Gauss' law & Dielectrics

Section 4.3.1

19.1.21

Polarisation gives rise to the bound charges.

Total electric field ≡ Field due to bound charges
+
Field due to free charges
(any charge that is not a result of polarisation)

Within the dielectric,

$$\rho = \rho_b + \rho_f$$

Use Gauss' law,

$$\underbrace{\epsilon_0 \vec{\nabla} \cdot \vec{E}}_{\text{total field}} = \rho = \rho_b + \rho_f$$

$$= -\vec{\nabla} \cdot \vec{P} + \rho_f$$

$$\Rightarrow \vec{\nabla} \cdot \left(\underbrace{\epsilon_0 \vec{E} + \vec{P}}_{} \right) = \rho_f$$

$$\Rightarrow \vec{\nabla} \cdot \vec{D} = \rho_f$$

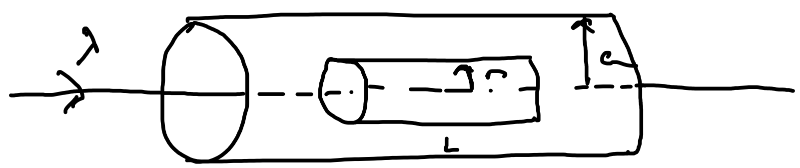
$\vec{D} \equiv$ Electric displacement

$$\Rightarrow \oint \vec{D} \cdot d\vec{a} = \int \rho_f \, d\tau$$

$\Rightarrow \oint \vec{D} \cdot d\vec{a} = Q_{f\,enc.}$ $\equiv$ Gauss' law in context of dielectrics

Only refers to free charges

<u>Ex:</u>



A long straight wire carrying uniform line charge λ surrounded by rubber insulation out to radius 'a'.

$\rightarrow$ using Gauss' law

$$D(2\pi s L) = \lambda L$$

$$\Rightarrow \vec{D} = \frac{\lambda}{2\pi s}\hat{s}$$

$\hookrightarrow$ Both for inside rubber tube and outside

Outside rubber tube, $\vec{P} = 0$

$$\Rightarrow \vec{E} = \frac{\vec{D}}{\epsilon_0} = \frac{\lambda}{2\pi\epsilon_0 s}\hat{s} \quad \text{for } s > a$$

Some Definitions:

Section 4.4.1

$$\vec{P} \propto \vec{E}$$

$$\Rightarrow \quad \vec{P} = \epsilon_0 \chi_e \vec{E}$$

$\chi_e \equiv$ Electric suceptibility (it depends on internal structure of substance)

$\epsilon_0 \rightarrow$ Extracted to make χ_e dimensionless

(x) Any material obeying $\vec{P} = \epsilon_0 \chi_e \vec{E}$ is called a linear dielectric.

$$\Rightarrow \vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$= \epsilon_0 (1 + \chi_e) \vec{E}$$

$$= \epsilon \vec{E}$$

$\epsilon = \epsilon_0 (1 + \chi_e) \equiv$ Permittivity of material

(x) In vacuum, there is no matter to Polarise.

$$\Rightarrow \chi_e = 0 \quad \Rightarrow \quad \epsilon = \epsilon_0$$

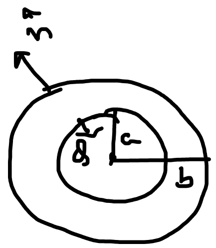
$\equiv$ Permittivity of free space

(*) $\epsilon_r = 1 + \chi_e = \dfrac{\epsilon}{\epsilon_0} \equiv$ Relative permittivity

(usually > 1) or

Dielectric constant

Ex:



A metal sphere of radius 'a' carries a charge 'q'. It is surrounded out to radius 'b' by a linear dielectric material of permittivity ϵ.

$$\vec{D} = \frac{q}{4\pi r^2}\,\hat{r} \qquad \text{for all points } r > a$$

Inside the sphere, $\vec{E} = \vec{P} = \vec{D} = 0$

Electric field,

$$\vec{E} = \frac{q}{4\pi\epsilon r^2}\,\hat{r} \qquad \text{for } a < r < b$$

$$\frac{q}{4\pi\epsilon_0 r^2}\,\hat{r} \qquad \text{for } r > b$$

Potential at the center,

$$V = -\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{b} \frac{q}{4\pi\epsilon_0 r^2}\,dr - \int_{b}^{a} \frac{q}{4\pi\epsilon r^2}\,dr - \int_{a}^{0} (0)\,dr$$

$$= \frac{q}{4\pi}\left[\frac{1}{\epsilon_0 b} + \frac{1}{\epsilon a} - \frac{1}{\epsilon b}\right]$$

Polarisation,

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$= \frac{\epsilon_0 \chi_e q}{4\pi \epsilon r^2}\hat{r} \quad \text{(inside dielectric)}$$

Charge density. $\rho_b = -\vec{\nabla}\cdot\vec{P} = 0$

$$\sigma_b = \vec{P}\cdot\hat{n} = \frac{\epsilon_0 \chi_e q}{4\pi \epsilon b^2} \quad \text{(at outer surface)}$$

$$-\frac{\epsilon_0 \chi_e q}{4\pi \epsilon a^2} \quad \text{(at inner surface)}$$

[$\hat{n}$ points outward w.r.t. the dielectric, $\hat{r}$ at b & $-\hat{r}$ at a]

(*) for a linear dielectric

$\vec{P} \propto \vec{E}$

$\vec{D} \propto \vec{E}$ $\Rightarrow$ Does this mean

$\vec{\nabla}\times\vec{D} = 0$?

$\vec{\nabla}\times\vec{P} = 0$?