E927500 Electric Circuit Analysis Spring 2017

Solution to Final Exam

1.

(a) $|H(\omega)| = \left|\dfrac{j\omega\omega_2}{(\omega_1+j\omega)(\omega_2+j\omega)}\right| = \dfrac{\omega\omega_2}{\sqrt{(\omega_1\omega_2-\omega^2)^2+(\omega(\omega_1+\omega_2))^2}} = \dfrac{\omega_2}{\sqrt{(\frac{\omega_1\omega_2}{\omega}-\omega)^2+(\omega_1+\omega_2)^2}} \le \dfrac{\omega_2}{\omega_1+\omega_2}$

The equality holds if

$$\frac{\omega_1\omega_2}{\omega_0} - \omega_0 = 0 \Rightarrow \omega_0 = \sqrt{\omega_1\omega_2}$$

(b) Let $|H(\omega)|^2 = \frac{1}{2}|H(\omega_0)|^2$

$$\Rightarrow \frac{\omega_2^2}{(\frac{\omega_1\omega_2}{\omega}-\omega)^2+(\omega_1+\omega_2)^2} = \frac{1}{2}\frac{\omega_2^2}{(\omega_1+\omega_2)^2}$$

$$2(\omega_1+\omega_2)^2 = (\frac{\omega_1\omega_2}{\omega}-\omega)^2 + (\omega_1+\omega_2)^2$$

$$\Rightarrow (\omega_1+\omega_2)^2 = (\frac{\omega_1\omega_2}{\omega}-\omega)^2 \Rightarrow \frac{\omega_1\omega_2}{\omega}-\omega = \pm(\omega_1+\omega_2)$$

$$-\omega^2+\omega_1\omega_2 \mp \omega(\omega_1+\omega_2) = 0$$

$$\omega^2 \pm \omega(\omega_1+\omega_2) - \omega_1\omega_2 = 0$$

$$\omega = \frac{(\omega_1+\omega_2)\pm\sqrt{(\omega_1+\omega_2)^2+4\omega_1\omega_2}}{2} \text{ (負不合)} \quad \text{or} \quad \frac{-(\omega_1+\omega_2)\pm\sqrt{(\omega_1+\omega_2)^2+4\omega_1\omega_2}}{2} \text{ (負不合)}$$

$$= \frac{(\omega_1+\omega_2)+\sqrt{\omega_1^2+6\omega_1\omega_2+\omega_2^2}}{2} \quad \text{or} \quad \frac{-(\omega_1+\omega_2)+\sqrt{\omega_1^2+6\omega_1\omega_2+\omega_2^2}}{2}$$

2. $|H(\omega)| = \left|\dfrac{j\omega B}{(j\omega)^2+j\omega B+\omega_0^2}\right| = \left|\dfrac{\omega B}{\frac{-\omega^2}{j}+\omega B+\frac{\omega_0^2}{j}}\right| = \left|\dfrac{\omega B}{\omega B+(\omega^2-\omega_0^2)j}\right|$

$$= \frac{\omega B}{\sqrt{(\omega B)^2+(\omega^2-\omega_0^2)^2}} = \frac{B}{\sqrt{B^2+(\omega-\frac{\omega_0^2}{\omega})^2}} \le \frac{B}{B} = 1$$

The equality holds if $\omega^2-\omega_0^2=0 \Rightarrow \omega = \omega_0$ #

Let $|H(\omega)|^2 = \frac{1}{2}|H(\omega_0)|^2$

$$\Rightarrow \frac{(\omega B)^2}{(\omega B)^2+(\omega^2-\omega_0^2)^2} = \frac{1}{2}$$

$$\Rightarrow (\omega B)^2 = (\omega^2-\omega_0^2)^2$$

$$\Rightarrow \omega B = \omega^2-\omega_0^2 \quad \text{or} \quad -\omega B = \omega^2-\omega_0^2$$

$$\omega^2-\omega B-\omega_0^2=0 \qquad \omega^2+\omega B-\omega_0^2=0$$

$$\omega = \frac{B\pm\sqrt{B^2+4\omega_0^2}}{2} \text{ (負不合)} \qquad \omega = \frac{-B\pm\sqrt{B^2+4\omega_0^2}}{2} \text{ (負不合)}$$

$$\text{Bandwith} = \omega_2-\omega_1 = \frac{B+\sqrt{B^2+4\omega_0^2}}{2} - \frac{-B+\sqrt{B^2+4\omega_0^2}}{2} = B$$ #

3

$$H(w) = \frac{V_o}{V_s} = \frac{\frac{1}{jw} \parallel (1+\frac{1}{jw})}{1+\frac{1}{jw} \parallel (1+\frac{1}{jw})} \times \frac{1}{1+\frac{1}{jw}} = \frac{1}{1+jw+\frac{jw}{1+jw}} \times \frac{1}{1+\frac{1}{jw}} = \frac{1+jw}{1-w^2+2jw+jw} \times \frac{jw}{1+jw}$$

$$= \frac{1}{3}\,\frac{3jw}{(jw)^2+3jw+1}$$

From the result of 2

it's a band-pass filter with banduith 3 rad/s and

Center frequency 1 rad/s

4.

