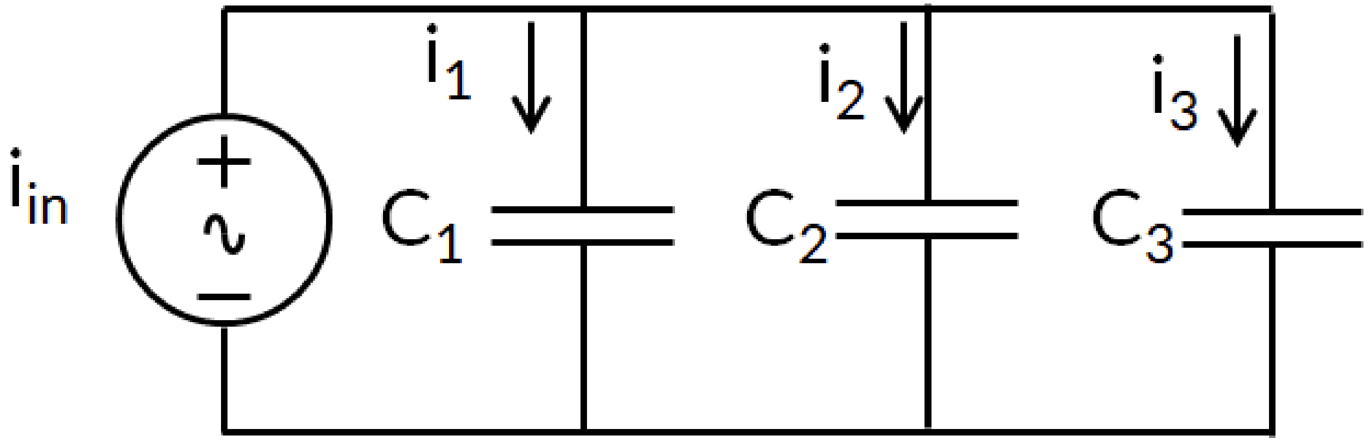
Capacitors in Parallel

- Consider capacitors connected in parallel configuration

– Voltage across the capacitors is equal

– Current is different



Writing KCL,

$$i_{in} = i_1 + i_2 + i_3$$

Noting the current voltage relation for a capacitor as

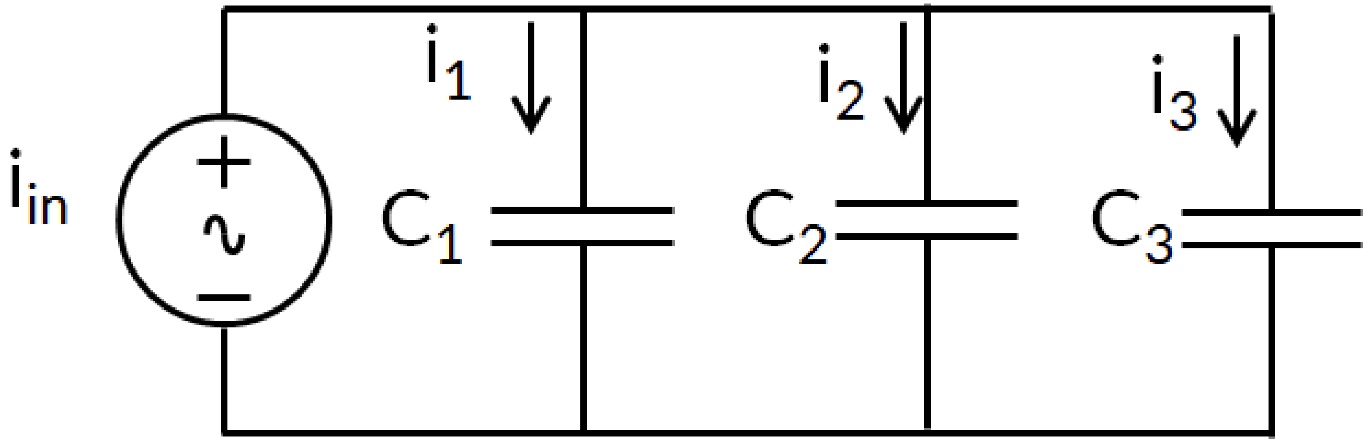
$$i = C\frac{dv}{dt}$$

If C_{eq} is the net capacitance, then $i_C = C_{eq}\frac{dv}{dt}$

Capacitors in Parallel

Writing current voltage relations for individual capacitances as

$$i_1 = C_1 \frac{dv}{dt} \qquad i_2 = C_2 \frac{dv}{dt} \qquad i_3 = C_3 \frac{dv}{dt}$$



