Robust Calculation of Frequency-Dependent Transmission-Line Transformation Matrices Using the Levenberg–Marquardt Method

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***Abstract*—This paper presents a new method for the calculation of smooth frequency-dependent transmission-line (TL) transformation matrices. The proposed method, based on the Levenberg–Marquardt algorithm, solves an equivalent real-valued approach of the generalized complex eigenproblem. The implemented formulation incorporates a robust convergence criterion and is applicable to all TL configurations, due to the included numerically well-defined computational scheme. Smooth modal transformation matrices are calculated for overhead and underground TL configurations under different representations of the imperfect earth. Results are compared and validated with the corresponding results obtained from the Newton–Raphson and the sequential quadratic programming methods, revealing the accuracy, efficiency, and robustness of the proposed formulation, even in cases where the other methods fail.**

***Index Terms*—Eigenproblem, Levenberg–Marquardt, matrix perturbation theory, modal transformation matrices, Newton–Raphson, sequential quadratic programming.**

I. Introduction

The majority of the modal- [1], [2] and phase- [3]–[6] domain transmission-line (TL) transient models are based on the modal decomposition theory [7], taking into account the frequency dependence of the modal transformation matrix. A key requirement for the accurate implementation of modal- and phase-domain models is the derivation of frequency-dependent transformation matrices as smooth functions of frequency, that is, without discontinuities and swaps between the elements in the frequency domain. An attempt to calculate the eigenvectors with conventional algorithms, such as the QR method [8], [9], results in "switchovers" in the frequency domain [10].

Several methods have been proposed in order to obtain smooth TL modal transformation matrices. They can be divided into two main categories, namely, 1) the postprocessing correlation and 2) the direct calculation methods [11].

In the postprocessing correlation category [11]–[13], the modal transformation matrices are initially calculated using a conventional algorithm, ignoring the possible switchovers between the columns in the frequency domain. Next, a correlation technique, based on the inner product between the eigenvectors of two subsequent frequencies, is used to track the correct order of the eigenvectors throughout the examined frequency range. Although this two-step procedure is straightforward, the implementation can be cumbersome in problems, including coalescing eigenvalues, such as the case of coaxial modes in polyphase cable systems at high frequencies.

In the direct calculation category, the eigenvectors are calculated in a one-step procedure, using an iterative method with a starting point. The success in obtaining smooth functions of frequency depends strongly on the selection of this starting point, which is readily taken from the final solution at the preceding frequency. The first method of this category was proposed in [1], based on a modified version of the Jacobi eigenvalue algorithm. However, this method is restricted to underground cable systems with inaccurate results obtained for multicircuit overhead transmission lines (OHTLs). Another method, based on the Newton–Raphson (NR) algorithm and suitable for OHTLs and underground cables, is proposed in [13] solving a set of nonlinear equations. Despite the computational efficiency of this method, the calculation routine is strongly dependent on the quadratic convergence assumption of the NR algorithm. Finally, a method formulated into a constrained minimization problem and based on the sequential quadratic programming (SQP) algorithm has been proposed in [14], permitting the use of a larger interval between the subsequent frequencies, but also presenting a slower convergence rate and possible numerical instabilities in the calculation routine.

In this paper, a new method is presented for obtaining smooth TL modal transformation matrices in the frequency domain. The proposed method is a direct calculation method and uses the Levenberg–Marquardt (LM) algorithm [15], [16] in order to solve a nonlinear real-valued formulation of the eigenvalue problem. Therefore, it can be considered as an extension of the NR method, but with numerical advantages similar to those of the SQP method. Specifically, the proposed method incorporates a robust convergence criterion which guarantees the decrease of the equation function in each iteration. Moreover, a positive definite approximation of the Hessian matrix is used, without the requirement of a nonsingular Jacobian matrix. With the aforementioned advantages, the use of a larger interval between the subsequent frequencies is allowed, significantly reducing the total execution time. Considering the convergence

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rate, the proposed method is slightly slower than the NR method but faster than the SQP method.

The performance of the proposed method is demonstrated in TL configurations located in the air, on the ground surface, and underground, assuming different representations of the imperfect earth [17]–[19], whereas a thorough investigation on its algorithm tracking ability is also conducted. The results reveal the accuracy and robustness of the proposed method in all examined cases, contrary to the corresponding results obtained from the NR and SQP methods.

II. Proposed Method

A. Complex Eigenvalue Problem

The eigenproblem on an N-phase arbitrary overhead or underground TL is considered. Let $\boldsymbol{Z}'$ and $\boldsymbol{Y}'$ be the $N \times N$ per-unit-length (pul) series impedance and shunt admittance matrices in the frequency domain [20], assuming any homogeneous [17]–[19] or stratified earth formulation [21], [22]. At each frequency, the matrix product $\boldsymbol{Y}' \cdot \boldsymbol{Z}'$ is scaled in order to avoid floating-point overflows; thus, the normalized matrix product $\boldsymbol{S}$ is formulated [23]

$$\boldsymbol{S} = \frac{\boldsymbol{Y}' \cdot \boldsymbol{Z}'}{-\omega^2 \mu_0 \varepsilon_0} - \boldsymbol{I} \tag{1}$$

where $\boldsymbol{I}$ is the $N \times N$ unit matrix, ω is the angular frequency at the examined frequency, and ε_0, μ_0 are the permittivity and permeability of the air, respectively.

The matrix product $\boldsymbol{S}$ can be decomposed into N-normalized eigenvalues $\bar{\lambda}$ and N eigenvectors $\boldsymbol{t}$ of order $N \times 1$, which synthesize the columns of the $N \times N$ current modal transformation matrix $\boldsymbol{T}_I$. Each eigenvalue/eigenvector pair satisfies (2), forming the generalized complex-valued eigenproblem.

$$\boldsymbol{S} \cdot \boldsymbol{t} - \bar{\lambda} \cdot \boldsymbol{t} = \boldsymbol{0}. \tag{2}$$

