Engineering Dynamics **Final exam**
E916100 **2021 Spring** **June 17 , 2021**

1. (30 points) A point/particle has a velocity $\mathbf{v}'$ measured relative to a rotated coordinate system with angular velocity $\boldsymbol{\omega}$.
   (a) What is the velocity $\mathbf{v}$ measured relative to a fixed coordinate system? Please give the detail about the derivation.
   (b) What is the acceleration $\mathbf{a}$ measured relative a fixed coordinate system? Please give the detail about the derivation.
   (c) A bullet fired from Bonds car with an initial velocity $\mathbf{v}' = v_2'\mathbf{e}_2'$ where $\mathbf{e}_2'$ denotes the direction of car which rotates with $\boldsymbol{\omega} = \omega_3\mathbf{e}_3$ where $\mathbf{e}_i$ denotes Cartesian base vector in a fixed coordinate system. What is the bullet's instantaneous velocity $\mathbf{v}$ and its instantaneous acceleration $\mathbf{a}$ expressed from the fixed coordinate system? Please detail every component of the velocity and the acceleration and please also point out the term "Coriolis acceleration." The position of the car and the fixed coordinate system $\mathbf{e}_i$ are plot in Figures 1 and 2.

2. (30 points) A motion of a particle with the mass m, has the position

$$\mathbf{x}(t) = r\cos\omega t\mathbf{e}_1 + r\sin\omega t\mathbf{e}_2 + \gamma t\mathbf{e}_3,$$

   where ω and γ are two constants. For this motion,
   (a) please show the velocity $\mathbf{v}$, the speed v, and the acceleration $\mathbf{a}$;
   (b) please show the arc length s derive base vectors of the natural coordinate system including the tangent vector $\mathbf{t}(s)$, the normal vector $\mathbf{n}(s)$, and the bi-normal vector $\mathbf{b}(s)$ and the curvature κ;
   (c) please find the force $\mathbf{f}$ which exerts the motion.

3. (40 points) The Falcon 9 designed and manufactured by SpaceX is a reusable and two-stage rocket that the first stage with the mass $m_1 = 400,000$ kg can be vertical loading after it is separated from the second state with the mass $m_2 = 150,000$ kg which can transport the payload into a low earth orbit or a geosynchronous transfer orbit.
   (a) At the time $t = 0$ s, the Falcon 9 launches and it reaches the stage separation at 147 s. Before the stage separation (please see the Falcon 9 sample mission profile in Figure 3), the Falcon 9 is being propelled from $t = 0$ s to $t = 147$ s by a thrust $\mathbf{f} = T(\cos\theta\mathbf{e}_1 + \sin\theta\mathbf{e}_2)$ where $T = 7000$ kN and $\theta = 90 - 0.3t$ (degree). Please plot the free body diagram and the impulse-momentum diagram and determine the velocity (function) and the position (function) of the Falcon 9 from the launch to the main engine cutoff before the stage separation.
   (b) The duration of the stage separation starts at $t = 147$ s that the first stage engine is cut off and the second stage engine ignites and provides an additional thrust $T_2 = 981$ kN. After four seconds, two stages separates completely. Please determine the instantaneous velocity of the first stage and the second stage at $t = 151$ s and also the position function of the second stage $\mathbf{x}^{2\text{nd}}(t)$.
   (c) After the stage separation, the first stage is flipped and the flip ends 10 seconds later of the stage separation. Then the the first stage moves along the orbit with instantaneous velocity $\mathbf{v} = v_1\mathbf{e}_1$ to the landing location. Please plot the inpulse-momentum diagram and use a rotation matrix to denote the rotation.
   (d) After the flip maneuver with 10 seconds, the first stage moves along the trajectory

$$\mathbf{x}^{1\text{st}}(t) = \{r + h\cos[\theta_1 - \alpha_1(t - t_1)]\}\,\mathbf{e}_1 + h\sin[\theta_1 - \alpha_1(t - t_1)]\mathbf{e}_3,$$

   where $\theta_1 = \frac{\pi}{2}$, $\alpha_1 = \frac{\pi}{300}$, and $t_1 = 161$. In above equation, r and h denote respectively the $\mathbf{e}_1$ and $\mathbf{e}_3$ components of position of the first stage at $t = 161$ s. Please determine the value of v_1 in 3(c); make a comment about whether the above trajectory is realistic; and show the relative position $\mathbf{x}^{1\text{st}/2\text{nd}}$, the relative velocity $\mathbf{v}^{1\text{st}/2\text{nd}}$, and the relative acceleration $\mathbf{a}^{1\text{st}/2\text{nd}}$.

