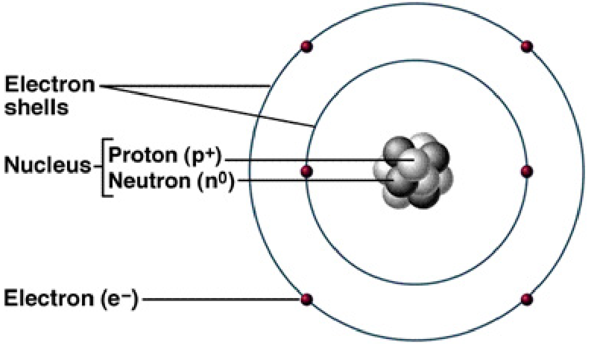
# 電子電工學
# Lecture 2

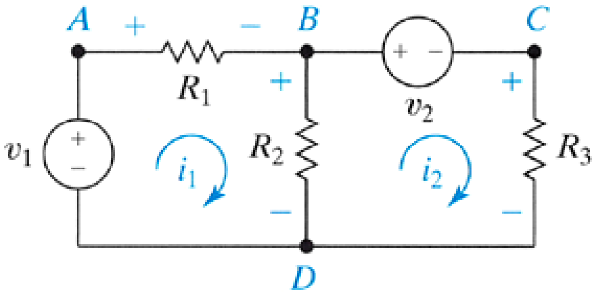
# Recap

- Basic quantities
  - Charge ✓ $Q$
  - Voltage ✓ $V$
  - Current ✓ $I$
  - Power ✓ $P = VI$
- Electrical components
  - Resistor $R$
  - Capacitor $C$
  - Inductor $L$
- Circuit diagrams

Interconnection of components.

# Recap: Electronics to circuits




Free $e^-$ flow

circuits.

**TABLE 7–2**
**Maxwell's Equations**

| Differential Form | Integral Form | Significance |
|---|---|---|
| $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ | $\oint_C \mathbf{E} \cdot d\boldsymbol{\ell} = -\frac{d\Phi}{dt}$ | Faraday's law |
| $\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$ | $\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = I + \int_S \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s}$ | Ampère's circuital law |
| $\nabla \cdot \mathbf{D} = \rho$ | $\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$ | Gauss's law |
| $\nabla \cdot \mathbf{B} = 0$ | $\oint_S \mathbf{B} \cdot d\mathbf{s} = 0$ | No isolated magnetic charge |

# Chapter 3. Circuit Laws and Equivalences

Goal: For each component, find

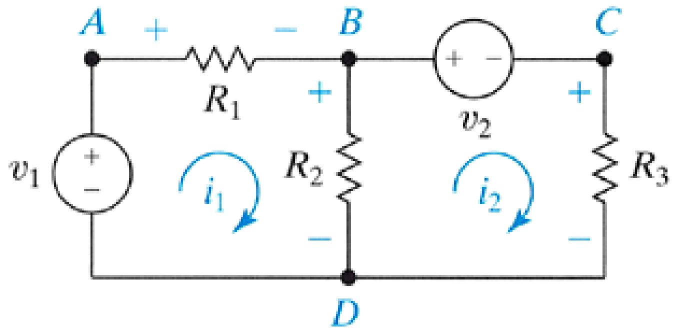
Voltage  V

Current  I

Power   P

Essential laws:

1. Ohm's law
2. Kirchhoff's laws



# Electrical component

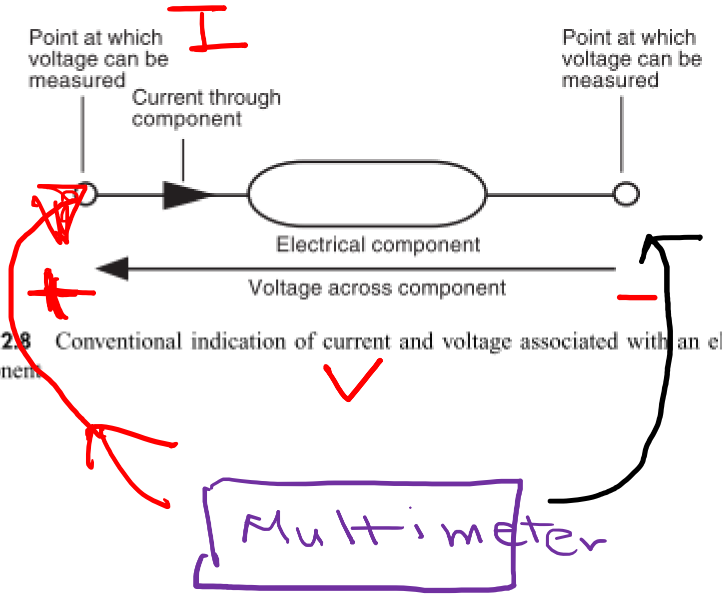
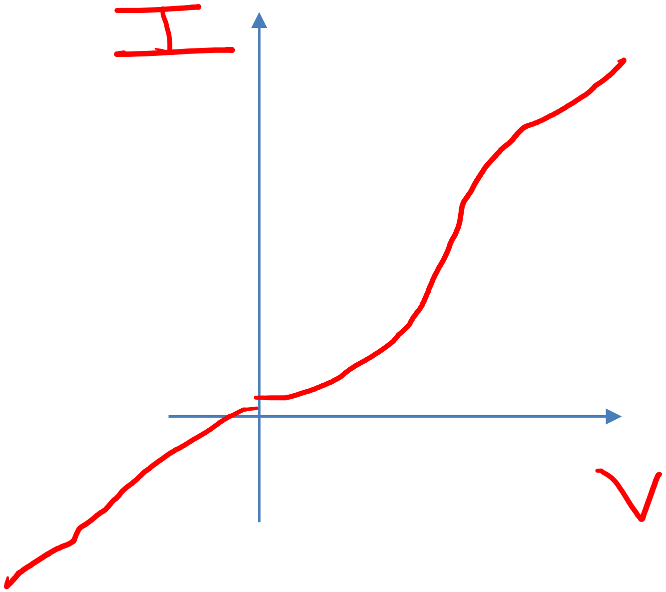



