Electrical Engineering
https://doi.org/10.1007/s00202-019-00807-8

ORIGINAL PAPER

Accuracy enhancement of the JMarti model by using real poles through vector fitting

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Received: 15 June 2018 / Accepted: 1 July 2019

Abstract

The JMarti model is used worldwide for electromagnetic transient simulations of transmission lines. This paper proposes increasing the accuracy of the model by using only real poles through vector fitting. The JMarti model uses a rational approximation process that ends in models that include only real poles and zeros. By using vector fitting to synthesize the functions of the JMarti model, one can have complex and real poles. In this work, a process in which complex poles are replaced by real ones, based on the non-dominance of complex poles for smooth functions, is implemented. The improvement in the accuracy is demonstrated through the ATP-EMTP (alternative transient program) program by modeling two cases: a single-phase transmission line configuration and an asymmetrical overhead line configuration. Additionally, the solutions of the test cases with the numerical Laplace transform are taken as references. The results show that, through the proposed methodology, not only an increase in accuracy possible but also the proposed methodology also can be used to improve the computational efficiency of the JMarti model. The presented methodology can easily be applied to any application in which the frequency response data present a smooth behavior. Finally, the presented methodology can be easily applied in an iterative way.

Keywords ATP-EMTP · JMarti model · Rational approximation · Real poles · Vector fitting

1 Introduction

The frequency-dependent JMarti model is used for the simulation of electromagnetic transient in overhead transmission lines and cables. The model was presented in 1982 by Jose Marti [1], and some improvements have recently been proposed in [2]. The JMarti model is mainly characterized by the rational approximation of the characteristic impedance ($\mathbf{Z}_c$) and by the elements of the propagation matrix (**H**). The fitting technique implemented in the JMarti model is based on the concept of asymptotic approximations of magnitude functions, which was introduced by Bode [3, 4]. This rational approximation technique leads to having only real poles and zeros and therefore real state variables. In this work, vector fitting (VF) [5] is used to obtain these rational function-based models. VF was developed in 1999 by Gustavsen and Semlyen [6], which including its different formulations and applications [7–10], has been positioned as one of the most popular techniques for rational approximation of frequency-dependent functions.

VF can deliver real and complex poles for a rational approximation of complex-curve functions. Specifically, in the case of the characteristic impedance ($\mathbf{Z}_c$), it is not common for complex poles to appear, unlike the fitting of the propagation function (**H**), where these complex poles are more common. This leads to having real and complex state

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variables for time-domain simulations. When complex states arise, a simple approach consists of treating all states as complex; however, it is computationally inefficient [11]. There are different techniques that have been proposed to treat these complex states more efficiently [11]. These techniques consist of a numerical treatment once complex states appear in the formulation of the model. Nevertheless, in this work, these techniques are not used since the rational approximation yields only real poles, and therefore, only real state variables in the time domain are obtained.

The aim of the present paper is to provide a general methodology to increase the numerical precision of the JMarti model by using rational approximations with only real poles obtained through vector fitting (VF). This leads to having real state variables where numerical techniques are not required to manipulate complex state variables. This paper presents the methodology by which it is possible to obtain a rational approximation of smooth frequency-domain functions, such as $\mathbf{Z}_c$ and $\mathbf{H}$, with only real poles.

Firstly, a review of the theory for the JMarti model is presented. Then, the methodology used for the approximation of smooth functions using only real poles through VF is described. Afterward, the concept of pole dominance is presented. Finally, advantages of the methodology are exposed by the modeling of a single-phase transmission line and by the modeling of a three-phase horizontal transmission line. In both cases, the ATPDraw and ATP-EMTP are used.

The transmission line is first simulated with the rational approximation procedure (Bode-ATP) used by the JMarti model in the ATP-EMTP program, and then the line parameters ($\mathbf{Z}_c$ and $\mathbf{H}$) are obtained from the ATPDraw program. These parameters are fitted through VF with the proposed methodology. The resulting rational function-based models substitute those obtained with Bode-ATP in the ATP-EMTP program. The solution through the numerical Laplace transform (NLT) for both cases is taken as a reference to show the increased accuracy of the model by using the proposed methodology.

2 The frequency-dependent or J. Marti line model

Electromagnetic wave propagation along an n-conductor line or cable is governed by Telegrapher's Eqs. (1) and (2), where $\mathbf{V}$ and $\mathbf{I}$ are the voltage and current vectors of length n. $\mathbf{Z} = \mathbf{R} + j\omega\mathbf{L}$ and $\mathbf{Y} = \mathbf{G} + j\omega\mathbf{C}$ are symmetrical matrices of dimensions $n \times n$:

$$\frac{\mathrm{d}^2\mathbf{V}}{\mathrm{d}x^2} = \mathbf{ZYV}; \tag{1}$$

$$\frac{\mathrm{d}^2\mathbf{I}}{\mathrm{d}x^2} = \mathbf{YZI}. \tag{2}$$

At ends $x = 0$ and $x = L$ of the line, the following equations relate incident current waves to those reflected from the other end:

$$\mathbf{I}_0 - \mathbf{Y}_c\mathbf{V}_0 = -\mathbf{H}[\mathbf{I}_L + \mathbf{Y}_c\mathbf{V}_L] \tag{3}$$

$$\mathbf{I}_L - \mathbf{Y}_c\mathbf{V}_L = -\mathbf{H}[\mathbf{I}_0 + \mathbf{Y}_c\mathbf{V}_0] \tag{4}$$

where

$$\mathbf{Y}_c = \mathbf{Z}^{-1}\sqrt{\mathbf{ZY}} = \sqrt{(\mathbf{YZ})^{-1}}\mathbf{Y} \quad \text{and} \quad \mathbf{H} = e^{-\sqrt{\mathbf{YZ}}l} \tag{5}$$

with $\mathbf{Z}$, $\mathbf{Y}$, $\mathbf{Y}_c$, $\mathbf{H}$ and l being the line series impedance per unit length, shunt admittance per unit length, characteristic admittance matrix, propagation function matrix, and length, respectively. Equations (3) and (4) represent a coupled system; however, it can be replaced by a decoupled system by introducing modal quantities [1],

$$\mathbf{I} = \mathbf{T}_I\mathbf{I}^m \quad \text{and} \quad \mathbf{V} = \mathbf{T}_V\mathbf{V}^m. \tag{6}$$

$\mathbf{T}_I$ and $\mathbf{T}_V$ are right eigenvectors matrices of $\mathbf{YZ}$ and $\mathbf{ZY}$, respectively. The superscript m indicates modal quantities. By substituting (6) into (3) and (4), a decoupled system is obtained. The diagonal matrices $\mathbf{H}^m$ and $\mathbf{Y}_c^m$ are related to their counterpart in the phase domain by:

$$\mathbf{H} = \mathbf{T}_I\mathbf{H}^m\mathbf{T}_I^{-1} \quad \text{and} \quad \mathbf{Y} = \mathbf{T}_I\mathbf{Y}_c^m\mathbf{T}_I^T. \tag{7}$$

