EXAM I
HEAT TRANSFER

May 15, 2015

I. The one-dimensional heat conduction equation can be written as

$$\rho C\frac{\partial T}{\partial t}=k\frac{\partial^2 T}{\partial x^2}$$

(1) What are $\rho C\frac{\partial T}{\partial t}$ and $k\frac{\partial^2 T}{\partial x^2}$? (4%)

(2) What are the units（單位）of C and k? (4%)

(3) In deriving this equation, what is the principle used? (2%)

II. Explain the following terms: (15%)

1. The zeroth law of thermodynamics.
2. The first law of thermodynamics.
3. Fourier law
4. Heat diffusion equation
5. Thermal diffusivity

III. Answer the following questions (36%)

1. In what conditions can the thermal resistance be applied?

2. What are the three material properties whose units are m^2/sec?

3. In what condition the lumped-heat-capacity system (the temperature in the solid body is uniform) can be applied in the analysis of a transient problem?

4. Which one has the higher thermal conductivity, wood or copper? How do you prove it?

5. From the view point of heat resistance, applying a fin to a high-power component would add thermal resistance to the component, which is similar to that wearing more clothes make us feel warmer. Why is it said that the fin can assist the heat dissipation（散熱）of the component?

6. How do people handle the boundary conditions of heat transfer at the interface between two different materials?

7. 傅立葉定律中之負號與那一個熱力學定律有關？請說明之。

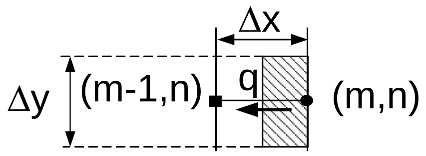
8. From the viewpoint of heat transfer, why is a child easier to catch cold than an adult when the weather turns cold?

9. What are the two kinds of error by using the finite difference to simulate the heat transfer problems?

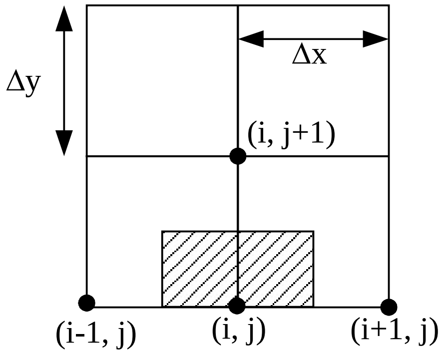
10. What’s the transport mechanism of heat conduction in solid?

11. 以熱傳觀點，冰箱為何需要除霜？

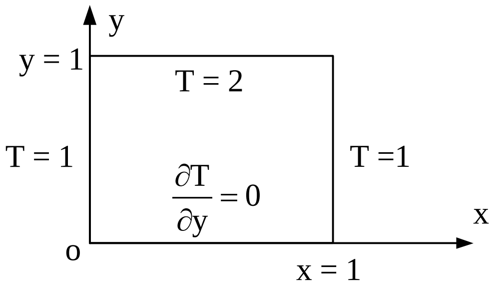
12. q is the heat transfer rate on the centeral plane between points (m, n) and (m-1, n), as shown in the following figure. Write down the finite-difference expression of q in terms of $T_{m-1,n}$ and $T_{m,n}$.



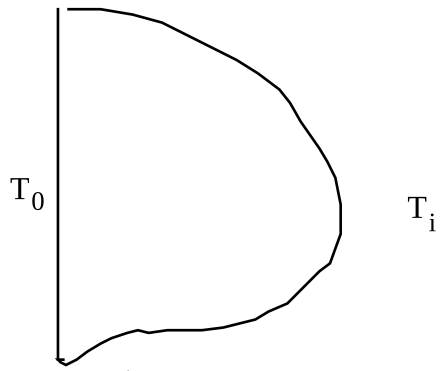
IV. Derive the finite difference equation of explicit formulation for a point (the point (i,j) shown in the following figure) on a heat-insulated boundary for a two-dimensional transient problem of heat transfer. (8%)



V. Consider a steady-state heat conduction problem of a rectangular plate (shown in the following figure). Its boundary conditions are indicated in the following figure. Find the temperature solution of the plate. (12%)

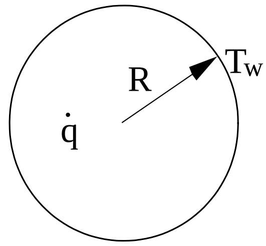


VI. Consider a semi-infinite solid shown in the following figure maintained at some initial temperature T_i. The surface temperature is suddenly lowered and maintain at a temperature T_0.



