

6. (10 points) Please write down the Fundamental Theorem of Calculus.

7. (20 points) Suppose that g is differentiable with $g'(x) < 0$ for all $x < 1$, $g'(1) = 0$, and $g'(x) > 0$ for all $x > 1$. And Suppose that $g(1) = 0$. Set

$$G(x) = \int_0^x g(t) dt.$$

Which of the following statements are true? (You need to prove your answer.)

- (a) G is twice-differentiable.
- (b) $x = 1$ is a critical number of G .
- (c) G is an increasing function.
- (d) The graph of G is concave up for all x .

8. (10 points) Please determine whether the integral is convergent or not.

(a)

$$\int_0^{\pi} x^{-1/2} dx \quad 2\sqrt{x}$$

(b)

$$\int_0^{\pi} \frac{\sin^2 x}{x^{1/2}} dx$$

9. (10 points) Prove that for all $x > 0$ and all positive integers n ,

$$e^x > 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}.$$

(Hint: $e^x = 1 + \int_0^x e^t dt$.)