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Fundamentals of Momentum, Heat, and Mass Transfer

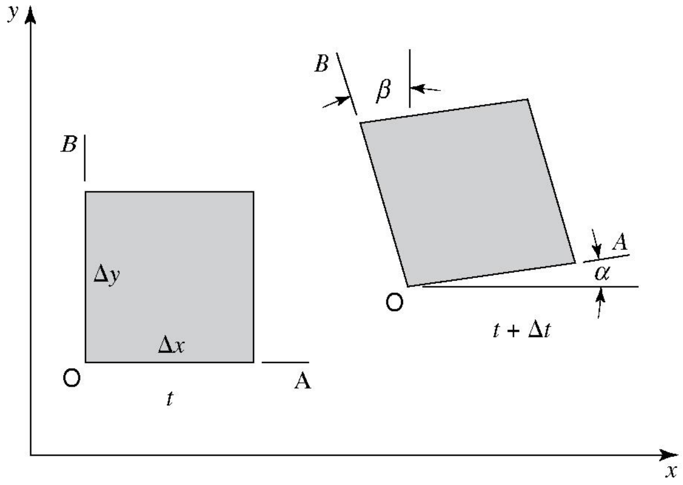
Sixth Edition

Welty • Rorrer • Foster

Chapter 10

Inviscid Fluid Flow

Vorticity



Definition from Stokes's theorem

$$\omega_z = \frac{1}{2}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right) \qquad (10\text{-}1)$$

$$\omega_y = \frac{1}{2}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) \qquad (10\text{-}2)$$

$$\omega_x = \frac{1}{2}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) \qquad (10\text{-}3)$$

$$\boldsymbol{\nabla} \times \mathbf{v} = 2\omega \qquad (10\text{-}4)$$

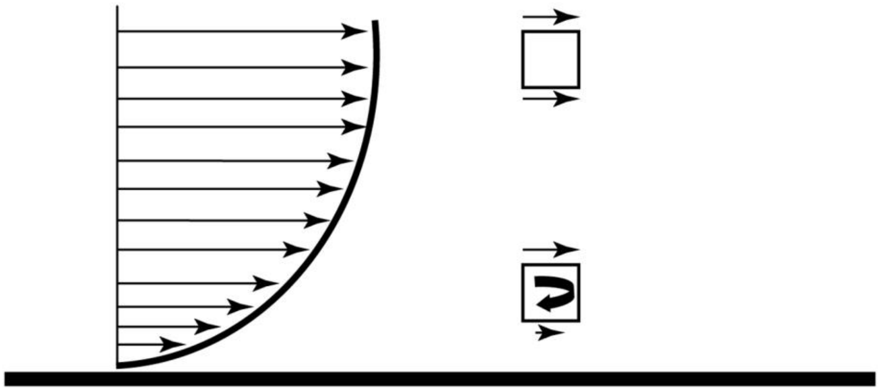
Fig_10-1a

Navier-Stokes Equation

$$\rho \frac{D\mathbf{v}}{Dt} = \rho\mathbf{g} - \nabla P + \mu\nabla^2\mathbf{v} \qquad (9\text{-}19)$$

For an **irrotational** flow (curl **v = 0**),

$$\rho \frac{Dv}{Dt} = -\nabla P + \rho\mathbf{g} - \mu[\nabla \times (\nabla \times v)] \qquad (9\text{-}29)$$



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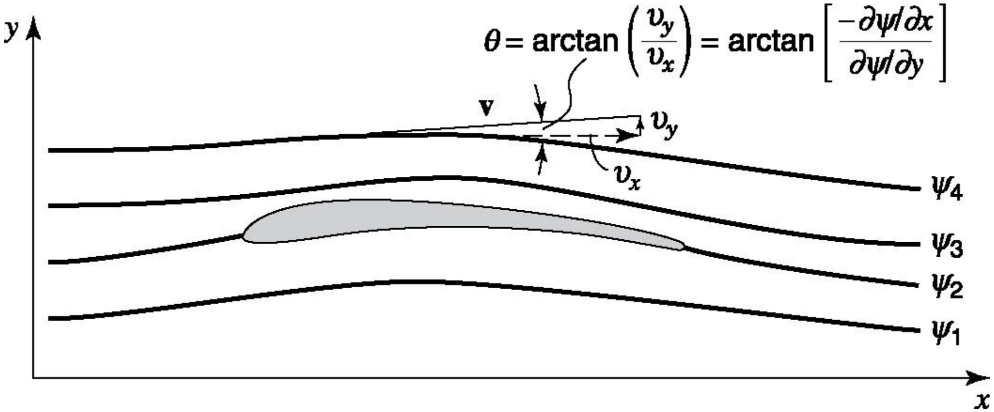
Streamlines and stream function



$$d\Psi = -v_y dx + v_x dy \qquad (10\text{-}5)$$

$$\frac{dy}{dx}\Big|_{\Psi=\text{constant}} = \frac{v_y}{v_x} \qquad (10\text{-}6)$$

