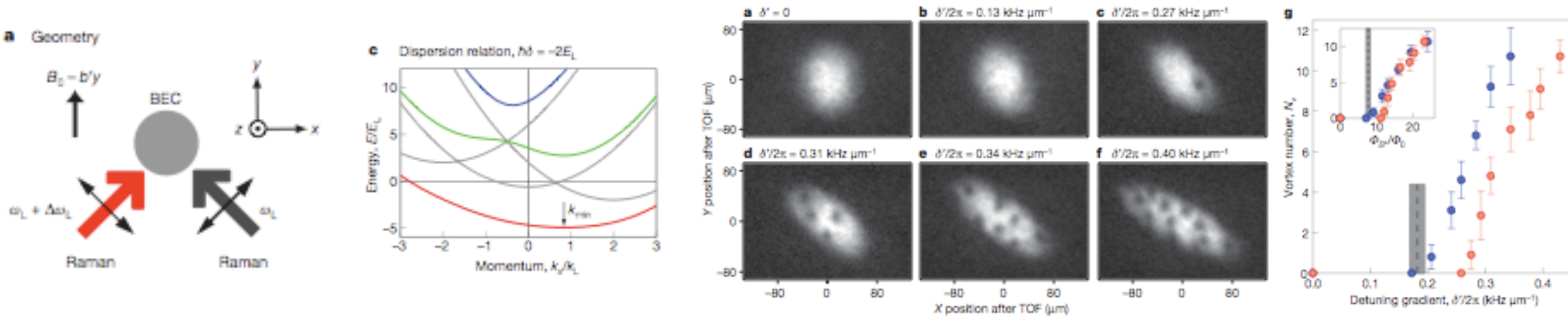
An Alternative to Rotation

Synthetic gauges in BECs



$$i\hbar\frac{\partial\psi}{\partial t} = \left[\frac{(-\hbar\nabla - q^*\mathbf{A}^*)}{2m} + V(\mathbf{r},t) + g|\psi|^2\right]\psi$$

$$\frac{\partial}{\partial t}\left[\begin{array}{c}\delta S\\ \delta n\end{array}\right] = \left[\begin{array}{cc}\nu\cdot\nabla & \frac{g}{\hbar}\\ \nabla\cdot n\left(\mathbf{r}\right)\nabla & \left[\left(\nabla\cdot\nu\right)+\nu\cdot\nabla\right]\end{array}\right]\left[\begin{array}{c}\delta S\\ \delta n\end{array}\right]$$

$$\frac{\Phi_{B_z^*}}{\Phi_0} = 11.1$$

Synthetic magnetic fields for ultracold neutral atoms, Y.-J. Lin *et al.*, Nature **462**, 628 (2009).

Synthetic magneto-hydrodynamics in Bose-Einstein condensates and routes to vortex nucleation, L.B. Taylor *et al.*, Physical Review A **84**, 021604(R) (2011).

LLL Condensate Wavefunction

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$$\psi_{LLL} = \sum_{m\geq 0} c_m \psi_m = f(\zeta) e^{-r^2/(2d_\perp^2)}$$

$$f(\zeta) \propto \prod_j (\zeta - \zeta_j)$$

- *$f(\zeta)$* vanishes at each of the points ζ_j which are the positions of the nodes of the condensate wave-function
- The phase of this wave-function increases by *2π* whenever ζ moves in the positive sense around any of these zeros
- Thus the points ζ_j are precisely the positions of the vortices in the trial state and minimization with respect to the constants c_m is effectively the same as minimization with respect to the position of the vortices

Energy Minimization

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$$E\left[\psi\right] = \int d^2r\psi^*\left(\frac{p^2}{2M} + \frac{1}{2}M\omega_\perp^2 r^2 - \Omega L_z + \frac{1}{2}g_{2D}|\psi|^2\right)\psi$$

$$E\left[\psi_{LLL}\right] = \hbar\Omega + \int d^2r\left[M\omega_\perp^2\left(1-\frac{\Omega}{\omega_\perp}\right)r^2\left|\psi_{LLL}\right|^2 + \frac{1}{2}g_{2D}|\psi_{LLL}|^4\right]$$

Unrestricted minimization

$$|\psi_{min}|^2 = n_{min}(0)\left(1-\frac{r^2}{R_0^2}\right) = \frac{\mu_{min}}{g_{2D}}\left(1-\frac{r^2M\omega_\perp^2(1-\tilde{\Omega})}{\mu_{\min}}\right)$$

$$\downarrow$$

$$\mu_{\min} = \sqrt{\frac{8aN(1-\tilde{\Omega})}{Z}}, \quad \text{where } Z = 2\pi d_z$$

LLL Condition (Unrestricted)

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$$\mu_{\min} \leq 2\hbar\omega_{\perp}$$

$$1 - \tilde{\Omega} \leq \frac{Z}{2Na}$$

Unrestricted minimization!!

What about vortices?

Highly Correlated States (ν)

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Mean field LLL regime:

$$1-\tilde{\Omega} \leq \frac{Z}{2N\beta a}, \quad \text{where } \beta = 1.1596$$

- At higher rotation frequencies the meanfield LLL regime should eventually disappear through a quantum phase transition, leading to a different, highly correlated, manybody ground state.
- For meanfield LLL regime

$$N_v \approx \frac{R_0^2}{d_\perp^2} = \sqrt{\frac{8Na\beta}{Z(1-\tilde{\Omega})}}$$

$$\nu = \frac{N}{N_v} = \sqrt{\frac{Z(1-\tilde{\Omega})N}{8a\beta}}$$

Exact Diagonalization ($v \geq v_c$)

- The equilibrium state in the meanfield LLL regime is a vortex array that breaks the rotational symmetry and is not an eigenstate of L_z
- Could use exact diagonalisation to study the ground state for increasing N_v
- Studies have investigated different filling fractions, v, from 0.5 to 9.
- Comparison between the meanfield LLL energy and exact diagonalization show that the meanfield vortex lattice is a ground state for $v \geq v_c$ ($v_c = 6$)
- Hence the meanfield LLL regime is valid for ($v_c = 1$)

$$1 - \frac{Z}{2N\beta a} \leq \tilde{\Omega} \leq 1 - \frac{8\beta a}{ZN}$$

Exact Diagonalization ($\nu < \nu_c$)

- The groundstates are rotationally symmetric incompressible vortex liquids that are eigenstates of L_z
- They have close similarities to the bosonic analogs of the Jain sequence of fractional quantum Hall states
- The simplest of these many body ground states is the bosonic Laughlin state

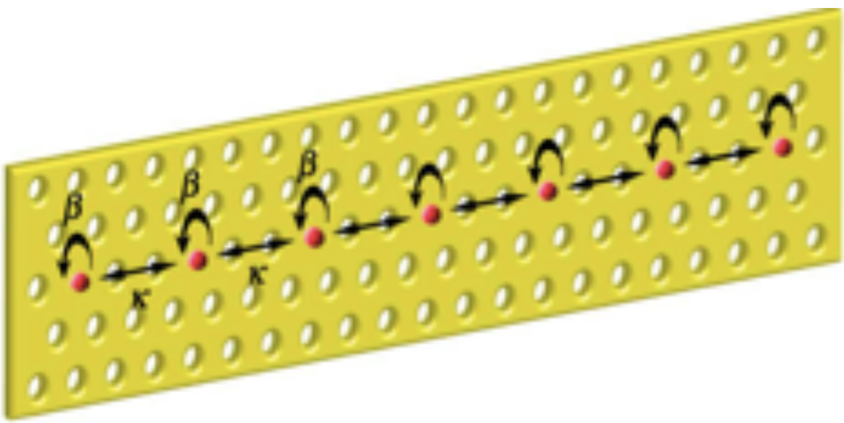
$$\Psi_{Laughlin}\left(\mathbf{r_1}, \mathbf{r_2}, ..., \mathbf{r_N}\right) \propto \prod_{n<n'}^{N} \left(z_n - z_{n'}\right)^2 e^{-\frac{1}{4}\sum_j |z_j|^2}$$

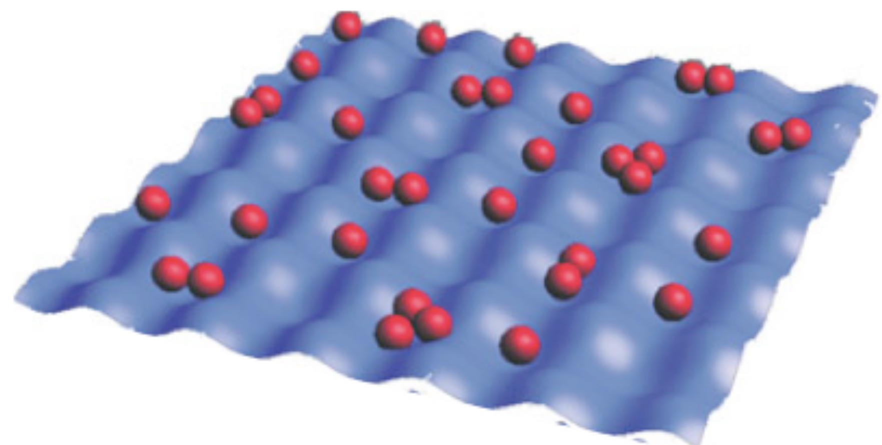
- No off-diagonal long range order and hence no BEC
- The Laughlin state vanishes whenever two particles come together, enforcing the many-body correlations
- The short range two body potential has zero expectation value in this correlated state
- Strong overlap between exact diagonalization and the Laughlin state ($\nu = 1/2$)

