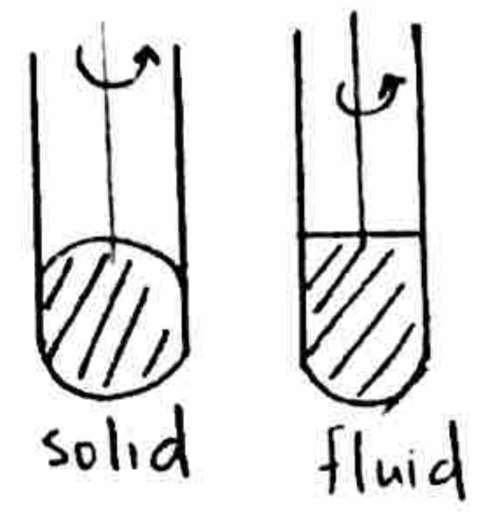
院系 College	工 學院 College	土木 系 三 年 班 Department Year Class	評閱成績 Score	93	of Instructor	

27

1、(a) 流體與固體最大的差別是固體可以承受剪應力，但流體不行。

Ex.



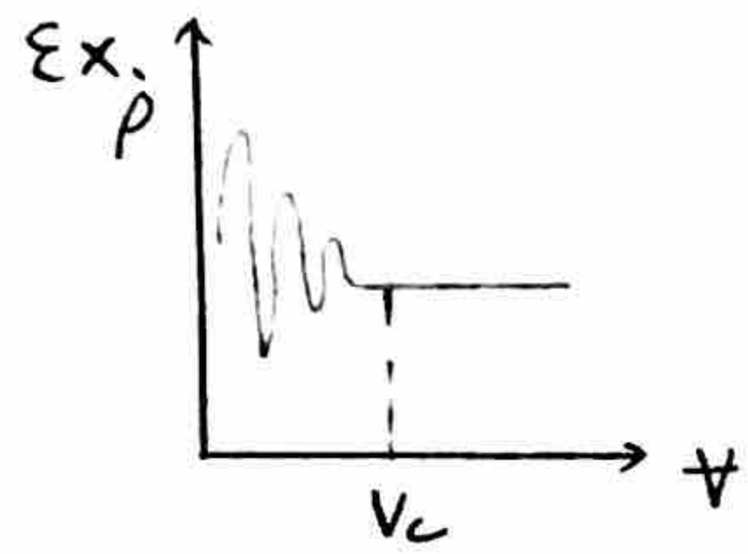
旋轉固體與流體可以發現，固體的形狀不變
但流體形狀會改變

3

(b) 流體粒子(fluid particle)是將一小團流體分子視為質量固定的粒子，其體積必須取的夠大也夠小，夠大才能忽略分子個別效應，但也必須比欲分析的流體空間小很多，使流體性質是可微分的。連續體是以「巨觀」角度分析流體，以流體粒子進行分析，如此可以忽略分子個別影響，僅考慮分子的平均效應。

3

Ex.



