Numerical method
Final Exam (2020/06/22 11:00~13:00)

程式題

Excel 及 Matlab 的檔名務必命名為"學號_題號"（ex: N96081234_9），如有違反者每題皆以**扣兩分**處理。

8. (20%) **(EXCEL only)** Solve the following initial-value problem:

$$y' = y + 2x + 1, \text{for } 0 \le t \le 1, \text{with } y(0) = 1 \text{ and } h = 0.1$$

(a) 3^{rd} order Improvements of Euler's method

$$y_{i+1} = y_i + h(y_i + 2x_i - 1) + \frac{h^2}{2!}(y_i + 2x_i + 1) + \frac{h^3}{3!}(y_i + 2x_i + 1)$$

(b) 4^{th} order Runge-Kutta

$$y_{i+1} = y_i + \frac{1}{6}h(k_1 + 2k_2 + 2k_3 + k_4) \qquad k_1 = f(x_i, y_i) \qquad k_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_1) \qquad k_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}hk_2) \qquad k_4 = f(x_i + h, y_i + hk_3)$$

(d) 5^{th} order Adams-Bashforth open formula

$$y_{i+1} = y_i + h\left[\frac{1901}{720}f_i - \frac{2774}{720}f_{i-1} + \frac{2616}{720}f_{i-2} - \frac{1274}{720}f_{i-3} + \frac{251}{720}f_{i-4}\right] + O(h^6)$$

(e) 4^{th} order Adams-Moulton closed formula.

$$y_{i+1} = y_i + h[\frac{9}{24}f_{i+1} + \frac{19}{24}f_i - \frac{5}{24}f_{i-1} + \frac{1}{24}f_{i-2}] + O(h^5)$$

9. (12%) **(MATLAB only)** Use TDMA with grid points (21, 21), (51, 51), and (101, 101) to solve the given equation

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 100 \cdot \sin 2x \cdot \cos y \qquad Lx = 4\pi \text{ m}, Ly = 2\pi \text{ m}$$

Boundary conditions: $T_{x=0} = 10,\ T_{y=0} = 10,\ T_{x=L_x} = 10,\ T_{y=L_y} = 10$

Use the Matlab command **"contour"** and **"axis equal"** to plot the contour plots at different meshes.

10. (8%) **(MATLAB only)** Use the Monte Carlo Method with samples of $N = 10^1,\ 10^2,\ 10^3,\ 10^4,\ 10^5,$ and 10^6 to estimate

(a) the area of the grey circle

(b) the π value

(c) the relative errors of the area and the π value.

Please define new variables **"C_area"** and **"C_pi"** to contain 6 different area and π values. Also plot the relative error distributions with respect to the number of samples in the loglog scale. The center coordinate of the circle is (1,1) and the diameter is 4.

(**Note: must use the Matlab command "rand" in your code.**)