Numerical method
Final Exam (2020/06/22 8:10~11:00)

學號：_____ 姓名：_____

手寫計算題

1. Given the data table.

x	1.0	1.3	1.6	1.9
$f(x)$	0.76	0.62	0.46	0.28

(7%) Use the Newton's divided-difference formula. Find $f(1.5)$.

2. (7%) Use the LU method to get the solution of x_1, x_2 and x_3.

$$\begin{cases} 2x_1 + 2x_2 + 4x_3 = -2 \\ 1x_1 + 5x_2 + 3x_3 = 2 \\ 3x_1 + 1x_2 + 1x_3 = 1.5 \end{cases}$$

3. (7%) Fit a least square curve of the form $y = ae^{bx}$ $(a > 0)$ to the data given below. Please calculate the values of a, b, and R^2.

x_i	1	2	3	4	5
y_i	1	3	5	7	9

4. (7%) Construct the cubic spline

$$S_i = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

where $i = 1, 2, 3$, using the following data and boundary condition $S_1'(0) = S_3'(3) = 7$.

x	0	1	2	3
$f(x)$	0	1	8	27

5. (7%) Use two-point Gauss quadrature rule to approximate the distance covered by a rocket from t = 8 to t = 30 as given by

$$x = \int_8^{30} \left(2000 \ln\left[\frac{140000}{140000 - 2100t}\right] - 9.8t\right) dt$$

6. (10%) Derive the Adams-Moulton four-step closed method.

7. (15%) Derive the 2^{nd} order Runge-Kutta method and show this method can include the modified Euler's method and the Heun's method.