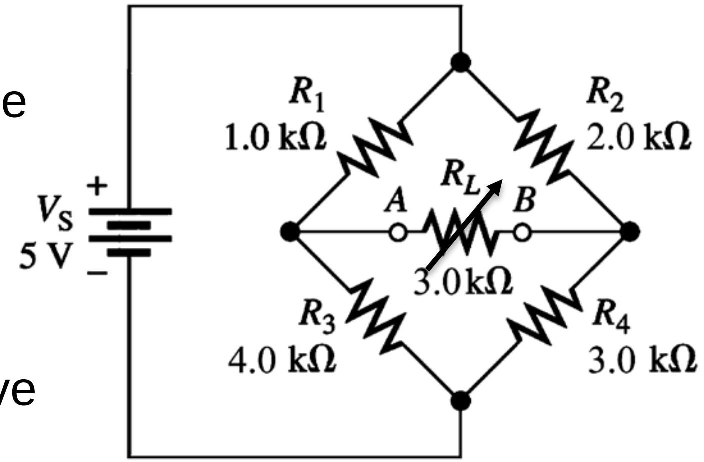
# 電子電工學
# Lecture 4

# Midterm Exam Information

Date: Oct 14, 2020
Time: 3:10 ~ 6:00 pm (in class)
Location: 化工系館柏林講堂/93156
Coverage: Textbook Chapters 1-4

# LAB 1. Circuit Analysis




Use the online simulator to build the circuit shown in the right figure. Change values of $V_s$, $R_1$, $R_2$, $R_3$, and $R_4$ according to your student ID (details as follows.)

1. Export the simulated circuit as a text file.
2. Replace $R_L$ with various resistor values: 1, 2, 3, …, 10 KΩ. Observe the voltages $V_{AB}$ and draw the curve $V_{AB}$ vs $R_L$

ID .txt

ID .JPG
GIF

Submit the above text file and the plot on Moodle.

Suppose your student ID is

*E(nn) 0 (abcde)*

Set the Voltage Source

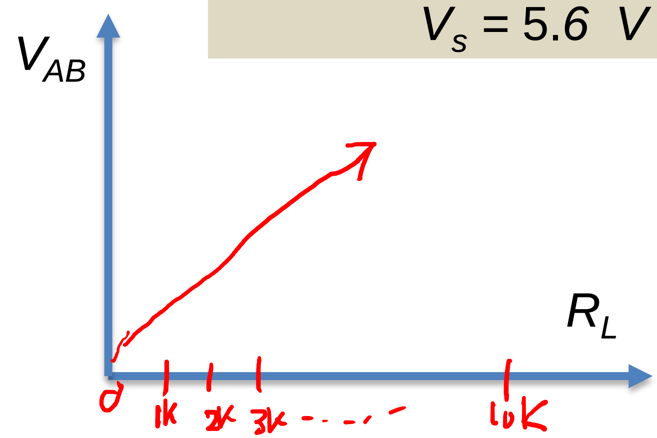
$$V_s = 5.e\ \ V$$

Set the resistors

$$R_1 = 1.a\ \text{K}\Omega$$
$$R_2 = 2.b\ \text{K}\Omega$$
$$R_3 = 4.c\ \text{K}\Omega$$
$$R_4 = 3.d\ \text{K}\Omega$$

Example: ID E34048156

$$R_1 = 1.4\ \text{K}\Omega$$
$$R_2 = 2.8\ \text{K}\Omega$$
$$R_3 = 4.1\ \text{K}\Omega$$
$$R_4 = 3.5\ \text{K}\Omega$$
$$V_s = 5.6\ \ V$$



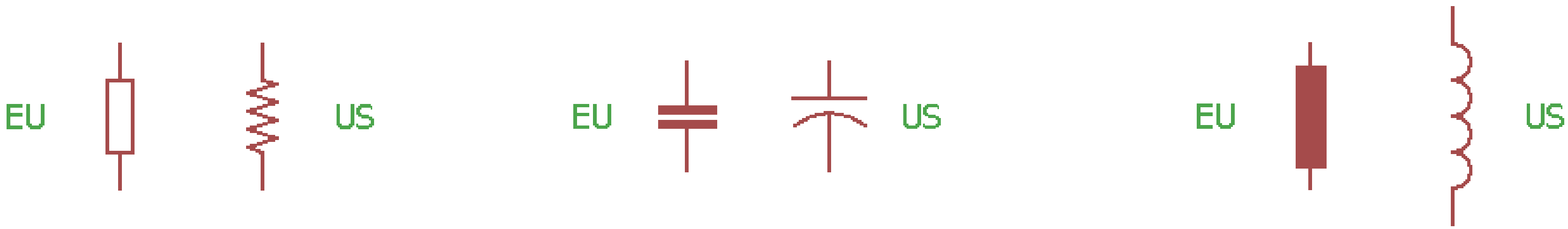
## Circuit simulation

- http://lushprojects.com/circuitjs/circuitjs.html
- http://www.falstad.com/circuit/circuitjs.html

## Matrix computation

- https://octave-online.net/

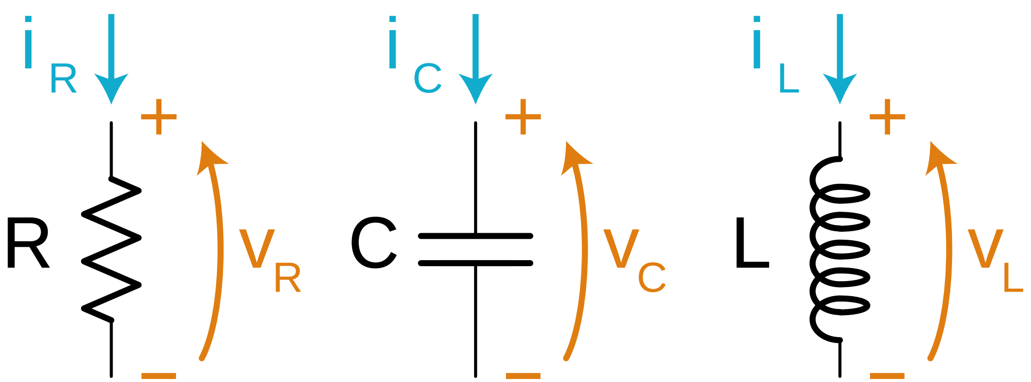
# Recap: Symbol US vs EU



# Recap: Voltage direction

$$V_R = i_R \times R$$

$-2V \qquad -1A \qquad 2\Omega$



# Recap: Circuit analysis

Ohm's law
KCL
KVL

Simplify →

NODAL ANALYSIS

1. Labeling Nodes
2. Identifying Ground (earth)
3. Applying KCL → eqn's
4. Solving system of equations
→ $V_A$ $V_B$ $V_C$