$$|H(w)| = \left|1-\frac{w_1w_2+j2ww_1-w^2}{(w_1+jw)(w_2+jw)}\right| = \left|1-\frac{w_1w_2-w^2+jw2w_1}{w_1w_2-w^2+jw(w_1+w_2)}\right|$$

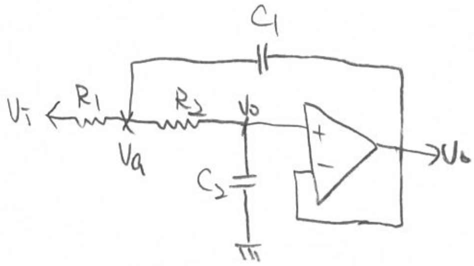
$$= \left|\frac{-jw(w_2-w_1)}{(w_1+jw)(w_2+jw)}\right| = \frac{w_2-w_1}{\sqrt{(\frac{w_1w_2}{w}-w)^2+(w_1+w_2)^2}} \le \frac{w_2-w_1}{w_1+w_2}$$

The equality holds if $\frac{w_1w_2}{w_0}-w_0=0$

$\Rightarrow w_0=\sqrt{w_1w_2}$

5.

$$\begin{cases} \frac{V_i-V_a}{R_1} = jwC_2V_o + jwC_1(V_a-V_o) & - \text{①} \\ \frac{V_a-V_o}{R_2} = \frac{V_o}{\frac{1}{jwC_2}} \Rightarrow V_a = V_o + jwR_2C_2V_o & - \text{②} \end{cases}$$



substituting ① into ②

$$\frac{V_i}{R_1} = \frac{1+jwR_2C_2}{R_1}V_o + jwC_2V_o + (jw)^2R_2C_1C_2V_o$$

$$V_i = (1+jwR_2C_2+jwR_1C_2-w^2R_1R_2C_1C_2)V_o$$

$$H(w) = \frac{V_o}{V_i} = \frac{1}{1-w^2R_1C_1R_2C_2+jwC_2(R_1+R_2)} \quad \#$$

$\Rightarrow w\to 0 \quad |H(w)|\to 1$

$\Rightarrow w\to\infty \quad |H(w)|\to 0 \qquad \Rightarrow$ low-pass filter #

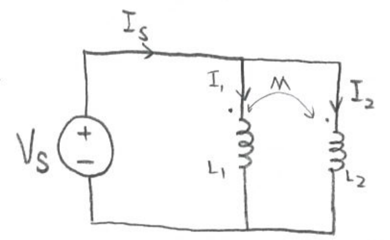
6. Let the current be I and Voltage be V.

$$V = (j\omega L_1 I + j\omega M_{12} I - j\omega M_{13} I) + (j\omega L_2 I + j\omega M_{12} I - j\omega M_{23} I) + (j\omega L_3 I - j\omega M_{13} I - j\omega M_{23} I)$$

$$= j\omega (L_1 + L_2 + L_3 + 2M_{12} - 2M_{13} - 2M_{23}) I$$

$$\Rightarrow L_{eq} = L_1 + L_2 + L_3 + 2M_{12} - 2M_{13} - 2M_{23}$$

7.



$$I_s = I_1 + I_2 \quad , \quad Z_{eq} = \frac{V_s}{I_s}$$

$$V_s = j\omega L_1 I_1 + j\omega M I_2 \quad \text{—— ①}$$

$$V_s = j\omega L_2 I_2 + j\omega M I_1 \quad \text{—— ②}$$

$$\begin{bmatrix} V_s \\ V_s \end{bmatrix} = \begin{bmatrix} j\omega L_1 & j\omega M \\ j\omega M & j\omega L_2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\Delta = -\omega^2 L_1 L_2 + \omega^2 M^2 \quad , \quad \Delta_1 = j\omega V_s (L_2 - M) \quad , \quad \Delta_3 = j\omega V_s (L_1 - M)$$

$$I_1 = \frac{\Delta_1}{\Delta} \qquad I_2 = \frac{\Delta_2}{\Delta}$$

$$I_s = I_1 + I_2 = \frac{\Delta_1 + \Delta_2}{\Delta} = \frac{(L_1 + L_2 - 2M) V_s}{j\omega (L_1 L_2 - M^2)}$$

$$Z_{eq} = \frac{V_s}{I_s} = \frac{j\omega (L_1 L_2 - M^2)}{L_1 + L_2 - 2M} = j\omega L_{eq}$$

$$\Rightarrow L_{eq} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

8.

