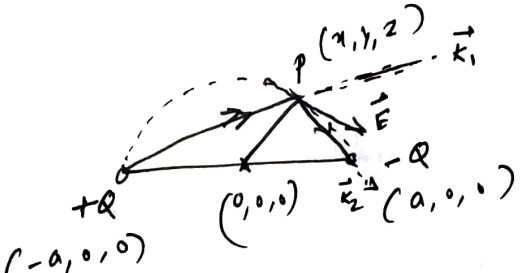
(1)

$\vec{r} = (x - x_0)\hat{i} + (y - y_0)\hat{j} + (z - z_0)\hat{k}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \left\{ \frac{Q}{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \right\} \hat{r}$$

2)



Electric field due to $+Q$

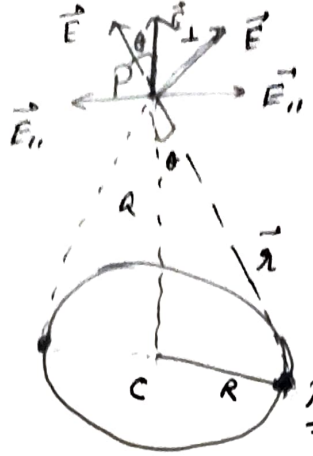
$$\vec{E}^{+} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{1}{(x+a)^2 + y^2 + z^2} \hat{r}_1$$

$$\vec{E}^{-} = \frac{-Q}{4\pi\epsilon_0} \frac{1}{(x-a)^2 + y^2 + z^2} \hat{r}_2$$

Total field $\vec{E} = \vec{E}^{+} + \vec{E}^{-}$

Calculate divergence of the field.

3)



'Horizontal' components cancel each other.

(λ = line-charge density = $\frac{Q}{2\pi R}$)

Therefore the electric field at P is just the vertical component ($\cos\theta$ component):

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl}{r^2}\cos\theta\ \hat{z}$$

($\hat{z}$ = unit vector along the vertical direction).

$$r^2 = a^2 + R^2$$
$$\cos\theta = \frac{a}{r}$$

and $\lambda = \frac{Q}{2\pi R}$

Hence $$\vec{E} = \frac{\lambda}{4\pi\epsilon_0}\int \frac{dl}{a^2+R^2}\cdot\frac{a}{\sqrt{a^2+R^2}}\,\hat{z}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\cdot\frac{a}{(a^2+R^2)^{3/2}}\int dl\ \hat{z}$$

$$= \frac{\lambda\cdot 2\pi R}{4\pi\epsilon_0}\cdot\frac{a}{(a^2+R^2)^{3/2}}\,\hat{z}$$

$$= \frac{Q}{4\pi\epsilon_0}\cdot\frac{a}{(a^2+R^2)^{3/2}}\,\hat{z}$$

4) Break it into the rings of radius 'r' and thickness dr, and use the previous problem's results.

Total charge of a ring = $\sigma \cdot 2\pi r\, dr$

(σ = surface charge density of circular disk = $\frac{Q}{\pi R^2}$)

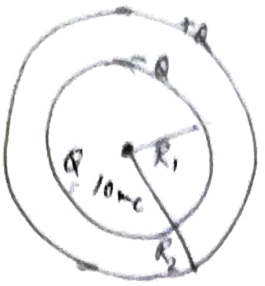
Electric field at a distance 'z' due to the ring = $\frac{\sigma \cdot 2\pi r\, dr}{4\pi\epsilon_0} \cdot \frac{z}{(r^2+z^2)^{3/2}}\,\hat{z}$

Therefore electric field for the disk at a distance z $\vec{E} = \frac{2\pi\sigma z}{4\pi\epsilon_0} \cdot \int_0^R \frac{r\, dr}{(r^2+z^2)^{3/2}}\,\hat{z}$

$= \frac{2\pi z}{4\pi\epsilon_0} \cdot \frac{Q}{\pi R^2} \left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}} \right] \hat{z}$

$= \frac{Qz}{2\pi\epsilon_0 R^2} \left[\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}} \right] \hat{z}$

5)



a) Total charges induced at the inner surface

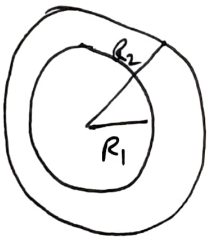
$= -Q = -10\,nc$

Total charges induced at the outer surface

$= +Q = +10\,nc$

b) Uniform

6)



vol. charge density $= \rho$.

