Chapter 2 Resistive Circuits (電阻性電路)

2.1 電阻串聯與並聯

- 以等效電路(equivalent resistances) 取代串聯或並聯電阻。
- 利用等效電路來分析電路。

Series Resistances (串聯電阻器)

Ohm's law

$$v_1 = R_1 i$$
$$v_2 = R_2 i$$
$$v_3 = R_3 i$$

KVL

$$v = v_1 + v_2 + v_3$$
$$= (R_1 + R_2 + R_3)i$$
$$= R_{eq} i$$

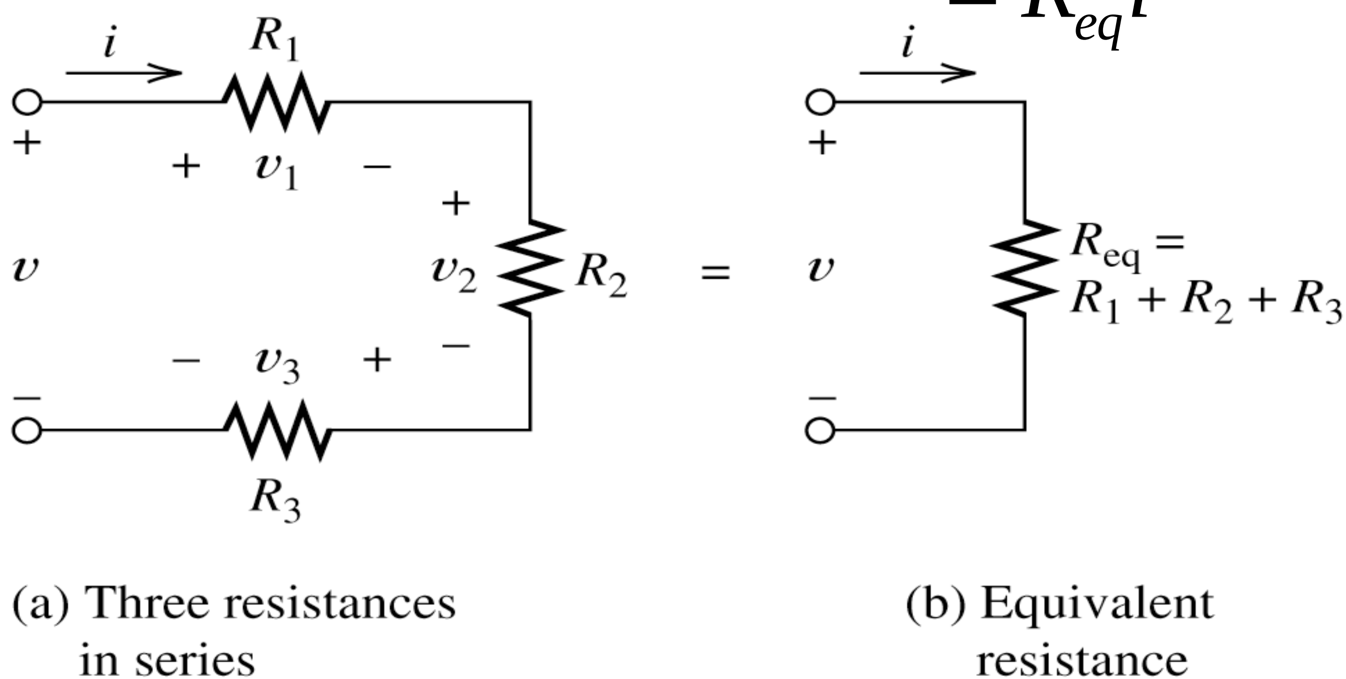


Figure 2.1 Series resistances can be combined into an equivalent resistance.

Series Resistances (串聯電阻器)

- 串聯的電阻器等效於所有電阻器的總和。

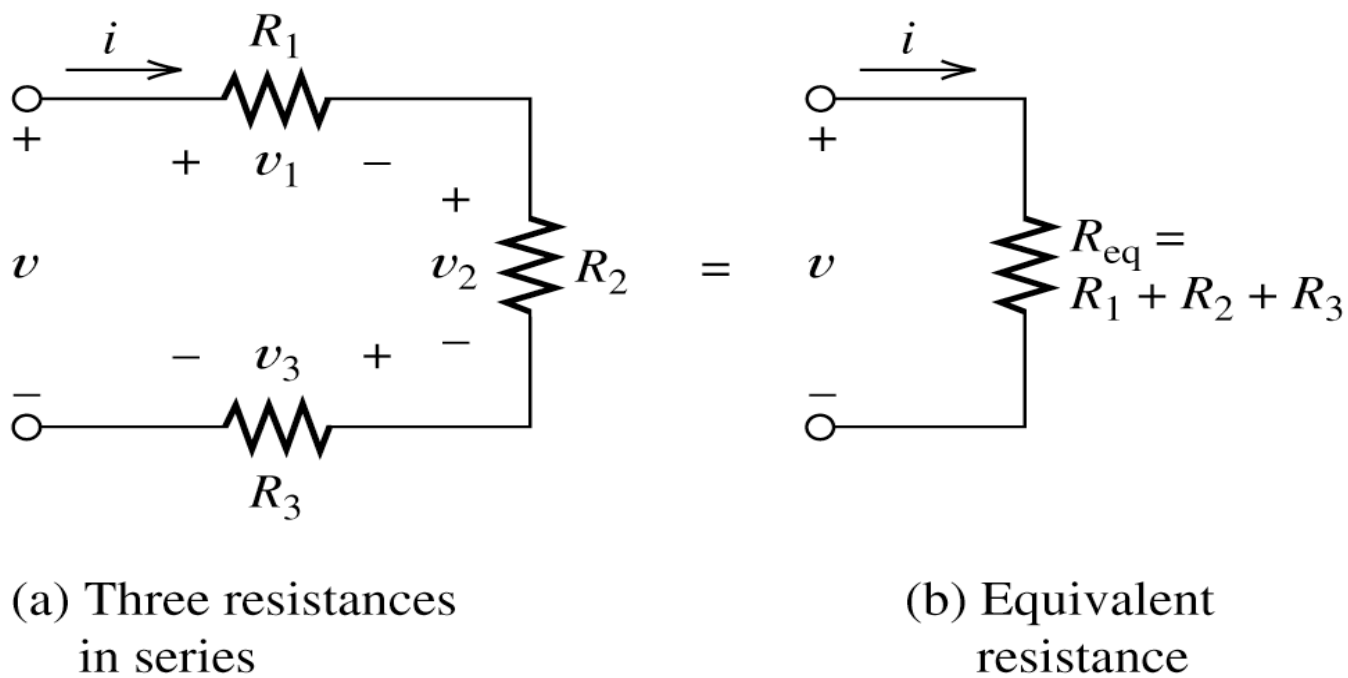


Figure 2.1 Series resistances can be combined into an equivalent resistance.