Physics of Transition

- Consider N bosonic particles in a plane, with $2N$ degrees of freedom
- Vortices appear as the system rotates and the corresponding vortex coordinates provide N_v collective degrees of freedom
- For slowly rotating systems the $2N$ particle coordinates provide a convenient description
- In principle, the N_v collective vortex degrees of freedom should reduce the original total $2N$ degrees of freedom to $2N - N_v$, but this is unimportant as long as $N_v << N$
- When N_v becomes comparable with N the depletion of the particle degrees of freedom becomes crucial
- This depletion on the particle degrees of freedom drives the phase transition to a wholly new ground state
- Hence when $\nu = N/N_v$ is small a transition is expected

Bose Hubbard Physics





Andy Martin
University of Melbourne

Outline

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- Bose Hubbard model (ultra-cold gases)
    - Simplistic description
    - Meanfield description
    - Perturbation analysis
    - Experiments with BECs
    - Dipolar Bose-Hubbard model: supersolids
- Coupled Atom Cavities
    - A single atom cavity
    - Hamiltonian for coupled atom cavities
    - Relation to the Bose Hubbard model
    - Fractional quantum Hall effect: again*
    - Solid-light
    - Supersolid-light

Reference Material

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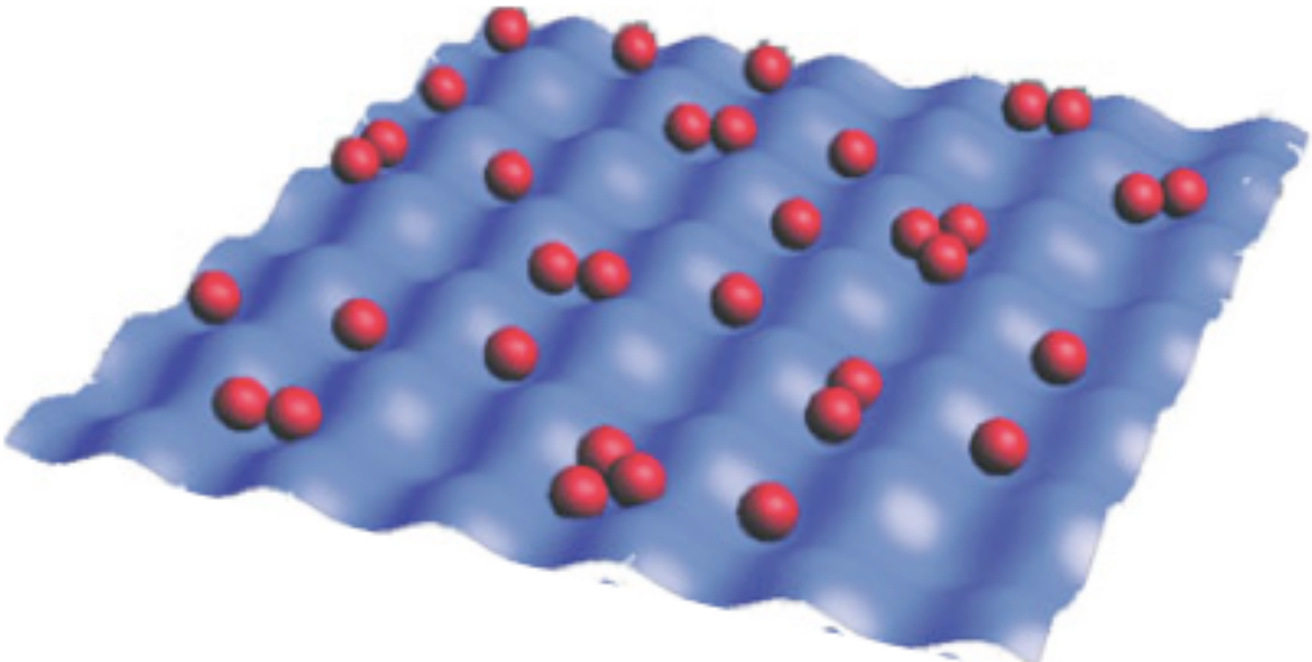
- Bose Hubbard model (ultra-cold gases)
  - *Quantum phases in an optical lattice*, D. van Oosten, P. van der Straten and H.T.C Stoof, Phys. Rev. A **63**, 053601 (2001)
  - M. Greiner *et al.*, Nature **415**, 39 (2002)
- Bose Hubbard model (coupled atom cavities)
  - *Quantum phase transitions of light*, A.D. Greentree *et al.*, Nature Physics **2**, 856 (2006)
  - *Strongly interacting polaritons in coupled arrays of cavities*, M.J. Hartmann *et al.*, Nature Physics **2**, 849 (2006)
- New correlated states in coupled atom cavities
  - *Fractional quantum Hall physics in Jaynes-Cummings-Hubbard lattices*, A.L.C. Hayward, A.M. Martin and A.D. Greentree, Phys. Rev. Lett. **108**, 223602 (2012)*
  - *Supersolid phases of light in extended Jaynes-Cummings-Hubbard systems*, B. Bujnowski *et al.*, arXiv:1310.4548

Bose-Hubbard Model



Hamiltonian

$$H = -J \sum_{i,j} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i \left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

Bose-Hubbard Model

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Assumptions

- The thermal and mean interaction energies at a single site are much smaller than the energy separation to the first excited band
- The Wannier functions decay essentially within a single lattice constant
- Under these assumptions:
    - Only the lowest energy band needs to be included in our description
    - The hoping matrix elements are only significant for nearest neighbours
    - The interactions are dominated by the on-site contribution only

Hamiltonian

$$H = -J\sum_{i,j} a_i^{\dagger} a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

Superfluid state ($U = 0$)

$$H = -J \sum_{i,j} a_i^\dagger a_j + \frac{U}{2} \sum_i n_i \left(n_i - 1 \right) + \sum_i \epsilon_i n_i$$

- The manybody ground state is simply an ideal BEC where all the atoms are in the $q = 0$ Bloch state of the lowest band

$$|\Psi_N^{U=0}\rangle = \frac{1}{\sqrt{N!}} \left(\frac{1}{\sqrt{N_L}} \sum_i a_i^\dagger \right)^N |0\rangle$$

- Hence the groundstate is a Gross-Pitaevskii type state with a condensate fraction equal to one
- However the critical temperature is significantly reduced (effective mass) as compared to the *free* case

Mott Insulator Phase ($U >> J$)

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$$H = -J\sum_{i,j} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

- Assume (for the moment) the number of atoms is equal to the number of lattice points

$$|\Psi_{N=N_L}^{J=0}\rangle = \left(\prod_i a_i^\dagger\right)|0\rangle$$

- With increasing J the atoms start to hop around, which involves double occupancy, increasing the energy by U. However, the ground state is no longer a simple product state
- Once J becomes of order or larger than U the gain in kinetic energy outweighs the repulsion due to double occupancy
- The atoms then undergo a transition, in the thermodynamic limit, to a superfluid state.

BHM (Phase Diagram)

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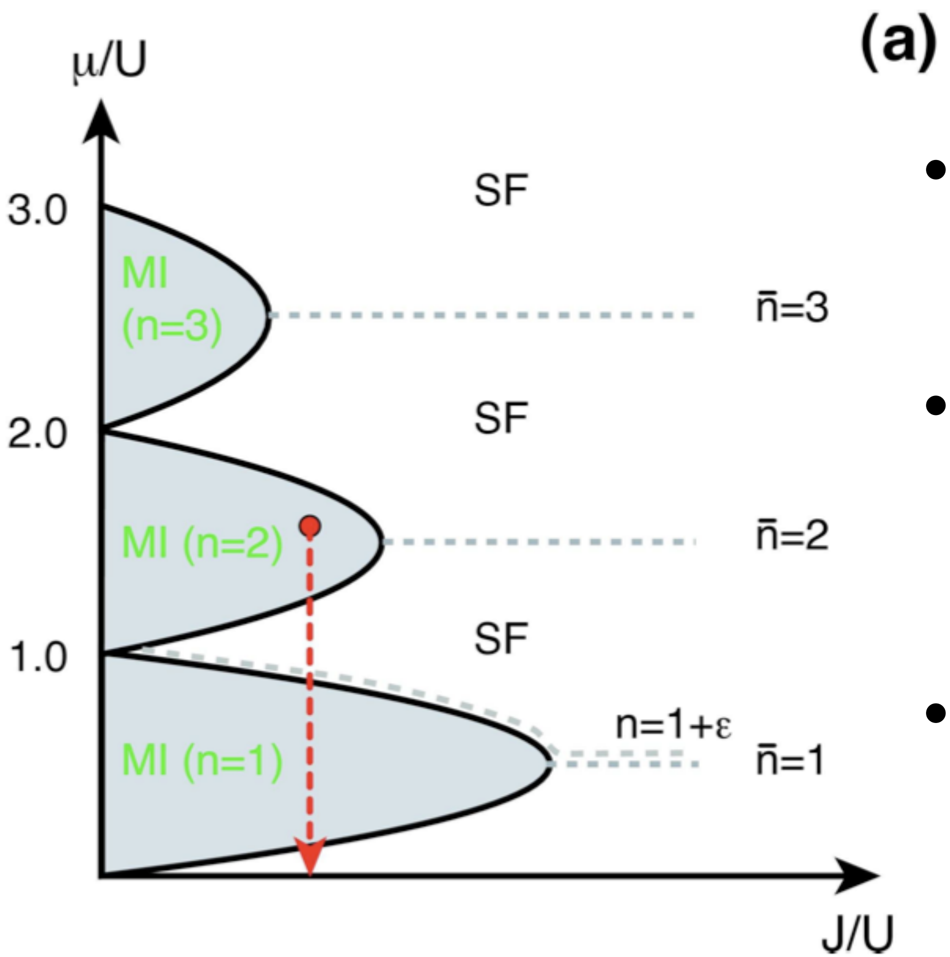
$$H = -J\sum_{i,j} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$



- *U/J* → *0*: the KE dominate and the ground state is a delocalized superfluid
- *U/J* is large: interactions dominate and one obtains a series of MI phases with fixed integer filling ($\partial n/\partial\mu = 0$)
- The transition between the SF and MI phases is associated with a loss of long-range order

