

# 電子電工學

## Lecture 5



# Midterm Exam Information

Date: Oct 14, 2020

Time: 3:10 ~ 6:00 pm (in class)

Location: 化工系館柏林講堂/93156

Coverage: Textbook Chapters 1-4

# Recap: Superposition

$\begin{bmatrix} \text{KCL} \\ \text{KVL} \\ \text{Ohm's} \end{bmatrix} \rightarrow \text{eqn's} \xrightarrow{\text{Solve}} V, I, P$  (10 eqn's)  
 $\xrightarrow{\text{Simplify}} \text{NODAL} \rightarrow \text{Voltages}$  (3 eq)

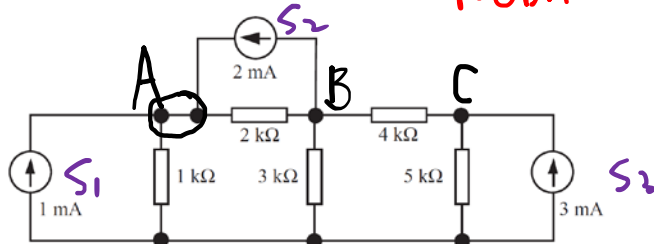
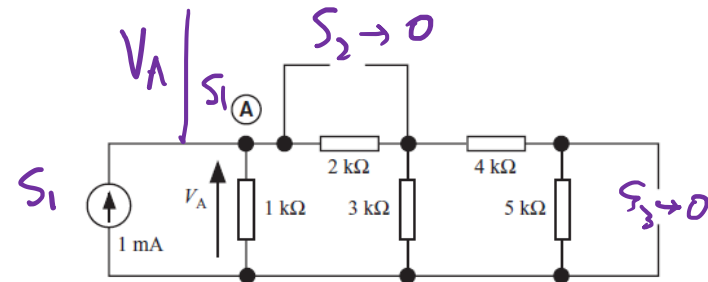


Figure 4.1 The circuit to be analysed

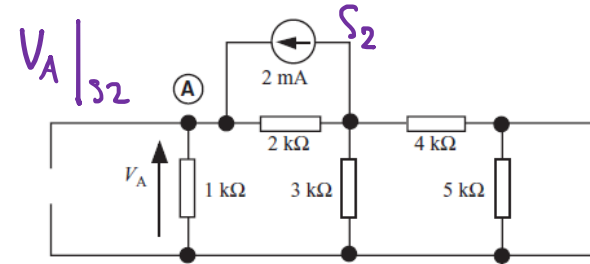
Want only  $V_A$   
 Apply only one src  
 Then sum-up

Zero sources

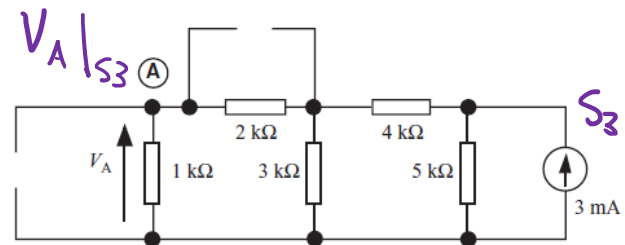
$I=0 \rightarrow$  Open ckt  
 $V=0 \rightarrow$  Short ckt



(a) Analyse to find the value of  $V_A$  due to the 1 mA source



(b) Analyse to find the value of  $V_A$  due to the 2 mA source



(c) Analyse to find the value of  $V_A$  due to the 3 mA source

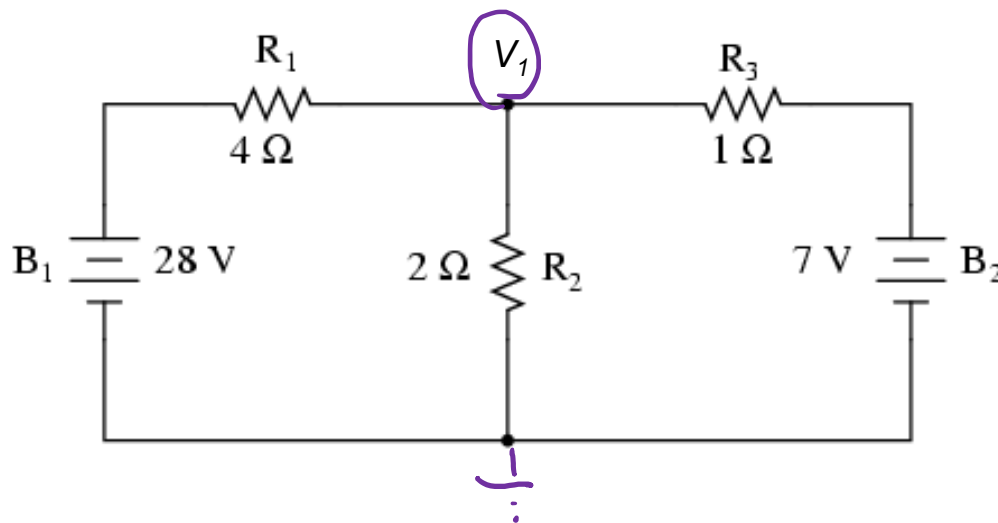
Figure 4.10 Illustration of the use of the superposition principle to find the voltage  $V_A$  at node A in the circuit of Figure 4.4. The three calculated voltages are added together to find the actual value of  $V_A$ .

# Quiz review

Find the voltage  $V_1$  by applying the principle of superposition.

$$V_1 = V_1 \Big|_{B_1} + V_2 \Big|_{B_2}$$

Voltage Src  $\rightarrow 0$   
Short Ckt.



# Quiz review

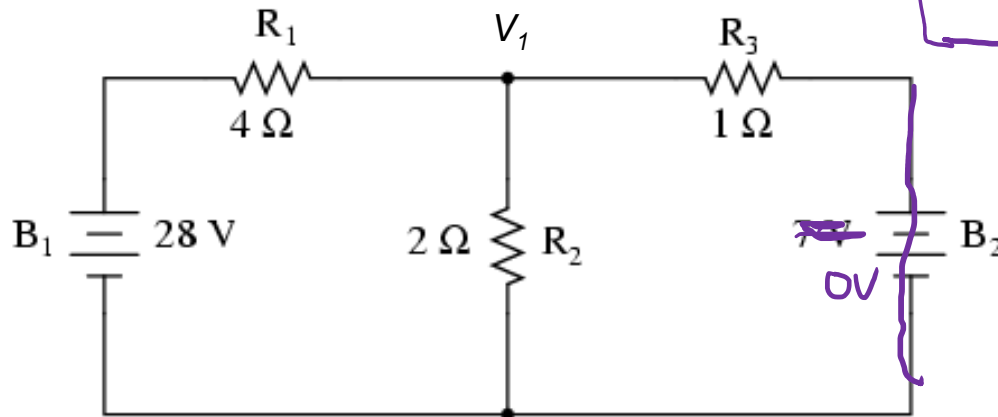
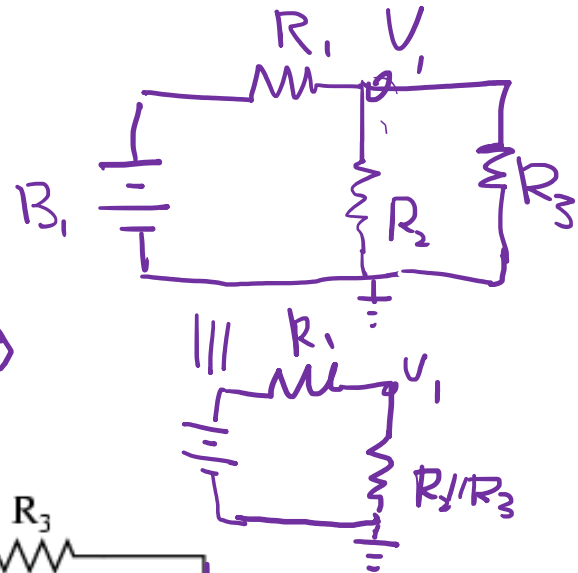
Apply only  $B_1$  28V

$B_2 \rightarrow 0V \rightarrow$  Short ckt

$$V_1|_{B_1} = 28 \times \frac{R_2 // R_3}{R_1 + (R_2 // R_3)}$$

$$= 28 \times \frac{2/3}{4 + 2/3}$$

$$= 4 \text{ (V)}$$

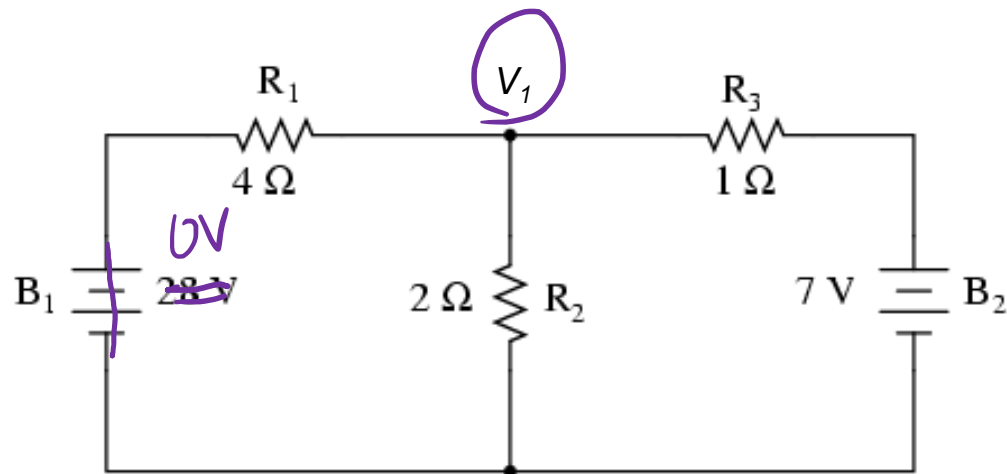


# Quiz review

Apply only  $B_2$

Set  $B_1 \rightarrow 0$

$$\begin{aligned} V_1|_{B_2} &= 7 \times \frac{R_1/R_2}{R_3 + (R_1/R_2)} \\ &= 7 \times \frac{4/3}{1 + 4/3} \\ &= 4 \text{ (V)} \end{aligned}$$

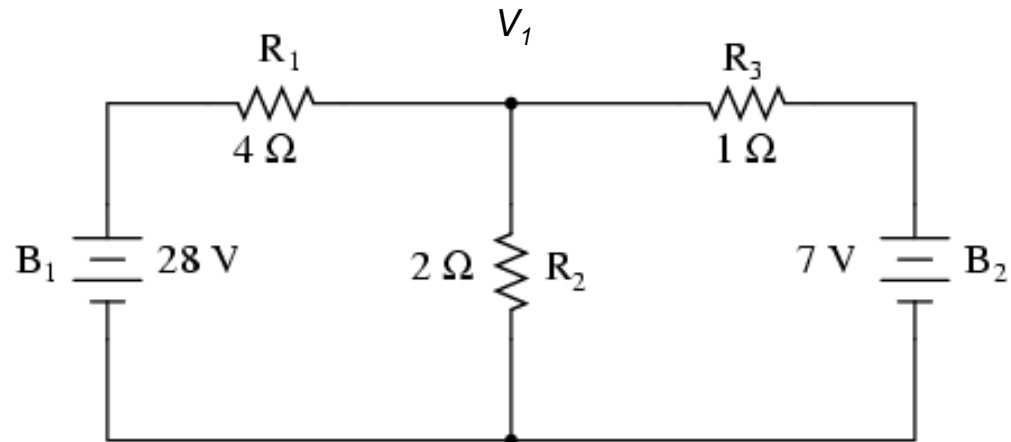


# Quiz review

$$V_1 = V_{1|B_1} + V_{1|B_2}$$

$$= 4 + 4$$

$$= 8$$

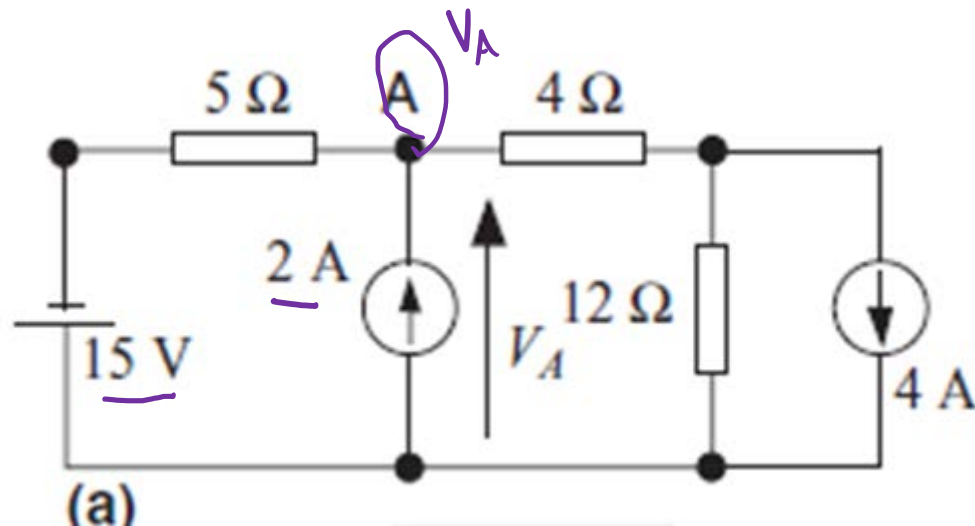


## Example 4.2 Superposition with voltage source

Aim: Find  $V_A$ .

