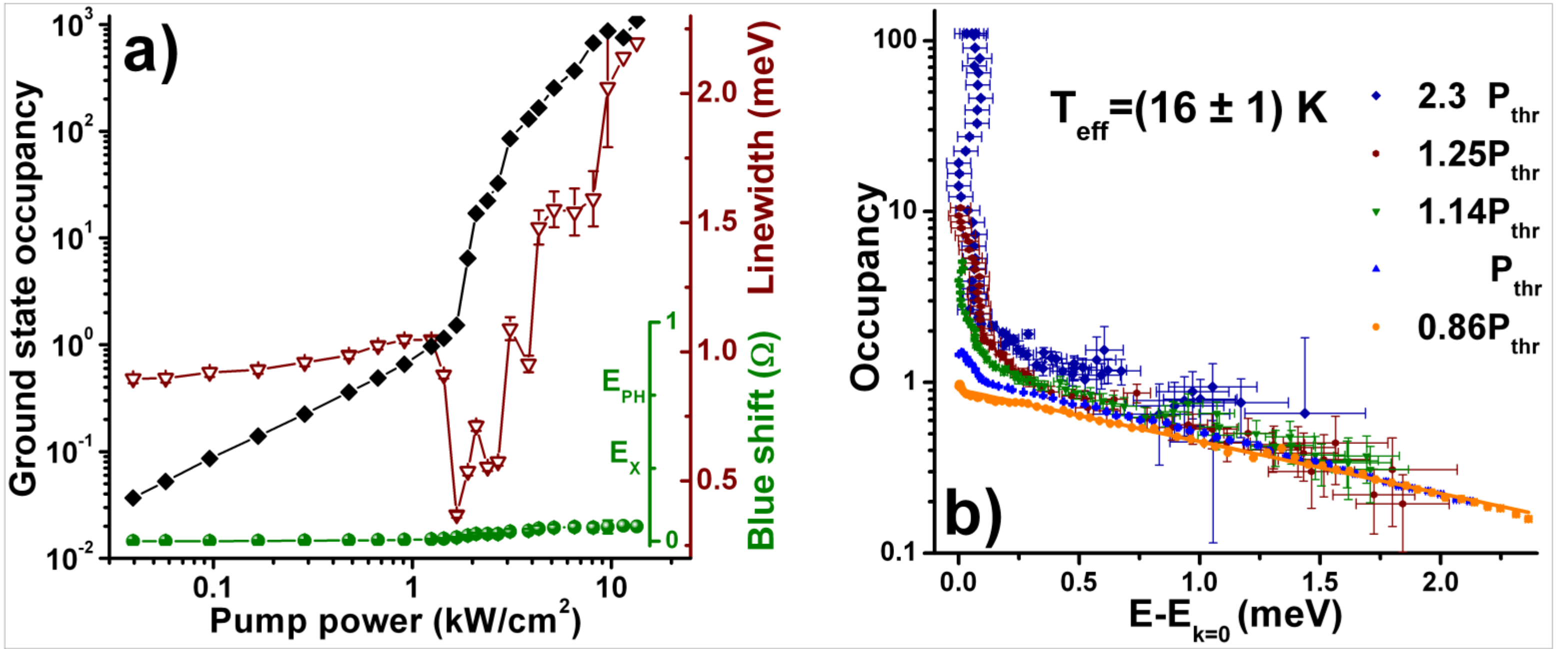
Polariton BEC signatures



Blue shift used to estimate density
High energy tail of distribution used to estimate temperature by fitting Maxwell-Boltzman distribution

$$f_E(E) = 2\sqrt{\frac{E}{\pi}}\left(\frac{1}{kT}\right)^{3/2}\exp\left(\frac{-E}{kT}\right)$$

Short polariton lifetimes and no equilibration with phonons

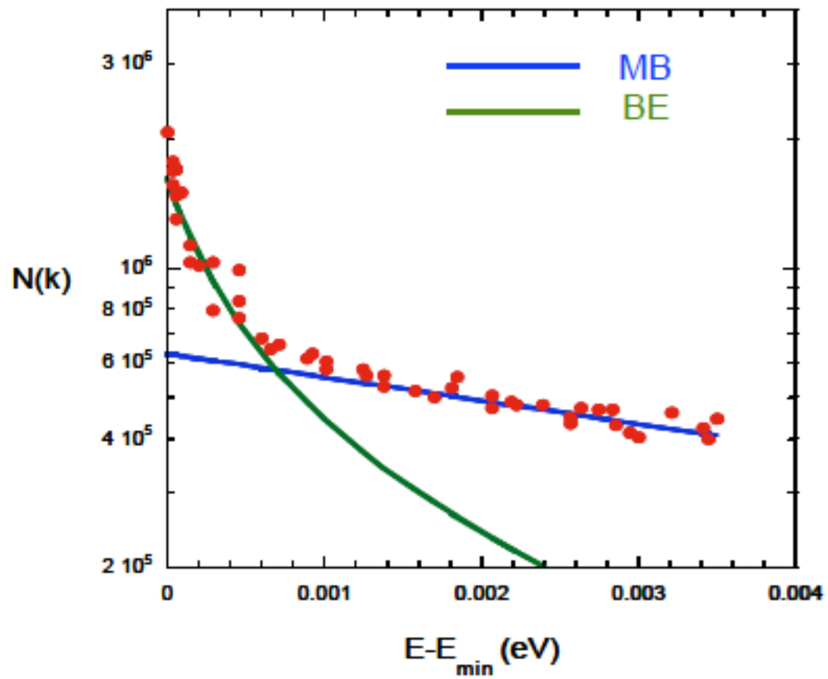
Kasprzak et al, Nature, 443, 409 (2006)

Equilibration issue

Ideal Bose-Einstein distribution:

$$N_k = \frac{1}{e^{(E_k - \mu)/k_B T} - 1}$$

The difference between MB and BE is the difference between classical and quantum



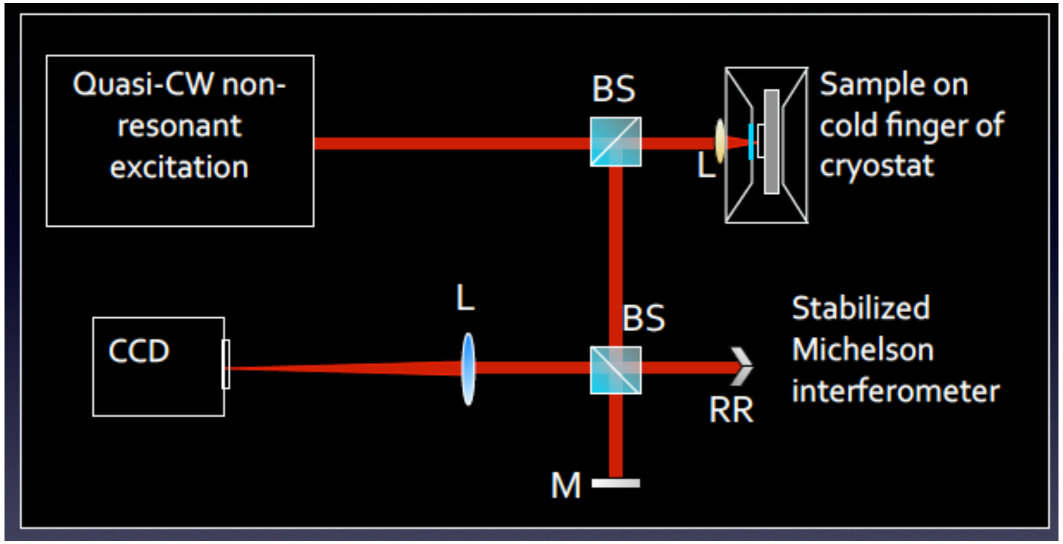
D. Snoke's group

Spatial coherence measurement

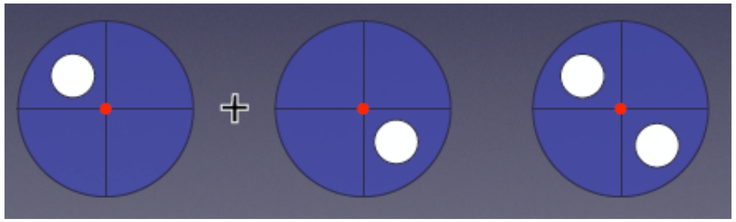
$$g^{(1)}(\mathbf{r},\mathbf{r}') = \frac{\langle E^{*}(\mathbf{r})E(\mathbf{r}')\rangle}{\langle E^{*}(\mathbf{r})\rangle\langle E(\mathbf{r}')\rangle}$$

