Problems and Solutions (Chapter 3)

1. A wireless receiver with an effective diameter of 250 cm is receiving signals at 20 GHz from a transmitter that transmits at a power of 30 mW and a gain of 30 dB.

   (a) What is the gain of the receiver antenna?

   (b) What is the received power if the receiver is 5 km away from the transmitter?

   [Solution]

   Given

   d_e = Effective diameter = 250 cm.

   f_c = Carrier frequency = 20 GHz.

   P_t = Transmitter power = 30 mW

   G_t = Transmitter gain = 30 dB = 1000.

   d = Distance of receiver = 5 km.

   A_e = Effective area = $(\frac{\pi d_e^2}{4}) = 4.91\ \text{m}^2$

   λ = Wavelength = $\frac{c \text{ (speed of light)}}{f_c} = 0.015$ m

   (a) G_r = Receiver antenna gain = $\frac{4\pi A_e}{\lambda^2} = 2.74 * 10^5 = 54.38$ dB.

   (b) P_r = Received power at distance of 5 km = $\frac{A_e G_t P_t}{4\pi d^2} = 4.69 * 10^{-7}$ Watts.

2. Consider an antenna transmitting a power of 5 W at 900 MHz. Calculate the received power at a distance of 2 km if propagation is taking place in free space.

   [*Solution*]

   Transmitted power = $P_t = 5$ W

   Carrier frequency = 900 MHz $\Longrightarrow \lambda = \frac{c}{f} = 0.33$ m

   $d = 2$ km

   Assuming unit gain in free space model, i.e., $G_t = G_r = 1$

   Received power can be calculated by the formula

$$
\begin{aligned}
P_r &= \frac{G_t G_r P_t}{\left(\frac{4\pi d}{\lambda}\right)^2} \\
&= 8.8 \times 10^{-10}\ \text{W}
\end{aligned}
$$

3. In a cellular system, diffraction, reflection, and direct path take different amount of time for the signal to reach a MS. How do you differentiate and use these signals? Explain clearly.Compute the level crossing rate with respect to the rms level for a vertical monopole antenna, assuming the Rayleigh faded isotropic scattering case. The receiver speed is 20 km/hr and the transmission occurs at 800 MHz.

   [Solution]

   Diffraction results in small-scale fading and reflection results in large scale fading. So on receiving the different signals, we can study the spatial and temporal variations of the signal strengths with respect to distance traveled to understand about the diffracted and reflected signals. If the variations change fast with respect to time and distance, then we conclude it to be diffraction and reflection otherwise.

4. The transmission power is 40 W, under a free space propagation model,

   (a) What is the transmission power in unit of dBm?

   (b) The receiver is in a distance of 1000 m, what is the received power, assuming that the carrier frequency $f_c = 900$ MHz and $G_t = G_r = 1$ dB?

   (c) Express the free space path loss in dB.

   [Solution]

   (a) $10 \times \log(40 \times 1000) = 46$ dBm.

   (b)

$$\begin{aligned} P_r &= \frac{G_t G_r P_t}{\left(\frac{4\pi d}{\lambda}\right)^2} \\ &= \frac{40 \times 1 \times 1 \times \left(\frac{1}{3}\right)^2}{(4 \times \pi \times 1000)^2} \\ &= 2.82 \times 10^{-8} \text{ W.} \end{aligned}$$

   (c) $\mathrm{Pr}\,(\mathrm{dB}) = 10 \times \log\left(2.82 \times 10^{-8}\right) = -75$ dB.

5. A receiver is tuned to 1 GHz transmission and receives signals with Doppler frequencies ranging from 10 Hz to 50 Hz when moving at a speed of 80 km/hr. What is the fading rate?

   [Solution]

   Moving speed $v = 80$ km/hr. $= 22.22$ m/sec

   $v = 1$ GHz

   Thus, $\lambda = c/v$,

   where c is free space propagation speed and equals to 3×10^8 m/sec.

$\lambda = 0.1$ m

Therefore, the fading rate

$$\begin{aligned} N(rm) &= \frac{2v}{\lambda} \\ &= \frac{2 \times 22.22}{0.1} \\ &= 444.4 \text{ Hz.} \end{aligned}$$

Here we are not considering the Doppler shift because the θ is not given.

6. What does a small delay spread indicate about the characteristics of a fading channel? If the delay spread is 1 microsecond, will two different frequencies that are 5 MHz apart, experience correlated fading?