Voltage Sources=0 -> *Short*

Current Sources=0 -> *Open*





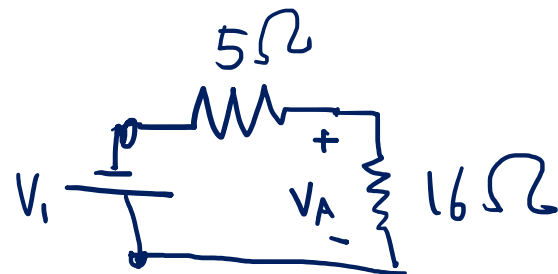
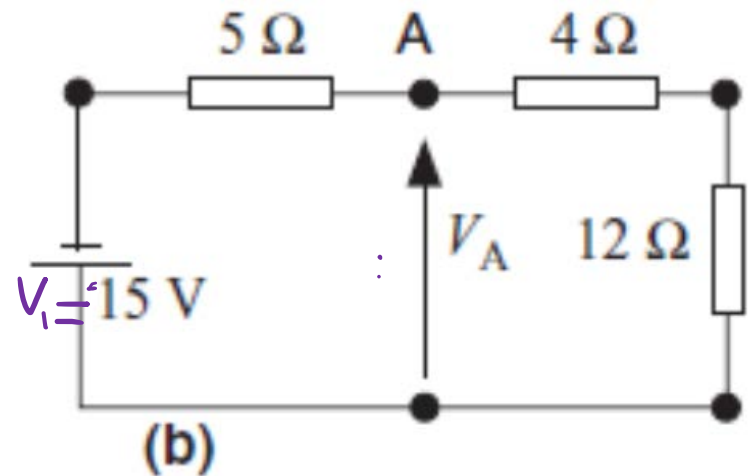
## Example 4.2 (source 1)

$$S_1: V_1 = 15V$$

$$\left. \begin{array}{l} S_2 \rightarrow 0 \\ S_3 \rightarrow 0 \end{array} \right\} U_{pen}$$

$$V_A|_{S_1} = (-15) \times \frac{4+12}{5+(4+12)}$$

$$= -\frac{80}{7} (V)$$



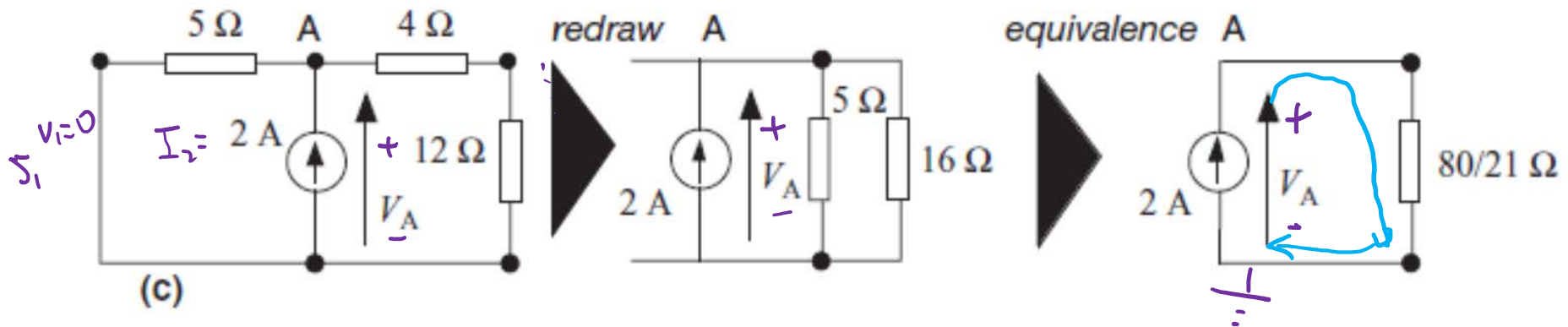
## Example 4.2 (source 2)

$$S_2 : I_2 = 2 \text{ A}$$

$$S_1 \rightarrow 0 \quad V_1 = 0 \quad \text{short ckt.}$$

$$S_3 \rightarrow 0 \quad I_3 = 0 \quad \text{Open ckt.}$$

$$V_A|_{S_2} = 2 \times \frac{80}{21} = \frac{160}{21} \text{ (V)}$$



# Example 4.2 (source 3)

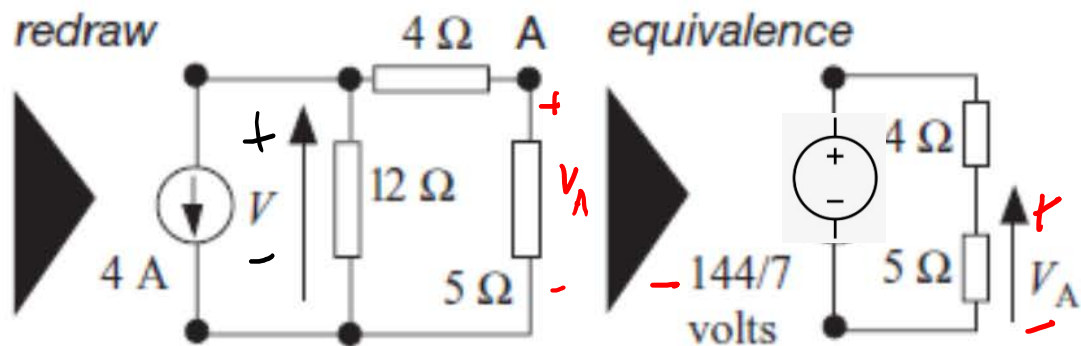
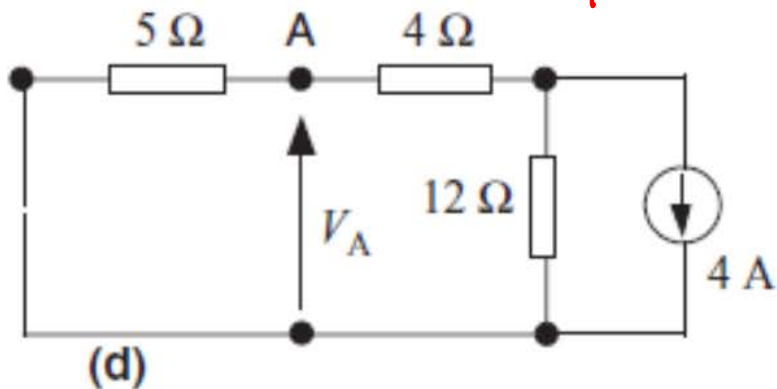
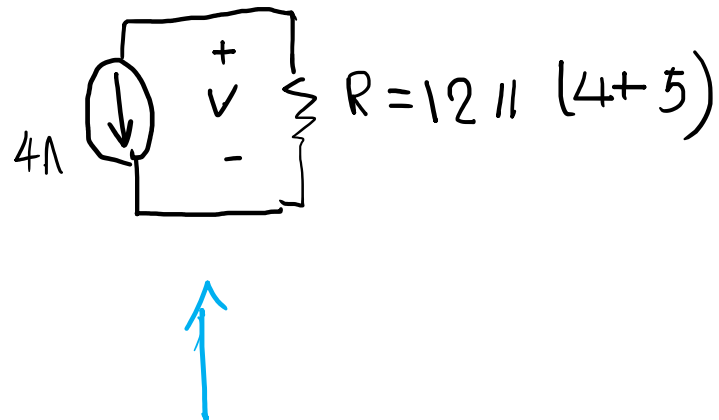
$$S_3 : I_3 = 4A$$

$$S_1 : V_1 = 0 \rightarrow \text{Short}$$

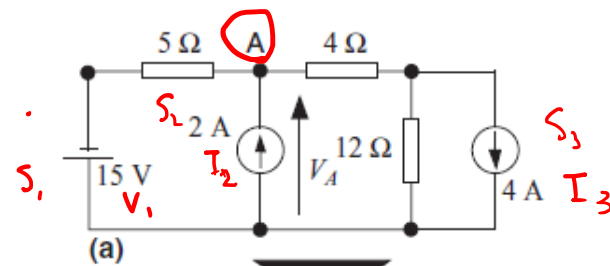
$$S_2 : I_2 = 0 \rightarrow \text{Open}$$

$$\begin{aligned} V_A|_{S_3} &= V \times \frac{5}{5+4} \\ &= \left(-\frac{144}{7}\right) \times \frac{5}{9} \\ &= -\frac{80}{7} \text{ (V)} \end{aligned}$$

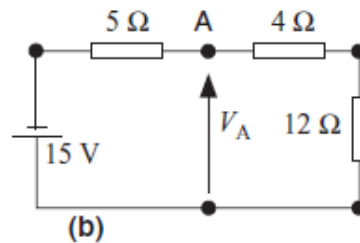
$$\begin{aligned} V &= -4 \times R \\ &= -\frac{144}{7} \text{ (V)} \end{aligned}$$



# Example 4.2 (superposition)



Apply superposition

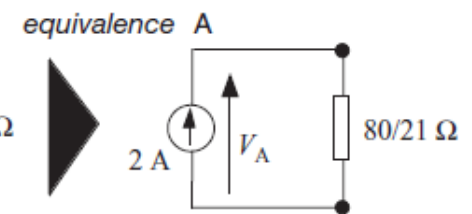
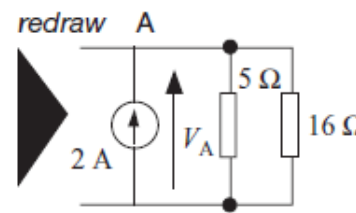
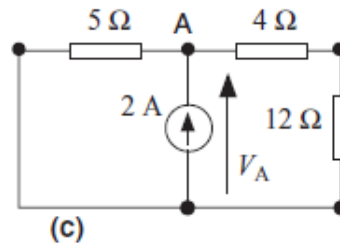


By the voltage divider principle:

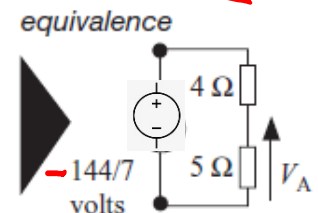
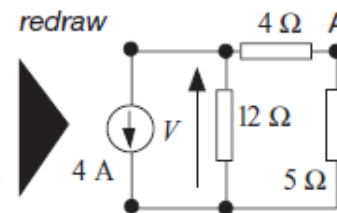
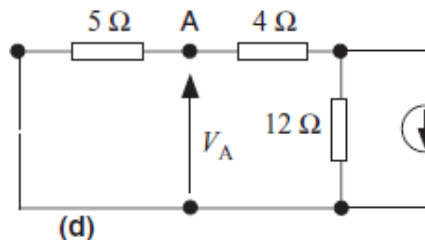
$$V_A = -15(16/21) = \underline{-80/7} \text{ volts}$$

$$V_A = V_A|_{S_1} + V_A|_{S_2} + V_A|_{S_3}$$

$$= -\frac{320}{21} \text{ (V)}$$



By Ohms law  $V_A = 2 \times 80/21$



By Ohms Law  $V = 4(108/21) = 144/7$  volts

By voltage divider principle  $V_A = (5/9)(144/7) = \underline{-80/7}$  volts

By Superposition, actual  $V_A$  in the circuit of Figure 4.6 =  $\underline{-80/7 + 160/21 - 80/7} = -320/21$  volts