System 10 eqn's
⇒ 10 variables

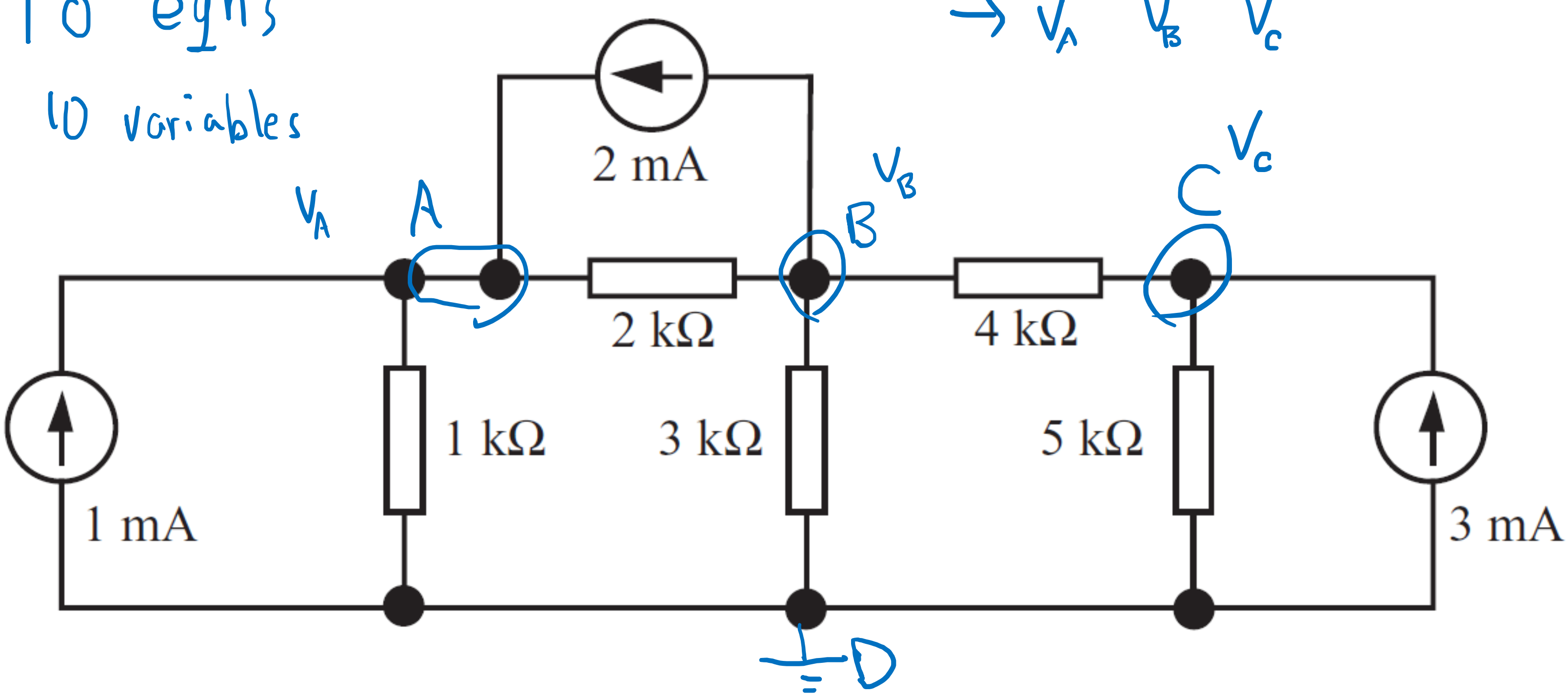



**Figure 4.1** The circuit to be analysed

# Recap: Nodal analysis with voltage sources

KCL @ A, B ⇒ 2 eqn's ⇒ Solve $V_A$ $V_B$

$$\begin{cases} V_A = -\frac{320}{21} V \\ V_B = -\frac{164}{7} V \end{cases}$$

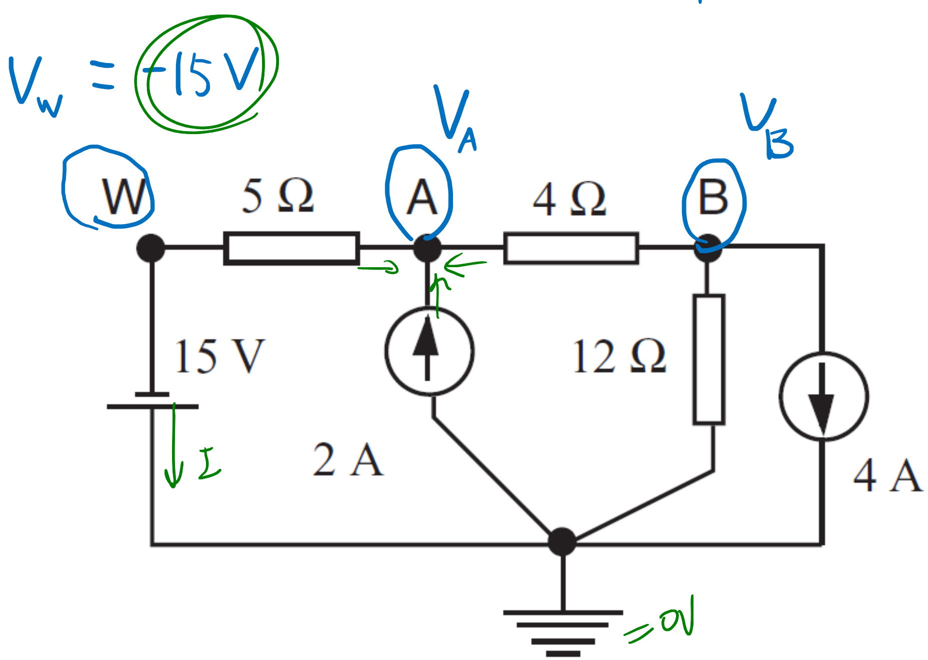



**Figure 4.7** A voltage reference node has been chosen and other circuit nodes labelled

# Example 4.1

Grounding A

KCL @ X: $\frac{0 - V_X}{4} + \frac{V_Y - V_X}{12} + (-4) = 0$

KCL @ Y:

$$\frac{0 - (V_Y - 15)}{5} + (-2) + \frac{V_X - V_Y}{12} + 4 = 0$$

Solve $\Rightarrow$ $\begin{bmatrix} V_X \\ V_Y \end{bmatrix} = \begin{bmatrix} -172/21 \\ 320/21 \end{bmatrix}$ $\quad V_X - V_Y = -\frac{164}{7}\ V$

