# Special Relativity

**Newton's laws of motion give us a complete description of the behaviour of moving objects in daily life.**

**However, subjects moving at very high speed behave differently and no longer abide by Newton’s laws!**

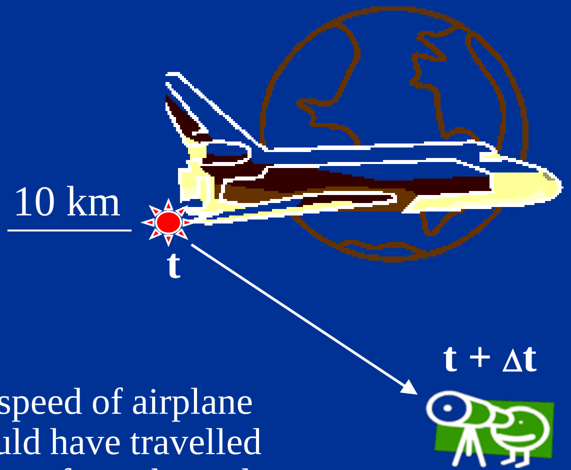
The trouble is that, when something travels at a speed comparable to the speed of light, we need to take into account of the time for the light to travel between the object and observer. The concept of simultaneity is in doubt. Space and time have to be defined more generally to explain events happening under the circumstances.

**Light speed: $c=3\times10^{8}$ m·s$^{-1}$, Sound speed: $v=350$ m·s$^{-1}$**

If an airplane, travelling at sound speed and 10 km above ground, flashes a light at time t, it takes

$$\Delta t = 10000 / 3\times10^{8} = 3\times10^{-5}\ \text{s}$$

for the light to travel to an observer on the ground.




In $3\times10^{-5}$ s, the airplane travels $350\times3\times10^{-5}$ =0.01 m. However, if the speed of airplane were 1/3 of the light speed, i.e. $v=1\times10^{8}$ m·s$^{-1}$, then in $3\times10^{-5}$ s, it would have travelled $1\times10^{8}\times3\times10^{-5}$=3000 m ! (so, the position we see is actually 3000 m away from the real position ! We now have a problem with “simultaneous”.)

# Frames of Reference (參考座標系)

Something, with which you can compare to see if your position is changing.

Physically speaking, reference frame is a coordinate system that allows description of time and position of points relative to a body.

<u>Inertial frames of reference</u> are the ones in which Newton's first law holds. Any frame of reference that is stationary or moves at constant velocity relative to an inertial fame is itself an inertial fame.

**There is no such thing as an absolute frame of reference in our universe**.

By saying *absolute*, what is actually meant is that there is no place in the universe that is completely stationary. This statement says that since everything is moving, all motion is relative. The earth itself is moving, so even though you are standing still, you are in motion. You are moving through both space and time at all times. Because there is no place or object in the universe that is stationary.

Special Relativity is applied only when the events happened in inertial frames of reference.

# Postulates of Special Relativity

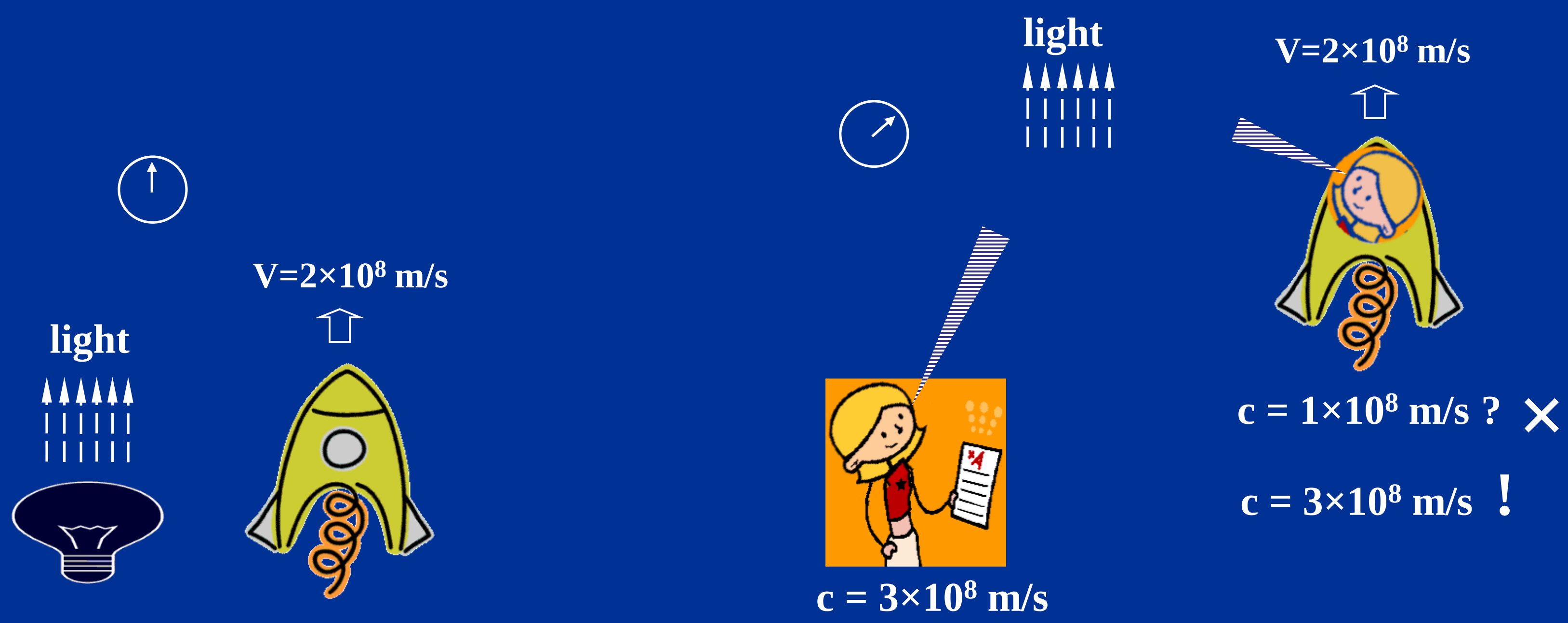
• The first, **the principle of relativity**, states:

**The laws of Physics are the same in all inertial frames of reference.**

• The second is based on the results of many experiments and it states:

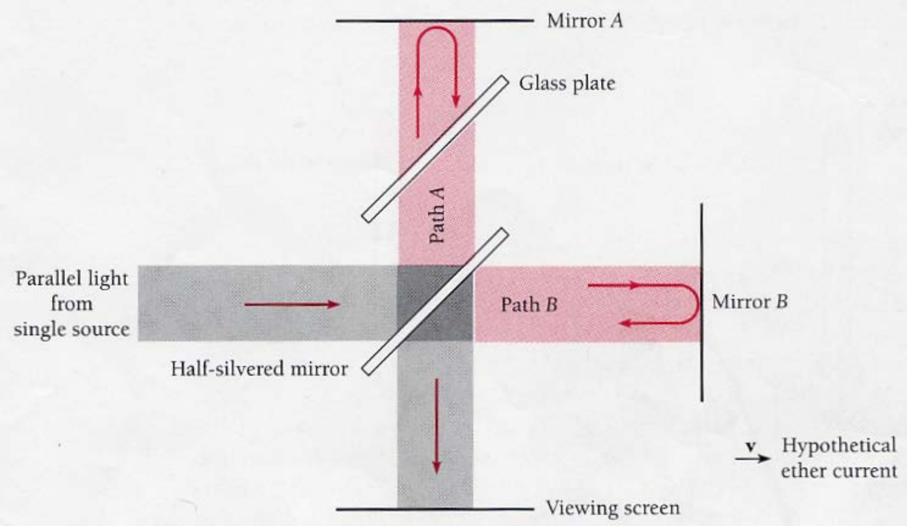
**The speed of light in free space has the same value in all inertial frames of reference.**

This speed is about $3\times10^8$ m/s.



# The Michelson-Morley experiment

In 19th century, scientists believed there existed a mysterious medium called "ether", in which light waves were assumed to occurred. This experiment was to look for the earth's motion through "ether".



The Michelson-Morley experiment.

The result:

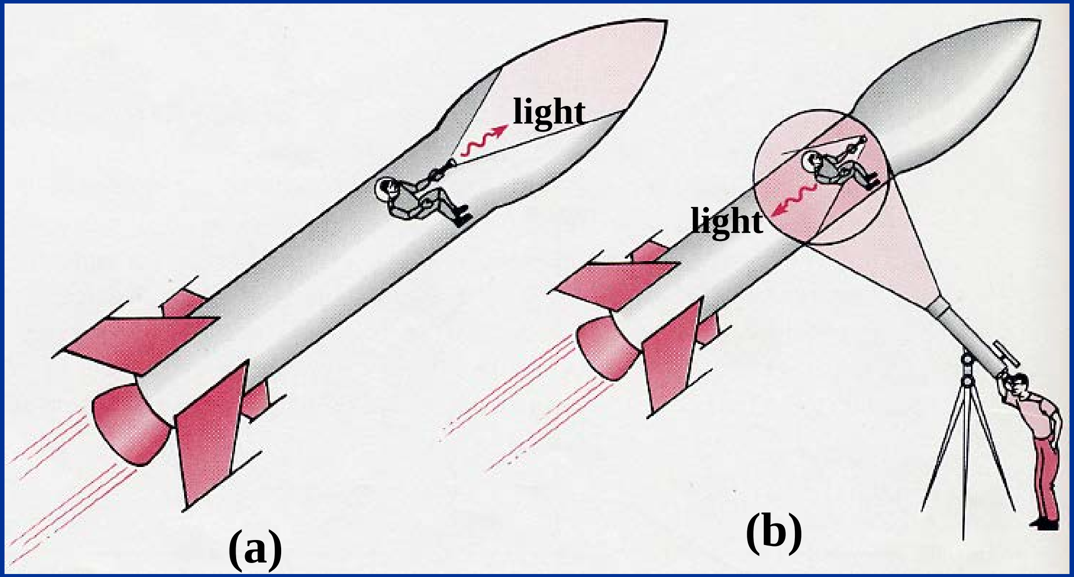
**<u>no either drift was found.</u>**

However, the negative result had two consequences:

1. It showed that ether does not exist and so there is no such thing as "absolute motion" relative to either.
2. It showed that the speed of light is the same for all observers, which is not true of waves that need a material medium in which to occur (like sound and water waves).

# The Ultimate Speed Limit

From the two postulates of special relativity, it follows that ***nothing can move faster than the speed of light in free space***




A flashlight is switched on in a spacecraft assumed to be moving relative to the earth in a speed faster than light.

(a) A person in the spacecraft will see the light goes to the front of the spacecraft. (the speed of light in the spacecraft frame is still $3\times10^8$ m/s — the second postulate)

(b) A person on the ground will see the light goes to the back of the spacecraft. (the spacecraft is assumed to be moving faster than light, therefore the light is falling behind the spacecraft)

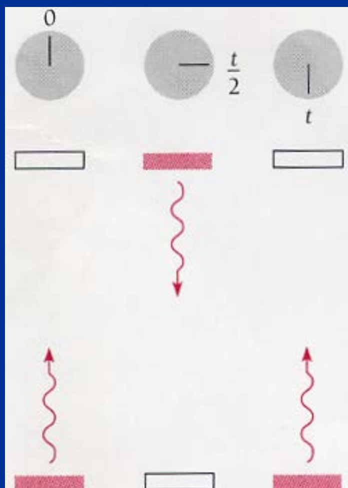
This is against the first postulate that the laws of Physics are the same in all inertial frames of reference, so the speed of light is the ultimate speed limit in free space.

