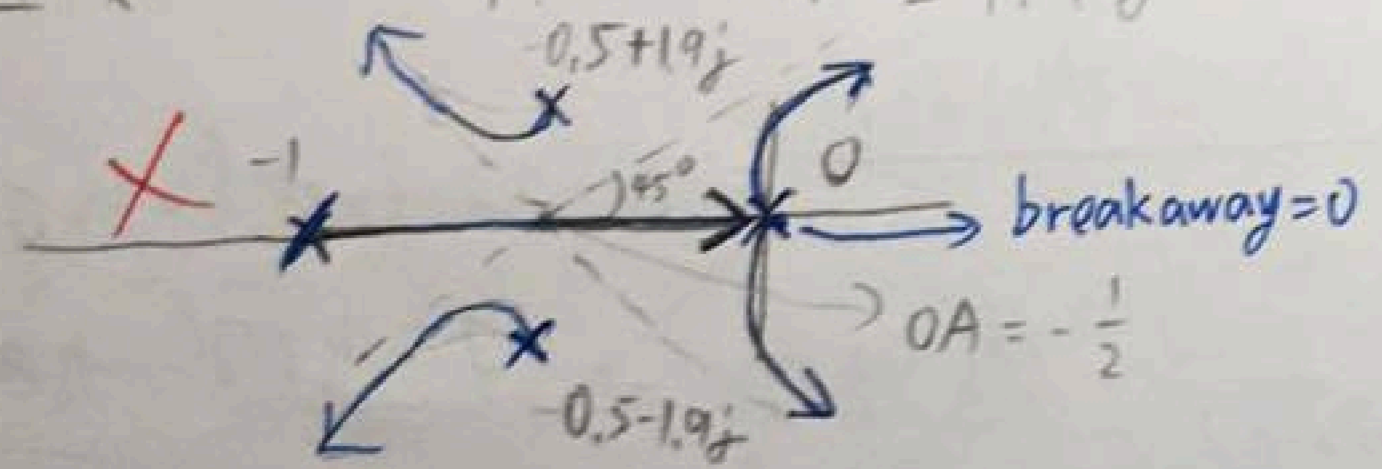
院系 College	工 學院 College	工科 系 Department	3 年 Year	班 Class	評閱成績 Score		of Instructor

1.(a) 極點：0、-1、$-0.5\pm1.9j$



$OA = \frac{0-1-0.5+1.9j-0.5-1.9j}{4-0} = -\frac{1}{2}$

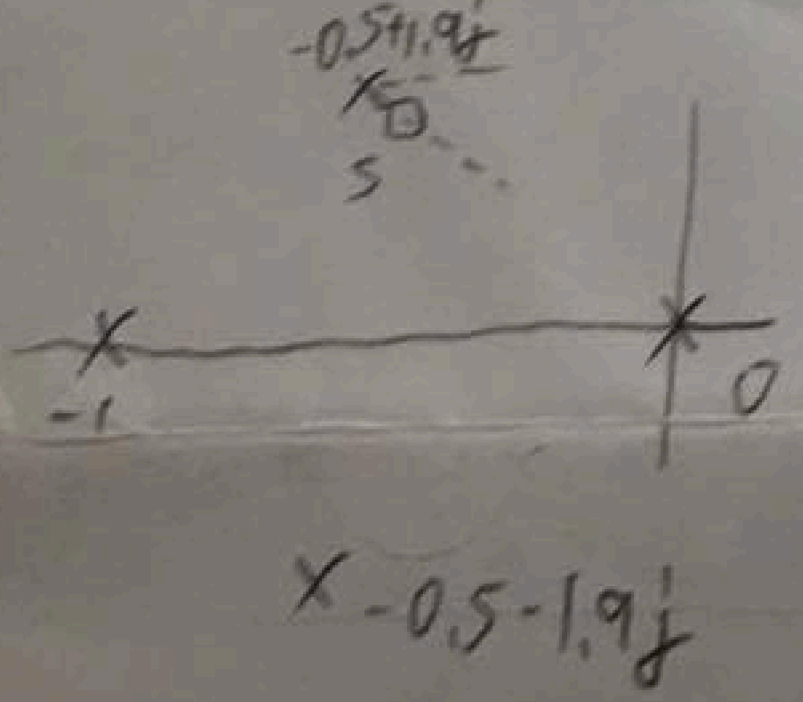
$\phi_{asy} = \frac{2\pi l+\pi}{4-0} = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} = \pm45^\circ, \pm135^\circ$

breakaway point: $\frac{k}{s(s+1)(s^2+s+4)} = -1 \Rightarrow k = -s(s^3+s^2+4s+s^2+s+4) = -(s^4+2s^3+5s^2+4)$