In the JMarti model, the modal quantities of the propagation function $\mathbf{H}$ and the characteristic admittance $\mathbf{Y}_c$ must be approximated by a rational fitting method. The methodology used by J. Marti [1, 2] is the Bode's method [3, 4], which implements the fitting of the characteristic impedance ($\mathbf{Z}_c = \mathbf{Y}_c^{-1}$) [1]. The rational approximation for the characteristic impedance $\mathbf{Z}_c$ (which also can be applied to the characteristic admittance $\mathbf{Y}_c$) in the modal domain is defined as [1]:

$$Z_{c,i}^m \cong \sum_{k=1}^{Nz_i} \frac{c_{i,k}}{s - p_{i,k}} + d_{i,k} \tag{8}$$

where Nz_i is the fitting order, $c_{i,k}$ is the kth residue corresponding to the ith modal quantity, $d_{i,k}$ is the kth constant term corresponding to the i-modal quantity, and $p_{i,k}$ is the kth pole corresponding to the fitting of the i-modal quantity. Similarly, the approximation for the modal quantity of the propagation function is defined as [1]:

$$H_i^m \cong \sum_{k=1}^{Nh_i} \frac{c_{i,k}}{s - p_{i,k}} e^{-s\tau_i} \tag{9}$$

where Nh_i is the fitting order, $c_{i,k}$ is the kth residue corresponding to the ith modal quantity, and $p_{i,k}$ is the kth pole corresponding to the fitting of the ith modal quantity. In addition, τ represents the travel time associated with the ith mode.

In the JMarti model, the $\mathbf{Z}_c$ and $\mathbf{H}$ are synthesized by rational function-based models in pole-zero form:

$$F(s) \cong h\frac{(s-z_1)(s-z_2)\cdots(s-z_n)}{(s-p_1)(s-p_2)\cdots(s-p_m)}. \tag{10}$$

In the JMarti fitting procedure, the poles (p_m) and zeros (z_n) are successively added to the total response function by tracing the resulting asymptote of the approximation to the magnitude function of the given response. In the numerical implementation, one starts at the lowest frequency sample and compares the function to be fitted with its rational approximation, adding a pole or a zero in (10) when the asymptote departs from the target function by a tolerance value.

For transmission line modeling, a relative tolerance criterion is normally used. It is also normal to enforce the zeros to lie in the left half-plane, thereby obtaining a minimum-phase shift function. In the implementation of the JMarti fitting, the accuracy is enhanced by shifting the poles and zeros around their first asymptotes. It should be noted that, by means of the Bode's method, it is not necessary to know the travelling time in the rational approximation process since it fits the magnitude of the function, as opposed to vector fitting, in which the time delay must be previously known to make the approach possible [12].

It can be noticed that the accuracy of the rational approximation depends on the performance of the numerical implementation. Commonly, this methodology tends to end in rational function-based models with higher orders compared with vector fitting. Therefore, the proposed methodology also may represent an increase in the computational efficiency for the JMarti model.

3 Rational approximation of smooth frequency-domain responses

In this section, a straightforward methodology for obtaining rational function-based models with real poles through VF is carefully described.

3.1 Vector fitting (VF) iteration

Vector fitting is used to synthesize models of frequency-domain responses, which can be measured or calculated [5, 6]. In this work, VF is used to obtain the rational models (8) and (9). VF searches a rational model in the pole-residue form (11)

$$F(s) \cong \sum_{n=1}^{N} \frac{c_n}{s-p_n} + d. \tag{11}$$

First, a set of heuristically calculated initial poles (p_i) is relocated to better positions by means of an iterative procedure. Then, the residues (c_n, d) are calculated with the given poles taken as known quantities.

3.2 Approximation of smooth functions by using real poles

In some applications, the frequency-domain responses to be fitted (*e.g.*, $\mathbf{Z}_c$ and $\mathbf{H}$) are smooth functions of frequency without resonance peaks. Usage of VF leads to a rational model with guaranteed stable poles; however, some complex conjugate poles and residues may result. Below, it will be observed that it is possible to modify the fitting procedure slightly to guarantee a model with only real poles and residues [13]. Consider that (11) has N_r real poles and N_c pairs of complex conjugate poles:

$$F(s) \cong \sum_{n=1}^{N_r} \frac{c_n}{s-p_n} + \sum_{n=1}^{N_c}\left(\frac{\gamma_n + j\eta_n}{s-(\alpha_n + j\beta_n)} + \frac{\gamma_n - j\eta_n}{s-(\alpha_n - j\beta_n)}\right) \tag{12}$$

where the coefficient d is zero. Each pair of complex conjugate poles can be written as

$$s^2 + 2\zeta\omega_n s + \omega_n^2 \tag{13}$$

where

$$\omega_n^2 = \alpha_n^2 + \beta_n^2 \quad \text{and} \quad \zeta\omega_n = \alpha_n. \tag{14}$$

The term ω_n is the corner frequency of the quadratic factor, and ζ is the damping coefficient of the quadratic term. The critical value of ζ is 1, defining two regions [13]: (1) if $\zeta < 1$, the poles of the quadratic factor are complex; and (2) if $\zeta = 1$, the poles of the quadratic factor are real. We are interested in the first case. It is considered that each complex conjugate pole is replaced with a repeated (double) real pole located at the corner frequency

$$j\omega - (\alpha \pm j\beta) \rightarrow (j\omega - \omega_n)^2. \tag{15}$$

Figure 1a shows the effect of ζ on the amplitude of $F(s)$ and the quality of the replacement (15). It can be observed that the quality of the approximation increases when ζ tends to 1. The use of a double pole is inconvenient because the implementation of recursive convolution in the JMarti model is based on first-order blocks [1, 2]. It is preferable to replace each complex pair with two real poles that are separated by a distance δ [13]:

$$j\omega - (\alpha \pm j\beta) \rightarrow j\omega - (\omega_n - \delta),\ j\omega - (\omega_n + \delta). \tag{16}$$

