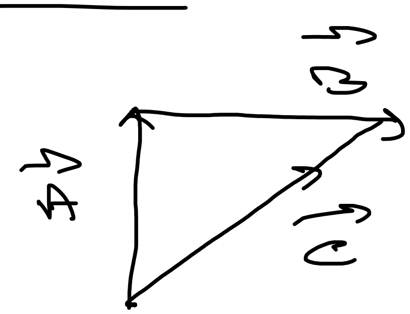
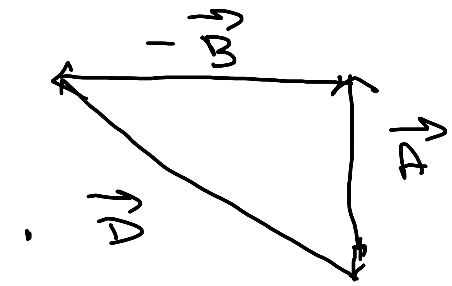
Ref: D. J. Griffiths. 27/10/2020

Vector operations:

(i) Addition: $\vec{A}$ & $\vec{B}$





$\vec{C} = \vec{A} + \vec{B}$ $\vec{D} = \vec{A} - \vec{B}$

Properties: ① Commutative

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

② Associative

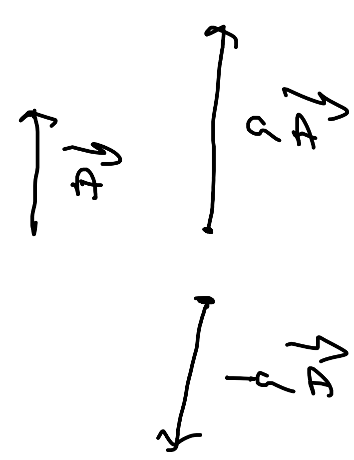
$$(\vec{A} + \vec{B}) + \vec{C} = \vec{A} + (\vec{B} + \vec{C})$$

(ii) Multiplication by a scalar:

$a \; \vec{A}$ | $a\vec{A} \Rightarrow$

↙
scalar

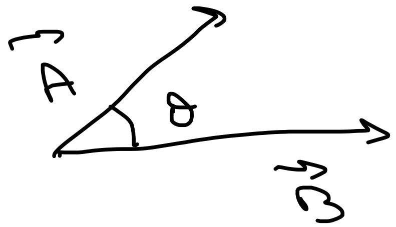
$-a\vec{A} \Rightarrow$



Properties: ① Distributive

$$a(\vec{A}+\vec{B}) = a\vec{A} + a\vec{B}$$

⊛ Dot product:



Definition: $\vec{A}\cdot\vec{B} = \underbrace{AB\cos\theta}_{\text{scalar}}$

$\Rightarrow$ scalar prod.

Properties: ① Commutative

$$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$$

② Distributive

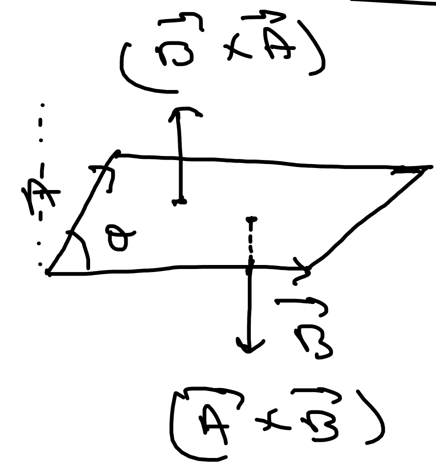
$$\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$$

Geometrically speaking $\Rightarrow$ product of A times projection of $\vec{B}$ along $\vec{A}$ or vice versa

We use: parallel $\Rightarrow$ $\vec{A}\cdot\vec{B} = AB$ $\to \theta = 0$

perpendicular $\Rightarrow$ $\vec{A}\cdot\vec{B} = 0$ $\to \theta = \pi/2$

⊛ **Cross product:**



Definition: $\vec{A}\times\vec{B} = AB\sin\theta\,\hat{n}$

$\hat{n}$: unit vector

Direction $\hat{n}$ determined by right-hand rule

⇓ vector product

Properties: ① Distributive

$$\vec{A}\times(\vec{B}+\vec{C}) = (\vec{A}\times\vec{B}) + (\vec{A}\times\vec{C})$$

② Not Commutative

$$\vec{B}\times\vec{A} = -\vec{A}\times\vec{B}$$

Geometrically: $|\vec{A}\times\vec{B}|$ ⟶ area of a parallelogram.

