THERMO SYPHON FLOW PREDICTION

Predicting Flow Reversals in a Computational Fluid Dynamics Simulated Thermosyphon using Data Assimilation

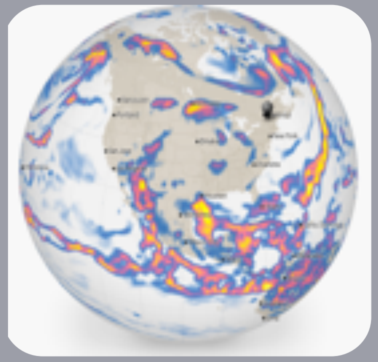
ANDREW REAGAN
DEPARTMENT OF MATH
UNIVERSITY OF VERMONT

#SIAMDS15

MAIN IDEAS

DATA ASSIMILATION

A brief overview

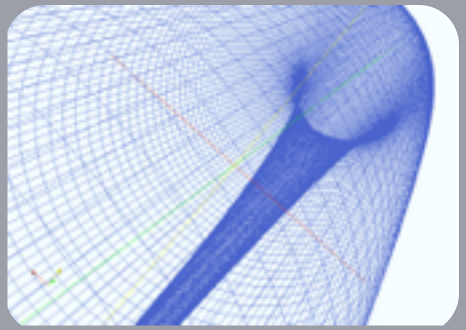


THERMOSYPHON AS MODEL ATMOSPHERE

An experiment that exhibits deterministic nonperiodic flow

MODELING THE THERMOSYPHON

A brief introduction to computational fluid dynamics



MAIN RESULTS

Using data assimilation to predict flow in the thermosyphon

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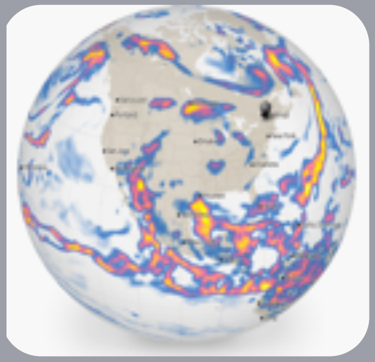

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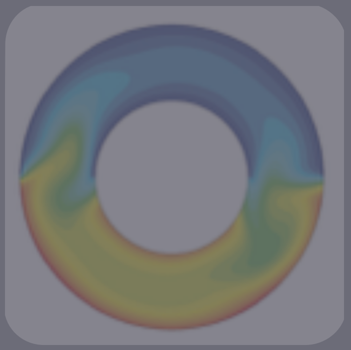

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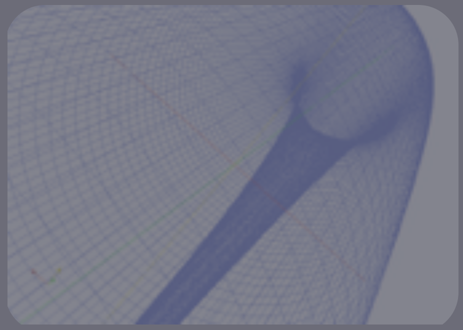

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Using data assimilation to predict flow in the thermosyphon

DATA ASSIMILATION

uses statistical combination of observations and short range forecasts to produce initial conditions

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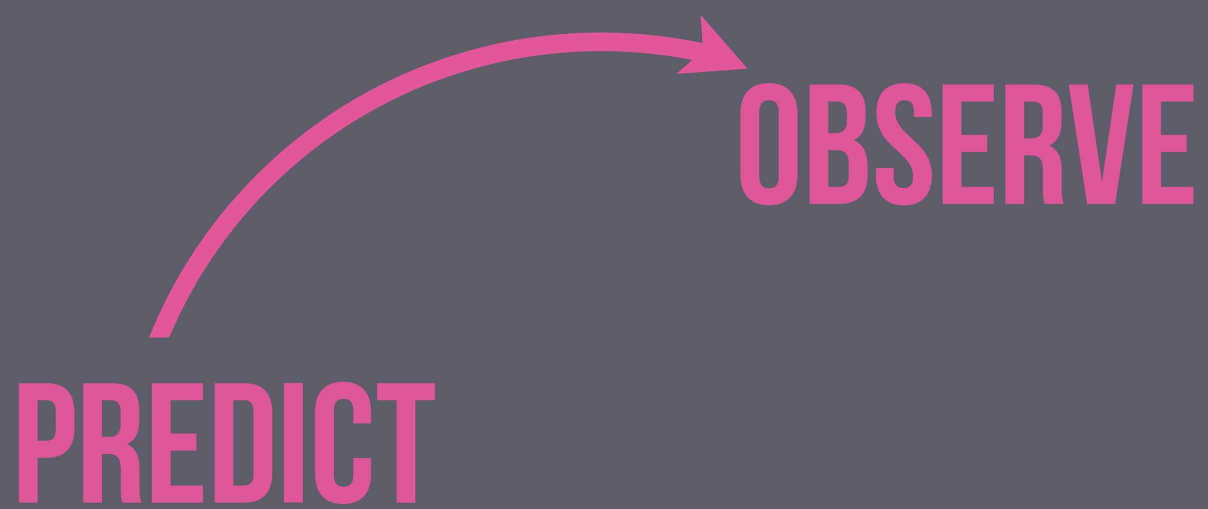

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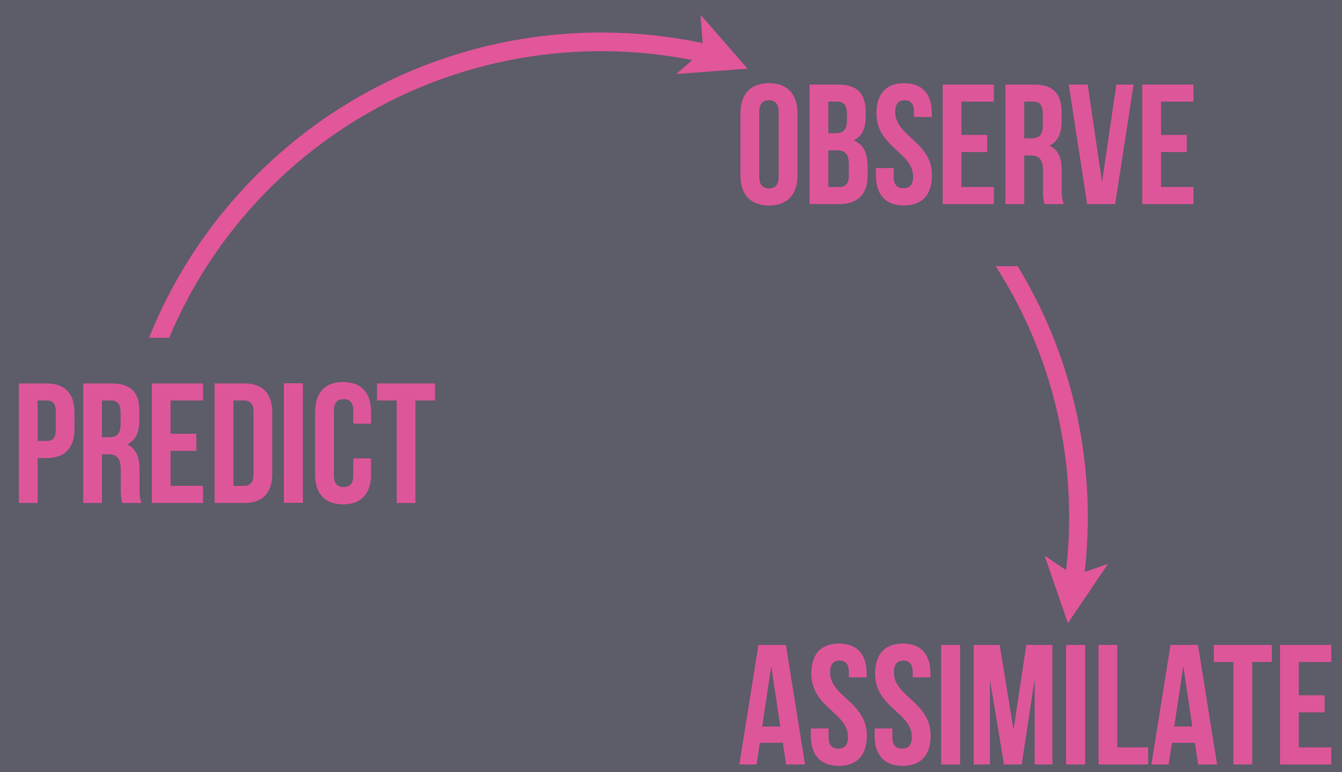

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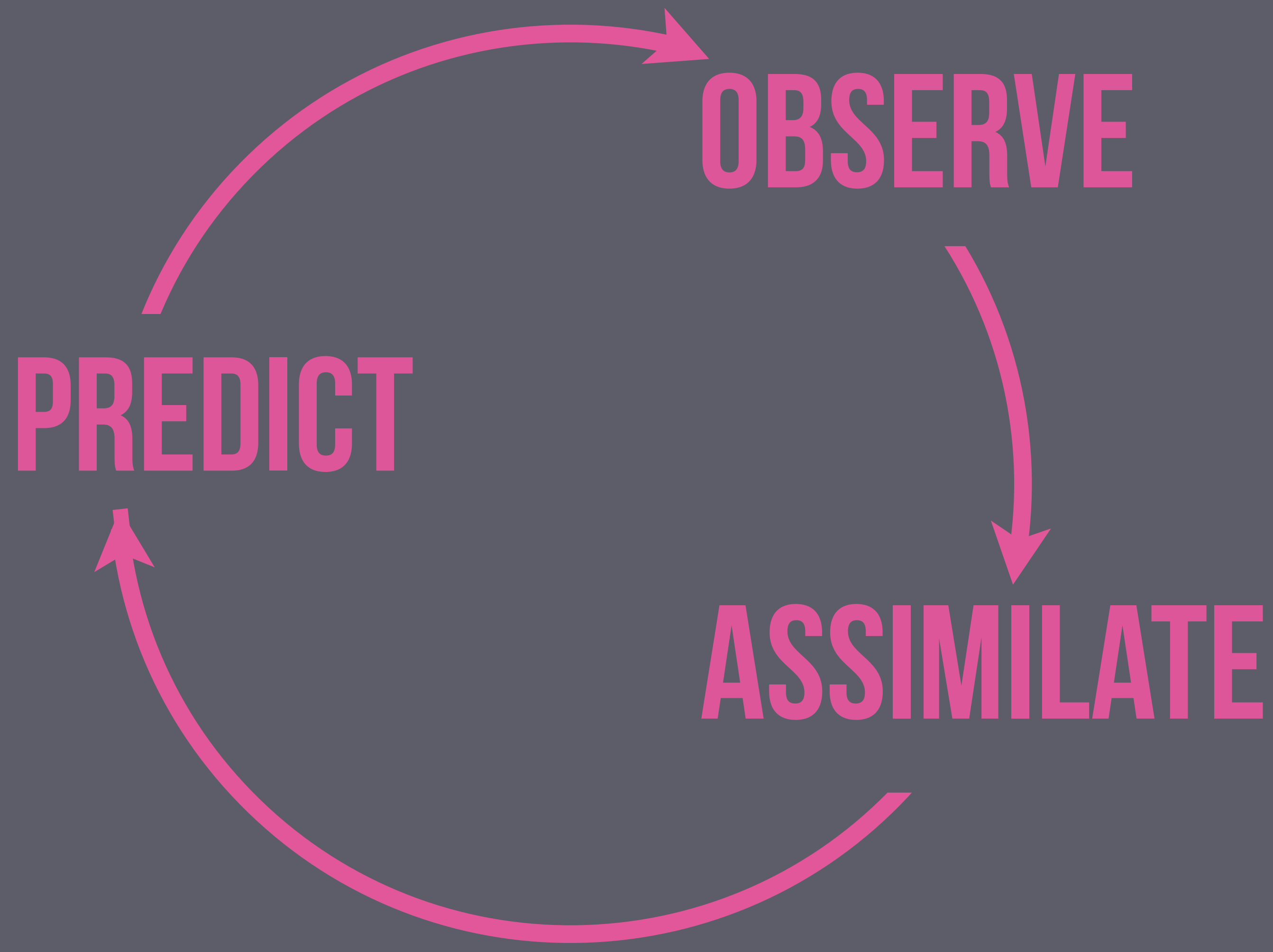
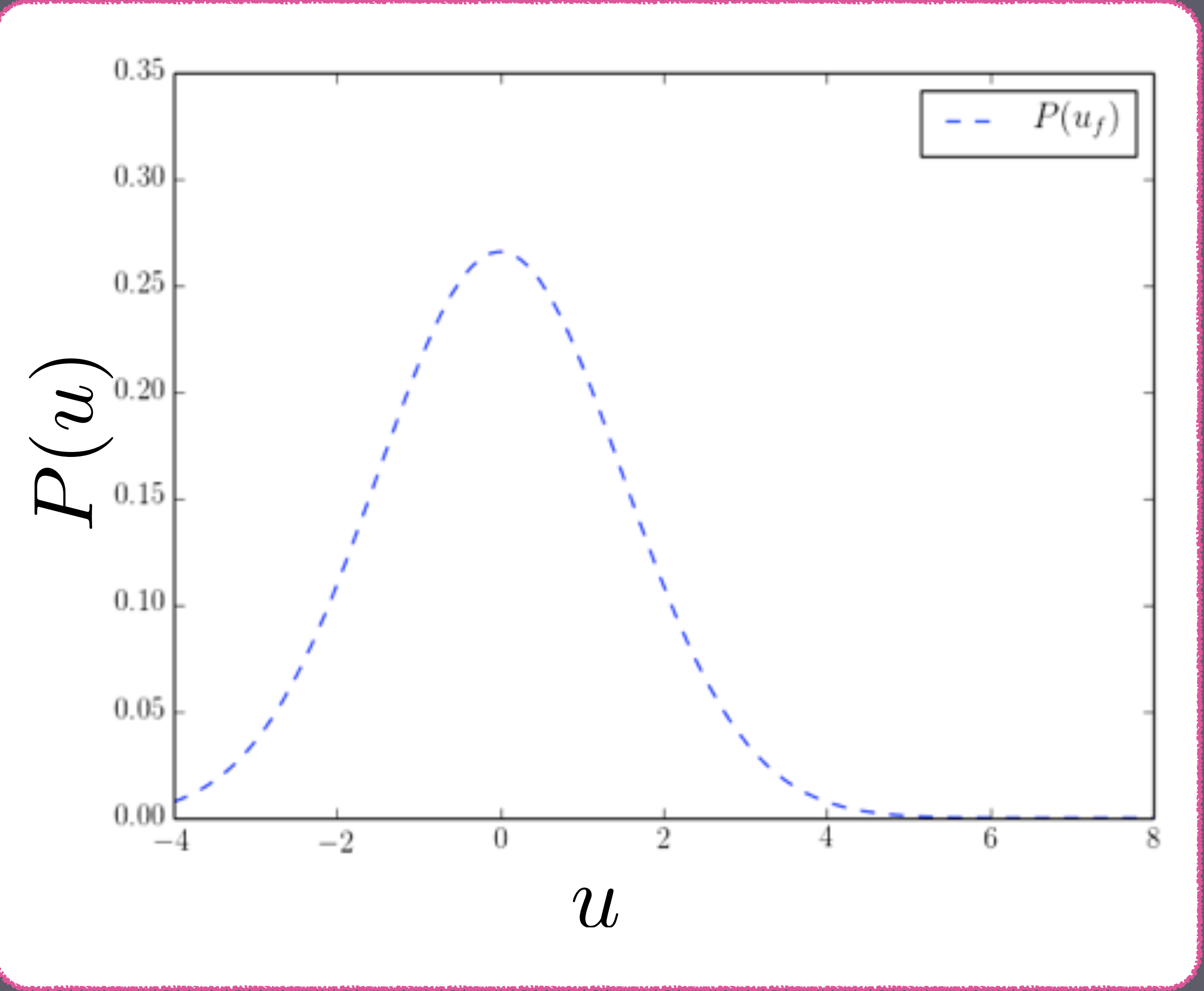


PREDICT

$$u_f = 0$$

$$\sigma_f^2 = 1$$



OBSERVE

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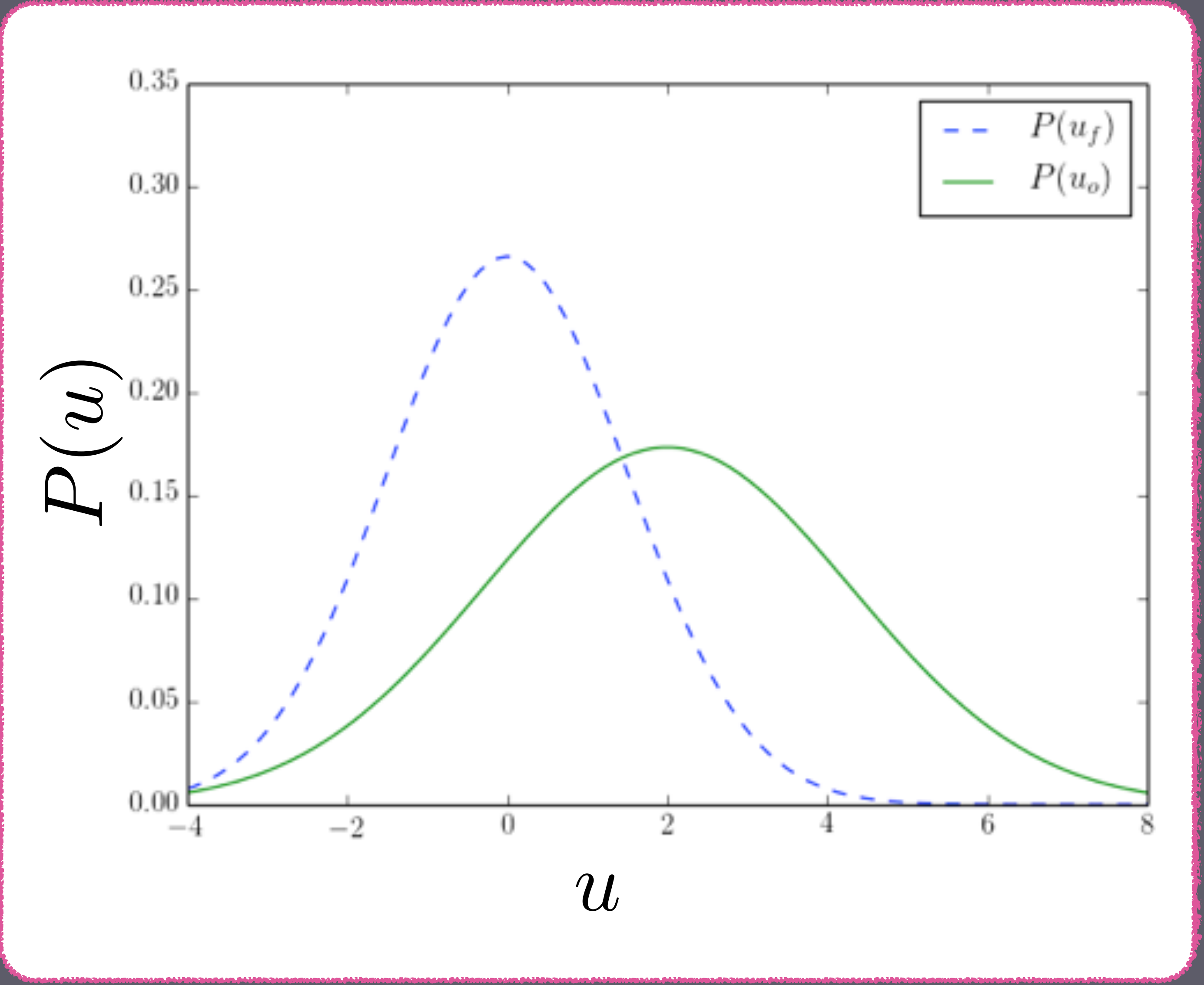

