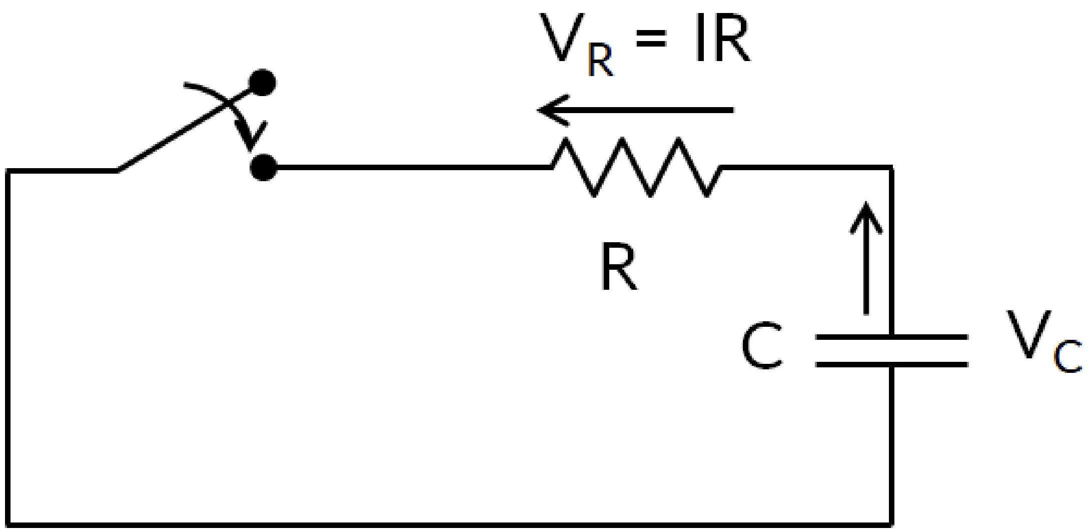
Discharging a Capacitor

- Coulombic repulsion between charges already existing the plates creates a force that lets charges to discharge out of the capacitor once the voltage on the charge in the capacitor is decreased
- Coulombic repulsion decreases as more charges are removed from the capacitor plates.
- Initially, voltage across the capacitor decreases rapidly as charge is removed from the plates.
- As more and more charge is removed, voltage across the capacitor decreases more slowly as it becomes difficult to force the remaining charge out of the capacitor.

