Problems and Solutions (Chapter 4)

1. Explain why channel coding reduces bandwidth efficiency of the link in high signal-to-noise ratio conditions.

   [Solution]

   Channel coding is a technique used to overcome the deficiencies of the noisy transmission medium. It basically adds redundancy to the transmitted bits. Due to redundant bits the effective bandwidth available for useful data transmission is less.

2. Can channel coding be considered as a post detection technique?

   [Solution]

   Since we have redundant bits we can use them to detect/correct errors. Thus it can be considered as a post detection technique.

3. What is the main idea behind channel coding? Does it improve the performance of mobile communication?

   [Solution]

   Channel coding is a technique used to overcome the deficiencies of the noisy transmission medium. It basically adds redundancy to the transmitted bits so that the receiver is able to detect and/or correct the errors in the received information using the additional bits. It improves the robustness but decreases the effective bandwidth.

4. If the code generator polynomial is $g(x) = 1 + x^2$ for a (5, 3) code. Find the linear block code generator matrix $\mathbf{G}$.

   [Solution]

   $g(x) = 1 + x^2$

   $n = 5,\ k = 3,\ n - k = 2,$

   $p^i = rem(\frac{(x^{n-k+i-1})}{g(x)})$ for $i = 1$ to k,

   $p^1 = rem(\frac{(x^2)}{(1+x^2)}) = 1 \rightarrow [1\ 0],$

   $p^2 = rem(\frac{(x^3)}{(1+x^2)}) = x \rightarrow [0\ 1],$

   $p^3 = rem(\frac{(x^4)}{(1+x^2)}) = 1 \rightarrow [1\ 0],$

   thus

   $$\mathbf{G} = [\mathbf{I}_k\ \mathbf{P}] = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

5. The following matrix represents a generator matrix for a (7, 4) block code.

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

What is the corresponding parity check matrix **H**?

[Solution]

$n = 7$, $k = 4$.

Given

$$\mathbf{G} = [\mathbf{I}_k\ \mathbf{P}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

Thus

$$\mathbf{P} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\mathbf{H}^T = \begin{bmatrix} \mathbf{P} \\ \mathbf{I}_{n-k} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore, parity check matrix

$$\mathbf{H} = [\mathbf{H}^T]^T = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

6. Find the linear block code generator matrix **G**, if the code generator polynomial is $g(x) = 1 + x^2 + x^3$ for a (7, 4) code.

[Solution]

$g(x) = 1 + x^2 + x^3$

$n = 7$, $k = 4$, $n - k = 3$,

$p^i = rem(\frac{(x^{n-k+i-1})}{g(x)})$ for $i = 1$ to k,

$p^1 = rem(\frac{(x^3)}{(1+x^2+x^3)}) = 1 + x^2 \rightarrow [1\ 0\ 1]$,

$p^2 = rem(\frac{(x^4)}{(1+x^2+x^3)}) = 1 + x + x^2 \rightarrow [1\ 1\ 1]$,

$p^3 = rem(\frac{(x^5)}{(1+x^2+x^3)}) = 1 + x \rightarrow [1\ 1\ 0]$,

$p^4 = rem(\frac{(x^6)}{(1+x^2+x^3)}) = x + x^2 \rightarrow [0\ 1\ 1]$,

thus

$$\mathbf{G} = [\mathbf{I}_k\ \mathbf{P}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

7. Repeat Problem 4.6 if $g(x) = 1 + x^3$ for a (7, 4) code.

[Solution]

$g(x) = 1 + x^3$

$n = 7,\ k = 4,\ n - k = 3,$

$p^i = rem(\frac{(x^{n-k+i-1})}{g(x)}$ for $i = 1$ to k,

$p^1 = rem(\frac{(x^3)}{(1+x^3)}) = 1 \rightarrow [1\ 0\ 0]$,

$p^2 = rem(\frac{(x^4)}{(1+x^3)}) = x \rightarrow [0\ 1\ 0]$,

$p^3 = rem(\frac{(x^5)}{(1+x^3)}) = x^2 \rightarrow [0\ 0\ 1]$,

$p^4 = rem(\frac{(x^6)}{(1+x^3)}) = 1 \rightarrow [1\ 0\ 0]$,

thus

$$\mathbf{G} = [\mathbf{I}_k\ \mathbf{P}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

8. Consider the rate $r = 1/2$ in the convolutional encoder shown below. Find the encoder output $(Y_1\ Y_2)$ produced by the message sequence 10111....

