E927500
Circuit Analysis
April 9, 2014

Mid Term I

A. (4×13=52 points) Explain or answer each of the following statements.

1. Compare "steady-state response" with "transient response".
2. Describe "superposition" based on linearity.

$L[f(t)] = f(x) \cdot e^{-st} dt$

3. Explain $|\mathbf{V}|^2 = \mathbf{V} \times \mathbf{V}^*$ based on complex conjugate.

sV $\quad e^{-st} \cdot dV$

4. $\dfrac{dv}{dt}$ (time domain) $\Leftrightarrow$ $j\omega\mathbf{V}$ (phasor domain) by Laplace or Fourier transform. $\quad u \quad dV$
5. Phasor analysis applies only when frequency is constant; when it is applied to two or more sinusoid signals only if they have the same frequency. $\quad -se^{-st} \quad V$
6. An inductor acts like an open circuit at high frequencies and a capacitor can reject a dc signal. ***Constraint:*** you must use the impedance concept in the phasor domain. $\quad du \quad V$
7. Compare "impedance" with "resistance".
8. In Fig. 1, the output is taken across the resistor. Does the output voltage $v_o(t)$ across the resistor lead the input voltage $v_i(t)$?
9. Compare "open-loop gain" with "closed-loop gain" in op amp circuit.
10. Compare "negative feedback" with "positive feedback" in op amp circuit.
11. The oscillator is a circuit produces an ac waveform as output when powered by dc input. Explain $f_o = \dfrac{1}{2\pi RC}$ in Fig. 2.
12. Capacitance multiplier is used to create a large capacitance. Explain $C_{eq} = \left(1 + \dfrac{R_2}{R_1}\right)C$ in Fig. 3.
13. Compare "instantaneous power" with "average power".

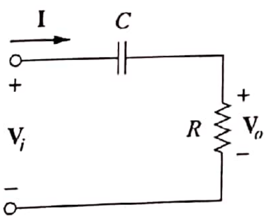

Fig. 1

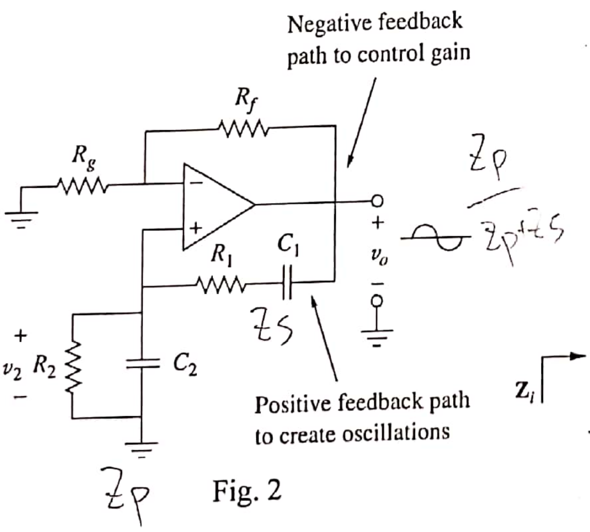

Fig. 2

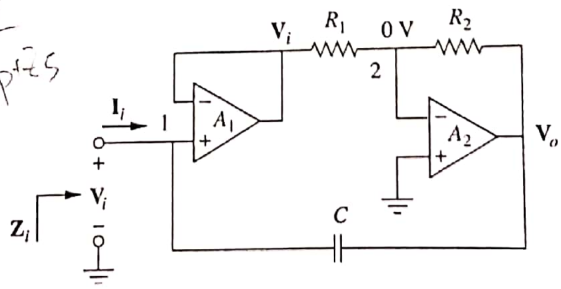

Fig. 3

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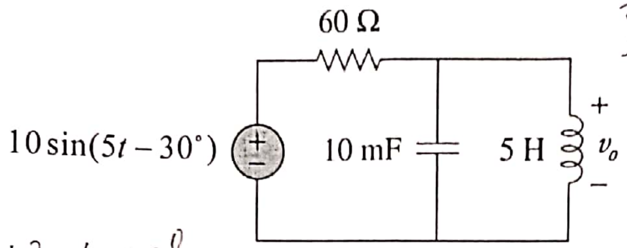
B. (10+5=15 points) Phasor:

(a) Derive the differential equations for the following circuit in order to solve for $v_o(t)$ in phase domain $\mathbf{V}_o$. See Fig. 4. (b) Explain "The solution only includes the steady-state response, and it does not require knowing the initial values".

