CHAPTER 6
Frequency Response, Bode Plots, and Resonance

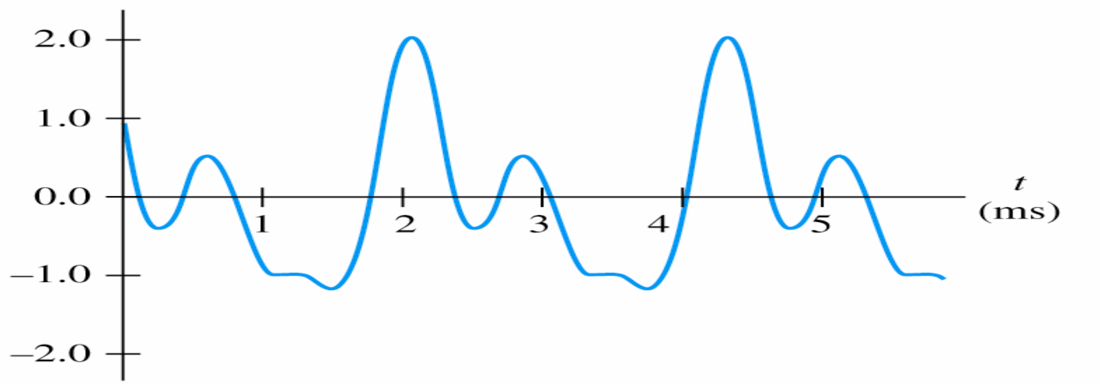
6.1 Fourier Analysis, Filters, and Transfer Functions

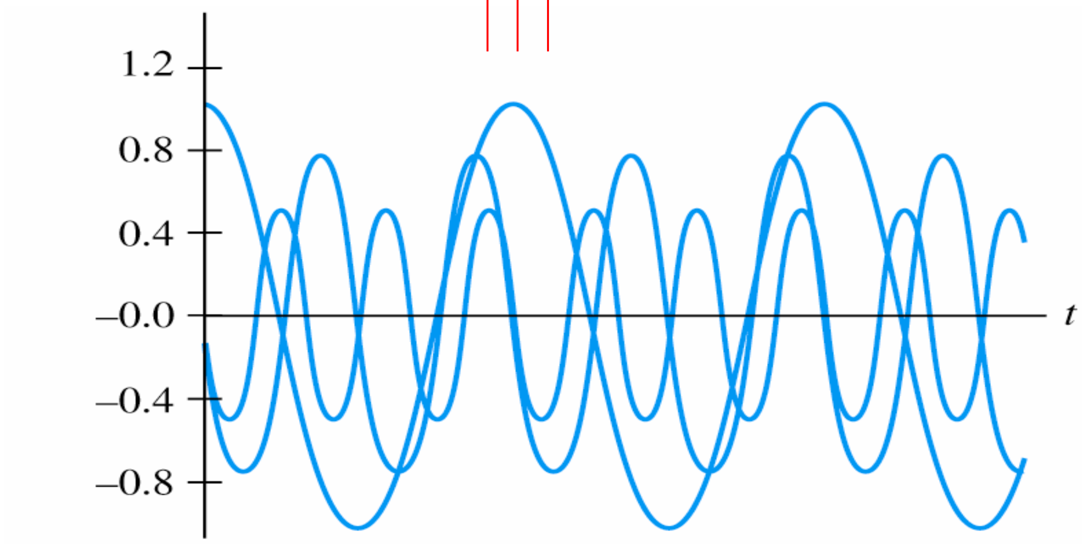
- 大部分蘊藏資訊的訊號都不是單純的 sinusoidal signals 。

- 但是我們可以藉由將不同大小(amplitude)、頻率(frequency)與相位(phase)的 sinusoidal signals 加(adding)在一起而獲得與這些訊號一模一樣的訊號 。

我們可以藉由結合不同sinusoids 成份(component)來組成(construct)所有訊號。

一小段music waveform





三個不同大小、頻率與相位的sinusoidal 成份(component)相加可獲得上面的 music waveform

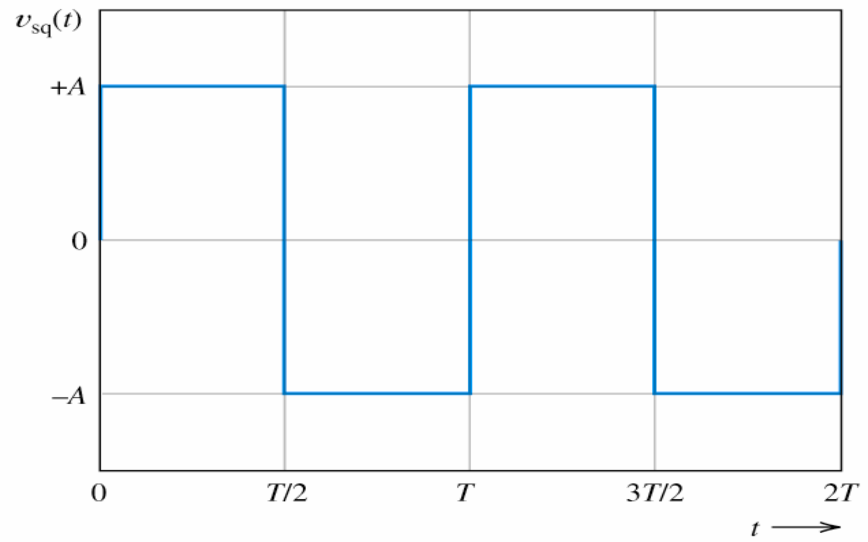
Fourier Analysis (傅利葉分析)

All real-world signals are sums of sinusoidal components having various frequencies, amplitudes, and phases.

Table 6.1. Frequency Ranges of Selected Signals

Electrocardiogram	0.05 to 100 Hz
Audible sounds	20 Hz to 15 kHz
AM radio broadcasting	540 to 1600 kHz
Video signals (U.S. standards)	Dc to 4.2 MHz
Channel 6 television	82 to 88 MHz
FM radio broadcasting	88 to 108 MHz
Cellular radio	824 to 891.5 MHz
Satellite television downlinks (C-band)	3.7 to 4.2 GHz
Digital satellite television	12.2 to 12.7 GHz

Fourier Series (傅利葉序列) of a Square Wave



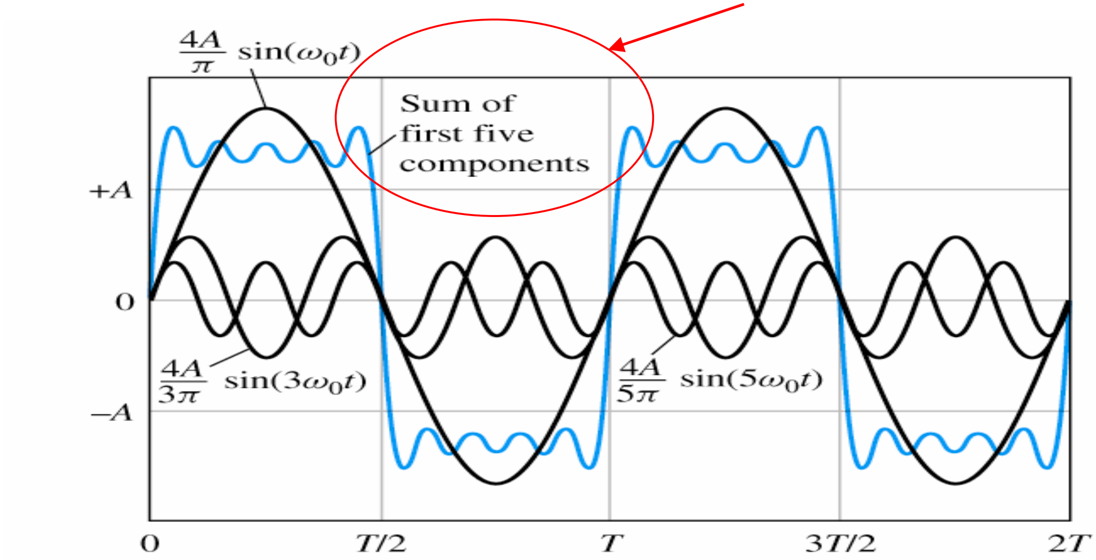
方波(square wave) $V_{sq}(t)$ 可由以下之sinusoids 來組成

$$v_{sq}(t) = \frac{44}{\pi}\sin(\omega_0 t) + \frac{44}{3\pi}\sin(3\omega_0 t) + \frac{44}{5\pi}\sin(5\omega_0 t) + \cdots$$

$\omega_0 = 2\pi / T$ 稱為fundamental angular frequency (基本角頻率).

$$v_{sq}(t) = \frac{44}{\pi}\sin(\omega_0 t) + \frac{44}{3\pi}\sin(3\omega_0 t) + \frac{44}{5\pi}\sin(5\omega_0 t) + \cdots$$

更多 Components 相加可更近似方波



方波為奇數倍基本角頻率($\omega_0, 3\omega_0, 5\omega_0 \cdots$)的sinusoidal components 組成。

•component角頻率越高時，其amplitude(大小) 越小。

•所有component 相位都是-90º($\sin(\omega t) = \cos(\omega t - 90^\circ)$) 。

Filters (濾波器)

Filters process the sinusoid components of an input signal differently depending on the frequency of each component.
(濾波器對輸入訊號之不同頻率成份進行不同處理)

Often, the goal of the filter is to retain (維持)the components in certain frequency ranges and to reject (去除)components in other ranges. (ex 收音機頻道、音響等化器、生理量測測儀,…)

Figure 6.5 Filters behave as if they separate the input into components, modify the amplitudes and phases of the components, and add the altered components to produce the output.

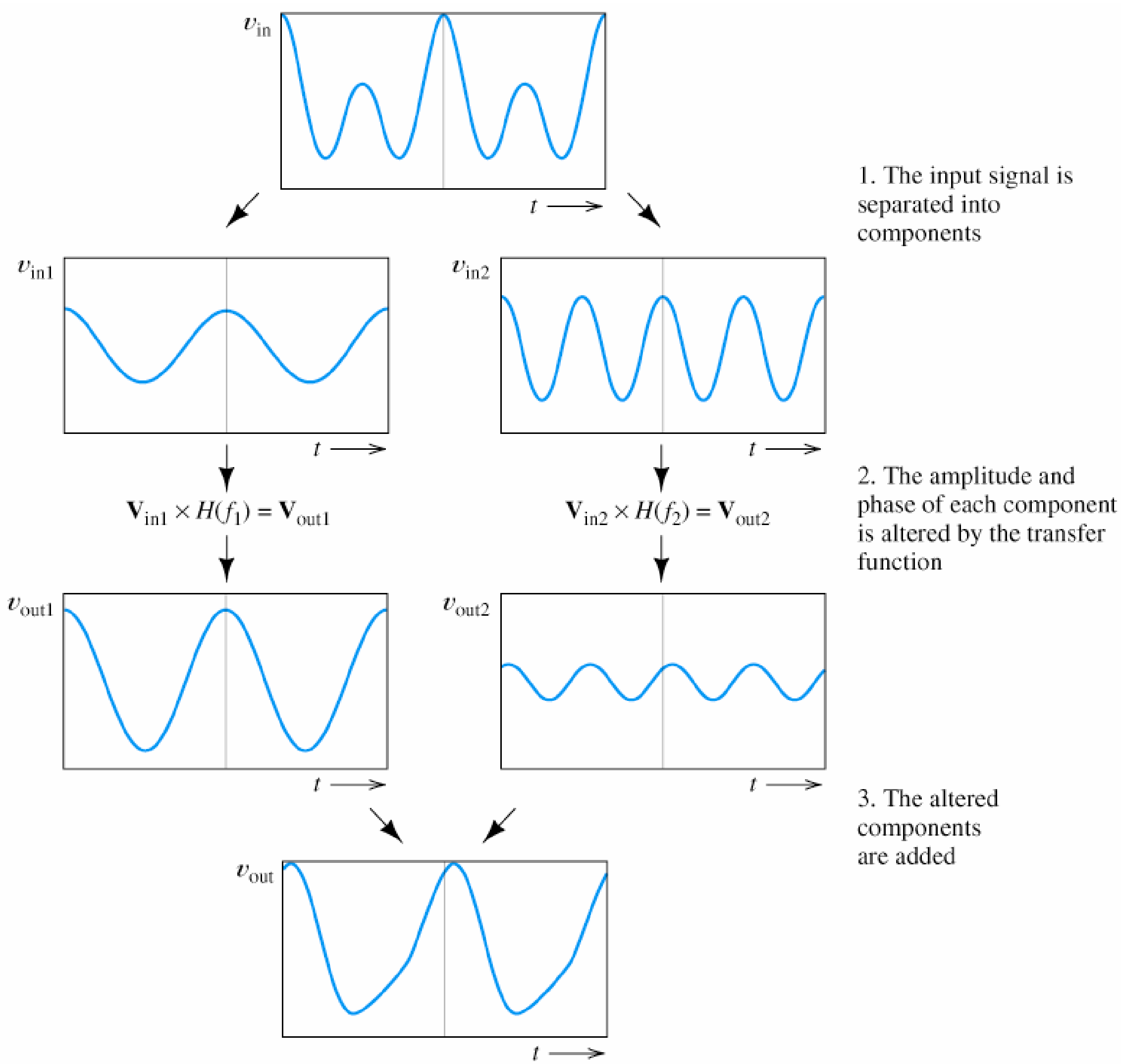
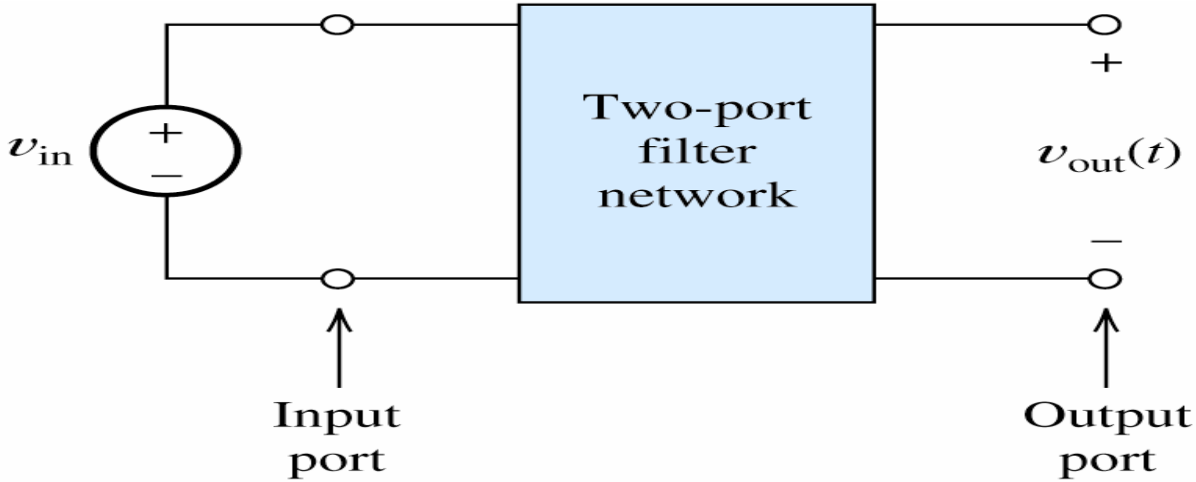