Observe

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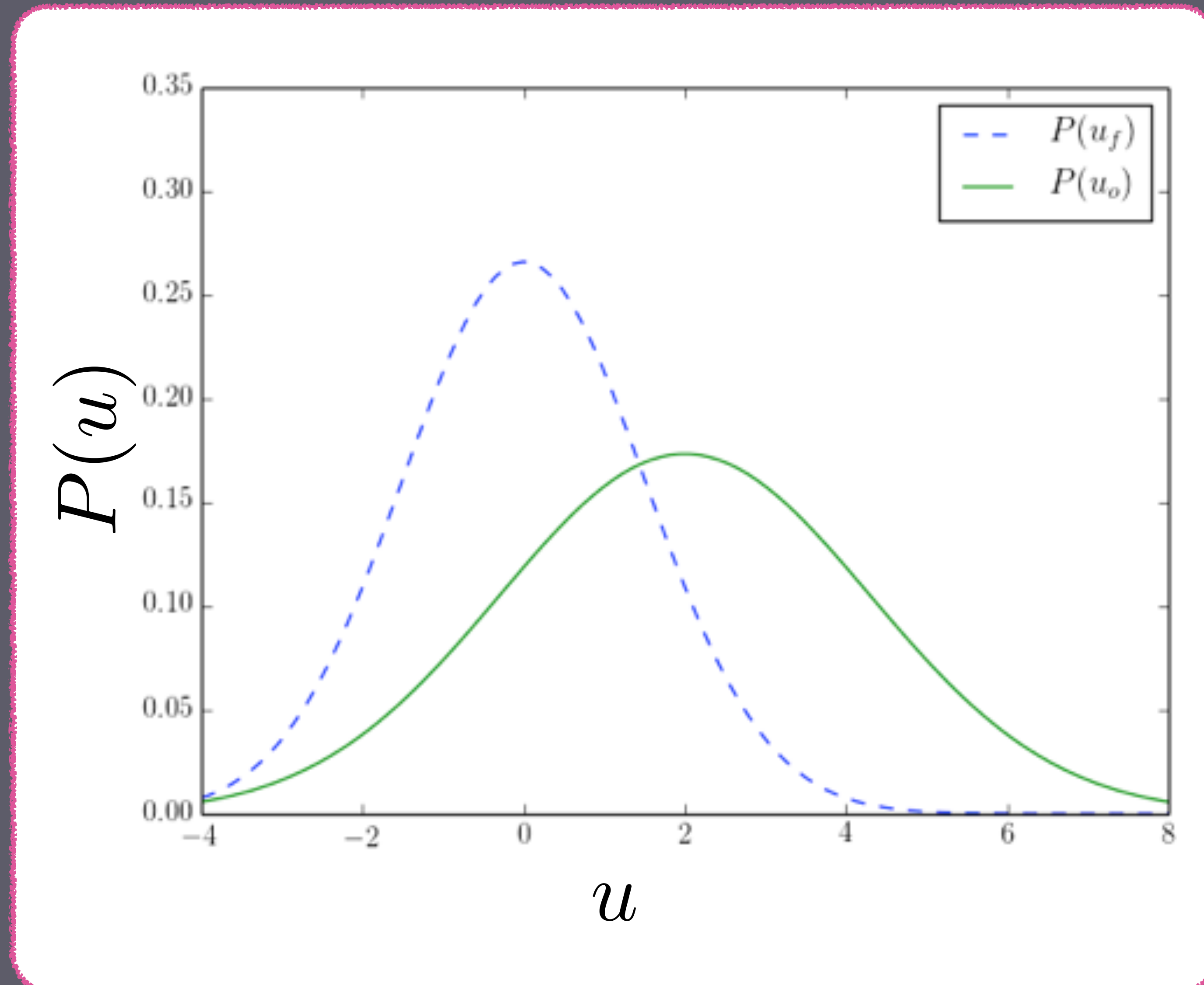
$$\sigma_f^2 = 1$$

$$u_o = 2$$

$$\sigma_o^2 = 2$$



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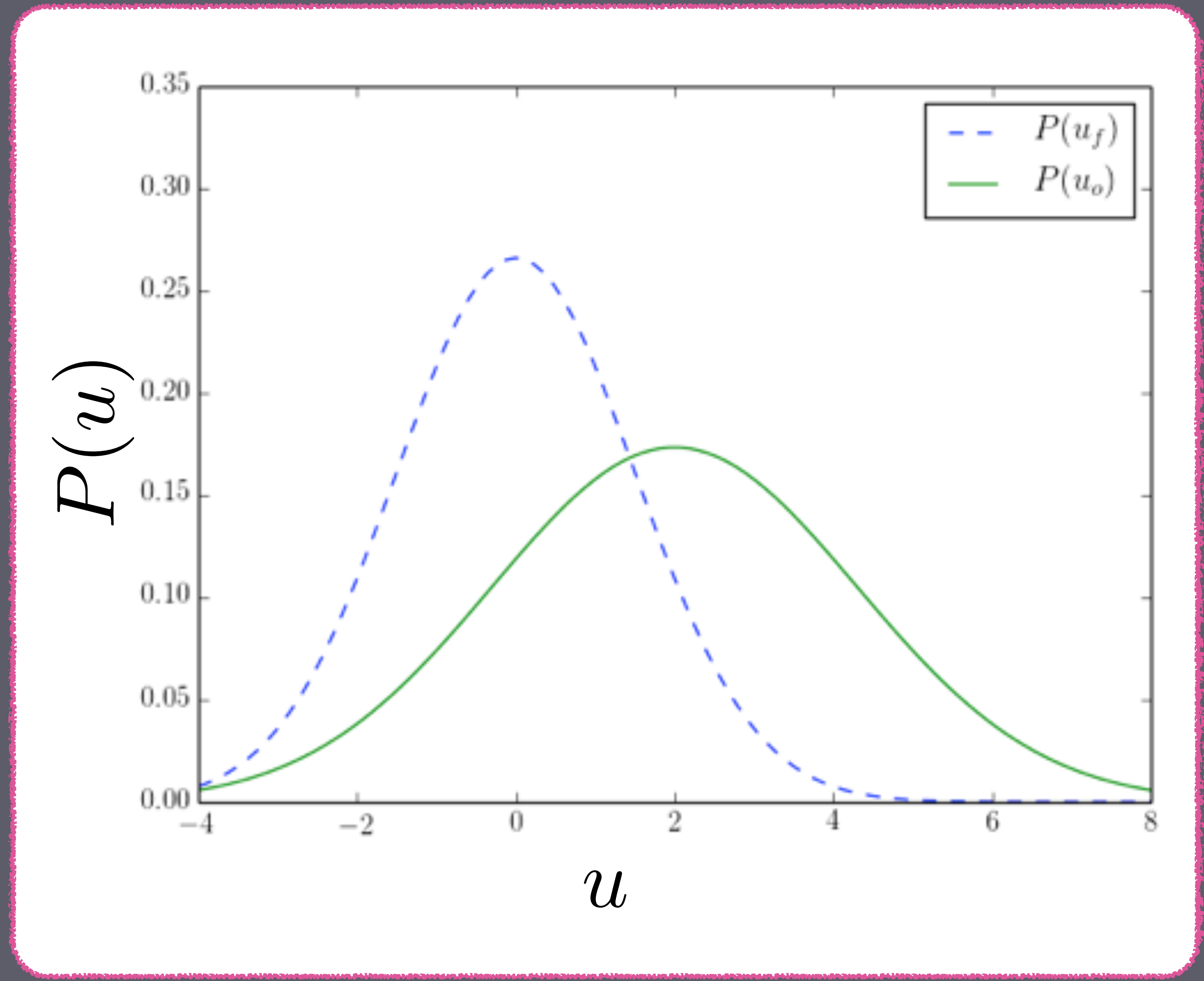

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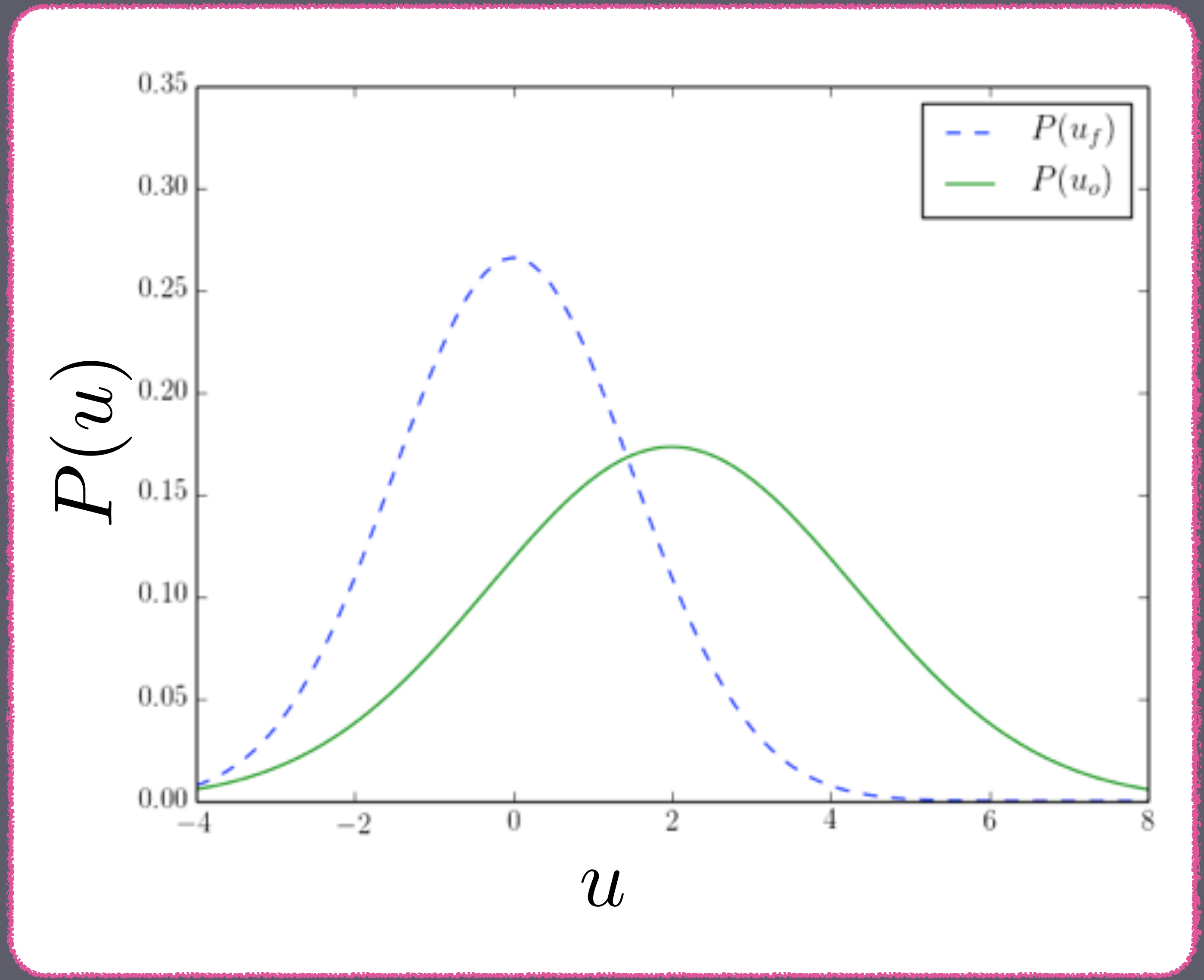

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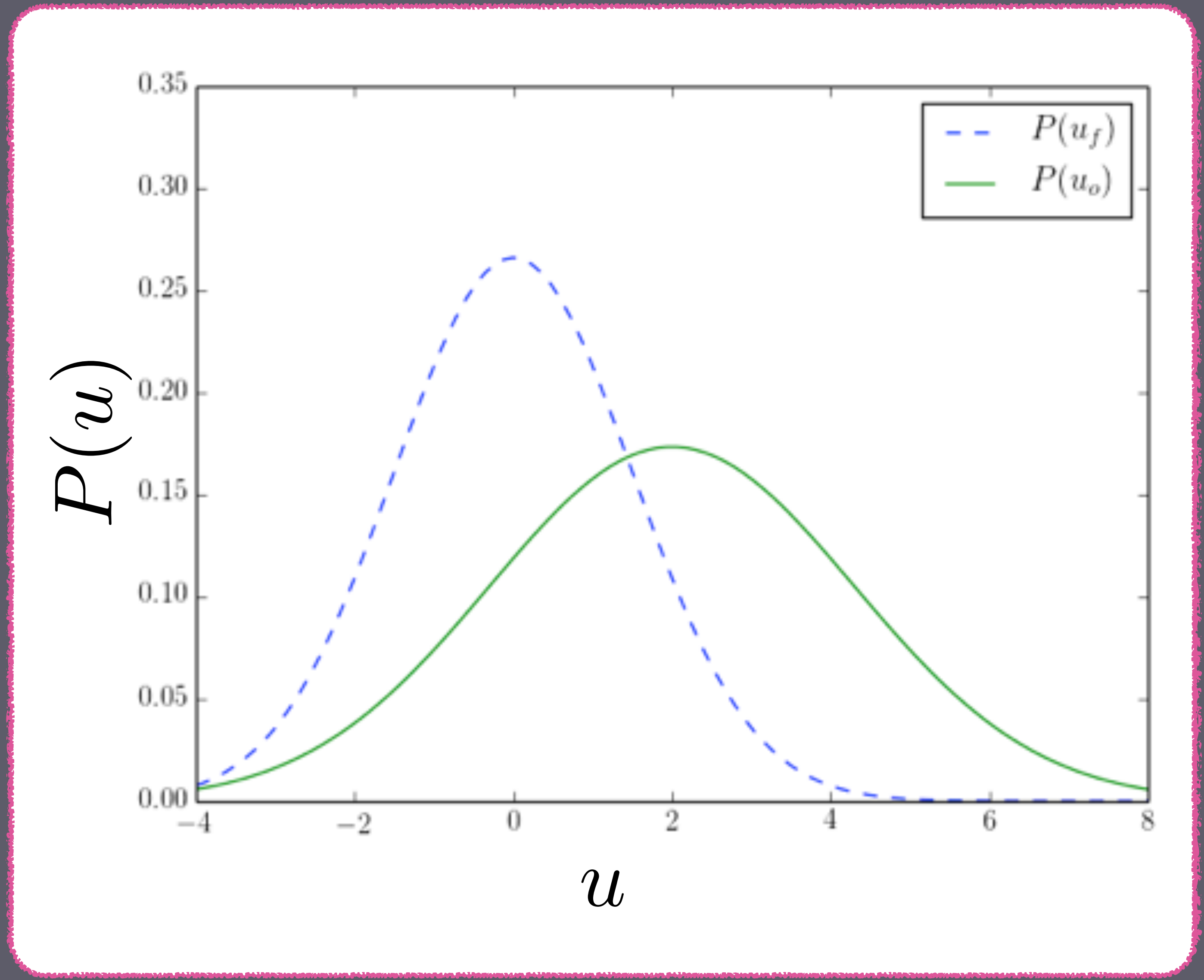

$$u_a = \alpha_1 u_f + \alpha_2 u_o$$

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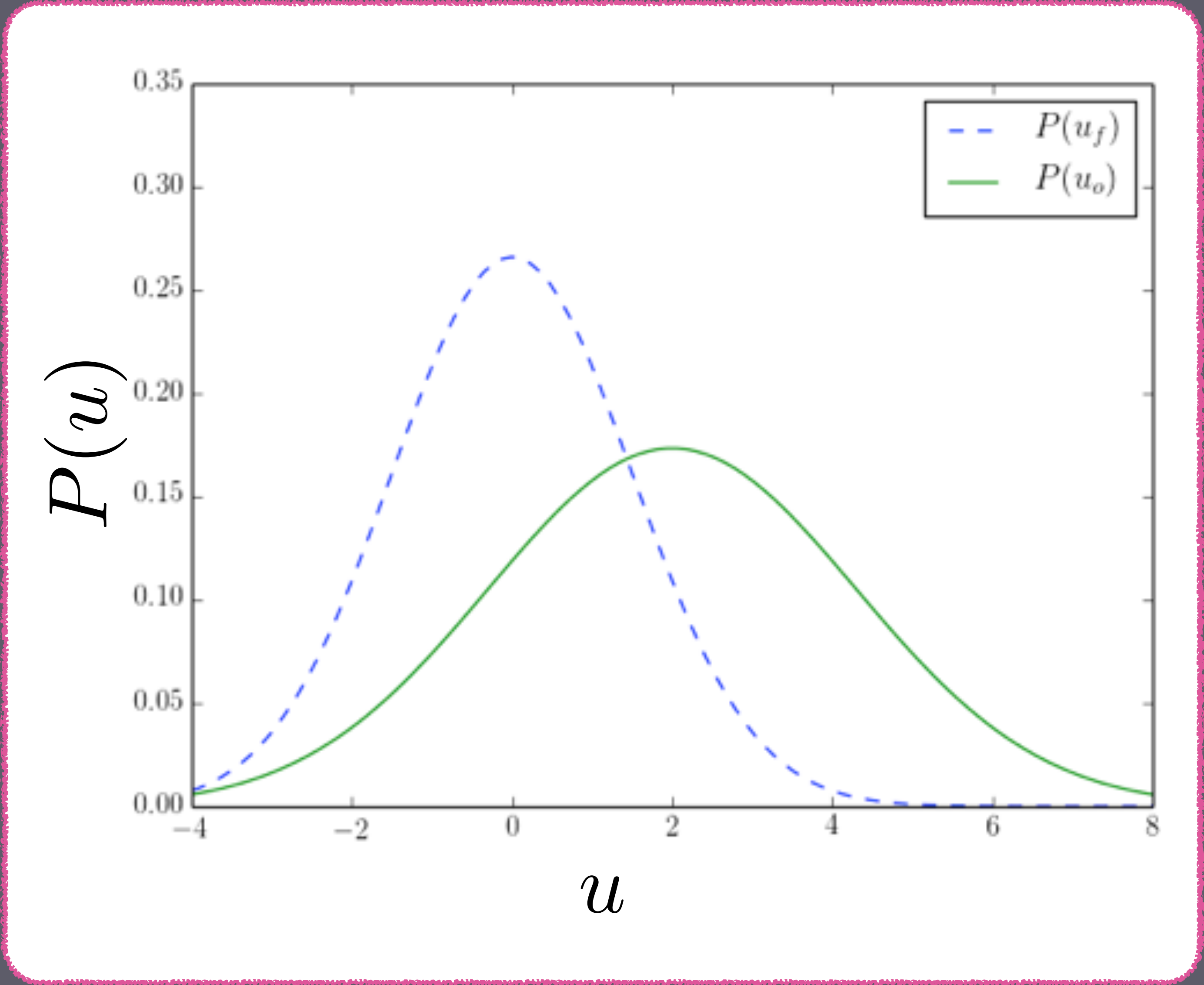

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$$\Rightarrow u_a = u_f + w(u_o - u_f)$$

