

The Mössbauer effect

21/09/2020

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Risk Assessment :

Risk factor	Risk Level	Control (existing)	Control (proposed)
High Voltage (120V) power supply	Low	<ul style="list-style-type: none">difficult to accessPower disconnected for maintenanceStudents not required to change set up	<ul style="list-style-type: none">If required, students inform lab technician of issues present in the controls set up and will not fix them independently
Radiation exposure from Sealed source ^{60}Co source $<0.1\text{ uSv/hr}$ (at surface of Pb box). 3.4 uSv/hr at 10cm from source inside box (90° from drive axis.)	Low	<ul style="list-style-type: none">operator opens box for ≤ 5 min to change samplesStudents must not touch the sourcewooden lead lined box covering apparatus and radiation sourceDosimeter use required	<ul style="list-style-type: none">Students must follow operator's instructions and not interfere with the change of samples process
Top lead lined wooden box lid (hinged) is heavy and can cause injury if dropped.	Low	<ul style="list-style-type: none">Design soft-top gas-spring brake to reduce impact speed if lid is dropped.correctly describe when lid is to be open	<ul style="list-style-type: none">automate the operation of the box by means of a microcontroller and a controlling program

Aim

objective ① is to observe the spectrum of photons emitted in the decay of ^{57}Co and learn about the processes in nuclear detectors

objective ② observing the Mössbauer absorption spectrum of a sample of potassium ferrocyanide and measure its linewidth and isomer shift

objective ③ to study the Mössbauer absorption spectrum of a sample of α-iron and calibrate the drive velocity scale, deduce the angular momentum and magnetic moment of the 14.4 keV level of ^{57}Fe and the isomer shift between α-iron and the source.

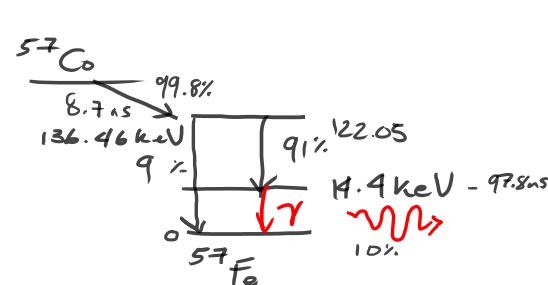
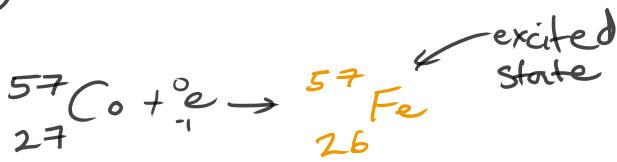
Introduction

In 1961 Rudolf Mössbauer received the Nobel prize for discovering the Mössbauer effect. [2] Mössbauer studied resonance in nuclei present in a solid sample of iridium. From his observations, he proposed that when atoms are bound in a solid under certain circumstances a fraction of the nuclear events could occur without recoil. He attributed the observed resonance to the recoil-free fraction (f) of nuclear events. [4]

very narrow linewidth nuclear transitions can be observed and consequently this enables to see the hyperfine energy splittings of the nucleus caused by interactions with surrounding electrons. [6]

Theory

Emitting and Absorbing Nuclei are ^{57}Fe
decay type is electron capture



Recoil energy [2]



Fig 2. emission of γ -ray from electron leads to recoil of electron in opposite direction to the γ -ray emission

$$\vec{P}_R = \frac{\vec{E}_\gamma}{c} \Rightarrow v = \frac{E_\gamma}{m_R c}$$

We know
The kinetic (recoil)-energy $E_R = \frac{1}{2} m_R v^2 = \frac{E_\gamma^2}{2 m_R c^2}$

where E_γ is gamma ray energy
 m_R is the mass of emitting nucleus
 c is speed of light

$$E_R = \frac{E_\gamma^2}{2 m_R c^2} \quad (1)$$

Mössbauer's great idea was
as $m_R \rightarrow \infty$ $E_R \rightarrow 0$

Then if m_R corresponds to the total mass of a crystal lattice we have no recoil energy!

[2]

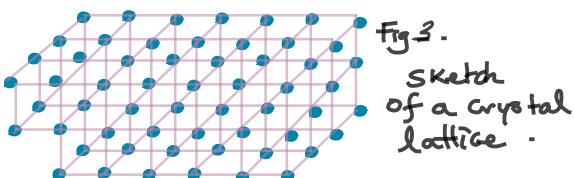


Fig 3.

Sketch of a crystal lattice.

Electromagnetic theory tells us that the intensity $I(E)$ of the emission line as a function of energy will follow [8]

$$I(E) = \frac{(\Gamma/2)^2}{(E - E_0)^2 + (\Gamma/2)^2}$$

Where E_0 is the centre of the emission line, Γ is the FWHM.

One of the most important parameters defining the usefulness of Mössbauer transitions is the fraction of gammas (f) emitted without recoil [8]

If we apply elementary theory to a Debye solid we find f

$$f = \exp\left(-\frac{6E_0}{k_B \Theta_D} \left(\frac{1}{4} + \left(\frac{T}{\Theta_D}\right)^2 \int_0^{\Theta_D/T} \frac{x}{e^x - 1} dx\right)\right), \quad (2)$$

where Θ_D is the Debye temperature of the lattice (atomic bonding)

T is the temperature at which the experiment is performed [a]
 k_B is Boltzmann constant

expression (2) can be integrated numerically to produce a plot of f vs T for different values of Θ_D [8]

* Mößbauer experiment Requirements * [8],[10]

Maximizing f requires that we minimize $\frac{E_\gamma}{E_R}$

(a) $E_\gamma < 100 \text{ keV} \Leftrightarrow E_R$ is small

for $T=0$:

$$f = \exp\left(\frac{-3E_\gamma}{2k\Theta_D}\right), E_R = \frac{E_\gamma^2}{2m_e c^2} \text{ where } \begin{cases} E_\gamma \text{ varies by a factor of } 10^3 \\ m_e \text{ varies by a factor of } 10^2 \end{cases} \\ \therefore E_\gamma^2 \text{ is the dominant factor} \\ (\text{we want } E_\gamma < 100 \text{ keV})$$

(b) Θ_D is large ($\Theta_D > 100 \text{ k}$)

(c) T is small \hookrightarrow difficult in practice

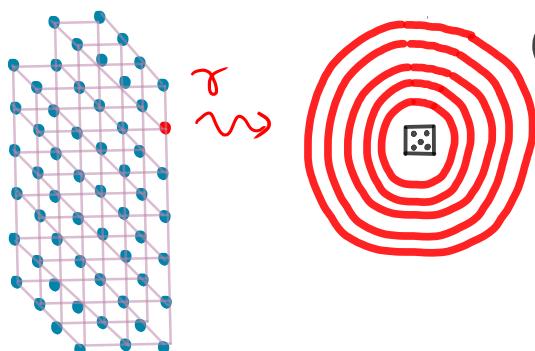


Fig 4. Mößbauer effect
Anchoring emitter nucleus in a crystal avoids energy lost to recoil. Incident γ -ray must be from a transition to the ground state
from refs [7][10]

(d) Any absorbing nucleus is going to be in the ground state
So it will only absorb γ -rays that correspond to a transition to the ground state. [10]

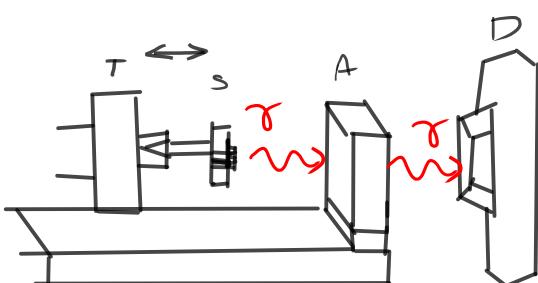


Fig 5. Schematic drawing (from left to right)

the transducer (T), the source (S), the absorber (A), and the defector (D).

Transducer moves the source back and forth to provide γ -ray doppler shift. from refs [7][10]

Obtaining a Mößbauer spectrum [10]

We need to provide γ -rays with energies that cover the desired energy range by:

(1) Doppler shifting the source

$$\left. \begin{array}{l} \text{The energy} \\ \text{Shift due} \\ \text{to doppler} \end{array} \right\} \Delta E = E_\gamma \frac{v}{c} \xleftarrow{\substack{\text{Velocity} \\ \text{of} \\ \text{source}}} \\ \xleftarrow{\substack{\text{rest} \\ \text{frame}}}$$

The natural linewidth of the 14.4 keV transition is only $4.6 \times 10^{-9} \text{ eV}$

A doppler shift of $1 \times 10^{-4} \text{ ms}^{-1}$ corresponds to this energy.

We use an electromechanical Transducer AKA voice coil drive.

Typical drive frequency is 10 Hz .

For ^{57}Fe the recoil energy is known to be $2.0 \times 10^{-3} \text{ eV}$ [8]

The half-life of the 14.4 KeV emitting level is 97.7 ns [8]

Resonance linewidth is determined by the lifetime of the excited state (Heisenberg uncertainty principle $\Delta E \Delta t \geq \hbar$)

(Δt) time uncertainty is interpreted as the mean life of the level emitting the γ -ray τ , related to the half-life by $\tau = T_{1/2} / \log_2 2$ [8]

(ΔE) is the uncertainty in the energy: $4.67 \times 10^{-9} \text{ eV}$

This uncertainty becomes the natural linewidth of the transition [8]

$$\Gamma = \frac{\hbar}{\tau} \quad (3)$$

This sets the best resolution obtained when we use the full γ -spectrum [8]

In practice this limit is not quite obtained [0]

- Reasons: [0]
- ① finite thickness of source & absorber
 - ② γ -rays don't necessarily travel parallel to the source-detector
 \therefore true velocity $\rightarrow v \cos \theta$ (where v is transducer velocity)
 - ③ Inhomogeneity and defects in the source & absorber (lattices)

Magnetic & electric Hyperfine fields at the ^{57}Fe nucleus may split the nuclear energy levels and cause the absorption line to split into more than one component

e.g. ^{57}Fe in a magnetic field: IF both the source and absorber lines are unsplit then the maximum absorption occurs at zero relative velocity between the source and absorber only if the source & absorber have very similar environments else: Resonance will occur at some non-zero velocity

The shift can result from the isomeric shift and/or from temperature differences between the source & the absorber (second-order Doppler shift) [0]

EXPERIMENTAL EQUIPMENT

22/09/2020

Mößbauer Drive System [0]

The transducer is driven so that the velocity changes in a triangular fashion with time.

Requiring a constant acceleration a for the first half of the cycle and a constant acceleration $-a$ for the second

thus the velocity changes linearly with time and hence each channel of the multichannel analyser corresponds to equal velocity increments

The spectrum will then consist of 2 mirror image spectra
(we fold them together after we finish counting)

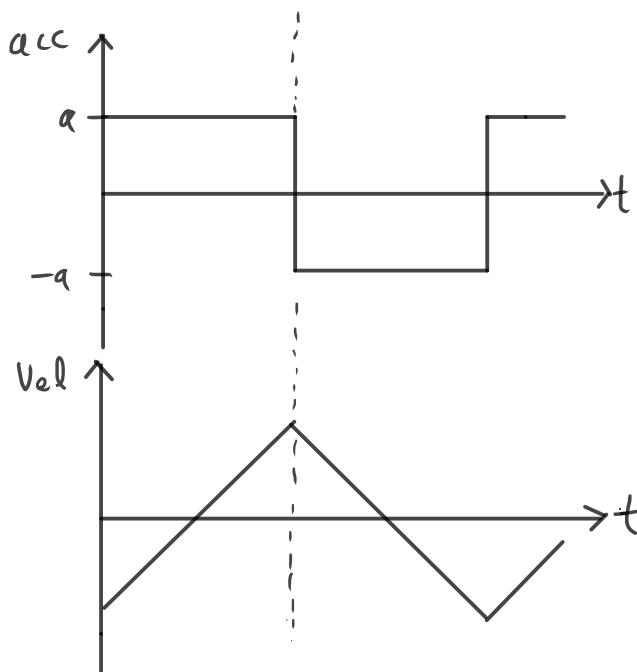


Fig 6 . from [0]

Mößbauer drive waveforms
a constant acceleration
is applied to the transducer
for the first half of the cycle
and the same acceleration
in the opposite directions
in the second half of the cycle

The corresponding velocity
of the transducer is
also shown .

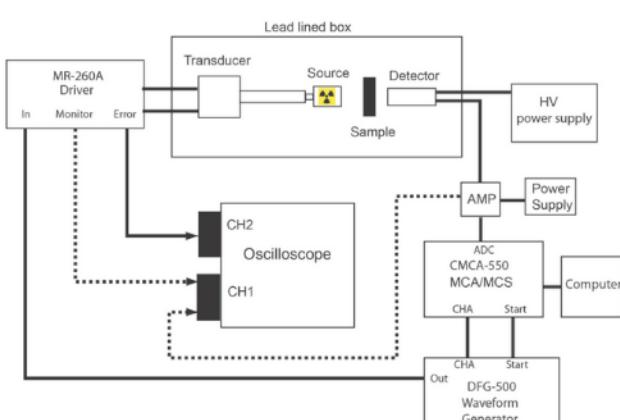


fig 7. Schematic diagram of the electrical connections for the Mößbauer experiment from ref . [0]

The transducer contains a driving coil
(loud-speaker magnet and coil) and a pick-up
coil (cylindrical bar magnet inside two
oppositely wound pickup coils)
The pick up coil produces a signal
proportional to the actual velocity
comparison of this with a reference
sawtooth produces an error signal
which is amplified, inverted and fed back
in as part of the drive signal
this signal is then monitored on the
DSG, as well as on the bar graph
display on the MR-260A drive unit

Experimental Method [0]

1. Turn on all the equipment
2. Undo lock on source box and the side catches and then open the front (you may need to lift the lid one cm or two)
3. IF The source is NOT mounted in the transducer speak to a demonstrator.
4. absorbers (2) are mounted on persplex slides if there is one in front of the source remove it and put it in the holder of the left hand side of the box.

