Estimation and Sensor Information Fusion

Lecture 5

Outline

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1. Implementation Methods
   - Computer Roundoff
   - Effects of Roundoff on Kalman Filters
   - Bierman-Thornton UD Filtering

2. Practical Considerations
   - Detecting and Correcting Anomalous Behavior
   - Prefiltering and Data Rejection Methods

Roundoff Errors

- **Cause**: Fixed number of bits for data representation
- **Result**: Values with very small differences could be set equal to each other
- **Example**:

$$\begin{aligned}\frac{1}{3} &= \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \cdots \\ &= 0_b010101010101010101010101010\ldots \\ &\approx 0_b010101010101010101010101011 \\ &\approx \frac{1}{3} - \frac{1}{100663296}\end{aligned}$$

Unit Roundoff Error

- Roundoff error $\varepsilon_{\text{roundoff}}$ is the largest number such that
  - $1 + \varepsilon_{\text{roundoff}} \equiv 1$, or
  - $1 + \varepsilon_{\text{roundoff}}/2 \equiv 1$

Terminology of Numerical Error Analysis

- **Robustness**: Quantifies the relative insensitivity of the solution to errors of some sort
- **Numerical Stability**: Robustness with regard to roundoff errors
- **Precision vs Numerical Stability**: More precision can increase numerical stability, but overall accuracy also depend on accuracy of initial parameters and implementation
- **Comparing Numerical Stability**: Some methods are considered more robust than others but can also depend on properties of the problem being solved
- **Conditioning**
  - *Ill-Conditioned*: Bad sensitivity to input/disturbances
  - *Well-Conditioned*: Not badly sensitive to input/disturbances

Ill-Conditioned Kalman Filtering Factors

- Large uncertainties in $\mathbf{\Phi}, \mathbf{Q}, \mathbf{H}, \mathbf{R}$
- Large ranges of variables
- Ill-conditioning of $\mathbf{H}\mathbf{P}_0\mathbf{H}^T + \mathbf{R}$
- Ill-conditioned theoretical solutions of the Riccati equation
- Large matrix dimensions
- Poor machine precision

Error Propagation in Kalman Filters

- Feedback in estimation

$$\hat{\mathbf{x}}(+) = \hat{\mathbf{x}}(-) + \mathbf{K}(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}(-)) \tag{1}$$

- No feedback for the gain

$$\mathbf{K} = \mathbf{P}(-)\mathbf{H}^T[\mathbf{H}\mathbf{P}(-)\mathbf{H}^T + \mathbf{R}]^{-1} \tag{2}$$

$$\mathbf{P}(+) = [\mathbf{I} - \mathbf{K}\mathbf{H}]\mathbf{P}(-) \tag{3}$$

$$\mathbf{P}(-) = \mathbf{\Phi}\mathbf{P}(+)\mathbf{\Phi}^T + \mathbf{Q} \tag{4}$$

Bierman UD Observational Update

- Covariance is partially factored in terms of UD factors

$$\mathbf{P}(-) = \mathbf{U}(-)\mathbf{D}(-)\mathbf{U}^T(-) \tag{5}$$
$$\mathbf{P}(+) = \mathbf{U}(+)\mathbf{D}(+)\mathbf{U}^T(+) \tag{6}$$

- Algorithm can be seen in table 6.15

Thornton UD Temporal Update

- Thornton temporal update is also known as modified weighted Gram-Schmidt (MWGS)
- Uses $\mathbf{Q} = \mathbf{G}\mathbf{D}_Q\mathbf{G}^T$ and

$$\mathbf{A} = \begin{bmatrix} \mathbf{U}^T(+)\mathbf{\Phi}^T \\ \mathbf{G}^T \end{bmatrix} \qquad \mathbf{D}_w = \begin{bmatrix} \mathbf{D}(+) & 0 \\ 0 & \mathbf{D}_Q \end{bmatrix}, \tag{7}$$

  such that

$$\mathbf{A}^T\mathbf{D}_w\mathbf{A} = \mathbf{P}(-) \tag{8}$$

- Algorithm can be seen in table 6.16

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Testing for Unpredictable Behavior

- Taking an average of the estimation error over many different simulations (using independent pseudo-random sequences) should result in an average estimation error of approximately 0
- Causes of non-convergence
  - Natural behavior of the dynamic equations
  - Non-observability with regard to the measurements

Rejecting Bad Data

- $\mathbf{P}$ and $\mathbf{K}$ are independent of data
- Important to reject bad data before they are processed
- Inspect the innovation vector $[\mathbf{z} - \mathbf{H}\hat{\mathbf{x}}]$ for sudden jumps/large values
- Possibly increase process noise to recover

Mismodeling Issues

- Unmodeled state variables
- Unmodeled process noise
- Errors in coefficients/state transition matrix
- Overlooked nonlinearities

Analysis and Repair of Covariance Matrices

- Covariance matrix must be positive definite
- Test all diagonal elements are greater than 0
- Use Cholesky decomposition or UDU (modified Cholesky) decomposition

Data Rejection Filters

- Excess amplitude

$$|(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}})| > A_{max} \tag{9}$$

- Excess rate

$$|(\mathbf{z} - \mathbf{H}\hat{\mathbf{x}})_{i+1} - (\mathbf{z} - \mathbf{H}\hat{\mathbf{x}})_i| > \delta A_{max} \tag{10}$$