Figure 4.11 Illustrating the use of superposition to analyse circuit behaviour

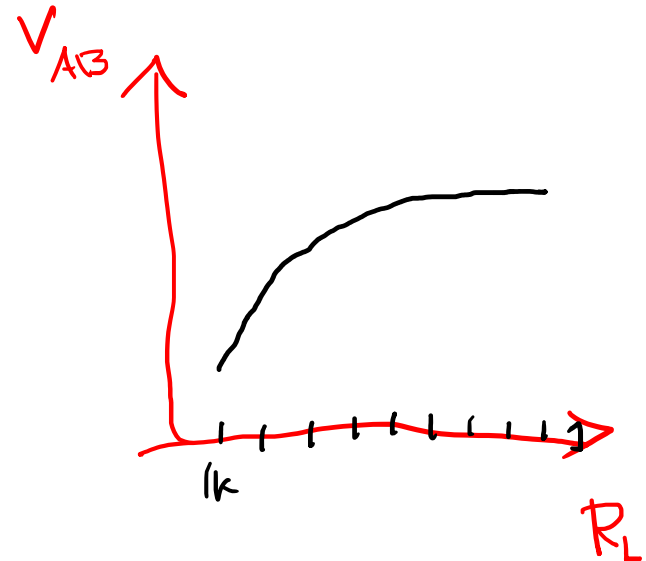
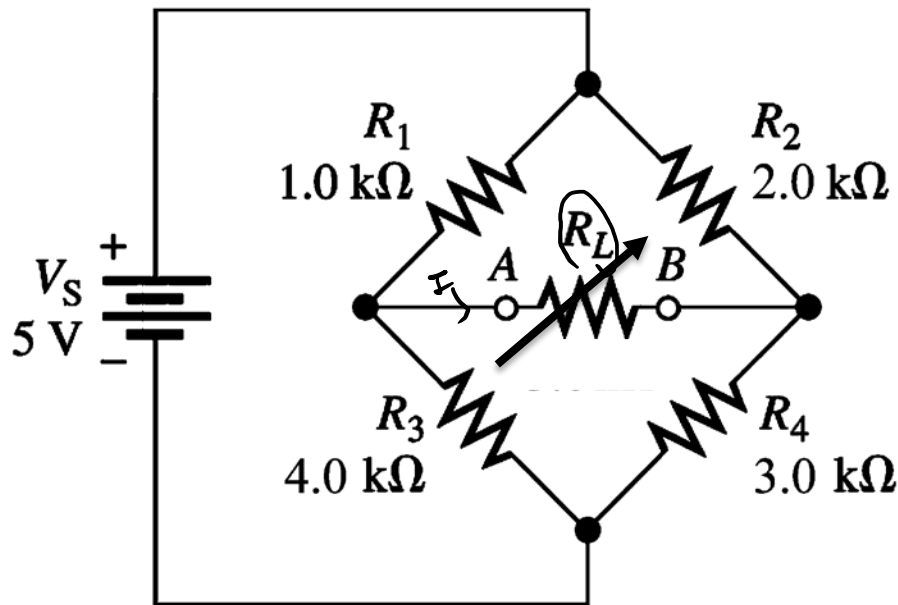
# Basic circuit analysis: variable loads

$R_L$  Load varies  $\rightarrow V_{AB}$  change  
 $I$

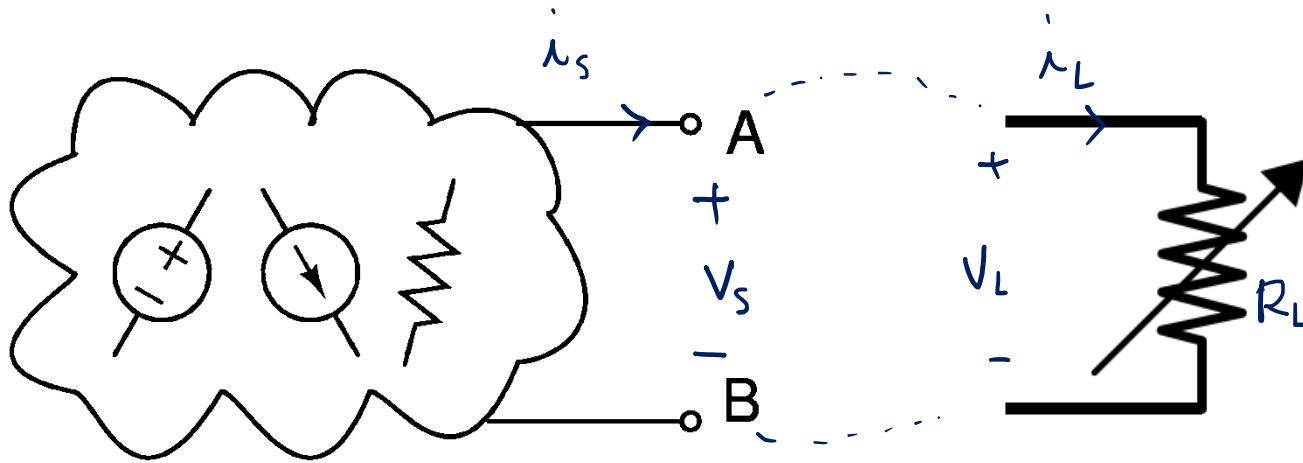
$$R_L = 1k \rightarrow V_{AB}$$

$$R_L = 2k \rightarrow V_{AB}$$

$$\vdots$$
$$R_L = 10k \rightarrow V_{AB}$$



# Equivalent circuit



Find  $i_s - V_s$  characteristic

$$V_s = f(i_s)$$

$$V_L = i_L \cdot R_L$$

$i_L - V_L$   
characteristic.

$V_L$  varies  
according to  $R_L$

$$R_L = 1k \rightarrow V_L = G_{1k}^{-1} \cdot S$$

$$R_L = 2k \rightarrow V_L = G_{2k}^{-1} \cdot S$$

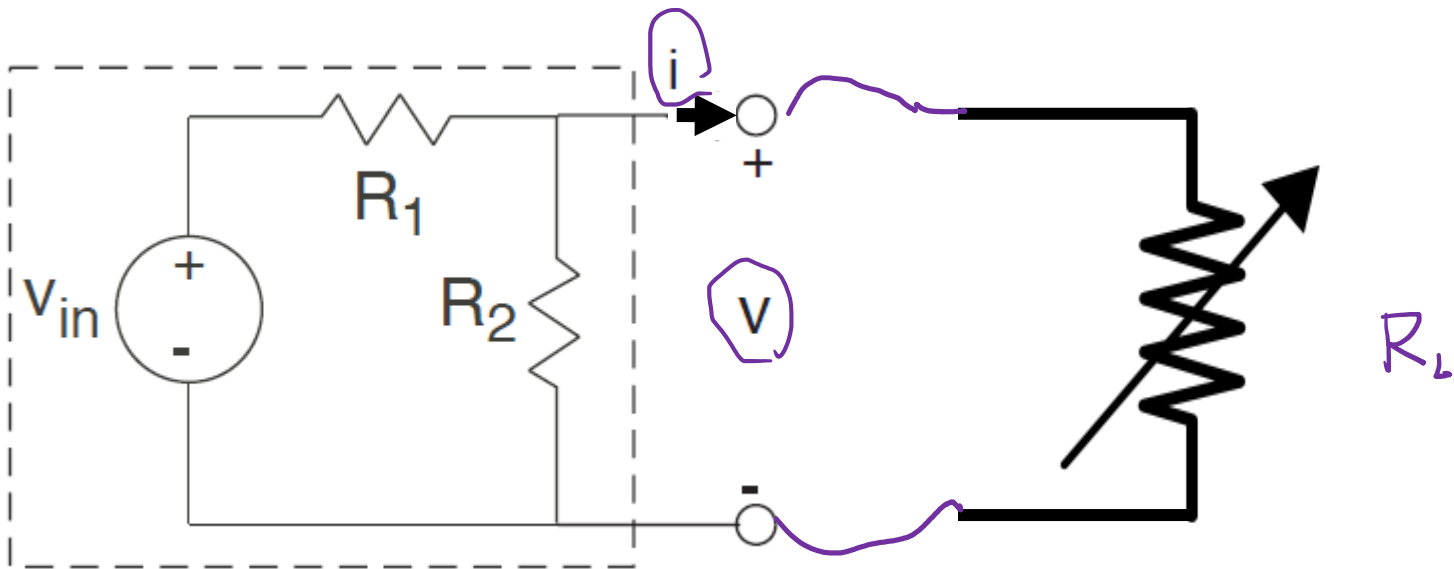
$$R_L = 3k \rightarrow V_L = G_{3k}^{-1} \cdot S$$

$\vdots$

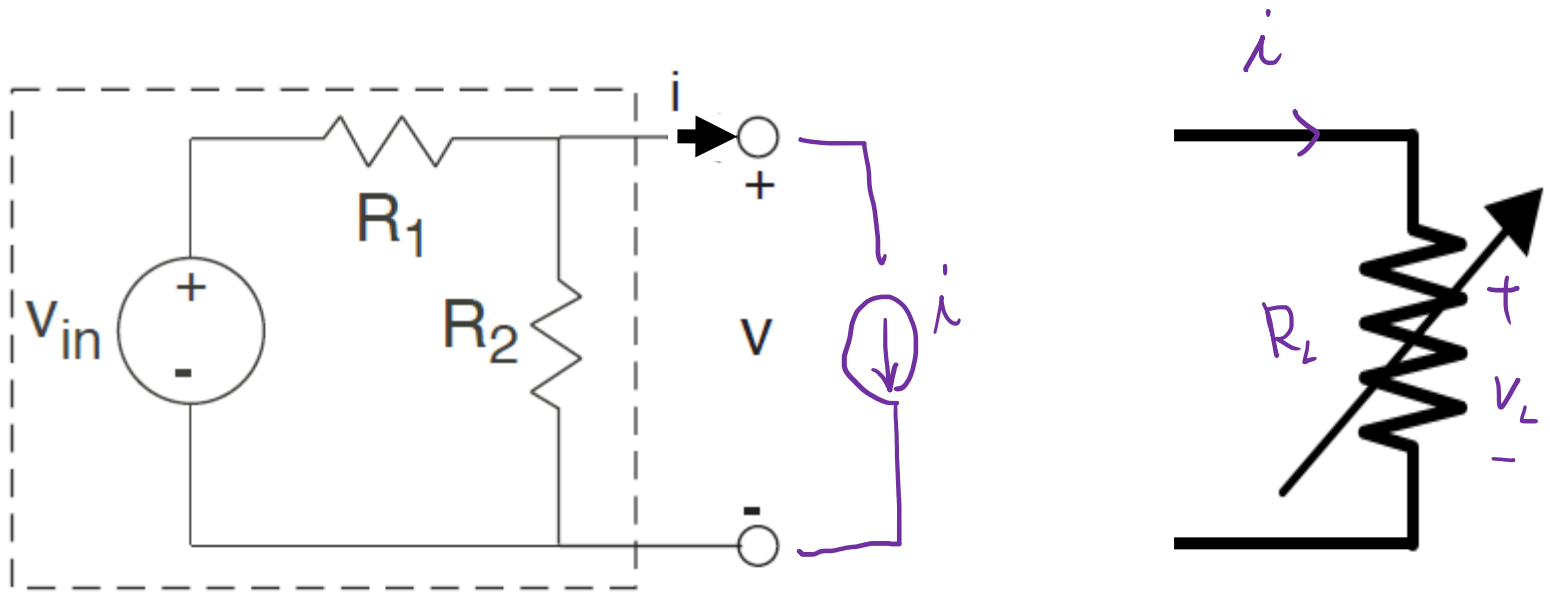
$$R_L = 10k \rightarrow V_L = G_{10k}^{-1} \cdot S$$

# Equivalent circuit

$$v = f(i) \quad ? \quad \text{for any } R_L$$



# Equivalent circuit



Apply a current source with  $i$  amp

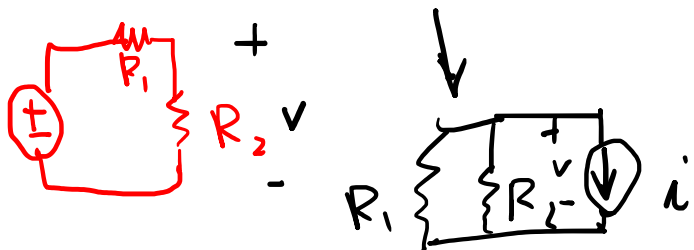
find  $V = \underline{\underline{f(i)}}$  ?