Figure 6.3 When an input signal $v_{in}(t)$ is applied to the input port of a filter, some components are passed to the output port while others are not, depending on their frequencies. Thus, $v_{out}(t)$ contains some of the components of $v_{in}(t)$ but not others. Usually, the amplitudes and phases of the components are altered in passing through the filter.



一般而言雜訊在高頻，因此低通率波器用來消除雜訊。Ex, 可藉由LC impedance 隨頻率改變的特性藉由RLC來實現慮波器。

$$Z_L = \omega L \angle 90^\circ = 2\pi f L \angle 90^\circ$$

$$Z_C = \frac{1}{\omega C} \angle -90^\circ = \frac{1}{2\pi f C} \angle -90^\circ$$

Transfer Functions (轉換函數)

The **transfer function** *H(f)* of the two-port filter is defined to be the ratio (比值) of the phasor output voltage to the phasor input voltage as a function of frequency:

$$H(f) = \frac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}$$

Transfer functions 是一個複數，包含 magnitude 與phase 且magnitude 與phase也都是頻率的函數(function of frequency)。

The magnitude of the transfer function shows how the amplitude of each frequency component is affected by the filter.

Similarly, the phase of the transfer function shows how the phase of each frequency component is affected by the filter.

Example 6.1 Using the Transfer Function to Determine the Output.

Input $v_{\text{in}}(t) = 2\cos(2000\pi t + 40°)$
Find the output of the filter ($v_{\text{out}}(t)$) .

$$H(f) = |H(f)| \angle H(f)$$

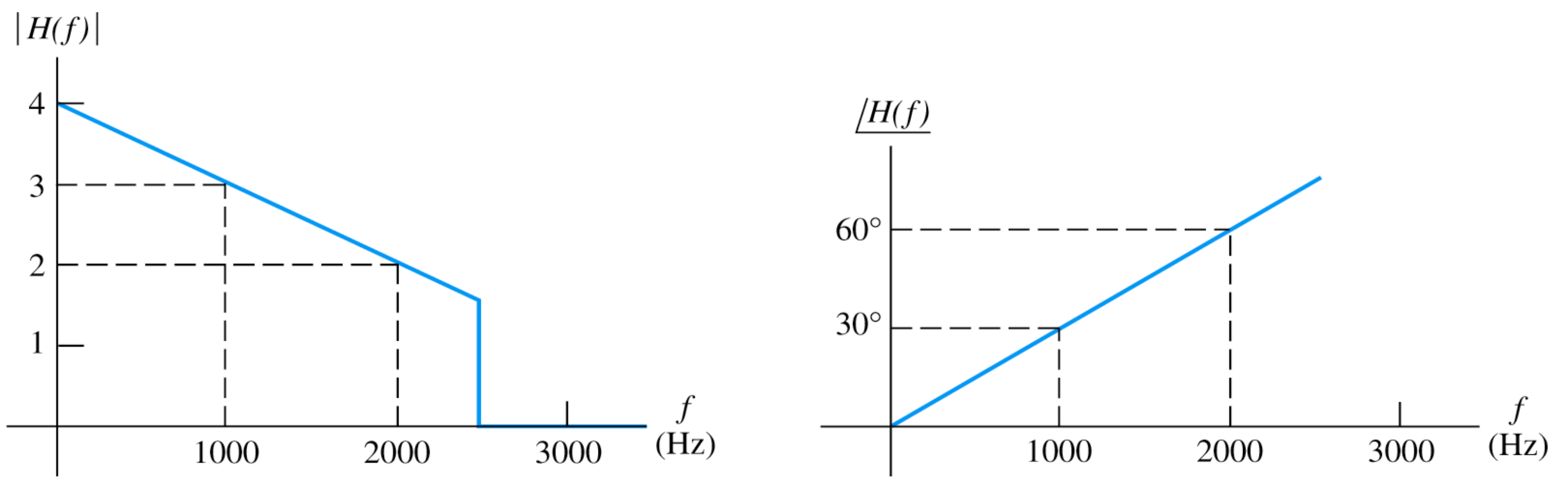


Figure 6.4 The transfer function of a filter. See Examples 6.1 and 6.2.

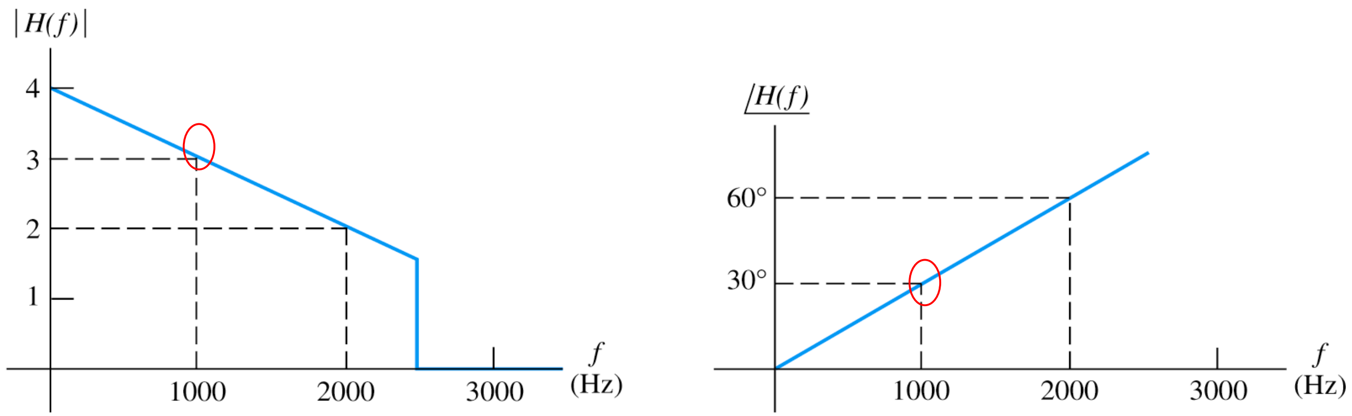


Figure 6.4 The transfer function of a filter. See Examples 6.1 and 6.2.

$$v_{\text{in}}(t) = 2\cos(2000\pi t + 40°) = 2\angle 40^\circ$$

$$\omega = 2\pi f = 2000\pi \quad \Rightarrow \quad f = 1000\text{Hz}$$

$$|H(1000)| = 3 \qquad \angle H(f) = 30^\circ$$

$$H(1000) = 3\angle 30^\circ = \frac{\mathrm{V}_{out}}{\mathrm{V}_{in}}$$

$$\Rightarrow \quad \mathrm{V}_{out} = H(1000)\times \mathrm{V}_{in} = 3\angle 30^\circ \times 2\angle 40^\circ = 6\angle 70^\circ$$

$$v_{\text{out}}(t) = 6\cos(2000\pi t + 70°)$$

Determining the output of a filter for an input with multiple components:

1. Determine the frequency and phasor representation for each input component.

2. Determine the (complex) value of the transfer function for each component.

3. Obtain the phasor for each output component by multiplying the phasor for each input component by the corresponding transfer-function value.

4. Convert the phasors for the output components into time functions of various frequencies. Add these time functions to produce the output.

Example 6.2 Using the Transfer Function with Several Input Components

$$v_{\text{in}}(t) = 3 + 2\cos(2000\pi t) + \cos(4000\pi t - 70°)$$

Find $v_{out}(t)$.

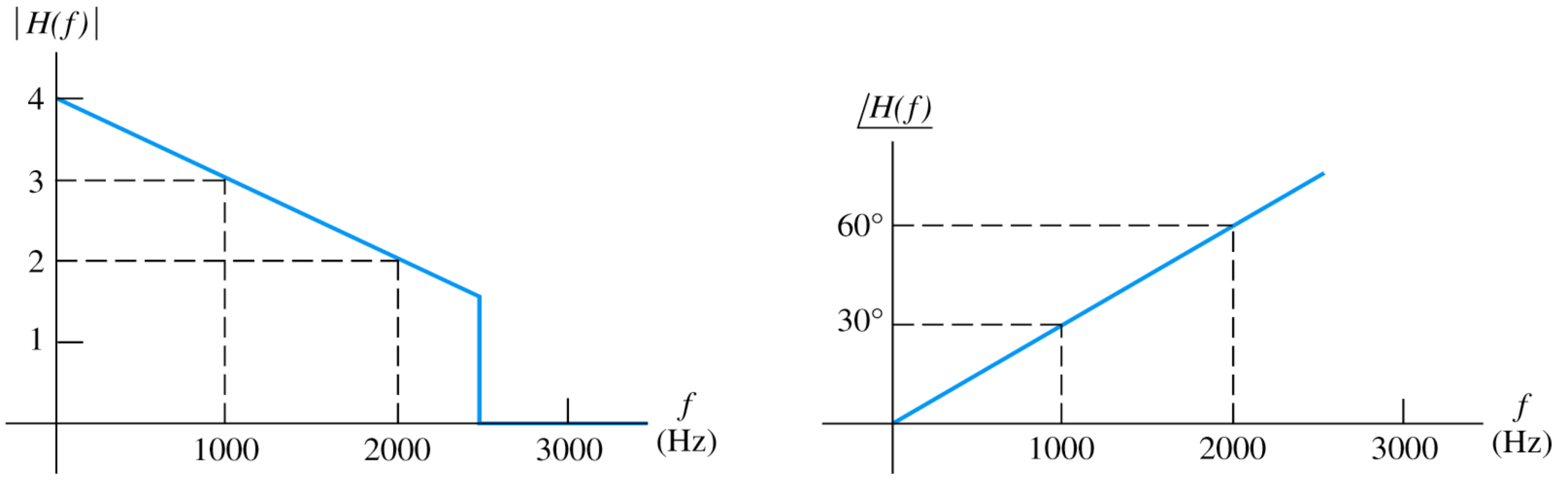


Figure 6.4 The transfer function of a filter. See Examples 6.1 and 6.2.

