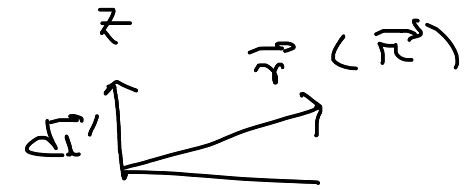
02.02.21

⊗ Magnetic field due to a long straight current carrying wire ($s \to$ distance from wire)

$$B = \frac{\mu_0 I}{2\pi s}$$

in cylindrical coordinate system,



$$\vec{B} = \frac{\mu_0 I}{2\pi s}\,\hat{\phi}$$

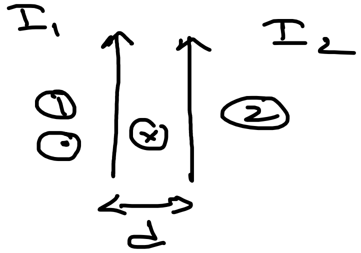
$$\hat{dl}' \times \hat{\mathscr{r}} = \hat{dl}' \times \hat{r}$$

$$= \hat{z} \times \hat{r} = \hat{\phi}$$

$\vec{r} = \vec{r}\,' + \vec{\mathscr{r}}$

$\Rightarrow \underline{\underline{\vec{\mathscr{r}}}} = \underline{\underline{\vec{r}}}$ (source at origin)

<u>Application:</u> Force of attraction betn. two long parallel wires a distance 'd' apart.



magnetic field at ② due to ①

$$B = \frac{\mu_0 I_1}{2\pi d} \quad \text{(points into page)}$$

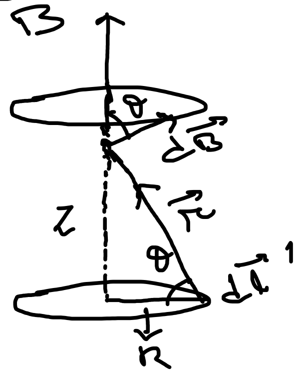
Resultant Force

$$F = \frac{\mu_0 I_1}{2\pi d} I_2 \int dl$$

Total force per unit length:

$$f = \frac{\mu_0}{2\pi} \frac{I_1 I_2}{d}$$

Ex:



Magnetic field a distance z above the centre of a circular loop of radius R carrying a steady current I

(1) Integrating over $d\vec{l}'$ around loop
$\Rightarrow$ $d\vec{B}$ sweeps out a cone

$d\vec{l}' \perp \vec{r}$

(2) Horizontal components cancel and vertical components add up.

$$B(z) = \frac{\mu_0 I}{4\pi} \int \left(\frac{dl'}{r^2} \right) \cos\theta$$

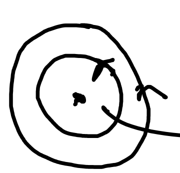
$$= \frac{\mu_0 I}{4\pi} \left(\frac{\cos\theta}{r^2} \right) 2\pi R$$

$$\cos\theta = \frac{R}{r}$$

$$r = (R^2 + z^2)^{1/2}$$

$$= \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}}$$

Divergence & Curl



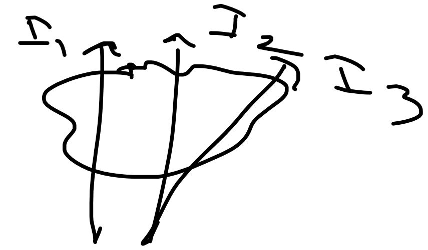
$\to$ current coming out of the page

Line integral of $\vec{B}$ around a circular path of radius 's' centered at the wire

$$\oint \vec{B} \cdot d\vec{l} = \oint \frac{\mu_0 I}{2\pi s} dl = \mu_0 I$$

(x) Any loop enclosing the wire gives same result.

(x) For a bundle of wires



$$\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{enc.}$$

$I_{enc.} = I_1 + I_2 + I_3$

↓

total current enclosed by the closed loop (Amperian loop)

(x) If the flow of charge is represented by a volume charge density, $\vec{J}$

$$I_{enc.} = \int \vec{J}\cdot d\vec{a}$$

→ surface bounded by the loop.

