國立成功大學
工科系統微積分(一) 期中考
11月 13 日, 2015

課程代碼: F115611
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Instructions:

1. There are **7 pages** (including the cover page), **20 problems** in this exam.
2. You have **110 minutes** to work on the exam.
3. Do **NOT** start the exam until you are told to do so.
4. Only the answers written above the answer lines will be graded.
5. Please have your **student ID** card ready.
6. No textbook, notes, calculator, or sketching sheets are allowed.
7. You may want to use the back of the exam pages for computations.

Page:	1	2	3	4	5	6	Total
Points:	25	10	15	25	15	10	100
Score:							

1. Evaluate the following limits.

(a) (5 points)

$$\lim_{t\to 0}\left(\frac{1}{t\sqrt{1+t}}-\frac{1}{t}\right)=$$

(a) _____

Solution: $\frac{-1}{2}$

(b) (5 points)

$$\lim_{x\to \pi}\frac{\sin(x+\cos x)}{x}=$$

(b) _____

Solution: $\frac{\sin(\pi-1)}{\pi}$

(c) (5 points)

$$\lim_{x\to 0}\frac{\cos^{10} x-1}{x}=$$

(c) _____

Solution: 0

(d) (5 points)

$$\lim_{x\to \infty} x\sin\frac{2}{x}=$$

(d) _____

Solution: 2

(e) (5 points)

$$\lim_{n\to\infty}\left(\sum_{i=1}^{n}\frac{32i^4}{n^5}\right)=$$

(e) _____

Solution: $\frac{32}{5}$

2. (5 points) Which of the following statement is **not** true?

A. $\lim_{x\to 0} \sin\frac{\pi}{x}$ doesn't exist;

B. $f(x) = \frac{x^2-x-2}{x-2}$ is not continuous at $x = 2$ but $\lim_{x\to 2} f(x)$ exists;

C. $f(x) = |x|$ is a continuous function that is not differentiable at $x = 0$;

D. $f(x) = \sqrt[3]{x}$ is differentiable at $x = 0$;

E. $\lim_{x\to 0^-} \left(\frac{1}{x} - \frac{1}{|x|}\right) = -\infty$.

2. _____

Solution: D.

3. (5 points) Which of the following formula is **not** correct?

A. $\int \frac{x}{\sqrt{2+2x}}\,dx = \frac{1}{3}(x-2)\sqrt{2+2x} + C$;

B. $\int \cos^2 x\,dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$;

C. $\int \cos^3 x\,dx = \sin x - \frac{1}{3}\sin^3 x + C$;

D. $\int \frac{\cos x}{\sin^2 x}\,dx = -\csc x + C$;

E. $\int \frac{x}{\sqrt{x^2+1}}\,dx = \sqrt{x^2+1} + C$.

3. _____

Solution: B.

4. (5 points) Find the horizontal asymptote of $f(x) = \frac{\sqrt{x}+x^2}{2x-x^2}$.

4. _____

Solution: $y = -1$

5. (5 points) Find the absolute maximum values of $f(x) = x\sqrt{4-x^2}$ on $[-1, 2]$.

5. _____

Solution: 2

6. (5 points) Find the tangent line to the curve $x^3 + y^3 = 4xy$ at the point $(2, 2)$.

6. _____

Solution: $y - 2 = -(x - 2)$.

7. Let

$$f(x) = \int_0^{x^2} \frac{1}{t^2+1}\, dt.$$

(a) (5 points) Find $f'(x)$.

(a) _____

Solution: $\frac{2x}{x^4+1}$

(b) (5 points) Find the critical number(s) of $f(x)$.

(b) _____

Solution: 0

(c) (5 points) Find the interval(s) on which $f(x)$ is increasing.

(c) _____

Solution: $(0, \infty)$

(d) (5 points) Find the interval(s) on which $f(x)$ is concave up.

(d) _____

Solution: $(-\frac{1}{\sqrt[4]{3}}, \frac{1}{\sqrt[4]{3}})$

(e) (5 points) Find the inflection point(s) of $f(x)$.

Hint: $\int_0^{\frac{1}{\sqrt{3}}} \frac{1}{t^2+1}\, dt = \frac{\pi}{6}$.

(e) _____

Solution: $(\pm\frac{1}{\sqrt[4]{3}}, \frac{\pi}{6})$

8. Evaluate the following definite integrals.

(a) (5 points)

$$\int_0^1 \frac{dx}{(1+\sqrt{x})^3} =$$

(a) _____

Solution: $\frac{1}{4}$

(b) (5 points)

$$\int_0^1 (x + \sqrt{1-x^2})\, dx =$$

(b) _____

Solution: $\frac{1}{2} + \frac{\pi}{4}$

9. (5 points) Evaluate the indefinite integral

$$\int \sec^4 x \tan^3 x \, dx.$$

9. _____

Solution: $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$

10. (5 points) Find the area of the region enclosed by $2x + y^2 = 3$ and $x = y$.

10. _____

Solution: $\frac{16}{3}$

11. (5 points) What is the minimal surface area (including top, bottom, and side) of a cylindrical can with volume 2?

11. _____

Solution: $6\pi^{\frac{1}{3}}$