$800mH \rightarrow j\omega L = j600 \times 800 \times 10^{-3} = j480$

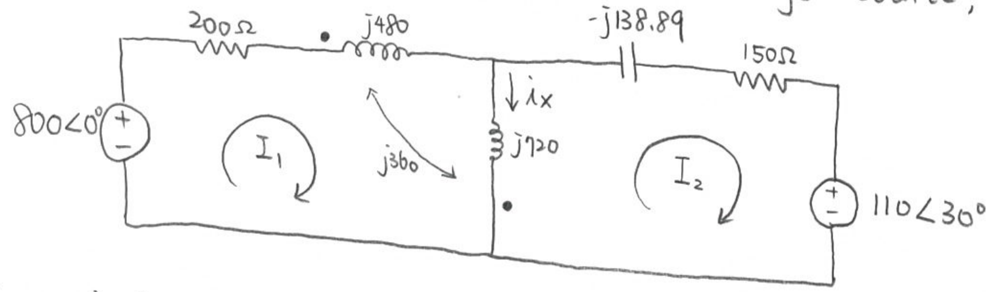
$600mH \rightarrow j360$

$1200mH \rightarrow j720$

$12\mu F \rightarrow \frac{1}{j\omega C} = -j138.89$

Transforming the current source to a voltage source,

$\Rightarrow$



For mesh 1

$$800 = (200 + j480 + j720)I_1 + j360(I_2 - I_1) - j720 I_2 - j360 I_1$$

$$800 = (200 + j1200) I_1 - 360 I_2$$

For mesh 2,

$$110\angle 30^\circ + (150 - j138.89 + j720)I_2 + j360 I_1 = 0$$

$$-95.2628 - j55 = -360 I_1 + (150 + j581.1)I_2$$

$$\Rightarrow \begin{bmatrix} 200 + j480 & -j360 \\ -j360 & 150 + j581.1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 800 \\ -95.2628 - j55 \end{bmatrix}$$

$\Delta = -119328 + j188220$ $\quad \Delta x = 139800 + j430585.392$ $\quad \Delta y = 7347.44 + j231273.856$

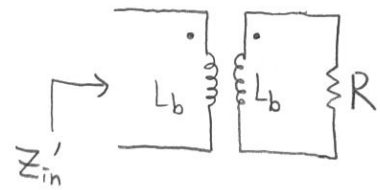
$$I_1 = \frac{\Delta x}{\Delta} = 1.2959 - j1.5643, \quad I_2 = \frac{\Delta y}{\Delta} = 0.8588 - j0.5835$$

$$i_x = I_1 - I_2 = 0.4371 - j0.9808 = 1.07\angle -65.97^\circ$$

$$\Rightarrow i_x = 1.07 \cos(600t - 65.97^\circ) A$$

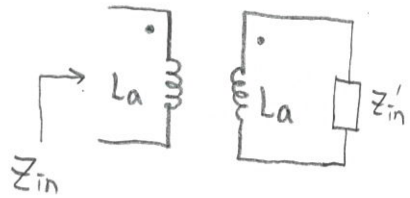
9.

We first find Z_{in} for the second stage using the concept of reflected impedance.



$$Z_{in}' = j\omega L_b + \frac{\omega^2 M_b^2}{R + j\omega L_b} = \frac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}$$

For the first stage, we have the circuit below.



$$Z_{in} = j\omega L_a + \frac{\omega^2 M_a^2}{j\omega L_a + Z_{in}'} = \frac{j\omega L_a Z_{in}' - \omega^2 L_a^2 + \omega^2 M_a^2}{Z_{in}' + j\omega L_a}$$

$$= \frac{-\omega^2 L_a^2 + \omega^2 M_a^2 + j\omega L_a \times \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}{j\omega L_a + \dfrac{j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2}{R + j\omega L_b}}$$

$$= \frac{-R\omega^2 L_a^2 + \omega^2 M_a^2 R - j\omega^3 L_a^2 L_b + j\omega^3 L_b M_a^2 + j\omega L_a (j\omega L_b R - \omega^2 L_b^2 + \omega^2 M_b^2)}{j\omega R L_a - \omega^2 L_a L_b + j\omega L_b R - \omega^2 L_a^2 + \omega^2 M_b^2}$$

$$Z_{in} = \frac{\omega^2 R (L_a^2 + L_a L_b - M_a^2) + j\omega^3 (L_a^2 L_b + L_a L_b^2 - L_a M_b^2 - L_b M_a^2)}{\omega^2 (L_a L_b + L_b^2 - M_b^2) - j\omega R (L_a + L_b)}$$

10.

(a) $$V_I \times \frac{\frac{1}{jwC_1}}{R_1 + \frac{1}{jwC_1}} = V \quad \text{—①}$$

$$V \times \frac{R_2}{\frac{1}{jwC_2} + R_2} = V_0 \quad \text{—②}$$

$$\frac{V_0}{V_I} = \frac{\frac{R_2}{\frac{1}{jwC_2} + R_2}}{\frac{R_1 + \frac{1}{jwC_1}}{\frac{1}{jwC_1}}} = \frac{\frac{R_2}{jwC_1}}{(R_2 + \frac{1}{jwC_2})(R_1 + \frac{1}{jwC_1})}$$

$$= \frac{R_2}{(R_2 + \frac{1}{jwC_2})(jwC_1R_1 + 1)}$$

$$= \frac{jwC_2R_2}{(jwC_2R_2 + 1)(jwC_1R_1 + 1)}$$

(b) when $w \to \infty$, $|H(w)| \to 0$

when $w \to 0$, $|H(w)| \to 0$

$\Rightarrow$ Band-pass filter