Measurement of β-ray spectra

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Abstract

Using a thin lens magnetic spectrometer, we measure the momentum spectrum of electrons emitted as β^- rays from a radioactive source of ^{137}Cs. The detected momentum of the radiated electrons is defined by the spectrometer's adjustable magnetic lens current and k a proportionality constant dependent on the geometry of the apparatus. The magnetic field of the lens is varied by changing the current passing though the lens coil which has the effect of modifying the trajectories of the electrons, focusing electrons with specific momenta onto the detector allowing us to measure their intensity. By converting the measured momentum to energy we are able to fit our data to a linear model based on the Fermi-Kurie plot. We find that the value of the kinetic energy of the nuclear transition is $T = 0.520 \pm 0.044$ MeV which is in agreement with the accepted value of $T = 0.512$ MeV[1].

1 Introduction

When Henri Becquerel first observed $\beta-$radiation, he determined that the observed radiated particle satisfied the same mass-to-charge ratio as the electron, discovered in 1897 by J.J Thompson[2].

Later experimental results showed that $\beta-$rays are detected with a continuous range of kinetic energies up to a maximum value[3]. The discovery of a continuous distribution of electron kinetic energies rather than a discrete predictable value led Wolfgang Pauli to propose in 1930 that the observed violation of conservation laws must be due emission of a yet unknown particle.

In 1934 Enrico Fermi called this apparently massless and undetectable particle the "neutrino", developing an advanced theory of beta decay. The neutrino was finally experimentally observed 1956.[4]

The process we currently know as β^- decay describes a neutron in a parent nucleus desintegrating into a proton in a daughter nucleus, an electron and an antineutrino.

In a β^-event, both nuclides (nuclear species) have the same number of nucleons. This means that the daughter nucleus will not experience a substantial change in kinetic energy (recoil) due to the decay event. Leaving most of the desintegration energy available to be carried-off by the leptons as kinetic energy.

A parent nucleus has a given initial energy w. The avalilable kinetic energy of the system is equal to the decrease in mass energy due to the creation of the radiated leptons:

$$T = w - mc^2, \tag{1}$$

where m is the difference in mass between the daughter and parent nuclides. In relativistic units:

$$T = w - 1, \tag{2}$$

The observable count of β^-electrons n as a function of energy is described by the Kurie–Fermi Theory of β^-decay.

2 Background Theory

In this experiment we measure the momentum spectrum of emitted $\beta-$rays from a radioactive source of ^{137}Cs into an excited state of ^{137}Ba. This transition occurs with a probability of 94.6% at a maximum energy value $T = 0.512$ MeV.[1].

A set of electrons with a specific momentum range is focused onto the spectrometer detector, while electrons outside this range undergo chromatic aberration.

The use of coordinates of momentum instead of energy in $\beta-$ray spectroscopy is partly due to the fact that it is the momentum of the focused electrons that is rigorously proportional to the axially symmetric magnetic field.[5, 6] In our experimental setup, the magnetic field is proportional to the adjustable current I_{lens} going through the lens coils.

The definition for the momentum of emitted electrons is:

$$p = e\rho B, \tag{3}$$

where B is the magnetic field strength, e is the electron charge, ρ is the gyroradius of the electrons due to B.

The magnetic rigidity P is a measure of the momentum of electrons[7]:

$$P = B\rho, \tag{4}$$

From this relation and the above definition of the momentum of electrons, we write:

$$p = kI_{lens}, \tag{5}$$

k is a constant determined by the geometry of the spectrometer alone[5].

2.1 K-peak Calibration

To calibrate the observed momentum distribution we use electrons emitted with a characteristic well-defined kinetic energy[1]. These electrons are named conversion electrons. In this experiment we study the most probable energy transition from ^{137}Cs to ^{137}Ba. in this transition ^{137}Ba is in an excited state. One way for the daughter atom to lose energy is by transferring the excess energy directly to an orbital electron[1].

The orbital will most likely be the K-shell since it is the lowest energy orbital. A higher energy group event is much rarer (probability of 6%), therefore little error is made by assuming that the peak is due to the K line only.[1].

The constant k in (3) is determined by calibrating the observed spectrum to the well-known K-conversion peak with kinetic energy $T_k = 624.21$ keV.

In relativistic units, the calibration calculation is as follows:

$$T = w - 1, \tag{6}$$

in terms of the momentum p_k

$$T_k = \sqrt{{p_k}^2 + 1} - 1, \tag{7}$$

$$\therefore p_k = \sqrt{(T_k + 1)^2 - 1}, \tag{8}$$

from equation (5)

$$\therefore kI_k = \sqrt{(T_k + 1)^2 - 1}, \tag{9}$$

$$\therefore k = \frac{\sqrt{(T_k + 1)^2 - 1}}{I_k}, \tag{10}$$

is the proportionality constant we are after.