B. Equivalent Real-Valued Formulation

The eigenproblem can be transformed into an explicitly real-valued formulation with simple matrix perturbation, by separating the real (re) and imaginary (im) parts. Therefore, $\boldsymbol{S}, \boldsymbol{t}$ and $\bar{\lambda}$ are rewritten as $\boldsymbol{S}_C, \boldsymbol{t}_C$ and $\bar{\boldsymbol{\lambda}}_C$ in (3)–(5), respectively, and the eigenproblem takes the equivalent matrix form of (6)

$$\boldsymbol{S}_C = \begin{bmatrix} \boldsymbol{S}_{\mathrm{re}} & -\boldsymbol{S}_{\mathrm{im}} \\ \boldsymbol{S}_{\mathrm{im}} & \boldsymbol{S}_{\mathrm{re}} \end{bmatrix} \tag{3}$$

$$\boldsymbol{t}_C = [\boldsymbol{t}_{\mathrm{re}} \quad \boldsymbol{t}_{\mathrm{im}}]^T \tag{4}$$

$$\bar{\boldsymbol{\lambda}}_C = \begin{bmatrix} \bar{\lambda}_{\mathrm{re}}\boldsymbol{I} & -\bar{\lambda}_{\mathrm{im}}\boldsymbol{I} \\ \bar{\lambda}_{\mathrm{im}}\boldsymbol{I} & \bar{\lambda}_{\mathrm{re}}\boldsymbol{I} \end{bmatrix} \tag{5}$$

$$\boldsymbol{S}_C \cdot \boldsymbol{t}_C - \bar{\boldsymbol{\lambda}}_C \cdot \boldsymbol{t}_C = \boldsymbol{0}. \tag{6}$$

Equation (6) contains $2N$ scalar equations, while (4) and (5) include $2N + 2$ unknown variables. Two additional constraint equations are added, in order to solve a determined system of nonlinear equations. These constraint equations specify the sum of the complex elements of $\boldsymbol{t}$ to unity and are expressed as

$$\sum_{i=1}^{N} \boldsymbol{t}_C^2(i) - \sum_{i=N+1}^{2N} \boldsymbol{t}_C^2(i) = 1 \tag{7}$$

$$\sum_{i=1}^{N} \boldsymbol{t}_C(i) \cdot \boldsymbol{t}_C(N+i) = 0. \tag{8}$$

After grouping (6)–(8), the real-valued vector function $\boldsymbol{f}$ of size $2N + 2$ is formulated which tends to zero for the solution pair of $\boldsymbol{t}_C$ and $\bar{\boldsymbol{\lambda}}_C$

$$\boldsymbol{f}\left(\boldsymbol{t}_C, \bar{\boldsymbol{\lambda}}_C\right) \to \boldsymbol{0} \Leftrightarrow \begin{bmatrix} \boldsymbol{S}_C \cdot \boldsymbol{t}_C - \bar{\boldsymbol{\lambda}}_C \cdot \boldsymbol{t}_C \\ \sum_{i=1}^{N} \boldsymbol{t}_C^2(i) - \sum_{i=N+1}^{2N} \boldsymbol{t}_C^2(i) - 1 \\ \sum_{i=1}^{N} \boldsymbol{t}_C(i) \cdot \boldsymbol{t}_C(N+i) \end{bmatrix} \to \boldsymbol{0}. \tag{9}$$

C. Application of the Levenberg–Marquardt Method

The function of (9) can be solved by using several techniques [13], [14]. In this paper, an iteration procedure is proposed, by applying the LM method as presented

$$\boldsymbol{x}_{\mathrm{new}} = \boldsymbol{x}_{\mathrm{old}} - \left(\boldsymbol{H}(\boldsymbol{x}_{\mathrm{old}}) + \sigma \cdot \mathbf{diag}\left(\boldsymbol{H}(\boldsymbol{x}_{\mathrm{old}})\right)\right)^{-1} \cdot \boldsymbol{J}^T(\boldsymbol{x}_{\mathrm{old}}) \cdot \boldsymbol{F}(\boldsymbol{x}_{\mathrm{old}}). \tag{10}$$

The formulation of (10) is expressed with respect to $\boldsymbol{x}$, which is a column vector including the $2N + 2$ unknowns. $\boldsymbol{F}(\boldsymbol{x})$ is the equation matrix based on (9), $\boldsymbol{J}(\boldsymbol{x})$ is the corresponding Jacobian matrix, that is, the first-order derivatives, and $\boldsymbol{H}(\boldsymbol{x})$ is the second-order approximation of the Hessian matrix, calculated by averaging the outer product of the first-order derivatives

$$\boldsymbol{x} = [\boldsymbol{t}_C \quad \bar{\lambda}_R \quad \bar{\lambda}_I]^T \tag{11}$$

$$\boldsymbol{H}(\boldsymbol{x}) = \langle \boldsymbol{J}^T(\boldsymbol{x}), \boldsymbol{J}(\boldsymbol{x}) \rangle. \tag{12}$$

The positive scalar combination coefficient σ controls the magnitude and the direction of the vector $\boldsymbol{x}_{\mathrm{new}} - \boldsymbol{x}_{\mathrm{old}}$ in (10). Starting from a small value, for example, 10^{-4}, σ is consecutively adjusted in order to ensure the decrease of $\boldsymbol{F}(\boldsymbol{x})$ in each iteration. If $\boldsymbol{F}(\boldsymbol{x})$ of the next iteration step is decreased, it implies that the quadratic assumption is satisfied and σ is reduced by a factor of 10 in order to reduce the influence of the gradient descent. On the other hand, if $\boldsymbol{F}(\boldsymbol{x})$ is increased, σ is increased by the same factor in order to follow the direction of the gradient more. A lower limit of σ, for example, 10^{-15} is also set in order to avoid floating-point overflows.

The algorithm iterates until all elements of $\boldsymbol{F}(\boldsymbol{x})$ become smaller than the tolerance ε, typically set to 10^{-8} or less. The starting value of $\boldsymbol{x}$ at each frequency is seeded from the final solution at the preceding frequency, which ensures smooth behavior of the eigenvectors in the frequency domain. At the initial frequency of the proposed method, a conventional eigenvalue

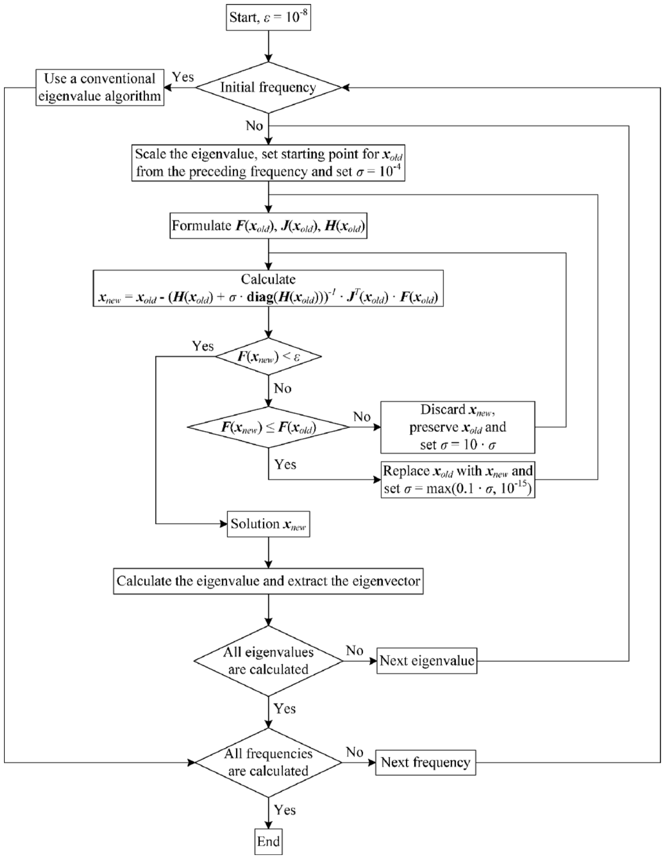

Fig. 1. Flowchart of the proposed method.

