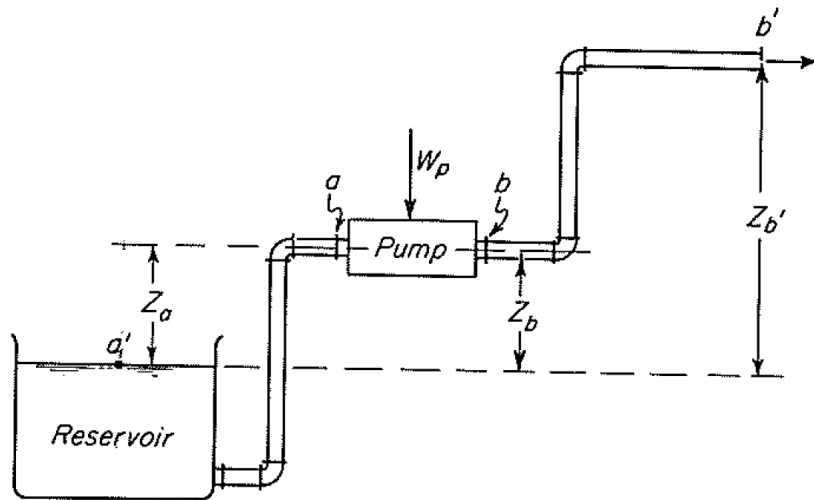


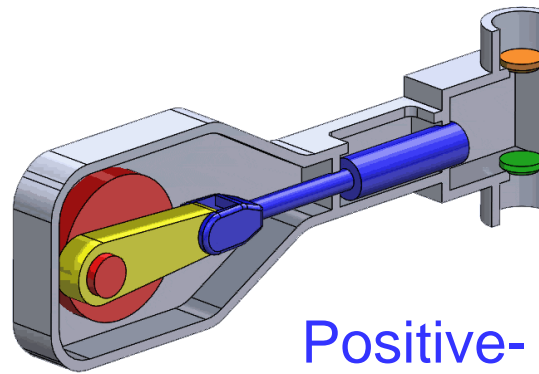
Transportation of fluids

- In this part, we will introduce pumps and compressors that used for transporting fluids.
- Pumps are used for liquid flow, and compressors are usually used for gas.
- It's cheaper to transport fluid than solid, thus solid is usually suspended in liquid or high-velocity gas stream.

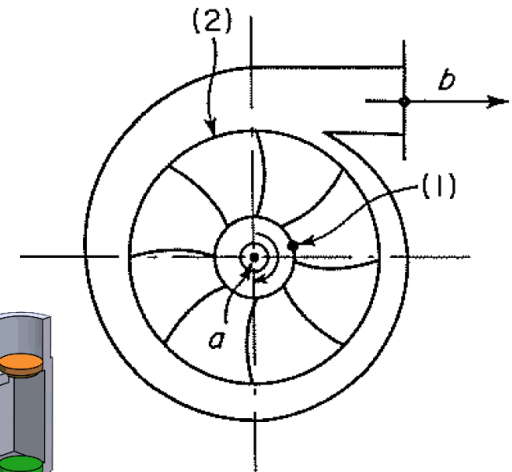


Power of the pump required?

NPSH?

