June 8. 2013.



Ⅰ

(1) $\mu\phi$ : viscous work

(2) (a) Reduction the numbers of control parameters
(b) Establishment of similarity conditions.

(3) liquid metal 之 $Pr \ll 1 \Rightarrow \delta \ll \delta_T$

所以 $u_s = U_\infty$.

(4) ?

(5) 不可，$Re = \frac{惯性项}{黏滞项}$，在 Reynold number 很大时，大部分的黏滞项可忽略，但在边界层内的黏滞效应不可忽略

(6) 自然对流的 continuity equation、momentum equation、energy equation 中之 $\rho$ 會隨溫度 T 而变動，所以 very difficult to solved these equations

(7) $Gr^* = Gr \times Nu = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \times \frac{hL}{k} = \frac{g\beta L^4 h(T_w - T_\infty)}{k\nu^2} = \frac{g\beta L^4 q_w''}{k\nu^2}$ ($\because q_w'' = h(T_w - T_\infty)$)

(8) 在 $y = \delta$，$\frac{\partial u}{\partial y} = 0$，$\tau = \mu\frac{\partial u}{\partial y} = \mu \times 0 = 0$，故沒有 shear stress

在 $y = \delta_T$，$T = T_\infty$，$q'' = h(T_\infty - T_\infty) = 0$，故沒有 heat flux.

(9) Nu is constant in the fully-developed region for both the temperature and velocity fields.

因為溫度梯度与速度方向垂直，所以沒有熱对流.

(10) (1) 真空 (2) 無溫度梯度 (3) 速度 = 0 (4) 溫度梯度与速度方向垂直.

(11) 白努力定律 $P + \frac{1}{2}\rho V^2 + \rho g x = constant$，因為 $V = V(y)$，对 x 微分後 $\frac{dp}{dx} + 0 + \rho g = 0$

$\Rightarrow \frac{dp}{dx} = -\rho g$

$y \leftarrow\!\!\uparrow x$

(12) ?

0.V

Ⅱ. natural convection

(1) Free convection：流体因溫度的改变，导致 density 改变，而使物体力 $\rho g$ 改变而(自然对流) ~~影响~~ 產生 浮力 (buoyancy force)，导致流体流動.

〔易錯考題：强制对流 ( forced convection：The fluid flow is caused by external forces，例：电扇使空氣流動)

(2) Bulk fluid temperature：容積溫度，是通过管路流体的能量平均溫度 $T_m$

$T_m = \frac{1}{A_c V}\int_{A_c} uT\, dA_c$，又稱為 mass-averaged temperature, mixing-cup temperature.

June 8, 2013 page 2

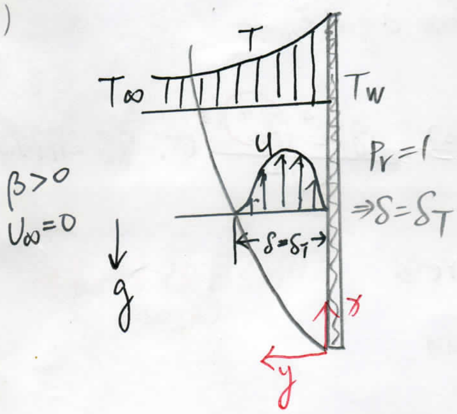
(3) Grashof number：$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2}$，是浮力與黏滯力的比值
它在自然對流的功用就類似 Reynold number 在強制對流的功用。

(4) Wind chill (temperature)：風寒效應，人体皮膚暴露在強風中，會感覺比當時氣溫更冷的效應。

(5) Boussinesq approximate：在自然對流的 governing equation 中，將密度 density 當作常數(constant)，除了浮力項(buoyancy term)之外

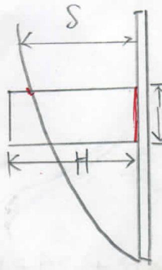
IV.

