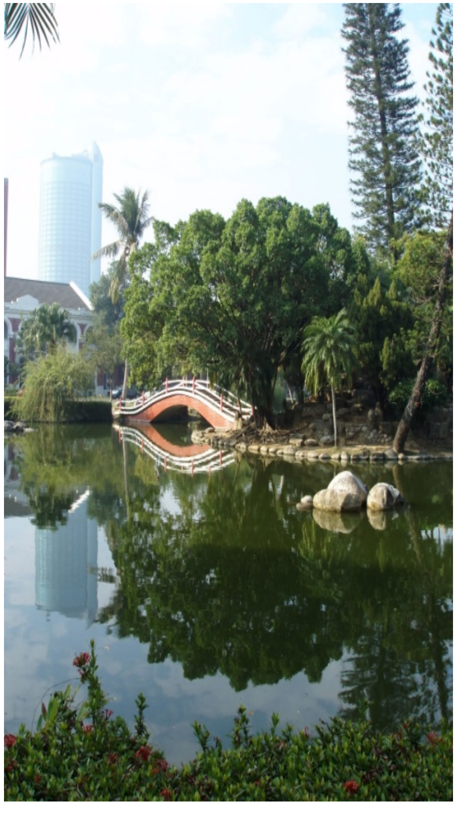
電子電工學
Lecture 3

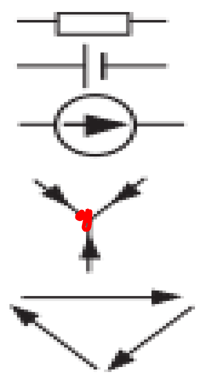


Recap: circuit diagram

- Interconnection of components

Table 3.1 Summary of the relations describing DC circuits

FEATURES OF A CIRCUIT	SYMBOLIC REPRESENTATION	RELATION
Components		Ohm's Law constant V constant I
Connection at a node		Kirchhoff's Current Law (KCL)
Connection to form a loop		Kirchhoff's Voltage Law (KVL)



- Equivalent circuits – same electrical characteristic

$R = R_1 + R_2$

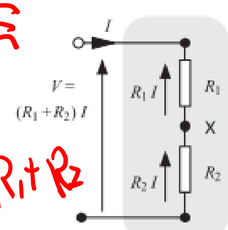


Figure 3.20 Derivation of the equivalent resistance of two resistors connected in series

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$G = G_1 + G_2$

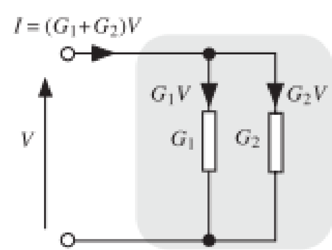


Figure 3.21 Derivation of the equivalent conductance of two resistors connected in parallel

Example 3.1

Find *I*, *V*.

KCL @ X (inward +)

$$(-I) + (-2) = 0$$

$$I = -2 \text{ A}$$

KVL in loop

$$Y \to X \to Y$$

$$(+6) + (-V) = 0$$

$$V = +6 \text{ V}$$

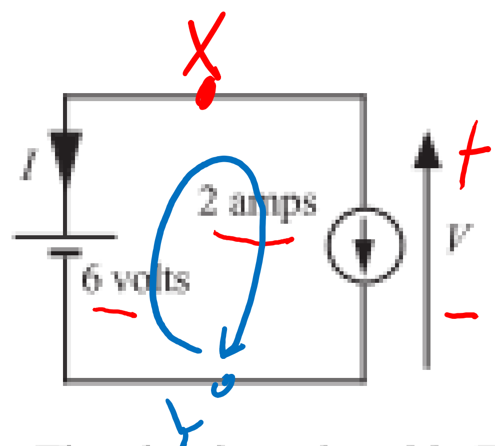


Figure 3.22 The circuit analysed in Example 3.1

Example 3.2

Find V.

KCL @ X (inward)

$5 + (-I) + (-3) = 0$

$\Rightarrow I = 2\ A$

Ohm's law

$V = -2 \times 2 = -4\ V$

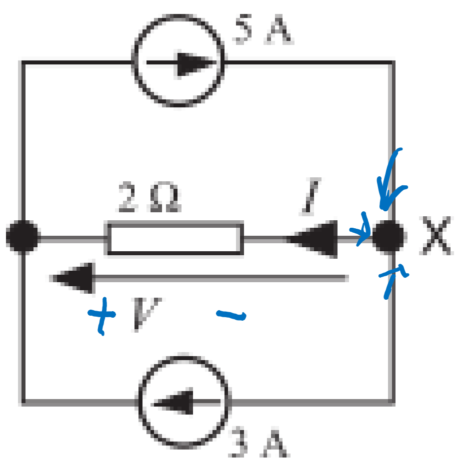


Figure 3.23 Pertinent to Example 3.2

Example 3.3

Find V_A.

[Voltage Divider]

Equiv. Ckt

$$I = \frac{V}{R_1+R_2}$$

Ohm's Law

$$V_A = I \cdot R_2$$

$$= V \frac{R_2}{R_1+R_2}$$

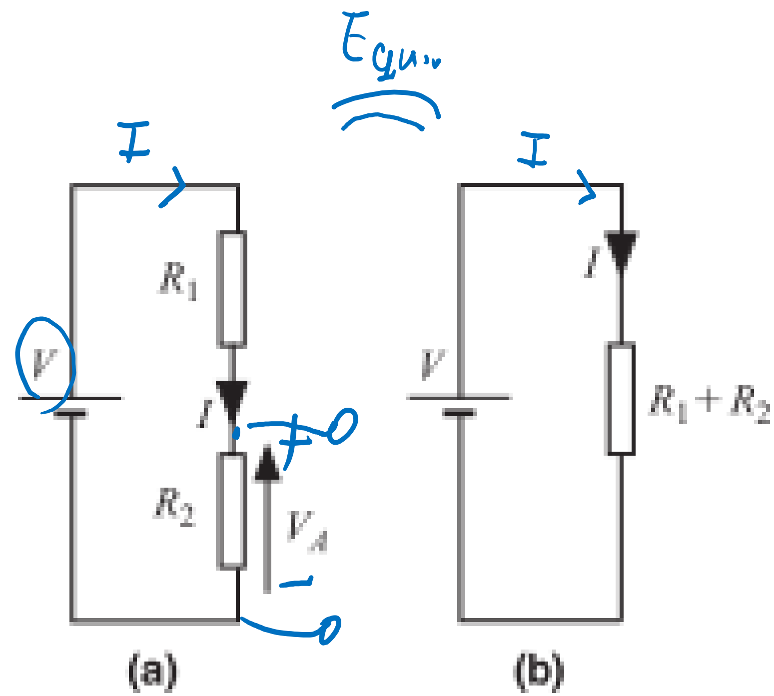


Figure 3.24 Pertinent to Example 3.3

Example 3.4

Find I_1 and I_2

Parallel

Equiv. Ckt.

$R = 6\Omega \parallel 3\Omega$

$= \dfrac{1}{\frac{1}{6}+\frac{1}{3}} = 2\Omega$

$V = I \cdot R = 6 \times 2 = 12$

$\Rightarrow I_1 = V/6 = 2\ (A)$

$I_2 = V/3 = 4\ (A)$

Equiv.