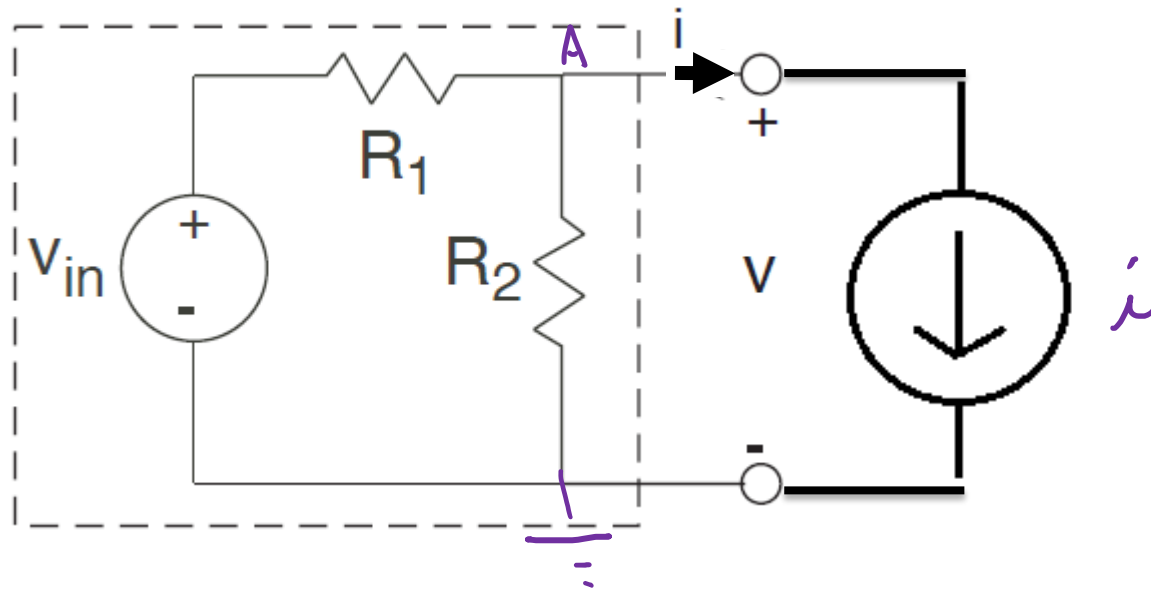


# Principle of superposition

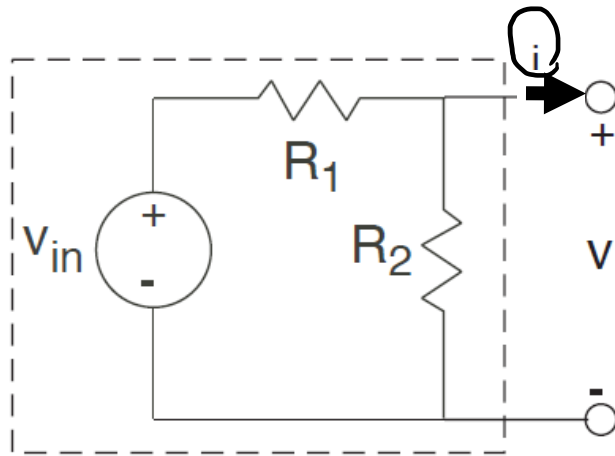
$$V = V_A|_{v_{in}} + V_A|i$$

$$= v_{in} \times \frac{R_2}{R_1 + R_2} + (-i) \times (R_1 || R_2)$$

$$\Rightarrow V = f(i)$$


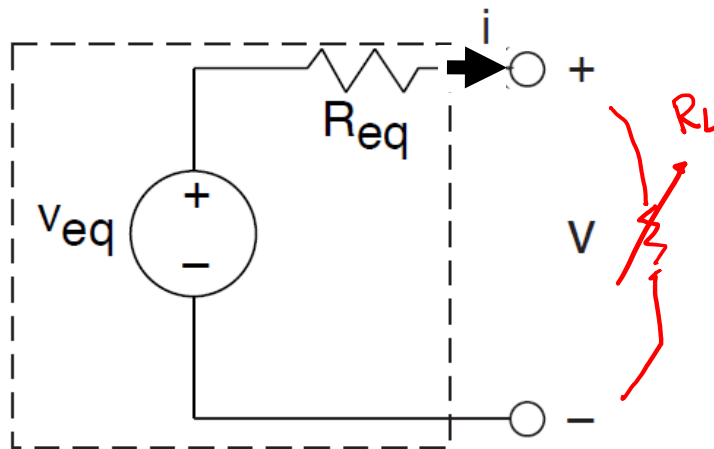


# Thevenin equivalent circuit



For any  $i$

$$v = \underbrace{v_{in} \times \frac{R_2}{R_1 + R_2}}_{v_{eq}} - i \times \underbrace{(R_1 \parallel R_2)}_{R_{eq}}$$



$$v = v_{eq} - i \cdot R_{eq}$$

$$v = f(i)$$

voltage divider  $v = v_{eq} \cdot \frac{R_L}{R_{eq} + R_L}$

# Multiple sources

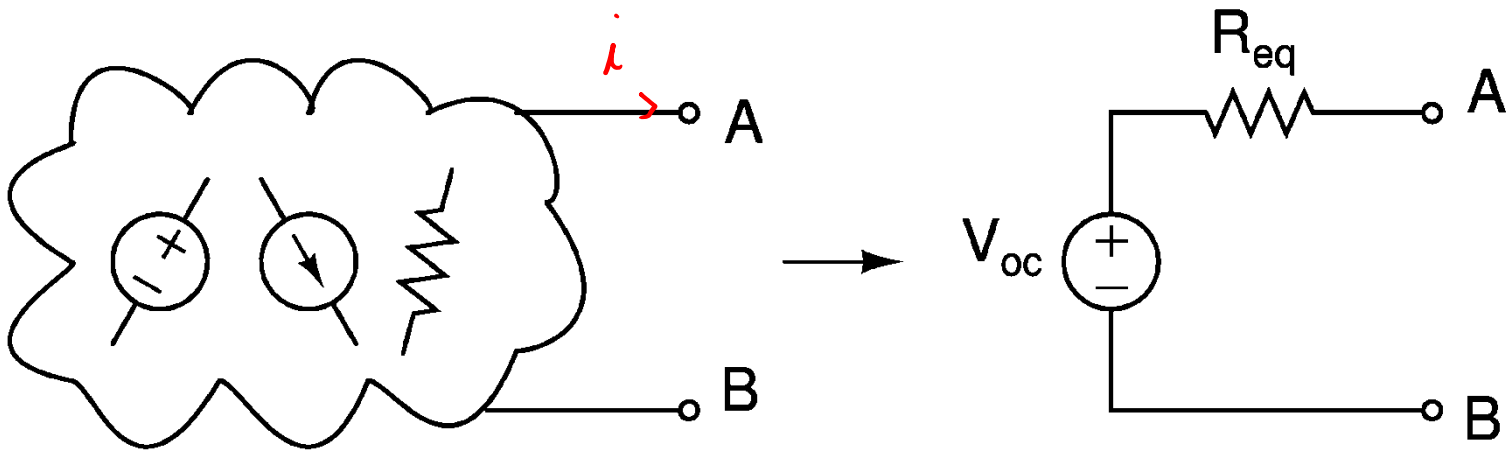
Apply superposition + virtual current src  $i'$

$$V_{AB} = V_{AB}|_{S_1} + V_{AB}|_{S_2} + \dots + V_{AB}|_{S_N} + V_{AB}|_{i'}$$

$$= (V_1 + V_2 \dots + V_N) - i' \cdot R_{eq}$$

$$= \frac{V_{eq} - i' R_{eq}}$$

$$= \underline{\underline{V_{oc}}} - i' \cdot R_{eq} \quad \text{Thevenin Equivalent}$$

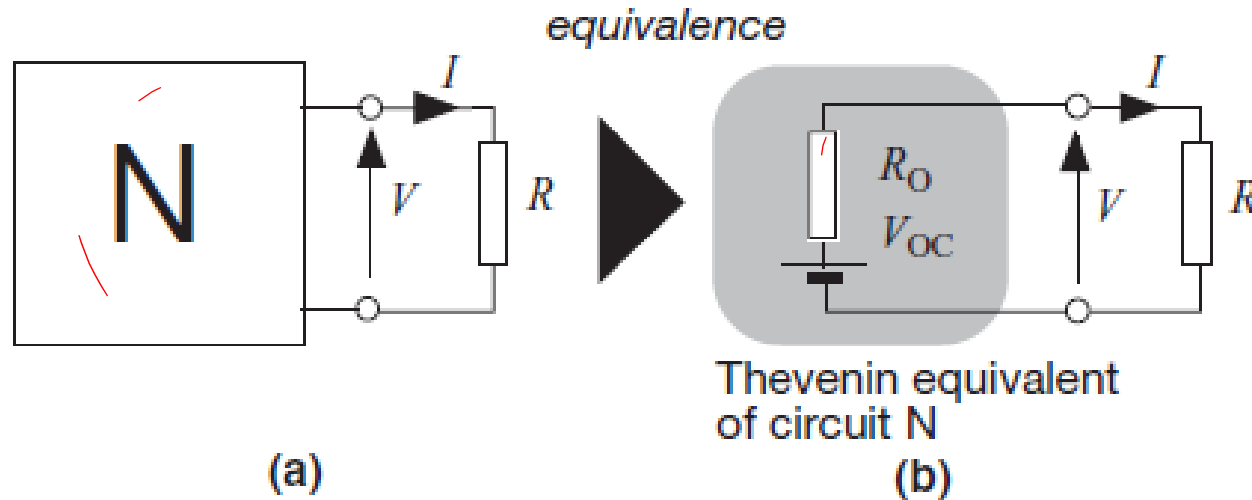


# Thevenin's theorem

Any Linear circuit  $N$

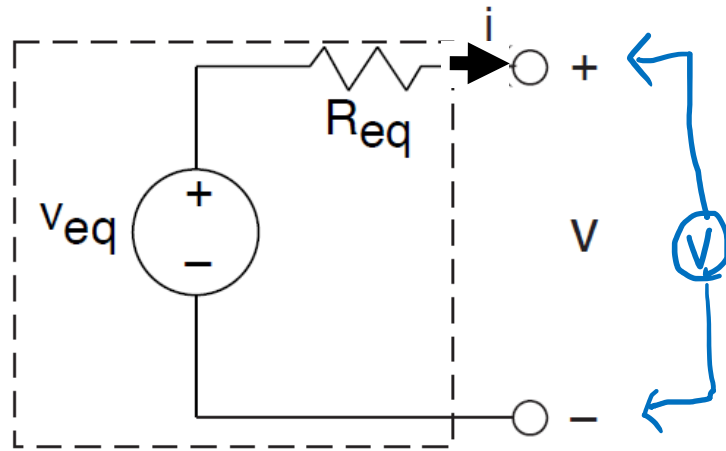
Can be modeled by

$$V = f(I) = V_{oc} - I \cdot R_o$$



**Figure 4.12** A linear circuit  $N$  can be represented by a Thevenin equivalent circuit consisting of a voltage source and a resistor