$\frac{1}{\frac{1}{jwC}}$ $5 \cdot 0.01$

$C\frac{dV}{dt} = I$

$L\frac{di}{dt} = V$



$10 \angle -30^0$

Fig. 4

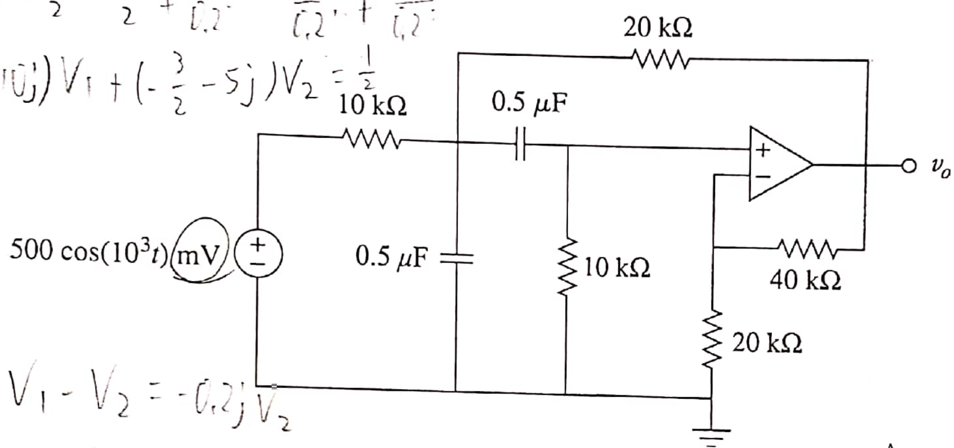
$10\cos(5t - 30 - 90)$

C. (12+5=17 points) Op Amp AC Circuits:

(a) Determine $v_o(t)$ in the op amp circuit in Fig. 5. (b) No inductor can be found in this op am circuit. Why?

$0.5 - V_1 = \frac{V_1}{2} - \frac{V_o}{2} + \frac{V_1 j}{0.2} - \frac{V_2 j}{0.2} + \frac{V_1 j}{0.2}$ $3V_2$

$(\frac{3}{2} + 10j)V_1 + (-\frac{3}{2} - 5j)V_2 = \frac{1}{2}$



$\frac{1}{\frac{1}{jwC}}$ $10^3 \cdot 0.5 \cdot 10^{-6}$

$-\frac{j}{5 \cdot 10^{-4}}$

$V_1 - V_2 = -0.2jV_2$

$V_1 + (-1 + 0.2j)V_2 = 0$

Fig. 5

D. (7+7+6+6=26 points) Maximum Average Power Transfer:

Determine the Thevenin equivalent of the circuit in Fig. 6 as seen from: (a) terminals *a-b* and (b) terminals *c-d*. (c) If a loading is added between terminals *a* and *b*, please find a matching impedance that absorbs the maximum average power and calculate P_{max}. (d) Find the average power of each source.

