Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35 °C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0 °C.

1

20%

a. Show the cycle on a T–s diagram.
b. Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0 °C.
c. Determine the COP for the cycle.
d. Show physical diagrams for these 4 Carnot processes

2

Oxygen gas in a piston/cylinder at 300 K, 100 kPa with a volume of 0.1 m³ is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using the 2 methods discussed in class

16%

3

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa and an isentropic efficiency of 87 %. Find the exit temperature using the 2 methods discussed in class.

18%

4

A desuperheater works by injecting liquid water into a flow of superheated steam. With 2 kg/s at 300 kPa, 200 °C, steam flowing in, what mass flow rate of liquid water at 20 °C should be added to generate saturated vapor at 300 kPa? We also want to know the rate of entropy generation in the process.

18%

5

One kilogram air at 100 kPa is mixed with 2 kg air at 200 kPa, both at 300 K, in a rigid, insulated tank. Find the final state (P, T) and the entropy generation in the process.

16%

6

Show that isentropic pumping of liquid does not change the temperature
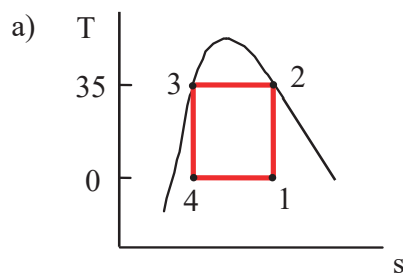
12%

Thermodynamics Final   Engr. Sci. Dept.
2017 0105

Borgnakke and Sonntag

6.32

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.
   a. Show the cycle on a *T–s* diagram.
   b. Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0°C.
   c. Determine the COP for the cycle.

Solution:
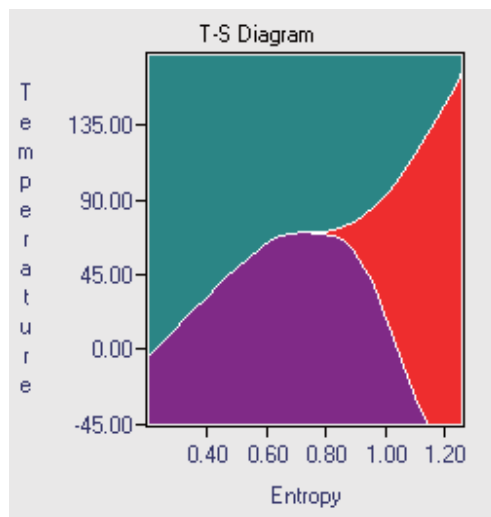
a)

b) From Table B.4.1, state 3 is saturated liquid

$$s_4 = s_3 = 0.4189 \text{ kJ/kg K}$$
$$= 0.2264 + x_4(0.8104)$$
$$\Rightarrow \quad x_4 = \textbf{0.2375}$$

State 2 is saturated vapor so from Table B.4.1
$$s_1 = s_2 = 0.9671 \text{ kJ/kg K} = 0.2264 + x_1(0.8104)$$
$$\Rightarrow \quad x_1 = \textbf{0.914}$$

c)
$$\beta' = \frac{q_H}{w_{IN}} = \frac{T_H}{T_H - T_L} = \frac{308.15}{35} = \textbf{8.8}$$


T-s diagram from CATT3 for R-410A

6.32

Consider a Carnot-cycle heat pump with R-410A as the working fluid. Heat is rejected from the R-410A at 35°C, during which process the R-410A changes from saturated vapor to saturated liquid. The heat is transferred to the R-410A at 0°C.
a. Show the cycle on a *T–s* diagram.
b. Find the quality of the R-410A at the beginning and end of the isothermal heat addition process at 0°C.
c. Determine the COP for the cycle.

Solution:

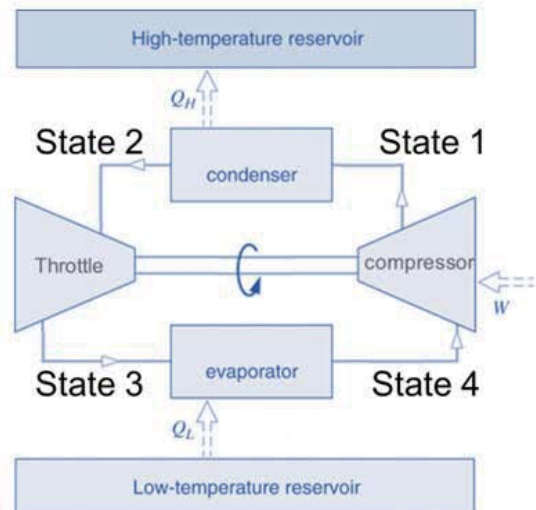

Carnot = reversible

Carnot heat engine cycle
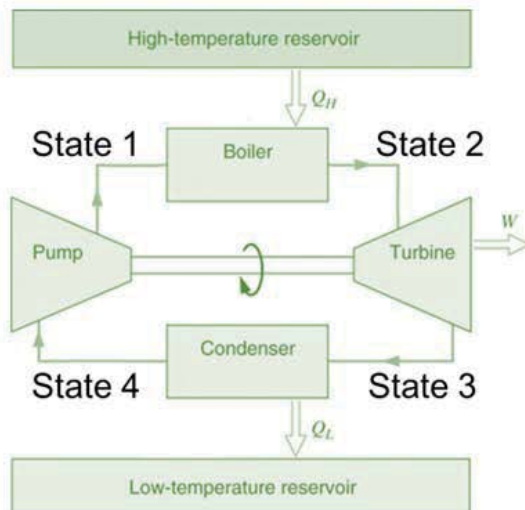
1→2: reversible isothermal (等溫) process heat is transferred from/expansion the high-temperature reservoir.

2→3: reversible adiabatic expansion (絕熱膨脹) process the temperature of the working fluid decreases

3→4: reversible isothermal (等溫) process heat is transferred to/compression the low-temperature reservoir.

4→1: reversible adiabatic compression (絕熱壓縮) process the temperature of the working fluid increases

Carnot refrigerator cycle

1→2: reversible isothermal (等溫) process heat is transferred to/compression the high-temperature reservoir.

2→3: reversible adiabatic compression (絕熱壓縮) process the temperature of the working fluid increases

3→4: reversible isothermal (等溫) process heat is transferred from/expansion the low-temperature reservoir.

4→1: A reversible adiabatic expansion (絕熱膨脹) process the temperature of the working fluid decreases

9

6.73

Oxygen gas in a piston cylinder at 300 K, 100 kPa with volume 0.1 m^3 is compressed in a reversible adiabatic process to a final temperature of 700 K. Find the final pressure and volume using Table A.5 and repeat the process with Table A.8.

Solution:

C.V. Air. Assume a reversible, adiabatic process.

Energy Eq.3.5: $u_2 - u_1 = 0 - {}_1w_2$;

Entropy Eq.6.37: $s_2 - s_1 = \int dq/T + {}_1s_{2\ gen} = 0$

Process: Adiabatic ${}_1q_2 = 0$ Reversible ${}_1s_{2\ gen} = 0$

Properties: Table A.5:   k = 1.393

With these two terms zero we have a zero for the entropy change. So this is a constant s (isentropic) expansion process. From Eq.6.23

$$P_2 = P_1(T_2 / T_1)^{\frac{k}{k-1}} = 100 \left(\frac{700}{300}\right)^{\frac{1.393}{1-1.393}} = \textbf{2015 kPa}$$

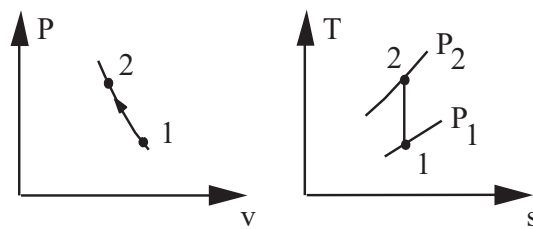Using the ideal gas law to eliminate P from this equation leads to Eq.6.24

$$V_2 = V_1(T_2 / T_1)^{\frac{1}{1-k}} = 0.1 \times \left(\frac{700}{300}\right)^{\frac{1}{1-1.393}} = \textbf{0.0116 m}^3 \qquad \textcolor{red}{method\ 1}$$

Using the ideal gas tables A.8 we get

$$s_2 - s_1 = s^o_{T2} - s^o_{T1} - R \ln(P_2/P_1) = 0 \quad or \quad P_2 = P_1 \exp[(s^o_{T2} - s^o_{T1})/R]$$

$$P_2 = 100 \exp[(7.2336 - 6.4168)/0.2598] = \textbf{2319.5 kPa}$$
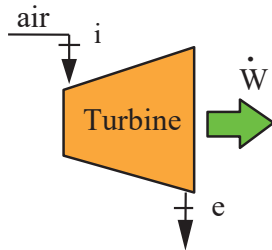
$$V_2 = V_1 (T_2 / T_1)(P_1/P_2) = 0.1 \text{ m}^3 \times \frac{700}{300} \times \frac{100}{2319.5} = \textbf{0.010 m}^3 \qquad \textcolor{red}{method\ 2}$$

7.112

A gas turbine with air flowing in at 1200 kPa, 1200 K has an exit pressure of 200 kPa and an isentropic efficiency of 87%. Find the exit temperature.

