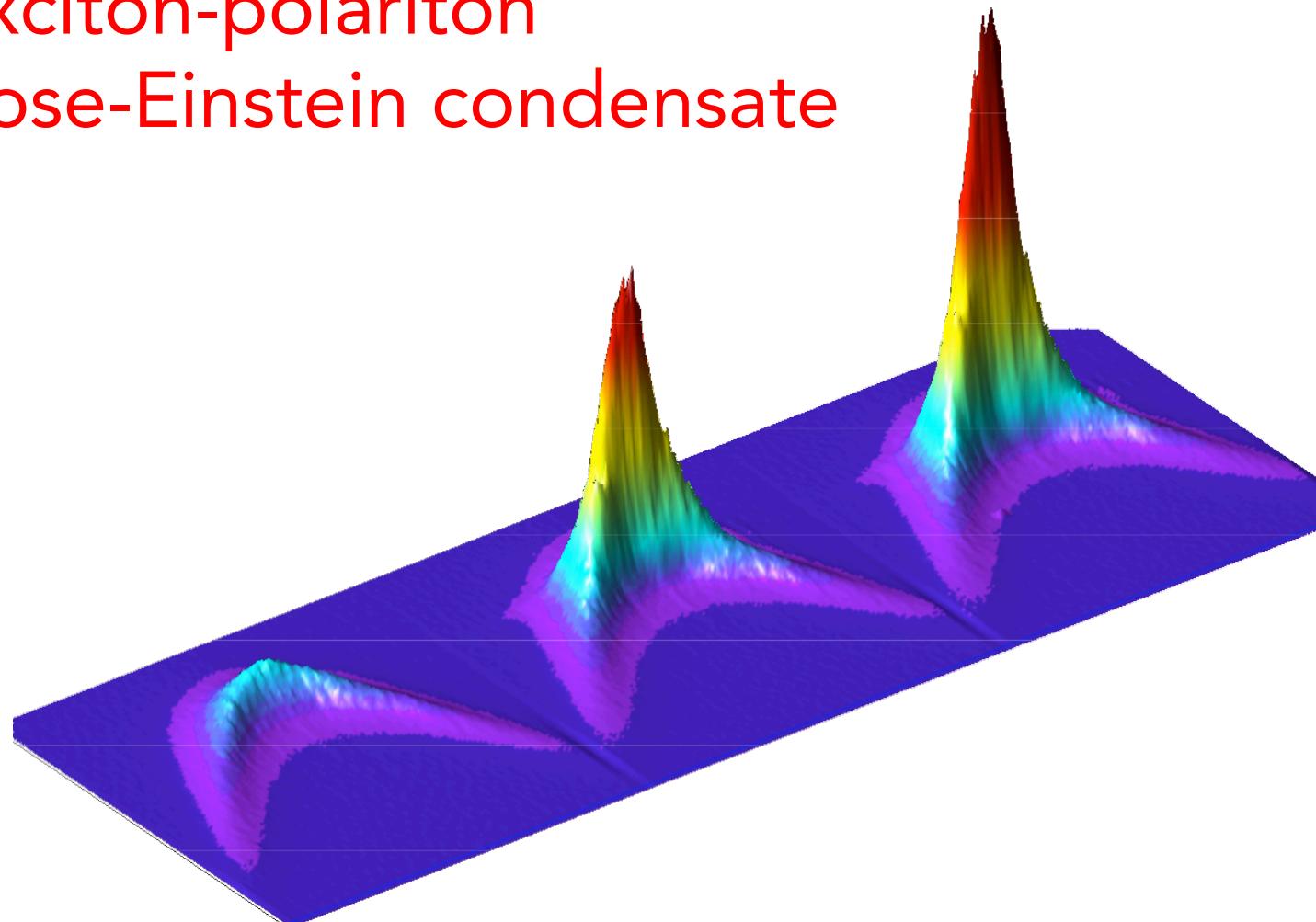
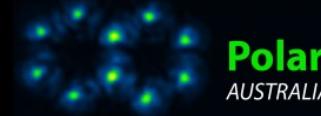


# Exciton-polariton Bose-Einstein condensate



Elena Ostrovskaya  
The Australian National University

<http://polaritonbec.org>

**Polariton BEC Research Group**  
AUSTRALIAN NATIONAL UNIVERSITY

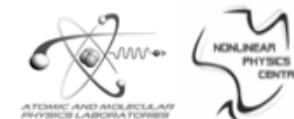
# Polariton BEC Laboratory

&

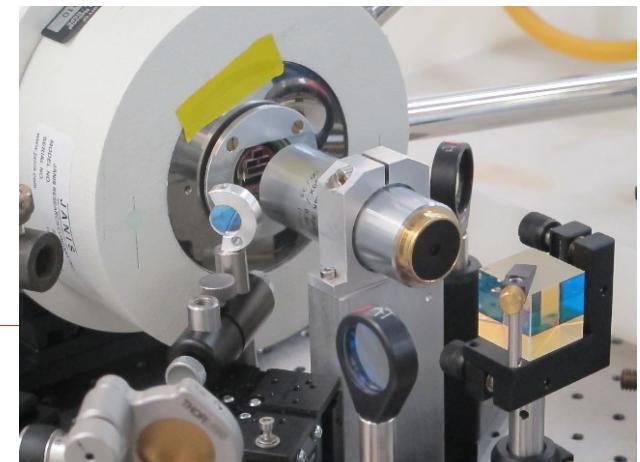


Australian  
National  
University

**Picosecond Imaging Facility**

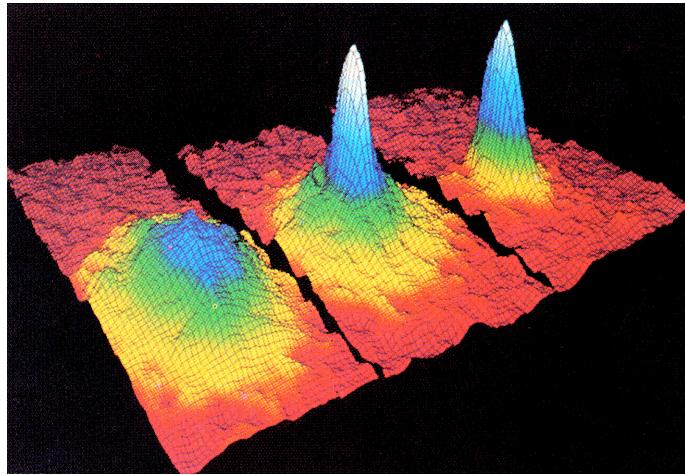


established in 2013



# outline of the lectures

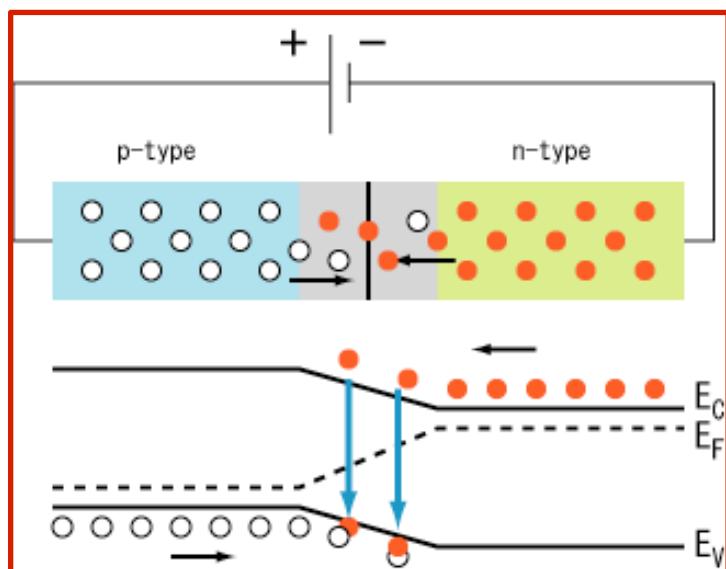
- Wannier-Mott excitons in QW
  - light-matter coupling in semiconductor cavities
  - (exciton-)polaritons
  
  - condensation of polaritons
  - superfluid dynamics
  - vortices in pBEC
  
  - trapping techniques
  - excitonic potentials
  - photonic potentials
  
  - pBEC in periodic potentials
  - polaritonic devices
-



## The Nobel Prize in Physics 2001

Eric A. Cornell, Wolfgang Ketterle  
and Carl E. Wieman

"for the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates"