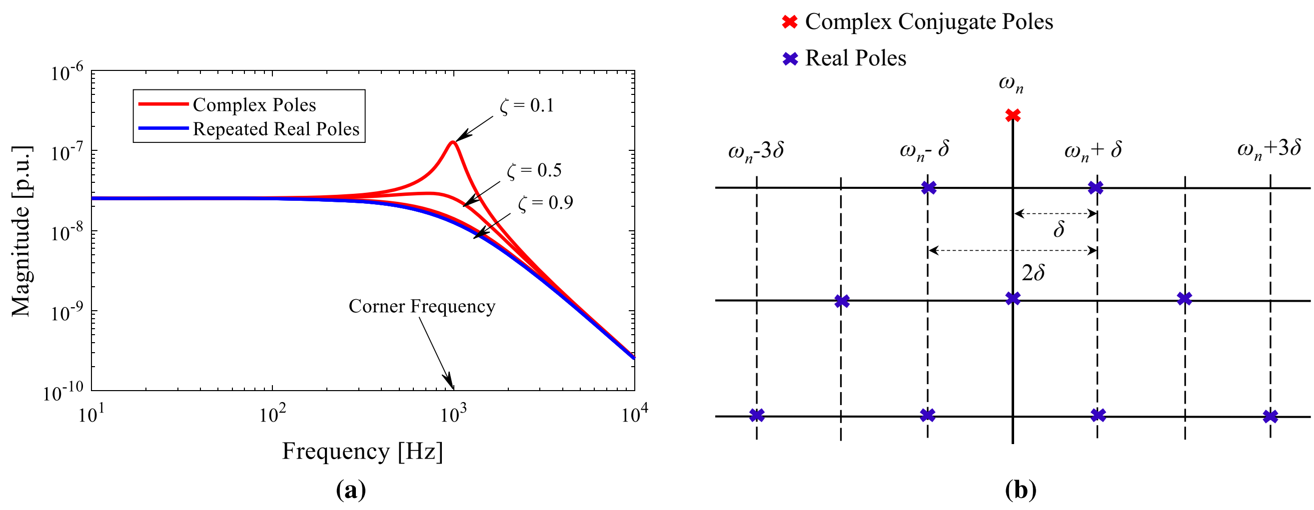

Fig. 1 **a** Effect of ζ on the magnitude of $F(s)$, **b** complex conjugate poles replaced by two, three or four real poles

Usage of very close poles as defined by the closeness parameter δ implies errors in the solution of the associated least square (LS) problem ($\mathbf{Ax} = \mathbf{b}$) to be solved in VF. The error is proportional to the condition number [13],

$$\|A\|\|A\|^{-1}. \tag{17}$$

In this work, we chose $\delta = 0.01$ to ensure negligible error magnifications [13]. Finally, Fig. 1b shows the strategy for replacing complex conjugate poles by two, three, or four real poles as a function of δ, in concordance with (16), (18), and (19), respectively.

$$j\omega - (\alpha \pm j\beta) \rightarrow j\omega - (\omega_n - 2\delta), (j\omega - \omega_n), j\omega - (\omega_n + 2\delta) \tag{18}$$

$$j\omega - (\alpha \pm j\beta) \rightarrow j\omega - (\omega_n - 3\delta), j\omega - (\omega_n - \delta), j\omega - (\omega_n + \delta), j\omega - (\omega_n + 3\delta). \tag{19}$$

This methodology can be used iteratively in VF, that is to say, it can be applied in each iteration that VF performs to find the poles of the fitting. In our experience, some of the complex poles can be iteratively eliminated before the last iteration. This characteristic is used in this work.

The rationale for using the aforementioned pole replacement strategy is based on two observations [13]:

1. In the case of a rational model with a smooth frequency response, any pole with ζ significantly smaller than 1 cannot be dominant.
2. Any complex pole that is dominant must have a damping coefficient ζ near 1.

The concept of pole dominance will now be described in more detail. A pole p_n in (11) is called dominant if $|c_n| > |c_k|$ for all $k \neq n$ [14]; this is the dominant pole measurement (DPM). Nevertheless, this measurement can weigh the peaks in the corresponding $F(s)$. Therefore, a relative dominant pole measurement (RDPM) is used: $|c_n|/|p_n|$. In this work, the dominant pole theory is used as a reference in some examples to show the contribution of each pole in the rational approximation.

4 Test cases

4.1 Test case 1: dominant poles of a synthetic function

In this example, the pole dominance of a synthetic function is analyzed through DPM and RDPM; in addition, they are compared with the calculation of the area under the curve for each term of the synthetic function. A synthetic response of order 18 is considered, defined by (11), and this function has been used in [6] as an example. The used poles, residues and constant term are given in Table 1. The function is represented by 400 samples separated logarithmically between 100 Hz and 10 MHz. Figure 2a shows the magnitude of each term given by (11). Figure 2b shows the results given by DPM, RDPM and the area under the curve (Area) for each term. Table 2 shows the results of these measurements together with the damping coefficient for each pole. The curves DPM and Area agree on the most dominant pole, number 13. However, it can be seen that, at low frequencies, the most dominant pole is the RDPM measurement, i.e., number 2. Clearly, DPM weighs the pole peak, unlike the RDPM measurement, which weighs a relative measurement. The dominant pole measurement justifies the procedure presented in Sect. 3.2 for the rational approximation of smooth

Table 1 Synthetic frequency response coefficients

Pole number	$p_i/2\pi$	c_i
1	− 4500	− 3000
2	− 41,000	− 83,000
3	− 100 + j5000	− 5 + j7000
4	− 100 − j5000	− 5 − j7000
5	− 120 + j15000	− 20 + j18000
6	− 120 − j15000	− 20 − j18000
7	− 3000 + j35000	6000 + j45000
8	− 3000 − j35000	6000 − j45000
9	− 200 + j45000	40 + j60000
10	− 200 − j45000	40 − j60000
11	− 1500 + j45000	90 + j10000
12	− 1500 − j45000	90 − j10000
13	− 500 + j70000	50,000 + j80000
14	− 500 − j70000	50,000 − j80000
15	− 1000 + j73000	1000 + j45000
16	− 1000 − j73000	1000 − j45000
17	− 2000 + j90000	− 5000 + j92000
18	− 2000 − j90000	− 5000 − j92000

functions. According to the test case, the best measurement corresponds to the RDPM. Finally, for reference in Table 2, the calculation of the real poles for each complex pair is shown using (14) and (16).

4.2 Test cases 2: ATPDraw and ATP-EMTP (testing methodology)

Different models have been implemented in the ATP-EMTP program for the modeling of overhead transmission lines. In this section, the modeling of a single-phase and three-phase transmission line is implemented by using the JMarti model through the ATPDraw and ATP-EMTP program, where the concept of rational approximation using only real poles by VF is applied.

In the following, the testing methodology implemented in these cases is described. A flowchart for the testing methodology is shown in Fig. 3. It can be seen that the preprocessor ATPDraw [15] is used to generate the input files for the ATP-EMTP program.

With the LCC (line/cable parameters) modules in the ATP-Draw, it is possible to specify the geometrical and material data for an overhead line. Then, the corresponding electrical data are calculated automatically by the LINE CONSTANTS routine of ATP-EMTP [16].

In the LCC module, all of the parameters for the modeling of the overhead line are selected and established, and the parameters for the rational approximation of $\mathbf{Z}_c$ and $\mathbf{H}$ by using the JMarti fitting procedure (Bode-ATP) are specified.

The specified line data and the rational function-based models obtained are described in a library file **.lib*. This text file has a pre-defined format and contains a header describing the positions of the parameters, the ATP cards, and finally a trailer containing the specification of the parameters. The library file is included in the ATP input file [16].

The line parameters are obtained through a temporary file (**.AFT*). These parameters are used to obtain the corresponding approximations for $\mathbf{H}$ and $\mathbf{Z}_c$ using VF. During this process, it is possible to obtain the fitting of the JMarti model (Bode-ATP) with the purpose of comparing the rational approximations.

The simulation results obtained with JMarti-ATP (Bode-ATP) are saved. Finally, as seen in Fig. 3, an alternate simulation is carried out using the rational function-based models obtained with VF-RP (JMarti-ATP (VF-RP-2)). In this manner, it is possible to compare both simulations.

This testing methodology will be applied for the modeling of the overhead transmission lines in the next sections.