# Time dilation（時間擴張）

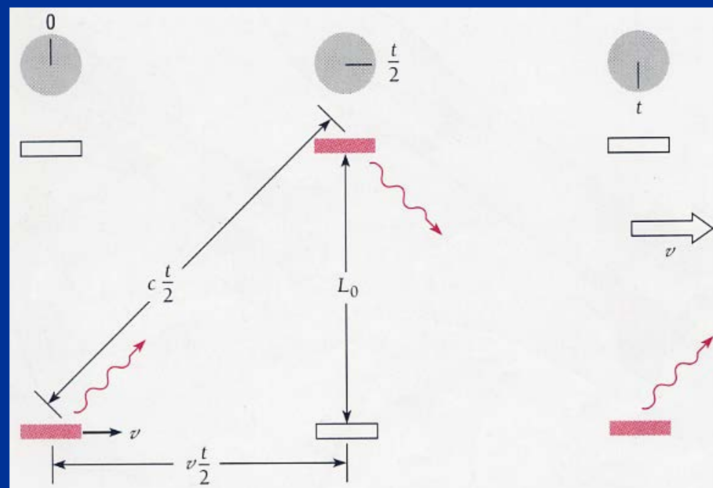
Measurement of time intervals are affected by the relative motion between an observer and what is observed

***A moving clock is ticks more slowly than a clock at rest.***

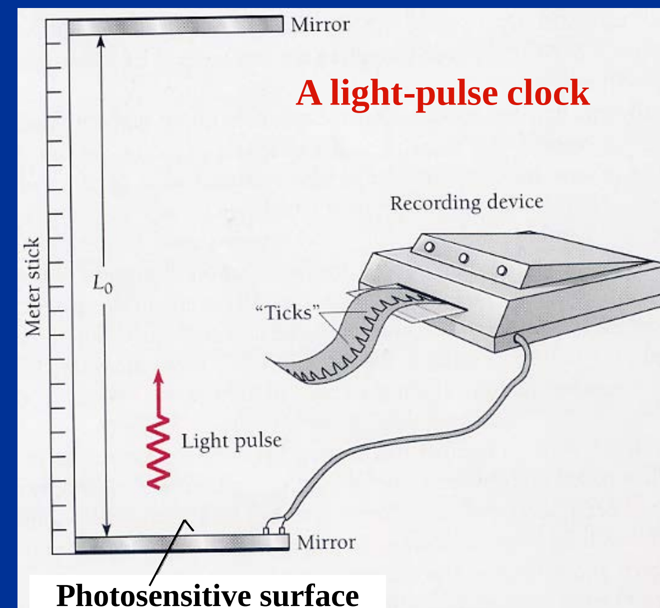
**How ?**




The light-pulse clock at rest on the ground as seen by an observer on the ground. The dial represent a conventional clock on the ground.




The light-pulse clock in a spacecraft as seen by an observer on the ground. The mirrors are parallel to the direction of motion of the spacecraft. The dial represent a conventional clock on the ground.




If the time interval between ticks is $t_0$,

then $$t_0 = \frac{2L_0}{c}$$

If the time interval between ticks is ***t***, the light pulse takes ***t/2*** to travel from the bottom to the top mirrors and in this time interval, the light pulse also travels a horizontal distance of ***v·(t/2)*** due to the motion of spacecraft.

So, the actual distance of light pulse travels in ***t/2*** is:

$$c \cdot \frac{1}{2} t = \sqrt{L_0^2 + (v \cdot \frac{1}{2} t)^2}$$

$$c \cdot \frac{1}{2} t = \sqrt{L_0^2 + (v \cdot \frac{1}{2} t)^2} \Rightarrow c^2 \cdot \frac{1}{4} t^2 = L_0^2 + v^2 \cdot \frac{1}{4} t^2 \Rightarrow t^2 = \frac{4L_0^2}{c^2 - v^2} \Rightarrow t = \frac{2L_0/c}{\sqrt{1 - v^2/c^2}}$$

Substitute $t_0 = \frac{2L_0}{c}$, which is the time interval on clock at rest, into above equation:

We have, $$\boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$$

$t_0$ is the time interval on the clock at rest relative to an observer and is called as ***proper time*** (特徵時間).

$t$ is the time interval on the clock in motion relative to an observer.

$v$ is the speed of relative motion to the observer.

$c$ is the speed of light,

Because $c > v$, $\sqrt{1 - v^2/c^2}$ is always smaller than 1. Therefore: $t > t_0$

*—A moving clock ticks more slowly than a clock at rest*

# Doppler Effect of Light

## A verification of time dilation

When the source of a wave is moving, the frequency of the wave changes. In the case of sound, we can hear the change in pitch. This is familiar to us.

Doppler effect in sound: $$\nu = \nu_0\left(\frac{1+\boldsymbol{v}/\boldsymbol{s}}{1-\boldsymbol{V}/\boldsymbol{s}}\right)$$

Here, $\nu$ = frequency heard by observer, $\nu_0$ = source frequency

$\boldsymbol{s}$ = speed of sound

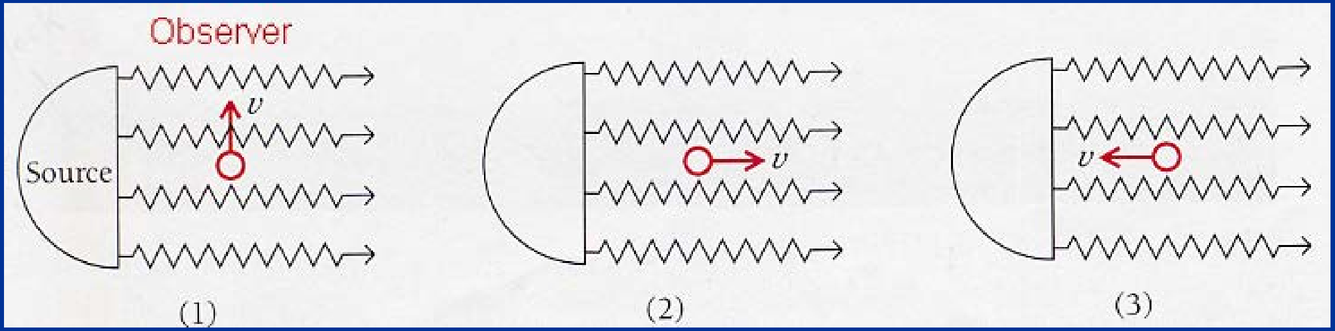
$\boldsymbol{v}$ = speed of observer (+/- for motion towards/away from the source)

$\boldsymbol{V}$ = speed of source (+/- for motion towards/away from the observer)

The Doppler effect in sound depends on whether the source, observer, or both are moving. Sound waves have to travel in a medium such as air or water, and this medium is itself a frame of reference in which the motions of source and observers are both measured.

