Mid Sem – Solutions

ECE – 250 (S&S)

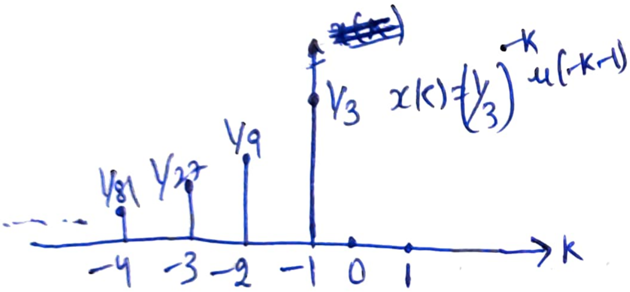
Sol 1: →

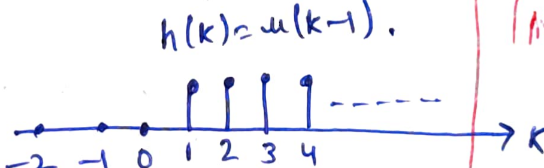
$$x(n) = \left(\tfrac{1}{3}\right)^{-n} u(-n-1)$$

$$h(n) = u(n-1)$$

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$x(k) = \left(\tfrac{1}{3}\right)^{-k} u(-k-1) \qquad h(k) = u(k-1)$$





1 Mark.

From the above figures, we can find that

$x(k) = 0$ for $k > -1$ and

$h(n-k) = 0$ for $k > n-1$

for $n-1 \le -1$, i.e. for $n < 0$, the interval of summation is

from $\boxed{k = -\infty \text{ to } n-1}$

$$y(n) = \sum_{k=-\infty}^{n-1} \left(\tfrac{1}{3}\right)^{-k} \quad \text{for } n-1 \le -1 \text{ or for } n \le 0$$

2 Marks

$$= \left(\tfrac{1}{3}\right)^{-(n-1)} + \left(\tfrac{1}{3}\right)^{-(n-2)} + \cdots\cdots$$

$$= \left(\tfrac{1}{3}\right)^{-(n-1)} \left[1 + \tfrac{1}{3} + \left(\tfrac{1}{3}\right)^2 + \cdots\right]$$

$$= 3^{n-1}\left[\frac{1}{1-1/3}\right] = 0.5(3)^n.$$

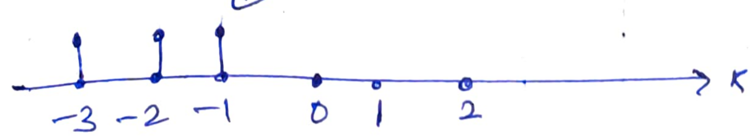
for $n-1 \geq -1$ i.e. for $n \geq 0$, the interval of summation is from $k = -\infty$ to -1

$$\therefore y(n) = \sum_{k=-\infty}^{-1} \left(\tfrac{1}{3}\right)^{-k}$$

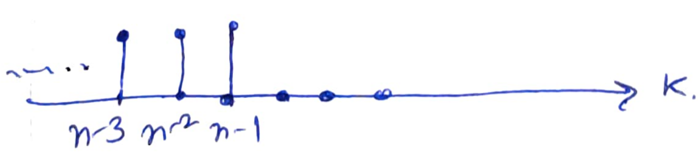
$$= \sum_{k=1}^{\infty} \left(\tfrac{1}{3}\right)^{k} = \frac{1/3}{1-1/3} = 0.5$$

2 marks

$$h(-k) = u(-k-1)$$

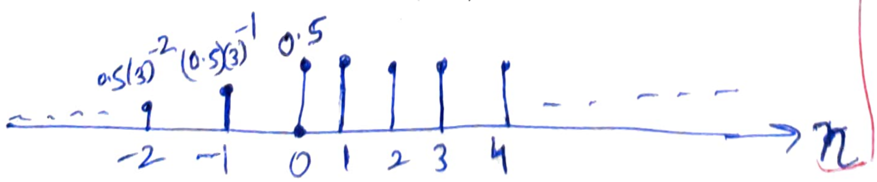


$h(n-k)$.



1 mark.

Plot $y(n)$.



Q2:→ $x(t) = 2e^{-3t}u(t-1)$

$$x(t) \longrightarrow y(t)$$

$$\frac{dx(t)}{dt} \longrightarrow -3y(t) + e^{-2t}u(t)$$

$$\frac{dx(t)}{dt} = 2e^{-3t}\delta(t-1) + -3\left(2e^{-3t}u(t-1)\right)$$

$$= 2e^{-3}\delta(t-1) + (-3x(t))$$

$$= -3x(t) + 2e^{-3}\delta(t-1).$$

2 marks

$$\Downarrow$$

$$= -3y(t) + 2e^{-3}h(t-1)$$

$$2e^{-3}h(t) = e^{-2(t+1)}u(t+1).$$

$$h(t) = \frac{1}{2e^{-3}}e^{-2(t+1)}u(t+1).$$

2 marks.

(3)

From part (i), we know $x(t)$ is a real signal. then,

we can say

$a_k = a_{-k}^*$ where (a_k are Fourier series coefficients) — (1)

From part (ii), we infer that

$T = 6$ ie $\omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$. — (2)

1 mark.