Pulse height γ -spectrum

1. Plug the output of the amplifier into CH1 of the DSO
2. examine the pulses: (unipolar with max height $\sim 6\text{-}8 \text{ V}$)
else: adjust amplifier, pulse height is LINEAR wrt photon energy
3. look up decay-scheme ^{57}Co

Top bright band \leftarrow 122 keV and 137 keV gammas
their Compton and escape peaks are poorly resolved
by their scintillation detector

The second bright band \leftarrow 14.4 keV
appears $\sim (0.5\text{-}1 \text{ V})$

The 6.3 keV iron K - x-rays are almost
completely absorbed by the Be window
on the detector and lost in the noise

4. The MCA mode of the data acquisition hardware is used

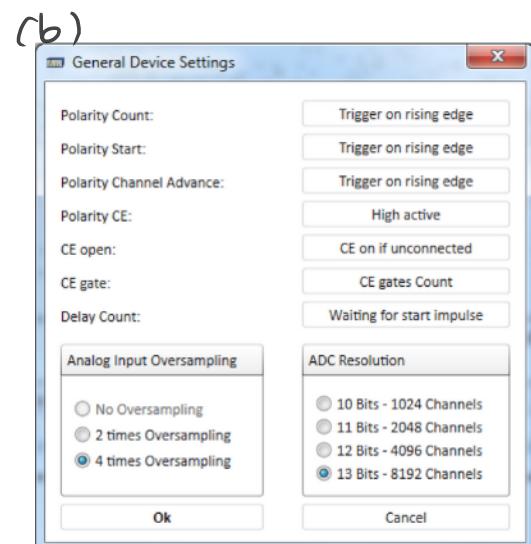
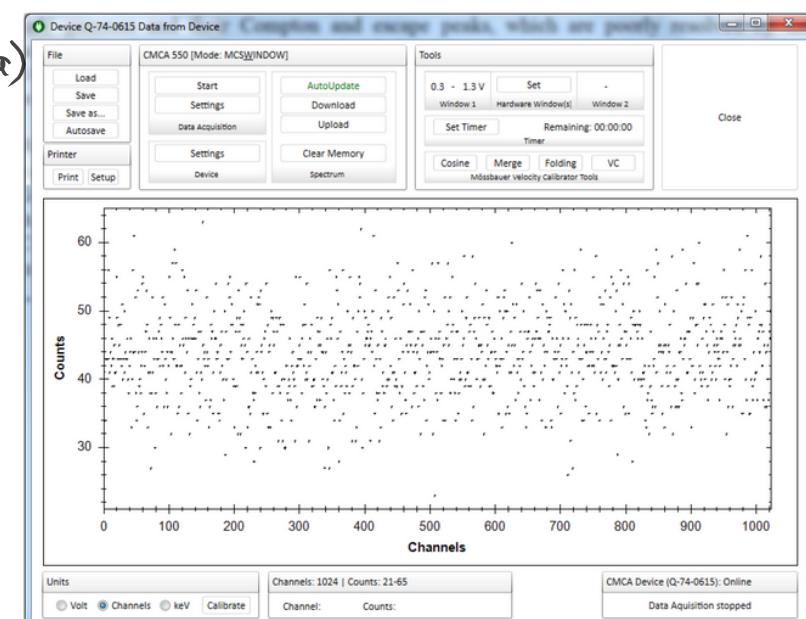


fig 8. (a) data acquisition screen (b) general device settings for CMCA-550 data acquisition device. from ref [0]

Data acquisition Algorithm [0]

1. set "PHA" button in the mode grouping
2. set "Single" in windows grouping
3. Set "Hysteresis" to 25 mV
4. In "Window1" grouping, set the range
50mV - 9990mV

 click **OK**
 (This will allow the scale of the window to display the full pulse size)
5. Click "Clear memory"
6. Click "Start" to begin data acquisition
7. You will see a large signal for channels < 200mV
this is electrical noise and must be eliminated.
8. Increase low level range from 50mV to an appropriate level by:
 - clicking "Tools" → "Set"
 - click display window at the upper edge of the region you wish to exclude ($> 200\text{mV}$)
 - click Save
9. Stop acquisition
10. Clear memory
11. Start acquisition

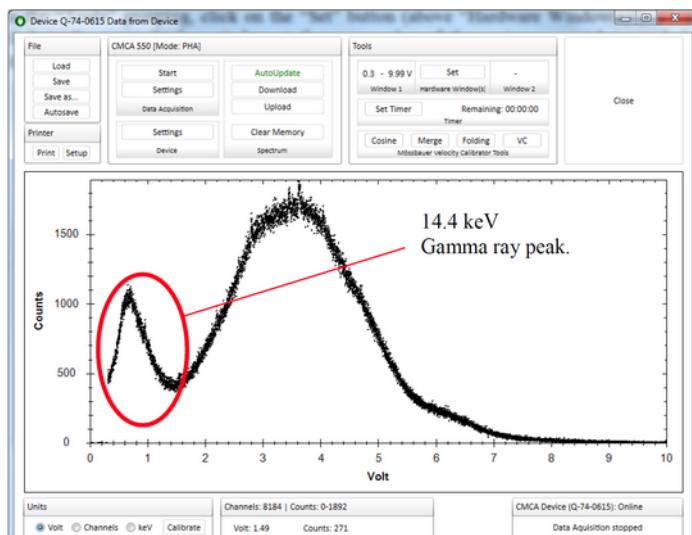


Fig 9. Full pulse height spectrum of the detected γ -ray signal
The circled peak is centered on the 14.4 keV transition
from [0]

12. we also need to exclude the higher energy peak
(repeat steps 7-11)

The width of the peak observed in the energy spectrum is due to measurement effects in the detector ($> 10 \text{ keV}$) and it is much wider than the energy spread that actually occurs in the transition which has a very narrow energy distribution.

Method

22/09/2020

Mößbauer Spectrum of potassium ferrocyanide [O]

We want to observe the absorption of 14.4 keV γ -photons as they are Doppler shifted in energy.

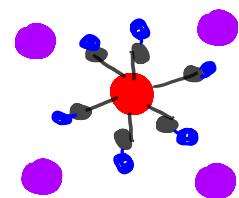


fig 1a molecule of
 $K_4[Fe(CN)_6] \cdot 3H_2O$

We choose to only count the gammas corresponding to 14.4 keV and plot the counts vs energy shift (occurring due to the moving source)

(AKA multichannel scalar (MCS) acquisition)

To ACTIVATE MCS mode

- 1- click "Settings"
- 2- tick "MCS [window]" button
- 3- Click "Single" button
- 4 Ensure PHA settings still apply
- 5- click "OK"

Set-up

1. Open box & place the KFCN absorber in the holder in front of the source
2. Set velocity range selector on the front of the Mößbauer drive unit to (0.4) and velocity switch to $\times 0.1$. The figure in the circular hole is the units the decimal places are on the rotating part.
3. Measure the magnitude of the velocity drive to the transducer by connecting the DSO to the monitor output of the MMR-260A drive unit (calibration is $\sim 30 \text{ mV/mm/s}$) we will determine the calibration more accurately from the spectra

Note: The signal indicates that the velocity ranges from negative to positive

4. Record the peak-to-peak voltage (V_{pp}) $\sim 160 \text{ mV}$. Also record the frequency

Note: Display at least 10 periods to get a sensible average as the signal is continuously modified by the error at the acceleration change points

5. click "Clear Memory"
6. Click "Start Acquisition"
7. Set display window to "Full range"
8. Display spectrum in terms of MCA channels by pressing "Channels"

Note: we want to monitor the error signal: diff (velocity, transducer motion) The amplitude of this signal should NOT be more than a few tens of mV. To do this we connect err output to DSO

Method for Möller Spectrum of α -Fe [0]

22/09/2020

1. Replace the absorber with the metallic iron absorber
2. Change velocity control helipot to 1.80
(keep range switch on "x 0.1")
3. Record amplitude of scan voltage ($\sim \mp 20 \text{ mV}$)
(measuring V_{pp} on "monitor" output of Drive unit)

Note: The iron is below its Curie temperature and so it has an internal magnetization $\propto J_z$ (for electrons)
The nucleus sees the electron cloud in this configuration as an effective magnetic field at room temperature
(33.0 T The magnetic hyperfine field)

Consequently its levels are split by the Zeeman effect
 \Rightarrow we should observe 6 lines in the spectrum

Note: The experiment will need to run overnight (at least)

Note: Using DSO: record velocity range & verify error signal $< 1\%$.

Note: When data acquisition is finished save raw spectrum and folded version separately

Feedback Signal

The mean error is indicated on the log bar-graph display it should be a few % for this low velocity and will be less for the next experiment

There are 2 sources of error signal

- constant spikes where "a" changes sign
- non-phase-locked signal (due to 50 Hz)

NOTE: If error signal becomes large

\rightarrow Drive is gone to oscillation

Consult a demonstrator to tune back the feedback loop.

Running the spectrum for 3 hrs should suffice to obtain data with an acceptable signal to noise ratio

Note: Save spectrum by:

1. "Stop" acquisition
2. "Save as": "file_name".unf

The spectrum is mirrored around one of the max velocity points (± 512)

Then, we can improve the signal-to-noise by "folding" the spectrum at this point

This also removes geometrical distortion due to changing solid angle subtended at the source by the detector

IMPORTANT:

1. Click "Möller Velocity calibrator tools"
2. Click "Folding"
3. "Save as": "file_name".fid

23/09/2020

PHTS3000 Data acquisition Video notes & screenshots.

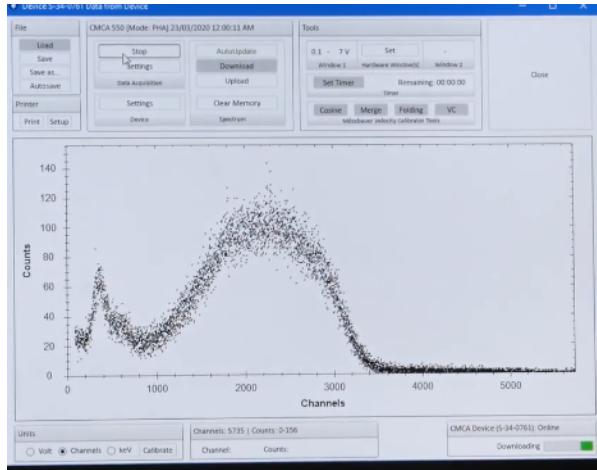
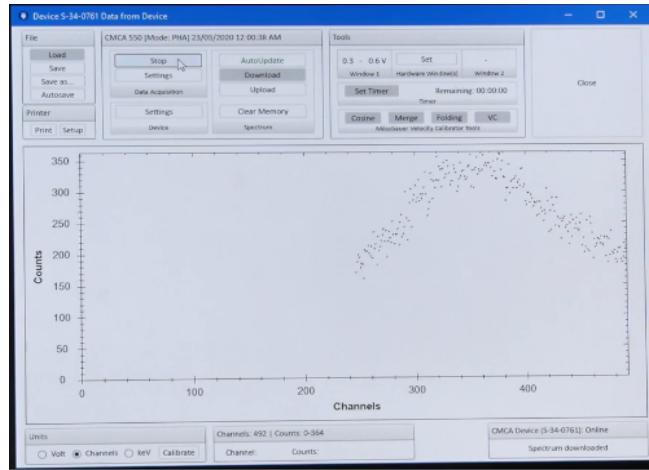
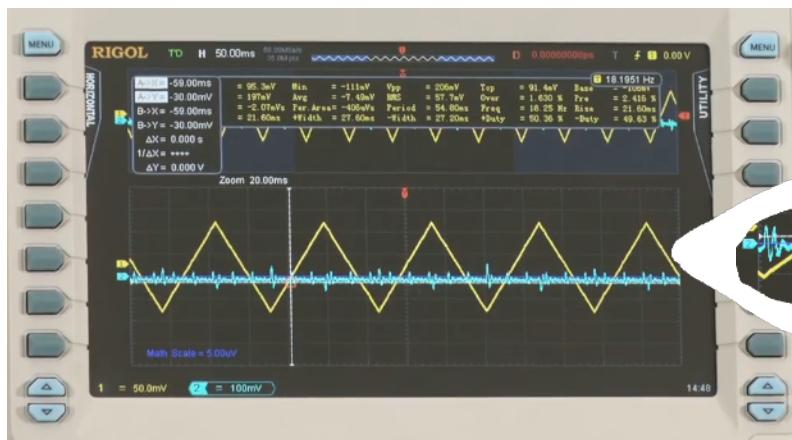


Fig11(a) full Mossbauer spectrum α -Fe
Taken from PHTS3000
Mossbauer experiment video



(b) trimmed Mossbauer spectrum α -Fe
(range adjustment)



enhanced view of signals

Fig12 Velocity of the source vs time (yellow) from equipment oscilloscope
producing 2 spectra for mossbauer data
Error signal (blue) difference between source settings & actual behaviour
Isomer : Potassium Ferricyanide
Taken from PHTS3000 Mossbauer experiment video

Hyperfine interactions [13]

1. Isomer (chemical shift)
(some non zero probability of finding electron within the nucleus for s-state and $p_{\frac{1}{2}}$ state electrons.)

resulting in energy shifting of all nuclear levels
mostly 1-s electrons.

