Scorecard Scaling

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The most commonly used method for developing a scorecard is logistic regression. In practice, this is closely linked to the presentation of its output through a process known as scaling. Scorecard scaling refers to transforming logistic regression outputs into a user-friendly scoring format. The objective is to map probabilities or odds into a numerical score range that is easy to interpret and use for decision-making. A standard industry approach uses a logarithmic scale, where the odds double every fixed number of points - known as "points to double the odds" (PDO).
The following equation gives the general relationship between score and odds:

$$\text{Score} = \text{Offset} + \text{Factor} \cdot \ln(\text{odds})$$

where the offset and factor are determined based on reference points chosen by the user.
For instance, if a score of 600 corresponds to odds of 50:1, and the odds double every 20 points, then the factor and offset can be calculated as:

$$\text{Factor} = \frac{\text{PDO}}{\ln(2)} = \frac{20}{\ln(2)} \approx 28.85$$

$$\text{Offset} = 600 - (28.85 \cdot \ln(50)) \approx 487.12$$

Each score in the scorecard is then derived using this formula, ensuring consistency and interpretability across different scorecards.

The example above demonstrates how to transform the model output into scores. However, practitioners often include an additional step in which each modality of a risk factor (i.e., each characteristic attribute) is assigned a specific score. The total sum of the individual modality scores should match the overall model score produced by the method described earlier. The following slides illustrate score allocation to risk factor modalities for the two most commonly used encoding methods: Weight of Evidence (WoE) and dummy encoding.

Scorecard Scaling - WoE Encoding

Since the scorecard is typically developed using WoE encoding, the final score for an applicant is determined by summing the points assigned to each modality of the risk factors in the scorecard.

By applying the Factor and Offset, points are allocated proportionally to the each modality. The final credit score is obtained by summing the points assigned to all relevant modalities. The point allocation follows the trends established in the WoE analysis. Expressed in terms of formulas, the steps below demonstrate the process of point allocation to the modalities of each risk factor:

$$\begin{gathered}
\text{Score} = \text{Offset} + \ln(\text{odds}) \cdot \text{Factor} = \\
-\left(\sum_{j=1}^{k}\sum_{i=1}^{n}(\text{WoE}_j \cdot \beta_i) + \alpha\right) \cdot \text{Factor} + \text{Offset} = \\
-\left(\sum_{j=1}^{k}\sum_{i=1}^{n}(\text{WoE}_j \cdot \beta_i + \frac{\alpha}{n})\right) \cdot \text{Factor} + \text{Offset} = \\
\sum_{j=1}^{k}\sum_{i=1}^{n}\left(-\text{WoE}_j \cdot \beta_i + \frac{\alpha}{n}\right) \cdot \text{Factor} + \frac{\text{Offset}}{n}
\end{gathered}$$

Scorecard Scaling - WoE Encoding cont.

where:

- WoE_j is the Weight of Evidence for each modality;
- β_i is the regression coefficient for each risk factor;
- α denotes the intercept term from logistic regression;
- n is the number of risk factors in the model;
- k denotes the number of modalities in each risk factor;
- Factor is a scaling parameter used to adjust the scores of each modality within a given risk factor;
- Offset is the constant used to shift the score into a desired range.

Scorecard Scaling - Dummy Encoding

Unlike WoE encoding, dummy encoding produces multiple logistic regression coefficients directly associated with the modalities of the risk factors. Although the logic of point allocation remains similar, the assignment process differs slightly. Since the impact of each risk factor's reference modality is absorbed into the model's intercept, the points allocated to these reference modalities will be the same across all risk factors. Expressed in terms of formulas, the steps below demonstrate the process of point allocation to the modalities of each risk factor using dummy encoding:

$$
\begin{gathered}
\text{Score} = \text{Offset} + \ln(\text{odds}) \cdot \text{Factor} = \\
-\left(\sum_{j=1}^{k}\sum_{i=1}^{n}(D_j \cdot \beta_j) + \alpha\right) \cdot \text{Factor} + \text{Offset} = \\
-\left(\sum_{j=1}^{k}\sum_{i=1}^{n}(D_j \cdot \beta_j + \frac{\alpha}{n})\right) \cdot \text{Factor} + \text{Offset} = \\
\sum_{j=1}^{k}\sum_{i=1}^{n}\left(-D_j \cdot \beta_j + \frac{\alpha}{n}\right) \cdot \text{Factor} + \frac{\text{Offset}}{n}
\end{gathered}
$$

