Q1.

Ans1 :

	S_4	S_3	S_2	S_1	R	G	
0	0	0	0	0	x	x	
1	0	0	0	1	0	0	
2	0	0	1	0	0	1	
3	0	0	1	1	0	1	(S_2: priority)
4	0	1	0	0	1	0	
5	0	1	0	1	x	x	
6	0	1	1	0	1	0	(S_3: priority)
7	0	1	1	1	1	0	
8	1	0	0	0	1	1	
9	1	0	0	1	x	x	
10	1	0	1	0	x	x	
11	1	0	1	1	x	x	
12	1	1	0	0	1	1	(S_4: priority)
13	1	1	0	1	x	x	
14	1	1	1	0	1	1	(S_4: priority)
15	1	1	1	1	1	1	

x → Don't Care Condn

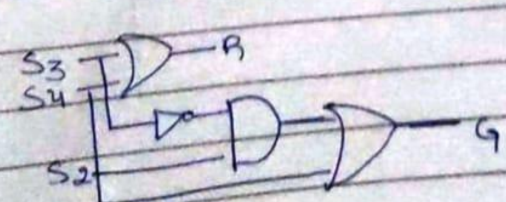
S_4S_3 \ S_2S_1	00	01	11	10
00	X_0	$_1$	$_3$	$_2$
01	1_4	X_5	1_7	1_6
11	1_{12}	X_{13}	1_{15}	1_{14}
10	1_8	X_9	X_{11}	X_{10}

$R \Rightarrow S_3 + S_4$ Ans

S_4S_3 \ S_2S_1	00	01	11	10
00	X_0	$_1$	1_3	1_2
01	0_4	X_5	$_7$	$_6$
11	1_{12}	X_{13}	1_{15}	1_{14}
10	1_8	X_9	X_{11}	X_{10}

$G = S_4 + \overline{S_3} S_2$ Ans

Combinational circuit:



Ans

Q2.

Ans 2 : Driver reaches on time $A = 1$

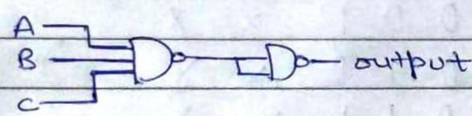
Total number of passengers are equal or greater than 10

↳ $B = 1$

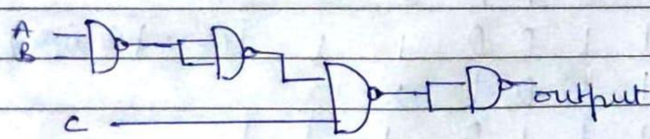
No rain $C = 1$

$P = ABC$ (traveling at 60kmph)

$P = \overline{\overline{ABC}}$



[using 3-input NAND Gate]



[2-input NAND Gate]

Q3.

Ans-3

$D_1 D_0$

00 → forward

01 → right

10 → left

11 → reverse

$S_1 S_0$

00 → zero

01 → low

10 → medium

11 → high

constraints →

high speed → for fwd movement only

rev. dirⁿ → at low speed only

D_1	D_0	S_1	S_0	P
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	
0	1	1	0	
0	1	1	1	1
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	1
1	1	0	0	
1	1	0	1	
1	1	1	0	1
1	1	1	1	1

$\Rightarrow P = f(D_1 D_0 S_1 S_0) = \Sigma(7, 11, 14, 15)$

$D_1D_0 \backslash S_1S_0$	00	01	11	10
00	0	1	3	2
01	4	5	1 (7)	6
11	12	13	1 (15)	1 (14)
10	8	9	1 (11)	10

$\Rightarrow P = D_1 D_0 S_1 + D_0 S_1 S_0 + D_1 S_1 S_0$

$\Rightarrow \boxed{P = D_1 D_0 S_1 + S_1 S_0 (D_0 + D_1)}$

Q4.

