Homework 1

1. Evaluate the limit, if it exists.
   (a) $\lim_{x \to -2}(3x - 7)$.
   (b) $\lim_{x \to 2} \dfrac{2 - x}{\sqrt{x + 2} - 2}$.
2. If $2x \leq g(x) \leq x^4 - x^2 + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.
3. Let
$$g(x) = \begin{cases} x & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2 - x^2 & \text{if } 1 < x \leq 2 \\ x - 3 & \text{if } x > 2 \end{cases}$$
   (a) i. $\lim_{x \to 1^-} g(x)$.
   ii. $\lim_{x \to 1} g(x)$.
   iii. $g(1)$.
   iv. $\lim_{x \to 2^-} g(x)$.
   v. $\lim_{x \to 2^+} g(x)$.
   vi. $\lim_{x \to 2} g(x)$.
   (b) Sketch the graph of g.
4. Prove the statement using the ϵ, δ definition of a limit.
   (a) $\lim_{x \to 4}(\frac{1}{2}x - 1) = 1$.
   (b) $\lim_{x \to 4} \dfrac{x^2 - 2x - 8}{x - 4} = 6$
   (c) $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ if $a > 0$.