

# 1. Dualling LPs [10 points]

Write a linear program in standard form for the following optimization problem and then write its LP dual:

Maximize  $4x_1 - 3x_2 + 6x_3$  such that  $4x_1 + x_2 - 4x_3 = 5$ ,  $2x_2 - 4x_3 \geq 2$ , and  $x_3 + 3x_1 + 4x_2 \leq 9$ , and none of the  $x_i$  are negative.

## Standard Form:

$$\text{maximize: } 4x_1 - 3x_2 + 6x_3$$

subject to:

$$4x_1 + x_2 - 4x_3 \leq 5$$

$$-4x_1 - x_2 + 4x_3 \leq -5$$

$$0x_1 - 2x_2 + 4x_3 \leq -2$$

$$3x_1 + 4x_2 + x_3 \leq 9$$

$$x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$$

$$\text{maximize: } [4 \ -3 \ 6] \vec{x}$$

subject to:

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 1 & -4 \\ -4 & -1 & 4 \\ 0 & -2 & 4 \\ 3 & 4 & 1 \end{bmatrix} \vec{x} \leq \begin{bmatrix} 5 \\ -5 \\ -2 \\ 9 \end{bmatrix}$$

$$\vec{x} \geq \vec{0}$$

## Duality Definition

$$\begin{array}{ll} \text{minimize} & b^T y \\ \text{subject to} & A^T y \geq c \\ & y \geq 0 \end{array}$$

The Dual  
"Buying Boba"

The Primal  
"Selling Boba"

$$\begin{array}{ll} \text{maximize} & c^T x \\ \text{subject to} & Ax \leq b \\ & x \geq 0 \end{array}$$

See slides for other formulation....

## LP Dual:

$$\text{minimize: } [5 \ -5 \ -2 \ 9] \vec{y}$$

subject to:

$$\vec{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -4 & 0 & 3 \\ 1 & -1 & -2 & 4 \\ -4 & 4 & 4 & 1 \end{bmatrix} \vec{y} \geq \begin{bmatrix} 4 \\ -3 \\ 6 \end{bmatrix}$$

$$\vec{y} \geq \vec{0}$$