

Solution - Quiz 1

①

What is Bolzano-Weierstrass theorem?

- ☐ (A) Every convergent sequence is bounded.
- ☐ (B) Every convergent sequence has a bounded subsequence.
- ☐ (C) All subsequence of a convergent sequence are bounded.
- ☒ (D) Every bounded sequence has a convergent subsequence.

②

Consider sequence whose n^{th} term is given as follows: $a_n = \frac{n^3}{3^n}$.

Then sequence a_n

- ☐ (A) is convergent to e .
- ☒ (B) is convergent to 0 .
- ☐ (C) is convergent to 1 .
- ☐ (D) divergent to ∞ .

Ans Tutorial Sheet 2 - Question 4 (i).

Put $\epsilon = 3$ and $k = 3$.

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An increasing sequence which is bounded below:

- (A) is always divergent.
- (B) is always convergent.
- (C) convergent to zero.
- ~~(D)~~ may or may not be convergent.

Ans: An increasing sequence is always bounded below.
However.

increasing sequence $\begin{cases} \nearrow \text{if bounded above} \Rightarrow \text{convergent to its supremum.} \\ \searrow \text{if not bounded above} \Rightarrow \text{divergent to infinity.} \end{cases}$

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Find limit of sequence $a_n = \sqrt{n^2+n} - \sqrt{n^2+1} \quad \forall n \in \mathbb{N}$.

- (A) converges to $\frac{1}{3}$.
- (B) converges to 1.
- ~~(C)~~ converges to $\frac{1}{2}$.
- (D) Divergent.

Ans Tutorial sheet 2 — Qus 2 (c).

⑤ Find $V_5(0.3)$.

Ⓐ $(-4.7, 5.3)$

Ⓑ $(0.7, 5.3)$

Ⓒ $(4.7, 5.3)$

Ⓓ $(-4.7, -5.3)$

Ans we know $V_\epsilon(a) = (a-\epsilon, a+\epsilon)$.

So
$$V_5(0.3) = (0.3-5, 0.3+5)$$
$$= (-4.7, 5.3)$$