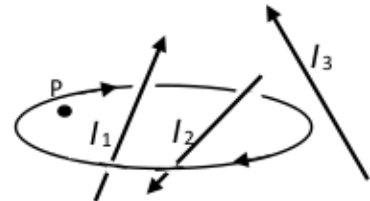


## Tutorial Set - 6

1. Which of the following functions cannot represent a magnetic field?

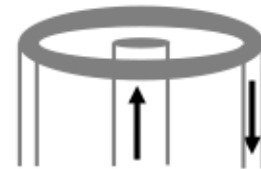
a)  $\vec{F}_1 = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$   
 b)  $\vec{F}_1 = xy\hat{i} + yz\hat{j} + 2xz\hat{k}$   
 c)  $\vec{F}_3 = \frac{\alpha}{(x^2+y^2)}(-y\hat{i} + x\hat{j})$

2. Three wires are carrying currents  $I_1$ ,  $I_2 = 2I_1$  and  $I_3 = 3I_1$  as shown in the figure.



- a) Write down the value of  $\oint \vec{B} \cdot d\vec{l}$  over the curved path shown.  
 b) Draw two different paths over which we will get (i) a value of  $2\mu_0 I_1$  for  $\oint \vec{B} \cdot d\vec{l}$ .  
 c) What will be the values of  $\nabla \cdot \vec{B}$  and  $\nabla \times \vec{B}$  at the point P?  
 d) State whether the following statement is true or false: "The magnetic field along the curved path shown in the figure is independent of the current  $I_3$ ."  
 3. A long cylindrical wire of radius  $R$  carries a current  $I$  with a volume current density of  $\vec{J} = \alpha r^2 \hat{z}$  where  $r$  is the distance from the axis of the cylinder and  $\hat{z}$  is the unit vector along the axis of the cylinder.  
 a) Obtain the magnetic field in all regions.  
 b) Obtain  $\nabla \cdot \vec{B}$  and  $\nabla \times \vec{B}$  in all regions.  
 4. Consider a straight cylindrical region of thickness  $(b - a)$  and having a circular cross section between inner radius  $a$  and outer radius  $b$ . A current  $I$  flows uniformly through the cross section of the cylinder.

- a) Calculate the magnetic field in all regions.  
 b) Obtain  $\nabla \cdot \vec{B}$  and  $\nabla \times \vec{B}$  in all regions.  
 5. Consider a coaxial configuration as shown in the figure.



- The inner solid cylinder carries a current in the upward direction while the outer annular cylinder (tube) carries the same current in the downward direction. Calculate the magnetic field in all regions. The radius of the inner cylinder is  $a$  and the inner and outer radii of the outer annular cylinder are  $b$  and  $c$  respectively. Calculate  $\nabla \cdot \vec{B}$  and  $\nabla \times \vec{B}$  in all regions.  
 6. A long cylindrical conductor of radius  $R$  has a cylindrical hole of radius  $a$  drilled out such that the axis of the hole is parallel to the axis of the cylinder. If  $b$  is the distance between the two axes and current  $I$  is passing through the remaining solid cylinder, show that the magnitude of the magnetic field is constant throughout the hole and is given by  $\frac{\mu_0 Ib}{2\pi(R^2 - a^2)}$ .  
 7. One can produce a reasonably uniform magnetic field by using two parallel current carrying coils of radii  $R$  placed at a distance  $d$  apart. For this configuration calculate  $B$  along the axis as a function of  $z$ , the distance from a point midway between the coils and find out under what conditions both the first and second derivative of  $B$  with respect to  $z$  will vanish at a point midway between the coils. Find the corresponding magnetic field at that point. Such an arrangement is referred to as Helmholtz coil.  
 8. A fine wire is used to wind six turns around an insulating sphere of radius  $a$  such that each turn makes an angle of  $30^\circ$  with the adjacent turn and that all the turns intersect each other at diametrically opposite points on the surface of the sphere. If a current  $I$  is passed through these turns, find the magnitude of  $\vec{B}$  at the center of the sphere. [Ans:  $B \approx 1.93\mu_0 I/a$ ].