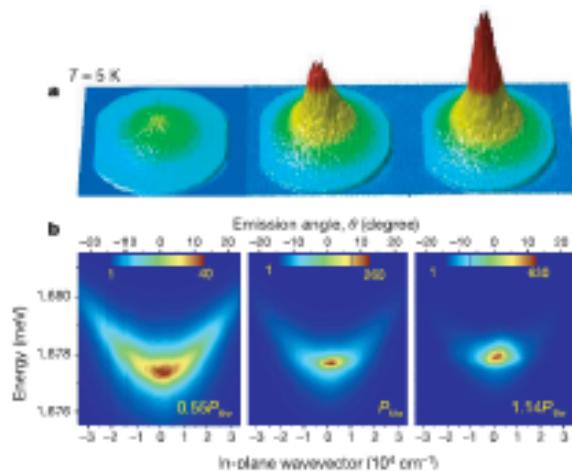


## The Nobel Prize in Physics 2000

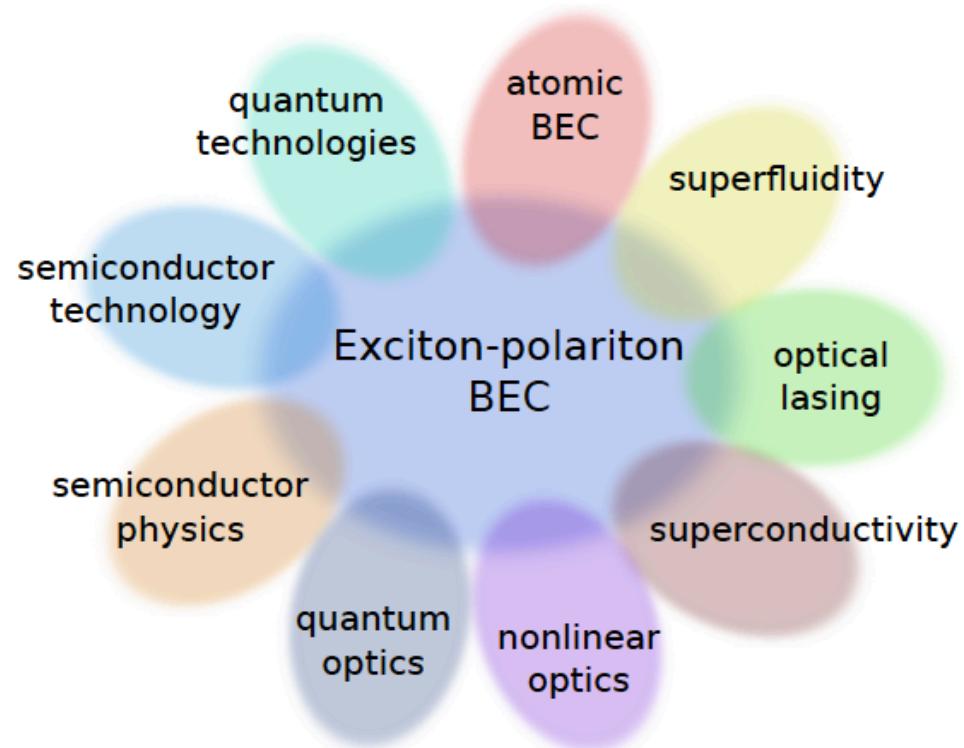
Zh. I. Alferov and H. Kroemer

"for developing semiconductor heterostructures used in high-speed- and opto-electronics"

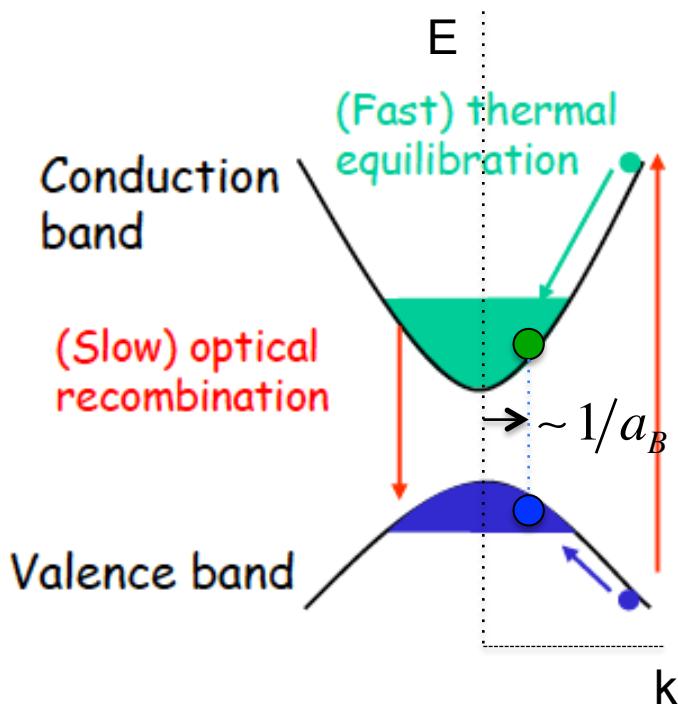
# 2006: Exciton-Polariton BEC



- Polariton mass  $10^9$  orders lighter than atom
- Transition temperature in 10s to 100s of K
- Ease of study using optical techniques
- Completely embedded within a semiconductor chip



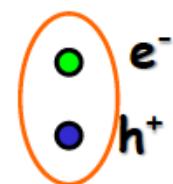
# Wannier-Mott excitons



In direct gap semiconductors, optical properties are governed by two bands above and below the Fermi level.

Absorption of a photon leads to electron excitation into conduction band.

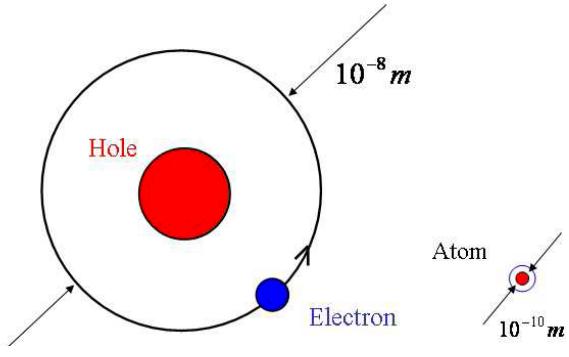
**Excitons** are quasi-particles formed by optically-excited electrons-hole pairs bound by Coulomb attraction.



Binding is weak and their size is larger than inter-atomic distance in the solid-state lattice.

At high densities  $n_{exc}a_B \sim 1$  and high temperatures electrons and holes unbind and form electron-hole plasma.

# Wannier equation



Schrödinger equation for the wavefunction of the relative e-h motion

$$-\frac{\hbar^2}{2\mu} \Delta f(r) - \frac{e^2}{4\pi\epsilon\epsilon_0 r} f(r) = Ef(r)$$

$$\mu = m_e m_h / (m_e + m_h) \quad \text{reduced mass}$$

Equivalent to Schrödinger equation for a hydrogen atom with:  $m_0 \rightarrow \mu$ ,  $e^2 \rightarrow e^2/\epsilon$

The first bound state (1s):  $f_{1s} = \frac{1}{\sqrt{\pi a_B^3}} e^{-r/a_B}$      $a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} \sim 0.529 \text{ \AA}$

Exciton Bohr radius and Rydberg (SI)

$$a_B = \frac{a_0 \epsilon}{\mu/m_e} \quad E_B = \frac{\hbar^2}{2\mu a_B^2}$$

e.g. in GaAs ( $m^* \sim 0.1 m_e$ ,  $\epsilon = 13$ )

Binding energy = 5 meV (13.6 eV for Hydrogen)

Bohr radius = 7 nm (0.05 nm for Hydrogen)

# Effective mass

$$\mu = m_e m_h / (m_e + m_h)$$

Schrödinger equation for the wavefunction of an electron in a periodic lattice potential  $V(r)$  has solutions in the form of Bloch waves:

$$\Psi_{n,\mathbf{k}} = U_{n,\mathbf{k}}(\mathbf{r}) e^{i\mathbf{kr}}$$
$$-\frac{\hbar^2}{2m_0} \Delta \Psi_{n,\mathbf{k}} + V(\mathbf{r}) \Psi_{n,\mathbf{k}} = E_{n,\mathbf{k}} \Psi_{n,\mathbf{k}}$$
$$\Delta = \nabla^2$$

Equation for Bloch amplitudes (in real space):

$$-\frac{\hbar^2}{2m_0} \Delta U_{\mathbf{k},n} + V(\mathbf{r}) U_{\mathbf{k},n} + \left( \frac{\hbar^2 \mathbf{k}^2}{2m_0} + \frac{\hbar}{m_0} (\mathbf{k} \cdot \mathbf{p}) \right) U_{\mathbf{k},n} = E_{\mathbf{k},n} U_{\mathbf{k},n}$$
$$\mathbf{p} = -i\hbar \nabla$$

Considering  $(\mathbf{k} \cdot \mathbf{p})$  terms as perturbation, we obtain energy near  $\mathbf{k}=0$  of all bands:

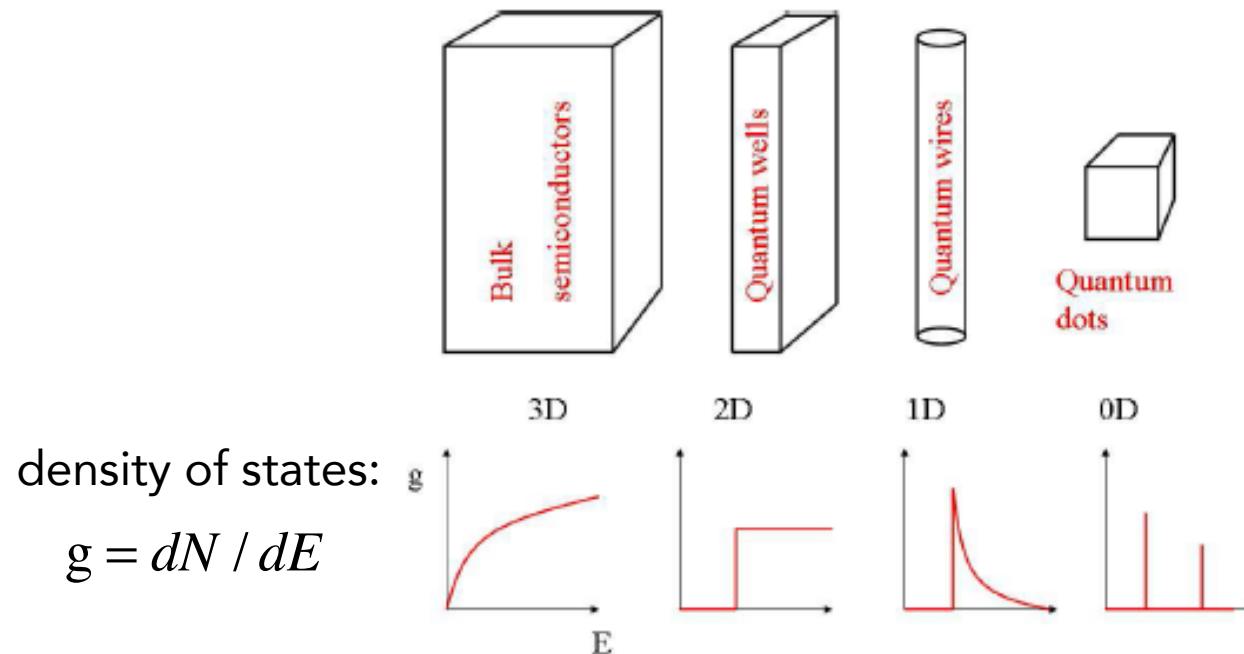
$$E_{\mathbf{k},n} \approx E_{0,n} + \frac{\hbar^2 k^2}{2m_n^*}$$

Electron's effective mass  $\frac{1}{m_n^*} = \frac{1}{m_0} + \frac{2}{m_0^2} \sum_{l \neq n} \frac{|\langle U_{0,l} | \mathbf{p} | U_{0,n} \rangle|^2}{E_{0,l} - E_{0,n}}$

In GaAs:  $m_e = 0.067m_0$ ,  $m_h = 0.45m_0$

# Spatial confinement

Semiconductor nanotechnology has enabled engineering of strongly confined electron states



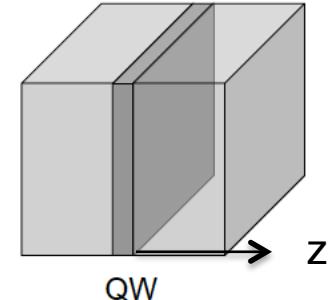
Confinement increases **binding energy** of excitons thus increasing their lifetime

Confinement enables **strong light-matter interaction**

---

# Exciton in a quantum well

$$-\frac{\hbar^2}{2m_e}\nabla_e^2 - \frac{\hbar^2}{2m_h}\nabla_h^2 + V_e(z_e) + V_h(z_h) - \frac{e^2}{4\pi\epsilon_0\epsilon|\mathbf{r}_e - \mathbf{r}_h|}\Psi = E\Psi$$

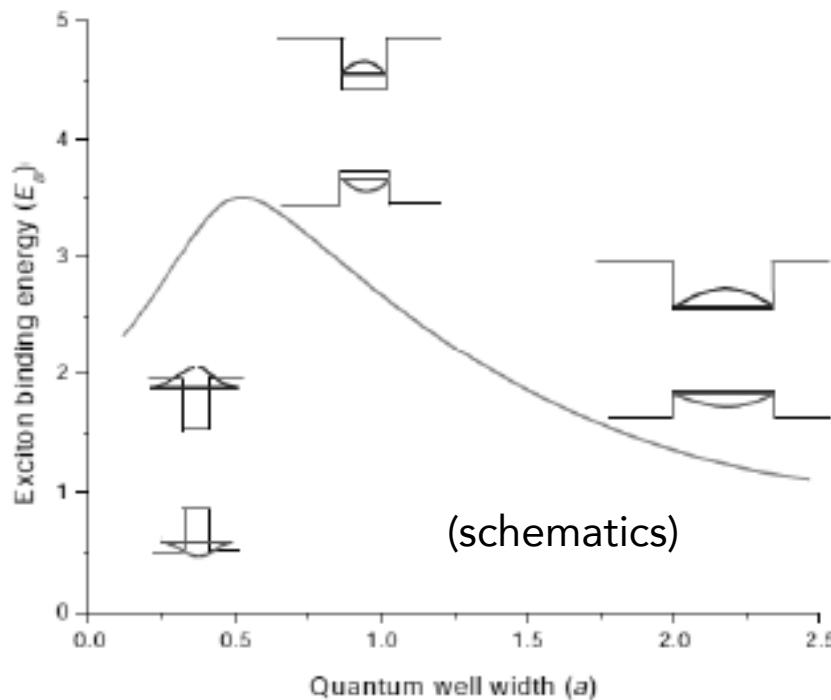


$$\Psi(\mathbf{r}_e, \mathbf{r}_h) = F(\mathbf{R})f(\boldsymbol{\rho})U_e(z_e)U_h(z_h)$$

$$\mathbf{R} = \frac{m_e \mathbf{r}_e + m_h \mathbf{r}_h}{m_e + m_h} \quad \boldsymbol{\rho} = \boldsymbol{\rho}_e - \boldsymbol{\rho}_h$$

In the ideal 2D case,  $|U_{e,h}(z_{e,h})|^2 = \delta(z_{e,h})$

$$\left\{ -\frac{\hbar^2}{2\mu} \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial}{\partial \rho} \right) - \frac{e^2}{\epsilon \rho} \right\} f(\rho) = E_B^{2D} f(\rho)$$



Exactly solvable  
hydrogen atom problem

$$f_{1S}(\rho) = \sqrt{\frac{2}{\pi}} \frac{1}{a_B^{2D}} \exp(-\rho/a_B^{2D})$$

$$a_B^{2D} = \frac{a_B}{2} \quad E_B^{2D} = 4E_B$$

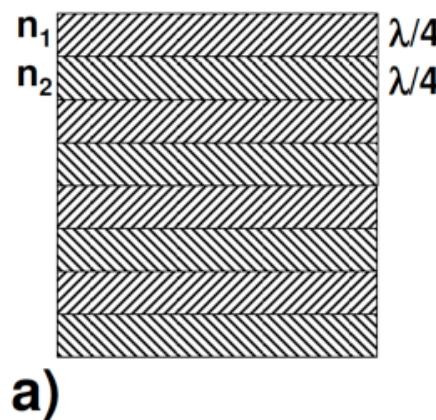
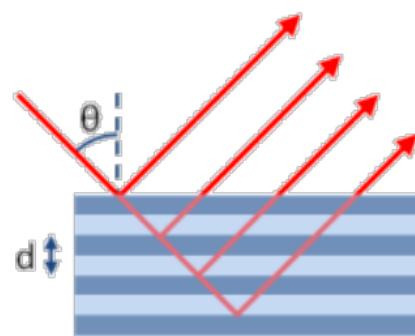
# QW excitons in a semiconductor microcavity – the birth of a polariton



LSDS semiconductor research group Sheffield University, UK

# Mirrors: Distributed Bragg Reflectors

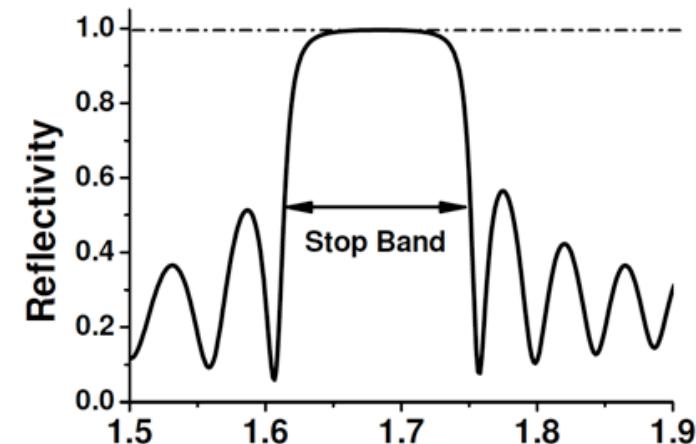
Alternative high and low dielectric layers with dimensions exactly  $\lambda/4n$



Reflectivity - depends on number of layers and index contrast

$$R = \left[ \frac{n_o(n_2)^{2N} - n_s(n_1)^{2N}}{n_o(n_2)^{2N} + n_s(n_1)^{2N}} \right]^2$$

