General Qualifying Exam Solutions: Physics

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Contents

1 Physics

1.1 Question 1

Draw the geometry of gravitational microlensing of one star by another, and estimate the angular displacement of the background star's image.

1.1.1 Short answer

Answer.

1.1.2 Additional context

Additional context.

1.1.3 Follow-up Questions

- Can you derive ϕ from a Newtonian approach?
- Why/how does gravitational lensing magnify a star?

1.2 Question 2

A two-element interferometer consists of two telescopes whose light is combined and interfered. Sketch the response of such an interferometer to a nearby red giant star, as a function of the (projected) separation between the two telescopes. The red giant subtends one-fiftieth of an arc second on the sky, and the telescope operates at a wavelength of 2 microns.

1.2.1 Short answer

Answer.

1.2.2 Additional context

Additional context.

1.2.3 Follow-up Questions

- What do the minima in the response function tell you?

1.3 Question 3

What's the minimum mass of a black hole you could survive a fall through the event horizon without being ripped to shreds? Why would you be ripped to shreds for smaller black holes? How does this relate to the BH mass range for which we expect tidal disruption flares caused by shredding main-sequence stars?

1.3.1 Short answer

Answer.

1.3.2 Additional context

Additional context.

1.3.3 Follow-up Questions

- How would you estimate the maximum tidal acceleration a star can withstand?
- Why is it enough to know if the surface of the star will be disrupted?

1.4 Question 4

How is synchrotron radiation generated, and how was it used to demonstrate the energy required to power radio galaxies?

1.4.1 Short answer

Answer.

1.4.2 Additional context

Additional context.

1.5 Question 5

What are "forbidden lines" of atomic spectra? In what conditions are they observationally important? In what conditions do they control the temperature of interstellar material?

1.5.1 Short answer

Answer.

1.5.2 Additional context

Additional context.

1.5.3 Follow-up Questions

- What are some common examples of forbidden lines?
- When do you get forbidden line absorption?
- What kind of radiation can be absorbed by forbidden line absorption?
- Can we observe any forbidden lines on Earth?
- What is the lifetime of the 21 cm line?
- At what redshifts do CMB photons get absorbed by 21 cm transitions?
- At what redshifts do you get neutral hydrogen?
- How would you estimate the lifetime from the maximum density where the line isn't washed out?
- Collisions happen much more frequently; why is the 21 cm line still visible?
- Why does forbidden line emission cool the gas?
- Why are they *so good* at cooling the gas?

1.6 Question 6

What is a polytropic equation of state? Give examples of objects for which this is a very good approximation, and explain why it is.

1.6.1 Short answer

Answer.

1.6.2 Additional context

Additional context.

1.7 Question 7

What was the solar neutrino problem, and how was it resolved?

1.7.1 Short answer

Answer.

1.7.2 Additional context

Additional context.

1.7.3 Follow-up Questions

- Can neutrinos account for all of the dark matter in the Universe?
- If there was a fourth neutrino flavour, how would we detect it?
- Why is the first step of the p-p chain not where we looked for neutrino signatures? (i.e., not energetic enough)

1.8 Question 8

Why is nuclear fusion stable inside a main-sequence star? Under what conditions is nuclear fusion unstable? Give examples of actual objects.

1.8.1 Short answer

Answer.

1.8.2 Additional context

Additional context.

1.8.3 Follow-up Questions

- Can main sequence stars have unstable nuclear fusion?
- If I suddenly increase the core temperature of a star by 20%, how quickly does the star respond?
- What happens when you ignite fusion in a white dwarf? Explain using explicit thermodynamics what happens when you turn up the temperature or reaction rate in the core of the star (i.e. explain adiabatic expansion and contraction).

1.9 Question 9

Why do neutrons inside a neutron star not decay into protons and electrons?

1.9.1 Short answer

Answer.

1.9.2 Additional context

Additional context.

1.9.3 Follow-up Questions

- What sort of products come from NS-NS mergers?
- What are typical photon energies emitted from each of these scenarios?

1.10 Question 10

What is the typical temperature of matter accreting on a star, a white dwarf, a neutron star, a stellar mass black hole, and a supermassive black hole? In what wavelength range would one best find examples of such sources?

1.10.1 Short answer

Answer.

1.10.2 Additional context

Additional context.

1.10.3 Follow-up Questions

- How does the accretion process work?
- If you don't assume a blackbody, and just dump a bunch of material onto a star and it can't radiate away, what is the maximum temperature it can reach?

1.11 Question 11

You don't usually need to cool down the detectors for short wavelength (e.g., X-ray) observations, but it's critical to cool down the detectors in long wavelength (e.g., far-IR) observations. Why is this, and why is it not necessary for radio observations?

1.11.1 Short answer

Answer.

1.11.2 Additional context

Additional context.

1.12 Question 12

Compare the S/N ratios between the following two cases where photon noise is dominant (assume an unresolved point source): [A] 1-minute exposure with a 10-m telescope; [B] 10-minute exposure with a 1-m telescope.

1.12.1 Short answer

Answer.

1.12.2 Additional context

Additional context.

1.13 Question 13

Describe linear and circular polarizations of electromagnetic waves and give examples of their relevance to astronomical observations.

1.13.1 Short answer

Answer.

1.13.2 Additional context

Additional context.

1.14 Question 14

What's the field of view of a 2K $\times$ 2K CCD camera on a 5 m telescope with $f/16$ focal ratio? The pixel size of the CCD is 20 μm. Now, lets bring this to a 10 m telescope with the same focal ratio. Explain how the field of view changes on the 10 m telescope (compared to that of the 5 m telescope) based on the Etendue conservation rule.

1.14.1 Short answer

Answer.

1.14.2 Additional context

Additional context.

1.14.3 Follow-up Questions

- If you wanted a smaller FOV with the same size telescope, what would you change?

1.15 Question 15

Sketch and give the equations for each of the following distributions: 1. Gaussian (Normal distribution); 2. Poisson distribution; 3. Log-normal distribution. Give two examples from astrophysics where each of these distributions apply.

1.15.1 Short answer

Answer.

1.15.2 Additional context

Additional context.