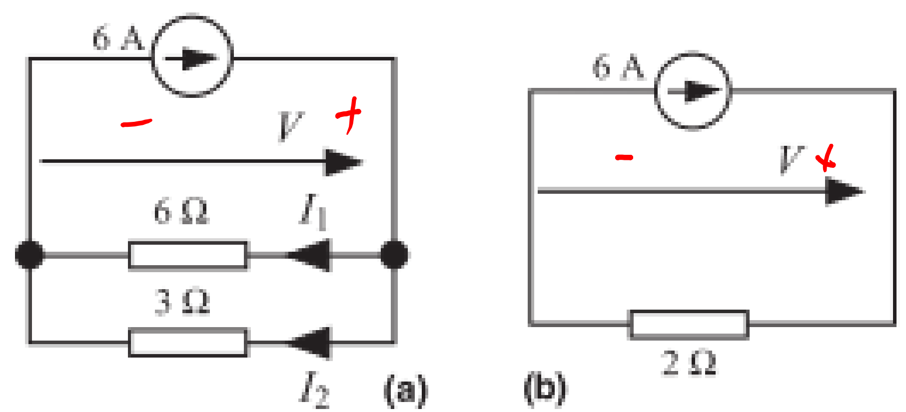
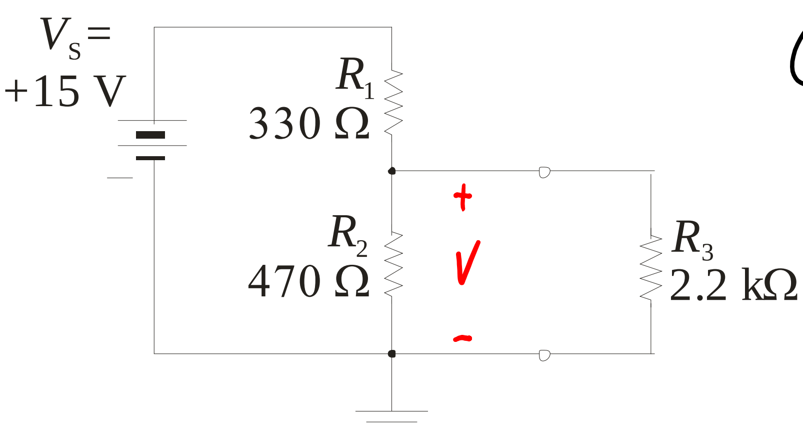


Figure 3.25 Pertinent to Example 3.4

Example

What is the voltage across R_3 ?



Remove R_3

$$V = V_s \cdot \frac{R_2}{R_1+R_2}$$

$$= 15 \times \frac{470}{330+470} \simeq 8.8V$$

Connect R_3

$$V = V_s \times \frac{R_2 /\!/ R_3}{R_1 + R_2 /\!/ R_3}$$

$470 /\!/ 2.2K \simeq 387\Omega$

$$\simeq 8.1V$$

DC Circuit Analysis

CHAPTER 4

Circuit analysis

Goal: Find all

Voltages V_1 V_2 V_3 V_4 V_5

Currents I_1 I_2 I_3 I_4 I_5

Powers P_1 P_2 P_3 P_4 $P_5 = V_5 \times I_5$

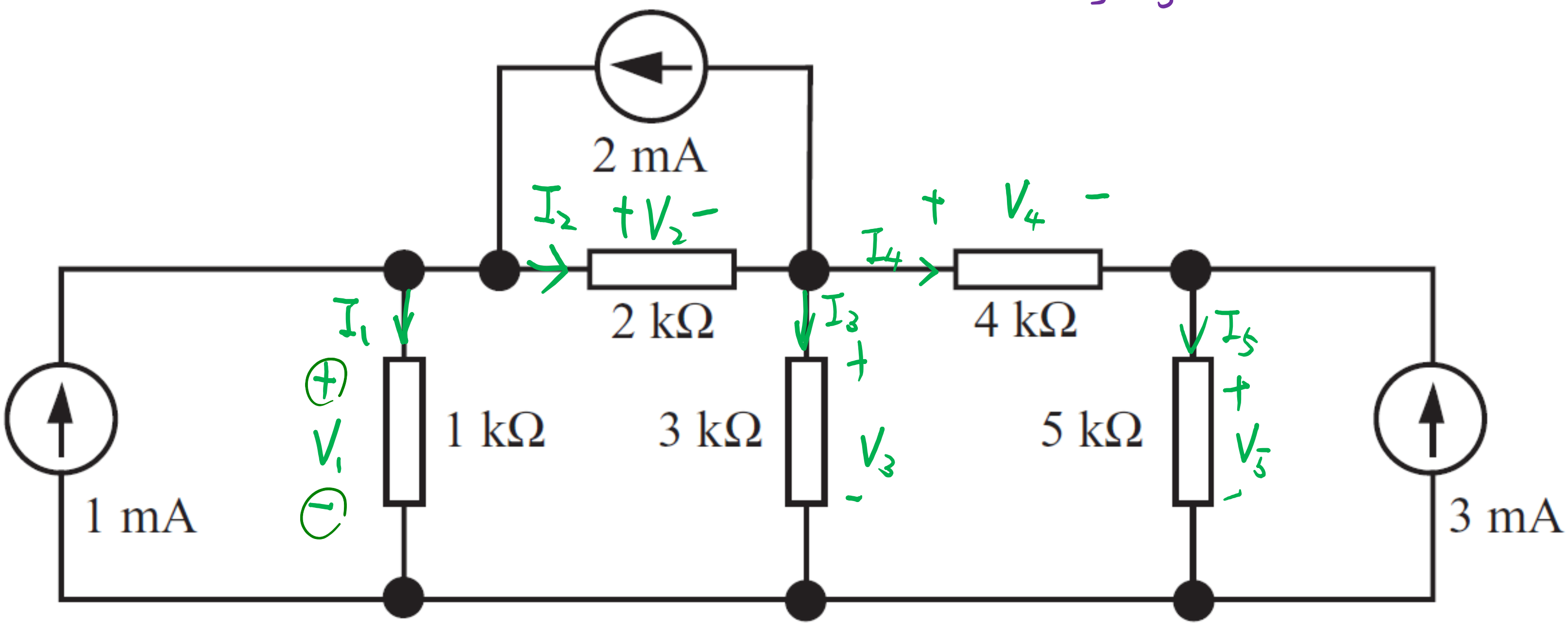


Figure 4.1 The circuit to be analysed

Circuit analysis

Ohm’s law

$$V_1 = I_1 \times 1\,k\Omega$$

$$V_2 = I_2 \times 2\,k\Omega$$

$$V_3 = I_3 \times 3\,k\Omega$$

$$V_4 = I_4 \times 4\,k\Omega$$

$$V_5 = V_5 \times 5\,k\Omega$$

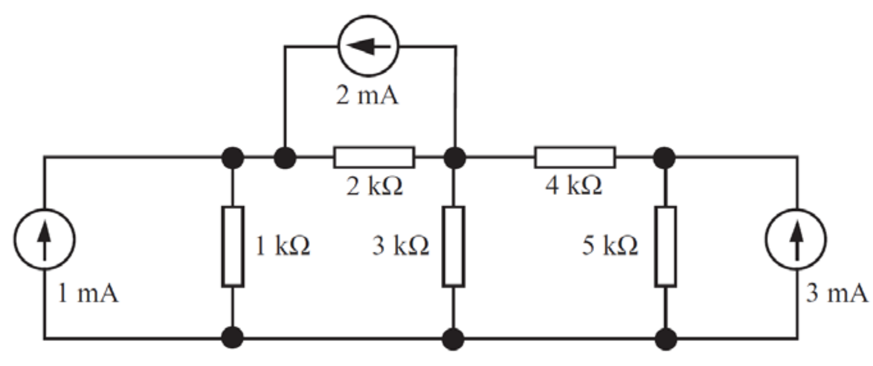


Figure 4.1 The circuit to be analysed

Circuit analysis

KCL (Inward currents as positive)

