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I. 简答题

1. ①真空 ② $v=0$ ③無溫度梯度 ④溫度梯度與流場方向垂直

2. $Pr=1$, $\delta_T=\delta$
   $Pr<1$, $\delta_T>\delta$
   $Pr>1$, $\delta_T<\delta$

   [註: $Pr=\nu/\alpha$
   $\frac{\delta}{\delta_T}\sim Pr^{\frac{1}{3}}$]

3. 因溫度梯度方向與流場方向垂直

4. (a) Reduction the numbers of control parameters.
   減少控制參數的個數
   (b) Establishment of similarity conditions
   建立相似情況

5. $$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial u^*}{\partial y^{*2}}\right)$$
   when (Reynolds number) Re is very large, the viscous term is neglected.

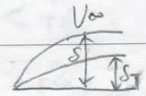
6.

7. $$\begin{cases} y=0,\ u=0 \\ y=\delta,\ u=U_\infty \\ y=\delta,\ \frac{\partial u}{\partial y}=0 \end{cases}$$

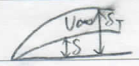
8.

9. film temperature $T_f = \frac{T_w + T_\infty}{2}$，流体的温度由平板表面温度 T_w 变化至边界層边緣温度 T_∞，为了適当地計算流体隨温度变化的性質，通常用薄膜温度(film temperature) T_f 来計算.

10. (a) 当 $Pr>1$，則 $\delta > \delta_T$，所以无法決定 U_s



(b) 当 $Pr \le 1$，則 $\delta \le \delta_T$，温度邊界層之 $U_s = U_\infty$



II. Explain the following terms

(1) Film temperature：膜膜温度 $T_f = \frac{T_w + T_\infty}{2}$，平板表面温度 T_w 与流体边界層温度 T_∞ 的平均，用以計算流体的性質.

(2) Prandtl number：$Pr = \frac{\nu}{\alpha}$，量測速度边界層的動量傳遞与熱边界層的能量傳遞的相对效率.

(3) Boussinesq approximation:
Treat density ρ as a constant in all terms in the equations governing natural convection, except in the buoyancy term.

(4) Mixed convection：結合 natural convection 与 force convection
(a) 通常，强制对流的作用比自然对流强，所以自然对流可忽略
(b) 在高温時，自然对流的作用會增強
(c) 当 $Gr/Re^2 > 10$，自然对流 is of primary importance.

(5) Nusselt number：$Nu = \frac{hL}{k}$

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III.

(1) 因為 $\frac{\partial^2 u^*}{\partial x^{*2}} \ll \frac{L^2}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}$，所以 $\frac{\partial^2 u^*}{\partial x^{*2}}$ 可忽略

(2) 因為 $\frac{\partial^2 v^*}{\partial x^{*2}} \ll \frac{L^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*}}$，$\frac{\partial^2 v^*}{\partial x^{*2}}$ 可省略

$\frac{L^2}{\delta^2} \gg 1 \Rightarrow \frac{\partial p^*}{\partial y^*} = 0$

$$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = \frac{1}{Re}\frac{L^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}$$

(3) $y=\delta$, $u=U_\infty$ $\left(\frac{\partial u}{\partial y}=0, \cdots\right)$, $V=0$, $P=P_\infty$

and $y=\delta_T$, $T=T_\infty$ $\left(\frac{\partial T}{\partial y}=0, \cdots\right)$

IV.

(a) ① velocity fields : $\begin{cases} y=0, & \frac{du}{dy}=0 \\ y=\frac{W}{2}, & u=0 \end{cases}$

② temperature fields : $\begin{cases} y=0, & \frac{\partial T}{\partial y}=0,\ T=T_c \\ y=\frac{W}{2}, & T=T_w \end{cases}$

(b) $\frac{T_w-T}{T_w-T_m}$ is invariant in the flow direction

and T_w, T_m are function of z only

$$\frac{\partial}{\partial z}\left[\frac{T_w-T}{T_w-T_m}\right]=0.$$

$$\frac{1}{T_w-T_m}\left[\frac{dT_w}{dz}-\frac{\partial T}{\partial z}\right]-\frac{T_w-T}{(T_w-T_m)^2}\left[\frac{dT_w}{dz}-\frac{dT_m}{dz}\right]=0$$

