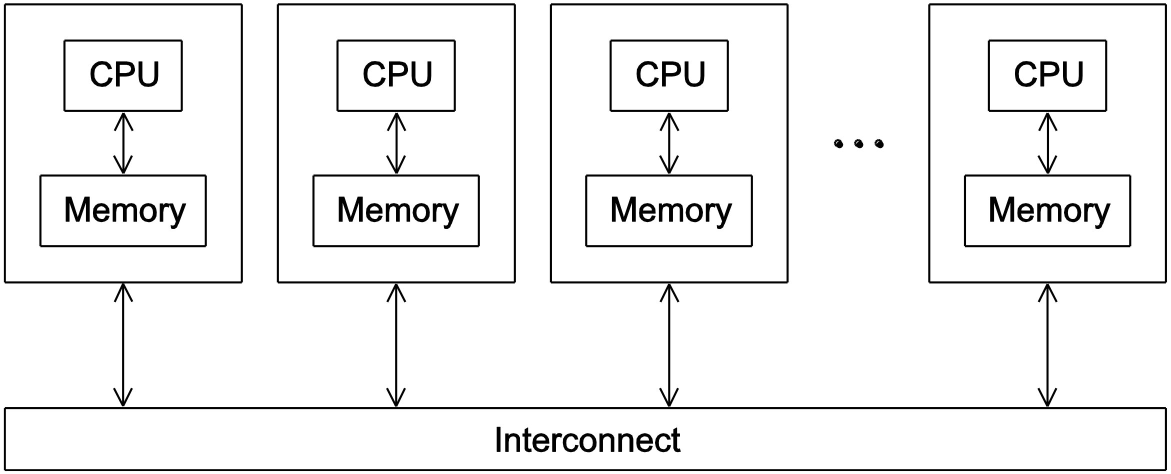
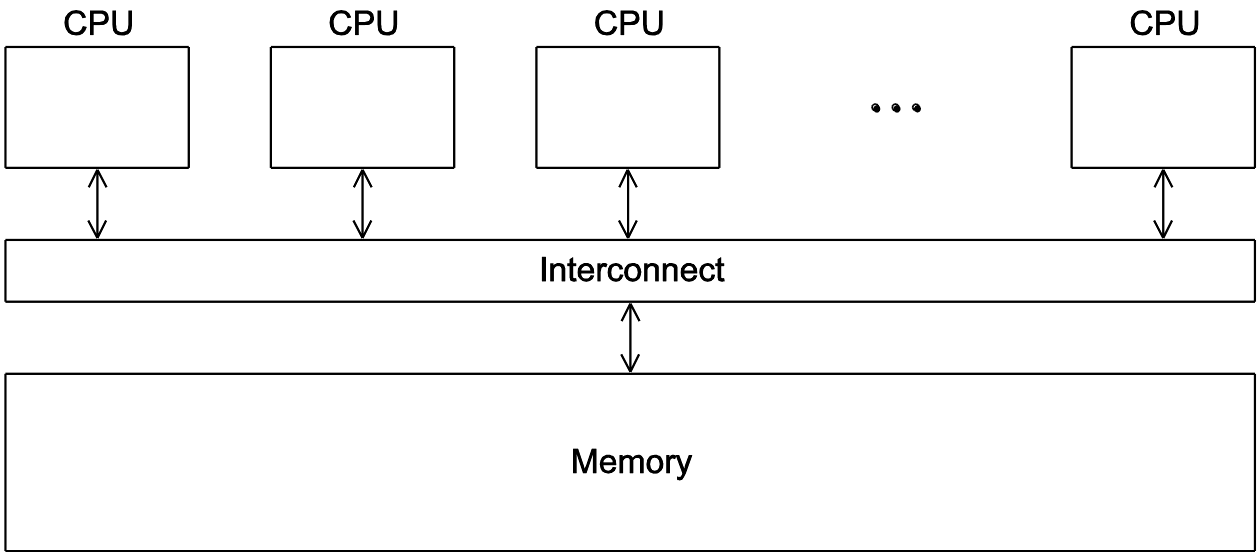
Chapter 3
Distributed Memory Programming with MPI

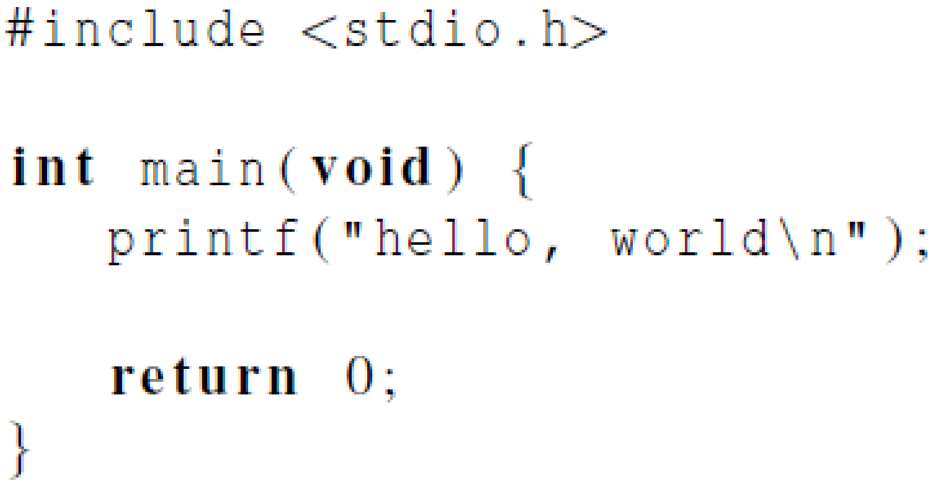
A distributed memory system



A shared memory system



Hello World!



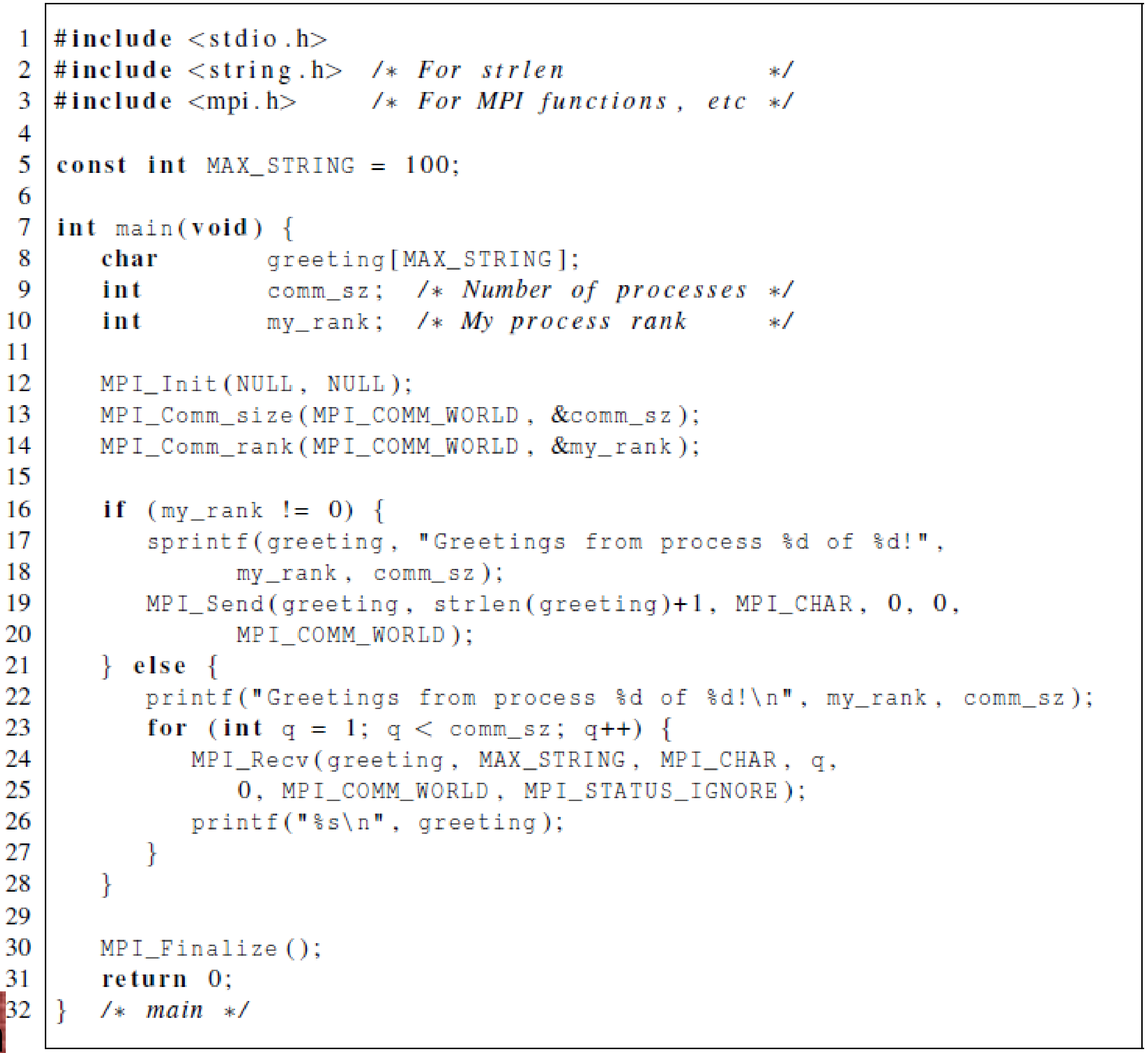
```
#include <stdio.h>

int main(void) {
    printf("hello, world\n");

    return 0;
}
```

(a classic)

Identifying MPI processes

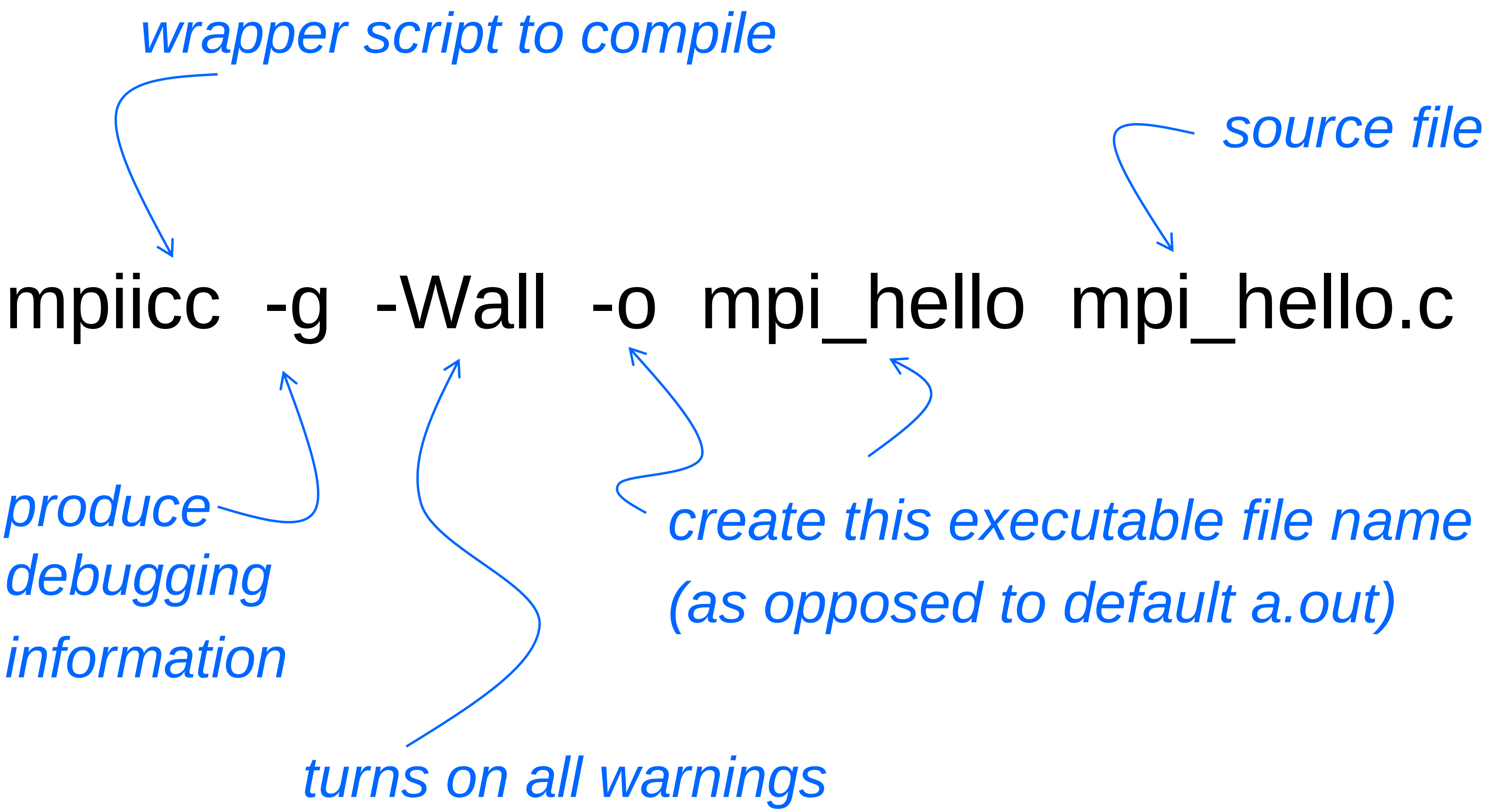
- Common practice to identify processes by nonnegative integer ranks.
- *p* processes are numbered *0, 1, 2, .. p-1*

Our first MPI program



```c
#include <stdio.h>
#include <string.h>  /* For strlen             */
#include <mpi.h>     /* For MPI functions, etc */

const int MAX_STRING = 100;

int main(void) {
   char       greeting[MAX_STRING];
   int        comm_sz;  /* Number of processes */
   int        my_rank;  /* My process rank     */

   MPI_Init(NULL, NULL);
   MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
   MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

   if (my_rank != 0) {
      sprintf(greeting, "Greetings from process %d of %d!",
            my_rank, comm_sz);
      MPI_Send(greeting, strlen(greeting)+1, MPI_CHAR, 0, 0,
            MPI_COMM_WORLD);
   } else {
      printf("Greetings from process %d of %d!\n", my_rank, comm_sz);
      for (int q = 1; q < comm_sz; q++) {
         MPI_Recv(greeting, MAX_STRING, MPI_CHAR, q,
            0, MPI_COMM_WORLD, MPI_STATUS_IGNORE);
         printf("%s\n", greeting);
      }
   }

   MPI_Finalize();
   return 0;
}  /* main */
```

Compilation



wrapper script to compile

source file

```
mpiicc  -g  -Wall  -o  mpi_hello  mpi_hello.c
```

produce debugging information

create this executable file name (as opposed to default a.out)

turns on all warnings

Execution

```
mpiexec  -n  <number of processes>   <executable>
```

```
mpiexec  -n  1  ./mpi_hello
```

run with 1 process

```
mpiexec  -n  4  ./mpi_hello
```

run with 4 processes

Execution

```
mpiexec  -n  1  ./mpi_hello

      Greetings from process 0 of 1 !
```

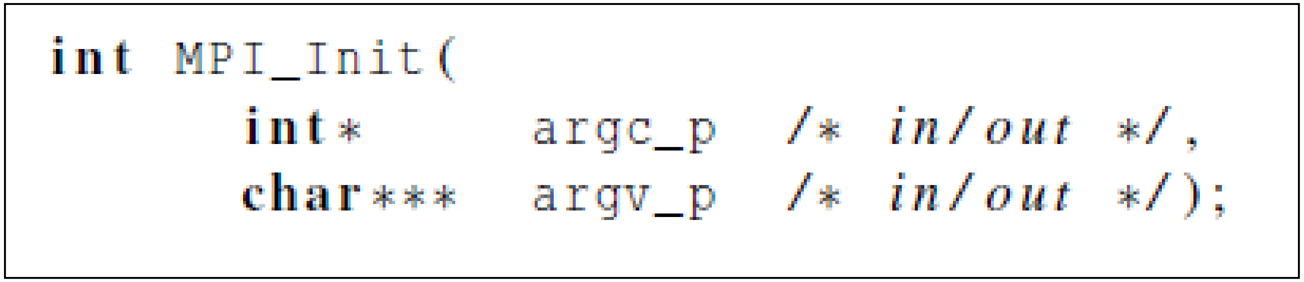
```
mpiexec  -n  4  ./mpi_hello

      Greetings from process 0 of 4 !
      Greetings from process 1 of 4 !
      Greetings from process 2 of 4 !
      Greetings from process 3 of 4 !
```

MPI Programs

- Written in C.
  - Has main.
  - Uses stdio.h, string.h, etc.
- Need to add mpi.h header file.
- Identifiers defined by MPI start with "MPI_" .
- First letter following underscore is uppercase.
  - For function names and MPI-defined types.
  - Helps to avoid confusion.

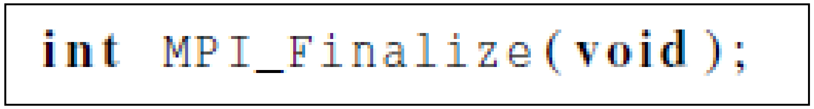
MPI Components

- MPI_Init
  - Tells MPI to do all the necessary setup.



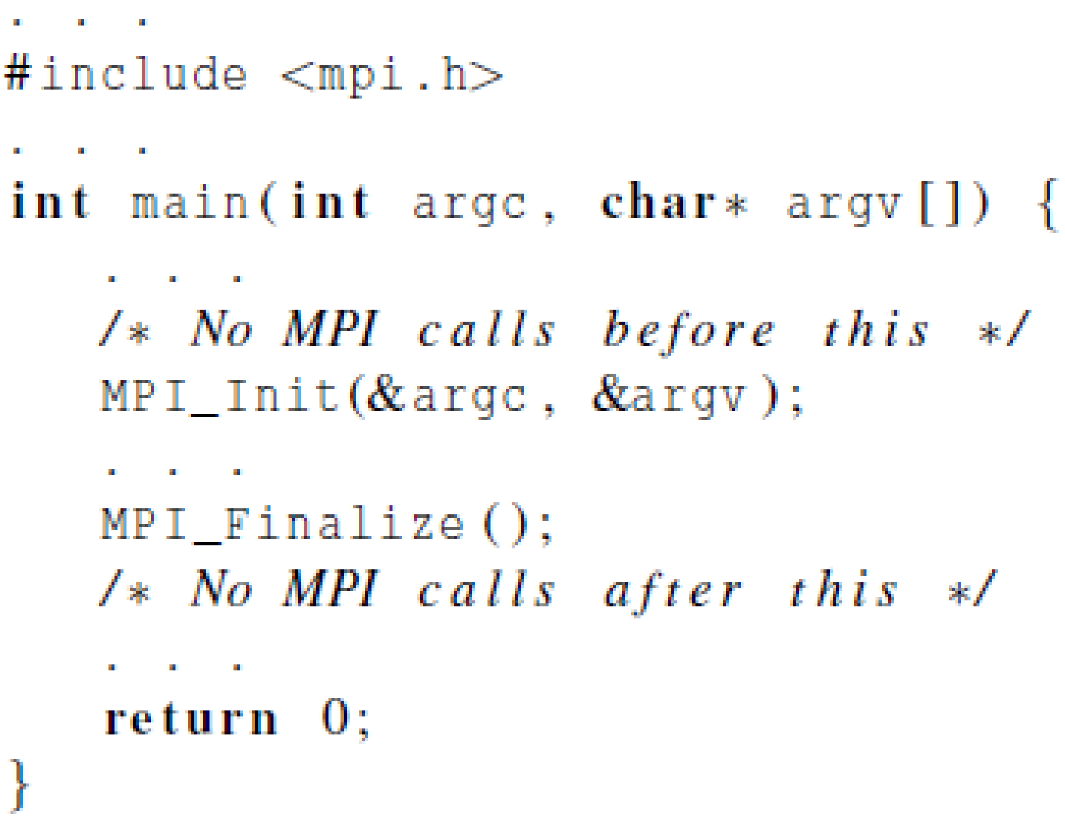
```
int MPI_Init(
        int*       argc_p  /* in/out */,
        char***    argv_p  /* in/out */);
```

- MPI_Finalize
  - Tells MPI we’ re done, so clean up anything allocated for this program.



```
int MPI_Finalize(void);
```

Basic Outline



```
. . .
#include <mpi.h>
. . .
int main(int argc, char* argv[]) {
   . . .
   /* No MPI calls before this */
   MPI_Init(&argc, &argv);
   . . .
   MPI_Finalize();
   /* No MPI calls after this */
   . . .
   return 0;
}
```

Communicators

- A collection of processes that can send messages to each other.
- MPI_Init defines a communicator that consists of all the processes created when the program is started.

- Called MPI_COMM_WORLD.

Communicators

```
int MPI_Comm_size(
      MPI_Comm   comm          /* in   */,
      int*       comm_sz_p     /* out  */);
```

number of processes in the communicator

```
int MPI_Comm_rank(
      MPI_Comm   comm          /* in   */,
      int*       my_rank_p     /* out  */);
```

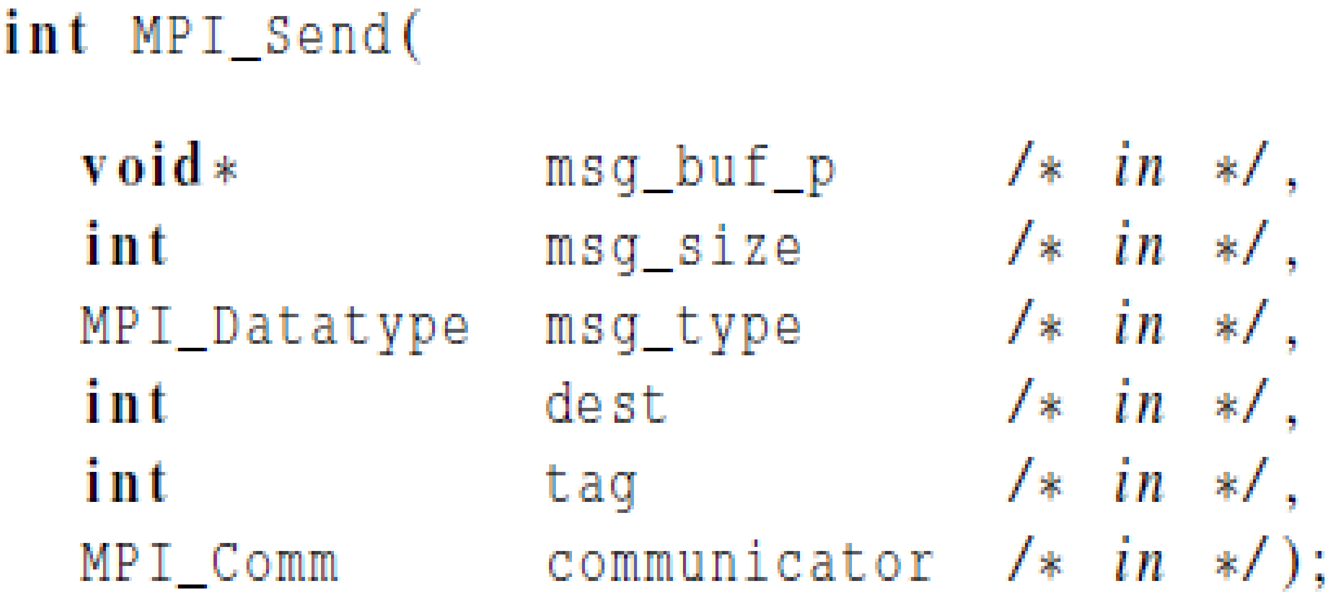
my rank
(the process making this call)

SPMD

- Single-Program Multiple-Data
- We compile one program.
- Process 0 does something different.
  - Receives messages and prints them while the other processes do the work.

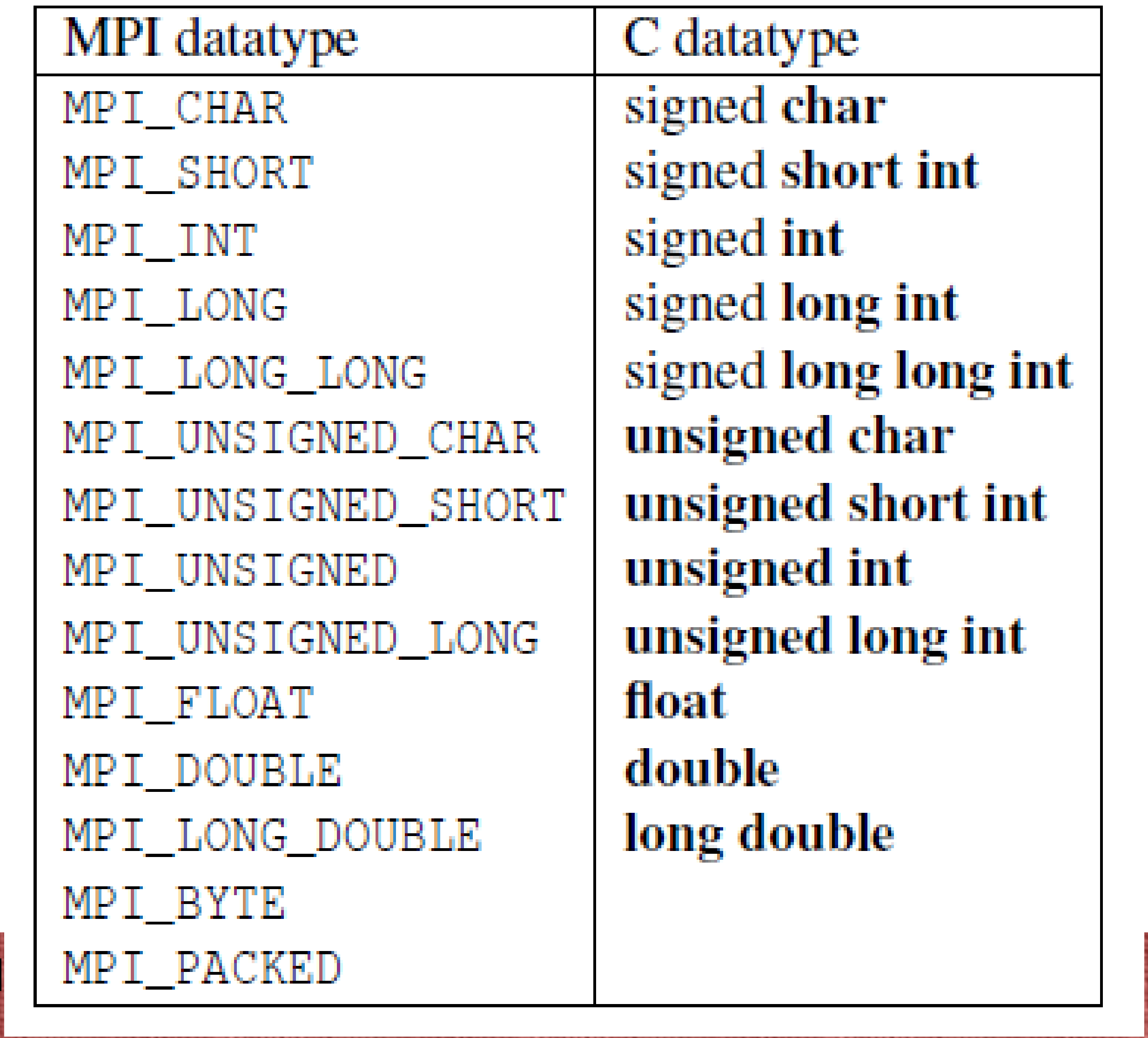
- The if-else construct makes our program SPMD.

Communication



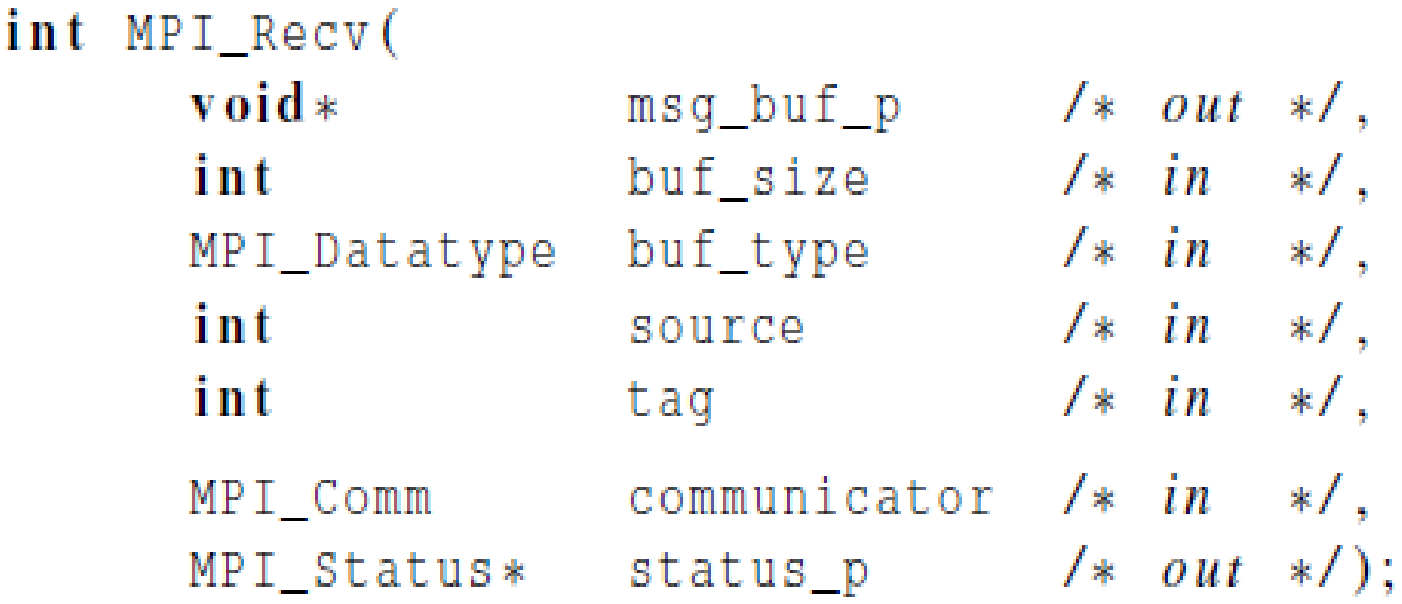
```
int MPI_Send(

   void*          msg_buf_p     /* in */,
   int            msg_size      /* in */,
   MPI_Datatype   msg_type      /* in */,
   int            dest          /* in */,
   int            tag           /* in */,
   MPI_Comm       communicator  /* in */);
```

Data types



MPI datatype	C datatype
MPI_CHAR	signed char
MPI_SHORT	signed short int
MPI_INT	signed int
MPI_LONG	signed long int
MPI_LONG_LONG	signed long long int
MPI_UNSIGNED_CHAR	unsigned char
MPI_UNSIGNED_SHORT	unsigned short int
MPI_UNSIGNED	unsigned int
MPI_UNSIGNED_LONG	unsigned long int
MPI_FLOAT	float
MPI_DOUBLE	double
MPI_LONG_DOUBLE	long double
MPI_BYTE	
MPI_PACKED	

Communication



```
int MPI_Recv(
      void*          msg_buf_p     /* out */,
      int            buf_size      /* in  */,
      MPI_Datatype   buf_type      /* in  */,
      int            source        /* in  */,
      int            tag           /* in  */,
      MPI_Comm       communicator  /* in  */,
      MPI_Status*    status_p      /* out */);
```

Message matching

```
MPI_Send(send_buf_p, send_buf_sz, send_type, dest, send_tag,
         send_comm);
```

r

MPI_Send

src = q

MPI_Recv

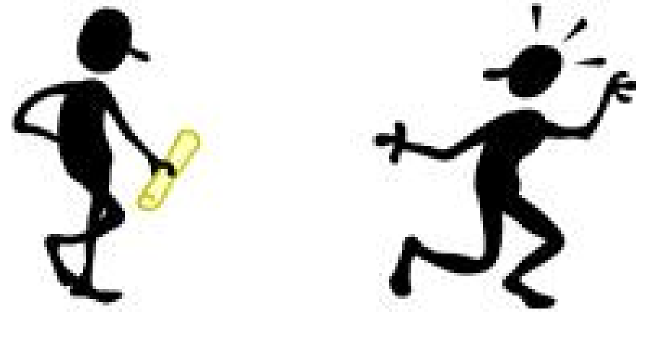
dest = r

```
MPI_Recv(recv_buf_p, recv_buf_sz, recv_type, src, recv_tag,
         recv_comm, &status);
```

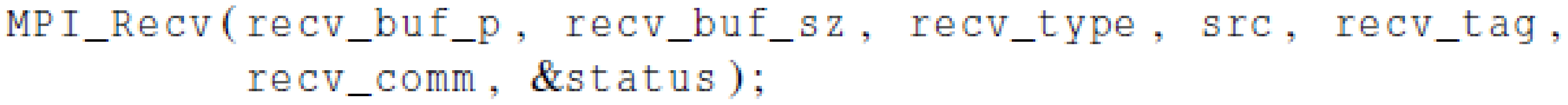
q

Receiving messages

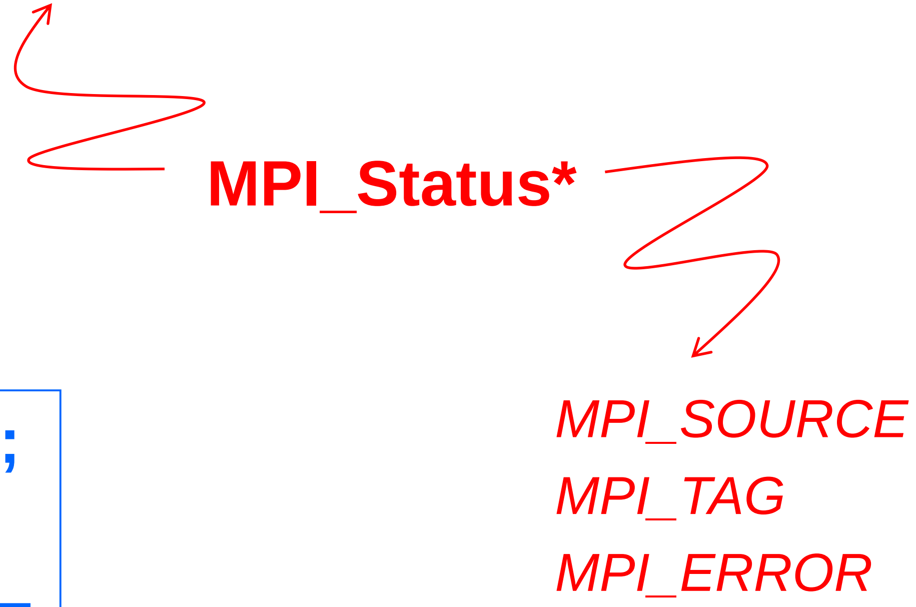
- A receiver can get a message without knowing:
  - the amount of data in the message,
  - the sender of the message,
  - or the tag of the message.



status_p argument



```
MPI_Recv(recv_buf_p, recv_buf_sz, recv_type, src, recv_tag,
         recv_comm, &status);
```



MPI_Status*

MPI_SOURCE

MPI_TAG

MPI_ERROR

MPI_Status* status;

status.MPI_SOURCE

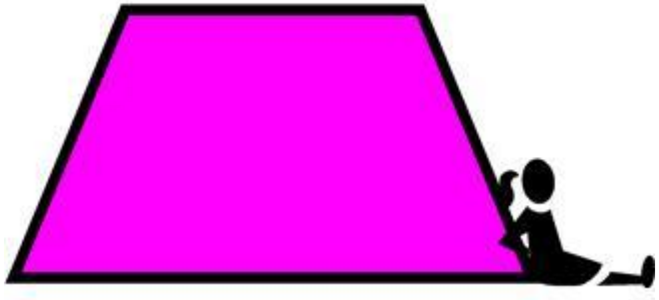
status.MPI_TAG

How much data am I receiving?

```
int MPI_Get_count(
        MPI_Status*   status_p  /* in  */,
        MPI_Datatype  type      /* in  */,
        int*          count_p   /* out */);
```

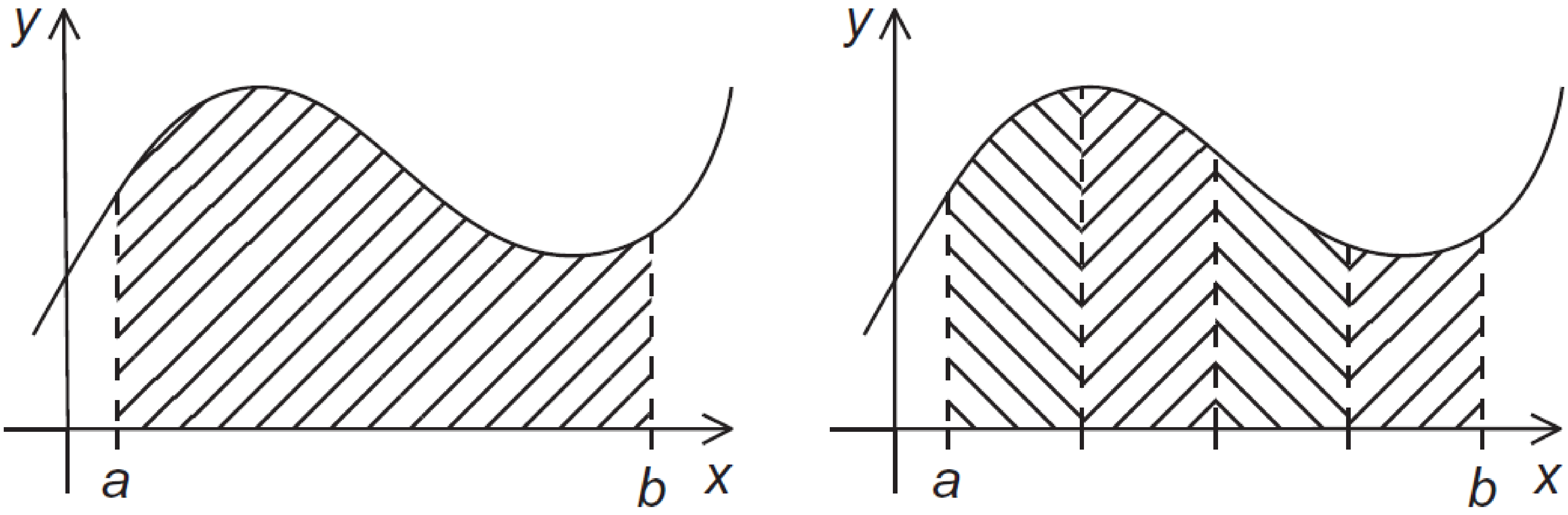
Issues with send and receive

- Exact behavior is determined by the MPI implementation.
- MPI_Send may behave differently with regard to buffer size, cutoffs and blocking.
- MPI_Recv always blocks until a matching message is received.
- Know your implementation; don’ t make assumptions!



