(1) Prandtl number : $Pr = \frac{\nu}{\alpha}$ 表示動黏滯傳播和熱擴散率的比例，也可視為動量傳輸（momentum transport）及熱量傳輸速率的比例.

Reynolds number : $Re = \frac{\rho u L}{\mu} = \frac{uL}{\nu}$，慣性力與黏滯力的比值

Grashof number : $Gr = \frac{g\beta L^3(T_w - T_\infty)}{\nu^2}$，浮力與黏滯力的比值

Nusselt number : $Nu = \frac{hL}{k}$ 說明 →

流體熱對流效應與流體熱傳導效應的比值.

Peclet number : $Pe = \frac{uD}{\alpha}$

註: $Pr = \frac{\nu}{\alpha}$ ← μ/ρ ← $\frac{k}{\rho c}$

$\frac{\delta}{\delta_T}$

δ_T大：表示熱擴散快

δ大：表示速度擴散快（動量傳輸）

助憶方式 $Re = \frac{uD}{\nu}$

$Pe = \frac{uD}{\alpha}$

$Pe = \frac{uD}{\alpha} = \frac{uD}{\nu} \cdot \frac{\nu}{\alpha} = Re \cdot Pr$

(2) Film temperature : $T_f = \frac{T_\infty + T_w}{2}$，在熱邊界層中，流體的溫度由壁溫 T_w 變化到邊界溫度 T_∞，為了適當計算流體的性質，其溫度通常用 film temperature $T_f = \frac{T_w + T_\infty}{2}$ 來計算。

bulk temperature :
(mass-average temperature)
(mixing-cup temperature)

管中流體的能量平均溫度，為整個流動區域積分的質量-能量平均溫度.

$$T_m = \frac{1}{A_c U}\int_{A_c} uT\,dA_c$$

8. D'Alembert's paradox : 物体在靜止或均速流動的不可壓縮、無粘性流體中作等速度運動，它所受到的外力和為零.

II.

(1) Biot number $= \frac{hL}{k_s}$ = $\boxed{\text{固体}}$之傳導熱阻 / 固体表面之對流熱阻

Nusselt number $= \frac{hL}{k_f}$ = $\boxed{\text{流体}}$之傳導熱阻 / 流体之對流熱阻

(2) 因為 $\zeta = \delta_T/\delta < 1$, $\zeta^4 \ll \zeta^2$, neglect ζ^4

(3) liquid metal 之 $Pr \ll 1$, $\delta \ll \delta_T$

water 之 $Pr = 7$. $\delta > \delta_T$

(4) Two or more problems are physically similar if they have
① similar geometric boundary，相似的幾何邊界
② the same control parameters，相同的控制參數，例如. Re, Pr, Nu

(5) $\tau = \mu \frac{\partial u}{\partial y}\Big|_{y=\delta}$，因為 $y=\delta$, $u=U_\infty$, $\frac{\partial u}{\partial y}=0$，所以 shear stress $\tau = 0$

$q'' = -k\frac{\partial T}{\partial y}\Big|_{y=\delta}$ 因為 $y=\delta$, $T=U_\infty$, $\frac{\partial T}{\partial y}=0$，所以 heat flux $= 0$.

(6) 將 density ρ 都視為常數，除了浮力項(buoyancy) $\rho\beta g(T-T_\infty)$ 之外

(7) (a) Reduction the numbers of control parameters 減少控制參數

(b) Establishment of similarity conditions 建立相似關係.

8. 因為在 tube 中流動的流體，其平均溫度無法表示流体的性質，其溫度是流動能量的指標，為整個流動區域積分的質量－能量平均溫度，$T_m = \frac{1}{\rho C A_c u}\int_{A_c} \rho c u T dA_c$

所以使用 T_m.

9.

白努力定律推導

10. $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + \nu\frac{\partial^2 u}{\partial y}$, 必須在 $y=0$, $u=0$, $v=0$, 且 $P=$constant 的情況下，才能推導出 $y=0$. $\frac{\partial^2 u}{\partial y^2}=0$, 若 $v\neq 0$ 或 P 不是定壓，則 $\frac{\partial^2 u}{\partial y^2}=0$ 不成立.

11. $\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$, $\frac{\partial^2 v}{\partial x^2} \ll \frac{\partial^2 v}{\partial y^2}$, $\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$, 所以 $\nu\frac{\partial^2 u}{\partial x^2}$, $\nu\frac{\partial^2 v}{\partial x^2}$, $k\frac{\partial^2 T}{\partial x^2}$ 可省略.

