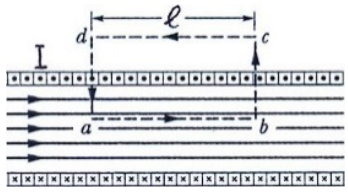
例：極緊密的無窮長螺管（電流 I 、每單位長度繞 n 圈）

內部 $\vec{B}$ 不能有徑向分量（否則有 $\oint_S \vec{B}\cdot d\vec{A} \neq 0$ ），也不能有圓切線分量（否則有 $\oint_{C'} \vec{B}\cdot d\vec{\ell} \neq 0$ 但 C' 內無電流）。

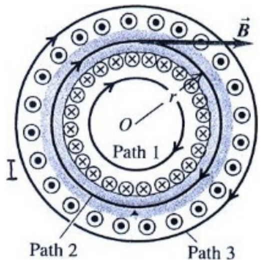


$$\oint_C \vec{B}\cdot d\vec{\ell} = \int_a^b \vec{B}\cdot d\vec{\ell} = B\ell = \mu_0(n\ell I) \;\Rightarrow\; B = \mu_0 nI \text{（與前同）。}$$

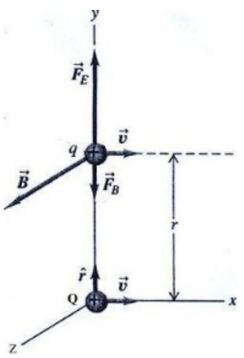
例：Toroid（電流 I 、共繞 N 圈）

$$\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 I_{encl} \Rightarrow B2\pi r = \mu_0(NI) \Rightarrow B(r) = \mu_0 NI/2\pi r\,.$$

當 $r\to\infty$ ， $N/2\pi r = n$ 時， $B = \mu_0 nI$ 。



$\vec{F}_E$ & $\vec{F}_B$ between charged particles



Q 在 q 處建立磁場 $\vec{B} = (\mu_0/4\pi)(Qv/r^2)\hat{z}$ ，

故 q 受磁力 $\vec{F}_B = qv\hat{x}\times\vec{B} = -(\mu_0/4\pi)(qQv^2/r^2)\hat{y}$ 。

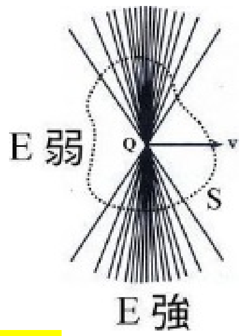
而 q 受電力 $\vec{F}_E = q\vec{E} \approx (1/4\pi\in_0)(qQ/r^2)\hat{y}$ ，

$\therefore F_B/F_E = -\mu_0\in_0 v^2 = -v^2/c^2$ 。

但總力 $\vec{F}_E + \vec{F}_B = md^2\vec{r}/dt^2$ 與慣性座標無關，而在它們的靜止座標中 $F_E' = qQ/4\pi\in_0 r^2$ ， $F_B' = 0$ ，

$\therefore F_E + F_B = F_E' + F_B' = qQ/4\pi\in_0 r^2$ ，

故 $F_E = qQ/4\pi\in_0 r^2 - F_B \approx (1+v^2/c^2)qQ/4\pi\in_0 r^2$ 。（右上圖）

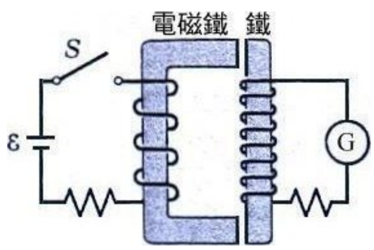


H.W.: Ex. 9, 14, 21, 22; Prob. 1, 3, 4, 5, 8, 9.

Ch. 31 Electromagnetic Induction

Oersted found $I \Rightarrow \vec{B}$ in 1820. Within weeks, electromagnet was found.

那磁場 $\vec{B}$ 能產生電流 I 嗎？Joseph Henry found in Aug. 1830 that a current was induced by a changing magnetic field. But he did not publish immediately.