$\frac{-V_1 + V_2}{4j} = \frac{jV_1 - jV_2}{4} + 4 = \frac{V_2}{4}$

$2 - \frac{V_1}{10} = \frac{j}{4}V_1 - \frac{j}{4}V_2 - \frac{j}{5}V_1$

$jV_1 - jV_2 + 16 = V_2$

$(\frac{1}{10} + \frac{j}{20})V_1 - \frac{j}{4}V_2 = 2$

$-j \quad V_1 + (1+j)V_2 = 16$

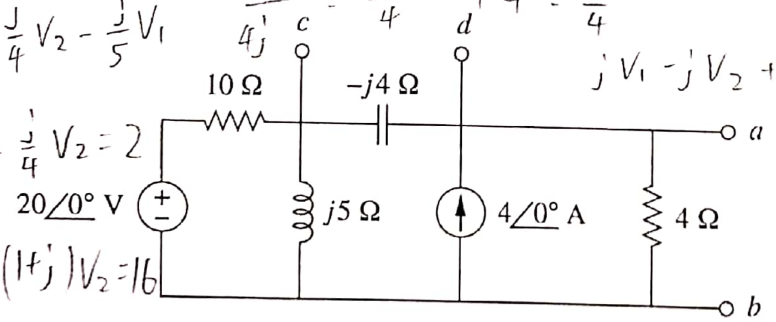


Fig. 6

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國立成功大學 學年度第 學期考試試卷 National Cheng Kung University Examination Sheet for Academic Year:			Semester:	年 Year 月 Month 日 Day
姓名 Name	高培瑄	科目名稱 Subject Name	电路分析	教師簽章 Signature of Instructor
學號 Student No.	E94011128			
院系 College	工 學院 College 工科 系 Department 2 年 Year 班 Class	評閱成績 Score	100+2=102	

A. 1. steady state response：穩態响应，指經过一段很長的时間，电流或电压不隨time↑而跟著改变，達到穩定的狀態

transient response：暫態响应，在电流或电压还沒達到穩定狀態前的形式，会隨time↑遞減

ex: $1+e^{-t}$ 穩 暫 一開始兩者皆存在，但time↑只剩穩態

其差異為時間的角度来观察

2. superposition 是將各独立电压(or电流)源独自看，並將各个結果加起來，才是最後的結果，∵V跟I会呈線性，故得到的slope都同(即R)，V跟I才能用線性疊加，如果是P就不行

3. 令 $V = V_m(\cos\phi + j\sin\phi) = V_m\angle\phi$

$V^* = V_m(\cos\phi - j\sin\phi) = V_m\angle-\phi$

$|V| = V_m\sqrt{(\cos\phi)^2+(j\sin\phi)^2} = V_m \Rightarrow |V|^2 = V_m^2$

而 $V\times V^* = V_m\angle\phi \times V_m\angle-\phi \Rightarrow V_m^2\angle\phi-\phi = V_m^2$

故 $|V|^2 = V\times V^*$

45
14
15
26
100

4. $\mathcal{L}\left[\frac{dv}{dt}\right] = \int_0^\infty \frac{dv}{dt}e^{-st}dt = \int_0^\infty e^{-st}\cdot dv = e^{-st}\cdot v - \int_0^\infty -se^{-st}v\cdot dt$

$= e^{-st}\cdot v + s\underbrace{\int_0^\infty v e^{-st}dt}_{V(s)} = sV(s) + \cancel{v(0^-)} = sV(s) = j\omega V$

initial condition 令為0　$s=j\omega$

5. ~~相位~~ Phasor 只能在相同頻率才可以疊加，∵我們將$\Sigma v(t)$換成$\Sigma V_m\cos(\omega t+\phi) = \Sigma Re(V_m e^{j\omega t+\phi})$ $= \Sigma Re(V_m e^{j\omega t}\cdot e^{j\phi})$ 要將 $Re(V_m e^{j\omega t})e^{j\phi}$ 化簡成 $Ve^{j\phi}$，即 w 皆同才提出，因此 $Re(V_m e^{j\omega t})\Sigma e^{j\phi} = V\cdot\Sigma e^{j\phi}$ 此時相位就可疊加

Laplace 形式

6. 在 phasor domain (frequency) L 可看成 $Z_L = j\omega L$　C可看成 $Z_C = \frac{1}{j\omega C}$

