Time Series Modelling of the Indian Monsoon Rainfall

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Final Report

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Contents

1 Introduction

The rainfall over the Inidan subcontinent functions in an annual cycle with most of it's rainfall coming in the months of June to September during the 'Indian Summer Monsoon'. The Indian summer Monsoon is one of the oldest and most anticipated weather phenomenon on earth. However, apart from being one of the most important weather phenomenas affecting the global weather patterns, it is of specific importance to the Indian subcontinent. India, being an agricultural nation, is heavily dependent on the Monsoons as it's primary source of water. However, the monsoons, despite being a consistently occuring annnual event, exhibits variability on some scale. Thus, a good monsoon turns out to be a good year for India, both agriculturally and economically, while a bad monsoon leads to floods or droughts. Thus, the prediction of the Indian Monsoons is of vital importance so we can prepare better for whatever the Monsoon has in store for us.

2 Explaining the Problem

2.1 The Monsoon

The monsoons are defined as a seasonal prevailing wind in the region of South and South East Asia, blowing from the south-west between May and September and bringing rain (the wet monsoon), or from the north-east between October and April (the dry monsoon).

There are four major Monsoon Domains on earth. They can be listed as follows -

- The Asian Monsoon
- The Australian Monsoon
- The North and South American Monsoons
- The African Monsoon

We are concerned with the Asian Monsoons, which in turn, has two parts - the Indian South West Monsoon or the south Asian Monsoon and the east Asian Monsoon. The south Asian Monsoon, affecting the Indian subcontinent, is the most intense among all the monsoons of the world. Under the influence of orography, the Indian South West Monsoon produces some of the world's highest precipitation amounts. Cherrapunji and Mawsynram in north-east India are the rainiest places of the world.

2.2 Onset and Withdrawal of the Indian SW Monsoon

More than half of the population in India are farmers. Thus, the Monsoon is looked forward to in India. The Indian Monsoon strikes Kerela in the south-eastern coast and then gradually makes its way towards the interior the country.

While the mean onset date of the Indian monsoon over Kerela is the 1^{st} of June, the north-western parts of the country can have monsoon onset as late as 15^{th} July. The standard deviation over these predictions is a week. The onset has occurred as early as 11^{th} May and also as late as 18^{th} June.

The withdrawal, on the other hand, commences from the extreme corners of northwest India by 1^{st} September. The process is gradual and is complete by around the 15^{th} of October. The southern peninsular region of India, however, receives rainfall till the end of December owing to the northeast monsoon which replaces the southwest monsoon.

2.3 Rainfall during the Indian SW Monsoon

The rainfall over India during the monsoon months of June to September accounts for almost 75-80% of the total annual rainfall. Over many parts of India, the rainfall in the remaining eight months is almost negligible in comparison. The average rainfall value, called the All India Summer Monsoon Rainfall, or AISMR, serves as a good index and for the overall Monsoon behaviour of the country. The average value of this AISMR is 89 cm.

2.4 Monsoon and Agriculture

India is largely an agricultural nation and with the Indian Monsoons as its primary source of water, the country has suffered in times of its faliures thoughout its history. Out of the 162 million hectares of arable land in India, the irrigated area is only about 52 million hectares. This is largely due to the meagre rainfall over many regions and topographical features that set a limit to the percentage of land that can be irrigated.

The monsoons being practically the only source of water and with 75-80% of it coming in a short period of four months, maximum use of the water has to be made in this period. At the same time, arrangements have to be made for storing the water for use in the remaining eight months of the year.

The influence of the Indian monsoons rains over the country's agriculture is further strengthened by the fact that in the years with very low or deficient rainfall, the the production has been very low. Besides directly impacting the agricultral production of the country, large scale deficiencies of the monsoon rainfall also cause other problems such as shortage of fodder for animals and

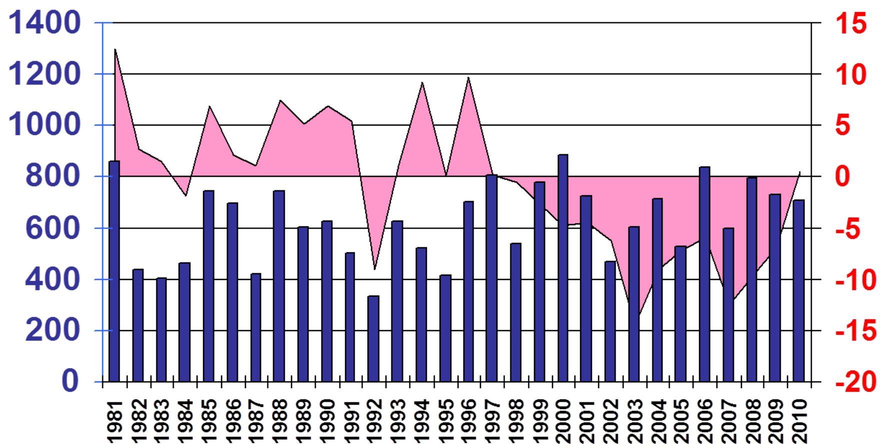

Figure 1: Agricultural Production over the years
The figure shows agricultural production in each year from 1981 - 2010. It is evident that the production drops in years if drought like 1982, 1987 and 2002.

scarcity of drinking water, while excess rains leads to flooding, disruption of normal life and loss of standing crops.

Also, since a large population of India is dependent on agriculture, the purchasing power of these people is also dependent on agriculture and hence, the purchasing power of the people is also dependent on the Monsoons. Thus, the Monsoons also play a role in the GDP or largely, the economy of India as a whole.

2.5 Need for Monsoon Predictions

As it is clear that the monsoons largely affects the agriculture and through it, the economy of the country, Monsoon Predictions can undoubtedly be of great value in any efforts aimed at minimizing agricultural risks and maximizing crop yields. Thus, various methods of prediction based on appropriate time and space scales are required to cater to the need of the millions of farmers or policy makers for different state and national levels.

3 Solving the Problem

3.1 Interannual variability of the Indian Monsoons

Predicting the physical behaviour of the Monsoon system and the resulting distribution of rainfall over thr whole of India is a challenging task. Although the people want a calendar of rainfall events to be drawn up well in advance so they can plan and organize their activities accordingly, this requirement is almost impossible to meet.

To simplify matters, meteorologists have started averging the rainfall over the country as a whole and make efforts towards predicting this one All India Summer Monsoon Rainfall(or AISMR) value. However, even the task of predicting the AISMR alone has not proved to be a simple task, the reason being the higher degree of interannual variability in the AISMR. The figure shows rainfall deficiency as high as 30% in the drought years while it shows rainfall excess as much as 20% in the higher rainfall years.