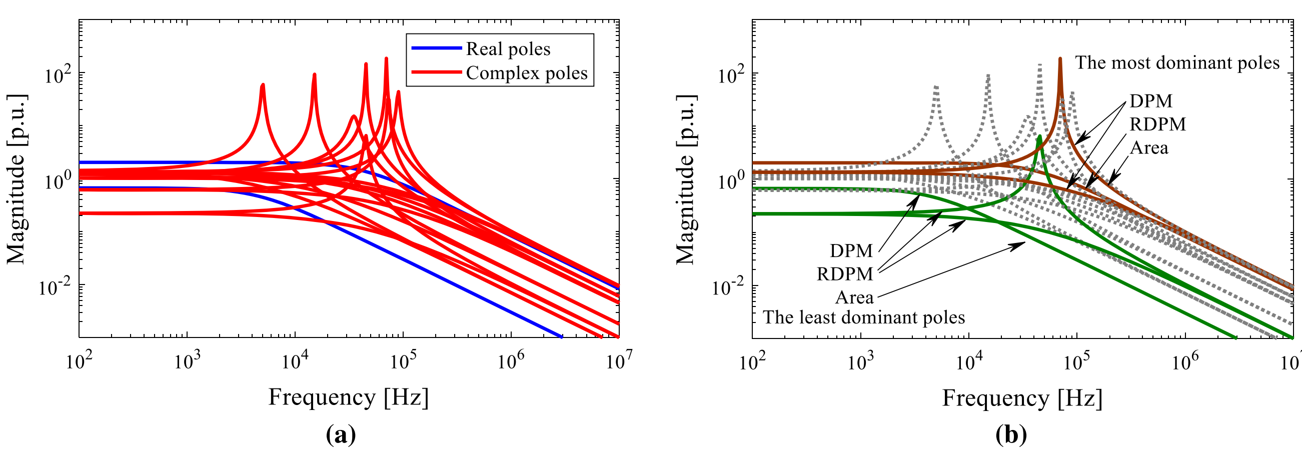

Fig. 2 **a** Each term for the synthetic frequency response, **b** dominant pole measurement for the synthetic function

Table 2 Damping coefficient, dominant pole measurement and real pole calculation

Pole number	ζ	Pole dominance			Real and complex Poles	Proposed real poles
		DPM $\lvert c_i \rvert$	RDPM $\lvert c_i \rvert / \lvert p_i \rvert$	Area under curve		
1	1.0000	0.0317	0.3293	0.0155	– 4500	– 4500
2	1.0000	0.8797	1.0000	0.3179	– 41,000	– 41,000
3	0.0199	0.0741	0.6914	0.0726	– 100 + j5000	– 5000.989
4	0.0199	0.0741	0.6914	0.0328	– 100 – j5000	– 5001.009
5	0.0079	0.1907	0.5927	0.1912	– 120 + j15000	– 15,000.469
6	0.0079	0.1907	0.5927	0.0724	– 120 – j15000	– 15,000.489
7	0.0854	0.4812	0.6383	0.3359	– 3000 + j35000	– 35,128.326
8	0.0854	0.4812	0.6383	0.1589	– 3000 – j35000	– 35,128.346
9	0.0044	0.6359	0.6586	0.6310	– 200 + j45000	– 45,000.434
10	0.0044	0.6359	0.6586	0.2008	– 200 – j45000	– 45,000.454
11	0.0333	0.1060	0.1097	0.0841	– 1500 + j45000	– 45,024.983
12	0.0333	0.1060	0.1097	0.0334	– 1500 – j45000	– 45,025.003
13	0.0071	1.0000	0.6657	1.0000	– 500 + j70000	– 70,001.775
14	0.0071	1.0000	0.6657	0.2901	– 500 – j70000	– 70,001.795
15	0.0136	0.4771	0.3045	0.4108	– 1000 + j73000	– 73,006.838
16	0.0136	0.4771	0.3045	0.1372	– 1000 – j73000	– 73,006.858
17	0.0222	0.9766	0.5055	0.7816	– 2000 + j90000	– 90,022.209
18	0.0222	0.9766	0.5055	0.2691	– 2000 – j90000	– 90,022.229

4.2.1 Single-phase transmission line modeling

The procedure presented in Sect. 3.2 for the rational approximation of smooth functions through VF is applied together with the testing methodology explained in the previous section for the modeling of a single-phase transmission line. Figure 4 shows the test case system used in the simulation where the current source is considered as an ideal 60 Hz.

Figure 5a shows the characteristic impedance data $\mathbf{Z}_{\mathrm{c}}$ and fitting deviations with Bode-ATP and VF in terms of absolute error with $N_{\mathrm{p}} = 14$. It is observed that VF has a better result in terms of this error. In this case, VF converges to only real poles in last iteration. This improvement in the fitting will be reflected in the time-domain simulation results of the event.

In Fig. 5b, the same result can be seen for the propagation function $\mathbf{H}$ with $N_{\mathrm{p}} = 11$ with Bode-ATP, VF (real and complex poles), VF-RP-2 (real and complex poles replaced by two), VF-RP-3 (real and complex poles replaced by three), and VF-RP-4 (real and complex poles replaced by four). In this case, VF converges to two pairs of complex poles. Once again, it is observed how VF has considerably improved the rational approximation in comparison with the asymptotic approach.

Figure 6a shows each term of the rational approximation given by VF for the fitting of the propagation function $\mathbf{H}$, while Fig. 6b shows the calculation of the dominant poles.

Table 3 presents a summary of the rational approximation of $\mathbf{H}$. It is observed that, for DPM, the most dominant pole is a complex pair; however, its damping factor ζ is very close to 1. Alternately, for the RDPM, the most dominant pole is a real one. This proves that the methodology for rational approximation of smooth functions with only real poles is a valid concept.