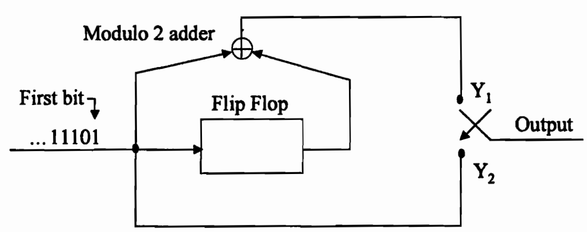


Figure for problem 4.8.

[Solution]

If we consider the input to be X and the flip flop to be D then from the figure we can derive the following transformation table.

We have $Y_1 = X \oplus D$

$Y_2 = X$

$D^n = X^{n-1}$

$D^1 = 0$

X	D	Y_1	Y_2
0	0	0	0
0	1	1	0
1	0	1	1
1	1	0	1

Therefore for the input string 10111....

The output string is 1110110101....

9. Find the state diagram for Problem 4.8.

[Solution]

The state diagram for the above problem is as follows

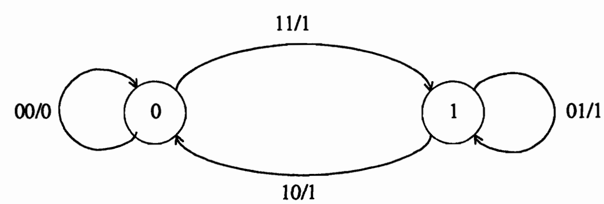


Figure for problem 4.9.

The states represent the value in the flip flop and the arrows represent the transitions from one state to another.

$\frac{YY}{X}$ represents that if X is the input then YY is the output.

Initially the state machine is in 0 state.

10. The following figure shows the encoder for a 1/2 rate convolutional code. Determine the encoder output produced by the message sequence 1011...

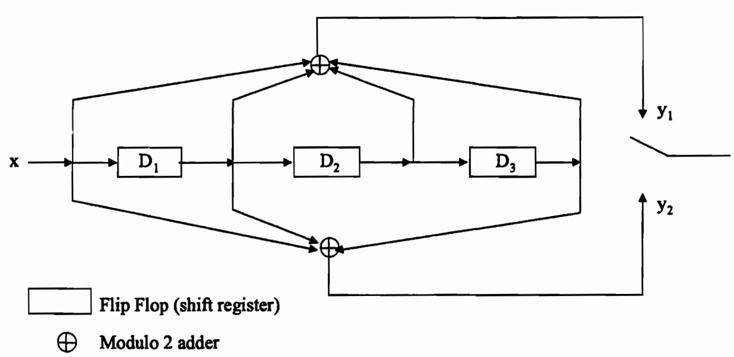


Figure for problem 4.10.

[Solution]

From the figure given we can derive the following equations

$Y_1 = X \oplus D_1 \oplus D_2 \oplus D_3$

$Y_2 = X \oplus D_1 \oplus D_3$

$D_1^n = X^{n-1}$

$D_2^n = D_1^{n-1}$

$D_3^n = D_2^{n-1}$

Using these equations we can construct the following table

X	D_1	D_2	D_3	Y_1	Y_2
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	1	0
0	0	1	1	0	1
0	1	0	0	1	1
0	1	0	1	0	0
0	1	1	0	0	1
0	1	1	1	1	0
1	0	0	0	1	1
1	0	0	1	0	0
1	0	1	0	0	1
1	0	1	1	1	0
1	1	0	0	0	0
1	1	0	1	1	1
1	1	1	0	1	0
1	1	1	1	0	1

For the given input sequence we can construct the following table

X	D_1	D_2	D_3	Y_1	Y_2
1	0	0	0	1	1
0	1	0	0	1	1
1	0	1	0	0	1
1	1	0	1	1	1
1	1	1	0	1	0
1	1	1	1	0	1
1	1	1	1	0	1

Thus for the input sequence 1011..., the output sequence is 11110111....