**Figure 2.8** Conventional indication of current and voltage associated with an electrical component




# I-V Characteristic

# Electrical component

L

R : Resistor Ω (ohm)

L : Inductor H (henry)

C : Capacitor F (Fara)

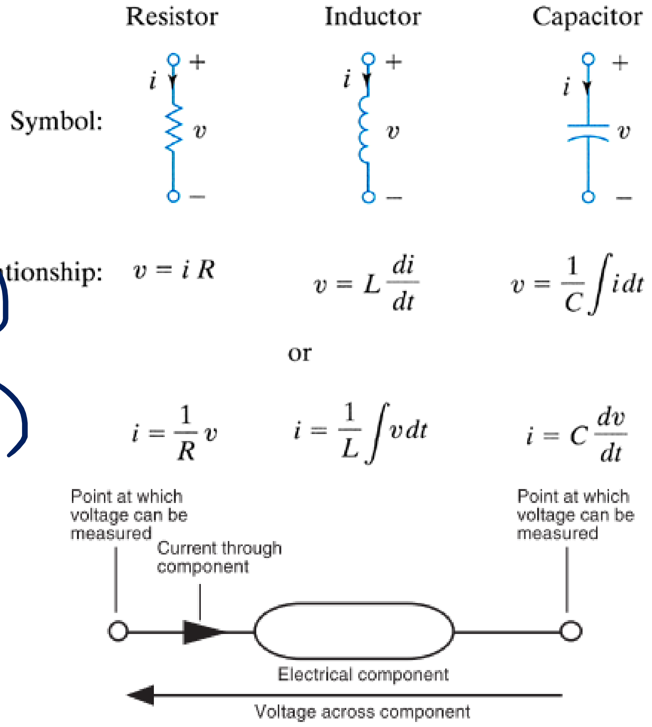


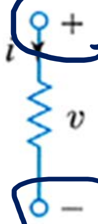

**Figure 2.8** Conventional indication of current and voltage associated with an electrical component

# Resistance

ANSI

US

Resistor




$$\boxed{v = i\,R}$$

Voltage drop

Ohm's Law

R : resistance (Ω)

$$i = \frac{1}{R} v$$

$= Gv$ → conductance

(S) ℧, mho

IEC

Europe

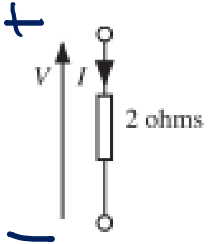



**Figure 3.2** Symbolic representation of the resistor whose voltage~current relation is shown in Figure 3.1

R = 2Ω

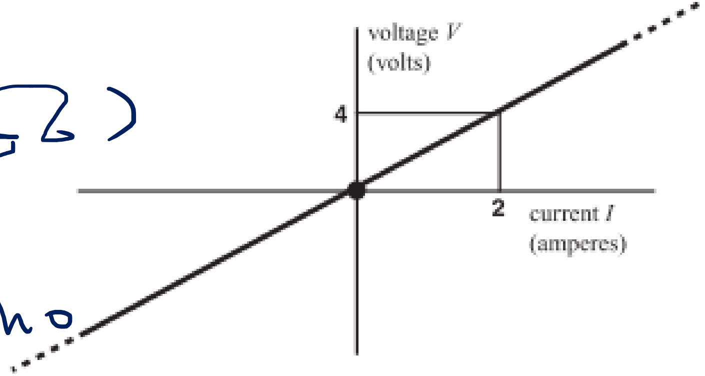
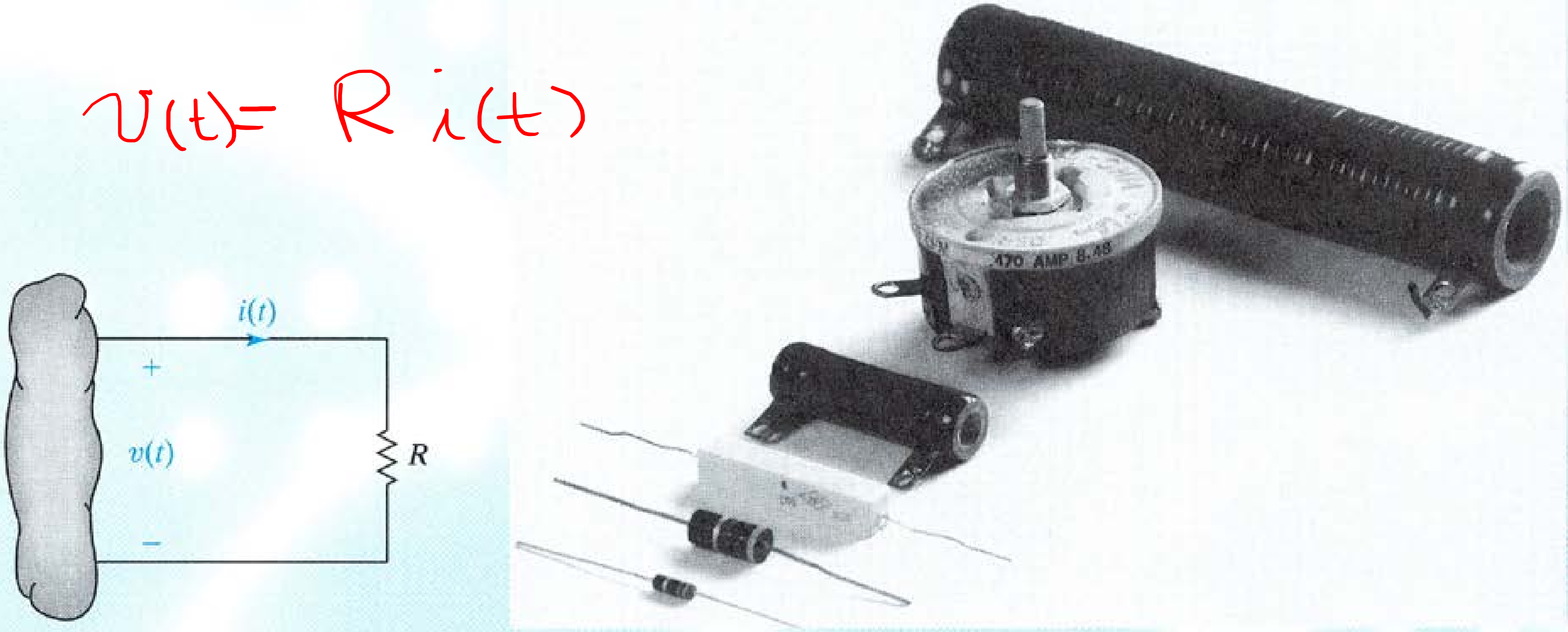