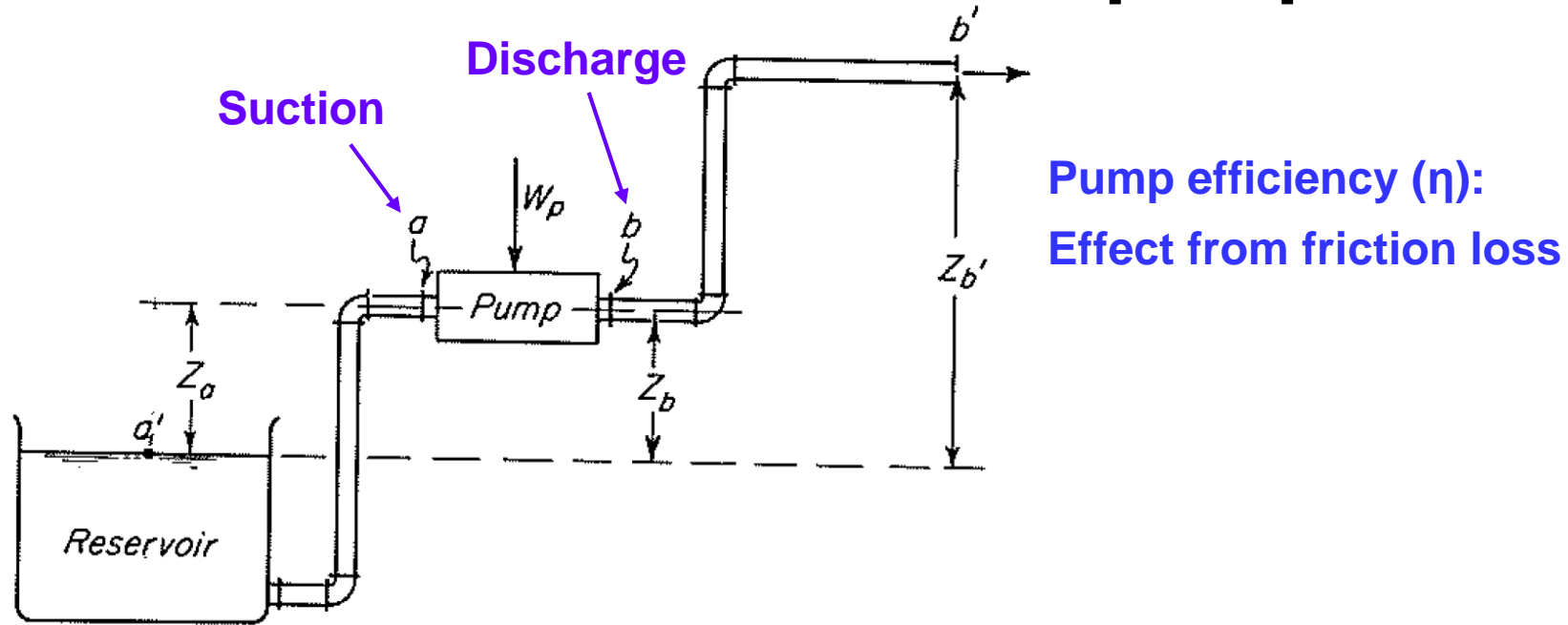


Positive-
displacement
pump



Centrifugal
pump

Power and virtual head of a pump



Recall:

$$\rho Av \left[F + \Delta\left(\frac{P}{\rho}\right) + \frac{1}{2} \Delta(\alpha v^2) + g \Delta z \right] = \text{Power required } (W_P)$$

- Let's define the system **from "a" to "b"**:

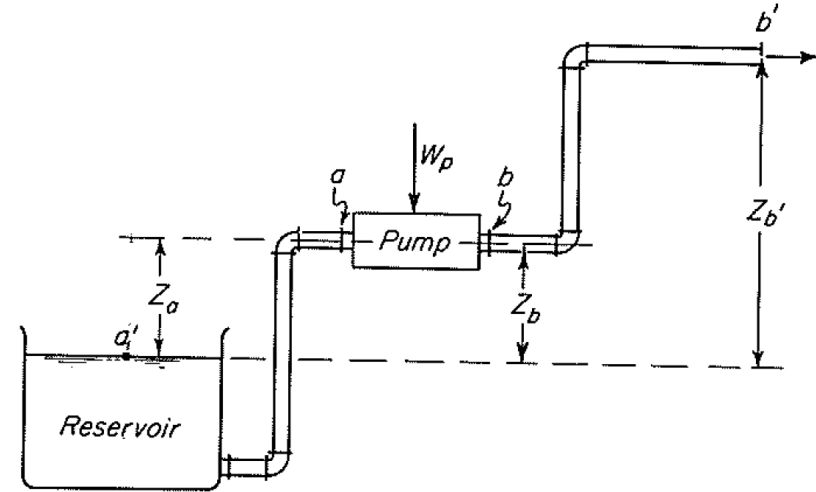
($\rho Av = G$ (mass flow rate))

$$\rho Av \left[\left(\frac{P_b - P_a}{\rho} \right) + \frac{\alpha_b v_b^2 - \alpha_a v_a^2}{2} + g(z_b - z_a) \right] = \eta W_P$$

$$\left(\frac{P_b - P_a}{\rho g} \right) + \frac{\alpha_b v_b^2 - \alpha_a v_a^2}{2g} + (z_b - z_a) = \text{virtual head of a pump } [m]$$

Net Positive Suction Head (NPSH)

- The suction pressure of a pump needs to be higher than the vapor pressure of the fluid. If not, the liquid may be vaporized inside the pump, a process called “**cavitation**”, which may damage the pump!



- Let's define the system from a' to a:

$$\left(\frac{F}{g} \right) + \Delta \left(\frac{P}{\rho g} \right) + \frac{1}{2g} \Delta(\alpha v^2) + \Delta z = 0; \quad h_f + \frac{P_a - P_{a'}}{\rho g} + \frac{\alpha_a v_a^2 - \cancel{\alpha_{a'} v_{a'}^2}}{2g} + Z_a = 0$$

No velocity at a'

H_f : friction head loss between a' and a [m]

$$\longrightarrow \underline{\frac{P_a}{\rho g} + \frac{\alpha_a v_a^2}{2g} - \frac{P_v}{\rho g}} = \frac{P_{a'}}{\rho g} - h_f - \Delta z - \frac{P_v}{\rho g} \equiv NPSH_{Available}$$

Suction pressure — vapor pressure
must be > 0

P_v : vapor pressure of the fluid

Net Positive Suction Head (NPSH)

To prevent cavitation in a pump:

$$NPSH_{Available} \equiv \frac{P_{a'}}{\rho g} - h_f - \Delta z - \frac{P_v}{\rho g} > 0$$

- In reality, the friction loss within the pump also needs to be considered. Thus, the “**NPSH_{required}**” (or “**NPSHR**”) is provided by the pump manufacturers for each pump. The value is around 2-3 m for small pump and can be up to 15 m for large pumps.

$$NPSH_{Available} > NPSHR$$

- To get high-enough $NPSH_{Available}$, a common strategy is to have a large enough negative Δz (by lifting the reservoir above the pump).

Example 8.1

Benzene at 37.8 °C is pumped through the system at the rate of 9.09 m³/h. The reservoir is at atmospheric pressure. The gauge pressure at the end of the discharge line is 345 kN/m². The discharge is 10 ft, and the pump suction is 4 ft above the level in the reservoir. The pipe has an inside diameter of 1.61 inch. The friction in the suction line is 3.45 kN/m², and that in the discharge line is 37.9 kN/m². The mechanical efficiency of the pump is 0.60. The density of benzene is 865 kg/m³, and its vapor pressure is 26.2 kN/m² at 37.8 °C.

