Smoothing Variational Objectives with Sequential Monte Carlo for Nonlinear Dynamics

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Abstract

Conductance based models of excitable cells are widely used in computational neuroscience to characterize the spiking activity of individual neurons. Recovering the multidimensional nonlinear dynamics that govern a cell from a single observation is a challenging problem motivating the development of novel techniques in time series analysis. Sequential Monte Carlo methods have been used to construct objective functions for variational inference on time series to perform simultaneous model inference and learning. We develop smoothed variational objectives analogous to forward-backwards message passing. We demonstrate that the use of information from the full time ordered sequence of observations improves both the state estimation and the dynamics learned. Experiments show that this method compares favorably against state of the art methods for variational inference in nonlinear dynamical systems. **TL;DR summary of paper**

1. Introduction

motivate the statistical problem with neuroscience question. introduce the method and outline the paper. Conductance based models of excitable cells are widely used in computational neuroscience to characterize the spiking activity of individual neurons. Recovering the multidimensional nonlinear dynamics that govern a cell from a single observation is a challenging problem motivating the development of novel techniques in time series analysis. Recently Sequential Monte Carlo methods have been used to construct objective functions for variational inference on time series to perform simultaneous model inference and learning. We develop smoothed variational objectives analogous to forward-backwards message passing. We demonstrate that the use of information from the full time ordered sequence of observations improves both the state estimation and the dynamics learned. Experiments show that this method compares favorably against state of the art methods for variational inference in nonlinear dynamical systems.

talk about dimensionality expansion, dimensionality reduction, prediction.

2. Related Work

Discuss SMC based methods including AESMC, VSMC, FIVO.

Contrast with pure VI methods including VRNN, DKF, LFADS, GfLDS, VIND.

3. Theory

3.1. Sequential Monte Carlo

Give a brief review of SMC and VI. Why is this approach promising as opposed to pure VI?

SMC methods factorize an ordered sequence of observations $\mathbf{X} \equiv \{\mathbf{x}_1, \dots \mathbf{x}_T\}$, $\mathbf{x}_t \in \mathbb{R}^{dx}$ governed by an ordered a sequence of latent variables $\mathbf{Z} \equiv \{\mathbf{z}_1, \dots \mathbf{z}_T\}$, $\mathbf{z}_T \in \mathbb{R}^{dz}$ that evolve according to stochastic dynamics. The target distribution $\{p_\theta(\mathbf{z}_t|\mathbf{x}_t)\}_{t=1}^T$ is assumed intractable due to

SMC factorizes the target distribution $\{p_\theta(\mathbf{z}_t|\mathbf{x}_t)\}_{t=1}^T$ into a sequence of distributions of increasing spaces.

3.2. Auto-Encoding Sequential Monte Carlo

Recap main idea from AESMC and FIVO.

$$\log p(\mathbf{X}) = \log \int p(\mathbf{X}, \mathbf{Z}) d\mathbf{Z} = \log \frac{p(\mathbf{X}, \mathbf{Z})}{p(\mathbf{Z}|\mathbf{X})} \,. \tag{1}$$

$$\log p(\mathbf{X}) \geq \mathscr{L}_{\text{ELBO}}(\mathbf{X}) = \mathop{\mathbb{E}}_q[\log p(\mathbf{X}, \mathbf{Z})] - \mathop{\mathbb{E}}_q[\log q(\mathbf{Z}|\mathbf{X})] \,. \tag{2}$$

$$\hat{\mathcal{Z}}_{SMC} \equiv \prod_{t=1}^T \Big[\frac{1}{N} \sum_{n=1}^N w_t^{(n)} \Big] \tag{3}$$

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3.3. Smoothing Variational Objectives

Emphasize novelty of the proposed model, novelty of the experiments and the results. Discuss the implementation details.

The big question to answer is, how is this different from AESMC or FIVO in terms of theory? what is unique with respect to the applications and results?

nonlinear time invariant function learned on a smoothed SMC objective.

4. Results

Define quantitative evaluation metric. Motivate the use of k-step MSE_k and k-step R_k^2.

$$\text{MSE}_k = \sum_{t=0}^{T-k} \left(\mathbf{x}_{t+k} - \hat{\mathbf{x}}_{t+k}\right)^2 \ , \quad R_k^2 = 1 - \frac{k\text{MSE}}{\sum_{t=0}^{T-k} \left(\mathbf{x}_{t+k} - \bar{\mathbf{x}}\right)^2} \tag{4}$$

4.1. Fitzhugh Nagumo

$$\begin{aligned} \dot{V} &= f(V) - W + I_{ext} \, , \\ \dot{W} &= a(bV - cW) \end{aligned} \tag{5}$$

4.2. Lorenz Attractor

$$\begin{aligned} \dot{z}_1 &= \sigma(z_2 - z_1) \, , \\ \dot{z}_2 &= z_1(\rho - z_3) - z_2 \, , \\ \dot{z}_3 &= z_1 z_2 - \beta z_3 \, . \end{aligned} \tag{6}$$

4.3. Single Cell Recordings

5. Discussion

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Figure 1. Summary of the Fitzhugh Nagumo results: (left) latent dynamics and paths for the original system (center) inferred 2D dynamics and paths from a noisy 1D observation (right) R_k^2 for various models.

Figure 2. Summary of the Lorenz results: (left) latent paths for the original system (center) inferred paths from a noisy 10D observation (right) R_k^2 for various models

Figure 3.