11. Consider a SAW ARQ system between 2 nodes A (transmitting node) and B (receiving node). Assume that data frames are all of the same length and require τ seconds for transmission. Acknowledgement frames require R seconds for transmission and there is a propagation delay P on

the link (in both directions). One in every 3 frames that A sends is in error at B. B responds to this with a NAK and this erroneous frame is received correctly in the (first) retransmission by A. Assume that nodes send new data packets and acknowledgements as fast as possible, subject to the rules of stop and wait. What is the rate of information transfer in frames/second from A to B?

[Solution]

The time taken to transmit a data frame from one node to the other is

$$Transmission\ Time + Propagation\ Delay = T + P$$

The time taken to transmit an ACK frame from one node to the other is

$$Transmission\ Time + Propagation\ Delay = R + P$$

Time required for successful transmission of a frame is

$$T + P + R + P = T + R + 2P$$

(time for data frame from A to B and positive ACK from B to A).

Time required for successful transmission of a frame after receiving a negative ACK is

$$2(T + R + 2P)$$

(Additional $T + R + 2P$ required for successful retransmission)

Given that one in every 3 frames is in error, the time required to transmit 3 consecutive frames is

$$T + R + 2P + T + R + 2P + 2(T + R + 2P) = 4(T + R + 2P)$$

This cycle repeats for every 3 frames so the information transfer rate is

$$\frac{3}{4\,(T + R + 2P)}$$

frames/second.

12. Compare and contrast GBN ARQ and SR ARQ schemes.

    [Solution]

GBN ARQ	SR ARQ
Provides better channel utilization as compared to SW ARQ	Provides better channel utilization as compared to SW ARQ
All frames transmitted after a frame for which an NAK is received must be retransmitted.	Only the frame for which an NAK is received is retransmitted
Better suited when burst errors are most possible during transmission	Better suited when single packet errors are most possible during transmission
The receiver does not maintain a buffer for the frames received as packets are always in order	The receiver has to maintain a buffer to order the packets explicitly

13. Consider the block diagram of a typical digital transmission system. Speculate where one would use source coding and channel coding. Differentiate between them. Would they increase or decrease the original message size? (Hint: we want to transmit most efficiently, i.e., message size should be the smallest possible but enough redundancy should be added to correct small errors so that retransmission is avoided as far as possible).

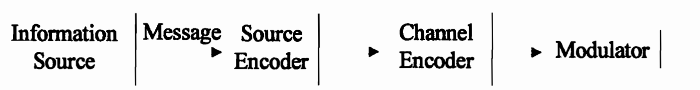


Figure for problem 4.13.

[Solution]

Source coding is a technique used to ensure that the information to be transmitted is coded using as few bits as possible. The basic purpose of source coding is to better utilize the available bandwidth by transmitting more information using fewer bits.

Channel coding is a technique used to overcome the deficiencies of the noisy transmission medium. It basically adds redundancy to the transmitted bits so that the receiver is able to detect and/or correct the errors in the received information using the additional bits.

Source coding decreases the original message size by using compression techniques whereas channel coding adds redundant information to overcome transmission errors thus increasing the transmitted message size.

14. Can you interleave an interleaved signal? What do you gain with such a system?

[Solution]

Yes, we can interleave an interleaved signal. It can increase the robustness and avoid burst errors. However there may be possibility of de-interleaving.

15. Why do you need both error correction capability and ARQ in a cellular system? Explain clearly.

    [Solution]

    In a cellular system, the last communication link is between the BS and the MS. The wireless link is exposed to loss and interference in the open air medium and hence the error rate is much higher than the wired link.

    Error correction techniques and ARQ schemes can provide an elegant way to implement end-to-end error-free transmission. Error correction capability can improve the link quality. It will decrease the probability of retransmission. However, channel coding can not correct all the errors in the transmission. Hence, the ARQ is also necessary if we need the error-free transmission.

