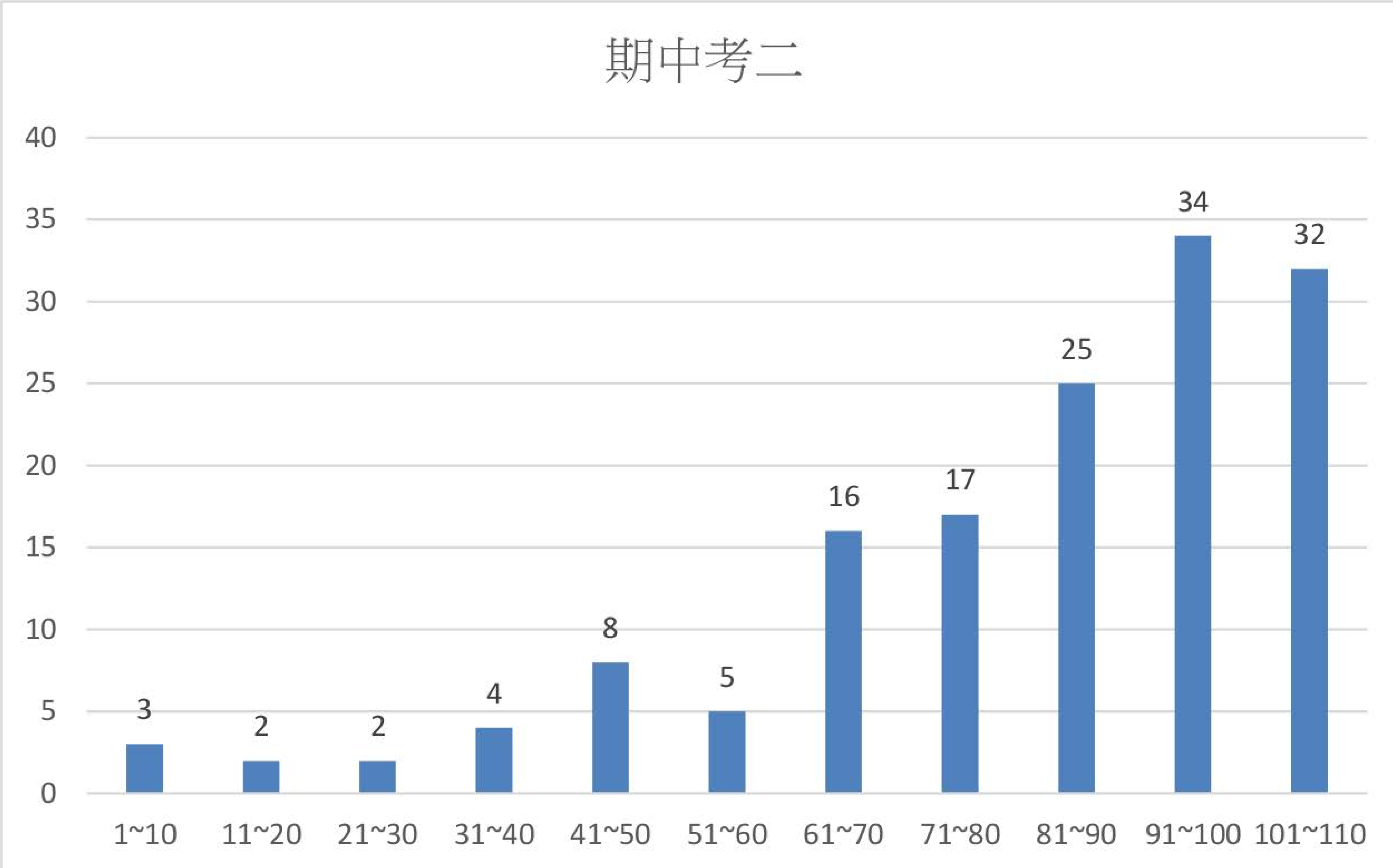
電子電工學
Lecture 10



Average 79.8
Std deviation 23.6

Chapter 9

AC Circuits

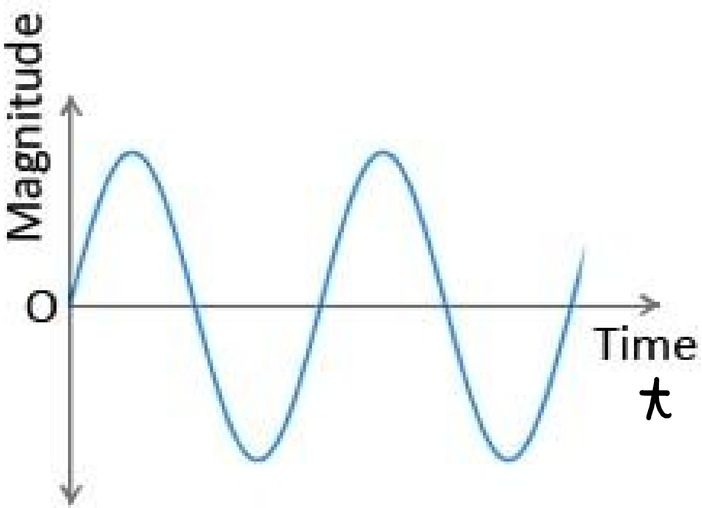
交流 直流

Alternating current vs Direct current

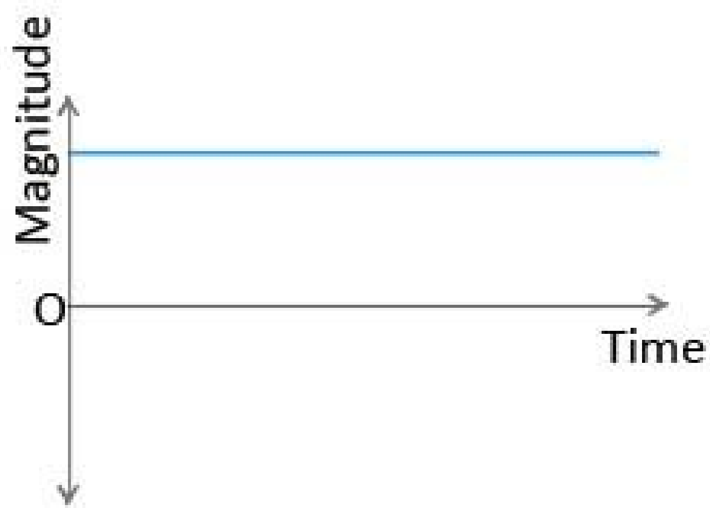
Periodically
Change direction

Constant
Flow in one direction

$\left.\begin{array}{l}\cos(\omega t)\\ \sin(\omega t)\end{array}\right\}$ sinusoidal wave