乘以(T_w-T_m) : $\frac{dT_w}{dz}-\frac{\partial T}{\partial z}-\frac{T_w-T}{T_w-T_m}\left(\frac{dT_w}{dz}-\frac{dT_m}{dz}\right)=0$

$$\frac{\partial T}{\partial z}=\frac{dT_w}{dz}-\frac{T_w-T}{T_w-T_m}\left(\frac{dT_w}{dz}-\frac{dT_m}{dz}\right)=0. \quad \text{——① 式}$$

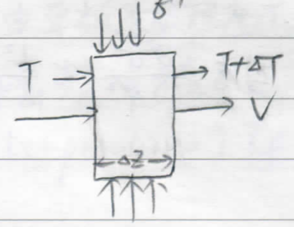
considering constant heat flux

$$\begin{cases} q'' = h(T_w - T_m) = \text{constand} \\ \text{and } h = \text{constant} \end{cases}$$

$\Rightarrow T_w - T_m = \text{constant}$

$\Rightarrow \dfrac{dT_w}{dz} - \dfrac{dT_m}{dz} = 0$

$\Rightarrow \dfrac{dT_w}{dz} = \dfrac{dT_m}{dz}$ —— ②



$\rho c V[(T+\Delta T) - T] =$

$\dfrac{\partial T}{\partial z} = \dfrac{2q''}{r_0 \rho c V} = \text{constant}$

①式代入②式，得 $\dfrac{\partial T}{\partial z} = \dfrac{dT_w}{dz}$ —— ③

由②,③式 得. $\dfrac{\partial T}{\partial z} = \dfrac{dT_w}{dz} = \dfrac{dT_m}{dz} = \boxed{\text{constant}}$

(C) $\boxed{u\dfrac{\partial T}{\partial x} + v\dfrac{\partial T}{\partial y} = \alpha \dfrac{\partial^2 T}{\partial y^2}}$ energy equation

$\because u = U_\infty,\ v = 0$

$\Rightarrow U_\infty \dfrac{\partial T}{\partial x} = \alpha \dfrac{\partial^2 T}{\partial y^2}$

$\Rightarrow \dfrac{\partial^2 T}{\partial y^2} = \dfrac{U_\infty \frac{\partial T}{\partial x}}{\alpha}$

積分 $\dfrac{\partial T}{\partial y} = (\alpha \dfrac{U_\infty}{2}\dfrac{\partial T}{\partial x})y + C_1$

Boundary condition: $y=0,\ \dfrac{\partial T}{\partial y} = 0$

$\Rightarrow C_1 = 0$

$\dfrac{\partial T}{\partial y} = (\alpha \dfrac{U_\infty}{2}\dfrac{\partial T}{\partial x})y$

積分 $T = \dfrac{1}{2}(\alpha \dfrac{U_\infty}{2}\dfrac{\partial T}{\partial x})y^2 + C_2$

Boundary condition $y=0,\ T=T_c$

$\Rightarrow C_2 = T_c$

得 $T = \dfrac{1}{2}(\alpha U_\infty \dfrac{\partial T}{\partial x})y^2 + T_c$

又 $y = \dfrac{W}{2},\ T = T_w$

得 $T_w = \dfrac{W^2}{8}(\alpha U_\infty \dfrac{\partial T}{\partial x}) + T_c$

$T_m = \dfrac{1}{A_c U_{ave}}\displaystyle\int_{A_c} uT\,dA_c$

$= \dfrac{1}{\frac{W}{2}\cdot U_\infty}\displaystyle\int_0^{\frac{W}{2}} U_\infty \cdot [\tfrac{1}{2}(\alpha U_\infty \tfrac{\partial T}{\partial x})y^2 + T_c]\,dy$

$= \dfrac{2}{W}[\tfrac{1}{2}(\alpha U_\infty \tfrac{\partial T}{\partial x})\tfrac{1}{3}y^3 + T_c y]\Big|_0^{\frac{W}{2}}$

