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Fundamentals of Momentum, Heat, and Mass Transfer

Sixth Edition

Welty • Rorrer • Foster

Chapter 7

Shear Stress in Laminar Flow

Newton's law of viscosity

The **shear modulus** of an elastic solid

$$\text{shear modulus} = \frac{\text{shear stress}}{\text{shear strain}} \qquad (7\text{-}1)$$

Resistance to deformation

The **viscosity** of a fluid

$$\boxed{\text{viscosity} = \frac{\text{shear stress}}{\text{rate of shear strain}}} \qquad (7\text{-}2)$$

Rate of deformation

The **viscosity** is the property of a fluid to resist the rate at which deformation takes place when the fluid is acted upon by shear forces

Eq_7-1

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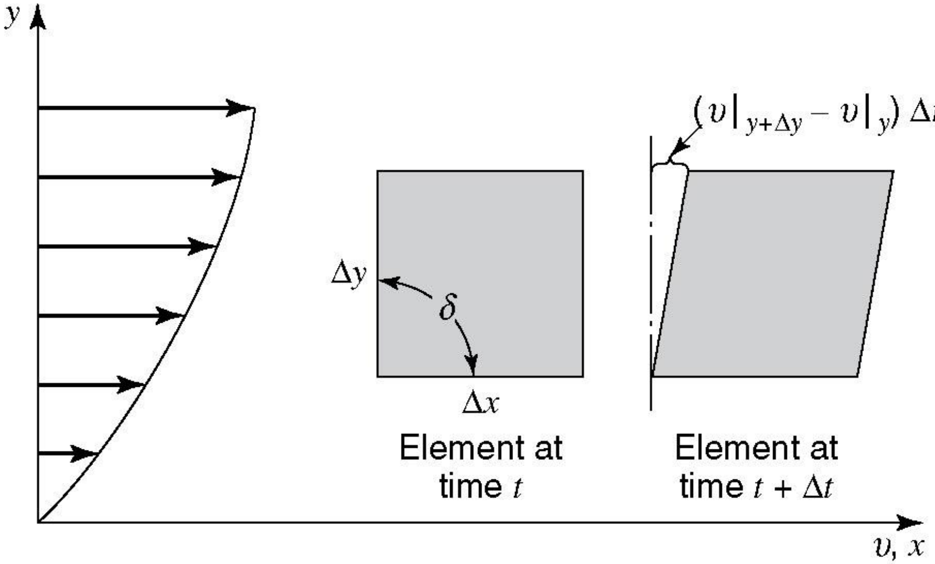
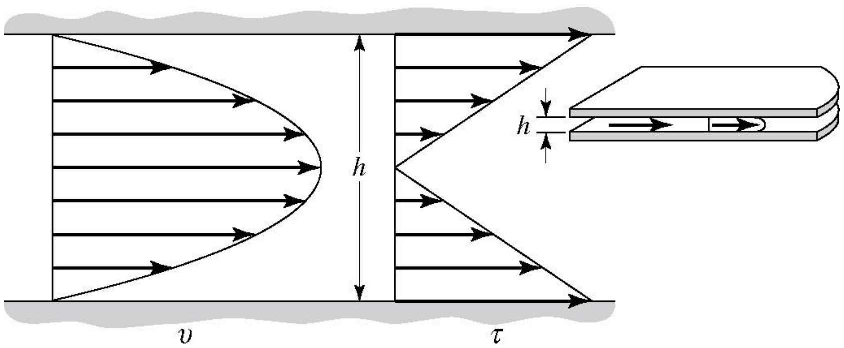
Newton's law of viscosity





$$-\frac{d\delta}{dt} = \lim_{\Delta x, \Delta y, \Delta t \to 0} - \frac{\delta|_{t+\Delta t} - \delta|_t}{\Delta t}$$

$$= \lim_{\Delta x, \Delta y, \Delta t \to 0} - \left(\frac{\{\pi/2 - \arctan[(v|_{y+\Delta y} - v|_y)\Delta t/\Delta y]\} - \pi/2}{\Delta t} \right) \tag{7-3}$$