1. Determine the frequency and phasorfor each input component.

$$v_{\text{in}}(t) = 3 + 2\cos(2000\pi t) + \cos(4000\pi t - 70°)$$

$$v_{\text{in1}}(t) = 3 \qquad \mathrm{V}_{\text{in1}} = 3\angle 0^\circ \qquad f_{\text{in1}} = 0\text{Hz}$$

$$v_{\text{in2}}(t) = 2\cos(2000\pi t) \qquad \mathrm{V}_{\text{in2}} = 2\angle 0^\circ \qquad f_{\text{in2}} = 1000\text{Hz}$$

$$v_{\text{in3}}(t) = \cos(4000\pi t - 70°) \qquad \mathrm{V}_{\text{in2}} = 1\angle -70^\circ \qquad f_{\text{in3}} = 2000\text{Hz}$$

2. Determine the (complex) value of the transfer function for each component.

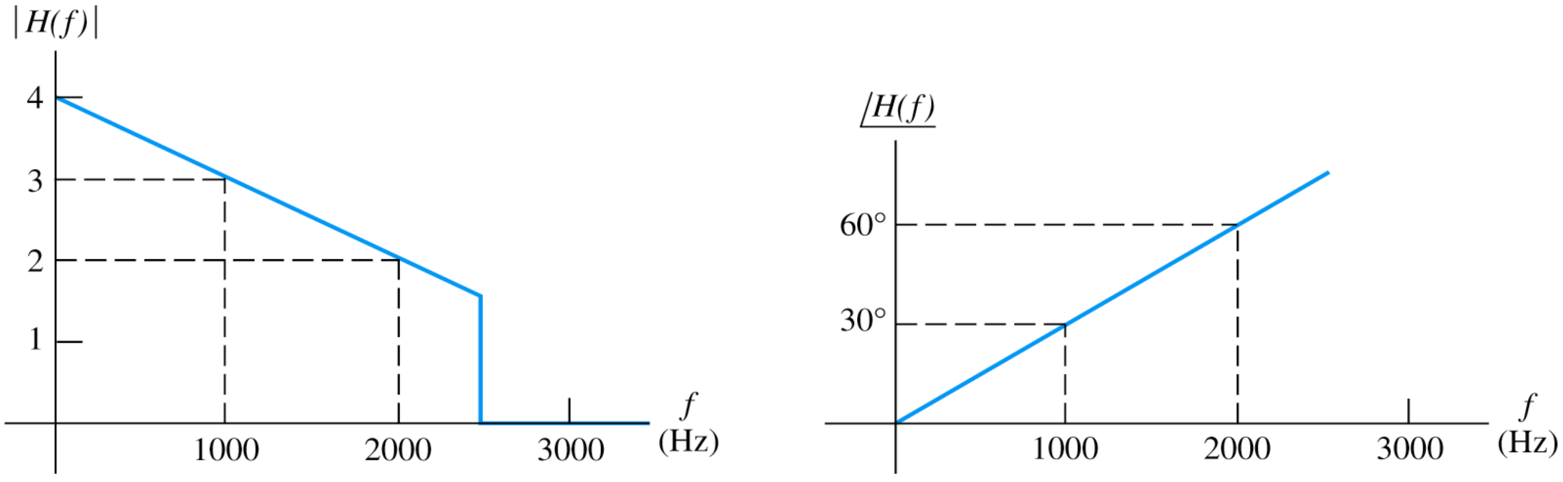


Figure 6.4 The transfer function of a filter. See Examples 6.1 and 6.2.

$$H(0) = 4$$

$$H(1000) = 3\angle 30°$$

$$H(2000) = 2\angle 60°$$

3. Obtain the phasor for each output component

$$v_{out1} = H(0)v_{in1} = 4 \times 3 = 12$$

$$\mathrm{V}_{out2} = H(1000) \times \mathrm{V}_{in2} = 3\angle 30° \times 2\angle 0° = 6\angle 30°$$

$$\mathrm{V}_{out3} = H(2000) \times \mathrm{V}_{in3} = 2\angle 60° \times 1\angle -70° = 2\angle -10°$$

4. Convert the phasors into time functions of various frequencies. Add these time functions to produce the output.

$$v_{out1}(t) = 12$$

$$v_{out2}(t) = 6\cos(2000\pi t + 30°)$$

$$v_{out3}(t) = 2\cos(4000\pi t - 10°)$$

$$\boxed{\begin{aligned} v_{out}(t) &= v_{out1}(t) + v_{out2}(t) + v_{out3}(t) \\ &= 12 + 6\cos(2000\pi t + 30°) + 2\cos(4000\pi t - 10°) \end{aligned}}$$

Linear circuits

1. Separate the input signal into components having various frequencies.

2. Alter (改變) the amplitude and phase of each component depending on its frequency.

3. Add the altered components to produce the output signal.

Figure 6.5 Filters behave as if they separate the input into components, modify the amplitudes and phases of the components, and add the altered components to produce the output.

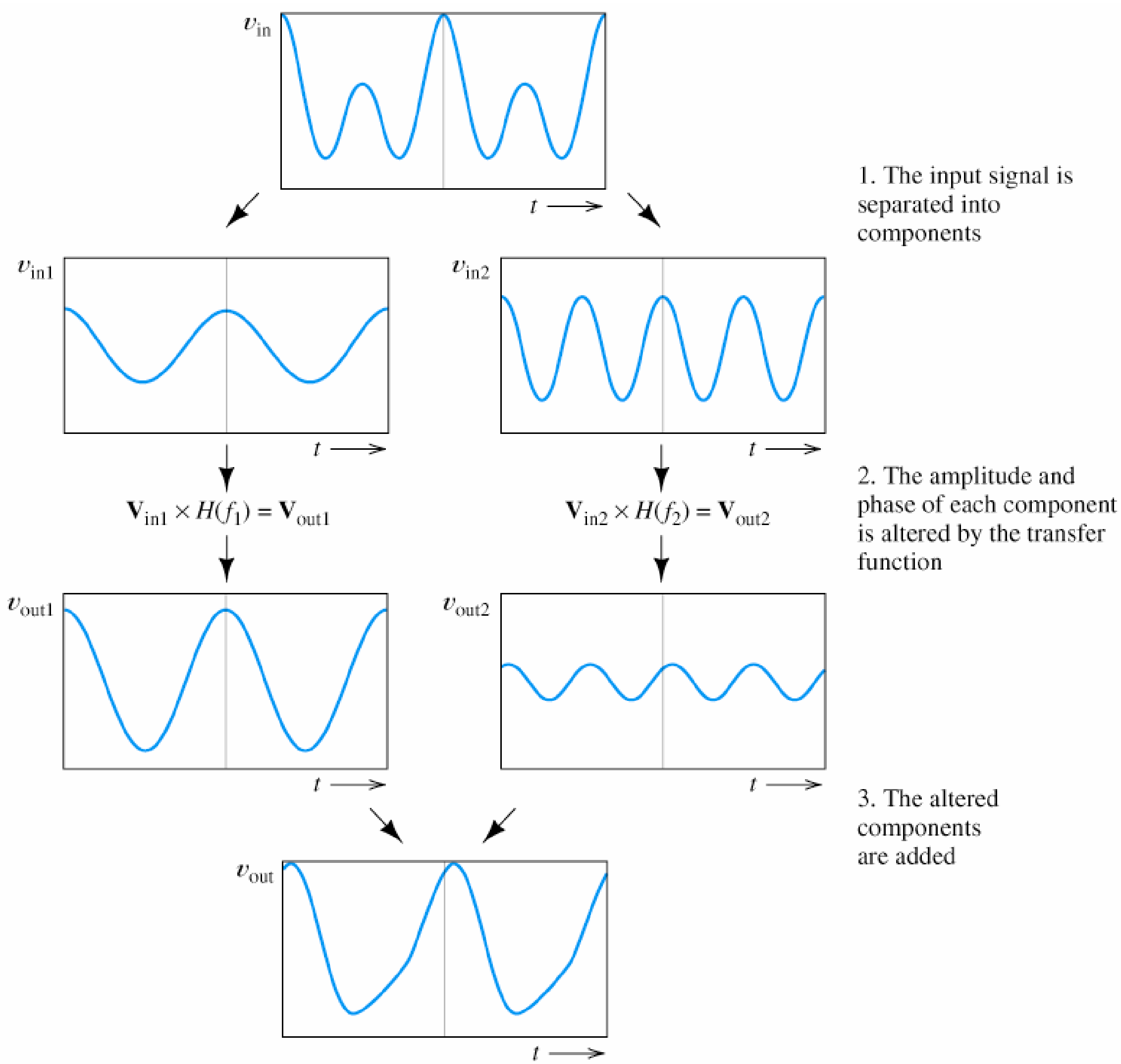
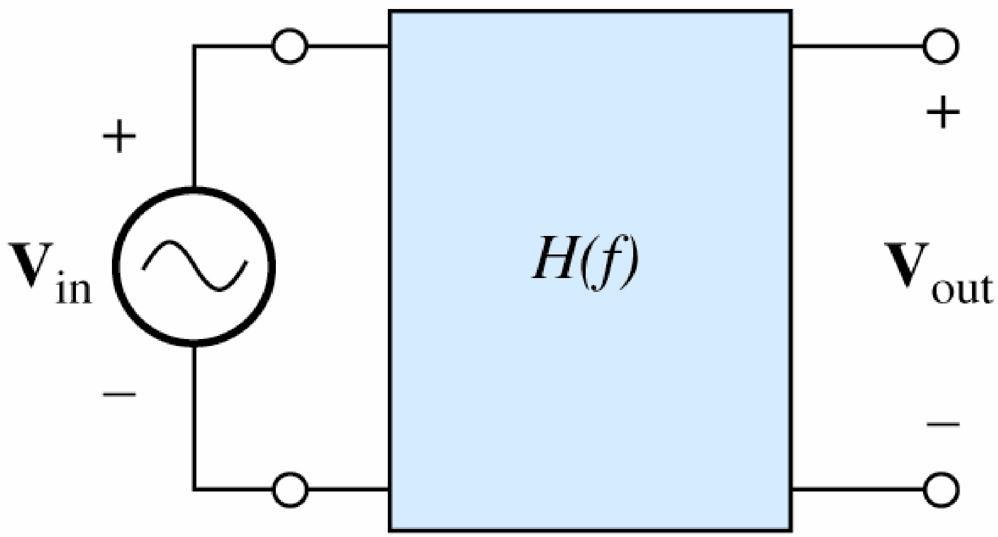


Figure 6.6 To measure the transfer function, we apply a sinusoidal input signal, measure the amplitudes and phases of input and output in steady state, and then divide the phasor output by the phasor input. The procedure is repeated for each frequency of interest.



6.2 FIRST-ORDER LOWPASS FILTERS (一階低通濾波器)

We can determine the transfer functions of RLC circuits by using steady-state analysis with complex impedances as a function of frequency

(可利用steady-state 分析並將阻抗表示為頻率函數的複數阻抗以決定RLC的轉換函數)

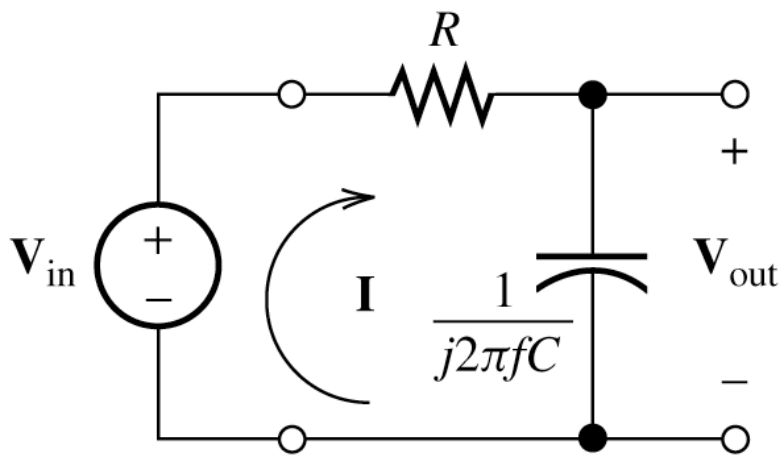


Figure 6.7 A first-order lowpass filter.

