CALCULUS 2 TEST 2

Please write down the detail for your answer.

Let ab be the final two digits of your student's ID. Define R and r as the following:

$$R = \begin{cases} 4 & \text{when } a \text{ is even} \\ 3 & \text{when } a \text{ is odd} \end{cases}$$

and

$$r = \begin{cases} 1 & \text{when } b \text{ is even} \\ 2 & \text{when } b \text{ is odd} \end{cases}.$$

For example, when your ID= $F99886672$, then $R = 3$ and $r = 1$ because $a = 7$ is odd and $b = 2$ is even.

Problem (20 points). Define the surface of a half-torus as the following

$$\mathcal{S} = \left\{ (x, y, z) \in \mathbb{R}^3 \middle| \left(\sqrt{x^2 + y^2} - R\right)^2 + z^2 = r^2, \text{ and } z \geq 0 \right\}$$

(R and r defined as above) and $\vec{n}$ be the unit normal vector points outward of the torus. Suppose $V = (y, -x, e^{xy} \sin z)$ be a vector field defined on $\mathbb{R}^3$. Compute the following integral:

$$\iint_{\mathcal{S}} \text{curl } V \cdot \vec{n} ds.$$

Hint: You have to use Stoke's theorem in a very different way. Notice that it is not necessary that the surface for the Stoke's theorem being smooth.