

Modern Physics Laboratory

Beta Spectroscopy Experiment

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Abstract

In this experiment, electrons emitted as a result of the radioactive beta decay of $^{137}_{55}\text{Cs}$ are measured as a function of their momentum by deflecting them in a magnetic field. The resulting spectrum is evidence for a complex decay scheme with several possible pathways.

1 Theory

1.1 Beta Decay

It is energetically favorable for stable nuclei of a given element to have the ratio of neutrons to protons in a relatively narrow range. A nucleus with too many or too few neutrons will generally be unstable and can transform into a more stable nucleus in a process known as beta decay. For a nucleus with an excess of neutrons the beta decay process can be represented schematically as $n \rightarrow p + e^- + \bar{\nu}$. Inside the nucleus, a neutron changes into a proton; in addition, two particles are emitted from the nucleus, a beta particle [either an electron (e^-) or a positron (e^+)—in this case an electron] and a neutrino (ν or $\bar{\nu}$). These particles carry away the energy released in the decay of the original nucleus into a more stable nucleus.

In practice, neutrinos cannot be detected with most apparatus. On the other hand, it is straightforward to observe electrons emitted in beta decay and to measure their speed. Examining the emitted electrons, it is found that they possess a range of kinetic energies from zero up to the maximum energy E_{max} released in the decay process. The broad spectrum of electron energies suggested that the energy released in the decay was being shared by the beta particle and an unseen companion particle. In 1930, the physicist Wolfgang Pauli analyzed the energy and momentum of the emitted beta particle and the recoiling nucleus and concluded that the unseen particle (which he named the “neutrino”) had mass that was very much smaller than the mass of the electron, and could be taken as essentially equal to zero. Direct observation of neutrinos did not occur until several decades later.

Nuclei with an excess of protons undergo a beta decay in which a proton is transformed into a neutron in the nucleus, and a positron e^+ (the antiparticle of the electron, with charge $+e$) and a neutrino are emitted and share the energy released in the decay.

1.2 Gamma Decay and Internal Conversion

When a new nucleus is formed in a nuclear reaction such as beta decay, the nucleus often has an excess of energy. Using the language of quantum mechanics, it is in an excited state. Typically, such a nucleus will then make a transition to the lowest-energy state (or ground state) while emitting a photon, which carries away the excess energy. These emitted photons have high energy and therefore very short wavelength, and are called gamma rays.

In the radioactive decay to be studied in this experiment, gamma rays (photons) are in fact produced. However, we cannot observe these photons directly in our beta spectrometer—it can only detect charged particles such as electrons or positrons. However, we *can* detect the gamma rays photons indirectly, via a process called **internal conversion**. When a nucleus emits a photon, there is some probability that the photon will interact with and be absorbed by one of the electrons surrounding the nucleus. The probability is largest for the K-shell electrons—which form the innermost shell and are closest to the nucleus. The probability is smaller for electrons in the L and higher-energy shells. The net effect is that a gamma ray photon is “converted” into an emitted electron. These converted electrons all have a definite energy—equal to the gamma ray energy minus the energy required for the electron to escape from the atom. The converted K electrons are the most numerous, and also have the lowest energy, since more energy is required for them to escape the nucleus. These electrons will produce a large, sharp peak in the spectrum of emitted electrons. The converted L electrons will produce a smaller peak and at slightly higher energy than the K electrons, etc.

1.3 $^{137}_{55}\text{Cs}$ Decay Scheme

The isotope of cesium, $^{137}_{55}\text{Cs}$ (also written as Cs-137) has an interesting decay scheme. As shown in Figure 1 below, $^{137}_{55}\text{Cs}$ can decay directly to $^{137}_{56}\text{Ba}$ via beta decay, releasing an energy of 1.18 MeV. However, only 8 % of the decays proceed that way. The great majority of the $^{137}_{55}\text{Cs}$ nuclei decay in a two step process, first with a beta decay to $^{137}_{56}\text{Ba}^*$, which is an excited state of $^{137}_{56}\text{Ba}$. This decay releases energy 0.512 MeV, which is shared between an emitted electron and a neutrino. In the second step, the $^{137}_{56}\text{Ba}^*$ makes a transition to the ground state, $^{137}_{56}\text{Ba}$ by emitting a gamma ray photon of energy 0.662 MeV. Some of these photons are internally converted—absorbed by an inner shell electron. These electrons are then emitted with energy approximately equal to 0.662 MeV (actually slightly less, since some of the absorbed energy is used in the process of the electron escaping from the $^{137}_{56}\text{Ba}$ atom).