Discharging a Capacitor

- Applying KVL



$$V_R + V_C = 0$$

$$iR + \frac{q}{C} = 0 \qquad \frac{dq}{dt}R + \frac{q}{C} = 0$$

$$\int \frac{dq}{q} = -\int \frac{dt}{RC} \qquad \ln q = -\frac{t}{RC} + C_1$$

$$q = e^{-\frac{t}{RC}+C_1} \qquad q = C_2 e^{-\frac{t}{RC}}$$

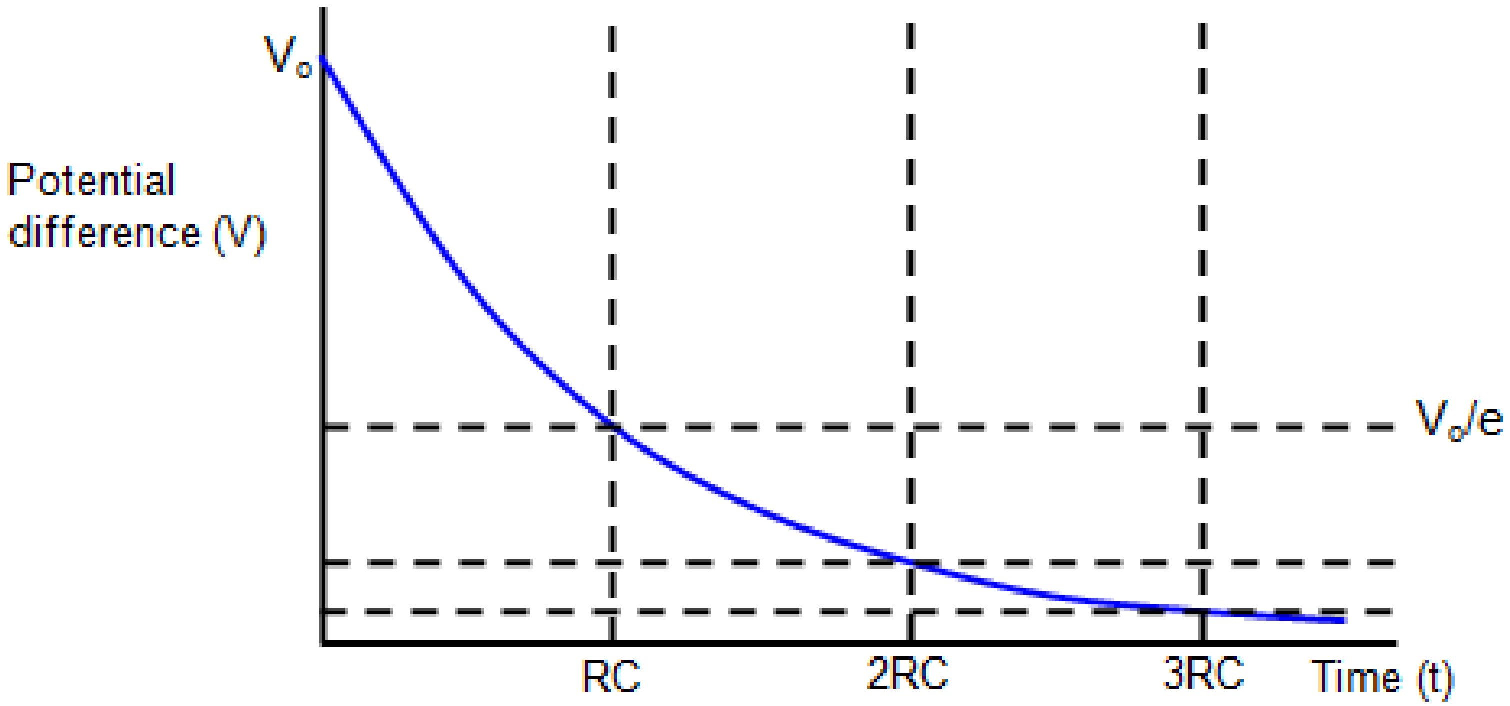
Substitute boundary condition: at t = 0, Voltage across C = V, q = VC

$$C_2 = VC \qquad q = VCe^{-\frac{t}{RC}}$$

$$\boxed{V_C = \frac{q}{C} = Ve^{-\frac{t}{RC}}} \qquad \boxed{i = \frac{dq}{dt} = -\frac{V}{R}e^{-\frac{t}{RC}}}$$

- **Note that the negative sign indicate that the current is opposite to the charging current's direction**

