

$$\frac{x^2}{e^x}$$

1. (10 points) Find the following limits.

(a) $\lim_{x \rightarrow \infty} x^2 e^{-x}$

$x \rightarrow \infty \quad x^2 \rightarrow \infty \quad e^{-\infty} \rightarrow 0 \quad \therefore = \lim_{x \rightarrow \infty} \frac{x^2}{e^x} =$

(b) $\lim_{x \rightarrow 0} \frac{\sqrt{2x+4} - 2}{x}$

$\frac{0}{0} \quad \therefore =$

2. (30 points) Calculate the integrals.

(a) $\int_0^1 x^3 - 2x + 5 \, dx$

$(2x+4)^{\frac{1}{2}} - 2 = \frac{1}{2}(2x+4)^{-\frac{1}{2}} - 2$
 $\frac{\sqrt{2x+4}}{1} \cdot x$

(b) $\int \frac{x^2 + 5x + 2}{(x+1)(x^2+1)} \, dx$

let $u = x^2 + 1$
 $du = 2x \, dx$

$x = \tan \theta \quad dx = \sec \theta \tan \theta \, d\theta$

(c) $\int_0^{\pi/2} x^2 \sin x \, dx$

$= \int \frac{1}{u} \, du$

$= 3 \int \frac{\sec \theta \tan \theta}{\sec^2 \theta} \, d\theta$

(d) $\int \frac{1}{x^{\frac{1}{3}} + x^{\frac{1}{2}}} \, dx$

$= \ln u$

$= 3 \int \frac{\sin \theta}{\frac{1}{\cos \theta}} \, d\theta$

(e) $\int_2^{\infty} \frac{1}{x \ln x} \, dx$

$\int_a^b f(x) \, dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n f(x_k) \Delta x$

$= 3 \int \sin \theta \, d\theta$

3. (10 points)

(a) Express the following limit as a definite integral:

$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{6k^2 - 6k + 6}{6k^5 + 6k^4}$

$\lim_{n \rightarrow \infty} \sum_{k=1}^n \sqrt{1 - \left(\frac{k}{n}\right)^2} \cdot \frac{1}{n}$

$= -3 \cos \theta$

$= \frac{-3}{\sqrt{x^2+1}}$

(b) Compute the integral of (a).

4. (10 points)

(a) Find the area of the region bounded by $y = \sqrt{x}$ and $y = x/2$, $x \geq 0$.

(b) Rotate the area in (a) about the x -axis, and compute the volume of the solid of rotation.

5. (10 points) Find the derivatives of the following functions.

(a)

$f(x) = \int_{\sin x}^1 u^2 \, du$

(b)

$f(x) = e^{x^2} + \ln(x^2 + 2x^4)$

