

Midterm Exam II

November 4, 2017

Rules and Regulations: It is permitted to bring one additional paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

9. There are three problems and their corresponding solutions, respectively, given as follows. But, the provided solutions are incorrect. Please give the correct solutions.

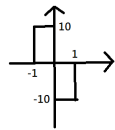
- (a) (5%) Please find the $x(t) = \frac{2}{1+t^2}$ Fourier transform by duality, known that,

$$e^{-|t|} \xleftrightarrow{F.T.} \frac{2}{1+\omega^2}$$

$$\frac{2}{1+t^2} \leftrightarrow \frac{e^{-|\omega|}}{2\pi}$$

$$X_2(j\omega) = \frac{e^{-|\omega|}}{2\pi}$$

- (b) (5%)



Find figure

Fourier transfer

$$X(j\omega) = \int_{-1}^1 x(t)e^{-j\omega t} dt = \int_{-1}^1 10e^{-j\omega t} dt = \frac{-10}{j\omega} e^{-j\omega t} \Big|_{-1}^1 = \frac{-10}{j\omega} (e^{-j\omega} - e^{j\omega}) = \frac{20}{\omega} \sin(\omega)$$

- (c) (5%) If $x(t)$ is real

$$\therefore a_k^* = a_{-k}$$

$$\therefore x(t) = a_0 + \sum_{k=1}^{\infty} [a_k e^{jk\omega_0 t} + a_{-k} e^{-jk\omega_0 t}]$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} [a_k e^{jk\omega_0 t} + a_k^* e^{-jk\omega_0 t}]$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2\Re[a_k e^{jk\omega_0 t}]$$

$$x(t) = a_0 + \sum_{k=1}^{\infty} 2a_k \cos(k\omega_0 t)$$