
CS21003 ALGORITHMS-1

WorkSheet 7

Date: 7 Nov 2020

Note: Make appropriate assumptions wherever required

1 Magic Distance

Consider a new metric to define distance between two points in a Cartesian plane as

$$\text{MagicDistance}((X_1, Y_1), (X_2, Y_2)) = \text{abs}((X_1 - X_2) - (Y_1 - Y_2))$$

Here, $\text{abs}()$ is the absolute value function.

Initially there is no point on the plane. You have to support two types of operations:

- **Type 1:** Add a new point or remove an existing point from the plane
- **Type 2:** Find the farthest point on the plane from a given point (X, Y) according to MagicDistance. i.e. find maximum $\text{MagicDistance}((X, Y), (X_i, Y_i))$ for all points (X_i, Y_i) in the plane

Explain how you can use heaps to solve this question, other approaches may lead to some penalty. Write pseudocodes for Add, Remove and FindFarthest functions. You can directly call the heap functions like `initialize_heap`, `heapify` etc. in your pseudocode. It is guaranteed that after each query, points will be unique on the plane i.e. $(X_i, Y_i) = (X_j, Y_j)$ only if $i = j$

Sample Input:

Type 1: Add(1, 2)

Type 1: Add(4, -5)

Type 2: FindFarthest(0, 0)

Type 1: Remove(4, -5)

Type 1: Add(10, -1)

Type 2: FindFarthest(3, -9)

Sample Output:

(4, -5)

(1, 2)

Explanation: Before the first Type 2 query, there are two points in the plane (1, 2) and (4, -5). The first query of Type 2 is to find the farthest distance from (0, 0). $\text{MagicDistance}((0, 0), (1, 2))$ is $\text{abs}((0 - 1) - (0 - 2)) = 1$ and $\text{MagicDistance}((0, 0), (4, -5))$ is $\text{abs}((0 - 4) - (0 - (-5))) = 9$.