

Magnetic Susceptibility

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Risk Assessment

Risk factor	Risk level	Control (existing)	Control (proposed)
0-5 A electromagnet may rise above ambient temperatures	Low	<ul style="list-style-type: none">Water cooling system is used to cool down the magnetCold water is set to flow through the cooling system before activating the magnet.	<ul style="list-style-type: none">If required, students inform lab technician of issues present in the controls set up.

Aim (from ref. [0])

To measure the magnetic susceptibility of a number of magnetic materials.

By combining theoretical expressions and experimental results information concerning the quantum numbers of the magnetic ions present in the materials is obtained.

Theory (from ref. [0])

Atomic magnetism

The magnetic moment of an atom or ion is

$$\mu = -g\beta J \quad (1)$$

Where g is the Lande g factor, $\beta = \frac{e\hbar}{(4\pi m_e)} = 9.274 \times 10^{-24} \text{ J/T}$ the Bohr magneton and J is the total angular momentum given by

$$J = L + S \quad (2)$$

L, S are the orbital and spin angular momentum respectively.

The magnetic moments associated with individual atoms or ions may be thought of as completely independent moments and non-interacting.

If there are N moments per unit volume in a material the vector sum of the individual moments (μ_i) produces a zero resultant magnetisation M .

$$M = \sum_i^N \mu_i = 0$$

Problem 1

This statement is true if there is no net magnetic moment polarisation in the system.

That is, if the material is not magnetic, the magnetic moments of the individual atoms or ions that comprise the material are mostly antialigned meaning that their individual vector sums cancel out.

Applying an external magnetic field on a material will cause both an alignment of the internal dipole moments and also an induced magnetic moment in the material.

Consequently, the magnetic flux density in the material will be different from the free space value \mathbf{B}

The magnetic intensity \mathbf{H} in the material is given by

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \quad (3)$$

where μ_0 is the permeability of free space, and \mathbf{M} is the resultant magnetisation.

Another way to write \mathbf{M} is

$$\mathbf{M} = \chi \mathbf{H} \quad (4)$$

where χ is the magnetic susceptibility.

The relation between the applied magnetic flux density and the magnetic susceptibility is:

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad (5)$$

using equation (4):

$$\Leftrightarrow \mathbf{B} = \mu_0 (\mathbf{H} + \chi \mathbf{H})$$

$$\Leftrightarrow \mathbf{B} = \mu_0 \mathbf{H} (1 + \chi) \quad (6)$$

where $\mu_r = (1 + \chi)$ is the relative permeability of the material.

It can be shown that for materials where the assumption we made about non-interacting moments is VALID.

The magnetic susceptibility follows Curie's law:

$$\chi = \frac{C}{T} \quad (7)$$

where T is the material's absolute temperature and the Curie constant is:

$$C = \frac{\mu_0 N g^2 \beta J(J+1)}{3K_B} \quad (8)$$

where the only quantity left to be defined here is K_B : Boltzmann's constant

Most of the magnetic salts used in this experiment follow Curie's law, specifically around room temperature.

Furthermore, if the magnetic moment of the atom, is due mainly to the spin contribution (because $L=0$, or the orbital angular momentum is quenched) we can write the Curie constant as:

$$C = \frac{\mu_0 N g^2 \beta S(S+1)}{3k_B} \quad (9)$$

There are a range of materials where, for example, the magnetic ions are close together or the temperatures are low enough that the Curie's Law will fail.

In this scenario an ion will not only experience an outside magnetic field but also an internal magnetic field.

We use this internal field to express interactions of neighbouring magnetic moments

We use a modified Curie's Law aka Curie-Weiss law:

$$\chi = \frac{C}{T - \Theta} \quad (10)$$

Where the Weiss constant Θ is a measure of the interaction energy (expressed as a temperature)

If Θ is positive and the temperature is below T_c the Curie temperature.

Then material becomes ferromagnetic.

$T_c = \Theta$ in this theory

* in practice they differ by 10-20 K *

If Θ is negative, the material becomes antiferromagnetic below the Néel temperature

$$T_N = |\Theta|$$

The Gouy Method [o]

The force in the z -direction of a dipole m in a magnetic field B is given by:

$$F_z = m_x \left(\frac{\partial B_x}{\partial z} \right) + m_y \left(\frac{\partial B_y}{\partial z} \right) + m_z \left(\frac{\partial B_z}{\partial z} \right) \quad (11)$$

for a volume V of magnetic material with Susceptibility χ , the magnetic flux density in the sample will be reduced due to the sample's magnetisation.

for paramagnetic and diamagnetic samples
the change is small (ie $M \approx 0$)

We approximate the magnetic flux density

$$B \approx \mu_0 H \quad (12)$$

Consequently,

$$F_z = \mu_0 V \left(M_x \left(\frac{\partial H_x}{\partial z} \right) + M_y \left(\frac{\partial H_y}{\partial z} \right) + M_z \left(\frac{\partial H_z}{\partial z} \right) \right) \quad (13)$$

$$\Leftrightarrow F_z = \mu_0 V \chi \left(H_x \left(\frac{\partial H_x}{\partial z} \right) + H_y \left(\frac{\partial H_y}{\partial z} \right) + H_z \left(\frac{\partial H_z}{\partial z} \right) \right)$$

$$\Leftrightarrow F_z = \frac{\mu_0 V}{2} \frac{\partial}{\partial z} H^2 \quad (14)$$

If the material is immersed in another medium of different susceptibility, then the force of the sample is reduced as any displacement in the z -direction requires an opposite displacement of the surrounding medium - for example in air:

$$F_z = \frac{1}{2} \mu_0 V (x - X_{air}) \frac{\partial}{\partial z} H^2 \quad (15)$$

In this experiment we can neglect X_{air} .

The magnetic flux density in the sample is approximately the same as the free space value outside of the sample -

$$\therefore F_z = \frac{1}{2} \mu_0 V (x - X_{air}) \frac{\partial}{\partial z} B^2 \quad (16)$$

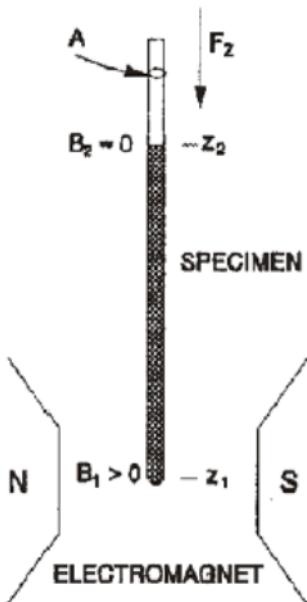


Figure 1: Geometry of the Gouy method.

Fig 1. from ref [0]

The geometry of the Gouy method. The downward force F_z (positive) is measured on a sample sufficiently long to extend from the position of maximum magnetic field strength ($z = -z_1$), to a position of essentially zero field strength B_2 ($z = z_2$).

The specimen is placed in a tube (gkiss) of cross-sectional area A so the total downward force (Neglecting X_{air}) is:

$$F_z = \frac{\chi}{2\mu_0} \int_{z_2}^{z_1} \frac{\partial B}{\partial z} Adz \quad (17)$$

where we have used $dV = Adz$ (a unit volume is 1 m^3)

Since $B_2 = 0$

$$\Rightarrow F_z = \frac{\chi A}{2\mu_0} B_1^2 \quad (18)$$

As long as A is kept constant

$$F_z = D \chi B_1^2 \quad (19)$$

where D is a constant

The magnetic susceptibility for a unit volume of material is:

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_B T}$$

Where N is the number of atoms or ions that form the sample.

Problem 2

in a material of molecular weight M (g/mol) and density ρ (kg/m³)

Show that the magnetic susceptibility / m³ is

$$\chi_2 = \frac{\mu_0 \rho N_0 g^2 \beta^2 J(J+1)}{3 k_B T M} \times 10^3$$

we know that the magnetic susceptibility for a unit volume is

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_B T}$$

Since $\frac{N}{N_0}$ = mol and $\frac{m}{mol} = M$

where N_0 is avogadro's number, mol is the number of moles
if m is the mass (in grams)

$$\frac{N_0}{N} m = M$$

$$\therefore N = \frac{N_0 m}{M}$$

Then

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_B T} = \frac{\mu_0 N_0 m g^2 \beta^2 J(J+1)}{3 k_B T M}$$

converting grams to kilograms and dividing by the unit volume V :

$$\chi_2 = \frac{\mu_0 \rho N_0 g^2 \beta^2 J(J+1)}{3 k_B T M} \times 10^3$$

□

Magnetic Susceptibility Data Set information:

24/10/2020

- Data is recorded on Gaussmeter set on the 3 kG range.
- Laboratory temperature is 294 +/- 1 K.
- Data for this experiment is in a single Excel file.
- Separate sheets contain the data for the magnetic field calibration, the sample tube calibration measurement and the measurements for each sample.
- Equipment measuring uncertainties are listed below.

