END SEM SOLUTIONS

Sol 1

(a) The Convolution is defined as

$$y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

→ Here $y_1(t) = 0$ at $t = 5$

So,

$$y_1(5) = \int_{-\infty}^{\infty} x_1(\tau)\, h(5-\tau)\, d\tau$$

$$= A \int_{5-T}^{5} x_1(\tau)\, d\tau = 0$$

1 mark.

If the lower limit is 1

The area of triangle b/w $\tau = 1$ and $\tau = 3$ is 2.

1 mark and cancels the area of rectangle b/w $\tau = 4$ & 5.

So, $\boxed{T = 4}$

→ from the $y_2(t) = 9$ at $t = 9$.

$$y_2(9) = A \int_{5}^{9} x_2(\tau) = 9$$

$$= A \int_{5}^{9} \sin\left(\frac{\pi \tau}{3}\right) d\tau$$

1 mark

$$= -\frac{A}{\pi/3} \cos\left(\frac{\pi\tau}{3}\right)\Bigg|_{0.5}^{9}$$

$$\cancel{9} = \frac{9\cancel{A}}{\cancel{\pi/3}} \qquad 9 = \frac{9A}{2\pi}$$

So, $\boxed{A = 2\pi}$

1 mark.

(b)

$y_2(t)$ is computed for all t in three ranges.

Case I :→ $t < 0$

No overlap b/w $x_2(\tau)$ and $h(t-\tau)$

$\therefore \boxed{y_2(t) = 0}$

0.5 mark.

Case II. $0 \leq t < 4$

Partial overlap b/w $x_2(\tau)$ and $h(t-\tau)$

$$y_2(t) = 2\pi \int_0^t \sin\left(\frac{\pi\tau}{3}\right) d\tau.$$

$$= -\frac{2\pi}{\pi/3} \cos\left(\frac{\pi\tau}{3}\right)\Bigg|_0^t$$

$$= 6\left(1 - \cos\left(\frac{\pi t}{3}\right)\right).$$

0.5 mark.

III : $\rightarrow$ $t \geq 4$

$$y_2(t) = 2\pi \int_{t-4}^{t} \sin\left(\frac{\pi\tau}{3}\right) d\tau$$

$$= 6\left(\cos\left(\frac{\pi(t-4)}{3}\right) - \cos\left(\frac{\pi t}{3}\right)\right).$$

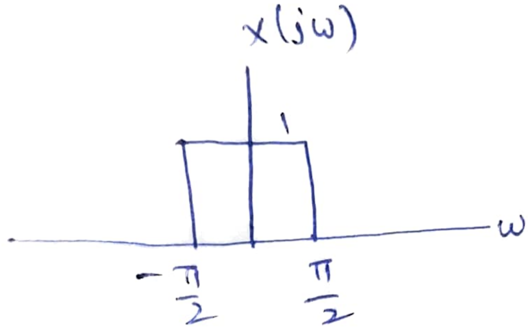
1 mark

So,

$$y_2(t) = \begin{cases} 0, & t < 0 \\ 6\left(1 - \cos\left(\frac{\pi t}{3}\right)\right); & 0 \leq t < 4 \\ 6\left(\cos\left(\frac{\pi(t-4)}{3}\right) - \cos\left(\frac{\pi t}{3}\right)\right); & t \geq 4. \end{cases}$$