Discharging a Capacitor

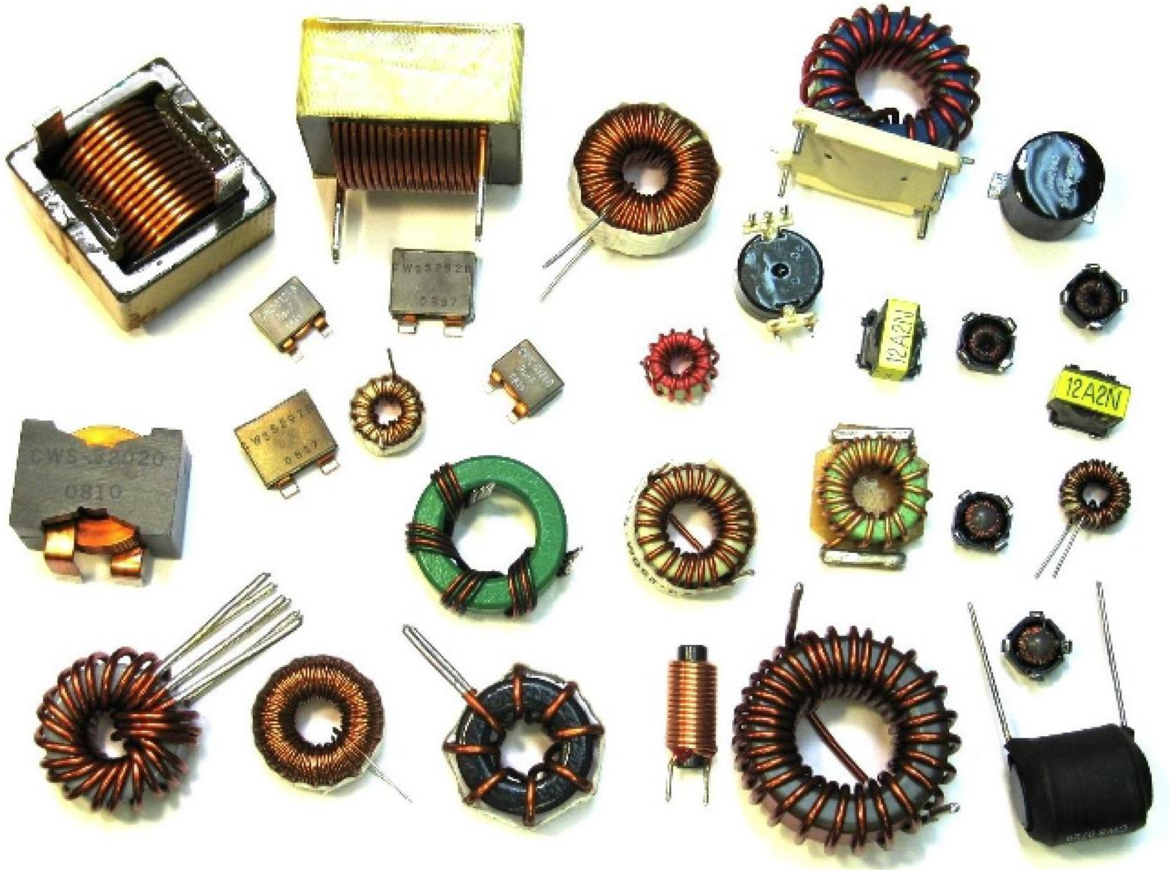


Inductors

- Inductor stores energy in a magnetic field created by an electric current flowing through it.

- Inductor opposes change (chokes) in current flowing through a conductor.

- Current through an inductor is continuous; voltage can be discontinuous.



Structure

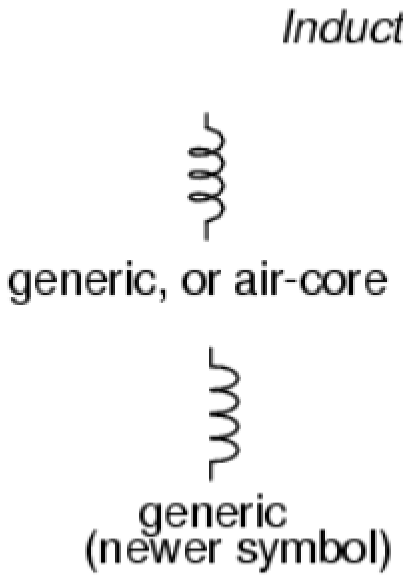
- Generally formed by a coil of conducting wire

- Conducting wire is usually wrapped around a solid core.

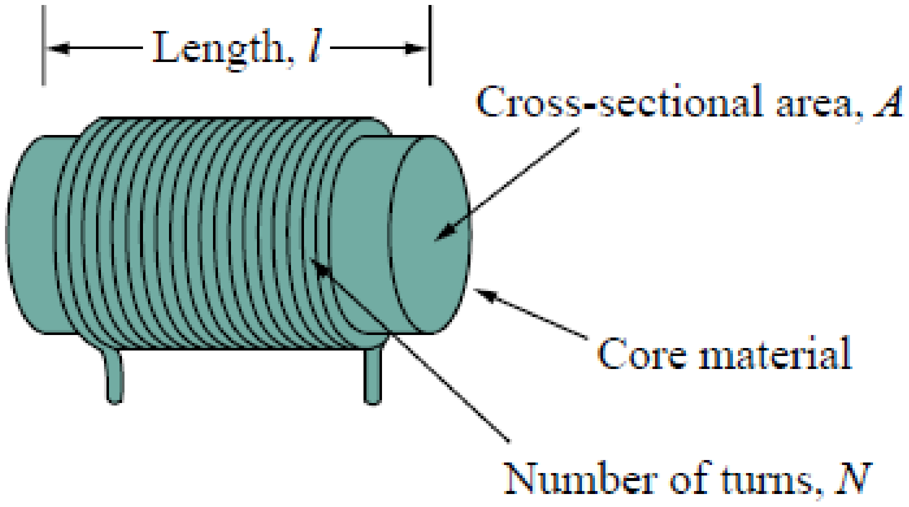
- In the absence of a core, the inductor is said to have an ‘air core’.