1.15.3 Follow-up Questions

- What sort of noise do you expect for these?
- What extra sources of noise can you get from a CCD?
- Where does the noise come from, and what does the variance/standard deviation actually tell you?
- What property of the Gaussian distribution makes it particularly useful? (i.e., Central limit theorem.)
- How do we get a log-normal distribution?
- Why do we quote n-sigma probabilities and confidence levels with a Gaussian assumption when the actual probability distribution might not be Gaussian?
- If I'm doing a cosmological survey and I say I assume a Poisson distribution for radio galaxy sources and a Gaussian distribution for dust sources, why would I use two different distributions?

1.16 Question 16

You are trying to determine a flux from a CCD image using aperture photometry, measuring source(+sky) within a 5-pixel radius, and sky within a 20-25 pixel annulus. Assume you find 10000 electrons inside the aperture and 8100 electrons in the sky region, and that the flux calibration is good to 1%. What is the fractional precision of your measurement? (Ignore read noise.) More generally, describe how you propagate uncertainties, what assumptions you implicitly make, and how you might estimate errors if these assumptions do not hold.

1.16.1 Short answer

Answer.

1.16.2 Additional context

Additional context.

1.17 Question 17

Suppose you measure the brightness of a star ten times (in a regime where source- noise dominates. (1) How do you calculate the mean, median, and mode and standard deviation? (2) How can you tell if any points are outliers? Say some points are outliers, what do you do now (i.e., how does this impact the calculation of the quantities in part 1)?

1.17.1 Short answer

Answer.

1.17.2 Additional context

Additional context.

1.17.3 Follow-up Questions

- How would you go about fitting a model to your data? (i.e., Bayesian approach.)
- Why is using a gaussian generally a good choice?

1.18 Question 18

Suppose you do an imaging search for binaries for a sample of 50 stars, and that you find companions in 20 cases. What binary fraction do you infer? Suppose a binary- star fraction of 50% had been found previously for another sample (which was much larger, so you can ignore its uncertainty). Determine the likelihood that your result is consistent with that fraction.

1.18.1 Short answer

Answer.

1.18.2 Additional context

Additional context.

1.19 Question 19

What are the primary wavelength bands at which searches for gravitational waves are conducted? What techniques are used to search in each band? What are the sources of gravitational waves in each band? What can we learn from detections (or non-detections)?

1.19.1 Short answer

Answer.

1.19.2 Additional context

Additional context.

1.20 Question 20

Self-similarity is a useful idealization of many astrophysical systems. Explain what self- similarity means, when it works, and why it is so useful, and provide two examples from any field.

1.20.1 Short answer

Answer.

1.20.2 Additional context

Additional context.

1.21 Question 21

Explain why diffraction-limited detectors tend to have sidelobes, and how sidelobes can be suppressed in optical and radio observations.

1.21.1 Short answer

Answer.

1.21.2 Additional context

Additional context.

1.22 Resources

- Astronomical Statistics, Taylor (2004)
- Statistical Methods for Astronomical Data Analysis, Chattopadhyay & Chattopadhyay (2014)
- Data Reduction and Error Analysis for the Physical Sciences, Bevington & Robinson (2003)
- Bayesian Logical Data Analysis for the Physical Sciences, Gregory (2005)
- Magnetic Fields in Diffuse Media, Lazarian (2015)
- Optics, Hecht (2002)
- Astronomical Optics, Schroeder (1987)
- Handbook of CCD Astronomy, Howell (2006)
- Gravitational Waves, Thorne (1994)
- Gravitational Bending of Light, Edwards (2007)
- Gravitational Lensing, Abdo
- Principles of Interferometry, Jackson (2008)

2 Acronyms

AGN: active galactic nuclei
AU: astronomical unit
BAOs: baryonic acoustic oscillations
BB: Big Bang or blackbody
BBN: Big Bang nucleosynthesis
bf: bound-free
BH: black hole
CCD: charged couple device
CCD: colour-colour diagram
CIB: Cosmic Infrared Background
CMB: Cosmic Microwave Background
CMD: colour-magnitude diagram
CNB: Cosmic Neutrino Background
COBE: COsmic Background Explorer
CDM: cold dark matter
CIB: cosmic infrared background
CMB: cosmic microwave background
CR: cosmic ray
CRB: cosmic radio background
CGB: cosmic gamma-ray background
CUVOB: cosmic ultraviolet/optical background
CXB: cosmic x-ray background
DE: dark energy
DM: dark matter
DM: distance modulus
EoR: epoch of reionization
ff: free-free
FIR: far infrared
GUT: grand unified theory
HRD: Hertzsprung-Russel diagram
ICM: intracluster medium
IGM: intergalactic medium
IMF: initial mass function
IR: infrared
ISM: interstellar medium
RJ: Rayleigh-Jeans
LMC: Large Magellanic Cloud
LTE: local thermodynamic equilibrium
MW: Milky Way
MWG: Milky Way Galaxy
NLTE: non-local thermodynamic equilibrium
NS: neutron star
pc: parsec
PL: period-luminosity
PSF point-spread function
QFT: quantum field theory
QSO: quasi-stellar object
RT: radiative transfer
RTE: radiative transfer equation
SED: spectral energy distribution
SF: star formation
SFR: star formation rate

SMBH: supermassive black hole
S/N: signal-to-noise
SNR: signal-to-noise ratio
TE: thermal equilibrium
TE: thermodynamic equilibrium
WD: white dwarf
ΛCDM: Λ cold dark matter
2dFGRS: 2 degree Field Galactic Redshift Survey

