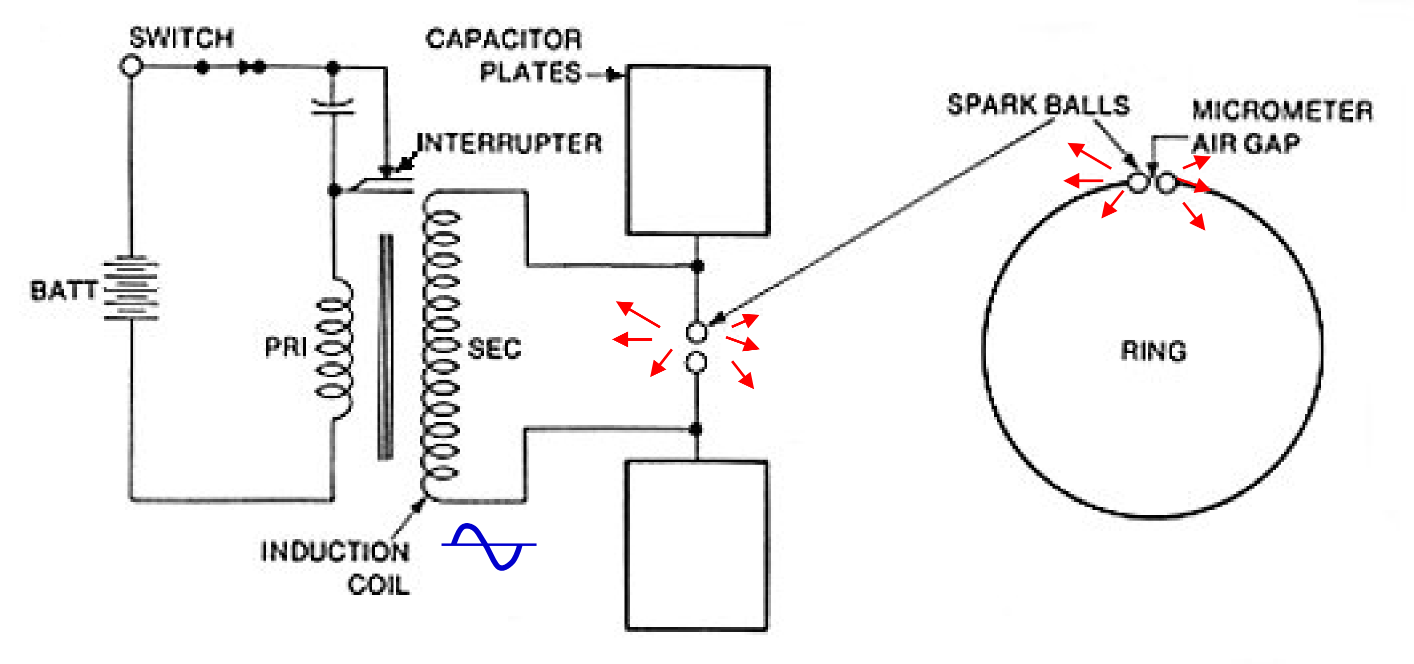
Photoelectric and Compton Effects

— the particle-like property of light

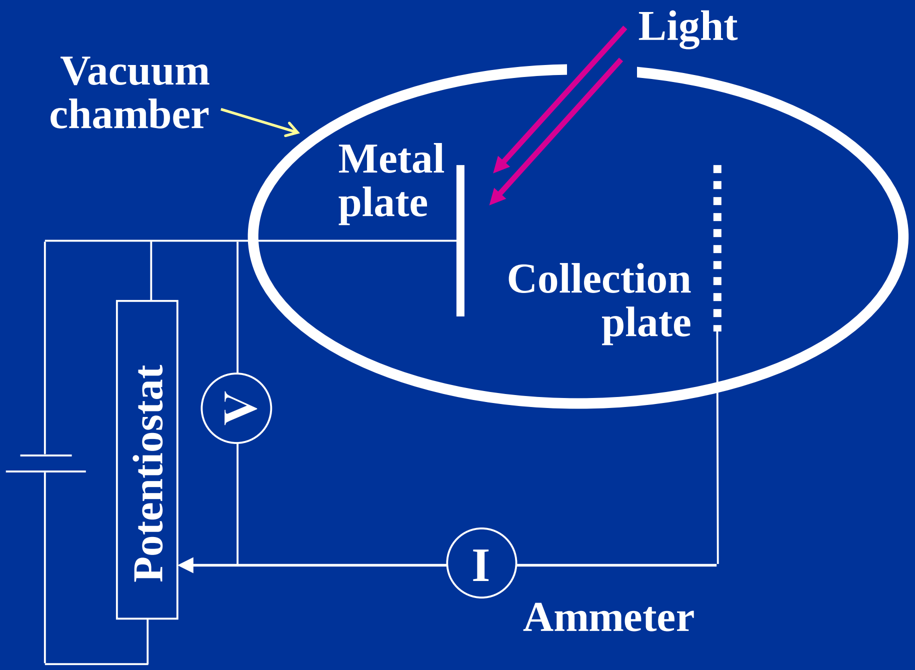
Photoelectric effect

During Hertz's experiment, he noticed that sparks occurred more readily in the air gap when ultraviolet light was directed at one of the metal balls.

Hertz



When ultra-violet (UV) lights impinge on a metal in a vacuum, charged particles are emitted (Hertz 1888), which were later shown to be electrons by J.J. Thomson (1899).




J.J. Thomson

Classical expectations

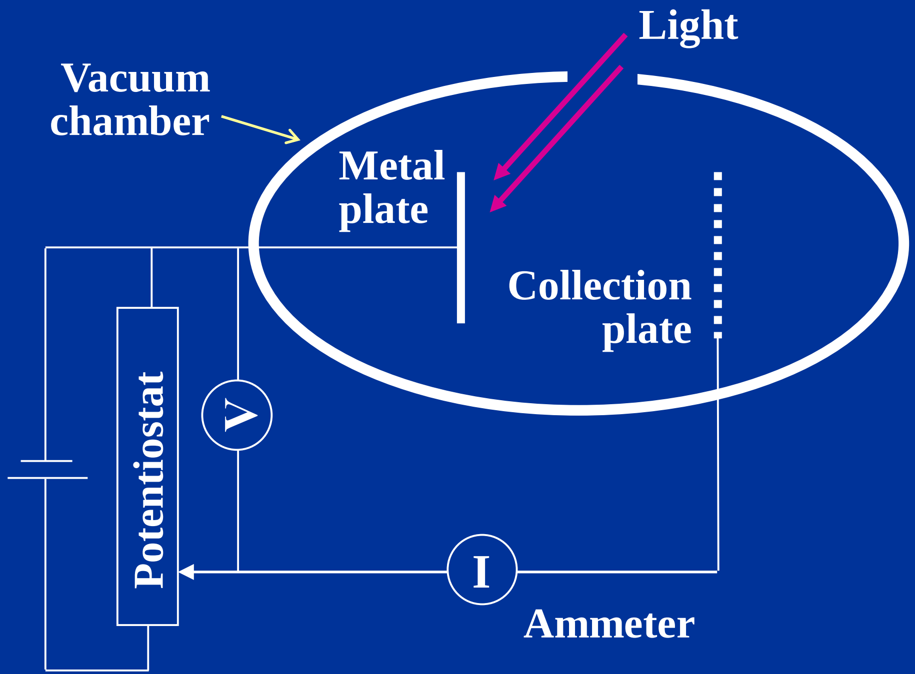
- Electric field E of light exerts force $\mathbf{F}=-e\mathbf{E}$ on electrons. As intensity of light increases, force increases, so kinetic energy of ejected electrons should increase.

