Number System

Decimal Number System

It contains ten unique symbol 0, 1, 2, ... 9. So, its base or radix is 10.

a_{n-1}	a_{n-2}	---	a_1	a_0	.	a_{-1}	a_{-2}	a_{-3}	---	a_{-m}

MSD   Radix Point   LSD

Integer Part   Fractional Part

$$N_r = a_{r-1}\, r^{n-1} + a_{n-2}\, r^{n-2} + a_{n-3}\, r^{n-3} + \cdots + a_1 r^1 + a_0 r^0 + a_{-1} r^{-1} + a_{-2} r^{-2} + \cdots + a_{-m} r^{-m}$$

Example:

$$(385.62)_{10} = 3\times10^2 + 8\times10^1 + 5\times10^0 + 6\times10^{-1} + 2\times10^{-2}$$

$$= (385.62)_{10}$$

$$= 300 + 80 + 5 + 0.6 + 0.02$$

Binary Number System

The binary number system is a base or radix 2 number system. Hence, only two independent symbols are present in this number system, '0' and '1'. A binary digit is called a bit.

Counting in Binary

Start counting with 0, the next count is 1. Now, all the symbols are exhausted; therefore, we put a 1 in the left column and continue to get 10, 11. Thus, 11 is maxim we can count using two bits. So, put 1 to the left of the next column and continue.

Decimal No.	Binary No.
0	0
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000

Decimal to Binary Conversion

Steps: i) Divide the decimal integer by 2, producing quotient and a remainder. The remainder is separated (LSB).

ii) Continue the above step 1, till the quotient is less than 2.

iii) Collect the remainders from bottom to top, to get equivalent binary integer.

Example.

		Remainder
2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
	1	1

$(45)_{10} = (101101)_2$.

For Fractional part of Decimal Number

1) Multiply the fractional part by 2 and collect the integer values.

2) Do the successive multiplication till the fractional part becomes zero or make a round-off

Example: Convert $(0.625)_{10}$ and $(0.8)_{10}$ to binary.

(i)

	Real Part
$0.625 \times 2 = 1.25$	1
$0.25 \times 2 = 0.50$	0
$0.5 \times 2 = 1.00$	1

$(0.625)_{10} = (0.101)_2$.

(ii)

	Real Part
$0.8 \times 2 = 1.6$	1
$0.6 \times 2 = 1.2$	1
$0.2 \times 2 = 0.4$	0
$0.4 \times 2 = 0.8$	1

$(0.8)_{10} = (0.11011101 \text{---})_2$

$= (0.1101)_2$ [After round-off]

Octal Number System

The octal number system has a base or radix 8. It has eight different digits 0, 1, 2, 3, 4, 5, 6, 7.

Hexadecimal Number System

The hexadecimal number system has a base of 16. It has sixteen different digits. 0 to 9, A, B, C, D, E and F.

Decimal to Octal Number System

- ☆ Successive division method is used to convert the decimal ~~digit~~ integer to octal integer.
- ☆ Successive multiplication method is used to convert the decimal fraction to octal fraction.

Decimal to Hexadecimal

Same as for other conversion. The number has to be divided or multiplied by 16.

Example: Convert $(243)_{10}$ into octal.

$$\begin{array}{r|l c}
 & & \text{Remainder} \\
8 & 243 & \\
8 & 30 & 3 \\
8 & 3 & 6 \\
 & 0 & 3
\end{array}$$

$(243)_{10} = (363)_8$

Example: Convert $(143.45)_{10}$ into octal.

$$\begin{array}{r|l c}
 & & \text{Remainder} \\
8 & 143 & \\
8 & 17 & 7 \\
8 & 2 & 1 \\
 & 0 & 2
\end{array}$$

	Real Part
$0.45 \times 8 = 3.6$	3
$0.6 \times 8 = 4.8$	4
$0.8 \times 8 = 6.4$	6
$0.4 \times 8 = 3.2$	3
	⋮

$(143.45)_{10} = (217.3463\ldots)_8$

Example: Convert $(473)_{10}$ into hexadecimal.

		R
16	473	9
16	29	13 (D)
	1	1

$(473)_{10} = (1D9)_{16}$

Example: Convert $(812.45)_{10}$ into hexadecimal.