**Figure 3.1** The result of measuring the current through, and the voltage across, a resistor

$v(t) = R\,i(t)$




(a) (b)

FIGURE 2.1 (a) The symbol for a resistor, and (b) some examples of typical carbon or wirewound resistors.

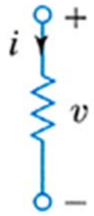
# Resistors

Color

Code

# Conductance

Resistor




$$v = i\,R$$

$$i = \frac{1}{R}v$$

$$i = G\,v$$

Conductance

(℧, mho)

(S, Siemens)

$$R = 2\,\Omega \leftrightarrow G = 0.5\,S$$

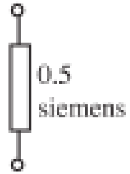



**Figure 3.3** Alternative symbolic representation of the resistor whose voltage~current relation is shown in Figure 3.1

# Current directions

Reference Direction

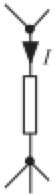



**Figure 3.4** The reference direction for current can be arbitrary, and does not necessarily indicate the actual direction of flow

Actual Direction

(Sign)

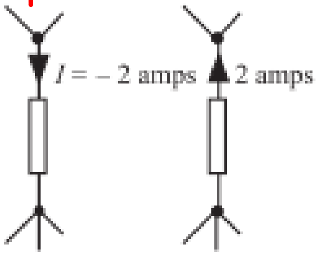
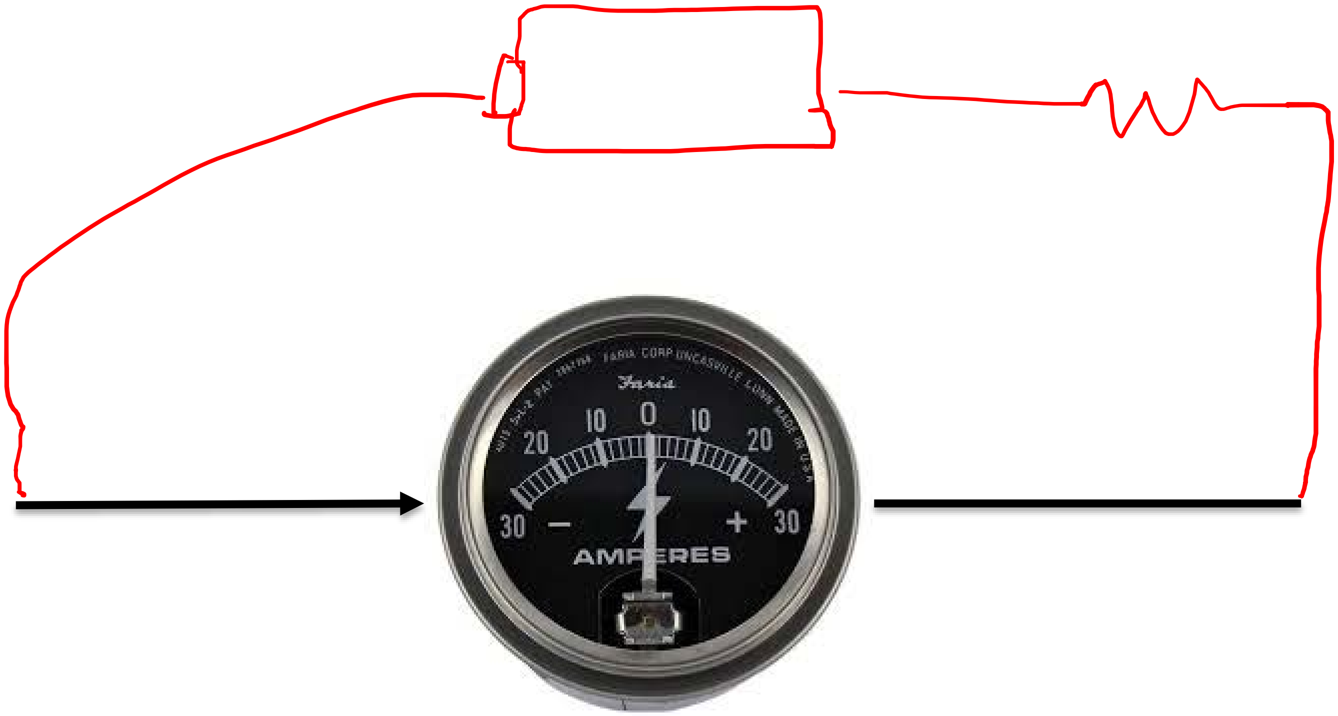



**Figure 3.5** If the value of *I* in Figure 3.4 is negative, that can be represented in either of the two ways shown here

# Current directions



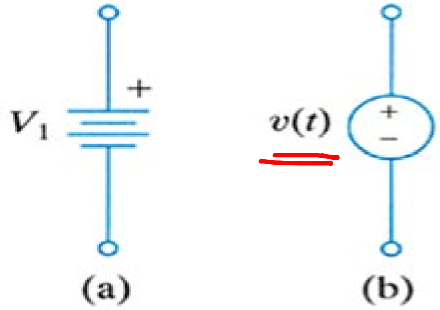
# Voltage source

直流

Direct Current (DC)

Ideal

⇒ Enforce constant voltage.



