1. (12%) Use the LU method to get the solutions of $x_1, x_2, x_3,$ and x_4.

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 + 3x_4 = -2 \\ 1x_1 + 5x_2 + 3x_3 + 5x_4 = 2 \\ 3x_1 + 1x_2 + 1x_3 + 2x_4 = 1.5 \\ 2x_1 + 4x_2 + 5x_3 + 4x_4 = -\dfrac{1}{3} \end{cases}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{vmatrix} \begin{vmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{22} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{vmatrix}$$

- ❑ $u_{1j} = a_{1j} \ (j = 1 \to 4)$
- ❑ $l_{i1} = a_{i1} / u_{11} \ (i = 2 \to 4)$
- ❑ $u_{2j} = a_{2j} - l_{21}u_{1j} \ (j = 2 \to 4)$
- ❑ $l_{i2} = (a_{i2} - l_{i1}u_{12})/u_{22} \ (i = 3 \to 4)$
- ❑ $u_{3j} = a_{3j} - l_{31}u_{1j} - l_{32}u_{2j} \ (j = 3 \to 4)$
- ❑ $l_{i3} = (a_{i3} - l_{i1}u_{13} - l_{i2}u_{23})/u_{33} \ (i = 4)$
- ❑ $u_{4j} = a_{4j} - l_{41}u_{1j} - l_{42}u_{2j} - l_{43}u_{3j} (j = 4)$

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 + 3x_4 = -2 \\ 1x_1 + 5x_2 + 3x_3 + 5x_4 = 2 \\ 3x_1 + 1x_2 + 1x_3 + 2x_4 = 1.5 \\ 2x_1 + 4x_2 + 5x_3 + 4x_4 = -\dfrac{1}{3} \end{cases} \Rightarrow \begin{vmatrix} 2 & 2 & 4 & 3 \\ 1 & 5 & 3 & 5 \\ 3 & 1 & 1 & 2 \\ 2 & 4 & 5 & 4 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1.5 \\ -1/3 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 2 & 4 & 3 \\ 1 & 5 & 3 & 5 \\ 3 & 1 & 1 & 2 \\ 2 & 4 & 5 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 1.5 & -0.5 & 1 & 0 \\ 1 & 0.5 & -0.11 & 1 \end{vmatrix} \begin{vmatrix} 2 & 2 & 4 & 3 \\ 0 & 4 & 1 & 3.5 \\ 0 & 0 & -4.5 & -0.75 \\ 0 & 0 & 0 & -0.83 \end{vmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 1.5 & -0.5 & 1 & 0 \\ 1 & 0.5 & -0.11 & 1 \end{vmatrix} \begin{vmatrix} 2 & 2 & 4 & 3 \\ 0 & 4 & 1 & 3.5 \\ 0 & 0 & -4.5 & -0.75 \\ 0 & 0 & 0 & -0.83 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1.5 \\ -1/3 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{vmatrix} 2 & 2 & 4 & 3 \\ 0 & 4 & 1 & 3.5 \\ 0 & 0 & -4.5 & -0.75 \\ 0 & 0 & 0 & -0.83 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 \\ 0.5 & 1 & 0 & 0 \\ 1.5 & -0.5 & 1 & 0 \\ 1 & 0.5 & -0.11 & 1 \end{vmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 1.5 \\ -1/3 \end{bmatrix} \qquad \xRightarrow{\textit{Forward substitution}} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \\ 0.83 \end{bmatrix}$$

$$\begin{vmatrix} 2 & 2 & 4 & 3 \\ 0 & 4 & 1 & 3.5 \\ 0 & 0 & -4.5 & -0.75 \\ 0 & 0 & 0 & -0.83 \end{vmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 6 \\ 0.83 \end{bmatrix} \qquad \xRightarrow{\textit{Backward substitution}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \mathbf{0.9167} \\ \mathbf{1.9167} \\ \mathbf{-1.1667} \\ \mathbf{-1.000} \end{bmatrix}$$

2. (10%) Fit a least square curve of the form $y = ae^{bx}$ $(a > 0)$ to the data given below. Please calculate the values of a, b, and R^2.

