OLS Regression in IRB Credit Risk Modeling

From Binary to Continuous: A WoE-Equivalent Encoding Method

Andrija Djurovic

www.linkedin.com/in/andrija-djurovic

OLS Regression and Encoding Methods

- In IRB credit risk modeling, Ordinary Least Squares (OLS) regression is one of the most commonly used methods for modeling Loss Given Default (LGD) and Exposure at Default (EAD) risk parameters.
- When developing these models, practitioners often deal with a mix of numeric and categorical risk factors. Although not a mandatory step, binning is usually used as part of the risk factor engineering process for OLS regression.
- After binning, practitioners must choose an appropriate encoding method to run the OLS regression. At this stage, they typically draw a comparison with Probability of Default (PD) modeling, where Weight of Evidence (WoE) is the predominant encoding method used in conjunction with binary logistic regression. Consequently, for LGD and EAD modeling, practitioners often adopt a WoE-based approach adjusted for a continuous target.
- When WoE encoding is used in PD modeling within logistic regression, the estimated coefficients have a specific interpretation. The intercept represents the baseline default rate, while each WoE value of a risk factor, multiplied by its corresponding estimated coefficient, provides an additional contribution to the overall default rate. Since WoE is expressed in terms of log-odds, it naturally fits within the logistic regression framework.
- Given the specific interpretation of coefficients in WoE logistic regression used for PD modeling, the question arises whether the same interpretation and advantages of WoE encoding, when adjusted for a continuous target, are preserved.
- This presentation provides a brief overview of the two most commonly used encoding methods for OLS regression in LGD and EAD modeling: WoE and mean target encoding. It also introduces an adjusted version of mean target encoding that fully aligns with the interpretability of WoE encoding used in PD modeling with logistic regression.

WoE and Mean Target Encoding Methods

WoE Encoding

In LGD and EAD IRB modeling, practitioners often draw comparisons with the WoE encoding used in PD modeling. Consequently, one of the commonly applied methods is WoE encoding adjusted for a continuous target. The following formula presents the calculation of WoE values for bin `i` of a categorical risk factor:

$$WoE_i = \ln\left(\frac{\bar{y}_i}{\bar{y}}\right)$$

where $\bar{y}_i$ denotes the average value of the target for bin `i` and $\bar{y}$ is the average value of the target for the full sample.

Mean Target Encoding

Another commonly used encoding method is mean target encoding. In essence, this approach assigns to each bin of a risk factor the average value of the target within that bin. Following the notation from the formula above, mean target encoding assigns the value $\bar{y}_i$ to bin `i` of the analyzed risk factor.

Notes on Interpretation

None of the above encoding methods provides an equivalent interpretation of the estimated coefficients from OLS regression to that of WoE in logistic regression.
The WoE encoding adjusted for a continuous target reflects the deviation of the average target per bin from the overall target average, but on a logarithmic scale, which also directly affects the interpretation of the estimated intercept.
Mean target encoding does not capture the additional contribution of a specific factor to the overall target mean, and its intercept therefore differs in interpretation from that of WoE logistic regression. However, the following slides will demonstrate that OLS regression with mean target encoding produces results equivalent to those obtained using an encoding method aligned with WoE in logistic regression.

Baseline-Adjusted Mean Target Encoding Method

To adjust mean target encoding so that OLS estimates share the same interpretation as coefficients from WoE logistic regression, practitioners can simply subtract the overall sample mean of the target from the mean target value of each bin of the risk factor.

The following table shows the results of baseline-adjusted mean target encoding applied in OLS regression. The simulation was performed using the dataset available at this link, assuming that the column `lgd` represents the target variable, while the risk factors used in the OLS regression are in columns `rf_1`, `rf_2`, and `rf_3`.

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.3686     0.0104 35.4878        0
## rf_1          0.5980     0.1368  4.3699        0
## rf_2          0.5966     0.1460  4.0862        0
## rf_3          0.7083     0.1242  5.7044        0
```

Notes on Interpretation

To confirm the interpretation of the estimated intercept from the table above with that from the WoE logistic regression, practitioners can calculate the average of the lgd column, which will yield the same value. Consequently, the product of the estimated coefficient for any risk factor and its corresponding value indicates the additional contribution to the mean of the target, equivalent to the interpretation in WoE logistic regression.

Baseline-Adjusted Mean Target vs. Mean Target Encoding

Although the baseline-adjusted mean target encoding fully aligns with the interpretation of coefficients from the WoE logistic regression, a model built using this encoding yields the same results as a model built using standard mean target encoding.
The following tables present the OLS regression summaries estimated using both encoding methods.

Baseline-Adjusted Mean Target Encoding

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)   0.3686     0.0104 35.4878        0
## rf_1          0.5980     0.1368  4.3699        0
## rf_2          0.5966     0.1460  4.0862        0
## rf_3          0.7083     0.1242  5.7044        0
```

Mean Target Encoding

```
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -0.3328     0.0656 -5.0726        0
## rf_1          0.5980     0.1368  4.3699        0
## rf_2          0.5966     0.1460  4.0862        0
## rf_3          0.7083     0.1242  5.7044        0
```

Baseline-Adjusted Mean Target vs. Mean Target Encoding cont.

Comparison of the Models' Outputs

As a final step in comparing the two encoding methods, the following overview shows that the model outputs are equivalent.

Sample of Model Output with Baseline-Adjusted Mean Target Encoding:

```
##         1         2         3         4         5         6
## 0.4140793 0.4140793 0.3095987 0.4140793 0.4140793 0.1733877
```

Sample of Model Output with Mean Target Encoding:

```
##         1         2         3         4         5         6
## 0.4140793 0.4140793 0.3095987 0.4140793 0.4140793 0.1733877
```

Discusion Points

- What are the benefits of using one encoding method over the other?
- How do these two encoding methods impact the analysis of risk factor importance?