16. In a two-stage coding system, the first stage provides (7, 4) coding while the second stage supports (11, 7) coding. Is it better to have such a two-stage coding scheme as compared to a single-stage (11, 4) complex coding? Explain your answer in terms of the algorithmic complexity and error correcting capabilities.

    [Solution]

    The two-stage coding system is equivalent to the single-stage coding in terms of algorithm complexity and error-correcting capabilities. For algorithm complexity, there are 3 XOR operations in the (4, 7) coding and 4 XOR operation in the (7, 11) coding. Hence, there are total 7 XOR operations in the two-stage coding system which is same as single-stage coding system. The error-correcting capabilities of two-stage and single-stage coding should be same, since only single bit error can be corrected in these two systems.

17. Under what scenarios would cyclic codes be preferred over interleaving and vice versa?

    [Solution]

    Interleaving is preferred for traffic where burst errors are more probable. Cyclic codes are used for noisy channels where individual bit errors are much more frequent.

18. Polynomial $1 + x^7$ can be factorized into three irreducible polynomials $(1+x)(1+x+x^3)(1+x^2+x^3)$ with $(1+x+x^3)$ and $(1+x^2+x^3)$ as primitive polynomials. Using $1+x+x^3$ as generator polynomial calculate the (7, 4) cyclic code word for given message sequence 1010.

    [Solution]

    Let $d(x)$ represents the polynomial for the message "1010" that we are trying to encode, then,

    $$d(x) = x + x^3$$

Let $g(x)$ represent the generating function, then

$$g(x) = 1 + x + x^3$$

Let the code be represented by its corresponding polynomial $c(x)$.

From the theory of cyclic codes, we have

$$\begin{aligned} c(x) &= g(x)\, d(x) \\ &= \left(1 + x + x^3\right)\left(x + x^3\right) \\ &= x + x^2 + x^3 + x^6 \end{aligned}$$

Therefore, the cyclic code is = (1 0 0 1 1 1 0).

19. Repeat Problem 18 with $1+x^2+x^3$ as generator polynomial and compare the results.

[Solution]

$b(x) = [x^3(1 + x^2)] \text{mod}[1 + x^2 + x^3] = x + x^2$

Therefore, $c(x) = x + x^2 + x^3 + x^5$ or 0111010, with 011 as parity and 1010 as message bits respectively.

20. Develop the encoder and syndrome calculator with $1+x^2+x^3$ as generator polynomial in Problem 18.

[Solution]

Encoder for the (7, 4) cyclic code generated by $g(x) = 1 + x^2 + x^3$:

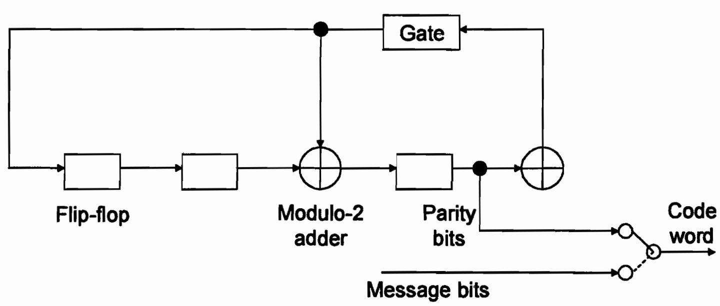


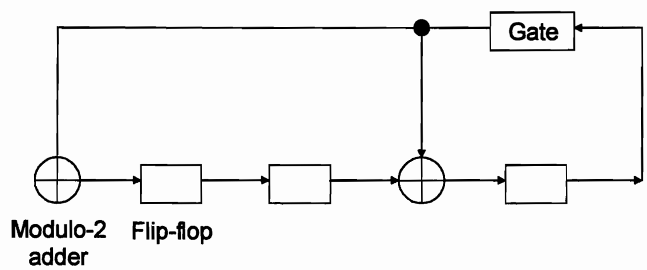
Figure of Answer 1 of Problem 20.

