國立成功大學

工科系統微積分(二) 期末考

6月 21 日, 2016

課程代碼: F115621

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Instructions:

1. There are **6 pages** (excluding the cover page), **11 problems** in this exam.
2. You have **110 minutes** to work on the exam.
3. Do **NOT** start the exam until you are told to do so.
4. Please have your **student ID** card ready.
5. No textbook, notes, calculator, or sketching sheets are allowed.
6. You may want to use the back of the exam pages for computations.

Page:	1	2	3	4	5	6	Total
Points:	20	10	10	20	20	20	100
Score:							

1. (10 points) If $z = e^{x+2y}$, where $x = s/t$ and $y = t/s$, find $\partial z/\partial s$ and $\partial z/\partial t$.

1. ____________

$$\frac{\partial z}{\partial s} = e^{x+2y} \cdot \frac{1}{t} - 2e^{x+2y} \cdot \frac{t}{s^2}$$ 5%

$$\frac{\partial z}{\partial t} = -e^{x+2y} \cdot \frac{s}{t^2} + 2e^{x+2y} \cdot \frac{1}{s}$$ 5%

2. (10 points) Find $\partial z/\partial x$ and $\partial z/\partial y$ if $yz + x\ln y = z^2$.

2. ____________

$$F(x,y,z) = yz + x\ln y - z^2$$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{\ln y}{y-2z}$$ 5%

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z+\frac{x}{y}}{y-2z}$$ 5%

3. (5 points) Find the directional derivative of $f(x,y) = \frac{x}{x^2+y^2}$ at the point $(1,2)$ in the direction of the vector $\langle 3,5\rangle$.

3. ___________

$\vec{u} = \frac{\langle 3,5\rangle}{|\langle 3,5\rangle|} = \frac{\langle 3,5\rangle}{\sqrt{34}}$

$\nabla f(x,y) = \left\langle \frac{(x^2+y^2)-x(2x)}{(x^2+y^2)^2}, \frac{-x(2y)}{(x^2+y^2)^2} \right\rangle$

$D_{\vec{u}} f(1,2) = \frac{1}{\sqrt{34}} \langle 3,5\rangle \cdot \left\langle \frac{3}{25}, \frac{-4}{25} \right\rangle = \frac{-11}{25\sqrt{34}}$ 5%

4. (5 points) Find the equations of the tangent plane and the normal line to the surface $x^4+y^4+z^4 = 3x^2y^2z^2$ at the point $(1,1,1)$.

4. ___________

$f(x,y,z) = x^4+y^4+z^4-3x^2y^2z^2$

$\nabla f = \langle 4x^3-6xy^2z^2, 4y^3-6x^2yz^2, 4z^3-6x^2y^2z \rangle$

$\nabla f(1,1,1) = \langle -2, -2, -2\rangle$

Tangent plane:

$(-2,-2,-2) \cdot \langle x-1, y-1, z-1\rangle = 0$

normal line:

$\vec{r}(t) = (1,1,1) + t(-2,-2,-2)$ → 對一個 3%

全對 5%

5. Let $f(x, y) = y^2 - 2y\cos x$.

(a) (5 points) Find the critical points of $f(x, y)$.

(a) ____________

$$\begin{cases} f_x = 2y\sin x \\ f_y = 2y - 2\cos x \end{cases}$$

$f_x = f_y = 0 \Rightarrow$ $x = \frac{\pi}{2} + n\pi$, $y = 0$

or $x = 2n\pi$, $y = 1$

or $x = \pi + 2n\pi$, $y = -1$

5%

(b) (5 points) Determine the local maximum/minimum values and saddle point(s) of $f(x, y)$.

(b) ____________

$f_{xx} = 2y\cos x$, $f_{xy} = 2\sin x$, $f_{yy} = 2$

$$D = \begin{vmatrix} 2y\cos x & 2\sin x \\ 2\sin x & 2 \end{vmatrix} = 4y\cos x - 4\sin^2 x$$

全討論 5%
只討論 min 3%

$D(\frac{\pi}{2} + n\pi, 0) = -4$ $\rightsquigarrow$ Saddle pts.

$D(2n\pi, 1) = 4$, $f_{xx}(2n\pi, 1) = 2$ $\rightarrow$ local min value

$D(\pi + 2n\pi, -1) = 4$, $f_{xx}(\pi + 2n\pi, -1) = 2$ $\nearrow$

$= f(2n\pi, 1)$

$= f(\pi + 2n\pi, -1)$

$= -1$

6. (10 points) Find the extreme values of the function $f(x, y) = 2x^2 + 3y^2 - 4x - 2$ subject to the condition $x^2 + y^2 \leq 16$.

