2. ① $y''+\lambda^2 y=0 \Rightarrow y=c_1\cos\lambda x+c_2\sin\lambda x$（二階ODE）

∵ 三角函數之正交性 ∴ solutions 必為 orthogonal

② $\int_a^b f(x)g(x)\underset{\text{weight function}}{\underline{w(x)}}dx=0 \Rightarrow f(x),g(x)$ 為 orthogonal

設 1

代入 boundary condition：$\begin{cases} y''=0 \\ y'=-\lambda^2 L \end{cases} \Rightarrow \lambda^2=0 \Rightarrow \int y''\lambda^2 y=0$ ∴ orthogonal ※

3. $f(x)=a_0+\sum_{n=1}^{\infty}\left(a_n\cdot\cos\frac{n\pi}{l}x+b_n\sin\frac{n\pi}{l}x\right)$

(a) $f(x)=a_0+a_1\cdot\cos\frac{\pi}{l}x+a_2\cos\frac{2\pi}{l}x+\cdots+b_1\sin\frac{\pi}{l}x+b_2\sin\frac{2\pi}{l}x+\cdots$

$\Rightarrow \int_{-l}^{l}f(x)\cos\frac{\pi}{l}x=\int_{-l}^{l}a_0\cdot\cos\frac{\pi}{l}x\,dx+\int_{-l}^{l}a_1\cos\frac{\pi}{l}x\,dx+\int_{-l}^{l}a_2\cos\frac{2\pi}{l}x\cdot\cos\frac{\pi x}{l}dx+\cdots$

$\therefore a_1=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{\pi}{l}x\cdot dx \Rightarrow a_n=\frac{1}{l}\int_{-l}^{l}f(x)\cos\frac{n\pi}{l}x\,dx$ ※

$\int_{-l}^{l}f(x)\sin\frac{\pi}{l}x=\cancel{\int_{-l}^{l}a_0\sin\frac{n\pi}{l}x\,dx}^{0}+\sum_{n=1}^{\infty}\left(\cancel{\int_{-l}^{l}a_n\cdot\cos\frac{n\pi}{l}x\sin\frac{n\pi}{l}x\,dx}^{0}+\int_{-l}^{l}b_n\sin\frac{n\pi}{l}x\cdot\sin\frac{n\pi}{l}x\,dx\right)$

$\Rightarrow b_n=\frac{1}{l}\int_{-l}^{l}f(x)\sin\frac{n\pi}{l}x\,dx$ ※

$\Rightarrow \int_{-l}^{l}f(x)dx=\int_{-l}^{l}a_0\,dx+\sum_{n=1}^{\infty}\left(\cancel{\int_{-l}^{l}a_n\cos\frac{n\pi}{l}x}^{0}+\cancel{\int_{-l}^{l}b_n\sin\frac{n\pi}{l}x}^{0}\right)=a_0\cdot 2l$

$\Rightarrow a_0=\frac{1}{2l}\int_{-l}^{l}f(x)dx$ ※

（續寫轉背頁）