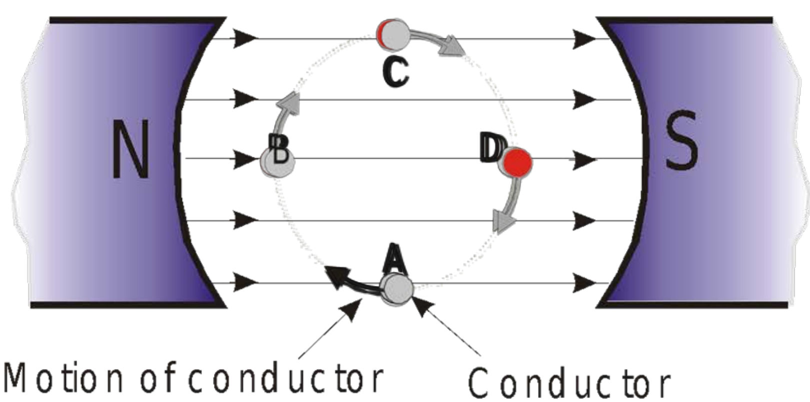
Alternating Current

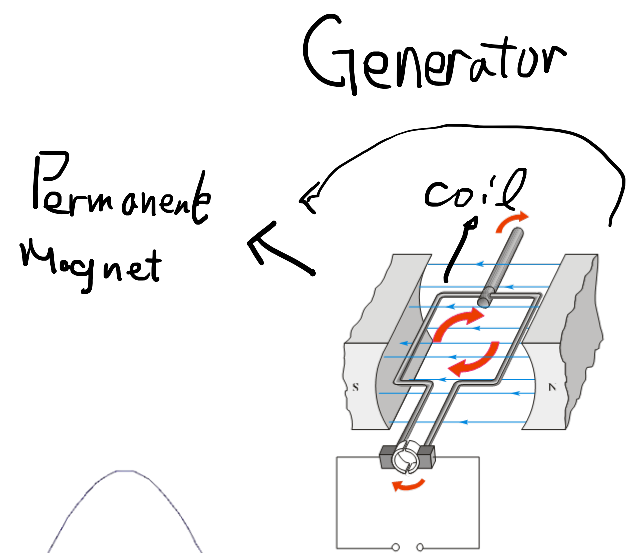


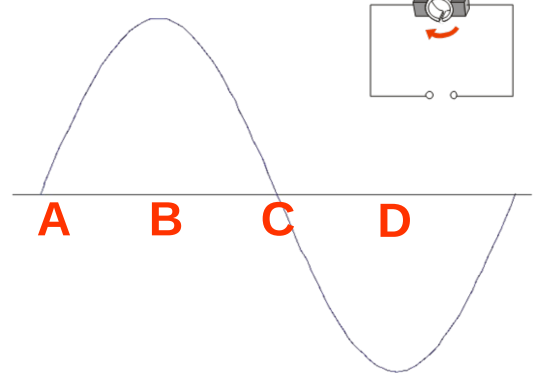
Direct Current

Alternating current

$I \propto \cos(\omega t)$

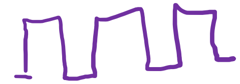






Sinusoidal waves

Square wave

$$\sum_{n=0}^{\infty} \frac{1}{2n+1} \sin[(2n+1)\omega t] =$$ 

$$\sin(\omega t) + \frac{1}{3}\sin(3\omega t) + \frac{1}{5}\sin(5\omega t) + \frac{1}{7}\sin(7\omega t)$$

$\omega = 2\pi f$

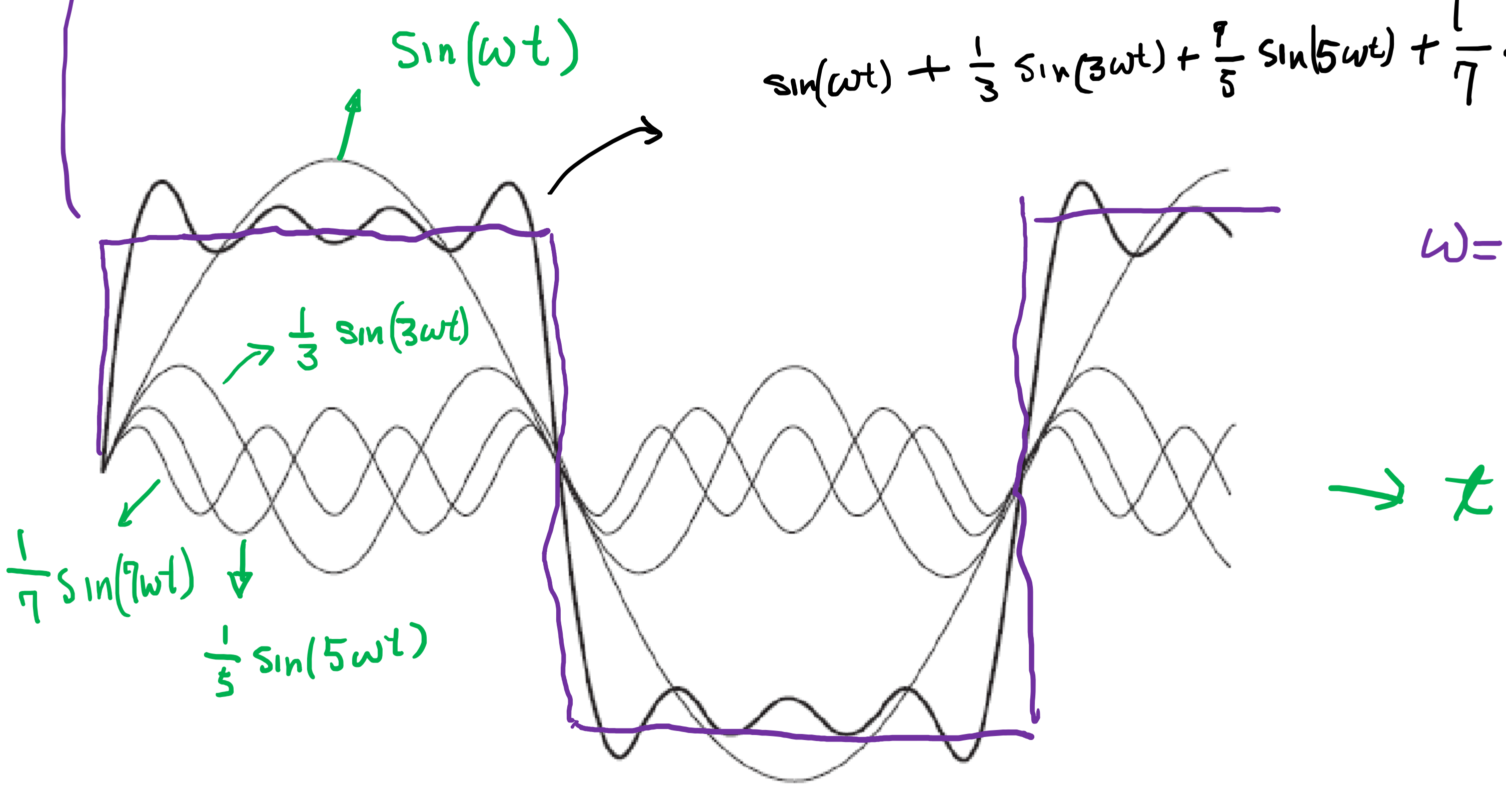
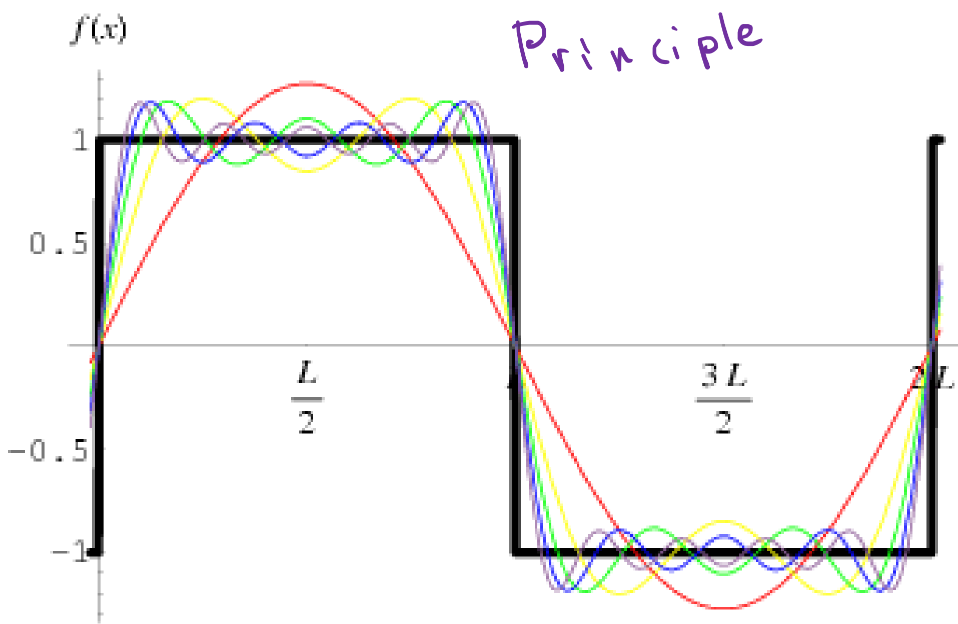
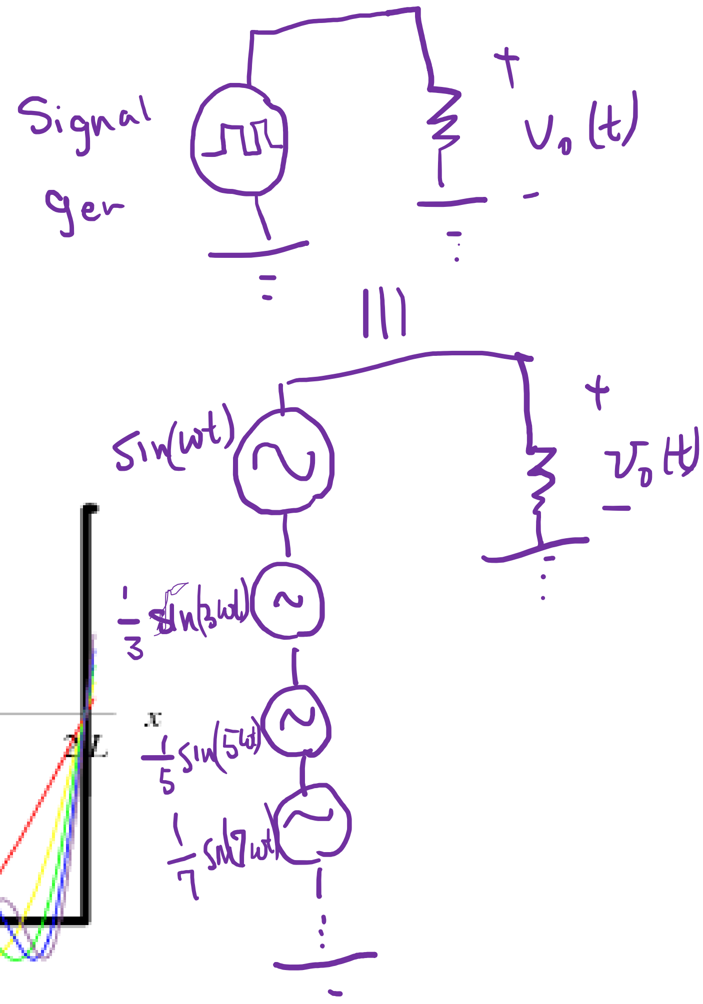


