Modeling a Simple Thermal System

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1 Introduction

This document presents the modeling of a 1-dimensional simplest thermal system using Hamiltonian mechanics and classical thermodynamics. A Simulink block diagram is also provided for simulation purposes.

2 Thermal Model Description

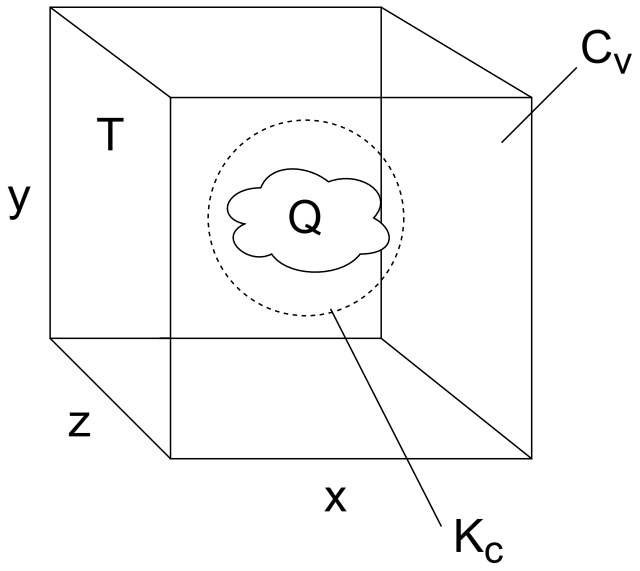

Figure 1: Overview of the thermal model

Presented thermal model is a simplified, most basic approach to modeling thermal system. It is flattened to a 1-dimensional equation, where temperature change over time is considered, neglecting the spread of heat flux in space. Therefore, we are modeling a lumped para-thermal capacitance system with heat input and loss to ambient that transforms thermal energy across generalized coordinate, which in this case is temperature.

3 Lagrangian Formulation

Since T represents the energy state, the time derivative $\dot{T}$ does not represent energy flow like velocity in mechanics. Instead, thermal systems are dissipative and do not have conservative potential energy like in mechanics. That would suggest thermal energy depends on T^2 like kinetic energy depends on velocity, which is not true for 1DoF thermal system. The classical Lagrangian approach expects second-order dynamics (like x, $\dot{x}$), but presented thermal system is first-order by nature - energy is proportional to T instead of $\dot{T}$. Forcing a Langrangian approach would violate its classical and co-energy form.

4 Port-Hamiltonian Formalism

Port-Hamiltonian formalism is perfect for thermodynamics and first-order energy storage systems.

4.1 State Variable

Internal energy ot the system is:

$$\begin{cases} x = U(T) \\ U(T) = C_V T \end{cases}$$

where:

$$C_V - \text{Thermal capacitance, } \left[\frac{J}{K}\right]$$
$$T - \text{Temperature, } [K] \text{ or } [^\circ C]$$

4.2 Time Derivative

$$\frac{d}{dt}(x) = \dot{x} = C_V \dot{T}$$

4.3 Hamiltonian

Total stored energy stored by the system is:

$$\mathcal{H}(x) = x$$

4.4 Dissipation

Dissipation in 1DoF thermal model is:

$$R = K_C \Delta T$$

where:

$$K_C - \text{Thermal conductance, } \left[\frac{W}{K}\right]$$

4.5 Power Port

$$G \cdot u = Q$$

where:

$$Q - \text{Input power, } [W]$$

4.6 Port-Hamiltonian Equation

For thermal systems (with no conservative interconnection structure), the port-Hamiltonian state equation becomes:

$$\frac{d}{dt}(x) = (J - R)\frac{\partial \mathcal{H}}{\partial x} + G \cdot u$$

In this case the Hamiltonian will yield:

$$\frac{d}{dt}(C_V T) = -(K_C \Delta T) \cdot \frac{\partial H(x)}{\partial x} + Q$$

$$C_V \cdot \frac{d}{dt}(T) = -(K_C \Delta T) \cdot \frac{\partial x}{\partial x} + Q$$

$$C_V \dot{T} = -K_C \Delta T + Q$$
$$C_V \dot{T} = Q - K_C \Delta T$$

5 Classical Thermodynamics

Directly from First Law of Thermodynamics for a thermal lumped system we can write:

$$\frac{d}{dt}U(T) = Q_{in} - Q_{loss}$$

$$\frac{d}{dt}(C_V T) = Q_{in} - K_C \Delta T$$

$$C_V \dot{T} = Q_{in} - K_C \Delta T$$

6 Simulink Model for Hamiltonian and Classic Thermodynamics

In order to model the equation properly, we must translate the dynamic equation into a form understandable for Simulink:

$$C_V \dot{T} = Q - K_C \Delta T$$

$$\dot{T} = \frac{1}{C_V}(Q - K_C \Delta T)$$

In the following, the Simulink model is presented.