$= \dfrac{W^2}{24}(\alpha U_\infty \dfrac{\partial T}{\partial x}) + T_c + T_c$

$h = \dfrac{k\frac{\partial T}{\partial y}\big|_{\frac{W}{2}}}{T_w - T_m} = \dfrac{k\cdot\frac{W}{2}(\alpha U_\infty \frac{\partial T}{\partial x})}{\frac{2}{24}W^2(\alpha U_\infty \frac{\partial T}{\partial x})}$

$= \dfrac{6k}{W}$

$Nu_w = \dfrac{h\cdot W}{k} = 6$

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V.

$$T_f = \frac{110^\circ C + 10^\circ C}{2} = 60^\circ C$$

由表知. $T_f = 60^\circ C$, $\nu = 1.896 \times 10^{-5}$, $k = 0.02808$, $Pr = 0.7202$

① $Re = \frac{VD}{\nu} = \frac{8 \times 0.1}{1.896 \times 10^{-5}} = 4.22 \times 10^4$

將 Re, Pr 代入公式求 ② Nu　　[題目有公式，按計算機求 Nu]

又 $Nu = \frac{hD}{k}$，求 ③ $h = \frac{Nu \times k}{D}$　　[$k = 0.02808$, $D = 0.1$, Nu]

$q = hA(T_w - T_\infty)$　　[$q = q'' \cdot A$]

$q = h \cdot [(2\pi r) \cdot L][T_w - T_\infty]$

$\frac{q}{L} = h \cdot \pi \cdot D \cdot (T_w - T_\infty)$　　[註. D 直徑 $= 0.1\,m$
$T_w = 110^\circ C$
$T_\infty = 10^\circ C$
h 由 $\frac{Nu \times k}{D}$ 求得]

[$\frac{q}{L}$ 的單位 W/m]

VI

(1) 物体力 body force $\rho\vec{g}$ 之 浮力 buoyancy force

?

(2)
$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{dp}{dx}-\rho g+\mu\frac{\partial^2 u}{\partial y^2} \quad \text{——①}$$

$$\frac{dp}{dx}=\frac{dp_\infty}{dx}=-\rho_\infty g \quad \text{——②}$$

②代入①：

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho_\infty g-\rho g+\mu\frac{\partial^2 u}{\partial y^2}$$

$$=(\rho_\infty-\rho)g+\mu\frac{\partial^2 u}{\partial y^2} \quad \text{——③}$$

又 $\beta=\frac{1}{V}\frac{\partial V}{\partial T}=\frac{1}{V_\infty}\frac{V-V_\infty}{T-T_\infty}=\frac{\frac{V}{V_\infty}-1}{T-T_\infty}$ （V为体積）

$$\beta(T-T_\infty)=\frac{V}{V_\infty}-1=\frac{\rho_\infty}{\rho}-1=\frac{\rho_\infty-\rho}{\rho}$$

$$\therefore \rho_\infty-\rho=\rho\beta(T-T_\infty) \quad \text{——④}$$

④代入③ 得

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho\beta g(T-T_\infty)+\mu\frac{\partial^2 u}{\partial y^2} \quad \text{得証}$$

(3) 令 $x^*=\frac{x}{L}$, $y^*=y/\delta$

$u^*=u/u_s$, $v^*=v/v_s$

代入連續方程式 $\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$

$$\frac{\partial(u^*u_s)}{\partial(x^*L)}+\frac{\partial(v^*v_s)}{\partial(y^*\delta)}=0$$

$$\frac{u_s}{L}\frac{\partial u^*}{\partial x^*}+\frac{v_s}{\delta}\frac{\partial v^*}{\partial y^*}=0$$

$$\frac{u_s}{L}=\frac{v_s}{\delta} \Rightarrow v_s=\frac{u_s}{L}\cdot\delta$$

$$\Rightarrow v=v^*v_s=v^*\frac{u_s}{L}\delta$$

令 $\theta=\frac{T-T_\infty}{T_w-T_\infty} \Rightarrow T=T_\infty+(T_w-T_\infty)\theta$

代入 $\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\rho\beta g(T-T_\infty)+\mu\frac{\partial^2 u}{\partial y^2}$

$$\rho\left[u_su^*\frac{\partial(u_su^*)}{\partial(x^*L)}+\frac{u_s}{L}\delta v^*\frac{\partial(u_su^*)}{\partial(y^*\delta)}\right]=\rho\beta g(T_w-T_\infty)\theta+\mu\frac{\partial^2(u_su^*)}{\partial(y^*\delta)^2}$$

$$\rho\frac{u_s^2}{L}\left[u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}\right]=\rho g\beta(T_w-T_\infty)\theta+\frac{\mu u_s}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

