

CMSC 330, Fall 2019 — Quiz 4

NAME \_\_\_\_\_

TEACHING ASSISTANT

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INSTRUCTIONS

- Do not start this quiz until you are told to do so.
- You have 15 minutes for this quiz.
- This is a closed book quiz. No notes or other aids are allowed.
- For partial credit, show all your work and clearly indicate your answers.

1. [4 pts] Circle all free variables in the following expressions. If there are no free variables, clearly indicate so by writing "no free variables" next to the corresponding problem.

(a)  $\lambda a. a \ b \ (\lambda b. a \ a \ b) \ a$

(b)  $\lambda y. (\lambda x. x \ y) \ (\lambda z. z \ x)y$

(a)  $\lambda a. a \ \underline{b} \ (\lambda b. a \ a \ b) \ a$

(b)  $\lambda y. (\lambda x. x \ y) \ (\lambda z. z \ \underline{x})y$

2. [8 pts] Reduce the following expressions to normal form. Remember to show **all** steps.

(a)

$(\lambda a. a \ a \ b)(\lambda b. b)$

$(\lambda a. a \ a \ b)(\lambda b. b) \Rightarrow_{\beta} (\lambda b. b) \ (\lambda b. b) \ b \Rightarrow_{\beta} (\lambda b. b) \ b \Rightarrow_{\beta} b$

(b)

$$(\lambda x. \lambda y. y \ x) (\lambda y. y) \ z$$

$$(\lambda x. \lambda y. y \ x) (\lambda y. y) \ z \Rightarrow_{\alpha} (\lambda x. \lambda y. y \ x) (\lambda w. w) \ z \Rightarrow_{\beta} (\lambda y. y \ (\lambda w. w)) \ z \Rightarrow_{\beta} z \ (\lambda w. w)$$

3. [8 pts] Recall the Church encodings for booleans.

$$\text{true} = \lambda a. \lambda b. a$$

$$\text{false} = \lambda a. \lambda b. b$$

$$\text{if } x \text{ then } y \text{ else } z = x \ y \ z$$

$$\text{not} = \lambda x. x \ \text{false} \ \text{true}$$

Prove that  $\text{if true then (not true) else true} = \text{false}$ .

$$\begin{aligned} \text{if true then (not true) else true} &= \text{true (not true) true} \\ &= (\lambda a. \lambda b. a) \ (\text{not true}) \ \text{true} \\ &\rightarrow_{\beta} \text{not true} \\ &= (\lambda x. x \ \text{false} \ \text{true}) \ \text{true} \\ &\rightarrow_{\beta} \text{true false true} \\ &= (\lambda a. \lambda b. a) \ \text{false} \ \text{true} \\ &\rightarrow_{\beta} \text{false} \end{aligned}$$