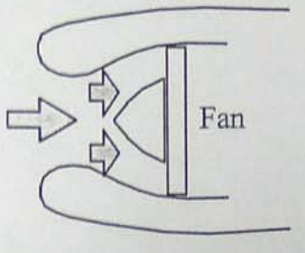
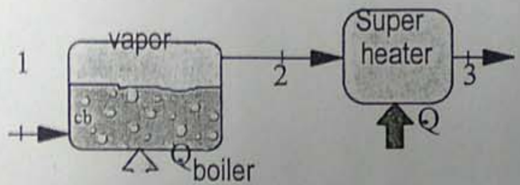
Thermodynamics 2nd midterm, Engineering Science Dept., 20161219

20% for each one of these five problems, total 100%.

1. The front of a jet engine acts similar to a diffuser, receiving air at 900 km/h, - 5°C, 50 kPa, bringing it to 80 m/s relative to the engine before entering the compressor. If the flow area is increased to 120% of the inlet area, find the temperature and pressure in the compressor inlet.



2. Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.008 kg/s and exits as saturated vapor, see below. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Find the rate of heat transfer in the boiler and the super heater. And plot both p-v and T-v diagrams. (Show interpolation calculation)



3. A room is heated with a 2000 W electric heater. How much power can be saved if a heat pump with a COP of 2.5 is used instead?

Draw the physical diagram of "3"

4. Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a heat pump operating from a low-temperature reservoir at 16°C with a COP of half that of a similar Carnot unit. Find the rate of work into the heat pump. Draw the physical diagram of "4"

5. What is the basis for the thermodynamic temperature scale? Prove this proposition given in class

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Thermodynamic Exam II 20161219 Solutions

1. This is C.V. steady state one-flow diffuser problem, where it is assumed $q_{cv}=0$, $W_{c.v}=0$ $\Delta PE=0$, Air is ideal gas with constant C_p $(T<500K)$

Continuity equation $\dot{m}_i = \rho_i A_i V_i = \dot{m}_e = \rho_e A_e V_e$

Energy equation $h_i + \frac{V_i^2}{2} = h_e + \frac{V_e^2}{2}$

$\therefore h_e - h_i = C_p(T_e - T_i) = \frac{1}{2}V_i^2 - \frac{1}{2}V_e^2 = \frac{1}{2}\left(\frac{900\times1000}{3600}\right)^2 - \frac{1}{2}(80)^2 \left[\frac{J}{Kg}\right]$

Note: $J = N.m = \frac{Kg\,m}{s^2}\cdot m \quad \therefore \frac{m^2}{s^2} = \frac{J}{Kg}$

$= 28050\ J/kg$

By Table A5 C_p of Air is $1.004\ KJ/KgK$

$\therefore T_e - T_i = T_e - \overset{-5°C}{268K} = \frac{28.05\ KJ/kg}{1.004\ Kg/kgK} = 27.9K \quad \therefore \underline{\underline{T_e = 296K}}$ Ans.

Now, use the continuity $\frac{\rho_e}{\rho_i} = \frac{A_i V_i}{A_e V_e} = \left(\frac{1}{1.2}\right)\left(\frac{250}{80}\right) = 2.604$

ideal gas $= \left(\frac{P_e}{RT_e}\right)\left(\frac{RT_i}{P_i}\right) = \frac{P_e}{P_i}\frac{268K}{296K} = 2.604$

$\therefore P_e = P_i \times 2.604 \cdot \frac{296K}{268K} = 50psi \times 2.876 = \underline{\underline{143.80\ psi}}$ Ans!

2. This is C.V. steady, one flow process neglecting KE, PE. energies (i.e. $\Delta KE = 0$, $\Delta PE = 0$) with zero works

Use N_2 Table B.6.1，内插求 $P = 600kpa$ 之 $\overset{h_1}{\overset{"}{h_f}}, \overset{h_2}{\overset{"}{h_g}}$, 及 T

$$\frac{779.2 - 541.1}{84.59 - 73.2} = \frac{600 - 541.1}{x} \quad x = 2.82 \quad \therefore h_1 = 84.59 - 2.82 = 81.77 \, KJ/kg$$

$$\frac{779.2 - 541.1}{87.48 - 86.47} = \frac{600 - 541.1}{y} \quad y = 0.25 \quad \therefore h_2 = 86.47 + 0.25 = 86.72 \, KJ/kg$$

$$\frac{792.1 - 541.1}{100 - 95} = \frac{600 - 541.1}{z} \quad z = 1.24K \quad \therefore T_{sat} = 95 + 1.24 = 96.24K$$