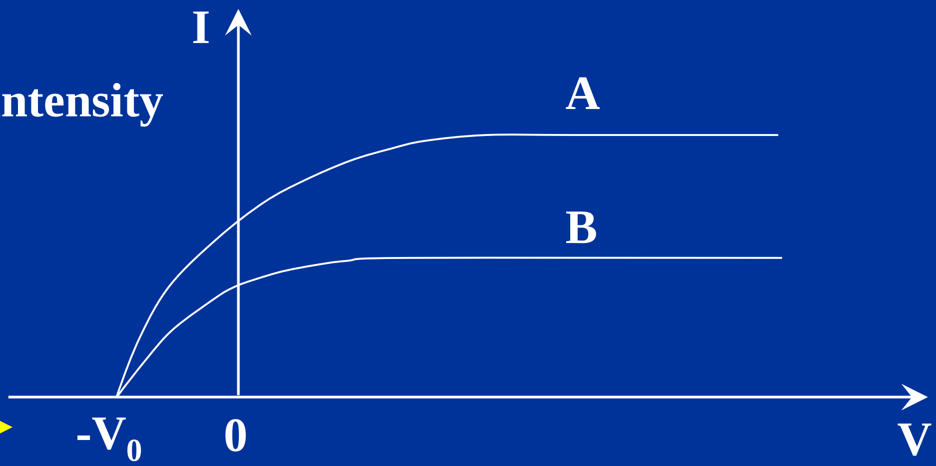
- Electrons should be emitted whatever the frequency ν of the light, so long as **E** is sufficiently large

- For very low intensities, expect a time lag between light exposure and emission, while electrons absorb enough energy to escape from material

Actual results:

- Maximum kinetic energy of ejected electrons is independent of intensity, but dependent on ν
- For $\nu < \nu_0$ (i.e. for frequencies below a cut-off frequency) no electrons are emitted
- There is no time lag. However, rate of ejection of electrons depends on light intensity.





Light intensity A>B

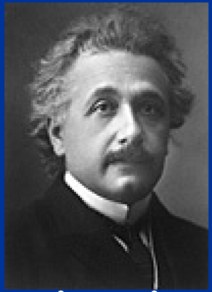
eV_0 — maximum kinetic energy of ejected photoelectrons

Einstein's interpretation (1905):

Light comes in packets of energy (***photons***)

$$E = h\nu$$

An electron absorbs a single photon to leave the material



Einstein

Millikan

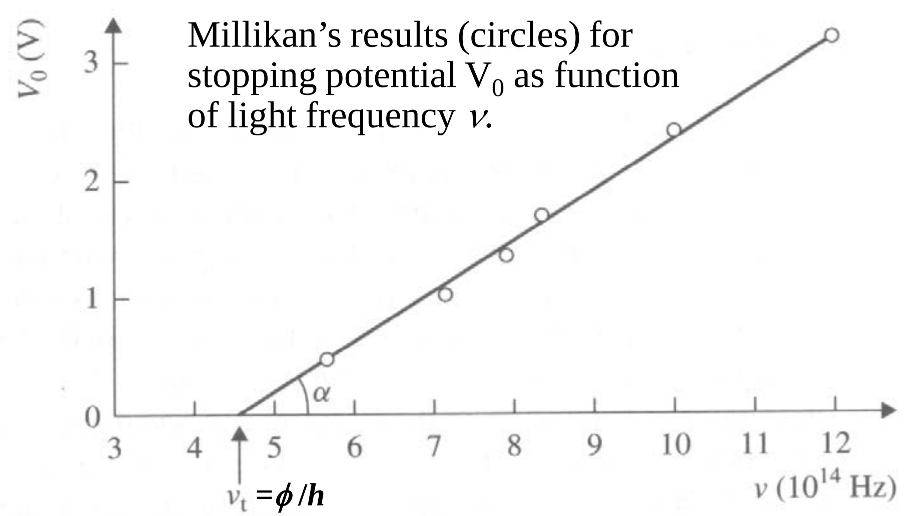
Maximum kinetic energy of an emitted electron is then

$$K_{\max} = h\nu - \phi$$

Planck constant

Work function: minimum energy needed for electron to escape from metal (depends on material, usually 2-5eV)

Verified in detail through experiments by Millikan



Millikan's results (circles) for stopping potential V_0 as function of light frequency ν.

$\nu_t = \phi/h$

$K_{max} = h\nu - \phi = eV_0$, so $V_0 = (h/e)\nu - \phi/e$ — a linear function of ν,

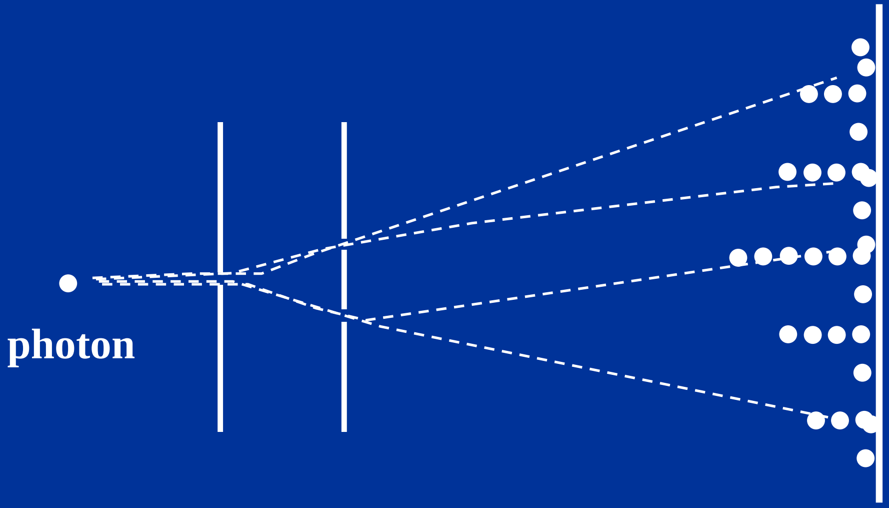
slope = $h/e \rightarrow h = 6.56\times10^{-34}$ J·s, Millikan's result agreed very well with Planck's result 6.55×10^{-34} J·s.

What is light ?

Light is wave as well as particle

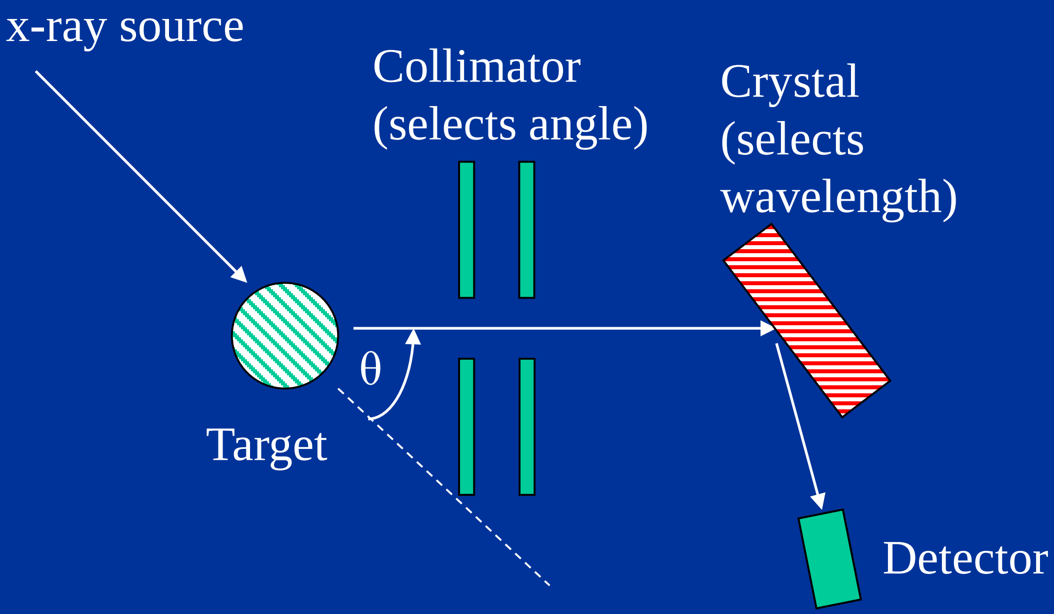
Light has a dual character !

