UNCERTAINTIES IN EXPERIMENTAL PHYSICS - Level 2

Handling uncertainties is often challenging as there is often a range of sources, and a variety of ways are used to estimate uncertainties. However, a consistent approach to uncertainties for all science and engineering disciplines has been adopted by the International Standards Organisation (ISO), and published in the *Guide to the Expression of Uncertainty in Measurement* (GUM). A good introduction is Kirkup and Frenkel (2006, see references), which is available from the 2nd-3rd Year Preparation Room (and 1st Year Prep Room) and online via the Hargrave-Andrews library.

The School of Physics aims to have a framework consistent with *GUM* with increasing detail and sophistication from level 1 to 3. If you are really good at it, you may find a job! - many scientists in Australia and throughout the world are employed in metrology both in national standards labs and in specialist instrumentation manufacturing.

As an integral part of your experimental skills you are expected to develop competency in handling uncertainties in the lab and in written reports. You should be able to :

- describe and quantify the main source(s) of uncertainty in measurements you carry out;
- use linear regression and find uncertainties in the two parameters;
- correctly combine uncertainties when more than one quantity is involved;
- appropriately use uncertainties when presenting and comparing results.

1. Key Concepts and Definitions

All measurements of physical quantities are subject to a range of variations arising from instruments used and from the object/quantity measured. **The result of a measurement or set of measurements is an <u>estimate</u> of the value of that quantity. A statement of the <u>uncertainty</u>** is necessary for the value to be used meaningfully.

Example: The temperature of a solar cell exposed to sunlight is measured using a thermocouple and digital multimeter. The measured temperature (or <u>measurand</u> - a general term for the quantity being measured) during one experiment is determined to be

T =	46.6	± 0.2	°C
↑	↑	↑	↑
Measurand	Value	Uncertainty $u(T)$	Unit

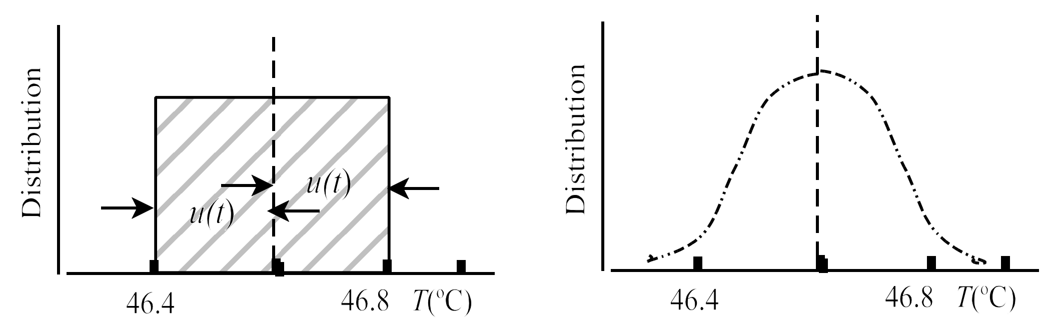

Figure 1. The likely range of values of the solar cell temperature T is represented in (a) by the rectangle, the best estimate is shown by the vertical dashed line, and the stated uncertainty is shown as $u(T)$. (b) shows an alternative probability distribution.

This statement means that the temperature is expected to lie between 46.4 and 46.8 °C. The essential idea here is **probability**. The graph shows a probability distribution **for the actual temperature** (it is not a distribution of the results of individual measurements). The shape may be rectangular as in Figure 1(a) (an equal probability of being anywhere in this range) or a bell-shaped gaussian distribution shown by the dashed curve Figure 1(b), or by some other shape.

When the results of repeated measurement of the same quantity lie in a relatively narrow range they are said to be **repeatable**; if different methods or equipment give the same results (within a relatively narrow range) they are **reproducible**. Figure 2 shows schematically the spread and closeness of sets of measurements. The adjective 'precise' is not used in *GUM* terminology (due to potential confusion with 'accurate'). Rather we may used the term 'repeatability'. Repeatability can be demonstrated by a set of measurements and does not require any other knowledge of the instruments used or the quantity under observation.

