

# **Chapter 3**

# **Inductance and Capacitance**

## **電感與電容**

# Inductance and Capacitance

## 電感與電容

- 電容與電感皆可用來儲存能量(energy-storage elements)，並將儲存的能量釋放回電路。
- 電容與電感無法自行產生能量，故稱為被動元件(passive elements,如同電阻)。
- 電容是根據電場現象製造的電路元件，將能量儲存於電場。
- 電感是依磁場現象製造的電路元件，將能量儲存於磁場。

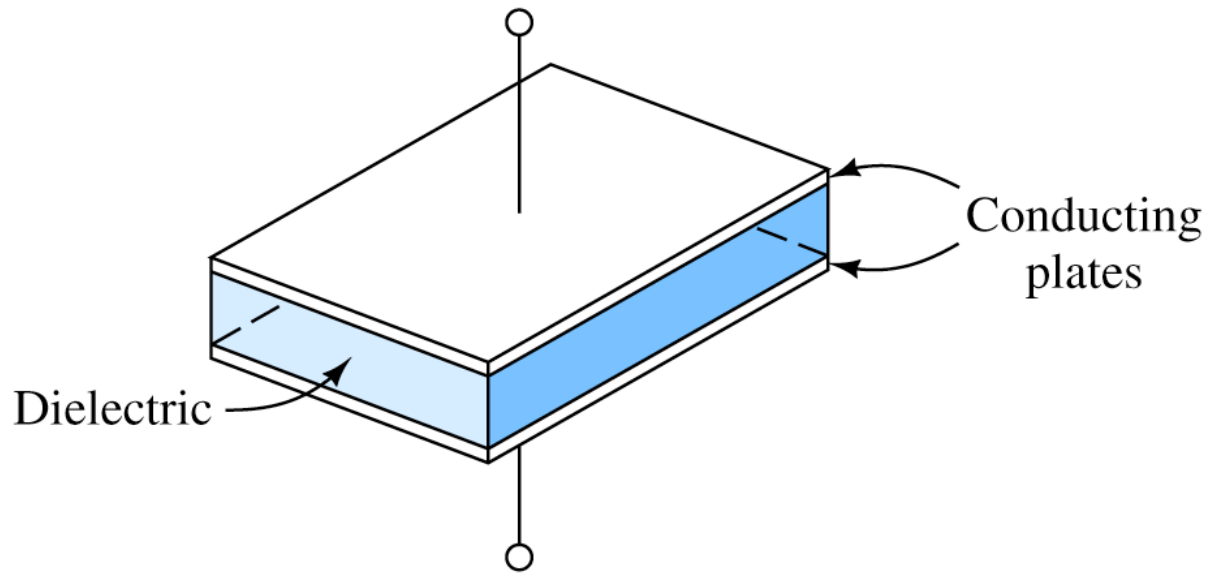
# Inductance and Capacitance

## 電感與電容

- 電容與電感皆具有線性微分的端點特性
  - 理想電容兩端電流與電壓對時間微分成正比
  - 理想電感兩端電壓與電流對時間微分成正比

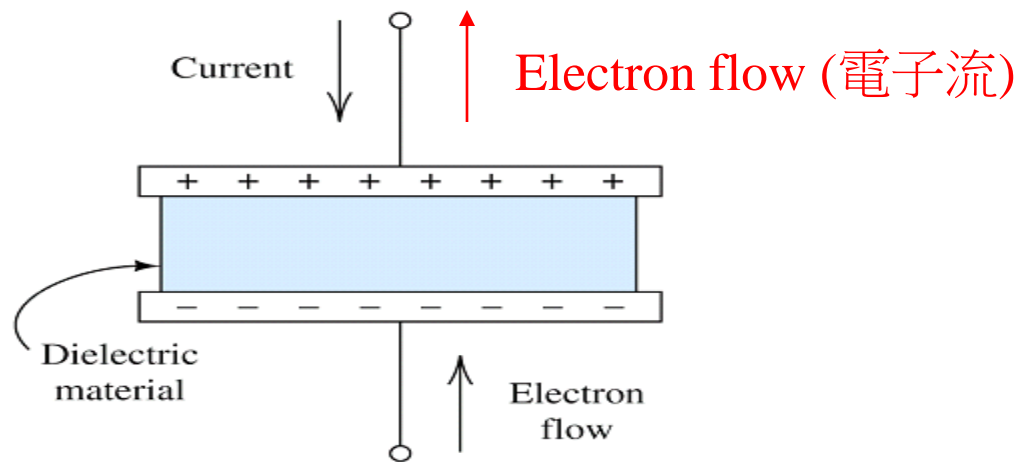
# 3.1 CAPACITANCE (電容)

- 電容器是由絕緣材料(dielectric, 介電質)隔開的兩片導電平板(conductive plates)形成。
- 介電質為mica (雲母), polyester(聚脂) 等絕緣材料。



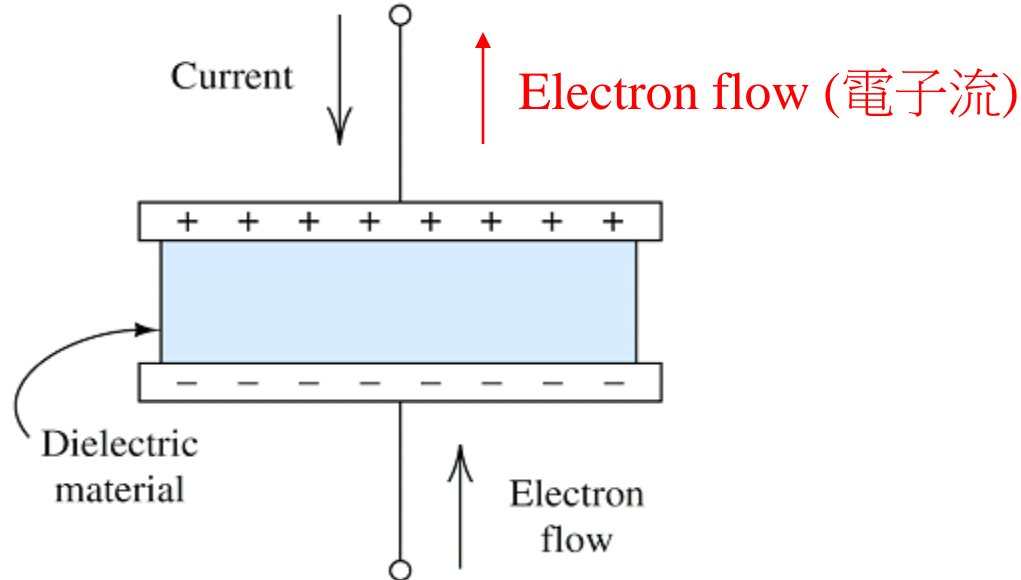
**Figure 3.1** A parallel-plate capacitor consists of two conductive plates separated by a dielectric layer.

- 在電容器的兩電板間**加上電壓**，因介電值的存在，**導線電荷**無法直接流經電板。
- 電荷聚集在兩導電平板上。如下圖，由下往上移動的**負電荷電子(electrons)**聚集在**下導電平板**，使下導電平板帶**負電**，而形成電場。
- 此電場使上導電平板的電子往上移動(**正電荷**聚集於**上導電平板**，而將電子往上推)。



(a) As current flows through a capacitor, charges of opposite sign collect on the respective plates

- 聚集在下導電平板的電子數等於離開上導電平板的電子數，可視為電流流過電容。
- 兩導電平板上電荷數相等，符號相反。



(a) As current flows through a capacitor, charges of opposite sign collect on the respective plates

# Stored Charge in Terms of Voltage

- 理想電容儲存的電荷(charge)  $q$  與其導電平板兩端電壓成正比。

$$q = Cv$$

- 常數 $C$ 為電容值(capacitance)，單位為法拉(F, farads)，等於coulombs/volt ( $C/V$ )。
- 電荷 $q$ 代表接電壓正端的導電平板上的淨(正)電荷。
- 一般電容值在數個pico法拉( $1\text{pF} = 10^{-12}\text{F}$ )到 $0.01\text{F}$ 之間

# Current in Terms of Voltage

Recall 電流：time rate of flow of electrical charge (單位時間通過電荷量)。電容兩端的電流為

$$i = \frac{dq}{dt} = \frac{d}{dt}(Cv)$$

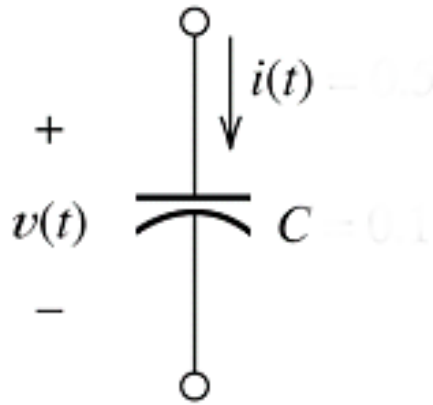
電容兩端電壓電流關係為

$$i = C \frac{dv}{dt}$$

若電壓上升，則有電流通過電容，而電荷累積在導電平板上。若電壓不變(DC)，則電流為0，視為斷路(open circuit)，平板電荷數不變。



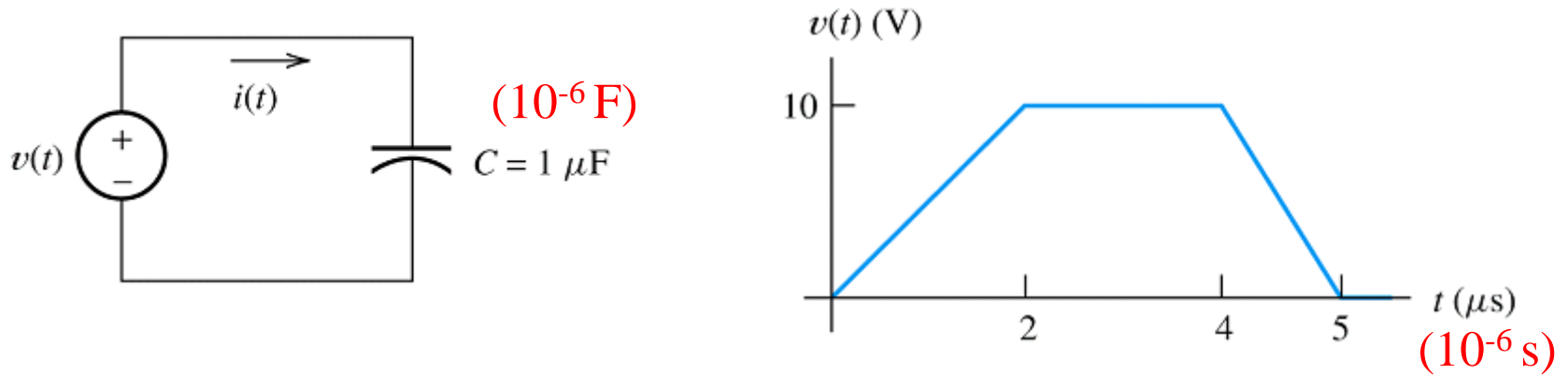
# Symbol for Capacitance



- 通過電容的電流參考方向為電容器兩端電壓降落的方向(由正極流入，負極流出)，具 passive configuration 特性。
- 若電容放電，則

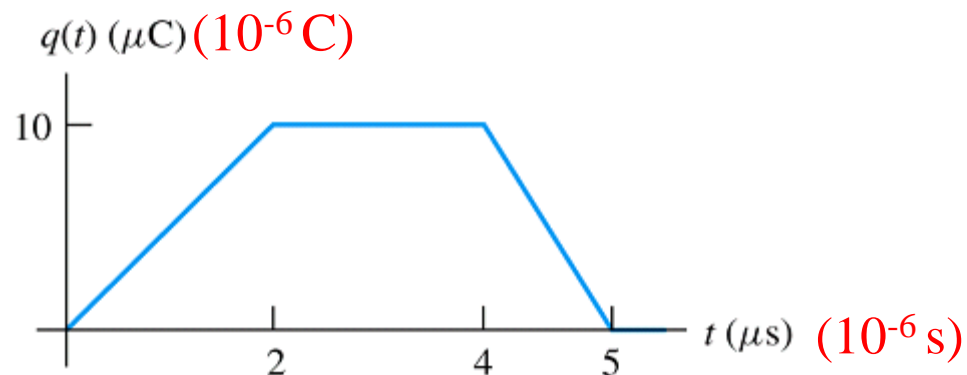
$$i = -C \frac{dv}{dt}$$

Example 3.1 Plot the **stored charge** and the **current** through the capacitance **versus time**.

