Magnetic Susceptibility

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Risk Assessment.

Risk factor	Risk level	Control (existing)	Control (proposed)
0-5 A electromagnet may rise above ambient temperatures	low	• Water cooling system is used to cool down the magnet • Cold water is set is set to flow throw the cooling system before activating the magnet.	• If required, students inform lab technitian of issues present in the controls set up.

Aim (from ref. [0])

To measure the magnetic susceptibility of a number of magnetic materials.

By combining theoretical expressions and experimental results information concerning the quantum numbers of the magnetic ions present in the materials is obtained.

Theory (from ref. [0])

Atomic magnetism

The magnetic moment of an atom or ion is

$$\boldsymbol{\mu} = -g\beta \mathbf{J} \qquad (1)$$

Where g is the Landé g factor, $\beta = \frac{e\hbar}{(4\pi m_e)} = 9.274 \times 10^{-24}$ J/T the Bohr magneton and $\mathbf{J}$ is the total angular momentum given by

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \qquad (2)$$

$\mathbf{L}$, $\mathbf{S}$ are the orbital and spin angular momentum respectively.

The magnetic moments associated with individual atoms or ions may be thought of as completely independent moments and non-interacting.

If there are N moments per unit volume in a material the vector sum of the individual moments (μ_i) produces a zero resultant magnetization $\mathbf{M}$.

$$\mathbf{M} = \sum_i^N \mu_i = 0$$

Problem 1

This statement is true if there is no net magnetic moment polarisation in the system.

That is, if the material is not magnetic, the magnetic moments of the individual atoms or ions that comprise the material are mostly antialigned meaning that their individual vector sums cancel out.

24/10/2020

Applying an external magnetic field on a material will cause both an alignment of the internal diapole moments and also an induced magnetic moment in the material.

Consequenty, the magnetic flux density in the material will be diferent from the free space value $\mathbf{B}$

The magnetic intensity $\mathbf{H}$ in the material is given by

$$\mathbf{H} = \frac{\mathbf{B}}{\mu_0} - \mathbf{M} \qquad (3)$$

where μ_0 is the permeability of free space, and $\mathbf{M}$ is the resultant magnetisation.

Another way to write $\mathbf{M}$ is

$$\mathbf{M} = \chi \mathbf{H} \qquad (4)$$

where χ is the magnetic susceptibilty.

The relation between the applied magnetic flux density and the magnetic suceptibility is:

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \qquad (5)$$

using equation (4):

$$\Leftrightarrow \mathbf{B} = \mu_0 (\mathbf{H} + \chi \mathbf{H})$$

$$\Leftrightarrow \mathbf{B} = \mu_0 \mathbf{H} (1 + \chi) \qquad (6)$$

where $\mu_r = (1 + \chi)$ is the relative permeability of the material.

It can be shown that for materials where the assumption we made about non-interacting moments is VALID.

The magnetic suceptibility follows Curie's law:

$$\chi = \frac{C}{T} \qquad (7)$$

where T is the material's absolute temperature and the Curie constant is:

$$C = \frac{\mu_0 N g^2 \beta J (J+1)}{3 k_B} \qquad (8)$$

where the only quantity left to be defined here is k_B: Boltzmann's constant.

24/10/2020

Most of the magnetic salts used in this experiment follow curie's law, specially around room temperature.

Furthermore, if the magnetic moment of the atom, is due mainly to the spin contribution (because **L** = 0, or the orbital angular momentum is quenched) we can write the curie constant as:

$$C = \frac{\mu_0 N g^2 \beta S(S+1)}{3K_B} \qquad (9)$$

There are a range of materials where, for example, the magnetic ions are close together or the temperatures are low enough that the Curie's Law will fail.

In this scenario an ion will not only experience an outside magnetic field but also an internal magnetic field.

we use this internal field to express interactions of neighbouring magnetic moments

We use a modified Curie's Law aka Curie's-Weiss law:

$$\chi = \frac{C}{T - \Theta} \qquad (10)$$

where the weiss constant Θ is a measure of this interaction energy (expressed as a temperature)

If Θ is positive and the temperature is <u>l below T_c</u> The curie temperature.

Then Material becomes <u>ferromagnetic</u>.

$T_c = \Theta$ in this theory

* in practice they differ by 10-20 K *

If θ is negative, the material becomes <u>antiferromagnetic</u> below the Néel temperature

$T_N = |\Theta|$

24/10/2020

The Gouy Method [o]

The force in the z-direction of a dipole $\mathbf{m}$ in a magnetic field $\mathbf{B}$ is given by:

$$F_z = m_x\left(\frac{\partial B_x}{\partial z}\right) + m_y\left(\frac{\partial B_y}{\partial z}\right) + m_z\left(\frac{\partial B_z}{\partial z}\right) \qquad (11)$$

for a volume V of magnetic material with susceptibility χ, the magnetic flux density in the sample will be reduced due to the sample's magnetisation.

for paramagnetic and diamagnetic samples the change is small (ie $M \approx 0$)

We approximate the magnetic flux density

$$\mathbf{B} \approx \mu_0 \mathbf{H} \qquad (12)$$

Consequently,

$$F_z = \mu_0 V\left(M_x\left(\frac{\partial H_x}{\partial z}\right) + M_y\left(\frac{\partial H_y}{\partial z}\right) + M_z\left(\frac{\partial H_z}{\partial z}\right)\right) \qquad (13)$$

$$\Leftrightarrow F_z = \mu_0 V \chi \left(H_x\left(\frac{\partial H_x}{\partial z}\right) + H_y\left(\frac{\partial H_y}{\partial z}\right) + H_z\left(\frac{\partial H_z}{\partial z}\right)\right)$$

$$\Leftrightarrow F_z = \frac{\mu_0 V \chi}{2} \frac{\partial}{\partial z} \mathbf{H}^2 \qquad (14)$$

If the material is immersed in another medium of different susceptibility, then the force of the sample is reduced as any displacement in the z-direction requires an opposite displacement of the surrounding medium. for example in air:

$$F_z = \frac{1}{2}\mu_0 V(\chi - \chi_{air}) \frac{\partial}{\partial z} \mathbf{H}^2 \qquad (15)$$

In this experiment we can neglect χ_{air}.

The magnetic flux density in the sample is approximately the same as the free space value outside of the sample.

$$\therefore F_z = \frac{1}{2\mu_0} V(\chi - \chi_{air}) \frac{\partial}{\partial z} \mathbf{B}^2 \qquad (16)$$

24/10/2020

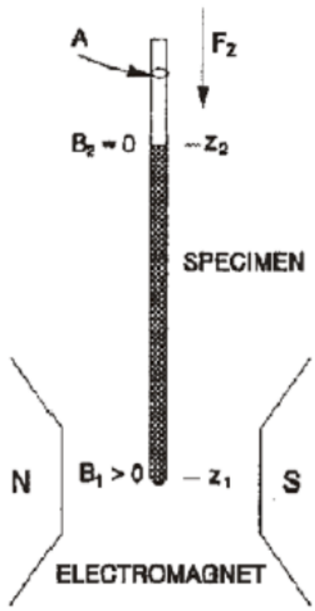


Figure 1: Geometry of the Gouy method.

Fig 1. from ref [0]
The geometry of the Gouy method.
The downward force F_z (positive) is measured on a sample sufficiently long to extend from the position of maximum magnetic field strength ($z = z_1$), to a position of essentially zero field strength B_2 ($z = z_2$).

The specimen is placed in a tube (glass) of cross-sectional area A so the total downward force (Neglecting χ_{air}) is:

$$F_z = \frac{\chi}{2\mu_0} \int_{z_2}^{z_1} \frac{\partial B}{\partial z} A dz \qquad (17)$$

where we have used $dV = Adz$ (a unit volume is 1 m3)

Since $B_2 = 0$

$$\Rightarrow F_z = \frac{\chi A}{2\mu_0} B_1^2 \qquad (18)$$

As long as A is kept constant

$$F_z = D \chi B_1^2 \qquad (19)$$

where D is a constant.

The magnetic susceptibility for a unit volume of material is:

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_b T}$$

Where N is the number of atoms or ions That form the sample.

24/10/2020

Problem 2

in a material of molecular weight M (g/mol) and density ρ (kg/m^3)

Show that the magnetic susceptibility/m^3 is

$$\chi_2 = \frac{\mu_0 \rho N_0 g^2 \beta^2 J(J+1)}{3 k_B T M} \times 10^3$$

we know that the magnetic susceptibility for a unit volume is

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_B T}$$

Since $\frac{N}{N_0} = \text{mol}$ and $\frac{m}{\text{mol}} = M$

where N_0 is avogadro's number, mol is the number of moles & m is the mass (in grams)

$$\frac{N_0}{N} m = M$$

$$\therefore N = \frac{N_0 m}{M}$$

Then

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3 k_B T} = \frac{\mu_0 N_0 m g^2 \beta^2 J(J+1)}{3 k_B T M}$$

converting grams to kilograms and dividing by the unit volume V:

$$\chi_2 = \frac{\mu_0 \rho N_0 g^2 \beta^2 J(J+1)}{3 k_B T M} \times 10^3$$

$\square$

24/10/2020

Magnetic Susceptibility Data Set information:

- Data is recorded on Gaussmeter set on the 3 kG range.
- Laboratory temperature is 294 +/- 1 K.
- Data for this experiment is in a single Excel file.
- Separate sheets contain the data for the magnetic field calibration, the sample tube calibration measurement and the measurements for each sample.
- Equipment measuring uncertainties are listed below.

Model 2100 Gaussmeter + probe accuracy:

1.5.1 Accuracy Specifications

DC Accuracy ±(% of reading + % of range)

Range	Typical	Maximum
≥ 30 KG	0.050 + 0.020	0.100 + 0.050
≥ 3 KG	0.050 + 0.020	0.110 + 0.050
≥ 300 G	0.060 + 0.020	0.150 + 0.050
≥ 30 G	0.080 + 0.020	0.200 + 0.050

Table 1.5.1 DC Accuracy Specifications

← $u(B)$ for calibration data

Ohaus weighing scale accuracy: ← $u(m)$

Repeatability +/- 0.0001 g

Linearity +/- 0.0002 g over full range (120 g) → fractional uncertainty

Current meter UNI-T UT803 accuracy:

For 10 A range +/-1.2 % ← $u(I)$ for all data sets: $u(I) = 0.012 \times I$

Note: $1G = 0.0001T$

on the data set info sheet we know that the Gaussmeter is set on the 3kG range.