Find: (a) The total power input.

(b) If the NPSHR of the pump is 3.05 m, will the pump be suitable for this process?

Solution:

Some notes:

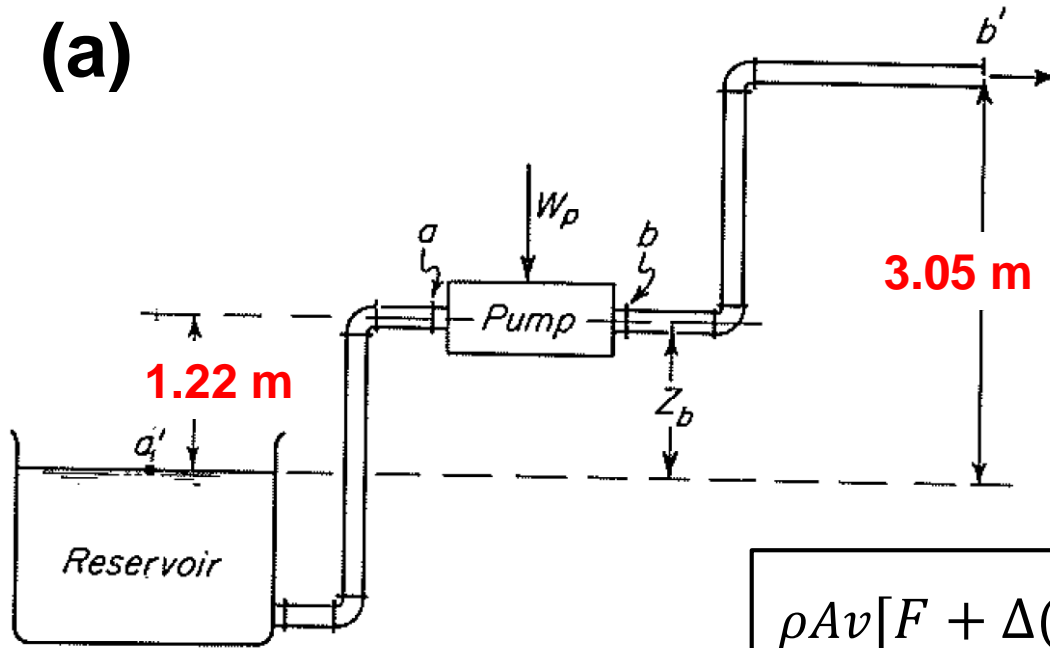
(1) Absolute pressure = Gauge pressure + Atmospheric pressure

(2) The unit of friction here is “kPa” → **F_p**

(3) **Plot the process first!**

Example 8.1

(a)



$$G = \rho A v = \frac{9.09 \times 865}{3600} = 2.18 \text{ (kg/s)}$$

$$v = \frac{2.18}{865(0.25\pi(0.041)^2)} = 1.91 \left(\frac{\text{m}}{\text{s}}\right)$$

$$v_{a'} = 0$$

$$P_{b'} - P_{a'} = 345000 \text{ (Pa)}$$

$$\rho A v \left[F + \Delta\left(\frac{P}{\rho}\right) + \frac{1}{2} \Delta(\alpha v^2) + g \Delta z \right] = \eta W_P$$

- Let's define the system from a' to b':

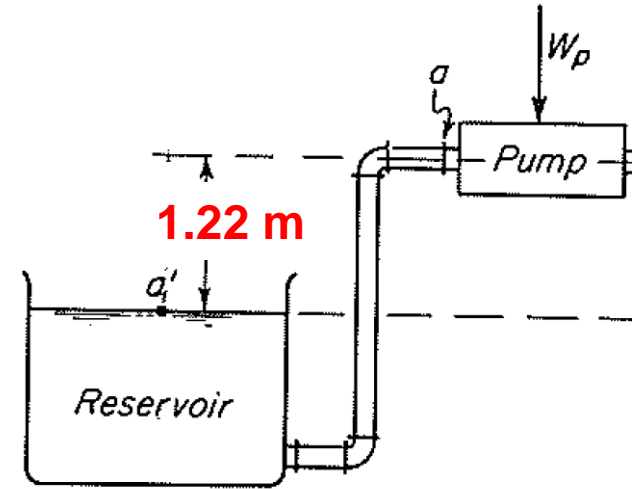
$$2.18 \left(\frac{3450 + 37900}{865} + \frac{345000}{865} + \frac{1(1.91)^2 - 0}{2} + 9.8 \times 3.05 \right] = 0.6 W_P$$

$$W_P = 1740 \text{ W}$$

Example 8.1

(b)

$$NPSH_{Available} = \frac{P_{a'}}{\rho g} - h_f - \Delta z - \frac{P_v}{\rho g}$$



$$NPSH_{Available} = \frac{101325}{865 \times 9.8} - \frac{3450}{865 \times 9.8} - 1.22 - \frac{26200}{865 \times 9.8} = 7.24 \text{ (m)}$$

$$(7.24 \text{ m} > 3.05 \text{ m})$$

Types of pumps

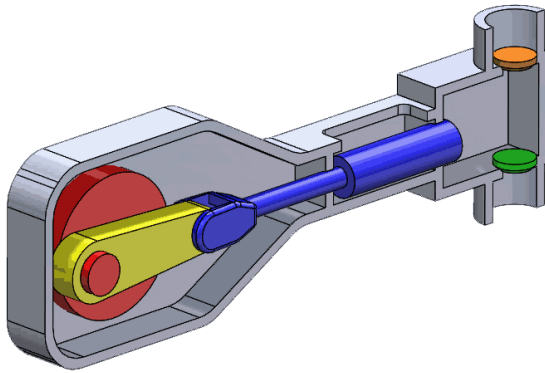
- Pumps can be classified into two categories:

