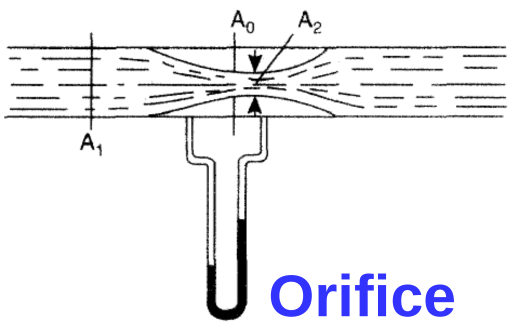
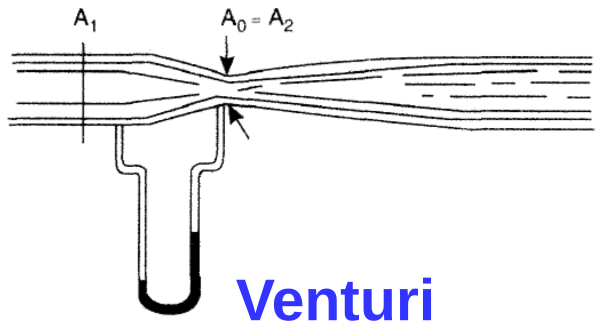
Flowmeters

- In this part, we will introduce common flowmeters that can be used to measure the flow rate/velocity of the fluid in pipes.

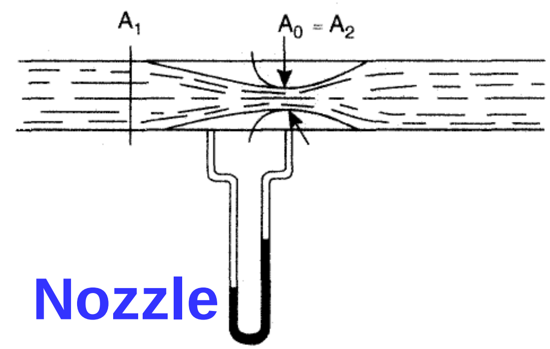
- “**Full-Bore Meters**”: Operate on all the fluid in the pipe.



Orifice

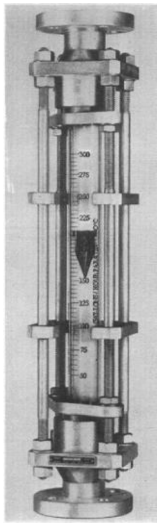


Venturi

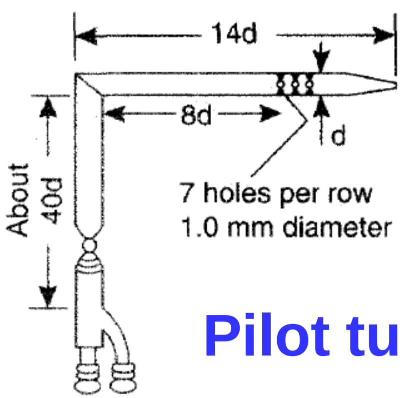


Nozzle

Rotameter

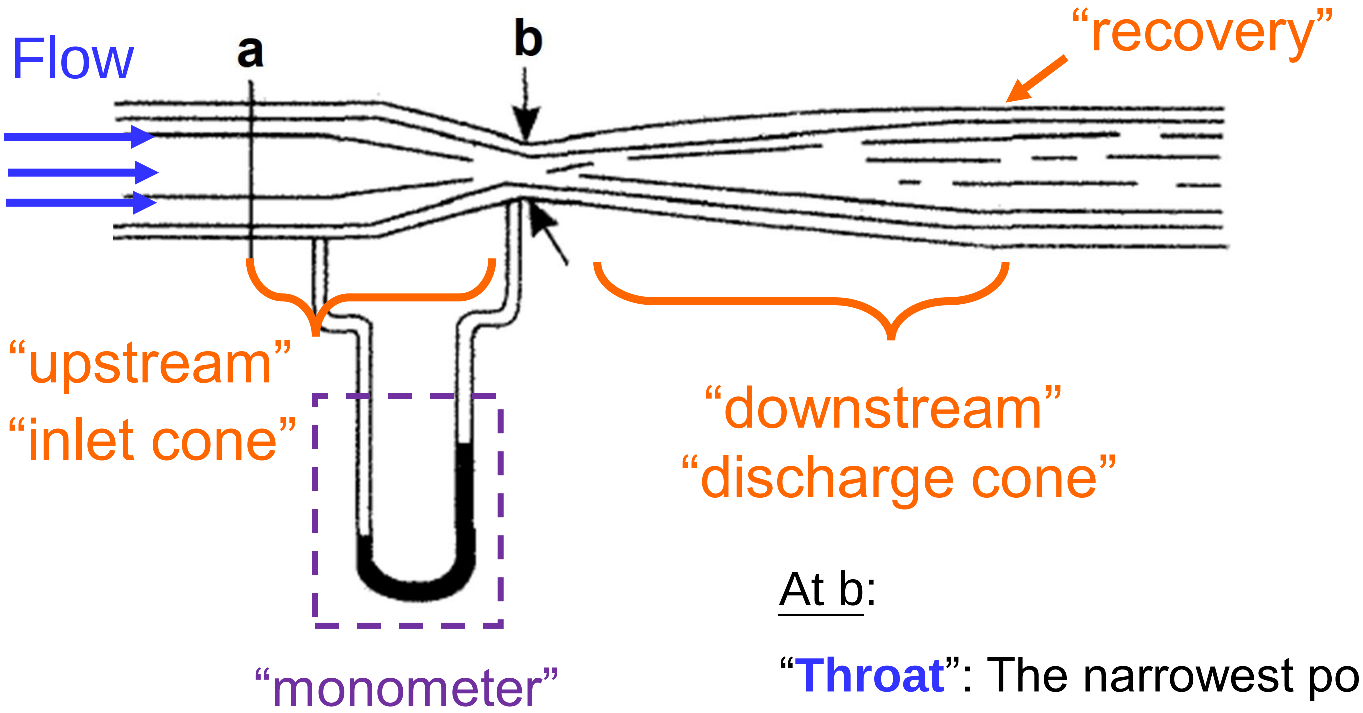


- “**Insertion Meters**”: Measure the flow rate, or more commonly the velocity, *at one point* only.



Pilot tubes

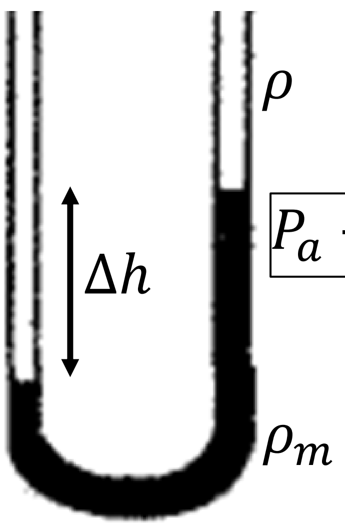
Venturi meter



At b:

"**Throat**": The narrowest point of the pipe

"**Vena contracta**": The narrowest point of the fluid



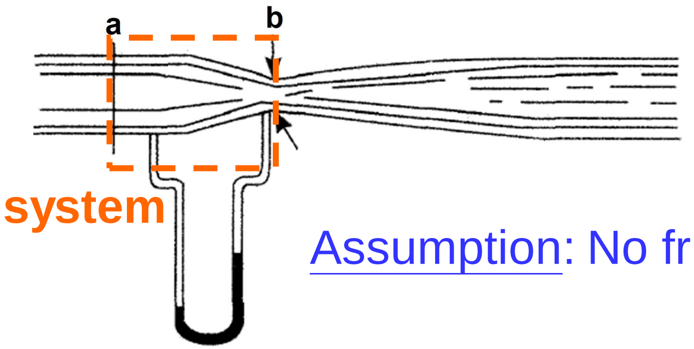
$$\boxed{P_a - P_b = (\rho_m - \rho)g\Delta h}$$

$$(\boldsymbol{A\downarrow, v\uparrow, P\downarrow})$$

Features of venturi:

- Long discharge cone to minimize friction loss
- Very good pressure recovery (>90%)
- Usually for liquid
- Expensive!

