Somers' D for Discriminatory Power in LGD and EAD Models: D_{yx} or D_{xy}?

ECB Instructions for Reporting the Validation Results of Internal Models

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Discriminatory Power in IRB Modeling

- Testing discriminatory power is one of the key aspects of the model structure investigated by credit risk practitioners.

- In simple terms, within the context of IRB modeling, discriminatory power testing aims to assess whether the analyzed model provides meaningful differentiation between facilities or obligors, based on the levels of risk parameters assigned and observed for the validation sample subject to testing.

- Considering that the three main risk parameters - Probability of Default (PD), Loss Given Default (LGD), and Exposure At Default (EAD) - can be modeled using different types of target variables (e.g., binary or continuous), the approach to testing discriminatory power can vary.

- However, as a common practice, Area Under the Receiver Operating Curve (AUROC), Gini, and Somers' D coefficient are widely adopted as standard metrics.

- Practitioners are generally aware of the relationships between these metrics: Gini can be derived from AUROC using the formula $(2 \cdot AUROC - 1)/2$, and for a binary target, Gini and Somers' D are equivalent.

- Furthermore, Somers' D is commonly used to assess discriminatory power in LGD and EAD models because the final outputs of these risk parameters are often ordinal, with a limited number of unique values. In practice, this typically corresponds to rating grades or pools.

- For regulatory reporting but also for internal validation purposes, practitioners frequently adopt the ECB approach presented in the Instructions for Reporting the Validation Results of Internal Models.

- Given that Somers' D is an asymmetric metric, it is worth questioning whether this approach effectively reflects the intended purpose of measuring discriminatory power.

- This presentation aims to challenge the most commonly used approach for measuring discriminatory power, as presented in the ECB Instructions for Reporting the Validation Results of Internal Models, by providing a clear, evidence-based assessment of specific methodological choices. It is not intended as a criticism of any institution's or practitioner's work; rather, it seeks to promote constructive dialogue and support the continued development of robust, transparent validation practices.

Pair of Asymmetric Somers' D Coefficients

In his paper, A New Asymmetric Measure of Association for Ordinal Variables, Robert H. Somers (1962) proposed a pair of asymmetric coefficients suitable for measuring association in ordered contingency tables.

Somers differentiates between situations in which one variable is considered independent and the other dependent. For a contingency table (X, Y), the coefficient D_{yx} reflects the situation where X is the independent variable and Y is the dependent variable. Conversely, for the same table, the coefficient D_{xy} treats Y as independent and X as dependent.

The formula for calculating Somers' D_{yx} coefficient for a contingency table (X, Y) is:

$$D_{yx} = \frac{P - Q}{n^2 - \sum (n_i)^2}$$

where:

- P is twice the number of concordant pairs;
- Q is twice the number of discordant pairs;
- n is the total number of observations in the contingency table;
- n_i denotes the row marginal totals.

To calculate Somers' D_{xy}, it is sufficient to replace n_i with the column marginal totals, typically denoted as n_j.

Note

With the progress and development of IRB modeling, Somers' D has become a standard measure of discriminatory power in LGD and EAD models. Specifically, the coefficient D_{yx} is presented in the ECB Instructions for Reporting the Validation Results of Internal Models and is often considered by practitioners as the standard metric for this purpose.

However, it remains questionable whether this version of Somers' D truly reflects rank-order discrimination as intended.

Interpretations of Somers' D

Newson, R. (2006), in his paper Confidence Intervals for Rank Statistics: Somers' D and Extensions, interprets the Somers' D pair of coefficients as follows:

1. D_{yx} as an effect size, measuring the effect of X on Y;
2. D_{xy} as a predictor performance indicator, measuring the performance of X as a predictor of Y.

Given the different approaches observed in practice, starting with the construction of contingency tables for Somers' D calculation, it is often unclear to practitioners whether D_{yx} or D_{xy} should be used to measure discriminatory power in LGD and EAD models, and which of these two coefficients should be used for calculating the generalized Area Under the Receiver Operating Curve ($gAUROC$).

The following slides present a simulation study based on the approach and terminology used in the ECB Instructions for Reporting the Validation Results of Internal Models, comparing results from the ECB template with other references on using Somers' D for rank discrimination, as well as its equivalence with Gini in binary target models.