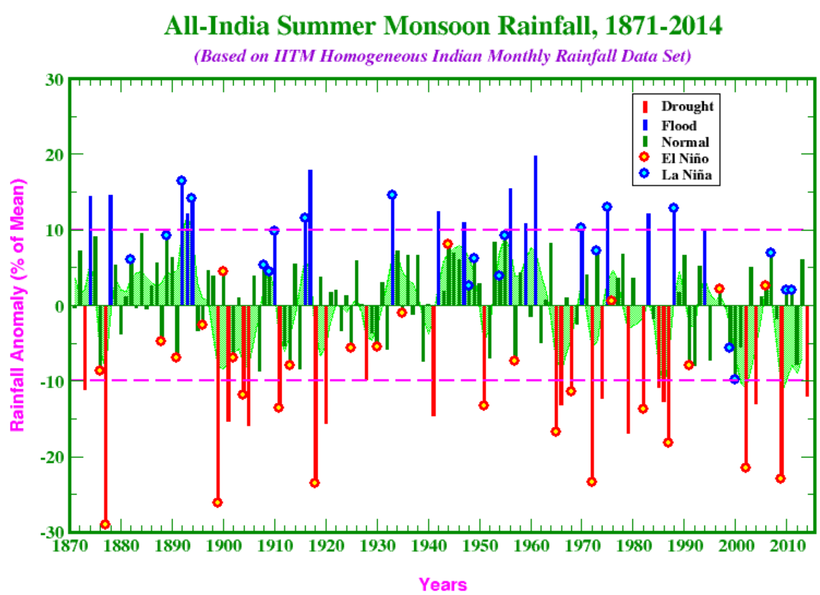

Figure 2: Interannual variability of the All-India Summer Monsoon Rainfall
The figure shows the high degree of variation in the value of the AISMR over the period 1870-2012.

Despite the variability however, the Indian monsoons are fairly regular in the sense that it has never failed to arrive altogether. The interannual variations also remain in bounds with the standard deviation of the AISMR being just 10%.

3.2 Types of Approaches

Various kinds of approaches have come up over the years to solve the problem of AISMR prediction. The various methods can be classified into three wide sections in general.

1. **Statistical Methods -** These methods include searching for potential parameters and predictors which influence the Monsoon Rainfalls and using these to develop predictive models. The various methods developed under this range from power regression to neural networks models.

   The statistical models are however, purely based on corellation between rainfall and the parameters without always having a causal element to it. At the same time, some of the features that can be explained using causal arguments are not used due to weak corellation with the rainfall.

2. **Dynamic Methods -** In this model, we consider the Monsoons as a dynamical system that is described by a system of ordinary differential equations and we try to solve these equations in order to predict Monsoon.

   Though this is expected to be an accurate prediction technique, it is not yet fully developed to give us a substantial result at present.

3. **Time Series Analysis -** Here, we assume that the future values of rainfall are in some way, dependent on the past values and models are developed based on these assumptions. The most common methods for such time analysis methods are AR, MA, ARMA and ARIMA models.

   Time series analysis comes with two distinct advantages. The first being that extrapolating the rainfall series eliminates the need for recording any oceanographic or meteorological data. The second advantage is the long lead time that it provides. However, it fails to explain our assumption on the dependence of rainfall on its previous values.

4 Existing Techniques and Models

After the faliures of Monsoon rainfall in the years 1866 and 1871, the british government in India set up an enquiry commision which recommended the setting up of an all-India meteorological organization. This gave birth to the IMD or the Indian Meteorological Department in 1875. In 1877, the monsoon rains had failed and the Government was anxious to know the prospects of the monsoon in 1878. John Eliot, who was the acting director of the IMD at that time, examined the previous meteorological statistics and submitted an opinion that the 1878 rainfall would be more equitably distributed than in 1877, a forecast which came quite correct. In the coming years, the IMD was to come up with vvarious models to predict the behaviour of the Monsoons.

Blanford, the first director of the IMD, had postulated that the Himalayan snow cover exercised a great influence on Indian rainfall. He initiated a system of snowfall observations in the Himalayas and made arrangements for obtaining this information. Between 1881 and 1885, Blanford made his first experiments at making long range forecasts of the monsoon rainfall on the basis of the snow reports. On 4 June 1886, he issued the first long range monsoon forecast of IMD, which was published in the Gazette of India. His successors continued the use of this method until it failed to predict the rainfall in the period 1899-1901.

Sir Gilbert Walker, the Director of the IMD from 1904-1924, also played a pioneering role in the field of Monsoon Predictions. He initiated the process of applying statisticl models to predict the rainfall quantitatively. In an era when the means of acquiring and processing data was primitive by all means, he found out about the relationship between the Monsoons and the El-Nino Southern Oscillation or the ENSO effect.

The IMD, in the subsequent years, came up with a number of models and techniques for predicting rainfall. Some models in brief are mentioned.

4.1 Parametric Models

These models are the simplest statistical models. Gowariker et al came up with this model in 1989. It consisted of 16 parameters which had significant correlation with the Monsoons.

If a parameter with positive corellation with the Monsoons showed positive deviation from its normal value, it was considered favorable. Otherwise, it was unfavourable. Similarly, if a parameter with negative corellation with the Monsoons showed negative deviation from its normal value, it was considered favourable. Otherwise, it was unfavourable. The Monsoon was predicted as normal, excess or deficient based on the percentage of favourable parameters.

This model, however, broke down as some of the parameters lost their corelation with the Monsoons. Also, the lack of a quantitative value was a drawback to the model.

4.2 Regression Models

These models also initially used the 16 parameters from the Parametric model to create a Regression Models. Let us assume the parameters to be X_j, j = 1,2, ...16. The initial model developed was a linear regression model with the formula-

$$\frac{Y + \alpha_o}{\beta_o} = C_o + \sum_{i=1}^{16} \frac{X_i + \alpha_i}{\beta_i}$$

The linear regression model was soon replaced by the power regression model.

$$\frac{Y + \alpha_o}{\beta_o} = C_o + \sum_{i=1}^{16} (\frac{X_i + \alpha_i}{\beta_i})^{p_i}$$

The parameters were regularly updated for this model due to the changing nature of the corellation of the parameters with the Monsoons. Eventually, the power regression model was scraped and two new models - Ensemble Multiple Linear Regression(EMR) and Projection Pursuit Regression(PPR) was used.

4.3 Neural Networks

In simple terms, the Neural Networks is a coupled input-output map constructed by means of an iterative process. The basic unit of the network is a neuron. As a segment of the input is presented to the network, the skill of the technique is determined by the manner in which the weights of the individual neurons are adjusted in order to predict the next point in the input data series. Once the desired level of accuracy has been reached in the training data set, it can be extended to make a prediction outside the training set.

A total of eight global and regional paremeters were used as predictors. Since some of the predictors were corellated, a principal component analysis of the predictors was carried out. This yielded five different principal components to develop a principal components neural networks model.

A lot of other methods such as prediction by probability, Extension of time series and ARIMA techniques have also been used. We discuss the ARIMA technique, which was the basis for the present analysis, in detail.

5 Time Series Analysis

A technique known as the Auto-Regressive Integrated Moving Average (ARIMA) method is a time series analysis method used for forecasting "stationary" time series. "Stationary" is a term used for time series which have a fairly constant mean and variance.

The ARIMA technique is made up of two different methods, the Auto-Regressive (AR) and the Moving Average(MA). The AR and MA techniques are similarly time series analysis techniques. We therefore begin by understanding the AR and MA techniques.