除以 $\frac{\rho u_s^2}{L}$，得

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=\frac{g\beta(T_w-T_\infty)L}{u_s^2}\theta+\frac{\nu}{u_sL}\left(\frac{L^2}{\delta^2}\right)\frac{\partial^2 u^*}{\partial y^{*2}}$$

令 $\frac{g\beta(T_w-T_\infty)L}{u_s^2}=1$

$$\Rightarrow u_s=\left[g\beta(T_w-T_\infty)L\right]^{\frac{1}{2}}$$

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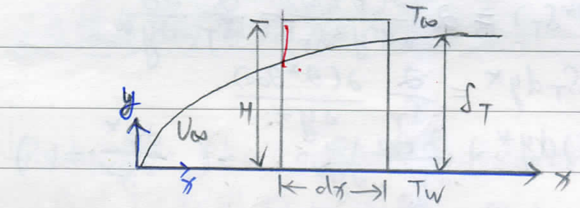
$$\frac{u_s L}{\nu} = \frac{[g\beta(T_w-T_\infty)L]^{\frac{1}{2}}\cdot L}{\nu} = \left[\frac{g\beta(T_w-T_\infty)L^3}{\nu^2}\right]^{\frac{1}{2}} = Gr^{\frac{1}{2}}$$

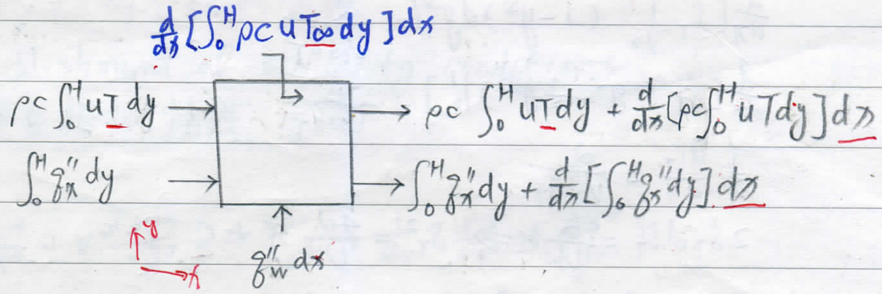
Grashof number $Gr = \dfrac{g\beta(T_w-T_\infty)L^3}{\nu^2}$

$$\Rightarrow u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*} = \theta + \frac{1}{\sqrt{Gr}}\left(\frac{L^2}{\delta^2}\right)\frac{\partial^2 u^*}{\partial y^*}$$

VII

(a)





$$\frac{d}{dx}\left[\rho c\int_0^H uT dy\right]dx + \frac{d}{dx}\left[\int_0^H q_x'' dy\right]dx = q_w'' dx + \frac{d}{dx}\left[\int_0^H \rho c u T_\infty dy\right]dx$$

去掉dx, 並代入 $u=U_\infty$

$$\frac{d}{dx}\left[\rho c\int_0^H U_\infty T dy\right] + \frac{d}{dx}\left[\int_0^H q_x'' dy\right] = q_w'' + \frac{d}{dx}\left[\int_0^H \rho c U_\infty T_\infty dy\right]$$

$$\frac{d}{dx}\left[\int_0^H \rho c U_\infty T_\infty dy\right] - \frac{d}{dx}\left[\rho c\int_0^H U_\infty T dy\right] = -q_w'' + \frac{d}{dx}\left[\int_0^H q_x'' dy\right]$$

$$\rho c\,\frac{d}{dx}\int_0^H U_\infty(T_\infty - T)dy = -q_w'' + \frac{d}{dx}\left[\int_0^H q_x'' dy\right]$$

☆ $\frac{d}{dx}\left[\int_0^H q_x'' dy\right]$ 遠小於其他項，可忽略.

$$\rho c\frac{d}{dx}\left[\int_0^H U_\infty(T_\infty - T)dy\right] = -q_w'' = -\left(-k\frac{\partial T}{\partial y}\Big)_w\right)$$