由此可以看出體積∀若取太小會考慮到分子個別行為，使得密度上下波動，代表Vc為流體粒子的最小體積

Kn (Knudson number) 是判別連續體的標準

$Kn = \frac{\lambda}{L} < 0.001 \Rightarrow$ continuum

λ: mean free path

L: characteristic length

(c) viscosity (黏滯性)：流體粒子間所能承受剪應力的能力，當黏滯性愈大，代表流體粒子層與層的作用力愈大，欲使流體移動所須施加的力愈大

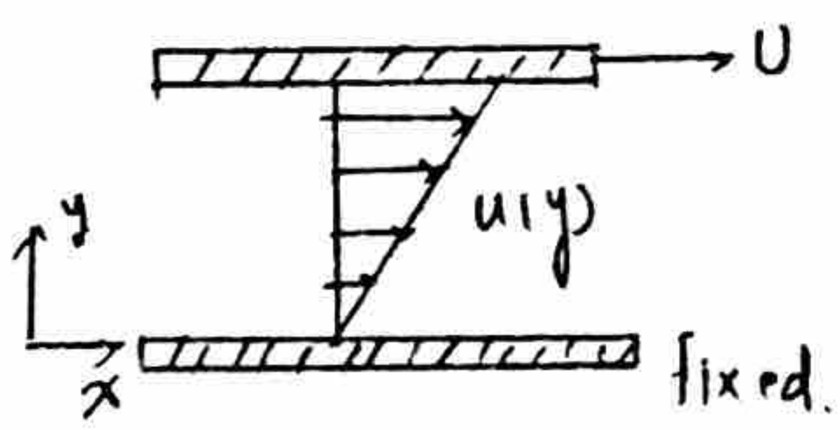
2

no slip condition (不滑移條件)：流體與固體接觸面上沒有相對速度。

(d) Newtonian Fluid (牛頓流體)：符合 Newtons Law of viscosity 的流體

3

而 Newton's Law of viscosity ⇒ 施加的剪應力與流體速度變化量成正比，即 $\tau = \mu\frac{\partial u}{\partial y}$.



(e) vorticity (渦量)：用來描述與判斷流體是否旋轉及其旋轉程度的物理量。

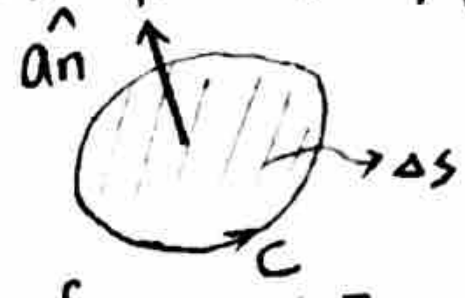
$\vec{\omega} = \nabla\times\vec{v}$ ← 流體速度　if $\vec{\omega}=0$ irrotational flow

$\vec{\omega}\neq 0$ rotational flow

3

circulation (環流量)：在流體中任意一個 contour 上，流體速度的線積分。

$C=\oint_c \vec{v}\cdot d\vec{l}$



$\vec{\omega}=\nabla\times\vec{v}=\lim_{\Delta s\to 0}\frac{[\hat{a}_n\oint_c\vec{v}\cdot d\vec{l}]_{max}}{\Delta s}$　由 vorticity 可以知道該點的最大的環流量及其方向

(f). shear force, pressure, surface tension.

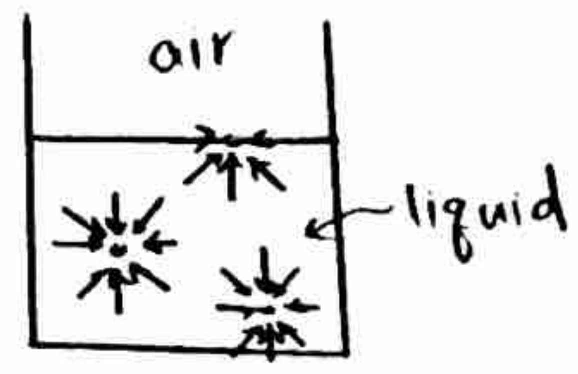
3

(續寫轉背頁)

(g) surface tension (表面張力)：在兩不同介質的界面上，因分子受力不平衡而造成的現象

air 與 liquid 為兩不同介質 ⇒ 在界面上會有表面張力的出現

3



在 liquid 中，液體分子四面八方受力相同，力平衡

但在 air-liquid interface 上，液體分子受力不平衡 ⇒ surface tension

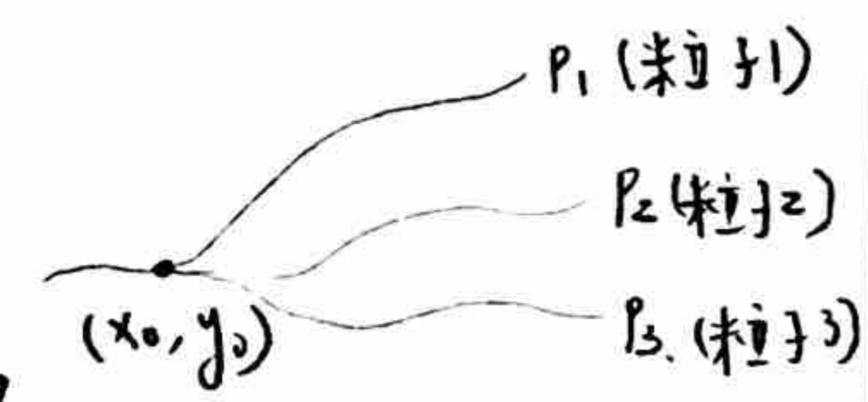
(h) streamline (流線)：流體上的切線大小及方向即代表該點的速度大小及方向。