TRAPEZOIDAL RULE IN MPI

The Trapezoidal Rule



(a)

(b)

The Trapezoidal Rule

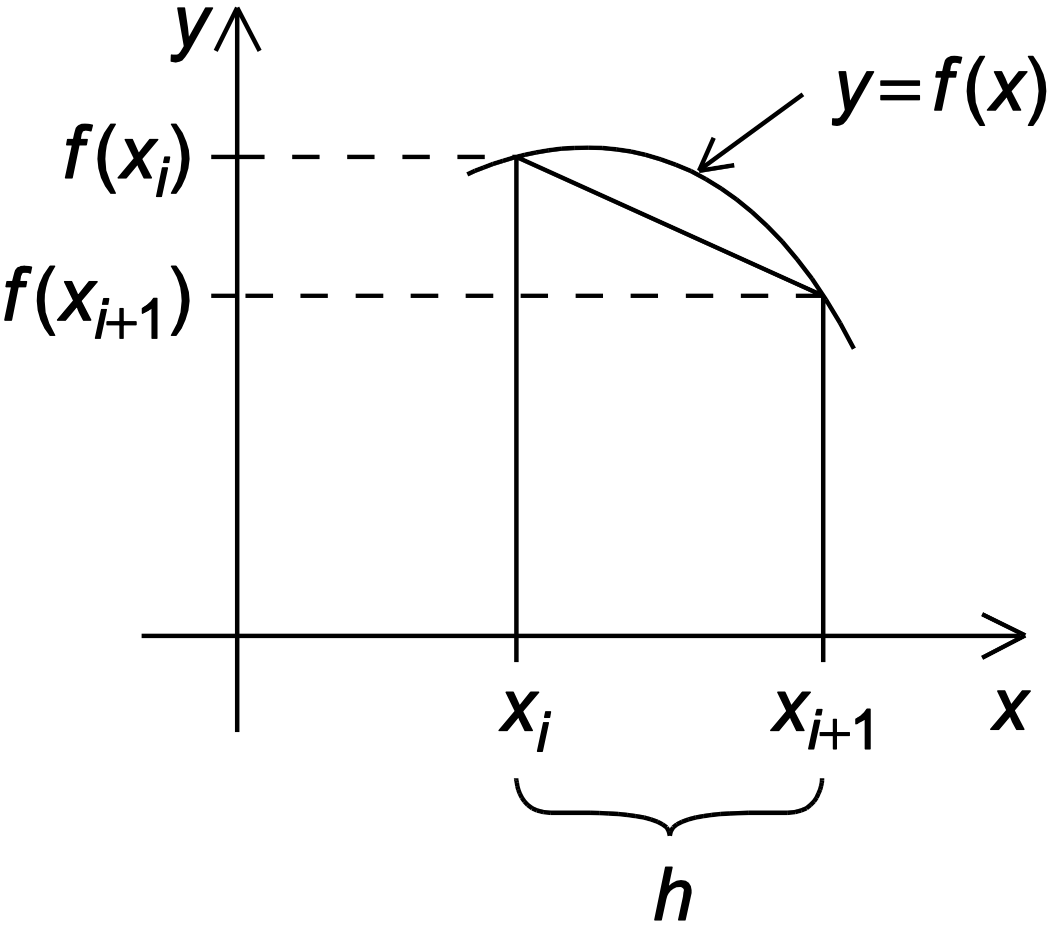
$$\text{Area of one trapezoid } = \frac{h}{2}[f(x_i) + f(x_{i+1})]$$

$$h = \frac{b-a}{n}$$

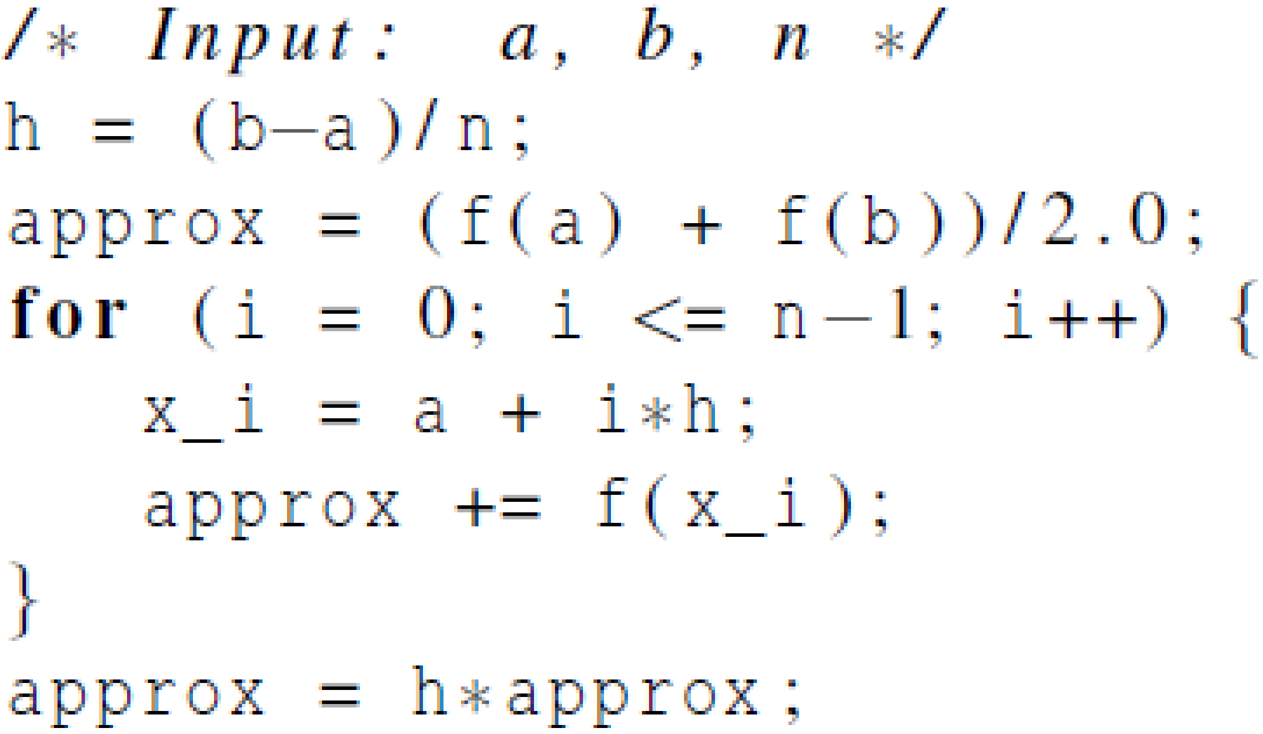
$$x_0 = a,\ x_1 = a + h,\ x_2 = a + 2h,\ \ldots,\ x_{n-1} = a + (n-1)h,\ x_n = b$$

$$\text{Sum of trapezoid areas } = h[f(x_0)/2 + f(x_1) + f(x_2) + \cdots + f(x_{n-1}) + f(x_n)/2]$$

One trapezoid



Pseudo-code for a serial program

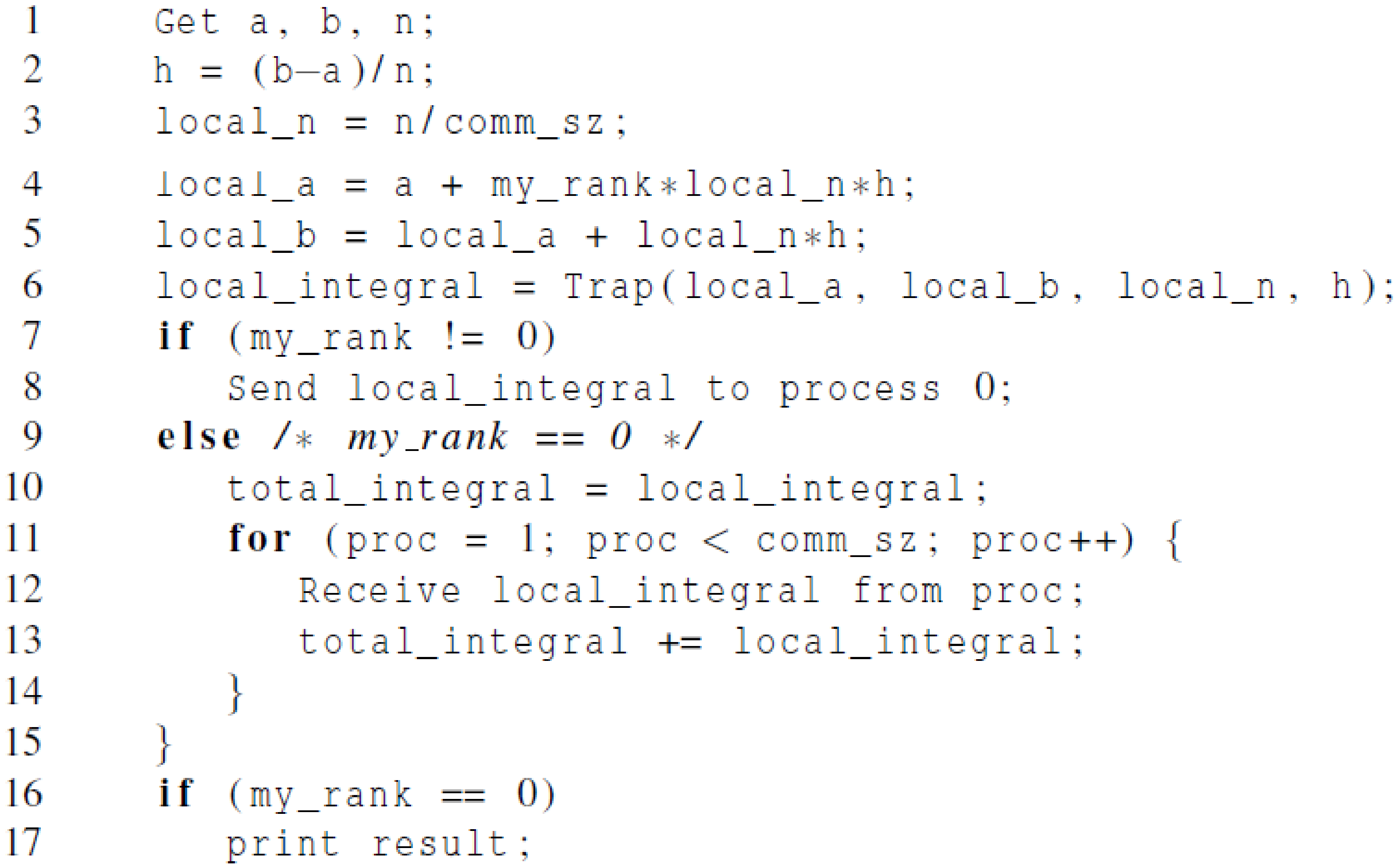


```
/* Input: a, b, n */
h = (b−a)/n;
approx = (f(a) + f(b))/2.0;
for (i = 0; i <= n−1; i++) {
    x_i = a + i*h;
    approx += f(x_i);
}
approx = h*approx;
```

Parallelizing the Trapezoidal Rule

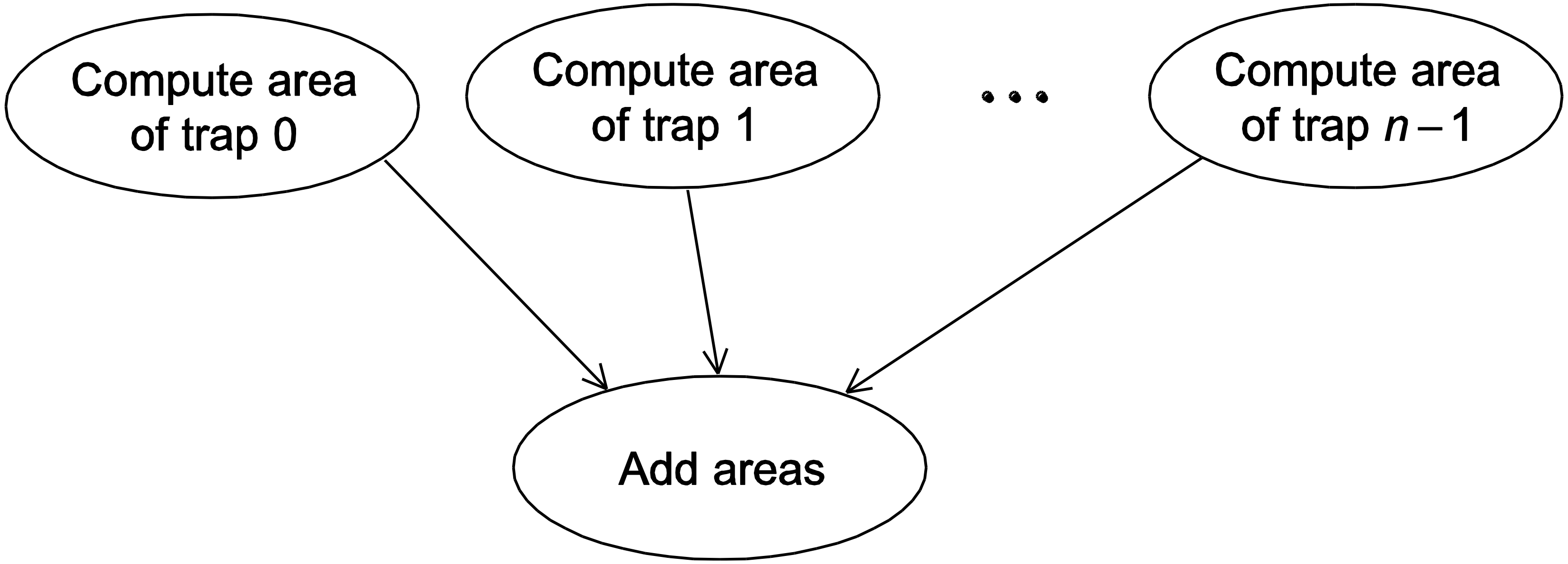
1. Partition problem solution into tasks.
2. Identify communication channels between tasks.
3. Aggregate tasks into composite tasks.
4. Map composite tasks to cores.

Parallel pseudo-code

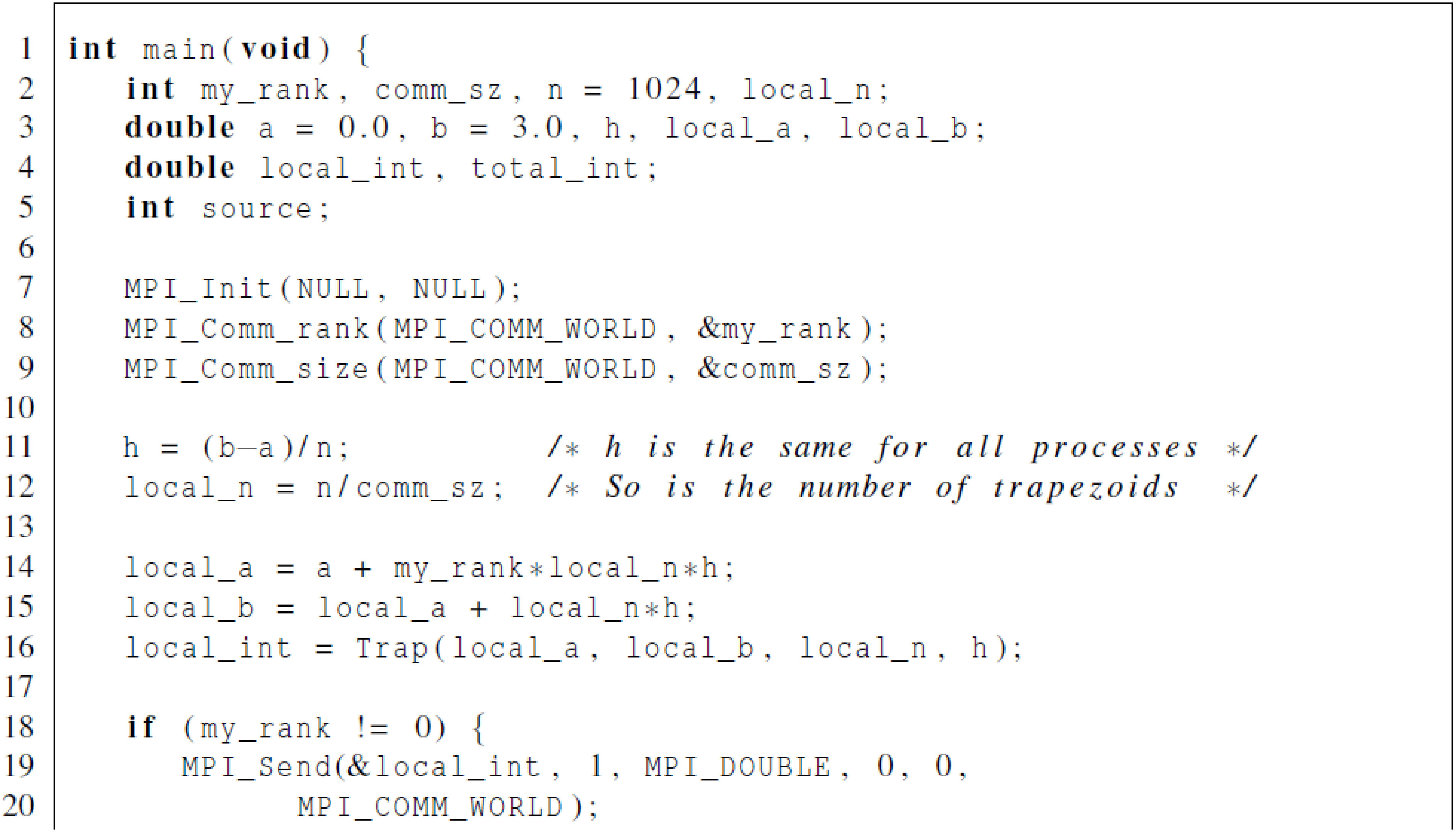


```
Get a, b, n;
h = (b−a)/n;
local_n = n/comm_sz;

local_a = a + my_rank*local_n*h;
local_b = local_a + local_n*h;
local_integral = Trap(local_a, local_b, local_n, h);
if (my_rank != 0)
   Send local_integral to process 0;
else /* my_rank == 0 */
   total_integral = local_integral;
   for (proc = 1; proc < comm_sz; proc++) {
      Receive local_integral from proc;
      total_integral += local_integral;
   }
}
if (my_rank == 0)
   print result;
```

Tasks and communications for Trapezoidal Rule

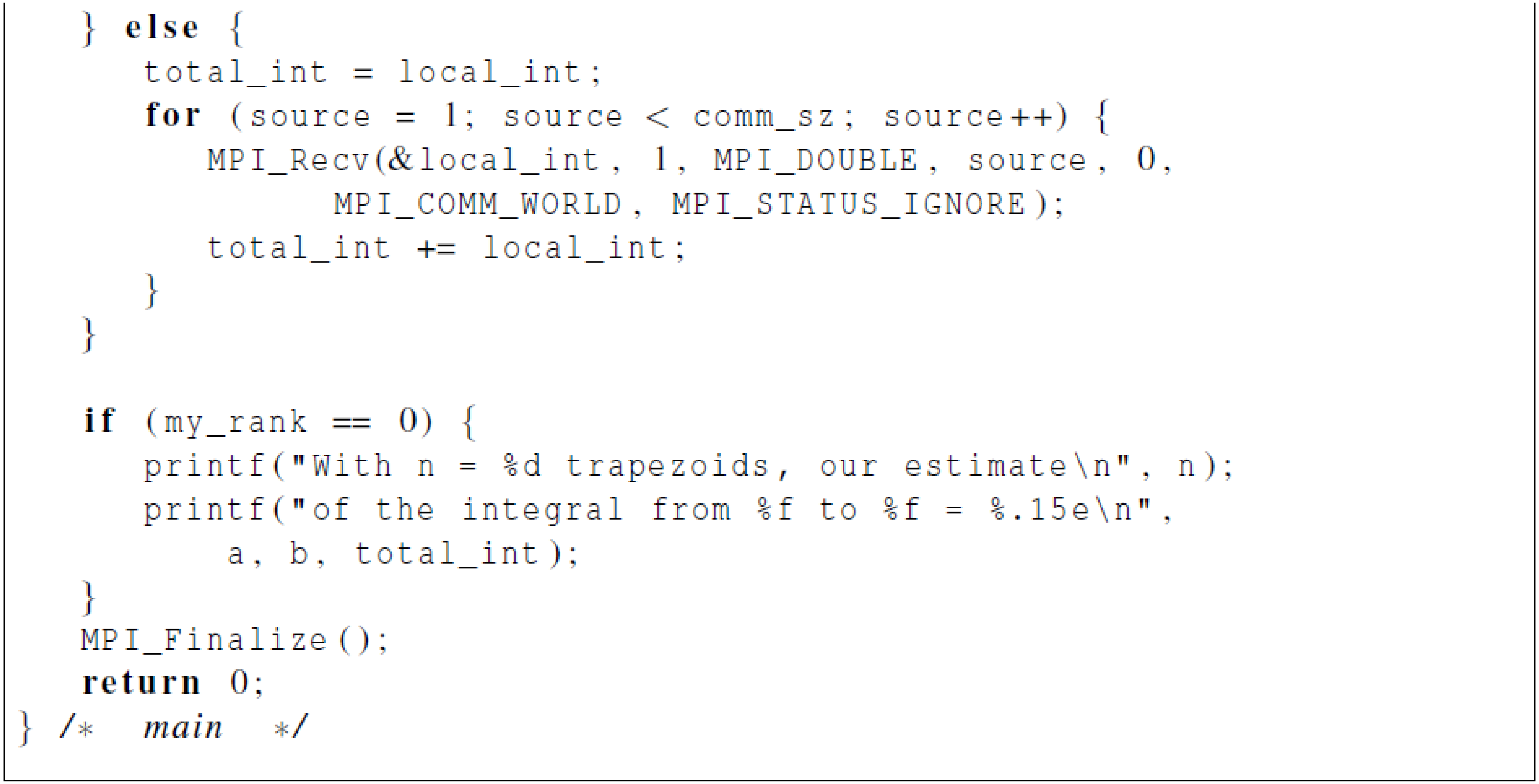


First version (1)



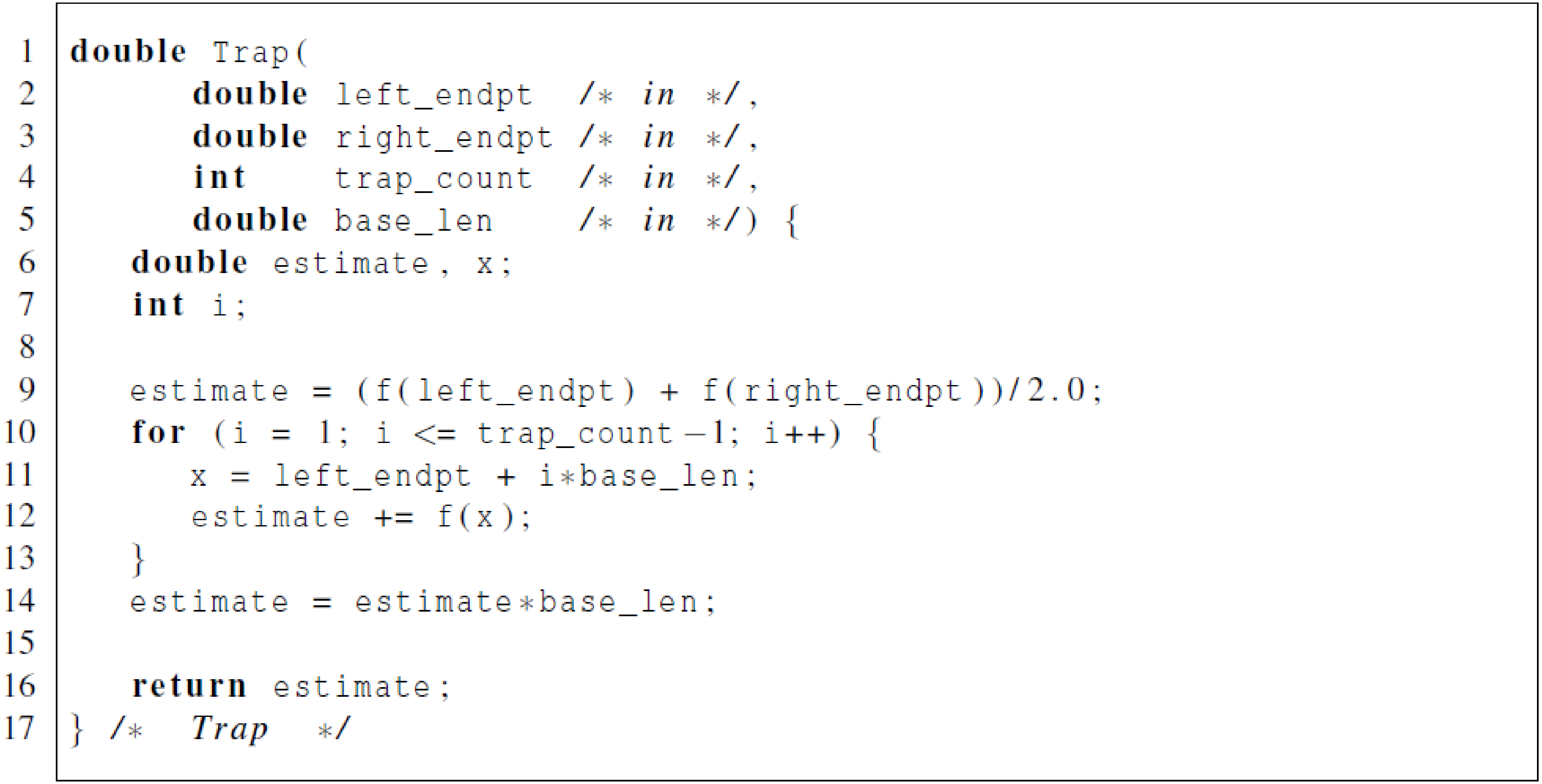
```
int main(void) {
   int my_rank, comm_sz, n = 1024, local_n;
   double a = 0.0, b = 3.0, h, local_a, local_b;
   double local_int, total_int;
   int source;

   MPI_Init(NULL, NULL);
   MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
   MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);

   h = (b-a)/n;            /* h is the same for all processes */
   local_n = n/comm_sz;    /* So is the number of trapezoids  */

   local_a = a + my_rank*local_n*h;
   local_b = local_a + local_n*h;
   local_int = Trap(local_a, local_b, local_n, h);

   if (my_rank != 0) {
      MPI_Send(&local_int, 1, MPI_DOUBLE, 0, 0,
            MPI_COMM_WORLD);
```

First version (2)



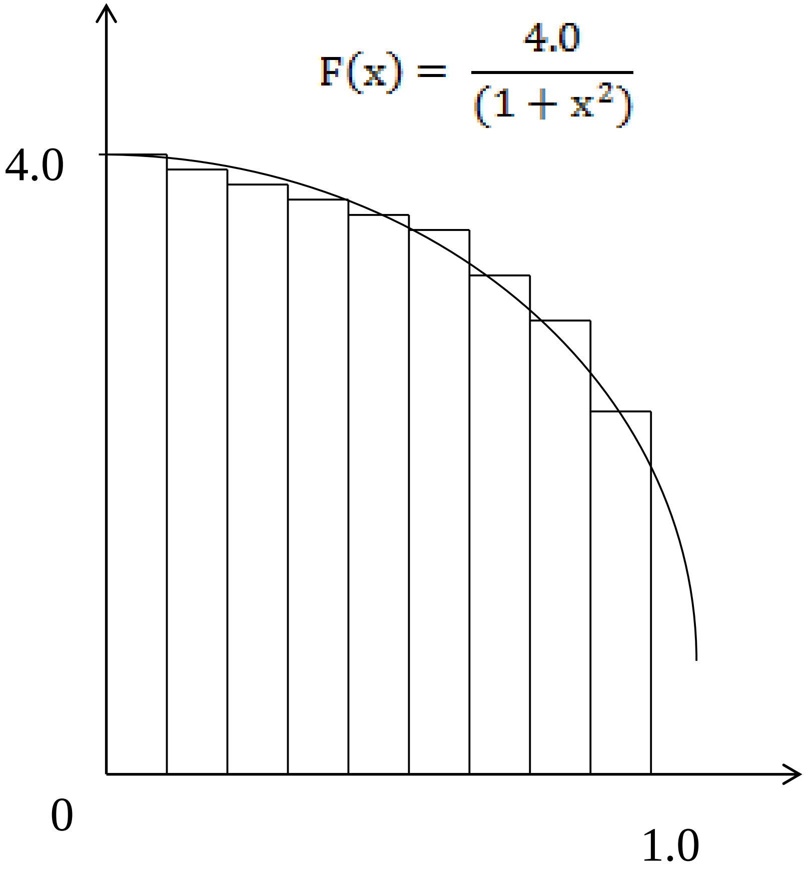
```
  } else {
    total_int = local_int;
    for (source = 1; source < comm_sz; source++) {
      MPI_Recv(&local_int, 1, MPI_DOUBLE, source, 0,
            MPI_COMM_WORLD, MPI_STATUS_IGNORE);
      total_int += local_int;
    }
  }

  if (my_rank == 0) {
    printf("With n = %d trapezoids, our estimate\n", n);
    printf("of the integral from %f to %f = %.15e\n",
        a, b, total_int);
  }
  MPI_Finalize();
  return 0;
} /* main */
```

First version (3)



```c
double Trap(
        double left_endpt  /* in */,
        double right_endpt /* in */,
        int    trap_count  /* in */,
        double base_len    /* in */) {
    double estimate, x;
    int i;

    estimate = (f(left_endpt) + f(right_endpt))/2.0;
    for (i = 1; i <= trap_count-1; i++) {
        x = left_endpt + i*base_len;
        estimate += f(x);
    }
    estimate = estimate*base_len;

    return estimate;
} /* Trap */
```

Program example



$$F(x) = \frac{4.0}{(1+x^2)}$$

$$\int_0^1 \frac{4.0}{(1+x^2)} dx = \pi$$

$$\sum_{i=0}^{N} F(x_i) * \Delta x \approx \pi$$

Program example: original code

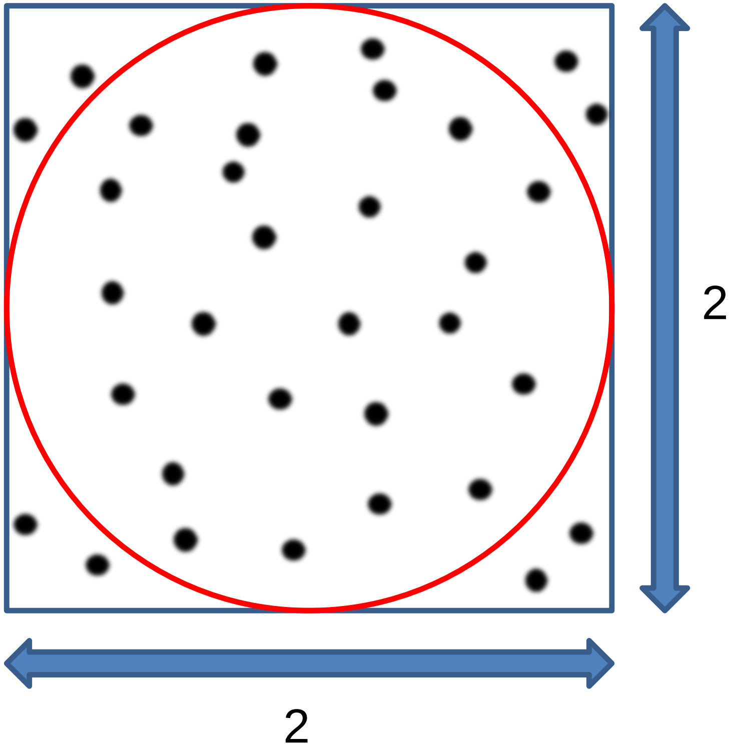
```
#include <stdio.h>
#define f(x)  (4.0/(1.0+x*x))
void main(int argc, char ** argv){
    int i, n = 10000000;
    double w, x, sum, pi;
    w = 1.0/n;
    sum = 0.0;
    for(i=1; i<= n; i++){
        x = w*(double)(i – 0.5);
        sum += f(x);
    }
    pi = w*sum;
    printf(“PI : %.16f”, pi);
}
```

Program example: parallel code(1/2)

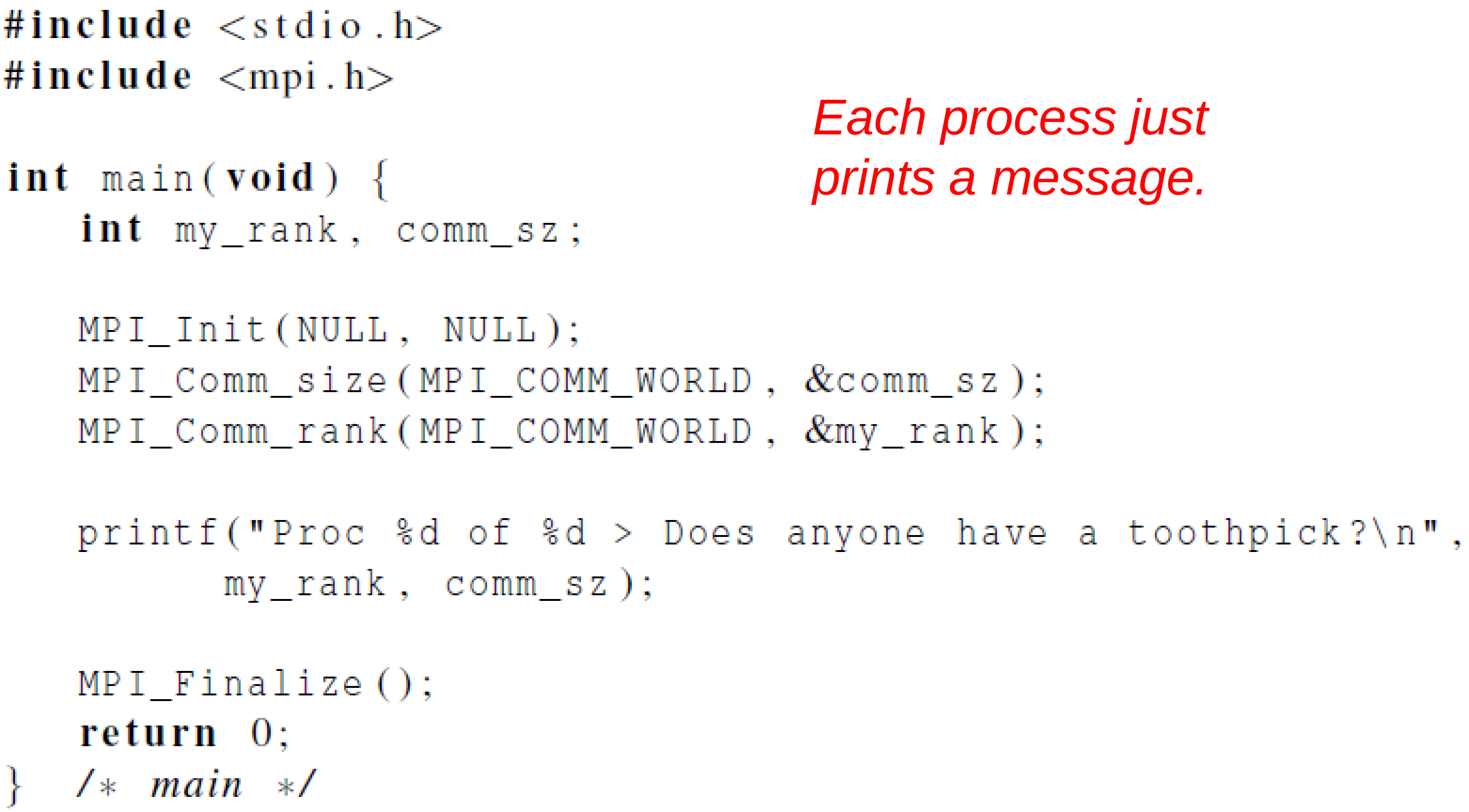
```
#include “mpi.h”
#include <stdio.h>
#define f(x) (4.0/(1.0+x*x))
void main(int argc, char *argv[]){
    int myid, t_process, i, n = 10000000;
    double mypi, pi, h, sum, x, recv_pi;
    MPI_Init(&argc, &argv);
    MPI_Comm_size(MPI_COMM_WORLD, &t_process);
    MPI_Comm_rank(MPI_COMM_WORLD, &myid);
    h = 1.0/(double) n;  sum = 0.0;
    for(i = myid +1 ; i <= n ; i+= t_process){
            x = h*(double) (i– 0.5);
            sum += f(x);
    }
```

Program example: parallel code(2/2)

```
     mypi = h*sum;
    if(myid == 0){
          pi = 0;
          MPI_Status status;
          for(i=1; i< t_process; i++){
                    MPI_Recv(&recv_pi, 1, MPI_DOUBLE, i, 1,
                    MPI_COMM_WORLD, &status);
                    pi += recv_pi;
          }
          printf("PI : %.16f\n", pi);
    }
    else MPI_Send(&mypi, 1, MPI_DOUBLE, 0, 1, MPI_COMM_WORLD);
    MPI_Finalize();
}
```


2
2

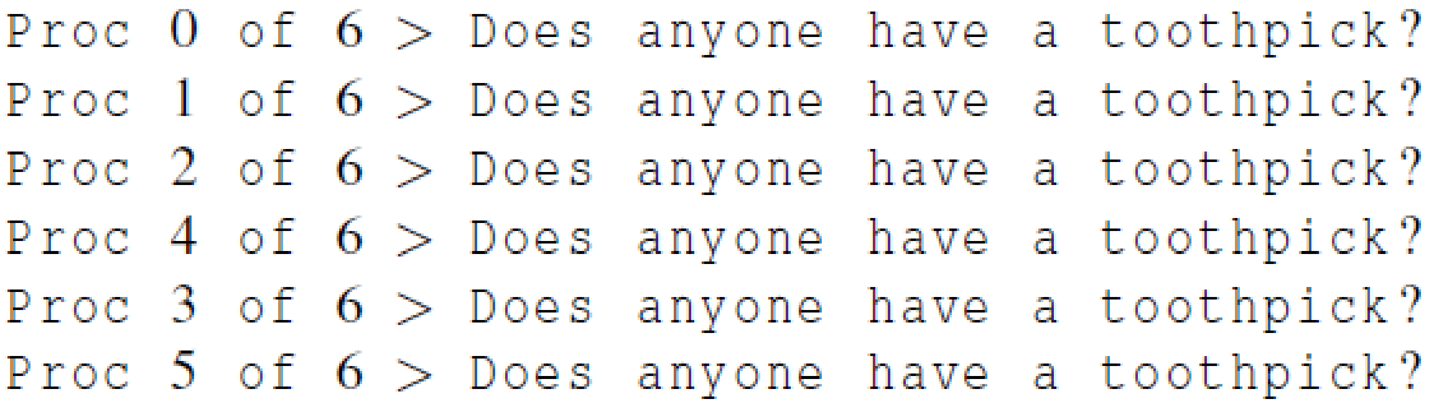
Dealing with I/O



```
#include <stdio.h>
#include <mpi.h>

int main(void) {
   int my_rank, comm_sz;

   MPI_Init(NULL, NULL);
   MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);
   MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);

   printf("Proc %d of %d > Does anyone have a toothpick?\n",
         my_rank, comm_sz);

   MPI_Finalize();
   return 0;
}  /* main */
```

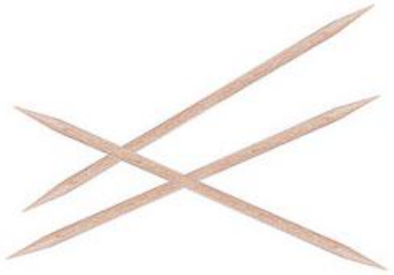
Each process just prints a message.

