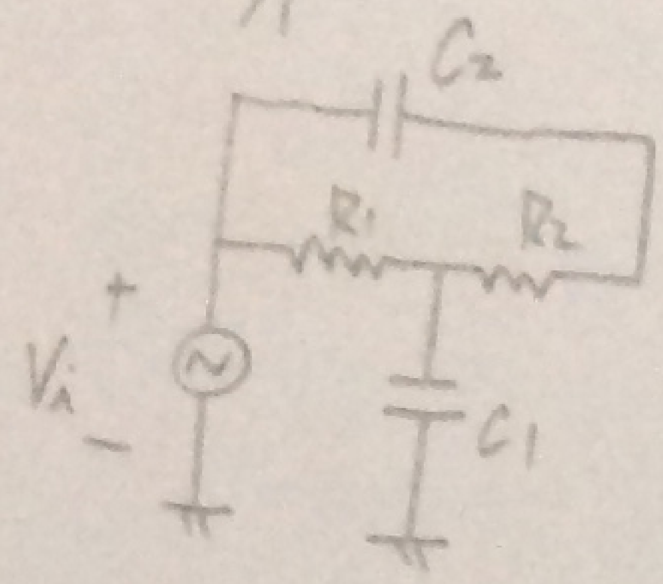
2009.

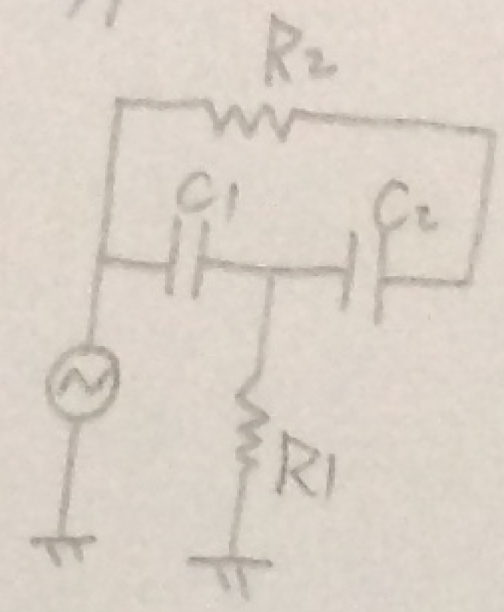
3. ① 試繪出3種 Bridg-T compensator (帶斥)之RC电路

② 說明 Bridg-T compsator 適合何種系統之控制補償設計

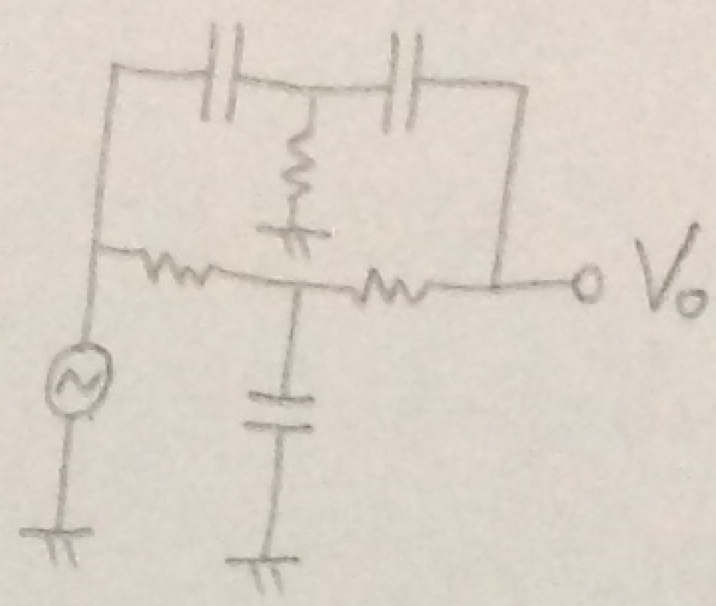
① type I



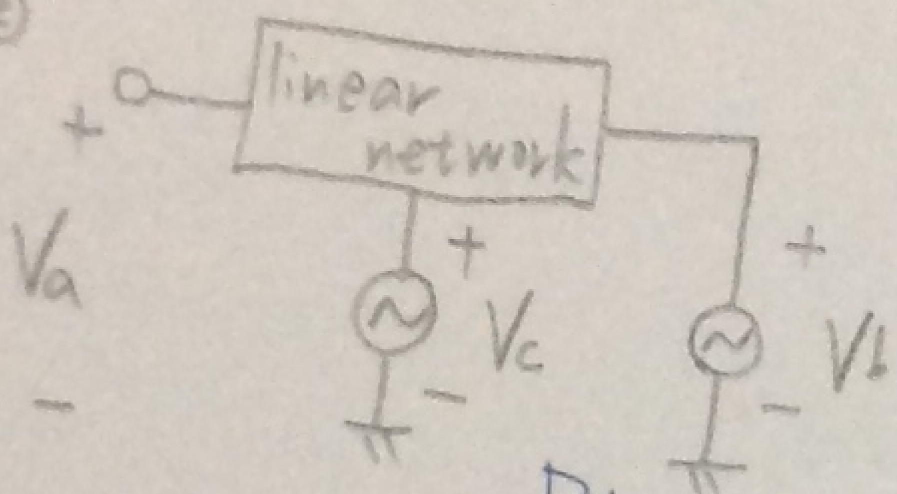
type II.