Substituting into KCL,

$$i_{in} = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

Hence,

$$C_{eq} = C_1 + C_2 + C_3$$

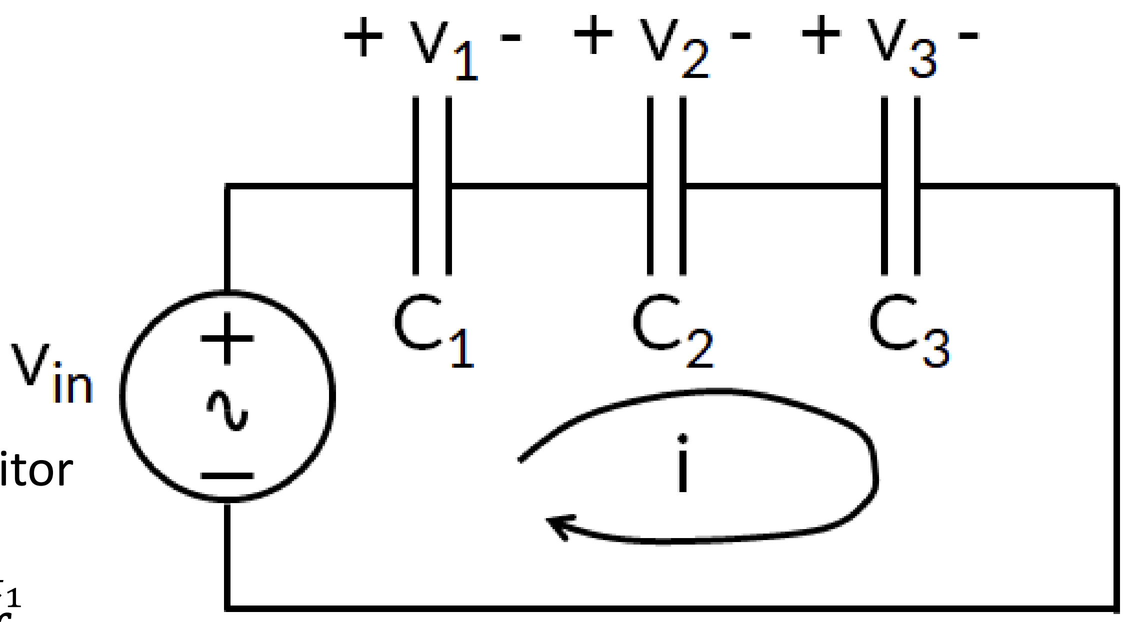
$$\boxed{C_{eq} = \sum_{p=1}^{m} C_p}$$

Capacitors in Series

- Consider capacitors connected in series configuration

– Current through the capacitors is same

– Voltage divides



Writing KVL

$$v_{in} = v_1 + v_2 + v_3$$

- Noting the relation between voltage across the capacitor and current through a capacitor as

$$v_1 = \frac{1}{C_1}\int_{t_0}^{t_1} idt \qquad v_2 = \frac{1}{C_2}\int_{t_0}^{t_1} idt \qquad v_3 = \frac{1}{C_3}\int_{t_0}^{t_1} idt$$