Each photon can be associated with a wave. The intensity of the wave at a location determines the probability that a photon will arrive there, not the energy. The energy of photon is determined by the frequency of the wave (i.e. *hv*).



Compton Effect

Compton (1923) measured intensity of scattered x-rays (lights with λ~ 0.1 nm) from solid target, as function of wavelength for different angles.



x-ray source

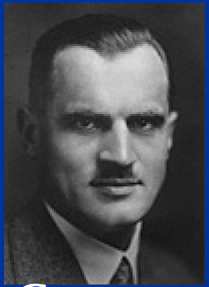
Collimator
(selects angle)

Crystal
(selects
wavelength)

θ

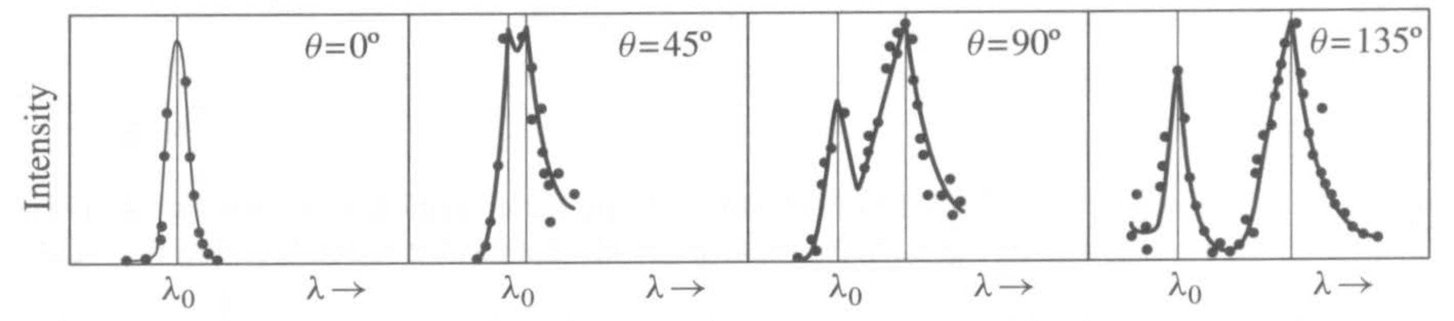
Target

Detector



Compton

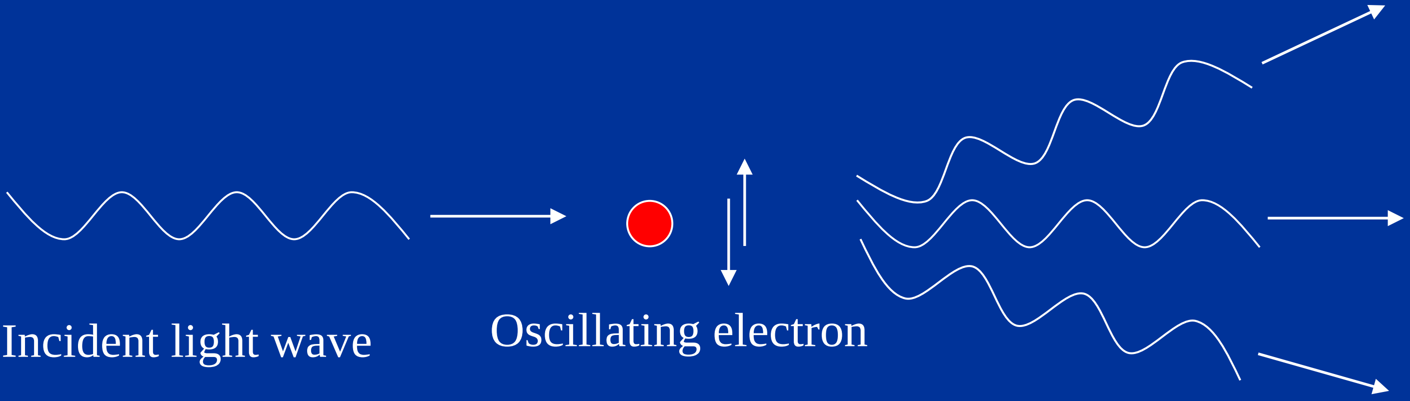
Result: peak in scattered radiation shifts to longer wavelength than source. Amount depends on *θ* (but not on the target material).



Compton's data (1923) for the scattering of X-rays by graphite

Classical picture: oscillating electromagnetic field causes oscillations in positions of charged particles, which re-radiate in all directions at *same frequency and wavelength* as incident radiation.

Change in wavelength of scattered light is completely unexpected classically

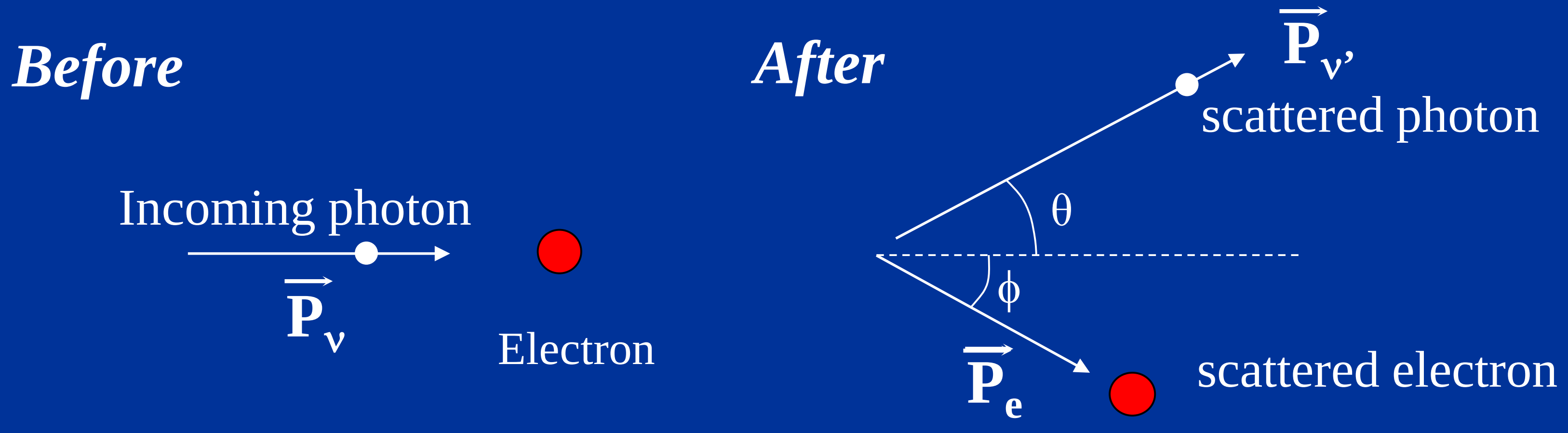


Incident light wave

Oscillating electron

Emitted light wave

Compton's explanation: "billiard ball" collisions between particles of light (x-ray photons) and electrons in the material