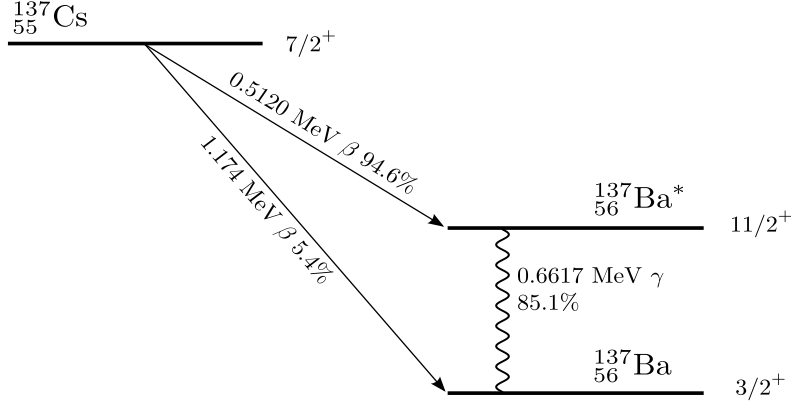


Figure 1: $^{137}_{55}\text{Cs}$ decay scheme. Decay energies listed are accepted values.

In this experiment, we observe the electrons emitted in the beta decay from $^{137}_{55}\text{Cs}$ to $^{137}_{56}\text{Ba}^*$. These electrons have kinetic energy E_K ranging from zero to a maximum $E_{K_m} = 0.512\text{MeV}$, the energy released in the decay. We also observe the internally converted electrons, which have kinetic energy approximately equal to $E_{K_c} = 0.662\text{MeV}$. If one plots the number of electrons emitted by $^{137}_{55}\text{Cs}$ as a function of their kinetic energy, one expects a plot that is illustrated schematically in Fig. 2 below. The broad peak is from the electrons emitted by beta decay of $^{137}_{55}\text{Cs}$ to $^{137}_{56}\text{Ba}^*$; the narrow peak is from the electrons internally converted from the gamma ray photons emitted in the decay from $^{137}_{56}\text{Ba}^*$ to $^{137}_{56}\text{Ba}$. In this figure, we have neglected the 8 % of the electrons emitted in the direct beta decay from $^{137}_{55}\text{Cs}$ to $^{137}_{56}\text{Ba}$.

1.4 Beta spectrum, Kurie Plot

The quantitative description of the beta decay spectrum is based on the following relations:

$$R = A(E_m - E)^2 p^2 = A(E_{K_m} - E_K)^2 p^2 \quad (1)$$

$$P = eBr \quad (2)$$

$$E = mc^2 + E_K \quad (3)$$

$$E^2 = p^2 c^2 + m^2 c^4 \quad (4)$$

where:

R = the number of electrons (or positrons) emitted per unit time in the beta decay process