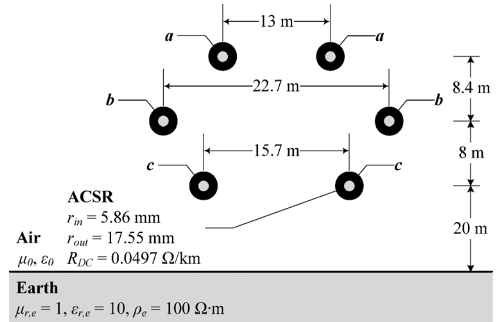

Fig. 2. Cross-section of the double-circuit 400-kV HV OHTL.

algorithm is used, such as the QR method, in order to provide the first starting pairs of the eigenvalue/eigenvector.

The aforementioned procedure is independently applied to each normalized eigenvalue $\bar{\lambda}$ for all frequencies. Finally, the corresponding original eigenvalues λ of the matrix product $\boldsymbol{Y}' \cdot \boldsymbol{Z}'$ are calculated from (13). The corresponding eigenvectors $\boldsymbol{t}$ do not have to be treated in a similar way, since they are eigenvectors of $\boldsymbol{Y}' \cdot \boldsymbol{Z}'$ and $\boldsymbol{S}$ [23]

$$\lambda = -\omega^2 \mu_0 \varepsilon_0 \cdot (1 + \bar{\lambda}). \tag{13}$$

The proposed method is summarized in Fig. 1 by means of a flowchart, presenting the steps for the calculation of smooth eigenvectors in the frequency domain.

D. Comparison With Other Methods

From (10), it is concluded that the proposed method can be considered as a combination of the NR method and the method of the gradient descent, exploiting the numerical advantages of both techniques. For σ being equal to zero, (10) is identical to the corresponding real-valued equation of the NR method, following the quadratic convergence [13]. For σ tending to infinity, the vector $\boldsymbol{x}_{\text{new}} - \boldsymbol{x}_{\text{old}}$ follows the steepest descent direction with a magnitude tending to zero. Thus, the proposed method contrary to the NR method uses a line-search criterion through the direct manipulation of the parameter σ, which ensures the decrease of $\boldsymbol{F}(\boldsymbol{x})$ in each iteration and results in an optimal step size.

Furthermore, from (10), it is shown that the inclusion of the main diagonal of (12), weighted with the combination coefficient σ, ensures that the approximation of the Hessian matrix $\boldsymbol{H}(\boldsymbol{x})$ is always positive definite and, therefore, invertible. This is not the case for the NR method where the existence of a nonsingular Jacobian matrix $\boldsymbol{J}(\boldsymbol{x})$ is only assumed.

The robust convergence rate of the proposed method, together with the lesser dependence on the starting value, enables the use of a lower sampling rate in the frequency domain. Therefore, the total execution time can be reduced with almost no loss in the accuracy of the results.

The SQP method offers similar numerical advantages to the proposed method [14]. However, the constrained minimization procedure in the SQP method results in a generally slower convergence rate than the direct solution of (9) with the NR and the proposed methods. Moreover, it has been found that the SQP method, contrary to the rest techniques, is very sensitive to the scaling technique applied to the matrix product $\boldsymbol{Y}' \cdot \boldsymbol{Z}'$ in (1). This has mainly been observed in cases of cable configurations and can lead to significant errors in the calculation of the corresponding eigenvalues and modal transformation matrices.

In this paper, the real-valued eigenproblem formulation is considered, since it can be implemented easily in any commercial numerical solvers, such as MATLAB, designed to adopt real variables only. However, the proposed method can also be applied to the equivalent complex-valued eigenproblem formulation by the direct application of (10) in (2).

III. Numerical Results

The proposed method is used in different TL configurations, and the calculated results are compared with the corresponding results obtained from the NR and the SQP methods. In all examined cases, the propagation characteristics and modal transformation matrices are calculated for the frequency range from 0.1 Hz up to 1 MHz with a sampling rate of 20 points/decade.

A. OHTLs

A double-circuit 400-kV high-voltage (HV) OHTL is considered, consisting of one conductor per phase. The line configuration and properties are shown in Fig. 2, whereas a homogeneous earth case is assumed [17] with earth resistivity and relative permittivity equal to 100 Ω·m and 10, respectively.

The examined OHTL can be decomposed into six modes, namely, one ground and five aerial modes, as shown in Fig. 3, where the propagation characteristics are presented. In Fig. 4,

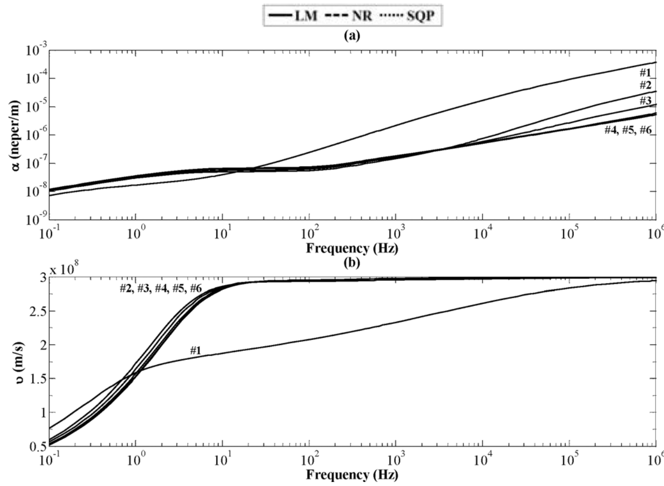

Fig. 3. Modal (a) attenuation and (b) propagation velocity for the OHTL.