Inductors…Continued

Circuit Symbols



Calculations of L



$$L = \frac{N^2 \mu A}{l} = \frac{N^2 \mu_0 \mu_r A}{l}$$

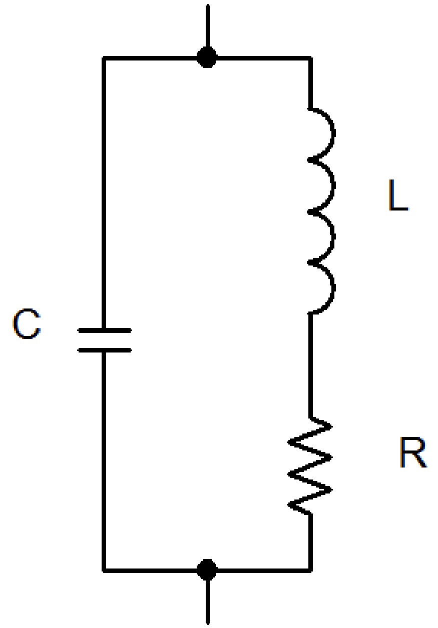
N is the number of turns of wire
A is the cross-sectional area of the toroid in m^2.
μ_r is the relative permeability of the core material
μ_o is the vacuum permeability ($4\pi \times 10^{-7}$ H/m)
l is the length of the wire used to wrap the toroid in meters

Properties of an Inductor

- Inductor acts like a short circuit in steady state.

- Current through an inductor must be continuous, meaning there are no abrupt changes to the current but there can be abrupt changes in the voltage across an inductor.

- No energy or power is dissipated by an ideal inductor. Ideal inductor absorbs energy or power from the circuit when storing energy and restores energy into circuit while discharging



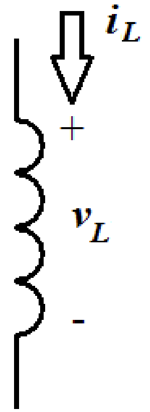
Properties of a Real Inductor

- Due to resistive losses and capacitive coupling between turns of wire

Sign Convention

- When current flows into the positive side of the voltage across the inductor, the current is positive, and the inductor is dissipating power

- When an inductor releases energy back into the circuit, the sign of the current is negative.



Current Voltage Relationship

- the voltage across the inductor is directly proportional to the time rate of change of the current.

$$v_L = L\frac{di}{dt}$$

$$i_L = \frac{1}{L}\int_{t_0}^{t_1} v_L \, dt$$

Current Voltage Relationship

Let $I_L = I_0 \sin(\omega t)$

$$v_L = L\frac{d}{dt}\left(I_0 \sin(\omega t)\right)$$

$$v_L = \omega L I_0 cos(\omega t)$$

$$= \omega L I_0 sin\left(\omega t + \frac{\pi}{2}\right)$$

- Voltage across inductor leads current through inductor by $\frac{\pi}{2}$

Let $I_L = I_0 e^{j\omega t}$ $\qquad v_L = L\frac{di}{dt}$

$$v_L = L\frac{d}{dt}\left(I_0 e^{j\omega t}\right) = j\omega L I_0 e^{j\omega t}$$

$$v_L = j\omega L i_L$$

Comparing the equation (similar to that of V= IR)

$$\boxed{j\omega L = jX_L = Z_L}$$

X_L is called as reluctance (in ohm) of an inductor

– When $\omega = 0$, $X_L = 0$, means reactance is zero and inductor behaves like a short at DC

– When $\omega = \infty$, $X_L = \infty$, means inductor behaves like an open circuit at higher frequencies

