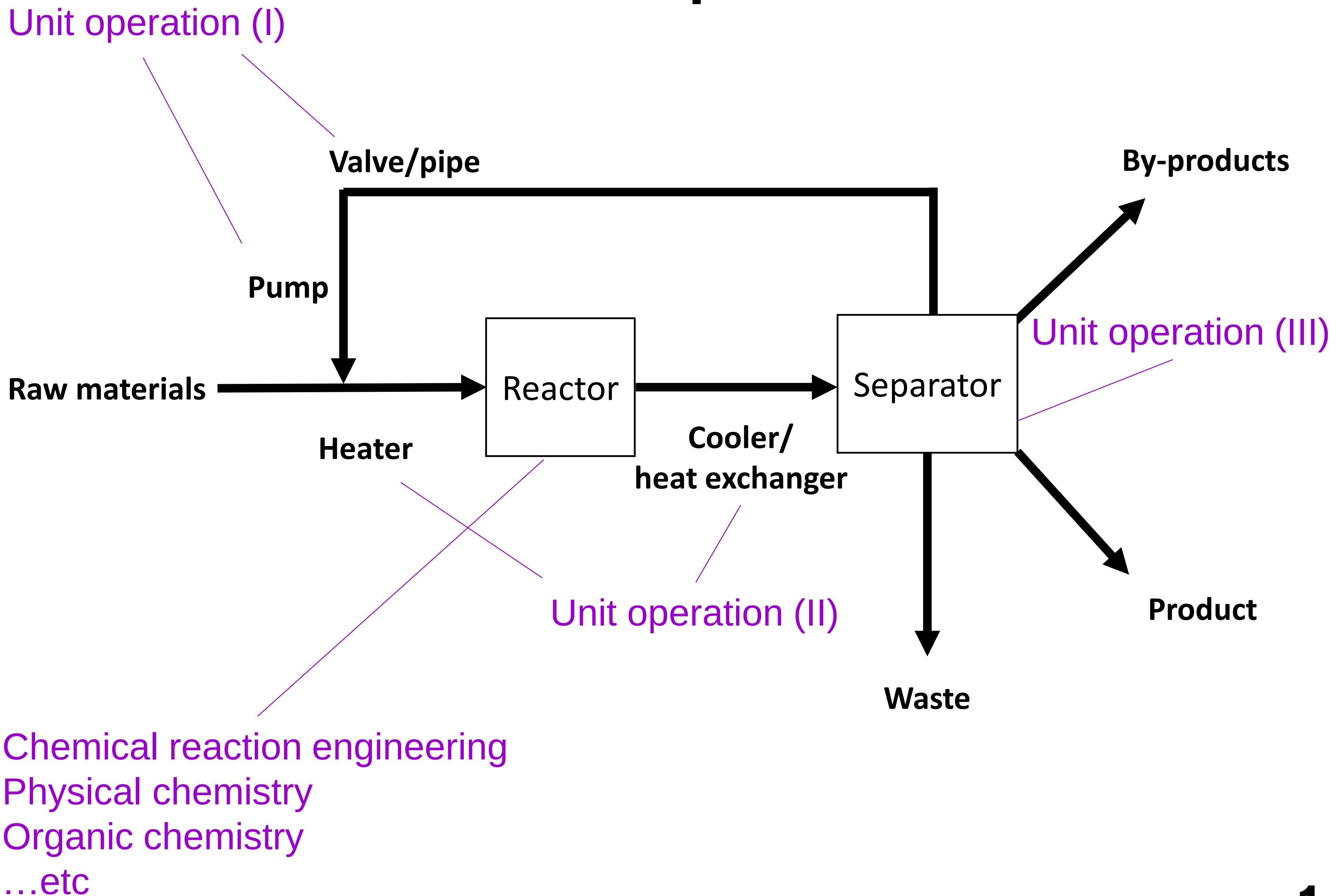
A chemical process…



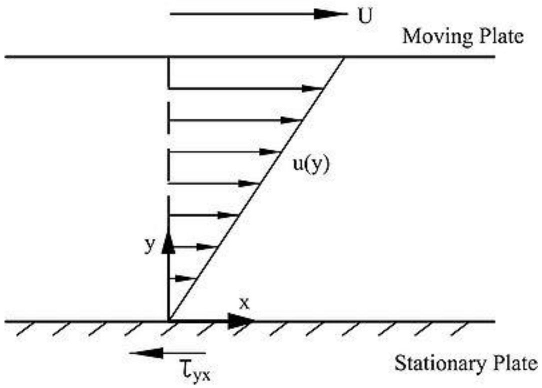
The scope of unit operation (I)