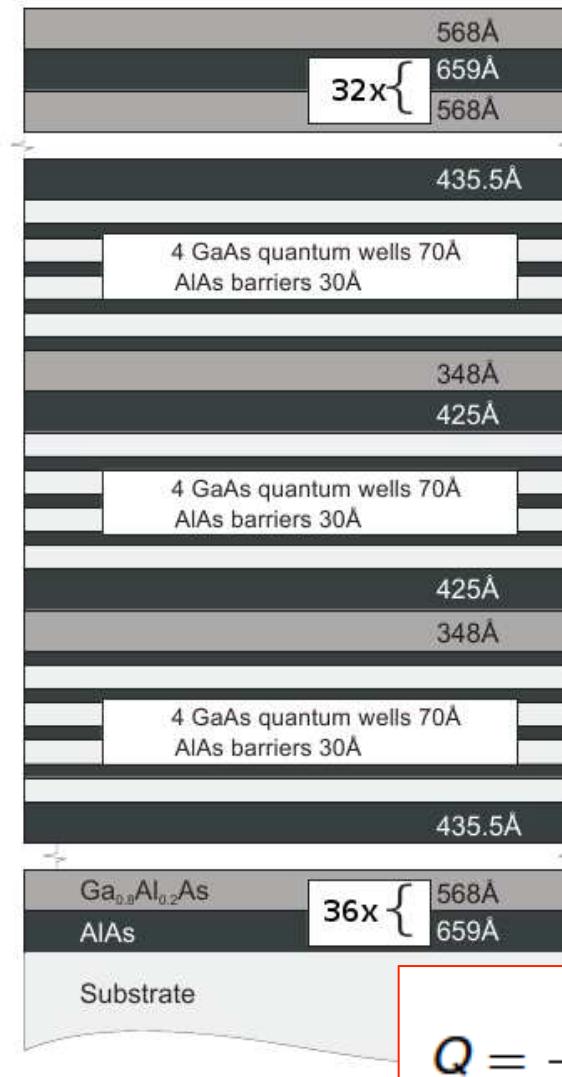
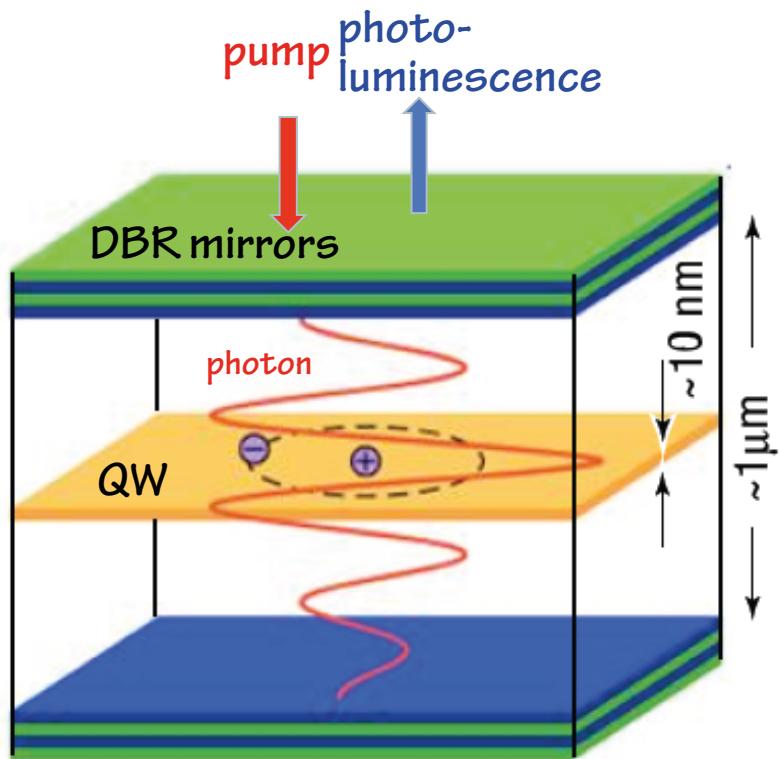
Reflectivity is angle and wavelength dependent



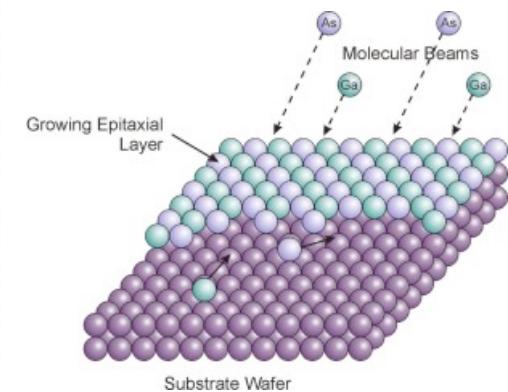
Bandwidth - fixed by index contrast

$$\Delta\lambda_0 = \frac{4\lambda_0}{\pi} \arcsin \left( \frac{n_2 - n_1}{n_2 + n_1} \right)$$

# Semiconductor microcavity



Fabrication:  
epitaxially grown QWs  
between DBR layers



DBR + QW  
photon and exciton  
confinement

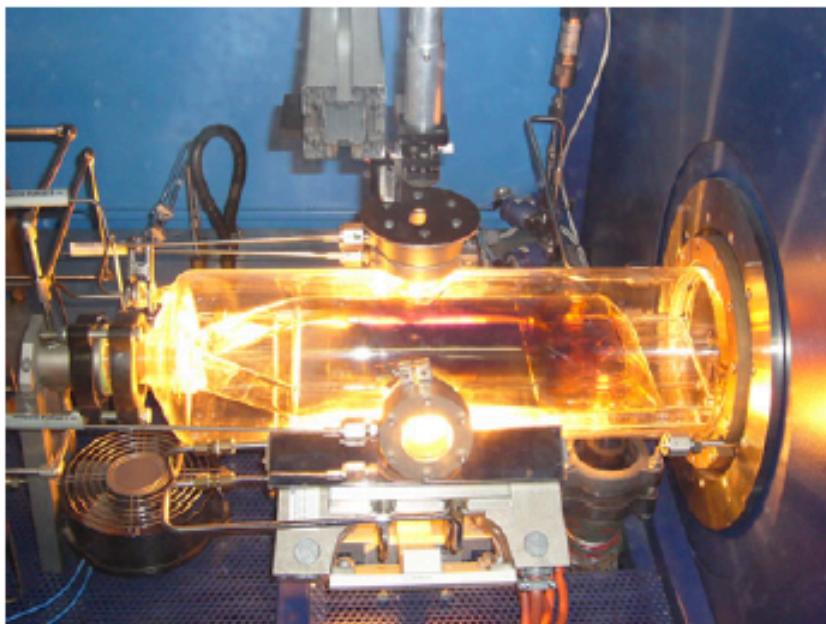
$$Q = \frac{\lambda_c}{\Delta\lambda_c} \simeq \frac{\pi(R_1 R_2)^{1/4}}{1 - (R_1 R_2)^{1/2}}$$

$Q \sim 10^5 - 10^6$

# Growing the heterostructures

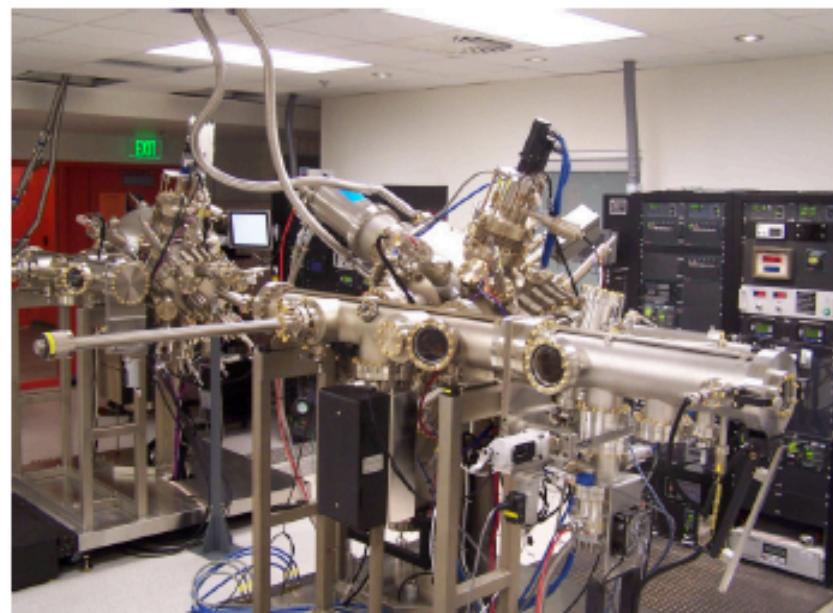
## Metal-organic chemical vapour deposition (MOCVD)

- dissociation of low pressure flow of molecular gases (e.g.  $As_3$ ,  $(CH_3)_3Ga$ )
- unwanted impurities (e.g. C, trace dopants)
- relatively fast - industry preferred



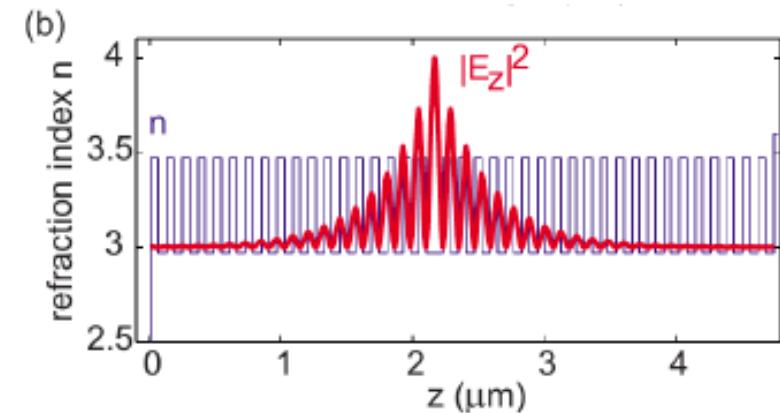
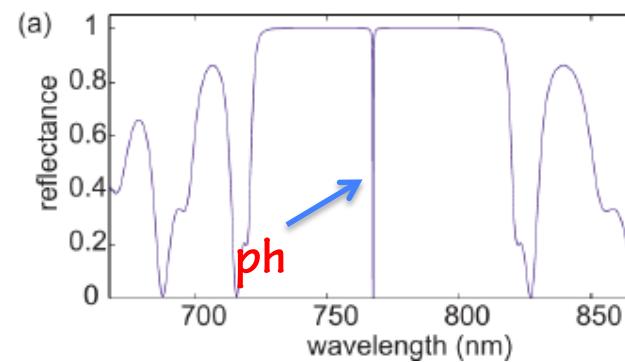
## Molecular-beam epitaxy (MBE)