Ans-4

Temperature & Pressure controlled by Heater(H) & inlet valve (V)

in such a way →

H = 1 ; T low & P not high
(heater on)

V = 1 ; P low & T not low
(valve open)

A = 1 when T & P either both low or both high
alarm

Temperature

TL	TH	y
0	0	Normal
0	1	high
1	0	low
1	1	Normal

Pressure

PL	PH	y
0	0	Normal
0	1	high
1	0	low
1	1	Normal

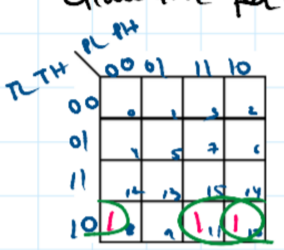
TL	TH	PL	PH	H	V	A
0	0	0	0			
0	0	0	1			
0	0	1	0		1	
0	0	1	1			
0	1	0	0			
0	1	0	1			1
0	1	1	0			
0	1	1	1			
1	0	0	0	1		
1	0	0	1			
1	0	1	0	1	1	1
1	0	1	1	1		
1	1	0	0			
1	1	0	1			
1	1	1	0		1	
1	1	1	1			

→ $H = \Sigma(8, 10, 11)$

$V = \Sigma(2, 10, 14)$

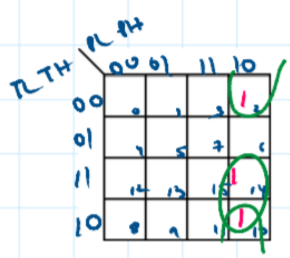
$A = \Sigma 5, 10]$

calculation for H



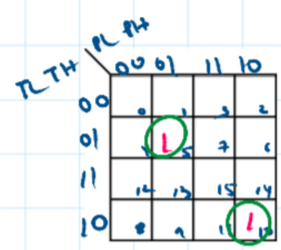
$H = L\overline{TH}(\overline{PH} + PL)$

calculation for V



$V = PL\overline{PH}(\overline{TH} + TL)$

calculation for A



$A = \overline{TL}\,TH\,\overline{PL}\,PH + TL\,\overline{TH}\,PL\,\overline{PH}$

Q5.

Sol^n: $x_1, x_2, x_3 \Rightarrow 3$ keys

Let output be "Y" (logic high when two or more keys are used).

Truth Table

x_1	x_2	x_3	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1 ✓
1	0	0	0
1	0	1	1 ✓
1	1	0	1 ✓
1	1	1	1 ✓

$\{x + \bar{x} = 1\}$; $\{x + \bar{x}y = x + y\}$

$$Y = \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 \cdot \bar{x}_2 \cdot x_3 + x_1 \cdot x_2 \cdot \bar{x}_3 + x_1 \cdot x_2 \cdot x_3$$

$$= \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 \cdot \bar{x}_2 \cdot x_3 + x_1 \cdot x_2 (\bar{x}_3 + x_3)$$

$$= \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 \cdot \bar{x}_2 \cdot x_3 + x_1 \cdot x_2$$

$$= \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 (\bar{x}_2 \cdot x_3 + x_2)$$

$$= \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 (x_2 + x_3)$$

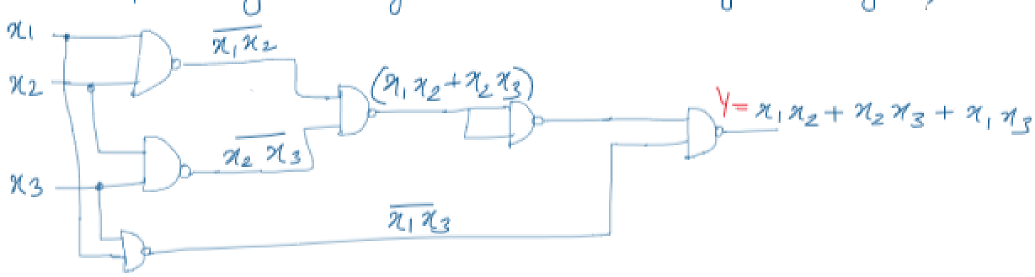
$$= \bar{x}_1 \cdot x_2 \cdot x_3 + x_1 \cdot x_2 + x_1 x_3$$

$$= x_2 (\bar{x}_1 \cdot x_3 + x_1) + x_1 x_3$$

$$= x_2 (x_3 + x_1) + x_1 x_3$$

$$Y = x_1 x_2 + x_2 x_3 + x_3 x_1$$

Implementing Y using minimum number of NAND gates,



Q6.