Model 2100 Gaussmeter + probe accuracy:

1.5.1 Accuracy Specifications

DC Accuracy \pm (% of reading + % of range)

Range	Typical	Maximum
$\geq 30 \text{ KG}$	$0.050 + 0.020$	$0.100 + 0.050$
$\geq 3 \text{ KG}$	$0.050 + 0.020$	$0.110 + 0.050$
$\geq 300 \text{ G}$	$0.060 + 0.020$	$0.150 + 0.050$
$\geq 30 \text{ G}$	$0.080 + 0.020$	$0.200 + 0.050$

$\leftarrow u(B)$ for calibration data

Table 1.5.1 DC Accuracy Specifications

Ohaus weighing scale accuracy: $\leftarrow u(m)$

Repeatability $\pm 0.0001 \text{ g}$

Linearity $\pm 0.0002 \text{ g}$ over full range (120 g) \rightarrow fractional uncertainty

Current meter UNI-T UT803 accuracy:

For 10 A range $\pm 1.2\%$ $\leftarrow u(I)$ for all data sets: $u(I) = 0.012 \times I$

Note: $1 \text{ G} = 0.0001 \text{ T}$

From the data set info sheet we know that the Gaussmeter is set on the 3 kG range.

$$\therefore 3 \text{ kG} = 0.3 \text{ T} = 300 \text{ mT}$$

Then, the DC typical accuracy (ie. magnetic field uncertainty) is :

$$\therefore u(B) = 0.050 (\pm \text{reading}) + 0.020 (\pm 300 \text{ mT})$$

Whilst, the respective maximum value for this uncertainty is:

$$u(B)_{\max} = 0.110 (\pm \text{reading}) + 0.050 (\pm 0.3 \text{ T})$$

Ouncertainty

we convert it to absolute by multiplying by the mass measured

* we also must consider the precision of our measurements

ie. All data provided to us contributes half the smallest digit to the total uncertainty.

\therefore The uncertainty in the mass

$$u(m) = \sqrt{0.0001^2 + \left(\frac{0.0002}{120} \cdot m\right)^2 + 0.00005^2}$$

Experiment [0]

24/10/2020

Note: ensure that water is flowing through the magnet windings before the magnet is turned on, and adjust the magnet current slowly.

Turn the water off at the conclusion of the experiment

Magnetic field Calibration

Method:

Using the Hall probe magnetic field strength meter, calibrate the magnet by measuring the magnetic field for current settings:

- $0A \rightarrow \underline{5A}$ (maximum)
in steps of $1A$ (minimum)
- Place the probe at the position of max $|B|$

Note: it's important that the probe is in the optimal position to obtain accurate current values.
These values will be used throughout the experiment.

Note: The meter is measuring the value of the magnetic flux density B [T] rather than magnetic field intensity H [$A\text{m}^{-1}$]

Uncertainty propagation (calibration data)

The typical uncertainty in the magnetic field readings is found by:

$$\therefore u(B) = 0.050(\% \text{ reading}) + 0.020(\% 300\text{mT})$$

Where after propagation, we scale the units to Teslas (T)

The uncertainty in the current is $u(I) = 0.012 \times I$

Magnet calibration

I (A)	$u(I)$ (A)	B (mT)	B (T)	$u(B)$ (T)
0	0	4.077	0.00408	0.006
0.97	0.01	71.15	0.07115	0.010
1.52	0.02	110.01	0.11001	0.012
2.04	0.02	147.66	0.14766	0.013
2.49	0.03	180.26	0.18026	0.015
2.99	0.04	215.62	0.21562	0.017
3.49	0.04	252.27	0.25227	0.018
4.06	0.05	292.61	0.29261	0.02
4.53	0.05	327.14	0.32714	0.02
4.98	0.06	359.35	0.35935	0.02

Table 1. Magnet calibration data with our calculated uncertainties.

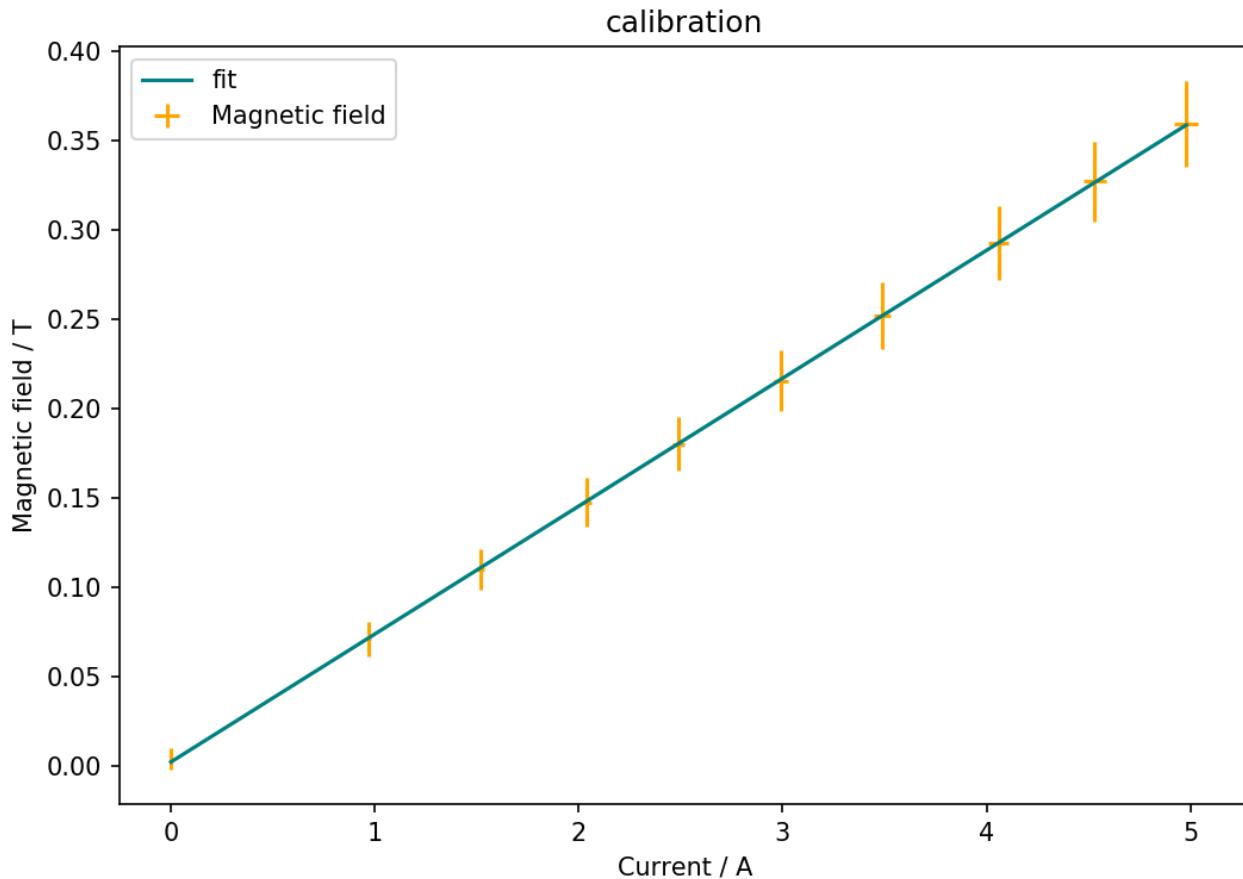


Fig 2. Magnet calibration data and linear fit
The relation between B and I is linear.

The parameters of this fit are :

gradient : 0.0716 ± 0.0002

intercept : 0.0023 ± 0.0006

fitting code

```

49 def line_fit(x, m, c):
50     return m * x + c
51
52 def fitting_calibration_data(xs, ys, initial_guess):
53     pars, pcov = scipy.optimize.curve_fit(line_fit, xs, ys, p0=initial_guess)
54     perr = np.sqrt(np.diag(pcov))
55     # print(f'{pars}, {perr}\n')
56     linear_fit = line_fit(xs, *pars)
57     return pars, perr, linear_fit
58

```

where the initial guess for the fit parameters is :

```
# from eyeballing calibration data
linear_guess_0 = [0.3, 0] # [gradient, intercept]
```

Measurements on the dummy sample

Method

1. Hang the dummy specimen, Specimen I, (glass tube + air) onto the balance and adjust it so it hangs centrally in the magnetic field. (see fig 2.)
2. Wait for the sample to steady.
3. Tabulate any mass.
4. Record change in mass Δm as a function of magnet current.
5. The mass change follows $F = g' \Delta m$, $g' = 9.8 \text{ ms}^{-2}$
6. These results may be assumed to be a valid correction for all glass specimen holders used in this experiment.

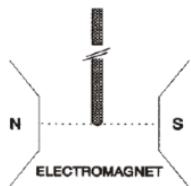


Figure 2: Dummy specimen alignment.

Fig 3. from ref [6]

Schematic
Drawing of
the position
of specimen
w.r.t the
electromagnet

Question 1

The observed Δm is negative for this sample -
Air and Glass are both diamagnetic materials
Diamagnetic materials are mildly repelled from magnets.