Incompressible fluid in venturi



$$\boxed{\frac{F}{g} + \Delta\left(\frac{P}{\rho g}\right) + \frac{1}{2g}\Delta(\alpha v_{avg}^2) + \Delta z = \frac{-W_S}{g}}$$

<u>Assumption</u>: No friction loss in the upstream cone

$$\Delta\left(\frac{P}{\rho g}\right) + \frac{1}{2g}\Delta(\alpha v_{avg}^2) = 0$$

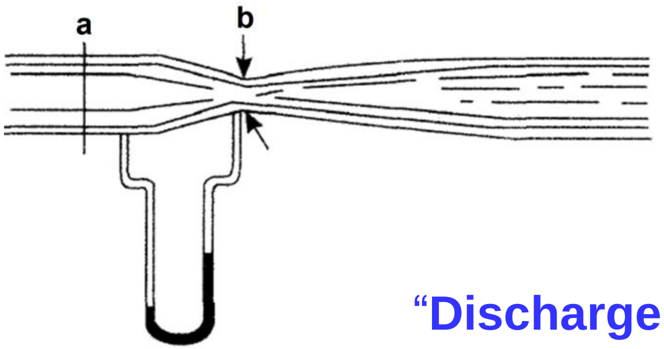
$$\begin{cases} \left(\dfrac{P_b - P_a}{\rho}\right) + \dfrac{1}{2}(\alpha_b v_b^2 - \alpha_a v_a^2) = 0 \\ A_a v_a = A_b\ v_b \quad \longrightarrow \quad v_b^2 = \dfrac{A_a^2 v_a^2}{A_b^2} \end{cases}$$

$$\alpha_b \frac{A_a^2 v_a^2}{A_b^2} - \alpha_a v_a^2 = 2\left(\frac{P_a - P_b}{\rho}\right) \qquad \frac{\alpha_b A_a^2 v_a^2 - \alpha_a A_b^2 v_a^2}{A_b^2} = 2\left(\frac{P_a - P_b}{\rho}\right)$$

$$v_a^2(\alpha_b A_a^2 - \alpha_a A_b^2) = 2A_b^2\left(\frac{P_a - P_b}{\rho}\right) \qquad v_a^2 = \frac{2}{\alpha_b \dfrac{A_a^2}{A_b^2} - \alpha_a}\left(\frac{P_a - P_b}{\rho}\right)$$

$$Mass\ flow\ rate = G = \rho A_a v_a = \rho A_a \sqrt{\frac{2}{\alpha_b \dfrac{A_a^2}{A_b^2} - \alpha_a}\left(\frac{P_a - P_b}{\rho}\right)} = \frac{A_b}{\sqrt{\alpha_b - \dfrac{A_b^2}{A_a^2}\alpha_a}}\sqrt{2\rho(P_a - P_b)}$$

Incompressible fluid in venturi & nozzle



$$Mass\ flow\ rate\ (G) = \frac{A_b}{\sqrt{\alpha_b - \frac{A_b^2}{A_a^2}\alpha_a}}\sqrt{2\rho(P_a - P_b)}$$

"Discharge coefficient (C_D)" for ventri (determined experimentally)

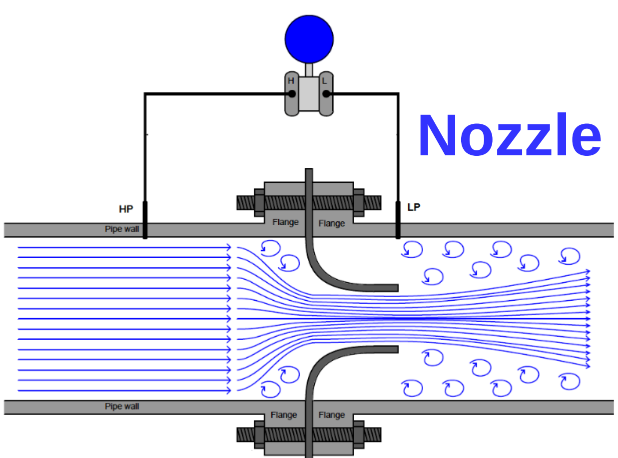
- To include the effect from kinetic energy correction factors
- To include the upstream friction loss

$$\boxed{G = \frac{C_D A_b}{\sqrt{1 - \frac{A_b^2}{A_a^2}}}\sqrt{2\rho(P_a - P_b)}}$$

(Or)

$$G = \frac{C_D A_b}{\sqrt{1 - \beta^4}}\sqrt{2\rho(P_a - P_b)} \qquad (\beta \equiv \frac{D_b}{D_a})$$

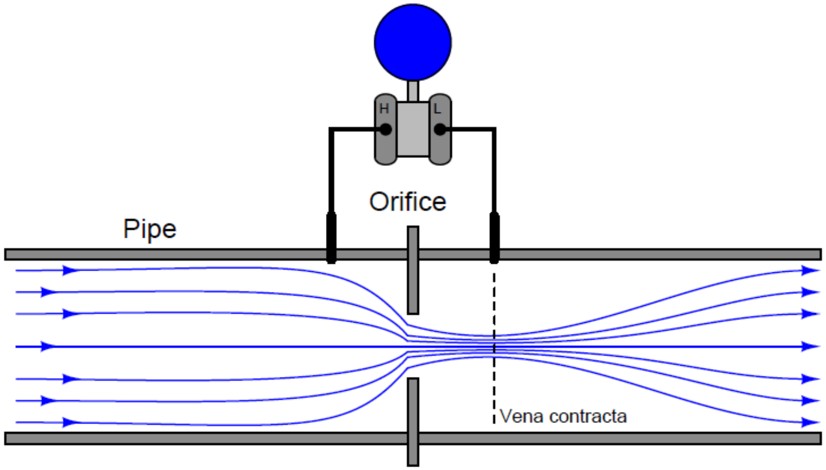
Nozzle



- For both venturi and nozzle, the locations of throat and vena contracta are the same with the area of $\mathbf{A_b}$.
- The same equation can also be used for nozzle.
- For a well-designed venturi or nozzle, C_D is around 0.98~1.

Incompressible fluid in orifice

- Both venturi and nozzle meters are expensive, and they occupy considerable space.
- A cheaper option is orifice meter (銳孔流量計).



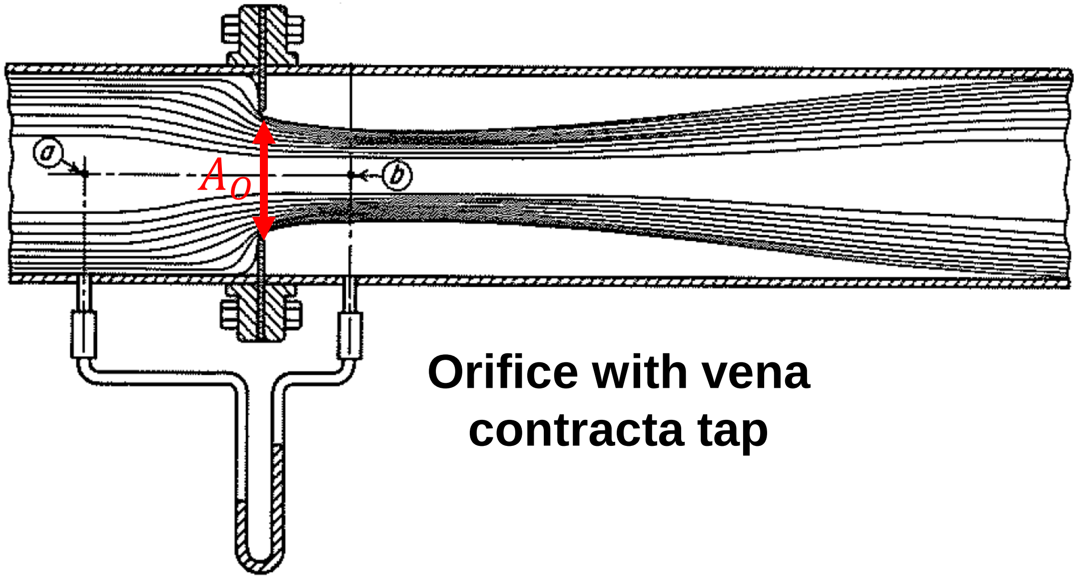
The key difference: **"The area of orifice throat (A_O) is not the same as A_b, and A_b is unknown!"**

