Astronomy Club, IITK
Hyperion 2023-24

'Echoes of Galactic Explosions'

Instructions

- You are supposed to type your answers in a `pdf` file and submit the same.
- Your solutions must be properly formatted. Any unconventional notation used must be explicitly stated.
- Attach any relevant code snippets and graphs which were utilized in analysis of the problems.
- You must submit all the solutions in English only.
- An introduction summary of not more than 200 words highlighting the basic workflow needs to be included at the beginning of the final report.
- The solutions must be your own and all references must be duly cited. Any form of plagiarism will not be tolerated under any circumstances and would lead to disqualification.
- You must have a valid college ID which would be verified at the time of prize distribution.
- The deadline of submission is **23:59 hrs of 28th January**. After the deadline, a 5 marks penalty will be imposed for every 15 minutes delay. No submission will be entertained post 1 hour of deadline.
- In case of any discrepancy the decision of the Astronomy Club IITK would be final and binding. Be sure that all the submissions would be evaluated with due diligence and utmost fairness.

All the best!
Cosmos Is Within Us

Let the quest begin...

Throughout the course of history, humans have sought refuge and storytelling in the night sky. These stars have always served as a beacon of hope and fascination. This dark night sky is not only a home for twinkling stars but also an abyss holding the universe's grand secrets. As part of our journey, it would be appropriate to visit the realm of chaos and jets. With you comes the bringer of wisdom, our cosmic compadre and Case Orator **"Hyperion"**.

A Brief Introduction to Hyperion: Today, our distinguished celestial maestro, Hyperion, has generously decided to pull a dazzling case study from his bag of cosmic conundrums, and what might that be, you ask? None other than the enigmatic Gamma-Ray Bursts! Now, brace yourselves, for Hyperion is not your average celestial speaker – oh no, he's the kind who won't settle for passive stargazers and demands your neurons to be as active as a supernova throughout our cosmic exploration.

Gamma-Ray Bursts (GRBs) are intriguing and captivating astrophysical phenomena, representing some of the universe's most explosive events. These quick and strong bursts of gamma rays, first observed unexpectedly by the U.S. Vela satellites in the 1960s, perplexed scientists for years. The following deployment of equipment, such as the Burst and Transient Source Experiment (BATSE) atop the Compton Gamma Ray Observatory, as well as advances in satellite technology, were critical in unlocking the riddles surrounding GRBs. The study of GRBs has not only provided insights into the extreme astrophysical processes behind these bursts but has also become a significant tool for probing the distant universe.

Now Hyperion needs your help, and may the odds ever be in your favour.

Help Hyperion..

1. The **Lorentz factor (τ)** quantifies the alteration in measurements of physical properties for an object in motion, a concept essential for comprehending Gamma Ray Burst (GRB) jets, where blast wave velocities are relativistic. Derive the expression for the Lorentz factor, given that v represents the speed of an object and c denotes the speed of light. Subsequently, calculate the Lorentz factor for a GRB-emitted blast wave traveling at a speed of 3×10^7m/s.

   [5 marks]

2. The **Blandford-McKee** model describes an adiabatic ultra-relativistic blast wave in the limit $\tau >> 1$, where τ is the Lorentz factor of the blast wave. Blandford-McKee solution describes a narrow shell in which the shocked material is concentrated. Approximating the Blandford-McKee solution with a thin homogeneous shell and considering varying ambient mass density $\rho = \rho_0 \left(\frac{R}{R_0}\right)^{-k}$, derive the energy expression E as given below, where R is the distance from the centre and Ω is solid angle of the afterglow.

$$E = \frac{\Omega}{3-k}(\rho_0 R_0^k) R^{3-k} \tau^2 c^2$$

[15 marks]

3. Let the expansion of the blast wave be conical with an opening angle θ such that $\Omega = 4\pi(1 - \cos\theta) \approx 2\pi\theta^2$ (assuming a double-sided jet). A generalized **Sedov scale** l is given by

$$l = \left[\frac{(3-k)E}{\rho_0 R_0^k c^2}\right]^{\frac{1}{3-k}}$$

Note: Assume Ω does not change with time for this question.

   i) Find R in terms of l where the ambient rest mass collected by blast wave equals the blast wave's initial energy.

   ii) Considering sideways expansion (after the jet break), we get $\tau \approx 1$ implying that the blast wave becomes Newtonian. Find R in terms of l where the blast wave becomes Newtonian.