		R
16	812	12 (C)
16	50	2
	3	3

	Integer
$0.45 \times 16 = 7.2$	7
$0.2 \times 16 = 3.2$	3
$0.2 \times 16 = 3.2$	3

$(812.45)_{10} = (32C.733)_{16}$

Octal to Decimal and Hexadecimal to Decimal

Octal to decimal and hexadecimal to decimal can be converted to their decimal equivalent by the positional weight and the product terms are added to obtain the decimal number.

Octal to Decimal

Example: $(273.61)_8$ into decimal.

$$2\times8^2+7\times8^1+3\times8^0+6\times8^{-1}+1\times8^{-2}$$

$$=2\times64+7\times8+3+\frac{6}{8}+\frac{1}{64}$$

$$=(187.7656)_{10}$$

Hexadecimal to Decimal

Example: convert $(A3E.4B)_{16}$ into decimal.

$$A\times16^2+3\times16^1+E\times16^0+4\times16^{-1}+B\times16^{-2}$$

$$=10\times256+3\times16+14+\frac{4}{16}+\frac{11}{256}$$

$$=(2622.29)_{10}.$$

Binary to Octal

- ☆ The conversion of binary to octal is performed by forming a 3-bit group and converting each 3-bit binary to its octal equivalent.
- ☆ The grouping starts from right to left in the form of 3-bit for integer part and the grouping starts from left to right in the form of 3-bit groups for the fractional part.

Octal digit	Binary Equivalent
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Example: Convert $(101011001101 0 \cdot 0101)_2$ into octal

$$\underbrace{001}_{1}\,\underbrace{010}_{2}\,\underbrace{110}_{6}\,\underbrace{011}_{3}\,\underbrace{010}_{2} \cdot \underbrace{010}_{2}\,\underbrace{100}_{4}$$

$(12632 \cdot 24)_8$.

Octal to Binary

★ The octal base is 8 and the base of binary no. is 2. The octal no. base is written in the form of powers of 2 (2^3), which is indicated by a group of 3-bits and is equal to one octal digit.

Example: Convert $(527 \cdot 135)_8$ into binary

5	2	7	·	1	3	5
101	010	111	·	001	011	101

$(527 \cdot 135) = (101010111 \cdot 001011101)_2$

Hexadecimal to Binary

☆ The base of hexadecimal is 16 and the base of binary no. is 2. The base of hexadecimal can be written as powers of 2 (2^4) to get the equivalent binary no. Each hexadecimal digit is replaced by 4-bit binary group.

Hexadecimal No.	Binary Equivalent
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
A	1010
B	1011
C	1100
D	1101
E	1110
F	1111

Example: Convert $(B29E \cdot 2A5)_{16}$ into binary.

B	2	9	E	.	2	A	5
1011	0010	1001	1110		0010	1010	0101

$(B29E \cdot 2A5) = (1011 0010 1001 1110 \cdot 0010 1010 0101)_2$

Binary to Hexadecimal

Conversion from binary to hexadecimal no. is performed by converting 4-bit binary to its equivalent hexadecimal digit.

Example: Convert $(1011010100011101 1 \cdot 10010011 1)_2$ into hexadecimal no.

$$\underbrace{0001}_{1}\underbrace{0110}_{6}\;\underbrace{1010}_{A}\;\underbrace{0011}_{3}\;\underbrace{1011}_{B}\cdot\underbrace{1001}_{9}\;\underbrace{0011}_{3}\;\underbrace{1000}_{8}$$

Ans: $(16A3B \cdot 938)_{16}$.

Octal to Hexadecimal

Steps: i) Convert octal into binary.
ii) Regroup in 4 bits group
iii) Each group is replaced by hexadecimal digit.

Example: $(627 \cdot 54)_8$ convert into octal.

$$\begin{array}{ccccccc} 6 & 2 & 7 & \cdot & 5 & 4 \end{array}$$

$$\underbrace{0001}_{1}\;\underbrace{1001}_{9}\;\underbrace{0111}_{7}\cdot\underbrace{1011}_{B}\;100$$

$(197 \cdot B)_{16}$.

