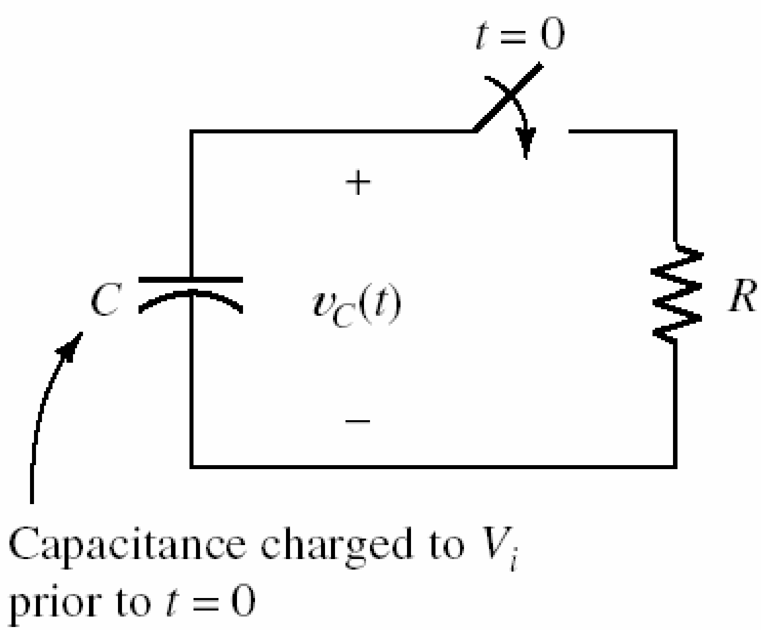
Chapter 4 Transients (暫態)

- 藉由 switch 開關突然對電路提供source 所造成的隨時間改變(時變, time-varying) 的電流或電壓稱為暫態(**transients**).
- 其電路方程式為積分微分方程式 (integrodifferential equations).

4.1 1st-Order RC Circuits

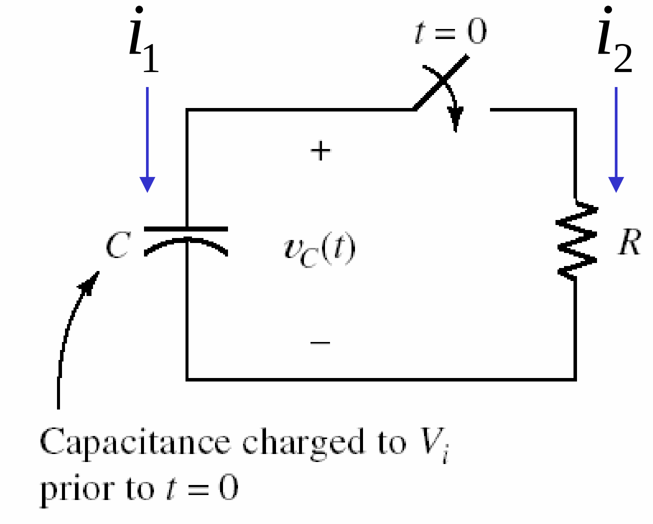
•Discharge of a Capacitance through a Resistance

•At $t<0$, the capacitor is charged to an initial voltage V_i.
•Switch closes at $t=0$, current flows through the resistor, discharging the capacitor.
•Find $v_C(t)$



Discharge of a Capacitance through a Resistance

1. $t<0$, $v_C(t) = V_i$.



2. $t = 0_+$, *Switch just closes*

電容電壓continuous(不可有瞬間的變化)。

$v_C(0_+) = V_i$.