   [Solution]

   A small delay spread indicates that smearing or spreading out effect is less. The delay spread determines to what extent the channel fading at two different frequencies f_1 and f_2 are correlated. Small delay spread indicates larger coherence bandwidth and hence correlated fading.

   1 microsecond delay spread indicates

   $B_c =$ Coherence bandwidth $= \frac{1}{(2\pi * 10^{-6})} = 159.15$ kHz.

   Since $|f_1 - f_2| > B_c$, correlated fading will not be experienced.

7. Consider an antenna transmitting at 900 MHz. The receiver is traveling at a speed of 40 km/h. Calculate its Doppler shift.

   [Solution]

   33.67 Hz

8. Repeat Problem 3.6. Calculate the average fading duration if $\rho = 0.1$.

   [Solution]

$$\begin{aligned} \overline{\tau} &= \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \\ &= \frac{e^{0.1^2} - 1}{0.1 \times f_m \sqrt{2\pi}} \\ &= \frac{0.01}{0.25 f_m} \\ &= \frac{1}{25 f_m}, \end{aligned}$$

where f_m is the Doppler frequency.

9. Describe the consequence of the Doppler effect on the receiver in an isotropic scattering environment. Based on your description, speculate on the meaning of the term "Doppler Spread".

   (a) Is the term "Doppler Spread" more appropriate in describing the channel than "Doppler Shift" in a scattering environment? Why?

   (b) Observe the inverse relationship that exists between "Coherence Bandwidth" and "Delay Spread" in a wireless channel. Attempt to similarly define a term "Coherence Time" that has an inverse relationship with the "Doppler Spread". What information does this term give about the channel?

   [Solution]

   The multiple reflections of signals causes a dynamic change in propagation time. Each reflection received is liable to be suffering a different Doppler shift. This means that a carrier transmitted on a single frequency will receive multiple reflections and each reflection will arrive at the receiver shifted in frequency by a different amount. The difference between the highest shift and the lowest shift gives the Doppler spread.

   (a) Since the term Doppler spread determines the difference between highest and lowest Doppler shifts, it doesn't inform about the maximum Doppler shift experienced. So Doppler spread is more appropriate in describing the channel than "Doppler shift" in a scattering environment.

   (b) Delay spread and coherence bandwidth are parameters which describe the time dispersive nature of the channel in a local area. However, information about the time varying nature of the channel caused by either relative motion between the mobile and base station, or by movement of objects in the channel is not given. Coherence time is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.

       Coherence time is the time duration over which two received signals have a strong potential for amplitude correlation. If the reciprocal bandwidth of the baseband signal is greater than the coherence time of the channel, then the channel will change during the transmission of the baseband message, thus causing distortion at the receiver.

10. How can you compensate for the impact of Doppler effect in a cellular system? Explain.

    [Solution]

    We can compensate the impact of Doppler effect at receiver by adaptive detection to track the channel variations.

11. How is radio propagation on land different from that in free space?

[Solution]

Propagation in free space does not have any obstacles and hence it characterizes the most ideal situation for propagation. Whereas, radio propagation on land may take place close to obstacles which cause reflection, diffraction, scattering.

12. What is the differences between fast fading and slow fading?

[Solution]

Slow fading is caused by movement over distances large enough to produce gross variations in overall path length between base station and mobile station. In other words, the long term variation in the mean level is known as slow fading.

Rapid fluctuations caused by local multipath are known as fast fading. It is short-term fading.

13. Path loss, fading, and delay spread are the three important radio propagation issues. Explain why those issues are important in a cellular system?

[Solution]

The wave propagation in multipath channel depends on the actual environment including factors like antenna height, profile of the buildings, road and the terrain. Therefore we need to describe the behavior of mobile radio channels using a good and relevant statistical mechanism. Thus this is provided by studying the path loss and fading phenomenon to understand about the characteristics of the land and terrain profile. The delay spread is also necessary to understand about the propagation channel, whether the signal is propagating through the frequency selective or the flat fading channels.

14. A BS has a 900 MHz transmitter and a vehicle is moving at the speed of 50 mph. Compute the received carrier frequency if the vehicle is moving

    (a) Directly toward the BS.
    (b) Directly away from the BS.
    (c) In a direction which is 60 degree to the direction of arrival of the transmitted signal.