Three regions —

Region 1 → $0 \leq r \leq R_1$

Region 2 → $R_1 < r \leq R_2$

Region 3 → $r > R_2$

} r = distance from the centre.

There is no charge in Region-1,

Hence electric field $\vec{E}_1 = 0 \quad 0 \leq r \leq R_1$

$\vec{\nabla} \cdot \vec{E}_1 = 0$

For regin -2 $(R_1 \leq r \leq R_2)$

$$\oint \vec{E} \cdot d\vec{s} = \frac{Q_{enc}}{\epsilon_0}$$

$$Q_{enc} = \rho \cdot \frac{4}{3}\pi (r^3 - R_1^3)$$

Henc $$\vec{E_2} = \frac{4\pi\rho}{3\epsilon_0} \cdot \frac{1}{4\pi r^2} (r^3 - R_1^3)\,\hat{r}$$

$$= \left(\frac{\rho r}{3\epsilon_0} - \frac{\rho R_1^3}{3\epsilon_0 r^2}\right)\hat{r}$$

$\vec{\nabla} \cdot \vec{E_2} = \rho/\epsilon_0$

For regin -3 $(r > R_2)$

$$Q_{enc} = \rho \cdot \frac{4\pi}{3}\left(R_2^3 - R_1^3\right)$$

$$\vec{E_3} = \frac{4\pi\rho}{3\epsilon_0} \cdot \frac{1}{4\pi r^2} (R_2^3 - R_1^3)$$

$$= \frac{\rho}{3\epsilon_0 r^2}(R_2^3 - R_1^3)\,\hat{r}$$

$$\vec{\nabla} \cdot \vec{E_3} = 0$$

7)

$$\rho(r) = \rho_0 + \alpha r \qquad 0 < r < R$$

$$= 0 \qquad r > R$$

a) $Q_{enc} = \int \rho(r)\, d\tau$

$d\tau$ in spherical polar coordinate

$= r^2 \sin\theta \, d\theta \, d\phi \, dr$

Total charge inside the sphere of radius R

$$Q_{en} = \int (\rho_0 + \alpha r)\, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= \rho_0 \int_0^R r^2 dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$+ \alpha \int_0^R r^3 dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$$= \frac{4\pi\rho_0}{3} R^3 + \frac{4\pi\alpha}{4} R^4$$

$$= \frac{4\pi R^3}{3}\left(\rho_0 + \frac{3\alpha}{4} R\right)$$

b) For $0 < r < R$

$$Q_{enc} = \frac{4\pi r^3}{3}\left(\rho_0 + \frac{3\alpha r}{4}\right)$$

Hence $\vec{E} = \frac{1}{4\pi r^2} \cdot \frac{4\pi r^3}{3\epsilon_0}\left(\rho_0 + \frac{3\alpha r}{4}\right)$

$$= \frac{r}{3\epsilon_0}\left(\rho_0 + \frac{3\alpha r}{4}\right)$$

for $r > R$

$$\vec{E} = \frac{1}{4\pi r^2} \frac{4\pi R^3}{3\epsilon_0}\left(\rho_0 + \frac{3\alpha R}{4}\right)$$

$$= \frac{R^3}{3\epsilon_0 r^2}\left(\rho_0 + \frac{3\alpha R}{4}\right)$$

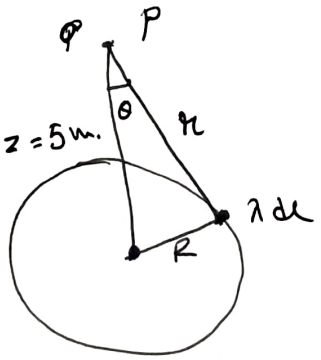
c) $\vec{\nabla}\cdot\vec{E} = \rho/\epsilon_0$ for $0 < r < R$

$\vec{r}\cdot\vec{E} = 0$ for $r > R$

d) $\vec{\nabla}\times\vec{E} = 0$ everywhere.