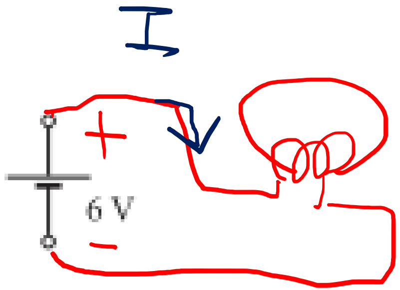




**Figure 3.7** Representation of an ideal voltage source of 6 V

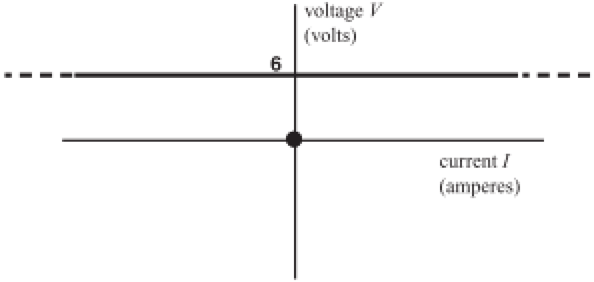



**Figure 3.6** The result of measuring the current through, and the voltage across, an ideal voltage source

# Short circuit

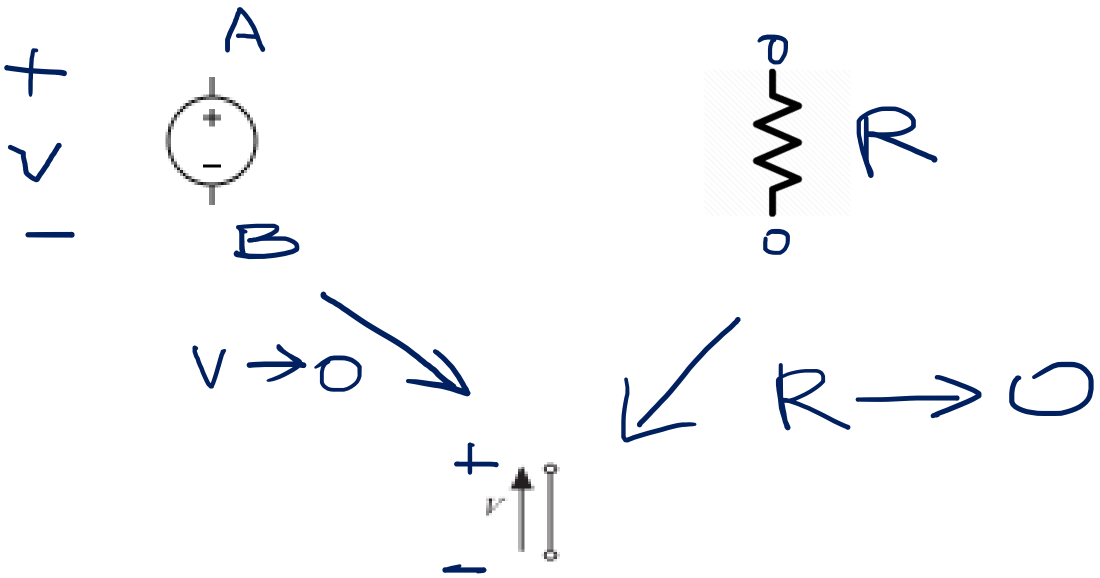



**Figure 3.8** A short-circuit. The voltage $V$ between the terminals is zero whatever the value of the current through it

# Current source

Ideal

Enforce a constant current, irrespective of the voltage across it.