Parallel Resistances (並聯電阻器)

Ohm's law

$$i_1 = \frac{v}{R_1},\ i_2 = \frac{v}{R_2},\ i_3 = \frac{v}{R_3}$$

KCL

$$i = i_1 + i_2 + i_3$$

$$= (1/R_1 + 1/R_2 + 1/R_3)v$$

$$= 1/R_{eq}\ v$$

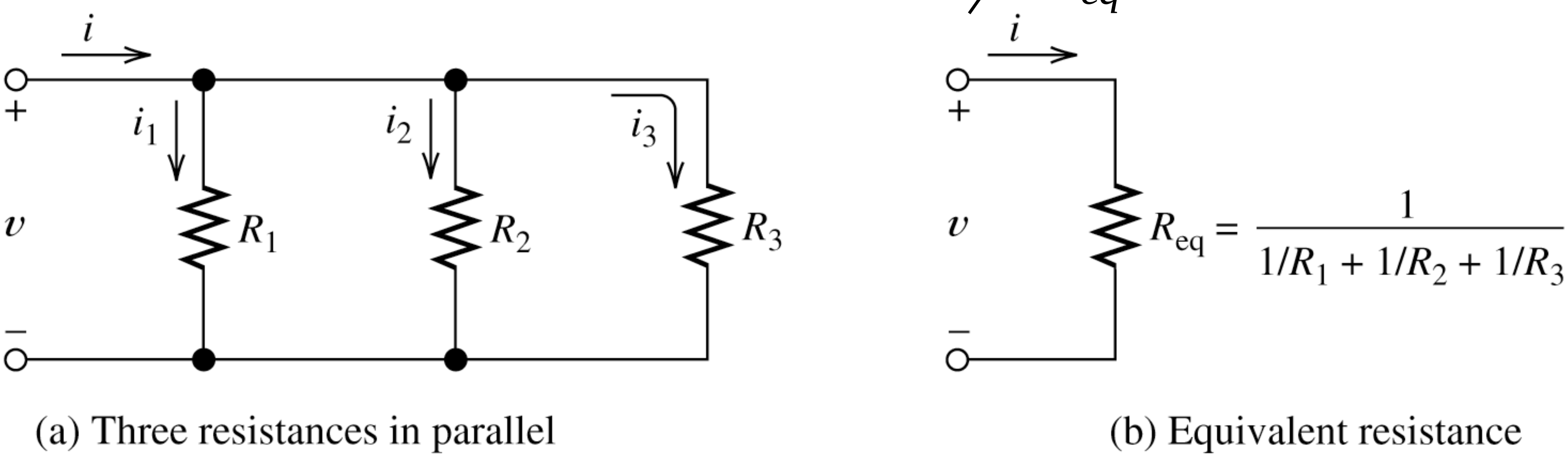
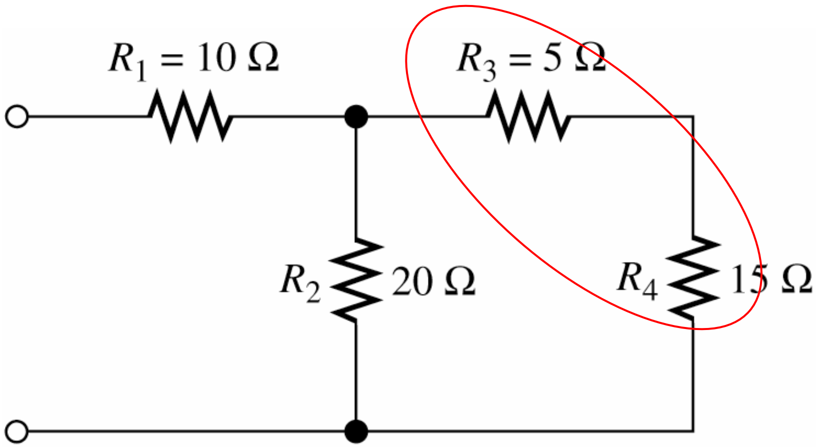
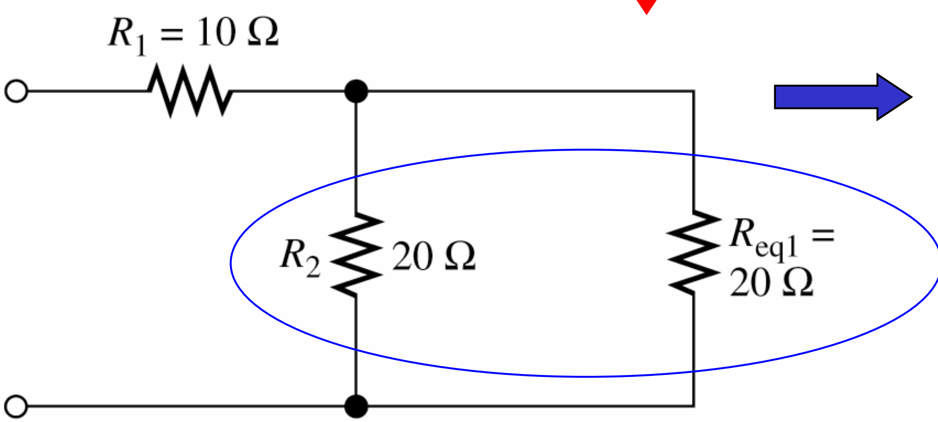


Figure 2.2 Parallel resistances can be combined into an equivalent resistance.

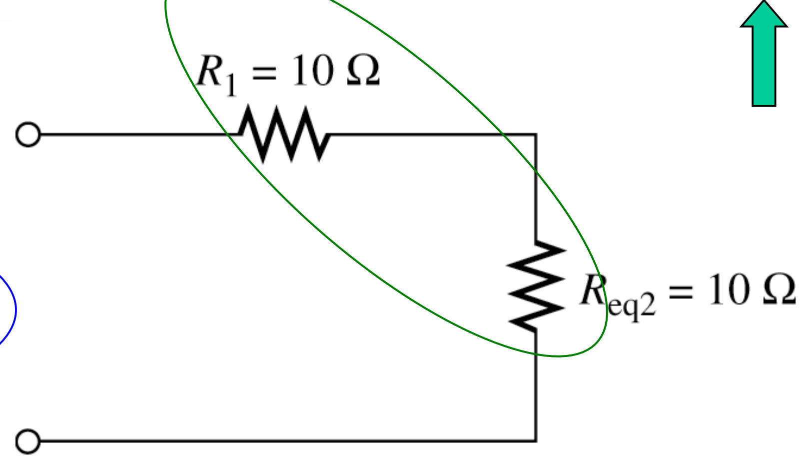
Example 2.1 Find a single equivalent resistance



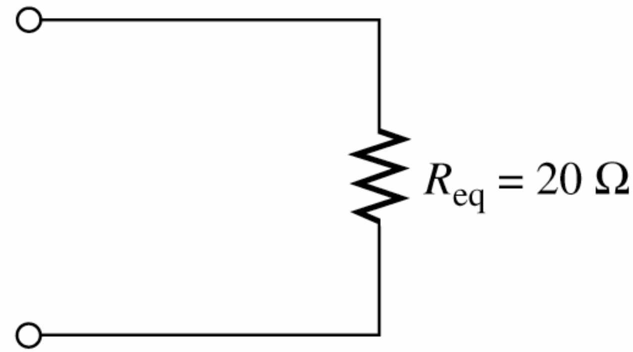
(a) Original network



(b) Network after replacing R_3 and R_4 by their equivalent resistance



(c) Network after replacing R_2 and R_{eq1} by their equivalent

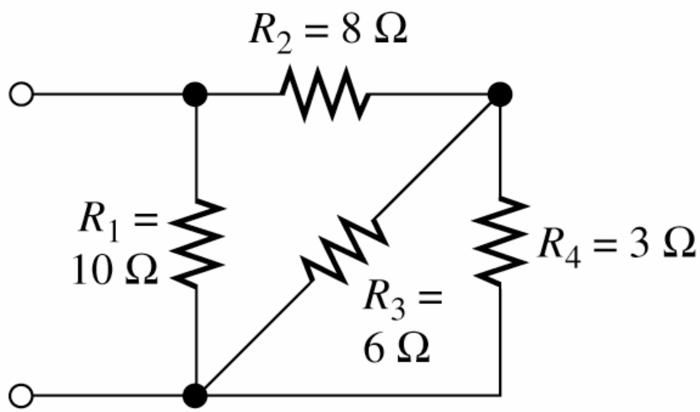


(d) Combining R_1 and R_{eq2} in series yields the equivalent resistance of the entire network

Series VS. Parallel Circuits

- Load (負載)：消耗能量的元件(如燈泡)稱為負載。
- 若要將單一電壓源分配給不同負載，通常使用並聯，因為單一負載故障不影響其他負載，但需要較多接線。
- 聖誕燈泡為了省接線，將燈泡串聯，若一個燈泡負載故障，則形成斷路，整個線路都無法運作。

Exercise 2.1 (b)



(b)

