E92/500 Electric Circuit Analysis Spring 2017

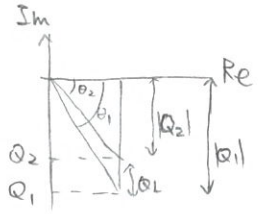
Solution to Midterm Exam II

1. (a) T (b) F (c) T (d) F (e) T

2.
$$S = \underline{V}_{rms}\underline{I}_{rms}^{*} = \underline{V}_{rms}\left(\frac{\underline{V}_{rms}}{Z}\right)^{*} = \frac{\underline{V}_{rms}\,\underline{V}_{rms}^{*}}{Z^{*}} = \frac{V_{rms}^{2}}{Z^{*}}$$

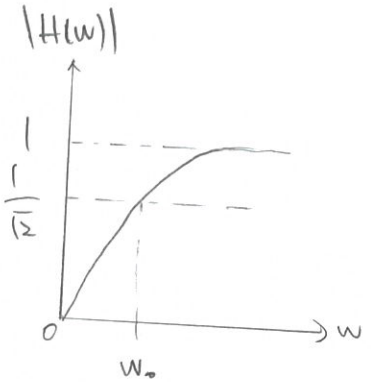
3.
$$P = \mathrm{Re}\left\{\frac{V_{rms}^{2}}{Z^{*}}\right\} = \mathrm{Re}\left\{\frac{V_{rms}^{2}}{R-jX}\right\} = \mathrm{Re}\left\{\frac{V_{rms}^{2}}{R^{2}+X^{2}}(R+jX)\right\}$$
$$= \frac{V_{rms}^{2}}{R^{2}+X^{2}}R$$

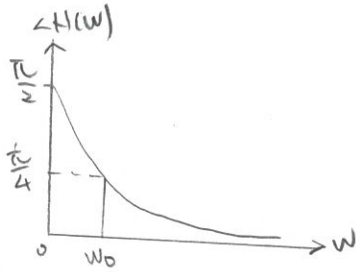
4.



$$Q_L = |Q_1| - |Q_2| = P(|\tan\theta_1| - |\tan\theta_2|) = \frac{V_{rms}^{2}}{X_L} = \frac{V_{rms}^{2}}{\omega L}$$
$$L = \frac{V_{rms}^{2}}{\omega P(|\tan\theta_1| - |\tan\theta_2|)}$$

5.
$$V_o(\omega) = V_s(\omega)\times\frac{R}{R+\frac{1}{j\omega C}} \Rightarrow H(\omega) = \frac{V_o(\omega)}{V_s(\omega)} = \frac{j\omega RC}{1+j\omega RC} = \frac{\frac{j\omega}{\omega_o}}{1+\frac{j\omega}{\omega_o}}$$
$$\text{where } \omega_o = \frac{1}{RC}$$





6. (a) Complex power : $S = \underline{V}_{rms}\,\underline{I}_{rms}^{*}$

$= (250\angle -10^\circ)(6.2\angle 25^\circ) = 1550\angle 15^\circ$ (VA)

Apparent power : $|\underline{S}| = 1550$ (VA)

real power : $\underline{S} = 1550\angle 15^\circ = 1497.185 + j401.17 = P + jQ$

$P = 1497.185$ (W)

reactive power : $Q = 401.17$ (VAR)

(b) Apparent power : $|\underline{S}| = V_{rms} I_{rms} = 220\frac{220}{40} = 1210$ (VA)

real power : $P = 1000$(W)

Reactive power : $\underline{S} = P + jQ = 1000 + jQ$

$\sqrt{1000^2 + Q^2} = 1210 \Rightarrow Q = 681.2488$ (VAR)

Complex power : $\underline{S} = P + jQ = 1000 + j681.2488$ (VA)

7. phase currents

$$I_{AB} = \frac{222\angle 0^\circ}{4+j3} = 35.2 - 26.4j = 44\angle -36.87^\circ \text{(A)}$$

$$I_{BC} = \frac{220\angle -120^\circ}{4+j3} = 44\angle -156.87^\circ \text{(A)}$$

$$I_{CA} = \frac{220\angle 120}{4+j3} = 44\angle 83.13^\circ \text{(A)}$$

line currents

$$I_a = I_{AB}\sqrt{3}\angle -30^\circ = 44\sqrt{3}\angle -66.87^\circ = 76.21\angle -66.87^\circ \text{(A)}$$

$$I_b = I_a\angle -120^\circ = 76.21\angle -186.87^\circ \text{(A)}$$

$$I_c = I_a\angle 120^\circ = 76.21\angle 53.13^\circ \text{(A)}$$

8.

$V_{ab} = \sqrt{3}\, V_{an} \angle 30^\circ = 120\sqrt{3} \angle 120^\circ$

$V_{bc} = V_{ab} \angle -120^\circ = 120\sqrt{3} \angle 0^\circ$

$I_{BC} = \dfrac{V_{bc}}{-j100} = 2.07846j = \underline{2.07846 \angle 90^\circ}$ (A)

$V_{ca} = V_{ab} \angle 120^\circ = 120\sqrt{3} \angle 240^\circ$

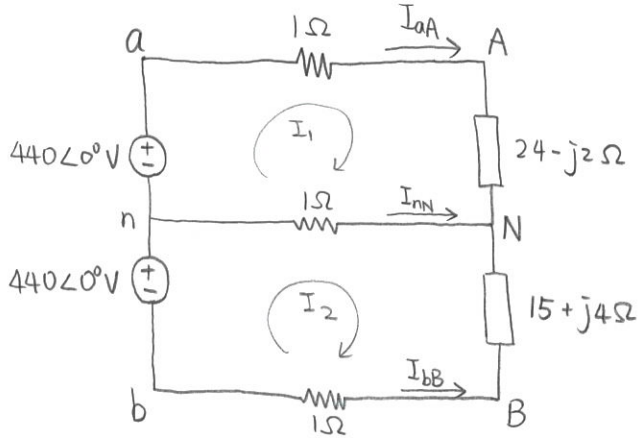
$I_{CA} = \dfrac{V_{ca}}{j100} = 2.07846 \angle 150^\circ$ (A)

$I_C = I_{CA} - I_{BC}$

$= 2.07846 \angle 150^\circ - 2.07846 \angle 90^\circ$

$= 2.07846 \angle -150^\circ$

9.



$$\begin{cases} 440 - I_1(25-j2) - (I_1 - I_2) = 0 \\ 440 - (I_2 - I_1) - I_2(16+j4) = 0 \end{cases}$$

$$(26-j2)I_1 - I_2 = 440 \quad \text{——①}$$

$$-I_1 + (17+j4)I_2 = 440 \quad \text{——②}$$

① + (26-j2)②:

$$I_2\left[(17+j4)(26-j2)-1\right] = 440(27-j2)$$

$$I_2 = 25.53 - j5.94 \text{ A}$$

$$I_1 = 17.82 + j1.14 \text{ A}$$

$$I_{aA} = I_1 = 17.86 \angle 3.66° \text{ A}$$

$$I_{nN} = I_2 - I_1 = 10.47 \angle -42.53° \text{ A}$$

$$I_{bB} = -I_2 = 26.21 \angle 166.91° \text{ A}$$