ACCURATE MODELLING OF FREQUENCY-DEPENDENT TRANSMISSION LINES IN ELECTROMAGNETIC TRANSIENT SIMULATIONS

J.R. Marti, Member IEEE

University of British Columbia
Department of Electrical Engineering
Vancouver, B.C. V6T 1W5

ABSTRACT

The parameters of transmission lines with ground return are highly dependent on the frequency. Accurate modelling of this frequency dependence over the entire frequency range of the signals is of essential importance for the correct simulation of electromagnetic transient conditions. Closed mathematical solutions of the frequency-dependent line equations in the time domain are very difficult. Numerical approximation techniques are thus required for practical solutions. The oscillatory nature of the problem, however, makes ordinary numerical techniques very susceptible to instability and to accuracy errors. The methods presented in this paper are aimed to overcome these numerical difficulties.

I. INTRODUCTION

It has long been recognized that one of the most important aspects in the modelling of transmission lines for electromagnetic transient studies is to account for the frequency dependence of the parameters and for the distributed nature of the losses. Models which assume constant parameters (e.g. at 60 Hz) cannot adequately simulate the response of the line over the wide range of frequencies that are present in the signals during transient conditions. In most cases the constant-parameter representation produces a magnification of the higher harmonics of the signals and, as a consequence, a general distortion of the wave shapes and exaggerated magnitude peaks.

The magnification of the higher harmonics in constant-parameter representations can readily be seen from figs. 13 and 14 (described in more detail in Section VIII). These figures show the frequency response of the zero sequence mode of a typical 100-mi, 500 kV 3-phase transmission line under short-circuit and open-circuit conditions. Curves (I) correspond to the "exact" response calculated analytically from frequency-dependent parameters obtained from Carson's equations [1]. Curves (II) represent the response with constant, 60 Hz parameters.

Much effort has been devoted over the last ten years to the development of frequency-dependent line models for digital computer transient simulations. Some of the most important contributions are listed in references [2] to [8].

In theory, many alternatives are possible for the formulation of the solution to the exact line equations. In practice, however, as it is illustrated in figs. 11 and 12, the nature of a transmission line is such that its response as a function of frequency is highly oscillatory. As a consequence, the numerical problems that can be encountered in the process of solution are highly dependent on the particular approach.

The routines described in this paper avoid a series of numerical difficulties encountered in previous formulations. These routines are accurate, general, and have no stability problems. In the tests performed, over a wide range of line lengths (5 to 500 miles) for the zero and positive sequence modes, the same routines could accurately model the different line lengths and modes over the entire frequency range, from 0 Hz (d.c. conditions) to, for instance, 10^6 Hz. This is achieved without user intervention, that is, the user of these routines does not have to make value judgements to force a better fit at certain frequencies, line lengths, or modes. In transient simulations, the frequency-dependent representation of transmission lines required only 10-30% more computer time than the constant-parameter simulation.

II. TIME DOMAIN TRANSIENT SOLUTIONS

Even though the modelling of transmission lines is much easier when the solution is formulated in the frequency domain, for the study of a complete system with switching operations, non-linear elements, and other phenomena, step by step time domain solutions are much more flexible and general than frequency domain formulations.

Probably the best known example of time domain transient solutions is the Electromagnetic Transients Program (EMTP) first developed at Bonneville Power Administration (B.P.A.) from Dommel's basic work [9]. The widespread use of this program has proven its value and flexibility for the study of a large class of electromagnetic transient conditions.

The new frequency-dependent line model described in this paper has been tested in the University of British Columbia Version of the EMTP.

In the EMTP, multiphase lines are first decoupled through modal transformation matrices, so that each mode can be studied separately as a single-phase circuit. Frequency-independent transformation matrices are assumed in these decompositions. This procedure is exact in the case of balanced line configurations and still very accurate for transposed lines. In the more general case of unbalanced, untransposed lines, however, the modal transformation matrices are frequency dependent. Nevertheless, as concluded by Magnusson [10] and Wasley [11], it seems that is still possible in this case to obtain a reasonably good approximation under the assumption of constant transformation matrices.

Frequency-independent transformation matrices have been assumed in the present work.

III. SIMPLIFIED LINE MODEL

In Dommel's basic work it is assumed that the line has constant parameters and no losses. Under these simplifying assumptions the line equations are written directly in the time domain. (To account for the losses Dommel splits the total line resistance into three lumped parts, located at the middle and at the ends of the line). From d'Alembert's solution of the simplified wave equations and Bergeron's concept of the constant relationship between voltage and current waves travelling along the line, Dommel arrives at the equivalent circuit shown in fig. 1 for the line as seen from node k. An analogous model is obtained for node m. In this model R_c is the line characteristic impedance and $I_{kh}(t)$ is a current source whose value at time step t is evaluated from the known history values of the current and voltage at node m τ units of time earlier (τ is the travelling time).

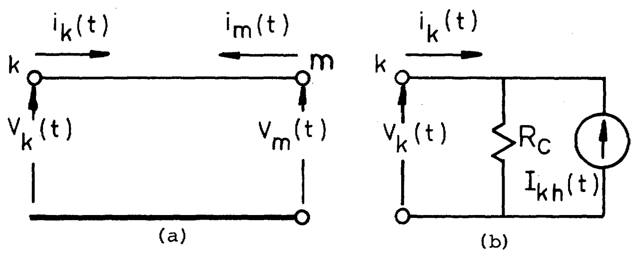

Fig. 1: Dommel's simplified line model. (a): Line mode. (b): Equivalent circuit at node k.

IV. FREQUENCY-DEPENDENT LINE MODEL: HISTORICAL REVIEW

When the frequency dependence of the parameters and the distributed nature of the losses are taken into account, it becomes very difficult, if not impossible in a practical way, to write the solution of the line equations directly in the time domain. This solution, however, can easily be obtained in the frequency domain, and is given by the well-known relations (e.g. Woodruff [12])

$$V_k(\omega) = \cosh[\gamma(\omega)\ell]V_m(\omega) - Z_c(\omega)\sinh[\gamma(\omega)\ell]I_m(\omega) \quad (1)$$

and

$$I_k(\omega) = \frac{1}{Z_c(\omega)}\sinh[\gamma(\omega)\ell]V_m(\omega) - \cosh[\gamma(\omega)\ell]I_m(\omega), \quad (2)$$

where

$$Z_c(\omega) = \sqrt{Z'(\omega)\cdot Y'(\omega)} = \text{characteristic impedance,} \quad (3)$$

$$\gamma(\omega) = \sqrt{\frac{Z'(\omega)}{Y'(\omega)}} = \text{propagation constant,} \quad (4)$$

$$Z'(\omega) = R'(\omega) + j\omega L'(\omega),\ Y'(\omega) = G'(\omega) + j\omega C'(\omega),$$

R' = series resistance, L' = series inductance,
G' = shunt conductance, C' = shunt capacitance
(primed quantities are in per unit length).

One of the first frequency-dependent line models for time-domain transient solutions was proposed by Budner [2], who used the concept of weighting functions in an admittance line model. The weighting functions in this model are, however, highly oscillatory and difficult to evaluate with accuracy.

