A UNIVERSAL MODEL FOR ACCURATE CALCULATION OF ELECTROMAGNETIC TRANSIENTS ON OVERHEAD LINES AND UNDERGROUND CABLES

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Abstract – This paper presents a transmission line model for the simulation of electromagnetic transients in power systems. The model can be applied to both overhead lines and cables, even in the presence of a strongly frequency dependent transformation matrix and widely different modal time delays. This has been achieved through a phase domain formulation where the modal characteristics have been utilized in the approximation for the propagation matrix. High computational efficiency is achieved by grouping modes with nearly equal velocities, and by columnwise realization of the matrices for propagation and characteristic admittance.

1 INTRODUCTION

A transmission line can be characterized by two matrix transfer functions : the propagation H and the characteristic admittance Y_c. These are frequency dependent quantities and can in practice only be calculated as discrete functions in the frequency domain. A time domain simulation can be carried out using convolutions involving the time domain counterparts of H and Y_c, which are obtained via an inverse Fourier transform. Numerical convolution can always be used [1], but this procedure is very time consuming. A much more efficient formulation is achieved if H and Y_c are replaced by low order rational function approximations in the frequency domain, as this permits a recursive implementation of the convolutions [2]. A critical part in transmission line modeling is therefore the accurate fitting of H and Y_c with rational functions.

The elements of Y_c are in general very smooth functions of frequency and can therefore easily be fitted. The fitting of H is more difficult because its elements are composed of modal components which in general have different time delays. In the case of overhead lines, the modal time delays are not very different, which makes it possible to take these into account by a common factor $\exp(-j\omega\tau)$, corresponding to the fastest mode. This procedure was used for the phase domain models in [3-5].

However, in the case of underground cables the modal time delays may be widely different. Compensating for a common time delay only will not permit a low order rational function fitting for H, due to the uncompensated part of the time delays. This problem can be avoided by using modal decomposition [6], because a separate time delay can then be assigned to each mode. However, this approach is made complicated by the frequency dependency of the transformation matrix T, which has to be taken into account. Although the elements of T can be fitted for most cable systems of interest [7], this in general is not possible for overhead lines [8].

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T. Noda et al. [9] recently introduced an interesting solution : They expressed H as a sum of modal contributions directly in the phase domain, which made it possible to assign a separate time delay for each mode. However, as a z-transform approach was used, the resulting model is dependent on Δt and is not directly applicable for an arbitrary time step [10].

This paper introduces a transmission line model applicable to all line configurations. Also, the resulting model is independent of Δt as the fitting is done in the frequency domain. The propagation matrix H is first fitted in the modal domain. The resulting poles and time delays are then used for fitting H in the phase domain, under the assumption that all poles contribute to all elements of H. The unknown residues are then calculated by solving an overdetermined linear equation as a least squares problem. As all elements in H get identical poles, a columnwise realization [8] can be used, which gives increased computational efficiency for the time domain simulation. Further increase in efficiency is achieved by lumping modal contributions with equal or nearly equal time delays, as has previously been done in a z-domain approach [9]. The characteristic admittance Y_c is fitted columnwise directly in the phase domain.

Application of the line model is demonstrated for underground single core and pipe-type cables, as well as for overhead lines.

2 APPROACH

2.1 Model formulation

The voltage/current relationship at the ends of a transmission line is given by

$$Y_c\, v - i = 2i_i = 2H\, i_{far} \tag{1}$$

where all quantities are in the frequency domain. i_i is the incident current wave and i_{far} is the reflected current wave from the opposite line end. The matrices for propagation H and characteristic admittance Y_c can be expressed as

$$H = \exp(-\sqrt{YZ}\,l) \tag{2}$$

$$Y_c = Z^{-1}\sqrt{ZY} \tag{3}$$

Z and Y are the series impedance and shunt admittance per unit length, and l is the length of the transmission line. For an n-conductor system, Z and Y are $n \times n$ matrices.

