（裝　　訂　　線）

國立成功大學	學年度第	學期第	次平時考試試卷		
評閱成績	100	教師簽章		學生	院系：工學院工科系三年級　班；學號：E94961365；姓名：洪千涵
科目	名稱：控制理論	開設班別	系　年級　班		

-8

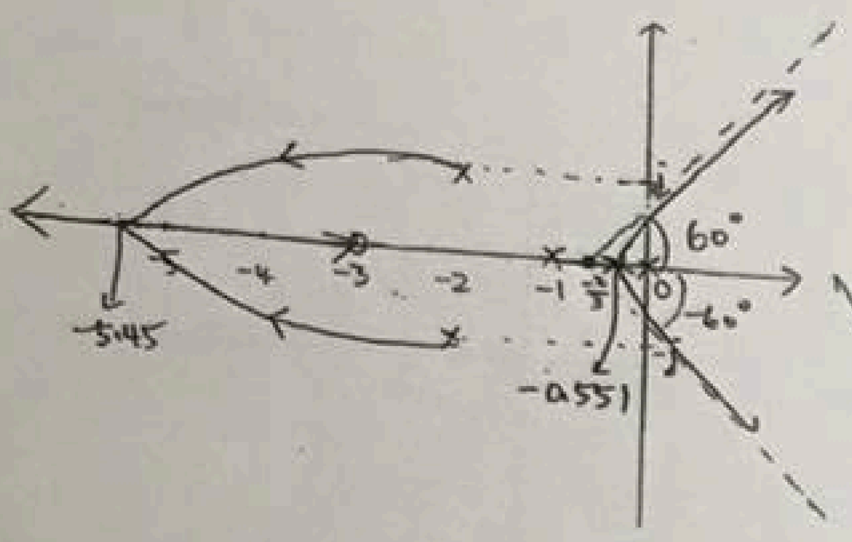
1、

$$G(s) = \frac{K(s+3)}{s(s+1)(s^2+4s+5)}$$

① zero $= -3$

pole $= 0, -1, -2+j, -2-j$

$K>0$，漸近線角度：$60°, 180°, 300°$，$\sigma_A = \frac{0-1-2+j-2-j-(-3)}{4-1} = \frac{-2}{3}$



$G(s) = -1$

breakaway point：

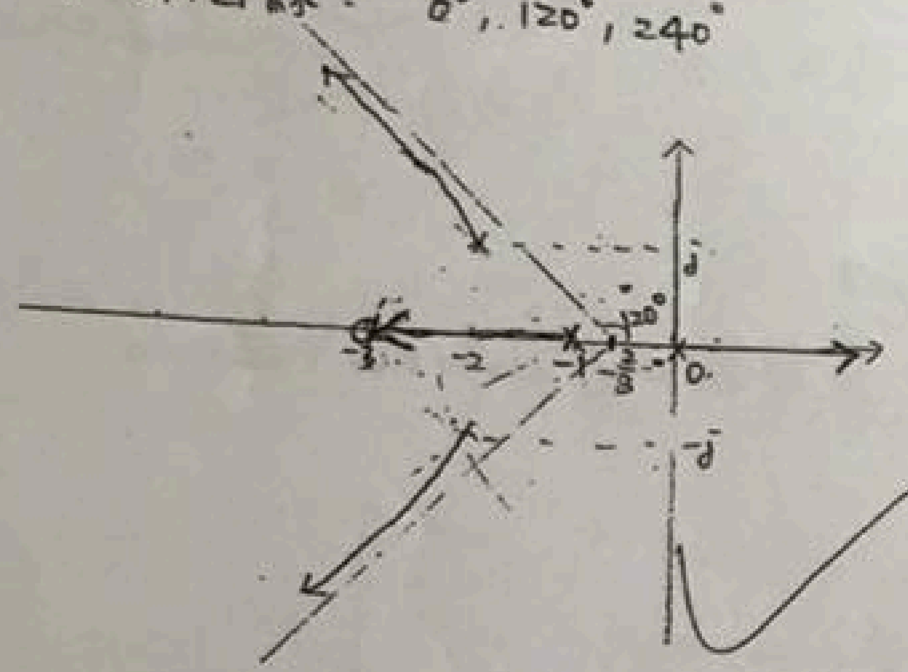
令實根為 σ.

