Mid-Semester Exam: Maths-I (Linear Algebra)

Indraprastha Institute of Information Technology, Delhi

21st February, 2021

Duration: 60 minutes **Maximum Marks:** 50

Question 1.

(a) (5 marks) Find the values of x for which the following is an augmented matrix corresponding to a consistent system:

$$\begin{bmatrix} 1 & -2 & 1 & x \\ 0 & 5 & -2 & x^2 \\ 4 & -23 & 10 & x^3 \end{bmatrix}$$

(b) (5 marks) Determine the RREF of the matrix formed by substituting x with π in the matrix in part (a).

Question 2.

(a) (5 marks) Let

$$A = \begin{bmatrix} 1 & 2 & x \\ 0 & 3 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Express the inverse of A as a function of x (i.e. a matrix whose entries are functions of x), without calculating the determinant or using Cramer's rule.

(b) Is the span of the columns of A^{-1} all of $\mathbb{R}^3$ (YES/NO) ? Justify your answer briefly.

(Note for Section A students: You may assume that x is a fixed scalar in Part (b).)

Question 3. Let

$$V = \{x \in \mathbb{R} \;:\; x > 0\},$$

and define addition for V by

$$x \oplus y := xy$$

and scalar multiplication by any $\alpha \in \mathbb{R}$ by

$$\alpha * x = x^{\alpha}.$$

(a) (7 marks) Verify the closure axioms, the commutative, zero and inverse properties for addition, and the property $1 * x = x$ for all $x \in V$.

(Remark: V is in fact a vector space over the field $\mathbb{R}$. You need not verify the other properties of a vector space.)

(b) (3 marks) Is V a subspace of $\mathbb{R}$ regarded as a vector space over itself (YES/NO) ? Justify your answer clearly.

Question 4 (10 marks)**.** Choose any four of the five sets below. For each set you choose, state whether or not it is a subspace of $M_{3\times 3}$ (the space of all 3×3 matrices having real entries). Justify each answer. All choices carry equal marks.

(a) The set of all invertible 3×3 invertible matrices

(b) The set of all 3×3 matrices whose trace is 0 (The trace of a square matrix A is the sum of its diagonal entries.)

(c) The set of all 3×3 echelon matrices

(d) The set of all symmetric 3×3 matrices (A square matrix A is said to be symmetric if $A^T = A$)

(e) The set of all skew-symmetric 3×3 matrices (A square matrix A is said to be skew-symmetric if $A^T = -A$)

(Note for Section B students: $M_{3\times 3}$ is the same as $\mathbb{R}^{3\times 3}$).

Question 5. (10 marks) Let V be a vector space over a field F. Suppose $v_1, v_2, \ldots, v_n$ are linearly independent in V and $w \in V$.

Show that if $v_1 + w, v_2 + w, \ldots, v_n + w$ are linearly dependent in V, then $w \in Span\{v_1, v_2, \ldots, v_n\}$.

(Note for Section A students: You may assume that $F = \mathbb{R}$.)