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TIRED LIGHT AND THE "MISSING MASS" PROBLEM

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I. Introductory remarks

The paradox of the discrepancy between the "number mass" and "dynamical mass" for clusters of galaxies has been known since the 1930's. All attempts to resolve this paradox to date must be considered inconclusive at best. The solution proposed by AMBARTSUMIAN — that clusters are expanding and thus the Virial Theorem cannot be used to calculate the dynamical mass — is generally held to be untenable [1, 2, 3, 4]. The other, most popular, hypothetical solution to the paradox is the well known "missing", i.e. non-luminous and as yet undetected, mass. A number of searches based on the assumption that this hypothetical "hidden mass" exists in familiar forms have all yielded negative results [5, 6, 7].

The "missing mass" hypothesis is appealing for cosmological reasons as well as that it appears to be the only feasible solution that will not require changes in accepted theory. The most popular relativistic cosmology, for which the cosmological constant is zero, requires a mass density for the universe which is substantially greater than that observed. In fact, 90 to 99 percent of the matter in the universe would have to be in the form of "missing mass". The most recent speculation is that perhaps the "missing mass" is in the form of black holes. However, the presence of a black hole has, as yet, never been detected and identified. Furthermore, the mass discrepancy for clusters of galaxies places another, rather stringent requirement on the form of the "missing mass". As BURBIDGE and BURBIDGE have pointed out [1], "If this material is distributed uniformly inside and outside clusters it will not affect the arguments concerning the stability of clusters. Only if it is also clustered will the argument be changed. If it were concentrated sufficiently to stabilize loose clusters, there would have to be a much higher density of intergalactic matter inside loose clusters than inside condensed clusters. This would appear to be improbable".

All currently popular cosmologies interpret the cosmological redshift as a genuine Doppler shift. However, when the systematic redshifts of the nebulae were first discovered it was proposed by MACMILLAN that perhaps the redshifts were due to a secular decrease in the frequency of light as it traveled through space [8] — the well known "tired light" hypothesis. When the mass discrepancy for clusters of galaxies was discovered in the thirties, and several times since then, it has been speculated in passing that the tired light hypothesis might resolve the paradox; but for various reasons it appears that this possibility has never been thoroughly investigated [2]. Since the paradox is still with us after almost forty years, perhaps a thorough, quantitative investigation of the "tired light" hypothesis is justified.

We must emphasize at this point, however, that we do not argue that the "tired light" hypothesis is the *only* acceptable solution to the mass discrepancy paradox. Our only intent is to demonstrate explicitly that it is a viable solution which is not at variance with any observational results. If one chooses to test the hypothesis, we suggest an experiment at the end of this paper which should unambiguously show whether or not the "tired light" hypothesis has any value.

II. Discussion

Unquestionably, the most popular and time-honored device used when one tinkers with a field theory is to introduce an exponential into the equation for the potential of the field. Aside from NEWTON, EULER and YUKAWA, others who have used this device are NEUMANN [9], MILNE [10], FREUND et al. [11], and GERASIM [12]. Let us assume then, as a formalism for the "tired light" hypothesis, that the frequency of light (ω) at a distance r from a source which has a frequency ω_0 is

$$\omega = \omega_0 \exp(-\alpha r),$$

where α is an attenuation coefficient. For r less than 10^3 megaparsecs (mpc) the approximation

$$\omega = \omega_0(1 - \alpha r)$$

is valid. We assume now that the cosmological redshift is entirely due to the exponential factor, i.e. that the cosmological redshift is purely a "tired light" phenomenon. This gives us

$$\omega = \omega_0(1 - Hr/c),$$

H being the Hubble constant and c the velocity of light.

The mass for a cluster of galaxies determined by the Virial Theorem (M_v) is given by the equation:

$$M_v = V^2 R/G,$$

where G is the constant of gravitation, R twice the harmonic weighted (by mass) mean distance between cluster members, and V the root-mean-square space velocity for cluster members determined from the redshift dispersion of the galaxies in the cluster. Reinterpretation of the cosmological redshift as due to a "tired light" effect will affect V in the above equation leading to a corrected Virial Theorem mass prediction (M_c) given by:

$$M_c = \frac{(V - 1.73HR')^2 R}{G},$$

where R' is the dynamical radius of the cluster [13]. Naturally M_c should be equal to the number mass (M_n); thus, defining the mass discrepancy as $A = M_v/M_n$, we have for the mass discrepancy

$$A^{1/2} = \frac{V}{V - 1.73HR'}.$$

As the dynamical radii of clusters, except in the case of the Coma cluster, are not well known, we shall rearrange the above equation to give R' in terms of A and V. We get:

$$R' = \frac{V}{1.73H}(1 - 1/A^{1/2}). \qquad (1)$$

It but remains to check and see if the above equation leads to reasonable dynamical radii for clusters of galaxies.

