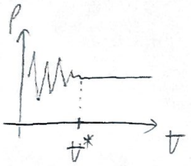
Student No.					of Instructor	
院系 College	工 學院 College 工科 系 Department 三 年 Year 班 Class	評閱成績 Score	95			

(1)

(a) 在尺度很小時，觀察到的流體密度會很不穩定，放大尺度到一定程度後，密度才會呈穩定的值，且 Knudsen number $K_n = \frac{\lambda}{L} < 0.1$，在此情況下可忽略流體的分子特性，稱為連續體(continuum)。 +5



尺度超過 V^* 後，密度 ρ 穩定 ✓

(b) 滿足 $\tau \propto \frac{du}{dy}$ 關係的流體，且 τ 與 $\frac{du}{dy}$ 的比值即為黏度 μ。線性 ✓ +4

(c) 液體內分子與分子間的作用力較強，而液體分子對空氣分子的作用力較弱，導致液體分子在液體與空氣的接觸面上的受力不平衡，因而產生表面張力(surface tension)。✓ +5

(2) +15

$\vec{V} = Ax\hat{i} + 2Ay\hat{j}$

Streamline: $\frac{dy}{dx} = \frac{v}{u}$ ✓

$\frac{dy}{dx} = \frac{2Ay}{Ax} = \frac{2y}{x}$

$\frac{dy}{y} = 2 \times \frac{dx}{x}$

$\int \frac{dy}{y} = 2\int \frac{dx}{x}$

$\ln y = 2\times(\ln x + C_1) = 2\ln x + C = \ln x^2 + C$

$y = e^{\ln x^2 + C} = C \times e^{\ln x^2} = Cx^2$

$(x,y) = (2,2) \rightarrow y = 2 = C\times 4,\ C = \frac{1}{2}$

$y = \frac{1}{2}x^2 \Big|_{\text{streamline}}$ ✓

Pathline: $\frac{dx}{dt} = Ax$ ✓ $\quad \frac{dy}{dt} = 2Ay$ ✓

$\frac{dx}{x} = A\,dt \qquad \frac{dy}{y} = 2A\,dt$

$\int\frac{dx}{x} = \int A\,dt \qquad \int\frac{dy}{y} = \int 2A\,dt$

$\ln x = At + C_1 \qquad \ln y = 2At + C_2$

$x = e^{At+C_1} = C_1 \times e^{At} \qquad y = e^{2At+C_2} = C_2 \times e^{2At}$

$y = C_2 \times e^{2At} = \frac{C_2}{C_1^2} \times C_1^2 \times e^{2At} = \frac{C_2}{C_1^2}x^2$

$(x,y) = (2,2)$ at $t=0 \rightarrow x = 2 = C_1 \times e^{A\times 0} = C_1,\ C_1 = 2$

$y = 2 = C_2 \times e^{2A\times 0} = C_2,\ C_2 = 2$

$y = \frac{C_2}{C_1^2}x^2 = \frac{2}{4}x^2 = \frac{1}{2}x^2,\quad y = \frac{1}{2}x^2 \Big|_{\text{Pathline}}$ ✓

Pathline: $y = \frac{1}{2}x^2$, streamline: $y = \frac{1}{2}x^2$，兩者相同 ✓

~~(3)~~ +13

$\vec{V} = ax\hat{i} + by\hat{j},\quad d\vec{A} = dydz\,\hat{i} + dxdy\,\hat{k}$

Volume flow rate: $\int_{cs} \vec{V}\cdot d\vec{A}$

$\iint (ax\hat{i} + by\hat{j})\cdot(dydz\,\hat{i} + dxdy\,\hat{k})$

$= \iint ax\,dydz$

$= a \times \int_0^3\int_0^5 x\,dy\,dz$

$= a \times \int_0^3 5x\,dz \quad (x = 4 - \frac{4}{3}z)$

$= 5a \times \int_0^3 4 - \frac{4}{3}z\,dz$

$= 5a \times \left(\int_0^3 4\,dz - \int_0^3 \frac{4}{3}z\,dz\right)$

$= 5a \times \left(4z\Big|_0^3 - \frac{2}{3}z^2\Big|_0^3\right)$

$= 5a \times (12 - 6) = 30a$

Momentum flux: $\int_{cs} \vec{V}\rho\vec{V}\cdot d\vec{A}$

$\iint \rho(ax\hat{i} + by\hat{j})\times(ax\hat{i} + by\hat{j})\cdot(dydz\,\hat{i} + dxdy\,\hat{k})$

$= \iint \rho(a^2x^2\hat{i}\hat{i} + 2axby\,\hat{i}\hat{j} + b^2y^2\hat{j}\hat{j})\cdot(dydz\,\hat{i} + dxdy\,\hat{k})$

$= \iint \rho a^2x^2\,dydz\ \hat{i}$

(續寫轉背頁)

$= \rho a^2 \int_0^3 \int_0^5 x^2 \, dy dz \ \hat{i}$

$= \rho a^2 \int_0^3 5x^2 dz \ \hat{i} \left(x = 4 - \frac{4}{3}z \ , \ x^2 = 16 - \frac{32}{3}z + \frac{16}{9}z^2 \right)$

$= 5\rho a^2 \int_0^3 16 - \frac{32}{3}z + \frac{16}{9}z^2 \, dz \ \hat{i}$

$= 5\rho a^2 \left(16z - \frac{16}{3}z^2 + \frac{16}{27}z^3 \Big|_0^3 \right) \hat{i} = 80\rho a^2 \hat{i}$

~~(4)~~

+8 $\frac{P}{\rho} + \frac{V_s^2}{2} + gZ = \text{constant}$

Assumptions: 1. steady flow, 2. incompressible flow

3. no friction

(5) +45 good!

