

the figure, to correct the velocity and height estimates. Values for the gains are selected to allow the baro-inertial system to follow the long-term variations in the barometric measurements, whilst filtering out any higher frequency fluctuations. Typically,  $K_1 = 2/T$  and  $K_2 = 1/T^2$ , where the value of  $T$  may be 30 s [14]. In the integrated system, a bias ( $B_z$ ) on the inertial estimate of vertical acceleration no longer propagates as a height error with time squared, but settles to a steady state value of  $T^2 B_z$ . Hence, a bias of 100 micro-g gives rise to a height error of approximately 1 m. The major limitation of this technique is that any longer-term errors in the barometric altimeter, resulting from weather conditions and the position of the device, persist in the integrated system.

As with all filtering techniques, the objective is to make use of the available knowledge about the long-term behaviour of a signal, contaminated with noise, in order to derive a better estimate of the signal than could be obtained using a single measurement. A practical form of filter, which is applicable for on-line estimation, relies on generating a mathematical model of the process that is producing the signal, and adjusting the parameters of the model to minimise the mean square deviation between the signal and the output of the model. A best estimate of the signal is derived based on knowledge of the expected errors in the model and the measured signal using a Kalman filter. Kalman filtering has become a well-established technique for the mixing of navigation data in integrated systems. It is particularly suitable for on-line estimation, being a recursive technique which lends itself to implementation in a computer.

The principles of Kalman filtering are described in Appendix A and its application is illustrated in the following section.

## **13.6 Application of Kalman filtering to aided inertial navigation systems**

### *13.6.1 Introduction*

As described in Appendix A, Kalman filtering involves the combination of two estimates of a variable to form a weighted mean, the weighting factors being chosen to yield the most probable estimate. One estimate is provided by updating a previous best estimate, in accordance with the known equations of motion, whilst the other is obtained from a measurement. In an integrated navigation system, the first estimate is provided directly by the inertial navigation system, that is, in filtering terms, the inertial system constitutes the model of the physical process that produces the measurement. The second estimate, the measurement, is provided by the navigation aid. The same technique can be applied irrespective of the source of measurement information.

A generalised block diagram representation is shown in Figure 13.14, and a design example is described in the following section.

However, the measurements provided by navigation aids are often non-linear combinations of the inertial navigation system estimates. Additionally, the inertial system equations themselves are non-linear, which means that a modified approach

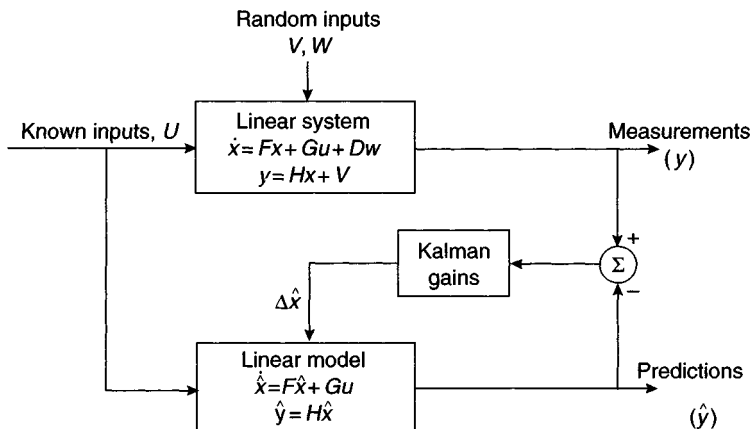


Figure 13.14 Kalman filter for linear systems

is needed. Consequently, it is customary to use an extended Kalman filter formulation for an aided inertial navigation system (see Appendix A).

### 13.6.2 Design example of aiding

In this section, a scheme for aiding a hypothetical missile on-board inertial navigation system is described, which relies upon tracking the missile during flight using a sensor on the launch platform. Suitable measurements may be provided by a multi-function radar or an infrared tracker in combination with a laser range-finder. In either case, it is assumed that measurements of a missile's range, elevation and bearing with respect to the chosen navigation frame will be provided. These measurements may be passed via an uplink transmitter to the missile and used to aid the on-board navigation system.

The transmitted measurements are combined, using a Kalman filter, with the measurements provided by the missile's inertial navigation system. This not only allows improved estimates of the missile's position to be derived from noisy measurement data, but also provides a means of estimating errors in the states of the navigation system which are not directly measurable; the velocity and attitude estimates for example. The form of the filter is as shown in Figure 13.15.

In order to provide estimates of the three attitude errors, the three velocity errors and the three position errors in the on-board inertial navigation system, a nine state Kalman filter is required. The associated system and measurement equations are described in the sections which follow.

#### 13.6.2.1 The system equations

To formulate an extended Kalman filter to update the on-board navigation system, it is necessary to develop a linear dynamic model of the errors that are to be estimated. For the purposes of this design example, a simplified version of the error model given

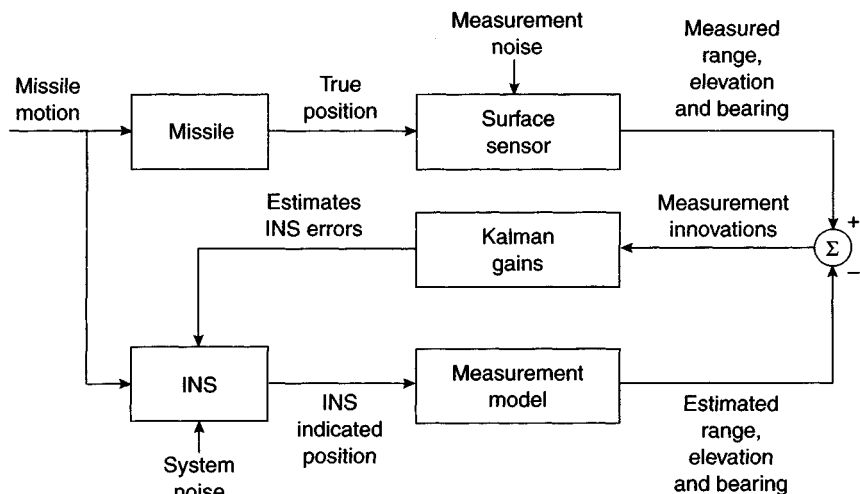


Figure 13.15 INS aiding using ground tracker measurements

in Chapter 12 may be used. The error model may be expressed in matrix form as:

$$\delta \dot{\mathbf{x}} = \mathbf{F} \delta \mathbf{x} + \mathbf{G} \mathbf{w} \quad (13.1)$$