$\therefore 3\,kG = 0.3\,T = 300\,mT$

Then, the DC typical accuracy (ie. magnetic field uncertainty) is:

$\therefore u(B) = 0.050(\%\text{ reading}) + 0.020(\%\ 300\,mT)$

Whilst, the respective maximum value for this uncertainty is:

$u(B)_{max} = 0.110\ (\%\text{ reading}) + 0.050(\%\ 0.3T)$

Ohaus uncertainty

we convert it to absolute by multiplying by the mass measured

* we also must consider the precision of our measurements

ie. All data provided to us contributes half the smallest digit to the total uncertainty.

$\therefore$ The uncertainty in the mass

$$u(m) = \sqrt{0.0001^2 + \left(\frac{0.0002}{120}\cdot m\right)^2 + 0.00005^2}$$

Experiment [0]

24/10/2020

Note: ensure that water is flowing through the magnet windings before the magnet is turned on, and adjust the magnet current slowly.
Turn the water off at the conclusion of the experiment

Magnetic field Calibration

Method:

Using the Hall probe magnetic field strength meter, calibrate the magnet by measuring the magnetic field for current settings:

- 0A → 5A (maximum) in steps of 1A (minimum)
- Place the probe at the position of max $|\vec{B}|$

Note: it's important that the probe is in the optimal position to obtain accurate current values. These values will be used throughout the experiment.

Note: The meter is measuring the value of the magnetic flux density **B** [T] rather than magnetic field intensity **H** [Am^{-1}]

Uncertainty propagation (calibration data)

The typical uncertainty in the magnetic field readings is found by:

$\therefore u(B) = 0.050(\%\ \text{reading}) + 0.020(\%\ 300\,mT)$

where after propagation, we scale the units to Teslas (T)

The uncertainty in the current is $u(I) = 0.012 \times I$

Magnet calibration				
I	u(I)	B	B	u(B)
(A)	(A)	(mT)	(T)	(T)
0	0	4.077	0.00408	0.006
0.97	0.01	71.15	0.07115	0.010
1.52	0.02	110.01	0.11001	0.012
2.04	0.02	147.66	0.14766	0.013
2.49	0.03	180.26	0.18026	0.015
2.99	0.04	215.62	0.21562	0.017
3.49	0.04	252.27	0.25227	0.018
4.06	0.05	292.61	0.29261	0.02
4.53	0.05	327.14	0.32714	0.02
4.98	0.06	359.35	0.35935	0.02

Table 1. Magnet calibration data with our calculated uncertainties.

24/10/2020

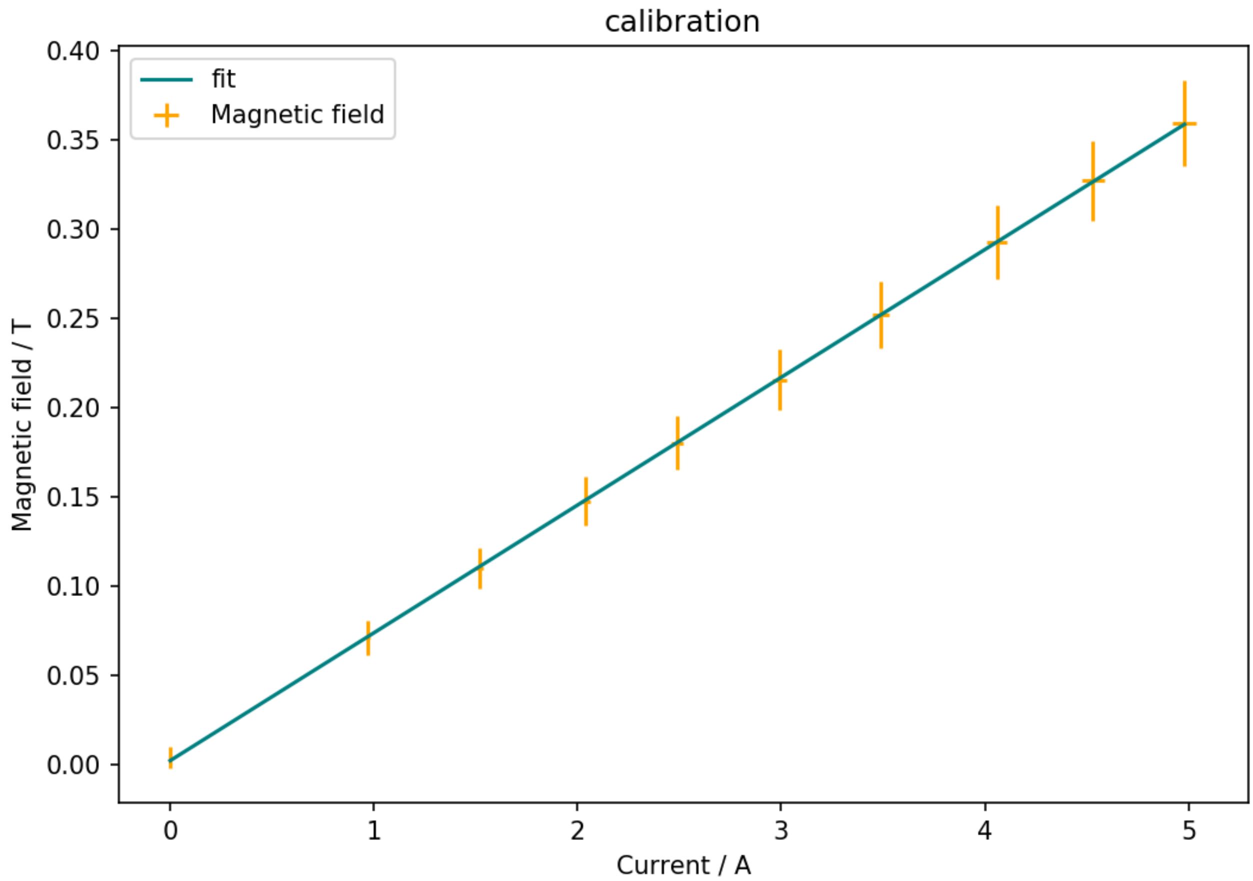


Fig 2. Magnet calibration data and linear fit
The relation between B and I is linear.

The parameters of this fit are:

gradient : 0.0716 ± 0.0002

intercept : 0.0023 ± 0.0006

fitting code

```
def line_fit(x, m, c):
    return m * x + c

def fitting_calibration_data(xs, ys, initial_guess):
    pars, pcov = scipy.optimize.curve_fit(line_fit, xs, ys, p0=initial_guess)
    perr = np.sqrt(np.diag(pcov))
    # print(f"{pars}, {perr}\n")
    linear_fit = line_fit(xs, *pars)
    return pars, perr, linear_fit
```

where the initial guess for the fit parameters is:

```
# from eyeballing calibration data
linear_guess_0 = [0.3, 0] # [gradient, intercept]
```

24/10/2020

Measurements on the dummy sample

Method

1. Hang the dummy specimen, specimen I, (glass tube + air) onto the balance and adjust it so it hangs centrally in the magnetic field. (see fig 2.)
2. Wait for the sample to steady.
3. Tabulate any mass.
4. Record change in mass Δm as a function of magnet current.
5. The mass change follows $F = g'\Delta m$, $g' = 9.8\,ms^{-2}$
6. These results may be assumed to be a valid correction for all glass specimen holders used in this experiment.

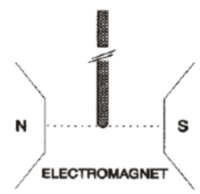


Figure 2: Dummy specimen alignment.

Fig 3. from ref [0] Schematic Drawing of the position of specimen w.r.t the electromagnet

Question 1

The observed Δm is negative for this sample.
Air and Glass are both diamagnetic materials
Diamagnetic materials are mildly repelled from magnets.

* The uncertainty in the current reading

$$u(I) = 0.012 \times I$$

* The uncertainty in the mass : $u(m) = \sqrt{0.0001^2 + \left(\frac{0.0002}{120}\cdot m\right)^2 + 0.00005^2}$

where after propagation we scale mass to [kg]

Dummy magnetization

I	u(I)	m	m	u(m)
(A)	(A)	(g)	(kg)	(kg)
5.02	0.06	-0.0017	-1.7E-06	0.11E-06
4.48	0.05	-0.0013	-1.3E-06	0.11E-06
3.99	0.05	-0.0011	-1.1E-06	0.11E-06
3.5	0.04	-0.0009	-9.0E-07	0.11E-06
2.99	0.04	-0.0007	-7.0E-07	0.11E-06
2.49	0.03	-0.0004	-4.0E-07	0.11E-06
1.98	0.02	-0.0002	-2.0E-07	0.11E-06
1.52	0.018	-0.0001	-1.0E-07	0.11E-06
0.95	0.011	0	0	0.11E-06
0	0	0	0	0.11E-06

Table 2. Empty glass tube sample data and calculated uncertainties.

29/10/2020

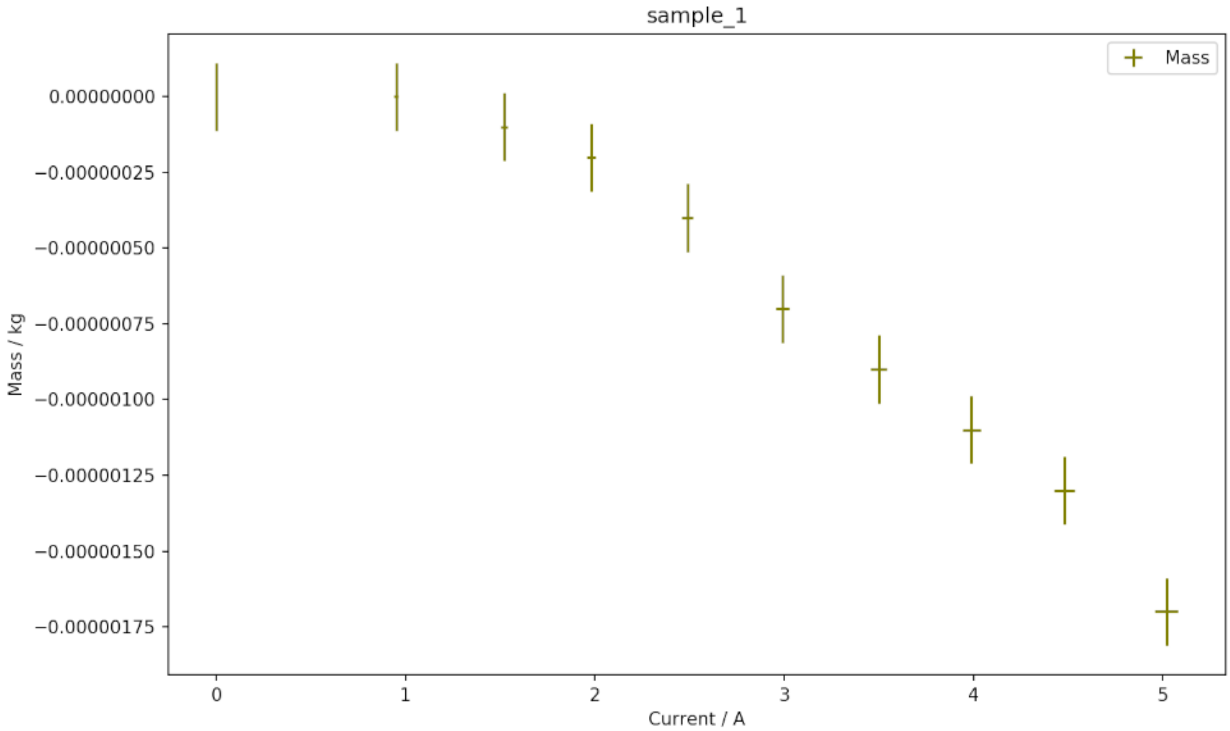


Fig 4. Empty glass tube sample data showing a negative trend for Δm.

