

Discrete Mathematics

HW4

共 12 題

Exercise 5.1

2. If $A = \{1, 2, 3\}$, and $B = \{2, 4, 5\}$, give examples of (a) three nonempty relations from A to B ; (b) three nonempty relations on A .

4. For which sets A, B is it true that $A \times B = B \times A$?

Exercise 5.2

2. Does the formula $f(x) = 1/(x^2 - 2)$ define a function $f: \mathbf{R} \rightarrow \mathbf{R}$? A function $f: \mathbf{Z} \rightarrow \mathbf{R}$?

18. Give an example of a function $f: A \rightarrow B$ and $A_1, A_2 \subseteq A$ for which $f(A_1 \cap A_2) \neq f(A_1) \cap f(A_2)$. [Thus the inclusion in Theorem 5.2(b) may be proper.]

Exercise 5.3

2. For each of the following functions $f: \mathbf{Z} \rightarrow \mathbf{Z}$, determine whether the function is one-to-one and whether it is onto. If the function is not onto, determine the range $f(\mathbf{Z})$.

a) $f(x) = x + 7$

b) $f(x) = 2x - 3$

c) $f(x) = -x + 5$

d) $f(x) = x^2$

e) $f(x) = x^2 + x$

f) $f(x) = x^3$

4. Let $A = \{1, 2, 3, 4\}$ and $B = \{1, 2, 3, 4, 5, 6\}$. (a) How many functions are there from A to B ? How many of these are one-to-one? How many are onto? (b) How many functions are there from B to A ? How many of these are onto? How many are one-to-one?

Exercise 5.4

6. Let $A = \{x, a, b, c, d\}$.
- a) How many closed binary operations f on A satisfy $f(a, b) = c$?
 - b) How many of the functions f in part (a) have x as an identity?
 - c) How many of the functions f in part (a) have an identity?
 - d) How many of the functions f in part (c) are commutative?
8. Let $A = \{2, 4, 8, 16, 32\}$, and consider the closed binary operation $f: A \times A \rightarrow A$ where $f(a, b) = \gcd(a, b)$. Does f have an identity element?

Exercise 5.5

2. Show that if eight people are in a room, at least two of them have birthdays that occur on the same day of the week.
4. Let $S = \{3, 7, 11, 15, 19, \dots, 95, 99, 103\}$. How many elements must we select from S to insure that there will be at least two whose sum is 110?

Exercise 5.6

2. a) For $A = (-2, 7] \subseteq \mathbf{R}$ define the functions $f, g: A \rightarrow \mathbf{R}$ by

$$f(x) = 2x - 4 \quad \text{and} \quad g(x) = \frac{2x^2 - 8}{x + 2}.$$

Verify that $f = g$.

- b) Is the result in part (a) affected if we change A to $[-7, 2)$?

22. If $|A| = |B| = 5$, how many functions $f: A \rightarrow B$ are invertible?