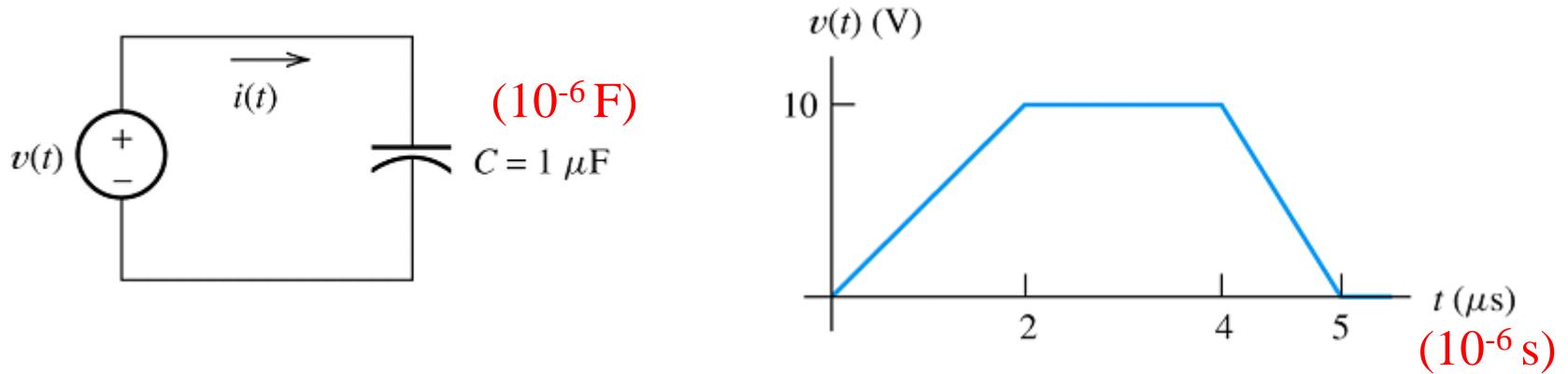


The stored charge

$$q(t) = Cv(t) = 10^{-6} v(t)$$



Example 3.1 Plot the **stored charge** and the **current** through the capacitance **versus time**.



The current

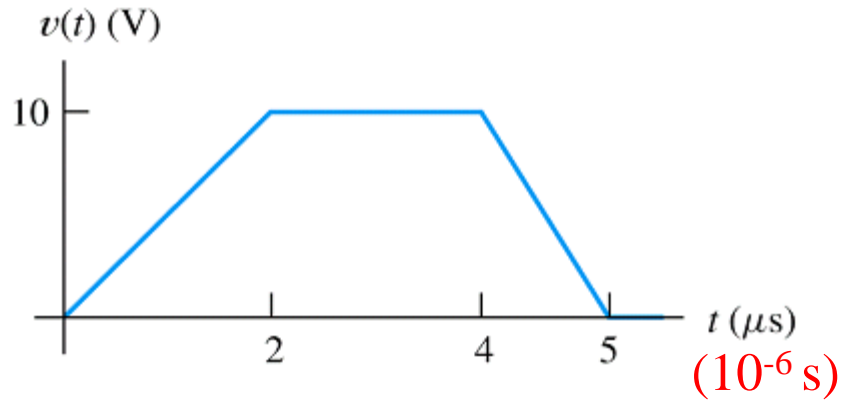
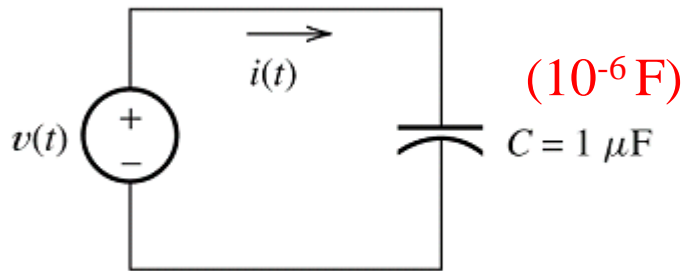
$$i(t) = \frac{dq(t)}{dt} = C \frac{dv(t)}{dt} = 10^{-6} \frac{dv(t)}{dt}$$

$$t = 0 \sim 2 \mu\text{s} \quad \frac{dv(t)}{dt} = \frac{10\text{V}}{2 \times 10^{-6} \text{ s}} = 5 \times 10^6 \text{ V/s} \quad i(t) = C \frac{dv(t)}{dt} = 10^{-6} \times 5 \times 10^6 = 5 \text{ A}$$

$$t = 2 \sim 4 \mu\text{s} \quad \frac{dv(t)}{dt} = 0 \quad i(t) = C \frac{dv(t)}{dt} = 0$$

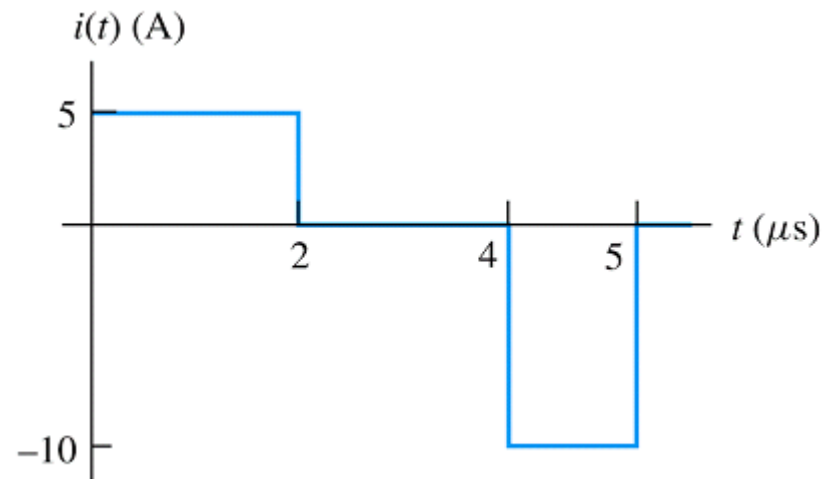
$$t = 4 \sim 5 \mu\text{s} \quad \frac{dv(t)}{dt} = \frac{-10\text{V}}{10^{-6} \text{ s}} = -10^7 \text{ V/s} \quad i(t) = C \frac{dv(t)}{dt} = 10^{-6} \times (-10^7) = -10 \text{ A}$$

Example 3.1 Plot the stored charge and the current through the capacitance versus time.



The current

$$\left\{ \begin{array}{ll} t = 0 \sim 2 \mu\text{s} & i(t) = 5 \text{ A} \\ t = 2 \sim 4 \mu\text{s} & i(t) = 0 \\ t = 4 \sim 5 \mu\text{s} & i(t) = -10 \text{ A} \end{array} \right.$$



# Voltage in Terms of Current

- 假設我們知道通過電容 $C$ 的電流 $i(t)$ ，如何求得電容器兩端電壓？假設初始時間 $t_0$ 時初始電荷為 $q(t_0)$ 。

$$i = \frac{dq}{dt} \quad \longrightarrow \quad q(t) = \int_{t_0}^t i(t) dt + q(t_0)$$

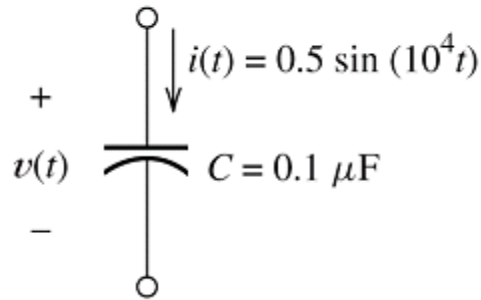
$$q(t) = Cv(t) \quad \longrightarrow \quad v(t) = \frac{1}{C} q(t) = \frac{1}{C} \int_{t_0}^t i(t) dt + \frac{q(t_0)}{C}$$

$$\text{Initial voltage} \quad v(t_0) = \frac{q(t_0)}{C}$$

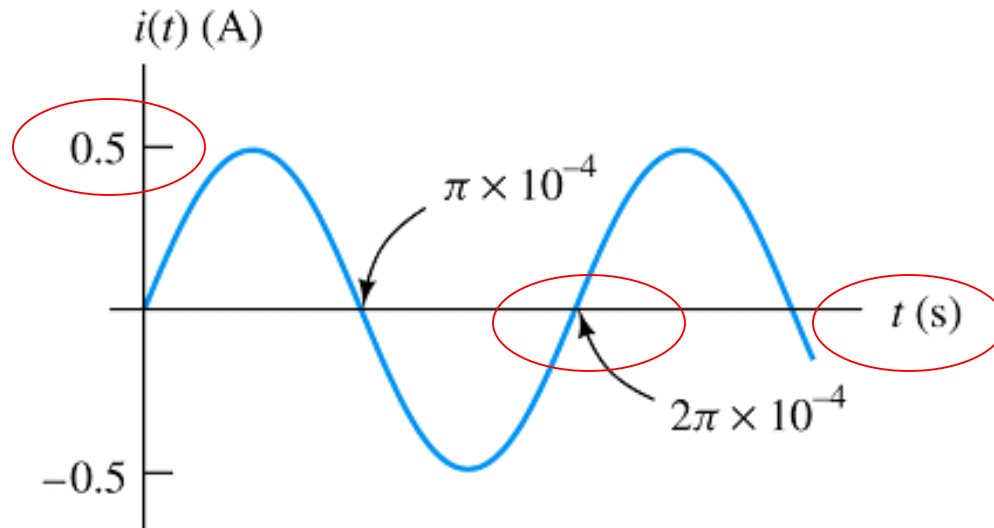
$$\longrightarrow \quad v(t) = \frac{1}{C} \int_{t_0}^t i(t) dt + v(t_0)$$

## Example 3.2 Determining Voltage Given Current

$C = 0.1 \mu\text{F}$ ,  $q(0) = 0$ . After  $t_0 = 0$ ,  $i(t) = 0.5 \sin(10^4 t)$ . Plot  $i(t)$ ,  $q(t)$  and  $v(t)$ .



