$\rho c \frac{\partial T}{\partial t} = \nabla \cdot (k \cdot \nabla T) + \dot{q}$

EXAM I
HEAT TRANSFER

April 28, 2017

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I. The energy equation of a straight fin can be written as

$$kA\frac{d^2T}{dx^2} - hP(T - T_\infty) = 0$$

(1) What are the assumptions made for the energy equation above? (3%)

(2) What are the physical meanings of $kA\frac{d^2T}{dx^2}$ and $hP(T - T_\infty)$? (4%)

(3) What are the units（單位）of $kA\frac{d^2T}{dx^2}$ and $hP(T - T_\infty)$? (4%)

II. Explain the following terms: (15%)

1. The zeroth law of thermodynamics.
2. The first law of thermodynamics.
3. Fourier law
4. Heat diffusion equation
5. Newton's cooling law

III. Answer the following questions (36%)

(1) What's the difference between heat transfer and thermodynamics?

(2) In what conditions can thermal resistance be applied?

(3) Why is free convection important in the design of heat transfer?

(4) When a fin is attached to a high-temperature body, can it be regarded as a thermal resistance to the body? Why or Why not?

(5) Why are the thermal conductivities of metal materials generally larger than those of non-metal ones?

(6) A heat-insulated layer is put between two solids with different temperatures. Why can the layer not completely block the heat transfer between the two solids?

(7) From the viewpoint of heat transfer, why is a child easier to catch cold than an adult when the weather turns cold?

(8) Why do tanks use the air cooling system instead of the water cooling one?

(9) What are the three material properties whose units are m^2/sec?

(10) The expression of forward difference for $\left(\frac{dT}{dx}\right)_i$ can written as

$$\left(\frac{dT}{dx}\right)_i = \frac{T_{i+1} - T_i}{\Delta x} + O(\Delta x)$$

What does $O(\Delta x)$ mean?

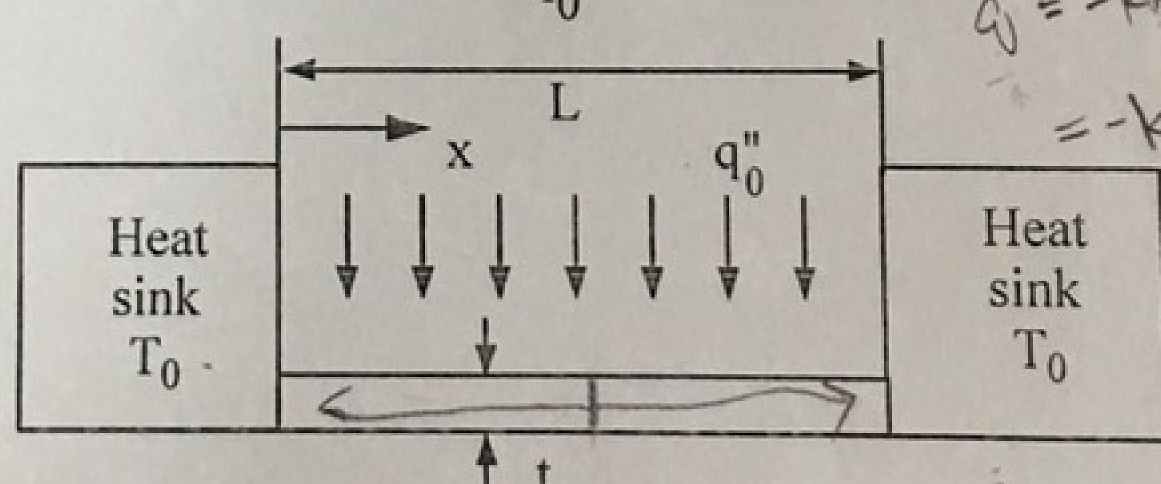
(11) Why is it said that the flowers and trees facing the south can easily feel the arrival of spring（向南花木易為春）?

(12) Based on the viewpoint of heat transfer, explain how does the cold-resistance effect of a sweater differ from that of a raincoat? (以熱傳觀點，來闡釋雨衣的禦寒效果與毛線衣的有何不同？)

IV. A thin flat plate of length L, thickness t, and width W >> L is thermally joined to two large heat sinks that are maintained at a temperature T_0, which is shown in the following figure. The bottom of the plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q''_0 .

