

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

DIVISIÓN DE ESTUDIOS DE LICENCIATURA



APPLYING HOMOMORPHIC COMPUTING IN THE  
CLOUD

POR

JESÚS ANTONIO SOTO VELÁZQUEZ

EN OPCIÓN AL GRADO DE

INGENIERO EN TECNOLOGÍAS DE SOFTWARE

SAN NICOLÁS DE LOS GARZA, NUEVO LEÓN

FECHA

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Los miembros del Comité de Tesis recomendamos que la Tesis «Applying homomorphic computing in the cloud», realizada por el alumno Jesús Antonio Soto Velázquez, con número de matrícula 1570031, sea aceptada para su defensa como opción al grado de Ingeniero en Mecatrónica.

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A mis padres . . .

No olvidar al asesor, sinodales, instituciones que dieron apoyo.

# RESUMEN

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Tu asesor

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## CHAPTER 1

# INTRODUCTION

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Because of the increasing people's needs, it has become more common to exchange information with our peers. Whether it is our e-mail address or current location, there are many scenarios where it is necessary to pass that information to another point. It becomes an issue when the information is considered to be sensitive, and thus, the confidentiality must be protected, so that no one other than the sender and receiver get to know about it. In order to protect the confidentiality of the information, cryptography could be used via encryption and decryption algorithms. The issue at hand becomes even more interesting when the information that needs to be passed somewhere does not go directly to another person, but is rather stored somewhere by someone in the cloud. The confidentiality is not broken if this scenario were to happen; however, it becomes more complicated if it were wished for the information to be modified while being stored in the cloud. Such goal calls for homomorphic encryption, which is an advanced technique able to modify the already encrypted data without losing its confidentiality.

There are many instances where it is required to send information to someone else. To sign up for a service, for instance. Usually, name and email address are required as basic pieces of information. Depending on the service, telephone number and personal address might also be required. And in such case, the subscriber might actually be worried about how safe his information is being kept. He would feel safer if he had some kind of proof that vouched for the confidentiality of his data. This also affects the service, as less people would sign up for the service if they had no

means to protect the data.

Cryptography is the study of ciphers, that is, encryption and decryption algorithms to be used on data. Commonly, these algorithms are used to protect the data from eavesdroppers that try to pry on it. Ciphers can be seen as the means of putting sensitive data into a box with a lock, and only those with the correct key are able to access the contents of the box.

It has become increasingly common to delegate computing tasks to cloud services in order to save resources. For instance, it might be employed when the *owner/company* cannot afford its own data center for its storage and computing needs. This is especially the case when the level of activity is seasonal: this is, during certain times of the year that require more processing and storage than usual. Therefore, a common solution is to make use of cloud storage and computing services. This way, the budget is spent as much as the resources are being employed, while saving configuration and maintenance costs. However, allowing the cloud service to make use of the data has raised concerns on security [reference], since it is hard to trust that the cloud provider will not look at the data and do something with it.

A typical solution to this scenario consists in encrypting the data before storing it in the cloud; therefore, assuring its confidentiality. It is a great solution when it is only needed to read the data, without actually making any changes on it. In order to view the encrypted data, it has to be downloaded and decrypted using the proper secret key. However, when it is required to modify it in some way, it still has to be downloaded and recovered to its plaintext form, before modifying it. Once it has been modified, it has to be re-encrypted and re-uploaded to the cloud, in a potentially slow manner. This approach turns out to have high overhead costs [reference] caused by the transfer of the encrypted data back and forth between the *owner/company* and the cloud.

In scenarios where the information is not considered to be sensitive, it would be appropriate to make use of cloud computing services directly, as it would cause

no harm to grant access to the data that is to be used if it were to be considered public. However, real life applications often imply sensitive data, and it should not be handled trivially, as there is no telling whether or not there is an eavesdropper listening on the communication lines.

A simple sounding, yet complicated solution to this problem would be to manipulate the encrypted data in some way such that the contents are not revealed, but on decryption, ends up being correct. It had been thought that doing something like that was not possible, as ciphers often consisted of permutations and substitutions. However, an interesting property found in the RSA algorithm proved otherwise [reference]. Such property is called homomorphic, and has since then gone through a lot of research, resulting in many schemes such as [...].

Homomorphic encryption is a cipher that makes use of an homomorphic property in order to alter the data while it is encrypted, by using some operation like addition or multiplication. It has been found that homomorphic encryption is not yet truly usable in real environments, mainly for two reasons: the size of the keys and the time it takes to decrypt or encrypt [reference]. An implementation of fully homomorphic encryption in the C++ language, HELib, has made an effort to make homomorphic evaluation runs faster.

Motivation

## 1.1 PROBLEM DEFINITION

## 1.2 MOTIVATION

There are many areas in which homomorphic cryptography could be used, such as the medical, marketing, and financial fields [reference]. Until now, there was no way to make a concrete implementation related to these areas, since most available schemes were either too limited or too slow. Using HELib, it would be possible to

reevaluate and show how feasible an implementation of a real life use case would be.

## 1.3 HIPOTHESIS

## 1.4 OBJECTIVES

### GENERAL OBJECTIVE

### SPECIFIC OBJECTIVES

1. Uno
2. Dos
3. Tres

## 1.5 STUDY CASE

## 1.6 ESTRUCTURA DE LA TESIS

Esta tesis se compone de los siguientes capítulos: introducción, marco teórico, trabajos relacionados, metodología, caso de estudio, experimentos y resultados y conclusiones.

En este capítulo se planteó la definición del problema y la motivación para trabajar con los reportes del CIC para implementar un sistema de detección de duplicados.

El capítulo ?? presenta la notación y las definiciones de conceptos necesarios para familiarizar el lector con el contenido técnico presentado en la tesis.

En el capítulo 3 se presentan algunos trabajos que abordan el tema de la detección de documentos duplicados y se describe brevemente que métodos y que documentos se utilizan para cada trabajo.

El capítulo ?? presenta el sistema de detección de duplicados de forma general; se explican cuáles son los distintos procesos que forman al sistema independientemente del contexto en el que se apliquen.

En el capítulo ?? el sistema propuesto se aplica a los reportes ciudadanos del CIC. Aquí se describen las implementaciones de cada uno de los procesos que forman el sistema de detección de duplicados.

El capítulo ?? presenta los experimentos realizados para probar el funcionamiento del sistema de detección de duplicados, muestra los resultados de desempeño obtenidos y presenta conclusiones en base a esos resultados.

Finalmente en el capítulo ?? se presentan las conclusiones generales obtenidas a partir de los resultados del sistema de detección de duplicados durante las pruebas y se presentan las posibles mejoras que serán incorporadas como trabajos a futuro.

## CHAPTER 2

# BACKGROUND

---

## 2.1 INFORMATION SECURITY AND DATA

### CONFIDENTIALITY

Information security is a broad topic that covers many different aspects, from which two of them are particularly important for this work: confidentiality and authentication. There are some aspects from cryptography that would be relevant to review, such as asymmetric and symmetric cryptography, as well as the concepts: plaintext, ciphertext and key. A brief description of cloud computing and its relevant aspects are to be described, and how can secure computations have an impact on it. Finally, the concept of homomorphic encryption will be explained, noting the mathematical properties and its categories of somewhat homomorphic encryption and fully homomorphic encryption, as well as possible applications of this type of encryption.

Seen from a general perspective, Whitman and Herbert describe security as “the quality or state of being secure—to be free from danger” [? ]. Danger would refer to a potential harming action that an adversary can do, whether it is intentionally or not. Computer security is defined by the NIST Computer Security Handbook as follows:

“The protection afforded to an automated information system in order to attain the applicable objectives of preserving the integrity, availability, and confidentiality of information system resources (includes hardware, software, firmware, informa-

tion/data, and telecommunications)”.<sup>1</sup>

This definition introduces two relevant concepts in the previous definition, namely: information and confidentiality. RFC 2828 defines information as “facts and ideas, which can be represented (encoded) as various forms of data,” and data as “information in a specific physical representation, usually a sequence of symbols that have meaning: especially a representation of information that can be processed or produced by a computer.” Although both terms have different connotations depending on what it represents, both words are used interchangeably in this work. Now that information has been described as a concept, confidentiality can be introduced. More specifically, data confidentiality refers to the assurance that private or confidential information is not made available or disclosed to unauthorized individuals [? ]. This term is closely related to *privacy*, which gives individuals the control of what information related to them may be accessed or stored by whom. Loss of confidentiality would imply unauthorized access or disclosure of information.

## 2.2 CRYPTOGRAPHY CONCEPTS

Cryptography is the study of protecting data and communications. It involves communicating messages or information between two or more parties by changing the appearance of the messages, so that it becomes very difficult for unauthorized parties to intercept or interfere with the transmission of the information [? ]. Cryptography should not be confused with cryptology, because even though they overlap with each other, cryptology includes cryptanalysis as well.

Other important aspects that have been linked to cryptography apart from confidentiality are *authentication* and *integrity*. Authentication gives the receiving parties the means to make sure that the source of the communication is really who it is thought to be, and not someone pretending to be the legitimate sender. On the other hand, integrity, among other things, ensures that the message received has not been tampered in any way. Different kind of cryptographic algorithms are required



to satisfy these goals, and the same goes for confidentiality.