$\frac{dk}{d\sigma} = 0 \Rightarrow \frac{d(-\sigma^4-2\sigma^3-5\sigma^2-4)}{d\sigma} = 0 \Rightarrow -4\sigma^3-6\sigma^2-10\sigma = 0$ $\quad \sigma = -0.75\pm1.392j, 0$

↑ breakaway point

(b) 令 s 在 $-0.5+1.9j$



$\angle G(s) = \angle k - \angle(s) - \angle(s+1) - \angle(s+0.5-1.9j) - \angle(s+0.5+$

$-\cancel{180^\circ} = 0^\circ - (\cancel{180^\circ} - \tan^{-1}\frac{1.9}{0.5}) - \tan^{-1}\frac{1.9}{0.5} - \angle dep - 90^\circ$

$\therefore \angle dep = -90^\circ$

(c) 令 jw 帶入 $G(s) = -1 \Rightarrow \frac{k}{s^4+5s^2+s(2s^2+4)} = -1 \Rightarrow \frac{k}{\ldots} = -1 \ldots$

$-0.5-1.9j$

(c) 令 jw 帶入 $G(s)=-1 \Rightarrow \dfrac{k}{s^4+5s^2+s(2s^2+4)}=-1 \Rightarrow \dfrac{k}{(w^4-5w^2)+jw(-2w^2+4)}=-1 \cdots ①$

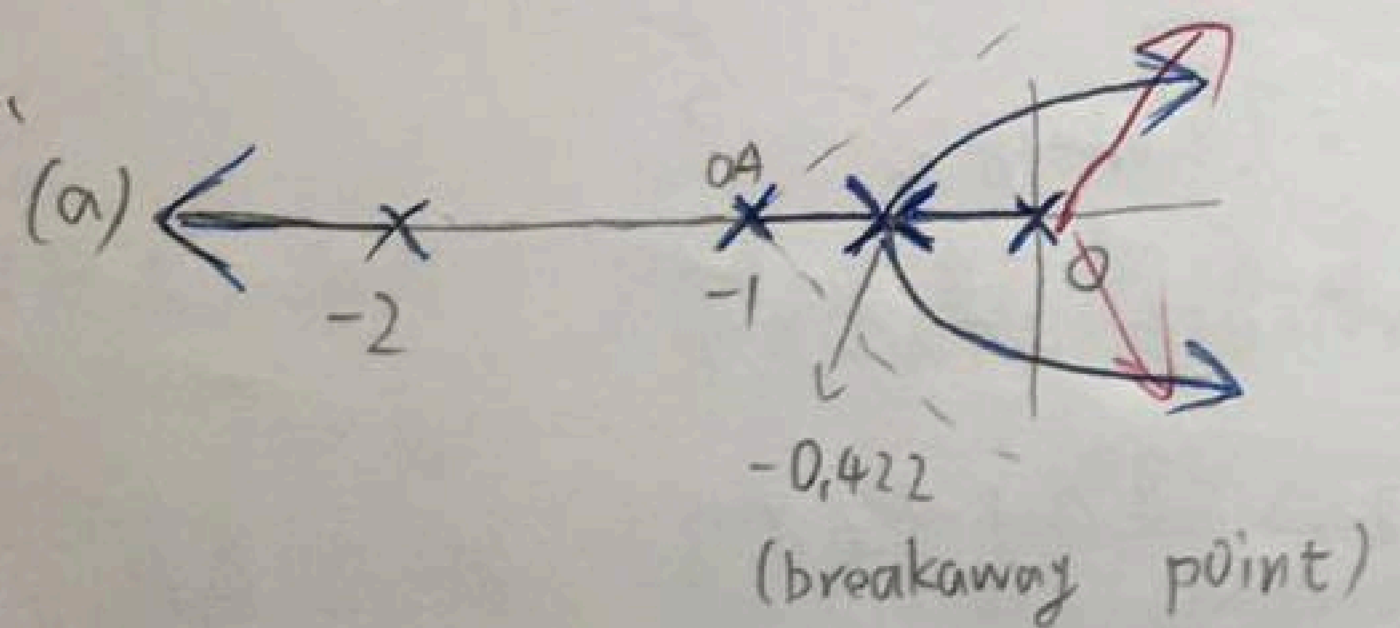
$\therefore w=\underset{(不合)}{0}$ or $\pm\sqrt{2}$ 再代回①式 $\Rightarrow \dfrac{k}{(4-10)+0}=-1 \Rightarrow k=6$