3

$$\vec{v} \parallel d\vec{r} \Rightarrow \vec{v}\times d\vec{r} = 0 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ u & v & w \\ dx & dy & dz \end{vmatrix} = \hat{i}\,(v\,dz - w\,dy) + \hat{j}\,(w\,dx - u\,dz) + \hat{k}\,(u\,dy - v\,dx)$$

$$\Rightarrow \frac{u}{dx} = \frac{v}{dy} = \frac{w}{dz}$$

path line：同一流體粒子移動的軌跡。



streak line (煙線)：經過同一點的所有流體粒子的軌跡集合 ↗

僅有在 steady state 時，streamline, path line, streak line 才會相等

(i). stream function

4

定義 $u = \frac{\partial \psi}{\partial y}$ $\quad v = -\frac{\partial \psi}{\partial x}$ $\quad \psi$ 自動滿足 continuity equation $\nabla\cdot\vec{v} = 0$

$$\nabla\cdot\vec{v} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial y \partial x} = 0$$

potential function

定義 $u = \frac{\partial \phi}{\partial x}$ $\quad v = \frac{\partial \phi}{\partial y}$ $\quad \phi$ 自動滿足 irrotational condition $\vec{\omega} = \nabla\times\vec{v} = 0$

$$\nabla\times\vec{v} = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)\hat{k} = 0 \qquad \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \frac{\partial^2 \phi}{\partial x \partial y} - \frac{\partial^2 \phi}{\partial x \partial y} = 0.$$

stream function 的物理意義可由其與 stream line 的關係中看出

2D $\vec{v}\times d\vec{r} = \hat{k}\,(u\,dy - v\,dx) = 0$

$$u\,dy - v\,dx = \frac{\partial \psi}{\partial y}dy + \frac{\partial \psi}{\partial x}dx = d\psi = 0 \qquad \therefore\ \psi = C$$

stream function is constant along stream line

potential function 的物理意義 ⇒ $\vec{v} \perp d\vec{r}$

$$\vec{v}\cdot d\vec{r} = u\,dx + v\,dy = 0$$

$$\frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = d\phi = 0 \qquad \phi = C$$

