Astronomy Club, IITK
Summer Projects 2021

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Space : The Final Frontier
Assignment 2

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←DEADLINE→
To Do Part:01→ 18th JUNE 23:59hours
To Do Part 02→ 25th JUNE 23:59hours

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[TRAJECTORY OPTIMIZATION]
"Generation of Baseline to start"

Complete Description of Rocket Dynamics consists of 3 Translational and 3 Rotational Degrees of Freedoms. However, during the initial phases of rocket launch, C.O.M trajectory is of great interest in comparison to its attitude motion**REFER WIKIPEDIA SOURCE FOR WHAT IS ATTITUDE IN CONTEXT OF ROCKETS.
True Attitude of Launcher just affects only the first phase of trajectory and its effect is minimal for overall trajectory. Hence considering this fact it is very reasonable sometimes to assume its rotational dynamics to be neglected and Modeling of Rocket is assumed to be point mass.

Note[1]: 1) ROCKET AXIS IS ALIGNED WITH THRUST DIRECTION AT ANY TIME.
2) Assume model configuration to be a planar 2D problem.

In order to make the problem statement easy we can look for some easy coordinate system like for position vector, define $(\mathrm{ECI})_{\text{earth-centred inertial}}$ coordinate system while velocity vector being in $(\mathrm{LVLH})_{\text{Local-Vertical-Local-Horizontal}}$ frame.
So, in overall define state vector being :

$$X = [\, r \;\; \theta \;\; v_r \;\; v_t \;\; m \,] \rightarrow (2.1)$$

where,
r = geocentric distance,
θ = right ascension (Angular Displacement from launch pads initial position)
v_r and v_t being radial and transverse velocity components.

Now, consider a two stage launch vehicle and its ascent trajectory could be divided into several phases :

1) Vertical Ascent
2) Pitch-over
3) $ZLGT_{\text{Zero Lift Gravity Turn}}$
4) Coasting
5) First Burn Stage
6) Coasting-2
7) Second Burn Stage-2

Note[2] : The time-lengths of the first stage phases and of the first coasting are *fixed*, while the other time-lengths have to be *optimized*, still retaining the *maximum overall burn time of the second stage.*

Consider the forces acting on Rocket to be *GRAVITY (***g***)*, *AERODYNAMIC DRAG(***D***)*, *ENGINE THRUST(***T***)*

→ Effect of gravitational perturbations to ascent trajectory is considered to be very minimal and more sophisticated expressions could be used in other very sophisticated models.

Hence,

→ The gravitational acceleration being :

$$\mathbf{g} = (\mu/r^3)\ [\boldsymbol{r}] \rightarrow \ (2.2)$$

where μ is Earth's gravitational parameter.

Extra Note : [-] is vectorial notation.

→ Drag Force being :

$$\mathbf{D = \tfrac{1}{2}\, C_d.A.\rho.\ v_{rel}^2} \rightarrow \ (2.3)$$

where C_d is coefficient of drag , A is considered as reference surface, ρ is atmospheric density and v is relative velocity w.r.t atmosphere

Also,

Consider the variation of **ρ** to be via isothermal exponential atmospheric model i.e.

$$\boldsymbol{\rho = \rho_o \exp(-(r-r_o)/H)} \rightarrow (2.4)$$

where ρ_o is the reference atmospheric density at the sea level ($r= r_0$) & H being the scaled height of 8.4 kms.

Also,

The relative velocity can be framed with the expression:

$$\mathbf{v_{rel}=v-\omega_E \times r} \rightarrow (2.5)$$

where ω_E is earth's angular velocity.

→ The thrust expression could be written with the contribution of Vacuum thrust (unique to each engine) and actual thrust due to pressure variations at nozzle.

Hence the expression goes as:

$$\mathbf{T\ = T_{vacc} - p.A_{exit}} \rightarrow (2.6)$$

Where, A_{exit} is exit nozzle area and p is the pressure at height h.

T_{vacc} is the usual definition of thrust that we have read in lectures dependent on exhaust velocity and mass flow rate.

Note[3] : Don't forget to consider thrust even in 2D structural force like :

$$\mathbf{[T] = [\ [T_r]\ [T_t]\ ] \rightarrow (2.7)}$$

$$\mathbf{s.t.\ [T_r]^2 + [T_t]^2 = 1 \rightarrow (2.8)}$$

During the gravity turn phase of the ascent trajectory the thrust direction is forced to be parallel to the relative velocity. In order to maintain the same equations of motion across all phases, the thrust magnitude **T** is *fictitiously* split into two attributes, $\mathbf{T_a}$ and $\mathbf{T_b}$.

$\mathbf{T_a}$ represents the optimally controlled thrust contribution, while $\mathbf{T_b}$ is always parallel to the relative velocity. It can be noticed that $\mathbf{T_a}$ and $\mathbf{T_b}$ are alternatively null: during the zero-lift arcs $\mathbf{T_a}$ is zero, while T_b is equal to the real thrust magnitude; conversely, $\mathbf{T_a = T}$ and $\mathbf{T_b = 0}$ during the other propelled arcs.

TO DO PART 01

Now, via all above concept utilization try to define The changes in the state variables of (2.1) and name it for another instant D-state Variable (D') and give it equation termed as (2.9).

Must ensure it should be D'x = f ([T], x, t)

TO DO PART 02

Do refer to the paper provided below in order to think of further steps to do after generating eqn. (2.9)

Few things to figure out :

[1] What is the main deliverable and optimization fn. Which is actually needed for optimizing trajectory.

[2] Look upon the paper with a different perspective and try to relate the above case study concept into it.

[3] Getting direct integration isn't possible So we have to get introduced with discretization techniques. Check the method used in the paper and also try to relate it.

LINK TO REFER : MIT- Aero_Multiobjective Optimization from basic

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