Sol^n: (a) $F_1(A,B,C,D) = \bar{A}B + A\bar{B} + \bar{C}D$

CSOP form,

$$= \bar{A}B(C+\bar{C})(D+\bar{D}) + A\bar{B}(C+\bar{C})(D+\bar{D}) + (A+\bar{A})(B+\bar{B})\bar{C}D$$

$$= \bar{A}B(CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + A\bar{B}(CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + (AB + A\bar{B} + \bar{A}B + \bar{A}\bar{B})\bar{C}D$$

$$= \underset{0111}{\bar{A}BCD} + \underset{0110}{\bar{A}BC\bar{D}} + \underset{0101}{\bar{A}B\bar{C}D} + \underset{0100}{\bar{A}B\bar{C}\bar{D}} + \underset{1011}{A\bar{B}CD} + \underset{1010}{A\bar{B}C\bar{D}} + \underset{1001}{A\bar{B}\bar{C}D} + \underset{1000}{A\bar{B}\bar{C}\bar{D}} + \underset{1101}{AB\bar{C}D} + \underset{1001}{A\bar{B}\bar{C}D} + \underset{0101}{\bar{A}B\bar{C}D} + \underset{0001}{\bar{A}\bar{B}\bar{C}D}$$

$$\boxed{F_1 = \bar{A}BCD + \bar{A}BC\bar{D} + \bar{A}B\bar{C}D + \bar{A}B\bar{C}\bar{D} + A\bar{B}CD + A\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}\bar{C}\bar{D} + AB\bar{C}D + \bar{A}\bar{B}\bar{C}D}$$

↳ CSOP form

(b) $F_1(A,B,C,D) = \underbrace{\bar{A}B + A\bar{B} + \bar{C}D}$

⇓

This is the minimized SOP form.

Converting SOP to POS (Dual form), → { AND ⟷ OR, OR ⟷ AND }

$$\boxed{F_1 = (\bar{A}+B)\cdot(A+\bar{B})\cdot(\bar{C}+D)}$$

↳ Minimized POS form.

(c) $F_2(A,B,C,D) = (A+B)\cdot(\bar{C}+\bar{D})\cdot(C+D)$

(c) $F2(A, B, C, D) = (A+B)\cdot(\bar{C}+\bar{D})\cdot(C+D)$

① CSOP form

Converting POS form to SOP form using Dual form.

$$F2 = A\cdot B + \bar{C}\bar{D} + CD$$

$$F2 = A\cdot B(C+\bar{C})(D+\bar{D}) + (A+\bar{A})(B+\bar{B})\bar{C}\bar{D} + (A+\bar{A})(B+\bar{B})CD$$

$$= AB(CD + C\bar{D} + \bar{C}D + \bar{C}\bar{D}) + (AB + A\bar{B} + \bar{A}B + \bar{A}\bar{B})\bar{C}\bar{D} + (AB + \bar{A}B + A\bar{B} + \bar{A}\bar{B})CD$$

$$= \underset{1111}{ABCD} + \underset{1110}{ABC\bar{D}} + \underset{1101}{AB\bar{C}D} + \underset{1100}{AB\bar{C}\bar{D}} + \underset{1100}{AB\bar{C}\bar{D}} + \underset{1000}{A\bar{B}\bar{C}\bar{D}} + \underset{0100}{\bar{A}B\bar{C}\bar{D}} + \underset{0000}{\bar{A}\bar{B}\bar{C}\bar{D}} + \underset{1111}{ABCD} + \underset{0111}{\bar{A}BCD} + \underset{1011}{A\bar{B}CD} + \underset{0011}{\bar{A}\bar{B}CD}$$

$$\boxed{F2 = ABCD + ABC\bar{D} + AB\bar{C}D + AB\bar{C}\bar{D} + A\bar{B}\bar{C}\bar{D} + \bar{A}B\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}BCD + A\bar{B}CD + \bar{A}\bar{B}CD}$$

→ CSOP form

② Minimized POS form

$$F2 = (A+B)\cdot(\bar{C}+\bar{D})\cdot(C+D)$$

↳ The expression is in minimized POS only.

Q7.