BHM (Meanfield)

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$$H = -J\sum_{i,j} a_i^\dagger a_j + \frac{U}{2}\sum_i n_i\left(n_i - 1\right) + \sum_i \epsilon_i n_i$$

Meanfield substitution

$$a_i^\dagger a_j = \langle a_i^\dagger \rangle a_j + a_i^\dagger \langle a_j \rangle - \langle a_i^\dagger \rangle \langle a_j \rangle = \psi(a_i^\dagger + a_j) - \psi^2$$

$$\downarrow$$

$$\begin{aligned} H_{MF} &= -zJ\psi\left(a^\dagger + a\right) + \frac{U}{2}n\left(n-1\right) - \mu n + zJ\psi^2 \\ &= -\psi\left(a^\dagger + a\right) + \frac{\overline{U}}{2}n\left(n-1\right) - \overline{\mu} n + \psi^2 = H_0 + \psi V \end{aligned}$$

BHM (Meanfield Perturbation)

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$$H_{MF} = H_0 + \psi V$$

Expansion in ψ (odd powers zero)

- Denote the unperturbed energy of the state with *n* particles by $E_n^{(0)}$

$$E_g^{(0)} = \left\{ E_n^{(0)} | n = 0, 1, 2, 3 ...\rangle \right\}_{min}$$
$$E_g^{(0)} = 0, \text{ if } \overline{\mu} = 0$$
$$E_g^{(0)} = \frac{1}{2}\overline{U}g(g-1) - \overline{\mu}g, \text{ if } \overline{U}(g-1) < \overline{\mu} < \overline{U}g$$

- Second order correction

$$E_g^{(2)} = \psi^2 \sum_{n \neq g} \frac{|\langle g|V|n\rangle|^2}{E_g^{(0)} - E_n^{(0)}} = \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U}g} \right) \psi^2$$

BHM (Meanfield Critical Points)

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$$E_g = E_g^{(0)} + E_g^{(2)} + O(\psi^4)$$

Minimize energy as a function of superfluid parameter

$$\psi = 0, \quad \text{when} \quad \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U} g} \right) > 0$$

$$\psi \neq 0, \quad \text{when} \quad \left(\frac{g}{\overline{U}(g-1) - \overline{\mu}} + \frac{g+1}{\overline{\mu} - \overline{U} g} \right) < 0$$

$$\downarrow$$

$$\overline{\mu}_{\pm} = \frac{1}{2}\left[\overline{U}(2g-1) - 1\right] \pm \frac{1}{2}\sqrt{\overline{U}^2 - 2\overline{U}(2g+1) + 1}$$

$$\overline{U}_c = 2g + 1 + \sqrt{(2g+1)^2 - 1}$$

BHM (Phase Diagram Revisited)

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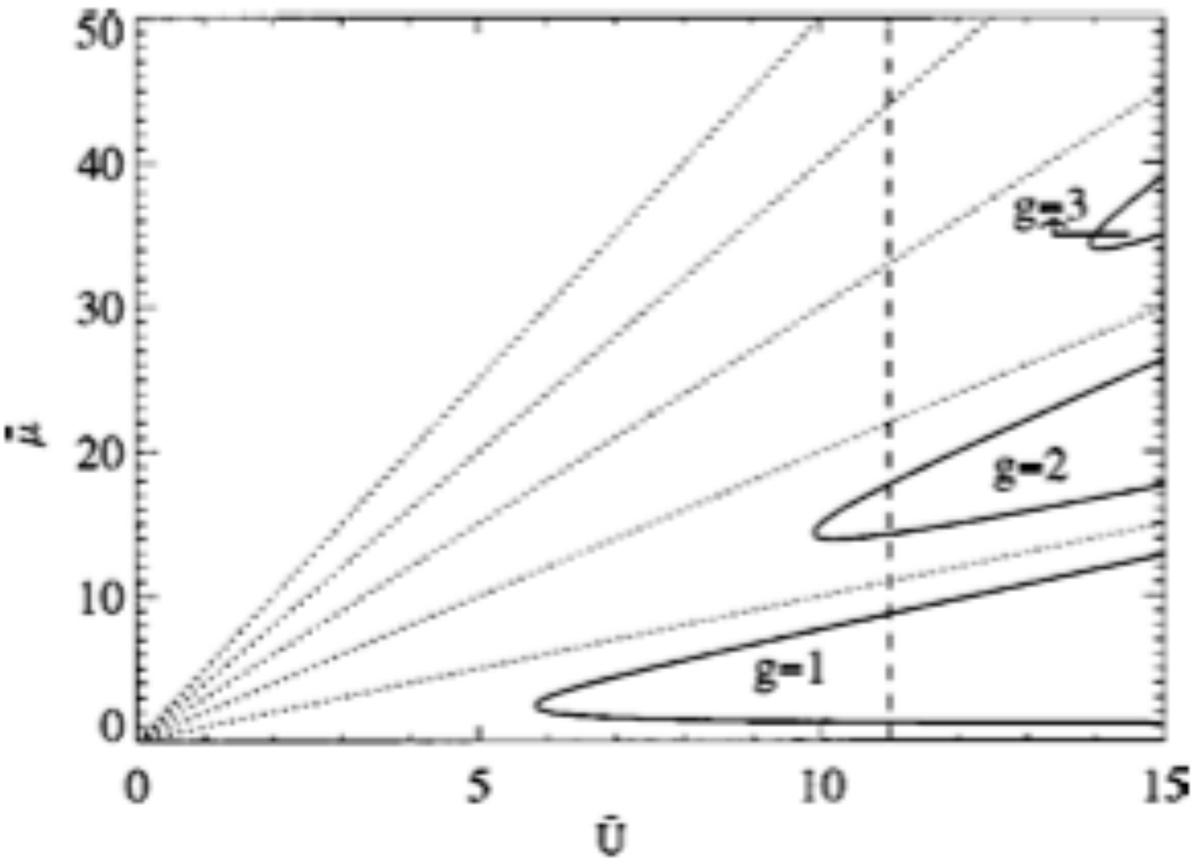
$$\overline{\mu}_{\pm} = \frac{1}{2}\left[\overline{U}(2g-1)-1\right] \pm \frac{1}{2}\sqrt{\overline{U}^2 - 2\overline{U}(2g+1)+1}$$

$$\overline{U}_c = 2g+1+\sqrt{(2g+1)^2-1}$$



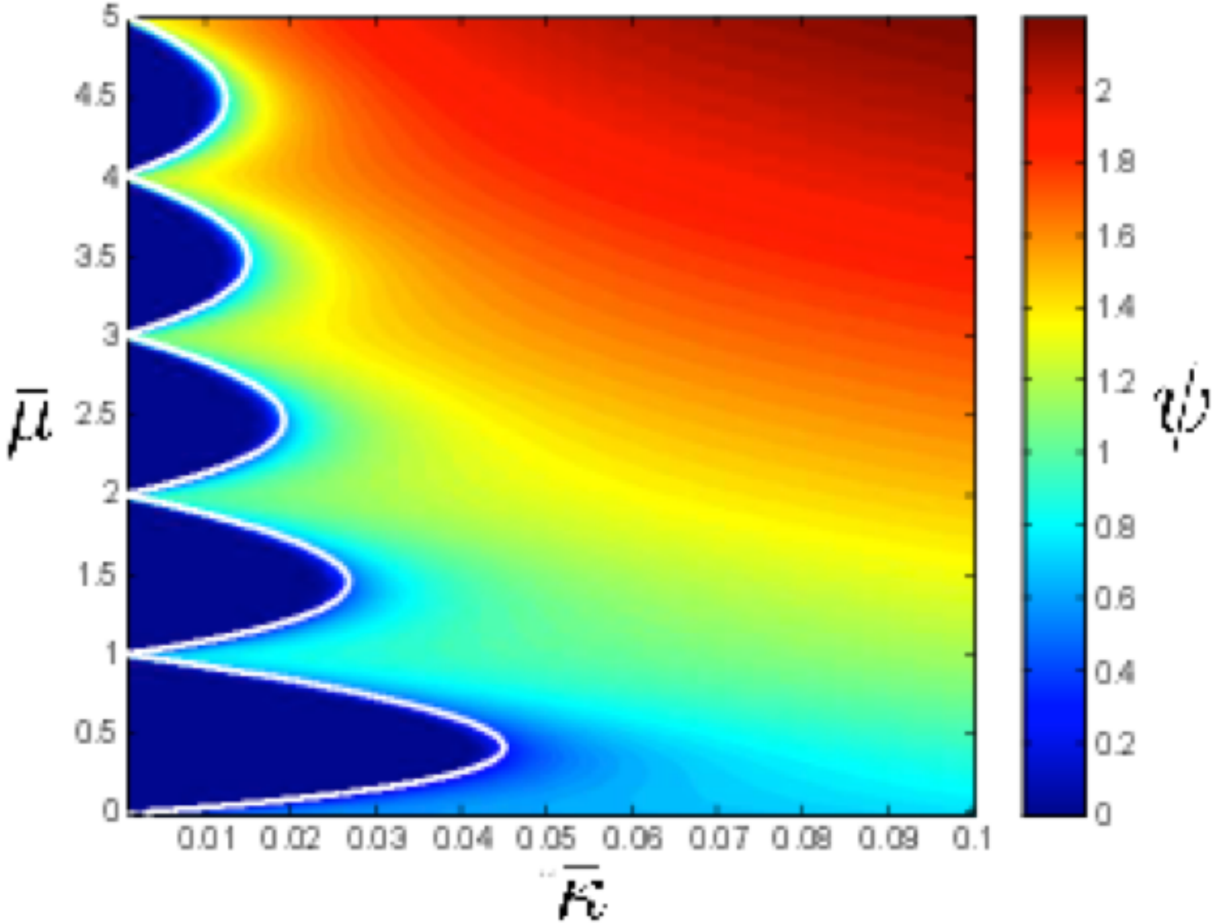
Meanfield

$$\overline{U}_c = 5.83, \quad \text{for } g = 1$$

QMC (*z=4*)

$$\overline{U}_c = 7.34, \quad \text{for } g = 1$$

BHM (Numerics)