Simulation Study

Assume that the following contingency table provides an overview of the estimated (rows) and realized (columns) pools of the LGD model, for which discriminatory power is to be assessed using the Somers' D coefficient.

```
##              Realised
## Estimated    [0%,5%) [5%,10%) [10%,20%) [20%,30%) [30%,40%) [40%,50%) [50%,60%) [60%,70%) [70%,80%) [80%,90%) [90%,100%) >=100%
##    [0%,5%)     1406      130        14         2        20         1         1         0         3        57          0      0
##    [5%,10%)     584      300       320        43         8         0         0         0         0         0        240      0
##   [10%,20%)     448      500        10         6        65         0         1         0         1        56        354      0
##   [20%,30%)    1530       50       140        70        45         0         5         9        20         0          9      0
##   [30%,40%)     200      100        10        10        80         7         3        15        34         5        604      0
##   [40%,50%)     584      120       104        50        40         1         7         2        87        67        364      0
##   [50%,60%)      25       30        20        19        55         7        15        20        35        15        699      8
##   [60%,70%)     200        0        30         8        37         3         7         5        80        77       1250     32
##   [70%,80%)      54       32         0        42        10         1         1        15       600       548       2008    564
##   [80%,90%)      30        0         2         0         0         0         3         0       100       854       2564    566
##  [90%,100%)      39        0        18         0         0         0         2         9       284       798       1286     56
##     >=100%      400      338         2         0         0         0         0         0        36      1223        122    774
```

For further clarification, assume that the table above represents the results of the LGD model segmentation as proposed in the ECB Instructions:

Segment 1:	facilities i with $0\% \leq LGD_i^{E/R} < 5\%$;
Segment 2:	facilities i with $5\% \leq LGD_i^{E/R} < 10\%$;
Segment 3:	facilities i with $10\% \leq LGD_i^{E/R} < 20\%$;
	...
Segments 4–11:	10% LGD steps
	...
Segment 12:	facilities i with $100\% \leq LGD_i^{E/R}$

Simulation Study cont.

Based on the contingency table from the previous slide, the first part of the simulation will calculate and compare the version of Somers' D from the ECB MS Excel template, as outlined in the ECB Instructions, with the approach used in Kazianka, H. et al. (2021), Assessing the Discriminatory Power of Loss Given Default Models.

In the second part, the simulation will compare the ECB approach to Somers' D with the Gini coefficient from the binary target model, based on the contingency table below.

```
##                       Realised
## Estimated              0   1
##    01 (-Inf,-2.3124)   181   7
##    02 [-2.3124,-1.2111)  237  47
##    03 [-1.2111,-0.2923)  160  72
##    04 [-0.2923,0.4614)   98  94
##    05 [0.4614,Inf)       24  80
```

Simulation Results

ECB Approach for Somers' D Calculation

Along with the Instructions for Reporting the Validation Results of Internal Models, the ECB provides an MS Excel template where certain calculations of reported values can be verified. Among other checks, practitioners can also review the Somers' D calculation performed within this template.

For the contingency table based on 12 pools, reported in the template, tab 2.0 and cell range `I10:T21`, the Somers' D result is reported in cell `AS11` of the same tab. For full replicability, the ECB MS Excel template with the populated contingency table is available for download here.

As shown by the reported result of 0.3485, the ECB version of Somers' D corresponds to D_{yx}, and the generalized AUROC is accordingly reported as 0.6743.

Kazianka, H. et al. (2021) Approach for Somers' D Calculation

Kazianka, H. et al. (2021) paper is supported by the `R` package `VUROCS`, which provides, among other functions, the function `SomersD`. This function computes Somers' D and its asymptotic standard errors, given inputs `y`, a vector of realized categories, and `fx`, the predicted values from the ranking function `f`. Transforming the contingency table into a set of realized and predicted vectors and supplying them as arguments to the `SomersD` function yields a value of 0.4056, corresponding to D_{xy} and a generalized AUROC of 0.7028.

Given Newson's (2006) interpretation of the Somers' D pair of coefficients, this value of Somers' D_{xy} appears to be the appropriate coefficient for measuring rank discriminatory power in LGD and EAD models.

To perform a final check, we compare the ECB Somers' D_{yx} and Somers' D_{xy} with the Gini from the binary target model.

Simulation Results cont.

Gini vs Somers' D for the Binary Target Models

Gini: 0.5671.
D_{yx}: 0.3044.
D_{xy}: 0.5671.

Conclusion

As the approach in Kazianka, H. et al. (2021) matches the Gini from the binary target model, aligns with the interpretation of Somers' D in Newson (2006), and is consistent with Harrell, F. E. (2015), Regression Modeling Strategies, practitioners can conclude that Somers' D_{xy} is the appropriate coefficient for measuring rank discriminatory power in LGD and EAD models.

Selected References

European Central Bank (2019). Instructions for Reporting the Validation Results of Internal Models: IRB Pillar i Models for Credit Risk

Newson, R. (2006). Confidence Intervals for Rank Statistics: Somers' D and Extensions. The Stata Journal, 6(3), 309-334.

Kazianka, H., Morgenbesser, A., & Nowak, T. (2021). Assessing the discriminatory power of loss given default models. Journal of Applied Statistics, 49(10), 2700–2716.

Harrell, F.E. (2015) Regression Modeling Strategies. 2nd Edition, Springer, Berlin.