E433200 Signals and Systems Solutions to Midterm Exam I   Full 2016.

1 (a) T   (f) F

(b) T   (g) T

(c) T   (h) F

(d) F   (i) F

(e) F   (j) T

(c)

suppose

$x[n]=0$ for all n. $\longrightarrow \boxed{\quad} \longrightarrow$ $y[n]\neq 0$ for some n

Let $y[n_0]=C\neq 0$.

Then, for a real number $a\neq 0$.

$x_1[n]=a\cdot x[n]=0$ for all n.

$\longrightarrow \boxed{\quad} \longrightarrow y_1[n_0]=C\neq a\cdot y[n_0]$

$\Rightarrow$ nonlinear.

(d) $x[n-5]\delta[n-5]=x[0]\delta[n-5]$

(e) $x(t)*\delta(t-t_0)=\int_{-\infty}^{\infty}x(\tau)\delta(t-\tau-t_0)d\tau=x(t-t_0)$

(f) Let $x_1[n]\longrightarrow y_1[n]=x_1[n]+1$

$x_2[n]\longrightarrow y_2[n]=x_2[n]+1$

and $x_3[n]=ax_1[n]+bx_2[n]$

$$\Rightarrow\quad y_3[n]=x_3[n]+1$$

$$=ax_1[n]+bx_2[n]+1$$

$$\neq \underbrace{ay_1[n]+by_2[n]}_{(ax_1[n]+a)+(bx_2[n]+b)=ax_1[n]+bx_2[n]+a+b}\quad(\text{when } a+b\neq 1)$$

(g) Let $z(t)=y(-t)$, $x(t)*z(t)=\int_{-\infty}^{\infty}x(\tau)z(t-\tau)d\tau$

$$=\int_{-\infty}^{\infty}x(\tau)y(\tau-t)d\tau \qquad \left(\begin{aligned}z(t-\tau)&=y(-(t-\tau))\\&=y(\tau-t)\end{aligned}\right)$$

(h) $\delta(ax)=\frac{1}{|a|}\delta(x)$ for $a\neq 0$

(i) Let $x[n]=x_r[n]+jx_i[n]\longrightarrow y[n]=R\{x[n]\}=x_r[n]$

Also let $x'[n]=jx[n]=-x_i[n]+jx_r[n]$

$\Rightarrow\ y'[n]=R\{x'[n]\}=-x_i[n]\neq jy[n]$

(j) $y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$

$y[-n]=\sum_{k=-\infty}^{\infty}x[k]h[-n-k]$

(let $m=-k$) $=\sum_{m=\infty}^{-\infty}x[-m]h[-n+m]$

$\left(\begin{aligned}\therefore\ x[n]&\text{ is even}\\ h[n]&\text{ is odd}\end{aligned}\right)=-\sum_{m=-\infty}^{\infty}x[m]h[n-m]=-y[n]$

2. Let $x(t)=x_{e1}(t)+x_{o1}(t)=x_{e2}(t)+x_{o2}(t)$ —— ①

$\Rightarrow\ x_{e1}(-t)+x_{o1}(-t)=x_{e2}(-t)+x_{o2}(-t)$

$\because\begin{cases}x_{e1}(-t)=x_{e1}(t)\\x_{e2}(-t)=x_{e2}(t)\end{cases}\quad\begin{cases}x_{o1}(-t)=-x_{o1}(t)\\x_{o2}(-t)=-x_{o2}(t)\end{cases}$

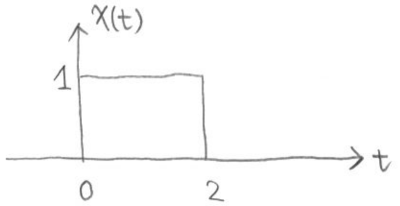
$\therefore\ x_{e1}(t)-x_{o1}(t)=x_{e2}(t)-x_{o2}(t)$ —— ②

①+② $2x_{e1}(t)=2x_{e2}(t)\Rightarrow x_{e1}(t)=x_{e2}(t)$

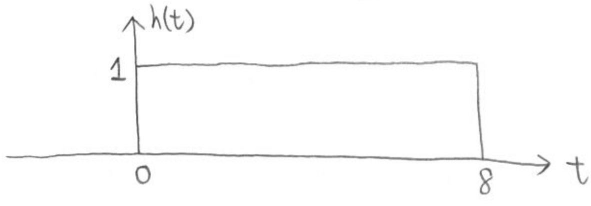
①−② $\Rightarrow x_{o1}(t)=x_{o2}(t)$ 得證

3.