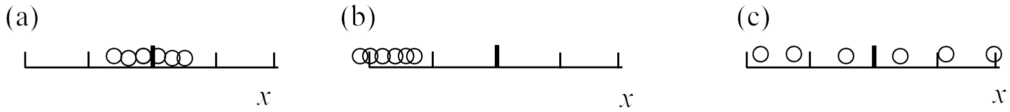

Figure 2. Small circles represent individual measurements of quantity x, and the thicker central tick represents the value of the quantity. The measurement results in (a) have a small spread and so are reasonably repeatable, and also accurate; in (b) the spread is still small but not close to the value of the quantity; results in (c) have a mean which is close to the value but the spread is large.

'Accurate' is a common adjective which means that the measurements are both repeatable within a small range, and are close to the value of the measurand. Note that we usually do not know the 'true' value of the measurand so accuracy can only be found through detailed procedures (for instance calibration, see below.) *GUM* avoids using 'accuracy' as a quantitative term.

2. Why are experimental uncertainties important ?

Uncertainties are essential for experimental measurements and physical quantities in general. The reasons are so that:

- users of the data may know its reliability and suitability for their intended purposes. You may want to use a chemical solution for an important analysis and the container label says the concentration is 0.1 mole/litre: is this good enough? To use a pendulum to make an accurate time-piece: is it enough to say the gravitational acceleration is g = 9.8 ms^{-2} ?
- hypotheses based on the data may be tested. In the history of science a small discrepancy between the expected and the measured value which would not go away with improved measurement, has often led to a deeper understanding or a better technology. For example a small annoying noise in radio-telescope signals led to the discovery of cosmic background radiation.

3. Sources of Uncertainty

There are likely to be several contributing causes of uncertainty for any measurement. These may relate to the 'object' of measurement, the measuring equipment, and the environment (including the person carrying out the procedure). It is valuable to be aware of the main sources of uncertainty and how the experimental method has sought to minimise uncertainties.

A few possible sources of uncertainty in measuring the solar cell's temperature are:

- The instrument itself has an inherent uncertainty (see below). This will normally be stated by the manufacturer. E.g., for a thermocouple-based digital thermometer it may be ±0.1 °C.
- The quantity to be measured (measurand) may vary across the object or sample. E.g., the base of the solar cell may be hottest on the upper side under an insulating covering (where the sun's rays hit it) . The thermocouple however can only be placed close to that region, not in

the very region itself. Instruments and methods are in general designed to reliably and accurately measure the desired quantity under specified conditions.

- Incorrect usage, e.g. the thermocouple junction should be placed within a small hole going into the base area of the solar cell.
- Time response. E.g. if a thermistor was used, it will take perhaps a minute to reach the temperature of the surrounding body even if there is good thermal contact.

ISO *GUM* uses two categories for sources of uncertainties, Type A and Type B.

Type A Uncertainties: can be dealt with by statistical methods

Variations in the measurand (measured quantity), e.g. the diameter of a wire will not be perfectly circular due to manufacturing or bending, or in applying uneven forces when using a micrometer to measure a wire diameter, or internal variations may occur in the method or instrument used. The term random uncertainties is sometimes applied to this.

Type B Uncertainties: require specific information about the instrument and/or method.

These would generally be referred to as "systematic uncertainties" in earlier terminology. They produce the same effect if the observation is repeated. Two common occurrences are:

(a) Poor calibration. E.g. a stroboscope has a scale which consistently shows 100 Hz when it is producing light pulses at an accurately measured frequency of 103 Hz. The deviation may be proportional, or may vary across the instrument's range.

(b) Zero Offset (or Zero Error). When an instrument does not show a zero reading for zero input, there will be a systematic difference between the value of the reading and the "actual" (accurately determined) value. Zero adjustments are necessary for most instruments with mechanical scales (such as a spring balance or analog electrical meter) - even including precision digital instruments such as electronic balances. Stray 'background' light may be detected in a spectrophotometer when there is no signal.

Calibration is the term used when the readings of one instrument are compared to a more accurate instrument or a method. Systematic uncertainties are usually detected when different methods or instruments or conditions are used.

A common example of the effect of systematic uncertainty is in the measurements of the gravitational constant *G* by various research groups using different methods, where the difference between groups typically exceeds the uncertainty estimated by the individual groups (e.g. Fixler 2007 in references).

4. Writing it Down - Significant Figures

When recording data, if you can measure to 1 mm, write 600 mm or 60.0 cm, not 60 cm, since the latter value implies that the result is most probably between 59 and 61 cm.