- If C_{eq} is the total capacitance, then

$$v_{in} = \frac{1}{C_{eq}}\int_{t_0}^{t_1} idt$$

Capacitors in Series…Continued

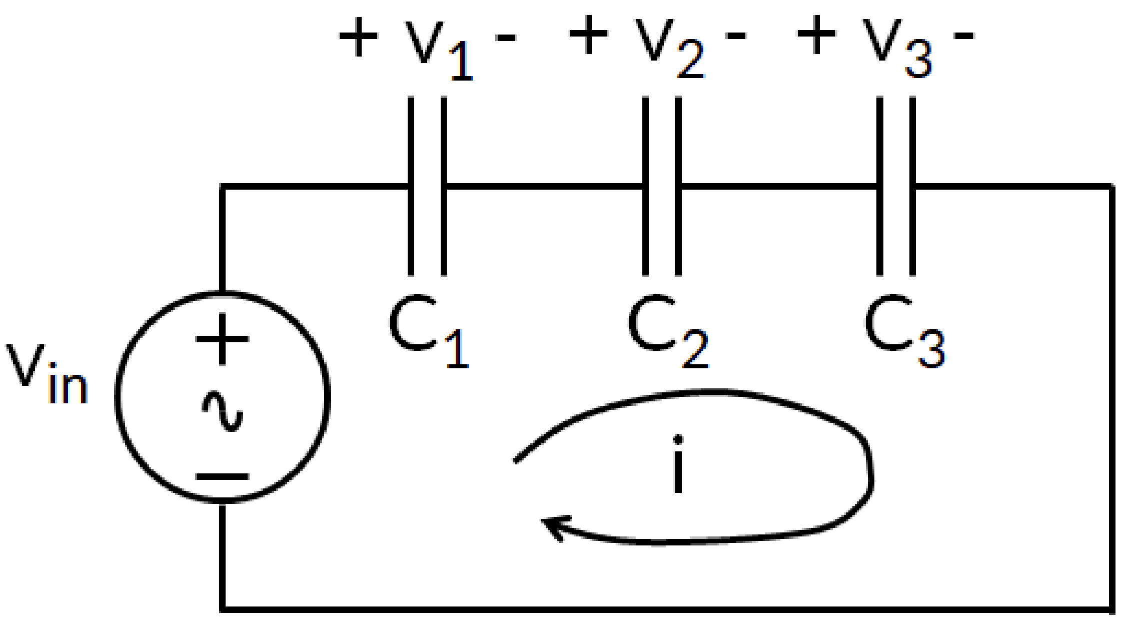
Substituting back into KVL

$$\frac{1}{C_{eq}}\int_{t_0}^{t_1} i dt = \frac{1}{C_1}\int_{t_0}^{t_1} i dt + \frac{1}{C_2}\int_{t_0}^{t_1} i dt + \frac{1}{C_3}\int_{t_0}^{t_1} i dt$$

Hence,

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

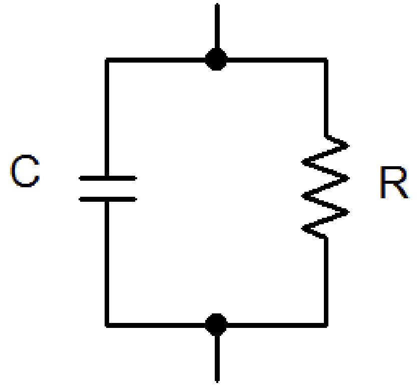
$$\boxed{C_{eq} = \left(\sum_{s=1}^{n} C_s^{-1}\right)^{-1}}$$



Properties of a Real Capacitor

- A real capacitor does dissipate energy due leakage of charge through its insulator.

- Real capacitor is modeled by keeping a resistor in parallel with an ideal capacitor.



Charging a capacitor

- One of the functions of capacitor is storing charge (and thus energy).
- Capacitor has an ability to store charge when a potential difference is applied across the capacitor plates.
- Energy is stored in the electric field between positive and negative plates.
- When a voltage is applied across a capacitor, current flows into the capacitor plates and develops a potential difference across the capacitor.
- With time, the potential difference between the battery and the capacitor become smaller and the flow rate of electrons (thus current flow) reduces .
- The charging process continues until the capacitor becomes fully charged.
- The charging current follows an exponential curve.

Charging a capacitor…Continued

- Initially, it is easy to store charge in the capacitor.

