## Model Shift and Model Risk Management

#### The R Package ModelShift

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# 1 The R Package ModelShift

The ModelShift package provides a framework for quantifying model shifts in logistic regression with categorical risk factors, one of the most commonly used methods in Probability of Default (PD) modeling.

The development version is currently available on the following GitHub page.

To install the ModelShift package, practitioners can run the following command:

The core of the package is the ms.pd function, which accepts the following arguments:

- db: A data frame containing the modeling dataset.
- target: A vector specifying the name of the target (dependent) variable.
- rf: A vector of risk factor names.
- data.shift: A data frame with column names from rf and the last column named "n".
- encoding: The encoding method, with available options: "dummy" and "WoE".
- method: The model shift method, with available options: "mm" (matrix multiplication), "wbr" (weighted binomial regression), and "wfr" (weighted fractional regression).
- woe.tbl: The manually created WoE table. A data frame with three columns: "rf", "bin", and "woe". The default value is NULL..
- lr.i: The glm object of the initial model. The default is NULL. If supplied, the function uses the given model instead of estimating it.

Before presenting the simulation study and demonstrating the functionalities of the ModelShift package, the following sections provide a brief overview of the model shift concept, its applications, and different methods for its quantification.

#### 2 Model Shift

Model shift refers to the quantitative change in a model's parameters and outputs resulting from shifts in the input data. It is particularly useful in credit risk modeling for understanding how models react to changes in their underlying assumptions, data, or environment. The concept of model shift enables practitioners to:

- efficiently address "What if...?" scenarios, quantifying how data shifts impact model outputs without needing complete model redevelopment;
- provide a systematic way to assess and respond to model sensitivities and weaknesses, enhancing model validation, monitoring, and risk management.

In this framework both data and models can be presented as observed and expected counts in a high-dimensional contingency table. These in turn are converted to points in a data space of high dimensional vectors. Here a data point is defined by the proportion of the observed population in each cell, viewed as a vector of real numbers indexed by the cells. Likewise, the model point is defined by the proportion of the expected population in each cell etc. We can keep track separately of the total population size for purposes of inference, but note that the data point and model point do not vary as population size changes. The maximum likelihood construction of logistic regression model from data depends solely on the proportions.

### 3 Applications of Model Shift

Model shift in response to data shift is more than an academic exercise. It is clearly closely related to data drift and concept drift, well known concerns in model management, and it underlies modeling techniques such as imputation, bootstrapping and rebalancing. We are interested in it as a method for important analyses in modern model risk management.

The following list outlines some use cases of the model shift:

- Dynamic Model Reweighting: Enables real-time updates of model parameters as new data streams in, ensuring agility in model adjustments without waiting for periodic reviews.
- Prioritizing Validation Investigations: Quickly computes model shifts for various data shift scenarios, enabling efficient triage and focus on the most impactful concerns.
- Quantifying Business Impacts: Links data shifts to business-relevant metrics like Probability of Default (PD) in Risk-Weighted Assets (RWA), ensuring sensitivity analyses are connected to actionable outcomes.
- Sensitive Data Shift Identification: Enables the identification of data shifts that have the most significant impact on models, enriching the validation narrative with actionable insights.
- Bespoke Model Monitoring: Defines monitoring metrics for sensitive data shifts, creating early warning systems, particularly for population shifts that do not immediately affect model outputs.
- Automated Validation and Monitoring: Streamlines validation and monitoring processes, integrating them with dynamic model updates for continuous, real-time risk management.

## 4 Methods for Quantifying Model Shift

The methods for quantifying model shift are:

- 1. matrix multiplication approach;
- 2. weighted binomial logistic regression;
- 3. weighted quasi-binomial regression (weighted fractional logistic regression).

Dr. Alan Forrest proposes a first-order approximation using a matrix multiplication approach to quantify changes in model parameters directly.

Andrija Djurovic introduces two exact alternative methods (weighted binomial and fractional logistic regression) based on re-estimating model parameters, both of which can address the same task.

All three approaches are related to the widely used binomial logistic regression method with categorized risk factors commonly employed in developing PD models.

