Discriminatory Power Shortfalls in IRB Credit Risk Models

Risk-Weighted Assets Impact Analysis

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Discriminatory Power in IRB Modeling

- One critical step in IRB model validation is examining the model's discriminatory power, also known as its ranking order power.
- Discriminatory power is evaluated throughout the entire model lifecycle, meaning practitioners assess it during model development, initial validation, and periodic validations.
- Often, discriminatory power is compared against a minimum threshold that the model must meet, and its changes are monitored over time to ensure no significant decline compared to the model development phase.
- The most commonly used metrics for measuring discriminatory power in IRB models are the Area Under the ROC (Receiver Operating Characteristic) Curve and Somers' D.

Consequences of Discriminatory Power Shortfalls

- Recognizing that discriminatory power is a critical aspect of model development and validation, the question often arises: What are the consequences of discriminatory power shortfalls?
- Although they heavily overlap, practitioners typically consider two main dimensions of discriminatory power shortfalls: business and regulatory.
- The business dimension often relates to adverse selection and its effect on portfolio returns, while the regulatory dimension focuses on the impact on Risk-Weighted Assets (RWA).
- Practitioners can refer to this document for further details on adverse selection and its effect on portfolio returns.
- The following slides present a simulation design for measuring the impact of discriminatory power shortfalls in the Probability of Default (PD) on RWA for a model with a discrete rating scale. Practitioners are encouraged to customize the simulation setup to reflect specific assumptions and to explore the combined effects of discriminatory power shortfalls and the potential impact of the model's lack of predictive ability.

Simulation Setup

Note:

As a general rule, improving discriminatory power is expected to decrease the RWA value. However, the opposite trend may occur for certain exposure distributions across rating grades.

Simulation Prerequisites:

- To assess discriminatory power shortfalls, practitioners must first define specific benchmarks.
- Two critical inputs are the current level of discriminatory power relative to the chosen benchmark and the definition of a "perfect" or optimal ranking order for the model.
- Validation procedures typically prescribe a minimum value for the metric used to measure discriminatory power (e.g., an AUC of 75%). Additionally, the impact of specific percentage improvements can be set as an input and used for if-else analysis.
- Practitioners can establish the best or perfect ranking order by independently sorting the target variable and model output in ascending order for the given model. This sorted order can then serve as a benchmark for evaluating the impact of improvements in discriminatory power.

Simulation Setup cont.

Simulation Steps:

Assuming practitioners measure the change in RWA based on a certain percentage of observations improving their ranking order, the following steps outline the simulation design:

1. Define benchmark inputs, including the “perfect” ranking order and the percentage of observations with improved rankings.
2. Cross-tabulate the observed rankings against the “perfect” ranking order.
3. (Optional) Define weights for each ranking mismatch based on the number of observations and the PD differences.
4. Determine the inflows and outflows for each rating based on the improved ranking order of specific observations.
5. Simulate a new allocation of observations per rating based on the calculated inflows and outflows.
6. Recalculate a simulated PD for the new allocation of observations.
7. Compute the RWA for both the original rating scale (RWA_i) and the simulated one (RWA_s).
8. Determine the RWA change using the formula: $\Delta\text{RWA} = \frac{\text{RWA}s - \text{RWA}i}{\text{RWA}_i}$.

The final step is to compare the RWA change against a predefined threshold to evaluate the significance of the discriminatory power shortfall.

Simulation Design:

The objective is to calculate the change in RWA for the rating scale below, with an AUC of 0.7942, assuming a 20% improvement in observations with mismatched ratings.

```
##                  rating   n     pd
## 1     01 (-Inf,-2.206) 223 0.0628
## 2  02 [-2.206,-1.4497) 191 0.1257
## 3 03 [-1.4497,-1.0122) 106 0.1887
## 4 04 [-1.0122,-0.1703) 230 0.3565
## 5  05 [-0.1703,0.5508) 163 0.5828
## 6      06 [0.5508,Inf)  87 0.7471
```

The simulation dataset is available here.

Simulation Results

1. The selected percentage of observations with improved ratings is 20%, while the "perfect" ranking order is summarized in the table below:

```
##          rating_best default_indicator   n
##      01 (-Inf,-2.206)            0.0000 223
##   02 [-2.206,-1.4497)            0.0000 191
##  03 [-1.4497,-1.0122)            0.0000 106
##  04 [-1.0122,-0.1703)            0.2174 230
##   05 [-0.1703,0.5508)            1.0000 163
##       06 [0.5508,Inf)            1.0000  87
```