BHM (Experiments: BECs)

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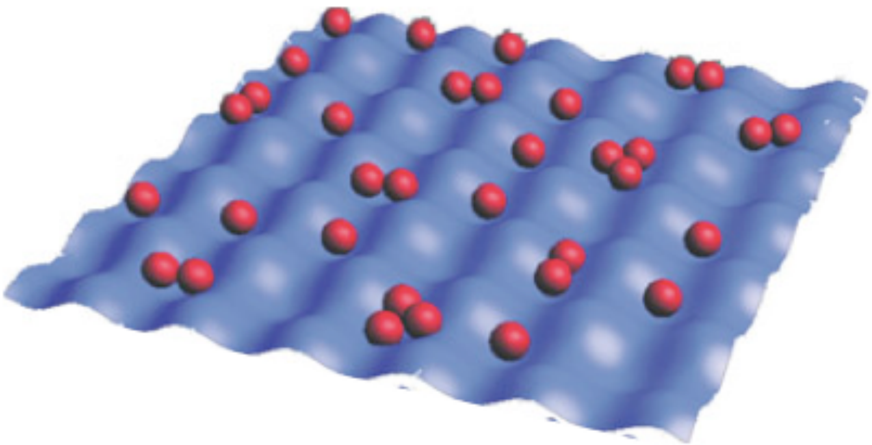


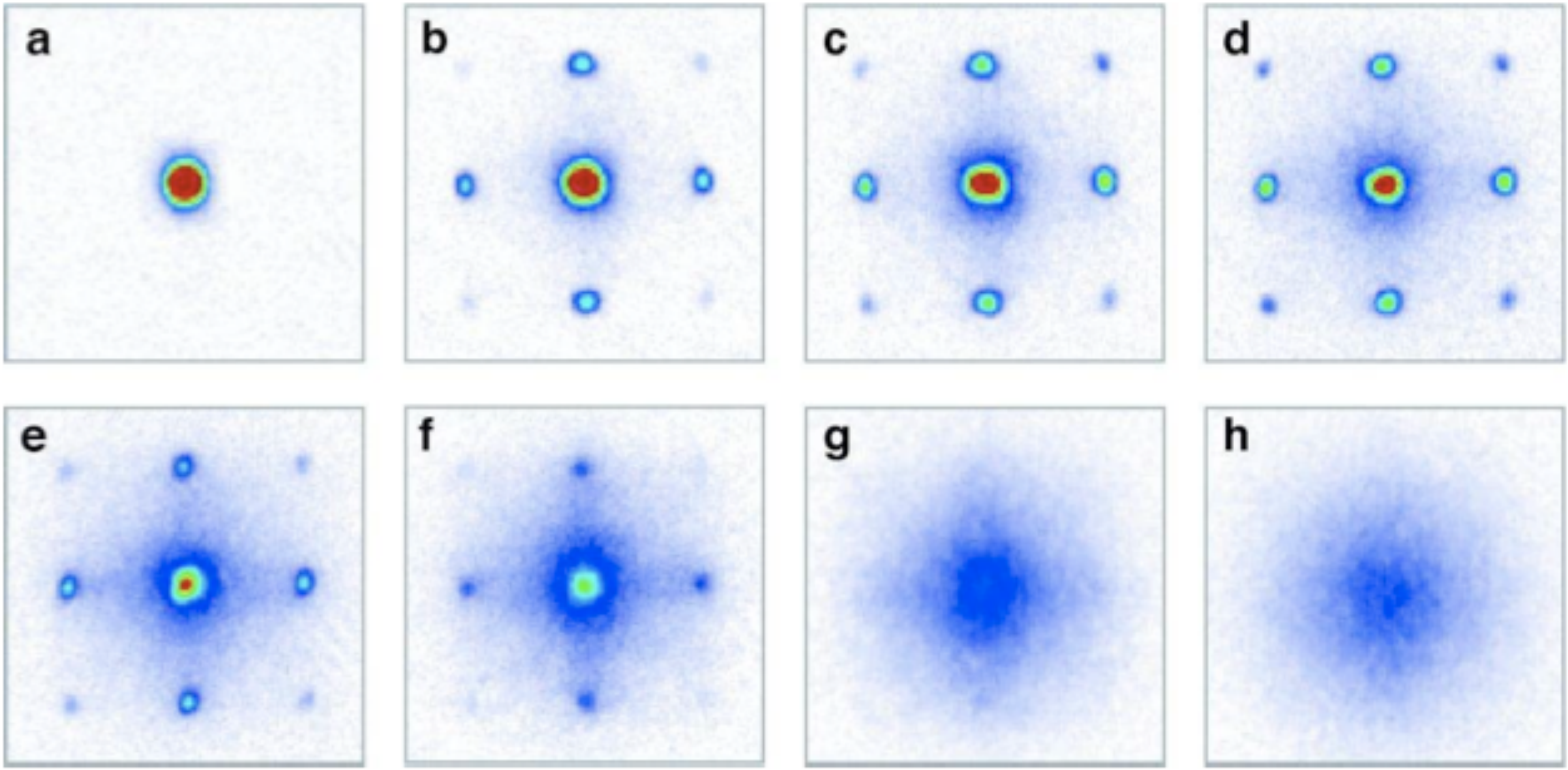
Translation to experimental parameters

$$(V_0/E_r)_c = \frac{1}{4}\ln^2\left(\frac{\sqrt{2}d}{\pi a}(U/J)_c\right),$$

where E_r is the recoil energy $E_r = h^2/(2m\lambda^2)$

BHM (Experimental Results)

Changing V



M. Greiner *et al.*, Nature **415**, 39 (2002)

Momentum distribution

$$n(\mathbf{k}) \sim |\tilde{w}(\mathbf{k})|^2 \sum_{R} e^{i\mathbf{k}\cdot\mathbf{R}} G^{(1)}(\mathbf{R})$$

- MI phase: the one particle density matrix decays to zero exponentially
- SF phase: is characterized by by a momentum distribution which exhibits sharp peaks at the reciprocal lattice vectors $\boldsymbol{k} = \boldsymbol{G}$ ($\boldsymbol{G}.\boldsymbol{R} = 2\pi n$)
- The peaks in the momentum distribution initially grow because of the decrease in the spatial extent of the Wannier function $w(r)$, which results in an increase in its Fourier transform at higher momentum
- In the MI regime the remnants of the interference peaks remain as long as $G^{(1)}(\boldsymbol{R})$ extends over several lattice spacings

Supersolids: History

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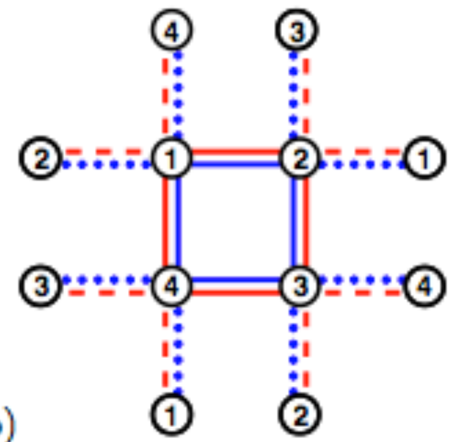
- Systems in a supersolid phase possess a spontaneously formed crystalline structure along with off diagonal long-range order which characterizes superfluidity.
- The investigation of supersolid phases in condensed matter systems has been a focus of research for more than half a century [Phys. Rev. **106**, 161 (1957), Sov. Phys. JETP **29**, 1107 (1969), Ann. Phys. **52**, 403 (1969), Phys. Rev. Lett. **25**, 1543 (1970)]
- . Until recently this effort has primarily focused on possible realization of a supersolid phase in ^{4}He [J. Low. Temp. Phys. **168**, 221 (2012)], with the most credible claim for observation [Nature **427**, 225 (2004), Science **305**, 1941 (2004)] now being withdrawn [Phys. Rev. Lett. **109**, 155301 (2012)]