# Thevenin equivalent circuit

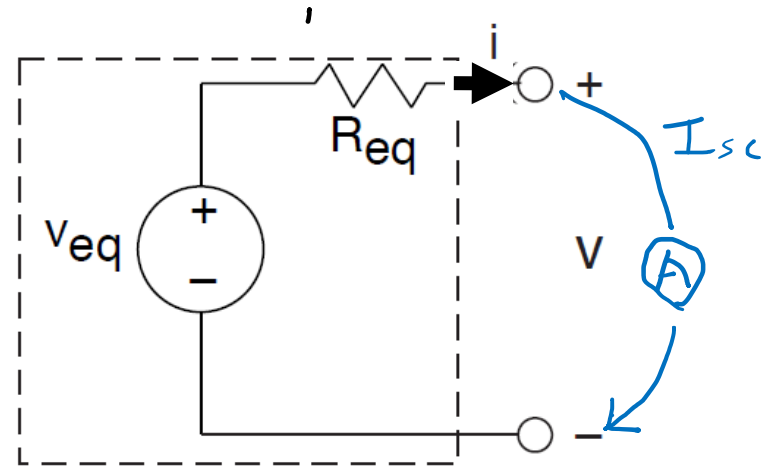


Apply

Voltmeter

Open-Circuit  
Voltage

$$V_{OC} = v_{eq}$$



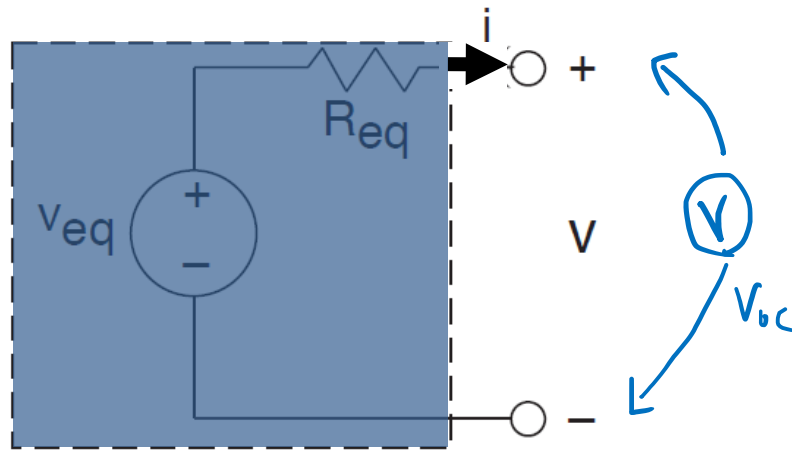
Apply

Amperemeter

$I_{sc}$  : Short-ckt  
current

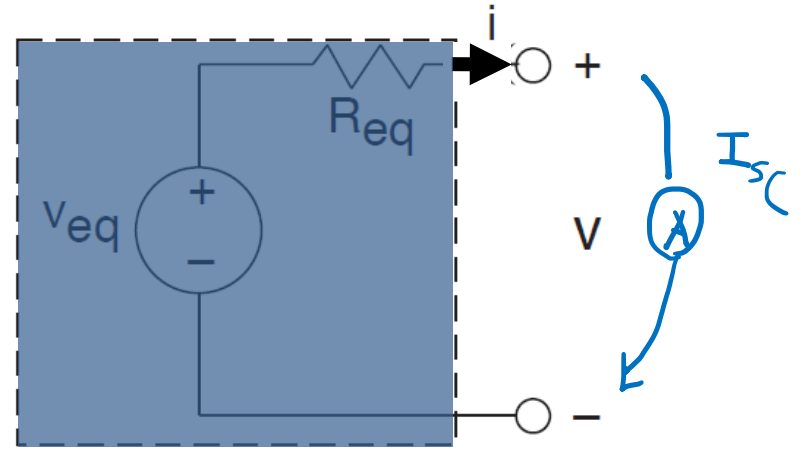
$$R_{eq} = V_{OC} / I_{sc}$$

# Thevenin equivalent circuit



Black-box

$$V_{eq} = V_{oc}$$

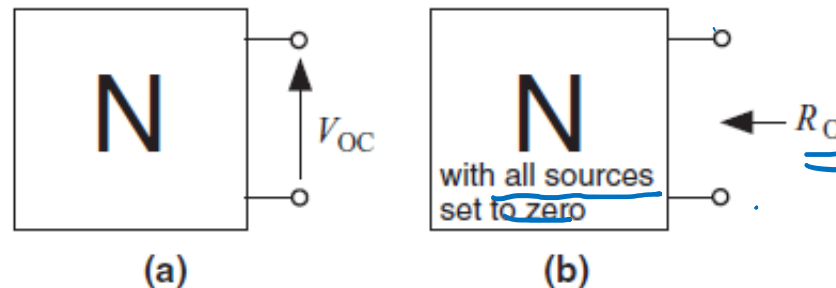


$$R_{eq} = V_{oc} / I_{sc}$$

# Thevenin equivalent circuit

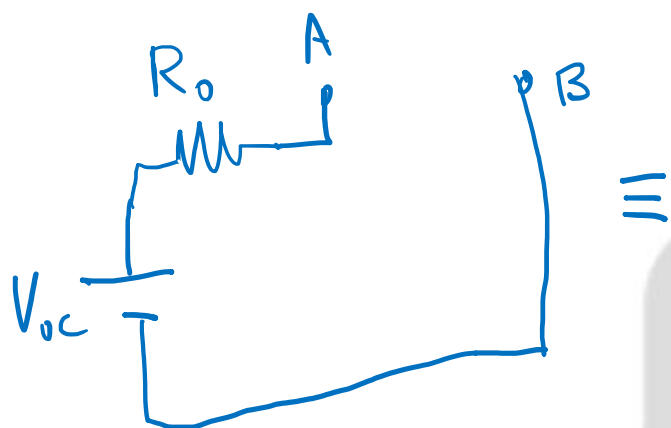
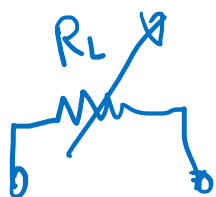
1. Find  
Open-circuit  
Voltage  
 $V_{OC}$

2. Find equiv. resistance  
Voltage  $S_{rc} \rightarrow$  Short.  
Current  $S_{rc} \rightarrow$  Open

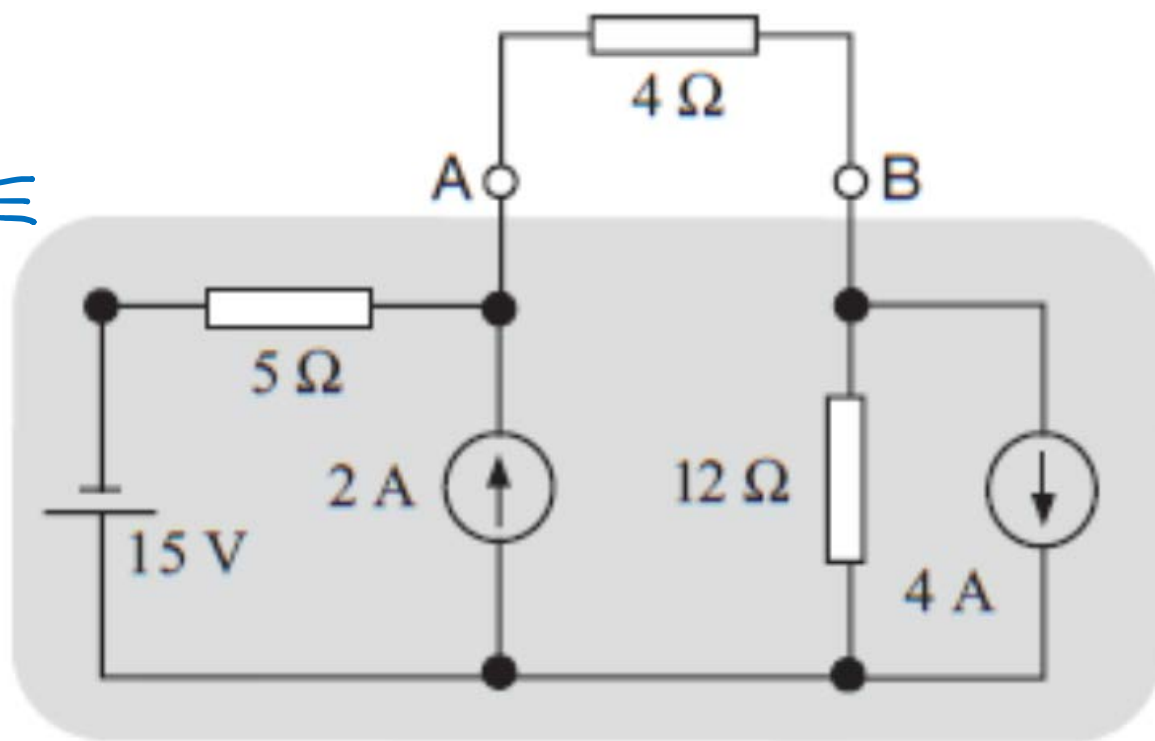


**Figure 4.13** Calculations required to find the two parameters defining a Thevenin equivalent circuit

## Example 4.3



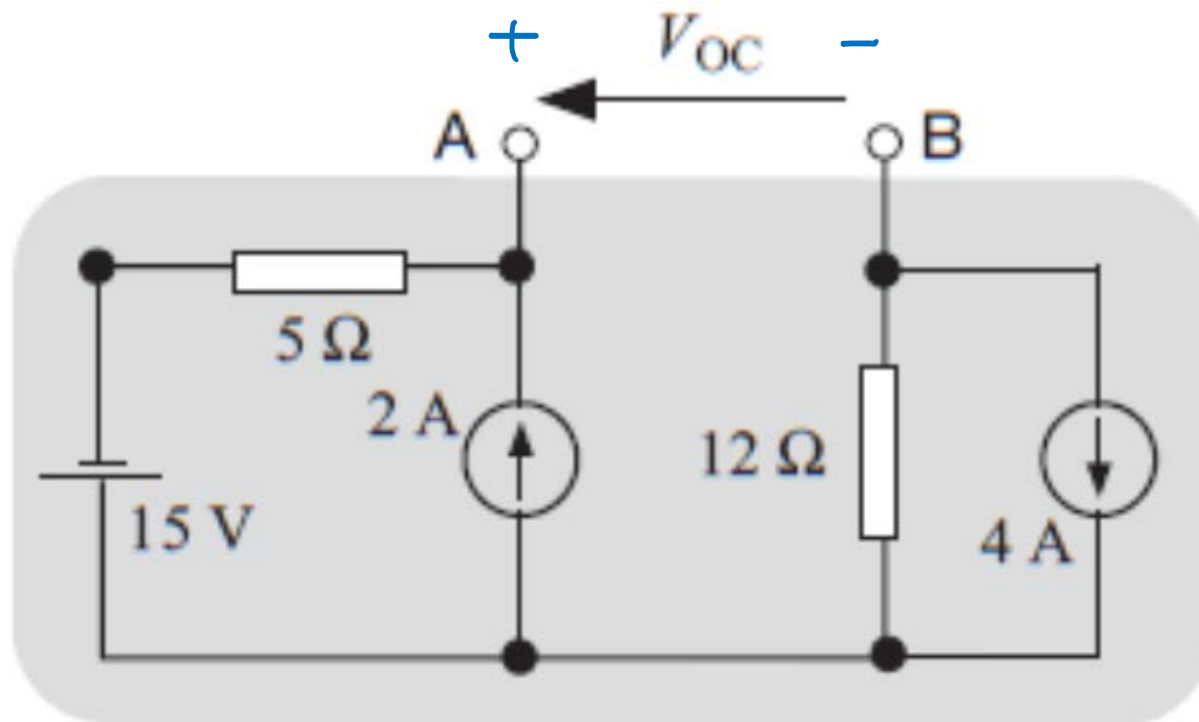
Find  $V_{oc} = ?$   
 $R_o = ?$





## Example 4.3

Nodal Analysis  
Superposition



## Example 4.3

Apply only  $S_1 = 15V$

Set  $S_2 \rightarrow 0A$        $S_3 \rightarrow 0A$

$$V_{AB} |_{S_1} = -15V$$

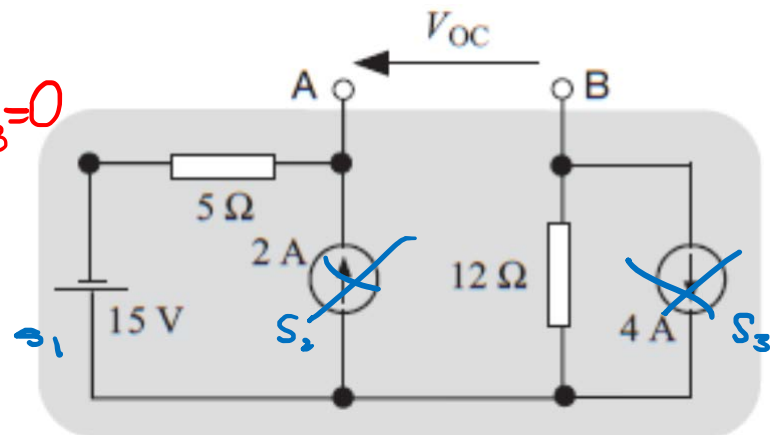
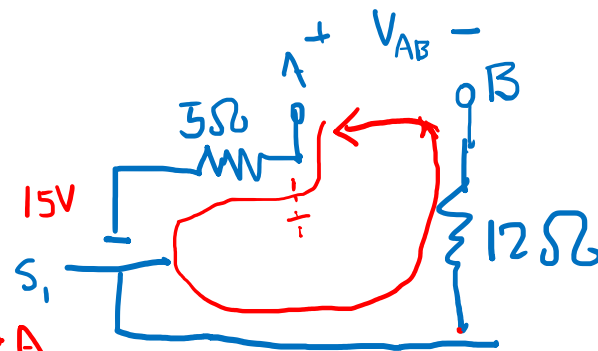
KVL:

$$A \rightarrow 5\Omega \rightarrow S_1 \rightarrow 12\Omega \rightarrow B \rightarrow A$$

$$I \approx 0$$

$$V_A - I \cdot 5 + 15V - I \cdot 12 + V_{AB} = 0$$

$$\Rightarrow V_{AB} = -15V$$



## Example 4.3

Apply only  $S_2 = 2A$

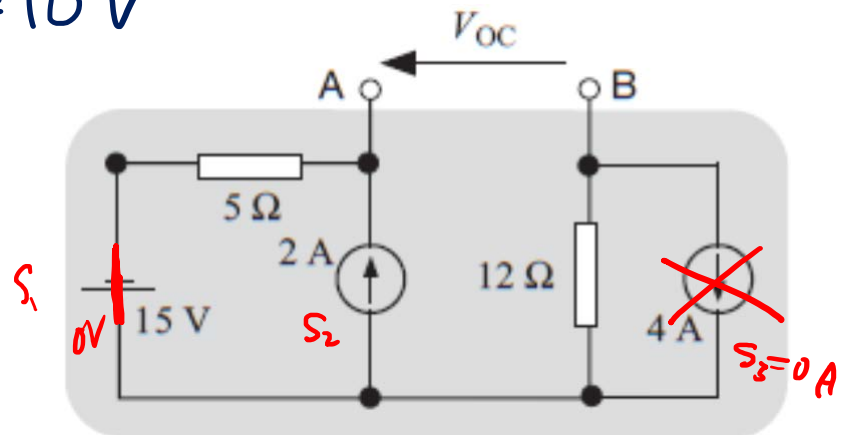
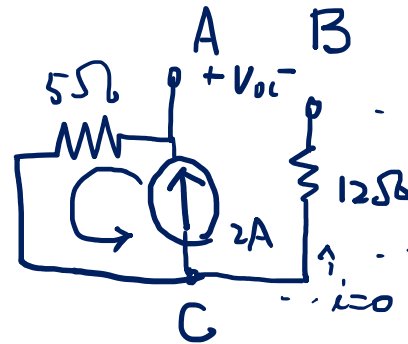
Set  $S_1 = 0V \rightarrow$  short

$S_3 = 0A \rightarrow$  Open

$$\Rightarrow V_{AC} = 2 \times 5 = 10V$$

$$V_B \approx V_C$$

$$\Rightarrow V_{AB} \big|_{S_2} \approx V_{AC} = 10V$$



## Example 4.3

Apply only  $S_3 = 4A$

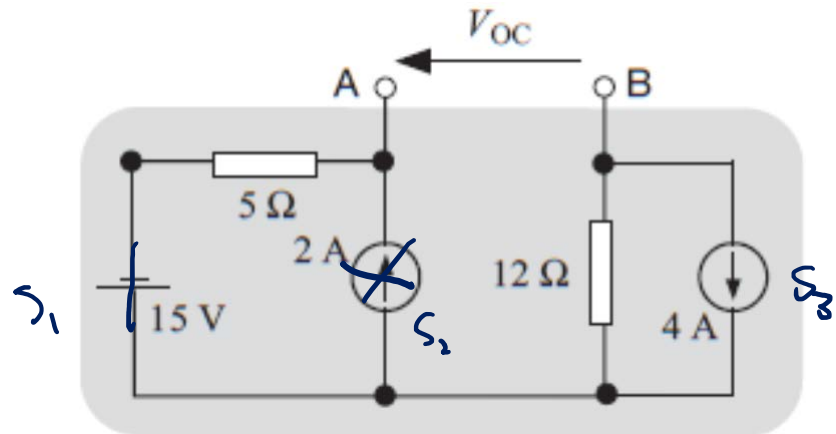
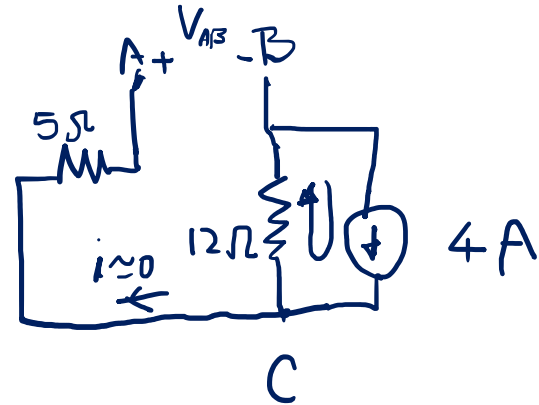
Set  $S_1 \rightarrow 0V \rightarrow \text{short}$

$S_2 \rightarrow 0A \rightarrow \text{open}$

$$V_{BC} = -4 \times 12 = -48V$$

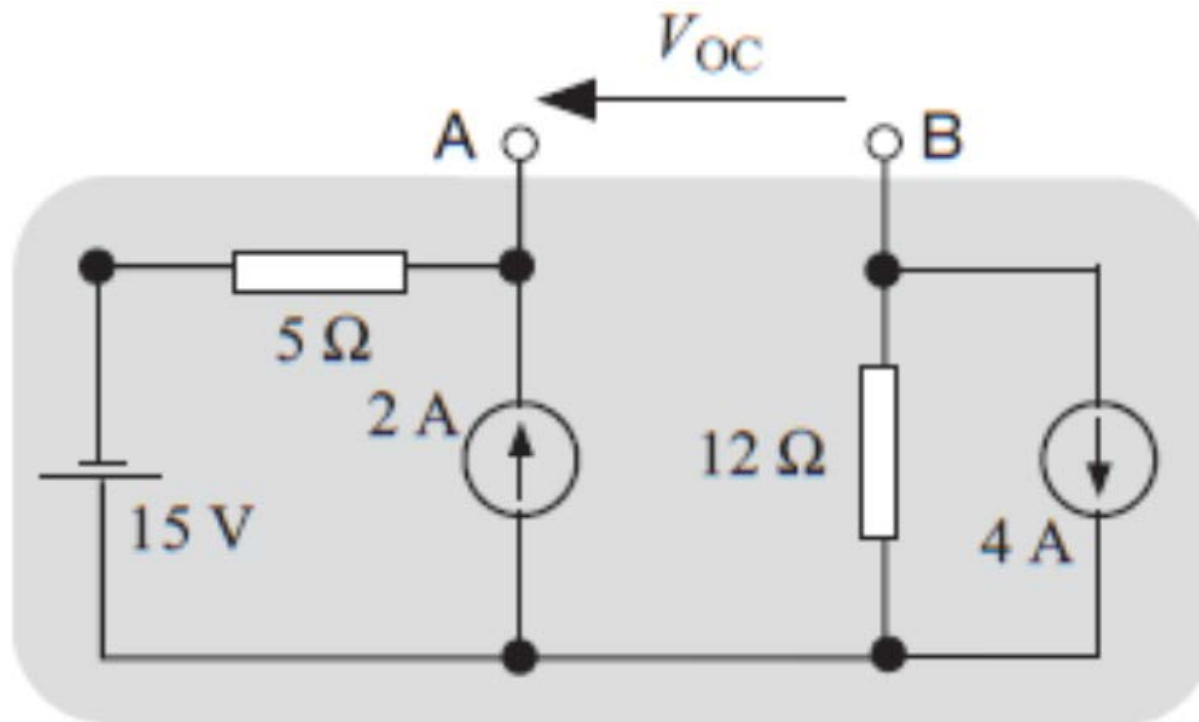
$$V_A \approx V_C$$

$$V_{AB} \big|_{S_3} = +48V$$



### Example 4.3

$$V_{oc} = \underset{\substack{\uparrow \\ S_1}}{-15} + \underset{\substack{\uparrow \\ S_2}}{10} + \underset{\substack{\uparrow \\ S_3}}{48} = 43 \text{ (V)}$$



## Example 4.3

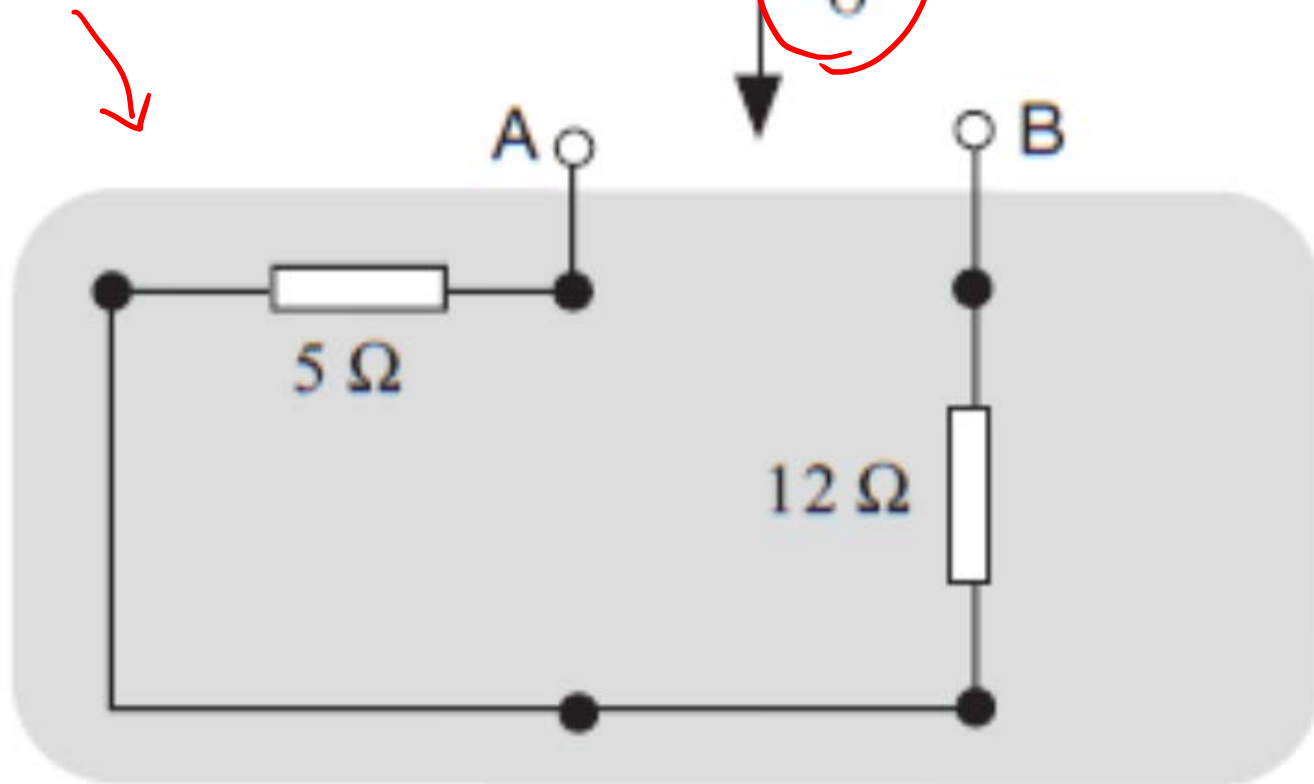
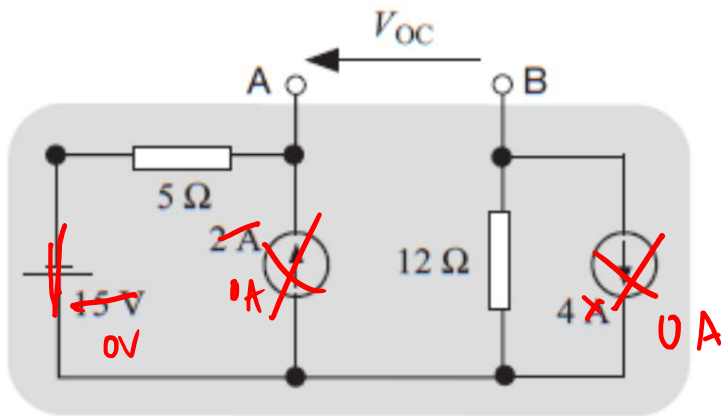
Find  $R_o$

