Tutorial Set-1 (EPHY105L)

1. Find the vector projection of $\vec{u} = 3\hat{x} - 4\hat{y}$ onto $\vec{v} = \hat{x} + \hat{y} + \hat{z}$.

2. Three vertices of a triangle are located at A(6,-1,2), B(-2,3,-4) and C(-3,1,5). Find
   (a) $\vec{R}_{AB}$ and $\vec{R}_{AC}$
   (b) The angle θ_{BAC} at vertex A
   (c) The vector projection of $\vec{R}_{AB}$ on $\vec{R}_{AC}$.

3. Find the area of a parallelogram determined by the vectors $\vec{a} = \hat{x} + 3\hat{y}$ and $\vec{b} = \hat{x} - 3\hat{y}$.

4. Find the volume of a parallelopiped generated by the vectors $\vec{u} = \hat{x} + 3\hat{y}$, $\vec{v} = \hat{x} - 3\hat{y}$ and $\vec{w} = -\hat{x} - \hat{y} - \hat{z}$.

5. Find the vector normal to the plane than contains the pointsi P(1,0,0), Q(1,2,3) and R(2,2,2).

6. Calculate the divergence of $\frac{\vec{r}}{r^3}$, where, $r = |\vec{r}|$ and $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$.

7. Find divergence of vector $\vec{D} = (2xyz - y^2)\hat{x} + (x^2z - 2xy)\hat{y} + x^2y\hat{z}$ at point P(2,3,-1).

8. Calculate the curl of $\vec{F}(x,y,z) = x^2\hat{x} + xyz\hat{y} - z\hat{z}$ at the point (2,1,-2).

9. Check that the following identity is correct: $\vec{\nabla}(\vec{A}.\vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A}.\vec{\nabla})\vec{B} + (\vec{B}.\vec{\nabla})\vec{A}$.

10. Find $\vec{\nabla}(\frac{f}{g})$, $\vec{\nabla}.(\frac{\vec{A}}{g})$ and $\vec{\nabla} \times (\frac{\vec{A}}{g})$. Here f and g are scalars.

11. Find $\vec{\nabla}.(\vec{\nabla} \times (\vec{\nabla}f))$ for $f(x,y,z) = x^3 + y^2 + z$.

Answer

1. $-\frac{1}{3}\hat{x} - \frac{1}{3}\hat{y} - \frac{1}{3}\hat{z}$

2. (a) $-8\hat{x} + 4\hat{y} - 6\hat{z}$, $-9\hat{x} + 2\hat{y} + 3\hat{z}$
   (b) $53.6°$
   (c) $-5.94\hat{x} + 1.319\hat{y} + 1.979\hat{z}$

3. 6

4. 6

5. $-2\hat{x} + 3\hat{y} - 2\hat{z}$

6. 0

7. -10

8. -10

9. Break into components and check

10. Find the expressions from Griffiths' book.

11. 0