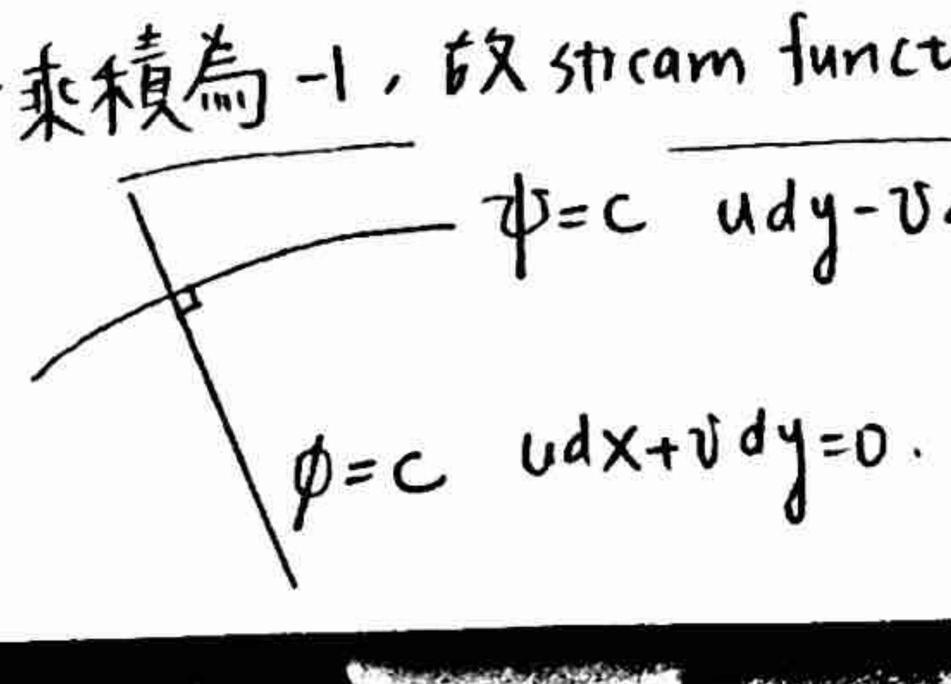
potential line 上每一點的切線方向都與流速垂直

potential function is constant along potential line

由上述分析可以看出 $d\psi = \frac{\partial \psi}{\partial x}dx + \frac{\partial \psi}{\partial y}dy = -v\,dx + u\,dy = 0$ $\quad \frac{dy}{dx} = \frac{v}{u}$

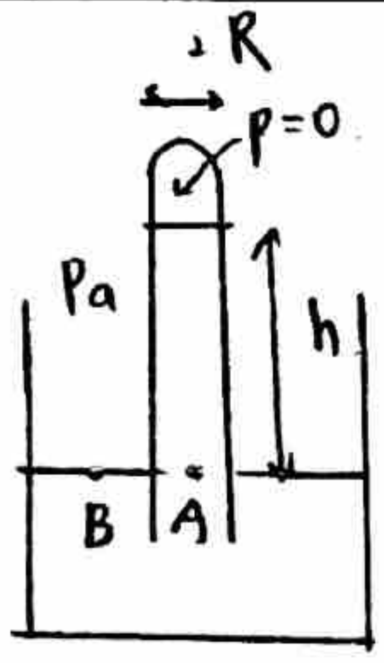
$$d\phi = \frac{\partial \phi}{\partial x}dx + \frac{\partial \phi}{\partial y}dy = u\,dx + v\,dy = 0 \qquad \frac{dy}{dx} = -\frac{u}{v}$$

兩者斜率乘積為 -1，故 stream function 與 potential function 正交



院系 College	工 學院 College	工科 系 Department	三 年 Year	班 Class	評閱成績 Score		of Instructor	

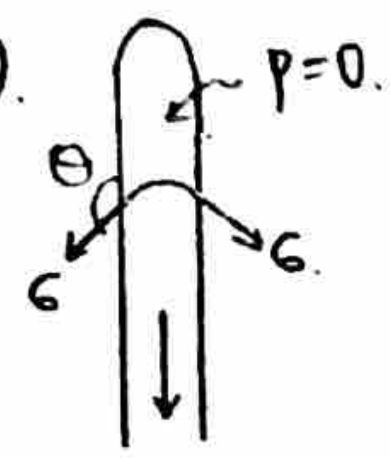
2、 20



(a) pressure at point A = pressure at point B

5 $\overset{0}{p} + \rho g h = P_a \Rightarrow \underline{P_a = \rho g h}$ ※

(b)



10

作用力 at point A = 作用力 at point B

$(\overset{0}{p} + \rho g h)(\cancel{\pi} R^2) + \sigma\cos(180-\theta)(2\cancel{\pi}\cancel{R}) = P_a(\cancel{\pi}R^2)$