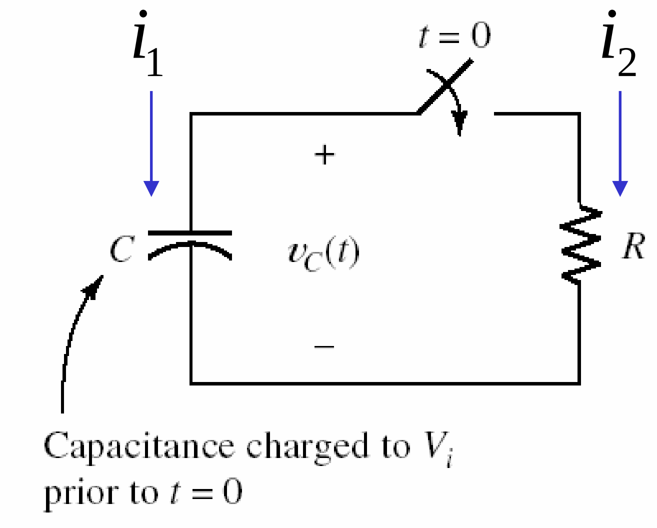
3. $t > 0$

KCL

$$i_1 + i_2 = C\frac{dv_C(t)}{dt} + \frac{v_C(t)}{R} = 0$$

$$\Rightarrow RC\frac{dv_C(t)}{dt} + v_C(t) = 0 \quad \text{(等號兩邊乘}R\text{)}$$

Recall $$\frac{de^t}{dt} = e^t$$



⇒ 預期 $$v_C(t) = Ke^{st}$$

代入 $$RC\frac{dv_C(t)}{dt} + v_C(t) = 0$$

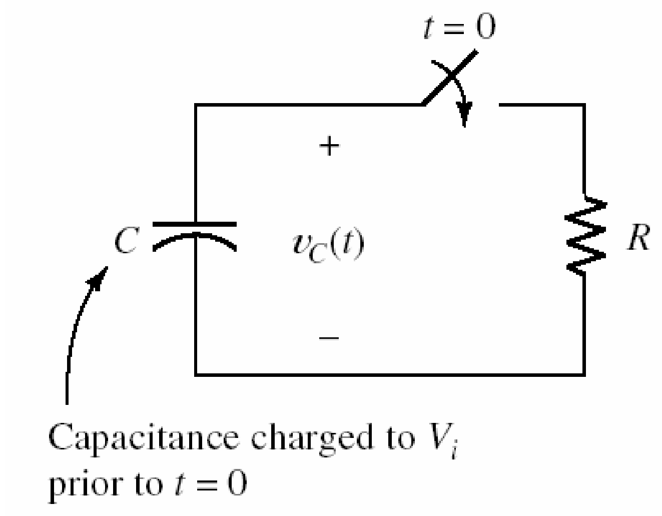
⇒

0

$$RCKse^{st} + Ke^{st} = K(RCs+1)e^{st} = 0$$

⇒ $$s = \frac{-1}{RC}$$ ⇒ $$v_C(t) = Ke^{-t/RC}$$

$\because \quad v_C(0_+) = V_i.$



$$v_c(0+) = V_i = Ke^0 = K$$

$$v_c(t) = V_i e^{-t/RC}$$

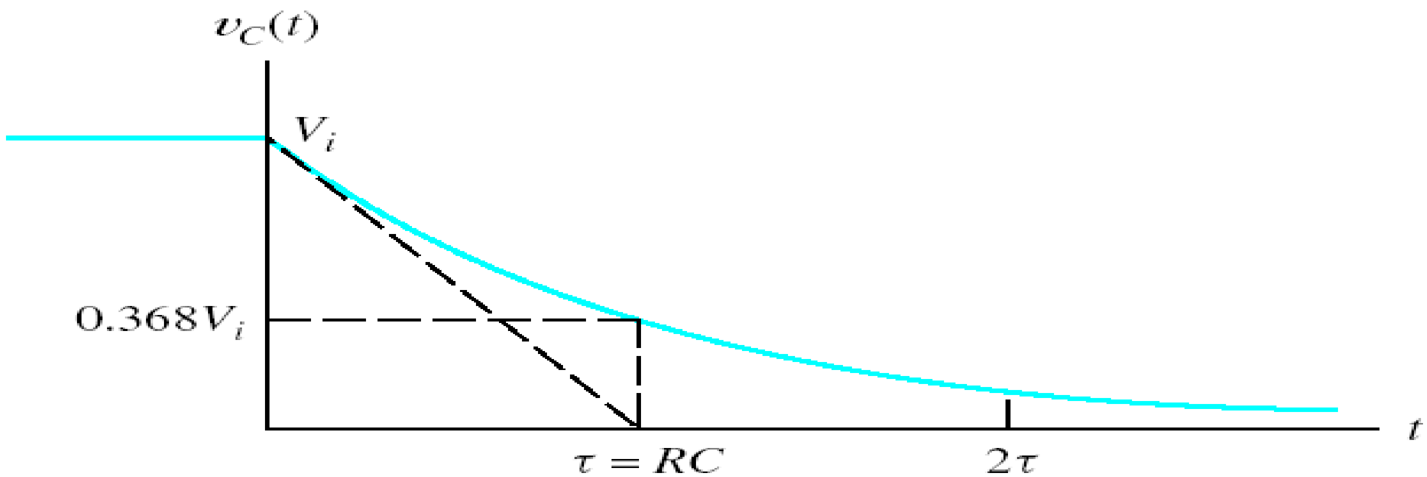


Figure 4.2 Voltage versus time for the circuit of Figure 4.1(a). When the switch is closed, the voltage across the capacitor decays exponentially to zero. At one time constant, the voltage is equal to 36.8 percent of its initial value.