決定 $H(f)=\dfrac{\mathbf{V}_{\text{out}}}{\mathbf{V}_{\text{in}}}$, 並隨frequency 改變畫出 $|H(f)|\ \&\ \angle H(f)$

$$Z_C=-j\frac{1}{\omega C}=\frac{1}{j\omega C}=\frac{1}{j2\pi fC} \quad\rightarrow\quad \frac{1}{Z_C}=j2\pi fC$$

$$V_{\text{out}}=Z_C\times I=Z_C\times\left(\frac{V_{\text{in}}}{R+Z_C}\right)$$

$$H(f)=\frac{V_{\text{out}}}{V_{\text{in}}}=\frac{Z_C}{Z_C+R}=\frac{1}{1+\dfrac{R}{Z_C}}=\frac{1}{1+j2\pi fRC}$$

定義 $f_B = \dfrac{1}{2\pi RC}$

$$\Longrightarrow \quad H(f) = \frac{1}{1 + j\left(f / f_B\right)}$$

Q: 如何隨frequency 改變畫出 $\left|H(f)\right| \,\&\, \angle H(f)$?

Magnitude $\left|H(f)\right|$ and Phase $\angle H(f)$ Plots

$$H(f) = \frac{1}{1 + j\left(f / f_B\right)}$$

$$\left|H(f)\right| = \frac{1}{\sqrt{1 + \left(f / f_B\right)^2}}$$

$$\angle H(f) = \frac{\angle 0^\circ}{\angle(1 + j(f / f_B))} = 0 - \arctan\left(\frac{f}{f_B}\right)$$

$$= -\arctan\left(\frac{f}{f_B}\right)$$

Magnitude $\left|H(f)\right|$ and Phase $\angle H(f)$ Plots

$$\left|H(f)\right| = \frac{1}{\sqrt{1+\left(f/f_B\right)^2}} \qquad \angle H(f) = -\arctan\left(\frac{f}{f_B}\right)$$

1. For low frequency $f \to 0$

$$\left|H(f)\right| \to 1 \qquad \angle H(f) \to 0^\circ$$

2. For high frequency $f >> f_B$

$$\left|H(f)\right| \to 0 \qquad \angle H(f) \to -90^\circ$$

Magnitude $|H(f)|$ and Phase $\angle H(f)$ Plots

3. For $f = f_B$

$$|H(f_B)| = 1/\sqrt{2} \cong 0.707 \qquad \angle H(f_B) = -45^\circ$$

$$|H(f_B)|^2 = 1/2 \longrightarrow |H(f_B)|^2 = \frac{|\mathbf{V}_{\text{out}}|^2}{|\mathbf{V}_{\text{in}}|^2} = 0.5$$

$f = f_B$ 稱為 half-power frequency

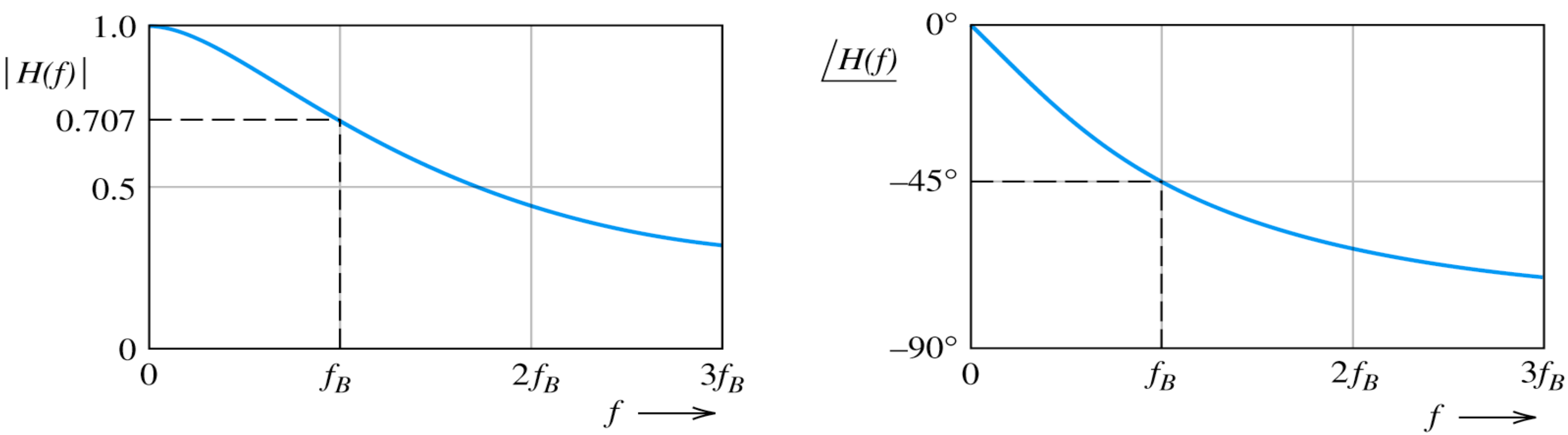


Figure 6.8 Magnitude and phase of the first-order lowpass transfer function versus frequency.

Example 6.3 Calculation of RC Lowpass Output

$$v_{\text{in}}(t) = 5\cos(20\pi t) + 5\cos(200\pi t) + 5\cos(2000\pi t)$$

The RC lowpass filter is shown in Figure 6.9.

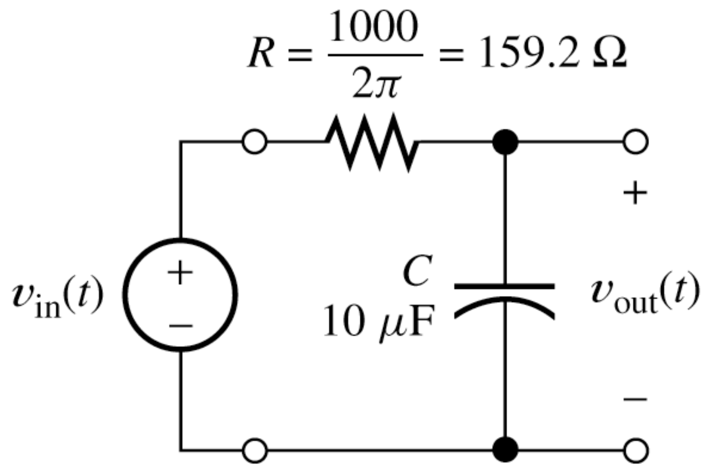
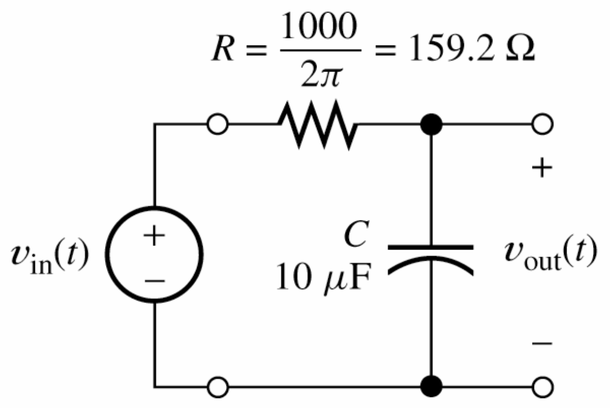


Figure 6.9 Circuit of Example 6.3. The resistance has been picked so the break frequency turns out to be a convenient value.

Find $v_{out}(t)$.

Example 6.3 Calculation of RC Lowpass Output



$$f_B = \frac{1}{2\pi RC} = \frac{1}{2\pi \times (1000/2\pi) \times 10 \times 10^{-6}} = 100\text{ Hz}$$

$$H(f) = \frac{1}{1 + j(f/f_B)}$$

1. The input components

$$v_{in1}(t) = 5\cos(20\pi t) \qquad \mathrm{V}_{in1} = 5\angle 0^\circ \qquad f_{in1} = \frac{\omega}{2\pi} = \frac{20\pi}{2\pi} = 10\text{Hz}$$

$$v_{in2}(t) = 5\cos(200\pi t) \qquad \mathrm{V}_{in2} = 5\angle 0^\circ \qquad f_{in2} = 100\text{Hz}$$

$$v_{in3}(t) = 5\cos(2000\pi t) \qquad \mathrm{V}_{in3} = 5\angle 0^\circ \qquad f_{in3} = 1000\text{Hz}$$

2. The transfer function

$$H(10) = \frac{1}{1 + j(10/100)} = \frac{1}{\sqrt{1 + (0.1)^2}} \angle -\arctan 0.1 = 0.9950\angle -5.71^\circ$$

$$H(100) = \frac{1}{1 + j(100/100)} = 0.7071\angle -45^\circ$$

$$H(1000) = \frac{1}{1 + j(1000/100)} = 0.0995\angle -84.29^\circ$$

3. The output components

$$\begin{cases} \mathrm{V}_{\text{out1}} = H(10)\times \mathrm{V}_{\text{in1}} = (0.9950\angle -5.71°)\times(5\angle 0°) = 4.975\angle -5.71° \\ v_{\text{out1}}(t) = 4.975\cos(20\pi t - 5.71°) \end{cases}$$

$$\begin{cases} \mathrm{V}_{\text{out2}} = H(100)\times \mathrm{V}_{\text{in2}} = (0.7071\angle -45°)\times(5\angle 0°) = 3.535\angle -45° \\ v_{\text{out2}}(t) = 3.535\cos(200\pi t - 45°) \end{cases}$$

$$\begin{cases} \mathrm{V}_{\text{out3}} = H(1000)\times \mathrm{V}_{\text{in3}} = (0.0995\angle -84.29°)\times(5\angle 0°) = 0.4975\angle -84.29° \\ v_{\text{out3}}(t) = 0.4975\cos(2000\pi t - 84.29°) \end{cases}$$

$\Rightarrow$

$$v_{\text{out}}(t) = 4.975\cos(20\pi t - 5.71°) + 3.535\cos(200\pi t - 45°) + 0.4975\cos(2000\pi t - 84.29°)$$

Note: We do not add the phasors for components with different frequencies

6.3 DECIBELS, THE CASCADE CONNECTION, AND LOGARITHMIC FREQUENCY SCALES

Decibel **(dB)** 分貝

$$\left|H(f)\right|_{\mathrm{dB}} = 20\log\left|H(f)\right|$$

$$\left|H(f)\right| > 1, \Rightarrow \left|H(f)\right|_{\mathrm{dB}} > 0$$

$$\left|H(f)\right| < 1, \Rightarrow \left|H(f)\right|_{\mathrm{dB}} < 0$$

$\lvert H(f)\rvert$	$\lvert H(f)\rvert_{\mathrm{dB}}$
100	40
10	20
2	6
$\sqrt{2}$	3
1	0
$1/\sqrt{2}$	-3
1/2	-6
0.1	-20
0.01	-40

Why using dB?

Very small and very large magnitudes can be displayed clearly on a single plot.

Linear scale

$|H(60)| < 10^{-4}$ 但圖上看不出大小

dB scale

可清楚看出 $|H(60)|_{dB} \approx -85\text{dB}$

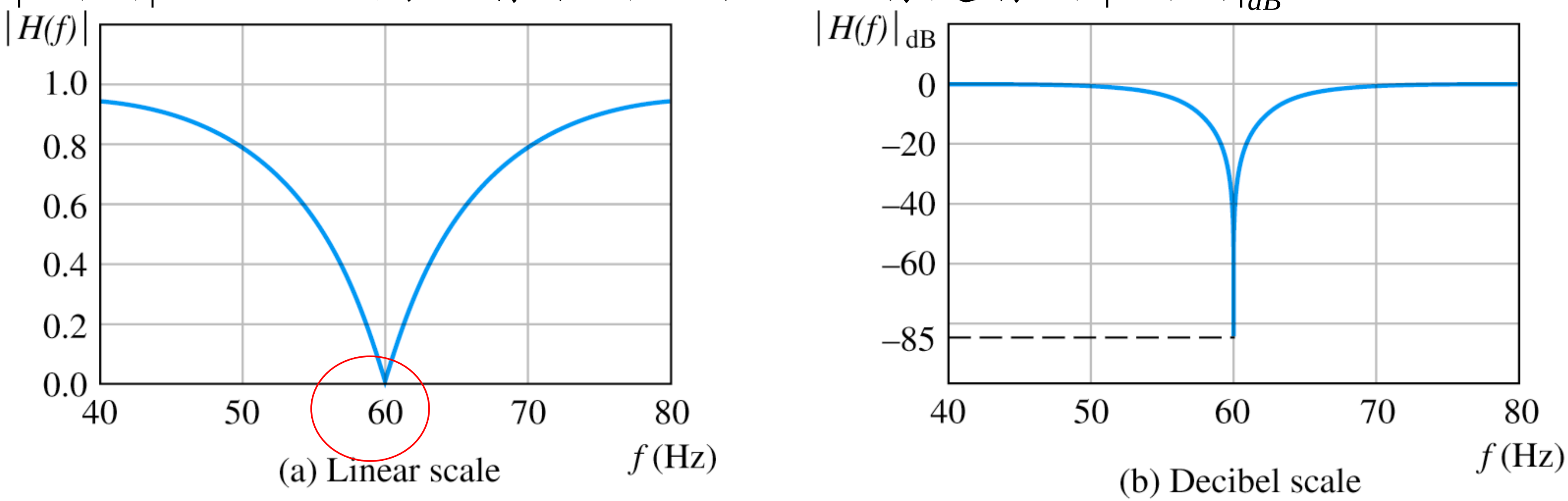


Figure 6.12 Transfer-function magnitude of a notch filter used to reduce hum in audio signals.

