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Fundamentals of Momentum, Heat, and Mass Transfer

Sixth Edition

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Chapter 6

Conservation of Energy: Control-Volume Approach

The first law of thermodynamics

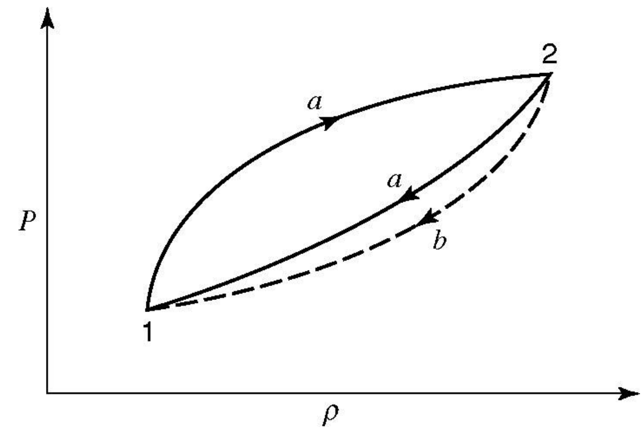
If a system is carried through a cycle, the total heat added to the system from its surroundings is proportional to the work done by the system on its surroundings.

$$\oint \delta Q = \frac{1}{J} \oint \delta W \qquad (6\text{-}1)$$

Differential heat transfer

Differential work done

$$\int_{1a}^{2} \delta Q + \int_{2a}^{1} \delta Q = \int_{1a}^{2} \delta W + \int_{2a}^{1} \delta W \tag{6-2a}$$



$$\int_{1a}^{2} \delta Q + \int_{2b}^{1} \delta Q = \int_{1a}^{2} \delta W + \int_{2b}^{1} \delta W \tag{6-2b}$$

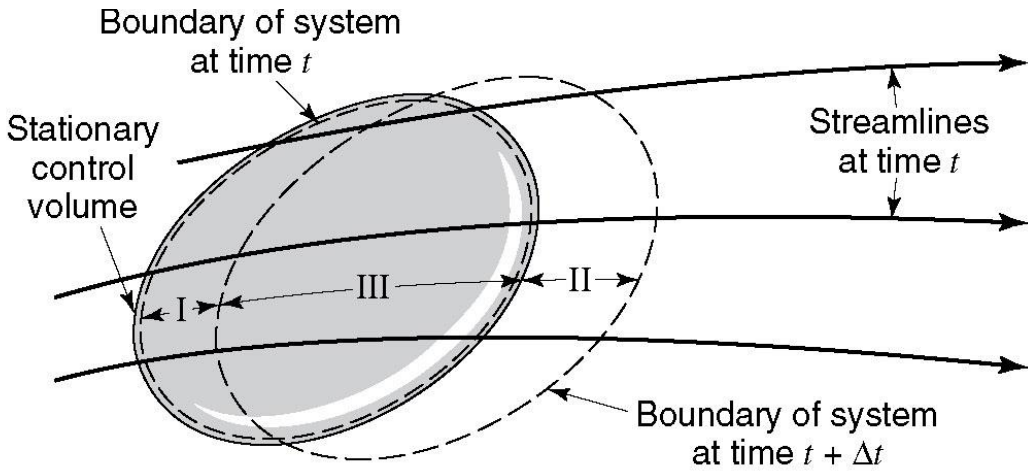
$$\int_{2a}^{1} (\delta Q - \delta W) = \int_{2b}^{1} (\delta Q - \delta W) \tag{6-3}$$

A point function

$$\delta Q - \delta W = dE \tag{6-4}$$

Total energy of the system

$$\boxed{\frac{\delta Q}{dt} - \frac{\delta W}{dt}} = \boxed{\frac{\delta E}{dt}} \tag{6-5}$$



$$\boxed{\lim_{\Delta t \to 0} \frac{E|_{t+\Delta t} - E|_t}{\Delta t}} = \lim_{\Delta t \to 0} \frac{E_{\mathrm{III}}|_{t+\Delta t} - E_{\mathrm{III}}|_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{E_{\mathrm{II}}|_{t+\Delta t} - E_{\mathrm{I}}|_t}{\Delta t} \tag{6-6}$$