x_i	1	2	3	4	5
y_i	1	3	5	7	9

$$y = ae^{bx} \xrightarrow{\text{取對數}} \ln(y) = \ln(a) + bx$$

$$\varepsilon_i = \ln(y_i) - \ln(a) - bx_i$$

$$\varepsilon_i{}^2 = [\ln(y_i) - \ln(a) - bx_i]^2$$

$$\varphi(a, b) = \sum \varepsilon_i{}^2 = \sum[\ln(y_i) - \ln(a) - bx_i]^2$$

$\boxed{\text{Let } \ln(a)=A}$

$$\frac{\varphi(A, b)}{\partial A} = 2\sum[\ln(y_i) - A - bx_i] \times (-1) = 0 \to \sum_{i=1}^{n} A + \sum_{i=1}^{n} bx_i = \sum_{i=1}^{n} \ln(y_i)$$

$$\frac{\varphi(A, b)}{\partial b} = 2\sum[\ln(y_i) - A - bx_i] \times (-x_i) = 0 \to \sum_{i=1}^{n} Ax_i + \sum_{i=1}^{n} bx_i{}^2 = \sum_{i=1}^{n} x_i \ln(y_i)$$

i	1	2	3	4	5
x_i	1	2	3	4	5
y_i	1	3	5	7	9

$$\frac{\varphi(\mathrm{A},\mathrm{b})}{\partial A} = 2\sum[\ln(y_i) - A - bx_i] \times (-1) = 0 \rightarrow \sum_{i=1}^{n} A + \sum_{i=1}^{n} bx_i = \sum_{i=1}^{n} \ln(y_i)$$

$$\frac{\varphi(\mathrm{A},\mathrm{b})}{\partial b} = 2\sum[\ln(y_i) - A - bx_i] \times (-x_i) = 0 \rightarrow \sum_{i=1}^{n} Ax_i + \sum_{i=1}^{n} b{x_i}^2 = \sum_{i=1}^{n} x_i\ln(y_i)$$

$$\begin{vmatrix} \sum_{i=1}^{5} {x_i}^2 & \sum_{i=1}^{5} x_i \\ \sum_{i=1}^{5} x_i & \sum_{i=1}^{5} 1 \end{vmatrix} \begin{bmatrix} b \\ A \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^{5} x_i\ln(y_i) \\ \sum_{i=1}^{5} \ln(y_i) \end{bmatrix} \rightarrow \begin{vmatrix} 55 & 15 \\ 15 & 5 \end{vmatrix} \begin{bmatrix} b \\ A \end{bmatrix} = \begin{bmatrix} 25.7953 \\ 6.8512 \end{bmatrix} \rightarrow b = 0.52417\,, A = -0.20227$$

$\mathrm{A} = \ln(\mathrm{a}) = -0.20227 \rightarrow a = 0.81687$ $\Longrightarrow$ $\boldsymbol{y = 0.81687e^{0.52417x}, R^2 = 0.8295}$

3. (12%) The following data (x_i, y_i, z_i) are points in the Cartesian coordinate. (X,Y,Z) is a center point and has almost the same distance to those points. According to the following data, please use the least square method to calculate the a_{ij} and b_i, and solve the X, Y, Z.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

x_i	y_i	z_i	subscript_i
99.9943	96.6325	9.9912	1
101.9789	96.6314	10.7004	2
102.9944	96.6357	12.8734	3
101.4784	96.6201	15.4812	4
100.0634	96.6189	15.8444	5
98.2554	96.6451	15.2208	6
97.1345	96.6146	12.9972	7
97.9674	96.6444	10.8775	8

$$a_{11} = \sum_{i=1}^{4} 2(x_{i+4} - x_i)(x_i - x_{i+4}) \qquad a_{12} = \sum_{i=1}^{4} 2(x_{i+4} - x_i)(y_i - y_{i+4}) \qquad a_{13} = \sum_{i=1}^{4} 2(x_{i+4} - x_i)(z_i - z_{i+4})$$

$$a_{21} = \sum_{i=1}^{4} 2(y_{i+4} - y_i)(x_i - x_{i+4}) \qquad a_{22} = \sum_{i=1}^{4} 2(y_{i+4} - y_i)(y_i - y_{i+4}) \qquad a_{23} = \sum_{i=1}^{4} 2(y_{i+4} - y_i)(z_i - z_{i+4})$$

$$a_{31} = \sum_{i=1}^{4} 2(z_{i+4} - z_i)(x_i - x_{i+4}) \qquad a_{32} = \sum_{i=1}^{4} 2(z_{i+4} - z_i)(y_i - y_{i+4}) \qquad a_{33} = \sum_{i=1}^{4} 2(z_{i+4} - z_i)(z_i - z_{i+4})$$