In an effort to improve Budner's weighting-functions method, Snelson [3] introduced a change of variables to relate currents and voltages in the time domain in a way which is analogous to Bergeron's interpretation of the simplified wave equations. The new variables are defined as follows:

forward travelling functions:

$$f_k(t) = v_k(t) + R_1 i_k(t), \quad (5)$$

$$f_m(t) = v_m(t) + R_1 i_m(t), \quad (6)$$

and backward travelling functions:

$$b_k(t) = v_k(t) - R_1 i_k(t), \quad (7)$$

$$b_m(t) = v_m(t) - R_1 i_m(t), \quad (8)$$

where R_1 is a real constant defined as $R_1 = \lim_{\omega\to\infty} Z_c(\omega)$.

Equations 5 to 8 are then transformed into the frequency domain and compared with the line solution as given by eqns. 1 and 2. This idea was further developed by Meyer and Dommel [4] and resulted in the weighting functions $a_1(t)$ and $a_2(t)$ shown in fig. 2, and the equivalent line representation shown in fig. 3 for node k. In this circuit the backward travelling

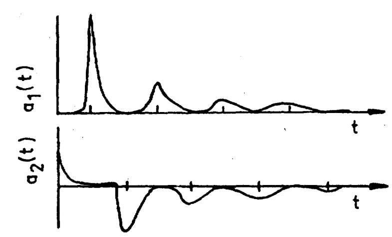

Fig. 2: Weighting functions in Meyer and Dommel's formulation.

function $b_k(t)$ is obtained from the "weighted" past history of the currents and voltages at both ends of the line and is given by the convolution integral

$$b_k(t) = \int_0^\infty \{f_m(t-u)a_1(u) + f_k(t-u)a_2(u)\}du\,. \quad (9)$$

An analogous equivalent circuit and convolution integral are obtained for node m.

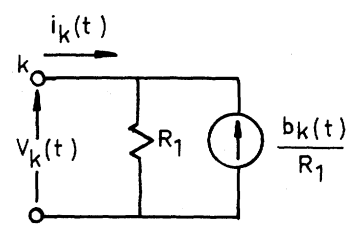

Fig. 3: Meyer and Dommel's frequency-dependent line model at node k.

Meyer and Dommel's formulation of the weighting function technique represented a considerable improvement over other weighting function methods, and has given reliable results in many cases of transient studies performed at B.P.A. This technique, however, still presents some numerical disadvantages. One of these disadvantages is the relatively time consuming process required to evaluate integral 9 at each time step of the solution. In the case study presented in [4], the running time per step for the case with frequency dependence was about three times longer than the time with no frequency dependence. Another disadvantage is the difficulty in evaluating the contribution of the tail portions of $a_1(t)$ and $a_2(t)$ to the convolution integral of eqn. 9. The successive peaks in these functions tend to become flatter and wider for increasing values along the t-axis.

Some of the main problems encountered with this method have been accuracy problems at low frequencies, including the normal 60 Hz steady state. These problems seem to be related to the evaluation of the tail portions of the weighting functions. Also, an error analysis seemed to indicate that the function $a_2(t)$ is more difficult to evaluate with sufficient accuracy than the function $a_1(t)$.

As suggested by Meyer and Dommel, the meaning of the weighting functions $a_1(t)$ and $a_2(t)$ can be visualized physically from the model shown in fig. 4. In this model the line is excited with a voltage impulse $\delta(t)$ and is terminated at both ends by the resistance R_1 of eqns. 5 to 8. Under these conditions $a_1(t)$ is directly related to the voltage at node m and $a_2(t)$ to the voltage at node k. From this model, it can be seen that the successive peaks in these functions (fig. 2) are produced by successive reflections at both ends of the line.

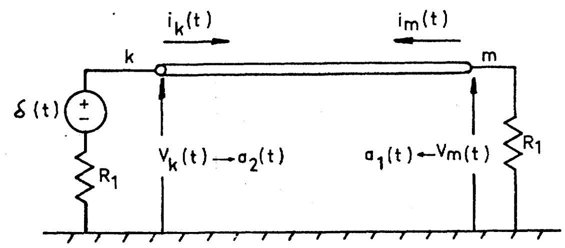

Fig. 4: Physical interpretation of Meyer and Dommel's weighting functions.

V. FREQUENCY-DEPENDENT LINE MODEL: NEW FORMULATION

The development of this model can be best explained from the physical interpretation of the concept of the weighting functions developed by Meyer and Dommel.

From the system shown in fig. 4 it can be seen that if the resistance R_1 is replaced by an equivalent network whose frequency response is the same as the characteristic impedance of the line, $Z_c(\omega)$, there will be no reflections at either end of the line. If such an equivalent network can be found, the new $a_1(t)$ weighting function will have only the first spike and the function $a_2(t)$ will become zero. This is shown in fig. 5. The form of the new weighting functions is shown in fig. 6. With this new model the problem of the tail portions and of the accurate determination of $a_2(t)$ are thus eliminated.

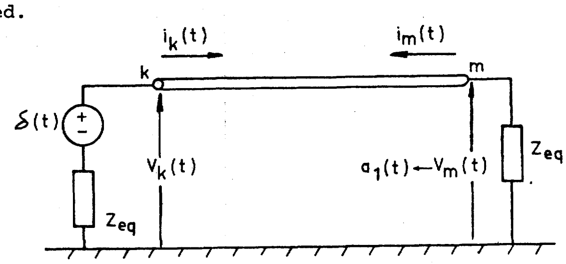

Fig. 5: Physical interpretation of the function $a_1(t)$ in the new formulation.

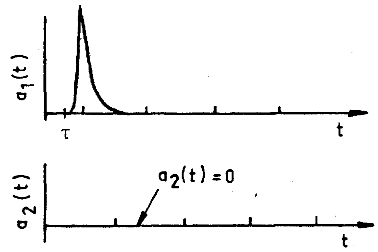

Fig. 6: Weighting functions $a_1(t)$ and $a_2(t)$ in the new formulation.

VI. MATHEMATICAL DEVELOPMENT OF THE NEW MODEL

In order to replace R_1 by Z_{eq} for the generation of the new weighting functions, the forward and backward travelling functions (eqns. 5 to 8) can be defined in the frequency domain as

$$F_k(\omega) = V_k(\omega) + Z_{eq}(\omega) I_k(\omega) \tag{10}$$

$$F_m(\omega) = V_m(\omega) + Z_{eq}(\omega) I_m(\omega) \tag{11}$$

and

$$B_k(\omega) = V_k(\omega) - Z_{eq}(\omega) I_k(\omega) \tag{12}$$

$$B_m(\omega) = V_m(\omega) - Z_{eq}(\omega) I_m(\omega) , \tag{13}$$

where $Z_{eq}(\omega)$ = impedance of linear network approximating $Z_c(\omega)$.

Comparing eqns. 10 to 13 with the general line solution in the frequency domain (eqns. 1 and 2), it follows that

$$B_k(\omega) = A_1(\omega) F_m(\omega) \tag{14}$$

and

$$B_m(\omega) = A_1(\omega) F_k(\omega) , \tag{15}$$

where

$$A_1(\omega) = e^{-\gamma(\omega)\ell} = \frac{1}{\cosh[\gamma(\omega)\ell] + \sinh[\gamma(\omega)\ell]} . \tag{16}$$

The time domain form of $A_1(\omega)$ is the function $a_1(t)$ shown in fig. 6. The time domain form of eqns. 14 and 15 is given by the convolution integrals

$$b_k(t) = \int_\tau^\infty f_m(t-u) a_1(u) du \tag{17}$$

and

$$b_m(t) = \int_\tau^\infty f_k(t-u) a_1(u) du. \tag{18}$$

The lower limit of these integrals is τ because, as it can be seen from fig. 6, $a_1(t)=0$ for $t<\tau$. (The time delay τ represents the travelling time of the fastest frequency component of the injected impulse.)