$V_Z = V_Y - 15$

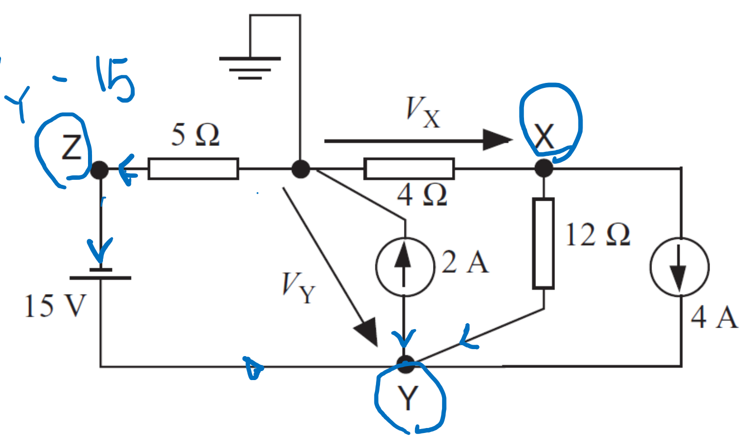



**Figure 4.9** Preparation for the analysis of the circuit of Figure 4.7, using a different choice of reference node

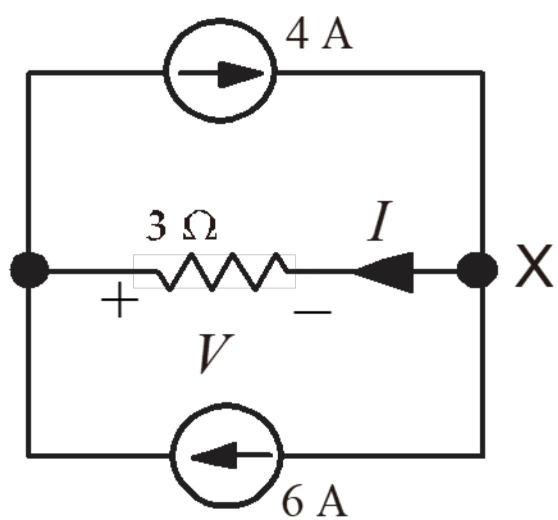
# Superposition principle

1. Decompose multiple sources

2. Apply only a single src each time
   * Easier to find an equiv. ckt.
   * Simpler analysis
   * Set other src's to 0

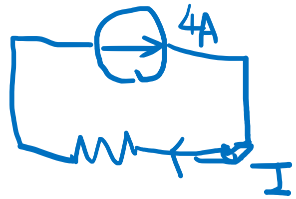
3. Sum up → Final case.