$\underline{P_a = \rho g h - \frac{2}{R}\sigma\cos\theta}$ ※

(c) 比較 $P_a = \rho g h$ 與 $P_a = \rho g h - \frac{2}{R}\sigma\cos\theta$

5 當管徑R很大時可以忽略表面張力的影響。

當surface tension σ 很小時，亦可忽略表面張力的影響。

3、 conservation of mass $b=1$ $B = \iiint_{c.v.} \rho \, d\forall$

23

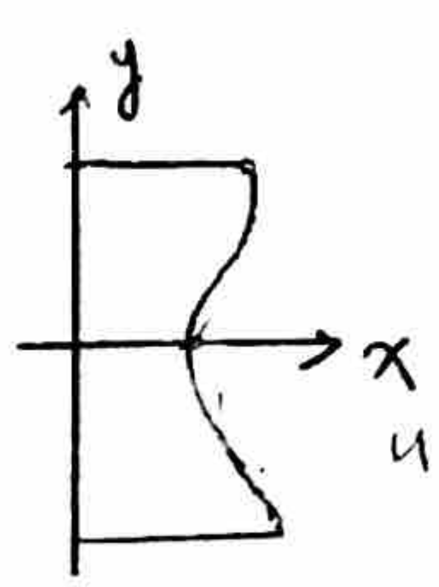
$\left(\frac{DB}{Dt}\right)_{system} = 0 = \cancel{\frac{d}{dt}\iiint_{c.v.}\rho\, d\forall}^{\,0} + \iint_{c.s.} \rho \vec{v}\cdot d\vec{A}$

steady state

$\iint_{c.s.} \rho\vec{v}\cdot d\vec{A} = \int_A^B \rho\vec{v}\cdot d\vec{A} + \int_B^C \rho\vec{v}\cdot d\vec{A} + \int_C^D \rho\vec{v}\cdot d\vec{A} + \int_D^A \rho\vec{v}\cdot d\vec{A} = 0$

(a)

5



$y=0 \quad u = 0.5U$

$y=H \quad u=U$

$y=-H \quad u=U$

令 $u = ay^2 + by + c$

$y=0 \Rightarrow u = c = 0.5U$

$y=H \Rightarrow u = aH^2 + bH + 0.5U = U$

$aH^2 + bH = 0.5U$

$y=-H \Rightarrow u = aH^2 - bH + 0.5U = U$

$aH^2 - bH = 0.5U$

$aH^2 + bH = 0.5U$

$+)\ aH^2 - bH = 0.5U$

$2aH^2 = U \quad a = \frac{U}{2H^2},\ b=0$

$\underline{u = \frac{U}{2H^2}y^2 + \frac{1}{2}U}$ ※

(b)

5 mass flux AB. $\left|\int_A^B \rho\vec{v}\cdot d\vec{A}\right| = \rho U(2H) = \underline{2\rho U H}$ ※

mass flux CD $\left|\int_C^D \rho\vec{v}\cdot d\vec{A}\right| = \rho\int_{-H}^{H} \frac{U}{2H^2}y^2 + \frac{1}{2}U\, dy = \rho\left[\frac{1}{6}\frac{U}{H^2}y^3 + \frac{1}{2}Uy \Big|_{-H}^{H}\right]$

$= \rho\left[\frac{1}{3}UH + UH\right] = \underline{\frac{4}{3}\rho U H}$ ※

(c). conservation of momentum $b=\vec{v}$ $B=\iiint_{c.v.}\rho\vec{v}\,d\forall$

13

$$\left(\frac{DB}{Dt}\right)_{system} = \underbrace{\cancel{\frac{d}{dt}\iiint_{c.v.}\rho\vec{v}\,d\forall}}_{\text{steady state}}^{0} + \iint_{c.s.}\rho\vec{v}\,(\vec{v}\cdot d\vec{A})$$

$$x\text{-dir}: P_1(2H) - P_2(2H) - F = \iint_{c.s.}\rho\vec{v}\,(\vec{v}\cdot d\vec{A}) \quad + \dot{m}_{AD}\times U + \dot{m}_{BC}\times U$$

進

$$\int_A^B \rho\vec{v}(\vec{v}\cdot d\vec{A}) = -\rho U^2(2H) = -2\rho U^2 H$$

出

$$\int_C^D \rho\vec{v}(\vec{v}\cdot d\vec{A}) = \rho\int_{-H}^{H}\left(\frac{U}{2H^2}y^2+\frac{U}{2}y\right)^2 dy = \frac{14}{15}\rho U^2 H$$

