7. Use the finite difference method and the TDMA method. Find the simultaneous equations in matrix form. The length of the rod is 5 and number of grid points is 6.

$$\frac{\partial^2 T}{\partial x^2} = 10$$

| i | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $T_i$ | $T_1$ | $T_2$ | $T_3$ | $T_4$ | $T_5$ | $T_6$ |

Boundary conditions : $T_1 = 100$ , $T_5 = T_6$ (10%)

8. Using the Taylor series in two variables $(x, y)$ of the form

$$f(x+h, y+k) = f(x,y) + hf_x(x,y) + kf_y(x,y) + \cdots$$

where $f_x = \partial f/\partial x$ and $f_y = \partial f/\partial y$, establish that Newton's method for solving the two simultaneous nonlinear equations

$$\begin{cases} f(x,y) = 0 \\ g(x,y) = 0 \end{cases}$$

can be described with the formulas

$$\begin{cases} x_{n+1} = x_n - \dfrac{fg_y - gf_y}{f_x g_y - g_x f_y} \\ y_{n+1} = y_n - \dfrac{f_x g - g_x f}{f_x g_y - g_x f_y} \end{cases}$$

Here the functions $f$, $f_x$, and so on are evaluated at $(x_n, y_n)$. (10%)

9. Solve the parameter x in this pair of simultaneous nonlinear equations by
(a) Newton's method (7%). Start with the initial value $x_0 = 2.0$ and iterate 3 times.
(b) Bisection method (7%). With the initial interval [2, 3] and iterate 3 times.

$$\begin{cases} x^3 - 2xy + y^7 - 4x^3 y = 5 \\ y \sin x + 3x^2 y + \tan x = 4 \end{cases}$$