Solution:

air i

$\dot{W}$

Turbine

e

C.V. Ideal air turbine.

Adiabatic: $q = 0$, reversible: $s_{gen} = 0$

Energy Eq.4.13: $w_T = h_i - h_e$,

Entropy Eq.7.9: $s_e = s_i$

Table A.7: $h_i = 1277.8$ kJ/kg, $s^o_{Ti} = 8.34596$ kJ/kg K

The constant s process is written from Eq.6.19 as

$$\Rightarrow s^o_{Te} = s^o_{Ti} + R \ln\left(\frac{P_e}{P_i}\right) = 8.34596 + 0.287 \ln\left(\frac{200}{1200}\right) = 7.83173 \text{ kJ/kg K}$$

Interpolate in A.7.1 $\Rightarrow T_{e\,s} = 761.9$ K, $h_{e\,s} = 780.52$ kJ/kg

$$w_{T\,s} = h_i - h_{e\,s} = 1277.81 - 780.52 = 497.3 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 497.3 = 432.65 \text{ kJ/kg} = h_i - h_{e\,ac}$$

$$h_{e\,ac} = h_i - w_{T\,ac} = 1277.81 - 432.65 = 845.16 \text{ kJ/kg}$$

Interpolate in A.7.1 $\Rightarrow$ $T_{e\,ac} =$ **820.8 K**

method 1

If constant specific heats are used we get

Table A.5: $C_{Po} = 1.004$ kJ/kg K, $R = 0.287$ kJ/kg K, $k = 1.4$, then from Eq.6.23

$$T_{e\,s} = T_i\,(P_e/P_i)^{\frac{k-1}{k}} = 1200\left(\frac{200}{1200}\right)^{0.286} = 719.2 \text{ K}$$

$$w_{T\,s} = C_{Po}(T_i - T_{e\,s}) = 1.004(1200 - 719.2) = 482.72 \text{ kJ/kg}$$

The actual turbine then has

$$w_{T\,ac} = \eta_T\, w_{T\,s} = 0.87 \times 482.72 = 419.97 \text{ kJ/kg} = C_{Po}(T_i - T_{e\,ac})$$

$$T_{e\,ac} = T_i - w_{T\,ac}/C_{Po} = 1200 - 419.97/1.004 = \textbf{781.7 K}$$

method 2

Use constant specific heat from Table F.4, $C_{P0} = 0.24$ Btu/lbm R, $k = 1.4$. The entropy equation gives constant s, which gives the relation in Eq. 6.23:

$$s_i = s_e \Rightarrow T_e = T_i \left(\frac{P_e}{P_i}\right)^{\frac{k-1}{k}}$$

$$T_e = 520 \left(\frac{147}{14.7}\right)^{0.2857} = 1003.9 \text{ R}$$

The energy equation per unit mass gives the work term

$$w_c = h_i - h_e = C_{P0}(T_i - T_e) = 0.24(520 - 1003.9) = -116.1 \text{ Btu/lbm}$$

Example 7.5

A de-superheater works by injecting liquid water into a flow of superheated steam. With 2 kg/s at 300 kPa, 200°C, steam flowing in, what mass flow rate of liquid water at 20°C should be added to generate saturated vapor at 300 kPa? We also want to know the rate of entropy generation in the process.

Solution

C.V. De-superheater (see Fig. 7.5), no external heat transfer, and no work.