[?] approaches the study of cryptography by explaining the classical problem of transmitting secret messages from a sender A to a receiver B. Both the sender and receivers, A and B, can be thought of persons, organizations, or various technical systems. They are formally described as abstract “parties” or “entities”, but it is often more convenient to identify both of them as human participants that go by the names of Alice and Bob, instead of using the letters A and B. For the messages to go from Alice to Bob, there has to be some kind of medium or channel where the information is transmitted. Generally, it is assumed that the channel can be potentially accessed by a third party that is neither the sender nor the receiver. On top of it, it is also assumed that either the receiver or sender has an adversary or enemy E, who is constantly trying to tamper or know the content of the messages passed between Alice and Bob. This attacker E who is constantly eavesdropping on the communication lines is usually called Eve, who is thought to have powerful computing facilities and is able to make use methods to learn of the message contents. It is clear that both Alice and Bob want to protect their messages so that their contents are unclear to Eve. This implies a confidentiality goal, and it can be reached by using encryption algorithms, or as they are commonly known: ciphers.

Before a message is sent from Alice to Bob or viceversa, the sender *encrypts* the message. In other words, a certain algorithm is applied on the message so that the content of it is unclear to a third party, such as Eve. Once the sent message has been received by the other party, he *decrypts* it to recover the original content of the message before encryption, known as *plaintext*. In order for this to work, Alice and Bob must have agreed in advance about various of the details needed, such as the algorithm and parameters used by the sender, so that the receiver knows how to perform the decryption properly. Since it would be troublesome that Eve learned of those details, Alice and Bob would have to use some sort of secure channel, such as trusted messengers or couriers, or set up a private meeting where no one else can eavesdrop. Making use of this secure channel, however, is more costly than using

the regular communication channel between Alice and Bob, and it might not be available at all times, which is why it is recommended to use it only to agree on some details in advance, like the parameters for the encryption algorithm, as well as a *private key*.

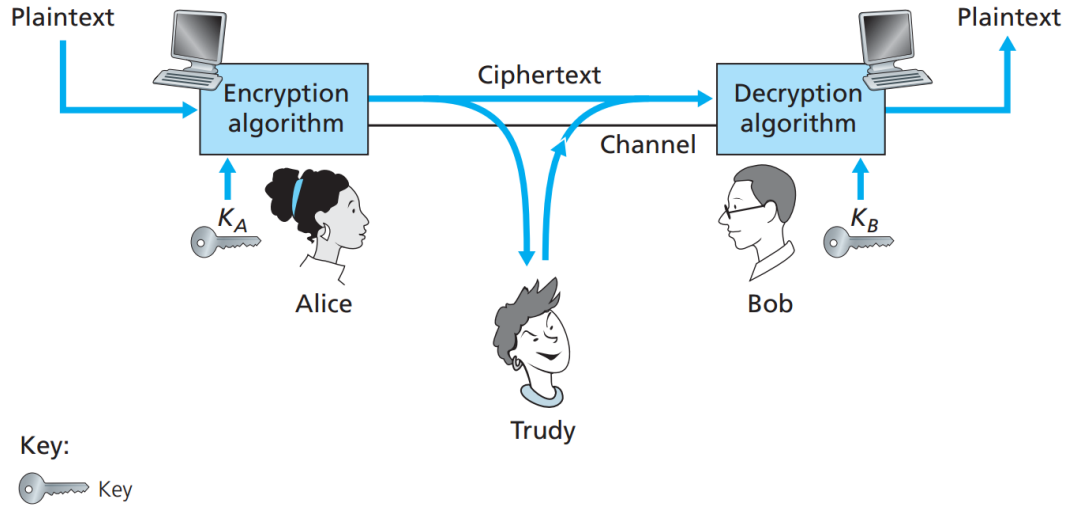
As Stanoyevitch [?] describes it on his Introduction to Cryptography book, the above scheme would make up a cryptosystem, since it makes of cryptographic algorithms. As it has been hinted, such a cryptosystem has two parts: encryption, which is done by the sender as means of putting or transforming the actual **plaintext** into **ciphertext**, and **decryption**, which is done once the message has been received at the other end and has the purpose of translating the ciphertext back into the original plaintext message. Both encryption and decryption are usually done using a **key**, and it is intended that only the sender and receiver learn of the nature and value of the key.

Stinson [?] formally defines a cryptosystem as a five-tuple  $(\mathcal{P}, \mathcal{C}, \mathcal{K}, \mathcal{E}, \mathcal{D})$ , where the following conditions are satisfied:

1.  $\mathcal{P}$  is a finite set of possible plaintexts;
2.  $\mathcal{C}$  is a finite set of possible ciphertexts;
3.  $\mathcal{K}$ , the keyspace, is a set of possible keys;
4. For each  $K \in \mathcal{K}$  there is an *encryption rule*  $e_K \in \mathcal{E}$  and a corresponding *decryption rule*  $d_K \in \mathcal{D}$ . Each  $e_K : \mathcal{P} \rightarrow \mathcal{C}$  and  $d_K : \mathcal{C} \rightarrow \mathcal{P}$  are functions such that  $d_K(e_K(x)) = x$  for every plaintext element  $x \in \mathcal{P}$ .

The following image depicts the process flow of a simple cryptosystem that is put in place. The plaintext and encryption key is used as input by an encryption algorithm, which results in a ciphertext that gets sent to the other end of the communication line. Then, the ciphertext is used as input by a decryption algorithm, which takes a decryption key  $H$ , finally resulting in the original plaintext. Depending

on the type of cryptosystem (symmetric or asymmetric), the encryption keys can be considered to be the same or different, i.e.  $K_A \neq K_B$  if asymmetric, or  $K_A = K_B$  if symmetric.



**Figure 2.1** – Basic cryptosystem diagram[? ]

### 2.2.1 THE FIRST FULLY HOMOMORPHIC ENCRYPTION SCHEME

The use of the plaintext, ciphertext, and key concepts are recurrent throughout this work, and given their relevance, they are explained as follows, along with other concepts, the same way Cameron [? ] and Stallings [? ] describe them.

**Plaintext** It is the original message that Alice wants to send to Bob. Neither Bob nor Alice want the message to be known to Eve. This piece of data is fed into the encryption algorithm before it is sent to the other end.

**Encryption algorithm** This algorithm uses a key that performs several substitutions and transformations on the plaintext. The result is a ciphertext that cannot be translated back to its original plaintext form without the appropriate key.

**Secret key** It is a value that both Alice and Bob agree upon, it is usually kept

secret, since it is used by the encryption algorithm to obtain the ciphertext. The encryption algorithm will produce a distinct output depending on the key being used. In other words, if the secret key is leaked to Eve, this would mean that Eve can now learn any messages that are being produced using that key.

**Ciphertext** This is the modified or scrambled message produced as output by the encryption algorithm. Its value depends on the algorithm and plaintext, as well as the chosen key.

**Decryption algorithm** It is like the encryption algorithm, except that its run in reverse. It has two inputs: the ciphertext and its corresponding key. It then performs the necessary substitutions and transpositions so that the ciphertext is translated back into the original plaintext.

It is assumed at all times that Eve has access to the communication lines, and can always know the ciphertext that is being sent between Alice and Bob. It is also a good practice to assume that the encryption and decryption algorithms are known to Eve, which leaves the key as the most important part of the scheme. As mentioned previously, Alice and Bob need to establish the means to agree on a private key, without anyone else knowing what this key is. As soon as the key is known to the attacker, i.e. Eve, all further communication is compromised. When that happens, another key has to be chosen and agreed upon once more by Alice and Bob.

The nature of the encryption and decryption can be classified in two broad categories: symmetric and asymmetric key encryption. [?] describes both categories by looking at the keys used. In **symmetric encryption**, the key used for encryption is virtually the same used in decryption. Therefore, key distribution has to be done before the transmission of the messages. The key length has a direct impact on the security offered by the encryption algorithm. Meanwhile, **asymmetric encryption** employs two different keys: public and private. The public key is closely tied to the receiver, and it is used for encryption of the data. It is called public because it

is available for general use; therefore, Alice can use Bob's public key to send him messages. On the other hand, the private is kept secret from the outside world, and is only available by an authorized person. The public and private keys always come in pairs, and this is because while the public key is used to encrypt data; its counterpart, the private key, is used to decrypt the data.

## 2.3 INTRODUCTION TO NUMBER THEORY

In order to understand more topics pertaining cryptosystems, number theory and its notation, as well as some relevant theorems must be introduced. Number theory consists of the study of the properties of whole numbers, and it is considered to be inextricably linked to cryptography [? ]. Wiley & Sons [? ] introduces number theory and most of its relevant aspects in his Algorithm Design book. The information is briefly summarized as follows.

### 2.3.1 FUNDAMENTAL THEOREMS

Given positive integers  $a$  and  $b$ , we use the notation

$$a|b \tag{2.1}$$

to indicate that  $a$  **divides**  $b$ , that is,  $b$  is a multiple of  $a$ . If  $a|b$ , then we know that there is some integer  $k$ , such that  $b = ak$ . From this definition, the following properties are found:

**Theorem on Divisibility:** *Let  $a$ ,  $b$  and  $c$  be arbitrary integers. Then*

- If  $a|b$  and  $b|c$ , then  $a|c$ .
- If  $a|b$  and  $a|c$ , then  $a|(ib + jc)$ , for all integers  $i$  and  $j$ .
- If  $a|b$  and  $b|a$ , then  $a = b$  or  $a = -b$ .

An integer  $p$  is said to be **prime** if  $p \geq 2$  and its only divisors are the trivial divisors 1 and  $p$ . Therefore, in the case  $p$  is a prime,  $p|q$  implies  $d = 1$  or  $d = p$ . An integer greater than 2 that is not prime is said to be **composite**. Also, two integers which only share a common divisor of 1 are considered to be **relatively prime**.