Running with 6 processes



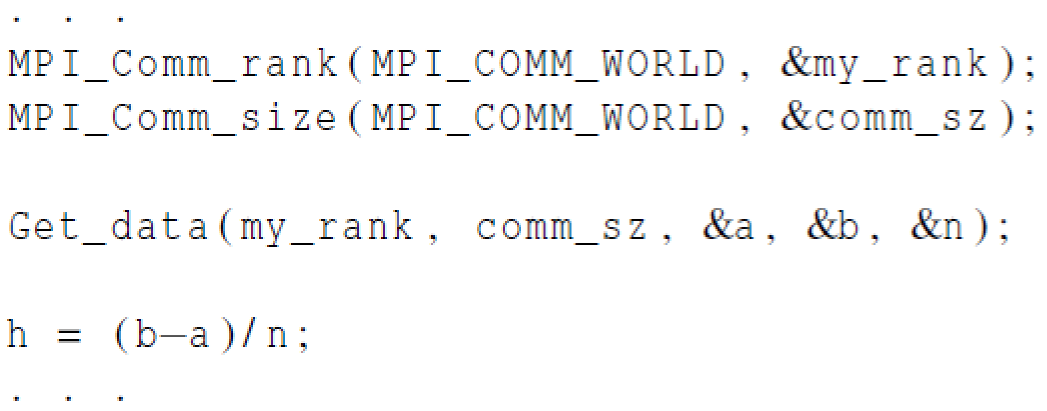
```
Proc 0 of 6 > Does anyone have a toothpick?
Proc 1 of 6 > Does anyone have a toothpick?
Proc 2 of 6 > Does anyone have a toothpick?
Proc 4 of 6 > Does anyone have a toothpick?
Proc 3 of 6 > Does anyone have a toothpick?
Proc 5 of 6 > Does anyone have a toothpick?
```

unpredictable output



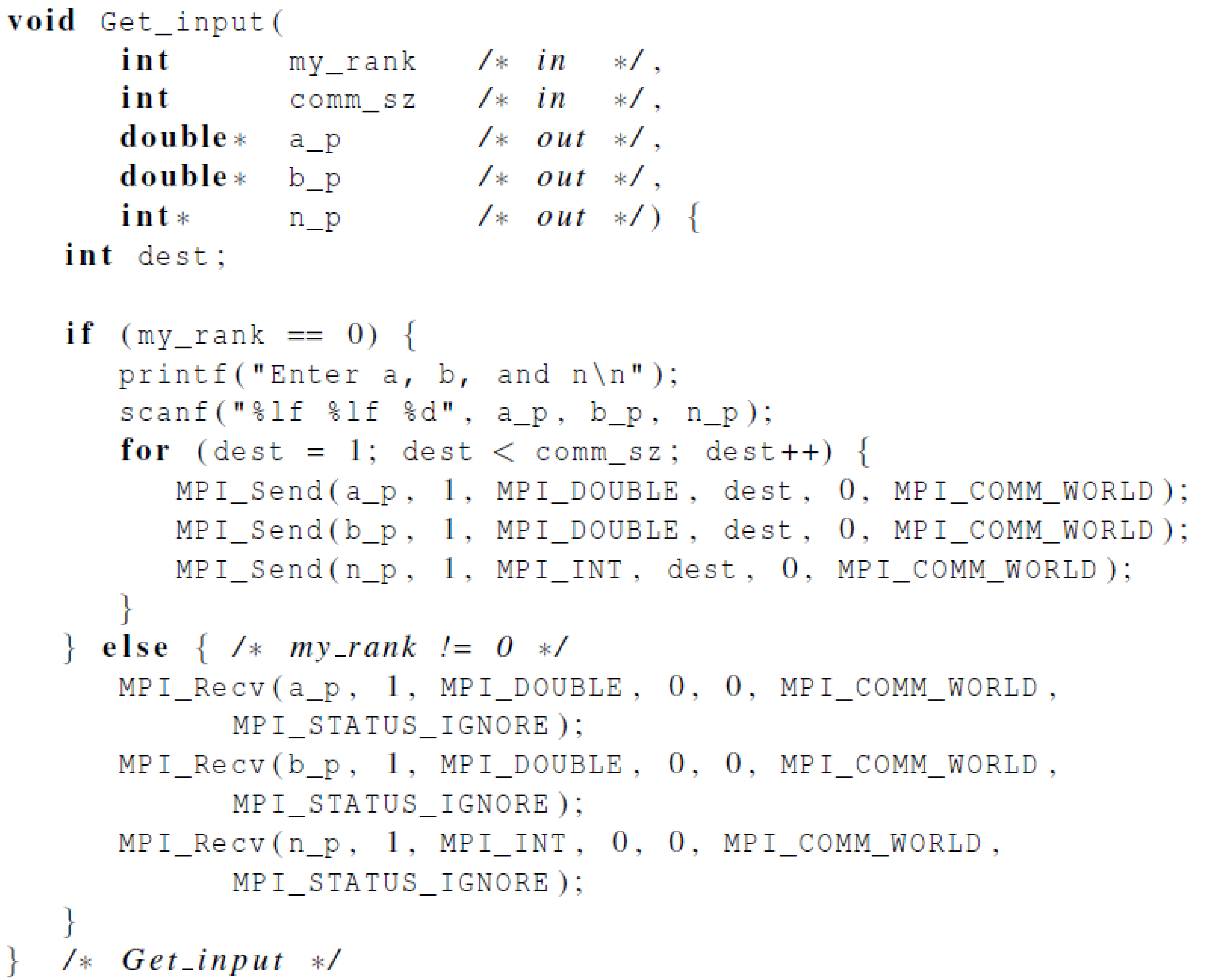
Input

- Most MPI implementations only allow process 0 in MPI_COMM_WORLD access to stdin.
- Process 0 must read the data (scanf) and send to the other processes.



```
. . .
MPI_Comm_rank(MPI_COMM_WORLD, &my_rank);
MPI_Comm_size(MPI_COMM_WORLD, &comm_sz);

Get_data(my_rank, comm_sz, &a, &b, &n);

h = (b-a)/n;
. . .
```

Function for reading user input



```c
void Get_input(
      int      my_rank   /* in  */,
      int      comm_sz   /* in  */,
      double*  a_p       /* out */,
      double*  b_p       /* out */,
      int*     n_p       /* out */) {
   int dest;

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);
      for (dest = 1; dest < comm_sz; dest++) {
         MPI_Send(a_p, 1, MPI_DOUBLE, dest, 0, MPI_COMM_WORLD);
         MPI_Send(b_p, 1, MPI_DOUBLE, dest, 0, MPI_COMM_WORLD);
         MPI_Send(n_p, 1, MPI_INT, dest, 0, MPI_COMM_WORLD);
      }
   } else { /* my_rank != 0 */
      MPI_Recv(a_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD,
            MPI_STATUS_IGNORE);
      MPI_Recv(b_p, 1, MPI_DOUBLE, 0, 0, MPI_COMM_WORLD,
            MPI_STATUS_IGNORE);
      MPI_Recv(n_p, 1, MPI_INT, 0, 0, MPI_COMM_WORLD,
            MPI_STATUS_IGNORE);
   }
}  /* Get_input */
```

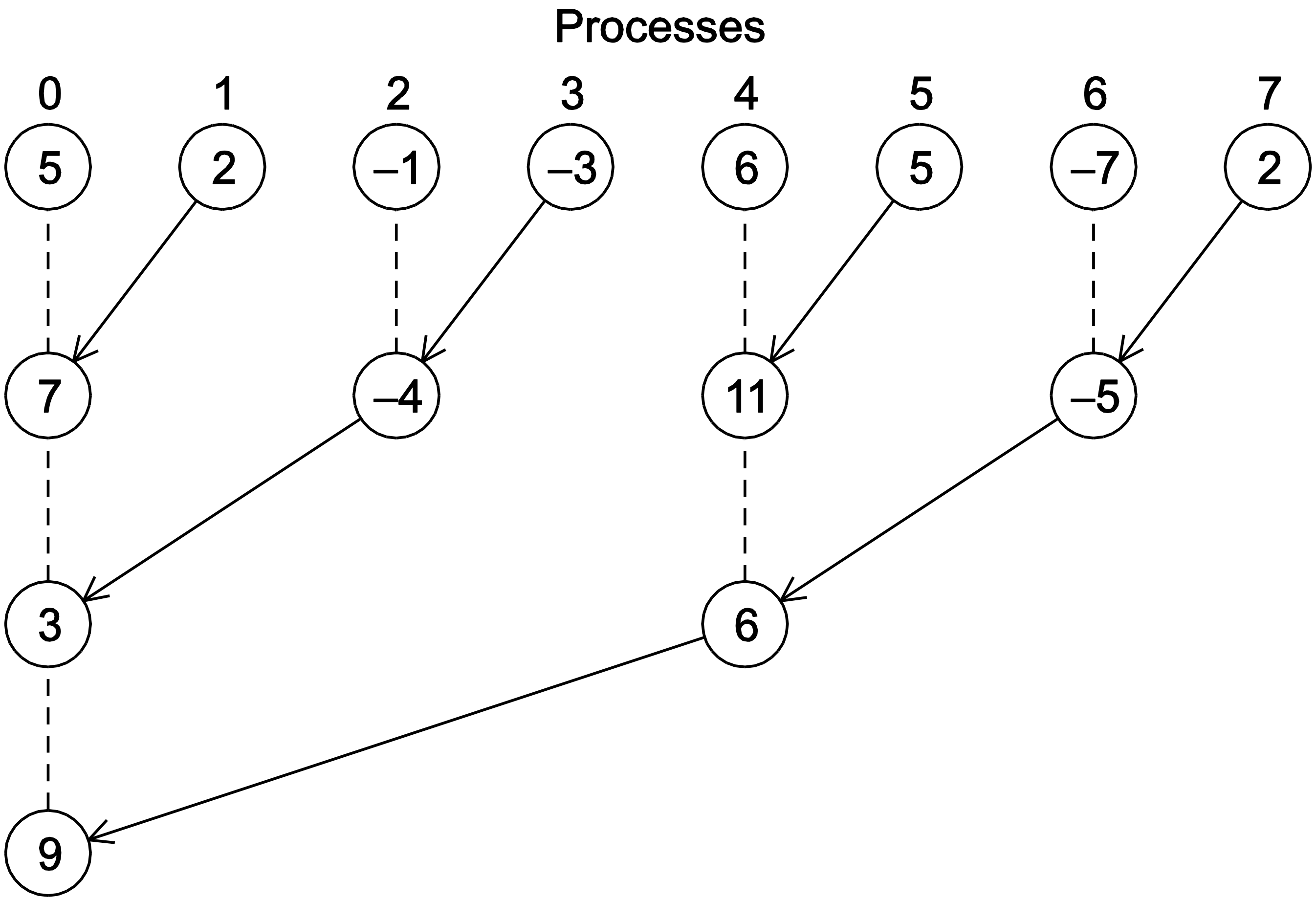
COLLECTIVE COMMUNICATION

Tree-structured communication

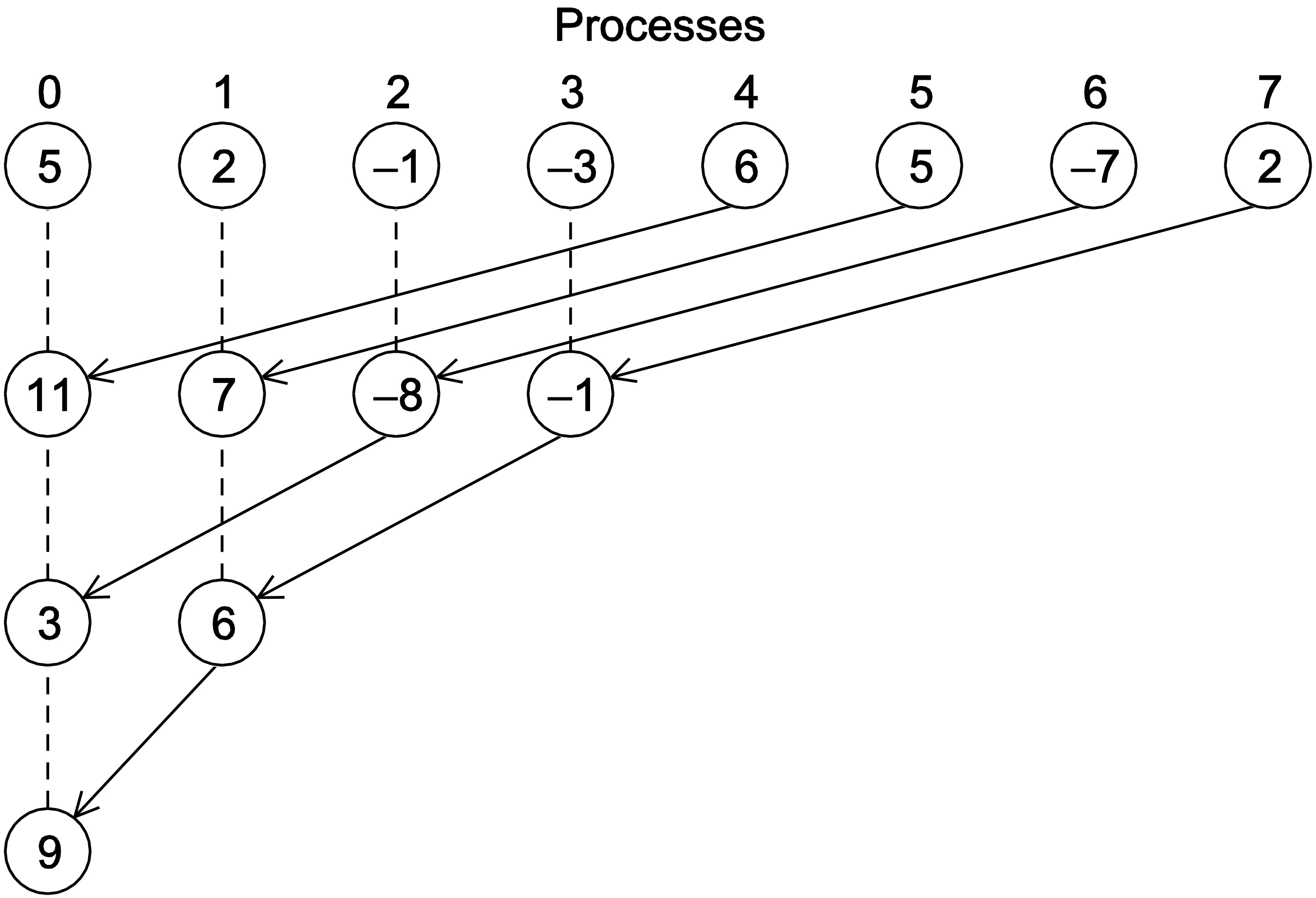
1. In the first phase:
   (a) Process 1 sends to 0, 3 sends to 2, 5 sends to 4, and 7 sends to 6.
   (b) Processes 0, 2, 4, and 6 add in the received values.
   (c) Processes 2 and 6 send their new values to processes 0 and 4, respectively.
   (d) Processes 0 and 4 add the received values into their new values.

2. (a) Process 4 sends its newest value to process 0.
   (b) Process 0 adds the received value to its newest value.

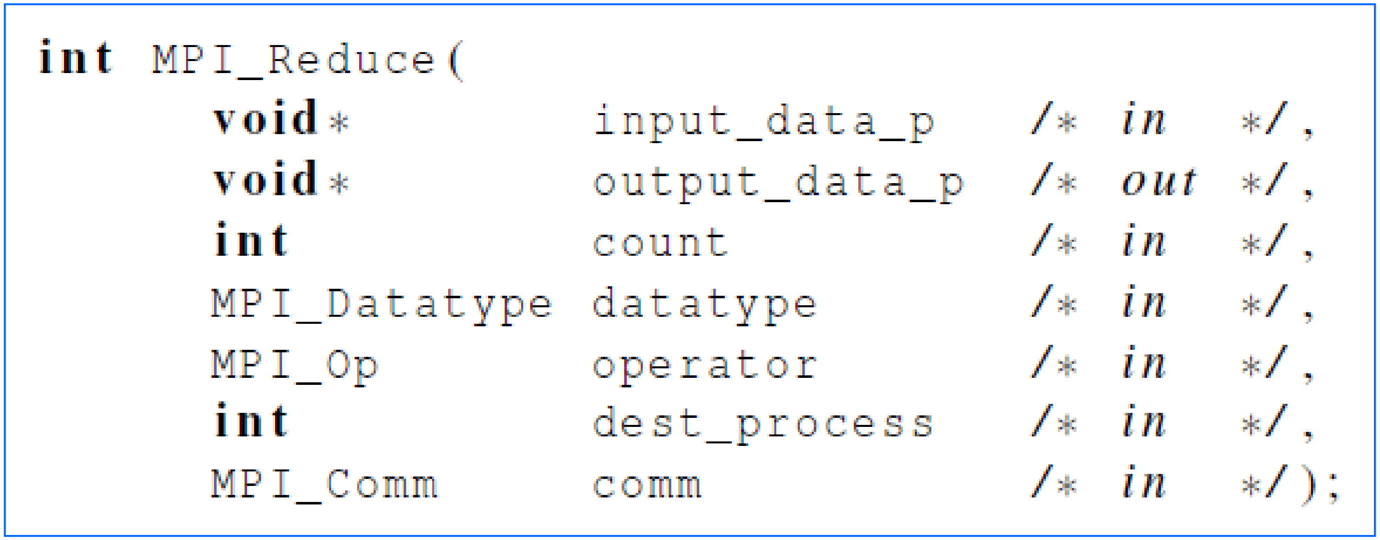
A tree-structured global sum



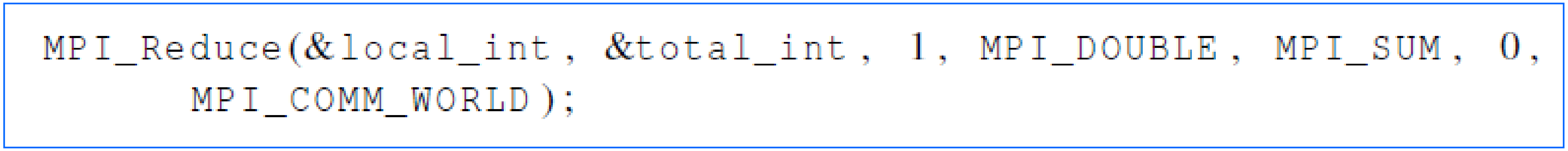
An alternative tree-structured global sum



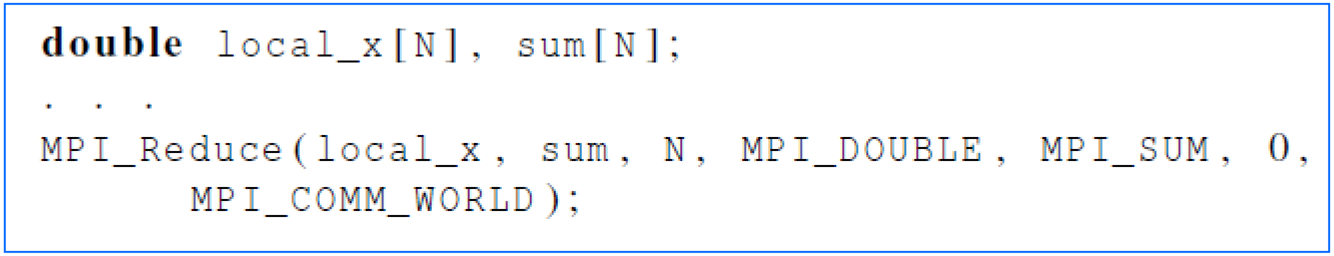
MPI_Reduce



```
int MPI_Reduce(
      void*         input_data_p    /* in   */,
      void*         output_data_p   /* out  */,
      int           count           /* in   */,
      MPI_Datatype  datatype        /* in   */,
      MPI_Op        operator        /* in   */,
      int           dest_process    /* in   */,
      MPI_Comm      comm            /* in   */);
```

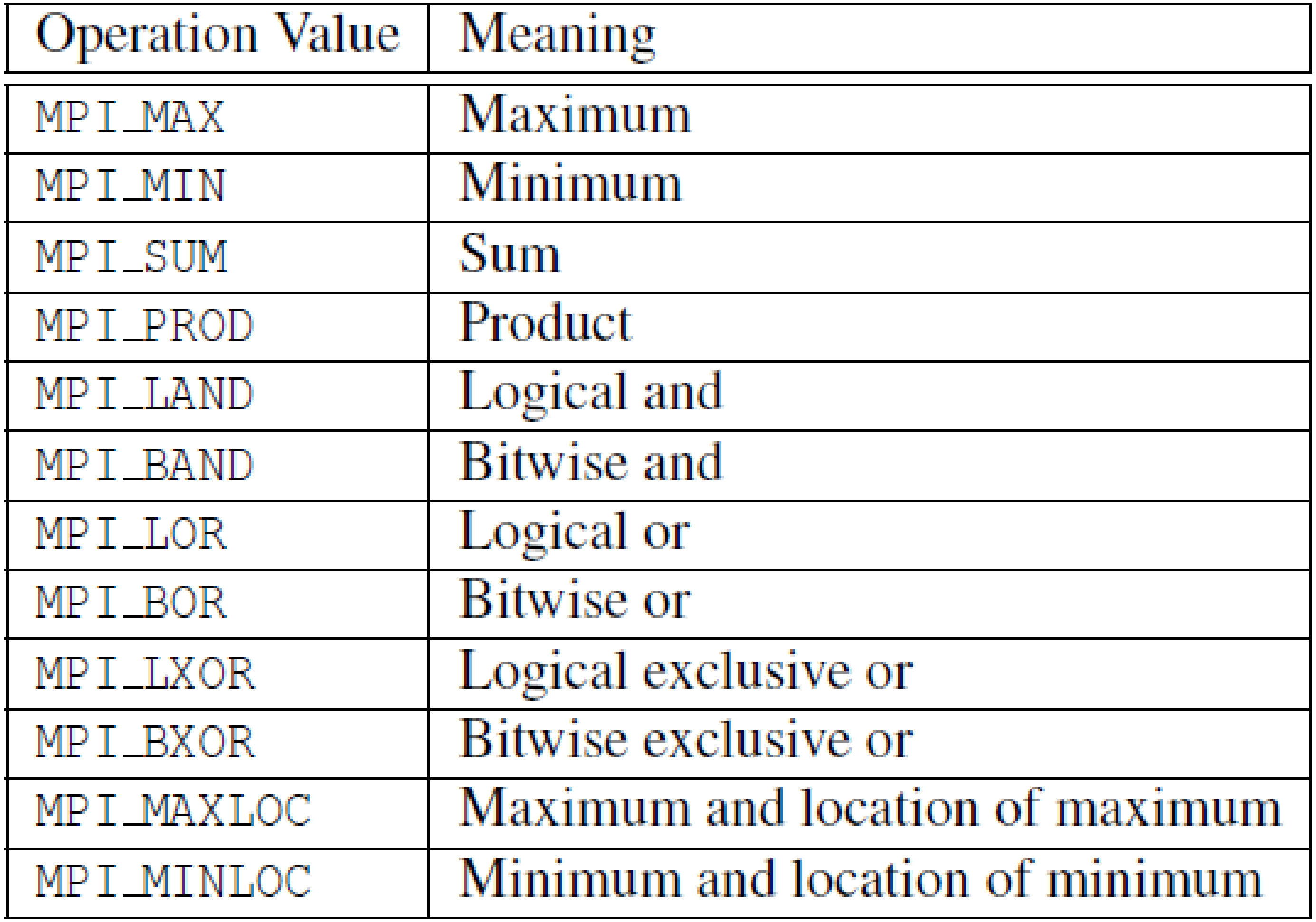


```
MPI_Reduce(&local_int, &total_int, 1, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```



```
double local_x[N], sum[N];
. . .
MPI_Reduce(local_x, sum, N, MPI_DOUBLE, MPI_SUM, 0,
      MPI_COMM_WORLD);
```

Predefined reduction operators in MPI



Operation Value	Meaning
MPI_MAX	Maximum
MPI_MIN	Minimum
MPI_SUM	Sum
MPI_PROD	Product
MPI_LAND	Logical and
MPI_BAND	Bitwise and
MPI_LOR	Logical or
MPI_BOR	Bitwise or
MPI_LXOR	Logical exclusive or
MPI_BXOR	Bitwise exclusive or
MPI_MAXLOC	Maximum and location of maximum
MPI_MINLOC	Minimum and location of minimum

Collective vs. Point-to-Point Communications

- All the processes in the communicator must call the same collective function.

- For example, a program that attempts to match a call to MPI_Reduce on one process with a call to MPI_Recv on another process is erroneous, and, in all likelihood, the program will hang or crash.

Collective vs. Point-to-Point Communications

- The arguments passed by each process to an MPI collective communication must be "compatible."

- For example, if one process passes in 0 as the dest_process and another passes in 1, then the outcome of a call to MPI_Reduce is erroneous, and, once again, the program is likely to hang or crash.

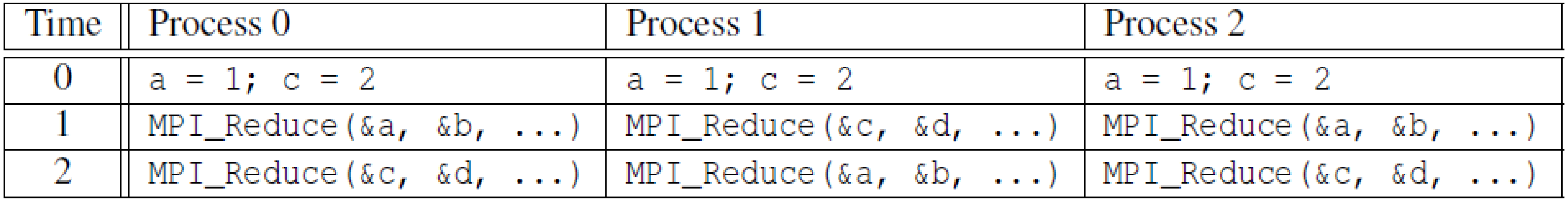
Collective vs. Point-to-Point Communications

- The output_data_p argument is only used on dest_process.
- However, all of the processes still need to pass in an actual argument corresponding to output_data_p, even if it’ s just NULL.

Collective vs. Point-to-Point Communications

- Point-to-point communications are matched on the basis of tags and communicators.
- Collective communications don’ t use tags.
- They’ re matched solely on the basis of the communicator and the order in which they’ re called.

Example (1)



Time	Process 0	Process 1	Process 2
0	`a = 1; c = 2`	`a = 1; c = 2`	`a = 1; c = 2`
1	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`
2	`MPI_Reduce(&c, &d, ...)`	`MPI_Reduce(&a, &b, ...)`	`MPI_Reduce(&c, &d, ...)`

Multiple calls to MPI_Reduce

Example (2)

- Suppose that each process calls MPI_Reduce with operator MPI_SUM, and destination process 0.
- At first glance, it might seem that after the two calls to MPI_Reduce, the value of b will be 3, and the value of d will be 6.

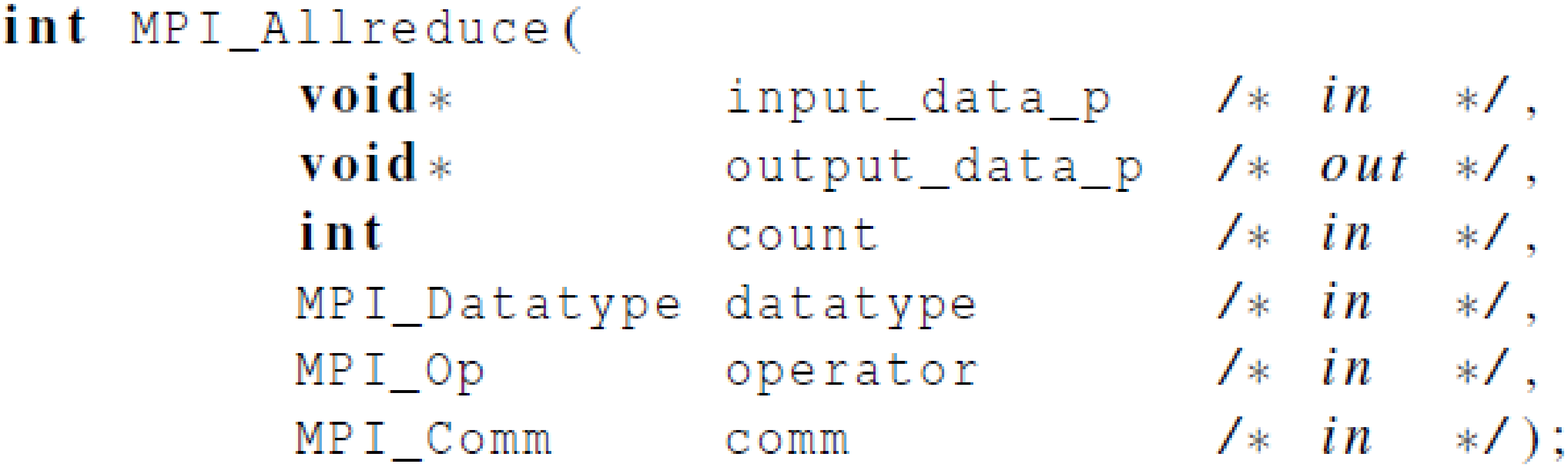
Example (3)

- However, the names of the memory locations are irrelevant to the matching of the calls to MPI_Reduce.

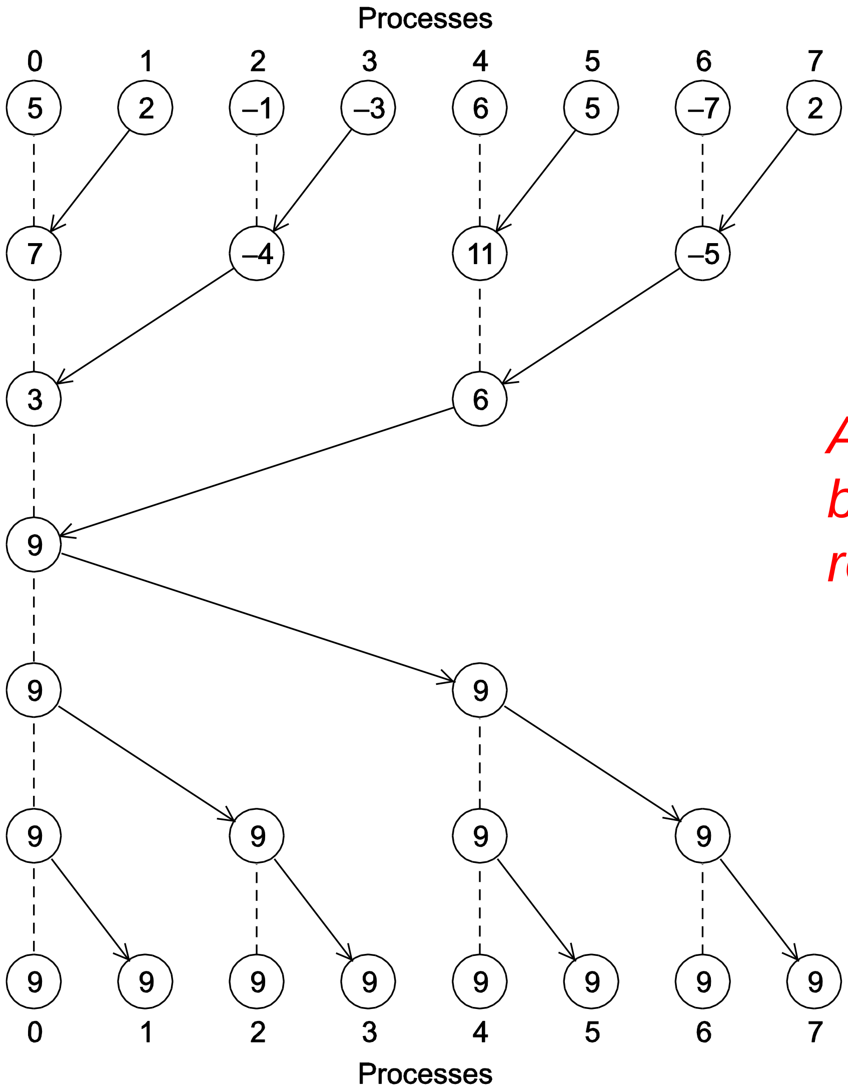
- The order of the calls will determine the matching so the value stored in b will be 1+2+1 = 4, and the value stored in d will be 2+1+2 = 5.

MPI_Allreduce

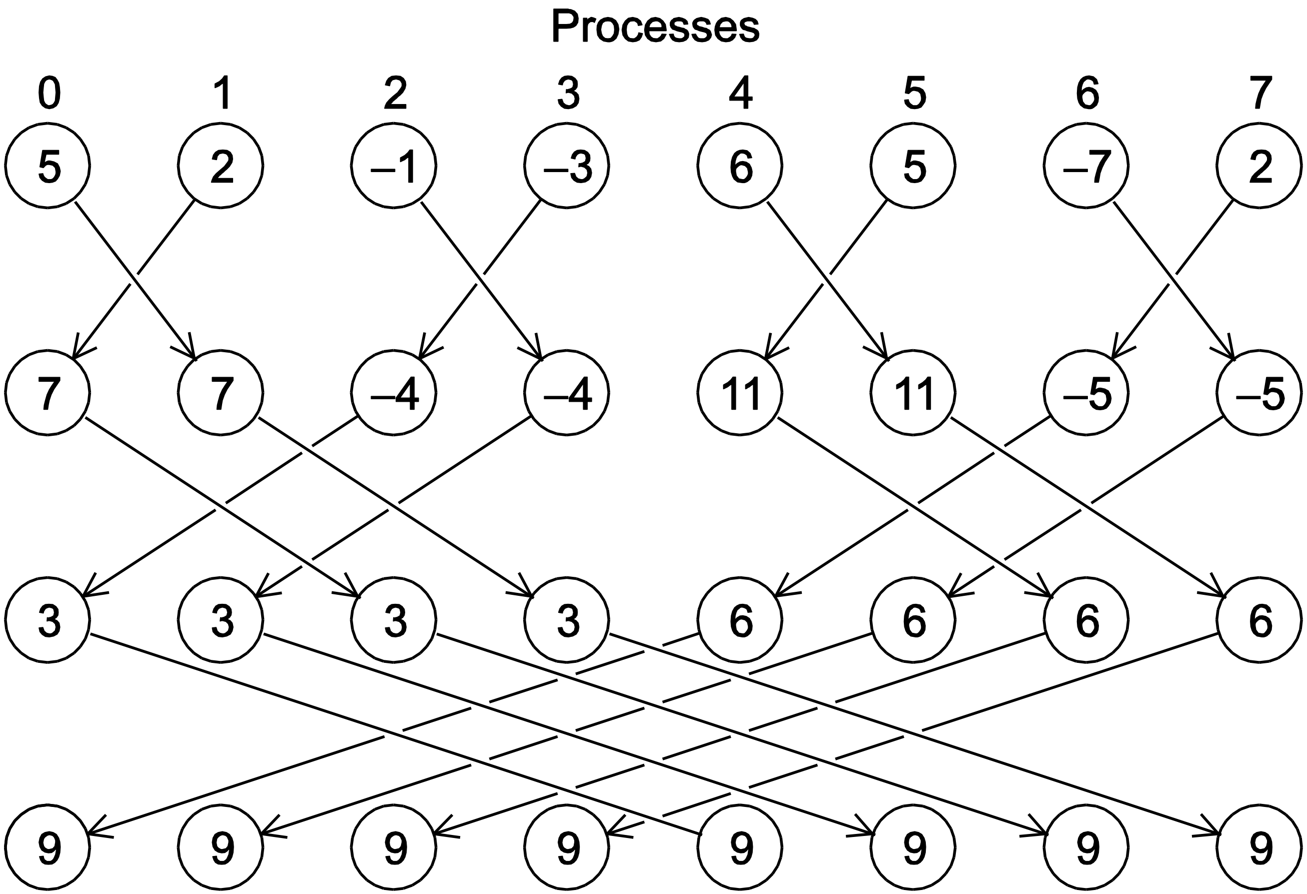
- Useful in a situation in which all of the processes need the result of a global sum in order to complete some larger computation.



```
int MPI_Allreduce(
        void*         input_data_p   /* in  */,
        void*         output_data_p  /* out */,
        int           count          /* in  */,
        MPI_Datatype  datatype       /* in  */,
        MPI_Op        operator       /* in  */,
        MPI_Comm      comm           /* in  */);
```



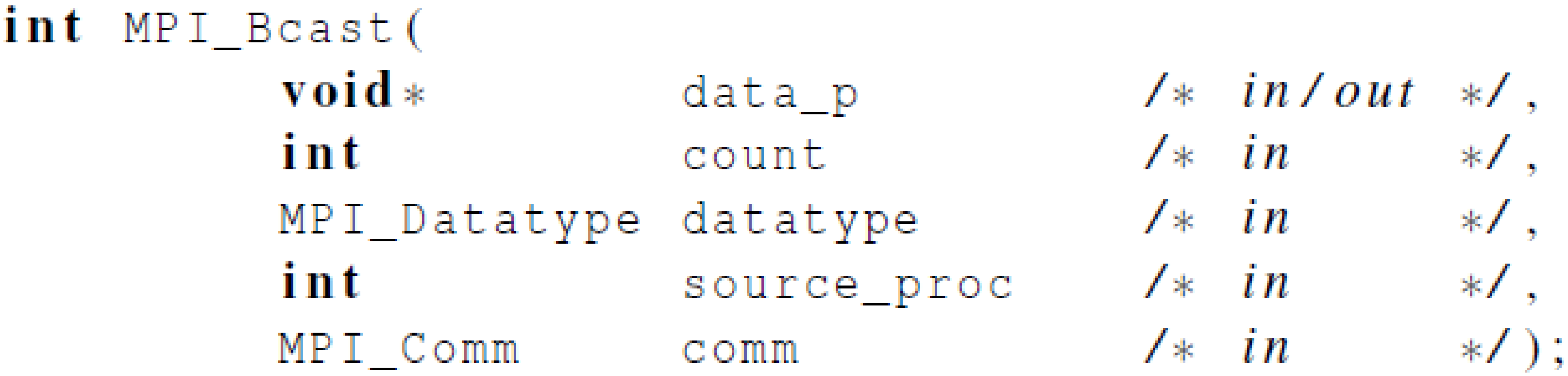
A global sum followed by distribution of the result.



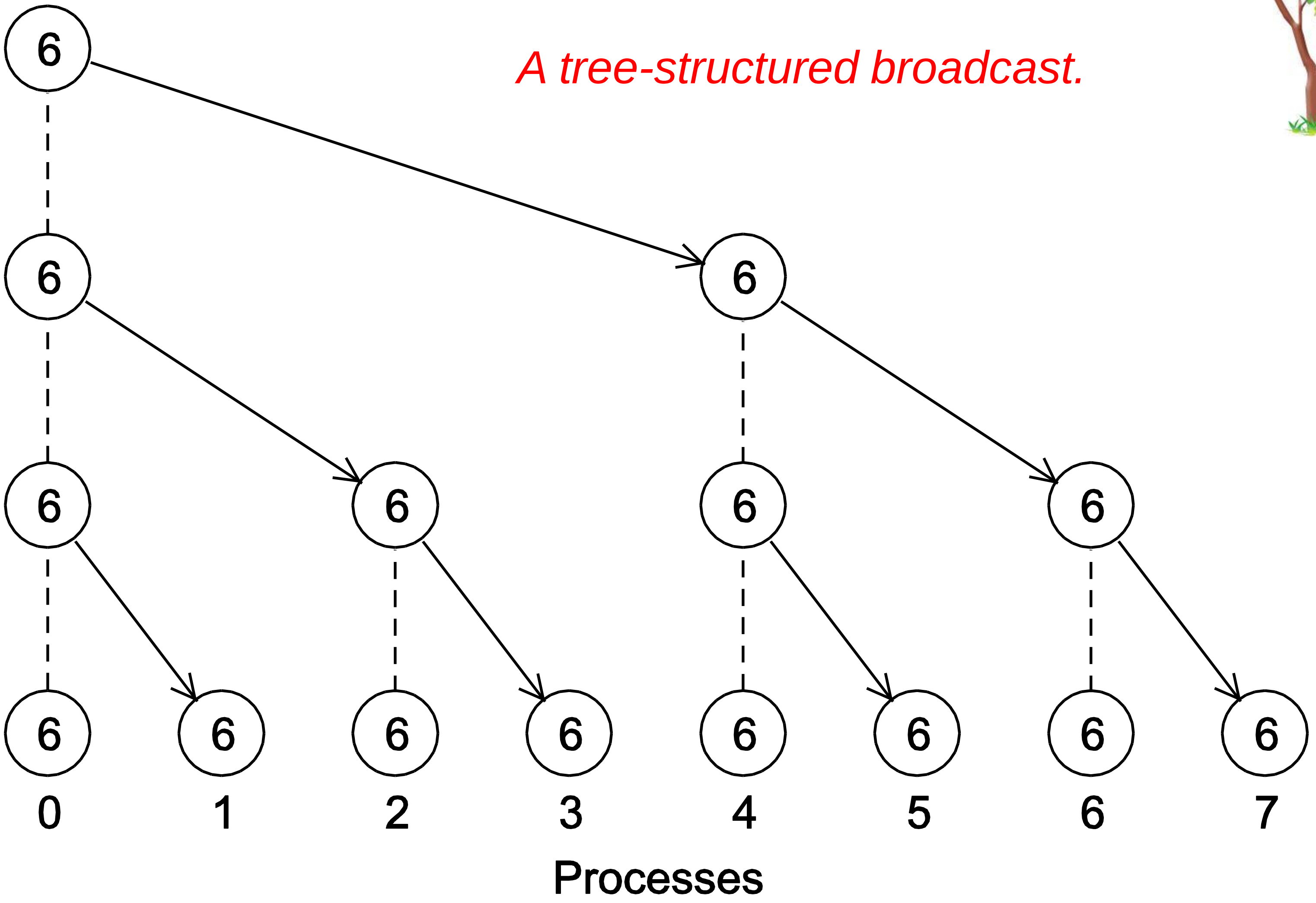
A butterfly-structured global sum.

Broadcast

- Data belonging to a single process is sent to all of the processes in the communicator.

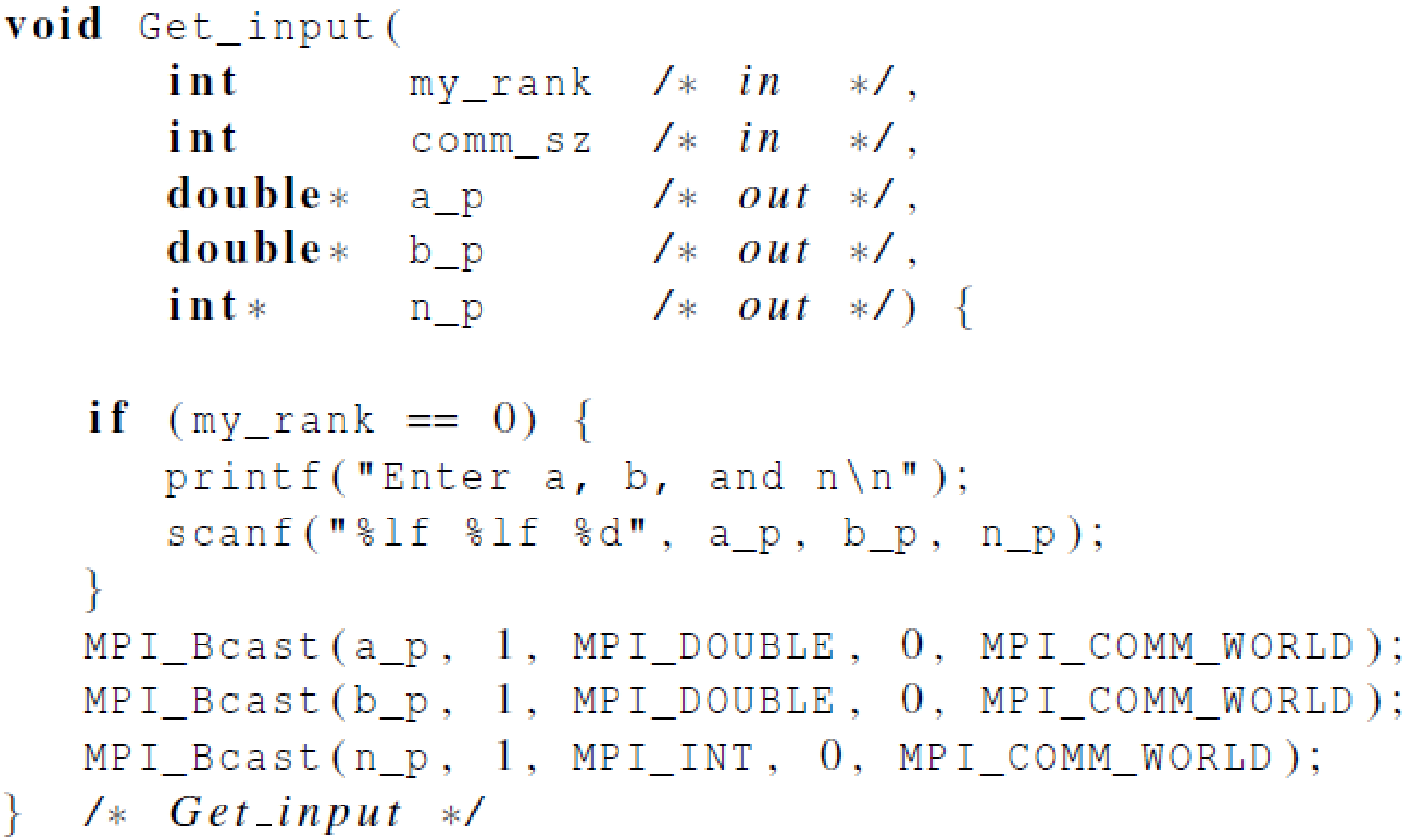


```
int MPI_Bcast(
        void*          data_p        /* in/out */,
        int            count         /* in     */,
        MPI_Datatype   datatype      /* in     */,
        int            source_proc   /* in     */,
        MPI_Comm       comm          /* in     */);
```



A tree-structured broadcast.

A version of Get_input that uses MPI_Bcast



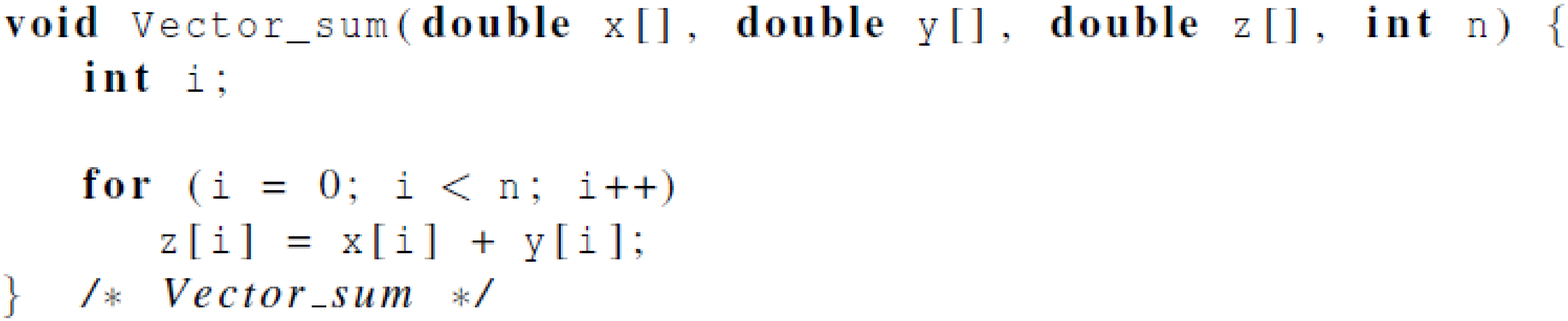
```
void Get_input(
      int      my_rank  /* in  */,
      int      comm_sz  /* in  */,
      double*  a_p      /* out */,
      double*  b_p      /* out */,
      int*     n_p      /* out */) {