Figure 9.1 The addition of four sinewaves of relative amplitudes 1, 1/3, 1/5 and 1/7, and relative frequencies ω, 3ω, 5ω and 7ω (light-lines) yields an approximation (bold line) to a square waveform.

Sine waves vs square wave

Superposition Principle





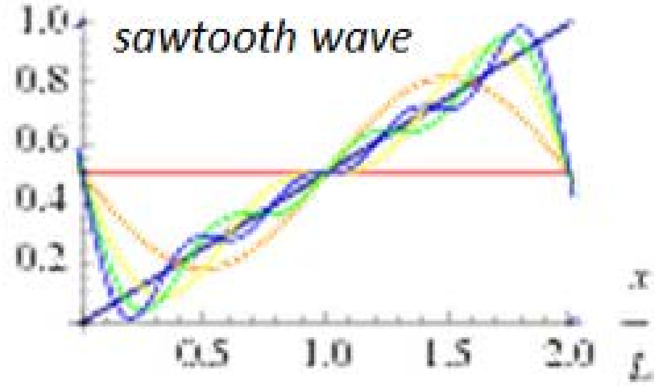
Fourier series expansion

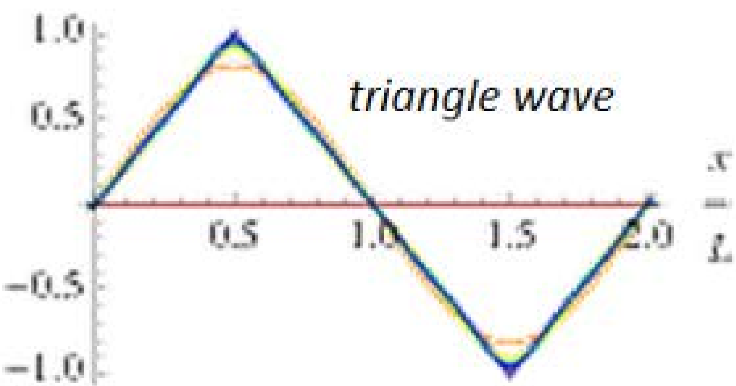
$$f(x) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right)\right]$$

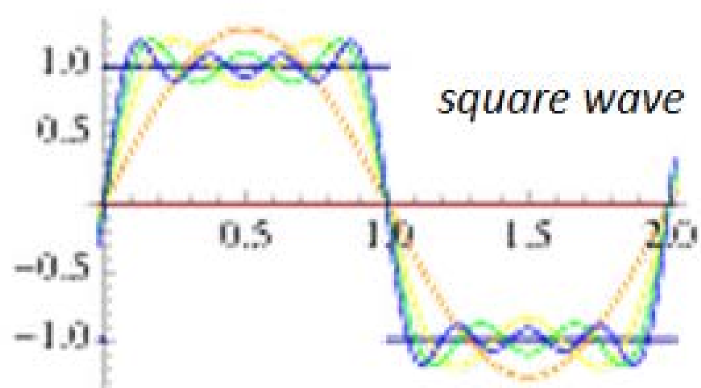
$$a_0 = \frac{1}{L}\int_0^L f(x)dx$$

$$a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx$$

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$







Basic Waves

$$A \cos(\overset{\omega}{\underline{2\pi f}}\, t) \longrightarrow A \cos(2\pi f t + \phi)$$