Measurements on a Nickel magnetic salt

31/10/2020

Method:

1. Replace the dummy specimen with specimen 2
   Material : $NiSO_4 6H_2O$
2. Correct the readings for the original values obtained using the dummy specimen.

* To do this correction:

Here I use x, y as current and mass respectively

1. obtain the values for the x's and y's of the dummy sample and their respective uncertainties
2. subtract the dummy x's and ys from the x's and y's recorded for this sample.
3. propagate uncertainty on the corrected x's and y's.

(These 3 steps apply for every subsequent sample)

#2 Ni sample		**NiSO4.6H2O**			
I	u(I)	M	m	correct m	u(m)
(A)	(A)	(g)	(kg)	(kg)	(kg)
0	0	0	0	1.7E-06	0.16E-6
0.935	0.011	0.0021	2.1E-06	3.4E-06	0.16E-6
1.54	0.018	0.0057	5.7E-06	6.8E-06	0.16E-6
1.99	0.02	0.0094	9.4E-06	0.00001	0.16E-6
2.5	0.03	0.0148	0.00001	0.00002	0.16E-6
3.01	0.04	0.0215	0.00002	0.00002	0.16E-6
3.56	0.04	0.0302	0.00003	0.00003	0.16E-6
4	0.05	0.038	0.00004	0.00004	0.16E-6
4.48	0.05	0.0478	0.00005	0.00005	0.16E-6
5	0.06	0.0591	0.00006	0.00006	0.16E-6

Table 3 Specimen 2 raw data, corrected mass values (m − dummy mass) and calculated uncertainties for data

The propagation of uncertainty follows this algorithm

$$m = (m_{raw} - m_{dummy})$$

where $u(m)_{raw} = \sqrt{0.0001^2 + \left(\frac{0.002}{120} m\right)^2 + 0.00005^2}$

$\therefore u(m) = \sqrt{u(m)_{raw}^2 + u(m_{dummy})^2}$

after propagating uncertainty we convert to [kg].

Question 2

Δm is positive

from the measurements of Δm against current check that $\Delta m \propto B_1^2$:

* we know that in our experiment $B_1 \propto I$ as can be seen in fig2.

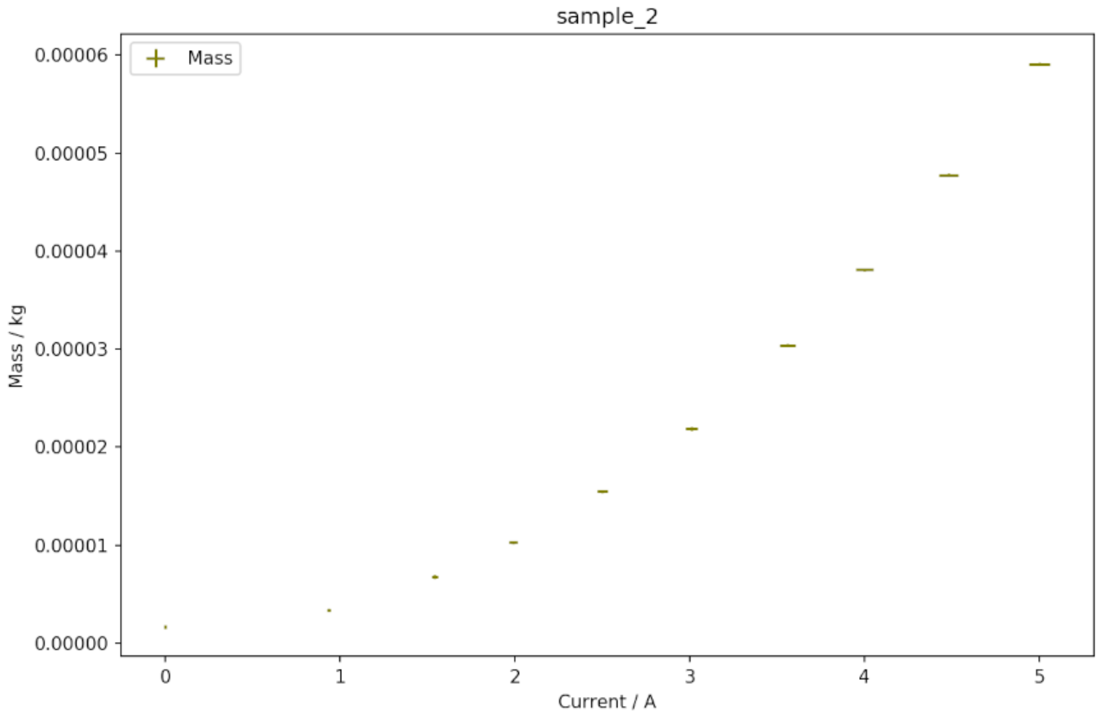


Fig 5. Specimen 2: corrected $NiSO_4$ data with uncertainties

in this plot there appears to be quadratic relationship between Δm and I

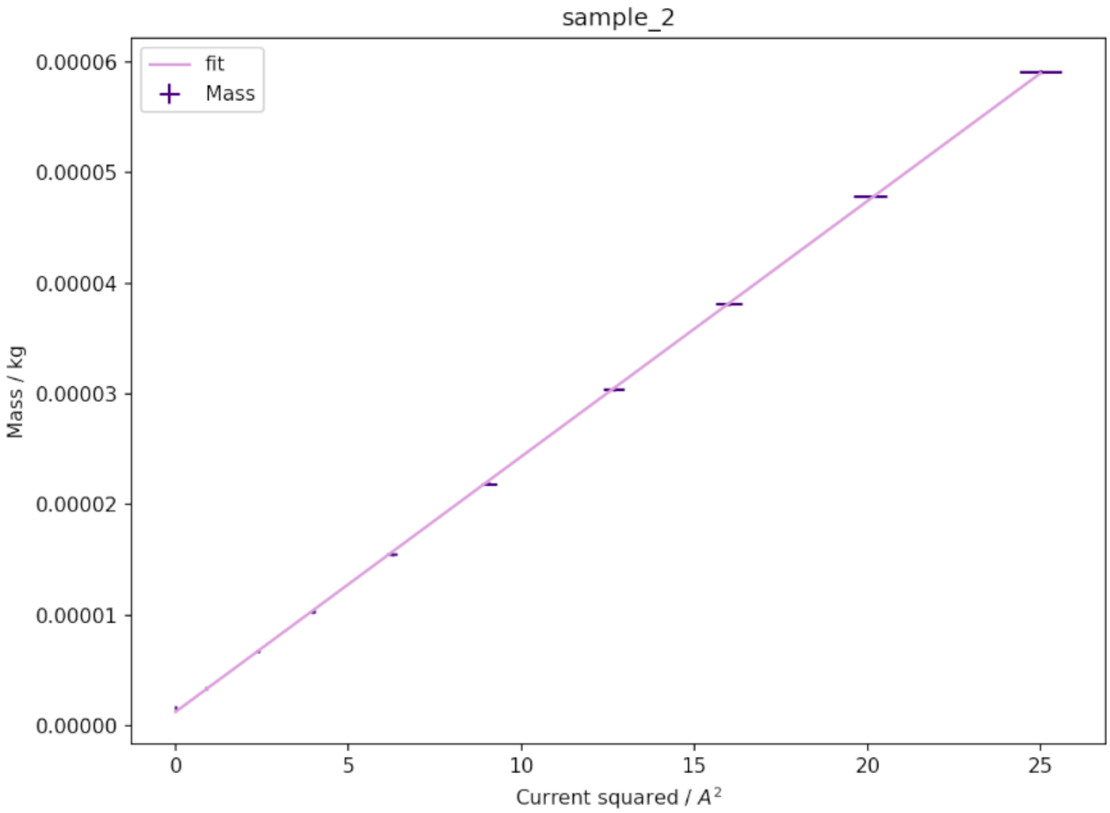


Fig 6. Plot of Δm vs I^2. from this, we conclude that $\Delta m \propto B_1^2$ which indicates that χ is field independent [0]

The fit parameters for sample 2

(m) gradient : $(2.307 \pm 0.009) \times 10^{-6}$

c) intercept : $(1.297 \pm 0.111) \times 10^{-6}$

31/10/2020

where the initial guess for the fit parameters is

```
# from eyeballing linearised magnetic sample data (I**2)
linear_guess_2 = [2e-6, 0] # [gradient, intercept]
```

Problem 3

Show that $$\chi = \frac{2\mu_0 g'}{A} \frac{\Delta m}{B_1^2}$$

from the calibration fit I found $B = mI + c$ (as above)

$\therefore B^2 = (mI + c)^2$

The force on the samples is

$$F_z = g'\Delta m, \quad \text{where } g' = 9.8\, ms^{-2}$$

and from equation (19):

$$F_z = D' B_1^2, \quad \text{where } D' = D\chi, \; D \in \mathbb{R}$$

$$\therefore g'\Delta m = D' B_1^2$$

$$\therefore g'\Delta m = D\chi (mI + c)^2$$

$$\therefore \chi = \frac{g'\Delta m}{D(mI+c)^2} \quad \square$$

from equation (18))

we deduce that $D = \frac{A}{2\mu_0}$, where A is the crossectional area of the tube holding the sample material:

we are told that $A = (38.5 \pm 0.05) 10^{-6}$ m

$$\therefore \chi = \frac{2\mu_0}{A} \frac{g'\Delta m}{(mI+c)^2}$$

Since we are after a single value of χ. we will fit equation (19) to our data for the Nickel sample. In the form:

$$\Delta m = \frac{A\chi (mI+c)^2}{2\mu_0 g'} = D(mI+c)^2, \quad \text{where } D' = \frac{A\chi}{2\mu_0 g'} =$$

