## Topics to discuss

Solve $T(n) = \begin{cases} 2T(n-1) - 1 & , \text{if } n > 0 \\ 1 & \text{otherwise} \end{cases}$

by substitution Method

## Problem 2 :

$$T(n) = \begin{cases} 2T(n-1)-1 & , \text{if } n>0 \\ 1 & \text{otherwise} \end{cases}$$

$T(n)=1 \quad , n=0$

$T(0)=1$

$S = \frac{a(r^n-1)}{r-1}$

assume

$n-K=0 \quad \Rightarrow K=n.$

Solution :- $T(n) = 2T(n-1)-1$ —— ①

$n \to n-1$

$T(n-1) = 2T(n-2)-1$

$n \to n-2$

$T(n-2) = 2T(n-3)-1$

Substitute, in eqn. ①

$$\begin{aligned} T(n) &= 2T(n-1)-1 \\ &= 2\{2T(n-2)-1\}-1 \\ &= 2^2T(n-2)-2-1 \\ &= 2^2\{2T(n-3)-1\}-2-1 \\ &= 2^3T(n-3)-2^2-2-1 \\ &\vdots \\ &= 2^KT(n-K)-2^{K-1}-2^{K-2}\ldots\ldots-2^2-2-1 \end{aligned}$$

$$\begin{aligned} T(n) &= 2^KT(n-K)-2^{K-1}-2^{K-2}\ldots\ldots-2-1 \\ &= 2^nT(0)-2^{n-1}-2^{n-2}-\ldots-2-2^0 \\ &= 2^n-2^{n-1}-2^{n-2}-\ldots\ldots-2^2-2-2^0 \\ &= 2^n-(2^0+2^1+2^2+\ldots\ldots+2^{n-2}+2^{n-1}) \\ &= 2^n-\left\{\frac{2^0(2^n-1)}{2-1}\right\} \\ &= \cancel{2^n}-\cancel{2^n}+1 \\ &= 1 \end{aligned}$$

$$\boxed{T(n)=O(1)}$$

**Follow Now**

Start Practicing

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