(b). f(x) is even function

$$a_0 = \frac{1}{2l}\left[\int_{-l}^{0} f(x)\,dx + \int_0^l f(x)\,dx\right] = \frac{1}{2l}\left(\int_l^0 \overset{f(y)=f(-y)}{f(-y)}(-dy) + \int_0^l f(x)\,dx\right]$$

令 $x=-y$
$dx=-dy$

$$= \frac{1}{2l}\left[\int_0^l f(x)\,dx + \int_0^l f(x)\,dx\right] = \frac{1}{l}\cdot\int_0^l f(x)\,dx \text{ ※}$$

$$a_n = \frac{1}{l}\left[\int_{-l}^{0} f(x)\cos\frac{n\pi}{l}x\,dx + \int_0^l f(x)\cos\frac{n\pi}{l}x\,dx\right] = \frac{1}{l}\left[\int_l^0 f(-y)\cos\frac{n\pi}{l}(-y)(-dy) + \int_0^l f(x)\cos\frac{n\pi}{l}x\,dx\right]$$

$$= \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi}{l}x\,dx \text{ ※}$$

$$b_n = \frac{1}{l}\left[\int_{-l}^{0} f(x)\sin\frac{n\pi}{l}x\,dx + \int_0^l f(x)\sin\frac{n\pi}{l}x\,dx\right] = \frac{2}{l}\left[\int_l^0 f(-y)\sin\frac{n\pi}{l}(-y)(-dy) + \int_0^l f(x)\sin\frac{n\pi}{l}x\,dx\right]$$

$$=0 \text{ ※}$$

f(x) is odd function:

$$a_0 = \frac{1}{2l}\int_{-l}^{l} f(x)\,dx = \frac{1}{2l}\left[\int_{-l}^{0} f(\underset{x=-y}{x})\,dx + \int_0^l f(x)\,dx\right] = \frac{1}{2l}\left[-\int_0^l f(x)\,dx + \int_0^l f(x)\,dx\right] = 0 \text{ ※}$$

$$a_n = \frac{1}{l}\left[\int_{-l}^{0} f(x)\cos\frac{n\pi}{l}x\,dx + \int_0^l f(x)\cos\frac{n\pi}{l}x\,dx\right] = \frac{1}{l}\left[\int_l^0 \underset{-f(y)}{f(-y)}\cos\frac{n\pi}{l}(-y)(-dy) + \int_0^l f(x)\cdot\cos\frac{n\pi}{l}x\cdot dx\right]$$

$$=0 \text{ ※}$$

$$b_n = \frac{1}{l}\left[\int_{-l}^{0} f(x)\sin\frac{n\pi}{l}x\,dx + \int_0^l f(x)\sin\frac{n\pi}{l}x\,dx\right] = \frac{1}{l}\left[\int_l^0 f(-y)\underbrace{\sin\frac{n\pi}{l}(-y)}_{-\sin\frac{n\pi}{l}y}(-dy) + \int_0^l f(x)\sin\frac{n\pi}{l}x\,dx\right]$$

$$= \frac{2}{l}\int_0^l f(x)\sin\frac{n\pi}{l}x\,dx \text{ ※}$$

4. $f(x,y) = h(x)\,g(y)$

$$\Rightarrow h(x) = a_{h0} + \sum_{n=1}^{\infty}\left(a_{hn}\cos\frac{n\pi}{l_1}x + b_{hn}\sin\frac{n\pi}{l_1}x\right) \Rightarrow \begin{cases} a_{h0} = \frac{1}{2l_1}\int_{-l_1}^{l_1} h(x)\,dx \\ a_{hn} = \frac{1}{l_1}\int_{-l_1}^{l_1} h(x)\cos\frac{n\pi}{l_1}x\,dx \\ b_{hn} = \frac{1}{l_1}\int_{-l_1}^{l_1} h(x)\sin\frac{n\pi}{l_1}x\,dx \end{cases}$$

$$g(y) = a_{g0} + \sum_{n=1}^{\infty}\left(a_{gn}\cos\frac{n\pi}{l_2}x + b_{gn}\sin\frac{n\pi}{l_2}x\right) \quad \begin{cases} a_{g0} = \frac{1}{2l_2}\int_{-l_2}^{l_2} g(y)\,dy \\ a_{gn} = \frac{1}{l_2}\int_{-l_2}^{l_2} g(y)\cos\frac{n\pi}{l_2}y\,dy \end{cases}$$