Our initial guess for the fit gradient D':

```
# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

we find $D' = (0.000459 \pm 0.00004)\ s^4 A^2/kg$

We rearrange the obtained D' to find $\rightarrow \chi_{Ni} = \frac{2\mu_0 g' D}{A} = 0.000293$

propagating the uncertainty we obtain:

$$u(\chi_{Ni}) = \chi_{Ni} \sqrt{\cancel{\left(\frac{u(\mu_0)}{\mu_0}\right)^2} + \cancel{\left(\frac{u(g')}{g'}\right)^2} + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} = u(\chi_{Ni}) = \chi_{Ni} \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}$$

$$\therefore u(\chi_{Ni}) = 0.000459 \sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.00004}{0.000459}\right)^2} = 0.000003$$

$$\therefore \chi_{Ni} = 0.000293 \pm 0.000003$$

3/11/2020

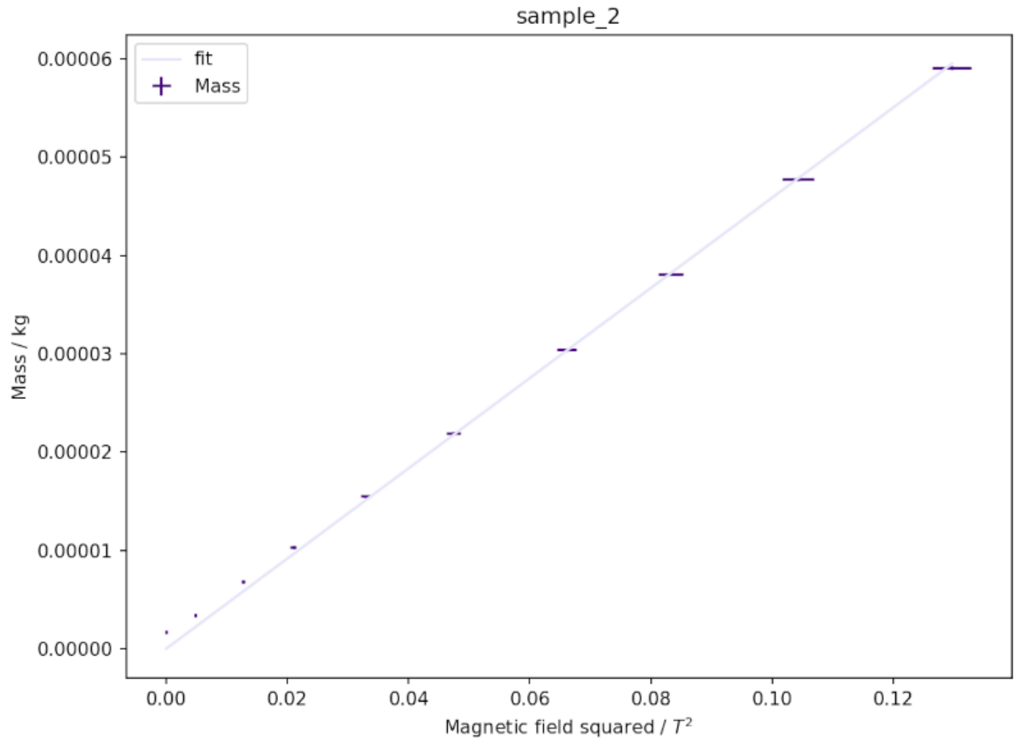


Fig 7. A plot of Δm against $D'B^2$, we found D' (the gradient) by fitting the data using equation (19)

from our Δm against B^2 graph we calculated the gradient,

$$D' = \frac{Ax}{2\mu_0 g'} = \frac{ANg^2\beta^2 J(J+1)}{6g'k_BT}$$

$$\therefore 0.000459\ s^4A^2/kg = \frac{ANg^2\beta^2 J(J+1)}{6g'k_BT}$$

Note: $N_{Ni} = \frac{N_0 \rho}{\text{molar mass}} = \frac{\#\text{molecules}}{\text{unit volume}} = \frac{\#\text{molecules}}{\text{mole}}\ \frac{\text{mole}}{\text{mass}}\ \frac{\text{mass}}{\text{unit volume}}$

$$\therefore N_{Ni} = \frac{11640.47\ kg/m^3}{2.454\times10^{-24}\ kg/\text{molecules}} = 4.74\times10^{27}\ \text{molecules}/m^3$$

from ref. [10] Hexahydrate values

$\therefore$ we determine $g^2\beta^2 J(J+1)$ for the Ni ion present in $NiSO_4 6H_2O$

$$\therefore g^2\beta^2 J(J+1) = \frac{(6g'k_BT)D'}{A\,N_{Ni}} = 5.996\times10^{-46}$$

Assuming $g=2$ and $J=S$ ie $L=0$

$$\therefore S(S+1) = \frac{1}{4\beta^2}\frac{(6g'k_BT)D'}{A}$$

$$\therefore S^2 + S - \frac{1}{4\beta^2}\frac{(6g'k_BT)D}{A} = 0$$ (we solve this as a quadratic)

we obtain $S^+ = 3.677$ or $S^- = -4.677$

propagating uncertainty in 2 steps:

1. $u(S(S+1)) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(T)}{T}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}\ (S(S+1))$

2. $u(S^+) = \frac{1}{2}\ \frac{u(S(S+1))}{S(S+1)}\ S^+$

Then our possible spins are

$S^{+} = 3.677 \pm 0.017$

$S^{-} = -4.677 \pm 0.02$

How do I decide which one is good?

Measurements on Samples 3 and 4

4/11/2020

Method:

1. Replace the $NiSO_4 6H_2O$ specimen with specimen 3
2. Correct the readings for the original values obtained using the dummy specimen.
3. Determine χ and $g^2\beta^2 J(J+1)$ and comment on the value of the magnetic quantum numbers associated with the ion present.
4. Repeat steps 1→3 for specimen 4

#3 Mn sample	MnSO4.4H2O				
I	u(I)	M	m	correct m	u(m)
(A)	(A)	(g)	(kg)	(kg)	(kg)
0	0	0	0	1.7E-06	0.16E-6
0.94	0.011	0.0075	7.5E-06	8.8E-06	0.16E-6
1.505	0.018	0.019	0.00002	0.00002	0.16E-6
1.99	0.02	0.0333	0.00003	0.00003	0.16E-6
2.51	0.03	0.053	0.00005	0.00005	0.16E-6
3.02	0.04	0.0767	0.00008	0.00008	0.16E-6
3.58	0.04	0.108	0.00011	0.00011	0.16E-6
4.01	0.05	0.1356	0.00014	0.00014	0.16E-6
4.5	0.05	0.17	0.00017	0.00017	0.16E-6
4.99	0.06	0.2088	0.00021	0.00021	0.16E-6

Table 4. Specimen 3 raw data, corrected mass values (m − dummy mass) and calculated uncertainties for data

The propagation of uncertainty follows the same algorithm as sample 2.

4 / 11/2020

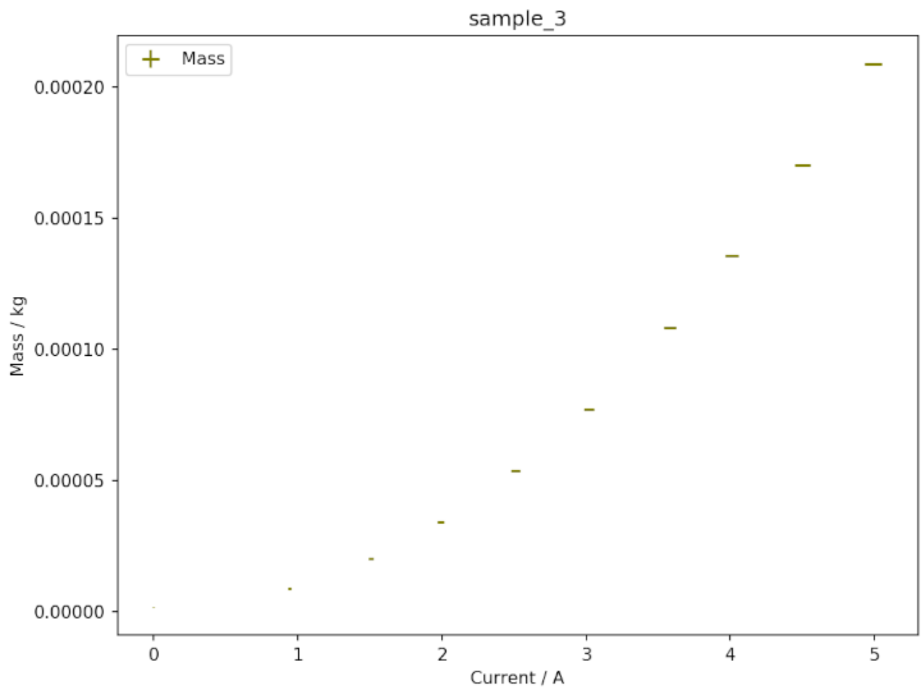


Fig 8. Specimen 3: corrected $MnSO_4 \, 4H_2O$ data with uncertainties
in this plot there appears to be quadratic relationship between Δm and I

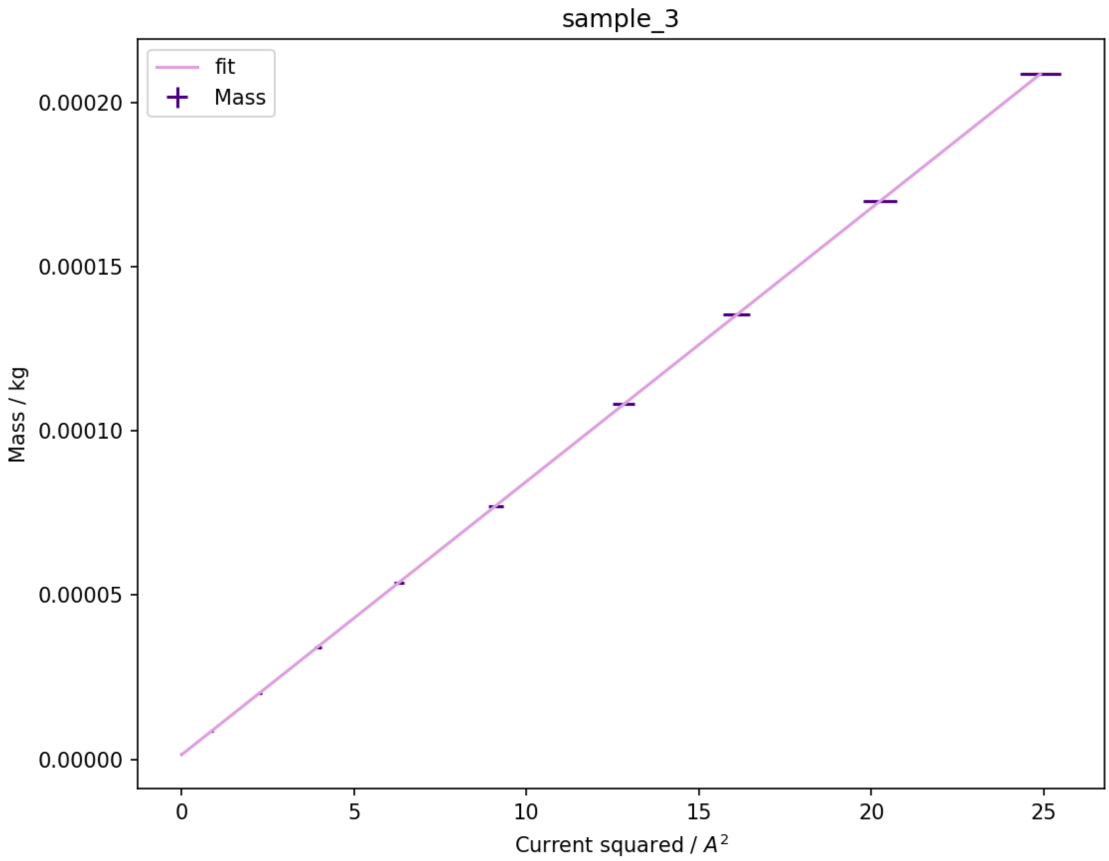


Fig 9. Plot of Δm vs I^2. from this, we conclude that $\Delta m \propto B_1^2$ which indicates that χ is field independent [0]

The fit parameters are: (m) gradient : $(8.331 \pm 0.009) \times 10^{-6}$
(c) intercept : $(1.367 \pm 0.117) \times 10^{-6}$

4/11/2020

using our new fit parameters

Once again, we fit the data with equation(19)

$$\Delta m = \frac{A\chi(mI+c)^2}{2\mu_0 g'} = D'(mI+c)^2, \quad \text{where } D' = \frac{A\chi}{2\mu_0 g'}$$

Since the same analysis applies for this data our initial guess for the fit gradient D' is once again

