

GENERAL QUALIFYING EXAM SOLUTIONS: APPENDIX

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1 Appendix

1.1 Acronyms

AGN: active galactic nuclei
AU: astronomical unit
BAOs: baryonic acoustic oscillations
BB: Big Bang or blackbody
BBN: Big Bang nucleosynthesis
bf: bound-free
BH: black hole
CCD: charged couple device
CCD: colour-colour diagram
CIB: Cosmic Infrared Background
CMB: Cosmic Microwave Background
CMD: colour-magnitude diagram
CNB: Cosmic Neutrino Background
COBE: COsmic Background Explorer
CDM: cold dark matter
CIB: cosmic infrared background
CMB: cosmic microwave background
CR: cosmic ray
CRB: cosmic radio background
CGB: cosmic gamma-ray background
CUVOB: cosmic ultraviolet/optical background
CXB: cosmic x-ray background
DE: dark energy
DM: dark matter
DM: distance modulus
EoR: epoch of reionization
ff: free-free
FIR: far infrared
GUT: grand unified theory
HRD: Hertzsprung-Russel diagram
ICM: intracluster medium
IGM: intergalactic medium
IMF: initial mass function
IR: infrared
ISM: interstellar medium
RJ: Rayleigh-Jeans
LMC: Large Magellanic Cloud
LTE: local thermodynamic equilibrium
MW: Milky Way
MWG: Milky Way Galaxy
NLTE: non-local thermodynamic equilibrium
NS: neutron star
pc: parsec
PL: period-luminosity
PSF: point-spread function
QFT: quantum field theory
QSO: quasi-stellar object
RT: radiative transfer
RTE: radiative transfer equation
SED: spectral energy distribution

SF: star formation
SFR: star formation rate
SMBH: supermassive black hole
S/N: signal-to-noise
SNR: signal-to-noise ratio
TE: thermal equilibrium
TE: thermodynamic equilibrium
WD: white dwarf
 Λ CDM: Λ cold dark matter
2dFGRS: 2 degree Field Galactic Redshift Survey

1.2 Variables

Absorption coefficient: α_ν [cm^{-1}]
Angström: \AA [unit]
Angular diameter distance: d_A [pc]
Angular size: θ [rad]
Binding energy of deuterium: E_D [MeV]
Binding energy of helium: E_{He} [MeV]
Binding energy of hydrogen: E_{H} [MeV]
Blackbody function: $B_\nu(T)$ [$\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1} \text{sr}^{-1}$]
Boltzmann constant: k_B [$\text{m}^2 \text{kg s}^{-2} \text{K}^{-1}$]
Chandrasekhar mass: M_C [M_\odot]
CMB power spectrum amplitude: C_ℓ [units??]
CMB temperature: T_{CMB} [K]
CMB temperature fluctuations: $\delta T/T$ [dimensionless]
CNB temperature: T_{CNB} [K]
Correlation function: $C(\theta)$ [dimensionless]
Cosmological constant: Λ [m^{-2}]
Critical density: ρ_c [g cm^{-3}]
Comoving sound horizon: $r_{\text{H,com}}$ [Mpc]
Cross section: σ [m^2]
Curvature term: $S_\kappa(r)$ [Mpc]
Dark energy density: ρ_Λ [g cm^{-3}]
Deceleration parameter: q_0 [dimensionless]
Degeneracy: *see statistical weight*
Density contrast: $\delta(\mathbf{x}, t)$ [dimensionless]
Density of baryons: ρ_b [m^{-3}]
Density of dark energy: ρ_Λ [kg m^{-3}]
Density of neutrons: ρ_n [m^{-3}]
Density of protons: ρ_p [m^{-3}]
Density of vacuum: ρ_{vac} [eV m^{-3}]
Density parameter: Ω [dimensionless]
Density parameter of baryonic matter: Ω_b [dimensionless]
Density parameter of dark energy: Ω_Λ [dimensionless]
Density parameter of dark matter: Ω_c [dimensionless]
Density parameter of matter: Ω_m [dimensionless]
Density parameter of radiation: Ω_r [dimensionless]
Density perturbation field: $\delta(\mathbf{x}, t)$ [dimensionless]
Deuterium: D [element]
Distance: d [m]
Distance modulus: $m - M$ [mag]
Electron: e^- [particle]
Electronvolt: eV [unit]
Electroweak age: t_{ew} [s]
Electroweak energy: E_{ew} [TeV]
Electroweak temperature: T_{ew} [K]
Electron radius (classical): $r_{e,0}$ [cm]
Elliptical D_n : D_n [pc]
Ellipticity: ϵ [dimensionless]
Energy: E [eV or J]
Energy density: *see density parameter*
Energy of CMB photons: E_{CMB} [eV]
Energy of photons: E_γ [eV]
Extinction coefficient: A_ν [mag]