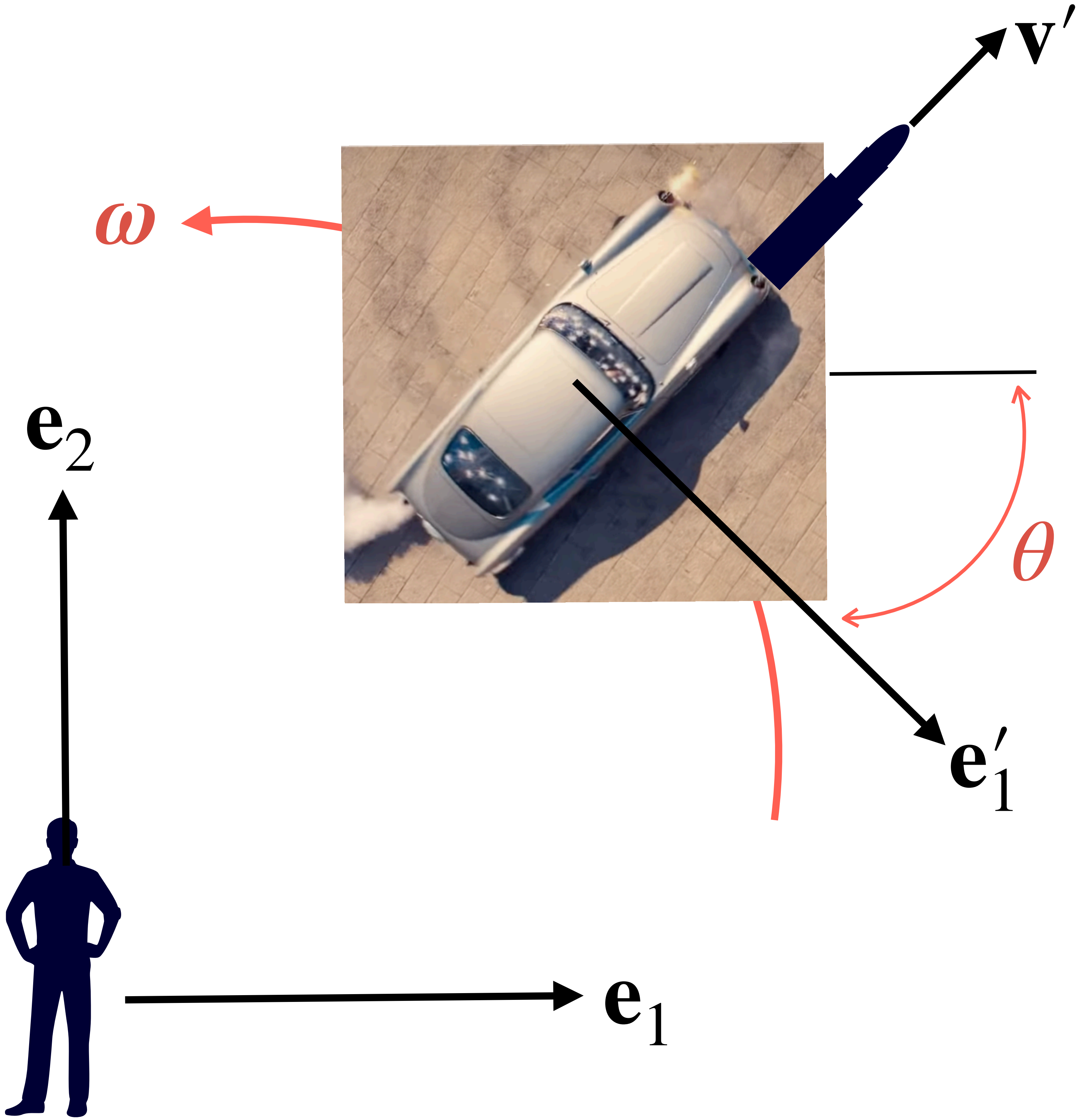


Figure 1

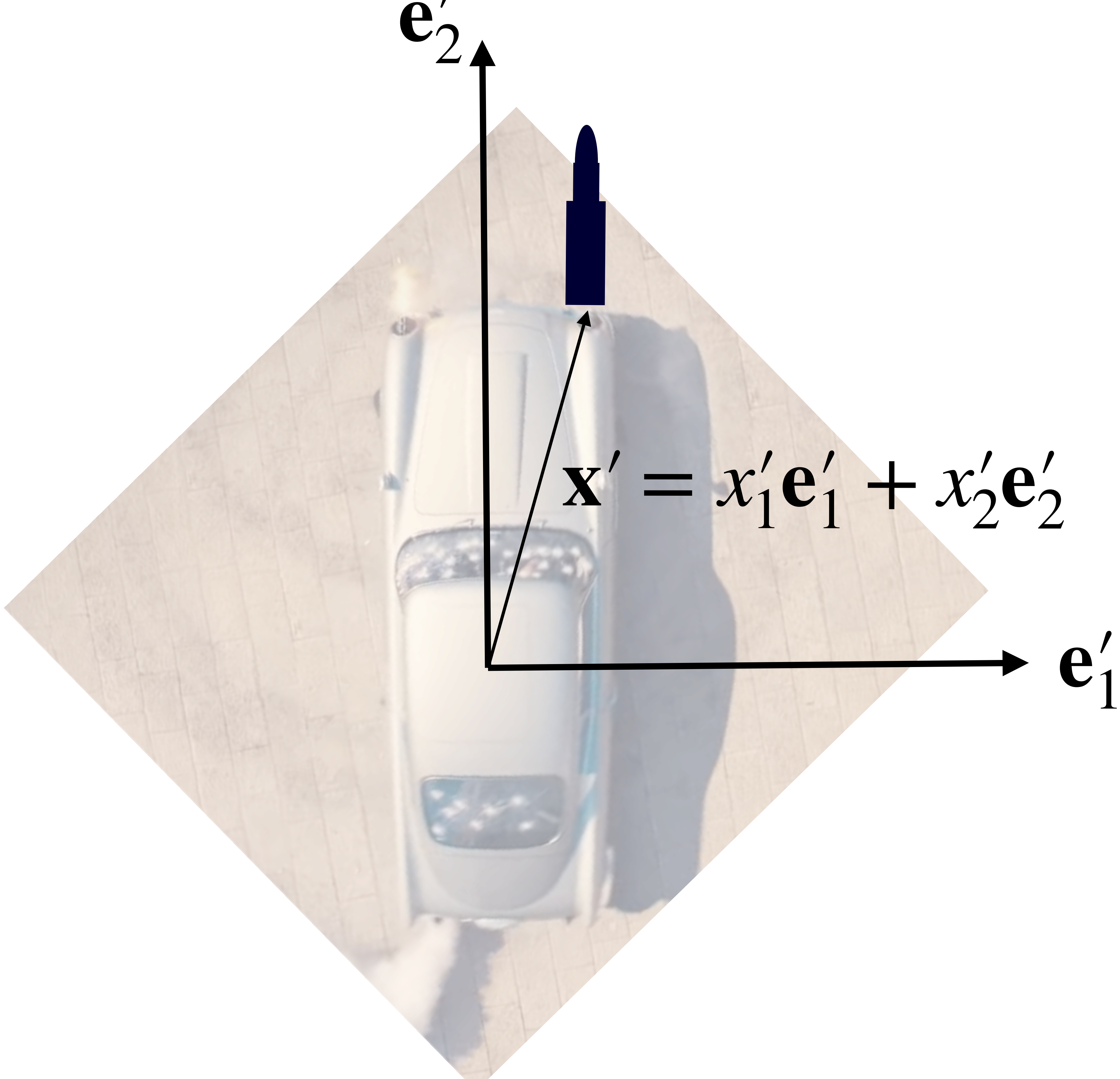
