Predictive safety filter using system level synthesis

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Motivation:

- Safety filters can augment any controller with safety guarantees
- Scalable predictive safety filters suffer from conservativeness in presence of disturbances

Contributions:

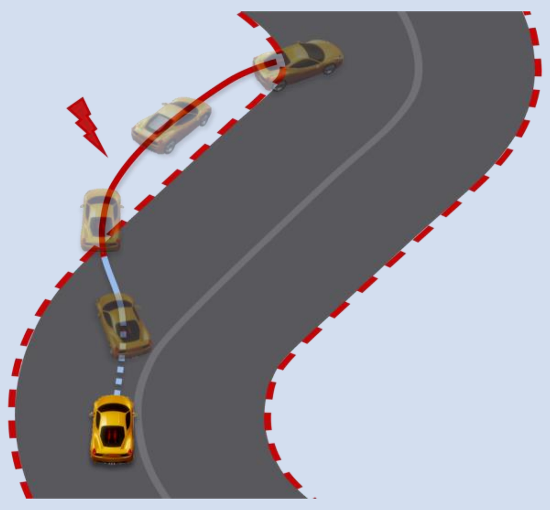
Improved safety filters based on system level synthesis.

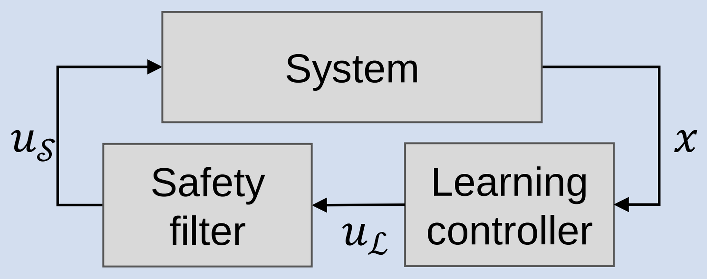
- Online method: Less interventions, and larger safe set.
- Offline method: Numerically efficient and scalable.

Website **Paper**

Safety filter background

Main idea:





Goal:

Guarantee constraint satisfaction while

- minimizing possible control intervention.
- maximizing size of the explorable (invariant) **safe set**: set of states $x(t)$ for which a safe backup controller.

Predictive safety filter

Main idea:

Formulate safety filter as a prediction problem

$$\min_{\boldsymbol{z},\boldsymbol{v}} ||\boldsymbol{v}_0 - u_{\mathcal{L}}(t)||_2^2$$
$$\text{s.t. } \boldsymbol{z}_{k+1} = A\boldsymbol{z}_k + B\boldsymbol{v}_k, \boldsymbol{z}_0 = x(t),$$
$$\boldsymbol{z}_k \in \mathcal{X}, \boldsymbol{v}_k \in \mathcal{U}, \boldsymbol{z}_N \in \mathcal{X}_{\mathrm{f}}.$$

If verified safe, $u_{\mathcal{L}}(t)$ is applied, $\boldsymbol{v}_0^\star$ otherwise.

- Addressing model uncertainty in the context of learning is key

$$x(t+1) = Ax(t) + Bu(t) + B_{\mathrm{w}}w(t).$$

- State-of-the-art [1] based on a **fixed** tube controller $u(t) = v(t) + K(x(t) - z(t))$, and robust constraint tightening techniques

$$\mathcal{X} \rightarrow \mathcal{X} \ominus \mathcal{F}_{\mathrm{x}}, \mathcal{U} \rightarrow \mathcal{U} \ominus K\mathcal{F}_{\mathrm{x}},$$

which can be conservative.

