

Scorecard Scaling

Andrija Djurovic

www.linkedin.com/in/andrija-djurovic

Scorecard Scaling

The most commonly used method for developing a scorecard is logistic regression. In practice, this is closely linked to the presentation of its output through a process known as scaling. Scorecard scaling refers to transforming logistic regression outputs into a user-friendly scoring format. The objective is to map probabilities or odds into a numerical score range that is easy to interpret and use for decision-making. A standard industry approach uses a logarithmic scale, where the odds double every fixed number of points - known as "points to double the odds" (PDO).

The following equation gives the general relationship between score and odds:

$$\text{Score} = \text{Offset} + \text{Factor} \cdot \ln(\text{odds})$$

where the offset and factor are determined based on reference points chosen by the user.

For instance, if a score of 600 corresponds to odds of 50:1, and the odds double every 20 points, then the factor and offset can be calculated as:

$$\text{Factor} = \frac{\text{PDO}}{\ln(2)} = \frac{20}{\ln(2)} \approx 28.85$$

$$\text{Offset} = 600 - (28.85 \cdot \ln(50)) \approx 487.12$$

Each score in the scorecard is then derived using this formula, ensuring consistency and interpretability across different scorecards.

The example above demonstrates how to transform the model output into scores. However, practitioners often include an additional step in which each modality of a risk factor (i.e., each characteristic attribute) is assigned a specific score. The total sum of the individual modality scores should match the overall model score produced by the method described earlier. The following slides illustrate score allocation to risk factor modalities for the two most commonly used encoding methods: Weight of Evidence (WoE) and dummy encoding.

In addition, for more details on scorecard scaling using WoE encoding, practitioners can refer to Siddiqi, N. (2012). Credit Risk Scorecards: Developing and Implementing Intelligent Credit Scoring. John Wiley & Sons, Inc.

Scorecard Scaling - WoE Encoding

Since the scorecard is typically developed using WoE encoding, the final score for an applicant is determined by summing the points assigned to each modality of the risk factors in the scorecard.

By applying the Factor and Offset, points are allocated proportionally to the each modality. The final credit score is obtained by summing the points assigned to all relevant modalities. The point allocation follows the trends established in the WoE analysis. Expressed in terms of formulas, the steps below demonstrate the process of point allocation to the modalities of each risk factor:

$$\begin{aligned}\text{Score} &= \text{Offset} + \ln(\text{odds}) \cdot \text{Factor} = \\ &- \left(\sum_{j=1}^k \sum_{i=1}^n (\text{WoE}_j \cdot \beta_i) + \alpha \right) \cdot \text{Factor} + \text{Offset} = \\ &- \left(\sum_{j=1}^k \sum_{i=1}^n \left(\text{WoE}_j \cdot \beta_i + \frac{\alpha}{n} \right) \right) \cdot \text{Factor} + \text{Offset} = \\ &\sum_{j=1}^k \sum_{i=1}^n \left(-\text{WoE}_j \cdot \beta_i + \frac{\alpha}{n} \right) \cdot \text{Factor} + \frac{\text{Offset}}{n}\end{aligned}$$

Scorecard Scaling - WoE Encoding cont.

where:

- WoE_j is the Weight of Evidence for each modality;
- β_i is the regression coefficient for each risk factor;
- α denotes the intercept term from logistic regression;
- n is the number of risk factors in the model;
- k denotes the number of modalities in each risk factor;
- Factor is a scaling parameter used to adjust the scores of each modality within a given risk factor;
- Offset is the constant used to shift the score into a desired range.

Scorecard Scaling - Dummy Encoding

Unlike WoE encoding, dummy encoding produces multiple logistic regression coefficients directly associated with the modalities of the risk factors. Although the logic of point allocation remains similar, the assignment process differs slightly. Since the impact of each risk factor's reference modality is absorbed into the model's intercept, the points allocated to these reference modalities will be the same across all risk factors. Expressed in terms of formulas, the steps below demonstrate the process of point allocation to the modalities of each risk factor using dummy encoding:

$$\begin{aligned}\text{Score} &= \text{Offset} + \ln(\text{odds}) \cdot \text{Factor} = \\ &- \left(\sum_{j=1}^k \sum_{i=1}^n (D_j \cdot \beta_j) + \alpha \right) \cdot \text{Factor} + \text{Offset} = \\ &- \left(\sum_{j=1}^k \sum_{i=1}^n (D_j \cdot \beta_j + \frac{\alpha}{n}) \right) \cdot \text{Factor} + \text{Offset} = \\ &\sum_{j=1}^k \sum_{i=1}^n \left(-D_j \cdot \beta_j + \frac{\alpha}{n} \right) \cdot \text{Factor} + \frac{\text{Offset}}{n}\end{aligned}$$

