Quantum Computation 2022/23

TPC-1

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Figure 1 presents the circuit respective to quantum teleportation. Analyse this circuit and then solve the two exercises below.

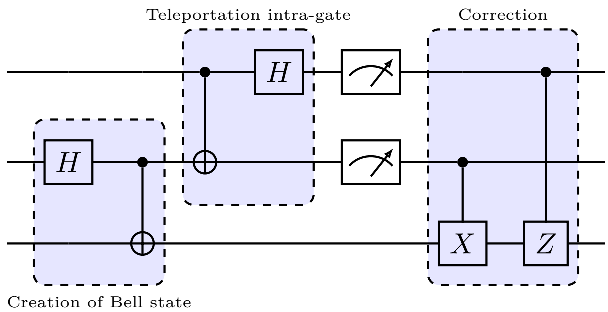

Figure 1: Quantum teleportation circuit.

Exercise 1. Write down the mathematical laws and definitions that were used at each step in the following computation. Note that the first step was already filled-in to serve as an example.

$$
\begin{aligned}
&(\alpha\,|0\rangle + \beta\,|1\rangle) \otimes \left(\frac{1}{\sqrt{2}}\,|00\rangle + \frac{1}{\sqrt{2}}\,|11\rangle\right)\\
&= \{ \text{ Distributivity of scaling over addition and the tensor law } v \otimes sw = s(v \otimes w) \ \}\\
&= \frac{1}{\sqrt{2}}\big(\,(\alpha\,|0\rangle + \beta\,|1\rangle) \otimes (|00\rangle + |11\rangle)\,\big)\\
&= \{ \ \dots \ \}\\
&= \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|100\rangle + \beta\,|111\rangle)\\
&\mapsto \{ \ \dots \ \}\\
&= \frac{1}{\sqrt{2}}(\alpha\,|000\rangle + \alpha\,|011\rangle + \beta\,|110\rangle + \beta\,|101\rangle)\\
&= \{ \ \dots \ \}\\
&= \frac{1}{\sqrt{2}}(|0\rangle \otimes \alpha\,|00\rangle + |0\rangle \otimes \alpha\,|11\rangle + |1\rangle \otimes \beta\,|10\rangle + |1\rangle \otimes \beta\,|01\rangle)\\
&= \{ \ \dots \ \}\\
&= \frac{1}{\sqrt{2}}\big(\,|0\rangle \otimes (\alpha\,|00\rangle + \alpha\,|11\rangle) + |1\rangle \otimes (\beta\,|10\rangle + \beta\,|01\rangle)\big)\\
&\mapsto \{ \ \dots \ \}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes (\alpha\,|00\rangle + \alpha\,|11\rangle) + \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes (\beta\,|10\rangle + \beta\,|01\rangle)\right) \\
&= \{\ \dots\ \} \\
&= \frac{1}{2}\left(((|0\rangle + |1\rangle) \otimes (\alpha\,|00\rangle + |11\rangle) + (|0\rangle - |1\rangle) \otimes (\beta\,|10\rangle + \beta\,|01\rangle)\right) \\
&= \{\ \dots\ \} \\
&= \frac{1}{2}\left(\alpha\,|000\rangle + \alpha\,|011\rangle + \alpha\,|100\rangle + \alpha\,|111\rangle + \beta\,|010\rangle + \beta\,|001\rangle - \beta\,|110\rangle - \beta\,|101\rangle\right) \\
&= \{\ \dots\ \} \\
&= \frac{1}{2}\left(\,|00\rangle \otimes (\alpha\,|0\rangle + \beta\,|1\rangle) + |01\rangle \otimes (\beta\,|0\rangle + \alpha\,|1\rangle) + |10\rangle \otimes (\alpha\,|0\rangle - \beta\,|1\rangle) + |11\rangle \otimes (-\beta\,|0\rangle + \alpha\,|1\rangle)\right)
\end{aligned}
$$

Exercise 2. The quantum teleportation protocol (Figure 1) starts by putting the qubits shared by Alice and Bob in the entangled state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. Show that after slight modifications the protocol will work equally well if these two qubits are put instead in the entangled state $\frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. Present the modified circuit in QISKIT and discuss how you can use the latter to test the circuit.

What to submit: A report in PDF containing the solutions to both exercises. Please send by email (nevrenato@gmail.com) your file named as "QC2223-N.PDF", where N is your student number. The subject of the email should be "QC2223-N".
Deadline: 24th October 2023 @ 23h59