Heterogeneity Shortfalls in IRB Credit Risk Models

Portfolio Returns Impact Analysis

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Model Heterogeneity

- A typical step in building credit risk models is discretizing the model output into ratings, pools, or buckets.
- Practitioners generally follow established principles for this discretization.
- These principles result in specific characteristics, some of which are mandatory, while others vary by model and are desirable but not essential.
- Monotonicity and heterogeneity are typically regarded as mandatory characteristics.
- In this context, heterogeneity refers to adequate differentiation in risk profiles across ratings, pools, or buckets. It is commonly tested in Probability of Default (PD), Loss Given Default (LGD), and Exposure at Default (EAD) models, often using tests like the two-proportion test and t-test.
- Heterogeneity is usually monitored over time, and practitioners are often challenged to assess the impact of potential heterogeneity shortfalls for thorough model validation.
- Besides the effect on the capital requirements, heterogeneity shortfall can impact the bank's pricing mehanism thus affecting the overall portfolio returns.
- The following slides first introduce the loan pricing mechanism and concept of the adverse selection. and then present simplified simulation for measuring the impact of heterogeneity shortfalls in the PD rating scale on the portfolio returns.
- Details on the economic value of the rating system can be found here.

The Loan Pricing Mechanism

Assuming the bank adopts risk-adjusted pricing, the loan pricing mechanism can be outlined as follows:

1. The bank must charge an interest rate r to cover all "general" costs unrelated to credit risk and expected losses.
2. Credit risk associated with expected losses is accounted for by adding a credit spread s.
3. The credit spread depends on the Probability of Default (PD) and the Loss Given Default (LGD) of the individual exposures. The bank receives $1 + \mathrm{r} + \mathrm{s}$ if no default occurs. If a default occurs, the bank receives $(1 + \mathrm{r} + \mathrm{s}) \cdot (1 - \mathrm{LGD})$.
4. The expected payoff of the loan must equal the risk-free payoff, leading to the equation:

$$1 + \mathrm{r} = (1 - PD) \cdot (1 + \mathrm{r} + \mathrm{s}) + PD \cdot (1 - LGD) \cdot (1 + \mathrm{r} + \mathrm{s})$$

.

5. Solving for the credit spread s gives:

$$s = (1 + \mathrm{r}) \cdot \frac{PD \cdot LGD}{1 - PD \cdot LGD}$$

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The Adverse Selection Concept

Building on the pricing mechanism outlined in the previous slide, the concept of adverse selection can be explained as follows:

1. Customers offered a spread that is too high are likely to leave the bank with a probability that depends on the magnitude m of the deviation from the spread corresponding to their true PD. The magnitude of this deviation is defined as:

$$m = s_{estimated} - s_{true} = (1+\mathrm{r}) \cdot \frac{PD_{estimated} \cdot LGD}{1 - PD_{estimated} \cdot LGD} - (1+\mathrm{r}) \cdot \frac{PD_{true} \cdot LGD}{1 - PD_{true} \cdot LGD}$$

2. Assuming $\mathrm{PD}_{\mathrm{estimated}}$ is available with a given measurement error σ, it is defined as:

$$PD_{estimated} = \frac{1}{1 + e^{-}(score_{true} + \sigma)}$$

   with the $score_{true}$ given by $ln(\frac{1-PD_{true}}{PD_{true}})$.

3. Based on customer elasticity (α) and the magnitude of the spread deviation (m), the probability of leaving can be defined as:

$$P(Leave) = 1 - e^{-\alpha \cdot \mathrm{m}}$$

   This probability models the impact of adverse selection.

4. For customers who remain with the bank after being offered the spread, the individual loan return r_i is defined as:

$$1 + \mathrm{r}_i = \begin{cases} 1 + \mathrm{r} + \mathrm{s} & : \ default = 0 \\ (1 + \mathrm{r} + \mathrm{s}) \cdot (1 - \mathrm{LGD}) & : \ default = 1 \end{cases}$$

5. Finally, given the individual customer returns, the portfolio return is calculated as:

$$r_{\mathrm{portfolio}} = \frac{1}{N} \sum_{i=1}^{N} r_i$$

   where N is the number of customers who stayed with the bank.