(1) Positive-displacement pumps

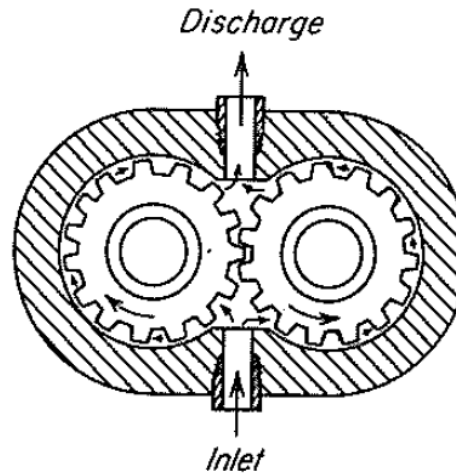
(2) Centrifugal pumps

- Positive-displacement pumps:

(1) Reciprocating pumps:

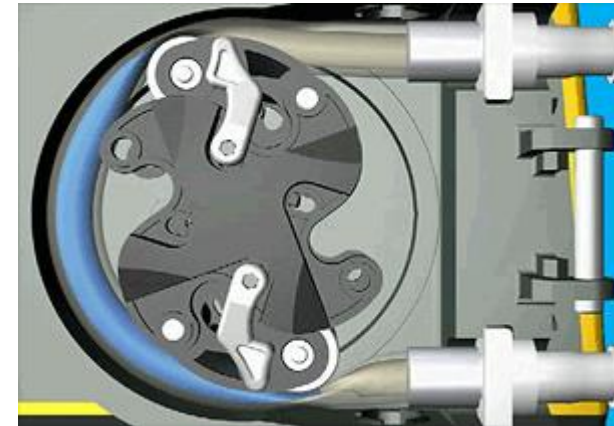


(2) Rotary pumps:



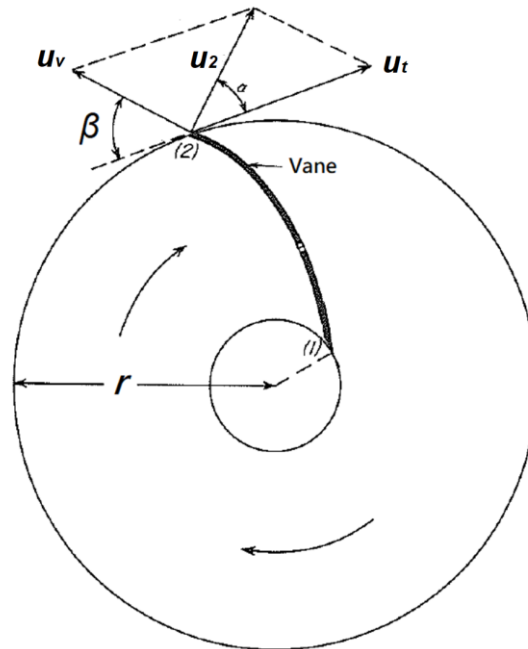
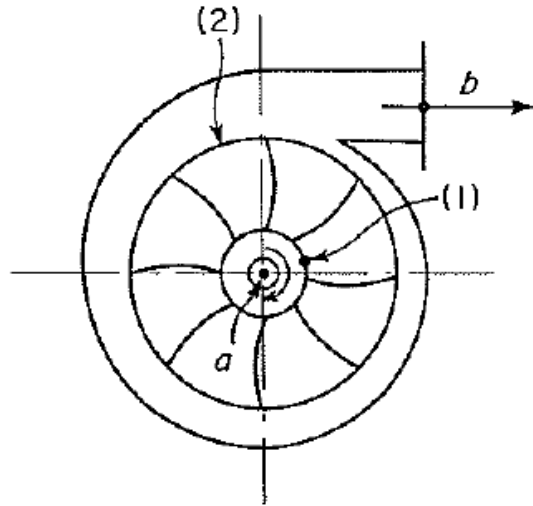
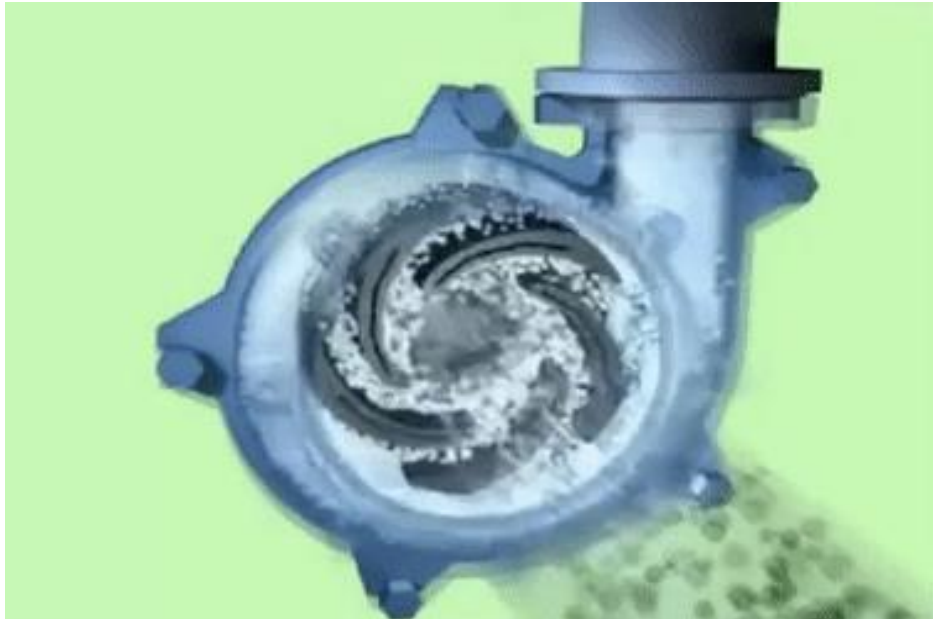
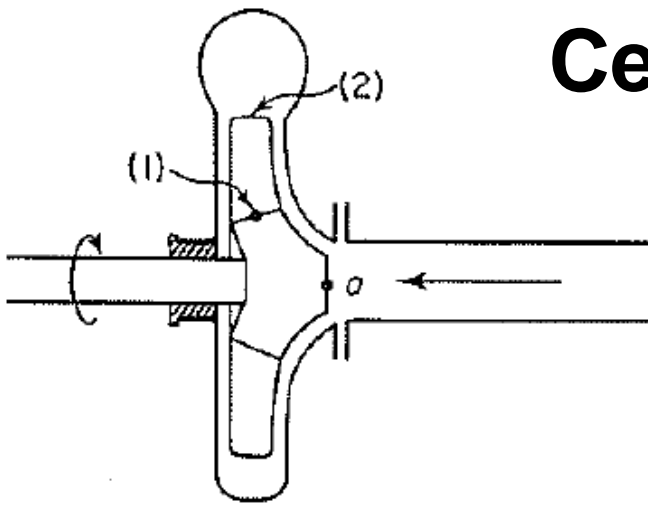
(Gear pump for
viscous fluid)