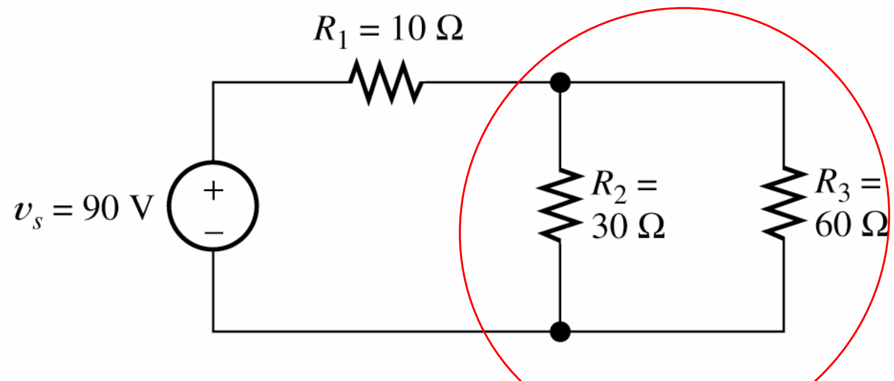
(b) R_3 and R_4 are in parallel. Furthermore, R_2 is in series with the combination of R_3, and R_4. Finally R_1 is in parallel with the combination of the other resistors. Thus we have:

$$R_{eq} = \frac{1}{1/R_1 + 1/[R_2 + 1/(1/R_3 + 1/R_4)]} = 5\,\Omega$$

2.2 Circuit Analysis using Series/Parallel Equivalents

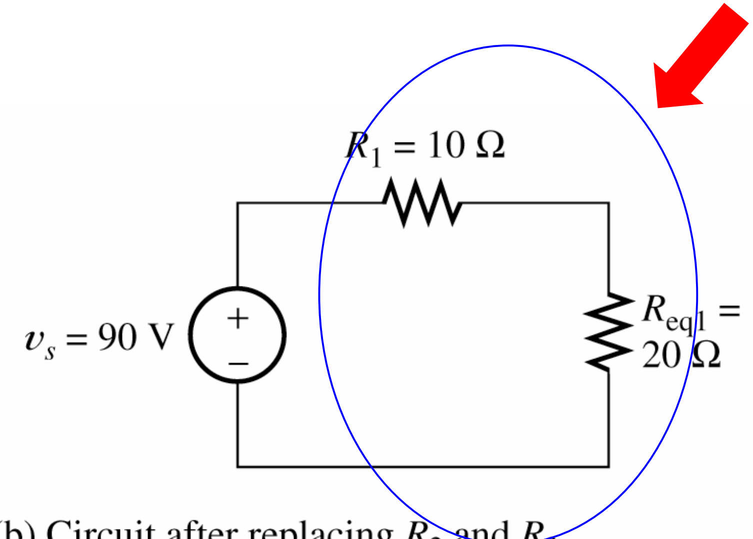
1. 找出電路中的串聯或並聯電阻器，通常於電壓/流源最遠處找起。
2. 將步驟1所找出串聯/並聯電阻器由等效電阻取代。
3. 重複步驟1&2盡可能將電路簡化。通常簡化至單一電壓/電流源與單一等效電阻。
4. 解出最後等效電路的電流電壓值。
5. 往回推，逐漸以原始電阻取代等效電阻，應用歐姆定律，KVL、KCL解出所有電路元件的電流電壓值。

Example 2.2 Circuit Analysis

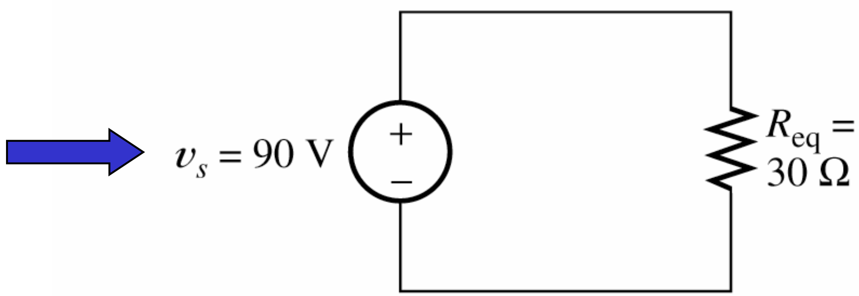


(a) Original circuit

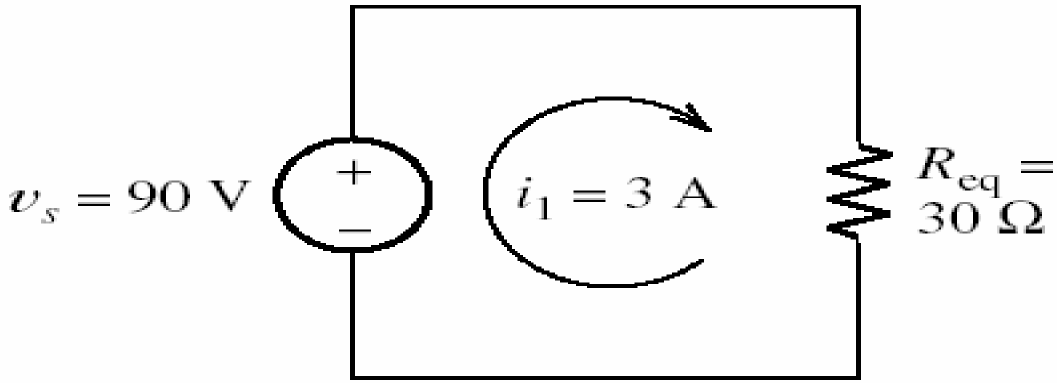
$$R_{eq1} = \frac{1}{1/R_2 + 1/R_3} = \frac{R_2 R_3}{R_2 + R_3} = \frac{1800}{90} = 20$$



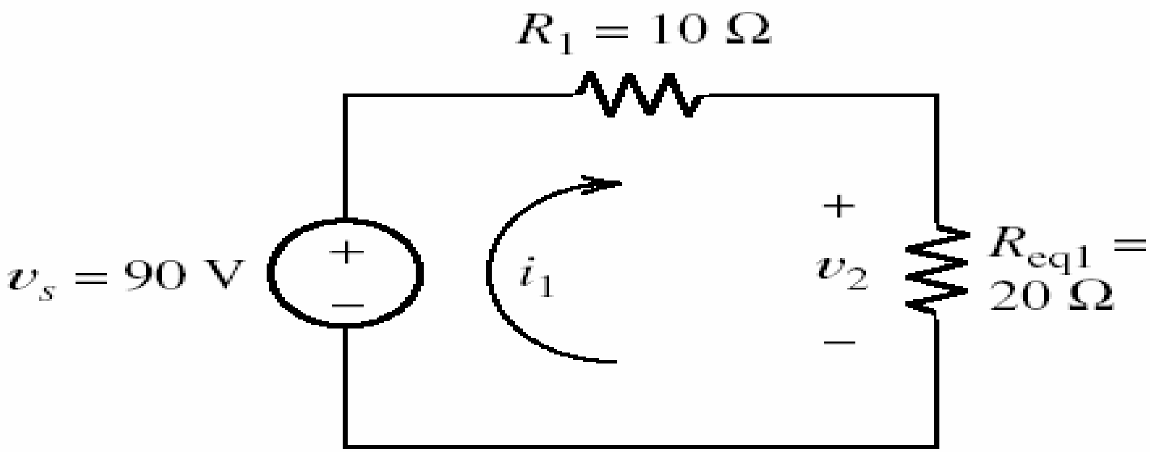
(b) Circuit after replacing R_2 and R_3 by their equivalent