$$b_1 = \sum_{i=1}^{4} (x_{i+4} - x_i)\left[\left(x_i^2 - x_{i+4}^2\right) + \left(y_i^2 - y_{i+4}^2\right) + \left(z_i^2 - z_{i+4}^2\right)\right]$$

$$b_2 = \sum_{i=1}^{4} (y_{i+4} - y_i)\left[\left(x_i^2 - x_{i+4}^2\right) + \left(y_i^2 - y_{i+4}^2\right) + \left(z_i^2 - z_{i+4}^2\right)\right]$$

$$b_3 = \sum_{i=1}^{4} (z_{i+4} - z_i)\left[\left(x_i^2 - x_{i+4}^2\right) + \left(y_i^2 - y_{i+4}^2\right) + \left(z_i^2 - z_{i+4}^2\right)\right]$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

$$\begin{bmatrix} -121.06955214 & 0.02725024 & 1.97823640 \\ 0.02725024 & -0.00281670 & 0.26431226 \\ 1.97823640 & 0.26431226 & -151.80669306 \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} -12086.597383 \\ 5.869274538 \\ -1737.510186 \end{Bmatrix}$$

$$\begin{bmatrix} -121.06955214 & 0.02725024 & 1.97823640 \\ 0.02725024 & -0.00281670 & 0.26431226 \\ 1.97823640 & 0.26431226 & -151.80669306 \end{bmatrix}^{-1} = \begin{bmatrix} -0.00829 & -0.10797 & -0.0003 \\ -0.10797 & -425.764 & -0.74271 \\ -0.0003 & -0.74271 & -0.00788 \end{bmatrix}$$

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} -0.00829 & -0.10797 & -0.0003 \\ -0.10797 & -425.764 & -0.74271 \\ -0.0003 & -0.74271 & -0.00788 \end{bmatrix} \begin{Bmatrix} -12086.597383 \\ 5.869274538 \\ -1737.510186 \end{Bmatrix} = \begin{Bmatrix} \mathbf{100.0646} \\ \mathbf{96.4836} \\ \mathbf{12.9175} \end{Bmatrix}$$

4. (10%)Construct the cubic spline

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

where $i = 1, 2, 3,$ using the following data and boundary condition $S_1'(0) = S_3'(3) = 7.$

x	0	1	2	3
$f(x)$	0	1	8	27

$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3,\ S_i'(x) = b_i + 2c_i(x - x_i) + 3d_i(x - x_i)^2,\ S_i''(x) = 2c_i + 6d_i(x - x_i)$

$S_i(x_i) = a_i + b_i(x_i - x_i) + c_i(x_i - x_i)^2 + d_i(x_i - x_i)^3 = y_i \Rightarrow S_i(x_i) = a_i = y_i$

Let $h_i = x_{i+1} - x_i,\ S_i(x_{i+1}) = a_i + b_i h_i + c_i {h_i}^2 + d_i {h_i}^3 = y_{i+1}$—①

$\boldsymbol{S_i'(x_{i+1}) = S_{i+1}'(x_{i+1})},\ S_i'(x_{i+1}) = b_i + 2c_i h_i + 3d_i {h_i}^2,\ S_{i+1}'(x_{i+1}) = b_{i+1} + 2c_{i+1}(x_{i+1} - x_{i+1}) + 3d_{i+1}(x_{i+1} - x_{i+1})^2$

$\Rightarrow b_i + 2c_i h_i + 3d_i {h_i}^2 = b_{i+1}$ —②

$\boldsymbol{S_i''(x_{i+1}) = S_{i+1}''(x_{i+1})},\ S_i''(x_{i+1}) = 2c_i + 6d_i(x_{i+1} - x_i) = 2c_i + 6d_i h_i,\ S_{i+1}''(x_{i+1}) = 2c_{i+1}$

$\Rightarrow 2c_i + 6d_i h_i = 2c_{i+1}$

Let $z_i = S_i''(x_i) = 2c_i,\ 2c_i + 6d_i h_i = 2c_{i+1} \Rightarrow z_i + 6d_i h_i = z_{i+1}$