Scorecard Scaling - Dummy Encoding cont.

where:

- D_j is the dummy variable indicator for modality j (1 if present, 0 otherwise);
- β_j is the regression coefficient for modality j ;
- α is the intercept term from the logistic regression;
- n is the number of risk factors in the model;
- k is the number of modalities in each risk factor;
- Factor is a scaling parameter used to adjust the scores of each modality within a given risk factor;
- Offset is the constant used to shift the score into a desired range.

Simulation Study - WoE Encoding

Assume that the model's binary target Creditability is estimated using only two categorical risk factors, Account_Balance and Maturity, with four and five unique modalities, respectively. A simulation dataset is available [here](#). Under the assumption that the model uses WoE encoding, this and the following slides present the necessary steps and elements for transforming risk factor modalities into scores.

- 1 After WoE encoding of Account_Balance and Maturity, the estimated model in the form $\text{Creditability} \sim \text{Account_Balance_WoE} + \text{Maturity_WoE}$ produces the following result:

	Estimate	Std. Error	z value	Pr(> z)
## (Intercept)	-0.8470	0.0774	-10.9364	0
## Account_Balance_WoE	-0.9913	0.0976	-10.1606	0
## Maturity_WoE	-0.9774	0.1489	-6.5664	0

- 2 Calculate the scaling inputs, Factor and Offset for a score of 600 at odds of 50:1, with 20 points to double the odds. Given these inputs, the value of the Factor is 28.85, and the Offset is 487.12.
- 3 Inputs from steps 1 and 2 are now sufficient to perform the score assignment process for each modality of the risk factors using the formula from slide 3.

	rf	bin	no	ng	nb	woe	intercept	beta	score
## 1	Account_Balance	01	274	139	135	-0.8181	-0.847	-0.9913	232
## 2	Account_Balance	02	269	164	105	-0.4014	-0.847	-0.9913	244
## 3	Account_Balance	03	63	49	14	0.4055	-0.847	-0.9913	267
## 4	Account_Balance	04	394	348	46	1.1763	-0.847	-0.9913	289
## 5	Maturity	01 (-Inf,8)	87	78	9	1.3122	-0.847	-0.9774	293
## 6	Maturity	02 [8,16)	344	264	80	0.3466	-0.847	-0.9774	266
## 7	Maturity	03 [16,36)	399	270	129	-0.1087	-0.847	-0.9774	253
## 8	Maturity	04 [36,45)	100	58	42	-0.5245	-0.847	-0.9774	241
## 9	Maturity	05 [45,Inf)	70	30	40	-1.1350	-0.847	-0.9774	224

Simulation Study - WoE Encoding cont.

The final step is to verify that the scaling has been performed correctly. This can be done by comparing the score produced directly from the model output (model's log odds) with the sum of the individual modality scores for each observation in the development sample. The table below presents the unique model outputs alongside the model scores and the corresponding sums of individual modality scores. As shown, the final scores match precisely.

##	Account_Balance	Maturity	Model_Score	Account_Balance_Score	Maturity_Score	Individual_Sum_Score
## 1	01	03 [16,36)	485	232	253	485
## 2	01	02 [8,16)	498	232	266	498
## 3	02	02 [8,16)	510	244	266	510
## 4	01	01 (-Inf,8)	525	232	293	525
## 5	04	03 [16,36)	542	289	253	542
## 6	02	03 [16,36)	497	244	253	497
## 7	02	05 [45,Inf)	468	244	224	468
## 8	02	04 [36,45)	485	244	241	485
## 9	04	02 [8,16)	555	289	266	555
## 10	03	04 [36,45)	508	267	241	508
## 11	03	03 [16,36)	520	267	253	520
## 12	04	04 [36,45)	530	289	241	530
## 13	04	05 [45,Inf)	513	289	224	513
## 14	04	01 (-Inf,8)	582	289	293	582
## 15	03	02 [8,16)	533	267	266	533
## 16	02	01 (-Inf,8)	537	244	293	537
## 17	03	01 (-Inf,8)	560	267	293	560
## 18	01	05 [45,Inf)	456	232	224	456
## 19	01	04 [36,45)	473	232	241	473

