Capacitors and Inductors

Complex Numbers

- Let $Z=a+jb$ be a complex number
- a is known as real part and b as imaginary part
- j is defined as $j = \sqrt{-1}$
- Complex conjugate of Z is $\overline{Z} = a - jb$
- Modulus of complex number is defined as $|Z| = \sqrt{Z\overline{Z}} = \sqrt{a^2 + b^2}$

Polar Representation of Complex Numbers

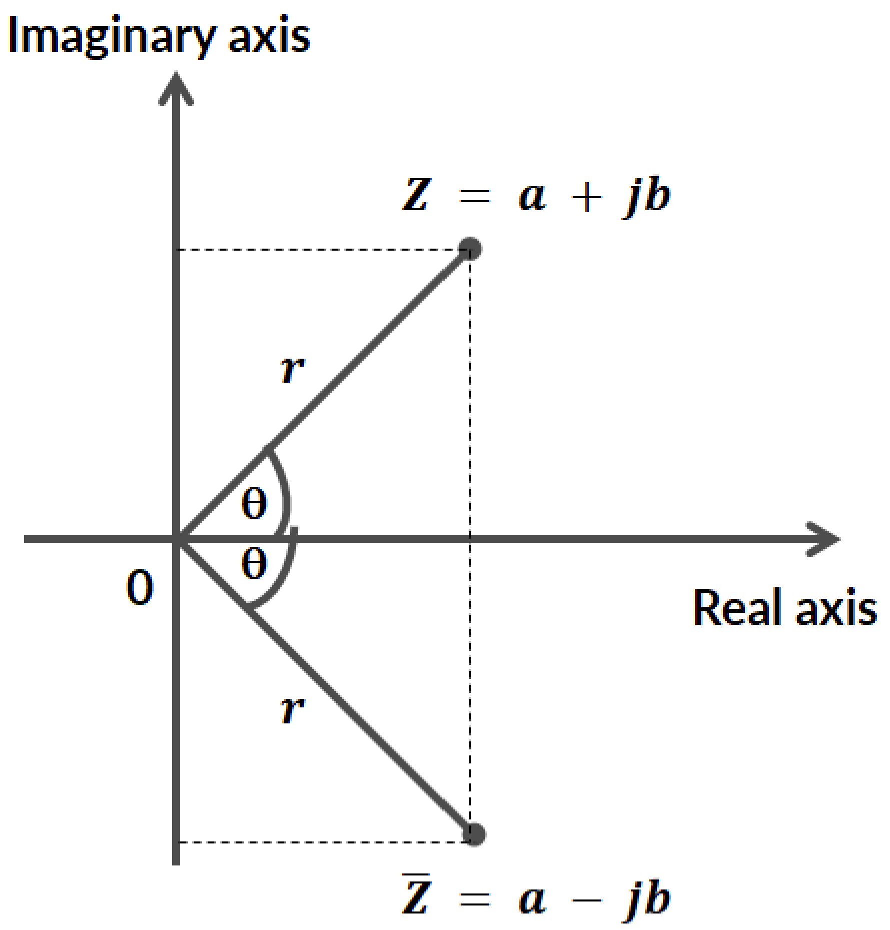
- A complex number *Z=a+jb* can be represented in a polar form
- In polar form, $Z = a + jb = re^{j\theta}$
- where $r = |Z| = \sqrt{a^2 + b^2}$ and $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

$$\overline{Z} = a - jb = re^{-j\theta}$$

$$|Z| = \sqrt{Z\overline{Z}} = \sqrt{re^{j\theta}re^{-j\theta}} = \sqrt{r^2} = r$$

- If we know r and θ, we can obtain a and b as

$$a = r\cos\theta \quad \text{and} \quad b = r\sin\theta$$



Arithmetic Operations of Complex Numbers: Cartesian Representation

Let $z_1 = a_1 + jb_1$ and $z_2 = a_2 + jb_2$

Addition: $$z_1 + z_2 = (a_1 + a_2) + j(b_1 + b_2)$$

Subtraction: $$z_1 - z_2 = (a_1 - a_2) + j(b_1 - b_2)$$

Multiplication: $$z_1 \times z_2 = (a_1 a_2 - b_1 b_2) + j(b_1 a_2 + b_2 a_1)$$

Division: $$\frac{z_1}{z_2} = \frac{(a_1 a_2 + b_1 b_2) + j(b_1 a_2 - b_2 a_1)}{{a_2}^2 + {b_2}^2}$$