$S_1 \rightarrow 0V$  Short

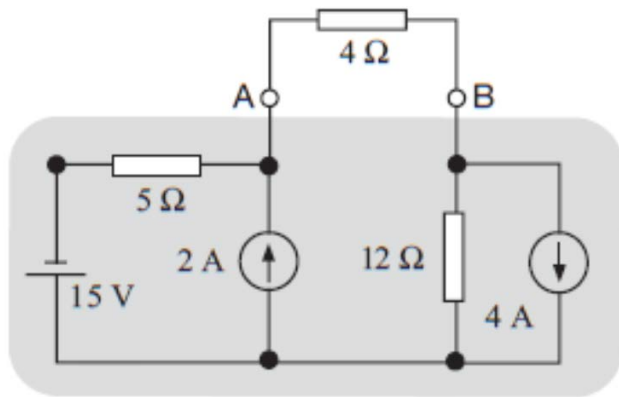
$S_2 \rightarrow 0A$  Open

$S_3 \rightarrow 0A$  Open

$$R_o = 17\Omega$$



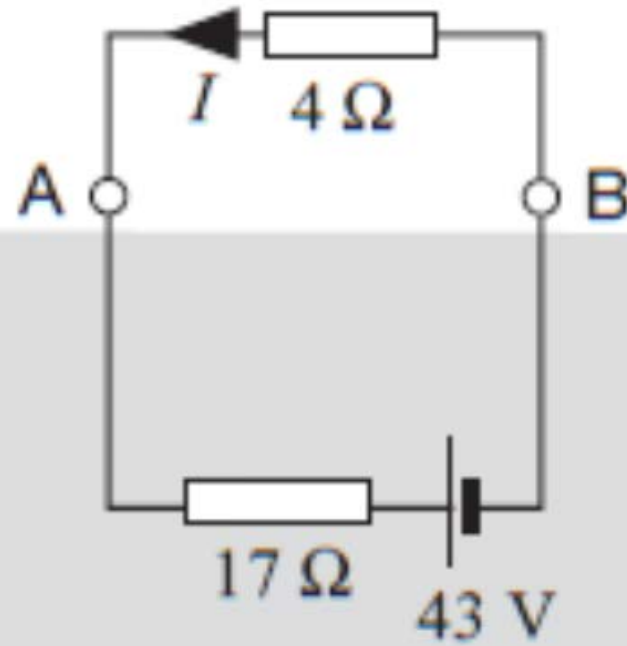
## Example 4.3



Equiv.  
ckt

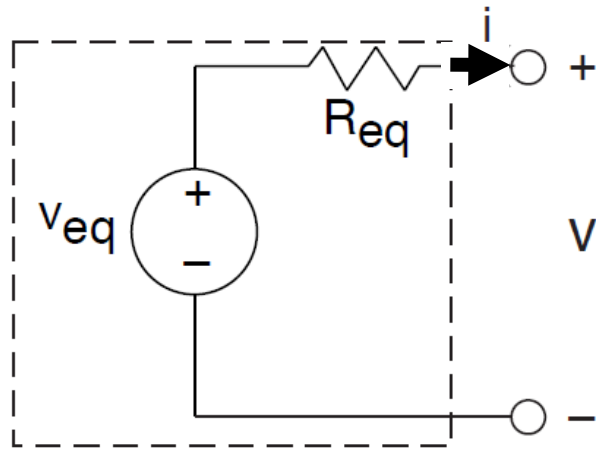
$$V_{AB} = 43 \times \frac{4}{17+4} = \frac{172}{21} \text{ (V)}$$

$$I = -\frac{43}{4+17} = -\frac{43}{21} \text{ (A)}$$



Thevenin model

# Norton Equivalent circuit



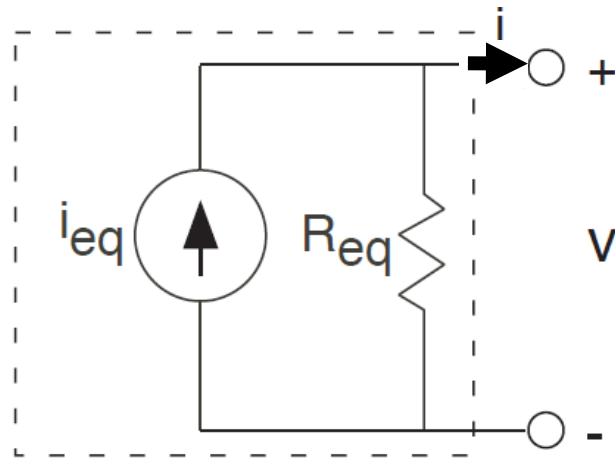
Thévenin

$$v = v_{eq} - i \cdot R_{eq}$$

$$\Rightarrow i = \left[ \frac{v_{eq}}{R_{eq}} \right] - \frac{v}{R_{eq}}$$

$\downarrow i_{eq}$

$$i = i_{eq} - \frac{v}{R_{eq}}$$



$\Rightarrow$  Norton equiv. circ.

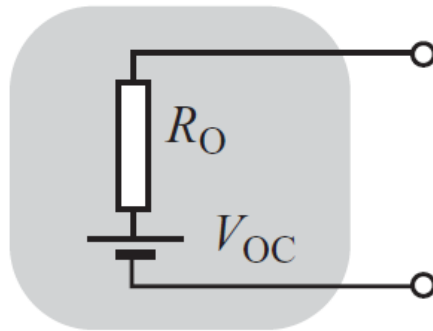


# Norton equivalent circuit

Short-Circuit Current

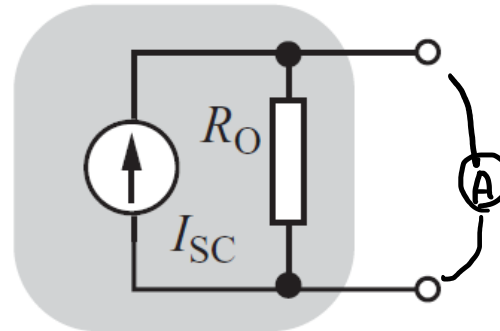
$$I_{sc}$$

$$R_o = \frac{V_{oc}}{I_{sc}}$$



(a)

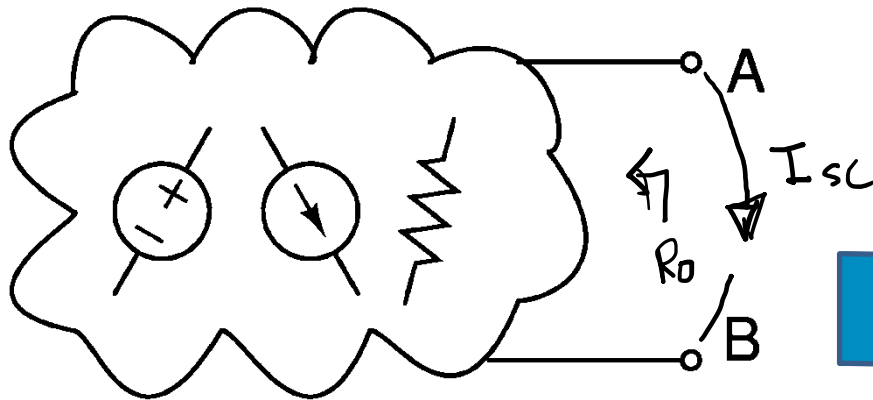
Thévenin



(b)