* The uncertainty in the current reading

$$u(I) = 0.012 \times I$$

* The uncertainty in the mass: $u(m) = \sqrt{0.0001^2 + \left(\frac{0.0002}{120} \cdot m\right)^2 + 0.00005^2}$

where after propagation we scale mass to [kg]

Dummy magnetization

I	$u(I)$	m	m	$u(m)$
(A)	(A)	(g)	(kg)	(kg)
5.02	0.06	-0.0017	-1.7E-06	0.11E-06
4.48	0.05	-0.0013	-1.3E-06	0.11E-06
3.99	0.05	-0.0011	-1.1E-06	0.11E-06
3.5	0.04	-0.0009	-9.0E-07	0.11E-06
2.99	0.04	-0.0007	-7.0E-07	0.11E-06
2.49	0.03	-0.0004	-4.0E-07	0.11E-06
1.98	0.02	-0.0002	-2.0E-07	0.11E-06
1.52	0.018	-0.0001	-1.0E-07	0.11E-06
0.95	0.011	0	0	0.11E-06
0	0	0	0	0.11E-06

Table 2. Empty glass tube sample data and calculated uncertainties.

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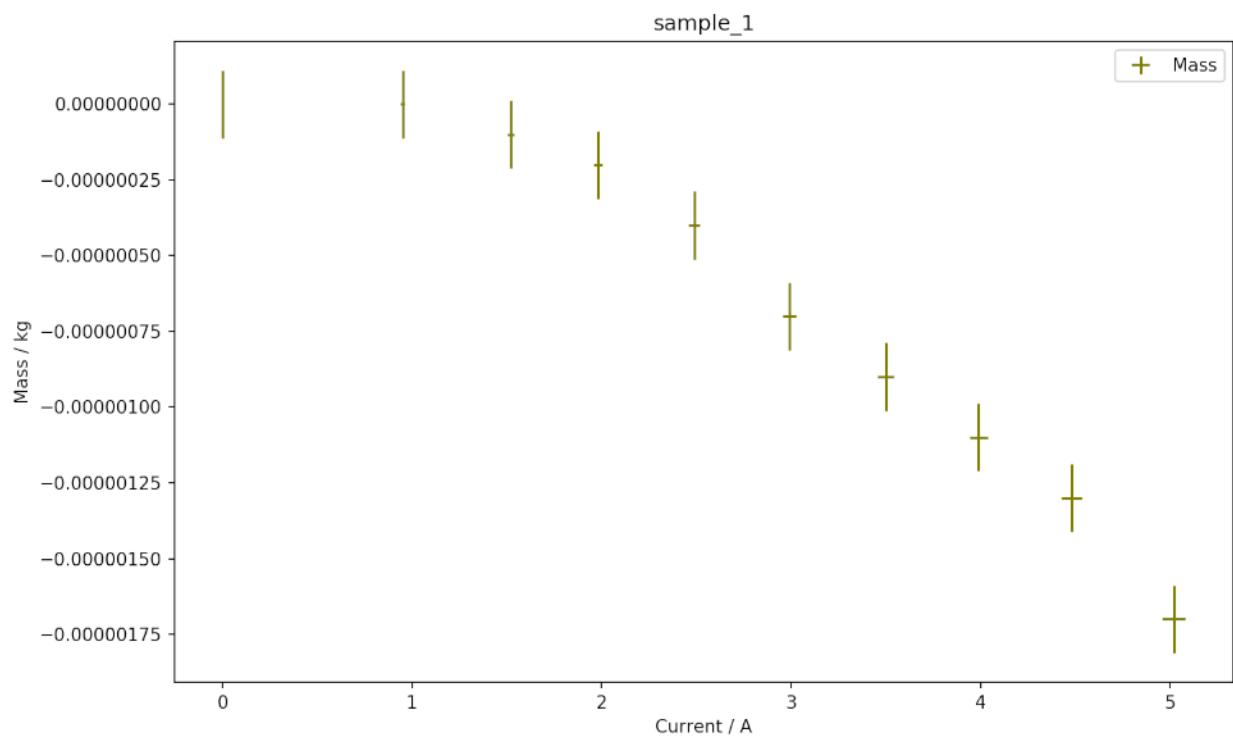


Fig 4. Empty glass tube sample data showing a negative trend for Δm .

Measurements on a Nickel magnetic salt

31/10/2020

Method:

1. Replace the dummy specimen with Specimen 2
Material : $\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$

2. Correct the readings for the original values obtained using the dummy specimen.

~~To do this correction:~~

Here I use x, y as current and mass respectively

1. obtain the values for the x's and y's of the dummy sample and their respective uncertainties
2. subtract the dummy x's and ys from the x's and ys recorded for this sample.
3. propagate uncertainty on the corrected x's and y's.
(These 3 steps apply for every subsequent sample)

#2 Ni sample $\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$

I	$u(I)$	M	m	correct m	$u(m)$
(A)	(A)	(g)	(kg)	(kg)	(kg)
0	0	0	0	1.7E-06	0.16E-6
0.935	0.011	0.0021	2.1E-06	3.4E-06	0.16E-6
1.54	0.018	0.0057	5.7E-06	6.8E-06	0.16E-6
1.99	0.02	0.0094	9.4E-06	0.00001	0.16E-6
2.5	0.03	0.0148	0.00001	0.00002	0.16E-6
3.01	0.04	0.0215	0.00002	0.00002	0.16E-6
3.56	0.04	0.0302	0.00003	0.00003	0.16E-6
4	0.05	0.038	0.00004	0.00004	0.16E-6
4.48	0.05	0.0478	0.00005	0.00005	0.16E-6
5	0.06	0.0591	0.00006	0.00006	0.16E-6

Table 3 Speciment 2 raw data, corrected mass values ($m - \text{dummy mass}$) and calculated uncertainties for data

The propagation of uncertainty follows this algorithm

$$m = (m_{\text{raw}} - m_{\text{dummy}})$$

$$\text{where } u(m) = \sqrt{0.0001^2 + \left(\frac{0.002}{120} m\right)^2 + 0.00005^2}$$

$$\therefore u(m) = \sqrt{u(m)_{\text{raw}}^2 + u(m_{\text{dummy}})^2}$$

after propagating uncertainty we convert to [kg].

Question 2

31/10/2020

Δm is positive

from the measurements of Δm against current check that $\Delta m \propto B_i^2$:

* we know that in our experiment $B_i \propto I$ as can be seen in fig 2.

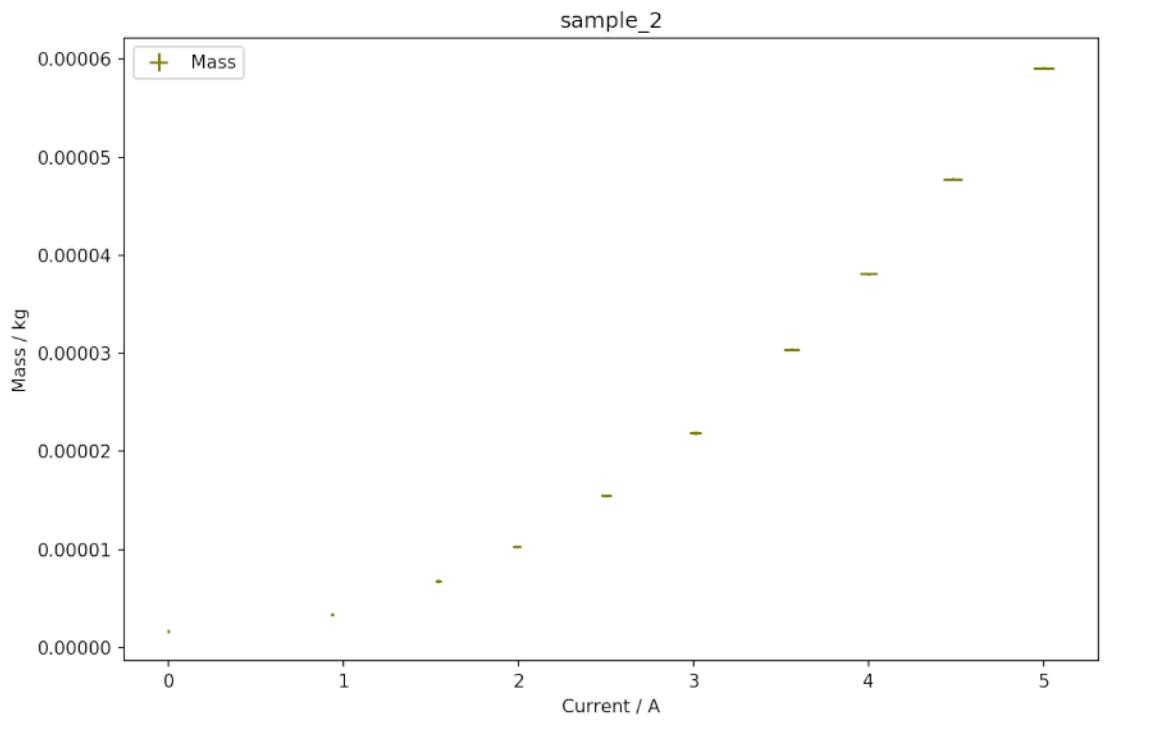


Fig 5. Specimen 2: corrected NiSO_4 data with uncertainties
in this plot there appears to be quadratic relationship between Δm and I