Hexadecimal to Octal

Step: i) Convert hexadecimal into binary.
ii) Regroup in 3-bits group
iii) Each group is replaced by octal digit.

Example: Convert $(D2FC \cdot ECE)_8$ into octal.

$$\begin{array}{cccccccccc} & D & 2 & F & C & \cdot & E & C & E \\ 001 & 101 & 0010 & 1111 & 1100 & \cdot & 1110 & 1100 & 1110 \end{array}$$

$$001\,101\,0010\,1111\,1100 \cdot 1110\,1100\,1110$$

$$\underline{001}\ \underline{101}\ \underline{0\,010}\ \underline{111}\ \underline{1\,11}\ \underline{00} \cdot \underline{111}\ \underline{0\,11}\ \underline{00\,1}\ \underline{110}$$

$$1 \quad 5 \quad 1 \quad 3 \quad 7 \quad 4 \cdot 7 \quad 3 \quad 1 \quad 6$$

$$(D2FC \cdot ECE)_8 = (151374 \cdot 7316)_8$$

Any Base/Radix to Other Base/Radix Conversion

Steps: i) Convert any base number into its equivalent decimal by using positional weight method.

ii) Convert decimal to other base by successive division method.

Example: Convert $(2A7)_{12}$ into base 7.

$$2 \times 12^2 + A \times 12^1 + 7 \times 12^0$$

$$= 2 \times 144 + 10 \times 12 + 7 = 288 + 120 + 7 = (415)_{10}$$

$$\begin{array}{r|l} 7 & 415 \\ \hline 7 & 59 \\ \hline 7 & 8 \\ \hline & 1 \end{array} \qquad \begin{array}{c} \text{Remainder} \\ 2 \\ 3 \\ 1 \\ 1 \end{array} \qquad (1132)_7.$$

$$(2A7)_{12} = (1132)_7.$$

Example: Convert $(532)_6$ into base 11.

$$5 \times 6^2 + 3 \times 6^1 + 2 \times 6^0 = 5 \times 36 + 3 \times 6 + 2$$

$$= 180 + 18 + 2 = (200)_{10}$$

$$\begin{array}{r|l} 11 & 200 \\ \hline 11 & 18 \\ \hline & 1 \end{array} \qquad \begin{array}{c} \text{Remainder} \\ 2 \\ 7 \\ 1 \end{array} \qquad (172)_{11} \qquad (532)_6 = (172)_{11}$$

Boolean Algebra & Logic Gates

Switching algebra is also called Boolean algebra. Boolean algebra differs from both ordinary algebra and binary no. system, In Boolean Algebra, $A+A \neq 2A$, $A.A \neq A^2$

Axioms and Laws of Boolean Algebra

$0.0=0$ $0+0=0$ $\overline{1}=0$

$0.1=0$ $0+1=1$ $\overline{0}=1$

$1.0=0$ $1+0=1$

$1.1=1$ $1+1=1$

Complementation Law

$\overline{0}=1$, $\overline{1}=0$, If $A=0$, $\overline{A}=1$

If $A=1$, $\overline{A}=0$, $\overline{\overline{A}}=A$

AND Laws

$A.0=0$, $A.1=A$, $A.A=A$, $A.\overline{A}=0$

OR Laws

$A+0=A$, $A+1=1$, $A+A=A$, $A+\overline{A}=1$.

Commutative Law

$A+B=B+A$, $A.B=B.A$

Associative Law

$$(A+B)+C = A+(B+C)$$

$$(A.B).C = A(B.C).$$

Distributive Law

$$A(B+C) = AB+AC$$

$$A+BC = (A+B)(A+C)$$

Proof: $RHS = (A+B)(A+C) = AA+AC+BA+BC$

$$= A+AC+AB+BC = A(1+C+B)+BC$$

$$= A+BC$$

Redundant Literal Rule

$$A+\bar{A}B = A+B.$$

Proof: $A+\bar{A}B = (A+\bar{A})(A+B)$ [Distributive Law]

