## 1. Find a Counterexample [10 points]

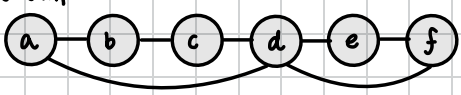
An independent set $I$ in a undirected graph $G = (V, E)$ is a subset $I \subseteq V$ of vertices such that no two vertices in $I$ are joined by an edge of $E$. Consider the following greedy algorithm to try to find a maximum size independent set which is based on the general idea that choosing vertices with small degree to be in $I$ will rule out fewer other vertices:

**function** GREEDYINDEPENDENTSET($G = (V, E)$)
  $I = \varnothing$
  **while** $G$ is not empty **do**
    Choose a vertex of smallest degree in $G$ ▷ Not counting deleted edges
    Add the vertex $v$ to $I$
    Delete $v$ and all of its neighbors and their incident edges from $G$
      ▷ None of the neighboring vertices can be included since $v$ is included
  **return** $I$

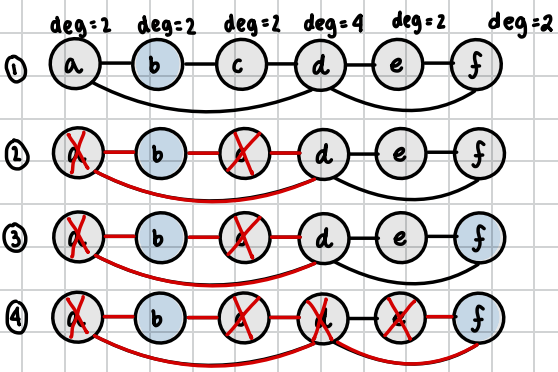
Prove that this algorithm is incorrect by counterexample. Specifically, give an example of a graph on which this algorithm does not produce a largest size independent set. Show both the independent set that the algorithm finds and a larger independent set.

Note: If you would like to see more discussion of independent sets, see section 1.2 of the text.

Counterexample:

$G =$ 


A independent set the algorithm could produce: {b,f}




An example of a larger independent set: {a,c,e}