From eqns. 17 and 18 it can be seen that the values of b_k and b_m at time step t are completely defined from the past history values of the functions f_m and f_k, as long as the integration step Δt of the network solution is smaller than τ. With b_k and b_m known, the time domain form of eqns. 12 and 13 leads directly to the desired equivalent circuits at the line ends. That is, let

$$b_k(t) = E_{kh} \text{ (from history)} \tag{19}$$

and

$$b_m(t) = E_{mh} \text{ (from history)}, \tag{20}$$

then from eqns. 12 and 13,

$$v_k(t) = e_k(t) + E_{kh} \tag{21}$$

and

$$v_m(t) = e_m(t) + E_{mh}, \tag{22}$$

where $e_k(t)$ and $e_m(t)$ are the voltages across the network Z_{eq}. After converting to a modal representation, eqns. 21 and 22 give at each time step t the equivalent line models shown in fig. 7.

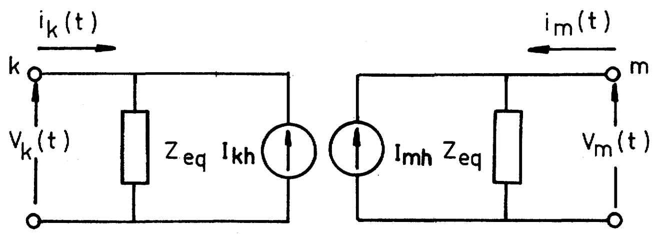

Fig. 7: New frequency-dependent line models at nodes k and m.

Synthesis of the Characteristic Impedance

The network Z_{eq} representing the line characteristic impedance $Z_c(\omega)$ is simulated by a series of Resistance-Capacitance (R-C) parallel blocks (Foster I network realization), as shown in fig. 8.

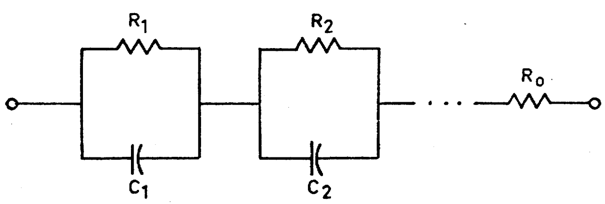

Fig. 8: Foster I realization of $Z_c(\omega)$.

The number of R-C building blocks is determined automatically by the approximating routine and depends on the particular line and mode being simulated. For the reference line studied in this paper the number of R-C blocks turned out to be 8 for the zero sequence and 9 for the positive sequence.

For the synthesis of Z_{eq} the tabular function $Z_c(\omega)$ evaluated from eqn. 3 (with the frequency dependent parameters obtained, for instance, from Carson's equations) is first approximated in the complex plane ($s=\sigma+j\omega$) by a rational function of the form

$$Z_{eq}(s) = \frac{N(s)}{D(s)} = H\,\frac{(s+z_1)(s+z_2)\ldots(s+z_n)}{(s+p_1)(s+p_2)\ldots(s+p_n)} \,. \tag{23}$$

The break points z_i and p_i of this function are real, positive, and simple. The values of the parameters of the R-C equivalent network are obtained by expanding eqn. 23 into a series of simple fractions:

$$Z_{eq}(s) = k_o + \frac{k_1}{s+p_1} + \frac{k_2}{s+p_2} + \ldots + \frac{k_n}{s+p_n} \,, \tag{24}$$

from where, in fig. 8,

$$R_o = k_o$$
$$R_i = k_i/p_i$$
$$C_i = 1/k_i$$

for i = 1, 2, ... , n.

The idea of approximating the line characteristic impedance by R-C combinations (even though by a much simpler model) has been suggested by Groschupf [13], who came to the conclusion that this approximation was not necessary, and that the frequency dependence of the characteristic impedance can usually be neglected. However, it has been found in the present work that, even though the accurate simulation of $Z_c(\omega)$ is not as essential for open-ended lines, it is actually very significant in the simulation of short circuits.

Weighting Function and Past History Convolution

In order to obtain the current sources in the equivalent circuits of fig. 7, it is necessary to evaluate the weighted history functions given by eqns. 17 and 18. As noted by Semlyen ([5], [7], and [8]) the process of evaluating convolution integrals having the form of eqns. 17 and 18 can greatly be accelerated if the corresponding weighting (or "transfer") functions can be expressed as a sum of exponential terms. That is, if in general the convolution integral at time step t has the the form

$$s(t) = \int_T^{\infty} f(t-u)\,k e^{-\alpha(u-T)}du, \tag{25}$$

then s(t) can be directly obtained from the known value $s(t-\Delta t)$ at the previous time step and the known history of f at T and $(T+\Delta t)$ units of time earlier:

$$s(t) = m s(t-\Delta t) + p f(t-T) + q f(t-T-\Delta t), \tag{26}$$

where m, p, and q are constants depending on k, α, the integration step Δt, and the numerical interpolation technique. (This property is also applied by Meyer and Dommel [4] to evaluate integral 9 in the tail portion of the weighting functions $a_1(t)$ and $a_2(t)$).

Recursive evaluation of the convolution integrals has also been adopted in this work. However, as explained in Section VII, the numerical process to approximate $a_1(t)$ as a sum of exponentials is different from the one proposed by Semlyen and avoids the numerical difficulties mentioned in reference [9].

From fig. 6 it can be seen that $a_1(t)$ can be expressed as

$$a_1(t) = p(t-\tau), \tag{27}$$

where p(t) has the same form as $a_1(t)$, but is displaced τ units of time towards the origin. From the shifting property of the Fourier Transform, the corresponding frequency domain form of eqn. 27 is

$$A_1(\omega) = P(\omega)e^{-j\omega\tau}. \tag{28}$$

The function P(s) corresponding to $P(\omega)$ in the complex plane is approximated by a rational function of the form

$$P_a(s) = \frac{N(s)}{D(s)} = H\,\frac{(s+z_1)(s+z_2)\ldots(s+z_n)}{(s+p_1)(s+p_2)\ldots(s+p_m)} \,, \tag{29}$$

where, since $A_1(\omega)$ corresponds to the response of a passive physical system and tends to zero when $\omega\to\infty$, the number of zeroes must be smaller than the number of poles, and the real part of the poles must lie in the left-hand side of the complex plane.

After a partial-fraction expansion of eqn. 29 and subsequent transformation into the time domain, the function approximating $a_1(t)$ becomes

$$a_{1a}(t) = [k_1 e^{-p_1(t-\tau)} + k_2 e^{-p_2(t-\tau)} + \ldots + k_m e^{-p_m(t-\tau)}]\,u(t-\tau), \tag{30}$$

from which the past history integrals (eqns. 17 and 18) can be evaluated recursively.

The number of exponentials in the approximation depends on the particular line and mode. Table 1 shows the number of exponentials used to simulate the reference line for the zero and positive sequence modes and for different lengths.

5 mi		30 mi		100 mi		500 mi	
zero	pos.	zero	pos.	zero	pos.	zero	pos.
14	12	15	14	13	15	12	13

Table 1: Number of exponentials for the simulation of $a_1(t)$ for reference line.

VII. NUMERICAL TECHNIQUES

As indicated earlier, in order to simulate the characteristic impedance by an R-C equivalent network and to allow recursive evaluations of the past history convolution integrals, the frequency domain functions $Z_c(\omega)$ and $A_1(\omega)$ are approximated by rational functions.