The vector  $\delta \mathbf{x}$  represents the error state of the system. For the purposes of this illustration,  $\delta \mathbf{x}$  consists of the three attitude errors ( $\delta \alpha \ \delta \beta \ \delta \gamma$ ), three velocity errors, ( $\delta v_x \ \delta v_y \ \delta v_z$ ) and three position errors ( $\delta x \ \delta y \ \delta z$ ).

In order to allow a discrete Kalman filter to be constructed, it is necessary to express the system error eqn. (13.1) in discrete form. If  $\delta \mathbf{x}_k$  represents the inertial navigation system error states at time  $t_k$ , and  $\delta \mathbf{x}_{k+1}$  the error states at time  $t_{k+1}$ , we may write:

$$\delta \mathbf{x}_{k+1} = \Phi_k \delta \mathbf{x}_k + \mathbf{w}_k \quad (13.2)$$

where  $\Phi_k$  is the system transition matrix at time  $t_k$ , which may be expressed in terms of the system matrix  $\mathbf{F}$  as follows:

$$\Phi_k = \exp[\mathbf{F}(t_{k+1} - t_k)] \quad (13.3)$$

### 13.6.2.2 The measurement equations

Measurements of the missile's position with respect to the radar are assumed to be available at discrete intervals of time throughout flight. The radar provides measurements in polar coordinates, that is, measurements of range ( $R$ ), elevation ( $\theta$ ) and bearing ( $\psi$ ). The polar quantities may be expressed in terms of the

Cartesian coordinates  $(x, y, z)$  as follows:

$$\begin{aligned} R^2 &= x^2 + y^2 + z^2 \\ \theta &= \arctan \left\{ \frac{z}{\sqrt{x^2 + y^2}} \right\} \\ \psi &= \arctan \left( \frac{y}{x} \right) \end{aligned} \quad (13.4)$$

Writing  $\mathbf{z} = [R \quad \theta \quad \psi]^T$ , the radar measurements, denoted by  $\tilde{\mathbf{z}}$ , may be expressed as:

$$\tilde{\mathbf{z}} = \mathbf{z} + \mathbf{n} \quad (13.5)$$

where  $\mathbf{n}$  represents the error in the measurements.  $\mathbf{n}$  is assumed to be a zero mean, Gaussian white-noise process.

Estimates of the radar measurements,  $\hat{\mathbf{z}}$ , may be obtained from the inertial navigation system estimates of position  $(x, y, z)$  as follows:

$$\hat{\mathbf{z}} = \begin{pmatrix} \hat{R} \\ \hat{\theta} \\ \hat{\psi} \end{pmatrix} = \begin{pmatrix} \sqrt{\hat{x}^2 + \hat{y}^2 + \hat{z}^2} \\ \arctan \left\{ \frac{\hat{z}}{\sqrt{\hat{x}^2 + \hat{y}^2}} \right\} \\ \arctan \left\{ \frac{\hat{y}}{\hat{x}} \right\} \end{pmatrix} \quad (13.6)$$

The difference between the radar measurements ( $\tilde{\mathbf{z}}$ ) and the estimates ( $\hat{\mathbf{z}}$ ) of those quantities is referred to as the filter measurement innovation ( $\delta\mathbf{z}$ ) and is generated as follows:

$$\delta\mathbf{z} = \tilde{\mathbf{z}} - \hat{\mathbf{z}} = \mathbf{H}\delta\mathbf{x} \quad (13.7)$$

where

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \frac{x}{R} & \frac{y}{R} & \frac{z}{R} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-xz}{R^2\sqrt{x^2 + y^2}} & \frac{-yz}{R^2\sqrt{x^2 + y^2}} & \frac{\sqrt{x^2 + y^2}}{R^2} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-y}{x^2 + y^2} & \frac{x}{x^2 + y^2} & 0 \end{bmatrix} \quad (13.8)$$

### 13.6.2.3 The Kalman filter

Equations (13.2) and (13.7) are the system and measurement equations needed to construct a Kalman filter. The equations for the Kalman filter, given in Appendix A, take the following form for the radar-aided inertial system considered here.

### Filter prediction step

Following each measurement update, the inertial navigation system is corrected using the current best estimates of the errors in position, velocity and attitude. Therefore, after an update, the best estimate of each of the inertial system errors becomes identically zero and the state prediction equation reduces to:

$$\delta \mathbf{x}_{k+1/k} = 0 \quad (13.9)$$

The covariance matrix is predicted forward in time using the expression:

$$\mathbf{P}_{k+1/k} = \Phi_k \mathbf{P}_{k/k} \Phi_k^T + \Delta \mathbf{Q}' \Delta^T \quad (13.10)$$

where  $\Phi_k$  is the transition matrix given by eqn. (13.3).  $\mathbf{P}_{k+1/k}$  denotes the expected value of the covariance matrix at time  $t_{k+1}$  predicted at time  $t_k$ . It is set up initially as a diagonal matrix, the individual elements being chosen according to the expected variances of the errors in the initial attitude, velocity and position passed to the missile navigation inertial system prior to launch.  $\mathbf{Q}'$ , the system noise matrix, is set up according to the expected level of noise on the inertial measurements of linear acceleration and angular rate.

