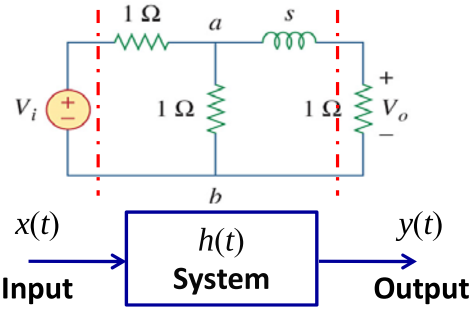
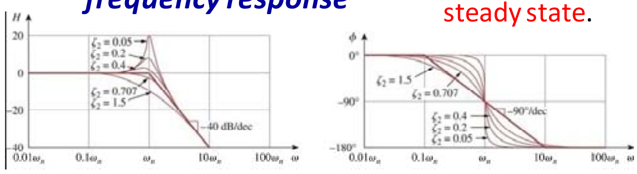
Final Exam

Circuit Analysis

June 25, 2014

A. Signal & System for Circuit (6×3=18 points): Describe the relativities among the (a) Figures, (b) Equations, and (c) Terms in the following table.



$y(t)=\int_0^t h(\lambda)x(t-\lambda)d\lambda$	**convolution** ⟹ *time-domain*	$x(t) * h(t) = y(t)$ ***impulse response***	**Convolution**: is to find the response $y(t)$ of a system to an excitation $x(t)$.
$F(s)=\int_{0^-}^{\infty} f(t)e^{-st}dt$ $s=\sigma+j\omega$ ↕ $s=0+j\omega$	**Laplace xform** ⟹ *s-domain*	$X(s) \times H(s) = Y(s)$ ***transfer function***	**Laplace xform**: is better for the analysis of transient response involving initial conditions.
$F(\omega)=\int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt$	**Fourier xform** ⟹ *frequency-domain*	$X(\omega) \times H(\omega) = Y(\omega)$ ***frequency response***	**Fourier xform**: is especially useful for problems in the steady state.
$H(\omega)=\lvert H(\omega)\rvert e^{j\angle H(\omega)}$ magnitude phase	**Bode plots** ⟹		

B. A RLC Circuit (10+7=17 points): The input $v_s(t)=2\sin 2t\, u(t)$ V in Fig. 1. Also, +1 A flows through the inductor and -5 V is across the capacitor at t = 0. (a) Find the output voltage $v(t)$ across the capacitor at t > 0. (b) Find the frequency response between the output voltage $v(t)$ and the input voltage $v_s(t)$.

C. An Op Amp Circuit (8+8+8+4+4=32 points): $v_i(t)=10\cos 10t\, u(t)$ V voltage is applied to the op amp circuit with R_1 = 100 kΩ, R_2 = 100 kΩ, and C = 0.5 μF from t = 0 in Fig. 2. Assume that the initial capacitor voltage was 1 V. (a) Find $v_o(t)$ in the circuit by using Laplace transform. (b) Derive the transfer function $H(s)$ between $V_o(s)$ and $V_i(s)$. And, find the impulse response $h(t)$ between $v_o(t)$ and $v_i(t)$. (c) Find the natural response and the forced response of $v_o(t)$ from (a). (d) Also, indicate the transient response and the steady-state response in $v_o(t)$. (e) Apply the final-value theorem to find the steady-state response in $v_o(t)$.

D. Convolution (6+6=12 points): (a) Please physically and mathematically discuss the "Convolution" according to the equation $y(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)d\lambda$! (b) Explain the "Convolution" according to the equation $y(t)=\int_{0}^{t}x(\lambda)h(t-\lambda)d\lambda$ with a causal input and a causal system. The following table is for your references.

- **Convolution (folding)**: is to find the response *y*(*t*) of a system to an excitation *x*(*t*).
- It is defined as $y(t)=\int_{-\infty}^{\infty}x(\lambda)h(t-\lambda)d\lambda=\int_{-\infty}^{\infty}h(\lambda)x(t-\lambda)d\lambda$

  $\Rightarrow y(t)=x(t)*h(t)=h(t)*x(t)$ ⟵ Physics if $x(t)$ is input & $y(t)$ is output

  The **convolution** of two signals consists of time-reversing one of the signals, shifting it, and multiplying it point by point with the second signal, and integrating the product. ⟵ mathematics

- It applies to any **linear system** and can be simplified

  $y(t)=\int_{0}^{\infty}x(\lambda)h(t-\lambda)d\lambda$ if $x(t)=0$ for $t<0$, i.e., a causal input

  $\boxed{=\int_{0}^{t}x(\lambda)h(t-\lambda)d\lambda}$ if $h(t)=0$ for $t<0$, i.e., a causal system

E. Fourier transform (5+6=11 points): (a) Derive the Fourier transform of $10\sin\omega_0 t$. (b) Plot the amplitude spectrum and phase spectrum.

F. MATLAB (5×4=20 points): (a) Find the frequency response $V_0(\omega)/I_i(\omega)$ for the circuit of Fig. 3. And write a MATLAB code to draw (b) a Bode plot, whose *x*-axis is 100 points from 10^{-2} to 10^{2} Hz, (c) a step response, and (d) a time response with a sinusoidal input at 500 rad/s. Function hint**:** bode(num,den), logspace(a,b,n), step(num,den), lsim(num,den,x,t).

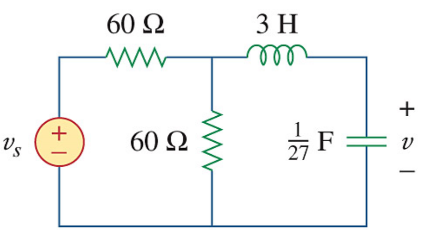


Fig. 1

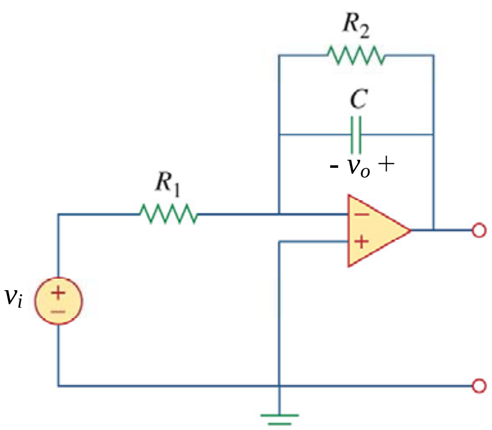


Fig. 2

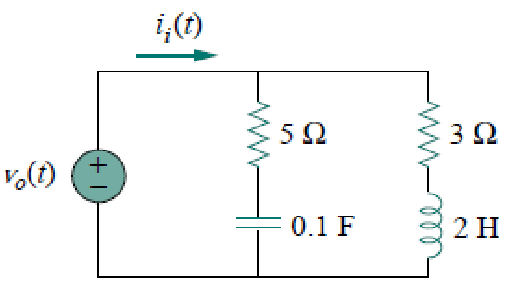


Fig. 3