故 $0<k<6$（系統穩定）

-3

2、

(a)



$OA=\dfrac{-0-2}{3}=-1$ $\phi_{asy}=\dfrac{2\pi l+\pi}{3-0}=\dfrac{\pi}{3}、\pi、\dfrac{5\pi}{3}$

$\pm 60^\circ、180^\circ$

breakaway point：$\dfrac{1}{s+2}+\dfrac{1}{s+1}+\dfrac{1}{s}=0$

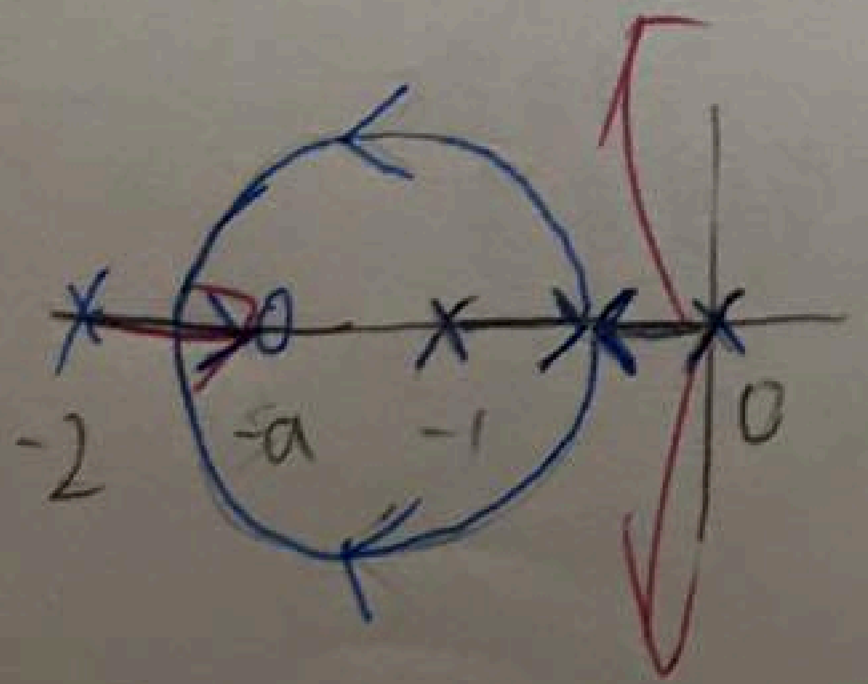
$\Rightarrow s^2+s+s^2+2s+s^2+3s+2=0 \Rightarrow 3s^2+6s+2=0$