- dissociated molecular beams under UHV
- slow, but very precise
- low impurities and disorder
- method of choice for polariton microcavities



# Microcavity spectrum

photon mode  
in a microcavity



$$E_{\text{cav}} = \frac{\hbar c}{n_c} \sqrt{k_\perp^2 + k_\parallel^2}$$

$$k_\perp = n_c (2\pi/\lambda_c)$$

In the region  $k_\parallel \ll k_\perp$ , we have

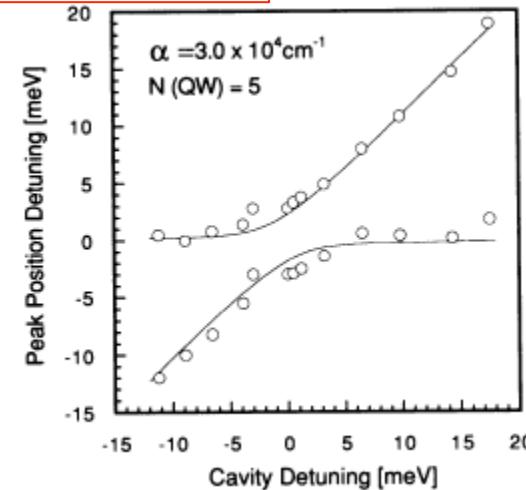
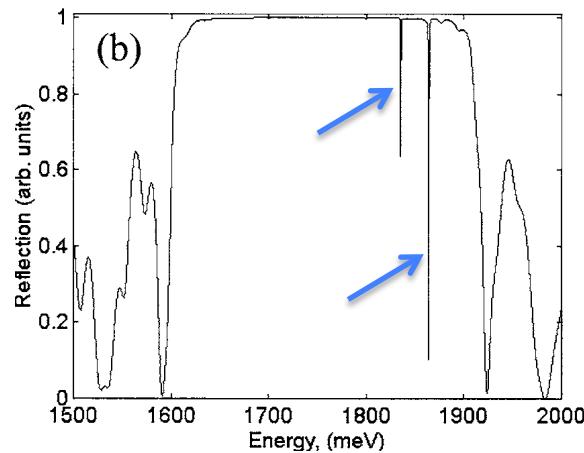
$$E_{\text{cav}} \approx \frac{\hbar c}{n_c} k_\perp \left( 1 + \frac{k_\parallel^2}{2k_\perp^2} \right) = E_{\text{cav}}(k_\parallel = 0) + \frac{\hbar^2 k_\parallel^2}{2m_{\text{cav}}}$$

$$m_{\text{cav}} = \frac{E_{\text{cav}}(k_\parallel = 0)}{c^2/n_c^2}$$

$\sim 10^{-5} m_e$

microcavity + QW:

normal mode  
splitting;  
avoided crossing



# Quantum picture of light-matter coupling

The linear Hamiltonian of the system in the second-quantization form:

$$\hat{H}_{\text{pol}} = \hat{H}_{\text{cav}} + \hat{H}_{\text{exc}} + \hat{H}_I = \sum E_{\text{cav}}(k_{\parallel}, k_c) \hat{a}_{k_{\parallel}}^{\dagger} \hat{a}_{k_{\parallel}} + \sum E_{\text{exc}}(k_{\parallel}) \hat{b}_{k_{\parallel}}^{\dagger} \hat{b}_{k_{\parallel}} + \sum g_0 (\hat{a}_{k_{\parallel}}^{\dagger} \hat{b}_{k_{\parallel}} + \hat{a}_{k_{\parallel}} \hat{b}_{k_{\parallel}}^{\dagger})$$

$\hat{a}_{k_{\parallel}}^{\dagger}$  and  $\hat{b}_{k_{\parallel}}^{\dagger}$  are photon and exciton creation operator with in-plane wavevector  $k_{\parallel}$

$k_c = \mathbf{k} \cdot \hat{\mathbf{z}}$  longitudinal wavevector defined by cavity resonance

$g_0$  strength of exciton-photon dipole interaction

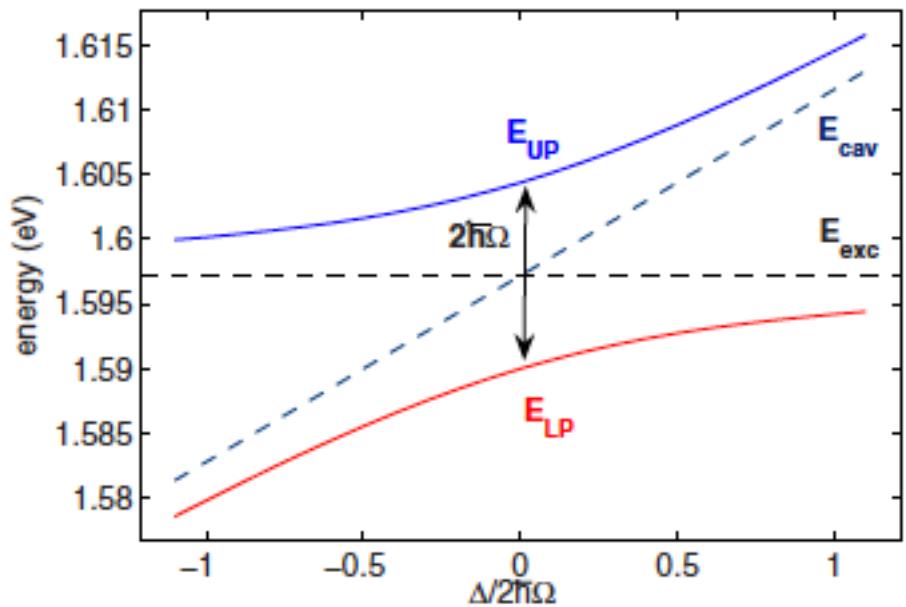
$$\begin{aligned} \hat{P}_{k_{\parallel}} &= X_{k_{\parallel}} \hat{b}_{k_{\parallel}} + C_{k_{\parallel}} \hat{a}_{k_{\parallel}}, & \text{Creation and annihilation operators of} \\ && \text{the new \textcolor{red}{bosonic} quasiparticles (eigenmodes) --} \\ \hat{Q}_{k_{\parallel}} &= -C_{k_{\parallel}} \hat{b}_{k_{\parallel}} + X_{k_{\parallel}} \hat{a}_{k_{\parallel}} & \text{upper and lower \textcolor{red}{polaritons}} \end{aligned}$$

$$\hat{H}_{\text{pol}} = \sum E_{\text{LP}}(k_{\parallel}) \hat{P}_{k_{\parallel}}^{\dagger} \hat{P}_{k_{\parallel}} + \sum E_{\text{UP}}(k_{\parallel}) \hat{Q}_{k_{\parallel}}^{\dagger} \hat{Q}_{k_{\parallel}}$$

The Hopfield coefficients define proportion of photon/exciton in a polariton:

$$|X_{k_{\parallel}}|^2 = \frac{1}{2} \left( 1 + \frac{\Delta E(k_{\parallel})}{\sqrt{\Delta E(k_{\parallel})^2 + 4g_0^2}} \right) \quad |C_{k_{\parallel}}|^2 = \frac{1}{2} \left( 1 - \frac{\Delta E(k_{\parallel})}{\sqrt{\Delta E(k_{\parallel})^2 + 4g_0^2}} \right) \quad \begin{aligned} \Delta E(k_{\parallel}) &= E_{\text{exc}}(k_{\parallel}) - E_{\text{cav}}(k_{\parallel}, k_c) \\ |X_{k_{\parallel}}|^2 + |C_{k_{\parallel}}|^2 &= 1 \end{aligned}$$

# Polariton: part-light and part-matter



$$E_{LP,UP}(k_{\parallel}) = \frac{1}{2}[E_{exc} + E_{cav} \pm \sqrt{4g_0^2 + (E_{exc} - E_{cav})^2}]$$

Rabi (normal mode) splitting

$$E_{UP} - E_{LP} = 2g_0$$

$$\hbar^2 k_{\parallel}^2 / 2m_{cav} \ll 2g_0$$

$$E_{LP,UP}(k_{\parallel}) \simeq E_{LP,UP}(0) + \frac{\hbar^2 k_{\parallel}^2}{2m_{LP,UP}}$$