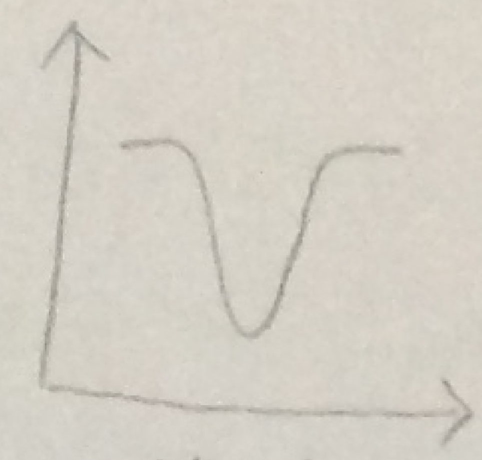
type III.



②

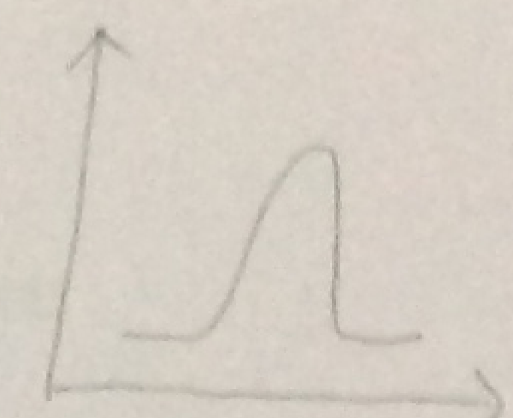


$$\frac{V_a}{V_b}+\frac{V_a}{V_c}=1$$



帶斥

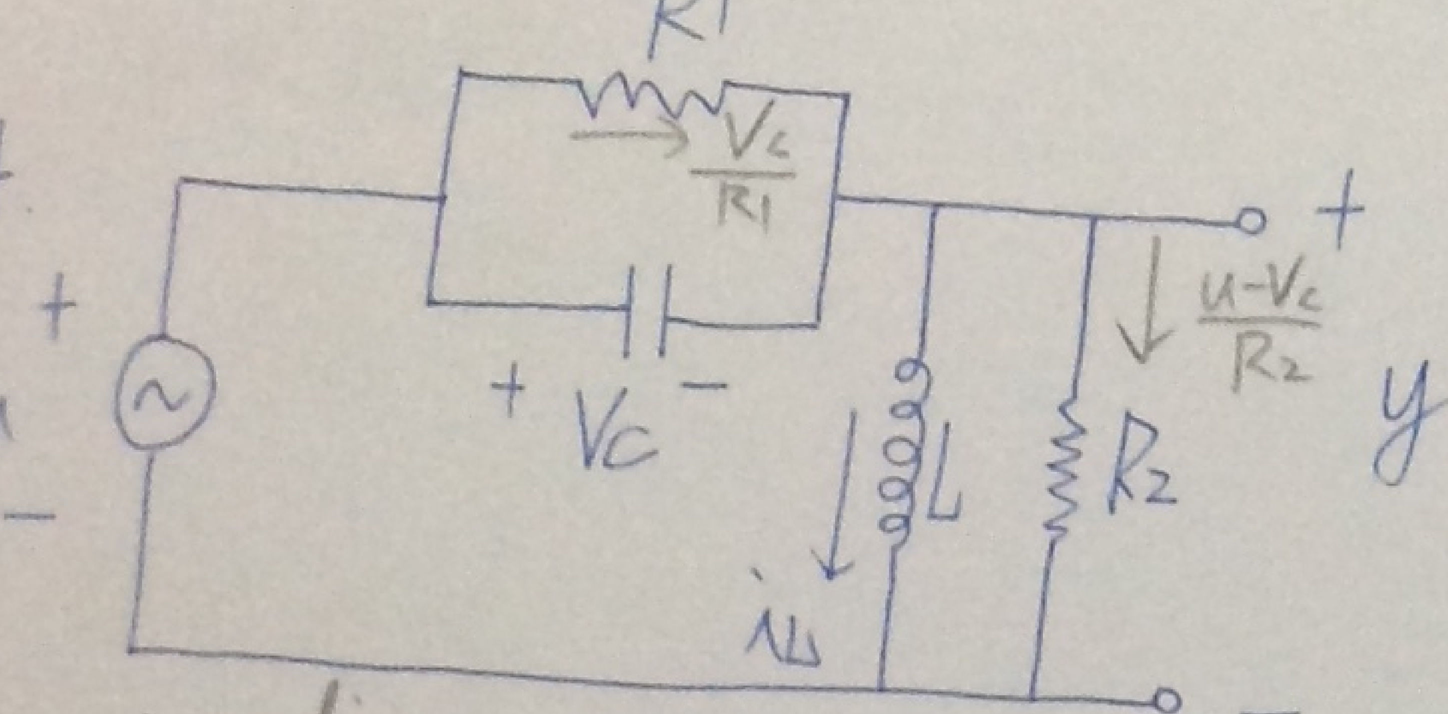
+



帶通

$=1$

4.



① 以 V_c、i_L 為狀態変數，輸入电压 u，輸出电压 y，試寫出狀態空間表示式

② 試討論此系統之可控性与可觀性.

①

$$\begin{cases} L\cdot\dfrac{di_L}{dt}=u-V_c \\ C\cdot\dfrac{dV_c}{dt}=\dfrac{u-V_c}{R_2}+i_L-\dfrac{V_c}{R_1}\end{cases}$$

$$\Rightarrow \begin{cases} \dfrac{di_L}{dt}=\dfrac{u}{L}-\dfrac{V_c}{L} \\ \dfrac{dV_c}{dt}=\dfrac{u-V_c}{CR_2}+\dfrac{i_L}{C}-\dfrac{V_c}{CR_1}\end{cases}$$

$$y=u-V_c$$

$$\Rightarrow \begin{bmatrix}\dfrac{di_L}{dt}\\ \dfrac{dV_c}{dt}\end{bmatrix}=\underbrace{\begin{bmatrix}0 & -\dfrac{1}{L}\\ \dfrac{1}{C} & -\left(\dfrac{1}{CR_1}+\dfrac{1}{CR_2}\right)\end{bmatrix}}_{A}\begin{bmatrix}i_L\\ V_c\end{bmatrix}+\underbrace{\begin{bmatrix}\dfrac{1}{L}\\ \dfrac{1}{CR_2}\end{bmatrix}}_{B}u$$

$$y=\underbrace{\begin{bmatrix}0 & -1\end{bmatrix}}_{C}\begin{bmatrix}i_L\\ V_c\end{bmatrix}+\underbrace{[1]}_{D}u$$

② 可控性 $[B\ \ AB\ \cdots]=$ full rank