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);
   }
   MPI_Bcast(a_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(b_p, 1, MPI_DOUBLE, 0, MPI_COMM_WORLD);
   MPI_Bcast(n_p, 1, MPI_INT, 0, MPI_COMM_WORLD);
}  /* Get_input */
```

Data distributions

$$\begin{aligned}
\mathbf{x}+\mathbf{y} &= (x_0, x_1, \ldots, x_{n-1}) + (y_0, y_1, \ldots, y_{n-1}) \\
&= (x_0+y_0, x_1+y_1, \ldots, x_{n-1}+y_{n-1}) \\
&= (z_0, z_1, \ldots, z_{n-1}) \\
&= \mathbf{z}
\end{aligned}$$

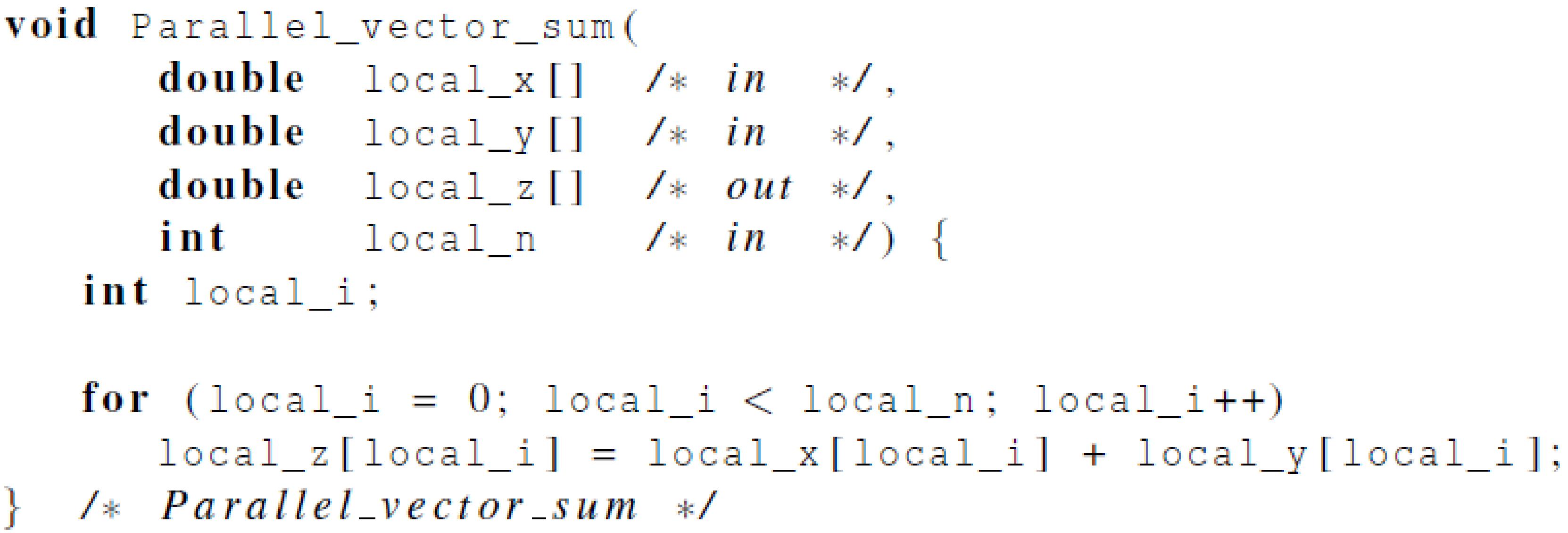
Compute a vector sum.

Serial implementation of vector addition



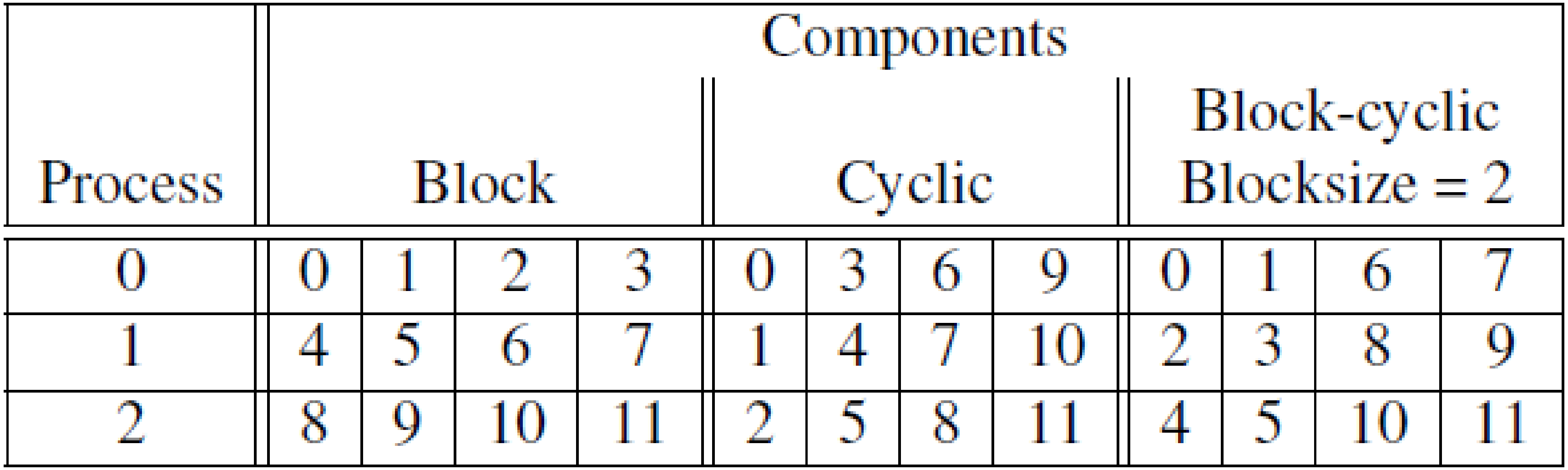
```
void Vector_sum(double x[], double y[], double z[], int n) {
   int i;

   for (i = 0; i < n; i++)
      z[i] = x[i] + y[i];
}  /* Vector_sum */
```

Parallel implementation of vector addition



```
void Parallel_vector_sum(
      double  local_x[]  /* in  */,
      double  local_y[]  /* in  */,
      double  local_z[]  /* out */,
      int     local_n    /* in  */) {
   int local_i;

   for (local_i = 0; local_i < local_n; local_i++)
      local_z[local_i] = local_x[local_i] + local_y[local_i];
}  /* Parallel_vector_sum */
```

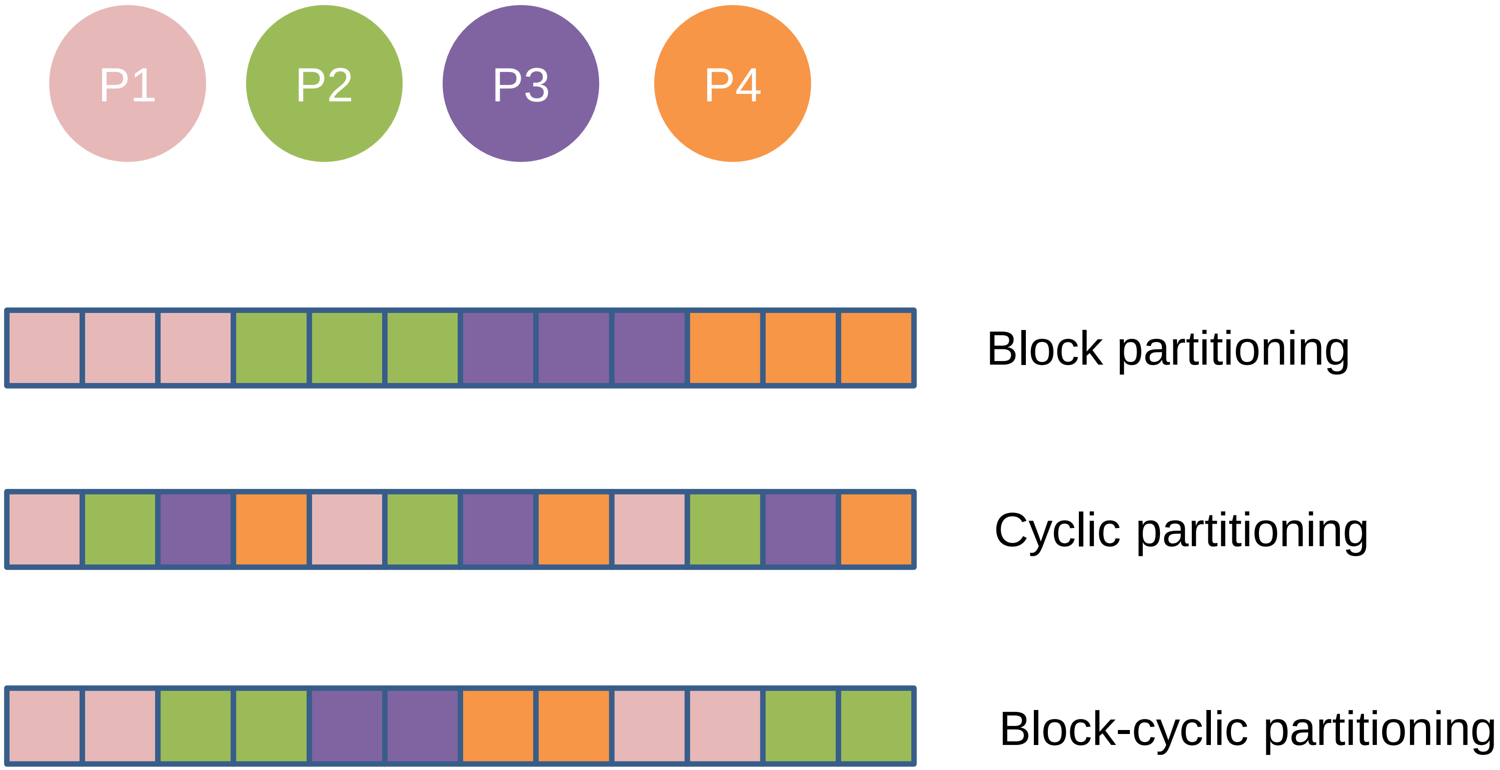
Different partitions of a 12-component vector among 3 processes



Process	Components											
	Block				Cyclic				Block-cyclic Blocksize = 2			
0	0	1	2	3	0	3	6	9	0	1	6	7
1	4	5	6	7	1	4	7	10	2	3	8	9
2	8	9	10	11	2	5	8	11	4	5	10	11

Partitioning options

- Block partitioning
  - Assign blocks of consecutive components to each process.
- Cyclic partitioning
  - Assign components in a round robin fashion.
- Block-cyclic partitioning
  - Use a cyclic distribution of blocks of components.

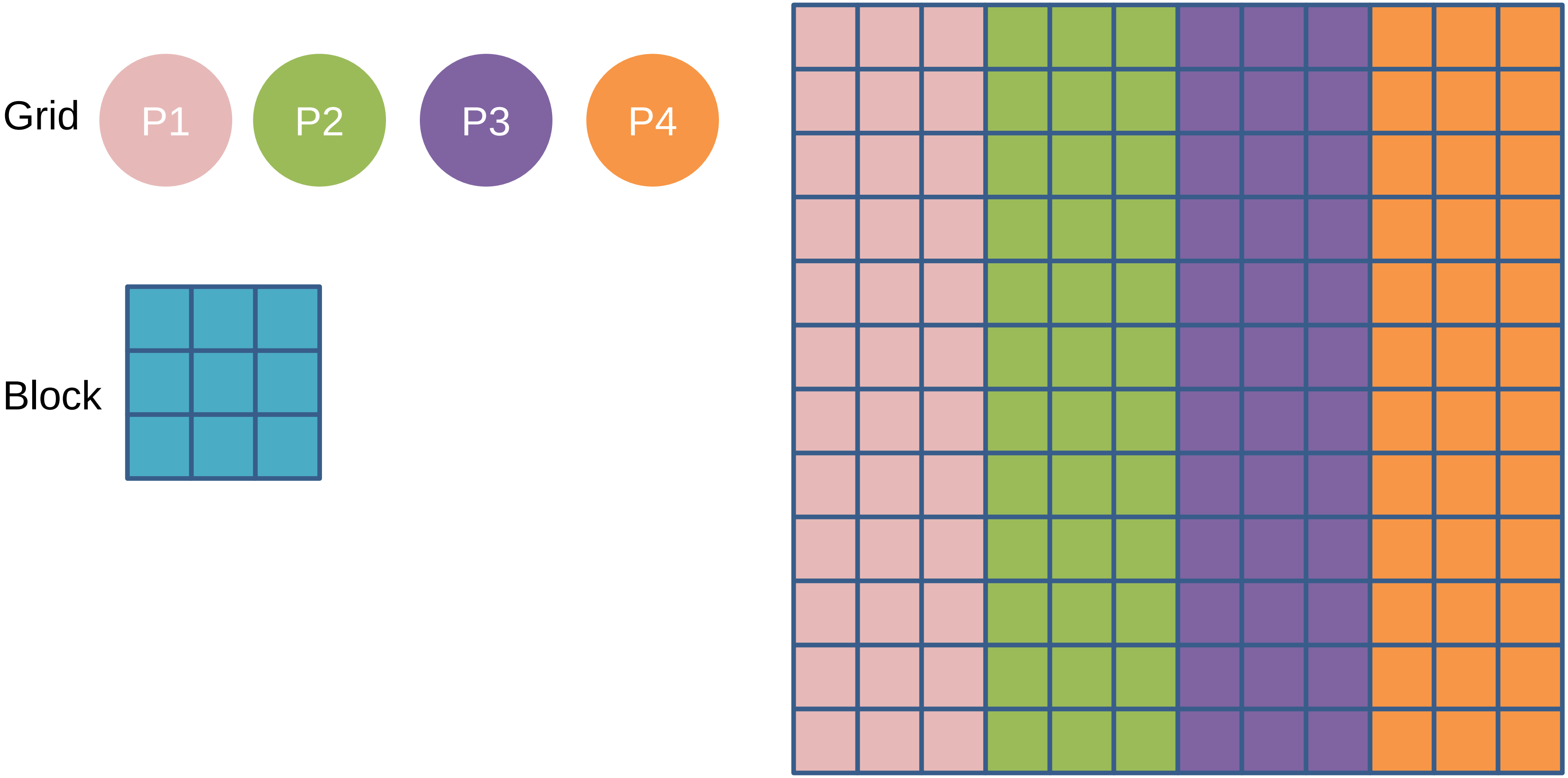


Block partitioning

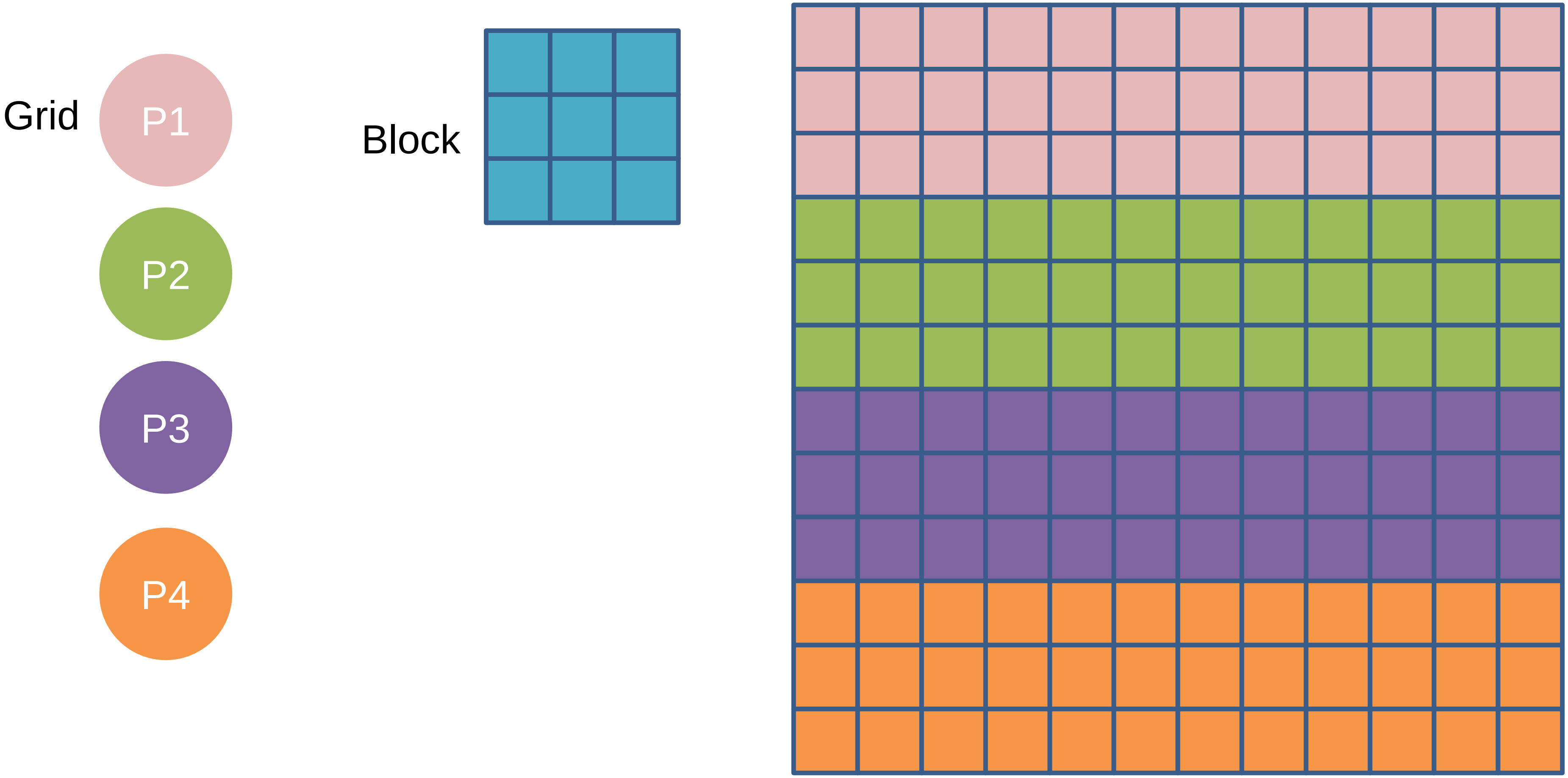
Cyclic partitioning

Block-cyclic partitioning

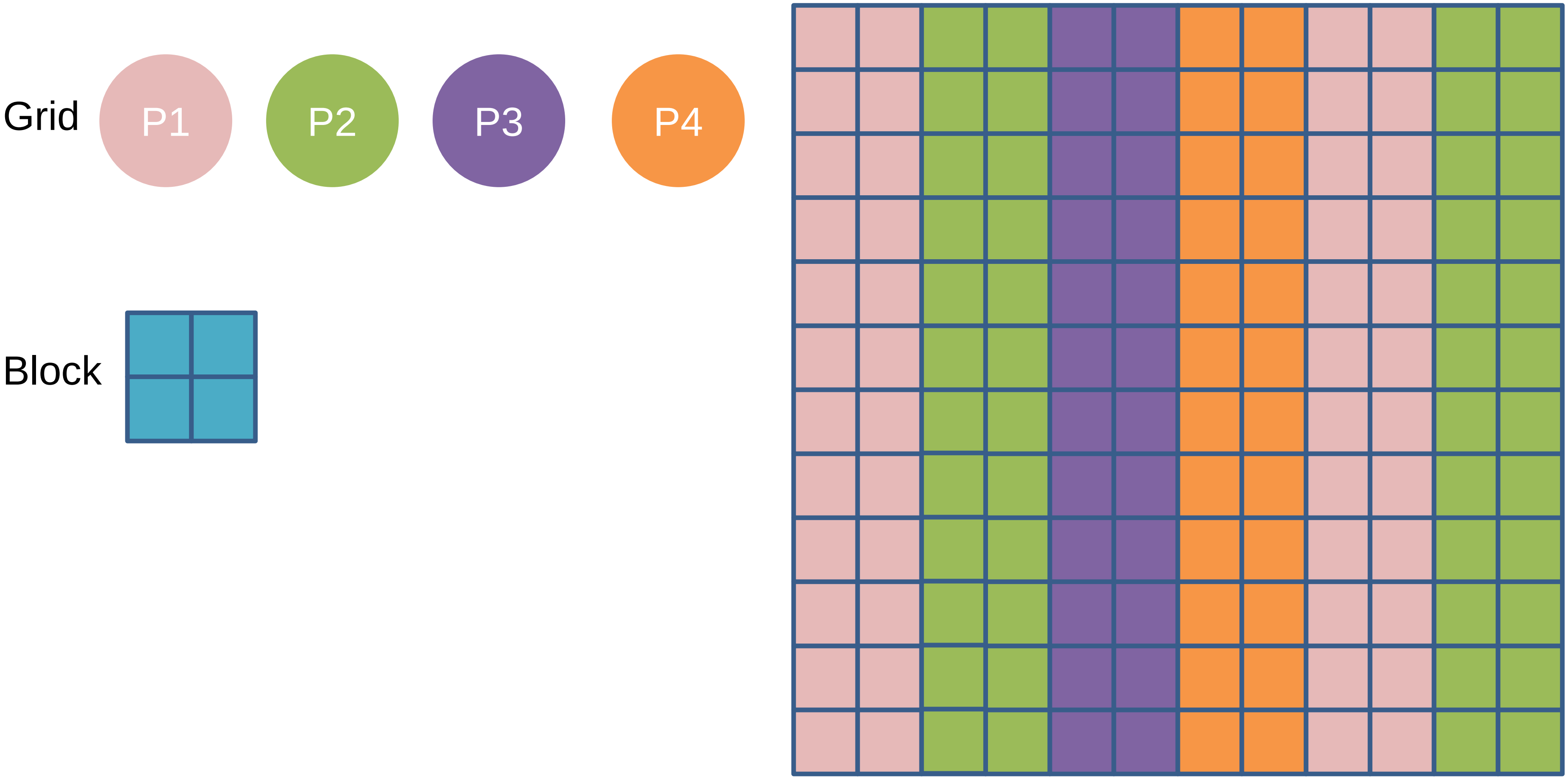
Two-Dimensional Block Cyclic Data Distribution



Two-Dimensional Block Cyclic Data Distribution

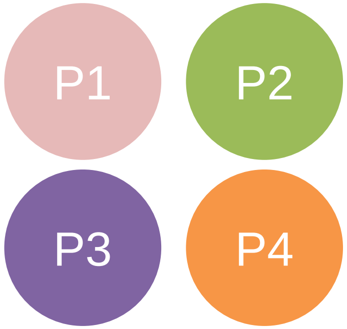


Two-Dimensional Block Cyclic Data Distribution

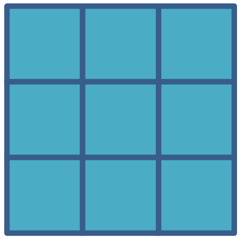


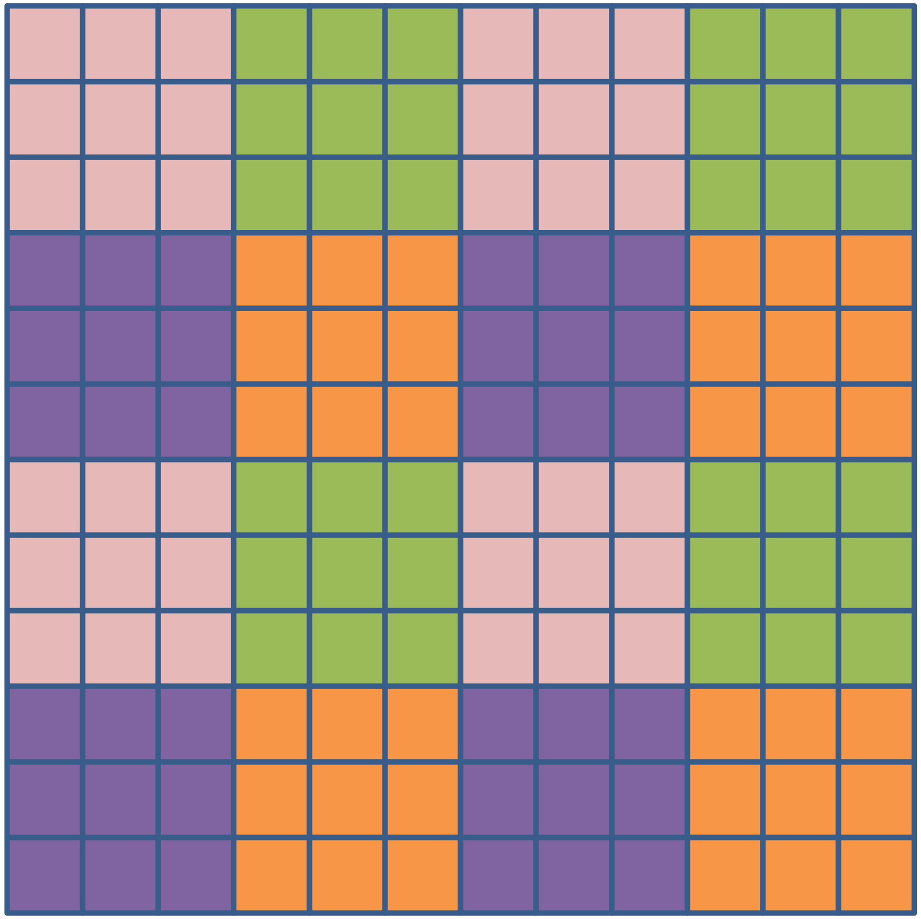
Two-Dimensional Block Cyclic Data Distribution

Grid

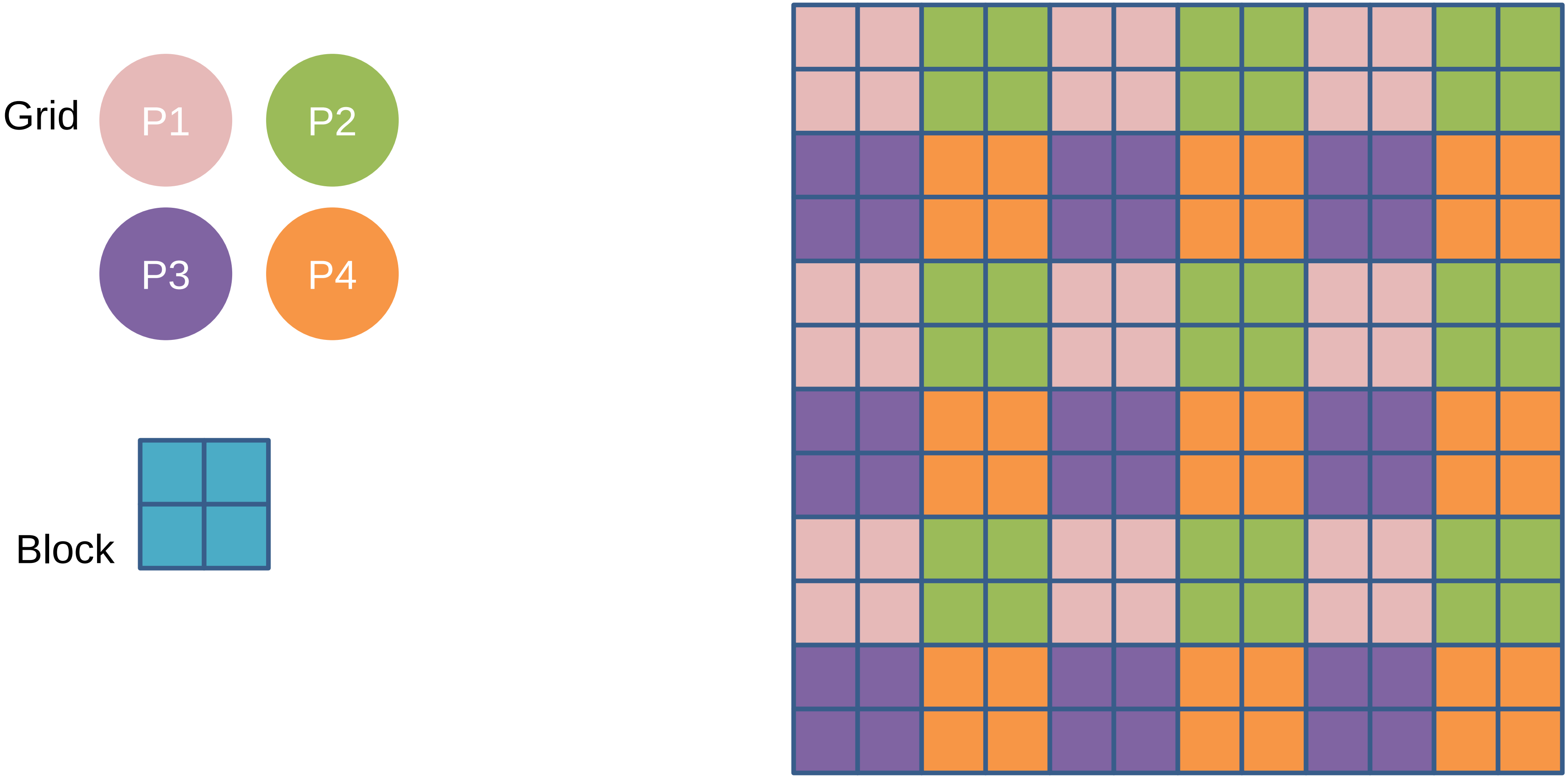


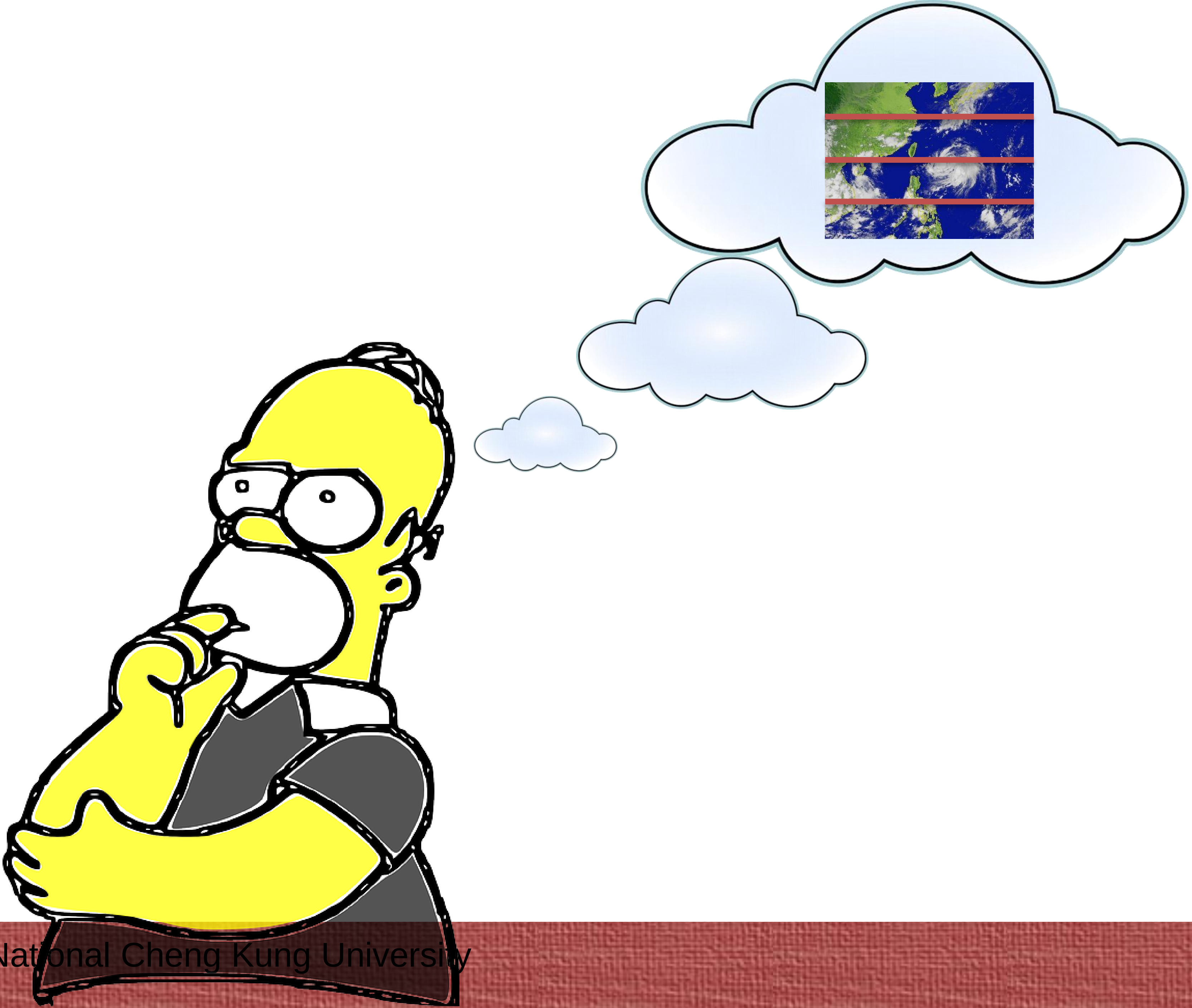
Block



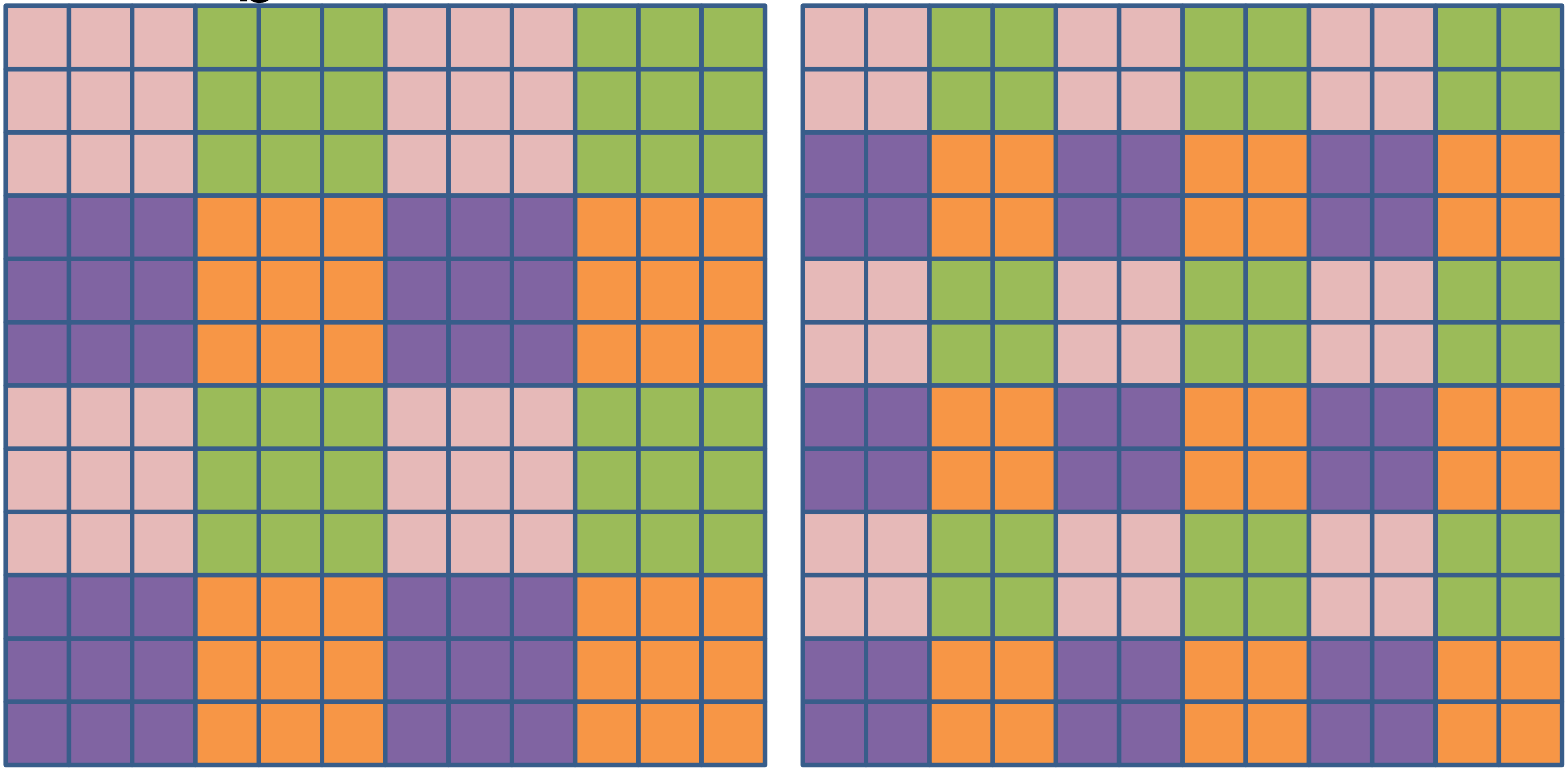


Two-Dimensional Block Cyclic Data Distribution





Message transmission time = l + n / b



$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 10 & 13 & 10 & 7 \\ 15 & 22 & 22 & 16 \\ 20 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 13 & 10 & 7 \\ 3 & 22 & 22 & 16 \\ 4 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 5 & 4 & 3 \\ 3 & 22 & 22 & 16 \\ 4 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 5 & 4 & 3 \\ 3 & 10 & 13 & 10 \\ 4 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 5 & 4 & 3 \\ 3 & 10 & 13 & 10 \\ 4 & 15 & 22 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & 2 & 13 & 10 \\ \cancel{4} & 3 & 22 & 22 \end{bmatrix}$$

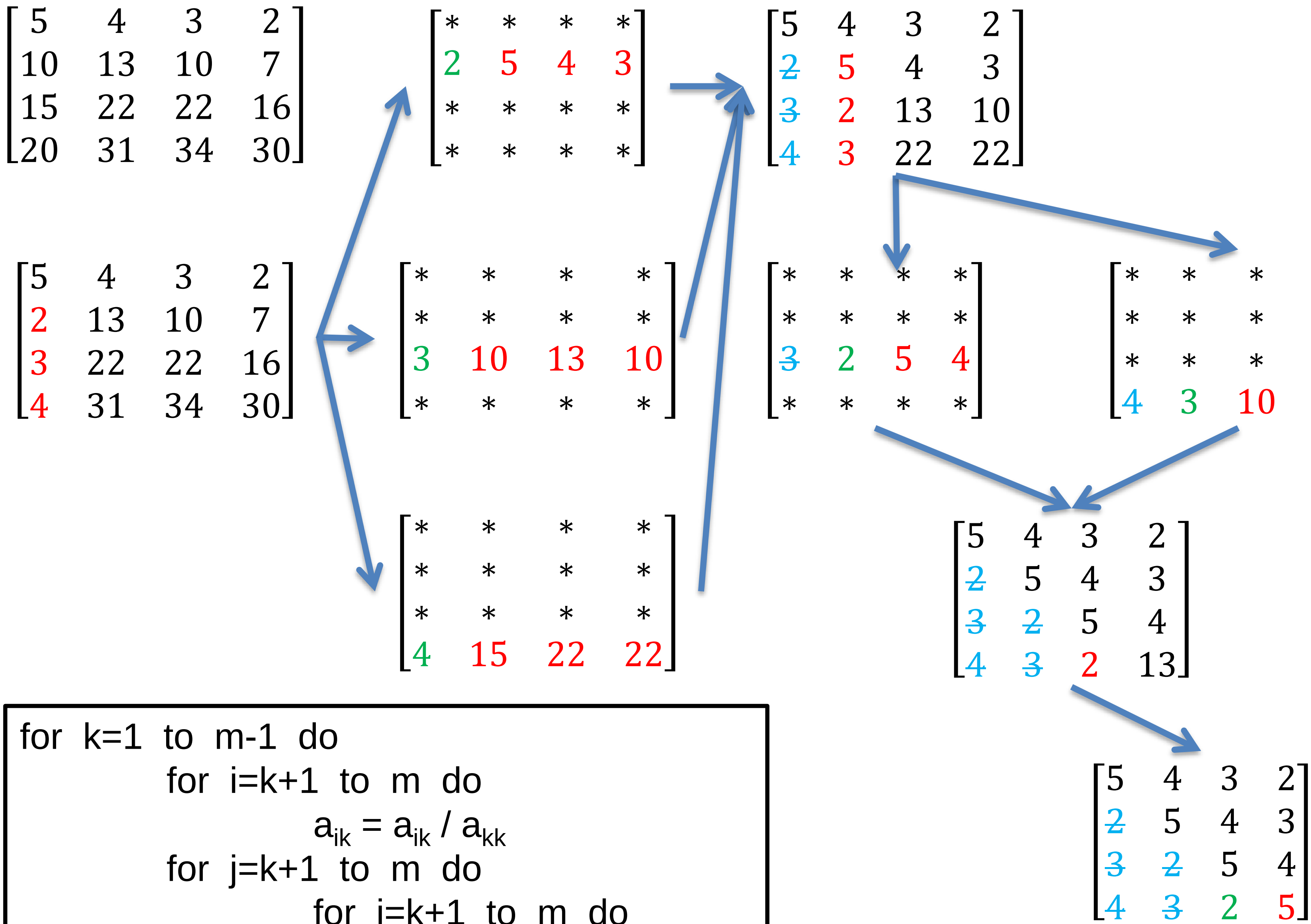
$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & 2 & 5 & 4 \\ \cancel{4} & 3 & 22 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & 2 & 5 & 4 \\ \cancel{4} & 3 & 10 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & \cancel{2} & 5 & 4 \\ \cancel{4} & \cancel{3} & 2 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & \cancel{2} & 5 & 4 \\ \cancel{4} & \cancel{3} & 2 & 5 \end{bmatrix}$$