Time constant (時間常數) $\tau = RC$

At $t = \tau = RC$

$$v_c(t) = v_c(\tau) = V_i e^{-\tau/\tau} = V_i e^{-1} \cong 0.368 V_i$$

• RC 電路電容放電至初始電壓36.8% 所需時間

At $t = 5\tau$, $\quad e^{-5\tau/\tau} \cong 0.0067 V_i \cong 0$

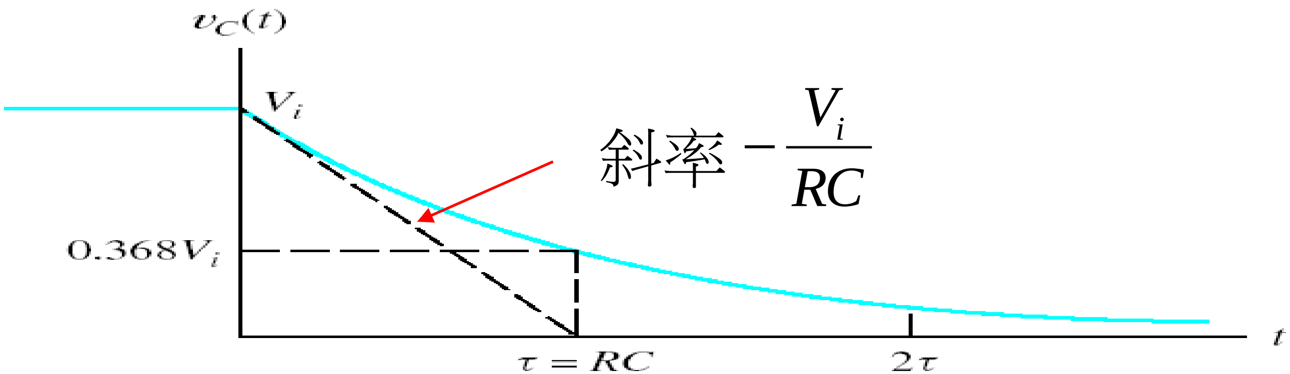


Figure 4.2 Voltage versus time for the circuit of Figure 4.1(a). When the switch is closed, the voltage across the capacitor decays exponentially to zero. At one time constant, the voltage is equal to 36.8 percent of its initial value.

Charge a Capacitance from a DC source through a Resistance

- Source voltage V*s* is constant (a dc source).
- Assume $v_C(0_-)=0$.
- Switch closes at t=0, current flows through the resistor, charging the capacitor.
- Find $v_C(t)$

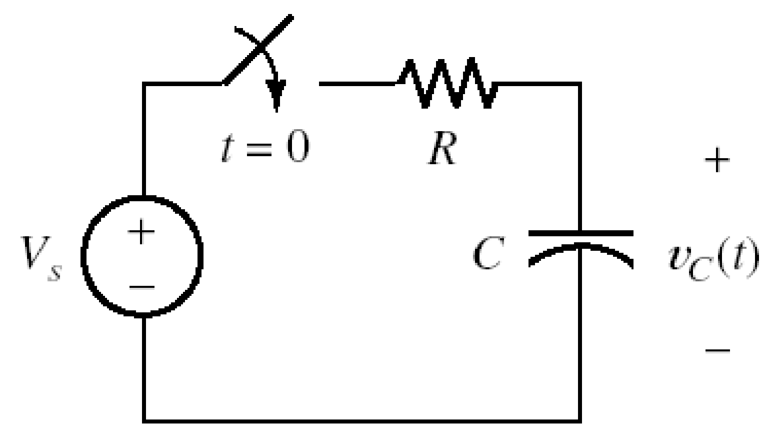


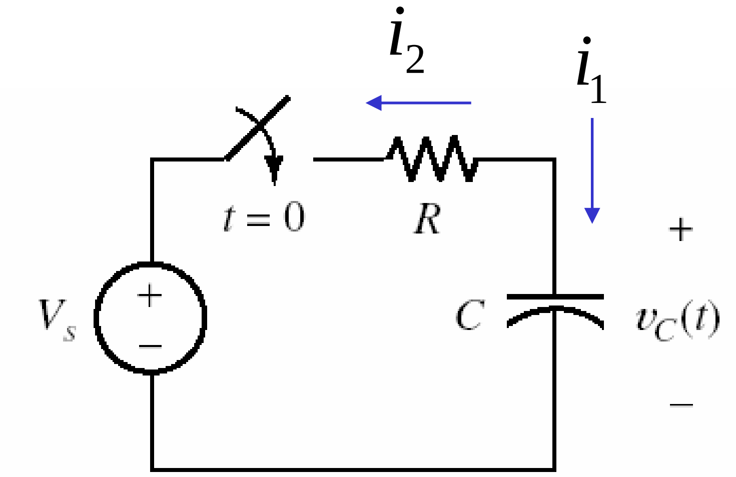
Figure 4.3 Capacitance charging through a resistance. The switch closes at $t = 0$, connecting the dc source V_s to the circuit.