2.2 Kurie-Fermy theory

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3 Experimental method

Our experimental apparatus is a thin magnetic-lens spectrometer. The operation of β spectrometers depends on the behaviour of electrons subject to magnetic fields.

The spectrometer is aligned parallel to the horizontal component of the Earth's magnetic field, whilst

The magnetic field of the spectrometer lens is varied by changing the current passing though the lens coils. Modifying a cone of electron trajectories diverging from the source along the spectrometer's axis, causing them to spiral around the axis of the instrument towards detector[1].

3.1 Background radiation

3.2 constant time vs constant counts

The counting controls of the experimental apparatus can be defined by the user. For constant time, the best estimation for the uncertainty in the number of counts n is $\sqrt{n}$[1] due to the poissonian nature of radioactive decay. The fractional uncertainty in counts is $\frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$ Fractional uncertainty is a preserved quantity. Therefore the uncertainty in time is :$\Delta t = \frac{t}{\sqrt{n}}$

By using constant counts, we are able to predetermine the fractional uncertainty for a data point. An obvious constraint from choosing this method is that the alloted time for the experiments is not as easily monitored as with constant time. Since both have pros and cons, the best option is a combination of both counting methods.

3.3 Resolution

In a magnetic spectrometer with fixed geometry and variable B the resolution

$$R = \frac{\Delta(B\rho)}{B\rho}, \tag{11}$$

is constant (in this experiment $R = (2 - 3\%)$). Where $\Delta(B\rho)$ is a measure of the accepted momentum band (Δp). When plotting the momentum distribution it is necessary to divide the number of counts n(p) at each current setting by the corresponding current in order to get the correct form of the spectrum[6].

3.4 Experimental procedure

Firstly, Decide what range and increments of lens current are adequate to resolve the momentum spectrum. Then, using the experimental control interface:

1. Calibrate the magnetic field probe. Coordinates of the probe must be set to: $(400, 190)$
2. Null Earth's magnetic field: Disable the lens current. Enable the bias coil current and increase this current until all displayed field values are as close to zero as possible. Note: (the x-component of the field will remain large compared to the other components.)
3. Set counting controls to either constant time or constant counts. Specify time interval or expected counts.
4. Background rate count: Close the source shutter. With the lens current disabled, proceed to run the experiment and count the background radiation (repeat this step 4 times).

3.5 Data acquisition algorithm

1. Enable the lens coil current.
2. Set the shutter status to open.
3. Run the experiment.
4. Increase the lens coil current.
5. Repeat previous steps until reaching the max coil current.

4 Results

The experimental parameters used are as follows: The lens current range is set from 0A to 3.6A Increments of $0.1A$ are chosen. Counting controls were set to constant time: The intervals $t = 360$s.

After we obtain the raw spectrum n_{raw} as a function of the lens current. The first processing step is to correct our data from the background radiation count. The 4 measurements of the background radiation counts are added, and averaged.

$$n_{b,avg} = \frac{685}{4} = 171, \tag{12}$$

The uncertainty in the average background count is:

$$u(n_{b,avg}) = \frac{\sqrt{n_{b,avg}}}{4} = 7 \tag{13}$$

Here we have converted the calculated values to the closest integer values. The average background count $n_{b,avg}$ is then subtracted from the measured data $n(I)$. We chose the uncertainty in the observed count to be: $u(n(I)) = 15$. Then, the uncertainty in the background corrected data is:

$$u(n(I)) = \sqrt{u(n_{b,avg})^2 + u(n_{raw})^2} \tag{14}$$

We now calibrate the spectrum by using the conversion peak energy T_k

Firstly, since there are only 2 data points visible in our conversion peak, it is unlikely that we have found the true value for the corresponding current I_k to the K-peak.

We perfom a lorentzian fit on the K-peak in order to find a good estimate of the true peak $n(I_k)$ value.

We proceed to find the constant of proportionality in $p = kI$ using equation (10).

We convert T_k into relativistic units $T_k = 1.22$ mc^2

From equation (10) we find $k = 1.049$. We define the uncertainty value for $I_k : u(I_k) = \frac{0.1}{2}$ since we estimated the true location of our peak based on a curve fit. The uncertainty in k is:

$$u(k) = k\frac{u(I_k)}{I_k} = 0.03 \tag{15}$$

References

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