System level synthesis

Main idea:

Optimization over the affine controller [2]

$$\boldsymbol{u} = \boldsymbol{v} + \mathbf{K}(\boldsymbol{x} - \boldsymbol{z}),$$

using the parametrization $\mathbf{K} = \boldsymbol{\Phi}_{\mathrm{u}}\boldsymbol{\Phi}_{\mathrm{x}}^{-1}$

$$[\mathbf{I} - \mathbf{ZA}, -\mathbf{ZB}]\begin{bmatrix}\boldsymbol{\Phi}_{\mathrm{x}} \\ \boldsymbol{\Phi}_{\mathrm{u}}\end{bmatrix} = \mathbf{B}_{\mathrm{w}},$$

with safety constraints

$$\mathrm{Tube}(x(t), \boldsymbol{\Phi}_{\mathrm{x}}) \subseteq \mathcal{X}.$$

System level model predictive safety filter (online)

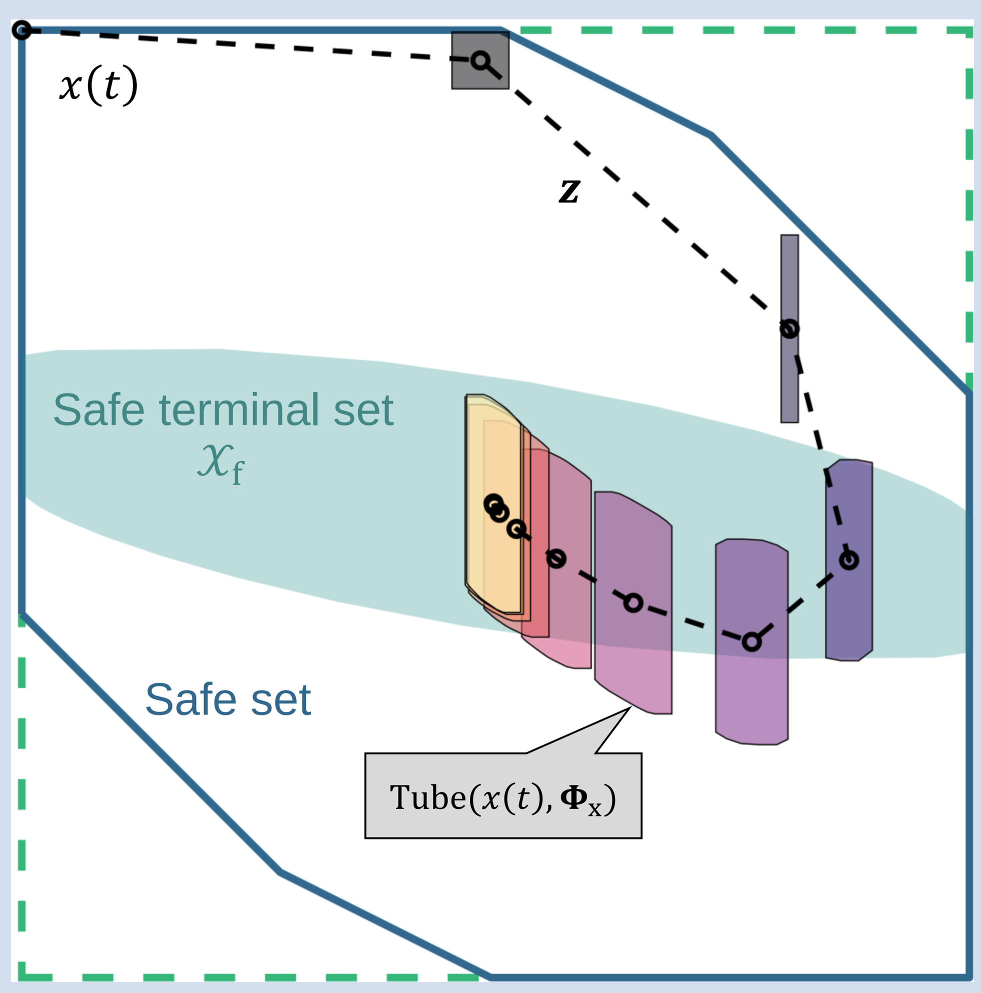
Main idea:

Optimize the prediction together with the tube controller using SLS [2]

$$\min_{\boldsymbol{z},\boldsymbol{v},\boldsymbol{\Phi}_{\mathrm{x}},\boldsymbol{\Phi}_{\mathrm{u}}} ||\boldsymbol{v}_0 - u_{\mathcal{L}}(t)||_2^2,$$
$$\text{s.t. dynamics, SLS constraints.}$$