**Fundamental Theorem of Arithmetic:** *Let  $n > 1$  be an integer. Then there is a unique set of prime numbers  $\{p_1, \dots, p_k\}$  and positive integer exponents  $\{e_1, \dots, e_k\}$ , such that*

$$n = p_1^{e_1} \cdots p_k^{e_k} \quad (2.2)$$

The above product  $p_1^{e_1} \cdots p_k^{e_k}$  is the prime decomposition of  $n$ . In other words, any positive integer is made up of the product between two prime numbers.

### 2.3.2 MODULAR ARITHMETIC

The remainder of  $a$  when divided by  $n$  is expressed as  $a \bmod n$ , that is:

$$r = a \bmod n \quad (2.3)$$

which means that there is some integer  $q$ , such that

$$a = qn + r \quad (2.4)$$

The remainder  $r$  is resulting from  $a \bmod n$  is always an integer in the set  $\{0, 1, 2, \dots, n-1\}$ , even when  $a$  is negative. The integer  $n$  is called the **modulus**.

**Congruence** modulo  $n$  is a relevant concept to mention as well. If

$$a \bmod n = b \bmod n, \quad (2.5)$$

then we say that  $a$  is **congruent** to  $b$  modulo  $n$ , so it can be written in this way:

$$a \bmod n \equiv b \pmod{n} \quad (2.6)$$

Therefore, if  $a \equiv b \pmod{n}$ , then  $a - b = kn$  for some integer  $k$ .

Congruences have the following properties:

1.  $a \equiv b \pmod{n}$  if  $n|(a - b)$ .
2.  $a \equiv b \pmod{n}$  implies  $b \equiv a \pmod{n}$ .
3.  $a \equiv b \pmod{n}$  and  $b \equiv c \pmod{n}$  imply  $a \equiv c \pmod{n}$ .

Suppose that there is a mapping of all integers by the  $\pmod{n}$  operator into the set of integers  $\{0, 1, \dots, (n - 1)\}$ . The technique used to perform arithmetic operations within the confines of this set is known as **modular arithmetic**. Modular arithmetic exhibits the following properties:

1.  $[(a \pmod{n}) + (b \pmod{n})] \pmod{n} = (a + b) \pmod{n}$
2.  $[(a \pmod{n}) - (b \pmod{n})] \pmod{n} = (a - b) \pmod{n}$
3.  $[(a \pmod{n}) \times (b \pmod{n})] \pmod{n} = (a \times b) \pmod{n}$

And so, the rules for ordinary arithmetic involving addition, subtraction, and multiplication carry over into modular arithmetic, except that the result is always within the boundaries of the defined set, which is between 0 and  $n - 1$ .

Having explained modulo operation, as well as having introduced modular arithmetic operations; a very convenient notation, called the set of residues, can be now described. Let  $Z_n$  denote the set of nonnegative integers less than  $n$ :

$$Z_n = \{0, 1, \dots, (n - 1)\} \tag{2.7}$$

The set  $Z_n$  is also called the set of **residues**, or **residue classes** modulo  $n$ , because if  $b = a \pmod{n}$ ,  $b$  is sometimes called the **residue** of  $a$  modulo  $n$ . The notation  $Z_n$  represents the integers between 0 and  $n - 1$ . Thus, when it says that

“Let  $a$  be a member of  $Z_n$ ”, it really means the same thing as “Let  $a$  be an integer between 0 and  $n - 1$ .”

Modular arithmetic in  $Z_n$ , where operations on the elements of  $Z_n$  are performed  $\text{mod } n$ , show properties similar to those in traditional arithmetic, such as the **associativity**, **commutativity**, **distributivity** of addition and multiplication, and the existence of **identity** elements 0 and 1 for addition and multiplication, respectively. However, other operations such as division and exponentiation, behave very differently than they do for normal arithmetic.

Every element  $x$  in  $Z_n$  has an **additive inverse**, that is, for each  $x \in Z_n$ , there is a  $y \in Z_n$  such that  $x + y \text{ mod } n = 0$ . In other words, it means there is an element  $a'$  that multiplies  $a$  modulo  $n$  which equals 1. A **multiplicative inverse** of  $x$  is an element  $z^{-1} \in Z_n$  such that  $xx^{-1} \equiv 1 \text{ mod } n$ . As well as in regular arithmetic, 0 does not have a multiplicative inverse in  $Z_n$ . There are some nonzero elements that do not have a multiplicative inverse in  $Z_n$ . However, for every  $n$  that is prime, every element  $x \neq 0$  of  $Z_n$  has a multiplicative inverse.

### 2.3.3 ORDER AND GENERATORS

Given a prime  $p$  and an integer  $a$  between 1 and  $p - 1$ , the **order** of  $a$  is the smallest exponent  $e > 1$  such that

$$a^e \equiv 1 \text{ mod } p \quad (2.8)$$

A **generator**, or primitive root, of  $Z_p$  is an element  $g$  of  $Z_p$  with order  $p - 1$ . It is called a generator, because the repeated exponentiation of it can generate all the elements in  $Z_p^*$ .  $Z_p^*$  is a subset of  $Z_p$ , defined to be the set of integers between 1 and  $n$  that are relatively prime to  $n$ .

**Theorem on generators:** *If  $p$  is a prime, then set  $Z_p$  has  $\phi(p - 1)$  generators.*



### 2.3.4 OTHER RELEVANT POINTS ON MODULAR ARITHMETIC

Cliff and Ken [?] review a several important concepts regarding modular arithmetic, and are thus listed as follows:

- *Fermat's Little Theorem* Let  $p$  be a prime, and let  $x$  be an integer such that  $x \bmod p \neq 0$ . In other words, that  $p$  does not divide the integer  $x$ . Then

$$x^{p-1} \equiv 1 \pmod{n} \tag{2.9}$$

- *Euclid's Division Theorem.* For every integer  $m$  and positive integer  $n$ , there exist unique integers  $q$  and  $r$  such that  $m = nq + r$  and  $0 \leq r < n$ . By definition,  $r$  is equal to  $m \bmod n$ .
- *Adding multiples of  $n$  does not change values mod  $n$ .* That is,  $i \bmod n = (i + kn) \bmod n$  for any integer  $k$ .
- *Mods (by  $n$ ) can be taken anywhere in calculation, as long as  $\bmod n$  is taken from the final result.*
- *Commutative, associative, and distributive laws.* Addition and multiplication  $\bmod n$  satisfy the commutative and associative laws, and multiplication distributes over addition.
- The expression " $x \in Z_n$ " is used to mean that  $x$  is a variable that can take on any of the integral values between 0 and  $n - 1$ .

## 2.4 ABSTRACT ALGEBRA: GROUPS AND RINGS

Stallings [?] describes groups and rings as some of the fundamental elements of a branch of mathematics known as abstract algebra. In abstract algebra, the focus is mostly on whose elements can be operated algebraically; in other words, how two

elements of a set can be combined to obtain a third element in the set. The nature of the set is defined by the operations that can be performed on the elements of the set. It has been chosen that, by convention, the two main classes of operations on set elements is usually the same as the notation for addition and multiplication on traditional arithmetic. However, there is not really a limitation on the kind of operations that can be defined using abstract algebra.

### 2.4.1 GROUPS

A **group**  $G$ , denoted by  $\{G, \cdot\}$  is a set of elements with a binary operation, which is denoted by the operator  $\cdot$ . This operator is generic, and can refer not only to multiplication, but also to addition or some other mathematical operation. It associates to each ordered pair  $(a, b)$  of elements in  $G$  an element  $(a \cdot b)$  in  $G$ , such that the following axioms are followed:

**(A1) Closure:** If  $a$  and  $b$  belong to  $G$ , then  $a \cdot b$  is also in  $G$ .

**(A2) Associative:**  $a \cdot (b \cdot c) = (a \cdot b) \cdot c$  for all  $a, b, c$  in  $G$ .

**(A3) Identity element:** There is an element  $e$  in  $G$  such that  $a \cdot e = e \cdot a = a$  for all  $a$  in  $G$ .

**(A4) Inverse element:** For each  $a$  in  $G$  there is an element  $a'$  in  $G$  such that  $a \cdot a' = a' \cdot a = e$

It is worth noting that if a group has a finite number of elements, it is called a **finite group**, and the order of the group is equal to the number of elements in the group. Otherwise, the group is an **infinite group**. Moreover, a group is said to be **abelian** if it satisfies the following condition, known as the commutative property:

**(A5) Commutative**  $a \cdot b = b \cdot a$  for all  $a, b$  in  $G$ .

When the group operation is addition, the identity element is 0; the inverse element of  $a$  is  $-a$ ; and subtraction is defined with the following rule:  $a - b = a + (-b)$ .