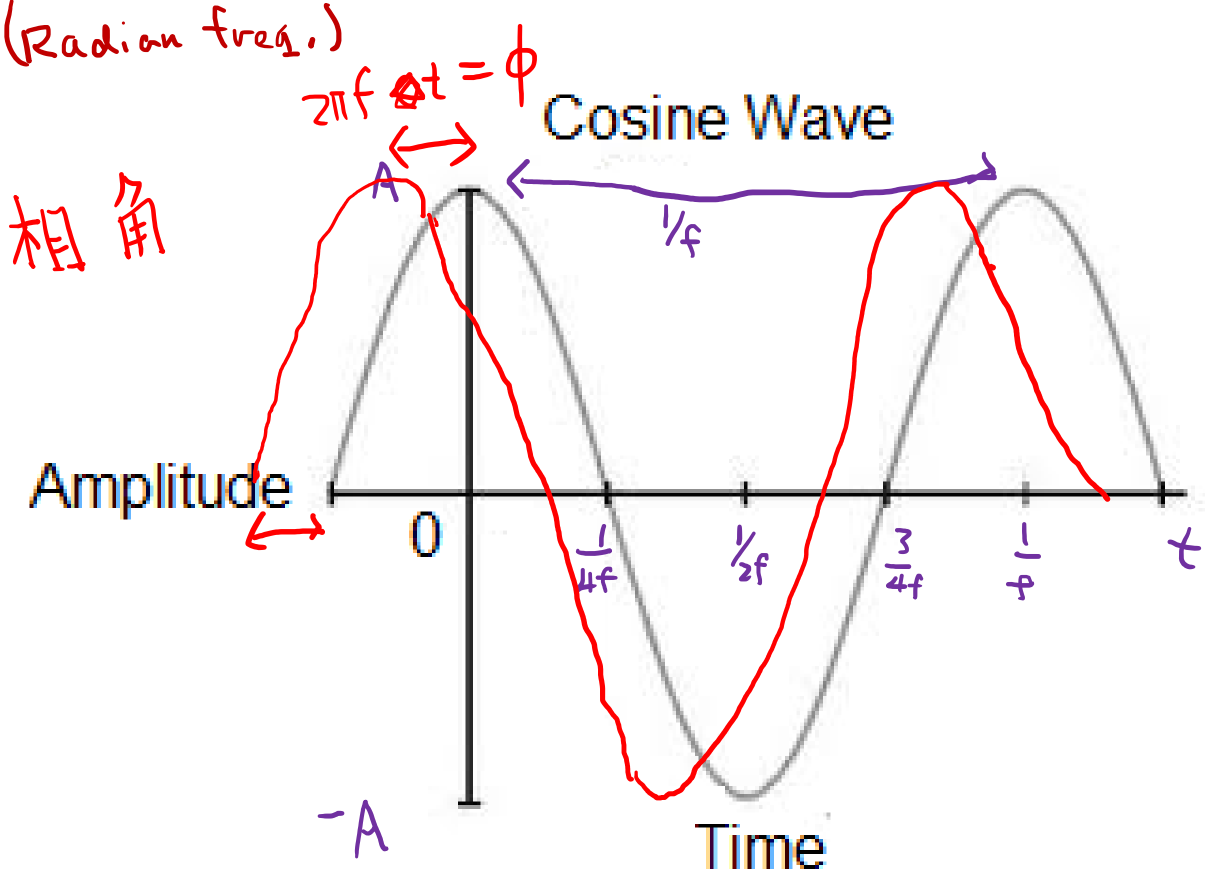
A : Amplitude

f : Frequency

$\omega = 2\pi f$ (Radian freq.)

1/f : period

ϕ : phase 相角



Capacitor

$$i(t) = C\,\frac{dv(t)}{dt} \longleftarrow i(t) = \frac{dq(t)}{dt}$$

Source $v(t) = V\cos(\omega t)$

$$\Rightarrow i(t) = CV\left[-\omega \sin(\omega t)\right]$$

$\frac{d}{dt}$

$$= -\omega CV \sin(\omega t)$$

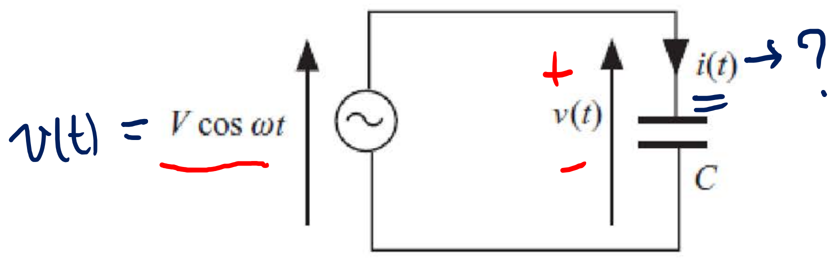


Figure 9.2 A capacitor with its voltage defined by a sinusoidal voltage source of radian frequency ω

Capacitor

$$-\sin(\omega t) = \cos\left(\omega t + \frac{\pi}{2}\right)$$

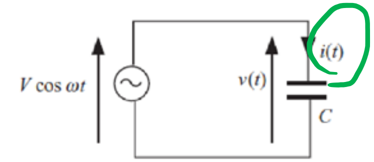


Figure 9.2 A capacitor with its voltage defined by a sinusoidal voltage source of radian frequency ω

$V \cos(\omega t)$

$$i(t) = -\omega C V \sin(\omega t)$$

$$= \omega C V \cos\left(\omega t + \frac{\pi}{2}\right)$$

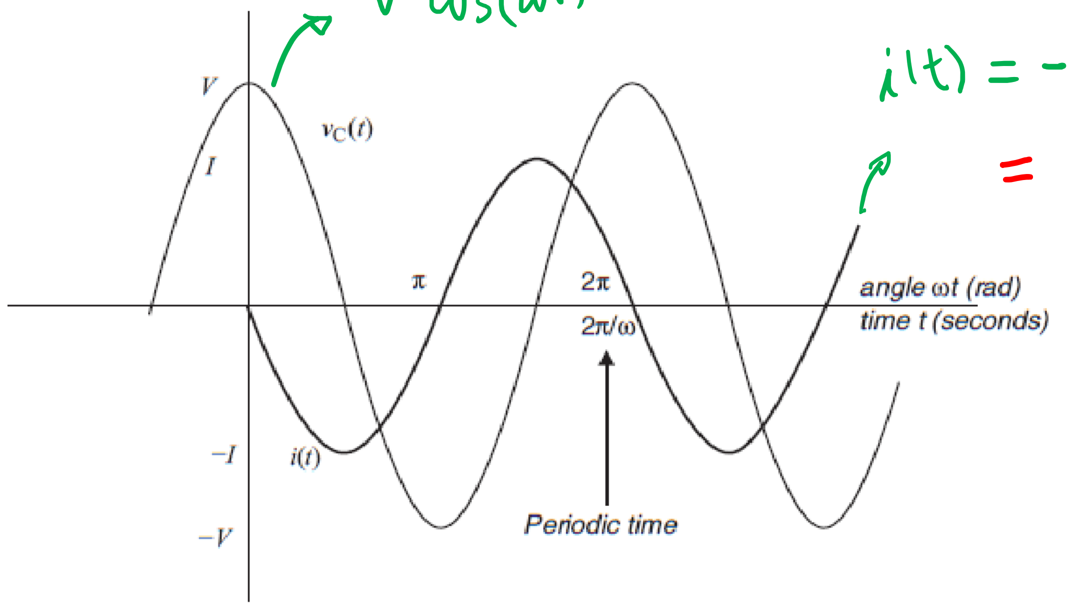
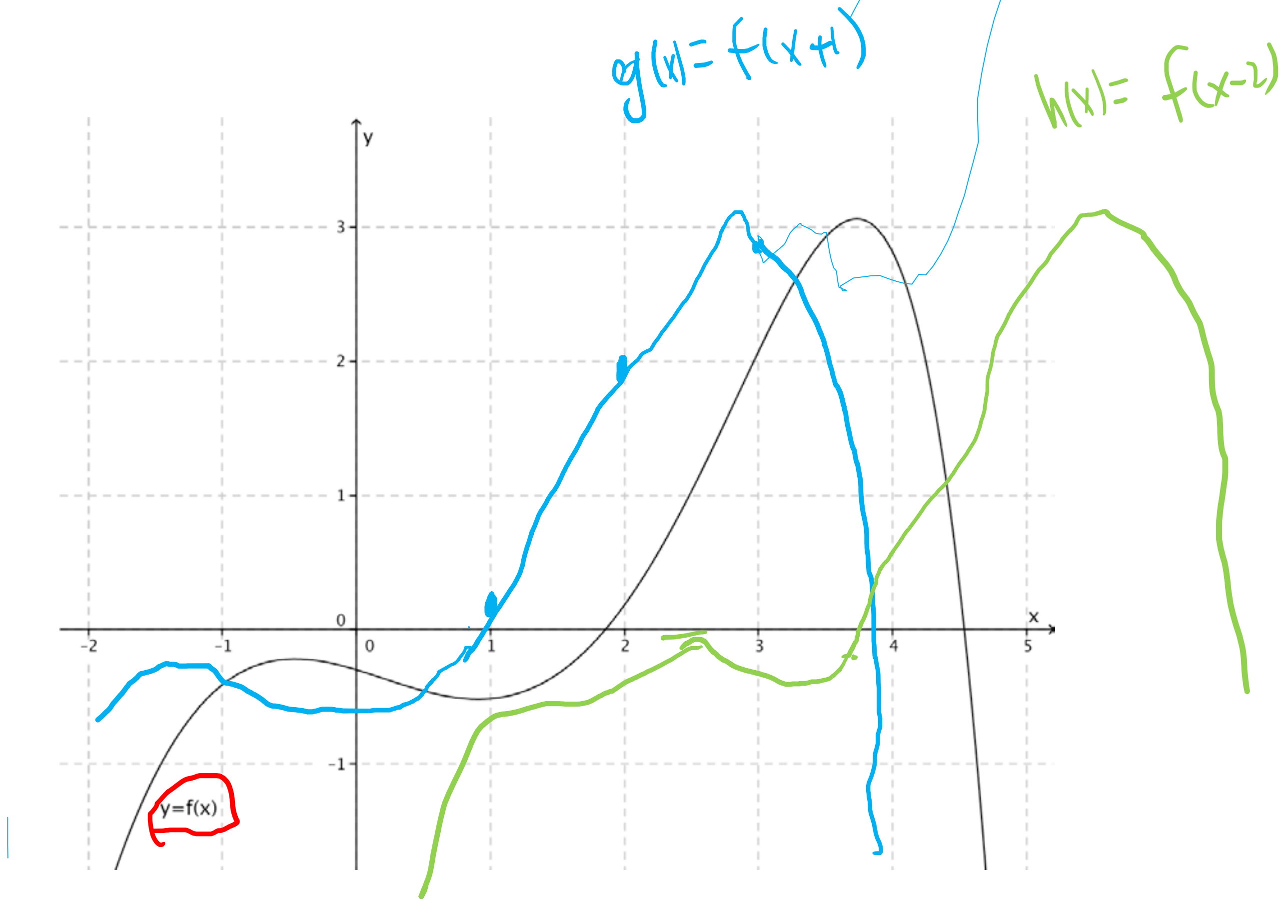