(3) Peristaltic pumps:



- Common for biochemicals that are not allowed to expose to air
- Only for small flow rates

Centrifugal pumps



- For unit mass of fluid:

$$W_p = \omega r_2 \left(u_t - \frac{Q}{A_p \tan \beta} \right)$$

“1”: Inlet

“2”: At the tip of the vane

Centrifugal pumps

Torque

$$d\tau = d(\text{force} \times r) = \frac{\partial [dm(ucos\theta)r]}{\partial t} = Q\rho \times d(urcos\theta)$$

$$\tau = Q\rho(u_2r_2cos\theta_2 - u_1r_1cos\theta_1)$$

$$\text{Power} = \tau\omega = \omega Q\rho(u_2r_2cos\theta_2 - u_1r_1cos\theta_1)$$

$[ML^2/t^2]$

$[1/t]$

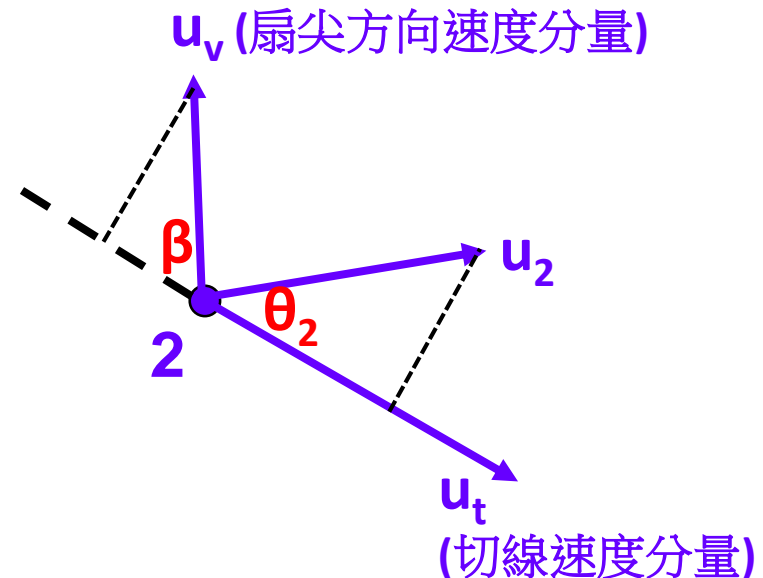
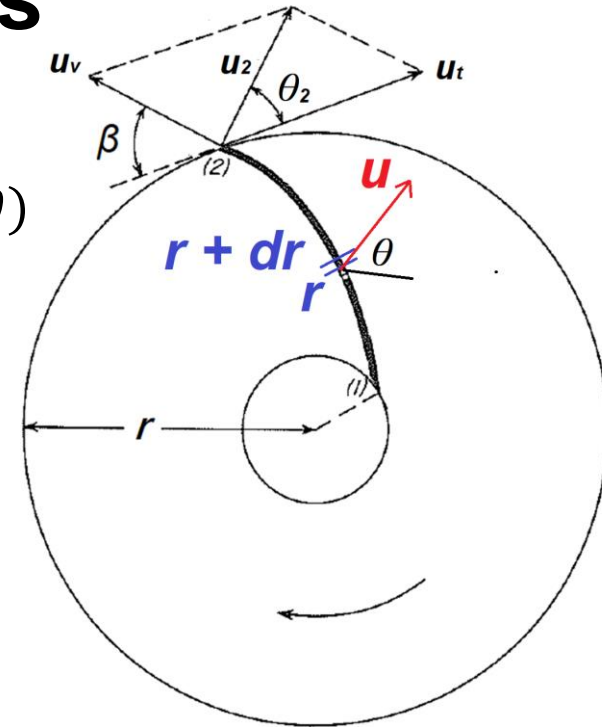
"1" is at the center (~0)