The flow of fluid in pipes, ducts, and units in between:

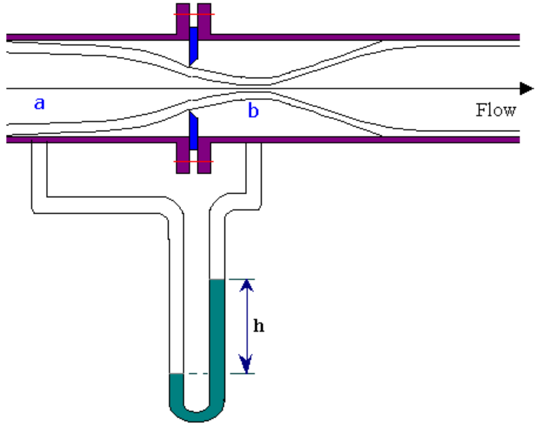
- The transfer of "momentum":

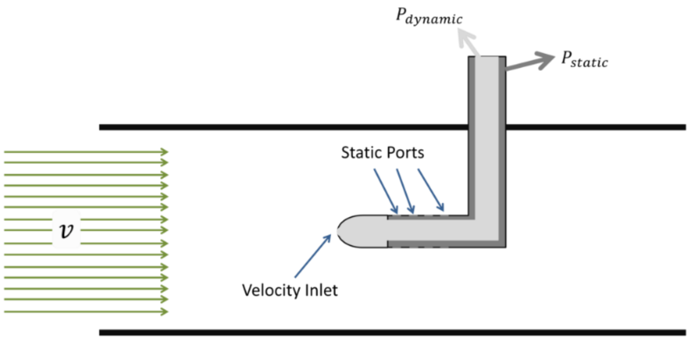
  Inviscid fluid *vs.* Viscous fluid

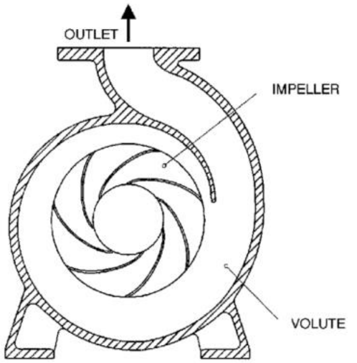
  Laminar flow *vs.* Turbulent flow



- The flow of fluid in flow meters and pumps





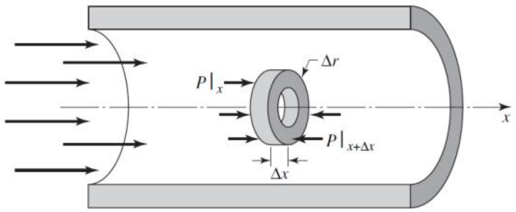


- "Scaling up" the system – Dimensional Analysis

What we have learned recently

- The transfer of "momentum":

1. Concept of "viscosity (μ)" (CH7)
2. Shear stress (τ) of a Newtonian fluid: $\tau = \mu \dfrac{dv}{dy}$
3. "Shell balance" approach for laminar flow (CH8):



4. **Equation of continuity** (mass balance; CH9):

$$\frac{\partial \rho}{\partial t} + v_x \frac{\partial \rho}{\partial x} + v_y \frac{\partial \rho}{\partial y} + v_z \frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}\right) = 0$$

5. **Equation of motion** (momentum balance; CH9):

$$\boxed{\rho \frac{D\mathbf{v}}{Dt} = \rho \mathbf{g} - \nabla P + \mu \nabla^2 \mathbf{v}}$$

