

1. (20 points) Differentiate the following functions.

(a) $f(x) = x^4 + 2x^2 + \ln(2x)$

(b) $f(x) = \cos(e^x)$

(c) $f(x) = \frac{1}{\sec x + \tan x}$

(d) $f(x) = \tan^{-1}(x)$

2. (10 points) Give the necessary and sufficient conditions on A and B for the function

$$f(x) = \begin{cases} Ax - B, & x \leq 1 \\ 3x, & 1 < x < 2 \\ Bx^2 - A, & 2 \leq x \end{cases}$$

to be continuous at $x = 1$ but discontinuous at $x = 2$.

3. (20 points) Consider the following function

$$f(x) = \sqrt{|x - 2|}, x \in \mathbb{R}.$$

(a) Is $f(x)$ differentiable? If it is differentiable, find its derivative.

(b) Determine where $f(x)$ is concave up or concave down.

(c) Sketch the graph of $f(x)$.

✓ 4. (10 points) Show that if $f(x)$ is continuous and $0 < f(x) < 1$ for all $x \in [0, 1]$, then there exists at least one $c \in [0, 1]$ for which $f(c) = c$.

5. (10 points) Please calculate the following limits.

(1) $\lim_{x \rightarrow \infty} \frac{\ln x}{x}$

(2) $\lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{\sin x}$

6. (20 points) Let

$$f(x) = \frac{x^3}{x^2 + 1}.$$

(1) Please calculate $f'(x)$ and $f''(x)$.

(2) Is $f(x)$ even or odd?

(3) Does $f(x)$ have asymptotes?

(4) Please sketch the graph of $f(x)$.

7. (10 points) The volume of a cube is increasing at a rate of $10 \text{ cm}^3/\text{min}$. How fast is the surface area increasing when the length of an edge is 30 cm ?