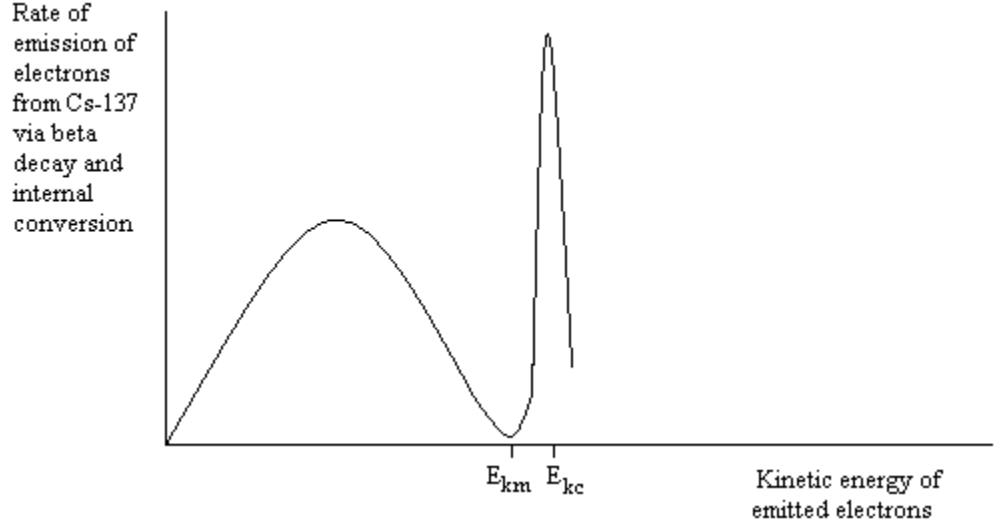


Figure 2: Schematic spectrum of electrons emitted from $^{137}_{55}\text{Cs}$

p = momentum of an emitted electron (or positron)

E = total energy of an emitted electron

E_m = maximum total energy of an emitted electron

E_K = kinetic energy of an emitted electron

E_{Km} = maximum kinetic energy of an emitted electron

B = magnitude of the uniform magnetic field in the beta spectrometer

r = radius of the circular path traveled by the electrons in the magnetic field B

e = magnitude of the charge on an electron

m = mass of electron

A is a constant.

Eq. 1 is a result obtained from the quantum mechanical theory of beta decay. The second equality in Eq. 1 is obtained by using Eq. 3. Eqs. 3 and 4 are familiar equations from special relativity describing the motion of a particle traveling at a speed that is an appreciable fraction of the speed of light.

Eq. 2 applies to an electron of momentum p traveling perpendicular to a uniform magnetic field of strength B . If the electron is non-relativistic (with $v \ll c$), we can apply Newton's 2nd law as follows. The magnetic force on the electron \vec{F}_B is given by:

$$\vec{F}_B = -e\vec{v} \times \vec{B} \quad (5)$$

and by Newton's second law,

$$\vec{F}_B = m\vec{a} = -e\vec{v} \times \vec{B} \quad (6)$$

The last equality in Eq. 6 implies that \vec{a} is perpendicular to \vec{v} , which means that the electron travels in a circle at constant speed with acceleration equal in magnitude to $a = v^2/r$. Therefore,

$$evB = mv^2/r \quad (7)$$

or

$$p = mv = eBr \quad (8)$$

This proves Eq. 2 for non-relativistic electrons. It can be shown (see Appendix) that Eq. 2 also holds for relativistic electrons (with $v < c$ and $v \approx c$).

In Eq. 1, while the electron kinetic energy E_K varies over its range, from 0 to $E_{K\text{m}}$, the electron momentum increases steadily from 0 to some maximum value. Examination of Eq. 1 shows that a plot of the number of beta decays/second = R , plotted as a function of E_K or of p , should have a single peak, approximately in the middle of the range of E_K or of p . This is illustrated in Fig. 2 (the broad-peaked part of the graph). In practice, we plot R as a function of the electromagnet current I ; this gives the same shape curve, since p is directly proportional to B , which is (to a good approximation) directly proportional to I .

In order to test the beta decay theory quantitatively, we combine Eqs. 1 and 2 to obtain the result (see Preliminary Question 2)

$$\frac{\sqrt{R}}{B} = A'(E_{K\text{m}} - E_K) \quad (9)$$

where A' is a constant. This means that if we plot \sqrt{R}/B vs E_K , we should obtain a straight line with negative slope and which intersects the horizontal axis at $E_K = E_{K\text{m}}$. This graph is called a Kurie plot, and allows one to determine $E_{K\text{m}}$, the energy released in the beta decay.

2 Preliminary Questions

1. Eq. 2 can be evaluated directly in a consistent set of units, such as SI units. However, for this experiment it is convenient to adapt Eq. 2 so that the momentum p is expressed in units of MeV/ c .

