18

4. Ω is the tetrahedron(四面體) with vertices $O(0, 0, 0)$, $B(1, 0, 0)$, $C(0, 2, 0)$ and $D(0, 0, 1)$. Suppose $\mathbf{n}$ is the outer unit normal of the surface of Ω, please compute the following:

[1] (8 %) $\iiint_{\Omega} y\,dV$.

[2] (10 %) $\iint_{OBC} \mathbf{F}\cdot\mathbf{n}\,d\sigma + \iint_{OBD} \mathbf{F}\cdot\mathbf{n}\,d\sigma + \iint_{OCD} \mathbf{F}\cdot\mathbf{n}\,d\sigma + \iint_{BCD} \mathbf{F}\cdot\mathbf{n}\,d\sigma$,

where $\mathbf{F} = \left(x+y, y^2+z, z+x\right)$

25

5. $\mathbf{F} = \left(x^3 - y, y^3 + x, z^3\right)$, Ω is the region (see the figure at right) above the cone $z = \sqrt{3x^2+3y^2}$ and below the sphere $x^2+y^2+z^2 = 4$. The curve Γ is the intersection of the cone and sphere. If $\mathbf{n}$ is the outer unit normal on the surface S (above Γ) and C (below Γ) of Ω,

[1] (10 %) Compute the volume integral $\iiint_{\Omega} \nabla\cdot\mathbf{F}\,dV$

[2] (5 %) Compute the surface integral $\iint_{S} (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$

[3] (5 %) Compute the surface integral $\iint_{C} (\nabla\times\mathbf{F})\cdot\mathbf{n}\,d\sigma$

[4] (5 %) Compute $\oint_{\Gamma} \mathbf{F}\cdot d\mathbf{r}$, (see the direction in the figure)

一個不能為自己面前事情努力的人

還能為什麼事情努力呢？

Don't give up when you are able to ~~fly, to dream and to love~~ write

雖然分數很重要，但更重要的是你們學到了什麼

以及在這過程中，所流下的汗水(不開冷氣的話就會有 XD)

Final exam ends but new challenges begin!

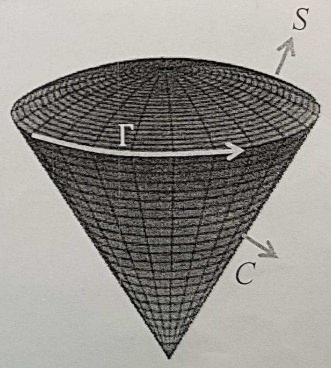

Figure for question 5

(More questions on the back 後面還有題目喔！)

$e^{-(x^2+y^2)}(x^2+2y^2)$ $\quad (x^2+2y^2)e^{-(x^2+y^2)} = (2x+2y^2)e^{-(x^2+y^2)} + (x^2+2y^2)e^{-(x^2+y^2)}(-2x)$

$= (2x+2y^2-2x^3-2y^2)e^{-(x^2+y^2)}$

32

$= (-2x^3-4y^2+2x)e^{-(x^2+y^2)}$

$= (-2x)(x$

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Final-Exam of Calculus II

Total: 115 points

Please write down **all the steps or reasons** for your answers.

要把答案的過程及原因寫下來喔！

5

1. (5 %) Determine the limit $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$.

33

2. Find the following answers for $f(x,y) = (x^2+2y^2)e^{-(x^2+y^2)}$

   [1] (10 %) $\dfrac{\partial f(x,y)}{\partial x}$, $\dfrac{\partial f(x,y)}{\partial y}$, $\dfrac{\partial^2 f(x,y)}{\partial x^2}$, $\dfrac{\partial^2 f(x,y)}{\partial y^2}$, $\dfrac{\partial^2 f(x,y)}{\partial x \partial y}$

   [2] (2 %) The directional derivative (方向導數) at (1, 1) along the direction (1, –2).

   [3] (13 %) Find all the maximum, minimum and saddle points (x, y).

   [4] (10 %) Under the condition $2x^2+y^2=3$, find all the extreme points (x, y). You don't need to check if it is a maximum, minimum, or saddle point.

3. Compute the following integrals

   [1] (8 %) $\displaystyle\iint_\Omega e^{-y^2}\,dA$, where Ω is the triangle with vertices (0, 0), (0, 1) and (1, 1).

   [2] (8 %) $\displaystyle\iint_\Omega (\ln y - \ln x)\,dA$,

   $\Omega = \left\{(x,y) \mid x \ge 0, y \ge 0, 1 \le xy \le 4, \frac{1}{4}x \le y \le 4x\right\}$

   [3] (8 %) $\displaystyle\int_\Omega \mathbf{F}\cdot d\mathbf{r}$ and $\displaystyle\oint_C \mathbf{F}\cdot d\mathbf{r}$, where $\mathbf{F} = (\cos x \sin y, \sin x \cos y)$, where Ω is the vector $\overline{AB}$, $A=(0,0)$, $B=\left(\frac{\pi}{2},\frac{\pi}{2}\right)$, and C is the unit circle (沿逆時針方向，counterclockwise).

   [4] (8 %) $\displaystyle\oint_C \mathbf{F}\cdot d\mathbf{r}$, $\mathbf{F} = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}\right)$, C is the curve defined in the polar coordinate $r = 2+\cos\theta, 0 \le \theta \le 2\pi$ (沿逆時針方向，counterclockwise).

(More questions on the back 後面還有題目喔！)