Simulation Study - Dummy Encoding

This and the next slide present a simulation study on the same dataset with two key differences. First, dummy encoding is used instead of WoE encoding. Second, the scaling inputs are set with the Factor equal to 10,000 and the Offset equal to 0. The following steps demonstrate the scaling process under these assumptions.

- 1 The estimated model in the form `Creditability ~ Account_Balance_WoE + Maturity_WoE` produces the following result:

```
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)    -1.3234    0.3720  -3.5579  0.0004
## Account_Balance02 -0.5064    0.1809  -2.7997  0.0051
## Account_Balance03 -1.0873    0.3332  -3.2629  0.0011
## Account_Balance04 -2.0194    0.2029  -9.9507  0.0000
## Maturity02 [8,16)  0.9783    0.3873   2.5262  0.0115
## Maturity03 [16,36) 1.4282    0.3809   3.7495  0.0002
## Maturity04 [36,45) 1.8817    0.4248   4.4297  0.0000
## Maturity05 [45,Inf) 2.4041    0.4491   5.3532  0.0000
```

- 2 The Factor and Offset are predefined with values of 10,000 and 0, respectively.
- 3 Inputs from steps 1 and 2 are now sufficient to perform the score assignment process for each modality of the risk factors using the formula from slide 5.

```
##           rf      bin intercept    beta  score
## 1 Account_Balance      01  -1.3234  0.0000   6617
## 2 Account_Balance      02  -1.3234 -0.5064  11681
## 3 Account_Balance      03  -1.3234 -1.0873  17490
## 4 Account_Balance      04  -1.3234 -2.0194  26811
## 5      Maturity 01 (-Inf,8)  -1.3234  0.0000   6617
## 6      Maturity 02 [8,16)  -1.3234  0.9783  -3166
## 7      Maturity 03 [16,36)  -1.3234  1.4282  -7665
## 8      Maturity 04 [36,45)  -1.3234  1.8817 -12200
## 9      Maturity 05 [45,Inf)  -1.3234  2.4041 -17424
```

Simulation Study - Dummy Encoding cont.

The final step is to verify that the scaling has been performed correctly. This can be done by comparing the score produced directly from the model output (model's log odds) with the sum of the individual modality scores for each observation in the development sample. The table below presents the unique model outputs alongside the model scores and the corresponding sums of individual modality scores. As shown, the final scores match precisely.

##	Account_Balance	Maturity	Model_Score	Account_Balance_Score	Maturity_Score	Individual_Sum_Score
## 1	01	03 [16,36)	-1048	6617	-7665	-1048
## 2	01	02 [8,16)	3451	6617	-3166	3451
## 3	02	02 [8,16)	8515	11681	-3166	8515
## 4	01	01 (-Inf,8)	13234	6617	6617	13234
## 5	04	03 [16,36)	19146	26811	-7665	19146
## 6	02	03 [16,36)	4016	11681	-7665	4016
## 7	02	05 [45,Inf)	-5743	11681	-17424	-5743
## 8	02	04 [36,45)	-519	11681	-12200	-519
## 9	04	02 [8,16)	23645	26811	-3166	23645
## 10	03	04 [36,45)	5290	17490	-12200	5290
## 11	03	03 [16,36)	9825	17490	-7665	9825
## 12	04	04 [36,45)	14611	26811	-12200	14611
## 13	04	05 [45,Inf)	9387	26811	-17424	9387
## 14	04	01 (-Inf,8)	33428	26811	6617	33428
## 15	03	02 [8,16)	14324	17490	-3166	14324
## 16	02	01 (-Inf,8)	18298	11681	6617	18298
## 17	03	01 (-Inf,8)	24107	17490	6617	24107
## 18	01	05 [45,Inf)	-10807	6617	-17424	-10807
## 19	01	04 [36,45)	-5583	6617	-12200	-5583