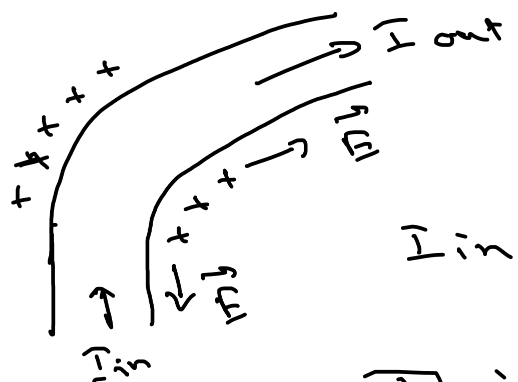
Electrodynamics:

11.2.21

Electromotive force:

$\boxed{\text{Section } 7.1.2}$

* Why current remains uniform all the way around a circuit?
  → Say, this was not the case.



There is an accumulation of charges in the bend. This gives rise to an electric field which resists I in but assists I out.

I in > I out

→ This goes on until $I_{in} = I_{out}$.

* Two forces that drive the current
  → the source, $\vec{f}_s$ (say, a battery)
  → the electrostatic field, $\vec{E}$

$$\vec{f} = \vec{f}_s + \vec{E}$$

The net effect of this force

$$\mathcal{E} \equiv \oint \vec{f} \cdot d\vec{l} = \oint \vec{f}_s \cdot d\vec{l} \qquad \left| \oint \vec{E} \cdot d\vec{l} = 0 \right.$$

↳ Electromotive force or emf of the circuit.

(x) If $\vec{f}_s = -\vec{E}$ (ideal source, $\vec{f} = 0$)

The potential difference between two terminals 'a' & 'b'

$$V = -\int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \vec{f}_s \cdot d\vec{l}$$

The integral can be extended to entire loop since outside, $\vec{f}_s = 0$

$$= \oint \vec{f}_s \cdot d\vec{l} = \mathcal{E}$$

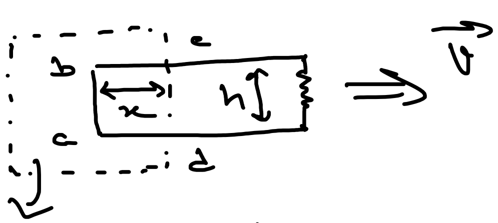
→ The battery maintains a voltage diff. equal to emf

→ The electrostatic force drives current around the circuit.

Motional emf:

Section 7.1.3

Motional emf generates when you move a wire through a magnetic field.



represents a magnetic field ($\vec{B}$) pointing into the page

The line segment 'ab' experiences a magnetic force qvB

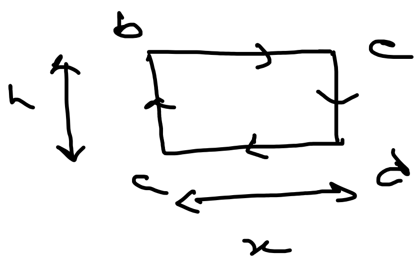
→ current flowing in clockwise direction.

The emf $\mathcal{E} = \oint \vec{f}_{mag.} \cdot d\vec{l}$

$$= vBh$$

* The 'bc' & 'ad' contributes nothing since the force is perpendicular in these segments.

* To express emf generated in a moving loop



→ Let Φ be the flux of $\vec{B}$ through loop

$$\Phi = \int \vec{B} \cdot d\vec{a}$$

For the rectangular loop, $\Phi = Bhx$

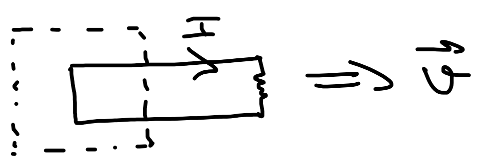
→ As the loop moves, the flux decreases.

$$\frac{d\Phi}{dt} = Bh\underbrace{\frac{dx}{dt}}_{\text{negative}} = -Bhv \equiv \text{emf}$$

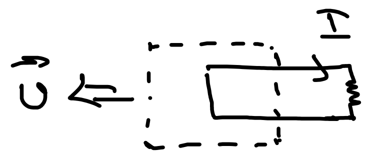
$\Rightarrow \mathcal{E} = -\frac{d\Phi}{dt} \Rightarrow$ flux rule of motional emf.

Faraday's law

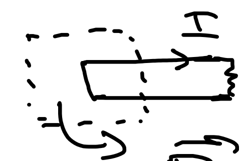
Section 7.2.1



Ex 1



Ex 2



$\vec{B}$ changes with time

Ex 3

Ex 1 $\quad \mathcal{E} = - \dfrac{d\Phi}{dt}$

Ex 2 Relative motion of the magnet & the circuit is the crucial factor

$\rightarrow$ charges are static $\Rightarrow$ they are not experiencing magnetic force

Faraday conjecture: A changing magnetic field induces an electric field. This induced electric field accounts for the emf.

$$\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi}{dt}$$

using Stoke's th:

$$\oint \vec{E} \cdot d\vec{l} = \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{a} = - \int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{a}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t}$$

* Remember, $\vec{\nabla} \times \vec{E} = 0$ if $\vec{B}$ is static.

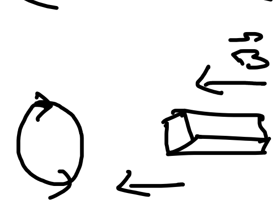
Ex.3 The magnetic field is changing for different reasons; but according to Faraday's law if flux changes, $\mathcal{E} = -\frac{d\Phi}{dt}$.

* Universal flux rule:

Whenever the magnetic flux through a loop changes, an emf, $\mathcal{E} = -\frac{d\Phi}{dt}$ appears in the loop.

→ Direction of this induced current flow is obtained using Lenz's law

(Nature abhors change in flux.)



→ magnet entering the loop

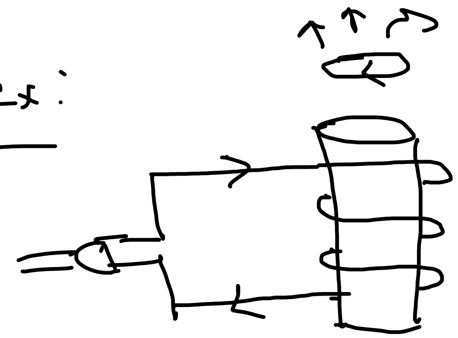
⇒ current clockwise

→ magnet leaving the loop

⇒ current anti-clockwise

The conducting loop intends to maintain a constant flux and if we change the flux, the response is to push the current in a way to negate that effect.

Ex:



Wind a solenoid coil around an iron core. Place a metal ring on top and let current pass through the solenoid. The ring will jump off.

→ Before turning the current on, flux through the ring is zero. Afterward, the flux is non-zero. The emf generated in the ring leads to a current. According to Lenz's law its direction tries to negate the change in flux. The current will flow in opposite direcⁿ. to current in solenoid. Opposite currents repel & the ring jumps off.