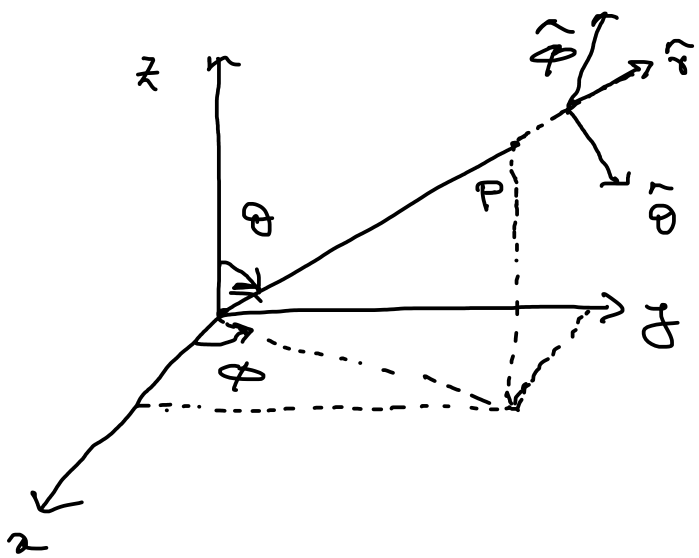
Spherical coordinates:

19.11.20



(r, θ, ϕ)

$$\begin{cases} x = r\sin\theta\cos\phi \\ y = r\sin\theta\sin\phi \\ z = r\cos\theta \end{cases}$$

unit vectors: $\hat{r}$, $\hat{\theta}$, $\hat{\phi}$

(*) Any vector: $\vec{A} = \underbrace{A_r}_{\text{radial comp.}} \hat{r} + \underbrace{A_\theta}_{\text{polar comp.}} \hat{\theta} + \underbrace{A_\phi}_{\text{azimuthal comp.}} \hat{\phi}$

What the unit vectors look like in terms of unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$= r\sin\theta\cos\phi\,\hat{x} + r\sin\theta\sin\phi\,\hat{y} + r\cos\theta\,\hat{z}$$

($\hat{r}$) $\quad d\vec{r} = \frac{\partial}{\partial r}(\vec{r})\, dr$

↳ vector pointing in r

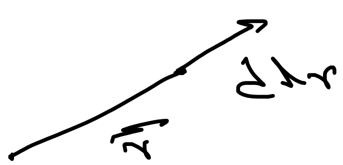
$$\hat{r} = \frac{\frac{\partial \vec{r}}{\partial r}}{\left|\frac{\partial \vec{r}}{\partial r}\right|}, \quad \hat{\theta} = \frac{\frac{\partial \vec{r}}{\partial \theta}}{\left|\frac{\partial \vec{r}}{\partial \theta}\right|}, \quad \hat{\phi} = \frac{\frac{\partial \vec{r}}{\partial \phi}}{\left|\frac{\partial \vec{r}}{\partial \phi}\right|}$$

$$\hat{r} = \sin\theta\cos\phi\, \hat{x} + \sin\theta\sin\phi\, \hat{y} + \cos\theta\, \hat{z}$$

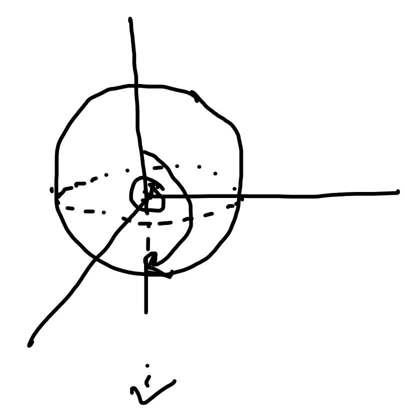
$$\hat{\theta} = \cos\theta\cos\phi\, \hat{x} + \cos\theta\sin\phi\, \hat{y} - \sin\theta\, \hat{z}$$

$$\hat{\phi} = -\sin\phi\, \hat{x} + \cos\phi\, \hat{y}$$

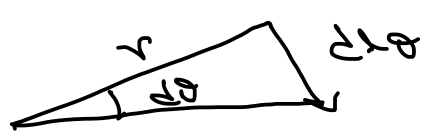
<u>Infinitesimal displacements:</u>

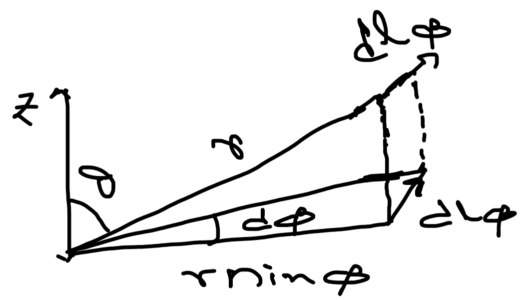
(r) 