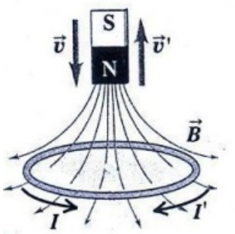
Magnetic flux $\phi_B \equiv \int_S \vec{B}\cdot d\vec{A}$ for any $\vec{B}(\vec{r},t)$ & surface S ,

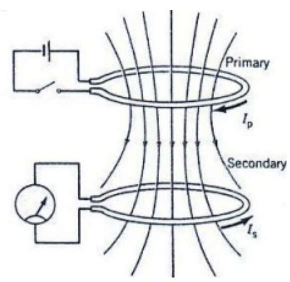
unit: $1\,\text{weber (W)} \equiv 1T\cdot 1m^2$.

Gauss's law for $\vec{B}$: $\oint_S \vec{B}\cdot d\vec{A} = 0$ ($= q_M$, but $q_M = 0$).

Faraday found (1831):

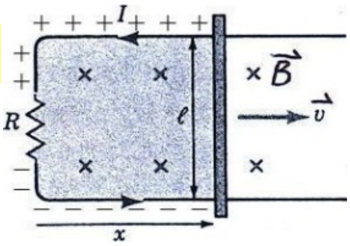
(1) Fixed coil, changing $\vec{B}(\vec{r},t)$ （右圖）





By a moving magnet or a changing I ,induced emf $\varepsilon \propto d\phi_B/dt$.

(2) Fixed $\vec{B}(\vec{r})$, moving coil（假設完美導体）



右圖，$qE = qvB \Rightarrow \Delta V = E\ell = vB\ell = \varepsilon$。若取 $\vec{A}$ 為出紙面，

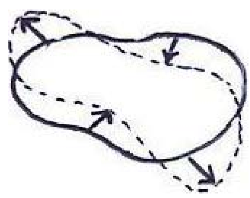
則 $\phi_B = -B\ell x$, $d\phi_B/dt = -B\ell\, dx/dt = -B\ell v = -\varepsilon$.

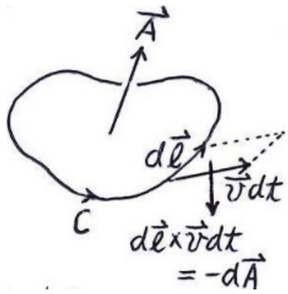
（註：作功率 $P = Fv = I\ell Bv = (vB\ell/R)\ell Bv = (vB\ell)^2/R$

$= \varepsilon^2/R$ ，等於 R 消耗的功率。）

任意形狀、任意運動的 coil 在 fixed $\vec{B}(\vec{r})$ 中，$\varepsilon = \oint_C \vec{v}\times\vec{B}\cdot d\vec{\ell}$

$$= \oint_C \begin{vmatrix} d\ell_x & d\ell_y & d\ell_z \\ v_x & v_y & v_z \\ B_x & B_y & B_z \end{vmatrix} = \oint_C \begin{vmatrix} B_x & B_y & B_z \\ d\ell_x & d\ell_y & d\ell_z \\ v_x & v_y & v_z \end{vmatrix}$$



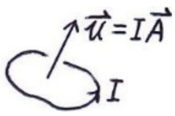


$$= \oint_C \vec{B}\cdot(d\vec{\ell}\times\vec{v}) = \oint_C \vec{B}\cdot(-d\vec{A}/dt) = -\oint_C (\vec{B}\cdot d\vec{A})/dt$$

$$= -(d/dt)\int_S \vec{B}\cdot d\vec{A} = -d\phi_B/dt \,.$$

Faraday's law $\varepsilon = -d\phi_B/dt$, i.e. $\oint_C (\vec{E}+\vec{v}\times\vec{B})\cdot d\vec{\ell} = -(d/dt)\int_S \vec{B}\cdot d\vec{A}$.