Multiple Sources



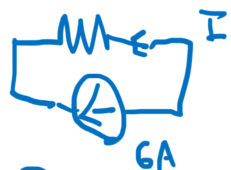
# Superposition example

* Apply $S_1 : 4A$ $S_2 : \rightarrow 0$

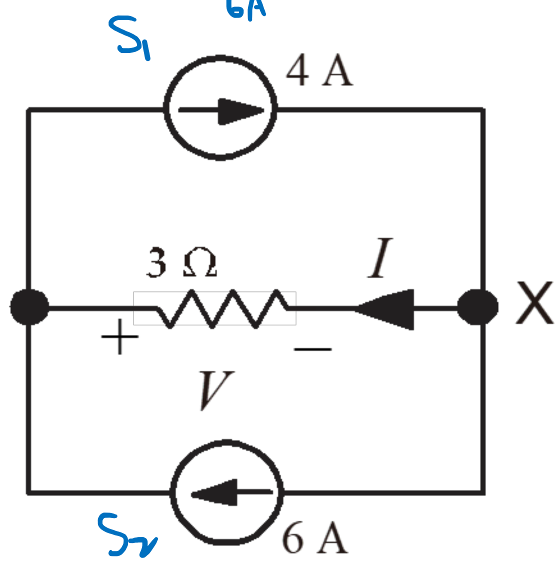



$I|_{S_1} = 4A$

* Apply $S_2 : 6A$ $S_1 \rightarrow 0$




$I|_{S_2} = -6A$




*

$I|_{S_1 S_2} = 4+(-6)$

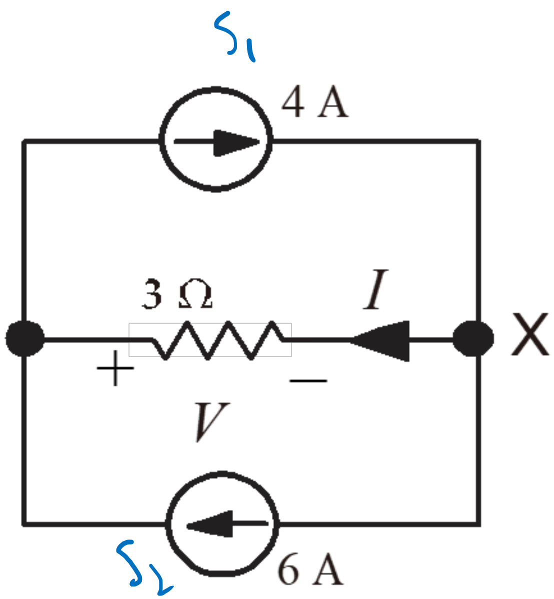
$= -2 A$

# Superposition example

$$V = V\Big|_{S_1} + V\Big|_{S_2}$$

$$= -12 + 18$$

$$= 6 \ (V)$$



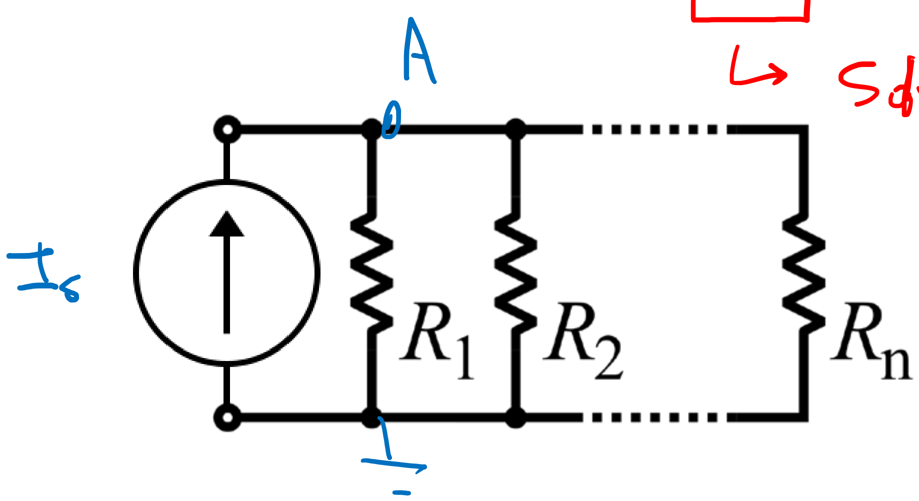
# Single source circuit

KCL @ A

$$I_s + \frac{0 - V_A}{R_1} + \frac{0 - V_A}{R_2} + \cdots + \frac{0 - V_A}{R_n} = 0$$

$$\underbrace{\left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}\right]}_{G} \cdot \underbrace{\left[V_A\right]}_{\vec{v}} = \underbrace{\left[I_s\right]}_{\vec{s}}$$

$$\vec{v} = \underbrace{G^{-1}}_{\hookrightarrow\ \text{Solver !}} \cdot \vec{s}$$



# Superposition

$$\underbrace{\begin{bmatrix} -3/2 & 1/2 & 0 \\ 1/2 & -13/12 & 1/4 \\ 0 & 1/4 & -9/20 \end{bmatrix}}_{G} \underbrace{\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix}}_{\vec{V}} = \begin{bmatrix} -I_1 - I_2 \\ I_2 \\ -I_3 \end{bmatrix} = \begin{bmatrix} -I_1 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -I_2 \\ I_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -I_3 \end{bmatrix}$$

$$G \quad \vec{V} = \vec{S} = \vec{S}_1 + \vec{S}_2 + \vec{S}_3$$

$$\vec{V} = \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \underbrace{G^{-1}}_{\text{Solver}} \cdot \vec{S}$$

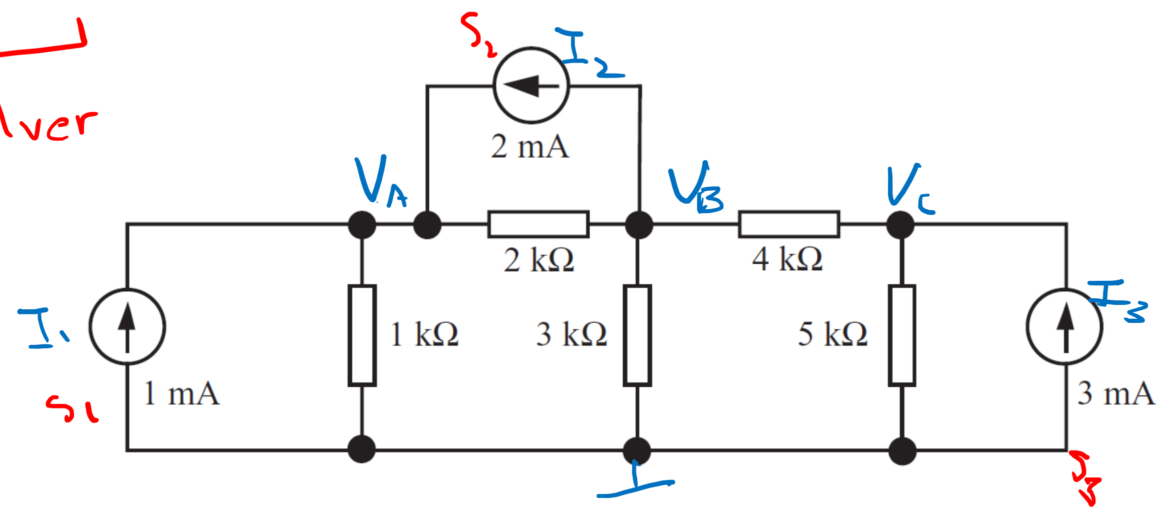



**Figure 4.1** The circuit to be analysed

# Superposition

Nodal eq

$$G \cdot \vec{v} = \underbrace{I_1 \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}}_{\vec{s}_1} + \underbrace{I_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}}_{\vec{s}_1} + \underbrace{I_3 \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}}_{\vec{s}_2}$$

$$\vec{V} = G^{-1} \vec{s}_1 + G^{-1} \vec{s}_2 + G^{-1} \vec{s}_3$$

$$\vec{V} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3$$

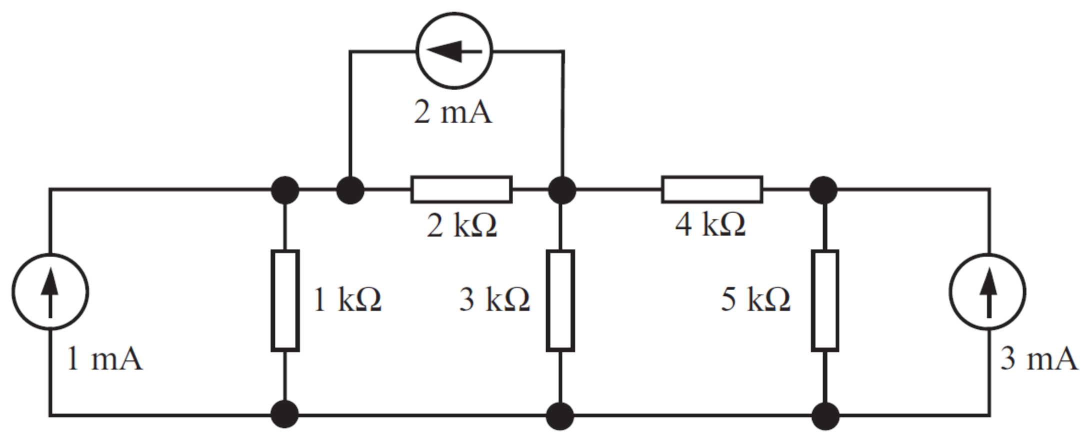



**Figure 4.1** The circuit to be analysed

# Superposition

Apply

$$\vec{V}_1 = \begin{bmatrix} V_{A,1} \\ V_{B,1} \\ V_{C,1} \end{bmatrix} = G^{-1} \vec{S}_1 \longrightarrow \text{Src 1 only}, I_2 = 0 \; I_3 = 0$$