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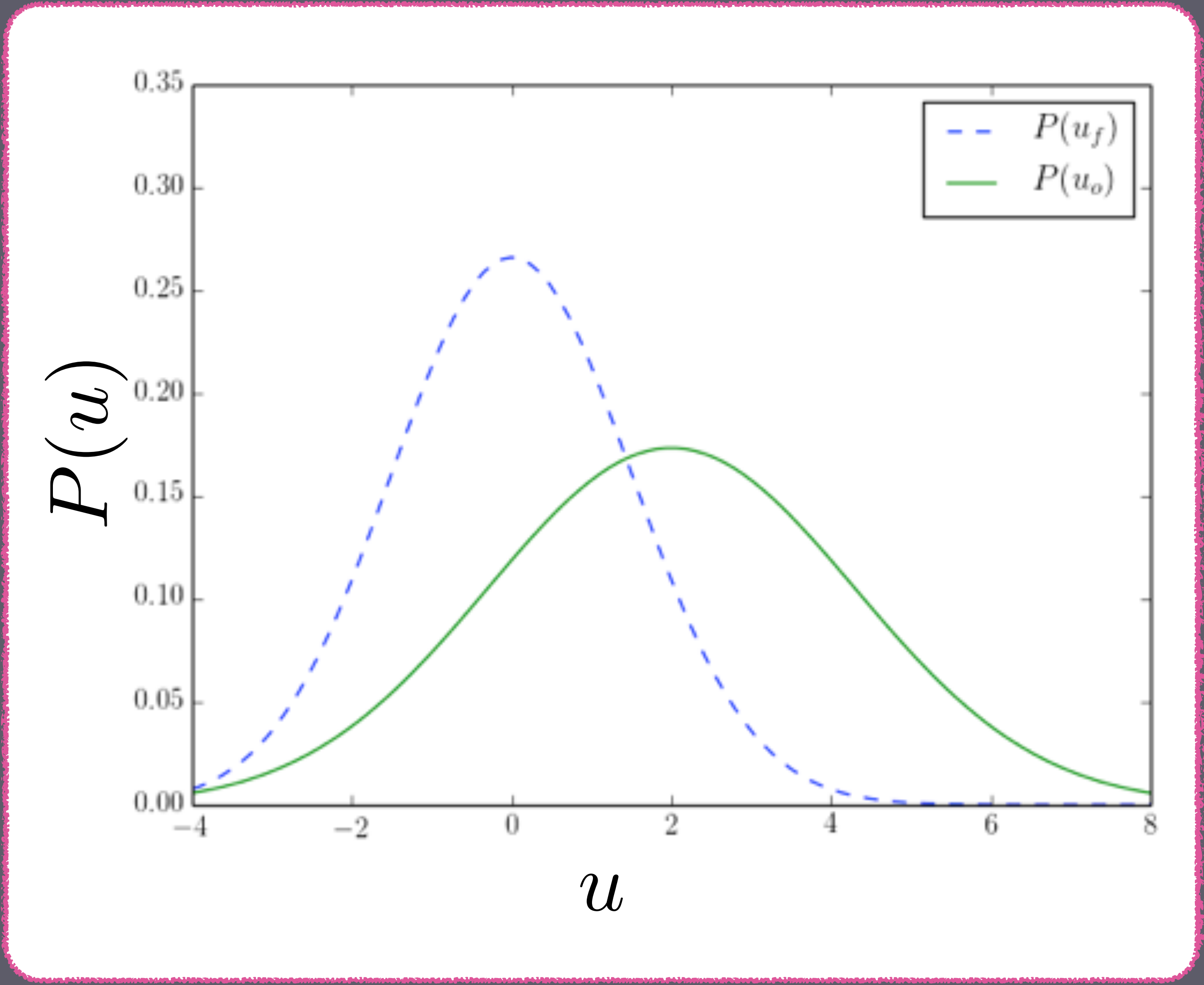

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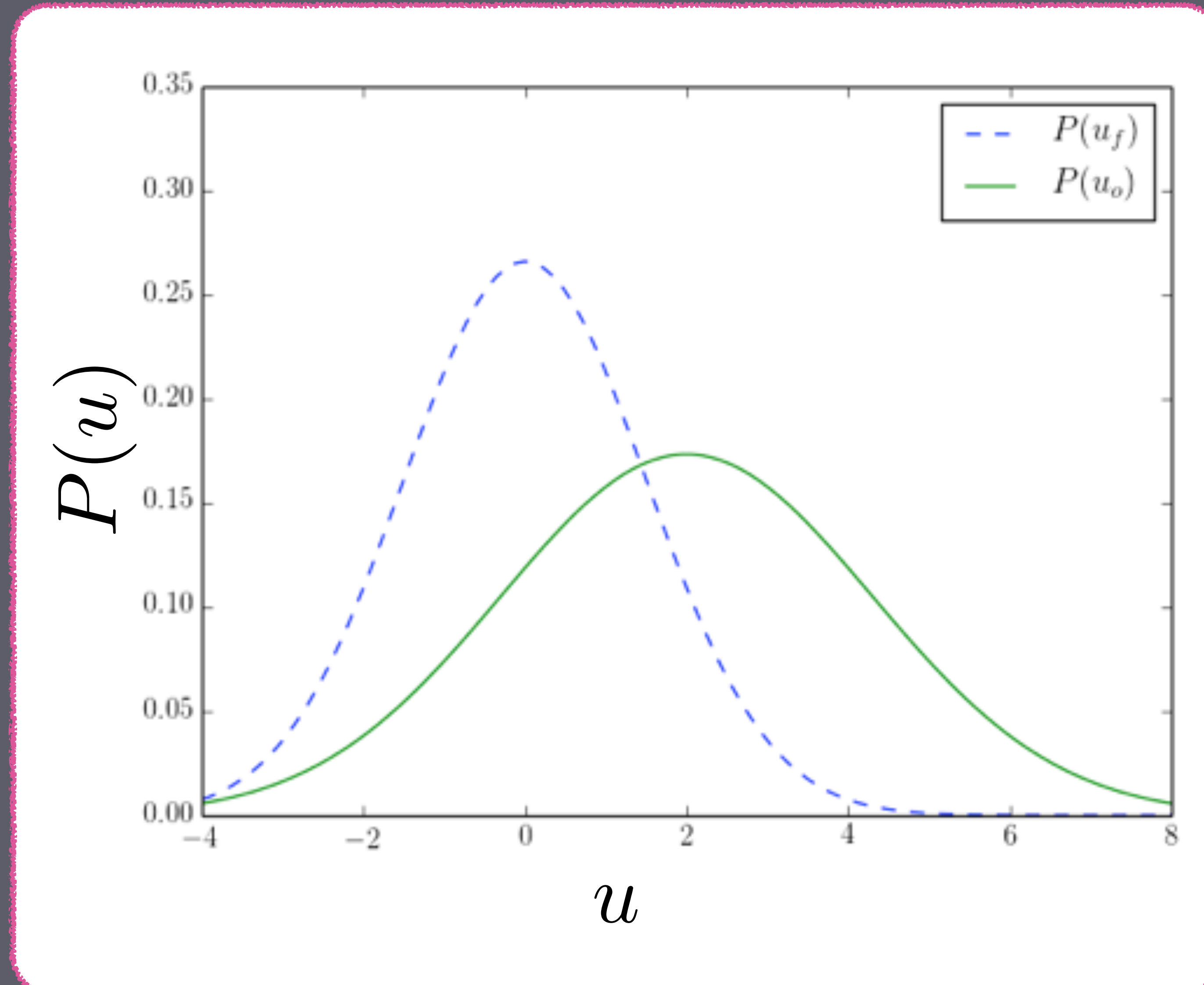

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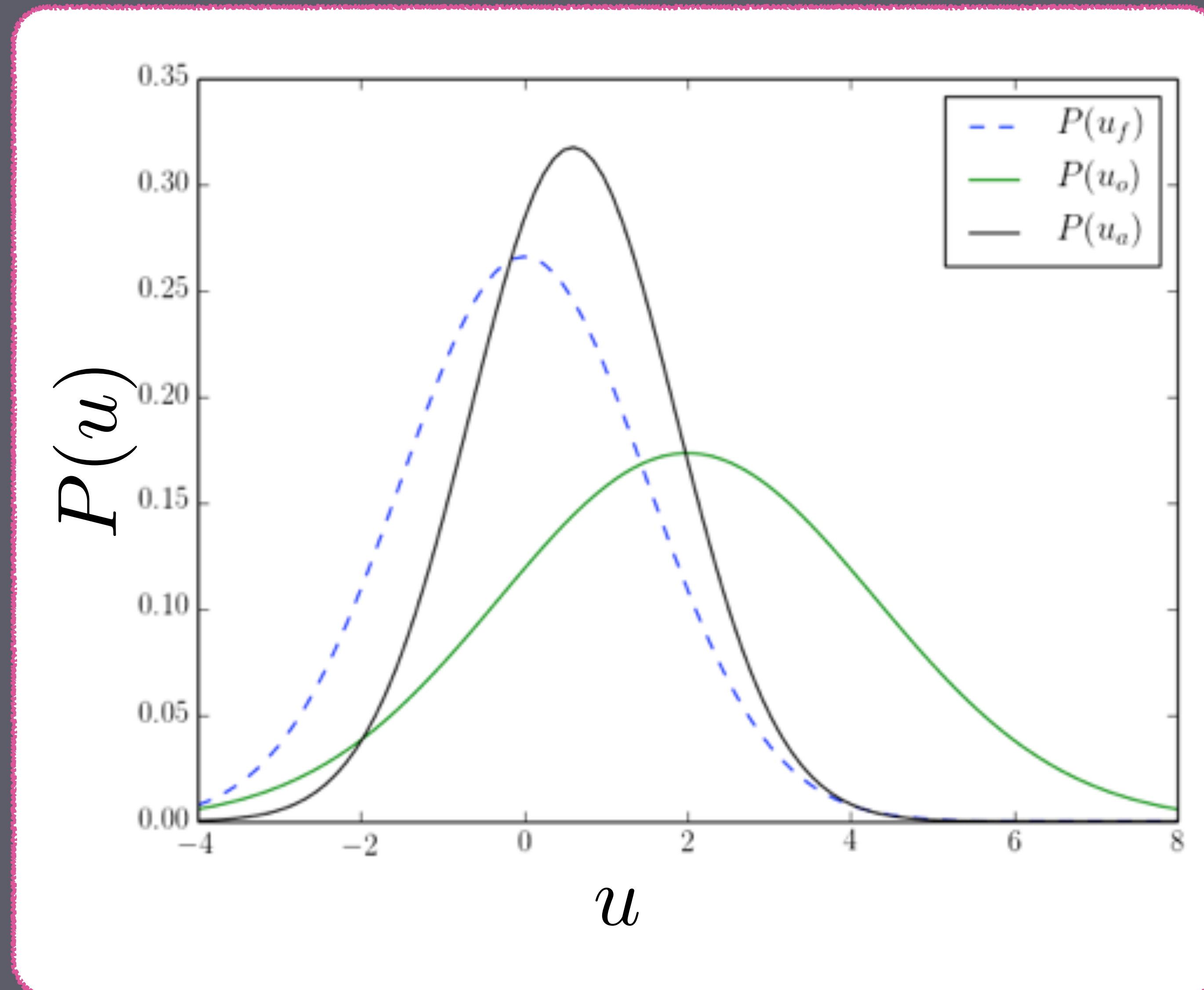

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EXTENDED KALMAN FILTER

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$$\mathbf{K} = \mathbf{P}^f \mathbf{H}^T (\mathbf{R} + \mathbf{H}\mathbf{P}^f \mathbf{H}^T)^{-1}$$

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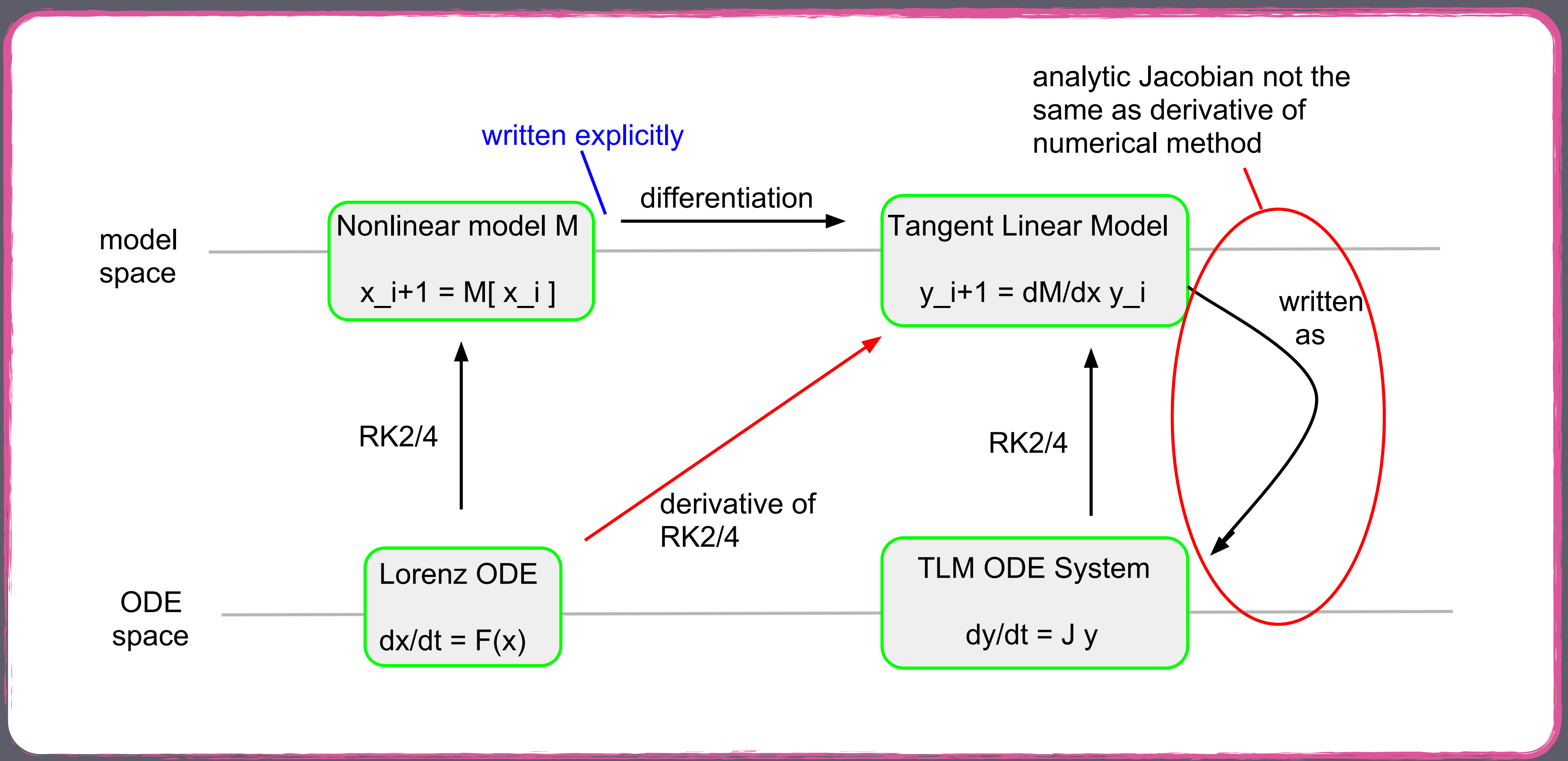
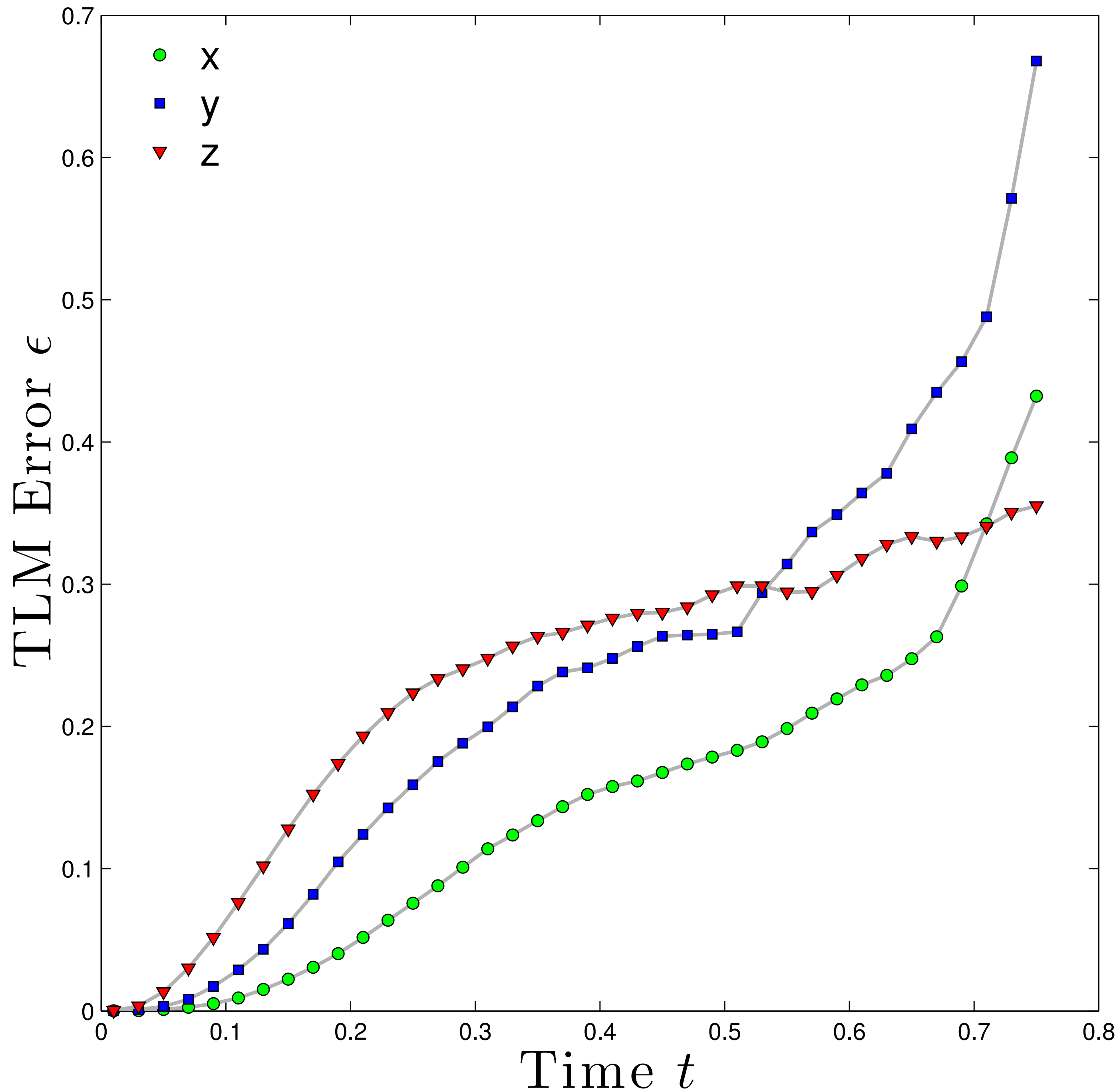


Extended Kalman Filter



ENSEMBLE APPROXIMATION

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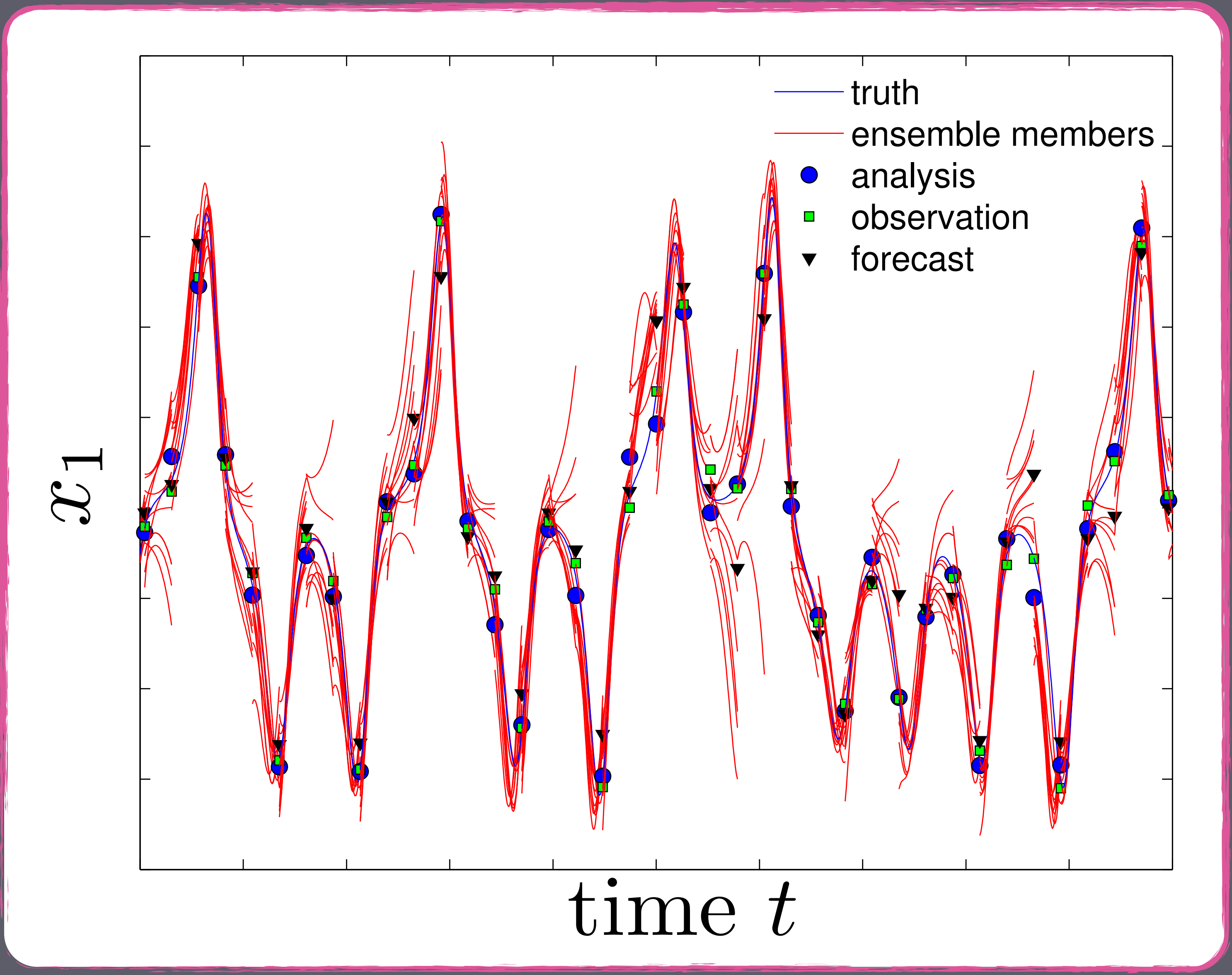