Simulation Setup

The following steps outline the simulation design for assessing the impact of a heterogeneity shortfall, measured by the change in portfolio returns:

1. Test the heterogeneity of adjacent ratings, pools, or buckets.
2. Identify adjacent pairs where heterogeneity testing fails.
3. Locate the pair with the closest risk profiles, indicating a lack of heterogeneity.
4. Merge the adjacent pair identified in step 4 into a single group.
5. After merging the pairs identified in step 5, calculate the weighted average calibrated PD for the merged ratings, pools, or buckets and aggregate any additional elements needed to reassess heterogeneity.
6. Reassess heterogeneity.
7. Repeat steps 3 to 6 as needed.
8. Calculate standard error of the score transformed PD values for the adjacent pairs for which heterogeneity shortfall is observed.
9. Simulate the effects of the adverse selection process adding estimation error only to the adjacent pair for which heterogeneity shortfall is observed and for the selected of general interest rate - r, customer elasticity α, LGD, calculate the average change of the portfolio returns.
10. Compare the average of the simulated portfolio returns to the expected portfolio return under the assumption of the true PDs given by the initial rating scale.

Besides comparing solely the average simulated portfolio return and the expected one, practitioners can inspect the distribution of this difference.

Simulation Results

1. PD rating scale (`rating` - rating, `no` - number of observations, `nb` - number of defaults, `pd` - calibrated PD, `odr` - observed default rate):

```
##   rating  no  nb     pd    odr
## 1    R01 170   3 0.0241 0.0176
## 2    R02 118  10 0.0937 0.0847
## 3    R03 274  47 0.1786 0.1715
## 4    R04 100  45 0.3194 0.4500
## 5    R05  91  43 0.4822 0.4725
## 6    R06 196 122 0.6277 0.6224
## 7    R07  51  44 0.8704 0.8627
```

2. Heterogeneity testing (`p-value` - p-value from the two-proportion test for adjacent ratings, `significance level` - selected test significance level, `test results` - test outcome):

```
##   rating rating.m   pd.w        p.val alpha                  res
## 1    R01      R01 0.0241           NA  0.05                 <NA>
## 2    R02      R02 0.0937 3.494655e-03  0.05  H1: DR(R02) > DR(R01)
## 3    R03      R03 0.1786 1.267879e-02  0.05  H1: DR(R03) > DR(R02)
## 4    R04      R05 0.3970 1.561927e-08  0.05  H1: DR(R04) > DR(R03)
## 5    R05      R05 0.3970 3.775379e-01  0.05 H0: DR(R05) <= DR(R04)
## 6    R06      R06 0.6277 8.407305e-03  0.05  H1: DR(R06) > DR(R05)
## 7    R07      R07 0.8704 5.645600e-04  0.05  H1: DR(R07) > DR(R06)
```

Simulation Results cont.

3. Heterogeneity testing failed for the pair R04 - R05.
4. The only pair that failed is the one with the closest risk profiles.
5. Merge ratings R04 and R05 into a single rating R05.
6. Recalculate elements needed for reassessing heterogeneity.
7. Retest heterogeneity on the rating scale with the merged ratings R04 and R05:

```
##   rating      p-value significance level          test results
## 1    R01           NA              0.05                   <NA>
## 2    R02 3.494655e-03              0.05 H1: DR(R02) > DR(R01)
## 3    R03 1.267879e-02              0.05 H1: DR(R03) > DR(R02)
## 4    R05 6.939438e-12              0.05 H1: DR(R05) > DR(R03)
## 6    R06 7.047683e-04              0.05 H1: DR(R06) > DR(R05)
## 7    R07 5.645600e-04              0.05 H1: DR(R07) > DR(R06)
```

   The heterogeneity test passes for all adjacent pairs.

8. Calculate standard error of the score transformed PD values for the adjacent pairs for which heterogeneity shortfall is observed - $\sigma = 0.34$
9. Run 1,000 simulations of the effects of the adverse selection process by adding estimation error only to the adjacent pair for which heterogeneity shortfall is observed and under assumptions of general interest rate $r = 0.03$, customer elasticity $\alpha = 500$, $LGD = 0.75$. Calculate the average change of the portfolio returns.
10. Compare the average simulated portfolio return and the expected one:
    Simulated average portfolio return: 0.0234349;
    Expected portfolio return: 0.0236373;
    Difference (in bps.): -2.02.

Simulation Results cont.