(a) $F_{buoyancy} = \rho g \forall$

$\Sigma F_Z = \rho_H g \forall - \rho_F g \forall - (\rho_H g h + P_a) A = 0$

$\rho_H g h A + P_a A = \rho_H g \forall - \rho_F g \forall$

$\rho_H g h A = \rho_H g \forall - \rho_F g \forall - P_a A$

$h = \dfrac{(\rho_H - \rho_F) g \forall - P_a A}{\rho_H g A}$ +15

(b)

(1) $u(y) = ay^2 + by + C$

$u(0) = C = \frac{U}{2}$

$u(H) = aH^2 + bH + \frac{U}{2} = U$

$u(-H) = aH^2 - bH + \frac{U}{2} = U$

$aH^2 - bH + \frac{U}{2} = U$

$+) \ aH^2 + bH + \frac{U}{2} = U$

$2aH^2 + U = 2U \ , \ 2aH^2 = U, \ a = \frac{U}{2H^2}$

$\frac{U}{2H^2} \times H^2 + bH + \frac{U}{2} = U = U, \ b = 0$

$u(y) = \frac{U}{2H^2} y^2 + \frac{U}{2}$ +5

(2) $\text{flux } AB = -\rho_H U \times 2H = -2\rho_H UH$

$\text{flux } CD = \int_{cs} \rho \vec{V} \cdot d\vec{A}$

$= \int_{-H}^{H} \rho_H \left(\frac{U}{2H^2} y^2 + \frac{U}{2} \right) dy$

$= \rho_H \left(\frac{U}{6H^2} y^3 + \frac{U}{2} y \Big|_{-H}^{H} \right)$

$= \rho_H \left(\frac{UH^3}{6H^2} + \frac{UH}{2} - \left(-\frac{UH^3}{6H^2} - \frac{UH}{2} \right) \right)$ +5

$= \rho_H \left(\frac{UH^3}{6H^2} + \frac{UH}{2} + \frac{UH^3}{6H^2} + \frac{UH}{2} \right)$

$= \rho_H \left(\frac{UH}{3} + UH \right)$

$= \frac{4}{3} \rho_H UH$

(c) conservation of momentum

$F_S + F_B = \frac{\partial}{\partial t} \int_{cv} \rho \vec{V} d\forall + \int_{cs} \vec{V} \rho \vec{V} \cdot d\vec{A}$

x-direction:

$F_B = 0$ (重力作用於 y 方向上)

$\frac{\partial}{\partial t} \int_{cv} \rho \vec{V} d\forall = 0$ (steady-state)

$F_S + (p_1 - p_2) 2H = \int_{cs} \vec{V} \rho \vec{V} \cdot d\vec{A}$

$= -U \times \rho_H \times U \times 2H + \rho_H \int_{-H}^{H} \left(\frac{U}{2H^2} y^2 + \frac{U}{2} \right)^2 dy$

$\left(\frac{U}{2H^2} y^2 + \frac{U}{2} \right)\left(\frac{U}{2H^2} y^2 + \frac{U}{2} \right) = \frac{U^2}{4H^4} y^4 + \frac{2U^2}{4H^2} y^2 + \frac{U^2}{4}$

$= \frac{U^2}{4H^4} y^4 + \frac{U^2}{2H^2} y^2 + \frac{U^2}{4}$

$= -2\rho_H U^2 H + \rho_H \int_{-H}^{H} \frac{U^2}{4H^4} y^4 + \frac{U^2}{2H^2} y^2 + \frac{U^2}{4} dy$

$= -2\rho_H U^2 H + \rho_H \left(\frac{U^2}{20H^4} y^5 + \frac{U^2}{6H^2} y^3 + \frac{U^2}{4} y \Big|_{-H}^{H} \right)$

$= -2\rho_H U^2 H + \rho_H \left(\frac{U^2H^5}{20H^4} + \frac{U^2H^3}{6H^2} + \frac{U^2H}{4} - \left(-\frac{U^2H^5}{20H^4} - \frac{U^2H^3}{6H^2} - \frac{U^2H}{4} \right) \right)$

$= -2\rho_H U^2 H + \rho_H \left(\frac{2}{20} U^2 H + \frac{2}{6} U^2 H + \frac{2}{4} U^2 H \right)$

$= -\frac{16}{15} \rho_H U^2 H$

$F_S = -(p_1 - p_2) 2H - \frac{16}{15} \rho_H U^2 H$

$F = (p_1 - p_2) 2H + \frac{16}{15} \rho_H U^2 H$

+20