Extinction coefficient for the MWG: $A_{V,\text{MWG}}$ [mag]
Flux: F [$\text{erg s}^{-1} \text{cm}^{-2}$]
Flux (spectral): F_ν [$\text{erg s}^{-1} \text{cm}^{-2} \text{Hz}^{-1}$]
Frequency: ν [Hz]
Fourier transform: $P(k)$ [function]
GUT age: t_{GUT} [s]
GUT energy: E_{GUT} [TeV]
GUT temperature: T_{GUT} [K]
Gravitational constant: G [$\text{m}^3 \text{kg}^{-1} \text{s}^{-2}$]
Growth factor: $D_+(t)$ [dimensionless]
Helium: He [element]
Helium mass fraction: Y [dimensionless]
Horizon angular size: θ_{hor} [rad]
Horizon angular size (at recombination): $\theta_{\text{hor,rec}}$ [rad]
Horizon distance: d_{hor} [Mpc]
Hubble constant: H_0 [$\text{km s}^{-1} \text{Mpc}$]
Hubble distance: d_H [Mpc]
Hubble time: t_H [Gyr]
Hydrogen (atomic): H [element]
Hydrogen (molecular): H_2 [element]
IMF: $\xi(m)$ [units??]
Inflationary period: t_{inflate} [s]
Joule: J [unit]
Lithium: Li [element]
Luminosity: L [erg s^{-1}]
Luminosity (Faber-Jackson): L_{FJ} [erg s^{-1}]
Luminosity (Tully-Fisher): L_{TF} [erg s^{-1}]
Luminosity distance: d_L [Mpc]
Luminosity of the Sun: L_\odot [W]
Luminosity of type Ia SN (at peak): L_{1a} [L_\odot]
Luminosity distance: d_L [Mpc]
Magnetic monopole density: n_M [m^{-3}]
Magnetic monopole mass: m_M [kg]
Magnitude (absolute): M [mag]
Magnitude (apparent): m [mag]
Mass: m or M [kg]
Mass absorption coefficient: κ_ν [$\text{cm}^2 \text{g}^{-1}$]
Mass of the Sun: M_\odot [kg]
Mean free path: ℓ [m]
Megaelectronvolt: MeV [unit]
Metallicity: Z [dimensionless]
Multipole moment: C_ℓ [units??]
Neutron: n [particle]
Neutrino: ν [particle]
Neutrino temperature: T_ν [K]
Neutrino velocity: v_ν [km s^{-1}]
Number density: n [m^{-3}]
Number density of baryons: n_b [m^{-3}]
Number density of electrons: n_e [m^{-3}]
Number density of neutrons: n_n [m^{-3}]
Number density of protons: n_p [m^{-3}]
Optical depth: τ [dimensionless]
Parallax: p [rad or $^\circ$]

Parsec: pc [unit]
Peculiar velocity: v_{pec} [km s⁻¹]
Period: P [yr]
Photon: γ [particle]
Photon scattering rate (ionized): Γ [s⁻¹]
Planck constant: h [m² kg s⁻¹]
Planck constant (reduced): \hbar [m² kg s⁻¹]
Planck energy: E_P [eV]
Planck function: $B_\nu(T)$ [erg s⁻¹ cm⁻² Hz⁻¹ sr⁻¹]
Planck length: ℓ_P [m]
Planck mass: m_P [kg]
Planck time: t_P [s]
Polarized angle: χ [rad]
Polarized intensity: P [Jy]
Positron: e^+ [particle]
Power spectrum: $P(k)$ [units??]
Proper distance: d_p [Mpc]
Proper motion: μ [″ year⁻¹]
Proton: p [particle]
Radius of curvature: R_0 [Mpc]
Radiative flux: F [W m⁻²]
Redshift: z [dimensionless]
Redshift (cosmological): z_{cos} [dimensionless]
Redshift (last scattering): z_{ls} [dimensionless]
Redshift of matter- Λ equality: $z_{m\Lambda}$ [dimensionless]
Redshift of radiation-matter equality: z_{rm} [dimensionless]
Reduced Hubble constant: h_0 [km s⁻¹ Mpc]
Reduced Planck constant: \hbar [m² kg s⁻¹]
Rest mass of electron: m_e [kg]
Rest mass of neutron: m_n [kg]
Rest mass of proton: m_p [kg]
Scale factor: a [dimensionless]
Semi-major axis: a [Mpc]
Semi-minor axis: b [Mpc]
Sound speed: v_s [m s⁻¹]
Spherical harmonics: $Y_{lm}(\theta, \phi)$ [dimensionless]
Sign of curvature: κ [dimensionless]
Silk mass: M_s [kg]
Solid angle: Ω [sr]
Specific intensity: I_ν [erg s⁻¹ cm⁻² Hz⁻¹ sr⁻¹]
Speed of light: c [m s⁻¹]
Statistical weight: g [dimensionless]
Stefan-Boltzmann constant: σ [W m⁻² K⁻⁴]
Steradian: sr [unit]
Stokes Q: Q [Jy]
Stokes U: Q [Jy]
Stress energy tensor: $T_{\alpha\beta}$ [unit]
Temperature: T [K]
Temperature of the CMB: T_{CMB} [K]
Temperature of recombination: T_{rec} [K]
Temperature of the Sun: T_\odot [K]
Thomson cross section: σ_T [m²]
Transfer function: $T(k)$ [??]