$$@A: \quad 1^{mA} + 2^{mA} + (-I_1) + (-I_2) = 0$$

$$@B: \quad I_2 + (-2^{mA}) + (-I_3) + (-I_4) = 0$$

$$@C: \quad I_4 + (-I_5) + 3^{mA} = 0$$

$$@D: \quad (-1^{mA}) + I_1 + I_3 + I_5 + (-3^{mA}) = 0$$

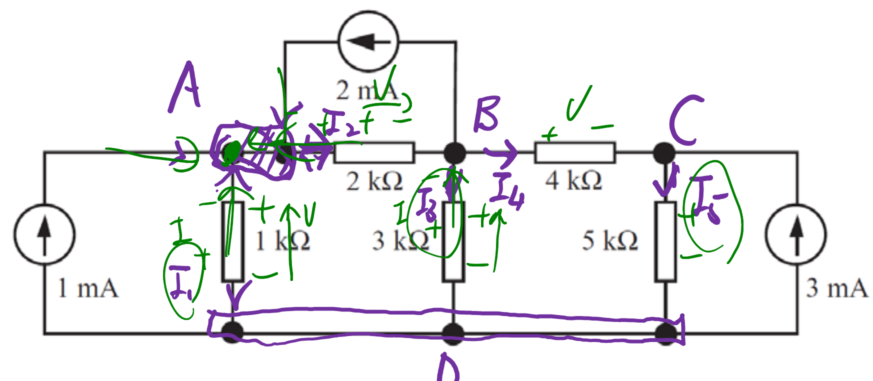


Figure 4.1 The circuit to be analysed

Circuit analysis

KVL

Loop 1 $D \to A \to B \to D$

$$+V_1 - V_2 - V_3 = 0$$

Loop 2. $D \to B \to C \to D$

$$+V_3 - V_4 - V_5 = 0$$

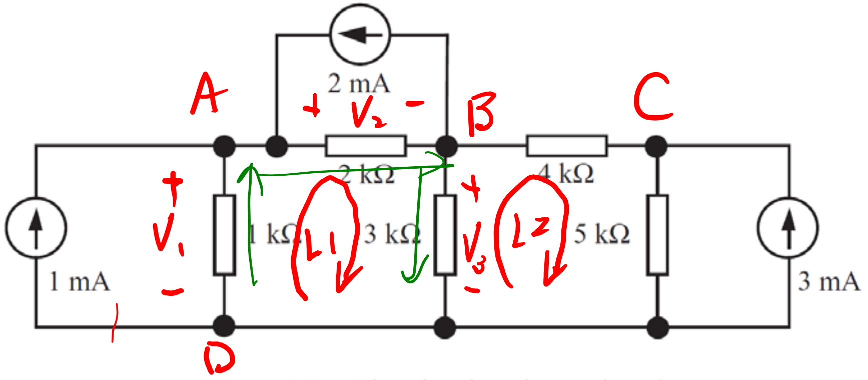


Figure 4.1 The circuit to be analysed

Circuit analysis

System of equations

$$
\left.
\begin{array}{l}
V_1 - I_1 \times 1 = 0 \\
V_2 - I_2 \times 2 = 0 \\
V_3 - I_3 \times 3 = 0 \\
V_4 - I_4 \times 4 = 0 \\
V_5 - I_5 \times 5 = 0
\end{array}
\right] \text{Ohm's}
$$

$$
\left.
\begin{array}{l}
I_1 + I_2 = 3 \\
I_2 - I_3 - I_4 = 2 \\
I_4 - I_5 = -3
\end{array}
\right] \text{KCL}
$$

$$
\text{KVL} \left[
\begin{array}{l}
V_1 - V_2 - V_3 = 0 \\
V_3 - V_4 - V_5 = 0
\end{array}
\right.
$$

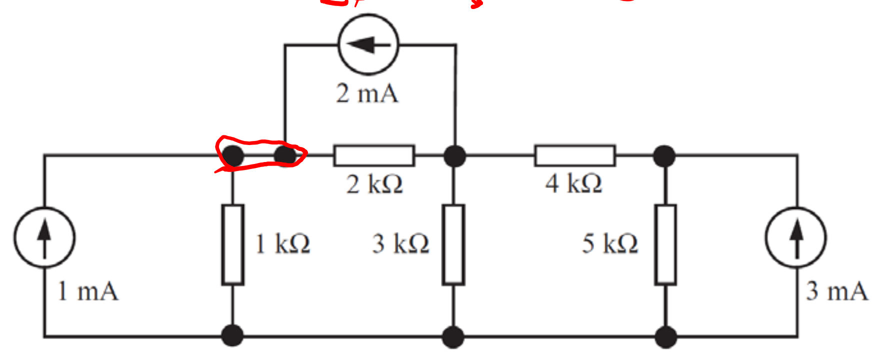


Figure 4.1 The circuit to be analysed

Circuit analysis

Matrix equation

$$
\begin{array}{l}
\text{Ohm} \\ \\ \\ \\ \\
\text{KCL} \\ \\ \\
\text{KVL} \\ \\
\end{array}
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -2 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & -3 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -4 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & -5 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\
1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5
\end{bmatrix}
=
\begin{bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 3 \\ 2 \\ -3 \\ 0 \\ 0
\end{bmatrix}
$$

$$A\vec{x} = \vec{s}$$

$$\Rightarrow \vec{x} = A^{-1}\vec{s}$$

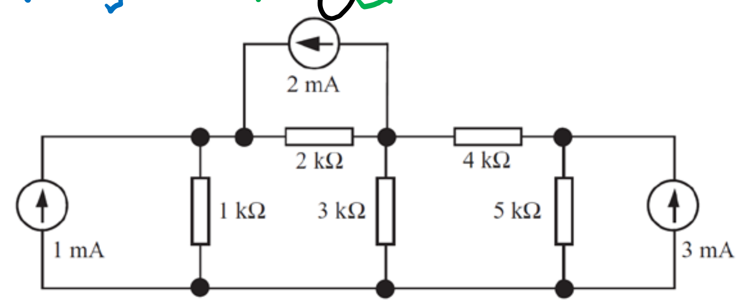


Figure 4.1 The circuit to be analysed

Circuit analysis

Matrix Eq

$$A\vec{x} = \vec{s}$$

unknowns

sources

⇒ Gaussian Elimination

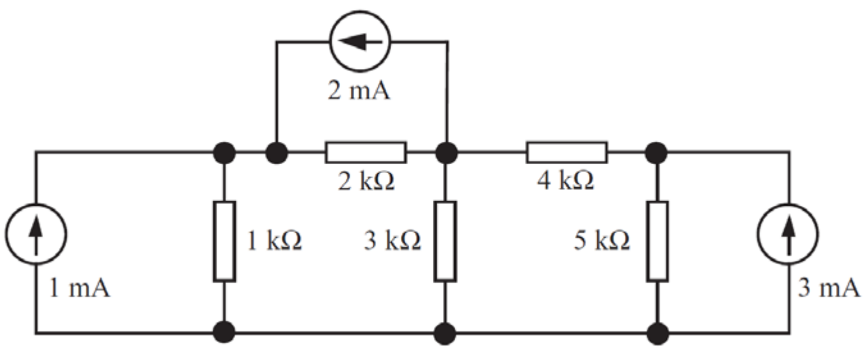


Figure 4.1 The circuit to be analysed

Circuit simulation

- http://lushprojects.com/circuitjs/circuitjs.html
- http://www.falstad.com/circuit/circuitjs.html