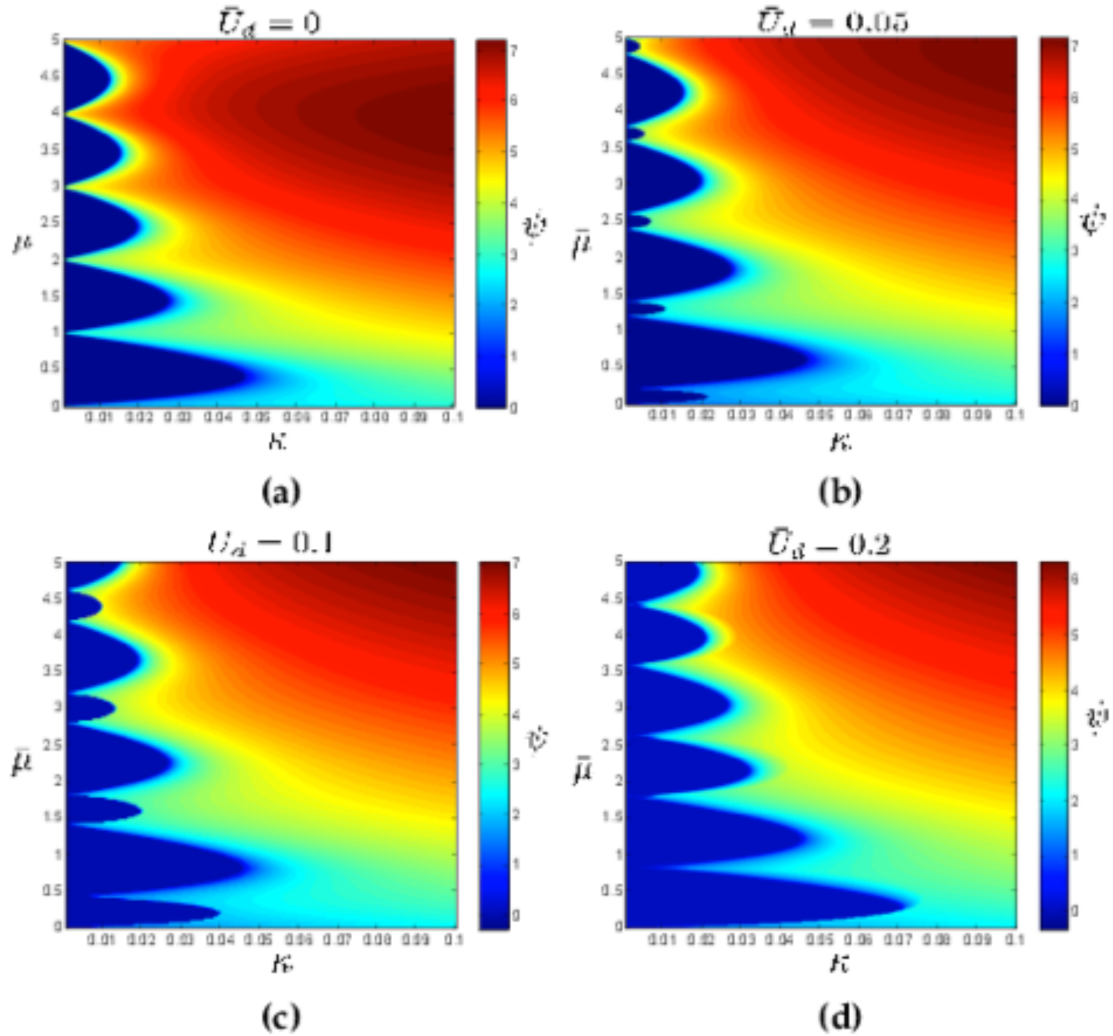
  In 2004, Kim and Chan carried out torsional oscillator measurements of solid helium confined in porous Vycor glass and found an abrupt drop in the resonant period below 200 mK. The period drop was interpreted as probable experimental evidence of nonclassical rotational inertia. This experiment sparked considerable activities in the studies of superfluidity in solid helium. More recent ultrasound and torsional oscillator studies, however, found evidence that shear modulus stiffening is responsible for at least a fraction of the period drop found in bulk solid helium samples. The experimental configuration of Kim and Chan makes it unavoidable to have a small amount of bulk solid inside the torsion cell containing the Vycor disk. We report here the results of a new helium in Vycor experiment with a design that is completely free from any bulk solid shear modulus stiffening effect. We found no measurable period drop that can be attributed to nonclassical rotational inertia.

Charge density wave

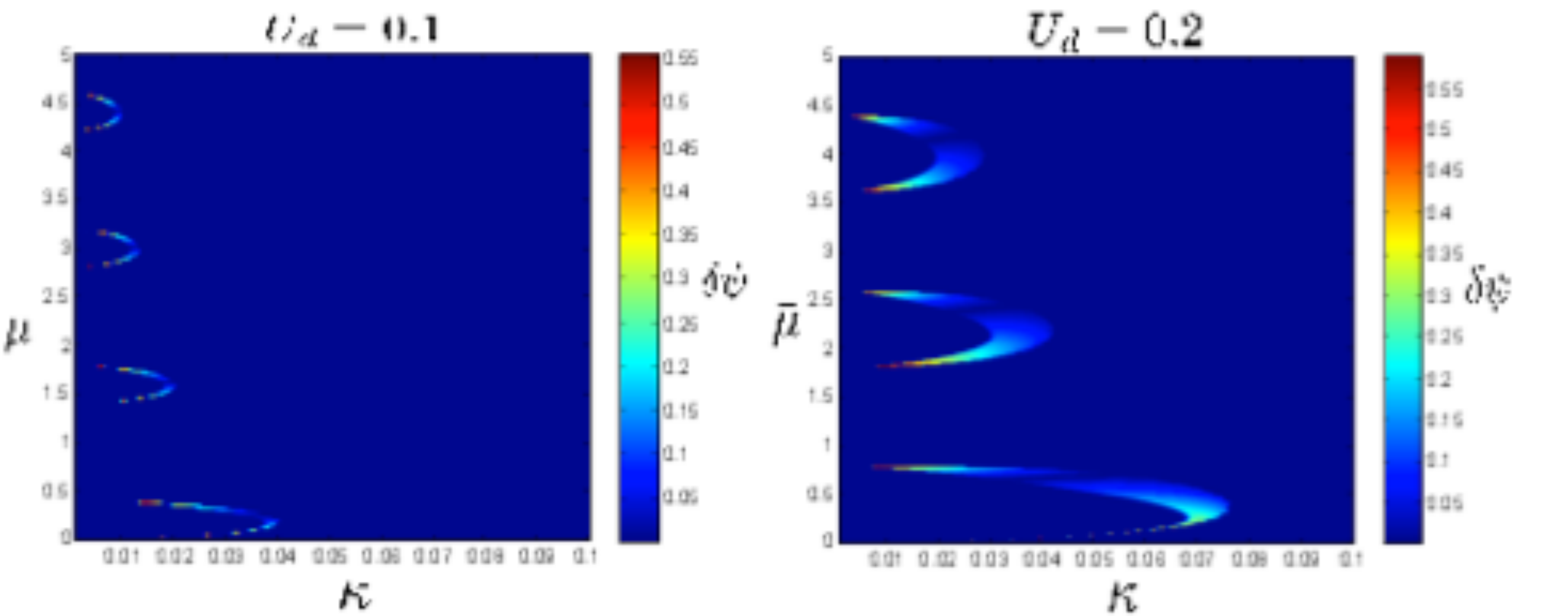
Extended Bose-Hubbard model: nearest neighbour



b)

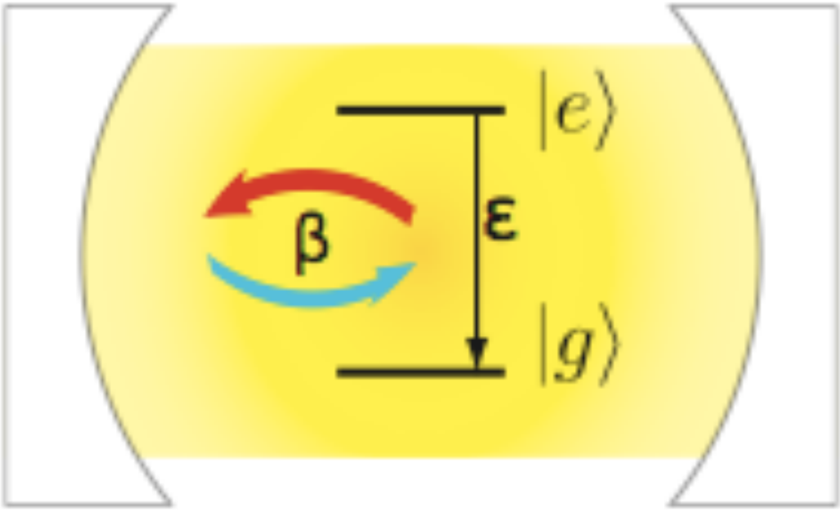


Extended Bose-Hubbard model: nearest neighbour



Single Atom Cavity

Jaynes-Cummings Model



- A two level atom interacts with a quantized cavity field
- The JC Hamiltonian consists of a single-moded quantized electromagnetic field, atomic excitation, and atom-field interaction terms

$$H^{\mathrm{JC}} = H_{\mathrm{field}} + H_{\mathrm{atom}} + H_{\mathrm{int}}$$

Jaynes-Cummings Model

$$H^{\mathrm{JC}} = H_{\mathrm{field}} + H_{\mathrm{atom}} + H_{\mathrm{int}}$$

- The Hamiltonian of the quantized free electromagnetic field for a single mode of frequency is

$$H_{\mathrm{field}} = \omega\left(a^{\dagger}a + \frac{1}{2}\right)$$

- The Hamiltonian of the atomic excitation is

$$H_{\mathrm{atom}} = \epsilon\sigma_{+}\sigma_{-}$$

- The Hamiltonian for the atom-photon interaction is derived from a classical description of a two-level transition of the electric dipole interaction (dipole approximation) and the rotating wave approximation:

$$H_{\mathrm{int}} = \beta\left(\sigma_{+}a + \sigma_{-}a^{\dagger}\right)$$

Single Atom Cavity

Jaynes-Cummings Model

$$H^{\mathrm{JC}} = \epsilon\sigma_+\sigma_- + \omega a^\dagger a + \beta\left(\sigma_+ a + \sigma_- a^\dagger\right)$$

$$\mathcal{H}^{\mathrm{JC}} = \begin{pmatrix} \boxed{0} & & & & & \\ & \epsilon & \beta & & & \\ & \beta & \omega & & & \\ & & & \epsilon+\omega & \sqrt{2}\beta & \\ & & & \sqrt{2}\beta & 2\omega & \\ & & & & & \ddots \end{pmatrix}, \quad \begin{pmatrix} |g,0\rangle \\ |e,0\rangle \\ |g,1\rangle \\ |e,1\rangle \\ |g,2\rangle \\ \vdots \end{pmatrix}$$