Tritium: T [element]
Two-point correlation function: $\xi(x)$ [dimensionless]
Velocity: v [m s^{-1}]
Velocity (radial): v_r [m s^{-1}]
Velocity (rotational): v_{rot} [m s^{-1}]
Velocity (tangential): v_t [m s^{-1}]
Wavelength: λ [m]
Wavenumber (comoving): k [m^{-1}]

1.3 Useful Values

Age of the Earth: $t_{\oplus} = 4.5$ [Gyr]
Age of the Solar System: $t_{\text{SS}} = 4.6$ [Gyr]
Age of the Universe: $t_{\text{universe}} = 13.7$ [Gyr]
Angström: $\text{\AA} = 10^{-10}$ [m]
Astronomical Unit: $1 \text{ AU} = 1.5 \times 10^{11}$ [m]
Binding energy of deuterium: $E_{\text{D}} = 1.1$ [MeV]
Binding energy of helium: $E_{\text{He}} = 7$ [MeV]
Binding energy of hydrogen: $E_{\text{H}} = 13.7$ [MeV]
Chandrasekhar mass: $M_{\text{C}} = 1.4$ [M_{\odot}]
CMB peak photon wavelength: $\lambda_{\text{CMB}} = 2$ [mm]
CMB temperature: $T_{\text{CMB}} = 2.725$ [K]
CMB temperature: $T_{\text{CMB}} = 1.9$ [K]
Cosmological constant: $\Lambda = 1.11 \times 10^{-52}$ [m^{-2}]
Density of dark energy: $\rho_{\Lambda} = 10^{-27}$ [kg m^{-3}]
Density parameter of baryons: $\Omega_b \approx 0.044$ [dimensionless]
Density parameter of CMB: $\Omega_{\text{CMB}} \approx 5 \times 10^{-5}$ [dimensionless]
Density parameter of curvature: $\Omega_k \approx 0.0$ [dimensionless]
Density parameter of cold dark matter: $\Omega_c \approx 0.23$ [dimensionless]
Density parameter of dark energy: $\Omega_{\Lambda} \approx 0.73$ [dimensionless]
Density parameter of matter: $\Omega_m \approx 0.27$ [dimensionless]
Density parameter of neutrinos: $\Omega_{\nu} < 0.0032$ [dimensionless]
Density parameter of radiation: $\Omega_r \approx 5 \times 10^{-5}$ [dimensionless]
Dipole: $\ell = 1$ [dimensionless]
Electronvolt in Joules: $1 \text{ eV} = 1.6 \times 10^{-19}$ J
Electron radius (classical): $r_{e,0} = 2.82 \times 10^{-13}$ [cm]
Electroweak age: $t_{\text{ew}} \sim 10^{-12}$ [s]
Electroweak energy: $E_{\text{ew}} \sim 1$ [TeV]
Electroweak temperature: $T_{\text{ew}} \sim 10^{16}$ [K]
Energy of CMB photons: $E_{\gamma, \text{CMB}} \sim 6 \times 10^{-4}$ [eV]
Energy of nuclei fission: $E_{\text{fission}} \sim 1$ [MeV]
Energy of nuclei ionization: $E_{\text{ion}} \sim 10$ [eV]
GUT age: $t_{\text{GUT}} \sim 10^{-36}$ [s]
GUT energy: $E_{\text{GUT}} \sim 10^{12} - 10^{13}$ [TeV]
GUT temperature: $T_{\text{GUT}} \sim 10^{28}$ [K]
Helium mass fraction: $Y = 0.25$ [dimensionless]
Hubble constant: $H_0 = 70$ [$\text{km s}^{-1} \text{ Mpc}^{-1}$]
IMF normalization: $\int_{m_L}^{m_U} m \xi(m) dm = 1 M_{\odot}$
Inflationary period: $t_{\text{inflation}} \sim 10^{-34}$ [s]
Hubble distance: $d_H \sim 0.2$ [Mpc]
Hubble time: $t_H \sim 14$ [Gyr]
Inflationary period: $t_{\text{inflation}} = 10^{-35}$ [s]
Large cosmological scale: ~ 100 [Mpc]
Legendre Polynomials: P_l
Luminosity of the Sun: $L_{\odot} = 3.8 \times 10^{26}$ [W]
Luminosity of type Ia SN (at peak): $L_{\text{Ia}} = 4 \times 10^9$ [L_{\odot}]
Lyman-alpha wavelength: $\lambda_{\text{Ly}\alpha} = 1216$ [\AA]
Magnitude (absolute) of the Sun: $M = 26.8$ [dimensionless]
Magnitude (apparent) of the Sun: $m = 4.74$ [dimensionless]
Mass of an electron: $m_e = 9.109 \times 10^{-31}$ [kg]
Mass of a neutron: $m_n = 1.67 \times 10^{-27}$ [kg]
Mass of a proton: $m_p = 1.67 \times 10^{-27}$ [kg]

