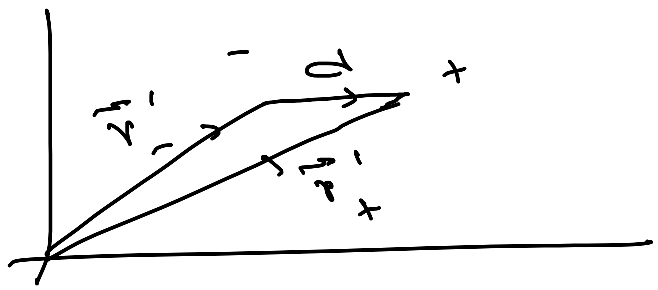
12.01.21

Electric Field of a Dipole:

Section 3.4.4



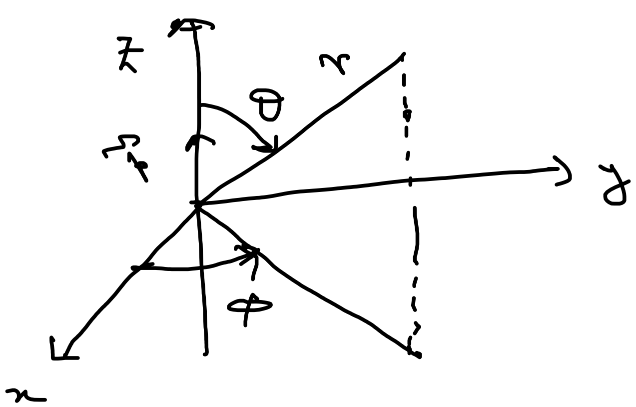
Dipole moment:

$$\vec{p} = q\vec{d}$$

$$\vec{d} = \vec{r}'_+ - \vec{r}'_-$$

At $\vec{r}$, the potential due to this dipole

$$V_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$$



$\vec{p}$ is at origin and points in z-direction.

$$V_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0 r^2}\, p\cos\theta$$

To get the electric field:

$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3}$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3}$$

$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} = 0$$

Thus, $\vec{E}_{dip.}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$

$$\vec{p} = (\vec{p}\cdot\hat{r})\,\hat{r} + (\vec{p}\cdot\hat{\theta})\,\hat{\theta}$$

$$= p\cos\theta\,\hat{r} - p\sin\theta\,\hat{\theta}$$

Hence $\quad 3(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}$

$$= 3p\cos\theta\,\hat{r} - p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}$$

$$= 2p\cos\theta\,\hat{r} + p\sin\theta\,\hat{\theta}$$

Hence,

$$E_{dip.}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\frac{1}{r^3}\left[3(\vec{p}\cdot\hat{r})\,\hat{r} - \vec{p}\right]$$

↳ A valid approximation for a physical dipole only in the limit $r \gg d$.

Polarization:

$\boxed{\text{Section 4.1.4}}$

Dielectric in an external electric field:

→ if the material is made up of neutral atoms (molecules) the external electric field will induce a tiny dipole in each, the direction being same as the electric field.

→ if the materials are made up of polar molecules, each permanent dipole experiences a torque, making it to align in the field direction.

(*) The net effect is a bunch of dipoles pointing along the direction of the external electric field.

↳ The material is <u>Polarised</u>

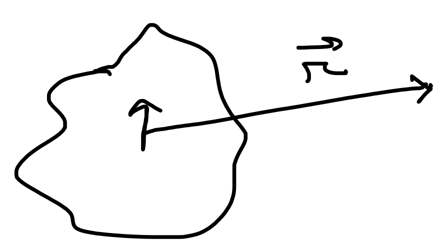
A measure:

$\vec{P} \equiv$ Dipole moment per unit volume

(called polarisation)

<u>The field of a polarised object:</u> $\boxed{\text{Section } 4.2.1}$

Suppose that we have a polarised object. (made up of a large number of microscopic dipoles)



For a single dipole $\vec{p}$,

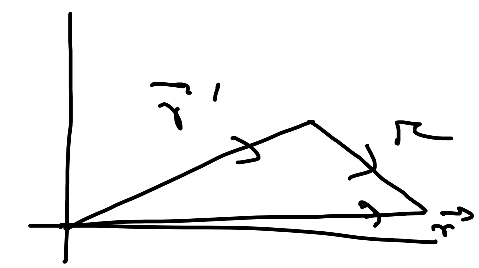
$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{p}\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}$$

$\vec{\mathscr{r}}$ = vector from the dipole to the point at which V is calculated.

Dipole moment: $\vec{p} = \vec{P}\, d\tau'$ in each volume element $d\tau'$.

Then the total potential

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\vec{P}(\vec{r}\,')\cdot\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$



We use, $\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\mathscr{r}}}{\mathscr{r}^2}$ (differentiation w.r.t. source coordinates $(\vec{r}\,')$)

$$\Rightarrow V = \frac{1}{4\pi\epsilon_0} \int_V \vec{P}\cdot\vec{\nabla}'\left(\frac{1}{\mathscr{r}}\right) d\tau'$$

Integrating by parts,

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_V \vec{\nabla}'\cdot\left(\frac{\vec{P}}{\mathscr{r}}\right) d\tau' - \int_V \frac{1}{\mathscr{r}}\left(\vec{\nabla}'\cdot\vec{P}\right) d\tau'\right]$$

Use divergence theorem:

$$V = \underbrace{\frac{1}{4\pi\epsilon_0}\oint_S \frac{1}{r}\vec{P}\cdot d\vec{a}'}_{\text{Potential of a surface charge density } \sigma_b = \vec{P}\cdot\hat{n}} - \underbrace{\frac{1}{4\pi\epsilon_0}\int_V \frac{1}{r}(\vec{\nabla}'\cdot\vec{P})\,d\tau'}_{\text{Potential of a volume charge density } \rho_b = -\vec{\nabla}'\cdot\vec{P}}$$

Potential of a surface charge density $\sigma_b = \vec{P}\cdot\hat{n}$ ($\hat{n}$ → normal unit vector)

Potential of a volume charge density $\rho_b = -\vec{\nabla}'\cdot\vec{P}$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0}\oint_S \frac{\sigma_b}{r}\,da' + \frac{1}{4\pi\epsilon_0}\int_V \frac{\rho_b}{r}\,d\tau'$$

→ Potential of a polarised object is the same as that produced by a volume charge density ρ_b plus a surface charge density σ_b. These are called <u>bound charges</u>.