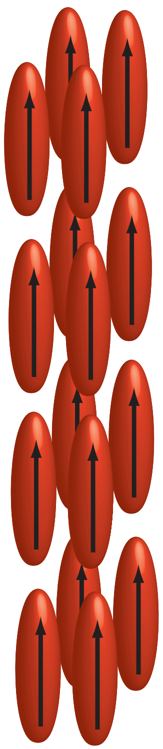
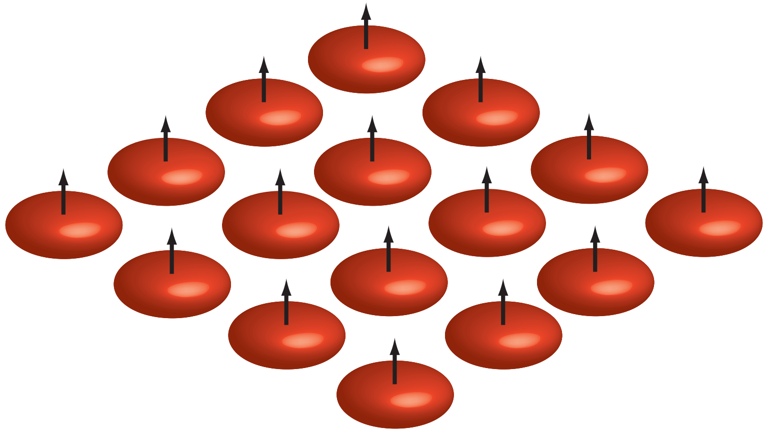
Ultracold Dipolar Gases





Andy Martin

University of Melbourne

Outline

- Dipolar BECs
  - Dipolar interactions
  - Stability
- Dipolar BECs: Vortex lattice
  - Non-dipolar BEC: vortex lattice
  - Dipolar BEC: vortex lattice
- Dipolar Fermi systems
  - Unconventional pairing
  - Novel superfluid state

Reference Material

THE UNIVERSITY OF MELBOURNE | TCMP

- Condensed matter emulation
    - *Ultracold atomic gases in optical lattices: mimicking condensed matter physics and beyond*, M. Lewenstein, A. Sanpera, V. Ahufinger, B. Damski, A. Sen and U. Sen, Advances in Physics **56**, 243 (2007)
    - *Many-body physics with ultracold gases*, I. Bloch, J. Dalibard and W. Zwerger, Rev. Mod. Phys. **80**, 885 (2008)
- Dipolar BECs
    - *Collective excitation frequencies and stationary states of trapped dipolar Bose-Einstein condensates in the Thomas Fermi regime*, R.M.W. van Bignen, N.G. Parker, S.J.J.M.F. Kokkelmans, A.M. Martin and D.H.J. O'Dell, Phys. Rev. A. **82**, 033612 (2010)
- Dipolar BECs and Dipolar Fermi Gases
    - *Theoretical progress in many-body physics with ultracold dipolar gases*, M.A. Baranov, Physics Reports **464**, 71 (2008)

Short-Range Interaction

THE UNIVERSITY OF MELBOURNE | TCMP

Two body collisions

- Dominated by s-wave (isotropic) scattering
- Short-ranged: $\sim 1/r^6$
- Effective interaction (psuedopotential)

$$U_{\mathrm{s}}(\mathbf{r}-\mathbf{r}') = \frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}-\mathbf{r}')$$

- Full Hamiltonian

$$H = \int d\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})\left[-\frac{\hbar^2}{2m}\nabla^2 + U_{\mathrm{T}}(\mathbf{r})\right]\hat{\Psi}(\mathbf{r}) + \int d\mathbf{r}'d\mathbf{r}\hat{\Psi}^\dagger(\mathbf{r})\hat{\Psi}(\mathbf{r})\frac{U_{\mathrm{s}}(\mathbf{r}-\mathbf{r}')}{2}\hat{\Psi}^\dagger(\mathbf{r}')\hat{\Psi}(\mathbf{r}')$$

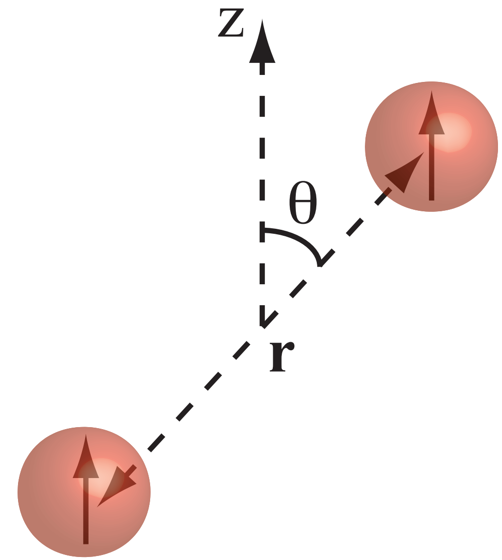
- Mean-field: Gross-Pitaevskii Equation

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left(-\frac{\hbar^2}{2m}\nabla^2 + U_T(\mathbf{r}) + g\left|\Psi(\mathbf{r},t)\right|^2\right)\Psi(\mathbf{r},t)$$

Long-Range Dipolar Interaction

$$U_{dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\left[\frac{1 - 3\cos^2\theta}{r^3}\right]$$

Long-Range   Anisotropic



Magnetic dipole-dipole interaction:

Atoms

^{52}Cr (Bosons)

^{164}Dy (Bosons)

^{168}Er (Bosons)

^{161}Dy (Fermions)

^{167}Er (Fermions)

Electrostatic dipole-dipole interaction:

Polar Molecules

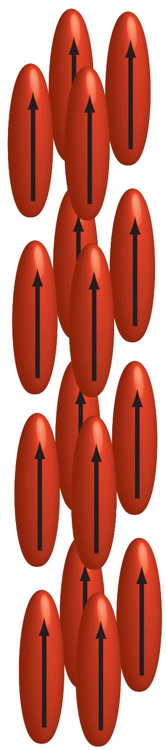
^{39}K^{87}Rb (Bosons)