Using modal decomposition we can write for H :

$$H(\omega) = T\, e^{-\Lambda(\omega)l}\, T^{-1} = \sum_{k=1}^{n} \Gamma_k(\omega)\, e^{-\lambda_k(\omega)l} = \sum_{k=1}^{n} H'_k(\omega)\, e^{-j\omega\tau_k} \tag{4}$$

where Λ is diagonal, and T is the eigenvector matrix. Γ_k is the square matrix which results from multiplying the kth column of T with the kth row of T^{-1}. Introducing (4) in (1) and using convolutions gives the time domain solution :

$$Y_c * V - i = 2\sum_{k=1}^{n} H'_k(t-\tau_k) * i_{far} \tag{5}$$

2.2 Rational function approximation

2.2.1 Approximation of H

Equation (4) showed that each modal contribution to H can be expressed as

$$H_k(\omega) = \Gamma_k(\omega)\, e^{-\lambda_k(\omega) l} = \Gamma_k(\omega)\, e^{-\lambda'_k(\omega) l}\, e^{-j\omega\tau_k} \qquad (6)$$

In principle, one could find an approximation for H by fitting each factor Γ_k and $\exp(-\lambda'_k l)$ separately. The poles of the model should then include the poles of all factors. However, the frequency variation in Γ_k is always accompanied by a frequency variation in $\exp(-\lambda'_k l)$, and we have found that it is possible to obtain an accurate fitting using only the poles of the factors $\exp(-\lambda'_k l)$. Thus, we postulate that *H can be accurately fitted using only the poles from the modes.*

The fitting technique in [8] is used to calculate a rational function approximation for the factors $\exp(-\lambda'_k l)$, as well as the time delays τ_k. The poles of the fitted functions and the time delays are then used in the phase domain fitting of H. At each frequency point ω_1, H can be written as :

$$H_{ij}(j\omega_1) \approx \sum_{k=1}^{n} \Big[\sum_{m=1}^{N_k} \frac{c_{mk\,ij}}{j\omega_1 - p_{mk}} \Big] e^{-j\omega_1\tau_k} \qquad (7)$$

where N_k is the number of poles for mode k. Writing (7) for several frequencies gives an overdetermined linear matrix equation of the form

$$AX = B \qquad (8)$$

where the unknown residues (c's) are in X. Each row in A and B corresponds to a frequency point, and each column in X and B corresponds to an element of H. Equation (8) is solved as a least squares problem. It should be noted that all poles p_{mk} are present in all elements (i,j) of the fitted H-matrix. This permits a columnwise realization for H, which leads to computational savings in the time domain.

Experience with the fitting method has shown that the eigenvalues Λ of H can be calculated with sufficient accuracy using a constant, real transformation matrix evaluated at a high frequency (e.g. 1MHz). The resulting eigenvalues will differ slightly from the accurate ones, which results in slightly different poles when fitting $\exp(-\lambda'_k l)$. However, this of minor importance for the end result as small displacements of the poles become compensated for by small changes in the residues when solving (8). The advantage of using a constant transformation matrix is that one eliminates the need for a "tracking routine" [11] or special diagonalization routines [7], [11] to avoid the problem of artificial mode switchovers.

2.2.2 Approximation of Y_c

The columns of Y_c are fitted directly in the phase domain, as described in [5].

2.3 Time domain implementation

The rational function approximation of both H and Y_c allows for a recursive convolution implementation in the time domain. The modal time delays in (5) will in general not be an integer of the time step used in the simulation. This problem was avoided by using linear interpolation of the vector i_{far}, as is done in EMTP.

3 CALCULATED RESULTS

The following examples demonstrate the applicability of the new transmission line model for different line geometries.

3.1 Coaxial cable system

Figure 1 shows data for a system of 3 single core coaxial cables. Also is shown the numbering of conductors used in the calculated examples. The cable length is 10 km.

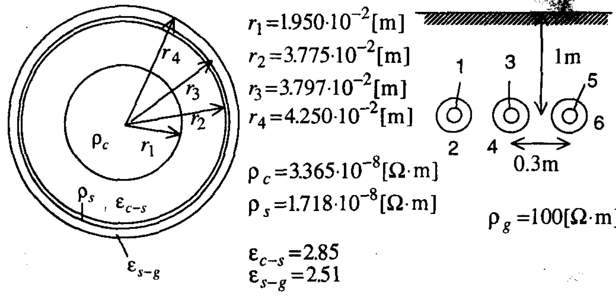

Fig. 1 66 kV single core coaxial cable system

A time delay τ_k was calculated for each of the six modes, and each mode $\exp(-\lambda'_k l)$ was fitted in the range 1Hz-1MHz using 6 poles. The resulting approximation is shown in figure 2.

