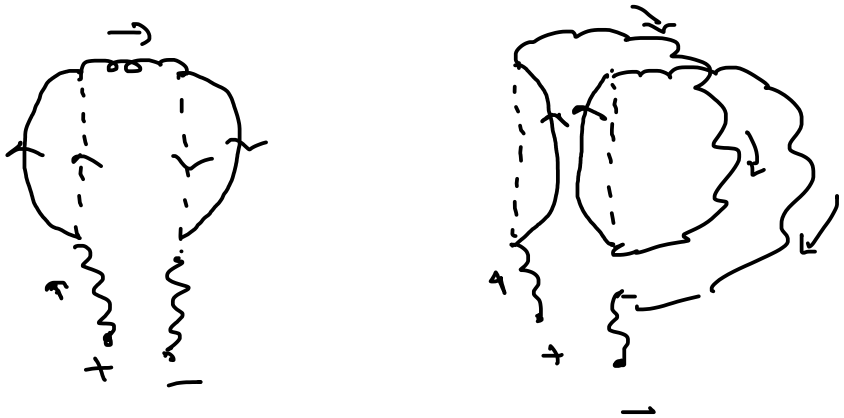
Magnetic fields:

$\boxed{\text{Section } 5.1.1}$

28.01.21



Two parallel wires

→ Repulsion when current flows in opposite direcⁿ

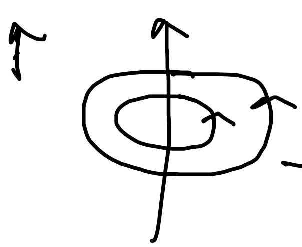
→ Attraction when current flows in same direcⁿ.

⇒ To describe this phenomena we need magnetic field.

⊗ A stationary charge ⇒ Electric field

A moving charge ⇒ Magnetic field + Electric field

⊗ Current carrying wire:



Right-hand-rule!

⊗ Magnetic force:

$\boxed{\text{Section } 5.1.2}$

For a charge Q moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$.

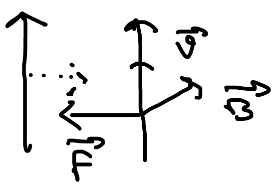
$$\vec{F} = q(\vec{v} \times \vec{B}) \equiv \text{Lorentz force law}$$

In presence of both $\vec{E}$ & $\vec{B}$:

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

⊗ → vector going into page

⊙ → vector going out of page



⊗ Magnetic force does no work

q moves an amount $d\vec{l} = \vec{v}\,dt$

Work done, $dW = q(\vec{v} \times \vec{B}) \cdot \vec{v}\,dt$

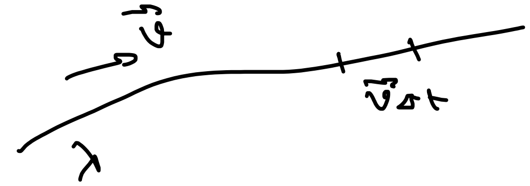
$$= 0$$

<u>Current:</u> $\boxed{\text{Section 5.1.3}}$

$I \equiv$ charge passing per unit time through a given point

$$dI = \frac{dq}{dt}$$

2 Line charge:



λ → line charge density

$v\Delta t \equiv$ length segment

$$dq = \lambda v \Delta t$$

$$\text{current} = \lambda v$$

3 Surface charge:

$\vec{K} \equiv$ Surface current density

→ Ribbon of width $dl_\perp$ running parallel to the flow

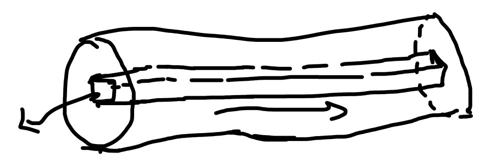
Current in the ribbon $= dI$

$$K = \frac{dI}{dl_\perp} \quad \text{(current per unit width)}$$

$$\vec{K} = \sigma \vec{v} \quad (\sigma \equiv \text{surface charge density})$$

(+) Volume charge:

$\vec{J}$ ≡ volume current density



$da_\perp$

↳ tube of infinitesimal cross-section $da_\perp$ running parallel to the flow

Current in the tube = dI

$$J = \frac{dI}{da_\perp} \quad \text{(current per unit area)}$$

$$\vec{J} = \rho \vec{v} \quad (\rho \equiv \text{volume charge density})$$

(+) Total current crossing a surface 'S'

$$I = \int_S J\, da_\perp = \int_S \vec{J} \cdot d\vec{a}$$

↳ dot product helps to pick the correct comp. of $d\vec{a}$

Charge per unit time leaving 'v'

$$\oint_S \vec{J} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{J})\, d\tau$$

Total charge must be conserved.

$$\Rightarrow \int_V (\vec{\nabla} \cdot \vec{J})\, d\tau = -\frac{d}{dt} \int_V \rho\, d\tau = -\int_V \left(\frac{\partial \rho}{\partial t}\right) d\tau$$

→ True for any arbitrary volume.

$$\Rightarrow \quad \vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t} \equiv \text{Continuity equation}$$

(*) Steady current: $\Rightarrow \quad \dfrac{\partial \rho}{\partial t} = 0$

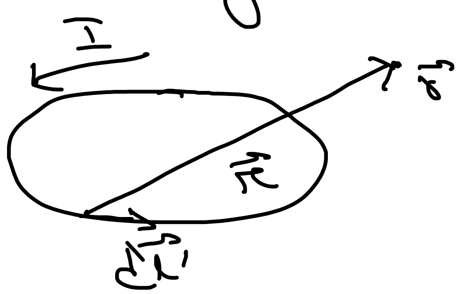
(Constant magnetic field)

$$\frac{\partial \vec{J}}{\partial t} = 0$$

Continuity eq: $\vec{\nabla} \cdot \vec{J} = 0$

Section 5.2

(*) Magnetic field due to a steady current



$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} I \int \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

Biot-Savart law

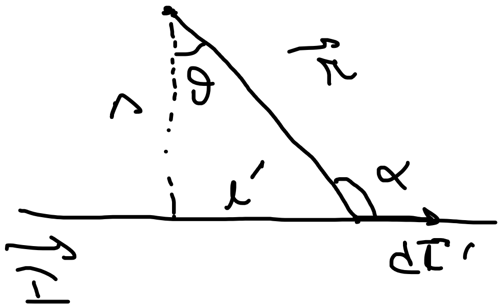
$d\vec{l}' \equiv$ Element of length along wire

$\vec{\mathscr{r}} \equiv$ vector from source to $\vec{r}$

$\mu_0 \equiv$ Permeability of free space

$= 4\pi \times 10^{-7}$ N/A^2

Ex: Magnetic field a distance away from a long straight wire carrying a steady current I



$d\vec{l}' \times \hat{r} \equiv$ pointing out of the page

Magnitude: $dl' \sin\alpha = dl' \cos\theta$

$$l' = s\tan\theta$$

$$\Rightarrow dl' = \frac{s}{\cos^2\theta}\, d\theta$$

$$s = r\cos\theta$$

$$\Rightarrow \frac{1}{r^2} = \frac{\cos^2\theta}{s^2}$$

⊕ Infinitely long wire:

$$\theta_1 = -\frac{\pi}{2}$$

$$\theta_2 = +\pi/2$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi s}$$

⊕ Right-hand rule $\Rightarrow \vec{B} = \dfrac{\mu_0 I}{2\pi s}\,\hat{\phi}$

$$B = \frac{\mu_0 I}{4\pi} \int_{\theta_1}^{\theta_2} \left(\frac{\cos^2\theta}{s^2}\right) \left(\frac{s}{\cos^2\theta}\right) \cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s} \int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s} (\sin\theta_2 - \sin\theta_1)$$