^{40}K^{87}Rb (Fermions)

Geometry Dependent Interaction

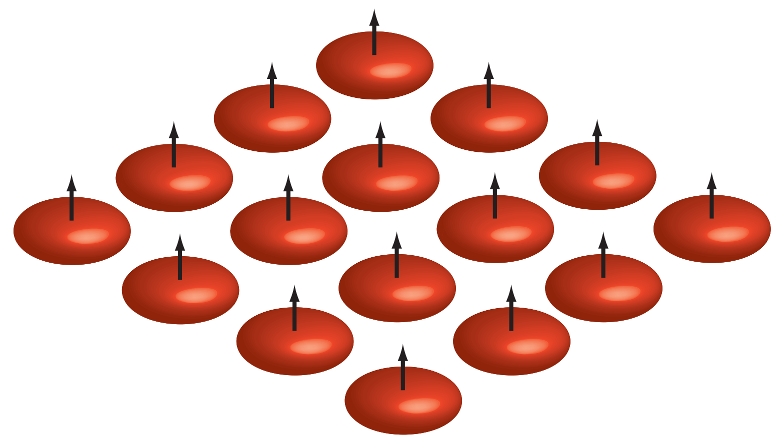
Prolate:

net top-to-tail
attractive interaction



Oblate:

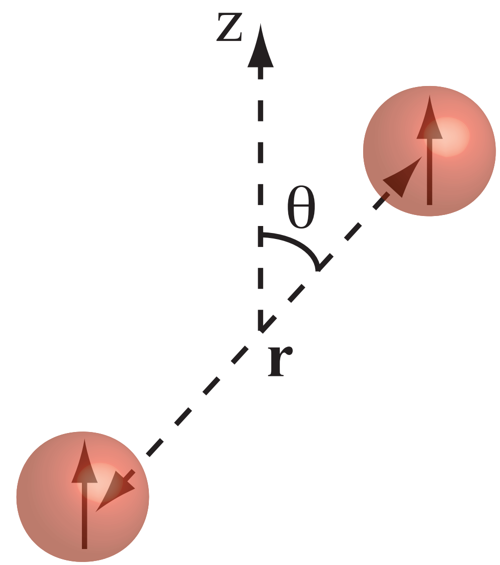
net side-by-side
repulsive interaction



Dipolar vs s-wave Interactions

long-range: $U_{dd}(\mathbf{r}) = \frac{C_{dd}}{4\pi}\left[\frac{1-3\cos^2\theta}{r^3}\right]$

short-range: $U_s(\mathbf{r}-\mathbf{r}') = g\delta(\mathbf{r}-\mathbf{r}')$



$$\varepsilon_{dd} \equiv \frac{C_{dd}}{3g}$$

$$\varepsilon_{dd} > 1 \to \text{collapse}$$

Magnetic dipole-dipole:

^{87}Rb	$\varepsilon_{dd} = 0.007$	^{52}Cr	$\varepsilon_{dd} = 0.16$
^{23}Na	$\varepsilon_{dd} = 0.004$	Feshbach	$\varepsilon_{dd} = 4.0$

A. Griesmaier *et al.*, PRL **94**, 160401 (2005)

Dipolar BEC (Mean Field [TF])

THE UNIVERSITY OF MELBOURNE | TCMP

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}) + \Phi_{dd}(\mathbf{r}) + g|\psi(\mathbf{r})|^2\right)\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

$$V(\mathbf{r}) = \frac{1}{2}m\omega_\perp^2\left[(1-\epsilon)x^2 + (1+\epsilon)y^2 + \gamma^2 z^2\right]$$

$$\Phi_{dd}(\mathbf{r}) = \int n(\mathbf{r}')U_{dd}(\mathbf{r}-\mathbf{r}')d\mathbf{r}'$$

$$V(\mathbf{r}) + gn(\mathbf{r}) + \Phi_{dd}(\mathbf{r}) = \mu$$

$$\kappa_i = R_i/R_z$$

$$n(\mathbf{r}) = n_0\left(1 - \frac{x^2}{R_x^2} - \frac{y^2}{R_y^2} - \frac{z^2}{R_z^2}\right)$$

$$\Phi_{dd}(\mathbf{r}) = -g\varepsilon_{dd}n(\mathbf{r}) + \frac{3g\varepsilon_{dd}n_0\kappa_x\kappa_y}{2}\left[\beta_{001} - \left(\beta_{101}x^2 + \beta_{011}y^2 + 3\beta_{002}z^2\right)R_z^{-2}\right]$$

$$\beta_{ijk} = \int_0^\infty \frac{ds}{(\kappa_x^2+s)^{i+1/2}(\kappa_y^2+s)^{j+1/2}(1+s)^{k+1/2}}$$

$$\kappa_x^2 = \frac{\omega_z^2}{\omega_x^2}\left(\frac{1+\varepsilon_{dd}\left[\frac{3}{2}\kappa_x^3\kappa_y\beta_{101} - 1\right]}{1-\varepsilon_{dd}\left[1-\frac{9}{2}\kappa_x\kappa_y\beta_{002}\right]}\right)$$

$$\kappa_y^2 = \frac{\omega_z^2}{\omega_y^2}\left(\frac{1+\varepsilon_{dd}\left[\frac{3}{2}\kappa_y^3\kappa_x\beta_{011} - 1\right]}{1-\varepsilon_{dd}\left[1-\frac{9}{2}\kappa_x\kappa_y\beta_{002}\right]}\right)$$