Rate of change of the total energy

Net rate of energy leaving across the control volume in the time interval

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$$\left\{\begin{matrix}\text{rate of addition}\\ \text{of heat to control}\\ \text{volume from}\\ \text{its surroundings}\end{matrix}\right\} - \left\{\begin{matrix}\text{rate of work done}\\ \text{by control volume}\\ \text{on its surroundings}\end{matrix}\right\} = \left\{\begin{matrix}\text{rate of energy}\\ \text{out of control}\\ \text{volume due to}\\ \text{fluid flow}\end{matrix}\right\}$$

$$- \left\{\begin{matrix}\text{rate of energy into}\\ \text{control volume due}\\ \text{to fluid flow}\end{matrix}\right\} + \left\{\begin{matrix}\text{rate of accumulation}\\ \text{of energy within}\\ \text{control volume}\end{matrix}\right\} \tag{6-7}$$

$$\lim_{\Delta t \to 0} \frac{E|_{t+\Delta t} - E|_t}{\Delta t} = \lim_{\Delta t \to 0} \frac{E_{\mathrm{III}}|_{t+\Delta t} - E_{\mathrm{III}}|_t}{\Delta t} + \lim_{\Delta t \to 0} \frac{E_{\mathrm{II}}|_{t+\Delta t} - E_{\mathrm{I}}|_t}{\Delta t} \tag{6-6}$$

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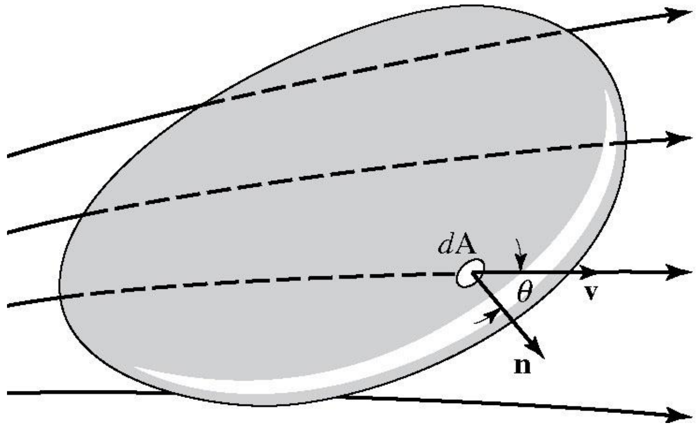

The **rate of energy leaving** the control volume through *dA*

$e(\rho\upsilon)(dA\cos\theta)$

e: specific energy

$(\rho\upsilon)(dA\cos\theta)$: the rate of mass efflux from the control volume

$$\frac{\delta Q}{dt} - \frac{\delta W}{dt} = \boxed{\iint_{\text{c.s.}} e\rho(\mathbf{v}\cdot\mathbf{n})\,dA} + \boxed{\frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dV} \qquad (6\text{-}8)$$

Net efflux of energy

Rate of accumulation of energy

$$\frac{\delta Q}{dt}\boxed{-\frac{\delta W_s}{dt}}+\boxed{\iint_{\text{c.s.}} \mathbf{v}\cdot\mathbf{S}\,dA}=\iint_{\text{c.s.}} e\rho(\mathbf{v}\cdot\mathbf{n})\,dA+\frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dv \qquad (6\text{-}9)$$

Rate of shaft work

Rate of Flow and shear work

W_s: Shaft work, which is that done by the control volume on its surroundings
It could cause a shaft to rotate or to raise a weight

W_σ: Flow work, which is that done on the surroundings to overcome normal stresses on the control surface

W_τ: Shear work, which is performed on the surroundings to overcome shear stresses on the control surface

S: is the force intensity (stress)

The rate of work done by the fluid flowing throught dA is ***S dA ·v***

Eq_6-9

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$$\frac{\delta Q}{dt} - \frac{\delta W_s}{dt} = \iint_{\text{c.s.}} \left(e + \boxed{\frac{P}{\rho}}\right)\rho(\mathbf{v}\cdot\mathbf{n})\,dA + \frac{\partial}{\partial t}\iiint_{\text{c.v.}} e\rho\,dv + \boxed{\frac{\delta W_\mu}{dt}} \qquad (6\text{-}10)$$

Rate of Flow and shear work

Rate of work accomplished in overcoming viscous effects at the control surface

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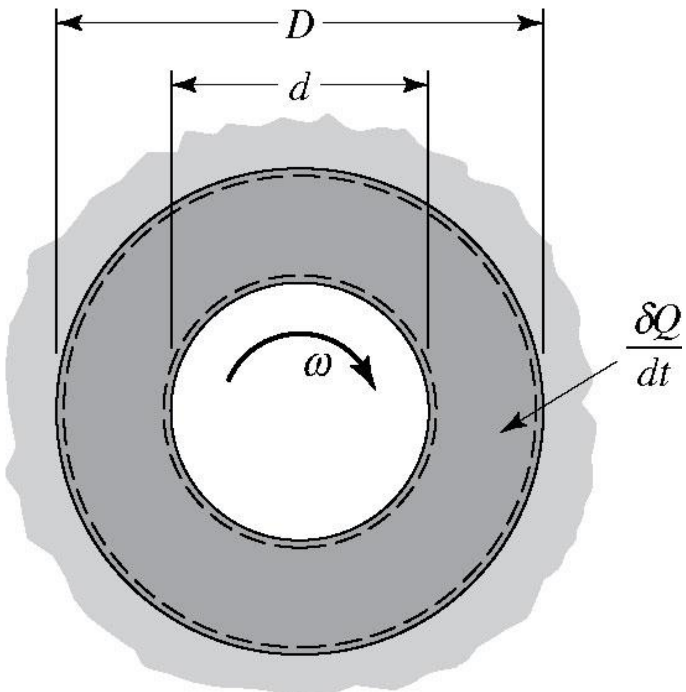