(c) Circuit after replacing R_1 and R_{eq1} by their equivalent

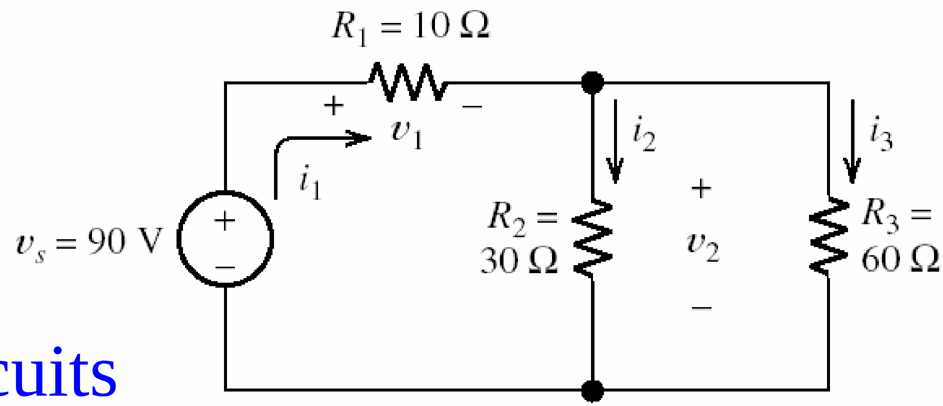


(c) First, we solve for $i_1 = \dfrac{v_s}{R_{eq}} = 3$ A



(b) Second, we find $v_2 = R_{eq1}\ i_1 = 60$ V we got

$i_1 = 3\text{A}, v_2 = 60\text{V}$



(a) Third, we use known values of i_1 and v_2 to solve for the remaining currents and voltages

Solve the remaining circuits

$$i_2 = \frac{v_2}{R_2} = 2\text{A}$$

$$i_3 = \frac{v_2}{R_3} = 1\text{A}$$

$$v_1 = R_1 i_1 = 30\text{V}$$

Check by KCL &KVL

Power?

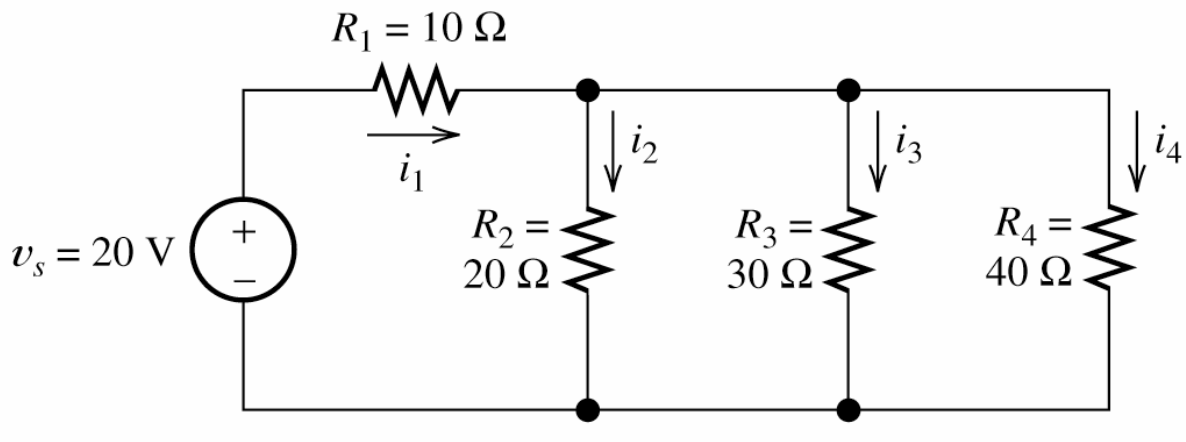
$$p_s = -v_s i_1 = -270\text{W}$$

$$p_1 = v_1 i_1 = 90\text{W}$$

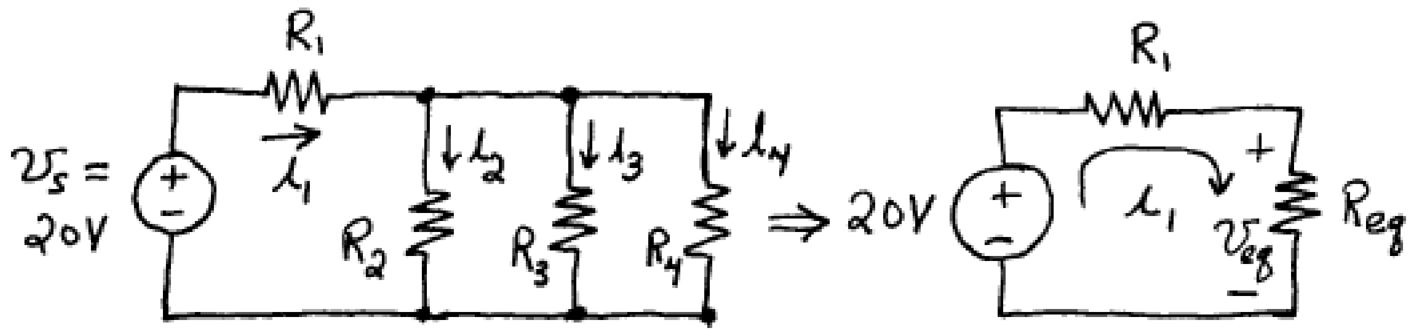
$$p_2 = v_2 i_2 = 120\text{W}$$

$$p_3 = v_2 i_3 = 60\text{W}$$

Exercise 2.2 (a)



(a)



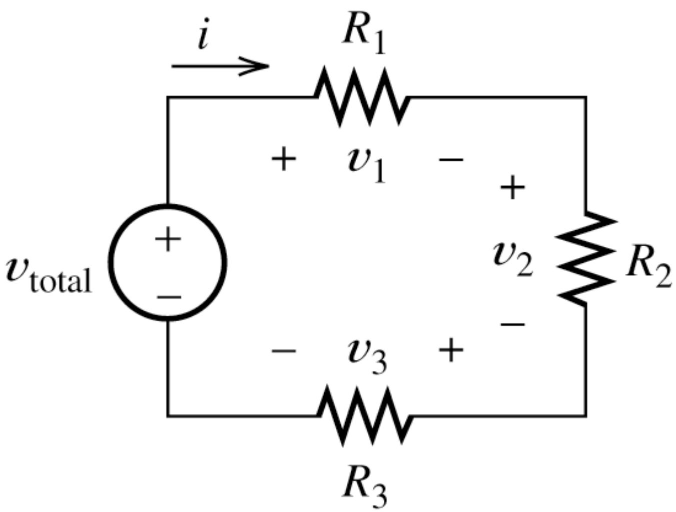
$$R_{eq} = \frac{1}{1/R_2 + 1/R_3 + 1/R_4} = 9.231\,\Omega \qquad i_1 = \frac{20\text{ V}}{R_1 + R_{eq}} = \frac{20}{10 + 9.231} = 1.04\ A$$

$$v_{eq} = R_{eq} i_1 = 9.600\text{ V} \qquad i_2 = v_{eq} / R_2 = 0.480\ A \qquad i_3 = v_{eq} / R_3 = 0.320\ A$$