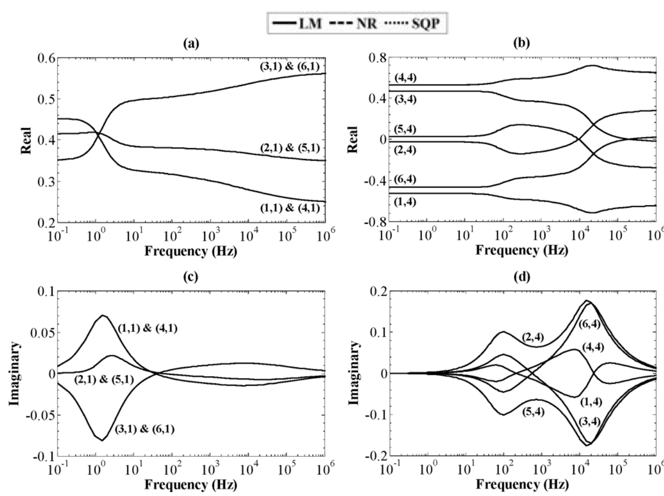

Fig. 4. (a) and (b) Real, and (c) and (d) imaginary parts of the first and fourth eigenvector for the OHTL.

the real and imaginary parts of the first and fourth column of the 6×6 modal transformation matrix are plotted, corresponding to the ground and one of the aerial modes. The curves of all eigenvector elements are smooth functions of frequency and overlap for the three methods, indicating the high accuracy of the proposed method. Moreover, it is shown that the elements of both eigenvectors appear in equal or opposite equal pairs, due to the symmetry of the examined OHTL. The same behavior is also observed in the rest columns of the modal transformation matrix.

B. Underground Single-Core Cables

In Fig. 5, three identical single-phase 20-kV medium-voltage (MV) single-core (SC) cables are in flat formation. The three-phase underground cable system is buried 1 m under the ground surface, assuming a homogeneous earth case [18] with the same earth characteristics as in the OHTL case.

The underground cable consists of one ground, two inter-sheaths, and three coaxial propagation modes [24]. Although the eigenvalues of the coaxial modes coalesce at high frequencies,

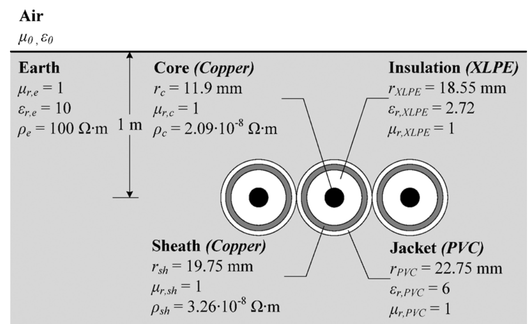

Fig. 5. Cross-section of the three-phase 20-kV MV underground cable.

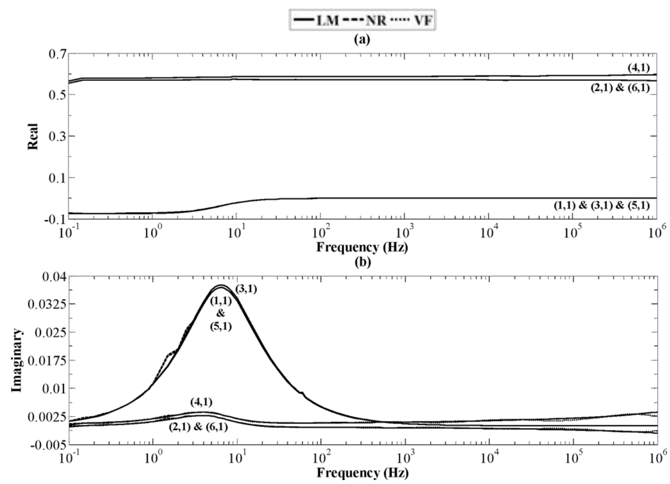

Fig. 6. (a) Real and (b) imaginary part of the first eigenvector for the underground cable using the earth formulation of [18].

the proposed method does not encounter any difficulty in calculating smooth eigenvectors. In Fig. 6, the real and imaginary part of the first column of the 6×6 modal transformation matrix is shown, corresponding to the ground propagation mode.

First, the results calculated with the proposed and the NR methods are compared. Small differences are observed in the eigenvector imaginary parts at low frequencies as well as on the corresponding real parts, not distinguishable due to the scaling of the y-axis. The eigenvector calculated with the NR method is also valid, presenting a scaled form, which is mainly attributed to the sampling rate and to the dependence on the starting point. Therefore, even in cases of high sampling rates (20 points per decade), the proposed method generally calculates smoother eigenvectors than the NR method. By further increasing the frequency resolution (e.g., 25 points per decade), the eigenvectors obtained from the NR method finally converge to the corresponding eigenvectors calculated with the proposed method. On the other hand, the SQP method fails to derive correct eigenvectors, even after numerous attempts, mainly due to the inherent sensitivity of the method to the scaling procedure of (1) in cable configurations.

Next, the results calculated with the proposed method are fitted using the vector-fitting (VF) technique [25]. The elements of each eigenvector share the same set of stable poles and are accurately approximated with rational functions of 10 poles maximum. Zeros located in the right-half plane are also required,

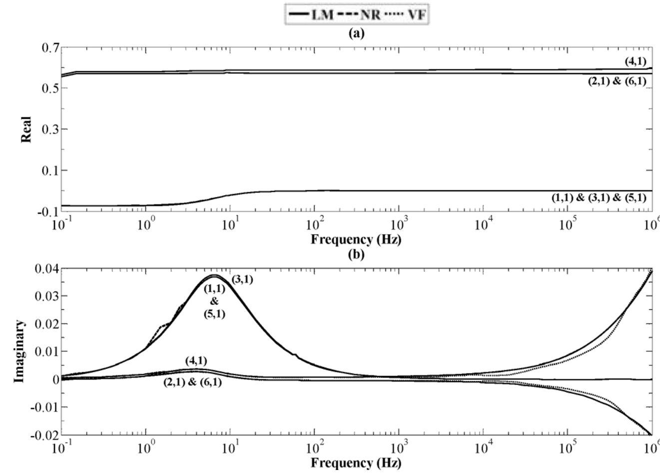

Fig. 7. (a) Real and (b) imaginary part of the first eigenvector for the underground cable using the earth formulation of [19].

