Engineering Mathematics II

Midterm Examination I **April 12, 2019**

1. Solve the P.D.E. (20%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - 2\frac{\partial u}{\partial x} + u \;;\; u(0,t) = u(1,t) = 0 \;;\; u(x,0) = e^x$$

2. Using <u>variable separation method</u> solves the P.D.E. (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \;;\; u_x(0,t) = u_x(\pi,t) = 0 \;;\; u(x,0) = 1+\cos^2 x$$

3. Solve the P.D.E. (20%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2 \;;\; u(0,t) = u(\pi,t) = 0 \;;$$

$u(x, 0) = 0$ when $x \in [0, \pi/2)$ and
$u(x, 0) = \sin 2x$ when $x \in (\pi/2, \pi]$

4. Solve the P.D.E. (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \;;\; u_x(0,t) = u(1,t) = 0 \;;\; u(x,0) = 1$$

5. Solve the <u>General solution</u> of P.D.E. (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \;;\; u(2,t) = u(3,t) = 2$$

6. Solve the P.D.E. (15%)

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \;;\; u(0,t) = 2,\ u_x(1,t) = 4; \;;\; u(x,0) = 4x + 2 + 3\sin\left(\frac{5}{2}x\right)$$

Note: $\sin(A-B) = \sin A\cos B - \cos A\sin B$