不論是 coil 動或 field lines 動，coil 切割 lines 造成 emf。 (Faraday 說法：Induced emf is proportional to the rate at which field lines cross the coil.)

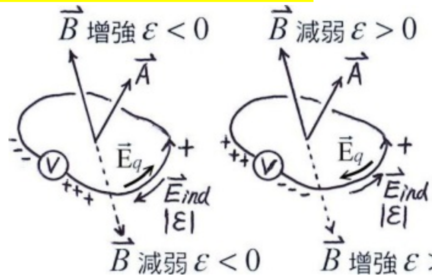
Right hand rule  same as  

只要有 $d\vec{B}/dt$ ，空間中必有 $\vec{E}_{ind}$ 存在，否則不動的 coil 中的電荷不受磁力，不可能產生電流。在完美導線上，total $\vec{E} = \vec{E}_{ind} + \vec{E}_q \perp d\vec{\ell}$ ，否則會有∞電流。

$\oint_C \vec{E}_q \cdot d\vec{\ell} = 0$（voltmeter 內外抵消），

$\oint_C \vec{E}_{ind} \cdot d\vec{\ell} = -d\phi_B/dt$（非保守），

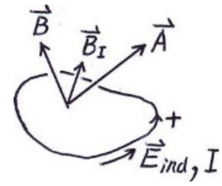
相加⇒ $\oint_C \vec{E}\cdot d\vec{\ell} = -d\phi_B/dt$ ，不管 $\vec{E}$ 的來源為何。



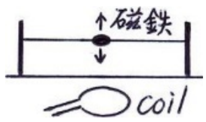
Lenz's law: Induced emf 的方向是要在 coil 產生電流抵抗 flux 的改變（無 coil 亦然）。

〝－〞的意義：能量守恒，因若是 $\oint_C \vec{E}\cdot d\vec{\ell} = +d\phi_B/dt$ ，

則如右圖 $B\uparrow \Rightarrow \phi_B\uparrow \Rightarrow \varepsilon > 0 \Rightarrow I\uparrow \Rightarrow B\uparrow$ ， $B, I \to \infty$ 。



例：磁頭 

例：電吉他 

例：AC 與 DC 發電機

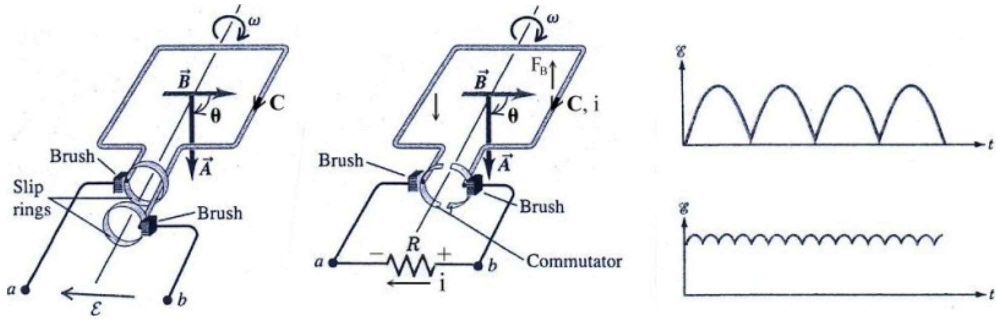
$\phi_B = BA\cos\theta$，

$\varepsilon = -N\,d\phi_B/dt$

$= NBA\sin\theta\,d\theta/dt$

$= NBA\omega\sin\theta$。

If $\omega = const.$，then



AC 發電機 $\varepsilon = \varepsilon_0 \sin\omega t$，DC 發電機 $\varepsilon = |\varepsilon_0 \sin\omega t|$，其中 $\varepsilon_0 = NBA\omega$。