### 2.4.2 RINGS

A *ring*  $R$ , which is sometimes denoted by  $\{R, +, \times\}$ , is a set of elements with two binary operations, called *addition* and *multiplication*, such that for all  $a, b, c$  in  $R$ , there are some axioms that must be obeyed. It must be pointed out that a ring fulfills the same axioms as an abelian group with respect to addition; that is, axioms A1 through A5. The rest of the axioms obeyed by a ring are listed as follows:

**(M1) Closure under multiplication:** If  $a$  and  $b$  belong to  $R$ , then  $ab$  is also in  $R$ .

**(M2) Associativity of multiplication:**  $a(bc) = (ab)c$  for all  $a, b, c$  in  $R$ .

**(M3) Distributive laws:**  $a(b + c) = ab + ac$  for all  $a, b, c$  in  $R$ .

$$(a + b)c = ac + bc \text{ for all } a, b, c \text{ in } R.$$

Essentially, a ring is a set in which addition, subtraction [ $a - b = a + (-b)$ ], and multiplication can be done without leaving the set. Furthermore, a ring is said to be **commutative** if it satisfies the following additional condition:

**(M4) Commutativity of multiplication:**  $ab = ba$  for all  $a, b, c$  in  $R$ .

Additionally, there are other axioms that, if obeyed, turn the commutative ring into an **integral domain**. The axioms are:

**(M5) Multiplicative identity:** There is an element 1 in  $R$  such that  $a1 = 1a = a$  for all  $a$  in  $R$ .

**(M6) No zero divisors:** If  $a, b$  in  $R$  and  $ab = 0$ , then either  $a = 0$  or  $b = 0$ .

### 2.4.3 HOMOMORPHISMS

As it has been previously noted, operations within a closed group always results in another element from the group. Only one operation is allowed in a group: addition

or multiplication, for instance. There may also be two groups or sets that are made up of different elements, each with its own operation. A *homomorphism* consists of the construction of a function that *translates* elements from one group to other with the same properties.

## GROUP HOMOMORPHISM

According to Beachy and Blair [? ], a homomorphism is defined as follows: Let  $G_1$  and  $G_2$  be groups, and let  $\mu : G_1 \rightarrow G_2$  be a function. Then  $\mu$  is said to be a **group homomorphism** if

$$\phi(a * b) = \phi(a) *' \phi(b) \quad (2.10)$$

for all  $a, b$  in  $G_1$ .

Consider the example presented by Sorzano [? ], where there are two sets:  $S = \{a, b, c\}$  and  $S' = \{A, B, C\}$  with the operations  $*$  :  $S \times S \rightarrow S$  and  $*'$  :  $S' \times S' \rightarrow S'$ . The operations within each group map two elements to another element from the same group. For example,  $b * c = a$  and  $a * a = a$ , and in the other group,  $A * A = A$  and  $B * C = A$ . So it can be seen that there is some similarity in the nature of the operations from both groups. Now consider the homomorphism between both groups, where:

$$\begin{aligned} \phi : S &\rightarrow S' \\ \phi(a) &= A \\ \phi(b) &= B \\ \phi(c) &= C \end{aligned} \quad (2.11)$$

Therefore, the following holds true using the above homomorphism:

$$b * c = a \rightarrow \phi(b) *' \phi(c) = \phi(a) \rightarrow B *' C = A \quad (2.12)$$

What this means is that an homomorphism serves as a link to translate operations from one group to the other, keeping the same properties.

## RING HOMOMORPHISM

Similarly to groups, there are also homomorphisms for rings, the main difference being that, instead of a single operation, two operations are considered.

Let  $R$  and  $S$  be rings with addition and multiplication. The map  $\phi : R \rightarrow S$  is a homomorphism if:

1.  $\phi$  is a group homomorphism on the additive groups  $(R, +)$  and  $(S, +)$
2.  $\phi(xy) = \phi(x)\phi(y) \forall x, y \in R$

## 2.5 HOMOMORPHIC ENCRYPTION

Homomorphic is a word that has its roots in Greek, and it means “the same shape”. It is used in various areas, such as abstract algebra, which then inspired its use in cryptography. This concept refers to the ability to make computations, i.e. operations such as addition and multiplication, on encrypted data without sharing the secret key to decrypt the data prior to the computations.

Before homomorphic encryption, any change to the secret value contained in a ciphertext required a secret key, so that it could be decrypted, applied the change, and encrypted once more. However, homomorphic encryption eliminates the need of sharing a secret, especially of those that are not to be completely trusted, in the event that there is some change that must be applied to encrypted data. Lange [?] mentions several applications that would benefit from a homomorphic encryption function, given its ability to manipulate encrypted data. Such applications fall in the domain of: e-cash, e-voting, private information retrieval (encrypted databases), and cloud computing.

### 2.5.1 WHAT IS HOMOMORPHIC ENCRYPTION?

According to Pötzelberger [? ], a *homomorphic cryptosystem* is a cryptosystem whose encryption function is a homomorphism, and thus preserves group operations performed on ciphertexts. Arithmetic addition and multiplication are the two group operations considered. A homomorphic encryption is additive if:

$$\mathcal{E}(x + y) = \mathcal{E}(x) \otimes \mathcal{E}(y) \quad (2.13)$$

where  $\mathcal{E}$  denotes an encryption function,  $\otimes$  denotes an operation depending on the used cipher and  $x$  and  $y$  are plaintext messages. A homomorphic encryption is multiplicative if:

$$\mathcal{E}(x \cdot y) = \mathcal{E}(x) \otimes \mathcal{E}(y) \quad (2.14)$$

where again  $\mathcal{E}$  denotes an encryption function,  $\otimes$  denotes an operation depending on the used cipher and  $x$  and  $y$  are plaintext messages.

The above conditions basically mean that the result of the operations between both inputs, in this case  $x$  and  $y$ , is going to be the same whether the operations are performed after or before encryption. This property is what makes it possible to perform computations on an encrypted value which later can be decrypted to the correct result.

### 2.5.2 CLASSIFICATION OF HOMOMORPHIC ENCRYPTION SCHEMES

Homomorphic encryption schemes are classified in two broad groups, depending on how they are defined, namely: partially homomorphic and fully homomorphic.

*Partially homomorphic encryption* schemes are defined over a group, and they support one operation at most: either addition or multiplication. On the other hand,

*fully homomorphic encryption* schemes are defined over a ring, and can support up to two operations: addition and multiplication.

According to Pötzelberger [? ], even though somewhat homomorphic cryptosystems support a limited number of homomorphic operations, they are the building blocks for fully homomorphic encryption, and provide much more efficiency and shorter ciphertexts than their fully homomorphic counterparts.

### 2.5.3 EXPLAINING HOMOMORPHIC ENCRYPTION

Craig Gentry [? ] explains the concept behind homomorphic encryption in a very simple way. Imagine that Alice has a jewelry store and wants her workers to assemble raw materials (diamonds, gold, silver, etc.) into finished products (necklaces, rings, etc.), but does not trust them, thinking they might run off with the raw materials at their first chance. So instead of giving them direct access to the materials, Alice places the materials inside a transparent glove boxes, which are then promptly locked with a key that only Alice has. This way, workers can put their hands inside the gloves that are connected to the box, and through them, work with the raw materials. Additionally, the workers are able to put in things inside the boxes, such as soldering iron to use on the raw materials, even though they cannot take anything out of it, since they do not have the key to open the box. Once the products are finished, Alice can then recover the products from the box using her key. The analogy is not perfect, because the workers are able to see the materials while working on them, but it is relevant because homomorphic encryption means that they cannot take the materials with them.

Hayes [? ] offers a proof of concept that accurately describes the very essence of a homomorphic cryptosystem. It is shown as follows: consider that the plaintext consists of integers; to encrypt a number, double it; to decrypt it, divide by 2. Using this scheme, addition and a nonstandard version of multiplication can be done on the encrypted data. Given the plaintext inputs  $x$  and  $y$ , we can encrypt each of them

separately, add the ciphertexts, and finally decrypt the result. This calculation would give the correct answer, expressed as  $2x + 2y = 2(x + y)$ . In other words, it is the same result as if each plaintext input were multiplied separately and then added with each other, because in the end, dividing it by two causes the same effect on both sides. Regarding multiplication, a little tweak has to be done for it to work. The product of the ciphertexts is defined as  $\frac{(C_x C_y)}{2}$ , while the plaintexts are multiplied by the regular formula  $xy$ . As an example, consider  $x = 5$  and  $y = 7$ . The result of the multiplication would come as  $x \times y = 35$ . To perform the same evaluation between ciphertexts, both inputs would be encrypted following the rule previously defined:

$$C_x = 5 \times 2 = 10 \quad (2.15)$$

$$C_y = 7 \times 2 = 14 \quad (2.16)$$

The multiplication is performed as  $\frac{10 \times 14}{2} = 70$ , and it is divided by 2 to decrypt the ciphertext:  $\frac{70}{2} = 35$ , which turns out to be the correct result of the multiplication.

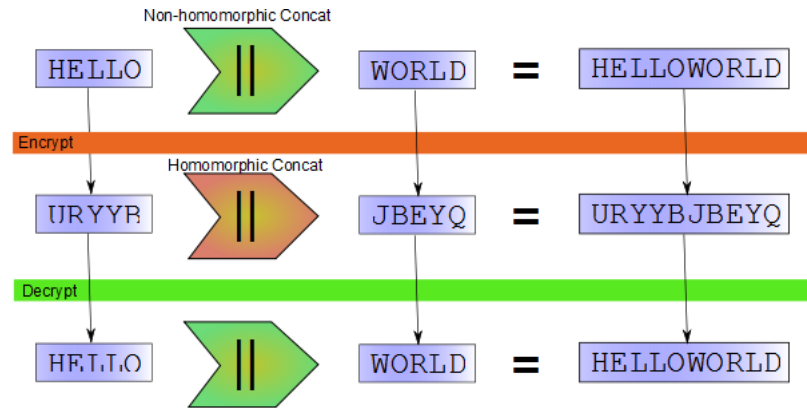
Certainly, the above cryptosystem is considered to be *fully homomorphic*, since it supports both addition and multiplication. However, it is definitely not a secure one, since anyone that knows of the workings of the cipher can easily get to learn the secret message. This is also partly the reason of why the encryption and decryption algorithms are not usually kept as a secret, because it would be quite impractical to make a different encryption and decryption algorithm every time it were discovered.

