Lecturer: Ryosuke Takahashi

1. Find the limit, if it exists, or show that the limit does not exist.
   (a) (5 points) $\displaystyle\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$
   (b) (5 points) $\displaystyle\lim_{(x,y)\to(0,0)} x \sin\frac{1}{x^2 + y^2} + y\cos\frac{1}{x^2 - y^2} + e^{xy}$
   (c) (5 points) $\displaystyle\lim_{(x,y)\to(1,2)} \frac{xy - 2x - y + 2}{\sqrt{x^2 + y^2 - 2x - 4y + 5}}$

2. Let $f(x, y) = x^4 + 3xy^3 + e^x \sin y + 2x - 7y + 4$.
   (a) (10 points) Find the tangent plane for $z = f(x, y)$ at $(0, 0, 4)$.
   (b) (10 points) Find the maximum and minimum of the $D_{\vec{v}}f(0, 0)$ with $|\vec{v}| = 1$. For which $\vec{v}$ the value $D_{\vec{v}}f(0, 0)$ attaches its maximum? For which $\vec{v}$ the value $D_{\vec{v}}f(0, 0)$ attaches its minimum?

3. (15 points) Show that the function $u(x, t) = \dfrac{t}{a^2t^2 - x^2}$ is a solution of equation $u_{tt} = a^2u_{xx}$.

4. (15 points) Let $f(x, y, z) = x^2 + 4y^2 + 2z^2 - 16$,
$$S = \{(x, y, z) \in \mathbb{R}^3 | f(x, y, z) = 0\},$$
and $\vec{p} = (5, 5, 5)$. Find the equation of the plane P such that for any $\vec{q} \in P \cap S$, the line passing through $\vec{p}$ and $\vec{q}$ is tangent to S at $\vec{q}$.

5. (15 points) Let $F(x, y) = 2x^3 + 3xy^2 - 3\sqrt{2}x^2 + 3\sqrt{2}y^2 - \dfrac{27\sqrt{2}}{2} = 0$ be a plane curve.
   (a) Find $\dfrac{dy}{dx}$ for the curve at $(0, \dfrac{3\sqrt{2}}{2})$.
   (b) Do we have $\dfrac{dy}{dx}$ at $(0, 0)$? Explain your answer.

6. (20 points) Find the absolute maximum and absolute minimum for the function $f(x, y, z) = x^2 + y + z^2 + 1$ defined on the region
$$\Omega = \{(x, y, z) \in \mathbb{R}^3 | 4x^2 + y^2 + 2xz + 4z^2 - 16 \le 0 \text{ and } x + z \ge 1\}.$$