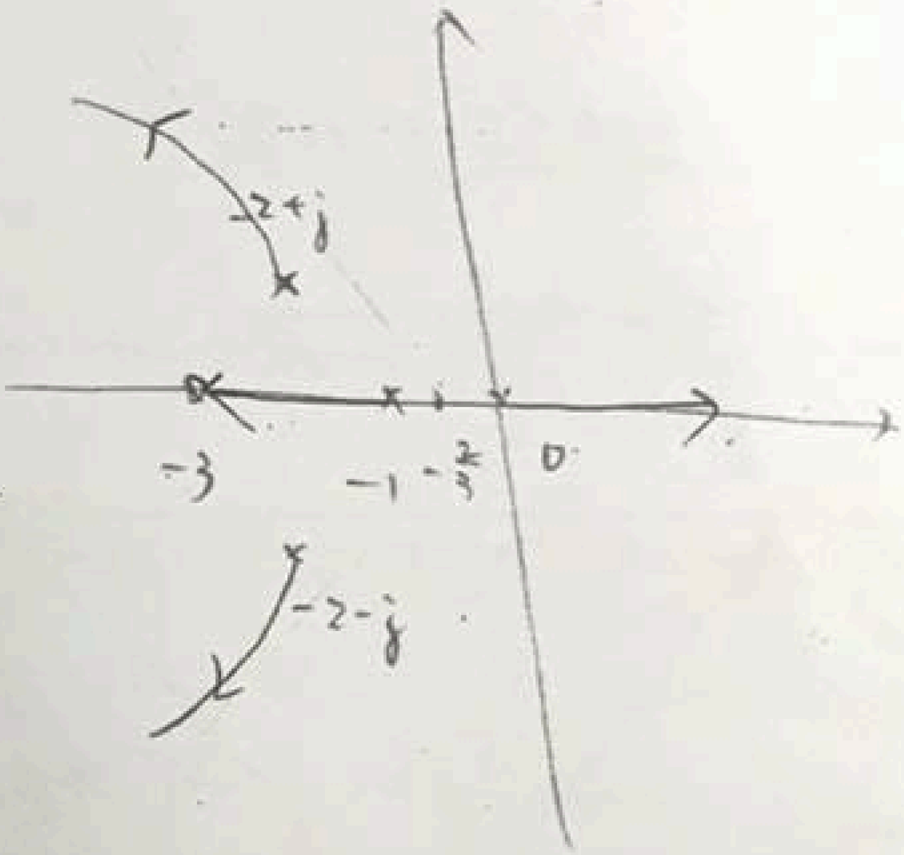
$\frac{1}{\sigma} + \frac{1}{\sigma+1} = \frac{1}{\sigma+3}$（共軛根可忽略）

$\Rightarrow \frac{2\sigma+1}{\sigma^2+\sigma} = \frac{1}{\sigma+3} \Rightarrow 2\sigma^2+7\sigma+3 = \sigma^2+\sigma$

$\Rightarrow \sigma^2+6\sigma+3=0 \Rightarrow \sigma = -0.551, -5.45.$

$K<0$，漸近線：$0°, 120°, 240°$





② $K<0$ 到右半平面 ∴ $K<0$ unstable

$X = -2$

$(-1)+(-2)+(-2) = (-3) + \cancel{jw+(-jw)} + X$

$K>0$

令在虛軸上根為 $s=jw$ 代入 $G(s)=-1$

令

$$G(s) = \frac{K(jw+3)}{(w^4-9w^2)+j(-5w^3+5w)} = -1$$

$$G(s) = -1 = \frac{K(2)}{(-2)(-1)(1)}$$

$\therefore \frac{3K}{w^4-9w^2} = \frac{Kw}{-5w^3+5w} \Rightarrow \frac{3}{w^2(w^2-9)} = \frac{w}{w(-5w^2+5)} \Rightarrow w^4-9w^2 = -15w^2+15$ $= -2$

$\Rightarrow w^4+6w^2-15=0$

$\therefore w = 1.378$ or -1.378 代回 $G(s)=-1$

$$\Rightarrow K = \frac{-(w^4-9w^2)}{3} = 4.49$$

$\therefore 0 < K < 4.49$ 為 stable（續寫轉背頁）

2.

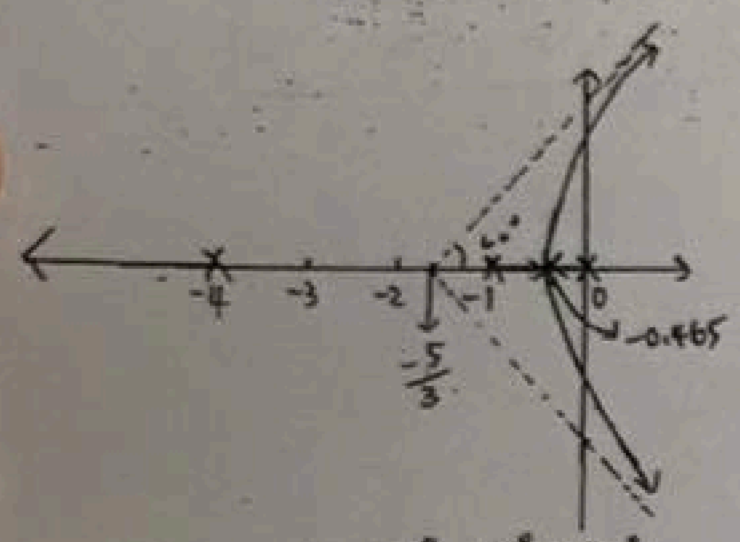
$G(s) = \dfrac{K}{s(s+1)(s+4)}$, pole: $0, -1, -4$