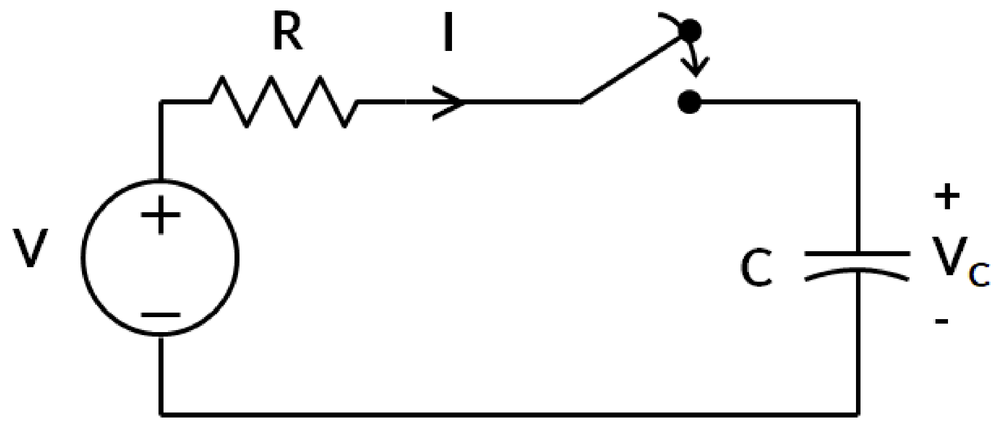
- As more charge is stored on the plates of the capacitor, it becomes increasingly difficult to place additional charge on the plates due to Coulombic repulsion.

- As charge is stored on the capacitor plates, the voltage across the capacitor increases rapidly.

- The charging voltage follows an exponential curve.

Charging a capacitor…Continued

Consider the circuit shown in figure:



Applying KVL

$$V = V_R + V_C$$

Noting that

$$V_C = \frac{q}{C} \quad \text{and} \quad V_R = IR$$

$$V = IR + \frac{q}{C}$$

$$VC = RC\frac{dq}{dt} + q$$

$$VC - q = RC\frac{dq}{dt}$$

$$\int \frac{dt}{RC} = \int \frac{dq}{VC - q}$$

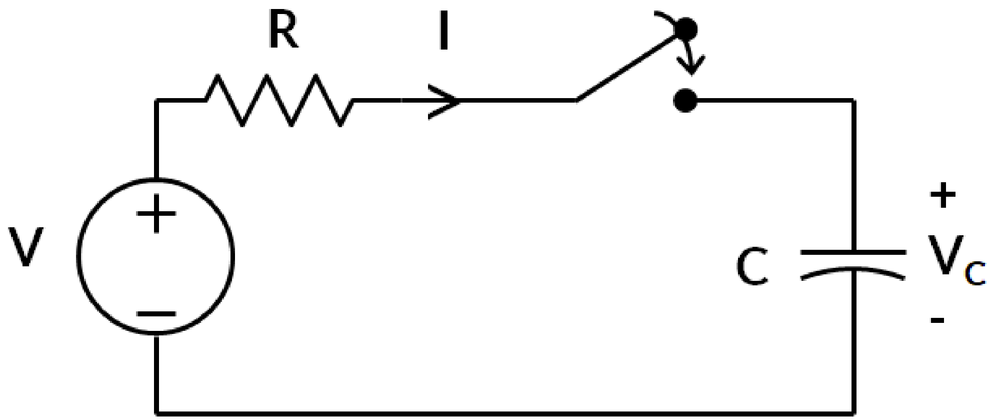
Charging a capacitor…Continued

$$\int \frac{dt}{RC} = \int \frac{dq}{VC - q}$$

$$-\int \frac{d(VC - q)}{VC - q} = \int \frac{dt}{RC}$$

$$-\ln(VC - q) = C_1 + \frac{t}{RC}$$

$$VC - q = C_2 e^{-\frac{t}{RC}}$$



Using the initial boundary condition: at time t = 0, when the capacitor is not initially charged, q = 0