The problem of finding a rational function to simulate the response of a network is studied in network synthesis theory. There are different numerical techniques to approximate a tabular function of frequency by means of a rational fraction of polynomials (e.g. Karni [14]).

However, most of the traditional techniques (for example, Butterworth's, Chebyshev's, Lagrange's) have mainly been applied to particular classes of problems, such as ideal filter responses. Of more recent development are more general numerical techniques, such as least-square optimizations and optimum search (e.g. gradient) algorithms.

Despite their merit for rational approximations of specific functions, programs using these routines require a series of control parameters and adjustments that depend on the particular function approximated. One of the main reasons for this is that the degree of the approximating polynomials is established beforehand and then the rational function is "forced" to fit the given curve. Specification of polynomials of larger or smaller degrees than actually required for the given function often results in numerical instability and accuracy problems. These problems are mentioned by Semlyen [8], who applies a least squares technique to simulate the system response function.

In the modelling of frequency-dependent transmission lines the form of the functions to be approximated depends on the particular line, its length, and the particular mode. An approximating function "tailor-cut" for a specific case will not generally represent the best solution for other cases.

The technique employed in this work avoids the above-mentioned problems by allowing the approximating function to "freely" adapt itself to the form of the function being approximated. This technique is based on an adaptation of the simple concept of asymptotic fitting of the magnitude function, first introduced by Bode [15]. During the process of approximation, the poles and zeros of the rational approximating function are successively allocated, as needed, while following the approximated function from zero frequency to the highest frequency at which the magnitude of the approximated function becomes practically zero or constant. The entire frequency range is thus considered and a uniformly accurate approximation is obtained. Since the poles and zeros are allocated when needed, the degree of the approximating polynomials is not pre-established, but determined automatically by the routine.

Another problem mentioned by Semlyen is the occurrence of ripples or local peaks in the approximating function. This problem is avoided here by allowing only real poles and zeros.

Some Analytical Considerations

Phase Functions:

The rational functions (23) and (29) determined by the method of asymptotic approximation have no zeros in the right-hand side of the complex plane. Under these conditions, it is shown in Fourier Transform Theory (e.g. Papoulis [16]) that the phase function is uniquely determined from the magnitude function and that the rational function belongs to the class of minimum-phase-shift functions. The agreement between the phases of $P(\omega)$ and $Z_c(\omega)$, and the phases of the corresponding rational approximations obtained in the present work shows the correctness of the minimum-phase-shift approximations.

Causality Condition:

The rational approximations $P_a(s)$ in eqn. 29 and $Z_{eq}(s)$ in eqn. 23 tend to a constant for $s=j\omega$ when $\omega\to\infty$, and have no poles in the right-hand side of the complex plane. These conditions are enough (e.g. Popoulis [16]) to assure that the corresponding time domain functions are causal (function=0 for $t<0$) and thus correspond to the response of a physical passive system.

VIII. NUMERICAL RESULTS

Simulation of the Characteristic Impedance and Weighting Function

Figs. 9 and 10 show the comparison between the magnitudes of $Z_c(\omega)$ and $A_1(\omega)$ for the zero and positive sequence modes of the reference 100-mi line, and the corresponding rational approximations obtained using the techniques described in this paper. The agreement shown in these figures was also found for other line lengths (5 to 500 mi) and for the corresponding phase functions.

Tests

The accuracy of the new frequency-dependent line model has been tested by analytical and field test comparisons.

Analytical Comparisons:

i) Frequency Domain Tests:

The accuracy of the new model can easily be tested in the frequency domain by connecting a single frequency voltage source at the sending end of the line, with the receiving end open or short circuited. The response of the line under these conditions can be calculated analytically from the functions $A_1(\omega)$ and $Z_c(\omega)$, and the results can be compared with those obtained using the corresponding rational approximations. These comparisons can be made over the entire frequency range.

From eqns. 1 and 2, the short-circuit current at the sending end of the line is given by

$$I_k = \frac{E_s}{Z_c}\frac{1 + A_1^2}{1 - A_1^2}, \tag{31}$$

where E_s is the applied voltage source. Similarly, the open-circuit voltage at the receiving end is given by

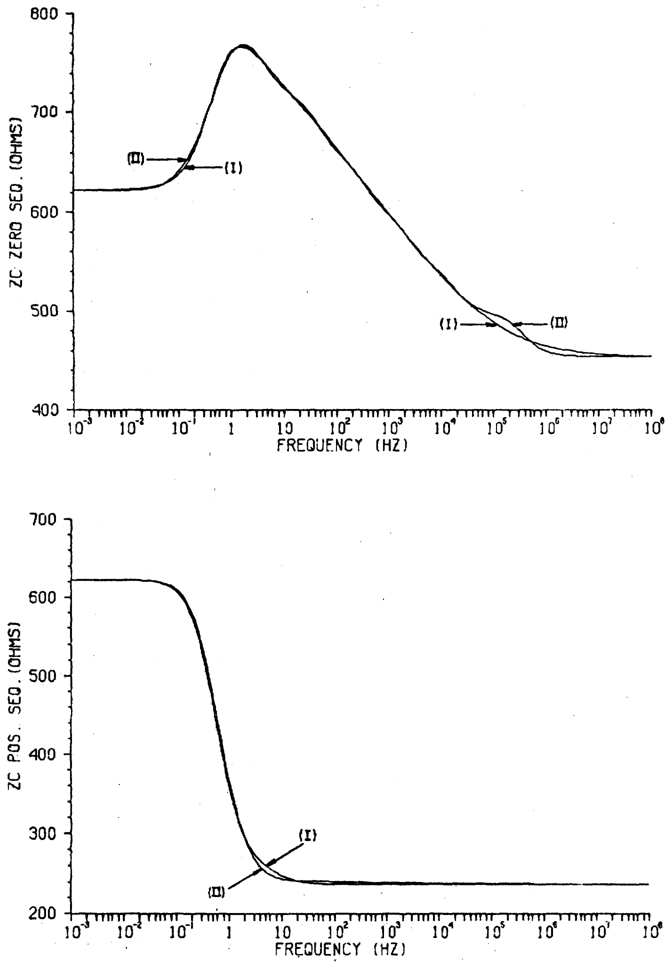

Fig. 9: Simulation of the characteristic impedance. Curves (I): Exact parameters. Curves (II): New model parameters.