Assumptions: constant ρ & constant μ

Navier-Stokes Equation

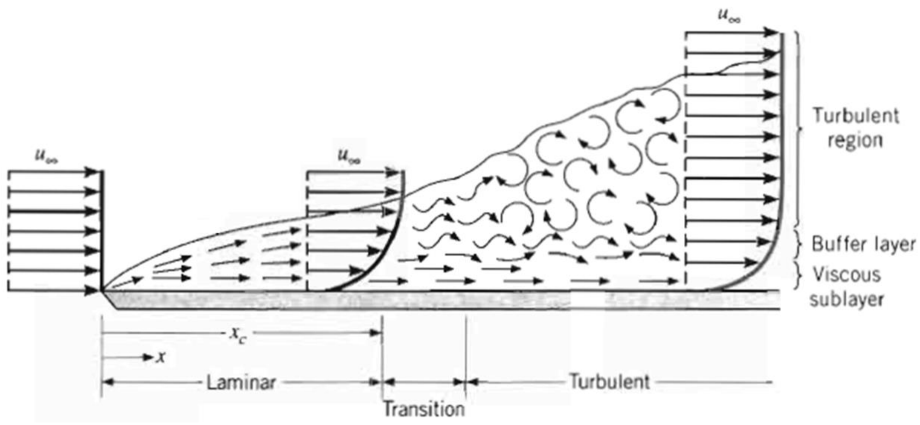
6. Inviscid fluid flow (no shear stress) (CH10)

Topics of the following weeks

1. "Scaling up" the system – Dimensional Analysis

2. The transfer of "momentum":

- Viscous flow and boundary layer
- Turbulent flow



3. Flow of fluid in pipes and pipe fittings

4. Flow meters and pumps

Dimensional analysis

- By combining the variables into a smaller number of dimensionless parameters, the work of experimental data reduction is considerably reduced.

Table 11.1 Important variables in momentum transfer

Variable	Symbol	Dimension
Mass	M	M
Length	L	L
Time	t	t
Velocity	v	L/t
Gravitational acceleration	g	L/t^2
Force	F	ML/t^2
Pressure	P	M/Lt^2
Density	ρ	M/L^3
Viscosity	μ	M/Lt
Surface tension	σ	M/t^2
Sonic velocity	a	L/t

"Fundamentals"

M, L, and t are the only three fundamentals for isothermal momentum-transfer system.

Important!

Dimensional analysis – For known equations

- For a two-dimensional incompressible flow:

Continuity:

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0$$

Momentum:

$$\rho\left(\frac{\partial \mathbf{v}}{\partial t} + v_x\frac{\partial \mathbf{v}}{\partial x} + v_y\frac{\partial \mathbf{v}}{\partial y}\right) = \rho\mathbf{g} - \boldsymbol{\nabla}\rho + \mu\left(\frac{\partial^2 \mathbf{v}}{\partial x^2} + \frac{\partial^2 \mathbf{v}}{\partial y^2}\right)$$

(Navier-Stokes Equation)

Let's define:
- reference length L
- reference velocity v_∞

$$x^* = x/L \qquad v_x^* = v_x/v_\infty$$

$$y^* = y/L \qquad v_y^* = v_y/v_\infty$$

$$t^* = \frac{t\,v_\infty}{L} \qquad \mathbf{v}^* = \mathbf{v}/v_\infty$$

$$\boldsymbol{\nabla}^* = L\,\boldsymbol{\nabla}$$

Dimensional analysis – For known equations

$$\frac{\partial v_x}{\partial x} = \frac{\partial v_x^*}{\partial x^*}\frac{\partial v_x}{\partial v_x^*}\frac{\partial x^*}{\partial x} = \frac{\partial v_x^*}{\partial x^*}(v_\infty)(1/L) = \frac{v_\infty}{L}\frac{\partial v_x^*}{\partial x^*}$$

$$\frac{\partial v_y}{\partial y} = \frac{\partial v_y^*}{\partial y^*}\frac{\partial v_y}{\partial v_y^*}\frac{\partial y^*}{\partial y} = \frac{v_\infty}{L}\frac{\partial v_y^*}{\partial y^*}$$

