Final Exam

January 15, 2018

Rules and Regulations: It is permitted to bring three pieces of paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

1. Please select the correct option (Y) or (N) for each statement.

   (a) (3%) The discrete-time Fourier transform of a periodic signal is (Y)discrete; (N)continuous in frequency-domain.

   (b) (3%) If we sample the signal $\cos(3t)$ with sampling frequency (Y)$\omega_s = 5$; (N)$\omega_s = 7$, then there is no aliasing.

   (c) (3%) The spectrum

   $$(Y)\, X(e^{j\omega}) = \frac{\sin(10\omega/2)}{\sin(\omega/2)}; \quad (N)\, X(e^{j\omega}) = \frac{\sin(11\omega/2)}{\sin(\omega/2)}$$

   is a valid discrete-time Fourier transform of a discrete-time signal.

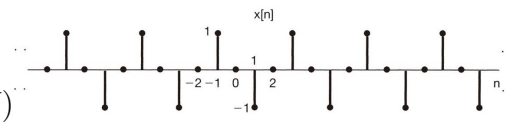
   (d) (3%) Whose spectrum $X(e^{j\omega})$ satisfies

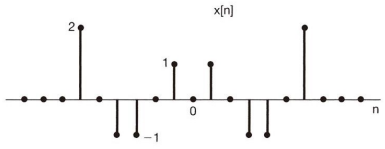
   $$\int_{-\pi}^{\pi} X(e^{j\omega}) d\,\omega = 0?$$

   (Y)$x[n] = u[n-1] + 3u[-n-1] + 100\delta[n+17]$;
   (N)$x[n] = u[n-1] + 3u[-n+1] + 100\delta[n+17]$.

   (e) (3%) The signal $x(t)$ and (Y)$y(t) = 2x(-t-1)$; (N)$y(t) = x(2t)$; have the same Nyquist rate.

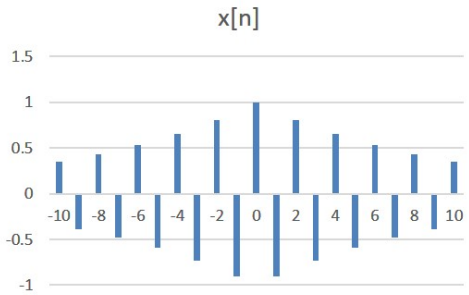
   (f) (3%) Whose spectrum $X(e^{j\omega})$ satisfies $\Re\{X(e^{j\omega})\} = 0$?

   (Y) 

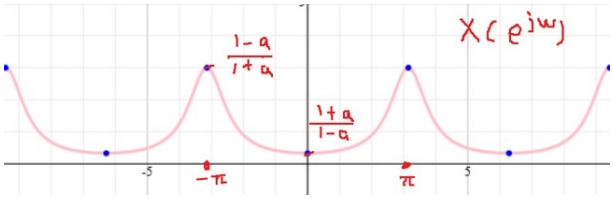
   (N) 

   (g) (3%) For a complex signal $x(t)$, the Fourier transform of $x_e(t)$ is $\Re\{X(j\omega)\}$. (Y)True; (N)False.

   (h) (3%) The discrete-time signal $x[n] = e^{j\pi n} \cdot a^{|n|}$ where $-1 < a < 0$ can be depicted below for $n = -10, -9, \ldots, 10$. (Y)True; (N)False.



(i) (3%) The spectrum $X(e^{j\omega})$ of the discrete-time signal $x[n] = e^{j\pi n} \cdot a^{|n|}$ where $-1 < a < 0$ can be depicted below for $\omega \in [-3\pi, 3\pi]$. (Y)True; (N)False.



(j) (3%) The spectrum of the discrete-time signal

$$x[n] = \begin{cases} -|n| + 1, & |n| \le 1; \\ 0, & |n| > 1 \end{cases}$$

is

$$X(e^{j\omega}) = \frac{\sin^2(3\omega/2)}{\sin^2(\omega/2)}.$$

(Y)True; (N)False.

2. (10%) For a signal $x[n]$ with the Fourier transform $X(e^{j\omega})$, please show that

$$\sum_{m=-\infty}^{n} x[m] \overset{\mathcal{F}}{\longleftrightarrow} \frac{X(e^{j\omega})}{1 - e^{-j\omega}} + \pi X(e^{j0}) \sum_{k=-\infty}^{\infty} \delta(\omega - 2\pi k).$$

3. (10%) Given that $x[n]$ has the Fourier transform $X(e^{j\omega})$, please express the Fourier transform of $y[n] = (x[n] + x^*[-n])/2$ in terms of $X(e^{j\omega})$.

4. (10%) Find the Fourier transform of $x(t) = e^{-t^2}$.

5. (10%) Consider a signal $g[n]$ with Fourier transform $G(e^{j\omega})$. Suppose

$$g[n] = x_{(3)}[n]$$

where the signal $x[n]$ has a Fourier transform $X(e^{j\omega})$. Determine a real number α such that $G(e^{j\omega}) = G(e^{j(\omega-\alpha)})$ and $0 < \alpha < \pi$.

6. (10%)感謝提供題目的張皓宣、黃威豪、鄭珮文、呂郁萱同學。 Find the Fourier transform of $x(t) = e^{-at}u(t)$ where a is a positive real number.

7. (10%) 感謝提供題目的賴建勳、張竣佑、張壹登、謝茹媛、陳芃文、李昱、宋婉瑄、刁兆瑜、潘佑欣同學。 Let the time-domain signal be

$$x(t) = \begin{cases} 1 - |t|, & |t| \le 1; \\ 0, & |t| > 1. \end{cases}$$

Please sketch the time-domain signal $x(t)$ and the frequency-domain signal $X(j\omega)$.

8. (10%) 感謝提供題目的劉彥成、鄧崴宸、李曼妤、古祐宗、蕭人豪同學。 For the frequency-domain signal

$$X(j\omega) = \frac{\sin^2(3\omega)}{\omega^2},$$

please sketch the time-domain signal $x(t)$ and the frequency-domain signal $X(j\omega)$.