12. ① 真空 ② 速度 $v=0$ ③ 無溫度梯度 ④ 溫度梯度與速度方向垂直

III 考古題

IV 考古題

1. 考古題

2. 令 $\theta = T - T_w$, $\theta_\infty = T_\infty - T_w$, $\theta^* = \frac{\theta}{\theta_\infty} = \frac{T-T_w}{T_\infty - T_w}$, $y^* = \frac{y}{\delta_t}$

則 $\frac{T-T_w}{T_\infty - T_w} = a + b\,(y/\delta_T)$

$\Rightarrow \quad \theta^* = a + b y^*$

boundary: $\begin{cases} y=0,\ T=T_w \\ y=\delta_T,\ T=T_\infty,\ \frac{\partial T}{\partial y}=0 \end{cases} \Rightarrow \begin{cases} y^*=0,\ \theta^* = \frac{T_w - T_w}{T_\infty - T_w} = 0 \\ y^*=1,\ \theta^*=1,\ \frac{\partial \theta^*}{\partial y^*}=0 \end{cases}$

代入得 $\begin{cases} a=0 \\ 1=a+b,\ b=1 \end{cases}$

$\therefore \quad \theta^* = y^*$ ⇓

$$\frac{d}{dx}\int_0^{\delta_T} U_\infty (T_\infty - T)\, dy = \alpha \frac{\partial T}{\partial y}\Big|_w \quad , \quad \left[\begin{array}{l} T_\infty - T = (T_\infty - T_w) - (T - T_w) = \theta_\infty - \theta \\ \theta^* = \frac{T - T_w}{T_\infty - T_w} \Rightarrow T = (T_\infty - T_w)\theta^* + T_w \end{array} \right] \text{代入}$$

$$\Rightarrow \frac{d}{dx}\int_0^{\delta_T} U_\infty (\theta_\infty - \theta)\, d(y^*\delta_T) = \alpha \frac{\partial [(T_\infty - T_w)\theta^* + T_w]}{\partial (y^*\delta_T)}\Big|_{y^*=0}$$

$$\Rightarrow \frac{d}{dx}\int_0^{1} U_\infty \theta_\infty \left(1 - \frac{\theta}{\theta_\infty}\right)\delta_T\, dy^* = \alpha \left(\frac{T_\infty - T_w}{\delta_T}\right)\frac{\partial \theta^*}{\partial y^*}\Big|_{y^*=0} \qquad \left[\theta^* = y^* \Rightarrow \frac{\partial \theta^*}{\partial y^*} = 1 \right.$$

$$U_\infty \theta_\infty \frac{d}{dx}\int_0^1 \delta_T (1 - \theta^*)\, dy^* = \frac{\alpha (T_\infty - T_w)}{\delta_T} \times 1$$

$$U_\infty \cancel{\theta_\infty} \frac{d}{dx}\left[\int_0^1 \delta_T (1 - y^*)\, dy^*\right] = \frac{\alpha \cdot \cancel{\theta_\infty}}{\delta_T}$$

$$U_\infty \frac{d}{dx}\left[\delta_T \left(y^* - \frac{1}{2}y^{*2}\right)\Big|_0^1\right] = \frac{\alpha}{\delta_T}$$

$$U_\infty \frac{d}{dx}\left[\delta_T \cdot \frac{1}{2}\right] = \frac{\alpha}{\delta_T}$$

$$2\delta_T\, d\delta_T = \frac{4\alpha}{U_\infty}$$

$$\delta_T^2 = \frac{4\alpha}{U_\infty}x + C$$

又 at $x=0$, $\delta_T = 0 \Rightarrow C = 0$

$$\therefore \delta_T^2 = \frac{4\alpha}{U_\infty}x$$

$$\delta_T = 2\sqrt{\frac{\alpha x}{U_\infty}}$$

$$\frac{\delta_T}{x} = 2\sqrt{\frac{\alpha}{U_\infty x}}$$

(3)

$$q'' = h(T_w - T_\infty) = -k\frac{\partial T}{\partial y}\Big|_{y=0} \quad , \quad \frac{\partial T}{\partial y}\Big|_{y=0} = \frac{\partial [(T_\infty - T_w)\theta^* + T_w]}{\partial (y^*\delta_T)}\Big|_{y^*=0} = \frac{T_\infty - T_w}{\delta_T}\frac{\partial \theta^*}{\partial y^*} = \frac{T_\infty - T_w}{\delta_T}$$

$$\therefore h = \frac{-k}{T_w - T_\infty}\frac{\partial T}{\partial y}\Big|_{y=0} = \frac{-k}{T_w - T_\infty} \times \frac{T_\infty - T_w}{\delta_T} = \frac{k}{\delta_T}$$

$$Nu_x = \frac{hx}{k} = \frac{x}{\delta_T} = \frac{1}{2}\sqrt{\frac{U_\infty x}{\alpha}}$$

V. 考古題

VI. $T_f = \frac{20+40}{2} = 30°C = 303k$

$$Ra_H = \frac{g\beta}{\alpha\nu}H^3(T_w - T_\infty)$$

$$= 90.7 \times (50)^3 \times (40-20)$$

$$= 2.27 \times 10^8$$

$$Nu = \frac{hH}{k} = \frac{h\,(0.5m)}{0.026\, W/m°C} = 0.517 \times (2.27 \times 10^8)^{\frac{1}{4}}$$

$$\Rightarrow \quad h = 7.6$$

$$q'' = h(T_w - T_\infty)$$
$$= h(40-20)$$
$$= h \cdot 20$$
$$= 7.6 \times 20$$
$$= 152$$

$$\dot{q} = A\, q''$$
$$= 0.5 \times 0.65 \times \cancel{152}\ 152 =$$

VII. 考古题

VIII

(1) 因為在自然对流中，密度 ρ 是温度的函数，随温度而变动，要解其方程式很難。

(2) since u_x is unknown, and assume $u_x \propto \delta^2$ 所以可以由 (92) → (91)