- The value of A_b also depends on the location of the pressure tap:

TABLE 8.2
Data on orifice taps

Type of tap	Distance of upstream tap from upstream face of orifice	Distance of downstream tap from downstream face
Flange	1 in. (25 mm)	1 in. (25 mm)
Vena contracta	1 pipe diameter (actual inside)	0.3–0.8 pipe diameter, depending on β
Pipe	$2\frac{1}{2}$ times nominal pipe diameter	8 times nominal pipe diameter

Incompressible fluid in orifice



Orifice with vena contracta tap

$$G = \frac{A_b}{\sqrt{\alpha_b - \frac{A_b^2}{A_a^2}\alpha_a}}\sqrt{2\rho(P_a - P_b)}$$

↓

$$G = \frac{C_D A_O}{\sqrt{1 - \frac{A_O^2}{A_a^2}}}\sqrt{2\rho(P_a - P_b)}$$

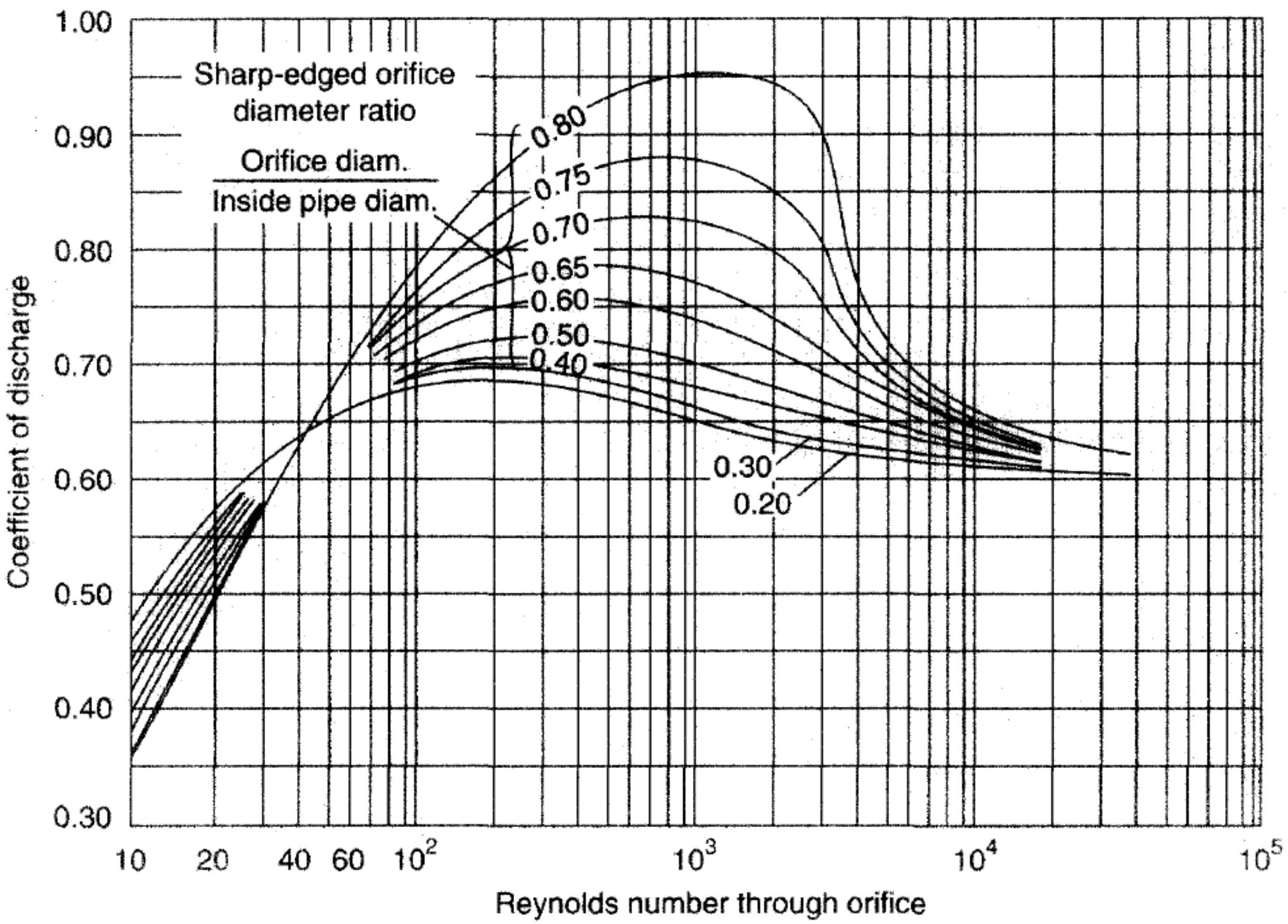
"Discharge coefficient (C_D)" for orifice (determined experimentally)

- To include the effect from kinetic energy correction factors
- To include the upstream friction loss
- To include the difference between A_b and A_O

- Definition of orifice Re: $$Re_O \equiv \frac{D_O \rho v_O}{\mu} = \frac{D_O G}{\mu A_O} = \frac{4G}{\mu \pi D_O}$$

Incompressible fluid in orifice

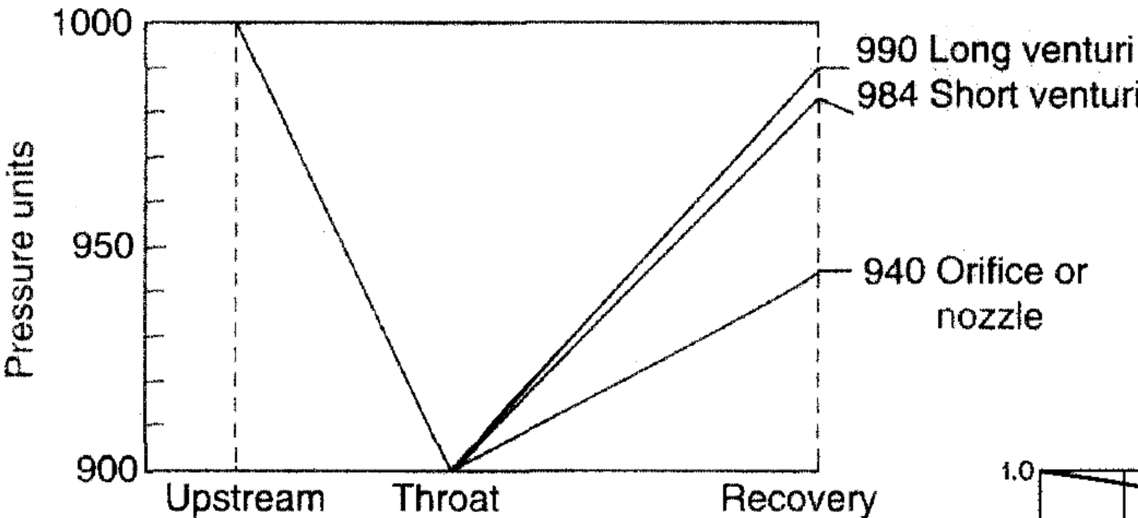
- From experiments, it was found that for both flange taps and vena contracta taps, the value of **C_D for orifice is 0.61** when the Re_O is larger than 30000.