$$\vec{V}_2 = \begin{bmatrix} V_{A,2} \\ V_{B,2} \\ V_{C,2} \end{bmatrix} = G^{-1} \vec{S}_2 \rightarrow \text{Src 2 only}, I_1 = 0, I_3 = 0$$

$$\vec{V}_3 = \begin{bmatrix} \; \end{bmatrix} = G^{-1} \vec{S}_3 \rightarrow \text{Src 3 only}, I_1 = 0 \; I_2 = 0$$

$$\vec{V}_1 + \vec{V}_2 + \vec{V}_3 = G^{-1}(\vec{S}_1 + \vec{S}_2 + \vec{S}_3)$$

$$\vec{V} = G^{-1} \vec{S}$$

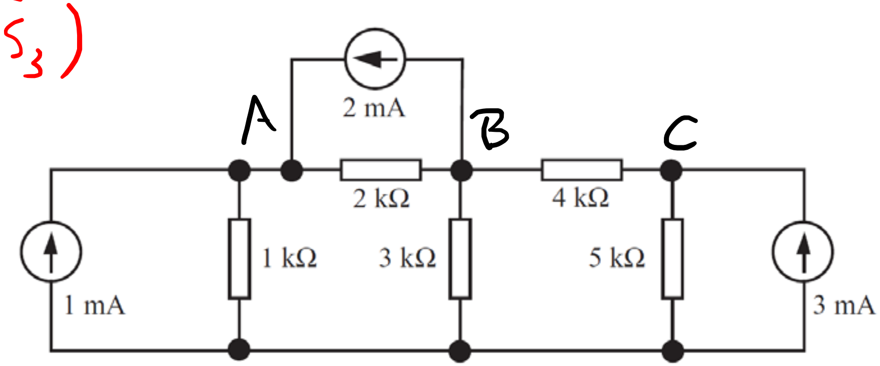



**Figure 4.1** The circuit to be analysed

# Superposition

Apply $S_1$, $S_2 \to 0$ mA $S_3 \to 0$ mA

$\Rightarrow V_A|_{S_1}$

Apply $S_2$, $S_1 \to 0$ mA $S_3 \to 0$ mA

$\Rightarrow V_A|_{S_2}$

Apply $S_3$, $S_1 \to 0$ mA $S_2 \to 0$ mA

$\Rightarrow V_A|_{S_3}$

$V_A = V_A|_{S_1} + V_A|_{S_2} + V_A|_{S_3}$

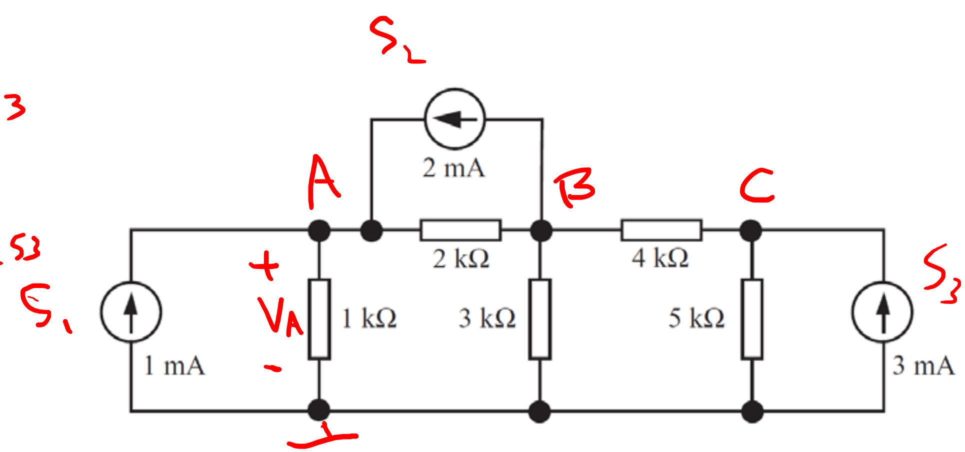



