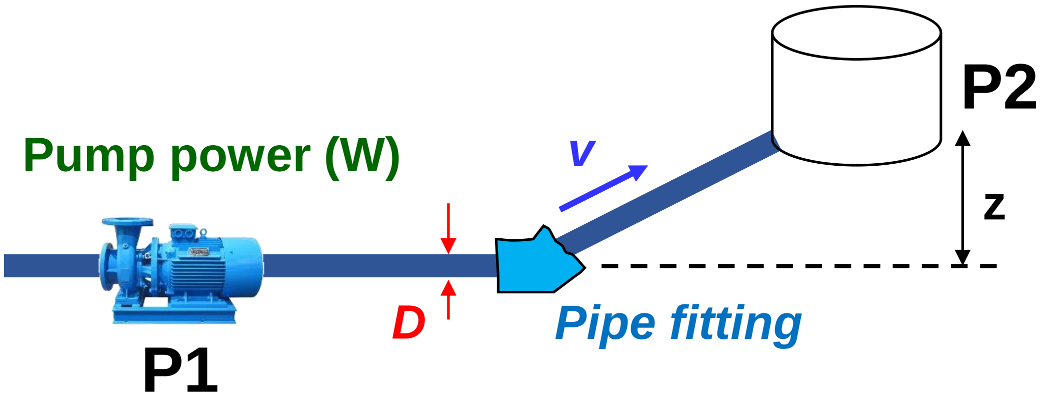
Flow of fluid in pipes and pipe fittings

- In the last Chapter, we have learned the effect of drag force on flowing fluid, the formation of boundary layers, and the characteristics of turbulent flow.

- Flow of fluid in closed conduits (pipes and pipe fittings) is more important for engineering applications:



We have to consider the **friction loss** of the pipe and pipe fittings!

- Without a pump, what is the pressure required to move the fluid from 1 to 2?
- At ambient pressure, what is the required pump power?
- At a given mass flow rate and pump power, what size of pipe is required?

From thermodynamics…

- For a **closed system**:

1st law of thermodynamics: $\Delta U = Q - W$

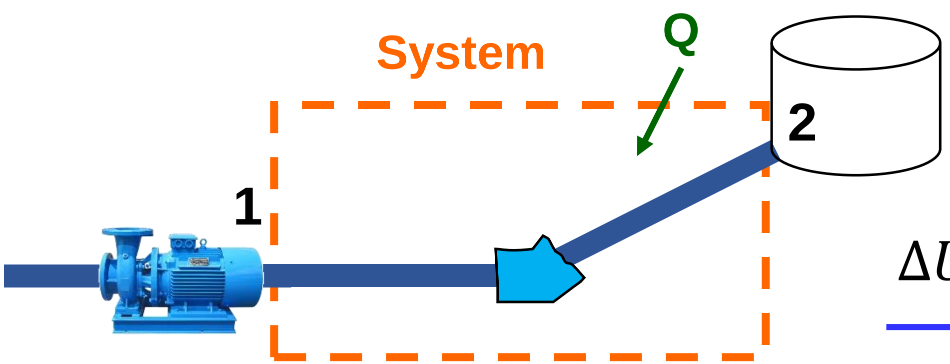
$$\mathrm{d}U = \delta Q - \delta W$$

Q: Heat ***added to*** the system

W: Work ***done by*** the system

δ: Used for path functions

- For a **flow system**:



- For **unit mass** of fluid:

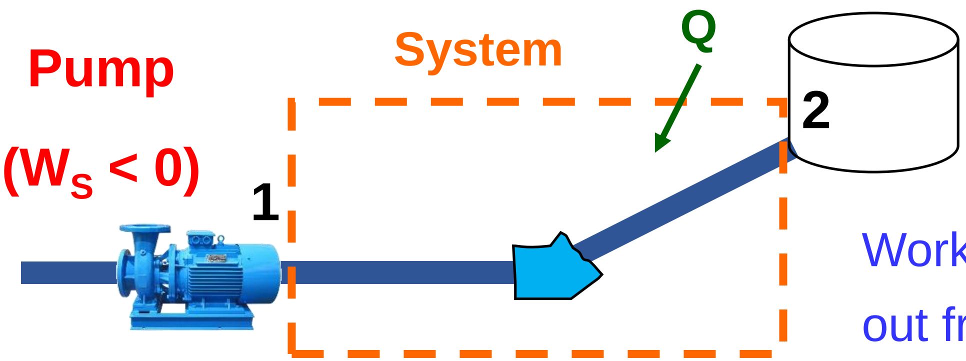
$$\underbrace{\Delta U}_{\text{Internal energy}} + \underbrace{\Delta\left(\frac{\alpha}{2}v_{avg}^2\right)}_{\text{Kinetic energy}} + \underbrace{g\Delta z}_{\text{Potential energy}} = Q - W$$

α: Kinetic energy correction factor

α ~ 1 for turbulent flow

α = 2 for laminar flow

Recall: types of work for flowing fluid



$$W = \underbrace{P_2V_2 - P_1V_1}_{\text{Work to allow the fluid to move out from the control volume}} + \underbrace{W_S}_{\text{Shaft work}}$$

Work to allow the fluid to move out from the control volume

(to overcome normal stress & shear stress)

Shaft work

"Positive W" = work done by the system!

$$\Delta U + \Delta\left(\frac{\alpha}{2} v_{avg}^2\right) + g\Delta z = Q - P_2V_2 + P_1V_1 - W_S$$

$$\Delta H + \Delta\left(\frac{\alpha}{2} v_{avg}^2\right) + g\Delta z = Q - W_S$$

$$dH + d\left(\frac{\alpha}{2} v_{avg}^2\right) + g\,dz = \delta Q - \delta W_S$$

From thermodynamics again…

2nd law of thermodynamics: $$dS = \frac{\delta Q}{T}$$ (For reversible system)

$$dS = \frac{\delta Q}{T} + \frac{\delta F}{T}$$ (For irreversible system)

F: Friction loss

- The loss of mechanical energy to generate heat
- A degree of irreversibility for the process

$$dH = TdS + VdP = \delta Q + \delta F + VdP$$

$$\longrightarrow \quad \delta Q + \delta F + VdP + d\left(\frac{\alpha}{2} v_{avg}^2\right) + gdz = \delta Q - \delta W_S$$

$$\longrightarrow \quad \delta F + VdP + d\left(\frac{\alpha}{2} v_{avg}^2\right) + gdz = -\delta W_S$$ (It's for unit mass!)

$$\longrightarrow \quad \delta F + \frac{dP}{\rho} + d\left(\frac{\alpha}{2} v_{avg}^2\right) + gdz = -\delta W_S$$

The mechanical equation for fluid

$$\delta F + \frac{dP}{\rho} + d\left(\frac{\alpha}{2} v_{avg}^2\right) + g dz = -\delta W_S$$