$\therefore s=-0.422、-1.5$

∵ root-locus上 poles 会往右半平面跑，∴当k大時，系統不穩定

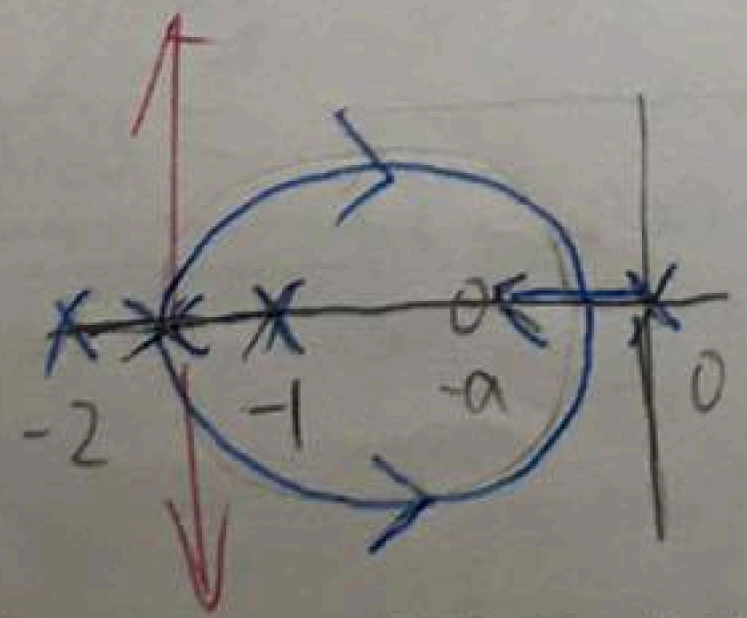
(b) 畫出 root locus 來：分兩种情形 ① $\overset{-a}{s}\in[-2,-1]$ ② $\overset{-a}{s}\in[-1,0]$

①



-3

②



由此2种情形來看，發現k增大，root-locus都不会跑到右半平面，∴会使系統穩定

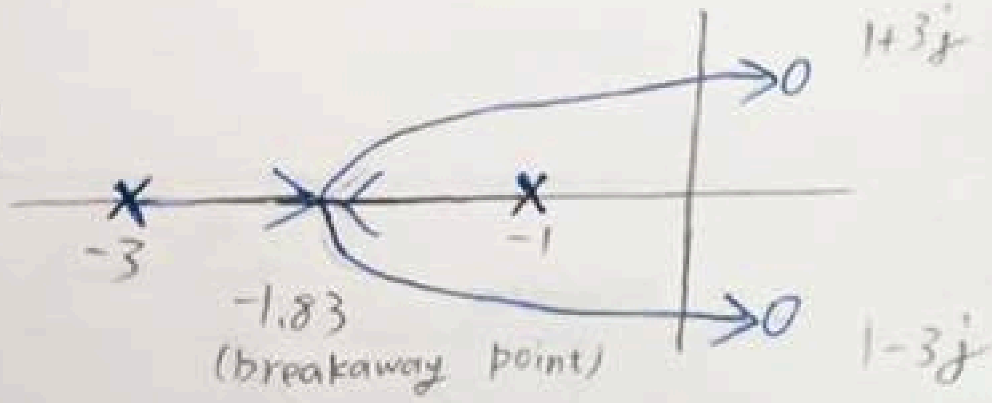
（續寫轉背頁）

學號：

姓名：

3.

(a)



k > 0

(会從 ±∞ 找到 zero)

breakaway point:

$k = \frac{s^2+4s+3}{-(s^2-2s+10)} \Rightarrow \frac{dk}{d\sigma} = 0$

(∵ 有zero点存在，pole 即使往 +∞ or -∞跑，最後要回到 zero)

$\Rightarrow \frac{d\left(\frac{\sigma^2+4\sigma+3}{-\sigma^2+2\sigma-10}\right)}{d\sigma} = 0 \Rightarrow \frac{(-\sigma^2+2\sigma-10)(2\sigma+4)-(\sigma^2+4\sigma+3)(-2\sigma+2)}{(-\sigma^2+2\sigma-10)^2} = 0 \Rightarrow 6\sigma^2-14\sigma-46=0$