$$\text{Power} = \omega Q\rho u_2r_2cos\theta_2 \quad (1)$$

$$u_t = u_2cos\theta_2 + u_vcos\beta \quad (2)$$

$$u_t = \omega r_2 \quad (3)$$

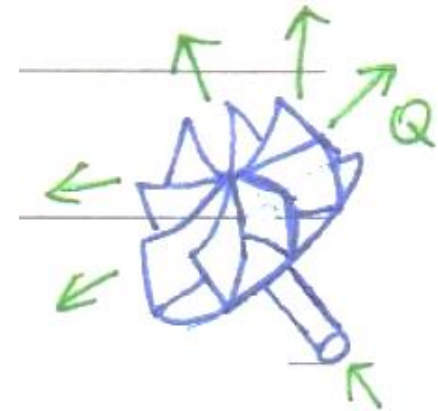
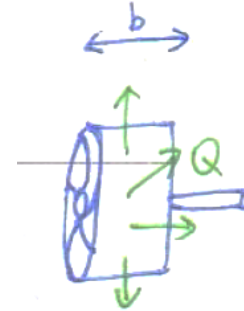
$$u_2sin\theta_2 = u_vsin\beta \quad (4)$$



Centrifugal pumps

$$Q = 2\pi r_2 b u_2 \sin\theta_2 \quad (5)$$

- From (1) to (5)...



$$\text{Power } (W_p) = Q\rho\omega u_2 r_2 \cos\theta_2 = Q\rho\omega r_2 (u_t - u_v \cos\beta)$$

$$= Q\rho\omega r_2 \left(\omega r_2 - \frac{u_2 \sin\theta_2}{\sin\beta} \cos\beta \right) = Q\rho\omega r_2 \left(\omega r_2 - \frac{Q}{2\pi r_2 b \tan\beta} \right)$$

$$= \underline{Q\rho} \left(\omega^2 r_2^2 - \frac{\omega Q}{2\pi b \tan\beta} \right)$$

Mass flow
rate

- Note:**

$$\omega = 2\pi f ;$$

f = revolution per second
($60f$ = RPM)

$$h \equiv \frac{W_p}{Q\rho g} = \frac{\omega^2 r_2^2}{g} - \frac{\omega Q}{2\pi b g \tan\beta}$$

**“Virtual head developed
by the pump”**

Centrifugal pumps – Key findings

$$\text{Power delivered by the pump } (W_p) = Q\rho \left(\omega^2 r_2^2 - \frac{\omega Q}{2\pi b \tan \beta} \right) \quad (\text{J/s})$$

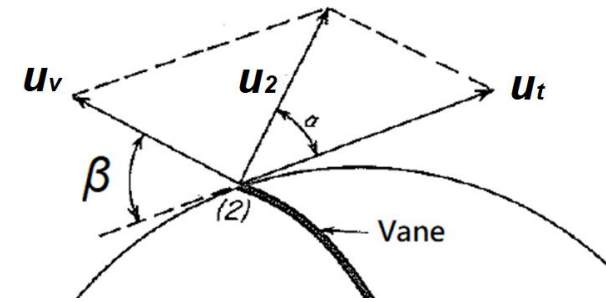
$$\text{Head developed by the pump } (h) = \frac{\omega^2 r_2^2}{g} - \frac{\omega Q}{2\pi b g \tan \beta} \quad (\text{m})$$

(1) The effect of β :

$\beta < 90^\circ \rightarrow$ Backward blades $\rightarrow \mathbf{Q\uparrow, h\downarrow}$

$\beta = 90^\circ \rightarrow$ Radical blades $\rightarrow \mathbf{h \neq f(Q)}$

$\beta > 90^\circ \rightarrow$ Forward blades $\rightarrow \mathbf{Q\uparrow, h\uparrow}$



Backward blades are commonly used; forward blades cause unstable flow!

(2) For the same pump:

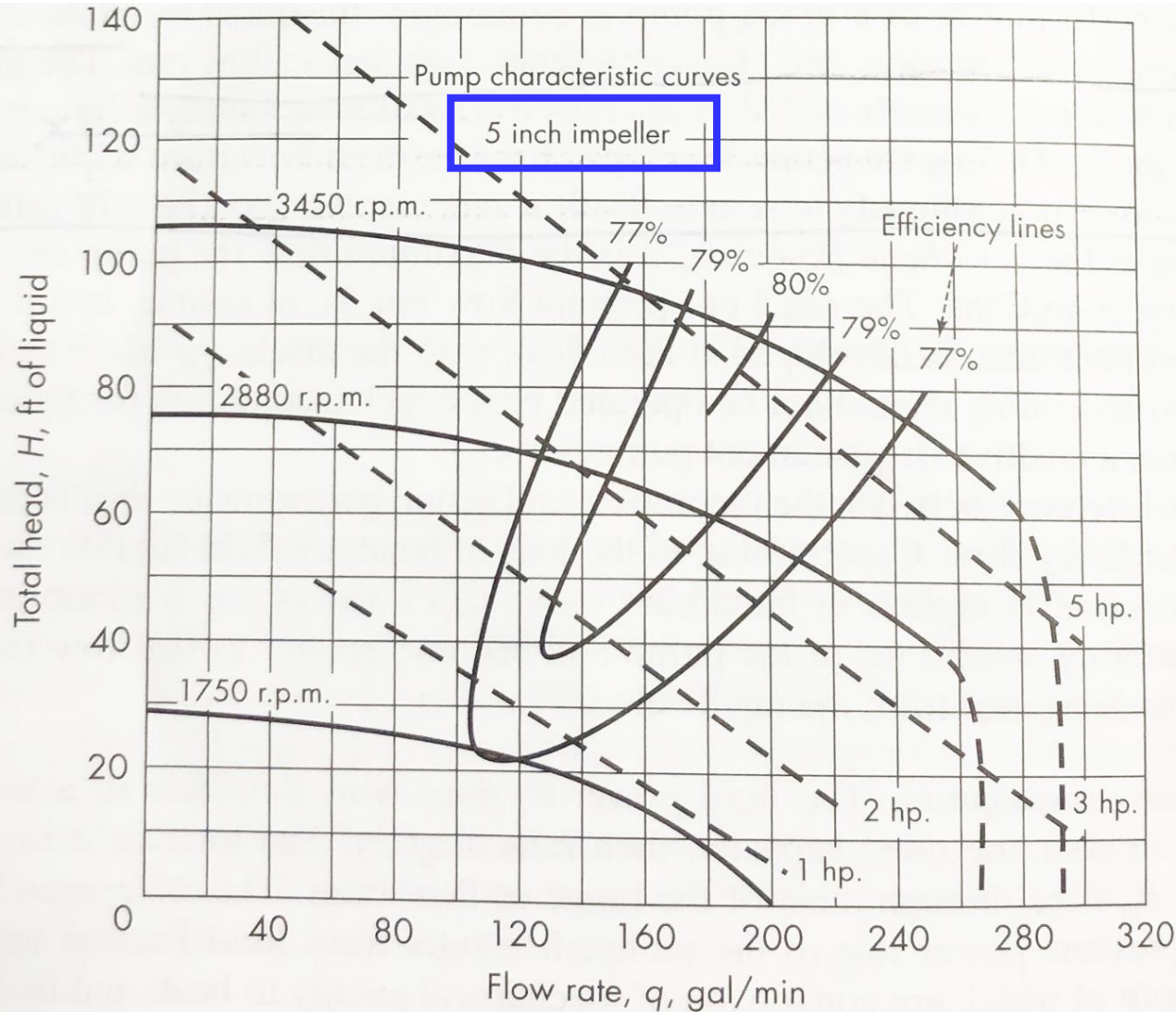
$$\mathbf{Q \propto \omega}$$

$$\mathbf{h \propto \omega^2}$$

$$\mathbf{W_p \propto \omega^3}$$

Centrifugal pumps – The efficiency

- The ideal pump head (h) plotted with volumetric flow rate (Q) should be a straight line, but in practice the head decreases a lot when Q is too large.



- Ex:

At 3450 rpm, the optimized Q is 200 gal/min, the total head is 88 ft, the power required is 5.5 hp, and the efficiency is 80%.

“Duty Point”

For gas transportation...

1. Fans: Low-velocity transportation of air; generated pressure < 0.04 atm
 2. Blowers: The maximum pressure is about 2 atm.
 3. Compressors: Above 2 atm
- Constant pressure can be assumed for centrifugal fans.

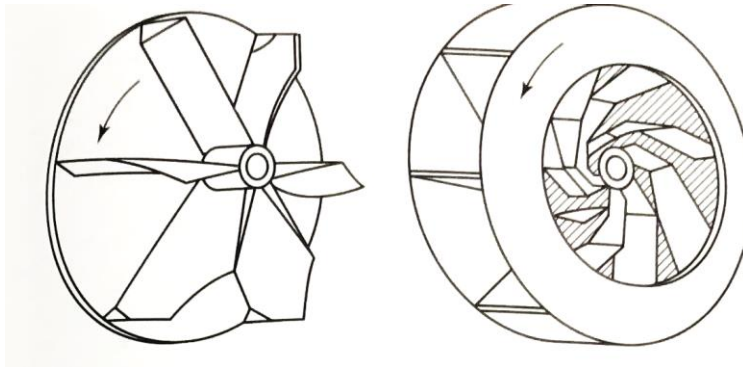


FIGURE 8.11
Impellers for centrifugal fans.

- For blowers and compressors, the thermodynamics of compressible fluids are necessary...

$$PV^{\frac{C_P}{C_V}} \equiv PV^\gamma = \text{constant}$$

Example 8.2

Question: A centrifugal fan is used to take flue gas at rest at 737 mmHg at 366 K and discharge it at 765 mmHg with a velocity of 45.7 m/s. Calculate the power needed to move 16990 std m³/h of gas. Use 101.32 kPa and 273 K as the standard conditions. The efficiency of the fan is 65%, and the molecular weight of the gas is 31.3.