– Frequency dependent electrical behavior of inductance on circuit

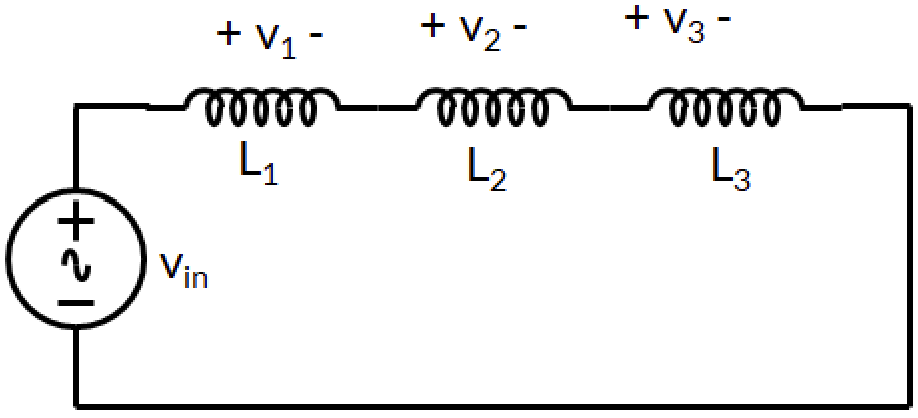
Power and Energy

$$p_L = v_L i_L = L i_L \int_{t_0}^{t_1} i_L dt$$

$$w_L = \int_{-\infty}^{t} p_L dt = \int_{-\infty}^{t} L \frac{di_L}{dt} i_L dt = L \int_{-\infty}^{t} i_L d\, i_L = \frac{1}{2} L i_L^2$$

Inductors in Series

- Consider inductors connected in a series configuration as shown in the circuit



- Applying KVL

$$v = v_1 + v_2 + v_3$$

- Noting the relation between voltage across and current through inductor,

$$v_1 = L_1 \frac{di}{dt} \qquad v_2 = L_2 \frac{di}{dt} \qquad v_3 = L_3 \frac{di}{dt}$$

- If L_{eq} is the total inductance of the circuit, then

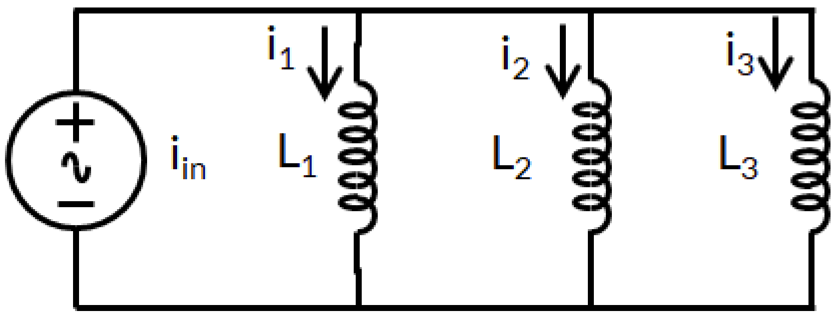
$$v_{in} = L_{eq} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

$$\boxed{L_{eq} = \sum_{s=1}^{N} L_s}$$

Inductors in Parallel

- Consider inductors connected in a parallel configuration as shown in the circuit



- Applying KCL

$$i_{in} = i_1 + i_2 + i_3$$

- Noting the relation between voltage across the inductor and current through inductor

$$i_1 = \frac{1}{L_1}\int_{t_0}^{t_1} vdt \qquad i_2 = \frac{1}{L_2}\int_{t_0}^{t_1} vdt \qquad i_3 = \frac{1}{L_3}\int_{t_0}^{t_1} vdt$$

- If Leq is the total inductance of the circuit, then

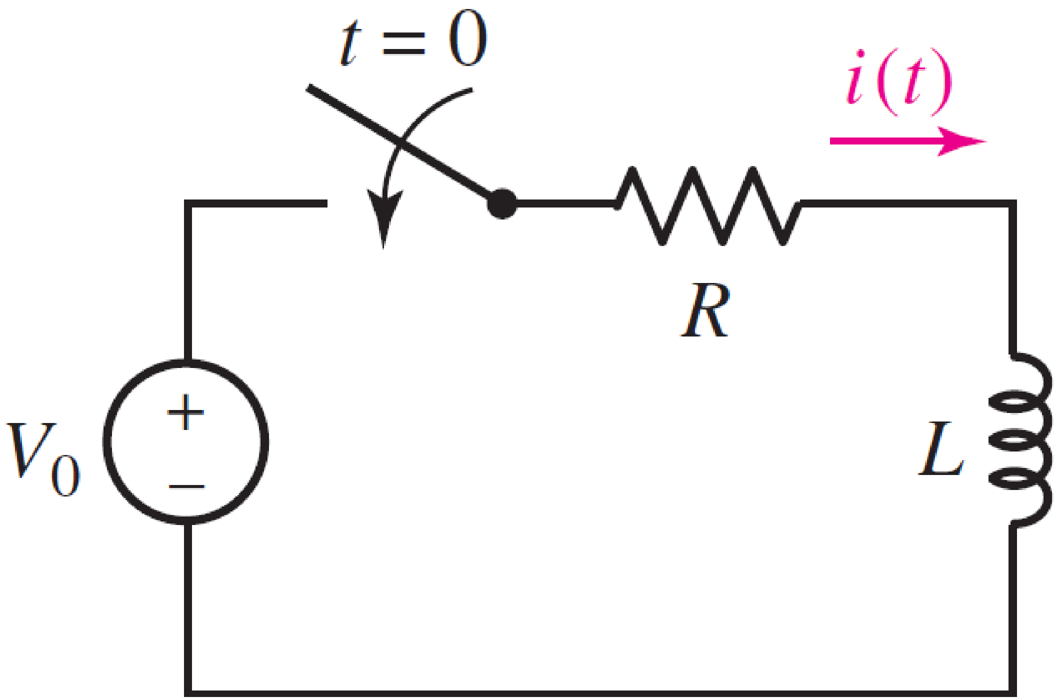
$$i_{in} = \frac{1}{L_{eq}}\int_{t_0}^{t_1} vdt = \frac{1}{L_1}\int_{t_0}^{t_1} vdt + \frac{1}{L_2}\int_{t_0}^{t_1} vdt + \frac{1}{L_3}\int_{t_0}^{t_1} vdt$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