$$i_4 = v_{eq} / R_4 = 0.240\ A$$

2.3 Voltage/Current-Divider Circuits

- Voltage-Division Principle (分壓定律)：電壓分配至串聯電阻之比例為其電阻值與總電阻值之比。

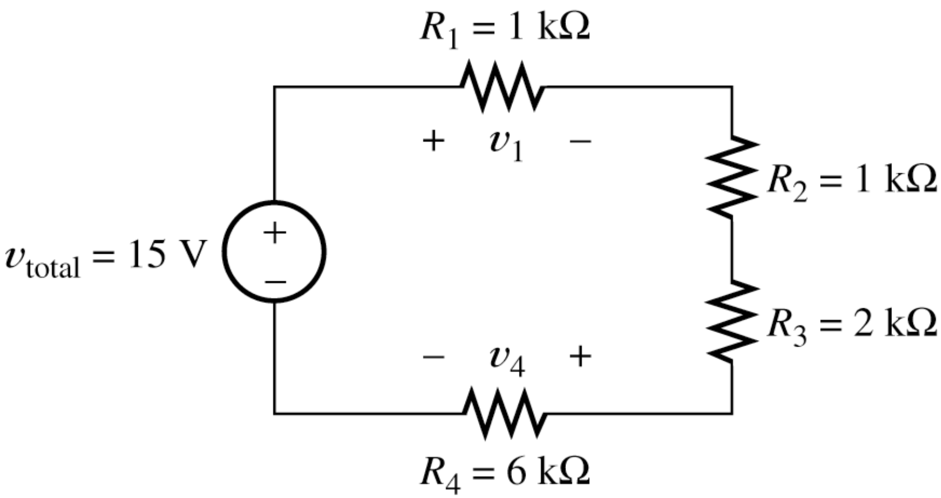


$$v_1 = R_1 i = \frac{R_1}{R_1 + R_2 + R_3} v_{\text{total}}$$

$$v_2 = R_2 i = \frac{R_2}{R_1 + R_2 + R_3} v_{\text{total}}$$

$$v_3 = R_3 i = \frac{R_3}{R_1 + R_2 + R_3} v_{\text{total}}$$

Example 2.3 Application of the Voltage-Division Principle

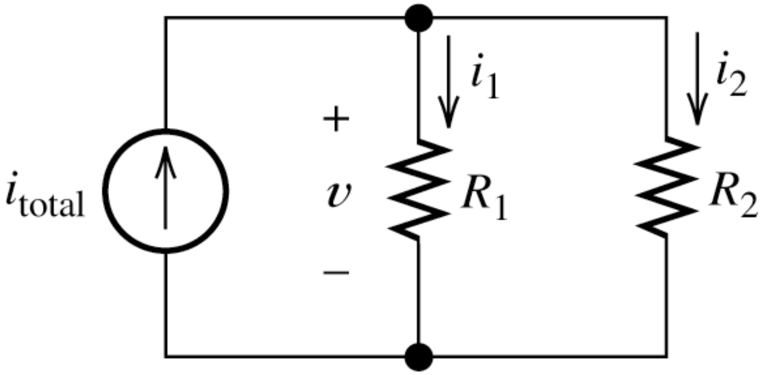


$$v_1 = \frac{R_1}{R_1 + R_2 + R_3 + R_4} v_{total}$$

$$= \frac{1000}{1000 + 1000 + 2000 + 6000} \times 15$$

$$= 1.5\text{V}$$

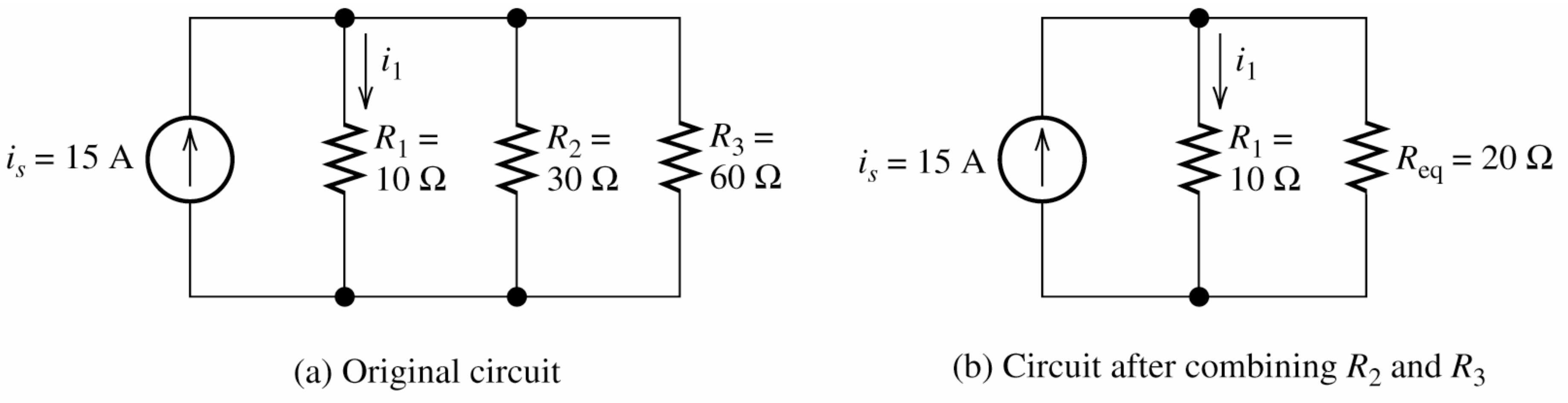
- Current-Division Principle (分流定律)：電流分配至兩並聯電阻比例為另一電阻與總電阻值之比。



$$i_1 = \frac{v}{R_1} = \frac{R_2}{R_1 + R_2} i_{total}$$

$$i_2 = \frac{v}{R_2} = \frac{R_1}{R_1 + R_2} i_{total}$$

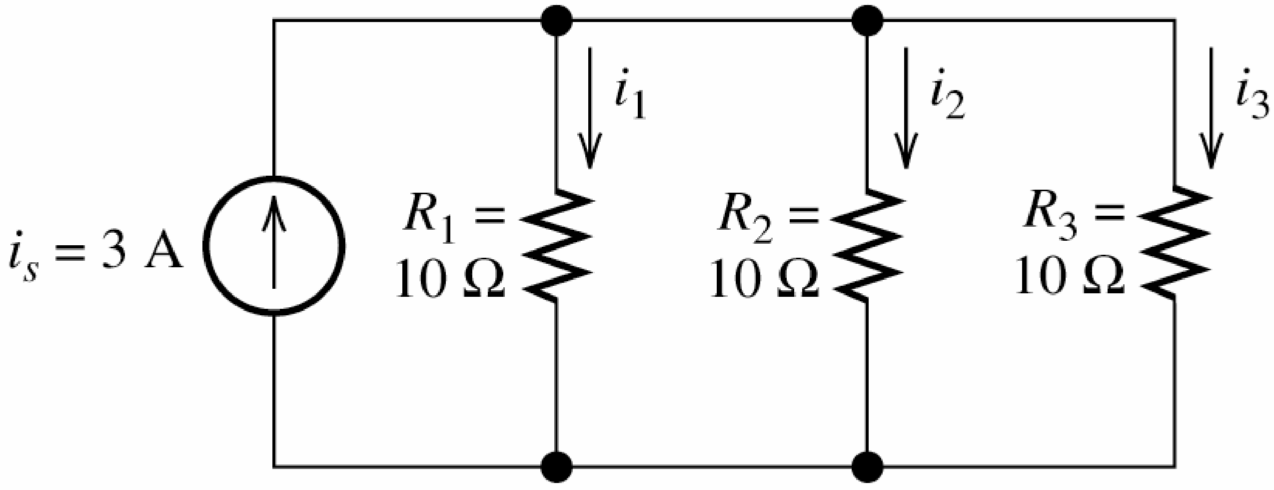
Example 2.5 Application of the Current-Division Principle



$$R_{eq} = \frac{R_2 R_3}{R_2 + R_3} = \frac{30 \times 60}{30 + 60} = 20\Omega$$

$$i_1 = \frac{R_{eq}}{R_1 + R_{eq}} i_s = \frac{20}{10 + 20} 15 = 10\text{A}$$

Exercise 2.4 (b)