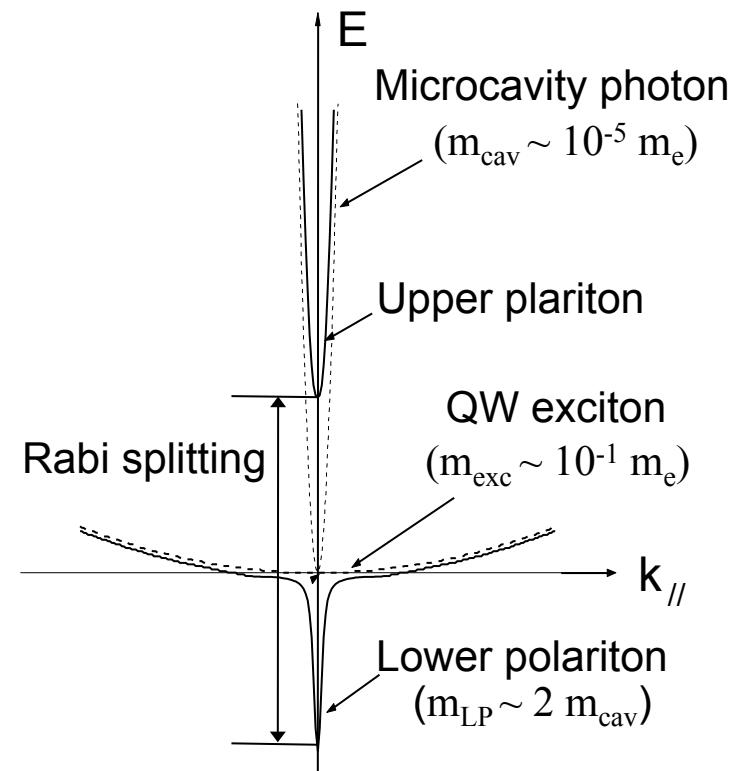
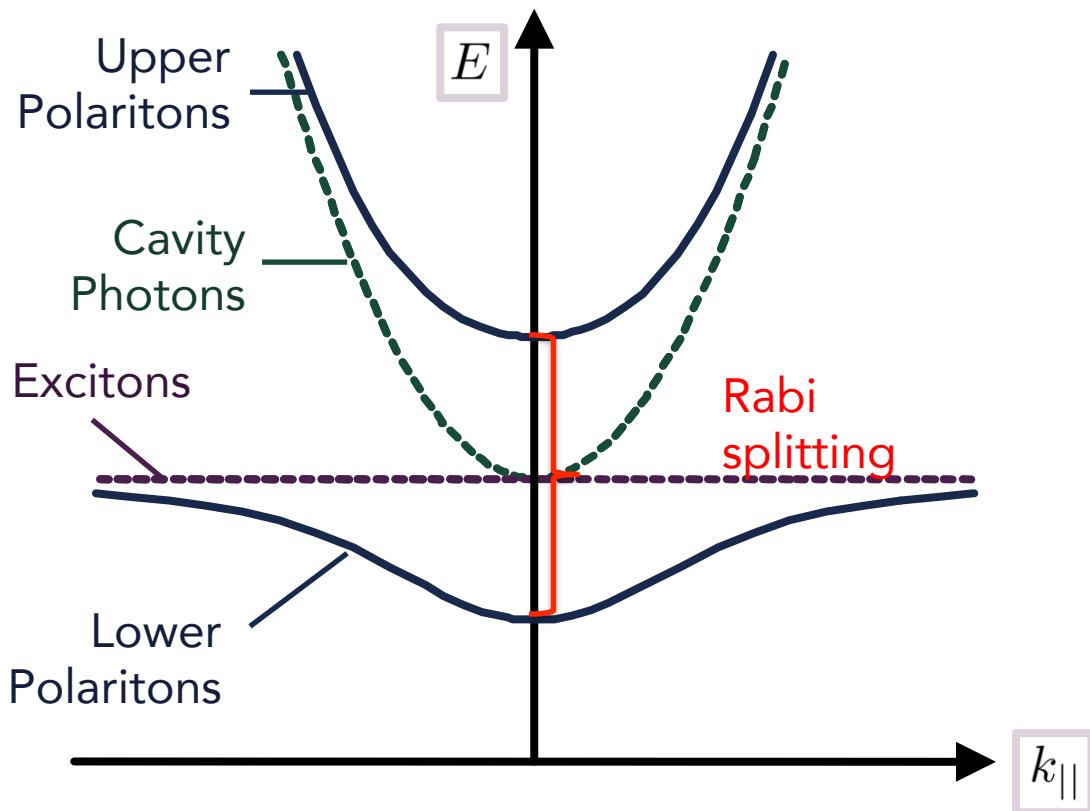
$$\frac{1}{m_{LP}} = \frac{|X|^2}{m_{exc}} + \frac{|C|^2}{m_{cav}}$$

$$\frac{1}{m_{UP}} = \frac{|C|^2}{m_{exc}} + \frac{|X|^2}{m_{cav}}$$

$$m_{LP}(k_{\parallel} \sim 0) \simeq m_{cav}/|C|^2 \sim 10^{-4} m_{exc}$$

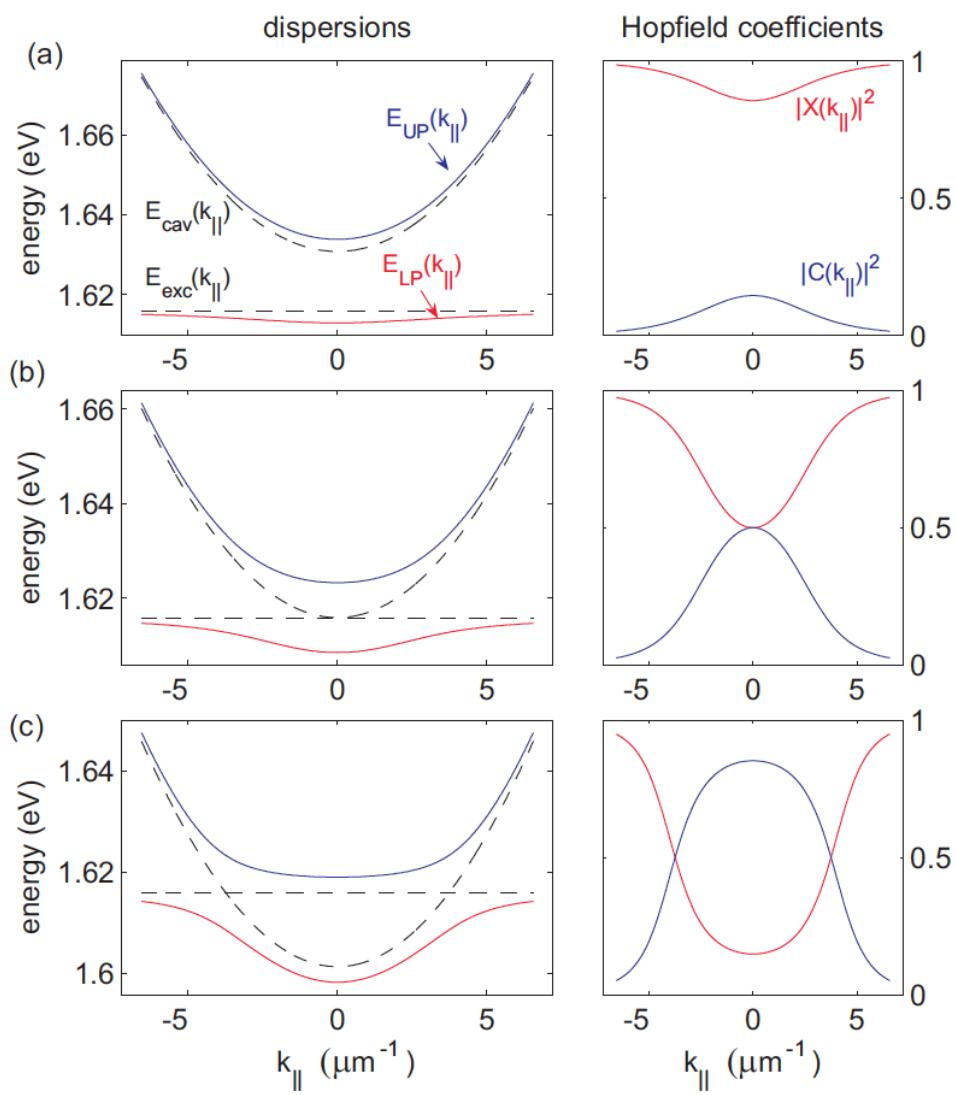
$$m_{UP}(k_{\parallel} \sim 0) \simeq m_{cav}/|X|^2.$$

# Polariton dispersion



$$E_{LP,UP}(k_{||}) = \frac{1}{2} \left[ E_{exc} + E_{cav} \pm \sqrt{4\hbar^2\Omega^2 + (E_{exc} - E_{cav})^2} \right]$$

# (Not quite) half-light half-matter



## Photon part (light):

- light effective mass ( $\sim 10^{-5} m_e$ )
- fast propagation ( $\sim \mu\text{m}/\text{ps}$ )
- short lifetime ( $\sim \text{ps}$ )
- low dephasing ( $\sim \text{ns}$ )