Applying Stoke's th.

$$\int (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0 \int \vec{J}\cdot d\vec{a}$$

$$\Rightarrow \underline{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}}$$

⇝ Generic formula for curl of $\vec{B}$

(Ampere's Law)

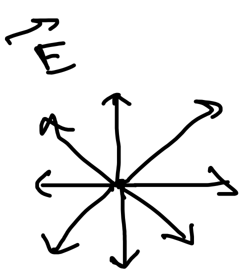
(x) $\vec{B} = \dfrac{\mu_0 I}{2\pi s}\hat{\phi}$ (cylindrical coordinate system)

$$\vec{\nabla}\cdot\vec{B} = \frac{1}{r}\frac{\partial}{\partial r}(r B_r) + \frac{1}{r}\frac{\partial B_\phi}{\partial \phi} + \frac{\partial B_z}{\partial z}$$

$$\vec{\nabla}\cdot\vec{B} = \vec{\nabla}\cdot\left(\frac{\mu_0 I}{2\pi s}\hat{\phi}\right)$$

$\Rightarrow \vec{\nabla}\cdot\vec{B} = 0 \rightarrow$ Generic formula for divergence of $\vec{B}$

(+) Maxwell's eq. for Electrostatics & Magnetostatics:

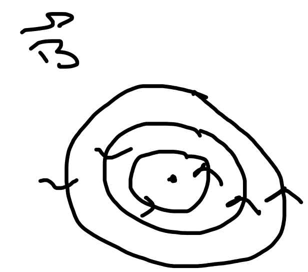


$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla}\times\vec{E} = 0$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$$



→ There are no point like sources for $\vec{B}$.
No magnetic analog to an electric charge.
In other words, no magnetic monopoles!!

→ $\int(\vec{\nabla}\cdot\vec{B})\,d\tau = \oint\vec{B}\cdot d\vec{a} = 0$

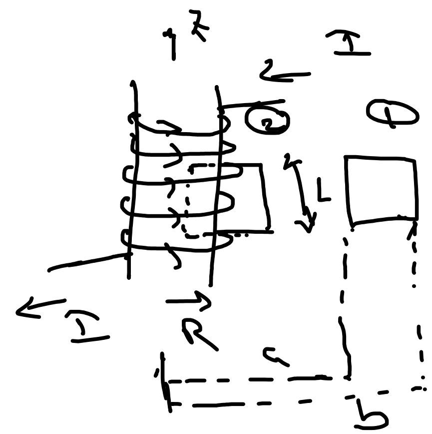
$\Rightarrow$ Magnetic flux through any closed surface is zero

Ampere's law: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ (differential form)

Directionality from right-hand rule $\leftarrow \oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc.}$ (integral form)

(*) For currents with appropriate symmetry, Ampere's law in integral form can be efficient in calculating magnetic field.

Ex:



Long solenoid with n number of closely wound wires per unit length each carrying a steady current I

→ Using right-hand rule, $\vec{B}$ points upward inside the solenoid.

For loop ①, $\oint \vec{B} \cdot d\vec{l} = \left(B(a) - B(b) \right) L = \mu_0 I_{enc.} = 0$

(independent of distance) $\Rightarrow B(a) = B(b)$

→ field goes to zero far away from source & we found out that the field outside is independent of distance $\Rightarrow$ outside at any point $\vec{B} = 0$

for loop ②

$$\oint \vec{B}\cdot d\vec{l} = BL = \mu_0 I_{enc.}$$ (B → field inside solenoid)

$$= \mu_0 n L I$$

$$\Rightarrow \vec{B} = \mu_0 n I \hat{z}$$

Magnetic field due to solenoid.

$$\vec{B} = \mu_0 n I \hat{z} \quad \text{(inside)}$$

$$= 0 \quad \text{(outside)}$$