**Figure 4.1** The circuit to be analysed

# Superposition

$$V_A|_{S_1} = G^{-1} \cdot \vec{S}_1$$

$$V_A|_{S_2} = G^{-1} \vec{S}_2$$

$$V_A|_{S_3} = G^{-1} \vec{S}_3$$

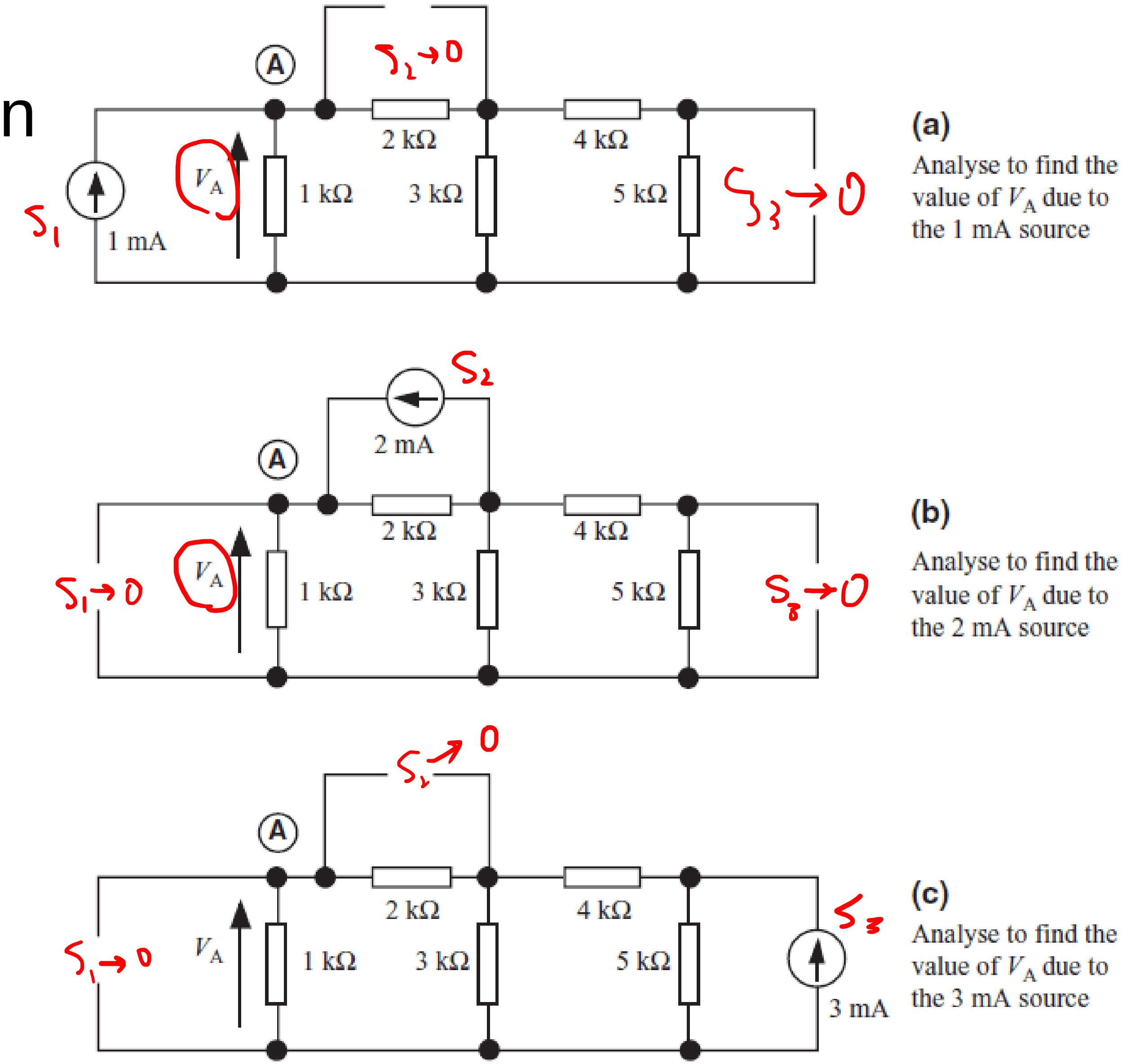



**Figure 4.10** Illustration of the use of the superposition principle to find the voltage $V_A$ at node A in the circuit of Figure 4.4. The three calculated voltages are added together to find the actual value of $V_A$.

# Review: voltage divider

$$V_A = V \cdot \frac{R_2}{R_1 + R_2}$$

$$R = R_1 + R_2$$

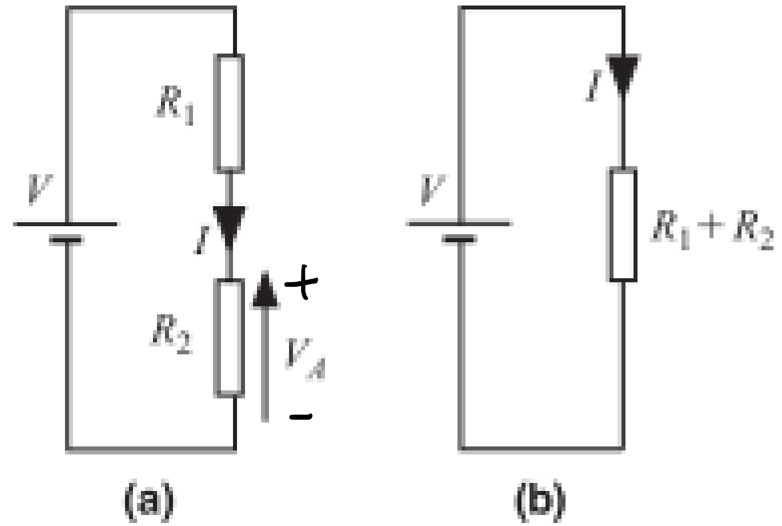



**Figure 3.24** Pertinent to Example 3.3

# Review: current divider

$$I_1 = I \cdot \frac{R_2}{R_1+R_2}$$

$$I_2 = I \cdot \frac{R_1}{R_1+R_2}$$

$$R = \underbrace{R_1 \parallel R_2} = \frac{1}{1/R_1 + 1/R_2}$$

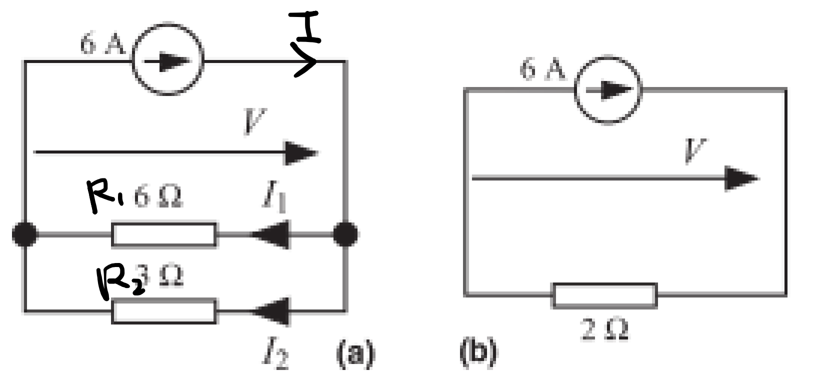
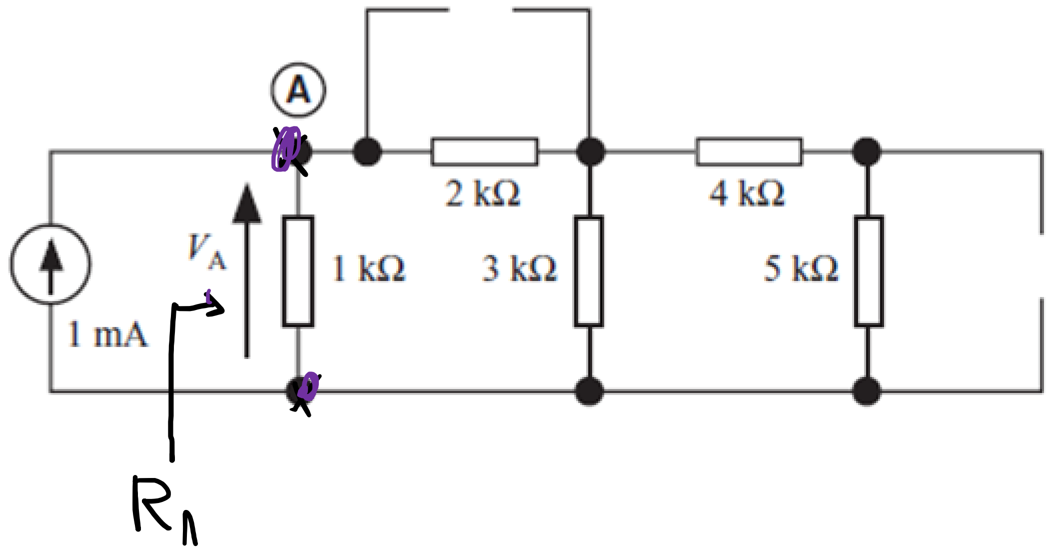