### 1. Plot $i(t)$

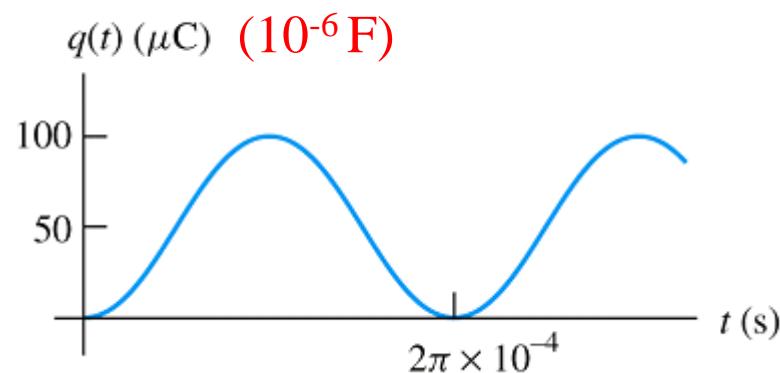


## Example 3.2 Determining Voltage Given Current

$C=0.1 \text{ } \mu\text{F}$ ,  $q(0)=0$ . After  $t_0=0$ ,  $i(t) = 0.5 \sin(10^4 t)$ . Plot  $i(t)$ ,  $q(t)$  and  $v(t)$ .

### 2. Determining $q(t)$

$$\begin{aligned} q(t) &= \int_0^t i(t) dt + q(0) \\ &= \int_0^t 0.5 \sin(10^4 t) dt \\ &= -0.5 \times 10^{-4} \cos(10^4 t) \Big|_0^t \\ &= 0.5 \times 10^{-4} [1 - \cos(10^4 t)] \end{aligned}$$

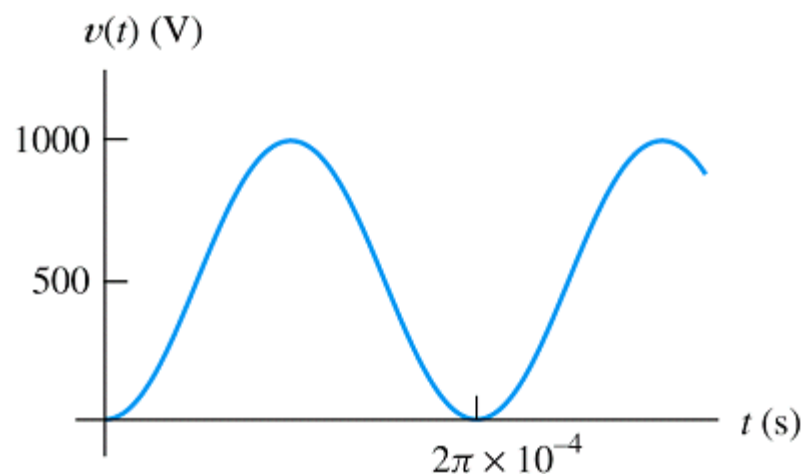


## Example 3.2 Determining Voltage Given Current

$C=0.1 \mu\text{F}$ ,  $q(0)=0$ . After  $t_0$ ,  $i(t) = 0.5 \sin(10^4 t)$  Plot  $i(t)$ ,  $q(t)$  and  $v(t)$ .

### 3. Determining $v(t)$

$$\begin{aligned} v(t) &= \frac{q(t)}{C} = \frac{q(t)}{10^{-7}} \\ &= 500[1 - \cos(10^4 t)] \end{aligned}$$





# Stored Energy

- 如何計算電容所儲存的能量(energy)? 假設電容 $C$ 的初始電壓 $v(t_0) = 0$ ， $q(t_0) = 0$ ，電容器兩端電壓在 $t_0$ 到 $t$ 由0上升 $v(t)$ 。

1. 計算power (單位時間消耗的能量 J/sec)

$$p(t) = v(t)i(t)$$

$$\because i = C \frac{dv}{dt} \quad \longrightarrow \quad p(t) = Cv \frac{dv}{dt}$$

# Stored Energy

## 2. 計算儲存能量

$$w(t) = \int_{t_0}^t p(t) dt$$

$$= \int_{t_0}^t C v \frac{dv}{dt} dt = \int_0^{v(t)} C v dv$$

$$= \frac{1}{2} C v^2 \Big|_0^{v(t)} = \frac{1}{2} C v^2(t)$$

# Stored Energy

## 3. 儲存能量其他表示式

$$w(t) = \frac{1}{2} C v^2(t)$$

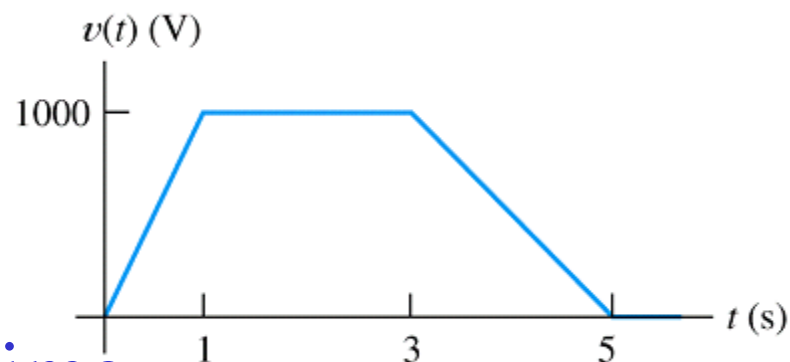
$$\because q(t) = C v(t) \quad \longrightarrow \quad w(t) = \frac{1}{2} v(t) q(t)$$

or

$$w(t) = \frac{q^2(t)}{2C}$$

## Example 3.3 Current, Power, and Energy for a Capacitance

$C=10\text{ }\mu\text{F}$ ,  $v(t)$  is given.  $q(0)=0$ . Plot current, the power delivered and energy stored for  $t = 0\sim 5\text{ s}$ .



1. Express  $v(t)$  as a function of time

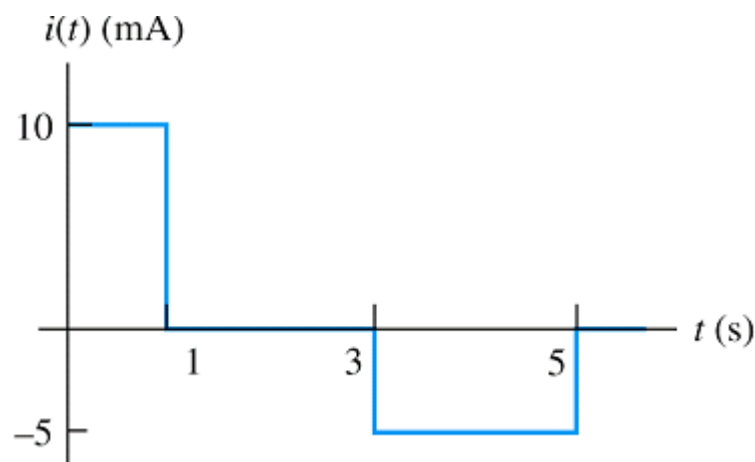
$$v(t) = \begin{cases} 1000tV & \text{for } 0 < t < 1 \\ 1000V & \text{for } 1 < t < 3 \\ 500(5-t)V & \text{for } 3 < t < 5 \end{cases}$$

2. Calculate  $i(t)$

$$i(t) = C \frac{dv(t)}{dt}$$

➡

$$i(t) = \begin{cases} 10 \times 10^{-3} \text{ A} & \text{for } 0 < t < 1 \\ 0 \text{ A} & \text{for } 1 < t < 3 \\ -5 \times 10^{-3} \text{ A} & \text{for } 3 < t < 5 \end{cases}$$



## Example 3.3 Current, Power, and Energy for a Capacitance

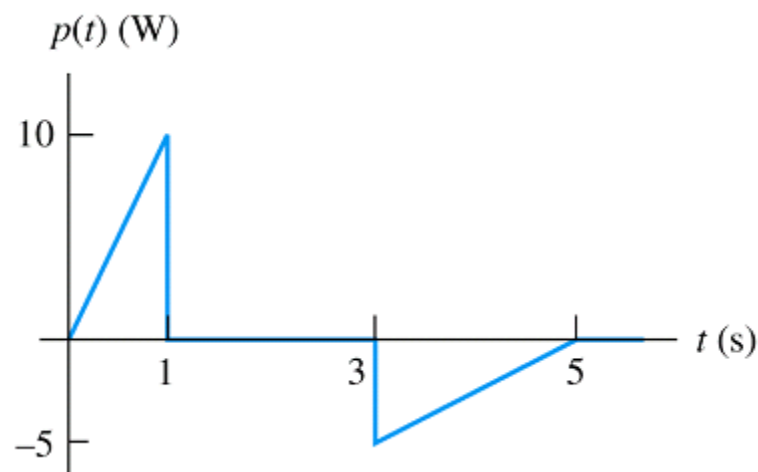
$$v(t) = \begin{cases} 1000tV & \text{for } 0 < t < 1 \\ 1000V & \text{for } 1 < t < 3 \\ 500(5-t)V & \text{for } 3 < t < 5 \end{cases} \quad i(t) = \begin{cases} 10 \times 10^{-3}A & \text{for } 0 < t < 1 \\ 0A & \text{for } 1 < t < 3 \\ -5 \times 10^{-3}A & \text{for } 3 < t < 5 \end{cases}$$

### 3. Calculate the power

$$p(t) = v(t)i(t)$$



$$p(t) = \begin{cases} 10tW & \text{for } 0 < t < 1 \\ 0W & \text{for } 1 < t < 3 \\ 2.5(t-5)W & \text{for } 3 < t < 5 \end{cases}$$



## Example 3.3 Current, Power, and Energy for a Capacitance

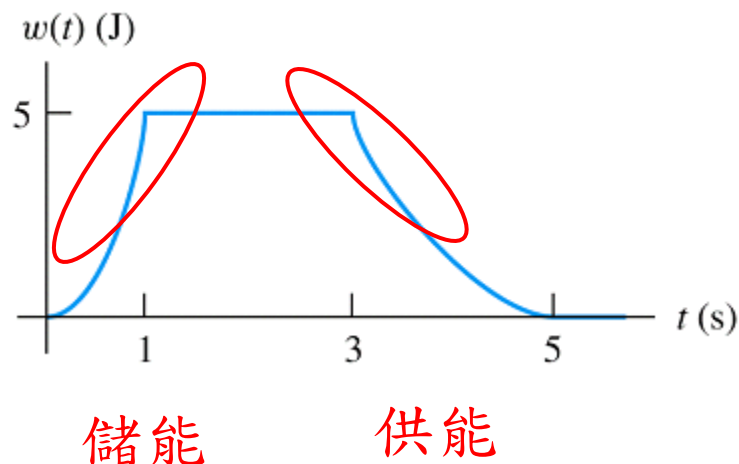
$$v(t) = \begin{cases} 1000tV & \text{for } 0 < t < 1 \\ 1000V & \text{for } 1 < t < 3 \\ 500(5-t)V & \text{for } 3 < t < 5 \end{cases}$$

