Midterm Exam I

November 8, 2017

Rules and Regulations: It is permitted to bring one paper of A4 size with handwritten formulas. There is a time limit of two hours and fifty minutes.

Problems for Solution:

1. Please determine whether each of the following statements is *True* or *False*.

   (a) (3%) If events A, B, and C are pairwise independent, then they are independent.

   (b) (3%) A random variable can be regarded as a function.

   (c) (3%) For a random variable X and a real-valued function g, $g(X)$ is also a random variable.

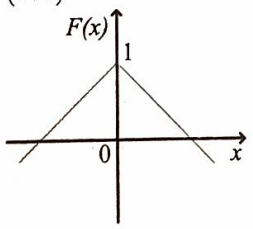
   (d) (3%) For two mutually exclusive events A and B, they must be independent. $P(A,B)=\phi$ $P(A\cap B)=P(A)P(B)$

   (e) (3%) The set of possible values of a discrete random variable should be finite.

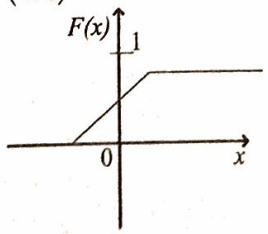
2. (5%) If events A, B, and C are independent, show that A and $B^C - C$ are independent.

3. Determine if each of the following figures is a valid cumulative distribution function. Reasons are required. T or F. Reason

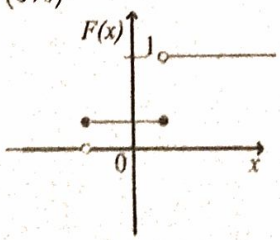
   (a) (5%)



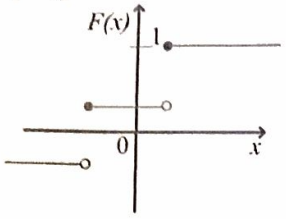
   (b) (5%)



   (c) (5%)



(d) (5%)



4. (5%) Consider the three alleles A, B, and O for the human blood types. Suppose that, in a certain population, the frequencies of these alleles are 0.45, 0.20, and 0.35, respectively. Mary and John are married and have a son named Peter. Marry and Peter both have blood types AB. John's blood type is B. What is the probability that John's genotype is BB? (Note: John's genotype is either BB or BO.)

5. (10%) For a random variable X with distribution function F, show that

$$\lim_{t \to -\infty} F(t) = 0.$$

6. If X is the number of heads in n flips of a fair coin, then the possible values of X are $0, 1, 2, 3, \ldots, n$.

   (a) (5%) What is the probability mass function $p(x)$ of X?

   (b) (5%) Show that $\sum_{x=0}^{n} p(x) = 1$.

   (c) (5%) Find $E(X)$.

7. (10%) Cards are drawn at random from an ordinary deck of 52, one by one and with replacement. What is the probability that the ace of hearts is drawn before the number cards (i.e., $2, 3, \ldots, 10$) of spades?

8. (10%) 感謝提供題目的陳欣怡、謝金芳同學。 推甄第二階段面試結束，有六位學生非常優秀但只能錄取兩名，於是面試官跟六位同學說: "這裡有一把左輪式六彈匣手槍，裡頭只裝了兩顆實彈，且兩顆實彈擺放在連續的彈匣位置，如果開槍射出實彈就表示你獲得錄取。" 第一位同學開槍後並沒有成功擊發實彈，而你是第二位同學。請問，"重新旋轉彈匣再開槍"還是"不選轉彈匣繼續開下一槍"，那一個讓你錄取的機率比較高呢？

9. (10%) 感謝提供題目的楊登宇、劉川榮、陳品叡同學。有數字0到9共10張牌，挑戰者挑選4張牌放在獎金版上，分別代表千位數、百位數、十位數、個位數。隨後，主持人打開個位數與十位數，分別是7和8。並把剩下的六張牌中打開四張，留下最大與最小的數字未開。挑戰者可以從千位數與百位數中挑選一張牌與蓋著的兩張牌中的一張做交換。請問，挑戰者如果選擇換牌，獎金會變多的機率為何？