Megaelectronvolt: $\text{MeV} = 10^{10.065} [\text{K}]$
Metallicity of the Sun: $Z \approx 0.02$ [dimensionless]
Milky Way luminosity: $L_{\text{MWG}} = 3.6 \times 10^{10} [L_{\odot}]$
Monopole: $\ell = 0$ [dimensionless]
Neutron half life: $t_n \sim 890 [\text{sec}]$ or $\sim 15 [\text{min}]$
Neutron-proton freezeout: $\frac{n_n}{n_p} \simeq \frac{1}{7} [\text{MeV}]$
Neutron-proton rest mass-energy difference: $m_n - m_p = 1.3 [\text{MeV}]$
Number density of baryons: $n_{b,0} = 0.22 [\text{m}^{-3}]$
Number density of CMB photons: $n_{\gamma} = 4.11 \times 10^8 [\text{m}^{-3}]$
Number density ratio of baryons to CMB photons: $\eta \equiv \frac{n_{b,0}}{n_{\gamma}} = 5 \times 10^{-10}$ [dimensionless]
Parsec: $1 \text{ pc} = 206265 \text{ AU} = 3.1 \times 10^{16} \text{ m}$
Photoionization energy of hydrogen: $E_{\text{ion}}^{\text{H}} = 13.6 [\text{eV}]$
Planck energy: $E_P = 10^{16} [\text{TeV}]$
Planck length: $\ell_P = 1.6 \times 10^{-35} [\text{m}]$
Planck mass: $M_P = 2.2 \times 10^{-8} [\text{kg}]$
Planck temperature: $T_P = 1.4 \times 10^{32} [\text{K}]$
Planck time: $t_P = 5.4 \times 10^{-44} [\text{s}]$
Proper surface area: $A_p(t_0) [\text{Mpc}^2]$
Recombination age of Universe: $t_{\text{rec}} \sim 380,000 [\text{yr}]$
Recombination redshift: $z_{\text{rec}} \sim 1,100$ [dimensionless]
Reduced Hubble constant: $\frac{H_0}{100} = 0.70 [\text{km s}^{-1} \text{ Mpc}^{-1}]$
Redshift of radiation-matter equality: $z_{rm} \approx 5399$ [dimensionless]
Redshift of matter- Λ equality: $z_{m\Lambda} \approx 0.39$ [dimensionless]
Rest energy of an electron: $m_e c^2 = 0.511 [\text{MeV}]$
Rest energy of a neutron: $m_n c^2 = 939.6 [\text{MeV}]$
Rest energy of a proton: $m_p c^2 = 938.3 [\text{MeV}]$
Scale factor (current): $a_0 = a(t_0) \equiv 1$ [dimensionless]
Seconds in a year: $1 \text{ yr} = 3.2 \times 10^7 [\text{s}]$
Seconds in a gigayear: $1 \text{ Gyr} = 3.2 \times 10^{16} [\text{s}]$
Sign of curvature (closer Universe): $\kappa > 0$ [dimensionless]
Sign of curvature (flat Universe): $\kappa = 0$ [dimensionless]
Sign of curvature (open Universe): $\kappa < 0$ [dimensionless]
Solar luminosity: $1 L_{\odot} = 3.8 \times 10^{26} [\text{W}]$
Solar mass: $1 M_{\odot} = 2.0 \times 10^{30} [\text{kg}]$
Solar mean photon energy: $\langle E_{\odot} \rangle = 1.3 [\text{eV}]$
Solar temperature: $T_{\odot} = 5800 [\text{K}]$
Speed of light: $c = 2.998 \times 10^8 [\text{m s}^{-1}]$
Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8} [\text{W m}^{-2} \text{ K}^{-4}]$
Temperature at CMB decoupling: $k_B T_{\text{CMB,dec}} = 1/4 [\text{eV}]$
Thomson cross section: $\sigma_T = 0.665 \times 10^{-24} [\text{cm}^2]$
Vacuum energy density: $\rho_{\text{vac}} = 10^{133} [\text{eV cm}^{-3}]$

1.4 Equations

Absorption coefficient: α_ν [cm^{-1}]

$$\alpha_\nu = n\sigma_\nu = \rho\kappa_\nu \text{ [cm}^{-1}\text{]}$$

Angular diameter distance:

$$D_A = D(1+z)^{-1} \text{ [pc]}$$

Apparent-absolute magnitude relation:

$$\begin{aligned} M &= m - 5 \log_{10} \left(\frac{d_L}{10 \text{ pc}} \right) \text{ [mag]} \\ &= m - 5 \log_{10} \left(\frac{d_L}{1 \text{ Mpc}} \right) - 25 \text{ [mag]} \end{aligned}$$

Blackbody function:

$$B_\nu(T) = \frac{2h\nu^2}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1} \text{ [erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}\text{]}$$

Blackbody mean photon energy:

$$\langle E_{\text{bb}} \rangle = 2.7 k_B T \text{ [eV]}$$

Blackbody peak photon energy:

$$E_{\text{bbpeak}} = 2.82 k_B T \text{ [eV]}$$

Boltzmann factor:

$$\frac{n_i}{n_j} = e^{-\Delta mc^2/k_B T} \text{ [dimensionless]}$$

CMB temperature fluctuations:

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{\ell=0}^{\infty} \sum_{m=-1}^{\ell} a_{\ell m} Y_{\ell m}(\theta, \phi) \text{ [dimensionless]}$$

Comoving sound horizon:

$$r_{\text{H,com}}(z) = \int_0^t \frac{cdt}{a(t)} \text{ [Mpc]} = \int_0^{(1+z)^{-1}} \frac{cda}{a^2 H(a)} \text{ [Mpc]}$$

Cosmological constant:

$$\Lambda = \frac{3H_0^2}{c^2} \Omega_\Lambda \text{ [m}^{-2}\text{]}$$

Correlation function of CMB temperature anisotropies:

$$\begin{aligned} C(\theta) &= \left\langle \frac{\delta T}{T}(\hat{n}) \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n} \cdot \hat{n}' = \cos \theta} \\ C(\theta) &= \frac{1}{4\pi} \sum_{\ell=0}^{\infty} (2\ell + 1) (C_\ell) P_\ell(\cos \theta), \end{aligned}$$

Critical density:

$$\rho_c \equiv \frac{3H_0}{8\pi G} [\text{g cm}^{-3}]$$

Curvature term:

$$S_\kappa(r) = \begin{cases} R \sin(r/R), & (\kappa = +1) \\ r, & (\kappa = 0) \\ R \sinh(r/R), & (\kappa = -1) \end{cases}$$

Dark energy density:

$$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} [\text{g cm}^{-3}]$$

Deceleration parameter:

$$q_0 = \Omega_{r,0} + \frac{1}{2}\Omega_{m,0} - \Omega_\Lambda [\text{dimensionless}]$$

Density contrast:

$$\delta(\mathbf{x}, t) \equiv \frac{\rho(\mathbf{x}, t) - \langle \rho(t) \rangle}{\langle \rho(t) \rangle} [\text{dimensionless}]$$

Density parameter of dark energy:

$$\begin{aligned} \Omega_\Lambda &\equiv \frac{\rho_\Lambda}{\rho_c} \\ &= \left(\frac{\Lambda}{8\pi G} \right) \left(\frac{8\pi G}{3H_0^2} \right) \\ &= \frac{\Lambda}{3H_0^2} [\text{dimensionless}] \end{aligned} \quad (???)$$

Density parameter of matter:

$$\Omega_m(a) \equiv \frac{\rho_m}{\rho_c} a(t)^{-3} [\text{dimensionless}]$$

Density parameter of radiation:

$$\Omega_r(a) \equiv \frac{\rho_r}{\rho_c} a(t)^{-4} [\text{dimensionless}]$$

Distance (moving cluster):

$$d = \frac{v_r \tan \theta}{\mu} [\text{pc}]$$

Distance (parallax):

$$d = \left(\frac{p}{1''} \right)^{-1} [\text{pc}]$$

Distance (photometric):

$$m - M = 5 \log \left(\frac{d}{1 \text{ pc}} \right) - 5 + A [\text{mag}]$$

Distance (spectroscopic):

$$m_V - A_V - M_V = 5 \log \left(\frac{d}{1 \text{ pc}} \right) - 5 [\text{mag}]$$

Distance modulus:

$$m - M \approx 43.17 - 5 \log_{10} \left(\frac{H_0}{70 \text{ km s}^{-1} \text{ Mpc}} \right) + 5 \log_{10} z + 1.086(1 - q_0)z \text{ [mag]}$$

Distance relation (low redshift):

$$d_p(t_0) \approx d_L \approx \frac{c}{H_0} z \text{ [Mpc]}$$

Distance-redshift relation (flat Universe):

$$d_L(\kappa = 0) = r(1 + z) = d_p(t_0)(1 + z) \text{ [Mpc]}$$

Electron radius (classical):

$$r_{e,0} = \frac{e^2}{m_e c^2} \text{ [cm]}$$

Elliptical $D_n - \sigma$ relation:

$$I_n \left(\frac{D_n}{2} \right)^2 \pi = 2\pi I_e R_e^2 \int_0^{D_n/(2R_e)} [\text{erg s}^{-1} \text{ m}^{-2}]$$
$$D_n = 2.05 \times \left(\frac{\sigma_v}{100 \text{ km s}^{-1}} \right) \text{ [kpc]}$$

Energy density:

$$\Omega \equiv \frac{\rho}{\rho_c} = nE \text{ [dimensionless]}$$

Energy-redshift relation:

$$E_0 = E(1 + z)^{-1} \text{ [eV]}$$

Extinction coefficient:

$$A_\nu \equiv m - m_0 = -2.5 \log(I_\nu / I_{\nu,0}) \text{ [mag]}$$
$$= 2.5 \log(e) \tau_\nu \text{ [mag]}$$
$$= 1.086 \tau_\nu \text{ [mag]}$$

