Engineering Science Engineering mathematics Test #2 2015/06/10

(10) 1. If integral transforms were used to solve PDEs, what choices do we have so far? How could we choose a proper one? Give your explanations.

(10) 2. Using the idea learned in exact ODEs, solve (a) $\frac{\partial u(x,,y)}{\partial x} = x + y$;

(b) $\frac{\partial u(x,y,z)}{\partial y} = 2 + xy + xz$

(10) 3. For second order linear PDEs, explain the concepts of elliptic, parabolic, and hyperbolic PDEs, and give one example for each type of PDEs.

(10) 4. Using the energy conservation law and Fourier's conduction law to derive the following equation and use the giving conditions to solve it.

$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2}, 0 \leq x \leq L, t \geq 0;\ \ T(0,t) = T(L,t) = T_0 = \text{constant};$ and

$T(x,0) = T_0[\frac{x(L-x)}{L^2} + 1]$

(10) 5.Solve$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}, 0 \leq x, 0 \leq t; u(0,t) = u_0, u(x,0) = 0$

(10) 6. Explain the procedure to solve $\nabla^2 u(x,y,z) = 0, 0 \leq x, y, z \leq L$ with different constant temperatures specified on the six boundaries. (DON'T solve it!)

(10) 7. Giving a wave form of $\sin mx$, if the wave travels to the left (i.e., in the direction of negative x) with a constant phase speed C, show that its general form is $\sin m(x + ct)$; and $m = \frac{2\pi}{\lambda}$, where λ is the wavelength. Derive its governing PDE.

(10) 8. Choose a wave phenomenon you like, derive its wave equation in PDE, try to solve it by providing proper conditions by yourself, and explain the solution you have.

(10) 9.$(1 - x^2)\frac{d^2 u}{dx^2} - 2x\frac{du}{dx} + 2u = 0;\ |x| < 1$

(10) 10. Solve $x^2 \frac{\partial^2 u}{\partial x^2} + x\frac{du}{dx} + \left(x^2 - \frac{1}{4}\right) u = 0$