$a_i = y_i, c_i = \frac{z_i}{2}, d_i = \frac{z_{i+1} - z_i}{6h_i}$, 代入①式可得 $b_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{2} z_i - \frac{h_i}{6}(z_{i+1} - z_i)$

$a_i = y_i, b_i = \frac{y_{i+1}-y_i}{h_i} - \frac{h_i}{2} z_i - \frac{h_i}{6}(z_{i+1} - z_i), c_i = \frac{z_i}{2}, d_i = \frac{z_{i+1}-z_i}{6h_i}$，將$a_i, b_i, c_i, d_i$代入②式可得

$$h_i z_i + 2(h_i + h_{i+1}) z_{i+1} + h_{i+1} z_{i+2} = 6\left(\frac{y_{i+2}-y_{i+1}}{h_{i+1}} - \frac{y_{i+1}-y_i}{h_i}\right)$$

$$S_1'(0) = 7 \Rightarrow b_1 = 7 \Rightarrow 7 = \frac{y_2 - y_1}{h_1} - \frac{h_1}{2} z_1 - \frac{h_1}{6}(z_2 - z_1) \Rightarrow 2h_1 z_1 + h_1 z_2$$
$$= 6\left(\frac{y_2 - y_1}{h_1} - 7\right)$$

$$S_3'(3) = 7 \Rightarrow b_3 + 2c_3 h_3 + 3d_3 h_3^2 = 7 \Rightarrow h_3 z_3 + 2h_3 z_4 = 6\left(7 - \frac{y_4-y_3}{h_3}\right)$$

$$\begin{bmatrix} 2h_1 & h_1 & 0 & 0 \\ h_1 & 2(h_1 + h_2) & h_2 & 0 \\ 0 & h_2 & 2(h_2 + h_3) & h_3 \\ 0 & 0 & h_3 & 2h_3 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = 6 \begin{bmatrix} \frac{y_2 - y_1}{h_1} - 7 \\ \frac{y_3 - y_2}{h_2} - \frac{y_2 - y_1}{h_1} \\ \frac{y_4 - y_3}{h_3} - \frac{y_3 - y_2}{h_2} \\ 7 - \frac{y_4 - y_3}{h_3} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 \\ 0 & 1 & 4 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -36 \\ 36 \\ 72 \\ -72 \end{bmatrix} \Rightarrow \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} -21.6 \\ 7.2 \\ 28.8 \\ -50.4 \end{bmatrix}$$，代回a_i, b_i, c_i, d_i可求得各係數

$$a_i = y_i \Rightarrow a_1 = 0, a_2 = 1, a_3 = 8$$

$$b_i = \frac{y_{i+1} - y_i}{h_i} - \frac{h_i}{2} z_i - \frac{h_i}{6}(z_{i+1} - z_i) \Rightarrow b_1 = 7,\ b_2 = -0.2,\ b_3 = 17.8$$

$$c_i = \frac{z_i}{2} \Rightarrow c_1 = -10.8,\ c_2 = 3.6,\ c_3 = 14.4$$

$$d_i = \frac{z_{i+1} - z_i}{6h_i} \Rightarrow d_1 = 4.8\,, d_2 = 3.6,\ d_3 = -13.2$$

$$\boldsymbol{S_1(x) = 7(x - x_1) - 10.8(x - x_1)^2 + 4.8(x - x_1)^3}$$

$$\boldsymbol{S_2(x) = 1 - 0.2(x - x_2) + 3.6(x - x_2)^2 + 3.6(x - x_2)^3}$$

$$\boldsymbol{S_3(x) = 8 + 17.8(x - x_3) + 14.4(x - x_3)^2 - 13.2(x - x_3)^3}$$

5. Evaluate $\int_0^6 \frac{2dx}{1+x^2}$ by using (a) (6%) Simpson's $\frac{1}{3}$ rule,
(b) (6%) Simpson's $\frac{3}{8}$ rule and compare the error with the exact solution separately.