Note: It is easy to add or subtract complex number in Cartesian representation

Arithmetic Operations of Complex Numbers: Polar Representation

Let $z_1 = r_1 e^{j\theta_1}$ and $z_2 = r_2 e^{j\theta_2}$

Multiplication:

$$z_1 \times z_2 = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

Division:

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

Note: It is easy to multiply or divide complex number in Polar representation

- The idea of phasor representation is based on Euler's identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \mathrm{Re}\left(e^{j\theta}\right) \qquad \sin\theta = \mathrm{Im}\left(e^{j\theta}\right)$$

Capacitors

- A capacitor is a passive element designed to store energy in its electric field.

- Capacitors are used extensively in electronics, communications, computers, and power systems. For example, they are used in the tuning circuits of radio receivers and as dynamic memory elements in computer systems.

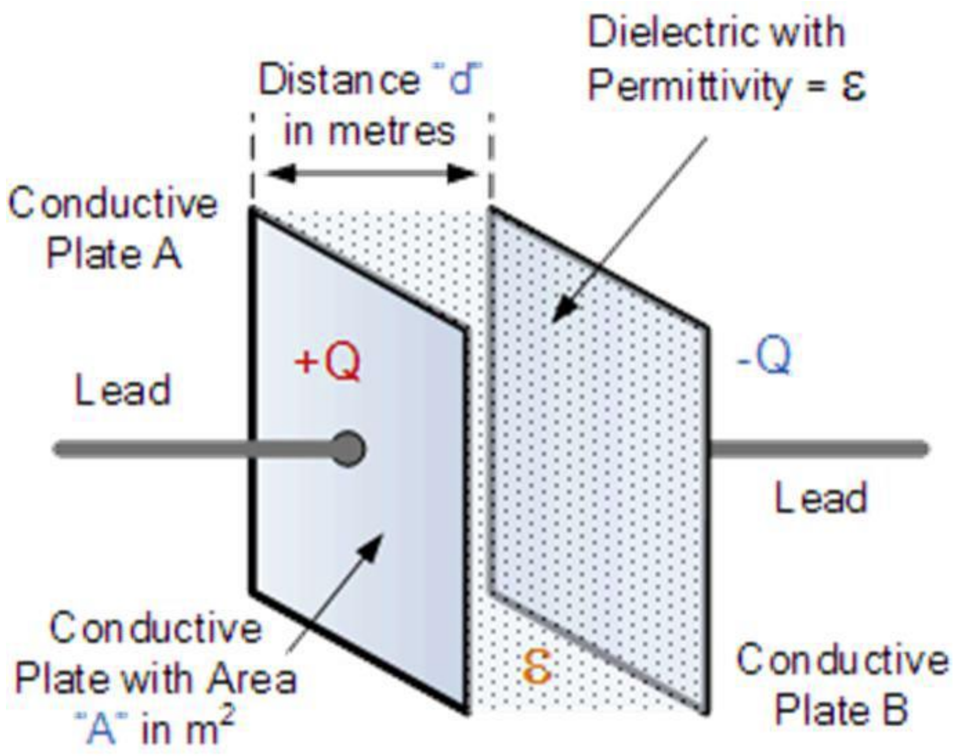
- Two conductive plates separated by an insulator (or dielectric) forms a capacitor. Commonly illustrated as two parallel metal plates separated by a distance, d.