First-order correlation function without time delay

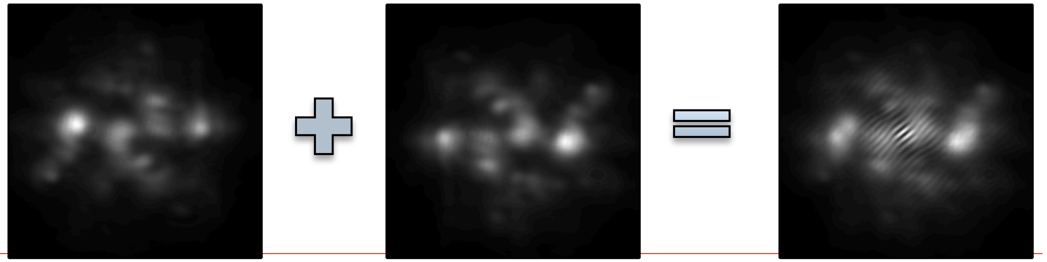
Expect correlation length = thermal de Broglie length below threshold and ~ condensate size above threshold



Principle of measurement:
Mirror image +
Retroreflected image=
Interferogram

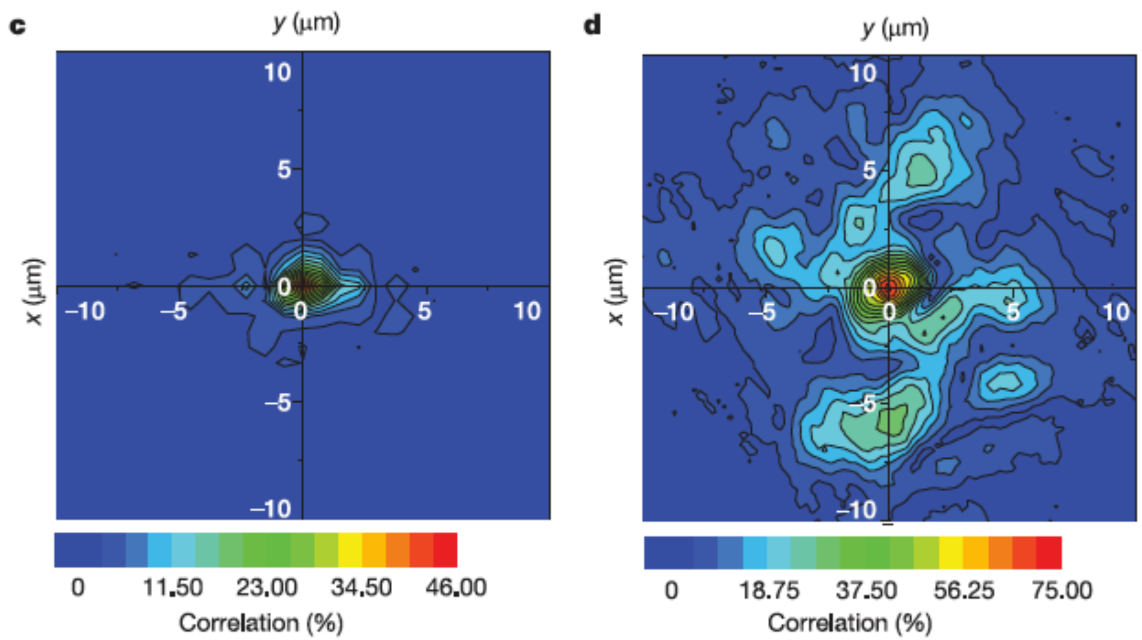


Visibility of interference is directly proportional to correlation



Kasprzak et al, Nature, **443**, 409 (2006); B. Deveaud “Superfluidity Colloquium”

Long-range spatial coherence



Below threshold:
spatial coherence length equal to thermal de Broglie length ~2.5 µm

Above threshold:
coherence length comparable to condensate size ~20 µm despite various impurities

Kasprzak et al, Nature, 443, 409 (2006)

Dimensionality issue

A uniform 2D system of bosons does not have a BEC phase transition at finite T in the thermodynamic limit since long-wavelength thermal fluctuations destroy long-range order. [Mermin and Wagner, Phys. Rev. Lett. 17, 1133 (1966); Hohenberg, Phys. Rev. 158, 383 (1967)]

If the Bose gas is confined by a spatially varying potential $U \sim r^{s}$, the constant density of states DOS of the uniform 2D system is dramatically modified by the potential, and a BEC phase at finite T is recovered. [Bagnato and Kleppner, Phys. Rev. A 44, 7439 (1991)]

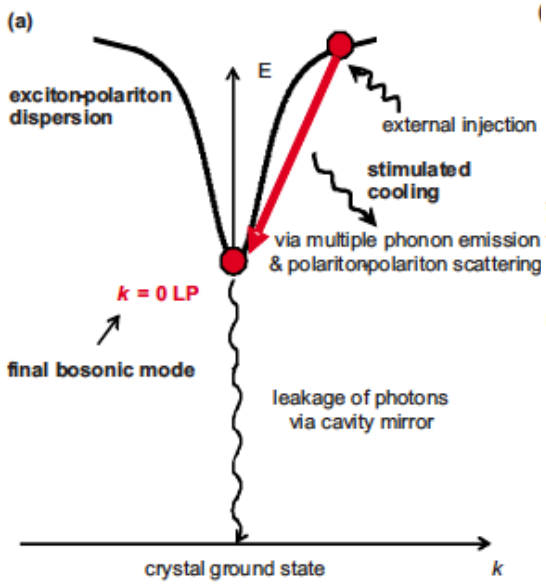
Any experimental 2D system has a finite size and a finite number of single-particle states. With discrete energy levels ε_i ($i=1,2,\ldots$) in a box with the size size $S=L^2$, the critical density for condensation can be defined as:

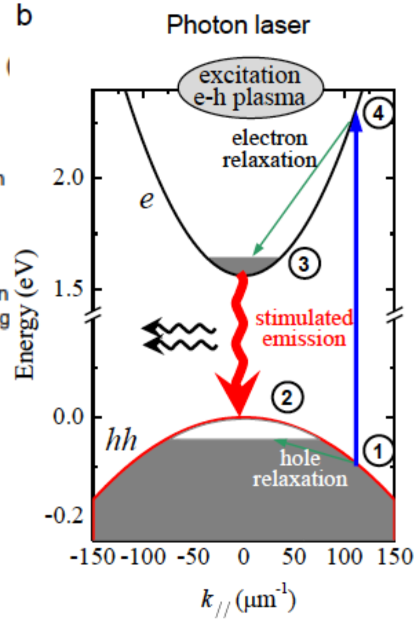
$$n_c = \frac{2}{\lambda_T^2} \ln\left(\frac{L}{\lambda_T}\right)$$