2. Electric quadrupole interaction [13]

Deviation of nuclear shape from spherical

- A crystal can be considered as having a negative charge distribution depending on unit cell structure
 \Rightarrow positively charged nucleus will have preferential orientations inside the field distribution of the crystal

If there is an asymmetrical electrical charge distribution in the crystal (an electric field gradient)
splitting of the nuclear energy levels will occur

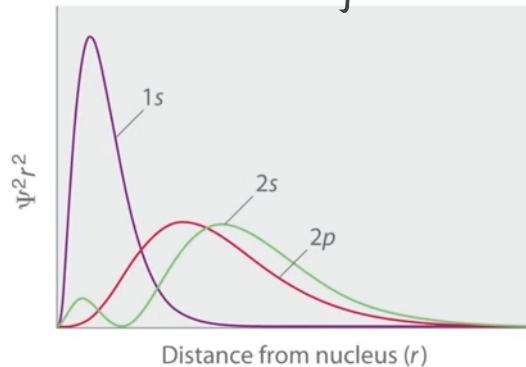
The energy shift due to the doppler effect is [13]

$$\Delta E = E_\gamma \frac{v}{c} \quad (4)$$

We know that the natural linewidth of the 14.4 keV transition is only 4.67×10^{-9} eV [13]

$$\frac{\Delta E}{E_\gamma} c = v$$

electron density vs distance



Analysis

24/09/2020

Spectrum fitting [0]

for $K_3[Fe(CN)_6] \cdot 3H_2O$:

$$I(E) = B - \frac{A \left(\frac{\Gamma}{2}\right)^2}{(E - E_0)^2 + \frac{\Gamma^2}{4}} \quad (5)$$

where E is the channel number

B is baseline

E_0 is centre of spectrum line

A is amplitude of Lorentzian (in counts) from the baseline

Γ is linewidth FWHM (in channels)

we want to change
the highlighted variables
in order to minimize
 χ^2

define a function in our code based on expression (5)
(playfully) call it

```
def not_a_Lorentzian(E, B, A, E_0, Γ):  
    return B - (A * (Γ / 2)**2 / ((E - E_0)**2 + (Γ / 2)**2))
```

where E is a linspace object that matches the number of bins
we have $E = np.linspace(1, 512, 512)$

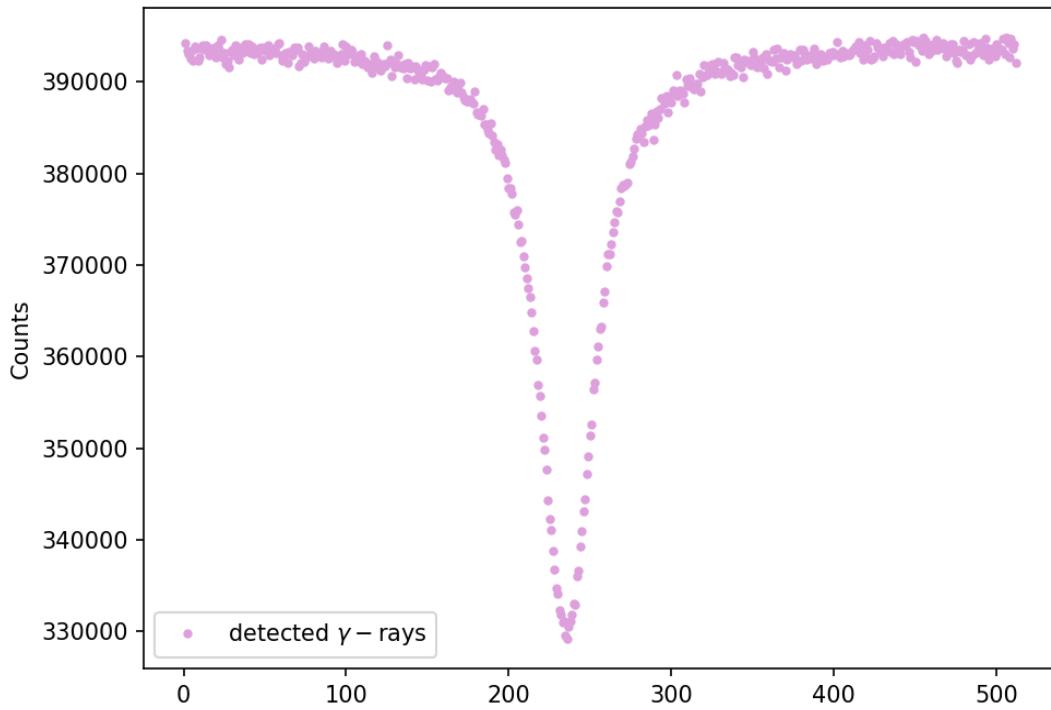


fig 13. Potassium Ferrocyanide data Spectrum of γ -rays
 $V = 160 \pm 1 \text{ mV}$ with correct folding of spectrum
The velocity waveform peak-to-peak amplitude is $160 \pm 1 \text{ mV}$
from this data our initial parameters guess is

$$B = 395 \times 10^3 \text{ counts}$$

$$A = 60 \times 10^3 \text{ counts}$$

$$E_0 = 239 \text{ (bin units)}$$

$$\Gamma = 80 \text{ (bin units)}$$

our fitting function:-

```
def KFe_spectrum_fitting(E, data_KFe):
    # Potassium Ferrocyanide data
    # initial parameters from eyeballing plot
    initial_guess = [395 * 1e3, 60 * 1e3, 234, 80]
    pars, pcov = scipy.optimize.curve_fit(not_a_Lorentzian, E, data_KFe, p0=initial_guess)
    perr = np.sqrt(np.diag(pcov))
    fit = not_a_Lorentzian(E, *pars)
    # return fit parameters
    return pars, fit
```

the fit we obtain is

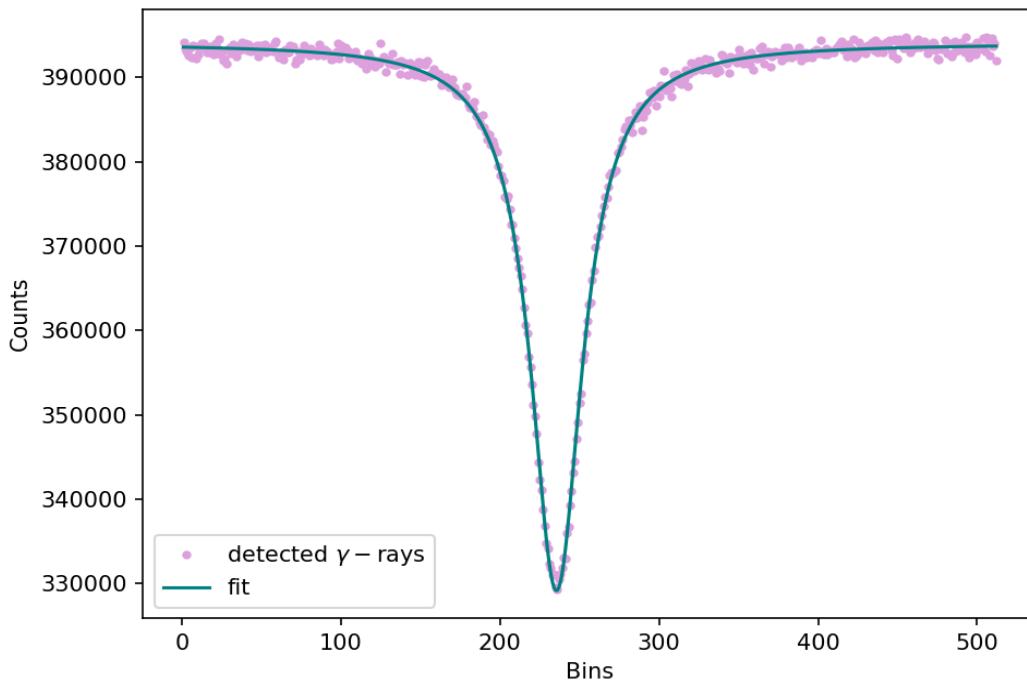


fig 14. Potassium Ferrocyanide data Spectrum of γ -rays
 $V = 160 \pm 1 \text{ mV}$ with correct folding of spectrum
with optimal fit obtained from `scipy.optimize.curve_fit`
the algorithm minimizes χ^2 by varying fit parameters.

fit parameters		parameter errors
B	394059	40
A	64947	170
E ₀	235.49	0.05
T	39.39	0.16

Table 1. obtained parameters from fit errors are calculated as per the code block above; using the square root of the diagonal of the obtained covariance matrix.
(units are specified in the previous page)

Möller spectrum of α -Fe [0]

24/09/2020

we use

$$I(E) = B - \sum_{i=1}^6 \frac{A_i (\frac{\Gamma_i}{2})^2}{(E - E_{0,i})^2 + \frac{\Gamma_i^2}{4}} \quad (6)$$

We will not assume Γ_i is fixed for all i .

Based on expression (6) we write:

```
def not_6_Lorentzians(E, B, A_1, E_0_1, Γ_1, A_2, E_0_2, Γ_2, A_3, E_0_3, Γ_3, A_4, E_0_4, Γ_4, A_5, E_0_5, Γ_5, A_6, E_0_6, Γ_6):
    return B - (A_1 * (Γ_1 / 2)**2 / ((E - E_0_1)**2 + (Γ_1 / 2)**2) + \
    A_2 * (Γ_2 / 2)**2 / ((E - E_0_2)**2 + (Γ_2 / 2)**2) + \
    A_3 * (Γ_3 / 2)**2 / ((E - E_0_3)**2 + (Γ_3 / 2)**2) + \
    A_4 * (Γ_4 / 2)**2 / ((E - E_0_4)**2 + (Γ_4 / 2)**2) + \
    A_5 * (Γ_5 / 2)**2 / ((E - E_0_5)**2 + (Γ_5 / 2)**2) + \
    A_6 * (Γ_6 / 2)**2 / ((E - E_0_6)**2 + (Γ_6 / 2)**2) )
```

Once again, to set initial guesses for our fit parameters we use a plot of our data, and visually estimate the values for our parameters.