**Figure 3.25** Pertinent to Example 3.4

## Source (1)

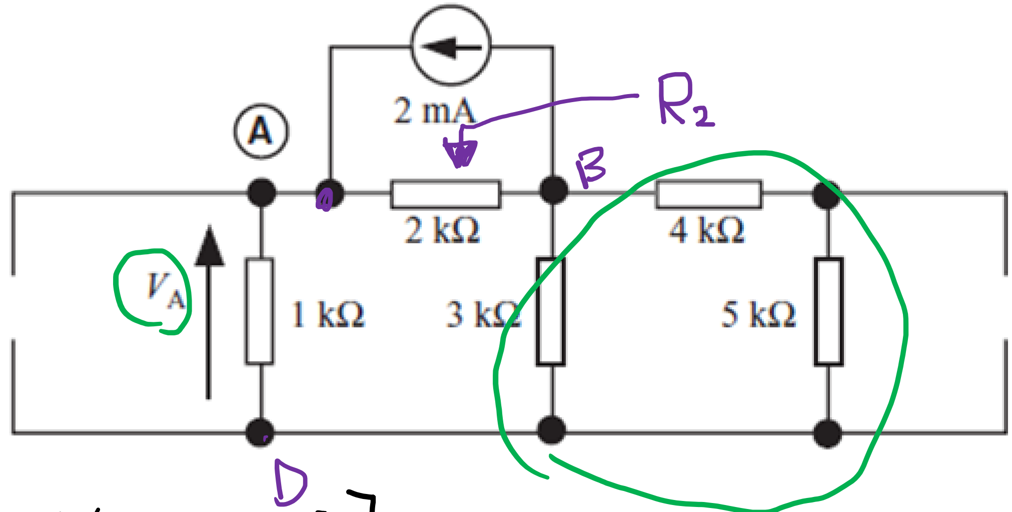



$$R_A = [[(4k+5k) \,//\, 3k] + 2k] \,//\, 1k$$

$$= 17/21 \ (k\Omega)$$

$$V_A|_{s1} = I_1 \cdot R_A = \frac{17}{21} (V)$$

# Source (2)



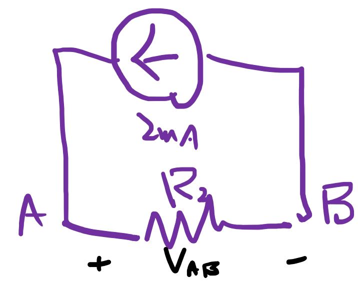

$$R_2 = 2^k /\!/ \left[ 1k + \left( 3k /\!/ (4k + 5k) \right) \right]$$

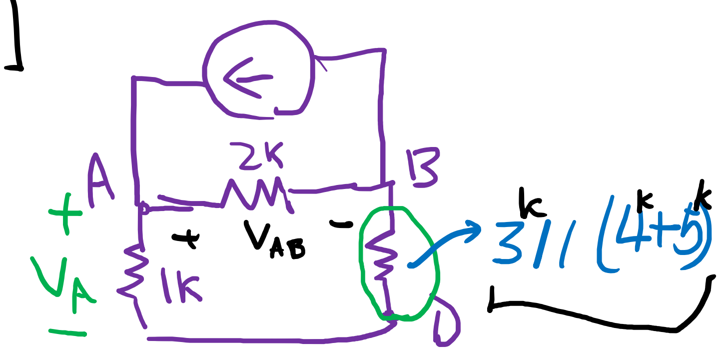
$$\Rightarrow V_{AB} = I_2 \times R_2 = 2^{mA} \times R_2$$

$$\Rightarrow V_{AD} = V_{AB} \times \frac{1^k}{1^k + \left[ 3^k /\!/ (4^k + 5k) \right]}$$