2. Observed versus "perfect" ranking order ratings:

```
##        rating_initial          rating_best default_indicator   n
##      01 (-Inf,-2.206)     01 (-Inf,-2.206)                 0 209
##      01 (-Inf,-2.206) 04 [-1.0122,-0.1703)                 1  14
##   02 [-2.206,-1.4497)     01 (-Inf,-2.206)                 0  14
##   02 [-2.206,-1.4497)  02 [-2.206,-1.4497)                 0 153
##   02 [-2.206,-1.4497) 04 [-1.0122,-0.1703)                 1  24
##  03 [-1.4497,-1.0122)  02 [-2.206,-1.4497)                 0  38
##  03 [-1.4497,-1.0122) 03 [-1.4497,-1.0122)                 0  48
##  03 [-1.4497,-1.0122) 04 [-1.0122,-0.1703)                 1  12
##  03 [-1.4497,-1.0122)  05 [-0.1703,0.5508)                 1   8
##  04 [-1.0122,-0.1703) 03 [-1.4497,-1.0122)                 0  58
##  04 [-1.0122,-0.1703) 04 [-1.0122,-0.1703)                 0  90
##  04 [-1.0122,-0.1703)  05 [-0.1703,0.5508)                 1  82
##   05 [-0.1703,0.5508) 04 [-1.0122,-0.1703)                 0  68
##   05 [-0.1703,0.5508)  05 [-0.1703,0.5508)                 1  73
##   05 [-0.1703,0.5508)      06 [0.5508,Inf)                 1  22
##       06 [0.5508,Inf) 04 [-1.0122,-0.1703)                 0  22
##       06 [0.5508,Inf)      06 [0.5508,Inf)                 1  65
```

Simulation Results cont.

3. Weights for each ranking mismatch based on the number of observations and the PD differences.

```
##               rating           rating.a  n dr  weight
##      01 (-Inf,-2.206) 04 [-1.0122,-0.1703) 14  1 0.02102
##   02 [-2.206,-1.4497)     01 (-Inf,-2.206) 14  0 0.09819
##   02 [-2.206,-1.4497) 04 [-1.0122,-0.1703) 24  1 0.04584
##  03 [-1.4497,-1.0122)  02 [-2.206,-1.4497) 38  0 0.26589
##  03 [-1.4497,-1.0122) 04 [-1.0122,-0.1703) 12  1 0.03153
##  03 [-1.4497,-1.0122)  05 [-0.1703,0.5508)  8  1 0.00895
##  04 [-1.0122,-0.1703) 03 [-1.4497,-1.0122) 58  0 0.15239
##  04 [-1.0122,-0.1703)  05 [-0.1703,0.5508) 82  1 0.15979
##   05 [-0.1703,0.5508) 04 [-1.0122,-0.1703) 68  0 0.13251
##   05 [-0.1703,0.5508)      06 [0.5508,Inf) 22  1 0.05905
##       06 [0.5508,Inf) 04 [-1.0122,-0.1703) 22  0 0.02484
```

4. Net migrations (inflows and outflows) for each rating based on the improved ranking order of specific observations (`nt`):

```
##               rating   n    net
##      01 (-Inf,-2.206) 223   5.59
##   02 [-2.206,-1.4497) 191   8.82
##  03 [-1.4497,-1.0122) 106 -11.15
##  04 [-1.0122,-0.1703) 230  -4.09
##   05 [-0.1703,0.5508) 163  -1.65
##       06 [0.5508,Inf)  87   2.48
```

5. New allocation of observations per rating based on the net migrations:

```
##               rating    n.s
##      01 (-Inf,-2.206) 228.59
##   02 [-2.206,-1.4497) 199.82
##  03 [-1.4497,-1.0122)  94.85
##  04 [-1.0122,-0.1703) 225.91
##   05 [-0.1703,0.5508) 161.35
##       06 [0.5508,Inf)  89.48
```

Simulation Results cont.

6. Simulated PD for the new allocation of observations.

```
##                rating    n.s   pd.s
##      01 (-Inf,-2.206) 228.59 0.0546
##   02 [-2.206,-1.4497) 199.82 0.1035
##  03 [-1.4497,-1.0122)  94.85 0.1800
##  04 [-1.0122,-0.1703) 225.91 0.3433
##   05 [-0.1703,0.5508) 161.35 0.6380
##       06 [0.5508,Inf)  89.48 0.7742
```

7. The following formula gives the RWA for Revolving Retail Exposures:

$$R = 0.04$$

$$K = LGD \cdot N\left[\frac{G(PD)}{\sqrt{1-R}} + \sqrt{\frac{R}{1-R}} \cdot G(0.999)\right] - PD \cdot \mathrm{LGD}$$

$$RWA = K \cdot 12.5 \cdot EAD$$

For an LGD of 75% and an EAD equal to the number of observations, the initial and simulated RWAs are:

Initial model (RWA_i): 1730.24.
Simulated model (RWA_s): 1625.58.

8. RWA change -6.05%.

Note:

Similar results can be achieved using a simulation approach with random sampling of observations, though this method is more computationally intensive.