假設surface BC, 與 surface AD的流速皆為y方向

$$\rho\int_{-H}^{H}\left(\frac{U}{2H^2}y^2+\frac{1}{2}U\right)^2 dy$$
$$=\rho\int_{-H}^{H}\frac{U^2}{4H^4}y^4+\frac{U^2}{2H^2}y^2+\frac{1}{4}U^2\,dy$$
$$=\rho\left[\frac{U^2}{20H^4}y^5+\frac{U^2}{6H^2}y^3+\frac{1}{4}U^2y\Big|_{-H}^{H}\right]$$
$$=\rho\left[\frac{1}{10}U^2H+\frac{1}{3}U^2H+\frac{1}{2}U^2H\right]$$
$$=\rho\left(\frac{14}{15}U^2H\right)=\frac{14}{15}\rho U^2 H$$

$$F = 2H(P_1-P_2)+2\rho U^2H-\frac{14}{15}\rho U^2H$$
$$= 2H(P_1-P_2)+\frac{16}{15}\rho U^2 H \quad \#$$

4. (a) Steady 2D incompressible flow.

23

5

由 RTT conservation of mass $\frac{d}{dt}\iiint_{c.v.}\rho\,d\forall + \iint_{c.s.}\rho\,\vec{v}\cdot d\vec{A}=0.$

$$\Rightarrow \iiint_{c.v.}\frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})\,d\forall=0 \Rightarrow \frac{\partial\rho}{\partial t}+\nabla\cdot(\rho\vec{v})=0$$

∵ incompressible ∴ ρ is constant. $\cancel{\frac{\partial\rho}{\partial t}}^{0}+\rho\nabla\cdot\vec{v}=0 \Rightarrow \nabla\cdot\vec{v}=0.$

Cartesian coordinate $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0.$ # $\vec{v}=u\hat{i}+v\hat{j}$

Momentum equation $\underbrace{\cancel{\frac{\partial\vec{v}}{\partial t}}}_{\text{steady state}}+\vec{v}\cdot\nabla\vec{v}=-\frac{1}{\rho}\nabla p+\nu\nabla^2\vec{v}.$

$$x\text{-dir}: u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \#$$

$$y\text{-dir}: u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \#$$

(b) ∵ $u=u(y)$.

12

$$\begin{cases}\cancel{\frac{\partial u}{\partial x}}^{0}+\frac{\partial v}{\partial y}=0. \Rightarrow \frac{\partial v}{\partial y}=0. \Rightarrow v=v(x).\\ u\cancel{\frac{\partial u}{\partial x}}^{0}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\cancel{\frac{\partial^2 u}{\partial x^2}}^{0}+\cancel{\frac{\partial^2 u}{\partial y^2}}^{0}\right)\\ u\frac{\partial v}{\partial x}+v\cancel{\frac{\partial v}{\partial y}}^{0}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\cancel{\frac{\partial^2 v}{\partial x^2}}^{0}+\cancel{\frac{\partial^2 v}{\partial y^2}}^{0}\right).\end{cases}$$

$$v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$u\frac{\partial v}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial y}.$$

$$\frac{\partial}{\partial y}\left(v\frac{\partial u}{\partial y}\right)=-\frac{1}{\rho}\frac{\partial^2 p}{\partial y\partial x}$$
$$\Rightarrow \cancel{\frac{\partial v}{\partial y}}^{0}\frac{\partial u}{\partial y}+v\frac{\partial^2 u}{\partial y^2}=-\frac{1}{\rho}\frac{\partial^2 p}{\partial y\partial x}.$$

$$\frac{\partial}{\partial x}\left(u\frac{\partial v}{\partial x}\right)=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y}$$
$$\cancel{\frac{\partial u}{\partial x}}^{0}\frac{\partial v}{\partial x}+u\frac{\partial^2 v}{\partial x^2}=-\frac{1}{\rho}\frac{\partial^2 p}{\partial x\partial y}.$$

$$\Rightarrow v\frac{\partial^2 u}{\partial y^2}=u\frac{\partial^2 v}{\partial x^2}. \Rightarrow \frac{1}{u}\frac{\partial^2 u}{\partial y^2}=\frac{1}{v}\frac{\partial^2 v}{\partial x^2}$$