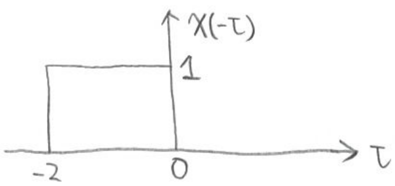
$x(t) = u(t) - u(t-2)$

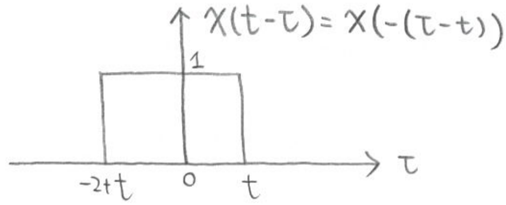


$h(t) = u(t) - u(t-8)$



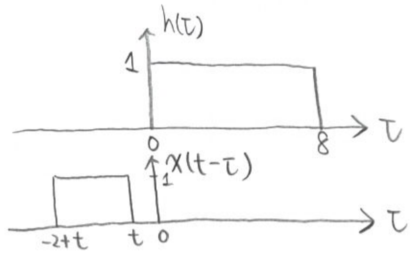
$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$



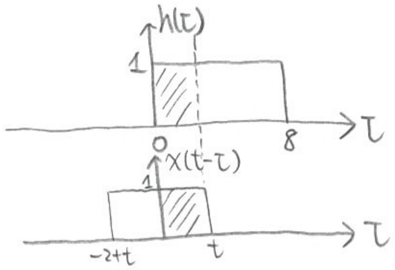


① $t<0$, $h(\tau)\cdot x(t-\tau) = 0$

$\Rightarrow y(t) = 0$

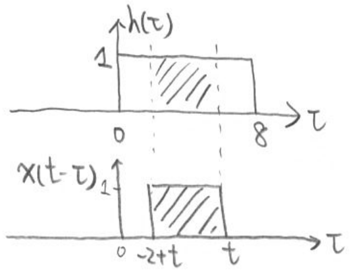


② $0 \le t < 2$, $y(t) = \int_0^t 1\cdot 1\, d\tau = t$



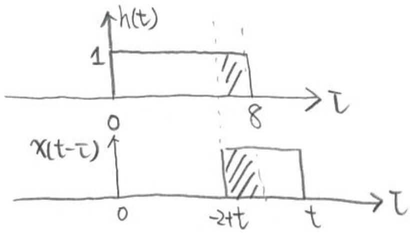
③ $2 \le t < 8$, $y(t) = \int_{-2+t}^{t} 1\cdot 1\, d\tau$

$= \tau \Big|_{-2+t}^{t} = t - (-2+t) = 2$

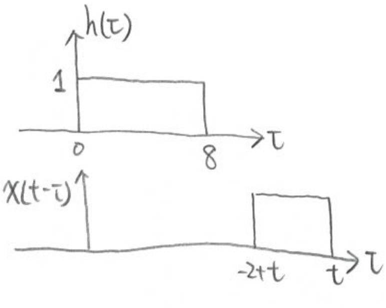


④ $8 \le t \le 10$, $y(t) = \int_{-2+t}^{8} 1\cdot 1\, d\tau$

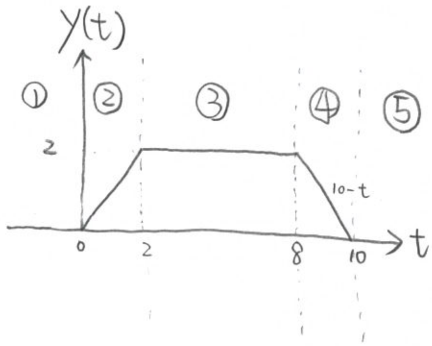
$= \tau \Big|_{-2+t}^{8} = 8 - (-2+t) = 10 - t$



⑤ $t > 10$, $y(t) = 0$



Ans:



4. $x(t) * \delta(t-t_0)$

$$= \int_{-\infty}^{\infty} x(\tau)\, \delta(t - t_0 - \tau)\, d\tau$$

$$= x(t-t_0) \int_{-\infty}^{\infty} \delta(t - t_0 - \tau)\, d\tau$$

$$= x(t-t_0).$$

5. Let

$$x(t) \longrightarrow \boxed{\text{system}} \longrightarrow y(t) = S[x(t)]$$

$$x(t+T) \rightarrow \boxed{\text{system}} \rightarrow y(t+T) \quad (\because \text{time-invariant})$$

Since $x(t+T) = x(t)$,

$$y(t+T) = S[x(t+T)]$$
$$= S[x(t)]$$
$$= y(t)$$

6. Let

$$x(t) \longrightarrow \boxed{A} \xrightarrow{y(t)=A[x(t)]} \boxed{B} \longrightarrow x(t) = B[y(t)]$$

(a) $B[a y_1(t) + b y_2(t)]$

$$= B[A[a x_1(t)] + A[b x_2(t)]]$$

$$= a_1 x_1(t) + b x_2(t)$$

(b) $B[y_1(t-\tau)]$

$$= B[A[x_1(t-\tau)]]$$

$$= x_1(t-\tau)$$

2.

(a)

$$sgn(s) = \begin{cases} 1 & s>0 \\ -1 & s<0 \\ 0 & s=0 \end{cases}$$

$|x(t)|$ 的範圍介在 $-1 \sim 1$ 之間

$$|x(t)| = |sgn(h(-t))| \leq 1 < \infty$$

$\therefore x(t)$ is bounded

(b)

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Then the ouput at $t=0$ is given by

$$y(0) = \int_{-\infty}^{\infty} h(\tau)\, sgn(h(-(0-\tau)))\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, sgn(h(\tau))\, d\tau$$

$$= \int_{-\infty}^{\infty} |h(\tau)|\, d\tau$$

Since this system is BIBO stable and $x(t)$ is bounded, $y(t)$ is also bounded $\Rightarrow |y(0)| = \int_{-\infty}^{\infty} |h(\tau)|\, d\tau < \infty$

8. $y[n] = (x[n] * h_1[n]) * h_2[n]$

$= x[n] * (h_1[n] * h_2[n])$ (associative)

$= x[n] * (h_2[n] * h_1[n])$ (Commutative)

$= (x[n] * h_2[n]) * h_1[n]$ (associative)

$\because x[n] * h_2[n] = (\delta[n] - a\delta[n-1]) * a^n u[n]$

$= a^n u[n] - a(\delta[n-1] * a^n u[n])$

$= a^n u[n] - a \cdot a^{n-1} \cdot u[n-1]$

$= a^n (u[n] - u[n-1])$

$= a^n \delta[n] = \delta[n]$

$\therefore y[n] = \delta[n] * h_1[n] = h_1[n] = \sin(\pi^n).$

9.

(a) $y(t) = \sin(x(t))$

1° Let $y_1(t) = \sin(x_1(t))$

$y_2(t) = \sin(x_2(t))$

Let $x_3(t) = a_1 x_1(t) + b x_2(t)$

$\Rightarrow y_3(t) = \sin(x_3(t))$

$= \sin(a x_1(t) + b x_2(t))$

$\neq a y_1(t) + b y_2(t)$

$\Rightarrow$ Not linear.

2° $y_1(t) = \sin(x_1(t))$

Let $x_2(t) = x_1(t-\tau)$

$\Rightarrow y_2(t) = \sin(x_2(t))$

$= \sin(x_1(t-\tau))$

$= y_1(t-\tau)$

$\therefore$ Time-invariant.

(b) $z(t) = x(\sin(t))$

1° Let $z_1(t) = x_1(\sin(t))$

$z_2(t) = x_2(\sin(t))$

Let $x_3(t) = a x_1(t) + b x_2(t)$

$\Rightarrow z_3 = x_3(\sin(t))$

$= a x_1(\sin t) + b x_2(\sin t)$

$= a z_1(t) + b z_2(t)$

$\Rightarrow$ Linear.

2° $z_1(t) = x_1(\sin(t))$

Let $x_2(t) = x_1(t-\tau)$

$\Rightarrow z_2(t) = x_2(\sin(t))$

$= x_1(\sin(t) - \tau)$.

$\neq z_1(t-\tau)$.

$\therefore$ Not time-invariant.