Using postulates,

a. find minimum SOP expression for the function,

$$F(A,B,C) = A\cdot C + A\cdot B' + A'\cdot B\cdot C + A'\cdot B'\cdot C'$$

b. Prove that $\mathbf{B \oplus (A\cdot B + B\cdot C + A'\cdot C) = A'\cdot(B\oplus C)}$

Sol^n: (a) $F(A,B,C) = AC + A\bar{B} + \bar{A}BC + \bar{A}\bar{B}\bar{C}$

$$= C(A + \bar{A}B) + \bar{B}(A + \bar{A}\bar{C})$$

$$= C(A + B) + \bar{B}(A + \bar{C})$$

$$= AC + BC + A\bar{B} + \bar{B}\bar{C}$$

$$\left\{ \because,\ \begin{array}{l} xy + yz + \bar{x}z = xy + \bar{x}z \\ x + \bar{x}y = x + y \end{array} \right\} \Rightarrow \text{Postulate}$$

$$\boxed{F(A,B,C) = BC + A\bar{B} + \bar{B}\bar{C}}$$

(b) $B \oplus (A \cdot B + B \cdot C + \bar{A} \cdot C) = \bar{A} \cdot (B \oplus C)$

$$\{\because,\ AB + BC + \bar{A}C = AB + \bar{A}C \Rightarrow \text{Postulate}\}$$

$\underline{\underline{LHS}}$: $\Rightarrow B \oplus (AB + \bar{A}C)$

$$= \bar{B} \cdot (AB + \bar{A}C) + B \cdot \overline{(AB + \bar{A}C)}$$

$$= AB \cdot \bar{B} + \bar{A}\bar{B}C + B(\overline{A \cdot B} \cdot \overline{\bar{A}C})$$

$$\left\{ \because,\ \begin{array}{l} x \cdot \bar{x} = 0 \\ \overline{x \cdot y} = \bar{x} + \bar{y} \\ \overline{x + y} = \bar{x} \cdot \bar{y} \end{array} \right\}$$

$$= 0 + \bar{A}\bar{B}C + B \cdot [(\bar{A} + \bar{B}) \cdot (A + \bar{C})]$$

$$= \bar{A}\bar{B}C + B[\cancel{\bar{A}A}^{0} + \bar{A}\bar{C} + A\bar{B} + \bar{B}\bar{C}]$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \cancel{AB\bar{B}}^{0} + \cancel{B \cdot \bar{B}\bar{C}}^{0}$$

$$= \bar{A}(\bar{B}C + B\bar{C})$$

$$= \bar{A}(B \oplus C) \Rightarrow R.H.S$$

Hence Proved

Q8.

Sol^n: (a)

Input			Outputs	
A	B	C_{in}	S	C_{out}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

for S,

(CSOP): $S = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$

(CPOS): $S = (A+B+C_{in}).(A+\bar{B}+\bar{C}_{in}).(\bar{A}+B+\bar{C}_{in}).(\bar{A}+\bar{B}+C_{in})$

Minimized SOP form,

$$S = \bar{A}\bar{B}C_{in} + \bar{A}B\bar{C}_{in} + A\bar{B}\bar{C}_{in} + ABC_{in}$$

$$= C_{in}(\bar{A}\bar{B}+AB) + \bar{C}_{in}(\bar{A}B+A\bar{B})$$

$$= C_{in}\overline{(A \oplus B)} + \bar{C}_{in}(A \oplus B)$$

$$= (A \oplus B) \oplus C_{in}$$

$$= \underbrace{A \oplus B}_{} \oplus C_{in}$$

$\{$ let $A \oplus B = P \}$

$$= P \oplus C_{in}$$

$$= P\bar{C}_{in} + \bar{P}C_{in}$$

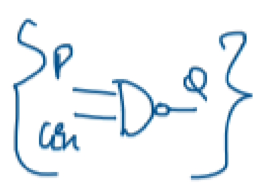
$$= (P + C_{in})(\bar{P} + \bar{C}_{in})$$