### Filter update

The estimates of the errors in the inertial navigation system states are derived using:

$$\delta \mathbf{x}_{k+1/k+1} = \mathbf{K}_{k+1} \delta \mathbf{z}_{k+1} \quad (13.11)$$

and the covariance matrix is updated according to:

$$\mathbf{P}_{k+1/k+1} = [\mathbf{I} - \mathbf{K}_{k+1} \mathbf{H}_{k+1}] \mathbf{P}_{k+1/k} \quad (13.12)$$

where

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1/k+1} \mathbf{H}_{k+1}^T [\mathbf{H}_{k+1} \mathbf{P}_{k+1/k} \mathbf{H}_{k+1}^T + \mathbf{R}']^{-1} \quad (13.13)$$

$\mathbf{H}$  is defined by eqn. (13.8) and  $\mathbf{R}'$ , the measurement noise is a  $3 \times 3$  diagonal matrix, the elements of which are selected in accordance with the anticipated level of radar measurement noise.

### Inertial navigation system correction

The inertial navigation states,  $\hat{\mathbf{x}}$ , are corrected immediately after each measurement update using the current best estimates of the errors. The correction equations are given below.

*Velocity and position correction.* Velocity and position may be corrected by simply subtracting the estimate error from the inertial system estimates of these quantities using:

$$\mathbf{x}_c = \hat{\mathbf{x}} - \delta \mathbf{x} \quad (13.14)$$

where  $\mathbf{x}_c$  is the corrected state.

*Attitude correction.* As described earlier (Chapter 11), the computed direction cosine matrix may be expressed in terms of the true matrix using:

$$\hat{\mathbf{C}} = [\mathbf{I} - \boldsymbol{\Psi}]\mathbf{C}$$

The corrected direction cosine matrix,  $\mathbf{C}_c$ , may therefore be expressed as follows:

$$\mathbf{C}_c = [\mathbf{I} + \boldsymbol{\Psi}]\hat{\mathbf{C}} \quad (13.15)$$

where  $\boldsymbol{\Psi} = \boldsymbol{\psi} \times$  and  $\boldsymbol{\psi} = [\delta\alpha \ \delta\beta \ \delta\gamma]^T$ .

By writing  $\mathbf{C}_c$  and  $\mathbf{C}$  in component form as functions of the corrected and estimated quaternion parameters, denoted  $[a_c \ b_c \ c_c \ d_c]$  and  $[\hat{a} \ \hat{b} \ \hat{c} \ \hat{d}]$ , respectively, and equating terms, it can be shown that the estimated quaternion parameters may be corrected directly using:

$$\begin{aligned} a_c &= \hat{a} + 0.5(\delta\alpha\hat{b} + \delta\beta\hat{c} + \delta\gamma\hat{d}) \\ b_c &= \hat{b} + 0.5(-\delta\alpha\hat{a} + \delta\beta\hat{d} - \delta\gamma\hat{c}) \\ c_c &= \hat{c} + 0.5(-\delta\alpha\hat{d} - \delta\beta\hat{a} + \delta\gamma\hat{b}) \\ d_c &= \hat{d} + 0.5(\delta\alpha\hat{c} - \delta\beta\hat{b} - \delta\gamma\hat{a}) \end{aligned} \quad (13.16)$$

#### 13.6.2.4 Results

The results presented in Figures 13.16 and 13.17 illustrate the effectiveness of the measurements provided by a surface sensor in improving the performance of a missile's on-board inertial navigation system when using the nine state Kalman filter, described in the previous section. Results are presented in graphical form showing the standard deviations of attitude and position errors, with and without aiding, over a typical short-range missile flight of 10 s duration. Over this period of time, the missile follows a boost/coast trajectory, accelerating at 20g for the first 4 s of flight and slowing under the influence of aerodynamic drag thereafter.

Figure 13.16 shows navigation performance when a high-grade inertial navigation system is used. The fixed biases for the gyroscopes and accelerometers used here were  $0.01^\circ/\text{h}$  ( $1\sigma$ ) and 100 micro-g ( $1\sigma$ ), respectively. For the purposes of this analysis, the standard deviations of the initial condition errors in the missile system were chosen to be as follows:

initial attitude errors: 10 mrad  
initial velocity errors: 1 m/s  
initial position errors: 1 m

Sensor measurement accuracies were set to values of 3 mrad ( $1\sigma$ ) in elevation and bearing and 10 m ( $1\sigma$ ) in range, and a data update rate of 1 Hz was assumed. The dramatic improvement in navigation performance with aiding is illustrated clearly in Figure 13.16. The position errors are corrected very rapidly and remain within 20 m for the remainder of the flight. The attitude errors settle to less than  $0.2^\circ$  after three measurements have been received. Because the instrument errors are small in this case, the system error model on which the Kalman filter is based provides

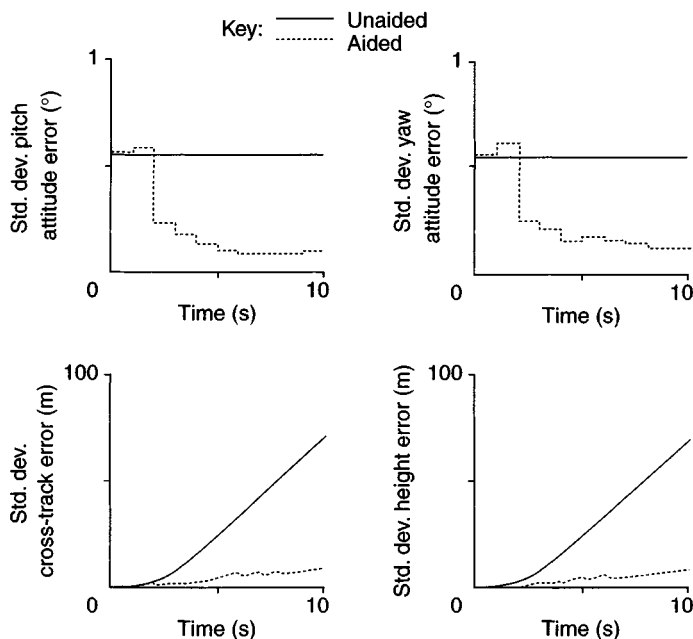


Figure 13.16 *Simulation results from aiding – high-grade INS*

a representative model of the actual system. This allows filter convergence to take place and accurate estimates of the inertial system errors to be derived.

