Calculus

Exam #2
June, 2021 **Name:**______________________________________

1. This exam is due on June 22.

2. Be sure to show all work and explain your reasoning as clearly as possible.

Problem	**Points**	**Score**
1	25	
2	25	
3	25	
4	25	
Total	100	

1. **(25 points)** Please state the divergence theorem in R^3 and prove it.

2. **(25 points)** Please use Green's theorem to prove the following theorem:

   Let Γ be a Jordan region in the uv-plane with a piecewise-smooth boundary C_Γ. A vector function $r(u,v) = x(u,v)i + y(u,v)j$ maps Γ onto a region Ω of the xy-plane.
   r is one-to-one and C^2 function, and the Jacobian J is non-zero on the interior of Γ. Then

$$\text{Area of } \Omega = \int\int_\Gamma |J(u,v)| \; dudv$$

3. **(25 points)** Please sketch the region of the integration and evaluate the integrals.

   (a)

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-y^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} y^2\sqrt{x^2+y^2+z^2}dzdxdy$$

   (b)

$$\int_0^1\int_x^1 \cos(y^2)dydx$$

   (c)

$$\int_C (y+\sin x)dx + (z^2+\cos y)dy + x^3 dz,$$

   where C is the curve $\mathbf{r}(t) = (\sin t, \cos t, \sin 2t), \quad 0 \le t \le 2\pi$. (in the clockwise direction)

4. **(25 points)**

   (a) Please state the Mean Value Theorem for the double integral: $\int\int_D f(x,y)\; dxdy$. Here, $D = \{(x,y) : x^2+y^2 \le 3\}$ is a closed disk. And f is a continuous function.

   (b) Please prove this theorem.