Problems and Solutions (Chapter 5)

1. An octagon shaped cell is closer to a circle as compared to a hexagon. Explain why such a cell is not used as an ideal shape of the cell?

   [Solution]

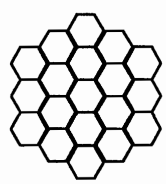
   Hexagonal modeling of cells achieves complete packing of space without any overlap or gaps between cells. Though octagons (or any polygon of sides 7 or more) are closer in shape to circles than hexagons, they cannot be packed without causing either overlap or gaps or both.

2. A new wireless service provider decided to employ a cluster of 19 cells as the basic module for frequency reuse.

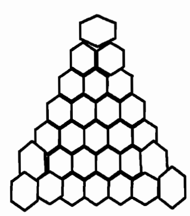
   (a) Can you identify one such cluster structure?
   (b) Repeat (a) for $N = 28$.
   (c) Can you get an alternative cluster structure for part (a)?
   (d) What is the reuse distance for the system of part (c)?
   (e) Can you find the worst case co-channel interference in such a system?

   [Solution]

   (a) 19-cell cluster $i = 3$, $j = 2 \Longrightarrow N = 4 + 9 + 6 = 19$

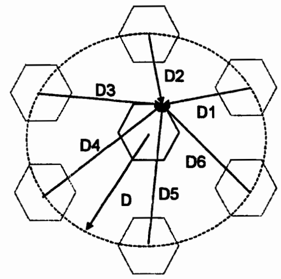
   (b) 28-cell cluster, $i = 4$, $j = 2 \Longrightarrow N = 16 + 4 + 8 = 28$

   (c) An alternative cluster will turn out to be the same, but seen from a different angle, i.e., with the shift parameters (i, j) interchanged. It will have the same reuse distance as the 19-cell cluster pattern shown above.

   (d) Reuse distance $D = \sqrt{3N}R = \sqrt{57}R = 7.55R$

(e) Let the reuse distance D and the radius of the cell R.



When $D_1 = D_2 = D - R$, $D_3 = D_6 = D$, $D_4 = D_5 = D + R$,

$$\frac{C}{I} = \frac{1}{(2(q-1)^{-\gamma} + 2(q)^{-\gamma} + 2(q+1)^{-\gamma})},$$

where $q = \frac{D}{R} = \sqrt{57} = 7.55$ (for $N = 19$).

3. Two adjacent BSs i and j are 30 kms apart. The signal strength received by the MS is given by the following expressions.

$$P(x) = \frac{G_t G_r P_t}{L(x)}$$

where

$$L(x) = 69.55 + 26.16 \log_{10} f_c(\text{MHz}) - 13.82 \log_{10} h_b(\text{m}) - a[h_m(\text{m})] \\ + [44.9 - 6.55 \log_{10} h_b(\text{m})] \log_{10}(x),$$

and x is the distance of the MS from BS i. Assume unity gain for G_r and G_t, given that $P_i(t) = 10$ Watts, $= 100$ Watts, $f_c = 300$ MHz, $h_b = 40$ m, $h_m = 4$ m, $\alpha = 3.5$, $x = 1$ km, and $P_j(t)$ is the transmission power of BS j.

(a) What is the power transmitted by BS j, so that the MS receives signals of equal strength at x?

(b) If the threshold value $E = 1$ dB and the distance where handoff is likely to occur is 2 km from BS j, then what is the power transmitted by BS j?

[Solution]

(a) Assume that the distance of the MS from base station i is x. Since $G_t = G_r = 1$. Since $G_R = G_T = 1$,

$$\frac{P_i}{L(x)} = \frac{P_j}{L(30 - x)} \tag{1}$$

$$P_i = 10\log(10) = 10 \text{ dB}$$

By substituting data given in $L(x)$, we get

$$\begin{aligned} L(x) &= 69.55 + 64.801 - 22.140 - 14 + 34.406\log(x) \\ &= 98.211 \end{aligned}$$

$$\begin{aligned} L(30-x) &= 98.211 + 34.406\log(30-x) \\ &= 98.211 + 50.315 \\ &= 148.526 \end{aligned}$$