### 4.1 Matrix Multiplication Approach

The first-order model shift  $(\Delta p)$  can be explicitly represented as a matrix multiplication of the data shift. The following formulas illustrate the process of approximating parameter changes given the data shifts  $(\Delta x^+$  and  $\Delta x^-)$ :

$$\Delta p = C^{-1}D^T \left[ (I+Z)^{-1}\Delta x^+ - (I+Z^{-1})^{-1}\Delta x^- \right]$$

where:

- *D* is the design matrix;
- Y<sup>+</sup> and Y<sup>-</sup> are the diagonal matrices of modeled frequencies restricted to binary output 1 and 0, respectively;
- $Z = Y^{+}(Y^{-})^{-1}$  is diagonal matrix of modeled odds ratios;
- *I* is identity matrix dimensions nrow(Z) x nrow(Z);
- $Y = (I + Z)^{-1}(I + Z^{-1})^{-1}(Y^{+} + Y^{-});$
- $C = D^T Y D$ ;
- $\Delta x^+$  and  $\Delta x^-$  are the shifts in the proportions of input factors for binary outputs 1 and 0, respectively.

### 4.2 Weighted Binomial Logistic Regression

Another way to quantify changes in the parameters of the logistic regression based on the data shift is to re-estimate the weighted binomial logistic regression.

The following formula presents the log-likelihood function used to estimate the parameters  $(\beta)$  of the weighted logistic regression::

$$\mathcal{L}(\beta) = \sum_{i=1}^{n} w_i \left[ y_i \log \left( \frac{1}{1 + \exp(-\mathbf{x}_i \boldsymbol{\beta})} \right) + (1 - y_i) \log \left( 1 - \frac{1}{1 + \exp(-\mathbf{x}_i \boldsymbol{\beta})} \right) \right]$$

where:

- $y_i$  is the binary response variable for the i-th observation (either 0 or 1);
- $\mathbf{x}_i$  is the vector of predictors for the i-th observation;
- $w_i$  is the associated weight of the i-th observation.

#### 4.3 Weighted Quasi-Binomial Regression

The third method for quantifying changes in logistic regression parameters, based on the data shift, is by re-estimating the weighted quasi-binomial regression. Unlike binomial logistic regression, which requires a dichotomous target (0/1), weighted quasi-binomial regression processes fractions between 0 and 1. The weighted fractional logistic regression parameters can be estimated similarly to binomial logistic regression by maximizing the log-likelihood function with an additional term to account for dispersion. Since the additional term affects only the standard error of estimates, the estimated coefficients between weighted binomial and weighted quasi-binomial regression are identical.

The following formula presents the log-likelihood function used to estimate the model parameters  $(\beta)$ , along with the adjustment of the variance-covariance matrix  $(\hat{\Sigma})$  based on the dispersion parameter:

$$\mathcal{L}(\beta) = \sum_{i=1}^{n} w_i \left[ y_i \log \left( \frac{1}{1 + \exp(-\mathbf{x}_i \boldsymbol{\beta})} \right) + (1 - y_i) \log \left( 1 - \frac{1}{1 + \exp(-\mathbf{x}_i \boldsymbol{\beta})} \right) \right]$$

$$\hat{\Sigma} = \hat{\Phi}\hat{V}$$

where:

- $y_i$  is the binary response variable for the i-th observation (either 0 or 1);
- $\mathbf{x}_i$  is the vector of predictors for the i-th observation;
- $w_i$  is the associated weight of the i-th observation;
- $\hat{\Phi}$  is the estimate of the dispersion parameter;
- $\hat{V}$  is the estimated variance-covariance matrix assuming a binomial distribution (the "naive" variance-covariance matrix).

### 5 Simulation Study

The following steps outline the simulation framework used to quantify changes in model parameters based on a simulated scenario:

1. Assume a simplified PD model consisting of the target variable Creditability and two categorical risk factors: Account\_Balance and Maturity. The following R code loads the ModelShift package, imports the simulation dataset, and checks the dataset structure:

```
## 'data.frame': 1000 obs. of 3 variables:
## $ Creditability : num 0 0 0 0 0 0 0 0 0 ...
## $ Account_Balance: chr "01" "01" "02" "01" ...
## $ Maturity : chr "03 [16,36)" "02 [8,16)" "02 [8,16)" "02 [8,16)" ...
```