Benefits:

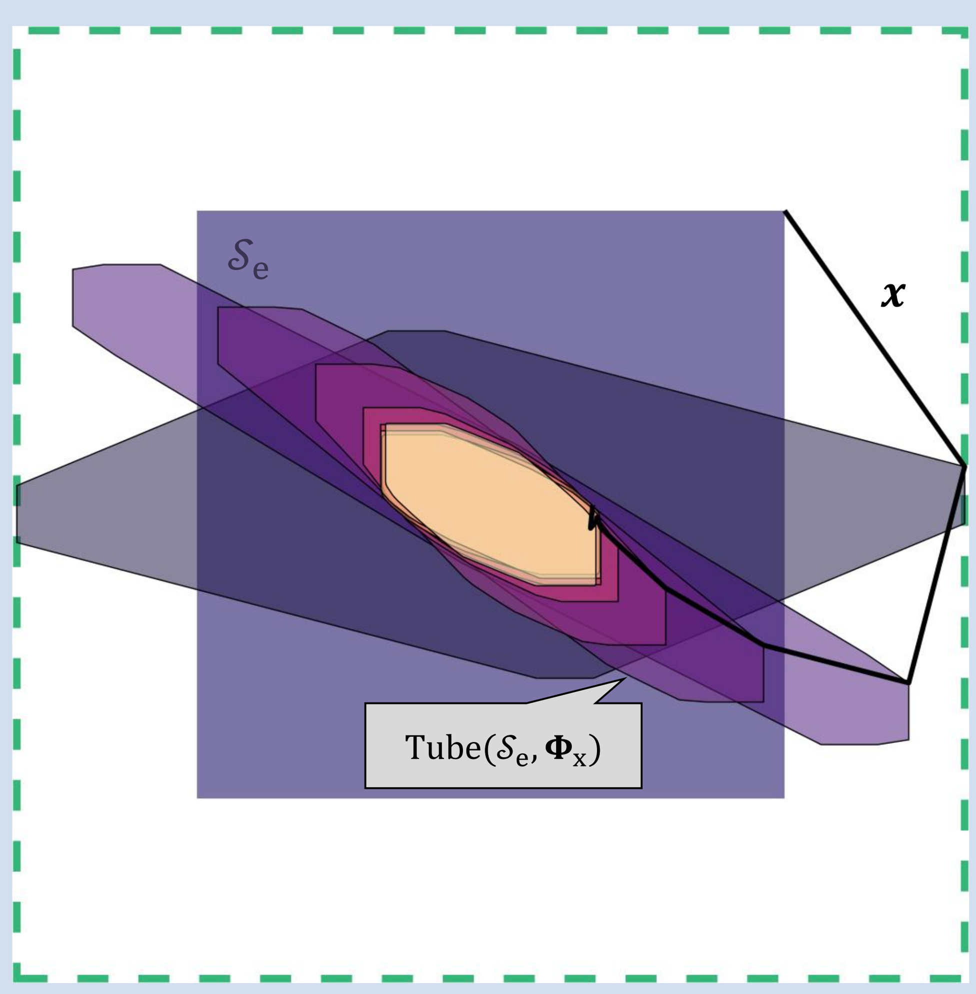
- Provably less conservative than state-of-the-art [1].
- Guaranteed robust safety.



Explicit safety filter (offline)

Main idea:

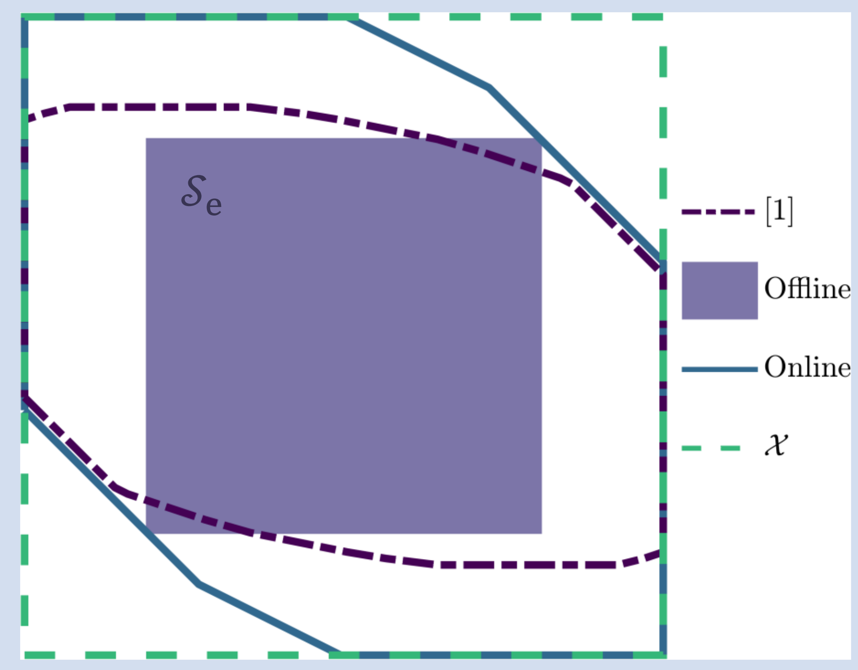
- Find $\mathbf{K}$ offline using (SLS) safety constraints and robustness to **largest** set of initial conditions $\mathcal{S}_{\mathrm{e}}$.
- Explicit safe set is (periodically) invariant by design.



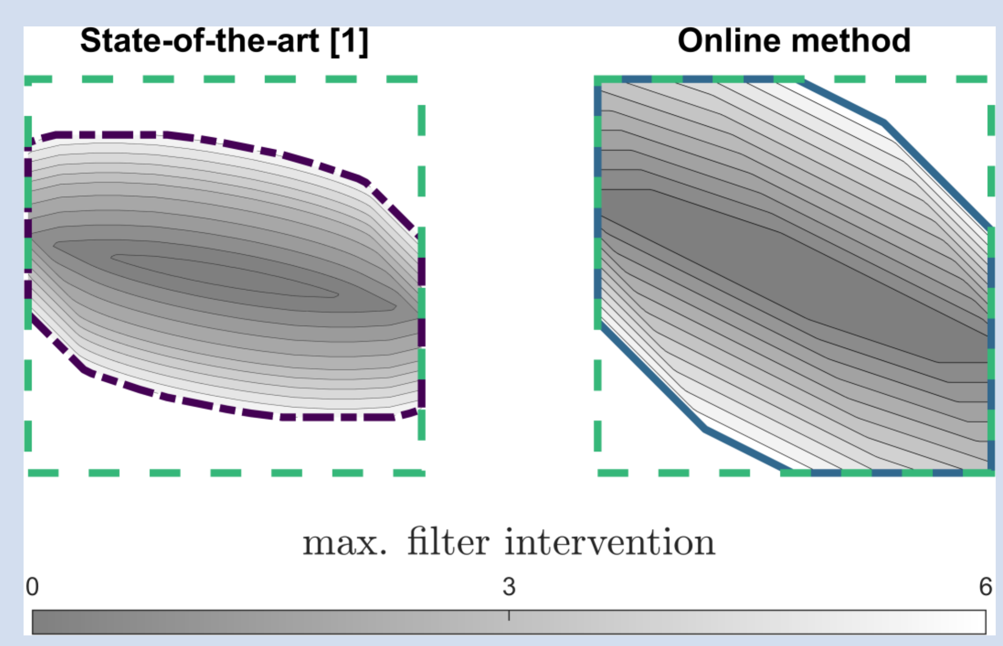
Numerical example

Constrained double integrator:

- Enlarged safe set for the online method.



- Reduced control interventions for the online method compared to state-of-the-art [1].



- Online complexity reduced for offline method.

	Online complexity [ms]	Rel. safe set size [%]
State-of-the-art [1]	4,8	~72,7
Online method	65,7	~99,6
Offline method	0,031	45,6

Conclusion

- Proposed predictive safety filter that interferes less with learning-based controller.
- Presented explicit variant that reduces computational complexity.
- Opens extensions, e.g. to nonlinear systems [3].

References

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