```
# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

This time, we find $D' = (0.001618 \pm 0.000003)\ s^4A^2/kg$

again, rearrange the obtained D' to find $\rightarrow \chi_{Mn} = \frac{2\mu_0 g' D'}{A} = 0.001035$

propagating the uncertainty we obtain:

$$u(\chi_{Mn}) = \chi_{Mn}\sqrt{\cancel{\left(\frac{u(\mu_0)}{\mu_0}\right)^2} + \cancel{\left(\frac{u(g')}{g'}\right)^2} + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} = u(\chi_{Ni}) = \chi_{Mn}\sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}$$

$$\therefore u(\chi_{Mn}) = 0.001035\sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.001618}{0.000003}\right)^2} = 0.000003$$

$$\therefore \chi_{Mn} = 0.001035 \pm 0.000003$$

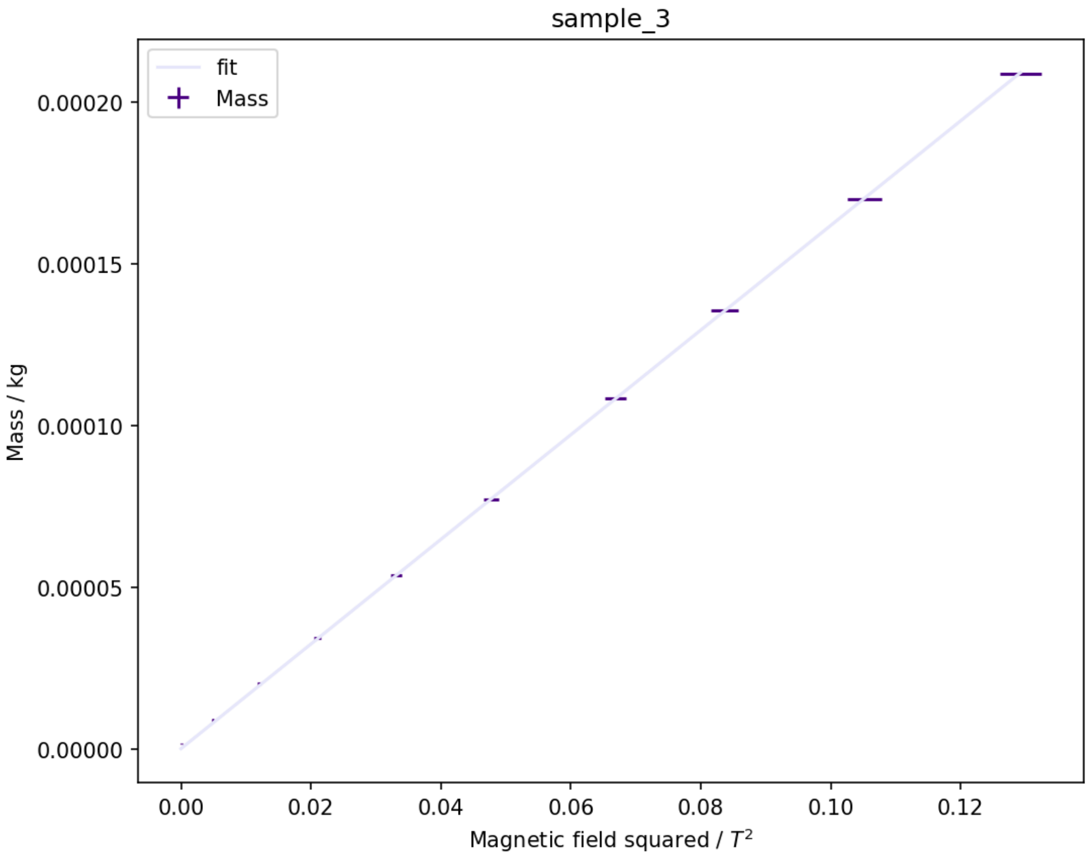


Fig10. A plot of Δm against DB^2, we found D' (the gradient) fitting the data using equation(19)

Fit parameter: $D' = (0.001618 \pm 0.000003)\ s^4A^2/kg$

4/11/2020

from our Δm against B^2 graph we calculated the gradient

$$D' = \frac{Ax}{2\mu_0 g'} = \frac{ANg^2\beta^2 J(J+1)}{6g'k_BT}$$

$$\therefore \quad 0.001618 \, s^4A^2/kg = \frac{ANg^2\beta^2 J(J+1)}{6g'k_BT}$$

Note: $N_{Mn} = \frac{N_0 \rho}{\text{molar mass}} = \frac{\#\text{molecules}}{\text{unit volume}} = \frac{\#\text{molecules}}{\text{mole}} \cdot \frac{\text{mole}}{\text{mass}} \cdot \frac{\text{mass}}{\text{unit volume}}$

from ref. [11]

(Tetrahydrate) values

$$\therefore N_{Mn} = \frac{11848.532 \ kg/m^3}{2.083\times10^{-24} \ kg/\text{molecules}} = 5.688\times10^{27} \text{ molecules}/m^3$$

$\therefore$ we determine $g^2\beta^2J(J+1)$ for the Mn ion present in $MnSO_4 4H_2O$

$$\therefore g^2\beta^2J(J+1) = \frac{(6g'k_BT)D'}{A N_{Ni}} = 1.763\times10^{-45}$$

once again, assuming $g=2$ and $J=S$ ie $L=0$

$$\therefore S(S+1) = \frac{1}{4\beta^2}\frac{(6g'k_BT)D'}{A}$$

$$\therefore S^2 + S - \frac{1}{4\beta^2}\frac{(6g'k_BT)D'}{A} = 0$$

we obtain $S^+ = 6.630$ or $S = -7.630$

propagating uncertainty in 2 steps

1. $u(S(S+1)) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(T)}{T}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}\,(S(S+1))$

2. $u(S^{\pm}) = \frac{1}{2}\,\frac{u(S(S+1))}{S(S+1)}\,S^{\pm}$

Then our possible spins are

$S^+ = 6.630 \pm 0.014$

$S^- = -7.630 \pm 0.016$

Measurements on $Co(NH_4)_2(SO_4)_2 6H_2$

5/11/2020

Method:

1. Record the results of Δm vs B^2 for specimen 5
2. Calculate χ

Note: The accepted experimental value of $g^2 J(J+1) = 23.0$
Note: This material has strongly interacting magnetic moments so that one must use the expression

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3k(T-\Theta)}$$

given this information and the details in the appendix
Determine Θ, which is a measure of this interaction

#5 Co sample		Co(NH4)2(SO4)2.6H2O			
				(m - dummy)	
I	u(I)	M	m	correct m	u(m)
(A)	(A)	(g)	(kg)	(kg)	(kg)
0	0	0	0	1.7E-06	0.16E-6
0.95	0.011	0.0024	2.4E-06	3.7E-06	0.16E-6
1.54	0.018	0.0063	6.3E-06	7.4E-06	0.16E-6
2	0.02	0.0105	0.00001	0.00001	0.16E-6
2.52	0.03	0.0167	0.00002	0.00002	0.16E-6
3.02	0.04	0.0241	0.00002	0.00002	0.16E-6
3.52	0.04	0.0325	0.00003	0.00003	0.16E-6
4.02	0.05	0.0426	0.00004	0.00004	0.16E-6
4.5	0.05	0.0533	0.00005	0.00005	0.16E-6
5.03	0.06	0.0663	0.00007	0.00007	0.16E-6

Table 5 Specimen 5 raw data, corrected mass values (m − dummy mass) and calculated uncertainties for data

