Modeling a Simple Fluid Flow

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1 Introduction

This document presents the modeling of a 1-dimensional fluid flow system using Lagrangian mechanics and co-energy analysis. A Simulink block diagram is also provided for simulation purposes.

2 Flow Model Description

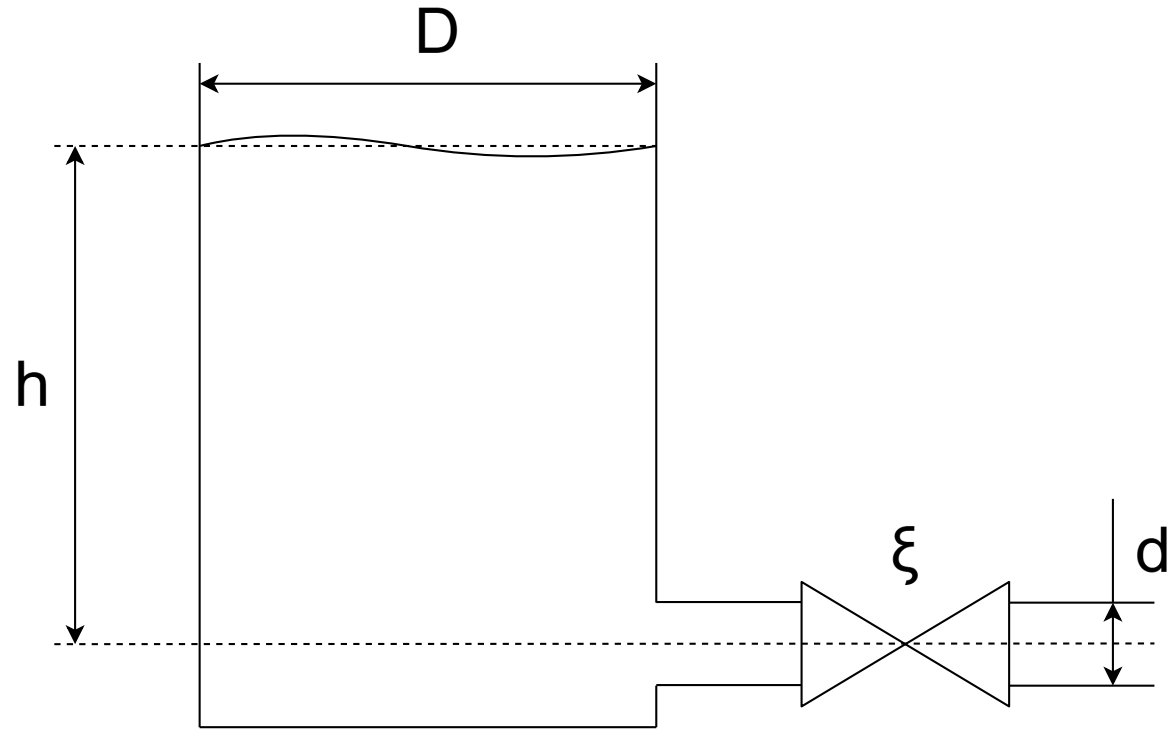

Figure 1: Overview of the flow model

3 Direct Fluid Mechanics Approach

3.1 Continuity Equation (Mass Conservation)

The rate of change of fluid volume in the tank must equal the outflow rate:

$$A_D \frac{dh}{dt} = -A_d \cdot (v)$$

where:

$$A_D = \frac{\pi D^2}{4}$$
$$A_d = \frac{\pi d^2}{4}$$

3.2 Modified Torricelli's Law (Energy Considerations)

From Bernoulli's principle with losses, the exit velocity v is given by:

$$\frac{1}{2}v^2 + \xi\frac{1}{2}v^2 = gh$$
$$v^2\left(1+\xi\right) = 2gh$$
$$v = \sqrt{\frac{2gh}{1+\xi}}$$

3.3 Combining the Equations

Substituting v into the continuity equation gives:

$$\frac{\pi D^2}{4}\frac{dh}{dt} = -\frac{\pi d^2}{4}\cdot\sqrt{\frac{2gh}{1+\xi}}$$

$$\frac{dh}{dt} = -\frac{d^2}{D^2}\sqrt{\frac{2gh}{1+\xi}} = -\left(\frac{d}{D}\right)^2\sqrt{\frac{2gh}{1+\xi}}$$

4 Simulink Model

Basically the system's equation can be written directly to Simulink as its representative model.