**However, in the case of light, no medium is involved and only the relative motions between the source and observer is meaningful. So we expect something different for Doppler effect in light.**

**How does the frequency change if there is a relative motion of light source and observer ?**




Considering a light source as a clock that ticks $\nu_0$ times per second and emits a wave of light with each click.

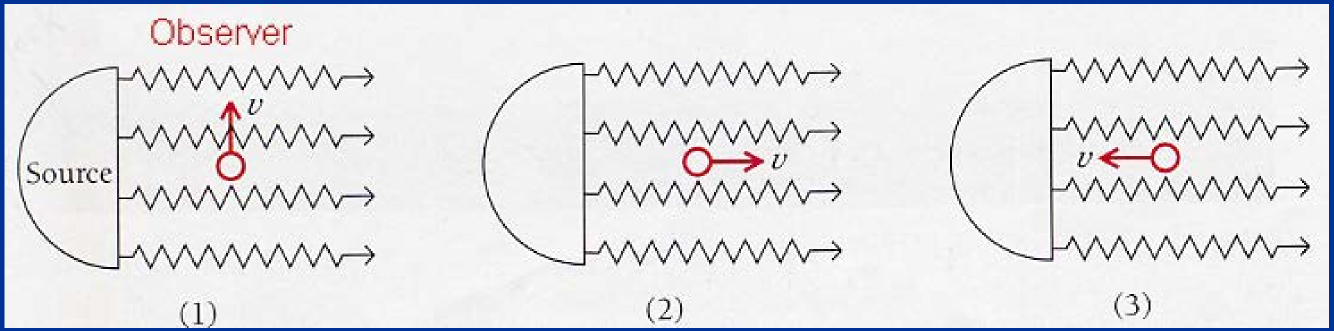
(1) Observer moving perpendicular to the light

In the reference frame of light source, the interval between ticks is $t_0=1/\nu_0$ . However, in the reference frame of observer, $t_0$ corresponds to a period of time $t = t_0 / \sqrt{1-v^2/c^2} = 1/\nu_0\sqrt{1-v^2/c^2}$ .

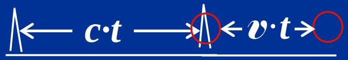
So, the frequency in the observer's frame of reference is:

$$\nu = 1/t = \nu_0\sqrt{1-v^2/c^2}$$

**In the transverse Doppler effect of light, the observed frequency $\nu$ is always lower than the source frequency $\nu_0$ .**




(2) Observer moving away from the light source




Now the observer travels a distance $\boldsymbol{v{\cdot}t}$ away from the source between the ticks, which means that the light wave from a given tick takes $(\boldsymbol{v{\cdot}t})/\boldsymbol{c}$ longer to reach the observer. Hence, the total time between the arrival of the successive waves is:

$$T = t + vt/c = t(1+v/c) = t_0\frac{1+v/c}{\sqrt{1-v^2/c^2}} = t_0\frac{\sqrt{1+v/c}\sqrt{1+v/c}}{\sqrt{1+v/c}\sqrt{1-v/c}} = t_0\sqrt{\frac{1+v/c}{1-v/c}}$$

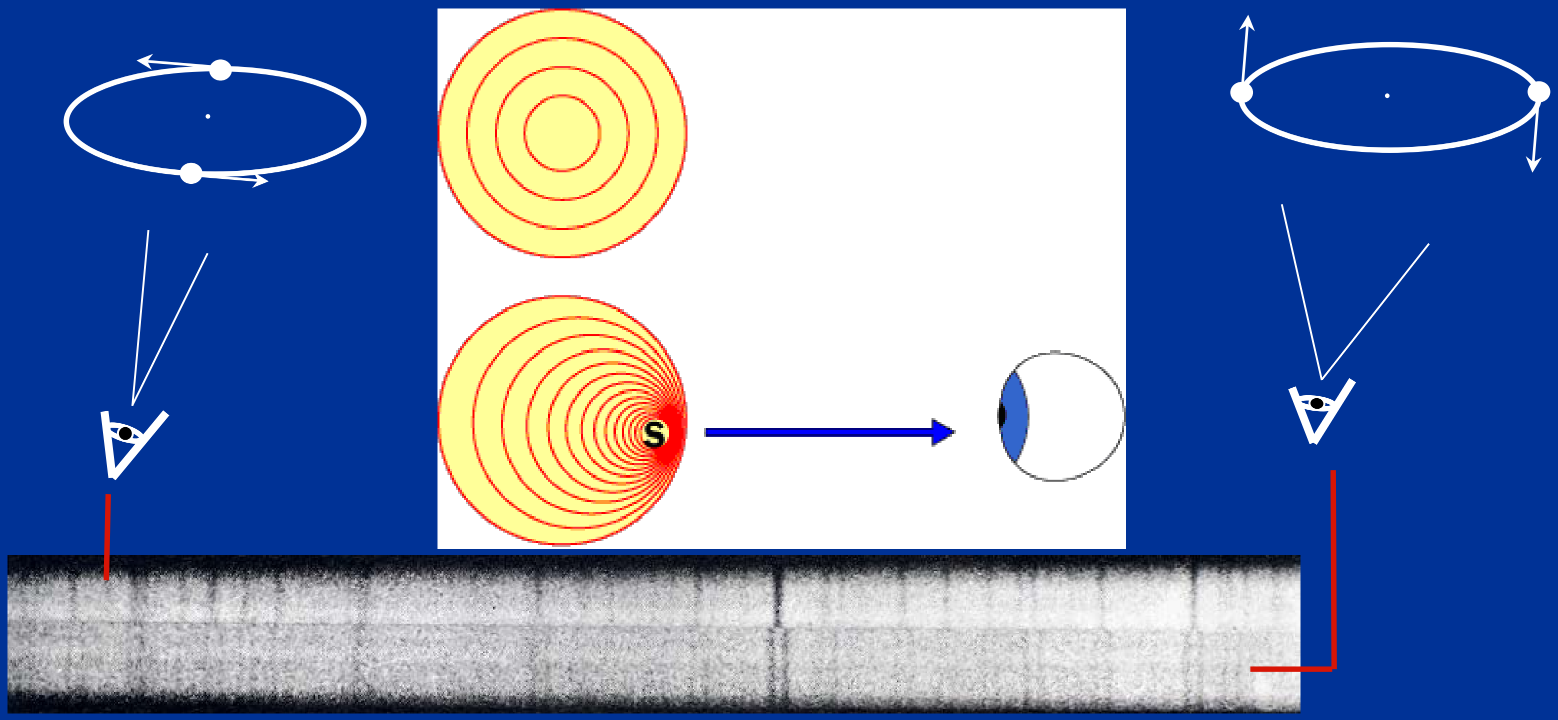
The observed frequency $\nu = \dfrac{1}{T} = \dfrac{1}{t_0}\sqrt{\dfrac{1-v/c}{1+v/c}} = \nu_0\sqrt{\dfrac{1-v/c}{1+v/c}}$