3 Parameters

Absorption coefficient: α_ν [cm^{-1}]
Angström: Å [unit]
Angular diameter distance: d_A [pc]
Angular size: θ [rad]
Binding energy of deuterium: E_{D} [MeV]
Binding energy of helium: E_{He} [MeV]
Binding energy of hydrogen: E_{He} [MeV]
Blackbody function: $B_\nu(T)$ [$\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}$]
Boltzmann constant: k_B [$\mathrm{m^2\,kg\,s^{-2}\,K^{-1}}$]
Chandrasekhar mass: M_C [$\mathrm{M_\odot}$]
CMB power spectrum amplitude: C_ℓ [units??]
CMB temperature: T_{CMB} [K]
CMB temperature fluctuations: $\delta T/T$ [dimensionless]
CNB temperature: T_{CNB} [K]
Correlation function: $C(\theta)$ [dimensionless]
Cosmological constant: Λ [m^{-2}]
Critical density: ρ_c [$\mathrm{g\,cm^{-3}}$]
Comoving sound horizon: $r_{\mathrm{H,com}}$ [Mpc]
Cross section: σ [m^2]
Curvature term: $S_\kappa(r)$ [Mpc]
Dark energy density: ρ_Λ [$\mathrm{g\,cm^{-3}}$]
Deceleration parameter: q_0 [dimensionless]
Degeneracy: *see statistical weight*
Density contrast: $\delta(\mathbf{x},t)$ [dimensionless]
Density of baryons: ρ_b [m^{-3}]
Density of dark energy: ρ_Λ [$\mathrm{kg\,m^{-3}}$]
Density of neutrons: ρ_b [m^{-3}]
Density of protons: ρ_b [m^{-3}]
Density of vacuum: ρ_{vac} [$\mathrm{eV\,m^{-3}}$]
Density parameter: Ω [dimensionless]
Density parameter of baryonic matter: Ω_b [dimensionless]
Density parameter of dark energy: Ω_Λ [dimensionless]
Density parameter of dark matter: Ω_c [dimensionless]
Density parameter of matter: Ω_m [dimensionless]
Density parameter of radiation: Ω_r [dimensionless]
Density perturbation field: $\delta(\mathbf{x},t)$ [dimensionless]
Deuterium: D [element]
Distance: d [m]
Distance modulus: $m-M$ [mag]
Electron: e^- [particle]
Electronvolt: eV [unit]
Electroweak age: t_{ew} [s]
Electroweak energy: E_{ew} [TeV]
Electroweak temperature: T_{ew} [K]
Electron radius (classical): $r_{e,0}$ [cm]
Elliptical D_n: D_n [pc]
Ellipticity: ϵ [dimensionless]
Energy: E [eV or J]
Energy density: *see density parameter*
Energy of CMB photons: E_{CMB} [eV]
Energy of photons: E_γ [eV]
Extinction coefficient: A_ν [mag]

Extinction coefficient for the MWG: $A_{V,\mathrm{MWG}}$ [mag]
Flux: F [$\mathrm{erg\,s^{-1}\,cm^{-2}}$]
Flux (spectral): F_ν [$\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}}$]
Frequency: ν [Hz]
Fourier transform: $P(k)$ [function]
GUT age: t_{GUT} [s]
GUT energy: E_{GUT} [TeV]
GUT temperature: T_{GUT} [K]
Gravitational constant: G [$\mathrm{m^3\,kg^{-1}\,s^{-2}}$]
Growth factor: $D_+(t)$ [dimensionless]
Helium: He [element]
Helium mass fraction: Y [dimensionless]
Horizon angular size: θ_{hor} [rad]
Horizon angular size (at recombination): $\theta_{\mathrm{hor,rec}}$ [rad]
Horizon distance: d_{hor} [Mpc]
Hubble constant: H_0 [$\mathrm{km\,s^{-1}\,Mpc}$]
Hubble distance: d_H [Mpc]
Hubble time: t_H [Gyr]
Hydrogen (atomic): H [element]
Hydrogen (molecular): $\mathrm{H_2}$ [element]
IMF: $\xi(m)$ [units???]
Inflationary period: $\mathrm{t_{inflate}}$ [s]
Joule: J [unit]
Lithium: Li [element]
Luminosity: L [$\mathrm{erg\,s^{-1}}$]
Luminosity (Faber-Jackson): L_{FJ} [$\mathrm{erg\,s^{-1}}$]
Luminosity (Tully-Fisher): L_{TF} [$\mathrm{erg\,s^{-1}}$]
Luminosity distance: d_L [Mpc]
Luminosity of the Sun: $L_\odot$ [W]
Luminosity of type Ia SN (at peak): $L_{1\mathrm{a}}$ [$\mathrm{L_\odot}$]
Luminosity distance: d_L [Mpc]
Magnetic monopole density: n_M [$\mathrm{m^{-3}}$]
Magnetic monopole mass: m_M [kg]
Magnitude (absolute): M [mag]
Magnitude (apparent): m [mag]
Mass: $m\,\mathrm{or}\,M$ [kg]
Mass absorption coefficient: κ_ν [$\mathrm{cm^2\,g^{-1}}$]
Mass of the Sun: $M_\odot$ [kg]
Mean free path: ℓ [m]
Megaelectronvolt: MeV [unit]
Metallicity: Z [dimensionless]
Multipole moment: C_ℓ [units???]
Neutron: n [particle]
Neutrino: ν [particle]
Neutrino temperature: T_ν [K]
Neutrino velocity: v_ν [$\mathrm{km\,s^{-1}}$]
Number density: n [$\mathrm{m^{-3}}$]
Number density of baryons: n_b [$\mathrm{m^{-3}}$]
Number density of electrons: n_e [$\mathrm{m^{-3}}$]
Number density of neutrons: n_n [$\mathrm{m^{-3}}$]
Number density of protons: n_p [$\mathrm{m^{-3}}$]
Optical depth: τ [dimensionless]
Parallax: p [rad or °]

Parsec: pc [unit]
Peculiar velocity: $v_{\rm pec}$ [$\rm km\,s^{-1}$]
Period: P [yr]
Photon: γ [particle]
Photon scattering rate (ionized): Γ [$\rm s^{-1}$]
Planck constant: h [$\rm m^2\,kg\,s^{-1}$]
Planck constant (reduced): $\hbar$ [$\rm m^2\,kg\,s^{-1}$]
Planck energy: E_P [eV]
Planck function: $B_\nu(T)$ [$\rm erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}$]
Planck length: ℓ_P [m]
Planck mass: m_P [kg]
Planck time: t_P [s]
Polarized angle: χ [rad]
Polarized intensity: P [Jy]
Positron: e^+ [particle]
Power spectrum: $P(k)$ [units???]
Proper distance: d_p [Mpc]
Proper motion: μ [$''\,\rm year^{-1}$]
Proton: p [particle]
Radius of curvature: R_0 [Mpc]
Radiative flux: F [$\rm W\,m^{-2}$]
Redshift: z [dimensionless]
Redshift (cosmological): $z_{\rm cos}$ [dimensionless]
Redshift (last scattering): $z_{\rm ls}$ [dimensionless]
Redshift of matter-Λ equality: $z_{m\Lambda}$ [dimensionless]
Redshift of radiation-matter equality: z_{rm} [dimensionless]
Reduced Hubble constant: h_0 [$\rm km\,s^{-1}\,Mpc$]
Reduced Planck constant: $\hbar$ [$\rm m^2\,kg\,s^{-1}$]
Rest mass of electron: m_e [kg]
Rest mass of neutron: m_n [kg]
Rest mass of proton: m_p [kg]
Scale factor: a [dimensionless]
Semi-major axis: a [Mpc]
Semi-minor axis: b [Mpc]
Sound speed: v_s [$\rm m\,s^{-1}$]
Spherical harmonics: $Y_{lm}(\theta,\phi)$ [dimensionless]
Sign of curvature: κ [dimensionless]
Silk mass: M_s [kg]
Solid angle: Ω [sr]
Specific intensity: I_ν [$\rm erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}$]
Speed of light: c [$\rm m\,s^{-1}$]
Statistical weight: g [dimensionless]
Stefan-Boltzmann constant: σ [$\rm W\,m^{-2}\,K^{-4}$]
Steradian: sr [unit]
Stokes Q: Q [Jy]
Stokes U: Q [Jy]
Stress energy tensor: $T_{\alpha\beta}$ [unit]
Temperature: T [K]
Temperature of the CMB: $T_{\rm CMB}$ [K]
Temperature of recombination: $T_{\rm rec}$ [K]
Temperature of the Sun: $T_\odot$ [K]
Thomson cross section: σ_T [$\rm m^2$]
Transfer function: $T(k)$ [???]