$\sigma = 4.17 \cdot \underline{-1.83}$

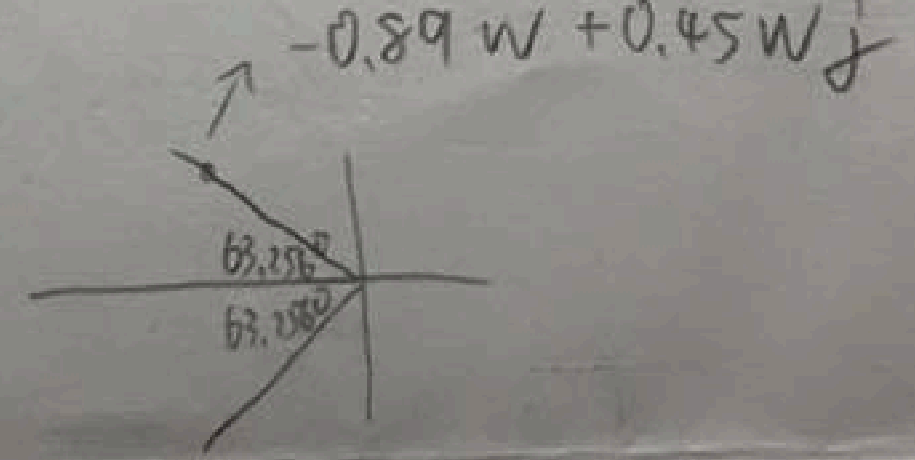
(b) 令 $s = jw$ 帶入 $G(s) = -1 \Rightarrow \frac{k(-w^2-2jw+10)}{(-w^2+4jw+3)} = \frac{k(-w^2+10)+jk(-2w)}{(-w^2+3)+j(4w)} = -1 \cdots ①$

$\therefore \frac{\cancel{k}(-w^2+10)}{-w^2+3} = \frac{\cancel{k}(-2w)}{4w} \Rightarrow -2w^2+20 = w^2-3 \Rightarrow 3w^2 = 23 \quad w = \pm 2.7689$

(帶回 ① 式)

$\Rightarrow \frac{k}{-2} = -1 \Rightarrow \therefore \underline{k=2}$

(c) $\because \zeta = 0.45 = \cos\theta \Rightarrow \therefore \theta = 63.256^0$



$\therefore s = -0.89w+0.45wj$ 帶入 $G(s) = -1 \Rightarrow \frac{k(0.89^2w^2-2\cdot0.89\cdot0.45w^2j-0.45^2w^2+2\cdot0.89w-2\cdot0.45wj+10)}{0.89^2w^2-2\cdot0.89\cdot0.45w^2j-0.45^2w^2-4\cdot0.89w+4\cdot0.45wj+3}$

$$0.89^2w^2-2\cdot0.89\cdot0.45w^2j-0.45^2w^2-4\cdot0.89w+4\cdot0.45wj+3$$

$$\Rightarrow \frac{k(0.5896w^2-0.801jw^2+1.78w-0.9jw+10)}{0.5896w^2-0.801jw^2-3.56w+1.8jw+3}=-1$$

$$\Rightarrow \frac{\cancel{k}(0.5896w^2+1.78w+10)}{0.5896w^2-3.56w+3}=\frac{\cancel{k}(-0.801w^2-0.9w)}{-0.801w^2+1.8w} \Rightarrow$$

$$\Rightarrow \cancel{-0.5896\cdot0.801w^3}-1.78\cdot0.801w^2-8.01w+0.5896\cdot1.8w^2+1.78\cdot1.8w+18=$$

-5

$$\cancel{-0.5896\cdot0.801w^3}+3.56\cdot0.801w^2-3\cdot0.801w-0.5896\cdot0.9w^2+3.56\cdot0.9w-2.7$$