Substituting in Equation (1), we have

$$\frac{10}{98.211} = \frac{P_J}{148.526}.$$

Solving for P_j, we get

$$P_j = 15.123 \text{ dB} = 32.512 \text{ Watts}$$

(b) Assume that the distance of the MS from the BS i is x, and if $E = 1$ dB then the point of hand off will be when

$$P_m(i) + 1dB = P_m(j)$$

$$P_m(i) = 10\log_{10} 10 = 10 \text{ dB}$$

Similar to (a), we get

$$\begin{aligned} L(x) &= 69.55 + 64.801 - 22.140 - 14 + 34.406\log_{10}(x) \\ &= 98.211 + 34.406\log_{10}(x) \\ &= 98.211 + 34.406\log_{10} 2 \\ &= 108.5682 \end{aligned}$$

$$\begin{aligned} L(30-x) &= 98.211 + 34.406\log_{10}(30-x) \\ &= 98.211 + 34.406\log_{10}(28) \\ &= 148.0019. \end{aligned}$$

Using Equation (1), we get

$$(10+1) * 148.0019 = 108.5682 * P_m(j)$$

Thus,

$$P_m(j) = 14.995 \text{ dB} = 31.589 \text{ Watts}.$$

4. If each user keeps a traffic channel busy for an average of 5% time and an average of 60 requests per hourm or generated, what is the Erlang value?

   [Solution]

   The request rate $r = 60/3600 = 1/60$ requests/sec

   Holding time $= 0.05 \times 3600 = 180$ sec

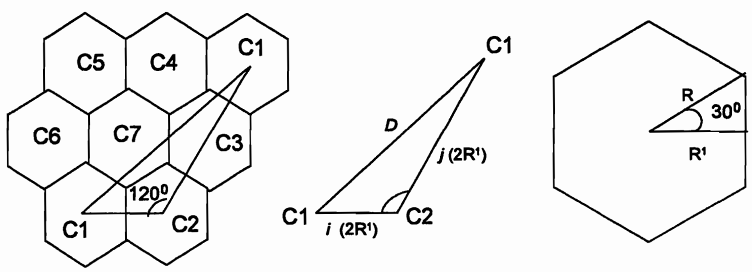
   Therefore the offered traffic load in Erlangs is =request rate $\times$ holding time $= (1/60) \times 180 = 3$ Erlangs.

5. Prove that $D = R\sqrt{3N}$.

   [Solution]

   To prove that $D = R\sqrt{(3N)}$

   Let R be the radius of the cell and D the reuse distance.



   where i and j are the number of cells in the corresponding directions. In the figure as we traverse from C1-C2-C1, we first move 1 cell in the i direction, from C1 to C2 and then 2 cells in the j direction from C2 to C1.

   We have

$$R^1 = R\cos 30 = \frac{\sqrt{3}}{2}R \tag{2}$$

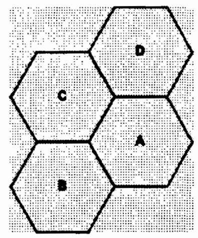
   The distance D can be found using the cosine law,

$$\begin{aligned} D^2 &= (i(2R^1))^2 + (j(2R^1))^2 - 2i(2R^1)j(2R^1)\cos 120 \\ &= (i^2 + j^2)(2R^1)^2 + ij(2R^1)^2 \\ &= (2R^1)^2(i^2 + j^2 + ij) \\ &= (2R^1)^2(N), \end{aligned}$$

   where N is the number of cells in the cluster.

   Using Equation (2) described above, we get

   $D^2 = 3NR^2 \Rightarrow D = \sqrt{3N}R$.

(a) 

6. Prove that $N = i^2 + j^2 + ij$.

   [Solution]

   Using the basic analysis in Problem 5.5. and the result obtained for D, we get

$$
\begin{aligned}
D^2 &= (i(2R^1))^2 + (j(2R^1))^2 - 2i(2R^1)j(2R^1)\cos 120 \\
&= (i^2 + j^2)(2R^1)^2 + ij(2R^1)^2 \\
&= (2R^1)^2(i^2 + j^2 + ij).
\end{aligned}
$$