(3) Observer moving toward the light source

$$T = t - vt/c \qquad \nu = \nu_0\sqrt{\frac{1+v/c}{1-v/c}}$$

## In the longitudinal Doppler effect of light

$$\nu = \begin{cases} \nu_0\sqrt{\dfrac{1-v/c}{1+v/c}} & \text{Moving away from the light source} \\[2em] \nu_0\sqrt{\dfrac{1+v/c}{1-v/c}} & \text{Moving towards the light source} \end{cases}$$



# The expanding Universe

The Doppler effect in light is an important tool in astronomy. Stars emit light of characteristic wavelengths called spectral lines, and motion of a star towards or away from the earth shows up as a Doppler shift in its spectrum. All distant galaxies of stars are found to shift towards the lower frequency, i.e. red side of the spectrum, and hence called “red shift”. Such shifts indicate the galaxies are moving away from us and from one another, that is the universe is expanding.

# Length Contraction

Measurements of lengths are also affected by the relative motion. The length $\boldsymbol{L}$ of an object in motion with respect to an observer always appears to the observer to be shorter than its length $\boldsymbol{L_0}$ when it is at rest. The length $\boldsymbol{L_0}$ of an object in its rest frame is called **proper length** of the object.

Here we'll show how this happen according to the time dilation and the principle of relativity.

Facts about muons in cosmic-ray:

•have speeds of about $2.994\times10^8$ m/s (0.998c)

•reach earth's surface (sea level) at 1 per centimeter squared per minute

•unstable with average lifetime of $\boldsymbol{t_0} = 2.2\ \mu s$

In the lifetime of a muon, it can only travel:

$$\boldsymbol{v}\cdot\boldsymbol{t} = (2.994\times10^8\ \text{m/s})\times(2.2\times10^{-6}\ \text{s}) = 0.66\ \text{km}$$

**How can the muons reach the earth surface from the sky (~10 km above) ?**

To resolve the paradox, we note that the muon lifetime of $t_0$ = 2.2 μs is what an observer at rest with respect to a muon would find. Because the muons are traveling **very rapidly** at the speed of 0.998c, their lifetimes are extended in our frame of reference (i,e, earth) by time dilation:

$$t = \frac{t_0}{\sqrt{1-v^2/c^2}} = \frac{2\cdot 2\times 10^{-6}}{\sqrt{1-(0\cdot 998c)^2/c^2}} = 34\cdot 8\times 10^{-6}\ \mathrm{s} = 34\cdot 8\ \mu\mathrm{s}$$

In 34.8 μs, a muon moving at speed of 0.998c will travel the distance:

$v \cdot t$ **= (2.994×10$^{8}$ m/s)×(34.8×10$^{-6}$ s) = 10.4 km**, and therefore can reach the earth.

What if somebody were travelling with the muon at $v$ = 0.998c, so that to him/her the muon is at rest? The observer and the muon are now in the same frame of reference and in this frame, the muon's lifetime is only 2.2 μs. To the observer in this frame, muons can travel only 0.66 km before decaying. However, according to the principle of relativity, muons do reach the surface of earth. (because they do in the earth frame of reference.) The only way to account for this is that to the observer in the moving frame of reference, the distance it travels is shortened to 0.66 km, i.e. by a factor of $\sqrt{1-v^2/c^2}$ .

So, the length contraction for a moving object at speed $v$ is:

$$L = L_0\sqrt{1-v^2/c^2}$$

*—Faster means shorter*

## Summary of last lecture:

- Two postulates of special relativity
  - The laws of Physics are the same in all inertial frames of reference.
  - The speed of light in free space has the same value in all inertial frames of reference.
- nothing can move faster than the speed of light in free space
- Time dilation $$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$
- Length contraction $$L = L_0\sqrt{1 - v^2/c^2}$$ (only along the direction of motion)

# Relativity of Mass


$\rightarrow V = (F/\mathrm{m}) \cdot t$

$V$ increases as $t$, at some point there will be $V > c$ !

We have known that ***c*** is the speed limit of universe, so the object's speed can not keep increasing in proportion as more work is done to it.

How about the conservation of energy ? (which is still valid in the world of relativity.)

The explanation is: **the mass of moving subject is changing.**

As the speed increase, so does the mass, so that the work done is turning into kinetic energy even though $V$ never exceeds $c$.

How does this happen?

Consider two small balls A and B in elastic collision:

- A and B are identical and at rest in the reference frame S and S', respectively.
- Frame S' is moving at constant speed $\boldsymbol{v}$ with respect to frame S.

At the moment that A and B cross each other in x-axis, A is thrown in the +y direction at the speed $V_A$, and at the same time B is thrown in the –y' direction at the speed $V'_B$, where

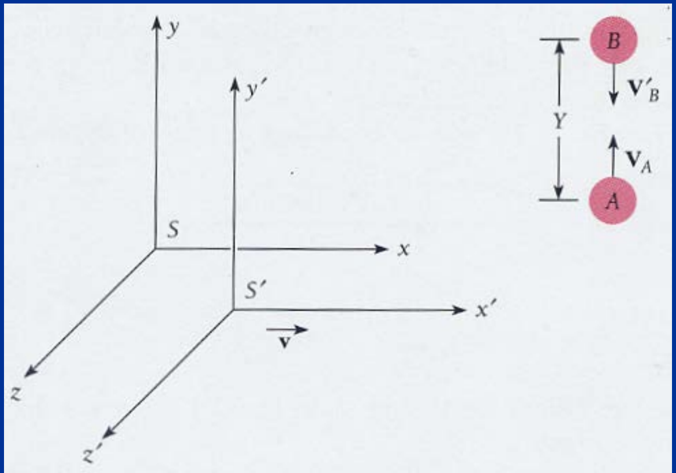
$$V_A = V'_B \quad (1)$$

When A and B collide, A rebounds in the –y direction at the speed $V_A$, while B rebounds in the +y' direction at the speed $V'_B$. If A and B are Y apart at beginning, an observer in S finds that the collision occurs at y=(1/2)Y. The round-trip time $T_0$ for A as measured in frame S is therefore:

$$T_0 = \frac{Y}{V_A} \quad (2)$$

The same will be observed for B by someone in S', and

$$T_0 = \frac{Y}{V'_B} \quad (3)$$




If linear momentum is conserved in the S frame, it must be true that

$$m_A V_A = m_B V_B \quad (4)$$

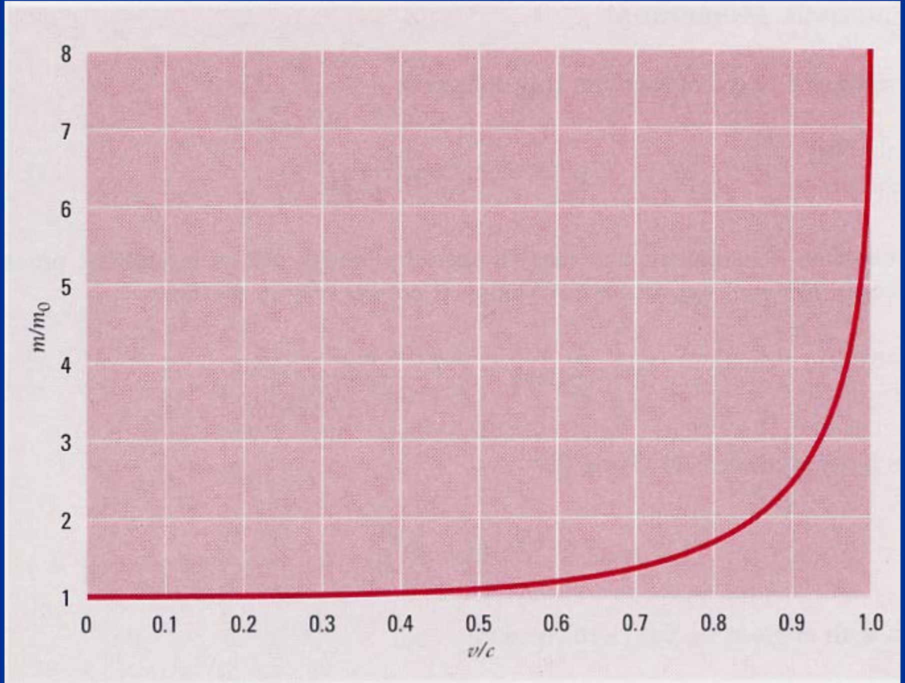
where $m_A$ and $m_B$ are the mass of A and B, and $V_A$ and $V_B$ are their speeds as measured in S frame, In S the speed $V_B$ is found from:

$$V_B = \frac{Y}{T} \quad (5)$$

where T is the time for B to make the round-trip as measured in S. But in its own frame S', B's trip time is $T_0$, and

$$T = \frac{T_0}{\sqrt{1 - v^2/c^2}} \quad (6)$$

Substitute (6) into (5): $V_B = \dfrac{Y\sqrt{1 - v^2/c^2}}{T_0}$ (7)



The relativity of mass. Since $m = \infty$ when $v = c$, no material object can equal the speed of light in free space.

From (2), we have $V_A = \dfrac{Y}{T_0}$ (8)

Insert (7) and (8) into (4), i.e. $m_A V_A = m_B V_B$

We have $m_A = m_B\sqrt{1 - v^2/c^2}$, i.e.

$$m_B = \frac{m_A}{\sqrt{1 - v^2/c^2}}$$

which is the mass of B observed in fame S.
(Note: in its own frame S', $m'_B = m_A$ )

If $V_A$, $V_B \ll \boldsymbol{v}$, we can ignore the motion of A in the frame S and of B in frame S'.

Then, in frame S: $m_A = m_0$ and $m_B = m$ So, $\boxed{m = \dfrac{m_0}{\sqrt{1 - v^2/c^2}}}$

*—Rest mass is least*

**Relativistic momentum**

$$\mathbf{p} = mv = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

conservation of momentum is valid in special relativity just as in classical physics. However, Newton's second law of motion is correct only in the form

**Relativistic second law**

$$F = \frac{d}{dt}(mv) = \frac{d}{dt}\left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}}\right)$$

This is *not* equivalent to saying that

$$F = ma = m\frac{dv}{dt}$$

because

$$\frac{d}{dt}(mv) = m\frac{dv}{dt} + v\frac{dm}{dt}$$

and $dm/dt$ does not vanish if the speed of the body varies with time. The resultant force on a body is always equal to the time rate of change of its momentum.

# Mass and Energy

$E = mc^2$ is the most famous equation obtained from the postulates of special relativity, which concerns the relation between mass and energy.

Let's see how it works out from what we already learn.

Kinetic energy $KE = \int_0^s F\, ds$, here $F = \frac{d}{dt}(mv)$, as we just discussed in previous section.

So, $KE = \int_0^s \frac{d(mv)}{dt} ds = \int_0^{mv} v\, d(mv) = \int_0^v v\, d\left(\frac{m_0 v}{\sqrt{1 - v^2/c^2}}\right)$

Integrating by parts ($\int x\, dy = xy - \int y\, dx$),

$$
\begin{aligned}
KE &= \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} - m_0 \int_0^v \frac{v\, dv}{\sqrt{1 - v^2/c^2}} \\
&= \frac{m_0 v^2}{\sqrt{1 - v^2/c^2}} + \left[ m_0 c^2 \sqrt{1 - v^2/c^2} \right]_0^v \\
&= \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 \\
&= mc^2 - m_0 c^2
\end{aligned}
$$

$$KE = mc^2 - m_0c^2 = (m - m_0)c^2$$

This result states that the kinetic energy of an object is equal to the increase in its mass due to its relative motion multiplied by the square of the speed of light.