Approximate the error covariance matrix with ensemble of model runs

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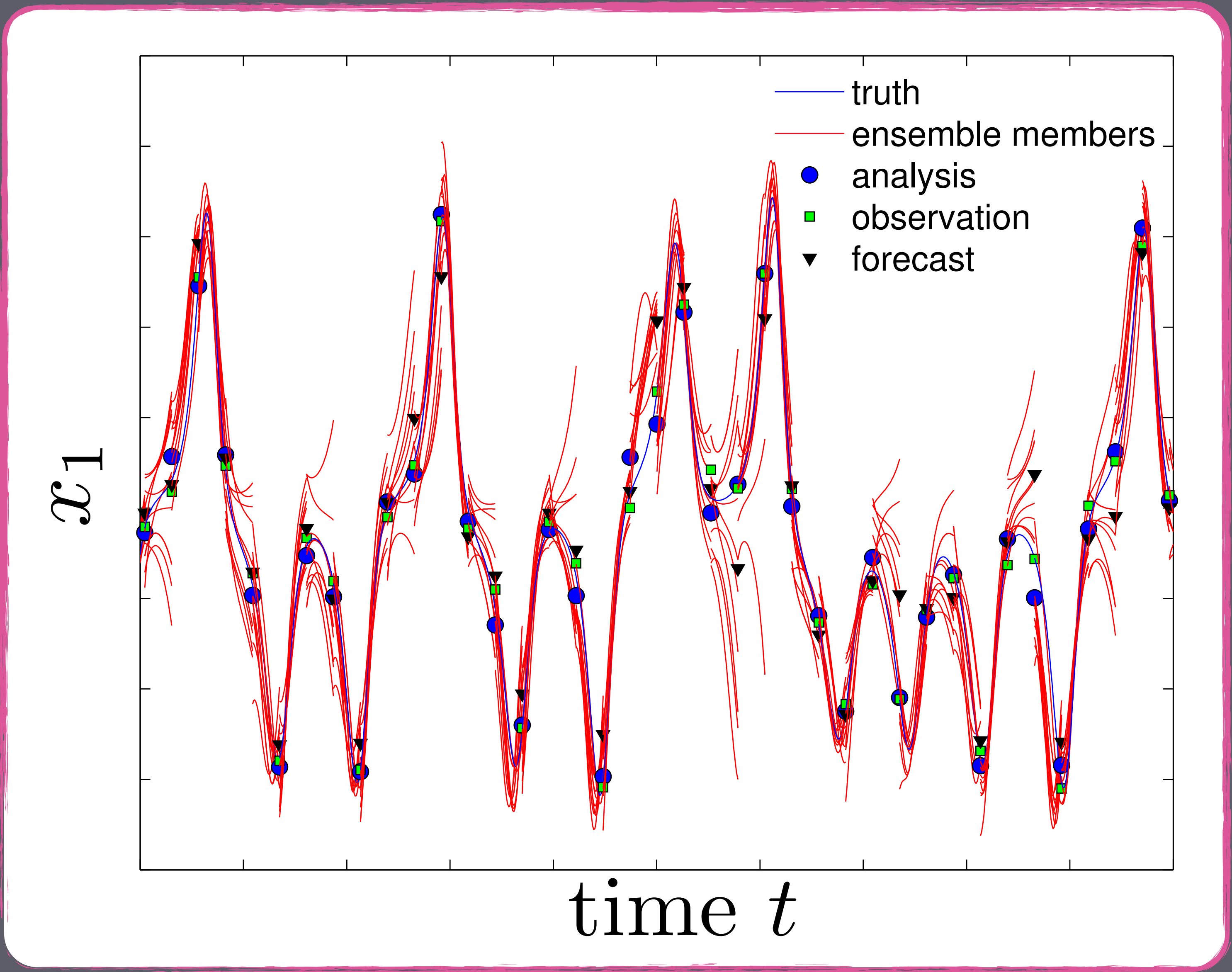
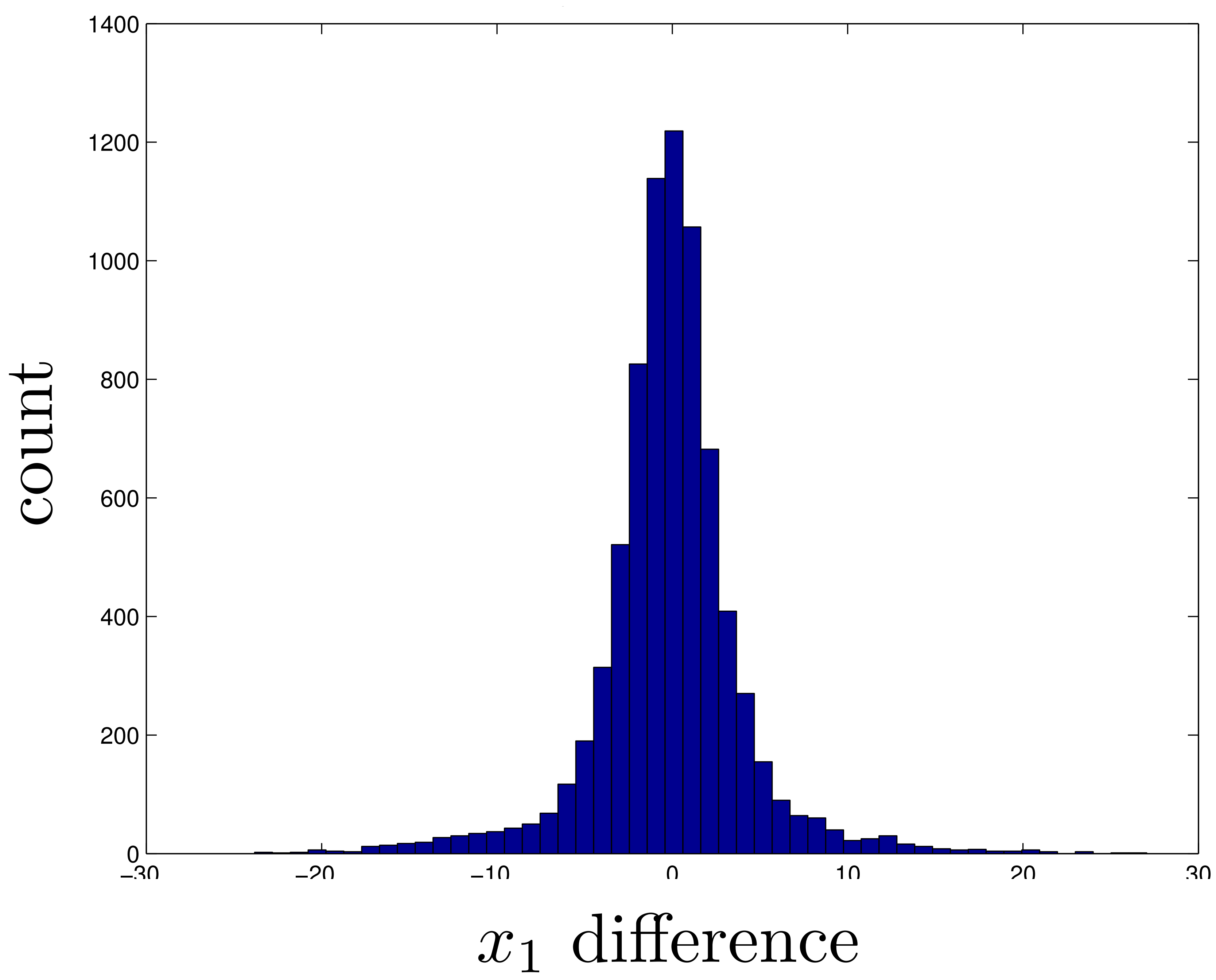


Approximate the error covariance matrix with ensemble of model runs

$$\mathbf{P}^f \approx \frac{1}{K-2} \sum_{k \neq l} (\mathbf{x}_k^f - \overline{\mathbf{x}}_l^f)(\mathbf{x}_k^f - \overline{\mathbf{x}}_l^f)^T$$

TUNING THE FILTERS

Tuning the Filters



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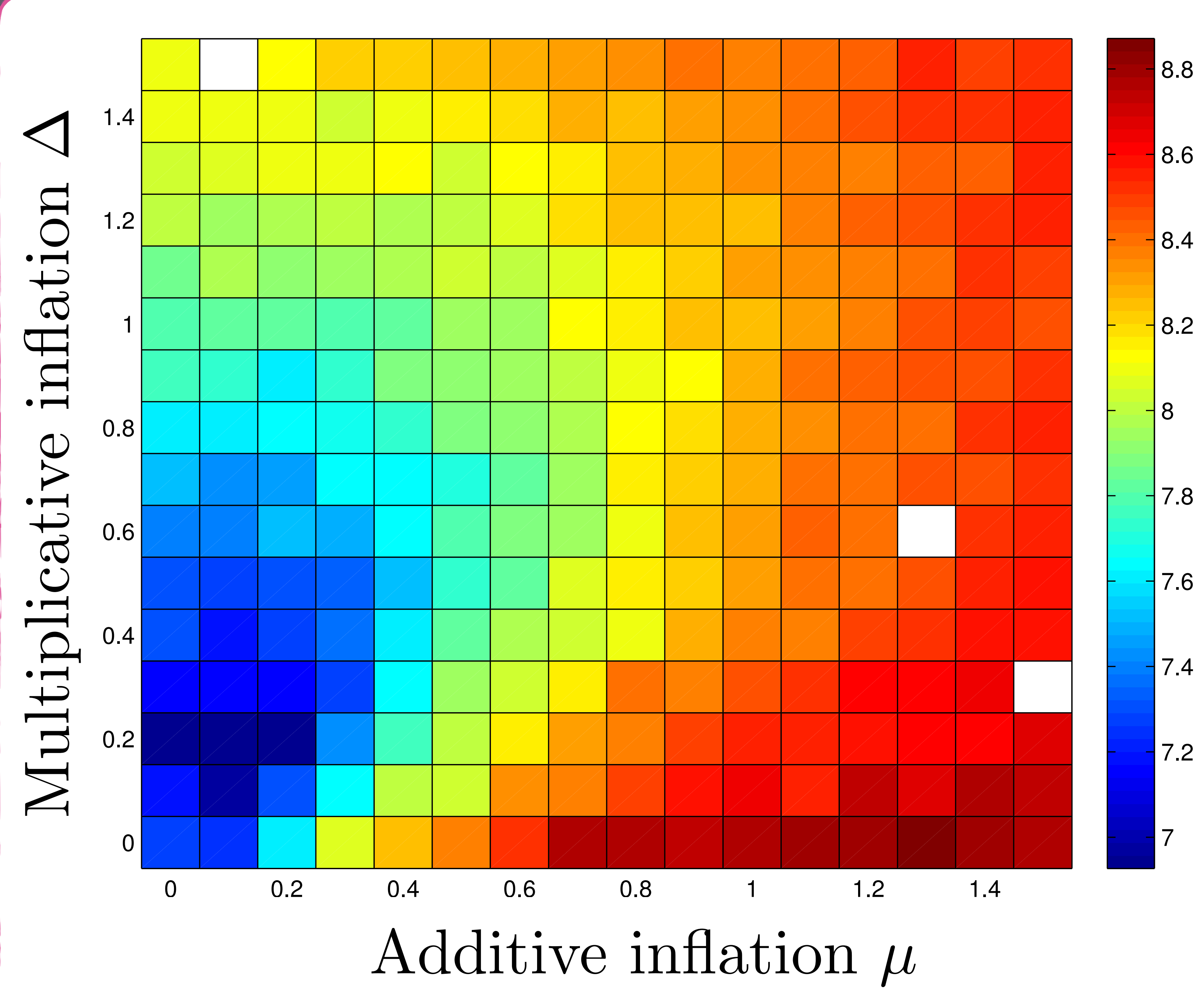
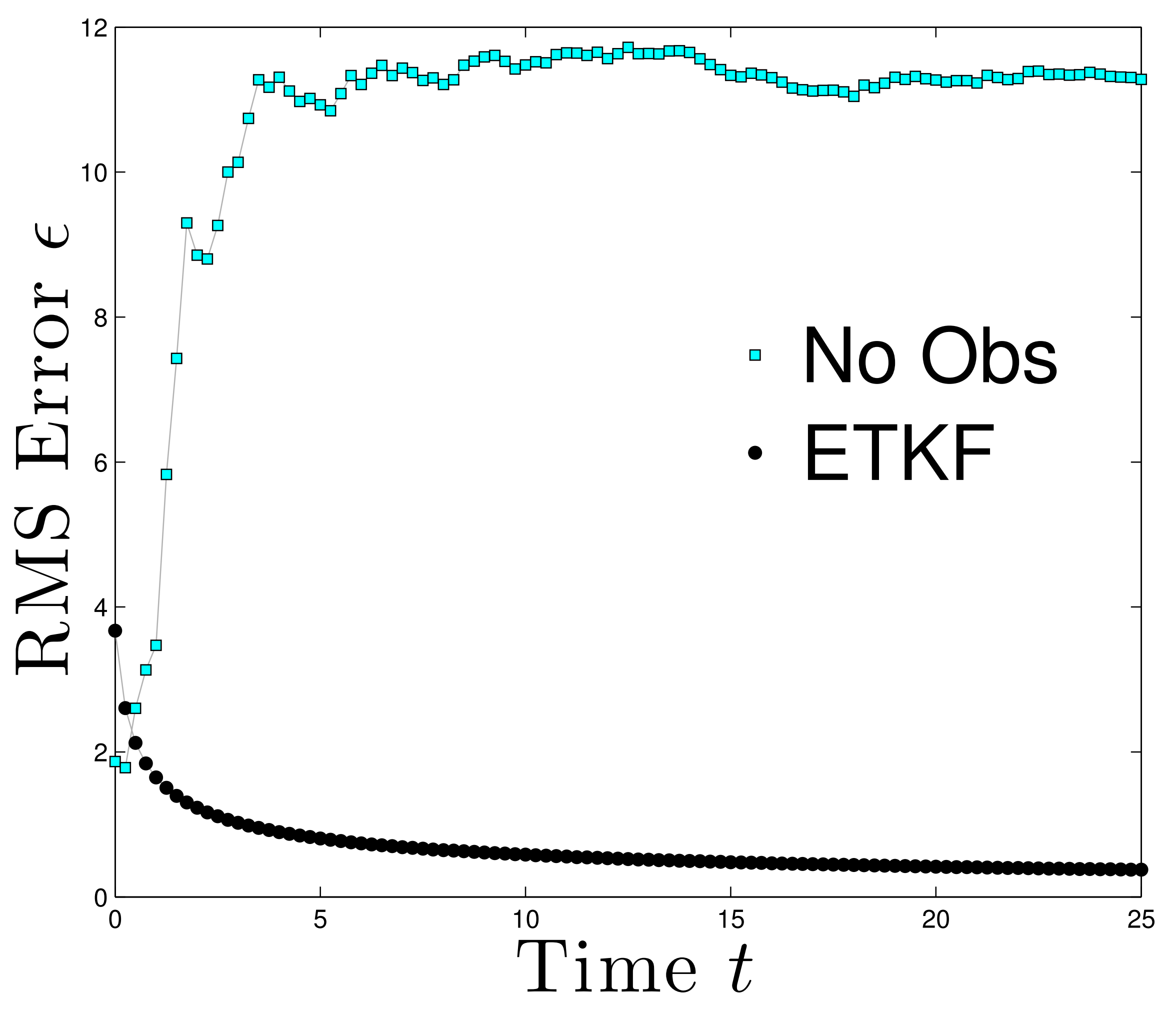
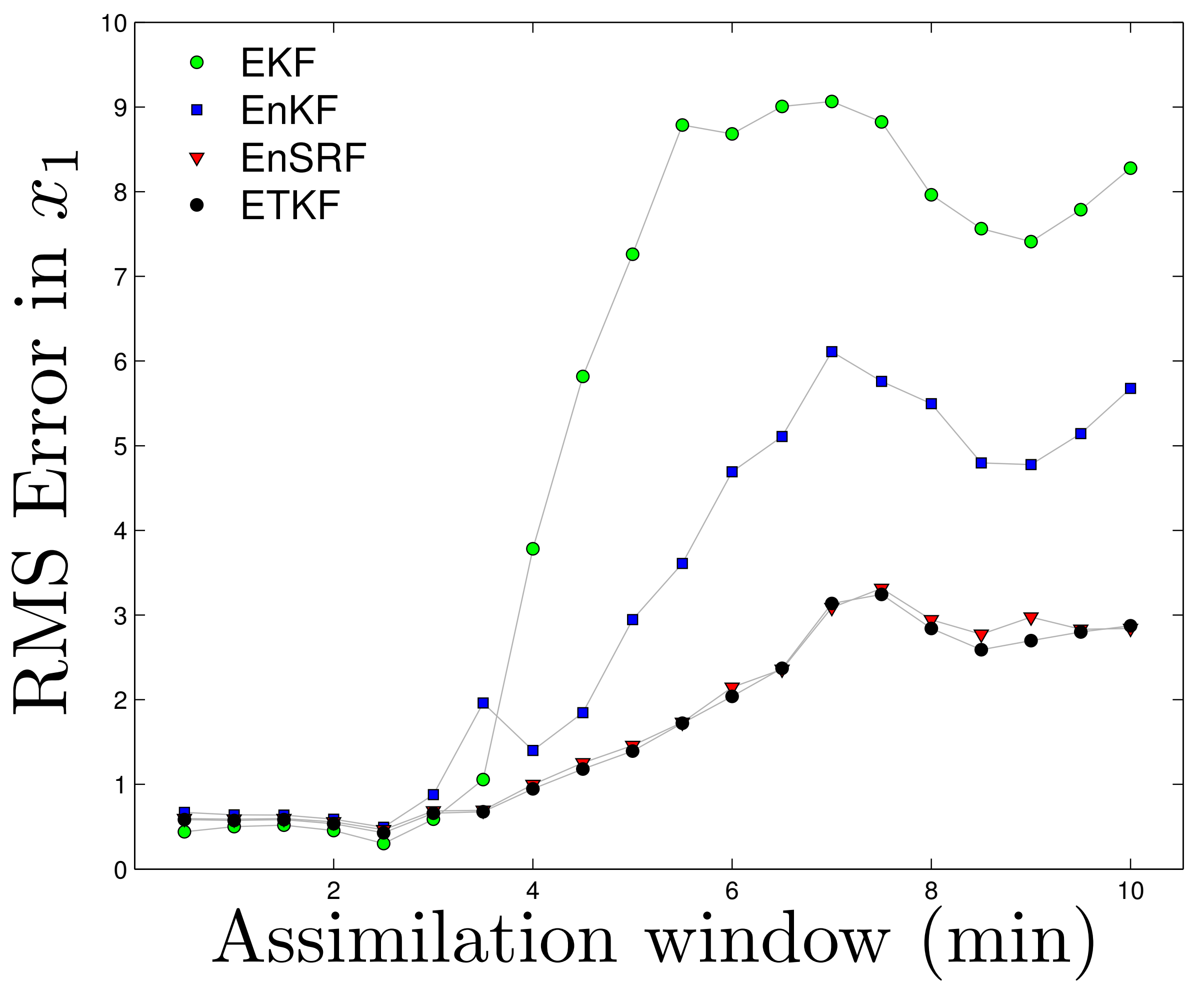