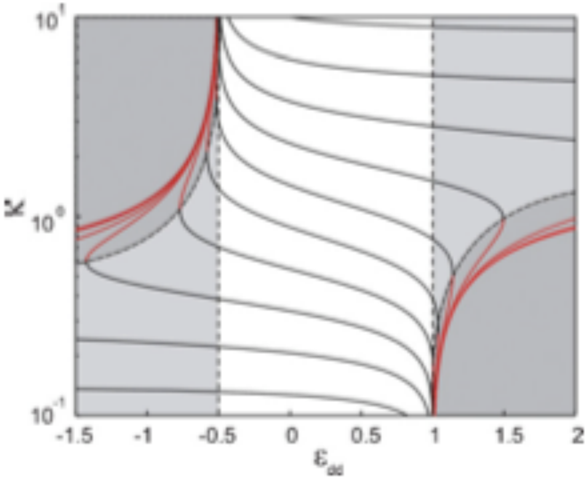
$$R_z^2 = \frac{2gn_0}{m\omega_z^2}\left(1-\varepsilon_{dd}\left[1-\frac{9}{2}\kappa_x\kappa_y\beta_{002}\right]\right) \qquad \varepsilon_{dd} = \frac{C_{dd}}{3g}$$

Condensate Aspect Ratio

$$\varepsilon_{dd} = 0 : \quad \kappa = \gamma$$

$$\varepsilon_{dd} > 0 : \quad \kappa < \gamma$$

$$\varepsilon_{dd} < 0 : \quad \kappa > \gamma$$



$$\varepsilon_{dd} = \frac{C_{dd}}{3g}$$

$$\kappa = \frac{R_x}{R_z}$$

$$\text{Global stability}: \quad -0.5 < \varepsilon_{dd} < 1$$

$$\text{Metastable for } \varepsilon_{dd} > 1 : \quad \gamma > 5.17$$

$$\text{Metastable for } \varepsilon_{dd} < 0.5 : \quad \gamma < 0.19$$

Many Vortices: No dipoles

THE UNIVERSITY OF MELBOURNE

Energy functional

$$E'[\Psi] = \underbrace{\int d\mathbf{r}\; \Psi(\mathbf{r})^* \, H_0' \, \Psi(\mathbf{r})}_{E_0'[\Psi]} + \underbrace{\frac{1}{2}\int d\mathbf{r}_1 \int d\mathbf{r}_2 \; \varrho(\mathbf{r}_1) \, V(\mathbf{r}_1 - \mathbf{r}_2) \, \varrho(\mathbf{r}_2)}_{E_{\mathrm{int}}[\Psi]},$$

$$H_0' = \frac{1}{2M}(\mathrm{i}\hbar\boldsymbol{\nabla} + \Omega M \hat{\mathbf{z}} \times \boldsymbol{\rho})^2 + \frac{1}{2}M(\omega_\rho^2 - \Omega^2)\rho^2 + \frac{1}{2}M\omega_z^2 z^2.$$

2D

$$\Psi(\mathbf{r}) = \frac{1}{(\pi l_z^2)^{\frac{1}{4}}} \exp\left(-\frac{z^2}{2l_z^2}\right) \Phi(\boldsymbol{\rho}),$$

Contact interactions

$$V(\mathbf{r}) = g\,\delta(\mathbf{r})$$

The important bit

$$E_{\mathrm{int}}[\Phi] = \frac{1}{2} g_{2\mathrm{D}} \int d\boldsymbol{\rho} \, n(\boldsymbol{\rho})^2,$$

Many Vortices: No dipoles

LLL: $n=0$

$$\Phi(\boldsymbol{\rho}) = \sum_{m=0}^{\infty} c_m u_{m,0}(\boldsymbol{\rho}) = \sum_{m=0}^{\infty} \frac{c_m}{\sqrt{2\pi m!}} \left(\frac{x+\mathrm{i}y}{l_\rho}\right)^m \exp\left(-\frac{x^2+y^2}{2l_\rho^2}\right),$$

$$\Phi(w) = f(w) \exp\left(-\frac{|w|^2}{2l_\rho^2}\right)$$

Jacobi theta function

$$f(w) = C^{\frac{1}{2}} h(w/b_1)\theta_1(w/b_1, \varsigma), \longrightarrow n(\boldsymbol{\rho}) = C\,|h(w/b_1)|^2\,|\theta_1(w/b_1,\varsigma)|^2 \exp\left(-\frac{|w|^2}{l_\rho^2}\right).$$

$$|\theta_1(w/b_1,\varsigma)|^2 = \mathrm{e}^{2\pi y^2/v_c} \sum_{\mathbf{q}} g_{\mathbf{q}} \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\boldsymbol{\rho}}; \qquad g_{\mathbf{q}} = \frac{(-1)^{m_1+m_2}\mathrm{e}^{-v_c \mathbf{q}^2/8\pi}}{\sqrt{\tau \sin\eta}}.$$

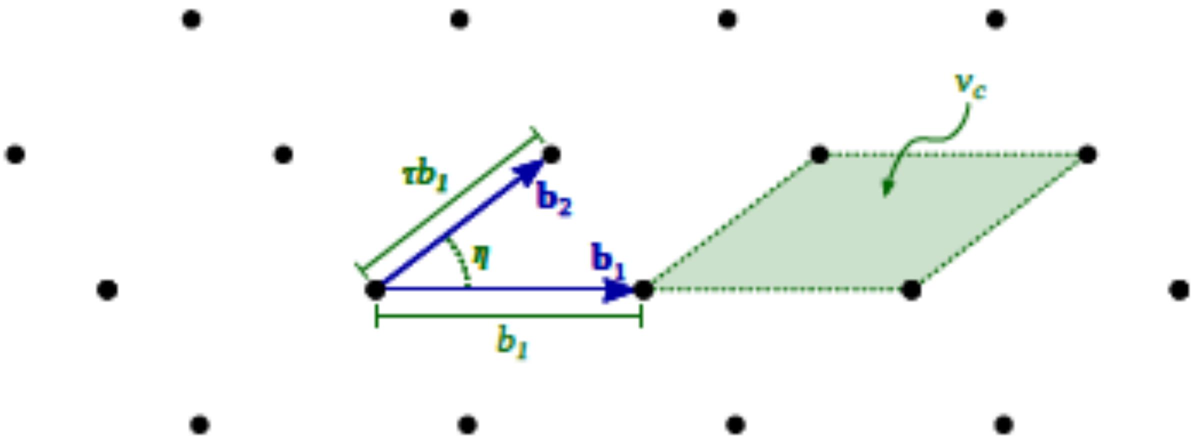
$$\downarrow$$

$$E_{\mathrm{int}}[n] = \frac{N^2 g_{2\mathrm{D}}}{2\pi\sigma^2}\mathcal{I}[n].$$

$$\mathcal{I}(\tau,\eta) = \frac{1}{2}\sum_{\mathbf{q},\mathbf{v}} \bar{g}_{\mathbf{q}}\bar{g}_{\mathbf{v}} \mathrm{e}^{-\frac{\sigma^2|\mathbf{q}+\mathbf{v}|^2}{8}}.$$