   Therefore,

$$3N * R^2 = (2R^1)^2(i^2 + j^2 + ij)$$

   However,

$$(2R^1)^2 = 3R^2$$

   Hence,

$$N * 3 * R^2 = 3 * R^2(i^2 + j^2 + ij)$$

   Therefore,

$$N = (i^2 + j^2 + ij)$$

7. The size and shape of each cluster in a cellular, need to be designed carefully so as to cover adjacent spoke in a non-overlapped manner. Define such pattens for the following cluster sizes:

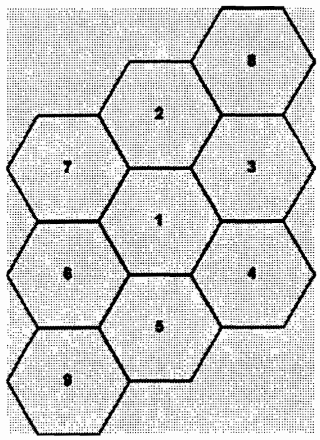
   (a) 4-cell

   (b) 9-cell

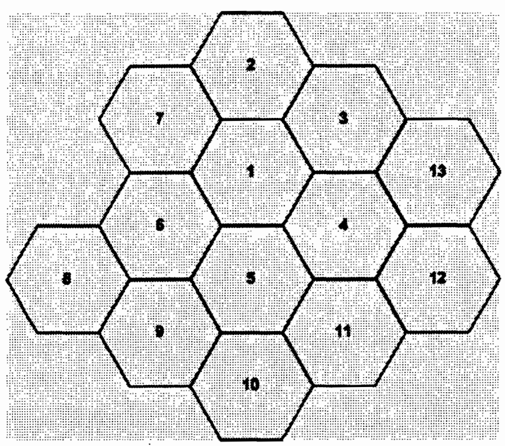
   (c) 13-cell

   (d) 37-cell

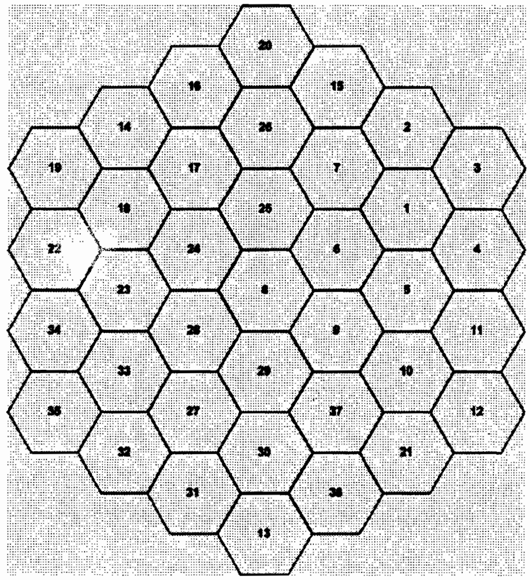
   [Solution]



(b)



(c)

(d) 

8. A cellular scheme employed a cluster of size 16 cells. Later on, it was decided to use two different clusters of size 7 and 9 cells. Is it possible to replace each original cluster by two new clusters? Explain clearly.

   [Solution]

   A 16-cell cluster cannot be substituted by two clusters of 7 and 9 cells each, as the resultant configuration of cells will not be as closely packed as the original 16-cell cluster and therefore will have gaps. This means that the original area covered by the 16-cell cluster, may not be fully covered in the new configuration. It is also possible that certain areas not covered earlier will be covered now, which is not acceptable.

9. For the following cell pattern,

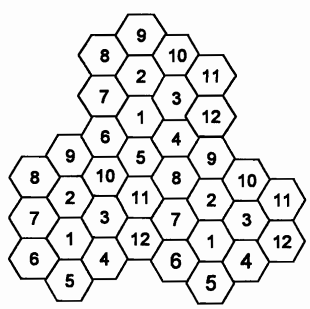


Figure for problem 5.9

(a) Find the reuse distance if radius of each cell is 2 kms.

(b) If each channel is multiplexed among 8 users, how many calls can be simultaneously processed by each cell if only 10 channels per cell are reserved for control, assuming a total bandwidth of 30 MHz available and each simplex channel of 25 kHz ?

(c) If each user keeps a traffic channel busy for an average of 5% time and an average of 60 requests per hour are generated, what is the Erlang value ?