5.1 Auto Regressive - AR Model

Autoregressive is a stochastic process used in statistical calculations in which future values are estimated based on a weighted sum of past values. An autoregressive process operates under the premise that past values have an effect on current values. A process considered AR(1) is the first order process, meaning that the current value is based on the immediately preceding value. A typical AR(1) equation is -

$$y_t = \beta_o + \beta_1 y_{t-1}$$

The Auto Regressive model is basically a linear regression model with the predictors being the past values of the response variable. An order two AR equation is as follows and is represented as AR(2).

$$y_t = \beta_o + \beta_1 y_{t-1} + \beta_2 y_{t-2}$$

The selection of the optimal order for the Auto Regressive model is best done with a PACF or a Partial Auto-corellation function graph.

5.2 Moving Average - MA Models

A moving average model also relies on past values to predict the future ones. The predictor values in the model are past errors. If w_t are identically, independently distributed, each with a normal distribution having mean - and same variance. Then, a first order MA model, MA(1) can be written as -

$$y_t = \beta_o + w_t + \beta_1 w_{t-1}$$

Similarly, an order two MA model, or MA(2) model can be written as -

$$y_t = \beta_o + w_t + \beta_1 w_{t-1} + \beta_2 w_{t-2}$$

To predict the order of a Moving Average model, we generally use the Autocorellation function graph.

5.3 Auto-Correlation and Partial Auto-Correlation Function

Autocorrelation is the linear dependence of a variable with itself at two points in time. For stationary processes, autocorrelation between any two observations only depends on the time lag h between them. If we define $Cov(y_y, y_{t-h})$ as Y_h, then the Lag-h Auto-Correlation is given by

$$\rho_h = Corr(y_t, y_{t-h}) = \frac{Y_h}{Y_o}$$

A Partial Auto-Correlation is a conditonal correlation. It is the correlation between two variables under the assumption that we know the value of some other set of variables. For example, considering a regression context in which y is the response variable dependent on the predictor variables x_1, x_2 and x_3, the Partial Auto-Correlation. The Partial correlation between y and x_3 will depend on the relationship of y and x_3 depends on their relationships with x_1 and x_2.

5.4 Auto Regressive integrated Moving Average - ARIMA Model

The ARIMA technique is one of the most widely used time series analysis method. It is an integration of the AR and MA models into one. The Integrated in ARIMA stands for order of differencing. Differencing techniques are used to convert a non - stationary time series into a stationary one.

Any ARIMA model has three different parameters- (p, d, q). The first parameter, p, signifies the order of the Auto Regressive part. The parameter q depicts the order of the Moving Average part. The parameter d, is the order of differencing that has to be applied to make the series stationary. Since our Rainfall model is already stationary, we can set the d parameter to be zero. The parameters p and q have to be tuned for every model.

A typical ARIMA model with parameters p and q can be written as ARIMA(p, d, q) -

$$y_t = \mu + \alpha_1 y_{t-1} + \alpha_2 y_{t-1} + ... + \alpha_p y_{t-p} + \beta_1 w_t + \beta_2 w_{t-1} + ... + \beta_q w_{t-q+1}$$

5.4.1 Checking if Model is Stationary

To check if a given time serie is stationary, we use the Dickey - Fuller test. The Dickey - Fuller test is a bit complicated to explain elaborately but it is frequently used in statistics to test stationarity of a time series. We use this test in our analysis to test the stationarity of our various time series before applying ARIMA models to them.

5.4.2 Applying the ARIMA model

We apply the ARIMA model in using the python programming language. After selecting the parameters, running the python model is very easy. But, it is also important to understnd what happens underneath when we run the model. When we run the ARIMA Model in using the pyton statsmodel library, python fits the AR coefficients easily. However, for the MA coefficients, it is neessary to estimate the w_t values. To understand how python does this, let us consider the following MA(1) process -

$$y_t = w_t + \theta_o w_{t-1}$$

We can also write this as

$$w_t = y_t - \theta_o w_{t-1}$$

So for t = 1,

$$w_1 = y_1 - \theta_o w_o$$

Therefore, we can recursively use this formula to obtain the subsequent values of w_t. For this however, we need to have an estimate of θ values. Barring the mathematical details, it has been shown that the autocorellations of the time series can be written in terms of the the θ values and their values can be solved through them.

Again, the requirement of knowing the initial estimate is still there. Barring the details once again, it has been shown that if the number of observations is large, it is perfectly safe to assume the initial estimate of the disturbance, i.e. $w_o = 0$. This is the basic picture of the algorithm python uses to fit an ARIMA model.

5.4.3 Tuning the p and q parameters

The ARIMA model comes with three different parameters, p, d and q. Since our data is largely stationary, a value d = 0 will suffice for our model. However, for the p and q parameters, we have to choose from a large collection of values. A natural choice for is to calculate the MSE (Mean Squared Error) from the fitted data.

If y_i is the original value and $y_i^{'}$ is the predicted value, The MSE is calculated as follows -

$$MSE = \frac{\sum_{i=1}^{n} (y_i^{'} - y_i)^2}{n}$$

However, if we do this by using the training data(the data using which this model was fitted), the model with higher complexity will always perform better. So, we calculate what is called the test error. For this, if the total number of sample data points is n, we select a number m = 0.75n. Now, we fit a model for the first m values($y_1, y_2, ...y_m$) and predict the $(m+1)^{th}$ value. The error here is e_m. Again, we fit the model for first m+1 values, and predict the $(m+2)^{th}$ value until m = n. And we calculate the mean error in the end for the model.

$$TestError = \frac{\sum_{i=m}^{n} e_i^2}{n-m}$$

The model having the least value of this test error is selected to yield the optimal p and q values.

6 Results

6.1 All India

We tried to predict Rainfall in India in different time scales - Annual, Monthly and daily scales. We fit various ARIMA models on these different scales to predict the Monsoon Rainfalls.

6.1.1 Annual Rainfall

Firstly, for the whole of India, we had monthly data from the 1901 to 2014. We used this to evaluate an yearly rainfall dataset and Monsoon Rainfall Dataset for India while assuming the June to September months to be Monsoon months.

After satisfying the Dickey-Fuller test for this, we checked for the optimal p and q parameters from a pool of 0-5 for both. Plotting the graph for the errors, we find (5, 1) to be the optimal value for p and q values.

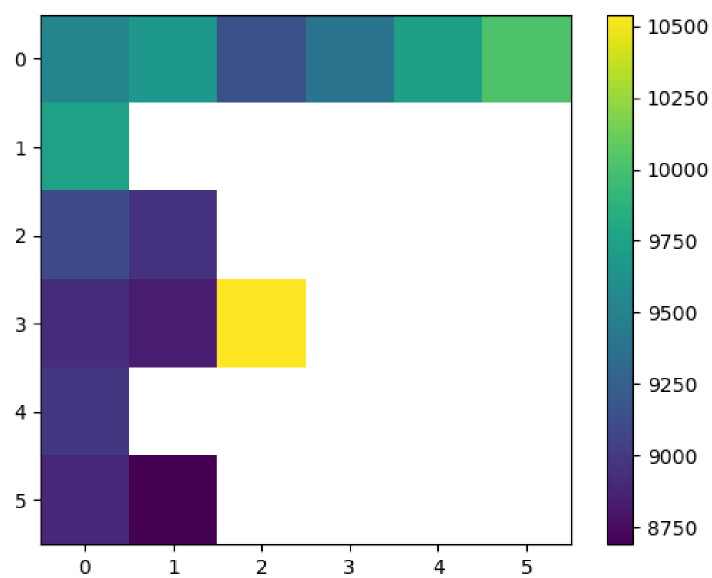

Figure 3: Errors for sets AR and MA parameters for Annual All India Rainfall
We use this colorplot to find the optimal p and q values. The white spaces are models which could not fit the time series model.