$$\boxed{\tau = \mu \frac{dv}{dy}} \tag{7-4}$$

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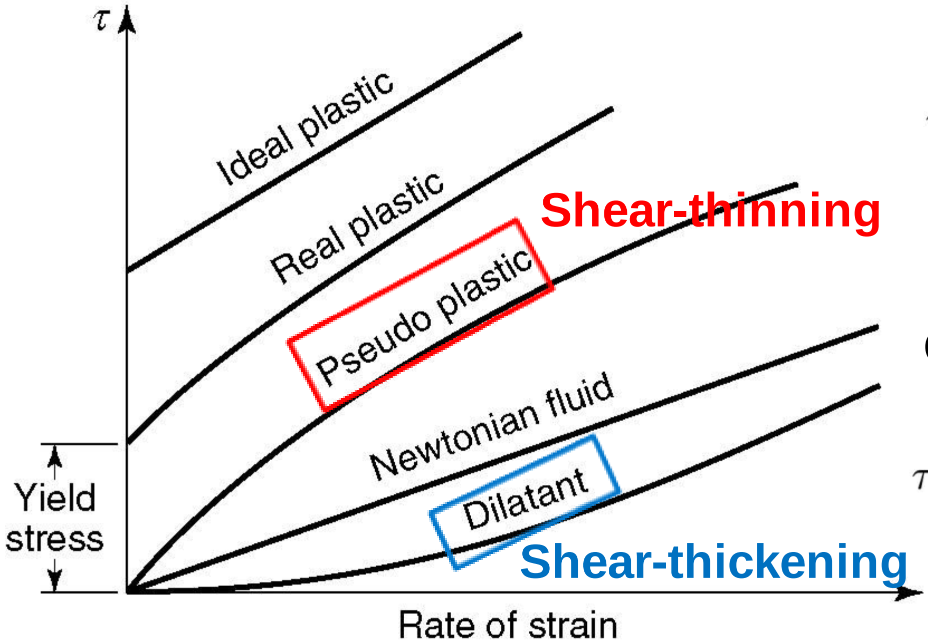


Bingham (Ideal) plastic

$$\tau = \mu \frac{dv}{dy} \pm \tau_0 \qquad (7\text{-}5)$$

Toothpaste, mayonnaise, ketchup

Ostwald-De Waele or power law model

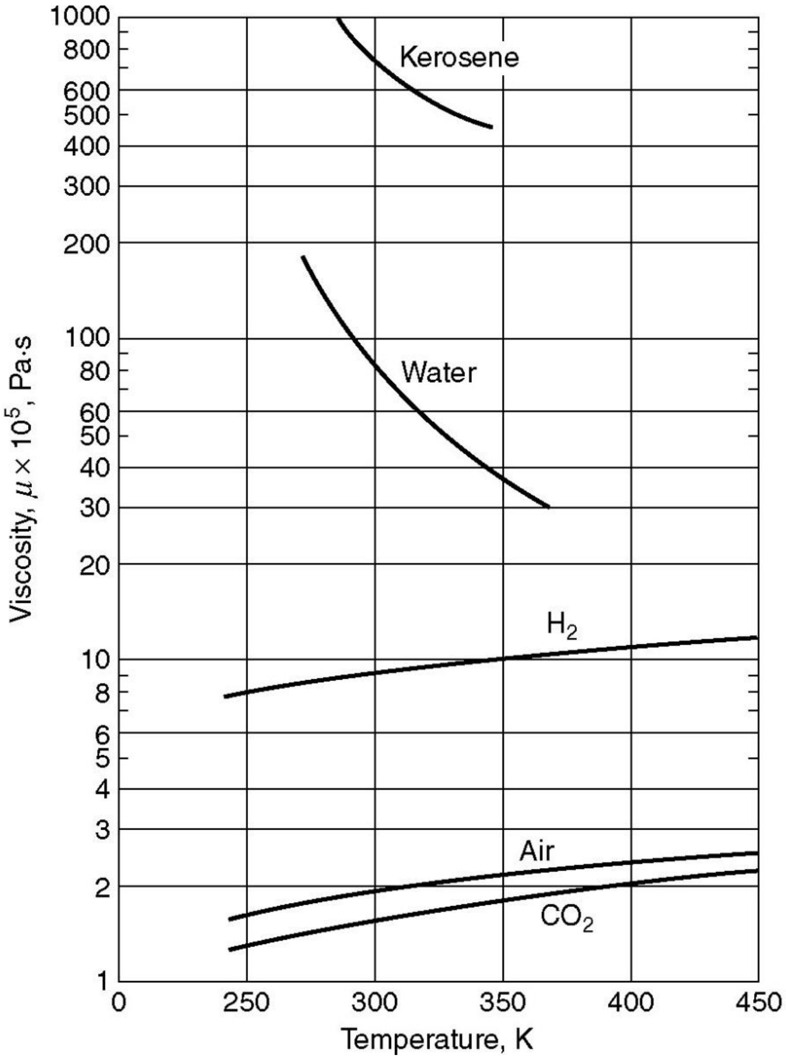
$$\tau = m \left| \frac{dv}{dy} \right|^{n-1} \frac{dv}{dy} \qquad (7\text{-}6)$$

