Linear Dielectrics (contd.)

$\boxed{\text{Section } 4.4.1}$

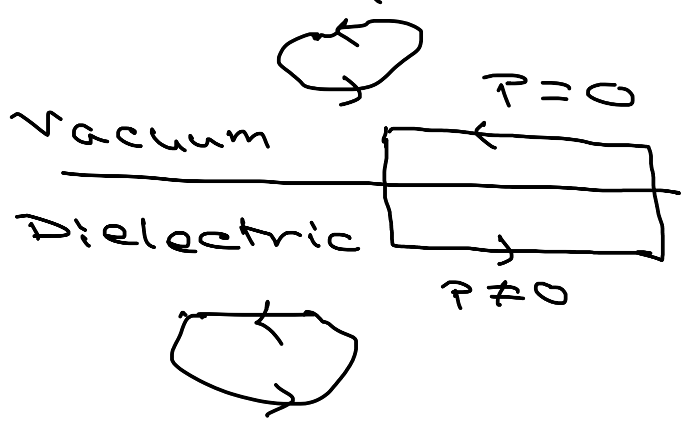
21.01.21

$$\left.\begin{array}{l}\vec{P} \propto \vec{E} \\ \vec{D} \propto \vec{E}\end{array}\right\} \Rightarrow$$ Since $\vec{\nabla} \times \vec{E} = 0$, does this mean that $\left.\begin{array}{l}\vec{\nabla} \times \vec{P} = 0 \\ \vec{\nabla} \times \vec{D} = 0\end{array}\right\}$?

<u>No !!</u>

$\vec{\nabla} \times \vec{P} = 0$ means everywhere $\oint \vec{P} \cdot d\vec{l} = 0$.

Say, we ~~take~~ an interface between a dielectric & vacuum.



$\Rightarrow$ Around this loop $\oint \vec{P} \cdot d\vec{l} \neq 0$

$\Rightarrow \vec{\nabla} \times \vec{P} \neq 0$

① If the space is filled with homogeneous linear dielectric material,

$$\vec{\nabla} \cdot \vec{D} = \rho_f \quad \& \quad \vec{\nabla} \times \vec{D} = 0$$

$$\vec{\nabla} \cdot \vec{P} = -\rho_b \quad \& \quad \vec{\nabla} \times \vec{P} = 0$$

$\rightarrow \vec{D}$ can be found from free charges

$$\vec{D} = \epsilon_0 \vec{E}_{vac.}$$ | $\vec{E}_{vac.} \equiv$ field the free charges produce in vacuum

Total electric field:

$$\vec{E} = \frac{1}{\epsilon}\vec{D}$$

$$= \frac{1}{\epsilon_r}\vec{E}_{vac}.$$

→ Same goes for potential as well.

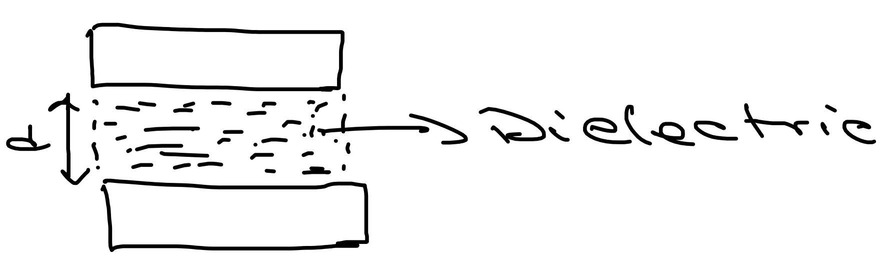
⊗ Ex: A free charge 'q' embedded inside a dielectric.

$$\vec{E} = \frac{1}{4\pi\epsilon}\frac{q}{r^2}\hat{r}$$

$$V = \frac{1}{4\pi\epsilon}\frac{q}{r}$$

Capacitance & Dielectric:

Say, we have a parallel plate capacitor.



⊗ In absence of dielectrics

$$C = \frac{A\epsilon_0}{d}$$

Capacitance, $C = \frac{Q}{V}$

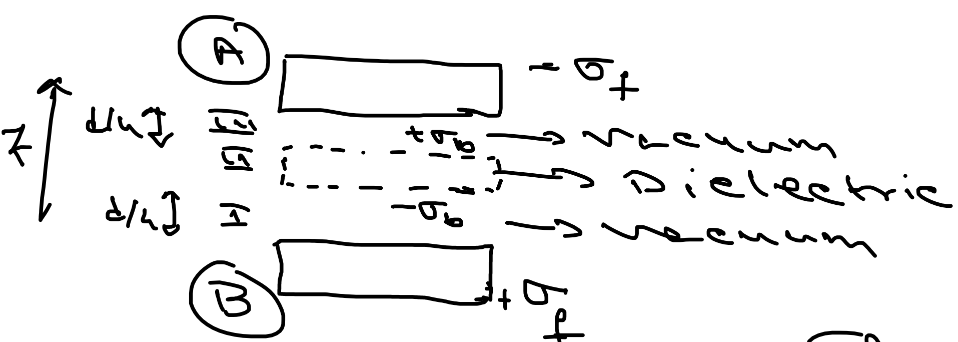
In presence of dielectric, $V' = \frac{1}{4\pi\epsilon}\frac{Q}{r}$