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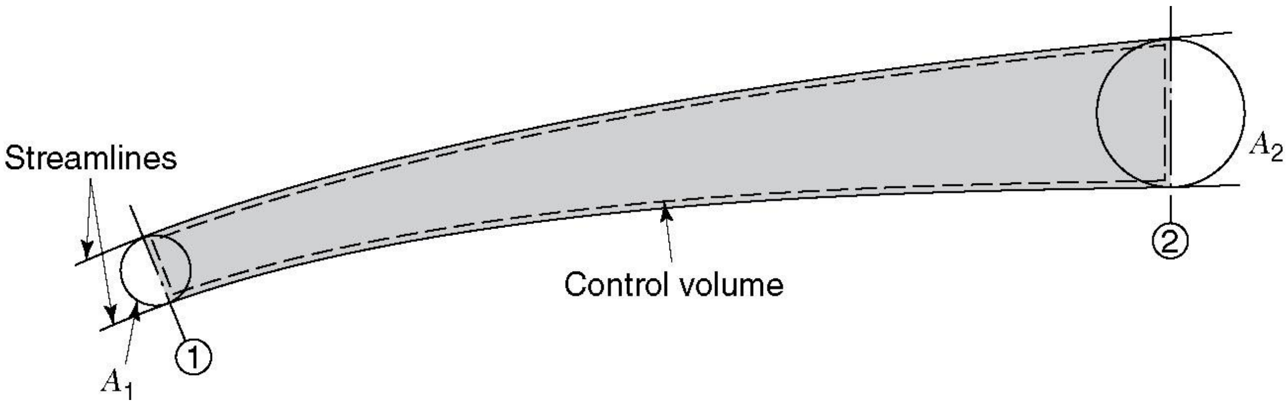

Bernoulli Equation

$$gy_1 + \frac{v_1^2}{2} + \frac{P_1}{\rho} = gy_2 + \frac{v_2^2}{2} + \frac{P_2}{\rho} \qquad \text{(6-11a)}$$

$$y_1 + \frac{v_1^2}{2g} + \frac{P_1}{\rho g} = y_2 + \frac{v_2^2}{2g} + \frac{P_2}{\rho g} \qquad \text{(6-11b)}$$

$$\frac{v_1^2}{2} + gy_1 + u_1 + \frac{P_1}{\rho} = \frac{v_2^2}{2} + gy_2 + u_2 + \frac{P_2}{\rho} \qquad \text{(6-14)}$$

$$\frac{P_1 - P_2}{\rho} = v_2^2 - v_1^2\left(\frac{A_1}{A_2}\right) \qquad \text{(6-13)}$$

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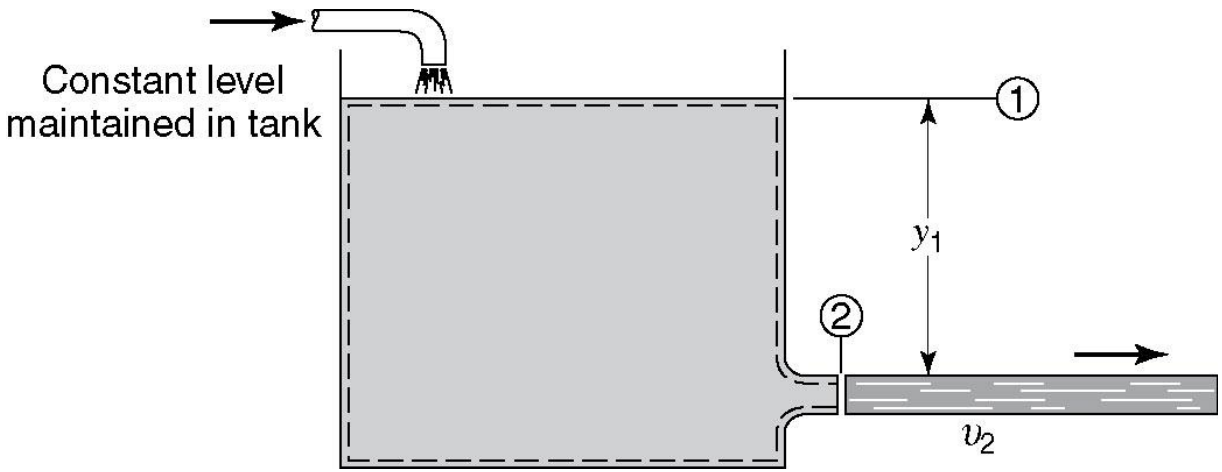

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