The propagation of uncertainty follows the same algorithm as sample 2 and 3

5/11/2020

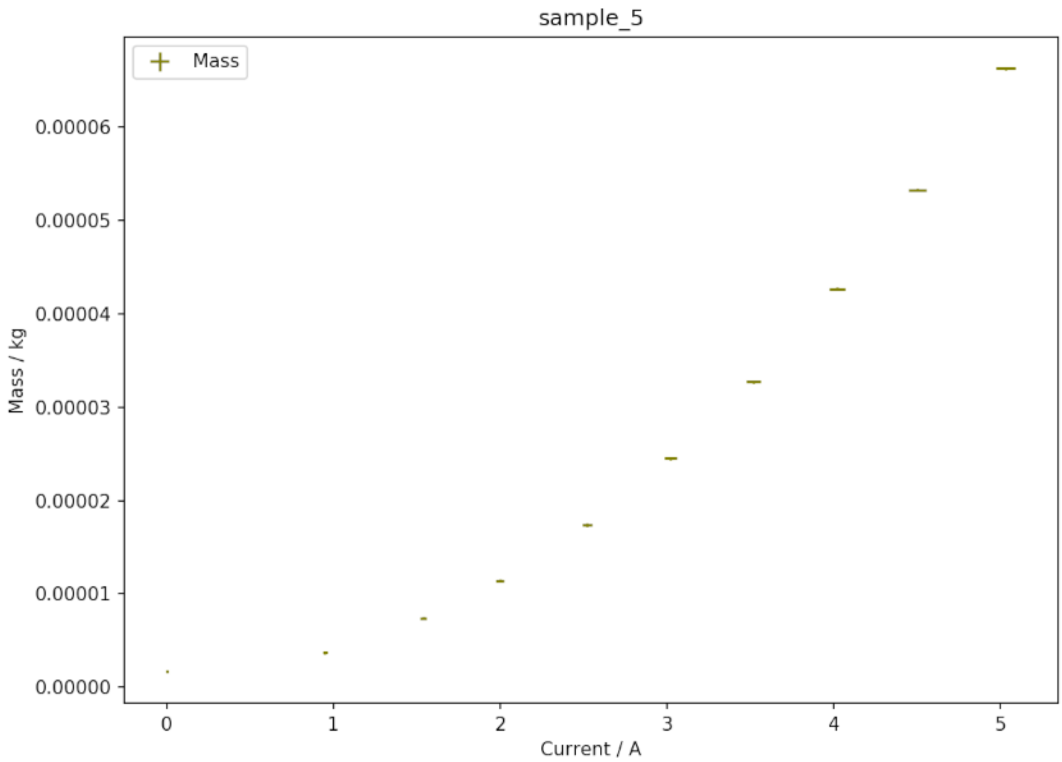


Fig 11. Specimen 4 : corrected Co(II) sulfate data with uncertainties
in this plot there appears to be quadratic relationship between Δm and I

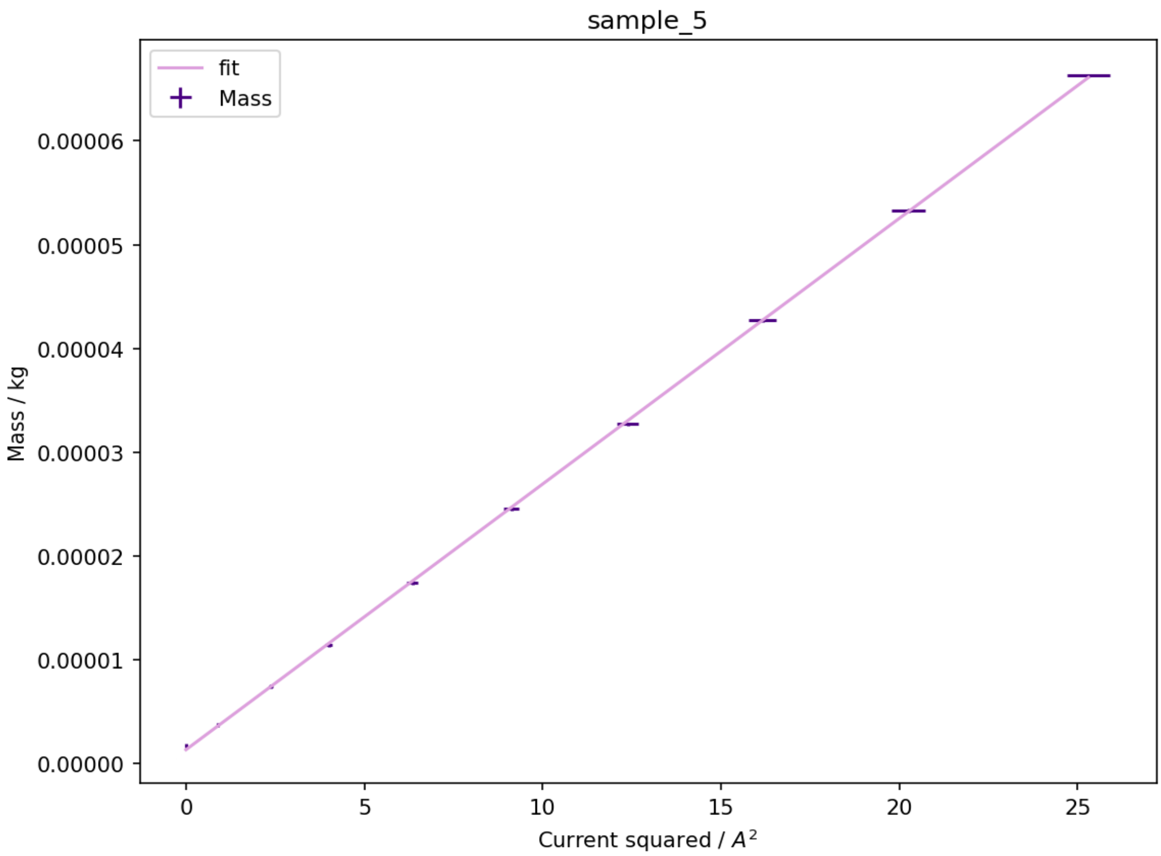


Fig 12. Plot of Δm vs I^2: from this, we conclude that $\Delta m \propto B_1^2$ which indicates that χ is field independent [0]

The fit parameters are:

(m) gradient : $(2.561 \pm 0.009) \times 10^{-6}$

(c) intercept : $(1.301 \pm 0.111) \times 10^{-6}$

5/11/2020

using our new fit parameters

Once again, we fit the data with equation(19)

$$\Delta m = \frac{A\chi(mI+c)^2}{2\mu_0 g'} = D'(mI+c)^2, \quad \text{where } D' = \frac{A\chi}{2\mu_0 g'}$$

Since the same analysis applies for this data our initial guess for the fit gradient D' is once again

```
# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient
```

This time, we find $D' = (0.000508 \pm 0.000004)\ s^4A^2/kg$

again, rearrange the obtained D' to find $\rightarrow \chi_{Mn} = \frac{2\mu_0 g' D'}{A} = 0.000325$

propagating the uncertainty we obtain:

$$u(\chi_{Mn}) = \chi_{Mn}\sqrt{\cancel{\left(\frac{u(\mu_0)}{\mu_0}\right)^2}^{\approx 0} + \cancel{\left(\frac{u(g')}{g'}\right)^2}^{\approx 0} + \left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} = u(\chi_{Ni}) = \chi_{Mn}\sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2}$$

$$\therefore u(\chi_{Mn}) = 0.000325\sqrt{\left(\frac{0.05}{38.5}\right)^2 + \left(\frac{0.000004}{0.000508}\right)^2} = 0.000003$$

$$\therefore \chi_{Co} = 0.000325 \pm 0.000003$$

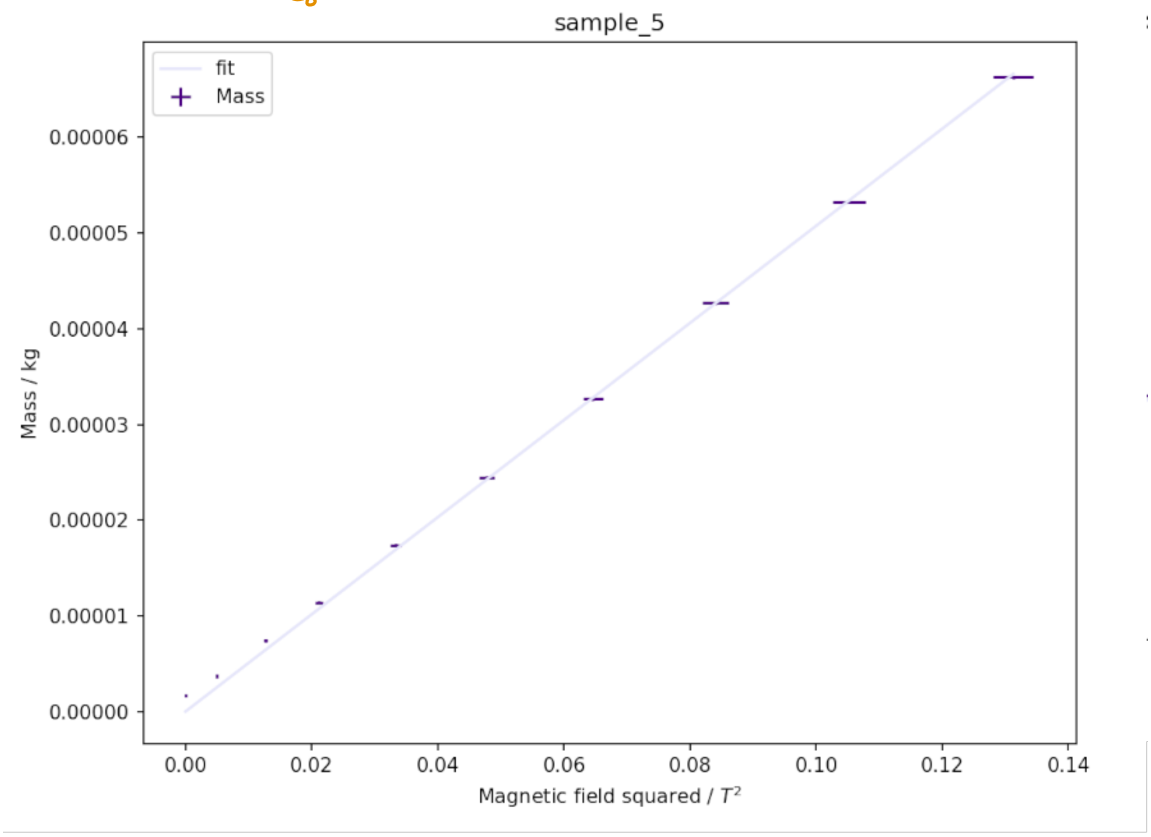


Fig13. A plot of Δm against DB^2, we found D' (the gradient) fitting the data using equation(19)