8)

a)



radius of ring $(R) = 2$ m

λ = line charge density

$$= \frac{50 \times 10^{-9}}{2\pi \times 2} \text{ Coulomb/m}$$

$$dV(r) = \frac{\lambda\, dl}{4\pi\epsilon_0 R} \qquad R = \sqrt{R^2 + z^2}$$

$$= \sqrt{2^2 + 5^2}$$

$$= \sqrt{29}$$

$$\Rightarrow V = \frac{\lambda}{4\pi\epsilon_0\sqrt{29}}\int dl$$

$$= \frac{2\pi\lambda \cdot R}{4\pi\epsilon_0\sqrt{29}}$$

$$= \frac{2\pi\lambda \cdot 2}{4\pi\epsilon_0\sqrt{29}}$$

b) Find the potential at $z = 0$ (V_1)
and at $z = 5$ m (V_2)

Work done $= Q(V_2 - V_1)$, Here $Q = 10$ nc.

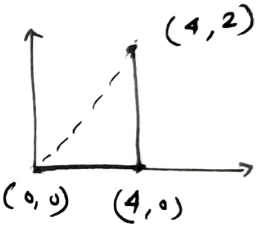
c) potential at $z = 5$ and $z = -5$ is same.

Hence in this case $V_1 = V_2$

Work done $= 0$.

g) $\vec{E} = 2(x+4y)\hat{i} + 8x\hat{j}$

$d\vec{l} = dx\hat{i} + dy\hat{j} + dz\hat{k}$



$\vec{E} \cdot d\vec{l} = 2(x+4y)\,dx + 8x\,dy$

potential difference $= \int \vec{E} \cdot d\vec{l}$

$$= \left[\int_0^4 2(x+4y)\,dx\right]_{y=0} + \left[\int_0^2 8x\,dy\right]_{x=4}$$

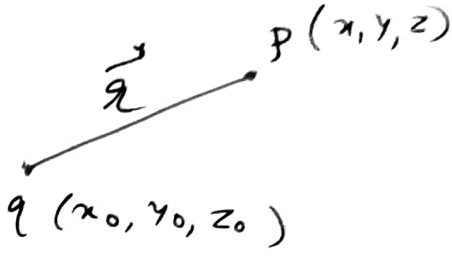
$$= x^2\Big|_0^4 + 8xy\Big|_{y=0}^{2}{}_{x=4}$$

$$= (16 + 64)\ V$$

$$= 80\ V.$$

10) ~~potential due to a point charge at (x₀~~

10) potential at (x, y, z) due to a point charge at (x_0, y_0, z_0) can be expressed as



$$V(x,y,z) = \frac{q}{4\pi\epsilon_0 |R|}$$

$$\vec{R} = (x-x_0)\hat{i} + (y-y_0)\hat{j} + (z-z_0)\hat{k}$$

$$|\vec{R}| = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2}$$

Calculate the potential at (2, 2, 3) where

$$|\vec{R}_1| = \sqrt{(2-2)^2 + (2-3)^2 + (3-3)^2}$$

$$= 1$$

$$V_1 = \frac{1.2 \times 10^{-9}}{4\pi\epsilon_0 \times 1}$$

Similarly $|\vec{R}|$ for (−2, 3, 3) is

$$|\vec{R}_2| = \sqrt{(-2-2)^2 + (3-3)^2 + (3-3)^2}$$

$$= 4$$

$$V_2 = \frac{1.2 \times 10^{-9}}{4\pi\epsilon_0 \times 4}$$

potential difference.

$$= (V_1 - V_2)$$