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Fundamentals of Momentum, Heat, and Mass Transfer

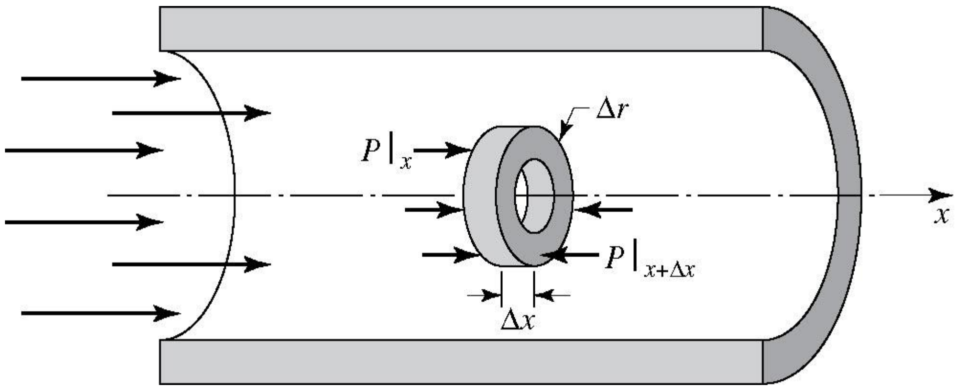
Sixth Edition

Welty • Rorrer • Foster

Chapter 8

Analysis of a Differential Fluid Element in Laminar Flow

Fully developed laminar flow in a circular conduit



Assumption:

1. The fluid behaves as a **continuum**.
2. The fluid is **incompressible** and **Newtonian**.
3. The flow is **laminar and fully developed**.
4. The flow is not influenced by entrance effects and represents a **steady-state** situation.

Fully developed: the velocity profile does not vary along the axis of flow

Conservation of **Mass** and **Momentum** should apply!!

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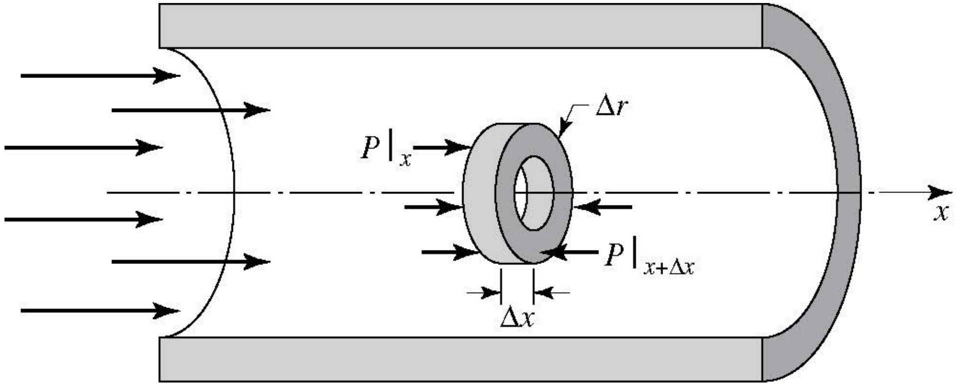
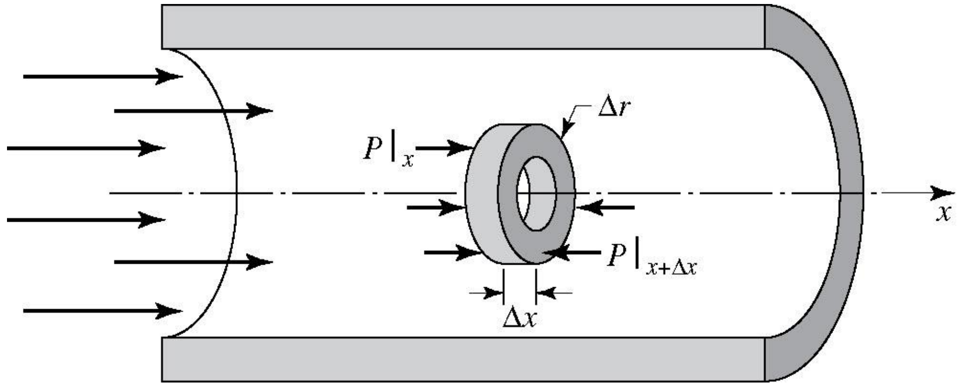


$$\Sigma F_x = \iint_{\text{c.s.}} \rho v_x (\mathbf{v} \cdot \mathbf{n}) dA + \frac{\partial}{\partial t} \iiint_{\text{c.v.}} \rho v_x dV \tag{5-5a}$$