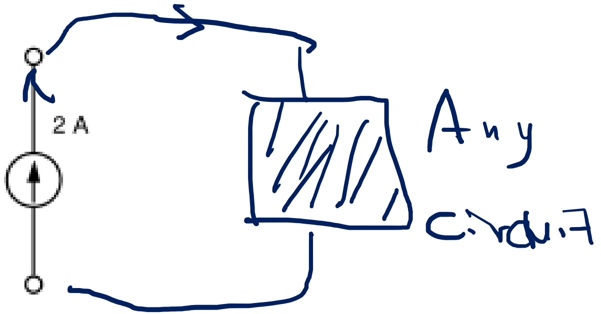



**Figure 3.10** Representation of an ideal current source of 2 A

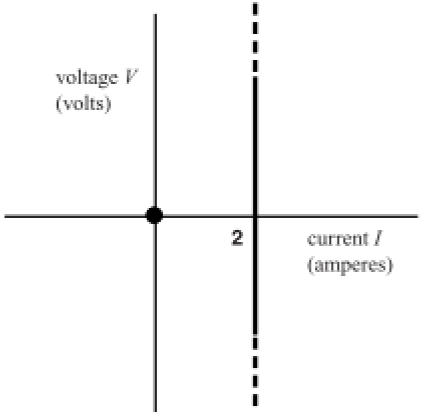



**Figure 3.9** The result of measuring the current through, and the voltage across, an ideal current source

# Open circuit

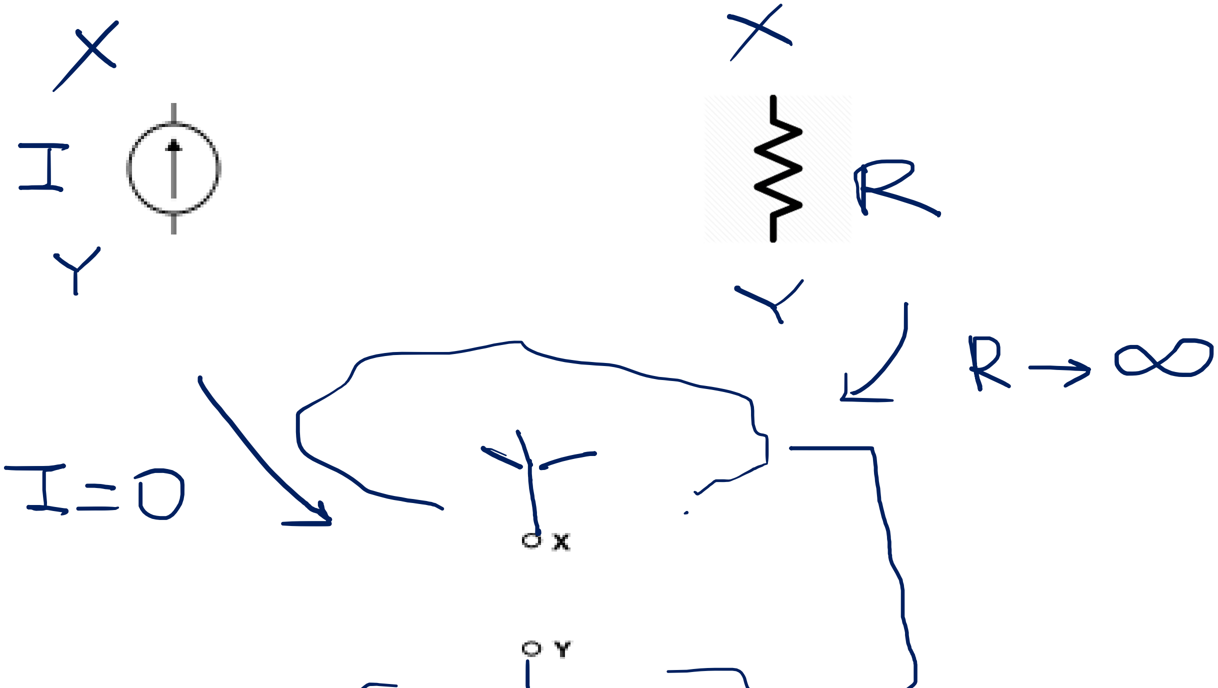
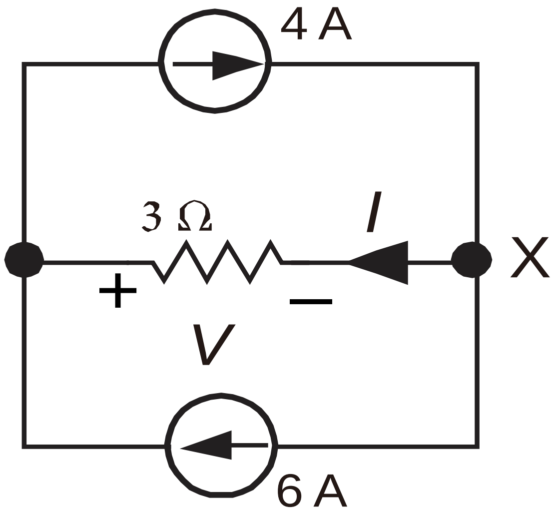



**Figure 3.11** There is an open-circuit between terminals X and Y, through which no current can flow

# Quiz 1

1. For the circuit shown in Fig. 1, find the current $I$ and $V$ across the resistor.
2. Let $i(t) = A\cos(\pi t + \theta)$, what is $\frac{di}{dt}$ ?
3. Evaluate $\int_0^4 e^{-t/2} dt$ .
4. Let $i = \sqrt{-1}$ , simplify $\frac{1+2i}{3+4i}$ to the form of $a+bi$.
5. Evaluate the absolute value $\left|\frac{1+2i}{3+4i}\right|$.

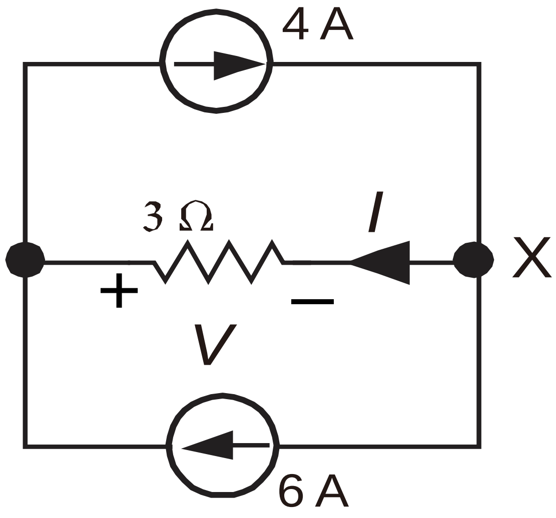


# Quiz 1 Review

1. For the circuit shown in Fig. 1, find the current *I* and *V* across the resistor.

$I = -2$ A

$V = 6$ V



# Quiz 1 Review

2. Let $i(t) = A\cos(\pi t + \theta)$, what is $\frac{di}{dt}$ ?
3. Evaluate $\int_0^4 e^{-t/2} dt$ .

2. $\frac{d\,i(t)}{dt} = -\pi A \sin(\pi t + \theta)$

3. $\int_0^4 e^{-t/2}\, dt = -2 e^{-t/2} \Big|_0^4$

$= -2(e^{-2} - 1)$

$= 2(1 - e^{-2})$

# Quiz 1 Review

4. Let $i = \sqrt{-1}$, simplify $\frac{1+2i}{3+4i}$ to the form of $a+bi$.
5. Evaluate the absolute value $\left|\frac{1+2i}{3+4i}\right|$.