2. The risk factor Account\_Balance includes four categories with the following distribution of observations:

```
table(db$Account_Balance)
```

```
## ## 01 02 03 04
## 274 269 63 394
```

3. The risk factor Maturity includes five categories with the following distribution of observations:

```
table(db$Maturity)
```

```
## ## 01 (-Inf,8) 02 [8,16) 03 [16,36) 04 [36,45) 05 [45,Inf) ## 87 344 399 100 70
```

4. The final PD model is estimated using binomial logistic regression and dummy encoding in the form: Creditability ~ Account\_Balance + Maturity with the following estimated coefficients:

```
glm(formula = Creditability ~ Account_Balance + Maturity,
    family = "binomial",
    data = db)
```

##

```
## Call: glm(formula = Creditability ~ Account_Balance + Maturity, family = "binomial",
##
       data = db)
##
## Coefficients:
##
           (Intercept)
                           Account Balance02
                                                Account Balance03
##
               -1.3234
                                     -0.5064
                                                           -1.0873
                          Maturity02 [8,16)
                                               Maturity03 [16,36)
##
     Account Balance04
##
               -2.0194
                                      0.9783
                                                            1.4282
##
    Maturity04 [36,45)
                        Maturity05 [45,Inf)
##
                1.8817
                                      2.4041
##
## Degrees of Freedom: 999 Total (i.e. Null); 992 Residual
## Null Deviance:
                         1222
## Residual Deviance: 1042 AIC: 1058
```

5. Assume the following scenario: the portfolio structure changes as the bank plans to increase loan approvals for riskier groups, specifically clients in the Account\_Balance category 01, by 40%. Simultaneously, loan approvals for clients in category 04 will decrease by the same number. Given this scenario, the new allocation of Account Balance modalities is:

**Note:** The function ms.pd accepts a data frame for the data.shift argument, which contains the names of some or all risk factors used to estimate the final model. The last column must be named "n".

6. Based on this scenario and the resulting portfolio structure changes, the objective is to quantify the change in the estimated parameters of the final PD model using the three methods available in the ms.pd function.

Let's start with the matrix multiplication method to demonstrate how model shift can be quantified using the ms.pd function, following the dummy encoding presented in the simulation design above. For this method, the function outputs a list of two elements: the first element (ms) quantifies the model parameter shift, while the second element (db.s) presents a summary table used for calculating the inputs for the matrix multiplication method.

```
names(res.mm)
## [1] "ms"
              "db.s"
#parameters shift
res.mm[["ms"]]
##
           (Intercept)
                          Account_Balance02
                                               Account_Balance03
                                                                    Account_Balance04
##
           0.014977152
                                0.005604455
                                                    -0.005485861
                                                                          0.004087598
                                              Maturity04 [36,45) Maturity05 [45,Inf)
##
     Maturity02 [8,16)
                         Maturity03 [16,36)
                                                    -0.015774832
##
          -0.002716596
                               -0.015953224
                                                                          -0.092029407
```

To compare the results of the approximate matrix multiplication method with the exact methods ("wbr", "wfr") for the same simulation design, practitioners can run the code below. Unlike the "mm" method, these two methods return a list of four elements: the initial glm model object (lr.i), the simulated glm model object (lr.s), the model parameter shift (ms), and the summary table (db.s).

1. Weighted Binomial Logistic Regression:

```
res.wb <- ms.pd(db = db,
                target = "Creditability",
                rf = c("Account Balance", "Maturity"),
                data.shift = data.shift,
                encoding = "dummy",
                method = "wbr")
#output names
names(res.wb)
## [1] "lr.i" "lr.s" "ms"
                             "db.s"
#parameters shift
res.wb[["ms"]]
##
           (Intercept)
                          Account_Balance02
                                               Account_Balance03
                                                                   Account_Balance04
##
           0.015796567
                                0.005588795
                                                    -0.005248640
                                                                          0.004220945
##
     Maturity02 [8,16)
                        Maturity03 [16,36)
                                             Maturity04 [36,45) Maturity05 [45,Inf)
##
          -0.004842827
                               -0.016517199
                                                    -0.015032368
                                                                         -0.092309301
```

