Test: Maths 1 - Linear Algebra

Maximum Marks: 10

6th February, 2021

Problem 1 (6 marks)**.** All parts carry equal marks.

(a) Let A be an $m \times n$ matrix. Show that if $A \neq 0$ then there exists $\mathbf{x} \in \mathbb{R}^n$ such that $A\mathbf{x} \neq 0$.

(b) State the contrapositive of the statement in the second sentence of part (a) (formed by removing the words "show that"). Also state its converse.

(c) Show that if U is an $n \times n$ upper triangular matrix having zeros as its diagonal entries, then $U^n = 0$.

Problem 2 (4 marks)**.** All parts carry equal marks.

(a) Show that if A is an $m \times n$ matrix having linearly independent columns, then the entries of the reduced echelon form of A consist of 1s and 0s.

(b) State the converse of the statement in part (a) (formed by removing the words "show that"). If this converse is true, give a proof. If false, provide a counterexample.