4. $$\frac{1+2i}{3+4i} = \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)} = \frac{11+2i}{25}$$

$$a = 11/25 \qquad b = 2/25$$

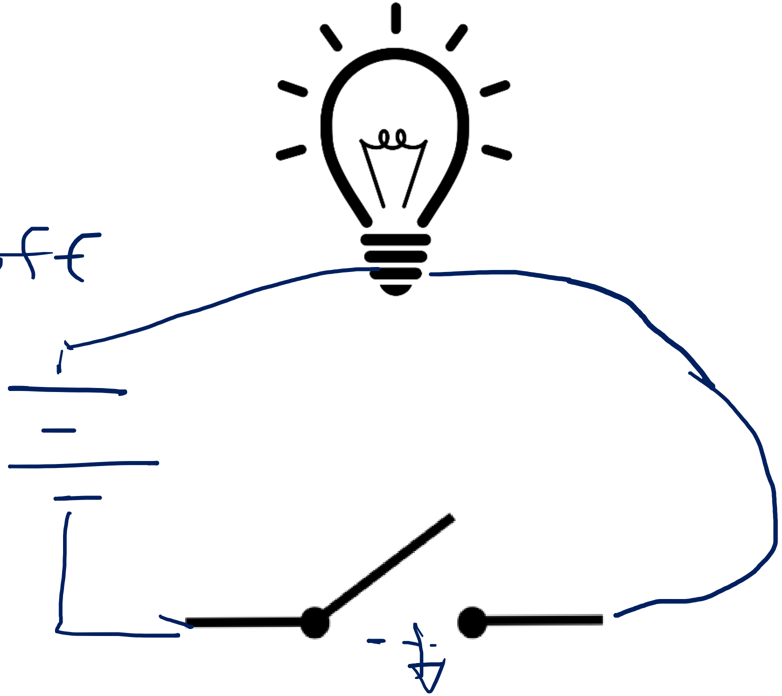
5. $$\left|\frac{1+2i}{3+4i}\right| = \frac{\sqrt{5}}{5} = 1/\sqrt{5}$$

# Switch

Turn-off

Open Switch → ?

Close Switch → ?



Turn-on

# Power

$$P = VI$$

$VI > 0$ Absorb power

(Passive component)

$VI < 0$ Supply power

(Active component)

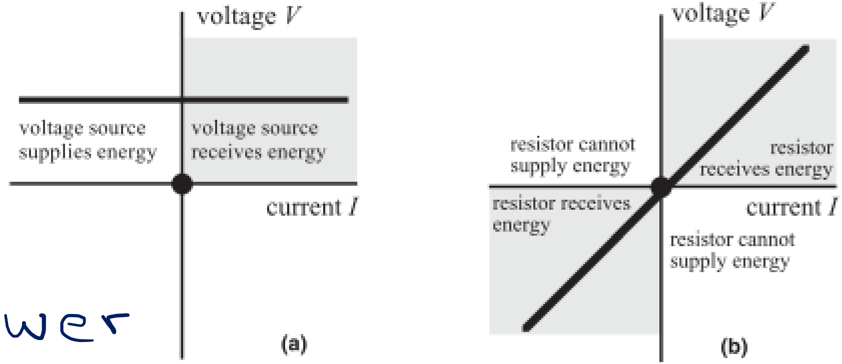



**Figure 3.13** A voltage source can supply energy because the product of $V$ and $I$ can be negative. With a resistor the product of $V$ and $I$ is always positive, so it can only receive energy

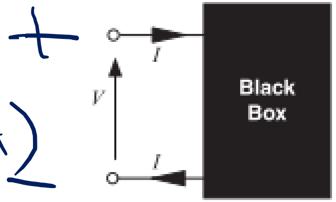



**Figure 3.12** The power supplied to a black box is the product of $V$ and $I$ provided the current $I$ enters at the terminal with the highest voltage (i.e., the positive reference for $V$)

# Kirchhoff's Current Law (KCL)

Interconnection

For any node (junction)

Inward as positive.

$$\sum_{n \in Branch(n)} I_n^{*} = 0$$

$$I_1 + I_2 + I_3 = 0$$

Simplify

Maxwell's

$$\nabla \times H = J + \frac{\partial D}{\partial}$$

H const (Ampere's Law)

Conservative of charge (Mass)

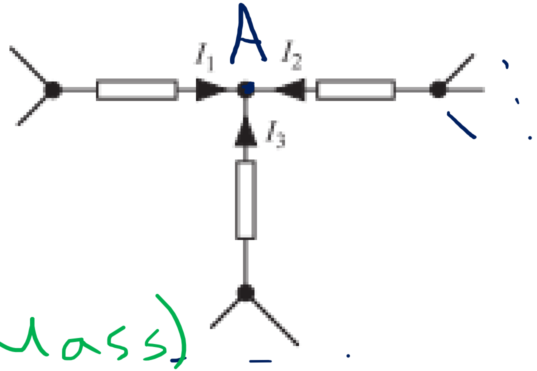



**Figure 3.14** Three resistors are connected to the same terminal

# Kirchhoff's Current Law (KCL)

$I_1 + I_2 + I_3$

$= 3 + 5 + (-8)$

$= 0$ A

$\sum$ Outward $I$ $= \sum$ Inward $I$

$I_3 = I_1 + I_2$

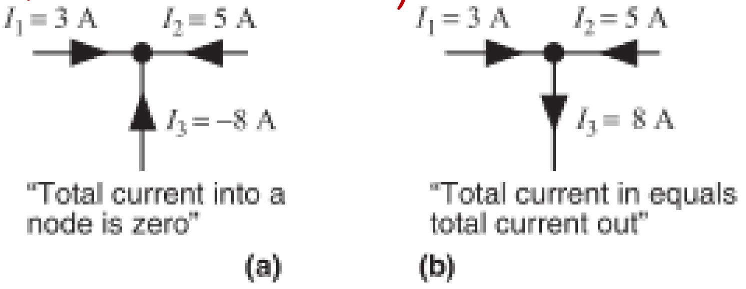



**Figure 3.15** Alternative expressions of Kirchhoff's current law

# Kirchhoff's Voltage Law (KVL)

Loop $\quad a \rightarrow b \rightarrow c \rightarrow d \rightarrow a$

$$V_{ba} + V_{cb} + V_{dc} + V_{ad} = 0$$

$$V_A + V_B + V_X + V_Y = 0$$

$$\sum_{i \in Loop} V_i = 0$$

↑

Voltage drop

Maxwell's

$$\nabla \times E = -\partial B / \partial t$$

Faraday's Law

[Conservative of Energy]