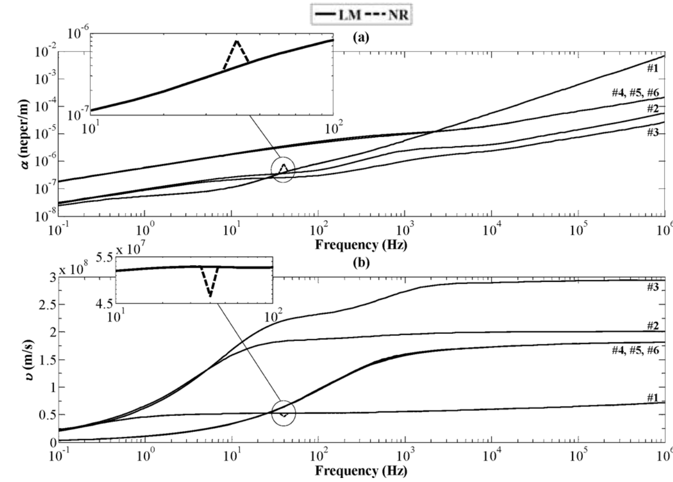

Fig. 8. (a) Modal attenuation and (b) propagation velocity for the cable lying on the ground surface.

since some of the elements are intrinsically described by nonminimum phase-shift functions [2].

The previously examined OHTL eigenvectors are also described by nonminimum phase-shift functions. However, the use of the VF technique results in an unstable modal decomposition, since the modal transformation matrix cannot be fitted with stable poles only. Generally, in OHTL configurations, similar eigenvalues can occur over a wide frequency range. As discussed in [2], this OHTL feature mainly prevents the VF technique from approximating the OHTL modal transformation matrix using only stable poles.

A similar analysis on the examined cable configuration is conducted assuming the earth formulation of [19], where a more accurate earth impedance formula is proposed and admittance earth correction terms are also included. In Fig. 7, the ground eigenvector is calculated by the proposed and the NR methods, presenting again small differences in the low-frequency range due to the selected sampling rate. The curves obtained with the proposed method are also fitted with the VF technique using rational functions of 20 poles maximum. The required higher order approximation is due to the different frequency behavior mainly observed in the imaginary part at high frequencies. The fit with the VF technique also leads to small divergences, mainly observed at high frequencies, which can be further reduced by performing a higher order rational approximation.

Comparing the results of Figs. 6 and 7, it is apparent that differences between the two earth formulations are observed in the imaginary part of the elements (2,1), (4,1) and (6,1) in the frequency range above 10 kHz. This is due to the different assumption of the earth conduction effects which are mainly associated with the ground mode and the cable sheaths, described by the specific eigenvector elements.

For most earth topologies, the approach of [19] must be considered for frequencies above some kilohertz, since in this frequency range, the effect of the earth admittance becomes significant and comparable to the insulation admittance [26]. The influence of the earth is characterized by the combined effect of the conducting and dielectric properties of the soil, resulting in a ground eigenvector which is not real at high frequencies [13], as is the case for the earth formulation of [18]. At very low frequencies, the earth behaves as a conductor and both formulations present the same results [26], leading to a real and constant ground eigenvector [13], as shown in Figs. 6 and 7.

This behavior of the ground eigenvector in the high-frequency range is well documented in literature [19], [22], [26]. Moreover, the approach of [19] for the simulation of the influence of the imperfect earth at high frequencies leads to results fully consistent with the initial and final value theorems of the Laplace transform.

C. Single-Core Cables on the Ground Surface

The three-phase MV SC cable system of Fig. 5 is located on the ground surface, assuming the homogeneous earth formulation of [17] and the same previous earth properties. Although the time-domain simulation of cables on the ground surface may not be such a demanding task as the corresponding simulation of multiple underground cables, it can pose certain difficulties in the calculation and rational approximation of their propagation characteristics.

In Fig. 8, the propagation characteristics are shown, namely, the modal attenuation and the modal propagation velocity, calculated by the proposed and the NR method. From the figure closeup, it is shown that the NR method presents a numerical error in the calculation of the ground eigenvalue at 40 Hz, whereas the proposed method exhibits a correct value. In Fig. 9, a detail of the corresponding eigenvector in the frequency domain is plotted, also indicating the error introduced by the NR method.

The numerical error is due to the fact that the convergence in the NR method is heavily dependent on the distance between the starting and final point in each iteration. If the initial estimation is not close enough to the final solution, convergence is not achieved, requiring a higher discretization between the examined frequencies in the frequency domain. On the other hand, the proposed method, incorporating the enhanced line-search criterion with the use of the combination coefficient σ, alleviates the aforementioned numerical error and allows the use of a larger calculation interval in the frequency domain.

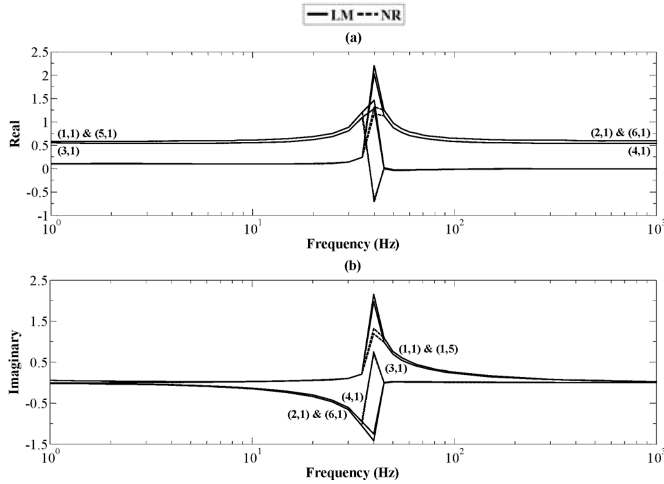

Fig. 9. Detail of the (a) real and (b) imaginary parts of the first eigenvector for the cable lying on the ground surface.

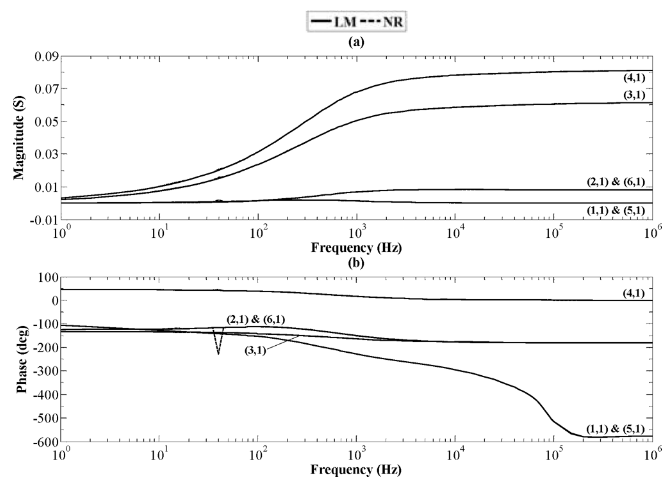

Fig. 10. (a) Magnitude and (b) phase of the first column of the phase-domain characteristic admittance for the cable lying on the ground surface.