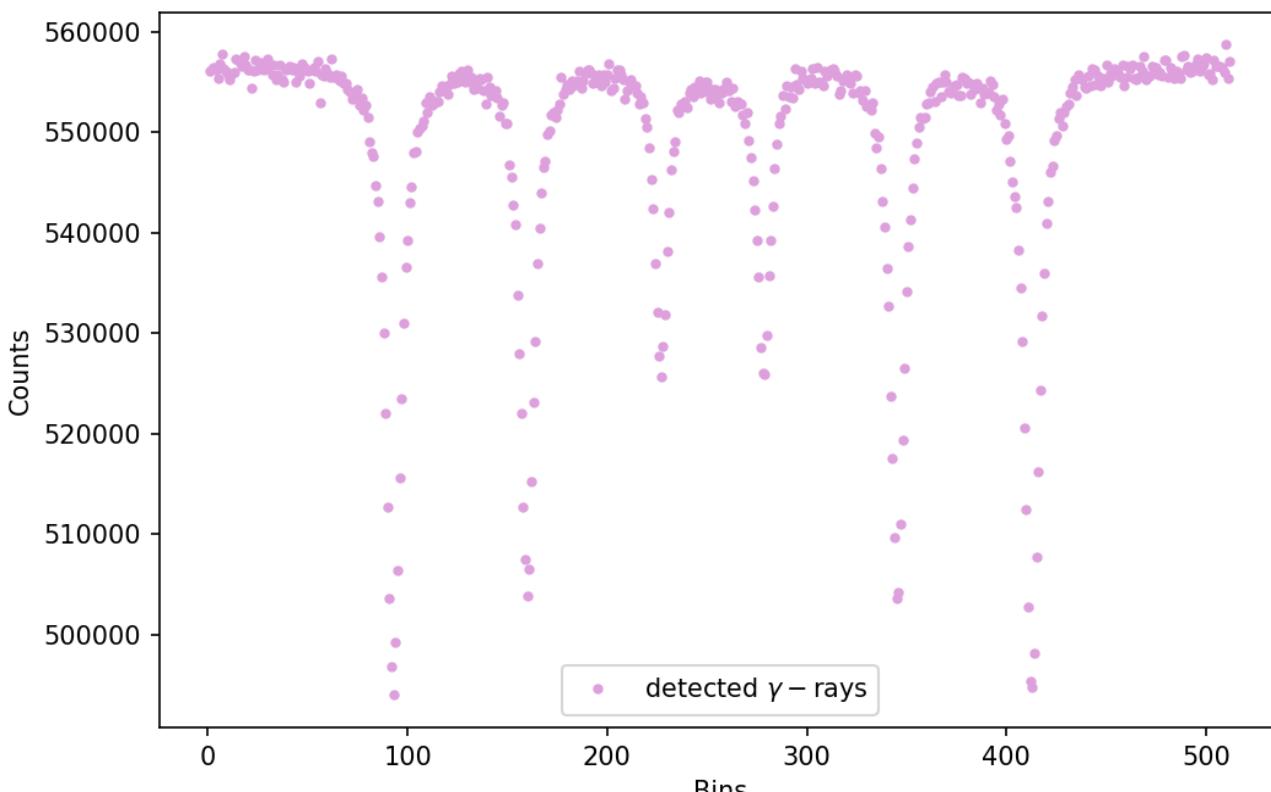


fig 15. α -Fe data Spectrum of γ -rays

$V = 711 \pm 1.4 \text{ mV}$ with correct folding of spectrum

The velocity waveform peak-to-peak amplitude is $711 \pm 2 \text{ mV}$

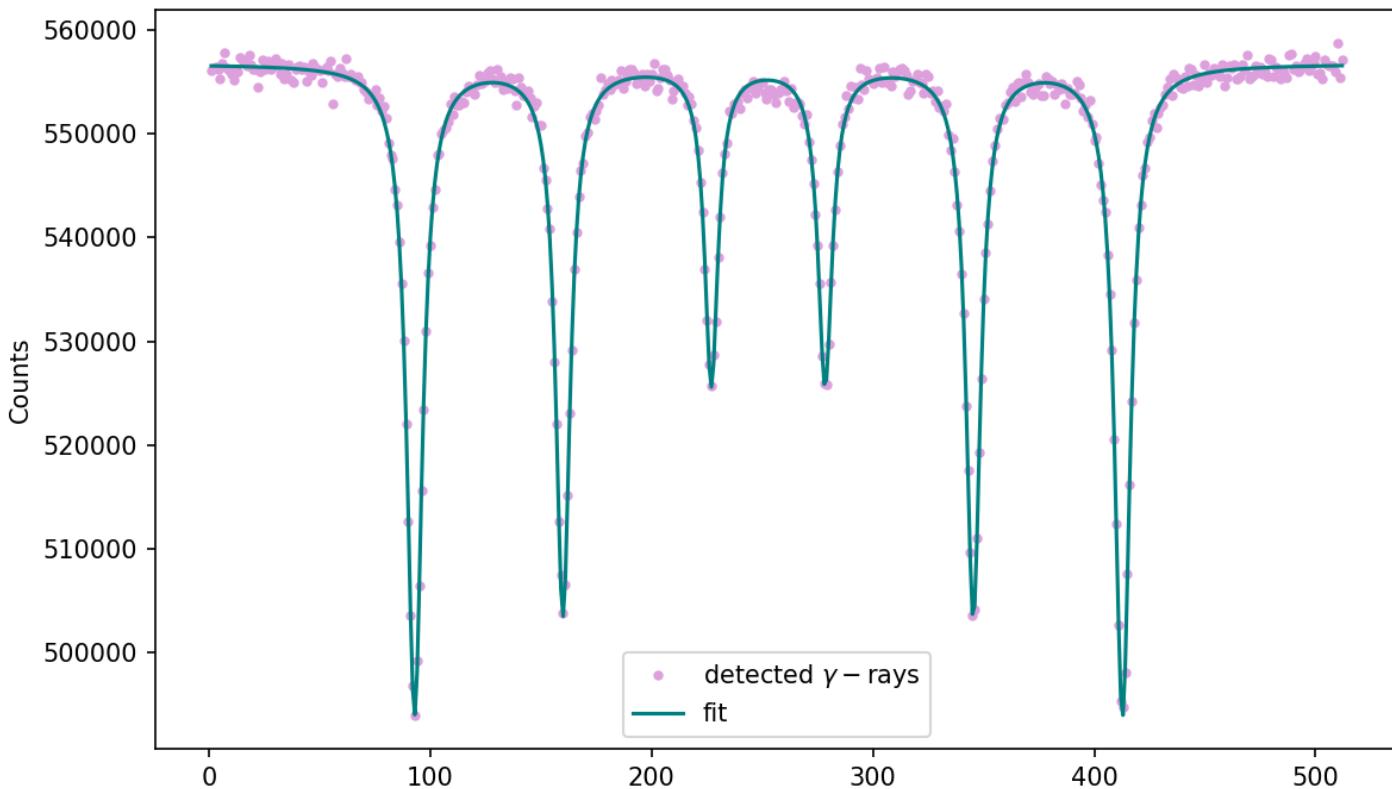
$A_1 = 60 \times 10^3$ counts $E_{0,1} = 90$ (bin units) $\Gamma_1 = 20$ (bin units)	$A_2 = 50 \times 10^3$ counts $E_{0,2} = 160$ (bin units) $\Gamma_2 = 20$ (bin units)	$A_3 = 30 \times 10^3$ counts $E_{0,3} = 225$ (bin units) $\Gamma_3 = 20$ (bin units)	$B = 560 \times 10^3$ counts
$A_4 = 30 \times 10^3$ counts $E_{0,4} = 280$ (bin units) $\Gamma_4 = 20$ (bin units)	$A_5 = 50 \times 10^3$ counts $E_{0,5} = 345$ (bin units) $\Gamma_5 = 20$ (bin units)	$A_6 = 60 \times 10^3$ counts $E_{0,6} = 410$ (bin units) $\Gamma_6 = 20$ (bin units)	

Table 2. our initial parameters guesses for each of the inverted lorentzians in the spectrum. we have 19 "optimisable" parameters

24/09/2020

our fitting function =

```
def alphaFe_spectrum_fitting(E, data_alphaFe):
    # α-Iron data
    # initial parameters from eyeballing plot
    initial_guess = [560 * 1e3, 60 * 1e3, 90, 20, 50 * 1e3, 160, 20, 30 * 1e3, 225, 20, 30 * 1e3, 280, 20, 50 * 1e3, 345, 20, 60 * 1e3, 410, 20]
    pars_6, pcov = scipy.optimize.curve_fit(not_6_Lorentzians, E, data_alphaFe, p0=initial_guess)
    perr = np.sqrt(np.diag(pcov))
    fit_6 = not_6_Lorentzians(E, *pars_6)
    # print(f"\n{pars_6}\n{pcov}\n{perr}")
    # return fit parameters
    return pars_6, fit_6
```

 \therefore the fit we obtain isfig 16. α -Fe data Spectrum of γ -rays Bins

$V = 711 \pm 1.4 \text{ mV}$ with correct folding of spectrum
with optimal fit obtained from `scipy.optimize.curve_fit`
the algorithm minimizes χ^2 by varying fit parameters.

fit parameters	Parameter errors	fit parameters	Parameter errors
$B = 55717000$	50		
$A_1 = 62482$	400	$A_4 = 30659$	400
$E_{01} = 92.88$	0.03	$E_{04} = 278.39$	0.05
$\Gamma_1 = 8.49$	0.08	$\Gamma_4 = 7.77$	0.16
$A_2 = 52805$	400	$A_5 = 52997$	400
$E_{02} = 159.91$	0.03	$E_{05} = 345.36$	0.03
$\Gamma_2 = 7.97$	0.1	$\Gamma_5 = 8.0$	0.1
$A_3 = 30607$	500	$A_6 = 62669$	400
$E_{03} = 226.97$	0.05	$E_{06} = 412.76$	0.03
$\Gamma_3 = 7.30$	0.16	$\Gamma_6 = 8.57$	0.08

Table 3 . obtained parameters from fit errors are calculated as per the code block above using the square root of the diagonal of the obtained covariance matrix . (units are specified in the previous page)

Analysis Questions

6/10/2020

1. The half life of the 14.4 keV level is 97.7 ns. Use the Heisenberg uncertainty principle formula, $\Delta E \Delta t \geq \hbar$, to calculate its natural linewidth in eV and in mm/s. (Use the simple Doppler shift formula $\Delta E/E_\gamma = \Delta v/v$, where the appropriate velocity, v , is the speed of light, to convert the units. Note that Δt must be the mean life, not the half life [why?]).

We use mean life because we are considering a lattice of nuclei in the sample (as opposed to a single unstable nucleus)

Heisenberg unc. princ: $\Delta E \Delta t \geq \hbar$

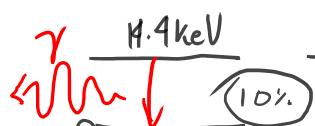
$$\therefore \Delta E \geq \frac{\hbar}{\Delta t} \text{ where } \Delta t = T_{1/2} / \log 2 = 141 \text{ ns}$$

$$\therefore \Delta E \geq 7.479 \times 10^{-28} \text{ J} = 9.668 \times 10^{-9} \text{ eV}$$

To convert the natural linewidth of the transition between eV and mm/s we use The doppler-shift formula: eqn(4) $\frac{\Delta E}{E_\gamma} = \frac{\Delta v}{c}$

2. The internal conversion coefficient of the 14.4 keV level is 8.2. How will this affect the number of 14.4 keV gamma-rays and x-rays hitting the detector in the Pulse height gamma spectrum section? (Most research Mössbauer experiments use a gas filled proportional counter, which picks up all the 6.3 keV x-rays and does not detect the 122 and 137 keV gamma rays.)

* we do not detect ~90% of decay energy emission from the 14.4 keV level

 Internal conversion is the process of emission of an atomic electron after it has absorbed energy. This type of decay often occurs simultaneously to γ -ray emission

3. What are two origins of the 6.3 keV Fe x-rays? (Hint – you should have one from Q2. If you can't think of the second – what did Einstein get his Nobel prize for?)

Internal conversion
photoelectric effect } emitted K-shell electrons

4. The broad band of intensity visible in the pulse height spectrum below the (122 + 137) keV peak is a combination of a Compton and an escape peak. You should know about the Compton peak from 2nd year physics. What is an escape peak?

incident photons "passing through" the detector volume can produce a photoelectron \rightarrow emitting an X-ray photon (K_α photon)

5. Radiation counting follows a Poisson distribution. To a good approximation, the total spread of data in the baseline is $\sim 5\sigma (\pm 2.5\sigma)$. How many counts do we need for a dip to be observable as \approx the spread of the data when the dip is (a) 10% and (b) 1% of the baseline? You need to consider the relation between σ and the number of counts.

spread $\sim 5\sigma \pm 2.5\sigma$, $\sigma = \sqrt{N}$

N is the baseline

(a) $0.1 \cdot N = 5\sigma$

$\therefore N = 50\sigma$

(b) $0.01 \cdot N = 5\sigma$

$\therefore N = 500\sigma$

THIS TELLS ME:

- How many counts are needed to get an analysable spectrum

\therefore How long I count for

6. Calculate the recoil energy of a free iron nucleus after emitting a 14.4 keV photon. How does this compare with your estimates of:

1. the binding energy of the atom in the solid (i.e. the chemical B.E.)
2. phonon energies? (If you have not yet encountered phonons, or quantized lattice waves, you can estimate the longest wavelength (lowest energy) phonon which can exist parallel to the gamma direction (and hence can be created by the gamma emission) in the 6 μm thick source foil. Velocity of sound in metals is ~ 1×10^4 m/s.)

What conclusions can one draw from these comparisons?

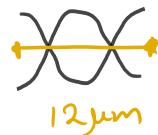
from equation(1)

$$E_R = \frac{E_\gamma^2}{2m_e c^2} = \frac{(14.4 \text{ keV})^2}{2(9.2733 \times 10^{-26} \text{ kg})(2.99 \times 10^8 \text{ m/s})^2} = 1.99 \text{ meV}$$

1. The BE of an outer electron in solid iron is 7.9024 eV [11]

2. The longest wavelength $\lambda = \frac{v}{f}$ and lowest energy $E = hf$
 $v \approx 1 \times 10^8 \text{ m/s}$

$\lambda \approx 12 \mu\text{m}$ since { we can have free BCS with phonons }



$$\therefore \frac{v}{\lambda} = f = 8.33 \times 10^8 \text{ s}^{-1} \quad \therefore E_{\text{phonon}} = hf \approx 6 \times 10^{-25} \text{ J}$$

$$\therefore E = 3 \times 10^3 \text{ meV}$$

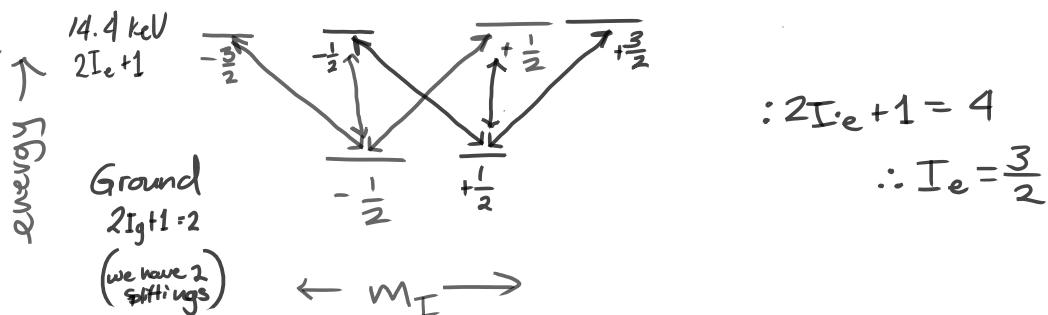
$E_R \ll \text{BE}$ \therefore Recoil from γ emission (14.4 keV) will not remove an iron atom from an iron crystal.

Energy of phonon $\ll E_R$ a large amount of the recoil energy is lost to the crystal environment.

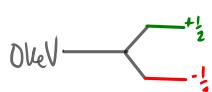
7. The ground state and the 14.4 keV excited level are split by the magnetic hyperfine field (or any other magnetic field) into $2I_g + 1$ and $2I_e + 1$ sublevels respectively. Quantum mechanical selection rules for dipole transitions tell us that transitions are only allowed which have $\Delta m = 0$ or ± 1 . Knowing that you observe 6 transitions and that $I_g = 1/2$, draw energy level diagrams and hence deduce the angular momentum I_e . (You should show that the spectrum would be different if the sign of the magnetic moments of the ground and excited states were the same or different.)

Transition Allowed
 $\Delta m = 0$ or ± 1

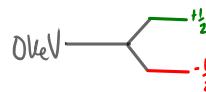
$$I_g = \frac{1}{2}$$



If their magnetic moment signs different



else their magnetic moment signs equal



• -m
 ♦ m

Fig 17. Different energy level diagrams considering the possible sign "matching" of the magnetic moments of the ground and excited states. We only included the energy level diagrams that consider the positive magnetic moment of the ground state.

8. Given that the magnetic moment of the ground state, μ_g , is +0.0904 nuclear magnetons, fit your spectrum with six Lorentzians, calibrate the velocity scale and calculate the value of μ_e (unlike electron moments, nuclear moments can be positive or negative).

$I_g = \frac{1}{2}$
 $I_e = \frac{3}{2}$ } Nuclear angular momentum for ground & excited states

The nuclear magneton:

- $\mu_N = 5.0507837461 \times 10^{-27} \text{ JT}^{-1}$ [12]

The Hyperfine field:

- $B = 33.04 \text{ T}$

The Ground State magnetic moment:

- $\mu_g = 0.0904 \mu_N = g_g \mu_N I_g \therefore \text{we can see that } g_g = \frac{\mu_g}{\mu_N I_g}$

- $E_i = g_g \mu_N B m_I$

using $m_g = (-\frac{1}{2}, \frac{1}{2})$

we calculate the energies of the ground state splittings:

* $E_g = (-1.51 \times 10^{-26}, 1.51 \times 10^{-26}) \text{ J}$

this is
* a list.

(So we can draw our spectrum knowing that
the value $m_g = -\frac{1}{2}$ yields less energy than $m_g = \frac{1}{2}$)

The uncertainty in the energy of the ground state splittings:

$$\sigma(E_g) = \sqrt{\left(\frac{\sigma(g_g)}{g_g}\right)^2 + \left(\frac{\sigma(\mu_N)}{\mu_N}\right)^2 + \left(\frac{\sigma(B)}{B}\right)^2} \times E_g$$

We will calculate this after we find $\sigma(g_g)$ (in the next couple of pages)

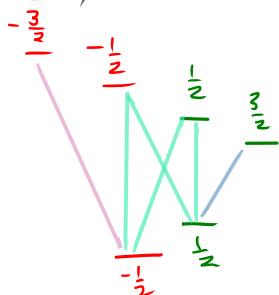
From your diagram from Q7, identify two of your six absorption lines which are separated by Δ .