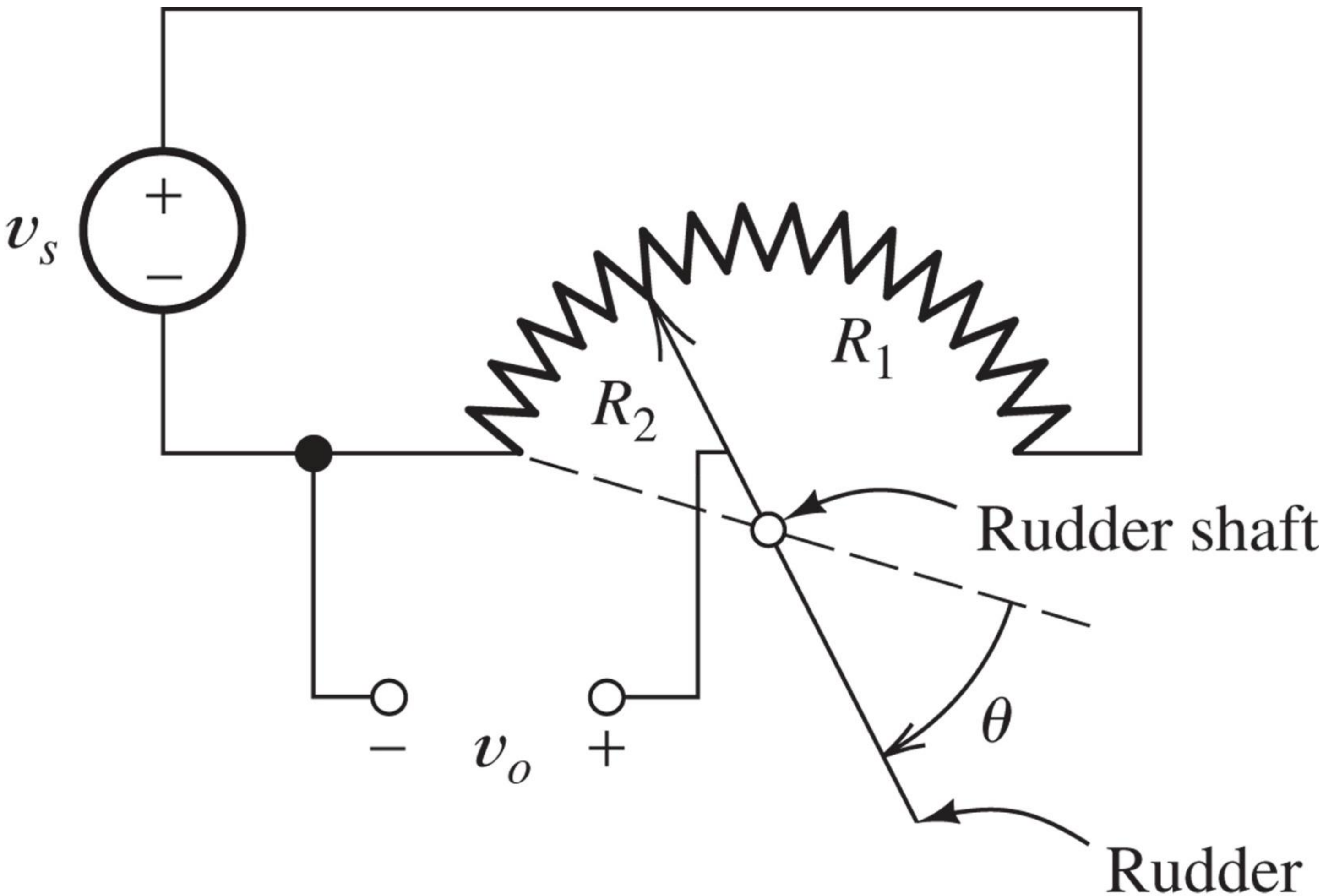


(b)

(b) The current division principle applies to two resistances in parallel. Therefore, to determine i_1, first combine R_2 and R_3 in parallel: $R_{eq} = 1/(1/R_2 + 1/R_3) = 5\ \Omega$. Then we have $i_1 = i_s \dfrac{R_{eq}}{R_1 + R_{eq}} = \dfrac{5}{10+5} = 1\ A$.

Similarly, $i_2 = 1\ A$ and $i_3 = 1\ A$.

Figure 2.13 The voltage-division principle forms the basis for some position sensors. This figure shows a transducer that produces an output voltage v_o proportional to the rudder angle θ.



2.4 Node-Voltage Analysis

- Although they are very important concepts, series/parallel equivalents and the current/voltage division principles are not sufficient to solve all circuits.