Tritium: T [element]
Two-point correlation function: $\xi(x)$ [dimensionless]
Velocity: v [$\mathrm{m\,s^{-1}}$]
Velocity (radial): v_r [$\mathrm{m\,s^{-1}}$]
Velocity (rotational): v_{rot} [$\mathrm{m\,s^{-1}}$]
Velocity (tangential): v_t [$\mathrm{m\,s^{-1}}$]
Wavelength: λ [m]
Wavenumber (comoving): k [$\mathrm{m^{-1}}$]

4 Useful Values

Age of the Earth: $t_\oplus = 4.5\,[\text{Gyr}]$
Age of the Solar System: $t_{\text{SS}} = 4.6\,[\text{Gyr}]$
Age of the Universe: $t_{\text{universe}} = 13.7\,[\text{Gyr}]$
Angström: $\text{Å} = 10^{-10}\,[\text{m}]$
Astronomical Unit: $1\ \text{AU} = 1.5 \times 10^{11}\,[\text{m}]$
Binding energy of deuterium: $E_{\text{D}} = 1.1\ [\text{MeV}]$
Binding energy of helium: $E_{\text{He}} = 7\ [\text{MeV}]$
Binding energy of hydrogen: $E_{\text{He}}13.7\ [\text{MeV}]$
Chandrasekhar mass: $M_C = 1.4\ [\text{M}_\odot]$
CMB peak photon wavelength: $\lambda_{\text{CMB}} = 2\,[\text{mm}]$
CMB temperature: $T_{\text{CMB}} = 2.725\,[\text{K}]$
CNB temperature: $T_{\text{CNB}} = 1.9\,[\text{K}]$
Cosmological constant: $\Lambda = 1.11 \times 10^{-52}\ [\text{m}^{-2}]$
Density of dark energy: $\rho_\Lambda = 10^{-27}\ [\text{kg}\,\text{m}^{-3}]$
Density parameter of baryons: $\Omega_b \approx 0.044$ [dimensionless]
Density parameter of CMB: $\Omega_{\text{CMB}} \approx 5 \times 10^{-5}$ [dimensionless]
Density parameter of curvature: $\Omega_\kappa \approx 0.0$ [dimensionless]
Density parameter of cold dark matter: $\Omega_c \approx 0.23$ [dimensionless]
Density parameter of dark energy: $\Omega_\Lambda \approx 0.73$ [dimensionless]
Density parameter of matter: $\Omega_m \approx 0.27$ [dimensionless]
Density parameter of neutrinos: $\Omega_\nu < 0.0032$ [dimensionless]
Density parameter of radiation: $\Omega_r \approx 5 \times 10^{-5}$ [dimensionless]
Dipole: $\ell = 1$ [dimensionless]
Electronvolt in Joules: $1\,\text{eV} = 1.6 \times 10^{-19}\,\text{J}$
Electron radius (classical): $r_{e,0} = 2.82 \times 10^{-13}\ [\text{cm}]$
Electroweak age: $t_{\text{ew}} \sim 10^{-12}\,[\text{s}]$
Electroweak energy: $E_{\text{ew}} \sim 1\,[\text{TeV}]$
Electroweak temperature: $T_{\text{ew}} \sim 10^{16}\,[\text{K}]$
Energy of CMB photons: $E_{\gamma,\text{CMB}} \sim 6 \times 10^{-4}\,[\text{eV}]$
Energy of nuclei fission: $E_{\text{fission}} \sim 1\,[\text{MeV}]$
Energy of nuclei ionization: $E_{\text{ion}} \sim 10\,[\text{eV}]$
GUT age: $t_{\text{GUT}} \sim 10^{-36}\,[\text{s}]$
GUT energy: $E_{\text{GUT}} \sim 10^{12} - 10^{13}\,[\text{TeV}]$
GUT temperature: $T_{\text{GUT}} \sim 10^{28}\,[\text{K}]$
Helium mass fraction: $Y = 0.25$ [dimensionless]
Hubble constant: $H_0 = 70\,[\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}]$
IMF normalization: $\int_{m_L}^{m_U} m\xi(m)\,\text{d}m = 1\,\text{M}_\odot$
Inflationary period: $\text{t}_{\text{inflate}} \sim 10^{-34}\ [\text{s}]$
Hubble distance: $d_H \sim 0.2\,[\text{Mpc}]$
Hubble time: $t_H \sim 14\,[\text{Gyr}]$
Inflationary period: $t_{\text{inflate}} = 10^{-35}\,[\text{s}]$
Large cosmological scale: $\sim 100\,[\text{Mpc}]$
Legendre Polynomials: P_l
Luminosity of the Sun: $L_\odot = 3.8 \times 10^{6}\ [\text{W}]$
Luminosity of type Ia SN (at peak): $L_{1\text{a}} = 4 \times 10^{9}\ [\text{L}_\odot]$
Lyman-alpha wavelength: $\lambda_{\text{Ly}\alpha} = 1216\,[\text{Å}]$
Magnitude (absolute) of the Sun: $M = 26.8$ [dimensionless]
Magnitude (apparent) of the Sun: $m = 4.74$ [dimensionless]
Mass of an electron: $m_e = 9.109 \times 10^{-31}\,[\text{kg}]$
Mass of a neutron: $m_n = 1.67 \times 10^{-27}\,[\text{kg}]$
Mass of a proton: $m_p = 1.67 \times 10^{-27}\,[\text{kg}]$

