E927500 Electric Circuit Analysis Spring 2017

Solution to Midterm Exam I

1 (a) T

(b) F

(c) F

(d) T

(e) F , $V_1 = -w^2 V$

2. $$\frac{dV(t)}{dt} + 5V(t) + 4\int V(t)dt = 20\sin(4t+10^\circ) = 20\cos(4t-80^\circ) \quad , W=4$$

$$\Rightarrow jwV + 5V + \frac{4V}{jw} = 20\angle -80^\circ$$

$$V(jw + 5 + \frac{4}{jw}) = 20\angle -80^\circ$$

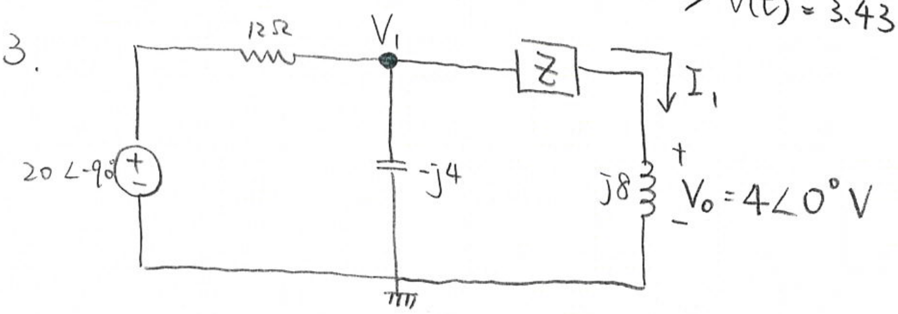
$$V(j4 + 5 + \frac{1}{j}) = 20\angle -80^\circ$$

$$V(j4 + 5 - j) = 20\angle -80^\circ$$

$$V = \frac{20\angle -80^\circ}{5+j3} = \frac{20\angle -80^\circ}{5.831\angle 30.96} = 3.43\angle -110.96^\circ$$

$$\Rightarrow V(t) = 3.43\cos(4t - 110.96^\circ)V$$

3.



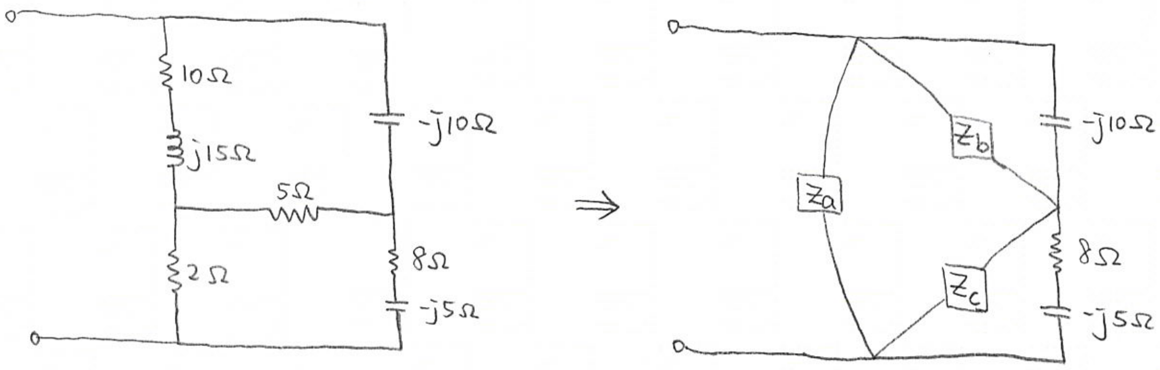
$$I_1 = \frac{4}{j8} = -0.5j$$

$$\frac{V_1 - 20\angle -90^\circ}{12} + \frac{V_1}{-j4} + (-0.5j) = 0 \Rightarrow V_1(\frac{1}{12} + j\frac{1}{4}) + j\frac{5}{3} - j0.5 = 0$$

$$V_1 = \frac{-j\frac{5}{3} + j0.5}{\frac{1}{12} + j\frac{1}{4}} = \frac{1.1667\angle -90^\circ}{0.2635\angle 71.56^\circ} = 4.425\angle -161.56^\circ = -4.2 - j1.4$$

$$Z = \frac{V_1 - V_o}{I_1} = \frac{-4.2 - j1.4 - 4}{-j0.5} = \frac{-8.2 - j1.4}{-j0.5} = 2.8 - j16.4 = 16.64\angle -80.31^\circ \ \Omega$$

4.



$$Z_a = \frac{(10+j5)\times 5 + (10+j15)\times 2 + 2\times 5}{5} = 16 + j21$$

$$Z_b = \frac{80 + j105}{2} = 40 + j52.5$$

$$Z_c = \frac{80 + j105}{10 + j15} = 7.31 - j0.46$$

$$Z_b' = Z_b // (-j10) = \frac{(40 + j52.5)\times(-j10)}{40 + j42.5} = 1.17 - j11.2$$

$$Z_c' = Z_c // (8 - j5) = \frac{(7.31 - j0.46)(8 - j15)}{15.31 - j5.46} = 4.1 - j1.2$$

$$Z_{eq} = Z_a // (Z_b' + Z_c')$$

$$= \frac{(16 + j21)(5.27 - j12.4)}{21.27 + j8.6}$$

$$= 12.5 - j9.2\ (\Omega)$$

or $= 15.53 \angle -36.33^\circ \Omega$

5. (a)

$$Z = \frac{1}{Y} = \frac{1}{G+jB} = \frac{G-jB}{G^2+B^2} = R+jX$$

$$R = \frac{G}{G^2+B^2}$$

$$X = \frac{-B}{G^2+B^2}$$

(b)

$$Z = \sqrt{\left(\frac{G}{G^2+B^2}\right)^2+\left(\frac{-B}{G^2+B^2}\right)^2} \angle \tan^{-1}\left(\frac{-B}{G}\right)$$

$$= \frac{1}{\sqrt{G^2+B^2}} \angle \tan^{-1}\left(-\frac{B}{G}\right)$$

6.