$$\Rightarrow -2.68542w^2-5.607w+20.7=0$$

$w=1.922$、-4.01（負不合） 代回 $G(s)=-1$

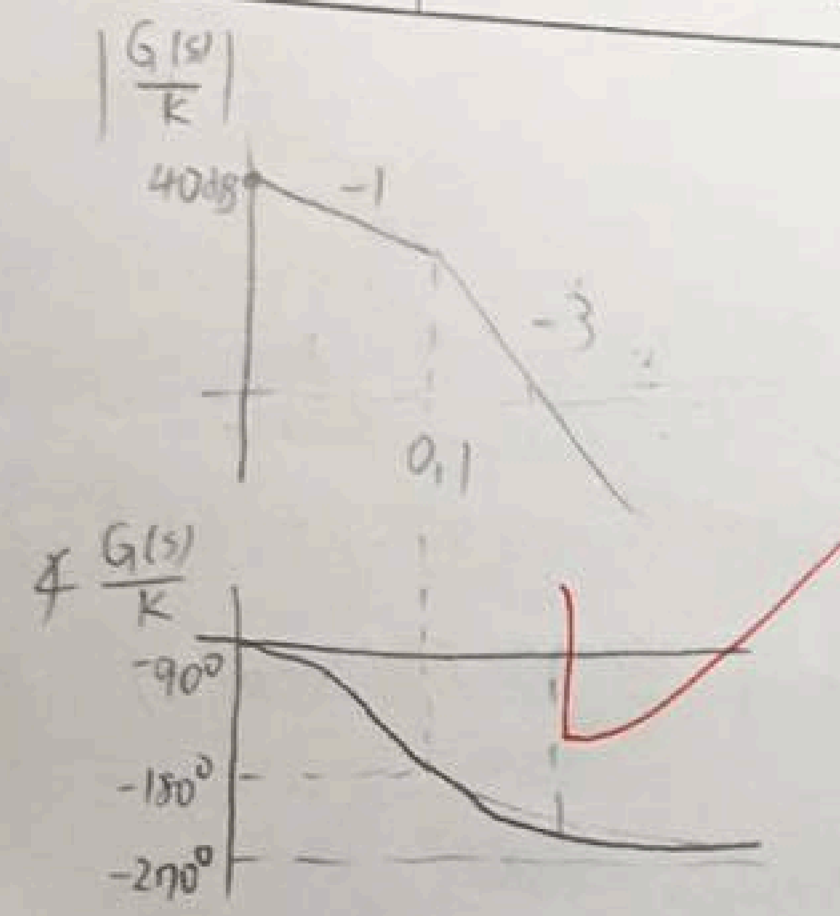
$\therefore k\cdot(-9.3728)=-1$ $\therefore K=0.10669$

4、(a) S在 0 & $\sqrt{0.01}=0.1$ 会有転折，令 $s=jw$，$w=0.1$帶入

$$G(s)=\frac{10}{0.1s^2+s(0.01s^2+1)}=\frac{10}{-0.1w^2+jw(-0.01w^2+1)}=100 \Rightarrow$$

転成dB 40 dB

College	Department	Year Class	評閱成績 Score			



⇐ 波德图

(b) $G(jw) = \dfrac{k \cdot 10}{-0.1w^2 + jw(-0.01w^2+1)}$　　∵ $-20\log|G(jw)| = 10\,dB$ ∴ $|G(jw)| = 0.31622$

∵ 虛部 $= 0$ ∴ $W_{pc} = 0$ or ± 10 ($W_{pc} = 10$)

∴ $\dfrac{|K \cdot 10|}{|-0.1W_{pc}^2 + jW_{pc}(-0.01W_{pc}^2+1)|} = 0.31622 \Rightarrow$ ∴ $K = 0.31622$

(c) $\measuredangle G(s) = -(180^0 - 45^0) = -135^0$

∴ $0^0 - \left(180^0 - \tan^{-1}\dfrac{-0.01w^3+w}{0.1w^2}\right) = -135^0 \Rightarrow W_{gc} = 6.18$

$|G(jw)| = 1 = \dfrac{|10K|}{\sqrt{(-0.1w^2)^2 + (-0.01w^3+w)^2}} \Rightarrow$ ∴ $K = 0.5401$

$$|G(jw)| = 1 = \frac{|10K|}{\sqrt{(1-0.1w^2)^2+(-0.01w^3+w)^2}} \Rightarrow \therefore \underline{K = 0.5401}$$

5.(a) 先求 GM、PM 才能畫出 Nichols Chart

(b)

(c) $G(s) = \dfrac{1000}{s(s^2+11s+10)} \underset{s=jw}{=} \dfrac{1000}{-11w^2+jw(-w^2+10)}$

$\because G(s) = -1 \Rightarrow \therefore$ 虛部 $= 0 \Rightarrow w = \underset{(不合)}{0}$ or $\underset{(負不合)}{\pm\sqrt{10}}$ 帶回原 $G(s)$

