EXAM II
HEAT TRANSFER

May 11, 2016

I. Explain the following terms: (9%)

(1) Similarity transformation

(2) Explicit Formulation

(3) Biot number

II. Answer the following questions: (21%)

(1) In what condition can the convective boundary condition be assumed as an insulated boundary condition?

(2) In solving a 2-D steady heat conduction problem, the variable separation is used. It is assumed T = X(x)Y(y). Put this relation into the energy equation and the equation can rearranged as

$$-\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{Y}\frac{d^2Y}{dy^2} = \text{constant}$$

Why are they equal to constant?

(3) What assupmtions should be made for the following heat diffusion equaiton?

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

(4) In the following finite difference formulation of the heat diffusion equation, the difference expression used for the time derivative $\partial T/\partial t$ is forward difference or backward difference? Why?

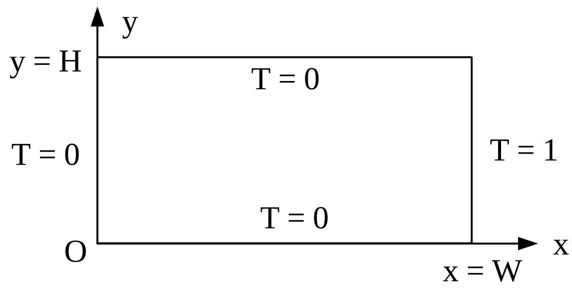
$$\frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} = \alpha\left[\frac{T_{i+1,j}^{n+1} - 2T_{i,j}^{n+1} + T_{i-1,j}^{n+1}}{\Delta x^2} + \frac{T_{i,j+1}^{n+1} - 2T_{i,j}^{n+1} + T_{i,j-1}^{n+1}}{\Delta y^2}\right]$$

(5) What are the two kinds of error by using the finite difference to simulate the heat transfer problems?

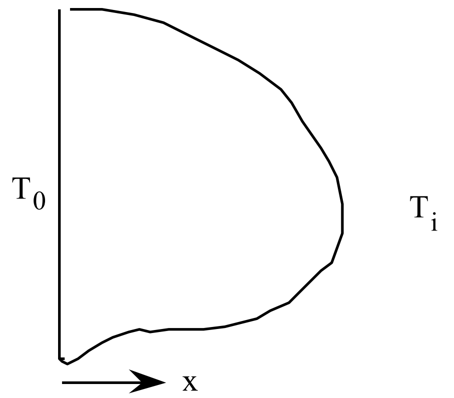
(6) What is the lumped-heat-capacity system? In what conditions can the system be applied?

(7) What are the advantage and disadvantage of the explicit method in solving transient problems with the finite difference method?

III. Consider a steady-state heat conduction problem in a rectangular plate. Its boundary conditions are shown in the following figure. Find the temperature solution of the plate. (12%)



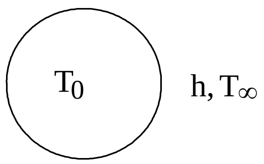
IV. Consider the semi-infinite solid shown in the following figure maintained at some initial temperature T_i. The surface temperature is suddenly lowered and maintain at a temperature T_0.



(a) Solve the temperature distribution for this transient problem. (12%)

(b) Draw the temperature profiles varying with x for several different time steps. (3%)

V. A solid body has a very high thermal conductivity, whose volume and surface area are V and A. It is put in a fluid, whose temperature is T_∞ and the convective heat transfer coefficient is h. The initial temperature of the solid body is T_0 and its thermal conductivity, density and specific heat are k, ρ and C. Derive the temperature expression of the body in terms of time. (8%)



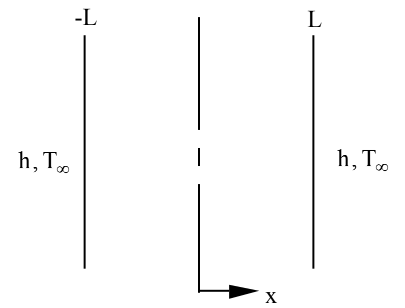
VI. A plane wall of thickness 2L, thermal conductivity k, and diffusivity α (shown in the following figure) is in thermal equilibrium with a surrounding fluid at T_∞. Suddenly, constant uniform generation (q''', W/m^3) begins in the wall. The fluid convective environment is such that h >> k/L. The governing heat conduction equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} - \frac{q'''}{k}$$

By using superposition, separation of variables, and appropriate boundary and initial conditions, obtain an expression for the transient, non-dimensional temperature profile

$$\theta = \frac{T(x^*, t^*) - T_\infty}{L^2 q''' / k}$$

where $t^* = t\alpha/L^2$ and $x^* = x/L$. (25%)



Hint:

(1) Write down the initial and boundary conditions. (3%)

(2) Non-dimensionalize the governing equation and the initial and boundary conditions with

$$\theta = \frac{T - T_\infty}{L^2 q''' / k},\ t^* = t\alpha/L^2 \text{ and } x^* = x/L. \ (5\%)$$

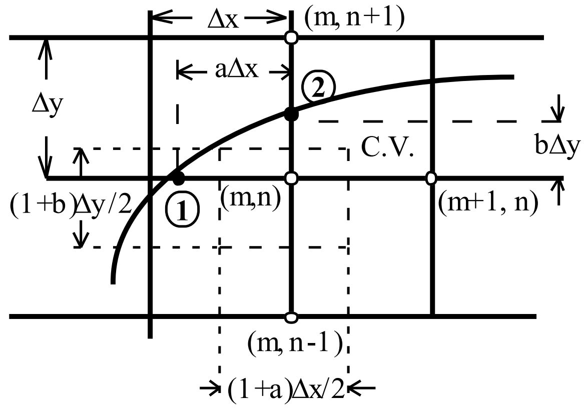
(3) Assume $\theta = \theta_{ss}(x^*) + \theta_t(x^*, t^*)$, where θ_{ss} is the steady solution and θ_t is the transient solution which will be approaching zero as time is close to the infinity. θ_{ss} is used to take care of the non-homogeneous term in the governing equation (i.e., the heat source term).

(i) Write down the governing equation (an ordinary differential equation of second order) and the boundary conditions of θ_{ss}. (3%)

(ii) Write down the governing equation and the initial and boundary conditions of θ_t. The governing equation is homogeneous and so are the boundary conditions. (4%)

(4) Solve θ_{ss} and θ_t. (10%)

VIII. Derive the finite difference equation for the interior node, (m, n), near the curve boundary. (15%)



C.V. : control volume

$$\frac{2}{b(b+1)}T_{②} + \frac{2}{a+1}T_{m+1,n} + \frac{2}{b+1}T_{m,n-1} + \frac{2}{a(a+1)}T_{①} - 2\left(\frac{1}{a} + \frac{1}{b}\right)T_{m,n} = 0$$

for $\Delta x = \Delta y$.

IX. Derive the finite difference equation of implicit formulation for an intersect point (the point (i,j) shown in the following figure) of two heat-insulated boundaries for a two-dimensional transient problem of heat transfer. (10%)