Scorecard Scaling - Dummy Encoding cont.

where:

- D_j is the dummy variable indicator for modality j (1 if present, 0 otherwise);
- β_j is the regression coefficient for modality j;
- α is the intercept term from the logistic regression;
- n is the number of risk factors in the model;
- k is the number of modalities in each risk factor;
- Factor is a scaling parameter used to adjust the scores of each modality within a given risk factor;
- Offset is the constant used to shift the score into a desired range.

Simulation Study - WoE Encoding

Assume that the model's binary target `Creditability` is estimated using only two categorical risk factors, `Account_Balance` and `Maturity`, with four and five unique modalities, respectively. A simulation dataset is available here. Under the assumption that the model uses WoE encoding, this and the following slides present the necessary steps and elements for transforming risk factor modalities into scores.

1. After WoE encoding of `Account_Balance` and `Maturity`, the estimated model in the form `Creditability ~ Account_Balance_WoE + Maturity_WoE` produces the following result:

```
##                     Estimate Std. Error  z value Pr(>|z|)
## (Intercept)          -0.8470     0.0774 -10.9364        0
## Account_Balance_WoE  -0.9913     0.0976 -10.1606        0
## Maturity_WoE         -0.9774     0.1489  -6.5664        0
```

2. Calculate the scaling inputs, Factor and Offset for a score of 600 at odds of 50:1, with 20 points to double the odds. Given these inputs, the value of the Factor is 28.85, and the Offset is 487.12.

3. Inputs from steps 1 and 2 are now sufficient to perform the score assignment process for each modality of the risk factors using the formula from slide 3.

```
##                rf         bin  no  ng  nb     woe intercept    beta score
## 1 Account_Balance          01 274 139 135 -0.8181    -0.847 -0.9913   232
## 2 Account_Balance          02 269 164 105 -0.4014    -0.847 -0.9913   244
## 3 Account_Balance          03  63  49  14  0.4055    -0.847 -0.9913   267
## 4 Account_Balance          04 394 348  46  1.1763    -0.847 -0.9913   289
## 5        Maturity 01 (-Inf,8)  87  78   9  1.3122    -0.847 -0.9774   293
## 6        Maturity   02 [8,16) 344 264  80  0.3466    -0.847 -0.9774   266
## 7        Maturity  03 [16,36) 399 270 129 -0.1087    -0.847 -0.9774   253
## 8        Maturity  04 [36,45) 100  58  42 -0.5245    -0.847 -0.9774   241
## 9        Maturity 05 [45,Inf)  70  30  40 -1.1350    -0.847 -0.9774   224
```

Simulation Study - WoE Encoding cont.

The final step is to verify that the scaling has been performed correctly. This can be done by comparing the score produced directly from the model output (model's log odds) with the sum of the individual modality scores for each observation in the development sample. The table below presents the unique model outputs alongside the model scores and the corresponding sums of individual modality scores. As shown, the final scores match precisely.

```
##    Account_Balance     Maturity Model_Score Account_Balance_Score Maturity_Score Individual_Sum_Score
## 1               01  03 [16,36)         485                   232            253                  485
## 2               01   02 [8,16)         498                   232            266                  498
## 3               02   02 [8,16)         510                   244            266                  510
## 4               01 01 (-Inf,8)         525                   232            293                  525
## 5               04  03 [16,36)         542                   289            253                  542
## 6               02  03 [16,36)         497                   244            253                  497
## 7               02 05 [45,Inf)         468                   244            224                  468
## 8               02  04 [36,45)         485                   244            241                  485
## 9               04   02 [8,16)         555                   289            266                  555
## 10              03  04 [36,45)         508                   267            241                  508
## 11              03  03 [16,36)         520                   267            253                  520
## 12              04  04 [36,45)         530                   289            241                  530
## 13              04 05 [45,Inf)         513                   289            224                  513
## 14              04 01 (-Inf,8)         582                   289            293                  582
## 15              03   02 [8,16)         533                   267            266                  533
## 16              02 01 (-Inf,8)         537                   244            293                  537
## 17              03 01 (-Inf,8)         560                   267            293                  560
## 18              01 05 [45,Inf)         456                   232            224                  456
## 19              01  04 [36,45)         473                   232            241                  473
```

Simulation Study - Dummy Encoding

This and the next slide present a simulation study on the same dataset with two key differences. First, dummy encoding is used instead of WoE encoding. Second, the scaling inputs are set with the Factor equal to 10,000 and the Offset equal to 0. The following steps demonstrate the scaling process under these assumptions.