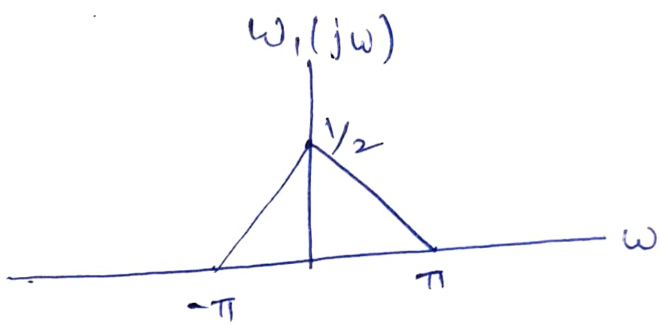
2. →

(a) $X(j\omega)$



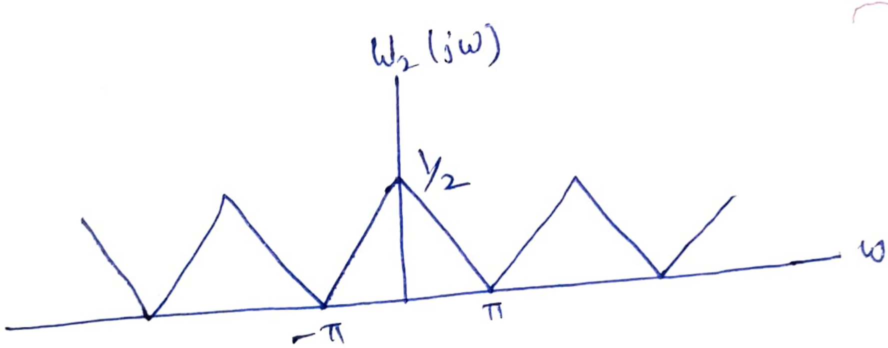
2 marks

(b)



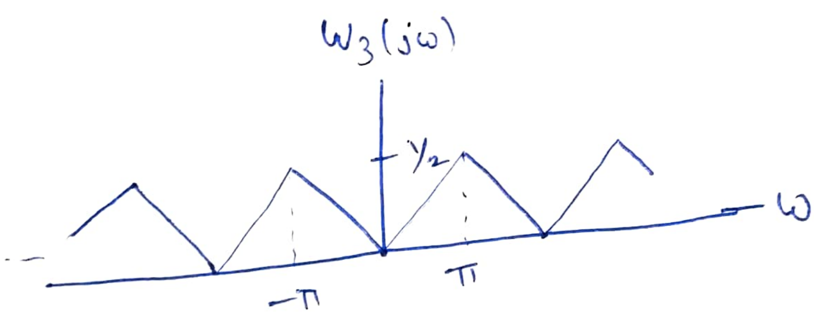
2 marks

(c)



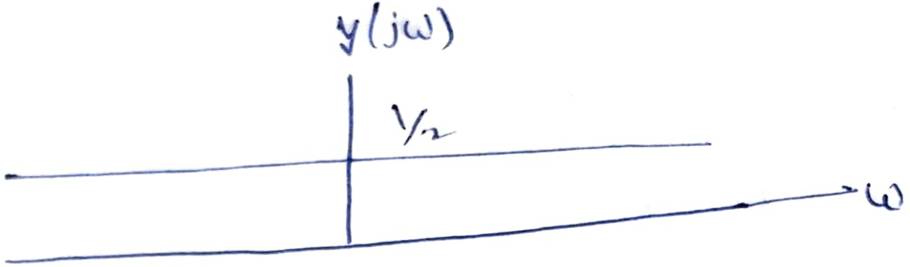
2 marks

(d)



2 marks

(e)



2 Marks

$y(t) = \frac{1}{2}\,\delta(t).$

END SEM SOLUTIONS

Q3

First of all, impulse sampling of $x_c(t) \to x_d[n]$

$$\therefore \; x_d[n] = x_c(nT)$$

(a)

$$x_d[n] = \frac{1}{2\pi} \int_{2\pi} X_d(e^{j\Omega}) e^{j\Omega n} \, d\Omega$$

$$= x_c(nT) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X_c(j\omega) e^{j\omega nT} \, d\omega$$

1 Mark.

$$\therefore \; X_d(e^{j\Omega}) = \frac{1}{T} X_c(j\omega) \Big|_{\omega = \frac{\Omega}{T}}$$

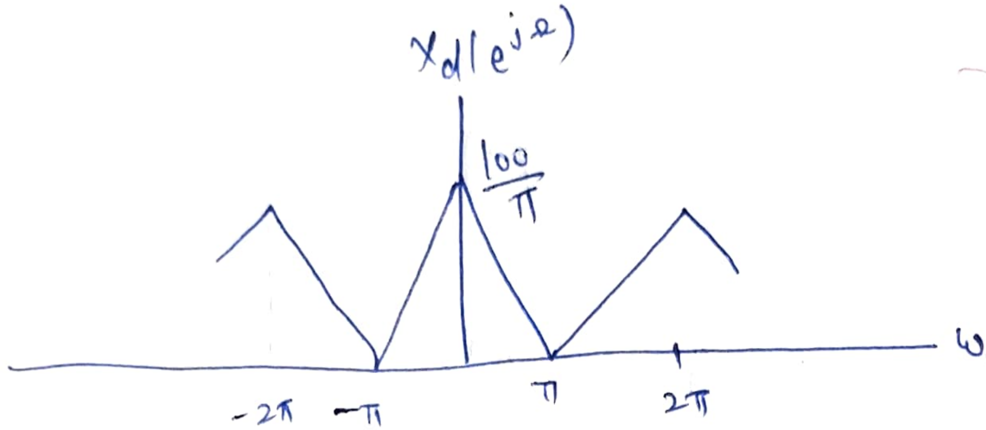
Since $d\omega = \frac{1}{T} d\Omega$

$\boxed{\text{Put } T = \pi/100}$

$$\therefore \; X_d(e^{j\Omega}) = \frac{1}{T} X_c\left(j\frac{\Omega}{T}\right) = \frac{100}{\pi} X_c\left(j\frac{\Omega}{\pi/100}\right)$$