Single Atom Cavity

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Jaynes-Cummings Model

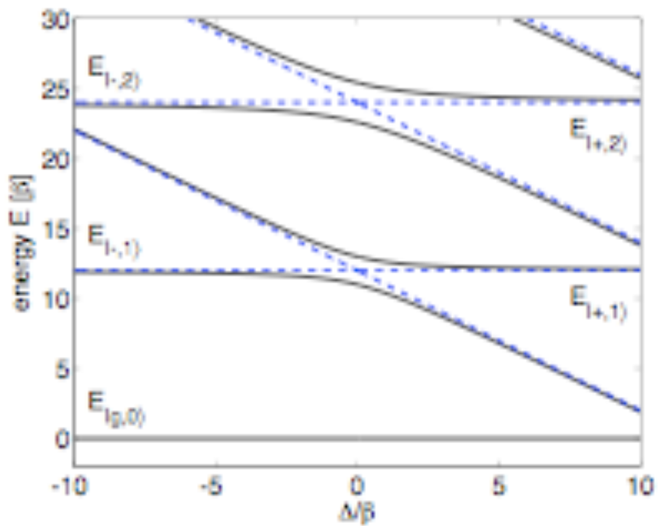
$$H^{\mathrm{JC}} = \epsilon\sigma_+\sigma_- + \omega a^\dagger a + \beta\left(\sigma_+ a + \sigma_- a^\dagger\right) \quad E_{|g,0\rangle} = 0 \qquad \Delta = \omega - \epsilon$$

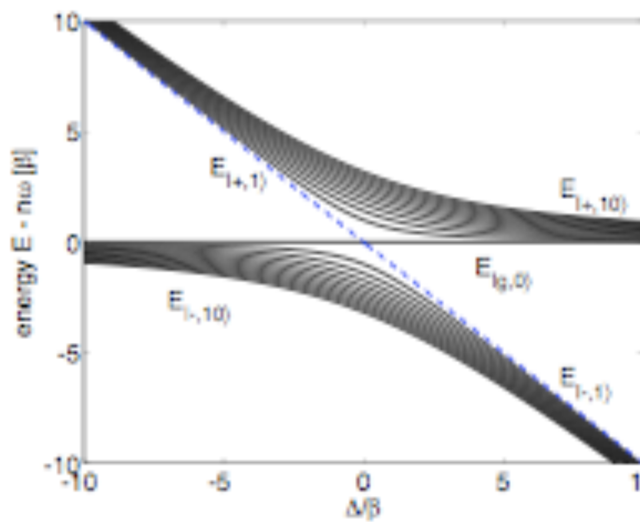
Bare basis	Dressed basis
$\lvert g,0\rangle$	$\lvert g,0\rangle$
$\lvert e,0\rangle$	$\lvert -,1\rangle$
$\lvert g,1\rangle$	$\lvert +,1\rangle$
$\lvert e,1\rangle$	$\lvert -,2\rangle$
$\lvert g,2\rangle$	$\lvert +,2\rangle$
$\vdots$	$\vdots$

$$E_{|\pm,n\rangle} = n\omega \pm \sqrt{n\beta^2 + \Delta^2/4} - \frac{\Delta}{2}$$

$$|\pm, n\rangle = \frac{\beta\sqrt{n}|g,n\rangle + [-\frac{\Delta}{2} \pm \chi(n)]|e,n-1\rangle}{\sqrt{2\chi^2(n) \mp \chi(n)\Delta}},$$

$$\chi(n) = \sqrt{n\beta^2 + \frac{\Delta^2}{4}}.$$





Single Atom Cavity

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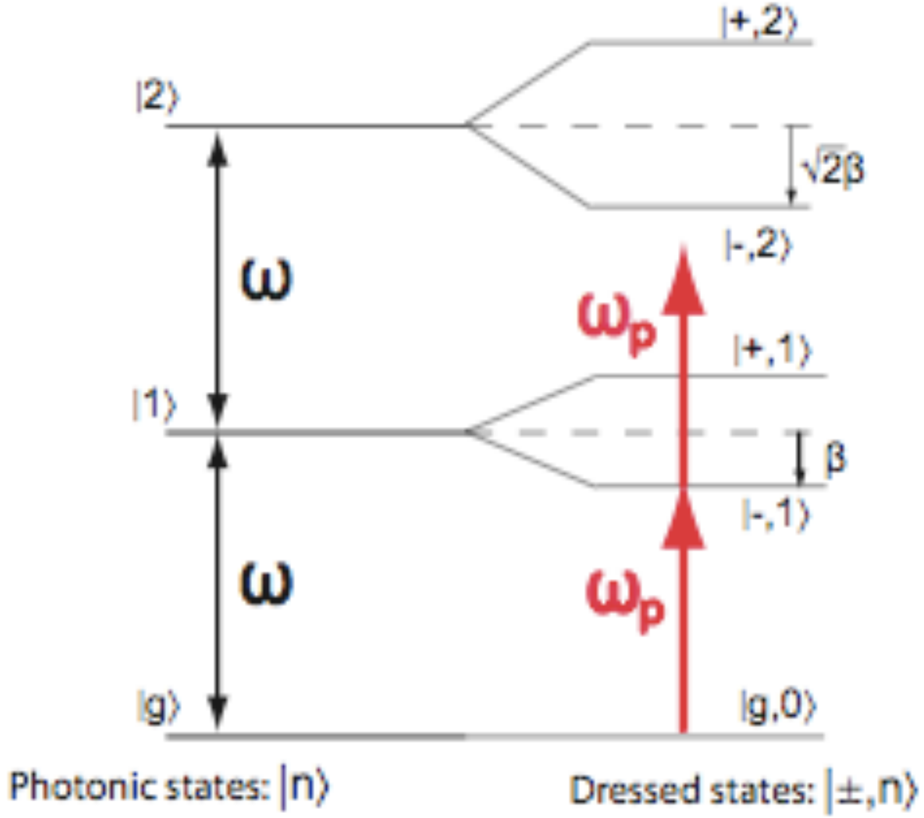
Jaynes-Cummings Model: Photon Blockade



Single Atom Cavity

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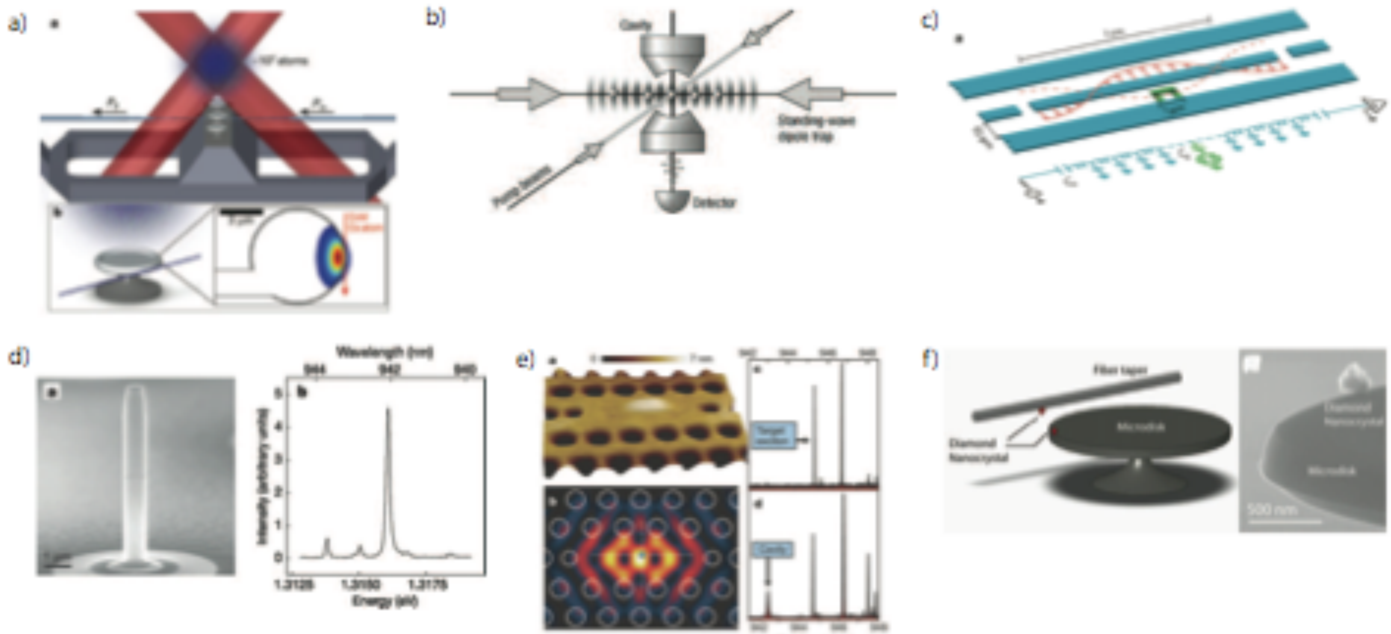
Experimental Possibilities (I)



Depictions of micro-cavities which have been coupled to high-dipole moment resonant transition systems: (a) cesium atoms in microtoroid cavity [1], (b) rubidium atom in Fabry-Perot resonators [2], (c) microstrip cavity with charge qubit [3] (d) quantum dots in Fabry-Perot resonator [4], (e) quantum dots in photonic bandgap cavity, and (f) diamond nitogen-vacancy centre in whispering gallery mode microdisk [5].

Coupled Atom Cavities

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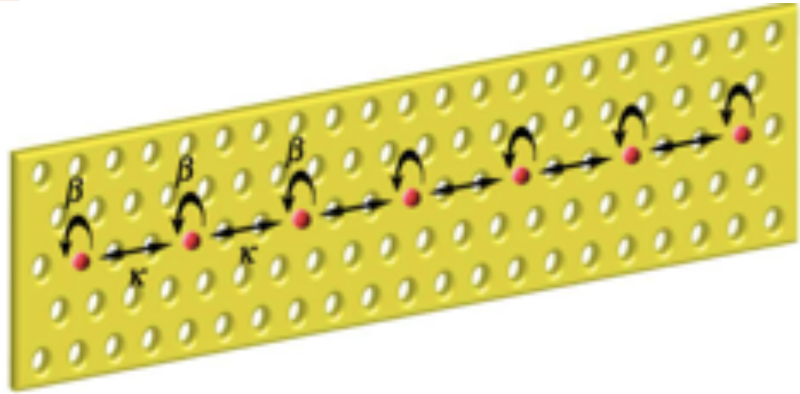
Experimental Possibilities (II)



- A possible realisation of a 1D solid-light systems. Here holes are drilled into a thin membrane and lattice defects serve as the optical cavities housing two-level atoms.