Charge a Capacitance from a DC source through a Resistance

1. $t = 0_+$, *Switch just closes*

電容電壓continuous(不可有瞬間的變化)

$$v_c(0+) = v_c(0-) = 0$$



2. $t > 0$

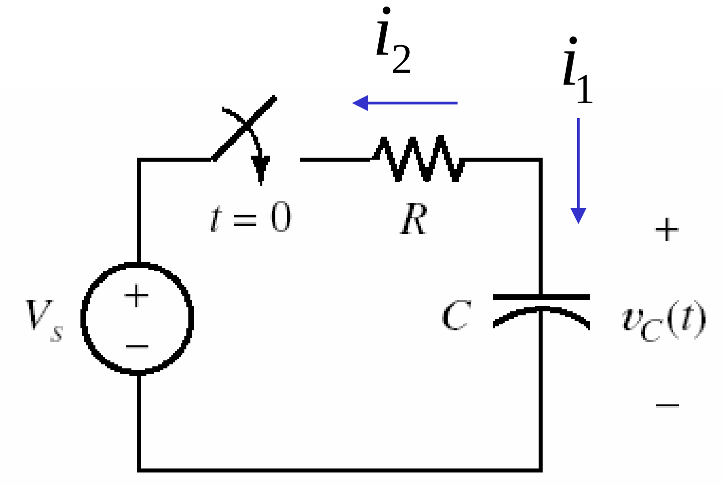
KCL $$i_1 + i_2 = C\frac{dv_c(t)}{dt} + \frac{v_c(t) - V_s}{R} = 0$$

$$\Rightarrow RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

(等號兩邊乘R)

Charge a Capacitance from a DC source through a Resistance

多了一項V_s



預期 $v_c(t) = K_1 + K_2 e^{st}$

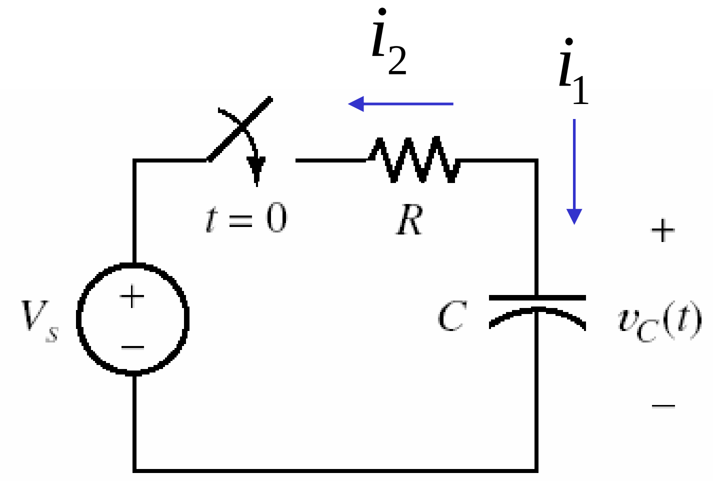
代入

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$

$$RCsK_2e^{st} + (K_1 + K_2e^{st}) = V_s \Rightarrow \underbrace{(1+RCs)}_{0}K_2e^{st} + K_1 = V_s$$

$$\begin{cases} K_1 = V_s \\ s = \dfrac{-1}{RC} \end{cases} \Rightarrow v_c(t) = V_s + K_2 e^{-t/RC}$$

$\because \quad v_c(0+) = v_c(0-) = 0$



$$\Rightarrow \quad v_c(0+) = 0 = V_s + K_2 e^0 = V_s + K_2$$

$$\Rightarrow \quad v_c(t) = V_s - V_s e^{-t/RC} = V_s - V_s e^{-t/\tau}$$

Steady-state response or forced response (穩態響應)

Transient response (暫態響應)