Figure 9.3 The waveforms of sinusoidal voltage across, and sinusoidal current through, a capacitor

Review: function shift

$y = f(x) \longrightarrow g(x) = f(x + \Delta x)$



Voltage vs Currents

$v(t) = \underline{V} \cos(\omega t)$ $i(t) = \underline{I} \cos\left(\omega t + \frac{\pi}{2}\right)$

1. Frequency : same $\omega = 2\pi f$
2. Amplitude : $I = \omega C V$
3. Phase : $i(t)$ leads $v(t)$ by $\pi/2$ ($90°$)

   $v(t)$ lags $i(t)$

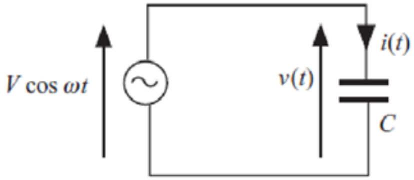


Figure 9.2 A capacitor with its voltage defined by a sinusoidal voltage source of radian frequency ω

Example 9.1

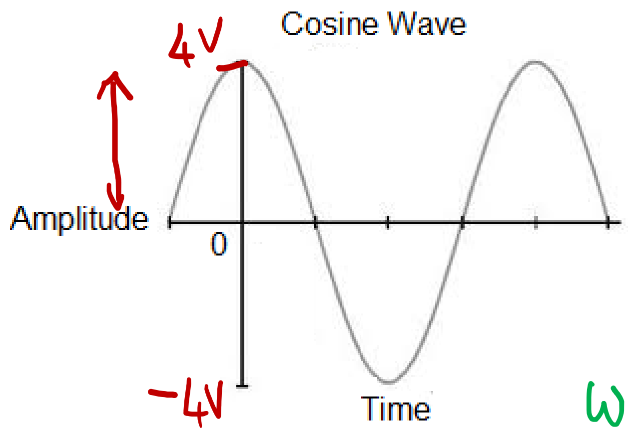
$$v(t) = 4\cos(2\pi \cdot 159 \cdot t)$$

$$= 4\cos(1000t)$$

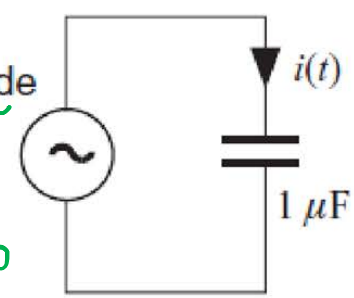
$$i(t) = C \cdot dv(t)/dt$$

$$= \omega C \cdot 4\cos(1000t + \pi/2)$$

$$= 4\times10^{-3}\cos\left(1000t + \frac{\pi}{2}\right)$$



[8V peak-to-peak]



$f = 159$ Hz

$\omega = 2\pi f \cong 1000$

$1\,\mu F = 10^{-6}\,F$

Figure 9.4 A capacitor with its voltage defined by a sinusoidal voltage source of frequency 159 Hz

Inductor

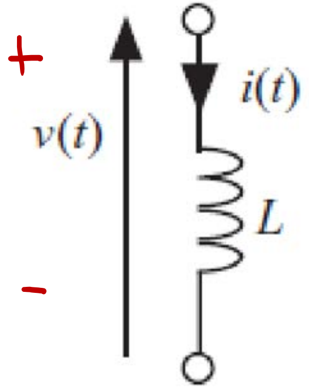


Figure 9.5 An inductor

$$\begin{cases} v(t) = \overset{(H)}{L} \cdot \dfrac{di(t)}{dt} \\ i(t) = \dfrac{1}{L} \displaystyle\int_{-\infty}^{t} v(\tau)\, d\tau \end{cases}$$

* An inductor resists a change in current.

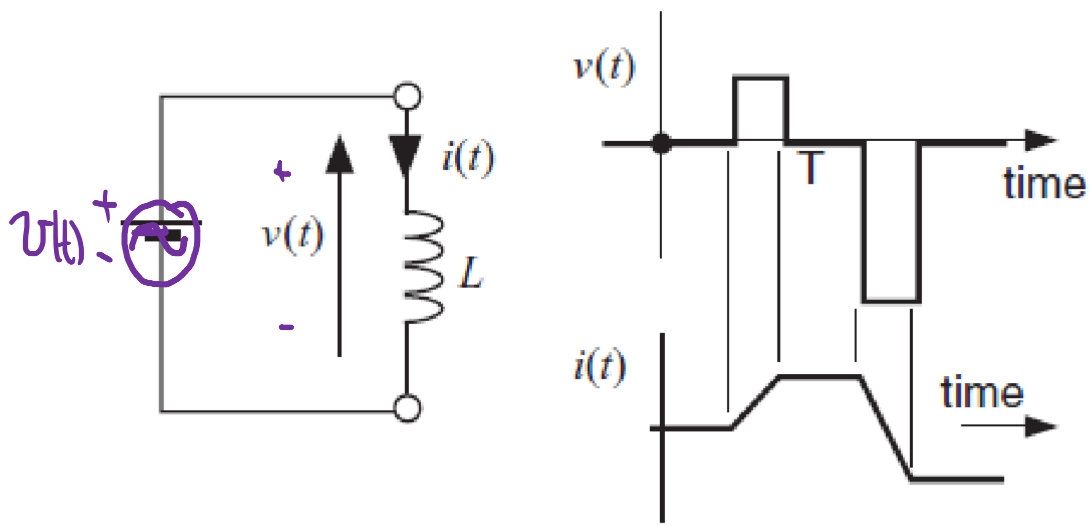


Figure 9.6 Reponse of an inductor to a time-varying voltage source

Inductor

Current src $i(t) = I\cos(\omega t+\theta)$

$$\Rightarrow v(t) = L\frac{di(t)}{dt}$$

$$= -\omega L I \sin(\omega t+\theta)$$

$$= \underbrace{\omega L I}_{V = \omega L I} \cos\left(\omega t+\theta+\frac{\pi}{2}\right)$$