Furthermore, as shown in Fig. 10, for the first column of the phase-domain characteristic admittance, the numerical error in the elements of the ground eigenvector leads to calculation errors in the characteristic admittance and the propagation function either in the modal- or in the phase-domain and eventually to errors in the time-domain simulation of transient phenomena [10].

In Table I, the variation of the combination coefficient σ is shown at the frequency of 40 Hz. In all propagation modes, apart from the ground mode, σ is reduced in each iteration indicating that the quadratic convergence is followed, whereas the final solution is derived after three iterations. However, this is not the case for the ground mode, where a change in the search direction occurs in the third iteration and the final solution is derived after seven iterations.

Also for the cable on the ground surface configuration, the calculated eigenvectors are generally expressed with nonminimum phase-shift functions. However, they cannot be fitted with stable poles only using the VF technique, resulting in an unstable modal decomposition. This may be attributed to the coaxial mode eigenvalues, shown in Fig. 8, that acquire

TABLE I
VARIATION OF THE COMBINATION COEFFICIENT AT 40 Hz

	Combination coefficient σ					
Iter.	*Ground*	*Intersh. 1*	*Intersh. 2*	*Coax. 1*	*Coax. 2*	*Coax. 3*
0	10^{-4}	10^{-4}	10^{-4}	10^{-4}	10^{-4}	10^{-4}
1	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}	10^{-5}
2	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}	10^{-6}
3	10^{-5}	10^{-7}	10^{-7}	10^{-7}	10^{-7}	10^{-7}
4	10^{-6}	-	-	-	-	-
5	10^{-7}	-	-	-	-	-
6	10^{-8}	-	-	-	-	-
7	10^{-9}	-	-	-	-	-

very close values over the entire frequency range similarly to the aerial mode eigenvalues of the OHTLs. Therefore, in both cases, the time-domain simulation of transient phenomena is prohibitive using modal-domain models that take into account the frequency dependence of the modal transformation matrix by the rational approximation of the calculated eigenvectors [1], [2]. In such cases, either phase-domain models that avoid the eigenvector rational approximations [3]–[6] or models with constant transformation matrices should be considered [27].

IV. Algorithm Performance

A. Algorithm Tracking Ability

A direct indicator of the robustness of the proposed method compared to the NR method is to identify the lowest sampling rate in the frequency domain, in order to accurately calculate the TL eigenvalues and smooth eigenvectors.

As an example, the convergence rate of the proposed method is examined for the three-phase underground cable system of Fig. 5 with the earth formulation of [18], by varying the total number of frequencies. In Fig. 11, the modal propagation velocity is shown for the proposed and the NR methods with 3 points per decade as well as for the proposed method with 20 points per decade, which is considered as reference. The NR method with 3 points per decade fails to calculate correctly the propagation velocity of the second and third mode above certain frequencies, as highlighted in the figure. On the other hand, the proposed method preserves its tracking ability, even in the case of this large frequency interval, mainly due to the enhanced line-search criterion that is incorporated in the calculation routine.

The aforementioned analysis is also performed considering the earth formulation of [19] and further extended to the OHTL case of Fig. 2. From the comparative results in all examined configurations, it is concluded that the sampling rate of 3 points/decade is the lowest frequency-domain resolution that can be safely used with the proposed method. The corresponding sampling rate for the NR method cannot be less than 5 points/decade, as indicated in Fig. 12, where the intersheath mode propagation velocity of the same underground cable is correctly calculated by the NR method only with 5 and 20 points/decade.

Apart from the improved algorithm tracking ability of the proposed method, the advantage of a lower sampling rate can also lead to the reduction of the total computational time of the

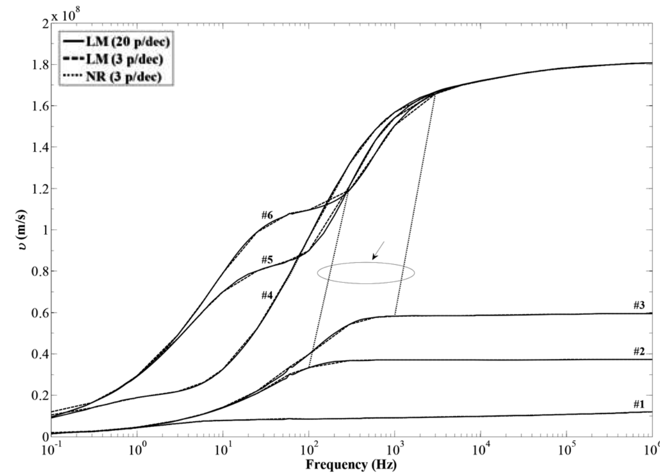

Fig. 11. Modal propagation velocity of the underground cable calculated by the proposed and the NR methods with 3 and 20 points/decade.

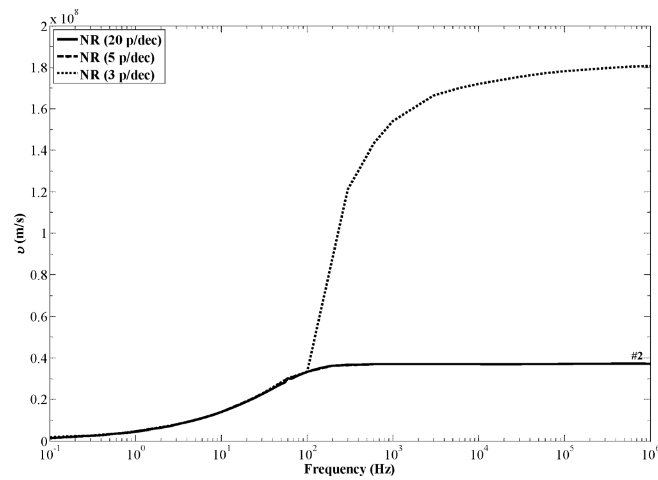

Fig. 12. Intersheath mode propagation velocity of the underground cable calculated by the NR method with 3, 5, and 20 points/decade.

preparatory calculation stage in an Electromagnetic Transients Program (EMTP)-like simulation program. This stage precedes the time-domain transient simulation and includes the calculation of the TL pul parameters as well as the derivation and rational approximation of the corresponding propagation characteristics and functions.