[1] **Observation of strong coupling between one atom and a monolithic microresonator**,
T. Aoki *et al.*, Nature **443**, 671 (2006)

[2] **Vacuum-stimulated cooling of single atoms in three dimensions**,
S. Nuβmann *et al.,* Nature Physics **1**, 122 (2005)

[3] **Superconducting quantum bits**,
J. Clarke and F. Wilhelm, Nature **453**, 1031 (2008)

[4] **Strong Coupling in a single quantum dot-semiconductor microcavity system**,
J.P. Reithmaier *et al.*, Nature **432**, 197 (2004)

[5] **Coherent interference effects in a nano-assembled diamond NV center cavity-QED system**,
P. Barclay *et al.*, Optics Express **17**, 8081 (2009)

Coupled Atom Cavities

JCH Hamiltonian

$$H^{\mathrm{JCH}} = \sum_i H_i^{\mathrm{JC}} - \kappa \sum_{\langle i,j \rangle} a_i^\dagger a_j - \sum_i \mu_i \left(\sigma_+^i \sigma_-^i + a_i^\dagger a_i \right)$$

Meanfield Hamiltonian

$$a_i^\dagger a_j = \langle a_i^\dagger \rangle a_j + a_i^\dagger \langle a_j \rangle - \langle a_i^\dagger \rangle \langle a_j \rangle = \psi (a_i^\dagger + a_j) - \psi^2$$

$$\downarrow$$

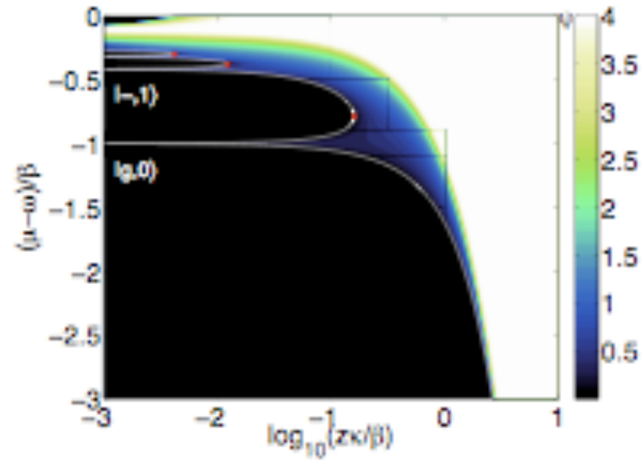
$$H^{\mathrm{JCH}} = H^{\mathrm{JC}} - z\kappa\psi(a^\dagger + a) + z\kappa\psi^2 - \mu \left(\sigma_+ \sigma_- + a^\dagger a \right)$$

Finding the Groundstate

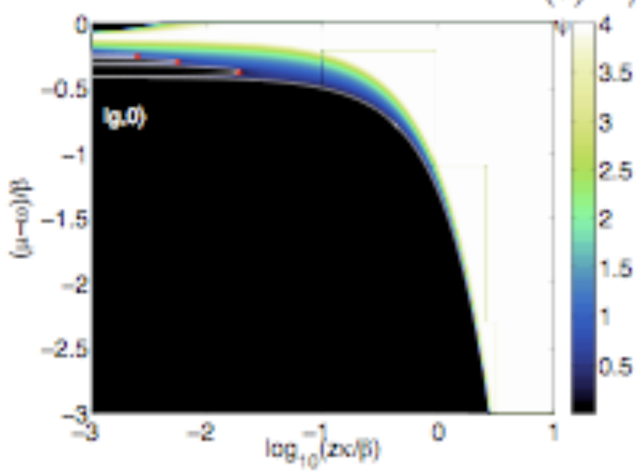
$$
\mathcal{H}^{\mathrm{MF}} = z\kappa|\psi|^2 I +
\begin{pmatrix}
\boxed{0} & & -z\kappa\psi & & & & & \\
 & \epsilon-\mu & \beta & -z\kappa\psi & & & & \\
-z\kappa\psi & \beta & \omega-\mu & & -\sqrt{2}z\kappa\psi & & & \\
 & -z\kappa\psi & & \epsilon+\omega-2\mu & \sqrt{2}\beta & -\sqrt{2}z\kappa\psi & & \\
 & & -\sqrt{2}z\kappa\psi & \sqrt{2}\beta & 2(\omega-\mu) & & -\sqrt{3}z\kappa\psi & \\
 & & & -\sqrt{2}z\kappa\psi & & \epsilon+2\omega-3\mu & \sqrt{3}\beta & \ddots \\
 & & & & -\sqrt{3}z\kappa\psi & \sqrt{3}\beta & 3(\omega-\mu) & \\
 & & & & & \ddots & & \ddots
\end{pmatrix}
$$

Coupled Atom Cavities

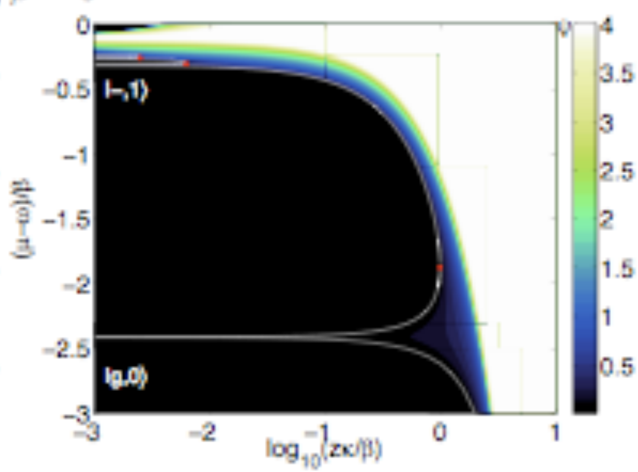
Meanfield Solution



(a) $\Delta/\beta = 0$



(b) $\Delta/\beta = -2$



(c) $\Delta/\beta = 2$

Long-range interactions in JCH

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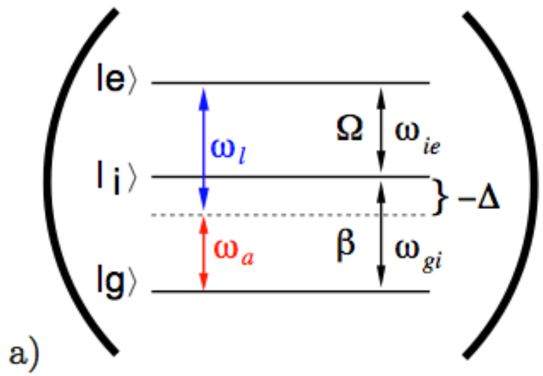

- To achieve long-range interactions in the coupled atom-cavity system we require the excited state of the atom to have a significant dipole moment.
- A possible realization of an atomic cavity that exhibits a dipole moment when excited, utilizes the Rydberg state of Rb atoms. The $5S_{1/2}$ ground state $|g\rangle$ of the Rb atom, that has been placed inside the cavity, is resonantly coupled to the Rydberg state $|e\rangle$ via a two photon process, by using the $5P_{3/2}$ state $|i\rangle$ as an intermediate step. By choosing appropriate detunings for the driving fields the intermediate state can be eliminated adiabatically as there are only small changes in its population over time.

Long-range interactions in JCH

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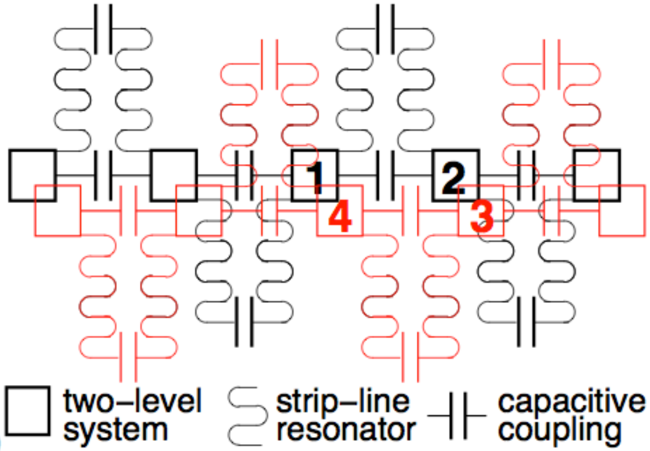


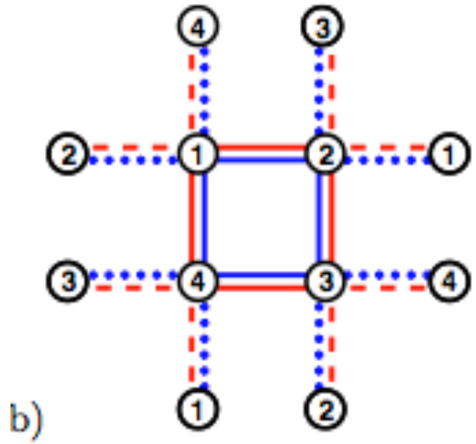
c)