$$C = \frac{\varepsilon A}{d}$$

where $\varepsilon = \varepsilon_0 \varepsilon_r$

ε_r is the relative dielectric constant

ε_0 is the vacuum permittivity



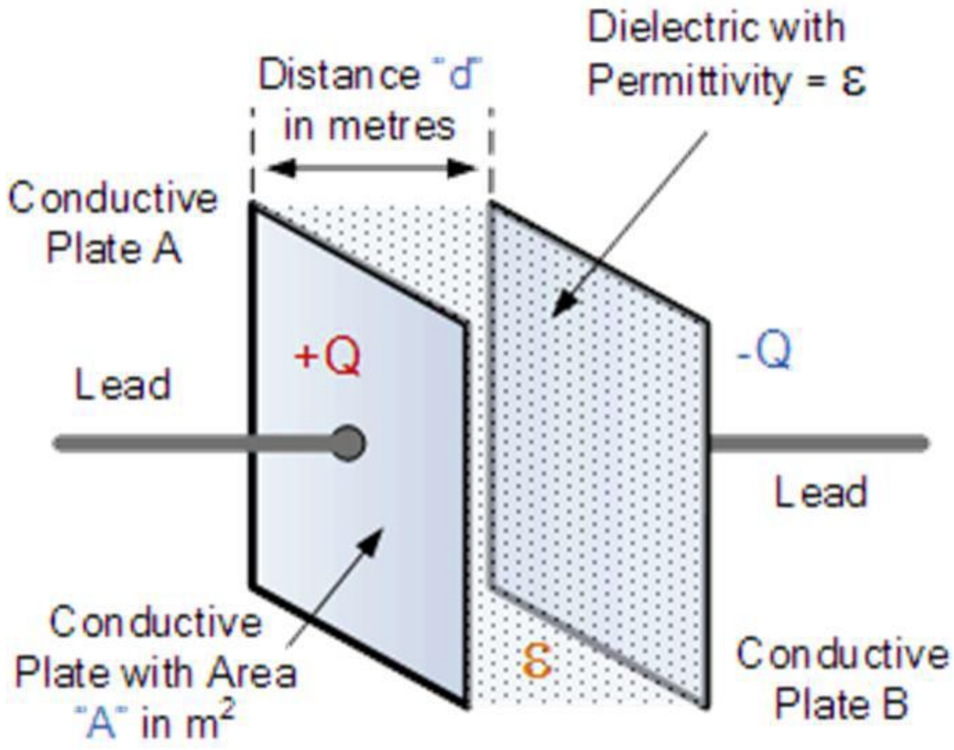
Effect of Dimensions on Capacitors

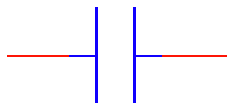
Capacitance increases with

- increasing surface area of the plates,
- decreasing spacing between plates
- increasing the relative dielectric constant of the insulator between the two plates.

$$C = \frac{\varepsilon A}{d}$$

Typically, capacitors have values in the picofarad (pF) to microfarad (μF) range.





Capacitor Circuit Symbol

Types of Capacitors

Fixed Capacitors

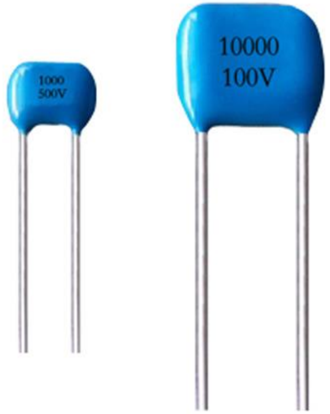
– Nonpolarized
- May be connected into circuit with either terminal of capacitor connected to the high voltage side of the circuit.
- Insulator: Paper, Mica, Ceramic, Polymer

– Polarized

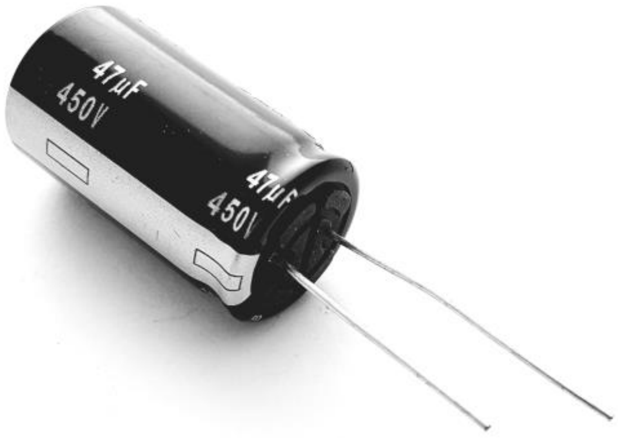
Electrolytic
- The negative terminal must always be at a lower voltage than the positive terminal
- Plates or Electrodes: Aluminum, Tantalum

- Difficult to make nonpolarized capacitors that store a large amount of charge or operate at high voltages.