### 4. Calculate the energy

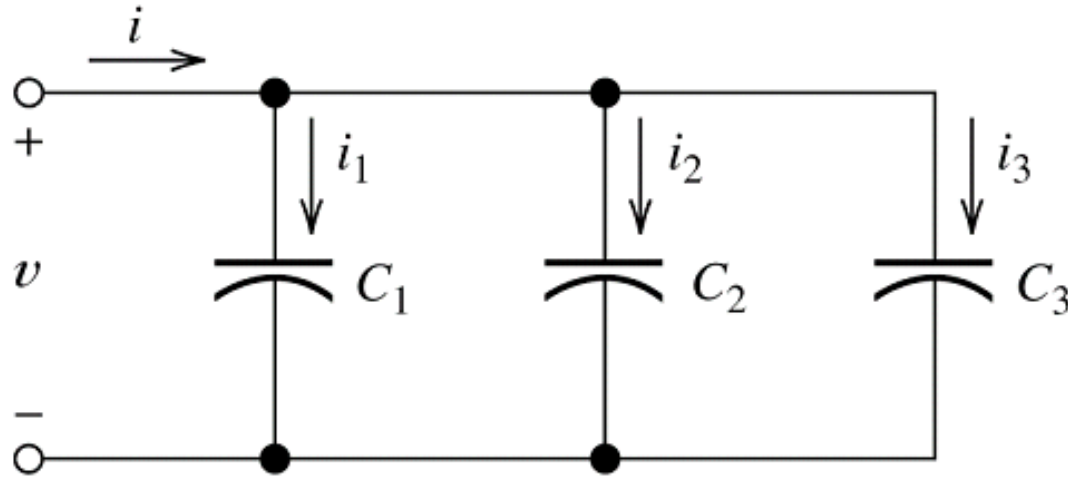
$$w(t) = \frac{1}{2} C v^2(t) = 5 \times 10^{-6} v^2(t)$$



$$w(t) = \begin{cases} 5t^2 J & \text{for } 0 < t < 1 \\ 5J & \text{for } 1 < t < 3 \\ 1.25(5-t)^2 J & \text{for } 3 < t < 5 \end{cases}$$



## 3.2 Capacitance in Series and Parallel



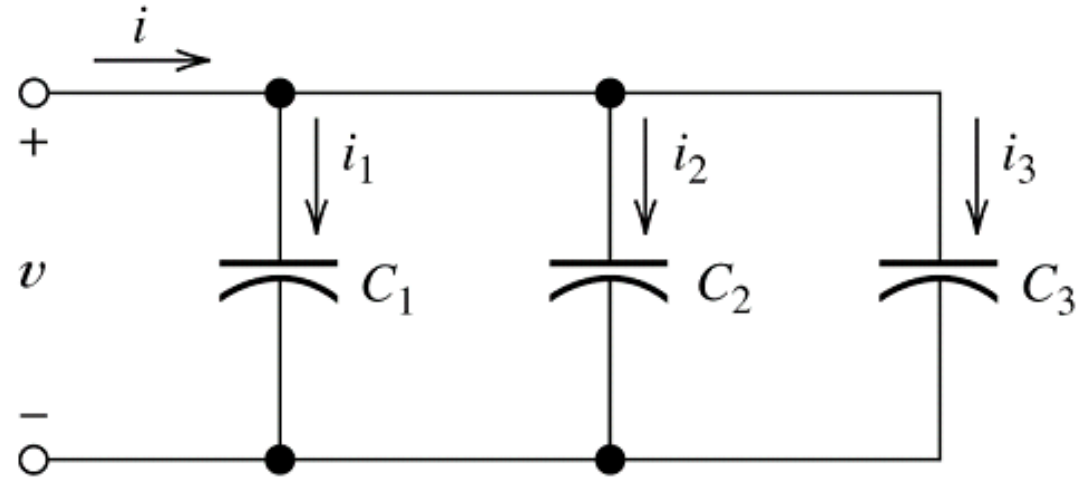
$$i = C \frac{dv}{dt}$$

$$i_1 = C_1 \frac{dv}{dt}$$

$$i_2 = C_2 \frac{dv}{dt}$$

$$i_3 = C_3 \frac{dv}{dt}$$

# Capacitance in Parallel



KCL

$$i = i_1 + i_2 + i_3$$

$$= C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt} + C_3 \frac{dv}{dt}$$

$$= (C_1 + C_2 + C_3) \frac{dv}{dt}$$

$$C_{eq} = C_1 + C_2 + C_3$$

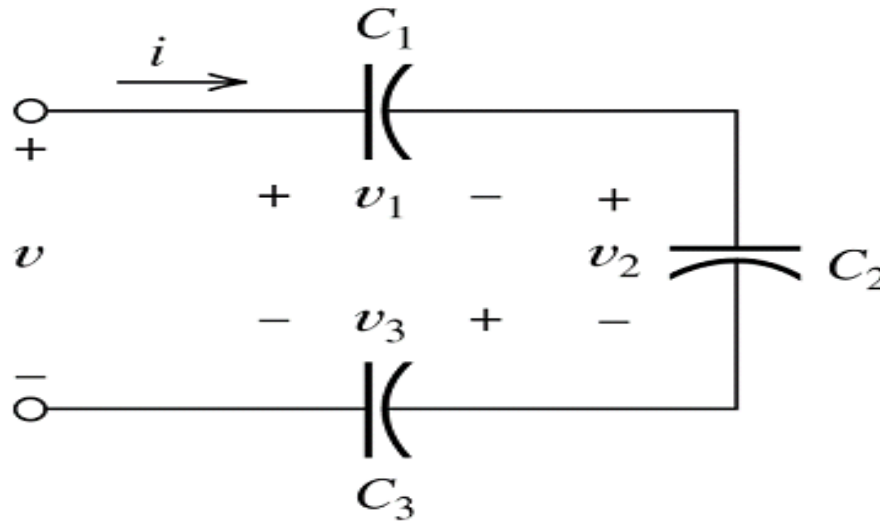
$$i = C_{eq} \frac{dv}{dt}$$

電容並聯，等效電容值相加

Recall: 電阻串聯，等效電阻值相加



# Capacitance in Series

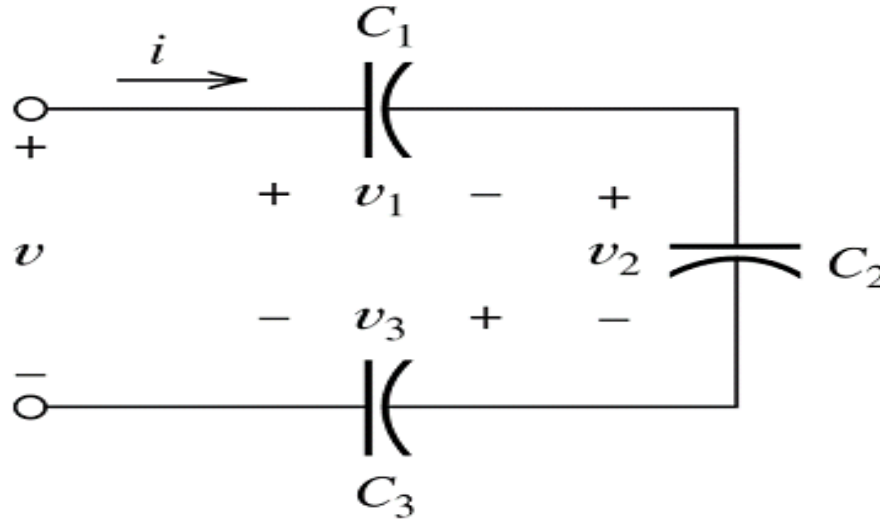


$$v(t) = \frac{1}{C} \int_0^t i(t) dt + v(t_0)$$

KVL

$$v = v_1 + v_2 + v_3$$

# Capacitance in Series

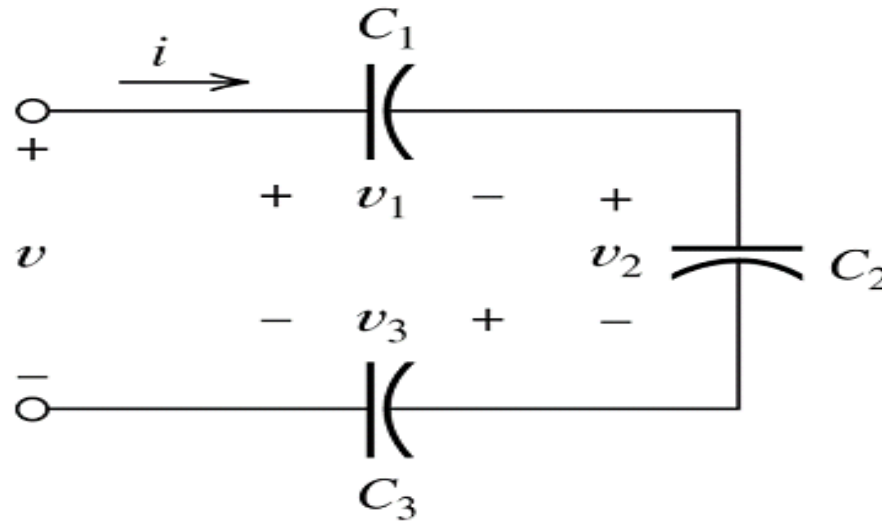


$$v(t) = \frac{1}{C_1} \int_0^t i(t) dt + v_1(0) + \frac{1}{C_2} \int_0^t i(t) dt + v_2(0) + \frac{1}{C_3} \int_0^t i(t) dt + v_3(0)$$

$$= \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \int_0^t i(t) dt + v_1(0) + v_2(0) + v_3(0)$$

$$\longrightarrow \frac{1}{C_{eq}} = \left( \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \quad \text{and} \quad v(0) = v_1(0) + v_2(0) + v_3(0)$$

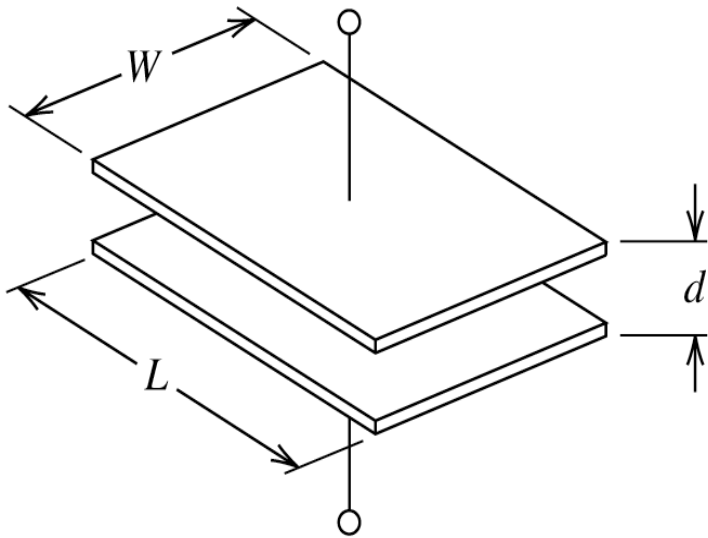
# Capacitance in Series



$$C_{eq} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$

Capacitances in series are combined like resistances in parallel.