Conservation of energy

$$\boxed{h\nu + m_e c^2 = h\nu' + \sqrt{p_e^2 c^2 + m_e^2 c^4}} \Rightarrow \boxed{h(\nu - \nu') + m_e c^2 = \sqrt{p_e^2 c^2 + m_e^2 c^4}} \Rightarrow$$

$$\boxed{\left[h(\nu - \nu') + m_e c^2\right]^2 = p_e^2 c^2 + m_e^2 c^4} \Rightarrow \boxed{p_e^2 = \frac{h^2}{c^2}(\nu - \nu')^2 + 2 m_e h(\nu - \nu')} \quad (1)$$

Conservation of momentum

$$\boxed{\vec{\mathbf{P}}_\nu = \vec{\mathbf{P}}_{\nu'} + \vec{\mathbf{P}}_e} \Rightarrow \boxed{\begin{aligned} p_\nu &= p_{\nu'} \cos\theta + p_e \cos\phi \\ 0 &= p_{\nu'} \sin\theta - p_e \sin\phi \end{aligned}} \Rightarrow \boxed{(p_\nu - p_{\nu'} \cos\theta)^2 + p_{\nu'}^2 \sin^2\theta = p_e^2}$$

$$(p_\nu - p_{\nu'}\cos\theta)^2 + p_{\nu'}^2 \sin^2\theta = p_e^2 \quad \Rightarrow \quad p_\nu^2 - 2p_\nu p_{\nu'}\cos\theta + p_{\nu'}^2 = p_e^2$$

Photon energy $E = mc^2 = h\nu$, $m = h\nu/c^2$

therefore, photon momentum $p = mc = h\nu/c$

Substitute $p_\nu = h\nu/c$ and $p_{\nu'} = h\nu'/c$ into the above equation

We have

$$p_e^2 = \frac{h^2}{c^2}\left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right) \qquad (2)$$

Put (2) into (1) $$p_e^2 = \frac{h^2}{c^2}(\nu-\nu')^2 + 2m_e h(\nu-\nu') = \frac{h^2}{c^2}\left(\nu^2 + \nu'^2 - 2\nu\nu'\cos\theta\right)$$

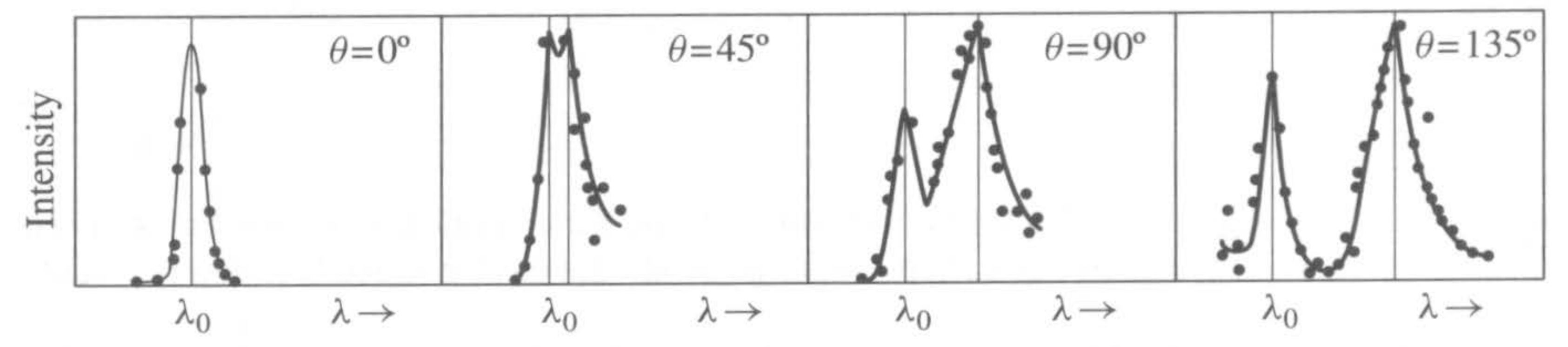
$$(\nu-\nu')^2 + 2m_e c^2(\nu-\nu')/h = \nu^2 + \nu'^2 - 2\nu\nu'\cos\theta$$

$$m_e c^2(\nu-\nu')/h = \nu\nu'(1-\cos\theta)$$

so $$\frac{\nu-\nu'}{\nu\nu'} = \frac{h}{m_e c^2}(1-\cos\theta)$$ replace $\nu = c/\lambda$, we finally have:

$$\lambda' - \lambda = \frac{h}{m_e c}(1-\cos\theta) = \lambda_c(1-\cos\theta) \geq 0$$

$$\lambda_c = \text{ Compton wavelength } = \frac{h}{m_e c} = 2.4\times10^{-12}\,\text{m}$$



Compton's data (1923) for the scattering of X-rays by graphite

$$\lambda' - \lambda = \lambda_c (1 - \cos\theta)$$

$\theta=0$, $\cos\theta=1$, $\lambda'-\lambda=0$

$\theta=45^o$, $\cos\theta=0.71$, $\lambda'-\lambda=0.39\lambda_c$

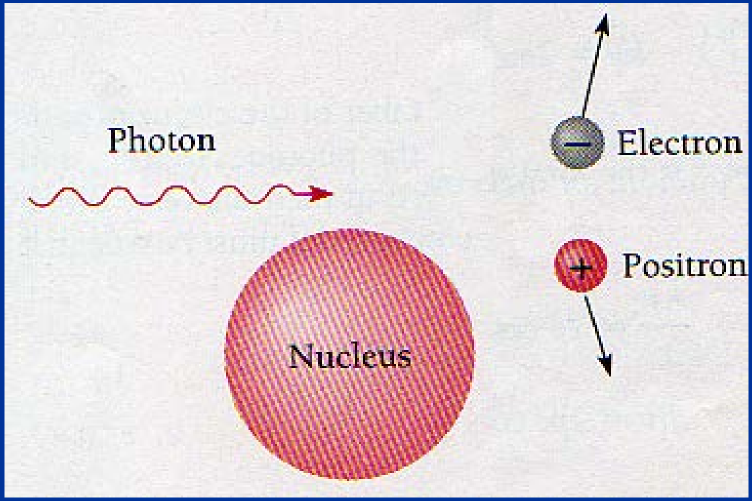
$\theta= 90^o$, $\cos\theta=0$, $\lambda'-\lambda= \lambda_c$

$\theta=135^o$, $\cos\theta=-0.71$, $\lambda'-\lambda=1.71\ \lambda_c$

Pair Production

A photon can give an electron all of its energy (as in photoelectric effect) or only a part (as in Compton effect)

A photon can also be materialized into an electron and a positron (positively charged electron)
— **pair production**



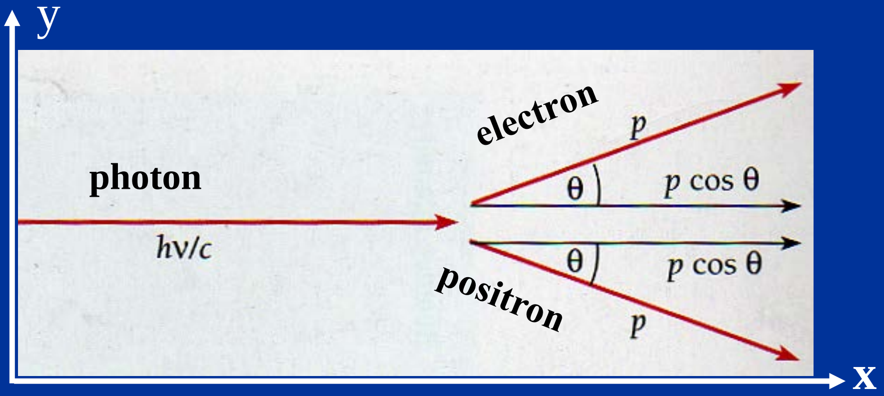
Charge conservation: **(photon) $\mathbf{0}$ = (electron) e^- + (positron) e^+**

Energy conservation: $h\nu = 2\times m_e c^2 + KE_e$

Momentum conservation: $h\nu/c = p_{elect.} + p_{posit} + \underset{\uparrow}{P}_{nucleus}$

Needs involvement of nucleus

Why the nucleus has to be involved?