Fit parameter: $D' = 0.000508 \pm 0.000004\ s^4A^2/kg$

5/11/2020

Note: The accepted experimental value of $g^2 J(J+1) = 23.0$

Note: This material has strongly interacting magnetic moments so that one must use the expression

$$\chi = \frac{\mu_0 N g^2 \beta^2 J(J+1)}{3k_B(T-\Theta)}$$

given this information and the details in the appendix Determine Θ which is a measure of this interaction

from ref. [12] Hexahydrate values

Note: $N_{Co} = \frac{N_0 \rho}{\text{molar mass}} = \frac{\#\text{molecules}}{\text{unit volume}} = \frac{\#\text{molecules}}{\text{mole}} \frac{\text{mole}}{\text{mass}} \frac{\text{mass}}{\text{unit volume}}$

$$\therefore N_{Co} = \frac{11353.671 \text{ kg/m}^3}{2.457\times10^{-24} \text{ k/molecules}} = 4.622\times10^{27} \text{ molecules/m}^3$$

Since $\chi_{Co} = 0.000325 \pm 0.000003$

$$\therefore \Theta = T - \frac{(23.0)\mu_0 N \beta^2}{3k_B \chi_{Co}}$$

$$\therefore \Theta = T - \frac{(23.0)\mu_0 N \beta^2}{3k_B \chi_{Co}} = 207.5 \text{ K}$$

This result is probably too high

I am expecting ~ -50 K given what John Cashion posted on moodle

$$\text{let } w = \frac{(23.0)\mu_0 N \beta^2}{3k_B \chi_{Co}} = 86.53 \text{ K}$$

The uncertainty in Θ is split into 2 steps:

1. $u(w) = \sqrt{\cancel{\left(\frac{u(\beta^2)}{\beta^2}\right)^2}^{0} + \left(\frac{u(A)}{A}\right)^2 + \cancel{\left(\frac{u(N_{Co})}{N_{Co}}\right)^2}^{0} + \cancel{\left(\frac{u(k_B)}{k_B}\right)^2}^{0} + \left(\frac{u(D')}{D'}\right)^2 + \cancel{\left(\frac{u(g')}{g'}\right)^2}^{0}} \cdot (86.53\text{ K})$

$$\therefore u(w) = \sqrt{\left(\frac{u(A)}{A}\right)^2 + \left(\frac{u(D')}{D'}\right)^2} \cdot w = 0.7 \text{ K}$$

2. $u(\Theta) = \sqrt{u(T)^2 + u(w)^2} = 1.2 \text{ K}$

$$\therefore \Theta = 207.5 \pm 1.2 \text{ K}$$

a quick check: The Curie Temperature of Co is 1400 K [13]

$$\chi = \frac{C}{T-\Theta} \rightarrow \therefore 0.000325 \overset{?}{=} \frac{1400 \text{ K}}{294\text{ K} - 207.5\text{ K}}$$

$$0.000325 \neq 16.19$$

* I am aware that this is not necessarily a precise check, but I might have a units issue.

5/11/2020

Question 3

what simple interpretation can be made of materials with

1. positive Θ ?

Curie-weiss Law $\chi = \dfrac{C}{T-\Theta}$ } If $\Theta > T > 0$ we have a singularity. The susceptibility is infinite

$\therefore$ material behaves like a permanent magnet (ferromagnetic)

2. negative Θ ?

Curie-weiss Law $\chi = \dfrac{C}{T-\Theta}$ } If $\Theta < 0 < T$ behaviour of material is weakly magnetic (Antiferromagnetic)

Question 4

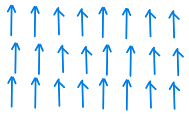
If you had the capabilities of reducing the temperature below $|\Theta|$ for a material of non-zero value of Θ. what would you expect to occur in the material? (Discuss the cases for positive & negative Θ.)

Brain Broken

4/11/2020

Problem 4

Distinguish between the following types of magnetic structure and give 2 examples of each.

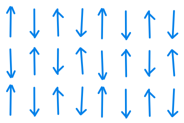


ferromagnetic ordering

1. Ferromagnetism [5]

materials can be divided into:
- Soft: not permanently magnetisable
  example: annealed iron.
- Hard: permanently magnetisable
  example: ferrite.

The atomic structure of these materials is crystalline



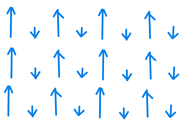
Antiferromagnetic ordering

2. Antiferromagnetism [6]

Magnetic moments of atoms or molecules align in a regular pattern with neighbouring spins in different sub-lattices.

This type of magnetism often occurs on transition metal compounds such as Hematite and Chromium.

Structures → Ising model, bipartite lattice

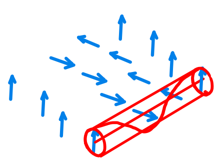


ferrimagnetic ordering

3. Ferrimagnetism [7]

Similar to antiferromagnetism but the antialigned magnetic moments are of unequal strength and so magnetisation remains active.
This occurs when populations consist of different materials or ions.

Example materials: Yttrium iron garnet (YIG)
Cubic Ferrites composed of iron oxides with other elements
Hexagonal ferrites such as $PbFe_{12}O_{19}$



Helimagnetic Ordering [9]

4. Helical (spiral) magnetism [8]

Spins of neighbouring magnetic moments align themselves in a spiral pattern with a characteristic angle in the range ($0 \to 180°$)
This results from a competition between ferromagnetic and antiferromagnetic exchange interactions.

Helical magnetism can be Right or left handed

Examples of this structure: Manganese dioxide
B20 crystal structure (Chiral cubic)

5/11/2020

Question 5

What is a Diamagnetic material and what would you expect to be the features of it's magnetic susceptibility χ_d ?

Diamagnetism is a quantum mechanical effect that occurs in all materials [4].

Diamagnetic materials are those nominally repelled by magnetic fields and do not simultaneously exhibit other kind of magnetic interactions with magnetic fields such as Paramagnetism, ferromagnetism or others [4].

The magnetic susceptibility of these materials is expected to be negative or nearly zero

Discussion

After checking my work and code several times I concluded that my uncertainty propagation for the Nickel Sample plots was adequate, even though I was very sceptical of the size of the vertical uncertainty (appearing so small)

Since I was not present in the data aquisition process, I was unable to verify how the windings of the electromagnet affect the accuracy of the measurement results

My weiss temperature was likely too high but I could not figure out the issue:
Some things that I checked where:
The molecule parameters I used where
- found on wikipedia: Cobalt (II) sulfate (Hexahydrate)

I might have a unit problem that I haven't found
- I Tried converting all the units to SI
- The problem could very well be my fits. I worked on possible issues but couldn't find the problem, I guess maybe a conceptual misunderstanding about the gradient D' I calculated.

5/11/2020

Conclusion

In this experiment I aimed to use measurements and theoretical expressions to find the spin quantum numbers of two compounds: Ni(II) sulfate: $S^+ = 3.677 \pm 0.017$ or $S^- = -4.677 \pm 0.02$ and of Mn(II) sulfate $S^+ = 6.630 \pm 0.014$ $S^- = -7.630 \pm 0.016$. In addition I calculated the magnetic susceptibility of these two compounds: $\chi_{Ni} = 0.000293 \pm 0.000003$ and $\chi_{Mn} = 0.001035 \pm 0.000003$ respectively and also the magnetic susceptibility of Co(II) Sulfate: $\chi_{Co} = 0.000325 \pm 0.000003$ and it's weiss temperature $\Theta = 207.5 \pm 1.2K$

24/10/2020

Note:

if the gradient of the graph of Δm against B^2 is written as $G\ [kg\,T^{-2}]$, M the molecular weight of the material of density $\rho\ [kg\,m^{-3}]$ and the absolute temperature $T\ [K]$

Then $$g^2 J(J+1) = 407\left(\frac{GTM}{\rho}\right)$$

Appendix [0]

Specimen details

No.	Specimen	Magnetic Ion	Molecular Weight	Density (kg/m^3)
1	Dummy	-	-	-
2	$NiSO_4.6H_2O$	Ni^{++}	262.86	1459
3	$MnSO_4 4H_2O$	Mn^{++}	223.06	1339
4	$CrK(SO_4)_2 12H_2O$	Cr^{+++}	499.4	877
5	$Co(NH_4)_2(SO_4)_2 6H_2O$	Co^{++}	395.2	1240
6	$CuSO_4.5H_2O$	Cu^{++}	249.68	1492
7	$MnSO_4.H_2O$	Mn^{++}	169.1	748

Table : From ref [0]

Note: the density values above are less than the true densities as the materials are not packed to totally exclude air.

5/11/2020

-

Logbook 4: References

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[12] Wikipedia (2020) Cobalt sulfate. Retrieved from https://en.wikipedia.org/wiki/Cobalt(II)_sulfate

[13] Wikipedia (2020) Curie Temperature. Retrieved from https://en.wikipedia.org/wiki/Curie_temperature

11/06/20 01:58:19 /home/ana/Documents/uni/PHS3000/code/log4/magnetic.py

```
# PHS3000
# Magnetic susceptibility
# Ana Fabela, 29/10/2020
import os
from pathlib import Path
import csv
import numpy as np
import matplotlib.pyplot as plt
import scipy.optimize
from physunits import *

plt.rcParams['figure.dpi'] = 150
read_folder = Path('data')
np.seterr(divide='ignore', invalid='ignore')

# Globals
# c = 299792458 # [m/s]
π = np.pi
hbar = 1.0545718e-34 * J * s
e_charge = 1.60217662e-19 * C
e_mass = 9.10938356e-31 * kg
gprime = 9.8 * m / s**2
μ0 = 1.25663706212e-6 * H / m # vacuum permeability
k = 1.38064852e-23 * m**2 * kg /(s**2 * K) # Boltzmann constant
β = e_charge * hbar / (2 * π * e_mass)

N_A = 6.0221409e+23 # Avogadro's number

# WIKIPEDIA VALUES
# hexahydrate values
M_Ni = 262.85 * g / N_A
ρ_Ni = 2.07 * g / cm**3
N_Ni = ρ_Ni / M_Ni
# tetrahydrate values
M_Mn = 223.07 * g / N_A
ρ_Mn = 1339 * g / cm**3
N_Mn = ρ_Mn / M_Mn
# hexahydrate values
M_Co = 263.08 * g / N_A
ρ_Co = 2.019 * g / cm**3
N_Co = ρ_Co / M_Co

g_S = 2
Temps = 294 * K
u_Temps = 1 * K
Area = 38.5e-6 * m**2
u_Area = 0.05e-6 * m**2