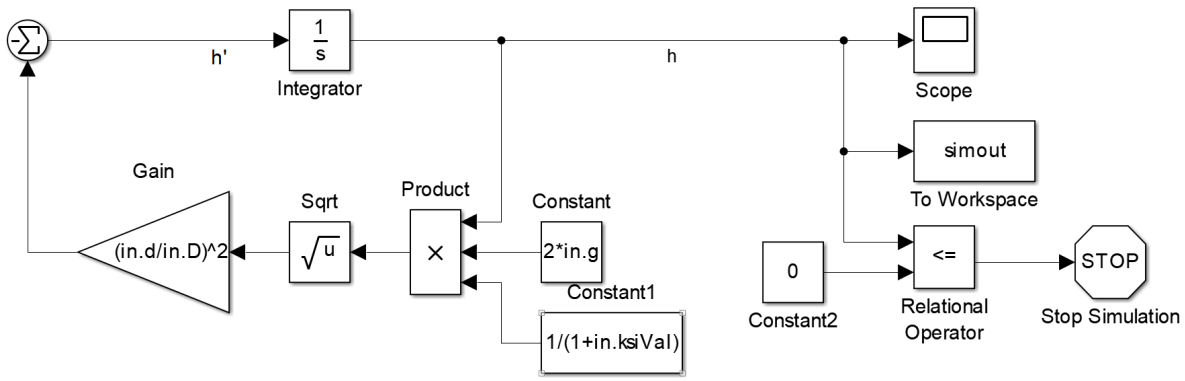

Figure 2: Simulink model of 1DoF fluid flow system

5 Matlab and Python script

For script-based system modeling, the equation will be written as:

$$\frac{d}{dt}\left[h\right] = \left[-(d/D)^2 * sqrt((2 * g * h)/(1 + ksi));\right]$$

Now, the equation can be coded into Matlab:

```
Dh = -(in.d / in.D)^2 * sqrt((2 * in.g * h) / (1 + in.ksiVal));
```

Equivalent Python code can be written as:

```
def fluid_flow(t, h):

    return [-(d / D) ** 2 * np.sqrt((2 * g * h[0]) / (1 + ksiVal)
       )]
```

6 Simulation Results

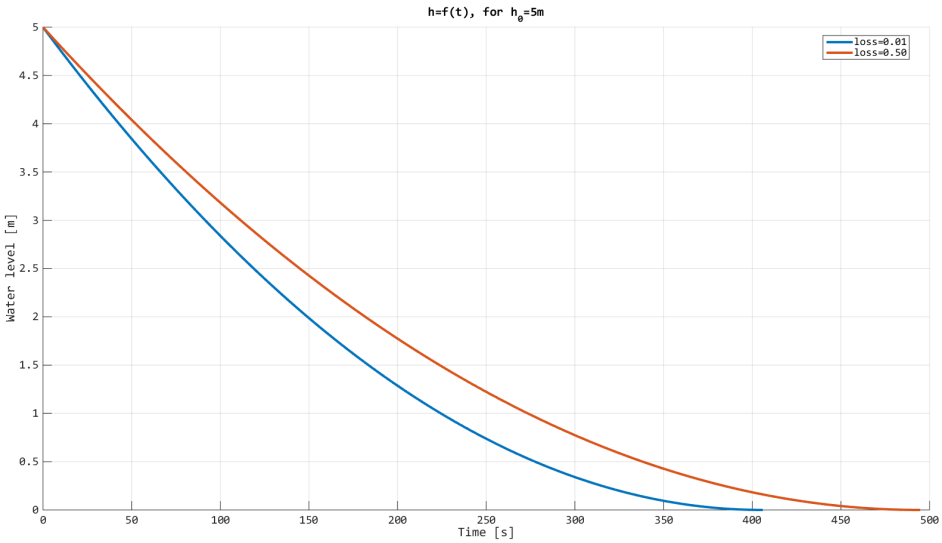

Figure 3: Result of simulation

7 Conclusion

This document demonstrates the modeling a 1-dimensional fluid flow. It also provides a visual block diagram suitable for simulation in Simulink and basic code for scripting.