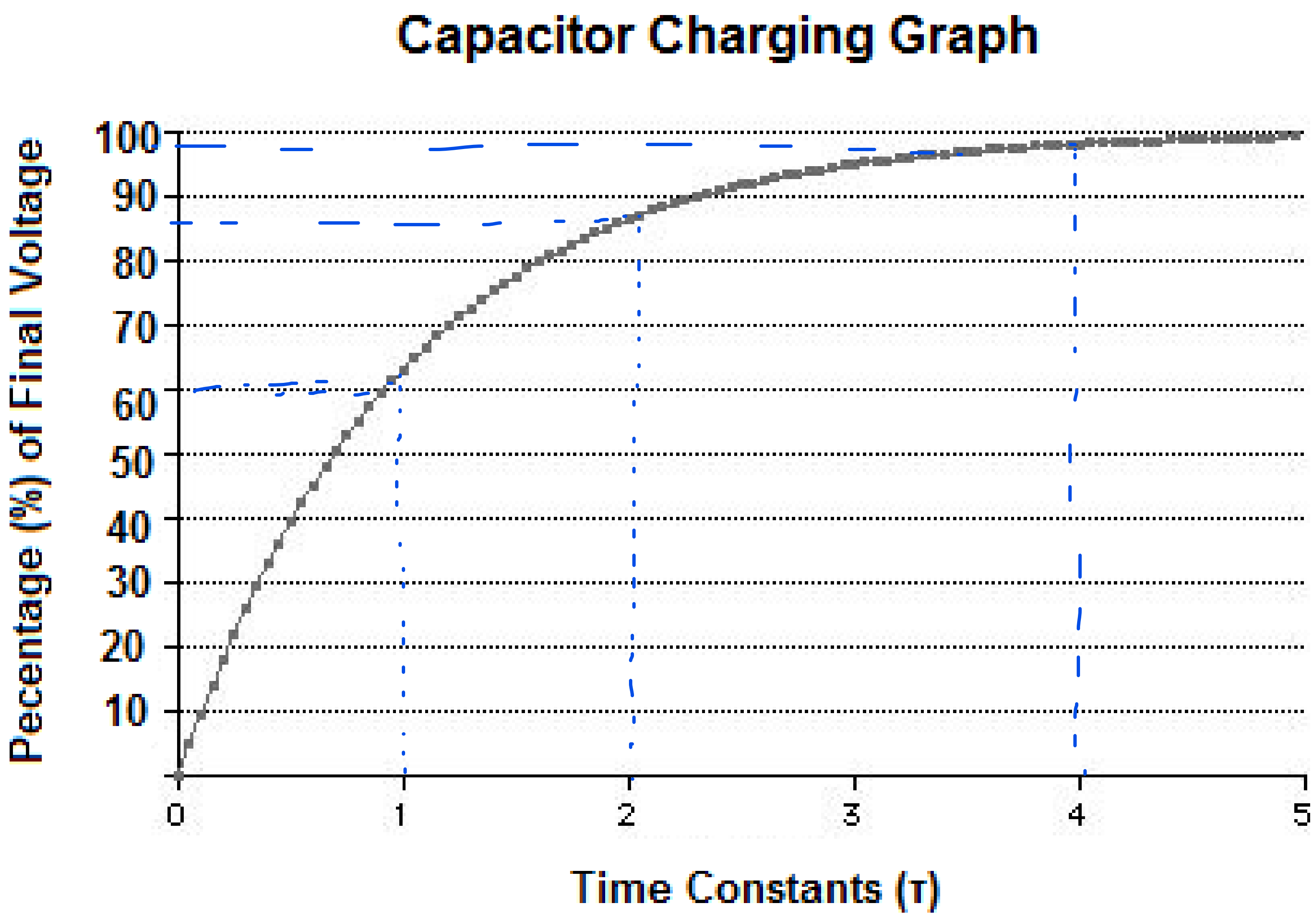
$$C_2 = VC$$

$$VC - q = VCe^{-\frac{t}{RC}}$$

$$q = VC\left(1 - e^{-\frac{t}{RC}}\right)$$

$$\boxed{v_C = \frac{q}{C} = VC\left(1 - e^{-\frac{t}{RC}}\right)}$$

$$\boxed{i = \frac{dq}{dt} = \frac{V}{R}e^{-\frac{t}{RC}}}$$


Capacitor Charging Graph
Pecentage (%) of Final Voltage
100
90
80
70
60
50
40
30
20
10
0
1
2
3
4
5
Time Constants (τ)

Time Constant

- The rate of charging is determined by the charging equation determined by the RC constant in the exponential term.

$$v_C = \frac{q}{C} = VC\left(1 - e^{-\frac{t}{RC}}\right)$$

- The term RC is termed the time constant (mostly RC time constant) since it affects the rate of charge.

- Mathematically, this is the time taken for the capacitor to reach 0.632 of the fully charged value.

- According to the charging equation, theoretically, capacitors takes infinite time to charge completely.

- For all practical purposes, it is assumed that a capacitor can be charged completely in only five times of the time constant, meaning the capacitor is said fully charged after 5×RC.