$$-r \frac{dP}{dx} + \frac{d}{dr}(r\tau_{rx}) = 0 \tag{8-1}$$

$$\tau_{rx} = \left(\frac{dP}{dx}\right)\frac{r}{2} \tag{8-2}$$

$$\tau_{rx} = \mu \frac{dv_x}{dr} \tag{8-3}$$



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Flux integral

ρ**v**: mass flux
ρ**vv**: convective momentum flux
τ: stress; momentum flux by molecular (viscous) transfer

$$v_x = -\left(\frac{dP}{dx}\right)\frac{1}{4\mu}(R^2 - r^2) \tag{8-4}$$

$$v_x = -\left(\frac{dP}{dx}\right)\frac{R^2}{4\mu}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{8-5}$$

$$v_{\max} = -\left(\frac{dP}{dx}\right)\frac{R^2}{4\mu} \tag{8-6}$$

$$v_x = v_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{8-7}$$

$$v_{\text{avg}} = \frac{v_{\max}}{2} = -\left(\frac{dP}{dx}\right)\frac{R^2}{8\mu} \tag{8-8}$$

Hagen-Poiseuille equation

$$-\frac{dP}{dx} = \frac{8\mu v_{\text{avg}}}{R^2} = \frac{32\mu v_{\text{avg}}}{D^2} \tag{8-9}$$

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Hagen-Poiseuille equation

$$-\frac{dP}{dx}=\frac{8\mu v_{\mathrm{avg}}}{R^2}=\frac{32\mu v_{\mathrm{avg}}}{D^2} \qquad (8\text{-}9)$$

Reynolds number for fluids flowing in tubes

$$\mathrm{Re}=\frac{D\ \upsilon_{avg}\rho}{\mu}$$

Re < 2100, laminar flow
Re > 2100, turbulent flow

The shear stress components in cylindrical coordinates

$$\tau_{r\theta}=\tau_{\theta r}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right)+\frac{1}{r}\frac{\partial \upsilon_r}{\partial\theta}\right]$$

$$\tau_{z\theta}=\tau_{\theta z}=\mu\left[\frac{\partial \upsilon_\theta}{\partial z}+\frac{1}{r}\frac{\partial \upsilon_z}{\partial\theta}\right]$$

$$\tau_{zr}=\tau_{rz}=\mu\left[\frac{\partial \upsilon_z}{\partial r}+\frac{\partial \upsilon_r}{\partial z}\right]$$

The shear stress components in spherical coordinates

$$\tau_{r\theta}=\tau_{\theta r}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{\upsilon_\theta}{r}\right)+\frac{1}{r}\frac{\partial \upsilon_r}{\partial\theta}\right]$$

$$\tau_{\phi\theta}=\tau_{\theta\phi}=\mu\left[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}\left(\frac{\upsilon_\phi}{\sin\theta}\right)+\frac{1}{r\sin\theta}\frac{\partial \upsilon_\theta}{\partial\phi}\right]$$

$$\tau_{\phi r}=\tau_{r\phi}=\mu\left[\frac{1}{r\sin\theta}\frac{\partial \upsilon_r}{\partial\phi}+r\frac{\partial}{\partial r}\left(\frac{\upsilon_\phi}{r}\right)\right]$$

Hagen-Poiseuille equation

$$-\frac{dP}{dx} = \frac{8\mu v_{\text{avg}}}{R^2} = \frac{32\mu v_{\text{avg}}}{D^2} \qquad (8\text{-}9)$$

Other forms?

Calculate the **force** of the fluid acting on the wetted surface of the pipe

Calculate the **mass rate of flow and average velocity**

Experimental observation

Reynolds number for fluids flowing in tubes

$$\mathrm{Re} = \frac{D\,\upsilon_{avg}\rho}{\mu}$$

Re < **2100**, laminar flow

Re > **2100**, turbulent flow

Physical meaning of Re: inertia force/viscous force; $\rho v^2 a^2/\mu v a$

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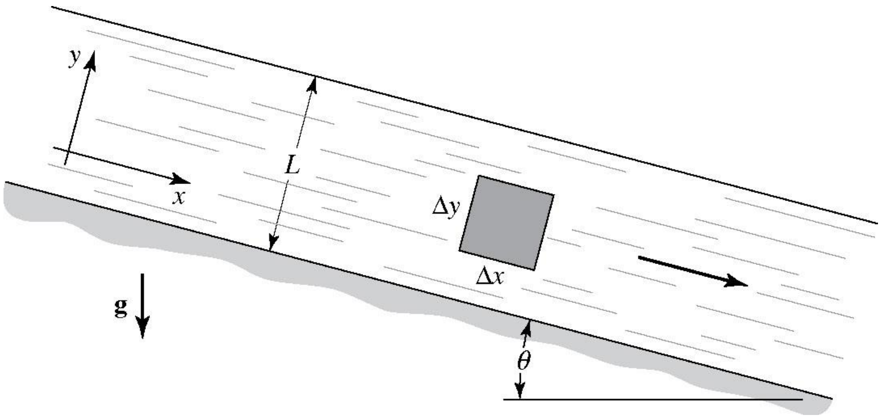
Laminar flow of a Newtonian Fluid down on inclined-plane surface



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Conservation of **Mass** and **Momentum** should apply!!