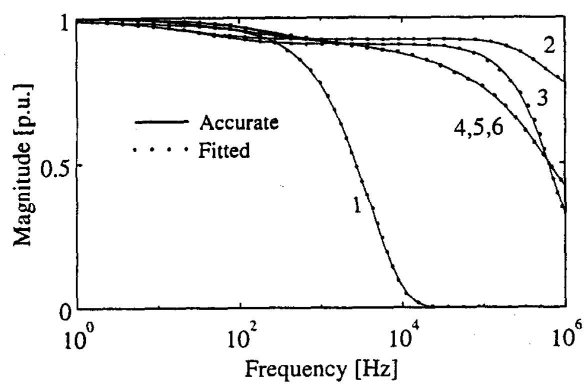

Fig. 2 Fitted modes $\exp(-\lambda'_k l)$ (6th order approximation)

Using the poles and time delays for the modal responses in figure 2, H was fitted directly in the phase domain using (7) and (8). Figures 3 - 5 compare the magnitude and phase angle of elements (2,2) and (1,4) of H with the exact functions.

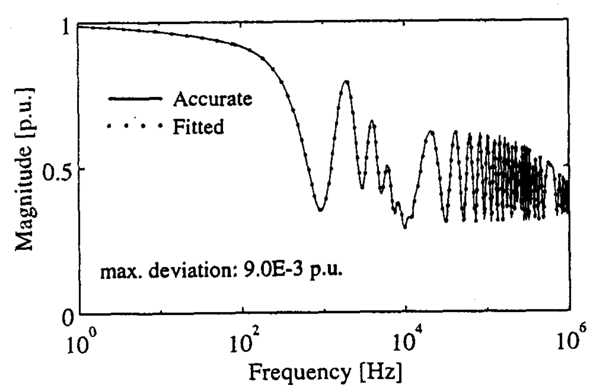

Fig. 3 Magnitude of element $H(2,2)$

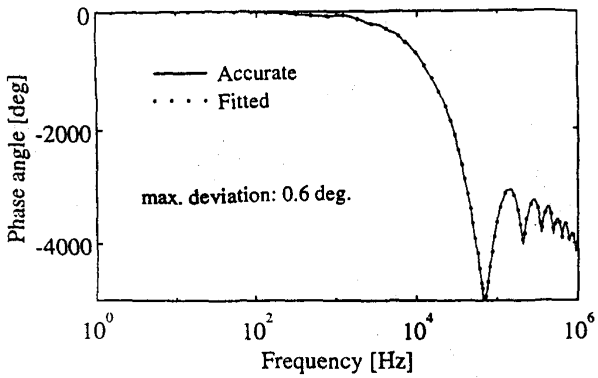

Fig. 4 Phase angle of element $H(2,2)$

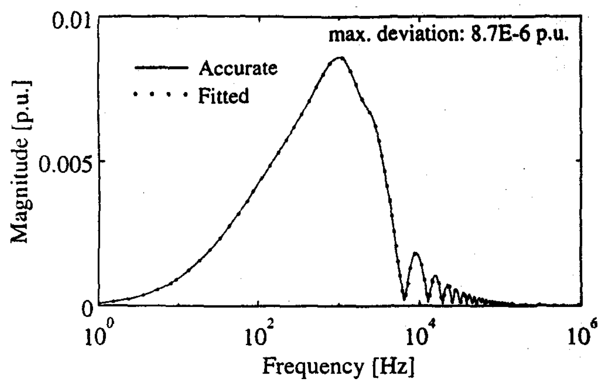

Fig. 5 Magnitude of element $H(1,4)$

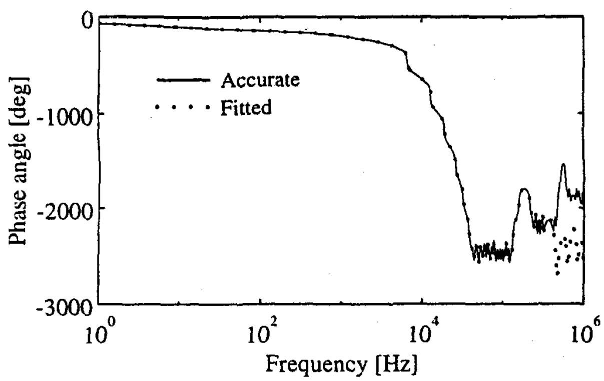

Fig. 6 Phase angle of element $H(1,4)$

The "oscillations" observed in the magnitude functions in figures 3 and 5 reflect contributions from modes having widely different time delays. Despite of this, the elements are seen to be *fitted to very high accuracy!* Similar accuracy was achieved for all 36 elements of H.

It is interesting to note that the accuracy of the fitted elements of H (figures 3-6) is significantly better than for the fitted modes $\exp(-\lambda_k' l)$ in figure 2, particularly at low frequencies. This result can be explained as follows : At low frequencies, the effect of the time delays in (7) is small ($\exp(-j\omega\tau_k) \approx 1$). Thus, while each of the modes in figure 2 is fitted with 6 poles, 36 poles