Momentum along x-axis: $h\nu/c = 2p\cdot\cos\theta \Rightarrow h\nu = 2pc\cdot\cos\theta$

Energy: $h\nu = 2\sqrt{m_e^2c^4 + p^2c^2}$

$$2pc\cdot\cos\theta = 2\sqrt{m_e^2c^4 + p^2c^2}$$

$$\cos\theta = \frac{\sqrt{m_e^2c^4 + p^2c^2}}{pc} = \sqrt{m_e^2/p^2 + 1} > 1$$

There is not such a θ.

Energy and momentum can not both be conserved if pair production were to occur in empty space, so it dose not occur there.

$2 \times m_e c^2$ = 1.02 MeV, hence pair production requires a photon energy at least 1.02 MeV, corresponding to the wavelength 1.2×10^{-12} m — **γ-rays**

The inverse of pair production (pair annihilation):

$$e^+ + e^- \rightarrow \gamma + \gamma$$

Both electron and positron vanish simultaneously, with the lost mass becoming energy in the form of two **γ-rays** photons.

Unlike pair production, **no nucleus or other particle is needed for pair annihilation.**

X-ray

Roentgen

Photoelectric effect shows that photons of light can transfer energy to electrons.

Is the inverse process possible?

The answer is yes. It was actually discovered before the works of Plank and Einstein, but was not very well understood at the time it was found.

Wilhelm Konrad Roentgen, 1895

When very fast electrons hit on matter, a highly penetrating radiation of unknown nature was generated, which was named as **x-ray**.

Some properties of x-ray

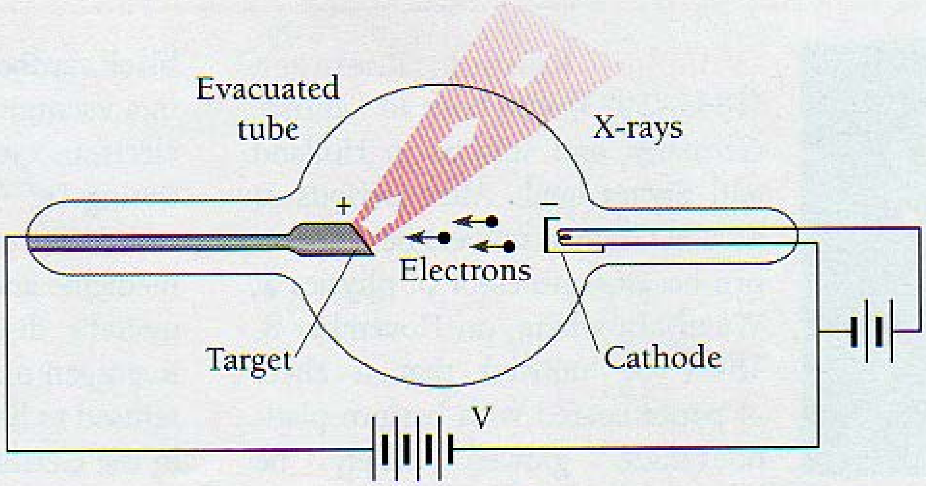
•travel in straight light

•unaffected by electric and magnetic fields

• readily pass through opaque materials to cause phosphors to glow and to expose photographic plate

An X-ray picture taken by Röntgen

Generation of X-ray



An x-ray tube. The higher the accelerating voltage *V*, the faster the electrons and the shorter the wavelengths of the x-rays.

The operating voltage of this modern x-ray tube is 150 kV. Circulating oil carries heat away from the target and releases it to the outside air through a heat exchanger.

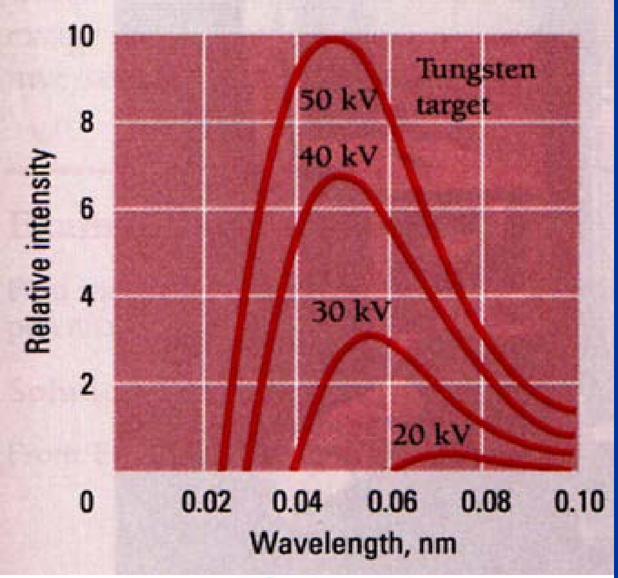
X-ray was soon recognize as electromagnetic waves

Accelerated electric charges will radiate electromagnetic waves
—— Maxwell’s electromagnetic theory.

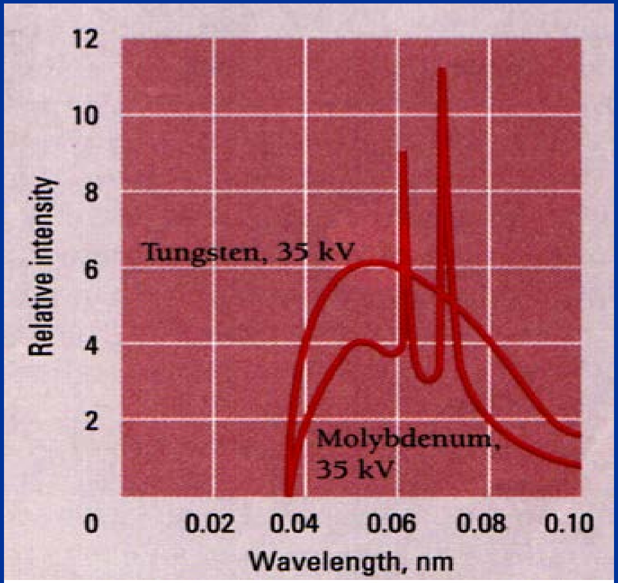
Fast moving electron suddenly brought to rest is certainly accelerated.

So, the generation of x-ray was understandable by classic physics.

But the theory and experiment did not agree very well in certain important respects.



X-ray spectra of tungsten at various accelerating potentials.



X-ray spectra of tungsten and molybdenum at 35 kV accelerating potential.