Notch filter:消除某些頻率component, 60Hz in this case.

THE CASCADE CONNECTION (串接)

In cascade connection, the output of one filter is connected to the input of a second filter
(前級filter的輸出是後級filter 的輸入)

$$H(f)=\frac{V_{out}}{V_{in}}=\frac{V_{out1}}{V_{in1}}\times\frac{V_{out2}}{V_{out1}}=H_1(f)\times H_2(f)$$

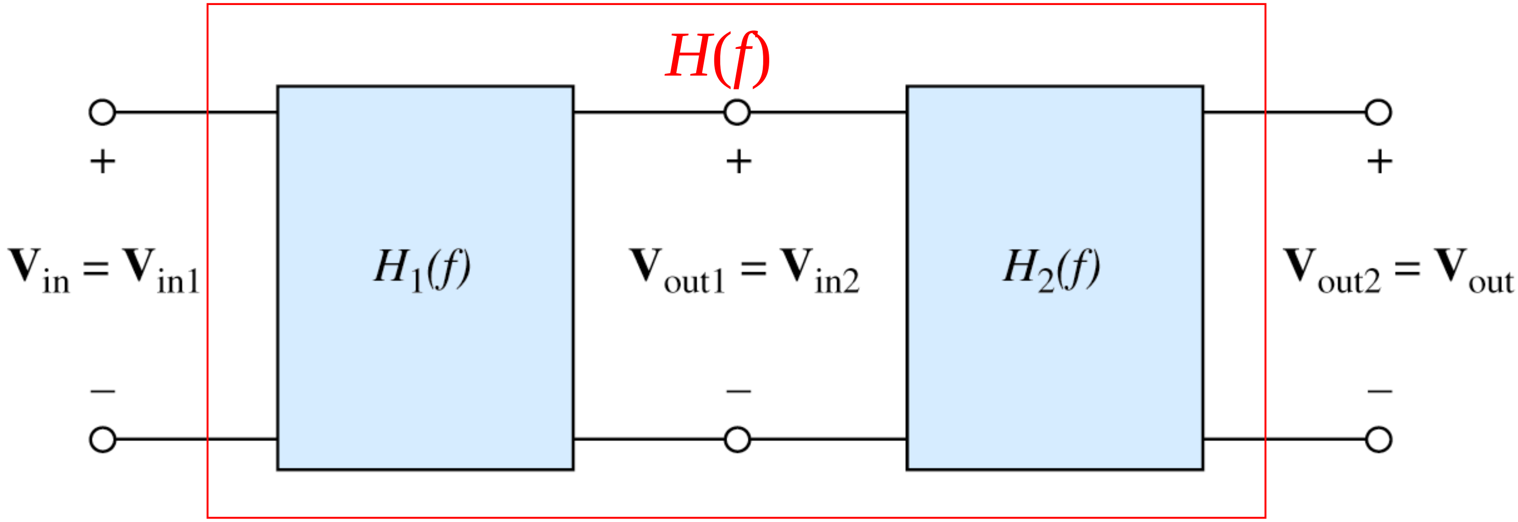


Figure 6.13 Cascade connection of two two-port circuits.

THE CASCADE CONNECTION (串接)

In dBs, the individual transfer-function magnitudes are added to find the overall transfer-function magnitude

$$H(f) = H_1(f) \times H_2(f)$$

$$\Rightarrow$$

$$20\log\left|H(f)\right| = 20\log\left[\left|H_1(f)\right| \times \left|H_2(f)\right|\right]$$

$$= 20\log\left|H_1(f)\right| + 20\log\left|H_2(f)\right| \qquad \boxed{\because \log(a \times b) = \log a + \log b}$$

$$\Rightarrow$$

$$\left|H(f)\right|_{dB} = \left|H_1(f)\right|_{dB} + \left|H_2(f)\right|_{dB}$$

Logarithmic Frequency Scales

•On a logarithmic scale, the variable is multiplied by a given factor for equal increments of length along the axis.

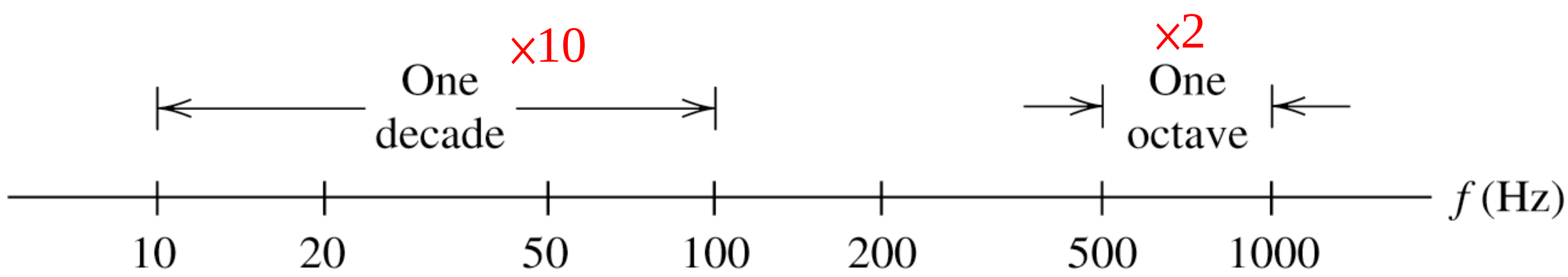


Figure 6.14 Logarithmic frequency scale.

•The advantage of a logarithmic frequency scale compared with linear scale is that the variations of a transfer function for a low range of frequency and the variations in a high range can be shown on a single plot.

A **decade** is a range of frequencies for which the ratio of the highest frequency to the lowest is 10.

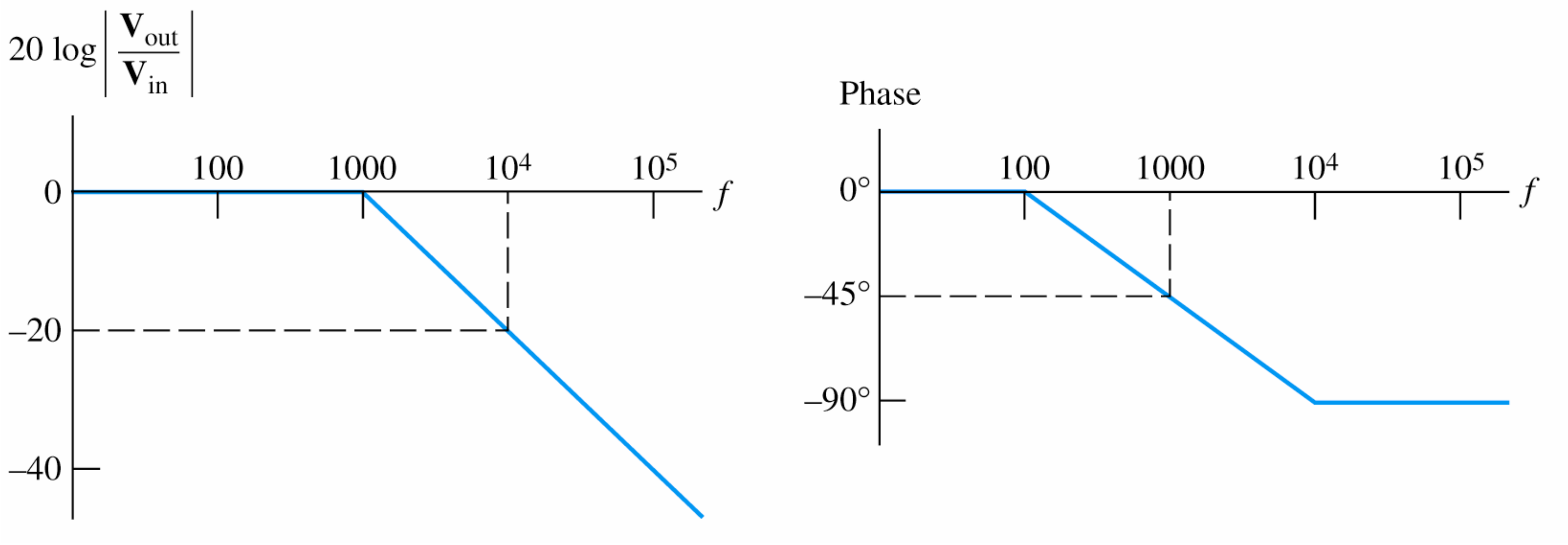
$$\text{number of decades} = \log\left(\frac{f_2}{f_1}\right)$$

An **octave** is a two-to-one change in frequency.

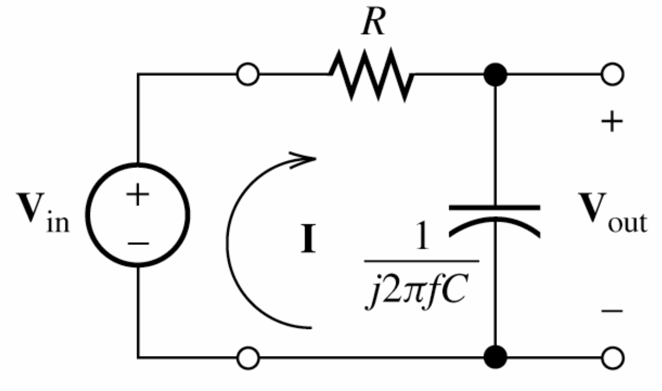
$$\text{number of octaves} = \log_2\left(\frac{f_2}{f_1}\right) = \left(\frac{\log\left(f_2/f_1\right)}{\log(2)}\right)$$

6.4 BODE PLOTS

A Bode plot shows the magnitude of a network function in decibels versus frequency using a logarithmic scale for frequency.



A Bode plot of the first-order lowpass transfer function (先R後C)



$$H(f) = \frac{1}{1 + j(f/f_B)}$$

$$|H(f)|_{dB} = 20\log\frac{1}{\sqrt{1+(f/f_B)^2}} = 20\log 1 - 20\log\sqrt{1+(f/f_B)^2}$$

$$= 0 - 20\log\sqrt{1+(f/f_B)^2} = -\frac{20}{2}\log\left[1+(f/f_B)^2\right] = -10\log\left[1+(f/f_B)^2\right]$$

$$\boxed{\because \log\sqrt{x} = \log x^{1/2} = \frac{1}{2}\log x}$$

$$\boxed{|H(f)|_{\mathrm{dB}} = -10\log\left[1+\left(\frac{f}{f_B}\right)^2\right]}$$

A Bode plot of the first-order lowpass transfer function- Magnitude plot

1. For low frequency $f \ll f_B$ (or $f \to 0$)

$$\left|H(f)\right|_{\text{dB}} = -10\log\left[1+\left(\frac{f}{f_B}\right)^2\right] \cong -10\log 1 = 0$$

低頻漸近線(low-frequency asymptote)為水平直線

2. For high frequency $f \gg f_B$

$$\left|H(f)\right|_{\text{dB}} = -10\log\left[1+\left(\frac{f}{f_B}\right)^2\right] \cong -10\log\left(\frac{f}{f_B}\right)^2 = -20\log\left(\frac{f}{f_B}\right)$$

高頻漸近線(high-frequency asymptote)為斜率 -20dB/decade之直線，起始於$f=f_B$, 故f_B稱為corner frequency or break frequency.