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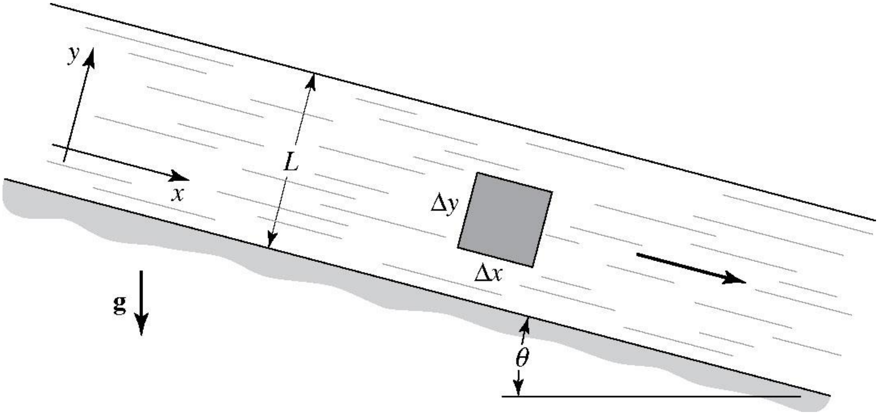

$$\frac{d}{dy}\tau_{yx} + \rho g \sin\theta = 0 \tag{8-10}$$

$$\tau_{yx} = \rho g L \sin\theta \left[1 - \frac{y}{L}\right] \tag{8-11}$$

$$v_x = \frac{\rho g L^2 \sin\theta}{\mu}\left[\frac{y}{L} - \frac{1}{2}\left(\frac{y}{L}\right)^2\right] \tag{8-12}$$

$$v_{\max} = \frac{\rho g L^2 \sin\theta}{2\mu} \tag{8-13}$$

Calculate the **force** of the fluid acting on the wetted surface of the wall

Calculate the **mass rate of flow and film thickness**

Experimental observation

Reynolds number for fluids flowing in tubes

$$\mathrm{Re} = \frac{4L\upsilon_{avg}\rho}{\mu}$$

Re < **20**, laminar flow with negligible rippling
20 < Re < **1500**, laminar flow with pronounced rippling
Re > **1500**, turbulent flow