Unit: Energy of unit mass [J/kg];

$L^2 t^{-2}$

$$\boxed{\frac{\delta F}{g} + \frac{dP}{\rho g} + \frac{1}{2g} d(\alpha v_{avg}^2) + dz = \frac{-\delta W_S}{g}}$$

Unit: Length [m]; ***L***

Mechanical equation

(General form)

- For an ***incompressible flow*** in a tube:

$$\boxed{\frac{F}{g} + \Delta\left(\frac{P}{\rho g}\right) + \frac{1}{2g} \Delta(\alpha v_{avg}^2) + \Delta z = \frac{-W_S}{g}}$$

Head loss (h_L) | Pressure head | Velocity head | Potential head

Definition of "Head"

Mass flow rate = $\rho A v_{avg}$

$$\longrightarrow \boxed{\rho A v_{avg}\left[F + \Delta\left(\frac{P}{\rho}\right) + \frac{1}{2} \Delta(\alpha v_{avg}^2) + g\Delta z\right] = \frac{-W_S \times m}{t}}$$

Power required from the pump

How about compressible fluid?

$$\boxed{\frac{\delta F}{g}+\frac{dP}{\rho g}+\frac{1}{2g}d(\alpha v_{avg}^2)+dz=\frac{-\delta W_S}{g}}$$

Mechanical equation

(General form)

= ***VdP/g***; *but V is also a function of P !*

- For ideal gas (unit mass):

$$PV=\frac{RT}{M}$$

(M: molecular weight)

(1) For isothermal process:

$$\int_1^2\frac{VdP}{g}=\int_1^2\frac{RTdP}{PMg}=\frac{RT}{Mg}\ln\left(\frac{P_2}{P_1}\right)$$

"Pressure head"

(2) For isentropic process ($\Delta S=0$; $\delta Q=-\delta F$):

We have to learn from thermodynamics…

H=U+PV

$$C_V\equiv\left(\frac{\partial U}{\partial T}\right)_V$$

$$C_P\equiv\left(\frac{\partial H}{\partial T}\right)_P$$

$$dH=\left(\frac{\partial H}{\partial P}\right)_V dP+\left(\frac{\partial H}{\partial V}\right)_P dV=\left(\frac{\partial H}{\partial T}\right)_V\left(\frac{\partial T}{\partial P}\right)_V dP+\left(\frac{\partial H}{\partial T}\right)_P\left(\frac{\partial T}{\partial V}\right)_P dV$$

$$=\left(C_V+V\left(\frac{\partial P}{\partial T}\right)_V\right)\left(\frac{\partial T}{\partial P}\right)_V dP+C_P\left(\frac{\partial T}{\partial V}\right)_P dV=\underbrace{TdS}_{0}+VdP$$

How about compressible fluid?

(2) For isentropic process ($\Delta S = 0;\ \ \delta Q = -\delta F$):

$$VdP = \left(C_V \underline{\left(\frac{\partial T}{\partial P}\right)_V} + V\right)dP + C_P \underline{\left(\frac{\partial T}{\partial V}\right)_P} dV = \left(C_V \frac{T}{P} + V\right)dP + C_P \frac{T}{V} dV$$

For ideal gas = T/P *For ideal gas = T/V*

$$C_V \frac{T}{P} dP + C_P \frac{T}{V} dV = 0 \longrightarrow C_V lnP + C_P lnV = constant \longrightarrow \boxed{PV^{\frac{C_P}{C_V}} \equiv PV^\gamma = constant}$$

Let's set: $PV^\gamma = P_1 V_1^\gamma = constant$

$$\int_1^2 \frac{VdP}{g} = \int_1^2 \frac{(\frac{P_1 V_1^\gamma}{P})^{\frac{1}{\gamma}} dP}{g} = \frac{P_1^{\frac{1}{\gamma}} V_1}{g} \int_1^2 (\frac{1}{P})^{\frac{1}{\gamma}} dP = \underline{\frac{P_1^{\frac{1}{\gamma}} V_1}{\left(1 - \frac{1}{\gamma}\right) g} (P_2^{1-\frac{1}{\gamma}} - P_1^{1-\frac{1}{\gamma}})}$$

$$\boxed{\frac{F}{g} + \underline{(???)} + \frac{1}{2g} \Delta(\alpha v_{avg}^2) + \Delta z = \frac{-W_S}{g}}$$

OK……Let's go back to incompressible fluid

$$\boxed{\rho A v_{avg}[F + \Delta(\frac{P}{\rho}) + \frac{1}{2}\Delta(\alpha v_{avg}^2) + g\Delta z] = \frac{-W_S \times m}{t}}$$

- For a special case:

1. No work is applied to the system.
2. Δz = 0
3. $v_1 = v_2$

$$\Rightarrow \quad F = -\Delta\left(\frac{P}{\rho}\right) = \frac{(P_1 - P_2)}{\rho}$$

(For incompressible fluid)

The "pressure drop" is usually served as an indicator for friction loss of a pipe.

Now the question becomes: How to estimate the *friction loss (F)*?

Dimensional analysis for the flow in a pipe

Variable	Symbol	Dimensions
Shear stress	τ	M/Lt^2
Viscosity	μ	M/Lt
Density	ρ	M/L^3
Velocity	v	L/t
Length	l	L
Diameter	D	L
Roughness	e	L

- Again, let's choose ***D*, *v*, and *ρ*** *as the recurring set:*

$$\boxed{L = D} \qquad \boxed{t = \frac{D}{v}} \qquad \boxed{M = \rho D^3}$$

$$\longrightarrow \quad D1 = \frac{\tau}{\rho v^2} \qquad D2 = \frac{\mu}{\rho D v} \qquad D3 = \frac{l}{D} \qquad D4 = \frac{e}{D}$$

$\longrightarrow$ **Definition:**

Fanning friction factor (f_f) = 2D1 = $2\frac{\tau}{\rho v^2}$

Darcy friction factor (f_D) = 8D1 = $8\frac{\tau}{\rho v^2}$

The friction head loss of *a pipe*

- For a pipe flow in a circular pipe: $\tau(\pi D)(l) = -\Delta P(\frac{1}{4}\pi D^2) \longrightarrow -\Delta P = \frac{4\tau l}{D}$

  Shear Pressure

  And: $f_f = \frac{2\tau}{\rho v^2}$

- Let's consider the pressure drop caused by friction loss ($-\Delta P_{\mathrm{f}}$):

$$h_L = \frac{F}{g} = \frac{-\Delta P_{\mathrm{f}}}{\rho g} = \frac{4\tau l}{D\rho g} = \underline{2f_f \frac{l}{D}\frac{v^2}{g}}$$

Head loss of a pipe

$$\boxed{F = 2f_f \frac{l}{D} v^2}$$

Friction loss in a circular pipe

$\longrightarrow$ Once ***Re*** and ***e/D*** are known, $\boldsymbol{f_f}$ can be determined from theory or experimental data. Thereafter, the head loss can be calculated.