- After 5 time constant, q, Vc and current will be over 99% ($1-e^{-5} = 0.9932$) to their final values.

Charging an Initially Charged Capacitor

- The ability to add charge to a capacitor depends on:

--the amount of charge already on the plates of the capacitor and the force (voltage) driving the charge towards the plates (i.e., current).

- If at the start of charging, the capacitor is charged to a voltage of V_1 Volts, then the initial condition gets modified as at t = 0, q = CV_1

- Thus, applying boundary condition

$$C_2 = C(V - V_1) \qquad VC - CV_1 = C_2$$

$$VC - q = C(V - V_1)e^{-\frac{t}{RC}} \qquad q = VC(1 - e^{-\frac{t}{RC}}) + V_1 C e^{-\frac{t}{RC}}$$

$$V_C = \frac{q}{C} = V - Ve^{-\frac{t}{RC}} + V_1 C e^{-\frac{t}{RC}} \qquad \boxed{V_C = \frac{q}{C} = V + (V_1 - V)e^{-\frac{t}{RC}}}$$

Discharging a Capacitor

- Coulombic repulsion between charges already existing the plates creates a force that lets charges to discharge out of the capacitor once the voltage on the charge in the capacitor is decreased

- Coulombic repulsion decreases as more charges are removed from the capacitor plates.

- Initially, voltage across the capacitor decreases rapidly as charge is removed from the plates.

- As more and more charge is removed, voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

Discharging a Capacitor

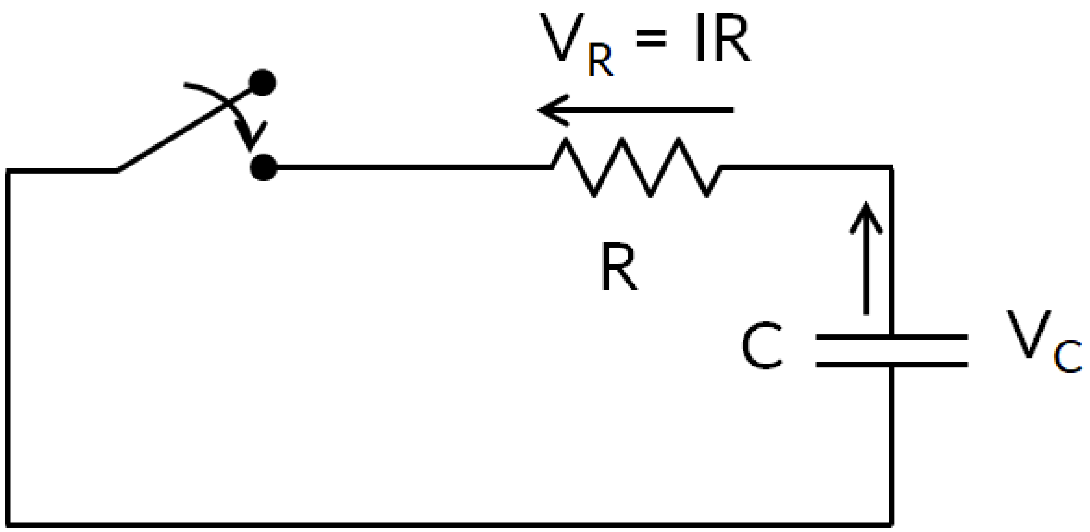
- Applying KVL

$$V_R + V_C = 0$$

$$iR + \frac{q}{C} = 0 \qquad \frac{dq}{dt}R + \frac{q}{C} = 0$$

$$\int \frac{dq}{q} = -\int \frac{dt}{RC} \qquad \ln q = -\frac{t}{RC} + C_1$$

$$q = e^{-\frac{t}{RC}} + C_1 \qquad q = C_2 e^{-\frac{t}{RC}}$$



Substitute boundary condition: at t = 0, Voltage across C = V, q = VC

$$C_2 = VC \qquad q = VCe^{-\frac{t}{RC}}$$

$$\boxed{V_C = \frac{q}{C} = Ve^{-\frac{t}{RC}}} \qquad \boxed{i = \frac{dq}{dt} = -\frac{V}{R}e^{-\frac{t}{RC}}}$$

- **Note that the negative sign indicate that the current is opposite to the charging current's direction**

Discharging a Capacitor