2. Weighted Quasi-Binomial Regression (Weighted Fractional Logistic Regression):

```
res.wf \leftarrow ms.pd(db = db,
                target = "Creditability",
                rf = c("Account_Balance", "Maturity"),
                data.shift = data.shift,
                encoding = "dummy",
                method = "wfr")
#output names
names(res.wf)
## [1] "lr.i" "lr.s" "ms"
                              "db.s"
#parameters shift
res.wf[["ms"]]
##
           (Intercept)
                          Account_Balance02
                                               Account_Balance03
                                                                    Account_Balance04
##
           0.015796567
                                0.005588795
                                                    -0.005248640
                                                                           0.004220945
##
     Maturity02 [8,16)
                         Maturity03 [16,36)
                                              Maturity04 [36,45) Maturity05 [45,Inf)
##
          -0.004842827
                               -0.016517199
                                                    -0.015032368
                                                                          -0.092309301
```

As can be seen from the above results, the first-order approximation - matrix multiplication method - produces results comparable to those of the exact methods.

The previous simulation demonstrated an example of dummy encoding, while a more common approach is Weights of Evidence (WoE) encoding for risk factors. To run the same exercise using WoE encoding, practitioners can change the encoding argument to "WoE" and compare the results. The following R code uses a different encoding method to compare the parameter shift results for the same data shift.

1. Matrix Multiplication with WoE Encoding:

2. Weighted Quasi-Binomial Regression with WoE Encoding:

The following data frame compares the results from the above runs.

```
data.frame(mm.woe = res.mm$ms,
     wf.woe = res.wf$ms)
```

The previous simulation runs assumed that WoE is calculated based on the observed data supplied in the db argument. However, practitioners often adjust WoE values during model development for specific bins or modalities of risk factors. The most common example involves adjustments for bins containing missing or special values.

The ms.pd function accepts a manually supplied WoE table to accommodate this. This table, provided via the woe.tbl argument, undergoes specific checks, so practitioners should prepare it as a data frame with three columns: "rf", "bin", and "woe".

To demonstrate this feature of the ms.pd function, we first define a function to calculate WoE, apply it to the analyzed risk factors Account\_Balance and Maturity, and then manually adjust one of the observed WoE values.

```
pct.b = nb / sum(nb, na.rm = TRUE),
                       dr = nb / no,
                      so = sum(no, na.rm = TRUE),
                      sg = sum(ng, na.rm = TRUE),
                      sb = sum(nb, na.rm = TRUE),
                      dist.g = ng / sg,
                      dist.b = nb / sb,
                      woe = log(dist.g / dist.b),
                      iv.b = (dist.g - dist.b) * woe,
                       iv.s = sum(iv.b)) %>%
               as.data.frame()
      woe.v <- tbl.s$woe
      names(woe.v) <- tbl.s$bin</pre>
      woe.trans <- unname(woe.v[db[, x]])</pre>
return(list(summary.tbl = tbl.s, x.trans = woe.trans))
}
#risk factors encoding
rf <- c("Account_Balance", "Maturity" )</pre>
rf.l <- length(rf)
woe.tbl <- vector("list", rf.1)</pre>
for (i in 1:rf.l) {
      rf.i <- rf[i]
      woe.tbl.i <- woe.calc(db = db,
                             x = rf.i,
                             y = "Creditability")
      woe.tbl[[i]] <- cbind.data.frame(rf = rf.i,</pre>
                                        woe.tbl.i[[1]][, c("bin", "woe")])
      }
woe.tbl <- bind_rows(woe.tbl)</pre>
#print observed woe table
woe.tbl
                              bin
                              01 -0.8180987
## 1 Account_Balance
## 2 Account_Balance
                               02 -0.4013918
## 3 Account_Balance
                              03 0.4054651
## 4 Account_Balance
                              04 1.1762632
## 5
            Maturity 01 (-Inf,8) 1.3121864
            Maturity
## 6
                      02 [8,16) 0.3466246
## 7
            Maturity 03 [16,36) -0.1086883
            Maturity 04 [36,45) -0.5245245
## 8
## 9
            Maturity 05 [45, Inf) -1.1349799
#manual woe adjustment
woe.tbl$woe[3] \leftarrow 0
#print adjusted woe table
woe.tbl
##
                              bin
                               01 -0.8180987
## 1 Account_Balance
## 2 Account_Balance
                               02 -0.4013918
## 3 Account_Balance
                               03 0.0000000
## 4 Account_Balance
                              04 1.1762632
## 5
            Maturity 01 (-Inf,8) 1.3121864
## 6
            Maturity 02 [8,16) 0.3466246
```

```
## 7 Maturity 03 [16,36) -0.1086883
## 8 Maturity 04 [36,45) -0.5245245
## 9 Maturity 05 [45,Inf) -1.1349799
```

With the above inputs prepared, practitioners can run the same model shift simulations and compare the results. In this run, we will again compare the matrix multiplication and the weighted quasi-binomial methods.