At $t = \tau = RC$

$$v_c(t) = v_c(\tau) \cong 0.632 V_s$$

•*RC* 電路電容充電至dc電壓63.2% 所需時間

At $t = 5\tau$,

$$v_c(t) = v_c(5\tau) \cong V_s$$

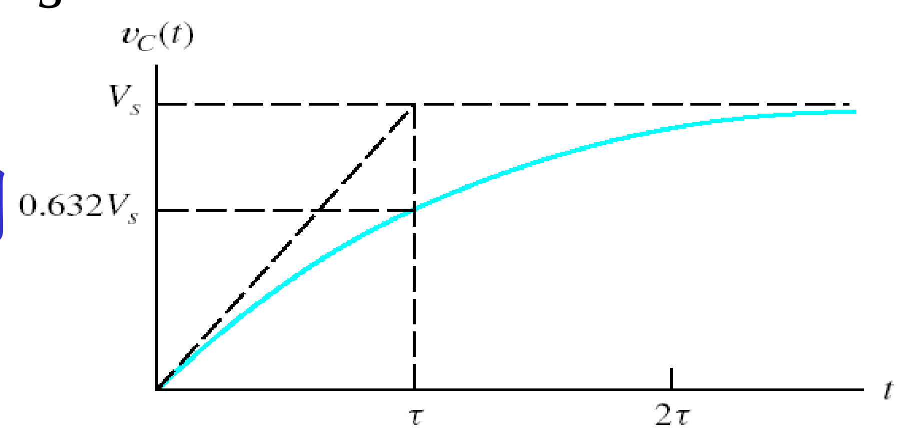


Figure 4.4 The charging transient for the *RC* circuit of Figure 4.3.

由於電容充放電需要數個time constant的時間(ex 5τ)，使得RC transients (RC 暫態)成為限制數位系統反應速度的主要原因。

4.2 DC STEADY STATE (DC 穩態)

- 在dc source 下，RLC 電路的暫態項(transient terms) 會隨著時間而減低至0。
- 因此在穩態(steady-state)時，流過電容的電流為0，電容可視為斷路(open circuits)。

$$i_c(t) = C\frac{dv_c(t)}{dt} \longrightarrow 0$$

- 在穩態(steady-state)時，電感兩端的電壓為0，電感可視為短路(short circuits)。

$$v_L(t) = L\frac{di_L(t)}{dt} \longrightarrow 0$$

4.2 DC STEADY STATE (DC 穩態)

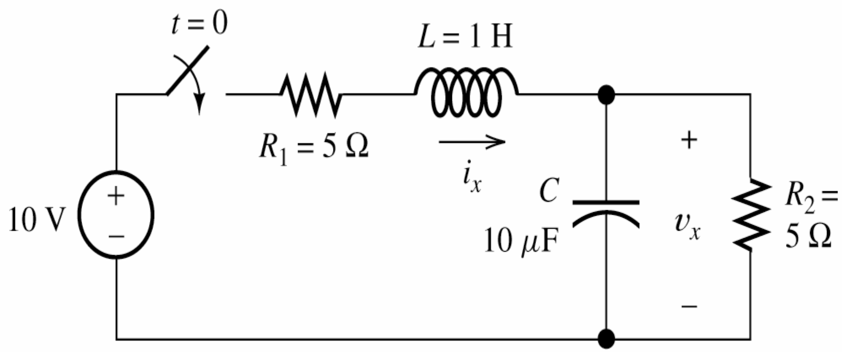
The steps in determining the forced response for *RLC* circuits with dc sources are:

1. Replace capacitances with open circuits.

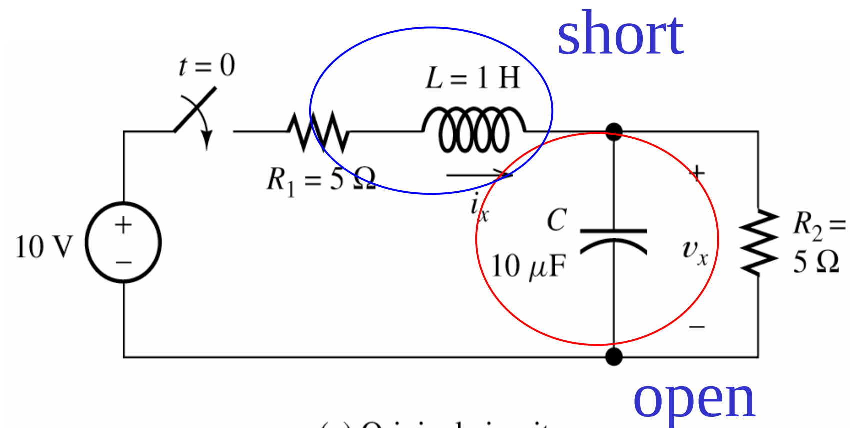
2. Replace inductances with short circuits.

