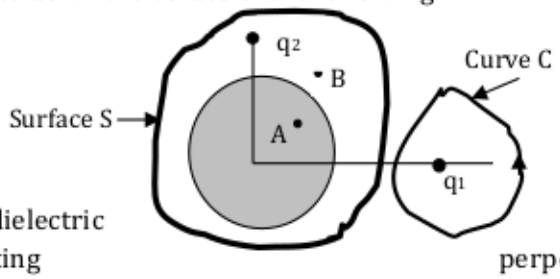
Department of Physics, Bennett University
EPHY105L (I Semester 2020-21)

Tutorial Set - 5

1. Consider a set of two point charges q_1 and q_2 and a dielectric sphere of radius R and uniform dielectric constant K (shown shaded in the figure). Write down the values of the following:
   i) $\oiint \vec{E}.d\vec{a}$ and $\oiint \vec{D}.d\vec{a}$ over surface S
   ii) $\oint \vec{E}.d\vec{l}$ over curve C
   iii) $\nabla.\vec{D}$ at point A and
   iv) $\nabla.\vec{E}$ at point B



2. An infinite dielectric slab of thickness d and dielectric constant K is placed in a uniform electric field $\vec{E}_0$ pointing perpendicular to the surface. Calculate the electric field inside the dielectric, the bound surface and bound volume charge densities. What is the total bound charge? [Ans: $\vec{E} = \frac{\vec{E}_0}{K}; \sigma_b = \epsilon_0 \frac{(K-1)}{K} E_0; \rho_b = 0$]
3. The space between the plates of a parallel plate capacitor is filled with two slabs of linear dielectric material. Each slab is of thickness a and the separation between the plates is $2a$. The dielectric constants of the slabs are K_1 and K_2. The free charge density on the upper plate is $+\sigma_f$ and that on the lower plate is $-\sigma_f$.
   a) Find the electric displacement $\vec{D}$ in each slab.
   b) Find the electric polarization $\vec{P}$ in each slab.
   c) Find the location and amount of all bound charges.
4. Consider a parallel plate capacitor filled with a linear dielectric with dielectric permittivity described by the following equation:

$$\epsilon = \epsilon_1 + \frac{(\epsilon_2 - \epsilon_1)}{d} x$$

   where d is the separation between the plates and x-direction is perpendicular to the plates of the capacitor. If the charge densities on the plates are $+\sigma$ and $-\sigma$,
   a) Obtain the bound volume charge density within the dielectric.
   b) Obtain the bound surface charge density on the two surfaces of the dielectric.
   c) Obtain the electric field variation within the dielectric.
   d) Obtain the potential difference between the capacitor plates.
5. Two parallel conducting plates 2 cm apart are given an equal and opposite surface charge densities of 30 μC/m². The space between the plates is occupied by two sheets of dielectric, one 8 mm thick (with dielectric constant 2) and the other 1.2 cm thick (with dielectric constant 3). Calculate the electric field and the electric displacement in each dielectric as well as the induced charge density on the surface of each dielectric.
   [Ans: $D_1 = D_2 = 30$ μC/m². $E_1 = 1680$ V/mm, $E_2 = 1120$ V/mm, $\sigma_1 = 12$ μC/m² and $\sigma_1 = 20$ μC/m².]
6. Consider a uniformly polarized dielectric sphere of radius R with the polarization given by $\vec{P} = P_0\hat{z}$. (a) Obtain the bound surface and volume charge densities. (b) Obtain the total bound charge in the dielectric. (c) Calculate the electric field at the center of the sphere due to this charge distribution.
7. A point charge Q is placed at the center of a dielectric sphere of radius R and dielectric constant K. Obtain the bound surface charge density on the surface of the sphere. [Ans: $\sigma_b = Q(K-1)/4\pi K R^2$].
8. A point charge Q is placed at the center of a sphere which has free space in the region $0 < r < R_1$ and a linear, homogeneous dielectric with susceptibility χ_e for $R_1 < r < R_2$ and free space for $r > R_2$.
   a) Find the electric field in all regions.
   b) Obtain all the bound surface and volume charge densities in the dielectric.
   c) What is the value of $\nabla.\vec{D}$ at a point r_0 with $R_1 < r_0 < R_2$?