If $\omega\to\infty$, $Z_L\to\infty$ ∴ L可視作 open circuit

而 If $\omega\to 0$ (即DC直流)，$Z_C\to\infty$ ∴ C也是open circuit，表电容沒有讓DC通过

7. impedence 比 resistance 还多考慮 L 或 C 的關係，∵L或C会造成一个相位差，还有要多考慮頻率因素，故要用複数來表示，而 resistance 單純只有实部的 R

impedence：$R+jZ$ (Z可為 ωL or $\frac{1}{\omega C}$)（續寫轉背頁）

resistance：R

8、$Z = R + \frac{1}{jwc} = R + \frac{-j}{wc}$

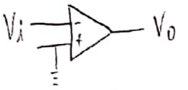
$$V_0 = V_i \frac{R}{R + \frac{-j}{wc}} = V_i \frac{R}{R^2 + (\frac{1}{wc})^2} (R + \frac{j}{wc}) = \frac{V_i R}{R^2 + (\frac{1}{wc})^2} (R + j\frac{1}{wc})$$

ψ ∴ $\tan^{-1} \frac{\frac{1}{wc}}{R} = \tan^{-1} \frac{1}{wcR}$ ∵ $w \cdot c \cdot R$ 皆為正数

∴ $\tan^{-1}$ 落在第一象限

故 $V_0(t)$ 超前 $V_i(t)$ ⇒ 即 $V_0(t)$ lead $V_i(t)$

9、



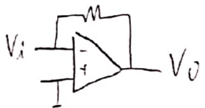
open loop，沒有形成一个迴路

closed-loop

有一个負回饋，讓 circuit 形成通路

主要差别在於有沒有形成迴路

10、



negative feedback

Vo 端線路接回 Vi - input 端

positive feedback

Vo 端線路接回 Vi + input 端

主要差別在 feedback 接回是 + 端 还是 - 端

11、$\frac{V_2}{V_0} = \frac{(R_2 // C_2)}{(R_2 // C_2) + (R_1 + C_1)} = \frac{wR_1C_1}{w(R_2C_1 + R_1C_1 + R_2C_2) + j(w^2R_1C_1R_2C_2 - 1)}$

ψ ∵ oscillator 特徵：沒有相位差 ∴ $j(w^2R_1C_1R_2C_2-1)$ 這項等於 0

$w^2R_1C_1R_2C_2 = 1 \Rightarrow w = \frac{1}{\sqrt{R_1C_1R_2C_2}} \xRightarrow[\substack{R_1=R_2=R \\ C_1=C_2=C}]{令} w = \frac{1}{RC}$ ∵ $2\pi f = w = \frac{1}{RC}$

故 $f = \frac{1}{2\pi RC}$

12、用 KCL 求 node ① node ②

① $\frac{V_i - V_0}{\frac{1}{jwc}} = I_i$ ② $\frac{V_i - 0}{R_1} = \frac{0 - V_0}{R_2} \Rightarrow V_0 = -\frac{R_2}{R_1} V_i$

↳ 代回①式的 V_0

ψ $(V_i + \frac{R_2}{R_1} V_i) jwc = I_i \Rightarrow V_i (1 + \frac{R_2}{R_1}) jwc = I_i \Rightarrow \frac{V_i}{I_i} = \frac{1}{jw(1+\frac{R_2}{R_1})c}$

$\frac{1}{jwC_{eq}}$ 相当於一个电容的形式，∴ $\frac{1}{jw(1+\frac{R_2}{R_1})c}$ 可化成一等效电容 $C_{eq} = (1 + \frac{R_2}{R_1}) C$

13、instantaneous power 指單一瞬間的 V 跟 I 相乘

ψ average power 是將各瞬間的 power 作積分再除以經過時間 T，所得到的大致 power

兩者差在取的範圍一个是瞬間，另一个是一段時間来看其總和 P/T.

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（裝　　訂　　線）

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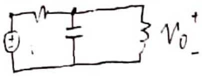
B (a) 利用 KCL



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node1: $\dfrac{10\sin(5t-30)-V_0}{60} = \dfrac{1}{L}\int V_0\,dt + C\dfrac{dV_0}{dt}$

15

$\therefore$ 同時微分 $\frac{1}{6}\cdot 5\cos(5t-30) - \frac{1}{60}\frac{dV_0}{dt} = \frac{V_0}{L} + C\frac{d^2V_0}{dt^2}$

