Topics to discuss

Master Theorem / Master Method

Some More Examples.

Master Theorem or Master Method :

General Form , $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, where $f(n) = \theta(n^k \log^p n)$

So, it become, $T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$

where, $a \geq 1$, $b > 1$, $k \geq 0$ and p is real number.

Case I : If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

Case II : If $a = b^k$

a) If $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

b) If $p = -1$ then $T(n) = \theta(n^{\log_b a} \log \log n)$

c) If $p < -1$ then $T(n) = \theta(n^{\log_b a})$

Case III : If $a < b^k$

a) If $p \geq 0$ then $T(n) = \theta(n^k \log^p n)$

b) If $p < 0$ then $T(n) = \theta(n^k)$

Solve $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$ using master theorem.

Solution :- $T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$.

$a = 6$, $b = 3$, $f(n) = \theta(n^2 \log^1 n)$

$K = 2$, $p = 1$

$a = 6$

$b^K = 3^2 = 9 \Rightarrow a < b^K$ & $p \geq 0$

Case III (a) :

$$T(n) = \theta(n^k \log^p n)$$

$$= \theta(n^2 \log^1 n)$$

$$\boxed{T(n) = \theta(n^2 \log n)}$$

Solve $T(n) = \sqrt{2}\, T\left(\frac{n}{2}\right) + \log n$ using Master Theorem

$$T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$$

$a = \sqrt{2}$, $b = 2$, $f(n) = \theta(n^0 \log^1 n)$

$K = 0$, $p = 1$

$$\boxed{T(n) = \theta(\sqrt{n})}$$

So,

$a = \sqrt{2}$

$b^k = 2^0 = 1 \Rightarrow a > b^k$

Then,

$$T(n) = \theta\left(n^{\log_b a}\right)$$

$$= \theta\left(n^{\log_2 \sqrt{2}}\right)$$

$$= \theta\left(n^{\frac{1}{2}}\right)$$

Solve $T(n) = 2^n T\left(\frac{n}{2}\right) + n^n$ using masters theorem.

$a = 2^n$ so, master theorem is not applicable.

Q:- $T(n) = 0.7\,T\left(\frac{n}{4}\right) + \frac{1}{n}$

$a = 0.7$ Not applicable.

Q:- $T(n) = 4T\left(\frac{n}{8}\right) - n^2$

Not applicable.

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