Where the instrument errors become large, failure to model them correctly in the Kalman filter means the error model does not provide an accurate representation of what is happening in the actual system. Under such conditions, there is said to be a mis-match between the filter error model and the actual system. The sensor errors now introduce additional contributions to the measurement differences, which will be interpreted incorrectly as alignment errors. As a result, the Kalman filter estimates of attitude, velocity and position will be in error.

This effect is illustrated in Figure 13.17. The simulation described above was repeated under identical conditions with the exception that the high-grade inertial navigation system has been replaced by a system of more modest performance. The sensors have been replaced by gyroscopes and accelerometers having biases of  $30^\circ/\text{h}$  and 10 milli-g, respectively, typical of the grade of sub-inertial quality instruments often specified for use in tactical missile systems.

Whilst a substantial improvement in navigation performance is still achieved, when compared with the unaided system, the resulting accuracy is reduced and the rate of convergence increased, in comparison with the previous set of results shown in Figure 13.16. The build up of attitude errors that occurs between the measurement updates, is indicative of the mis-match which now exists in the Kalman filter. A major source of error in this particular case is from gyroscopic mass unbalance, which

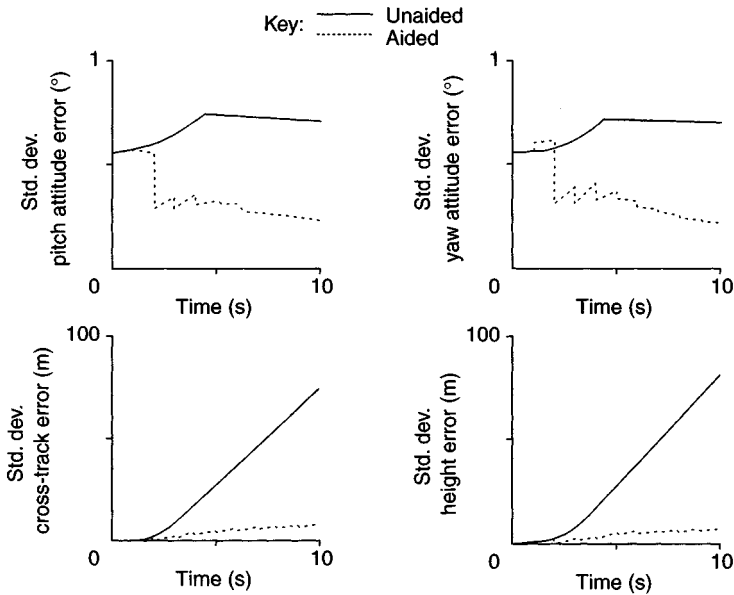


Figure 13.17 Simulation results from aiding – low-grade INS

introduces a rate bias that varies with the applied acceleration. This accounts largely for the shape of the attitude error curves shown in Figure 13.17.

In theory, there may be scope to improve the performance of the aided system by including additional filter states to model explicitly the dominant sensor errors [15]. By adopting this approach, the Kalman filter can be used to achieve some degree of in-flight sensor calibration, so leading to even greater enhancements of overall navigation performance. However, such a system will usually require time for the effects of modelling the sensor errors to become apparent.

In conclusion, the Kalman filter described earlier allows the effect of initial alignment errors to be reduced dramatically, and so provides sufficiently accurate inertial data for many short range missile applications, without recourse to much higher quality and more expensive inertial sensors. It is noted that for some applications, the in-flight aiding scheme described above may well allow some relaxation in the accuracy of the pre-flight alignment required. Pre-flight alignment methods are discussed in Chapter 10.

### 13.7 INS–GPS integration

As discussed earlier in this chapter, integrated navigation systems attempt to take advantage of the complementary attributes of two or more navigation systems to yield a system that provides greater precision than either of the component systems operating in isolation. Nowhere is this more true than for systems that combine inertial



navigation system (INS) measurements with satellite navigation data, the latter being provided by GPS, GLONASS or the Galileo system. A substantial level of effort has been focused on this topic in recent years, and work is continuing in an effort to produce integration schemes that are resistant to the effects of jamming of the satellite signals.

An INS exhibits relatively low noise, but tends to drift over time. For example, the position errors arising in a typical aircraft system will grow at between 1 and 10 nautical miles per hour of operation. In contrast, satellite navigation system estimates of position are relatively noisy, but exhibit no long-term drift.

Inertial and satellite navigation system measurements are complementary for two reasons:

- their error characteristics are radically different;
- they measure different quantities.

Satellite systems provide measurements of position and velocity, or more specifically pseudo-range and pseudo-range rate, whilst inertial systems measure specific force acceleration, which must be compensated for gravity and resolved into a known co-ordinate reference frame before being integrated twice to yield estimates of position.

Satellite position measurement accuracy is limited as a result of low signal strength, the length of the pseudo-random code and errors in the code tracking loop. Further errors arise as a result of multi-path, variations in the satellite geometry, changes in propagation conditions and user clock instability. Satellite velocity measurements are also noisy, again owing to variations in signal strength, the effects of changing multi-path and user clock instability.

