EQ33200 Signals and Systems Fall 2017

Solutions to Midterm Exam I.

1. (a) O

(b) O

(c) O

(d) X $\rightarrow$ $x[n+3]\cdot\delta[n+3] = x[0]\cdot\delta[n-3]$.

(e) O $\rightarrow$ The inverse system is $x[n] = \sum_{k=-\infty}^{n} y[k]$

(f) X $\rightarrow$ $y_1[n] = 3x_1[n] - 3$

$y_2[n] = 3x_2[n] - 3$

Let $x_3[n] = x_1[n] + x_2[n]$

$\Rightarrow y_3[n] = 3x_3[n] - 3$

$= 3x_1[n] + 3x_2[n] - 3 \neq y_1[n] + y_2[n]$

(g) O $\rightarrow$ Let $z(t) = y(-t)$

$\because x(t) * z(t) = \int_{-\infty}^{\infty} x(\tau) z(t-\tau) d\tau$

$\Rightarrow x(t) * y(-t) = \int_{-\infty}^{\infty} x(\tau) y(-(t-\tau)) d\tau = \int_{-\infty}^{\infty} x(\tau) y(\tau - t) d\tau$

(h) O $\rightarrow$ Assume $x(t) = x_{e_1}(t) + x_{o_1}(t)$ and $x(t) = x_{e_2}(t) + x_{o_2}(t)$

$\Rightarrow x_{e_1}(t) + x_{o_1}(t) = x_{e_2}(t) + x_{o_2}(t)$ —— ①

$\Rightarrow x_{e_1}(-t) + x_{o_1}(-t) = x_{e_2}(-t) + x_{o_2}(-t)$ —— ②

$\because x_{e_i}(t) = x_{e_i}(-t)$, $x_{o_i}(t) = -x_{o_i}(-t)$ for $i = 1, 2$.

$\therefore$ ② $\Rightarrow x_{e_1}(t) - x_{o_1}(t) = x_{e_2}(t) - x_{o_2}(t)$ —— ③

$\Rightarrow$ ① + ③ $\Rightarrow x_{e_1}(t) = x_{e_2}(t)$

$\Rightarrow$ ① − ③ $\Rightarrow x_{o_1}(t) = x_{o_2}(t)$

(i) X $\rightarrow$ Let $x(t) \rightarrow y(t) = t\,x(t)$

If $x(t) = \cos(t)$

then $y(t) = t\cdot\cos(t)$ is not periodic

(j) X $\longrightarrow$ Let $n = n' + 94$

$$y[n] = \cos^2[(n-87)^2]\, x[n-94]$$

$$\Rightarrow y[n'+94] = \cos^2[[(n'+94)-87]^2]\, x[(n'+94)-94]$$

$$\Rightarrow \frac{y[n'+94]}{\cos^2[(n'+7)^2]} = x[n']$$

2. $y[n] = x[n] * h[n]$

$$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$$

$$\Rightarrow y[-n] = \sum_{k=-\infty}^{\infty} x[k]\, h[-n-k]$$

(Let $m = -k$)

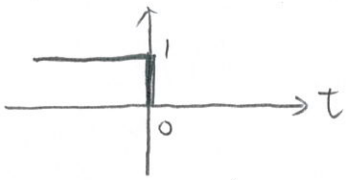
$$\Rightarrow \sum_{m=-\infty}^{\infty} x[-m]\, h[-n+m]$$

$$= -\sum_{m=-\infty}^{\infty} x[m]\, h[n-m] \quad \left(\because \begin{array}{l} x[n] \text{ is even} \\ h[n] \text{ is odd} \end{array}\right)$$

$$= -y[n]$$

3.

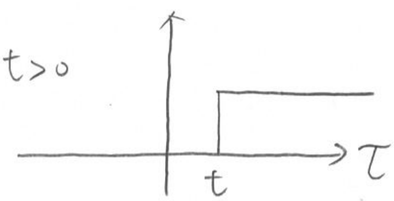
$x(t) = u(-t)$



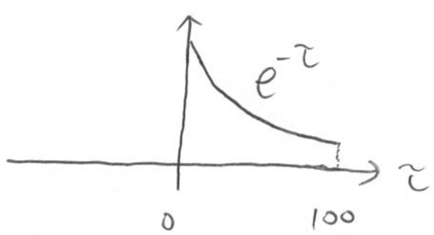
$h(t) = e^{-t}(u(t) - u(t-100))$

$x(t-\tau) = u(-t+\tau)$

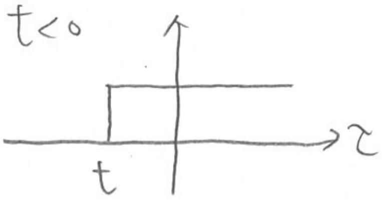
$t>0$



$h(\tau) = e^{-\tau}(u(\tau) - u(\tau-100))$



$t<0$



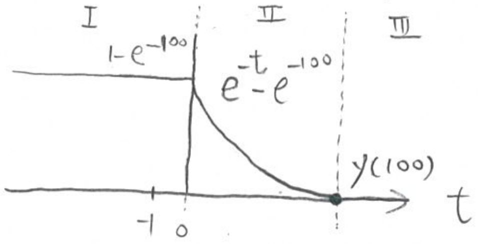
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