Megaelectronvolt: $\mathrm{MeV} = 10^{10.065}\,[\mathrm{K}]$
Metallicity of the Sun: $Z \approx 0.02$ [dimensionless]
Milky Way luminosity: $\mathrm{L_{MWG}} = 3.6 \times 10^{10}\,[\mathrm{L}_\odot]$
Monopole: $\ell = 0$ [dimensionless]
Neutron half life: $t_n \sim 890\,[\mathrm{sec}] or \sim 15\,[\mathrm{min}]$
Neutron-proton freezeout: $\frac{n_n}{n_p} \simeq \frac{1}{7}\,[\mathrm{MeV}]$
Neutron-proton rest mass-energy difference: $m_n - m_p = 1.3\,[\mathrm{MeV}]$
Number density of baryons: $n_{b,0} = 0.22\,[\mathrm{m}^{-3}]$
Number density of CMB photons: $n_\gamma = 4.11 \times 10^8\,[\mathrm{m}^{-3}]$
Number density ratio of baryons to CMB photons: $\eta \equiv \frac{n_{b,0}}{n_\gamma} = 5 \times 10^{-10}$ [dimensionless]
Parsec: $1\,\mathrm{pc} = 206265\,\mathrm{AU} = 3.1 \times 10^{16}\,\mathrm{m}$
Photoionization energy of hydrogen: $E_{\mathrm{ion}}^{\mathrm{H}} = 13.6\,[\mathrm{eV}]$
Planck energy: $E_P = 10^{16}\,[\mathrm{TeV}]$
Planck length: $\ell_P = 1.6 \times 10^{-35}\,[\mathrm{m}]$
Planck mass: $M_P = 2.2 \times 10^{-8}\,[\mathrm{kg}]$
Planck temperature: $T_P = 1.4 \times 10^{32}\,[\mathrm{K}]$
Planck time: $t_P = 5.4 \times 10^{-44}\,[\mathrm{s}]$
Proper surface area: $A_p(t_0)$ $[\mathrm{Mpc}^2]$
Recombination age of Universe: $t_{\mathrm{rec}} \sim 380,000\,[\mathrm{yr}]$
Recombination redshift: $z_{\mathrm{rec}} \sim 1,100$ [dimensionless]
Reduced Hubble constant: $\dfrac{H_0}{100} = 0.70\,[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$
Redshift of radiation-matter equality: $z_{rm} \approx 5399$ [dimensionless]
Redshift of matter-Λ equality: $z_{m\Lambda} \approx 0.39$ [dimensionless]
Rest energy of an electron: $m_e c^2 = 0.511\,[\mathrm{MeV}]$
Rest energy of a neutron: $m_n c^2 = 939.6\,[\mathrm{MeV}]$
Rest energy of a proton: $m_p c^2 = 938.3\,[\mathrm{MeV}]$
Scale factor (current): $a_0 = a(t_0) \equiv 1$ [dimensionless]
Seconds in a year: $1\,\mathrm{yr} = 3.2 \times 10^7\,[\mathrm{s}]$
Seconds in a gigayear: $1\,\mathrm{Gyr} = 3.2 \times 10^{16}\,[\mathrm{s}]$
Sign of curvature (closer Universe): $\kappa > 0$ [dimensionless]
Sign of curvature (flat Universe): $\kappa = 0$ [dimensionless]
Sign of curvature (open Universe): $\kappa < 0$ [dimensionless]
Solar luminosity: $1\,\mathrm{L}_\odot = 3.8 \times 10^{26}\,[\mathrm{W}]$
Solar mass: $1\,\mathrm{M}_\odot = 2.0 \times 10^{30}\,[\mathrm{kg}]$
Solar mean photon energy: $\langle E_\odot \rangle = 1.3\,[\mathrm{eV}]$
Solar temperature: $T_\odot = 5800\,[\mathrm{K}]$
Speed of light: $c = 2.998 \times 10^8\,[\mathrm{m\,s^{-1}}]$
Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ $[\mathrm{W\,m^{-2}\,K^{-4}}]$
Temperature at CMB decoupling: $k_B T_{\mathrm{CMB,dec}} = 1/4\,[\mathrm{eV}]$
Thomson cross section: $\sigma_T = 0.665 \times 10^{-24}$ $[\mathrm{cm}^2]$
Vacuum energy density: $\rho_{\mathrm{vac}} = 10^{133}$ $[\mathrm{eV\,cm^{-3}}]$

5 Useful Equations

Absorption coefficient: α_ν [cm^{-1}]

$$\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu \text{ [cm}^{-1}\text{]}$$

Angular diameter distance:

$$D_A = D(1+z)^{-1} \text{ [pc]}$$

Apparent-absolute magnitude relation:

$$\begin{aligned} M &= m - 5\log_{10}\left(\frac{d_L}{10\,\mathrm{pc}}\right) \text{ [mag]} \\ &= m - 5\log_{10}\left(\frac{d_L}{1\,\mathrm{Mpc}}\right) - 25 \text{ [mag]} \end{aligned}$$

Blackbody function:

$$B_\nu(T) = \frac{2h\nu^2}{c^2}\frac{1}{\exp(h\nu/k_BT)-1} \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}\text{]}$$

Blackbody mean photon energy:

$$\langle E_{\mathrm{bb}}\rangle = 2.7\,k_BT \text{ [eV]}$$

Blackbody peak photon energy:

$$E_{\mathrm{bb_{peak}}} = 2.82\,k_BT \text{ [eV]}$$

Boltzmann factor:

$$\frac{n_i}{n_j} = \mathrm{e}^{-\Delta mc^2/k_BT} \text{ [dimensionless]}$$

CMB temperature fluctuations:

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-1}^{\ell} a_{\ell m}Y_{\ell m}(\theta,\phi) \text{ [dimensionless]}$$

Comoving sound horizon:

$$r_{\mathrm{H,com}}(z) = \int_0^t \frac{c\mathrm{d}t}{a(t)} \text{ [Mpc]} \qquad = \int_0^{(1+z)^{-1}} \frac{c\mathrm{d}a}{a^2H(a)} \text{ [Mpc]}$$