Moreover, there is the possibility of jamming a GPS receiver with modest power jammers. Hence, any system that is dependent on a GPS-based navigation approach is vulnerable, and consequently, its availability may be compromised. The latter fact being a major concern for many users, particularly the armed forces.

The main features of inertial and satellite systems in terms of their respective advantages and disadvantages are summarised in Table 13.1.

Operating the two systems together yields benefits over operating either system alone. By making use of this basic synergy between inertial and satellite systems, it is possible to produce an integrated system that yields low noise and low drift estimates of vehicle position. There are also other relative attributes, which add to the benefits of integrating the two system approaches as discussed next.

The wide availability of GPS satellite navigation updates, coupled with the low cost of GPS receivers, has provided much of the impetus for the continuing development of techniques for the integration of INS and GPS. Given the availability of uninterrupted GPS access, there is considerable scope to combine low-accuracy inertial systems with GPS in order to produce low-cost precision navigation systems capable of operating under a wide range of conditions. The grade of inertial sensors required for such systems is determined to a large extent by the duration of GPS interruptions expected; such interruptions may be expected to occur in military applications as a result of jamming of the satellite signals and more generally as a result of 'signal shading' when attempting to operate in an urban environment. The accuracy

*Table 13.1 Comparison of features of inertial and satellite navigation systems*

	Advantages	Disadvantages
Inertial navigation systems	High data rate Provides both translational and rotational data Autonomous – not susceptible to jamming	Unbounded errors Knowledge of gravity required
Satellite navigation systems	Errors are bounded	Low data rate No attitude information Susceptible to jamming – both intentional and unintentional

of the inertial sensors is also a factor in applications calling for precision estimates of velocity and attitude, in addition to positional data. For systems in which the threat of interference is minimal, future inertial systems incorporating GPS updates are expected to provide 1 m (CEP)<sup>4</sup> navigation accuracy.

A number of different integration architectures have been developed to allow INS and GPS to be combined; the level of integration depending in part on whether one is dealing with the creation of a new system, or the addition of GPS updates as a retro-fit to an existing system. A number of INS–GPS integration schemes that are in use or under development for the future are described below. Four main classes of integration architecture may be defined [16, 17], viz.

*Uncoupled systems* in which GPS estimated position is used simply to reset the INS indicated position at regular intervals of time;

*Loosely coupled systems* in which the INS and GPS estimates of position and velocity are compared, the resulting differences forming the measurement inputs to a Kalman filter;

*Tightly coupled systems* in which the GPS measurements of pseudo-range and pseudo range rate are compared with estimates of these quantities generated by the inertial system;

*Deep or ultra-tightly coupled systems* which combine the GPS signal tracking function and the INS/GPS integration into a single algorithm.

### *13.7.1 Uncoupled systems*

This is the simplest method of deriving the respective benefits of GPS and INS, whilst maintaining the two systems operating independently and providing system redundancy. By using GPS position and velocity estimates to reset the INS, the growth

<sup>4</sup> Circular error probable (50%).

of errors in the INS estimates of position and velocity is bounded. Whilst this approach involves minimal changes to either system, it does not provide the opportunities for performance enhancement and jamming avoidance that are possible with the coupled systems described in the following section.

### 13.7.2 *Loosely coupled integration*

This approach allows the GPS to function autonomously, whilst simultaneously providing measurement updates to the inertial system. The two systems are effectively operated in cascade with the position and/or velocity estimates provided by the GPS navigation calculation forming measurement inputs to an INS–GPS integration Kalman filter. A simplified representation of a loosely coupled INS–GPS integration architecture is given in Figure 13.18.

The two main advantages of loosely coupled integration are simplicity and redundancy. This approach can be used with any INS and any GPS receiver, and is therefore well suited to retro-fit applications. In a loosely coupled configuration, it is usual to provide a stand-alone GPS navigation solution, in addition to the integrated solution. The redundant navigation solution can be used to monitor the integrity of the integrated solution and to facilitate a filter failure recovery process should the need arise.

In the scheme described here, the integration filter provides estimates of the INS errors, which may be used to correct the inertial system following each measurement update. GPS uses the INS purely to aid the satellite signal acquisition process with this level of system integration. As in all of the integration architectures described here, aiding the satellite receiver's code-tracking loops with inertial-sensor information allows the effective bandwidth of these loops to be reduced. This feature allows an

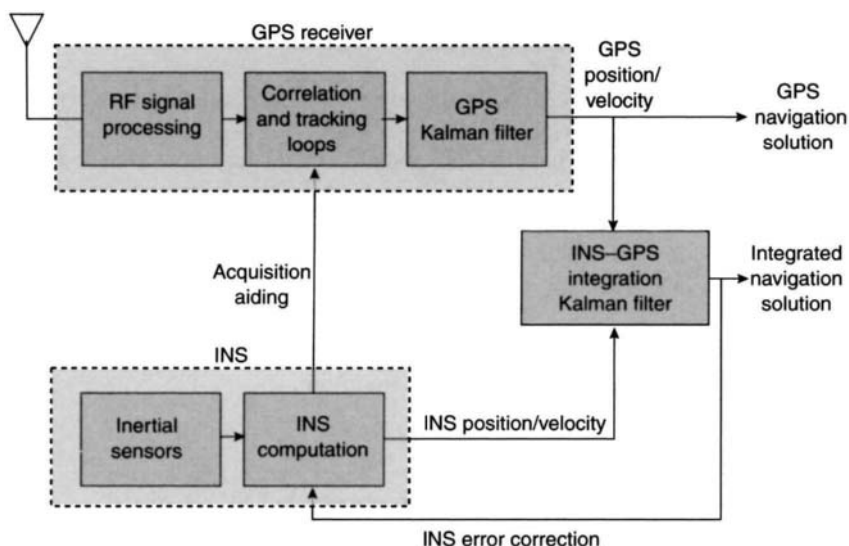


Figure 13.18 *Loosely coupled INS–GPS integration architecture*

improvement in the ability of the receiver to track signals in a noisy environment, as will exist in the presence of signal jamming.