(b) $\underline{GM} = -20\log|G(s)| = -20\log 9.0909 = \underline{-19\,(dB)}$

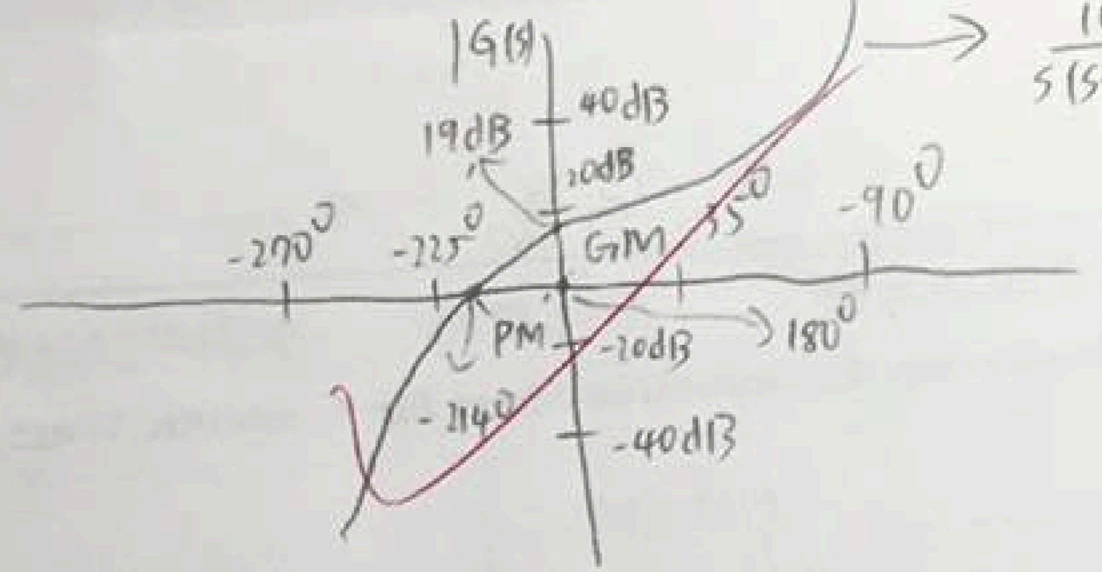
再求 PM：先求 $w_{gc} \Rightarrow \dfrac{1000}{\sqrt{(-11w^2)^2+(-w^3+10w)^2}} = 1 \Rightarrow w_{gc} = 8.664$

$$\angle G(jw_{gc}) = 0^\circ - \left(180^\circ + \tan^{-1}\frac{w^3-10w}{11w^2}\right) = -214^\circ$$

（續寫轉背頁）

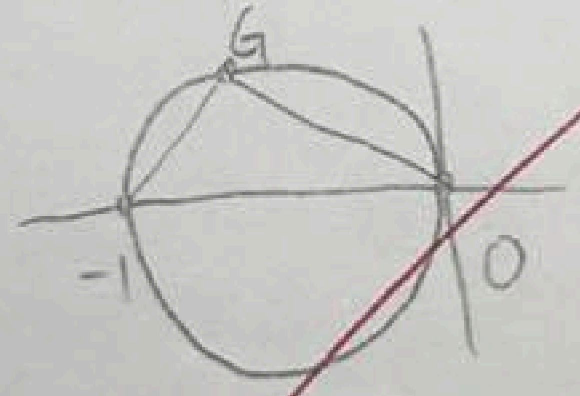
$\therefore \underline{PM} = 180^\circ - 214^\circ = \underline{-34^\circ}$

3. Nichols Chart



(d) 由 (a) 題的 Nichols Chart 可看出 GM < 0 & PM < 0，∵ 交在 19 dB 的位置 (即 GM = -19 dB) & 交在 -214° (即 PM = -34°)，∴ 对於一个系統，只要 GM or PM < 0，就会不穩定，故此系統不穩定

6. (a) $|M| = \frac{|G-(-0)|}{|G-(-1)|}$ ⇒ 图示



所有符合的点形成1个圆

表示一个集合：G，(G到0的距離) ÷ (G到-1的距離) 呈一个常数 (constant)

⇒ 而此常数用 M 来表示

(b) 令 $s = x+jy$ ⇒ $|M| = \frac{|x+jy|}{|1+x+jy|} = \frac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$

$\Rightarrow [(1+2x+x^2)+y^2]\, M^2 = x^2+y^2$

呈一个常数 (constant)

⇒ 而此常数用 M 来表示

(b) 令 $s = x + jy \Rightarrow |M| = \dfrac{|x+jy|}{|1+x+jy|} = \dfrac{\sqrt{x^2+y^2}}{\sqrt{(1+x)^2+y^2}}$

