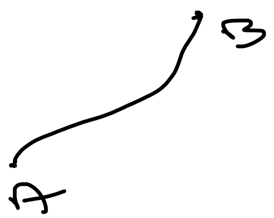
17.11.20

Line integral example:

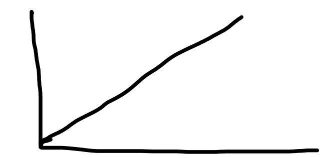
$$\vec{v} = x^2 y\,\hat{x} + y^2 x\,\hat{y} + xyz\,\hat{z}$$

↳ Do the line integral over path $x = y$



$$\int_a^b \vec{v} \cdot d\vec{l}$$

↳ $dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$



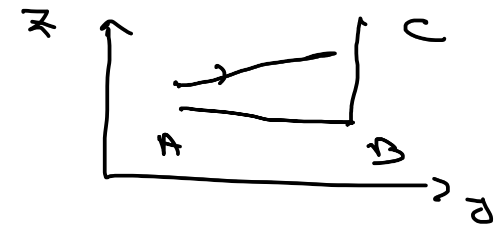
from $A(1, 1, 0)$ to $B(2, 2, 0)$

$$\vec{v} \cdot d\vec{l} = x^2 y\,dx + y^2 x\,dy + xyz\,dz$$

$$\int_1^2 x^2 y\,dx + \int_1^2 y^2 x\,dy$$

$\begin{cases} y = x \\ dy = dx \end{cases}$

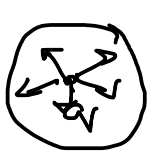
$=$



$\begin{cases} d\vec{l} = \hat{y}\,dy \\ d\vec{l} = \hat{z}\,dz \\ d\vec{l} = \hat{y}\,dy + \hat{z}\,dz \end{cases}$

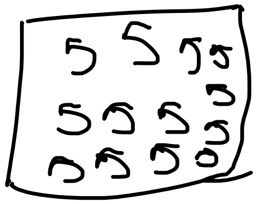
Divergence Th:

$$\int_V (\vec{\nabla} \cdot \vec{v})\,d\tau = \oint_S \underbrace{\vec{v} \cdot d\vec{a}}_{\text{flux}}$$

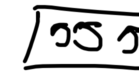


$\vec{E}$

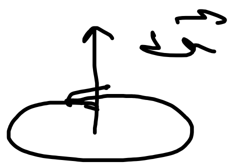
Stoke's Th:

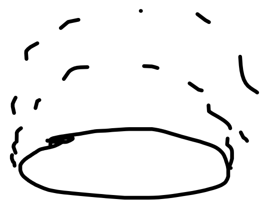


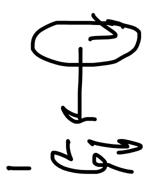
$$\int_S (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = \oint_P \vec{v}\cdot d\vec{l}$$

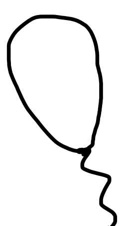


Total amount of swirl → can also be determined by just going around the edge









closed

(*) $\int (\vec{\nabla}\times\vec{v})\cdot d\vec{a}$ depends only on boundary line and independent of the shape of the surface

(*) $\oint (\vec{\nabla}\times\vec{v})\cdot d\vec{a} = 0$ for any closed surface since the boundary shrinks to a point.

For visual demonstration, go to the following link: https://mathinsight.org/stokes_theorem_idea#:~:text=(Recall%20that%20a%20surface%20integral,field%20perpendicular%20to%20the%20surface.)&text=Stokes%20theorem%20says%20the%20surface,where%20C%3D%E2%88%82S%20)

Spherical coordinates:

cartesian $\equiv (x, y, z)$

Spherical $\equiv (r, \theta, \phi)$
(polar)

$r \rightarrow$ distance from origin

$\theta \rightarrow$ angle from z-axis (polar angle)

$\phi \rightarrow$ angle around from x-axis

(azimuthal angle)