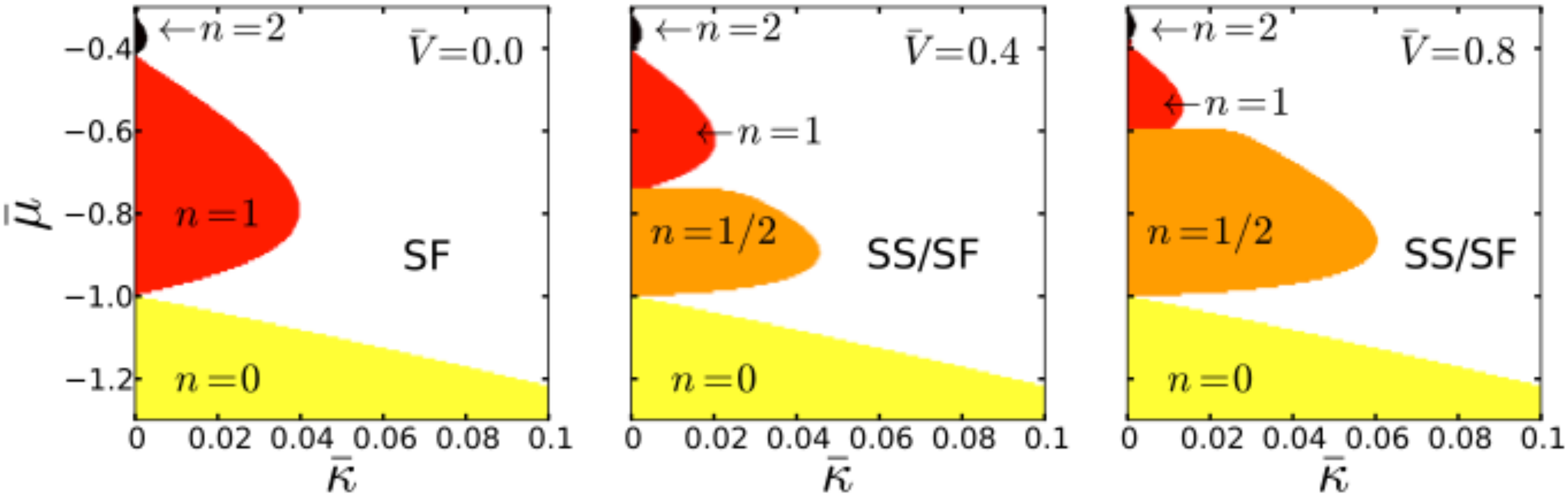
- For the cQED, arrays of coupled cavities can be fabricated with either capacitive or inductive coupling linking the resonators. This architecture provides an entirely equivalent realization of the JCH model.
- The figure provides a schematic for two layers of a possible multiple layer circuit. Each layer in the circuit consists of a one dimensional array of Josephson junction based two-level systems coupled via strip-line resonators and capacitors. The photonic components of the JCH model are now microwave excitations in the strip-line resonators and the long-range interactions (in this case nearest-neighbour) arise from capacitive coupling between adjacent Josephson junction two-level systems.
- For a multi-layered system capacitive coupling between strip-line resonators in adjacent layers enables microwave excitations to couple between layers. Additionally, capacitive coupling between Josephson junctions in adjacent layers mediates a long-range interaction between two-level systems.

The Hamiltonian

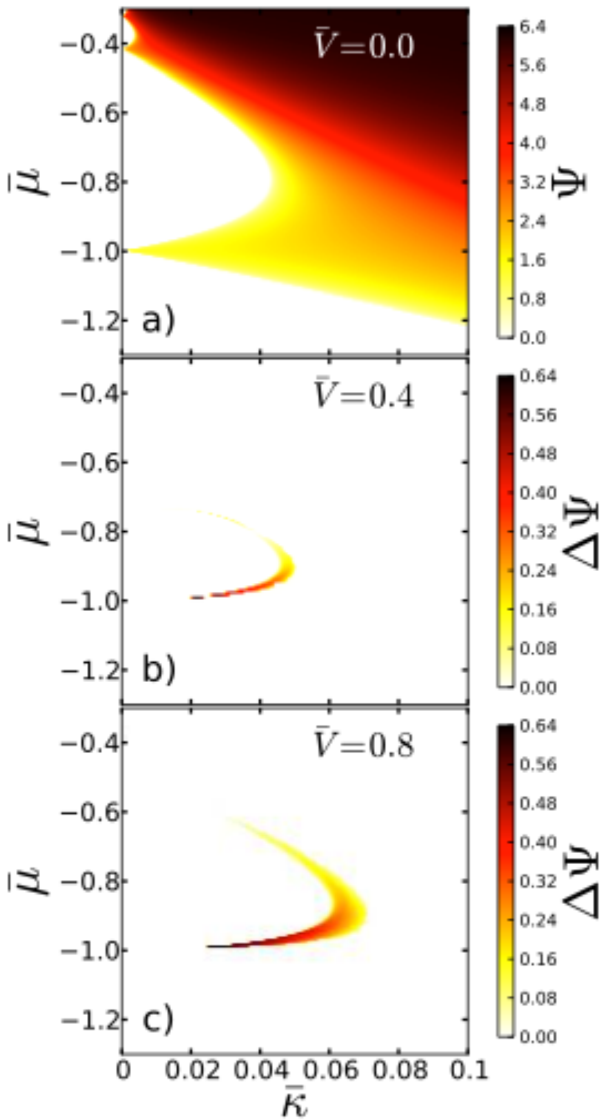
$$\hat{\mathcal{H}} = -\kappa \sum_{\langle i,j \rangle} \hat{a}_i^\dagger \hat{a}_j + \tilde{\beta} \sum_i \left(\hat{a}_i^\dagger \hat{\sigma}_i + \hat{a}_i \hat{\sigma}_i^\dagger \right) + \sum_i \left(\epsilon \hat{n}_i^\sigma + \omega \hat{n}_i^a \right)$$
$$+ \frac{V}{2} \sum_i \sum_{j \neq i} \frac{\hat{n}_i^\sigma \hat{n}_j^\sigma}{|\mathbf{r}_i - \mathbf{r}_j|^3} - \mu \sum_i \hat{l}_i. \qquad (3)$$

Meanfield solutions: CDW





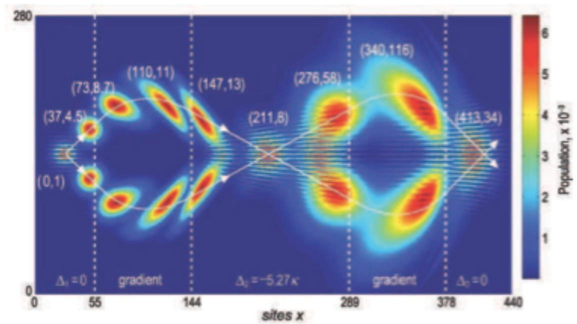
Meanfield solutions: SS

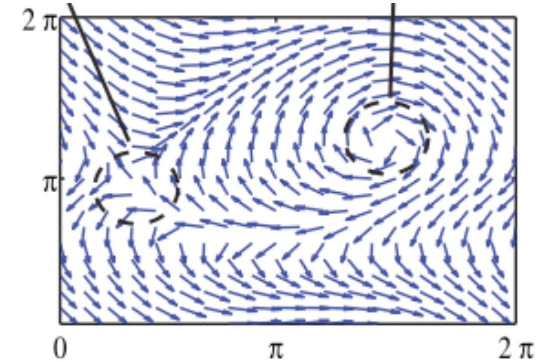


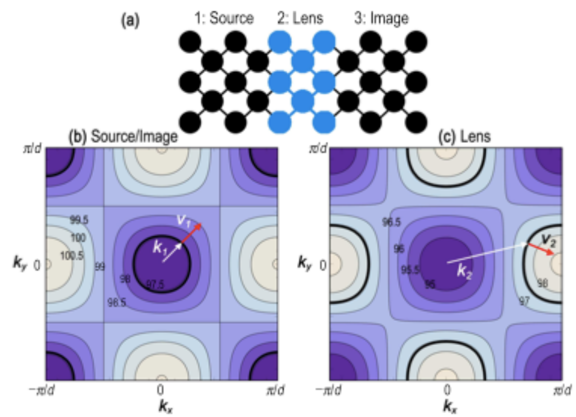
- The nature of the nearest neighbour interaction in the JCH model is qualitatively different to that found in other lattice systems with long-range interactions, such as ultra-cold dipolar gases in optical lattices, where the extended Bose-Hubbard model is appropriate.
- Specifically in the extended JCH model the interaction is mediated via a two-level system. Thus the interaction depends on the simultaneous excitation of neighbouring atoms which favours anti-ferromagnetic correlations between the atomic states.
- Indeed, at $\kappa = 0$ the JCH system maps to a quantum Heisenberg model, in contrast to the Bose Hubbard case, which lies in the classical Ising universality class.

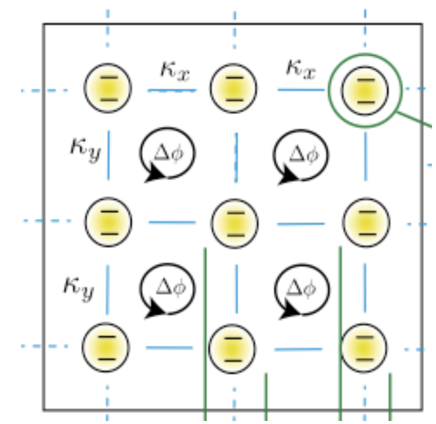
Other Stuff

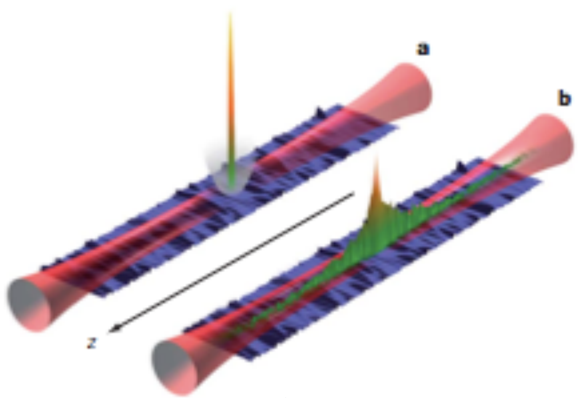
- Anderson localisation
- Synthetic gauge fields
- Spin-orbit coupling
- Fields on lattices (cold gases and CAC)
- Exotic superfluidity (Dipolar)
- Quantum metamaterials
- BKT + lots more …











$$\langle L=1, M=0|1-3\cos^2\theta|L=1, M=0\rangle = -\frac{4\pi}{5} < 0$$