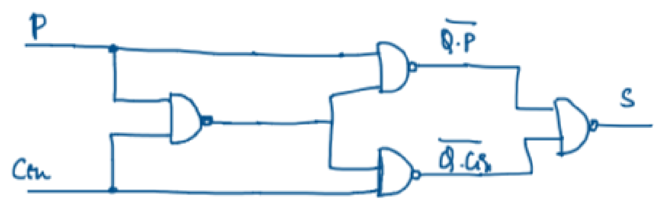
$\{$ Let $\bar{P} + \bar{C}_{in} = Q \}$ ⇒ for simplicity

$$= Q(P + C_{in})$$

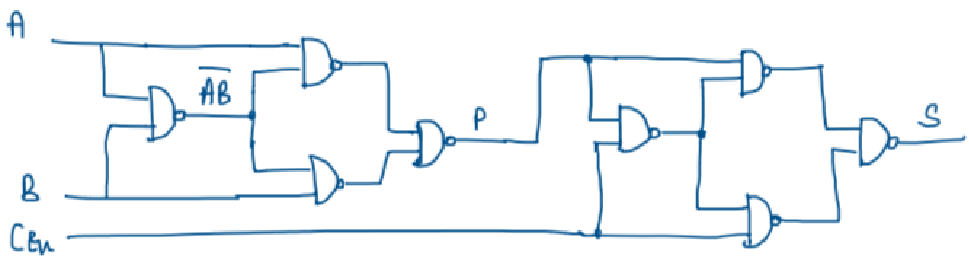
$$= \overline{\overline{QP + QC_{in}}}$$

$$S = \overline{\overline{QP} \cdot \overline{QC_{in}}}$$





Now, $P = A \oplus B$ so we can repeat the above steps for A & B.



for C_{out},

CSOP: $C_{out} = \bar{A}BC_{in} + A\bar{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$

CPOS: $C_{out} = (A+B+C_{in}).(A+B+\overline{C_{in}}).(A+\bar{B}+C_{in}).(\bar{A}+B+C_{in})$

Minimizing in SOP form,

$\Rightarrow \bar{A}BC_{in} + A\bar{B}C_{in} + AB\overline{C_{in}} + ABC_{in}$

$\Rightarrow \bar{A}BC_{in} + A\bar{B}C_{in} + AB(\overline{C_{in}} + C_{in})$

$\Rightarrow \bar{A}BC_{in} + A\bar{B}C_{in} + AB(1)$

$\Rightarrow \bar{A}BC_{in} + A(B + \bar{B}C_{in})$

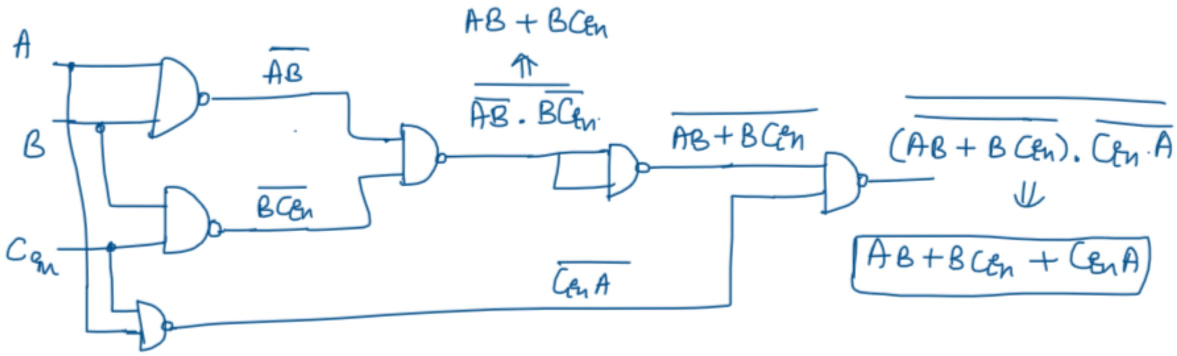
$\Rightarrow \bar{A}BC_{in} + A(B + C_{in})$ $\{\because X + \bar{X}Y = X + Y\}$

$\Rightarrow \bar{A}BC_{in} + AB + AC_{in}$

$\Rightarrow C_{in}(\bar{A}B + A) + AB$

$\Rightarrow C_{in}(A+B) + AB$

$\boxed{C_{out} \Rightarrow AB + BC_{in} + C_{in}A}$



(b)

Inputs				Output
A	B	C	D	F
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	1
0	1	0	0	0
0	1	0	1	0
0	1	1	0	0
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

CSOP

$$F = \bar{A}\bar{B}CD + \bar{A}BCD + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$$

