1. Dualling LPs [10 points]

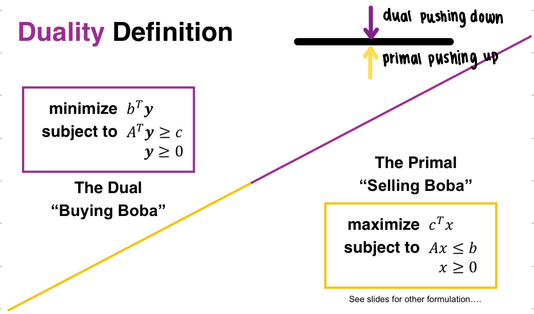
Write a linear program in standard form for the following optimization problem and then write its LP dual: Maximize $4x_1 - 3x_2 + 6x_3$ such that $4x_1 + x_2 - 4x_3 = 5$, $2x_2 - 4x_3 \geq 2$, and $x_3 + 3x_1 + 4x_2 \leq 9$, and none of the x_i are negative.

Standard Form:

maximize: $4x_1 - 3x_2 + 6x_3$

subject to:

$$4x_1 + x_2 - 4x_3 \le 5$$
$$-4x_1 - x_2 + 4x_3 \le -5$$
$$0x_1 - 2x_2 + 4x_3 \le -2$$
$$3x_1 + 4x_2 + x_3 \le 9$$
$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0$$



maximize: $[4\ \ -3\ \ 6]\ \vec{x}$

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

subject to:

$$\overbrace{\begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array}}^{A}$$
$$\begin{bmatrix} 4 & 1 & -4 \\ -4 & -1 & 4 \\ 0 & -2 & 4 \\ 3 & 4 & 1 \end{bmatrix} \vec{x} \le \overset{\vec{b}}{\begin{bmatrix} 5 \\ -5 \\ -2 \\ 9 \end{bmatrix}}$$

$$\vec{x} \ge \vec{0}$$

LP Dual:

minimize: $[5\ \ -5\ \ -2\ \ 9]\ \vec{y}$

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

subject to:

$$\overbrace{\begin{array}{cccc} y_1 & y_2 & y_3 & y_4 \\ \end{array}}^{A^T}$$
$$\begin{bmatrix} 4 & -4 & 0 & 3 \\ 1 & -1 & -2 & 4 \\ -4 & 4 & 4 & 1 \end{bmatrix} \vec{y} \ge \overset{\vec{c}}{\begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix}}$$

$$\vec{y} \ge \vec{0}$$