國立成功大學
工科系統微積分(一) 期末考
1月 12 日, 2016

課程代碼: F115611
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Instructions:

1. There are **6 pages** (including the cover page), **15 problems** in this exam.
2. You have **110 minutes** to work on the exam.
3. Do **NOT** start the exam until you are told to do so.
4. Only the answers written above the answer lines will be graded.
5. Please have your **student ID** card ready.
6. No textbook, notes, calculator, or sketching sheets are allowed.
7. You may want to use the back of the exam pages for computations.

Page:	1	2	3	4	5	Total
Points:	28	28	20	14	10	100
Score:						

1. Evaluate the following integrals.

(a) (7 points)

$$\int_1^e \frac{\ln x}{x}\,dx =$$

A. $\frac{1}{e}$ **B.** 1 **C.** $\frac{1}{2}$ **D.** $e-1$ **E.** $e+1$.

(a) _____

Solution: **C.** $\frac{1}{2}$

(b) (7 points)

$$\int_1^2 3^t\,dt =$$

A. $\frac{6}{\ln 3}$ **B.** $2\ln 3$ **C.** $\frac{2}{\ln 3}$ **D.** $6\ln 3$ **E.** 6.

(b) _____

Solution: **A.** $\frac{6}{\ln 3}$

(c) (7 points)

$$\int_0^{\frac{1}{2}} \frac{1}{x-1}\,dx =$$

A. $-\ln 2 - 1$ **B.** $1 - \ln 2$ **C.** $\ln 2$ **D.** $-\ln 2$ **E.** $\ln 2 - 1$.

(c) _____

Solution: **D.** $-\ln 2$

(d) (7 points)

$$\int_0^{\ln 2} e^x\sqrt{e^x-1}\,dx =$$

A. $\frac{1}{3}$ **B.** $\frac{3}{2}$ **C.** $\frac{1}{2}$ **D.** 2 **E.** $\frac{2}{3}$.

(d) _____

Solution: **E.** $\frac{2}{3}$

2. Evaluate the following integrals.

(a) (7 points)

$$\int_0^{\frac{\pi}{4}} \sec x \tan x \, dx =$$

A. $\sqrt{2}-1$ **B.** $\frac{1}{2}(\sqrt{2}-1)$ **C.** 1 **D.** $\frac{\sqrt{2}}{2}-1$ **E.** $\frac{\sqrt{2}}{2}$.

(a) _____

Solution: **A.** $\sqrt{2}-1$

(b) (7 points)

$$\int_0^{\frac{\pi}{2}} \cos^5 x \sin x \, dx =$$

A. $-\frac{1}{6}$ **B.** $\frac{1}{6}$ **C.** $-\frac{1}{4}$ **D.** $\frac{1}{4}$ **E.** $\frac{1}{5}$.

(b) _____

Solution: **B.** $\frac{1}{6}$

(c) (7 points)

$$\int_0^{\frac{\pi}{4}} \cos^2 x \, dx =$$

A. $\frac{1}{4}+\frac{\pi}{4}$ **B.** $\frac{1}{2}+\frac{\pi}{4}$ **C.** $\frac{1}{2}+\frac{\pi}{8}$ **D.** $\frac{1}{4}+\frac{\pi}{8}$ **E.** $\frac{1}{8}+\frac{\pi}{4}$.

(c) _____

Solution: **D.** $\frac{1}{4}+\frac{\pi}{8}$

(d) (7 points)

$$\int_1^{\sqrt{3}} \frac{1}{1+x^2} \, dx =$$

A. $\frac{\pi}{12}$ **B.** $\frac{\pi}{4}$ **C.** $\frac{\pi}{6}$ **D.** $\frac{\pi}{2}$ **E.** $\frac{\pi}{3}$.

(d) _____

Solution: **A.** $\frac{\pi}{12}$

3. (6 points) If $f(x) = x + e^x$, find $(f^{-1})'(1)$.

A. 1 **B.** $\frac{1}{2}$ **C.** $e+1$ **D.** $\frac{1}{e+1}$ **E.** 2.

3. _____

Solution: **B.** $\frac{1}{2}$

4. Let $f(x) = -xe^{-x}$.

(a) (7 points) Find the limit

$$\lim_{x\to\infty} f(x) =$$

A. 1 **B.** -1 **C.** 0 **D.** ∞ **E.** $-\infty$.

(a) _____

Solution: **C.** 0

(b) (7 points) Find the value of the improper integral (if it is convergent)

$$\int_0^\infty f(x)\,dx =$$

A. 1 **B.** -1 **C.** 0 **D.** ∞ **E.** $-\infty$.

(b) _____

Solution: **B.** -1

5. (7 points)

$$\int_{-4}^{1} \frac{x-9}{(x+5)(x-2)}\,dx =$$

A. $3\ln 6 - 3$ **B.** $4\ln 6$ **C.** $2\ln 6$ **D.** $4\ln 6 - 2$ **E.** $3\ln 6$.

5. _____

Solution: **E.** $3\ln 6$

6. (7 points) Find the length of the curve $y = \ln(\sec x)$, $0 \le x \le \frac{\pi}{6}$.

A. $\ln \frac{2}{\sqrt{3}}$ **B.** $\ln \frac{1}{\sqrt{3}} - 1$ **C.** $\ln \frac{2}{\sqrt{3}} - 1$ **D.** $\ln \sqrt{3}$ **E.** $\ln \frac{1}{\sqrt{3}}$.

6. _____

Solution: **D.** $\ln \sqrt{3}$

7. (a) (5 points)

$$\int \tan^{-1} x \, dx =$$

(a) _____

Solution: $x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$

(b) (5 points)

$$\int \sqrt{1 - x^2} \, dx =$$

(b) _____

Solution: $\frac{1}{2}(x\sqrt{1 - x^2} + \sin^{-1} x) + C$