Enough straight pipe must be provided both before and after the orifice.

Downstream friction loss

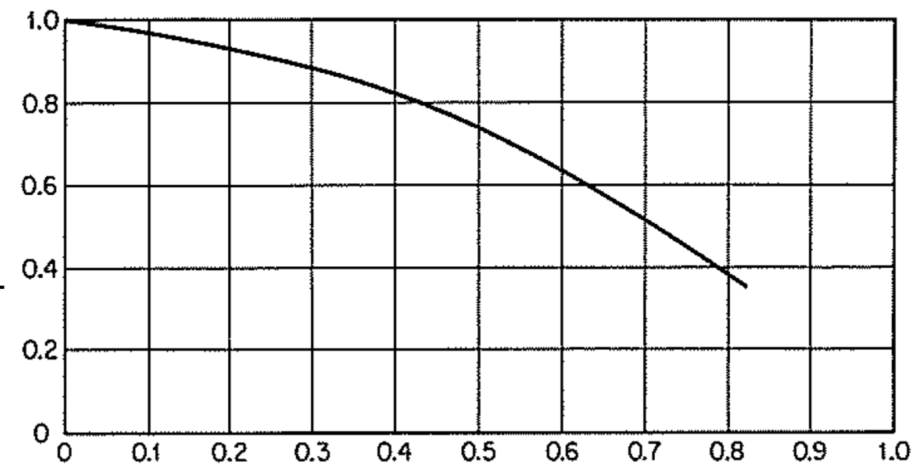
- <u>Orifice</u> and <u>nozzle</u> do not provide gradual increase in flow area after vena contracta, which causes serious downstream friction loss due to the formation of eddies.



P_a P_b P_c

For orifice:

$$\frac{P_a - P_c}{P_a - P_b}$$



$\beta \equiv D_O/D_a$

How about compressible fluid?

Recall for the isentropic process:

$$\int_1^2 \frac{VdP}{g} = \frac{P_1^{\frac{1}{\gamma}} V_1}{\left(1-\frac{1}{\gamma}\right) g} \left(P_2^{1-\frac{1}{\gamma}} - P_1^{1-\frac{1}{\gamma}}\right)$$

$$\begin{cases} \dfrac{P_a^{\frac{1}{\gamma}}}{\rho_a\left(1-\frac{1}{\gamma}\right)} \left(P_b^{1-\frac{1}{\gamma}} - P_a^{1-\frac{1}{\gamma}}\right) + \dfrac{1}{2}(\alpha_b v_b^2 - \alpha_a v_a^2) = 0 \\ A_a v_a = A_b\, v_b \end{cases}$$

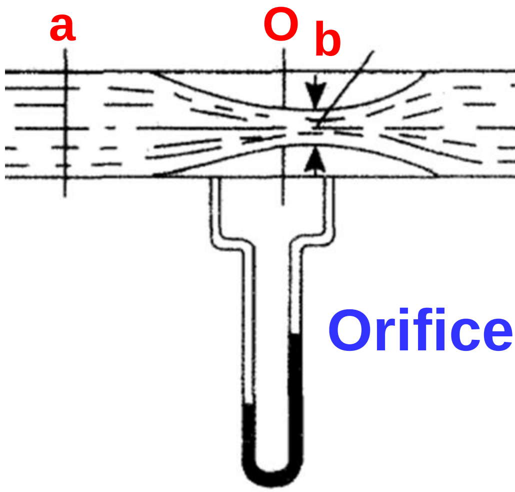
……

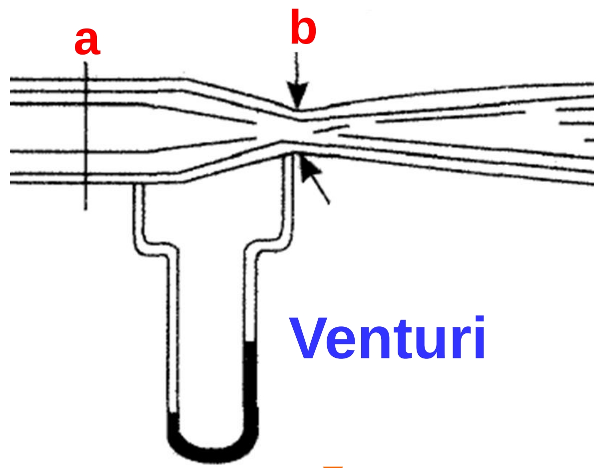
$$G = \frac{Y C_D A_b}{\sqrt{1-\frac{A_b^2}{A_a^2}}} \sqrt{2\rho(P_a - P_b)}$$

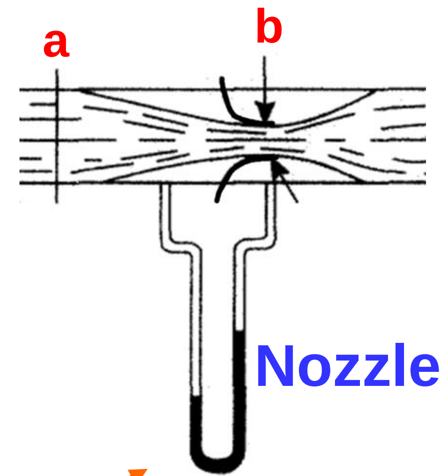
$$Y = \left(\frac{p_b}{p_a}\right)^{1/\gamma} \left\{ \frac{\gamma(1-\beta^4)[1-(p_b/p_a)^{1-1/\gamma}]}{(\gamma-1)(1-p_b/p_a)[1-\beta^4(p_b/p_a)^{2/\gamma}]} \right\}^{1/2}$$

Or the empirical equation for orifice: $Y = 1 - \dfrac{0.41 + 0.35\beta^4}{\gamma}\left(1-\dfrac{p_b}{p_a}\right)$

A quick summary for incompressible fluid







$$\boxed{G = \frac{C_D A_O}{\sqrt{1-\beta^4}}\sqrt{2\rho(P_a - P_b)}}$$

$$\beta \equiv \frac{D_O}{D_a}$$

$$C_D = 0.61$$

$$when\ Re_O > 30000$$

$$\boxed{G = \frac{C_D A_b}{\sqrt{1-\beta^4}}\sqrt{2\rho(P_a - P_b)}}$$

$$\beta \equiv \frac{D_b}{D_a}$$

$$C_D \sim 1$$

Example 8.4

An orifice meter with flange taps is to be installed in a 100-mm line to measure the flow of water. The maximum flow rate is expected to be 50 m^3/h at 288 K. The manometer used to measure the differential pressure is to be filled with mercury (ρ=13600 kg/m^3), and water (ρ=999 kg/m^3) is to fill the leads above the surfaces of the mercury.

(a) If the maximum manometer reading is 1.25 m, what diameter, to the nearest millimeter, should be specified for the orifice?

(b) What will be the power to operate the meter at full load?