Friction factor in laminar flow

$$-\frac{dP}{dx} = \frac{8\mu v_{\text{avg}}}{R^2} = \frac{32\mu v_{\text{avg}}}{D^2}$$

Hagen–Poiseuille equation (CH8)

$$\Delta P = \frac{32\mu v_{avg} l}{D^2}$$

$$h_L = \frac{F}{g} = \frac{\Delta P}{\rho g} = \frac{32\mu v_{avg} l}{\rho g D^2} = 2 f_f \frac{l}{D} \frac{v_{avg}^2}{g}$$

$$\longrightarrow \quad f_f = \frac{16\mu}{\rho D v_{avg}} = \frac{16}{Re}$$

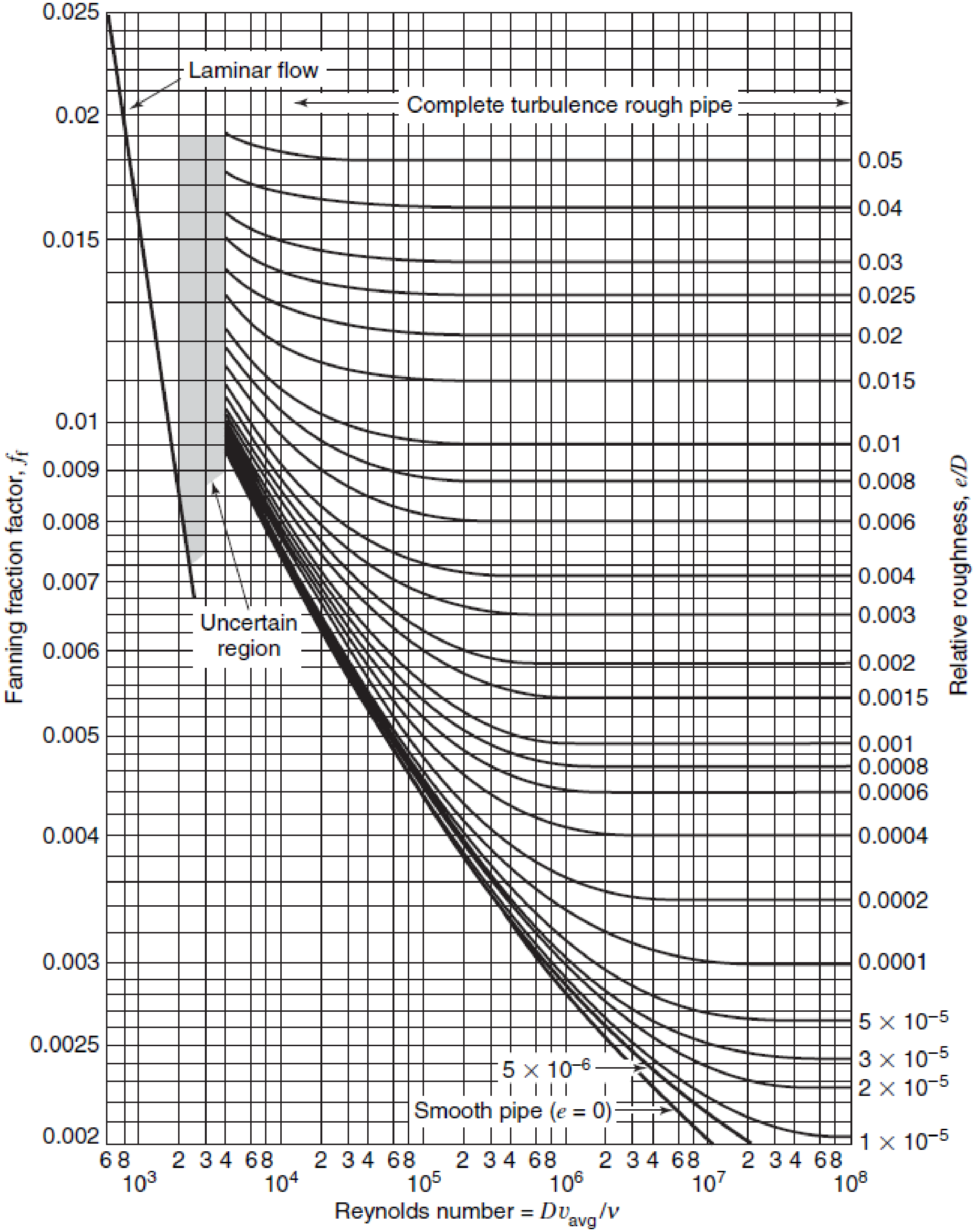
- In laminar flow region (Re < 2300), the friction factor is not a function of roughness!

Friction factor in turbulent flow

- No simple relationship is available! Empirical correlations are needed.
- Ex: For **10^8 > Re > 4 x 10^4** and **0.05 > e/D > 0**, the following eq. may be used:

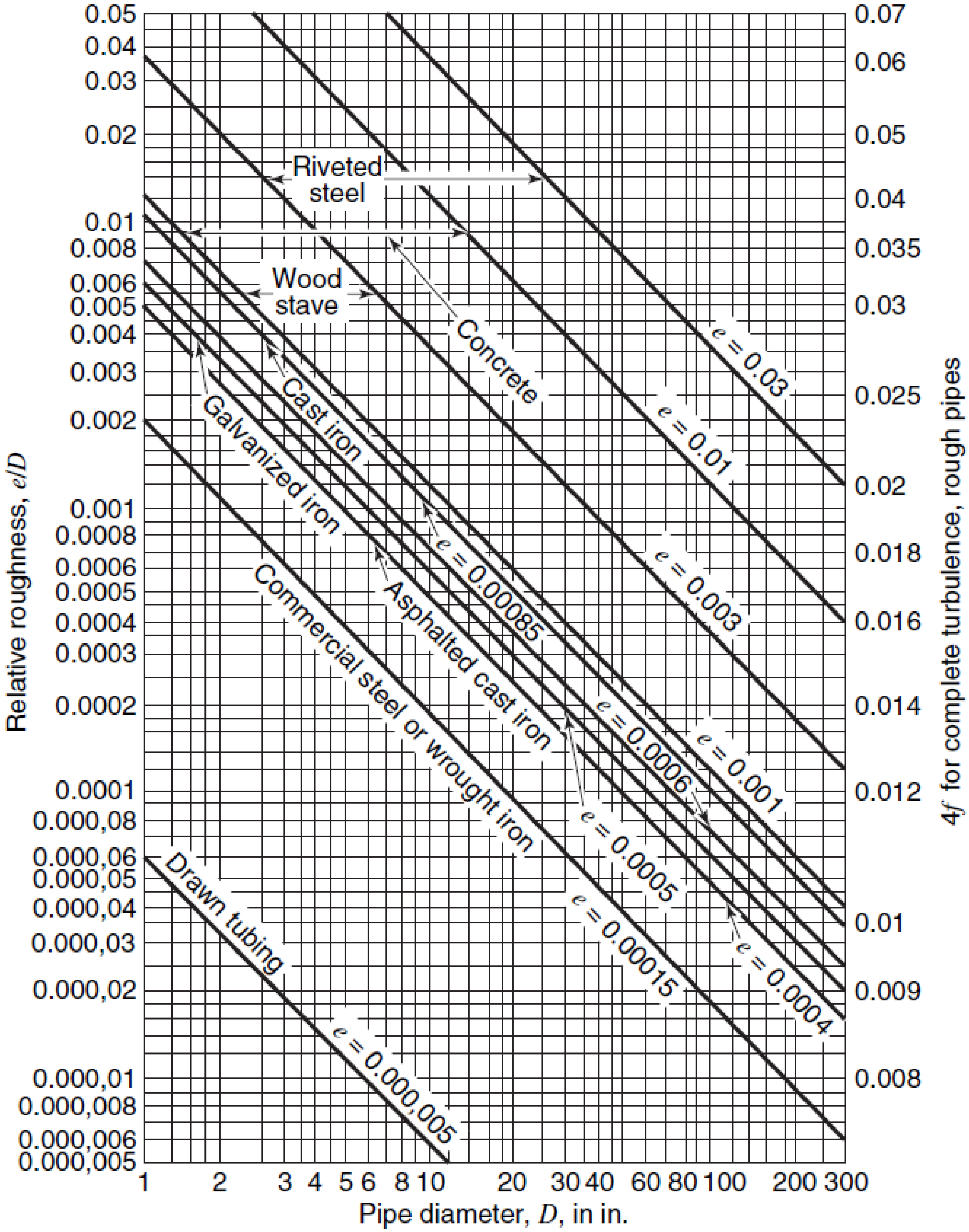
$$\frac{1}{\sqrt{f_f}} = -3.6 \log_{10}\left[\frac{6.9}{\text{Re}} + \left(\frac{e}{3.7D}\right)^{10/9}\right]$$

Friction factor: information from chart



"Moody diagram"

Friction factor: information from chart



The friction head loss of ***pipe fittings***

- There are lots of pipe fittings between pipes, which also cause head loss:

Elbow

Union

Swagelok

Gate valve

Globe valve

$$h_L = \frac{F}{g} = \frac{\Delta P_{\mathrm{f}}}{\rho g} \equiv K \frac{v^2}{2g}$$

or

$$h_L = 2 f_f \frac{L_{eq}}{D} \frac{v^2}{g}$$

K: Friction factor for pipe fitting

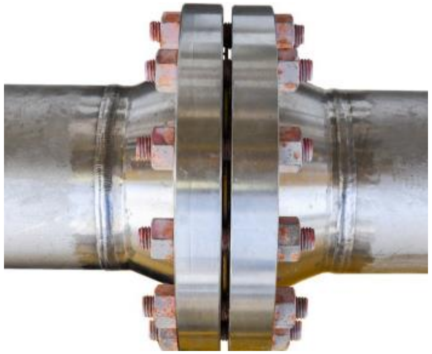
L_{eq}: Equivalent length

Fitting	K	L_{eq}/D
Globe valve, wide open	7.5	350
Angle valve, wide open	3.8	170
Gate valve, wide open	0.15	7
Gate valve, $\frac{3}{4}$ open	0.85	40
Gate valve, $\frac{1}{2}$ open	4.4	200
Gate valve, $\frac{1}{4}$ open	20	900
Standard 90° elbow	0.7	32
Short-radius 90° elbow	0.9	41
Long-radius 90° elbow	0.4	20
Standard 45° elbow	0.35	15
Tee, through side outlet	1.5	67
Tee, straight through	0.4	20
180° Bend	1.6	75

Types of pipe fittings (*McCabe CH8*)

1. Joints and fittings:

Screwed fittings



Flange joint

Welding joint

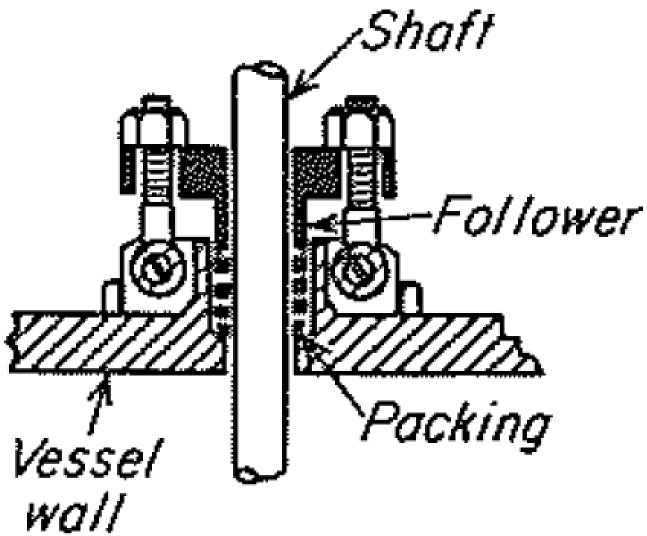
- For small pipe (< 3 inch)
- It weakens the pipe wall; thus, a thick pipe wall is usually needed.

- For high-pressure fluid
- The only disadvantage is that it cannot be opened without destroying it.