3. Solve the remaining circuit.

Example 4.1 Steady-state DC Analysis

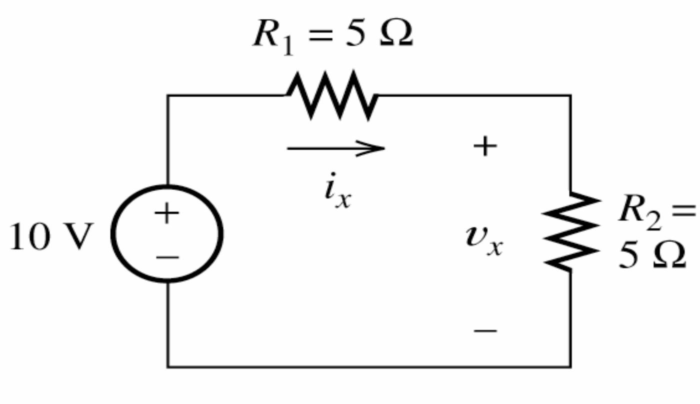


(a) Original circuit



(a) Original circuit

Equivalent circuit for steady state



(b) Equivalent circuit for steady state

$$i_x = \frac{10}{R_1 + R_2} = 1A$$

$$v_x = R_2 i_x = 5V$$

4.3 *RL* CIRCUITS

- 與RC Circuit 類似，探討包含dc sources, resistances 及一個電感的電路特性。
- RL or RC 電路分析步驟歸納如下：

1. Apply Kirchhoff's current and voltage laws to write the circuit equation.(RC: KCL, RL: KVL)

2. If the equation contains integrals, differentiate each term in the equation to produce a pure differential equation.

3. Assume a solution of the form $K_1 + K_2 e^{st}$.

4. Substitute the solution into the differential equation to determine the values of K_1 and s . (Alternatively, we can determine K_1 by solving the circuit in steady state as discussed in Section 4.2.)

5. Use the initial conditions to determine the value of K_2.

6. Write the final solution.

Example 4.2 RL Transient Analysis

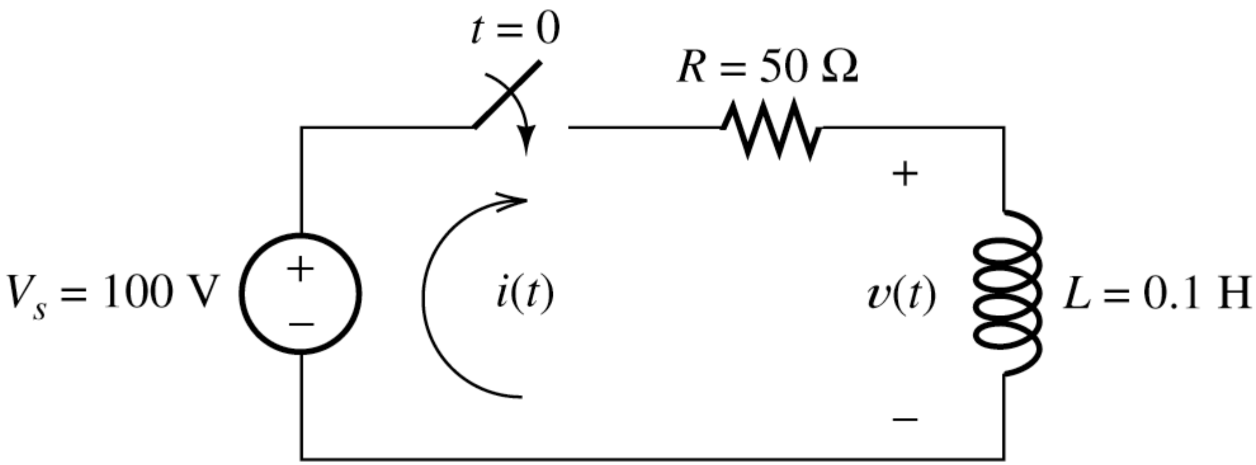


Figure 4.7 The circuit analyzed in Example 4.2.

1. $t<0$, $i(t) = 0$.