[Solution]

(a) $D = \sqrt{3N}R \Rightarrow D = 2 * (3 * 12)^{0.5}$
The reuse distance = 12 kms

(b) One duplex channel = 2 (BW of one simplex channel) = $2 * 25 = 50$ kHz
Number of channels = $(\frac{30*10^3}{50}) - 10 * 12 = 600 - 120 = 480$ channels
Number of channels per cell = $\frac{480}{12} = 40$ /cell
Total number of calls per cell = $8 * 40 = 320$ calls/cell

(c) The request rate $\alpha = \frac{60}{3600} = \frac{1}{60}$ requests/second
Holding time = 5% = $.05 * 3600 = 5 * 36 = 180$ seconds
Therefore the offered traffic load in Erlangs is
a = request rate $*$ holding time = $(\frac{1}{60}) * 180 = 3$ Erlangs.

10. A TDMA-based system shown in the Figure, has a total bandwidth of 12.5 MHz and contains 20 control channels with equal channel spacing of 30 kHz. Here, the area of each cell is equal to 8 km^2 and cells required to cover a total area of 3600 km^2. Calculate the following:

(a) Number of traffic channels/cell

(b) Reuse distance

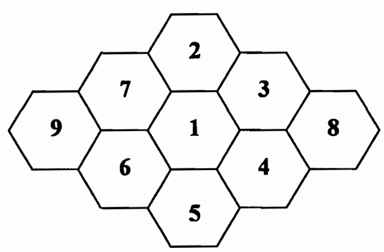


Figure of Problem 5.10.

[Solution]

(a)

$$\frac{\frac{12.5\times10^6}{30\times10^3} - 20}{9} \approx 44 \text{ traffic channels/cell}$$

(b)

$$
\begin{aligned}
D &= \sqrt{3N}R \\
&= \sqrt{3 \times 9} \times 4 \\
&= 9.12 \text{ Km}
\end{aligned}
$$

11. During a busy hour, the number of calls per hour for each of the 12 cells of a cellular cluster is 2220, 1900, 4000, 1100, 1000, 1200, 1800, 2100, 2000, 1580, 1800 and 900. Assume that 75% of the car phones in this cluster are used during this period and that one call is made per phone.

    (a) Find the number of customers in the system.

    (b) Assuming the average hold time of 60 seconds, what is the total Erlang value of the system ?

    (c) Find the reuse distance D if $R = 5$ kms.

    [Solution]

    (a) Number of customers in the system $= (2220 + 1900 + 4000 + 1100 + 1000 + 1200 + 1800 + 2100 + 2000 + 1580 + 1800 + 900)/0.75 = 16200$

    (b) Average number of calls in each cell $= (2220 + 1900 + 4000 + 1100 + 1000 + 1200 + 1800 + 2100 + 2000 + 1580 + 1800 + 900)/12 = 1800$
    Holding time $= 60$ seconds
    Request rate $\alpha = \frac{1800}{3600} = 0.5$ calls/second
    Therefore the offered traffic load in Erlangs is
    $a =$ request rate $*$ holding time $= 0.5 * 60 = 30$ Erlangs

    (c) $R = 5$ km
    Reuse distance $= \sqrt{3N}R = \sqrt{3 * 12} * 5 = 30$ km

12. Given a bandwidth of 25 MHz and a frequency reuse factor of 1 and RF channel size of 1.25 MHz and 38 calls per RF channel, find:

    (a) The number of RF channels for CDMA.

    (b) The number of permissible calls per cell (CDMA).

    [Solution]

    (a) The number of RF channels $= \frac{25}{1.25} = 20$ channels

    (b) The number of permissible calls per cell $= 20 * 38 = 760$ calls per cell.

13. If a wireless service provider has 20 cells to cover the whole service area, with each cell having 40 channels, how many users can the provider support if a blocking probability p of 2 % is required? Assume that each user makes an average of three calls/hour and each call duration is an average of three minutes. (**Erlang B** values are given in the Appendix A)

    [Solution]

    For 40 channels and a blocking probability of 2% (i.e., 0.02), we get an

$$
\begin{aligned}
A &= 30.997 \text{ Erlangs (from the } \textbf{Erlang B} \text{ table)} \\
A &= \left(\frac{\text{number of calls per hour}}{\text{time in seconds}}\right) * \text{holding time in seconds} \\
&= \left(\frac{\text{number of customers} * 3}{3600}\right) * 3 * 60 = \frac{n * 3}{20} = 30.997
\end{aligned}
$$

    solving for n, we get,

$$n = 206.67 \text{ customers per cell}$$

    Total number of users in the system $= 206.67 * 20 = 4133$ users (approx)

14. The following figure shows a cellular architecture. Is there some specific reason why it could have been designed this way ?