We use: parallel $\Rightarrow \vec{A}\times\vec{B} = 0$ → $\theta = 0$

perpendicular $\Rightarrow \vec{A}\times\vec{B} = AB$ → $\theta = \pi/2$

In component notation:

Cartesian: Point (x, y, z)

Basis vectors / unit vectors : $\hat{x}, \hat{y}, \hat{z}$

$$\Rightarrow \quad \vec{A} = A_x \hat{x} + A_y \hat{y} + A_z \hat{z}$$

$A_i \Rightarrow$ components

$\hookrightarrow$ projection of $\vec{A}$ on different axes

$$A_x = \vec{A} \cdot \hat{x}$$
$$A_y = \vec{A} \cdot \hat{y}$$
$$A_z = \vec{A} \cdot \hat{z}$$

* Addition:

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{x} + (A_y + B_y)\hat{y} + (A_z + B_z)\hat{z}$$

* Multiplication by scalar:

$$a\vec{A} = (aA_x)\hat{x} + (aA_y)\hat{y} + (aA_z)\hat{z}$$

* Dot product :

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

$$\left.\begin{aligned} \hat{x}\cdot\hat{x} = \hat{y}\cdot\hat{y} = \hat{z}\cdot\hat{z} = 1 \\ \hat{x}\cdot\hat{y} = \hat{x}\cdot\hat{z} = \hat{y}\cdot\hat{z} = 0 \end{aligned}\right\}$$

* Cross product:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{x} + (A_z B_x - A_x B_z)\hat{y} + (A_x B_y - A_y B_x)\hat{z}$$

$$\hat{x} \times \hat{x} = \hat{y} \times \hat{y} = \hat{z} \times \hat{z} = 0$$

$$\hat{x} \times \hat{y} = -\hat{y} \times \hat{x} = \hat{z}$$

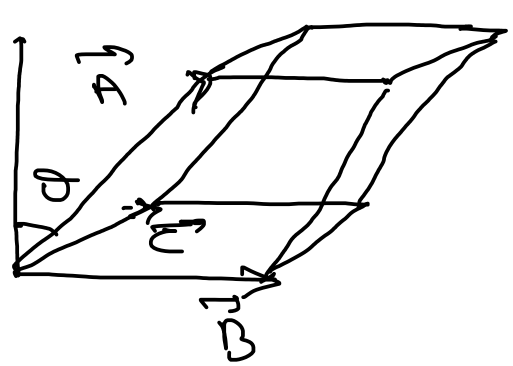
$$\hat{y} \times \hat{z} = -\hat{z} \times \hat{y} = \hat{x}$$

$$\hat{z} \times \hat{x} = -\hat{x} \times \hat{z} = \hat{y}$$

<u>Triple products</u>:

* Scalar triple product:

$\underset{\sim}{\vec{A}} \cdot (\underset{\sim}{\vec{B} \times \vec{C}})$ $\Rightarrow$ Geometrically this volume of a parallelopiped.



$|\vec{B} \times \vec{C}|$ $\Rightarrow$ Area of base

$A\cos\theta$ $\Rightarrow$ Altitude

<u>Property</u>:

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B}) \quad \#$$

$$\vec{A}\cdot(\vec{B}\times\vec{C}) \neq \cancel{(\vec{A}\cdot\vec{B})\times\vec{C}} \Rightarrow \text{Parentheses is important!!}$$

In component form:

$$\vec{A}\cdot(\vec{B}\times\vec{C}) = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

* Vector triple product:

$\vec{A}\times(\vec{B}\times\vec{C}) = \vec{B}\,(\vec{A}\cdot\vec{C}) - \vec{C}\,(\vec{A}\cdot\vec{B})$

$\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$

simplify complicated expression.

$$(\vec{A}\times\vec{B})\cdot(\vec{C}\times\vec{D}) = (\vec{A}\cdot\vec{C})(\vec{B}\cdot\vec{D}) - (\vec{A}\cdot\vec{D})(\vec{B}\cdot\vec{C})$$

$$\vec{A}\times[\vec{B}\times(\vec{C}\times\vec{D})] = \vec{B}[\vec{A}\cdot(\vec{C}\times\vec{D})] - (\vec{A}\cdot\vec{B})(\vec{C}\times\vec{D})$$

$\rightarrow \vec{A}\times(\vec{B}\times\vec{F})$

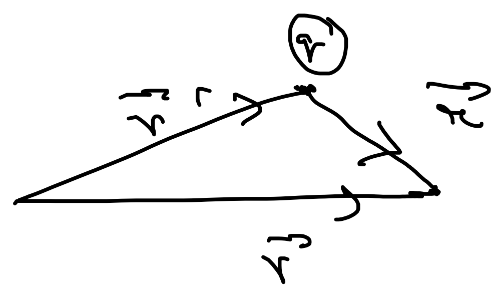
Position vector: $(\vec{r})$

Cartesian: $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$

$$r = (x^2 + y^2 + z^2)^{1/2}$$

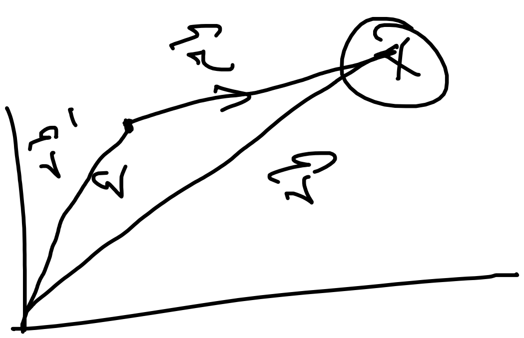
$$\hat{r} = \frac{\vec{r}}{r}$$

(x) separation vector: $(\vec{\mathscr{r}})$



$\vec{r}' \to$ source point vector

$\vec{r} \to$ field point vector



$$\Rightarrow \vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

$$\mathscr{r} = |\vec{r} - \vec{r}'|$$

$$\hat{\mathscr{r}} = \frac{\vec{\mathscr{r}}}{\mathscr{r}}$$

Introduction to Electrodynamics

$\to$ D. J. Griffiths.