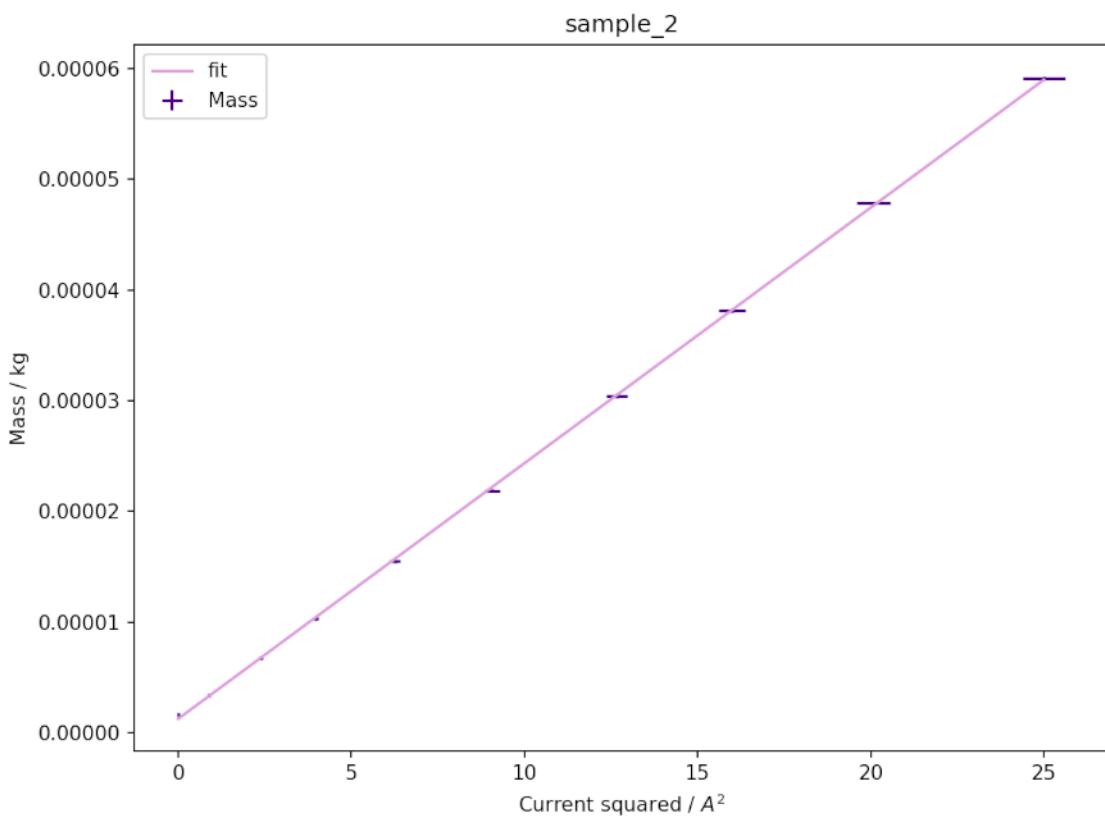


Fig 6. Plot of Δm vs I^2 . from this, we conclude that
 $\Delta m \propto B^2$, which indicates that X is field independent [o]

The fit parameters for sample 2

31/10/2020

$$(m) \text{ gradient: } (2.307 \pm 0.009) \times 10^{-6}$$

$$(c) \text{ intercept: } (1.297 \pm 0.111) \times 10^{-6}$$

where the initial guess for the fit parameters is

```
# from eyeballing linearised magnetic sample data (I**2)
linear_guess_2 = [2e-6, 0] # [gradient, intercept]
```

Problem 3

Show that

$$X = \frac{2\mu_0 g'}{A} \frac{\Delta m}{B^2}$$

from the calibration fit

$$\begin{aligned} I \text{ found } B &= mI + c \\ \therefore B^2 &= (mI + c)^2 \quad (\text{as above}) \end{aligned}$$

The force on the samples is

$$F_z = g' \Delta m, \text{ where } g' = 9.8 \text{ ms}^{-2}$$

and from equation (19):

$$F_z = D' B_i^2, \text{ where } D' = Dx, D \in \mathbb{R}$$

$$\therefore g' \Delta m = D' B_i^2$$

$$\therefore g' \Delta m = D x (mI + c)^2$$

$$\therefore x = \frac{g' \Delta m}{D (mI + c)^2} \quad \square$$

from equation (18))

we deduce that $D = \frac{A}{2\mu_0}$, where A is the cross-sectional area of the tube holding the sample material:

$$\text{we are told that } A = (38.5 \pm 0.05) \times 10^{-6} \text{ m}$$

$$\therefore x = \frac{2\mu_0 g' \Delta m}{A (mI + c)^2}$$

Since we are after a single value of x .

We will fit equation (19) to our data for the Nickel sample.

In the form:

$$\Delta m = \frac{A x (mI + c)^2}{2\mu_0 g'} = D (mI + c)^2, \text{ where } D' = \frac{A x}{2\mu_0 g'} =$$