Many Vortices: No dipoles

THE UNIVERSITY OF MELBOURNE

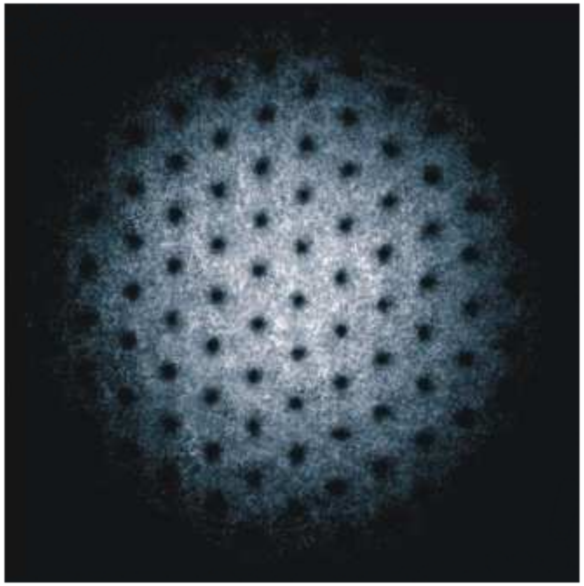


$$E_{\text{int}}[n] = \frac{N^2 g_{\text{2D}}}{2\pi\sigma^2}\mathcal{I}[n].$$

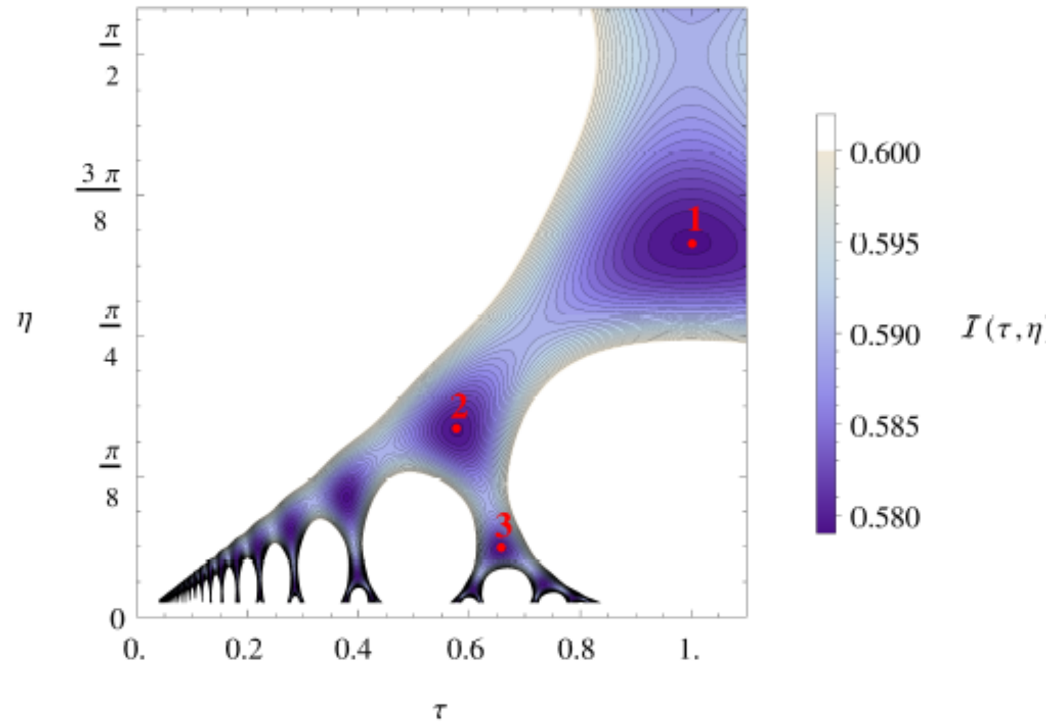
$$\mathcal{I}(\tau,\eta) = \frac{1}{2}\sum_{\mathbf{q},\mathbf{v}} \bar{g}_{\mathbf{q}}\bar{g}_{\mathbf{v}} \mathrm{e}^{-\frac{\sigma^2|\mathbf{q}+\mathbf{v}|^2}{8}}.$$

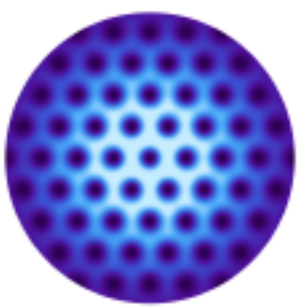
$$\begin{aligned}\mathcal{I}(\tau,\eta) &\approx \frac{1}{2}\sum_{\mathbf{q}} (\bar{g}_{\mathbf{q}})^2 \\ &= \sum_{m_1,m_2=-\infty}^{\infty} \frac{1}{2}\exp\left[\pi\left(2m_1m_2\cot\eta - m_1^2\tau\csc\eta - \frac{m_2^2\csc\eta}{\tau}\right)\right]\end{aligned}$$