$n = 1$, Newtonian fluid
$n < 1$, pseudo plastic (**Shear-thinning**)
$n > 1$, dilatant (**Shear-thickening**)

Shear-thinning materials: hair gel, plasma, syrup, latex paint

Shear-thickening materials: mixture of cornstarch and water

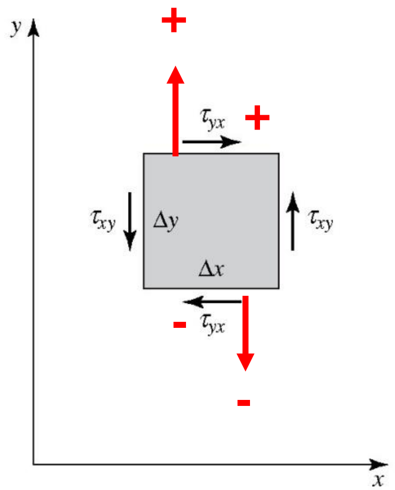
Fig_7-3



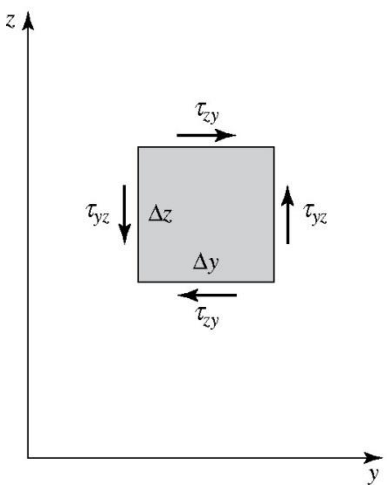
Fluid	Viscosity (cP) at 20°C
Ethanol	1.194
Mercury	15.47
H_2SO_4	19.15
Water	1.0019
Air	0.018
CO_2	0.015
Blood	2.5 (at 37°C)
SAE 40 motor oil	290
Corn oil	72
Ketchup	50,000
Peanut butter	250,000
Honey	10,000

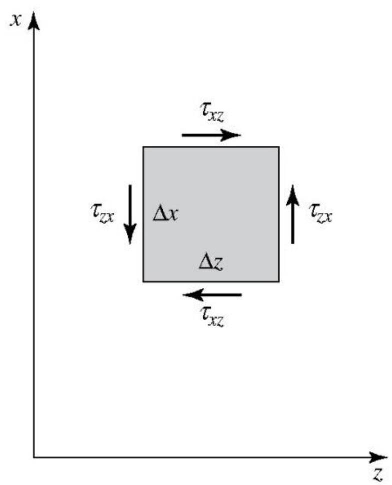
1 centipoise (cP) = 0.001 kilogram/meter second.
1 centipoise (cP) = 0.001 Pascal second.

Shear stress



(a)





Shear stress components τ_{ij}

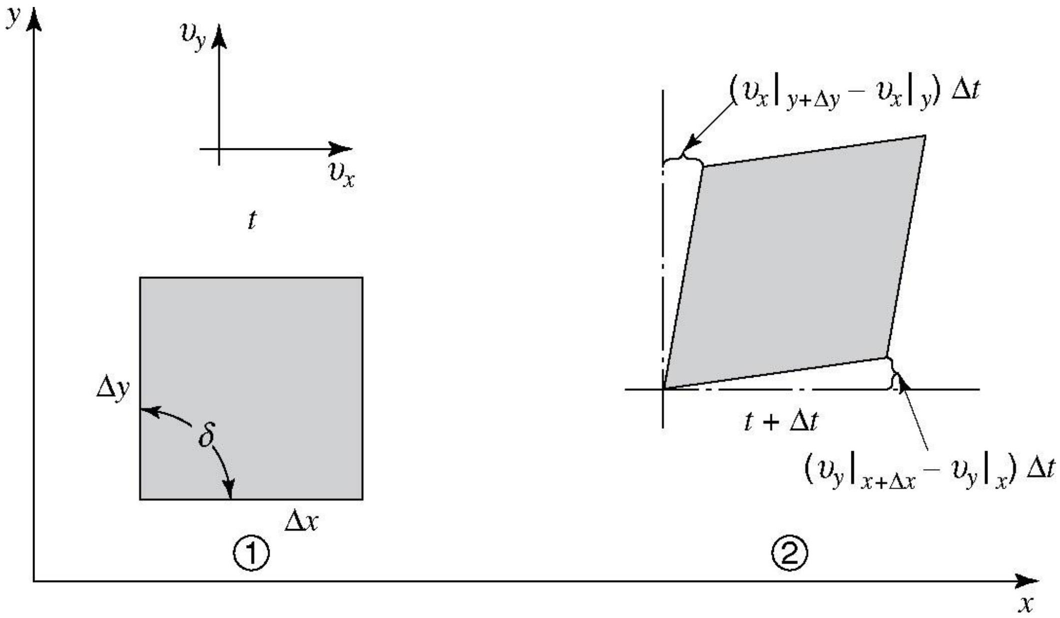
Normal stress components σ_{ij}

The first subscript i: **orientation**, direction of axis to which plane of action of shear stress is normal

