EXAM I
HEAT TRANSFER

April 20, 2016

I. The heat conduction equation is written as

$$\rho C\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}$$

(1) What are the assumptions or conditions for the heat conduction equation above? (3%)

(2) What are the physical meanings of $\rho C\frac{\partial T}{\partial t}$ and $k\frac{\partial^2 T}{\partial x^2}$? (4%)

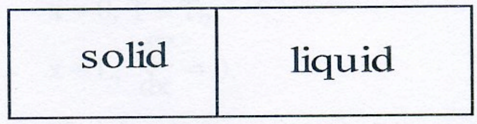
(3) What are the units（單位）of $\rho C\frac{\partial T}{\partial t}$ and $k\frac{\partial^2 T}{\partial x^2}$? (4%)

II. Explain the following terms: (15%)

(1) Fourier law
(2) Biot number
(3) Heat diffusion equation
(4) Steady state
(5) Centered-difference

III. Answer the following questions (32%)

1. In what conditions can the thermal resistance be applied?
2. What are the three material properties whose units are m^2/sec?
3. 鑽石與銅，那一個熱傳導係數較高？這兩種材料之熱傳導機制有何不同？(5%)
4. For a house, the inside temperature is higher than the outside one. Is it possible to make a completely heat-insulated wall? Why? or Why not?
5. For a solid body in a fluid, when the Biot number is very large, why is the surface temperature of the body close to the fluid temperature?
6. From the view point of heat resistance, applying a fin to a high-power component would add thermal resistance to the component, which is similar to that wearing more clothes make us feel warmer. Why is it said that the fin can assist the heat dissipation（散熱）of the component?
7. 戰車為何不使用水冷式散熱，而使用氣冷式？
8. Write down the boundary condition at the liquid-solid interface, as shown in the following figure. (Note: 1-D)

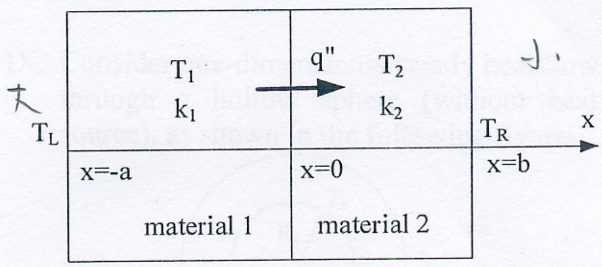


9. In calculating the heat lost by the fin, why can it be computed by either of the following two equations?

$$q_b=\int_0^L hP(T_b-T_\infty)dx \quad \text{or} \quad q_b=hPL(T_b-T_\infty)$$

10. Diamond and graphite are two different allotropes of carbon. What is the difference between their heat transfer behaviors?

IV. In the following figure, two materials with k_1 and k_2 thermal conductivities have perfect contact and their corresponding temperatures are T_1 and T_2. Assume T_L is larger than T_R.



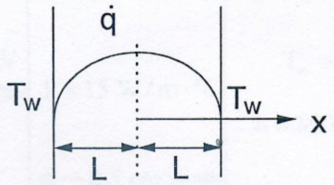
(i) Write down the boundary conditions at x = 0, x = -a and x = b. (4%)

(ii) Find the heat flux, q”, going through these two materials under the assumptions of 1-D steady state with no heat source and constant k_1 and k_2. (5%)

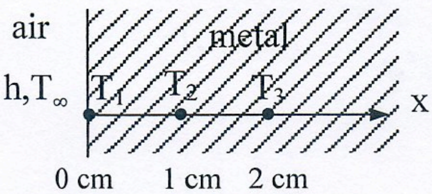
(iii) If these two materials have non-perfect contact, write down the boundary conditions at x = 0 and find the heat flux, q”, going through these two materials under the assumptions of 1-D steady state with no heat source and constant k_1 and k_2. (7%)

Hint: $q = \Delta T/R_{th}$, $R_{th} = \Delta x/(kA)$, $q'' = q/A$.

V. A one-dimensional steady heat transfer problem of a plane wall with a uniform heat source $\dot{q}$ is shown in the following figure. What is the heat flux on the wall surface (i.e., at x = L)? (7%)

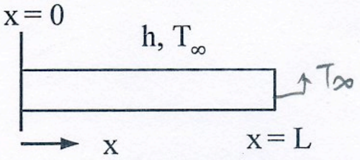


VI. As shown in the following figure, $T_1 = 22°C$, $T_2 = 27°C$, $T_3 = 32°C$ and $T_\infty = 20°C$. If the thermal conductivity of the metal is 1 W/(m·°C). What is the convective heat transfer coefficient h? (8%)



22 27 32

VII. (1) In what conditions can a fin be regarded as a one-dimensional problem mathematically? (3%)



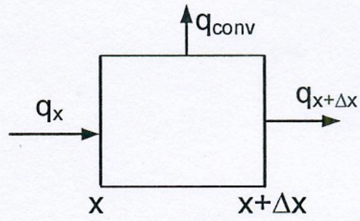
(2) Prove that the (one-dimensional) energy equation of a straight fin with constant cross-sectional area is

$\frac{d^2T}{dx^2} = \frac{hP}{kA}(T-T_\infty)$.

$$\frac{d^2T}{dx^2} - \frac{hP}{kA}(T - T_\infty) = 0,$$

where A is the cross-sectional area and P is the perimeter. (6%)

Hint: Use the following control volume taken from the fin to prove the energy equation.



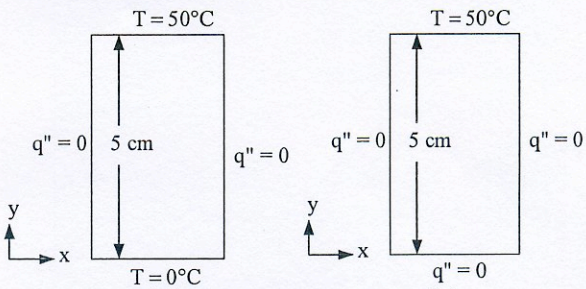
(3) The boundary conditions of the fin are

$x = 0$, $T = T_b$;

$x = L$, $\frac{dT}{dx} = 0$.

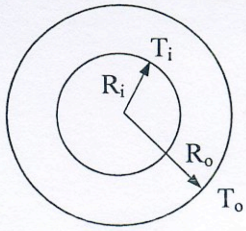
Find the temperature solution. (5%)

VIII. According to the following boundary conditions, draw the isotherms for the steady solutions or write down the solutions. (No calculation required) (8%)



(a) (b)

IX. Consider one-dimensional steady heat flow thrugh a hollow sphere (without heat source), as shown in the following figure.



Derive the expression of the thermal resistance for the heat flow from R_i to R_o. (8%)

X. Consider a plate with the thickness of 0.5 cm. The cross-sectional area is 300 cm^2 and its thermal conductivity is 15 W/m·°C. One surface of the plate is subjected to uniform heat flux, whose total heat transfer rate is 1200 W. The other surface loses heat to the surroundings at T_∞ by convection, as shown in the following figure. The convection heat transfer coefficient (h) is 80 $W/m^2 \cdot °C$. Evaluate the temperatures at these two surfaces (i.e., T(0) and T(L)). (10%)