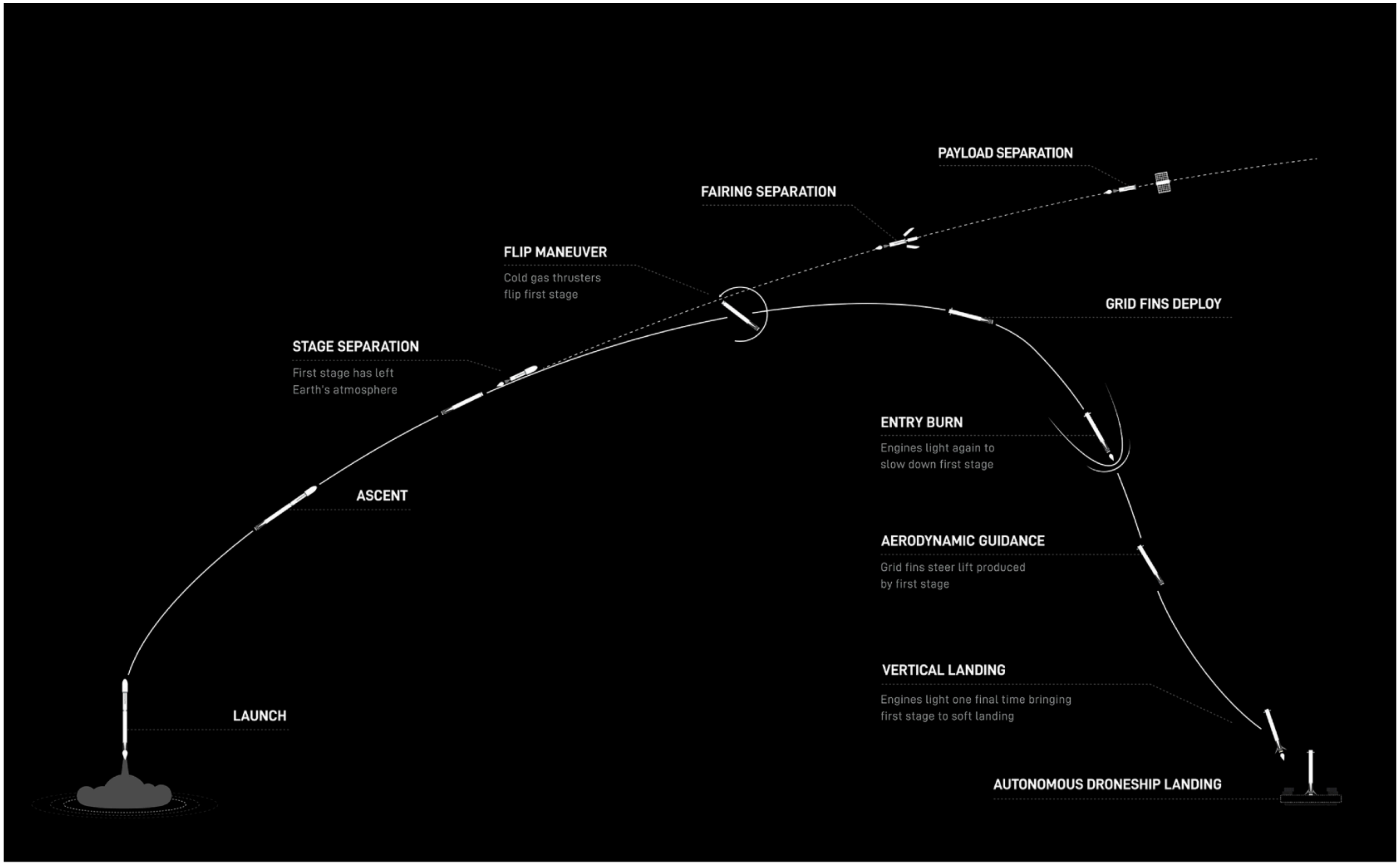
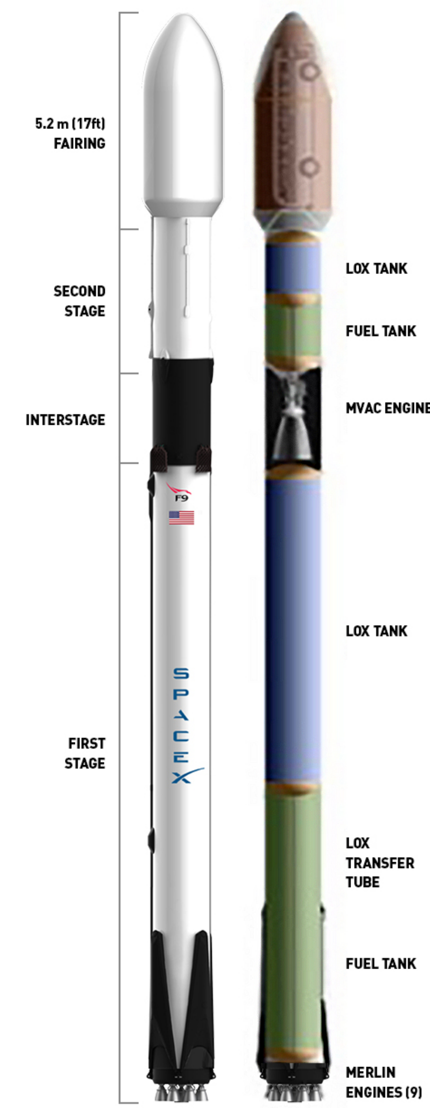


Figure 2





Falcon 9

Overview of Falcon 9: https://youtu.be/Z4TXCZG_NEY

Figure 3