The result of fitting this model to our rainfall data was not very satisfactory. Though it was able to predict the rising and falling behaviour, it failed to capture the large amount of variation in the model.

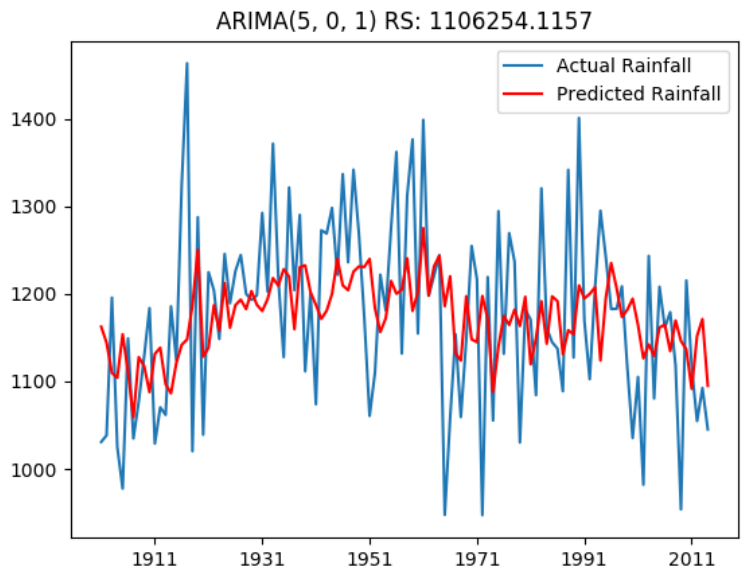

Figure 4: All-India Annual Rainfall ARIMA(5, 0, 1) Model
The model fails to capture the variation present in the actual data.

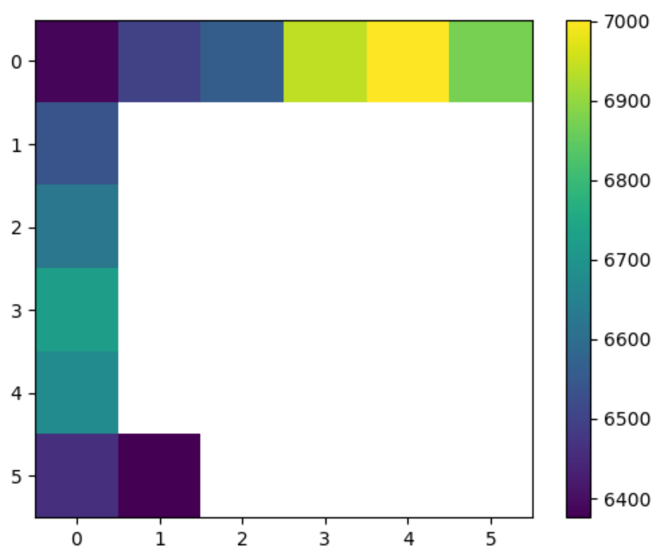

Figure 5: Errors for AR and MA parameters for Monsoons ARIMA model

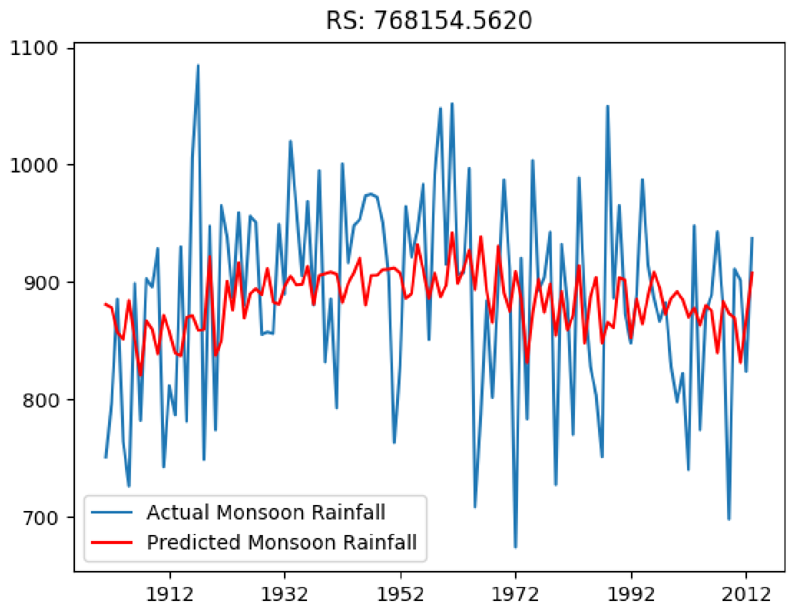

Figure 6: ARIMA(5, 0, 1) model for monsoon rainfall in India

While estimating the Monsoon Rainfall for the same period, we get the same results as for thr Annual Predictions.

6.1.2 Monthly Rainfall

We used the monthly rainfall dataset from 1971 to 2013 for predicting rainfall here. The rainfall is low in the months of November to May while most of it comes in the period June to September. The estimates for the order of the AR and the MA terms are tested from 0 to 8. And the best model ARIMA(8, 0, 1) is obtained from the error graph in Figure 7.

This model, as shown in Figure. 8, describes the variation in the rainfall to a certain extent but it also has large amount of errors to be treated as an acceptable modelling technique.

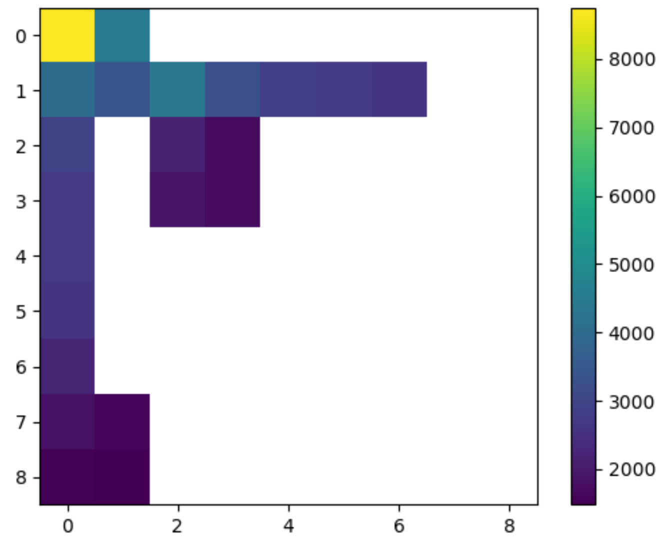

Figure 7: The errors for the monthly ARIMA parameters, the least being for the parameters(8, 1)

