CS21003 ALGORITHMS-1
(Tutorial 2: Divide and Conquer)
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1 Unimodal Search

An array A[1 . . n] is unimodal if it consists of an increasing sequence followed by a decreasing sequence, or more precisely, if there is an index m $\in \{1, 2, \ldots, n\}$ such that

A[i] $<$ A[i + 1] for all 1 $\leqslant$i $<$ m, and A[i] $>$ A[i + 1] for all m $\leqslant$ i $<$ n. In particular, A[m] is the maximum element, and it is the unique "locally maximum" element surrounded by smaller elements (A[m - 1] and A[m + 1]).

▷ Give an algorithm to compute the maximum element of a unimodal input array A[1 . .n] in $\mathcal{O}$ (log n) time. Provide the bound on its running time.

2 Convex Polygon

A polygon is convex if all of its internal angles are less than 180° (and none of the edges cross each other). We represent a convex polygon as an array V [1 . .n] where each element of the array represents a vertex of the polygon in the form of a coordinate pair (x, y). We are told that V [1] is the vertex with the minimum x coordinate and that the vertices V [1..n] are ordered counterclockwise. You may also assume that the x coordinates of the vertices are all distinct, as are the y coordinates of the vertices.

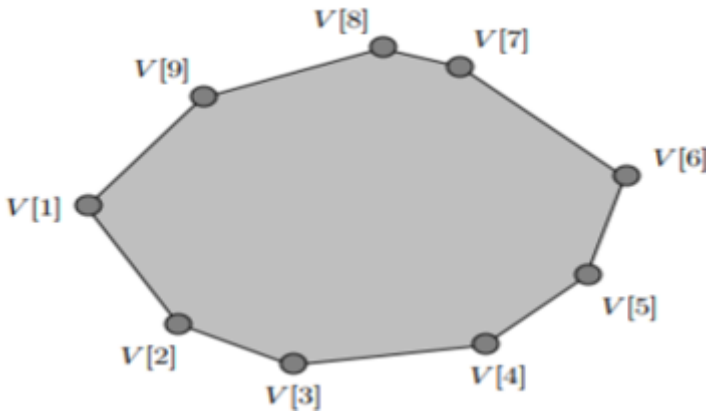

Figure 1: Caption

▷ Give an algorithm to find the vertex with the maximum x coordinate in O(lg n) time.

▷ Give an algorithm to find the vertex with the maximum y coordinate in O(lg n) time.

3 Median Of Sorted Arrays

There are two sorted arrays A and B of size n each. Write an algorithm to find the median of the array obtained after merging the above 2 arrays (i.e. Array of length 2n). Please note that you are not allowed to use any additional array. Also, the merged array is not required; only the median of the merged array is required. The complexity should be O(log(n)).

4 A bag of rice

You are provided with a large bag that can hold C kilogram of rice. As you are staying in a village far away, only one delivery truck comes to the village at the start of each day. It provides you with l kilogram of rice and in the case of overflow, it stops. Now, on the i-th day, i kg of rice is required by the village. We need to find out the day at which the bag will become empty for the first time. Your solution should use divide and conquer. Note that initially, the bag is filled as per its capacity.

Examples:
Initial bag Capacity (C) = 5
delivery truck refill (l) = 2
Output : 4
At the start of 1st day, rice in bag = 5
and at the end of the 1st day = (5 - 1) = 4
At the start of 2nd day, rice in bag = 4 + 2 = 6
but bag capacity is 5 so rice = 5
and at the end of the 2nd day = (5 - 2) = 3
At the start of 3rd day, rice in bag = 3 + 2 = 5
and at the end of the 3rd day = (5 - 3) = 2
At the start of 4th day, rice in bag = 2 + 2 = 4
and at the end of the 4th day = (4 - 4) = 0
So final answer will be 4