Gr - 1 Monday 7 Dec 2020.

Differentiability:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad \text{exist.}$$

Chain Rule

$$\begin{cases} f(x) = \sin x. \checkmark \\ g(x) = e^x \checkmark \\ h(x) = x^2 \checkmark \end{cases}$$

$\underline{f(g(h(x)))}$ ✓

$= f(g(x^2))$

$= f(e^{x^2})$

x^2

Algebra

If f and g are diff. func^n

then $f \pm k \cdot g$ is also diff.

ex: $f(x) = x^2$

$f'(x) = 2x.$

$g(x) = \sin x.$

$g'(x) = \cos x.$

$f(x) + 5 \cdot g(x)$

$= x^2 + 5 \cdot \sin x$

$= \underline{\sin e}$

$(\sin e^x)^2$

$$h\left(f\left(g(x)\right)\right)$$

$h \circ f \circ g$

$$\frac{d}{dx}\left(f \circ g \circ h(x)\right)$$

$$f'(x)\left(\frac{d}{dx}\, g \circ h(x)\right)$$

$$= f'(g(h(x)))\left(g'(h(x))\frac{d}{dx}\, h(x)\right)\Big)$$

$$\frac{d}{dx}\left(\sin e^{x^2}\right)$$

$$= \cos e^{x^2} \cdot \frac{d}{dx} e^{x^2}.$$

$(f+5g)'(x)$

$= 2x + 5\cos x.$

$\dfrac{f(x)}{g(x)}$

Polynomial func^{n}s.

exponential func^{n}s

e^x, $\begin{cases}\sin x, \\ \cos x.\end{cases}$

$x^5 + 3x^2 + 7.$

$5x^4 + 6x$

$20x^3 + 6$

$60x^2.$

$\vdots$

0

$\dfrac{f(x)}{g(x)}.$

x^2+5

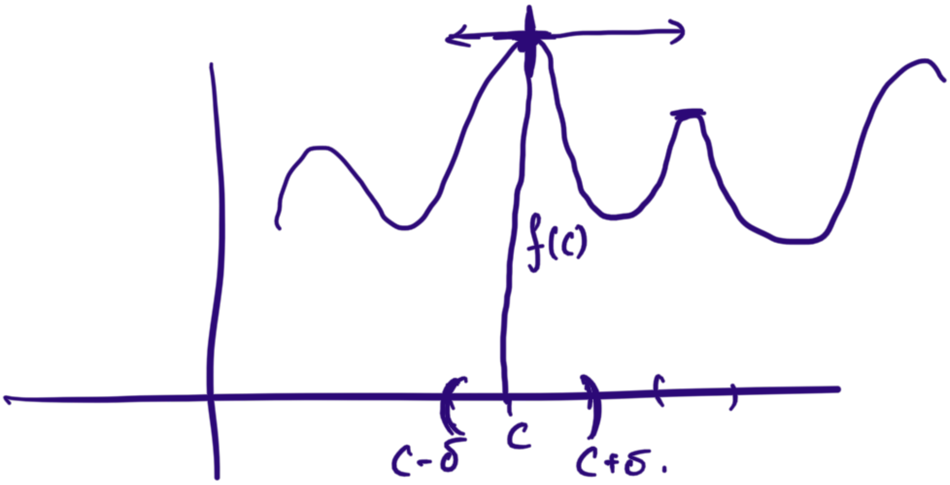
$$= \cos e^{x^2} \cdot e^{x^2} \cdot \frac{d}{dx} x^2$$

