

國立成功大學				學年度第	學期考試試卷	年	月	日
National Cheng Kung University Examination Sheet for Academic Year:				Semester:		Year	Month	Day
姓名 Name			科目名稱 Subject Name			教師簽章 Signature of Instructor		
學號 Student No.			學院 College	系 Department	年 班 Year Class			
院系 College						評閱成績 Score		

5. cont.

Inverse of Laplace Transform $F(s) = \int_0^{\infty} f(t) e^{-st} dt$

$$f(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} f(t) e^{-i\omega t} dt \right) e^{i\omega t} d\omega \Rightarrow g(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\int_{-\infty}^{\infty} g(t) e^{-i\omega t} dt \right) e^{i\omega t} d\omega$$

$$\frac{1}{2} g(t) = f(t) e^{-\alpha t} H(t) \quad \begin{cases} = 0, & t < 0 \\ = f(t) e^{-\alpha t}, & t > 0 \end{cases}$$

$$\therefore g(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\int_0^{\infty} f(t) e^{-\alpha t} e^{-i\omega t} dt \right) e^{i\omega t} d\omega = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\int_0^{\infty} f(t) e^{-(\alpha + i\omega)t} dt \right) e^{i\omega t} d\omega$$

$$\text{令 } S = \alpha + i\omega \Rightarrow \frac{1}{2\pi i} \int_{-\infty}^{\infty} \left(\int_0^{\infty} f(t) e^{-St} dt \right) e^{i\omega t} d\omega = \frac{1}{2\pi i} \int_{-\infty}^{\infty} F(s) e^{i\omega t} d\omega = f(t) e^{-\alpha t} \cdot H(t)$$

$$S = \alpha + i\omega$$

$$\therefore f(t) \cdot H(t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} F(s) \cdot e^{(\alpha + i\omega)t} d\omega = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} F(s) \cdot e^{st} \cdot \frac{ds}{i}$$

Laplace 轉換 $t > 0$ 時, $H(t) = 1$ when $t > 0$

$$\Rightarrow f(t) = \frac{1}{2\pi i} \int_{\alpha - i\infty}^{\alpha + i\infty} F(s) \cdot e^{st} ds$$