are effectively used when fitting the phase domain functions. At high frequencies the time delay factors $\exp(-j\omega\tau_k)$ are very different but some redundancy still exists because several time delays are almost equal. Table 1 shows the time delays for the system.

Table 1 Time delays τ_k [μs]

663	261	212	56	56	56

The time delays for the 3 coaxial modes of propagation are virtually identical (56μs) as is also the their magnitude functions at high frequencies. In section 4 we will show how to take advantage of this to produce a much lower order model, which results in computational savings in the time domain simulation.

The characteristic admittance Y_c is easily fitted directly in the phase domain, as no time delays exist for this matrix. The elements of the first column is shown in figure 7, when using an eighth order approximation. The accuracy is seen to be very good.

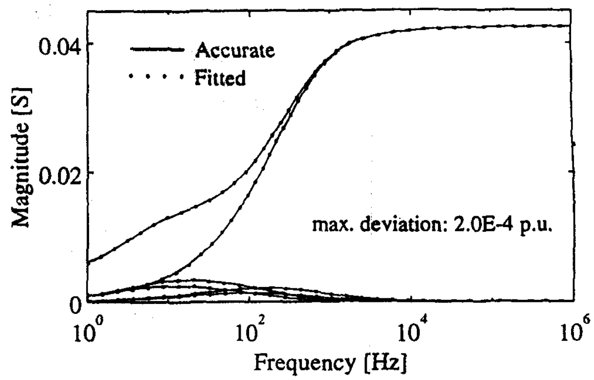

Fig. 7 First column of Y_c. (8th order approximation)

Based on the calculated rational function approximation for H and Y_c, a time domain model was developed, as explained in section 2.3. Figure 9 shows the simulated open end voltage at the core of cable b and at the sheath of cable c, when energizing the core of cable a with a step voltage (figure 8). The simulated voltages are seen to agree closely with those by a Fourier method.