① $K>0$, 漸近線角度：$60°, 180°, 300°$, $OA = \dfrac{-1+0-4-0}{3-0} = \dfrac{-5}{3}$

breakaway point：

$G(s) = \dfrac{K}{s(s+1)(s+4)} = -1 \Rightarrow K = -[s^3+5s^2+4s]$

$\dfrac{dK}{ds} = 0 \Rightarrow 3s^2+10s+4=0 \Rightarrow \boxed{s = -0.465, -2.869}$

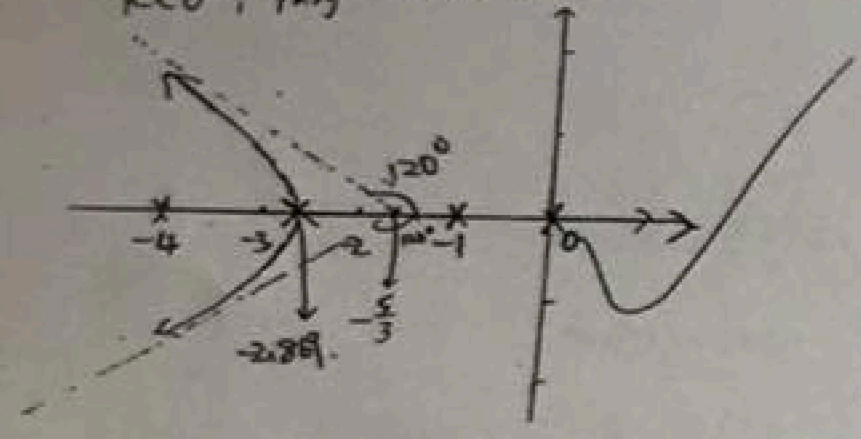


$\dfrac{1}{\sigma} + \dfrac{1}{\sigma+1} + \dfrac{1}{\sigma+4} = 0$

$\sigma^2+5\sigma+4+\sigma^2+4\sigma+\sigma^2+\sigma = 0$

$3\sigma^2+10\sigma+4=0$

$K<0$, $\phi_{asy} = 0°, 120°, 240°$



② $K<0 \Rightarrow$ unstable $\therefore K>0$

$\zeta > 0.707 \Rightarrow \cos\theta = \zeta \Rightarrow$ 代表 $\theta < 45°$，求 $\theta = 45°$ 時，pole 為 $-w+jw$ 代入 $G(s) = -1$

$G(s) = \dfrac{K}{s^3+5s^2+4s} \Rightarrow G(-w+jw) = \dfrac{K}{(-w+jw)^3+5(-w+jw)^2+4(-w+jw)} = -1$

$\Rightarrow \dfrac{K}{w^3(2+2j)+5w^2(-2j)+4w(-1+j)} = \dfrac{K}{(2w^3-4w)+j(2w^3-10w^2+4w)} = -1$ 分母 ∴ 虛部為 0 $\Rightarrow 2w^3-10w^2+4w=0$

$\because K \in R$

$\Rightarrow w = 4.562, 0.438$

$K = -(2w^3-4w)$, $w = 4.562 \Rightarrow K = -177.639$ 不合理.

$w = 0.438 \Rightarrow K = 1.584$ 合理.

$\therefore \quad 0 < K < 1.584$ #

下頁.

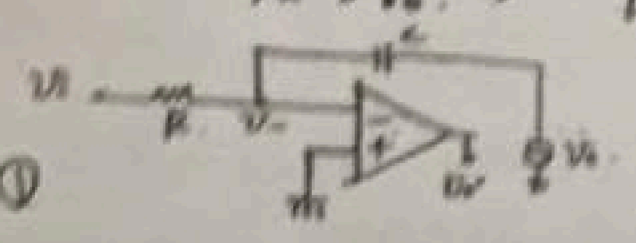
(裝 訂 線)

國立成功大學 學年度第 學期第 次平時考試試卷

評閱成績		教師簽章		學生	院系	工學院 工科系三年級	科目	名稱	
					學號	E94961365		開課系別	系 年級
					姓名	[illegible]			

3.