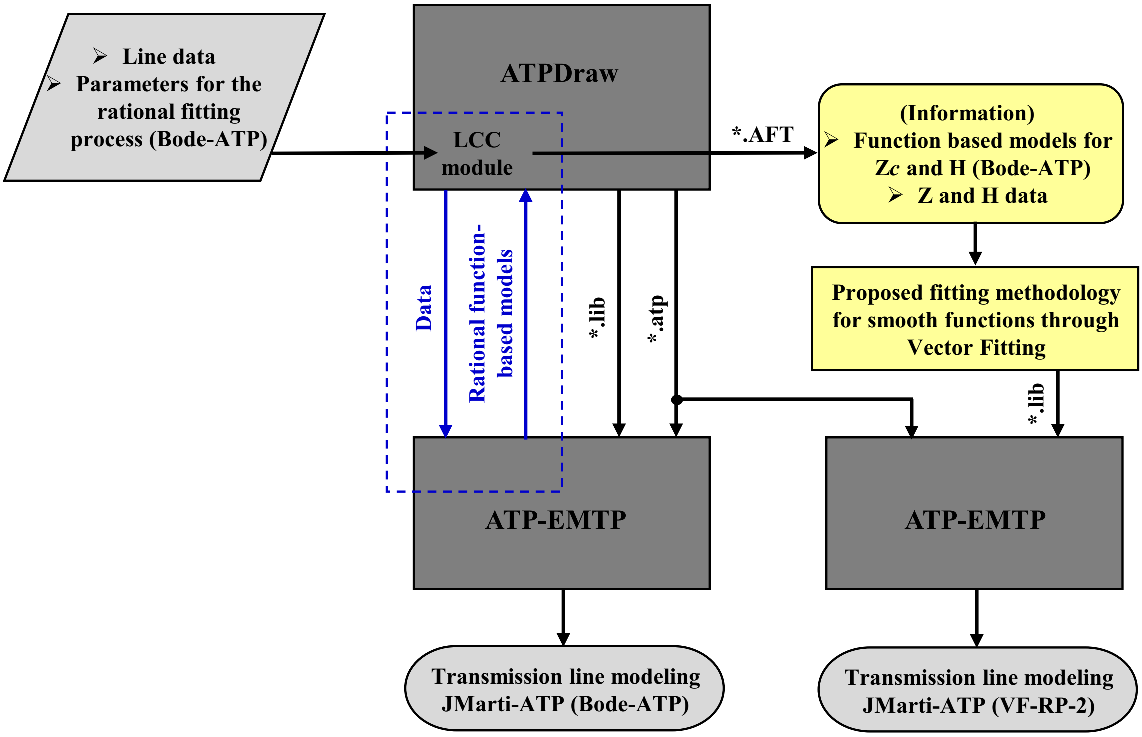

Fig. 3 Flowchart of the testing methodology

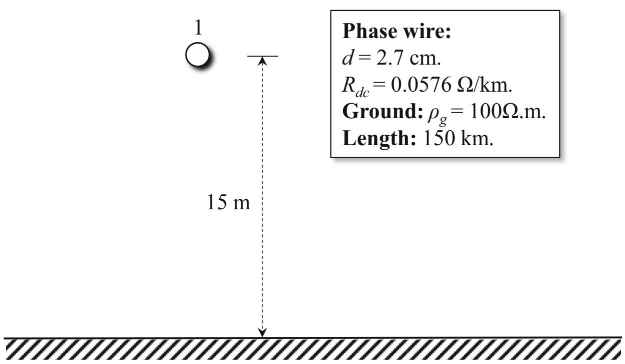

Fig. 4 Single-phase transmission line

The transient simulation of an open circuit in the time domain is presented in Fig. 7a. In this figure, the solution given by the NLT [17, 18] (V_2), the simulation with the ATP-EMTP program with Bode-ATP and the solution given by the ATP-EMTP using the rational approximation obtained with VF-PR-2 are presented. In addition, in Fig. 7b, the absolute error of the simulations is shown, taking as reference the solution given by the NLT. It can be seen that the precision of the JMarti model is increased by one order.

Finally, Fig. 8a, b shows a zoom of the transient response of the line in the open circuit. In these figures, it is possible to appreciate the accuracy enhancement given by the proposed methodology.

4.2.2 Three-phase horizontal transmission line modeling

In this section, the procedure presented in Sect. 3.2 for the rational approximation of smooth functions through VF is applied together with the testing methodology explained in Sect. 4.2 for the modeling of a three-phase horizontal transmission line. Figure 9 shows the test case system used in the simulation, where the current source is considered as an ideal 60 Hz.

Figure 10a shows the characteristic impedance data $\mathbf{Z}_{\mathrm{c}}$ and the fitting deviations with Bode-ATP and VF in terms of absolute error for each mode with $N_{\mathrm{p}} = 14$, 19 and 19. In all cases, the fitting order is set in VF according to the order obtained by the Bode-ATP routine. It is observed that VF has a better result in terms of this error. In these cases, VF converges to only real poles. A substantial accuracy enhancement is observed when VF is used.

Figures 10b and 11a, b show the rational approximations for the propagation function data $\mathbf{H}$ and the fitting deviations in terms of absolute error for each mode with $N_{\mathrm{p}} =$ 20, 17 and 16, respectively. These fitting orders are set in VF according to the order obtained by the Bode-ATP routine. In this figure, it can also be noticed that, for mode number 3, VF converges to only real poles. Once again, it is observed how VF has considerably improved the rational approximations in comparison with the asymptotic approach.

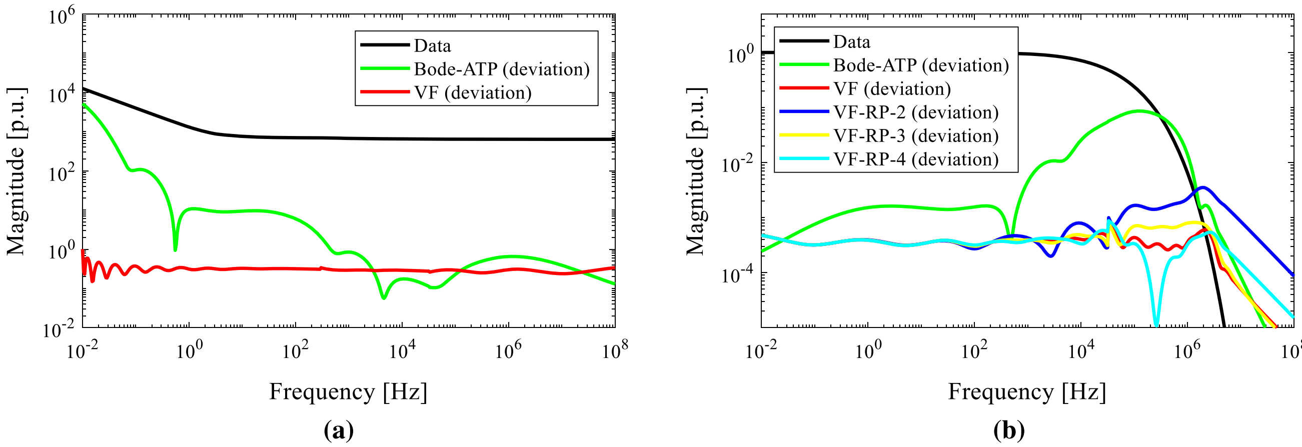

Fig. 5 **a** Characteristic impedance data $\mathbf{Z}_c$ and fitting deviations, **b** propagation function data **H** and fitting deviations