Continuity Eq. 4.9: $\dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Energy Eq. 4.10: $\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 = (\dot{m}_1 + \dot{m}_2)h_3$

Entropy Eq. 7.7: $\dot{m}_1 s_1 + \dot{m}_2 s_2 + \dot{S}_{gen} = \dot{m}_3 \dot{s}_3$

Process: $P = $ constant, $\dot{W} = 0$, and $\dot{Q} = 0$

All the states are specified (approximate state 2 with saturated liquid at 20°C)

B.1.3: $h_1 = 2865.54 \dfrac{\text{kJ}}{\text{kg}}$, $s_1 = 7.3115 \dfrac{\text{kJ}}{\text{kg K}}$; $h_3 = 2725.3 \dfrac{\text{kJ}}{\text{kg}}$, $s_3 = 6.9918 \dfrac{\text{kJ}}{\text{kg K}}$

B.1.2: $h_2 = 83.94 \dfrac{\text{kJ}}{\text{kg}}$, $s_2 = 0.2966 \dfrac{\text{kJ}}{\text{kg K}}$
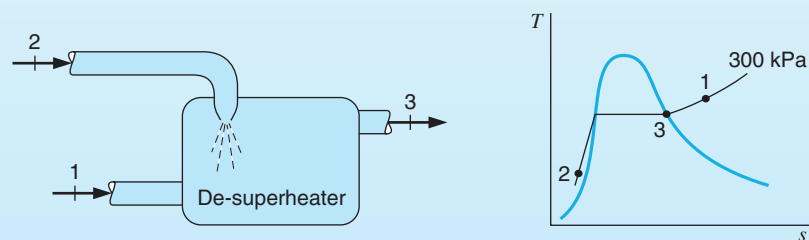

FIGURE 7.5   Sketch and diagram for Example 7.5.

Now we can solve for the flow rate $\dot{m}_2$ from the energy equation, having eliminated $\dot{m}_3$ by the continuity equation

$$\dot{m}_2 = \dot{m}_1 \frac{h_1 - h_3}{h_3 - h_2} = 2 \frac{2865.54 - 2725.3}{2725.3 - 83.94} = 0.1062 \text{ kg/s}$$

$$\dot{m}_3 = \dot{m}_1 + \dot{m}_2 = 2.1062 \text{ kg/s}$$

Generation is from the entropy equation

$$\dot{S}_{gen} = \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2$$

$$= 2.1062 \times 6.9918 - 2 \times 7.3115 - 0.1062 \times 0.2966 = 0.072 \text{ kW/K}$$

Transient Process

For the transient process, which was described in Section 4.6, the second law for a control volume, Eq. 7.2, can be written in the following form:

$$\frac{d}{dt}(ms)_{c.v.} = \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \sum \frac{\dot{Q}_{c.v.}}{T} + \dot{S}_{gen} \qquad (7.11)$$

If this is integrated over the time interval t, we have

$$\int_0^t \frac{d}{dt}(ms)_{c.v.} \, dt = (m_2 s_2 - m_1 s_1)_{c.v.}$$

$$\int_0^t \left(\sum \dot{m}_i s_i \right) dt = \sum m_i s_i, \quad \int_0^t \left(\sum \dot{m}_e s_e \right) dt = \sum m_e s_e, \quad \int_0^t \dot{S}_{gen} \, dt = {}_1 S_{2gen}$$

Therefore, for this period of time t, we can write the second law for the transient process as

$$(m_2 s_2 - m_1 s_1)_{c.v.} = \sum m_i s_i - \sum m_e s_e + \int_0^t \sum_{c.s.} \frac{\dot{Q}_{c.v.}}{T} \, dt + {}_1 S_{2gen} \qquad (7.12)$$

Since in this process the temperature is uniform throughout the control volume at any instant of time, the integral on the right reduces to

$$\int_0^t \sum_{c.s.} \frac{\dot{Q}_{c.v.}}{T} \, dt = \int_0^t \frac{1}{T} \sum_{c.s.} \dot{Q}_{c.v.} \, dt = \int_0^t \frac{\dot{Q}_{c.v.}}{T} \, dt$$

and therefore the second law for the transient process can be written

$$(m_2 s_2 - m_1 s_1)_{c.v.} = \sum m_i s_i - \sum m_e s_e + \int_0^t \frac{\dot{Q}_{c.v.}}{T} \, dt + {}_1 S_{2gen} \qquad (7.13)$$

Example 7.6

Assume an air tank has 40 L of 100 kPa air at ambient temperature 17°C. The adiabatic and reversible compressor is started so that it charges the tank up to a pressure of 1000 kPa and then it shuts off. We want to know how hot the air in the tank gets and the total amount of work required to fill the tank.