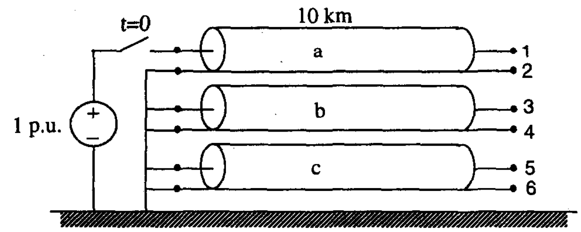

Fig. 8 Open circuit test

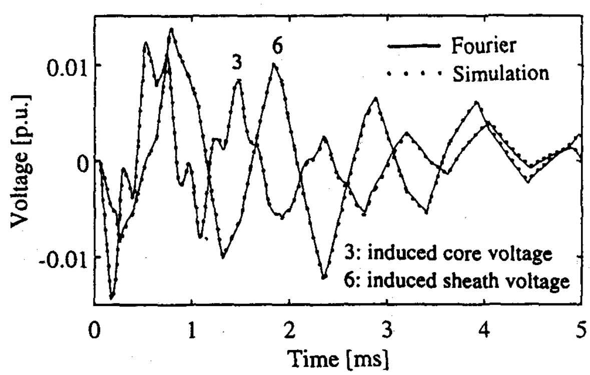

Fig. 9 Simulated open end step responses

The long term stability of the model under trapped charge conditions was tested by applying balanced voltage sources (50Hz) to the cores at the sending end of the cable. The sources were disconnected after 15ms, leaving the cores floating. Figure 10 shows the resulting core voltages at the receiving end. It is seen that the trapped charge voltage shows no sign of instability.

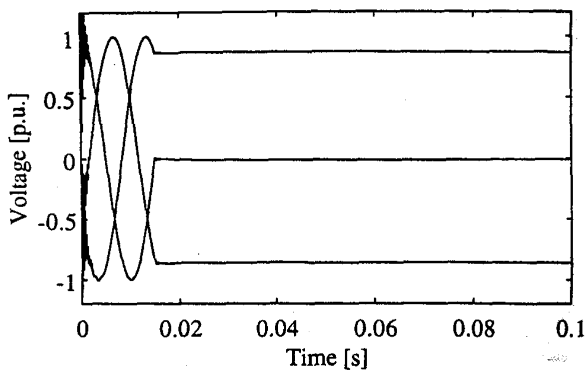

Fig. 10 Simulated trapped charge voltage

3.2 Pipe type cable

Figure 11 shows a pipe type cable consisting of three single core coaxial cables enclosed within an oil-filled plastic pipe. Each cable has skid wires which gives galvanic contact between the sheath conductors. Because the sheaths are at the same potential, we have only four independent modes of propagation.

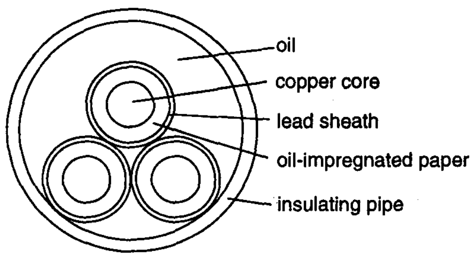

Fig. 11 115 kV pipe type cable.

Assuming a cable length of 10km, the modes of propagation $\exp(-\lambda_k' l)$ were fitted in the range 1Hz - 1MHz using 6 poles. The fitted modes are shown in figure 12.

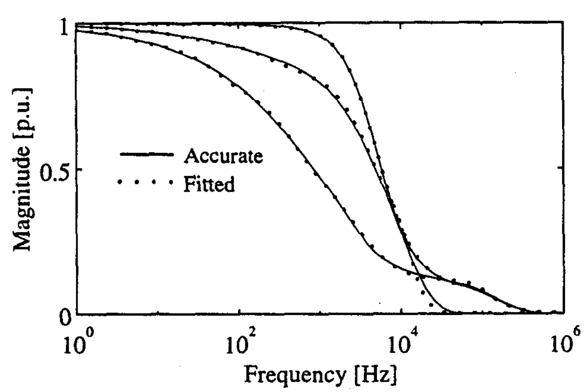

Fig. 12 Fitted modes $\exp(-\lambda_k' l)$ (6th order approximation)

Using the poles for $\exp(-\lambda_k' l)$ and the associated time delays, H was fitted in the phase domain using (7) and (8). The diagonal and off-diagonal elements of H is shown in figures 13 and 14, respectively. It is seen that the resulting fitting is very good. We also note that some elements display strong "oscillations" in magnitude, due to modes with widely different time delays (see table 2).