$\because$ given $T_3 = 280K > 96.24K$ $\therefore$ State "3" is superheated N_2

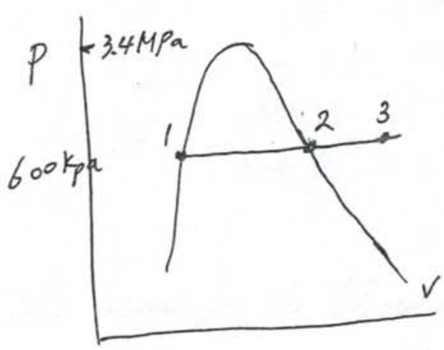
$\therefore$ Use Table B.6.2 get $h_3 = 289.05 \, KJ/kg$

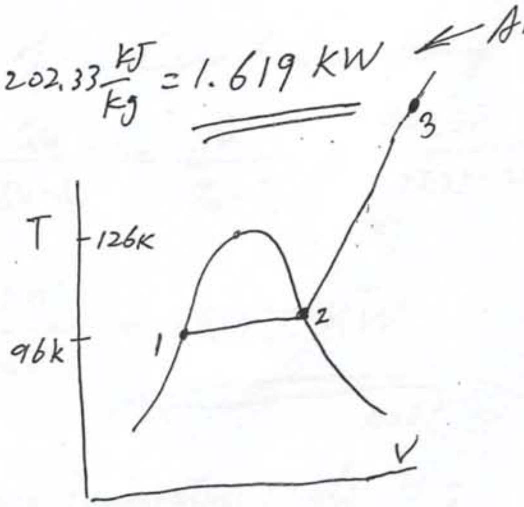
Energy equation for the boiler $q_{boiler} = h_2 - h_1 = 86.72 - 81.77 = 168.49 \, KJ/kg$

$$\dot{Q}_{boiler} = \dot{m} \, q_{boiler} = 0.008 \frac{kg}{s} \times 168.49 \, KJ/kg = \underline{\underline{1.348 \, KW}}$$ Ans!

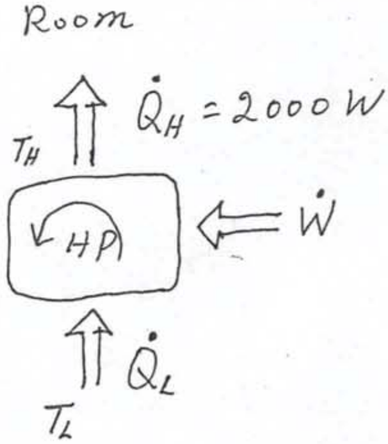
Energy equation for superheater $q_{\substack{super\\heater}} = h_3 - h_2 = 289.05 - 86.72 = 202.33 \, KJ/kg$

$$\dot{Q}_{\substack{super\\heater}} = \dot{m} \, q_{\substack{super\\heater}} = 0.008 \frac{kg}{s} \times 202.33 \frac{KJ}{kg} = \underline{\underline{1.619 \, KW}}$$ Ans!





3. First, draw a heat pump diagram and note that 2000W is its $\dot{Q}_H$



The COP of a heat pump is $\beta' = \dfrac{\dot{Q}_H}{\dot{W}}$

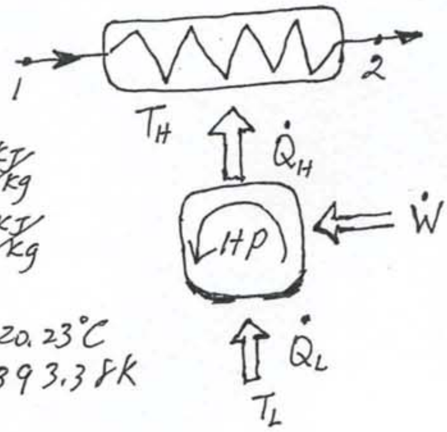
$$\therefore \dot{W} = \dot{Q}_H / \beta' = \frac{2000\,W}{2.5} = 800\,W$$

$\therefore$ The amount of power saved:

$$2000\,W - 800\,W = \underline{1200\,W}$$

Ans!

