E933200 Signals and Systems Fall 2016

Final Exam

January 9, 2017

Rules and Regulations: It is permitted to bring three pieces of paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

1. Please determine whether each of the following statements is *True* or *False*.

   (a) (3%) The discrete-time Fourier transform of a periodic signal is discrete in frequency-domain.

   (b) (3%) For a real signal $x(t)$, the Fourier transform of $x_o(t)$ is $j\Im\{X(e^{j\omega})\}$.

   (c) (3%) If we sample the signal $\cos(3t)$ with sampling frequency $\omega_s = 5$, then there is no aliasing.

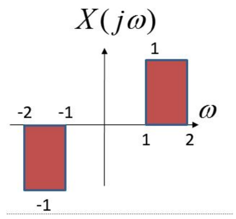
   (d) (3%) The spectrum given by

   $$X(e^{j\omega}) = \frac{\sin(10\omega/2)}{\sin(\omega/2)}$$

   is a possible discrete-time Fourier transform of a discrete-time signal.

   (e) (3%) For a complex signal $x(t)$, the Fourier transform of $x_e(t)$ is $\Re\{X(e^{j\omega})\}$.

   (f) (3%) The inverse Fourier transform of the following spectrum is real and odd in time-domain.



   (g) (3%) The spectrum $X(e^{j\omega})$ of the discrete-time signal $x[n] = u[n-1] + 3u[-n-1] + 100\delta[n+17]$ satisfies

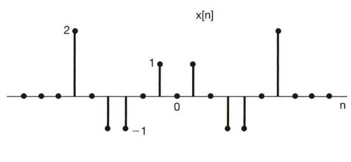
   $$\int_{-\pi}^{\pi} X(e^{j\omega})d\,\omega = 0.$$

   (h) (3%) The spectrum $X(e^{j\omega})$ of the discrete-time signal $x[n] = 5u[n-1] - 5u[-n-1]$ satisfies

   $$X(e^{j0}) = 0.$$

   (i) (3%) The signal $x(t)$ and $y(t) = x(t-1)$ have the same Nyquist rate.

   (j) (3%) The spectrum $X(e^{j\omega})$ of the following discrete-time signal $x[n]$ satisfies $\Re\{X(e^{j\omega})\} = 0$.



2. (10%) Find and sketch the inverse continuous-time Fourier transform of

$$X(j\omega) = \mathrm{sinc}^2\left(\frac{\omega}{2\pi}\right).$$

3. (10%) Please find the Fourier transform of the continuous-time signal

$$x(t) = \frac{1}{\pi t}.$$

4. (a) (10%) Please find the Fourier transform of $u[n]$.
   (Hint: $\mathrm{sgn}[n] \xleftrightarrow{\mathcal{F}} 2/(1-e^{-j\omega})$.)

   (b) (10%) If we have the Fourier transform pair $x[n]$ and $X(e^{j\omega})$, then please show that

$$\sum_{m=-\infty}^{n} x[m] \xleftrightarrow{\mathcal{F}} \frac{1}{1-e^{-j\omega}}X(e^{j\omega}) + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k).$$

5. (10%) Please determine $x[n]$ according to the following four facts given about a real signal $x[n]$ with Fourier transform $X(e^{j\omega})$:

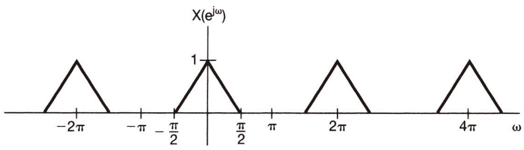
   (i) $x[n] = 0$ for $n > 0$.

   (ii) $x[0] > 0$.

   (iii) $\Im\{X(e^{j\omega})\} = \sin(\omega) - \sin(2\omega)$.

   (iv)
$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega = 3.$$

6. Let $x[n]$ be a signal with Fourier transform $X(e^{j\omega})$ as illustrated in the figure below.



Let

$$w[n] = x[n]p[n]$$

where

$$p[n] = \sum_{k=-\infty}^{\infty} \delta[n-4k].$$

   (a) (5%) Find the Fourier transform $P(e^{j\omega})$ of $p[n]$.

   (b) (5%) Please sketch the Fourier transform of $w[n]$.

7. (10%) Please show that the Fourier transform of

$$x(t) = A \cdot \mathrm{rect}\left(\frac{t}{T}\right) \cdot \cos(\omega_c t)$$

is

$$X(j\omega) = \frac{AT}{2}\mathrm{sinc}\left(\frac{(\omega-\omega_c)T}{2\pi}\right) + \frac{AT}{2}\mathrm{sinc}\left(\frac{(\omega+\omega_c)T}{2\pi}\right).$$