Solution: (a) $P_a - P_b = (13600 - 999)\,\mathrm{g}\Delta h = 154300\ (Pa)$

$$\boxed{G = \frac{C_D A_O}{\sqrt{1-\beta^4}}\sqrt{2\rho(P_a - P_b)}}$$

$$\frac{50}{3600} \times 999 = \frac{C_D A_O}{\sqrt{1-\beta^4}}\sqrt{2 \times 999 \times (154300)}$$

$$7.90 \times 10^{-4} = \frac{C_D \pi D_O^2}{4\sqrt{1-\beta^4}}$$

Let's make two assumptions:

$$\begin{cases} \beta^4 \ll 1 \\ Re_O > 30000 \end{cases}$$

$$7.90 \times 10^{-4} = \frac{0.61\pi D_O^2}{4}; \quad D_O = 40.6\ mm$$

Example 8.4

<u>*Let's check the two assumptions*</u>:

$$\beta^{4} = \left(\frac{40.6}{100}\right)^{4} = 0.027 \ll 1$$

$$Re_O = \frac{4G}{\mu\pi D_O} = \frac{4\times 999\times 50}{3600\times \mu\pi D_O} = \frac{4\times 999\times 50}{3600\times(1.15\times 10^{-3})\pi(0.0406)} = 378000 \gg 30000$$

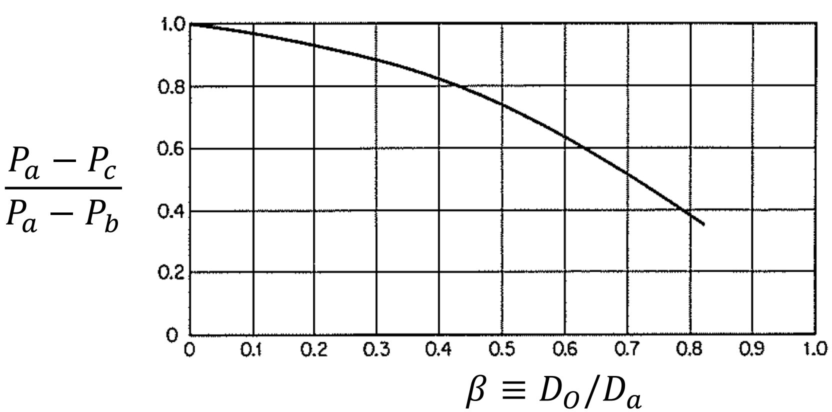
$Answer$: D_O $should\ be\ 41\ mm.$

(b) Recall the original mechanical equation:

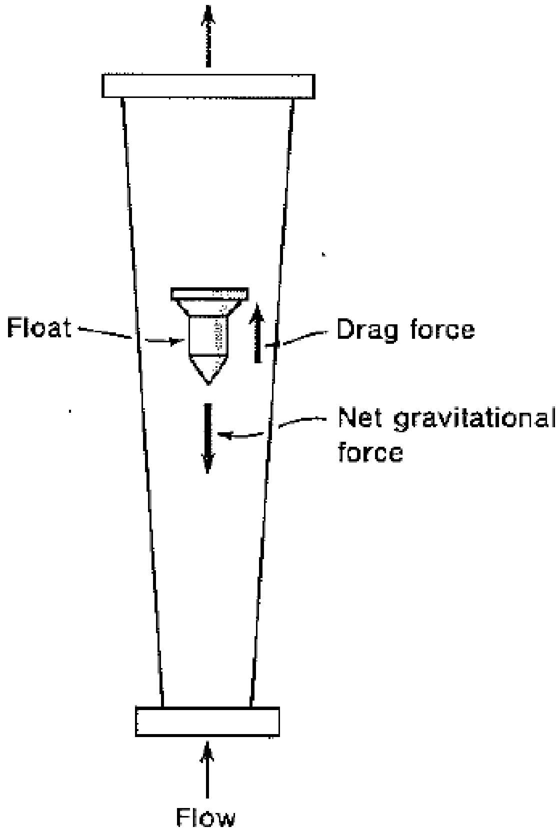
$$\boxed{\rho A v_{avg}\left[F + \Delta\left(\frac{P}{\rho}\right) + \frac{1}{2}\Delta(\alpha v_{avg}^{2}) + g\Delta z\right] = \frac{-W_S\times m}{t}}$$

Power is required to overcome the friction loss of the meter!

$$Power = \rho A v\times F = G\times\frac{P_a - P_c}{\rho} = G\times\frac{0.81(P_a - P_b)}{\rho} = 1736\ (W)$$

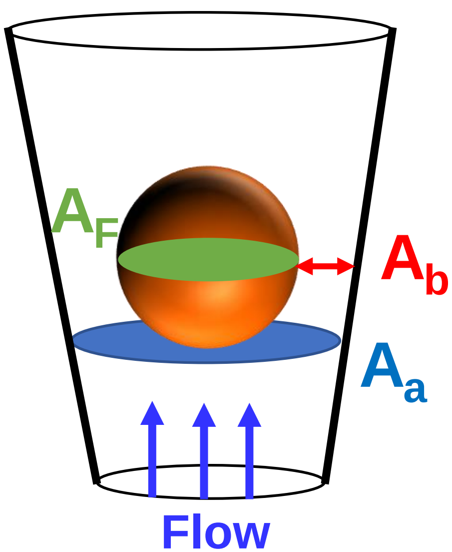


Rotameter

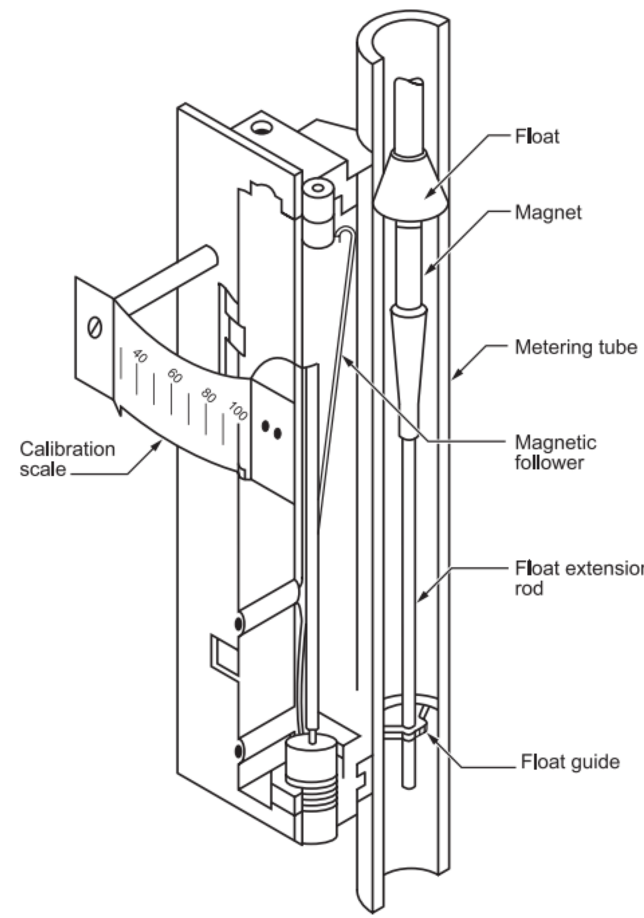


- Rotameters have a nearly linear relationship between flow and position of the float.