The above equation can be written as:

**Total energy** $$mc^2 = m_0c^2 + KE$$

If we interpret $mc^2$ as the **total energy** $E$ of the object, we see that when it is at rest and KE = 0, it nevertheless possesses the energy $m_0c^2$. Accordingly $m_0c^2$ is called the **rest energy** $E_0$ of something whose mass at rest is $m_0$. We therefore have

$$E = E_0 + KE$$

where

**Rest energy** $$E_0 = m_0c^2$$

If the object is moving, its total energy is

**Total energy** $$\boxed{E = mc^2} = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}}$$

## Kinetic Energy at Low Speeds

When the relative speed $v$ is small compared with $c$, the formula for kinetic energy must reduce to the familiar $\frac{1}{2}m_0v^2$, which has been verified by experiment at such speeds. Let us see if this is true. The relativistic formula for kinetic energy is

**Kinetic energy**

$$\mathrm{KE} = mc^2 - m_0c^2 = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} - m_0c^2 \tag{1.23}$$

Since $v^2/c^2 \ll 1$, we can use the binomial approximation $(1 \pm x)^n \approx 1 \pm nx$, valid for $|x| \ll 1$, to obtain

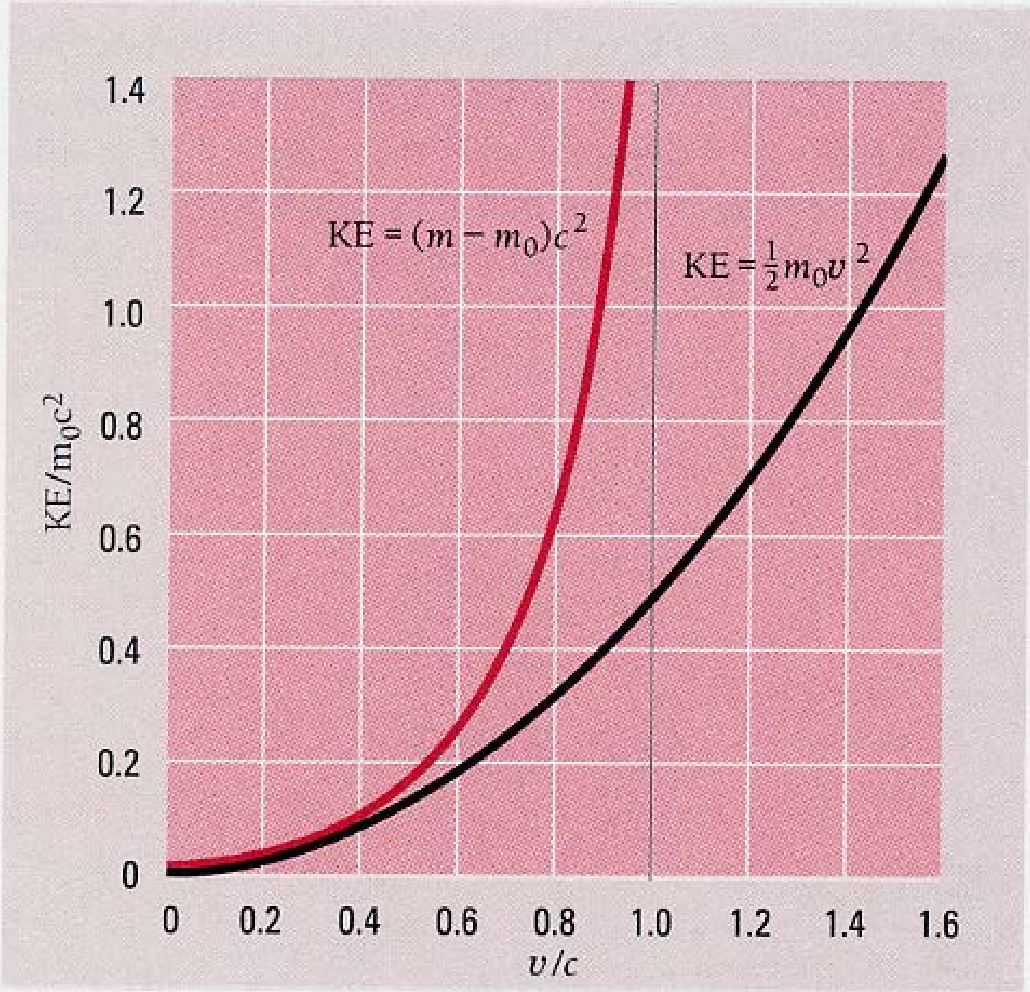
$$\frac{1}{\sqrt{1 - v^2/c^2}} \approx 1 + \frac{1}{2}\frac{v^2}{c^2} \qquad v \ll c$$

Thus we have the result

$$\mathrm{KE} \approx \left(1 + \frac{1}{2}\frac{v^2}{c^2}\right)m_0c^2 - m_0c^2 \approx \frac{1}{2}m_0v^2 \qquad v \ll c$$

At low speeds the relativistic expression for the kinetic energy of a moving object does indeed reduce to the classical one. So far as is known, the correct formulation of mechanics has its basis in relativity, with classical mechanics representing an approximation that is valid only when $v \ll c$.

A comparison between the classical and relativistic formulas for the ratio between kinetic energy KE of a moving body and its rest energy $m_0c^2$. At low speeds the two formulas give the same results, but they diverge at speeds approaching that of light. According to relativistic mechanics, a body would need an infinite kinetic energy to travel with the speed of light, whereas in classical mechanics it would need only a kinetic energy of half its rest energy to have this speed.



## MASSLESS PARTICLES

Can a massless particle exist? To be more precise, can a particle exist which has no rest mass but which nevertheless exhibits such particlelike properties as energy and momentum? In classical mechanics, a particle must have rest mass in order to have energy and momentum, but in relativistic mechanics this requirement does not hold.