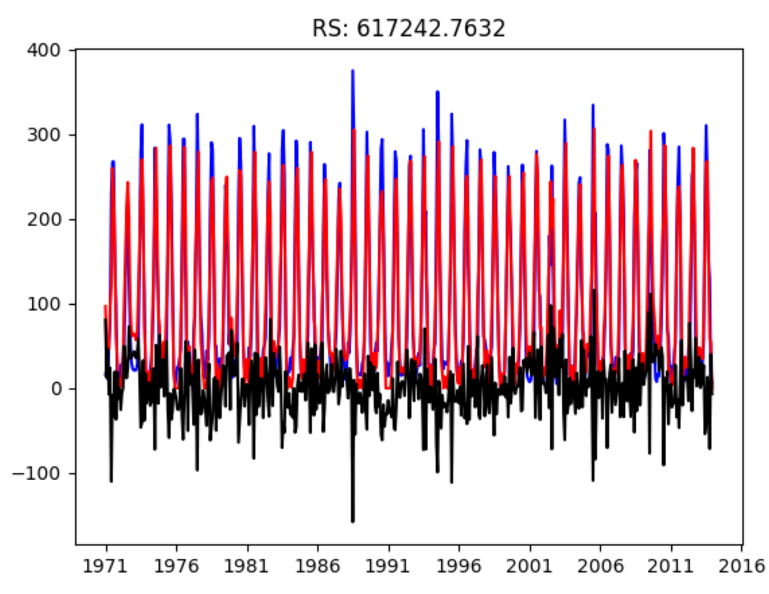

Figure 8: The Monthly All India ARIMA(8, 0, 1) model
The Monsoon and Annual Rainfall variation according to this model is 4.080% and 7.9124% against the standard deviation of 9% in the Annual Rainfall.

6.1.3 Daily Rainfall

The Daily Rainfall Data for the years 2008 to 2014 for the period between 1^{st} April to 30^{th} November was analysed. Since the data for the intermediate 4-month period was not accessible, the data for the years was analysed separately and the best set of parameters for the collective models was (4, 0, 1)

