

Power Play: Probability of Default Predictive Ability Testing

Andrija Djurovic

www.linkedin.com/in/andrija-djurovic

Probability of Default Predictive Ability Tests

Three of the most commonly used tests:

- Exact binomial
- Z-score test
- Jeffreys test.

Testing the hypothesis that the calibrated probability of default (PD) is not underestimated.

Exact binomial

$$Pr(X \geq d) = \binom{n}{d} PD^d (1 - PD)^{n-d}$$

where:

- n is the number of observations
- d is the number of defaults
- PD is the calibrated PD.

Z-score test:

$$Z_{score} = \frac{ODR - PD}{\sqrt{\frac{PD(1-PD)}{n}}}$$

where:

- ODR is the observed default rate
- PD is the calibrated PD
- n is the number of observations.

Under the assumption that the Z_{score} test statistic follows the standard normal distribution, a p - *value* is calculated accordingly.

Jeffreys test

$$Pr(X \geq d) = \beta(q = PD, shape1 = d + 0.5, shape2 = n - d + 0.5)$$

where:

- β is the beta distribution
- PD is the calibrated PD
- d is the number of defaults
- n is the number of observations.

What if. . . ?

What if the tests lead to the opposite conclusion?

Example:

```
#number of defaults  
nb = 15  
#number of observations  
no = 99  
#calibrated pd  
pdc = 0.09656014  
#significance level  
alpha = 0.05
```

Testing results:

##	Exact binomial	Z-score test	Jeffreys test
## 1	5.30%	3.21%	3.87%

Monte Carlo Simulations for Statistical Power

It helps identify which test has higher statistical power.

Steps:

- 1 The observed default rate is the true calibrated PD ($ODR = nb / no$)
- 2 Simulate random numbers from the binomial distribution with parameters ODR and no
- 3 Calculate the simulated ODR and nb
- 4 Collect the results of the tests
- 5 Repeat steps 2 to 4 N times
- 6 Calculate the average number of simulations for which tests reject the null hypothesis for an alpha significance level (a higher average indicates greater power)

Simulation results:

##	Exact binomial	Z-score test	Jeffreys test
## 1	43.10%	54.17%	54.17%