Use the standard value of e and perform the appropriate unit conversions to show that Eq. 2 can be written as:

$$p \text{ [MeV}/c] = 300B \text{ [T]} r \text{ [m]} \quad (10)$$

or

$$pc \text{ [MeV]} = 300B \text{ [T]} r \text{ [m]} \quad (11)$$

Note: $1\text{MeV} = 10^6\text{eV}$.

Hint: Start with Eq. 2 and show that if $B = 1.0\text{T}$ and $r = 1.0\text{m}$, then $pc = 3.0 \times 10^8\text{eV} = 300\text{MeV}$. Hence, show that $pc \text{ [MeV]} = 300B \text{ [T]} r \text{ [m]}$.

2. Combine Eqs. 1 and 2 to obtain Eq. 9, which is the basis of the Kurie plot.

3 Equipment

Beta spectrometer, vacuum pump, $^{137}_{55}\text{Cs}$ source, Geiger-Muller (G-M) tube, scalar.

3.1 Notes on Equipment

The beta spectrometer consists of a steel chamber in the shape of a rectangular solid. The chamber is mounted between the poles of an electromagnet, which produces a uniform horizontal magnetic field inside the chamber. There are three openings into the front of the chamber. The bottom one is for the $^{137}_{55}\text{Cs}$ source, and the top one is for the G-M tube which detects the emitted electrons. The path of the electrons which enter the G-M tube lies in a plane perpendicular to the magnetic field direction, as shown in Figure 3 below. The distance between the upper and lower port is 3.0 inches, and we see from the figure that the radius r of the circular path followed by the electrons must be $3.0 \text{ inches}/2 = 1.5 \text{ inches} = 0.0381 \text{ m}$.

Energetic electrons travel only a few cm in air, due to collisions with air molecules. In order to detect a significant fraction of the electrons emitted by the $^{137}_{55}\text{Cs}$ source, the spectrometer chamber is evacuated with a vacuum pump.

4 Procedure

Initially, we carry out a preliminary step. We use the gaussmeter to measure the magnetic field produced in the beta spectrometer, as a function of the electromagnet current (part A). After this step, we determine the Geiger-Muller (G-M) tube plateau and the background count (part B), and finally, measure the spectrum of electrons emitted from $^{137}_{55}\text{Cs}$ (part C).

4.1 A. Calibration curve for the electromagnet

Note: In the measurement done with the gaussmeter, the flat side of the gaussmeter probe must be oriented perpendicular to the direction of the magnetic field. You can check the orientation by rotating the probe slightly. In the proper orientation, the gaussmeter reading will be at a maximum.

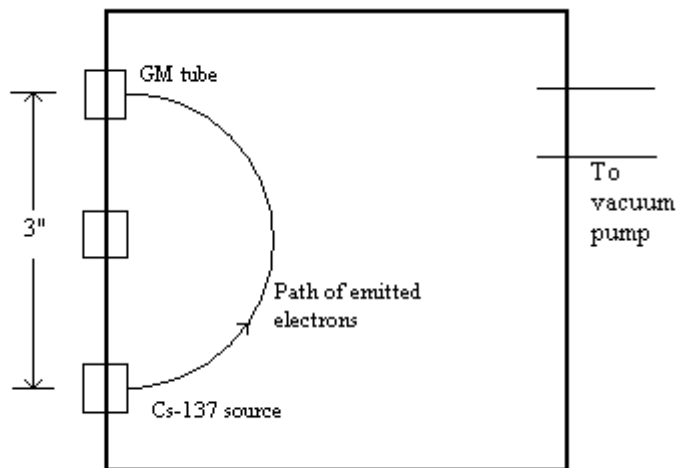


Figure 3: Beta spectrometer chamber. Magnetic field is perpendicular to the plane of the figure.

The magnetic field in the spectrometer is measured by inserting the gauss-meter probe in the center port of the spectrometer.