Solution:

$$\rho_{in} = \frac{31.3 \times 10^{-3} \times P}{RT} = \frac{0.0313 \times 98260}{8.3145 \times 366} = 1.01 \text{ kg/m}^3$$

$$\rho_{out} = \frac{31.3 \times 10^{-3} \times P}{RT} = \frac{0.0313 \times 101990}{8.3145 \times 366} = 1.05 \text{ kg/m}^3$$

Assume: $\rho = 1.03 \text{ kg/m}^3$

$$G \left[F + \Delta \left(\frac{P}{\rho} \right) + \frac{1}{2} \Delta (\alpha v_{avg}^2) + g \Delta z \right] = \text{Power required}$$

Example 8.2

$$G = \rho_{std} Q = \frac{P(0.0313)}{RT} Q = \left(\frac{101325 \times 0.0313}{8.3145 \times 273} \right) \left(\frac{16990}{3600} \right) = 6.594 \left(\frac{kg}{s} \right)$$

$$\begin{aligned} \text{Power required} &= G \left[\left(\frac{101990 - 98260}{\rho} \right) + \frac{1}{2} (45.7)^2 \right] \\ &= 30770 \text{ (J/s)} \end{aligned}$$

$$\frac{30770}{0.65} = 47340 \left(\frac{J}{s} \right) = 47.34 \text{ kW}$$

Scaling laws for pumps/fans (WRF CH14)

Variable	Symbol	Dimensions
Head x g	gh	L^2/t^2
Viscosity	μ	M/Lt
Density	ρ	M/L^3
Flow rate	Q	L^3/t
Shaft speed	ω	$1/t$
Impeller diameter	D	L
Power	W	ML^2/t^3

- Let's choose D , ω , and ρ as the recurring set:

$$\longrightarrow \underline{D1 = \frac{gh}{D^2 \omega^2}}$$

Head coefficient
(C_H)

$$\underline{D2 = \frac{Q}{\omega D^3}}$$

Flow coefficient
(C_Q)

$$\underline{D3 = \frac{W}{\rho \omega^3 D^5}}$$

Power coefficient
(C_P)

$$D4 = \frac{\mu}{D^2 \omega \rho} \quad Re^{-1}$$

Scaling laws for pumps/fans (WRF CH14)

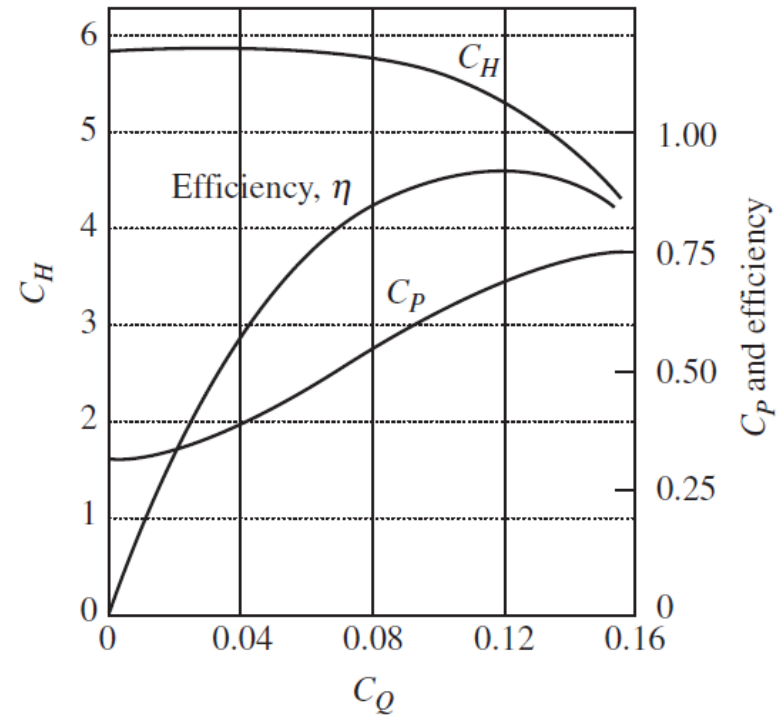
$$\underline{\eta = \frac{C_H C_Q}{C_P} = \frac{gh}{D^2 \omega^2} \frac{Q}{\omega D^3} \frac{\rho \omega^3 D^5}{W} = \frac{\rho gh Q}{W}}$$

- To scale up the pumps/fans with the same geometry:

$$\boxed{C_{H1} = C_{H2}} \quad \boxed{C_{Q1} = C_{Q2}} \quad \boxed{C_{P1} = C_{P2}}$$

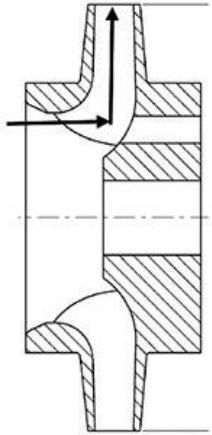


$$\frac{h_1}{D_1^2 \omega_1^2} = \frac{h_2}{D_2^2 \omega_2^2} \quad \frac{Q_1}{\omega_1 D_1^3} = \frac{Q_2}{\omega_2 D_2^3} \quad \frac{W_1}{\rho_1 \omega_1^3 D_1^5} = \frac{W_2}{\rho_2 \omega_2^3 D_2^5}$$

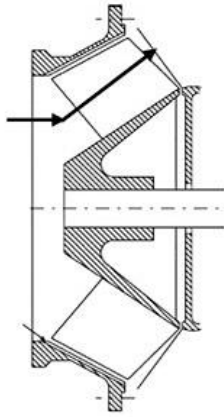


(See details in Example 3 in section 14.2)

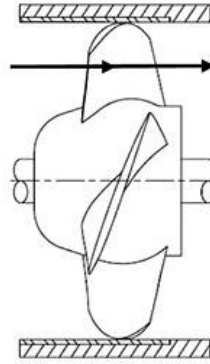
Selection of pumps with maximized efficiency



Centrifugal pump



Mixed-flow pump



Axial pump

$$N_s \equiv \frac{C_Q^{0.5}}{C_H^{0.75}} = \frac{\omega Q^{0.5}}{(gh)^{0.75}}$$

“Specific speed”

- High flow rate & low head
→ Axial pump
- Large head & low flow rate
→ Centrifugal pump

