch on surface



$da = dx\,dy$

$\displaystyle\oint_S \vec{v}\cdot d\vec{a}$ = if surface is closed (e.g. sphere)

<u>Flux</u>: Say, $\vec{v}$ describes flow of a liquid
(mass per unit area per unit time)

$\int \vec{v} \cdot d\vec{a}$ $\Rightarrow$ total mass per unit time passing through the surface

$\Downarrow$

<u>Flux</u>

<u>Volume integral:</u>

$$\int_V T \, d\tau$$

$d\tau$ ↳ infinitesimal volume element

in cartesian:

$$d\tau = dx \, dy \, dz$$

(*) if $T \equiv$ density of a substance

$$\int_V T \, d\tau \equiv \text{Mass}$$

vol. integral of a vector:

$$\int \vec{v} \, d\tau = \hat{x} \int v_x \, d\tau + \hat{y} \int v_y \, d\tau + \hat{z} \int v_z \, d\tau$$

<u>Gauss' law</u>

$$\int_V (\vec{\nabla} \cdot \vec{v}) \, d\tau = \oint_S \vec{v} \cdot d\vec{a}$$

$\vec{v} \equiv$ flow of an incompressible liquid

$\Rightarrow$ Flux of $\vec{v}$ is the total amount of fluid passing through the surface per unit time. The divergence measures the spreading out of a vector