3. For $f = f_B$

$$\left|H(f)\right|_{\mathrm{dB}} = -10\log\left[1+\left(\frac{f_B}{f_B}\right)^2\right] = -10\log(2) = -10\times 0.301 \cong -3\mathrm{dB}$$

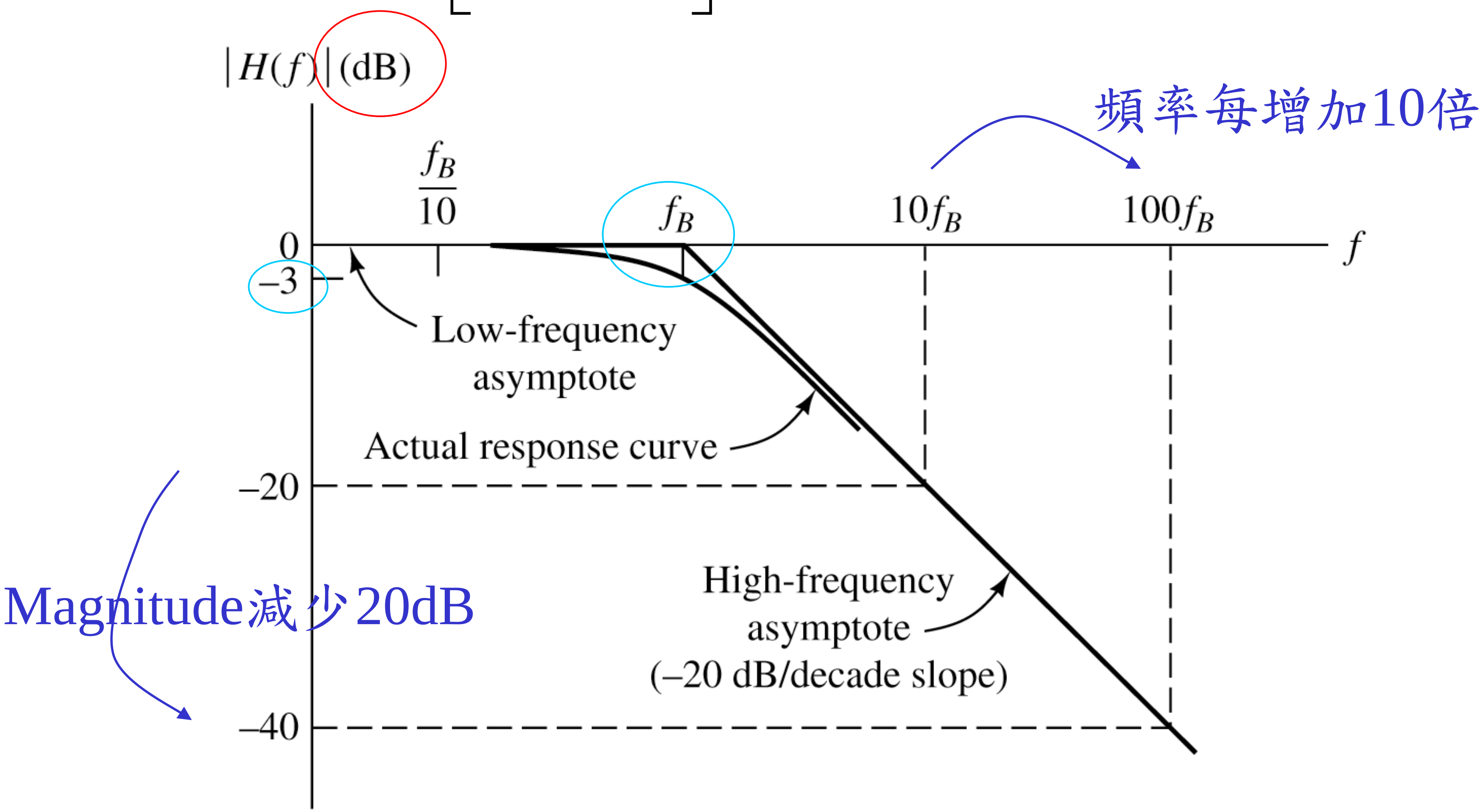
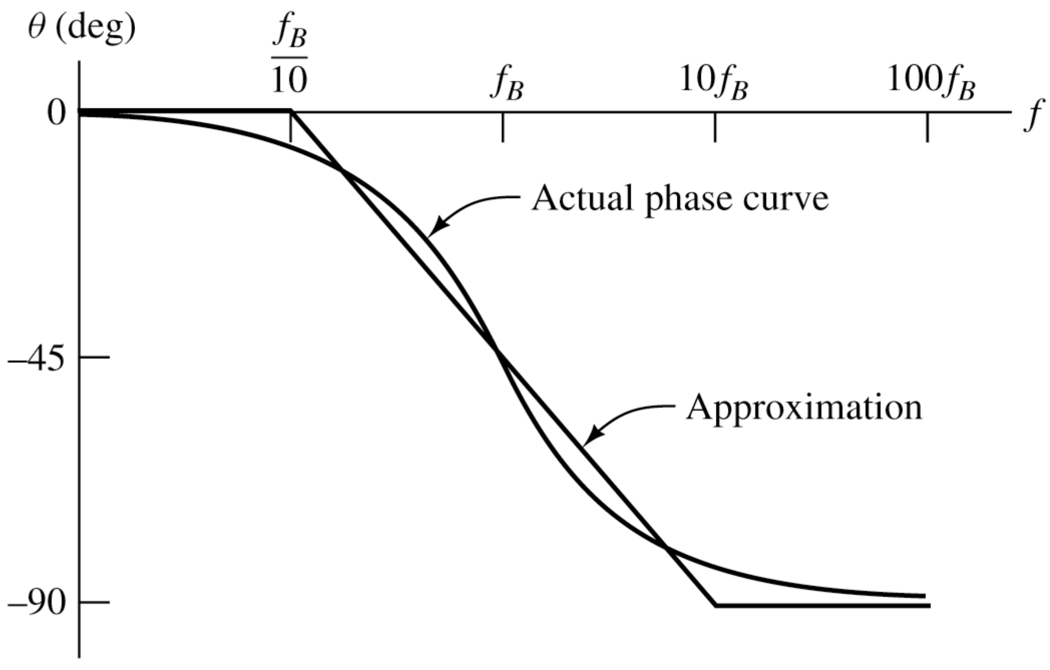


Figure 6.15 Magnitude Bode plot for the first-order lowpass filter.

A Bode plot of the first-order lowpass transfer function- Phase plot

$$\angle H(f) = -\arctan\left(\frac{f}{f_B}\right)$$

1. A horizontal line at zero for $f < (f_B/10)$.
2. A sloping line from 0 phase at $f_B/10$ to -90° at $10f_B$.
3. A horizontal line at -90° for $f > 10f_B$.



Exercise 6.11 Sketch approximate straight-line Bode magnitude and phase plots

A 先R後C circuit

➡ Lowpass filter

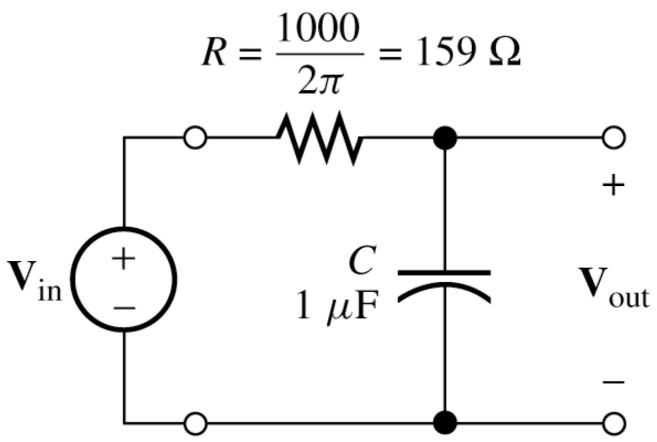


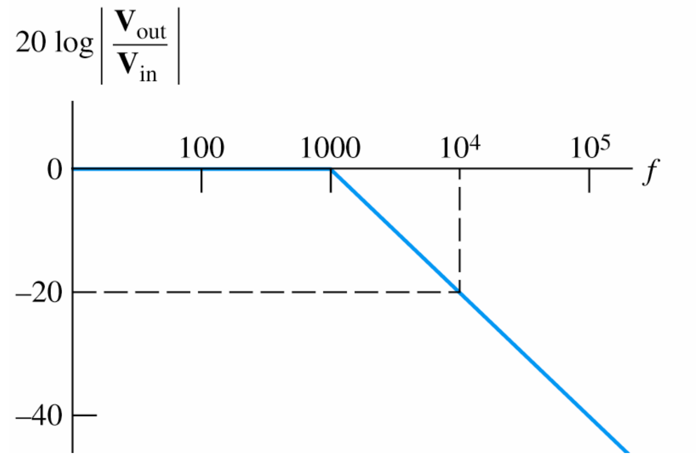
Figure 6.17 Circuit for Exercise 6.11.

$$H(f) = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{1/(j2\pi fC)}{R + 1/(j2\pi fC)} = \frac{1}{1 + j2\pi RCf} = \frac{1}{1 + jf/f_B}$$

$$f_B = \frac{1}{2\pi RC} = \frac{1}{2\pi \times (1000/2\pi) \times 10^{-6}} = 1000\text{Hz}$$

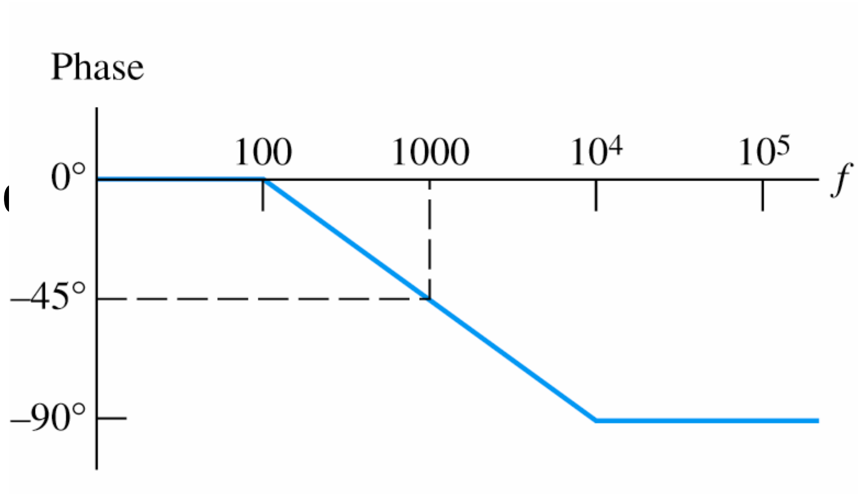
The magnitude plot is approximated by

1. 0 dB below 1000 Hz
2. A straight line sloping downward at 20 dB/decade above 1000 Hz.



The phase plot is approximated by

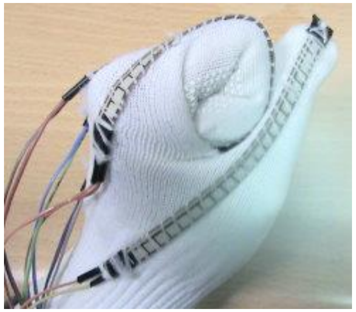
1. 0°below 100 Hz
2. -90° above 10 kHz
3. a line sloping downward betwe
   at 10 kHz.



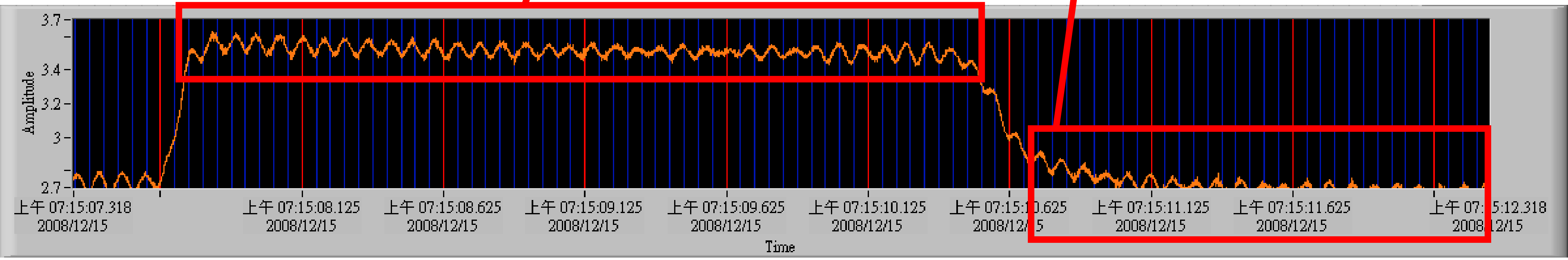
Applications of Electronics and Circuits

Human Computer Interface-Flex Sensor

手攤平(手指彎曲度最小)



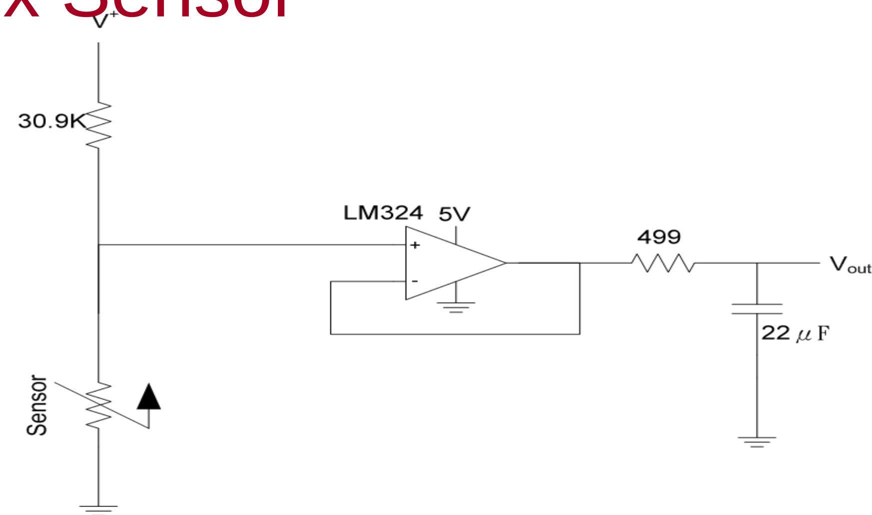
手握拳(手指彎曲度最大)

