Bias Variance Experiment

December 1, 2018

0.0.1 Problem 2.24

- For each final hypothesis g1, g2, ..., gk, estimate the average hypothesis for a single x with the following equation:

$$\bar{g}(x) = \frac{1}{K}\sum_{k=1}^{K} g_k(x)$$

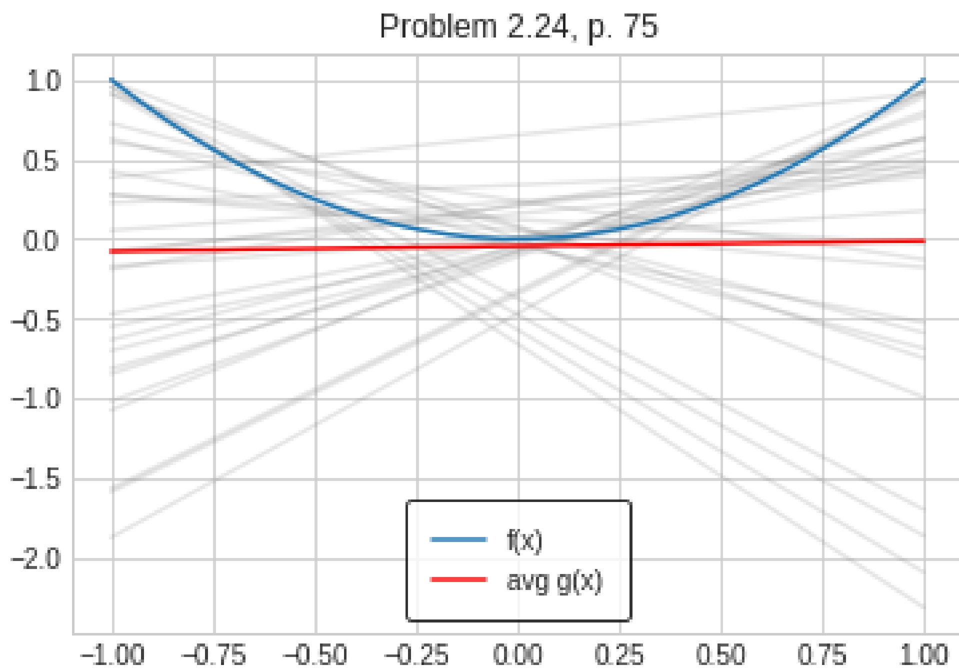
- Generate K datasets. Then, for each x can calculate gbar(x) using the above equation. Next, calculate bias for each x: bias(x) = (gavg(x) - f(x))2. After that, calculate variance for each x: var(x) = (g(D)(x) - gavg(x))2]. Finally, calculate the expected value of the out of sample error ED[Eout(g(D))] with the following equation: ED[Eout(g(D))] = Ex[bias(x) + var(x)].
- See results of experiment and plot below.
- See analytic computation of Eout, bias, and var in experiment results.

```
In [1]: %matplotlib inline

        import numpy as np
        import matplotlib.pyplot as plt

        def main():
            num_datasets = 30
            target_fn = np.square

            p1, p2 = get_experiment_data(num_datasets, target_fn)
            m = get_slope(p1, p2)
            b = get_y_intercept(p1, m)

            bias = calculate_bias(get_x(num_datasets), m, b, target_fn)
            var = calculate_var(get_x(num_datasets), m, b)

            print_data('bias', bias)
            print_data('var', var)
            print_data('e_out', round(bias + var, 2))

            plot_exp(m, b, target_fn)
```

```python
def print_data(label, value, width=10):
    print('{:{}}: {}'.format(label, width, value))


def get_experiment_data(num_datasets, target_fn):
    num_samples = num_datasets * 2
    s = np.random.uniform(-1, 1, num_samples)
    s = s.reshape(num_datasets, 2)

    x1, x2 = s[:, 0], s[:, 1]
    y1, y2 = target_fn(x1), target_fn(x2)

    p1 = np.column_stack((x1, y1))
    p2 = np.column_stack((x2, y2))

    return p1, p2


def get_slope(p1, p2):
    # slope = (y2 - y1) / (x2 - x1)
    return (p2[:, 1] - p1[:, 1]) / (p2[:, 0] - p1[:, 0])


def get_y_intercept(p, m):
    # y - y1 = m(x - x1)
    # y - y1 = mx - mx1
    # y = mx - mx1 + y1
    # let b = -mx1 + y1
    return -m * p[:, 0] + p[:, 1]


def calculate_bias(x, m, b, target_fn):
    g_avg = get_g_avg_vect(x, m, b)
    f_x = target_fn(x)
    return round(mean_sum_squared_error(g_avg, f_x), 2)


def calculate_var(x, m, b):
    g = hypothesis_fn(m, x, b)
    g_avg = get_g_avg_vect(x, m, b)
    return round(mean_sum_squared_error(g, g_avg), 2)


def get_g_avg_vect(x, m, b):
    g_avg = np.full_like(x, 1)
    for i, this_x in enumerate(x):
        g_avg[i] = calculate_g_avg(m, this_x, b)
    return g_avg
```

```
def calculate_g_avg(m, x, b):
    # this must take in a single x
    x_vect = np.full_like(m, x)
    return np.average(hypothesis_fn(m, x_vect, b))


def mean_sum_squared_error(x1, x2):
    return np.average(np.square(x1 - x2))


def get_x(num_pts):
    return np.linspace(-1, 1, num_pts)


def hypothesis_fn(m, x, b):
    return m * x + b


def plot_exp(m, b, target_fn):
    plt.style.use('seaborn-whitegrid')
    fig, ax = plt.subplots()

    ax.set(title='Problem 2.24, p. 75')

    x = np.linspace(-1, 1, 30)

    # plot each hypothesis fn
    for this_m, this_b in zip(m, b):
        ax.plot(x, hypothesis_fn(this_m, x, this_b), color='gray', alpha=0.2)

    ax.plot(x, target_fn(x), label='f(x)')
    ax.plot(x, get_g_avg_vect(x, m, b), color='r', label='avg g(x)')

    ax.legend(facecolor='w', fancybox=True, frameon=True, edgecolor='black', borderpad=
    plt.show()


if __name__ == '__main__':
    main()
```

```
bias      : 0.26
var       : 0.22
e_out     : 0.48
```


Problem 2.24, p. 75
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
−1.00
−0.75
−0.50
−0.25
0.00
0.25
0.50
0.75
1.00
f(x)
avg g(x)