- Tolerance on capacitance values is very large: can be as high as ±20%



Mica Capacitor

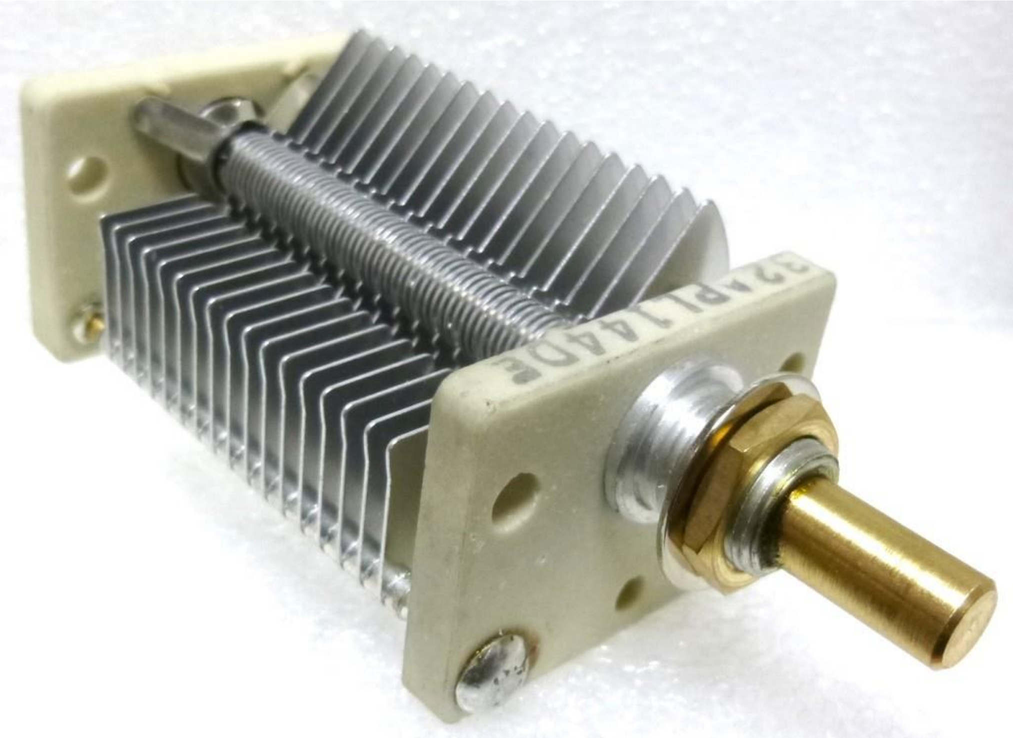


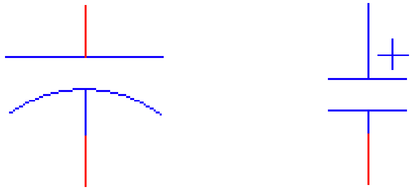
Electrolytic Capacitor

Types of Capacitors...Continued

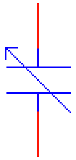
Variable Capacitors

- Cross-section area of capacitor plate is changed as one set of plates are rotated with respect to the other.





Polarized Capacitor Symbol



Variable Capacitor Symbol

https://www.rfparts.com/review/product/list/id/13998/category/1453/

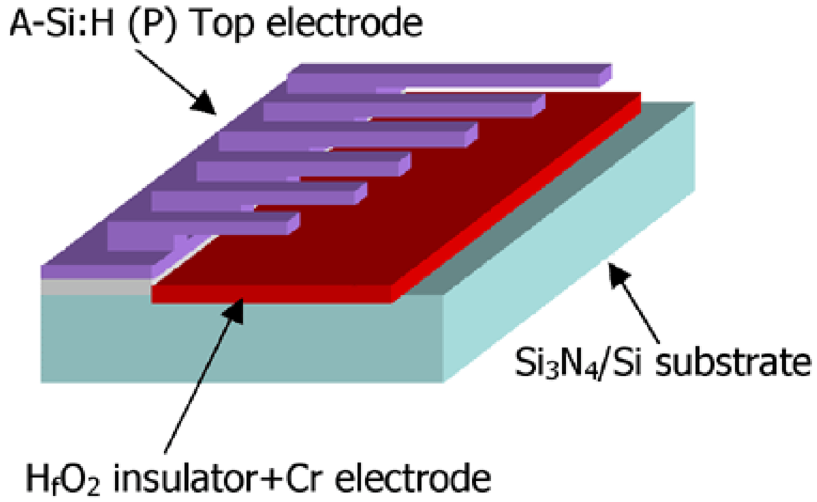
MEMS Capacitors

MEMS (Microelectromechanical systems)