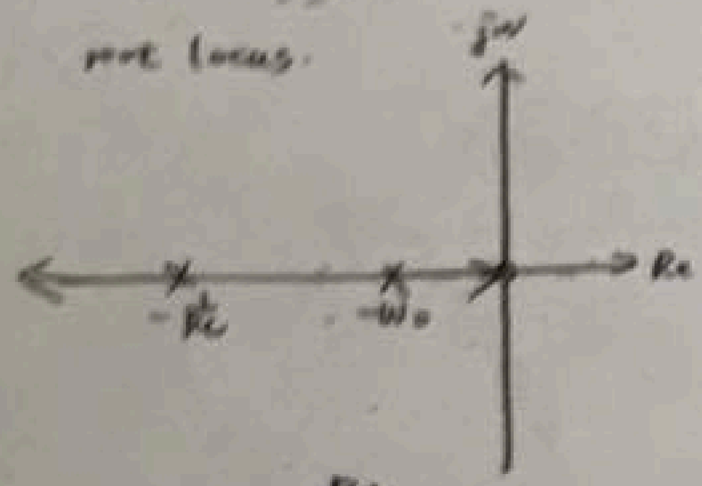
ideal 積分器. ⇒ $A_v = \dfrac{20萬}{1+\frac{s}{\omega_o}}$



①

$$olf(s) = \frac{-v_o'}{v_-} \times \frac{v_-}{v_t} = -\left(\frac{-20萬}{1+\frac{s}{\omega_o}}\right) \times \frac{R}{R+\frac{1}{sC}} = \frac{20萬R}{(1+\frac{s}{\omega_o})(R+\frac{1}{sC})} = \frac{20萬\omega_o s}{(s+\omega_o)(s+\frac{1}{RC})}$$

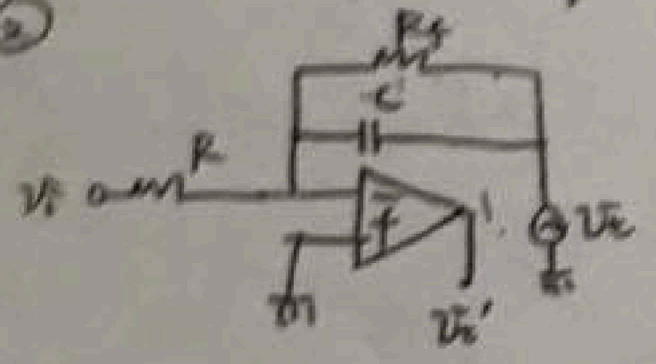
root locus.



∵ $20萬\omega_o > 0$ ∴ 定義 $K = 20萬\omega_o$, $K>0$.

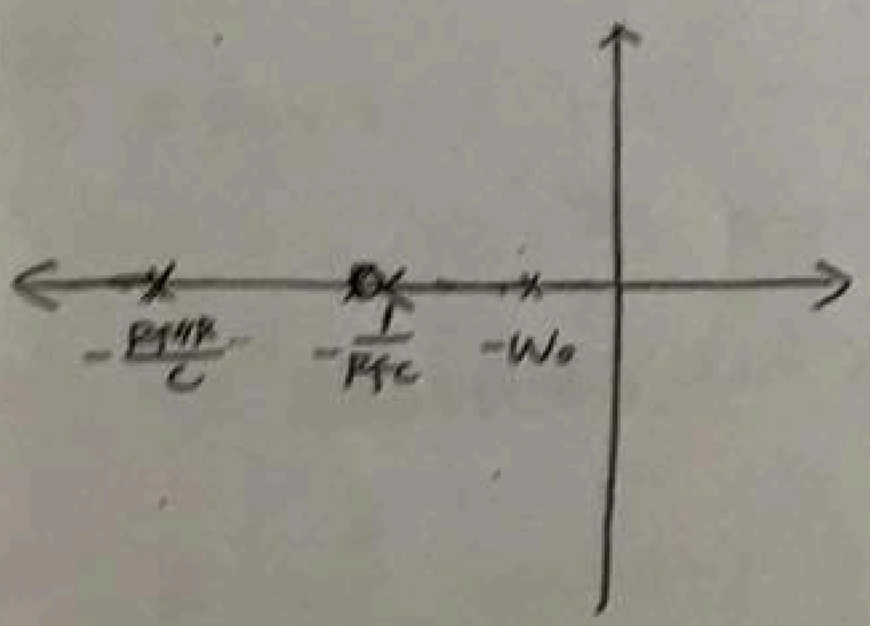
當 $K\to\infty$ 時, pole 會靠近 $j\omega$ 軸, 當一有其它變化, pole 就會跑到右半平面, ∴ 穩定性不好.