Matrix computation

- https://octave-online.net/

Circuit analysis

Ohm's → 5 eqn's
KCL → 3 eqn's
KVL → 2 eqn's

10 eqns → System of eqn's

Solve ⇒ 5 voltages, 5 currents

Simplify → Nodal Analysis, Loop Analysis

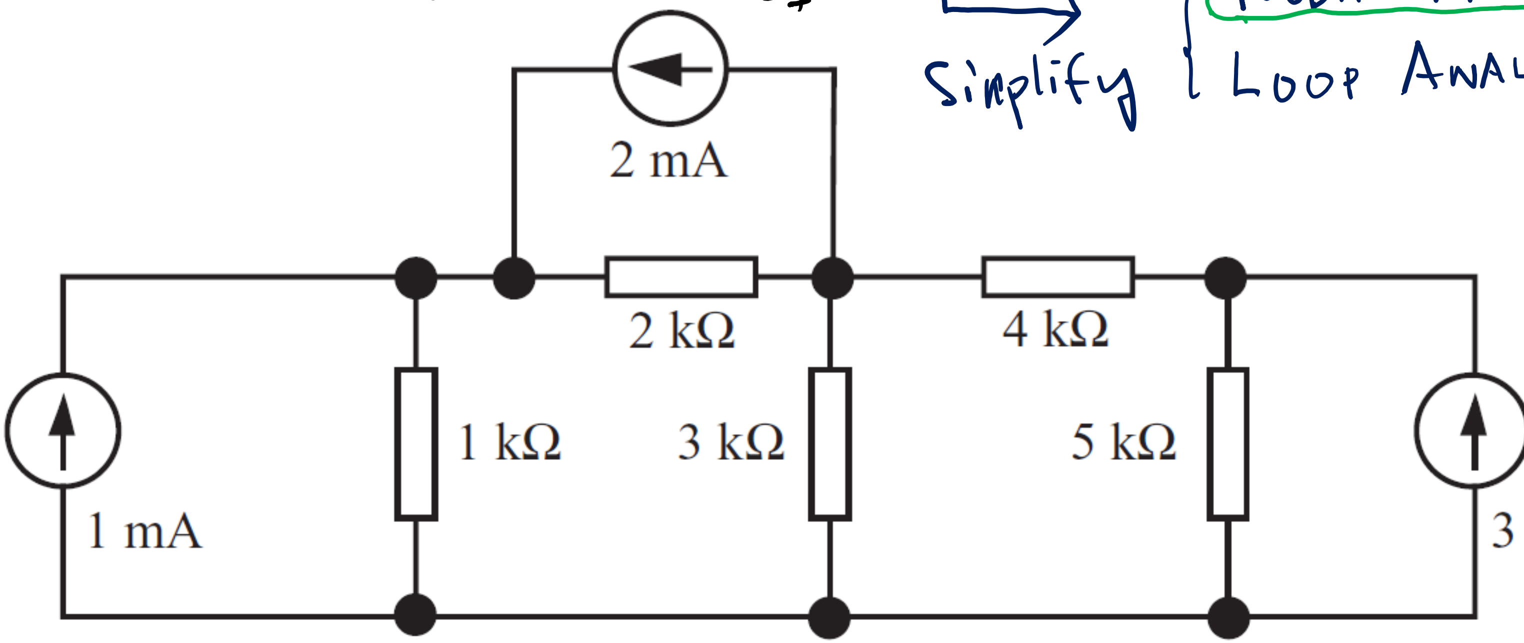


Figure 4.1 The circuit to be analysed

Nodal analysis

A systematic approach

Unknowns: node voltages

Equations: KCL

→ 3 eqn's

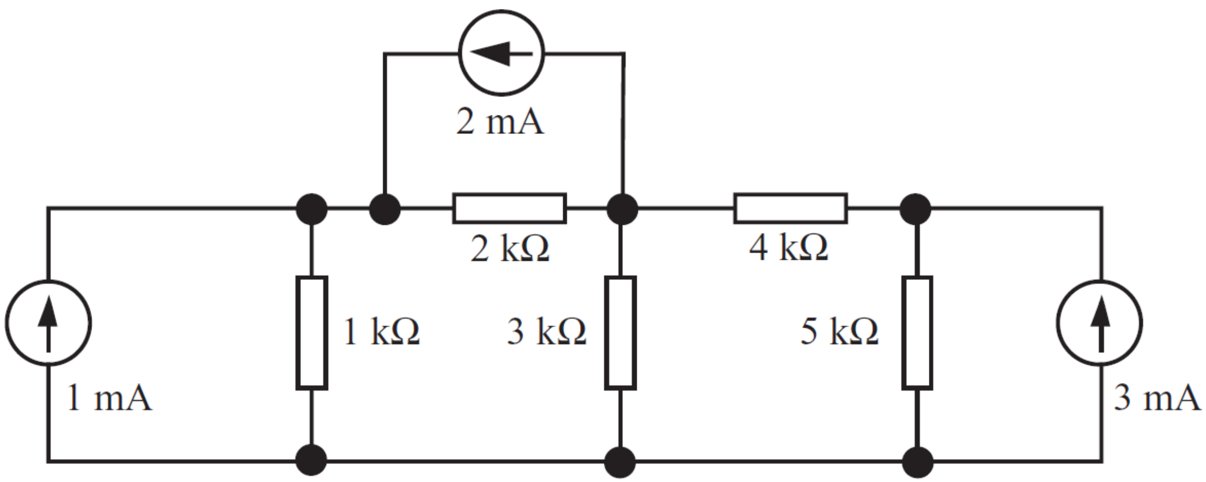


Figure 4.1 The circuit to be analysed

Nodal analysis

Step 1

Assign nodes.

V_A V_B V_C V_D

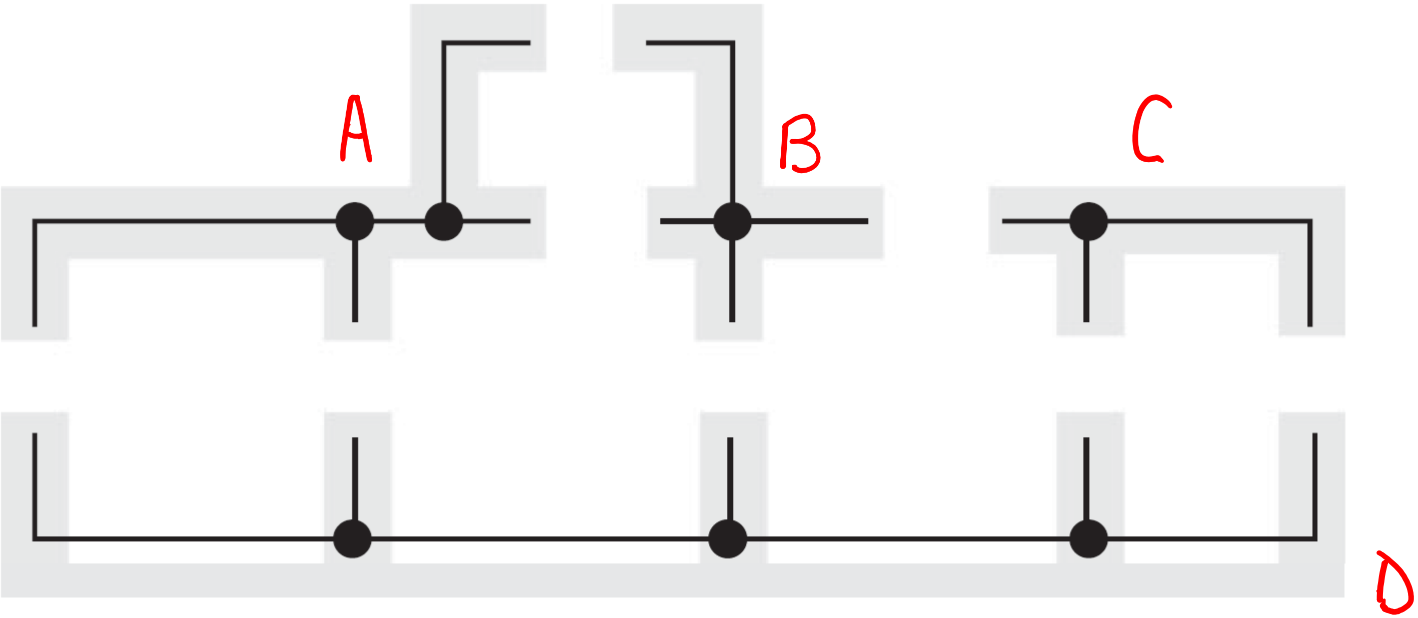


Figure 4.2 The identification of circuit nodes for the circuit of Figure 4.1

Nodal analysis

(KVL Implied)