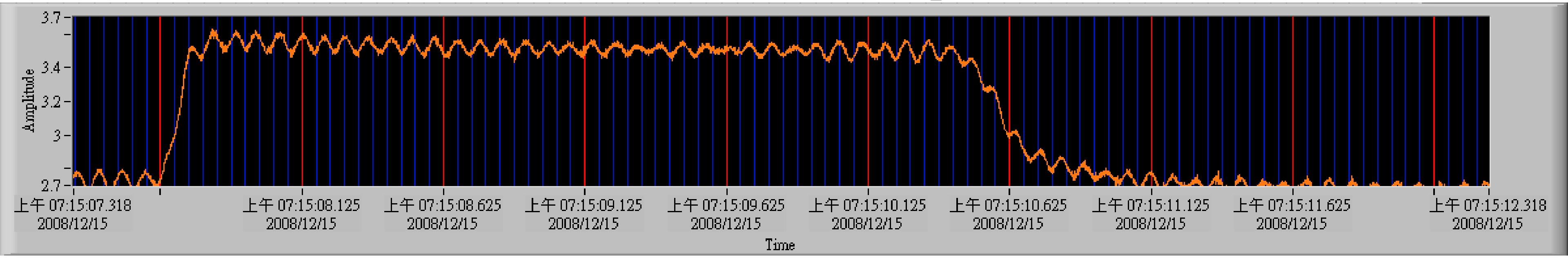


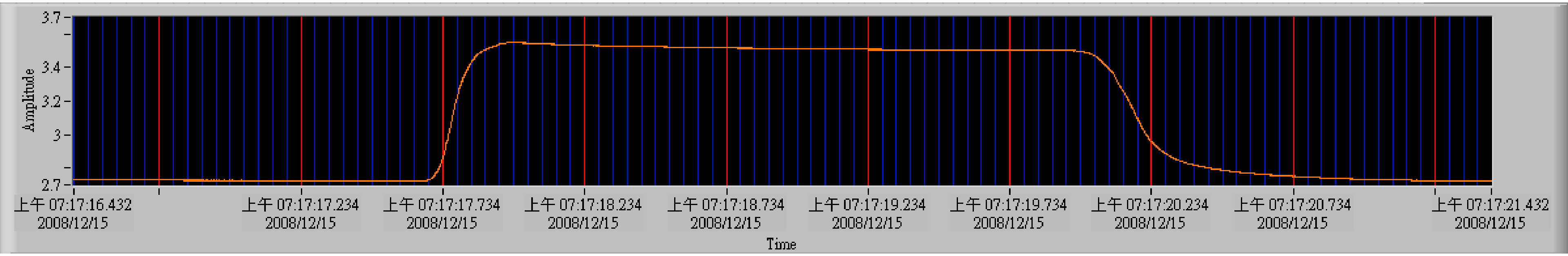
Applications of Electronics and Circuits

Human Computer Interface-Flex Sensor

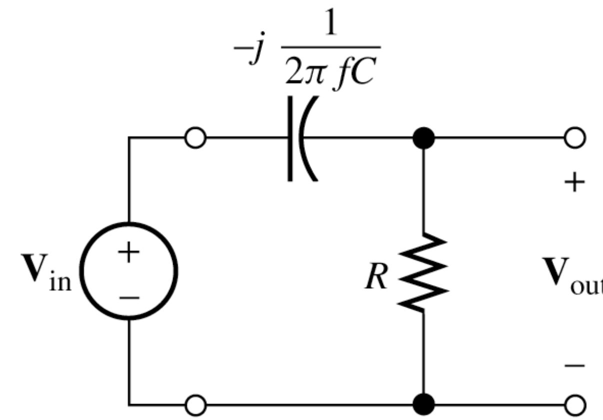
Low Pass Filter







6.5 FIRST-ORDER HIGHPASS FILTERS (先C後R)



$$\mathrm{V_{out}} = R \times \mathrm{I} = R \times \left(\frac{\mathrm{V_{in}}}{\mathrm{R} + Z_C}\right)$$

$$\frac{R}{\mathrm{Z}_C} = j2\pi fRC = j\frac{f}{f_B} \quad f_B = \frac{1}{2\pi RC}$$

$$H(f) = \frac{\mathrm{V_{out}}}{\mathrm{V_{in}}} = \frac{R}{Z_C + R} = \frac{R/Z_C}{1 + (R/Z_C)} = \frac{j(f/f_B)}{1 + j(f/f_B)}$$

Magnitude $\left|H(f)\right|$ and Phase $\angle H(f)$ Plots

$$\left|H(f)\right| = \frac{f/f_B}{\sqrt{1+\left(f/f_B\right)^2}}$$

$$\angle H(f) = \frac{\angle j(f/f_B)}{\angle(1+j(f/f_B))} = 90^\circ - \arctan\left(\frac{f}{f_B}\right)$$

1. For low frequency $f \to 0$

$$\left|H(f)\right| \to 0 \qquad \angle H(f) \to 90^\circ$$

2. For high frequency $f >> f_B$

$$\left|H(f)\right| \to 1 \qquad \angle H(f) \to 0^\circ$$

Magnitude $|H(f)|$ and Phase $\angle H(f)$ Plots

3. For $f = f_B$

$$\left|H\left(f_B\right)\right| = 1/\sqrt{2} \cong 0.707 \quad \angle H\left(f_B\right) = 90^\circ - 45^\circ = 45^\circ$$

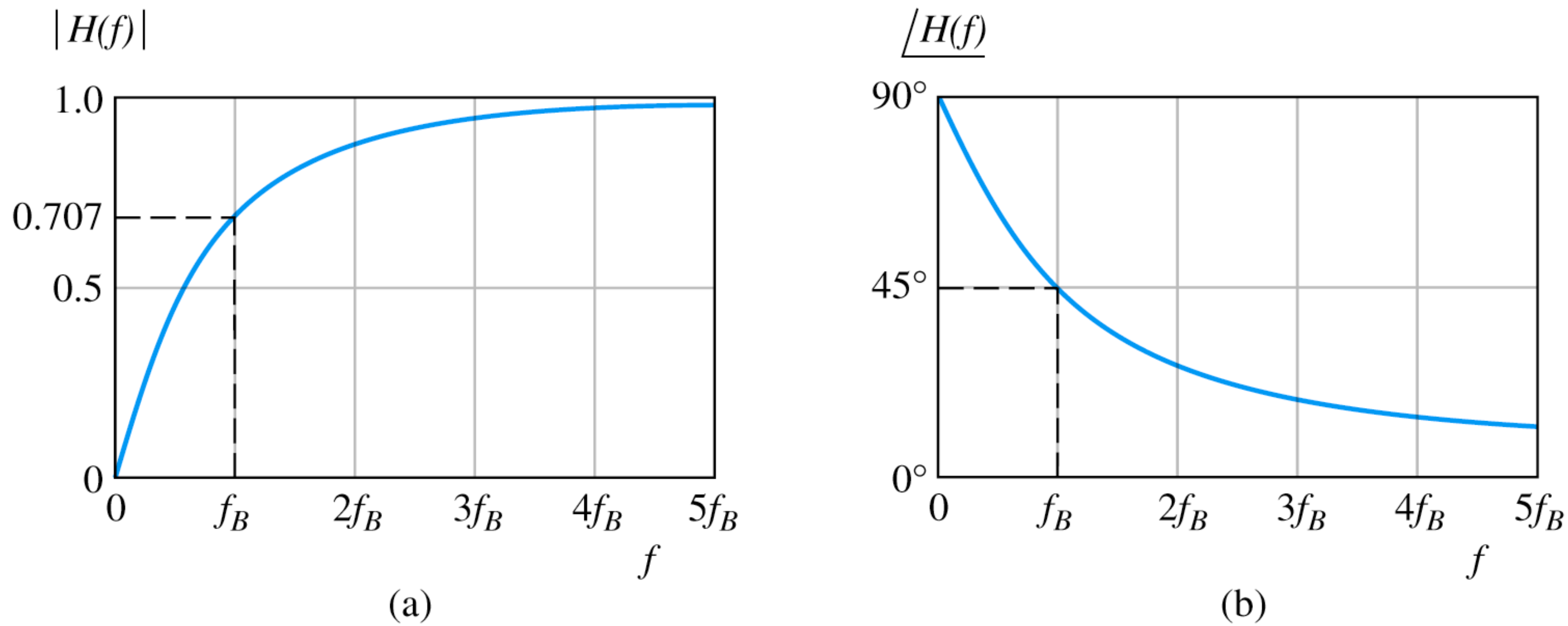


Figure 6.20 Magnitude and phase for the first-order highpass transfer function.

A Bode plot of the first-order highpass transfer function- Magnitude plot

$$\left|H(f)\right|_{dB} = 20\log\frac{f/f_B}{\sqrt{1+(f/f_B)^2}} = 20\log\left(\frac{f}{f_B}\right) - 10\log[1+\left(\frac{f}{f_B}\right)^2]$$

1. For low frequency $f \ll f_B (\text{or } f \to 0)$

$$\left|H(f)\right|_{dB} \cong 20\log\left(\frac{f}{f_B}\right)$$

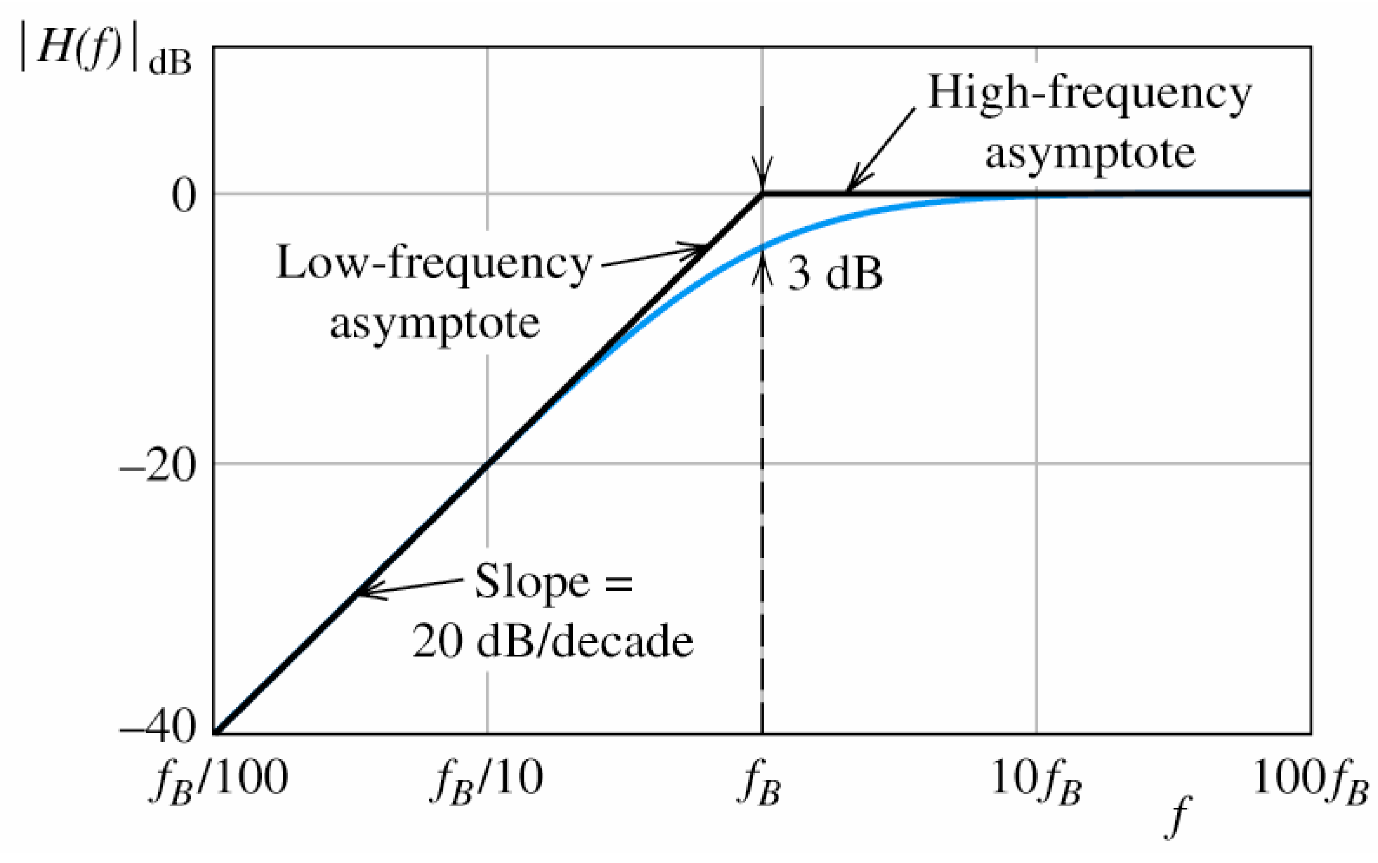
低頻漸近線(low-frequency asymptote)為斜率 +20dB/decade之直線