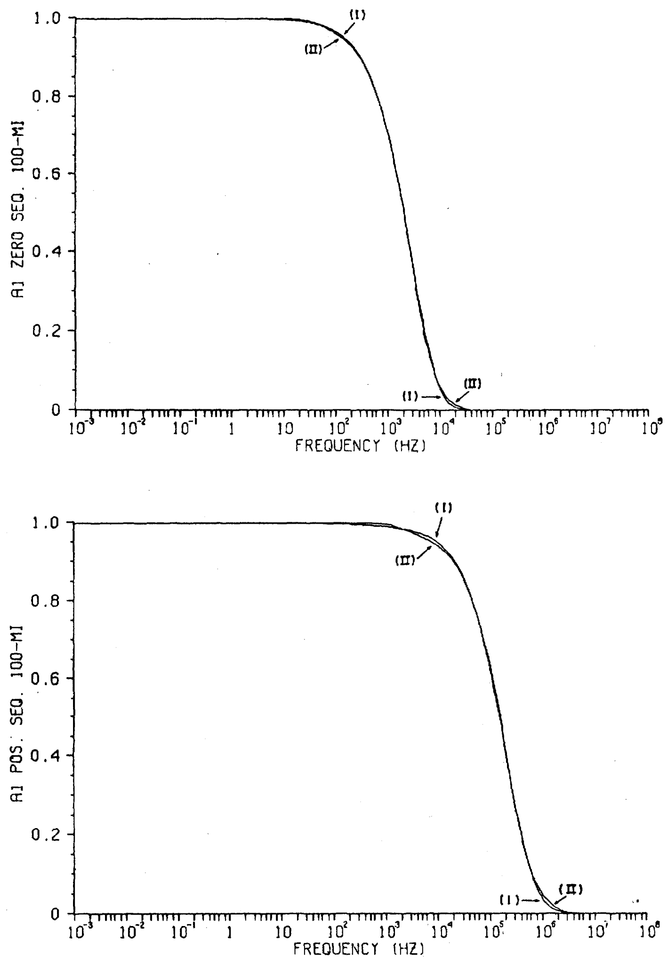

Fig. 10: Simulation of the weighting function. Curves (I): Exact parameters. Curves (II): New model parameters.

$$V_m = E_s \frac{2A_1}{1 + A_1^2} . \qquad (32)$$

It is interesting to note from this last equation that the open circuit voltage is independent of the characteristic impedance. This explains why some frequency dependence models that neglect the frequency dependence of the characteristic impedance can give acceptable results if they are only tested for open-circuit conditions. On the other hand, as can be seen from eqn. 31, the correct modelling of Z_C is very important for short-circuit conditions.

The results of these comparisons are shown in figs. 11 and 12 for the zero sequence mode and a length of 100 miles. These comparisons were also made for other line lengths (from 5 to 500 miles), as well as for the positive sequence mode, with similarly good agreements. The same agreement was also found for the corresponding phase angles.

Figs. 13 and 14 show the comparison between the responses obtained using exact parameters and those assuming constant 60 Hz parameters. The limitations of the constant-parameter model for the simulation of the lower and higher frequencies is clearly illustrated in these figures.

The magnification of the higher harmonics by the constant-parameter model can also clearly be seen in the transient simulations shown in figs. 15 and 16. These figures compare the simulations using the new line model and the constant-parameter model for two cases of open-circuited line energizations. In fig. 15 the zero sequence mode of the 100-mi reference line is energized with a sinusoidal, 60 Hz, voltage source, with the peak voltage applied at t=0. In fig. 16 the line mode is energized with a unit voltage step.

ii) Time Domain Tests:

The validity and accuracy of the new line model in time domain simulations can also be assessed from single frequency open and short circuit conditions. For this purpose, the line represented by its frequency-dependent transient model was energized by a single frequency sinusoidal voltage source. Starting from the correct a.c. initial conditions (so that no disturbances exist) transient simulations using the EMTP were run. Under the indicated conditions, the time domain solutions must be perfectly sinusoidal waves with magnitude and phase as given by eqns. 31 and 32. These tests were performed for the different line lengths and modes and for frequencies along the entire frequency range. The results had the correct sinusoidal waveforms and were in complete agreement with the magnitude and phase values previously obtained in the frequency tests.

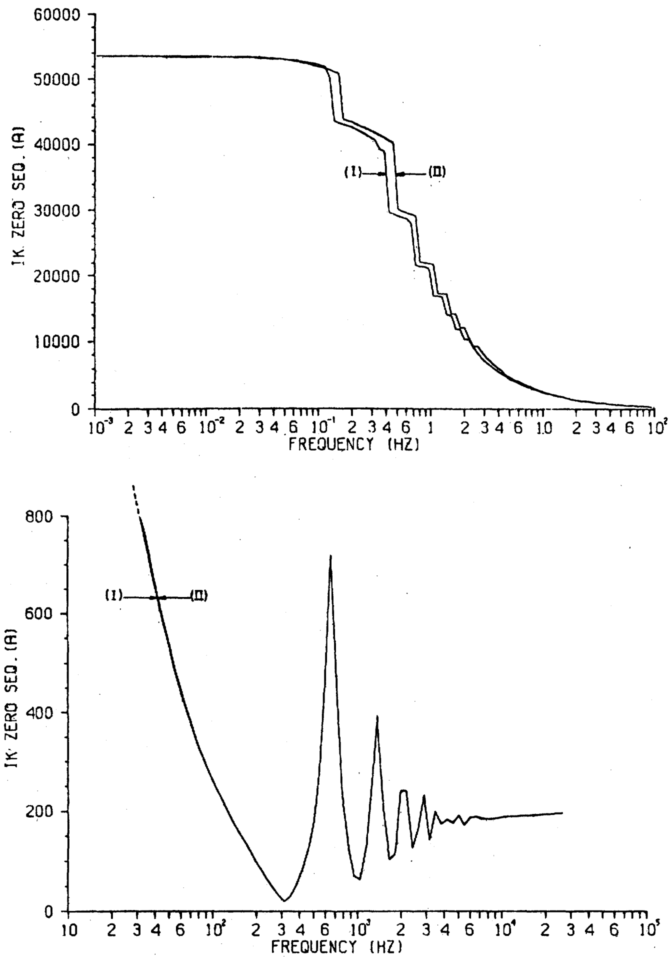

Fig. 11: Short-circuit frequency response. (Source voltage = 100 kV, I_k in amperes) Curves (I): Exact parameters. Curves (II): New model parameters.

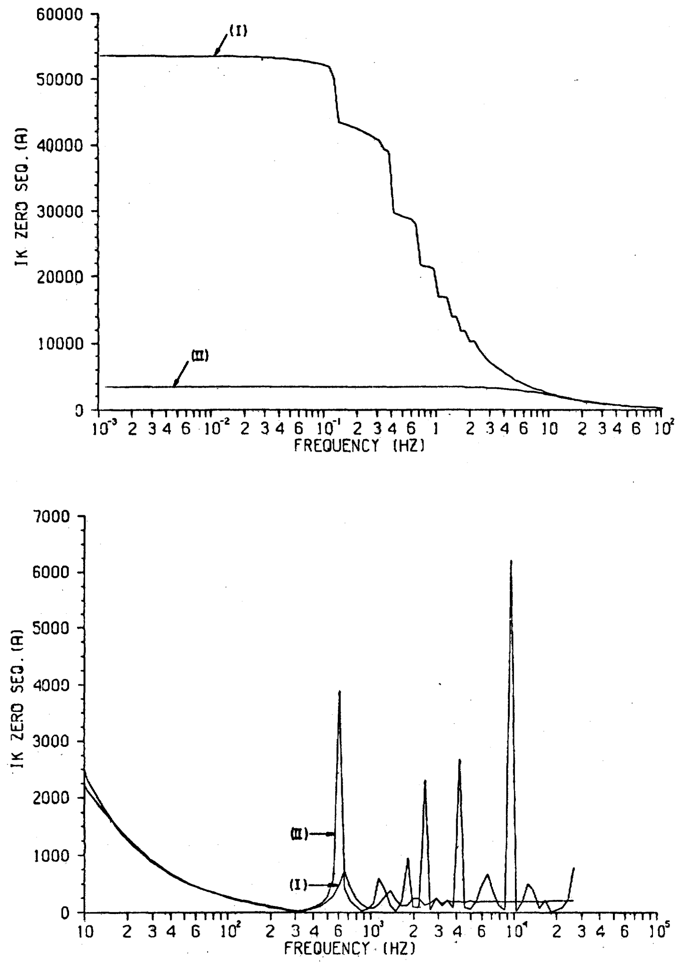

Fig. 13: Short-circuit frequency response. (Source voltage = 100 kV, I_k in amperes) Curves(I): Exact Parameters. Curves (II): Constant, 60 Hz parameters.