Vorticity

$$-2\omega_z = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} \qquad (10\text{-}7)$$

For an **irrotational** flow (curl **v = 0**),

$$\nabla^2\Psi = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} = 0 \qquad (10\text{-}8)$$

Laplace's equation

Fig_10-2

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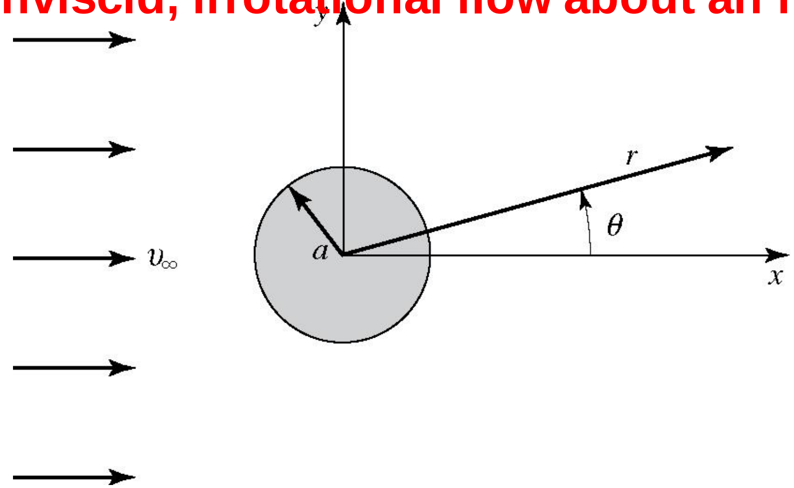
Inviscid, irrotational flow about an infinite cylinder



$$\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{r}\frac{\partial\Psi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Psi}{\partial\theta^2}=0 \qquad (10\text{-}9)$$

$$v_r=\frac{1}{r}\frac{\partial\Psi}{\partial\theta} \qquad v_\theta=-\frac{\partial\Psi}{\partial r} \qquad (10\text{-}10)$$

Fig_10-3

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$$\Psi(r,\theta) = v_\infty r \sin\theta \left[1 - \frac{a^2}{r^2}\right] \qquad (10\text{-}11)$$

$$v_r = \frac{1}{r}\frac{\partial \Psi}{\partial \theta} = v_\infty \cos\theta \left[1 - \frac{a^2}{r^2}\right] \qquad (10\text{-}12)$$

$$v_\theta = -\frac{\partial \Psi}{\partial r} = -v_\infty \sin\theta \left[1 + \frac{a^2}{r^2}\right] \qquad (10\text{-}13)$$

$$v_\theta = -2 v_\infty \sin\theta \qquad (10\text{-}14)$$

Eq_10-12

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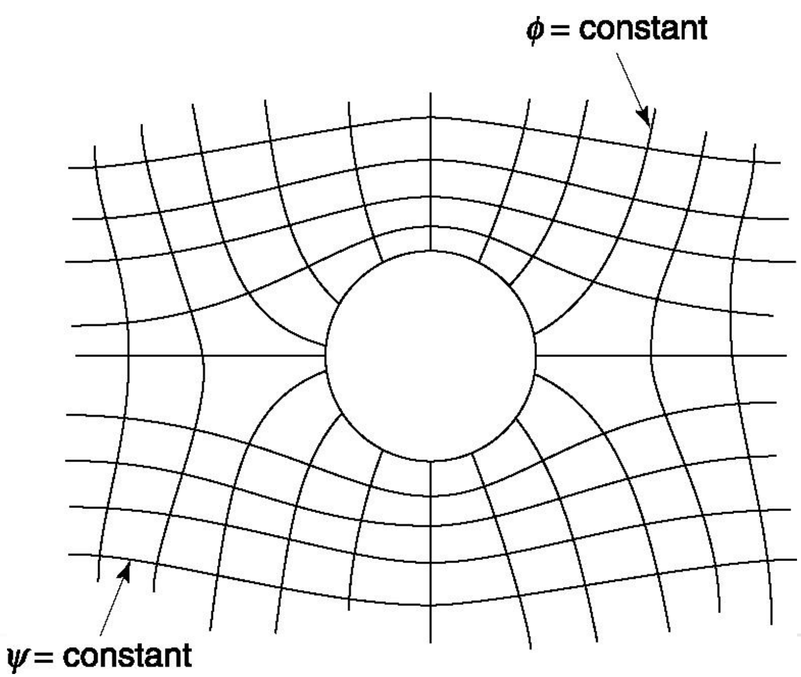
Velocity potential function

$$\mathbf{v} = \nabla\phi \qquad (10\text{-}15)$$

$$\nabla \cdot \nabla\phi = \nabla^2\phi = 0 \qquad (10\text{-}16)$$

$$dy/dx|_{\phi=\text{constant}} = -\frac{1}{dy/dx}\bigg|_{\Psi=\text{constant}} \qquad (10\text{-}17)$$