$i(t) = I\cos(\omega t+\theta)$

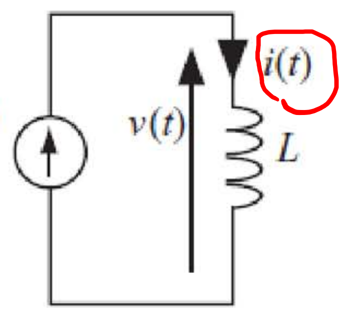
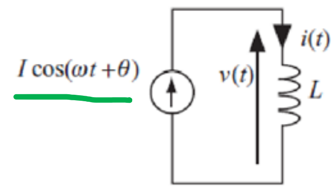


Figure 9.7 An inductor with its current defined by a sinusoidal current source of radian frequency ω

Inductor

Voltage vs Current

1. Frequency : same $\omega = 2\pi f$
2. Amplitude : $V = \omega L I$
3. Phase: $v(t)$ leads $i(t)$ by $\pi/2$



$i(t) = I\cos(\omega t + \theta)$

$v(t) = \omega L I \cos(\omega t + \theta + \frac{\pi}{2})$

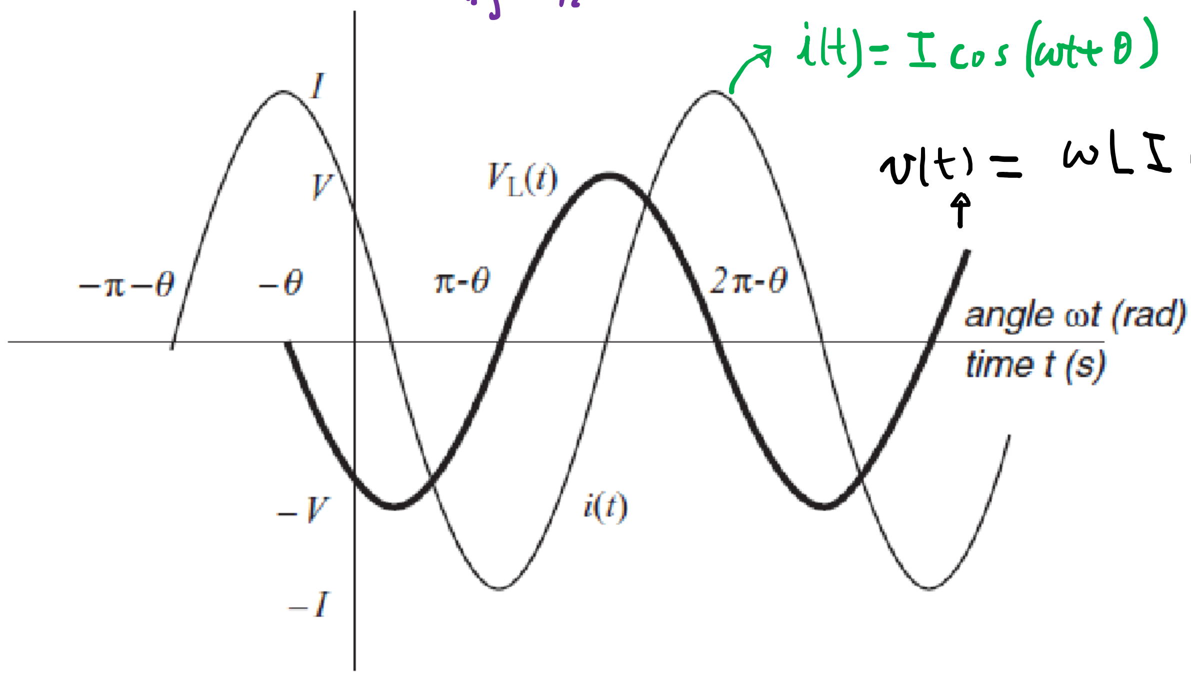


Figure 9.8 The waveforms of sinusoidal current through, and sinusoidal voltage across, an inductor

Resistor

Source $v(t) = V \cos \omega t$

By Ohm's Law: $i(t) = \frac{1}{R} v(t)$

$= \frac{V}{R} \cos(\omega t)$

Voltage vs Current

Freq: same $\omega = 2\pi f$

Amp: $I = V/R$

Phase: same $\Delta\phi = 0$

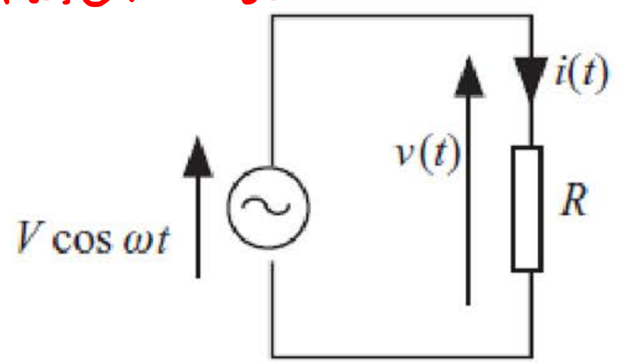
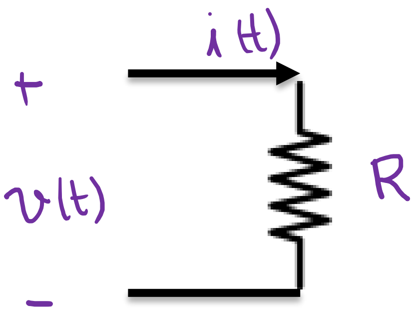
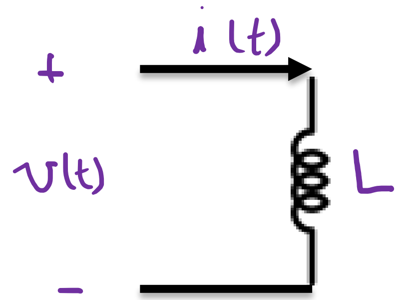
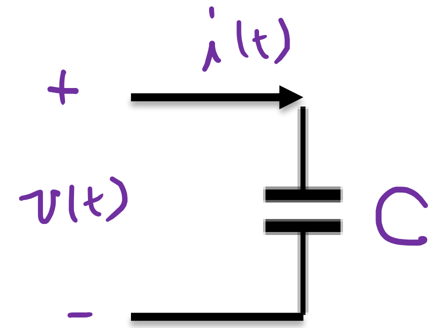


Figure 9.9 A resistor with its voltage defined by a sinusoidal voltage source of radian frequency ω

Summary: passive components







$v(t) = V\cos(\omega t)$

$i(t) = I\cos(\omega t)$

$$\boxed{V = RI}$$

$i(t) = I\cos(\omega t)$

$v(t) = V\cos(\omega t + \frac{\pi}{2})$

$$\boxed{V = \omega L I}$$

$v(t) = V\cos(\omega t)$

$i(t) = I\cos(\omega t + \frac{\pi}{2})$

$I = \omega C V$

$$\boxed{V = \left(\frac{1}{\omega C}\right) I}$$

Example 9.2

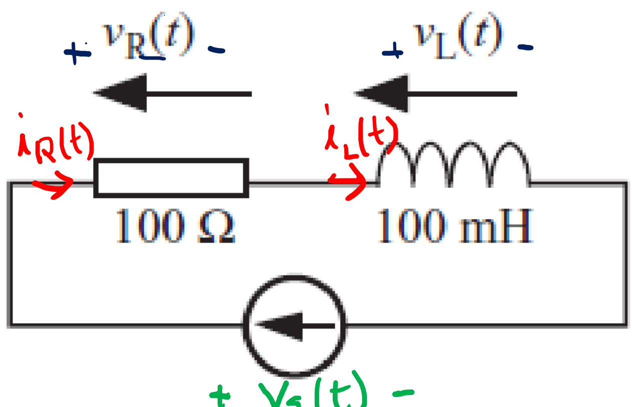
KCL: $i_R(t) = i_L(t) = i_s(t) = 2\cos(1000t + 20^\circ)$