$q'' = -kA\frac{dT}{dx}$

$= -kA \frac{\quad}{L}$



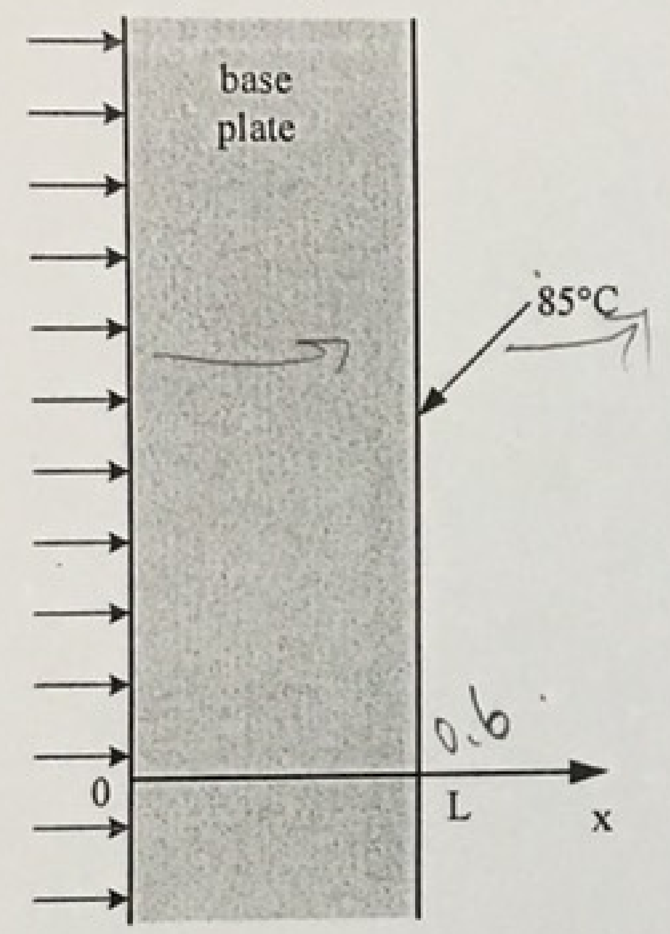
Obtain an expression for the rate of heat transfer from the plate to these two sinks. (7%)

Hint: Apply the energy-conservation concept to the flat plate.

V. Consider the base plate of a 800-W household iron（800 家用熨斗）with a thickness of L = 0.6 cm, base area of A =

$Q = kA \frac{dT}{dx}$

160 cm^2, and thermal conductivity of $k = 20$ $W/m \cdot °C$. The inner surface of the base plate is subjected to uniform heat flux generated by the resistance heaters inside. When steady operating conditions are reached, the outer surface temperature is measured to be 85°C. Disregarding any heat loss through the upper part of the iron, (a) express the differential equation and the boundary conditions for steady one-dimensional heat conduction through the plate, (b) obtain a relation for the variation of temperature in the base plate by solving the differential equation, and (c) evaluate the inner surface temperature. (12%)

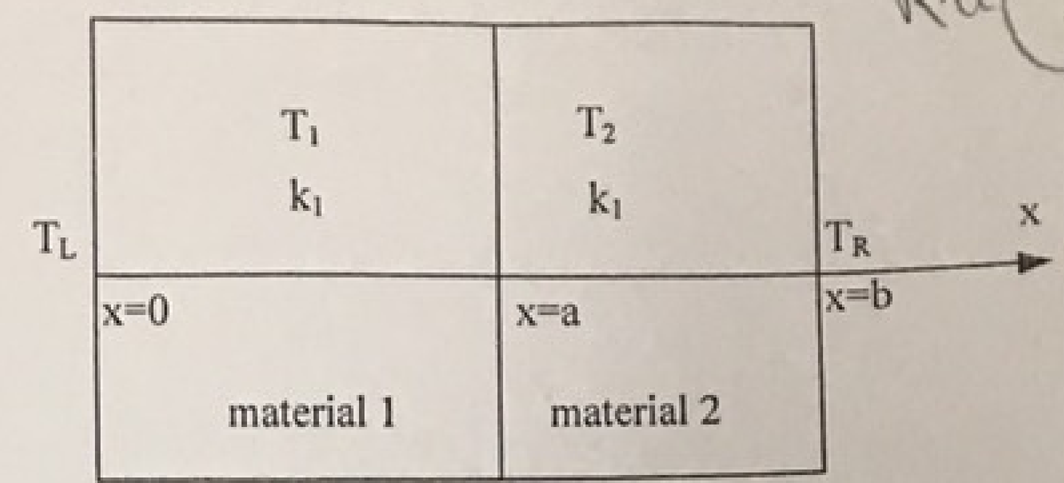


VI. One side of a plane wall is maintained at 100°C, while the other side is exposed to a convection environment having T = 10°C and h = 12 W/($m^2 \cdot$°C). The wall has k = 1.2 W/(m·°C) and is 40 cm thick. Calculate the heat flux through the wall. (8%)

VII. In a Cartesian coordinate, prove that the heat flux (q”) is constant for the one-dimensional steady heat conduction with constant thermal conductivity and without heat source. (8%)

VIII. In the following figure, two materials with k_1 and k_2 thermal conductivities have perfect contact and their corresponding temperatures are T_1 and T_2.

$R_{th} = \frac{1}{\Delta x A}$



(i) Write the boundary conditions are x=0, x=a and x=b. (4%)

(ii) Solve the T_1 and T_2 under the assumptions of 1-D steady state with no heat source and constant k_1 and k_2. (6%)

(iii) If these two materials do not have perfect contact at their interface, what are the boundary conditions at x = a? (4%)

XI. The energy equation and boundary conditions of a fin can be written as

$$\frac{d}{dx}\left[kA(x)\frac{dT}{dx}\right] - h(T - T_\infty)\frac{dS}{dx} = 0,$$

$T = T_b$, at $x = 0$

$-k\frac{dT}{dx} = h(T - T_\infty)$, at $x = \ell$.

(a) Non-demensionalize the energy equation and boundary conditions by using
$x^* = x/\ell$, $T^* = (T - T_\infty)/(T_b - T_\infty)$
$S^* = S/S_\ell$, $A^* = A/A_b$. (7%)

(b) Why are the dimensionless energy equation and boundary conditions obtained above helpful to the heat transfer analysis of a fin problem? (5%)