Our initial guess for the fit gradient D' :

```
# from eyeballing delta m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

$$\text{we find } D' = (0.000459 \pm 0.00004) \text{ s}^4 \text{ A}^2 / \text{kg}$$

$$\text{we rearrange the obtained } D' \text{ to find } \rightarrow x_{\text{Ni}} = \frac{2\mu_0 g' D}{A} = 0.000293$$

propagating the uncertainty we obtain:

$$u(x_{\text{Ni}}) = x_{\text{Ni}} \sqrt{\left(\frac{u(g')}{g'}\right)^2 + \left(\frac{u(D)}{D}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(\mu_0)}{\mu_0}\right)^2} = u(x_{\text{Ni}}) = x_{\text{Ni}} \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D)}{D}\right)^2}$$

$$\therefore u(x_{\text{Ni}}) = 0.000459 \sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.00004}{0.000459}\right)^2} = 0.000003$$

$$\therefore x_{\text{Ni}} = 0.000293 \pm 0.000003$$

3/11/2020

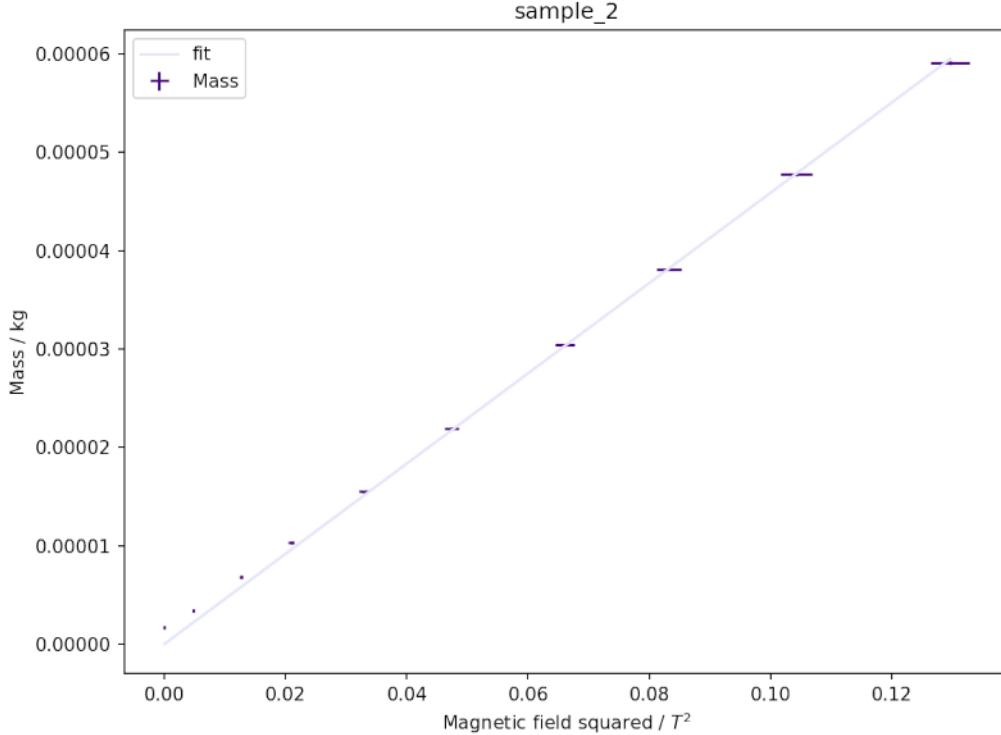


Fig 7. A plot of Δm against B^2 , we found D' (the gradient) by fitting the data using equation(19)

from our Δm against B^2 graph we calculated the gradient,

$$D' = \frac{\Delta x}{2\mu_0 g} = \frac{ANg^2 \beta^2 J(J+1)}{6g'k_B T}$$

$$\therefore 0.000459 \text{ s}^4 \text{A}^2/\text{kg} = \frac{ANg^2 \beta^2 J(J+1)}{6g'k_B T}$$

Note: $N_{Ni} = \frac{N_A p}{\text{molar mass}} = \frac{\# \text{molecules}}{\text{unit volume}} = \frac{\# \text{molecules}}{\text{mole}} \frac{\text{mole}}{\text{mass}} \frac{\text{mass}}{\text{unit volume}}$

$$\therefore N_{Ni} = \frac{11640 \cdot 47 \text{ kg/m}^3}{2.454 \times 10^{-24} \text{ kg/molecules}} = 4.74 \times 10^{27} \text{ molecules/m}^3$$

from ref.
[18]
Hexahyd rate values

\therefore we determine $g^2 \beta^2 J(J+1)$ for the Ni^{+2} ion present in $NiSO_4 \cdot 6H_2O$

$$\therefore g^2 \beta^2 J(J+1) = \frac{(6g'k_B T)D'}{AN_{Ni}} = 5.996 \times 10^{-46}$$

Assuming $g=2$ and $J=5$ ie $L=0$

$$\therefore S(S+1) = \frac{1}{4\beta^2} \frac{(6g'k_B T)D'}{A}$$

$$\therefore S^2 + S - \frac{1}{4\beta^2} \frac{(6g'k_B T)D'}{A} = 0 \quad (\text{we solve this as a quadratic})$$

We obtain $S^+ = 3.677$ or $S^- = -4.677$

propagating uncertainty in 2 steps:

$$1. u(S(S+1)) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(T)}{T}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} (S(S+1))$$

$$2. u(S^\pm) = \frac{1}{2} \frac{u(S(S+1))}{S(S+1)} S^\pm$$

Then our
possible spins
are

$$S^+ = 3.677 \pm 0.017$$

$$S^- = -4.677 \pm 0.02$$

How do I decide
which one is good?

Measurements on Samples 3 and 4

4/11/2020

Method:

1. Replace the $\text{NiSO}_4 \cdot 6\text{H}_2\text{O}$ specimen with Specimen 3
2. Correct the readings for the original values obtained using the dummy specimen.
3. Determine X and $g^2\beta^2J(J+1)$ and comment on the value of the magnetic quantum numbers associated with the ion present.
4. Repeat steps 1→3 for specimen 4

#3 Mn sample $\text{MnSO}_4 \cdot 4\text{H}_2\text{O}$

I	u(I)	M	m	correct m	u(m)
(A)	(A)	(g)	(kg)	(kg)	(kg)
0	0	0	0	1.7E-06	0.16E-6
0.94	0.011	0.0075	7.5E-06	8.8E-06	0.16E-6
1.505	0.018	0.019	0.00002	0.00002	0.16E-6
1.99	0.02	0.0333	0.00003	0.00003	0.16E-6
2.51	0.03	0.053	0.00005	0.00005	0.16E-6
3.02	0.04	0.0767	0.00008	0.00008	0.16E-6
3.58	0.04	0.108	0.00011	0.00011	0.16E-6
4.01	0.05	0.1356	0.00014	0.00014	0.16E-6
4.5	0.05	0.17	0.00017	0.00017	0.16E-6
4.99	0.06	0.2088	0.00021	0.00021	0.16E-6

Table 4. Specimen 3 raw data, corrected mass values ($m - \text{dummy mass}$) and calculated uncertainties for data

The propagation of uncertainty follows the same algorithm as Sample 2.

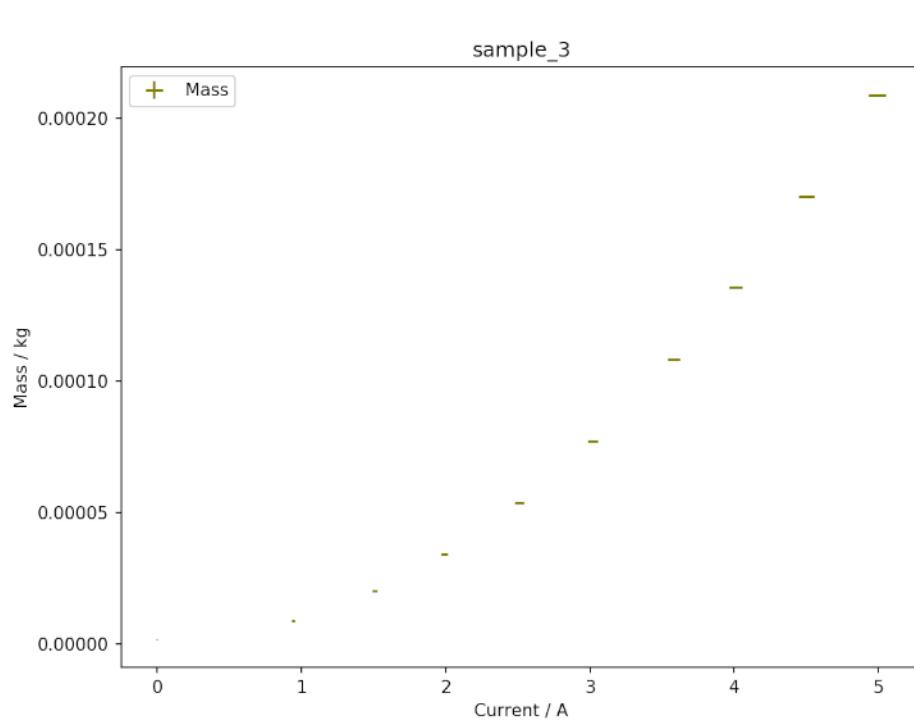


Fig 8. Specimen 3: corrected $\text{MnSO}_4 \cdot 4\text{H}_2\text{O}$ data with uncertainties
in this plot there appears to be quadratic relationship between Δm and I

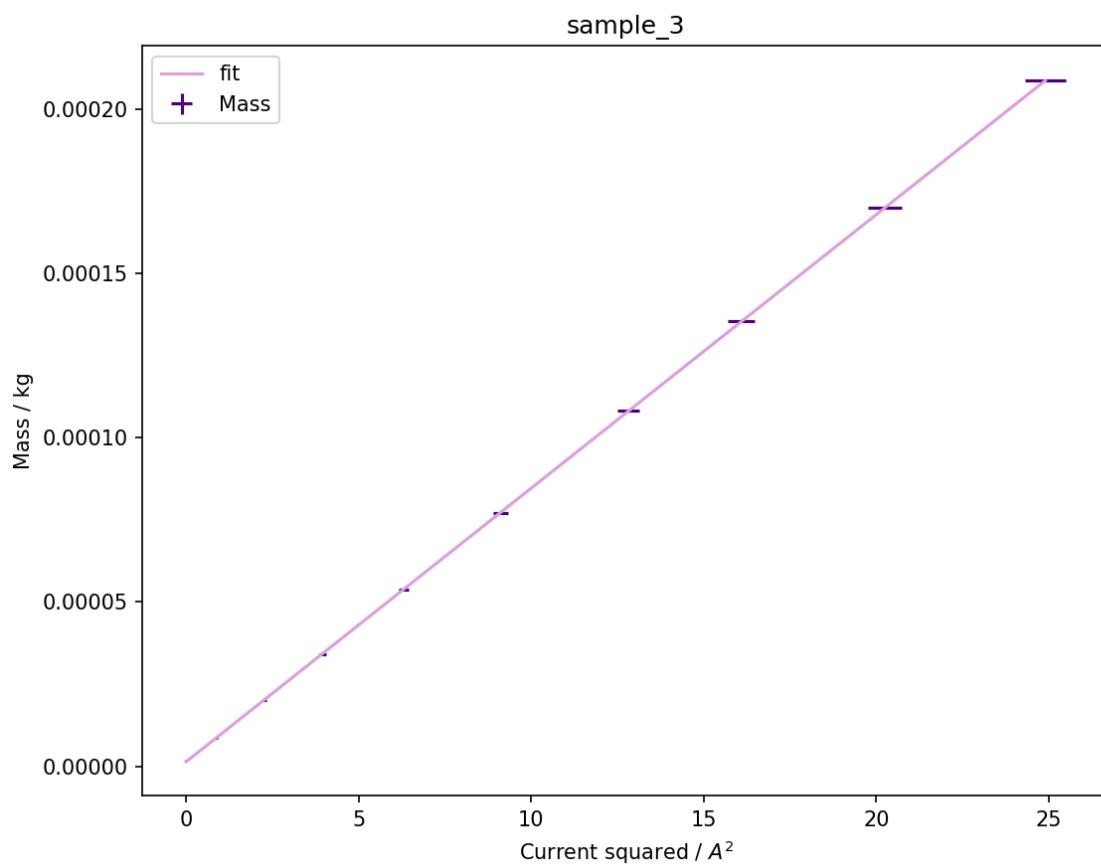


Fig 9. Plot of Δm vs I^2 . from this, we conclude that
 $\Delta m \propto B^2$, which indicates that X is field independent [0]

The fit parameters are:

- (m) gradient: $(8.331 \pm 0.009) \times 10^{-6}$
- (c) intercept: $(1.367 \pm 0.111) \times 10^{-6}$

using our new fit parameters

Once again, we fit the data with equation (19)

$$\Delta m = \frac{A \times (mI + c)^2}{2\mu_0 g} = D(mI + c)^2, \text{ where } D' = \frac{\Delta x}{2\mu_0 g}$$

Since the same analysis applies for this data
Our initial guess for the fit gradient D' is once again