Extinction coefficient for the MWG:

$$A_{V,\text{MWG}} = (3.1 \pm 0.1) E(B - V) \text{ [mag]}$$

Flux:

$$F = \frac{L}{4\pi S_\kappa(r)^2} (1 + z)^{-2} \text{ [erg s}^{-1} \text{ cm}^{-2}]$$

Flux (spectral):

$$F_\lambda(t) = \int_0^t \text{SFR}(t - t') S_{\lambda, Z(t-t')}(t') dt' \text{ [erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1}]$$

Friedmann equation:

$$\left(\frac{\dot{a}(t)}{a(t)} \right)^2 = \left(\frac{H}{H_0} \right)^2 = \frac{8\pi G}{3} \rho - \frac{\kappa c^2}{R_0^2 a(t)^2} + \frac{\Lambda}{3}$$
$$\left(\frac{H}{H_0} \right)^2 = \frac{\Omega_{r,0}}{a^4} + \frac{\Omega_{m,0}}{a^3} + \Omega_{\Lambda,0} + \frac{1 - \Omega_0}{a^2}$$

Growth factor:

$$D_+(t) \propto \frac{H(a)}{H_0} \int_0^a \frac{da'}{[\Omega_m/a' + \Omega_\Lambda a'^2 - (\Omega_m + \Omega_\Lambda - 1)]^{3/2}} \text{ [dimensionless]}$$

Helium mass fraction:

$$Y = \frac{4 \times n_n/2}{n_n + n_p} = \frac{2}{1 + n_n/n_p} \text{ [dimensionless]}$$

Horizon angular size at recombination:

$$\theta_{H,\text{rec}} \approx \sqrt{\frac{\Omega_m}{z_{\text{rec}}}} \sim \frac{\sqrt{\Omega_m}}{30} \sim \sqrt{\Omega_m} 2 \text{ [}^\circ\text{]}$$

Hubble constant:

$$H_0(t) \equiv \frac{\dot{a}_0(t)}{a_0(t)} = \dot{a}_0(t) \text{ [km s}^{-1} \text{ Mpc]}$$

Hubble distance:

$$d_H \equiv \frac{c}{H(t)} \text{ [Mpc]}$$

Hubble time:

$$t_H \equiv ct_H = \frac{1}{H_0(t)} \text{ [Gyr]}$$

IMF (Salpeter):

$$\xi(m) \propto m^{-2.35} \text{ [units??]}$$

Kepler's third law:

$$P = \sqrt{\frac{4\pi^2}{G(m_1 + m_2)}} a^3 \text{ [yr]}$$

Legendre Polynomials:

$$\begin{aligned} P_0(x) &= 1 \text{ [dimensionless]} \\ P_1(x) &= x \text{ [dimensionless]} \\ P_2(x) &= \frac{1}{2}(3x^2 - 1) \text{ [dimensionless]} \end{aligned}$$

Luminosity (Faber-Jackson):

$$L_{\text{FJ}} \propto \sigma_v^\alpha \text{ [erg s}^{-1}\text{]}$$

Luminosity (Tully-Fisher):

$$\begin{aligned} L_{\text{TF}} &= \propto v_{\text{max}}^\alpha \text{ [erg s}^{-1}\text{]} \\ &= \left(\frac{M}{L}\right)^{-2} \left(\frac{1}{G^2 \langle I \rangle}\right) v_{\text{max}}^4 \text{ [erg s}^{-1}\text{]} \end{aligned}$$

Luminosity distance:

$$\begin{aligned} d_L &\equiv \sqrt{\frac{L}{4\pi F}} \text{ [Mpc]} \\ d_L(z \ll 1) &\approx \frac{c}{H_0} z \left(1 + \frac{q_0}{2} z\right) \text{ [Mpc]} \\ d_L(\kappa = 0) &\approx \frac{c}{H_0} z \left(1 + \frac{1 - q_0}{2} z\right) \text{ [Mpc]} \end{aligned}$$

Luminosity distance-redshift relation:

$$d_L = d(1 + z) \text{ [Mpc]}$$

Magnetic monopole density:

$$n_M(t_{\text{GUT}}) \sim \frac{1}{(2ct_{\text{GUT}})^3} \sim 10^{82} \text{ [m}^{-3}\text{]}$$

Magnitude (absolute):

$$M \equiv -2.5 \log_{10} \left(\frac{L}{L_0} \right) \text{ [mag]}$$

Magnitude (apparent):

$$m \equiv -2.5 \log_{10} \left(\frac{F}{F_0} \right) \text{ [mag]}$$

Matter density-scale factor relation:

$$\rho_m = \rho_{m,0} a^{-3} \text{ [g cm}^{-3}\text{]}$$

Maxwell-Boltzmann equation:

$$\begin{aligned} n &= g \left(\frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} \exp \left(-\frac{E}{k_B T} \right) \text{ [m}^{-3}\text{]} \\ &= g \left(\frac{mk_B T}{2\pi\hbar^2} \right)^{3/2} \exp \left(-\frac{h\nu}{k_B T} \right) \text{ [m}^{-3}\text{]} \end{aligned}$$