[5 + 5 = 10 marks]

4. Assuming that the shell moves at constant velocity given by relation $T_R = \frac{R}{2c\tau^2}$, obtain R and τ as a function of time as follows:

$$R(t) = \left(\frac{2l^{3-k}}{\omega}\right)^{\frac{1}{4-k}} \times t^{\frac{1}{4-k}}$$

$$\tau(t) = \left(\frac{l^{3-k}}{2^{3-k}\omega}\right)^{\frac{1}{2(4-k)}} \times t^{-\frac{3-k}{2(4-k)}}$$

[15 marks]

5. For this question, you are presented with two `.csv` files containing data collected from 10000 GRBs, one set for the adiabatic phase and the other for the radiative phase. Each dataset contains measurements for v_c, v_m and $F_{v,max}$ with the parameters ϵ_B (energy associated with magnetic field), E_{52} (blast wave energy in units of 10^{52} ergs), n_1 (number density of the ambient medium in cm^3), t_d (time in days), ϵ_e (energy associated with electrons), D_{28} (Luminosity distance in units of 10^{28} cm) and τ (Lorentz factor).

   Note that v_c, v_m, and $F_{v,max}$ are each proportional to the other variables within their respective datasets but do not have proportional relationships among themselves. For instance, v_c is not proportional to v_m or $F_{v,max}$.

   i) **Light Curve during Adiabatic Phase:**
   Derive the expressions for v_c (in Hz), v_m (in Hz) and $F_{v,max}$ (in μJ) using `adiabatic_light_curve.csv`. Explain the steps used to obtain the relations and state the equations of v_c, v_m and $F_{v,max}$ for adiabatic phase. Plot the light curve.

   ii) **Light Curve during Radiative Phase:**
   Derive the expressions for v_c (in Hz), v_m (in Hz) and $F_{v,max}$ (in μJ) using `radiative_light_curve.csv`. Explain the steps used to obtain the relations and state the equations of v_c, v_m and $F_{v,max}$ for radiative phase. Plot the light curve.

   [15 + 15 = 30 marks]

6. For this question, you are presented with two datasets, each in the form of a `.csv` file with two columns: v (frequency) and F_v (flux due to radiated electrons). Your task is to formulate expressions for F_v in the following cases. Parameters v_c (cut-off frequency), v_m (peak frequency), and $F_{v,\max}$ (maximum flux) are provided. Hyperion would like to tell you that considering three distinct regions takes you forward. Specific attention should be given to the designated frequency regions for both cases.

   i) **Case 1: Fast Cooling ($v_c < v_m$)**

   Given $F_{v,\max} = 10^{-2}$ J, $v_c = 10^{14}$ Hz and $v_m = 10^{16}$ Hz.

   Consider the frequency range $v < v_c$ and $v_c < v < v_m$. In both regions, the contribution of v_m is negligible. The impact of v_c, derived for $v_c < v < v_m$, seamlessly extends to $v > v_m$. Use `fast_cooling.csv` dataset to formulate F_v.

ii) **Case 2: Slow Cooling** ($v_m < v_c$)

Given $F_{v,\text{max}} = 10^{-2}$ J, $v_m = 10^{10}$ Hz and $v_c = 10^{16}$ Hz.

Consider the same three frequency regions ($v < v_m$, $v_m < v < v_c$, and $v > v_c$) for a comprehensive analysis. In both the former regions, the influence of v_c is negligible. The effect of v_m, established for $v_m < v < v_c$, seamlessly extends to $v > v_c$. Use `slow_cooling.csv` dataset to formulate F_v.

Assume F_v is proportional to v, v_m, v_c and $F_{v,max}$ (not necessarily linearly or directly proportional) in both cases. Additionally, plot the graphs depicting the relationship between F_v and v.

[10 + 10 = 20 marks]

7. Consider the derived relations for v_c and v_m in both the adiabatic and radiative phases, as obtained in question 5.

   i) It is observed that at sufficiently early times, $v_c < v_m$, implying fast cooling, whereas at later times $v_c > v_m$, indicative of slow cooling. Determine the time instance t_0 where this transition occurs in the adiabatic phase.

   ii) Find the transition time from the radiative to the adiabatic phase characterized by t_{rad}, representing the moment when radiation losses become negligible.

[6 + 9 = 15 marks]

8. Several methods are used to detect Gamma Ray Bursts. BeppoSAX is a satellite that detected a GRB, specifically GRB970508, on May 8, 1997. The spectrum analysis of this GRB revealed a redshift of $z = 0.835$. Based on this information, calculate the distance of this GRB source from Earth. Does the source lie in the Milky Way galaxy? What would be the consequences if a GRB directed towards the Earth, occurred in the Milky Way?

[10 marks]

Link for data files

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Hyperion would like to emphasize that
amidst the demise of stars lies the jets of hope. ☺
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