Loop $D \to A \to B \to D$

KVL: $V_{AD} + V_{BA} + V_{DB}$

$= V_A - V_D + V_B - V_A + V_D - V_B$

$= 0$

$V_{BD} = V_B - V_D$

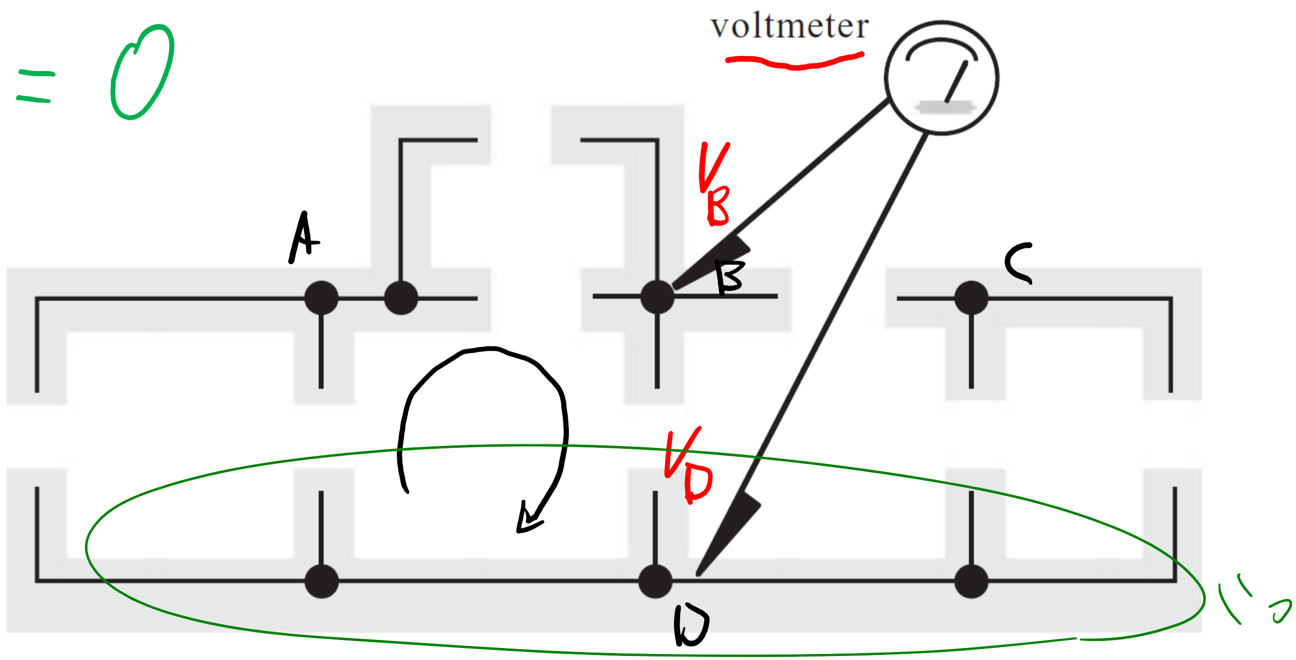


Figure 4.3 The measurement of a voltage

Nodal analysis

Step 2

Assign a reference node.

(Ground) (Earth)

⇒ Unknown V_A V_B V_C

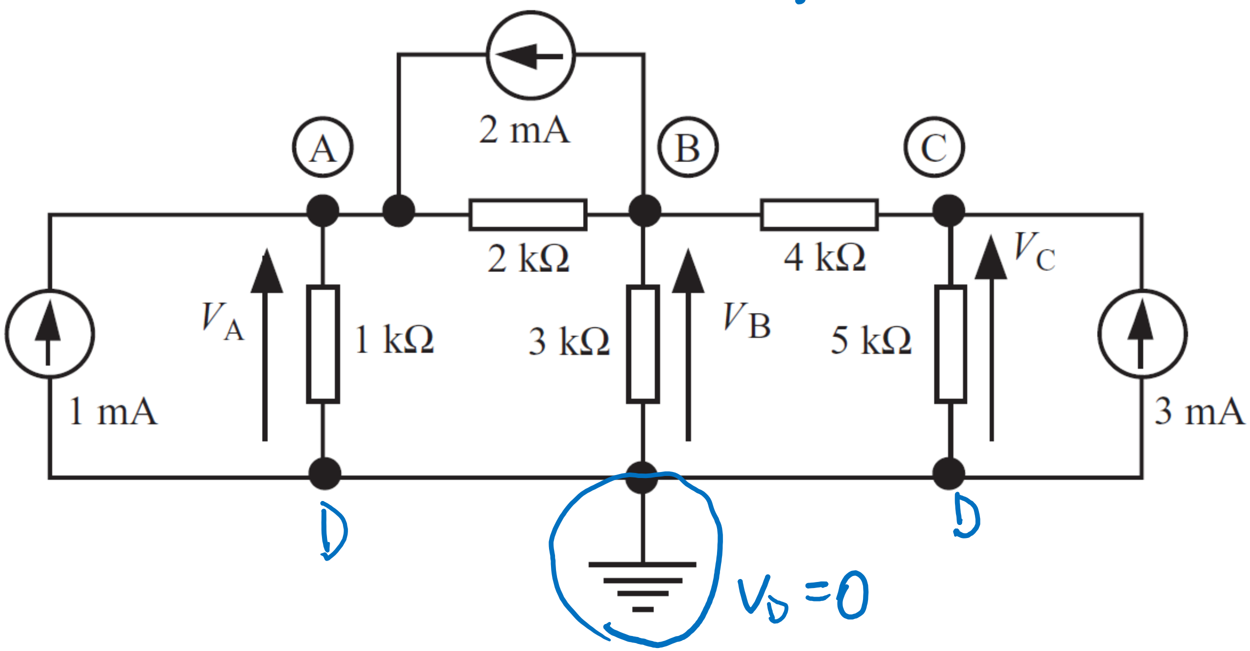


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages

Nodal analysis

Step 3

KCL ⇒ eqn's

@A: $1\,mA + \frac{0 - V_A}{1K} + \frac{V_B - V_A}{2K} + 2\,mA = 0$

@B: $\frac{V_A - V_B}{2k} + \frac{0 - V_B}{3k} + \frac{V_C - V_B}{4} + (-2\,mA) = 0$

@C: $\frac{V_B - V_C}{4k} + \frac{0 - V_C}{5k} + 3\,mA = 0$

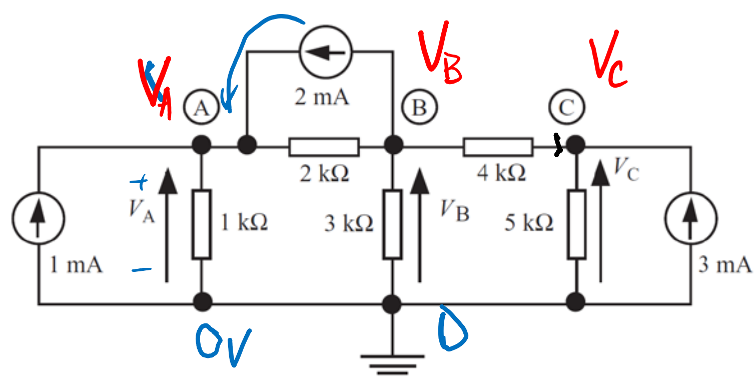


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages

Nodal analysis

Step 4

Solve eqn' $\Rightarrow V_A \quad V_B \quad V_C$

$$\begin{cases} -\frac{3}{2} V_A + \frac{1}{2} V_B + 0 V_C = -3 & \text{— ①} \\ \frac{1}{2} V_A - \frac{13}{12} V_B + \frac{1}{4} V_C = 2 & \text{— ②} \\ 0 V_A + \frac{1}{4} V_B - \frac{9}{20} V_C = -3 & \text{— ③} \end{cases}$$