Eq. of continuity

$$\Rightarrow \quad \frac{\partial v_x^*}{\partial x^*} + \frac{\partial v_y^*}{\partial y^*} = 0$$

Then, the Eq. of motion becomes:

$$\frac{\rho v_\infty^2}{L}\left(\frac{\partial \mathbf{v}^*}{\partial t^*} + v_x^*\frac{\partial \mathbf{v}^*}{\partial x^*} + v_y^*\frac{\partial \mathbf{v}^*}{\partial y^*}\right) = \rho\mathbf{g} + \frac{1}{L}\nabla^* P + \frac{\mu v_\infty}{L^2}\left(\frac{\partial^2 \mathbf{v}^*}{\partial x^{*2}} + \frac{\partial^2 \mathbf{v}^*}{\partial y^{*2}}\right)$$

$$\Rightarrow \quad \frac{\partial \mathbf{v}^*}{\partial t^*} + v_x^*\frac{\partial \mathbf{v}^*}{\partial x^*} + v_y^*\frac{\partial \mathbf{v}^*}{\partial y^*} = \mathbf{g}\underline{\frac{L}{v_\infty^2}} - \frac{\nabla^* P}{\underline{\rho v_\infty^2}} + \underline{\frac{\mu}{L v_\infty \rho}}\left(\frac{\partial^2 \mathbf{v}^*}{\partial x^{*2}} + \frac{\partial^2 \mathbf{v}^*}{\partial y^{*2}}\right)$$

3 dimensionless groups!

Dimensional analysis – For known equations

$$\frac{\partial \mathbf{v}^*}{\partial t^*} + v_x^* \frac{\partial \mathbf{v}^*}{\partial x^*} + v_y^* \frac{\partial \mathbf{v}^*}{\partial y^*} = \mathbf{g} \underline{\frac{L}{v_\infty^2}} - \underline{\frac{\nabla^* P}{\rho v_\infty^2}} + \underline{\frac{\mu}{L v_\infty \rho}} \left(\frac{\partial^2 \mathbf{v}^*}{\partial x^{*2}} + \frac{\partial^2 \mathbf{v}^*}{\partial y^{*2}} \right)$$

$$Fr \equiv v_\infty^2 / gL$$

Froude number (Fr) = $\dfrac{Inertial\ Force}{Gravitational\ Force}$

$$\mathrm{Eu} \equiv P / \rho v_\infty^2$$

Euler number (Eu) = $\dfrac{Pressure\ Force}{Inertial\ Force}$

$$\mathrm{Re} \equiv L v_\infty \rho / \mu$$

Reynolds number (Re) = $\dfrac{Inertial\ Force}{Viscous\ Force}$

Dimensional analysis – A general approach

"Buckingham pi theorem"

Number of "*independent*" fundamentals

$$\boxed{i = n - r}$$

Number of dimensionless groups

Number of variables

Select *r* variables to **represent** *r* fundamentals.

"Recurring set"

Each of the other *i* variables was **divided** by the group of fundamentals to get *i* dimensionless groups.

Dimensional analysis – A general approach

Example: Force is known to be a function of viscosity, density, velocity, and length. Perform dimensional analysis to find the dimensionless groups.

Solution:

Variable	Symbol	Dimensions
Force	F	ML/t^2
Viscosity	μ	M/Lt
Density	ρ	M/L^3
Velocity	v	L/t
Length	l	L

	M	*L*	*t*
F	1	1	-2
μ	1	-1	-1
ρ	1	-3	0
v	0	1	-1
l	0	1	0

- All fundamentals are independent!

  → 5 - 3 = 2 dimensionless groups

  → We need 3 variables as recurring set.