2. $t > 0$

KVL

$$Ri(t) + L\frac{di}{dt} = V_s \qquad \left(v(t) = L\frac{di}{dt} \right)$$

Example 4.2 RL Transient Analysis

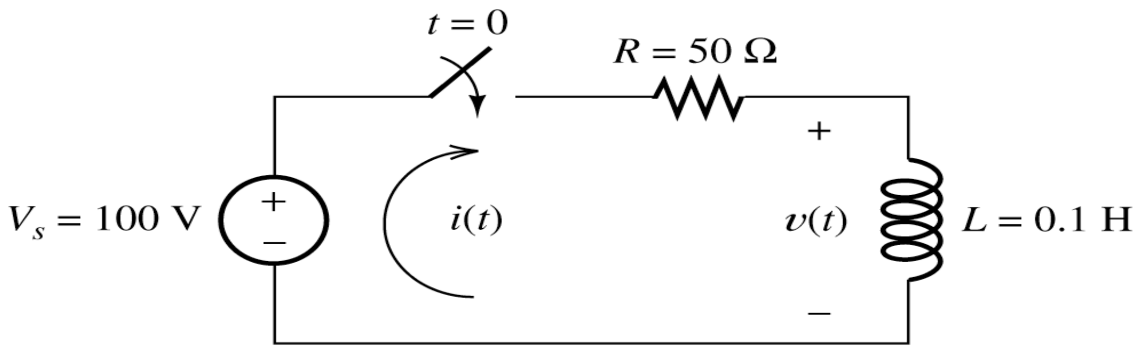


Figure 4.7 The circuit analyzed in Example 4.2.

Assume

$$i(t) = K_1 + K_2 e^{st}$$

➡

$$RK_1 + (RK_2 + sLK_2)e^{st} = V_s$$

➡

$$K_1 = \frac{V_s}{R} = 2 \qquad s = \frac{-R}{L}$$

Example 4.2 RL Transient Analysis

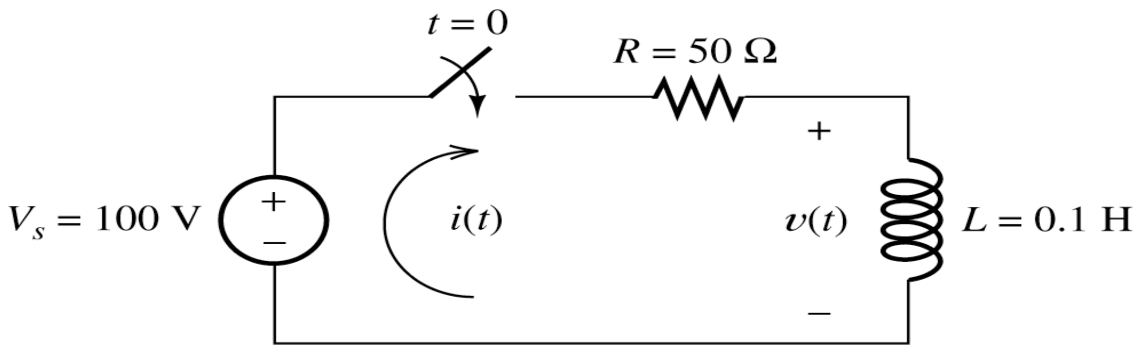


Figure 4.7 The circuit analyzed in Example 4.2.

$$i(t) = 2 + K_2 e^{-tR/L}$$

$$\because \quad i(0_+) = 0 = 2 + K_2 e^0 = 2 + K_2 \quad \Rightarrow \quad K_2 = -2$$

$$i(t) = 2 - 2e^{-t/\tau} \quad for \;\; t > 0 \qquad \tau = \frac{L}{R}$$

Example 4.2 RL Transient Analysis

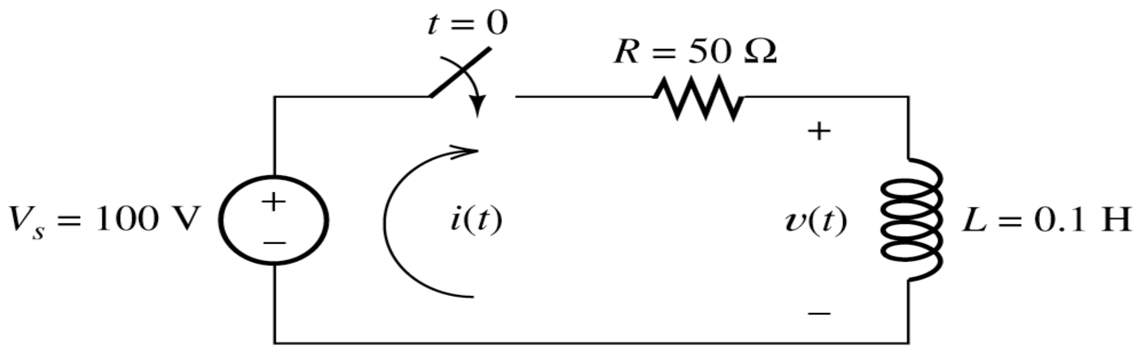
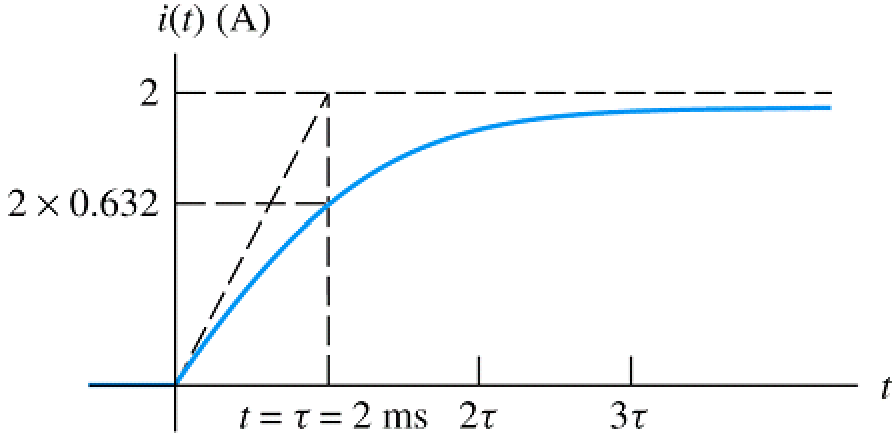