$$= 1.(A+B). = A+B.$$

Idempotence Law

$$A+A=A, \quad A.A=A$$

Absorption Law

$$A+AB = A.$$

Proof: $A+AB = A(1+B) = A.1 = A.$

Consensus Theorem

$$AB + \bar{A}C + BC = AB + \bar{A}C.$$

Proof: $LHS = AB + \bar{A}C + BC(A + \bar{A})$

$$= AB + \bar{A}C + ABC + \bar{A}BC$$

$$= AB(1+C) + \bar{A}C(1+B) = AB + \bar{A}C.$$

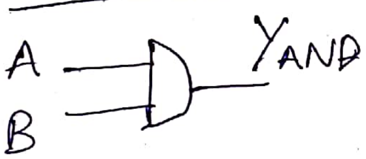
De Morgan's Theorem

$$\overline{A+B} = \bar{A}.\bar{B}$$

$$\overline{AB} = \bar{A} + \bar{B}$$

Logic Gates

AND gates

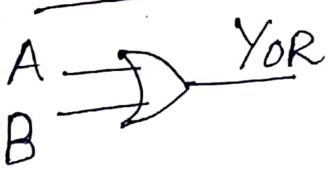


$$Y_{AND} = A.B$$

Truth Table

A	B	Y_{AND}
0	0	0
0	1	0
1	0	0
1	1	1

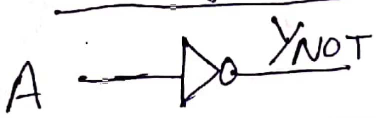
OR gate



$$Y_{OR} = A + B$$

A	B	Y_{OR}
0	0	0
0	1	1
1	0	1
1	1	1

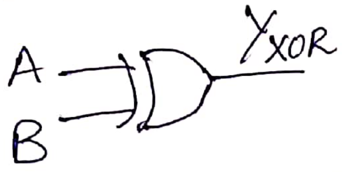
NOT gate/Inverter



$$Y_{NOT} = \bar{A}$$

A	Y_{NOT}
0	1
1	0

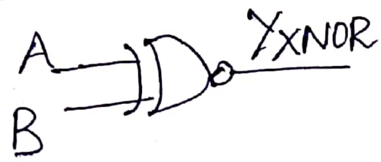
EX-OR gate



A	B	Y_{XOR}
0	0	0
0	1	1
1	0	1
1	1	0

$$Y_{XOR} = \overline{A}B + A\overline{B}$$
$$= A \oplus B$$

X-NOR gate

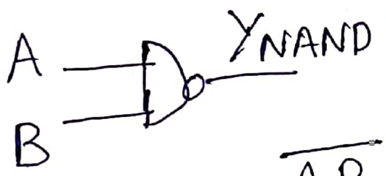


A	B	Y_{XNOR}
0	0	1
0	1	0
1	0	0
1	1	1

$$Y_{XNOR} = \overline{A}\,\overline{B} + AB$$
$$= A \odot B$$

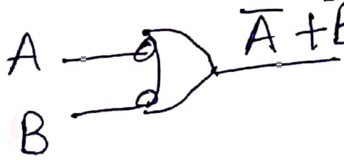
Universal Gates

NAND gate



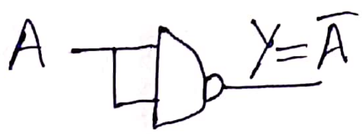
A	B	Y_{NAND}
0	0	1
0	1	1
1	0	1
1	1	0

$$Y_{NAND} = \overline{AB}$$
$$= \overline{A} + \overline{B}$$



$\overline{A} + \overline{B} = Y_{NAND}$

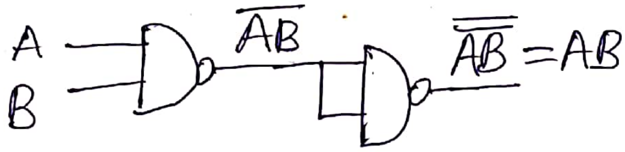
Inverter using NAND



$$Y = \overline{A \cdot A} = \overline{A}$$

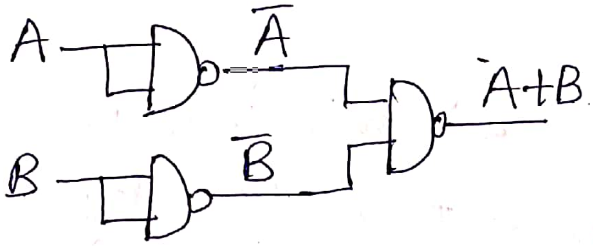
AND using NAND

$$Y = AB = \overline{\overline{AB}}$$

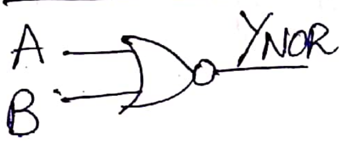


OR using NAND

$$Y = A+B = \overline{\overline{A+B}} = \overline{\bar{A} \cdot \bar{B}}$$