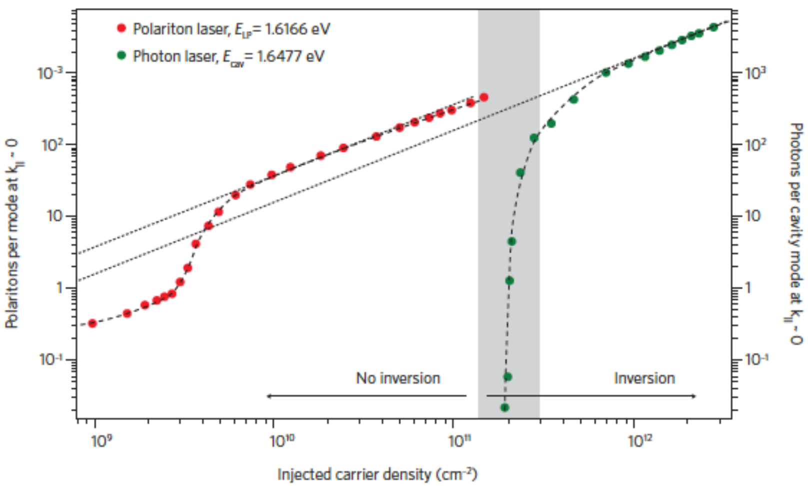
If the particle number N is sufficiently large and/or T is sufficiently low, the phase transition shows features similar to a BEC phase transition at the thermodynamic limit. [Ketterle and van Druten, Phys. Rev. A 54, 656 (1996)]

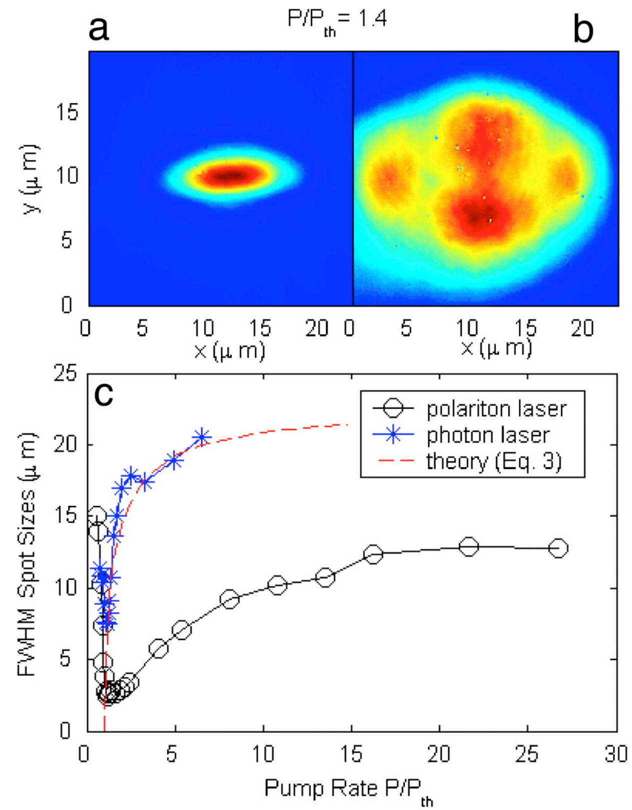
Griffin, A., D. W. Snoke, and S. Stringari, ‘Bose-Einstein Condensation’, Cambridge University Press, Cambridge (1995)

BEC vs photon laser









Deng, Weihs, Snoke, Bloch, & Yamamoto, Proc. Natl Acad. Sci. USA 100, 15318 (2003).

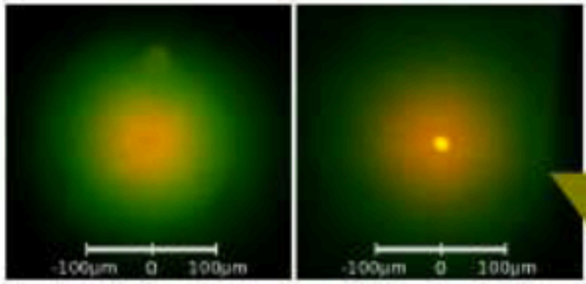
Imamoglu, A., R. J. Ram, S. Pau, and Y. Yamamoto, Phys. Rev. A 53, 4250 (1996)

Why do we need another BEC?

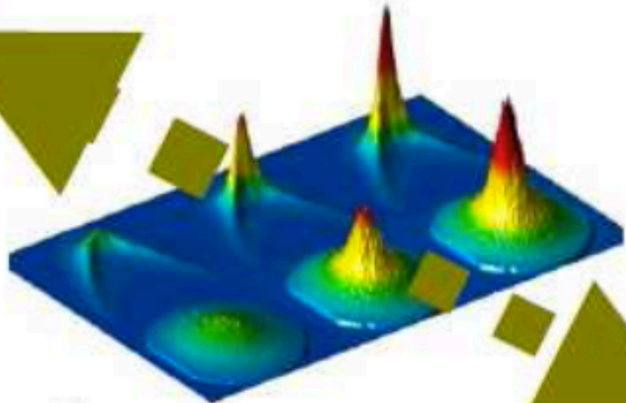
Photon BEC
Klaers J. et al.
Nature **468**, 545-548 (2010)

Polariton BEC
Kasprzak, J. et.al.
Nature, **443**, 409 (2006)

Atom (or exciton) BEC
Anderson, M.H. et.al.
Science, **269**, 198-201 (1995)

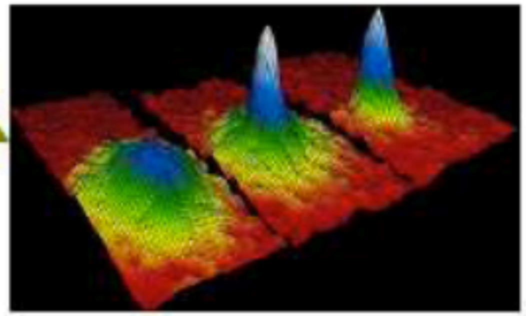


- Very short lifetime
- Extremely small mass
- > RT transition temperature
- Photon number conserved



- Short lifetime ~1-100 ps
- Variable effective mass
- Cryogenic to RT
- Dissipative, non-equilibrium

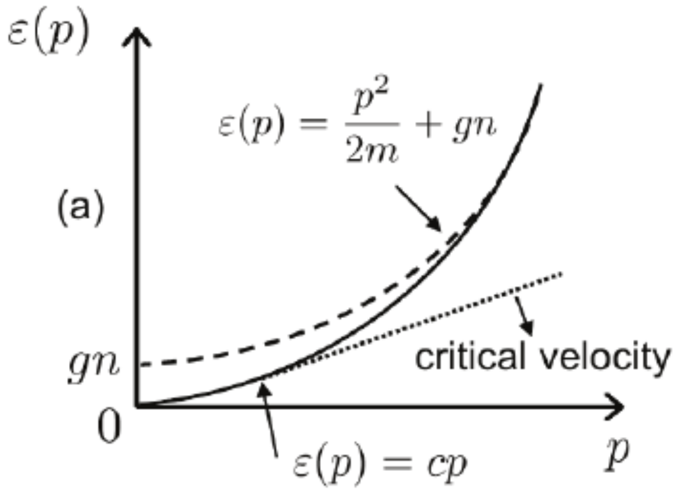
- Effective infinite lifetime
- Large mass
- Low transition temperature
- 'ideal' Bose gas



100% photon-like — or distinctly unique? — 100% particle-like

Is polariton BEC a superfluid?

Superfluidity is defined as frictionless and persistent flow – absence of excitations below a critical velocity



Excitation spectrum for weakly interacting BEC of ultracold atoms

Landau criterion - superfluid critical velocity

$$v_c = \min_p \frac{\epsilon(p)}{p}$$

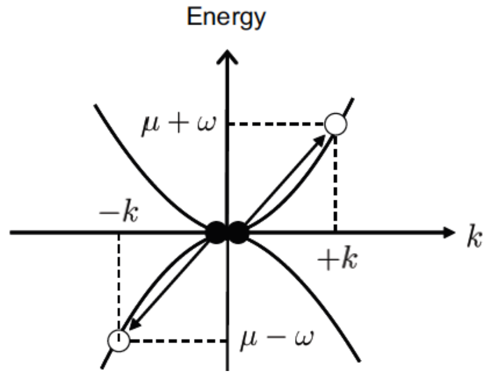
Does this hold for polaritons?