III. Observation

As the value of A depends on M_n, which in turn depends on the mass to luminosity (M/L) ratios of the various types of galaxies, we shall briefly consider the M/L ratios deduced from observation. Perhaps the most reliable M/L ratios are those obtained by PAGE from observations of double galaxies [14]. However, two corrections must be applied to PAGE's values. As VAN DEN BERGH has pointed out [3], the values obtained by PAGE are almost certainly in error as many of the pairs used by PAGE are located in clusters where the application of the Virial Theorem is highly questionable. He recomputed the M/L ratios excluding the pairs that are cluster members. The exclusion of cluster members reduces PAGE's value for E and SO systems by about 50%. Also, as POVEDA has noted [15], the assumption of circular orbits for pairs is, by analogy with diatomic molecules, unlikely to be valid. Accordingly, VAN DEN BERGH's values of M/L (which assume circular orbits) should be reduced by about 20%. Making these modifications, one obtains 40 for the M/L ratio for E and SO galaxies and 3.5 for S or Irr systems. (M/L is in units of solar mass and luminosity.)

From the literature one may, for particular clusters, get the values of the velocity dispersion (V), the percentage of E and SO galaxies in the cluster, and the average M/L ratio computed from the Virial Theorem $(\overline{M/L})_v$. ([1, 3,

Table I

The predicted values of the dynamical radius (R') for four clusters with more than fifty members and the quantities for each cluster necessary to obtain the predictions

Cluster	V (km/sec)	%E + SO	$\overline{M/L}$	$(\overline{M/L})$	A	R' (mpc)
Hercules	1093	30	15	100	6.7	3.9
Coma	1730	approx. 100	40	350	9.0	6.7
Virgo	1040	40	18	600	33	5.0
NGC 541	703	50	22	240	11	2.8

4, 16, 17 and 18].) The value of A may then be determined by dividing $(\overline{M/L})_v$ by the average M/L ratio computed from the percentage of E and SO galaxies and the values of M/L given above. Using the values of V and A so obtained (and $H = 100$ km/sec per mpc) the predicted dynamical radii (R') may be computed using Eq. (1). All of the relevant values appear in Table I for four clusters with more than fifty members.

The only cluster for which R' is observationally well known is Coma where R' is about 6.3 mpc. The agreement for this cluster is quite good considering that A and H are only accurate to a factor of about 1.5. As for the other clusters, it is known that there are members, usually much fainter and less massive, outside of the central area of condensation where most of the bright, heavy galaxies are located; e.g. the "southern extension" of the Virgo cluster. Thus it would seem that the values of R' in Table I are at least not unreasonable.

One of the two serious objections that can be made against the use of the "tired light" hypothesis in the above manner is that it reinterprets most of the normal velocity dispersion (V), calculated from the redshifts, as being simply an indication of distance and not of velocity of recession. Consequently, the relaxation time for mass segregation, which is approximately proportional to the cube of the actual velocity, must in general be reduced. For instance, in the case of the Coma cluster the relaxation time must be reduced by a factor of 28. Thus one should observe extensive mass segregation if the "tired light" hypothesis is to be an acceptable explanation of the mass discrepancy paradox.

But one can claim that extensive mass segregation is observed. Let us take as an example the cluster NGC 541. Zwicky and Humason report that there are three distinct levels of mass segregation present in the cluster [4]. 1) a central region with five pairs of galaxies actually in contact, 2) a region with a radius of about 0.47 mpc where practically all the bright, massive galaxies are located, and 3) a less readily delimitable region where there are many faint galaxies of small mass associated with the cluster. In the Coma cluster virtually all of the massive galaxies are located within a radius of one mpc of the center, while small-mass cluster members are observed at distances of up to 6.3 mpc from the center.