(I) for $t<0$, $y(t) = \int_0^{100} e^{-\tau} d\tau = -e^{-\tau}\Big|_0^{100} = 1 - e^{-100}$.

(II) $0 \le t \le 100$, $y(t) = \int_t^{100} e^{-\tau} d\tau = -e^{-\tau}\Big|_t^{100} = -e^{-100} + e^{-t} = e^{-t} - e^{-100}$

(III) $t \ge 100$, $y(t) = 0$

$\Rightarrow y(t)$



4.

(1) Let $x_1(t) \to y_1(t)$

and $x_2(t) \to y_2(t)$

Then $a x_1(t) + b x_2(t) \to a y_1(t) + b y_2(t)$

Also let $y_1(t) \to \boxed{\text{inverse system}} \to x_1(t)$

$y_2(t) \to \boxed{\text{inverse system}} \to x_2(t)$

Then $a y_1(t) + b y_2(t) \to \boxed{\text{inverse system}} \to a x_1(t) + b x_2(t)$

$\therefore$ Linear

(2) Let $y_1(t) \to \boxed{\text{inverse system}} \to x_1(t)$

Since we have

$x_1(t) \to y_1(t)$

and

$x_1(t-\tau) \to y_1(t-\tau)$,

we can obtain

$y_1(t-\tau) \to \boxed{\text{inverse system}} \to x_1(t-\tau)$

$\therefore$ time-invariant.

5. (a) $h[n] = (-0.5)^n u[n] + (1.01)^n u[n-1]$

$\because$ $h[n] = 0$ for $n < 0$

$\therefore$ causal

(b) $\displaystyle\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} \left|(-\tfrac{1}{2})^k u[k] + (1.01)^k u[k-1]\right| = \frac{1}{2} + \sum_{k=1}^{\infty} \left|(-\tfrac{1}{2})^k + 1.01^k\right|$

$\displaystyle = \frac{1}{2} + \sum_{k=odd} |1.01^k - 0.5^k| + \sum_{k=even} |1.01^k + 0.5^k| \to \infty$ $\therefore$ unstable

6.
(a) $|x[n]| = |sgn(h[-n])| \le 1$

so $x[n]$ is bounded

(b) $y[n] = x[n] * h[n]$

$\displaystyle = \sum_{k=-\infty}^{\infty} x[k] \cdot h[n-k]$

$\displaystyle y[0] = \sum_{k=-\infty}^{\infty} x[k] \cdot h[-k] = \sum_{k=-\infty}^{\infty} sgn(h[-k])\, h[-k]$

$\displaystyle = \sum_{k=-\infty}^{\infty} |h[-k]| = \sum_{k=-\infty}^{\infty} |h[k]|$

7. $y[n] = x[n] * h_1[n] * h_2[n]$

$= (x[n] * h_2[n]) * h_1[n]$

$\because$ $x[n] * h_2[n] = (e^2 \delta[n] - \delta[n-1]) * e^{-2n} u[n]$

$= e^{-2(n-1)} u[n] - e^{-2(n-1)} u[n-1]$

$= e^{-2(n-1)} (u[n] - u[n-1])$

$= e^{-2(n-1)} \delta[n]$

$= e^2 \delta[n]$

$\therefore$ $y[n] = e^2 \delta[n] * h_1[n]$

$= e^2 \delta[n] * e^{\sin(\pi n^2)}$

$= e^{\sin(\pi n^2) + 2}$

8. ① $y[n] = x_o[n]$

$= \frac{1}{2}[x[n] - x[-n]]$

$y[n-n_0] = \frac{1}{2}(x[n-n_0] - x[-n+n_0])$

Let $x'[n] = x[n-n_0] \rightarrow y'[n] = \frac{1}{2}(x'[n] - x'[-n])$

$= \frac{1}{2}(x[n-n_0] - x[-n-n_0])$

$\therefore$ time-Varying $\neq y[n-n_0]$

②

$y(t) = e^{t+1}\sin(x(2t-1))$

$y(2) = e^{3}\sin(x(3))$ depends on $x(3)$

$\therefore$ noncausal