1. Capillary viscometer

$$V_{avg} = \rho g \frac{R^2}{8\mu}$$

$$\mu = \frac{\rho g R^2}{8 V_{avg}}$$

$$\mu \propto \frac{\rho}{V_{avg}} \propto \rho t.$$

2. Falling sphere viscometer

$$\sum F = 0$$

$$\underset{\downarrow}{F_{gravity}} + \underset{\uparrow}{\underset{\text{Fluid-Solid}}{F_{drag}}} + \underset{\uparrow}{F_{buoyancy}} = 0$$

$$\frac{4}{3}\pi R^3 \rho_s g - \underbrace{6\pi \mu V_t R}_{\text{Stokes drag}} \cancel{=} -$$
$$\frac{4}{3}\pi R^3 \rho_l g = 0$$

$$\Rightarrow \mu = \frac{2R^2(\rho_s - \rho_l)g}{9 V_t}$$

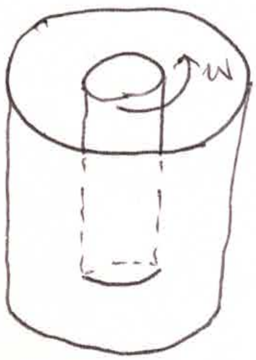
3. Falling rod viscometer

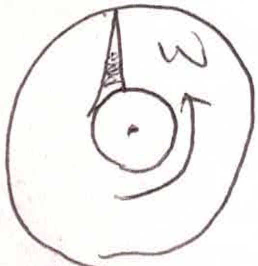
$$\sum F = 0$$

$$F_{drag} = \left(\tau_{rz}\big|_{r=R}\right) \cdot 2\pi R L$$

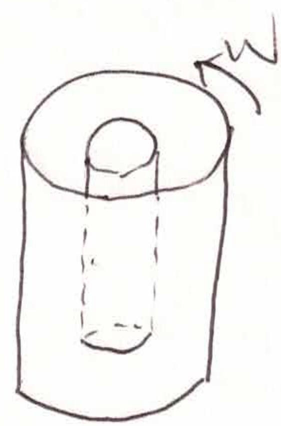
$$\underset{\downarrow}{F_{gravity}} + \underset{\uparrow}{F_{drag}} + \underset{\downarrow}{F_{buoyancy}} = 0$$

4. Stormer





Couette-Hatschek



$r: \quad -\rho \frac{v_\theta^2}{r} = -\frac{\partial p}{\partial r}$

$\theta: \quad 0 = \frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}(r v_\theta)\right)$

$z: \quad 0 = -\frac{\partial p}{\partial z} + \rho g$

B.C.

1. $r = KR, \; v_\theta = 0$
2. $r = R, \; v_\theta = \Omega_0 R$

$$v_\theta = \Omega_0 R \frac{\left(\frac{KR}{r} - \frac{r}{KR}\right)}{\left(K - \frac{1}{K}\right)}$$

$$\tau_{r\theta} = \mu\left\{ r \frac{d}{dr}\left[\frac{\Omega_0 R}{r} \frac{\left(\frac{KR}{r} - \frac{r}{KR}\right)}{\left(K - \frac{1}{K}\right)}\right]\right\}$$

$$= 2\mu\Omega_0 R^2 \left(\frac{1}{r^2}\right)\left(\frac{K^2}{1-K^2}\right)$$

$$T = 2\pi R L \cdot \left|\tau_{r\theta}|_{r=R}\right| \cdot R$$

$$= 4\pi\mu L \Omega_0 R^2 \left(\frac{K^2}{1-K^2}\right)$$

torque required to turn the outer shaft

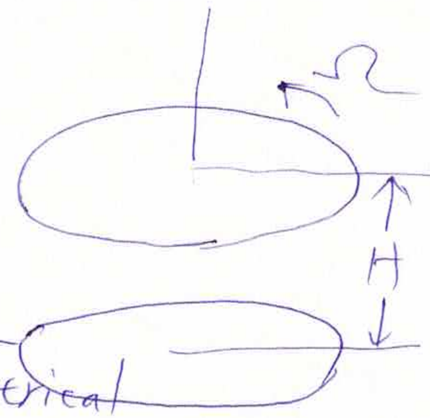
4

5. Cone-Plate rheometer

viscosity

elasticity

"Parallel Disk"



6. Microrheology

solution

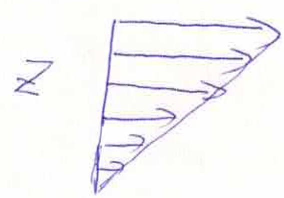
$v_r = 0$ no radial flow θ symmetrical

$v_z = 0$. $v_\theta(r, z)$

$$\theta: \quad 0 = \mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right) + \frac{\partial^2 v_\theta}{\partial z^2}\right]$$

B.C. $r = 0$ v_θ finite

$$\begin{cases} v_\theta = 0 \text{ at } z = 0 \\ v_\theta = r\Omega \text{ at } z = H \end{cases} \Rightarrow \begin{cases} f(0) = 0 \\ f(H) = 1 \end{cases}$$



$\Rightarrow$ Trial solution

$$v_\theta = r\Omega f(z)$$

$$0 = \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r^2 \Omega f(z))\right) + \frac{\partial^2}{\partial z^2}(r\Omega f(z))$$

$$0 = \underbrace{\frac{\partial}{\partial r}(2\Omega f(z))}_{0} + r\Omega \frac{\partial^2 f(z)}{\partial z^2}$$

$$0 = \frac{\partial^2 f(z)}{\partial z^2}$$

$$f(z) = az + b.$$

$0 = b$

$aH = 1$

$a = \frac{1}{H}$

$$\Rightarrow v_\theta = \frac{rz\Omega}{H}$$

$$\tau_{z\theta}\Big|_{z=H} = \mu \frac{\partial v_\theta}{\partial z} = \frac{\mu r \Omega}{H}$$

The force exerted by the liquid on the surface

$$F = \int_0^R dF = \int_0^R \tau_{\theta z}\Big|_{z=H} = \int_0^R \frac{\mu r \Omega}{H} 2\pi r\, dr = \frac{2\pi}{3} \cdot \frac{\mu \Omega R^3}{H}$$

The torque required to turn the rotating disk 5.

$$T = \int_0^R \tau_{z\theta}\big|_{z=H} \cdot r\,dA$$

$$= \int_0^R \frac{\mu \Omega r^2}{H} \cdot 2\pi r\,dr$$

$$= \frac{\pi \Omega \mu R^4}{2H}$$