Final results should not be given with more significant figures than are warranted by the uncertainty (not all 9 digits shown on your calculator!) For working calculations, always use sufficient figures to avoid inaccuracy due to rounding (especially for subtraction of two values when the difference is small.)

The final value for uncertainty may be written with two significant figures, however _one significant figure is usually acceptable for class experiments. Two significant figures are warranted in research papers and reports from scientific labs where the uncertainty analysis is thorough and accurate, and where rounding to one figure would lose some significant information.

For example from many data points, the slope of a graph is found to be $(1.93 \pm 0.14)\times10^{-3}\ K^{-1}$. There is no good reason to round this uncertainty to 1 significant figure as you have confidence in the data and your linear regression analysis. On the other hand if you used a budget multimeter with AC Volt accuracy of 2%, you may decide to round up an uncertainty of ±0.14 volt to ±0.2 volt.

5. Methods for Experimental Uncertainties

In the level 2 laboratory , you will use more accurate methods than used in level 1. For a single quantity in general the procedure is the same, with the following terminology to be used.

- Type A: random uncertainty found from repeated measurements, graphs, etc.
- Type B: dependent on the inherent characteristics of the instrument/method.

(a) Repeated Measurements - Type A uncertainty

When a quantity is subject to random uncertainty, repeated measurements enables an estimate of the standard deviation, quantifying the spread or dispersion. The “standard uncertainty” (or “standard error”) gives the uncertainty estimate.

For repeated readings of the same quantity; if s = Standard Deviation

then the Standard Uncertainty $u(x) = \frac{s}{\sqrt{n-1}}$

Example. For the following 11 observed values of x:

12.5, 12.3, 12.7, 12.6, 12.4, 12.7, 12.5, 12.2, 12.6, 12.4, 12.6

The mean value of x is 12.50 and the standard deviation is $s = 0.16$.

The standard uncertainty $u(x) = 0.16/\sqrt{(11)} = 0.049 \approx 0.05$

(b) Reading Uncertainty - Type B uncertainty.

Some uncertainty arises from reading an analog scale. For example, on a good quality ruler marked in millimeters, you may be able to estimate a position as 211.7 mm whereas all you can say with full confidence is that the reading lies between 211.5 mm and 212 mm. It is common to quote such a reading as 211.7 ± 0.5 mm where the 0.5 mm is half the smallest scale division. This rule will be applied to digital instruments also.

Reading Uncertainty = ½ (Smallest Scale Division)

(c) Uncertainty due to instrument characteristics - Type B Uncertainty

Many instruments have specifications giving an estimated “accuracy” or **fractional uncertainty**:

Fractional Uncertainty in x $= \frac{u(x)}{x}$

Example An oscilloscope has an fractional uncertainty of 2% (i.e. 0.02).

When a signal of V = 2.3 V is measured, the uncertainty in V is

$u(V) = V\{u(V)/V\}$ = (2.3 V)(0.02) = 0.046 V ≈ 0.05 V.

(d) More than one source of uncertainty for a single measurement

If there are two or more sources of uncertainty for the one measured value, then they are combined according to the rules given in section 6 below. Often the resultant uncertainty can be well estimated by simply taking the largest of the two or more uncertainties.

Example . A 3.5 digit multimeter has fractional uncertainty 1%. It displays a current reading of 50.2 mA. The uncertainty due to the meter accuracy is 0.01 x 50.2 mA = 0.5 mA, and the reading uncertainty is ± 0.05 mA.

Hence we take the overall uncertainty as 0.5 mA.

6. Combining Uncertainties and Uncertainties in Functions

When measured quantities are combined to produce a final result, uncertainties in the measured quantities all influence the uncertainty in the final result. Assume R is a function of independent measured quantities x, y, z, which have uncertainties $u(x)$, $u(y)$ and $u(z)$ respectively. All uncertainties here are positive quantities.

Since each source of uncertainty contributes independently, they can be considered as each being one dimension in a multi-dimensional space. Hence the total uncertainty in R is given by

$$u(R)=\left[\left(\frac{\partial R}{\partial x}u(x)\right)^2+\left(\frac{\partial R}{\partial y}u(y)\right)^2+\left(\frac{\partial R}{\partial z}u(z)\right)^2\right]^{\frac{1}{2}}$$

A few simple cases (also used in the level 1 lab manuals) can be then deduced from the above.