1. Adjust the electromagnet current I in steps of 0.10 A, from 0 to 1.50 A. Measure the magnetic field B for each value of I . After the measurements are complete, replace the cover on the center port.
2. Tabulate the values of I and the corresponding measured values of B . Express B in Tesla (the SI unit). Recall that $1\text{T} = 10^4\text{G} = 10\text{kG}$. Plot a graph of B vs I . Do a curve fit which expresses B as a function of I . This will be your calibration curve for the electromagnet.
3. Set the electromagnet current at 0.50 A. Use a compass to determine the direction of the magnetic field \vec{B} in the immediate vicinity of the beta spectrometer. Assume that the $^{137}_{55}\text{Cs}$ sample will be located in the lower port of the spectrometer, and the detector—a G-M tube—will be in the upper port. Sketch the path that the electrons follow from source to detector, and include the direction of \vec{B} in your sketch. Use the right-hand rule for magnetic force to determine the predicted direction of the force on the electrons after they are emitted by the $^{137}_{55}\text{Cs}$. Is the predicted direction of the force correct in order for the electrons to be deflected toward the detector (the G-M tube)? Explain briefly, referring to your sketch.

4.2 B. G-M tube characteristics and background count rate

CAUTION: The instructor will handle and position the $^{137}_{55}\text{Cs}$ source. This source is in the form of a powder on a thin membrane surface, and is therefore delicate and easily disturbed. Contamination is a concern. **Students must not handle the $^{137}_{55}\text{Cs}$ source.**

1. The instructor will mount the G-M tube over the $^{137}_{55}\text{Cs}$ source.
2. Measure the number of counts in a 30 second time interval as the tube voltage is varied from 0 to 620 volts. Increase the voltage in increments of 50 volts until you begin to record counts. After counts appear, reduce the voltage increment to 20 volts. **Do not exceed 650 volts!**
3. Sketch a graph of counts/30 second interval as a function of tube voltage. Determine a suitable plateau voltage and set the tube voltage to that value for the remainder of the experiment. Include this graph with the data for this experiment.
4. The instructor will place the $^{137}_{55}\text{Cs}$ source and the G-M tube in the appropriate openings in the spectrometer chamber, and then turn on the vacuum pump. If, after a few seconds, the pump makes relatively soft clicking sounds, the vacuum inside the spectrometer chamber should be acceptable. If the pump makes relatively loud gurgling sounds, then the vacuum seal of one or more of the spectrometer ports is inadequate. In this event, the pump should be shut off and the seals rechecked.
5. With the source in the spectrometer chamber under vacuum but no magnetic field, measure the background count for 10 minutes.

4.3 C. Beta spectrum and internal conversion peaks

1. With vacuum in chamber, measure counts for 2.0 minute intervals. Increase the electromagnet current I starting from zero in steps of 0.10 Amps. Do not exceed 1.50 Amps.

Refer to the schematic plot of Fig. 2. Note that after the broad beta decay peak, a much sharper internal conversion peak should appear. For measurements in the vicinity of the internal conversion peak, use smaller electromagnet current steps, *e.g.*, 0.05 Amps.

2. After the first sharp internal conversion peak, there should be a second, smaller peak at slightly higher values of I (see discussion in the Theory section, under Internal Conversion). Continue to increase I by small increments to see if you can detect it.

5 Analysis

1. Subtract the background count rate (Procedure B5) from the count rate data (Procedure C1 & C2) to yield the emitted-electron count rate R . Be sure to use a consistent set of units (e.g. counts/minute).
2. Plot carefully R vs electromagnet current I .
3. Tabulate for the beta decay count data (which gives the broad peak) the following quantities for each value of electromagnet current I for which counts were measured. Do *not* include in this table the count data taken around the (narrow) internal conversion peak.