- Let's choose ***l*, v, and ρ** *as the recurring set:*

$$\boxed{L = l} \qquad \boxed{t = \frac{l}{v}} \qquad \boxed{M = \rho l^3}$$

Dimensional analysis – A general approach

Recurring set: $\boxed{M = \rho l^3}$ $\boxed{L = l}$ $\boxed{t = \dfrac{l}{v}}$

Other variables: $F\ [\frac{ML}{t^2}]$ ⇨ $$D1 = \frac{F}{(ML/t^2)} = \frac{Fl^2}{\rho l^4 v^2} = \frac{F/l^2}{\rho v^2} = \frac{P}{\rho v^2} = Eu$$

$\mu\ [\frac{M}{Lt}]$ ⇨ $$D2 = \frac{\mu}{(M/Lt)} = \frac{\mu l^2}{\rho l^3 v} = \frac{\mu}{\rho l v} = Re^{-1}$$

$F = f(\mu, \rho, v, l)$ ⟶ $Eu = f(Re)$

5 variables → 2 variables

Dimensional analysis – A general approach

Table 11.2 Common dimensionless parameters in momentum transfer

Name/Symbol	Dimensionless group	Physical meaning	Area of application
Reynolds number, Re	$L v \rho / \mu$	$\dfrac{\text{Inertial force}}{\text{Viscous force}}$	Widely applicable in a host of fluid flow situations
Euler number, Eu	$P / \rho v^2$		
Coefficient of skin friction, C_f	$\dfrac{F/A}{\rho v^2/2}$	$\dfrac{\text{Pressure Force}}{\text{Inertial force}}$	Flows involving pressure differences due to frictional effects
Froude number, Fr	v^2/gL	$\dfrac{\text{Inertial force}}{\text{Gravitational force}}$	Flows involving free liquid surfaces
Weber number, We	$\dfrac{\rho v^2 L}{\sigma}$	$\dfrac{\text{Inertial force}}{\text{Surface tension force}}$	Flows with significant surface tension effects
Mach number, M	v/C	$\dfrac{\text{Inertial force}}{\text{Compressibility force}}$	Flows with significant compressibility effects

Notes for dimensional analysis

- Choosing the recurring set:

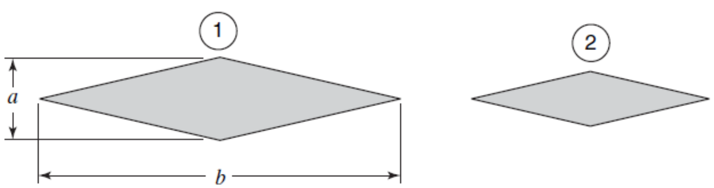
  1. The selected variable needs to contain the targeted fundamental in its dimensions.

  2. In general, the selected variable should not be the main focus of the system.

     Ex: In the previous example, both force and viscous force are crucial.

- Experimental results obtained using models can be used to ***predict*** the performance of full-size prototypical systems. But such scaling requires:

  (1) Geometric similarity; (2) Kinematic similarity; (3) Dynamic similarity



$$\left(\frac{v_x}{v_y}\right)_1 = \left(\frac{v_x}{v_y}\right)_2$$

Ratios of significant forces (ex: Re, Eu, or Fr)

- All variables must be considered when one is performing dimensional analysis.

Scaling up the system

Step 1. Build a small model geometrically similar to the large system.

Step 2. Experimental data obtained from the small model are then scaled to predict the performance of the large prototype according to the similarity requirements.

Example: A 1/6-scale model of a torpedo is tested in a water tunnel to determine drag characteristics. What model velocity corresponds to a torpedo velocity of 20 knots? If the model resistance is 10 lb, what is the prototype resistance?

Solution: For dynamic similarity:

$$Re|_m = Re|_p; \quad \frac{d_m \rho v_m}{\mu} = \frac{6 d_m \rho (20)}{\mu}; \quad v_m = 120\ knots$$

Also for dynamic similarity:

$$Eu|_m = Eu|_p;$$

$$\frac{(10)/A_m}{\rho v_m^2} = \frac{F_p/A_p}{\rho 20^2}; \quad \frac{(10)/d_m^2}{\rho (120)^2} = \frac{F_p/(36 d_m^2)}{\rho 20^2}; \quad F_p = 10\ lb$$