9、 $x(t) = \frac{2}{1+t^2}$ known that $e^{-|t|} \overset{F.T.}{\longleftrightarrow} \frac{2}{1+w^2}$, Find Fourier transform by duality

錯誤

By Duality

$$\frac{2}{1+t^2} \Leftrightarrow \frac{e^{-|w|}}{2\pi}$$

$$X_2(jw) = \frac{e^{-|w|}}{2\pi}$$

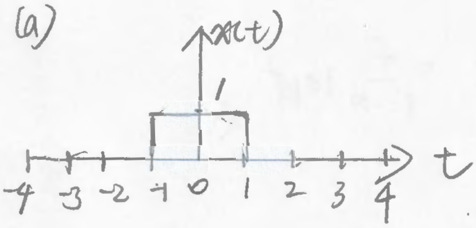
正確

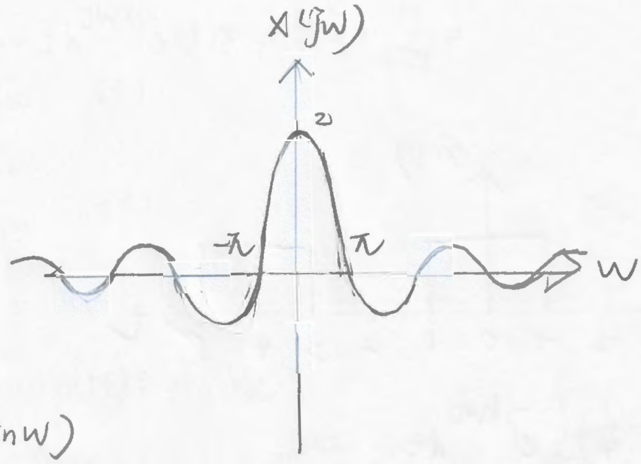
By Duality

$$\frac{2}{1+t^2} \Leftrightarrow 2\pi e^{-|w|}$$

$$X_2(jw) = 2\pi e^{-|w|}$$

8. (a)





$$X(jw) = \int_{-\infty}^{\infty} x(t) e^{-jwt} dt$$

$$= \int_{-1}^{1} e^{-jwt} dt = \frac{1}{-jw} e^{-jwt}\Big|_{-1}^{1}$$

$$= \frac{1}{-jw}(e^{-jw} - e^{jw}) = \frac{j}{w}(-2j\sin w)$$

$$= \frac{2}{w}\sin w = 2\mathrm{sinc}\, w$$

(b) $\int_0^\infty \frac{\sin w}{w} dw = \frac{2\pi}{2\cdot 2} = \frac{\pi}{2}$

9.) $x(t)$ 的FS為 $\sum_{k=-\infty}^{+\infty} a_{-k}^* e^{jk\omega_0 t}$，請問还有無其它形式？有

($x(t)$ is real)

$\because a_k^* = a_{-k}$

$\therefore x(t) = a_0 + \sum_{k=1}^{\infty} [a_k e^{jk\omega_0 t} + a_{-k} e^{-jk\omega_0 t}]$

$x(t) = a_0 + \sum_{k=1}^{\infty} [a_k e^{jk\omega_0 t} + a_k^* e^{-jk\omega_0 t}]$

$x(t) = a_0 + \sum_{k=1}^{\infty} 2Re\{a_k e^{jk\omega_0 t}\}$

錯了！$\because a_k$ is complex

$x(t) = a_0 + \sum_{k=1}^{\infty} 2 a_k \cos k\omega_0 t$

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答案應為

$x(t) = a_0 + 2\sum_{k=1}^{\infty} A_k \cos(k\omega_0 t + \theta_k)$

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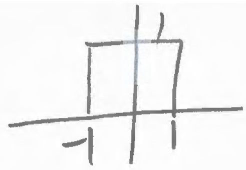
8.

$$x(t) = \text{rect}\left(\frac{t}{2}\right) = \begin{cases} 1, & |t| \le 1 \\ 0, & |t| > 1 \end{cases}$$



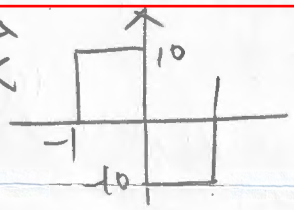
(a) $$X(j\omega) = \int_{-1}^{1} x(t) e^{-j\omega t} dt = \int_{-1}^{1} e^{-j\omega t} dt = \frac{1}{j\omega}\left(-e^{-j\omega t}\right)\Big|_{-1}^{1}$$

$$= \frac{-1}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \frac{1}{j\omega}\left(e^{j\omega} - e^{-j\omega}\right)$$

$$= \frac{2}{\omega}\sin\omega = 2\,\text{sinc}\,\omega$$

(b) $$\int_0^{\infty} \frac{\sin(\omega)}{\omega} d\omega = \frac{\pi}{2} ※$$

9. 組員:陳虹衣

求 

的 Fourier transfer.

wrong solution:

$$X(j\omega) = \int_{-1}^{1} x(t) e^{-j\omega t} dt = \int_{-1}^{1} 10 e^{-j\omega t} dt$$

$$= \frac{-10}{j\omega} e^{-j\omega t}\Big|_{-1}^{1} = \frac{-10}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \frac{20}{\omega}\sin\omega.$$

correct solution:

$$X(j\omega) = \int_{-1}^{1} x(t) e^{-j\omega t} dt.$$

<要拆開>

$$\Rightarrow X(j\omega) = \int_{-1}^{0} 10 e^{-j\omega t} dt + \int_{0}^{1} -10 e^{-j\omega t} dt$$

$$= \frac{-10}{j\omega} e^{-j\omega t}\Big|_{-1}^{0} + \frac{10}{j\omega} e^{-j\omega t}\Big|_{0}^{1}$$

$$= \frac{-10}{j\omega}\left(1 - e^{j\omega}\right) + \frac{10}{j\omega}\left(e^{-j\omega} - 1\right) = \frac{10}{j\omega}\left(e^{-j\omega} + e^{j\omega} - 2\right)$$

$$= \frac{20}{j\omega}(\cos\omega - 1) ※$$