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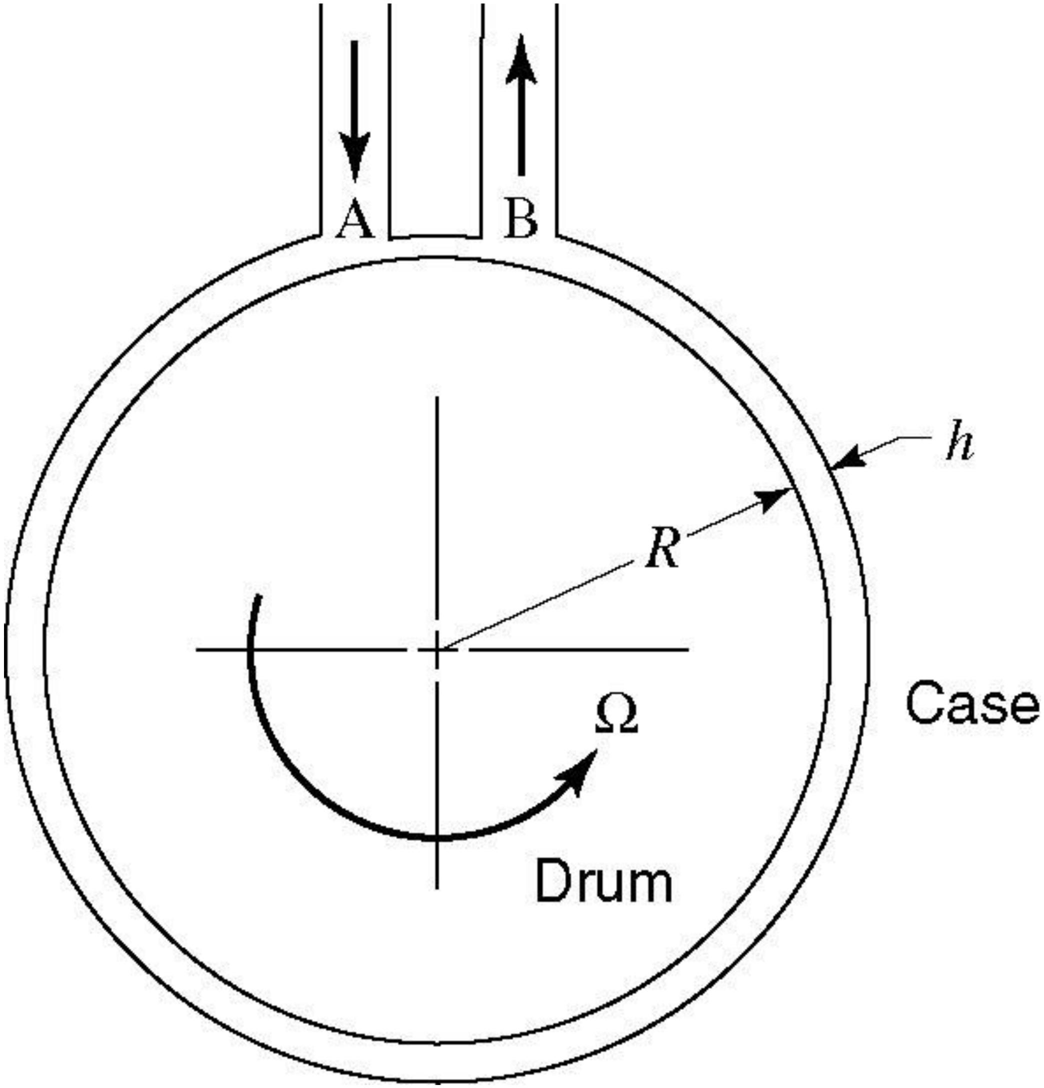
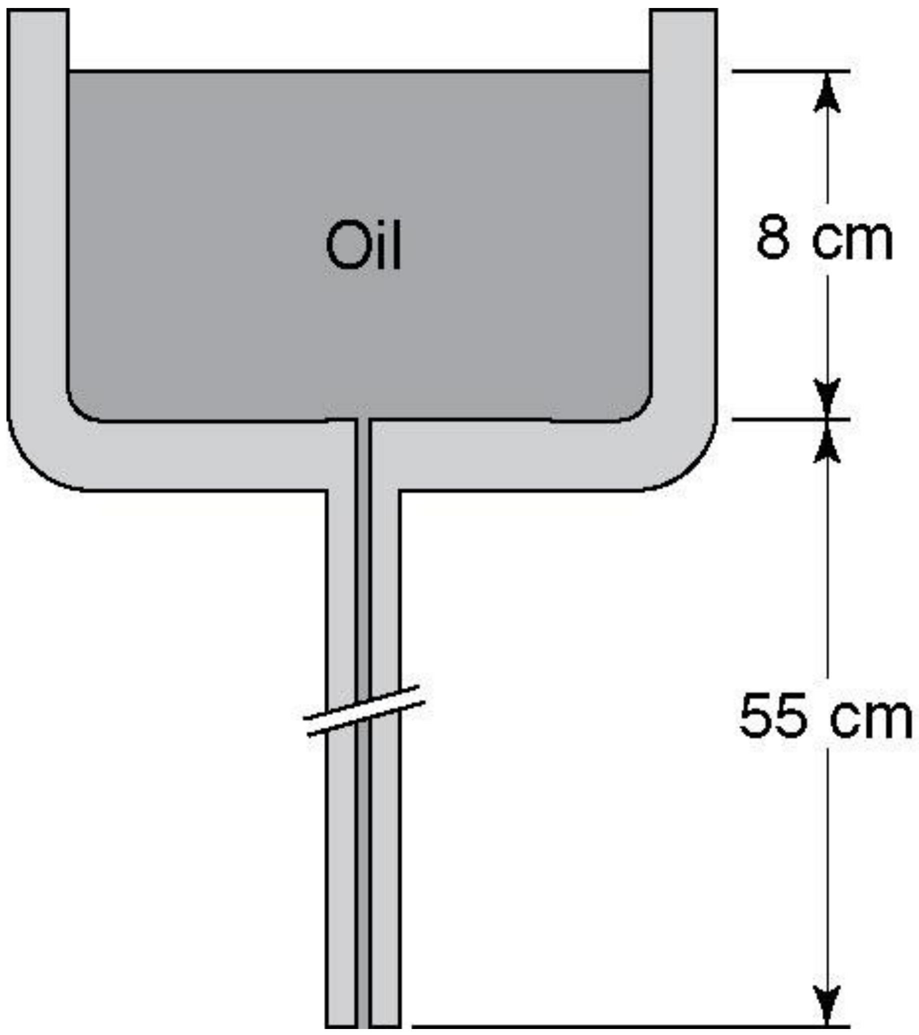

A
B
h
R
Ω
Case
Drum


Oil
8 cm
55 cm