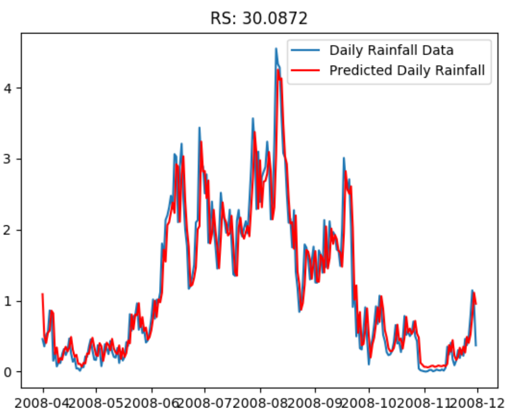

Figure 9: The ARIMA(4, 0, 1) All India model fitted for the year 2008

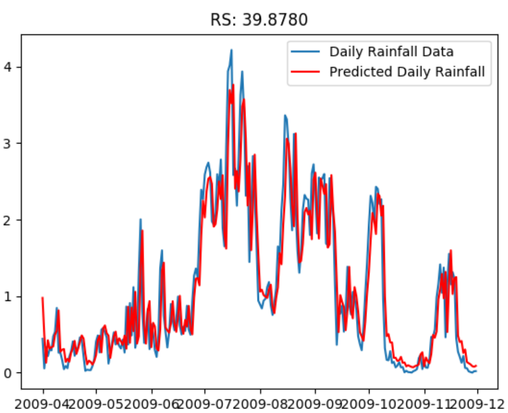

Figure 10: The ARIMA(4, 0, 1) All India model fitted for the year 2009

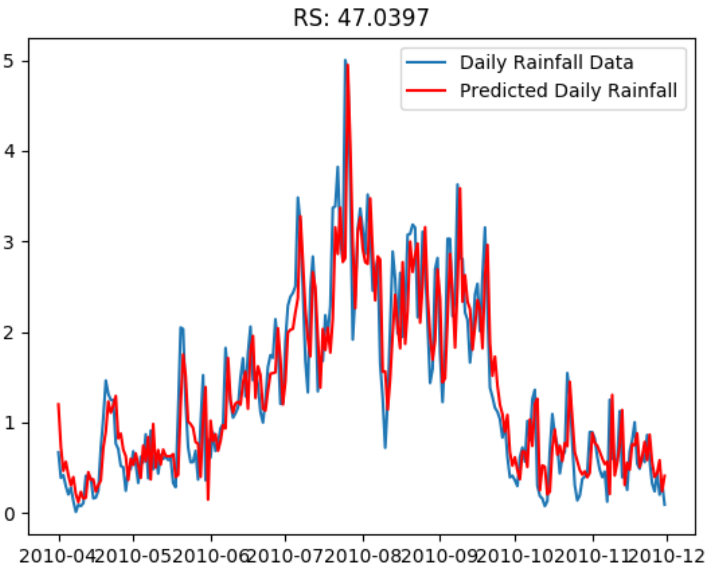

Figure 11: The ARIMA(4, 0, 1) All India model fitted for the year 2010

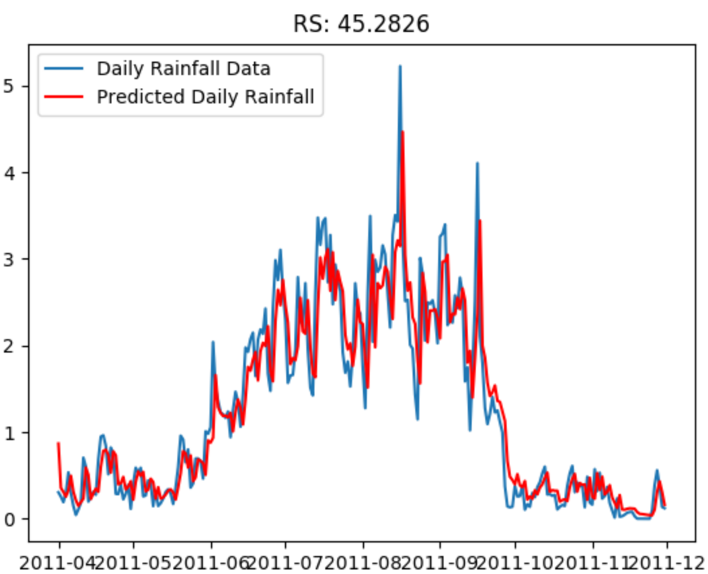

Figure 12: The ARIMA(4, 0, 1) All India model fitted for the year 2011

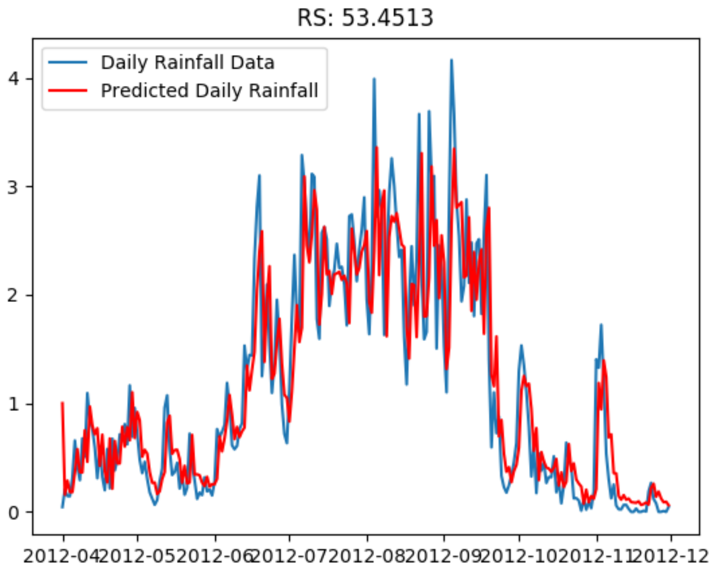

Figure 13: The ARIMA(4, 0, 1) All India model fitted for the year 2012

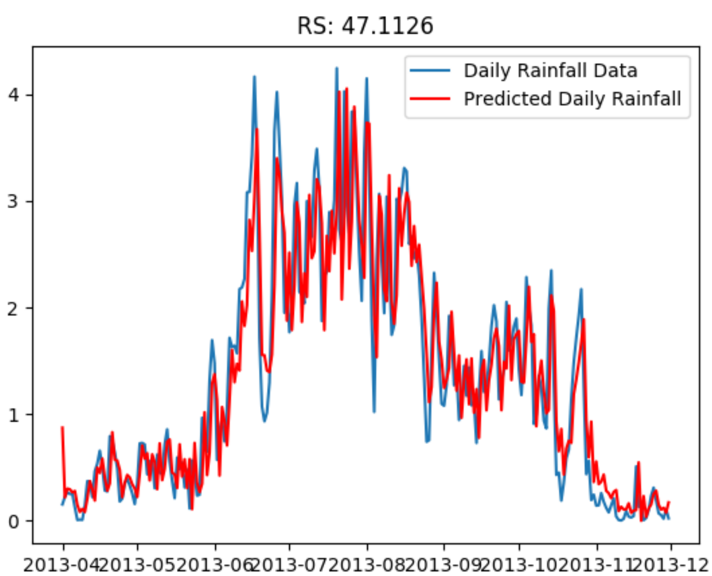

Figure 14: The ARIMA(4, 0, 1) All India model fitted for the year 2013

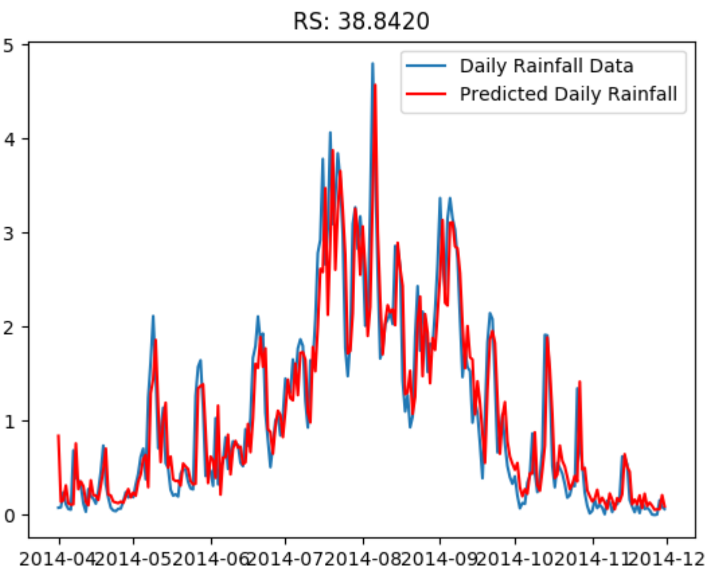

Figure 15: The ARIMA(4, 0, 1) All India model fitted for the year 2014

Here, we can see that the fit for the Daily data explains a fair bit of the actual time series in each of the years. There is a lag between the actual and the predicted time series but the error is sufficiently minimal in each of the cases. However, when we look at the coefficients of the different years, for each of the years, the coefficients vary a lot without apparently following any kind of pattern. This daily time scale ARIMA model for All India rainfall however, may have potential to be turned into a good predictive model with some modifications.

6.2 Bangalore

The rainfall data for the city of Bangalore has also been analysed for Annual, Monthly and Daily scales. However, since we are looking at a much smaller scale compared to the whole country, the degree of variability is much higher and the model fails at the fundamental levels in the Annual and Monthly scales.

6.2.1 Annual Scale

Here, we have used Annual Rainfall data for the years 1901 to 2002. The best parameters for the Annual Rainfall and the Monsoon Rainfalls are (2, 0, 2) and (1, 0, 0) respectively.

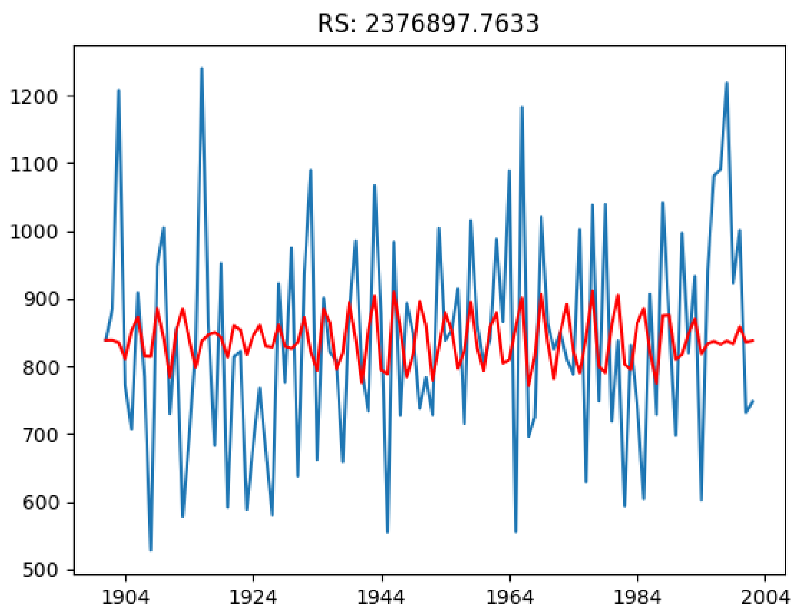

Figure 16: Annual Rainfall ARIMA(2, 0, 2) Model Bangalore
The model fails to explain both the variability and the rising and falling trends of the rainfall in this model and gives almost no information on the Rainfall.

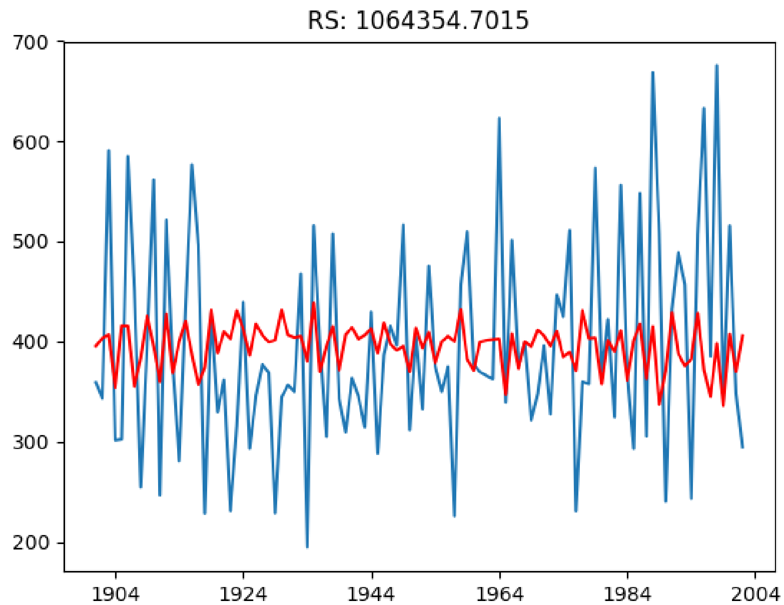

Figure 17: Annual Rainfall ARIMA(1, 0, 0) Model Bangalore
The ARIMA model similarly fails for the Monsoon Rains as for the Annual Rainfall and describes nothing for the actual time series.