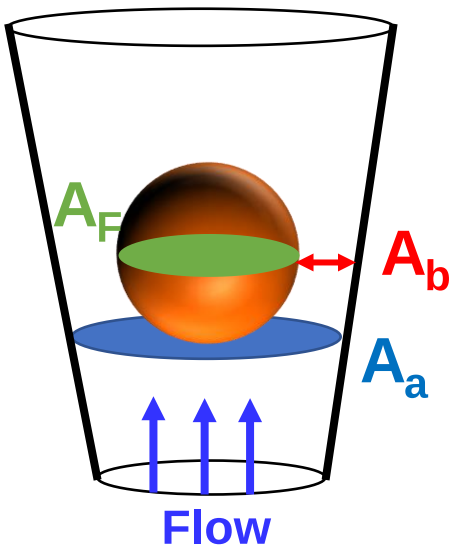
- For high temperature/high pressure conditions or the use with opaque liquids, an *extension rod* is usually used.



$$A_b \approx A_a - A_F$$



Rotameter with incompressible fluid



- Let's use the same equation:

$$G = \frac{C_D A_b}{\sqrt{1 - \frac{A_b^2}{A_a^2}}} \sqrt{2\rho(P_a - P_b)}$$

V_F: volume of the float
ρ_F: density of the float

And: $Drag\ force = (P_a - P_b)A_F$
$= gravity\ force\ - buoyancy\ force$

$$(P_a - P_b)A_F = V_F(\rho_F - \rho)g$$

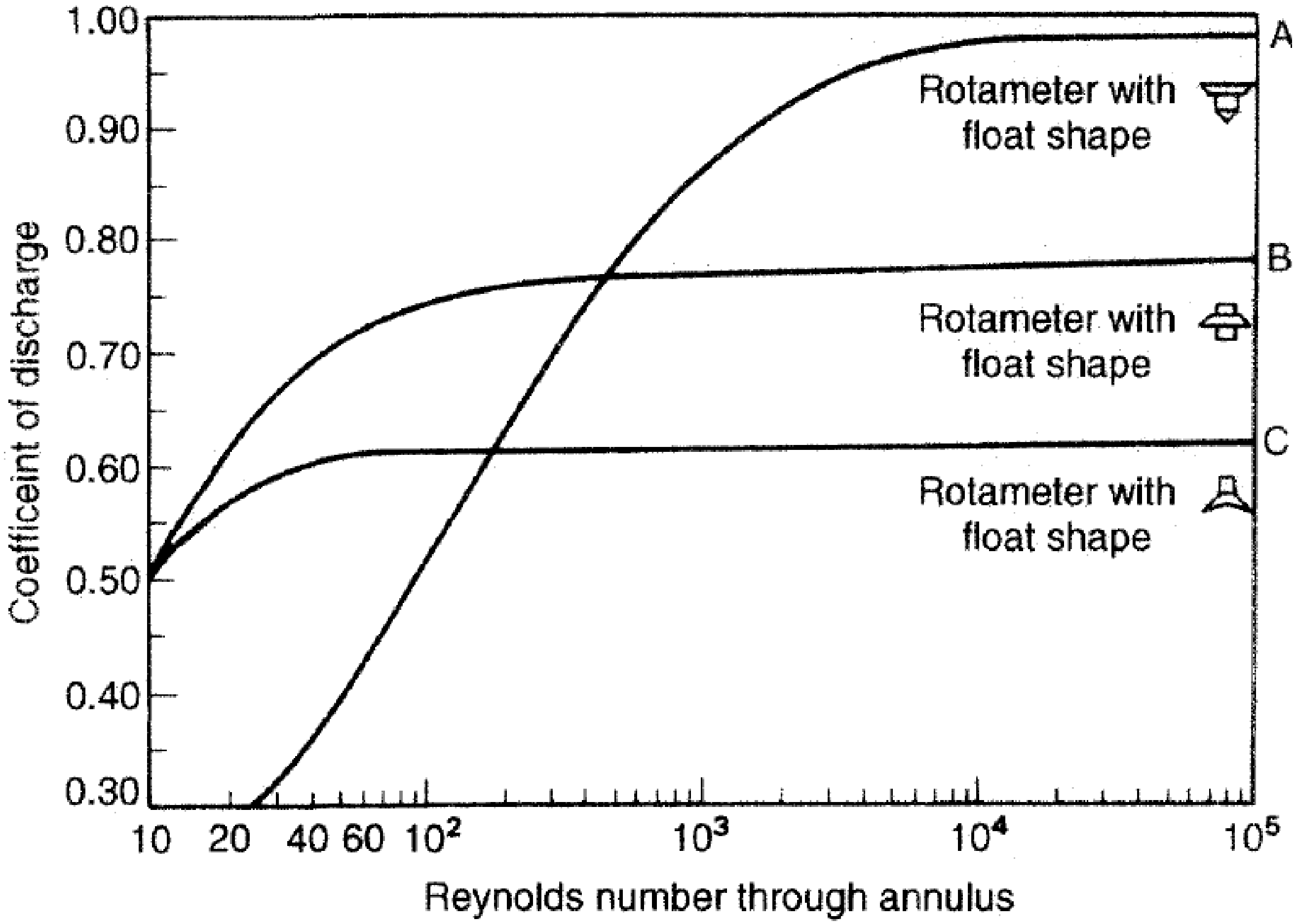
$$G = \frac{C_D A_b}{\sqrt{1 - \frac{A_b^2}{A_a^2}}} \sqrt{2\rho \frac{V_F(\rho_F - \rho)g}{A_F}} = C_D(A_a - A_F)\sqrt{\frac{2\rho V_F(\rho_F - \rho)g}{A_F[1 - \left(\frac{A_a - A_F}{A_a}\right)^2]}}$$

A_F, V_F, ρ_F and ρ are all known!

→ Once $\boldsymbol{C_D}$ is determined, the relationship between $\boldsymbol{A_a}$ and $\boldsymbol{G}$ can be obtained.

Rotameter with incompressible fluid

- C_D can be found on chart:

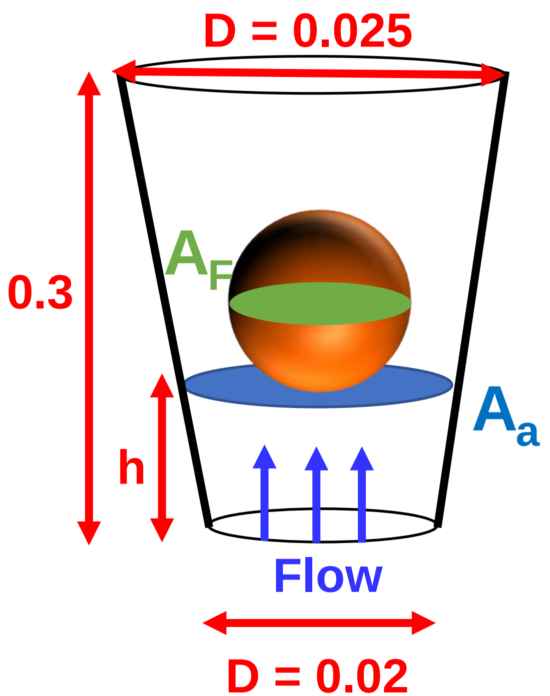


Rotameter - Example

A rotameter has a 0.3 m-long tube which has an internal diameter of 25 mm at the top and 20 mm at the bottom. The diameter the float is 20 mm, its effective specific gravity is 4.80, and its volume is 6.6 cm^3. The coefficient of discharge is 0.72. Find the height of the float when measuring a water flow at 50 cm^3/s.

Solution:

$$A_a = \frac{1}{4}\pi(0.02 + 0.005\frac{h}{0.3})^2 \qquad A_F = \frac{1}{4}\pi(0.02)^2 = 3.14 \times 10^{-4}\ (m^2)$$



$$\boxed{G = C_D(A_a - A_F)\sqrt{\frac{2\rho V_F(\rho_F - \rho)g}{A_F[1 - \left(\frac{A_a - A_F}{A_a}\right)^2]}}}$$

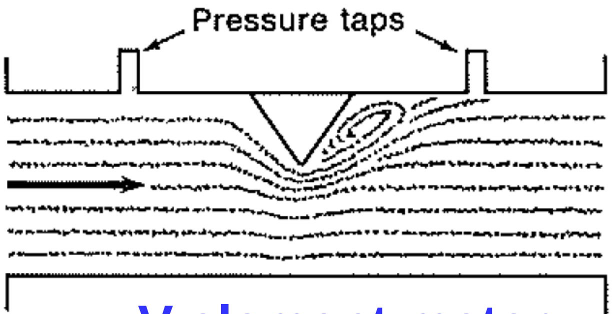
$$50 \times 10^{-6} \times 1000 = 0.72(A_a - A_F)\sqrt{\frac{2000 \times 6.6 \times 10^{-6}(4800 - 1000)g}{A_F\left[1 - \left(\frac{A_a - A_F}{A_a}\right)^2\right]}}$$