$$\det\begin{bmatrix}\dfrac{1}{L} & -\dfrac{1}{LCR_2}\\ \dfrac{1}{CR_2} & \dfrac{1}{LC}-\dfrac{1}{C^2R_2}\left(\dfrac{1}{R_1}+\dfrac{1}{R_2}\right)\end{bmatrix}\neq 0$$

∴ 可控

可觀性：$\begin{bmatrix}C\\ CA\\ \vdots\end{bmatrix}=$ full rank

$$\det\begin{bmatrix}0 & -1\\ -\dfrac{1}{C} & \dfrac{1}{CR_1}+\dfrac{1}{CR_2}\end{bmatrix}\neq 0$$

∴ 可觀

5. $\dot{x} = \overset{A}{\begin{bmatrix} 1 & 3 & 4 \\ 2 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}} x + \overset{B}{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} u$

$y = \overset{C}{[1 \quad 4 \quad 0]} x$

試討論此系統之
① 穩定性
② 可控性
③ 可觀性
④ 將此系統寫成 controllable canonical form

① 穩定性，求其特徵值：

$$sI - A = \begin{bmatrix} s-1 & -3 & -4 \\ -2 & s-1 & -1 \\ -3 & -2 & s-1 \end{bmatrix}$$

$$\det(sI-A) = (s-1)^3 - 9 - 16 - 6(s-1) - 2(s-1) - 12(s-1) = s^3 - 3s^2 - 17s - 6 = (s-6)(s^2+3s+1)$$

⇒ 有根在右半平面，∴ unstable

② 可控性：

$[B \quad AB \quad \cdots]$，$\det[B \quad AB \quad \cdots] = \det\begin{bmatrix} 1 & 5 & 30 \\ 0 & 3 & 17 \\ 1 & 4 & 25 \end{bmatrix} = 2 \neq 0$ ⇒ ∴可控