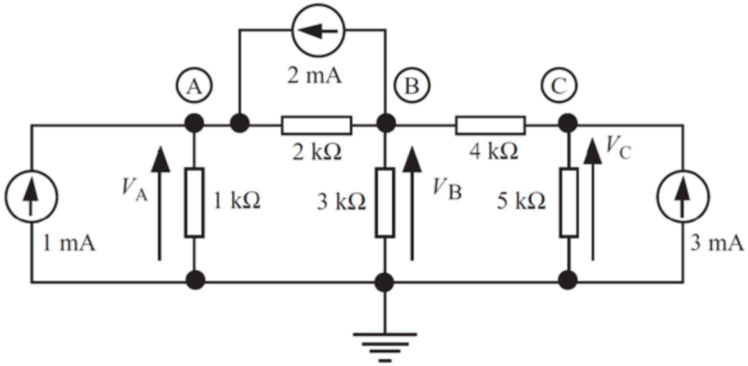


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages

Nodal analysis

$$\begin{bmatrix} -\frac{3}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{13}{12} & \frac{1}{4} \\ 0 & \frac{1}{4} & -\frac{9}{20} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_c \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \\ -3 \end{bmatrix}$$

$$A \qquad \vec{V} = \vec{S}$$

$$\Rightarrow \quad \vec{V} = (A^{-1})\vec{S}$$

$$\vec{V} = \begin{bmatrix} 16/7 \\ 6/7 \\ 50/7 \end{bmatrix} \begin{array}{l} \to V_A \\ \to V_B \\ \to V_c \end{array}$$

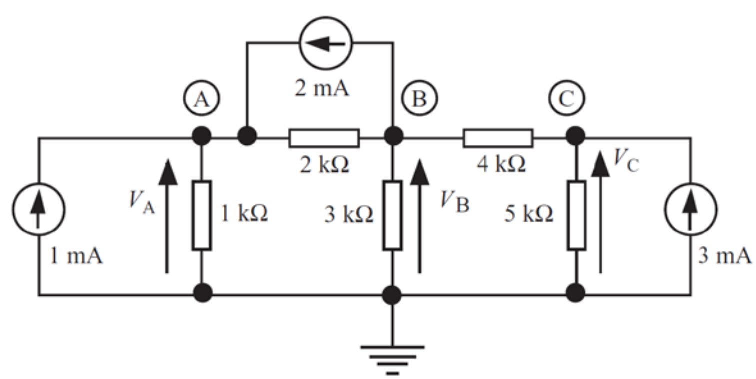


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages

Nodal analysis

Step 1. Find all nodes.

Step 2. Assign Ground.

Step 3. KCL ⇒ Nodes' equ's.

Step 4. Solve ⇒ Nodes' voltages.

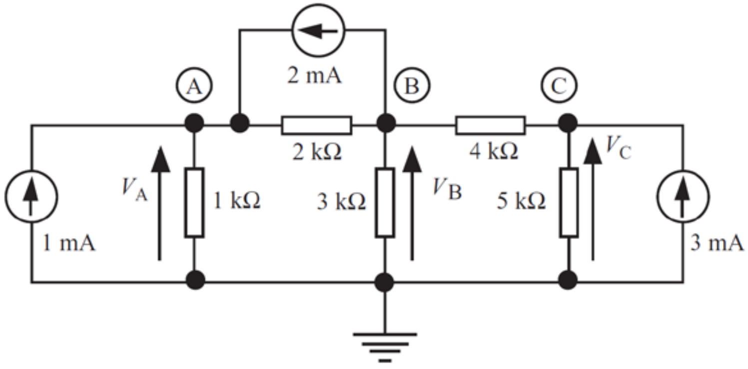


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages

Circuit analysis

Ohm’s law
KCL
KVL

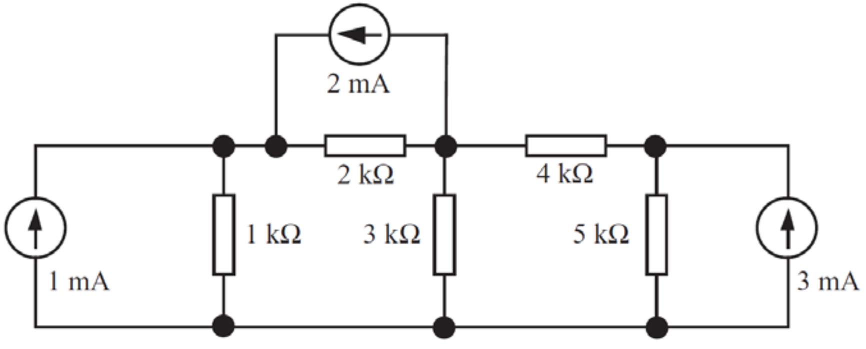
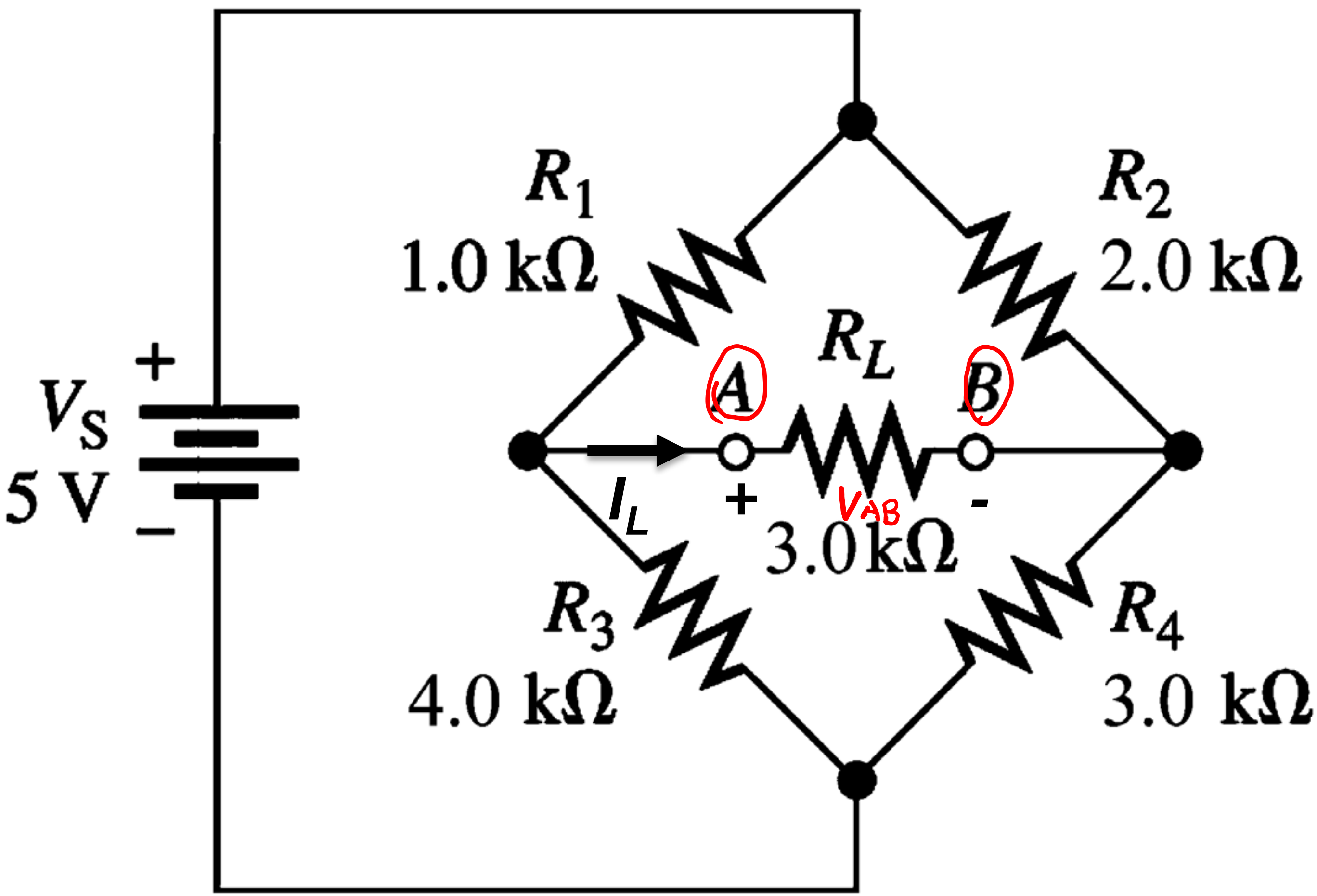


Figure 4.1 The circuit to be analysed

Quiz: find the current I_L through R_L (from A to B) and the voltage V_{AB} by using Nodal Analysis.