IV. Maxwell's equations and photon rest mass

The second serious objection to the "tired light" hypothesis is that Maxwell's equations must be altered. The hypothesis, as formulated above, implies that the Poynting vector ($\vec{S}$) normally given by

$$\vec{S} \sim \vec{E} \times \vec{H},$$

must be changed to

$$\vec{S} \sim e^{-Hr/c}(\vec{E} \times \vec{H}), \tag{2}$$

where $\vec{E}$ and $\vec{H}$ are the electric and magnetic field vectors for, in this case, an isotropic, spherically symmetric field. The exponential factor in Eq. (2) gives the electromagnetic field a finite range and consequently photons with finite rest mass. The modification of Maxwell's equations which includes finite rest mass photons is the Proca equation [19]:

$$\partial^{\lambda} F_{\lambda\nu} + \mu^2 A_\nu = (4\pi/c) J_\nu,$$

which yields a Poynting vector of the form

$$\vec{S} \sim e^{-2\mu r}(\vec{E} \times \vec{H}),$$

μ being the photon rest mass. Thus, reinterpretation of the cosmological redshift as a "tired light" effect implies that the photon should have a rest mass:

$$\mu = H/2c.$$

It is empirically established that the value of μ in terms of the characteristic length of the field is less than 10^{-10} cm^{-1} [19]. The value of $H/2c$ is about 10^{-26} cm^{-1}; thus the above interpretation of the cosmological redshift is not at variance with any empirical observations. Looking at the other side of the coin, we may say that the cosmological redshift sets an upper limit of about 10^{-63} grams to the photon rest mass — a value 15 orders of magnitude better than that of GOLDHABER and NIETO [19].

V. A testable consequence

As we have already noted, the "tired light" hypothesis is highly speculative and the principal aim of this paper is to demonstrate that this hypothesis can resolve the mass discrepancy paradox for clusters of galaxies, though it may very well not be the correct explanation. Astronomically there is no direct method of distinguishing between the "tired light" hypothesis and those theories which interpret the cosmological redshift as the result of a real velocity of recession. Interestingly, there is, however, a consequence of the tired light hypothesis that may be terrestrially testable.

The above interpretation of the cosmological redshift demands that the universe be static. Concomitantly, due to the energy loss of electromagnetic radiation (the tired light effect), the mass-energy density of the universe must be decreasing. This, of course, is a violation of both the perfect cosmological principle and the principle of conservation of mass-energy. There are two ways of avoiding this dilemma. One is to assume that the energy lost by radiation is replaced by the "continuous creation" of matter *ex nihilo* throughout space (a mechanism similar to that of Steady State cosmology). This we tentatively reject because of its metaphysical nature. The other is to assume that the lost energy does not cease to exist, but is reconstituted into intergalactic hydrogen [8].

The recent experiments designed to test the "continuous creation" hypothesis of Steady State cosmology suggest that an analogous experiment might be done to test this "continuous reconstitution" hypothesis. One would simply pump electromagnetic radiation into a confining cavity to see if it decays into hydrogen. The most effective way to do this would be to use radio-frequency radiation and a cavity with superconducting walls to maximize the lifetime of the photons. In a straight-forward manner it may be shown that the number of hydrogen atoms (n) produced in a spherical cavity of radius r with a skin depth δ in a time t is:

$$n \simeq \frac{HrPt}{3\delta\omega E_0},$$

where P is the power of the source of radiation, ω its frequency, H the Hubble constant, and E_0 the rest energy of the hydrogen atom. If we take $r = 3$ meters, $\omega = 10^8$ Hz, and $P = 10^3$ watts, this should result in a production rate of several hydrogen atoms per day. If the experiment could be done, the results would be of interest and it would either corroborate or lay to rest forever the "tired light" hypothesis.

Addendum

While this paper was in the press, the work of VIGIER [20] and DUCHESNE and VIGIER [21] has come to our attention. They furnish empirical evidence of a convincing nature which indicates that, contrary to current belief, the photon has a finite rest-mass. Unfortunately, the experiment does not admit of a quantitative determination of the rest-mass.

However, the "tired light" interpretation of the cosmological red-shift is certainly more plausible and appealing if one can show from independent observations (as one evidently is able to do) that the photon has a finite rest-mass. In any event, as we noted in our paper, the cosmological red-shift demands a value limit of 10^{-63} grams for the photon rest-mass. Clearly, in the laboratory it will prove quite difficult to obtain an accurate value for so small a quantity. But if it turns out to be 10^{-63} grams, then the "tired light hypothesis" is in fact the correct solution to the missing mass problem.

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$$(v_r)_i = s_i - s_0 ,$$

where s_0 is the redshift of the center of the cluster which is just the mean of the values of $(v_r)_i$. But according to the "tired light" hypothesis the true space velocity in one dimension of the ith galaxy $(u_r)_i$ will be $(v_r)_i$ minus the apparent velocity of recession due to the cosmological redshift, i.e.

$$(u_r)_i = (v_r)_i - H_i .$$

It is impossible to know r_i for any particular galaxy, but this is not important as we are only interested in the mean of all of the values of r_i. It is easy to see that the mean of the r_i's is approximately equal to the dynamical radius of the cluster (R').
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