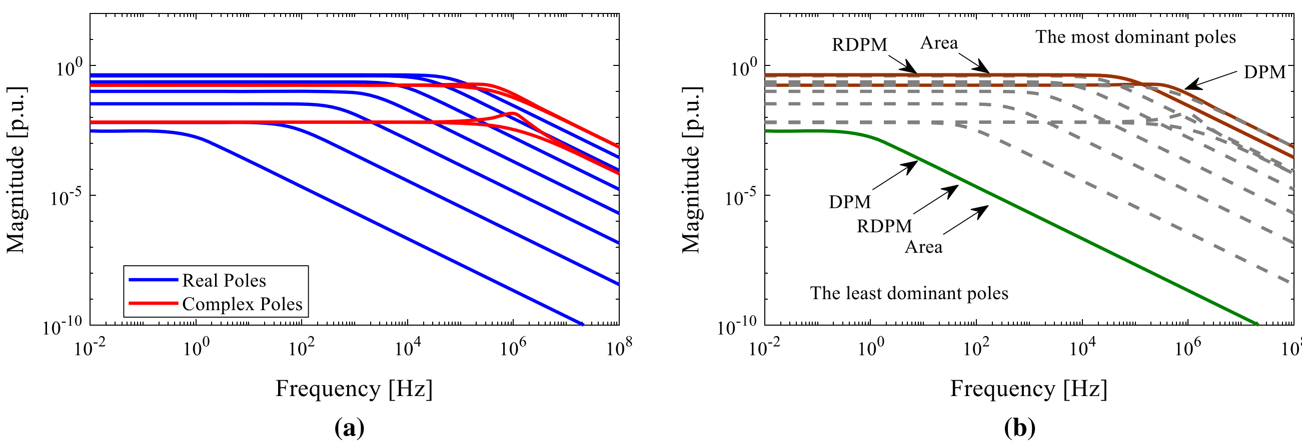

Fig. 6 **a** Each term of the rational approximation given by VF, **b** dominant pole measurement for **H**

Table 3 Damping coefficient and dominant pole measurement for **H**

Pole number	ζ	Pole dominance		
		DPM $\lvert c_i \rvert$	RDPM $\lvert c_i \rvert / \lvert p_i \rvert$	Area under curve
1	1.0000	2.96e−08	6.90e−03	2.09e−03
2	1.0000	5.04e−06	1.46e−02	8.33e−03
3	1.0000	1.98e−04	7.78e−02	5.33e−02
4	1.0000	2.77e-03	2.31e-01	1.82e-01
5	1.0000	2.33e-02	5.38e-01	4.65e-01
6	1.0000	1.26e-01	9.18e-01	8.58e-01
7	1.0000	3.95e-01	1.0000	1.0000
8	0.9048	1.0000	0.4001	0.45857
9	0.9048	1.0000	0.4001	0.43639
10	0.4527	0.0946	0.0150	0.01957
11	0.4527	0.0946	0.0150	0.01695

The transient simulations of the open circuit in the time domain are presented in Figs. 12a, 13a and 14a. In these figures, the solution given by the NLT [17, 18] (V_2), the simulation with the ATP-EMTP program with Bode-ATP and the solution given by the ATP-EMTP using the rational approximation obtained with VF-PR-2 for each phase are presented.

Finally, in Figs. 12b, 13b and 14b, the absolute error for each simulation is shown, taking as reference the solution given by the NLT. It can be seen that the precision of the JMarti model is increased by one order. In these figures, it is possible to appreciate the accuracy enhancement given by the proposed methodology.

5 Discussion

The calculated results in Sect. 4 show that the implementation of the VF method in the JMarti model as a fitting tool is possible. The implementation of VF in the model could increase the computational effort of the model because VF can deliver real and complex poles, which would yield complex states in the time domain. As known, the JMarti fitting technique leads to having only real poles and zeros and there-

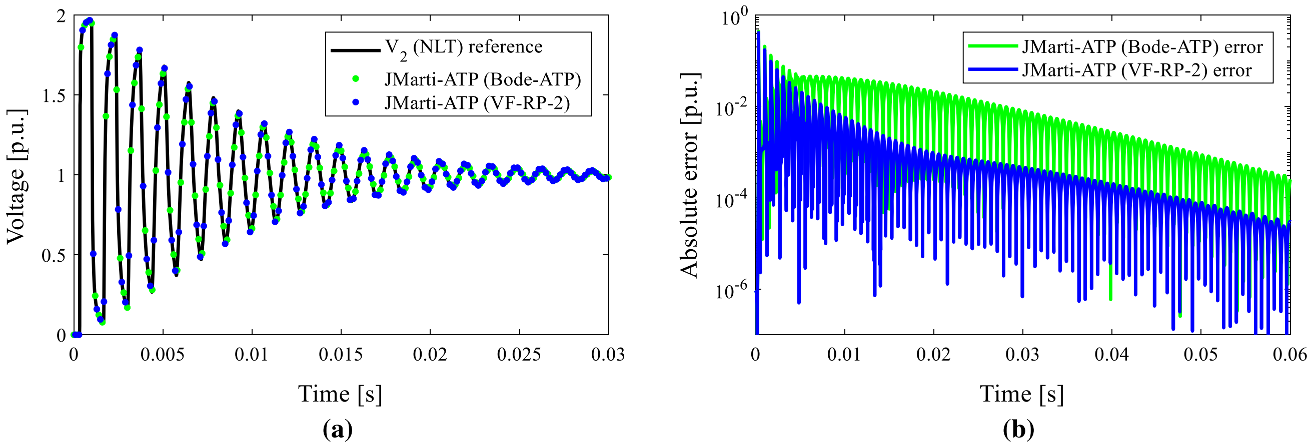

Fig. 7 **a** Open-circuit transient simulation, **b** absolute error in the time domain

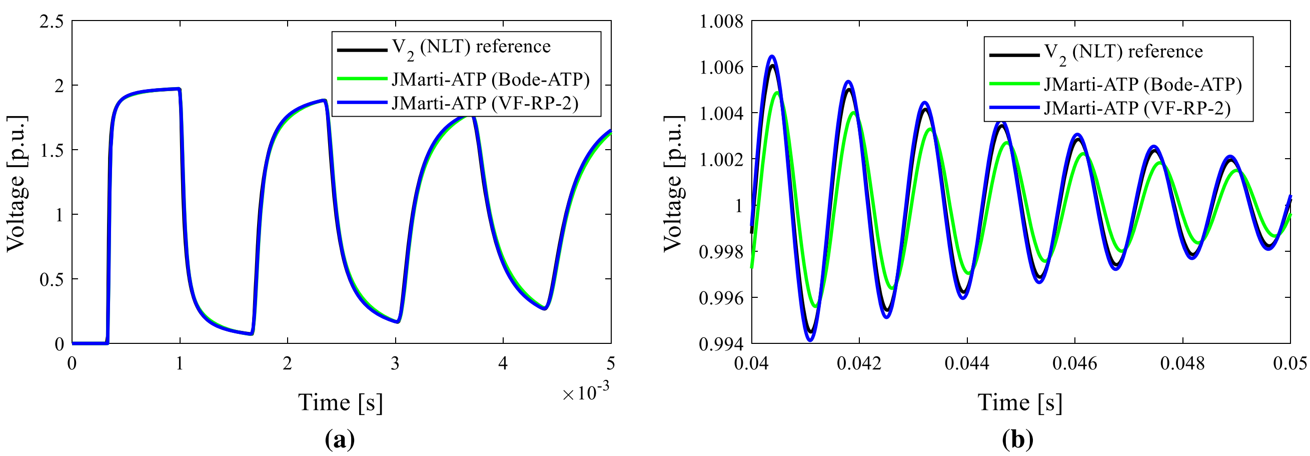

Fig. 8 **a** Zoom of the transient simulation of the open circuit, **b** zoom of the transient simulation of the open circuit

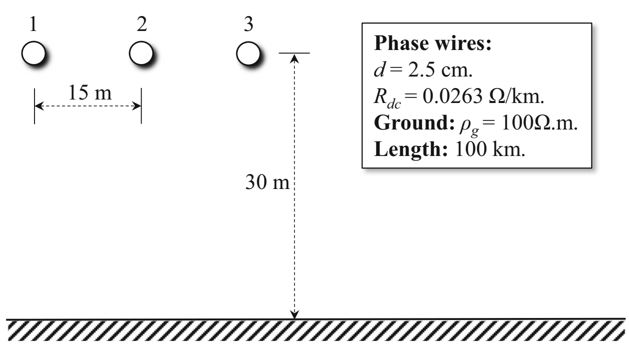

Fig. 9 Three-phase horizontal transmission line

fore real state variables. Therefore, the challenge was to find a fitting methodology that would maintain the attributes of the JMarti fitting and also improve the accuracy of the fitting.