1. The estimated model in the form `Creditability ~ Account_Balance_WoE + Maturity_WoE` produces the following result:

```
##                      Estimate Std. Error z value Pr(>|z|)
## (Intercept)           -1.3234     0.3720 -3.5579   0.0004
## Account_Balance02     -0.5064     0.1809 -2.7997   0.0051
## Account_Balance03     -1.0873     0.3332 -3.2629   0.0011
## Account_Balance04     -2.0194     0.2029 -9.9507   0.0000
## Maturity02 [8,16)      0.9783     0.3873  2.5262   0.0115
## Maturity03 [16,36)     1.4282     0.3809  3.7495   0.0002
## Maturity04 [36,45)     1.8817     0.4248  4.4297   0.0000
## Maturity05 [45,Inf)    2.4041     0.4491  5.3532   0.0000
```

2. The Factor and Offset are predefined with values of 10,000 and 0, respectively.

3. Inputs from steps 1 and 2 are now sufficient to perform the score assignment process for each modality of the risk factors using the formula from slide 5.

```
##                rf          bin intercept    beta  score
## 1 Account_Balance           01   -1.3234  0.0000   6617
## 2 Account_Balance           02   -1.3234 -0.5064  11681
## 3 Account_Balance           03   -1.3234 -1.0873  17490
## 4 Account_Balance           04   -1.3234 -2.0194  26811
## 5        Maturity 01 (-Inf,8)   -1.3234  0.0000   6617
## 6        Maturity   02 [8,16)   -1.3234  0.9783  -3166
## 7        Maturity  03 [16,36)   -1.3234  1.4282  -7665
## 8        Maturity  04 [36,45)   -1.3234  1.8817 -12200
## 9        Maturity 05 [45,Inf)   -1.3234  2.4041 -17424
```

Simulation Study - Dummy Encoding cont.

The final step is to verify that the scaling has been performed correctly. This can be done by comparing the score produced directly from the model output (model's log odds) with the sum of the individual modality scores for each observation in the development sample. The table below presents the unique model outputs alongside the model scores and the corresponding sums of individual modality scores. As shown, the final scores match precisely.

```
##    Account_Balance     Maturity Model_Score Account_Balance_Score Maturity_Score Individual_Sum_Score
## 1               01  03 [16,36)       -1048                  6617          -7665                -1048
## 2               01   02 [8,16)        3451                  6617          -3166                 3451
## 3               02   02 [8,16)        8515                 11681          -3166                 8515
## 4               01 01 (-Inf,8)       13234                  6617           6617                13234
## 5               04  03 [16,36)       19146                 26811          -7665                19146
## 6               02  03 [16,36)        4016                 11681          -7665                 4016
## 7               02 05 [45,Inf)       -5743                 11681         -17424                -5743
## 8               02  04 [36,45)        -519                 11681         -12200                 -519
## 9               04   02 [8,16)       23645                 26811          -3166                23645
## 10              03  04 [36,45)        5290                 17490         -12200                 5290
## 11              03  03 [16,36)        9825                 17490          -7665                 9825
## 12              04  04 [36,45)       14611                 26811         -12200                14611
## 13              04 05 [45,Inf)        9387                 26811         -17424                 9387
## 14              04 01 (-Inf,8)       33428                 26811           6617                33428
## 15              03   02 [8,16)       14324                 17490          -3166                14324
## 16              02 01 (-Inf,8)       18298                 11681           6617                18298
## 17              03 01 (-Inf,8)       24107                 17490           6617                24107
## 18              01 05 [45,Inf)      -10807                  6617         -17424               -10807
## 19              01  04 [36,45)       -5583                  6617         -12200                -5583
```