$\Rightarrow [(1+2x+x^2)+y^2]\,M^2 = x^2+y^2$

$\Rightarrow (1-M^2)x^2 - 2M^2x + (1-M^2)y^2 = M^2$

同除 $1-M^2$

$\Rightarrow x^2 - \dfrac{2M^2}{1-M^2}x + y^2 = \dfrac{M^2}{1-M^2}$

同加 $\left(\dfrac{M^2}{1-M^2}\right)^2$

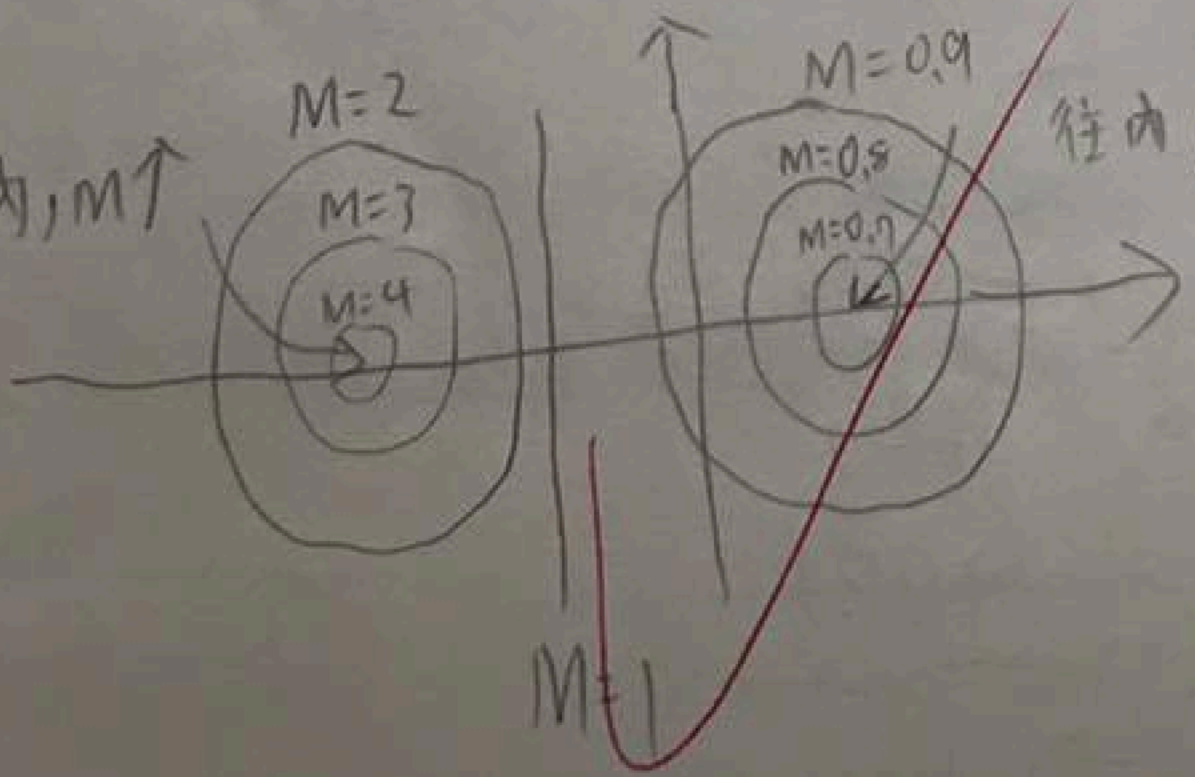
$\Rightarrow x^2 - \dfrac{2M^2}{1-M^2} + \left(\dfrac{M^2}{1-M^2}\right)^2 + y^2 = \dfrac{M^2}{1-M^2} + \left(\dfrac{M^2}{1-M^2}\right)^2$

$\Rightarrow \left(x - \dfrac{M^2}{1-M^2}\right)^2 + y^2 = \dfrac{M^2 \cancel{-M^4} \cancel{+M^4}}{(1-2M^2+M^4)} = \left(\dfrac{M}{1-M^2}\right)^2$

∴ 由上式：圆的方程式可得知 圆心在 $\left(\dfrac{M^2}{1-M^2}, 0\right)$

半径 $\left|\dfrac{M}{1-M^2}\right|$

(c) 往内, M↑



4.