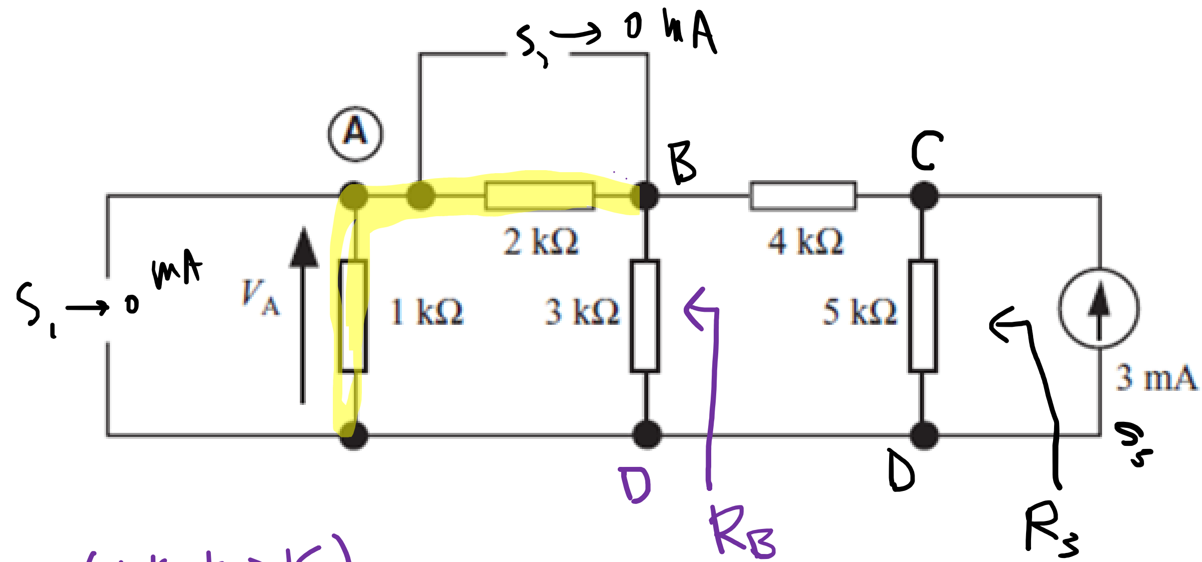
$$\downarrow$$

$$V_A \big|_{S2}$$



# Source (3)



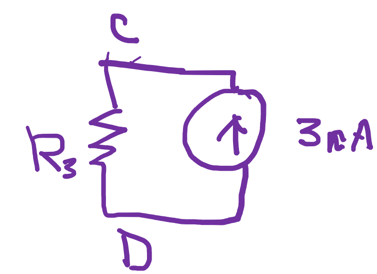

$R_B = 3K \,//\, (1K + 2K)$

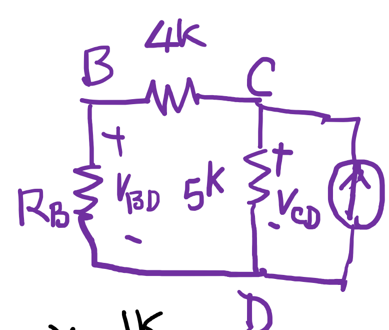
$R_3 = 5K \,//\, (4K + R_B)$

$V_{CD} = I_3 \times R_3 = 3^{mA} \times R_3$

$V_{BD} = V_{CD} \times \frac{R_B}{R_B + 4K}$

$V_{AD} = V_{BD} \times \frac{1K}{1K + 2K}$




$V_A \big|_{S3} = I_3 \times R_3 \times \frac{R_B}{R_B + 4K} \times \frac{1K}{1K + 2K}$

# Superposition

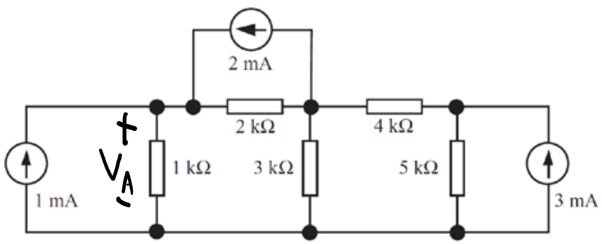
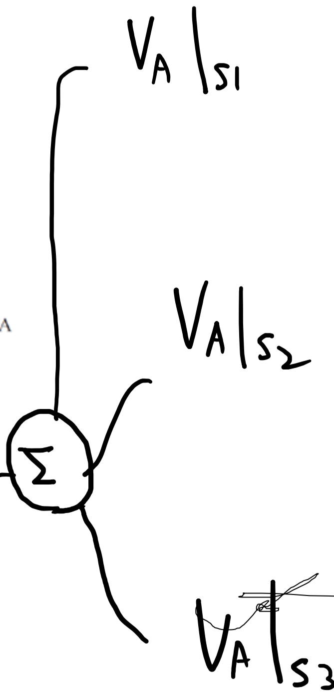
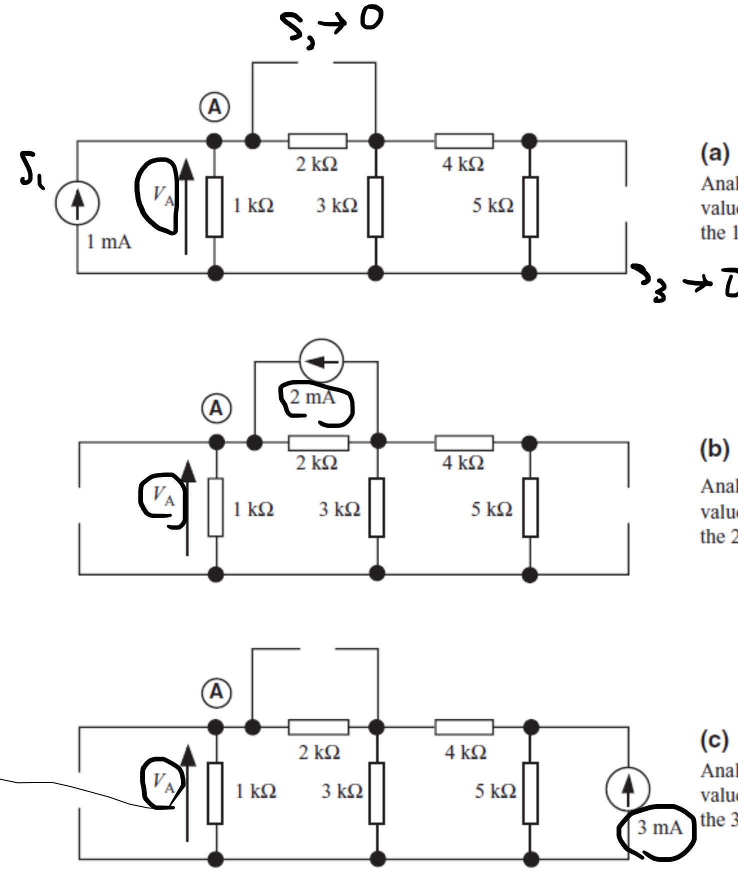



**Figure 4.1** The circuit to be analysed



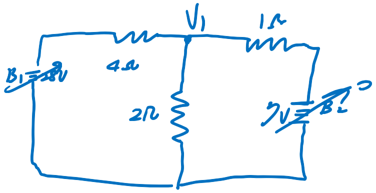




**(a)** Analyse to find the value of $V_A$ due to the 1 mA source

**(b)** Analyse to find the value of $V_A$ due to the 2 mA source

**(c)** Analyse to find the value of $V_A$ due to the 3 mA source

**Figure 4.10** Illustration of the use of the superposition principle to find the voltage $V_A$ at node A in the circuit of Figure 4.4. The three calculated voltages are added together to find the actual value of $V_A$.




B1

$$\frac{2}{1+2} = \frac{2}{3}$$

$$\frac{2}{3} + 4 =$$

28 x