```
for  k=1  to  m-1  do
          for  i=k+1  to  m  do
                    $a_{ik} = a_{ik} / a_{kk}$
          for  j=k+1  to  m  do
                    for  i=k+1  to  m  do
                              $a_{ij} = a_{ij} - a_{ik} * a_{kj}$
```



$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 10 & 13 & 10 & 7 \\ 15 & 22 & 22 & 16 \\ 20 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ 2 & 13 & 10 & 7 \\ 3 & 22 & 22 & 16 \\ 4 & 31 & 34 & 30 \end{bmatrix}$$

$$\begin{bmatrix} * & * & * & * \\ 2 & 5 & 4 & 3 \\ * & * & * & * \\ * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ 3 & 10 & 13 & 10 \\ * & * & * & * \end{bmatrix}$$

$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ 4 & 15 & 22 & 22 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & 2 & 13 & 10 \\ \cancel{4} & 3 & 22 & 22 \end{bmatrix}$$

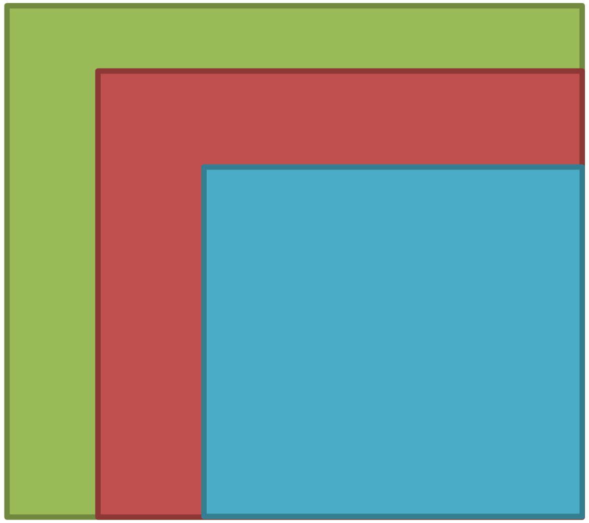
$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ \cancel{3} & 2 & 5 & 4 \\ * & * & * & * \end{bmatrix}$$

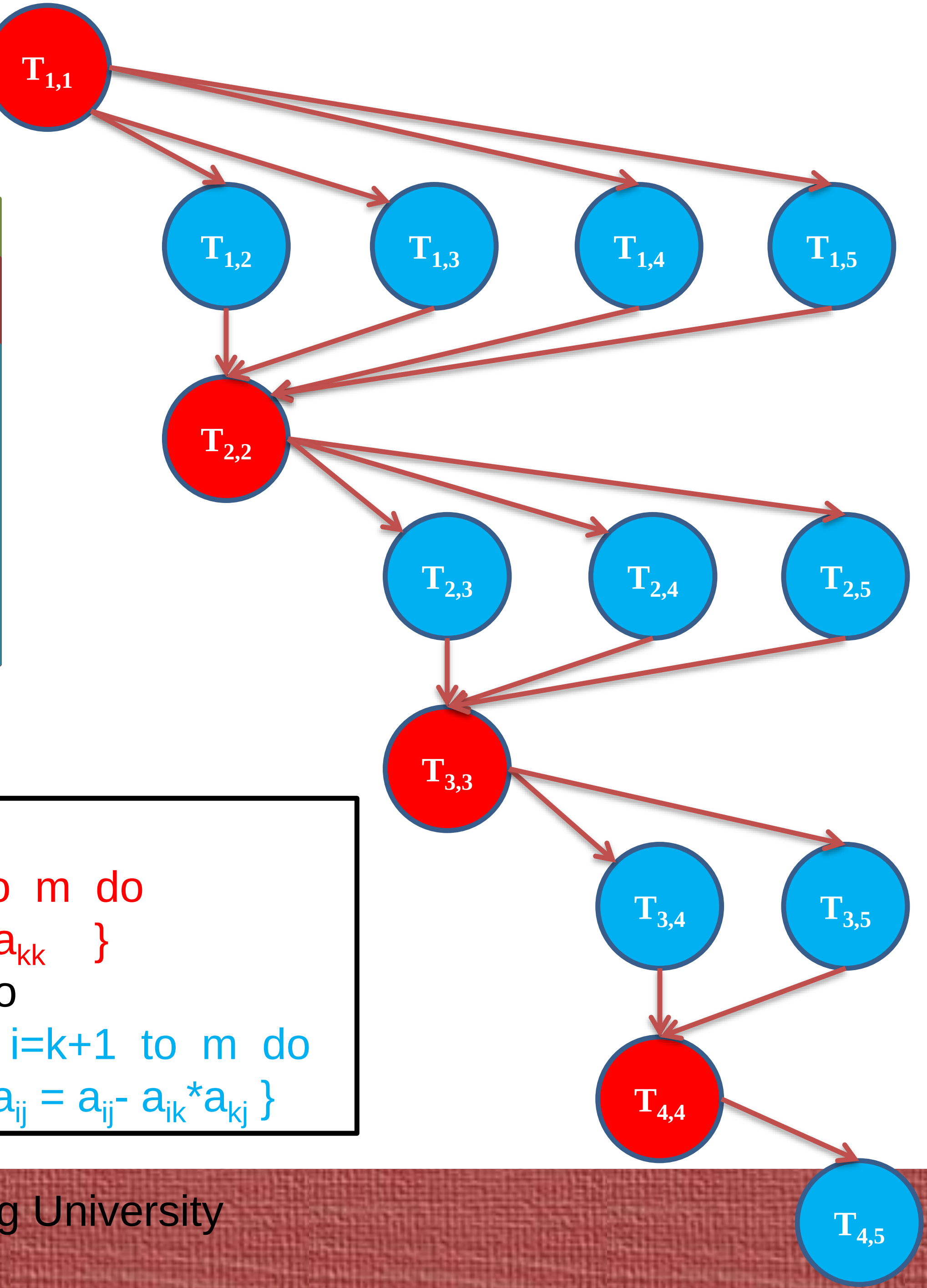
$$\begin{bmatrix} * & * & * & * \\ * & * & * & * \\ * & * & * & * \\ \cancel{4} & 3 & 10 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & \cancel{2} & 5 & 4 \\ \cancel{4} & \cancel{3} & 2 & 13 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 4 & 3 & 2 \\ \cancel{2} & 5 & 4 & 3 \\ \cancel{3} & \cancel{2} & 5 & 4 \\ \cancel{4} & \cancel{3} & 2 & 5 \end{bmatrix}$$

```
for  k=1  to  m-1  do
          for  i=k+1  to  m  do
                    $a_{ik} = a_{ik} / a_{kk}$
          for  j=k+1  to  m  do
                    for  i=k+1  to  m  do
                              $a_{ij} = a_{ij} - a_{ik} * a_{kj}$
```

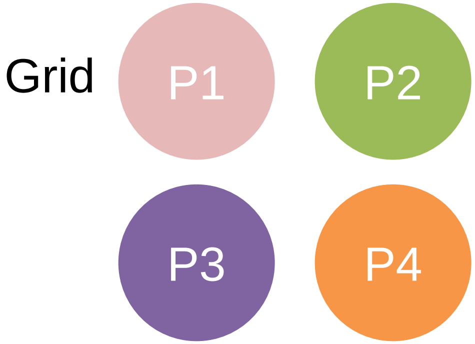




```
for  k=1  to  m-1  do
        $T_{k,k}$ : { for  i=k+1  to  m  do
                        $a_{ik} = a_{ik} / a_{kk}$    }
        for  j=k+1  to  m  do
                $T_{k,j}$ : { for  i=k+1  to  m  do
                                $a_{ij} = a_{ij} - a_{ik} * a_{kj}$ }
```

Linpack Benchmark

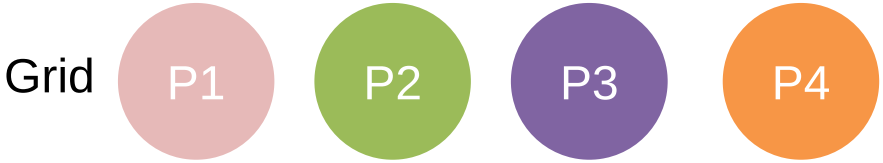
- Problems sizes
- Block sizes
- Specify the number of process rows and columns (Ps × Qs)



Grid

P1 P2

P3 P4

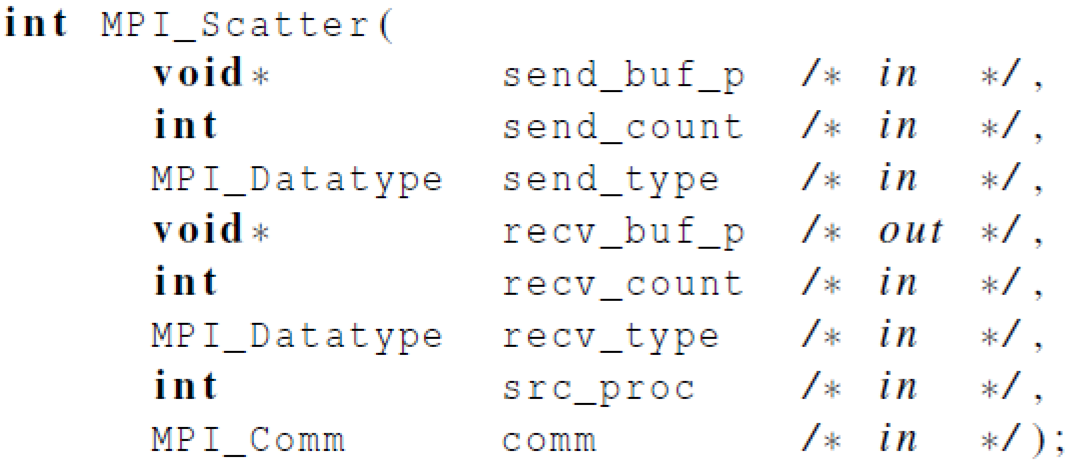


Grid

P1 P2 P3 P4

Scatter

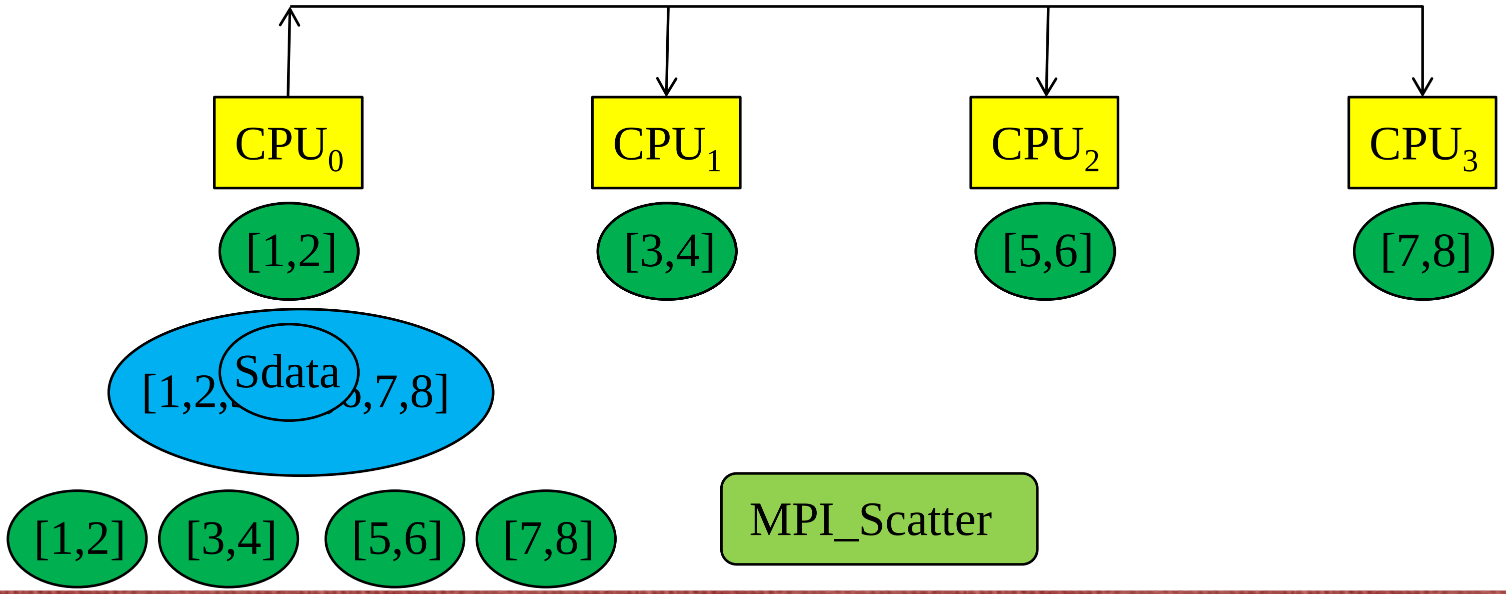
- MPI_Scatter can be used in a function that reads in an entire vector on process 0 but only sends the needed components to each of the other processes.



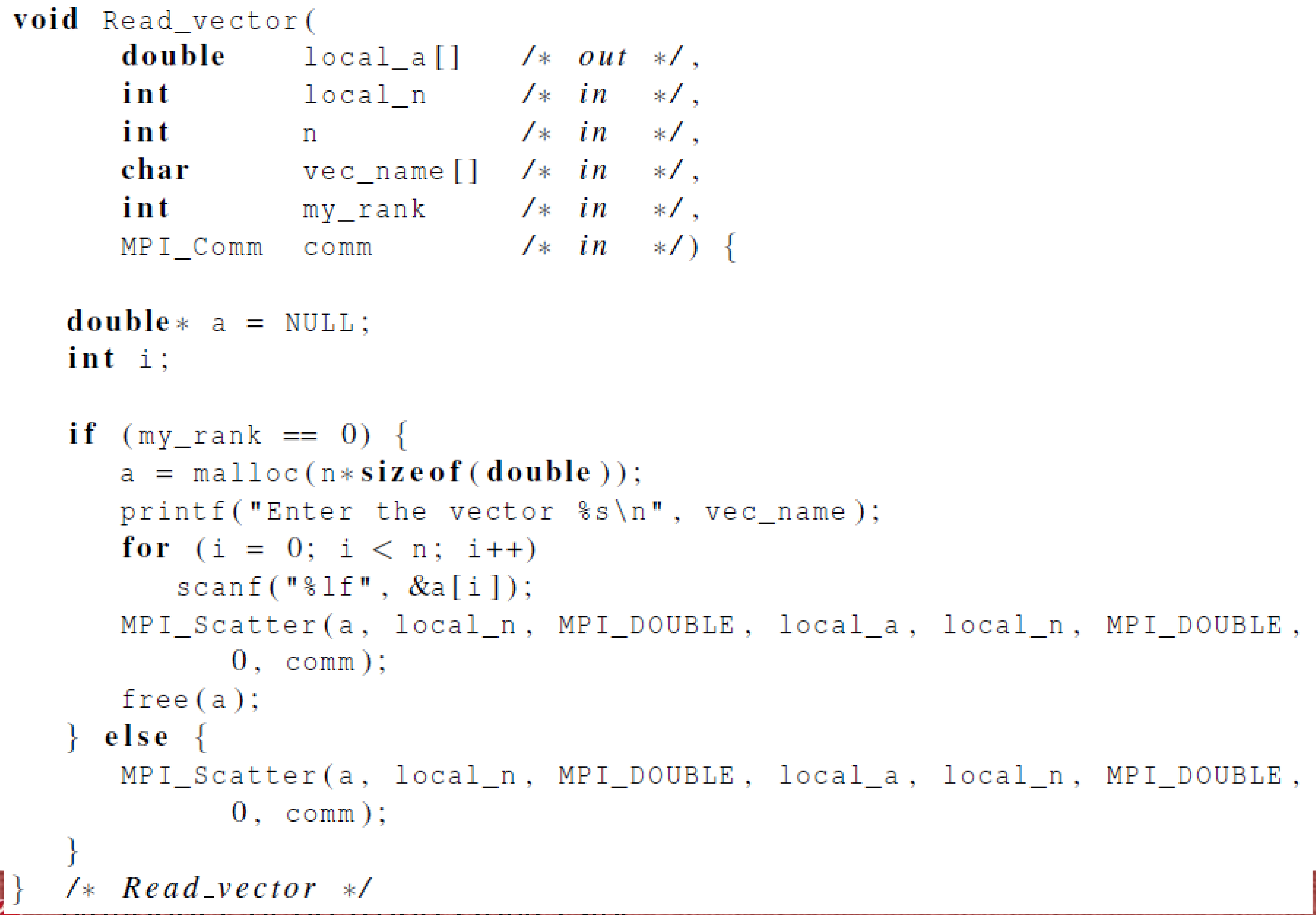
```
int MPI_Scatter(
      void*         send_buf_p  /* in   */,
      int           send_count  /* in   */,
      MPI_Datatype  send_type   /* in   */,
      void*         recv_buf_p  /* out  */,
      int           recv_count  /* in   */,
      MPI_Datatype  recv_type   /* in   */,
      int           src_proc    /* in   */,
      MPI_Comm      comm        /* in   */);
```

MPI_Scatter

int Sdata[8] = {1,2,3,4,5,6,7,8}, Rdata[2];
int Send_cnt = 2, Recv_cnt = 2, src = 0;
MPI_Scatter(Sdata, Send_cnt, MPI_INTEGER, Rdata, Recv_cnt, MPI_INTEGER , src, MPI_COMM_WORLD);



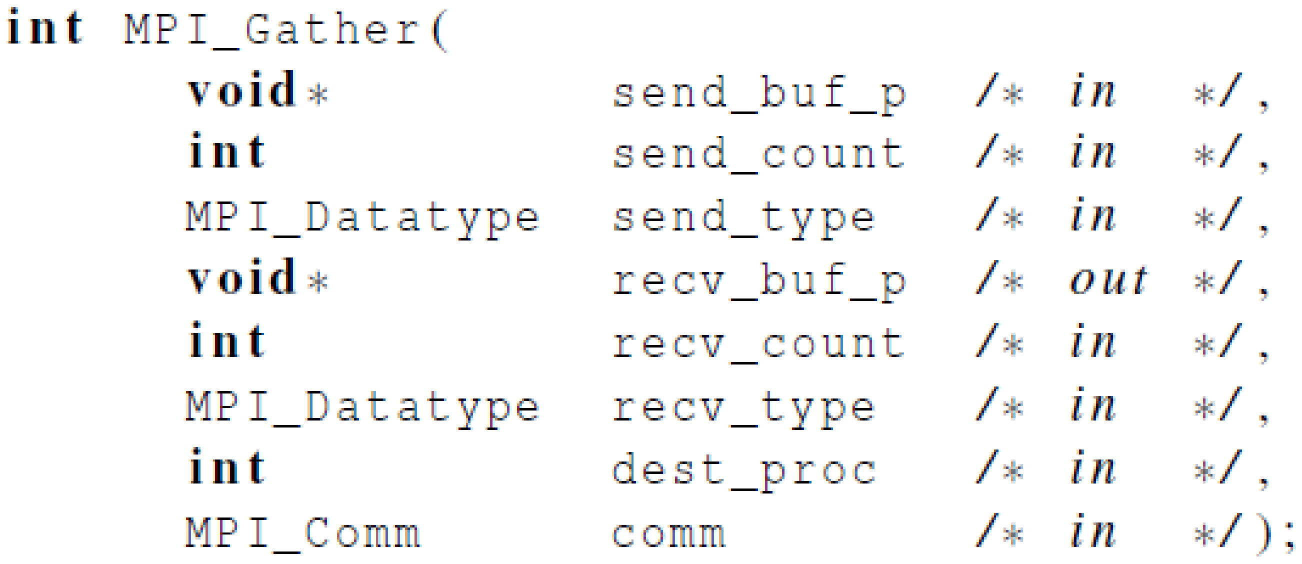
Reading and distributing a vector



```
void Read_vector(
      double    local_a[]   /* out */,
      int       local_n     /* in  */,
      int       n           /* in  */,
      char      vec_name[]  /* in  */,
      int       my_rank     /* in  */,
      MPI_Comm  comm        /* in  */) {

   double* a = NULL;
   int i;

   if (my_rank == 0) {
      a = malloc(n*sizeof(double));
      printf("Enter the vector %s\n", vec_name);
      for (i = 0; i < n; i++)
         scanf("%lf", &a[i]);
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
      free(a);
   } else {
      MPI_Scatter(a, local_n, MPI_DOUBLE, local_a, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Read_vector */
```

Gather

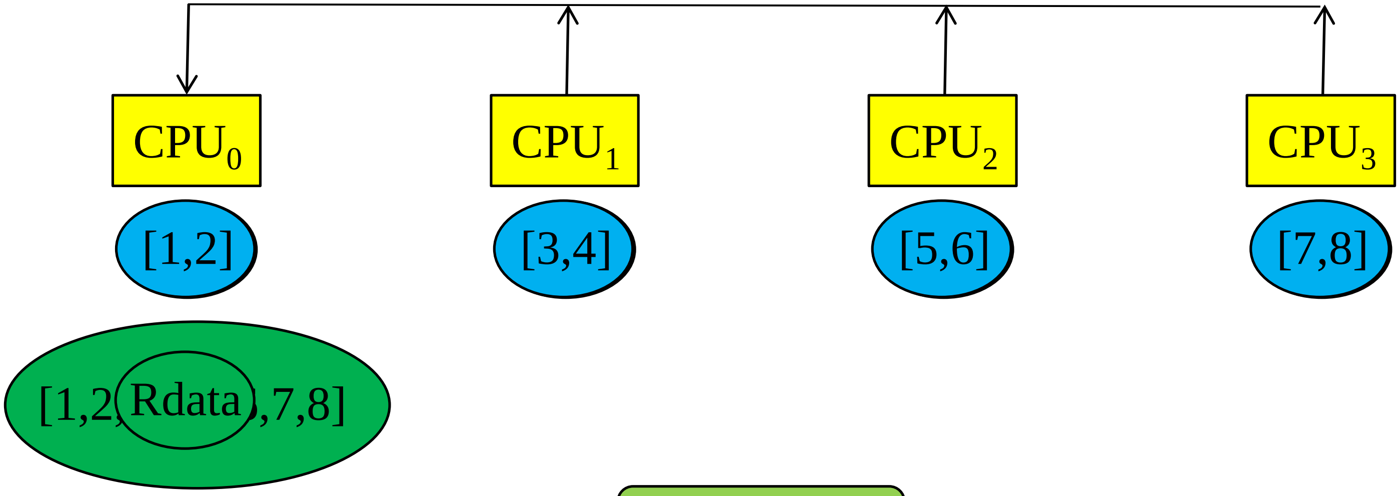
- Collect all of the components of the vector onto process 0, and then process 0 can process all of the components.



```
int MPI_Gather(
      void*          send_buf_p   /* in   */,
      int            send_count   /* in   */,
      MPI_Datatype   send_type    /* in   */,
      void*          recv_buf_p   /* out  */,
      int            recv_count   /* in   */,
      MPI_Datatype   recv_type    /* in   */,
      int            dest_proc    /* in   */,
      MPI_Comm       comm         /* in   */);
```

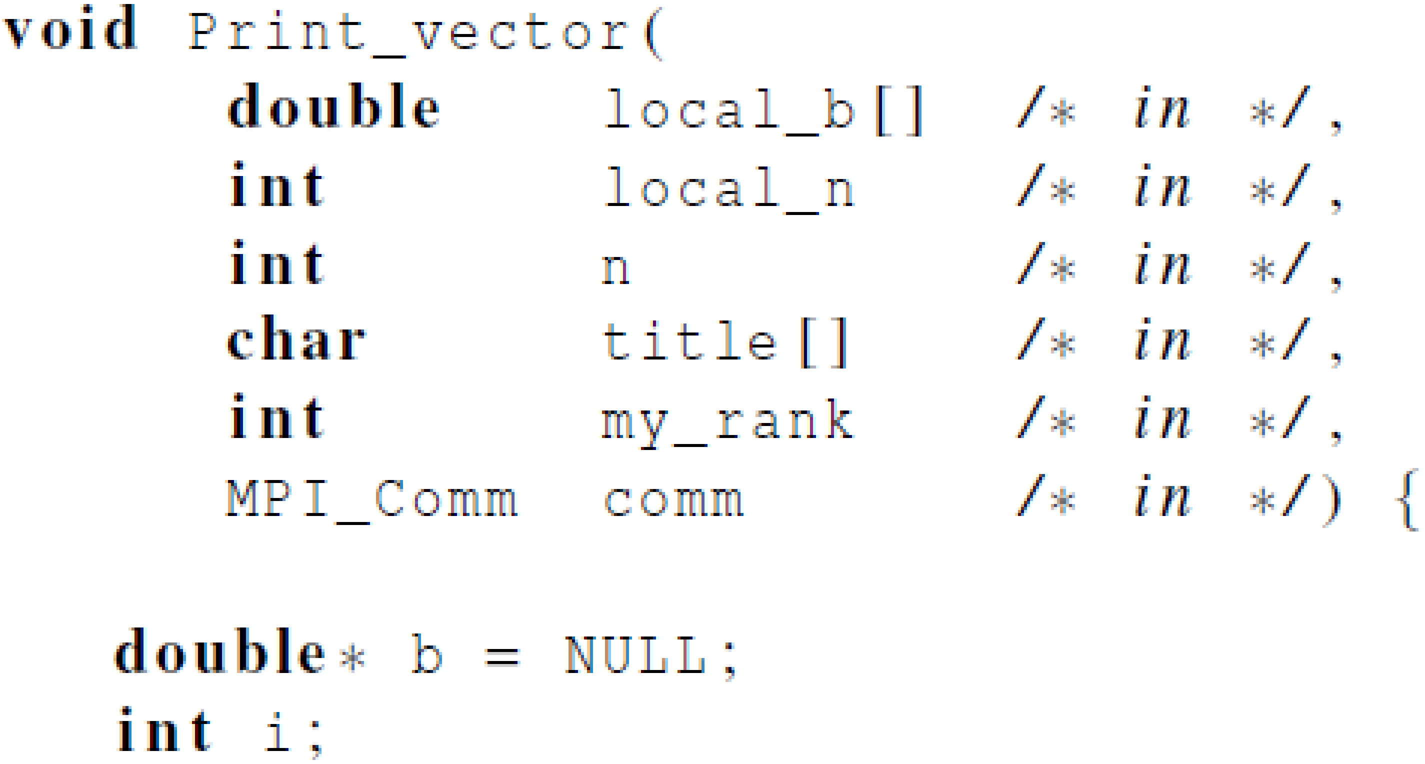
MPI_Gather

```
int Send_cnt = 2, Recv_cnt = 2, dest = 0;
MPI_Gather ( Sdata, Send_cnt, MPI_INTEGER, Rdata, Recv_cnt,
MPI_INTEGER , dest, MPI_COMM_WORLD);
```



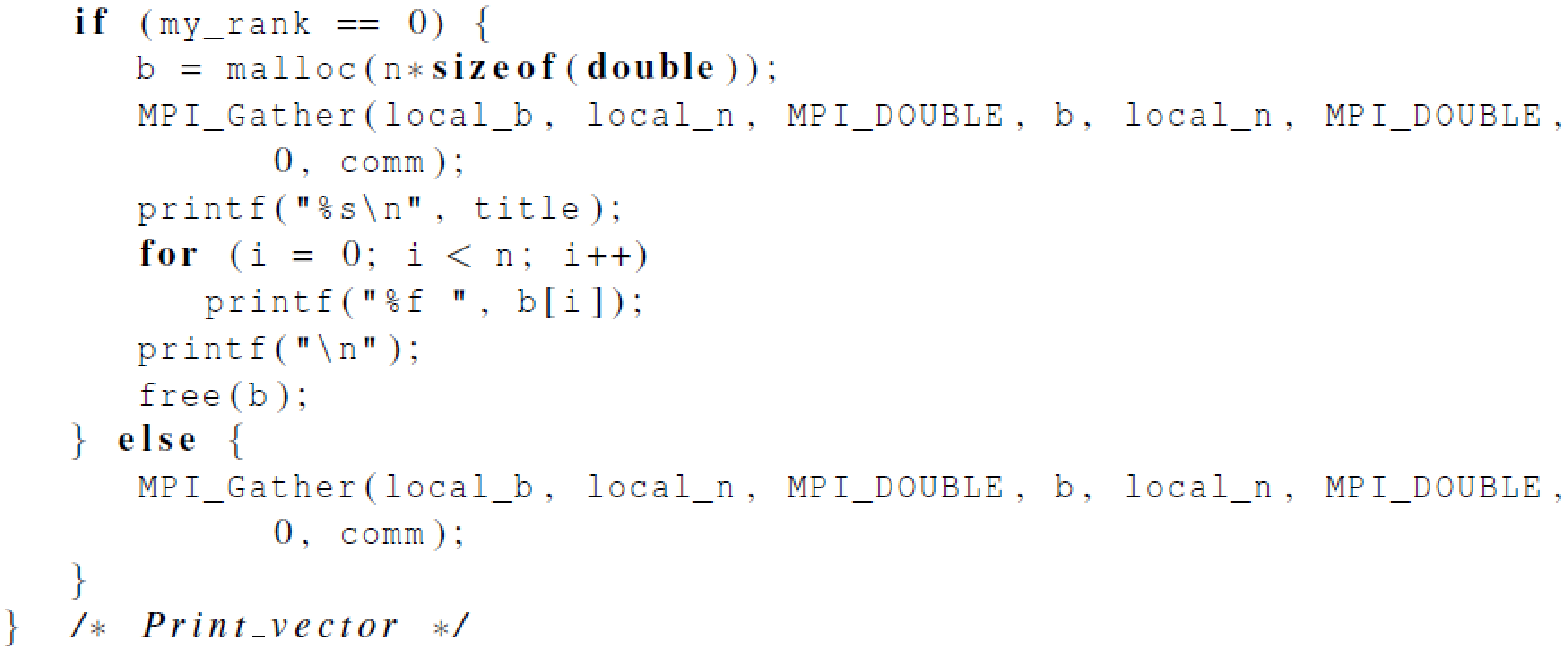
MPI_Gather

Print a distributed vector (1)



```
void Print_vector(
      double    local_b[]  /* in */,
      int       local_n    /* in */,
      int       n          /* in */,
      char      title[]    /* in */,
      int       my_rank    /* in */,
      MPI_Comm  comm       /* in */) {

   double* b = NULL;
   int i;
```

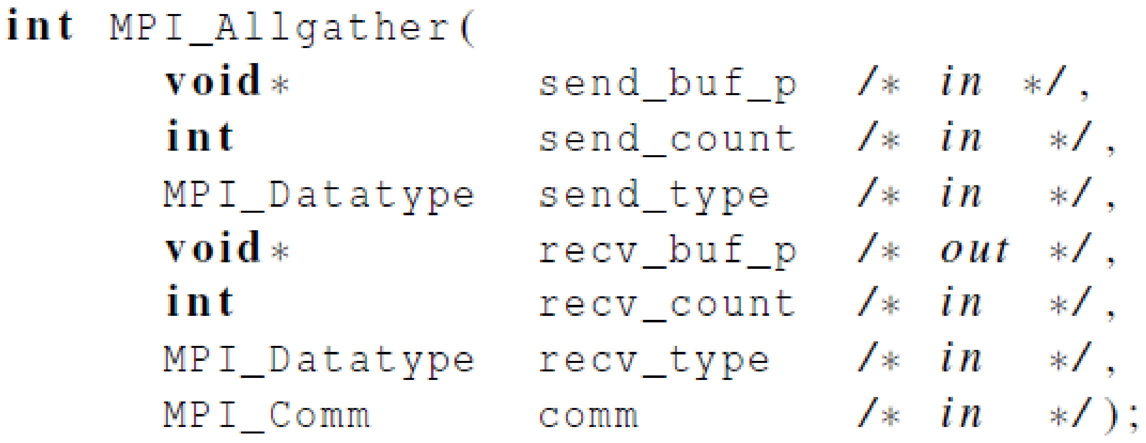
Print a distributed vector (2)



```
   if (my_rank == 0) {
      b = malloc(n*sizeof(double));
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
      printf("%s\n", title);
      for (i = 0; i < n; i++)
         printf("%f ", b[i]);
      printf("\n");
      free(b);
   } else {
      MPI_Gather(local_b, local_n, MPI_DOUBLE, b, local_n, MPI_DOUBLE,
            0, comm);
   }
}  /* Print_vector */
```

Allgather

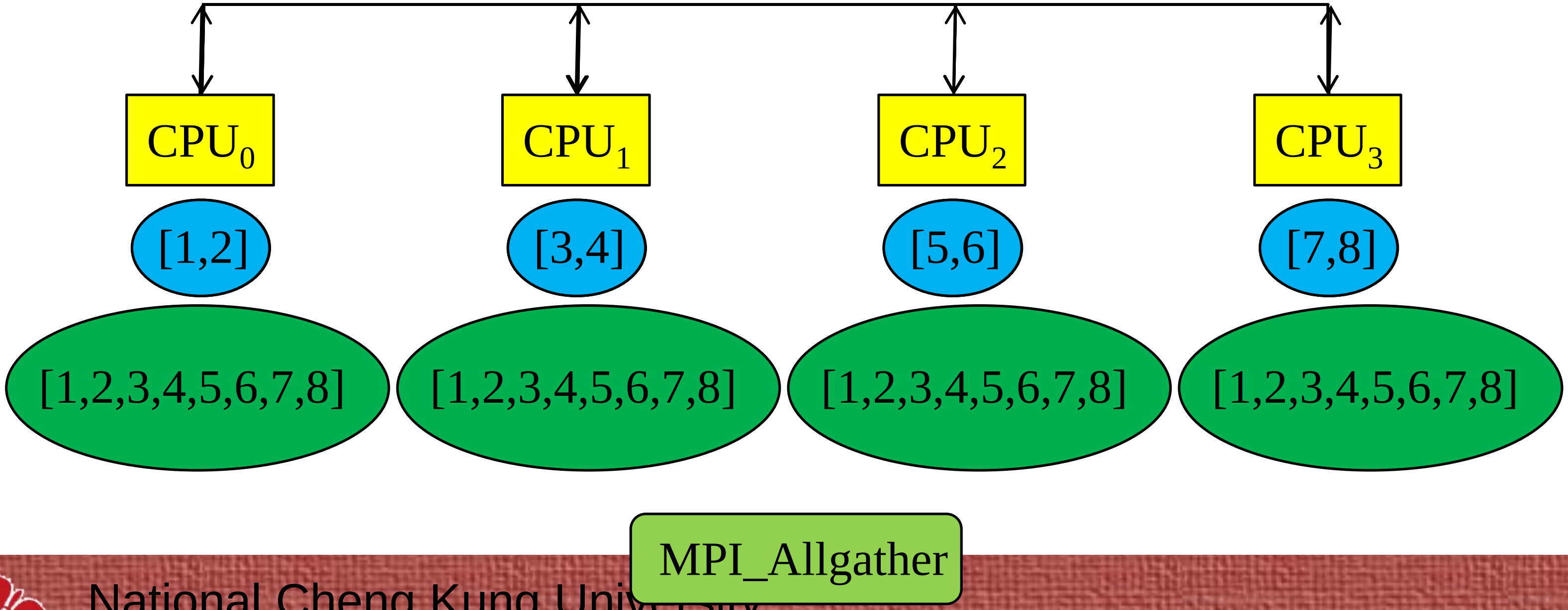
- Concatenates the contents of each process' send_buf_p and stores this in each process' recv_buf_p.
- As usual, recv_count is the amount of data being received from each process.