Table 2 Time delays τ_k [μs]

226.1	62.7	62.7	62.3

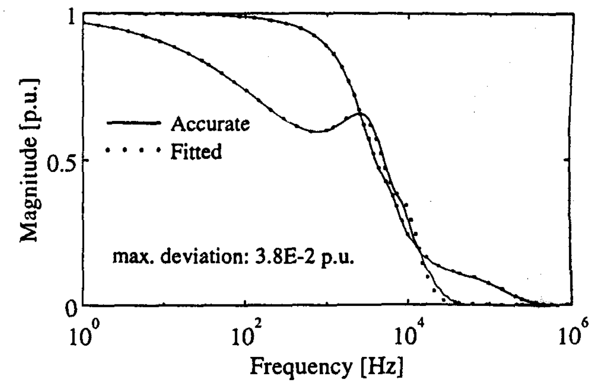

Fig. 13 Fitted diagonal elements of H (four elements)

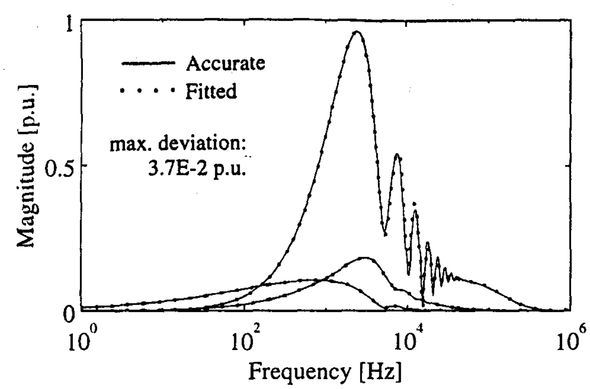

Fig. 14 Fitted off-diagonal elements of H (twelve elements)

Accurate results were also achieved when the plastic pipe was replaced by a steel pipe. In that case we have only 3 modes of propagation with nearly equal time delays.

4 MODAL GROUPING

4.1 Procedure

In the previous examples we have seen that in many instances, several of the modal time delays are nearly equal. From (5) it follows that the computational effort in the time domain due to H is proportional to the number of modes, n. However, modes with nearly equal time delays can be lumped together in order to increase the computational efficiency of the resulting line model.

Assume that the H-matrix of a two-conductor line is to be fitted up to a frequency Ω. Time delays τ_1 and τ_2 are first calculated. If the elements of H are multiplied by the factor $\exp(j\omega\tau_1)$, then the uncompensated part of τ_2 gives a phase shift $ang = \omega(\tau_1 - \tau_2) = \omega\Delta\tau$. Thus, it follows that the maximum uncompensated phase shift is equal to $\Omega\Delta\tau$. A practical approach is to assume that two modes may be compensated by the same factor $\exp(j\omega\tau)$ if $\Omega\Delta\tau$ is less than 10 degrees.

A practical approach is as follows :

1. Calculate the differences $\Delta\tau$ between the modal time delays of the line. Modes that fulfill the criterion

$$\Omega\Delta\tau < 2\pi \cdot 10/360 \tag{9}$$

are lumped together and compensated by a common time delay τ^*

2. The lumping is done by fitting the average of the modal terms $\exp(-\lambda_k l)$. The time delay τ^* for the group is selected equal to the smallest of the individual time delays.

This procedure may result in several modal groups for a line. The groups replace ordinary modes when fitting H (section 2.2.1).

4.2 Application to six-circuit overhead line

The significance of lumping modes increases with the number of modes (conductors) of the line. Figure 15 shows an example with 6 circuits (18 phase conductors) on the same right-of-way.

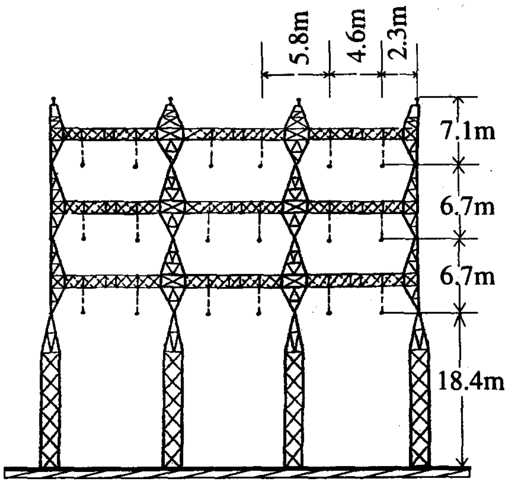

Fig. 15 230 kV six-circuit overhead line

For this line the $\Delta\tau$'s (relative to the smallest τ) were calculated to be :