$I \quad R \quad B \quad p \quad E \quad E_K$

Notes:

- (a) The first three quantities are obtained directly from your count data corrected for background (Analysis step 1) and your electromagnet calibration curve (Procedure step A2).
 - (b) The remaining three quantities refer to the beta decay electron, and are calculated as follows: First find the electron momentum p from the magnetic field B , using Eq. 2. Then use Eqs. 3 and 4 to find E and, finally, E_K .
 - (c) It is convenient in these calculations to express all energies in units of MeV and to express the momentum p in MeV/c. The result of Preliminary Question 1 will be helpful here. Also, recall that the rest energy of the electron, $mc^2 = 0.511\text{MeV}$.
4. (Kurie plot). The theoretical prediction for the beta decay counting rate is given by Eq. 1. In the Theory section and Preliminary Question 2 it was shown that Eqs. 1 and 2 could be combined to yield Eq. 9, which provides a straightforward test of the theory. If the count rate data obey Eq. 1, then a plot of \sqrt{R}/B vs E_K should yield a straight line.

Using the values tabulated in step 5 above, plot \sqrt{R}/B vs E_K . Comment on the shape of the graph.

Use the graph to determine E_{K_m} , the maximum kinetic energy of an electron emitted in the beta decay. Hint: See the discussion just preceding and following Eq. 9.

5. In the Theory section it was claimed that, in a beta decay process, the maximum kinetic energy of an emitted electron was equal to the energy released in the decay. See the text just preceding and following Fig. 1.

Compare your experimental value of E_{K_m} found in step 4 above to the accepted value of the energy released in the beta decay of $^{137}_{55}\text{Cs}$ to $^{137}_{56}\text{Ba}^*$ (see the $^{137}_{55}\text{Cs}$ decay scheme in Fig. 1).

6. Find the current at which the narrow internal conversion peak occurs. From your electromagnet calibration curve (or the curve-fit function), determine the corresponding value of the magnetic field B . Using that value of B , calculate the total energy E_c for an internally converted electron emitted at the internal conversion peak. From this, compute the kinetic energy E_{Kc} of the electron. Use the same method of calculation as described in the Notes for Analysis step 3 above.
7. Explain qualitatively the physical origin of the internal conversion electrons. In the decay scheme shown in Fig. 1, which energy should approximately equal E_{Kc} ? Explain briefly. Compare your experimentally-determined value of E_{Kc} (from step 6 above) to the appropriate energy value given in the decay scheme.

6 Appendix

Derivation of Eq. 2 for an electron moving at relativistic speed.

In relativistic mechanics, Newton's 2nd law is replaced by

$$\vec{F}_B = \frac{d\vec{p}}{dt} = -e\vec{v} \times \vec{B} \quad (12)$$

for an electron moving in a uniform magnetic field. We note that $\vec{F}_B \cdot \vec{v} = 0$, since the cross product $\vec{v} \times \vec{B}$ is perpendicular to \vec{v} .

Now recall that the change in relativistic kinetic energy of a particle is defined to be equal to the work done by the net force on the object. The work done in time dt is just

$$dW = \vec{F}_B \cdot d\vec{r} = \vec{F}_B \cdot \vec{v}dt \quad (13)$$

and this in turn is equal to the change in kinetic energy dE_K . Therefore, we have the familiar result,

$$\vec{F}_B \cdot \vec{v} = \frac{dE_K}{dt} \quad (14)$$

However, since $\vec{F}_B \cdot \vec{v} = 0$ in this case, it follows that the electron's kinetic energy E_K is constant. It then follows from Eqs 3 and 4 that the total energy E and the magnitude p of the electron momentum are themselves constant. Since the relativistic momentum p defined as

$$\vec{p} = \gamma m \vec{v} \quad (15)$$

and has a constant magnitude, we see that the magnitude of the electron velocity \vec{v} must also be constant. Hence, the relativistic counterpart of Newton's 2nd law becomes:

$$\vec{F}_B = \frac{d\vec{p}}{dt} = \gamma m \vec{a} = -e\vec{v} \times \vec{B} \quad (16)$$

since v is constant. Note that this equation is the same as Eq. 6, except that the mass m is replaced by γm . Hence, we conclude, as in the non-relativistic case,

that \vec{a} is perpendicular to \vec{v} and the electron's motion is circular. Following the steps following Eq. 6, we again arrive at Eq. 2, $p = eBr$, except that p is now the magnitude of the *relativistic* momentum.

7 References

1. Tipler and Llewellyn, Modern Physics, 5th ed., pp. 499-501, 504-505