②



在 C 上面並一大電阻 R_f.

$$olf(s) = -\frac{v_o'}{v_-} \times \frac{v_-}{v_t} = \frac{20萬}{1+\frac{s}{\omega_o}} \times \frac{R}{R+(R_f /\!/ \frac{1}{sC})} = \frac{20萬\omega_o}{(s+\omega_o)} \times \frac{R(1+sCR_f)}{sCRR_f+(R_f+R)} = \frac{20萬\omega_o}{s+\omega_o} \times \frac{s+\frac{1}{R_fC}}{s+\frac{(R_f+R)}{C}}$$



root locus 會變成如右.

此時 pole 會遠離 $j\omega$ 軸與右半平面,

∴ 穩定.

(續寫轉背頁)

4.

$$G(s) = \frac{w_n^2}{s(s+2\zeta w_n)}$$

① 求 PM = ?

$s = jw$ 代入 $G(jw) = \dfrac{w_n^2}{-w^2 + j2\zeta w w_n}$

當 $|G(jw)| = 1$ 時，$w = w_{gc}$

$$\left| \frac{w_n^2}{-w_{gc}^2 + j2\zeta w_{gc} w_n} \right| = \frac{w_n^2}{\sqrt{w_{gc}^4 + 4\zeta^2 w_{gc}^2 w_n^2}} = 1 \Rightarrow w_{gc}^4 + 4\zeta^2 w_{gc}^2 w_n^2 = w_n^4$$

$$\Rightarrow w_{gc}^4 + 4\zeta^2 w_{gc}^2 w_n^2 - w_n^4 = 0 \xrightarrow{\text{令 } w_{gc}^2 = x} x^2 + 4\zeta^2 w_n^2 x - w_n^4 = 0$$

$$\Rightarrow x = \frac{-4\zeta^2 w_n^2 \pm \sqrt{16\zeta^4 w_n^4 + 4w_n^4}}{2} = \frac{-4\zeta^2 w_n^2 \pm 2w_n^2\sqrt{4\zeta^4+1}}{2}$$

$\because w_{gc}^2 > 0$ $(w_{gc} > 0)$ $\therefore x = \dfrac{-4\zeta^2 w_n^2 + 2w_n^2\sqrt{4\zeta^4+1}}{2} = -2\zeta^2 w_n^2 + w_n^2\sqrt{4\zeta^4+1}$

$$\Rightarrow w_{gc} = w_n\sqrt{-2\zeta^2 + \sqrt{4\zeta^4+1}}$$

$$PM = 180^\circ - \angle s - \angle(s+2\zeta w_n) = 90^\circ - \tan^{-1}\frac{w_{gc}}{2\zeta w_n} = 90^\circ - \tan^{-1}\frac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta} \quad \#$$

求 GM = ?

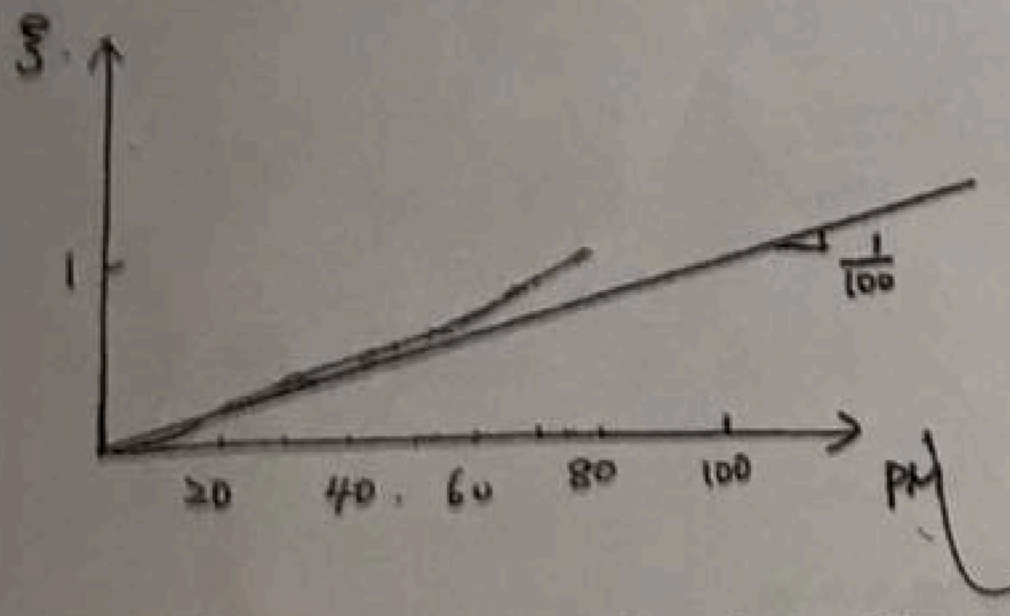
$G(jw) = \dfrac{w_n^2}{-w^2 + j2\zeta w w_n}$ 當 $\angle G(jw) = -180^\circ$ 時，$G(jw) \in R$ $\therefore w_{pc} = 0$