4. First, draw the physical diagram of this problem showing the heat exchanger whose heat is supplied by a Carnot heat pump.



Use Table B.1.2 for water

given 200 kpa saturated liquid $h_1 = 504.68 \ kJ/kg$

" " saturated vapor $h_2 = 2706.63 \ kJ/kg$

T_H is the saturated temperature for 200 kpa $= 120.23°C = 393.38K$

T_L given : $16°C = 289.15K$

Now it's a C.V. steady one flow heater process with zero work neglecting $\Delta KE, \Delta PE$

Continuity for heat exchanger $\dot{m}_1 = \dot{m}_2 = 60 \ kg/hr = \dot{m}$

Energy equation for this exchanger $\dot{m} h_1 + \dot{Q}_H = \dot{m} h_2$

$$\therefore \dot{Q}_H = \frac{60}{3600}(2706.63 - 504.68) = 36.70 \ kW$$

Now, calculate the COP of a Carnot heat pump

$$\beta' = \frac{\dot{Q}_H}{\dot{W}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_L} = \frac{T_H}{T_H - T_L} = \frac{393.38}{393.38 - 289.15} = 3.77$$

$$\therefore \dot{W} = \frac{\dot{Q}_H}{\text{half } \beta'} = \frac{36.70 kW}{3.77 \times 0.5} = 19.47 \ KW$$

Ans!

5. The basis for the thermodynamic temperature scale is :

First and (or) Second Propositions of Carnot cycle Efficiency

Proposition I : $\eta_{any} \leq \eta_{rev}$ It's impossible to construct an engine that operates between two given reservoirs and is more efficient than a reversible engine operating between the same two reservoirs.

Proposition II : $\eta_{rev1} = \eta_{rev2}$ All engines that operate on the Carnot cycle between two given constant-temperature reservoirs have the same efficiency.

Proof of Proposition I given in Class Notes!

4.26

The front of a jet engine acts similar to a diffuser, receiving air at 900 km/h, −5°C, 50 kPa, bringing it to 80 m/s relative to the engine before entering the compressor. If the flow area is increased to 120% of the inlet area, find the temperature and pressure in the compressor inlet.

Solution:

C.V. Diffuser, Steady state, 1 inlet, 1 exit flow, no q, no w.

Continuity Eq.4.3: $\dot{m}_i = \dot{m}_e = (A\mathbf{V}/v)$

Energy Eq.4.12: $\dot{m}\,(h_i + \frac{1}{2}\mathbf{V}_i^2) = \dot{m}\,(\frac{1}{2}\mathbf{V}_e^2 + h_e)$

$$h_e - h_i = C_p\,(T_e - T_i) = \frac{1}{2}\mathbf{V}_i^2 - \frac{1}{2}\mathbf{V}_e^2 = \frac{1}{2}\left(\frac{900 \times 1000}{3600}\right)^2 - \frac{1}{2}(80)^2$$

$$= \frac{1}{2}(250)^2 - \frac{1}{2}(80)^2 = 28050 \text{ J/kg} = 28.05 \text{ kJ/kg}$$

$\Delta T = 28.05/1.004 = 27.9 \quad \Rightarrow \qquad T_e = -5 + 27.9 = \mathbf{22.9°C}$

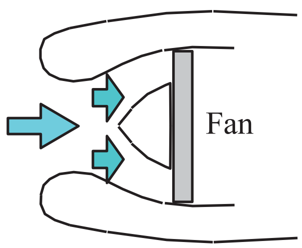
Now use the continuity eq.:

$$A_i\mathbf{V}_i / v_i = A_e\mathbf{V}_e / v_e \quad \Rightarrow \qquad v_e = v_i \left(\frac{A_e\mathbf{V}_e}{A_i\mathbf{V}_i}\right)$$

$$v_e = v_i \times \frac{1.2 \times 80}{1 \times 250} = v_i \times 0.384$$

Ideal gas: $Pv = RT \quad => \quad v_e = RT_e/P_e = RT_i \times 0.384/P_i$

$$P_e = P_i\,(T_e/T_i)/0.384 = 50 \text{ kPa} \times 296/(268 \times 0.384) = \mathbf{143.8 \text{ kPa}}$$



4.55

Saturated liquid nitrogen at 600 kPa enters a boiler at a rate of 0.008 kg/s and exits as saturated vapor, see Fig. P4.55. It then flows into a super heater also at 600 kPa where it exits at 600 kPa, 280 K. Find the rate of heat transfer in the boiler and the super heater.