Let us see what we can learn from the relativistic formulas for total energy and linear momentum:

**Total energy**

$$E = \frac{m_0c^2}{\sqrt{1 - v^2/c^2}} \quad (1)$$

**Relativistic momentum**

$$p = \frac{m_0v}{\sqrt{1 - v^2/c^2}} \quad (2)$$

When $m_0 = 0$ and $v < c$, it is clear that $E = p = 0$. A massless particle with a speed less than that of light can have neither energy nor momentum. However, when $m_0 = 0$ and $v = c$, $E = 0/0$ and $p = 0/0$, which are indeterminate: $E$ and $p$ can have any values. Thus Eqs. (1) and (2) are consistent with the existence of massless particles that possess energy and momentum *provided that they travel with the speed of light.*

**Total energy**

$$E = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} \qquad (1)$$

**Relativistic momentum**

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} \qquad (2)$$

From Eq. ( 1 ) $E^2 = \dfrac{m_0^2 c^4}{1 - v^2/c^2}$

From Eq. ( 2 ) $p^2 = \dfrac{m_0^2 v^2}{1 - v^2/c^2}$ $\qquad p^2 c^2 = \dfrac{m_0^2 v^2 c^2}{1 - v^2/c^2}$

Subtracting $p^2c^2$ from $E^2$ yields

$$E^2 - p^2c^2 = \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{1 - v^2/c^2} = \frac{m_0^2 c^4 (1 - v^2/c^2)}{1 - v^2/c^2} = m_0^2 c^4$$

$$E^2 = m_0^2 c^4 + p^2 c^2$$

**All particles** $\boxed{E = \sqrt{m_0^2 c^4 + p^2 c^2} = \sqrt{E_0^2 + p^2 c^2}}$

**All particles** $$E = \sqrt{m_0^2c^4 + p^2c^2} = \sqrt{E_0^2 + p^2c^2}$$

According to this formula, if a particle exists with $m_0 = 0$, the relationship between its energy and momentum must be given by

**Massless particles** $$E = pc$$

All the above means not that massless particles necessarily occur, only that the laws of mechanics do not exclude the possibility provided that $v = c$ and $E = pc$ for them. In fact, massless particles of two different kinds—the photon and the neutrino—have indeed been discovered and their behavior is as expected.

## Electronvolts

In atomic physics the usual unit of energy is the **electronvolt** (eV), where 1 eV is the energy gained by an electron accelerated through a potential difference of 1 volt. Since $W = QV$,

$$1 \text{ eV} = (1.602 \times 10^{-19} \text{ C})(1.000 \text{ V}) = 1.602 \times 10^{-19} \text{ J}$$

The rest energies of elementary particles are often expressed in MeV and GeV and the corresponding rest masses in MeV/$c^2$ and GeV/$c^2$. The advantage of the latter units is that the rest energy equivalent to a rest mass of, say, 0.938 GeV/$c^2$ (the rest mass of the proton) is just $E_0 = m_0c^2 = 0.938$ GeV. If the proton's kinetic energy is 5.000 GeV, finding its total energy is simple:

$$E = E_0 + \text{KE} = (0.938 + 5.000) \text{ GeV} = 5.938 \text{ GeV}$$

$$(1 \text{ MeV} = 10^6 \text{ eV} \qquad 1 \text{ GeV} = 10^9 \text{ eV})$$

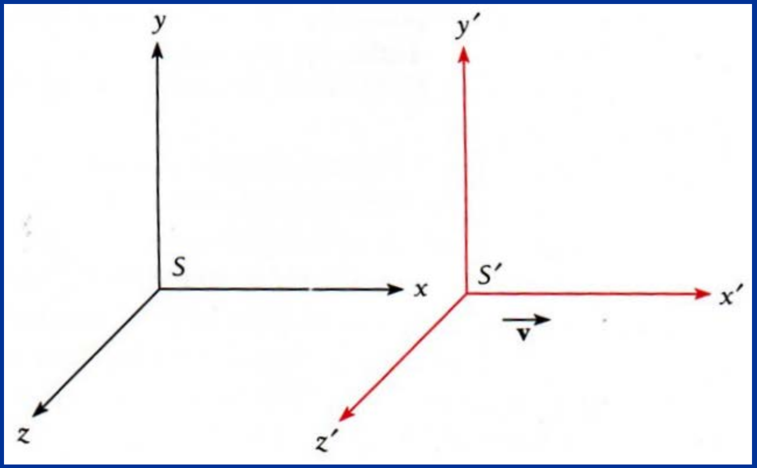
# The Lorentz Transformation

**Galilean Transformation**

$$x' = x - vt$$

$$y' = y$$

$$z' = z \qquad t' = t$$




$$v'_x = \frac{dx'}{dt'} = v_x - v$$

$$v'_y = \frac{dy'}{dt'} = v_y$$

$$v'_z = \frac{dz'}{dt'} = v_z$$

$$c' = c - v \qquad ?$$

**Lorentz Transformation**

$x' = A(x - vt), \;\; x = A(x' + vt')$

$t_0 = t_0' = 0 \;\Rightarrow\; x' = c\,t', \;\; x = c\,t$

$x' = A\,(ct - vt)$ $\boxed{= Act\,(1 - v/c) = c\,t'}$

$x = A\,(ct' + vt')$ $\boxed{= Act'(1 + v/c) = c\,t}$

$$\Rightarrow\; A = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}}$$

$x = A(x' + vt')$
$\;\;\; = A\,(A(x - vt) + vt')$

$$\Rightarrow\; \boxed{t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}}$$

## Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = \frac{t - \dfrac{vx}{c^2}}{\sqrt{1 - v^2/c^2}}$$

## Relativistic velocity transformation

$$x = \frac{x' + vt'}{\sqrt{1 - v^2/c^2}}$$

$$V_x = \frac{dx}{dt} = \frac{dx}{dt'} \cdot \frac{dt'}{dt}$$

$$V_x = \frac{V'_x + v}{1 + \dfrac{vV'_x}{c^2}}$$

$$V_y = \frac{V'_y\sqrt{1 - v^2/c^2}}{1 + \dfrac{vV'_x}{c^2}}$$

$$V_z = \frac{V'_z\sqrt{1 - v^2/c^2}}{1 + \dfrac{vV'_x}{c^2}}$$

# Summary of Special Relativity

- Two postulates of special relativity
  - The laws of Physics are the same in all inertial frames of reference.
  - The speed of light in free space has the same value in all inertial frames of reference.
- nothing can move faster than the speed of light in free space
- Time dilation $$t = \frac{t_0}{\sqrt{1 - v^2/c^2}}$$
- Length contraction $$L = L_0\sqrt{1 - v^2/c^2}$$ (only along the direction of motion)
- Mass of a moving object $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$
- Energy of matters $$E=mc^2$$