$$= \frac{V}{\epsilon_r}$$

$$C' = \frac{Q}{\frac{V}{\epsilon_r}} = \epsilon_r \frac{Q}{V} = \epsilon_r C_{vac.}$$

↳ usually $\epsilon_r > 1$

Ex: Parallel plate capacitor half filled with dielectrics.



Distance betn plates $= d$

Thickness of dielectric slab $= \frac{d}{2}$

(*) Inside, if there were no dielectrics,

$$\vec{E} = \frac{\sigma_f}{\epsilon_0}\hat{z}$$

(*) Electric displacement

$$\int \vec{D}\cdot d\vec{a} = Q_{fenc.}$$

$$\Rightarrow DA = \sigma_f A$$

$$\Rightarrow \vec{D} = \sigma\hat{z} \quad \text{(in all regions)}$$

(*) Electric field:

$$\vec{E} = \frac{\sigma_f}{\epsilon_0}\hat{z} \quad (I)$$

$$= \frac{\sigma_f}{\epsilon}\hat{z} \quad (II)$$

$$= \frac{\sigma_f}{\epsilon_0}\hat{z} \quad (III)$$

⊛ Polarization (region II)

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

$$= \frac{\epsilon_0 \chi_e \sigma_f}{\epsilon} \hat{z} = \frac{\chi_e \sigma_f}{1+\chi_e} \hat{z}$$

⊛ Volume charge density, $\rho_b = -\vec{\nabla} \cdot \vec{P} = 0$

Surface charge density, $\sigma_b = \vec{P} \cdot \hat{n}$

$$= \frac{\chi_e \sigma_f}{1+\chi_e} \hat{z} \cdot \hat{z}$$

$$= \frac{\chi_e \sigma_f}{1+\chi_e}$$

⊛ Potential difference:

$$V_B - V_A = -\int_B^A \vec{E} \cdot d\vec{l} = \int_A^B \vec{E} \cdot d\vec{l}$$

$$= \int_{\mathrm{I}} \frac{\sigma_f}{\epsilon_0} dz + \int_{\mathrm{II}} \frac{\sigma_f}{\epsilon} dz + \int_{\mathrm{III}} \frac{\sigma_f}{\epsilon_0} dz$$

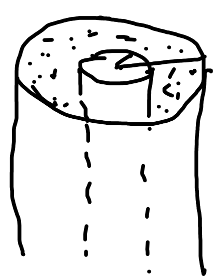
$$\Rightarrow \nabla V = \frac{\sigma_f}{\epsilon_0}\left(\frac{d}{4}\right) + \frac{\sigma_f}{\epsilon}\left(\frac{d}{2}\right) + \frac{\sigma_f}{\epsilon_0}\left(\frac{d}{4}\right)$$

$$= \frac{\sigma_f d}{2\epsilon_0} + \frac{\sigma_f d}{2\epsilon} = \frac{Q d}{2 A \epsilon_0}\left(1 + \frac{\epsilon_0}{\epsilon}\right)$$

Capacitance, $C = \frac{Q}{\Delta V} = \frac{\epsilon_0 A}{d}\left(\frac{2}{1+\frac{1}{\epsilon_r}}\right)$

$$= \frac{\epsilon_0 A}{d}\left(\frac{2\epsilon_r}{1+\epsilon_r}\right)$$

Ex: Coaxial cable consisting copper wire & surrounded by a copper tube. Space in between filled with dielectric. (dielectric const. $= \epsilon_r$)



= Inner radius $= a$
Outer radius $= b$
Line charge density $= \lambda_f$.

Electric field inside dielectric

$$\vec{E} = \frac{\lambda_f}{2\pi\epsilon r}\hat{r} \qquad b > r > a$$

r ↳ radial distance.

$$\Delta V = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b \frac{\lambda_f}{2\pi\epsilon r}\,dr = \frac{\lambda_f}{2\pi\epsilon}\ln\left(\frac{b}{a}\right)$$

$Q = \lambda_f l \implies C = \frac{2\pi\epsilon l}{\ln\left(\frac{b}{a}\right)}$

capacitance per unit length:

$$\frac{C}{l} = \frac{2\pi\epsilon}{\ln(b/a)}$$