....

$$8.12 \times 10^5 A_a^4 - 510A_a^3 + 0.080A_a^2 - 1.57 \times 10^{-6}A_a = -2.47 \times 10^{-10}$$

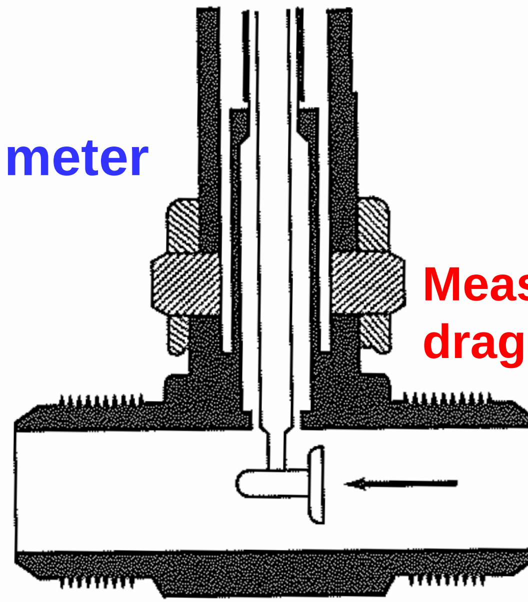
Use "CALC" function for trial and error .... h = 0.102 (m)

