HEAT TRANSFER

EXAM II June 8, 2013

I. 簡答題 (36%)

1. The complete equation of convective heat transfer (energy equation) can be written as

$$\rho c_p \frac{DT}{Dt} - \rho T\left[\frac{\partial(1/\rho)}{\partial T}\right]\frac{DP}{Dt} = \nabla\cdot\left(k\nabla T\right) + u''' + \mu\Phi_v$$

What is the term of $\mu\Phi_v$?

2. What are the objectives of non-dimensionalization?
3. In talking about the boundary layer theory of Prandtl, what is the special feature of liquid metal?
4. How does the boundary layer theory simplify the momentum and energy equations?
5. Is the boundary layer theory useful for a flow field of low Reynolds number? Why? or Why not?
6. Why is the problem of natural convection more difficult to solve than that of forced convection generally?
7. For a constant wall temperature, $Gr = g\beta\left(T_w - T_\infty\right)L^3 / \nu^2$. In the case of constant wall heat flux (q''_w = constant), how would you modify Gr to make it include q''_w ?
8. Why are the shear stress at y = δ and heat flux (in the y-direction) at y = δ_T are zero?
9. In a tube flow problem with contant wall heat flux or wall temperature, the Nusselt number is constant in the fully-developed region for both the temperature and velocity fileds, which means the effect of convective heat transfer is not significant. Why is that?
10. In what conditions is there no convection heat transfer?
11. Considering a boundary layer of natural convection along a vertical plate, why is dp/dx equal to -ρg? The x direction is the vertical one.
12. How do people judge whether natural convection is important in a convection problem?

II. Explain the following terms: (15%)

(1) Free convection
(2) Bulk fluid temperature
(3) Grashof number
(4) Wind chill (temperature)
(5) Boussinesq approximation

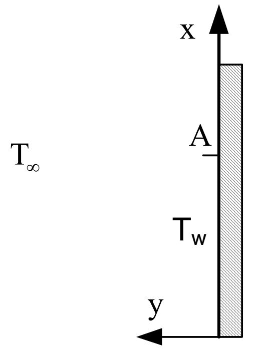
III. The following problems are related to the convective heat transfer inside a tube.

(1) Prove that h is constant for the fully temperature profile. (5%)

(2) For the constant wall heat flux, please prove the following equation (5%)

$$\frac{\partial T}{\partial z} = \frac{2}{r_0}\frac{q''}{\rho c V}$$

IV. Consider the natural convection along a vertical plate with a uniform temperature T_w in an environment at temperature T_∞. Prandtl number is equal to one.



(1) Draw the boundary layer thickness δ vs. x and temperature and velocity distributions vs. y (i.e. T vs. y and u vs. y) at x = x_A. x_A is the x-coordinate of point A. Explain why the velocity profile is like that you draw. (7%)

(2) Prove that the momentum integral equation is (10%)

$$\frac{d}{dx}\left[\int_0^\delta \rho u^2 dy\right] = -\mu\left.\frac{\partial u}{\partial y}\right)_{y=0} + \int_0^\delta \rho g\beta\left(T - T_\infty\right)dy$$

V. The following problems are related to the dimensional analysis of the momentum and energy equations inside the boundary layer over a flat plate.

(1) In the following equation, why is $\partial^2u^*/\partial x^{*2}$ neglected, and why is $(L^2/\delta^2)/Re$ set to be one? (3%)

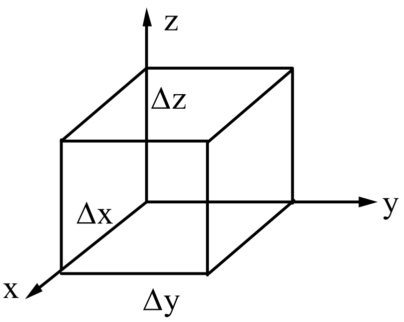
$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{\partial p^*}{\partial x^*}+\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{L^2}{\delta^2}\frac{\partial^2 u^*}{\partial y^{*2}}\right)$$

(2) What can the following equation be simplified to? Why? (3%)

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{L^2}{\delta^2}\frac{\partial p^*}{\partial y^*}+\frac{1}{Re}\left(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{L^2}{\delta^2}\frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

(3) The boundary conditions for $y\to\infty$ are $u = U_\infty$, $v = 0$, $T = T_\infty$, $P = P_\infty$. When the boundary layer theory is applied, how can these conditions be modified? (3%)

VI. For a one-dimensional transient problem of heat convection, derive the expression of the energy equation. To derive this equation, consider a small control volume as shown in the following figure. The temperature is function of t and y, i.e., $T = T(t, y)$. Only the velocity in the y direction, v, exists, i.e., $u = 0$ and $w = 0$. (8%)



VII. Consider the heat transfer in a parallel plate duct with constant wall heat flux.

(a) What's meaning of "fully-developed" of the temperature and velcoity fields inside the duct ? (4%)

(b) Prove $\frac{\partial T}{\partial z}=\frac{dT_w}{dz}=\frac{dT_m}{dz}=\text{constant}$, where T_m is the bulk fluid temperature and T_w is the wall temperature. (6%)

(c) Derive the expression of Nu for the case of constant wall heat flux in the fully-developed region if it is assumed that $u = U_\infty$ and $v = 0$. (10%)

VIII. Heat loss from a steam pipe in windy air. A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected agrainst the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s. (10%)

Hint:

$$Nu = 0.3+\frac{0.62\,Re^{1/2}\,Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

$$Nu=\frac{hD}{k},\quad Re=\frac{VD}{\nu}$$

T, °C	$\nu\times10^5$ m^2/s	k, W/mK	Pr
50	1.798	0.02735	0.7228
60	1.896	0.02808	0.7202
70	1.995	0.02881	0.7177