2. For high frequency $f \gg f_B$

$$\left|H(f)\right|_{\mathrm{dB}} \cong 20\log\left(\frac{f}{f_B}\right) - 20\log\left(\frac{f}{f_B}\right) = 0$$

3. For $f = f_B$

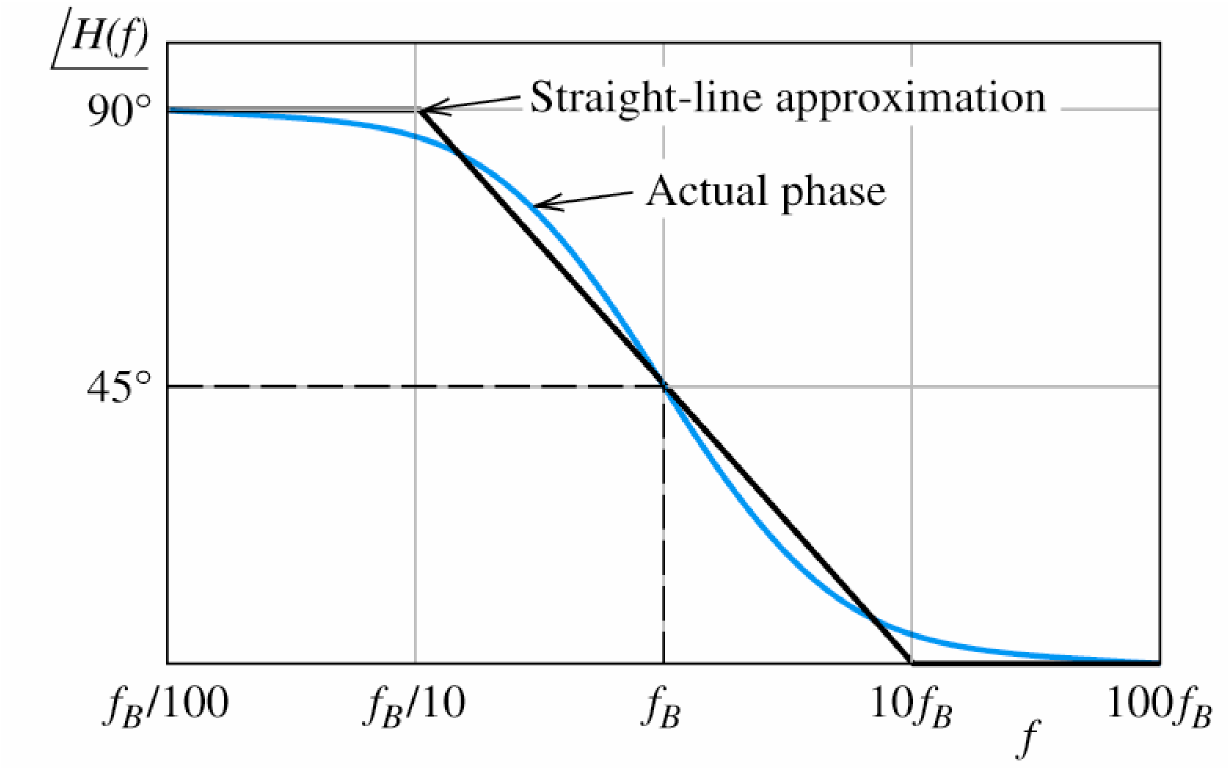
$$\left|H(f)\right|_{\mathrm{dB}} = 0 - 10\log\left[1 + \left(\frac{f_B}{f_B}\right)^2\right] = 0 - 10\log(2) = -10\times 0.301 \cong -3\mathrm{dB}$$



A Bode plot of the first-order highpass transfer function- Phase plot

$$\angle H(f) = \frac{\angle j(f/f_B)}{\angle(1 + j(f/f_B))} = 90^\circ - \arctan\left(\frac{f}{f_B}\right)$$

與lowpass filter 類似, 只是加了90º.



Example 6.4 Determine the break frequency of a highpass filter

Determine the break frequency of a first-order highpass filter such that the transfer-function magnitude at 60 Hz is -30 dB.

When $f \ll f_B (\text{or } f \to 0)$ $\quad \left|H(f)\right|_{dB} \cong 20\log\left(\frac{f}{f_B}\right)$

$$-30 \approx 20\log\left(\frac{60}{f_B}\right)$$

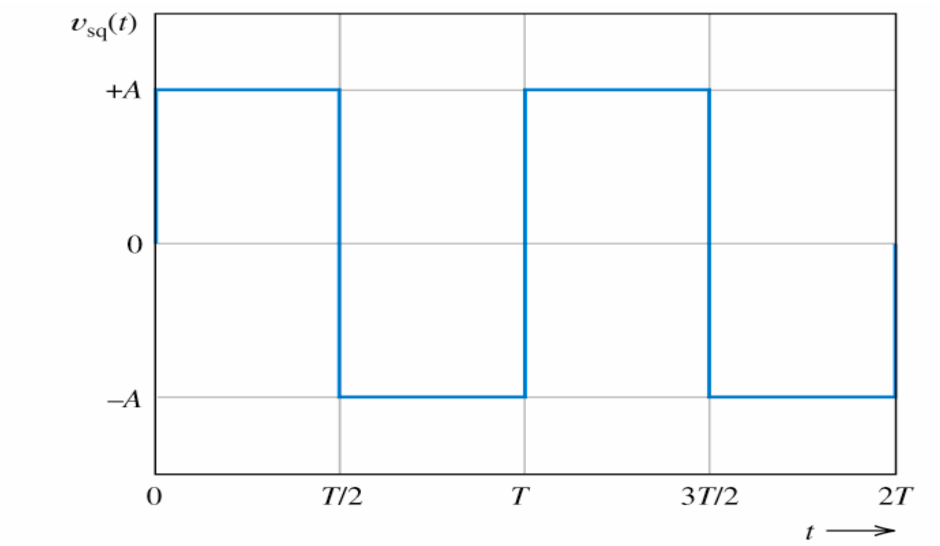
$$\Rightarrow$$

$$\frac{f_B}{60} \approx 10^{1.5} = 31.6$$

$$\Rightarrow$$

$$f_B \cong 1900 \text{ Hz}$$

Appendix-Fourier Series of a Square Wave



方波(square wave) $V_{sq}(t)$ 可由以下之sinusoids 來組成

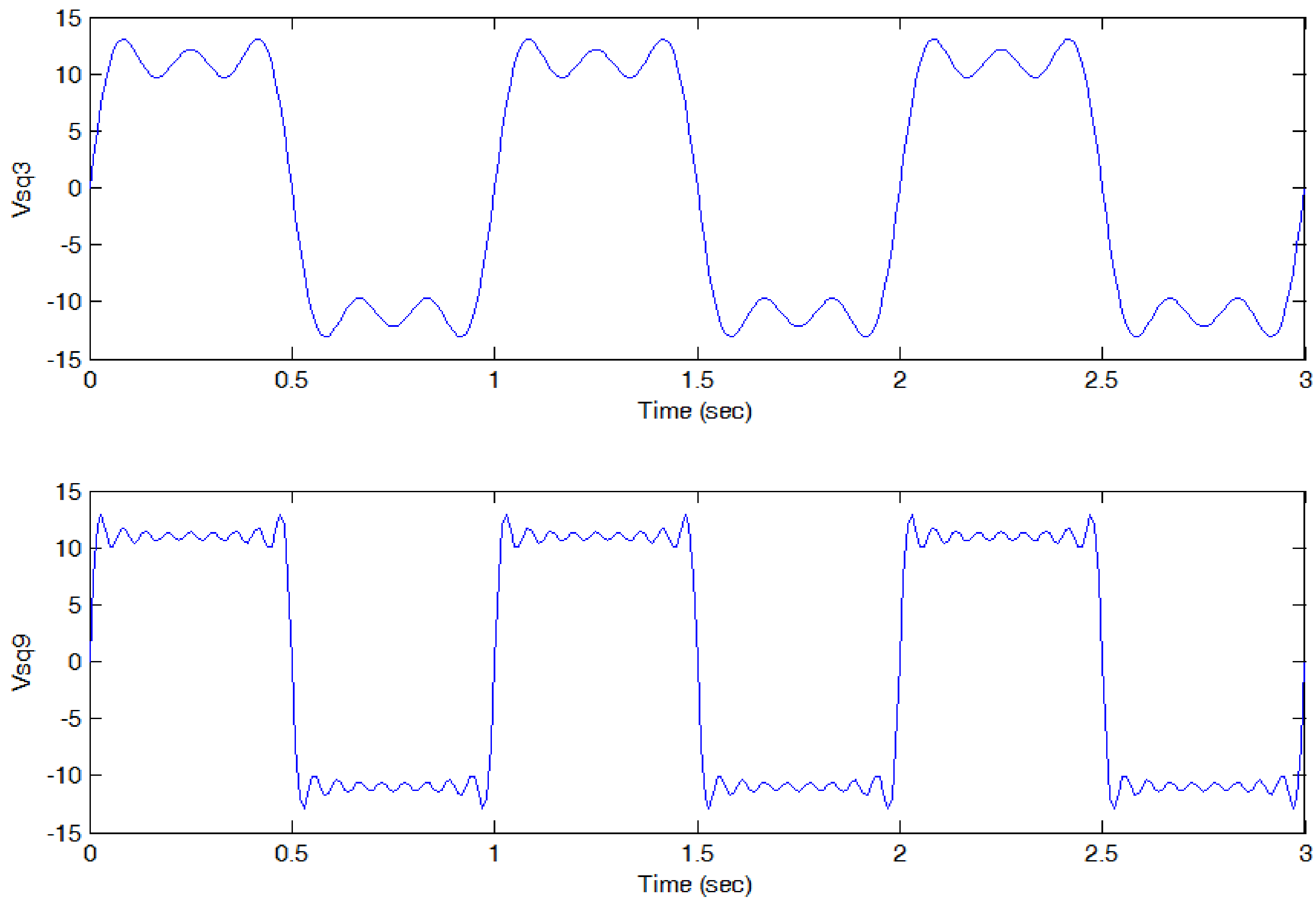
$$v_{sq}(t) = \frac{44}{\pi}\sin(\omega_0 t) + \frac{44}{3\pi}\sin(3\omega_0 t) + \frac{44}{5\pi}\sin(5\omega_0 t) + \cdots$$

$$v_{sq}(t) = \frac{44}{\pi}\sin(\omega_0 t) + \frac{44}{3\pi}\sin(3\omega_0 t) + \frac{44}{5\pi}\sin(5\omega_0 t) + \cdots$$

```
% Matlab Code
T = 1;                          % Period = 1s
t=0:0.01:3*T;                   % t=0-3s, delta t=0.1s
Omega0 = 2*pi/T;                % Omega0=2*pi*f=2*pi
Vsq_3=[zeros(1,length(t))];  % vector for the sum of the first 3 terms
Vsq_9 =[zeros(1,length(t))]; % vector for the sum of the first 9 terms

for i=1:3
Vsq_3=Vsq_3+(44/((2*i-1)*pi))*sin(Omega0*(2*i-1)*t);
end
for i=1:9
Vsq_9=Vsq_9+(44/((2*i-1)*pi))*sin(Omega0*(2*i-1)*t);
end

subplot(2,1,1)
plot(t, Vsq_3);
xlabel('Time (sec)');
ylabel('Vsq3');
subplot(2,1,2)
plot(t, Vsq_9);
xlabel('Time (sec)');
ylabel('Vsq9');
```



Vsq3
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
Time (sec)
Vsq9
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3
Time (sec)