Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 4 (Limit and Continuity)

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1. Using $\epsilon - \delta$ definition, show following limits/continuity:

   (a) $\lim\limits_{x\to 0} x^2 \cos\left(\dfrac{1}{x}\right) = 0$ $\quad$ (b) $\lim\limits_{x\to a} x^2 = a^2$ $\quad$ (c) $\lim\limits_{x\to 3} x^2 + 5x + 4 = 28.$

2. Show that each of the following limits does not exist:

   (a) $\lim\limits_{x\to 0} \cos\left(\dfrac{1}{x}\right)$ $\quad$ (b) $\lim\limits_{x\to 0} \dfrac{1}{x}$ $\quad$ (c) $\lim\limits_{x\to a} \sin\left(\frac{1}{(x-a)^{1/k}}\right),\ k \geq 1.$

3. Using intermediate value theorem, show that there exists $c \in (-2, 0)$ such that

$$c^{179} + \frac{163}{1 + c^2 + \sin^2 c} = 119.$$

4. Determine if the following equations admits solutions in the interval mentioned.

   (a) $x^5 - 3x^2 = -1,\ [0, 1]$ $\quad$ (b) $\sin^2 x - 2\cos x = -1,\ \left[0, \frac{\pi}{2}\right]$

5. a) Give an example of a function which is continuous only at one point.

   (b) Give an example of a function which is continuous everywhere.