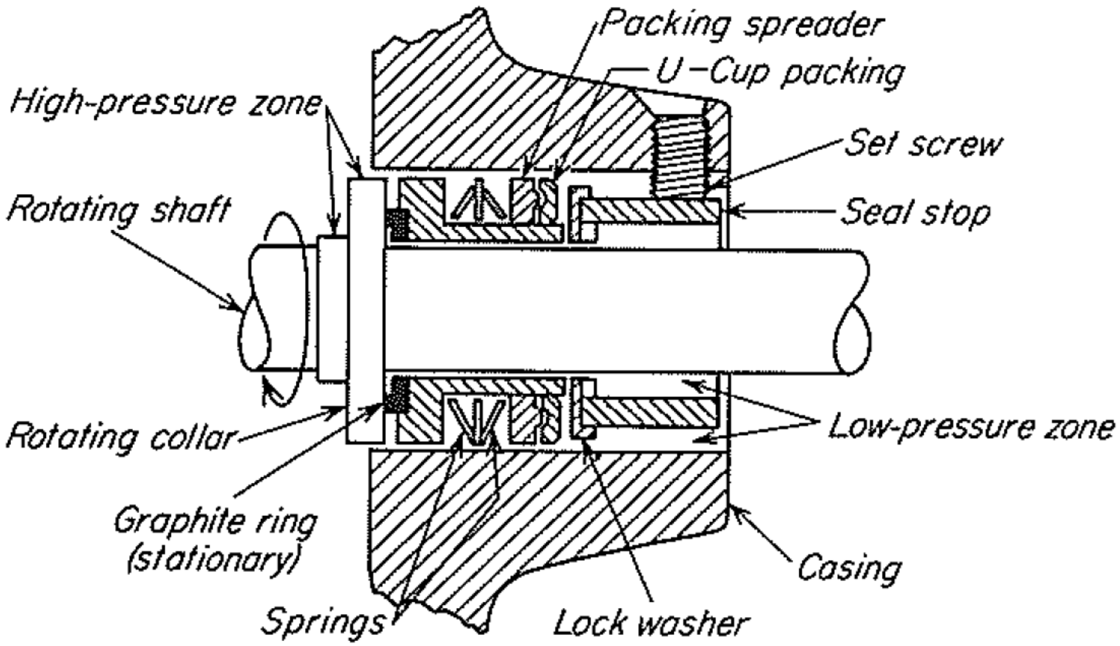
Expansion joints

Types of pipe fittings (*McCabe CH8*)

2. Leakage prevention:



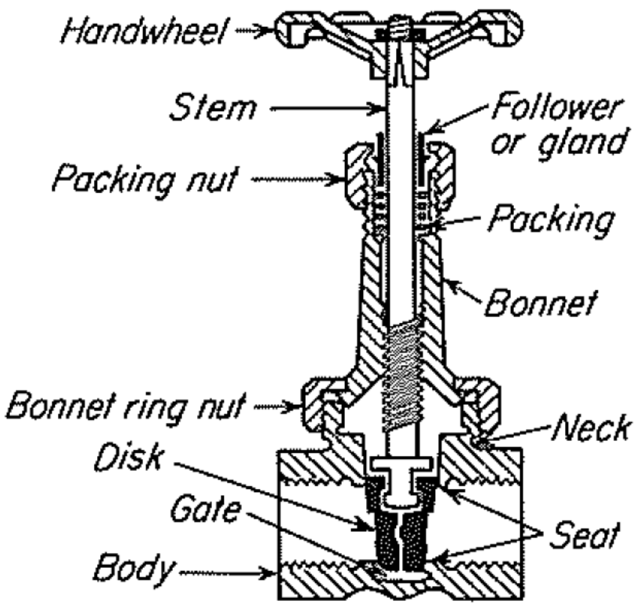
Stuffing boxes



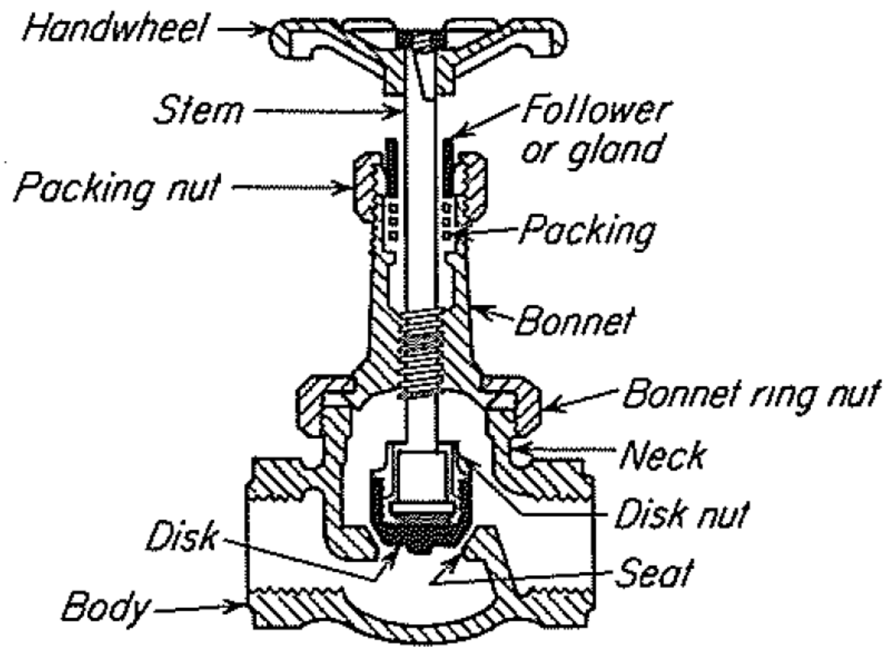
Mechanical seal

Types of pipe fittings (*McCabe CH8*)

3. Valves: Valves are the final control elements in the control loops!



Gate valve



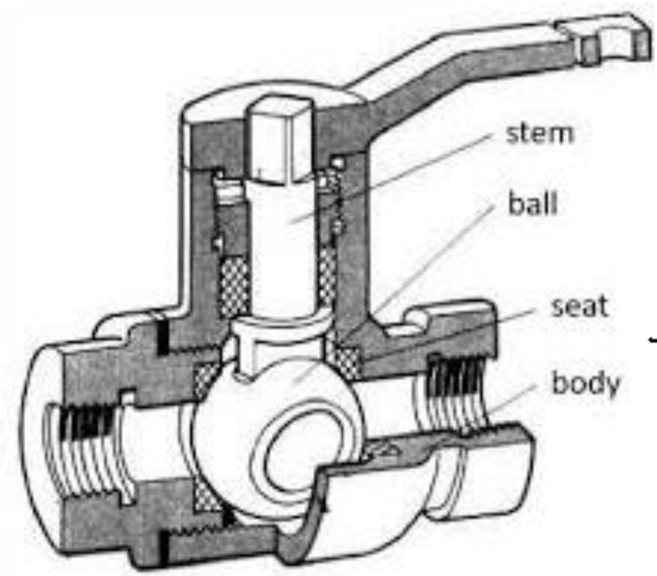
Globe valve

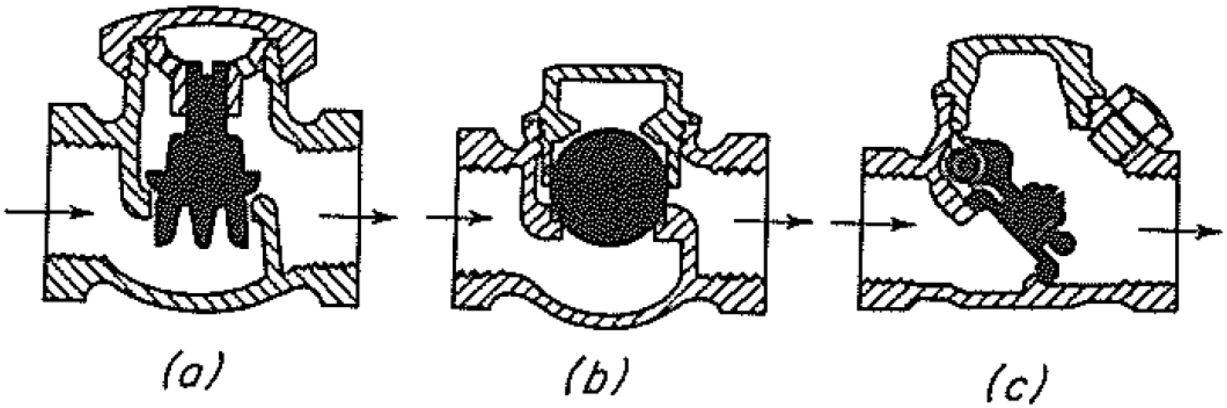
- No direction change for flow
- Small pressure drop
- On/Off only

