ECE250: Signals and Systems
Monsoon 2022

End-Semester Exam

17/12/2022

Max marks: 34 | Duration: 120 mins

Total number of questions: 05

Instructions

1. **Please do not plagiarize. Any act of plagiarism will be dealt with strictly as per the institute's policy.**
2. Please provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.

Questions

1. (6 points) We are given that the impulse response of a continuous-time LTI system is of the form as shown below, where A and T are unknown.

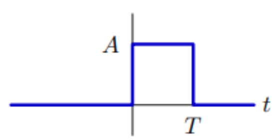

Figure 1: Impulse response (problem-1).

- When the system is subjected to the input $x_1(t)$ as shown in Fig. 2, the output $y_1(t)$ is zero at $t = 5$.

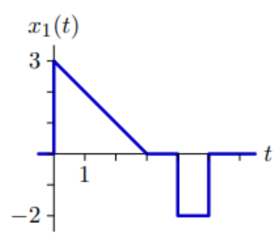

Figure 2: Input $x_1(t)$ (problem-1).

- When the input is $x_2(t) = sin\left(\frac{\pi t}{3}\right)u(t)$, the output $y_2(t)$ is equal to 9 at $t = 9$.

(a) (4 pts) Determine A and T.

(b) (2 pts) Also determine $y_2(t)$ for all values of t.

2. (10 points) Consider the following transformations from $x(t)$ to $y(t)$ as shown in Fig. 3, where:

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-k). \tag{1}$$

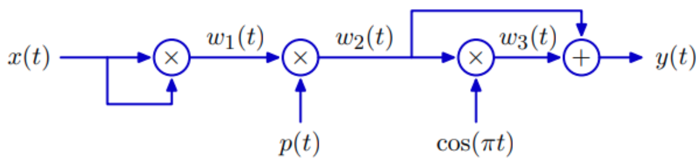

Figure 3: Transformations (problem-2).

Sketch the following signals when $x(t) = sin(\pi t/2)/(\pi t)$:

(a) (2 pts) $X(j\omega)$

(b) (2 pts) $W_1(j\omega)$

(c) (2 pts) $W_2(j\omega)$

(d) (2 pts) $W_3(j\omega)$

(e) (2 pts) $Y(j\omega)$

3. (8 points) Sampling and reconstruction allow us to process continuous time signals using digital electronics as shown in the following figure. The "impulse sampler" and "impulse reconstruction"

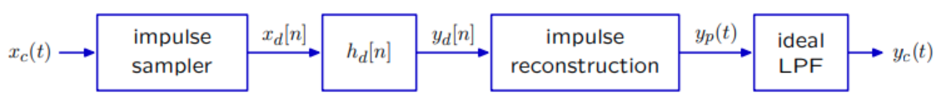

Figure 4: Sampling and reconstruction (problem-3).

use sampling interval $T = \pi/100$. The function $h_d[n]$ represents the unit-impulse response of an ideal discrete time low-pass filter with gain of 1 for frequencies in the range $-\frac{\pi}{2} < \Omega < \frac{\pi}{2}$. The "ideal LPF" passes frequencies in the range $-100 < \omega < 100$. It also has a gain of T throughout its pass band. Assume that the Fourier transform of the input $x_c(t)$ is $X_c(j\omega)$ shown below:

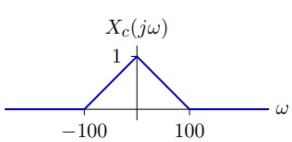

Figure 5: Sampling and reconstruction (problem-3).

Compute the following:

(a) (2 pts) $X_d(e^{j\Omega})$

(b) (2 pts) $Y_d(e^{j\Omega})$

(c) (2 pts) $Y_p(j\Omega)$

(d) (2 pts) $Y_c(j\Omega)$

Hint: Use relation $\Omega = \omega T$ for appropriate axes scaling from continuous to discrete frequencies.

4. (6 points) For a real signal $x(t)$ with the Laplace transform $X(s)$ the following information is given:

   - $X(s)$ has exactly two poles
   - $X(s)$ has no zeros in the finite s-plane
   - $X(s)$ has one pole at $s = -2 + 3j$.
   - $e^{-5t}x(t)$ is not absolutely integrable.
   - $X(0) = 2$.

   (a) (4 Points) Determine $X(s)$ and specify its region of convergence.

   (b) (2 points) Determine $x(t)$.

5. (4 points) Following information is provided for a discrete time signal $x[n]$:

   - $x[n]$ has a rational z-transform $X(z)$.
   - $X(z)$ is known to have a pole at $z = 1/2$.
   - $x_1[n] = 4^n x[n]$ is absolutely summable.
   - $x_2[n] = 8^n x[n]$ is not absolutely summable.

   (a) (3 Points) Determine whether $x[n]$ is left sided, right sided, or two sided. Support your answer with proper justification.

   (b) (1 Point) Is $x[n]$ Fourier transformable? Justify your answer based on the analysis in part (a) above.