$= V_A - V_B$



Quiz review

Step 1. Nodes A, B, X, Y

Step 2. Ground Y $\Rightarrow V_Y = 0$

Src $V_x = 5V$

Step 3. KCL @A

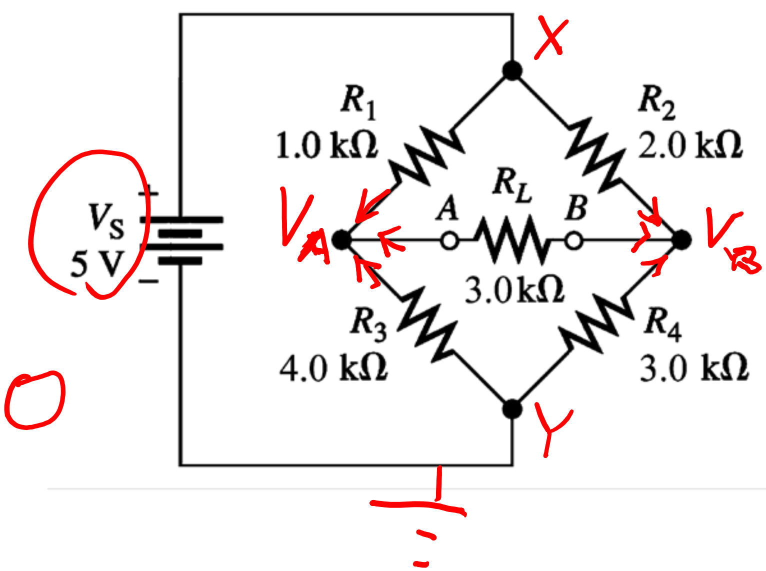
$$\frac{5 - V_A}{R_1} + \frac{V_B - V_A}{R_L} + \frac{0 - V_A}{R_3} = 0$$

1kΩ 3kΩ 4kΩ

KCL @ B

$$\frac{5 - V_B}{R_2} + \frac{V_A - V_B}{R_L} + \frac{0 - V_B}{R_4} = 0$$

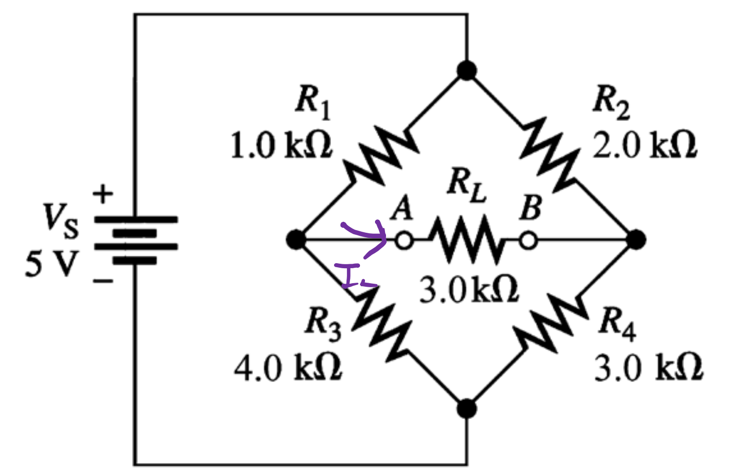
2kΩ 3kΩ 3kΩ



Quiz review

Step 4.

$$\begin{bmatrix} 19/12 & 1/3 \\ 1/3 & -7/6 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} -5 \\ -5/2 \end{bmatrix}$$



Solve : $\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3.84 \\ 3.24 \end{bmatrix}$

$$V_{AB} = 3.84 - 3.24 = 0.6V$$

$$I_L = \frac{V_{AB}}{R_L} = \frac{0.6}{3k} = 0.2\,mA$$

Nodal analysis with voltage sources (1)

Step 1. Nodes A, B, C, X & Y

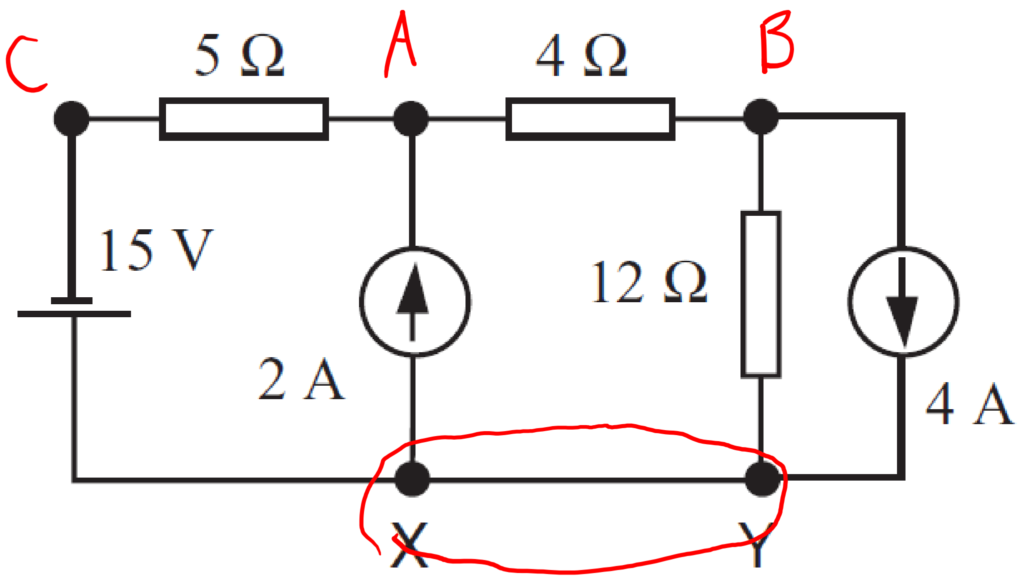


Figure 4.5 A circuit containing a voltage source

Nodal analysis with voltage sources (2)

Step 2. Ground X & Y

$V_X = V_Y = 0$

$\Rightarrow$ Voltage Src

$V_c = -15V$

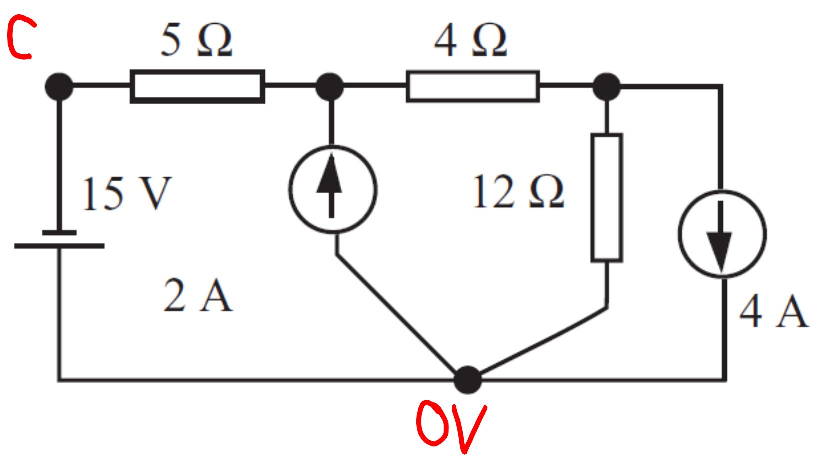


Figure 4.6 The circuit of Figure 4.5 redrawn to emphasize the circuit nodes

Nodal analysis with voltage sources (3)