Full-bore meters - Others

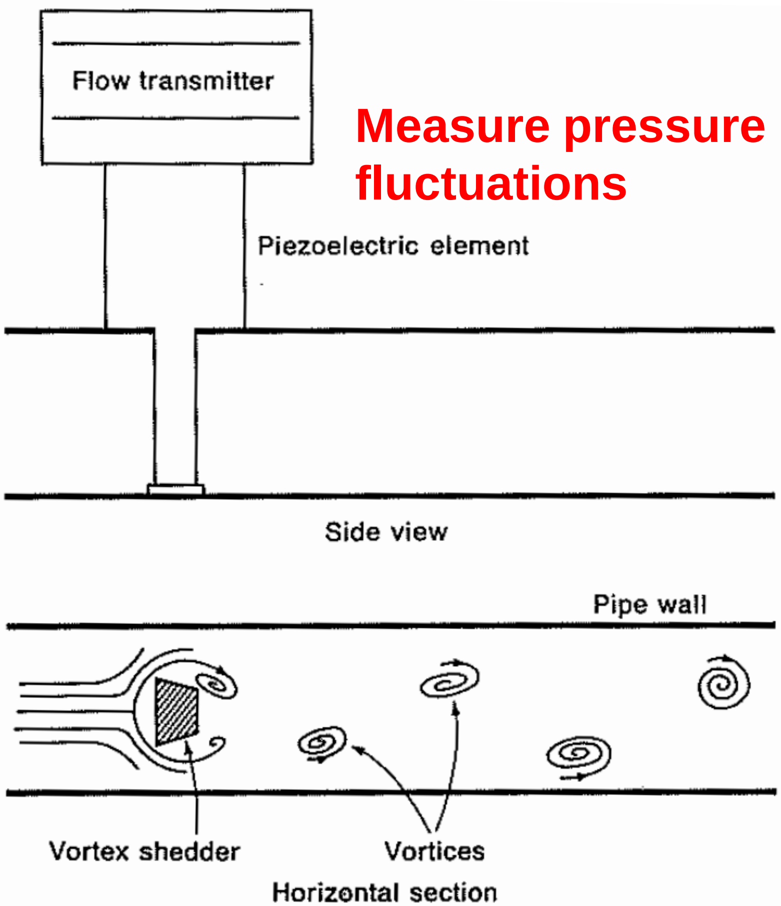


V-element meter

Target meter

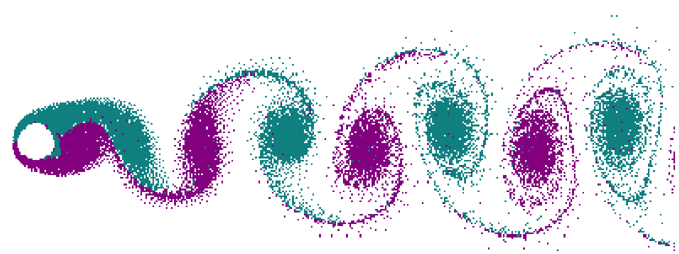


Measure drag force

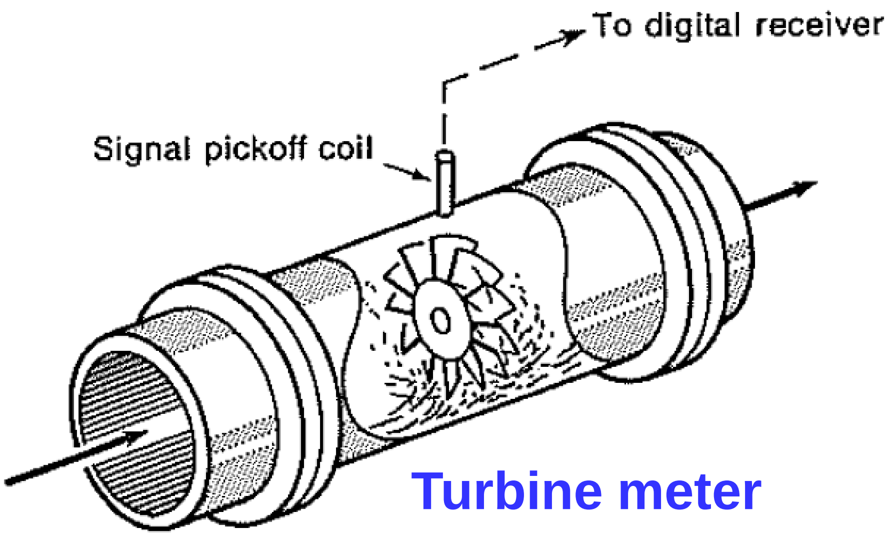


Measure pressure fluctuations

Vortex-shedding meter

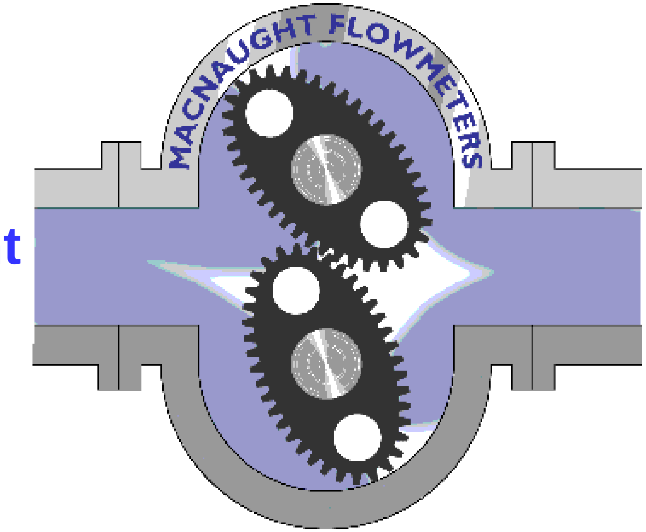


Full-bore meters - Others

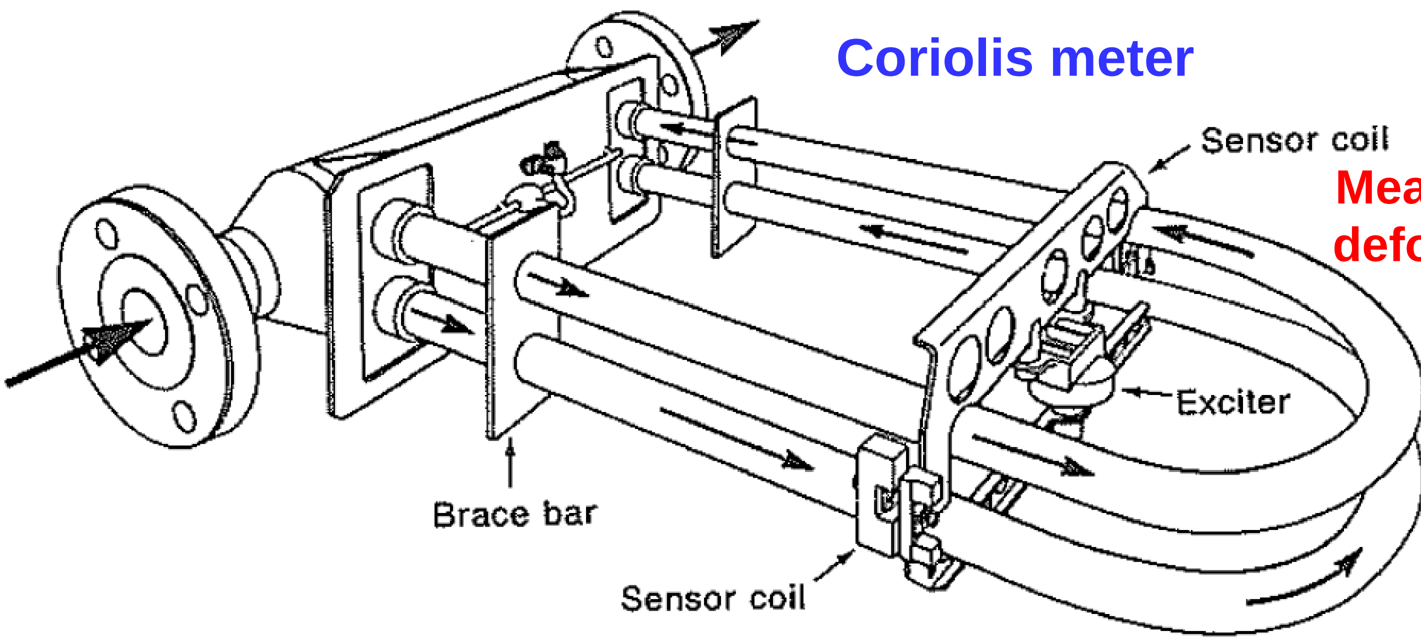


Turbine meter

Positive-displacement meter



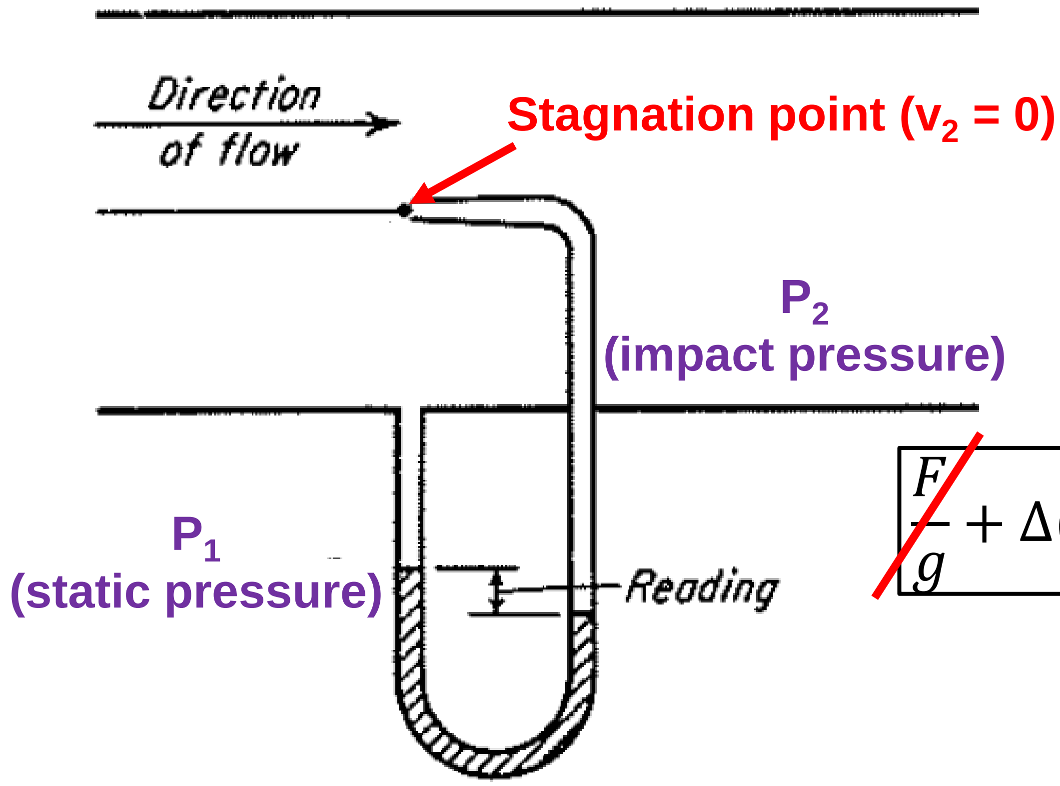
Coriolis meter



Measure the elastic deformations of the tubes

Insertion meters – Pitot tube

- It can only measure the **"velocity"** of the flow at one point in the pipe.



$$\boxed{\cancel{\frac{F}{g}} + \Delta\left(\frac{P}{\rho g}\right) + \frac{1}{2g}\Delta(\alpha v^2) + \cancel{\Delta z} = \cancel{\frac{-W_s}{g}}}$$

- It cannot measure average velocity directly.
- Readings for gases are extremely small.
- Cheap option for large pipes

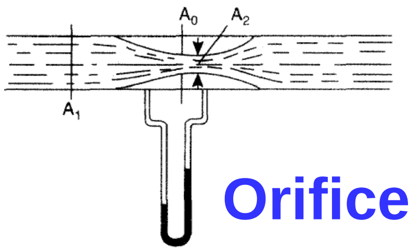
- For incompressible fluid:

$$\frac{P_1 - P_2}{\rho g} + \frac{1}{2g}\alpha v_1^2 = 0$$

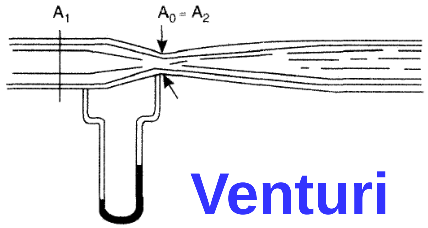
$$\boxed{v_1 = \sqrt{\frac{2(P_2 - P_1)}{\alpha \rho}}}$$

Insertion meters vs. Full-bore meters

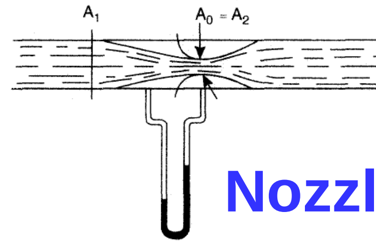
- **<u>Full-Bore Meters</u>**: They can measure the *<u>mass flow rate (G)</u>* directly.



Orifice

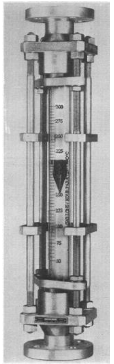


Venturi



Nozzle

Rotameter



- **<u>Pitot tube</u>**: The maximum velocity (v_{max}) in the tube can be measured.

Laminar flow ➡ $v = v_{max}[1 - (\frac{r}{R})^2]$; $v_{avg} = \frac{1}{2} v_{max}$; $G = \rho A v_{avg}$

Turbulent flow ➡ $v = v_{max}(\frac{R-r}{R})^{1/7}$; $v_{avg} = \frac{49}{60} v_{max}$; $G = \rho A v_{avg}$

(if Re~100000)