Node Voltage Analysis

- 選定參考節點(reference node)並標示其他結點的電壓符號。
- 一般參考節點為電壓源的一端，並用接地符號(ground symbol)表示。

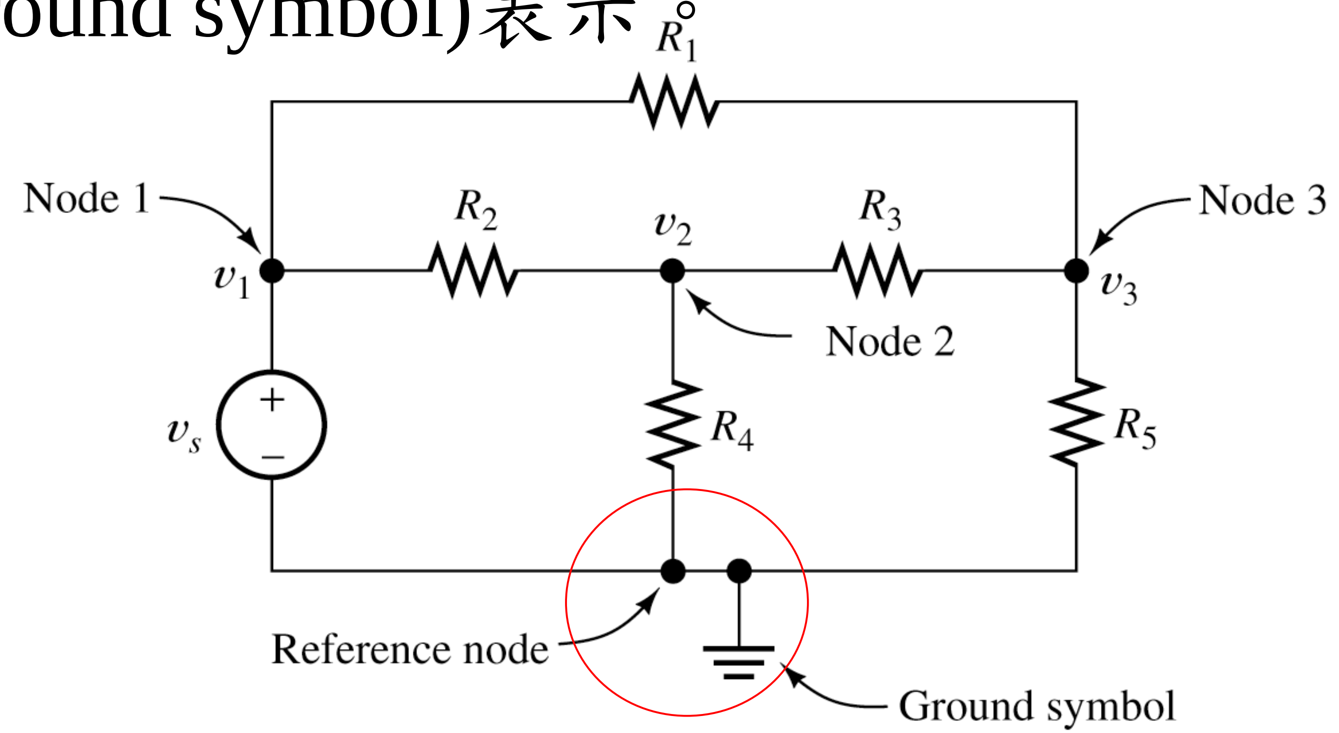
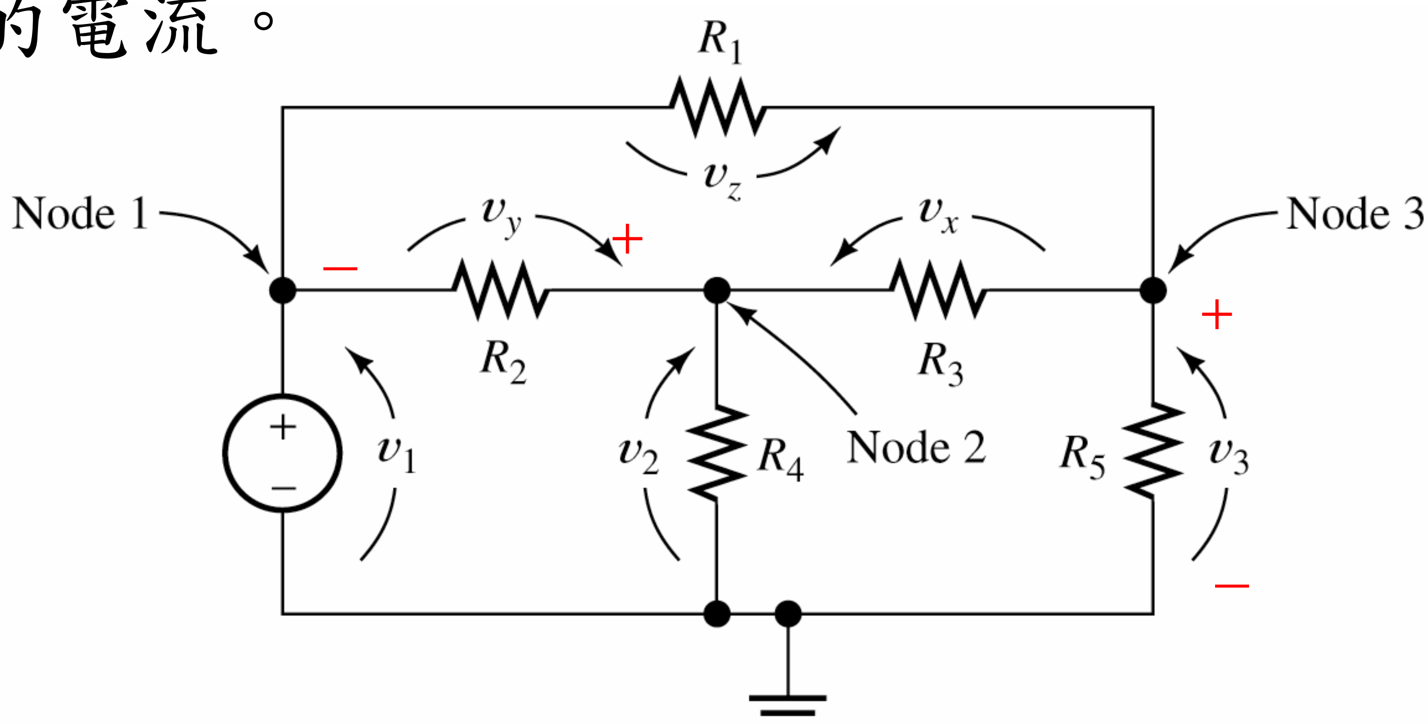
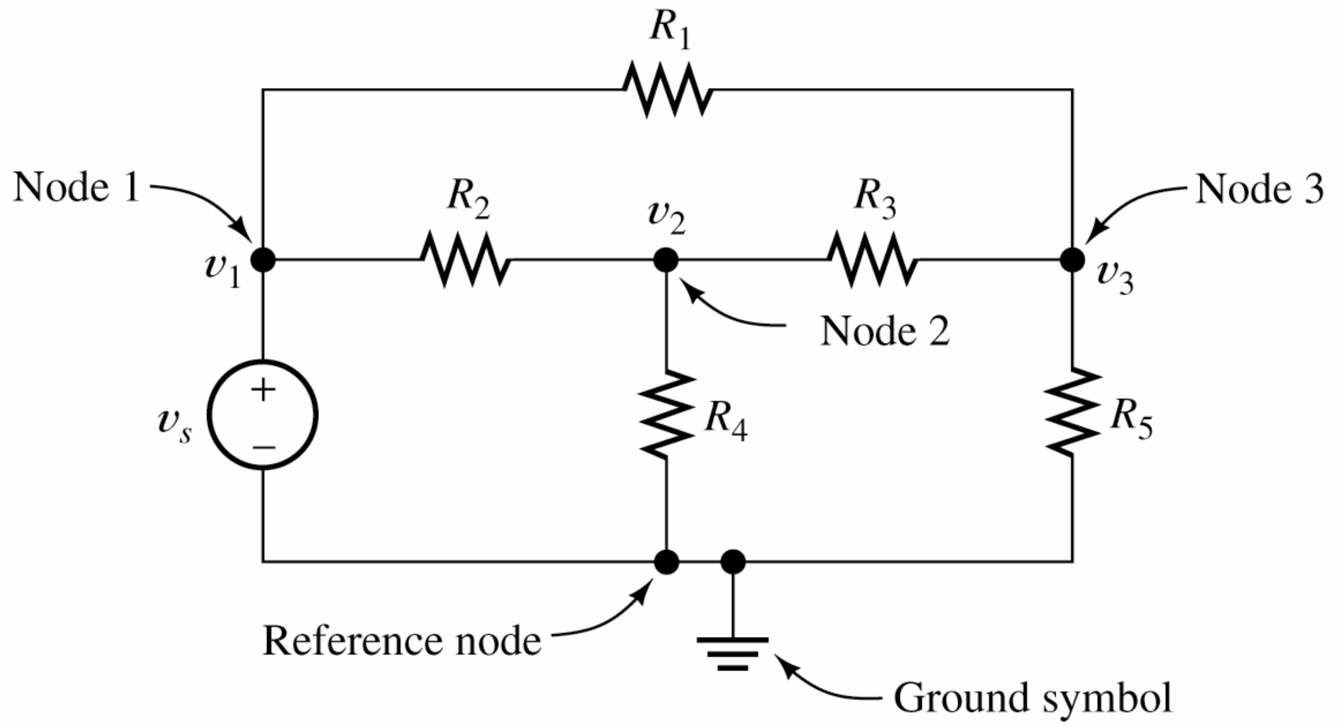


Figure 2.16 The first step in node analysis is to select a reference node and label the voltages at each of the other nodes.

- 假設我們可以決定節點電壓(v_1, v_2, v_3),則可透過KVL來決定v_x, v_y, v_z
- 以節點電壓(v_1, v_2, v_3) 為未知數，對節點寫出KCL equation， 並求解。
- 最後，可透過Ohm’s law 來決定各個流過各電阻的電流。





$$v_1 = v_s$$

KCL: Node 2
(流出 node 2 的淨電流為0)

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_4} + \frac{v_2 - v_3}{R_3} = 0$$

