Topics to discuss

Master Theorem / Master Method

Master Theorem or Master Method :

General Form , $T(n) = aT\left(\frac{n}{b}\right) + f(n)$, where $f(n) = \theta(n^k \log^p n)$

so, it become, $T(n) = aT\left(\frac{n}{b}\right) + \theta(n^k \log^p n)$

where, $a \geq 1$, $b > 1$, $k \geq 0$ and p is real number.

Case I : If $a > b^k$, then $T(n) = \theta(n^{\log_b a})$

Case II : If $a = b^k$

a) If $p > -1$ then $T(n) = \theta(n^{\log_b a} \log^{p+1} n)$

b) If $p = -1$ then $T(n) = \theta(n^{\log_b a} \log \log n)$

c) If $p < -1$ then $T(n) = \theta(n^{\log_b a})$

Case III : If $a < b^k$

a) If $p \geq 0$ then $T(n) = \theta(n^k \log^p n)$

b) If $p < 0$ then $T(n) = \theta(n^k)$

Solve $T(n) = 2T\left(\frac{n}{2}\right) + 1$ using Master Theorem.

Solution :- $T(n) = aT\left(\frac{n}{b}\right) + f(n)$

$= aT\left(\frac{n}{b}\right) + \theta\left(n^k \log^p n\right)$

$\boxed{T(n) = \theta(n)}$

Here, $a = 2$, $b = 2$

$f(n) = \theta\left(n^k \log^p n\right) = \theta(1)$

$= \theta\left(n^0 \log^0 n\right) = \theta(1)$

So, $k = 0$ and $p = 0$

Now,

$a = 2$

$b^k = 2^0 = 1 \implies a > b^k \implies$ Case I

$T(n) = \theta\left(n^{\log_b a}\right) = \theta\left(n^{\log_2 2}\right)$

$= \theta\left(n^1\right)$

Follow Now

Start Practicing

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