信号与系统课程笔记：Lecture 5

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1 复习

1.1 Guideline

(1) $\delta[k] \longrightarrow \boxed{h[k]} \longrightarrow h[k]$

$\delta(t) \longrightarrow \boxed{h(t)} \longrightarrow h(t)$

(2) $f[k] = \sum_{\tau=-\infty}^{+\infty} f[\tau]\delta[k-\tau]$

$f(t) = \int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau)\mathrm{d}\tau$

(3) $y[k] = f[k] * h[k]$

$y(t) = (f*h)(t) = f(t)*h(t) = \int_{-\infty}^{+\infty} f(\tau)h(t-\tau)\mathrm{d}\tau$

1.2 Review

$$12312 \times 321 \equiv [1,2,3,1,2] \bigotimes [3,2,1] \triangleq f[k] * h[k]$$

1.3 Base function

Option 1:

$$
\begin{aligned}
[1,2,3,2,1] =& 1 \times [1,1,1,1,1,1\cdots]+ \\
& 1 \times [0,1,1,1,1,1\cdots]+ \\
& 1 \times [0,0,1,1,1,1\cdots]+ \\
& (-2) \times [0,0,0,1,1,1\cdots]+ \\
& 1 \times [0,0,0,0,1,1\cdots]+ \\
& (-1) \times [0,0,0,0,0,1\cdots]
\end{aligned}
$$

Option 2:

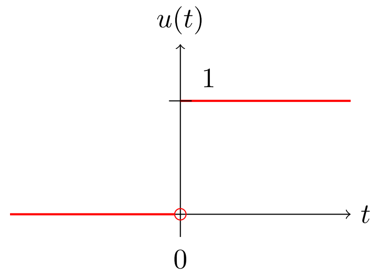
$$
\begin{aligned}
[1,2,3,2,1] =& 1 \times [1,1,1,0,0,0]+ \\
& 1 \times [0,1,1,1,0,0]+ \\
& 1 \times [0,0,1,1,1,0]+ \\
& (-2) \times [0,0,0,1,1,1]+ \\
& \cdots
\end{aligned}
$$

因此，转换为通用格式为：

$$\begin{aligned}[1,2,3,2,1] =& a_1 \times [a,b,c,d,e,f\cdots]+\\ & a_2 \times [0,a,b,c,d,e\cdots]+\\ & a_3 \times [0,0,a,b,c,d\cdots]+\\ & \cdots\end{aligned}$$

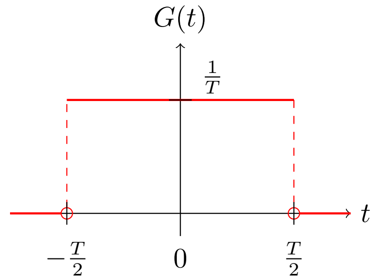
2 特殊信号

2.1 阶跃函数



$$u(t) = \begin{cases} 1 & t \geqslant 0 \\ 0 & t < 0 \end{cases} \tag{1}$$

2.2 门函数



$$G(t) = \begin{cases} \frac{1}{T} & t \in [-\frac{T}{2}, \frac{T}{2}] \\ 0 & 其他 \end{cases} \tag{2}$$

注意：不同书籍内的门函数表达式会有所不同，但其本质不变。

2.3 Dirac delta 函数

$$\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & 其他 \end{cases} \tag{3}$$

并且：

$$\int_{-\infty}^{+\infty} \delta(\tau)\mathrm{d}\tau = 1 \tag{4}$$

2.4 Dirac delta 函数性质

(1) $\int_{-\infty}^{t} \delta(\tau)\mathrm{d}\tau = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$

(2) $\delta'(t) \quad \delta''(t) \quad \delta'''(t) \quad \cdots$

(3) $\delta(t-t_0) = \begin{cases} +\infty & t = t_0 \\ 0 & \text{其他} \end{cases}$ and $\int_{-\infty}^{+\infty} \delta(\tau - t_0)\mathrm{d}\tau = 1$

(4) $A\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & \text{其他} \end{cases}$ and $A\int_{-\infty}^{+\infty} \delta(\tau)\mathrm{d}\tau = A$

(5) $f(t)\delta(t) = \begin{cases} +\infty & t = 0 \\ 0 & \text{其他} \end{cases}$ and $\int_{-\infty}^{+\infty} f(\tau)\delta(\tau)\mathrm{d}\tau = f(0)$

(6) 采样特性（sifting property）：$f(t)\delta(t-t_0) = f(t_0)\delta(t-t_0)$ and

$\int_{-\infty}^{+\infty} f(t)\delta(t-t_0)\mathrm{d}t = f(t_0)\int_{-\infty}^{+\infty} \delta(t-t_0)\mathrm{d}t = f(t_0)$

(7) 对偶性：$\delta(t-t_0) = \delta(t_0-t)$

问题：$f(t) = \int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau)\mathrm{d}\tau$ 如何推导而来？

① 令 $t=\tau$，$\int_{-\infty}^{+\infty} f(\tau)\delta(\tau-t_0)\mathrm{d}\tau = f(t_0)$

② 令 $t_0 = t$，$\int_{-\infty}^{+\infty} f(\tau)\delta(\tau-t)\mathrm{d}\tau = f(t)$

③ 根据性质 (7)，$\int_{-\infty}^{+\infty} f(\tau)\delta(t-\tau)\mathrm{d}\tau = f(t)$

(8) $[f(t)\delta(t)]' = f'(t)\delta(t) + f(t)\delta'(t)$

$[f(t)\delta(t)]' = [f(0)\delta(t)]' = f'(0)\delta(t) + f(t)\delta'(t)$

$f(t)\delta'(t) = f(0)\delta'(t) - f'(0)\delta(t)$

(9) $\int_{-\infty}^{+\infty} f(t)\delta'(t)\mathrm{d}t = -f'(0)$

(10) $\delta(at) = \frac{1}{|a|}\delta(t)$

3 卷积的性质

(1) 交换律：$u * v = v * u$

(2) 分配律：$(u+v) * w = u * w + v * w$

(3) 结合律：$(u * v) * w = u * (v * w)$

(4) $u(t-t_0) * v(t-t_1) = (u * v)(t - t_0 - t_1)$

(5) $f(t) = f(t) * \delta(t)$

(6) $f(t) * \delta'(t) = f'(t) * \delta(t) = f'(t)$

(7) $f(t)*u(t)=\int_{-\infty}^{t}f(\tau)\mathrm{d}\tau$

(8) 如果 $f_1(-\infty)=0$，$f_2(-\infty)=0$，那么 $f_1(t)*f_2(t)={f_1}'(t)*{f_2}^{-1}(t)$

(9) $(u*v)'(t)=[u(t)*v(t)]'=u'(t)*v(t)=u(t)*v'(t)$

(10) $\int_{-\infty}^{t}[u(\tau)*v(\tau)]\mathrm{d}\tau=\int_{-\infty}^{t}u(\tau)\mathrm{d}\tau*v(\tau)=u(\tau)*\int_{-\infty}^{t}v(\tau)\mathrm{d}\tau$

4 卷积的理解

$(f*h)(t)=\int_{-\infty}^{\infty}f(\tau)h(-\tau+t)\mathrm{d}\tau$