Midterm Exam II

November 4, 2017

Rules and Regulations: It is permitted to bring one additional paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

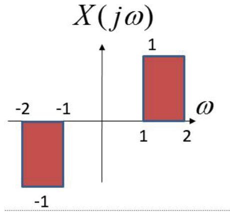
1. Please determine whether each of the following statements is *True* or *False*.

   (a) (4%) Let $X(j\omega)$ be the spectrum of the signal $x(t)$. We have

   $$jtx(t) \xleftrightarrow{\mathcal{F}} \frac{d\,X(j\omega)}{d\,\omega}.$$

   (b) (4%) For a real signal $x(t)$, the Fourier transform of $x_o(t)$ is $\Im\{X(j\omega)\}$ where $X(j\omega) = \Re\{X(j\omega)\} + j\Im\{X(j\omega)\}$ is the spectrum of $x(t)$.

   (c) (4%) The inverse Fourier transform of the following spectrum is real and odd in time-domain.

   (d) (4%) The spectrum $X(j\omega)$ of the signal $x(t) = 3u(t-1) - 3u(-t-1)$ satisfies

   $$X(j0) = 0.$$

   (e) (4%) 感謝提供題目的何若慈、邱莉雯好朋友。 Let $X(j\omega)$ be the Fourier transform of $x(t) = \delta(t)$. Since

   $$e^{j\omega_0 t}x(t) \xleftrightarrow{\mathcal{F}} X(j(\omega - \omega_0)),$$

   we have $F^{-1}\{X(j(\omega - \omega_0))\} = x(t)e^{j\omega_0 t} = \delta(t)e^{j\omega_0 t} = \delta(t) = x(t)$.

2. (10%) Please show that

   $$\text{sinc}(at) * \text{sinc}(t) = \text{sinc}(at)$$

   for $0 < a \leq 1$.

3. (10%) For a periodic signal

   $$x(t) = \cos\left(\frac{2\pi}{3}t\right) + \sin\left(\frac{7\pi}{3}t\right),$$

   please determine the fundamental frequency ω_0 and the Fourier series coefficients a_k of $x(t)$.

4. (10%) Please find the inverse transform of

$$X(j\omega) = \frac{1}{(1+j\omega)^3}.$$

5. (10%) Let $x(t)$ be a periodic signal with period 6 and the Fourier series coefficients for $x(t)$ are specified as

$$a_k = \begin{cases} jk, & |k| < 4; \\ 0, & \text{otherwise.} \end{cases}$$

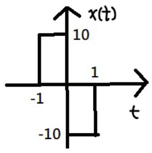
Please determine the signal $x(t)$.

6. (10%) Please find the Fourier transform of the continuous-time signal

$$x(t) = \frac{1}{\pi t}.$$

7. Please provide the correct solutions to the following incorrect statements.

(a) (10%) 感謝提供題目的陳虹衣、李羿同學。 A continuous-time signal $x(t)$ is depicted below.



The Fourier transform of $x(t)$ is

$$\begin{aligned} X(j\omega) &= \int_{-1}^{1} x(t)e^{-j\omega t}dt = \int_{-1}^{1} 10e^{-j\omega t}dt \\ &= \frac{-10}{j\omega}e^{-j\omega t}\mid_{-1}^{1} = \frac{-10}{j\omega}(e^{-j\omega} - e^{j\omega}) = \frac{20}{\omega}\sin(\omega). \end{aligned}$$

(b) (10%) 感謝提供題目的呂郁萱、鄭珮文同學。 We known that

$$e^{-|t|} \overset{\mathcal{F}}{\longleftrightarrow} \frac{2}{1+\omega^2}.$$

According to the duality, we have

$$\frac{2}{1+t^2} \overset{\mathcal{F}}{\longleftrightarrow} \frac{e^{-|-\omega|}}{2\pi} = \frac{e^{-|\omega|}}{2\pi}.$$

(c) (10%) 感謝提供題目的陳芃文、張竣佑、張壹登同學。 For a real signal

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t},$$

we have $a_k^* = a_{-k}$. Hence,

$$\begin{aligned} x(t) &= a_0 + \sum_{k=1}^{\infty}[a_k e^{jk\omega_0 t} + a_{-k}e^{-jk\omega_0 t}] = a_0 + \sum_{k=1}^{\infty}[a_k e^{jk\omega_0 t} + a_k^* e^{-jk\omega_0 t}] \\ &= a_0 + \sum_{k=1}^{\infty} 2\Re\{a_k e^{jk\omega_0 t}\} = a_0 + \sum_{k=1}^{\infty} 2a_k\cos(k\omega_0 t). \end{aligned}$$