Stuntz [?] provides an example of a cipher scheme that has an inherent homomorphic property. Consider the popular, yet insecure, ROT13 cipher, also known as the Caesar cipher. The ROT13 is a simple substitution cipher which changes each letter with the letter that is thirteen positions ahead on encryption. To do decryption, it does the opposite: each character goes back thirteen positions in the alphabet. ROT13 is partially homomorphic with respect to the concatenation operation, because it is possible to concatenate two different pieces of ciphertexts. At decryption, the concatenation of the two pieces of ciphertexts ends up to be same as the concatenation of the original messages, the plaintext. Therefore, it is partially



homomorphic with respect to concatenation.

Consider, for instance, the plaintext pair  $P_1 = \text{HELLO}$  and  $P_2 = \text{WORLD}$ , which both concatenate to “HELLOWORLD”. If both plaintexts were encrypted using ROT13, the ciphertexts would be  $C_1 = \text{URYYB}$  and  $C_2 = \text{JBHEYQ}$ , and if concatenated, “URYYBJBEYQ”. So if the result of the ciphertext concatenation was decrypted, it would result in “HELLOWORLD”, which is exactly the same message obtained when the pair of plaintexts were concatenated originally.



**Figure 2.2** – Example of the homomorphic property in ROT13[?] ]

#### 2.5.4 THE FIRST FULLY HOMOMORPHIC ENCRYPTION SCHEME

In an article published by the research staff of the NSA [?] ], it is discussed how the term of fully homomorphic encryption has been around for about 30 years, and only recently, in 2009 to be exact, a very important breakthrough regarding fully homomorphic encryption was made by Craig Gentry. Thanks to this breakthrough, the first fully homomorphic scheme was created, opening the doors of research to this topic that was thought to be at a dead end.

As Gentry [?] ] himself points out, by “fully” it means that there are no limitations on what manipulations can be performed on the ciphertext. Given ciphertexts  $c_1, \dots, c_t$  that encrypt  $m_1, \dots, m_t$  with a fully homomorphic encryption scheme under some key, and given any efficiently computable function  $f$ , anyone can efficiently

compute a ciphertext (or set of ciphertexts) that encrypts  $f(m_1, \dots, m_t)$  under that key. This permits all kinds of computations on the encrypted data. Of course, no information about the plaintexts  $m_1, \dots, m_t$  or the value of  $f(m_1, \dots, m_t)$  is leaked.

One of the main problems that fully homomorphic encryption faces is the managing of “noise” in the ciphertext. In this case, noise refers to the gradual distortion of ciphertexts that is caused by doing operations (e.g., addition or multiplication) on them. As more operations are performed on the ciphertext, the larger the noise becomes, rendering the ciphertext undecipherable, as it would end up with the wrong result after attempting to decrypt. As Hayes [?] points out, each homomorphic addition doubles the noise, and each multiplication squares it. Thus, the number of operations must be limited, should one not wish to “corrupt” the ciphertext.

An obvious solution to the problem of accumulating noise is to decrypt the data and decrypt it whenever the noise begins to approach the critical threshold. Thus, the noise is reset to its original level, allowing for more operations. The problem with this approach is that decryption needs the secret key, which is usually not going to be available to whoever is performing the operations on the ciphertext.

Instead, Gentry developed a process called *bootstrapping* to overcome the problem of noise accumulation. The *evaluate* function used in the cryptosystem can virtually perform any computation, as long as it has not reached the critical threshold of noise. Therefore, that very function is used to run the *decrypt* function, which takes as input the noise-heavy ciphertext that has gone through several operations, and the *encrypted secret key*. This makes sense because the decrypt function relies on the evaluate function, which was made to work on encrypted values. So when the *decrypt* function is run, the decrypted result is not the plaintext, but rather a new encryption, with reduced noise. This process of re-encrypting and refreshing the noisy ciphertext can be repeated as many times as needed, because then it technically has no limitations on the number of operations it can perform. However, extra layer of encryption will consequently increase the overall computational effort required to complete the set of operations desired to be performed. As the research

staff from the NSA exemplifies [? ], if the process were to be used by Google to search the web homomorphically, the normal computing time required would be multiplied by about a trillion. This is the reason why this scheme is not practical enough as a solution for implementation.

### 2.5.5 RING-LEARNING WITH ERROR PROBLEMS

[? ] More recently, Brakerski, Vaikuntanathan and Gentry have developed a variant of the learning-with-errors system that takes a different approach to noise management. Instead of stopping the computation at intervals to re-encrypt the data, they incrementally adjust parameters of the system after every computational step in a way that prevents the noise level from ever approaching the limit.

### 2.5.6 BGV SCHEME

### 2.5.7 APPLICATIONS

As it was previously mentioned, various individuals and corporations are hesitant to make use of cloud services, mainly because of the security risks it poses, such as privacy loss and tampering of the data. Traditional cryptosystems can protect confidentiality if the only goal is remote storage; but it is restricted to just that. However, if the encryption employed were homomorphic, the cloud server where the data is stored would be able to perform meaningful computations on the encrypted data. The scenarios where homomorphic encryption can be applied are many, and some of the most important areas are: medical applications, financial applications, advertising and pricing, electronic voting, data mining, and biometric authentication.

**Medical applications** A straightforward application of homomorphic encryption in a medical context is the storage and processing of medical records of patients. Most of the time, these records are considered private, but a hospital would be greatly benefited if it could rely on a cloud service rather than setting

up its own data center. If homomorphic encryption has been used as a cipher to protect the medical records, then there could be meaningful pieces of information that could be extracted from it. For example, if a doctor in another clinic was granted access to the patient's medical record, he could make use of a function that makes homomorphic evaluations on the encrypted record to obtain statistical data from the patient. And this would be possible without the cloud server from learning what information the medical record contained.

**Financial applications** Data about corporations, such as their stock price, performance or inventory is often considered to be private, and also very relevant towards making investment decisions. There are many kinds of computations that could be done on this private data, such as running predictive simulations on stock price performance.

**Advertising and pricing** Nowadays, user activity is heavily monitored by the websites that he browses, and uses this data as to build a profile and direct to him appropriate ads. Take Amazon as an example: amazon keeps a history on what items the user is looking at, even if they were not bought, for further advertising. The user might get emails advertising the same or similar products, or might even see the same ads on unrelated sites, such as Facebook [? ]. The above example is just the tip of the iceberg. There is a lot of contextual information that can be retrieved from the user to gain insight on his interests and current status, without him having to be active. For example, he could be subscribed to a service that sends alerts to his smartphone whenever he is walking nearby a shop that could be of his interest. Geolocation and user profile are pieces of information that are considered to be private, the user might not subscribe to the service if it just shared around his information. If, however, he is guaranteed that his information is being kept encrypted, and that it stays that way whenever a computation is made to see if there is a nearby store of his interest, then he might not be as reluctant to make use of the service. The applications on advertising using homomorphic encryption

are diverse, because there are pieces of data that can be obtained passively, such as the user's geolocation or browsing habits.

**Electronic Voting** Although not necessarily related to cloud computing, electronic voting is another area where homomorphic encryption can to make such voting scheme feasible and convenient. Voting could be done from anywhere as long as the participant is provided with Internet access. Simply put, homomorphic encryption can be used to hide the content of a ballot by calculating the total number of votes, without having to decrypt any of the ballots. However, electronic voting is not as trivial as it seems, because apart of confidentiality, there are other security parameters that are equally important, such as: correctness of the tally, democracy, robustness, verifiability, fairness, and verifiable participation. Therefore, applying homomorphic encryption would only satisfy one of the many requirements for electronic voting to be feasible.

**Data mining** Closely related to advertisement, data mining has several issues on privacy. Using homomorphic encryption would mean that certain sensitive attributes of customers data can be protected from unauthorized parties. Yang, Zhong and Wright [?] have proposed a solution that allows a data miner to compute frequencies of values or tuples of values in the customers' data in a privacy preserving manner. The advantage of this solution is that, unlike other existing attempts to make data mining more private, is that this solution is both fully private and fully accurate, without compromising one in exchange of the other.

**Biometric Authentication** Biometric authentication is an area of application quite different from the rest. This is because biometric data is assumed to be public, since fingerprints virtually on every object touched by a person. Therefore, according to Pötzelsberger [?], what is considered to be private is the relationship between the biometric data and the person from whom it comes. It can be applied on the scenario where the user tries to authenticate using one of his biometric features: the biometric feature would be stored and encrypted

before it looks for a match in the database. The homomorphic evaluation would consist in the comparison between the user's biometric feature and all of the existing rows in the database. Traditional cryptographic would not be appropriate for this task, since biometric data can vary slightly each time, so an exact match would not work out. Assuming that the biometric feature is stored as a binary string, the homomorphic evaluation would consist in computing the Hamming distance between two feature vectors, and a predefined threshold to achieve the comparison. This is how the biometric data itself can stay public, but the relationship of whom it belongs to is kept private.

## 2.6 SERVICES IN THE CLOUD

The cloud, a trendy term used to describe a network of servers usually accessible through the Internet, currently provides two important services: storage and computation. As Xu et al. in [?] point out, cloud storage services, such as Dropbox, Skydrive, Google Drive, and Amazon S3 have become very popular in recent years. As the use of these services become more common, more sensitive information is kept in the cloud, such as emails, health records, private videos and photos, company finance data, government documents, etc.