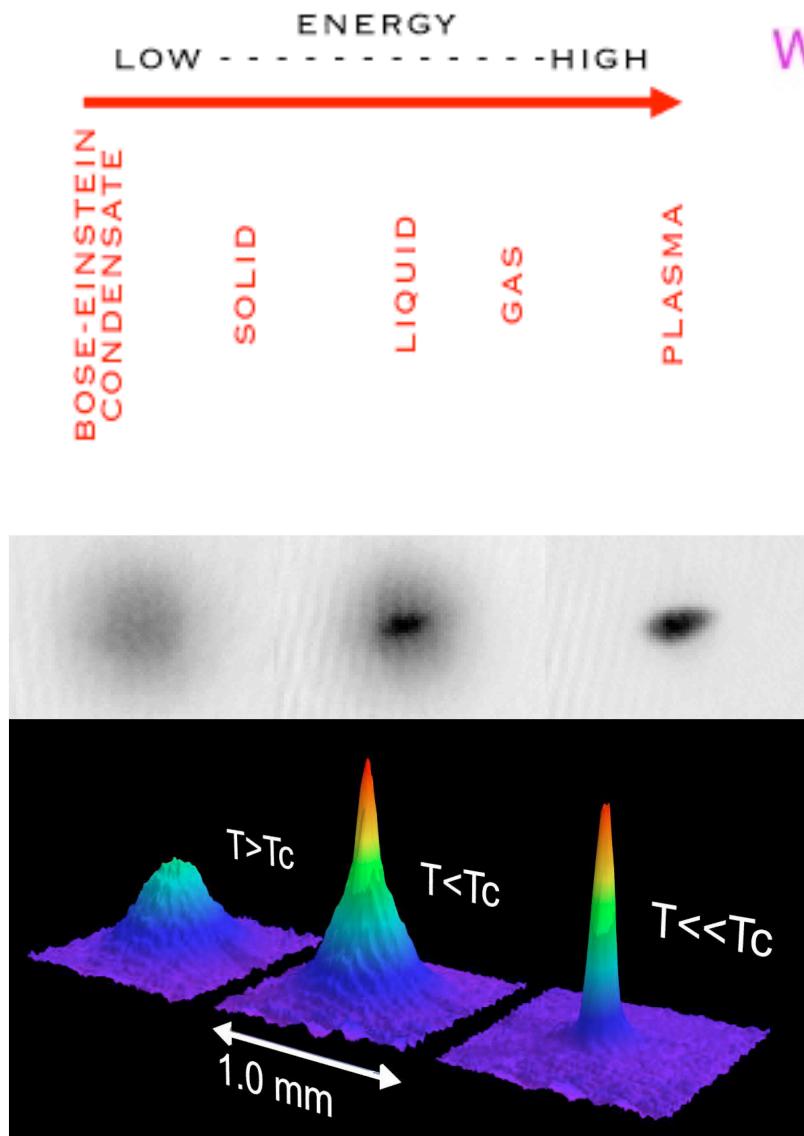
## Exciton part (matter):

- nonlinear interactions
- electric + magnetic field dependence

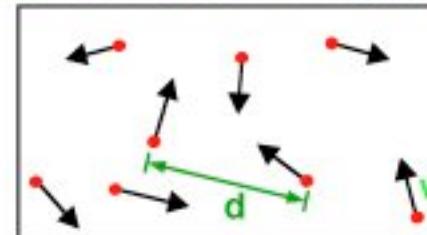
## Hybrid property:

- spin degree of freedom

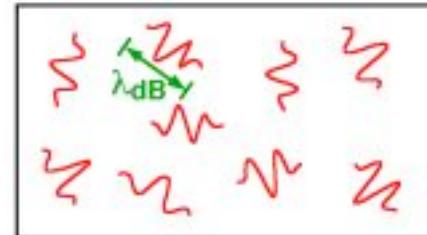
# Bose-Einstein condensation



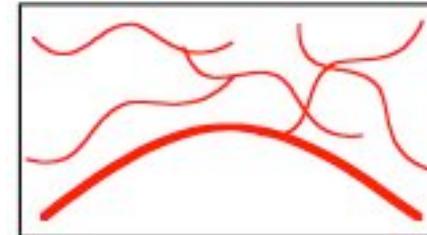
What is Bose-Einstein condensation (BEC)?



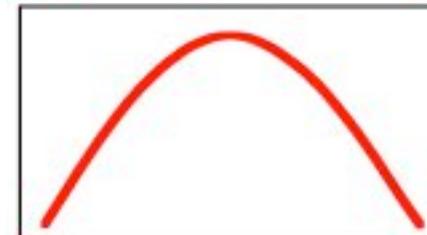
**High Temperature  $T$ :**  
thermal velocity  $v$   
density  $d^{-3}$   
"Billiard balls"



**Low Temperature  $T$ :**  
De Broglie wavelength  
 $\lambda_{dB} = h/mv \propto T^{-1/2}$   
"Wave packets"



$T = T_{\text{crit}}$ :  
Bose-Einstein Condensation  
 $\lambda_{dB} = d$   
"Matter wave overlap"



$T = 0$ :  
Pure Bose condensate  
"Giant matter wave"

# BEC of neutral alkaline atoms

- Created in 1995, Nobel Prize in 2001
  - Access to quantum physics on a macroscopic scale
  - Studies of fundamental physics (quantum many-body physics and nonlinear dynamics)
  - Clean, very controllable system
  - Basis for ultra-precise sensing and measurement, quantum simulation
  - Atom laser
  - On-chip integration (?)
  - Ultracold (~nK) temperatures, ultra-high vacuum
  - How does the polariton BEC compare?
-

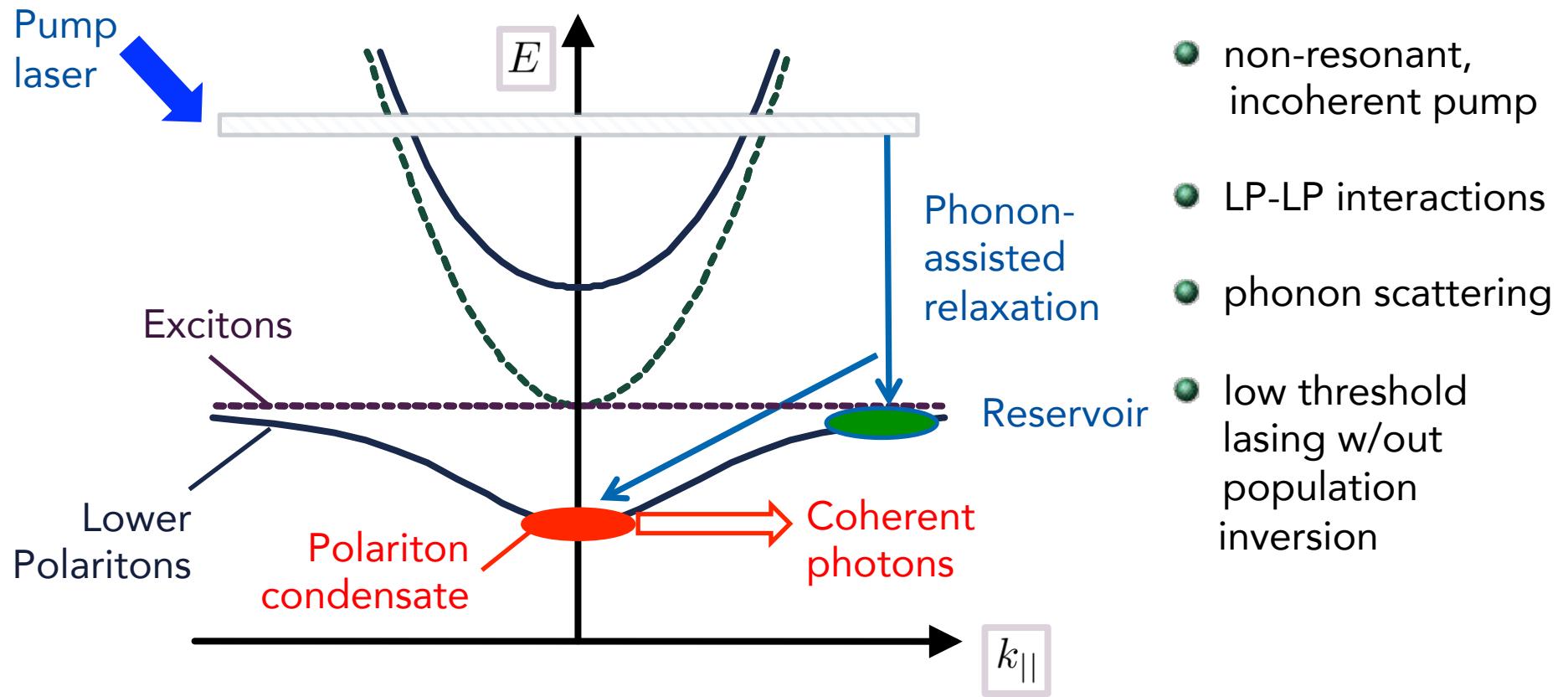
# Can polaritons condense?

TABLE I. Parameter comparison of BEC systems.