You should also find another pair. Thus the spectrum gives you two independent values of Δ . Determine these values, in channels, from your fitting of the spectrum.

Now calculate Δ from the values of μ_g and B and convert it from J to eV to mm/s.

If we expand our diagram in Q7 we can identify two of our 6 absorption lines which are separated by Δ :

If their magnetic moment signs different
(a)



else: their magnetic moment signs equal
(b)

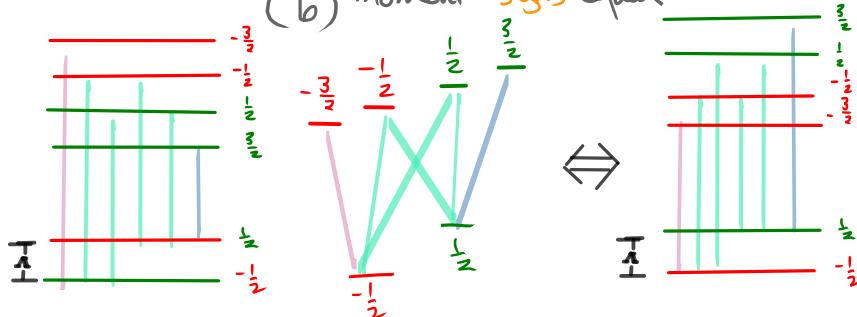
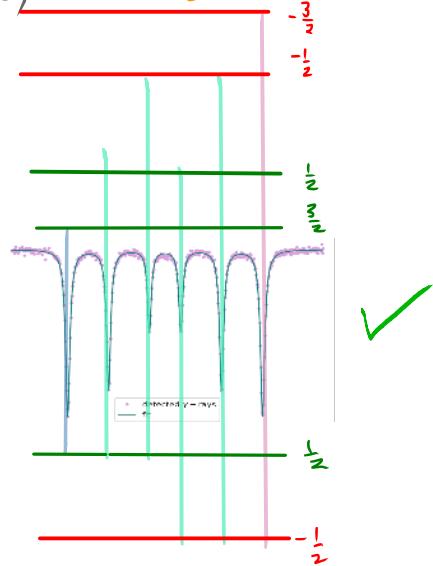


Fig 18. Expanded view of Fig 17 Possible configuration of the energy levels. Here we include the transition lines and mark in green the lines that are separated by the distance between the ground state splittings (Δ_g)

If: their magnetic moment signs different
 (a) moment signs different



else: their magnetic moment signs equal
 (b) moment signs equal

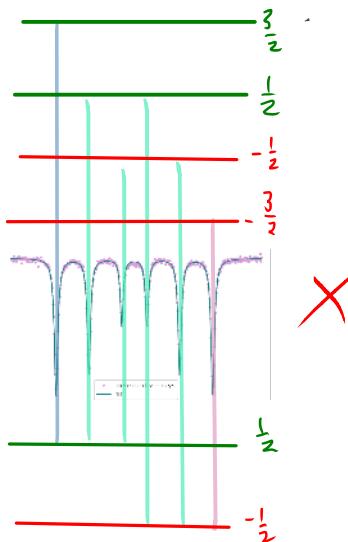


Fig 19. The energy level diagrams superimposed on the spectrum of α -Fe (including the Lorentzian fit.)

After inverting the transition lines (horizontally) to match the lines to the highest energy dip in the spectrum, we can identify that only figure (a) matches the transition line of highest energy to the largest energy difference peak. Therefore we can conclude that the magnetic moments of the ground and excited states have different signs.

We calculate the energy differences between the centres of the dips in our fit of the absorption spectrum.

Fit: Centers of absorption spectrum	Fit error	ΔE (adjacent)	$n(\Delta E)$
E_{01} 92.88	0.03	67.04	0.04
E_{02} 159.91	0.03	67.06	0.06
E_{03} 226.97	0.05	51.42	0.07
E_{04} 278.39	0.05	66.97	0.06
E_{05} 345.36	0.03	67.40	0.04
E_{06} 412.76	0.03		

Table 4 - Position of 6 Lorentzian dips centres (originally displayed in Table 3) and Energy difference between adjacent dips (Bm units) from these numbers we verify the symmetry of our spectrum. We have marked the two transitions we identified in Fig 19. using green (on the left)

From the symmetry of our spectrum we can deduce that the dip pairs differences (i.e. Δ) are :

$$\Delta_1 = E_{05} - E_{03} = 345.36 - 226.97 = 118.39 \text{ chan}$$

$$\Delta_2 = E_{04} - E_{02} = 278.39 - 159.91 = 118.48 \text{ chan}$$

The uncertainty propagation is :

$$u(\Delta_1) = \sqrt{u(E_{05})^2 + u(E_{03})^2} = \sqrt{u(0.03)^2 + u(0.05)^2} = 0.06 \text{ chan}$$

$$u(\Delta_2) = \sqrt{u(E_{04})^2 + u(E_{02})^2} = \sqrt{u(0.05)^2 + u(0.03)^2} = 0.06 \text{ chan}$$

(our results are within uncertainty of each other.)

$$\Delta_1 = (118.39 \pm 0.06) \text{ chan}$$

$$\Delta_2 = (118.48 \pm 0.06) \text{ chan}$$

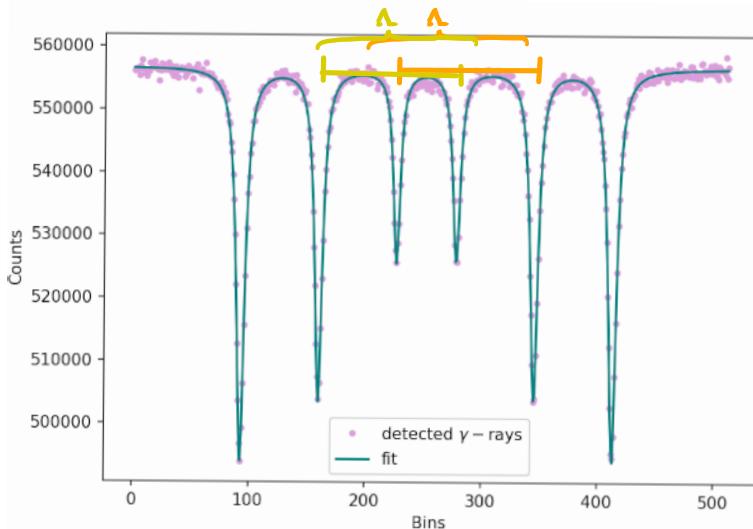


Fig 20. our α -Fe spectrum and fit
we have drawn a cartoon
of the approximate
separations Δ_1, Δ_2
as a visual aid in
our calculations.

Calculating Δ_g

14/10/2020

$$u(\mu_N) = 0.000000015 \times 10^{-27} \text{ JT}^{-1} [12]$$

The Hyperfine field has no reported uncertainty

- $B = 33.04 \text{ T}$ \therefore assume 0.005 T is the uncertainty

The ground state magnetic moment has uncertainty equal to:

$$u(\mu_g) = \sqrt{\frac{u(\mu_N)^2}{\mu_N}} \times \mu_g = \sqrt{\left(\frac{0.000000015}{5.0507837461}\right)^2} \times (4.57 \times 10^{-28}) = 1.4 \times 10^{-37} \text{ JT}^{-1}$$

$$\therefore \mu_g = (4.5659085065 \pm 0.0000000014) \times 10^{-28} \text{ JT}^{-1}$$

The ratio $g_g = \frac{\mu_g}{\mu_N I_g} = 0.180800000000$ has uncertainty:

$$\frac{u(g_g)}{g_g} = \sqrt{\left(\frac{u(\mu_g)}{\mu_g}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2 + \left(\frac{u(I_g)}{I_g}\right)^2} \rightarrow$$

$$\therefore u(g_g) = \sqrt{\left(\frac{u(\mu_g)}{\mu_g}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2} \times g_g = \sqrt{\left(\frac{0.000000014}{4.5659085065}\right)^2 + \left(\frac{0.000000015}{5.0507837461}\right)^2} \times 0.18 = 8 \times 10^{-11}$$

$$\therefore g_g = 0.180800000000 \pm 0.10000000008$$

* (Brief interlude) ... so we finally, calculate $u(E_g)$ *

$$u(E_g) = \sqrt{\left(\frac{u(g_g)}{g_g}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2 + \left(\frac{u(B)}{B}\right)^2} \times E_g \\ = \sqrt{\left(\frac{8 \times 10^{-11}}{0.18}\right)^2 + \left(\frac{0.000000015}{5.0507837461}\right)^2 + \left(\frac{0.005}{33.04}\right)^2} \times (-1.51 \times 10^{-26}, 1.51 \times 10^{-26}) \text{ J} = 8 \times 10^{-36} \text{ J}$$

The values of the the energies of the ground state splitting are:

$$\therefore E_{\pm} = -1.51 \times 10^{-26} \text{ J and } E_{\pm} = 1.51 \times 10^{-26} \text{ both with uncertainty } \pm 8 \times 10^{-36}$$

The ground state energy splitting separation Δ_g is:

$$\Delta_g = g_g \mu_N B = 0.180800000000 \times 5.0507837461 \times 33.04 = 3.01715 \times 10^{-26} \text{ J}$$

where the uncertainty in Δ_g is

$$u(\Delta_g) = \sqrt{\left(\frac{u(g_g)}{g_g}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2 + \left(\frac{u(B)}{B}\right)^2} \times \Delta_g = \sqrt{\left(\frac{8 \times 10^{-11}}{0.18}\right)^2 + \left(\frac{0.000000015}{5.0507837461}\right)^2 + \left(\frac{0.005}{33.04}\right)^2} \times 3.01715 \times 10^{-26} \text{ J}$$

$$\therefore u(\Delta_g) = 0.00005 \times 10^{-26} \text{ J}$$

\therefore the value of the ground state splitting separation

$$\Delta_g = (3.01715 \pm 0.00005) \times 10^{-26} \text{ J}$$

We convert Δ_g to eV by dividing by $1.602176639 \times 10^{-19} \text{ J eV}^{-1}$

$$\therefore \Delta_g = (1.88316 \pm 0.00003) \times 10^{-7} \text{ eV}$$

To convert the natural linewidth of the transition between eV and mm/s we use The doppler-shift formula (where $\Delta E = \Lambda_g$)

$$\nu = \frac{\Delta E}{E_g} c = \frac{1.88316 \times 10^7 \text{ eV}}{14.9 \text{ keV}} (299792458 \text{ m s}^{-1}) = 3.92 \text{ mm/s}$$

To calculate this uncertainty, we estimate $u(E_g) = 0.05 \text{ keV}$.

$$\therefore u(\nu) = \sqrt{\left(\frac{u(\Delta E)}{\Delta E}\right)^2 + \left(\frac{u(E_g)}{E_g}\right)^2} \times \nu = \sqrt{\left(\frac{0.00005}{1.88316}\right)^2 + \left(\frac{0.05}{14.9}\right)^2} \times 3.92 \text{ mm/s} = 0.01 \text{ mm/s}$$

$$\therefore \nu = 3.92 \pm 0.01 \text{ mm/s}$$

Then, the calibration constant for this spectrum is [0]

$$K_{1.8} = \frac{\Delta (\text{mm/s})}{\Delta (\text{channels})} \quad \text{Here we will use } \Delta_1 = (118.39 \pm 0.06) \text{ chan}$$

Since both $\Delta_1 \neq \Delta_2$ are within uncertainty of each other.

$$\therefore K_{1.8} = \frac{3.92 \text{ mm/s}}{118.39 \text{ chan}} = 0.0331 \text{ mm/s/chan}$$

with uncertainty :

$$\therefore u(K_{1.8}) = \sqrt{\left(\frac{u(\nu)}{\nu}\right)^2 + \left(\frac{u(\Delta)}{\Delta}\right)^2} \times K_{1.8} = \sqrt{\left(\frac{0.01}{3.92}\right)^2 + \left(\frac{0.06}{118.39}\right)^2} \times 0.0331 \text{ mm/s/chan}$$

$$\therefore u(K_{1.8}) = 0.0001 \text{ mm/s/chan}$$

$$\therefore K_{1.8} = 0.0331 \pm 0.0001 \text{ mm/s/chan}$$

The excited state has magnetic moment [0]

$$\mu_e = g_e \mu_N I_e$$

The spectrum has 4 independent measures of λ , where λ is the splitting between each of the 4 levels of the excited state. Using the mean value of λ from your fits, and the calibration constant, use the reverse procedure to the ground state calculation to determine the magnetic moment of the excited state in units of μ_N .

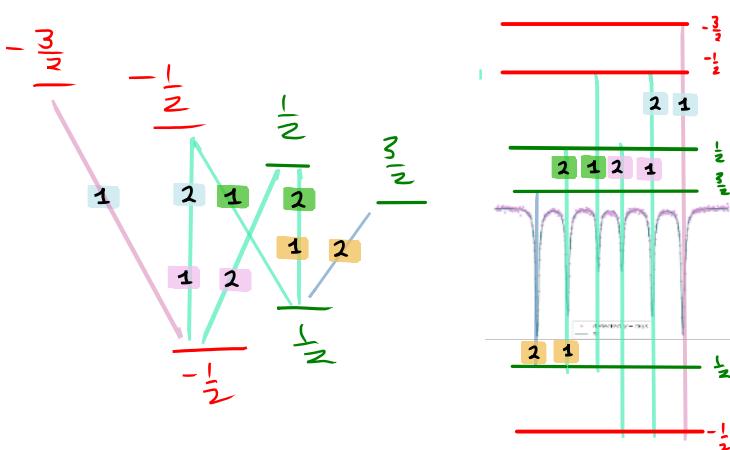


Fig 21. Energy level diagrams with possible transitions. We have labelled the pairs of transitions that are different by the same measure of λ ; (There are 4 possible measures)

λ_1	λ_2	λ_3	λ_4
$-\frac{3}{2} \rightarrow -\frac{1}{2}$	$-\frac{1}{2} \rightarrow -\frac{1}{2}$	$-\frac{1}{2} \rightarrow \frac{1}{2}$	$\frac{1}{2} \rightarrow \frac{3}{2}$
$-\frac{1}{2} \rightarrow -\frac{1}{2}$	$\frac{1}{2} \rightarrow -\frac{1}{2}$	$\frac{1}{2} \rightarrow \frac{1}{2}$	$\frac{3}{2} \rightarrow \frac{1}{2}$

from fig 21 we were able to identify the dips locations in our data and fit.