Many Vortices: No dipoles



Coddington et al. *Phys. Rev. A.* **70** 063607)





$$1: \quad (1.0000\ldots, 1.0472\ldots) = \left(1, \frac{\pi}{3}\right)$$

$$2: \quad (0.5773\ldots, 0.5235\ldots) = \left(\frac{1}{\sqrt{3}}, \frac{\pi}{6}\right)$$

$$3: \quad (0.6546\ldots, 0.1901\ldots) = \left(\frac{\sqrt{3}}{\sqrt{7}}, \arcsin\left(\frac{\sqrt{3}}{\sqrt{7}}\right) - \frac{\pi}{6}\right)$$

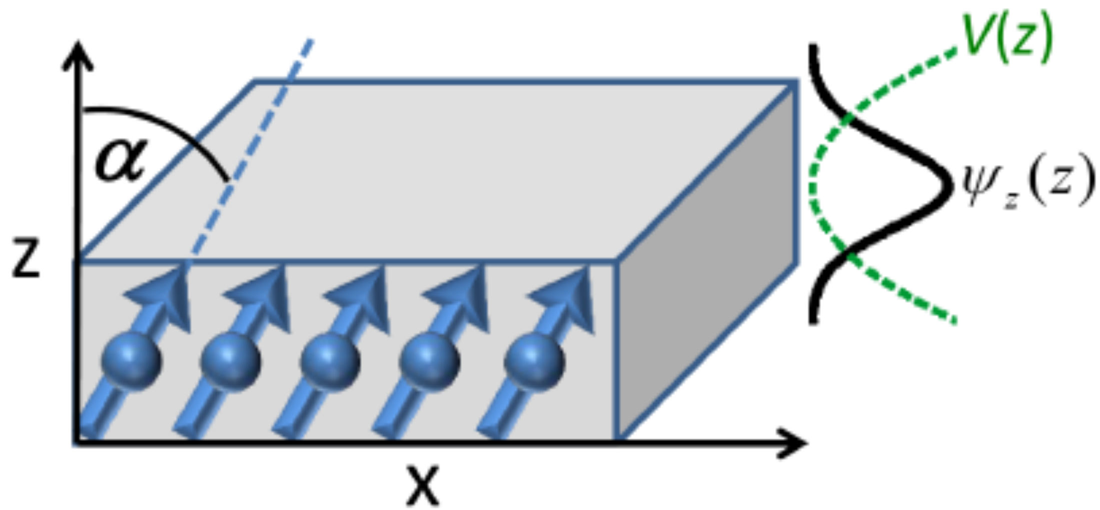
Many Vortices: Dipoles

Repeat but include dipole interactions

$$E_{\mathrm{dd}}(\tau,\eta) = \frac{2N^2C_{\mathrm{dd}}}{3(2\pi)^{\frac{3}{2}}l_z\sigma^2}\mathcal{I}(\tau,\eta) + \frac{N^2C_{\mathrm{dd}}}{3(2\pi)^{\frac{3}{2}}l_z^3}\mathcal{W}(\tau,\eta),$$

$$\mathcal{W}(\tau,\eta) \equiv \sum_{\mathbf{q},\mathbf{v}} \bar{g}_{\mathbf{q}}\bar{g}_{\mathbf{v}}\mathrm{e}^{-\frac{\sigma^2}{4}(\mathbf{q}^2+\mathbf{v}^2)}A_2(\mathbf{v}-\mathbf{q}).$$

$$\mathcal{W}(\tau,\eta) \approx \sum_{\mathbf{q}}(\bar{g}_{\mathbf{q}})^2\left(\mathrm{e}^{-\frac{\sigma^2\mathbf{q}^2}{2}}A_2^a(2\mathbf{q}) + \mathrm{e}^{-\frac{\sigma^2\mathbf{q}^2}{2}}A_2^b(2\mathbf{q})\right)$$

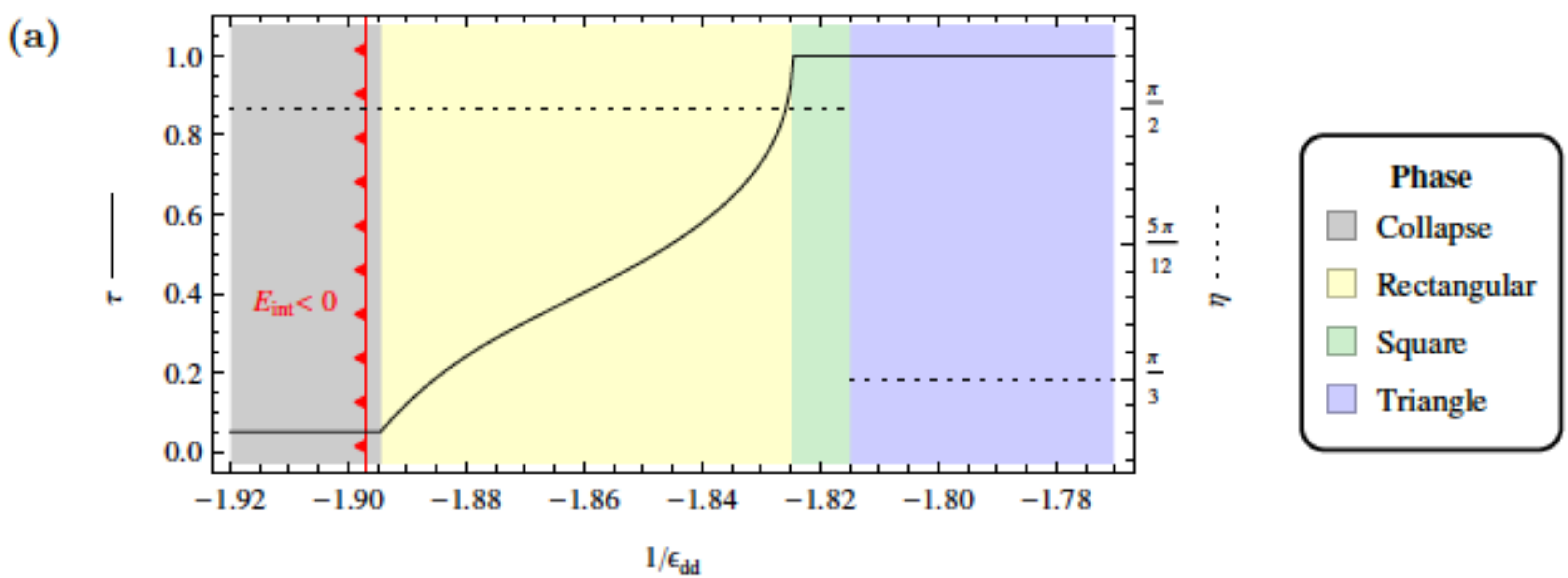


Many Vortices: Dipoles - z

$$\bar{E}_{\text{int}}(\tau,\eta) \equiv \left(2+\frac{1}{\epsilon_{\text{dd}}}\right)\left(\frac{l_z}{\sigma}\right)^2 \mathcal{I}(\tau,\eta) + \mathcal{W}(\tau,\eta),$$