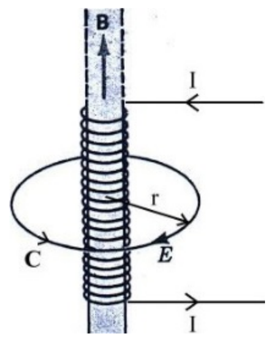
DC 發電機的 commutator 分得愈細，ε 愈接近定值，如右上圖。

DC 發電機也可作為 DC motor，只需將電阻 換成電池 ，ω 方向不變，但電流 i 反向。此事實在 1873 年 Vienna 展覽會上因二機並排而工人誤接而被發現。DC motor 的 back emf $\varepsilon = |NBA\omega\sin\theta|$，$\therefore\ i = (V - |NBA\omega\sin\theta|)/r$，剛起動時 $\omega = 0$，start-up $i = V/r$ 很大，然後 i 漸小。

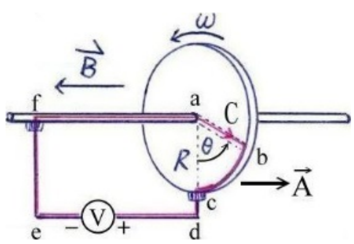
例：Induced $\vec{E}$ field around a solenoid with changing I

$$\oint_C \vec{E}\cdot d\vec{\ell} = -d\phi_B/dt \Rightarrow E2\pi r = \begin{cases} -(\pi r^2)\,dB/dt & \text{for } r < R \\ -(\pi R^2)\,dB/dt & \text{for } r > R \end{cases},$$

$$\therefore\ E = \begin{cases} -(r/2)\,dB/dt = -(r/2)\mu_0 n\,dI/dt & \text{for } r < R \\ -(R^2/2r)\,dB/dt = -(R^2/2r)\mu_0 n\,dI/dt & \text{for } r > R \end{cases}.$$



例：Homopolar generator (Faraday's dynamo)



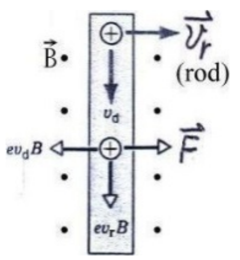
Conducting disk 中心與邊緣間的 emf = ?

法(1)：$\varepsilon = \oint_C \vec{v}\times\vec{B}\cdot d\vec{\ell} = \int_0^R vBdr = \int_0^R (r\omega)Bdr = \omega BR^2/2$.

法(2)：考慮 loop abcdef，$\phi_B = -BR^2\theta/2$，

$\varepsilon = -d\phi_B/dt = (BR^2/2)\,d\theta/dt = BR^2\omega/2$。

$\vec{B}$（不作功）的作用是把外力作功的方向轉 90 度

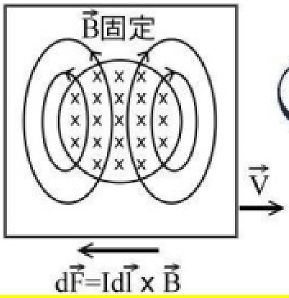


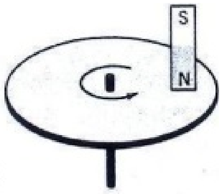
Conducting rod moving in $\vec{B}$ at $\vec{v}_r$

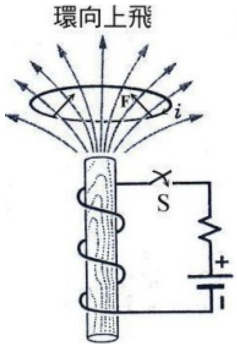
$\vec{B}$ 作功率為 $0 = q(\vec{v}_r + \vec{v}_d)\times\vec{B}\cdot(\vec{v}_r + \vec{v}_d) = q\vec{v}_r\times\vec{B}\cdot\vec{v}_d + q\vec{v}_d\times\vec{B}\cdot\vec{v}_r$。