Li, Jin et. al [?] introduce cloud computing as a new term that views computing as an utility which enables convenient, on-demand access to computing resources that can be rapidly deployed with minimal management overhead and great efficiency. Cloud computing has the potential to benefit its users in avoiding large capital expenditure used for deployment and management of both software and hardware, such as setting up a data center.

All of these services are opening up a new era powered by the software as a service (SaaS) computing architecture. One of the main advantages of storing this information in the cloud is that the owners are relieved from the burden of data storage and maintenance at all times. There are many plans that adjust to the on-

demand needs of the owner of the data, which is way less costly than building and maintaining a whole data center. This implies that the clients of these services can expect to gain reliability and availability from having the data stored remotely. On the other hand, the data is necessarily brought out of their control and protection. This becomes a risk for the owner of the data, since the information stored in the cloud could potentially be accessed by unauthorized individuals, such as competitors or malicious attackers. There also exists the possibility that the information stored could be tampered in some way, thus compromising its integrity. Therefore, the computing and storage service providers are usually not trusted completely, which is why it has been difficult for the public and organizations to fully adopt the use of these services.

The aforementioned issue brings up the need of having a mechanism that verifies that a cloud provider is storing the whole database intact, even the portions that are rarely accessed. Ateniese et al. [?] point out that, fortunately, a series of *proofs-of-storage* protocols have been proposed to solve this problem related to data integrity. Xu et al. [?] define a proof of storage (POR or PDP) as a cryptographic tool, which enables a data owner or third party auditor to audit integrity of data stored remotely in a cloud storage server, without keeping a local copy of data or downloading data back during auditing. In other words, applying a proof of storage in a cloud server can guarantee that the data has not been tampered with; and in the event that it is, most likely the service could be held liable for damages to the data. A huge advantage of using a proof of storage is that the data does not have to be downloaded to confirm its integrity, which makes it ideal to make use of it routinely.

One of the simplest use cases for cloud computing is when a user independent from the cloud wants to store some data in it for later retrieval. In this simple case, data confidentiality and integrity can be trivially ensured. For that, typical cryptography primitives can be used by encrypting the user's data before it is sent for storage in the cloud. To ensure confidentiality, the key used for encryption

is kept secret from the cloud provider. However, real life applications are not as simple. Damgard et al. [?] mentions that the cloud is more than a storage medium; in particular, computation over the stored data is outsourced to the cloud. Sometimes it might even be distributed among distinct cloud servers geographically apart, operated by different parties. Wang et al [?] describe the nature of the problem at length, because usually a cloud service is not solely restricted as a data warehouse. The data stored in the cloud may be frequently updated by the users, including insertion, deletion, modification, appending, reordering, etc. This kind of complex scenario is what makes the use of traditional cryptographic primitives so difficult to adapt, especially because the data is not stored in a single cloud server, but is instead redundantly stored in multiple physical locations to further reduce the data integrity threats. Therefore, making use of cloud services highly improve data availability and storage flexibility, data confidentiality and integrity is put at risk.

In order to tackle these issues, there have been many attempts to make cloud services, especially cloud computing, more secure. These attempts are very different in nature, though very similar regarding its goals. These are *secure multi-party computation* and *homomorphic encryption*. David, Nishimaki, et al. [?] describes secure multiparty computation (MPC) as the means to allow mutually distrustful parties to compute functions on private data that they hold, without revealing data to each other. There are many methods and approaches proposed to perform secure multiparty computation, although only recent attempts are considered to be efficient enough for application. On the other hand, homomorphic encryption is based on the idea of performing computations on the ciphertext that has gone through an encryption algorithm, without having to decrypt it beforehand. This detail makes homomorphic encryption a good technique for cloud services, since only one authorized user is able to decrypt the ciphertext after the necessary computations have been performed on it.



## CHAPTER 3

# TRABAJOS RELACIONADOS

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En este capítulo se presentan cuatro trabajos relacionados con el sistema de detección de reportes duplicados desarrollado en esta tesis. Se realiza una breve descripción de cada uno de los trabajos y se explican las similitudes y diferencias entre esos trabajos y este trabajo de tesis.

## CHAPTER 4

# METHODOLOGY

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This work looks at a case study which presents an issue directly related to the confidentiality of information when it is being stored in the cloud. It discusses the situation where it is important to be able securely store a secret value, as well as make modifications on it. A solution to the problem is proposed by employing homomorphic encryption. Then, it describes in a general way a proof of concept as a stand-alone solution. Afterwards, the scenario of using a client-server architecture is considered, discussing some decisions that were made regarding the communication needs. Relevant points on key management are mentioned, and also, how the tasks previously mentioned are being split by each component.

The methodology in this work describes all the considerations and steps taken to build a solution to the problem analyzed from the case study. As such, the methodology has a very close relationship with the case study. Even though the object of study is the application of homomorphic encryption in the cloud, it is not trivial to find a fitting situation where it can be applied. The search becomes more complex as the limitations of state-of-the-art homomorphic encryption tools are considered. The case study that was thought of is simple enough so that the currently available tools can be used effectively, and complex enough so that applying homomorphic encryption is compelling and better than other alternatives.

### **Case study**

The case study is explained as follows: a household commonly has more than

one member, e.g. partner and children. The responsible members of the household (i.e. the parents) might want to keep an eye on the house while being away, especially if children are left behind. They would like to ensure that their children stay inside during this time, and that nobody else, except for a babysitter, enters the house. And if it were the case that somebody got in or out, they would probably wish to be promptly notified of it. This is especially true for the case when they are away for long periods of time, during a vacation, for example. A traditional solution consists in setting up a surveillance system throughout the house, which would certainly work to prevent robbery, but wouldn't necessarily work as a measure to know whether or not the children have left the house. Setting a surveillance system might be too costly for some families, and not be quite adaptive to their needs. It might be more efficient to deploy a system that detects when people enter and exit the building through the main door, and keep a counter of it. The main idea is that the *surveillance count* gets initialized at some point in time, and, as people go in and out, the counter increases or decreases, respectively. Setting up the sensors and other pieces of hardware can be considered as a Do-It-Yourself project, as resources could easily be found. The issue comes out when the recorded data, that is, the counter, is to be accessed remotely via some kind of web application.

[INSERT SAMPLE FIGURE]

### **Proposed solution**

The proposed solution addresses the issue hinted at the end of the study case description. This refers to making available the counter data to the householder without compromising confidentiality. Assuming that there might be more than one person who would have access to this data per household, it might not be appropriate to set up a Peer to Peer communication scheme. It makes more sense to have a server in the cloud that stores the counter data, waiting for requests from the clients to deliver the information. The data stored in the server is to be accessed from anywhere else; however, one of the most important aspects of the whole scheme remains unaddressed: confidentiality.

Usually, the householder would not allow others to know about the actual counter, including the server, as this information is considered to be sensitive. The traditional approach is to make use of public-key cryptography to encrypt the data and prevent from anybody else to know the counter. The counter value would then have to be encrypted using a certain cipher before it is uploaded to the cloud. However, this would imply that every time that there is a change in the counter, the whole ciphertext would have to be reuploaded to the server. Considering that person comes in and out of a building quite frequently, it would turn out to be a heavy burden which has an overhead cost that keeps accumulating every time there is an update.

An alternative approach is to simply notify the cloud service of any changes that occur, so that it performs the addition or subtraction itself instead of relying on receiving the whole encrypted result. Even though this would not be possible to do with traditional cryptography, it can be done using homomorphic cryptography. The concept behind this approach is that, while initial counter value is being stored on the server, the client part that resides on the household sends over any change of individuals that are inside the building, and so the cloud service, i.e. the server, performs the appropriate homomorphic computations on the currently stored value. This occurs without the need of anybody reuploading the counter data again, and the server has no need of decrypting the data in the first place. Since the full ciphertext is not being resent, the overhead cost is reduced significantly.

Whenever the householder needs to know the value of the counter, he can authenticate with the cloud service to download the current ciphertext that represents the counter. The ciphertext can be decrypted using the private key generated in the beginning. This aspect remains the same whether homomorphic encryption is used or not, and there is no way around it without compromising confidentiality. Considering that a system that detects when people come in and out might not be perfect, and if somehow the counter gets to an incorrect value, it could be reset to the correct amount. In this case, it would be unavoidable to send a newly encrypted counter

value.

The solution itself consists in an implementation of a client-server software that aims to store and modify a secure counter by making use of homomorphic encryption. The various tasks that help reach this goal are distributed by the client and server components of the software. The client focuses initialization of the homomorphic encryption scheme, generation of public and private keys, encryption of initial counter data, among other things. On the other hand, the server is dedicated to store and manage the public key of the user and his current counter value. Both parts of the software utilize a library called HELib, which makes it possible to run homomorphic evaluations. In other words, this library is used perform basic operations on it, such as addition, subtraction, and multiplication. Consequently, it is also used to generate the public and private keys, encrypt data, and decrypt it. Once the encrypted counter is stored in the server, a different kind of client can then ask to download and decrypt it. It makes sense to say that it will be a different kind of client, since most likely the interested person would not be at the place where the counter was initialized.

### **Details of the HELib library**

An implementation of an homomorphic encryption scheme, the Brakerski-Gentry-Vaikuntanathan (BGV), is openly available as a C++ library, called HELib. Using this library, it is possible to encrypt integer values and perform operations on them while being encrypted. There are many parameters to be considered when using this implementation, and these usually define how the public and private keys are going to be like. For this particular application, most parameters were left in their default values, as they seemed to serve the purpose.