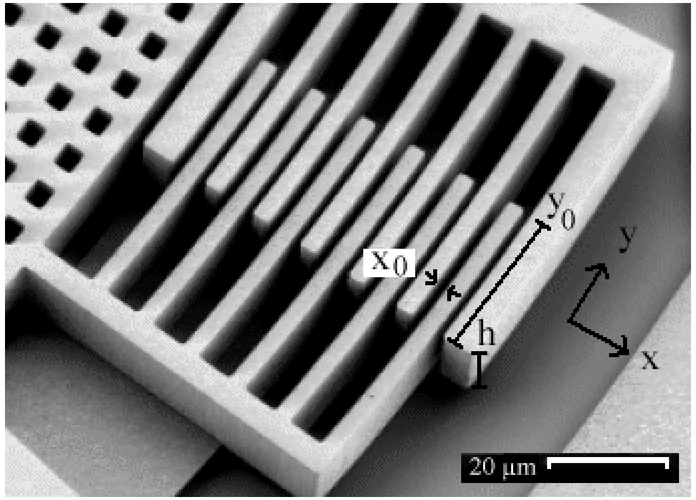
- Fabricated on Silicon Substrate

– Can be a variable capacitor by changing the distance between electrodes.

– Use in sensing applications as well as in RF electronics.



https://www.silvaco.com/tech_lib_TCAD/simulationstandard/2005/aug/a3/a3.html



https://www.tf.unikiel.de/matwis/amat/semitech_en/kap_7/backbone/r713.html

Advantages

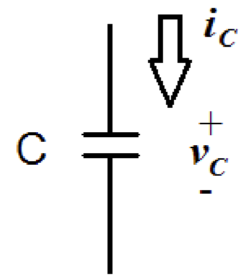
- Small in size
- Compact
- Consume low power
- Show faster response

Electrical Properties of a Capacitor

- Acts like an open circuit at steady state when connected to a d.c. voltage or current source.
- Voltage on a capacitor must be continuous. There are no abrupt changes to the voltage, but there may be discontinuities in the current.
- An ideal capacitor does not dissipate energy, it uses power when charging energy and returns power when discharging energy.

Sign Conventions

The sign convention used with a capacitor is the same as for a power dissipating device.



- When current flows into the positive side of the voltage across the capacitor, it is positive and the capacitor is dissipating power.
- When the capacitor releases energy back into the circuit, the sign of the current will be negative.

Current-Voltage Relationships

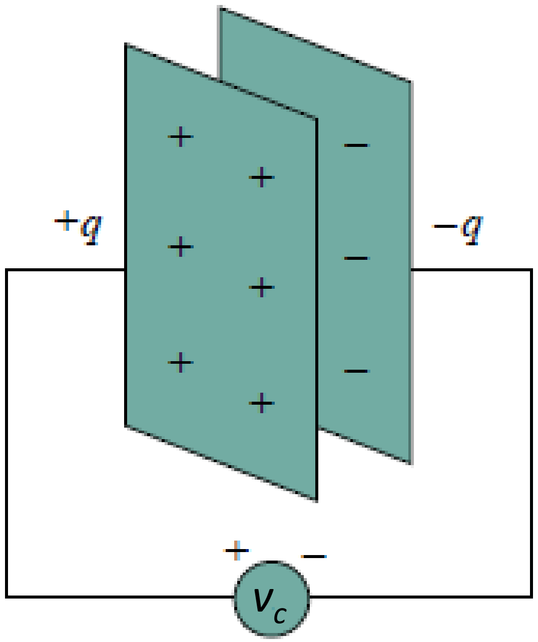
When a voltage source v_c is applied across a capacitor, a positive charge $+q$ is stored on one plate and a negative charge $-q$ on the other. The amount of charge stored, represented by q, is directly proportional to the applied voltage v_C.