```
# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

This time, we find $D' = (0.001618 \pm 0.000003) \text{ s}^4 \text{ A}^2/\text{kg}$

again, rearrange the obtained D' to find $\chi_{mn} = \frac{2\mu_0 g' D'}{A} = 0.001035$

propagating the uncertainty we obtain:

$$u(\chi_{mn}) = \chi_{mn} \sqrt{\left(\frac{u(\chi_{mn})}{\mu_0}\right)^2 + \left(\frac{u(g')}{g'}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} = u(\chi_{mn}) = \chi_{mn} \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}$$

$$\therefore u(\chi_{mn}) = 0.001035 \sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.001618}{0.000003}\right)^2} = 0.000003$$

$$\therefore \chi_{mn} = 0.001035 \pm 0.000003$$

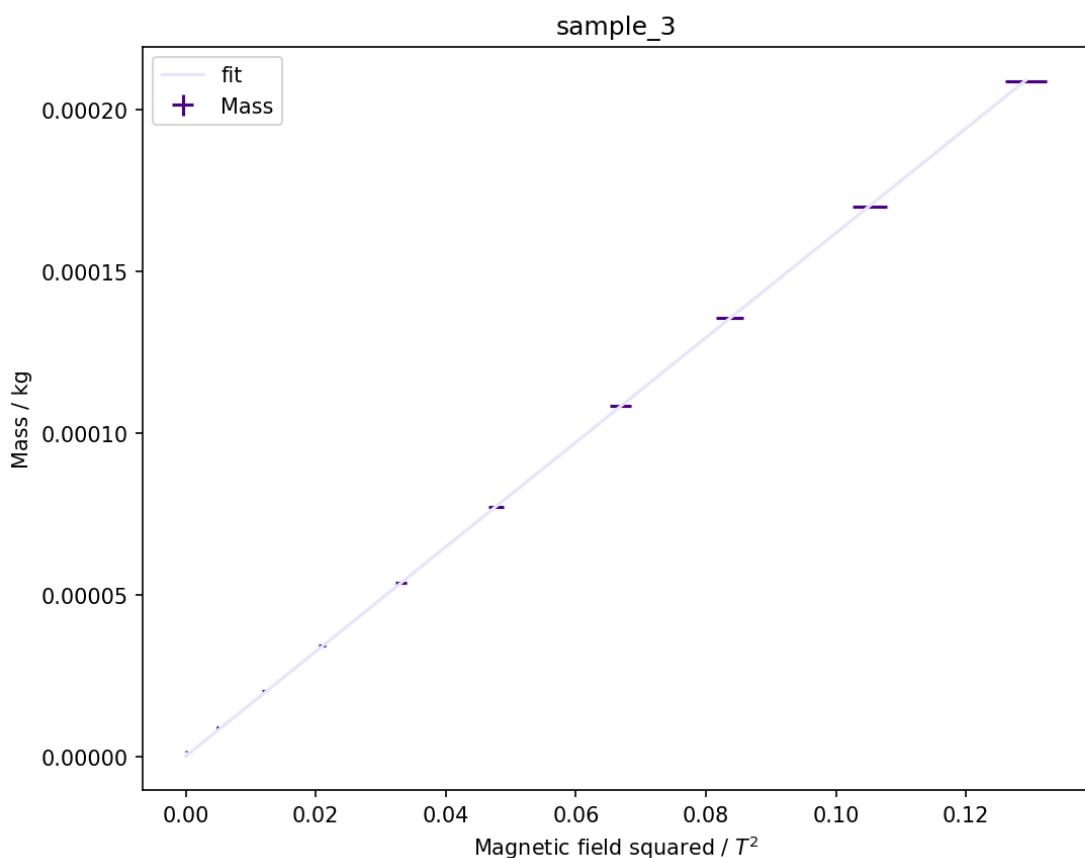


Fig 10. A plot of Δm against DB^2 , we found D' (the gradient) fitting the data using equation(19)

Fit parameter: $D' = (0.001618 \pm 0.000003) \text{ s}^4 \text{ A}^2/\text{kg}$

from our Δm against B^2 graph we calculated the gradient

$$\begin{aligned} D' &= \frac{\Delta X}{2\mu_0 g'} = \frac{ANg^2 B^2 J(J+1)}{6g'k_B T} \\ \therefore 0.001618 \text{ s}^4 \text{A}^2/\text{kg} &= \frac{ANg^2 B^2 J(J+1)}{6g'k_B T} \end{aligned}$$

Note: $N_{M_n} = \frac{N_0 P}{\text{molar mass}} = \frac{\# \text{molecules}}{\text{unit volume}} = \frac{\# \text{molecules}}{\text{mole}} \frac{\text{mole}}{\text{mass}} \frac{\text{mass}}{\text{unit volume}}$ from ref. [11]

$$\therefore N_{M_n} = \frac{11848.532 \text{ kg/m}^3}{2.083 \times 10^{-24} \text{ kg/molecules}} = 5.688 \times 10^{27} \text{ molecules/m}^3$$
(Tetrahydrate values)

\therefore we determine $g^2 \beta^2 J(J+1)$ for the M_n ion present in $MnSO_4 \cdot 4H_2O$

$$\therefore g^2 \beta^2 J(J+1) = \frac{(6g'k_B T)D'}{AN_{M_n}} = 1.763 \times 10^{-45}$$

once again, assuming $g=2$ and $J=5$ ie $L=0$

$$\therefore S(S+1) = \frac{1}{4\beta^2} \frac{(6g'k_B T)D'}{A}$$

$$\therefore S^2 + S - \frac{1}{4\beta^2} \frac{(6g'k_B T)D'}{A} = 0$$

we obtain $S^+ = 6.630$ or $S^- = -7.630$

propagating uncertainty in 2 steps

$$1. u(S(S+1)) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(T)}{T}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} (S(S+1))$$

$$2. u(S^\pm) = \frac{1}{2} u(S(S+1)) S^\pm$$

Then our
possible spins
are

$$S^+ = 6.630 \pm 0.014$$

$$S^- = -7.630 \pm 0.016$$



Measurements on $\text{Co}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$

5/11/2020

Method:

1. Record the results of Δm vs B^2 for specimen 5
2. Calculate χ

Note: The accepted experimental value of $g^2 J(J+1) = 23.0$

Note: This material has Strongly interacting magnetic moments so that one must use the expression

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3k(T-\Theta)}$$

given this information and the details in the appendix

Determine Θ , which is a measure of this interaction

#5 Co sample		$\text{Co}(\text{NH}_4)_2(\text{SO}_4)_2 \cdot 6\text{H}_2\text{O}$				
I	$u(I)$	M	m	correct m	$u(m)$	(m - dummy)
(A)	(A)	(g)	(kg)	(kg)	(kg)	
0	0	0	0	1.7E-06	0.16E-6	
0.95	0.011	0.0024	2.4E-06	3.7E-06	0.16E-6	
1.54	0.018	0.0063	6.3E-06	7.4E-06	0.16E-6	
2	0.02	0.0105	0.00001	0.00001	0.16E-6	
2.52	0.03	0.0167	0.00002	0.00002	0.16E-6	
3.02	0.04	0.0241	0.00002	0.00002	0.16E-6	
3.52	0.04	0.0325	0.00003	0.00003	0.16E-6	
4.02	0.05	0.0426	0.00004	0.00004	0.16E-6	
4.5	0.05	0.0533	0.00005	0.00005	0.16E-6	
5.03	0.06	0.0663	0.00007	0.00007	0.16E-6	

Table 5 Specimen 5 raw data, corrected mass values (m - dummy mass) and calculated uncertainties for data

The propagation of uncertainty follows the same algorithm as Sample 2 and 3

5/11/2020

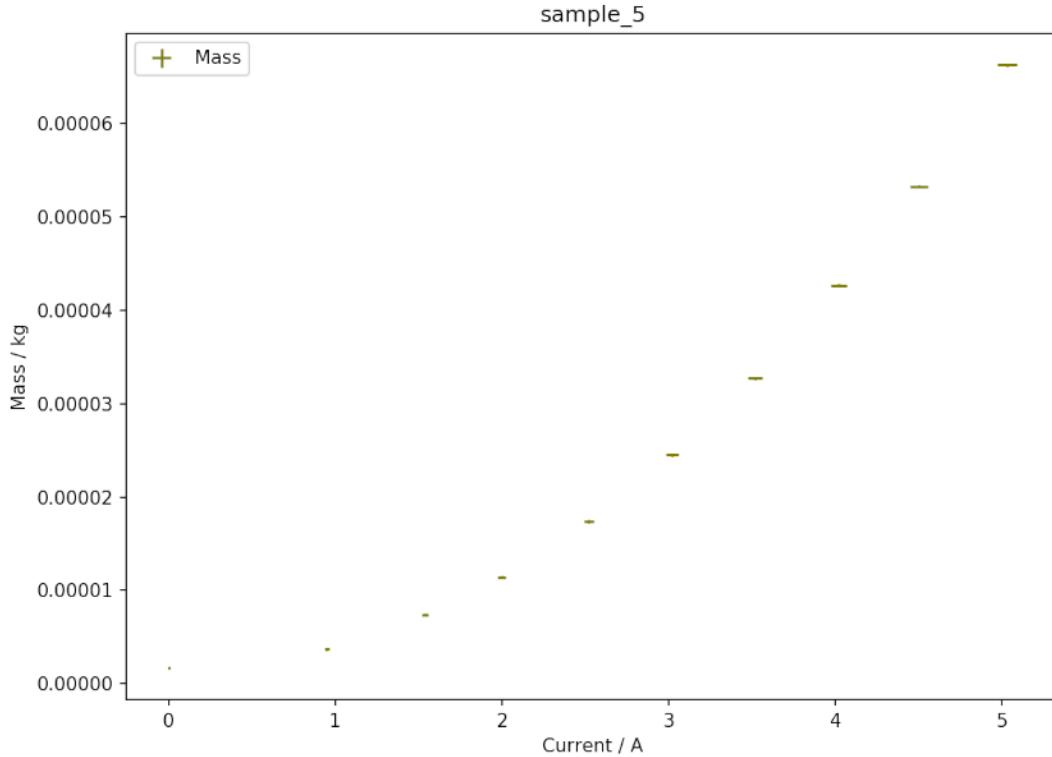


Fig 11. Specimen 4: corrected $\text{Co}(\text{II})$ sulfate data with uncertainties
in this plot there appears to be quadratic relationship between Δm and I