6.119

One kg of air at 100 kPa is mixed with 2 kg air at 200 kPa, both at 300 K, in a rigid insulated tank. Find the final state (P, T) and the entropy generation in the process.
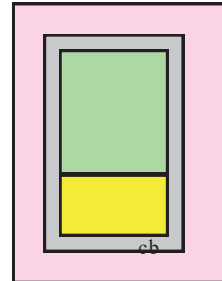
C.V. All the air.

Energy Eq.: $U_2 - U_1 = Q - W$

Entropy Eq.: $S_2 - S_1 = Q/T + {}_1S_2{}_{gen}$

Process Eqs.: $V = C;$ $W = 0,$ $Q = 0$

States A1, B1: $u_{A1} = u_{B1}$

$V_A = m_A RT_1/P_{A1};$ $V_B = m_B RT_1/P_{B1}$

$U_2 - U_1 = m_2 u_2 - m_A u_{A1} - m_B u_{B1} = 0$ $\Rightarrow$ $u_2 = (u_{A1} + 2u_{B1})/3 = u_{A1}$

State 2: **$T_2 = T_1 = 300$ K** (from u_2); $m_2 = m_A + m_B = 3$ kg;

$V_2 = m_2 RT_1/P_2 = V_A + V_B = m_A RT_1/P_{A1} + m_B RT_1/P_{B1}$

Divide with $m_A RT_1$ and get

$$3/P_2 = 1/P_{A1} + 2/P_{B1} = \frac{1}{100} + \frac{2}{200} = 0.02 \text{ kPa}^{-1} \Rightarrow P_2 = \textbf{150 kPa}$$

Entropy change from Eq. 6.16 with the same T, so only P changes

$${}_1S_2{}_{gen} = S_2 - S_1 = -m_A R \ln \frac{P_2}{P_{A1}} - m_B R \ln \frac{P_2}{P_{B1}}$$

$$= -0.287 \left[1 \times \ln \frac{150}{100} + 2 \times \ln \frac{150}{200} \right]$$

$$= -0.287 (0.4055 - 0.5754) = \textbf{0.049 kJ/K}$$

If we consider substances of fixed composition other than a simple compressible substance, we can write "$T\,dS$" equations other than those just given for a simple compressible substance. In Eq. 3.47 we noted that for a reversible process we can write the following expression for work:

$$\delta W = P\,dV - \mathcal{T}\,dL - \mathcal{S}\,dA - \mathcal{E}\,dZ + \cdots$$

It follows that a more general expression for the thermodynamic property relation would be

$$T\,dS = dU + P\,dV - \mathcal{T}\,dL - \mathcal{S}\,dA - \mathcal{E}\,dZ + \cdots \qquad (6.9)$$

6.6 ENTROPY CHANGE OF A SOLID OR LIQUID

In Section 3.10 we considered the calculation of the internal energy and enthalpy changes with temperature for solids and liquids and found that, in general, it is possible to express both in terms of the specific heat, in the simple manner of Eq. 3.30, and in most instances in the integrated form of Eq. 3.31. We can now use this result and the thermodynamic property relation, Eq. 6.7, to calculate the entropy change for a solid or liquid. Note that for such a phase the specific volume term in Eq. 6.7 is very small, so that substituting Eq. 3.30 yields

$$ds \simeq \frac{du}{T} \simeq \frac{C}{T}\,dT \qquad (6.10)$$

Now, as was mentioned in Section 3.10, for many processes involving a solid or liquid, we may assume that the specific heat remains constant, in which case Eq. 6.10 can be integrated. The result is

$$s_2 - s_1 \simeq C \ln \frac{T_2}{T_1} \qquad (6.11)$$

If the specific heat is not constant, then commonly C is known as a function of T, in which case Eq. 6.10 can also be integrated to find the entropy change. Equation 6.11 illustrates what happens in a reversible adiabatic ($dq = 0$) process, which therefore is isentropic. In this process, the approximation of constant v leads to constant temperature, which explains why pumping liquid does not change the temperature.

Example 6.3

One kilogram of liquid water is heated from 20°C to 90°C. Calculate the entropy change, assuming constant specific heat, and compare the result with that found when using the steam tables.

Control mass:	Water.
Initial and final states:	Known.
Model:	Constant specific heat, value at room temperature.

Solution

For constant specific heat, from Eq. 6.11,

$$s_2 - s_1 = 4.184 \ln\left(\frac{363.2}{293.2}\right) = 0.8958 \text{ kJ/kg K}$$