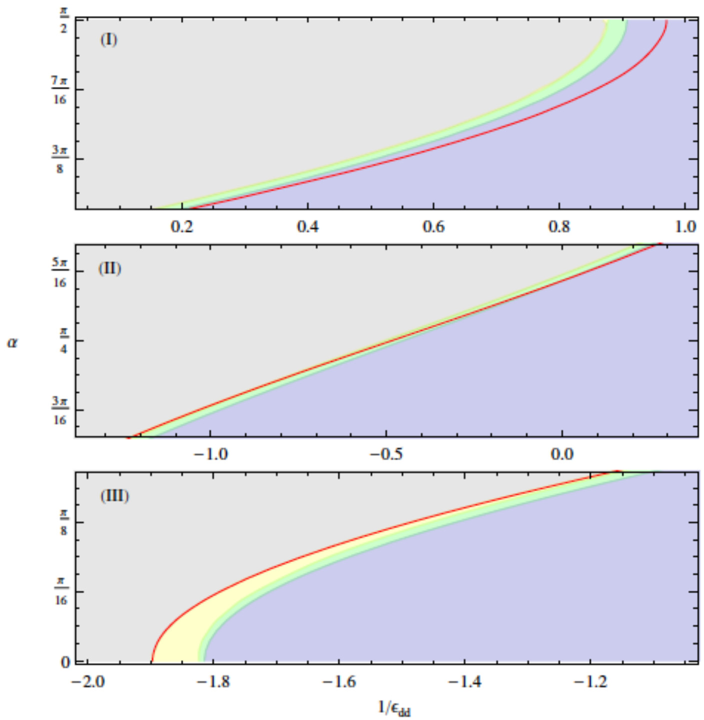
	$v_c/\pi l_\rho^2$	l_ρ/l_z	σ/l_z
(a)	1.01908	40	292.331 ...
(b)	1.01908	80	584.662 ...

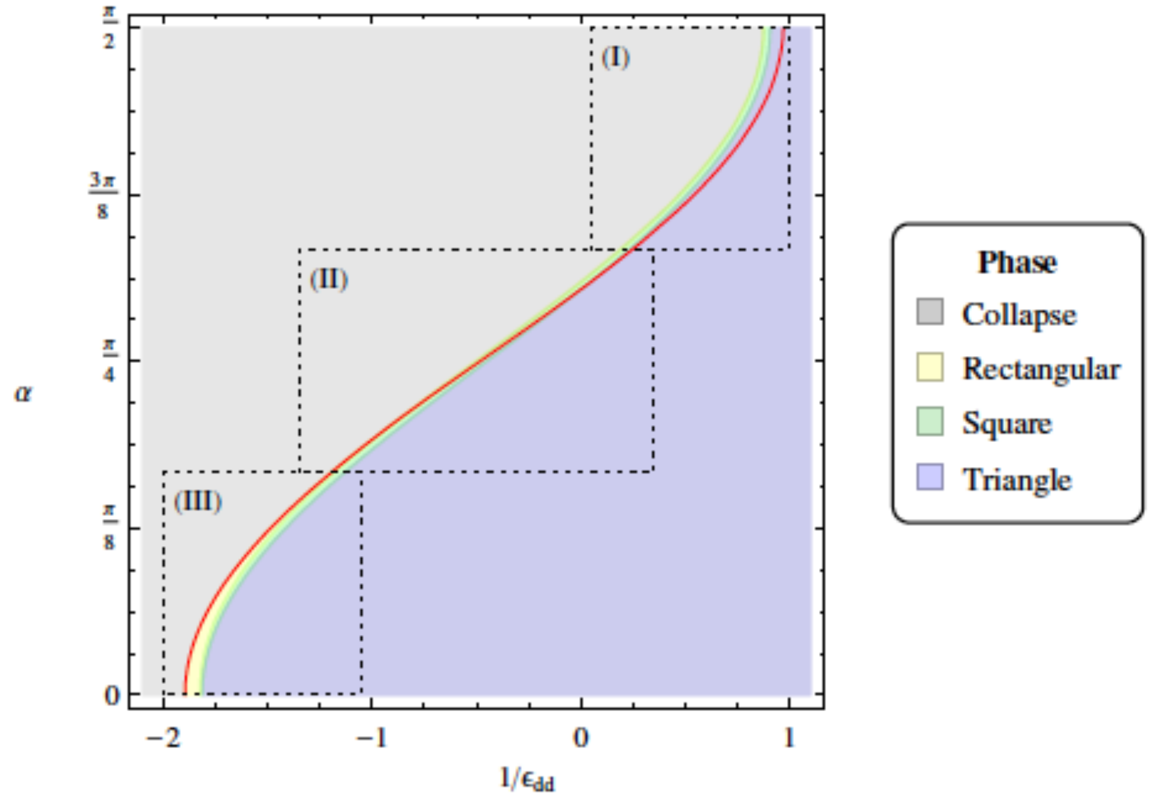
$$\mathcal{W}(\tau,\eta) \approx \sum_{\mathbf{q}} (\bar{g}_{\mathbf{q}})^2 \left(\mathrm{e}^{-\frac{\sigma^2 \mathbf{q}^2}{2}} A_2^a(2\mathbf{q}) + \mathrm{e}^{-\frac{\sigma^2 \mathbf{q}^2}{2}} A_2^b(2\mathbf{q}) \right)$$