Leggett, Superfluidity, Rev. Mod. Phys. 71, S318 (1999) ;
Stringari & Pitaevskii, 'Bose-Einstein Condensate', Clarendon Press, 2003

Excitation spectrum

The mean field Gross-Pitaevskii equation for the condensate wavefunction:

$$i\hbar\frac{d}{dt}\psi(r,t) = \left[-\frac{\hbar}{2m}\nabla^2 + g|\psi(r,t)|^2\right]\psi(r,t)$$



$$\psi(r,t) = \psi_0(r)e^{-i\mu t} + \psi_k e^{i[k\cdot r-(\mu+\omega)t]} + \psi_{-k}e^{-i[k\cdot r-(\mu-\omega)t]}$$

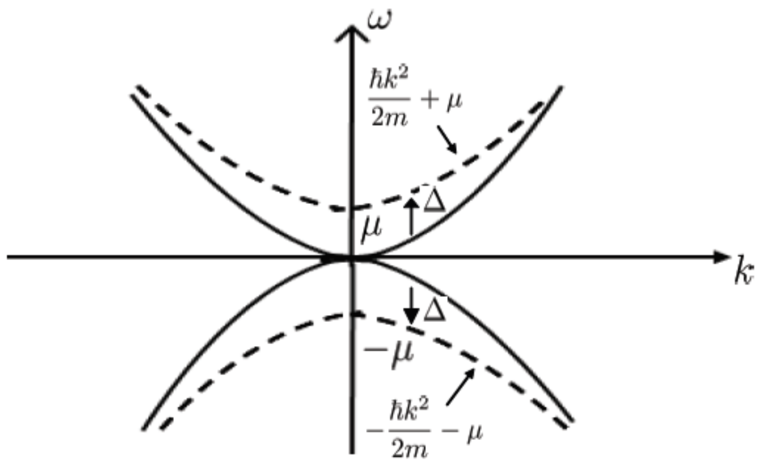
$$e^{-i\mu t} \longrightarrow \mu = g\psi_0^2$$

$$\begin{matrix} e^{i[k\cdot r-(\mu+\omega)t]} \\ e^{-i[k\cdot r-(\mu-\omega)t]} \end{matrix} \longrightarrow [A]\begin{pmatrix}\psi_k \\ \psi_{-k}\end{pmatrix} = \begin{pmatrix}\frac{\hbar k^2}{2m}+\mu & \mu \\ -\mu & \frac{\hbar k^2}{2m}-\mu\end{pmatrix}\begin{pmatrix}\psi_k \\ \psi_{-k}\end{pmatrix} = \omega\begin{pmatrix}\psi_k \\ \psi_{-k}\end{pmatrix}$$

$$\boxed{\omega = \pm\sqrt{\left(\frac{\hbar k^2}{2m}\right)\left[\left(\frac{\hbar k^2}{2m}\right)+2\mu\right]}}$$

$$\Delta \simeq \mu \quad \begin{aligned} \psi^{(+)}_{r,t} &= i\sqrt{2}\sin(k\cdot r)e^{-i(\mu+\omega_+)t} \\ \psi^{(-)} &= \sqrt{2}\cos(k\cdot r)e^{-i(\mu+\omega_-)t} \end{aligned}$$

$$\Delta \simeq 0 \quad \begin{aligned} \psi^{(+)}_{r,t} &= e^{i[k\cdot r-(\mu+\omega_+)t]} \\ \psi^{(-)}r,t &= e^{i[k\cdot r-(\mu+\omega_-)t]} \end{aligned}$$

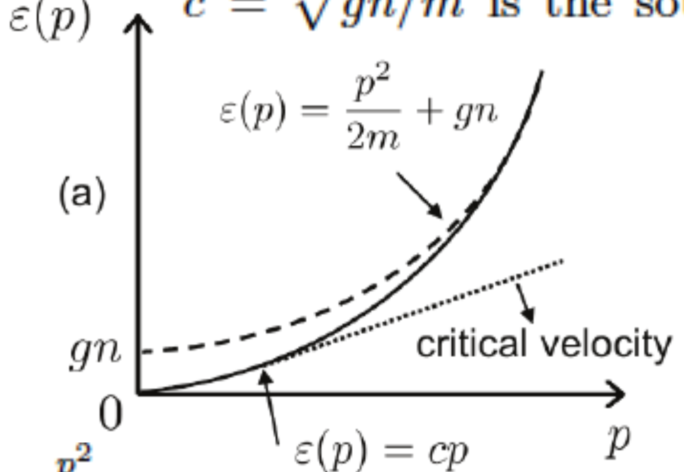


$$\Delta = \frac{\hbar k^2}{2m} + \mu - \omega_+$$

Observation of Bogoliubov spectrum

$$\varepsilon(p) = \left[\frac{gn}{m}p^2 + \left(\frac{p^2}{2m}\right)^2\right]^{1/2}$$

$c = \sqrt{gn/m}$ is the sound velocity

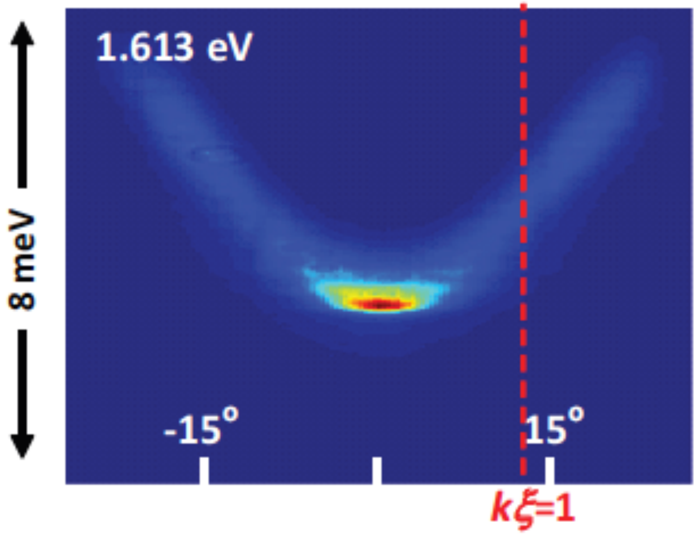


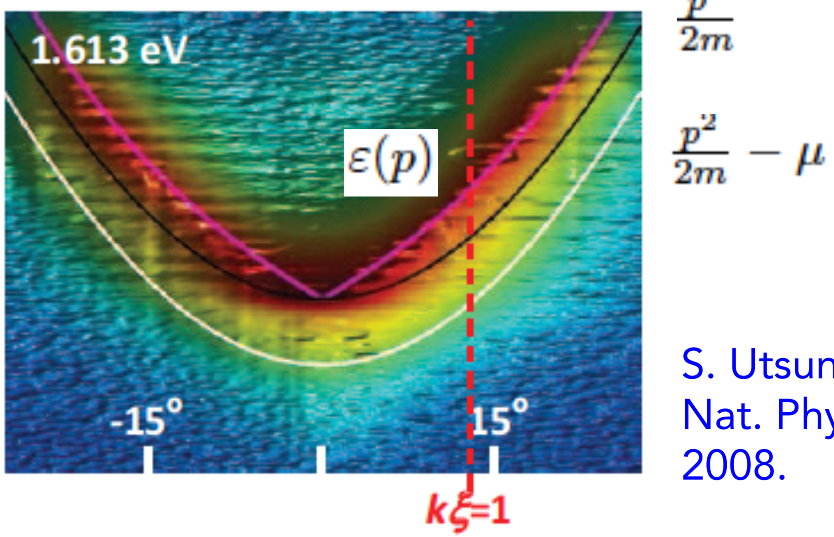
$p \ll mc = \sqrt{mgn}$ $\qquad \varepsilon(p) = cp$

$p \gg mc$ $\qquad \varepsilon(p) = \frac{p^2}{2m} + gn$

From the phonon to free-particle regime: $\frac{p^2}{2m} = gn$

$p = \hbar/\xi \longrightarrow \xi = \sqrt{\frac{\hbar^2}{2mgn}} = \frac{1}{\sqrt{2}}\frac{\hbar}{mc}$ the "healing length"





S. Utsunomiya et al, Nat. Phys., 4, 700, 2008.

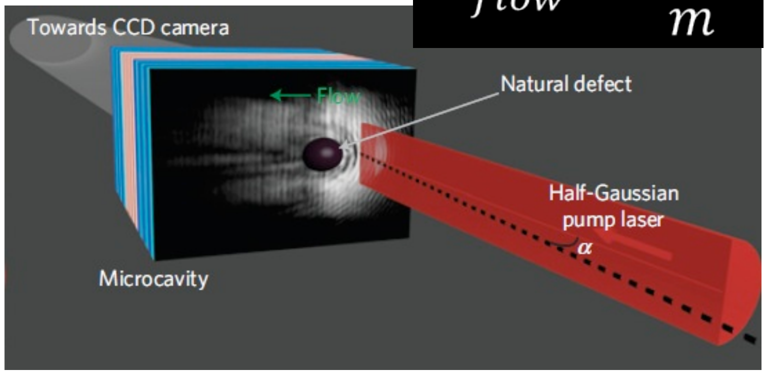
Is polariton BEC a superfluid?