Then, the proposed methodology of increasing the accuracy of the JMarti model by using only real poles through vector fitting represents an improvement in the model performance. Moreover, the rational fitting process inherits the main characteristics of VF: robustness, reliability, and accuracy. Another important characteristic is that, unlike JMarti fitting with VF, it is possible to set the order of the approximation; this gives the possibility of improving not only the accuracy of the model but also the numerical performance.

An important feature of the proposed methodology is that it can be used iteratively in the pole identification step of VF for the rational approximation of smooth frequency-domain functions with real poles. In our experience, some of the complex poles can be iteratively eliminated before the last iteration by replacing them with real poles at each iteration.

Regarding to the computational efficiency of the proposed methodology; in [13] the operation counts are shown for realizations considering *N* real poles, *N* complex poles and *N* complex poles using the real arithmetic technique [11]. It is concluded that the realization based on real poles is the most efficient. It is worth mentioning that the real poles can provide a solution with fewer oscillations than the one obtained with complex poles.

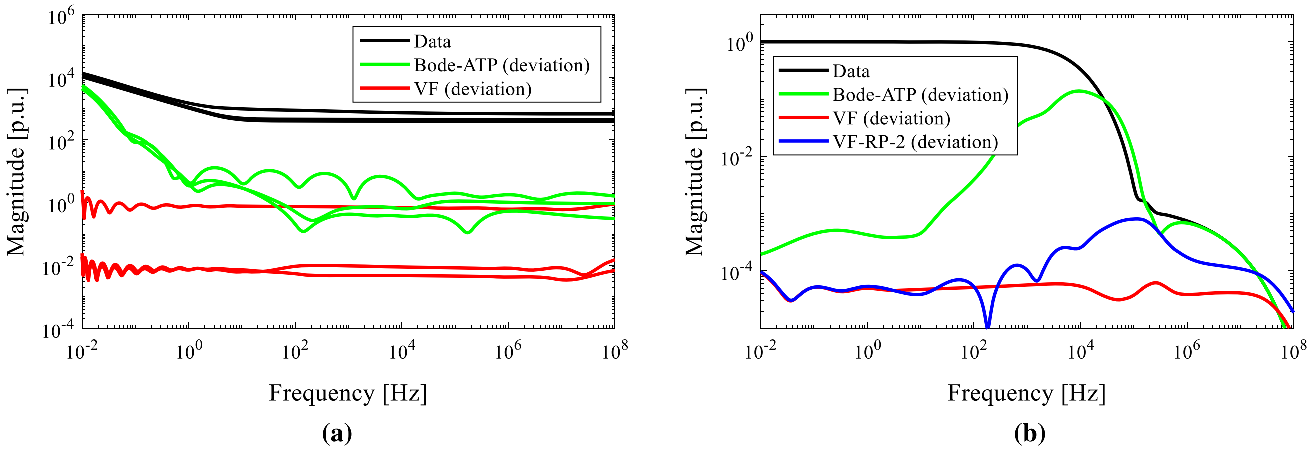

Fig. 10 **a** Characteristic impedance data $\mathbf{Z}_c$ and fitting deviations for each mode, **b** propagation function data $\mathbf{H}$ and fitting deviations for mode number 1

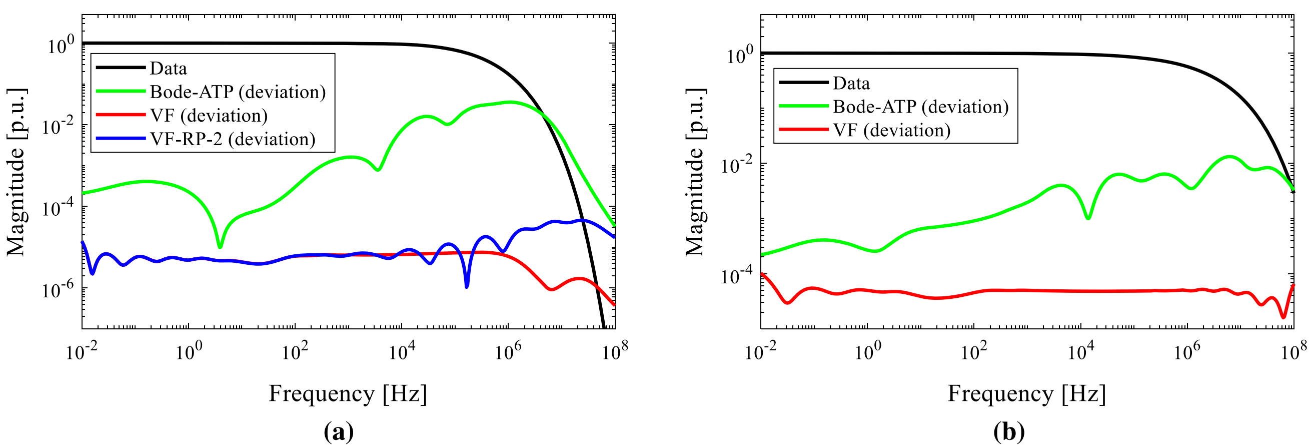

Fig. 11 **a** Propagation function data $\mathbf{H}$ and fitting deviations for mode number 2, **b** propagation function data $\mathbf{H}$ and fitting deviations for mode number 3