Table 3 Difference between time delays : $(\tau - \tau_{\min})$ $[s]$

0	2.53E-10	4.60E-10	5.36E-10	9.53E-10	1.72E-7
2.06E-10	3.19E-10	4.88E-10	5.52E-10	1.38E-8	3.40E-6
2.36E-10	4.20E-10	5.10E-10	6.74E-10	1.70E-8	2.87E-5

Assuming we want to fit H up to 1MHz, it follows from (9) that the criterion is $\Delta\tau = 2.77E-8\,s$. Thus, the first 15 modes can be lumped together, reducing the total number of modes from 18 to 4. The lumped mode was very smooth and could easily be fitted.

The 4 modes were each approximated with a 7th order realization. Using the resulting poles and time delays, H was fitted in the phase domain by (7) and (8). Figure 16 shows the fitted elements of the first column of H. (Note that the magnitude of element (1,1) is outside the figure scale. Also, the phase angles have been backwound by a common time delay prior to plotting).

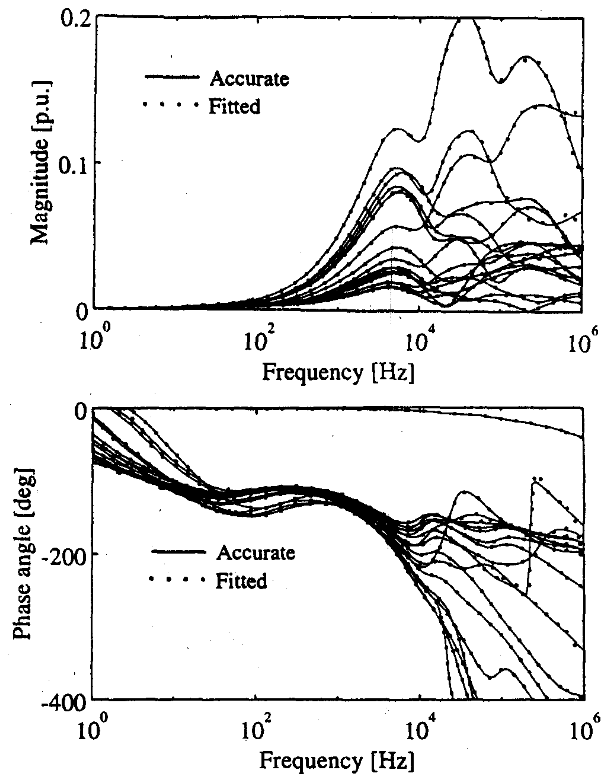

Fig. 16 First column of H. (Line length : 100km).

5 DISCUSSION

5.1 Model accuracy

The calculated results in sections 3 and 4 demonstrate that H can be accurately fitted directly in the phase domain, using precalculated poles and time delays from the modes. It was shown that the model gives highly accurate results for cable systems and overhead lines, also in the presence of widely different modal time delays and a strongly frequency dependent transformation matrix. The accuracy was demonstrated both in the frequency domain and in time domain simulations for the especially challenging case of underground cable sheath transients. A trapped voltage was simulated without instability. Accurate frequency domain results were also shown for a pipe type cable and the fitting method worked well even for an 18-conductor overhead line.

5.2 Model efficiency

Equation (5) suggests that the execution of the convolution for H involves n^3 scalar convolutions. However, the actual number of convolutions will in practice be at most $4n^2$, due to the technique of grouping. An additional n^2 convolutions result from Y_c, giving at most $5n^2$ convolutions per line end. For comparison, there are $(2n^2 + 2n)$ convolutions per line end in the L.Marti model [7]. However, the new model is expected to be at least as efficient because columnwise realization is used for both H and Y_c. This increases the efficiency by about a factor of 2 [8]. In addition, very high accuracy has been achieved using relatively few poles in comparison to currently available models. (In most calculated examples we used 6 poles per mode).