BEC of atoms is shown to be superfluid. Similar experiments were designed and repeated in polariton condensates

Known signatures: no drag force and 'laminar' flow below the critical velocity, overcritical flow excites Cherenkov waves, solitons and vortices

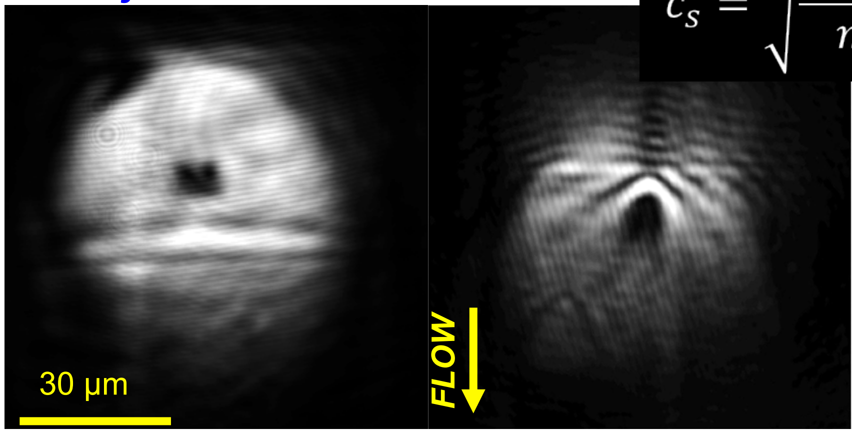
Superfluid critical velocity $v_c = \min_p \frac{\epsilon(p)}{p}$

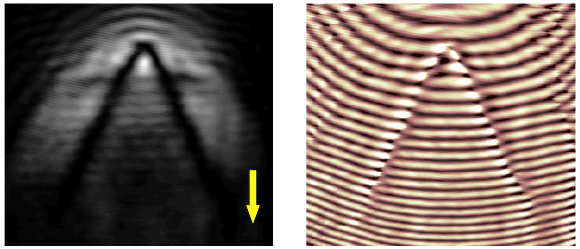
$$v_{flow} = \frac{\hbar k_{\parallel}}{m}$$



Superfluid and supersonic flows for a resonant excitation with pump (density) controlled sound velocity

$$c_s = \sqrt{\frac{g|\psi|^2}{m}}$$





Typical overcritical behavior

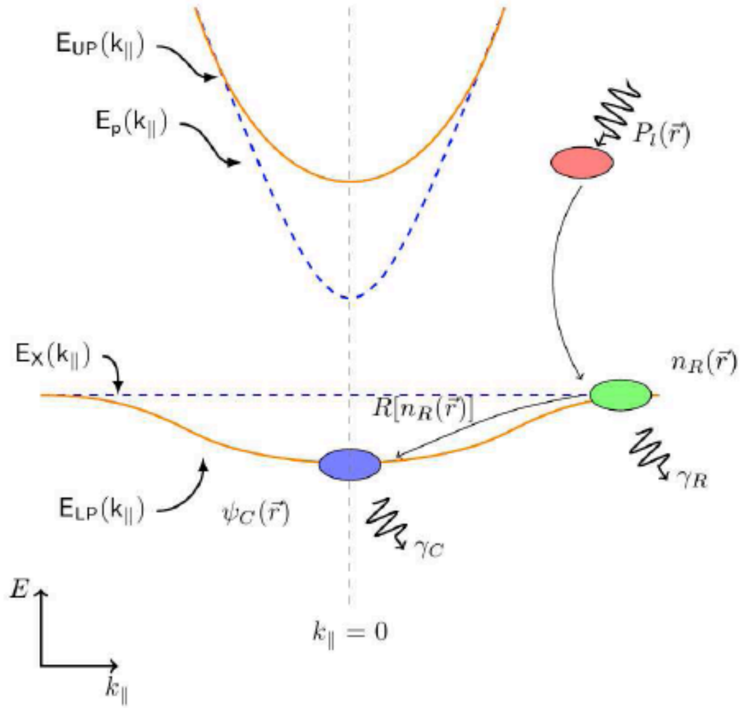
Amo et al., Nature Physics 5, 805 (2009); Science 332, 1167 (2011)

Non-resonant excitation

Open-dissipative
Gross-Pitaevskii equation (GPE)

Condensate distribution is uniquely determined by pump profile + potential + repulsive interactions

Wouters & Carusotto, PRL, 99, 140402 (2007)
Kneer et al. PRA 58, 4841 (1998)



Condensate order parameter time evolution

External potential — Stimulated scattering gain — Reservoir interaction

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) - \frac{i\hbar}{2}[\,\gamma_C - R(n_R(\mathbf{r},t))\,] + g_C|\psi(\mathbf{r},t)|^2 + g_R n_R(\mathbf{r},t)\right)\psi(\mathbf{r},t)$$

Condensate loss — Condensate interaction

Reservoir rate equation

Laser profile

$$\frac{\partial n_R(\mathbf{r},t)}{\partial t} = P_l(\mathbf{r},t) - \gamma_R n_R(\mathbf{r},t) - R(n_R(\mathbf{r},t))|\psi(\mathbf{r},t)|^2$$

Reservoir loss — Stimulated scattering loss

Homogeneous steady state

Disregard all gradients

$$i\frac{\partial\psi}{\partial t} = \left(g_c|\psi|^2 + g_R n_R + \frac{i}{2}(Rn_R - \gamma_c)\right)\psi$$

Multiply by complex conjugate to obtain the equation for the condensate density:

$$\frac{\partial n_c}{\partial t} = -\left(\gamma_c + Rn_R\right)n_c; \qquad \frac{\partial n_R}{\partial t} = -\left(\gamma_R + Rn_c\right)n_R + P$$

Supercritical bifurcation of a steady state:

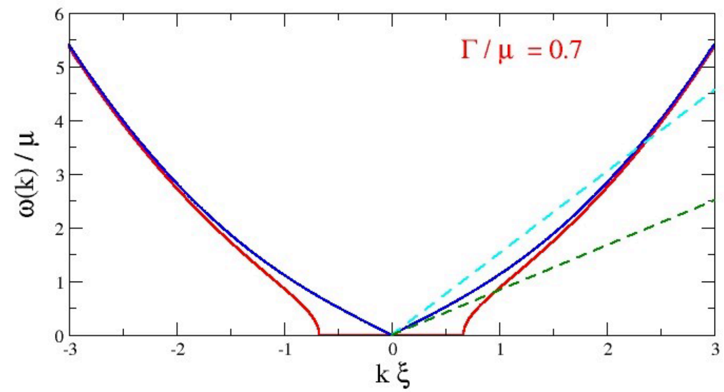
$$\boxed{P_{th} = \frac{\gamma_c\gamma_R}{R}}$$