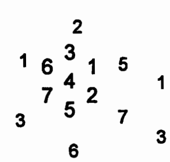


Figure for problem 5.14

    [Solution]

    The cellular architecture is designed in a way to cater to increased traffic in some areas. Smaller cells can handle a larger number of users per square kilometer as the number of allocated channels to each cell is the same. In addition, interference among cells using the same frequencies is minimized. This way there is a greater frequency reuse and hence greater capacity within the limited bandwidth provided.

15. The following figure shows the cell structure of a metro area. Can you explain why this might have been designed so?

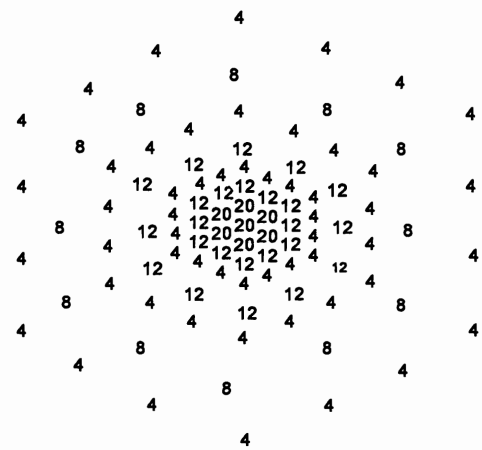


Figure for problem 5.15

[Solution]

The smaller cells and the greater number of channels assigned to the cells at the center of the cluster indicate regions of very high traffic (like the business district of a large metropolis). This provides better service in terms of coverage and capacity, in an area where the greatest traffic is expected to be generated. Moving away from the business areas, outwards towards the suburbs, the number of channels available per cell also decreases, in expectation with the lesser number of calls made from the residential areas of a city. It is also possible that the inner cells may be activated/deactivated based on dynamic conditions like traffic or time of the day. This is a way of using limited resources frequency.

16. Prove the following for a hexagonal cellular system with radius R, reuse distance D and given the value of N:

    (a) $N = 3$, prove $D = 3R$.
    (b) $N = 4$, prove $D = \sqrt{12}R$.
    (c) $N = 7$, prove $D = \sqrt{21}R$.

    [Solution]

    Using reuse distance $D = R\sqrt{3N}$

    (a) $N = 3, \ D = \sqrt{3 * 3}R = 3R$
    (b) $N = 4, \ D = \sqrt{3 * 4}R = \sqrt{12}R$
    (c) $N = 7, \ D = \sqrt{3 * 7}R = \sqrt{21}R$

17. In Figure 5.14, calculate the co-channel interference ratio in the worst case for the forward channel, given $N = 7$, $R = 3$ kms, and $\gamma = 2$.

    [Solution]

    $$D = \sqrt{3N}R$$

    $$q = \frac{D}{R} = \frac{\sqrt{3N}R}{R} = \sqrt{3N} = \sqrt{3 * 7} = 4.58$$

    $$\begin{aligned}\frac{C}{I} &= \frac{1}{2(q-1)^{-\gamma} + 2(q)^{-\gamma} + 2(q+1)^{-\gamma}} \\ &= 3.17\end{aligned}$$

18. What is meant by handoff interval and handoff region? Explain their usefulness with appropriate diagrams.

    [Solution]

    Handoff interval is the time duration required for completing a handoff. Handoff region is region where the mobile station requests the base station for handoffs.

19. What are the differences between adjacent channel interference and cochannel interference? Explain with suitable diagrams.

    [Solution]

    Adjacent channel interference is caused when adjacent channels affect the signal in the current channel due to side bands. Co-channel interference is the interference caused due to ongoing transmissions in different cells which are using the same channel.

20. What are the advantages of cell-sectoring? Explain with suitable diagrams.

    [Solution]

    Cell sectoring helps in reducing co-channel interference and improve spatial reuse of the frequencies.