$$\boxed{L_{eq} = \left(\sum_{p=1}^{N} \frac{1}{L_p}\right)^{-1}}$$

Charging of an Inductor

- Consider the circuit shown in figure
- Applying KVL



$$V_0 = V_R + V_L$$

$$V_0 = iR + L\frac{di}{dt}$$

$$\frac{Ldi}{V_0 - Ri} = dt$$

$$\int \frac{Ldi}{V_0 - Ri} = \int dt$$

$$-\frac{L}{R}\ln(V_0 - Ri) = t + k$$

Using the initial boundary condition: at time *t* = 0, *i(t)*=0 $\quad k = -\frac{L}{R}\ln V_0$

$$-\frac{L}{R}\ln[(V_0 - Ri) - \ln(V_0)] = t \Rightarrow \frac{V_0 - Ri}{V_0} = e^{-\frac{Rt}{L}} \Rightarrow i = \frac{V_0}{R}\left(1 - e^{-\frac{Rt}{L}}\right)$$

Discharge Current through an Inductor

Applying KVL to the circuit:

$$v(t) + Ri(t) = 0$$

$$L\frac{di(t)}{dt} + Ri(t) = 0$$

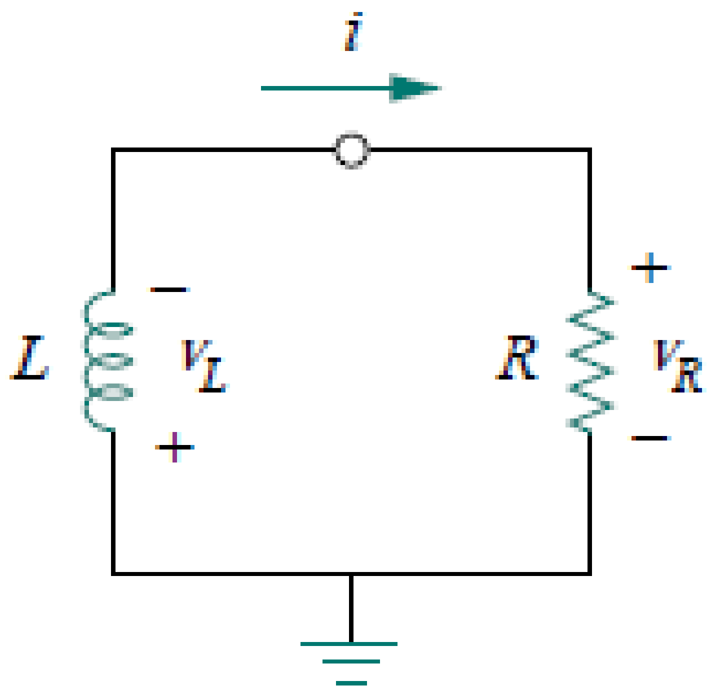
$$\frac{di(t)}{i(t)} = -\frac{R}{L}dt$$

$$\ln\frac{i(t)}{i(0)} = -\frac{R}{L}t$$

$$i(t) = I_0 e^{-\frac{R}{L}t}$$

Time constant τ is given by

$$\tau = \frac{L}{R}$$



Discharge Current through an Inductor