Engineering Mathematics (2) Test #1 2015/4/27

(8)1. The origin of Fourier series: (a) Derive the heat conduction equation for a thin rod as $\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$ starting from the First Law of Thermodynamics (conservation of energy); (b) By using the boundary conditions $u(0,t) = u(l,t) = 0$ and the initial condition $u(x,0) = f(x)$ to show that $u(x,t)$ can be given as a combination of sine and cosine functions.

(8)2. Show that the solutions of $y'' + \lambda^2 y = 0; y(0) = y(l) = 0$ are orthogonal by TWO different methods.

(8)3. For a periodic function $f(x)$ with period 2l, with its Fourier series written as $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi}{l}x + b_n \sin\frac{n\pi}{l}x)$, (a) show that $a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)dx$, $a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cos\frac{n\pi}{l}x\,dx$, and $b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi}{l}x\,dx$; (b) simplify the corresponding Fourier series and coefficients for both even and odd periodic functions with period 2l.

(8)4. Extend the above Fourier series to $f(x,y)$ and $f(x,y,z)$ situations.

(8)5. Extend the period 2l to ∞ for the period function f(x) to show that $f(x) = \frac{1}{2\pi}\iint_{-\infty}^{\infty} f(u)e^{-i\omega u}e^{i\omega x}du\,d\omega$ and the inverse Laplace transform is $f(t) = \frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} F(s)e^{st}ds$

(8)6. What is Parseval theorem? Show it for both Fourier series and transform.

(8)7. Obtain the Fourier transform of the following periodic function f(x) with period 2l, $f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos\frac{n\pi}{l}x + b_n \sin\frac{n\pi}{l}x)$.

(8)8. Find the Fourier series of $|\sin x|$ and explain the meaning of each term.

(8)9. Find the Fourier series of (a) f(x)=x, -1<x<1, 2l=2; (b) f(x)=|x|, -1<x<1, 2l=2; (c) Compare their convergent rates and explain the possibility of Gibbs phenomena.

(8)10. Find the Fourier transform of f(x)=1, -1<x<1, f(x)=0 otherwise, and explain some possible meaning in optics.

(8)11. What might be the difference between Laplace transform, Fourier transform, Fourier sine transform, and Fourier cosine transform?

(8)12. Give two examples to explain how we can use Fourier series and Fourier transform to solve engineering problems.

(4)13. Give two relatives of Fourier transform and explain why.