$dl_r = dr$



$dl_\theta = r\, d\theta$

(θ) 

(ϕ) 

$dl_\phi = r\sin\theta\, d\phi$

- Displacement vector:

$$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$$

- Infinitesimal volume element:

$$d\tau = dl_r\,dl_\theta\,dl_\phi$$

$\boxed{d\tau = dx\,dy\,dz}$

Ranges

$r = (0 \to \infty)$

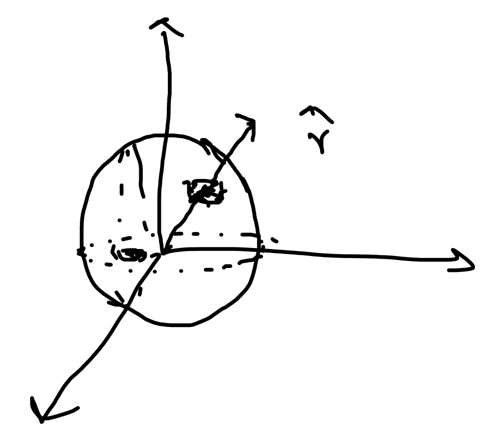
$\theta = (0 \to \pi)$

$\phi = (0 \to 2\pi)$

$$= r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$V = \int d\tau = \int_0^R \int_0^\pi \int_0^{2\pi} r^2 \sin\theta\,dr\,d\theta\,d\phi$$

$$= \frac{4}{3}\pi r^3$$



- Surface element $(d\vec{a})$

⊛ on surface of a sphere

$$d\vec{a}_1 = dl_\theta\,dl_\phi\,\hat{r}$$

$$= r^2 \sin\theta\,d\theta\,d\phi\,\hat{r}$$

$r = R$

⊛ surface lying on x, y plane.

$$d\vec{a}_2 = dl_r\,dl_\phi\,\hat{\theta}$$

$$= r\,dr\,d\phi\,\hat{\theta} \quad (\sin\theta = 1)$$

(*) Vector Derivatives:

Gradient $\vec{\nabla} T = \frac{\partial T}{\partial x}\hat{x} + \frac{\partial T}{\partial y}\hat{y} + \frac{\partial T}{\partial z}\hat{z}$

$$\frac{\partial T}{\partial x} = \frac{\partial T}{\partial r}\left(\frac{\partial r}{\partial x}\right) + \frac{\partial T}{\partial \theta}\left(\frac{\partial \theta}{\partial x}\right) + \frac{\partial T}{\partial \phi}\left(\frac{\partial \phi}{\partial x}\right)$$

$$\frac{\partial T}{\partial y} = \frac{\partial T}{\partial r}\left(\frac{\partial r}{\partial y}\right) + \cdot - - -$$

$$\frac{\partial T}{\partial z} = \cdot - \cdot$$

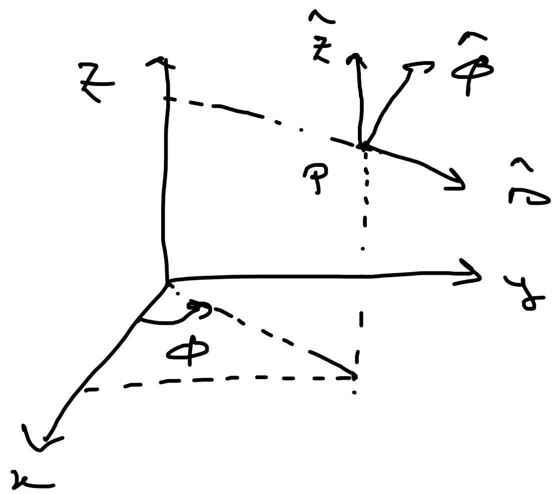
$$\vec{\nabla} T = \frac{\partial T}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial T}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\hat{\phi}$$

$$\vec{\nabla}\cdot\vec{v} = - -$$

$$\vec{\nabla}\times\vec{v} = \cdot\ \cdot$$

$$\nabla^2 T = \cdot\ \cdot\ \cdot$$

Cylindrical coordinates:



$P(r, \phi, z)$