Homework 2

Michael Anderson

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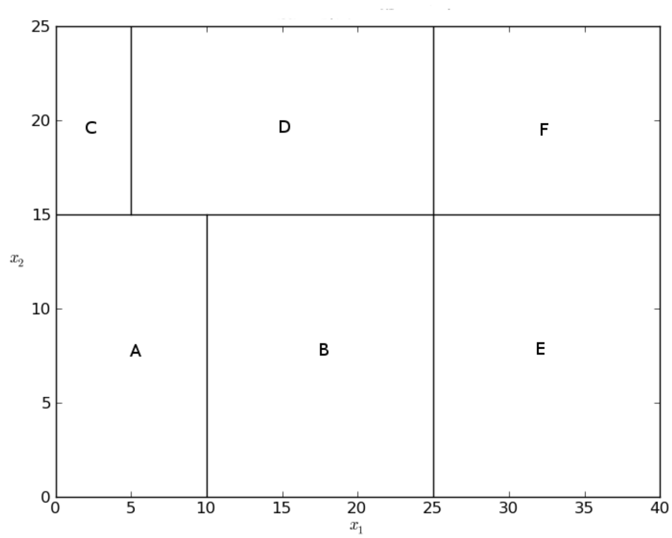
CS534

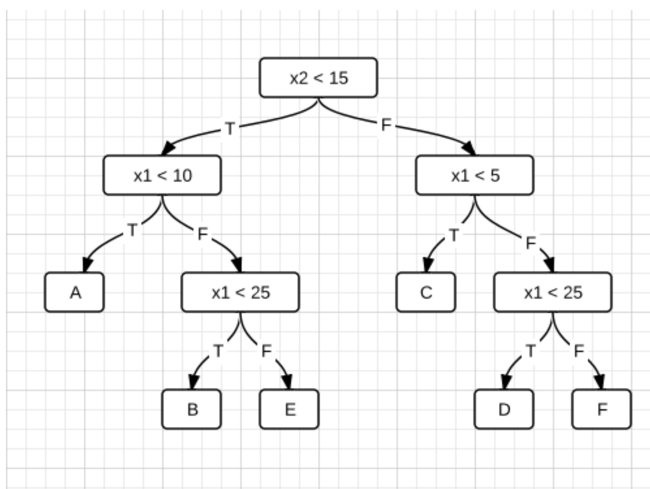
Prof. Fern

1

(a) Decision boundaries for the given tree:



(b) Different tree for same set of boundaries:



The syntactic redundancy is a computational problem, because it creates a larger search space of possible trees. We do not get anything extra of value from the larger search space, just more possibilities to wade through until we find a correct tree for the correct decision boundary.

2

(a) Suppose that some split about a feature value $X_n = x$ sends no training examples to one of its children. This means it sends all of its training examples to the other child. In terms of splitting up the examples, we are back where we started when we go to split the child with all of the training examples, and so this is equivalent to saying that the split did not decrease our uncertainty about Y, and that $H(Y) - H(Y|X_n = x) = 0$.

Since $I(Y; X_n = x) = H(Y) - H(Y|X_n = x)$, this further implies that the mutual information between the two variables is 0. Iff a split about a feature sends no training examples to one of its children, the feature has 0 mutual information with the training set; so a split about a feature that has non-zero mutual information with the training set must send at least one training example to each of its children.

(b) It is possible that each split could assign one training example to one child, and the remaining training examples to the other child, which would create a decision class for each training example. So there could be as many as m leaf nodes.

(c) It seems that here as well the upper bound is m, but I'm honestly not really sure how to compute this.

(d) According to my guess, the leaf nodes of both are upper bounded by m. It seems plausible that the maximum mutual information tree could have an upper bound $< m$. If that is the case, then the maximum mutual information tree would be more accurate because there would be fewer classification bins used to classify the training data, and a smaller chance of overfitting the data with more bins that are no more accurate.

3

(a) $P(Y = 0) = 1/2$
$P(A = 0|Y = 0) = 2/3$
$P(B = 0|Y = 0) = 1/3$
$P(C = 0|Y = 0) = 1/3$

In this particular example 0 and 1 occur equally often in each of the four variables, so I do not need to count any further.

