

Tutorial Sheet 8
Limit and Continuity of a Function of Several Variables

1. Examine the limits of the below mentioned functions as $(x, y) \rightarrow (0, 0)$.

(a) $f_1(x, y) = \begin{cases} \frac{x^2+y^2}{x^2-y^2}, & x \neq y \\ 0, & x = y \end{cases}$

(b) $f_2(x, y) = xy \left(\frac{x^2 - y^2}{x^2 + y^2} \right)$

(c) $f_3(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x}, & xy \neq 0 \\ 0, & xy = 0 \end{cases}$

(d) $f_4(x, y) = \frac{\sin(xy)}{x^2 + y^2}$

2. Find the repeated limits of the following functions.

(a) $f(x, y) = \frac{3x - 4y}{5x + 8y}$

(b) $g(x, y) = \left(\frac{2+x}{3+y} \right) \left(\frac{y-x}{y+x} \right)$

3. Show that $\lim_{x \rightarrow 0} \lim_{y \rightarrow 0} f(x, y)$ exist, but the double limit does not exist at the origin, where

$$f(x, y) = \begin{cases} y \sin \frac{1}{x} + \frac{xy}{x^2+y^2}, & x, y \neq 0 \\ 0, & x = y = 0. \end{cases}$$

4. Examine the continuity of the following functions at $(0, 0)$.

(a) $f_1(x, y) = \begin{cases} \frac{xy^3}{x^2 + y^6}, & (x, y) \neq (0, 0) \\ 0, & \text{otherwise.} \end{cases}$

(b) $f_3(x, y) = \begin{cases} \frac{\sin^2(x-y)}{|x| + |y|}, & (x, y) \neq (0, 0) \\ 0, & \text{otherwise.} \end{cases}$

(c) $f_4(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 y^2 + (x-y)^2}, & (x, y) \neq (0, 0) \\ 0, & \text{otherwise.} \end{cases}$