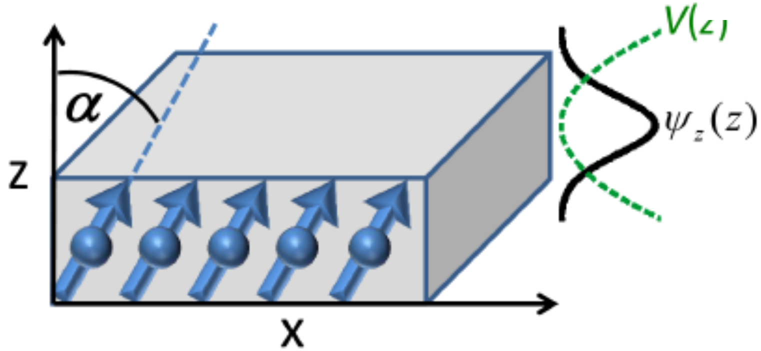


Many Vortices: Dipoles

THE UNIVERSITY OF MELBOURNE







Rectangular solution?

Comparison

z-direction

- Compares well with previous analytic methods:

J. Zhang and H. Zhai, Physical Review Letters **95**, 200403 (2005).

N. R. Cooper, E. H. Rezayi, and S. H. Simon, Physical Review Letters **95**, 200402 (2005).

- Does not agree with numerical approach: only “observe” triangular lattice.

S. Yi and H. Pu, Physical Review A **73**, 061602 (2006).

off-axis

- No previous analytic calculations
- Does agree with numerical approach (in-plane)

S. Yi and H. Pu, Physical Review A **73**, 061602 (2006).

Dipolar Fermi systems

THE UNIVERSITY OF MELBOURNE TCMP

Dipolar Interaction Partially Attractive

$$\langle L=1, M=0|1-3\cos^2\theta|L=1, M=0\rangle = -\frac{4\pi}{5} < 0$$

$$\Downarrow$$

BCS Pairing?

- In a single-component Fermi gas the Pauli principle allows pairing with only odd angular momentum L of the relative motion of two particles in a Cooper pair.
- On the other hand, the anisotropy of the dipole-dipole interaction results in coupling between different angular momenta.
- Hence the problem of superfluid pairing requires consideration of all odd angular momentum L.

BCS State

BCS Order Parameter

$$\Delta(\mathbf{r_1} - \mathbf{r_2}) = U_{dd}(\mathbf{r_1} - \mathbf{r_2})\langle\hat{\psi}(\mathbf{r_1})\hat{\psi}(\mathbf{r_2})\rangle$$

Gap Equation

$$\Delta(\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} V(\mathbf{p},\mathbf{p}') \frac{\tanh(E(\mathbf{p}')/2T)}{2E(\mathbf{p}')}\Delta(\mathbf{p}')$$

$$E(\mathbf{p}) = \sqrt{\Delta^2(\mathbf{p}) + (p^2/2m - \mu)^2}$$

$$V(\mathbf{p},\mathbf{p}') = V_d(\mathbf{p} - \mathbf{p}') + \delta V(\mathbf{p}\mathbf{p}')$$

Many body correction, via creation of virtual particle-hole pairs

Near the Critical Temperature

THE UNIVERSITY OF MELBOURNE | TCMP

$$\Delta(\mathbf{p}) = -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} V(\mathbf{p},\mathbf{p}') \left[K(p')\Delta(\mathbf{p}') + \frac{\partial K(p')}{\partial \xi'} \frac{\Delta^3(\mathbf{p}')}{2\xi'} \right]$$

$$K(p) = \tanh(\xi/2T)/2\xi, \ \ \text{and} \ \ \xi = p^2/2m - \mu$$

- Cooper pairing is associated with the existence of a nontrivial solution
- Linearize

$$\begin{aligned} \Delta(\mathbf{p}) &= -\int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \Gamma_d(\mathbf{p},\mathbf{p}') \left[K(p') - K_0(p') \right] \Delta(\mathbf{p}') \\ &- \int \frac{d\mathbf{p}'}{(2\pi\hbar)^3} \delta V(\mathbf{p},\mathbf{p}') K(p') \Delta(\mathbf{p}') \end{aligned}$$

$$\Gamma_d(\mathbf{p},\mathbf{p}') = V_d(\mathbf{p}-\mathbf{p}') - \int \frac{d\mathbf{q}}{(2\pi\hbar)^3} V_d(\mathbf{p}-\mathbf{q}) K_0(q) V_d(\mathbf{q}-\mathbf{p}')$$

Properties of the BCS State

THE UNIVERSITY OF MELBOURNE

TCMP

Critical Temperature

$$T_c = 1.44 E_F \exp\left\{-\frac{\pi^2 E_F}{3nC_{dd}}\right\} \qquad E_F \approx (\hbar^2/2m)(6\pi^3 n)^{2/3}$$

$$T_c \approx 100nK$$

Order Parameter

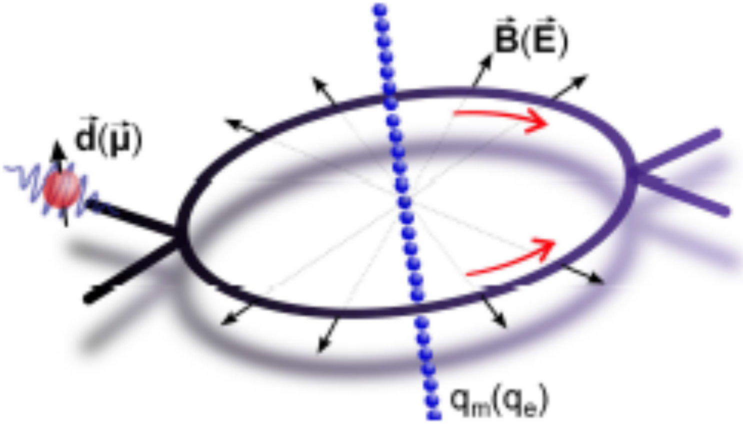
$$\Delta(p_F \hat{\mathbf{p}}) \approx 2.5 T_c \sqrt{1 - \frac{T}{T_c}} \sqrt{2} \sin\left(\frac{\pi}{2}\cos\theta_{\mathbf{p}}\right)$$