$P(Y = 0|(A = 0, B = 0, C = 0)) = 0.5 * 2/3 * 1/3 * 1/3$
$P(Y = 1|(A = 0, B = 0, C = 0)) = 0.5 * 1/3 * 2/3 * 2/3$
$P(Y = 0|(A = 0, B = 0, C = 1)) = 0.5 * 2/3 * 1/3 * 2/3$
$P(Y = 1|(A = 0, B = 0, C = 1)) = 0.5 * 1/3 * 2/3 * 1/3$
$P(Y = 0|(A = 0, B = 1, C = 0)) = 0.5 * 2/3 * 2/3 * 1/3$
$P(Y = 1|(A = 0, B = 1, C = 0)) = 0.5 * 1/3 * 2/3 * 2/3$

$P(Y=0|(A=0,B=1,C=1)) = 0.5*2/3*2/3*2/3$
$P(Y=1|(A=0,B=1,C=1)) = 0.5*1/3*2/3*1/3$
$P(Y=0|(A=1,B=0,C=0)) = 0.5*1/3*1/3*1/3$
$P(Y=1|(A=1,B=0,C=0)) = 0.5*2/3*2/3*2/3$
$P(Y=0|(A=1,B=0,C=1)) = 0.5*1/3*1/3*2/3$
$P(Y=1|(A=1,B=0,C=1)) = 0.5*2/3*2/3*1/3$
$P(Y=0|(A=1,B=1,C=0)) = 0.5*1/3*2/3*1/3$
$P(Y=1|(A=1,B=1,C=0)) = 0.5*2/3*2/3*2/3$
$P(Y=0|(A=1,B=1,C=1)) = 0.5*1/3*2/3*2/3$
$P(Y=1|(A=1,B=1,C=1)) = 0.5*2/3*2/3*1/3$

For (A = 1, B = 0, C = 0) predict Y = 1, because $0.5(2/3)^3 > 0.5(1/3)^3$.

(b) Yes, because saying that two random variables are independent is strictly stronger than saying that they are conditionally independent. If A and B are not conditionally independent over Y, then they cannot be independent because knowing the value of one would influence the value of the other through the relationship they share with Y.

(c) We have that the feature X_k chosen at each split is given by minimizing the entropy of $(Y|X_k)$, where Y is the set of training examples given to the node to split. In other words:

$$X_k = \underset{X}{\operatorname{argmax}}[-\sum_x P(X_i = x) \sum_y P(Y=y|X_i=x) \log P(Y=y|X_i=x)]$$

For the data given:

$$H(Y|A) = H(Y|C) = -\frac{1}{2}[\frac{2}{3}log(\frac{2}{3})+\frac{1}{3}log(\frac{1}{3})]-\frac{1}{2}[\frac{1}{3}log(\frac{1}{3})+\frac{2}{3}log(\frac{2}{3})] \approx .63$$

$$H(Y|B) = -\frac{1}{3}[\frac{1}{2}log(\frac{1}{2}) + \frac{1}{2}log(\frac{1}{2})] - \frac{2}{3}[\frac{1}{3}log(\frac{1}{3}) + \frac{2}{3}log(\frac{2}{3})] \approx .69$$

$H(Y|A) = H(Y|C)$ and $H(Y|A) < H(Y|B)$, There is a tie between a split about A and C. I will arbitrarily choose a split about A.

If $A = 0$:

$$H(Y|B) = -\frac{1}{3}[0+0] - \frac{2}{3}[\frac{1}{2}log(\frac{1}{2}) + \frac{1}{2}log(\frac{1}{2})]$$

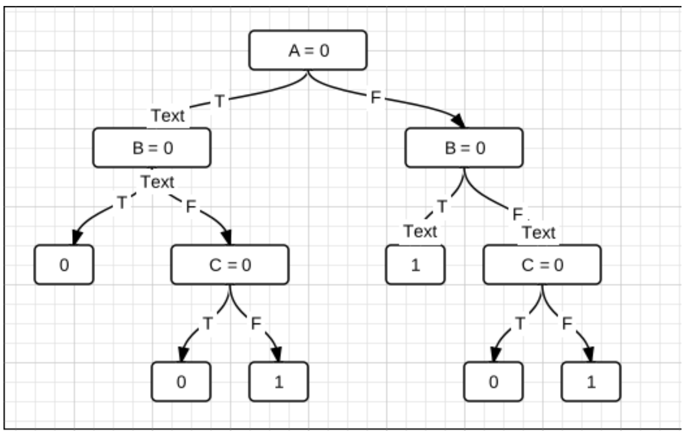
$$H(Y|C) = -\frac{2}{3}[\frac{1}{2}log(\frac{1}{2})] - \frac{1}{3}[0+0]$$