6. ____________

$f_x = 4x - 4$

$f_y = 6y$ → critical pt = $(1, 0)$

$f(1,0) = -4$ ~ $\boxed{\text{min}}$ 5%

$g(x,y) = x^2 + y^2 - 16$

$\nabla g = \langle 2x, 2y \rangle$

$$\begin{cases} 4x - 4 = \lambda(2x) \\ 6y = \lambda(2y) \\ x^2 + y^2 - 16 = 0 \end{cases} \Longrightarrow \begin{cases} \lambda = 3 \\ x = -2 \\ y = \pm 2\sqrt{3} \end{cases}$$

$f(-2, \pm 2\sqrt{3})$

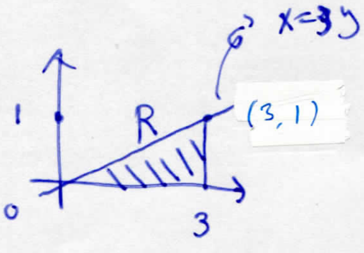
$= 8 + 36 + 8 - 2 = 50$

$\boxed{\text{max}}$ 5%

7. (10 points) Evaluate the double integral

$$\int_0^1 \int_{3y}^3 e^{x^2}\, dx\, dy.$$

7. ____________



$$= \iint_R e^{x^2}\, dA$$

$$= \boxed{\int_0^3 \int_0^{\frac{x}{3}} e^{x^2}\, dy\, dx} \quad 5\%$$

$$= \int_0^3 \frac{x}{3} e^{x^2}\, dx$$

$$= \frac{1}{6} e^{x^2} \Big|_0^3 = \frac{1}{6}(e^9 - 1) \quad 5\%$$

8. (10 points) Find the volume of the solid that is bounded by the cylinder $x^2 + y^2 = 4$ and the ellipsoid $4x^2 + 4y^2 + z^2 = 64$.

8. ____________

$$z = \pm\sqrt{64 - 4x^2 - 4y^2}$$

$$\text{Volume} = \boxed{\iint_{\{x^2+y^2\le 4\}} 2\sqrt{64-4x^2-4y^2}\, dA} \quad 3\%$$

$$= \boxed{2\int_0^{2\pi}\int_0^2 \sqrt{64-4r^2}\, r\,dr\,d\theta} \quad 4\%$$

$$= 8\pi \int_0^2 \sqrt{16-r^2}\, r\,dr$$

$$\begin{pmatrix} u = 16 - r^2 \\ du = -2r\,dr \end{pmatrix}$$

$$= -4\pi \int_{16}^{12} u^{\frac{1}{2}}\, du$$

$$= \boxed{-\frac{8}{3}\pi\left(12^{\frac{3}{2}} - 16^{\frac{3}{2}}\right)} \quad 3\%$$

9. (10 points) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$.

9. ____________

$$\begin{cases} z = 1 \\ x^2 + y^2 + z^2 = 4 \end{cases} \Rightarrow x^2 + y^2 = 3$$

$$z = \sqrt{4 - x^2 - y^2}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\cancel{2}}(4 - x^2 - y^2)^{-\frac{1}{2}}(-\cancel{2}x)$$

$$\frac{\partial z}{\partial y} = \frac{1}{2}(4 - x^2 - y^2)^{-\frac{1}{2}}(-2y)$$

$$\sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} = \sqrt{1 + (4 - x^2 - y^2)^{-1}(x^2 + y^2)}$$

$$\text{area} = \boxed{\int_0^{2\pi}\int_0^{\sqrt{3}} \boxed{\sqrt{1 + \frac{r^2}{4-r^2}}}\, r\,dr\,d\theta} = -2\pi\int_4^1 u^{-\frac{1}{2}}\, du$$

列式 5%

$$= -4\pi(1 - \sqrt{4}) = 4\pi \quad 5\%$$

10. (10 points) Evaluate the integral

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} y\,dz\,dy\,dx.$$

$$= \boxed{\int_0^{\pi}\int_0^{\frac{\pi}{4}}\int_0^{\sqrt{2}} \rho\sin\phi\sin\theta\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta} \quad 5\%$$

10. ______

or $\int_0^{\pi}\int_0^{1}\left(\sqrt{2-r^2}-r\right)\cdot r\sin\theta\cdot r\,dr\,d\theta$

$$= \left(\int_0^{\sqrt{2}}\rho^3\,d\rho\right)\cdot\left(\int_0^{\pi}\sin\theta\,d\theta\right)\cdot\left(\int_0^{\frac{\pi}{4}}\sin^2\phi\,d\phi\right)$$

$$= \left[\frac{\rho^4}{4}\right]_0^{\sqrt{2}}\cdot\left[-\cos\theta\right]_0^{\pi}\cdot\left[\frac{1}{2}\phi-\frac{\sin 2\phi}{4}\right]_0^{\frac{\pi}{4}} = \boxed{\frac{\pi}{4}-\frac{1}{2}} \quad 5\%$$

11. (10 points) Evaluate the integral

$$\iint_R (x+y)e^{x^2-y^2}\,dA,$$

where R is the rectangle enclosed by the lines $x-y=0, x-y=2, x+y=0, x+y=3$.

11. ______

$$\begin{cases} u=x+y \\ v=x-y \end{cases} \Rightarrow \begin{cases} x=\frac{u+v}{2} \\ y=\frac{u-v}{2} \end{cases} \qquad \frac{\partial(x,y)}{\partial(u,v)} = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} = -\frac{1}{2}$$

$$\iint_R (x+y)e^{x^2-y^2}\,dA = \boxed{\int_0^3\int_0^2 u e^{uv}\cdot\left|-\frac{1}{2}\right|\,dv\,du} \quad 5\%$$

$\not= \int_0^2\int_0^3 u e^{uv}\cdot\frac{1}{2}\,du\,dv.$

$$= \frac{1}{2}\int_0^3\left[e^{uv}\right]_0^2\,du$$

$$= \frac{1}{2}\int_0^3\left(e^{2u}-1\right)du = \boxed{\frac{1}{4}\left(e^6-7\right)} \quad 5\%$$