Mean free path:

$$\ell_{\text{mfp}} = \frac{1}{n\sigma} \text{ [pc]}$$

Metallicity index:

$$[\text{X}/\text{H}] \equiv \log \left(\frac{n(\text{X})}{n(\text{H})} \right)_* - \log \left(\frac{n(\text{X})}{n(\text{H})} \right)_\odot \text{ [dimensionless]}$$

Neutrino velocity:

$$v_\nu \sim 150(1 + z) \left(\frac{m_\nu}{1 \text{ eV}} \right)^{-1} \text{ [km s}^{-1}\text{]}$$

Peculiar velocity:

$$v_{\text{pec}} = c \left(\frac{z - z_{\text{cos}}}{1 + z} \right) \text{ [km s}^{-1}\text{]}$$

Photon energy:

$$E_\gamma = h\nu \text{ [eV]}$$

Photon scattering rate (ionized):

$$\Gamma = \frac{c}{\ell_{\text{mfp}}} = n_e \sigma_e c = n_b \sigma_e c = \left(\frac{n_{b,0}}{a^3} \right) \sigma_e c = \frac{4.4 \times 10^{-21} a^3}{\text{s}^{-1}}$$

Planck function:

$$B_\nu(T) = \frac{2h\nu^2}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1} \text{ [erg s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1}\text{]}$$

Polarized angle:

$$\chi = \frac{1}{2} \arctan \left(\frac{U}{Q} \right) \text{ [rad]}$$

Polarized intensity:

$$P = \sqrt{Q^2 + U^2} = pI \text{ [Jy]}$$

Proper distance:

$$d_p(t_0) = c \int_t^{t_0} \frac{dt}{a(t)} \text{ [Mpc]}$$
$$d_p(t_0, z \ll 1) \approx \frac{c}{H_0} z \left(1 - \frac{1+q_0}{2} z \right) \text{ [Mpc]}$$

Proper surface area:

$$A_p(t_0) = 4\pi S_\kappa(r)^2 \text{ [Mpc}^2\text{]}$$

Radiation density-scale factor relation:

$$\rho_r = \rho_{r,0} a^{-4} \text{ [g cm}^{-3}\text{]}$$

Random walk:

$$d = \sqrt{N} \ell \text{ [m]}$$

Redshift relation:

$$\frac{1}{1+z} = \frac{\lambda}{\lambda_0} \text{ [dimensionless]}$$

Robertson-Walker metric:

$$ds^2 = cdt^2 - a(t)^2 \left(\frac{dx^2}{1 - \kappa x^2/R^2} + x^2 \Omega^2 \right)$$

Saha equation:

$$\frac{n_{i+1}}{n_i} = T^{3/2} e^{(-1/T)} \text{ [dimensionless]}$$

Scale factor evolution (radiation-dominated):

$$a_m(t) \propto t^{1/2} \text{ [dimensionless]}$$

Scale factor evolution (matter-dominated):

$$a_m(t) \propto t^{2/3} \text{ [dimensionless]}$$

Scale factor evolution (Λ -dominated):

$$a_\Lambda(t) \propto e^{Ht} \text{ [dimensionless]}$$

Scale factor-redshift relation:

$$a = \frac{1}{1+z} \text{ [dimensionless]}$$

Solid angle:

$$\Omega = \frac{A}{r^2} \text{ [sr]}$$

Sound speed:

$$c_s = \sqrt{\frac{\partial P}{\partial \rho}} \text{ [m s}^{-1}\text{]}$$

Star formation rate:

$$\text{SFR} = -\frac{dM_{\text{gas}}}{dt} \text{ [M}_{\odot}\text{ yr}^{-1}\text{]}$$

Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k^4}{15c^2 h^3} \text{ [W m}^{-2}\text{ K}^{-4}\text{]}$$

Stefan-Boltzmann Law:

$$F = \sigma T^4 \text{ [W m}^{-2}\text{]}$$

Temperature-redshift relation:

$$T = (1 + Z)T_0 \text{ [K]}$$

Temperature-scale factor relation:

$$T = aT_0 \text{ [K]}$$

Thomson cross section:

$$\sigma_T = \frac{8\pi}{3} r_{e,0}^2 \text{ [m}^2\text{]}$$

Tired Light Hypothesis energy scaling:

$$E = E_0 e^{(-r/R_0)} \text{ [eV]}$$

Uncertainty Principle:

$$\sigma_x \sigma_p \geq \frac{\hbar}{2} \text{ [J s]}$$
$$\Delta E \Delta t \geq \frac{\hbar}{2} \text{ [J s]}$$

Velocity (radial):

$$v_r = \left(\frac{\Delta \lambda}{\lambda_0} \right) c \text{ [m s}^{-1}\text{]}$$

Velocity (tangential):

$$v_t = d\mu = 4.74 \left(\frac{d}{1 \text{ pc}} \right) \left(\frac{\mu}{1'' \text{ year}^{-1}} \right) \text{ [m s}^{-1}\text{]}$$