Cosmological constant:

$$\Lambda = \frac{3H_0^2}{c^2}\Omega_\Lambda \text{ [m}^{-2}\text{]}$$

Correlation function of CMB temperature anisotropies:

$$C(\theta) = \left\langle \frac{\delta T}{T}(\hat{n})\frac{\delta T}{T}(\hat{n}')\right\rangle_{\hat{n}\cdot\hat{n}'=\cos\theta}$$

$$C(\theta) = \frac{1}{4\pi}\sum_{\ell=0}^{\infty}(2\ell+\ell)(C_\ell)P_\ell(cos\theta),$$

Critical density:

$$\rho_c \equiv \frac{3H_0}{8\pi G} \ [\mathrm{g\,cm^{-3}}]$$

Curvature term:

$$S_\kappa(r) = \begin{cases} R\sin(r/R), & (\kappa = +1) \\ r, & (\kappa = 0) \\ R\sinh(r/R), & (\kappa = -1) \end{cases}$$

Dark energy density:

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} \ [\mathrm{g\,cm^{-3}}]$$

Deceleration parameter:

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_\Lambda \ [\text{dimensionless}]$$

Density contrast:

$$\delta(\mathbf{x}, t) \equiv \frac{\rho(\mathbf{x}, t) - \langle\rho(t)\rangle}{\langle\rho(t)\rangle} \ [\text{dimensionless}]$$

Density parameter of dark energy:

$$\begin{aligned} \Omega_\Lambda &\equiv \frac{\rho_\Lambda}{\rho_c} \\ &= \left(\frac{\Lambda}{\cancel{8\pi G}}\right)\left(\frac{\cancel{8\pi G}}{3H_0}\right) \\ &= \frac{\Lambda}{3H_0} \ [\text{dimensionless}] \ (???) \end{aligned}$$

Density parameter of matter:

$$\Omega_m(a) \equiv \frac{\rho_m}{\rho_c} a(t)^{-3} \ [\text{dimensionless}]$$

Density parameter of radiation:

$$\Omega_r(a) \equiv \frac{\rho_r}{\rho_c} a(t)^{-4} \ [\text{dimensionless}]$$

Distance (moving cluster):

$$d = \frac{v_r \tan\theta}{\mu} \ [\mathrm{pc}]$$

Distance (parallax):

$$d = \left(\frac{p}{1''}\right)^{-1} \ [\mathrm{pc}]$$

Distance (photometric):

$$m - M = 5\log\left(\frac{d}{1\,\mathrm{pc}}\right) - 5 + A \ [\mathrm{mag}]$$

Distance (spectroscopic):

$$m_V - A_V - M_V = 5\log\left(\frac{d}{1\,\mathrm{pc}}\right) - 5 \ [\mathrm{mag}]$$

Distance modulus:

$$m - M \approx 43.17 - 5\log_{10}\left(\frac{H_0}{70\,\mathrm{km\,s^{-1}\,Mpc}}\right) + 5\log_{10} z + 1.086(1-q_0)z \text{ [mag]}$$

Distance relation (low redshift):

$$d_p(t_0) \approx d_L \approx \frac{c}{H_0} z \text{ [Mpc]}$$

Distance-redshift relation (flat Universe):

$$d_L(\kappa = 0) = r(1+z) = d_p(t_0)(1+z) \text{ [Mpc]}$$

Electron radius (classical):

$$r_{e,0} = \frac{e^2}{m_e c^2} \text{ [cm]}$$

Elliptical $D_n - \sigma$ relation:

$$I_n\left(\frac{D_n}{2}\right)^2 \pi = 2\pi I_e R_e^2 \int_0^{D_n/(2R_e)} \text{ [erg s}^{-1}\text{ m}^{-2}]$$

$$D_n = 2.05 \times \left(\frac{\sigma_v}{100\,km\,s^{-1}}\right) \text{ [kpc]}$$

Energy density:

$$\Omega \equiv \frac{\rho}{\rho_c} = nE \text{ [dimensionless]}$$

Energy-redshift relation:

$$E_0 = E(1+z)^{-1} \text{ [eV]}$$

Extinction coefficient:

$$\begin{aligned} A_\nu &\equiv m - m_0 = -2.5\log(I_\nu/I_{\nu,0}) \text{ [mag]} \\ &= 2.5\log(e)\tau_\nu \text{ [mag]} \\ &= 1.086\tau_\nu \text{ [mag]} \end{aligned}$$

Extinction coefficient for the MWG:

$$A_{V,\mathrm{MWG}} = (3.1 \pm 0.1)E(B-V) \text{ [mag]}$$

Flux:

$$F = \frac{L}{4\pi S_\kappa(r)^2}(1+z)^{-2} \text{ [erg s}^{-1}\text{ cm}^{-2}]$$

Flux (spectral):

$$F_\lambda(t) = \int_0^t \mathrm{SFR}(t-t') S_{\lambda, Z(t-t')}(t')\,\mathrm{d}t' \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}]$$

Friedmann equation:

$$\left(\frac{\dot{a(t)}}{a(t)}\right)^2 = \left(\frac{H}{H_0}\right)^2 = \frac{8\pi G}{3}\rho - \frac{\kappa c^2}{R_0^2 a(t)^2} + \frac{\Lambda}{3}$$

$$\left(\frac{H}{H_0}\right)^2 = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1-\Omega_0}{a^2}$$

Growth factor:

$$D_+(t) \propto \frac{H(a)}{H_0} \int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a'^2 - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}} \text{ [dimensionless]}$$

Helium mass fraction:

$$Y = \frac{4 \times n_n/2}{n_n + n_p} = \frac{2}{1 + n_n/n_n} \text{ [dimensionless]}$$

Horizon angular size at recombination:

$$\theta_{H,\mathrm{rec}} \approx \sqrt{\frac{\Omega_m}{z_{\mathrm{rec}}}} \sim \frac{\sqrt{\Omega_m}}{30} \sim \sqrt{\Omega_m} 2 \text{ [}^\circ\text{]}$$

Hubble constant:

$$H_0(t) \equiv \frac{\dot{a_0}(t)}{a_0(t)} = \dot{a_0}(t) \text{ [km s}^{-1}\text{ Mpc]}$$