1. Matrix Multiplication with Manual WoE Encoding:

2. Weighted Quasi-Binomial Regression with Manual WoE Encoding:

The following data frame compares the results from the above runs.

To conclude this section, let's run another simulation for the model with multiple risk factors. For this purpose, we will import a new dataset and run the simulation. To align with the design most commonly used in practice, we will opt for "WoE" encoding, while for the data shift, we will use the same scenario as in the previous simulations.

```
1000 obs. of 7 variables:
## 'data.frame':
## $ Creditability
                                       : num 0 0 0 0 0 0 0 0 0 \dots
                                              "01" "01" "02" "01" ...
## $ Account_Balance
                                       : chr
                                             "03 [16,45)" "02 [8,16)" "02 [8,16)" "02 [8,16)" ...
## $ Duration_of_Credit__month
                                       : chr
## $ Payment_Status_of_Previous_Credit: chr
                                             "04" "04" "02" "04" ...
                                              "02" "00" "09" "00" ...
## $ Purpose
                                       : chr
                                             "01" "01" "02" "01" ...
## $ Value_Savings_Stocks
                                       : chr
```

```
## $ Length_of_current_employment : chr "02" "03" "04" "03" ...
#print data shift input
data.shift
```

1. Matrix Multiplication with WoE Encoding:

2. Weighted Quasi-Binomial Regression with WoE Encoding:

The following data frame compares the results from the above runs.

```
data.frame(mm.woe = res.mm$ms,
    wf.woe = res.wf$ms)
```

```
##
                                               mm.woe
                                                            wf.woe
## (Intercept)
                                         -0.006661372 -0.004677474
## Account_Balance.woe
                                          0.012746237 0.008321911
## Duration_of_Credit__month.woe
                                          0.022230188 0.017521925
## Payment_Status_of_Previous_Credit.woe 0.029643169 0.027173187
## Purpose.woe
                                         -0.075207284 -0.068414196
## Value_Savings_Stocks.woe
                                          0.046837955 0.038668358
## Length_of_current_employment.woe
                                         -0.171941549 -0.158032535
```

## 6 Future Development

The ModelShift package is currently in the development phase. Although the current version supports the most commonly used methods, future versions can still be improved.

One feature currently not supported by the package is data shift based on combinations of risk drivers that do not appear in the supplied modeling dataset (db). This means if such a combination exists in the dataset provided for the data.shift argument, it will be ignored when quantifying the model parameter shift.

Practitioners should also consider performance optimization when running a very high number of simulations. For example, the current version of the package allows supplying an already estimated initial model via the <code>lr.i</code> argument, which is recommended to avoid re-estimating the initial model in every simulation. Certain

conditions within the existing function implementation can be removed or adjusted depending on the design to optimize the simulation process.

Another improvement being considered for the upcoming version involves introducing statistical hypothesis testing for the shift in quantified parameters.

Additionally, similar methods can be extended to Loss Given Default (LGD) and Exposure at Default (EaD) models, which often use linear regression to estimate the risk differentiation function.

Finally, a Python version of this R package is in the pipeline and will be developed in the coming months.

Beyond these improvements, practitioners should also be aware of specific data requirements when using the existing model. For instance, the data frame supplied to the data.shift argument must contain the same modalities as those in the modeling dataset (db). Otherwise, the function will return an error.

Another aspect worth mentioning relates explicitly to the mm (matrix multiplication) method. To enable the matrix multiplication process, zero model points (the diagonal of matrices  $Y^+$  and  $Y^-$ ) were replaced with 1e-10. If practitioners prefer a different approach or wish to assess the bias introduced by this imputation step, they are encouraged to modify the source code of this method directly.