KVL: $v_s(t) = v_R(t) + v_L(t)$

$= R \cdot i_s(t) + L \cdot \dfrac{d\,i_s(t)}{dt}$

$R \rightarrow 100\,\Omega$

$L \rightarrow 100\text{ mH} = 0.1\text{ H}$



$i_S(t) = 2\cos(1000t + 20^\circ)$ mA

Figure 9.10 The circuit discussed in Example 9.2

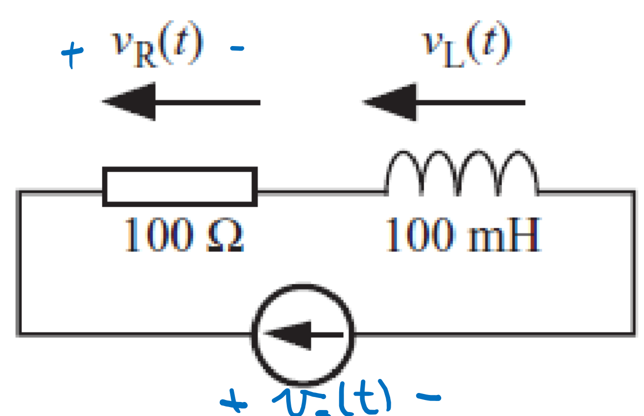
Example 9.2

$$v_R(t) = R \cdot i_s(t) = 200 \cos(1000t + 20^\circ)$$

$$v_L(t) = L \frac{d i_s(t)}{dt} = 200 \cos(1000t + 20^\circ + 90^\circ)$$

$$v_s(t) = v_R(t) + v_L(t)$$

$$= 200 \left[\underbrace{\cos(1000t + 20^\circ) + \cos(1000t + 110^\circ)}_{\text{Apply Sum-to-product rule}} \right]$$



$i_S(t) = 2\cos(1000t + 20^\circ)$ mA

Figure 9.10 The circuit discussed in Example 9.2

Trigonometric Identities

$\cos(1000t + 20^\circ) + \cos(1000t + 110^\circ)$

$= 2\cos(1000t + 65^\circ)\cos(45^\circ)$

$= \sqrt{2}\cos(1000t + 65^\circ)$

Sum to Product Formula

$$\boxed{\cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2}}$$

$$\cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}$$

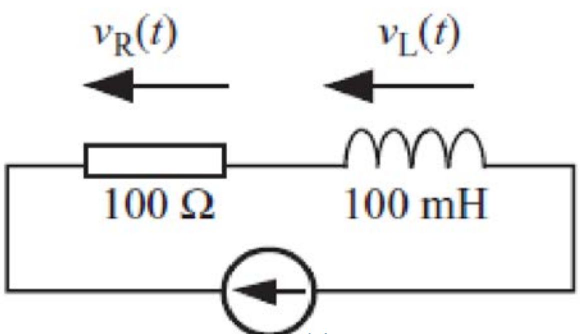
Product to Sum Formula

$$2\cos x\cos y = \cos(x+y) + \cos(x-y)$$
$$-2\sin x\sin y = \cos(x+y) - \cos(x-y)$$
$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$2\cos x\sin y = \sin(x+y) - \sin(x-y).$$

Example 9.2

$$v_s(t) = 200 \times \sqrt{2} \cos(1000t + 65^\circ)$$

$$\approx 282.8 \cos(1000t + 65^\circ)$$



$i_S(t) = 2 \cos(1000t + 20^\circ)$ mA

Figure 9.10 The circuit discussed in Example 9.2

R-L circuit

Given $V(t) = V \cos(\omega t)$

Find $i(t) = ?$ unknown

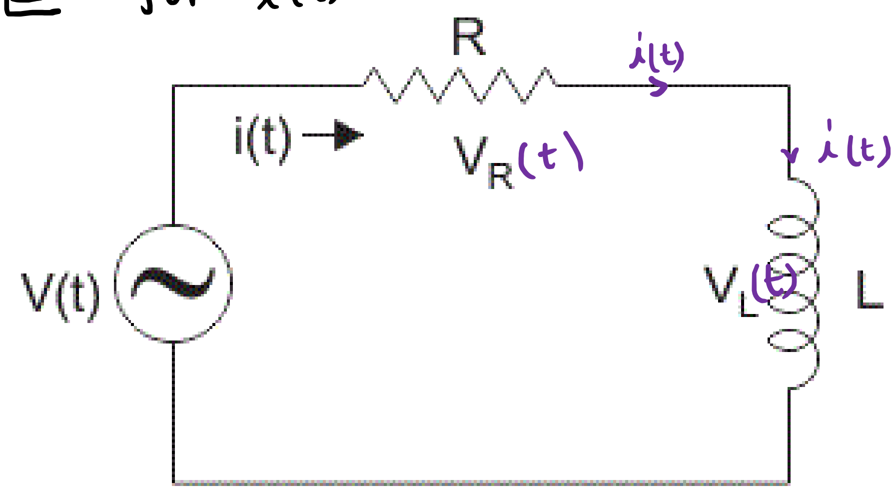
KCL, KVL

$$v(t) = v_R(t) + v_L(t)$$

$$= R \cdot i(t) + L \frac{di(t)}{dt}$$

$$V \cos(\omega t) = R\, i(t) + L\, i'(t)$$

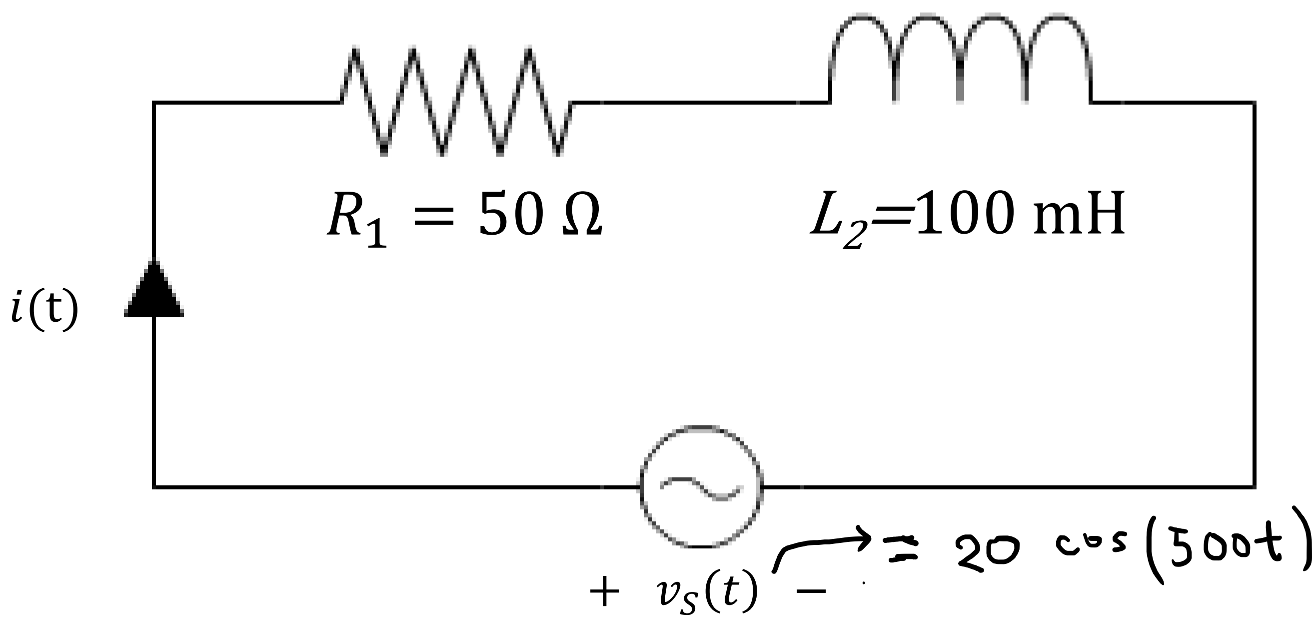
ODE for $i(t) \Rightarrow$ $i'(t) + \frac{R}{L}\, i(t) = \frac{V}{L} \cos(\omega t)$



Quiz

Suppose the initial current is *i*(0)=0 for the circuit shown in the figure below.