CPOS

$$F = (A+B+C+D)\cdot(A+B+C+\bar{D})\cdot(A+B+\bar{C}+D)\cdot(A+B+\bar{C}+\bar{D})\cdot(A+\bar{B}+C+D)\cdot(\bar{A}+B+C+D)\cdot(\bar{A}+\bar{B}+C+D)$$

Minimizing,

$$F = \bar{A}\bar{B}CD + \bar{A}BCD + A\bar{B}CD + AB\bar{C}\bar{D} + AB\bar{C}D + ABC\bar{D} + ABCD$$

$$= \bar{B}CD(\bar{A}+A) + BCD(\bar{A}+A) + AB\bar{C}(\bar{D}+D) + ABC\bar{D}$$

$$= \bar{B}CD + BCD + AB\bar{C} + ABC\bar{D}$$

$$= CD(\bar{B}+B) + AB\bar{C} + ABC\bar{D} \qquad \left\{ \begin{array}{l} x + \bar{x}y = x + y \\ x + \bar{x} = 1 \end{array} \right\}$$

$$= CD + AB\bar{C} + ABC\bar{D}$$

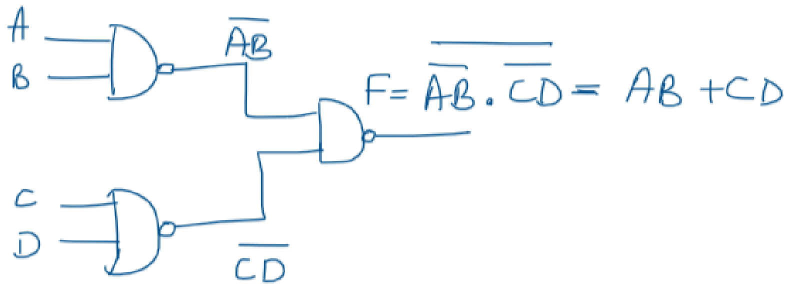
$$= AB\bar{C} + C(D + AB\bar{D})$$

$$= AB\bar{C} + C(D + AB)$$

$$= AB\bar{C} + CD + ABC$$

$$= AB(\bar{C} + C) + CD$$

$$\boxed{F = AB + CD}$$



Q9. Prove that A'B(D' + C'D) + B(A + A'CD) = B

$\underline{\underline{\text{Sol}^n}}$: $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD) = B$

$\underline{\underline{\text{LHS}}}$: $\bar{A}B(\bar{D} + \bar{C}D) + B(A + \bar{A}CD)$

$\{\text{using } x + \bar{x}y = x + y\}$

$= \bar{A}B(\bar{D} + \bar{C}) + B(A + CD)$

$= \bar{A}B\bar{D} + \bar{A}B\bar{C} + AB + BCD$

$= B[\bar{A}\bar{D} + (\bar{A}\bar{C} + A) + CD]$

$= B[\bar{A}\bar{D} + \bar{C} + A + CD]$

$= B[(\bar{A}\bar{D} + A) + (\bar{C} + CD)]$

$= B[\bar{D} + A + \bar{C} + D]$

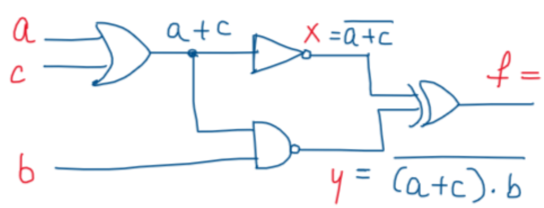
$= B[1 + A + \bar{C}]$ $\{\text{using } x + \bar{x} = 1\}$

$= B[1]$

$= B \Rightarrow \underline{\underline{\text{R.H.S}}}$ ✓

Q10. Assuming zero delay between input and output of each gate, complete the timing diagram of the following circuit. (In answer sheet, show all 6 timing diagrams corresponding to boolean variables a,b,c,x,y,f on the same page).

Sol^n:



$f = \overline{(a+c)} \oplus \overline{(a+c).b}$

$y = \overline{(a+c).b}$

Truth-Table

a	b	c	x $\overline{a+c}$	y $\overline{(a+c).b}$	f $x \oplus y$
0	0	0	1	1	0
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	0	0	0
1	0	0	0	1	1
1	0	1	0	1	1
1	1	0	0	0	0
1	1	1	0	0	0