The measures for λ_i are:

$$\lambda_1 = E_{0.6} - E_{0.5} = 412.76 - 345.36 = 67.40$$

$$\lambda_2 = E_{0.5} - E_{0.4} = 345.36 - 278.39 = 66.97$$

$$\lambda_3 = E_{0.3} - E_{0.2} = 226.97 - 159.91 = 67.06$$

$$\lambda_4 = E_{0.2} - E_{0.1} = 226.97 - 92.88 = 67.09$$

$\delta\lambda$ uncertainty propagation is:

$$u(\lambda_1) = \sqrt{u(E_{0.6})^2 + u(E_{0.5})^2} = \sqrt{u(0.03)^2 + u(0.03)^2} = 0.04 \text{ chan}$$

$$u(\lambda_2) = \sqrt{u(E_{0.5})^2 + u(E_{0.4})^2} = \sqrt{u(0.03)^2 + u(0.05)^2} = 0.06 \text{ chan}$$

$$u(\lambda_3) = \sqrt{u(E_{0.3})^2 + u(E_{0.2})^2} = \sqrt{u(0.03)^2 + u(0.05)^2} = 0.06 \text{ chan}$$

$$u(\lambda_4) = \sqrt{u(E_{0.2})^2 + u(E_{0.1})^2} = \sqrt{u(0.03)^2 + u(0.03)^2} = 0.04 \text{ chan}$$

We now must calculate the mean value of

$$\lambda_1 = 67.40 \pm 0.04 \text{ chan}$$

$$\lambda_2 = 66.97 \pm 0.06 \text{ chan}$$

$$\lambda_3 = 67.06 \pm 0.06 \text{ chan}$$

$$\lambda_4 = 67.09 \pm 0.04 \text{ chan}$$

$$\left. \begin{array}{l} \lambda_1 = 67.40 \\ \lambda_2 = 66.97 \\ \lambda_3 = 67.06 \\ \lambda_4 = 67.09 \end{array} \right\} \begin{aligned} \mu_\lambda &= \frac{\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4}{4} = 67.12 \text{ chan} \\ u(\mu_\lambda) &= \sqrt{\frac{1}{4} \left[u(\lambda_1)^2 + u(\lambda_2)^2 + u(\lambda_3)^2 + u(\lambda_4)^2 \right]} \\ &= \sqrt{\frac{1}{4} \left[(0.04)^2 + (0.06)^2 + (0.06)^2 + (0.04)^2 \right]} \\ &= 0.03 \text{ chan} \end{aligned}$$

$$\therefore \mu_\lambda = 67.12 \pm 0.03 \text{ chan}$$

from the energy of the excited state splittings, we obtain the separation of the splittings

$$E_i - E_f = g_e \mu_N B m_{I_e} \Rightarrow \Delta E = g_e \mu_N B = \lambda$$

Here we use the mean value of $\lambda = \mu_\lambda$

To change the units of μ_λ , we use our calibration constant $K_{1.8} = 0.0331 \pm 0.0001 \text{ mm/s/chan}$ to find the velocity.

$$\lambda_{\mu_\lambda} = \mu_\lambda \times K_{1.8} = 67.12 \text{ chan} \times 0.0331 \text{ mm/s/chan}$$

$$\lambda_{\mu_\lambda} = 2.223 \text{ mm/s}$$

$$\text{with } u(\lambda_{\mu_\lambda}) = \sqrt{\left(\frac{u(\mu_\lambda)}{\mu_\lambda}\right)^2 + \left(\frac{u(K_{1.8})}{K_{1.8}}\right)^2} \times \lambda_{\mu_\lambda} = \sqrt{\left(\frac{0.03}{67.12}\right)^2 + \left(\frac{0.0001}{0.0331}\right)^2} \times 2.223 \text{ mm/s} = 0.008$$

$$\therefore \lambda_{\mu_\lambda} = 2.223 \pm 0.008 \text{ mm/s}$$

We proceed by using the Doppler Shift formula: $\Delta E = \frac{E_8 V_{\mu_\lambda}}{c}$

$$\Delta E = \frac{(14.4 \text{ keV} \times 2.223 \text{ mm/s})}{299792458 \text{ m/s}} = \frac{(14.4 \text{ keV} \times 0.00223 \text{ m/s})}{299792458 \text{ m/s}} = 1.068 \times 10^{-10} \text{ keV}$$

We convert ΔE to [J] by multiplying by $1.602176634 \times 10^{-16} \text{ J keV}^{-1}$

$$\therefore \Delta E = 1.711 \times 10^{-26} \text{ J}$$

with uncertainty

$$u(\Delta E) = \sqrt{\left(\frac{u(E_8)}{E_8}\right)^2 + \left(\frac{u(V_{\mu_\lambda})}{V_{\mu_\lambda}}\right)^2} \times \Delta E$$

$$\therefore u(\Delta E) = \sqrt{\left(\frac{0.05}{14.4}\right)^2 + \left(\frac{0.008}{2.223}\right)^2} \times 1.711 \times 10^{-26} \text{ J} = 0.008 \times 10^{-26} \text{ J}$$

$$\therefore \Delta E = 1.711 \pm 0.008 \times 10^{-26} \text{ J}$$

Recall that $\Delta E_e = g_e \mu_N B$

$$\therefore g_e = \frac{\Delta E}{\mu_N B} = \frac{1.711 \times 10^{-26} \text{ J}}{5.0507837461 \times 10^{27} \text{ JT}^{-1} \times 33.04} = 0.1025$$

with uncertainty

$$\therefore u(g_e) = \sqrt{\left(\frac{u(\Delta E)}{\Delta E}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2 + \left(\frac{u(B)}{B}\right)^2} = \sqrt{\left(\frac{0.008}{1.711}\right)^2 + \left(\frac{0.0000000015}{5.0507837461}\right)^2 + \left(\frac{0.005}{33.04}\right)^2}$$

$$\therefore u(g_e) = 0.0005$$

$$\therefore g_e = 0.1025 \pm 0.0005$$

To find the magnetic moment of the excited state we use:

$$\mu_e = g_e \mu_N I_e$$

$$\therefore \mu_e = 0.1025 \times 5.0507837461 \times 10^{27} \text{ JT}^{-1} \times \frac{3}{2} = 7.77 \text{ JT}^{-1}$$

with uncertainty

$$\therefore u(\mu_e) = \sqrt{\left(\frac{u(g_e)}{g_e}\right)^2 + \left(\frac{u(\mu_N)}{\mu_N}\right)^2} \times \mu_e$$

$$\therefore u(\mu_e) = \sqrt{\left(\frac{0.0005}{0.1025}\right)^2 + \left(\frac{0.000000015}{5.0507837461}\right)^2} \times \mu_e = 0.04 \text{ JT}^{-1}$$

$$\text{Then } \mu_e = (7.77 \pm 0.04) \text{ JT}^{-1}$$

which in units of the nuclear magneton is

$$\mu_e = (0.1538 \pm 0.0008) \mu_N$$

9. The maximum velocity is linear with helipot setting, so the calibration constant of the KFCN spectrum is:

$$K_{0.4} = \frac{0.4 \times K_{1.8}}{1.80}.$$

Note that you should replace the ratio (0.4/1.80) with the measured ratio of velocities calculated from your V_{pp} values (drive unit "monitor" voltages).

Take the fitted linewidth of your potassium ferrocyanide spectrum and convert it to mm/s. How does this compare with the expected value from Q1? Note that the expected experimental linewidth is the convolution of the source and absorber linewidths from Q1, with a ~30% increase for absorber thickness and imperfections.

we obtain the V_{pp} values from the infosheet about our data set

* We must replace the ratio (0.4/1.80) with 711/160

Our fwhm for KFCN spectrum is

$$\tau = 39.39 \pm 0.16 \text{ chan}$$

To convert our units into mm/s we use our calibration constant: $K_{0.4} = \frac{711 \times K_{1.8}}{160} = 1.47 \times 10^{-4} \text{ mm/s/channel}$

The uncertainty in $K_{0.4}$

$$\therefore u(K_{0.4}) = \sqrt{\left(\frac{2}{711}\right)^2 + \left(\frac{1}{160}\right)^2 + \left(\frac{0.0001}{0.0331}\right)^2} \times 1.47 \times 10^{-4} \text{ mm/s/channel}$$

$$\therefore u(K_{0.4}) = 0.01 \times 10^{-4} \text{ mm/s/channel}$$

$$\therefore k_{0.4} = (1.47 \pm 0.01) \times 10^{-4} \text{ mm/s / chan}$$

Then we can calculate a converted linewidth

$$\Gamma_v = k_{0.4} \Gamma$$

$$\therefore \Gamma_v = 1.47 \times 39.39 \text{ chan} = 5.79 \mu\text{m/s}$$

with uncertainty:

$$u(\Gamma_v) = \sqrt{\left(\frac{u(k_{0.4})}{k_{0.4}}\right)^2 + \left(\frac{u(\Gamma)}{\Gamma}\right)^2} \times \Gamma_v$$

$$\therefore u(\Gamma_v) = \sqrt{\left(\frac{0.01}{1.47}\right)^2 + \left(\frac{0.16}{39.39}\right)^2} \times 5.79 \mu\text{m/s}$$

$$\therefore u(\Gamma_v) = 0.05 \mu\text{m/s}$$

$$\text{Then } \Gamma_v = (5.79 \pm 0.05) \mu\text{m/s}$$

In Question 1 we calculated
the predicted natural linewidth using
the Heisenberg uncertainty principle.

$$\therefore \Delta E \geq 7.479 \times 10^{-28} \text{ J} = 9.668 \times 10^{-9} \text{ eV}$$

We convert this ΔE into a velocity by the doppler shift formula.

$$v = \frac{\Delta E}{E_8} c = \frac{9.668 \times 10^{-6} \text{ keV}}{14.9 \text{ keV}} (2.99792458 \text{ m ms}^{-1}) = 0.1 \text{ mm s}^{-1}$$

Note: The experimental linewidth is
the convolution of the source & the absorber
linewidths, with a ~30% increase for absorber
thickness & imperfections [o]

* Doubling this natural linewidth allows us to
roughly account for the convolution mentioned above.
We can also account for the ~30% increase by scaling
our result by 1.3, but given our precision its enough to double (~).

$$\therefore v_* = 0.2 \text{ mm s}^{-1} \quad (\text{vs}) \quad \Gamma_v = (5.79 \pm 0.05) \mu\text{m/s}$$

Γ_v is much smaller than v_*



10. Determine the isomer shift of potassium ferrocyanide w.r.t. α -iron (which is the reference for all ^{57}Fe isomer shifts – see Murad and Cashion 2004, p22). These do not appear in the same spectrum, so it must be a two-stage process.

To find C_0 (the channel at the centre of both spectra) we must consider off-by-1 errors ~~*~~
we have channel range [0-511]

$$\therefore C_0 = \frac{511}{2} = 255.50 \pm 0.05$$

(we estimated this uncertainty by the half-the-smallest-digit rule)

① Isomer shift of KFCN w.r.t source [0]

$$\text{IS}_{\text{KFCN}} = (C_{\text{KFCN}} - C_0) K_{0.4}$$

where C_{KFCN} is the fitted centre of the absorption line

We found $C_{\text{KFCN}} = 234.95 \pm 0.05$ channels

$$\therefore \text{IS}_{\text{KFCN}} = (235.49 - 255.50) \text{chan} \times 1.47 \times 10^{-4} \text{ mm/s / chan}$$

$$\therefore \text{IS}_{\text{KFCN}} = -2.94 \text{ mm/s}$$

Here we need to split the uncertainty calculation:

$$u(C_{\text{KFCN}} - C_0) = \sqrt{u(C_{\text{KFCN}})^2 + u(C_0)^2} = \sqrt{\left(\frac{0.05}{235.49}\right)^2 + \left(\frac{0.05}{255.50}\right)^2} = 0.1$$

$$\therefore u(\text{IS}_{\text{KFCN}}) = \sqrt{\left(\frac{u(C_{\text{KFCN}} - C_0)}{C_{\text{KFCN}} - C_0}\right)^2 + \left(\frac{u(K_{0.4})}{K_{0.4}}\right)^2} \times \text{IS}_{\text{KFCN}}$$

$$\therefore u(\text{IS}_{\text{KFCN}}) = \sqrt{\left(\frac{0.1}{-20.0}\right)^2 + \left(\frac{0.01}{1.47}\right)^2} \times (-2.94 \text{ mm/s}) = -0.03$$

Then the value of the isomer shift of KFCN w.r.t source is

$$\text{IS}_{\text{KFCN}} = -(2.94 \pm 0.03) \text{ mm/s}$$

(2) Similarly, the isomer shift of α Fe w.r.t the source is found by [0]

$$\text{IS}_{\alpha\text{Fe}} = (C_{\alpha\text{Fe}} - C_0) K_{1.8}$$

where we find $C_{\alpha\text{Fe}}$ by taking an average of every transition in the spectrum.

$$C_{\alpha\text{Fe}} = \frac{\sum_i^6 E_{oi}}{6} = 92.88 + 159.91 + 226.97 + 278.39 + 345.36 + 412.76 = 252.713 \text{ chan}$$

with uncertainty

$$u(C_{\alpha\text{Fe}}) = \sqrt{\sum_i^6 u(E_{oi})^2} \times \frac{1}{6} = \sqrt{0.03^2 + 0.03^2 + 0.05^2 + 0.05^2 + 0.03^2 + 0.03^2} \times \frac{1}{6}$$

$$\therefore u(C_{\alpha\text{Fe}}) = 0.016 \text{ chan}$$

$$\therefore C_{\alpha\text{Fe}} = (252.713 \pm 0.016) \text{ chan}$$

we then calculate $C_{\alpha\text{Fe}} - C_0 = (252.713 - 255.50) \text{ chan} = -2.79 \text{ chan}$
with uncertainty:

$$u(C_{\alpha\text{Fe}} - C_0) = \sqrt{u(C_{\alpha\text{Fe}})^2 + u(C_0)^2} = \sqrt{0.016^2 + 0.05^2} = 0.05$$