又 $y \geq \delta_T$, $T = T_\infty$

$$\Rightarrow \rho c\frac{d}{dx}\int_0^{\delta_T} U_\infty(T_\infty - T)dy = k\frac{\partial T}{\partial y}\Big)_w$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_T} U_\infty(T_\infty - T)dy = \frac{k}{\rho c}\frac{\partial T}{\partial y}\Big)_w$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_T} U_\infty(T_\infty - T)dy = \alpha\frac{\partial T}{\partial y}\Big)_w \quad \left[\because \alpha = \frac{k}{\rho c}\right]$$

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(b) $\frac{T-T_w}{T_\infty-T_w} = y/\delta_T$

令 $\theta = T-T_w,\ \theta_\infty = T_\infty - T_w,\ y^* = y/\delta_T,\ \theta^* = \frac{\theta}{\theta_\infty}$

則 $\frac{T-T_w}{T_\infty-T_w} = y/\delta_T \Rightarrow \frac{\theta}{\theta_\infty} = \frac{y}{\delta_T} \Rightarrow \theta^* = y^*$,

$T_\infty - T = (T_\infty - T_w) - (T - T_w) = \theta_\infty - \theta$

$\because \frac{d}{dx}\int_0^{\delta_T} U_\infty (T_\infty - T)dy = \alpha \left.\frac{\partial T}{\partial y}\right)_w$

$U_\infty \frac{d}{dx}\int_0^{\delta_T}(\theta_\infty - \theta)\, d(y^*\delta_T) = \alpha \frac{\partial(\theta + T_w)}{\partial(y^*\delta_T)} = \frac{\alpha}{\delta_T}\frac{\partial\theta}{\partial y^*}$

$U_\infty \frac{d}{dx}\int_0^1 \theta_\infty(1-\frac{\theta}{\theta_\infty})\delta_T dy^* = \frac{\alpha}{\delta_T}\frac{\partial(\theta^*\theta_\infty)}{\partial y^*}$

$U_\infty \frac{d}{dx}\int_0^1 \theta_\infty \delta_T (1-\theta^*)dy^* = \frac{\alpha\theta_\infty}{\delta_T}\frac{\partial\theta^*}{\partial y^*}$ $\quad (\frac{\partial\theta^*}{\partial y^*} = 1)$

$\frac{d}{dx}[\delta_T\int_0^1 (1-y^*)dy^*] = \frac{\alpha}{U_\infty\delta_T}$

$\frac{d}{dx}[\delta_T (y^* - \frac{1}{2}y^{*2})|_0^1] = \frac{\alpha}{U_\infty\delta_T}$

$\frac{1}{2}\frac{d\delta_T}{dx} = \frac{\alpha}{U_\infty\delta_T}$

$2\delta_T d\delta_T = \frac{4\alpha}{U_\infty}dx \Rightarrow \delta_T^2 = \frac{4\alpha}{U_\infty}x + C$

when $x=0,\ \delta_T = 0 \Rightarrow C=0$

$\therefore \delta_T = (\frac{4\alpha x}{U_\infty})^{\frac{1}{2}} \Rightarrow \frac{\delta_T}{x} = 2(\frac{\alpha}{U_\infty x})^{\frac{1}{2}}$

$q'' = h(T_w - T_\infty) = -k\left.\frac{\partial T}{\partial y}\right)_w$ $\quad [\left.\frac{\partial T}{\partial y}\right)_w = \frac{T_\infty - T_w}{\delta_T}]$

$h = \frac{-k\left.\frac{\partial T}{\partial y}\right)_w}{T_w - T_\infty} = \frac{-k(\frac{T_\infty - T_w}{\delta_T})}{T_w - T_\infty} = \frac{k}{\delta_T}$

$Nu_x = \frac{hx}{k} = \frac{x}{\delta_T} = \frac{1}{2}(\frac{U_\infty x}{\alpha})^{\frac{1}{2}}$

$\frac{U_\infty x}{\alpha} = \frac{U_\infty x}{\nu}\cdot\frac{\nu}{\alpha} = Re_x \cdot Pr$

$\therefore Nu_x = \frac{1}{2}Pr^{\frac{1}{2}} Re_x^{\frac{1}{2}}$