Eq_10-15

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$$\boldsymbol{\nabla}\left\{\frac{P}{\rho}+\frac{v^2}{2}+gy\right\}=\mathbf{v}\times(\boldsymbol{\nabla}\times\mathbf{v})-\frac{\partial\mathbf{v}}{\partial t}. \tag{10-18}$$

$$\frac{P}{\rho}+\frac{v^2}{2}+gy=\text{constant} \tag{10-19}$$

$$\frac{P}{\rho}+\frac{v^2}{2}=\text{constant} \tag{10-20}$$

$$P+\frac{\rho v^2}{2}=P_\infty+\frac{\rho v_\infty^2}{2}=P_0 \tag{10-21}$$

$$P=P_0-2\rho v_\infty^2\sin^2\theta \tag{10-22}$$

Eq_10-18

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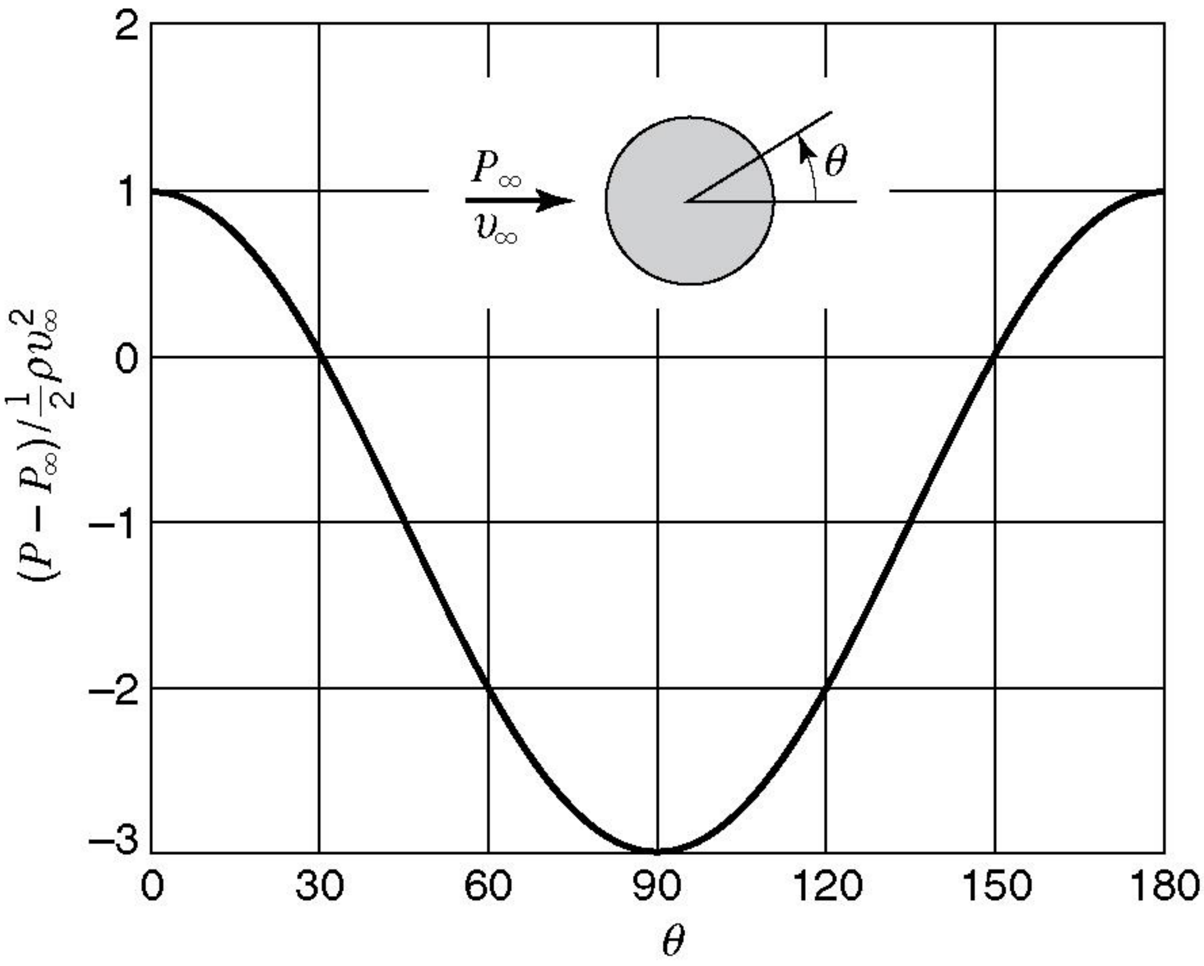

Fig_10-5

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