Properties of the BCS Gap

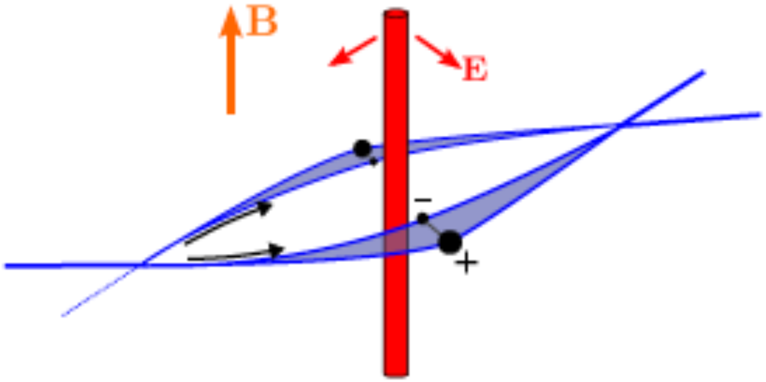
- The anisotropy of the gap provides the major difference of the properties of the superfluid dipolar Fermi gas from the more conventional two component fermionic gases, with s-wave pairing
- As a consequence excitations with momenta in the direction of the dipoles acquire the largest gap. In contrast eigenenergies of excitations with momenta perpendicular to the dipoles remain unchanged
- The nodes in the gap lead to significant changes in the properties of the specific heat, i.e. well below the transition the single particle contribution to the specific heat is proportional to T^2.
- This also has a consequence for how disorder effects the critical temperature.

He, McKellar, Wilkens Phase

Theory Perspective



Experimental Perspective



Dipolar Superfluid in a Ring

Some Recent Work: 2010-

THE UNIVERSITY OF MELBOURNE

NV Centres

The non-Abelian geometric phase in the diamond nitrogen-vacancy centre, M.A. Kowarsky *et al.*, Physical Review A **90**, 042116 (2014)

Nanoscale magnetometry through quantum control of nitrogen-vacancy centres in rotationally diffusing nanodiamonds, D. Maclaurin *et al.*, New Journal of Physics **15**, 013041 (2013)

Measurable quantum geometric phase from a rotating single spin, D. Maclaurin *et al.*, Physical Review Letters **108**, 240403 (2012)

Atomtronics

Negative refraction of excitations in the Bose-Hubbard model, R. A. Henry *et al.*, Physical Review A **90**, 043639 (2014)

Interferometry using adiabatic passage in dilute gas Bose-Einstein condensates, M. Rab *et al.*, Physical Review A **86**, 063605 (2012)

Coherent tunnelling via adiabatic passage in a three well Bose-Hubbard system, C.J. Bradly *et al.*, Physical Review A **85**, 053609 (2012)

Synthetic magnetohydrodynamics in BECs and routes to vortex nucleation, L.B. Taylor *et al.*, Physical Review A **84**, 021604(R) (2011)

Quantum reflection of ultracold atoms from thin films, graphene and semiconductor heterostructures, T.E. Judd *et al.*, New Journal of Physics **13**, 083020 (2011): **selected as a 2011 highlight**

Zone-plate focusing of BECs for atom optics and erasable high speed lithography of quantum electronic components, T.E. Judd *et al.*, New Journal of Physics **12**, 063033 (2010)

Other BEC stuff

Vibrations of a columnar vortex in a trapped Bose-Einstein condensate, L. Koens, T.P. Simula and A.M. Martin, Physical Review A **87**, 063614 (2012)

Perturbative behavior of a vortex in a trapped Bose-Einstein condensate, L. Koens and A.M. Martin, Physical Review A **86**, 013605 (2012)

Other Recent Work: 2010-

THE UNIVERSITY OF MELBOURNE

Strongly Interacting Cold Gases

Coupled-pair approach for strongly interacting trapped fermionic atoms, C.J. Bradly *et al.*, Physical Review A **90**, 023626 (2014)

Universality and itinerant ferromagnetism in rotating strongly interacting Fermi gases, B.C. Mulkerin *et al.*, Physical Review A **86**, 053631 (2012)

Universality in rotating strongly interacting gases, B.C. Mulkerin *et al.*, Physical Review A **85**, 053636 (2012)

Dipolar BECs

Vortices in the two-dimensional dipolar Bose gas, B.C. Mulkerin *et al.*, Journal of Physics: Conference Series **497**, 012025 (2014)

Anisotropic and long-range vortex interactions in two-dimensional dipolar bose gases, B.C. Mulkerin *et al.*, Physical Review Letters **111**, 170402 (2013)

Collective excitation frequencies and stationary states of trapped dipolar BECs in the Thomas Fermi regimes, R.M.W. van Bijnen *et al.*, Physical Review A **82**, 033612 (2010)

Jaynes Cummings Hubbard Model

Supersolid phases of light in extended Jaynes-Cummings-Hubbard systems, B. Bujnowski *et al.* Physical Review A **90**, 043801 (2014)

Fractional quantum Hall physics in Jaynes-Cummings-Hubbard lattices, A.L.C. Hayward *et al.*, Physical Review Letters **108**, 223602 (2012)

Reconfigurable quantum metamaterials, J.Q. Quach *et al.*, Optics Express **19**, 11018 (2011)

Quantum Graphity

Domain structures in quantum graphity, J.Q. Quach *et al.*, Physical Review D **86**, 044001 (2012)

The Future

THE UNIVERSITY OF MELBOURNE

TCMP

Say Hello to Flynn

Contact

Dr Andy Martin
martinam@unimelb.edu.au