(a) (5%) Simpson's $\frac{1}{3}$ rule

$$\mathrm{f}\,(\mathrm{x}) = \frac{2}{1+x^2} \Rightarrow \begin{cases} f_{i+1} = f(b) = f(6) = \frac{2}{37} \\ f_i = f\left(\frac{a+b}{2}\right) = f(3) = \frac{2}{10} \\ f_{i-1} = f(a) = f(0) = 2 \end{cases}$$

$$\mathrm{h} = \frac{6-0}{2} = 3$$

$$\mathrm{I} = \frac{\mathrm{h}(f_{i+1}+4f_i+f_{i-1})}{3} = \frac{3(2/37+4\times0.2+2)}{3} = \frac{3(528/185)}{3} = \mathbf{2.8541}$$

$$\text{relative error} = \left|\frac{2.8541-2.8113}{2.8113}\right| \times 100\% = \mathbf{1.5224}\%$$

(b) (5%) Simpson's $\frac{3}{8}$ rule

$$f(x) = \frac{2}{1+x^2} \Rightarrow \begin{cases} f_{i+2} = f(6) = \frac{2}{37} \\ f_{i+1} = f(4) = \frac{2}{17} \\ f_i = f(2) = \frac{2}{5} \\ f_{i-1} = f(0) = 2 \end{cases}$$

$$h = \frac{6-0}{3} = 2$$

$$I = \frac{3h(f_{i+2} + 3f_{i+1} + 3f_i + f_{i-1})}{8}$$

$$= \frac{3 \times 2[(\frac{2}{37}) + 3 \times (\frac{2}{17}) + 3 \times (\frac{2}{5}) + 2]}{8} = \frac{6(3.607)}{8} = \mathbf{2.705}$$

$$\text{relative error} = \left|\frac{2.705 - 2.8113}{2.8113}\right| \times 100\% = \mathbf{3.7812}\%$$

6. (a) (4%) Derive the two-point Gauss-quadrature method.
(b) (7%) Use two-point Gauss-quadrature rule to approximate the distance covered by a rocket from t = 8 to t = 30 as given by

$$x = \int_{8}^{30} \left(2000 \ln \left[\frac{140000}{140000 - 2100t} \right] - 9.8t \right) dt$$

(c) (3%) Find the relative error.

(a) n = 2 $\int_{-1}^{+1} f(x)dx = \sum_{i=1}^{2} w_i f(x_i)$

Let $f(x) \to 1,\ x,\ x^2,\ x^3$

$$f(x) = 1 \to \int_{-1}^{+1} 1\,dx = 2 = w_1 + w_2 \qquad \to w_1 = w_2 = 1$$

$$f(x) = x \to \int_{-1}^{+1} x\,dx = 0 = w_1 x_1 + w_2 x_2 \qquad \to x_1 = -x_2$$

$$f(x) = x^2 \to \int_{-1}^{+1} x^2 dx = \frac{2}{3} = w_1 x_1^2 + w_2 x_2^2 \qquad \to x_1 = -\frac{1}{\sqrt{3}}, x_2 = \frac{1}{\sqrt{3}}$$

$$f(x) = x^3 \to \int_{-1}^{+1} x^3 dx = 0 = w_1 x_1^3 + w_2 x_2^3 \qquad \to \text{satisfy}$$

(b) Let $\mathrm{I} = \int_{-1}^{1} F(Y)dY = \int_{8}^{30} f(x)dx,\ f(x) = 2000\ln\left[\frac{140000}{140000-2100x}\right] - 9.8x$

$\mathrm{Y}=\frac{(b-a)x+a+b}{2},\ \mathrm{dY} = \frac{(b-a)}{2}dx,\ a = 8,\ b = 30$

$\rightarrow \mathrm{Y} = 11x + 19, \qquad \mathrm{dY} = 11dx$

$$I = \int_{-1}^{1} f(11x + 19)\cdot 11dx \approx 11[w_1 f(11x_1 + 19) + w_2 f(11x_2 + 19)]$$

$$f(11x_1 + 19) = f(12.649) = 2000\ln\left[\frac{140000}{140000 - 2100(12.649)}\right] - 9.8(12.649) = 296.832$$

$$f(11x_2 + 19) = f(25.351) = 2000\ln\left[\frac{140000}{140000 - 2100(25.351)}\right] - 9.8(25.351) = 708.481$$

$\rightarrow I = 11[296.832 + 708.481] = \mathbf{11058.44}$ [m]

(c) exact solution $= \int_{8}^{30}\left(2000\ln\left[\frac{140000}{140000-2100t}\right] - 9.8t\right)dt = 11061.33551$

$\rightarrow$ relative error $= \left|\frac{11061.33551 - 11058.44}{11061.33551}\right| \times 100\% = \mathbf{0.0262}\%$