Norton

# Norton Equivalent circuit



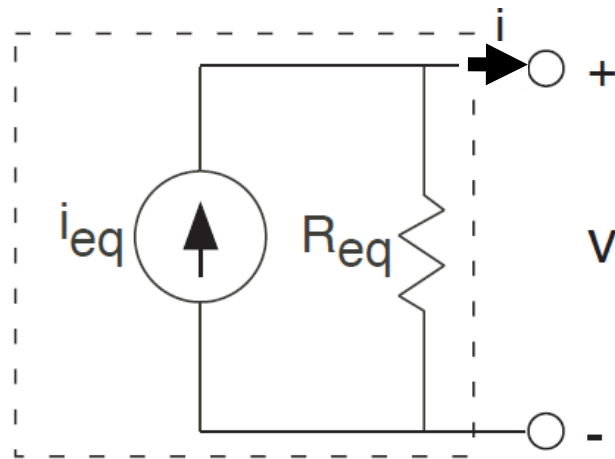
1. Find  $i_{eq}$

$$I_{sc} = i_{eq}$$

2. Find  $R_{eq}$

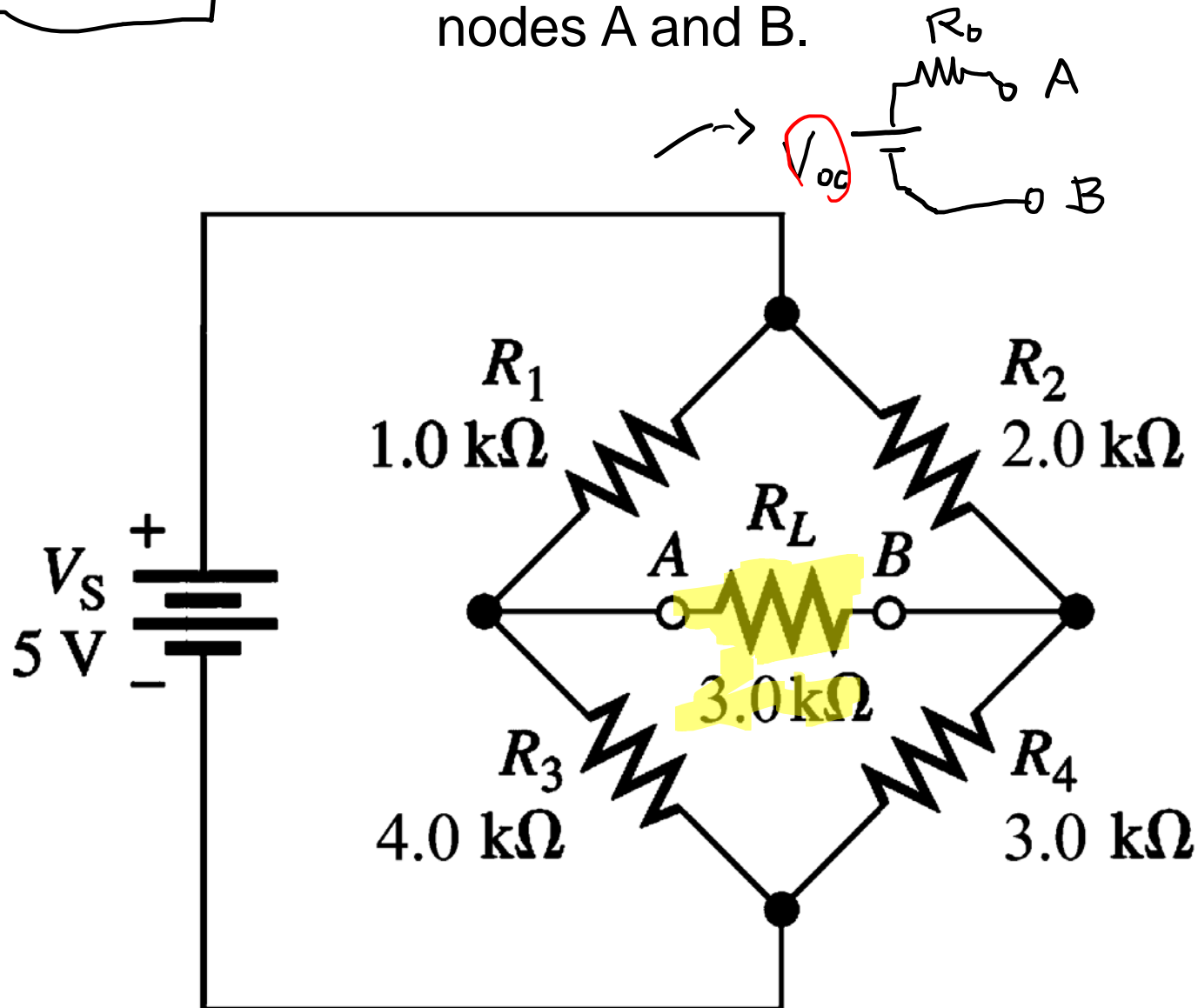
$$R_{eq} = R_0$$

Source  $\rightarrow 0$



## Quiz

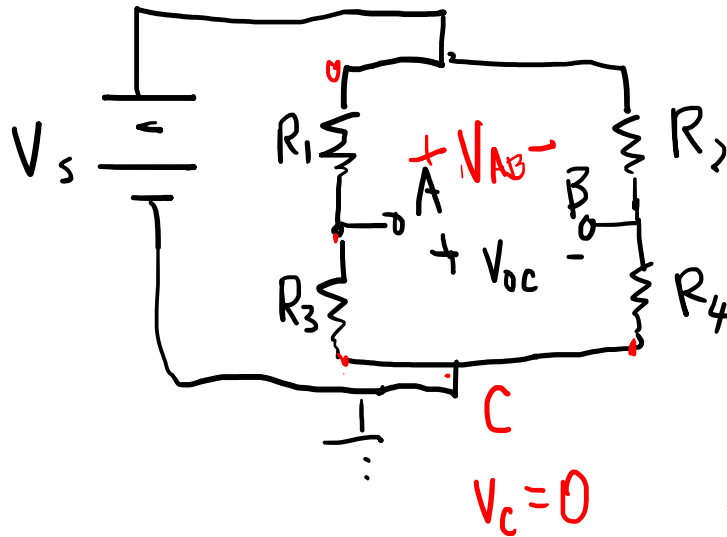
Remove  $R_L$ , draw the Thevenin equivalent circuit between nodes A and B.



$$V_{AB} = V_{AC} - V_{BC}$$

$$= V_A - V_B$$

# Quiz Review



$$V_A = V_s \cdot \frac{R_3}{R_1 + R_3}$$

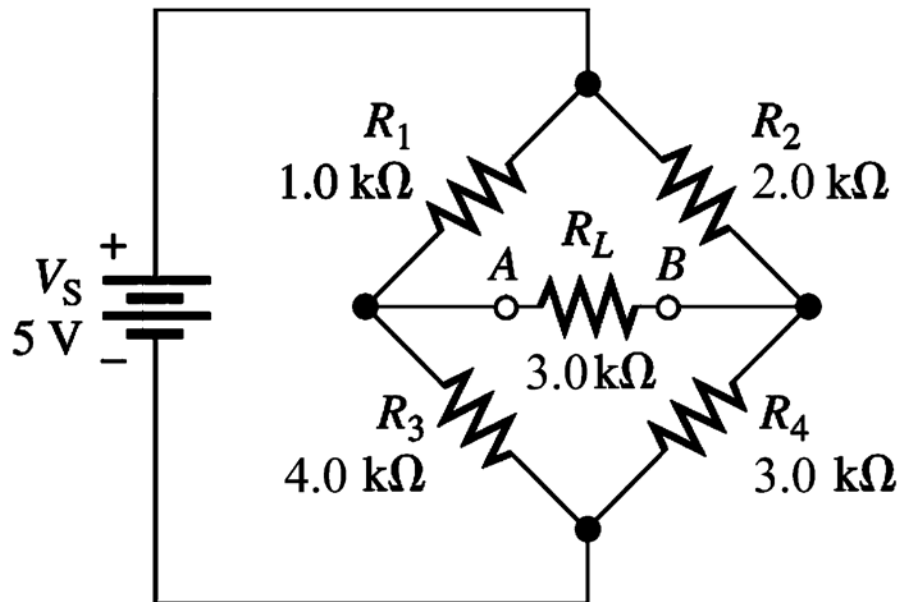
$$V_B = V_s \cdot \frac{R_4}{R_2 + R_4}$$

$$V_{OC} = V_{AB}$$

$$= V_s \left[ \frac{R_3}{R_1 + R_3} - \frac{R_4}{R_2 + R_4} \right]$$

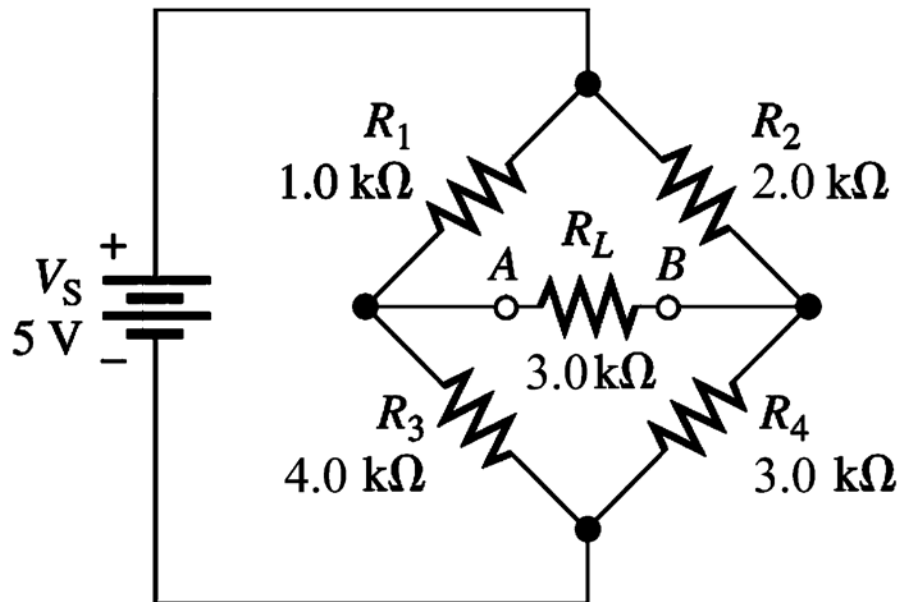
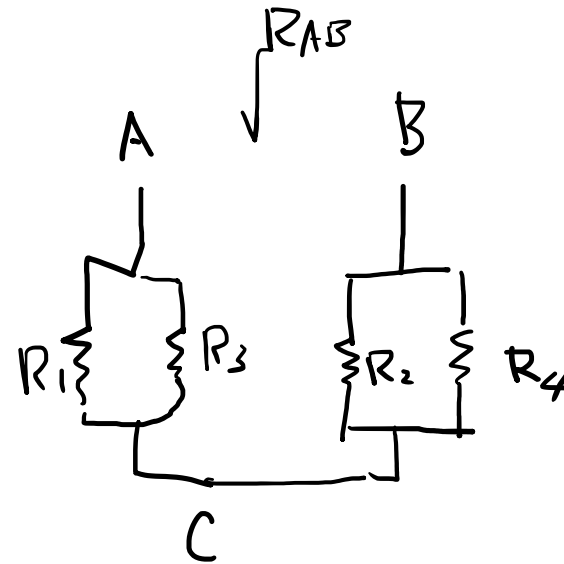
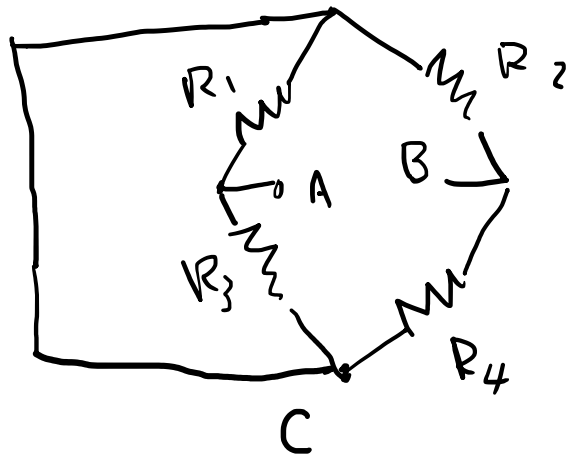
$$= V_s \left[ \frac{4}{5} - \frac{3}{5} \right]$$

$$= V_s \cdot \frac{1}{5} = 1(V)$$



$$V_S \rightarrow 0$$

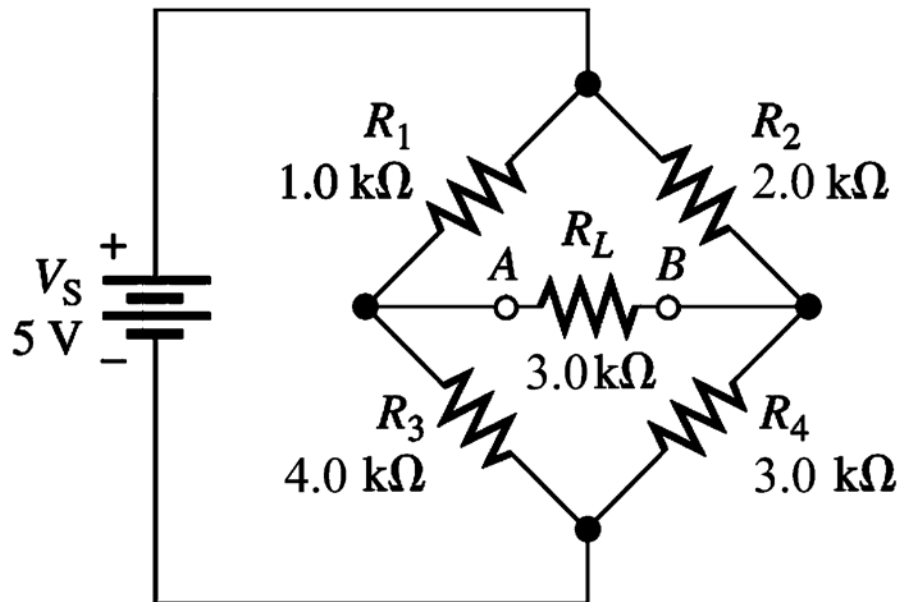
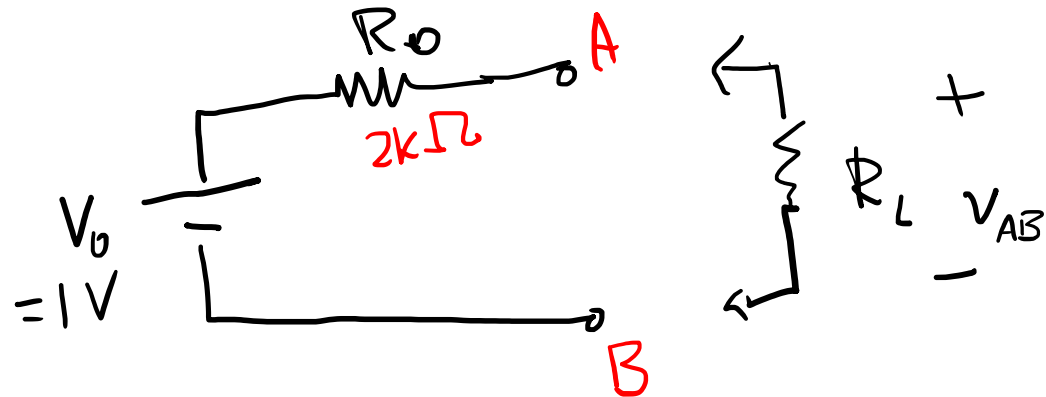
# Quiz Review



$$\begin{aligned} R_0 &= R_{AB} \\ &= (R_1 \parallel R_3) + (R_2 \parallel R_4) \\ &= 2 \text{ k}\Omega \end{aligned}$$

# Quiz Review

Thevenin Equiv. Ckt



$$V_{AB} = V_0 \cdot \frac{R_L}{R_0 + R_L}$$