Whilst GPS position updates alone may be used to aid the inertial system, it is more usual to use both position and velocity measurements for a more robust solution. Because there are fewer integration steps between attitude errors and sensor biases, these errors propagate more rapidly as velocity errors. Therefore, velocity measurements allow more immediate estimates of sensor biases and attitude errors to be obtained. However, the use of velocity measurements alone reduces the observability of position errors in the INS. For these reasons, it is customary to use both GPS position and velocity updates to aid the inertial system in most integration algorithms of this type.

The main problems with loosely coupled INS–GPS integration stem from the use of cascaded Kalman filters; the fact that the output of the GPS Kalman filter is used as a measurement input to the integration filter. In formulating a Kalman filter, the implicit assumption is made that the measurement errors are uncorrelated, that is, that the measurement noise is ‘white’. For the system configuration considered here, such an assumption is not necessarily true. For example, situations can arise where the integration algorithm samples the GPS data faster than the tracking loops can supply independent measurements causing the Kalman filter measurement errors to be time correlated. Further time correlation can arise through multi-path effects, the process whereby several delayed copies of a signal reach the antenna following reflection from nearby surfaces.

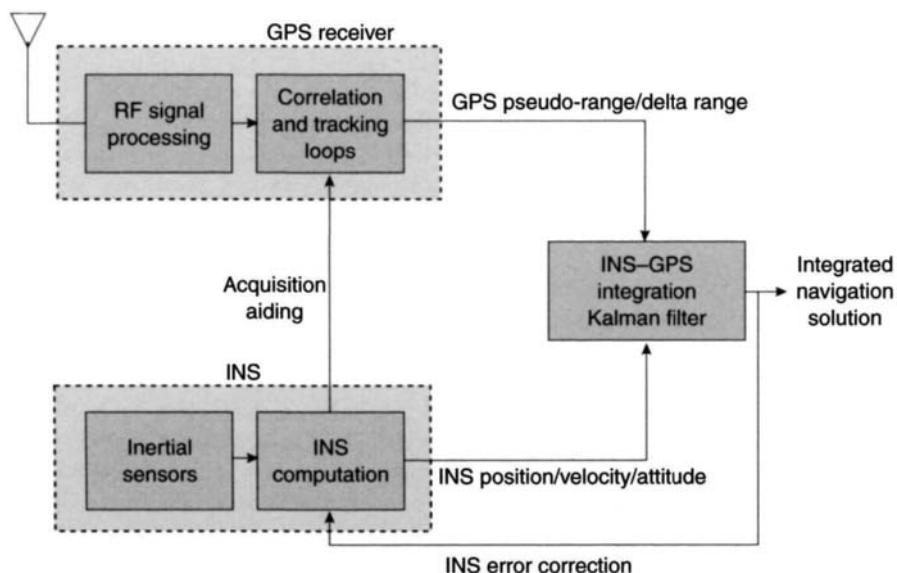
The correlation time of the GPS Kalman filter outputs varies with the tracking-loop bandwidths, and can be up to 100 s on position and 20 s on velocity and in a dynamic receiver can be 10 s on position and 0.1–1.0 s on velocity. This is too short for the correlated errors to be estimated, but long enough to slow down the process of estimating the INS errors within the integration filter. As a result of these issues, the selection of the Kalman filter measurement update interval becomes critical.

To overcome this problem, the measurement update interval may be increased until the measurement errors are no longer correlated. Alternatively, estimates of the correlated errors can be derived by modelling them as Markov processes and including the correlation time as an additional state in the Kalman filter system model.

Other factors that should be borne in mind when attempting to use loosely coupled integration are as follows. Signals from at least four separate satellites are required to form and maintain a GPS navigation solution, although degraded navigation can be maintained for short periods using only three satellites. Therefore, in situations where fewer satellites are ‘in view’, the GPS cannot be used to aid the INS. In addition, the integration filter requires knowledge of the covariance of the GPS filter outputs. This varies with satellite geometry and availability, and for many GPS receivers, covariance data are unreliable or not available at all.

### 13.7.3 *Tightly coupled integration*

Figure 13.19 shows a simplified representation of a tightly coupled INS–GPS scheme; also referred to as closely coupled, centralised or direct integration architecture. In



*Figure 13.19 Tightly coupled INS–GPS integration architecture*

this approach, the GPS Kalman filter becomes an integral part of the integration filter, which accepts measurements of pseudo-range and pseudo-range rate provided by the GPS tracking loops. These measurements are used to generate estimates of the errors in the INS. The corrected INS navigation solution forms the integrated navigation solution, and either the corrected or the raw INS data may be used to aid the GPS tracking loops. Signal timing is critical if this system is to operate successfully.

Whilst either pseudo-range or pseudo-range rate measurements may be used, it is common practice to use both. The two measurements are complimentary in that pseudo-range comes from the GPS code-tracking loop, whilst the pseudo-range rate is derived mainly from the more accurate, but less robust carrier-tracking loop.

The benefits of adopting the tightly coupled approach stem mainly from combining the two Kalman filters used in the loosely coupled system as summarised below:

- the statistical problems arising through using the output of one Kalman filter as the measurement input to the second filter are eliminated;
- the handover of GPS position and velocity covariance is done implicitly;
- the system does not require a full GPS solution to aid the INS, GPS data being input to the filter even if only a single satellite signal is being tracked, but accuracy degrades rapidly.