CHECK





data > no data

more > less

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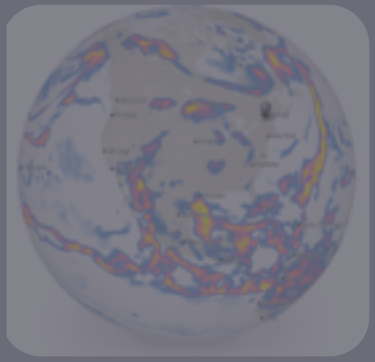

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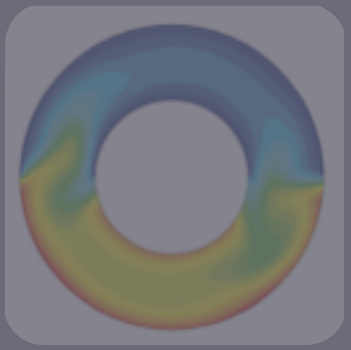

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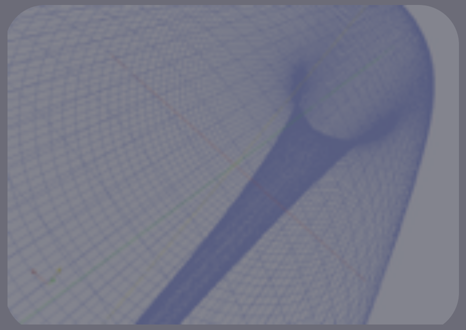

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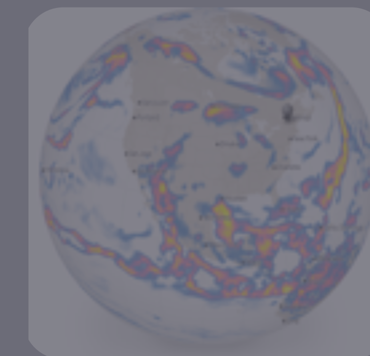

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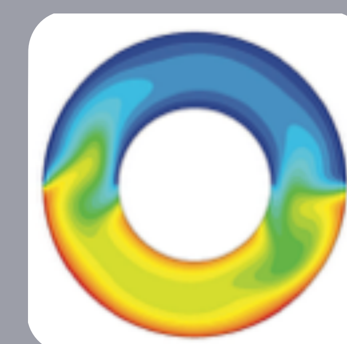

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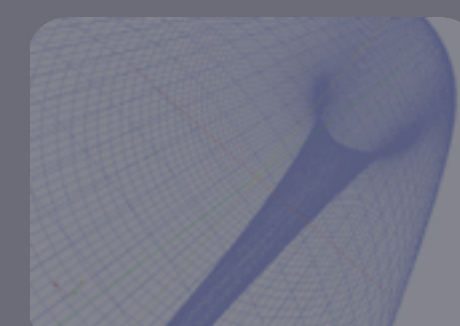

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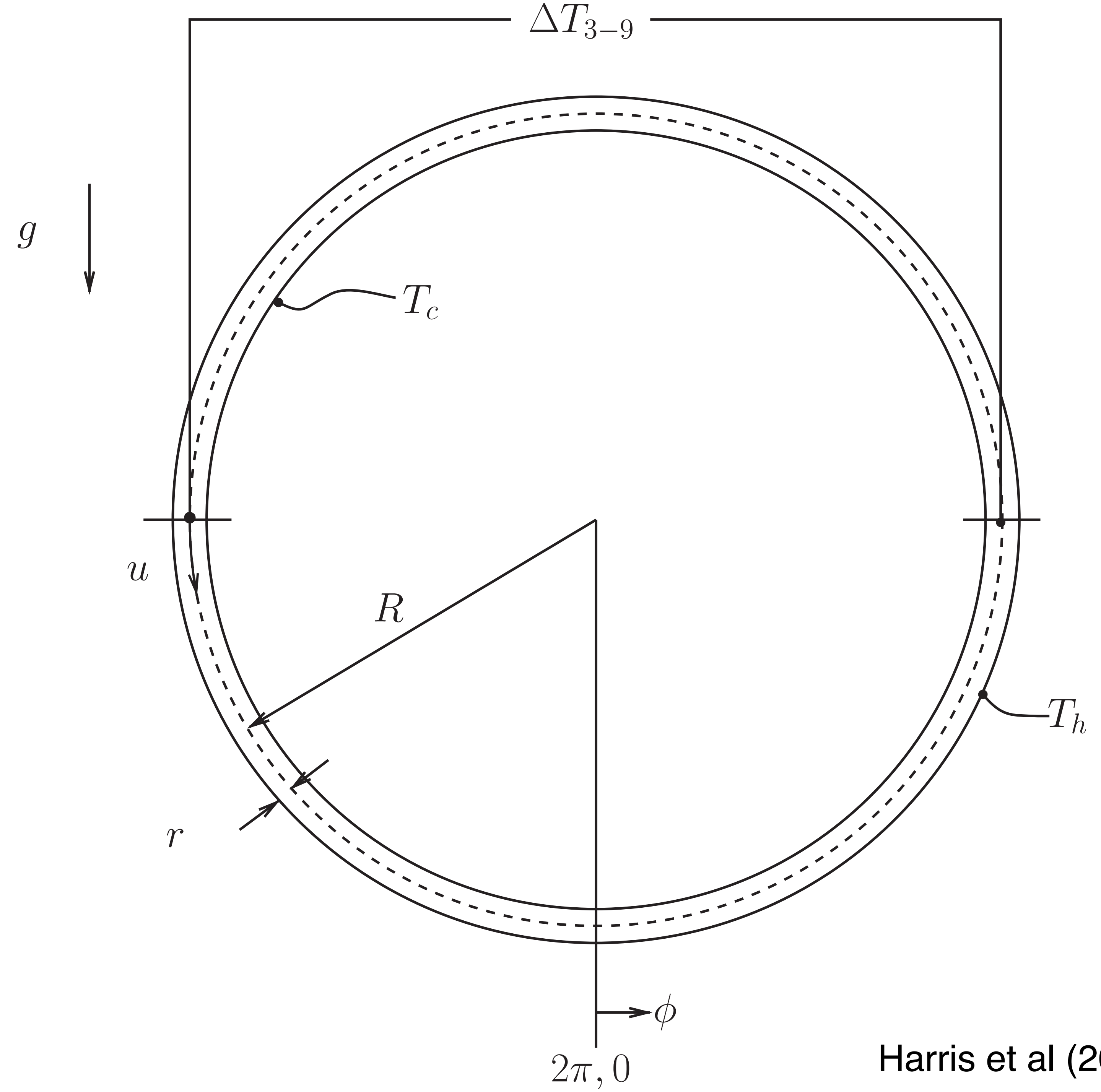
GENERAL SCHEMATIC



Harris et al (2012)

THERMOSYPHONS IN THE WILD

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Professor Chris Danforth with the loop

IT'S CHAOTIC


Computational Story Lab
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Chaos in an Atmosphere Hanging on a Wall
Thermal Convection Loop
University of Vermont
Played at x10 speed

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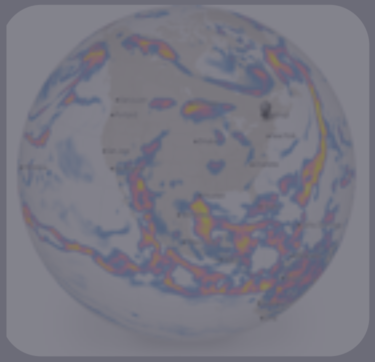

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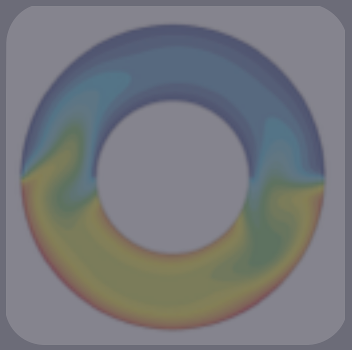

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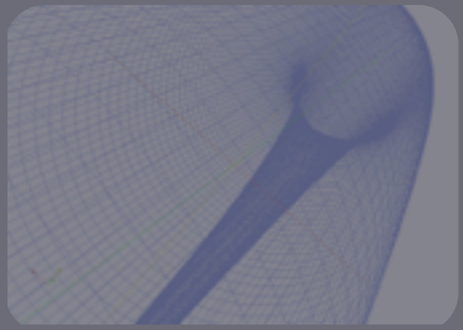

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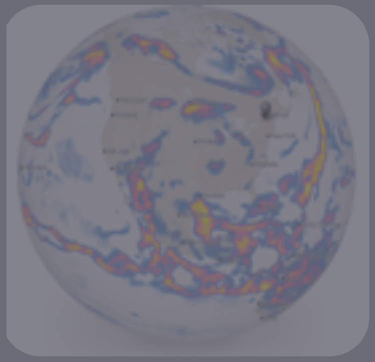

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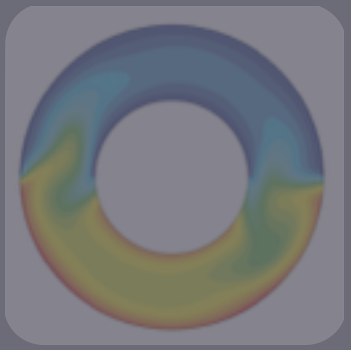

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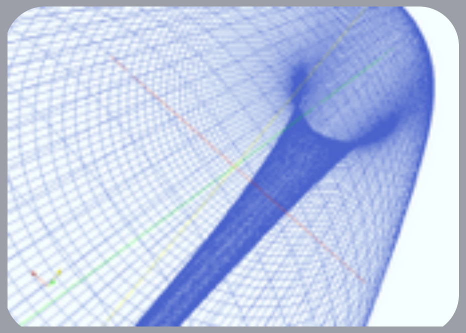

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NAVIER-STOKES EQUATIONS

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$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_j}\left[\rho u_j\right] = 0$$

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Navier-Stokes Equations

$$\frac{\partial}{\partial x_j} u_j = 0$$

$$\rho \frac{\partial u_i}{\partial t} + \rho \frac{\partial}{\partial x_j}(u_j u_i) = -\frac{\partial p}{\partial x_i} + \mu \frac{\partial u_i}{\partial x_j^2} + \rho g_i$$

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$$\frac{\partial}{\partial t}(\rho e) + \frac{\partial}{\partial x_j}(\rho e u_j) = -k \frac{\partial T}{\partial x_k^2}$$

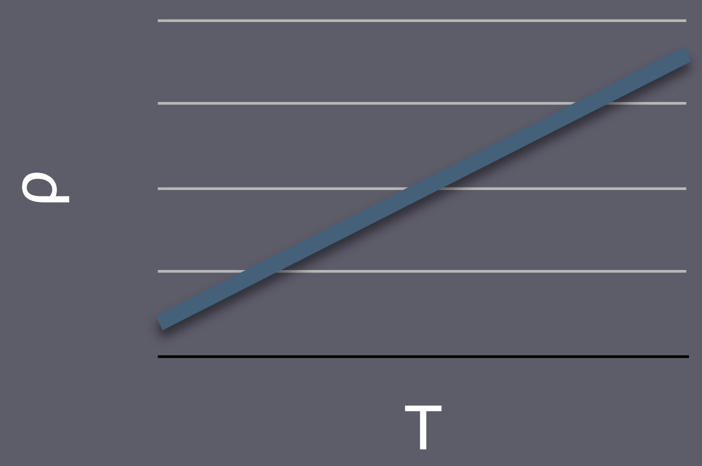
BOUSSINESQ APPROX

J. Boussinesq, Gauthier-Villars et fils (1897)

BOUSSINESQ APPROX

(1) density changes are linear with temperature

$$\rho_k = 1 - \beta(T - T_{\mathrm{ref}})$$



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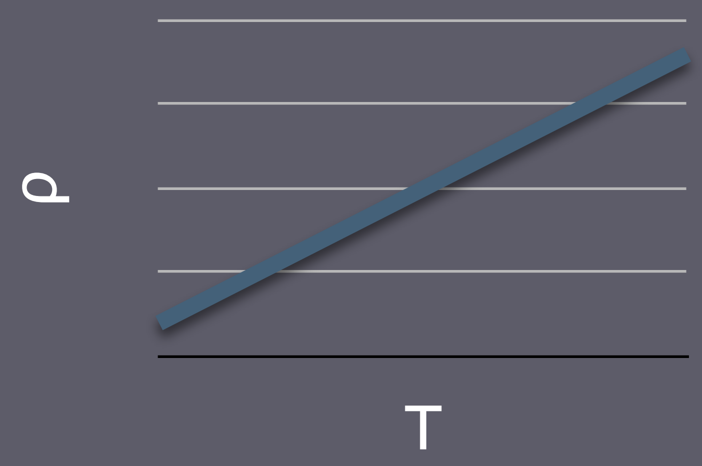

(2) we neglect their effect except when multiplied by gravity, so $\rho = \rho_0$

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SPATIAL DISCRETIZATION: MESHING

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blockMeshDict

```
convertToMeters 0.1; // specifications in .1 meters

vertices
(
    // 16 vertices for the loop (x y z)
    // top four
    (0 0 34.5) //0
    (1.5 0 36) //1
    (0 0 37.5) //2
    (-1.5 0 36) //3
...

blocks
(
    // block 1, upper left
    hex (6 5 4 7 2 1 0 3) (30 30 300) simpleGrading (1 1 1)
...

// define the arcs of the loop
edges
(
    // top circle python blockMeshDictHelper.py arc 0 0 34.5 1.5 0 36 0 0 37.5 -1.5 0 36 0 1 2 3
    arc 0 1 (1.0606601717798212,0.0,34.93933982822018)
    arc 1 2 (1.0606601717798212,0.0,37.06066017177982)
    arc 2 3 (-1.0606601717798212,0.0,37.06066017177982)
    arc 3 0 (-1.0606601717798212,0.0,34.93933982822018)
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...
```
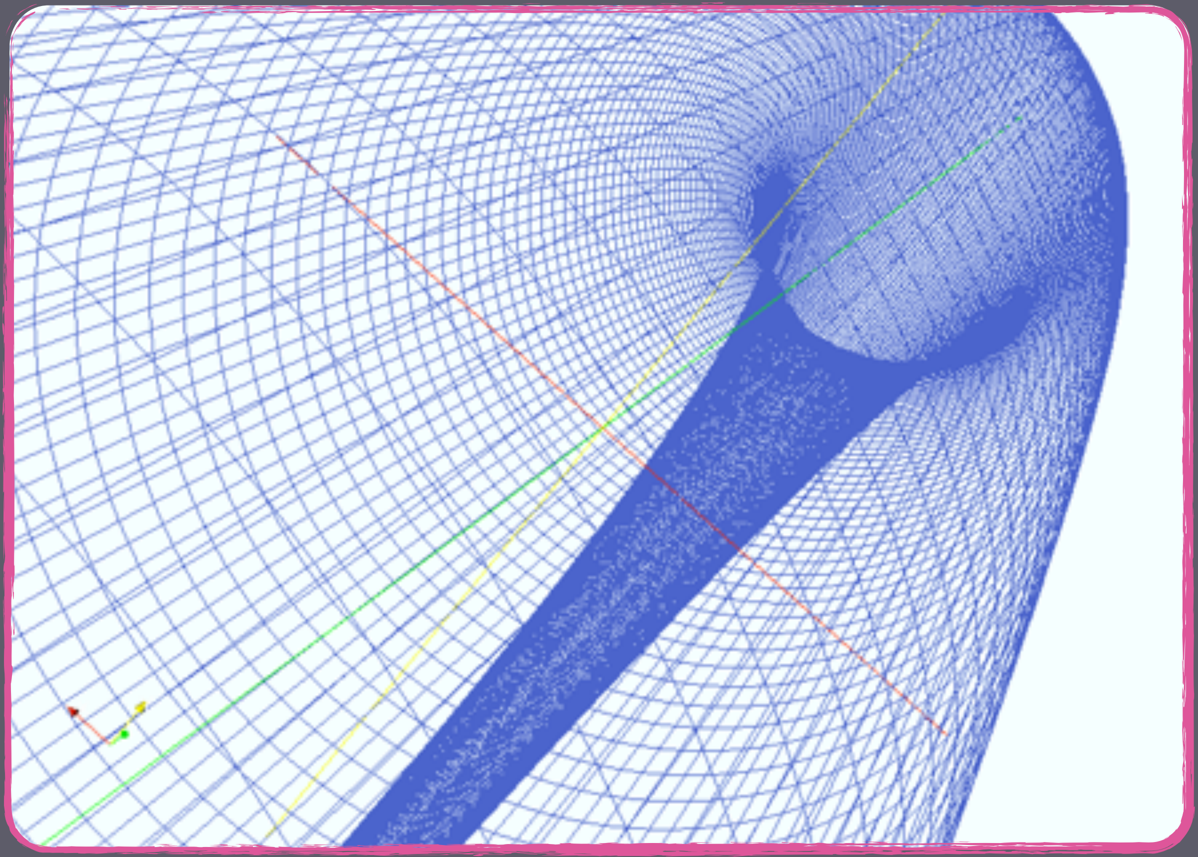
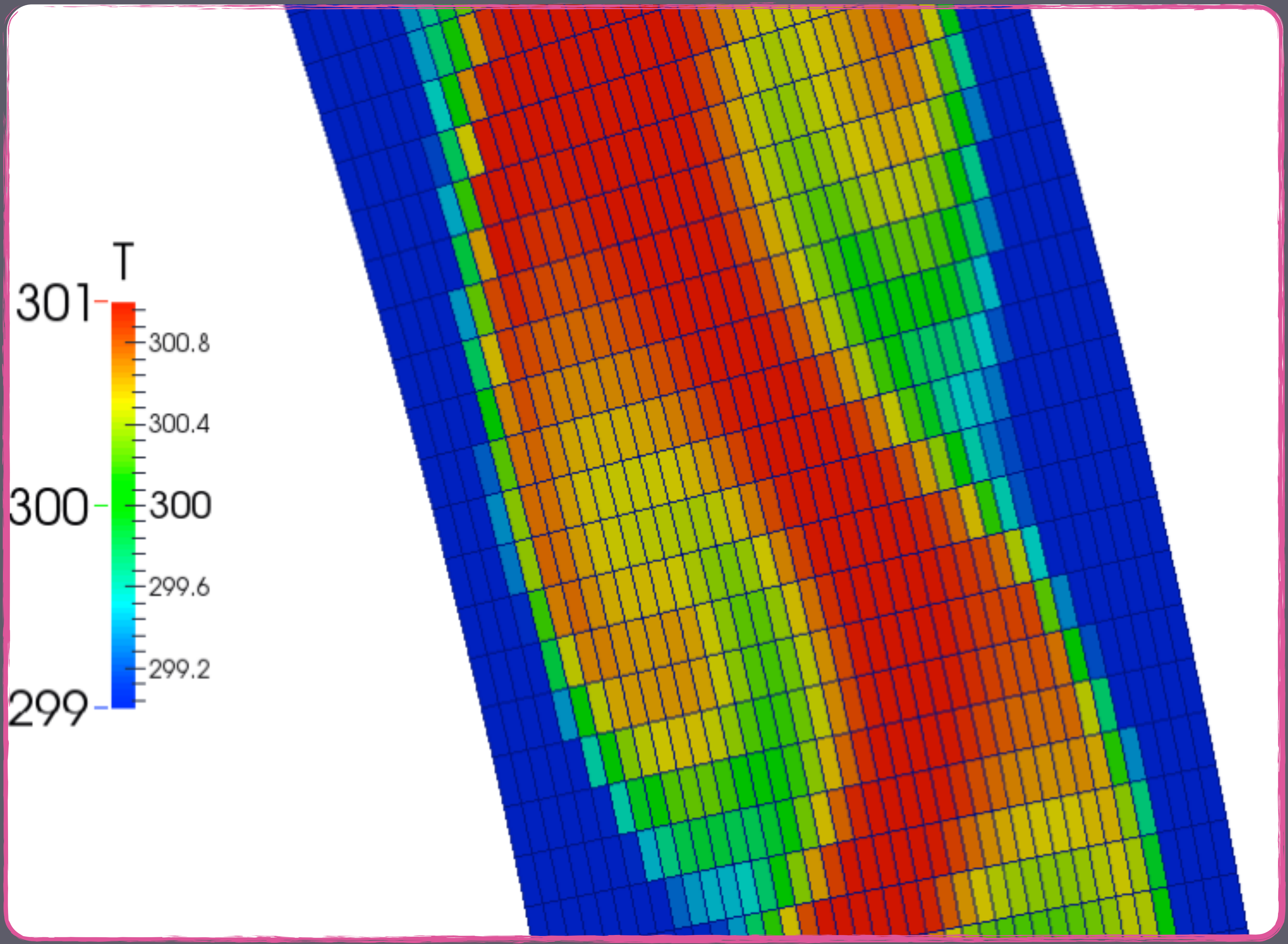