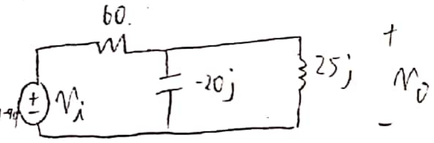
對t　　10^{-2}　　5

整理為 $5\frac{d^2V_0}{dt^2} + \frac{1}{60}\frac{dV_0}{dt} + 100V_0 = \frac{5}{6}\cos(5t-30)$

$\Rightarrow \frac{d^2V_0}{dt^2} + \frac{1}{12}\frac{dV_0}{dt} + 20V_0 = \frac{1}{6}\cos(5t-30)$

0

化成 phase domain 求解



$10\cos(5t-30-90)$

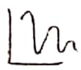
$10\angle -120$

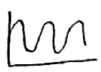
$V_0 = V_i \cdot \dfrac{(25j \,//\, -20j)}{60+(25j\,//\,-20j)} = V_i\left(\dfrac{25-15i}{34}\right)$

$= 8.575\angle -150.96^0$

$\Rightarrow 8.575\cos(5t-150.96^0) = 8.575\sin(5t-60.96^0)$

(b) $\because$ 一个 sinsouid 型式的电压源，並不会隨時間有衰退的現象，而振幅会一致





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$\therefore$ 不用求 initial condition，只要找到穩態的方程式即可

$\because$ 大自然當中的初始條件並不会影响其过程，过程会經过一段很長的time

$\therefore$ 常可以忽略 initial condition

（但嚴謹考慮的話，还是要存在）

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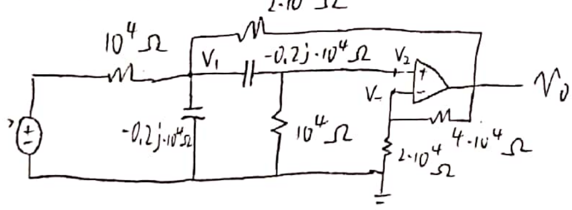
15 (a) 換成 phase domain

2



利用KCL：① $\dfrac{0.5-V_1}{10^4} = \dfrac{V_1-V_0}{2\cdot10^4} + \dfrac{V_1-V_2}{-0.2j\cdot10^4} + \dfrac{V_1-0}{-0.2j\cdot10^4}$

② $\dfrac{V_1-V_2}{-0.2j\cdot10^4} = \dfrac{V_2-0}{10^4}$

（續寫轉背頁）

③ $V_2 = \dfrac{2\cdot10^4}{2\cdot10^4+4\cdot10^4}V_0 \Rightarrow V_2 = \frac{1}{3}V_0$

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$$\Rightarrow \begin{cases} \left(\frac{3}{2}+10j\right)V_1 + \left(-\frac{3}{2}-5j\right)V_2 = \frac{1}{2} \\ V_1 + \left(-1+\frac{1}{5}j\right)V_2 = 0 \end{cases} \Rightarrow \text{利用克拉瑪} \quad V_1 = \frac{\begin{vmatrix} \frac{1}{2} & -\frac{3}{2}-5j \\ 0 & -1+\frac{1}{5}j \end{vmatrix}}{\begin{vmatrix} \frac{3}{2}+10j & -\frac{3}{2}-5j \\ 1 & -1+\frac{1}{5}j \end{vmatrix}} = 0.0998\angle -78.258$$

$$\begin{vmatrix} \frac{1}{2} & -\frac{3}{2}-5j \\ 0 & -1+\frac{1}{5}j \end{vmatrix} = -\frac{1}{2}+\frac{1}{10}j \qquad \begin{vmatrix} \frac{3}{2}+10j & -\frac{3}{2}-5j \\ 1 & -1+\frac{1}{5}j \end{vmatrix} = -2-\frac{47}{10}j$$

$$V_2 = \frac{\begin{vmatrix} \frac{3}{2}+10j & \frac{1}{2} \\ 1 & 0 \end{vmatrix}}{-2-\frac{47}{10}j} = 0.0978\angle -66.948$$

$$\Rightarrow V_0 = 3V_2 = 0.2936\angle -66.948 \Rightarrow \underbrace{V_0(t) = 0.2936\cos(10^3 t - 66.948)}$$

