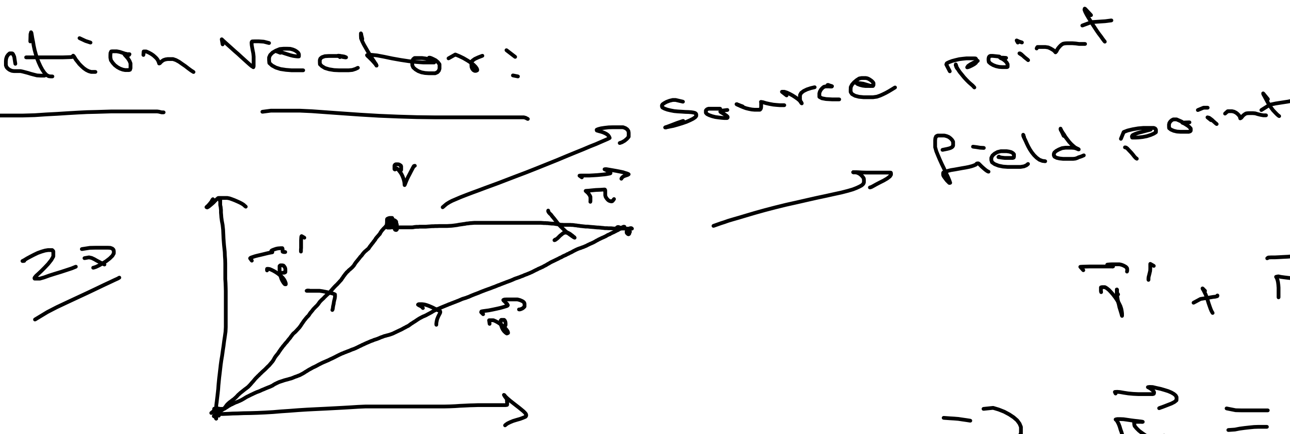
29.10.20

Seperation vector:



$$\vec r' + \vec{\mathscr{r}} = \vec r$$

$$\Rightarrow \vec{\mathscr{r}} = \vec r - \vec r'$$

$$\mathscr{r} = |\vec r - \vec r'|$$

$$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}}$$

(*) $\vec A \times (\vec B \times \vec C)$

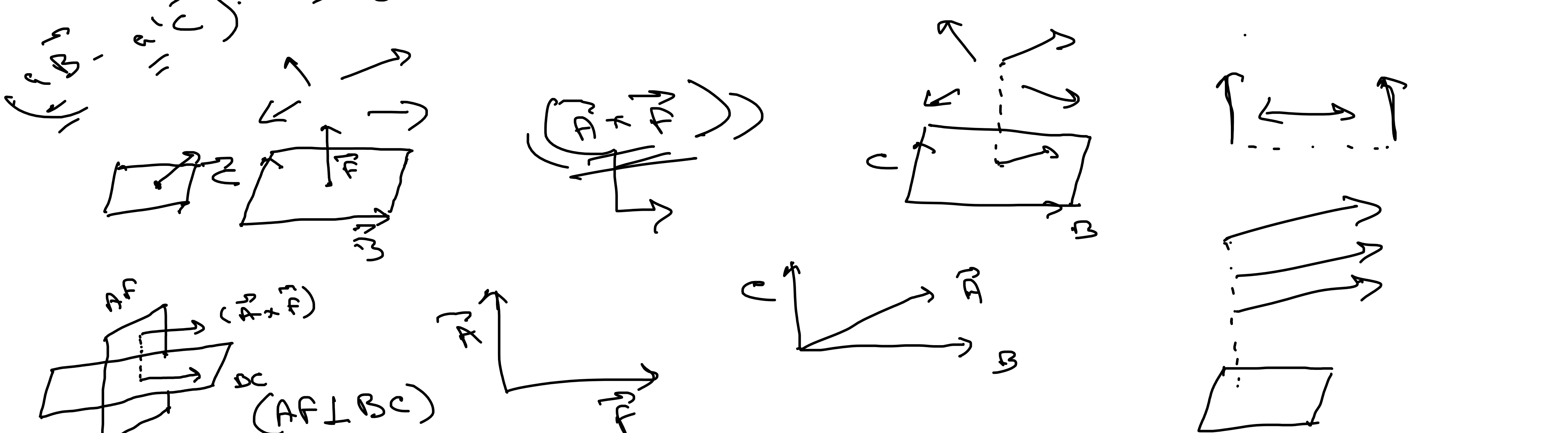
$= \vec B\,(\vec A \cdot \vec C) - \vec C\,(\vec A \cdot \vec B)$

↳ lies on BC plane

$(a\vec B + a'\vec C)$

$\vec A \cdot (\vec B \times \vec C)$

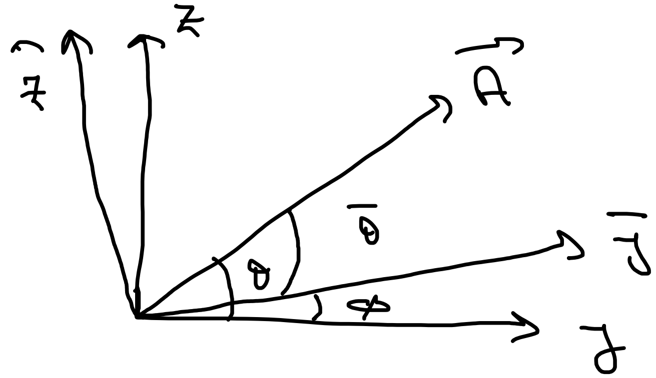
↳ volume of a parallelopiped.



(Added after class: I think this will help to visualise)

Vector transformation:

Cartesian



$$(x, y, z) \xrightarrow[\text{about } x\text{-axis}]{\text{Rotate by } \phi} (\bar{x}, \bar{y}, \bar{z})$$

$$x = \bar{x}$$

$$A_y = A\cos\theta$$

$$A_z = A\sin\theta$$

$$\begin{cases} \vec{A} = \hat{x} A_x + \hat{y} A_y + \hat{z} A_z \\ \vec{A} = \hat{\bar{x}} \bar{A}_x + \hat{\bar{y}} \bar{A}_y + \hat{\bar{z}} \bar{A}_z \end{cases}$$

$$\bar{A}_y = A\cos\bar{\theta} = A\cos(\theta - \phi)$$

$$= A(\cos\theta\cos\phi + \sin\theta\sin\phi)$$

$$= A_y\cos\phi + A_z\sin\phi$$

$$\bar{A}_z = A\sin\bar{\theta} = A\sin(\theta - \phi)$$

$$= -A_y\sin\phi + A_z\cos\phi$$

In matrix notation:

$$\begin{pmatrix} \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} A_y \\ A_z \end{pmatrix}$$

More generally,

$$\begin{pmatrix} \bar{A}_x \\ \bar{A}_y \\ \bar{A}_z \end{pmatrix} = \begin{pmatrix} R_{xx} & R_{xy} & R_{xz} \\ R_{yx} & R_{yy} & R_{yz} \\ R_{zx} & R_{zy} & R_{zz} \end{pmatrix} \begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}$$

Compactly,

$$\bar{A}_i = \sum_{j=1}^{3} R_{ij} A_j \quad \Big| \quad i, j \equiv x, y, z$$

New definition of vector: A vector is any set of three components that transforms as

$$\bar{A}_i = \sum_{j=1}^{3} R_{ij} A_j$$

under change of coordinates.