Solve the temperature distribution for this transient problem. (12%)

VII. A one-dimensional steady heat transfer problem of a cylinder with a uniform heat source $\dot{q}$ is shown in the following figure. What is the heat flux on the cylinder surface (i.e., at r = R)? (8%)



Hint: $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$

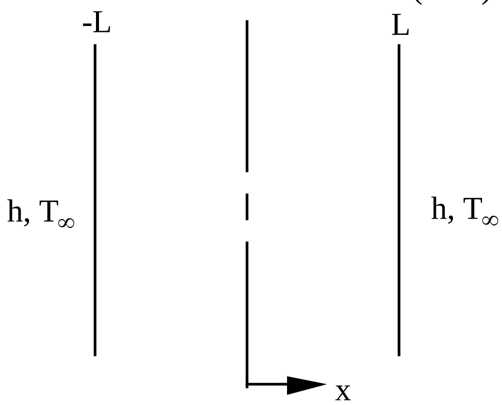
VIII. A plane wall of thickness 2L, thermal conductivity k, and diffusivity α (shown in the following figure) is in thermal equilibrium with a surrounding fluid at T_∞. Suddenly, constant uniform generation (q''', W/m^3) begins in the wall. The fluid convective environment is such that h >> k/L. The governing heat conduction equation is

$$\frac{\partial^2 T}{\partial x^2}=\frac{1}{\alpha}\frac{\partial T}{\partial t}-\frac{q'''}{k}$$

By using superposition, separation of variables, and appropriate boundary and initial conditions, obtain an expression for the transient, non-dimensional temperature profile

$$\frac{T(x^*,t^*)-T_\infty}{L^2 q'''/k}$$

where $t^* = t\alpha/L^2$ and $x^* = x/L$. (15%)



Hint:

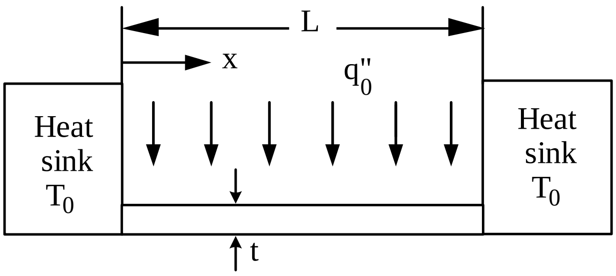
(1) Non-dimensionalize the governing equation and initial and boundary conditions with

$$\theta=\frac{T-T_\infty}{L^2 q'''/k},\ t^* = t\alpha/L^2 \text{ and } x^* = x/L.$$

(2) Assume $\theta = \theta_{ss}(x^*) + \theta_t(x^*, t^*)$, where θ_{ss} is the steady solution and θ_t is the transient solution which will be approaching zero as time is close to the infinity.

XI. A truncated cone 30 cm high is constructed of aluminum. The diameter at the top is 7.5 cm, and the diameter at the bottom is 12.5 cm. The lower surface is maintained at 93 °C; the upper surface, at 540 °C. The other surface is insulated. Assuming one-dimensional steady heat flow, what is the rate of heat transfer in watts. k = 204 W/m°C. (8%)

X. A thin flat plate of length L, thickness t, and width W >> L is thermally joined to two large heat sinks that are maintained at a temperature T_0. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q''_0 .



(a) Derive the differential equation that determines the steady-state temperature distribution T(x) in the plate. (7%)

(b) Solve the foregoing equation for the temperature distribution for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the sink. (8%)