Part of the description in the code repository of HELib, it is stated that the library is considered to be low-level, and given its difficulty and constant changes, not that appropriate to use to build big applications with it. [quote] 'At its present state, this library is mostly meant for researchers working on HE and its uses. Also

currently it is fairly low-level, and is best thought of as "assembly language for HE". That is, it provides low-level routines (set, add, multiply, shift, etc.), with as much access to optimizations as we can give.'

It is important to note that the library recently started to support multi-threading, which would considerably speed up the evaluation times of the homomorphic operations. Not too long ago it started to support bootstrapping as well, a technique that prevents the ciphertext from getting too much noise and results in the incorrect evaluation of the operations.

### **Setting up the parameters and context**

The HElib library is an implementation of a homomorphic encryption scheme, and as such, it has certain requirements before it can encrypt data or even generate keys. First of all, it has a list of parameters which must be manually set. Depending on the parameters applied, some aspects that directly affect the keys and ciphertext are altered. The suggested values are used for the majority of the parameters. Altering the values of the parameters could be an interesting way to do experimentation with the key size and computation time required.

The following table briefly describes each parameter used in HElib. It does not go into much depth, as those details are mostly pertaining to the algorithms in the library.

R	number of rounds	default=1
p	plaintext base	default=2
r	lifting	default=1
d	degree of the field extension	default=1
c	number of columns in the key-switching matrices	default=2
k	security parameter	default=80
L	number of levels in the modulus chain	default=heuristic
s	minimum number of slots	default=0
repeat	number of times to repeat the test	default=1
m	use specified value as modulus	optional
mvec	use product of the integers as modulus	optional
gens	use specified vector of generators	optional
ords	use specified vector of orders	optional

### Key generation and serialization

As this library requires the use of public and private keys, the user is required to create his own set of keys, and it is only the public key which he should be willing to share. The user then shares his public key with the cloud service by some means of registration, so he does not have to share it every time he makes use of the service. The public key is used to encrypt the counter data for the first time before sending it over to the server, and the server itself requires this public key in order to perform the homomorphic operations. The generation of the keys is pseudorandom, so extra care should be taken as to not to use a predictable seed, so that the public and private keys are not recreated by someone else.

After instantiating context with all the necessary parameters, the set of keys can be generated. Firstly, the public key is obtained by simply instantiating an object of the class *FHESecKey*. Then, a copy of the public key is made as the foundation for the private key. This copy makes use of a method called *GenSecKey(w)*, which takes  $w$  as a seed to create the real private key. Finally, it goes through a process that computes key-switching matrices, which are used in the internal algorithms of

the library.

```
publicKey = new FHESecKey(*context);
secretKey = publicKey;

secretKey->GenSecKey(w);
addSome1DMatrices(*secretKey);
```

Before proceeding to encrypt the data, it is important to have the public key serialized and ready to send it to the server. Serialization is a popular term used to describe the encoding of objects and the objects reachable from them, into a string of bytes. It is commonly used for lightweight persistence and for communication via sockets [reference: <http://docs.oracle.com/javase/8/docs/technotes/guides/serialization/index.htm>]. In this case, it is used as a means of temporarily saving the public key in a stream of bytes so it can be shared with a remote server that will be able to read such stream. Only after the contents of the saved object have been fully read, it is possible to reconstruct the public key. Fortunately, the HElib provides a very simple way to do serialization, since the class *FHESecKey* supports the «operator which works quite nicely with a couple popular serialization class in C++ called *istream* and *ostream*. The use of this class is showed as follows:

```
ostream pkstream;
pkstream << *publicKey;
```

This way, the data of the attributes found in the public key object has been stored as a stream of bytes in a highly portable object from the *ostream* class. Using this newly populated object, the public key can be shared using sockets.

### Encryption of the Counter Value

In order to protect the value of the counter from being known from other parties, including the server itself, it must be encrypted. This means that an encryption algorithm, provided by the HElib, is to be used on the plaintext that represents the counter. The result of this process is a ciphertext, which, unless provided with the



corresponding private key, cannot be brought back to its plaintext form.

Following the conventions defined in the HELib, an object from the *EncryptedArray* is initialized using the *context* and *G* which were previously set. *EncryptedArray* can be seen as a container for the plaintext, where the value of the counter is placed. Once it has been done, a method called *encode* of the same class can be used to prepare the data for encryption. The ciphertext is stored in a different kind of container, which comes from the class *Ctxt*. This container is initialized using the public key as argument. Afterwards, the encryption of the plaintext data can be done using a method of the *EncryptedArray* object simply called *encrypt*. This method receives as arguments the objects of the plaintext and ciphertext containers, and the public key.

```
EncryptedArray ea(*context , G);
PlaintextArray counter(ea);
counter.encode(people);
Ctxt& encryptedCounter = *(new Ctxt(*publicKey));
ea.encrypt(encryptedCounter , *publicKey , counter);
```

This ciphertext can be used in several ways: its value can be modified by adding, subtracting or multiplying it by some arbitrary value, it could also be decrypted at any time using the private key, or it could be serialized as it was done with the public key, so another party, e.g. the server, can receive and store it.

### Single component operation flow

Before attempting to break down the whole implementation in two parts: the server and client side, a proof of concept was implemented which combined the tasks of both parts into a single program. The flow of this program represents how the data gets transformed through several tasks.

1. Parameters to use with the HELib are arbitrarily chosen, while other required values are computed.

2. Context of the HELib library is set.
3. Public and private keys are created using the context.
4. Optionally, public key can be serialized into a file so it can be used later.
5. The structures to store plaintext and ciphertext are declared.
6. The initial counter value is set into the plaintext structure.
7. The plaintext is encrypted using the public key and stored into the ciphertext structure.
8. Values are arbitrarily added or subtracted from the ciphertext.
9. Ciphertext is decrypted using the private key, and its value is stored in another plaintext structure.
10. Newly decrypted plaintext is printed to verify correct result from operations.

### **Software Architecture and Data transmission**

Even though the case study started out considering cloud services, an actual implementation of the application can be addressed with a client-server architecture. The client-server architecture is a popular model that consists of a two parts: a client and a server. Usually, the server just waits for any kind of request from a client to do some kind of task. Meanwhile, the client usually starts some kind of task, but depends on the server to do it completely. Most of the time, the client depends on the server because it might have something that the client does not, like a database.

In this case, the implementation of the counter application is approached using a client-server model. The server represents the cloud service that stores and modifies the encrypted data on request, while the client plays the part of obtaining and encrypting the sensitive data. The client also takes care of tasks such as initializing the homomorphic encryption context, generating the public and private keys, encrypting the data, and serializing the encrypted counter before sending it

over to the server. Meanwhile, the server takes care of establishing the communication details via TCP sockets, reconstructing the received ciphertext, and performing operations on it. Although a more secure implementation would consider using the Secure Socket Layer (SSL) over TCP, it was simplified to cover only the details pertaining the use of homomorphic encryption.

Once the user has obtained the initial data, i.e. by counting the number of people currently in the house at a given time, he proceeds to feed the data to the client program, so it is encrypted using the previously generated public key. After the counter data has been encrypted successfully, the data is then sent to the server via sockets. There were two possible protocols that could have been used for this part: UDP and TCP. UDP stands for User Datagram Protocol and is used in conjunction with the Internet Protocol. UDP works best when the data units that are being sent are very small. It is also a little bit problematic considering that it does not reassembles data packets once they have arrived at the destination. The other option that was considered was TCP, which stands for Transmission Control Protocol. It is a little friendlier to use in the sense that it reassembles datagrams in the correct order once they arrive at their destination. However, it adds a little bit of overhead cost since it adds a header section per segment. In the end, it was chosen to use TCP over UDP, especially since it might become complicated to keep track of the order of the data packets.

### **Sending and reading serialized data**

The serialized data that is sent through sockets was very large to be done with a single function call, which is why a few considerations have to be made on the implementation. It is recommended to have some kind of supporting function that continuously is attempting to send data to the server, until there are no bytes left.

The following snippet of code shows how the ciphertext is serialized using the *ostream* class, and then a function call is made to a supporting function which continuously attempts to send all the remaining bytes to the server. It is relatively

simple to keep track of how many bytes are being sent and how many are left, which is why it stops sending data at the appropriate time.

```

    ostreamstream oss;
    oss << encryptedCounter;

    if(sendalldata(sockfd, oss.str().c_str(), &msgsize) == -1) {
        printf("ERROR. Only %d bytes sent!\textbackslash n", msgsize);
    }

int sendalldata(int s, char const* buffer, int *len)
{
    int total = 0; // bytes sent
    int bytesleft = *len; // bytes left to send
    int n;

    while(total < *len) {
        n = send(s, buffer+total, bytesleft, 0);
        if (n == -1) { break; }
        total += n;
        bytesleft -= n;
    }
    *len = total;
    if(n == -1)
        return -1;
    else
        return 0;
}

```

As it cannot be expected that the serialized ciphertext of great size will be succesfully sent with a single send function call, the same goes for the receiving end.

Certain measures have to be taken so that all the data is received complete and in the appropriate order.