$$q = Cv_C$$

$$i_C = \frac{dq}{dt}$$

$$i_C = C\frac{dv_C}{dt}$$

$$v_C = \frac{1}{C}\int_{t_0}^{t_1} i_C dt$$



Current-Voltage Relationships…Continued

Let $$v_C = V_0 \sin \omega t$$

$$i_C = C \frac{d}{dt}\left(V_0 \sin \omega t\right)$$

$$i_C = \omega C V_0 \cos \omega t$$

$$i_C = \omega C V_0 \sin\left(\frac{\pi}{2} - \omega t\right)$$

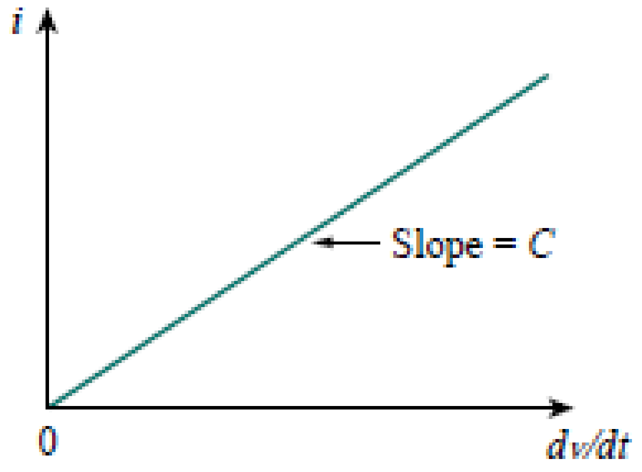
Current through capacitor leads voltage across capacitor by $\frac{\pi}{2}$

$$i_C = C\frac{dv_C}{dt}$$

Let $$v_C = V_0 e^{j\omega t}$$

$$i_C = C\frac{d}{dt}\left(V_0 e^{j\omega t}\right) = j\omega C V_0 e^{j\omega t}$$

$$v_C = \frac{1}{j\omega C} i_C$$



The current-voltage relationship is illustrated in the Figure for a capacitor whose capacitance is independent of voltage. Capacitors that satisfy Equation $i_C = C\frac{dv_C}{dt}$ are said to be *linear.* For a *nonlinear capacitor*, the plot of the current-voltage relationship is not a straight line.

Reactance and Impedance of a Capacitor

$$v_C = \frac{1}{j\omega C} i_C = -jX_C i_C = Z_C i_C$$

$$\boxed{X_C = \frac{1}{\omega C}}$$

$$\boxed{Z_C = \frac{1}{j\omega C}}$$

- X_C is called the reactance of the capacitor and Z_C, the impedance of the capacitor.

– When ω= 0, X_C = ∞, means reactance is infinite i.e., Capacitor blocks DC

– When ω = ∞, X_C = 0, means capacitor behaves like a short at higher frequencies

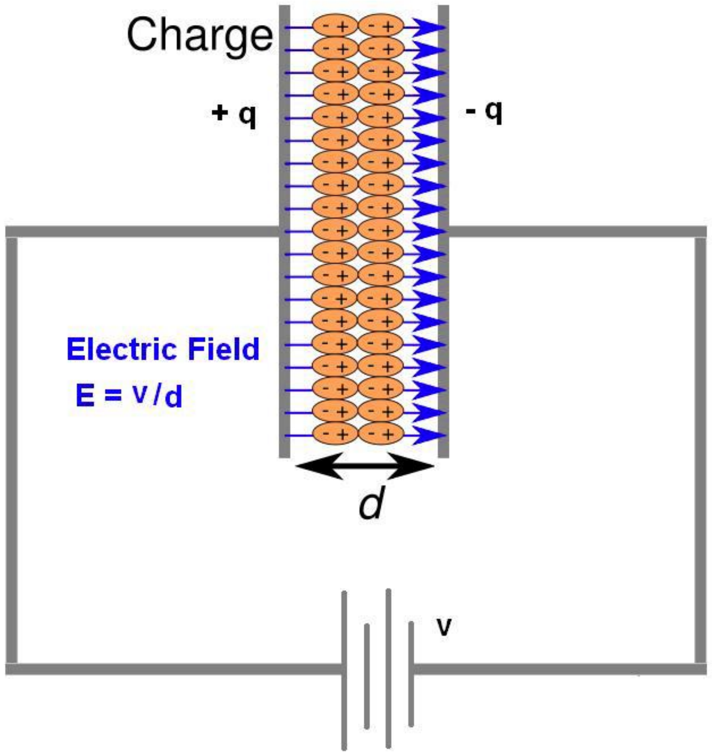
– Frequency dependent electrical behavior of capacitance on circuit

Power and Energy

Charge is stored on the plates of the capacitor.

Equation: Q = CV

Units:
Farad = Coulomb/Voltage; Farad is abbreviated as F



- A capacitor stores energy when charged and gives away energy while discharging.

- The instantaneous power delivered to the capacitor is

$$p_c = v_c i_c = C v_c \frac{dv_c}{dt}$$

- The energy stored in the capacitor is therefore

$$w_C = \int_{-\infty}^{t} p_C dt = C \int_{-\infty}^{t} v_c \frac{dv_c}{dt} dt = C \int_{-\infty}^{t} v_c dv_c = \frac{1}{2} C\, v_C^2$$