From part (iii), we know that

$a_k = 0$ for $k = 0$ and $k > 2$. — (3)

From (1) and (3), we inferred that only a_1, a_{-1}, a_2 and a_{-2} are the unknowns to be determined, all other coefficients are zero.

1 mark.

From part (iv), we get that

$x(t) = -x(t-3)$

computing Fourier series for both the signals..

$F(x(t)) \longrightarrow a_1 e^{-j\omega_0 t} + a_{-1} e^{j\omega_0 t} + a_2 e^{-2j\omega_0 t} + a_{-2} e^{2j\omega_0 t}$ — (4)

$F(-x(t-3)) \longrightarrow -\left[a_1 e^{-j\omega_0(t-3)} + a_{-1} e^{j\omega_0(t-3)} + a_2 e^{-2j\omega_0(t-3)} + a_{-2} e^{2j\omega_0(t-3)}\right]$ 1 mark.

$$F(-x(t-3)) \rightarrow -\left[a_1 e^{-jw_0 t} . e^{+j\pi} + a_{-1} e^{+jw_0 t} e^{-j\pi} + a_2 e^{-2jw_0 t} e^{+j2\pi} + a_{-2} e^{+2jw_0} e^{-j2\pi}\right]$$

$\because$ $e^{\pi} = e^{-\pi} = (-1)$ and $e^{2\pi} = e^{-2\pi} = 1$

$$\rightarrow -\left[-a_1 e^{-jw_0 t} - a_{-1} e^{jw_0 t} + a_2 e^{-2jw_0 t} + a_{-2} e^{+2jw_0 t}\right] \quad -(4)$$

$$\rightarrow \left[a_1 e^{-jw_0 t} + a_{-1} e^{jw_0 t} - a_2 e^{-2jw_0 t} - a_{-2} e^{2jw_0 t}\right] \quad -(5)$$

1 marks

comparing coefficients in (4) and (5)

we get $a_2 = a_{-2} = 0$

From (v)

From Parseval's Relation.

we get

$$|a_1|^2 + |a_{-1}|^2 = \frac{1}{2} \quad -(6)$$

1 mark

From (vi)

a_1 is positive real., ~~so~~.

also from (1)

$a_1 = a_{-1}$

$|a_1|^2 + |a_1|^2 = \frac{1}{2}$

$a_1^2 + a_1^2 = \frac{1}{2}$ $\boxed{a_1 = \frac{1}{2} = a_{-1}}$

1 mark

Finally,

$$x(t) = \frac{1}{2} e^{j\omega_0 t} + \frac{1}{2} e^{-j\omega_0 t}$$

$$= \cos(\omega_0 t)$$

comparing with $A \cos(Bt + C)$

$$\boxed{\begin{aligned} A &= 1 \\ B &= \frac{\pi}{3} \\ C &= 0 \end{aligned}}$$

2 marks

4.

a). If $x[n]$ is real than from fourier series coefficient c_K is defined as.

$$c_K = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-jK\frac{2\pi}{N_0}n}$$

1.5 marks

$$c_{-K} = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{jK\frac{2\pi}{N_0}n}$$

$$= \left[\frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-jK\frac{2\pi}{N_0}n} \right]^* = c_K^*$$

Thus

$$c_{-K} = a_{-K} + j b_{-K} = (a_K + jb_K)^* = a_K - jb_K.$$

1.5 marks

and we have

$$\boxed{a_{-K} = a_K \text{ and } b_{-K} = -b_K.}$$

(b). If N_0 is even then.

$$c_{N_0/2} = \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j(N_0/2)(2\pi/N_0)n}$$

1 mark

$$= \frac{1}{N_0} \sum_{n=0}^{N_0-1} x[n] e^{-j\pi n}$$

1 mark

$$= \frac{1}{N_0} \sum_{n=0}^{N_0-1} (-1)^n x[n] = \boxed{\text{Real.}}$$

1 mark

(5)

$$x(t) = e^{-3|t-2|}$$

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-3|t-2|} e^{-j\omega t} dt$$

$$= \int_{2}^{\infty} e^{-3(t-2)} e^{-j\omega t} dt + \int_{-\infty}^{2} e^{3(t-2)} e^{-j\omega t} dt$$ 1 Mark

$$= e^{6} \int_{2}^{\infty} e^{-3t} e^{-j\omega t} dt + e^{-6} \int_{-\infty}^{2} e^{3t} e^{-j\omega t} dt$$

$$= e^{6} \int_{2}^{\infty} e^{-(3+j\omega)t} dt + e^{-6} \int_{-\infty}^{2} e^{(3-j\omega)t} dt$$ 1 Marks

$$= e^{6} \frac{\left[0 - e^{-6} e^{-2j\omega}\right]}{-(3+j\omega)} + e^{-6} \frac{\left[e^{6} e^{-2j\omega} - 0\right]}{3-j\omega}$$

$$= \frac{e^{-2j\omega}}{3+j\omega} + \frac{e^{-2j\omega}}{3-j\omega}$$ 1 Mark

$$= \frac{6e^{-2j\omega}}{9+\omega^2}$$ — 1 Mark.