Tutorial Sheet 9

Derivative of a Function of Several Variables

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1. Find the first order partial derivatives of the following functions at the given points

   $(a)\ f(x,y) = (x^2+xy)^3\,,$ at $(1,0)$ $\qquad$ $(b)\ g(x,y) = \left(\dfrac{x^2+y^2}{xy}\right),$ at $(\sqrt{2},\sqrt{2})$.

2. Prove that if f is a function such that $f_x(x,y) = f_y(x,y) = 0$, for all (x,y), then $f(x,y)$ is a constant function.

3. Discuss the differentiability of the following functions at $(0,0)$.

   $(a)\ f(x,y) = \begin{cases} x\sin\dfrac{1}{x} + y\sin\dfrac{1}{y}, & xy \neq 0 \\ 0, & xy = 0 \end{cases}$ $\qquad$ $(b)\ g(x,y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}, & x^2+y^2 \neq 0 \\ 0, & x = y = 0 \end{cases}$

4. Let $f(x,y) = \Big|\,|x|-|y|\,\Big| - |x| - |y|$. Is f continuous at $(0,0)$? Which directional derivatives of f exist at $(0,0)$? Is f differentiable at $(0,0)$? Give reasons.

5. If $z = x^5e^{9y}$, then find the value of dz.

6. Find the total differential of $z = x^3y + xy$ at the point $(1,2)$.

7. If $z = x^3y + xy + 4$ and $x = \cos t,\ y = \sin 2t$, then compute $\frac{dz}{dt}$ and evaluate it at $t = \frac{\pi}{4}$.

8. Find the direction where the directional derivative is greatest for the function $f(x,y) = 3x^2y^2 - x^4 - y^4$ at the point $(1,2)$.

9. Let $f(x,y) = \dfrac{1}{2}\ln(x^2+y^2) + tan^{-1}\left(\dfrac{y}{x}\right),\ P = (1,3)$. Find the direction in which $f(x,y)$ is increasing the fastest at P. Find the derivative of $f(x,y)$ in this direction.

10. Let $\sin(xyz) = x + 3z + y$. Then find $\dfrac{\partial z}{\partial x}$.

11. Let $f(x,y) = \sqrt{x}\sin y$. Find the approximate value of $f(4.1, 0.2)$.

12. Given that $f(2,-3) = 6, f_x(2,-3) = 1.3$ and $f_y(2,-3) = -0.6$. Approximate the value of $f(2.1,-3.03)$.

13. An ant travels along a path on a surface. The exact path and surface are not known, but at time $t = t_0$, it is known that :

$$\frac{\partial z}{\partial x} = -7, \ \frac{\partial z}{\partial y} = 2, \ \frac{dx}{dt} = -1 \quad \text{and} \quad \frac{dy}{dt} = 3.$$

Find $\dfrac{dz}{dt}$ at time t_0.

14. Let $\sin(xy) + y^2 + x = 5$. Find $\dfrac{dy}{dx}$.

15. Find the equation of the plane tangent to the ellipsoid $\dfrac{x^2}{16} + \dfrac{y^2}{8} + \dfrac{z^2}{4} = 1$ at $P = (2, 2, 1)$.

16. Find the tangent plane and normal line to $x^2 + y^2 + z^2 = 10$ at the point $(1, 0, 3)$.