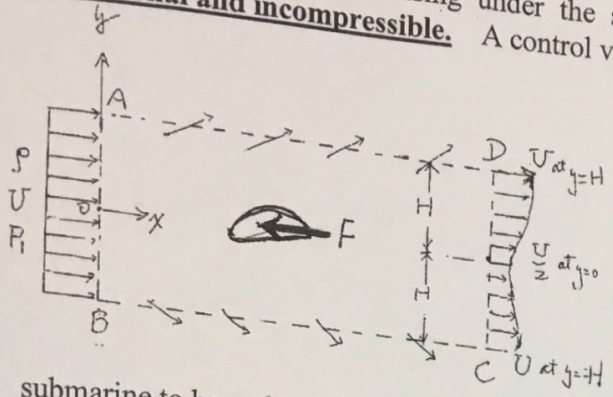
Consider a submarine is cruising under the sea. Assume the flow field is **steady-state, two-dimensional and incompressible.** A control volume is chosen as shown below.



The submarine is located and fixed in the center of the control volume and the flow field is symmetric with respect to y=0. The in-flow has uniform velocity U, density ρ and pressure P_1. The out-flow has uniform pressure P_2, a parabola velocity profile (抛物線)with 0.5U at center and U at top and bottom of the control volume. Find (a) the velocity profile u(y) at exit plane. (5%) (b) the mass flux across AB and CD, (5%) and (c) the thrust (F) of the submarine to have this flow conditions (15%).

(4) The incompressible Navier-Stoke equation can be written as following:

Continuity equation: $\nabla \cdot \vec{V} = 0$

Momentum equation: $\frac{\partial \vec{v}}{\partial t} + \vec{V} \cdot \nabla \vec{V} = -\frac{1}{\rho}\nabla P + \upsilon \nabla^2 \vec{V}$, where ρ is fluid density and $\upsilon = \mu/\rho$ is kinematic viscosity .

(a) Spell out the conservation of mass and conservation of momentum equations for a **steady 2-D incompressible** flow in Cartesian coordinate system. (6%)

(b) Analyze the viscous flow using the PDE from (a) for the flow between two infinite parallel **stationary** plates shown in the following figure. Assume the flow is **laminar** and pumped by a pressure at inlet. Obtain the fully-developed x-component velocity profile u=? The so-called fully-developed velocity u implies that the u-profile will no longer change along x-direction, i.e. u is function of y only. What is the maximum velocity? What's the mass flow rate across the channel? (20%)

(c) Draw the velocity profile u(y) and calculate its vorticity across the y-direction. If we put two circles at y= +0.8h and y= -0.8h and let the two circles flow down along x-direction, indicate the rotation direction of the two circles. (Assume the two circles are extremely light so that they will be simply convective down the stream without producing disturbance to the original flow field. (9%)