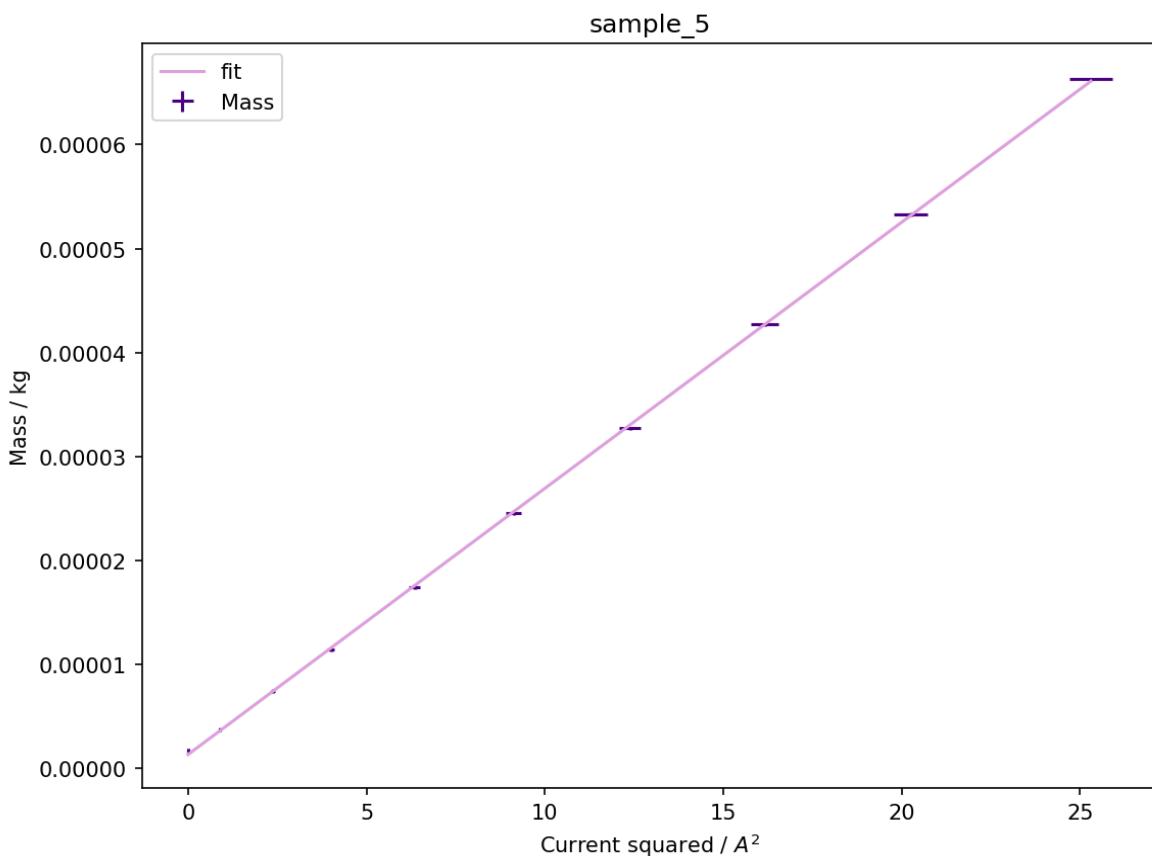


Fig 12. Plot of Δm vs I^2 . from this, we conclude that
 $\Delta m \propto B^2$, which indicates that χ is field independent [o]

The fit parameters are: (m) gradient: $(2.561 \pm 0.009) \times 10^{-6}$
(c) intercept: $(1.301 \pm 0.111) \times 10^{-6}$

using our new fit parameters

Once again, we fit the data with equation (19)

$$\Delta m = \frac{A \times (mI + c)^2}{2\mu_0 g} = D(mI + c)^2, \text{ where } D' = \frac{\Delta X}{2\mu_0 g}$$