```
int MPI_Allgather(
      void*          send_buf_p   /* in  */,
      int            send_count   /* in  */,
      MPI_Datatype   send_type    /* in  */,
      void*          recv_buf_p   /* out */,
      int            recv_count   /* in  */,
      MPI_Datatype   recv_type    /* in  */,
      MPI_Comm       comm         /* in  */);
```

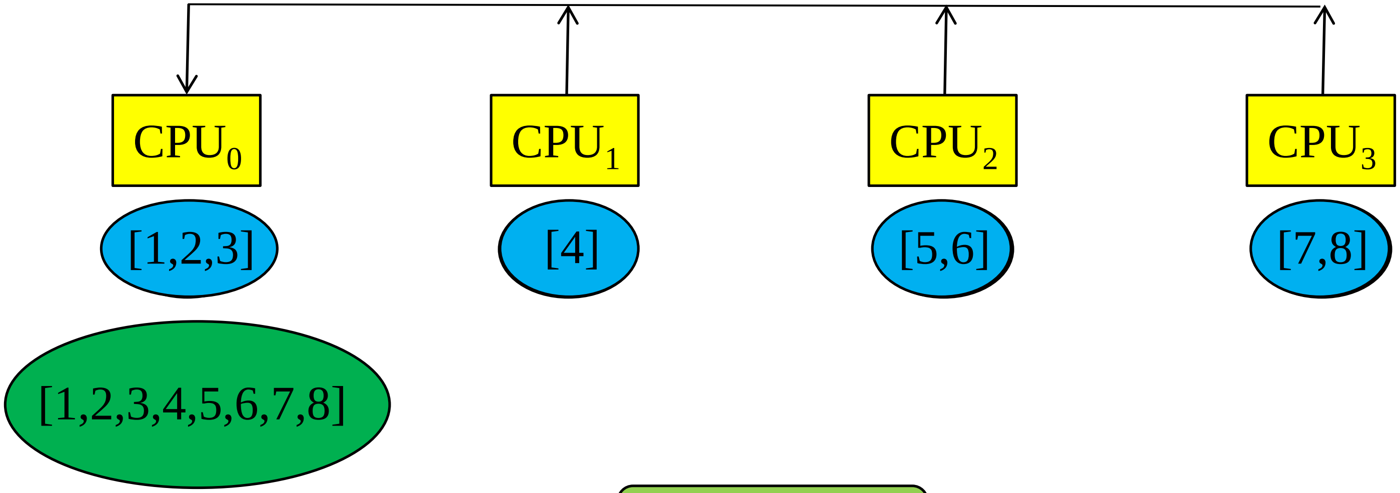
MPI_Allgather

int Send_cnt = 2, Recv_cnt = 8;
MPI_Allgather (Sdata, Send_cnt, MPI_INTEGER, Rdata, Recv_cnt, MPI_INTEGER , MPI_COMM_WORLD);



MPI_Gatherv

int dest = 0, Send_cnt = sizeof(Sbuf);
int rc[4] = {3,1,2,2}, disp[4] = {0,3,4,6};
MPI_Gatherv (Sbuf, Send_cnt, MPI_INTEGER, Rbuf, rc, disp, MPI_INTEGER , dest, MPI_COMM_WORLD);



MPI_Gatherv

MPI_Scatterv

- int MPI_Scatterv(const void *sendbuf, const int *sendcounts, const int *displs, MPI_Datatype sendtype, void *recvbuf, int recvcount, MPI_Datatype recvtype, int root, MPI_Comm comm)

Matrix-vector multiplication

$A = (a_{ij})$ is an $m \times n$ matrix

$\mathbf{x}$ is a vector with n components

$\mathbf{y} = A\mathbf{x}$ is a vector with m components

$$y_i = a_{i0}x_0 + a_{i1}x_1 + a_{i2}x_2 + \cdots a_{i,n-1}x_{n-1}$$

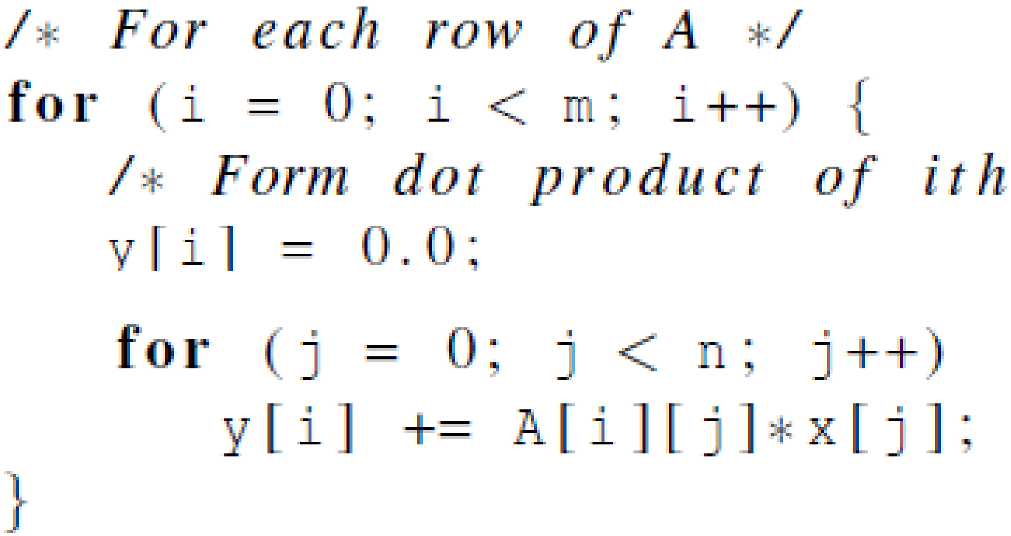
i-th component of y

Dot product of the ith row of A with x.

Matrix-vector multiplication

$$
\begin{array}{|c|c|c|c|}
\hline
a_{00} & a_{01} & \cdots & a_{0,n-1} \\
\hline
a_{10} & a_{11} & \cdots & a_{1,n-1} \\
\hline
\vdots & \vdots & & \vdots \\
\hline
a_{i0} & a_{i1} & \cdots & a_{i,n-1} \\
\hline
\vdots & \vdots & & \vdots \\
\hline
a_{m-1,0} & a_{m-1,1} & \cdots & a_{m-1,n-1} \\
\hline
\end{array}
\begin{array}{|c|}
\hline
x_0 \\
\hline
x_1 \\
\hline
\vdots \\
\hline
x_{n-1} \\
\hline
\end{array}
=
\begin{array}{|c|}
\hline
y_0 \\
\hline
y_1 \\
\hline
\vdots \\
\hline
y_i = a_{i0}x_0 + a_{i1}x_1 + \cdots a_{i,n-1}x_{n-1} \\
\hline
\vdots \\
\hline
y_{m-1} \\
\hline
\end{array}
$$

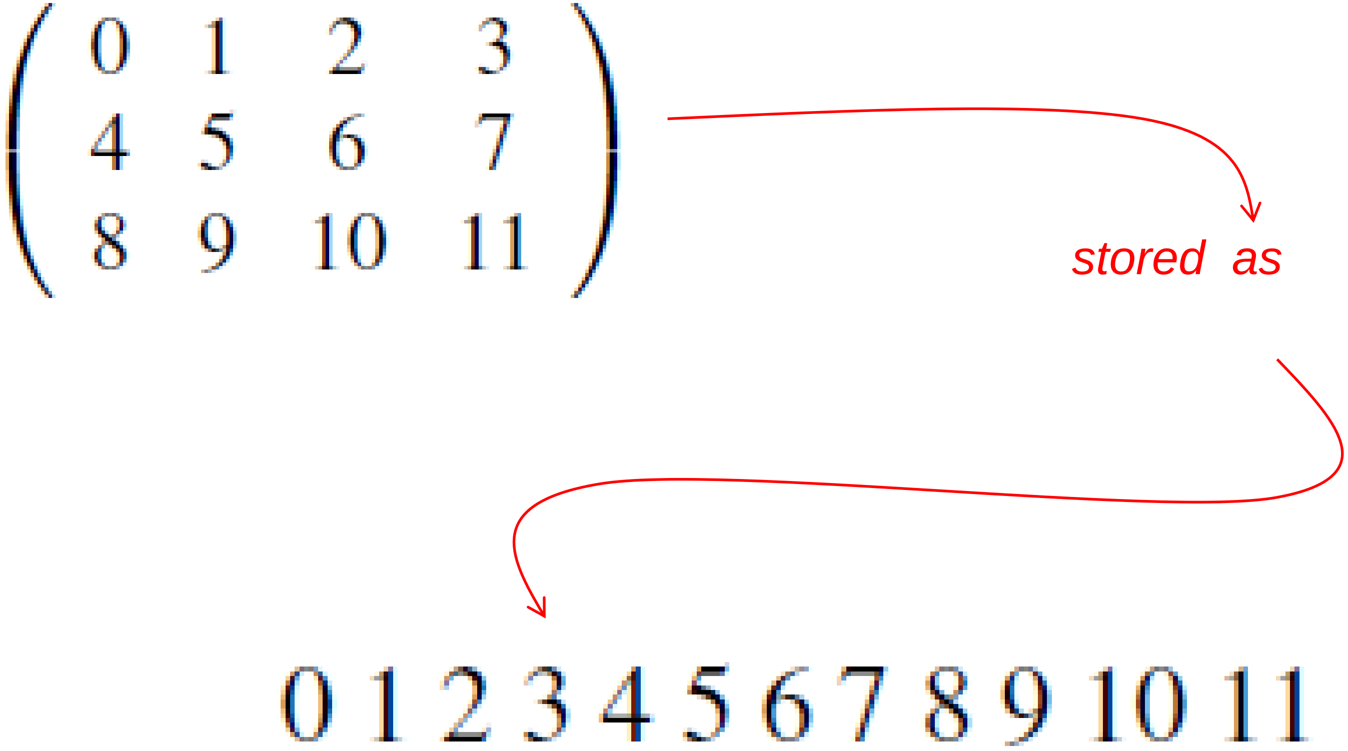
Multiply a matrix by a vector



```
/* For each row of A */
for (i = 0; i < m; i++) {
   /* Form dot product of ith row with x */
   y[i] = 0.0;
   for (j = 0; j < n; j++)
      y[i] += A[i][j]*x[j];
}
```

Serial pseudo-code

C style arrays

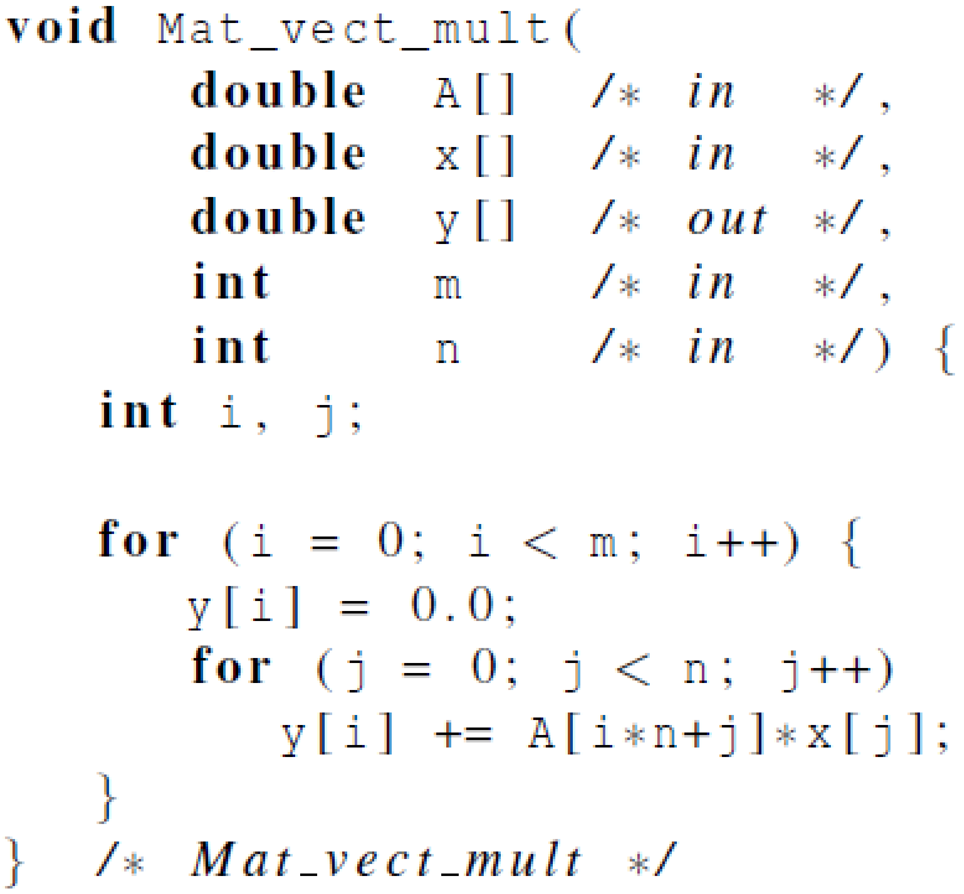


$$\begin{pmatrix} 0 & 1 & 2 & 3 \\ 4 & 5 & 6 & 7 \\ 8 & 9 & 10 & 11 \end{pmatrix}$$

stored as

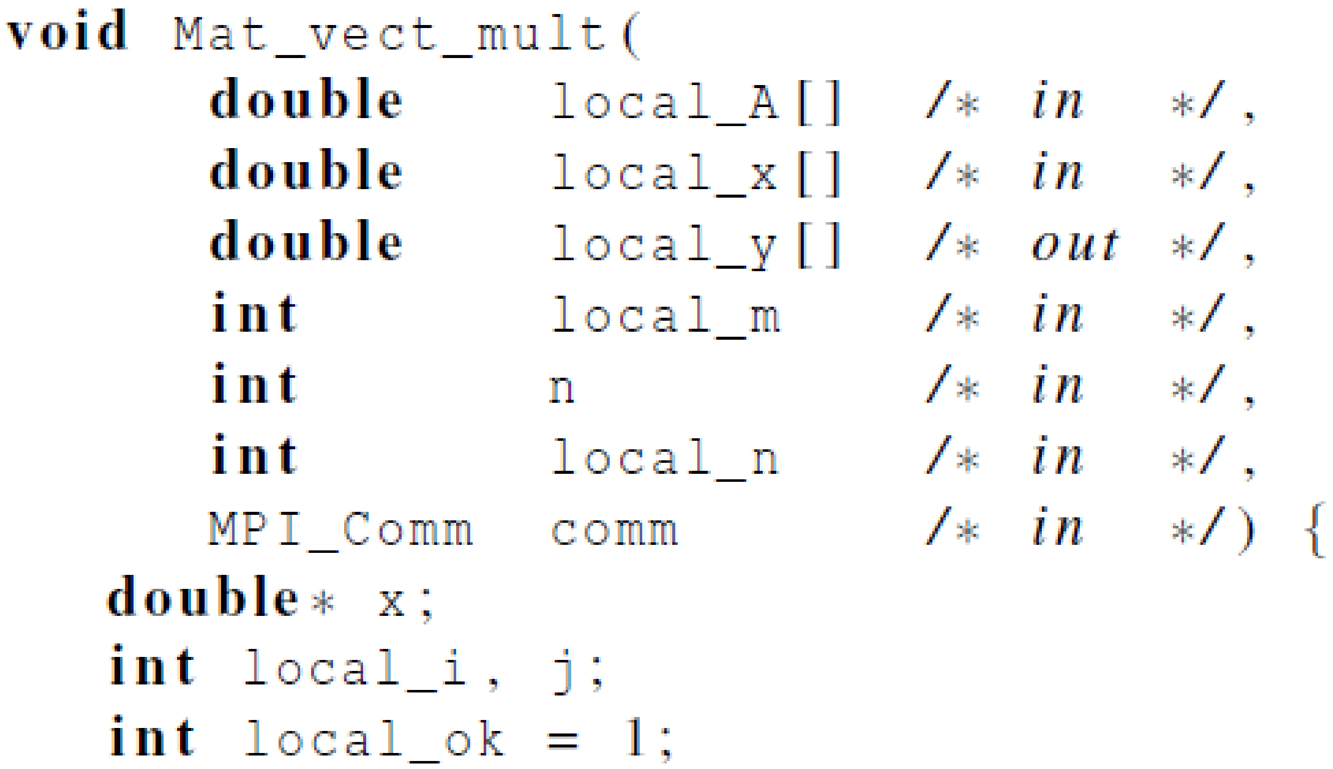
0 1 2 3 4 5 6 7 8 9 10 11

Serial matrix-vector multiplication



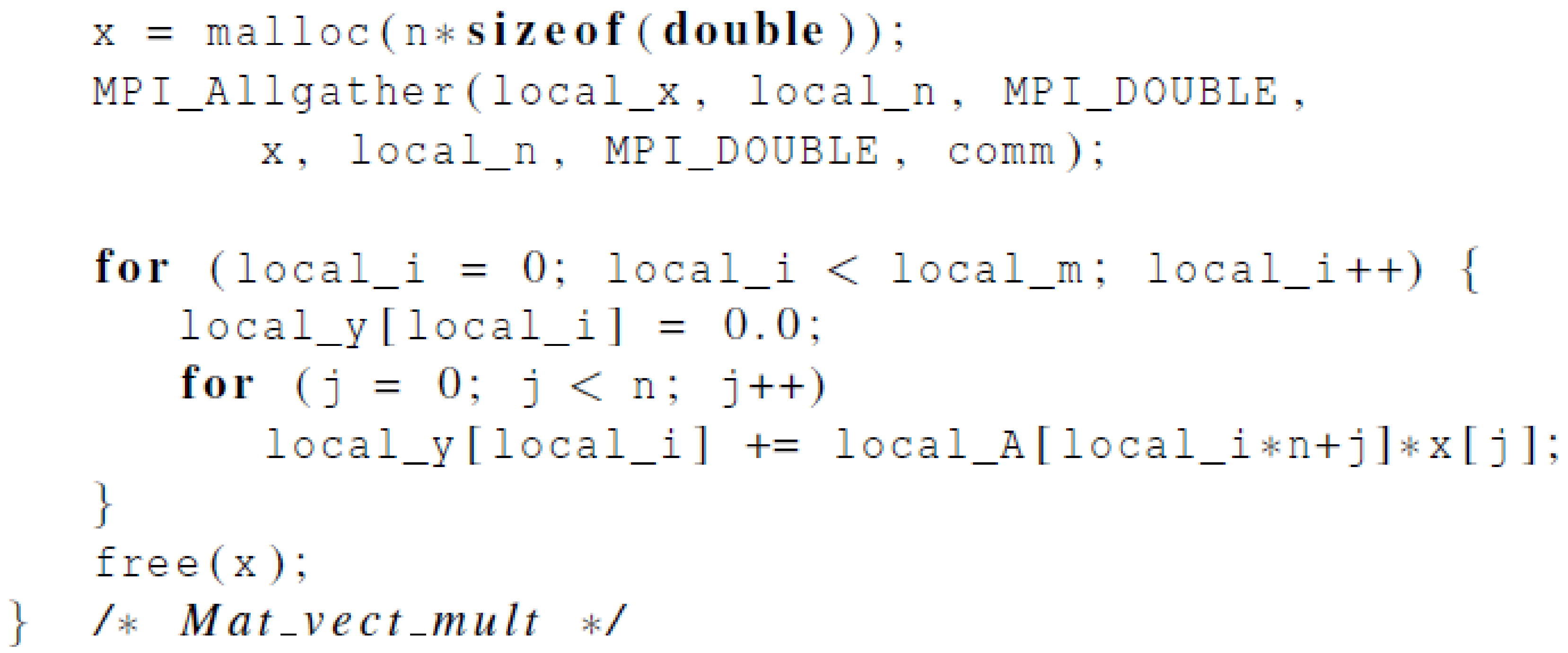
```
void Mat_vect_mult(
      double A[]  /* in  */,
      double x[]  /* in  */,
      double y[]  /* out */,
      int    m    /* in  */,
      int    n    /* in  */) {
   int i, j;

   for (i = 0; i < m; i++) {
      y[i] = 0.0;
      for (j = 0; j < n; j++)
         y[i] += A[i*n+j]*x[j];
   }
}  /* Mat_vect_mult */
```

An MPI matrix-vector multiplication function (1)



```
void Mat_vect_mult(
      double    local_A[]  /* in  */,
      double    local_x[]  /* in  */,
      double    local_y[]  /* out */,
      int       local_m    /* in  */,
      int       n          /* in  */,
      int       local_n    /* in  */,
      MPI_Comm  comm       /* in  */) {
   double* x;
   int local_i, j;
   int local_ok = 1;
```

An MPI matrix-vector multiplication function (2)

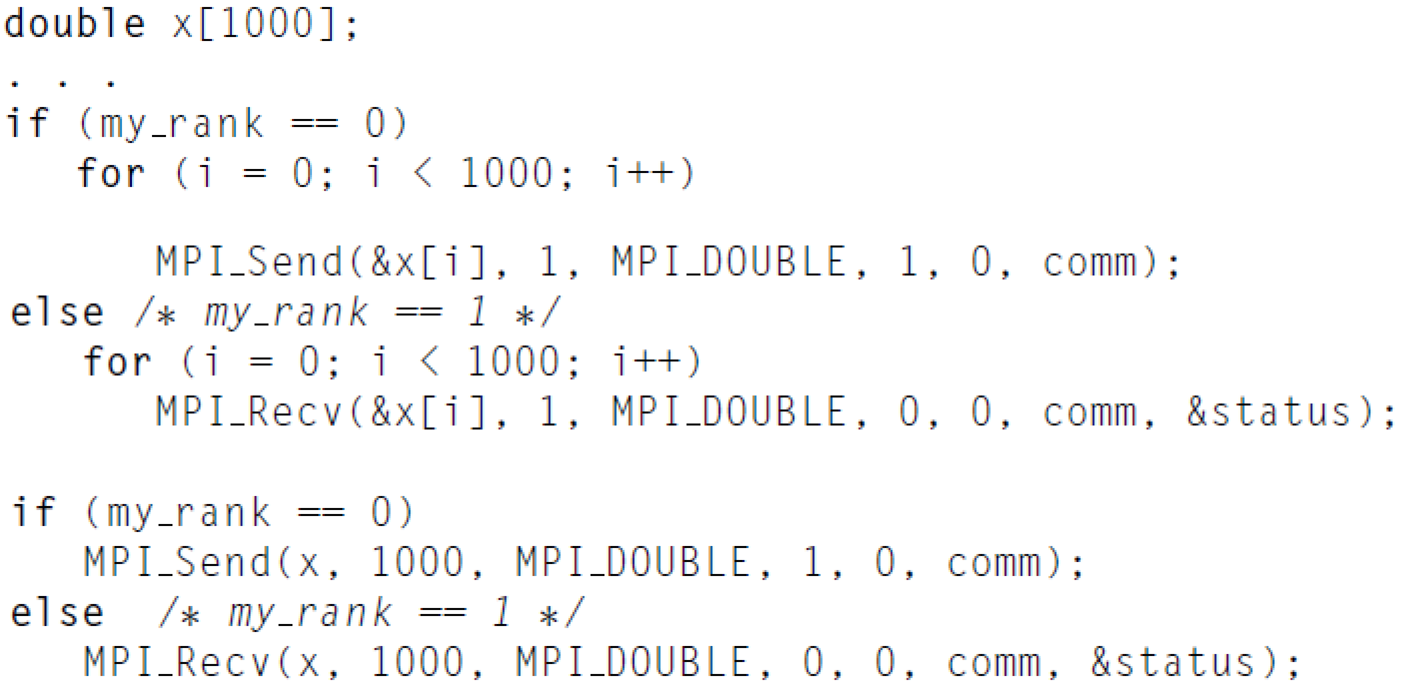


```
   x = malloc(n*sizeof(double));
   MPI_Allgather(local_x, local_n, MPI_DOUBLE,
         x, local_n, MPI_DOUBLE, comm);

   for (local_i = 0; local_i < local_m; local_i++) {
      local_y[local_i] = 0.0;
      for (j = 0; j < n; j++)
         local_y[local_i] += local_A[local_i*n+j]*x[j];
   }
   free(x);
}  /* Mat_vect_mult */
```

MPI DERIVED DATATYPES

Derived datatypes

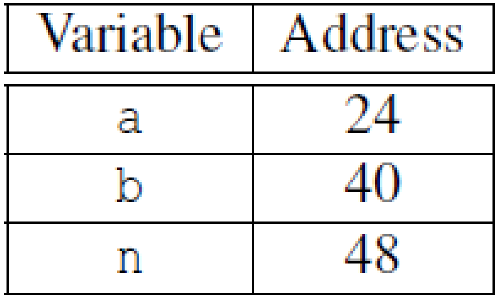
- Used to represent any collection of data items in memory by storing both the types of the items and their relative locations in memory.
- The idea is that if a function that sends data knows this information about a collection of data items, it can collect the items from memory before they are sent.
- Similarly, a function that receives data can distribute the items into their correct destinations in memory when they’ re received.



```
double x[1000];
. . .
if (my_rank == 0)
   for (i = 0; i < 1000; i++)

      MPI_Send(&x[i], 1, MPI_DOUBLE, 1, 0, comm);
else /* my_rank == 1 */
   for (i = 0; i < 1000; i++)
      MPI_Recv(&x[i], 1, MPI_DOUBLE, 0, 0, comm, &status);

if (my_rank == 0)
   MPI_Send(x, 1000, MPI_DOUBLE, 1, 0, comm);
else  /* my_rank == 1 */
   MPI_Recv(x, 1000, MPI_DOUBLE, 0, 0, comm, &status);
```

Derived datatypes

- Formally, consists of a sequence of basic MPI data types together with a displacement for each of the data types.
- Trapezoidal Rule example:

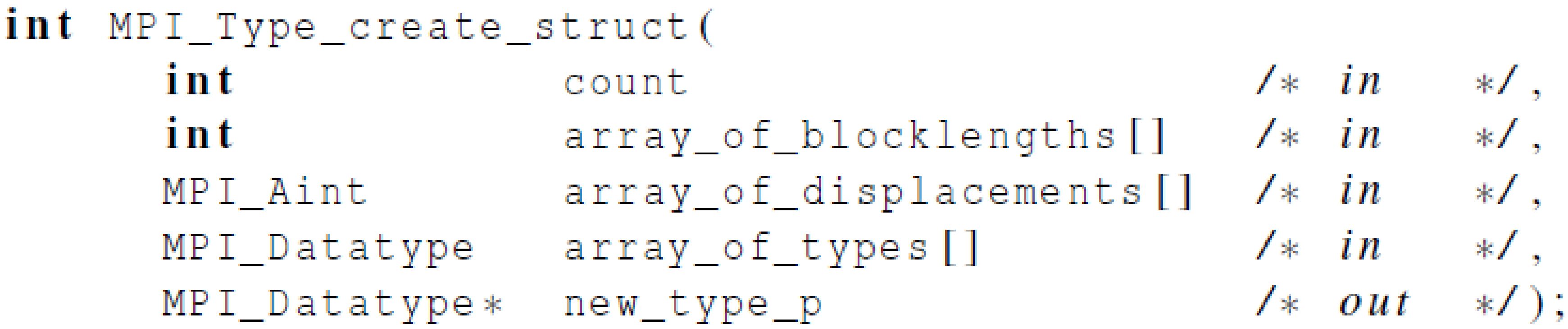


Variable	Address
a	24
b	40
n	48

{(MPI_DOUBLE, 0), (MPI_DOUBLE, 16), (MPI_INT, 24)}

MPI_Type create_struct

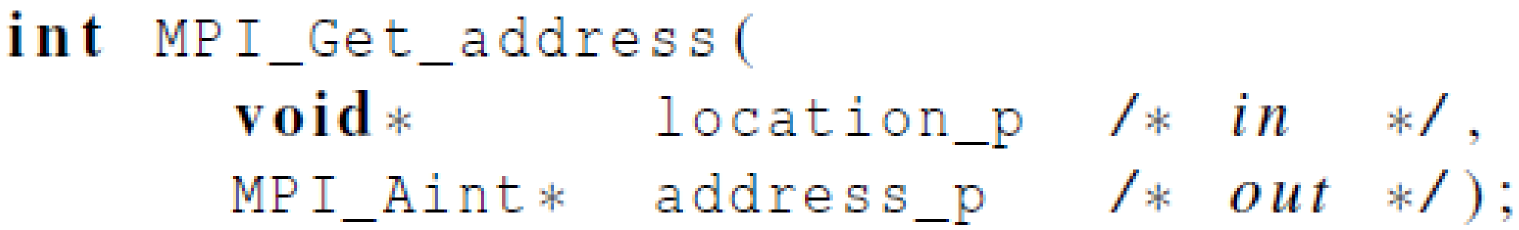
- Builds a derived datatype that consists of individual elements that have different basic types.



```
int MPI_Type_create_struct(
      int            count                     /* in  */,
      int            array_of_blocklengths[]   /* in  */,
      MPI_Aint       array_of_displacements[]  /* in  */,
      MPI_Datatype   array_of_types[]          /* in  */,
      MPI_Datatype*  new_type_p                /* out */);
```

MPI_Get_address

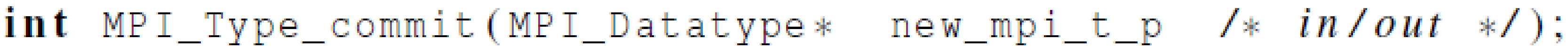
- Returns the address of the memory location referenced by location_p.
- The special type MPI_Aint is an integer type that is big enough to store an address on the system.



```
int MPI_Get_address(
      void*        location_p  /* in   */,
      MPI_Aint*    address_p   /* out  */);
```

MPI_Type_commit

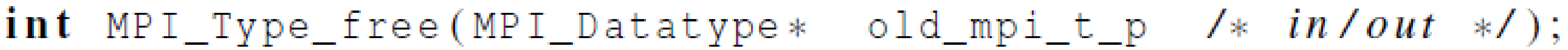
- Allows the MPI implementation to optimize its internal representation of the datatype for use in communication functions.



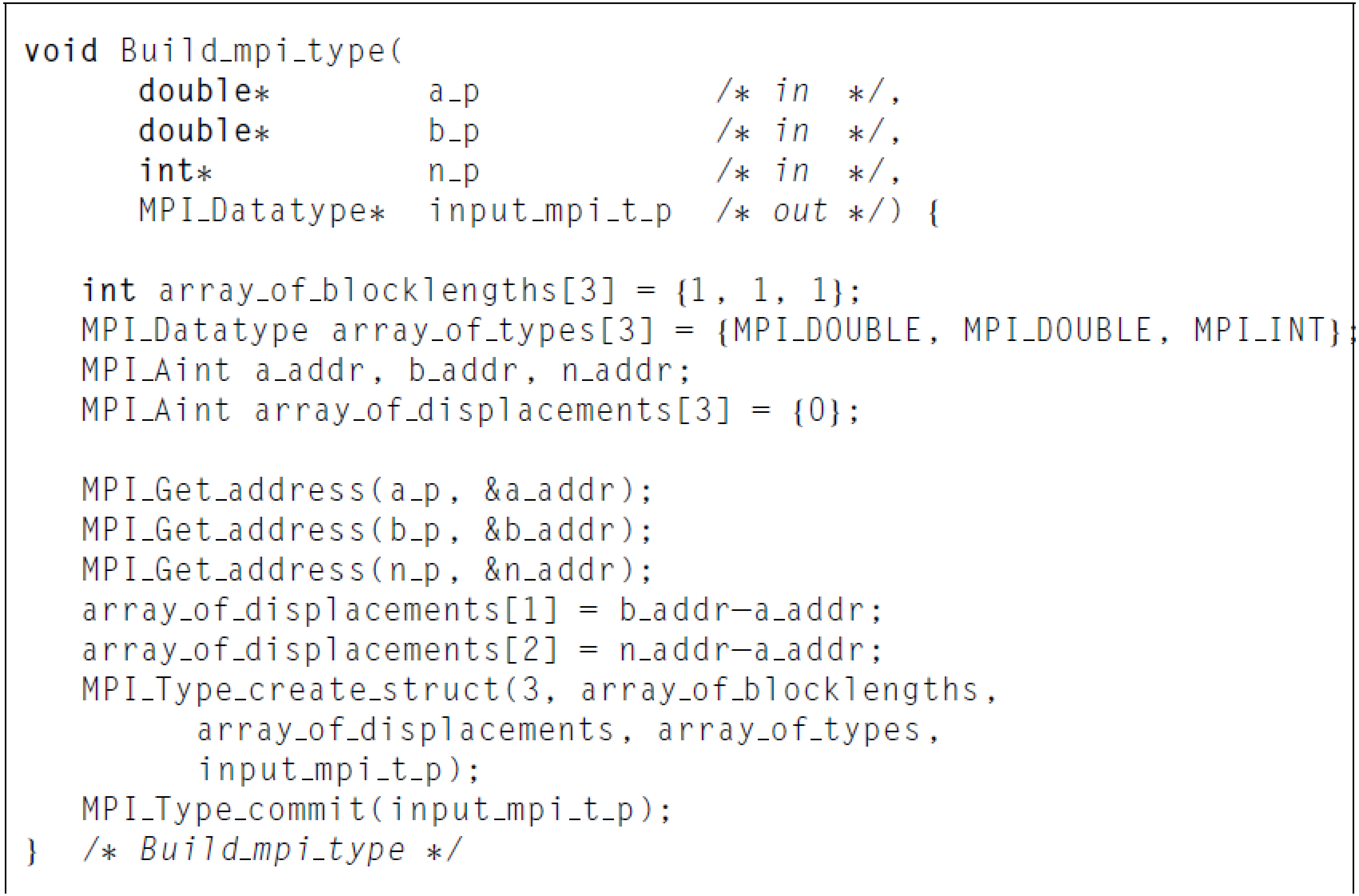
```
int MPI_Type_commit(MPI_Datatype*  new_mpi_t_p  /* in/out */);
```

MPI_Type_free

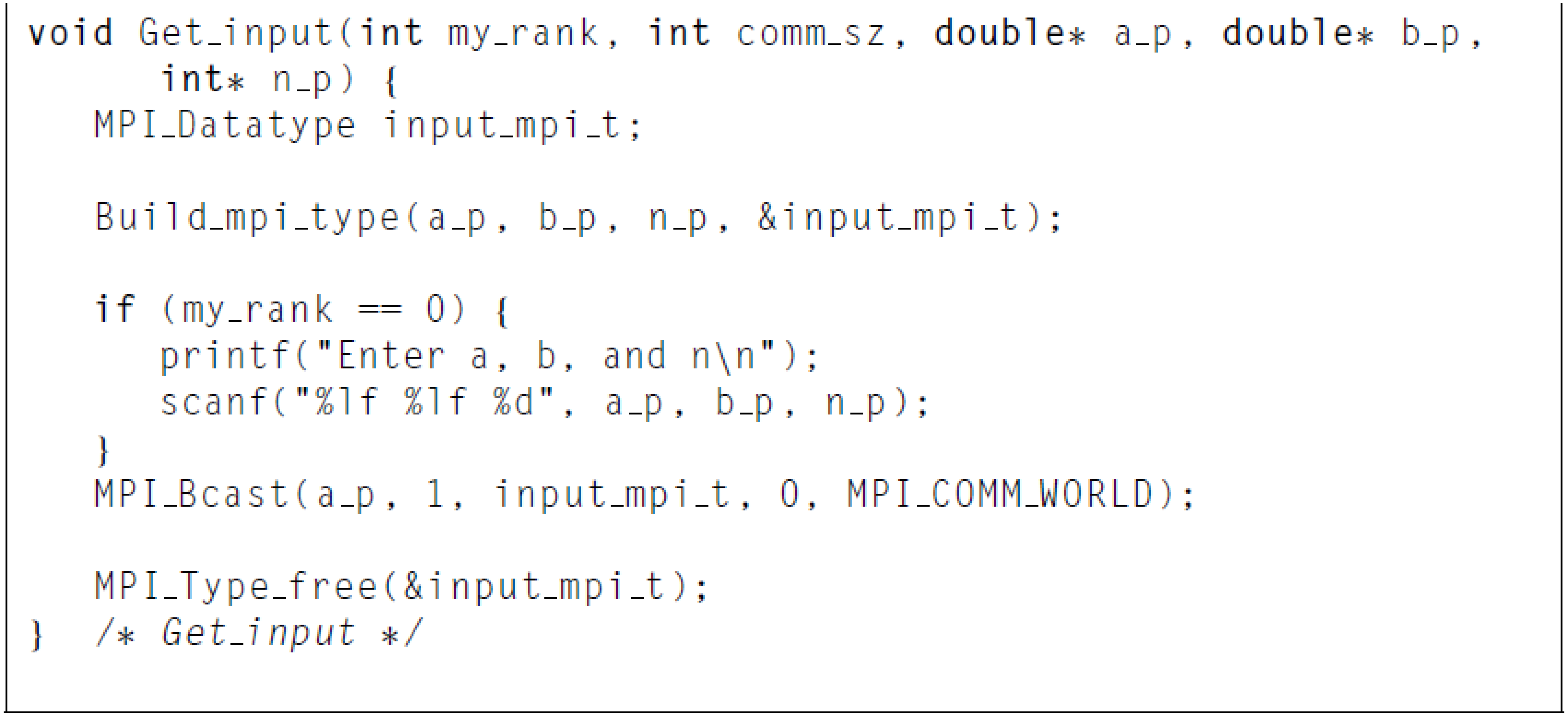
- When we’ re finished with our new type, this frees any additional storage used.



```
int MPI_Type_free(MPI_Datatype*  old_mpi_t_p  /* in/out */);
```



```
void Build_mpi_type(
      double*        a_p          /* in  */,
      double*        b_p          /* in  */,
      int*           n_p          /* in  */,
      MPI_Datatype*  input_mpi_t_p  /* out */) {

   int array_of_blocklengths[3] = {1, 1, 1};
   MPI_Datatype array_of_types[3] = {MPI_DOUBLE, MPI_DOUBLE, MPI_INT};
   MPI_Aint a_addr, b_addr, n_addr;
   MPI_Aint array_of_displacements[3] = {0};

   MPI_Get_address(a_p, &a_addr);
   MPI_Get_address(b_p, &b_addr);
   MPI_Get_address(n_p, &n_addr);
   array_of_displacements[1] = b_addr-a_addr;
   array_of_displacements[2] = n_addr-a_addr;
   MPI_Type_create_struct(3, array_of_blocklengths,
         array_of_displacements, array_of_types,
         input_mpi_t_p);
   MPI_Type_commit(input_mpi_t_p);
}  /* Build_mpi_type */
```



```c
void Get_input(int my_rank, int comm_sz, double* a_p, double* b_p,
      int* n_p) {
   MPI_Datatype input_mpi_t;

   Build_mpi_type(a_p, b_p, n_p, &input_mpi_t);

   if (my_rank == 0) {
      printf("Enter a, b, and n\n");
      scanf("%lf %lf %d", a_p, b_p, n_p);
   }
   MPI_Bcast(a_p, 1, input_mpi_t, 0, MPI_COMM_WORLD);

   MPI_Type_free(&input_mpi_t);
}  /* Get_input */
```

MPI_Type_contiguous

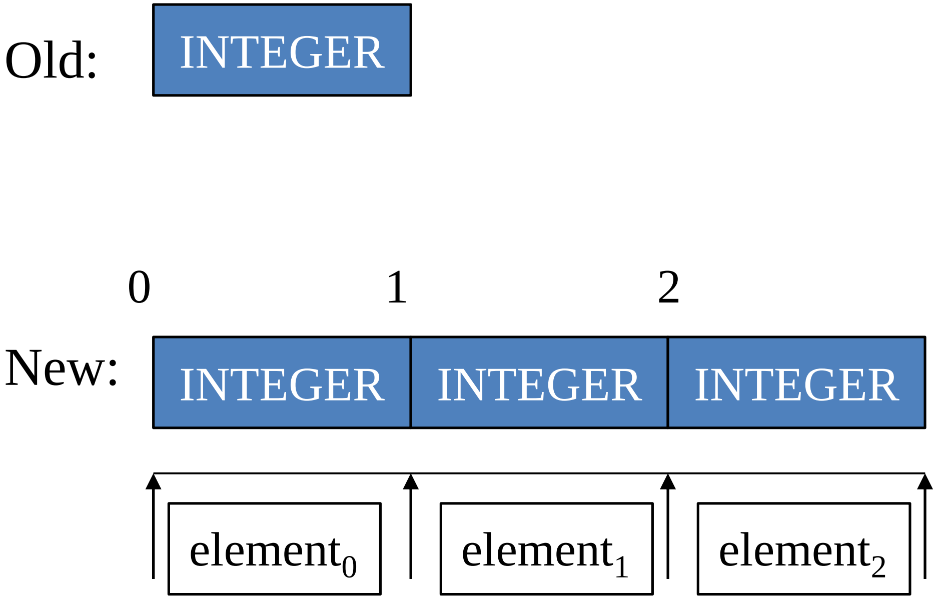
- Include a fixed amount of continuous array which have the same data type.

```
int err, count;
MPI_Datatype oldtype, newtype;
err = MPI_Type_contiguous( count, oldtype, &newtype) ;
```

count	The size of array.
oldtype	The old data type.
newtype	The new data type.

MPI_Type_contiguous

```
int count = 3;
MPI_Datatype newtype;
err = MPI_Type_contiguous( count, MPI_INTEGER, &newtype);
```



MPI_Type_vector

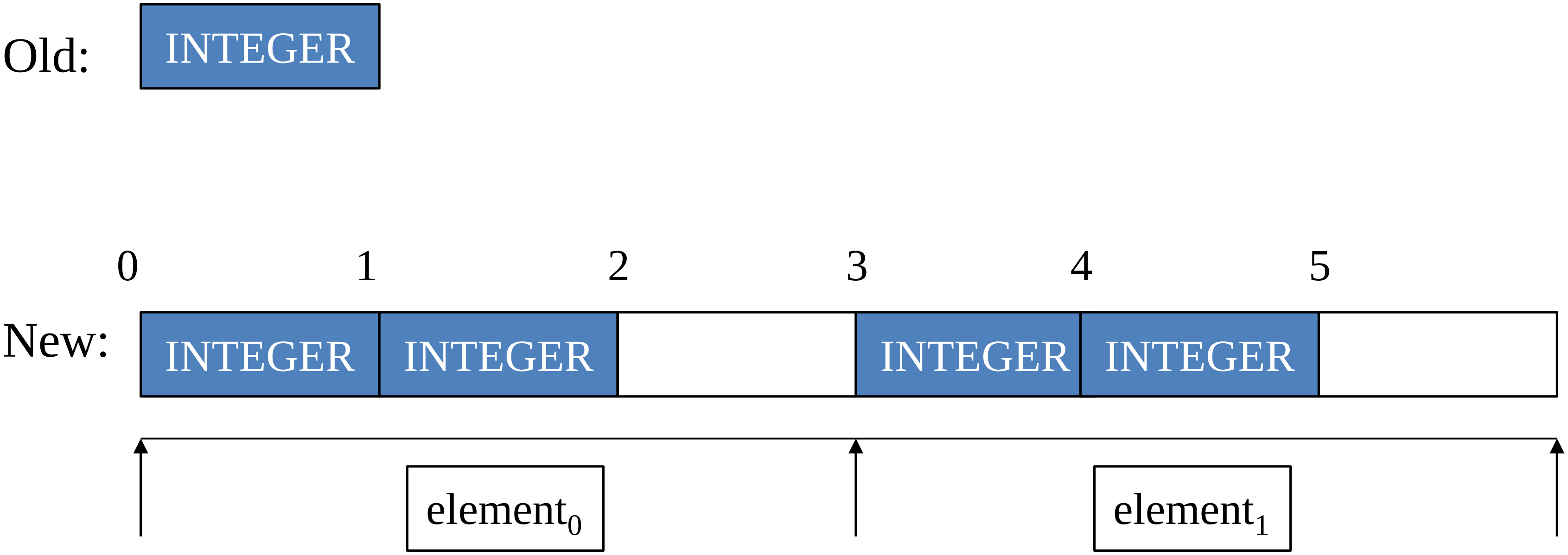
- Include a fixed size of interval of discontinuous array which have the same data type.

```
int err, count, blocklength, stride;
MPI_Datatype oldtype, newtype;
err = MPI_Type_vector( count, blocklength, stride, oldtype, &newtype);
```

count	The amount of block.
blocklength	The amount of data with old data type at a block.
stride	The distance of block, and using old data type as unit.
oldtype	The old data type.
newtype	The new data type.

MPI_Type_vector

```
int count = 2, blocklength = 2, stride = 3;
MPI_Datatype newtype;
MPI_Type_vector( count, blocklength, stride, MPI_INTEGER, &newtype);
```



MPI_Type_indexed

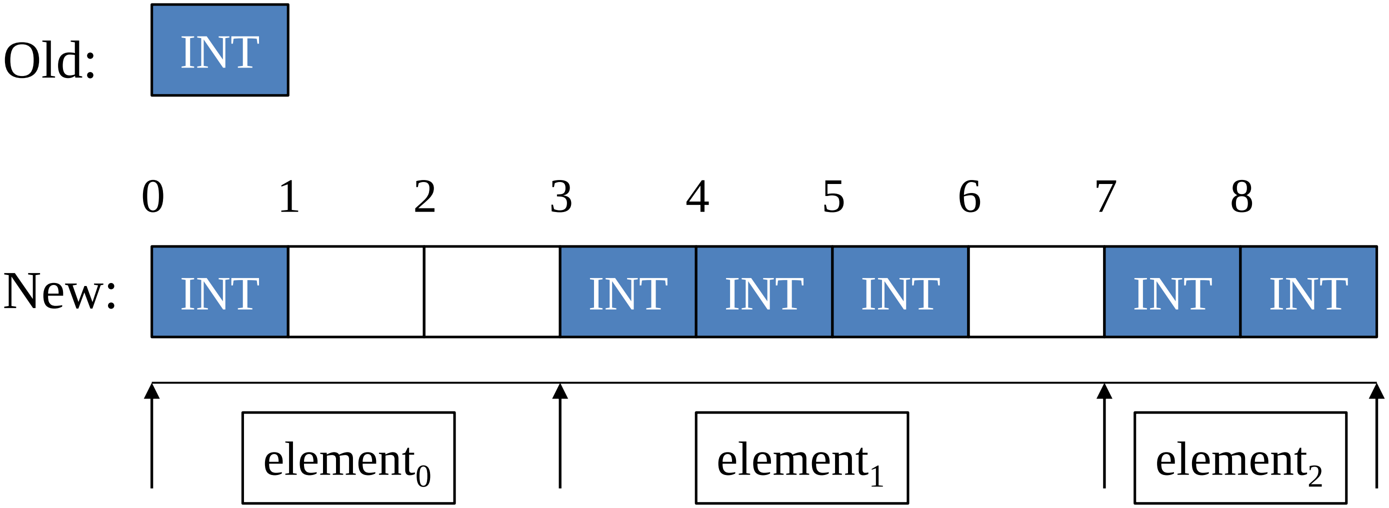
- Include an any distance and discontinuous array which have the same data type.

