08.12.20

Work done to move a charge:

We have a stationary configuration of charges. We have a test charge 'q' that moves from 'a' to 'b'.

Say, the electric field is $\vec{E}$

Force on test charge: $\vec{F} = q\vec{E}$

The force to be exerted $= -q\vec{E}$

Work done

$$W = \int_a^b \vec{F} \cdot d\vec{l} = -q\int_a^b \vec{E} \cdot d\vec{l}$$

$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l}$

$$= q\left[V(b) - V(a)\right]$$

$\downarrow$

Independent of path.

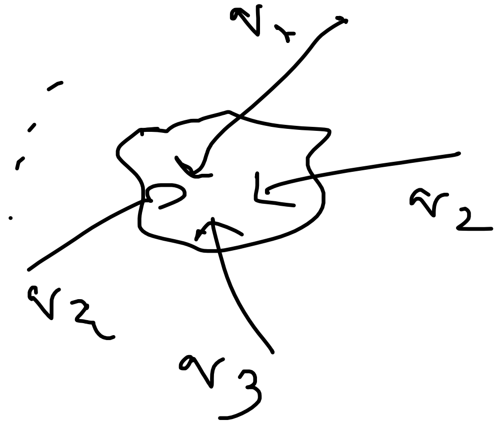
$$\Rightarrow V(b) - V(a) = \frac{W}{q}$$

In general, to bring 'q' from ∞ to $\vec{r}$

$$\Rightarrow W = q\left[V(\vec{r}) - V(\infty)\right] = qV(\vec{r})$$

Energy of a point charge distribution:

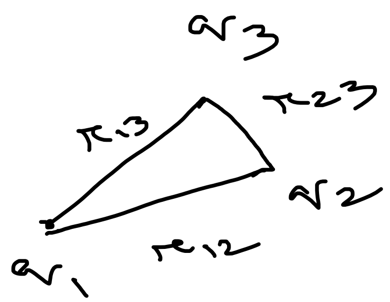
We are assembling a collection of point charges



* For q_1, it doesn't face any resistance.

* for q_2, the work done

$$W_2 = \frac{1}{4\pi\epsilon_0} q_2 \left(\frac{q_1}{r_{12}}\right)$$



* for q_3,

$$W_3 = \frac{1}{4\pi\epsilon_0} q_3 \left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

Hence, total work necessary to assemble:

$$W = \frac{1}{4\pi\epsilon_0}\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

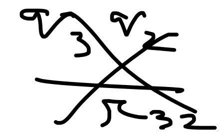
In general, for 'n' number of charges

$$W = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j>i}^{n} \frac{q_i q_j}{r_{ij}}$$

$$\sum_{i=1}^{3} \sum_{j\neq i}^{3} \frac{q_i q_j}{r_{ij}} = \frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \cancel{\frac{q_2 q_1}{r_{21}}} + \cancel{\frac{q_3 q_1}{r_{31}}} +$$

<u>OR</u>

$$W = \frac{1}{2} \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \sum_{j\neq i}^{n} \frac{q_i q_j}{r_{ij}}$$



$$\Rightarrow W = \frac{1}{2} \sum_{i=1}^{n} q_i \underbrace{\left(\sum_{j\neq i}^{n} \frac{1}{4\pi\epsilon_0} \frac{q_j}{r_{ij}} \right)}_{V(\vec{r}_i)}$$

$V(\vec{r}_i)$ ↳ Potential at $\vec{r}_i$ due to all other charges.

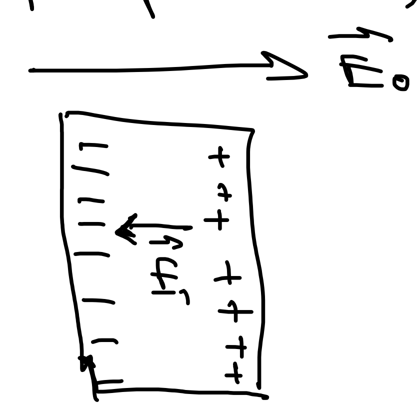
$$\Rightarrow W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\vec{r}_i)$$

→ Work done to assemble a configuration of point charges

→ Work required to dismantle the system

→ Energy stored in the system.

Conductor (Basic Electrostatic properties)



→ $\vec{E} = 0$ inside a conductor

Inside the conductor
$|\vec{E}_i| = |\vec{E}_0|$

⇒ Net electric field inside is zero.

→ $\rho = 0$ inside the conductor

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} = 0$$

$$\Rightarrow \rho = 0$$

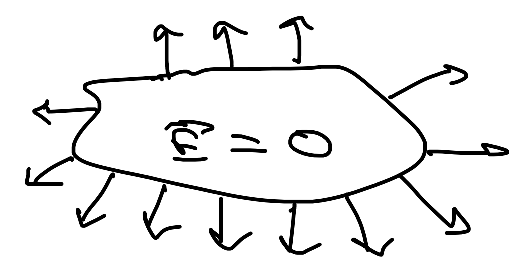
→ Any net charge only resides on surface
(⇒ Electrostatic energy in this configuration is at its minimum.)

→ Conductor is equipotential

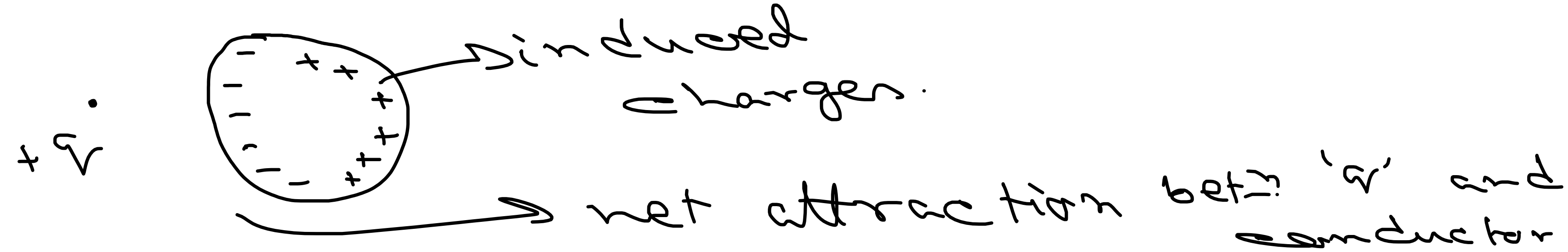
$$V(b) - V(a) = -\int_a^b \vec{E} \cdot d\vec{l} = 0$$

$$\Rightarrow V(b) = V(a)$$

→ $\vec{E}$ is perpendicular to surface



<u>Induced charges:</u>



Cavity