- It can control the flow rate.
- By changing the handwheel to electric motor, it becomes a *control valve*.
- Large pressure drop

Types of pipe fittings (*McCabe CH8*)

3. Valves: Valves are the final control elements in the control loops!





Ball valve **Check valves**

The "equivalent diameter"

- Now the head loss of a flow system can be estimated by considering all pipes and pipe fitting:

$$\frac{F}{g} \equiv h_L = 2f_f \frac{l}{D}\frac{v^2}{g}\bigg|_{pipe} + \sum \underline{2f_f \frac{L_{eq}}{D}\frac{v^2}{g}}$$

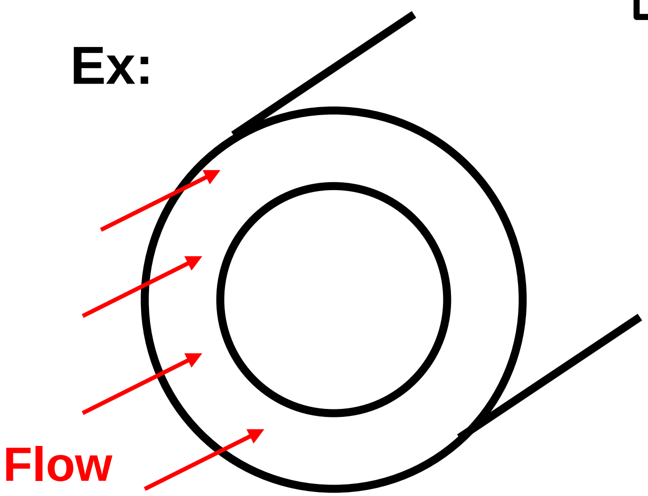
For all pipe fittings

- For noncircular pipes:

Only for turbulent flow

$$\boxed{D_{\mathrm{eq}} = 4\,\frac{\text{cross-sectional area of flow}}{\text{wetted perimeter}}}$$

Ex:



$$\text{Cross-sectional area} = \frac{\pi}{4}(D_0^2 - D_i^2)$$

$$\text{Wetted perimeter} = \pi(D_0 + D_i)$$

$$D_{\mathrm{eq}} = 4\,\frac{\pi/4}{\pi}\,\frac{(D_0^2 - D_i^2)}{(D_0 + D_i)} = D_0 - D_i$$

Example 13.1

Q: Water at 59 °F flows through a straight section of a 6-in.-ID cast-iron pipe with an average velocity of 4 fps. The pipe is 120 ft long, and there is an increase in elevation of 2 ft from the inlet of the pipe to its exit.

Find the power required to produce this flow rate.

Sol: (1)

$$\rho A v_{avg}[F + \cancel{\Delta(\frac{P}{\rho})} + \frac{1}{2}\cancel{\Delta(\alpha v_{avg}^2)} + g\Delta z] = power\ required$$

No pressure change

Constant cross-section area

$$\rho A v_{avg} g(2f_f \frac{l}{D}\frac{v^2}{g} + \Delta z) = power\ required$$

(2) Be careful of the units!

$$\rho = 999\ \frac{\text{kg}}{m^3}; \quad \mu = 1.19 \times 10^{-3}\ Pa - s$$

$$D = 0.152\ m; \quad A = 0.0181\ m^2\ ; \ v_{avg} = \frac{4ft}{s} = 1.22\ \frac{m}{s}; \quad l = 36.6\ m; \quad \Delta z = 0.61\ m$$

Example 13.1

$$\rho A v_{avg} g \left(2 f_f \frac{l}{D} \frac{v^2}{g} + \Delta z \right) = 999 \times 0.0181 \times 1.22 \times 9.8 \times (2 f_f \frac{36.6}{0.152} \frac{1.22^2}{9.8} + 0.61)$$

$$= 216.2 \times (73.14 f_f + 0.61)$$

(3) Find ***e/D*** and $\boldsymbol{f_f}$ from the charts:

- For 6-in.-ID cast-iron pipe, **e/D** = 0.0017
- Re = 0.152 x 999 x 1.22 / (1.19 x 10^{-3}) ~ 156,000
- For e/D = 0.0017 and Re = 156,000, $\boldsymbol{f_f}$ = 0.006

$$Pump\ power = 216.2 \times (73.14 \times 0.006 + 0.61) = \mathbf{226}\ [\boldsymbol{J/s}] \ = 0.3\ [hp]$$

Both ***v*** and ***D*** are known → **Re** is known → Trial & error is not required!

Example 13.2

Q: A heat exchanger is required, which will be able to handle 0.0567 m^3/s of water through a smooth pipe with an equivalent length of 122 m. The total pressure drop is 103,000 Pa. What size pipe is required for this application?

Sol:

$$\rho A v_{avg}[F + \Delta(\frac{P}{\rho}) + \cancel{\frac{1}{2}\Delta(\alpha v_{avg}^2)} + g\cancel{\Delta z}] = \cancel{Power\ input}$$

Constant cross-section area

0

No work applied to the system

$$F = 2 f_f \frac{l}{D} v^2 = -\frac{\Delta p}{\rho} = \frac{103{,}000}{1000} = 103; \qquad v = \frac{0.0567}{A} = \frac{0.0722}{D^2}$$

$\longrightarrow$ $f_f = 81.0 \times D^5$ <u>and</u> $Re = \frac{D\rho v}{\mu} = \frac{72000}{D}$

Trial & error with chart!