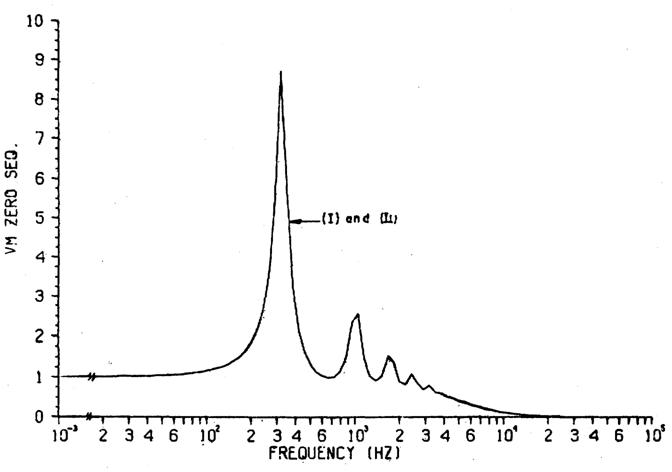

Fig. 12: Open-circuit frequency response. (Source voltage = 1.0). Curve (I): Exact parameters Curve (II): New model parameters.

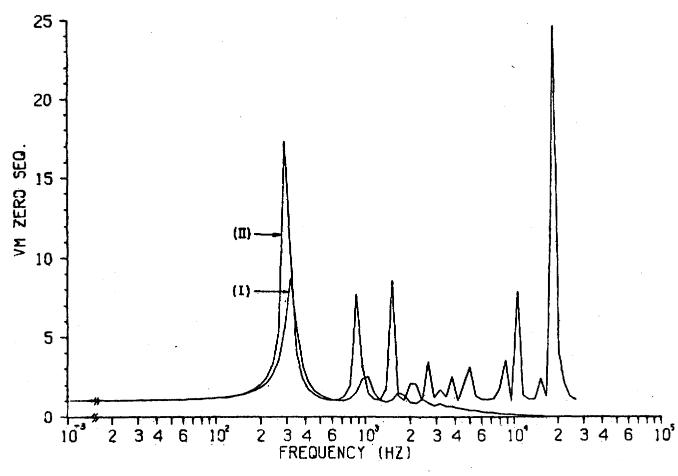

Fig. 14: Open-circuit frequency response. (Source voltage = 1.0). Curve (I): Exact parameters. Curve (II): constant, 60 Hz parameters.

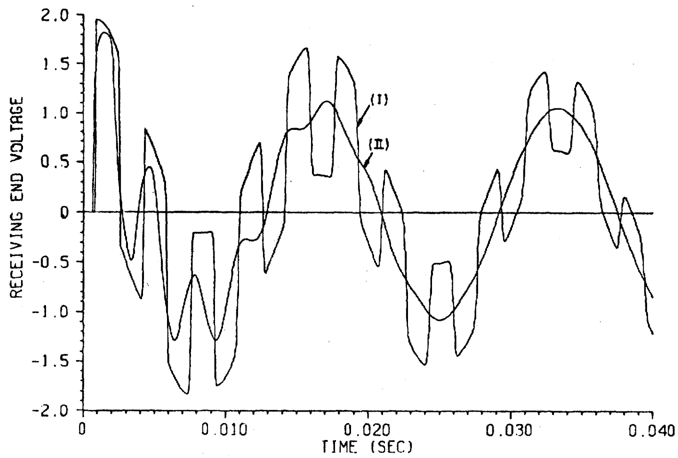

Fig. 15: Sinusoidal energization of open-circuited line (peak voltage at t=0). Curve (I) Constant, 60 Hz parameters. Curve(II):New model parameters.

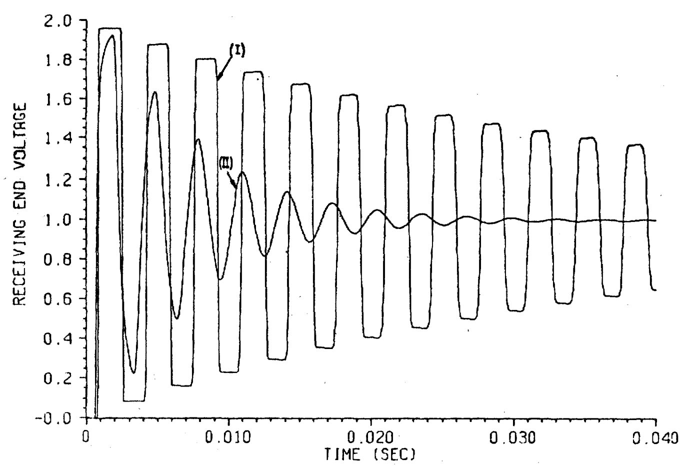

Fig. 16: Step function energization of open-circuited line. Curve (I): Constant, 60 Hz parameters. Curve (I): New model parameters.

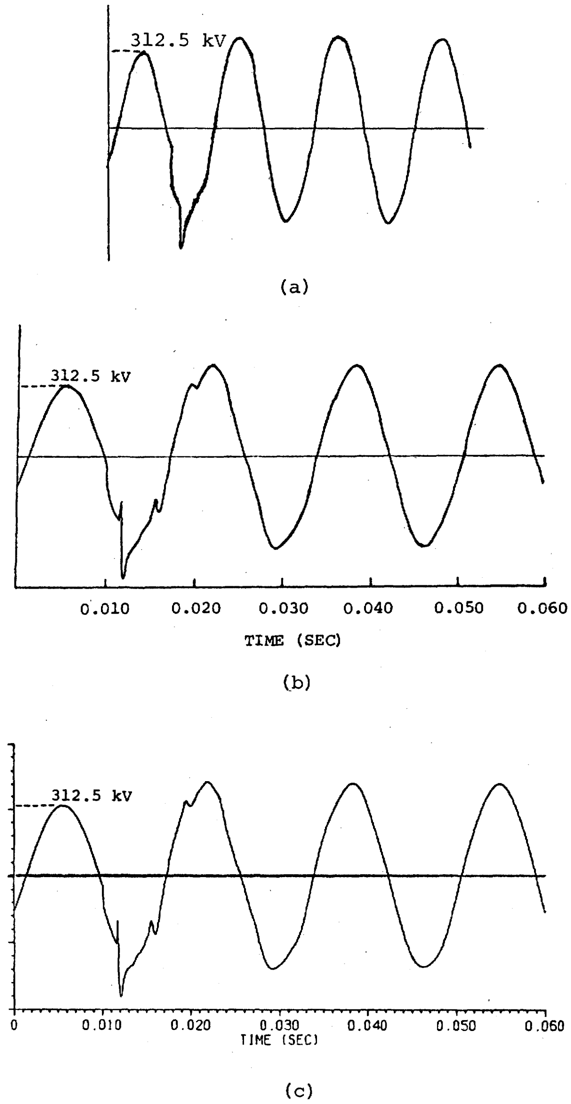

Fig. 17: Field test simulation. (a): BPA field test oscillograph. (b) BPA simulation. (c) New model simulation.

