EXAM II
HEAT TRANSFER

May 24, 2017

I. Explain the following terms: (9%)

(1) Similarity transformation
(2) Implicit formulation
(3) Biot number

II. Answer the following questions: (21%)

(1) In what condition can the convective boundary condition be assumed as a fixed temperature boundary condition?

(2) In solving a 1-D transeint heat conduction problem, the variable separation method is used. It is assumed T = X(x)H(t). Put this relation into the energy equation and the equation can rearranged as

$$-\frac{1}{\alpha H}\frac{dH}{dt} = -\frac{1}{X}\frac{d^2X}{dx^2} = \text{constant}$$

Why are they equal to constant?

(3) What assupmtions should be made for the following heat diffusion equaiton?

$$\frac{\partial T}{\partial t} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

(4) In the following finite difference formulation of the heat diffusion equation, the difference expression used for the time derivative $\partial T/\partial t$ is forward difference or backward difference? Why?

$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \alpha\left[\frac{T_{i+1,j}^{n} - 2T_{i,j}^{n} + T_{i-1,j}^{n}}{\Delta x^2} + \frac{T_{i,j+1}^{n} - 2T_{i,j}^{n} + T_{i,j-1}^{n}}{\Delta y^2}\right]$$

(5) What are the two kinds of error by using the finite difference method to simulate heat transfer problems?

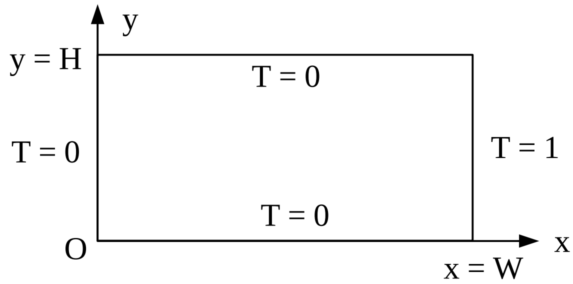
(6) What is the lumped-heat-capacity system? In what conditions can the system be applied?

(7) The energy equation of a heat transfer problem can be written as

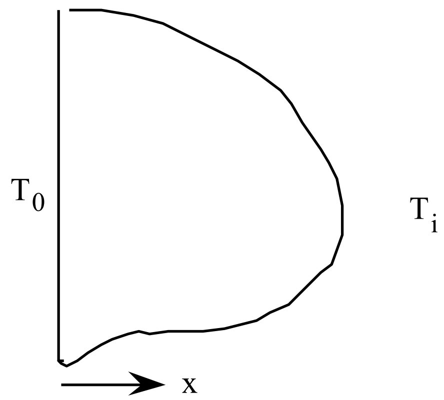
$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$

In solving the problem by using the variable separation method, what kind of boundary conditions are needed?

III. Consider a steady-state heat conduction problem in a rectangular plate. Its boundary conditions are shown in the following figure. Find the temperature solution of the plate. (12%)

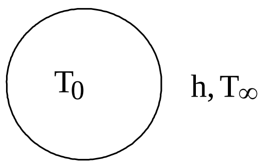


IV. Consider a semi-infinite solid shown in the following figure maintained at some initial temperature T_i. The surface temperature is suddenly lowered and maintain at a temperature T_0.

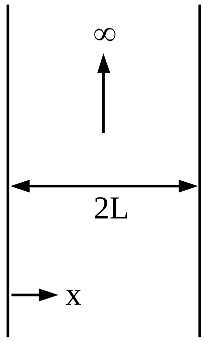


Solve the temperature distribution for this transient problem. (12%)

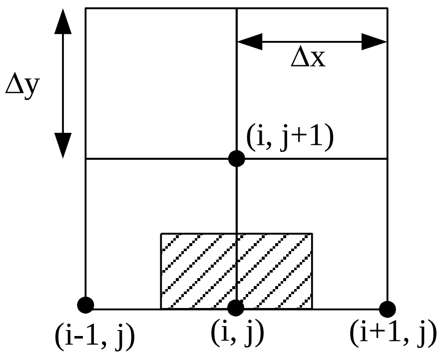
V. A solid body has a very high thermal conductivity, whose volume and surface area are V and A. It is put in a fluid, whose temperature is T_∞ and the convective heat transfer coefficient is h. The initial temperature of the solid body is T_0 and its thermal conductivity, density and specific heat are k, ρ and C. Derive the temperature expression of the body in terms of time. (9%)



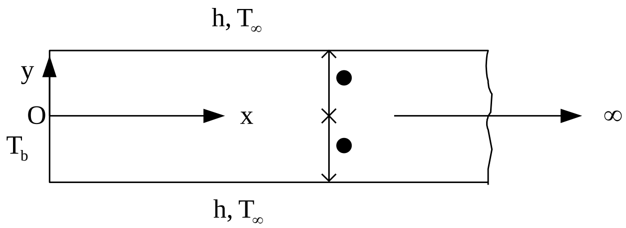
VI. Consider a 1-D transient problem, shown in the following figure .
$t = 0,\ 0 \le x \le 2L,\ T = T_i$
$t > 0,\ x = 0,\ T = T_1,\ x = 2L,\ T = T_1$
Find the temperature solution with the separation-of-variable method. (12%)



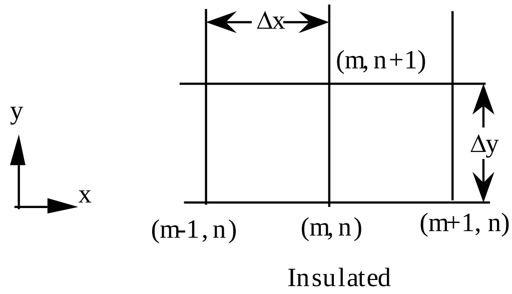
VII. Derive the finite difference equation of explicit formulation for a point (the point (i,j) shown in the following figure) on a heat-insulated boundary for a two-dimensional transient problem of heat transfer. (10%)



VIII. Consider an infinitely long two-dimensional fin of thickness 2λ. The base temperature of the fin is T_b, the ambient temperature is T_∞. The heat transfer coefficient h is very large. Find the steady temperature of the fin. (15%)



IX. (a) In steady-state condition, derive the expression of the finite difference formula for insulated boundary as shown in the following figure. ($\Delta x = \Delta y$) (7%)



(b) Using finite difference method to compute the temperatures at nodes 1, 2, 3 and 4. (assume steady state) (8%)