$\longrightarrow$ $D = 0.13\ m$

Some useful charts

For a horizontal pipe with a constant cross-section area:

- **For a known mass flow rate (G):**

G can be extracted from here!

$$Re^{5/3}{f_f}^{1/3} = \frac{D^{5/3}\rho^{5/3}v^{5/3}}{\mu^{5/3}}\left(\frac{D(-\Delta P)}{2\rho l v^2}\right)^{1/3} = \frac{(-\Delta P)^{1/3}\boxed{D^2\rho^{4/3}v}}{1.26 l^{1/3}\mu^{5/3}} = \frac{4(-\Delta P)^{1/3}\quad \rho^{1/3}G}{1.26\pi l^{1/3}\mu^{5/3}}$$

- **For a known velocity (v) but unknown d:**

$$f_f Re^{-1} = \frac{D(-\Delta P)}{2\rho l v^2}\frac{\mu}{D\rho v} = \frac{(-\Delta P)\mu}{2\rho^2 l v^3}$$

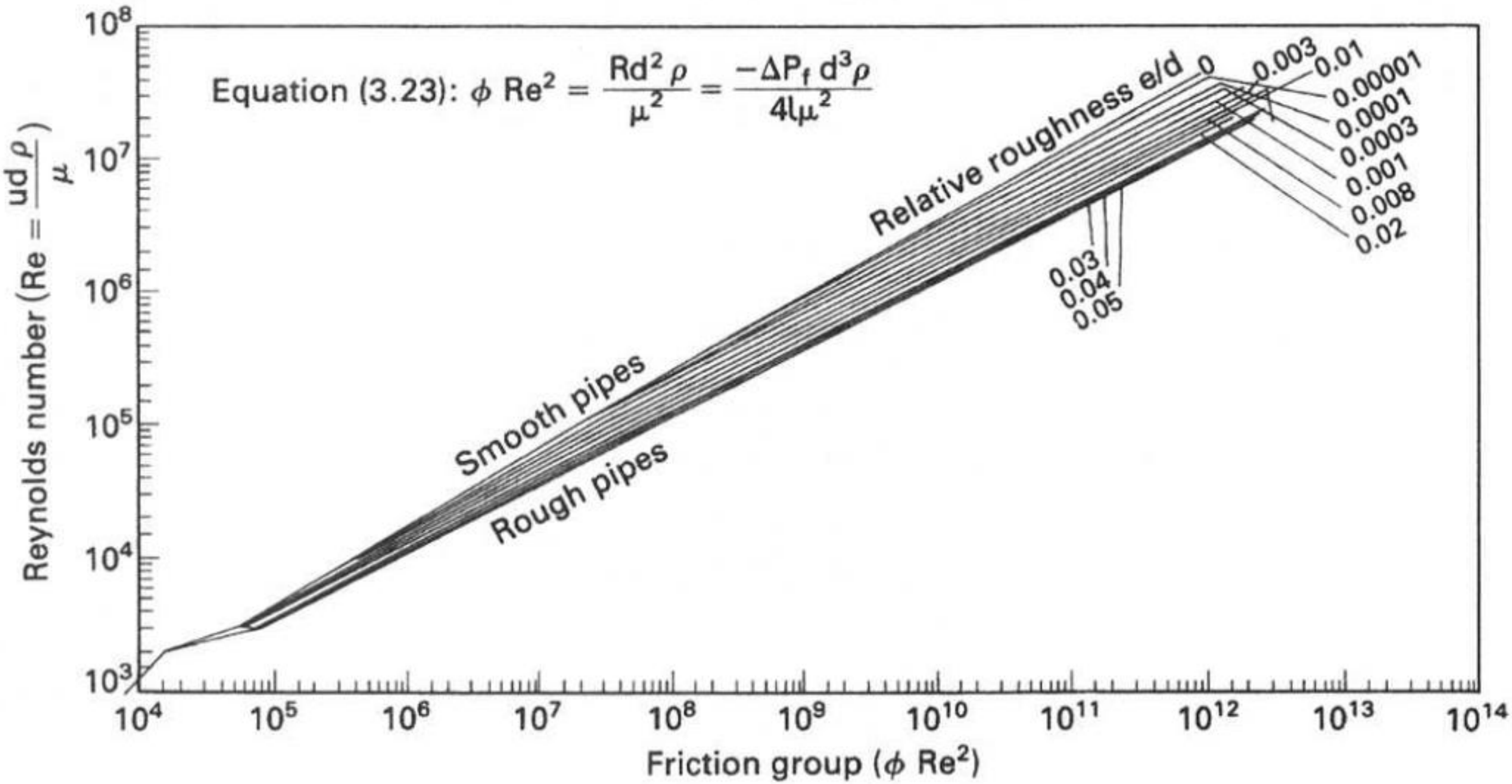
- **For a known diameter (d) but unknown v :**

$$f_f Re^2 = \frac{D(-\Delta P)}{2\rho l v^2}\frac{D^2\rho^2 v^2}{\mu^2} = \frac{(-\Delta P)\rho D^3}{2 l \mu^2}$$

Find more details in: Moody, L.F., "Friction Factors for Pipe Flow", *Transactions of the American Society of Mechanical Engineers*, 66 (1944) 671-684, if you are interested in…

Some useful charts

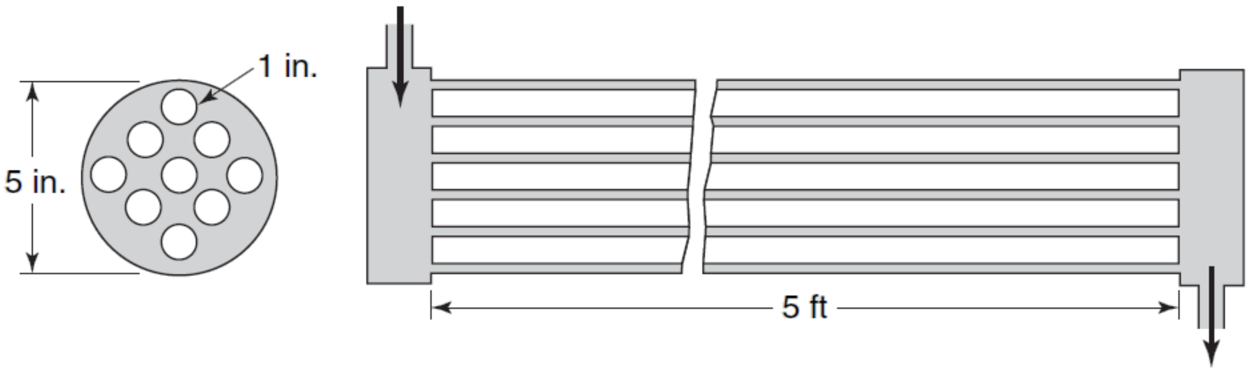
One example (from Coulson & Richardson, *Chemical Engineering*, Volume 1):



$(f_f \equiv 2\phi)$

The "equivalent diameter" in heat exchanger

(1) Flow parallel to the tubes:



(*see example 13.3*)

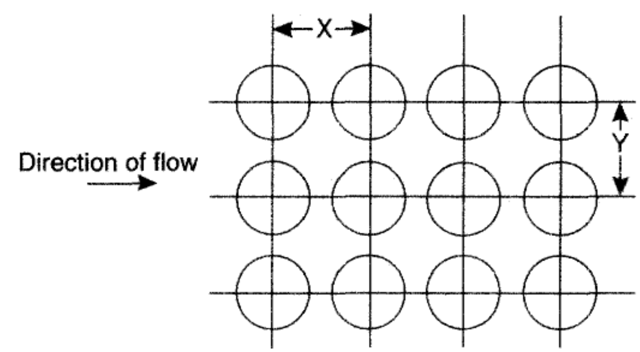
$$\text{Flow area} = \frac{\pi}{4}(25 - 9) = 4\pi \text{ in.}^2$$

$$\text{Wetted perimeter} = \pi(5 + 9) = 14\pi \text{ in.}$$

$$D_{\text{eq}} = 4\frac{4\pi}{14\pi} = 1.142 \text{ in.}$$

(2) Flow vertical to the tubes:

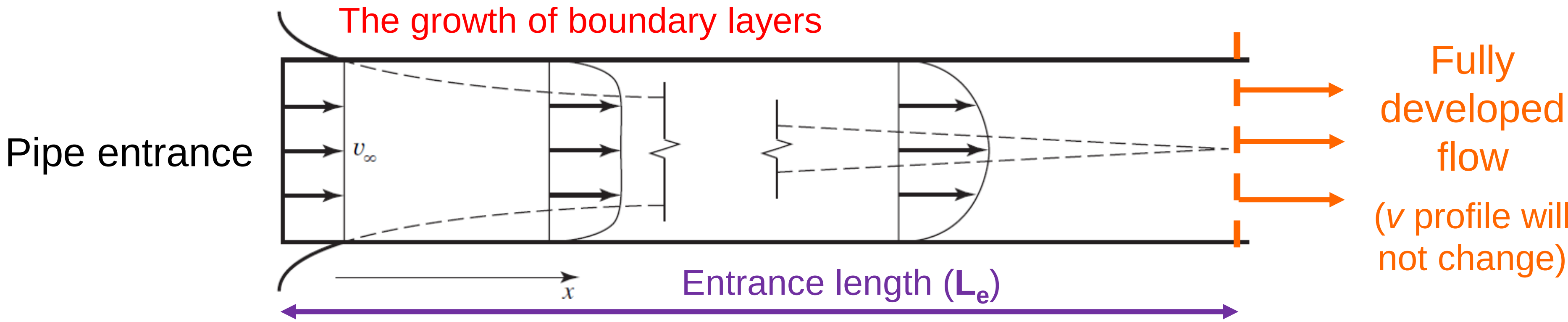
- The cross-sectional area is continually changing, and the problem may be treated as one involving a series of sudden enlargements and sudden contractions.



- One must use empirical equation to find the friction loss…

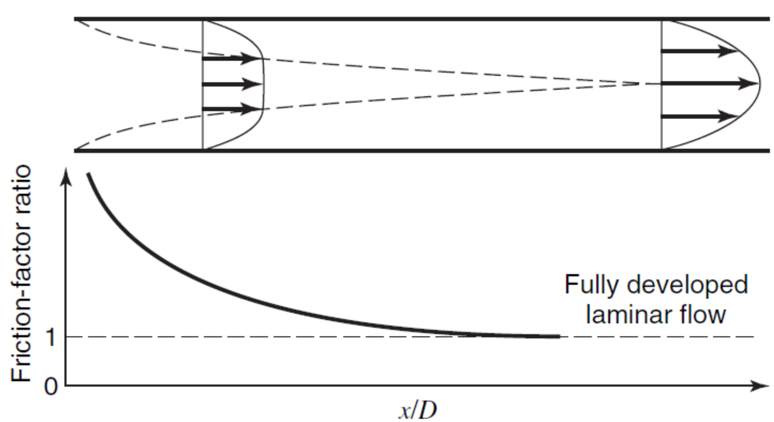
The "entrance length"

- It should be noted that all the f_f from charts and previous discussion are for "fully developed" flow in a pipe!



- For laminar flow:

$$\frac{L_e}{D} = 0.0575\,\mathrm{Re}$$

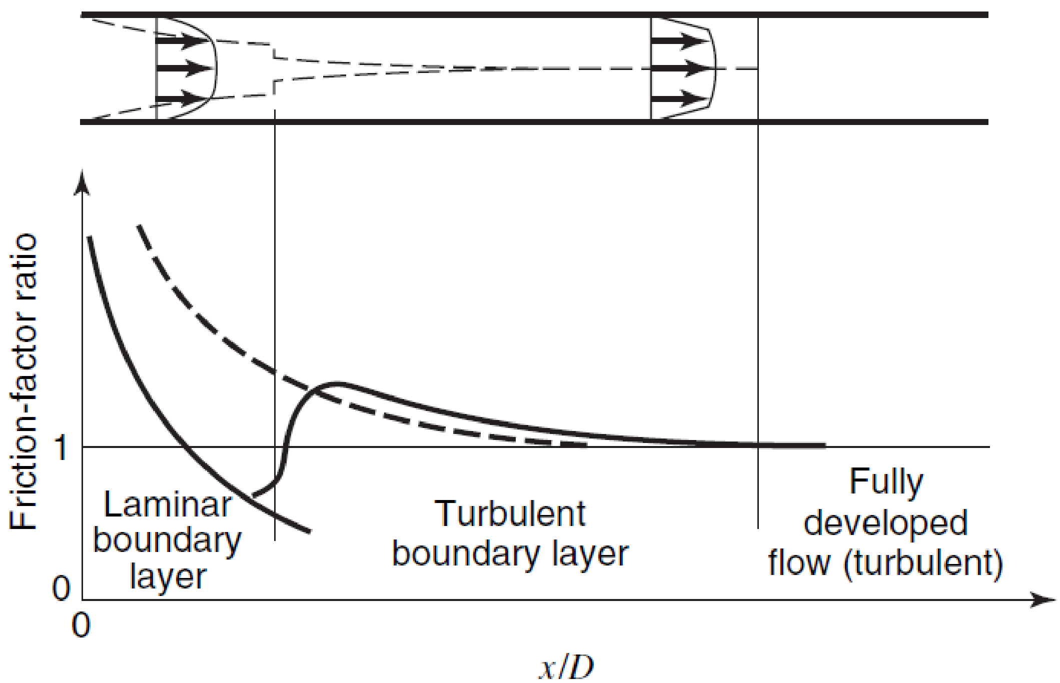


$\frac{x/D}{\mathrm{Re}}$	$f_f\left(\frac{x}{D}\right)$
0.000205	0.0530
0.000830	0.0965
0.001805	0.1413
0.003575	0.2075
0.00535	0.2605
0.00838	0.340
0.01373	0.461
0.01788	0.547
0.02368	0.659
0.0341	0.845
0.0449	1.028
0.0620	1.308
0.0760	1.538

The "entrance length"

- For turbulent flow: No relation available!

- Even for a very high velocity, the region very near the entrance should still be laminar, and there should be a transition region near the entrance.



- If the flow in the pipe is never fully developed, or the *entrance length is not negligible* compared to the pipe length, the real friction loss should be higher than what we estimated based on previous charts.