Since the same analysis applies for this data
Our initial guess for the fit gradient D' is once again

```
# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

This time, we find $D' = (0.000508 \pm 0.000004) \text{ s}^4 \text{ A}^2 / \text{kg}$

again, rearrange the obtained D' to find $\chi_{mn} = \frac{2\mu_0 g' D'}{A} = 0.000325$

propagating the uncertainty we obtain:

$$u(\chi_{mn}) = \chi_{mn} \sqrt{\left(\frac{u(g')}{g'}\right)^2 + \left(\frac{u(D')}{D'}\right)^2 + \left(\frac{u(A)}{A}\right)^2} = u(\chi_{mn}) = \chi_{mn} \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}$$

$$\therefore u(\chi_{mn}) = 0.000325 \sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.000004}{0.000508}\right)^2} = 0.000003$$

$$\therefore \chi_c = 0.000325 \pm 0.000003$$

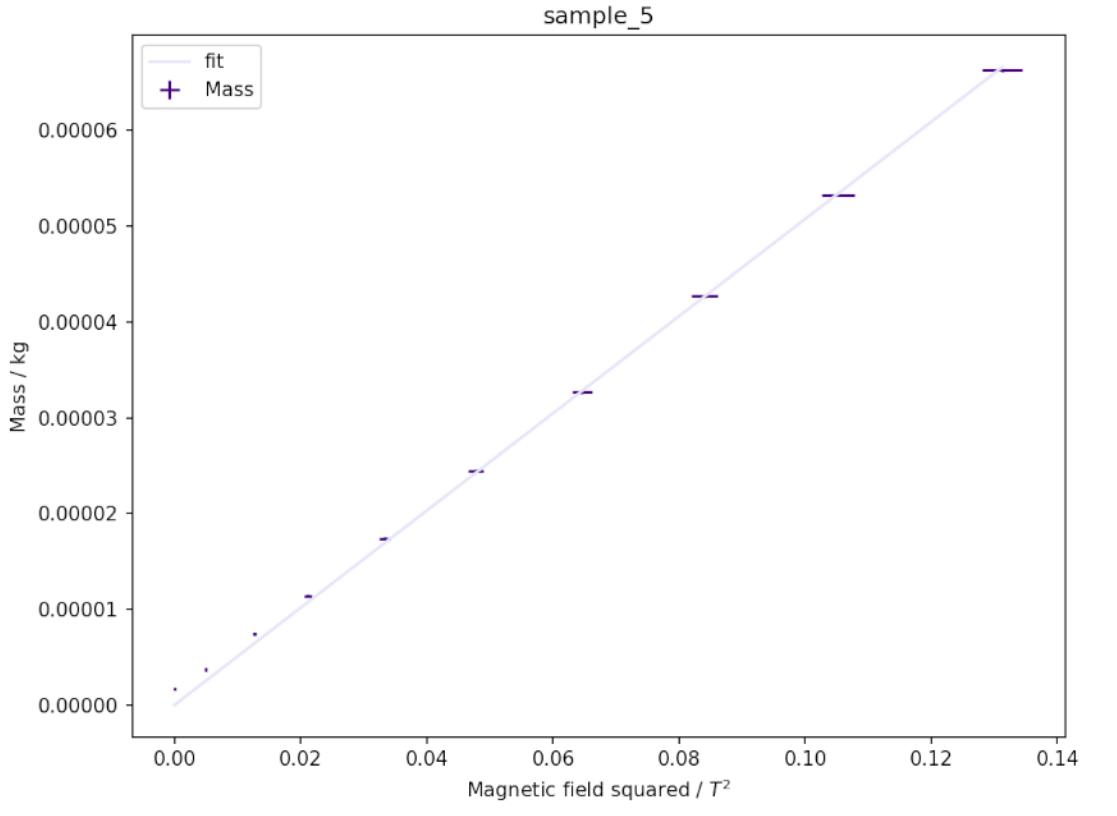


Fig 13. A plot of Δm against DB^2 , we found D' (the gradient) fitting the data using equation(19)

Fit parameter: $D' = 0.000508 \pm 0.000004 \text{ s}^4 \text{ A}^2 / \text{kg}$

Note: The accepted experimental value of $g^2 J(J+1) = 23.0$

Note: This material has strongly interacting magnetic moments so that one must use the expression

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3k_B(T-\Theta)}$$

given this information and the details in the appendix

Determine Θ which is a measure of this interaction

Note: $N_{Co} = \frac{N_0 P}{\text{molar mass}} = \frac{\# \text{molecules}}{\text{unit volume}} = \frac{\# \text{molecules}}{\text{mole}} \frac{\text{mole}}{\text{mass}} \frac{\text{mass}}{\text{unit volume}}$

from ref.
[12]

Hexahydrate
values

$$\therefore N_{Co} = \frac{11353.671 \text{ kg/m}^3}{2.457 \times 10^{-24} \text{ k/molecules}} = 4.622 \times 10^{27} \text{ molecules/m}^3$$

$$\text{Since } \chi_{Co} = 0.000325 \pm 0.000003$$

$$\therefore \Theta = T - \frac{(23.0) \mu_0 N \beta^2}{3k_B \chi_{Co}}$$

$$\therefore \Theta = T - \frac{(23.0) \mu_0 N \beta^2}{3k_B \chi_{Co}} = 207.5 \text{ K}$$

This result is probably
too high
I am expecting $\sim -50 \text{ K}$
given what John Cashion
posted on moodle

The uncertainty in Θ is split into 2 steps:

$$1. u(w) = \sqrt{\left(\frac{u(\beta^2)}{\beta^2}\right)^2 + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(N_{Co})}{N_{Co}}\right)^2 + \left(\frac{u(k_B)}{k_B}\right)^2 + \left(\frac{u(D')}{D'}\right)^2 + \left(\frac{u(g')}{g'}\right)^2} \cdot (86.53 \text{ K})$$

$$\therefore u(w) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} \cdot w = 0.7 \text{ K}$$

$$2. u(\Theta) = \sqrt{u(T)^2 + u(w)^2} = 1.2 \text{ K}$$

$$\therefore \Theta = 207.5 \pm 1.2 \text{ K}$$

a quick check: The Curie Temperature of Co is 1400 K [13]

$$\chi = \frac{C}{T-\Theta} \rightarrow \therefore 0.000325 = ? \frac{1400 \text{ K}}{294 \text{ K} - 207.5 \text{ K}}$$

$$0.000325 \neq 16.19$$

* I am aware that this is not necessarily a precise check, but I might have a units issue.

Question 3

What simple interpretation can be made of materials with
1. positive Θ ?

Curie-Weiss Law
$$\chi = \frac{C}{T-\Theta} \begin{cases} \text{if } \Theta > T > 0 \\ \text{we have a singularity} \\ \text{The susceptibility is infinite} \end{cases}$$

$$\therefore \text{material behaves like a permanent magnet} \\ (\text{ferromagnetic})$$

2. negative Θ ?

Curie-Weiss Law
$$\chi = \frac{C}{T-\Theta} \begin{cases} \text{if } \Theta < 0 < T \\ \text{behaviour of material} \\ \text{is weakly magnetic} \\ (\text{Antiferromagnetic}) \end{cases}$$

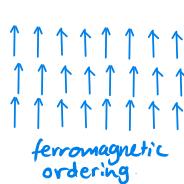
Question 4

If you had the capabilities of reducing the temperature below $|\Theta|$ for a material of non-zero value of Θ , what would you expect to occur in the material? (Discuss the cases for positive & negative Θ)

Brain
Broken

Problem 4

Distinguish between the following types of magnetic structure and give 2 examples of each.



1. Ferromagnetism [5]

materials can be divided into :

- Soft: not permanently magnetisable
example: annealed iron.
- Hard: permanently magnetisable
example: ferrite.

The atomic structure of these materials is crystalline



2. Antiferromagnetism [6]

Magnetic moments of atoms or molecules align in a regular pattern with neighbouring spins in different sub-lattices.

This type of magnetism often occurs on transition metal compounds such as Hematite and Chromium.

Structures → Ising model, bipartite lattice



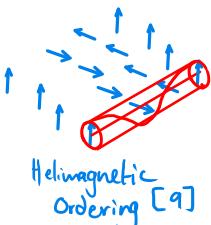
3. Ferrimagnetism [7]

Similar to antiferromagnetism but the antialigned magnetic moments are of unequal strength and so magnetisation remains active.
This occurs when populations consist of different materials or ions.

Example materials : Yttrium iron garnet (YIG)

Cubic Ferrites composed of iron oxides with other elements

Hexagonal ferrites such as $\text{PbFe}_{12}\text{O}_{19}$



4. Helical (Spiral) magnetism [8]

Spins of neighbouring magnetic moments align themselves in a spiral pattern with a characteristic angle in the range ($0 \rightarrow 180^\circ$).
This results from a competition between ferromagnetic and antiferromagnetic exchange interactions.

Helical magnetism can be Right or left handed

Examples of this structure : Manganese dioxide

$\text{B}2\text{O}$ crystal structure (Chiral cubic)

Question 5

What is a Diamagnetic material and what would you expect to be the features of its magnetic susceptibility χ_d ?

Diamagnetism is a quantum mechanical effect that occurs in all materials [4].

Diamagnetic materials are those non-natively repelled by magnetic fields and do not simultaneously exhibit other kind of magnetic interactions with magnetic fields such as paramagnetism, ferromagnetism or others [4].

The magnetic susceptibility of these materials is expected to be negative or nearly zero

Discussion

After checking my work and code several times I concluded that my uncertainty propagation for the Nickel Sample plots was adequate, even though I was very sceptical of the size of the vertical uncertainty (appearing so small)

Since I was not present in the data acquisition process, I was unable to verify how the windings of the electromagnet affect the accuracy of the measurement results

My Weiss temperature was likely too high but I could not figure out the issue:

Some things that I checked were:

The molecule parameters I used were

- found on wikipedia: Cobalt (II) sulfate (Hexahydrate)
I might have a unit problem that I haven't found
- I tried converting all the units to SI
- The problem could very well be my fits.
I worked on possible issues but couldn't find the problem, I guess maybe a conceptual misunderstanding about the gradient D' I calculated.

5/11/2020

Conclusion

In this experiment I aimed to use measurements and theoretical expressions to find the spin quantum numbers of two compounds: Ni(II) sulfate: $S = 3.677 \pm 0.017$ or $S = -4.677 \pm 0.02$ and of Mn(II) sulfate $S = 6.630 \pm 0.014$ $S = -7.630 \pm 0.016$. In addition I calculated the magnetic susceptibility of these two compounds: $\chi_{Ni} = 0.000293 \pm 0.000003$ and $\chi_{Mn} = 0.001035 \pm 0.000003$ respectively and also the magnetic susceptibility of Co(II) sulfate: $\chi_C = 0.000325 \pm 0.000003$ and its weiss temperature $\Theta = 207.5 \pm 1.2K$

24/10/2020

Note:

if the gradient of the graph of Δm against B^2 is written as $G [kg T^{-2}]$, M the molecular weight of the material of density $\rho [kg m^{-3}]$ and the absolute temperature $T [K]$

$$\text{then } g^2 J(J+1) = 407 \left(\frac{G T M}{\rho} \right)$$

Appendix [o]

Specimen details

No.	Specimen	Magnetic Ion	Molecular Weight	Density (kg/m ³)
1	Dummy	-	-	-
2	NiSO ₄ .6H ₂ O	Ni ⁺⁺	262.86	1459
3	MnSO ₄ .4H ₂ O	Mn ⁺⁺	223.06	1339
4	CrK(SO ₄) ₂ .12H ₂ O	Cr ⁺⁺⁺	499.4	877
5	Co(NH ₄) ₂ (SO ₄) ₂ .6H ₂ O	Co ⁺⁺	395.2	1240
6	CuSO ₄ .5H ₂ O	Cu ⁺⁺	249.68	1492
7	MnSO ₄ .H ₂ O	Mn ⁺⁺	169.1	748

Table

From ref [o]

Note: the density values above are less than the true densities as the materials are not packed to totally exclude air.

Logbook 4: References

- [0] Monash SPA (2020) 2.4 Magnetic susceptibility. Retrieved from Moodle (student site)
- [1] Wikipedia (2020). Curie's law. Retrieved from https://en.wikipedia.org/wiki/Curie%27s_law
- [2] Wikipedia (2020). Magnetic field. Retrieved from https://en.wikipedia.org/wiki/Magnetic_field
- [3] Wikipedia (2020). Gouy balance. Retrieved from https://en.wikipedia.org/wiki/Gouy_balance
- [4] Wikipedia (2020) Diamagnetism. Retrieved from <https://en.wikipedia.org/wiki/Diamagnetism>
- [5] Wikipedia (2020) Ferromagnetism. Retrieved from <https://en.wikipedia.org/wiki/Ferromagnetism>
- [6] Wikipedia (2020) Antiferromagnetism. Retrieved from <https://en.wikipedia.org/wiki/Antiferromagnetism>
- [7] Wikipedia (2020) Ferrimagnetism. Retrieved from <https://en.wikipedia.org/wiki/Ferrimagnetism>
- [8] Wikipedia (2020) Helimagnetism. Retrieved from <https://en.wikipedia.org/wiki/Helimagnetism>
- [9] London Centre for Nanotechnology (2018) Escaping the quantum critical singularity; modulated magnetism in PrPtAl. Retrieved from <https://www.london-nano.com/research/escaping-quantum-critical-singularity-modulated-magnetism-prptal>
- [10] Wikipedia (2020) Nickel sulfate. Retrieved from [https://en.wikipedia.org/wiki/Nickel\(II\)_sulfate](https://en.wikipedia.org/wiki/Nickel(II)_sulfate)
- [11] Wikipedia (2020) Manganese sulfate. Retrieved from [https://en.wikipedia.org/wiki/Manganese\(II\)_sulfate](https://en.wikipedia.org/wiki/Manganese(II)_sulfate)
- [12] Wikipedia (2020) Cobalt sulfate. Retrieved from [https://en.wikipedia.org/wiki/Cobalt\(II\)_sulfate](https://en.wikipedia.org/wiki/Cobalt(II)_sulfate)
- [13] Wikipedia (2020) Curie Temperature. Retrieved from https://en.wikipedia.org/wiki/Curie_temperature