1.

a) (1) no assumption → general form.

(2) incompressible flow $\nabla \cdot \vec{v} = 0$, $\mu = \text{const}$ (Newtonian)

(3) inviscid flow $\mu = 0$.

b) Newton's second Law of motion. $(\vec{F} = m\vec{a})$

2.

(($\boxed{\text{convective}}$ momentum flux) due to motion!)

a) momentum flux through control volume, $\frac{M}{Lt^2} = \frac{kg}{ms^2}$

b) external forces acting on the control volume due to normal and $\underbrace{\text{shear stress}}_{\text{per unit volume}}$, $\frac{M}{L^2t^2} = \frac{kg}{m^2s^2}$ (~~momentum flux~~ rate of momentum due to viscious transfer)

c) local and convective acceleration, $\frac{L}{t^2} = \frac{m}{s^2}$

3. Steady, irrotational, incompressible flow.

4.

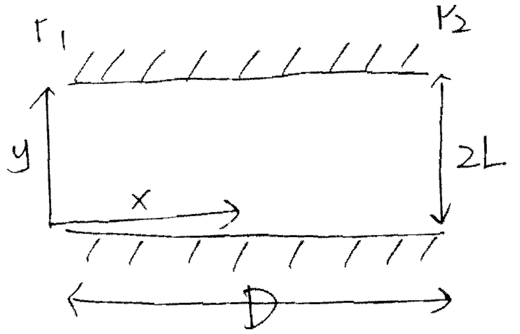
a) $V_{avg} = \frac{100}{\pi(0.125)^2}\left(\times \frac{1}{60}\right)$, $\mu = \frac{1}{8} \times \frac{2.15\times10^6}{30} \times (0.125)^2 \times \frac{\pi(0.125)^2}{100}\left(\times\frac{60}{1}\right)$

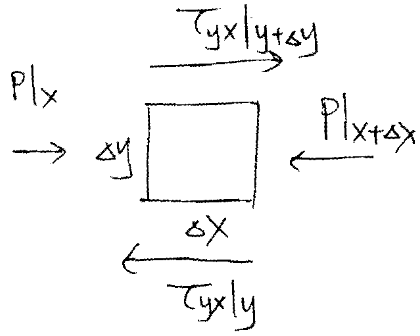
$= 0.0878$ dyne·min·cm^{-2}

$= 5.268$ dyne·s·cm^{-2} (g/cm·s)

b) $Re = \frac{0.25\times100\times1.261}{5.268\times\pi(0.125)^2\times60} = 2.03$ ⟶ It's a liminar flow.

5





Acuording to incompressible, larminar flow $\Rightarrow \begin{cases} V_x = f(y) \\ V_y = 0 \\ V_z = 0 . \end{cases}$

where only exist τ_{yx}.

a)

$$\Sigma F_x = P|_x \Delta y - P|_{x+\Delta x} \Delta y + \tau_{yx}|_{y+\Delta y} \Delta X - \tau_{yx}|_y \Delta X = 0$$

$$\Rightarrow -\frac{dP}{dx} + \frac{d\tau_{yx}}{dy} = 0 \quad , \quad \tau_{yx} = \mu\left(\frac{dV_x}{dy}\right) \quad , \quad \frac{dP}{dx} = \frac{P_2 - P_1}{D}$$

B.C.

$$\Rightarrow \begin{cases} y = 0 \ , \ V_x = 0 \\ y = 2L \ , \ V_x = 0 \end{cases} \Rightarrow V_x = \frac{(P_2 - P_1)L^2}{\mu D}\left[\frac{1}{2}\left(\frac{y}{L}\right)^2 - \left(\frac{y}{L}\right)\right]$$

We can also easily find that when $y = L$,

$$V_{max} = -\frac{(P_2 - P_1)L^2}{2\mu D}$$

b)

$$V_{avg} = \frac{1}{A}\iint_A V_x dA = \frac{\int_0^{2L}\int_0^{W} V_x dz\, dy}{2LW} = -\frac{(P_2 - P_1)L^2}{3\mu D} = \frac{2}{3} V_{max}$$

⑥

which is due to incompressible, laminar flow.

a) We can deal with $V_r = 0$, $V_z = 0$, $V_\theta \neq 0$ in laminar flow, and $V_\theta = f(r)$. Elimating the ~~navior~~ navier - Stokes Eq into θ direction only, we get:

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r V_\theta)\right] = 0$$

, Know that $\begin{cases} D_1 = \frac{R_1}{2} \\ D_2 = \frac{R_2}{2} \end{cases}$

$$\text{B.C.} \begin{cases} V_\theta|_{D_1} = \omega D_1 \\ V_\theta|_{D_2} = 0 \end{cases}$$

$$\Rightarrow V_\theta = \frac{\omega r}{\left(1 - \frac{R_2^2}{R_1^2}\right)} - \frac{\omega R_2^2}{4\left(1 - \frac{R_2^2}{R_1^2}\right) r}$$

\#

b) $$\tau_{r\theta} = \mu\left[r \frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)\right]$$

$$F_z = \int_0^L \int_0^{2\pi} \left(\tau_{r\theta}|_{\frac{R_2}{2}}\right) \frac{R_2}{2}\, d\theta\, dz = \frac{2\pi L \omega R_2 \mu}{\left(1 - \frac{R_2^2}{R_1^2}\right)}$$

\#

7

(1) $\mu \nabla^2 \vec{v} = 0$. due to. $V_r = 0$, $V_z = 0$, $V_\theta = \frac{\omega R^2}{r} = f(r)$ in ~~in~~ incompressible fluid.

(2) $\begin{cases} V_r = 0 \\ V_\theta = \frac{\omega R^2}{r} = -\frac{\partial \Psi}{\partial r} = \frac{1}{r}\frac{\partial \phi}{\partial \theta} \end{cases}$

$$\Rightarrow \begin{cases} \Psi = -\omega R^2 \ln r \\ \phi = \omega R^2 \theta \end{cases}$$

(3)