Department of Mathematics, Bennett University
Engineering Calculus (EMAT101L)
Tutorial Sheet 2

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1. Find the limit of the following sequences.
   (a) $a_n = \frac{4}{1+n^2}$ (b) $a_n = (-1)^n \left(\frac{2}{n+2}\right)$ (c) $a_n = \frac{n+1}{2n+3}$

2. Examine whether the following sequences are convergent. Also, determine their limits if they are convergent.
   (a) $a_n = \frac{1}{n}\sin^2 n \ \forall\ n \in \mathbb{N}$.
   (b) $a_n = \frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \cdots + \frac{1}{(n+n)^2} \ \forall\ n \in \mathbb{N}$.
   (c) $a_n = \frac{n}{n^3+1} + \frac{2n}{n^3+2} + \cdots + \frac{n^2}{n^3+n} \ \forall\ n \in \mathbb{N}$.
   (d) $a_n = \sqrt{4n^2+n} - 2n \ \forall\ n \in \mathbb{N}$.
   (e) $a_n = \sqrt{n^2+n} - \sqrt{n^2+1} \ \forall\ n \in \mathbb{N}$.

3. Examine the convergence of the following sequences using Monotone Convergence Theorem.
   (a) $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{n+n} \ \forall\ n \in \mathbb{N}$.
   (b) $a_1 = 1$ and $a_{n+1} = 1 + \sqrt{a_n} \ \forall\ n \in \mathbb{N}$.

4. Discuss the convergence of the following sequences. Also, find their limits if they are convergent.
   (i) $a_n = \dfrac{n^k}{\alpha^n}$, where $|\alpha| > 1$ and $k > 0$. Ratio test
   (ii) $a_n = \dfrac{m(m-1)(m-2)....(m-n+1)}{n!}x^n$, where $|x| < 1$ and m is a fixed positive integer.

5. State whether the following statements are true/false. Give proper justifications.
   (a) A sequence can have exactly two limits. 0
   (b) A sequence must have at least one limit. 0
   (c) A bounded sequence must have a limit. 0 (must also be monotone)
   (d) An unbounded sequence will never have a limit. 1
   (e) A monotone sequence must have a limit. 0
   (f) A bounded monotone sequence must have a limit. 1