The second subscript j: **direction** of action of shear stress

$$\tau_{ij} = \tau_{ji}$$

Shear stress



Cartesian coordinates

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right) \tag{7-15a}$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) \tag{7-15b}$$

$$\tau_{zx} = \tau_{xz} = \mu\left(\frac{\partial v_z}{\partial x} + \frac{\partial v_x}{\partial z}\right) \tag{7-15c}$$

Fig_7-7

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Normal stress

Generalized Hooke's law

$$\sigma_{xx} = \mu\left(2\frac{\partial v_x}{\partial x} - \frac{2}{3}\nabla\cdot\mathbf{v}\right) - P \qquad \text{(7-16a)}$$

$$\sigma_{yy} = \mu\left(2\frac{\partial v_y}{\partial y} - \frac{2}{3}\nabla\cdot\mathbf{v}\right) - P \qquad \text{(7-16b)}$$

$$\sigma_{zz} = \mu\left(2\frac{\partial v_z}{\partial z} - \frac{2}{3}\nabla\cdot\mathbf{v}\right) - P \qquad \text{(7-16c)}$$

The shear stress components in cylindrical coordinates

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right) + \frac{1}{r}\frac{\partial \upsilon_r}{\partial \theta}\right]$$

$$\tau_{z\theta} = \tau_{\theta z} = \mu\left[\frac{\partial \upsilon_\theta}{\partial z} + \frac{1}{r}\frac{\partial \upsilon_z}{\partial \theta}\right]$$

$$\tau_{zr} = \tau_{rz} = \mu\left[\frac{\partial \upsilon_z}{\partial r} + \frac{\partial \upsilon_r}{\partial z}\right]$$

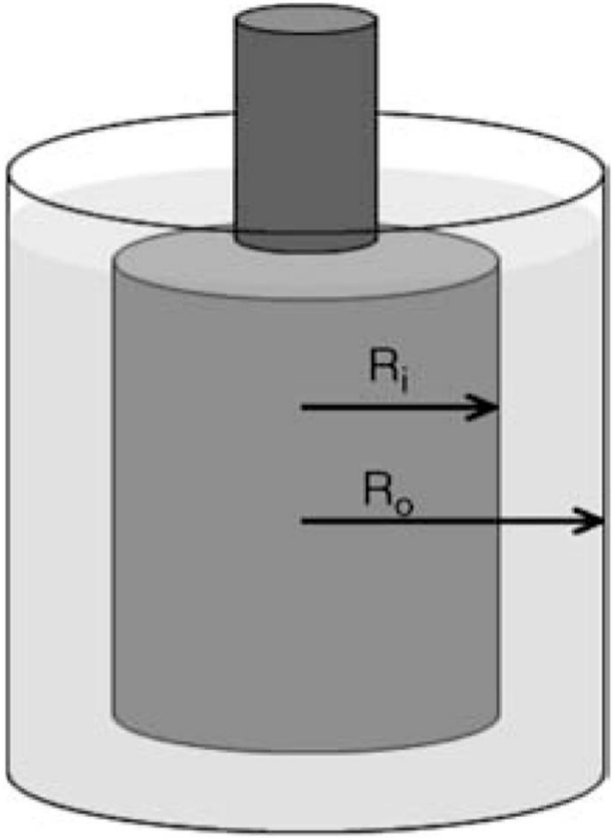
The shear stress components in spherical coordinates

$$\tau_{r\theta} = \tau_{\theta r} = \mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right) + \frac{1}{r}\frac{\partial \upsilon_r}{\partial \theta}\right]$$

$$\tau_{\phi\theta} = \tau_{\theta\phi} = \mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial \theta}\left(\frac{\upsilon_\phi}{\sin\theta}\right) + \frac{1}{r\sin\theta}\frac{\partial \upsilon_\theta}{\partial \phi}\right]$$

$$\tau_{\phi r} = \tau_{r\phi} = \mu\left[\frac{1}{r\sin\theta}\frac{\partial \upsilon_r}{\partial \phi} + r\frac{\partial}{\partial r}\left(\frac{\upsilon_\phi}{r}\right)\right]$$

Eq_7-16a



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