Here again the two values are completely equal, so arbitrarily split about B

Now by looking at the data we see that if $A = 0$ and $B = 0$ predict 0. If $A = 0$ and $B = 1$ then predict $\neg C$.

Still have to handle $A = 1$. Since I'm practiced at this by this point, can see from the data that $H(Y|B) = H(Y|C)$, so arbitrarily split about B.

Now by looking at the data we see that if $A = 1$ and $B = 0$ predict 1. If $A = 1$ and $B = 1$ then predict $\neg C$. So here's what the final tree looks like:



4

(a)

$$\hat{y} = \sigma(W_i \cdot A^T) = \frac{e^{x_i}}{\sum_{j=1}^{3} e^{x_j}}(W_i \cdot A_T)$$

$$J(w) = \frac{1}{2}\sum_{i=1}^{N}\left(\frac{e^{x_i}}{\sum_{j=1}^{3} e^{x_j}}(W_i \cdot A_T) - y_i\right)^2$$

(b)

$$\frac{\partial J(w)}{\partial w_{9,6}} = (\hat{y}_i - y) \cdot \frac{\partial}{\partial w_{9,6}}(\sigma(W_9 \cdot A^T) - y_i)$$

(c)

$$\frac{\partial J(w)}{\partial w_{6,3}} = (\hat{y}_i - y) \cdot \frac{\partial}{\partial w_{6,3}}(\sigma(W_6 \cdot X^T) - y_i)$$

(d)
```
epsilon <- acceptable level of error
while error > epsilon
    error = 0
```

```
    for each training example y:
        error <- error + (yHat - y)
    for each output node v:
        for each hidden node u:
            w(v,u) <- w(v,u) - nu[error([sigma]W(v)A^T) - y(i)]'
    for each hidden node j:
        w(j,v) <- w(j,v) - nu[delta(u)x(i,j)]
do until error < epsilon
```

5

$$(x^i + x^j + 1)^3 = (x^i + x^j + 1)^2 (x^i + x^j + 1) =$$

$$(x_1^{i^2} x_1^{j^2} + 2x_1^i x_2^i x_1^j x_2^j + x_2^{i^2} x_2^{j^2} + 2x_1^i x_1^j + 2x_2^i x_2^j + 1)(x^i + x^j + 1) =$$

$$x_1^{i^3} x_1^{j^3} + 3x_1^{i^2} x_2^i x_1^{j^2} x_2^j + 3x_1^{i^2} x_1^{j^2} + 3x_1^i x_2^{i^2} x_1^j x_2^{j^2} + 6x_1^i x_2^i x_1^j x_2^j + 3x_1^i x_1^j + x_2^{i^3} x_2^{j^3} + 3x_2^{i^2} x_2^{j^2} + 3x_2^i x_2^j + 1 =$$

$$(x_1^{i^3}, \sqrt{3}x_1^{i^2} x_2^i, \sqrt{3}x_1^{i^2}, \sqrt{3}x_1^i x_2^{i^2}, \sqrt{6}x_1^i x_2^i, \sqrt{3}x_1^i, x_2^{i^3}, \sqrt{3}x_2^{i^2}, \sqrt{3}x_2^i, 1) +$$

$$(x_1^{j^3}, \sqrt{3}x_1^{j^2} x_2^j, \sqrt{3}x_1^{j^2}, \sqrt{3}x_1^j x_2^{j^2}, \sqrt{6}x_1^j x_2^j, \sqrt{3}x_1^j, x_2^{j^3}, \sqrt{3}x_2^{j^2}, \sqrt{3}x_2^j, 1) =$$

$$\phi(x^i) \cdot \phi(x^j)$$