Midterm Exam I

October 30, 2017

Rules and Regulations: It is permitted to bring one paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.
Problems for Solution:

1. Please determine whether each of the following statements is *True* or *False*.

   (a) (3%) A signal can be represented by a function.

   (b) (3%) A system can be represented by a function.

   (c) (3%) For a linear system, if the input is $x[n] = 0$ for all n, then the output must be $y[n] = 0$ for all n.

   (d) (3%) For a discrete-time signal $x[n]$, we have $x[n+3]\delta[n+3] = x[0]$.

   (e) (3%) 感謝提供題目的呂郁萱、鄭珮文同學。 Consider a discrete-time system with input $x[n]$ and output $y[n]$ related by

   $$y[n] = x[n] - x[n-1].$$

   This system is invertible.

   (f) (3%) The system with input and output relationship $y[n] = 3x[n] - 3$ is a linear system.

   (g) (3%) For continuous-time signals $x(t)$ and $y(t)$, we have

   $$x(t) * y(-t) = \int_{-\infty}^{\infty} x(\tau)y(\tau - t)d\tau.$$

   (h) (3%) The even-odd decomposition of any signal is unique.

   (i) (3%) If the input $x(t)$ to a linear system is periodic, then the output $y(t)$ is also periodic.

   (j) (3%) 感謝提供題目的何若慈、邱莉雯同學。 For a discrete-time system with input $x[n]$ and output $y[n]$ related by

   $$y[n] = \cos^2[(n-87)^2]x[n-94],$$

   the inverse system is given by

   $$x[n] = \frac{y[n+94]}{\cos^2[(n-87)^2]}.$$

2. (10%) If $x[n]$ is an even signal and $h[n]$ is an odd signal, then show that $y[n] = x[n] * h[n]$ is an odd signal.

3. (10%) Find and sketch $y(t) = x(t) * h(t)$ where

$$x(t) = u(-t)$$

and

$$h(t) = e^{-t}\left(u(t) - u(t-100)\right).$$

4. (10%) Show that the inverse system of an invertible LTI system is also an LTI system.

5. The impulse response of a discrete-time LTI system is $h[n] = (-0.5)^n u[n] + (1.01)^n u[n-1]$.

   (a) (5%) Is this system causal?

   (b) (5%) Is this system stable?

   Please provide your answers with reasons.

6. If a discrete-time LTI system with the impulse response $h[n]$ is BIBO stable, then we want to show that

$$\sum_{k=-\infty}^{\infty} |h[k]| < \infty.$$

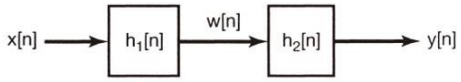
   Let the input $x[n] = \text{sgn}(h[-n]))$ where

$$\text{sgn}(s) = \begin{cases} 1, & s > 0; \\ -1, & s < 0; \\ 0, & s = 0. \end{cases}$$

   (a) (5%) Show that $x[n]$ is bounded.

   (b) (5%) Let $y[n]$ be the output. Find $y[0]$.

7. (10%) Consider the cascade of two LTI systems shown as below.



   Let $h_1[n] = e^{\sin(\pi n^2)}$ and $h_2[n] = e^{-2n}u[n]$. Please find the output $y[n]$ if the input $x[n] = e^2\delta[n] - \delta[n-1]$.

8. (10%) 感謝提供題目的楊登宇、張嘉詠同學以及賴建勳、李昱、謝茹媛同學。 In the following table, there is one property given incorrectly for each system. Please indicate which property is incorrect and provide your reasons. Note that $x_o[n]$ denotes the odd component of $x[n]$.

System	Stable	Causal	Invertible	Linear	Time-invariant
$y[n] = x_o[n]$	Yes	No	No	Yes	Yes
$y(t) = e^{t+1}\sin(x(2t-1))$	No	Yes	No	No	No