Midterm Exam I
October 24, 2016

Rules and Regulations: It is permitted to bring one paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

1. Please determine whether each of the following statements is *True* or *False*.

   (a) (3%) A signal can be represented by a function.

   (b) (3%) A system can be represented by a function.

   (c) (3%) For a linear system, if the input is $x[n] = 0$ for all n, then the output must be $y[n] = 0$ for all n.

   (d) (3%) For a discrete-time signal $x[n]$, we have $x[n-5]\delta[n-5] = x[0]$.

   (e) (3%) For a continuous-time signal $x(t)$, we have $x(t)*\delta(t-t_0) = x(t_0)*\delta(t-t_0)$.

   (f) (3%) The system with input and output relationship $y[n] = x[n]+1$ is a linear system.

   (g) (3%) For continuous-time signals $x(t)$ and $y(t)$, we have
   $$x(t) * y(-t) = \int_{-\infty}^{\infty} x(\tau)y(\tau - t)d\tau.$$

   (h) (3%) For a real number a, we have
   $$\delta(ax) = \frac{1}{a}\delta(x).$$

   (i) (3%) Consider a discrete-time system where input $x[n]$ and output $y[n]$ are related by $y[n] = \Re\{x_{[n]}\}$ and $\Re\{\cdot\}$ denotes the real part. This system is linear.

   (j) (3%) If $x[n]$ is even and $h[n]$ is odd, then $y[n] = x[n] * h[n]$ is odd.

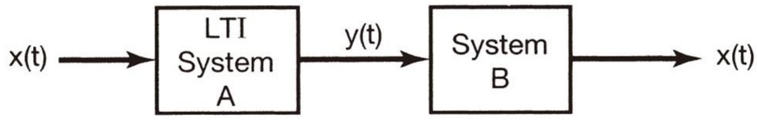
2. (10%) Please show that the even-odd decomposition of a signal is unique. Take a continuous-time signal $x(t)$ as an example, if there exist even functions $x_{e_1}(t), x_{e_2}(t)$ and odd functions $x_{o_1}(t), x_{o_2}(t)$ such that $x(t) = x_{e_1}(t) + x_{o_1}(t)$ and $x(t) = x_{e_2}(t) + x_{o_2}(t)$, then show that $x_{e_1}(t) = x_{e_2}(t)$ and $x_{o_1}(t) = x_{o_2}(t)$.

3. (10%) Let $x(t) = u(t) - u(t-2)$ and $h(t) = u(t) - u(t-8)$. Please sketch $y(t) = x(t) * h(t)$.

4. (5%) Please show that
   $$x(t) * \delta(t - t_0) = x(t - t_0).$$

5. (5%) Please show that if the input $x(t)$ to a time-invariant system is periodic with period T, then the output $y(t)$ is also periodic with the same period T.

6. Consider the cascade of two systems given below. The first system, A, is LTI. The second system, B, is the inverse of system A. Let $y_1(t)$ denote the response of system A to $x_1(t)$, and let $y_2(t)$ denote the response of system A to $x_2(t)$.



   (a) (5%) Find the response of system B to the input $ay_1(t)+by_2(t)$, where a and b are constants.

   (b) (5%) Find the response of system B to the input $y_1(t-\tau)$.

7. If a continuous-time LTI system with the impulse response $h(t)$ is BIBO stable, then we want to show that

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty.$$

   Let the input $x(t) = \operatorname{sgn}(h(-t))$ where

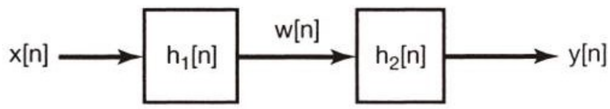
$$\operatorname{sgn}(s) = \begin{cases} 1, & s > 0; \\ -1, & s < 0; \\ 0, & s = 0. \end{cases}$$

   (a) (5%) Show that $x(t)$ is bounded.

   (b) (5%) Let $y(t)$ be the output. Find $y(0)$ and show that

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty.$$

8. (10%) Consider the cascade of two LTI systems shown as below.



   Let

$$h_1[n] = \sin(\pi^n)$$

   and

$$h_2[n] = a^n u[n], \quad |a| < 1.$$

   We also let the input $x[n] = \delta[n] - a\delta[n-1]$. Please find the output $y[n]$. (Hint: Please use the associative and commutative properties of convolution.)

9. (10%) Thank E94036128 for providing this problem extended from the example given in class. Consider system, A, where the input $x(t)$ and output $y(t)$ are related by $y(t) = \sin(x(t))$ and system, B, where the input $x(t)$ and output $z(t)$ are related by $z(t) = x(\sin(t))$. Please determine which system is linear and which system is time-invariant.