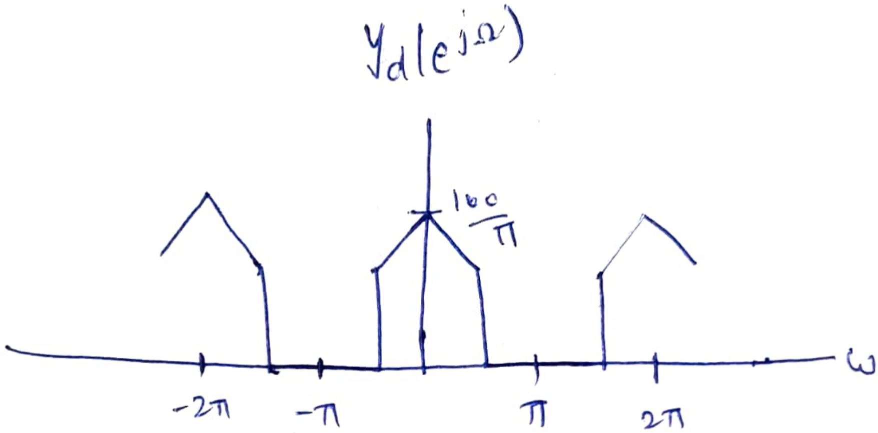
$\rightarrow$ Cut-off freq. $\omega = 100 \rightarrow \Omega = \omega T = 100 \times \frac{\pi}{100} = \pi$

$\therefore \; X_d(e^{j\Omega})$ is periodic with 2π as DTFT.



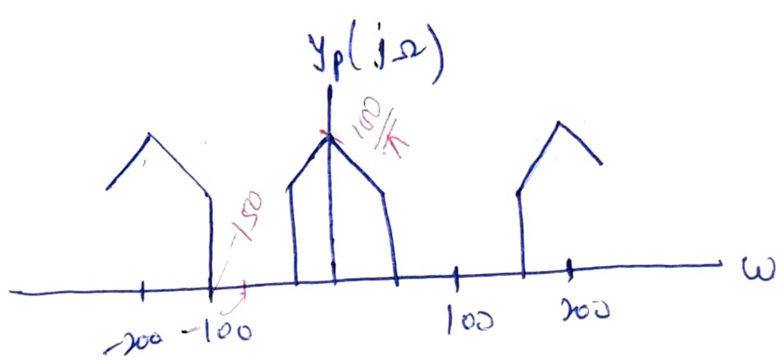
1 Mark

(b) o/p of ideal low pass filter



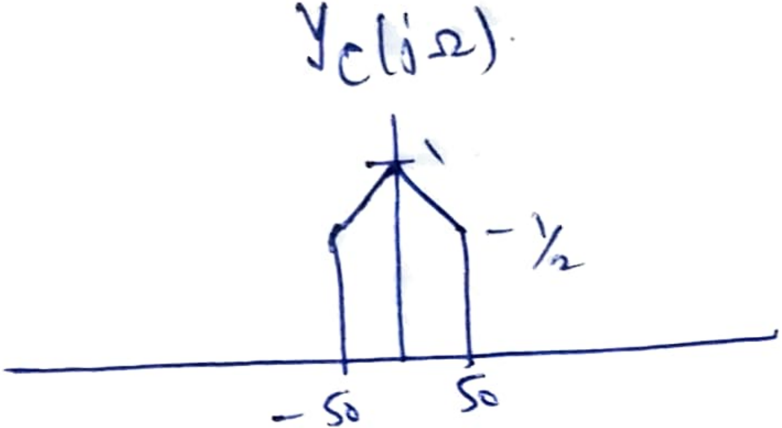
2 marks

(c) o/p of Impulse Reconstruction



2 marks

(d) final o/p, with DC value is $\frac{100}{\pi} \times T = 1$



2 Marks.

SOLUTIONS

4.

