Numerical method
Midterm Exam (2020/06/01 9:10~12:00)

[redacted]

手寫計算題

1. (12%) Use the LU method to get the solution of $x_1, x_2, x_3,$ and x_4.

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 + 3x_4 = -2 \\ 1x_1 + 5x_2 + 3x_3 + 5x_4 = 2 \\ 3x_1 + 1x_2 + 1x_3 + 2x_4 = 1.5 \\ 2x_1 + 4x_2 + 5x_3 + 4x_4 = -\frac{1}{3} \end{cases}$$

2. (10%) Fit a least square curve of the form $y = ae^{bx}$ $(a > 0)$ to the data given below. Please calculate the values of a, b, and R^2.

x_i	1	2	3	4	5
y_i	1	3	5	7	9

3. (12%) The following data (x_i, y_i, z_i) are points in the Cartesian coordinate. (X, Y, Z) is a center point and has almost the same distance to those points. According to the following data, please use the least square method to calculate the a_{ij} and b_i, and solve the X, Y, Z.

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} b_1 \\ b_2 \\ b_3 \end{Bmatrix}$$

x_i	y_i	z_i	subscript_i
99.9943	96.6325	9.9912	1
101.9789	96.6314	10.7004	2
102.9944	96.6357	12.8734	3
101.4784	96.6201	15.4812	4
100.0634	96.6189	15.8444	5
98.2554	96.6451	15.2208	6
97.1345	96.6146	12.9972	7
97.9674	96.6444	10.8775	8

4. (10%) Construct the cubic spline

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

where $i = 1, 2, 3$, using the following data and boundary condition $S_1'(0) = S_3'(3) = 7$.

x	0	1	2	3
$f(x)$	0	1	8	27

（背面尚有試題，請翻面作答）

5. Evaluate $\int_0^6 \frac{2dx}{1+x^2}$ by using (a) (6%) Simpson's $\frac{1}{3}$ rule, (b) (6%) Simpson's $\frac{3}{8}$ rule and compare the relative error with the exact solution separately.

6. (a) (4%) Derive the two-point Gauss-quadrature method.
(b) (7%) Use two-point Gauss-quadrature rule to approximate the distance covered by a rocket from t = 8 to t = 30 as given by

$$x = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$$

(c) (3%) Find the relative error.