3D Mesh



SIMPLIFY SLIGHTLY



SOLVING: OPENFOAM

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OpenFOAM is an open source C++ library of CFD solvers

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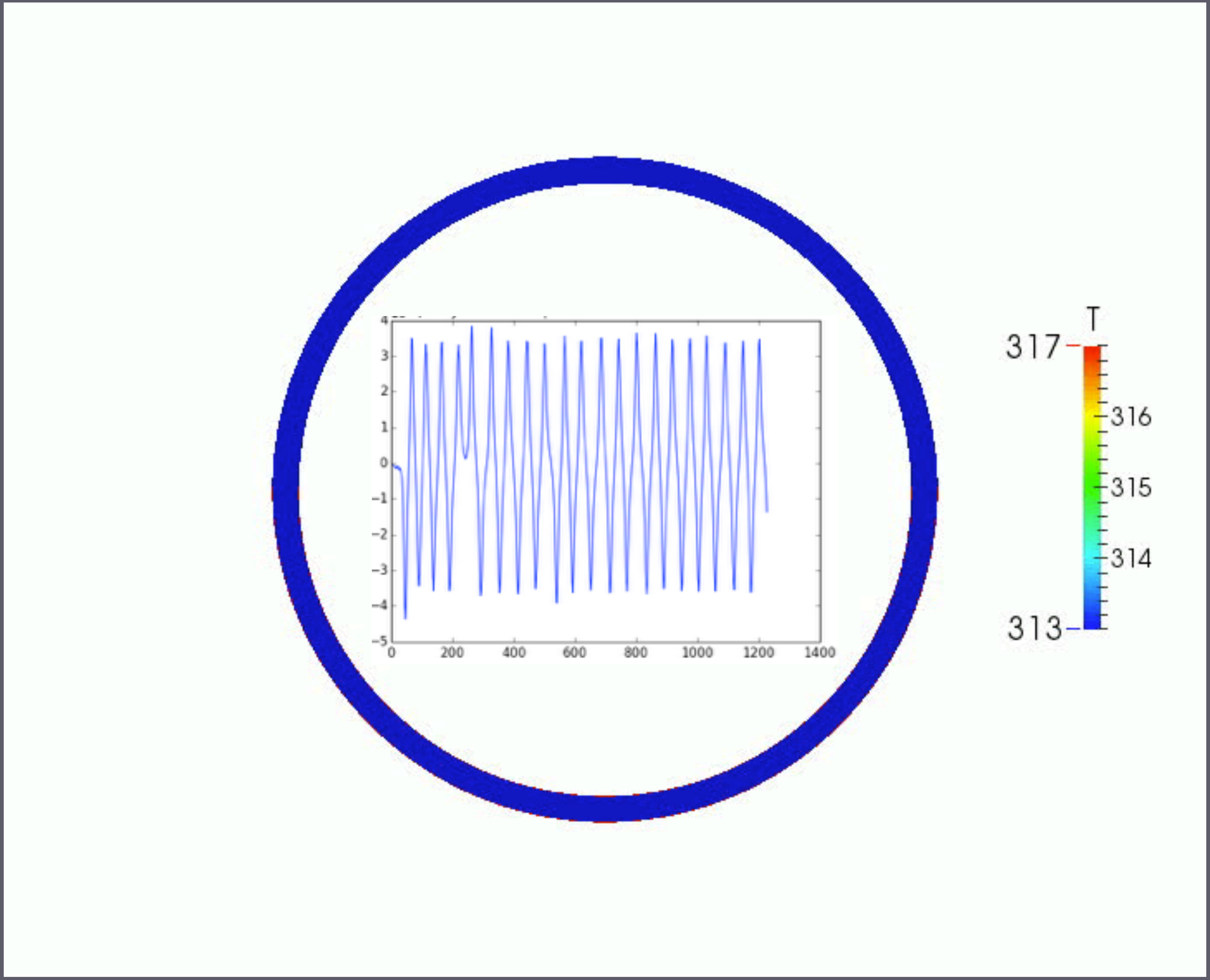
BUOYANTBOUSSINESQ PIMPLEFOAM

READY, SET, ACTION



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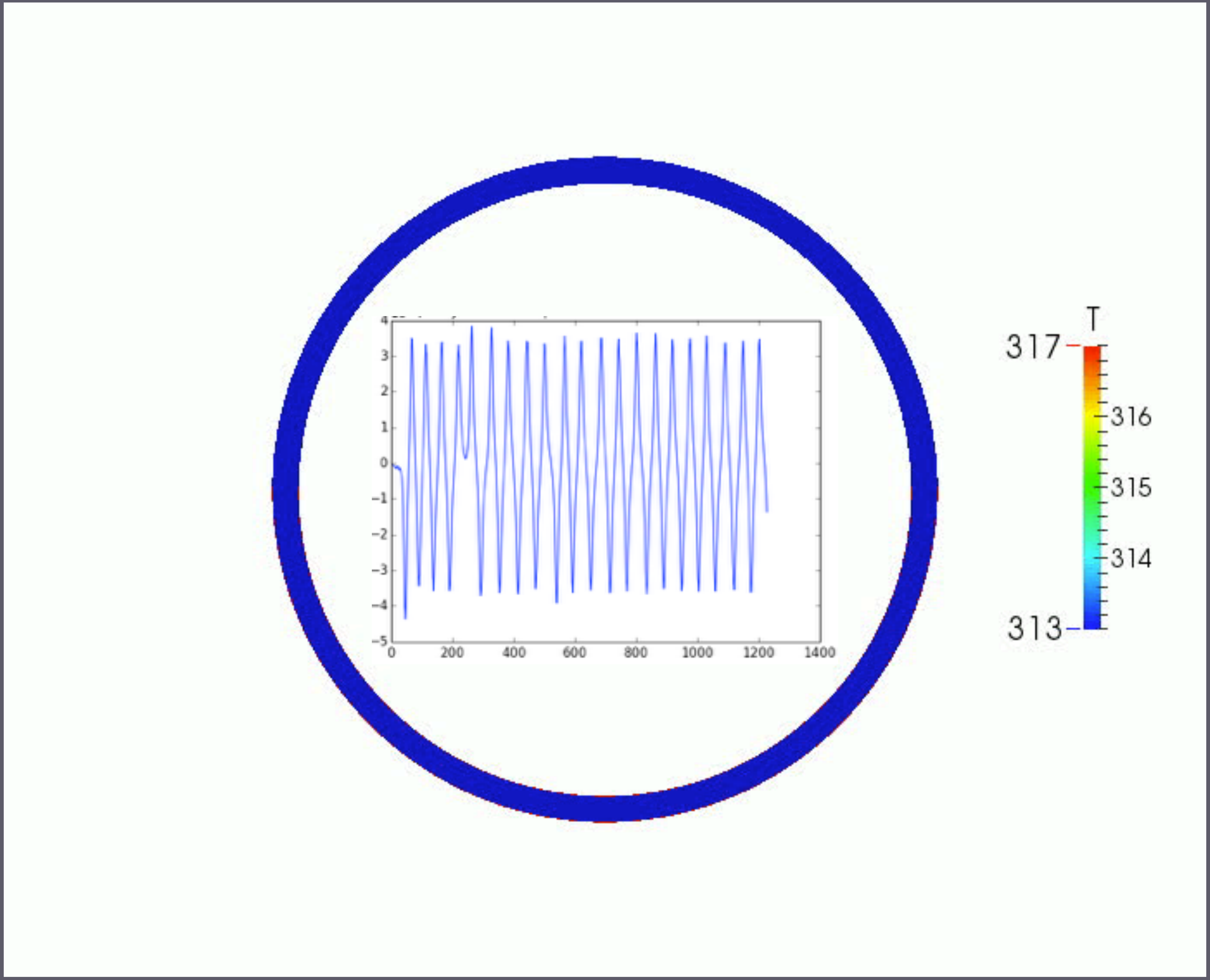
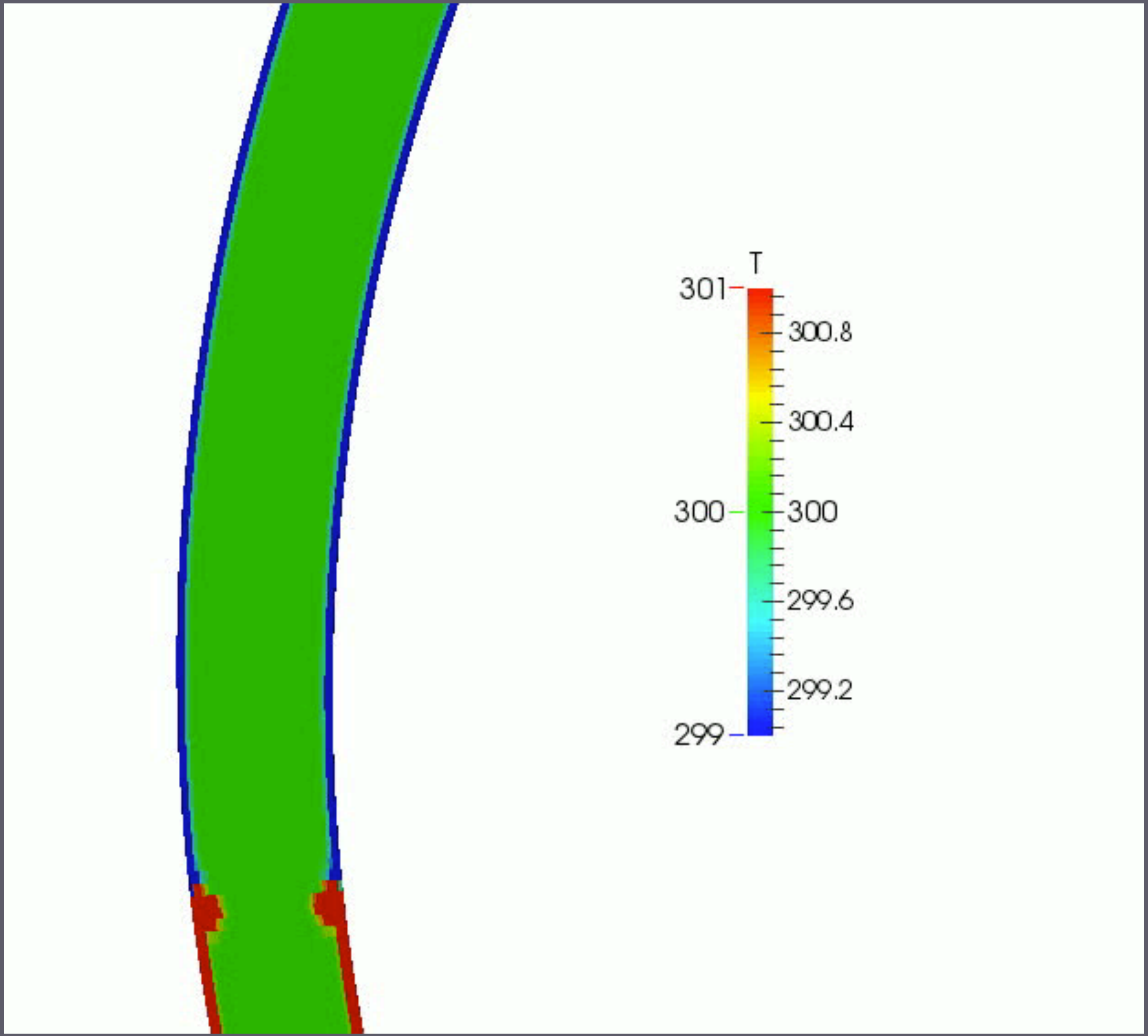
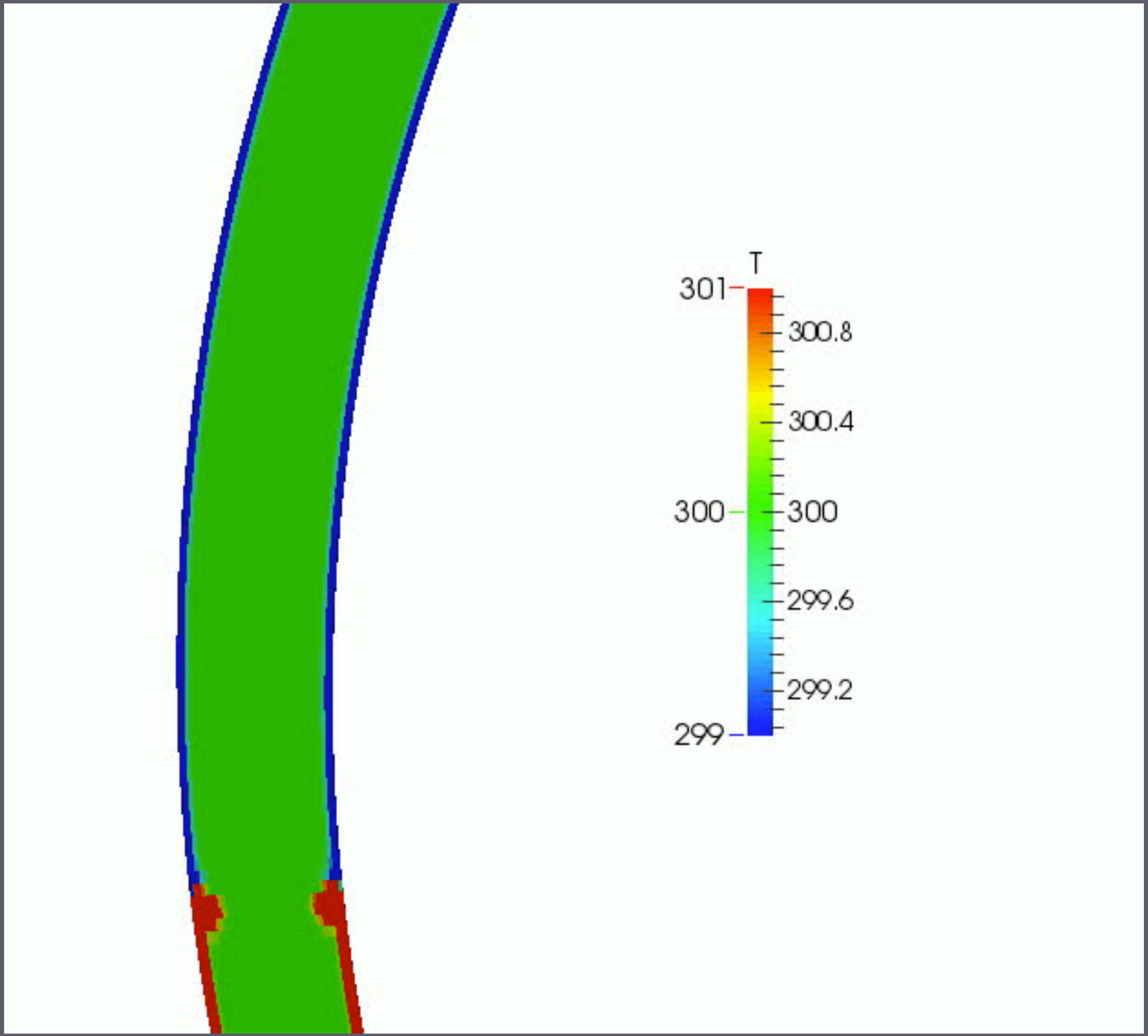



T
301
300.8
300.4
300
300
299.6
299.2
299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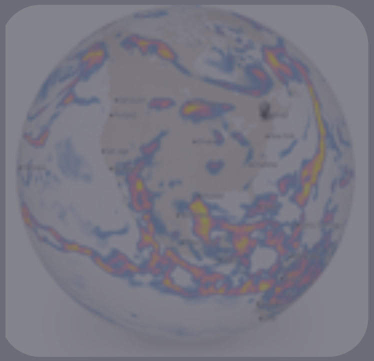
300
299.6
299.2
299