There is no inherent stand-alone navigation solution available from the tightly coupled system as described. However, a GPS only solution may be generated

in parallel with the integrated solution. This solution may be used, when required, for integrity monitoring and failure recovery.

The tightly coupled approach is preferable to the loosely coupled system, giving the better performance in terms of both accuracy and system robustness.

### 13.7.4 Deep integration

Deep integration, also known as ultra-tightly coupled integration, combines GPS signal tracking and INS–GPS integration into a single Kalman filter, as illustrated in Figure 13.20. Deep integration methods are presently under development. Although many authors have published theory and simulation results [17], a fully working hardware implementation has yet to be published in the open literature.

By tracking the GPS signals together, instead of using independent tracking loops, the tracking of each signal is aided by the others and by the inertial data, bringing three main benefits:

- as fewer independent quantities are tracked using the same data, the effective signal to noise ratio is improved, the more satellites tracked, the greater the improvement;
- multi-path resistance is improved;
- the reacquisition of a signal following a brief interruption as a result of signal obstruction or jamming can be much faster.

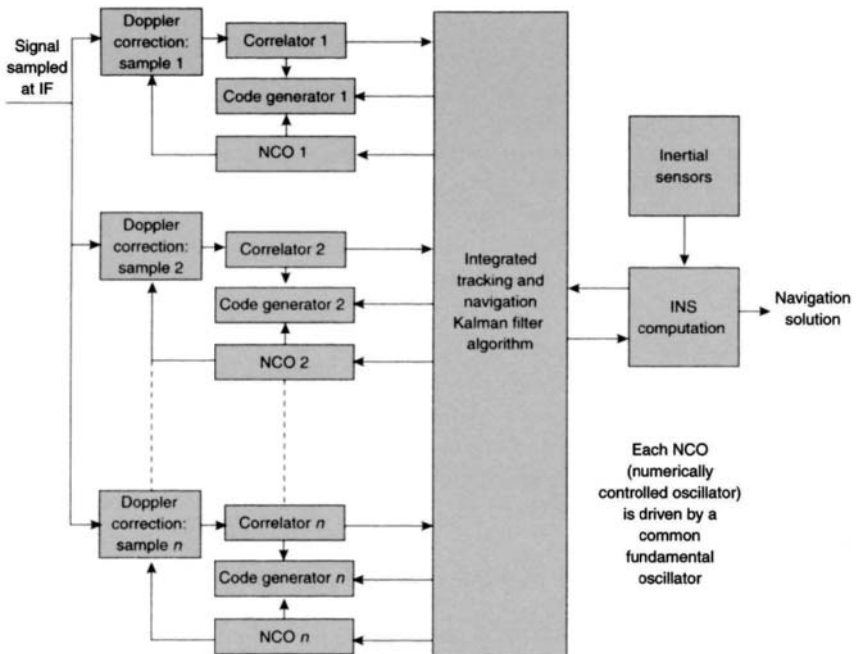


Figure 13.20 Simplified diagram of deep INS–GPS integration architecture

The potential benefits described here may be achieved at the expense of greatly increased complexity; increased computational load and tight time synchronisation requirements for some configurations, as well as high performance INS data to maintain tracking loop lock. A major difference between deep integration and other integration architectures is the need for a very fast update rate, typically 200 to 1 kHz, in order to keep the signal tracking functions in lock.

### 13.7.5 *Concluding remarks*

It is not possible to provide hard and fast rules about the performance of one technique over another, because of the range of countermeasures or other techniques that a system may encounter. As a consequence, the resulting performance of any technique is usually critically dependent on the actual scenarios where the platforms are expected to perform, for example where the jammers are positioned or where the obscurations may occur.

Some generalised comments are possible [18]:

- GPS measurement data dominate navigation estimates in an integrated system when the GPS data are available. The steady state navigation error will be reduced through the inertial aiding by effectively averaging out the noise in the GPS measurements.
- Enhancement to navigation accuracy is not critically dependent on the quality of the inertial system, so using a high-grade system has limited benefits.
- Navigation systems with higher-grade sensors benefit more from the in-flight calibration that is possible with an integrated system, owing to the superior compensation that is possible with low-noise sensors, thus the high-grade system provides superior performance during a sustained loss of GPS to the integrated system.
- Tight coupling appears superior for maintaining lock-on to satellites during operation in a jamming environment when compared with loose coupling techniques, however, the benefit is difficult to quantify except in terms of jam to signal ratio.
- The different types of coupling architectures can have an impact on navigation accuracy after short-term losses of the GPS data, owing to the way the data are used for in-flight calibration of the sensors and alignment of the system, however, for long-term loss of GPS it is the sensor quality that will dominate system accuracy.

It is therefore recommended that simulation and hardware evaluation studies are undertaken, paying particular attention to the scenario in which the system is required to operate.

### 13.7.6 *INS aiding of GPS signal tracking*

Aiding of GPS carrier tracking by the corrected inertial navigation solution is difficult. This is because very tight time synchronisation is required to keep the carrier tracking locked under high dynamics and to follow the receiver oscillator noise. The development of receivers with software correlators is making this easier as the incoming

GPS signals can be stored and retrieved to match the processing lags in the rest of the system. The tolerances for inertial aiding of GPS code tracking are much wider, so this is an established technique.

Selection of the GPS tracking loop bandwidths is a trade-off between noise resistance and dynamical response. Narrower bandwidths are more tolerant of interference; wider bandwidths respond better to the effects of dynamic motion. By aiding the GPS tracking loops with the corrected INS velocity, the aiding information handles the vehicular motion dynamics, enabling narrow bandwidths to be used to resist noise.