$$Z_1 = R_2 + \frac{1}{jwC_2}$$

$$Z_2 = R_4 \parallel \frac{1}{jwC_4}$$

$$\because R_1 Z_2 = R_3 Z_1$$

$$\therefore R_1 \times \frac{\frac{R_4}{jwC_4}}{R_4 + \frac{1}{jwC_4}} = R_3 \times \left(R_2 + \frac{1}{jwC_2}\right)$$

$$\Rightarrow \frac{R_1R_4}{jwC_4R_4+1} = R_3\left(R_2 - \frac{j}{wC_2}\right)$$

$$\Rightarrow \frac{R_1R_4(1-jwR_4C_4)}{w^2R_4^2C_4^2+1} = R_3\left(R_2 - \frac{j}{wC_2}\right) = R_3R_2 - \frac{jR_3}{wC_2}$$

Equating the real and imaginary parts

$$\begin{cases} \dfrac{R_1R_4}{w^2R_4^2C_4^2+1} = R_2R_3 \Rightarrow w^2R_4^2C_4^2+1 = \dfrac{R_1R_4}{R_2R_3} \quad - ① \\[2ex] \dfrac{wR_1R_4^2C_4}{w^2R_4^2C_4^2+1} = \dfrac{R_3}{wC_2} \quad - ② \end{cases}$$

Substituting ① into ②

$$w R_4 C_4 R_2 R_3 = \frac{R_3}{wC_2} \Rightarrow w^2 = \frac{1}{R_2R_4C_2C_4} \Rightarrow f = \frac{1}{2\pi\sqrt{R_2R_4C_2C_4}}$$

7.

1° For 15 V voltage source,

$v_1(t) = 15\,V$

2° For $6\sin 2t$ current source,

$$V_2 = 6 \times (6 \parallel \frac{12}{j\omega} \parallel 2j\omega) = 6(4.8 + 2.4j) = 32.2\angle 26.57^\circ$$

$$\Rightarrow v_2(t) = 32.2\sin(2t + 26.57^\circ)\,V$$

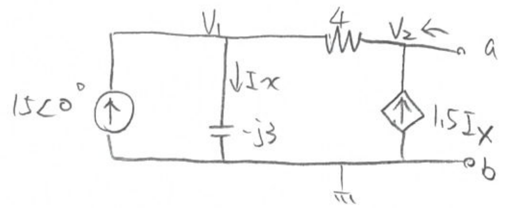
3° For $18\cos 3t$ voltage source

$$V_3 = 18 \times \frac{\frac{12}{j\omega} \parallel 2j\omega}{6 + (\frac{12}{j\omega} \parallel 2j\omega)} = 18(0.8 - 0.4j) = 16.1\angle -26.57^\circ$$

$$\Rightarrow v_3(t) = 16.1\cos(3t - 26.57^\circ)\,V$$

$$v_o(t) = v_1(t) + v_2(t) + v_3(t) = 15 + 32.2\sin(2t + 26.57^\circ) + 16.1\cos(3t - 26.57^\circ)\,V$$

8. To find V_{th},



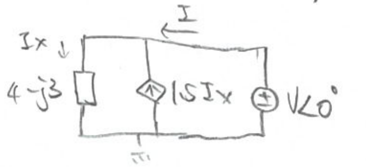
$$15 + \frac{V_2 - V_1}{4} = I_x = \frac{V_1}{-3j}$$

and $$\frac{V_2 - V_1}{4} = 1.5I_x$$

$$\therefore 15 + 1.5I_x = I_x \Rightarrow I_x = -30 \quad V_1 = -90j \quad V_2 = -180 + 90j$$

We have $V_{th} = V_2 = -180 + j90 = 201.2\angle 153.44^\circ\,V$

To find Z_{th},

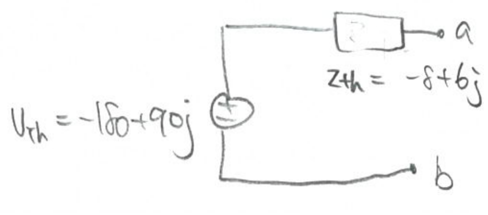


$$I_x = 1.5I_x + I \Rightarrow I = -0.5I_x$$

$$V = I_x(4 - j3) = -2I(4 - j3) = (-8 + j6)I$$

$$Z_{th} = Z_{eq} = \frac{V}{I} = -8 + j6$$

The Thevenin equivalent is given as



9.

$$V_o = \frac{j\omega\frac{1}{4}}{1+j\omega\frac{1}{4}} \cdot V \rightarrow V \quad \text{as } \omega \rightarrow \infty$$

Hence, the answer is (d).