Then we proceed to calculate our Isomer shift for α Fe

$$\text{IS}_{\alpha\text{Fe}} = (C_{\alpha\text{Fe}} - C_0) K_{1.8} = -0.0923 \text{ mm/s}$$

The uncertainty in this value is

$$\therefore u(\text{IS}_{\alpha\text{Fe}}) = \sqrt{\left(\frac{u(C_{\alpha\text{Fe}} - C_0)}{C_{\alpha\text{Fe}} - C_0}\right)^2 + \left(\frac{u(K_{1.8})}{K_{1.8}}\right)^2} \times \text{IS}_{\alpha\text{Fe}}$$

$$\therefore u(\text{IS}_{\alpha\text{Fe}}) = \sqrt{\left(\frac{0.05}{-2.79}\right)^2 + \left(\frac{0.0001}{0.0331}\right)^2} \times -0.0923 \text{ mm/s} = -0.0018 \text{ mm/s}$$

Then the value of the Isomer shift of α Fe w.r.t source is

$$\text{IS}_{\alpha\text{Fe}} = -(0.0923 \pm 0.0018) \text{ mm/s}$$

The isomer shift of KFCN w.r.t α Fe is obtained by subtracting $\text{IS}_{\alpha\text{Fe}}$ from IS_{KFCN}

$$\therefore \text{IS}_{\text{KFCN}} - \text{IS}_{\alpha\text{Fe}} = -(2.94 - 0.0923) \text{ mm/s} = -2.85 \text{ mm/s}$$

with uncertainty:

$$u(\text{IS}_{\text{KFCN}} - \text{IS}_{\alpha\text{Fe}}) = \sqrt{u(\text{IS}_{\text{KFCN}})^2 + u(\text{IS}_{\alpha\text{Fe}})^2} = \sqrt{0.03^2 + 0.0018^2} = 0.03 \text{ mm/s}$$

$$\therefore \text{IS}_{\text{KFCN} \rightarrow \alpha\text{Fe}} = -2.85 \pm 0.03 \text{ mm/s}$$

Comparing our value for the
isomer shift between our samples to
Murad and Cashion (2004) $IS_{MC} = -0.035 \pm 0.007 \text{ mm/s}$

15/10/2020

$IS_{KFCN \rightarrow \alpha Fe} = -2.85 \pm 0.03 \text{ mm/s}$? I don't understand

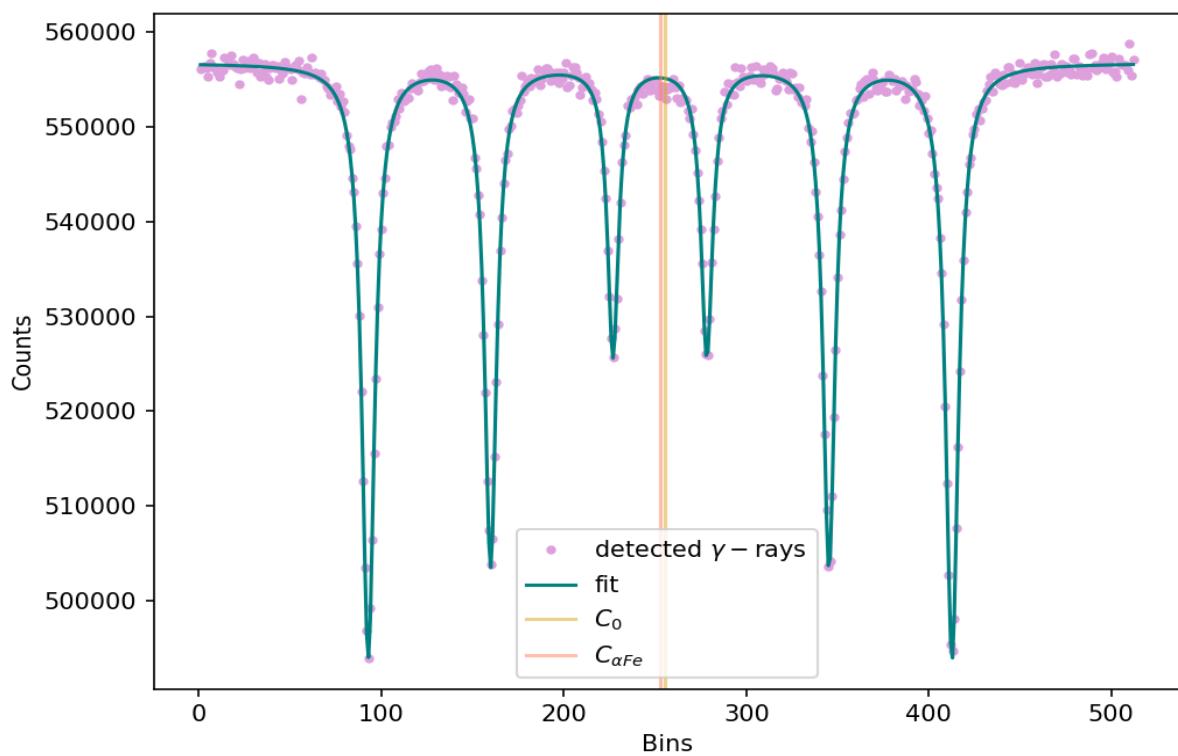
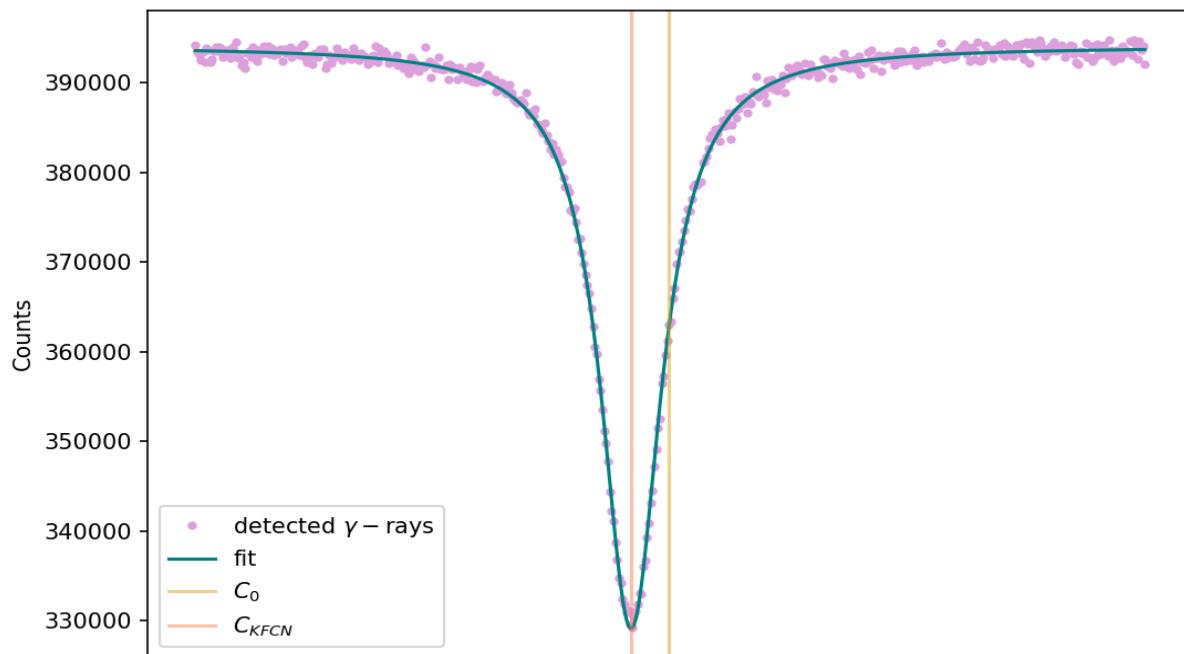


Fig 22. Our Absorption spectra with fits and an added set of lines marking C_0 , C_{KFCN} and $C_{\alpha Fe}$ correspondingly in each plot.

Discussion

Comparing the predicted linewidth in Q1 to the experimental linewidth Γ_e in Q9 demonstrated how much variability occurs in calculations due to the physical properties of the material being studied.

The difference observed in this experiment is due to a line broadening effect known as "Resonance broadening"

This type of pressure broadening is described in Wikipedia as a perturbation due to a particle that is of the same type as the emitting particle leading to an energy exchange process which we know is due to γ -rays.

which can then be analysed by nuclear resonance spectroscopy. [14]

Unfortunately we were unable to compare our value for isomer shift w.r.t. α Fe to Murad and Cashior's result. I did not understand the table presented in the recommended text.

I did notice that:

$$\frac{IS_{\alpha Fe}}{IS_{KFCN \rightarrow \alpha Fe}} = \frac{0.0923}{2.85} = 0.032$$

which has similar magnitude to the reported value in Murad/Cashior but the sign is wrong and so I decided against this method.

Conclusion

We observed the spectra of γ -ray photons emitted by decaying ^{57}Co into ^{57}Fe and studied the absorption spectra of two different samples (potassium ferrocyanide and α -iron) using Mössbauer Spectroscopy.

We measured the linewidths of our samples and also the isomer shifts of our samples relative to the source and each other. We also calculated the angular momentum and magnetic moment of the ^{57}Fe source matrix.

(Sorry I didn't include all the values in the conclusion. There are too many.)

Logbook 3: References

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Appendix - Terminology

The Mössbauer effect

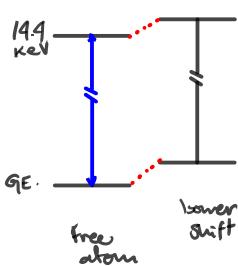
Nuclei in crystals are unable to recoil during emission and absorption of γ -rays, then the energy is lost due to the recoil is observed in discrete wavepackets (**phonons**)

A significant number of emission and absorption events are recoil-free (Lamb-Mössbauer factor) γ -rays emitted by a nucleus can be resonantly absorbed by a nuclei of the same isotope and this absorption can be measured. [1]

Isomer shift (δ)

is a relative measure describing a shift in the resonance energy of a nucleus due to transition of electrons within its s orbitals, the whole spectrum is shifted in either a positive or negative direction depending on the s electron charge density in the nucleus. The shift arises due to the alterations in electrostatic response between the non-zero probability s orbital electrons and the non-zero volume nucleus they orbit. [1]

Shift in absorption like proportional to the difference in electron density between the source & the absorber. [10]

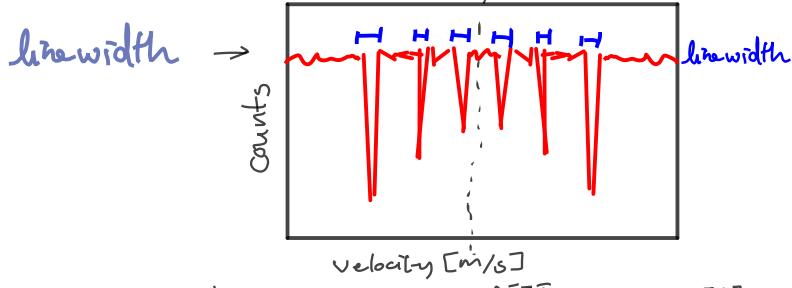


Fig? The isomer shift [10]
is a measure of the bonding with
the nearest neighbours
(sensitive to): valency, ionicity, covalency
high spin/low spin, neighbour type

α -iron reference material for ^{57}Fe chosen for isomer shift

Dalloye solid

The Dalloye model estimates phonon contribution to specific heat [4]



Fig? Absorption spectrum of ^{57}Fe from ref. [4]

Phonon collective excitation in a periodic, elastic arrangement of atoms or molecules in condensed matter (often designated quasiparticle) [5]

* **Momentum conservation** - when a recoil-free event occurs conservation of momentum is satisfied by the momentum of the crystal \Rightarrow No energy loss.