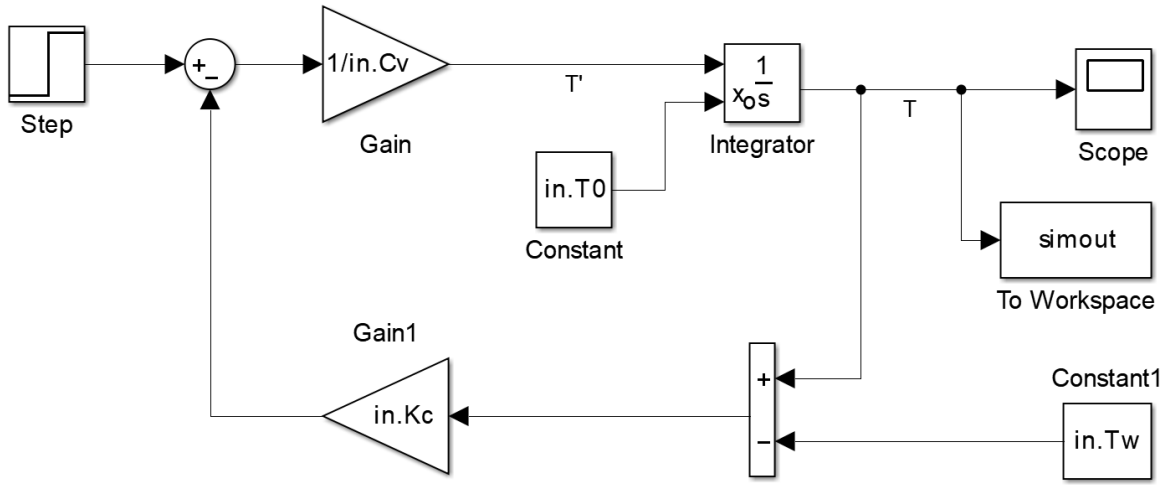

Figure 2: Simulink model of 1DoF thermal system

7 Matlab and Python script

For script-based system modeling, we must take a slightly different approach to translating mathematical equations into code. The matrix of thermal system equation will be written as:

$$\frac{d}{dt}\left[T\right] = \left[(Q - (T - Tw) * Kc)/Cv\right]$$

Now, the equation can be coded into Matlab:

```
DT = (Q - (T - in.Tw) * in.Kc) / in.Cv;
```

Equivalent Python code can be written as:

```
def thermal(T, t):
    if t > t_start:
        step = Qin
    else:
        step = 0

    return (step - (T - Tw) * Kc) / Cv
```

8 Simulation Results

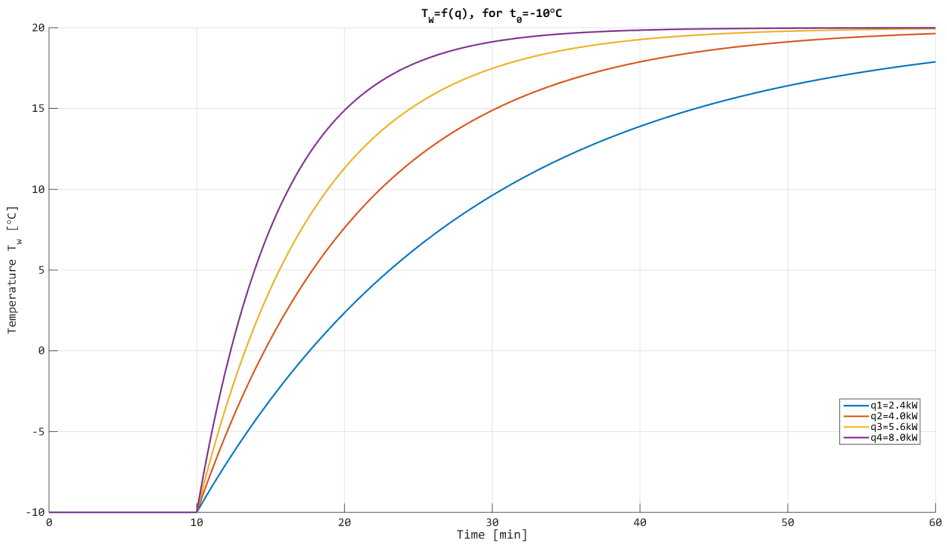

Figure 3: Result of simulation

9 Conclusion

This document demonstrates the Lagrangian and co-energy approach to modeling a 1-dimensional thermal system. It also provides a visual block diagram suitable for simulation in Simulink and basic code for scripting.