Solution:

C.V.: boiler steady single inlet and exit flow, neglect KE, PE energies in flow

Continuity Eq.: $\dot{m}_1 = \dot{m}_2 = \dot{m}_3$

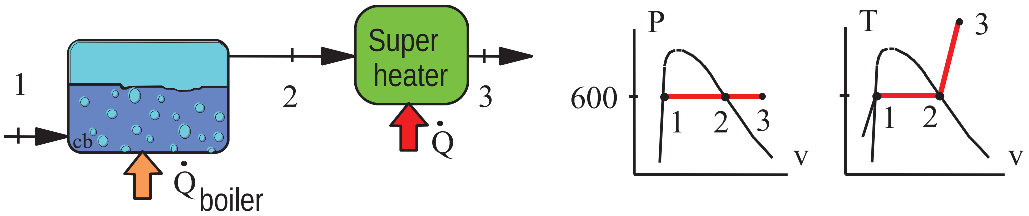


Table B.6.1: h_1 = -81.469 kJ/kg, Table B.6.2: h_2 = 86.85 kJ/kg,

Table B.6.2: h_3 = 289.05 kJ/kg

Energy Eq. 4.13: $q_{boiler} = h_2 - h_1 = 86.85 - (- 81.469) = 168.32$ kJ/kg

$\dot{Q}_{boiler} = \dot{m}_1 q_{boiler} = 0.008 \text{ kg/s} \times 168.32 \text{ kJ/kg} = \mathbf{1.346 \text{ kW}}$

C.V. Superheater (same approximations as for boiler)

Energy Eq. 4.13: $q_{sup\ heater} = h_3 - h_2 = 289.05 - 86.85 = 202.2$ kJ/kg

$\dot{Q}_{sup\ heater} = \dot{m}_2 q_{sup\ heater} = 0.008 \text{ kg/s} \times 202.2 \text{ kJ/kg} = \mathbf{1.62 \text{ kW}}$

5.18

A room is heated with a 2000 W electric heater. How much power can be saved if a heat pump with a COP of 2.5 is used instead?

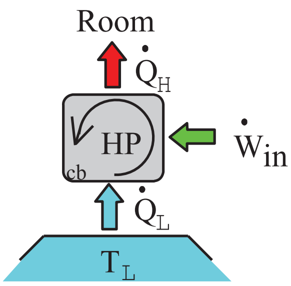
Assume the heat pump has to deliver 2000 W as the $\dot{Q}_H$.

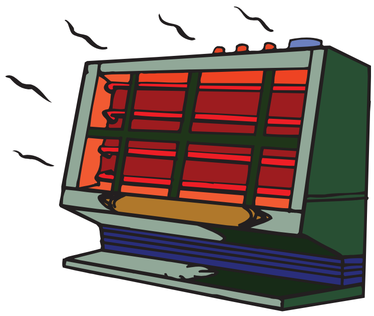
Heat pump: $\beta' = \dot{Q}_H/\dot{W}_{IN}$

$$\dot{W}_{IN} = \dot{Q}_H/\beta' = \frac{2000}{2.5} = 800 \text{ W}$$

So the heat pump requires an input of 800 W thus saving the difference

$$\dot{W}_{saved} = 2000 \text{ W} - 800 \text{ W} = \mathbf{1200\ W}$$





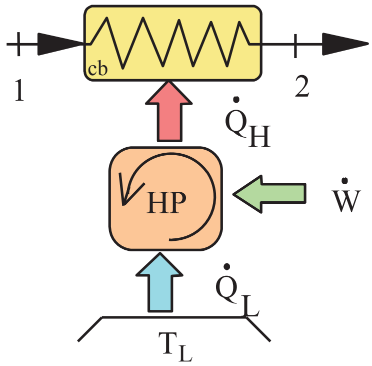
Microsoft clipart.