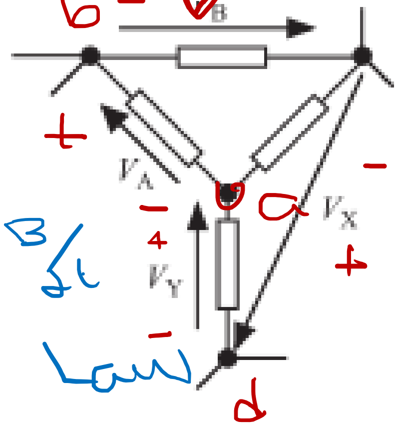



**Figure 3.16** A closed loop is formed by the voltages $V_A$, $V_B$, $V_X$ and $V_Y$

# Kirchhoff's Voltage Law (KVL)

Loop  a – b – c – d – a

KVL: $V_{ba} + V_{cb} + V_{dc} + V_{ad} = 0$

$\Rightarrow V_C + (-V_D) + V_R + (-V_S) = 0$

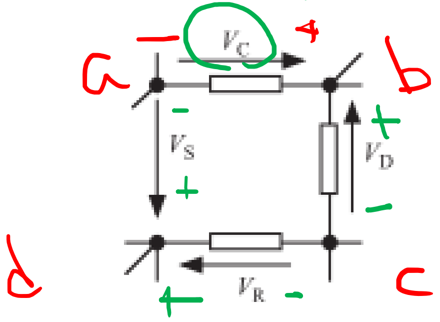
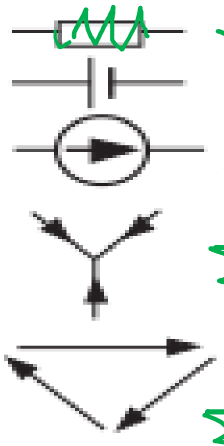



**Figure 3.17** Four voltages forming a closed loop within a circuit

# DC circuits

**Table 3.1** Summary of the relations describing DC circuits



| FEATURES OF A CIRCUIT | SYMBOLIC REPRESENTATION | RELATION |
|---|---|---|
| Components | | Ohm's Law<br>constant V<br>constant I |
| Connection at a node | $\sum I_n = 0$ | Kirchhoff's Current Law (KCL) |
| Connection to form a loop | $\sum V_n = 0$ | Kirchhoff's Voltage Law (KVL) |

