Assignment A4 – Recursion

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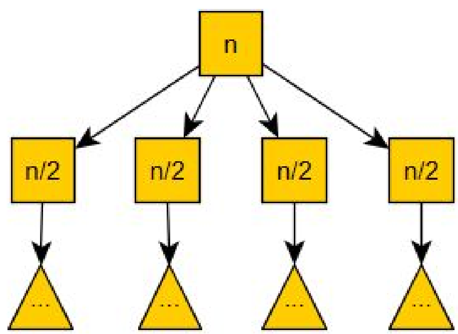
1 T(n) = 4T $(\frac{n}{2})$ + n^2lg(n)



»> Apologies for some discrepancies in the diagrams «<

height of tree is log(n)

T(n) = $\sum_{i=0}^{logn}$ n^2 log($\frac{n}{2}$i)

= n^2 $\sum_{i=0}^{logn}$ log($\frac{n}{2}$i)

= n^2 [$\sum_{i=0}^{logn}$ log n - $\sum_{i=0}^{logn}$ $log_2 i$]

= n^2 [log n * log n - $\sum_{i=0}^{logn}$ i]

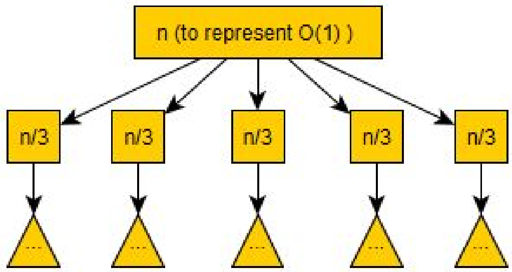
= n^2 [log^2 n - $\sum_{i=0}^{logn}$ i]

= n^2 [log^2 n - $\frac{logn(logn+1)}{2}$]

= n^2 [log^2 n - $\frac{log^2 n}{2}$ - $\frac{logn}{2}$]

T(n) = O(n^2 log^2 n)

2 T(n) = 5T $(\frac{n}{3})$ + O(1)



T(n) = O(1) + 5($\frac{n}{3}$ + 5T($\frac{n}{6}$))

= n + 5^2(5T($\frac{n}{12}$) + ($\frac{n}{6}$)) + 5($\frac{n}{3}$)

= ....

T(n) = 5^k T($\frac{n}{3^k}$) + n $\sum_{i=0}^{k=1}$ ($\frac{5}{3}^i$)

n = 3^k for k:k = log_3n

T(n) = 5^k T($\frac{3^k}{3^k}$) + n $\sum_{i=0}^{k=1}$ ($\frac{5}{3}^i$)

= n $log_3$5 + n $\Theta(\frac{5^k}{3^k})$

= $n^{log_3 5}$ + n $\Theta(\frac{5}{k})$

= $n^{log_3 5}$ + n $\Theta(n^{log_3 5})$

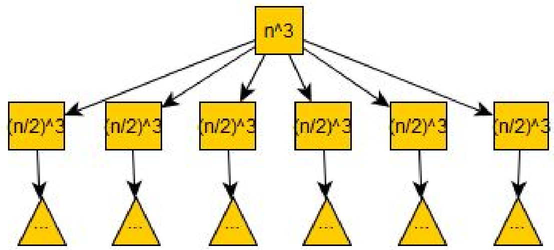
= $\Theta(n^{log_3 5})$

3 T(n) = 6T $(\frac{n}{2})$ + n^3

Subproblem size at i = $\frac{n}{2^i}$

size = 1 when 2^i = n | i = $log_2 n$

Height = $log_2 n$ + 1 | Each level has 6^i blocks Level i = $6^{log_2 n}$ = $n^{log_2 6}$