Comparison with Field Test:

The new line model was used to simulate the BPA field test described in reference [4]. This test simulates a single line to ground short circuit on an open-ended 222 km, 500 kV, 3-phase transmission line. The short circuit was applied to phase-c. The field test oscillograph for the voltage at phase-b at the end of the line is shown in fig. 17(a). To compare with BPA's digital simulation in ref. [4], the same integration step Δt=50 μsec was used, and the zero sequence mode of the line was represented by the new model described in this paper. The result of this simulation is shown in fig. 17(c). This result compares well with the field test and with BPA's simulation (fig. 17(b)). In BPA's simulation the average time per step, as compared with the solution with constant parameters, was 3.13 times longer. In the simulation with the new model this time was only 1.19 times longer.

IX. CONCLUSIONS

A new, fast, and reliable approach has been developed for the accurate modelling of transmission lines over the entire frequency range. The routines for obtaining the parameters of the model do not present the numerical difficulties encountered with previous formulations. These routines are easy to use because they do not require value judgements on the part of the user. Further work is needed in connection with the representation of unbalanced, untransposed lines with frequency-dependent modal transformation matrices.

X. ACKNOWLEDGEMENTS

The author would like to express his gratitude to Dr. H.W. Dommel, whose clear and practical thinking are always the best encouragement; to the University of British Columbia Computer Centre for its convenient and easy to use facilities; to Central University of Venezuela for their financial support during the author's leave of absence at U.B.C.; and to the Bonneville Power Administration for their constant cooperation, and for allowing the reproduction of the field test result used in this paper.

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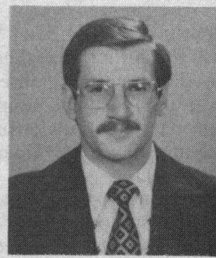

José R. Martí (M'71) was born in Spain on June 15, 1948. He received the degree of Electrical Engineer from Central University of Venezuela, Caracas, Venezuela, in 1971, and the degree of M.E. in Electric Power Engineering from Rensselaer Polytechnic Institute, N.Y., in 1974. He is presently a Ph.D. candidate at the University of British Columbia, Canada.

From 1970 to 1971 he worked for Exxon in Venezuela in coordination of protective relays. From 1971 to 1972 he worked for a Consulting Engineering firm in Caracas, Venezuela, in relaying and substation design projects. In 1974 he joined the Central University of Venezuela as a professor in Power System Analysis. He is presently at U.B.C. on a leave of absence from Central University of Venezuela.

Discussion

Adam Semlyen (University of Toronto, Ontario, Canada): This is a timely and interesting paper. It shows the path the author has taken to perfect the weighting function method[4] so that the impulse responses of Fig. 2 are replaced by something more manageable. The end result is identical to the approach which uses a wave propagation transfer function and a frequency dependent characteristic impedance.[5,8,A] I would like to make the following remarks related to this fact.

a. Fundamentally, there exists a single set of transfer functions for transmission line transient analysis which yield smooth step or impulse responses. There are $e^{-\gamma(\omega)\ell}$ and $Z_c(\omega)$. This fact is related to the decomposition of voltages and currents into travelling waves and has led us to use, since 1970, these transfer functions for simulation of transients on transmission lines with frequency dependent parameters.[B] It is therefore not surprising that the author is now using this same approach as the result of perfecting a different method.
b. There are several sets of transfer functions which one can use in transient analysis on a transmission line. They represent relations between the two terminal voltages and currents, and can be expressed by 2×2 transfer function matrices with only two independent elements. Examples are the two-port transfer functions of eqns. (1) and (2), driving port and transfer admittances, and the transfer functions related to the weighting function $a_1(t)$ and $a_2(t)$ of Fig. 2. All these contain hyperbolic functions of $\gamma(\omega)\ell$ and, consequently, have complex poles in the s-plane. Therefore, an oscillating behaviour is unavoidable unless the hyperbolic functions are combined in an exponential function.
c. The forward and backward travelling functions of eqns. (5) to (8) are not very different from the travelling wave components V' and V'' ($V' + V'' = V$). They are related by the factor 2; if R_1 is replaced by Z_c as in eqns. (10) to (13):

   F = twice the voltage V' of the outgoing wave
   B = twice the incident wave voltage V''

   It is then clear that replacing R_1 by Z_c actually replaces the method which uses the weighting functions $a_1(t)$ and $a_2(t)$ of Fig. 2 by the approach which uses the propagation transfer function.
d. Often, in previous calculations, the characteristic impedance has been considered constant. This appears to be justified for positive sequence, according to Fig. 9. When the approximation is acceptable the weighting function approach and the travelling wave transfer function approach become identical, except for numerical procedures.

A useful contribution of this paper is the RC realization of the characteristic impedance. It permits easy implementation in the EMTP and reflects the fact that the rational approximation (23) satisfies all essential physical requirements.

It is interesting to note that the d.c. value of Z_c, as shown in Fig. 9, is not very large ($\sim 620\ \Omega$). It indicates a relatively large shunt conductance G adopted in the calculation. Could the author please comment on the way it has been selected; whether it is related to attenuation of trapped charges or to losses? If $G \neq 0$ is the assumption in the calculation of Z_c, has it been considered in the calculation of the propagation transfer function (16) as well? Does its magnitude affect significantly the calculated overvoltages?

In previous calculations [5] we have considered $G = 0$. This has had the effect that a step voltage arrived at the other end distorted but in full magnitude (at $t = \infty$). The computational effect was that if a line has been disconnected at both ends in some cases the trapped voltage tended to take off. This indicates some instability resulting, apparently, from the basic difference equation (26). A numerical analysis of the stability of (26) in a closed loop condition indicates indeed the possibility of a slow numerical instability if $G = 0$. The remedy consists in adopting non-zero conductance. The author's comment on this topic would be appreciated. One should of course mention that this type of unstable behaviour is related to the nature of information transfer from one end of the line to the other, as shown in the recursion formula (26). This shows that new values of s are related to values of f, and therefore of s itself, many time steps back; and only two past values are used to calculate the new value. A method using full, i.e. non-recursive, or partial convolutions is therefore expected to have better stability characteristics.

For the fitting of the propagation transfer function the author uses a "backwinding" of A_1 by $e^{j\omega\tau}$. Is τ based on light velocity? We used to add a $\Delta\tau$ to take into account the toe portion of the propagation step response curves. The result was that our original time domain fitting[5] was good even with only two exponentials. Later, for frequency domain fitting and very accurate (and smooth) results we went up to six (real and/or complex) exponentials. This is less than half of the numbers shown in the present paper.