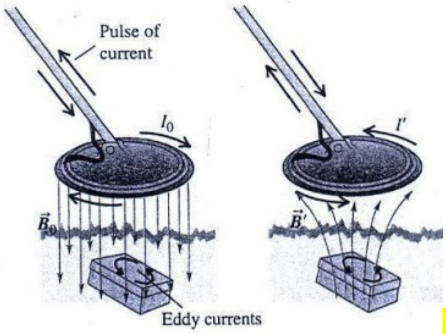
故 $\vec{B}$ 的作用是把「外力 $\vec{F} = -q\vec{v}_d\times\vec{B}$，向右作功率 $-q\vec{v}_d\times\vec{B}\cdot\vec{v}_r$」轉變為「電荷被向下作功率 $q\vec{v}_r\times\vec{B}\cdot\vec{v}_d$」。

Eddy Currents



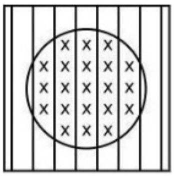






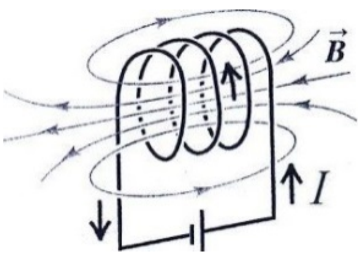
例：電磁爐　　例：阻泥、煞車、車速表　　魔術？　　metal detector

防止 eddy：切成條或片如右圖



H.W.：Ex. 14; Prob. 1, 3, 6, 7, 8, 9.

Ch. 32 Inductance and Magnetic Materials



$\varepsilon = -d\phi/dt$，ϕ是通過「一端直線前進、一端螺旋前進的橡皮筋所掃出的波浪狀曲面」的磁通量。

但若繞得很緊密，則可用$\varepsilon = \sum(-d\phi_i/dt) = -(d/dt)\sum\phi_i$。

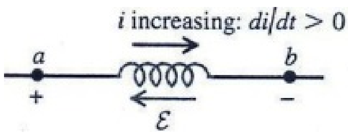
$\because$ each $\phi_i \propto I$，$\therefore$ flux linkage $\sum\phi_i \propto I$。

寫成$\sum\phi_i = LI$，則$\varepsilon = -(d/dt)(LI) = -L\,dI/dt$，

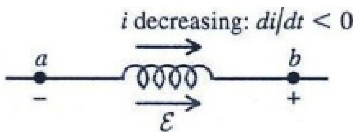
(self) inductance $L \equiv \left(\sum\phi_i\right)/I$ 或 $L \equiv -\varepsilon/(dI/dt)$，

unit: $1\,Henry\,(H) \equiv 1\,Weber/A$ or $1\,V\cdot\sec/A$。

If all $\phi_i = \phi$, then $\sum\phi_i = N\phi$, $L \equiv N\phi/I$.

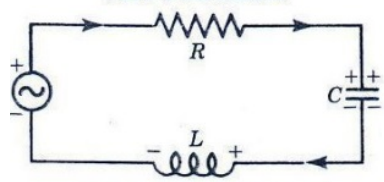


$V_{ab} = L\,di/dt > 0$



$V_{ab} = L\,di/dt < 0$

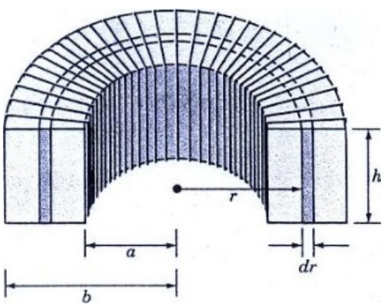
阻力是碰撞



電荷被阻絕

阻力是感應電場

例：



Toroid 內 $B(r) = \mu_0 NI/2\pi r$。

$\phi_B \equiv \int_S \vec{B}\cdot d\vec{A} = \int_a^b Bhdr = \int_a^b (\mu_0 Ni/2\pi r)hdr$

$= (\mu_0 Nih/2\pi)\ln(b/a)$，

$\therefore L \equiv N\phi/i = (\mu_0 N^2 h/2\pi)\ln(b/a)$。