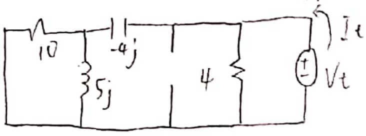
(b) ∵ inductor 換成 phase domain $\Rightarrow j\omega L$ 而 capacitor $\Rightarrow \frac{1}{j\omega C}$

因此可將 $j\omega L$ 転換成類似 $\frac{1}{j\omega C}$ 的型式 $\left(\frac{1}{j\frac{-1}{\omega L}}\right.$，故 $\left.\frac{-1}{\omega L} = \omega C\right)$

這是想法上，但實際不是如此

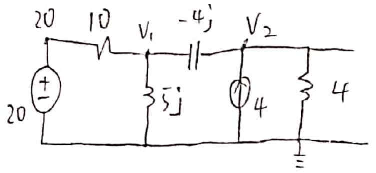
但在實際IC製造上，做一個線狀的L比兩塊板子的C還要難做，常用C而不是L

26 D (a) 先求 Z_{Th}



$$\underset{\sim}{Z_{Th}} = \left[(10 \,//\, 5j) + (-4j)\right] // 4 = \left(\frac{50j}{10+5j} - 4j\right) // 4 = \frac{\left(\frac{50j}{10+5j}-4j\right)\cdot 4}{\frac{50j}{10+5j}-4j+4} = \frac{4}{3}(\Omega)$$

再求 V_{Th} 利用 KCL



$$\frac{20-V_1}{10} = \frac{V_1-V_2}{-4j} + \frac{V_1-0}{5j} \quad \cdots ①$$

$$\frac{V_1-V_2}{-4j} + 4 = \frac{V_2-0}{4} \quad \cdots ②$$

$$\begin{cases} \left(\frac{1}{10}+\frac{j}{20}\right)V_1 + \left(-\frac{j}{4}\right)V_2 = 2 \\ (-j)V_1 + (1+j)V_2 = 16 \end{cases}$$

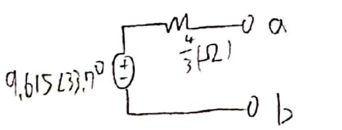
$$\Rightarrow \text{利用克拉瑪} \quad V_1 = \frac{\begin{vmatrix} 2 & -\frac{j}{4} \\ 16 & 1+j \end{vmatrix}}{\begin{vmatrix} \frac{1}{10}+\frac{j}{20} & -\frac{j}{4} \\ -j & 1+j \end{vmatrix}} = \frac{40}{3}+\frac{40}{3}j = 18.856\angle 45^\circ$$

$$\begin{vmatrix} 2 & -\frac{j}{4} \\ 16 & 1+j \end{vmatrix} = 2+6j \qquad \begin{vmatrix} \frac{1}{10}+\frac{j}{20} & -\frac{j}{4} \\ -j & 1+j \end{vmatrix} = \frac{3}{10}+\frac{3}{20}j$$

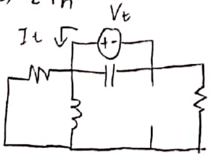
$$V_2 = \frac{\begin{vmatrix} \frac{1}{10}+\frac{j}{20} & 2 \\ -j & 16 \end{vmatrix}}{\frac{3}{10}+\frac{3}{20}j} = 8+\frac{16}{3}j = 9.615\angle 33.7$$

$$\begin{vmatrix} \frac{1}{10}+\frac{j}{20} & 2 \\ -j & 16 \end{vmatrix} = \frac{8}{5}+\frac{14}{5}j$$

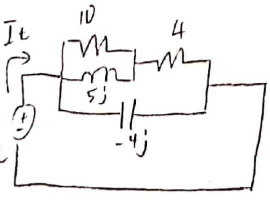
$$\therefore V_{Th} = V_2 = 9.615\angle 33.7^\circ$$



(b) 先求 Z_{Th}



$\Rightarrow$



$$Z_{Th} = \left[(10 \,//\, 5j) + 4\right] // -4j = \frac{\left[\frac{50j}{10+5j}+4\right](-4j)}{\frac{50j}{10+5j}+4+(-4j)} = \frac{8}{3}-4j = 4.81\angle -56.31$$

