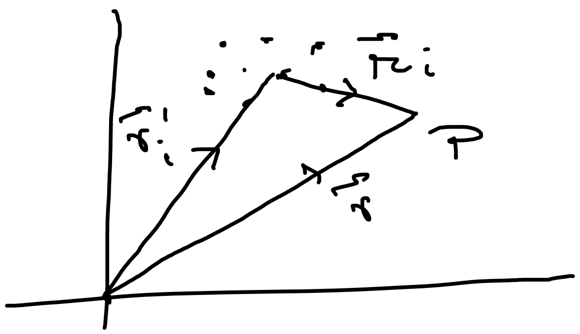
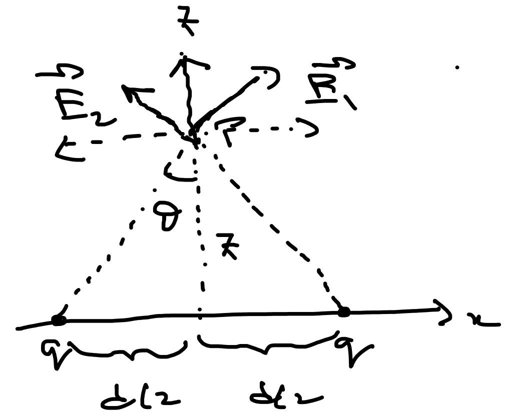
Electric Field

26.11.20



$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{r}_i$$

Ex:



$$\vec{E} = \vec{E}_1 + E_2$$

→ Horizontal components cancel.

$$\hat{r} \equiv \frac{\vec{r}}{|\vec{r}|}$$

$$E_z^{(1)} = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \hat{r}$$

(vertical direction)

$$|\vec{r}| = \left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}$$

$$E = 2\frac{1}{4\pi\epsilon_0} \frac{q\, z\, \hat{z}}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{3/2}}$$

$$\cos\theta = \frac{z}{r}$$

$$\left[\hat{r} = \frac{z\,\hat{z}}{\left[z^2 + \left(\frac{d}{2}\right)^2\right]^{1/2}} \right]$$

Far away from origin: $z \gg d$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{2q}{z^2} \hat{z}$$

Continuous distribution of charge:

Replace $\Sigma \to \int \Rightarrow \vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\hat{r}}{r^2} dq$

(*) Line charge distribution

$$dq = \lambda \, dl'$$

dl' → infinitesimal line element

λ → charge per unit length

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\lambda(\vec{r}\,')}{\mathscr{r}^2} \hat{\mathscr{r}} \, dl'$$

(*) Surface charge distribution:

$$dq = \sigma \, da'$$

da' → infinitesimal surface element

σ → charge per unit area

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma(\vec{r}\,')}{\mathscr{r}^2} \hat{\mathscr{r}} \, da'$$

(*) Volume charge distribution:

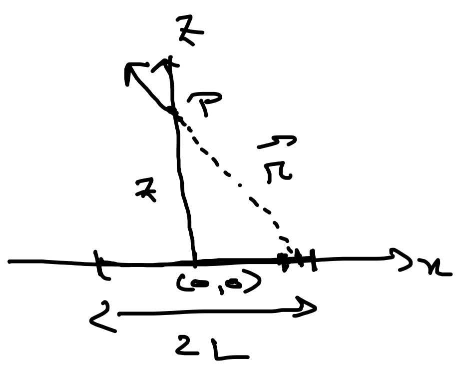
$$dq = \rho \, d\tau'$$

$d\tau'$ → infinitesimal volume element

ρ → charge per unit volume

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}\,')}{\mathscr{r}^2} \hat{\mathscr{r}} \, d\tau'$$

Ex



$\vec{r} = z\hat{z}$ (position vec. for ref. point (r))

$\vec{r}\,' = x\hat{x}$ (- - - - - - for source point)

$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$ $\Rightarrow$ $\hat{\mathscr{r}} = \dfrac{z\hat{z} - x\hat{x}}{[z^2 + x^2]^{1/2}}$

separation vector $= z\hat{z} - x\hat{x}$

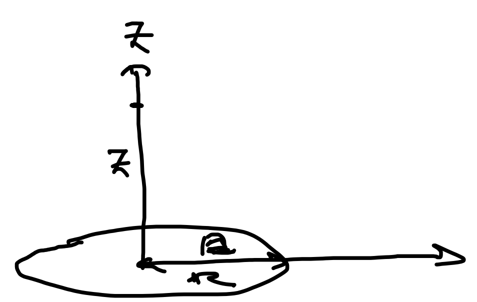
$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_{-L}^{+L} \frac{\lambda}{(z^2+x^2)} \frac{z\hat{z} - x\hat{x}}{(z^2+x^2)^{1/2}}\, dx$$

$$= \frac{\lambda}{4\pi\epsilon_0}\left[z\hat{z}\left(\frac{x}{z^2(z^2+x^2)^{1/2}}\right)\Bigg|_{-L}^{+L} - \hat{x}\left(-\frac{1}{(x^2+z^2)^{1/2}}\right)\Bigg|_{-L}^{+L}\right]$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z(z^2+L^2)^{1/2}}\hat{z}$$

for $z \gg L$, $|\vec{E}| \cong \frac{1}{4\pi\epsilon_0}\frac{2\lambda L}{z^2}$

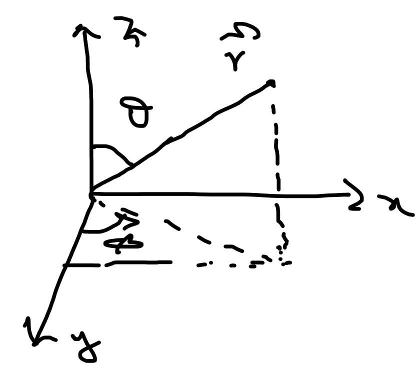
Ex Surface ch. density $= \sigma$



infinitesimal surface element $= dl_r\, dl_\phi$

$= r\,dr\,\sin\theta\, d\phi$

$= r\,dr\,d\phi$

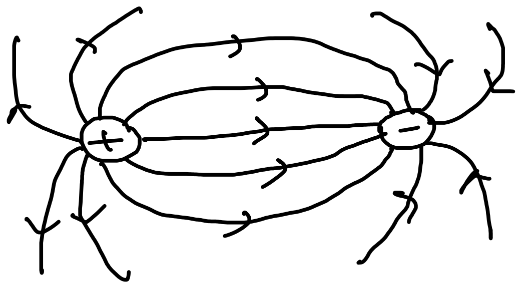


$\frac{\hat{r}}{r^2}$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int_0^R\int_0^{2\pi} \frac{r\,dr\,d\phi\;\sigma\, z}{(z^2+r^2)^{3/2}}\hat{z}$$

$$= \frac{1}{4\pi\epsilon_0} 2\pi\sigma z \left(\frac{1}{z} - \frac{1}{\sqrt{R^2+z^2}} \right) \hat{z}$$

Divergence & curl of electric fields



Flux of $\vec{E}$ through a surface S:

$$\Phi_E = \int_S \vec{E} \cdot d\vec{a}$$

measurement of number of field lines passing through S