(a) Addition and Subtraction

If $R = x + y$ or $R = x - y$,

$$u(R) = [\ \{u(x)\}^2 + \{u(y)\}^2]^{1/2}$$

Example. If $A = B + C$ and $D = B - C$,

where $B = 10.0 \pm 0.2$ m, $C = 5.0 \pm 0.1$ m,

then A = (15.0 ± 0.23) m and D = (5.0 ± 0.23) m

(b) Multiplication and Division

If $R = xy$ or $R = x/y$ then by simple calculus,

$$\frac{u(R)}{R} = \left[\left(\frac{u(x)}{x}\right)^2 + \left(\frac{u(y)}{y}\right)^2\right]^{\frac{1}{2}}$$

Example: A bike travels distance $x = 50.0 \pm 0.1$ m in time $t = 4.0 \pm 0.1$ s. Speed v has uncertainty

$$\frac{\delta v}{v} = \left[\left(\frac{u(x)}{x}\right)^2+\left(\frac{u(t)}{t}\right)^2\right]^{\frac{1}{2}} = \left(\frac{0.1}{50}\right)^2+\left(\frac{0.1}{4.0}\right)^2 = 0.025$$

(c) Powers

If $R = x^n$ then $$\frac{u(R)}{R} = n\frac{u(x)}{x}$$

Example. Power P due to a wind-turbine is proportional to v^3 where v = wind-speed

If the fractional uncertainty in v is $u(v)/v = 0.1$

then the fractional uncertainty in power P is 3 × 0.1 = 0.3.

(d) Functions of one variable

If $R = R(x)$ is any function of x, then by basic calculus

$$u(R) = \frac{dR}{dx}u(x)$$

Example. $A = \sin\Theta$ and $\Theta = 30° \pm 0.5°$

Radians must be used for angles in calculus formulae, π radian is equivalent to 180°.

So Θ = 30°(π/180°) = π/6 radian, and u(Θ)= 0.5°(π/180°) radian = 0.0088 radian.

Then $u(A) = \left(\frac{dA}{d\theta}\right) u(\theta) = \cos\theta\, u(\theta)$ = 0.866 × 0.0088 = 0.0076.

7. Finally, things to avoid

"Error" should not be used to describe "uncertainty". If you mean a "mistake" then use "mistake" It is true that much literature still uses "error". And one valid possibility (used in Kirkup and Frenkel) is where "error" refers to a particular contribution to a deviation or discrepancy, hence a number of sources of "error" lead to an overall "uncertainty".

The difference between an expected value and its measured value is often called the "deviation". However, when comparing your measured result with an expected value it is not helpful to include the term "deviation". E.g. "We found the density of aluminium was 2690 kg/m^3 which has a deviation of only 10 kg/m^3 from the accepted value of 2700 kg/m^3." What is wrong with this?

'Human error' is often mentioned by students in discussing their results. However, lab staff who read this may interpret it as "I can't think of what to say". If you mean "we made a mistake", or "our results were rushed and not careful", you should say exactly what you mean or what happened. As you will always attempt to identify the major sources of uncertainty, you do not need to add 'human error' o a list of otherwise meaningful potential sources.

The term "true value" can be misleading as it suggests that when we make measurements, one value is correct and all others are in error. Where appropriate, we may say 'standard value' for physical constants, or when the value is from a particular source, "the value reported by XX and YYY" or "the commonly accepted value".

A bad way to obtain agreement between your result and the expected value is deliberately over-estimating the uncertainty! It effectively says you have done your experiment poorly. You should have objective reasons for your estimate of the uncertainties. Inconsistency in a result may alert you to a mistake or some uncertainty which has not yet been considered.

8. References

J. B. Fixler, G. T. Foster, J. M. McGuirk, M. A. Kasevich, (2007) Atom Interferometer Measurement of the Newtonian Constant of Gravity, Science Vol. 315. no. 5808, pp. 74 - 77,

Kirkup L and Frenkel RB (2006) *An Introduction to Uncertainty in Measurement* (Cambridge: University Press) - Available from Monash Hargrave-Andrews Library, also as an E-book.

National Institute of Standards and Technology, *Uncertainty of Measurement Results,* http://physics.nist.gov/cuu/Uncertainty/index.html A comprehensive website for *GUM*.