[Solution]

Given

f_c = Carrier frequency = 900 MHz

λ = Wavelength $= \frac{c}{f_c} = 0.3333$ m

v = Velocity = 50 mph = 22.22 m/s.

Let Doppler shift frequency be denoted by f_d.

(a) $\theta = 180$, (direction is towards BS.)
$f_d = \frac{v}{\lambda}\cos\theta = -67.06$
Received carrier frequency:
$f_r = f_c - f_d = 900 * 10^6 + 67.06 = 900.000067 * 10^6$ MHz.

(b) $\theta = 0$, (direction away from BS.)
$f_d = \frac{v}{\lambda}\cos\theta = 67.06$
Received carrier frequency:
$f_r = f_c - f_d = 900 * 10^6 - 67.06 = 899.99993 * 10^6$ MHz.

(c) $\theta = 60$, (direction is towards BS.)
$f_d = \frac{v}{\lambda}\cos\theta = -33.53$
Received carrier frequency:
$f_r = f_c - f_d = 900 * 10^6 + 33.53 = 900.000033 * 10^{10}$ MHz.

15. What is the diversity reception? How can it be used to combat multipath?

[Solution]

Diversity reception: Two frequencies, that are larger than the coherence bandwidth, fade independently. This concept is used in diversity reception. Therefore, it is a radio reception in which a resultant signal is obtained by combining or selecting signals, from two or more independent sources. They are modulated with identical information-bearing signals. (i.e., multiple copies of same message are sent using different frequencies.)

Combating with multipath: The signals are chosen such that the frequencies are larger than coherence bandwidth, therefore their fading characteristics are independent at any given instant. This minimizes effects of correlated fading.

16. What is the role or usefulness of reflected and diffracted radio signals in a cellular system? Explain with suitable examples.

[Solution]

Role of reflected and diffracted waves help in signal propagation even in worst cases when there are no LOS component between Tx and Rx. Ex. Tx and Rx separated by the hilly terrains, communication in urban areas with high rise buildings.

17. What is the intersymbol interference (ISI)? Does it affect the transmission rate of a digital channel? Explain clearly.

[Solution]

Inter-symbol interference (ISI) is caused by time delayed multipath signals, where in the second multipath signal is delayed such that it is received during the next symbol. Therefore, the received signals tend to get elongated and smeared into each other. Thus, ISI is caused by time delayed multipath signals.

The time for which second multipath signal is delayed is delay spread T_d. In time-dispersive medium, transmission rate R for digital transmission is limited by this delay spread T_d. Therefore, for a low-error-rate performance,

$$R < \frac{1}{2T_d}$$

ISI also affects burst error rate of channel.

18. A MS is not in the direct line of sight of a BS transmitting station. How is the signal received? Explain.

    [Solution]

    With the help of multipath fading signals can reach the places inside buildings and places which are hidden by tall buildings and trees. With multipath fading signals are reflected, diffracted and scattered which results in multiple low power signals traveling in different directions.

19. Consider two random variables X and Y that are independent and Gaussian with identical variances. One is of zero mean and the other is of mean μ. Prove that the density function of $Z = \sqrt{X+Y}$ is Rician distributed.

    [Solution]

    Suppose $T = X^2 + Y^2$. Since X and Y are statistically independent Gaussian random variables with mean 0 and μ, and common variance σ^2, T has a noncentral chi-square distribution with oncentrality parameter $s^2 = 0 + \mu^2 = \mu^2$. The pdf of T is given by

    $$p_T(t) = \frac{1}{2\sigma^2} e^{-\frac{t+\mu^2}{2\sigma^2}} I_0\left(\frac{\sqrt{t}\mu}{\sigma^2}\right), \qquad t \geq 0.$$

    Now we define a new random variable $R = \sqrt{T}$. The pdf of R is given by

    $$p_R(t) = \frac{t}{\sigma^2} e^{-\frac{t^2+\mu^2}{2\sigma^2}} I_0\left(\frac{t\mu}{\sigma^2}\right), \qquad t \geq 0.$$

    This is the pdf of a Ricean-distributed random variable.

20. What causes the intersymbol interference and how can you reduce the intersymbol interference in the wireless communication system?

    [Solution]

    Intersymbol interference is caused by time-delayed multipath signals. The most common methods to reduce intersymbol interference are: guard time, pulse shaping, signal encoding and equalization.