(1)



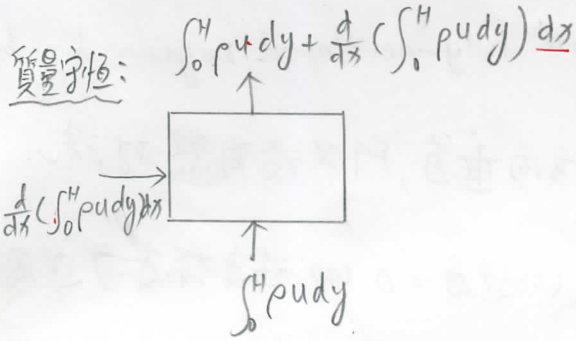

速度曲線：由於不滑動(no-slip)條件，使得平板壁上的速度=0，並逐漸增加至最大值，然後逐漸減小，直到邊界層邊緣，其速度又為零，因為自然對流中，流體是靜止的。

(2)

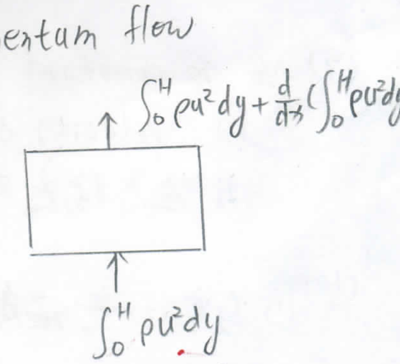



$H > \delta$

① 質量守恆：

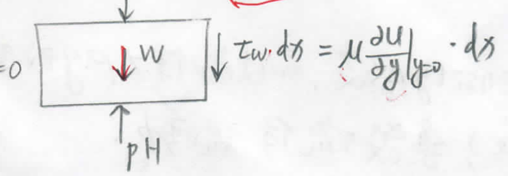



② momentum flow




$\frac{d}{dx}\left[\int_0^H \rho u^2 dy\right]dx \uparrow$ ---- (A)式

③ force balance




$W = \left(\int_0^H \rho g\, dy\right)dx$

$\frac{dp}{dx}\cdot H\,dx + W + \tau_w dx \downarrow$

$= \left(\frac{dp}{dx}H\right)dx + \left(\int_0^H \rho g\,dy\right)dx + \left(\mu \frac{\partial u}{\partial y}\Big|_{y=0}\right)dx \downarrow$ --- (B)式

(A)式 = (B)式

$\frac{d}{dx}\left(\int_0^H \rho u^2 dy\right)dx - \left(\frac{dp}{dx}H\right)dx - \left(\int_0^H \rho g\,dy\right)dx - \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

去掉 $dx$，並代入 $\frac{dp}{dx} = -\rho_\infty g$

$\frac{d}{dx}\left(\int_0^H \rho u^2 dy\right) = \rho_\infty g H - \int_0^H \rho g\,dy - \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

$= \int_0^H \rho_\infty g\,dy - \int_0^H \rho g\,dy - \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

$= \int_0^H (\rho_\infty - \rho) g\,dy - \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

將 $\rho_\infty - \rho = \rho\beta(T - T_\infty)$ 代入

得 $\frac{d}{dx}\left[\int_0^H \rho u^2 dy\right] = \int_0^H \rho\beta(T - T_\infty) g\,dy - \mu\frac{\partial u}{\partial y}\Big|_{y=0}$

III.