I-V curve

# Equivalent circuits

2 resistors → 1 resistor

Same

I-V characteric

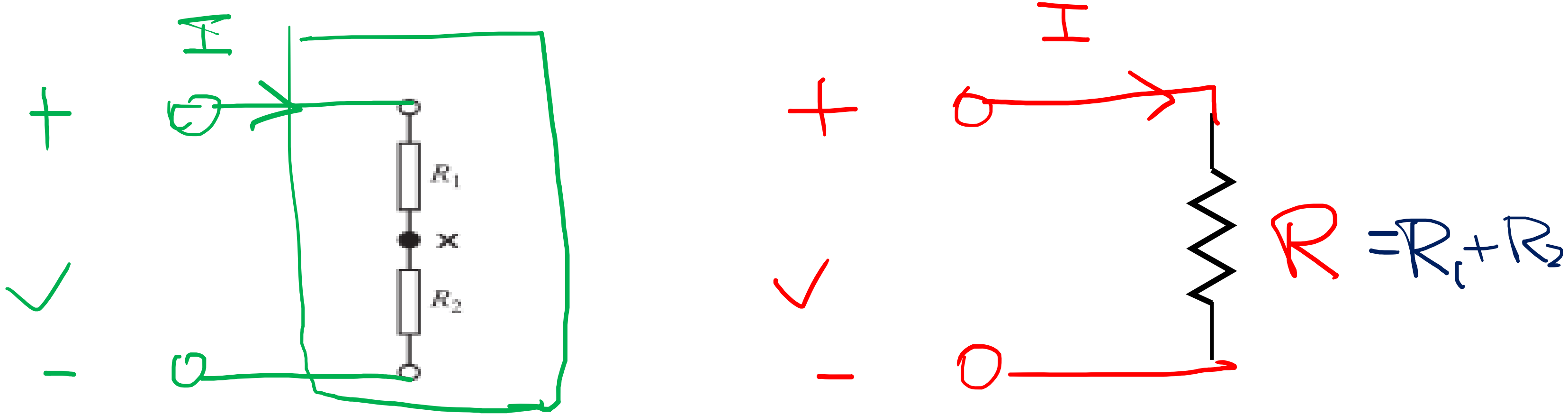



**Figure 3.18** The series connection of two resistors. Note that there is nothing else connected to point X

# Equivalent circuits

Same
I-V
Characteristic

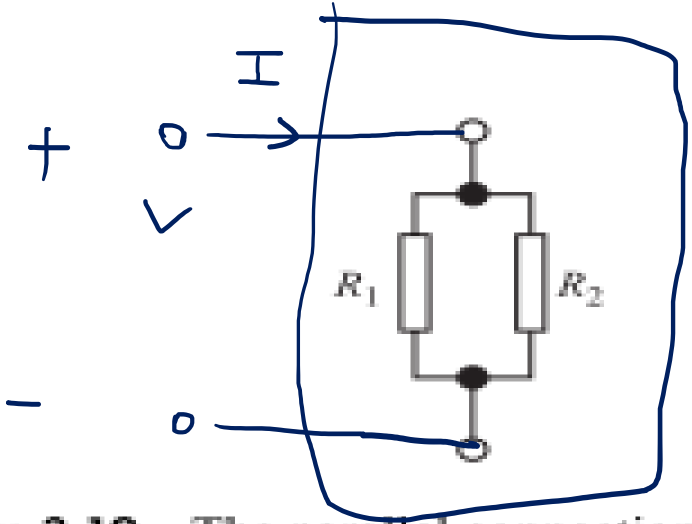
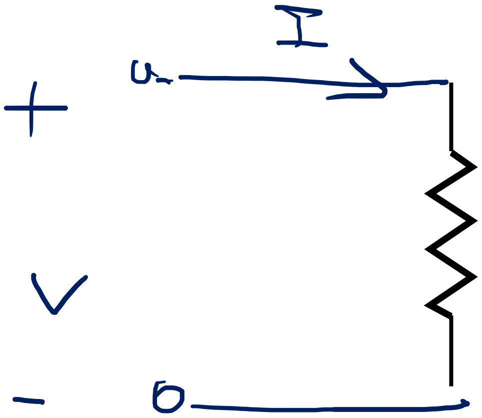



Figure 3.19 The parallel connection of two resistors




$= R_1 // R_2$

$$R = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

# Equivalent circuits

Apply source V

Ohm's Law  $I, I_1, I_2, I_3$

$R_1$ : $V_{R_1} = R_1 I_1$
$R_2$ : $V_{R_2} = R_2 I_2$

$V_{R_1}$ $V_{R_2}$

KCL

node Y: $I = I_1$
node X: $I_1 = I_2$
node Z: $I = I_3$

$\Big\} I = I_1 = I_2 = I_3$

$$\boxed{V = I(R_1 + R_2)}$$

KVL : Loop Z-A-B-Y-X-Z

$$\Rightarrow V - V_{R_1} - V_{R_2} = 0$$

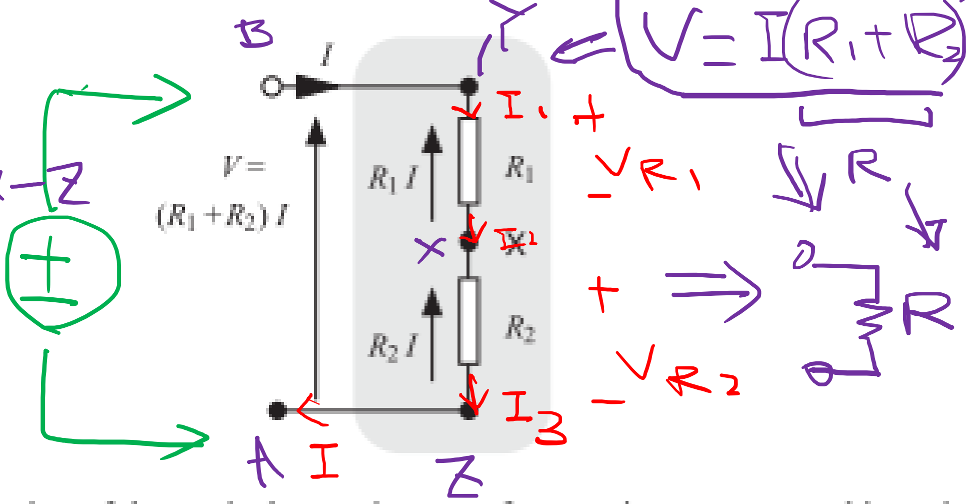



**Figure 3.20** Derivation of the equivalent resistance of two resistors connected in series

# Equivalent circuits

$I = G V$

Ohm's Law

$G = 1/R$

[Apply current src]

KCL @ A

$I = I_1 + I_2$

$\begin{cases} I_1 = G_1 V \\ I_2 = G_2 V \end{cases}$

$= G_1 V + G_2 V$

$\boxed{I = (G_1 + G_2) V}$

$R = R_1 // R_2$

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

$G = G_1 + G_2$

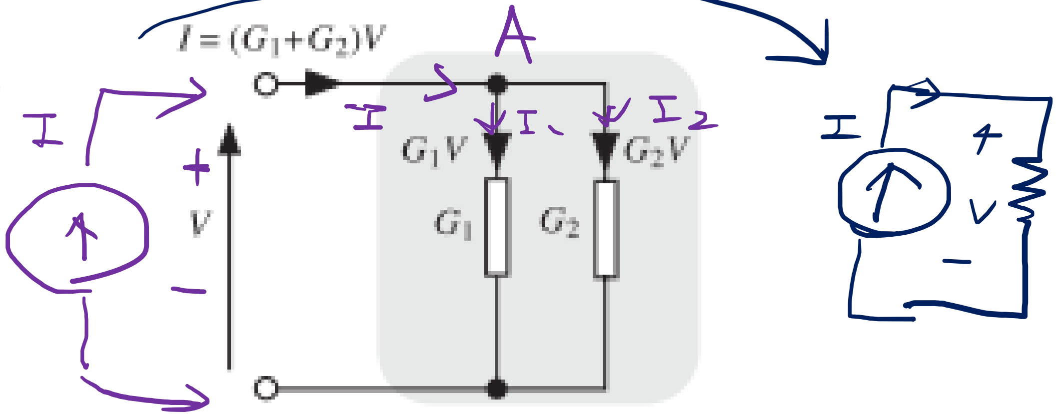



**Figure 3.21** Derivation of the equivalent conductance of two resistors connected in parallel