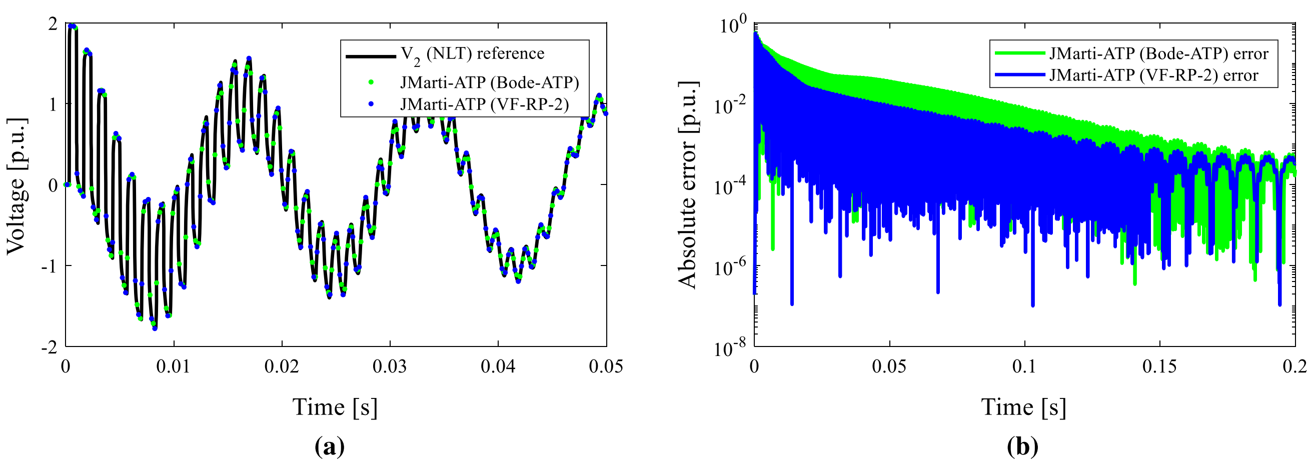

Fig. 12 **a** Open-circuit transient simulation for phase number 1, **b** absolute error in the time domain

In addition to the JMarti fitting procedure, only iterative methods such as Levenberg–Marquardt (LM) [19], the Gauss–Newton (GN) method [12], among others, can guarantee rational approximations with only real poles; because they can be implemented in pole-zero form, polynomial-form or pole-residue form. However, this type of methods

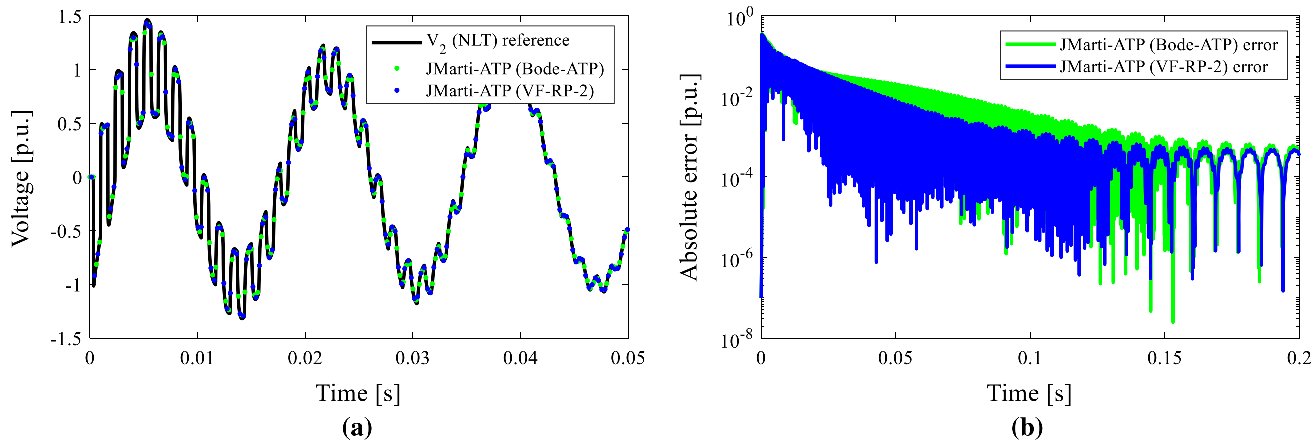

Fig. 13 **a** Open-circuit transient simulation for phase number 2, **b** absolute error in the time domain

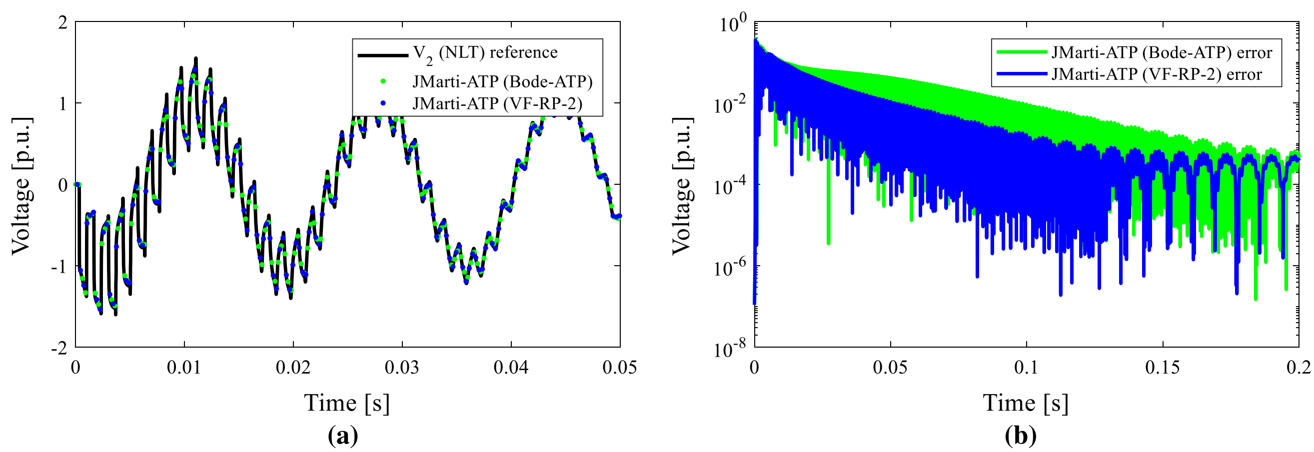

Fig. 14 **a** Open-circuit transient simulation for phase number 3, **b** absolute error in the time domain

can get easily stuck in a local minimum and take a lot of iterations. Moreover, the LM or GN efficiency is not better than the efficiency of relaxed vector fitting (RVF); in [12] we present a comparison between RVF and the damped GN (DGN) method. Commonly both methods end in the same global minimum.

Finally, we conclude that the presented methodology can be easily included in an EMTP-type program because it inherits the efficiency and robustness of VF; which has been demonstrated in the paper.

6 Conclusions

A general methodology to improve the accuracy of the JMarti model by using only real poles through VF was presented. The calculated results of the transmission line examples by means of the ATPDraw and ATP-EMTP program show that the methodology introduced for rational approximation with real poles through VF is a valid approach. The main conclusions are listed below:

1. By using only the real poles in the rational approximations, we obtain systems with only real states and therefore higher efficiencies from a computational point of view. Hence, it is important to have rational approximations that contain only real poles.
2. It is possible to include rational approximations of the transmission line parameters ($\mathbf{Z}_c$ and $\mathbf{H}$) in the ATP-EMTP program by means of VF, and thus, the accuracy of the JMarti model in the program can be increased.
3. Through the simulation cases in Sect. 4, it is verified that the methodology used for the rational approximation with only real poles is a valid and useful concept.
4. The use of VF leads to having rational approximations with lower errors compared to the JMarti fitting method.
5. The use of the proposed methodology for the rational approximation of smooth functions leads to negligible errors compared to the fitting of VF with real and complex poles.
6. Some of the complex poles can be iteratively eliminated before the last iteration by using the presented method-

ology in each iteration that VF performs to find the final poles of the fitting.

Acknowledgements Funding was provided by PRODEP (Grant No. UDG-PTC-1389).

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