Syndrome calculator for the (7, 4) cyclic generated by $g(x) = 1 + x^2 + x^3$:

21. What is an RSC code? Why are these codes called be systematic?

[Solution]

RSC code is Recursive Systematic Convolutional (RSC) codes. Since the parity sequence is appended at the end of the information sequence in the encoded sequence, it is called systematic.



22. Describe briefly syndrome decoding and incomplete decoding?

[Solution]

Syndrome decoding:

In linear block encoding, $n-k$ parity bits are added to the original message vector $\mathbf{m}$ which is k bits long. Hence, the transmitted vector $\mathbf{c}$ and the received vector $\mathbf{x}$ at the decoder are n bits sequences. Syndrome decoding consists of calculating the $1 \times (n-k)$ vector s which would either uniquely map the received vector $\mathbf{x}$ to a possible match in $\mathbf{c}$ or if it is not possible to correctly decode the received vector $\mathbf{x}$, a non-null vector value of $\mathbf{s}$ would indicate errors in transmission. In a linear block coding transmission, the syndrome vector $\mathbf{s}$ can be calculated by

$$\mathbf{s} = \mathbf{x}\mathbf{H}^T,$$

where

$$\mathbf{H} = \left[\mathbf{P}^T \mathbf{I}_{n-k}\right]$$

and $\mathbf{H}$ is called the parity check matrix and is given by concatenating the transpose of the parity matrix $\mathbf{P}$ (k by $n-k$ matrix) and the identity matrix $\mathbf{I}_{n-k}$ ($n-k$ by $n-k$ matrix).

Incomplete decoding:

Incomplete decoding is not an independent decoding method. Actually, it is combined with existing decoding methods to improve their error correction capability. For example, the target of the maximum likelihood (ML) decoding is to find the likely codeword after a codeword is received. However, it is possible to have more than one such codeword. If using the existing ML decoding, the decoder will arbitrarily select one of them. However, it will request a retransmission in the incomplete ML decoding. It can improve the error correction capability since the probability of continuous decoding failure is much smaller than probability of single decoding failure. Furthermore, incomplete decoding can be used in the other probability decoding method.

23. Prove that the average transmission time in terms of block duration, T_{SR}, for Selective-Repeat ARQ is given by:

$$T_{\mathrm{SR}} = 1.P_{\mathrm{ACK}} + 2.P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}}) + 3.P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}})^2 + \cdots,$$

where P_{ACK} is the probability to return a ACK in the transceiver side. Also, solve the above equation for $P_{\mathrm{ACK}} = 0.5$.

[Solution]

The probability of receiving an ACK with no losses $= P_{\mathrm{ACK}}$

The probability of receiving an ACK after one packet loss $= P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}})$

The probability of receiving an ACK after two packet losses $= P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}})^2$

...

Thus, total time is given by

$$T_{\mathrm{SR}} = 1.P_{\mathrm{ACK}} + 2.P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}}) + 3.P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}})^2 + \cdots,$$

For $P_{\mathrm{ACK}} = 0.5$, $T_{\mathrm{SR}} = 2$.

24. In Stop-and-Wait ARQ, let the probability of the transmitting side receiving a ACK after exactly one loss of ACK be $P = 0.021$. Find the Average transmission time in terms of a block duration if :

$D =$ (Round trip propagation delay)

$R_b =$ bit rate,

$n =$ number of bits in a block.

(Hint: The probability for the considered case $= P_{\mathrm{ACK}}(1 - P_{\mathrm{ACK}})$)

[Solution]

$$\frac{70}{9}\left(1 + \frac{DR_b}{n}\right).$$

25. Compare a block with a convolutional interleaver.

[Solution]

A block interleaver accepts a set of symbols and rearranges them, without repeating or omitting any of the symbols in the set. The number of symbols in each set is fixed for a given interleaver. The interleaver's operation on a set of symbols is independent of its operation on all other sets of symbols. Whereas a convolutional interleaver consists of a set of shift registers, each with a fixed delay. Each new symbol from the input signal feeds into the next shift register and the oldest symbol in that register becomes part of the output signal.