```

```python

# from eyeballing calibration data
linear_guess_0 = [0.3, 0] # [gradient, intercept]

# from eyeballing linearised magnetic sample data (I**2)
linear_guess_2 = [2e-6, 0] # [gradient, intercept]

# from eyeballing delta_m = Dprime*B**2 for magnetic sample data
linear_guess_3 = 2e-3 # gradient

def load_data(filename):
    xs = []
    ys = []
    for line in filename:
        x, y = line.split(',')
        xs.append(float(x))
        ys.append(float(y))
    return np.array(xs), np.array(ys)

def line_fit(x, m, c):
    return m * x + c

def fitting_calibration_data(xs, ys, initial_guess):
    pars, pcov = scipy.optimize.curve_fit(line_fit, xs, ys, p0=initial_guess)
    perr = np.sqrt(np.diag(pcov))
    # print(f"{pars}, {perr}\n")
    linear_fit = line_fit(xs, *pars)
    return pars, perr, linear_fit

def plot_data(xs, ys, u_xs, u_ys, fit, filename, field=False, squared=False, calibration=False):
    if calibration:
        plt.plot(xs, fit, color='teal',label=r"fit")
        plt.errorbar(xs, ys,
            xerr=u_xs,yerr=u_ys,color='orange',
            marker='None',linestyle='None', label="Magnetic field"
        )
        plt.ylabel('Magnetic field / T')
        plt.xlabel("Current / A")

    elif squared:
        # plot of data and fit of equation (19)
        plt.errorbar(xs, ys,
                    xerr=u_xs, yerr=u_ys, color='indigo',
                    marker='None',linestyle='None', label="Mass"
        )
        plt.ylabel('Mass / kg')

        if field:
```

```
            plt.plot(xs, fit, color='lavender',label=r"fit")
            plt.xlabel(r"Magnetic field squared / $T^2$")
        else:
            plt.plot(xs, fit, color='plum',label=r"fit")
            plt.xlabel(r"Current squared / $A^2$")

    else:
        plt.errorbar(xs, ys,
            xerr=u_xs, yerr=u_ys, color='olive',
            marker='None',linestyle='None', label="Mass"
        )
        plt.ylabel('Mass / kg')
        plt.xlabel("Current / A")

    plt.title(f"{filename}")
    plt.legend()
    plt.show()


def propagate_uncertainty(i, xs, ys, calibration=False):
    u_xs = []
    u_ys = []
    # print("New sample")
    for x in xs:
        u_x = 0.012 * x
        u_xs.append(u_x)
        # print(f"{u_x = }")
    if calibration:
        # uncertainty in Magnetic field measurements: using typical uncertainty
        for y in ys:
            u_y = 0.050 * y + 0.020 * 300 # [mT]
            # print(f"{u_y = }")
            u_ys.append(u_y)
    else:
        # uncertainty in mass measurements: using repeatability and linearity
        for y in ys:
            # print("New sample")
            u_y = np.sqrt(0.0001**2 + (0.0002/120 * y)**2 + 0.00005**2) # [g]
            # print(f"{u_y = }")
            u_ys.append(u_y)
    return np.array(u_xs), np.array(u_ys)

def equation_19(B, Dprime):
    return Dprime * B

def fitting_data(B_squared, ys, initial_guess):
```

```
    Dprime, pcov = scipy.optimize.curve_fit(equation_19, B_squared, ys, p0=initial_guess)
    perr = np.sqrt(np.diag(pcov))
    # print(f"{Dprime}, {perr}\n")
    eqn_19_fit = equation_19(B_squared, Dprime)
    return Dprime, perr, eqn_19_fit

def main():
    files = list(os.listdir(path=read_folder))
    files.sort(key=lambda name: int(name.strip('.csv').split('_')[-1]) if name[-5].isdigit() else -1)
    # print(files)
    file_names = []

    for i, file in enumerate(files):
        name = file.split(".")[0]
        file_names.append(name)
        # print(name)
        # print(i, file)
        file = open(read_folder / file)
        xs, ys = load_data(file)
        if i == 0:
            u_xs, u_ys = propagate_uncertainty(i, xs, ys, calibration=True)
            ys = ys * mT # convert to Teslas
            u_ys = u_ys * mT # convert to Teslas
            pars0, perr0, fit0 = fitting_calibration_data(xs, ys, linear_guess_0)
            # plot_data(xs, ys, u_xs, u_ys, fit0, name , field=False, squared=False, calibration=True)
            print(f"\nCalibration data:\nI={xs} ± {u_xs}\nB={ys} ± {u_ys}")
            print(f"\nfit parameters for calibration:\n    m = {pars0[0]:.4f} ± {perr0[0]:.4f}\n    c = {pars0[1]:.4f} ± {perr0[1]:.4f}")

        elif i == 1:
            u_xs, u_ys = propagate_uncertainty(i, xs, ys, calibration=False)
            dummy_ys = ys
            u_dummy_ys = u_ys
            kg_ys = ys / 1000
            u_kg_ys = u_ys / 1000
            print(f"\nSample {i}: I=\n{xs} ± {u_xs}\nm={ys} ± {u_ys}")
            # plot_data(xs, kg_ys, u_xs, u_kg_ys, fit0, name , field=False, squared=False, calibration=False)


```

```
        elif i > 1:
            u_xs, u_ys = propagate_uncertainty(i, xs, ys, calibration=False)
            corrected_ys = (ys - dummy_ys) # to correct for dummy values. Units: [g]

            total_ys = corrected_ys / 1000 # convert to kg
            u_total_ys = np.sqrt(u_ys**2 + u_dummy_ys**2) / 1000 # Units: [kg]
            # print(f"\nSample {i}:\nu(I)={xs} ± {u_xs}\nu(m)={u_total_ys}")

            # quadratic plot
            # plot_data(xs, total_ys, u_xs, u_total_ys, fit0, name, field=False, squared=False, calibration=False)

            u_xs_squared = []
            for x in xs:
                u_x_squared = 2 * 0.012 * x**2
                u_xs_squared.append(u_x_squared)

            # current squared - Linearised plot and fit
            pars2, perr2, fit2 = fitting_calibration_data(xs**2, total_ys, linear_guess_2)
            # plot_data(xs**2, total_ys, u_xs_squared, u_total_ys, fit2, name, field=False, squared=True, calibration=False)

            # print(f"Current squared u(I**2)={u_xs_squared}")
            print(f"\nfit parameters for sample {i}:\n    m = {pars2[0] * 1e6:.4f} ± {perr2[0] * 1e6:.4f} * 1e-6\n    c = {pars2[1]* 1e6:.4f} ± {perr2[1]* 1e6:.4f} * 1e-6")

            # calculating B and propagating uncertainty
            mI = pars0[0] * xs
            u_mI = np.sqrt((perr0[0] / pars0[0])**2 + (u_xs/ xs)**2) * mI
            B = mI + pars0[1]
            u_B = np.sqrt(u_mI**2 + perr0[1]**2)

             # squaring B and propagating uncertainty
            B_squared = (mI + pars0[1])**2
            u_B_squared = 2* u_B * B
            # print(f"Magnetic field squared u(B**2)={u_B_squared}")

            # fiting equation (19)
            Dprime, perr, eqn_19_fit = fitting_data(B_squared, total_ys, linear_guess_3)
            print(f" Equation (19) gradient:\n    D' = {Dprime[0]:.6f} ± {perr[0]:.6f}")
            χ = 2 * μ0 * gprime * Dprime[0] / Area
```

```
            u_χ = χ * np.sqrt((perr[0] / Dprime[0])**2 + (u_Area / Area)**2)
            print(f"\nMagnetic susceptibility for sample {i}:\n{χ = :.6f} ± {u_χ:.6f}")

            # plot of data and fit of equation (19)
            # plot_data(B_squared, total_ys, u_B_squared, u_total_ys, eqn_19_fit, name, field=True, squared=True, calibration=False)
            if i == 2:
                # print(f"{M_Ni=}\n{ρ_Ni=}\n{N_Ni=}\n")
                RHS = 6 * gprime  * k * Temps * Dprime / (Area * N_Ni)
                print(f"\nSample {i}g^2 * β^2 * J(J+1) = {RHS[0]}")
                SSplus1 = RHS / (4 * β**2)
                u_SSplus1 = np.sqrt((u_Temps / Temps)**2 + (u_Area / Area)**2 + (perr[0]/Dprime)**2) * SSplus1
                S_plus = (-1 + np.sqrt(1 + 4 * SSplus1 )) / 2
                S_minus = (-1 - np.sqrt(1 + 4 * SSplus1 )) / 2
                u_S_plus = (1/2 * u_SSplus1 / SSplus1) * S_plus
                u_S_minus = (1/2 * u_SSplus1 / SSplus1) * S_minus

                print(f"S = {S_plus} ± {u_S_plus} or \n    {S_minus} ± {u_S_minus}")

            elif i == 3:
                # print(f"{M_Mn=}\n{ρ_Mn=}\n{N_Mn=}\n")
                RHS = 6 * gprime  * k * Temps * Dprime / (Area * N_Mn)
                print(f"\nSample {i} g^2 * β^2 * J(J+1) = {RHS[0]}")
                SSplus1 = RHS / (4 * β**2)
                u_SSplus1 = np.sqrt((u_Temps / Temps)**2 + (u_Area / Area)**2 + (perr[0] / Dprime)**2) * SSplus1
                S_plus = (-1 + np.sqrt(1 + 4 * SSplus1 )) / 2
                S_minus = (-1 - np.sqrt(1 + 4 * SSplus1 )) / 2
                u_S_plus = (1/2 * u_SSplus1 / SSplus1) * S_plus
                u_S_minus = (1/2 * u_SSplus1 / SSplus1) * S_minus

                print(f"S = {S_plus} ± {u_S_plus} or \n    {S_minus} ± {u_S_minus}")

            elif i == 5:
                print(f"{M_Co=}\n{ρ_Co=}\n{N_Co=}\n")
                w = 23.0 * (μ0 * N_Co * β**2 / (3 * k * χ))
                u_w = w * np.sqrt((u_Area / Area)**2 + (perr[0] / Dprime)**2)
                𝚯 = Temps - w
                u_𝚯 = np.sqrt(u_Temps**2 + u_w**2)
                print(f"\nSample {i} w = {w} ± {u_w} K")
                print(f"\nWeiss temperature 𝚯 = {𝚯} ± {u_𝚯} K\n")
```

```
                    assert(0)


main()

```