Two things do not go along with the classic theory

1. Intensity peaks occurred at certain peaks can not be explained.
2. X-rays produced at a given potential V vary in wavelength, but there is a minimum wavelength λ_{min}. Why?

   Increasing V decreases λ_{min}. At a particular V, λ_{min} is the same for both targets.

   It was found experimentally: $$\lambda_{\mathbf{min}} = \frac{1.24\times10^{-6}}{V}$$ (V·m)

How does the quantum theory of radiation explain?

Most of electrons gradually lose their energy by many glancing collisions and turn the energy into heats.

A few electrons lose most or all of their energy in single collision —— this is the energy that becomes x-ray

Similar to the photoelectric effect, we can write the energy conservation of the striking electron and the released x-ray photon as:

$$h\nu_{max} = K + \phi$$

frequency of x-ray

kinetic energy of electron

work function

$K = V\cdot e >> \phi$ so $h\nu_{max} \approx K = V\cdot e$ $\quad \nu_{max} = V\cdot e/h$

~ keV

~ 2-5eV

$$\lambda_{min} = c / \nu_{max} = hc / Ve = \frac{1.24\times 10^{-6}}{V}$$

X-ray diffraction

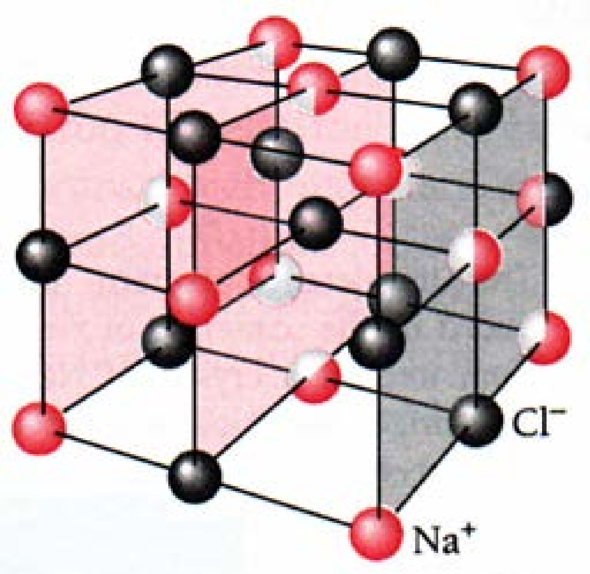
X-ray is an electromagnetic wave (light), so it should have the common properties that other waves have — **interference and diffraction**

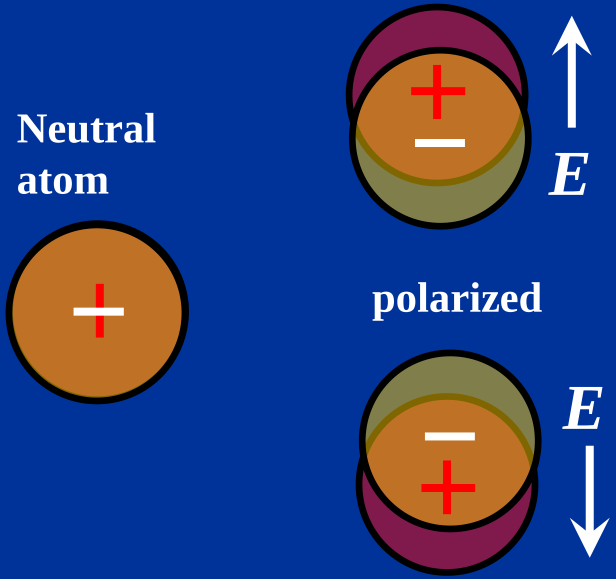
However, the wavelengths of x-rays are so short (~0.1nm) that it is very difficult to find a diffraction grating, which has the line spacing similar to the wavelength of x-ray.

In 1912, Laue realized that the wavelengths of x-ray were comparable to atomic spacing in crystals and proposed to use crystal for x-ray diffraction.

Max von Laue

A crystal consists of a regular array of atoms





Neutral atom

polarized

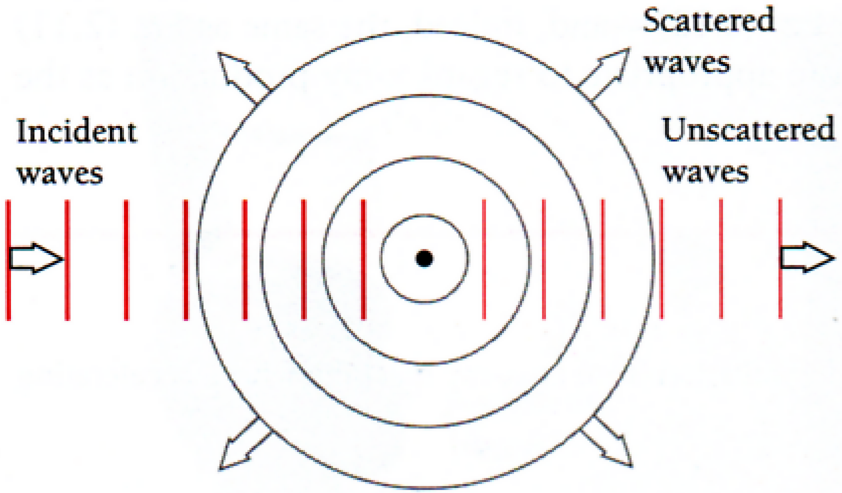
The mechanism of scattering

In a constant electric field, an atom will become polarized (separation of negative and positive centre), forming an electric dipole.

In the alternating electric field of electromagnetic wave, the polarization direction of the dipole will change at the same frequency as the incoming electromagnetic wave *v*.

The oscillating dipole will in turn radiate electromagnetic waves of frequency *v* and the secondary radiation will go out in all direction.

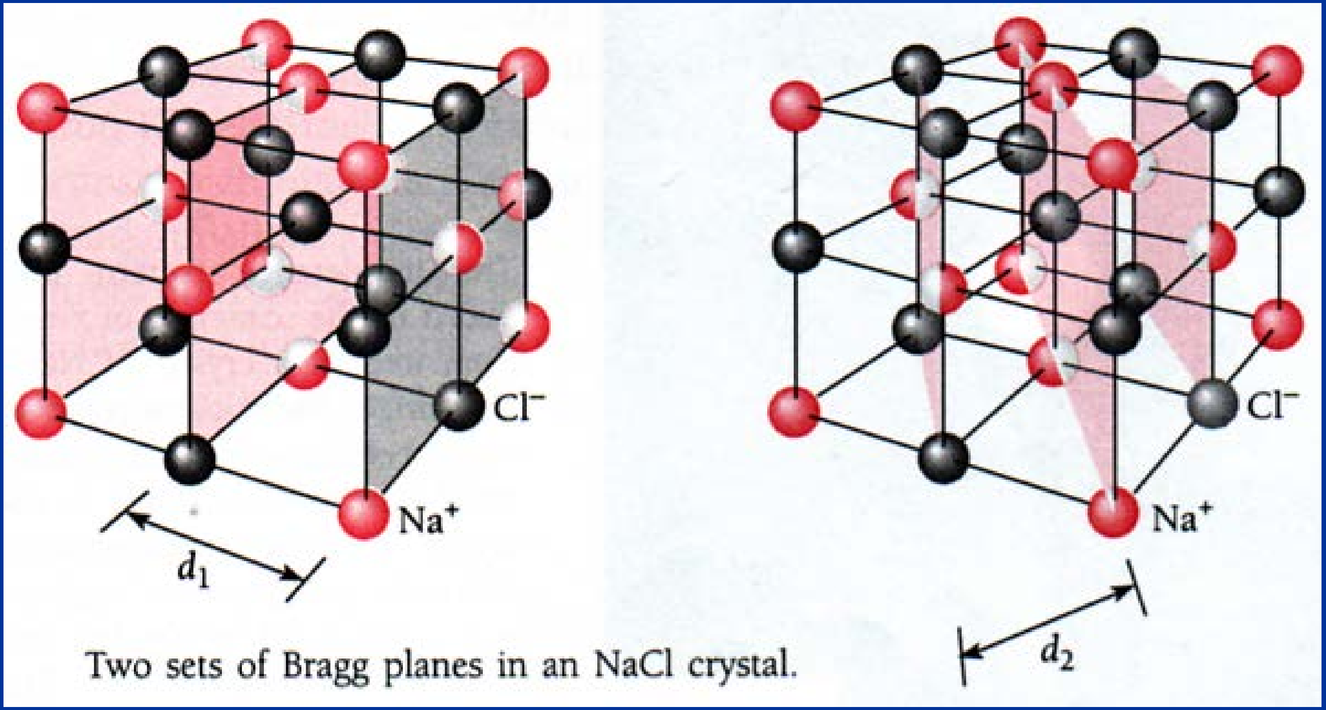
Atoms absorb incoming plane waves and re-emit spherical waves of the same frequency, which will travel in all the directions.