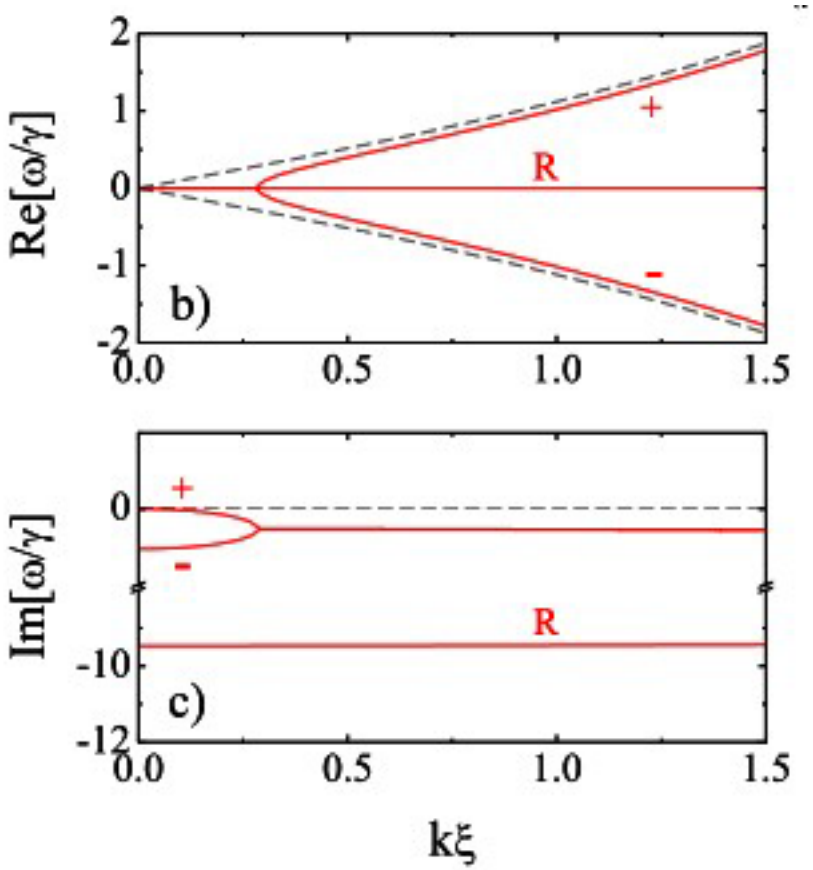
$$P < P_{th} \quad \rightarrow \quad n_c = 0$$

$$P > P_{th} \quad \rightarrow \quad n_c = n_c^0 = \frac{\gamma_R}{R}\left(1 + \frac{P}{P_{th}}\right), \quad n_R = n_R^0 \frac{\gamma_c}{R}$$

Condensate excitation spectrum

+ mode has diffusive nature at low k
$Re[\omega(k)] = 0$, $Im[\omega(k)] = -ak^2$
but recovers Bogoliubov sound at high k



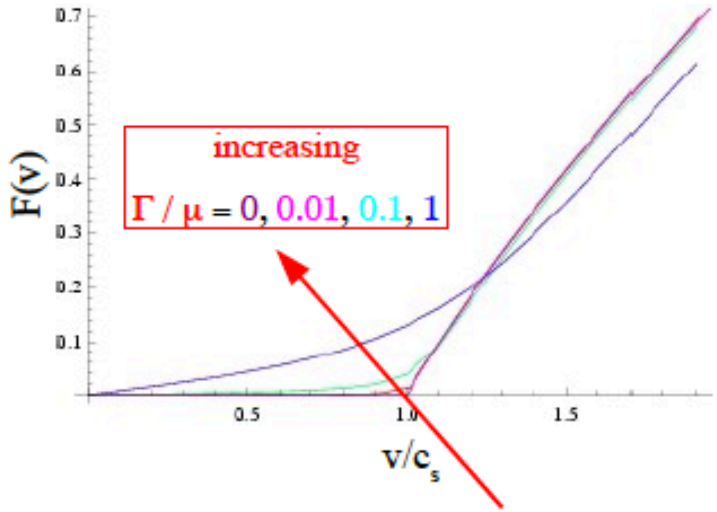


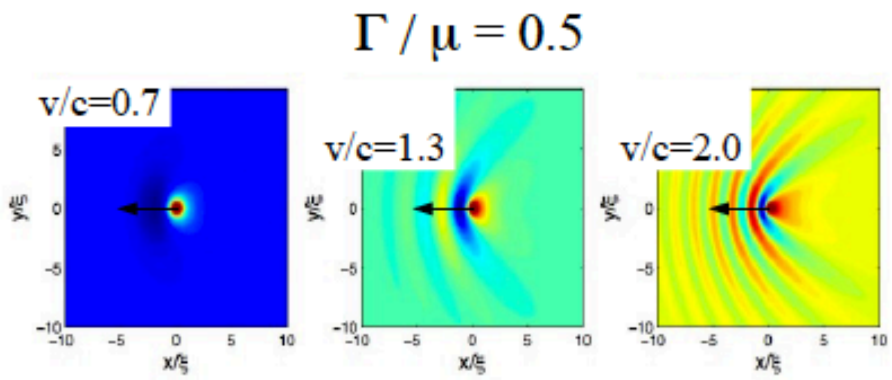
Naïf Landau argument:

- Landau critical velocity $v_c = \min_k \frac{\omega(k)}{k} = 0$ for non-equilibrium BEC
- Any moving defect expected to emit phonons

M. H. Szymanska, J. Keeling, P. B. Littlewood, Phys. Rev. Lett. 96, 230602 (2006)
M. Wouters and I. Carusotto, Phys. Rev. Lett. 99, 140402 (2007)

Dissipative superfluidity





- drag force F(v): $\Gamma / \mu = 0$ recovers Landau criterion, $F(v<v_c)=0$

  $\Gamma / \mu > 0$ smoothened, still crossover behaviour

- real-space "Cerenkov" wake: localized perturbation for small v

  propagating phonons for large v

Landau criterion redefined

Low v :

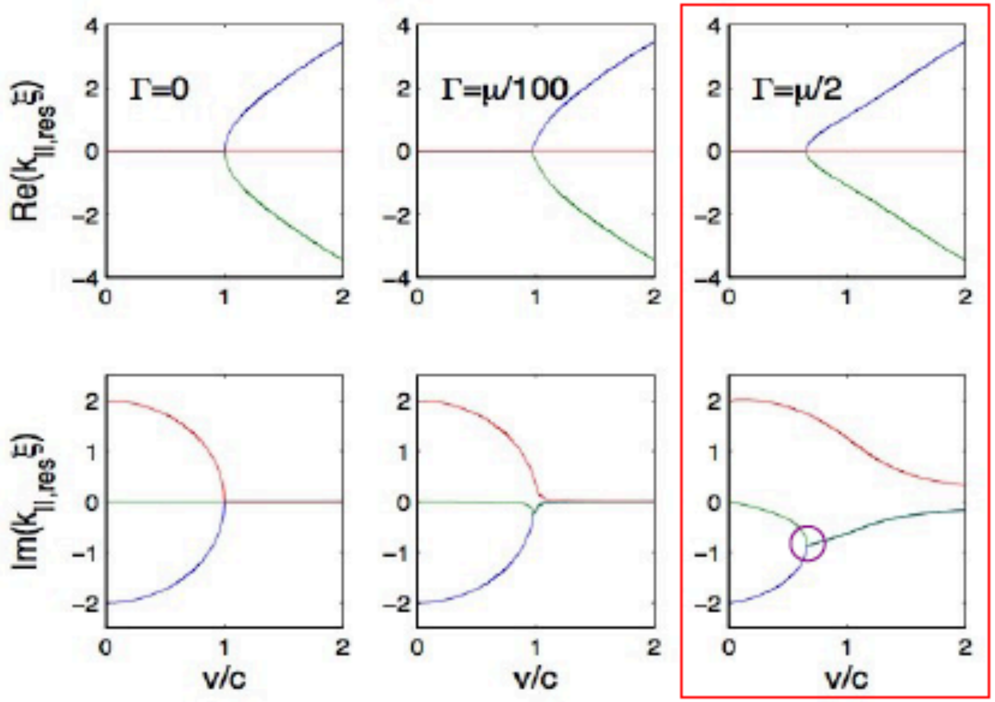
- emitted $k_{\parallel}$ purely imaginary
- no real propagating phonons
- localized perturbation around defect