Hubble distance:

$$d_H \equiv \frac{c}{H(t)} \text{ [Mpc]}$$

Hubble time:

$$t_H \equiv ct_H = \frac{1}{H_0(t)} \text{ [Gyr]}$$

IMF (Salpeter):

$$\xi(m) \propto m^{-2.35} \text{ [units???]}$$

Kepler's third law:

$$P = \sqrt{\frac{4\pi^2}{G(m_1 + m_2)} a^3} \text{ [yr]}$$

Legendre Polynomials:

$$\begin{aligned} P_0(x) &= 1 \text{ [dimensionless]} \\ P_1(x) &= x \text{ [dimensionless]} \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) \text{ [dimensionless]} \end{aligned}$$

Luminosity (Faber-Jackson):

$$L_{\mathrm{FJ}} \propto \sigma_v^\alpha \text{ [erg s}^{-1}\text{]}$$

Luminosity (Tully-Fisher):

$$\begin{aligned} L_{\mathrm{TF}} &= \propto v_{\max}^\alpha \text{ [erg s}^{-1}\text{]} \\ &= \left(\frac{M}{L}\right)^{-2} \left(\frac{1}{G^2 \langle I \rangle}\right) v_{\max}^4 \text{ [erg s}^{-1}\text{]} \end{aligned}$$

Luminosity distance:

$$\begin{aligned} d_L &\equiv \sqrt{\frac{L}{4\pi F}} \text{ [Mpc]} \\ d_L(z \ll 1) &\approx \frac{c}{H_0} z \left(1 + \frac{q_0}{2} z\right) \text{ [Mpc]} \\ d_L(\kappa = 0) &\approx \frac{c}{H_0} z \left(1 + \frac{1 - q_0}{2} z\right) \text{ [Mpc]} \end{aligned}$$

Luminosity distance-redshift relation:

$$d_L = d(1+z) \text{ [Mpc]}$$

Magnetic monopole density:

$$n_M(t_{\rm GUT}) \sim \frac{1}{(2ct_{\rm GUT})^3} \sim 10^{82} \text{ [m}^{-3}\text{]}$$

Magnitude (absolute):

$$M \equiv -2.5\log_{10}\left(\frac{L}{L_0}\right) \text{ [mag]}$$

Magnitude (apparent):

$$m \equiv -2.5\log_{10}\left(\frac{F}{F_0}\right) \text{ [mag]}$$

Matter density-scale factor relation:

$$\rho_m = \rho_{m,0}a^{-3} \text{ [g cm}^{-3}\text{]}$$

Maxwell-Boltzmann equation:

$$\begin{aligned} n &= g\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{E}{k_BT}\right) \text{ [m}^{-3}\text{]} \\ &= g\left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3/2}\exp\left(-\frac{h\nu}{k_BT}\right) \text{ [m}^{-3}\text{]} \end{aligned}$$

Mean free path:

$$\ell_{\rm mfp} = \frac{1}{n\sigma} \text{ [pc]}$$

Metallicity index:

$$[{\rm X/H}] \equiv \log\left(\frac{n({\rm X})}{n({\rm H})}\right)_* - \log\left(\frac{n({\rm X})}{n({\rm H})}\right)_\odot \text{ [dimensionless]}$$

Neutrino velocity:

$$v_\nu \sim 150(1+z)\left(\frac{m_\nu}{1\,{\rm eV}}\right)^{-1} \text{ [km s}^{-1}\text{]}$$

Peculiar velocity:

$$v_{\rm pec} = c\left(\frac{z - z_{\rm cos}}{1+z}\right) \text{ [km s}^{-1}\text{]}$$

Photon energy:

$$E_\gamma = h\nu \text{ [eV]}$$

Photon scattering rate (ionized):

$$\Gamma = \frac{c}{\ell_{\rm mfp}} = n_e\sigma_e c = n_b\sigma_e c = \left(\frac{n_{b,0}}{a^3}\right)\sigma_e c = \frac{4.4\times10^{-21}a^3}{}\text{[s}^{-1}\text{]}$$

Planck function:

$$B_\nu(T) = \frac{2h\nu^2}{c^2}\frac{1}{\exp(h\nu/k_BT)-1} \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}\text{]}$$

Polarized angle:

$$\chi = \frac{1}{2}\arctan\left(\frac{U}{Q}\right)\ \text{[rad]}$$

Polarized intensity:

$$P = \sqrt{Q^2 + U^2} = pI\ \text{[Jy]}$$

Proper distance:

$$d_p(t_0) = c\int_t^{t_0} \frac{\mathrm{d}t}{a(t)}\ \text{[Mpc]}$$
$$d_p(t_0, z \ll 1) \approx \frac{c}{H_0} z\left(1 - \frac{1+q_0}{2}z\right)\ \text{[Mpc]}$$

Proper surface area:

$$A_p(t_0) = 4\pi S_\kappa(r)^2\ [\text{Mpc}^2]$$

Radiation density-scale factor relation:

$$\rho_r = \rho_{r,0} a^{-4}\ [\text{g cm}^{-3}]$$

Random walk:

$$d = \sqrt{N}\ell\ \text{[m]}$$

Redshift relation:

$$\frac{1}{1+z} = \frac{\lambda}{\lambda_0}\ \text{[dimensionless]}$$

Robertson-Walker metric:

$$\mathrm{d}s^2 = c\mathrm{d}t^2 - a(t)^2\left(\frac{\mathrm{d}x^2}{1 - \kappa x^2/R^2} + x^2\Omega^2\right)$$

Saha equation:

$$\frac{n_{i+1}}{n_i} = T^{3/2}\mathrm{e}^{(-1/T)}\ \text{[dimensionless]}$$

Scale factor evolution (radiation-dominated):

$$a_m(t) \propto t^{1/2}\ \text{[dimensionless]}$$

Scale factor evolution (matter-dominated):

$$a_m(t) \propto t^{2/3}\ \text{[dimensionless]}$$

Scale factor evolution (Λ-dominated):

$$a_\Lambda(t) \propto e^{Ht}\ \text{[dimensionless]}$$

Scale factor-redshift relation:

$$a = \frac{1}{1+z}\ \text{[dimensionless]}$$

Solid angle:

$$\Omega = \frac{A}{r^2}\ [\mathrm{sr}]$$

Sound speed:

$$c_s = \sqrt{\frac{\partial P}{\partial \rho}}\ [\mathrm{m\,s^{-1}}]$$

Star formation rate:

$$\mathrm{SFR} = -\frac{\mathrm{d}M_{\mathrm{gas}}}{\mathrm{d}t}\ [\mathrm{M_\odot\,yr^{-1}}]$$

Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3}\ [\mathrm{W\,m^{-2}\,K^{-4}}]$$

Stefan-Boltzmann Law:

$$F = \sigma T^4\ [\mathrm{W\,m^{-2}}]$$

Temperature-redshift relation:

$$T = (1+Z)T_0\ [\mathrm{K}]$$

Temperature-scale factor relation:

$$T = aT_0\ [\mathrm{K}]$$

Thomson cross section:

$$\sigma_T = \frac{8\pi}{3} r_{e,0}^2\ [\mathrm{m^2}]$$

Tired Light Hypothesis energy scaling:

$$E = E_0 \mathrm{e}^{(-r/R_0)}\ [\mathrm{eV}]$$

Uncertainty Principle:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2}\ [\mathrm{J\,s}]$$
$$\Delta E \Delta t \geq \frac{\hbar}{2}\ [\mathrm{J\,s}]$$

Velocity (radial):

$$v_r = \left(\frac{\Delta\lambda}{\lambda_0}\right) c\ [\mathrm{m\,s^{-1}}]$$

Velocity (tangential):

$$v_t = d\mu = 4.74 \left(\frac{d}{1\,\mathrm{pc}}\right)\left(\frac{\mu}{1''\,\mathrm{year}^{-1}}\right)\ [\mathrm{m\,s^{-1}}]$$

Wavelength-redshift relation:

$$\lambda_0 = \frac{1}{a(t)}\lambda = (1+z)\lambda\ [\mathrm{m}]$$

Wien's Law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_BT)-1}\ [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}]$$
$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\frac{1}{\exp(h\nu/k_BT)}\ [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}]$$
$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\exp\left(\frac{-h\nu}{k_BT}\right)\ [\mathrm{erg\,s^{-1}\,cm^{-2}\,Hz^{-1}\,sr^{-1}}]$$

6 General Notes

- $k_B(300\,\mathrm{K}) \sim \frac{1}{40}\,\mathrm{eV}$
- $k_B(11,000\,\mathrm{K}) \sim 1\,\mathrm{eV}$
- $k_B(10^7\,\mathrm{K}) \sim 1\,\mathrm{keV}$
- If a particle scatters with a rate greater than the expansion rate of the Universe, that particle remains in equilibrium.
- Multipole moment $\ell = 1$ and $\ell = 2$ are the dipole and quadrupole moment, respectively.
- The greatest baryon contribution to the density comes not from stars in galaxies, but rather from gas in groups of galaxies; in these groups, $\Omega \sim 0.02$.
- The ratio of the neutrino temperature T_ν to the CMB temperature T_{CMB} is $\frac{T_\nu}{T_{\mathrm{CMB}}} = \left(\frac{4}{11}\right)^{1/3}$.
- We expect photons to decouple from matter when the Universe is already well into the matter-dominated era (i.e., not in the radiation-dominated era).
- When the Universe was only one second old, the mean-free-path of a photon was about the size of an atom.
- Nuclear binding energies are typically in the MeV range, which explains why Big Bang nucleosynthesis occurs at temperatures a bit less than 1 MeV even though nuclear masses are in the GeV range.
- The electron-positron energy threshold is $\sim 10^{10}$ K, above which there exist thermal populations of both electrons and neutrinos to make the reaction $p + e^- \leftrightarrow n + \nu$ go equally well in either direction.
- Themodynamically, nucleons with greater binding energies are more energetically favourable.
- Simplest way to form helium is via deuterium fusion rather than the improbable coincidence of 2 protons and 2 neutrons all arriving at the same place simultaneously to make ^{4}He in one go.
- Nucleosynthesis starts at about 10^{10} K when the Universe was about 1 s old, and effectively ends when it has cooled by a factor of 10, and is about 100 times older.
- Primordial elemental abundances: 75% H, 25% He, trace Li.
- A normal population of states in LTE satisfies $(n_1/g_1)/(n_2/g_2) > 1$ via the Maxwell-Boltzmann equation; inverted populations such as masers satisfy $(n_1/g_1)/(n_2/g_2) < 1$.
- The solid angle subtended by a complete sphere is 4π sr.
- Just as the period of a Cepheid tells you its luminosity, the rise and fall time of a type Ia SN tells you its peak luminosity.
- The Hubble distance d_H is the distance between the Earth and any astrophysical object receding away from us at the speed of light.
- At $z = 10^9$, the Jeans mass is $M_J \sim M_\odot$; at $z = 10^6$, the Jeans mass is $M_J \sim M_{\mathrm{gal}}$.
- Apparent magnitudes $0 < m < 6$ are typically visible to the naked eye.

- The absolute magnitude of a light source M is defined as the apparent magnitude m that it would have if it were at a luminosity distance of $d_L = 10\,\mathrm{pc}$.
- The apparent magnitude is really nothing more than a logarithmic measure of the flux, and the absolute magnitude is a logarithmic measure of the luminosity.
- When space is positively curved, the proper surface area is $A_p(t_0) < 4\pi r^2$, and the photons from distance r are spread over a smaller area than they would be in flat space. When space is negatively curved, $A_p(t_0) > 4\pi r^2$, and photons are spread over a larger area than they would be in flat space.
- The Benchmark Model has a deceleration parameter of $q_0 \approx 0.55$.
- The angular distribution of the CMB temperature reflects the matter inhomogeneities at the redshift of decoupling of radiation and matter.
- CMB polarization probes the epoch of last scattering directly as opposed to the temperature fluctuations which may evolve between last scattering and the present. Moreover, different sources of temperature anisotropies (scalar, vector, and tensor) give different patterns in the polarization: both in its intrinsic structure and in its correlation with the temperature fluctuations themselves.
- The sound speed in the photon-dominated fluid of the early Universe is given by $c_s \approx c/\sqrt{3}$. Thus, the sound horizon is about a factor of $\sqrt{3}$ smaller than the event horizon at this time.
- At recombination, the free electrons recombined with the hydrogen and helium nuclei, after which there are essentially no more free electrons which couple to the photon field. Hence, after recombination the baryon fluid lacks the pressure support of the photons, and the sound speed drops to zero – the sound waves do no longer propagate, but get frozen in.
- Due to the steep slope of the IMF, most of the stellar mass is contained in low-mass stars; however, since the luminosity of main-sequence stars depends strongly on mass, approximately as $L \propto M^3$, most of the luminosity comes from high-mass stars.