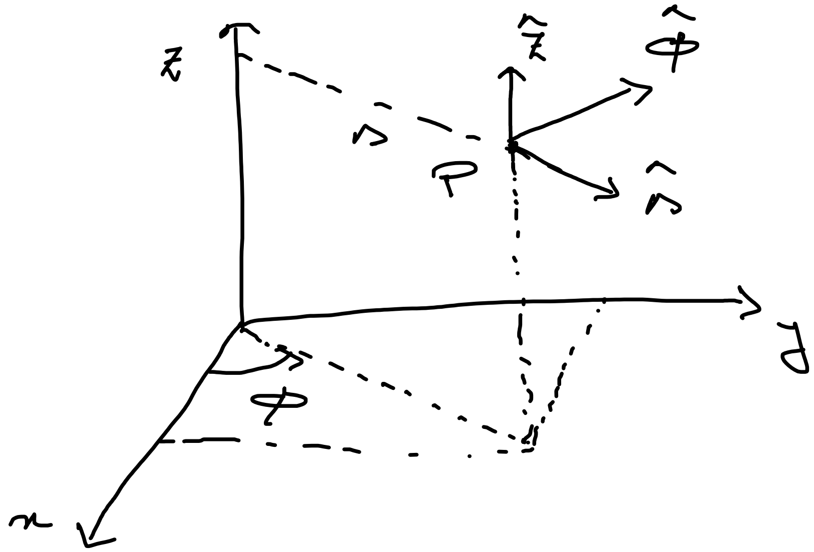
Cylindrical coordinates

24.11.20



$P(s, z, \phi)$

s → distance from z-axis.

z → same as Cartesian

ϕ → Azimuthal angle

$$\left.\begin{aligned} x &= s\cos\phi \\ y &= s\sin\phi \\ z &= z \end{aligned}\right\} \quad \begin{aligned} \hat{s} &= \cos\phi\,\hat{x} + \sin\phi\,\hat{y} \\ \hat{\phi} &= -\sin\phi\,\hat{x} + \cos\phi\,\hat{y} \\ \hat{z} &= \hat{z} \end{aligned}$$

Infinitesimal displacements:

$$\boxed{\begin{aligned} \vec{r} &= A_s\hat{s} + A_\phi\hat{\phi} + A_z\hat{z} \\ &= s\cos\phi\,\hat{x} + s\sin\phi\,\hat{y} + z\,\hat{z} \end{aligned}}$$

$$\left.\begin{aligned} dl_s &= ds \\ dl_\phi &= s\,d\phi \\ dl_z &= dz \end{aligned}\right\} \quad d\vec{l} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}$$

In general, s from $0 \to \infty$

ϕ ... $0 \to 2\pi$

z ... $-\infty \to +\infty$

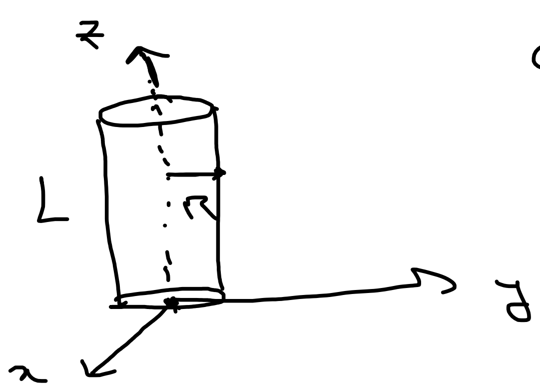
Define volume element:

$$d\tau = s\,ds\,d\phi\,dz$$

$V = \pi r^2 l$

l → height

r → radius.



$$\left.\begin{aligned} s &\equiv (0, R) \\ \phi &\equiv (0, 2\pi) \\ z &\equiv (0, L) \end{aligned}\right\} \quad V = \int_0^R\int_0^{2\pi}\int_0^L s\,ds\,d\phi\,dz$$

$$= \pi R^2 L$$

The Electric Field:

* Electric charge is an intrinsic property
  ↳ determines how a particle is going to interact with electromagnetic force

* Electric charge is a real number
  ↳ can be positive/negative/zero

* Charge is quantised; charge of a particle is a multiple of charge carried by an electron. $|e| = 1.602.. \times 10^{-19}\ C$
  (charge $= -e$)

  → usually the multiple is an integer
  $2|e|$, $10|e|$, etc.
  (Quarks $-2/3\,e$, $\frac{1}{3}e$)

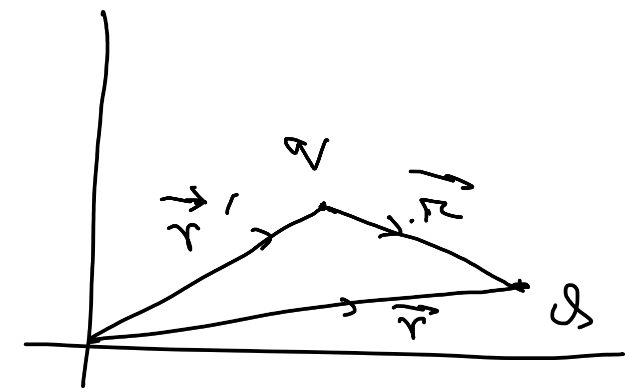
$$q = ne$$

Electrostatics = All charges are stationary

Coulomb's law:

Force on a test charge (Q) due to a single point charge (q) at rest and at a distance ($\mathscr{r}$)

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}'$$



$$\vec{F} = \frac{1}{4\pi\epsilon_0} \frac{qQ}{\mathscr{r}^2} \hat{\mathscr{r}}$$

↳ Permittivity of free space (characteristic strength of electric interaction)

SI unit ≡ $\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}$

(*) Force can be attractive / repulsive depending on the charges.

(*) Say, we have several point charges $q_1, q_2, \ldots q_n$ at distances $\mathscr{r}_1, \mathscr{r}_2, \ldots, \mathscr{r}_n$ from Q

Total force on q:

$$\vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots \vec{F}_n$$

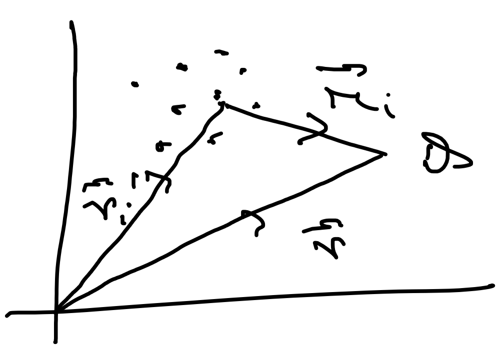
$$= \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q}{r_1^2}\hat{r}_1 + \frac{q_2 q}{r_2^2}\hat{r}_2 + \cdots\cdots\right]$$

$$= \frac{q}{4\pi\epsilon_0}\left[\frac{q_1}{r_1^2}\hat{r}_1 + \frac{q_2}{r_2^2}\hat{r}_2 + \cdots\right]$$

$$= q\vec{E}$$

Where, $$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i^2}\hat{r}_i$$

↳ Electric field of source charges.



⊛ Depends on position ($\vec{r}$) of field point

⊛ Independent of test charge q.