（裝　　訂　　線）

國立成功大學　　學年度第　　學期考試試卷
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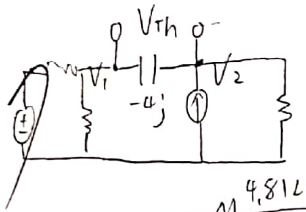
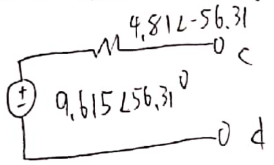
學號 Student No.					
院系 College	學院 College　系 Department　年 Year　班 Class	評閱成績 Score			

再求 V_{Th}，從 (a) 的 V_1、V_2 可知 ∵ $V_{Th} = V_1 - V_2$



$$\therefore V_{Th} = \frac{40}{3}+\frac{40j}{3} - \left(8+\frac{16}{3}j\right) = 9.615\angle 56.31^{\circ}$$

∴



(c) $P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(9.615)^2}{8\cdot\frac{4}{3}} = 8.667\ (W)$

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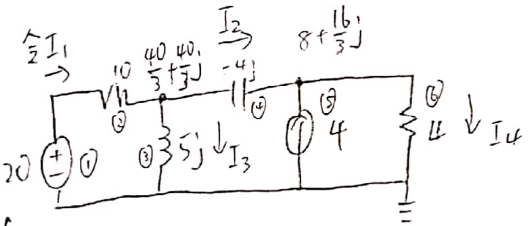
(d)



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$$I_1 = \frac{20-\left(\frac{40}{3}+\frac{40}{3}j\right)}{10} = 1.49\angle -63.43 \quad , I_3 = \frac{\frac{40}{3}+\frac{40}{3}j}{5j} = 3.77\angle -45$$

$$I_2 = \frac{\left(\frac{40}{3}+\frac{40}{3}j\right)-\left(8+\frac{16}{3}j\right)}{-4j} = 2.4\angle 146.31 \quad I_4 = \frac{8+\frac{16}{3}j}{4} = 2+\frac{4}{3}j = 2.4\angle 33.7$$

① $P = -\frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = -\frac{1}{2}\,20\cdot I_1 \cos(\theta_v - \theta_1)$
$= -\frac{1}{2}\cdot 20\cdot 1.49 \cos(0+63.43) = -6.66\ (W)$

② $V = 20 - \left(\frac{40}{3}+\frac{40}{3}j\right) = 14.91\angle -63.43$

$P = \frac{1}{2}\cdot 14.91\cdot 1.49 \cos(-63.43+63.43) = 11.11\ (W)$

③ $V = \frac{40+40j}{3} = 18.86\angle 45$　　$P = \frac{1}{2}\cdot 18.86\cdot 3.77\cos(45+45) = 0\ (W)$

④ $V = \left(\frac{40+40j}{3}\right) - \left(8+\frac{16}{3}j\right) = 9.61\angle 56.31$　　$P = \frac{1}{2}\cdot 9.61\cdot 2.4 \cos(56.31-146.31)$
$= 0\ (W)$

（續寫轉背頁）

⑤ $V = 8 + \frac{16}{3}j = 9.61 \angle 33.7$

$P = \frac{1}{2} \cdot 9.61 \cdot (-4) \cos(33.7 - 0) = -15.99\ (W)$

⑥ $V = 8 + \frac{16}{3}j = 9.61 \angle 33.7$

$P = \frac{1}{2} \cdot 9.61 \cdot 2.4 \cos(33.7 - 33.7) = 11.532\ (W)$

$\therefore P$: $20\angle 0^\circ$: $-6.66\ (W)$ ✓

$10\,\Omega$: $11.11\ (W)$

$5j\,\Omega$: $0\ (W)$

$-4j\,\Omega$: $0\ (W)$

$4\angle 0^\circ$: $-15.99\ (W)$

$4\,\Omega$: $11.532\ (W)$

excellent!

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