ECE250: Signals and Systems
Monsoon 2022

Mid-Semester Exam

18/10/2022

Max marks: 24 + 4(Bonus) Duration: 60 mins

Total number of questions: 05

Instructions

1. **Please do not plagiarize. Any act of plagiarism will be dealt with strictly as per the institute's policy.**
2. Please provide proper mathematical justifications with your answers. No marks will be awarded without a valid justification.

Questions

1. (C02) (6 points) Evaluate the response $y[n]$ of a system with the impulse response $h[n]$ and input $x[n]$ defined as:

$$x[n] = \left(\frac{1}{3}\right)^{-n} u(-n-1)$$

$$h[n] = u(n-1)$$

2. (C01) (4 points) Consider an LTI system S and a signal $x(t) = 2e^{-3t}u(t-1)$. If

$$x(t) \to y(t)$$

and

$$\frac{dx(t)}{dt} \to -3y(t) + e^{-2t}u(t),$$

determine the impulse response $h(t)$ of S.

3. (C03) (8 points) For a signal $x(t)$, the following information is given:

   i) $x(t)$ is a real signal

   ii) $x(t)$ is periodic with period $T = 6$ and has Fourier coefficients a_k.

   iii) $a_k = 0$ for $k = 0$ and $k > 2$.

   iv) $x(t) = -x(t-3)$

   v) $\frac{1}{6}\int_{-3}^{3} |x(t)|^2 dt = \frac{1}{2}$

   vi) a_1 is a positive real number

Here, each part describes some characteristics of the signal $x(t)$. Using the available information, infer signal $x(t)$. Show that $x(t) = Acos(Bt + C)$, and determine the values of constants A, B and C.

4. (C03) (6 points) Let $x[n]$ be a real discrete periodic signal with fundamental period N_0 and Fourier coefficients $c_k = a_k + jb_k$, where a_k and b_k are both real.

   (a) (3 pts) Show that $a_{-k} = a_k$ and $b_{-k} = -b_k$

   (b) (3 pts) Show that $c_{N_0/2}$ is real if N_0 is even.

5. (CO3)(4 points)(Bonus) Compute the Fourier transform of the signal $x(t)$, where:

$$x(t) = e^{-3|t-2|}$$