Systems	Atomic gases	Excitons	Polaritons
Effective mass $m^*/m_e$	$10^3$	$10^{-1}$	$10^{-5}$
Bohr radius $a_B$	$10^{-1}$ Å	$10^2$ Å	$10^2$ Å
Particle spacing: $n^{-1/d}$	$10^3$ Å	$10^2$ Å	$1 \mu\text{m}$
Critical temperature $T_c$	1 nK–1 $\mu\text{K}$	1 mK–1 K	$1 \rightarrow 300$ K
Thermalization time/Lifetime	$1 \text{ ms}/1 \text{ s} \sim 10^{-3}$	$10 \text{ ps}/1 \text{ ns} \sim 10^{-2}$	$(1\text{--}10 \text{ ps})/(1\text{--}10 \text{ ps}) = 0.1\text{--}10$

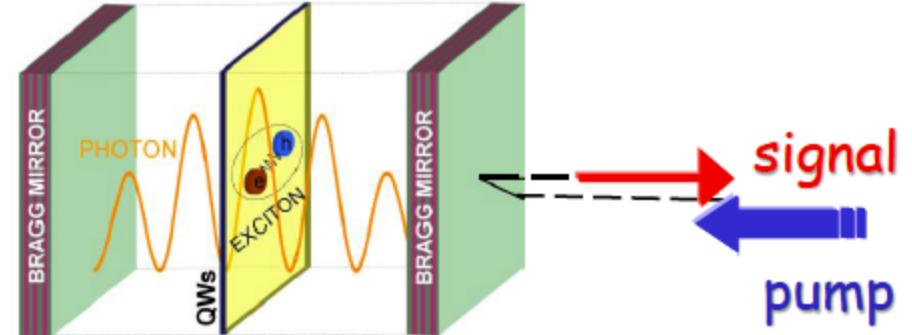
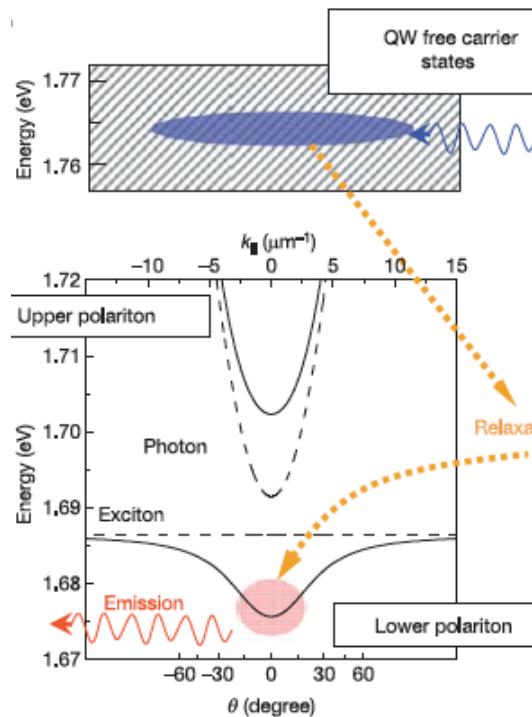
Critical temperature is inversely proportional to the mass of the condensing particles!

# Polariton condensation

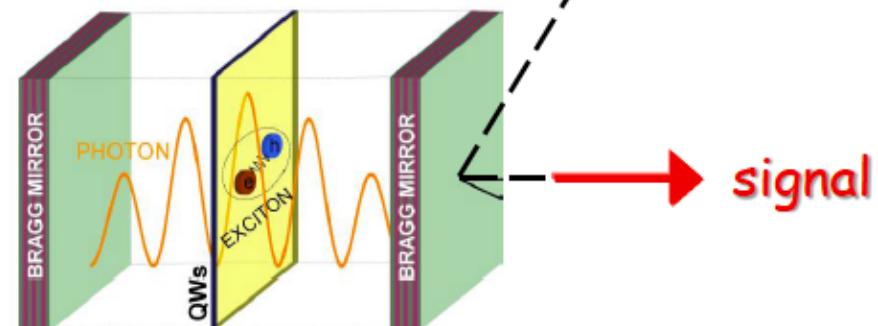
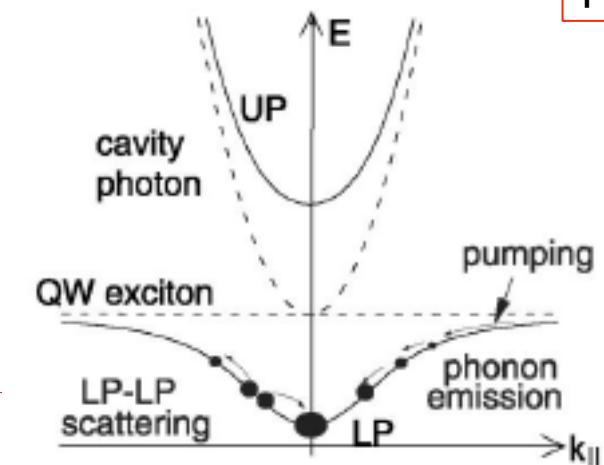


Condensate forms by stimulated scattering of polaritons from an incoherent reservoir – **reservoir plays a strong role in condensate dynamics**

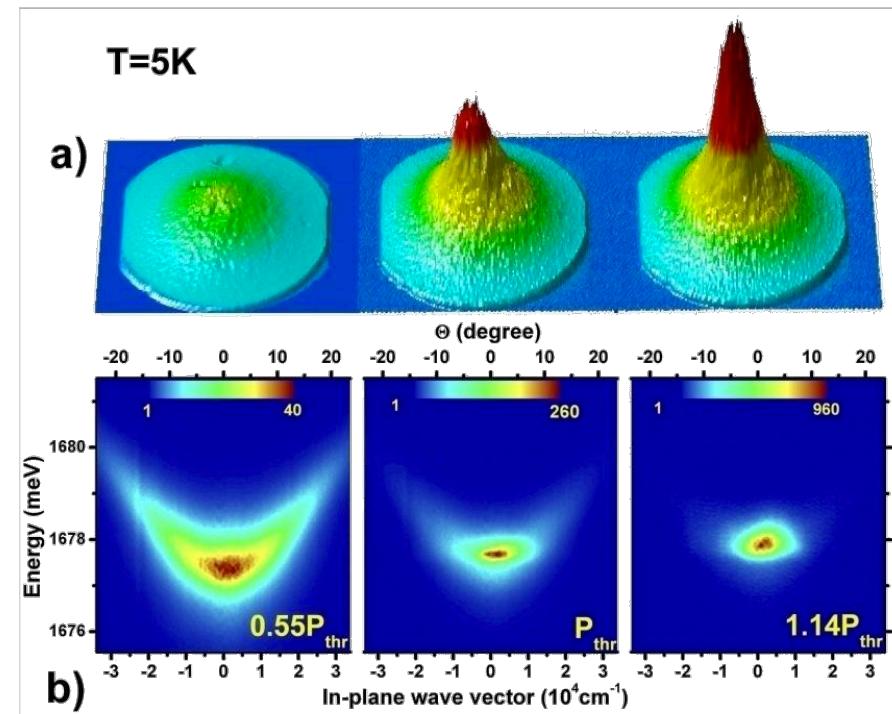
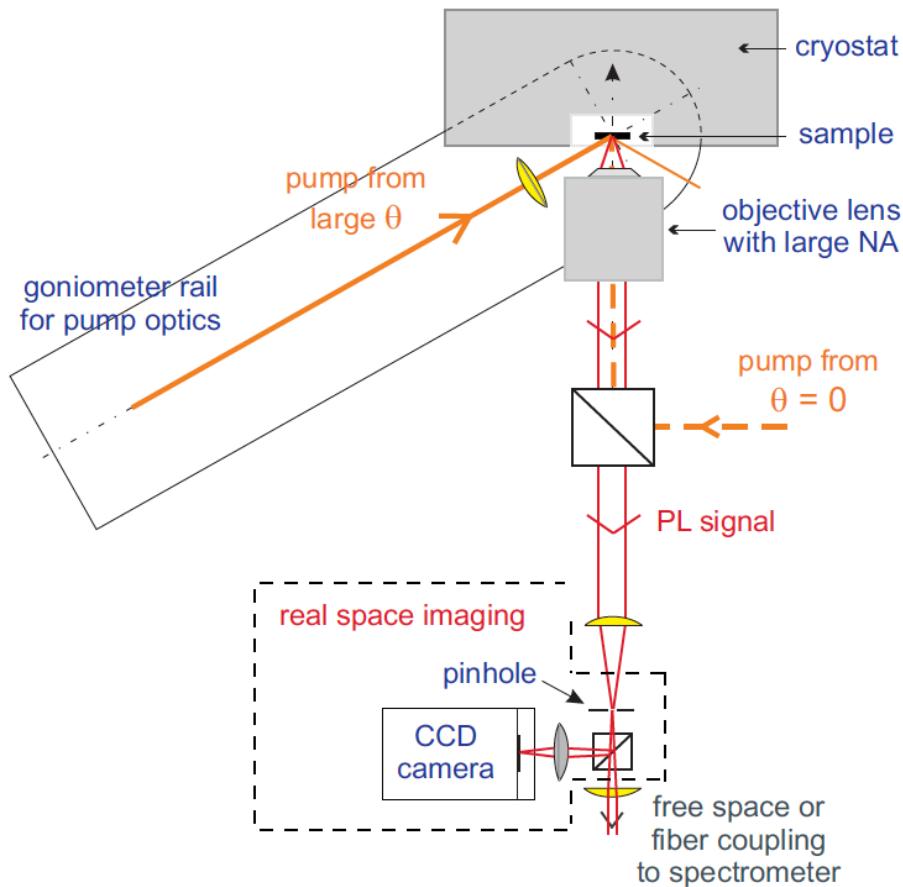
# Incoherent pumping schemes



angle-resolved photoluminescence:  
1:1 mapping between the polariton and the emitted photon  $k_{\parallel}$



# Polariton BEC experiment



Transition to condensation with increasing laser power @  $T=5\text{K}$

From cavity emission we extract: energy, momentum, polarization, coherence, noise properties, spatial distribution of probability density (wavefunction)

Kasprzak et al, Nature, 443, 409 (2006)