# 3.3 Physical Characteristics of Capacitors



$$C = \frac{\epsilon A}{d}$$

$$A = W \times L$$

$d$  上下極板間的距離

$\epsilon$  介電常數

Figure 3.11 A parallel-plate capacitor including dimensions.

真空(vacuum)  $\epsilon = \epsilon_0 \cong 8.85 \times 10^{-12} \text{ F/m}$

其他材質 (Table 3.1)  $\epsilon = \epsilon_r \epsilon_0$

Example 3.4 A parallel-plate capacitor have rectangular plates  $10\text{ cm} \times 20\text{ cm}$ . Distance is  $0.1\text{ mm}$ . Calculate the capacitance.

1. The dielectric is air.
2. The dielectric is mica.

1. The dielectric is air.

$$A = L \times W = (10 \times 10^{-2}) \times (20 \times 10^{-2}) = 0.02\text{m}^2$$

$$\epsilon = \epsilon_r \epsilon_0 = 1.00 \times 8.85 \times 10^{-12} \text{ F/m}$$

$$C = \frac{\epsilon A}{d} = \frac{8.85 \times 10^{-12} \times 0.02}{10^{-4}} = 1770 \times 10^{-12} \text{ F}$$

Example 3.4 A parallel-plate capacitor have rectangular plates  $10\text{ cm} \times 20\text{ cm}$ . Distance is  $0.1\text{ mm}$ . Calculate the capacitance.

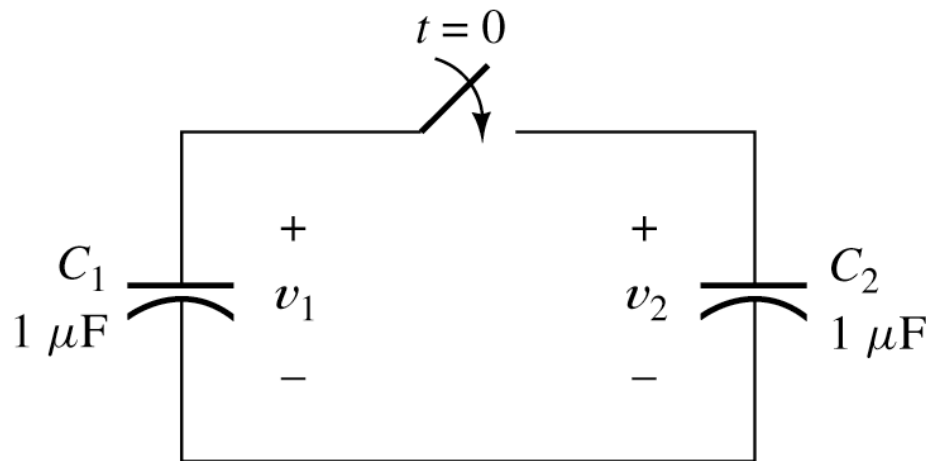
2. The dielectric is mica.

$$\epsilon_r = 7.0$$

$$C = \frac{\epsilon A}{d} = \frac{\epsilon_r \epsilon_0 A}{d} = \frac{7.0 \times 8.85 \times 10^{-12} \times 0.02}{10^{-4}} = 12,390 \times 10^{-12} F$$

### Example 3.5

1. Prior to  $t=0$ ,  $C_1$  is charged to  $v_1=100\text{V}$  and  $C_2$  has no charge(i.e.,  $v_2=0$ ).
2. At  $t=0$ , the switch closes.
3. Compute the total energy stored by  $C_1$  and  $C_2$  before and after the switch closed.



**Figure 3.14** See Example 3.5.

## Example 3.5

1. Before switch closes ( $t < 0$ )

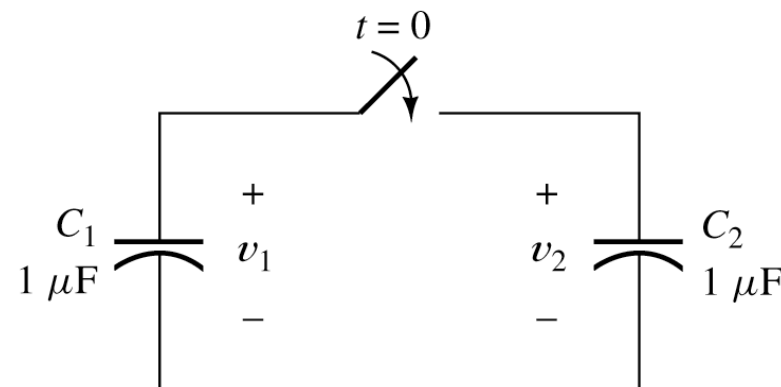


Figure 3.14 See Example 3.5.

$$w_1 = \frac{1}{2} C_1 v_1^2 = \frac{1}{2} (10^{-6}) (100)^2 = 5mJ$$

$$w_2 = 0$$

$$w_{total} = w_1 + w_2 = 5mJ$$

$$q_1 = C_1 v_1 = 1 \times 10^{-6} \times 100 = 100\mu C$$

$$q_2 = 0$$

$$q_{eq} = q_1 + q_2 = 100\mu C$$



## Example 3.5

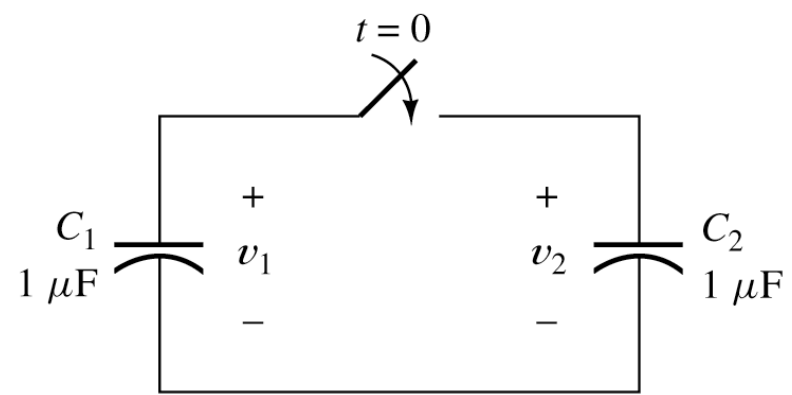


Figure 3.14 See Example 3.5.

2. After switch closes ( $t \geq 0$ )

$C_1, C_2$ , 並聯

→  $C_{eq} = C_1 + C_2 = 2\mu F$

等效電容兩端電壓

$$v_{eq} = \frac{q_{eq}}{C_{eq}} = \frac{100\mu C}{2\mu F} = 50V \quad (\text{假設儲存於兩電容之總電荷數不變})$$

$$v_1 = v_2 = v_{eq}$$

## Example 3.5

### 儲存與兩電容之能量

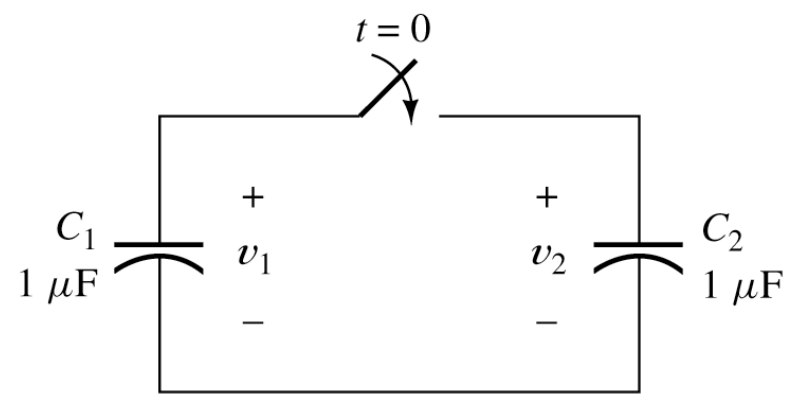


Figure 3.14 See Example 3.5.

$$w_1 = \frac{1}{2} C_1 v_{eq}^2 = \frac{1}{2} (10^{-6}) (50)^2 = 1.25\text{mJ}$$

$$w_2 = \frac{1}{2} C_2 v_{eq}^2 = \frac{1}{2} (10^{-6}) (50)^2 = 1.25\text{mJ}$$

→  $w_{total} = w_1 + w_2 = 2.5\text{mJ}$

儲存於兩電容之總能量為switch closes 前之一半



Parasitic resistances or parasitic inductance  
(寄生電阻消耗或寄生電感儲存能量)

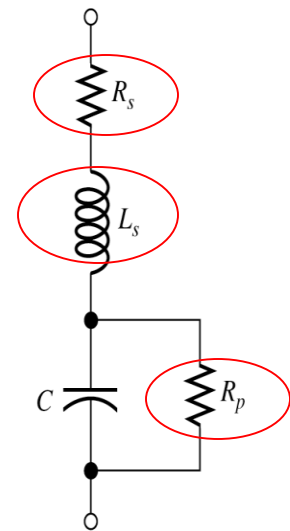
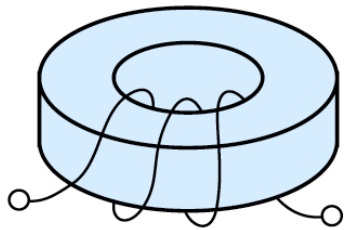


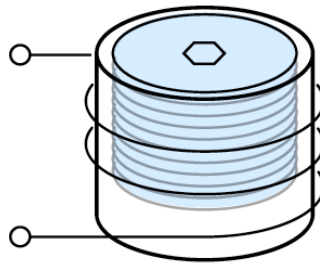
Figure 3.13 The circuit model for a capacitor including the parasitic elements  $R_s$ ,  $L_s$ , and  $R_p$ .