MAIN IDEAS

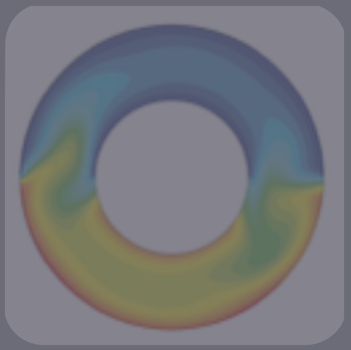
DATA ASSIMILATION

Prediction and where data assimilation fits in



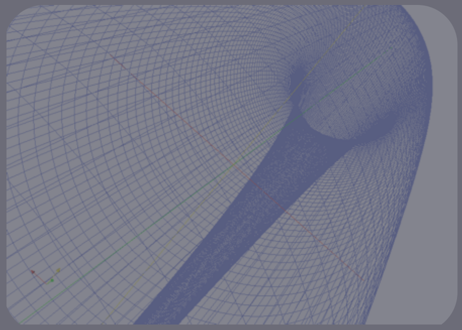
THERMOSYPHON AS MODEL ATMOSPHERE

An experiment that exhibits deterministic nonperiodic flow



MODELING THE THERMOSYPHON

A brief introduction to computational fluid dynamics



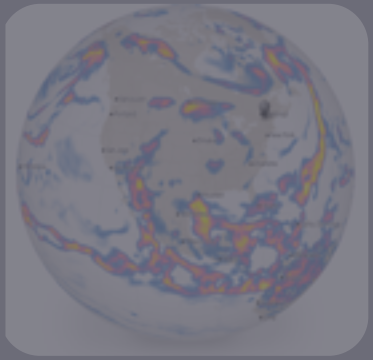
MAIN RESULTS

Using data assimilation to predict flow in the thermosyphon

MAIN IDEAS

DATA ASSIMILATION

Prediction and where data assimilation fits in

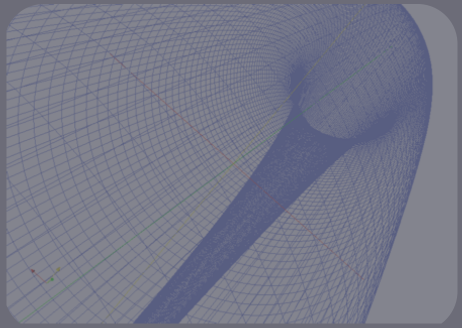


THERMOSYPHON AS MODEL ATMOSPHERE

An experiment that exhibits deterministic nonperiodic flow

MODELING THE THERMOSYPHON

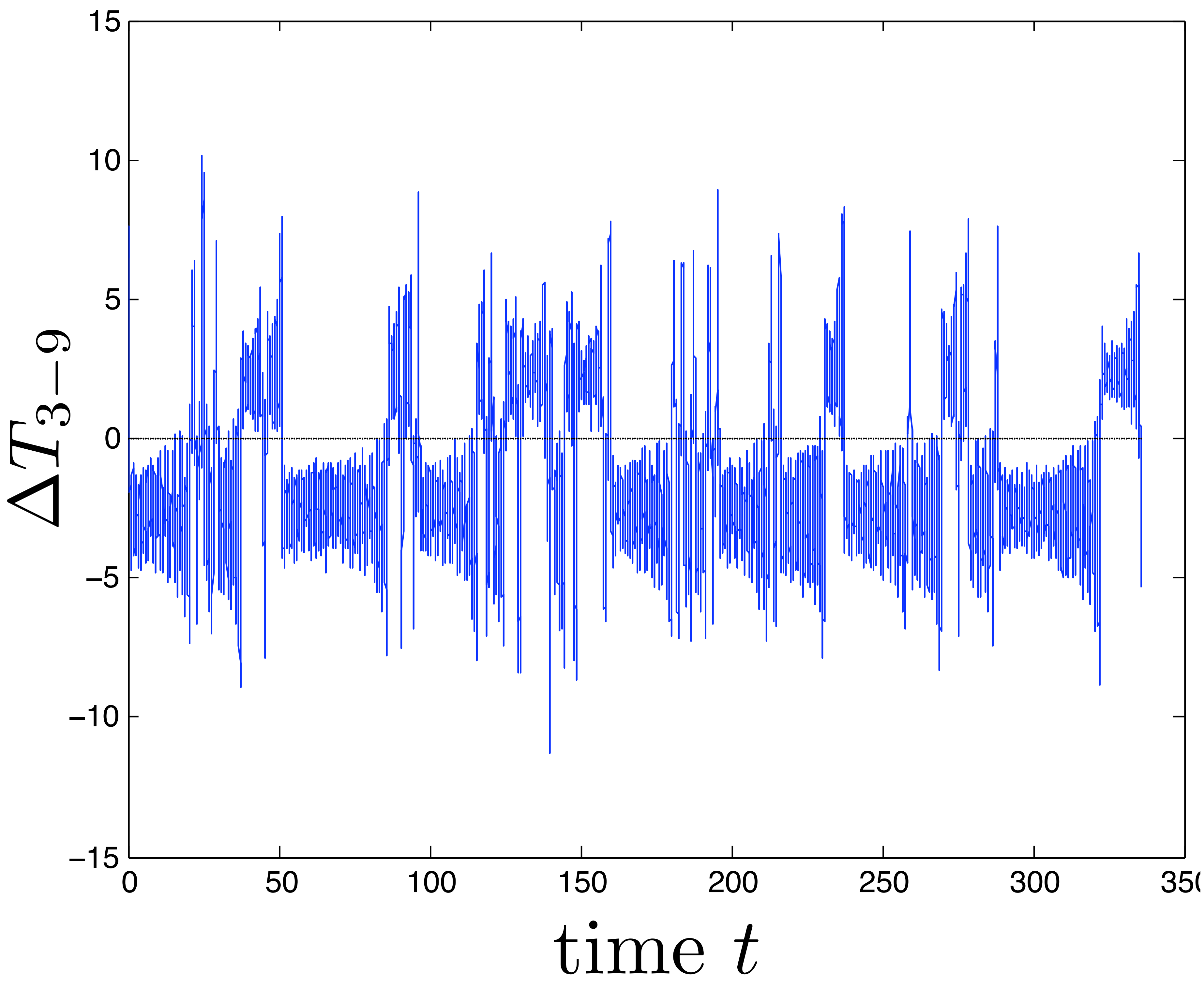
A brief introduction to computational fluid dynamics



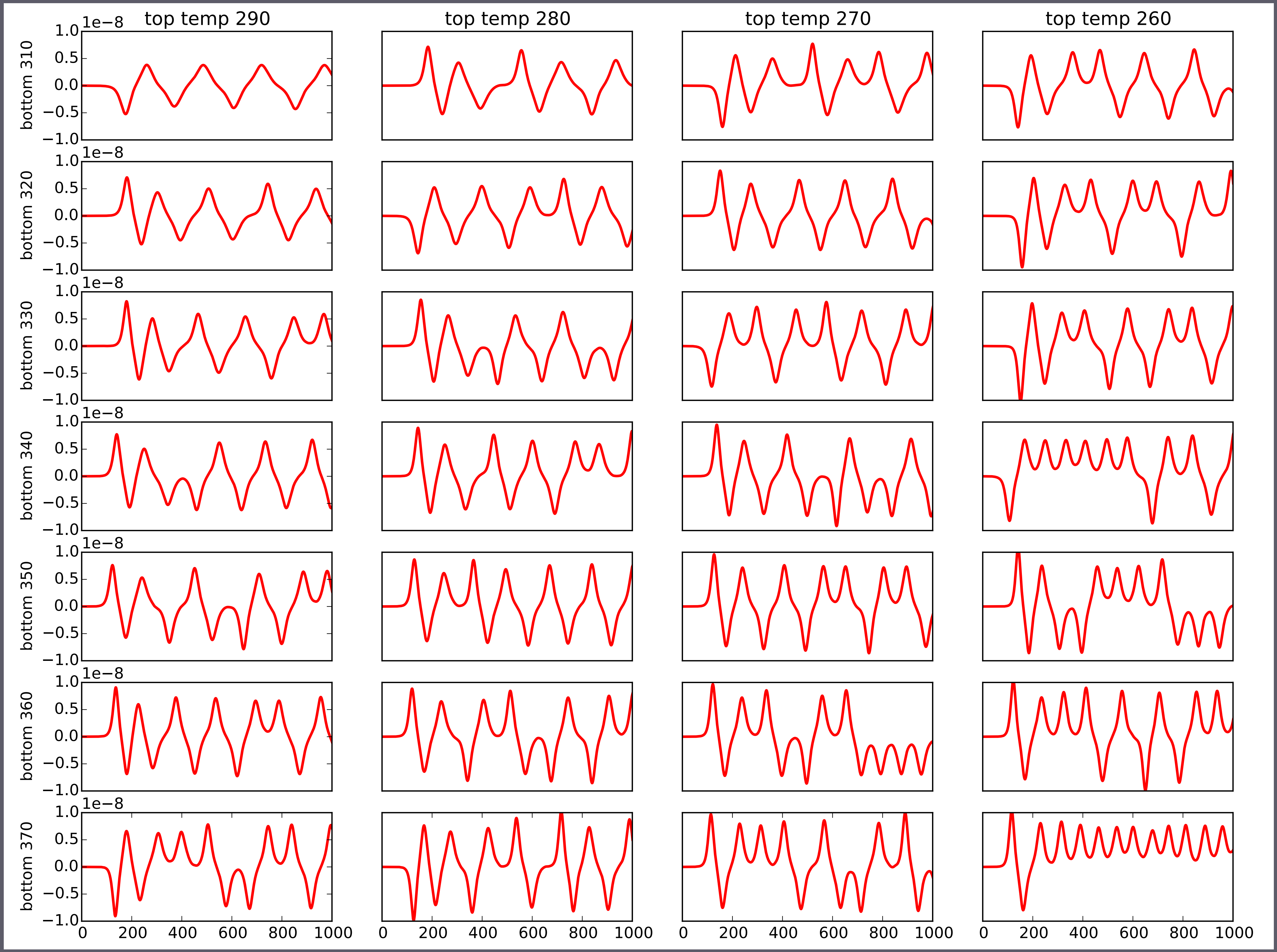
MAIN RESULTS

Using data assimilation to predict flow in the thermosyphon

EXPERIMENTAL DATA



SYNTHETIC TEMP DATA



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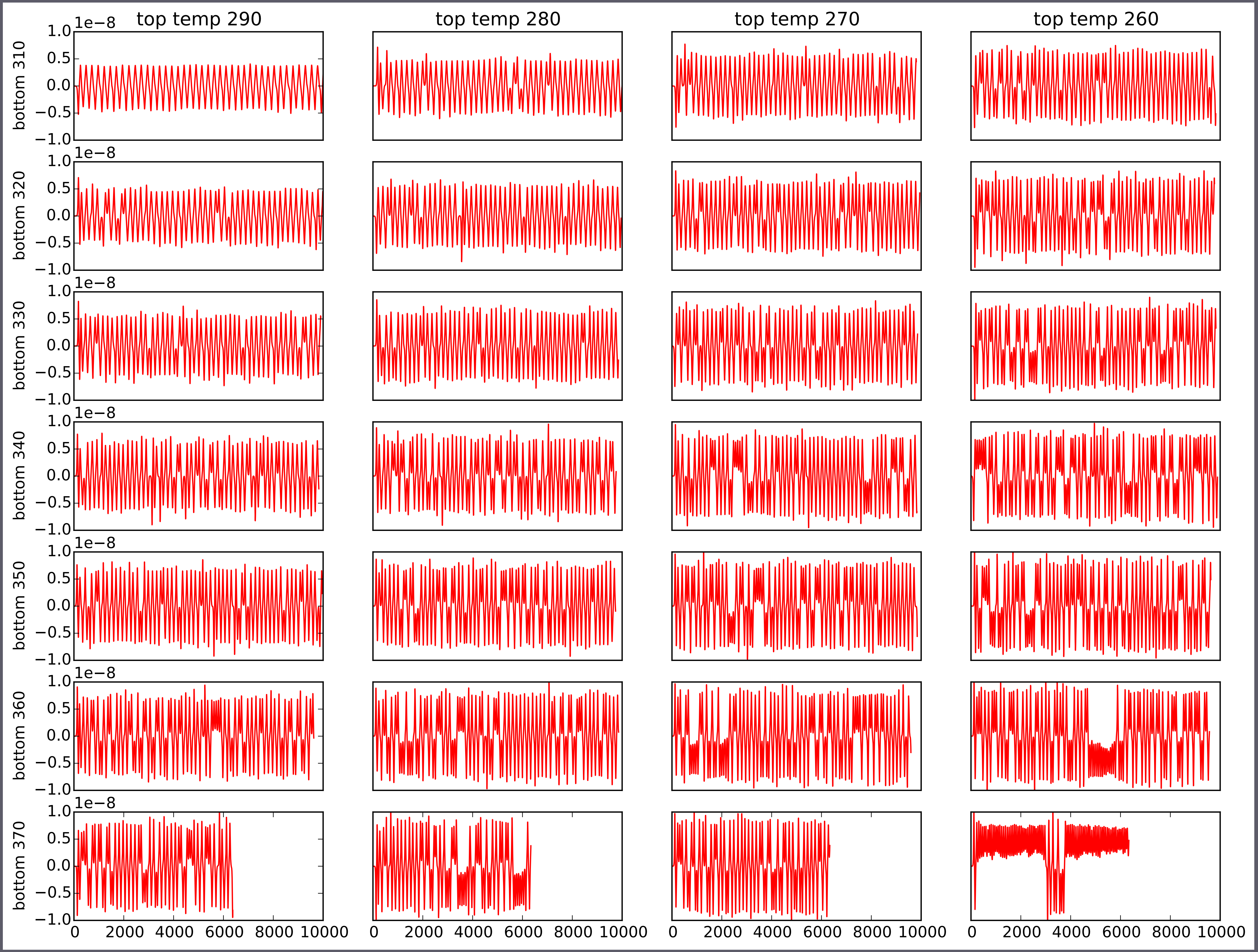
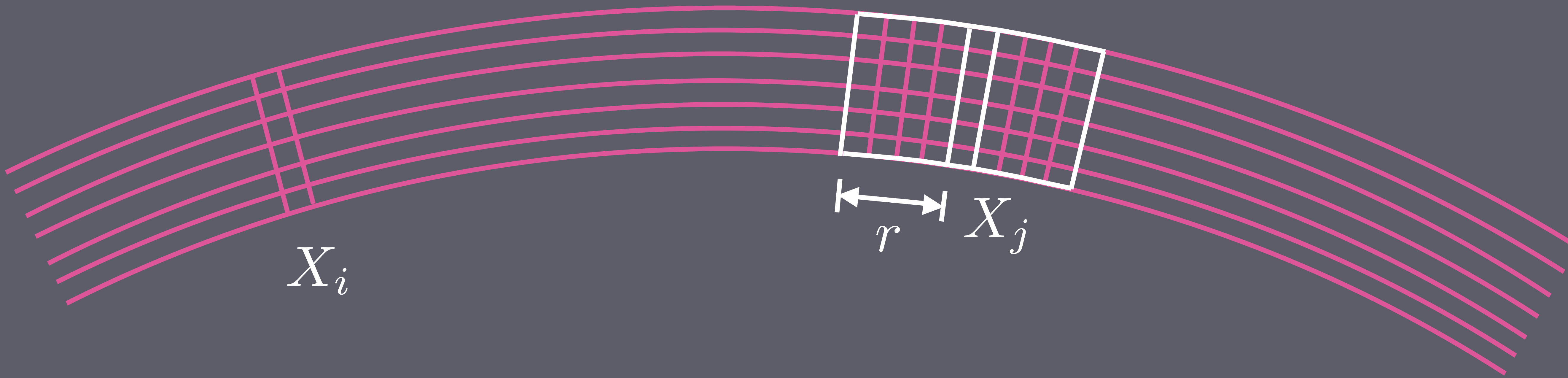
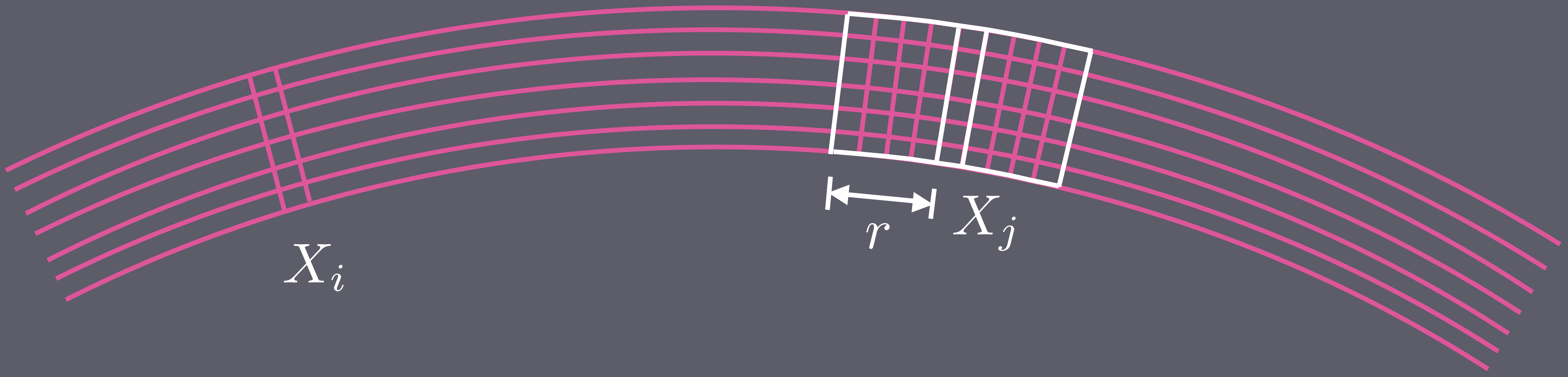



X_i
r
X_j



X_i
r
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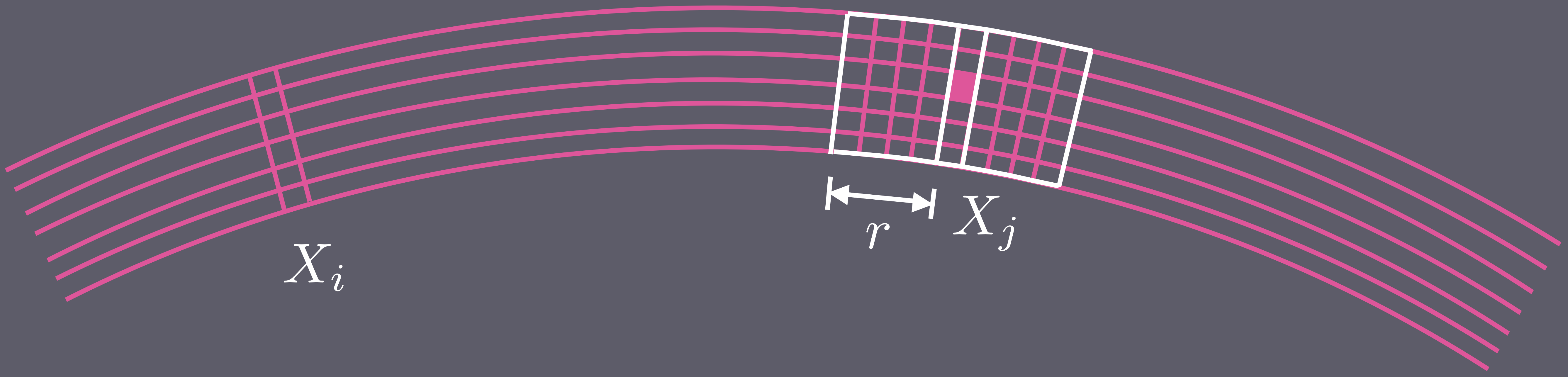
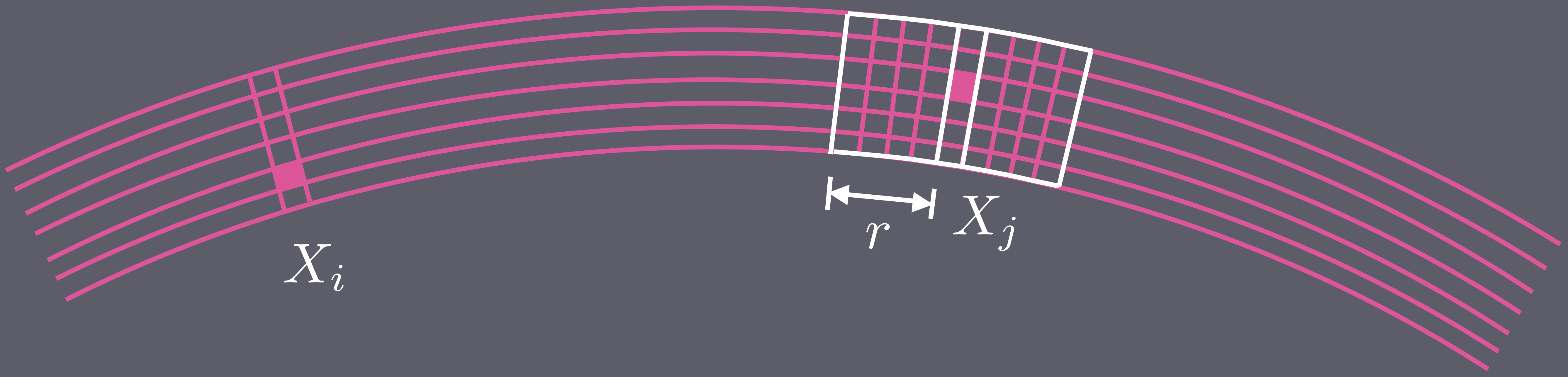


