

數值方法 Midterm exam 1

1. 解釋名詞
 - (a) [3%]何謂 Taylor expansion?請說明其數值意義。
 - (b) [3%]何謂超級電腦?請說明其組成基本架構。
2. [10%]Determine the single-precision and double-precision machine representation of the following decimal numbers: (a)-285.75 (b) 10^{-2}
3. [4%]A decimal number: 10^{-100} . Convert the decimal number to a 32-bit binary number.
4. [4%]What is the Taylor series of the function $f(x) = 1+x+x^2$ at the point $c = 2$?
5. [6%]If $2/3$ is approximated to 0.6667. Find (a) absolute error; (b) relative error; (c) percentage error.
6. (a) [6%]Find the root of the equation $e^x = 4x$, which is approximately 2, correct to three places of decimals by the Bisection, Newton, and Secant methods.
(b) [6%]Find the root of the equation $\sin x = 1+x^3$, which is between -2 and -1, correct to three places of decimals by the Bisection, Newton, and Secant methods.
7. [5%]Use Gauss elimination to decompose the following system. Please show all the steps in the computation.

$$\begin{cases} x_1 + 7x_2 - 4x_3 = -51 \\ 4x_1 - 4x_2 + 9x_3 = 62 \\ 12x_1 + x_2 + 3x_3 = 8 \end{cases}$$

8. [8%]請使用 LU 分解法計算 x_1, x_2, x_3, x_4

$$\begin{cases} x_1 + 3x_2 + 2x_3 + x_4 = -2 \\ 4x_1 + 2x_2 + x_3 + 2x_4 = 2 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ x_1 + 2x_2 + 4x_3 + x_4 = -1 \end{cases}$$

9. [5%]用 Taylor series 證明 Euler's Formula
10. [5%]Fit a least square curve of the form $y = ae^{bx}$ ($a > 0$) to the data given below and compute the R^2 value

x_i	1	2	3	4
y_i	1.65	2.70	4.50	7.35

11. [5%]請使用下面 5 個點，求得其回歸方程式 $y=A/x+B$ ，請計算出 A、B、 R^2 的數值。

x_i	0	1	3	4	7
y_i	9.31	7.99	5.78	5.88	5.51

12. [10%]Using a calculator, observe the sluggishness with which Newton's method converges in the case of $f(x) = (x - 1)^m$ with $m = 8$ or 12. Reconcile this with the theory. Use $x_0 = 1.1$.

13. [10%]Use the TDMA method to solve the given equation:

$$\frac{\partial^2 T}{\partial x^2} = 3, L = 10m, n = 11$$

i	1	2	3	4	5	6	7	8	9	10	11
T_i	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	T_{10}	T_{11}

(a) Boundary conditions: $T_1 = 12, T_{11} = 38$ **Find** $T_i (i = 2 \text{ } 10)$

(b) Boundary conditions: $T_1 = 12, T_{11} = T_{10}$ **Find** $T_i (i = 2 \text{ } 10)$

14. [10%] Write a m file to execute the LU decomposition of matrix B