$x - 1$

$x = 1$

$$= \cos e^{x^2} \cdot e^{x^2} \cdot 2x.$$

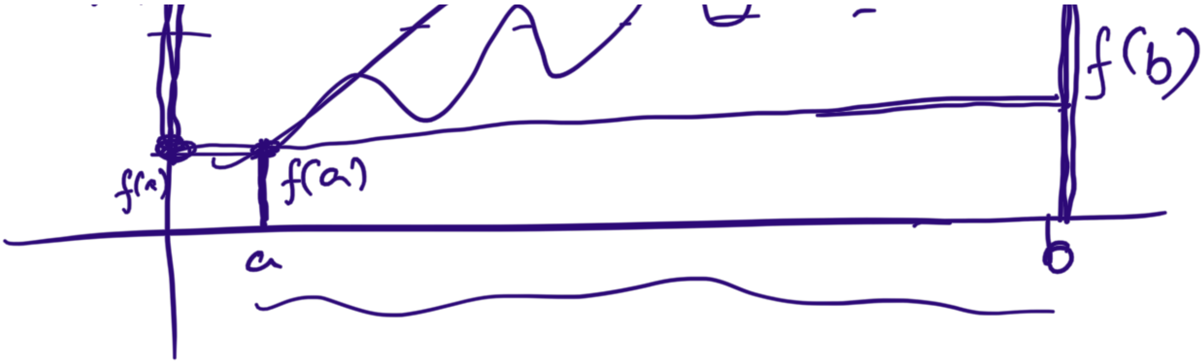
Local maxima

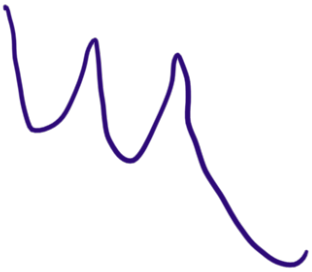


$\delta > 0.$

$x \in (c-\delta, c+\delta).$

$f(c) \geq f(x).$ ✓

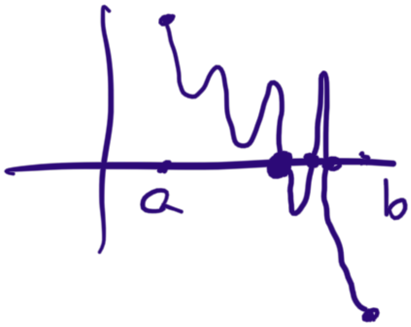




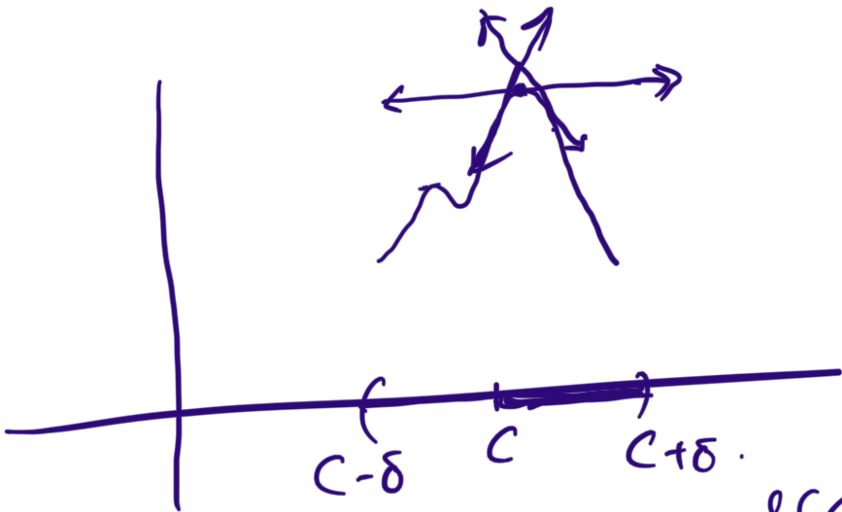
f : cont.

$f(a) > 0$

$f(b) < 0$.



$f(x) = 0$ ✓



local maximum

$f(c) \geq f(x)$.

$x \neq c$.

$x \in (c, c+\delta)$ ✓.

✓ $$\frac{f(x) - f(c)}{x - c} \leq 0.$$

$x - c > 0$.

$f(x) - f(c) \leq$

$$x \in (c-\delta, c) \qquad x \neq c.$$

$$x - c < 0.$$

$$f(x) \leq f(c)$$

$$f(x) - f(c) \leq 0.$$

$$\checkmark \quad \frac{f(x)-f(c)}{x-c} \geq 0.$$

$$\lim_{x \to c^-} \frac{f(x)-f(c)}{x-c}$$

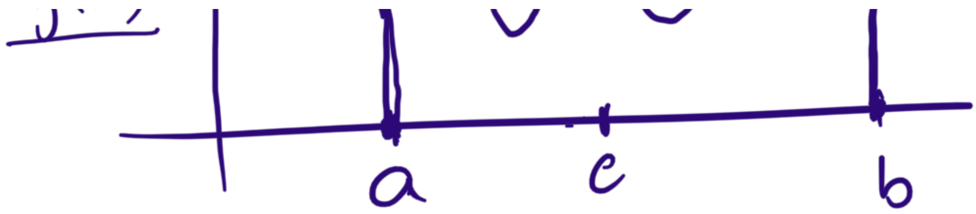
$$f'(c) = 0.$$

Rolle's theorem

f: cont. $[a,b]$.

diff. (a,b)

$f(a) = f(b)$.



$f'(c) = 0.$