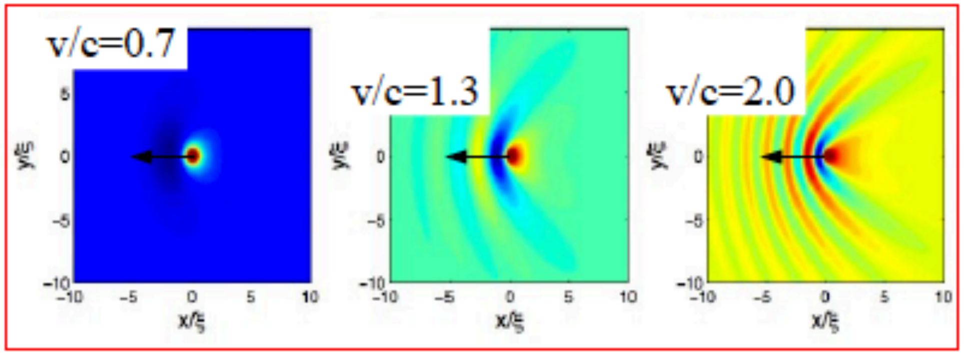
Critical velocity v_c < c:

- corresponds to bifurcation point
- decreases with Γ / μ

High v:

- propagating phonons are emitted :
  - → Cerenkov cone
  - → parabolic precursors
- spatial damping of Cerenkov cone





Wouters & Carusotto, Phys. Rev. Lett. 105, 020602 (2010)

Quantized vortices in superfluids

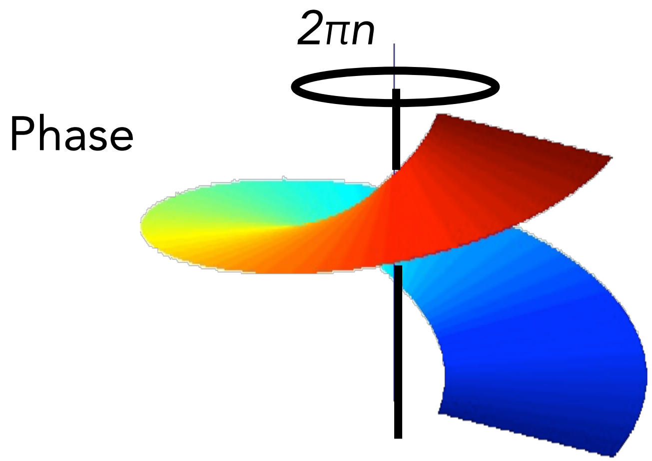
Another signature of superfluidity are persistent (without stirring) circular flows.
Orbital angular momentum carried by a circular flow of a superfluid is quantized (Onzager, 1947).
Superfluid velocity is proportional to phase gradient:

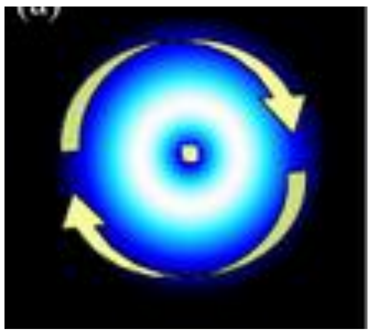
$$\oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{\hbar}{m} \oint_C \nabla\phi \cdot d\mathbf{l} = \frac{\hbar}{m} \Delta\phi, \qquad \oint_C \mathbf{v} \cdot d\mathbf{l} = \frac{2\pi\hbar}{m} n$$

Macroscopic wavefunction is continuous and single-valued, so should re-gain its value after completing a close loop. Therefore: $\Delta\phi = 2\pi n$

Silver Spoon [Wiki]

Vortices: structure and detection





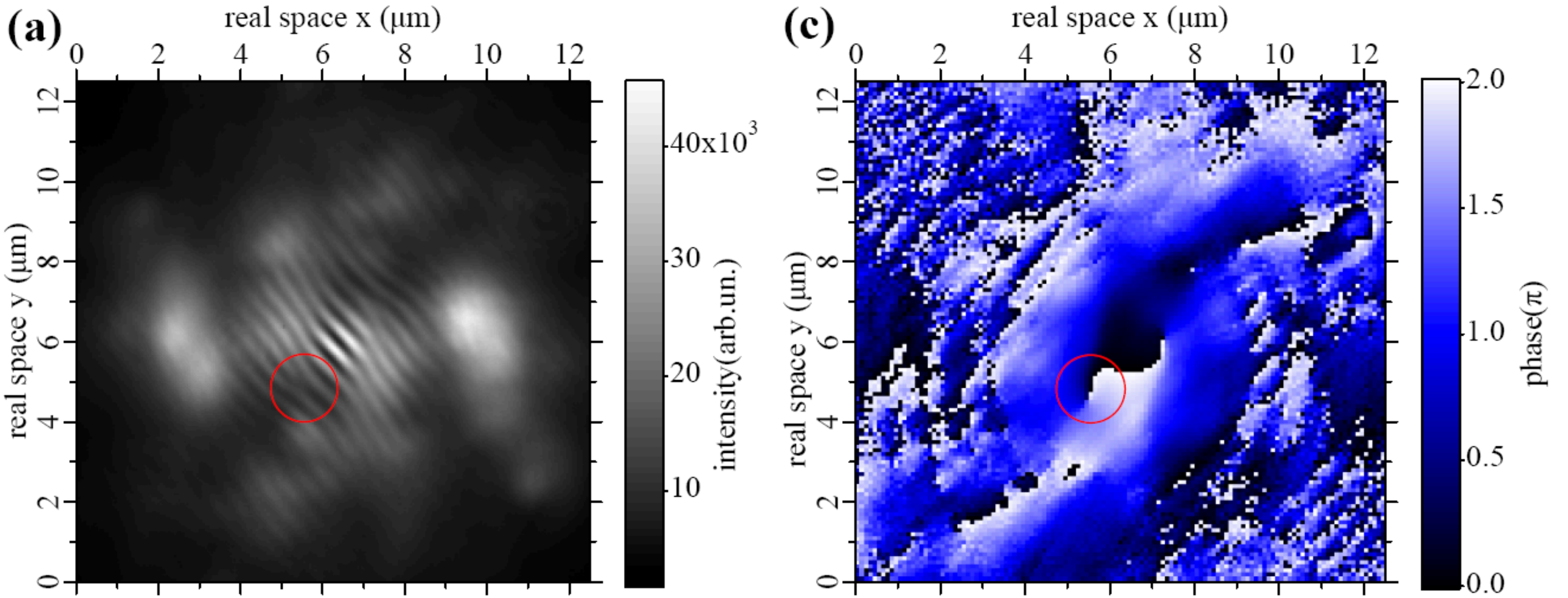
$$\psi(\mathbf{r}) = \sqrt{\rho(\mathbf{r})}e^{i\varphi(\mathbf{r})}$$

Quantized vortices – steady states of a rotating superfluid

- In experiment, should observe a phase change by 2π and a density minimum at the core
- Michelson interferometry
- Fork-like dislocation in interference pattern
- Phase may be retrieved through a Fourier transform