$X(s)$ has only two Poles.

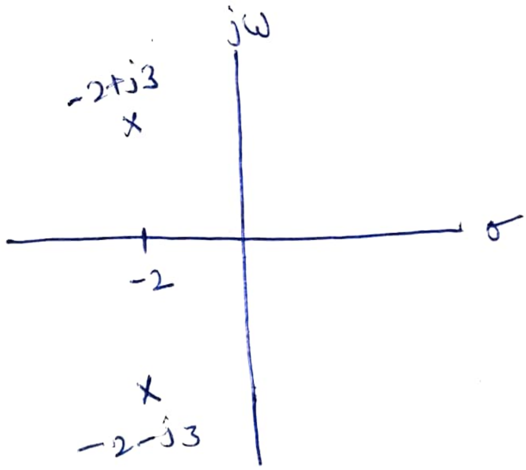
$X(s)$ is in the form

$$X(s) = \frac{A}{(s+a)(s+b)}$$

Given that one pole at $s = -2+3j$

Here, $x(t)$ is real, then poles of $X(s)$ must exist in complex conjugate

1 Mark.



$\therefore\ a = 2-j3$ and $b = 2+j3$

$$\therefore\ X(s) = \frac{A}{(\underbrace{s+2}_{a}\underbrace{-j3}_{b})(\underbrace{s+2}_{a}\underbrace{+j3}_{b})}$$

$$X(s) = \frac{A}{(s+2)^2 - (j3)^2}$$

$$= \frac{A}{s^2+4s+4+9}$$

2 marks

$$X(s) = \frac{A}{s^2+4s+13}$$

$$X(0) = \frac{A}{13} \Rightarrow 2 = \frac{A}{13} \Rightarrow \boxed{A = 26}$$

$$\boxed{X(s) = \frac{26}{s^2+4s+13}}$$

ROC is either

$Re\{s\} > -2$

or

$Re\{s\} < -2$

Inplace of e^{-st_0}

$x(t) \longleftrightarrow X(s+5) \rightarrow$ Not absolutely integrable.

Shifting to left by 5 and not contain $j\omega$-axis

$\boxed{\text{The ROC has to be } Re\{s\} < -2.}$

1 Mark

$$\frac{26}{(s+2-3j)(s+2+3j)} = \frac{a}{(s+2-3j)} + \frac{b}{(s+2+3j)}$$

Using Partial fraction.

$$a(s+2+3j) + b(s+2-3j) = 26.$$

$$(a+b)s + (2+3j)a + (2-3j)b = 26.$$

$$a+b=0$$

$$(2+3j)a - (2-3j)a = 26$$

$$\cancel{2a} + 3ja - \cancel{2a} + 3ja = 26$$

1 mark

$$a = \frac{26}{6j} = \frac{13}{3j}, \quad b = -\frac{13}{3j}$$

$$\frac{13}{3j(s+2-3j)} - \frac{13}{3j(s+2+3j}$$

$$-\frac{13j}{3(s+2-3j)} + \frac{13j}{3(s+2+3j)}.$$

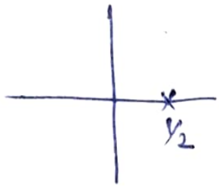
$$= -\frac{13j}{3} e^{-(2-3j)t} u(-t) + \frac{13j}{3} e^{-(2+3j)t} u(-t)$$

1 mark

$$= \frac{13j}{3} u(-t) \left[e^{-(2+3j)t} - e^{-(2-3j)t} \right].$$

Given

(a) # $X(z)$ is known to have a pole at $z = 1/2$



let R be the region of convergence.

$x_1[n] = 4^n x[n]$ is absolutely integrable:
ie. it includes the unit circle.
the updated ROC is $4R$.

Similarly, we know that
$x_2[n] = 8^n x[n]$ is not absolutely integrable
ie. it doesn't includes the unit circle.
the updated ROC is $8R$.

1 Mark.

This implies that there is ROC, R lying inside the unit circle and is not extending to infinity. It extends inwards from pole at $1/2$.
But since, both $4^n(x[n])$ and $8^n(x[n])$ are not absolutely integrable.
ie ROC is a ring lying within unit circle.

1 mark

ie $x[n]$ is two sided.

1 marks

(b) $\because$ $x[n]$ is not absolutely integrable
it is not Fourier transformable.

1 marks