

$$= \frac{1}{\pi} \left[ \frac{-\cos(1+2n)x}{1+2n} + \frac{-\cos(1-2n)x}{1-2n} \right] \Big|_0^{\pi}$$

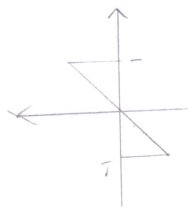
$$= \frac{4}{\pi} \left[ \frac{1}{1-4n^2} \right] \quad n=1, 2, 3, \dots$$

$b_n$  是偶函数  $\therefore b_n = 0$

$$f(x) = \frac{2}{\pi} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{1-4n^2} \cos nx$$

函数的平均值

9. a)  $f(x) = x, -1 < x < 1, L=1$



$\therefore$  奇函数  $\therefore a_0 = 0, a_n = 0$

$$b_n = \frac{1}{L} \int_{-L}^L f(x) \sin \frac{n\pi}{L} x \, dx$$

$$= \int_{-1}^1 x \sin(n\pi x) \, dx$$

$$= \frac{-x}{n\pi} \cos(n\pi x) \Big|_{-1}^1 - \int_{-1}^1 \frac{-1}{n\pi} \cos(n\pi x) \, dx$$

$$= \frac{-2\cos(n\pi)}{n\pi}$$

$$= \frac{2(-1)^n}{-n\pi}$$

$$f(x) = \sum_{n=1}^{\infty} \frac{-2(-1)^n}{n\pi} \sin(n\pi x)$$