```
int err, count, length[], disp[];
MPI_Datatype oldtype, newtype;
err = MPI_Type_indexed( count, length, disp, oldtype, &newtype);
```

count	The amount of block.
length	The amount of data with old data type at a block.
disp	The location of block, and using old data type as unit.
oldtype	The old data type.
newtype	The new data type.

MPI_Type_indexed

```
int count = 3, length[3] = {1,3,2}, disp[3] = {0,3,7};
MPI_Datatype newtype;
MPI_Type_indexed( count, length, disp, MPI_INTEGER, &newtype);
```



MPI_Datatype_struct

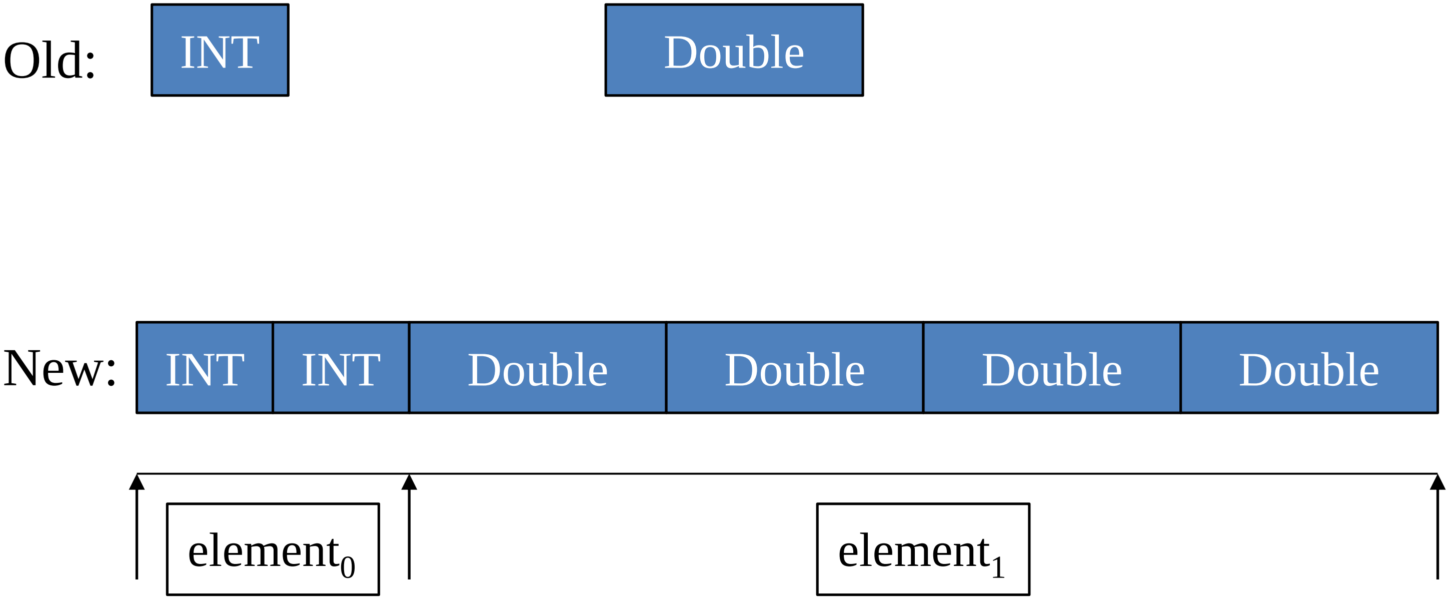
- Any combination of data types.

```
int err, count, length[];
MPI_Aint disp[];
MPI_Datatype oldtype[], newtype;
err = MPI_Type_struct( count, length, disp, oldtype, &newtype);
```

count	The amount of block.
length	The amount of data with old data type at a block.
disp	The location of block, and using type as unit.
oldtype	The old data types.
newtype	The new data type.

MPI_Datatype_struct

```
int count = 2, length[2] = {2,4}, disp[2] = {0, extent(MPI_INTEGER)*2};
MPI_Datatype oldtype[2] = {MPI_INTEGER, MPI_DOUBLE}, newtype;
MPI_Type_struct( count, length, disp, oldtype, &newtype);
```



MPI_Type_extent

- For using to know the memory size of a specifies data type .

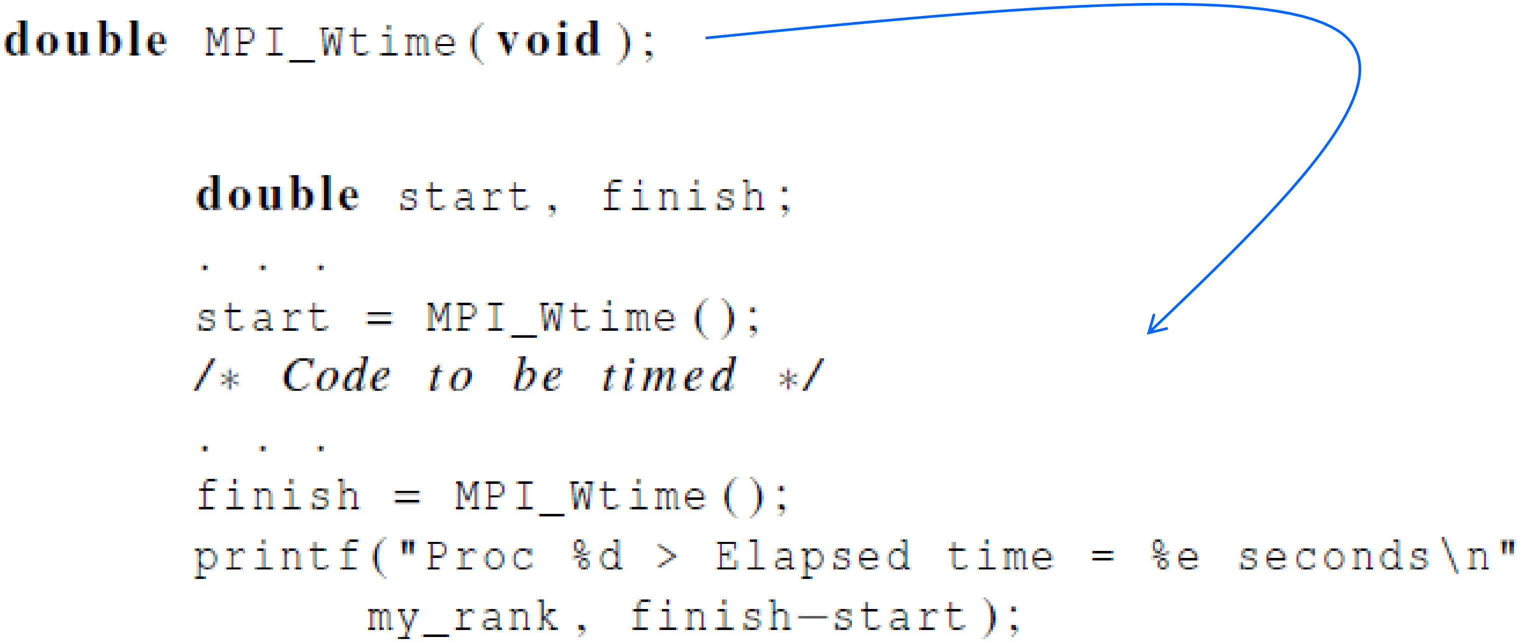
```
MPI_Datatype type;
MPI_Aint extent;
int err = MPI_Type_extent( type, &extent);
```

type	Data type.
extent	The memory size of an data type unit.

PERFORMANCE EVALUATION

Elapsed parallel time

- Returns the number of seconds that have elapsed since some time in the past.

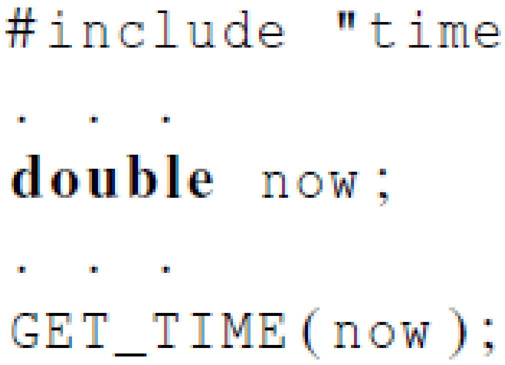


```
double MPI_Wtime(void);
```

```
double start, finish;
. . .
start = MPI_Wtime();
/* Code to be timed */
. . .
finish = MPI_Wtime();
printf("Proc %d > Elapsed time = %e seconds\n"
      my_rank, finish-start);
```

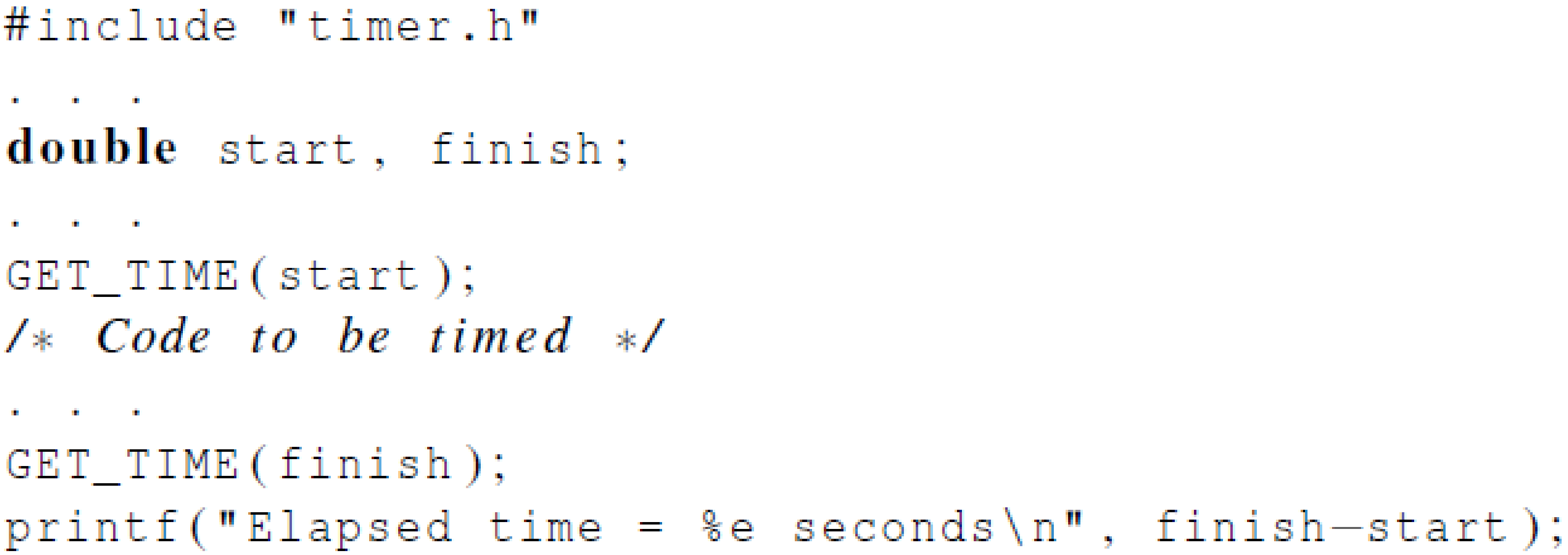
Elapsed serial time

- In this case, you don’t need to link in the MPI libraries.
- Returns time in microseconds elapsed from some point in the past.



```
#include "timer.h"
. . .
double now;
. . .
GET_TIME(now);
```

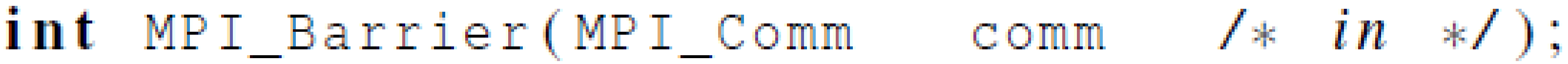
Elapsed serial time



```
#include "timer.h"
. . .
double start, finish;
. . .
GET_TIME(start);
/* Code to be timed */
. . .
GET_TIME(finish);
printf("Elapsed time = %e seconds\n", finish-start);
```

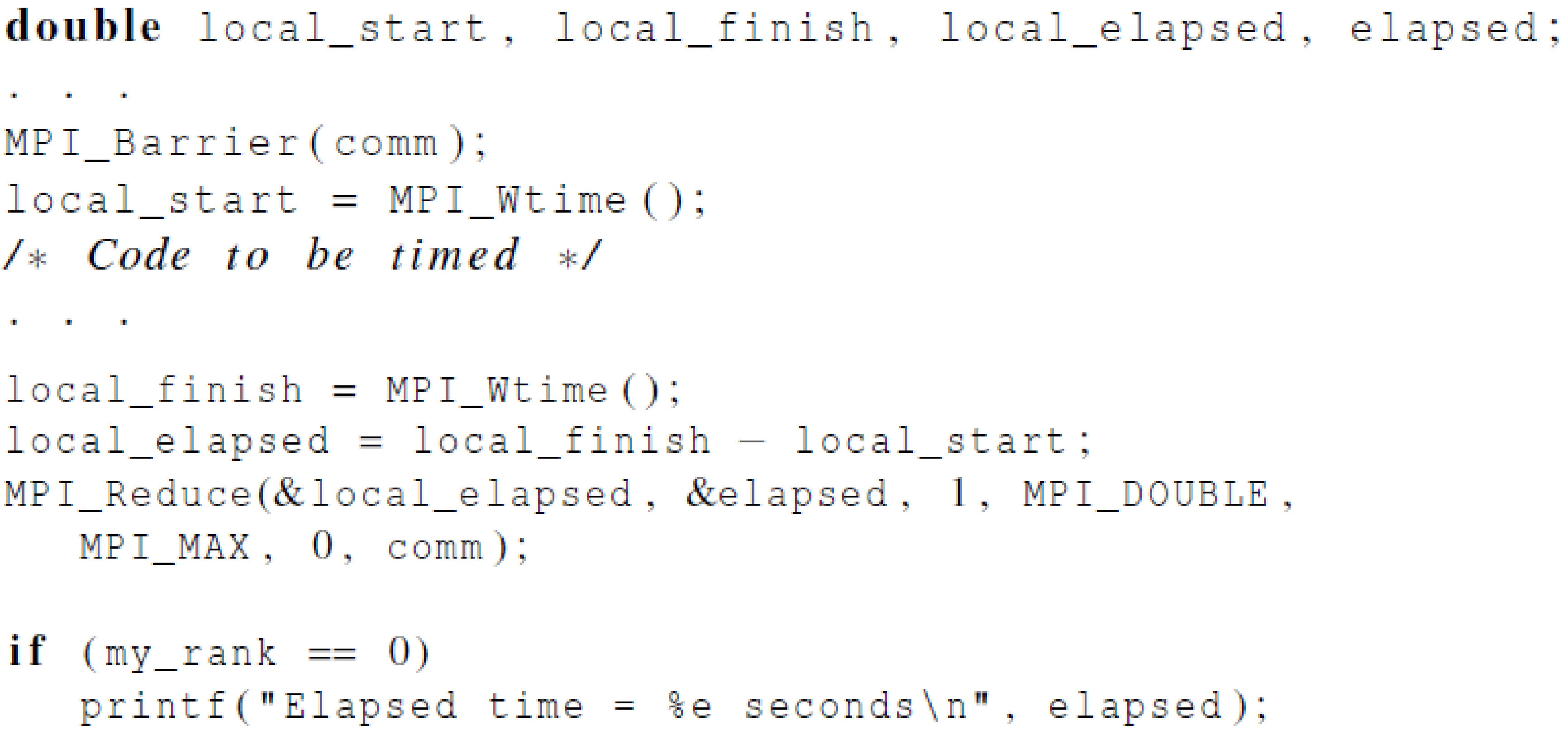
MPI_Barrier

- Ensures that no process will return from calling it until every process in the communicator has started calling it.



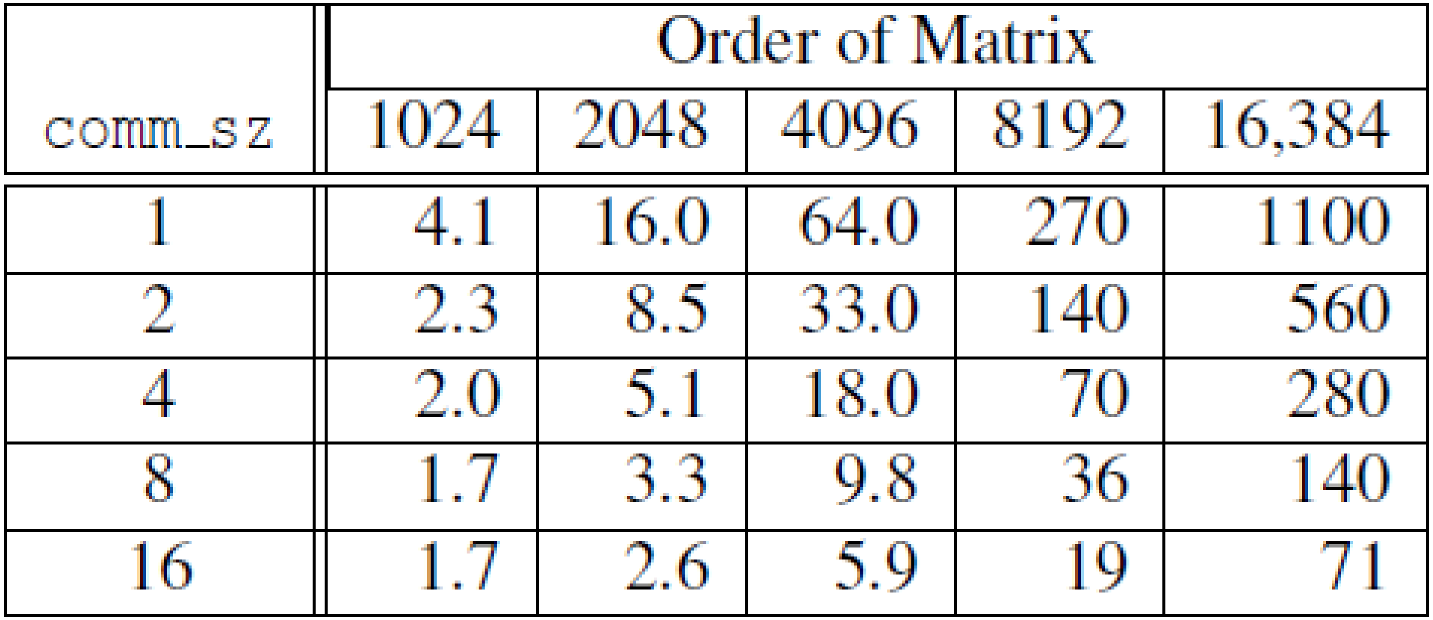
```
int MPI_Barrier(MPI_Comm    comm    /* in */);
```

MPI_Barrier



```
double local_start, local_finish, local_elapsed, elapsed;
. . .
MPI_Barrier(comm);
local_start = MPI_Wtime();
/* Code to be timed */
. . .
local_finish = MPI_Wtime();
local_elapsed = local_finish - local_start;
MPI_Reduce(&local_elapsed, &elapsed, 1, MPI_DOUBLE,
   MPI_MAX, 0, comm);

if (my_rank == 0)
   printf("Elapsed time = %e seconds\n", elapsed);
```

Run-times of serial and parallel matrix-vector multiplication



	Order of Matrix				
comm_sz	1024	2048	4096	8192	16,384
1	4.1	16.0	64.0	270	1100
2	2.3	8.5	33.0	140	560
4	2.0	5.1	18.0	70	280
8	1.7	3.3	9.8	36	140
16	1.7	2.6	5.9	19	71

(Seconds)

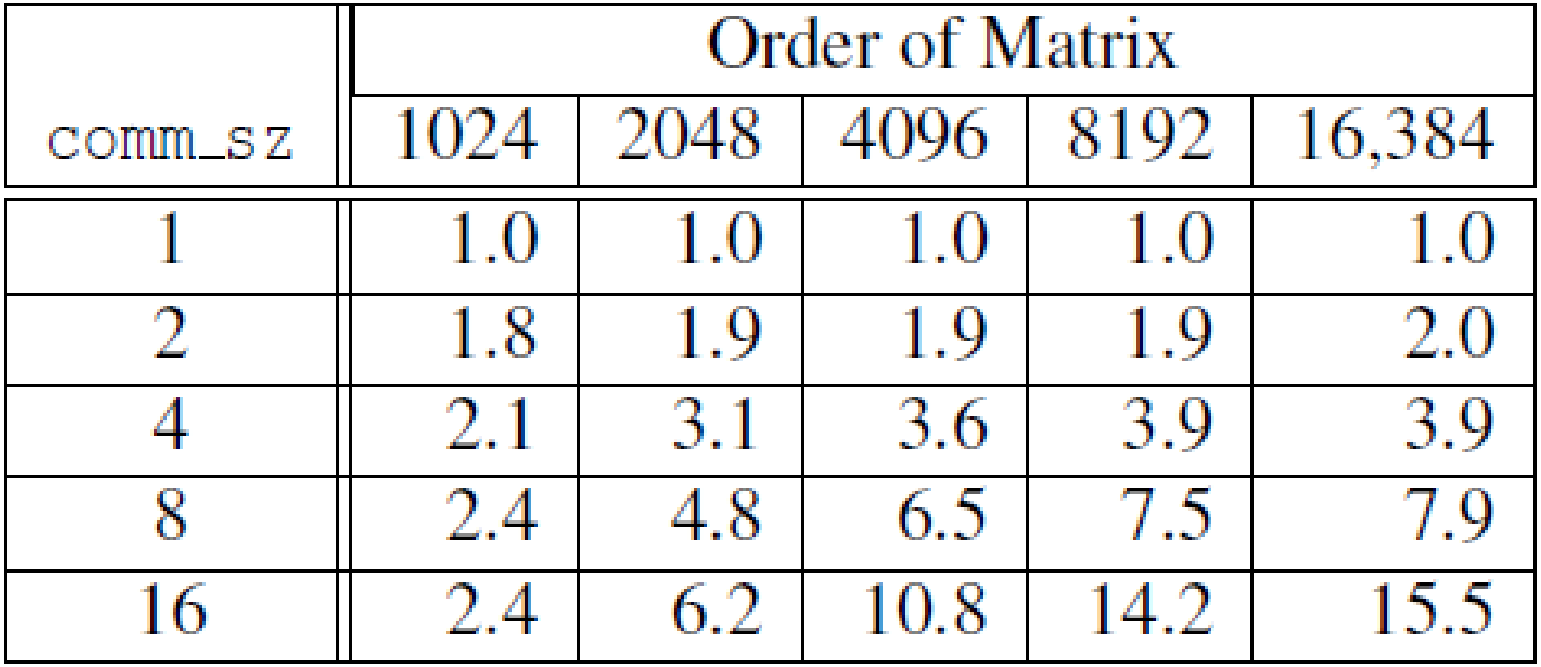
Speedup

$$S(n,p) = \frac{T_{\text{serial}}(n)}{T_{\text{parallel}}(n,p)}$$

Efficiency

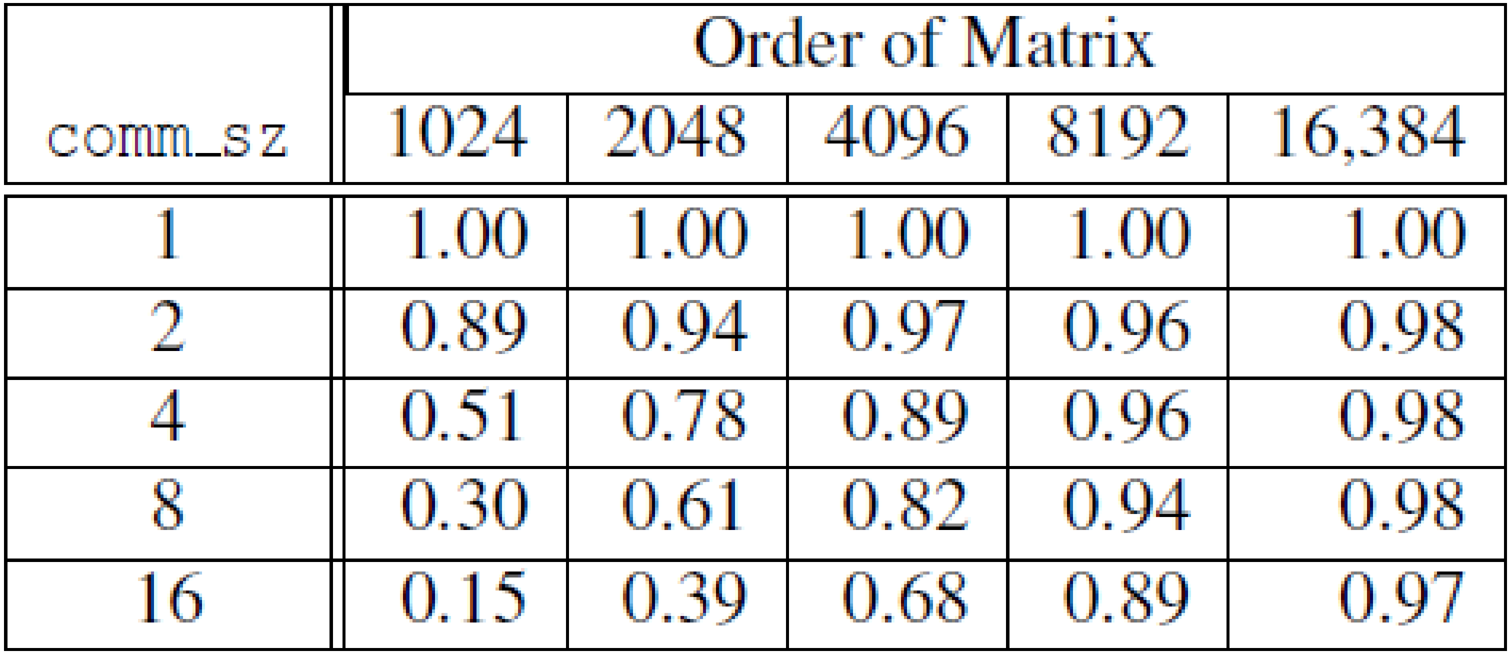
$$E(n,p) = \frac{S(n,p)}{p} = \frac{T_{\mathrm{serial}}(n)}{p \times T_{\mathrm{parallel}}(n,p)}$$

Speedups of Parallel Matrix-Vector Multiplication



comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.0	1.0	1.0	1.0	1.0
2	1.8	1.9	1.9	1.9	2.0
4	2.1	3.1	3.6	3.9	3.9
8	2.4	4.8	6.5	7.5	7.9
16	2.4	6.2	10.8	14.2	15.5

Efficiencies of Parallel Matrix-Vector Multiplication



comm_sz	Order of Matrix				
	1024	2048	4096	8192	16,384
1	1.00	1.00	1.00	1.00	1.00
2	0.89	0.94	0.97	0.96	0.98
4	0.51	0.78	0.89	0.96	0.98
8	0.30	0.61	0.82	0.94	0.98
16	0.15	0.39	0.68	0.89	0.97

A PARALLEL SORTING ALGORITHM

Sorting

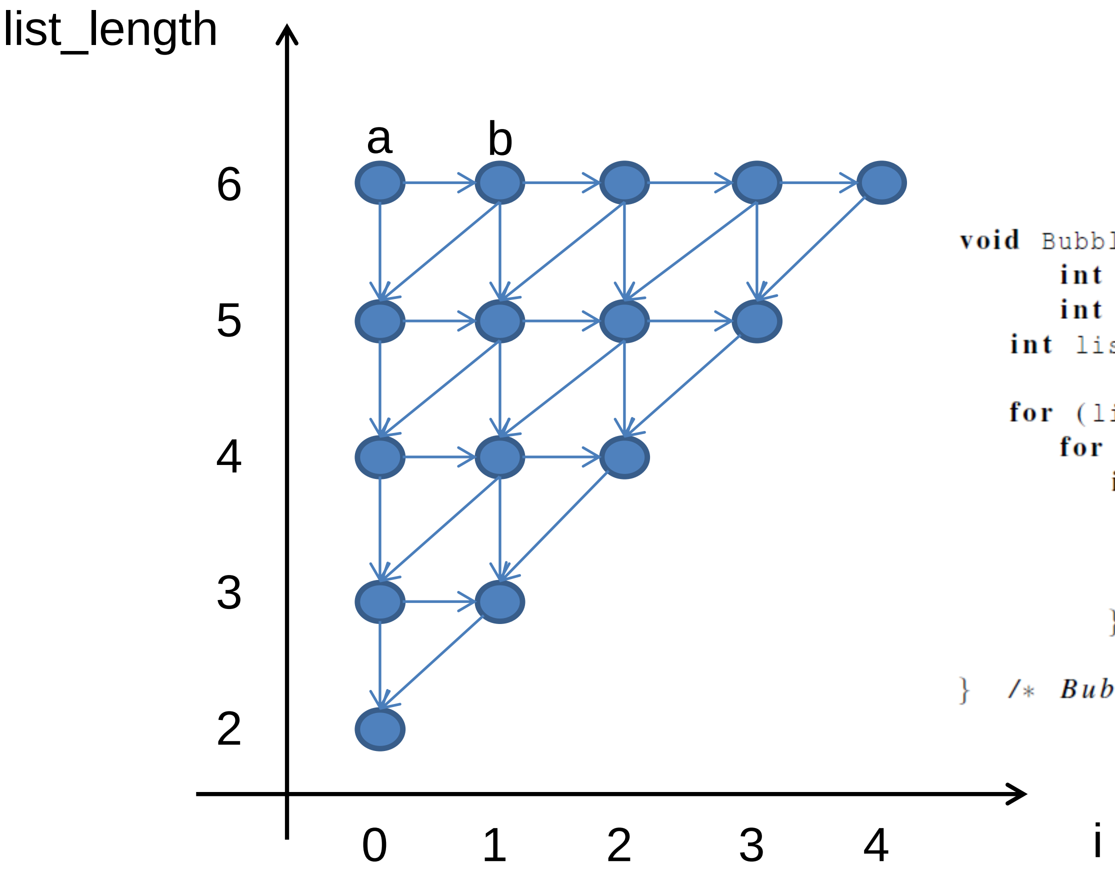
- n keys and p = comm sz processes.
- n/p keys assigned to each process.
- No restrictions on which keys are assigned to which processes.
- When the algorithm terminates:
  - The keys assigned to each process should be sorted in (say) increasing order.
  - If $0 \leq q < r < p$, then each key assigned to process q should be less than or equal to every key assigned to process r.

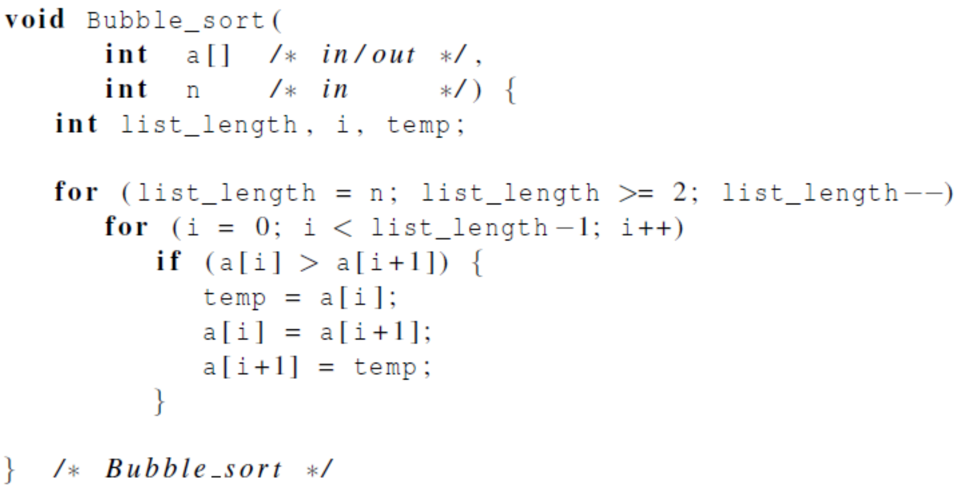
Serial bubble sort

```
void Bubble_sort(
      int  a[]  /* in/out */,
      int  n    /* in     */) {
   int list_length, i, temp;

   for (list_length = n; list_length >= 2; list_length--)
      for (i = 0; i < list_length-1; i++)
         if (a[i] > a[i+1]) {
            temp = a[i];
            a[i] = a[i+1];
            a[i+1] = temp;
         }

}  /* Bubble_sort */
```

6 5 3 1 8 7 2 4

Data dependence (bubble sort)





```
void Bubble_sort(
        int   a[]   /* in/out */,
        int   n     /* in     */) {
    int list_length, i, temp;

    for (list_length = n; list_length >= 2; list_length--)
        for (i = 0; i < list_length-1; i++)
            if (a[i] > a[i+1]) {
                temp = a[i];
                a[i] = a[i+1];
                a[i+1] = temp;
            }

}  /* Bubble_sort */
```

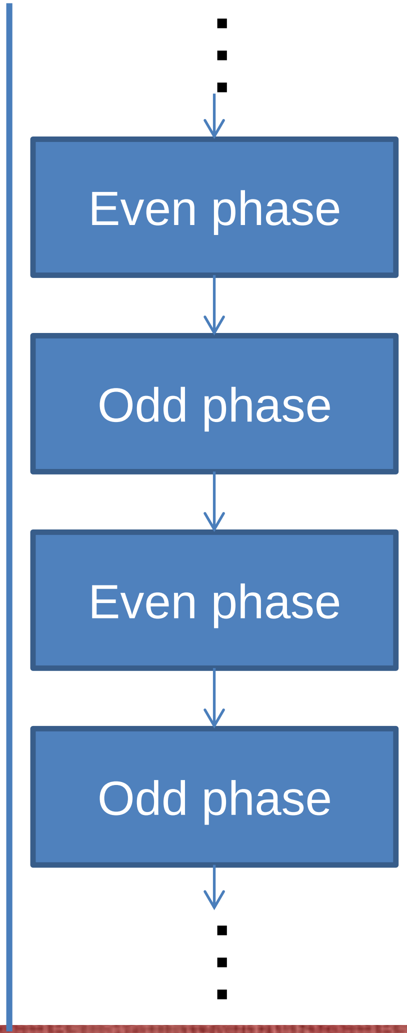
Odd-even transposition sort

- A sequence of phases.
- Even phases, compare swaps:

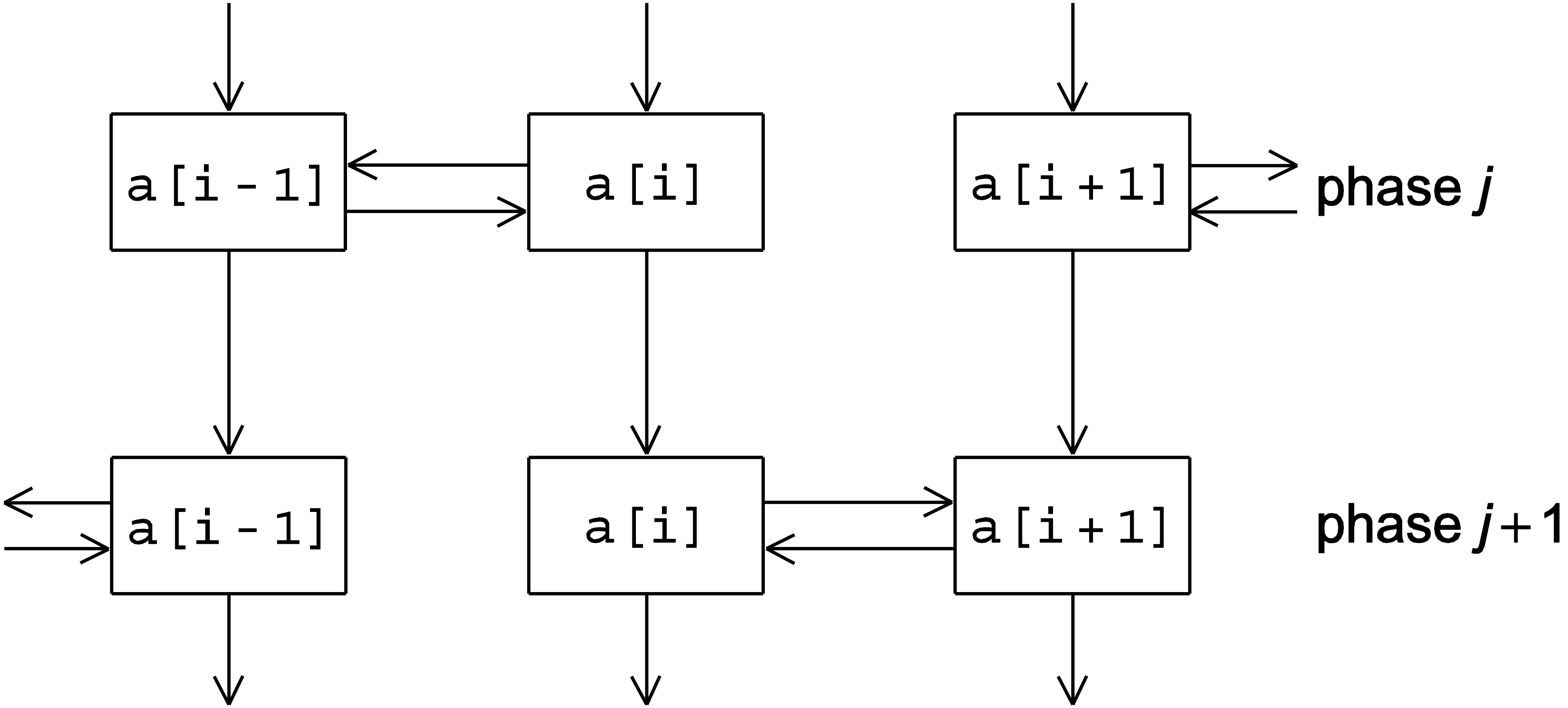
$$(a[0],a[1]),(a[2],a[3]),(a[4],a[5]),\ldots$$

- Odd phases, compare swaps:

$$(a[1],a[2]),(a[3],a[4]),(a[5],a[6]),\ldots.$$



Communications among tasks in odd-even sort



Tasks determining a[i] are labeled with a[i].

Example

Start: 5, 9, 4, 3

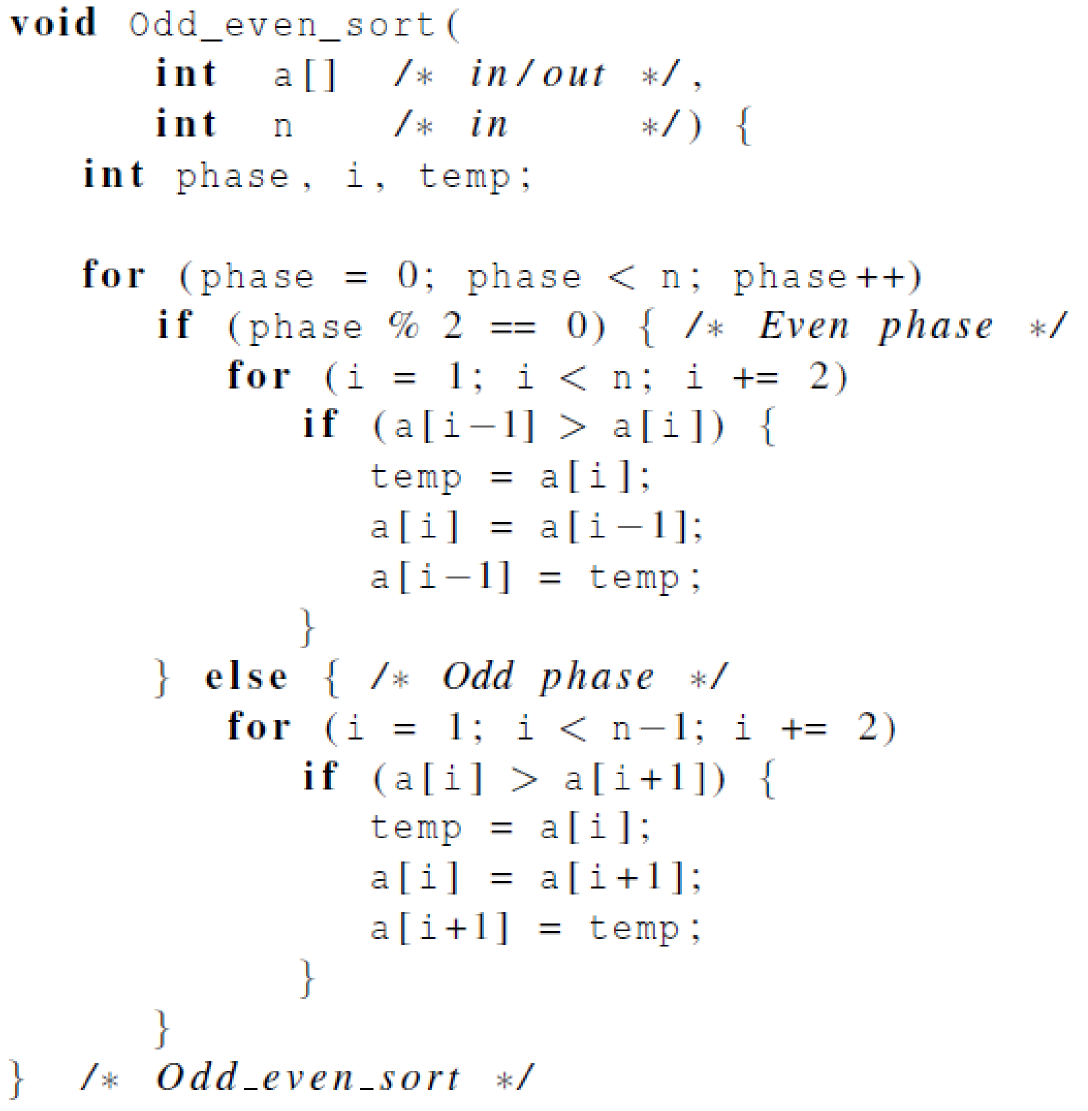
Even phase: compare-swap (5,9) and (4,3) getting the list 5, 9, 3, 4

Odd phase: compare-swap (9,3) getting the list 5, 3, 9, 4

Even phase: compare-swap (5,3) and (9,4) getting the list 3, 5, 4, 9

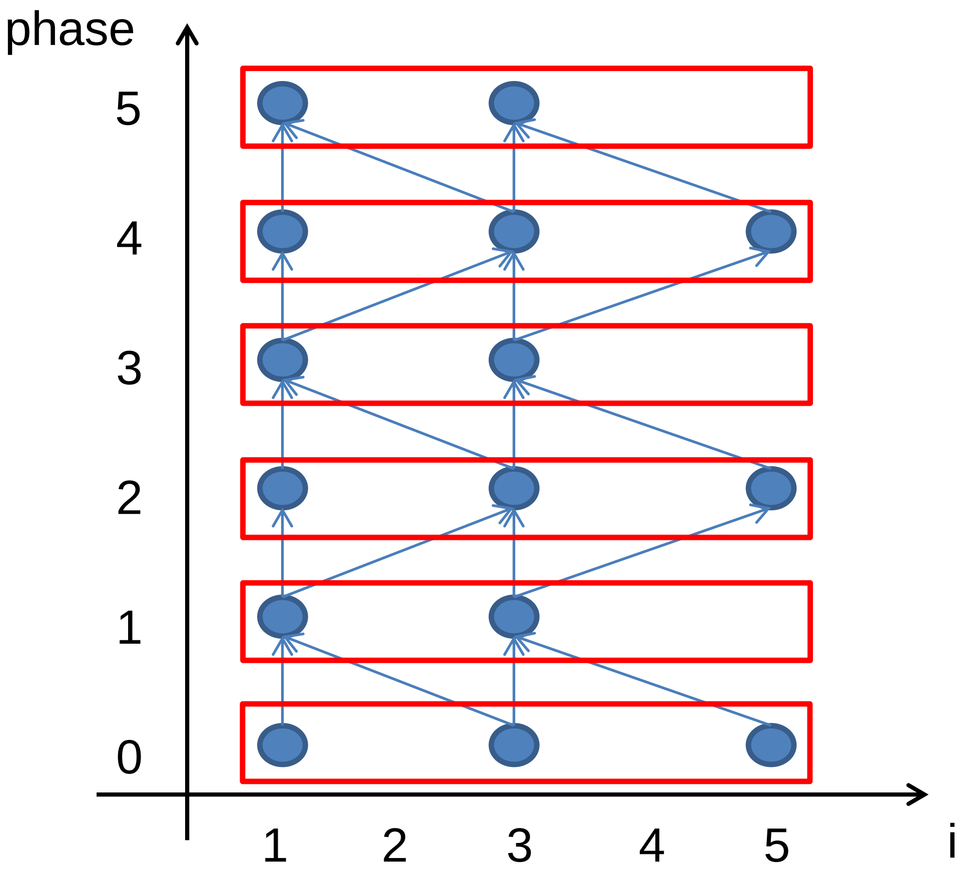
Odd phase: compare-swap (5,4) getting the list 3, 4, 5, 9

Serial odd-even transposition sort