$\therefore GM = -20\log|G(jw_{pc})| = -20\log\left|\dfrac{w_n^2}{\sqrt{w_{pc}^4 + 4\zeta^2 w_{pc}^2 w_n^2}}\right|$ $\because w_{pc} = 0$

$\therefore GM = \infty \quad \#$

② 由①推得 $PM = 90^\circ - \tan^{-1}\dfrac{\sqrt{-2\zeta^2+\sqrt{4\zeta^4+1}}}{2\zeta}$

當
$\zeta = 0.01$，$PM = 1.45 \Rightarrow \frac{PM}{100}$ 0.0145
$\zeta = 0.1$，$PM = 11.311$
$\zeta = 0.2$，$PM = 21.83$
$\zeta = 0.3$，$PM = 31.14$
$\zeta = 0.5$，$PM = 46.28 \rightarrow 0.4628$
$\zeta = 0.6$，$PM = 52.60 \rightarrow 0.5260$
$\zeta = 0.8$，$PM = 64.04 \rightarrow 0.6404$
$\zeta = 1.0$，$PM = 76.345$
$\zeta = 1.09$，$PM = 86.088$
$\zeta = 0$ 與 $\zeta = 1.1$ 皆無值



$\therefore$ 此近似方式合理，但在 $\zeta > 0.5$ 後，誤差 $\left|\dfrac{0.5-0.4628}{}\right| \times 100\% = 3.72\%$ 尚可接受

在 $\zeta = 1$ 時，誤差 $= 23.655\%$ 太大

在 $\zeta > 0.5$ 後誤差愈來愈大，∴要小心使用。

5.

oltf: $G(s) = \frac{1}{s(s+1)(s^2+2s+25)} = \frac{1}{(s^2+s)(s^2+2s+25)} = \frac{1}{s^4+3s^3+27s^2+25s}$

pole = $0, -1, -1+4.9j, -1-4.9j$

(1)

