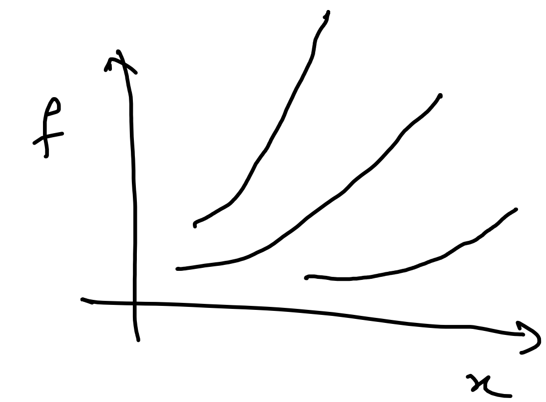
3.11.20

Gradient

$f(x) \xrightarrow[\text{derivative}]{} \frac{df}{dx}$ (variation of 'f' w.r.t. x)

$$df = \left(\frac{df}{dx}\right) dx$$

$\hookrightarrow$ slope



(*) Function has more than one variable:

$$T(x, y, z)$$

'Partial derivative' : $\frac{\partial T}{\partial x}$, $\frac{\partial T}{\partial y}$, $\frac{\partial T}{\partial z}$

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}\right) \cdot \left(dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}\right)$$

$\vec{\nabla} T$ — Gradient of 'T'

Displacement vector ($d\vec{l}$)

$T = x^2 + y^2 + z^2$

$\vec{\nabla} T$ fixed.

$$dT = (\vec{\nabla} T) \cdot (d\vec{l}) = |\vec{\nabla} T|\,|d\vec{l}| \cos\theta$$

$\vec{\nabla} T$ ↓ vector

<u>Geometric interpretation</u>: (for a fixed $|d\vec{l}|$)

$$dT\big|_{max.} = |\vec{\nabla} T|\,|d\vec{l}|$$

→ alligned.

(*) Gradient of T points in direcn. of max. increase in 'T'.

(*) The magnitude $|\vec{\nabla} T|$ given by the slope.

- $\vec{\nabla} T = 0 \Rightarrow$ what does it mean?

for a fn. of one variable,

$$\frac{df}{dx} = 0 \quad \text{(at one particular point)}$$

↳ condn. for extrema.

$\Rightarrow$ In order to locate extrema of a fn. with more than one variable, put $\underline{\vec{\nabla} T = 0}$.

Del operator: Derivative op: $\frac{d}{dx}$

Definition: $\vec{\nabla} = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$.

(*) Not a vector in the usual sense

(*) Not a quantity unless it operates on something.

Correspondence with usual vector

multiplication ⟶ act upon

(for vector) (for del)

(*) Consider an ordinary vector $(\vec{A})$:

multiplication: (i) $a\vec{A}$

(ii) $\vec{A}\cdot\vec{B}$

(iii) $\vec{A}\times\vec{B}$

Similarly for Del operator:

(i) $\vec{\nabla}T$ (Gradient) ←

(ii) $\vec{\nabla}\cdot\vec{A}$ (Divergence)

(iii) $\vec{\nabla}\times\vec{A}$ (Curl)

Divergence:

Definition: $(\vec{\nabla}\cdot\vec{v}) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot$

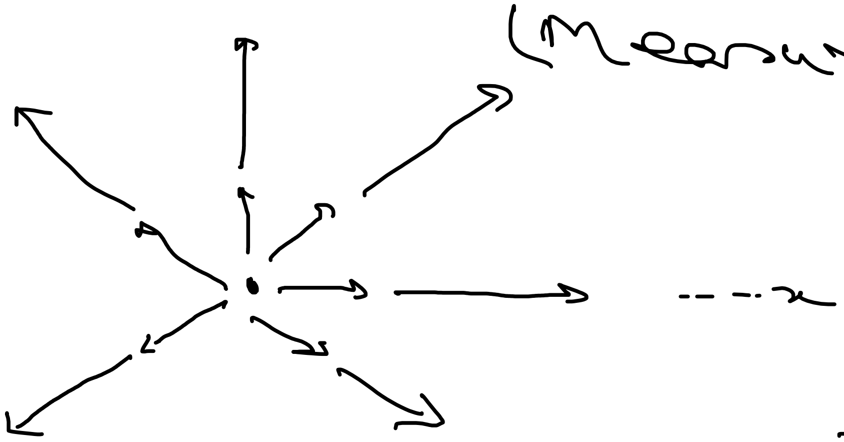
$(v_x\hat{x} + v_y\hat{y} + v_z\hat{z})$

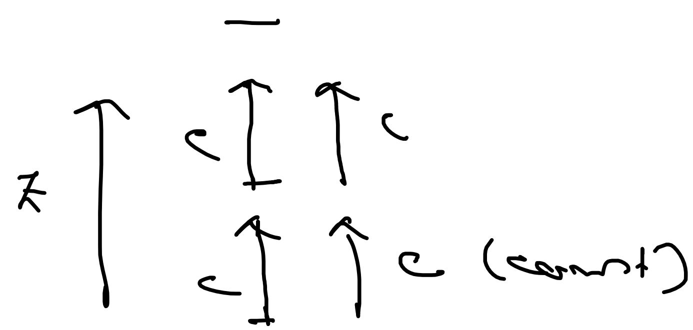
↙ scalar.

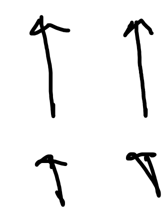
$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}.$$

Geometric interpretation: (measurement of how the vector spreads out)

(Measure of divergence)







$v(x,y,z) = x^2\hat{x} + y^2\hat{y} + z^2\hat{z}$

$\vec{\nabla}\cdot\vec{v} = 2x + 2y + 2z$

$\vec{v}(x,y,z) = c\hat{z}$

$\vec{\nabla}\cdot\vec{v} = 0$

$\vec{v}(x,y,z) = z\hat{z}$

$\vec{\nabla}\cdot\vec{v} = 1z$