The "toe" of the propagation step response is not just an empirical fact but it is a theoretically expected extension $\Delta\tau$ of τ, intrinsically related to the concept of penetration depth, and, as discussed, important for efficient rational approximation. This flat portion of the step response is due to the fact that f(t) and all its derivatives of finite order are zero at $t = 0$. In the frequency domain the transfer function P(s) and also $s^nP(s)$ for $n = 1, 2, \ldots N$ must all be zero for $\omega = \infty$ in $s = j\omega$. P(s) is given in equation (28) of the paper:

$$P(s) = e^{-\gamma(s)\ell} \times e^{s\tau} = e^{-(\gamma(s)\ell - s\tau)} \tag{a}$$

The expression in the exponent is:

$$G(s) = \gamma(s)\ell - s\tau \tag{b}$$

where

$$\gamma(s) = \sqrt{Z(s)Y(s)}$$

$$Z(s) = Z_c(s) + sL(s) \tag{c}$$

$$Y(s) = sC$$

In (c), Z_c pertains to the conductor and L(s) is the complex inductance of the earth return.[C] We express it for a single conductor:

$$L(s) = \frac{\mu_o}{2\pi} \ln \frac{2(h + p)}{r} \tag{d}$$

where p is the complex penetration depth: $p = \dfrac{1}{\sqrt{s\mu_o\sigma}}$

At high frequencies $p \rightarrow 0$, and equation (d) becomes:

$$L(s) = \frac{\mu_o}{2\pi}\left(\ln \frac{2h}{r} + \frac{p}{h}\right)$$

Then $Z_c(s)$ of (c) has the simple expression:

$$Z_c(s) = \frac{1}{\sigma_c \ell_c p_c}$$

where

$$p_c = \frac{1}{\sqrt{s\mu_o\sigma_c}}$$

and ℓ_c is the perimeter of the conductor along the surface of the outside wires.

Consequently G(s) becomes:

$$G(s) = \ell\sqrt{\left(\frac{1}{\sigma_c\ell_c p_c} + s\frac{\mu_o}{2\pi}\left(\ln\frac{2h}{r} + \frac{p}{h}\right)\right)s\,\frac{2\pi\varepsilon_o}{\ln\frac{2h}{r}}} - s\tau$$

$$= s\tau\left(\sqrt{1 + \frac{p}{h\ln\frac{2h}{r}} + \frac{2\pi}{s\mu_o\sigma_c\ell_c p_c \ln\frac{2h}{r}}} - 1\right)$$

$$\cong \frac{s\tau}{\ln\frac{2h}{r}}\left(\frac{p}{2h} + \frac{\pi}{s\mu_o\sigma_c\ell_c p_c}\right)$$

$$= \frac{\sqrt{s}\,\tau}{\sqrt{\mu_o}\ln\frac{2h}{r}}\left(\frac{1}{2h\sqrt{\sigma}} + \frac{\pi}{\ell_c\sqrt{\sigma_c}}\right) = K\sqrt{s}$$

This expression shows that the value of $G(j\infty) = (j\infty) - (j\infty)$ of equation (b) is in fact itself infinite. Then, of course, P(s) of (a) and all products $s^nP(s)$ become zero for $s = j\omega$, $\omega = \infty$.

Finally, since $f(0), f'(0), f''(0), \ldots f^N(0)$ are all zero, a Taylor series expansion of f(t) around $t = 0$ yields $f(t) \cong 0$. This will be valid for a small value of t ($\leqslant \Delta\tau$) for which the assumptions implicit in the above frequency domain derivations are valid.

I would like to conclude by commending the author for his valuable job in clarifying essential aspects of calculating transients on transmission lines with frequency dependent parameters. There is an implied emphasis on simulation based on rational approximation of transfer functions and this technique is advanced by specific applications.

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Manuscript received June 3, 1981.

J. R. Marti: The author would like to express his appreciation for the detailed review contributed by Prof. Semlyen and for the opportunity to comment on some important aspects of the formulation presented in the paper.

As Prof. Semlyen notes in the first part of his Discussion, the formulation of the transient problem in transmission lines in terms of the "natural" system functions, $Z_c(\omega)$ and $e^{-\gamma(\omega)\ell}$, greatly simplifies the numerical manipulations involved in the solution of the problem. The main reason why our approach was developed from the concept of weighting functions resulting from Bergeron's type of relationships (eqns. 10 to 13) was because these relationships allowed us to more easily visualize the physical significance (fig. 5) of the functions involved in the formulation of the problem, and, as a result, to arrive at very important conclusions regarding the nature of their mathematical synthesis (e.g. R-C realization of $Z_c(\omega)$).

As the Discussor notes, the use of a constant value (simple resistance) instead of a more complete model (e.g. R-C network) to simulate the line characteristic impedance would appear to be justified for the aerial mode (fig. 9, botton), though only if the frequencies of interest are higher than about 20 Hz for the line studied in the paper. However, in most cases of transient studies (asymmetric conditions in the system), the accurate modelling of the ground return mode is the most critical one. The strong frequency dependence of the characteristic impedance for the ground mode is clearly seen in fig. 9 (top).

The need for a finite value of the shunt conductance G ($G \neq 0$) arises from the fundamental mathematical description of the travelling-wave phenomena. In its most basic form, the current travelling wave is related to the voltage travelling wave through the characteristic impedance Z_c; for instance, for a forward-travelling wave

$$I^+(x,\omega) = \frac{V^+(x,\omega)}{Z_c(\omega)}$$

and similarly, for a backward-travelling wave. If these relations are to be valid for any frequency, they should also be valid for $\omega = 0$ (dc conditions). But, for $\omega = 0$, $Z_c(dc) = \sqrt{\frac{R(dc)}{G(dc)}}$. If G(dc) is taken to be zero, then $Z_c(dc) = \infty$, that is, the basic relationship between the current and voltage waves will present a singularity at $\omega = 0$. It can then be understood that a numerical solution based on the fundamental travelling wave equations can give extraneous results when trying to simulate dc conditions (e.g. trapped charge) if G is assumed to be zero. With the formulation presented in the paper, which considers a finite value of G, no problems have been encountered in the simulation of trapped charge or other dc conditions (e.g. exponentially decaying dc components in asymmetric short-circuit currents and final dc levels in open-circuit or short-circuit step responses).

For most cases of transient studies, as long as it is chosen within a reasonable order of magnitude, the actual finite value used for G is not very critical and its effect upon the simulation is practically negligible. (A possible exception would be specific studies of the corona phenomenon, where a more detailed representation of the non-linear corona characteristics would be necessary.) The value of G used in the simulations presented in the paper was 0.3×10^{-7}mho/km, which represents a "rule of thumb" average value for shunt losses (leakage through the insulation plus corona losses) in high voltage overhead lines. Since G is considered as one of the line parameters, the same value is used for both the evaluation of the characteristic impedance and the evaluation of the propagation function.

In the formulation presented in the paper, the phase displacement factor τ in eqn. 28 is not evaluated from the travelling time at the speed of light, but it is directly obtained in the frequency domain by comparing the phase angle of the rational function $P_a(s)$ in eqn. 29 (which is a minimum-phase-shift function) with the phase angle of the propagation function $A_1(\omega)$. It can be seen from eqn, 28 that $\tau = \frac{\phi Pa - \phi A1}{\omega}$. In this way, the effect of the particular line configuration, propagation mode, and what Prof. Semlyen refers to as the "toe" of the propagation function are automatically taken into account, without the need for additional calculations (since all the information is intrinsically contained in $A_1(\omega)$). (For 100 miles of the line studied in the paper, the value of τ was 0.598 ms for the zero sequence mode and 0.539 ms for the positive sequence mode.) Also, in connection with this point, we would like to emphazise that the method of asymptotic tracing used to obtain the rational function $P_a(s)$ in eqn. 29 only requires the magnitude function of $A_1(\omega)$ ($|A_1(\omega)| = |P_a(\omega)|$). Due to the analytical properties of $P_a(s)$, the phase angle of this function is "fixed" by its magnitude. Therefore, the order of the approximation is not determined by the phase angle or by τ, but only by the shape of the magnitude of $A_1(\omega)$ and by the accuracy with which the approximation is desired.

Finally, we would like, in closing these comments, to reiterate our appreciation of Prof. Semlyen's extensive discussion on a subject in which he has been one of the main pioneers.

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