# Midterm Exam II

## December 13, 2017

**Rules and Regulations:** It is permitted to bring one paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

**Problems for Solution:**

1. Please determine whether each of the following statements is *True* or *False*.

   (a) (4%) For a standard normal random variable $X$, we have $E(X^3) = 0$.

   (b) (4%) The function $f(x) = 0.5 - e^{-x}, x > 0$, is the probability density function of a random variable.

   (c) (4%) For any random variable $X$, we have $E(X^2) \geq (E(X))^2$.

   (d) (4%) For a Poisson random variable with parameter $\lambda = 3.9$, its probability mass function $p(x)$ has the largest value at $x = 4$. x

   (e) (4%) A box contains 20 apples, of which 5 are black. The expected number of black apples among 4 apples selected randomly is 1. $\frac{4\cdot 5}{20} = 1$

2. (10%) If $X$ is a Poisson random variable with parameter $\lambda > 0$, please show that

$$\frac{P(X = i+1)}{P(X = i)} = \frac{\lambda}{i+1}.$$

3. (10%) Suppose that, from a box containing $D$ defective and $N - D$ good fuses, fuses are drawn one by one, at random and without replacement. Show that the probability that the $k$th item drawn is defective equals $D/N$ where $k \leq \min(D, N - D)$.
   (Hint: Let $E_j$ be the event of obtaining exactly $j$ defective fuses among the first $(k-1)$ draws. Let $A_k$ be the event that the $k$th item drawn is defective. Then the desired probability is $P(A_k) = \sum_{j=0}^{k-1} P(A_k|E_j)P(E_j)$.)

4. An absentminded professor does not remember which of his 12 keys will open his office door. If he tries them at random and with replacement:

   (a) (5%) On average, how many keys should he try before his door opens? 試n次 才打開 E(X).

   (b) (5%) What is the probability that he opens his office door after only three tries?

5. (10%) The average grade for Midterm Exam is 80, and the standard deviation is 9. If 5.05% of the class is awarded a certificate of merit, and the grades are curved to follow a normal distribution, what is the lowest possible grade to get a certificate of merit? Note that $\Phi(1.64) = 0.9495$.

6. (a) (5%) If $X$ is a binomial random variable with parameters $(n, p)$, find $E(X^3)$.

   (b) (5%) Let $X$ be a geometric random variable with parameter $p$. Find $E(X^3)$.

7. (10%) Suppose the number of fish that a fisherman catches follows a Poisson process with rate $\lambda = 0.5$ per hour. The fisherman will keep fishing for at least two hours. If he has caught at least one fish, he quits. Otherwise, he continues until he catches at least one fish. Find the probability that he catches at least 4 fish. Note that $e^{-1} = 0.3679$.

8. (10%) Let $X$ be a uniform random number over the interval $(0, 1)$. Find the density function of $Y = -2\ln(1 - X)$.

9. (10%) 感謝提供題目的楊登宇、劉川榮同學。工科系最近舉辦了導生盃九宮格投球大賽，每位同學可投十顆球。參賽同學小宇投球命中率高達90%。請問，小宇在最後一顆投球(第十次投球)時，完美擊破九格的機率爲多少？提示: Negative binomial random variable.