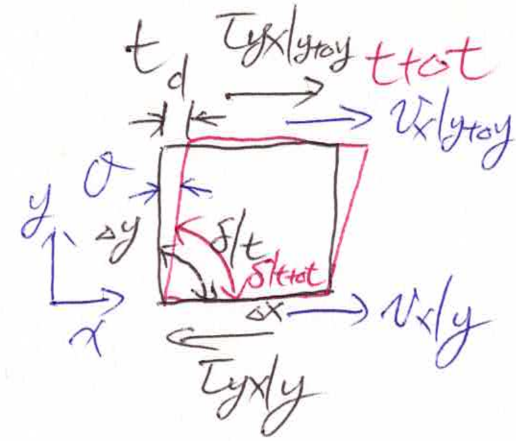
Chap 7.

Newton's law of viscosity

$$\text{Viscosity } (\mu) = \frac{\text{shear stress } (\tau)}{\text{rate of shear strain } \left(\frac{d\delta}{dt}\right)}$$

In a 2-D flow,
the rate of shear strain



$$-\frac{d\delta}{dt} = \lim_{\Delta t, \Delta x, \Delta y \to 0} \frac{-(\delta|_{t+\Delta t} - \delta|_t)}{\Delta t}$$

$$= \lim_{\Delta t, \Delta x, \Delta y \to 0} \frac{\theta}{\Delta t}$$

$$= \lim_{\Delta t, \Delta x, \Delta y \to 0} \frac{(v_x|_{y+\Delta y} - v_x|_y)\Delta t}{\Delta y \Delta t}$$

$$= \frac{dv_x}{dy}$$

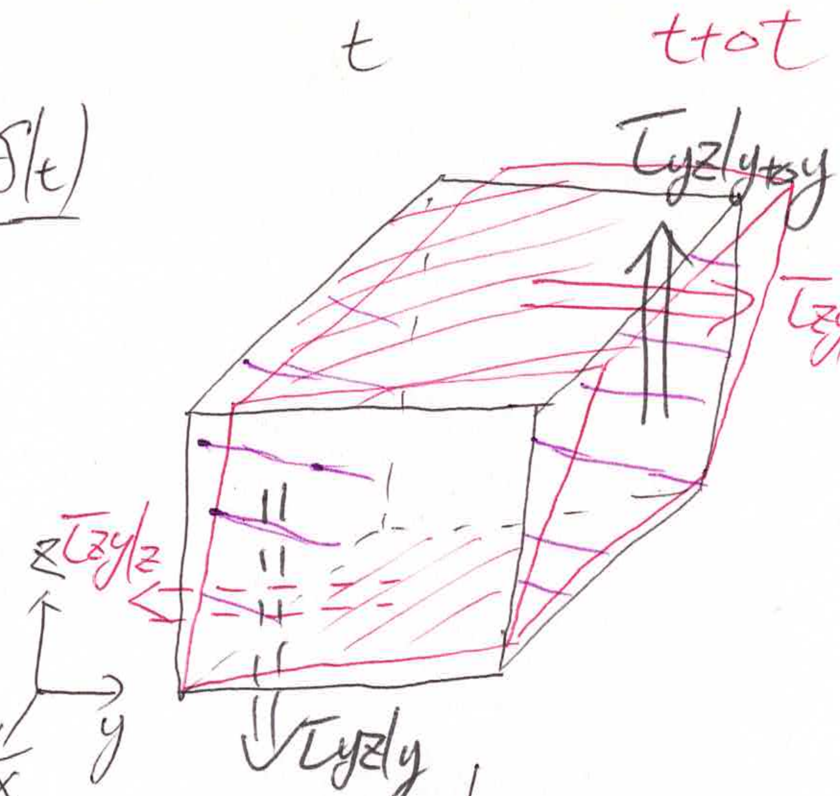
$$\tan\theta = \frac{d}{\Delta y} = \frac{(v_x|_{y+\Delta y} - v_x|_y)\Delta t}{\Delta y}$$

$$\lim_{\Delta \to 0} \tan\Delta = \Delta$$

$$\Rightarrow \mu = \frac{\tau_{yx}}{\frac{dv_x}{dy}} \quad \text{or} \quad \tau_{yx} = \mu\left(\frac{dv_x}{dy}\right) \quad (7\text{-}4)$$

In a 3-D flow, the rate of shear strain 2

on yz plane

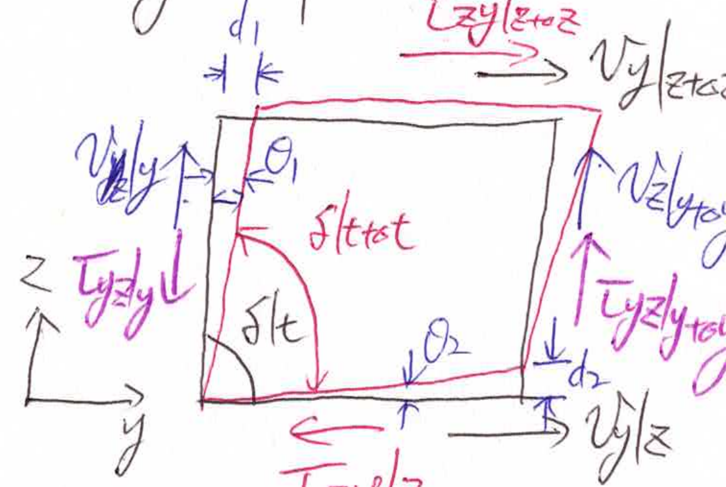


$$-\frac{d\delta}{dt} = \lim_{\Delta t,\Delta x,\Delta y,\Delta z\to 0} \frac{-(\delta|_{t+\Delta t} - \delta|_t)}{\Delta t}$$

$$= \lim_{\Delta t,\Delta x,\Delta y,\Delta z\to 0} \frac{(\theta_1+\theta_2)}{\Delta t}$$

$$= \lim_{\Delta t,\Delta x,\Delta y,\Delta z\to 0} \left[\frac{(v_y|_{z+\Delta z} - v_y|_z)}{\Delta z} + \frac{(v_z|_{y+\Delta y} - v_z|_y)}{\Delta y} \right]$$

yz plane



$$= \frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}$$

rate of shear stain in y direction with shear stress (τ_{zy}) acting on xy plane

rate of shear strain in z direction with shear stress (τ_{yz}) acting on xz plane

$$\tan\theta_1 = \frac{d_1}{\Delta z} = \frac{(v_y|_{z+\Delta z} - v_z|_z)}{\Delta z}$$

$$\tan\theta_2 = \frac{d_2}{\Delta y} = \frac{(v_z|_{y+\Delta y} - v_z|_y)}{\Delta y}$$

$$\lim_{\Delta\to 0} \tan^{-1}\Delta = \Delta$$

Stress 2nd order vector

$$\vec{\vec{\tau}} = \begin{array}{cl} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix} & \begin{array}{l} \text{yz plane} \\ \text{xz plane in cartesian} \\ \text{acting on xy plane} \end{array} \\ \begin{array}{ccc} \cancel{\text{actin}}\ \text{x direction} & y & z \end{array} & \end{array}$$

$\tau_{ij} = \tau_{ji}$ <u>**magnitude**</u> the same

$i \neq j$