(1) if $\frac{T_w - T}{T_w - T_\infty}$ is invariant in the flow direction.

$$\frac{\partial}{\partial r}\left(\frac{T_w - T}{T_w - T_\infty}\right)_{r=r_0} = \text{constant}$$

$$\frac{-\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_\infty} = \text{constant}$$

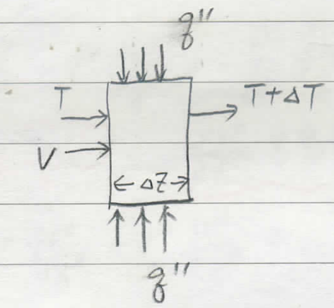
$T_w$, $T_\infty$ are function of $z$ only

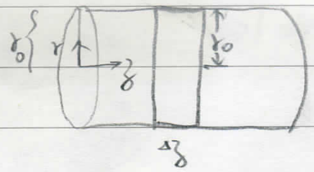
$$\Rightarrow \frac{\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_\infty} = \text{constant}$$

$$q'' = h(T_w - T_m) = k \left.\frac{\partial T}{\partial r}\right|_{r=r_0}$$

$$h = \frac{k \left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_m} = k \cdot \frac{\left.\frac{\partial T}{\partial r}\right|_{r=r_0}}{T_w - T_m} = k \cdot \text{constant} = \text{constant}.$$

(2)




$$\rho C V[(T+\Delta T) - T]\pi r_0^2 = q''(2\pi r_0)\Delta z$$

$$\rho c V \Delta T \cdot r_0 = 2 q'' \Delta z$$

$$\frac{\Delta T}{\Delta z} = \frac{2}{r_0}\frac{q''}{\rho c V}$$

$k \frac{\partial T}{\partial y}) \Delta y \Delta z$

$$\frac{\partial V}{\partial t} = (q_{conv1} - q_{conv2}) + (q_{cond1} - q_{cond2})$$

$$\frac{\partial V}{\partial t} = \frac{\partial(\rho c T)}{\partial t}\Delta y \Delta z$$

$$q_{conv1} = \rho c v T \Delta z$$

$$q_{conv2} = \left[\rho c v T + \frac{\partial}{\partial y}(\rho c v T)\Delta y\right]\Delta z$$

$$C\left[\frac{\partial(\rho T)}{\partial t} + \frac{\partial}{\partial y}(\rho v T)\right] = k\frac{\partial^2 T}{\partial y^2}$$

$$C\left[T\frac{\partial \rho}{\partial t} + \rho\frac{\partial T}{\partial t} + \rho v\frac{\partial T}{\partial y} + T\frac{\partial(\rho v)}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2}$$

$$CT\underbrace{\left[\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y}\right]}_{0} + c\rho\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} \quad \text{—①}$$

$\Downarrow$

因為 $y \ge \delta$, $T = T_\infty$ 且 $u=0$, 所以

$$\frac{d}{dx}\left[\int_0^\delta \rho u^2 dy\right] = -\mu \frac{\partial u}{\partial y}\Big|_{y=0} + \int_0^\delta \rho\beta g(T-T_\infty)dy$$

V.

(1) 因為 $\frac{\partial^2 u^*}{\partial x^{*2}}$ 之 order ≪ $\frac{L^2}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}$ 之 order, 所以 $\frac{\partial^2 u^*}{\partial x^*}$ 可省略

$$\Rightarrow \underbrace{\overset{O(1)}{u^*}\overset{O(1)}{\frac{\partial u^*}{\partial x^*}}}_{O(1)} + \underbrace{\overset{O(1)}{v^*}\overset{O(1)}{\frac{\partial u^*}{\partial y^*}}}_{O(1)} = \underbrace{-\overset{O(1)}{\frac{\partial p^*}{\partial x^*}}}_{O(1)} + \underbrace{\frac{1}{Re}\frac{L^2}{\delta^2}}_{O\left(\frac{1}{Re}\frac{L^2}{\delta^2}\right)}\underbrace{\overset{O(1)}{\frac{\partial^2 u^*}{\partial y^{*2}}}}_{O(1)}$$

上式各項之order為1

$\therefore O\left(\frac{1}{Re}\frac{L^2}{\delta^2}\right)$ 必須等於 $O(1)$

$\therefore \frac{1}{Re}\frac{L^2}{\delta^2} = 1$

(2)

$$u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = -\frac{L^2}{\delta^2}\frac{\partial p^*}{\partial y^*} + \frac{1}{Re}\left[\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{L^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}\right]$$