$G(jw) = \frac{1}{(w^4-27w^2)+j(-3w^3+25w)}$

$|G(jw)| = \frac{1}{\sqrt{(w^4-27w^2)^2+(-3w^3+25w)^2}}$

$w=0.01 \rightarrow |G(jw)| = 12\text{dB}$

$w=0.1 \rightarrow |G(jw)| = -8\text{dB}$

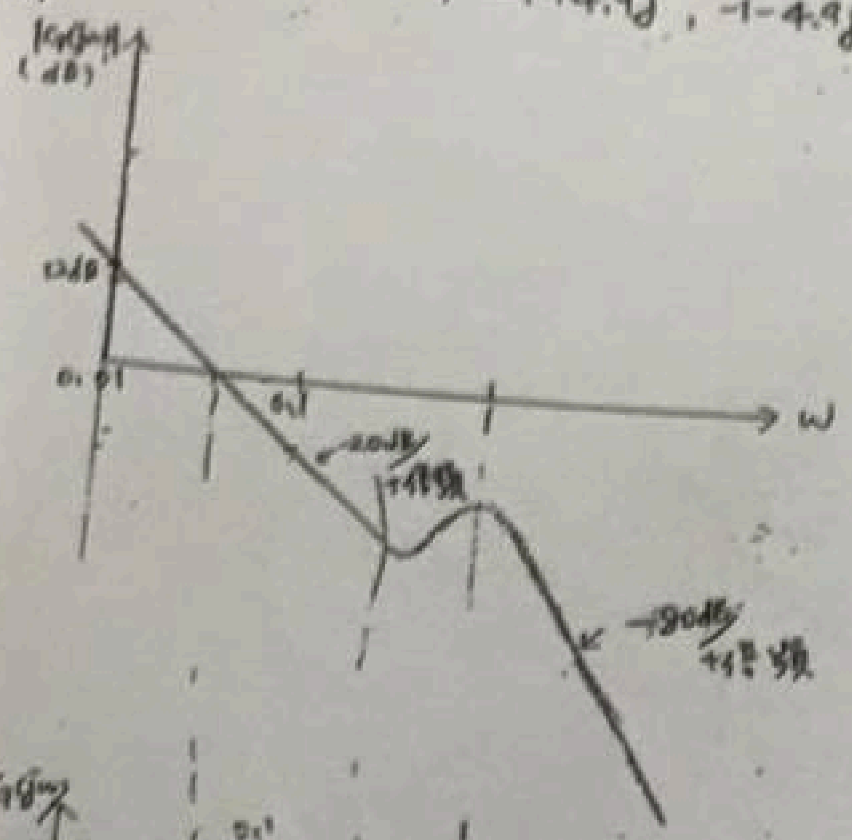
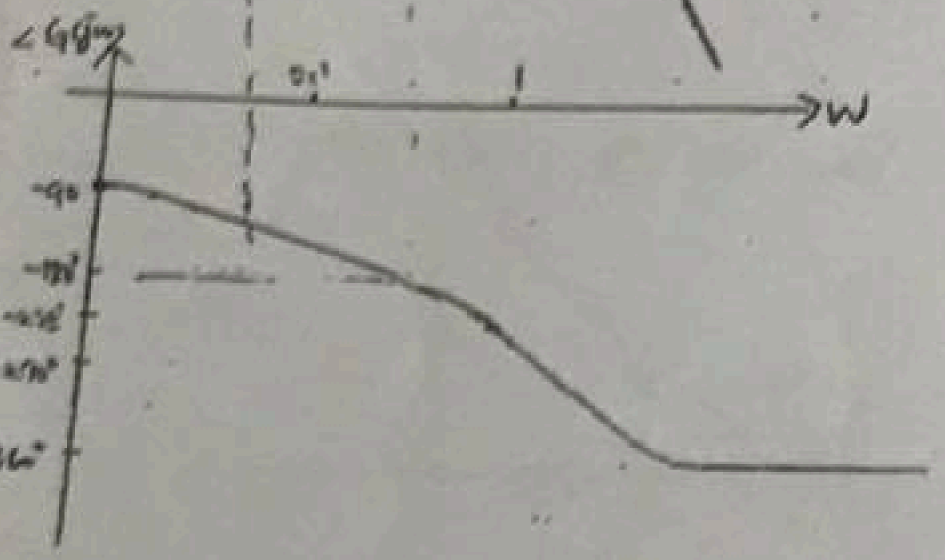


圖 B

-1



②

由 Bode Plot ⇒ $|G(jw)| = 0\text{dB}$ 時, $\angle G(jw) < -180^\circ$ ∴ PM > 0

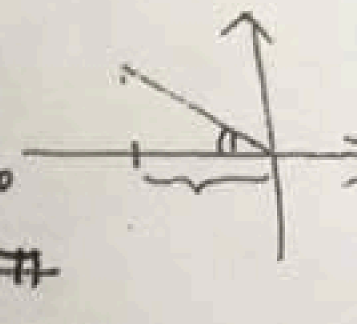
當 $\angle G(jw) = -180^\circ$ 時, $|G(jw)| < 0\text{dB}$ ∴ GM > 0

∴ 其為 stable

$-20\log\left|\quad\right|$

③ 求 PM $|G(jw)| = 1$

$\frac{1}{\sqrt{(w^4-27w^2)^2+(-3w^3+25w)^2}} = 1 \Rightarrow (w^4-27w^2)^2+(-3w^3+25w)^2 = 1 \Rightarrow w = 0.04$



∴ $PM = 180^\circ - \angle[(w^4-27w^2)+j(-3w^3+25w)] = 180^\circ - (180^\circ - \tan^{-1}\frac{-3w^3+25w}{27w^2-w^4}) = 87.526^\circ$ #

求 GM, oltf(s) 與實軸有交點 ⇒ $-3w_{pc}^3+25w_{pc} = 0 \Rightarrow w_{pc} = 2.887$

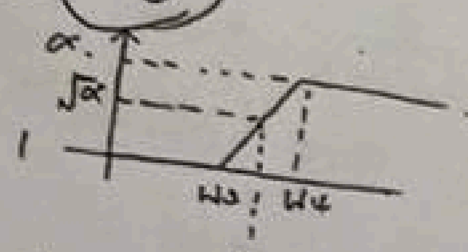
$GM = -20\log\left|\frac{1}{\sqrt{(w_{pc}^4-27w_{pc}^2)^2}}\right| = -20\log\frac{1}{155.57} = 43.84\text{dB}$ #

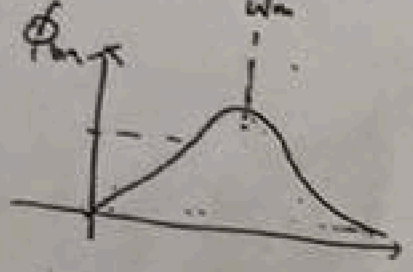
(續寫轉背頁)

國立成功大學 ＿＿ 學年度第 ＿＿ 學期第 ＿＿ 次平時考試試卷 （裝 訂 線）

成績	命題教師	學生	院系	工學院 工科 系 三年級 班	科目	名稱
			學號	E94951027		考試日期
			姓名	林孝[illegible]		

6. (d)





$\alpha = \frac{s+w_3}{s+w_4}$

不影響低頻 gain = 1.