6 CONCLUSIONS

In this paper we have presented a new transmission line model based on fitting the matrices for propagation H and characteristic admittance Y_c in the phase domain. The main characteristics of the model are :

1. The fitting of H is based on poles and time delays obtained from the modes. With these quantities known, H is fitted directly in the phase domain by solving a linear matrix equation as a least squares problem.
2. The resulting model is fully general : it gives highly accurate results for both overhead lines and underground cables. It has no problem in the presence of a frequency dependent transformation matrix or widely different modal time delays. In the paper we have demonstrated the suitability of the model for coaxial type and pipe type cable systems, as well as for overhead lines.
3. The computational efficiency in the time domain simulations is high because columnwise realization for H is utilized, and because modes with nearly equal time delays are grouped together.
4. Very high accuracy can be achieved with a low number of poles per mode, as compared to currently used models.

7 ACKNOWLEDGMENTS

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9 BIOGRAPHIES

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Discussion

Adam Semlyen (University of Toronto): I would like to congratulate the authors for their interesting and useful paper. It presents a new methodology for the simulation of transients on transmission lines with strongly frequency dependent modal transformations that is both very accurate and computationally efficient. Because of these merits, it could well be used as a blueprint for practical EMTP implementation.

The basic underlying idea to the new approach is intuitively easy to accept: it is the fact that the set of partial fractions with real poles needed for fitting smooth transfer functions is not uniquely and precisely defined but can be modified somewhat without reducing the quality of the approximation. Conversely, a given set of poles can be used for the fitting of a slightly changed transfer function.

The authors have used this fact to fit the elements of the phase domain H matrix that result from the modal components modified by the frequency dependence of the transformation matrices, by using only the poles of the modal propagation transfer functions. This, together with combining modes having nearly equal delays into modal groups and a few other skillful procedures, has lead to the efficiency and accuracy of the new method. The idea of the non-uniqueness of poles is also the basis of the vector fitting used in the paper and documented in detail in [5] and [8].

The paper is very instructive by its presentation of some fundamental methods currently used in the simulation of transmission line transients. In relation to recursive convolutions, I would like to point out that this widely used approach, first applied to the calculation of transmission line transients in [2], is essentially, but not in all its details, equivalent to traditional, implicit numerical integration (see [2], where both approaches are discussed), and has no particular advantages. Therefore, state equation realization of the transfer functions together with numerical (e.g., trapezoidal) integration is preferable. Recursive convolutions are sometimes used in name only to designate a procedure where traditional convolutions are not used. In that sense, recursive convolutions can at present be viewed as obsolete. Which approach did the authors use in their work in the time domain implementation?

The readers of this paper may have the impression that the authors are attributing the initial use of modal decompositions to reference [6]. In fact, modes have been used in the context of transmission lines at least since 1963, see [A], and have widely been applied in computations ever since.

The views and answers of the authors would be highly appreciated.

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Atef Morched and Bjørn Gustavsen: The authors would like to thank Professor Semlyen for his interest in this paper and his kind remarks regarding the accuracy and efficiency of the proposed line model. These attributes are the main reasons that this model is being currently implemented in several electromagnetic transients programs. Professor Semlyen offered a number of questions and remarks for which we offer the following comments:

We agree with Professor Semlyen that pole identification processes based on optimization techniques will not produce unique poles for fitting the transmission transfer function.

However, we point out the fact that in the frequency range of interest, the number of poles of H in (2) are finite, and their location is uniquely defined. This is true even when the longitudinal impedance Z is frequency dependent. The better the fitting process, the closer the fitted poles to the actual poles. We still believe that only the poles of the modal propagation transfer functions are needed to characterize the behavior of the system. The frequency dependence of the transformation matrices is accounted for by adjusting the amount of contribution of each of these poles in the fitting of H. As shown in the paper, this seems to be enough to produce accurate fittings and results in lower order approximations.

We agree with Professor Semlyen that the use of recursive convolutions may not provide particular advantages over the use of the trapezoidal rule of integration in the evaluation of the line transients and that these methods can be applied alternately. This, however, was not the topic of the discussed paper.

There was no intention of detracting from the importance of Professor Wedepohl's pioneering work on the use of modal decomposition for transmission line solutions as outlined in the discussor's reference [A]. Any different impression we may have left the reader with is inadvertent and regrettable.

In conclusion, we thank Professor Semlyen again for his interest and thoughtful ideas and look forward to seeing this method widely used in transmission line transient calculations.