$$b_{gn} = \frac{1}{l_2}\int_{-l_2}^{l_2} g(y)\sin\frac{n\pi}{l_2}y\,dy$$

$$\therefore f(x,y) = h(x)g(y) = \left[a_{h0} + \sum_{n=1}^{\infty}\left(a_{hn}\cos\frac{n\pi}{l_1}x + b_{hn}\sin\frac{n\pi}{l_1}x\right)\right]\cdot\left[a_{g0} + \sum_{n=1}^{\infty}\left(a_{gn}\cos\frac{n\pi}{l_2}y + b_{gn}\sin\frac{n\pi}{l_2}y\right)\right]$$

同理: $f(x,y,z) = h(x)\cdot g(y)\,k(z)$

$$k(z) = a_{k0} + \sum_{n=1}^{\infty}\left(a_{kn}\cos\frac{n\pi}{l_3}z + b_{kn}\sin\frac{n\pi}{l_3}z\right) \quad \begin{cases} a_{k0} = \frac{1}{2l_3}\int_{-l_3}^{l_3} k(z)\,dz \\ a_{kn} = \frac{1}{l_3}\int_{-l_3}^{l_3} k(z)\cos\frac{n\pi}{l_3}z\,dz \\ b_{kn} = \frac{1}{l_3}\int_{-l_3}^{l_3} k(z)\sin\frac{n\pi}{l_3}z\,dz \end{cases}$$

5. $2L \to \infty \Rightarrow L \to \infty$

$$f(x) = \frac{1}{2l}\int_{-l}^{l} f(u)\,du + \sum_{n=1}^{\infty}\left[\left(\frac{1}{l}\int_{-l}^{l} f(u)\cdot\cos\frac{n\pi}{l}u\,du\right)\cdot\cos\frac{n\pi}{L}x + \left(\frac{1}{l}\int_{-l}^{l} f(u)\sin\frac{n\pi}{l}u\,du\right)\sin\frac{n\pi}{L}x\right]$$

$$\frac{n\pi}{l} = \frac{2n\pi}{2l} = \frac{2\pi n}{T} = w\cdot n \equiv w_n \quad (n+1)\frac{\pi}{l} - n\frac{\pi}{l} = \frac{\pi}{l} \equiv \Delta w_n,\ \therefore \frac{1}{l} = \frac{\Delta w_n}{\pi}$$

$$\Rightarrow f(x) = \cancel{\frac{1}{2l}\int_{-\infty}^{\infty} f(u)\,du}^{\to 0}_{\text{finite}} + \frac{1}{\pi}\sum_{n=1}^{\infty}\left[\left(\int_{-\infty}^{\infty} f(u)\cos w_n u\,du\right)\cos w_n x + \left(\int_{-\infty}^{\infty} f(u)\sin w_n u\,du\right)\cdot\sin w_n x\right]\cdot\Delta w_n$$

$\because l \to \infty \quad \therefore \frac{1}{l} = \frac{\Delta w_n}{\pi} \Rightarrow \Delta w_n \simeq dw_n \to$ 黎曼和

$$f(x) = 0 + \frac{1}{\pi}\int_0^{\infty}\left(\int_{-\infty}^{\infty} f(u)\cdot\cos wu\,du\right)\cdot\cos wx\,dw + \frac{1}{\pi}\int_0^{\infty}\left(\int_{-\infty}^{\infty} f(u)\sin wu\,du\right)\sin wx\,dw$$

$$= \frac{1}{\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(u)\cdot\cos w(u-x)\,du\,dw, \quad \cos w(u-x) = \frac{1}{2}\left(e^{iw(u-x)} + e^{-iw(u-x)}\right)$$

$$= \frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(u)e^{iw(u-x)}\,du\,dw + \frac{1}{2\pi}\int_0^{\infty}\int_{-\infty}^{\infty} f(u)e^{-iw(u-x)}\,du\,dw$$

令 $w = -\Omega \quad dw = -d\Omega$ (dummy variable)

$$\Rightarrow \frac{1}{2\pi}\int_0^{-\infty}\int_{-\infty}^{\infty} f(u)e^{-i\Omega(u-x)}\,du(-d\Omega)$$

$$= \frac{1}{2\pi}\int_{-\infty}^{0}\int_{-\infty}^{\infty} f(u)e^{-iw(u-x)}\,du\,dw$$

$$\therefore f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(u)e^{-iwu}e^{iwx}\,du\,dw$$