```c
void Odd_even_sort(
      int  a[]  /* in/out */,
      int  n    /* in     */) {
   int phase, i, temp;

   for (phase = 0; phase < n; phase++)
      if (phase % 2 == 0) { /* Even phase */
         for (i = 1; i < n; i += 2)
            if (a[i-1] > a[i]) {
               temp = a[i];
               a[i] = a[i-1];
               a[i-1] = temp;
            }
      } else { /* Odd phase */
         for (i = 1; i < n-1; i += 2)
            if (a[i] > a[i+1]) {
               temp = a[i];
               a[i] = a[i+1];
               a[i+1] = temp;
            }
      }
}  /* Odd_even_sort */
```

Data dependence (odd-even transposition sort)



1, 2, 3, 4, 5, 6

1, (3, 2), (5, 4), 6

(3, 1), (5, 2), (6, 4)

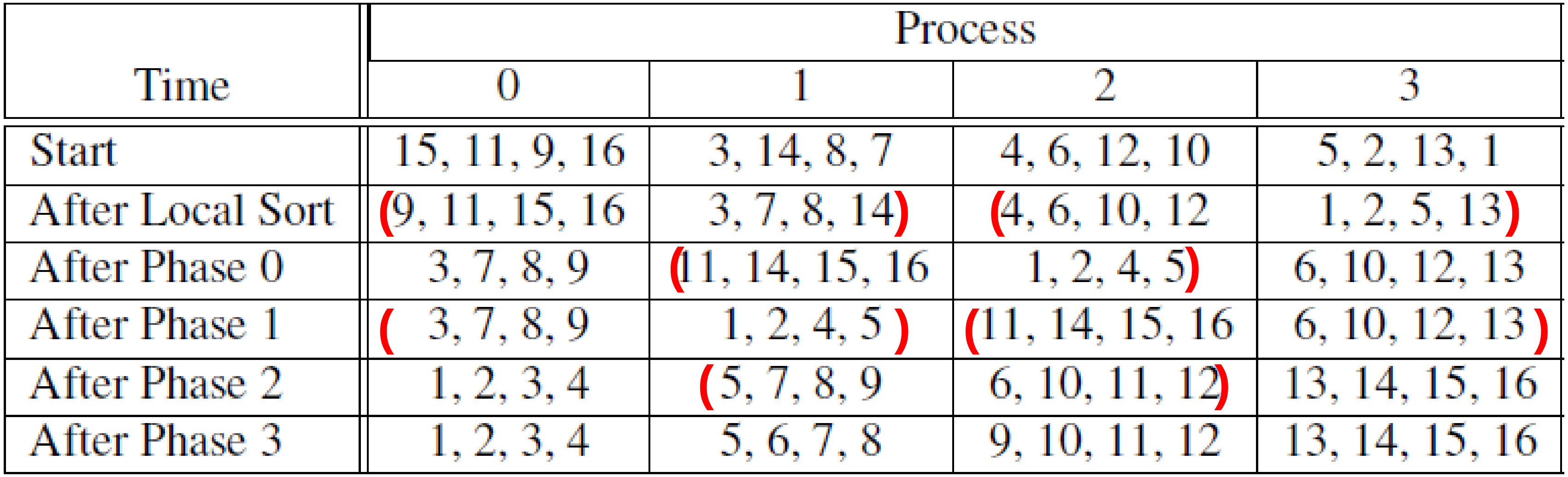
3, (5, 1), (6, 2), 4

(5, 3), (6, 1), (4, 2)

5, (6, 3), (4, 1), 2

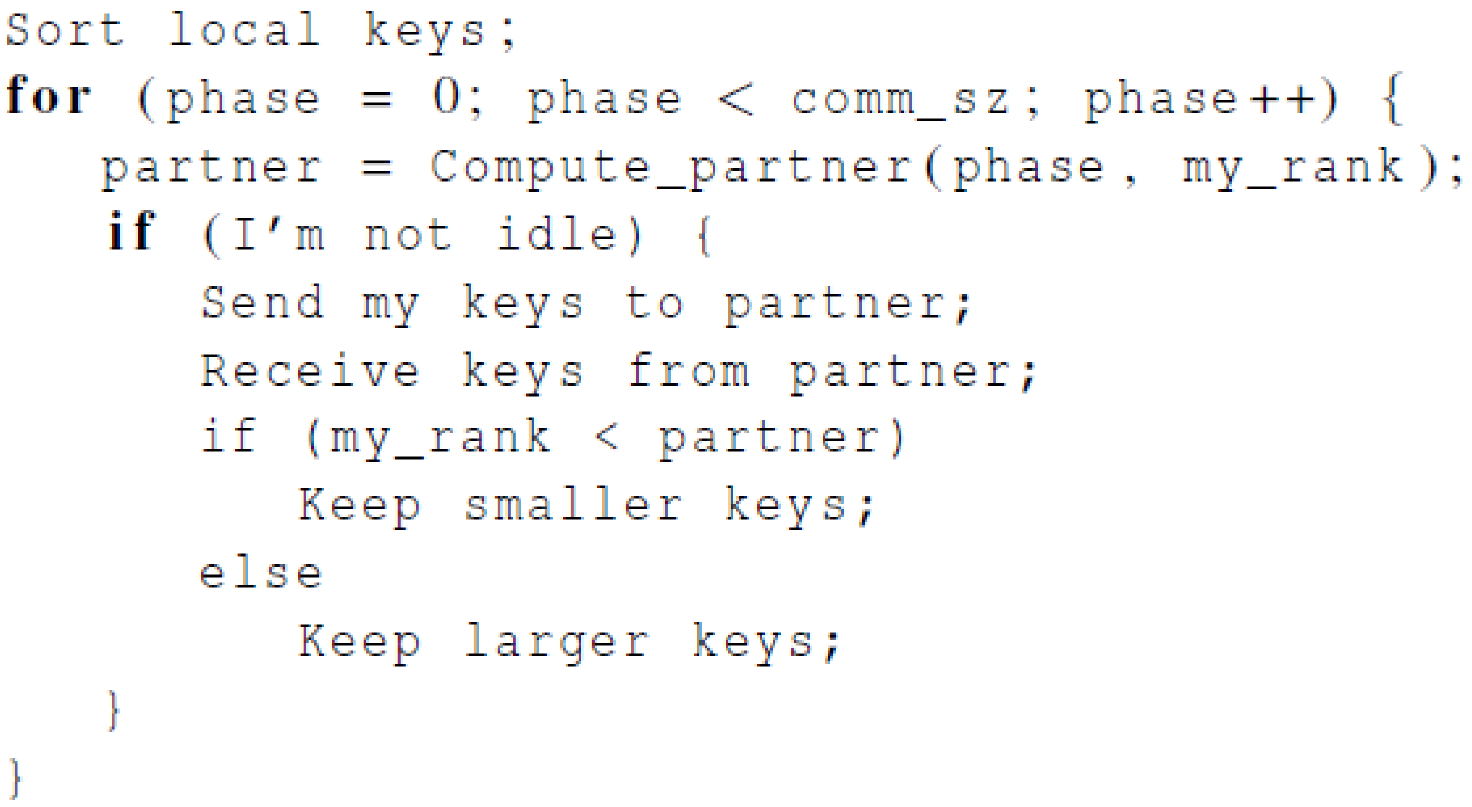
(6, 5), (4, 3), (2, 1)

Parallel odd-even transposition sort



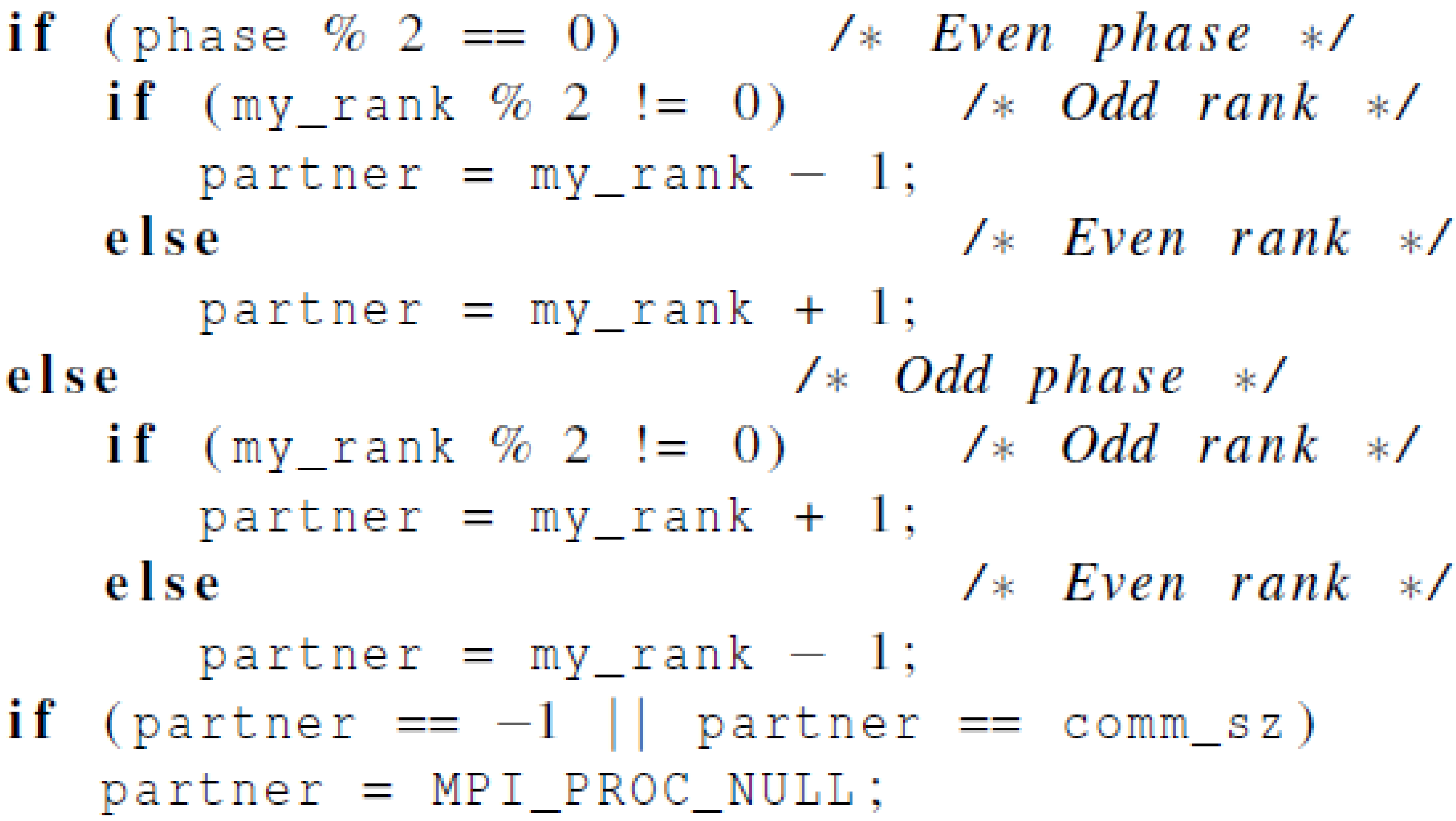
Time	Process			
	0	1	2	3
Start	15, 11, 9, 16	3, 14, 8, 7	4, 6, 12, 10	5, 2, 13, 1
After Local Sort	(9, 11, 15, 16	3, 7, 8, 14)	(4, 6, 10, 12	1, 2, 5, 13)
After Phase 0	3, 7, 8, 9	(11, 14, 15, 16	1, 2, 4, 5)	6, 10, 12, 13
After Phase 1	(3, 7, 8, 9	1, 2, 4, 5)	(11, 14, 15, 16	6, 10, 12, 13)
After Phase 2	1, 2, 3, 4	(5, 7, 8, 9	6, 10, 11, 12)	13, 14, 15, 16
After Phase 3	1, 2, 3, 4	5, 6, 7, 8	9, 10, 11, 12	13, 14, 15, 16

Pseudo-code



```
Sort local keys;
for (phase = 0; phase < comm_sz; phase++) {
   partner = Compute_partner(phase, my_rank);
   if (I'm not idle) {
      Send my keys to partner;
      Receive keys from partner;
      if (my_rank < partner)
         Keep smaller keys;
      else
         Keep larger keys;
   }
}
```

Compute_partner



```
if (phase % 2 == 0)           /* Even phase */
   if (my_rank % 2 != 0)         /* Odd rank */
      partner = my_rank - 1;
   else                          /* Even rank */
      partner = my_rank + 1;
else                          /* Odd phase */
   if (my_rank % 2 != 0)         /* Odd rank */
      partner = my_rank + 1;
   else                          /* Even rank */
      partner = my_rank - 1;
if (partner == -1 || partner == comm_sz)
   partner = MPI_PROC_NULL;
```

Safety in MPI programs

- The MPI standard allows MPI_Send to behave in two different ways:
  - it can simply copy the message into an MPI managed buffer and return,
  - or it can block until the matching call to MPI_Recv starts.

Safety in MPI programs

- Many implementations of MPI set a threshold at which the system switches from buffering to blocking.
- Relatively small messages will be buffered by MPI_Send.
- Larger messages, will cause it to block.

Safety in MPI programs

- If the MPI_Send executed by each process blocks, no process will be able to start executing a call to MPI_Recv, and the program will hang or deadlock.

- Each process is blocked waiting for an event that will never happen.

(see pseudo-code)

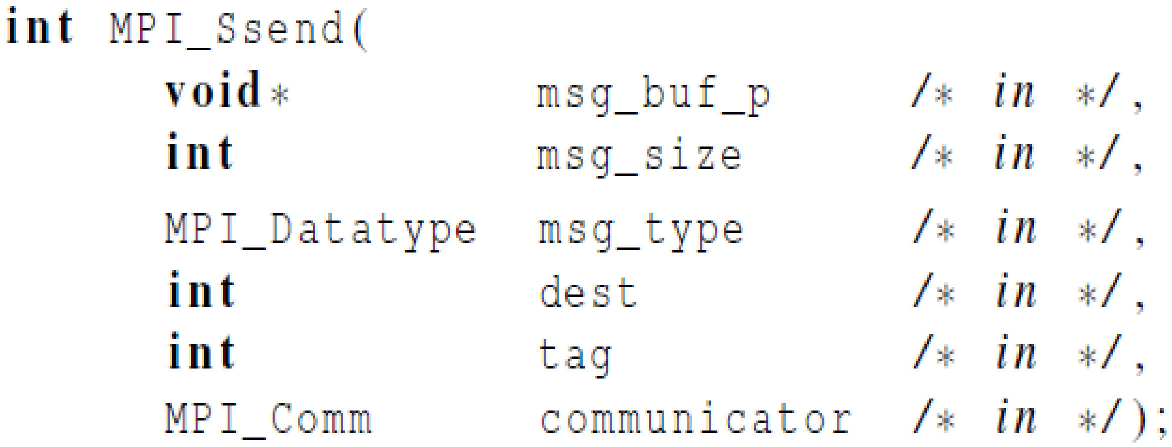
Safety in MPI programs

- A program that relies on MPI provided buffering is said to be unsafe.

- Such a program may run without problems for various sets of input, but it may hang or crash with other sets.

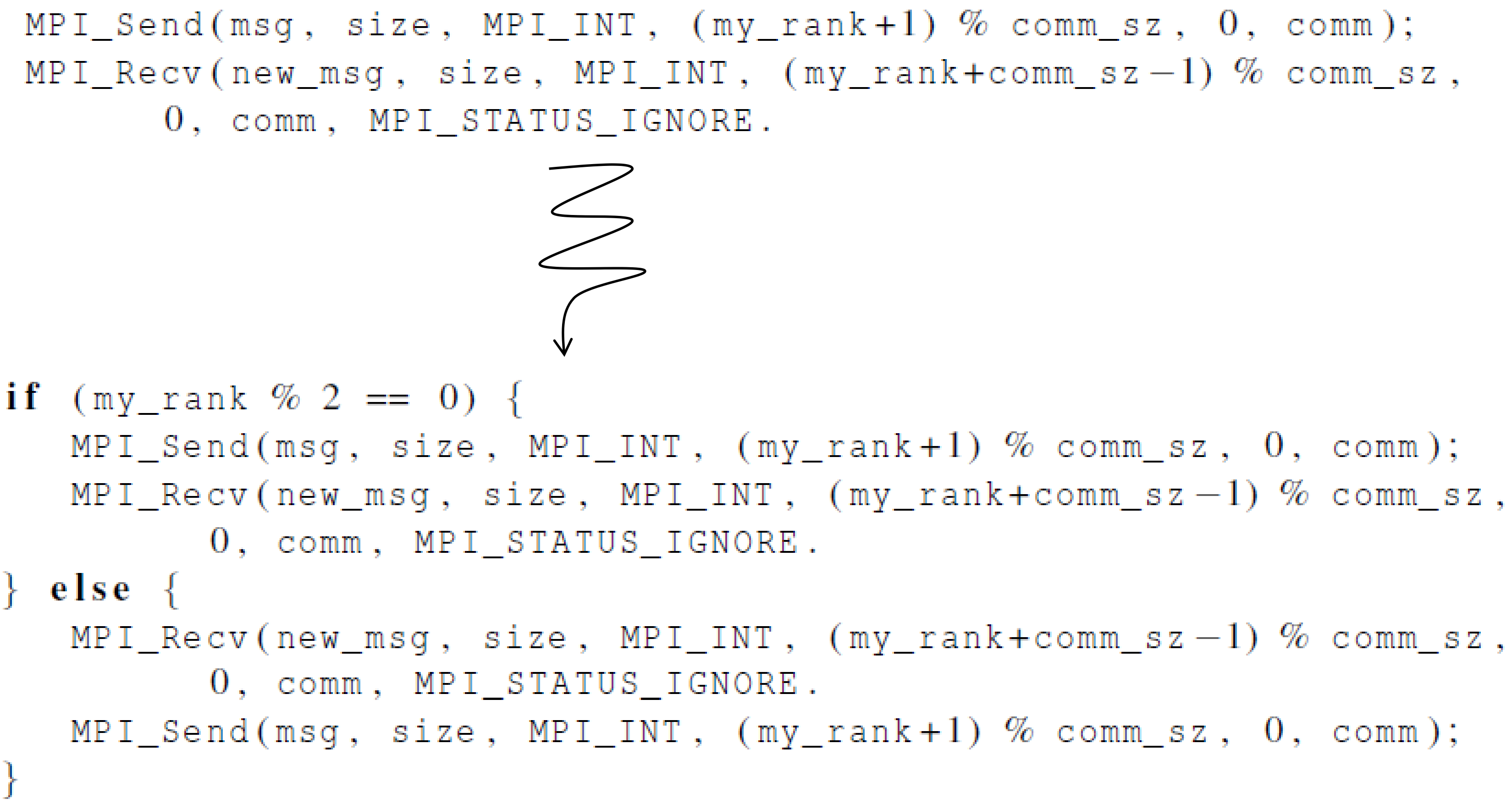
MPI_Ssend

- An alternative to MPI_Send defined by the MPI standard.
- The extra "s" stands for synchronous and MPI_Ssend is guaranteed to block until the matching receive starts.



```
int MPI_Ssend(
        void*          msg_buf_p      /* in */,
        int            msg_size       /* in */,
        MPI_Datatype   msg_type       /* in */,
        int            dest           /* in */,
        int            tag            /* in */,
        MPI_Comm       communicator   /* in */);
```

Restructuring communication



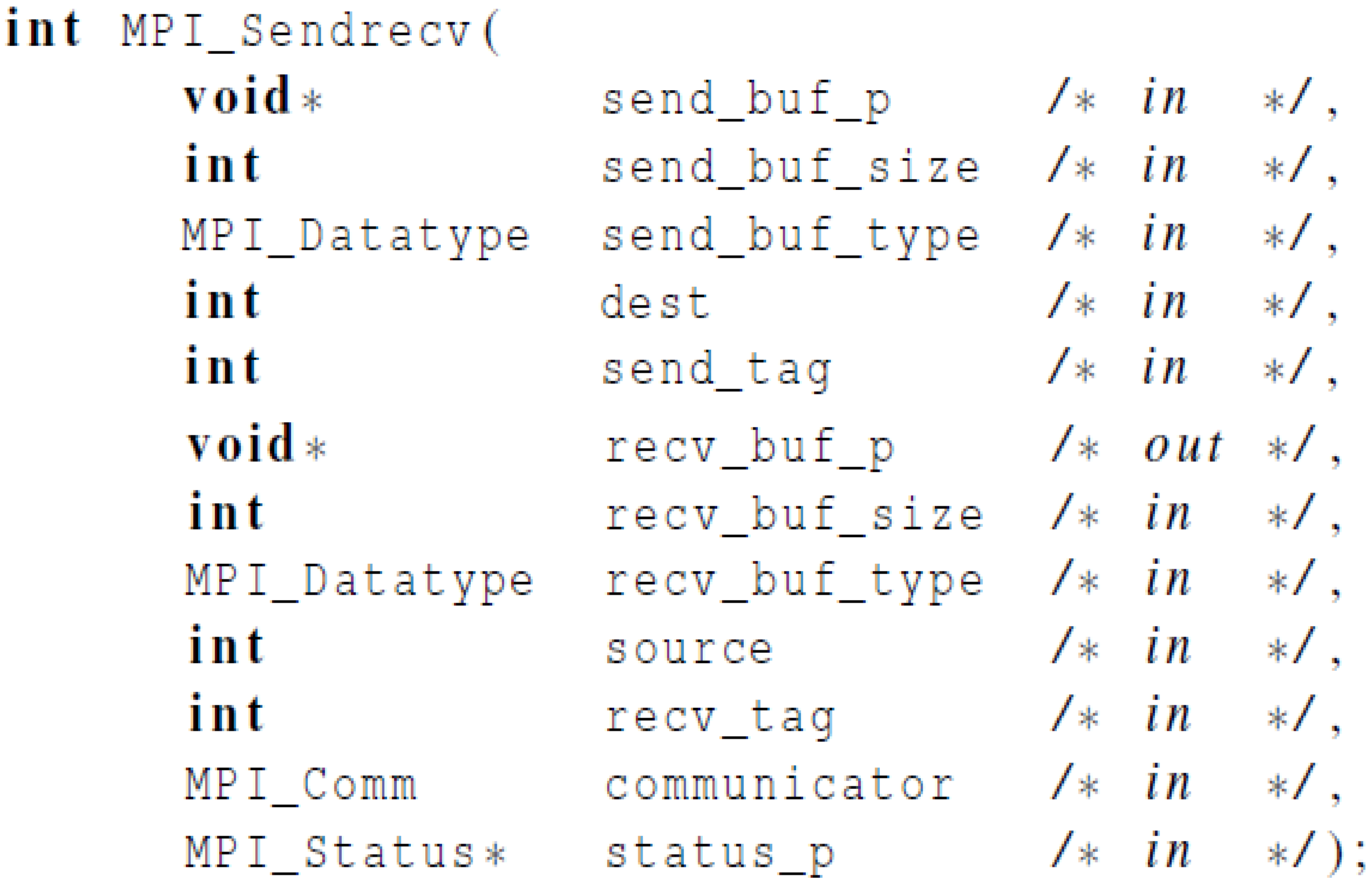
```
MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
      0, comm, MPI_STATUS_IGNORE.
```

```
if (my_rank % 2 == 0) {
   MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
   MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
         0, comm, MPI_STATUS_IGNORE.
} else {
   MPI_Recv(new_msg, size, MPI_INT, (my_rank+comm_sz-1) % comm_sz,
         0, comm, MPI_STATUS_IGNORE.
   MPI_Send(msg, size, MPI_INT, (my_rank+1) % comm_sz, 0, comm);
}
```

MPI_Sendrecv

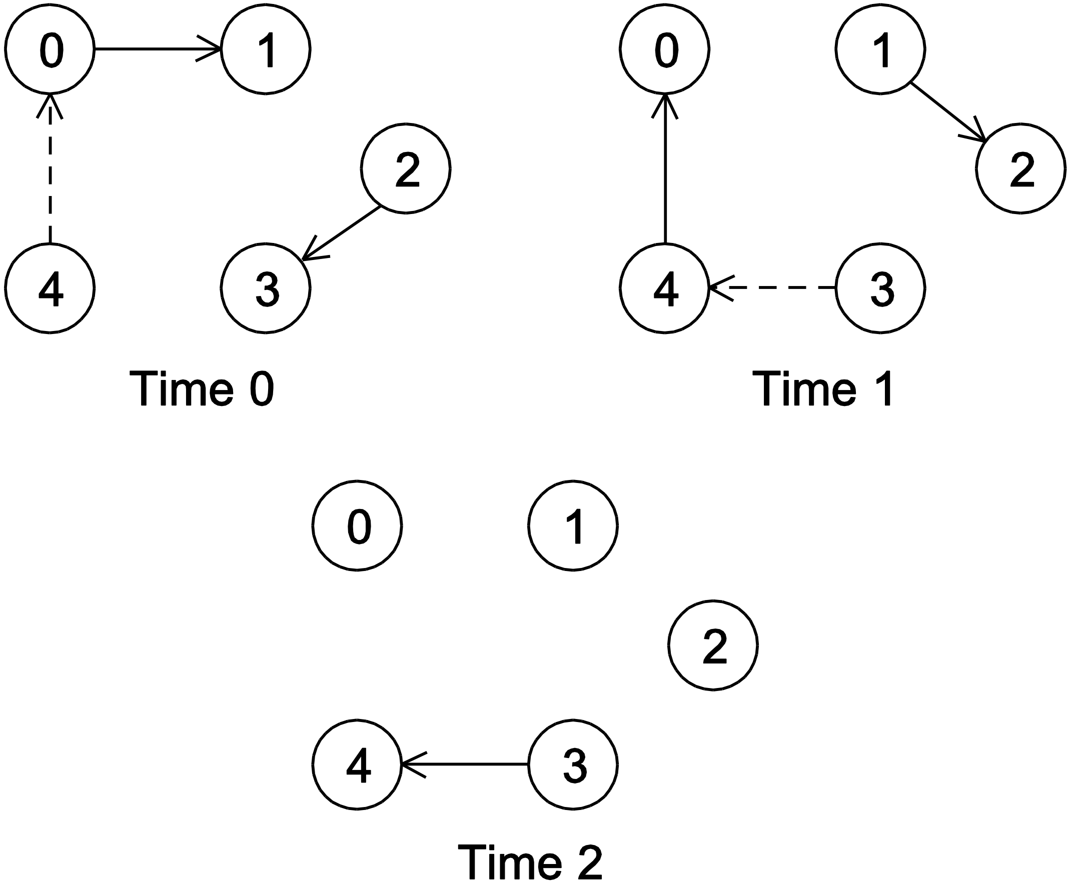
- An alternative to scheduling the communications ourselves.
- Carries out a blocking send and a receive in a single call.
- The dest and the source can be the same or different.
- Especially useful because MPI schedules the communications so that the program won’ t hang or crash.

MPI_Sendrecv



```
int MPI_Sendrecv(
      void*          send_buf_p      /* in   */,
      int            send_buf_size   /* in   */,
      MPI_Datatype   send_buf_type   /* in   */,
      int            dest            /* in   */,
      int            send_tag        /* in   */,
      void*          recv_buf_p      /* out  */,
      int            recv_buf_size   /* in   */,
      MPI_Datatype   recv_buf_type   /* in   */,
      int            source          /* in   */,
      int            recv_tag        /* in   */,
      MPI_Comm       communicator    /* in   */,
      MPI_Status*    status_p        /* in   */);
```

Safe communication with five processes

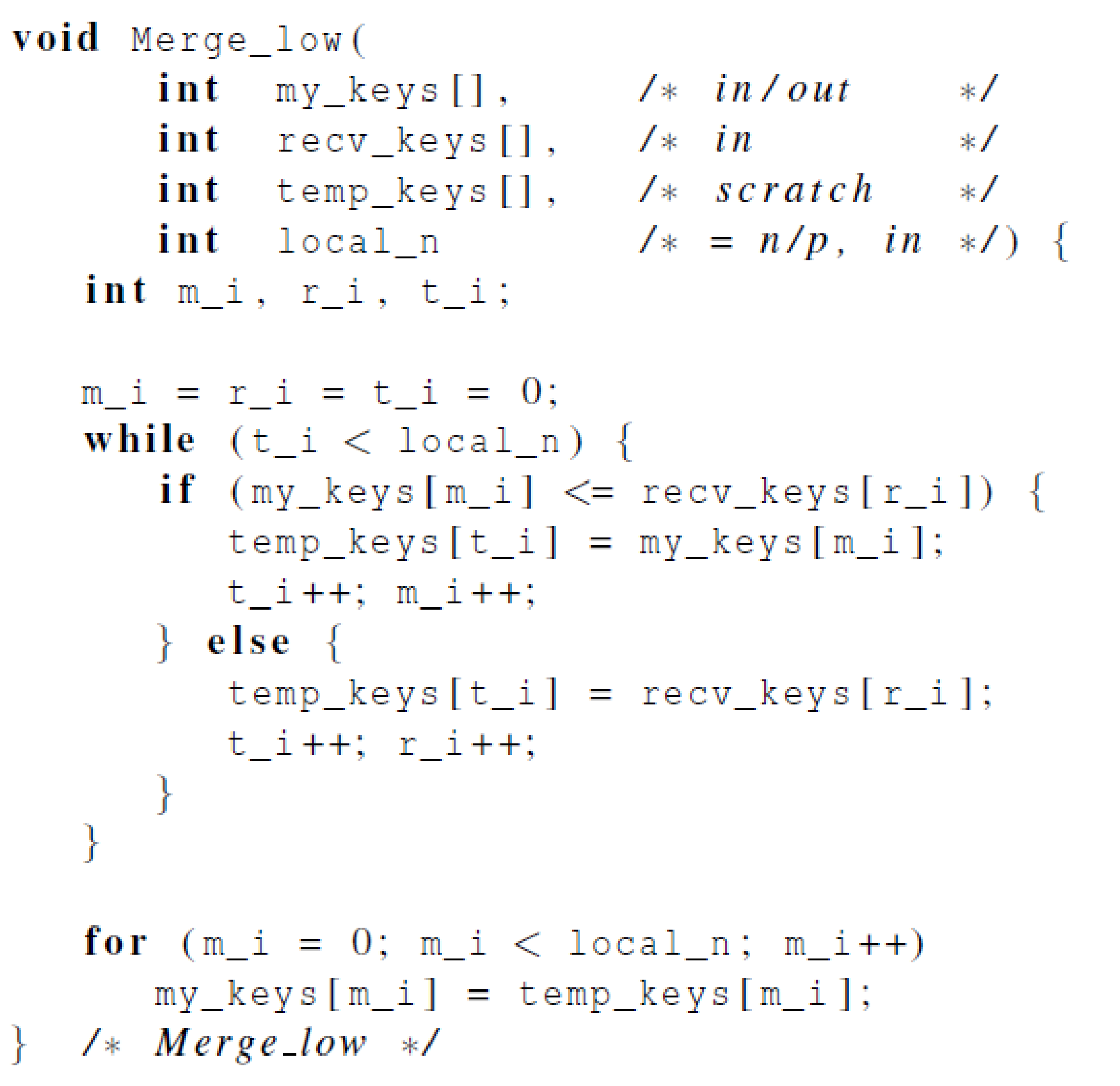


Time 0

Time 1

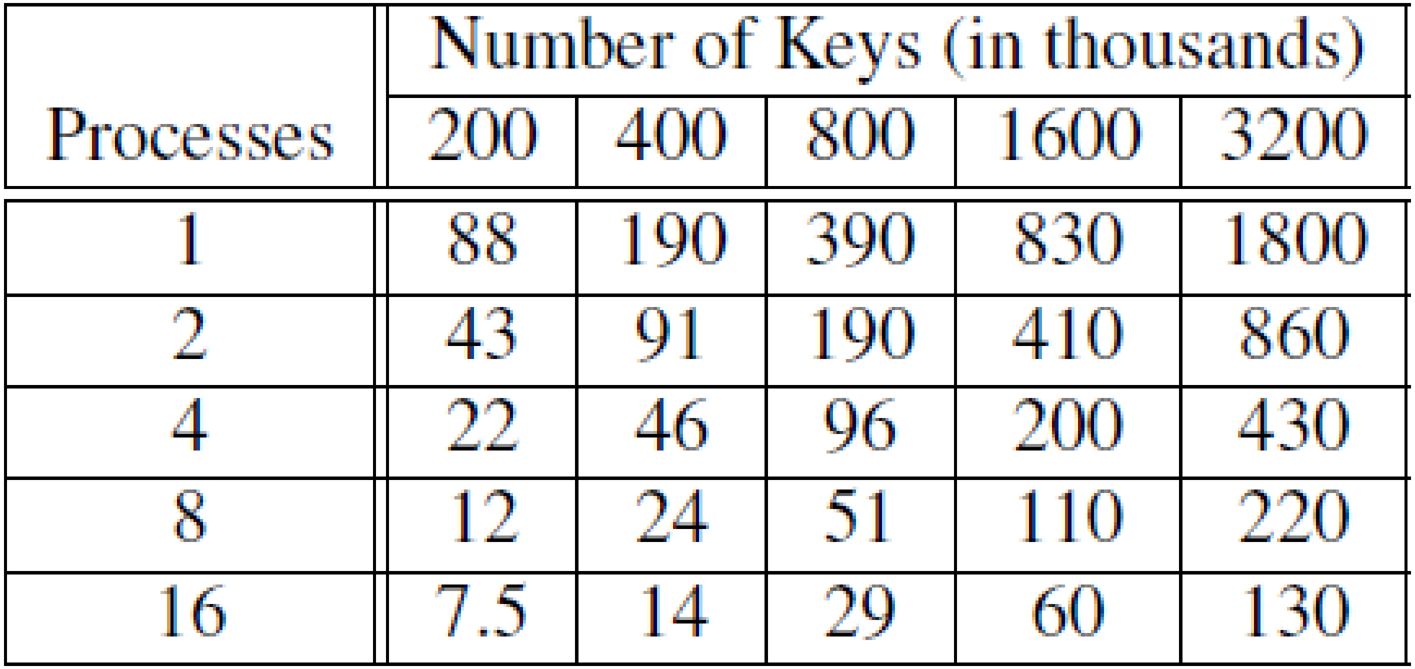
Time 2

Parallel odd-even transposition sort



```
void Merge_low(
        int  my_keys[],     /* in/out     */
        int  recv_keys[],   /* in         */
        int  temp_keys[],   /* scratch    */
        int  local_n        /* = n/p, in */) {
    int m_i, r_i, t_i;

    m_i = r_i = t_i = 0;
    while (t_i < local_n) {
        if (my_keys[m_i] <= recv_keys[r_i]) {
            temp_keys[t_i] = my_keys[m_i];
            t_i++; m_i++;
        } else {
            temp_keys[t_i] = recv_keys[r_i];
            t_i++; r_i++;
        }
    }

    for (m_i = 0; m_i < local_n; m_i++)
        my_keys[m_i] = temp_keys[m_i];
}  /* Merge_low */
```

Run-times of parallel odd-even sort



Processes	Number of Keys (in thousands)				
	200	400	800	1600	3200
1	88	190	390	830	1800
2	43	91	190	410	860
4	22	46	96	200	430
8	12	24	51	110	220
16	7.5	14	29	60	130

(times are in milliseconds)

Non-blocking Communication

- Call returns immediately without knowing if the operation has been completed.
- Allows you to overlap other computation while testing for the operation to be completed.
- Less possibility of deadlocking code.
- Used with MPI_Wait or MPI_Test.
-

Type of communication	MPI function
Non-blocking send Nob-blocking receive	MPI_Isend MPI_Irecv

MPI_Isend

```
int data[], count, dest, tag, err;
MPI_Datatype, type;  MPI_Comm comm;  MPI_Request request;
err = MPI_Isend( &data, count, type, dest, tag, comm, &request);
```

Data	*Data* which can be a scalar variable or an array.
count	An amount of data, if *count* >1, then *Data* must be an array.
type	Data type.
dest	CPU id which receive data.
tag	Message identifier.
comm	Communicator
request	The serial number of this transmission.

MPI_Irecv

```
int data[], count, src, tag, err;
MPI_Datatype type;  MPI_Comm comm;  MPI_Request request;
err = MPI_Irecv( &data, count, type, src, tag, comm, &request);
```

Data	*Data* which can be a scalar variable or an array.
count	An amount of data, if *count* >1, then *Data* must be an array.
type	Data type.
src	CPU id which send data.
tag	Message identifier.
comm	Communicator
request	The serial number of this transmission.

MPI_Wait

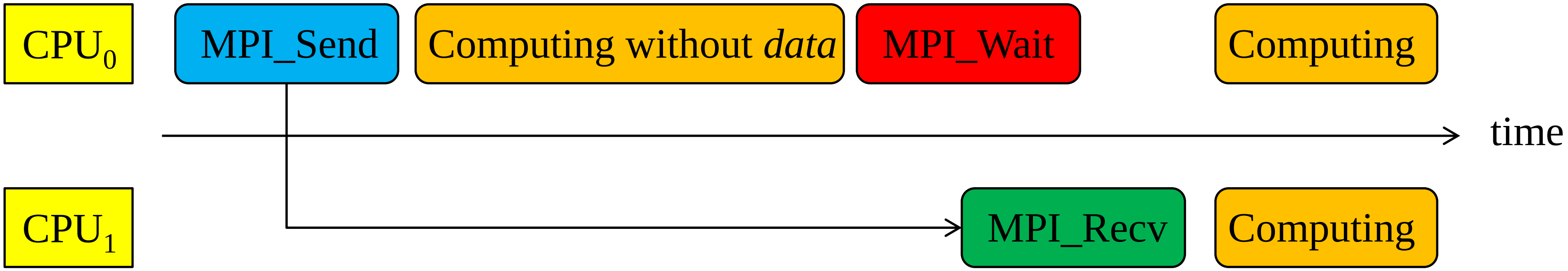
- Used for both sender and receiver of non-blocking communications.
- On the receive side, receiving process blocks until message is received, under programmer control.
- On the sending side, sending process blocks until send operation completes, at which time the message buffer is available for re-use.

```
MPI_Request request;  MPI_Status status;
int err = MPI_Wait( &request, &status);
```

request	The *request* of using by *Isend* and *Irecv*.
status	The result of transmission.

Non-blocking Communication

```
int data = 0, recv; MPI_Status stauts;  MPI_Request request;
if(id == 0)
  MPI_Isend(&data, 1, MPI_INTEGER, 1, 1, MPI_COMM_WORLD, & request);
else if(id == 1)
  MPI_Irecv(&recv, 1, MPI_INTEGER, 0, 1, MPI_COMM_WORLD, & request);
……    Must compute unrelated with data.
MPI_Wait(&request, &statuts);
……    Allow to compute with data.
```



Concluding Remarks (1)

- MPI or the Message-Passing Interface is a library of functions that can be called from C, C++, or Fortran programs.
- A communicator is a collection of processes that can send messages to each other.
- Many parallel programs use the single-program multiple data or SPMD approach.

Concluding Remarks (2)

- Most serial programs are deterministic: if we run the same program with the same input we'll get the same output.
- Parallel programs often don't possess this property.
- Collective communications involve all the processes in a communicator.

Concluding Remarks (3)

- When we time parallel programs, we’ re usually interested in elapsed time or “wall clock time” .
- Speedup is the ratio of the serial run-time to the parallel run-time.
- Efficiency is the speedup divided by the number of parallel processes.

Concluding Remarks (4)

- If it’ s possible to increase the problem size (n) so that the efficiency doesn’ t decrease as p is increased, a parallel program is said to be scalable.
- An MPI program is unsafe if its correct behavior depends on the fact that MPI_Send is buffering its input.