6.2.2 Monthly Scale

We also performed time series analysis for monthly rainfall in the city of Bangalore. The data used was the monthly data from the years 1971 to 2002. (8, 0, 2) was the best model and was selected for modelling. However, the errors in this case were again very high and it is therefore, not very useful in predictions.

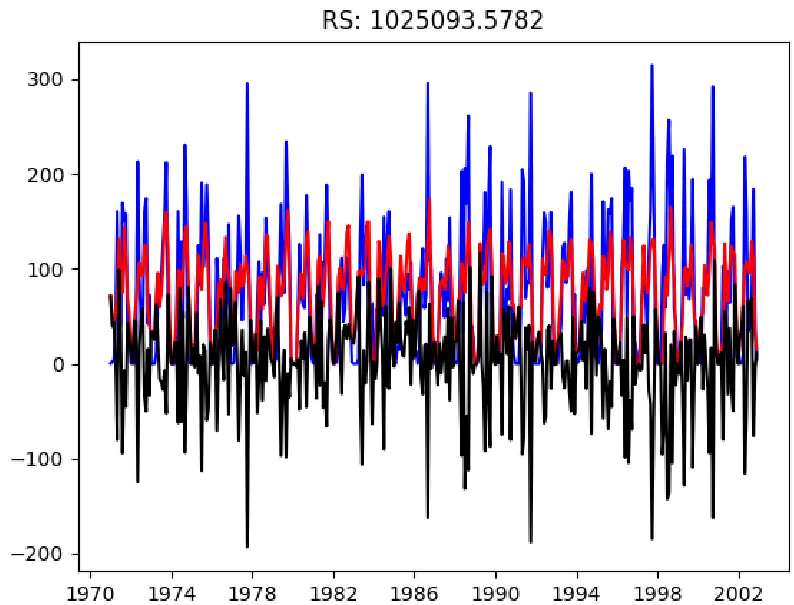

Figure 18: Monthly ARIMA(8, 0, 2) Model for Bangalore

6.2.3 Daily Scale

For the daily scale, we again had data for the years 2008 - 2014 from April to November. We tried to apply an optimal model for all of the years and we found that the best model was of the order (2, 0, 0). However, unlike the daily rainfall model for the All India Rainfall, we do not find the ARIMA(2, 0, 0) model accurately describing any of the years except 2008.

The fitted ARIMA models for the years 2008 - 2014 follow. We can see that the model largely fails to capture any kind of variation in the model. However, it succeeds in pointing out the peaks in the graphs. The lower values are also not very well described here.