5.58

Sixty kilograms per hour of water runs through a heat exchanger, entering as saturated liquid at 200 kPa and leaving as saturated vapor. The heat is supplied by a Carnot heat pump operating from a low-temperature reservoir at 16°C with a COP of half that of a similar Carnot unit. Find the rate of work into the heat pump.

Solution:

C.V. Heat exchanger



$\dot{m}_1 = \dot{m}_2$; $\dot{m}_1 h_1 + \dot{Q}_H = \dot{m}_1 h_2$

Table B.1.2: $h_1 = 504.7$ kJ/kg,

$h_2 = 2706.7$ kJ/kg

$T_H = T_{sat}(P) = 120.93 + 273.15$

$= 394.08$ K

$$\dot{Q}_H = \frac{60}{3600}(2706.7 - 504.7) = 36.7 \text{ kW}$$

First find the COP of a Carnot heat pump.

$$\beta' = \dot{Q}_H/\dot{W} = T_H / (T_H - T_L) = 394.08 / (394.08 - 289.15) = 3.76$$

Now we can do the actual one as $\beta'_H = 3.76/2 = 1.88$

$$\dot{W} = \dot{Q}_H/\beta'_H = 36.7 \text{ kW} / 1.88 = \mathbf{19.52 \text{ kW}}$$

5. 的第 1 小題，寫任一個 Proposition 都給對

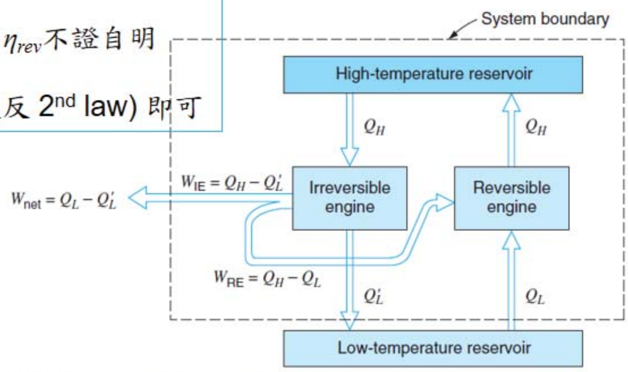
5. 的第 2 小題，是要寫 1st Proposition 的證明，不是去推出 $Q_H/Q_L = T_H/T_L$

圖要畫

Prove Carnot cycle 1st Proposition: $\eta_{any} \le \eta_{rev}$

(same T_H and T_L)

觀念釐清 (不用寫)
η any = η irreversible & η reversible
η_{rev} 一定= η_{rev} (根本一樣的東西)， $\eta_{rev} \le \eta_{rev}$ 不證自明
剩下的 $\eta_{irr} \le \eta_{rev}$
只要去證明 $\eta_{irr} > \eta_{rev}$ 是違反自然的 (違反 2nd law) 即可



在 $\eta_{irr} > \eta_{rev}$ 的情況下
RE機器有 Q_H 進，Q_L 出；IE機器有 Q_H 進，Q_L'出；兩個 Q_H 設定成一樣
因為 $\eta_{irr} > \eta_{rev}$，所以 W_{irr} 會> W_{rev}
W_{irr} 是QH-QL'
W_{rev} 是QH-QL
由此可知 Q_L 會> Q_L'

整體來看，輸入的淨熱傳 Q_{net} 全部變成淨功 W_{net} 做出去
這違反了2nd law (違反KP)
得證 「$\eta_{irr} \le \eta_{rev}$」

(ps rev 就是 carnot)

Carnot cycle 2nd Proposition $\eta_{rev\ 1} = \eta_{rev\ 2}$ (same T_H and T_L)

看過就好，免證明

absolute temperature scale $\frac{Q_H}{Q_L} = \frac{T_H}{T_L}$

這推導，等其他地方都OK了
再去課本看

由此可衍伸出：

$$\beta = \frac{Q_L}{Q_H - Q_L} \underset{\text{Carnot}}{=} \frac{T_L}{T_H - T_L}$$

$$\beta' = \frac{Q_H}{Q_H - Q_L} \underset{\text{Carnot}}{=} \frac{T_H}{T_H - T_L}$$

$$\eta_{\text{thermal}} = 1 - \frac{Q_L}{Q_H} = 1 - \frac{T_L}{T_H}$$

另外，移項一下，後面Clausius inequality會用到：

$$\frac{Q_H}{T_H} = \frac{Q_L}{T_L}$$