The following piece of code extracted from the server side of the software goes through the part where the ciphertext is being received after it has been serialized. First of all, a buffer is declared and initialized with the size of the ciphertext, which was previously received. A function called *bzero()* is used to prepare and empty the buffer. Then, a while loop is done as long as there are bytes remaining to be received. As more iterations go by, more data is appended to the buffer. A couple of variables are used to keep in check how many bytes have been read and how many are still pending to be read.

```
char* responseBuffer = new char[responseBufferSize];
bzero(responseBuffer, responseBufferSize);
bytes_read = 0;
int bytes_remaining = responseBufferSize;
int this_recv;
while(bytes_remaining > 0) {
    this_recv = recv(newsockfd, responseBuffer+bytes_read,
                    bytes_remaining, 0);
    if(this_recv <=0) error("error on receive");
    else {
        bytes_remaining -= this_recv;
        bytes_read += this_recv;
    }
}
```

### Reconstructing the ciphertext

It was mentioned before that serialization allowed persistence and made it easier for object data to be sent via sockets. However, the serialized objects cannot be used the way they are received. In this case, the ciphertext was serialized before

being sent to the server, and once it is properly received, the data must be used to reconstruct the original ciphertext object. If this is not done, there is no way to perform operations on the ciphertext, or even decrypt it to know its value.

The following piece of code describes the steps to be taken in order to reconstruct the object that can contain the ciphertext. First of all, the buffer that was previously filled with data is then copied into a string variable, then it goes through a couple of steps so it can be used as a *istream* object. The preparation is certainly more complicated, but the reconstruction of the object is quite simple itself. Since `HElib` is friendly with the `>>` operators, it can be done in one step. This operator basically takes the data from one variable and puts it in another kind of variable, one being a *istream* object, and the other a *Ctxt* object.

```
string strBuffer((const char*) responseBuffer , bytes_read);
istream serialCipher;
serialCipher.str(strBuffer);

Ctxt receivedCipher(publicKey);
serialCipher >> receivedCipher;
```

### Changes in the counter

Once the initial value of the counter has been received by the server, there are two possible scenarios that could occur next: the value gets viewed by the householder, or a change in the counter occurs. Whenever a significant change is observed by the client program, the change is promptly notified to the server, which could be either positive or negative. When the cloud service receives the value of change, it either adds or subtracts it from the previously stored value. The change is immediately visible to the householder who might wish to know the current value of the counter.

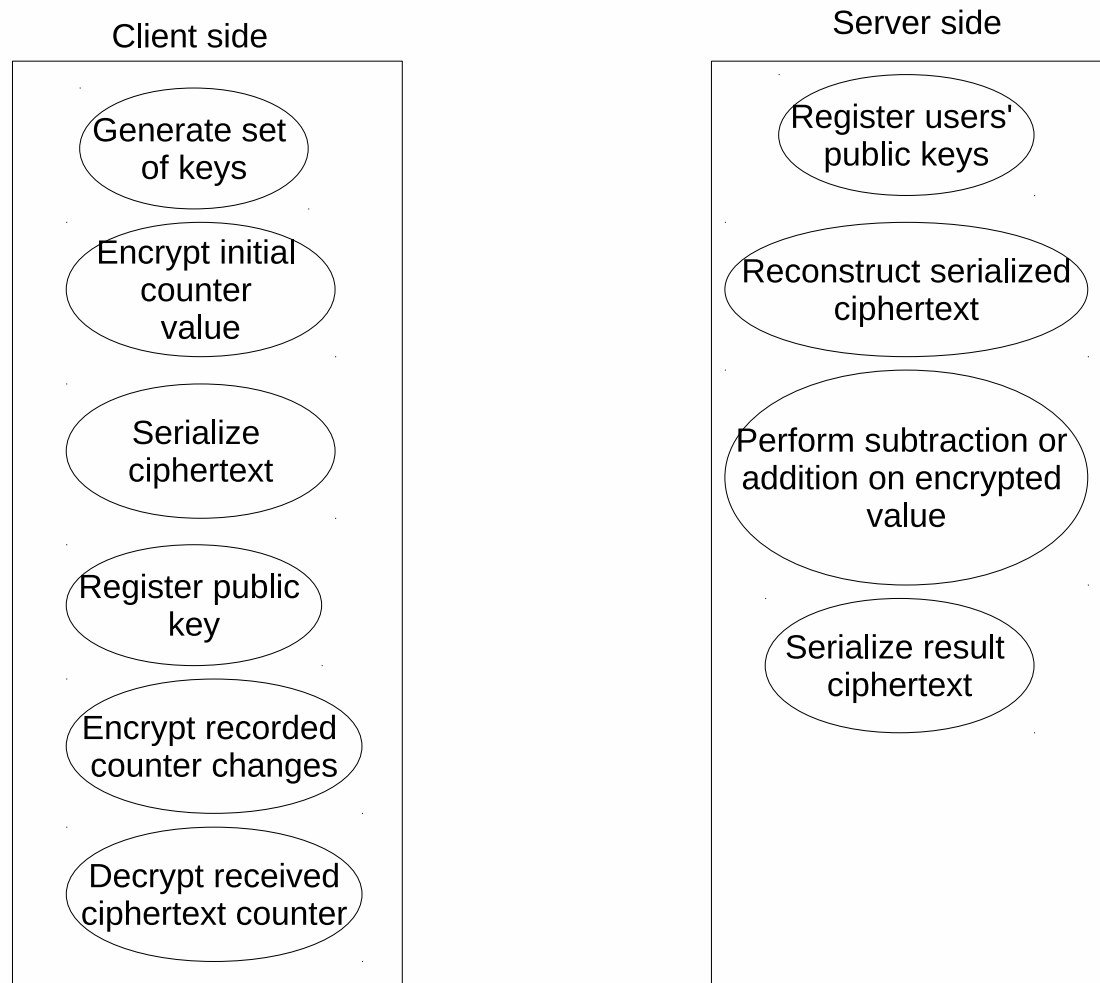
As the implementation of the counter was started before December 2014, bootstrapping was not considered to use. Bootstrapping is an advanced technique in fully

homomorphic encryption which allows more homomorphic evaluations to be run on a ciphertext. As it has been mentioned, whenever a homomorphic evaluation is performed on the ciphertext, a small amount of noise is added to it, and after a certain threshold, the ciphertext becomes unusable, in the sense that it no longer decrypts back to the correct value. As an alternative to bootstrapping, however, it is possible to reset the counter, i.e. upload a newly created ciphertext with an initial counter value, so that more homomorphic evaluations can be performed on it. Considering that the operations that will be done most of the time are addition and subtraction, this is not something that will occur often, but eventually will.

### **Architecture breakdown**

The proof of concept that makes use of homomorphic encryption ends up being completely linear in the sense that it has no communication at all with different parts of the outside world. The considered case study implies that there is both a household where the counter is initialized and another location where the current counter value can be accessed. Considering this aspect, it makes sense to have a server in the cloud that can be accessed securely which will contain the current value of the counter for each household. This requires the server to store the public key as well, since to be able to modify the ciphertext of the counter data, such a key is needed.

In the following figure, a breakdown of the client-model architecture employed is depicted. As previously mentioned, the solution is split into two parts: client and server sides. Each side has specific tasks that must be performed independently. The figure was conceived as an attempt to organize the software structure and assign a functionality to one of the components. The client side is in charge of the tasks that are performed at the place where the counter is initialized, i.e. the household. Such tasks include: setting up the parameters and context required by the HElib, generating the set of public and private keys, registering the public key, encrypting the initial counter value, serializing the ciphertext, recording changes in the counter value, and decrypting the received ciphertext.



**Figure 4.1** – Architecture breakdown into client and server.

[ADD SEQUENCE DIAGRAM AND DESCRIPTION]

### Discussion

The part of user registration is not part of the planned implementation itself, but it is recommended to consider how to handle different users. There might many more than a couple of households that make use of such a counting service. This is why each householder would have to register his public key and go through some kind of authentication mechanism every time they restart or download the counter data. It is especially important to take the appropriate measures so that somebody else



does not reset the counter value of a household that is not theirs, because it would be chaotic when the householder looks at the counter and finds a value that does not represent reality. To keep the registration under control, it might be ideal to keep a list of registered users either on a database or just a file for each household which would contain basic pieces of information such as an email address or username, along with the hash value of a password and the appropriate public key for the household that is being registered.

There were several problems at this stage that were directly related to the implementation. The transmission of serialized data between the client and server was especially problematic. The early attempts at sending serialized data to the server were not quite successful. Many different kind of errors were found, and the most common one was related to memory allocation. After going through several trial and error attempts, it was found the errors started appearing as the message got larger. This seemed to be inevitable as the default settings of a TCP blocking socket were not adequate to send large pieces of data.

After doing the right adjustments, the problem seemed to go away, except that *the result was not quite as expected*. It was known immediately that something was up with the received ciphertext, as it could not decrypt properly. It helped to look at the data that the server received, and it was quite evident that it was not complete. Indeed, only certain fragments of the ciphertext were received successfully, which led to an incorrect decryption. Several adjustments were made to the process of receiving data, so that a couple of control variables were used to keep in check how many bytes were being received. It was set so that it would only stop reading data from the client until all of the bytes had been read and assembled in a buffer. Both the functionalities of reading and writing the ciphertext on a socket were relatively more complicated considering that was done without the support of a framework that specialized on it.

[TALK ABOUT DECRYPTION]

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In summary, a description of the case study was given, a solution based on homomorphic encryption was proposed to tackle the problem found in the case study, details of the HElib were discussed, and all of the relevant considerations pertaining to the implementation were explained. These considerations took on points such as the parameters and context, key generation and serialization, software architecture, transmission of serialized data, reconstruction of the ciphertext, and dealing with changes of the counter.