Tutorial Sheet 7

Improper Integral, Beta and Gamma Function

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1. Discuss the convergence/divergence of improper integrals of first kind

$$(a)\ \int_0^\infty e^{-x}cosx\ dx, \qquad (b)\ \int_1^\infty \frac{dx}{x^2(1+e^x)} \qquad (c)\ \int_1^\infty \frac{x+1}{\sqrt{x^3}}dx$$

2. Discuss the convergence/divergence of improper integrals of second kind

$$(a)\ \int_1^2 \frac{\sqrt{x}}{\ln x}dx \qquad (b)\ \int_0^1 \frac{\sin(\frac{1}{x})}{x^3}dx \qquad (c)\ \int_1^{\pi/2} \frac{\tan x}{x^{3/2}}dx$$

3. Discuss the convergence/divergence of improper integrals

$$(a)\ \int_0^\infty x^{\frac{-1}{2}}e^{x^2}dx \qquad (b)\ \int_{-\infty}^\infty \frac{dx}{|x|^p(1+x^2)}dx, p>0 \qquad (c)\ \int_0^\infty \frac{1+x}{1+x^3}dx$$

4. Show that $\int_0^1 \frac{\sin x}{x^p}dx$ converges if and only if $0 < p < 2$.

5. Show the following:

$$(a)\ \int_0^\infty e^{-tx}\frac{\sin x}{x}dx = \frac{\pi}{2} - \arctan t, \qquad (b)\ \int_0^1 \frac{x^t - 1}{\log x}dx = \log(1+t).$$

6. Using Beta and Gamma functions, evaluate the following:

$$(a)\ \int_0^\infty e^{-x^2}dx \qquad (b)\ \int_0^{\pi/2} \sqrt{\tan x}dx \qquad (c)\ \int_0^\infty x^{2/3}e^{-\sqrt{x}}dx$$

7. Find the values of the gamma function at the given points.

$$(a)\ \frac{5}{2} \qquad (b)\ \frac{9}{2} \qquad (c)\ -\frac{1}{2}$$