Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Solutions for Tutorial Sheet 5

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1. (a) $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{f(h)}{h}=\begin{cases}\lim\limits_{h\to 0}\frac{h}{h}=1, & h\in\mathbb{Q}\\ \lim\limits_{h\to 0}\frac{\sin h}{h}=1, & h\notin\mathbb{Q}.\end{cases}$ Thus $f'(0)=1$.

   (b) $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{\sin\frac{1}{h}}{\sqrt{h}}$ doesn't exist. So f is not differentiable at $x=0$.

   (c) $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}h\cos\dfrac{1}{h}=0$. Therefore f is differentiable at $x=0$.

   (d) $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{e^{-\frac{1}{h^2}}}{h}=\lim\limits_{k\to\infty}\dfrac{k}{e^{k^2}}=0$. Thus f is differentiable at 0.

   (e) $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}\cos\dfrac{1}{h}$ doesn't exist. So f is not differentiable at 0.

   (f) $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{e^{-|h|}-1}{h}$ doesn't exist. So f is not differentiable at 0.

2. (a) $\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{h^3\sin\frac{1}{h}}{h}=\lim\limits_{h\to 0}h^2\sin\dfrac{1}{h}=0$. Thus f is differentiable at 0 and $f'(0)=0$. Now $f'(x)=3x^2\sin\frac{1}{x}-x\cos\frac{1}{x}$. So $\lim\limits_{x\to 0}f'(x)=0=f'(0)$. Therefore f' is continuous at $x=0$.

   (b) $f'(0)=\lim\limits_{h\to 0}\dfrac{f(0+h)-f(0)}{h}=\lim\limits_{h\to 0}\dfrac{h^2\cos\frac{1}{h}}{h}=0$. Therefore f is differentiable at 0 and $f'(0)=0$. Now $f'(x)=2x\cos\frac{1}{x}+\sin\frac{1}{x}$, $x\neq 0$. So limit does not exist as $x\to 0$. Thus f' is not continuous at $x=0$.

   (c) For $x>0$, $f'(x)=2x\ln\frac{1}{x}-x$ and $\lim\limits_{x\to 0^+}f'(x)=0$. Also for $x<0$, $f'(x)=2x\ln\frac{1}{|x|}-x$ and $\lim\limits_{x\to 0^-}f'(x)=0$. As $f'(0)=\lim\limits_{h\to 0}h\ln\frac{1}{|h|}=0$, thus f' is continuous at 0.

3. Use $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$.

4. Use L'Hospital rule. $(a)\ \frac{1}{2}$, $(b)\ -\frac{1}{24}$, $(c)\ -1$.