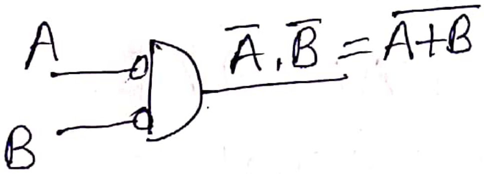
NOR Gate



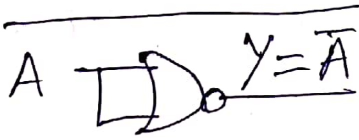
A	B	Y_{NOR}
0	0	1
0	1	0
1	0	0
1	1	0

$$Y_{NOR} = \overline{A+B}$$

$$= \bar{A} \cdot \bar{B}$$



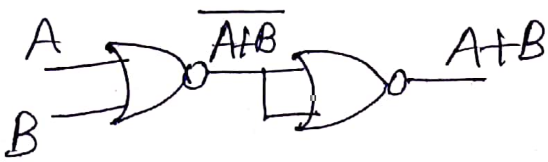
Inverter using NOR



$$Y = \overline{A+A} = \bar{A}$$

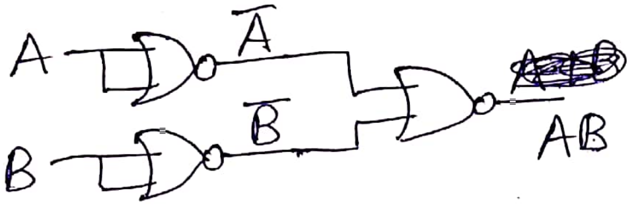
OR using NOR

$$Y = A+B = \overline{\overline{A+B}}$$



AND using NOR

$$Y = AB = \overline{\overline{AB}} = \overline{\bar{A} + \bar{B}}$$



Standard Sum-of-Product (SOP) Form

Steps to realize Standard SOP form:

1) Write down all the terms
2) If one or more variables is missing in any product term, expand that term by multiplying it with sum of each one of the missing variable and its complement.
3) Drop out redundant terms.

Example:

$$F(A,B,C) = \bar{A}B + \bar{A}B\bar{C} + \bar{B}C$$

$$= \bar{A}B(C+\bar{C}) + \bar{A}B\bar{C} + (A+\bar{A})\bar{B}C$$

$$= \bar{A}BC + \bar{A}B\bar{C} + \bar{A}B\bar{C} + A\bar{B}C + \bar{A}\bar{B}C$$

$$= 011 + 010 + 010 + 101 + 001$$

$$= \sum m(3,2,5,1) = \sum m(1,2,3,5).$$

Standard Product-of-Sum (POS) form

Steps: i) Write down all the terms.

ii) If one or more variable is missing in any sum term, expand that term by adding the products of each of the missing ~~term~~ variable and its complement.

iii) Drop out the redundant terms.

Example: $F(A,B,C) = (A+\overline{B})(\overline{A}+\overline{B}+C)(B+\overline{C})$

$= (A+\overline{B}+C\overline{C})(\overline{A}+\overline{B}+C)(A\overline{A}+B+\overline{C})$

$= (A+\overline{B}+C)(A+\overline{B}+\overline{C})(\overline{A}+\overline{B}+C)(A+B+\overline{C})(\overline{A}+B+\overline{C})$

$= (010)(011)(110)(110)(001)(101)$

$= \prod M(2,3,6,1,5) = \prod M(1,2,3,5,6).$