Figure 4.7 The circuit analyzed in Example 4.2.

$$i(t) = 2 - 2e^{-t/\tau} \quad for\ \ t > 0 \qquad \tau = \frac{0.1}{50} = 2 \times 10^{-3}\,(\text{sec})$$



Example 4.2 *RL* Transient Analysis

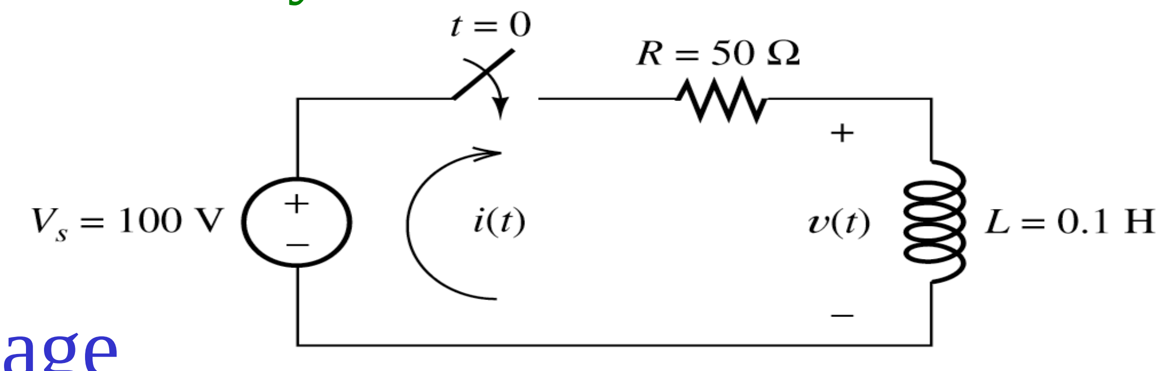


Figure 4.7 The circuit analyzed in Example 4.2.

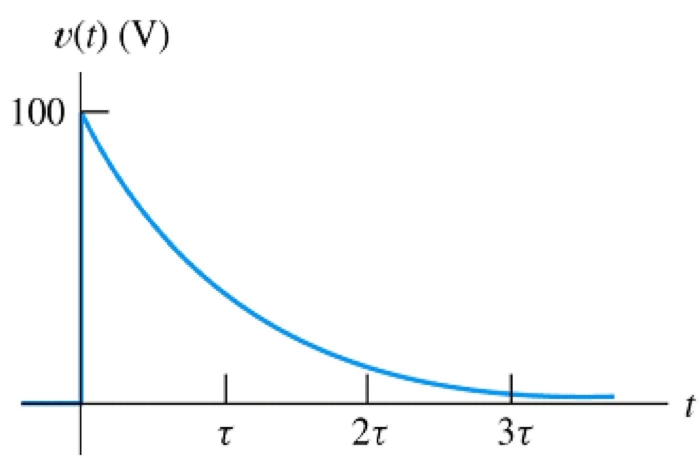
Consider the voltage

1. $t<0$, $v(t) = 0$.

2. $t > 0$

$$v(t) = 100 - 50i(t) \quad for\ \ t > 0 \quad \left(i(t) = 2 - 2e^{-t/\tau} \right)$$

$$\Rightarrow \quad v(t) = 100e^{-t/\tau}$$



or

$$v(t) = L\frac{di}{dt} = 0.1 \cdot (2/\tau)e^{-t/\tau}$$

$$= 0.1 \cdot 1000 \cdot e^{-t/\tau} = 100e^{-t/\tau}$$