## 3.4 Inductance 電感

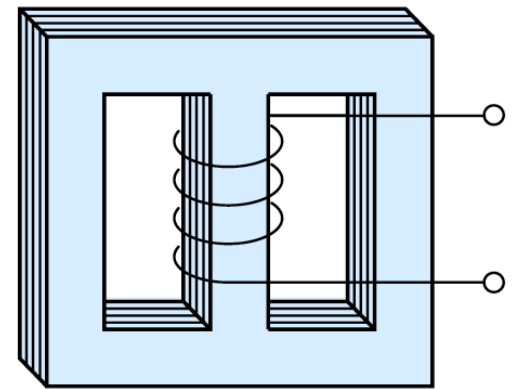
- 電感為線圈(coil)形式之導線圍繞磁性核心的電路元件。
- 電流流過線圈會產生磁場(magnetic field)或磁通量(magnetic flux)。



(a) Toroidal inductor



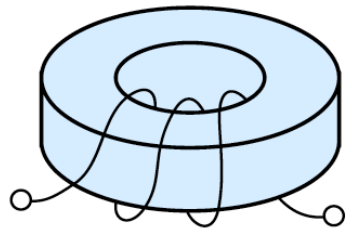
(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance



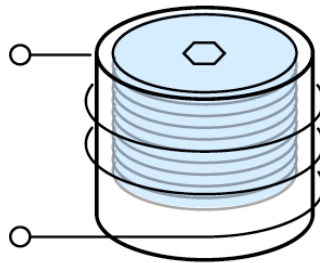
(c) Inductor with a laminated iron core

**Figure 3.15** An inductor is constructed by coiling a wire around some type of form.

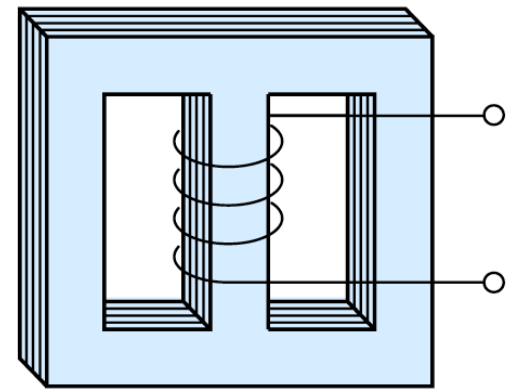
- 電流改變會產生磁場改變(Faraday's law) ，而磁場改變會產生一個電壓(感應電動勢)在線圈兩端，而此電壓值正比於產生磁場的電流改變率。



(a) Toroidal inductor



(b) Coil with an iron-oxide slug that can be screwed in or out to adjust the inductance

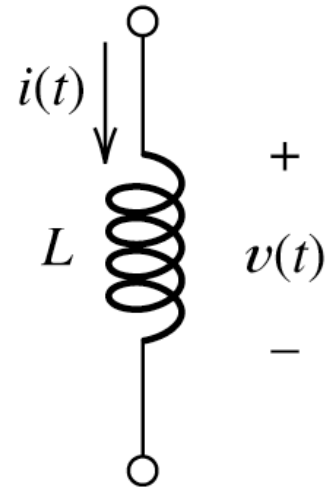


(c) Inductor with a laminated iron core

**Figure 3.15** An inductor is constructed by coiling a wire around some type of form.

# V-I Relationship

- 理想電感兩端電壓與電流對時間微分成正比。
- 常數 $L$ 為電感值(inductance)，單位為亨利(H, henries)，等於volt seconds/ampere ( $V \bullet \text{Sec}/A$ )。
- 一般電感值在數個micro H( $1\mu\text{H} = 10^{-6}$ )到數十H之間。
- 通過電感的電流參考方向為電感兩端電壓降落的方向(由正極流入，負極流出)，具 passive configuration 特性。



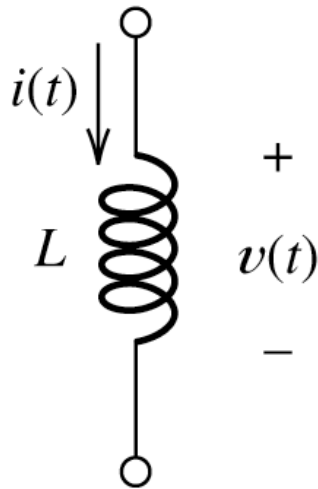
$$v(t) = L \frac{di}{dt}$$

# Current in Terms of Voltage

$$v(t) = L \frac{di}{dt} \longrightarrow di = \frac{1}{L} v(t) dt$$

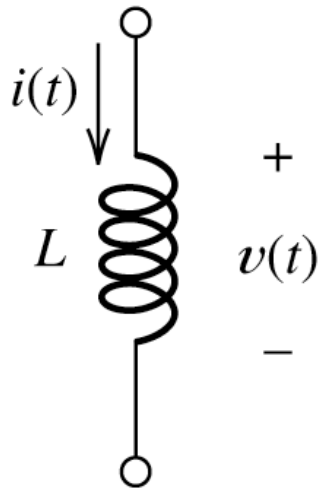
$$\int_{i(t_0)}^{i(t)} di = \frac{1}{L} \int_{t_0}^t v(t) dt$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$



$$v(t) = L \frac{di}{dt}$$

# Stored Energy



$$v(t) = L \frac{di}{dt}$$

$$\begin{aligned} p(t) &= v(t)i(t) \\ &= Li(t) \frac{di}{dt} \end{aligned}$$

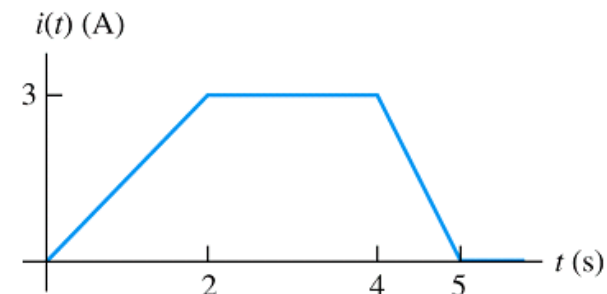
$$\begin{aligned} w(t) &= \int_{t_0}^t p(t) dt \\ &= \int_{t_0}^t Li(t) \frac{di}{dt} dt \end{aligned}$$

$$= \int_0^{i(t)} Lidi \quad (\text{Assume } i(t_0)=0)$$

$$= \frac{1}{2} Li^2(t)$$

## Example 3.6 Voltage, Power, and Energy for an Inductance

$L=5$  H,  $i(t)$  is given. Plot voltage, the power delivered and energy stored for  $t = 0 \sim 5$  s.



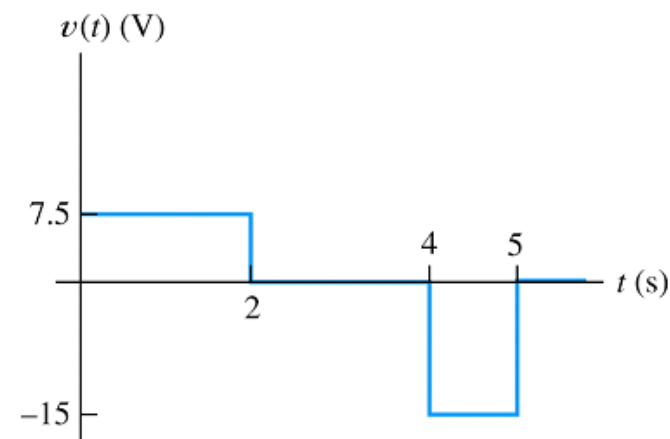
1. Express  $i(t)$  as a function of time

$$i(t) = \begin{cases} 1.5t \text{ A} & \text{for } 0 < t < 2 \\ 3 \text{ A} & \text{for } 2 < t < 4 \\ 3(5-t) \text{ A} & \text{for } 4 < t < 5 \end{cases}$$

2. Calculate  $v(t)$

$$v(t) = L \frac{di}{dt}$$

$$v(t) = \begin{cases} 7.5 \text{ V} & \text{for } 0 < t < 2 \\ 0 \text{ V} & \text{for } 2 < t < 4 \\ -15 \text{ V} & \text{for } 4 < t < 5 \end{cases}$$



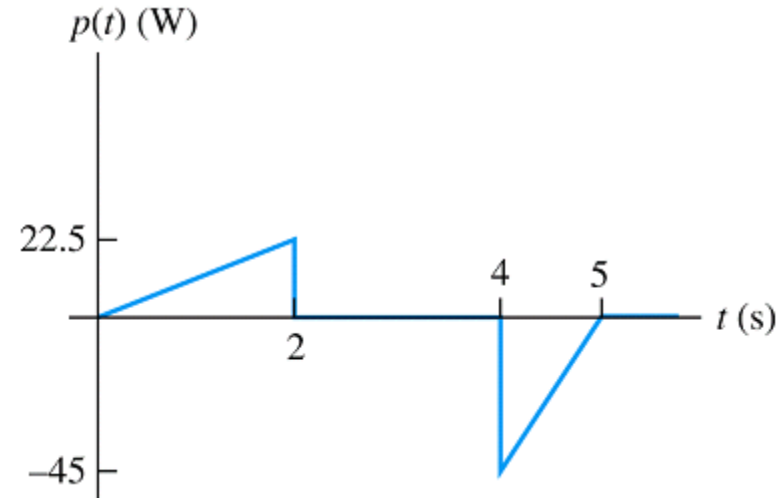


## Example 3.6 Voltage, Power, and Energy for a Inductance

### 3. $p(t)$

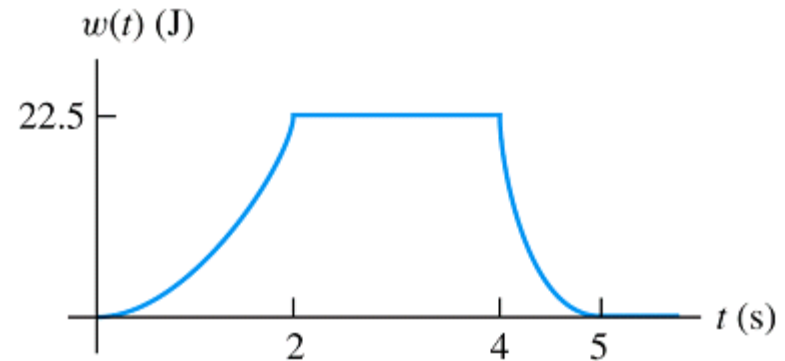
$$p(t) = v(t)i(t)$$

$$p(t) = \begin{cases} 11.25tW & \text{for } 0 < t < 2 \\ 0W & \text{for } 2 < t < 4 \\ -45(5-t)W & \text{for } 4 < t < 5 \end{cases}$$



### 4. Energy

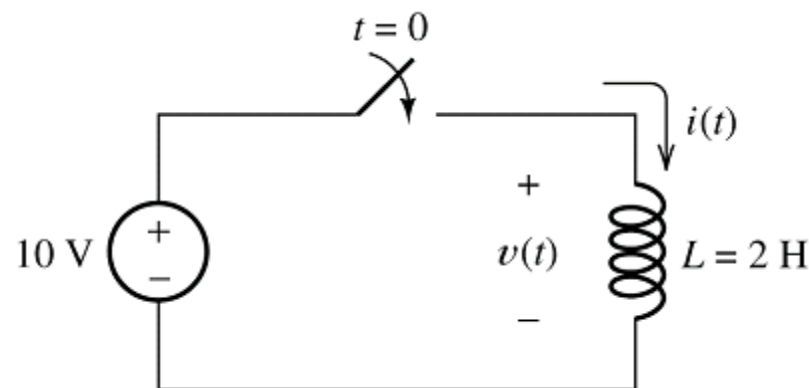
$$w(t) = \frac{1}{2}Li^2(t)$$



## Example 3.7 Inductor Current with Constant Applied Voltage

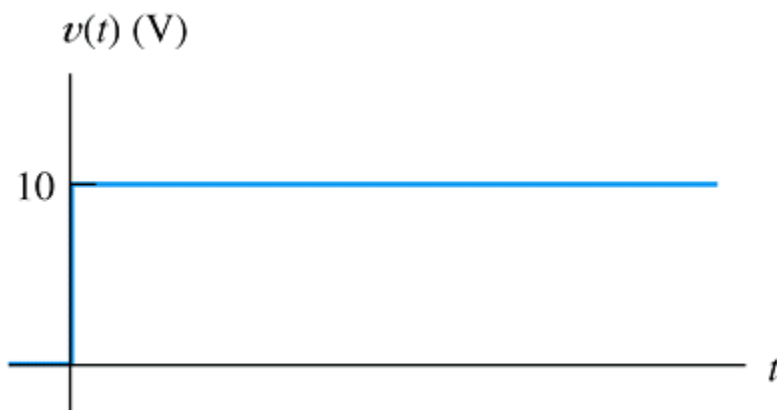
Switch closes at  $t=0$ , connecting a 10-V source to a 2-H inductance. Find  $i(t)$ .

$$v(t) = L \frac{di}{dt}$$



剛導通時 $i(t)=0$ .