In most practical applications, where simple algebraic formulas of the influence of the imperfect earth are used [28], the sampling rate in the frequency domain is seldom below 10 points/decade. However, a lower resolution (even less than 5 points/decade) may be desirable if earth formulations that require the numerical evaluation of complex integrals are adopted [18], [19], [21], [22]. In such cases, the additional computational time for the calculation of the pul parameters per frequency may become quite extensive, limiting the efficiency of the total simulation procedure.

This is highlighted in Fig. 13, where the required computational time per frequency for the calculation of the pul series impedance and shunt admittance matrices is presented with respect to the number of underground SC cables. In this test case,

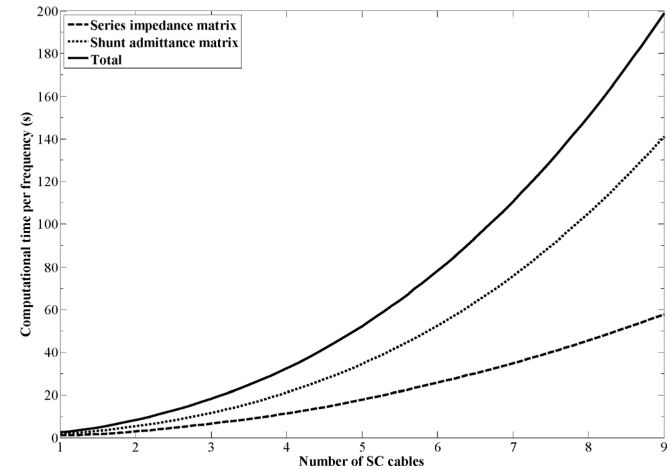

Fig. 13. Computational time per frequency of the pul parameters with respect to the number of SC cables.

TABLE II
INDICATIVE TOTAL EXECUTION TIME (IN PER UNIT)

	Method		
Configuration	*NR*	*LM*	*SQP*
OHTL with [17]	1	1.12	2.36
Underground cable with [18]	1	1.34	-
Underground cable with [19]	1	1.35	-
Cable on the ground surface with [17]	1	1.21	-

the earth formulation of [19] and the numerical integration of [29] are considered. As indicated, the calculation time strongly depends on the complexity of the examined system and can reach 200 s per frequency for a group of nine SC cables. In such cases, the use of the proposed method with a lower sampling rate can result in a considerable reduction of the computation burden of the preparatory calculation procedure.

B. Execution Time

Finally, the computational efficiency of the proposed method is investigated, comparing the indicative execution time, needed for the calculation of the TL modal transformation matrices by the three examined methods. The results are summarized in Table II for the examined OHTL and cable configurations. The execution time is expressed in per unit (p.u.), assuming as a base value the corresponding execution time of the NR method. In order to highlight the computational efficiency of the proposed method in all examined configurations, the case of the three-phase cable on the ground surface is also presented, although the eigenvectors are calculated correctly only by the proposed method.

Despite the uncertain efficiency of the different mathematical routines included in the three examined methods, from the results of Table II, it is shown that the NR method is the fastest in all examined cases. Although the proposed method seems to be slightly slower, the corresponding execution time is acceptable considering the numerical improvements. The SQP method offers similar advantages with the proposed method, but the calculation routine is significantly less efficient and numerically unstable.

V. Discussion

Based on the presented analysis and the examined configurations, the following conclusions can be summarized:

Proposed method

- The proposed method is based on a numerically well-defined computational scheme with the inclusion of the line-search criterion, which guarantees a robust convergence rate with the use of a larger frequency interval. Moreover, the proposed method is applicable to any type of TL without any numerical issues, since a positive definite approximation of the Hessian matrix always exists. On the contrary, the NR method does not support the aforementioned features. Although the SQP method offers similar numerical advantages, the corresponding optimization technique may result in eigenvalue calculation errors especially in complex cable configurations.
- A frequency sampling rate of 3 points/decade can be used in the proposed method, whereas the corresponding sampling rate in the NR method cannot be less than 5 points/decade. This is not only a direct indicator of the improved tracking ability of the proposed method, but can also be helpful in reducing the total computational time of the pul parameters when complex earth formulations that include semi-infinite integrals are considered.
- The proposed method generally provides smoother eigenvectors (Figs. 6 and 7) with no calculation errors (Fig. 8), compared to the NR method. This is also observed in cases of high sampling rates (e.g., 20 points per decade).
- The proposed and NR method execution times are comparable since both methods adopt a similar straightforward technique to solve the eigenproblem. The SQP method is less efficient, since a slower optimization scheme is utilized.

Eigenvector rational approximations

- In all examined configurations, the calculated eigenvectors intrinsically include elements which are described by non-minimum phase-shift functions.
- In underground cable configurations, the accurate rational approximation of the corresponding eigenvectors can be achieved with stable poles and zeros in the right-half plane, automatically introduced by the VF technique.
- In configurations of OHTLs and cables on the ground surface, the eigenvectors cannot be fitted accurately with stable poles only. The unstable modal decomposition of both configurations is mainly related to the fact that several eigenvalues (aerial or coaxial modes) acquire very close values over the entire frequency range. This is not the case for underground cables, since the coaxial modes coalesce only above 1 kHz.

VI. Conclusion

A new method is presented for the calculation of smooth TL modal transformation matrices in the frequency domain. The proposed method is based on the Levenberg–Marquardt algorithm in order to solve a real-valued formulation of the eigenproblem, and can be easily incorporated in the most recent versions of EMTP-like programs, such as the EMTP-RV or the PSCAD/EMTDC, improving the efficiency, tracking ability, and range of applicability of their preparatory calculation stage.

Modal transformation matrices for different types of TL configurations and earth formulations are calculated. Results obtained from the proposed method are compared and validated with the corresponding results from the NR and the SQP methods. Smooth eigenvectors are calculated in all examined cases, verifying the accuracy and efficiency of the proposed method, even in cases where the NR and SQP methods fail.

The distinct advantage of the proposed method is its ability to produce accurate results in the frequency domain using low sampling rates. This feature can be very useful in cases where enhanced, complex models are used for the calculation of TL parameters.

References

[1] L. Marti, "Simulation of transients in underground cables with frequency-dependent modal transformation matrices," *IEEE Trans. Power Del.*, vol. 3, no. 3, pp. 1099–1110, Jul. 1988.