Step 3. KCL

@ A : $\frac{-15 - V_A}{5} + 2 + \frac{V_B - V_A}{4} = 0$

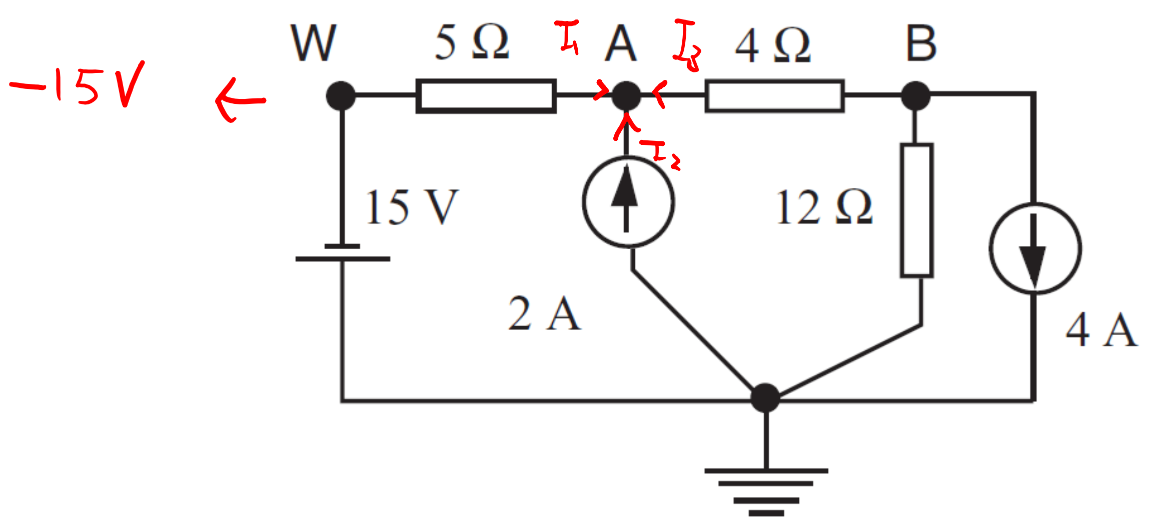


Figure 4.7 A voltage reference node has been chosen and other circuit nodes labelled

Nodal analysis with voltage sources (4)

KCL @ A $\quad I_1 + I_2 + I_3 = 0$

$$\Rightarrow \frac{-15 - V_A}{5} + 2 + \frac{V_B - V_A}{4} = 0 \quad — (1)$$

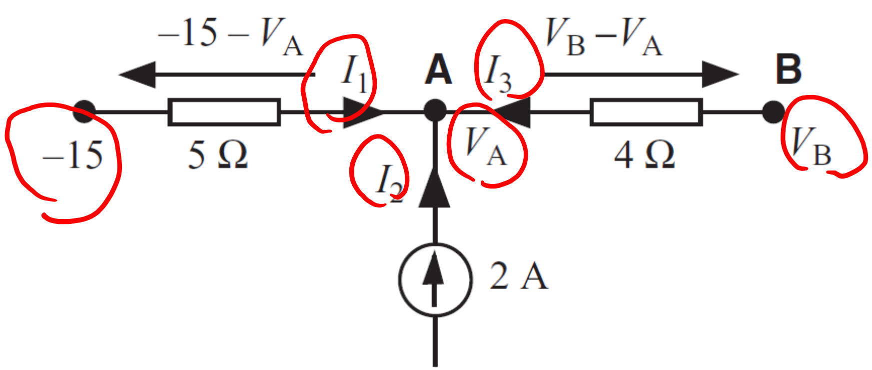


Figure 4.8 Illustration of the application of KCL at node A of the circuit of Figure 4.7

Nodal analysis with voltage sources (5)

KCL @ B

$$\frac{V_A - V_B}{4} + \frac{0 - V_B}{12} + (-4) = 0 \quad -(2)$$

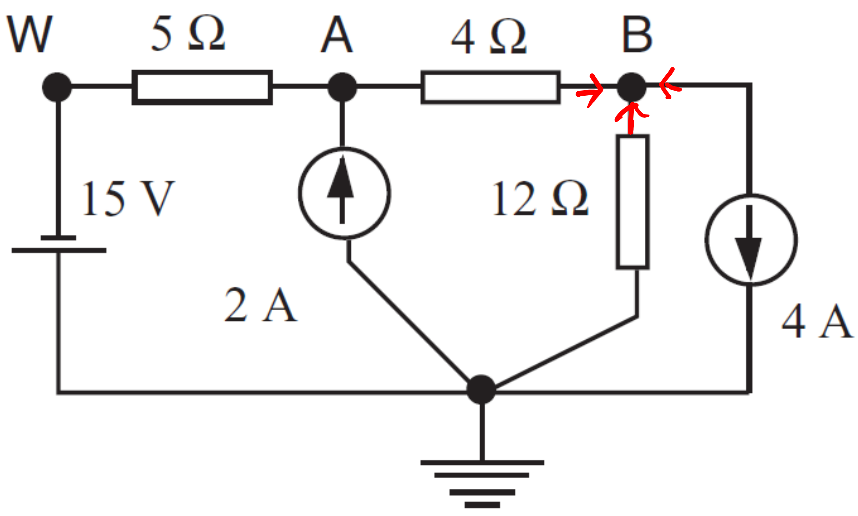


Figure 4.7 A voltage reference node has been chosen and other circuit nodes labelled

Nodal analysis with voltage sources (6)

Step 4. Solve

$$\begin{bmatrix} -9/20 & 1/4 \\ 1/4 & -1/3 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$$A \vec{V} = \vec{S}$$

$$\Rightarrow \vec{V} = A^{-1} \vec{S}$$

$$= \begin{bmatrix} -\frac{320}{21} \\ -\frac{164}{7} \end{bmatrix}$$

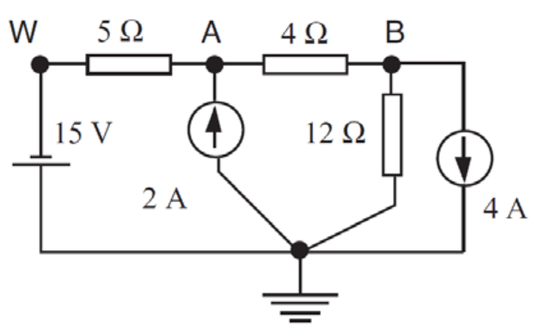


Figure 4.7 A voltage reference node has been chosen and other circuit nodes labelled

Nodal analysis
Summary

1. Labeling Nodes
2. Identifying Ground (earth)
3. Applying KCL
4. Solving system of equations

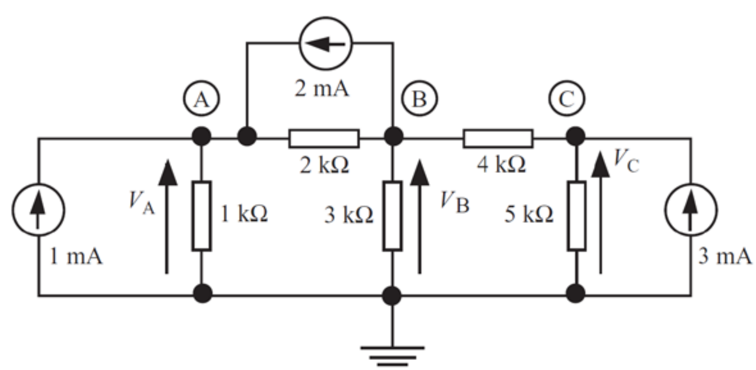


Figure 4.4 The selection of a voltage reference point and the identification of other nodal voltages