Internal conversion coefficient

describes the rate of internal conversion. decay process wherein a nucleus interacts electromagnetically with one of the orbital electrons of the atom. Causing electron ejection from its orbit. [9]

```
10/16/20 01:20:40 /home/ana/Documents/uni/PHS3000/code/log3/mossbauer.py

1 # PHS3000
2 # Mossbauer effect
3 # Ana Fabela, 21/09/2020
4 import os
5 from pathlib import Path
6 import monashspa.PHS3000 as spa
7 import numpy as np
8 import matplotlib.pyplot as plt
9 import scipy.optimize
10
11 plt.rcParams['figure.dpi'] = 150
12 # folder = Path('spectra')
13 # os.makedirs(folder, exist_ok=True)
14
15 # Globals
16 E = np.linspace(1, 512, 512)
17 C_0 = 255.5
18 u_C_0 = 0.05
19
20 hbar = 1.0545718e-34 # [Js]
21 c = 299792458 # [m/s]
22 eV = 1.602176634e-19 # [J]
23 meV = 1e-3 * eV
24 keV = 1e3 * eV
25
26 E_gamma = 14.4 * keV # [J]
27 u_E_gamma = 0.05 * keV
28 amu = 1.6605402e-27 # [kg]
29 Fe_A = 55.845 * amu # [kg]
30 u_Fe_A = 2e-3 * amu # [kg]
31 # print(f"atomic weight of Fe atom : ({Fe_A*1e26:.4f} ± {u_Fe_A*1e26:.4f}) ×10^26 kg")
32 B = 33.04 #[T]
33 u_B = 0.0005 #[T]
34 mu_N = 5.0507837461 * 1e-27 #[J/T]
35 u_mu_N = 0.0000000015 * 1e-27 #[J/T]
36 mu_g = 0.0904 * mu_N #[J/T]
37 u_mu_g = np.sqrt((u_mu_N / mu_N)**2) * mu_g #[J/T]
38
39 I_g = 1/2
40 I_e = 3 / 2
41 m_g = [-1/2, 1/2]
42 m_e = [-3/2, -1/2, 1/2, 3/2]
43
44 def recoil_energy():
45     # recoil energy of a single Fe atom
46     E_R = E_gamma**2 / (2 * Fe_A * c**2)
47     print(f"\nE_gamma**2= {E_gamma**2}keV^2, {Fe_A=}kg, {c**2=}(m/s)^2")
48     print(f"\nE_R : {E_R / meV} meV")
49     return E_R
50
51 def read_files():
52     # read data files and return numpy arrays
```

```
53     data_KFe = np.array([int(value) for value in open('KFe.txt')])
54     data_alphaFe = np.array([int(value) for value in
55         open('alpha_Fe.txt')])
56     x = np.arange(len(data_KFe))
57     return x, data_KFe, data_alphaFe
58
59 def plot_data(E, data_file, fit, C_KFCN, C_alphaFe, alphaFe=False):
60     plt.plot(E, data_file, linestyle='None', marker='.',
61             color='plum', label=r"detected $\gamma$-rays")
62     plt.plot(E, fit, color='teal', label=r"fit")
63
64     plt.axvline(x=C_0, color='goldenrod', alpha=0.5, label=r"$C_0$")
65     if alphaFe:
66         plt.axvline(x=C_alphaFe, color='coral', alpha=0.5,
67             label=r"$C_{\alpha Fe}$")
68     else:
69         plt.axvline(x=C_KFCN, color='coral', alpha=0.5,
70             label=r"$C_{KFCN}$")
71
72     plt.xlabel('Bins')
73     plt.ylabel('Counts')
74     plt.legend()
75     plt.show()
76
77 def not_a_Lorentzian(E, B, A, E_0, Gamma):
78     return B - (A * (Gamma / 2)**2 / ((E - E_0)**2 + (Gamma / 2)**2))
79
80 def not_6_Lorentzians(E, B, A_1, E_0_1, Gamma_1, A_2, E_0_2, Gamma_2, A_3,
81 E_0_3, Gamma_3, A_4, E_0_4, Gamma_4, A_5, E_0_5, Gamma_5, A_6, E_0_6, Gamma_6):
82     return B - (A_1 * (Gamma_1 / 2)**2 / ((E - E_0_1)**2 + (Gamma_1 / 2)**2) +
83     A_2 * (Gamma_2 / 2)**2 / ((E - E_0_2)**2 + (Gamma_2 / 2)**2) +
84     A_3 * (Gamma_3 / 2)**2 / ((E - E_0_3)**2 + (Gamma_3 / 2)**2) +
85     A_4 * (Gamma_4 / 2)**2 / ((E - E_0_4)**2 + (Gamma_4 / 2)**2) +
86     A_5 * (Gamma_5 / 2)**2 / ((E - E_0_5)**2 + (Gamma_5 / 2)**2) +
87     A_6 * (Gamma_6 / 2)**2 / ((E - E_0_6)**2 + (Gamma_6 / 2)**2) )
88
89 def KFe_spectrum_fitting(E, data_KFe):
90     # Potassium ferrocyanide data
91     # initial parameters from eyeballing plot
92     initial_guess = [395 * 1e3, 60 * 1e3, 234, 80]
93     pars, pcov = scipy.optimize.curve_fit(not_a_Lorentzian, E,
94     data_KFe, p0=initial_guess)
95     perr = np.sqrt(np.diag(pcov))
96     fit = not_a_Lorentzian(E, *pars)
97     # print(f"\n{pars} \n{perr} ")
98     # return fit parameters
99     return pars, perr, fit
100
101 def alphaFe_spectrum_fitting(E, data_alphaFe):
102     # α-Iron data
103     # initial parameters from eyeballing plot
104     initial_guess = [560 * 1e3, 60 * 1e3, 90, 20, 50 * 1e3, 160, 20,
105 30 * 1e3, 225, 20, 30 * 1e3, 280, 20, 50 * 1e3, 345, 20, 60 * 1e3,
106 410, 20]
```

```
99      pars_6, pcov = scipy.optimize.curve_fit(not_6_Lorentzians, E,
100     data_alpha_Fe, p0=initial_guess)
101     perr_6 = np.sqrt(np.diag(pcov))
102     fit_6 = not_6_Lorentzians(E, *pars_6)
103     # print(f"\n{pars_6}\n{perr_6}")
104     # return fit parameters
105     return pars_6, perr_6, fit_6
106
107 def energy_differences(pars_6, perr_6):
108     # extracting values of peak locations
109     E_0i = np.sort(pars_6)[6:12]
110     u_E_0i = np.sort(perr_6)[:6]
111     # print(f"\n{E_0i} ")
112     LAMBDA_1 = E_0i[-2] - E_0i[2]
113     u_LAMBDA_1 = np.sqrt(0.03**2 + 0.05**2)
114     LAMBDA_2 = E_0i[-3] - E_0i[1]
115     u_LAMBDA_2 = np.sqrt(0.03**2 + 0.05**2)
116
117     lambda_1 = E_0i[5] - E_0i[4]
118     u_lambda_1 = np.sqrt(0.03**2 + 0.03**2)
119     lambda_2 = E_0i[4] - E_0i[3]
120     u_lambda_2 = np.sqrt(0.03**2 + 0.05**2)
121     lambda_3 = E_0i[2] - E_0i[1]
122     u_lambda_3 = np.sqrt(0.03**2 + 0.05**2)
123     lambda_4 = E_0i[1] - E_0i[0]
124     u_lambda_4 = np.sqrt(0.03**2 + 0.03**2)
125
126     mean_lambda = np.mean([lambda_1, lambda_2, lambda_3, lambda_4])
127     u_mean_lambda = (1 / 4) * np.sqrt(0.04**2 + 0.06**2 + 0.04**2 +
128     0.06**2)
129
130     return LAMBDA_1, u_LAMBDA_1, mean_lambda, u_mean_lambda, E_0i,
131     u_E_0i
132
133 def Q8(LAMBDA_1, u_LAMBDA_1, mean_lambda, u_mean_lambda):
134     # g value for ground state
135     g_g = mu_g / (mu_N * I_g)
136     u_g_g = np.sqrt((u_mu_g / mu_g)**2 + (u_mu_N / mu_N)**2) * g_g
137     # Energy levels ground state
138     E_g = []
139     u_E_g = []
140     for i in m_g:
141         E_i = g_g * mu_N * B * i
142         E_g.append(E_i)
143         u_E_i = np.sqrt((u_g_g / g_g)**2 + (u_mu_N / mu_N)**2 + (u_B /
144         B)**2) * E_i
145         u_E_g.append(u_E_i)
146     LAMBDA_g = g_g * mu_N * B
147     u_LAMBDA_g = np.sqrt((u_g_g / g_g)**2 + (u_mu_N / mu_N)**2 + (u_B /
148     B)**2) * LAMBDA_g
149
150     v = (LAMBDA_g * c) / E_gamma
151     u_v = np.sqrt((u_LAMBDA_g / LAMBDA_g)**2 + (u_E_gamma / E_gamma)**2) * v
152     print(f"\n v = ({v * 1e3:.2f} ± {u_v * 1e3:.2f}) mm/s")
```

```

149     K_18 = v / LAMBDA_1
150     u_K_18 = np.sqrt((u_v / v)**2 + (u_LAMBDA_1 / LAMBDA_1)**2) *
151     K_18
152     print(f"\nK_18 = ({K_18 * 1e3:.4f} ± {u_K_18 * 1e3:.4f}) "
153     mm/s/channel")
154     v_mean_lambda = mean_lambda * K_18
155     u_v_mean_lambda = np.sqrt((u_mean_lambda / mean_lambda)**2 +
156     (u_K_18 / K_18)**2) * v_mean_lambda
157     print(f"\nv_mean_lambda = ({v_mean_lambda * 1e3:.3f} ±
158     {u_v_mean_lambda * 1e3:.3f}) mm/s")
159     Delta_E = ((E_gamma / keV)* v_mean_lambda) / c
160     u_Delta_E = np.sqrt((u_E_gamma / E_gamma)**2 + (u_v_mean_lambda /
161     v_mean_lambda)**2) * Delta_E
162     print(f"\nDelta_E = ({Delta_E * keV * 1e26:.3f} ± {u_Delta_E *
163     keV * 1e26:.3f}) * 1e-26 J")
164     g_e = Delta_E * keV / (mu_N * B)
165     u_g_e = np.sqrt((u_Delta_E / Delta_E)**2 + (u_mu_N / mu_N)**2 + +
166     (u_B / B)**2) * g_e
167     print(f"\ng_e = ({g_e:.4f} ± {u_g_e:.4f}) ")
168     mu_e = g_e * mu_N * (3 / 2)
169     u_mu_e = np.sqrt((u_g_e / g_e)**2 + (u_mu_N / mu_N)**2) * mu_e
170     print(f"\nmu_e = ({mu_e * 1e28:.2f} ± {u_mu_e * 1e28:.2f}) J/T")
171     print(f"\nmu_e = ({mu_e / mu_N:.4f} ± {u_mu_e / mu_N:.4f}) mu_N")
172     return K_18, u_K_18
173
174 def Q9(pars, perr, K_18, u_K18):
175     v_alphaFe = 711 * 1e-3 #[V]
176     u_v_alphaFe = 2 * 1e-3 #[V]
177     v_KFCN = 160 * 1e-3 #[V]
178     u_v_KFCN = 1 * 1e-3 #[V]
179     K_04 = v_alphaFe * K_18 / v_KFCN
180     u_K_04 = np.sqrt((u_v_alphaFe / v_alphaFe)**2 + (u_K_18 / K_18)**2
181     +(u_v_KFCN / v_KFCN)**2) * K_04
182     print(f"\nK_04 = ({K_04 * 1e4:.2f} ± {u_K_04 * 1e4:.2f}) * 1e-4
183     mm/s/channel")
184     GAMMA_v = K_04 * pars[3]
185     u_GAMMA_v = np.sqrt((u_K_04 / K_04)**2 +(perr[3] / pars[3])**2
186     * GAMMA_v)
187     print(f"\nGAMMA_v = ({GAMMA_v * 1e3:.2f} ± {u_GAMMA_v *
188     1e3:.2f}) * 1e-3 mm/s")
189     v_Q_1 = (7.479 * 1e-28 * c) / E_gamma
190     print(f"\nv_Q_1 = {v_Q_1 * 1e3:.1f} mm/s")
191     convolv_v_Q1 = v_Q_1 * 2.3
192     print(f"\nconvolv_v_Q1 = {convolv_v_Q1 * 1e3:.1f} mm/s")
193
194     return K_04, u_K_04

```

```
193 def Q10(pars,perr, pars_6, perr_6, K_04, u_K_04, K_18, u_K_18, E_0i,  
194 u_E_0i):  
195     C_KFCN = pars[2]  
196     u_C_KFCN = perr[2]  
197     print(f"\nC_KFCN = ({C_KFCN:.3f} ± {u_C_KFCN:.3f})")  
198     C_KFCN_C0 = C_KFCN - C_0  
199     u_C_KFCN_C0 = np.sqrt(u_C_KFCN**2 + u_C_0**2)  
200     print(f"\n{C_KFCN_C0} = :.1f" ± {u_C_KFCN_C0} = :.1f")  
201     IS_KFCN = C_KFCN_C0 * K_04  
202     u_IS_KFCN = np.sqrt((u_C_KFCN_C0/ C_KFCN_C0)**2 + (u_K_04 /  
203 K_04)**2) * IS_KFCN  
204     print(f"\nIS_KFCN = ({IS_KFCN * 1e3:.2f} ± {u_IS_KFCN *  
205 1e3:.2f}) mm/s")  
206  
207     C_alphaFe = np.mean(E_0i)  
208     u_C_alphaFe = (1/6) * np.sqrt(sum(x**2 for x in u_E_0i))  
209     print(f"\nC_alphaFe = {C_alphaFe:.3f} ± {u_C_alphaFe:.3f}")  
210     C_alphaFe_C0 = C_alphaFe - C_0  
211     u_C_alphaFe_C0 = np.sqrt(u_C_alphaFe**2 + u_C_0**2)  
212     print(f"\nC_alphaFe_C0 = {C_alphaFe_C0:.2f} ± {u_C_alphaFe_C0:.2f}")  
213  
214     IS_alphaFe = C_alphaFe_C0 * K_18  
215  
216     u_IS_alphaFe = np.sqrt((u_C_alphaFe_C0 / C_alphaFe_C0)**2 + (u_K_18 /  
217 K_18)**2) * IS_alphaFe  
218     print(f"\nIS_alphaFe = ({IS_alphaFe * 1e3:.4f} ± {u_IS_alphaFe * 1e3:.4f})  
219 mm/s")  
220  
221     diff = IS_KFCN - IS_alphaFe  
222     u_diff = np.sqrt(u_IS_KFCN**2 + u_IS_alphaFe**2)  
223     print(f"\ndiff = ({diff * 1e3:.2f} ± {u_diff * 1e3:.2f}) mm/s")  
224  
225     return C_KFCN, u_C_KFCN, C_alphaFe, u_C_alphaFe  
226  
227  
228  
229  
230  
231 #*~ function calls ~*#  
232 x, data_KFe, data_alphaFe = read_files()  
233 pars, perr, fit = KFe_spectrum_fitting(E, data_KFe)  
234  
235 pars_6, perr_6, fit_6 = alphaFe_spectrum_fitting(E, data_alphaFe)  
236  
237 LAMBDA_1, u_LAMBDA_1, mean_lambda, u_mean_lambda, E_0i, u_E_0i =  
238 energy_differences(pars_6, perr_6)  
239 K_18, u_K_18 = Q8(LAMBDA_1, u_LAMBDA_1, mean_lambda, u_mean_lambda)  
240  
241 K_04, u_K_04 = Q9(pars, perr, K_18, u_K_18)
```

```
242
243 C_KFCN, u_C_KFCN, C_alphaFe, u_C_alphaFe = Q10(pars,perr, pars_6, perr_6,
244 K_04, u_K_04, K_18, u_K_18, E_0i, u_E_0i)
245
246 plot_data(E, data_KFe, fit, C_KFCN, C_alphaFe, False)
247 plot_data(E, data_alphaFe, fit_6, C_KFCN, C_alphaFe, True)
```