院系 College	工學院 College	工科系 Department	三 年 Year	班 Class	評閱成績 Score		Signature of Instructor

$$\frac{1}{u}\frac{d^2u}{dy^2}=\frac{1}{v}\frac{d^2v}{dy^2} \Rightarrow \frac{1}{u}\frac{d^2u}{dy^2}=\frac{1}{v}\frac{d^2v}{dy^2}=0$$

$$\frac{1}{u}\frac{d^2u}{dy^2}=0 \quad \because \frac{1}{u}\neq 0 \quad \therefore \frac{d^2u}{dy^2}=0 \Rightarrow u=ay+b.$$

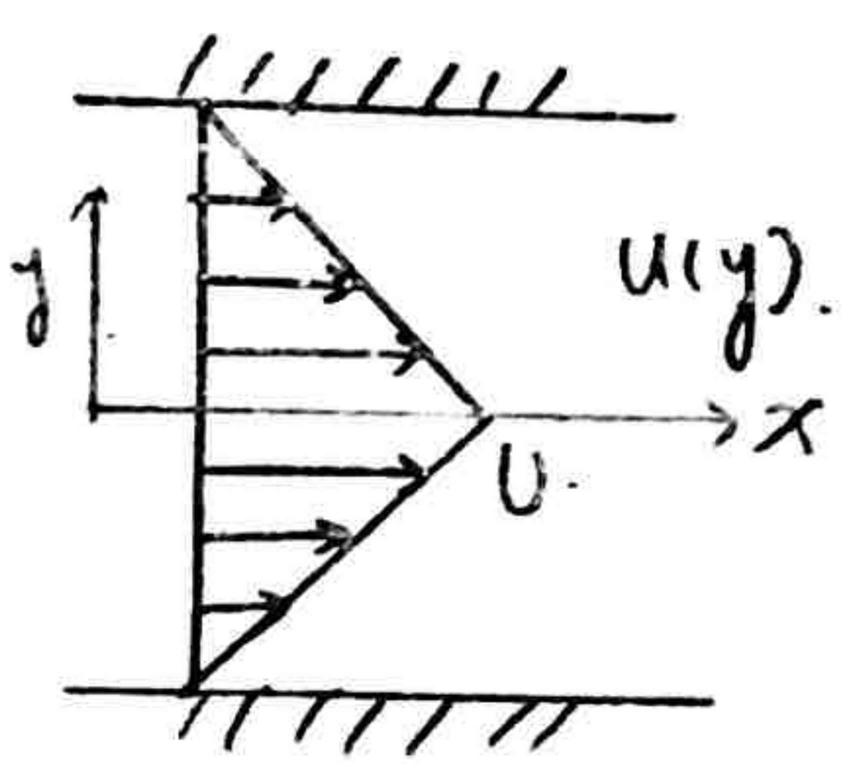
$y>0$ 時 $\Rightarrow$ $y=h \quad u=0$, $y=0 \quad u=U$ $\Rightarrow u=-\frac{U}{h}y+U.$

$y<0$ 時 $\Rightarrow$ $y=-h \quad u=0$, $y=0 \quad u=U$ $\Rightarrow u=\frac{U}{h}y+U.$

$\therefore$ 在 $y=0$ 時 u 有最大值 U.

mass flow rate $\rho(Uh)$.

(c).



$$\nabla\times\vec{v}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0\\ -\frac{U}{h}y+U & v & 0\end{vmatrix}=\begin{cases}\hat{k}\left(\frac{\partial v}{\partial x}-\frac{U}{h}\right) & y>0.\\ \hat{k}\left(\frac{\partial v}{\partial x}+\frac{U}{h}\right) & y<0.\end{cases}$$

y 方向的 vorticity 為 0.

在 $y=+0.8h$ 處的槳子會逆時針旋轉 ($\because \nabla\times\vec{v}>0$)

在 $y=0.8h$ 處的槳子會順時針旋轉. ($\because \nabla\times\vec{v}<0$)