1. Write a differential equation for the current *i*(*t*).
2. Apply the knowledge you learned from Engineering Math to solve *i*(*t*) for *t*>0.



(40 V peak-to peak amplitude, 500 rad/s)

Quiz Review

(1) KVL: $V_s(t) = R_1\, i(t) + L_2\, di(t)/dt$

$20\cos(500t)$ $\quad$ 50 $\quad$ 0.1

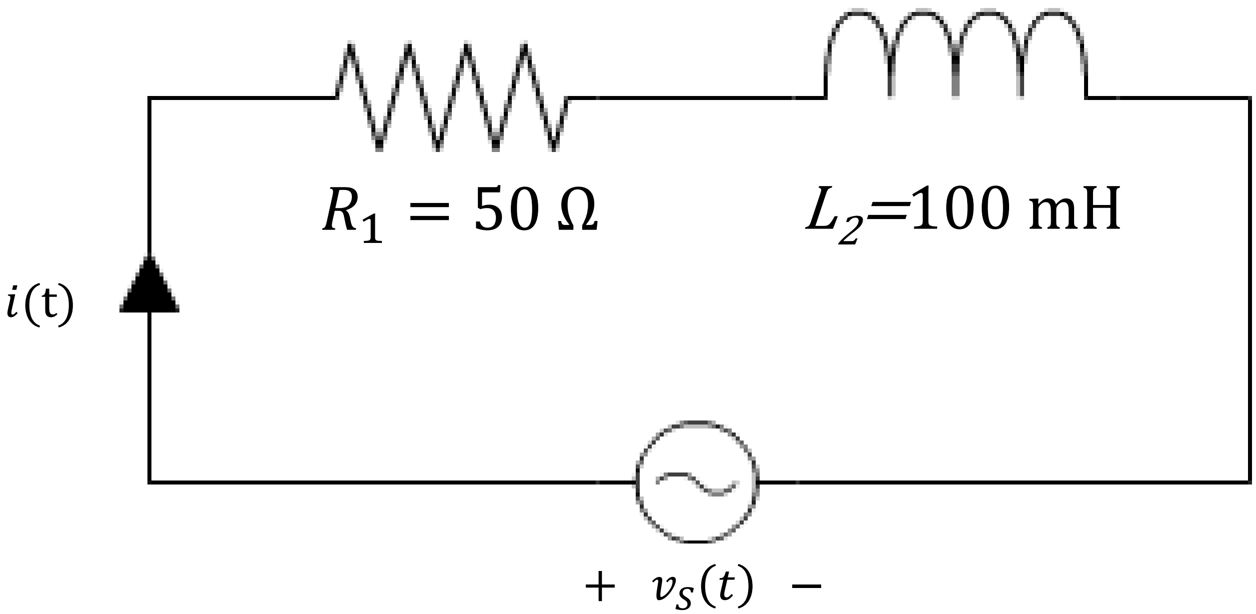
$\rightarrow \quad 0.1\, i'(t) + 50\, i(t) = 20 \cos(500t)$

$i'(t) + 500\, i(t) = 200 \cos(500t)$

(2) Solve

method 1. Integrating Factor $\times e^{-500t}$

2. Undetermined coefficient



(40 V peak-to peak amplitude, 500 rad/s)

Quiz Review

Let $i(t) = i_h(t) + i_p(t)$

$$\begin{cases} i_h'(t) + 500\, i_h(t) = 0 \\ i_p'(t) + 500\, i_p(t) = 200\cos(500t) \end{cases}$$

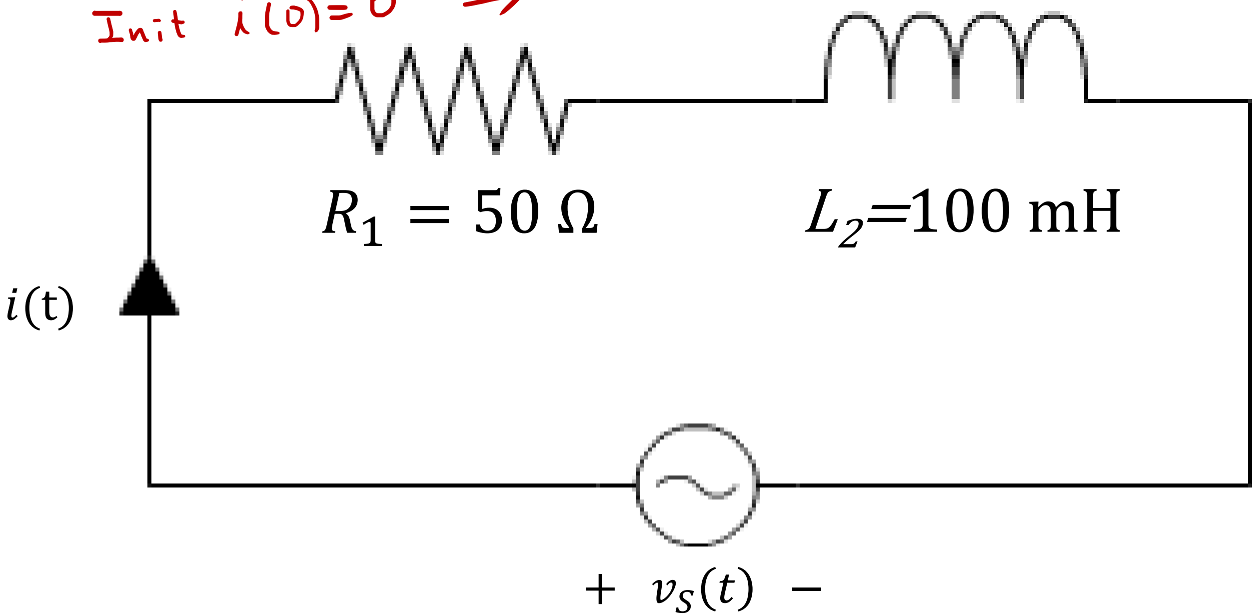
$\rightarrow i_h(t) = K e^{-500t}$

Let $i_p(t) = A\cos(500t) + B\sin(500t)$

$\hookrightarrow A = B = 0.2$

$\Rightarrow i(t) = Ke^{-500t} + 0.2\cos(500t) + 0.2\sin(500t)$

Init $i(0) = 0 \Rightarrow K + 0.2 = 0 \Rightarrow K = -0.2$



(40 V peak-to peak amplitude, 500 rad/s)

Summary

1. AC vs DC

2. $\cos(\omega t)$

3. Component

ω, $\phi = \pm\frac{\pi}{2}$, $\left[\begin{array}{l} V = \omega L I \\ V = RI \\ V = \frac{1}{\omega C} I \end{array}\right]$

4. KCL, KVL

↳ Sum-to-product → Phasor Diagram

Differential Equation

Simplify!