③ 可觀性：

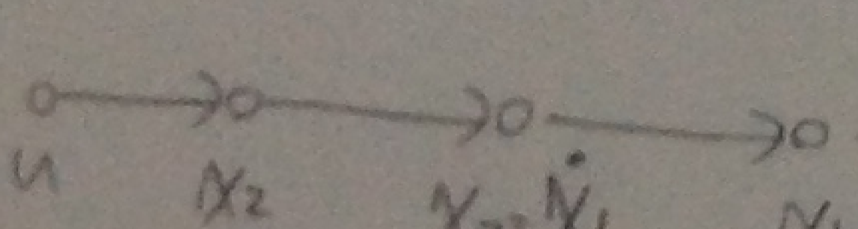
$\begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix}$，$\det\begin{bmatrix} C \\ CA \\ \vdots \end{bmatrix} = \det\begin{bmatrix} 1 & 4 & 0 \\ 9 & 7 & 8 \\ 47 & 50 & 51 \end{bmatrix} = -375 \neq 0$ ⇒ ∴可觀

④

$$C(sI-A)^{-1}B + D = [1 \quad 4 \quad 0] \cdot \begin{bmatrix} s-1 & -3 & -4 \\ -2 & s-1 & -1 \\ -3 & -2 & s-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$G(s) = \frac{Y(s)}{U(s)}$

$$= [1 \quad 4 \quad 0] \cdot \frac{1}{s^3-3s^2-17s-6} \cdot \begin{bmatrix} s^2-2s-1 & 3s+5 & 4s-1 \\ 2s+1 & s^2-2s-11 & s+7 \\ 3s+1 & 2s+7 & s^2-2s-5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$



$\dot{y}$

$$= \frac{1}{s^3-3s^2-17s-6} \cdot [s^2+6s+3 \quad 4s^2-5s-39 \quad 8s+27] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \frac{s^2+14s+30}{s^3-3s^2-17s-6} = \frac{\frac{1}{s} + \frac{14}{s^2} + \frac{30}{s^3}}{1 - \frac{3}{s} - 17\cdot\frac{1}{s^2} - 6\cdot\frac{1}{s^3}} = \frac{\frac{1}{s} + \frac{14}{s^2} + \frac{30}{s^3}}{1 - \left[\frac{3}{s} + \frac{17}{s^2} + \frac{6}{s^3}\right]}$$

$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & 17 & 3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$

$y = [30 \quad 14 \quad 1]$

6. $$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\underbrace{\begin{bmatrix}0&1&-1\\0&1&1\\1&-3&2\end{bmatrix}}_{A}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\underbrace{\begin{bmatrix}0\\0\\1\end{bmatrix}}_{B}u$$

試利用狀態回授法，將系統之極点置放於 -1，-2，-3

$$(sI-A)=\begin{bmatrix}s&-1&1\\0&s-1&-1\\-1&3&s-2\end{bmatrix}$$

$$\det(sI-A)=s^3\underbrace{-3}_{a_2}s^2\underbrace{+6}_{a_1}s\underbrace{-2}_{a_0}$$

$$S=[B\ \ AB\ \ A^2B]=\begin{bmatrix}0&-1&-1\\0&1&3\\1&2&0\end{bmatrix}$$

$$m=\begin{array}{c}\\ a_2\\ \\ \end{array}\begin{array}{c}\begin{array}{ccc}a_1&a_2&\end{array}\\ \begin{bmatrix}6&-3&1\\-3&1&0\\1&0&0\end{bmatrix}\end{array}$$

$$P=Sm=\begin{bmatrix}0&-1&-1\\0&1&3\\1&2&0\end{bmatrix}\begin{bmatrix}6&-3&1\\-3&1&0\\1&0&0\end{bmatrix}=\begin{bmatrix}2&-1&0\\0&1&0\\0&-1&1\end{bmatrix}$$

$$\underbrace{P^{-1}AP}_{\hat{A}}=\begin{bmatrix}0&1&0\\0&0&1\\2&-6&3\end{bmatrix}\qquad \underbrace{P^{-1}B}_{\hat{B}}=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$\dot{z}=(\hat{A}-\hat{B}k)z+Br$$

$$[\lambda I-(A-Bk)]$$

$$\hat{A}-\hat{B}\hat{k}$$

$$=\begin{bmatrix}0&1&0\\0&0&1\\2&-6&3\end{bmatrix}-\begin{bmatrix}0\\0\\1\end{bmatrix}[\hat{k}_1\ \ \hat{k}_2\ \ \hat{k}_3]$$

$$=\begin{bmatrix}0&1&0\\0&0&1\\2-\hat{k}_1&-6-\hat{k}_2&3-\hat{k}_3\end{bmatrix}$$

$$s^3-(3-\hat{k}_3)s^2-(-6-\hat{k}_2)s-(2-\hat{k}_1)$$

$$=s^3+6s^2+11s+6 \qquad (s-1)(s-2)(s-3)$$

$$\hat{k}_3=9,\ \hat{k}_2=5,\ \hat{k}_1=8,\quad \hat{k}=[8\ \ 5\ \ 9]$$

$$\therefore\ k_1=4,\quad k_2=18,\quad k_3=9$$