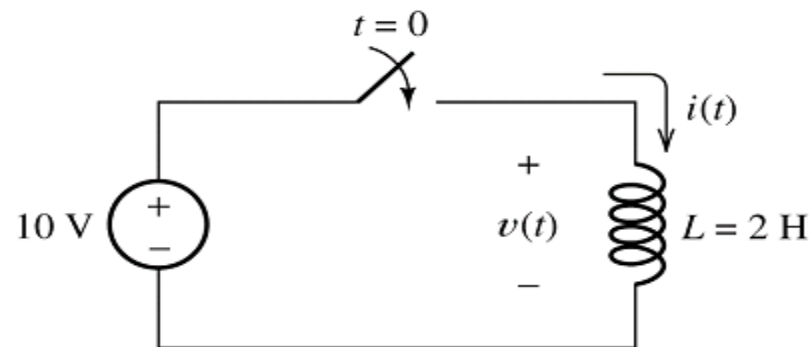
### 1. $v(t)$



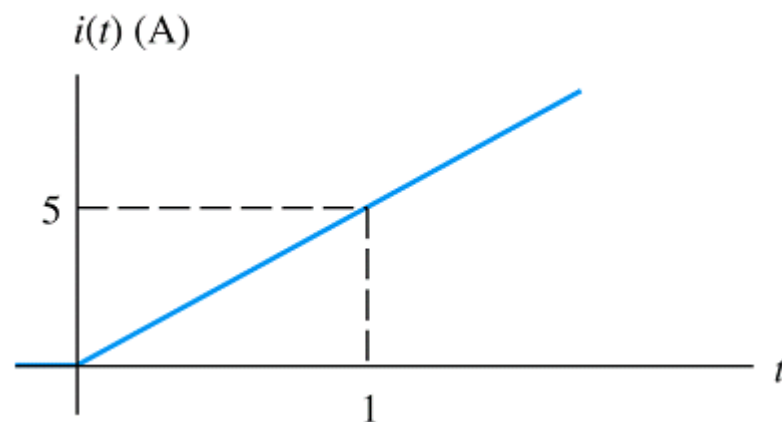
## Example 3.7 Inductor Current with Constant Applied Voltage

Switch closes at  $t=0$ , connecting a 10-V source to a 2-H inductance. Find  $i(t)$ .

2.  $i(t)$   $t > 0$



$$\begin{aligned} i(t) &= \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0) \\ &= \frac{1}{2} \int_0^t 10 dt \\ &= 5t \text{ A for } t > 0 \end{aligned}$$



If we open the circuit at  $t=1$ , since  $v(t) = L \frac{di}{dt}$  we got a large voltage.

➡ 開關含有電感的電路會產生大電壓。

# 3.5 Inductance in Series and Parallel

KVL

$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

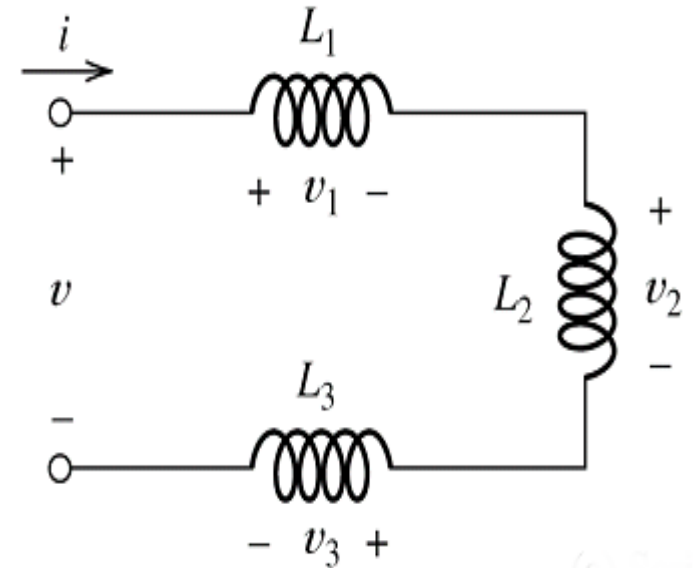
$$v(t) = L \frac{di}{dt}$$



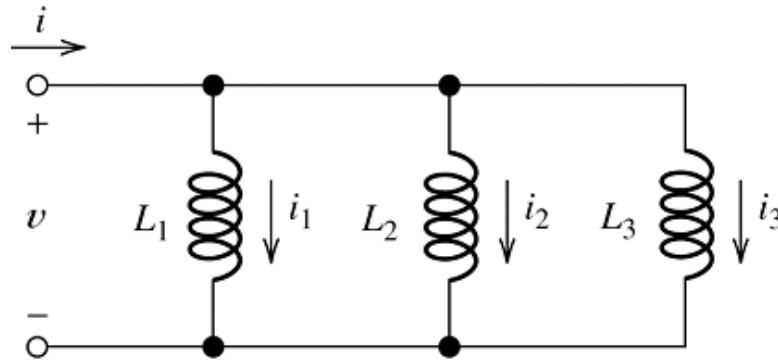
$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt}$$

$$= (L_1 + L_2 + L_3) \frac{di(t)}{dt} = L_{eq} \frac{di(t)}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$



# Inductance in Parallel



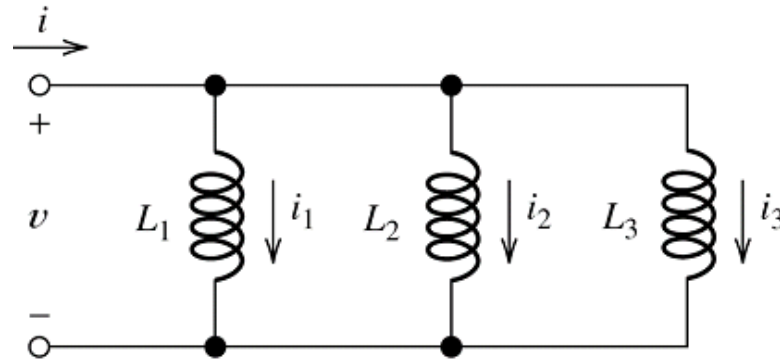
KCL

$$i(t) = i_1(t) + i_2(t) + i_3(t)$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(t) dt + i(t_0)$$

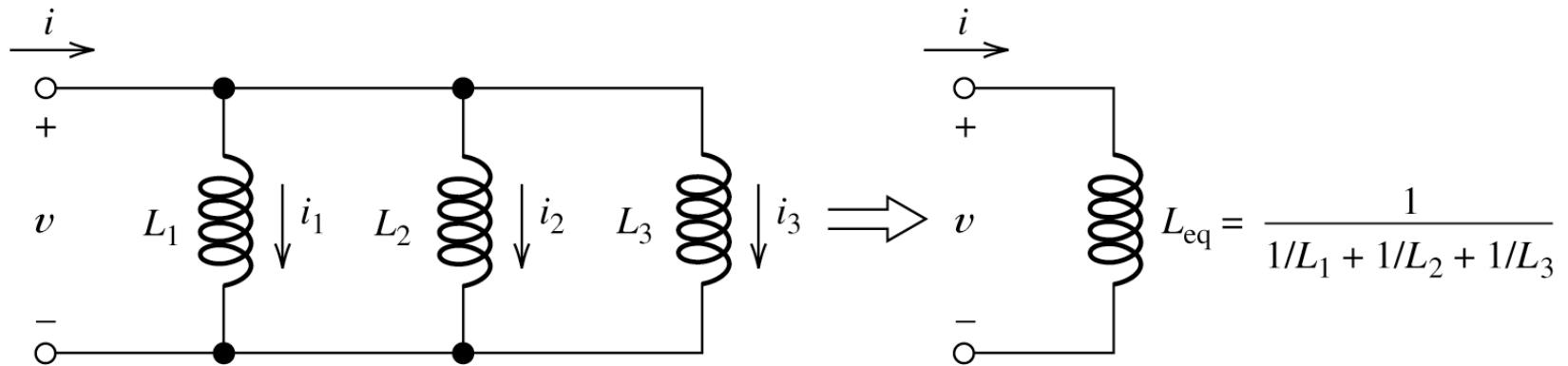
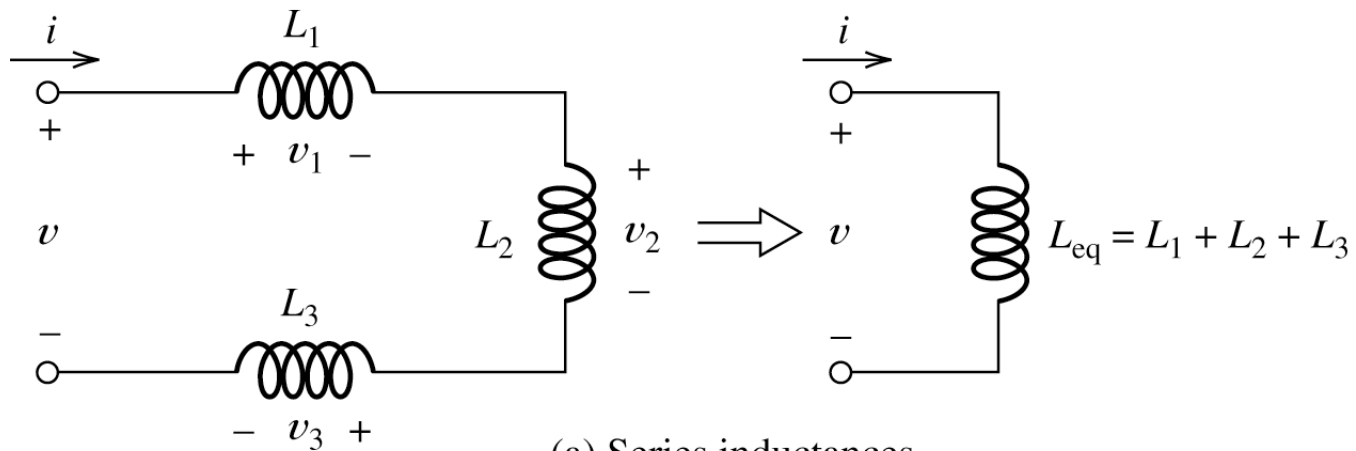
$$\begin{aligned} \Rightarrow i(t) &= \frac{1}{L_1} \int_0^t v(t) dt + i_1(0) + \frac{1}{L_2} \int_0^t v(t) dt + i_2(0) + \frac{1}{L_3} \int_0^t v(t) dt + i_3(0) \\ &= \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int_0^t v(t) dt + i_1(0) + i_2(0) + i_3(0) \end{aligned}$$