[2] B. Gustavsen and A. Semlyen, "Simulation of transmission line transients using vector fitting and modal decomposition," *IEEE Trans. Power Del.*, vol. 13, no. 2, pp. 605–614, Apr. 1998.

[3] T. Noda, N. Nagaoka, and A. Ametani, "Phase domain modeling of frequency-dependent transmission lines by means of an ARMA model," *IEEE Trans. Power Del.*, vol. 11, no. 1, pp. 401–411, Jan. 1996.

[4] H. V. Nguyen, H. W. Dommel, and J. R. Marti, "Direct phase-domain modelling of frequency-dependent overhead transmission lines," *IEEE Trans. Power Del.*, vol. 12, no. 3, pp. 1335–1342, Jul. 1997.

[5] B. Gustavsen and A. Semlyen, "Combined phase and modal calculation of transmission line transients based on vector fitting," *IEEE Trans. Power Del.*, vol. 13, no. 2, pp. 596–604, Apr. 1998.

[6] A. Morched, B. Gustavsen, and M. Tartibi, "A universal model for accurate calculation of electromagnetic transients on overhead lines and underground cables," *IEEE Trans. Power Del.*, vol. 14, no. 3, pp. 1032–1038, Jul. 1999.

[7] L. M. Wedepohl, "Application of matrix methods to the solution of travelling-wave phenomena in polyphase systems," in *Proc. Inst. Elect. Eng.*, 1963, vol. 110, no. 12, pp. 2200–2212.

[8] J. G. F. Francis, "The QR transformation, a unitary analogue to the LR transformation, Part I," *Comput. J.*, vol. 4, no. 3, pp. 265–271, 1961.

[9] J. G. F. Francis, "The QR transformation, a unitary analogue to the LR transformation, Part II," *Comput. J.*, vol. 4, no. 4, pp. 332–345, 1961.

[10] A. B. Fernandes and W. L. A. Neves, "Phase-domain transmission line models considering frequency-dependent transformation matrices," *IEEE Trans. Power Del.*, vol. 19, no. 2, pp. 708–717, Apr. 2004.

[11] S. Fan, Y. Li, X. Li, and L. Bi, "A method for the calculation of frequency-dependent transmission line transformation matrices," *IEEE Trans. Power Syst.*, vol. 24, no. 2, pp. 552–560, May 2009.

[12] T. Noda, "Numerical technique for accurate evaluation of overhead line and underground cable constants," *Inst. Elect. Eng. Jpn. Trans. Elect. Electron. Eng.*, vol. 3, no. 5, pp. 549–559, 2008.

[13] L. M. Wedepohl, H. V. Nguyen, and G. D. Irwin, "Frequency-dependent transformation matrices for untransposed transmission lines using Newton-Raphson method," *IEEE Trans. Power Del.*, vol. 11, no. 3, pp. 1538–1546, Jul. 1996.

[14] T. T. Nguyen and H. Y. Chan, "Evaluation of modal transformation matrices for overhead transmission lines and underground cables by optimization method," *IEEE Trans. Power Del.*, vol. 17, no. 1, pp. 200–209, Jan. 2002.

[15] K. Levenberg, "A method for the solution of certain problems in least squares," *Quart. Appl. Math.*, vol. 2, pp. 164–168, 1944.

[16] D. Marquardt, "An algorithm for least-squares estimation of nonlinear parameters," *SIAM J. Appl. Math.*, vol. 11, no. 2, pp. 431–441, 1963.

[17] W. H. Wise, "Potential coefficients for ground return circuits," *Bell Syst. Tech. J.*, vol. 27, pp. 365–372, 1948.

[18] E. D. Sunde, *Earth Conduction Effects in Transmission Systems*, 2nd ed. New York: Dover, 1968, pp. 99–139.

[19] T. A. Papadopoulos, D. A. Tsiamitros, and G. K. Papagiannis, "Impedances and admittances of underground cables for the homogeneous earth case," in *Proc. IET Gen. Transm. Distrib.*, 2011, vol. 5, no. 2, pp. 161–171.

[20] A. Ametani, “A general formulation of impedance and admittance of cables,” *IEEE Trans. Power App. Syst.*, vol. PAS-99, no. 3, pp. 902–910, May 1980.

[21] T. A. Papadopoulos, G. K. Papagiannis, and D. P. Labridis, “A generalized model for the calculation of the impedances and admittances of overhead power lines above stratified earth,” *Elect. Power Syst. Res.*, vol. 80, pp. 1060–1070, 2010.

[22] T. A. Papadopoulos, D. A. Tsiamitros, and G. K. Papagiannis, “Earth return admittances of underground cables in non-homogeneous earth,” *IEEE Trans. Power Del.*, vol. 25, no. 2, pp. 961–969, Apr. 2010.

[23] R. H. Galloway, W. B. Shorrocks, and L. M. Wedepohl, “Calculation of electrical parameters for short and long polyphase transmission lines,” in *Proc. Inst. Elect. Eng.*, 1964, vol. 111, no. 12, pp. 2051–2059.

[24] L. M. Wedepohl and D. J. Wilcox, “Transient analysis of underground power-transmission systems: System-model and wave-propagation characteristics,” in *Proc. Inst. Elect. Eng.*, 1973, vol. 120, no. 2, pp. 253–260.

[25] B. Gustavsen and A. Semlyen, “Rational approximation of frequency domain responses by vector fitting,” *IEEE Trans. Power Del.*, vol. 14, no. 3, pp. 1052–1061, Jul. 1999.

[26] T. A. Papadopoulos, A. I. Chrysochos, and G. K. Papagiannis, “Analytical study of the frequency-dependent earth conduction effects on underground power cables,” in *Proc. IET Gen. Transm. Distrib.*, 2013, vol. 7, no. 3, pp. 276–287.

[27] J. R. Marti, “Accurate modeling of frequency-dependent transmission lines in electromagnetic transient simulations,” *IEEE Trans. Power App. Syst.*, vol. PAS-101, no. 1, pp. 147–157, Jan. 1982.

[28] A. Deri, G. Tevan, A. Semlyen, and A. Castanheira, “The complex ground return plane: A simplified model for homogeneous and multi-layer earth return,” *IEEE Trans. Power App. Syst.*, vol. PAS-100, no. 8, pp. 3686–3693, Aug. 1981.

[29] G. K. Papagiannis, D. A. Tsiamitros, D. P. Labridis, and P. S. Dokopoulos, “Direct numerical evaluation of earth return path impedances of underground cables,” in *Proc. Inst. Elect. Eng.*, 2005, vol. 152, no. 3, pp. 321–327.

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