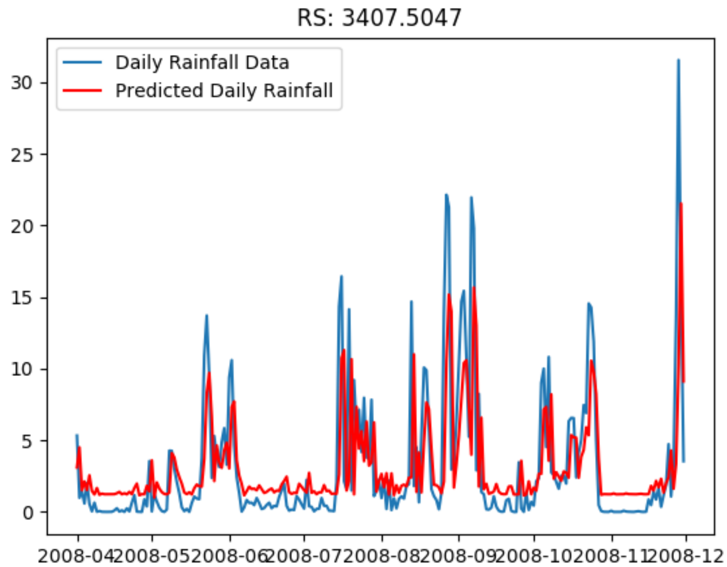

Figure 19: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2008

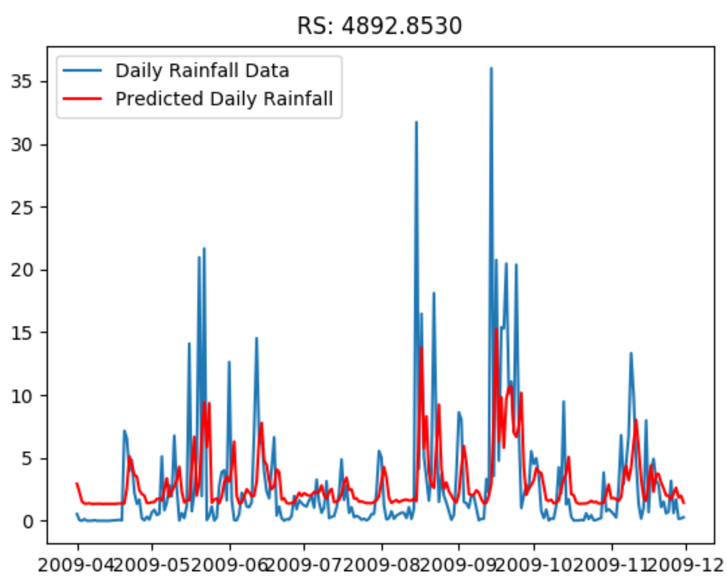

Figure 20: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2009

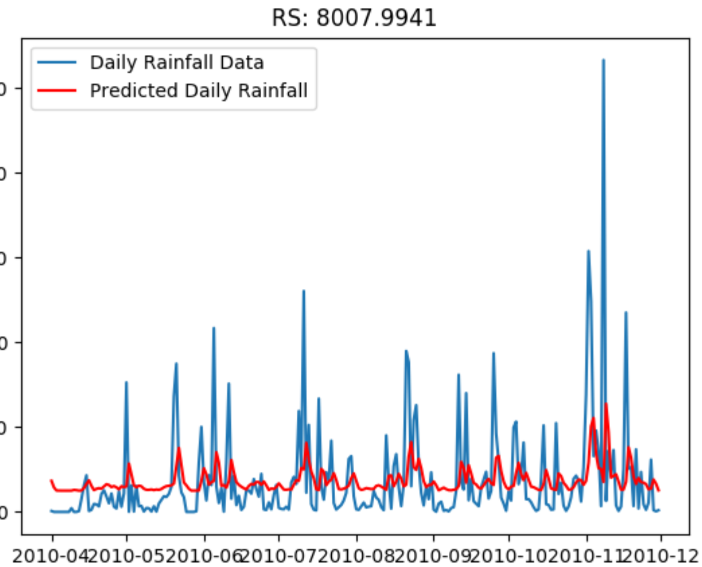

Figure 21: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2010

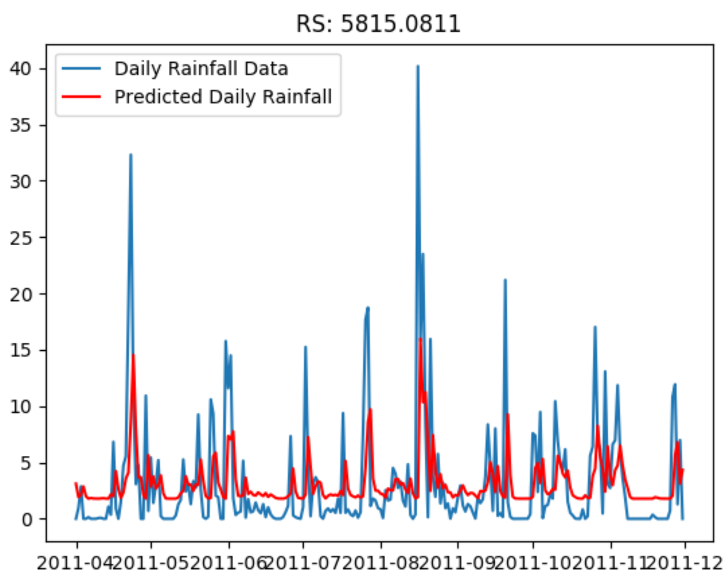

Figure 22: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2011

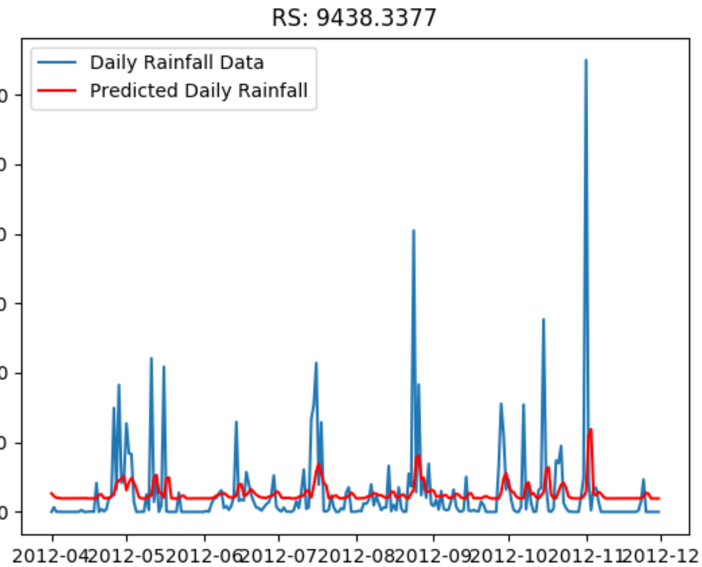

Figure 23: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2012

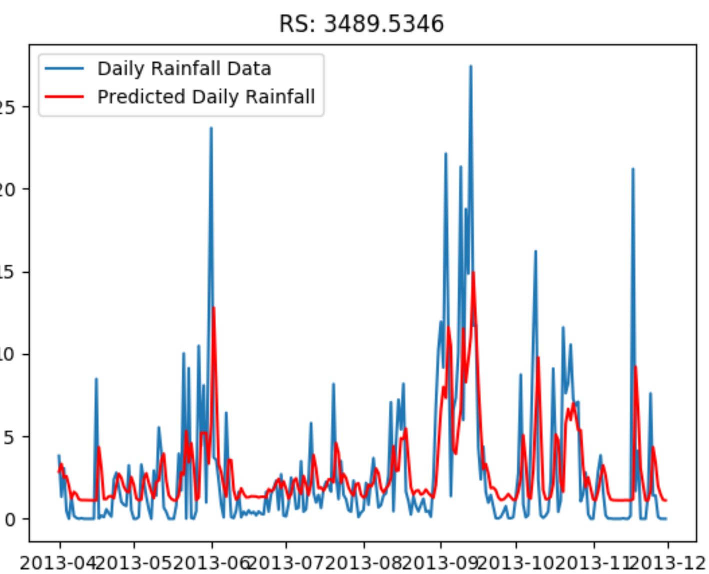

Figure 24: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2013

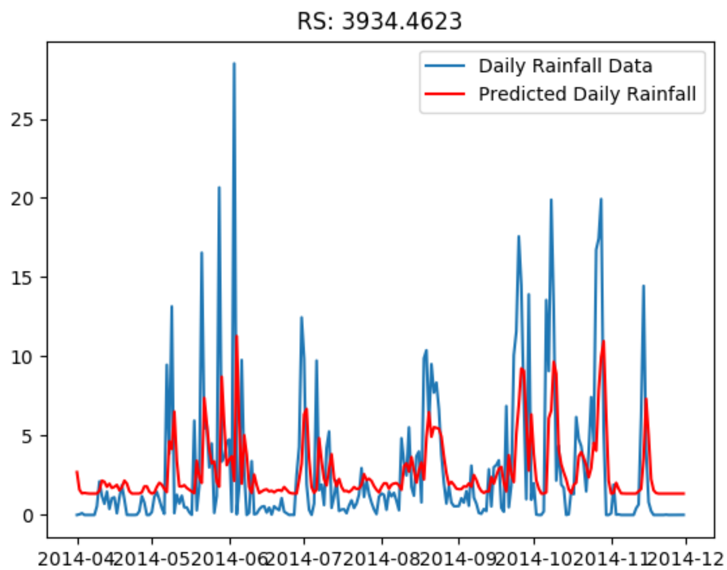

Figure 25: The ARIMA(2, 0, 0) Bangalore model fitted for the year 2014

7 Conclusion

It can be inferred from the graphs that the ARIMA model is not very efficient in estimating the variance in the Annual, monsoon or even the Monthly model. It has however been successful in doing so for the Daily All India Rainfall Data. I would, therefore, like to conclude from the given results that the ARIMA model may be useful in predicting rainfall on the All India Daily level, but only with some amount of modifications to the model.

At the smaller time scale however, the amount of variability is very high ad the ARIMA modelling technique breaks down altogether. At the Annual and Monthly levels, the ARIMA model fails to capture any information about the data while in the Daily level, the ARIMA model only succeeds in predicting the rising and falling behaviour to a small extent.

8 References

1. R. R. Kelkar, Monsoon Predictions
2. Tipping elements of the Indian monsoon: Prediction of onset and withdrawal, 2016
3. Forecasting of seasonal monsoon rainfall at subdivision level, 2006
4. Updated operational methods for long-range forecast of Indian summer monsoon rainfall, 2003
5. A power regression model for long range forecast of southwest monsoon rainfall over India, 1991
6. Parametric and power regression models: New approach to long range forecasting of monsoon rainfall in India, 1989
7. `http://www.scipy-lectures.org/`
8. `http://machinelearningmastery.com/category/time-series/`
9. `https://www.analyticsvidhya.com/blog/2016/02/time-series-forecasting-codes-python/`
10. `http://www.indiawaterportal.org/met_data/`