Wavelength-redshift relation:

$$\lambda_0 = \frac{1}{a(t)} \lambda = (1 + z) \lambda \text{ [m]}$$

Wien's Law:

$$B_\nu(T) = \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T) - 1} \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}\text{]}$$
$$B_\nu(T) \approx \frac{2h\nu^3}{c^2} \frac{1}{\exp(h\nu/k_B T)} \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}\text{]}$$
$$B_\nu(T) \approx \frac{2h\nu^3}{c^2} \exp\left(\frac{-h\nu}{k_B T}\right) \text{ [erg s}^{-1}\text{ cm}^{-2}\text{ Hz}^{-1}\text{ sr}^{-1}\text{]}$$

1.5 General Notes

- $k_B(300\text{ K}) \sim \frac{1}{40}\text{ eV}$
- $k_B(11,000\text{ K}) \sim 1\text{ eV}$
- $k_B(10^7\text{ K}) \sim 1\text{ keV}$
- If a particle scatters with a rate greater than the expansion rate of the Universe, that particle remains in equilibrium.
- Multipole moment $\ell = 1$ and $\ell = 2$ are the dipole and quadrupole moment, respectively.
- The greatest baryon contribution to the density comes not from stars in galaxies, but rather from gas in groups of galaxies; in these groups, $\Omega \sim 0.02$.
- The ratio of the neutrino temperature T_ν to the CMB temperature T_{CMB} is $\frac{T_\nu}{T_{\text{CMB}}} = \left(\frac{4}{11}\right)^{1/3}$.
- We expect photons to decouple from matter when the Universe is already well into the matter-dominated era (i.e., not in the radiation-dominated era).
- When the Universe was only one second old, the mean-free-path of a photon was about the size of an atom.
- Nuclear binding energies are typically in the MeV range, which explains why Big Bang nucleosynthesis occurs at temperatures a bit less than 1 MeV even though nuclear masses are in the GeV range.
- The electron-positron energy threshold is $\sim 10^{10}\text{ K}$, above which there exist thermal populations of both electrons and neutrinos to make the reaction $p + e^- \leftrightarrow n + \nu$ go equally well in either direction.
- Thermodynamically, nucleons with greater binding energies are more energetically favourable.
- Simplest way to form helium is via deuterium fusion rather than the improbable coincidence of 2 protons and 2 neutrons all arriving at the same place simultaneously to make ${}^4\text{He}$ in one go.
- Nucleosynthesis starts at about 10^{10} K when the Universe was about 1 s old, and effectively ends when it has cooled by a factor of 10, and is about 100 times older.
- Primordial elemental abundances: 75% H, 25% He, trace Li.
- A normal population of states in LTE satisfies $(n_1/g_1)/(n_2/g_2) > 1$ via the Maxwell-Boltzmann equation; inverted populations such as masers satisfy $(n_1/g_1)/(n_2/g_2) < 1$.
- The solid angle subtended by a complete sphere is $4\pi\text{ sr}$.
- Just as the period of a Cepheid tells you its luminosity, the rise and fall time of a type Ia SN tells you its peak luminosity.
- The Hubble distance d_H is the distance between the Earth and any astrophysical object receding away from us at the speed of light.
- At $z = 10^9$, the Jeans mass is $M_J \sim M_\odot$; at $z = 10^6$, the Jeans mass is $M_J \sim M_{\text{gal}}$.
- Apparent magnitudes $0 < m < 6$ are typically visible to the naked eye.

- The absolute magnitude of a light source M is defined as the apparent magnitude m that it would have if it were at a luminosity distance of $d_L = 10 \text{ pc}$.
- The apparent magnitude is really nothing more than a logarithmic measure of the flux, and the absolute magnitude is a logarithmic measure of the luminosity.
- When space is positively curved, the proper surface area is $A_p(t_0) < 4\pi r^2$, and the photons from distance r are spread over a smaller area than they would be in flat space. When space is negatively curved, $A_p(t_0) > 4\pi r^2$, and photons are spread over a larger area than they would be in flat space.
- The Benchmark Model has a deceleration parameter of $q_0 \approx 0.55$.
- The angular distribution of the CMB temperature reflects the matter inhomogeneities at the redshift of decoupling of radiation and matter.
- CMB polarization probes the epoch of last scattering directly as opposed to the temperature fluctuations which may evolve between last scattering and the present. Moreover, different sources of temperature anisotropies (scalar, vector, and tensor) give different patterns in the polarization: both in its intrinsic structure and in its correlation with the temperature fluctuations themselves.
- The sound speed in the photon-dominated fluid of the early Universe is given by $c_s \approx c/\sqrt{3}$. Thus, the sound horizon is about a factor of $\sqrt{3}$ smaller than the event horizon at this time.
- At recombination, the free electrons recombined with the hydrogen and helium nuclei, after which there are essentially no more free electrons which couple to the photon field. Hence, after recombination the baryon fluid lacks the pressure support of the photons, and the sound speed drops to zero – the sound waves do no longer propagate, but get frozen in.
- Due to the steep slope of the IMF, most of the stellar mass is contained in low-mass stars; however, since the luminosity of main-sequence stars depends strongly on mass, approximately as $L \propto M^3$, most of the luminosity comes from high-mass stars.