Vortices in polariton superfluid

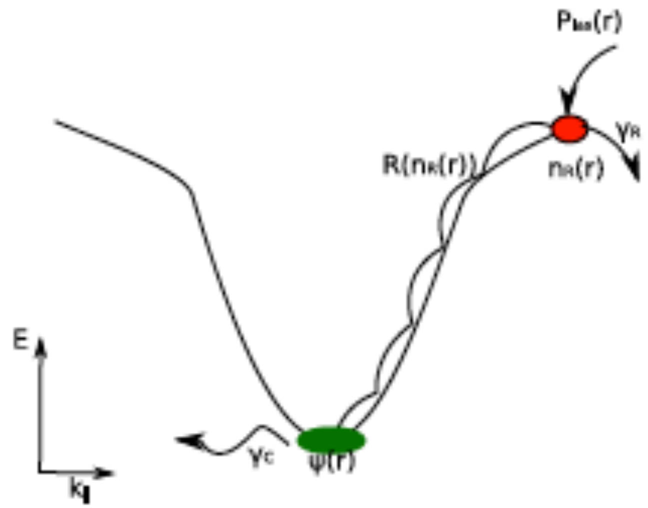
- Spontaneously created and pinned on impurities
- Form pairs, so that total AOM = 0
- Are NOT a signature of superfluidity
- Hard to create in spontaneously (incoherently) created BEC



Lagoudakis, Wouters, Richard, Baas, Carusotto,
André, Le Si Dang & B. Deveaud-Plédran, Nature Physics 4, 706 (2008)

Ground state of a polariton condensate

open-dissipative
Gross-Pitaevskii equation (GPE)



Condensate order parameter time evolution

External potential — Stimulated scattering gain — Reservoir interaction

$$i\hbar\frac{\partial\psi(\mathbf{r},t)}{\partial t} = \left(-\frac{\hbar^2\nabla^2}{2m} + V_{ext}(\mathbf{r}) - \frac{i\hbar}{2}[\,\gamma_C - R(n_R(\mathbf{r},t))\,] + g_C|\psi(\mathbf{r},t)|^2 + g_R n_R(\mathbf{r},t)\right)\psi(\mathbf{r},t)$$

Condensate loss — Condensate interaction

Reservoir rate equation

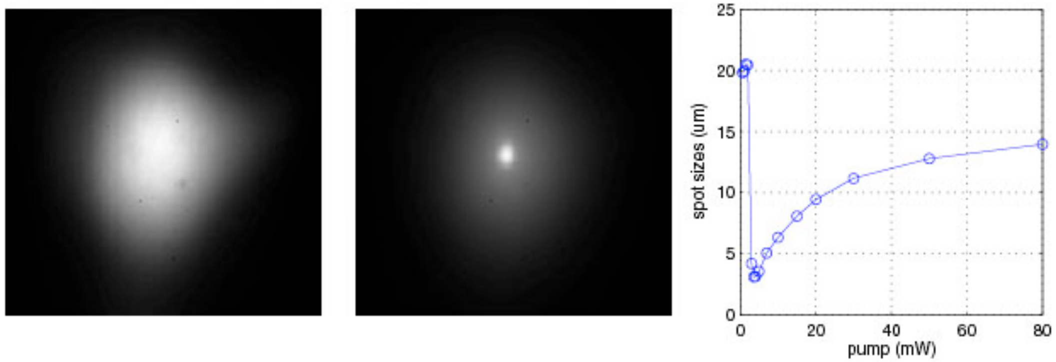
Laser profile

$$\frac{\partial n_R(\mathbf{r},t)}{\partial t} = P_l(\mathbf{r},t) - \gamma_R n_R(\mathbf{r},t) - R(n_R(\mathbf{r},t))|\psi(\mathbf{r},t)|^2$$

Reservoir loss — Stimulated scattering loss

M. Wouters and I. Carusotto, Phys. Rev. Lett. 99, 140402 (2007)

Spatial localisation without confinement?



$$V_{ext}(\vec{r}) = 0$$

repulsive polariton-polariton interactions

repulsive polariton-reservoir interactions

strong anti-trapping potentials

$$U(\vec{r},t) = g_c|\Psi|^2 + g_R n_R(\vec{r},t)$$

how can the condensate be spatially localised?

Superfluid current density

Re-cast the model in the dimensionless form and consider the stationary flux

$$\Psi(\vec{r},t) = \psi(\vec{r})e^{-i\mu t} \qquad \vec{j} = \mathrm{Im}(\Psi^* \nabla \Psi)$$

assume Gaussian pump profile

$$P(r) = P_0 \exp(-r^2/\sigma^2) \qquad P_{th} \approx \frac{\gamma_R \gamma_c}{R}$$

and in-plane radial symmetry of the condensate

$$\psi(\vec{r}) \longrightarrow \Phi(r)e^{i\phi(r)+im\theta}$$

Equations for the the radial component of the flux and condensate density:

$$\frac{1}{r}\frac{d}{dr}(rJ) - (Rn_R^0 - \gamma_c)\Phi^2 = 0$$

$$\nabla_r^2 \Phi - 2\left[U(r) - \mu\right]\Phi - V_J \Phi = 0$$

$$\boxed{J = \Phi^2(r)\frac{d\phi}{dr} \qquad n_R^0 = \frac{P(r)}{\gamma_R + R\Phi^2(r)}}$$

Trapping potential due to phase gradient: $V_J = (d\phi/dr)^2$

Properties of the stationary flux

Divergence of the flux: $D(r) = \nabla \vec{j} = \frac{1}{r}\frac{d}{dr}(rJ)$

measures local gain (D>0) or loss (D<0)

Localised steady state exists if generation of superfluid

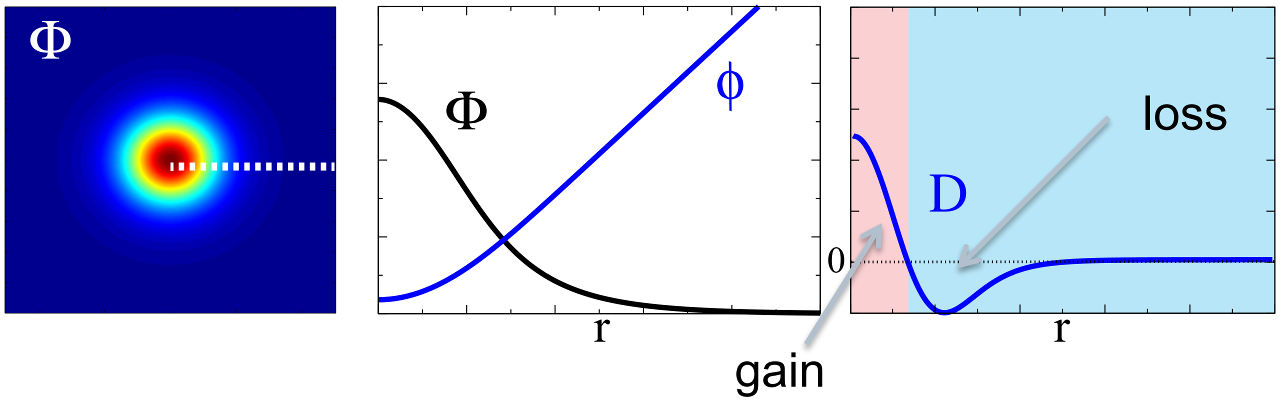
is balanced by its dissipation:

$$\boxed{\int_0^\infty D(r) r dr = 0}$$

This condition can be satisfied for both repulsive and attractive nonlinearities

*Open-dissipative optical systems (**Ginzburg-Landau model**): Rosanov et al, JETP 98, 427 (2003); Ankiewicz et al., PRA, 77, 033840 (2008) Zezyulin et al., Opt. Lett. 36, 1200 (2011)*

Steady-state condensate: gain trapping



The ground state of a repulsive polariton BEC is a bright dissipative, continuously self-defocusing soliton localised due to the balance of superfluid currents.

Asymptotic behaviour of the order parameter and phase

$r \longrightarrow 0$

$$\Phi(r) \sim \exp(-q_- r^2), \quad \phi(r) \sim q_+ r^2$$

$$q_- = [U(0) - \mu]/2, \quad q_+ = [R n_R^0(0) - \gamma_c]/4$$

$r \longrightarrow \infty$

$$\Phi(r) \sim \exp(-p_- r), \quad \phi(r) \sim p_+ r$$

$$p_\pm = \left[(\mu^2 + \gamma_c^2/4)^{1/2} \pm \mu\right]^{1/2}$$