

1 Even truncation

One-sided p-value:

$$Y \sim N(0, 1), \quad c > 0$$
$$P(Y > t | Y > c) = \frac{1 - \Phi(t)}{1 - \Phi(c)}$$

Two-sided p-value:

$$P(Y > t | |Y| > c) = \frac{1 - \Phi(t)}{1 - \Phi(c) + \Phi(-c)} = \frac{1 - \Phi(t)}{2(1 - \Phi(c))}$$

So the p-value in the two-sided case is half of that of the one-sided p-value, but the test needs but if testing is done at a level α , the two-sided p-value needs to be lower than $\alpha/2$ and the one-sided p-value only needs to be lower than α .

2 Uneven truncation

One-sided p-value:

$$Y \sim N(0, 1), \quad u > 0, l < 0, |u| > |l|, t > 0.$$

$$P(Y > t | Y > u) = \frac{1 - \Phi(t)}{1 - \Phi(u)}$$

Two-sided p-value:

$$P(Y > t | Y > u \cup Y < l) = \frac{1 - \Phi(t)}{1 - \Phi(u) + \Phi(l)} > \frac{1 - \Phi(t)}{2(1 - \Phi(u))}$$

For a positive realization, conditioning on one-sided screening and performing a one-sided test will lead to more rejections than conditioning on a two-sided event and performing a two-sided test. So it is likely, that one-sided conditioning will be more powerful for $\mu > 0$ and less powerful for $\mu < 0$.