# Inductance in Parallel



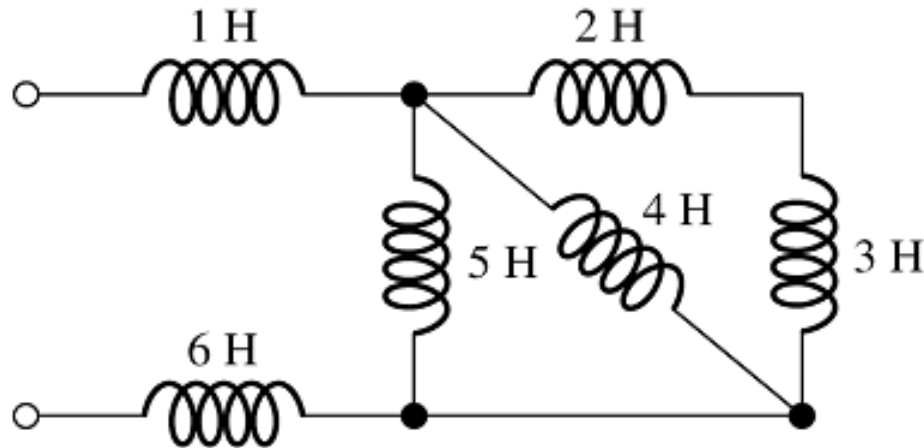
$$i(t) = \frac{1}{L_{eq}} \int_0^t v(t) dt + i(0)$$

$$\frac{1}{L_{eq}} = \left( \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \quad \text{and} \quad i(0) = i_1(0) + i_2(0) + i_3(0)$$



**Figure 3.20** Inductances in series and parallel are combined in the same manner as resistances.

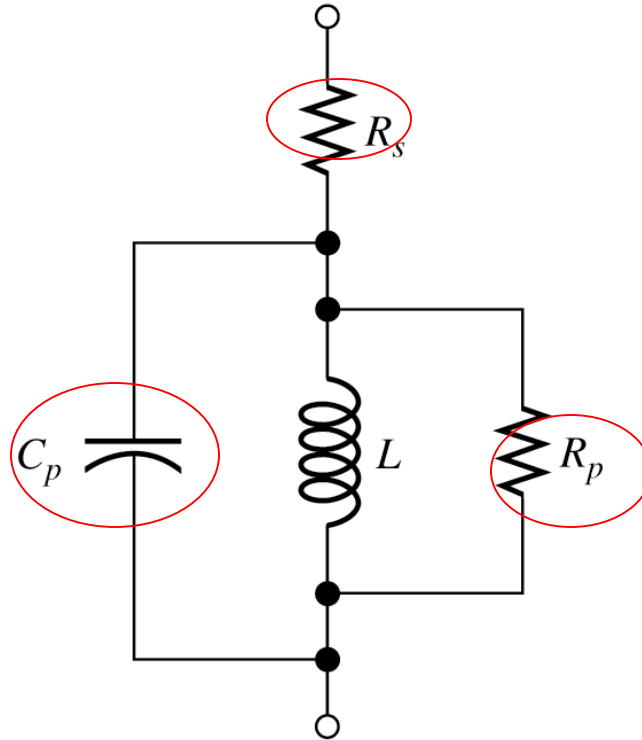
## Exercise 3.10



- The 2-H and 3-H are in series  $\Rightarrow 5\text{ H}$ .
- The equivalent 5H is in parallel with the 5-H and 4-H  $\Rightarrow 1/(1/5 + 1/4 + 1/5) = 1.538\text{ H}$ .
- The equivalent 1.538 H is in series with the 1-H and 6-H  $\Rightarrow 1.538 + 1 + 6 = 8.538\text{ H}$ .



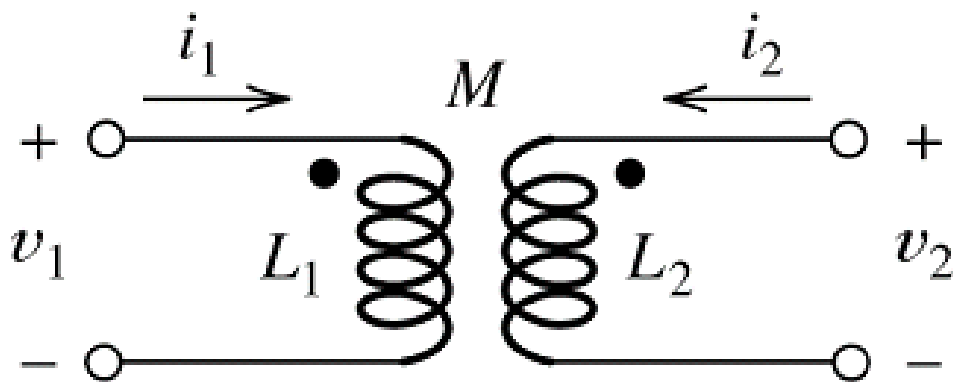
# 3.6 Practical Inductors



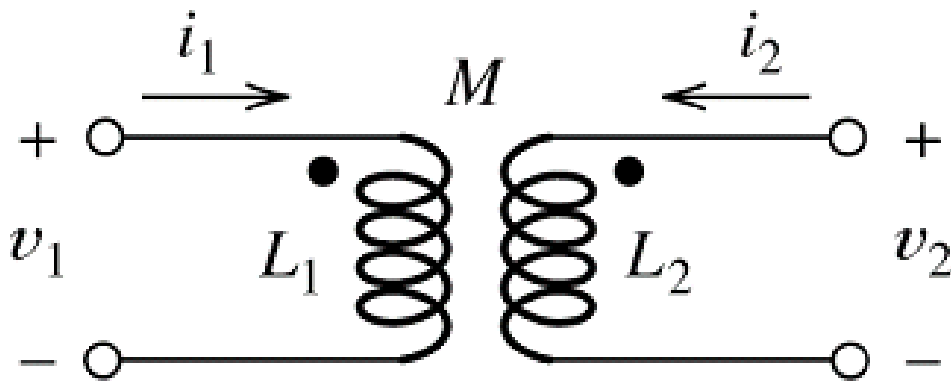
**Figure 3.22** Circuit model for real inductors including several parasitic elements.

## 3.7 Mutual Inductance (互感)

- 若數個線圈圍繞相同的磁性核心，則不同線圈通過電流所引發的磁通量會彼此影響。
- 自感(self inductance)  $L_1$  &  $L_2$  表示自身的電感效應，互感(mutual inductance)  $M$  表示彼此相互影響所產生的電感效應。



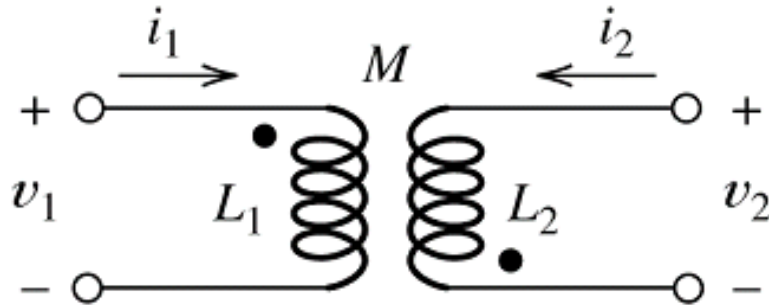
- 不同線圈電感效應產生的磁力線方向可能相同(增加)或相反(減弱)。
- 以兩端黑點為參考方向，兩線圈電流皆流入或流出黑點則電感效應增強。



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

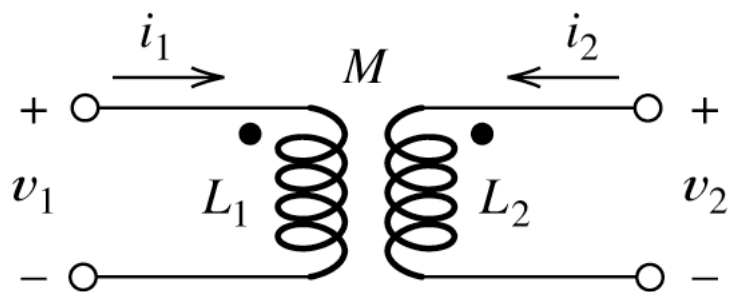
$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

- 兩線圈電流一流入，一流出黑點則電感效應減弱。



$$v_1 = L_1 \frac{di_1}{dt} \ominus M \frac{di_2}{dt}$$

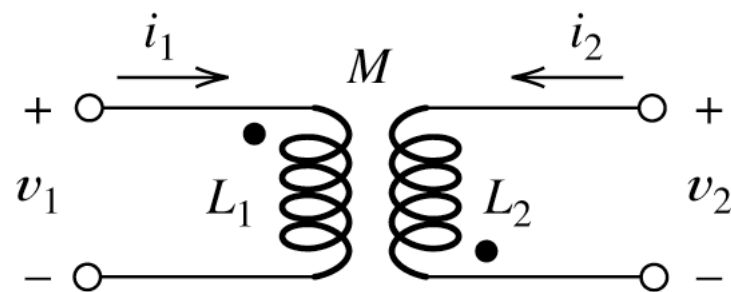
$$v_2 = \ominus M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$



$$v_1 = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$v_2 = M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

(a)



$$v_1 = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$$

$$v_2 = -M \frac{di_1}{dt} + L_2 \frac{di_2}{dt}$$

(b)

**Figure 3.23** Circuit symbols and  $v - i$  relationships for mutually coupled inductances.

# 電容與電感的相對關係

電容

$$i(t) = C \frac{dv(t)}{dt}$$

$$v(t) = \frac{1}{C} \int_{t_0}^t i(x) dx + v(t_0)$$

$$p(t) = Cv(t) \frac{dv(t)}{dt}$$

$$w(t) = \frac{1}{2} Cv(t)^2$$

電感

$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L} \int_{t_0}^t v(x) dx + i(t_0)$$

$$p(t) = Li(t) \frac{di(t)}{dt}$$

$$w(t) = \frac{1}{2} Li^2(t)$$

電容串聯	$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_N}$	類似電阻並聯
電容並聯	$C_p = C_1 + C_2 + \cdots + C_N$	類似電阻串聯
電感串聯	$L_s = L_1 + L_2 + \cdots + L_N$	類似電阻串聯
電感並聯	$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}$	類似電阻並聯