For carrier tracking, the minimum bandwidth is limited by the need to track the oscillator noise. Thus, INS–GPS systems can track GPS code at lower signal to noise levels under dynamical motion than a stand-alone GPS receiver. However, lower-grade inertial systems require constant calibration from the GPS receiver and narrower tracking-loop bandwidths reduce the rate at which independent measurements are provided by the GPS receiver, and hence the INS calibration accuracy. To prevent positive feedback, the gains in the INS–GPS integration algorithm must be matched to the GPS tracking bandwidths. Whilst deep integration does this implicitly [18, 19, 20], this may also be achieved using the tightly coupled integration architecture; a technique known as adaptive tightly coupled (ATC) integration [21]. Recent simulation studies have shown that both deep and ATC integration techniques enable GPS signals to be tracked under noise levels at least 10 dB higher than obtainable with a fixed bandwidth INS–GPS system, when lower-grade inertial sensors are used.

### 13.8 Multi-sensor integrated navigation

As indicated at the start of Section 13.5, the measurements provided by two or more complementary navigation systems may be combined to yield a navigation solution for a given application; a best estimate of a vehicle's position, velocity and attitude. Prime examples of applications in which it is often desirable, if not essential, to incorporate multiple sources of navigation data are modern military aircraft, long-range missile systems and, more recently, precision guided munitions. Whilst such systems may rely upon an integrated INS–GPS system as their primary source of navigation data, use is frequently made of terrain-reference navigation systems to obtain a more robust navigation solution over the full range of operational conditions anticipated. Terrain-based navigation systems that are used in such applications are terrain contour matching, scene matching area correlation (SMAC) and, more recently, continuous visual navigation (CVN) as described earlier in this chapter.

Issues to be considered when designing such systems are:

- dealing with ambiguous measurement updates;
- choice of integration architectures.

Terrain referenced navigation systems are liable to produce false position fixes on occasions. CVN systems are designed to generate multiple hypothesis position fixes where there is not a unique match between the measurements and the database. Each candidate fix will be accompanied by an associated covariance and an estimated



probability. Ambiguities between these fixes can be treated in different ways. The system may be designed either to accept the position fix that has the highest probability, to compute a weighted fix based upon the estimated probabilities of each, or to reject a fix altogether in the event that its probability is too low.

The multiple hypothesis technique maintains several hypotheses, as the term suggests; each hypothesis is contained in the form of an alternative position-fix Kalman filter, and providing a different set of navigation estimates and an associated probability. Subsequent fixes are used to clarify which of the hypotheses are unlikely to be true, and these are ignored thereafter. Hence, a list of hypotheses is held in order of likelihood, the list being constantly updated as the flight proceeds. At all times, the system has an absolute favourite hypothesis, which is taken to provide the best estimate of position at that time. This method has been proposed as a method for dealing with terrain referenced navigation measurements in an optimal manner [21], although the processing requirement can be very large, but is feasible with modern technology.

Various integration architectures may be applied to combine the data generated using multiple sensor navigation. One possibility is to combine the INS data with the data from each measurement source using a separate Kalman filter. It is postulated that a navigation solution that combines all of the available measurement sources may subsequently be derived by taking a weighted sum of the solutions from each filter based on the covariances of the individual solutions. This approach ignores any unmodelled correlations that may exist between the solutions generated by the individual filters and can result in a false navigation solution.

An additional master Kalman filter may be used to combine the outputs of the individual filters. However, the presence of correlated noise on the outputs of the individual filters can present problems causing the master filter to become unstable. An alternative is required to give a robust solution over a broad range of inputs.

A preferred approach to the integration methods outlined above, is to adopt a centralised architecture [22] in which, as the name suggests, all of the measurement sources are processed by a single Kalman filter. The effectiveness of this approach is, of course, reliant on the availability of representative error models for the individual measurement sources.

To minimise the possibility of a false fix corrupting the integrated navigation solution, integrity monitoring techniques are frequently recommended for sensitive applications. This may be achieved by implementing parallel filters and by monitoring the residual measurement errors throughout the filtering process. This so-called federated scheme was proposed some years ago, but has yet to be implemented in a system.

## 13.9 Summary

There are many sources of navigation data, which may be used to correct inertial system estimates to provide enhanced navigation performance. These include external measurements derived from equipment outside the vehicle, such as radio navigation aids and satellites, and measurements derived from additional sensors on-board the

vehicle such as various types of altimeters and Doppler radar. The various navigation aids often provide attitude, velocity or position updates, any of which may be used to bound the drift errors arising in an inertial navigation system, and so improve its performance.

The resulting integrated navigation systems often permit substantial improvements in navigation accuracy compared with the performance that may be achieved purely from using inertial systems, even when the inertial navigation system uses very accurate inertial sensors. Whilst very accurate navigation performance may be achieved through the use of higher quality inertial sensors and more precise alignment techniques, the use of integrated systems, such as those described here, often provide a more cost-effective solution.

Techniques for mixing inertial and other measurement data have been described, culminating in the description of an algorithm specifically for aided inertial systems. A Kalman filter algorithm may be used for the integration of different measurement data with inertial measurements. The design example described has shown how external measurement data can be combined with the inertial system information to bound the growth of navigation errors. As a result, it may be possible to allow some minor relaxation in pre-flight alignment accuracy and in the precision of the inertial sensors. Such techniques can be extended to achieve a measure of sensor calibration as part of the aiding process.

The increased accuracy and availability of satellite navigation systems (GPS, GLONASS) have encouraged designers of modern combined systems to become more reliant for good performance on the satellite component. So, now military concerns about vulnerability to interference, reliability and receiver jamming are being investigated by a variety of methods including the use of controlled radiation pattern antenna and integration algorithm developments. For particularly sensitive applications, reversionary modes of operation incorporating additional navigation aids are frequently adopted.

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