r
X_j
X_i



X_i
r
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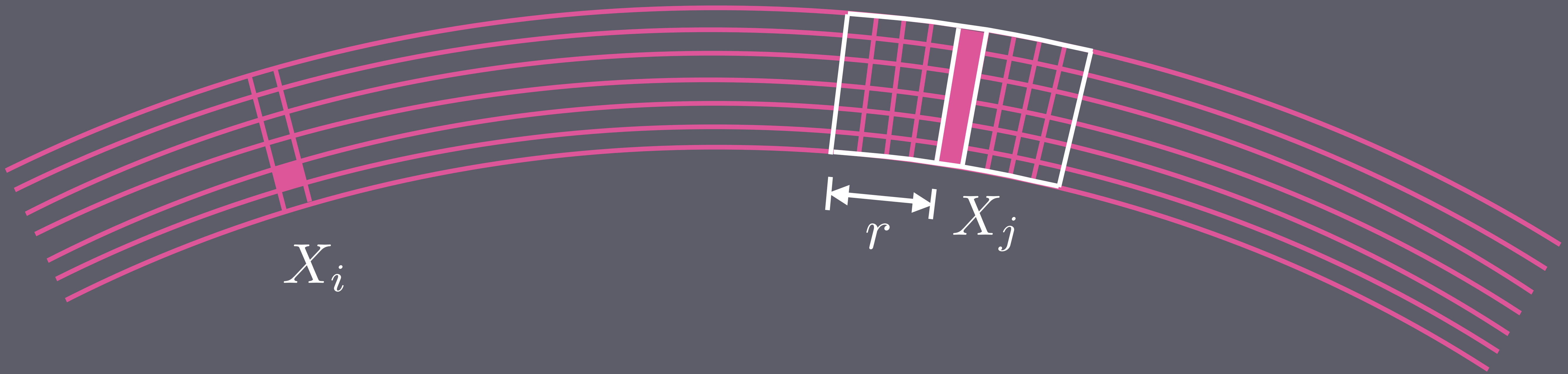
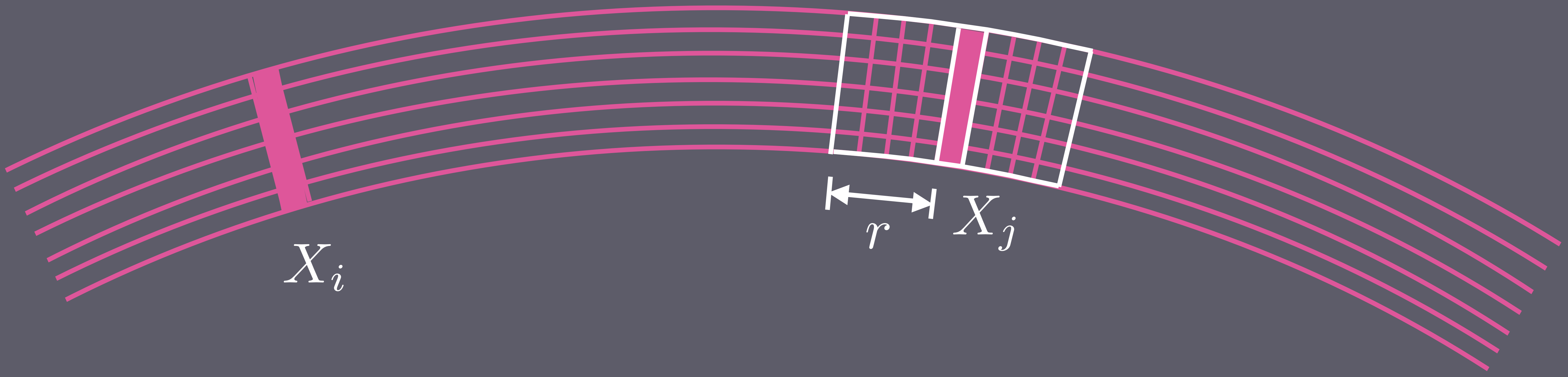
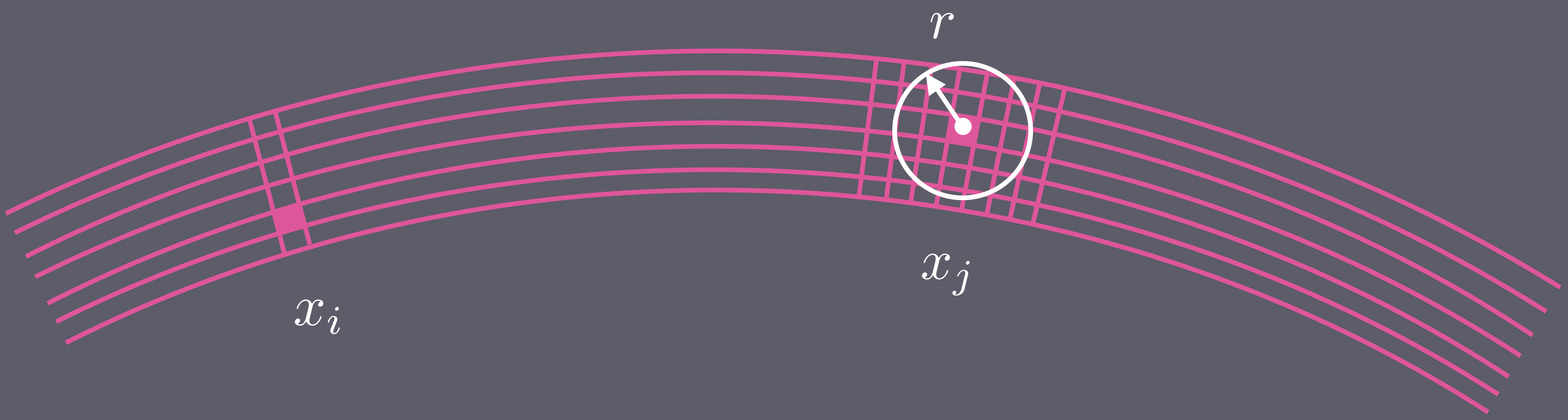


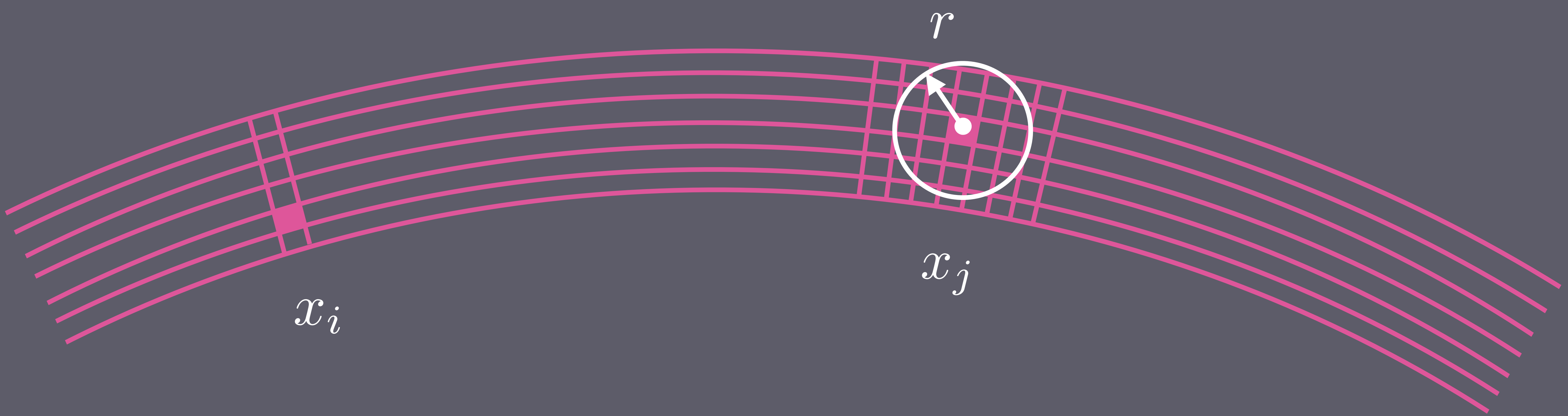
X_i
r
X_j



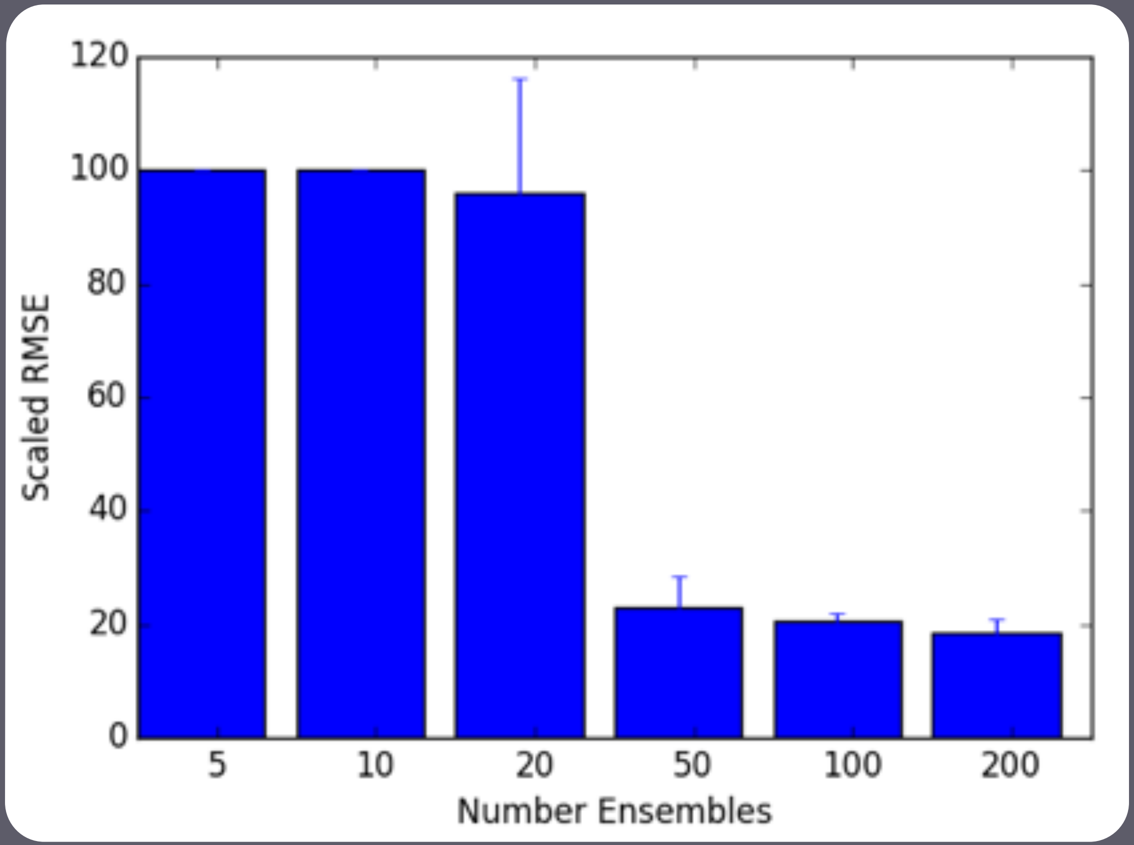
X_i
r
X_j

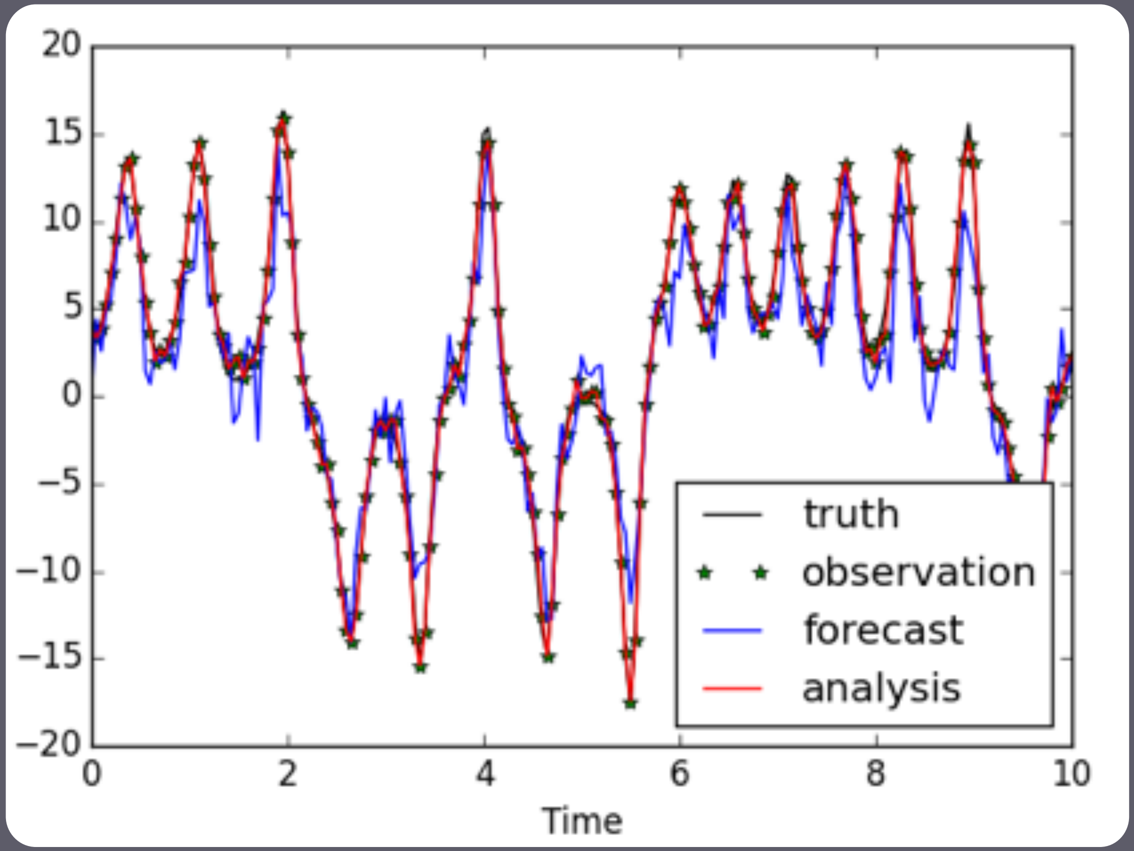


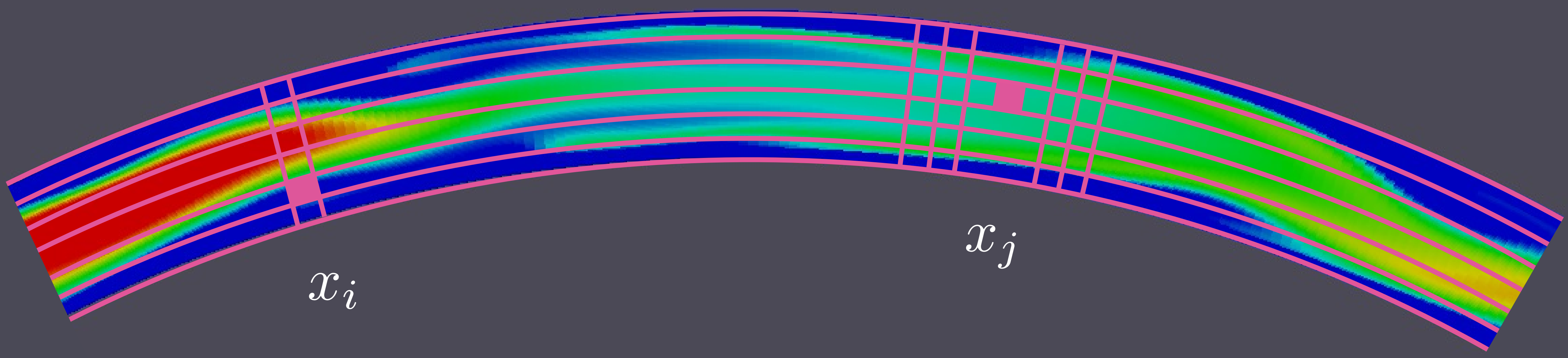
r
x_j
x_i


r
x_j
x_i

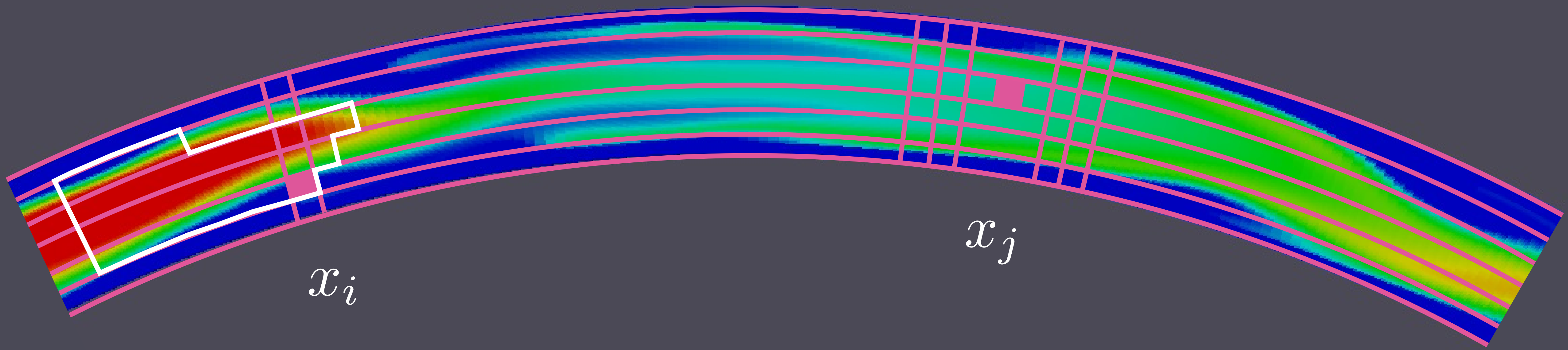
TWIN MODEL EXPERIMENT





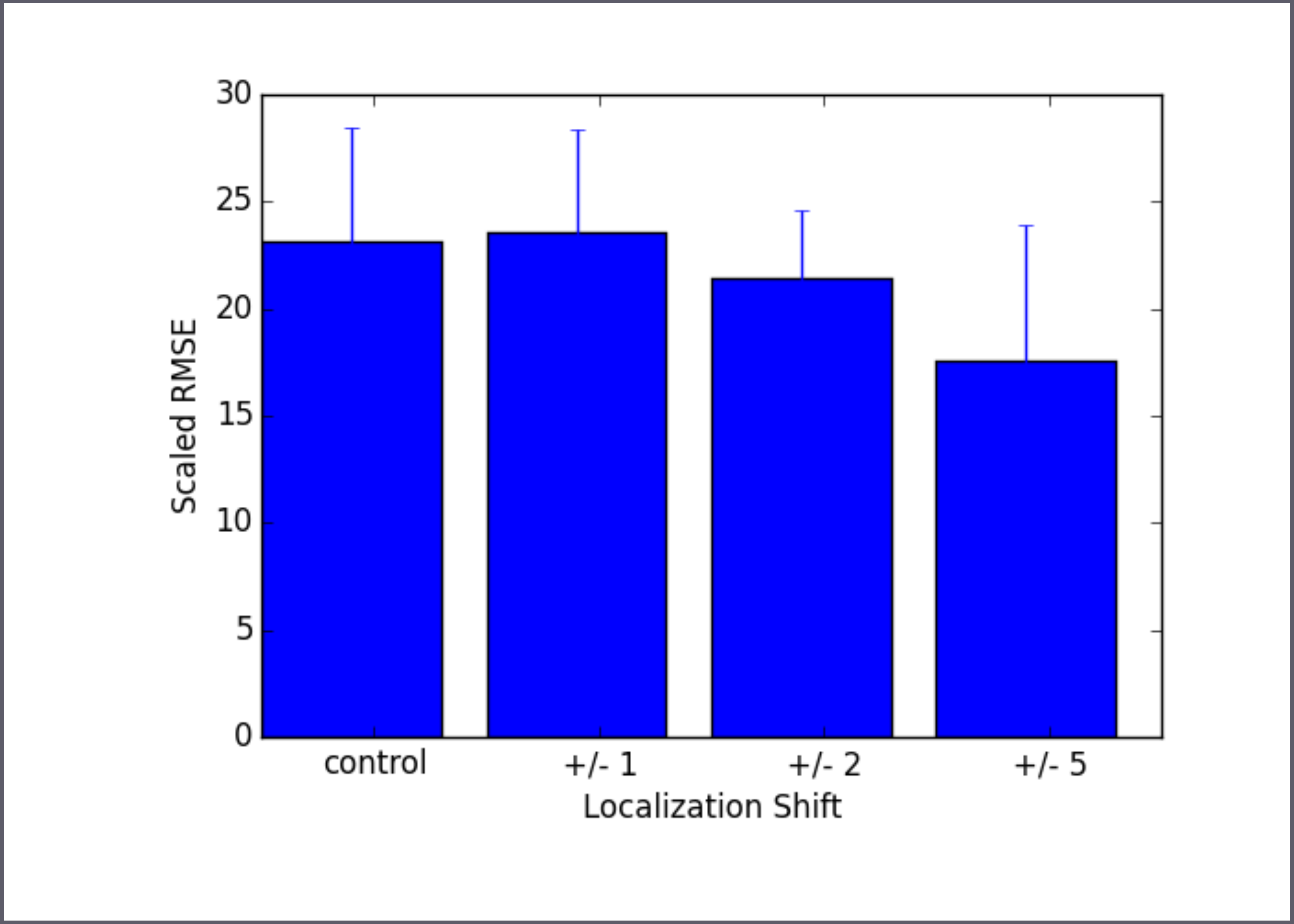


x_i
x_j



x_i
x_j

Adaptive Localization Helps



FUTURE WORK

FUTURE WORK

Tune adaptive localization

FUTURE WORK

Tune adaptive localization

Work towards real-time prediction

THANK YOU