Topics to discuss

Master Theorem / Master Method

Some More Examples.

Master Theorem or Master Method :

General Form, $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, where $f(n) = \theta(n^k \log^p n)$

so, it become, $T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$

where, $a \geq 1$, $b > 1$, $k \geq 0$ and p is real number.

Case I : If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

Case II : If $a = b^k$

a) If $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

b) If $p = -1$ then $T(n) = \theta(n^{\log_b a} \log \log n)$

c) If $p < -1$ then $T(n) = \theta(n^{\log_b a})$

Case III : If $a < b^k$

a) If $p \geq 0$ then $T(n) = \theta(n^k \log^p n)$

b) If $p < 0$ then $T(n) = \theta(n^k)$

Solve $T(n) = 4T\left(\frac{n}{2}\right) + n^2$ using Master Theorems

Solution :- $T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$

So, $a = 4$, $b = 2$, $f(n) = \theta(n^2 \log^0 n)$

and, $k = 2$ and $p = 0$

$a = 4$

$b^k = 2^2 = 4 \Rightarrow a = b^k$ & $p > -1$

Case - II (a)

$T(n) = \theta(n^{\log_b^a} \log^{p+1} n)$

$= \theta(n^{\log_2^4} \log^{0+1} n)$

$= \theta(n^2 \log^1 n)$

$T(n) = \theta(n^2 \log n)$

Solve $T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\log n}$ using Master theorem.

Solution :- $T(n) = a\,T\left(\frac{n}{b}\right) + \Theta(n^k \log^p n)$

$a = 2$, $b = 2$, $f(n) = \Theta(n^1 \log^{-1} n)$

So, $k = 1$, $p = -1$

Now, $a = 2$

$b^k = 2^1 = 2$

$\Rightarrow a = b^k$ and $p = -1$ $\Rightarrow$ Case II (b)

$T(n) = \Theta(n^{\log_b^a} \log\log n)$

$= \Theta(n^1 \log\log n)$

$\boxed{T(n) = \Theta(n \log\log n)}$

Follow Now

Start Practicing

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