$\alpha = \frac{w_4}{w_3}$.

$\log w_m = \frac{\log w_3 + \log w_4}{2}$

$w_m = \sqrt{w_3 w_4}$

利用 GBP

$w_4 \cdot x = \sqrt{w_3 w_4} \cdot \alpha$.

$x = \sqrt{\alpha}$

$w_3 \cdot \sqrt{\alpha} = w_m \cdot 1 \Rightarrow w_3$

$w_4 \sqrt{\alpha} = w_m \cdot \alpha \Rightarrow w_4 = w_m$

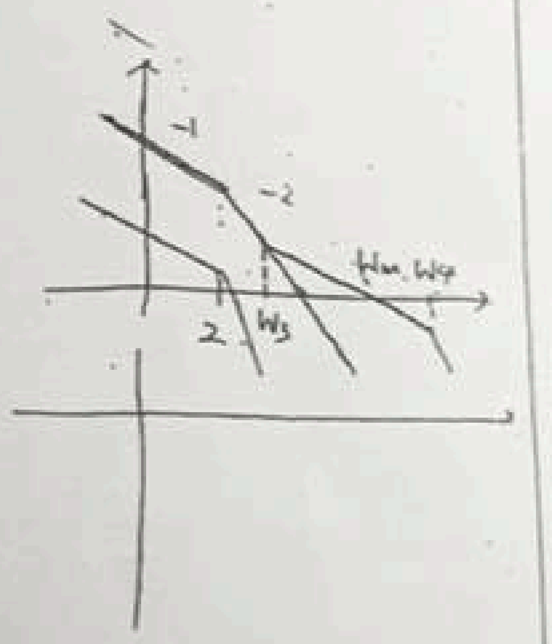
$\therefore oltf(s) = G_c(s) \frac{8}{s(s+2)}$

原來：

當 $G(s) = 1$.

$e_{ss}|_{ramp} = \frac{1}{k_v}$ $k_v = \lim_{s\to 0} s \cdot oltf(s) = 4$. $e_{ss}|_{ramp} = 25\% > 5\%$

故 $G(s) = 5 \cdot \frac{1+\frac{s}{w_3}}{1+\frac{s}{w_4}}$

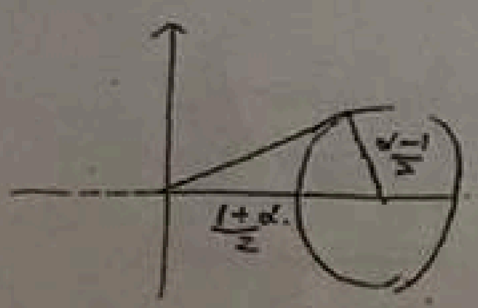


決定 PM ⇒

$|oltf(s)| = \left|\frac{40}{s(s+2)}\right| \Rightarrow$

$\left|\frac{40}{jw(jw+2)}\right| = 1 \rightarrow w = 6.486$

$PM = 180 - (90 + \tan^{-1}\frac{w}{2}) = 17.14°$ 不夠 28° ⇒ 補 40°



$\because \phi_m = \sin^{-1}\frac{\alpha-1}{1+\alpha}$ $40° = \sin^{-1}\frac{\alpha-1}{1+\alpha}$ $\alpha = 4.6$

$\therefore \frac{1}{\sqrt{\alpha}} = |oltf(s)| = \frac{1}{\sqrt{4.6}} = \frac{40}{\sqrt{(w^2)^2+(2w)^2}}$ $w_{gc} \Rightarrow 9.37$

$\therefore w_3 = \frac{9.37}{\sqrt{4.6}} = 4.369$ $w_4 = 9.37 \times \sqrt{4.6} = 20.1$ $G_c(s) = 5 \times \frac{1+\frac{s}{4.}}{1+\frac{s}{20.}}$

$PM = 180° + \angle oltf(w_{gc}) + 40°$

$= 180° + (-168) + 40 \geq 45°$ （請寫於背頁）