$\frac{\partial^2 v^*}{\partial x^{*2}} \ll \frac{L^2}{\delta^2}$

$\frac{\partial^2 v^*}{\partial x^{*2}}$ 可省略, 又 order$\left(\frac{L^2}{\delta^2}\right) \gg 1 \Rightarrow \frac{\partial p^*}{\partial y^*} = 0$

$$\Rightarrow u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = \frac{1}{Re}\frac{L^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}$$

set $\frac{1}{Re}\frac{L^2}{\delta^2} \simeq 1$

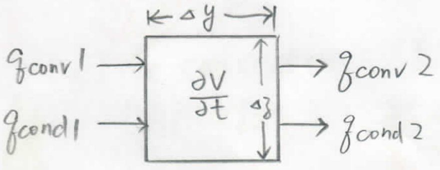
$$\Rightarrow u^*\frac{\partial v^*}{\partial x^*} + v^*\frac{\partial v^*}{\partial y^*} = \frac{\partial^2 v^*}{\partial y^{*2}}$$

(3) $y \to \infty$, $u = U_\infty$, $v=0$, $T = T_\infty$, $P = P_\infty$

apply boundary layer theorey.

$$\Rightarrow \begin{cases} y=\delta, & \frac{\partial u}{\partial y}=0,\ v=0,\ P=P_\infty \\ y=\delta_T, & \frac{\partial T}{\partial y}=0 \end{cases}$$

VI.




$$\frac{\partial v}{\partial t} = (q_{conv1} - q_{conv2}) + (q_{cond1} - q_{cond2})$$

$$\frac{\partial v}{\partial t} = \frac{\partial(\rho cT)}{\partial t}\Delta y\Delta z$$

$$q_{conv1} = \rho cvT\Delta z$$

$$q_{conv2} = \left[\rho cvT + \frac{\partial}{\partial y}(\rho cvT)\Delta y\right]\Delta z$$

$$q_{cond1} = -k\frac{\partial T}{\partial y}\Delta z$$

$$q_{cond2} = \left[-k\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left(-k\frac{\partial T}{\partial y}\right)\Delta y\right]\Delta z$$

$$\therefore \frac{\partial(\rho cT)}{\partial t}\Delta y\Delta z = -\frac{\partial}{\partial y}(\rho cvT)\Delta y\Delta z + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)\Delta y\Delta z$$

c, k is constant

$$c\left[\frac{\partial(\rho T)}{\partial t} + \frac{\partial}{\partial y}(\rho vT)\right] = k\frac{\partial^2 T}{\partial y^2}$$

$$c\left[T\frac{\partial \rho}{\partial t} + \rho\frac{\partial T}{\partial t} + \rho v\frac{\partial T}{\partial y} + T\frac{\partial(\rho v)}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2}$$

$$cT\underbrace{\left[\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial y}\right]}_{0} + c\rho\left[\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial y}\right] = k\frac{\partial^2 T}{\partial y^2} \quad \text{—①}$$

$\Downarrow$

连续方程式 $\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \vec{V} = 0$

$$\Rightarrow \frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial y} = 0 \quad \text{②}$$

②式代入①式，得

$$\rho c \left[ \frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} \right] = k \frac{\partial^2 T}{\partial y^2}$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c} \frac{\partial^2 T}{\partial y^2}$$

$$= \alpha \frac{\partial^2 T}{\partial y^2} \quad \left[ \alpha = \frac{k}{\rho c} \right]$$

VII. (与另一份题目相同)(2012)

(a) $\begin{cases} \frac{\partial u}{\partial x} = 0 \\ \frac{\partial}{\partial x} \left( \frac{T_w - T}{T_w - T_\infty} \right) = 0 \end{cases}$

VIII (与另一份题目相同)(2012)