The scattering of electromagnetic radiation by a group of atoms. Incident plane waves are reemitted as spherical waves.

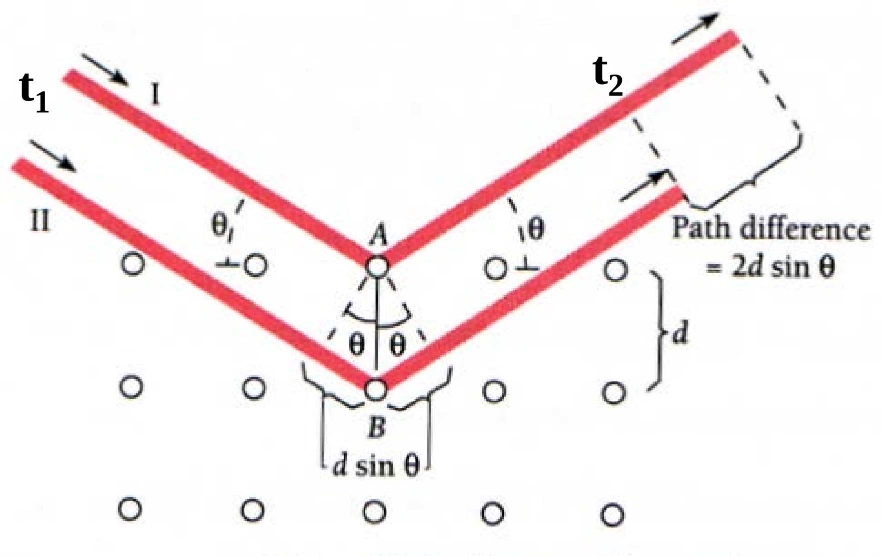
The re-emitted waves from different atoms in the crystal will obey the principle of the superposition. Owing to the regular arrangements of atoms in the crystal, in some directions constructive interference will be observed, while in other directions, destructive interference will be observed.

We can imagine that the 3-dimensinal atomic lattice in a crystal is built up by a set of parallel planes of equal spacing. You can choose another set of parallel planes to build the lattice, but the spacing of the planes will be different. Therefore the inter-plane spacing, *d*, is a characteristic parameter for a particular set of parallel planes. Such planes are called **Bragg planes.**



Two sets of Bragg planes in an NaCl crystal.

We can analyze x-ray scattering according to Bragg planes.



X-ray scattering from a cubic crystal.

Constructive interference occurs when:

- scattered rays are parallel to each other, with the common scattering angle equal to the incident angle θ
- the path difference of scattered rays, ***ΔL*****,** equals the integral multiples of incident x-ray wavelength, i.e. *nλ*.

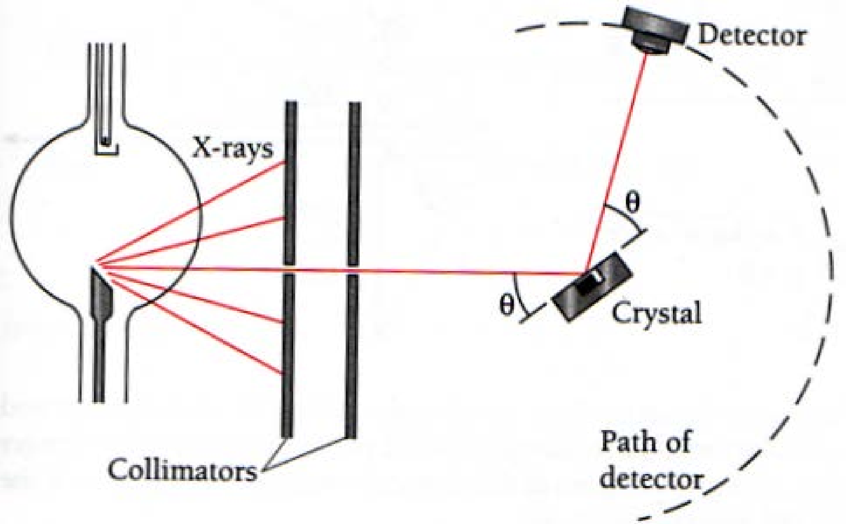
The path difference for the ray I (scattered by the atoms in first plane) and the ray II (scattered by the atoms in the second plane) is:

$$\Delta L = 2d \cdot \sin\theta$$

So, constructive interference happens, when

$$2d \cdot \sin\theta = n\lambda \quad (n = 1, 2.3 \ldots.)$$

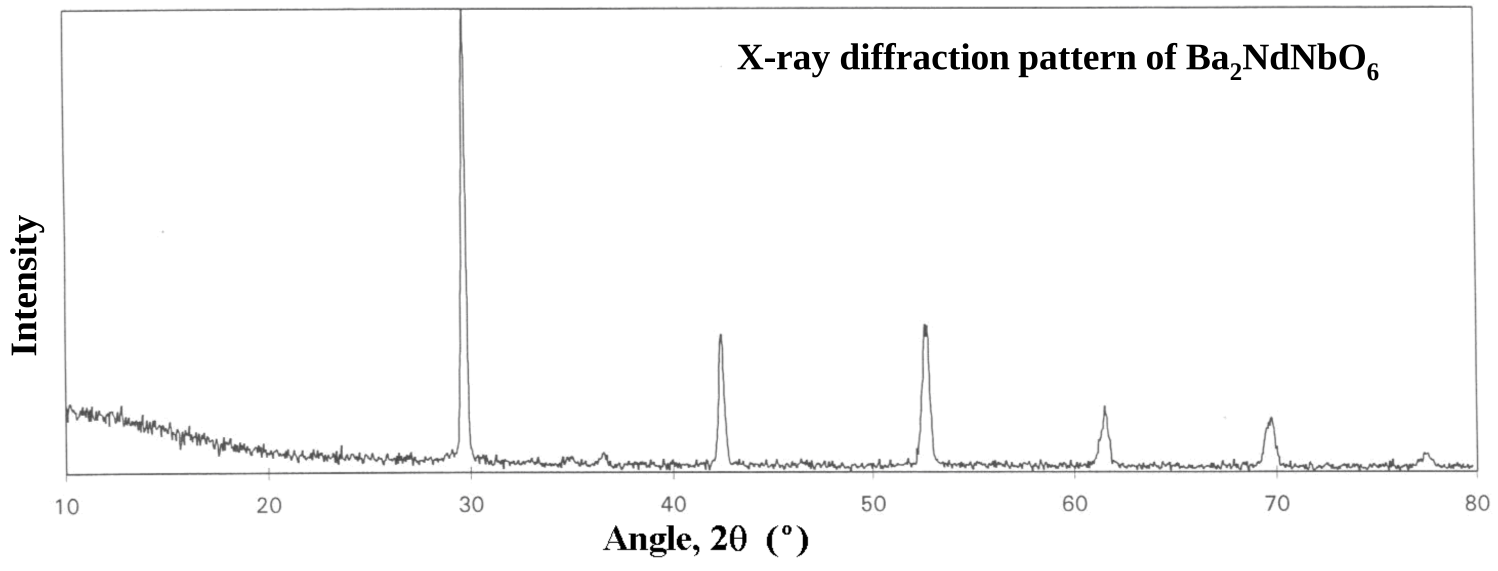
X-ray spectrometer



$$2d \cdot \sin\theta = n\lambda$$

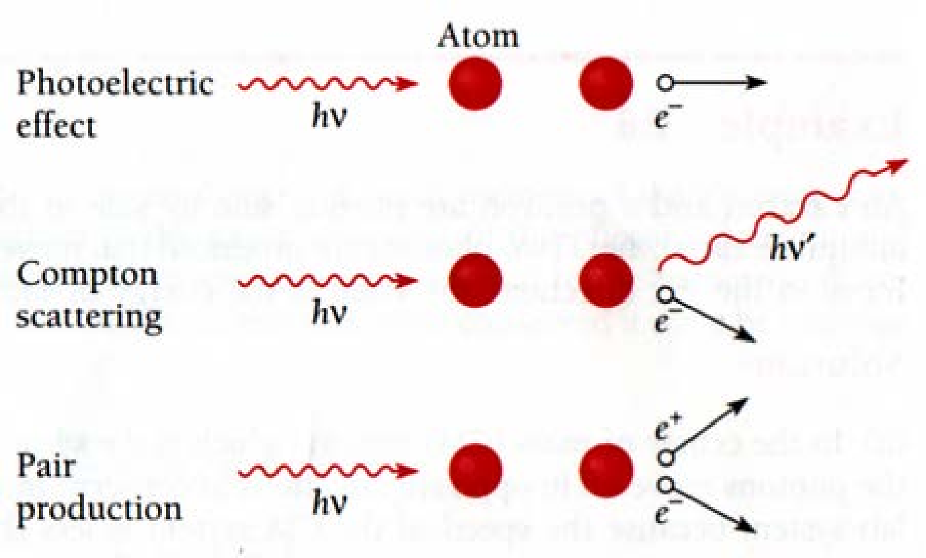
If the spacing ***d*** is known, the x-ray wavelength, λ, can be calculated.

For x-ray, λ falls between 0.01–10nm



Photon Absorption

Photons of high-energy radiations (x-ray & γ-ray) interact with matters in three chief ways



Importance of energy loss mechanism

photon energy low

photon energy high

photoelectric

Compton
(10s keV)

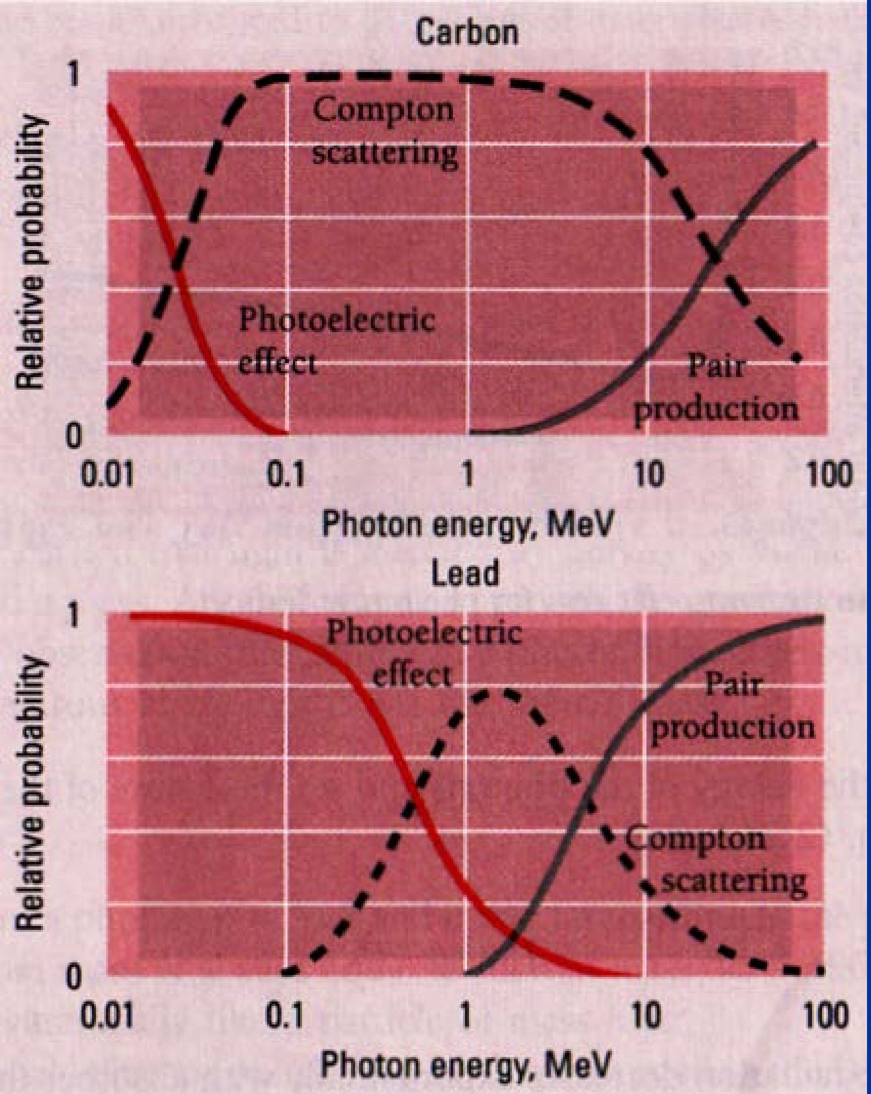
pair-production
(>1.02 MeV)

heavier absorber

Remains significant for higher energy photons

Not happen until 1 MeV

Take over at lower energy (4Mev for heaviest, 10 MeV for lighter ones)



The relative probabilities of photoelectric effect, Compton scattering, and pair production as functions of photon energy in carbon (a light element) and lead (a heavy element).

Linear Absorption

The intensity of x- or γ-ray beam is defined as the rate at which it transports energy per unit cross-sectional area o the beam.

The fractional energy – *dI/I* lost by the beam in passing a thickness *dx* of a matter is found to be proportional to *dx*, i.e.

$$\boxed{-\frac{dI}{I} = \mu \, dx}$$

The proportional constant μ is called the **linear attenuation coefficient** and its value depends on the energy of the photons as well as the nature of the matter.

Integrating above equation gives: $\boxed{I = I_0 e^{-\mu \cdot x}}$

We can use the above equation to work out the thickness of absorbing material required to reduce the intensity of x- or γ-ray to a specific value:

$$\boxed{x = \frac{\ln(I_0 / I)}{\mu}}$$