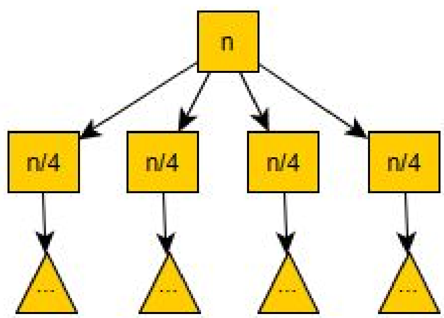


T(n) = $\sum_{i=0}^{log_2 n-1}$ $(\frac{6}{8})^i$ n^3 + $\Theta(n^{log_2 6})$ < $\sum_{i=0}^{\infty}$ $(\frac{6}{8})^i$ n^3 + $\Theta(n^{log_2 6})$

= $\frac{1}{1-(\frac{6}{8})}$ n^3 + $\Theta(n^{log_2 6})$

= O(n^3)

4 T(n) = 4T ($\frac{n}{4}$) + n



Let number of levels be from 0 extending through i with h being the final level

$\frac{n}{4^h}$ = 1 | 4^h = n | h = log_4n

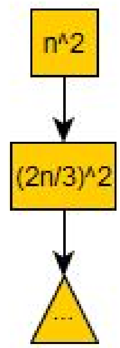
T(n) = 4^h * T(1) + $\sum_{i=0}^{h-1}$n

= $4^{log_4 n}$ T(1) + n log_4n

= n T(1) + n log_4n

= Θ(n logn)

5 T(n) = T($\frac{2n}{3}$) + n^2



Let number of levels be from 0 extending through i with h being the final level

Subproblem size at i = $\frac{2^i n}{3^i}$

$\frac{n}{(\frac{3}{2})^h}$ = 1 | h = $log_{3/2}$ n

T(n) = $\sum_{i=0}^{h-1}$ $(\frac{4}{9})^i$ n^2 + T(1) < $\sum_{i=0}^{\infty}$ $(\frac{4}{9})^i$ n^2 + T(1)

= $\frac{1}{1-\frac{4}{9}}$ n^2 + T(1)

= $\frac{9}{5}$ n^2 + T(1)

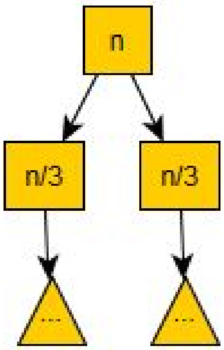
= O(n^2)

6 T(n) = 2T ($\frac{n}{3}$) + n

Let number of levels be from 0 extending through i with h being the final level

Subproblem size at i = $\frac{n}{3^i}$

$\frac{n}{3^h}$ = 1 | 3^h = n | h = log_3 n



T(n) = $2^{log_3 n}$ T(1) + $\sum_{i=0}^{h-1}$ $(\frac{2}{3})^i$n

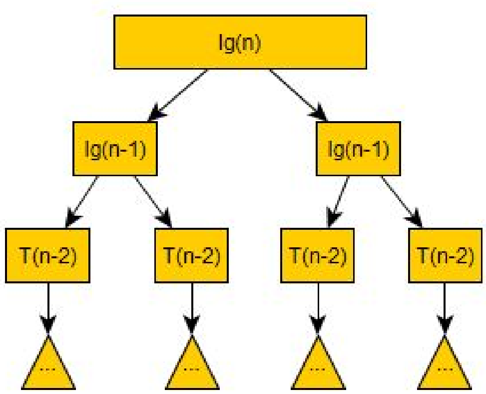
= $\Theta(n^{log_3 2})$ + $\sum_{i=0}^{h-1}$ $(\frac{2}{3})^i$n < $\Theta(n^{log_3 2})$ + $\sum_{i=0}^{\infty}$ $(\frac{2}{3})^i$n

= $\frac{1}{1-\frac{2}{3}}$n + $\Theta(n^{log_3 2})$

= 3n + $\Theta(n^{log_3 2})$

= O(n)

7 T(n) = 2T(n-1) + lg(n)



T(n) = lg(n) + 2 lg(n-1) + 2^2lg(n-2) + ... + 2^{n-2}lg(2)

<= lg(n)(1+2+2^2+2^3+....+2^{n-2})

<= (2^{n-1}-1)(lg (n))

<= O(2^nlg(n))

= Θ lg(n)

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