5.11.20

Gradient:

$$\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$$

scalar

Change in 'T'

$d\vec{l} \equiv$ displacement vector

$= dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$

$$dT = (\vec{\nabla} T) \cdot (d\vec{l})$$

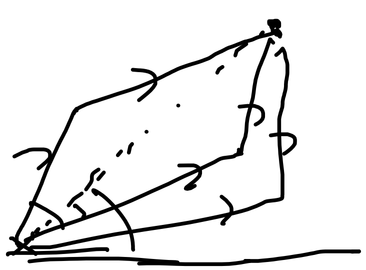
$$= |\vec{\nabla} T| \times |d\vec{l}| \cos\theta$$

$$\boxed{dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz}$$

interpretation:

$T = xyz$

$$\vec{\nabla} T = yz\,\hat{x} + xz\,\hat{y} + xy\,\hat{z} \quad \#$$



For fixed $|d\vec{l}|$

$$dT|_{max} = |\vec{\nabla} T| \times |d\vec{l}| \times 1 \quad \#$$

$y = mx + c$

D. J. Griffiths

Introduction to Electrodynamics.

$\vec{\nabla} T \Rightarrow$ direction points to max. increase of fn. 'T'.

$\Rightarrow$ magnitude given by the slope

Curl:

Definition: $$\underbrace{\vec{\nabla} \times \vec{v}}_{\text{vector}} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$

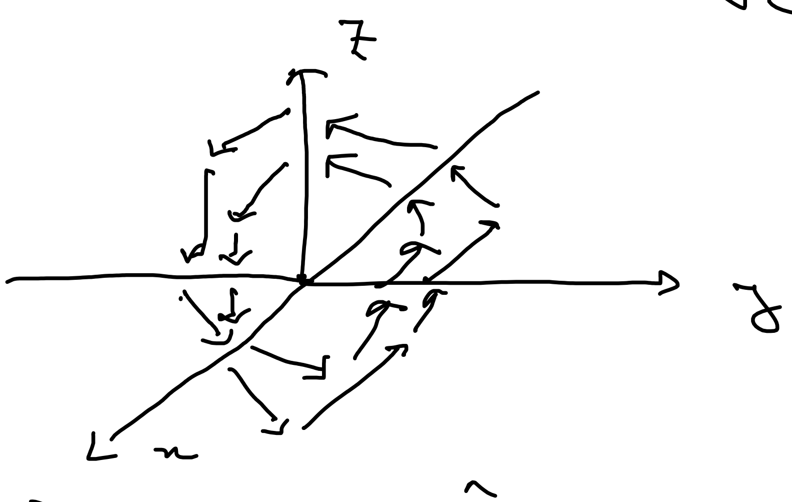
$$= \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Del op.:

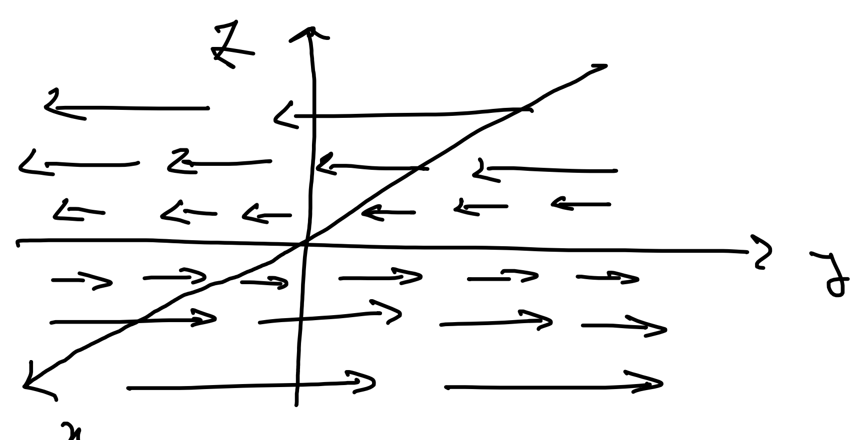
$$\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$$\left\{ \begin{array}{l} \vec{\nabla} T \\ \vec{\nabla} \cdot \vec{v} \\ \vec{\nabla} \times \vec{v} \end{array} \right\}$$

interpretation: measure of how much a vector swirls around a poit.



$\vec{v} = -y\hat{x} + x\hat{y}$



$\vec{v} = x\hat{y}$

Product Rules:

For vector derivatives:

$$\left\{\begin{array}{l} f \equiv f(x,y,z) \\ g \equiv g(x,y,z) \end{array}\right.$$

$k = \text{const.}$

$$\vec{\nabla}(f+g) = \vec{\nabla}f + \vec{\nabla}g$$

$$\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B}$$

$$\vec{\nabla}\times(\vec{A}+\vec{B}) = \vec{\nabla}\times\vec{A} + \vec{\nabla}\times\vec{B}$$

$$\vec{\nabla}(kf) = k\vec{\nabla}f$$

$$\vec{\nabla}\cdot(k\vec{A}) = k(\vec{\nabla}\cdot\vec{A})$$

$$\vec{\nabla}\times(k\vec{A}) = k(\vec{\nabla}\times\vec{A})$$

Product of scalars / vectors:

$\underline{fg}$ $\quad \underline{\vec{A}\cdot\vec{B}}$ $\quad \underline{f\vec{A}}$ $\quad \underline{\vec{A}\times\vec{B}}$

$A_xB_x + A_yB_y + A_zB_z$

$$\vec{\nabla}(fg) = f\vec{\nabla}g + g\vec{\nabla}f$$

$$\vec{\nabla}(\vec{A}\cdot\vec{B}) = \vec{A}\times(\vec{\nabla}\times\vec{B}) + \vec{B}\times(\vec{\nabla}\times\vec{A}) + (\vec{A}\cdot\vec{\nabla})\vec{B} + (\vec{B}\cdot\vec{\nabla})\vec{A}$$