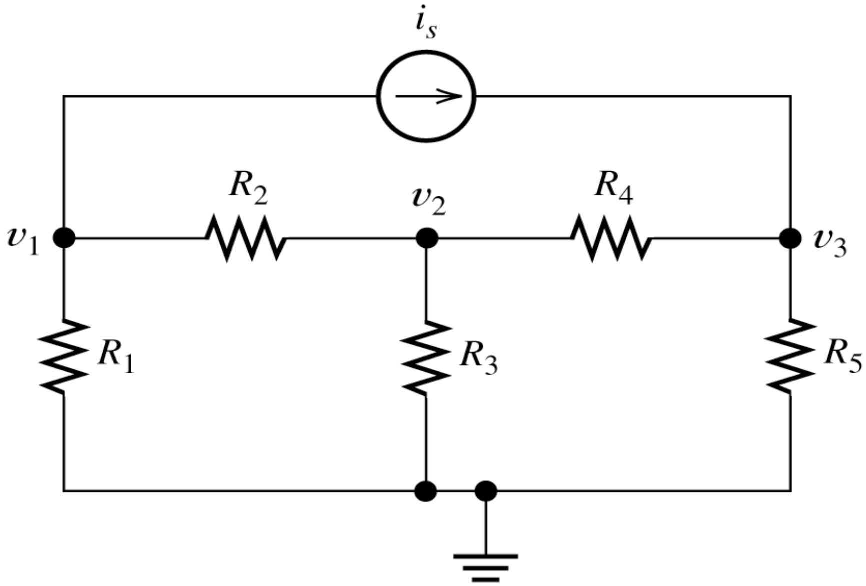
KCL: Node 3

$$\frac{v_3 - v_1}{R_1} + \frac{v_3}{R_5} + \frac{v_3 - v_2}{R_3} = 0$$

Example 2.6

KCL: Node 1

$$\frac{v_1}{R_1} + \frac{v_1 - v_2}{R_2} + i_s = 0$$



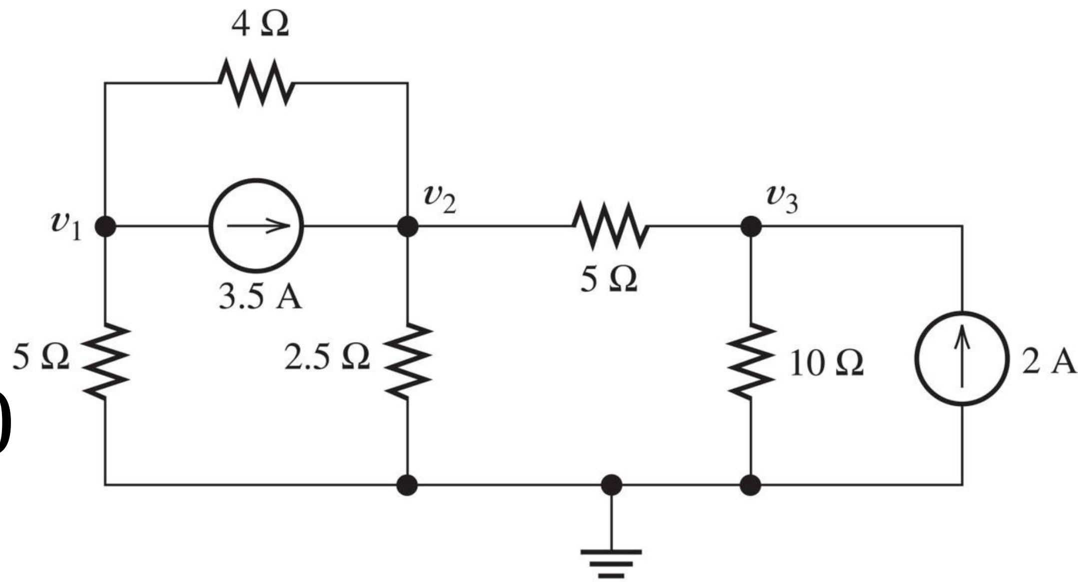
KCL: Node 2

$$\frac{v_2 - v_1}{R_2} + \frac{v_2}{R_3} + \frac{v_2 - v_3}{R_4} = 0$$

KCL: Node 3

$$\frac{v_3}{R_5} + \frac{v_3 - v_2}{R_4} = i_s$$

Example 2.7



KCL: Node 1

$$\frac{v_1}{5} + \frac{v_1 - v_2}{4} + 3.5 = 0$$

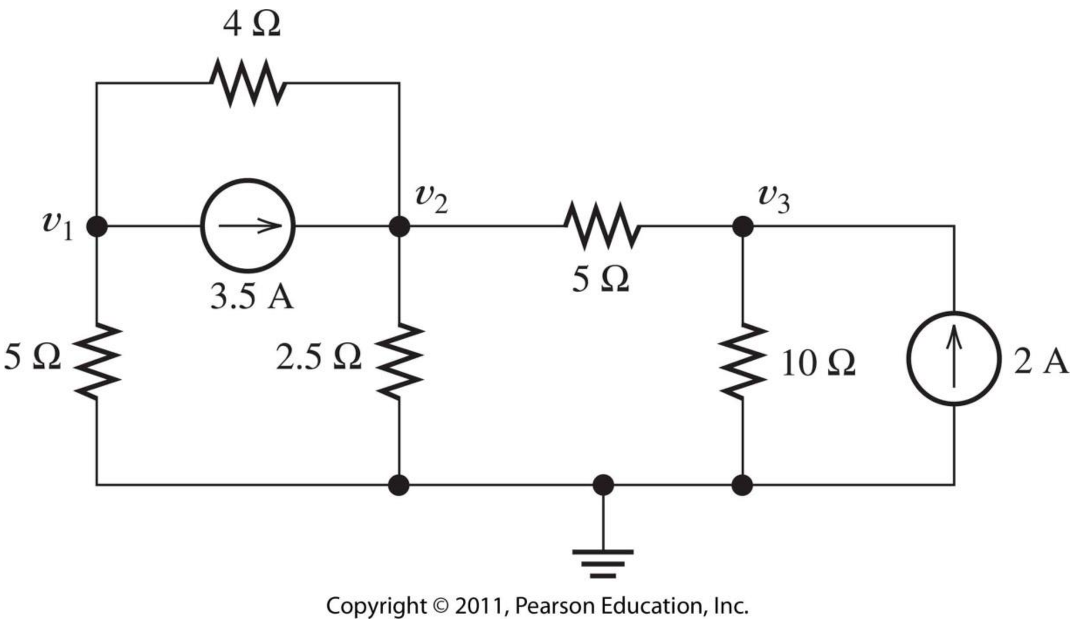
KCL: Node 2

$$\frac{v_2 - v_1}{4} + \frac{v_2}{2.5} + \frac{v_2 - v_3}{5} = 3.5$$

KCL: Node 3

$$\frac{v_3 - v_2}{R_5} + \frac{v_3}{10} = 2$$

Example 2.7



- 三元一次方程組

$$0.45v_1 - 0.25v_2 = -3.5$$

$$-0.25v_1 + 0.85v_2 - 0.2v_3 = 3.5$$

$$-0.2v_2 + 0.35v_3 = 2$$

- Matrix Form

$$\begin{bmatrix} 0.45 & -0.25 & 0 \\ -0.25 & 0.85 & -0.20 \\ 0 & -0.20 & 0.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 3.5 \\ 2 \end{bmatrix}$$

Example 2.7

- Solve inverse matrix

$$\begin{bmatrix} 0.45 & -0.25 & 0 \\ -0.25 & 0.85 & -0.20 \\ 0 & -0.20 & 0.30 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} -3.5 \\ 3.5 \\ 2 \end{bmatrix}$$

G V I

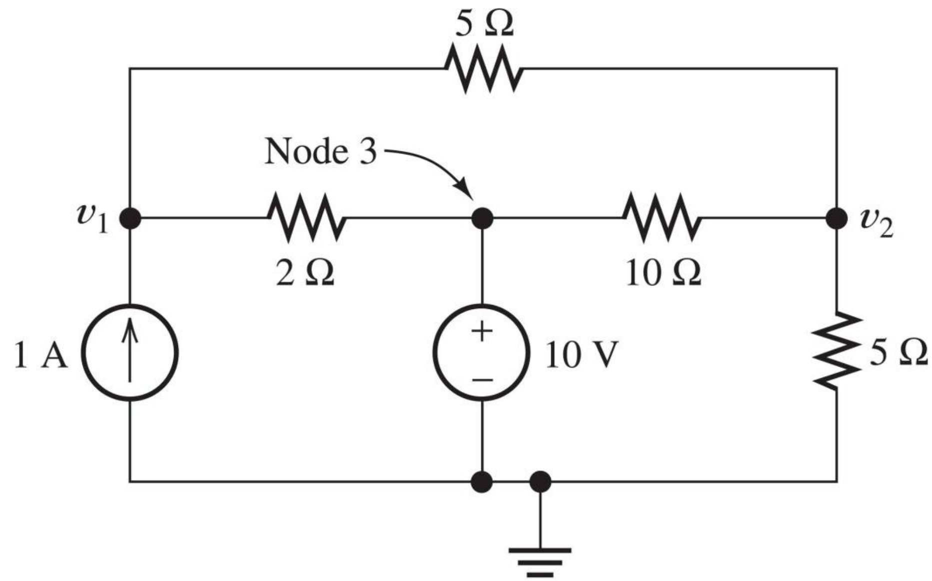
$$V = G^{-1} I$$

- Matlab

```
clear

G=[0.45 -0.25 0; -0.25 0.85 -0.20; 0 -0.20 0.30];
I= [-3.5; 3.5;2];
V=G\I
```

Example 2.9



KCL: Node 1

$$\frac{v_1 - v_2}{5} + \frac{v_1 - 10}{2} = 1$$

KCL: Node 2

$$\frac{v_2}{5} + \frac{v_2 - 10}{10} + \frac{v_2 - v_1}{5} = 0$$

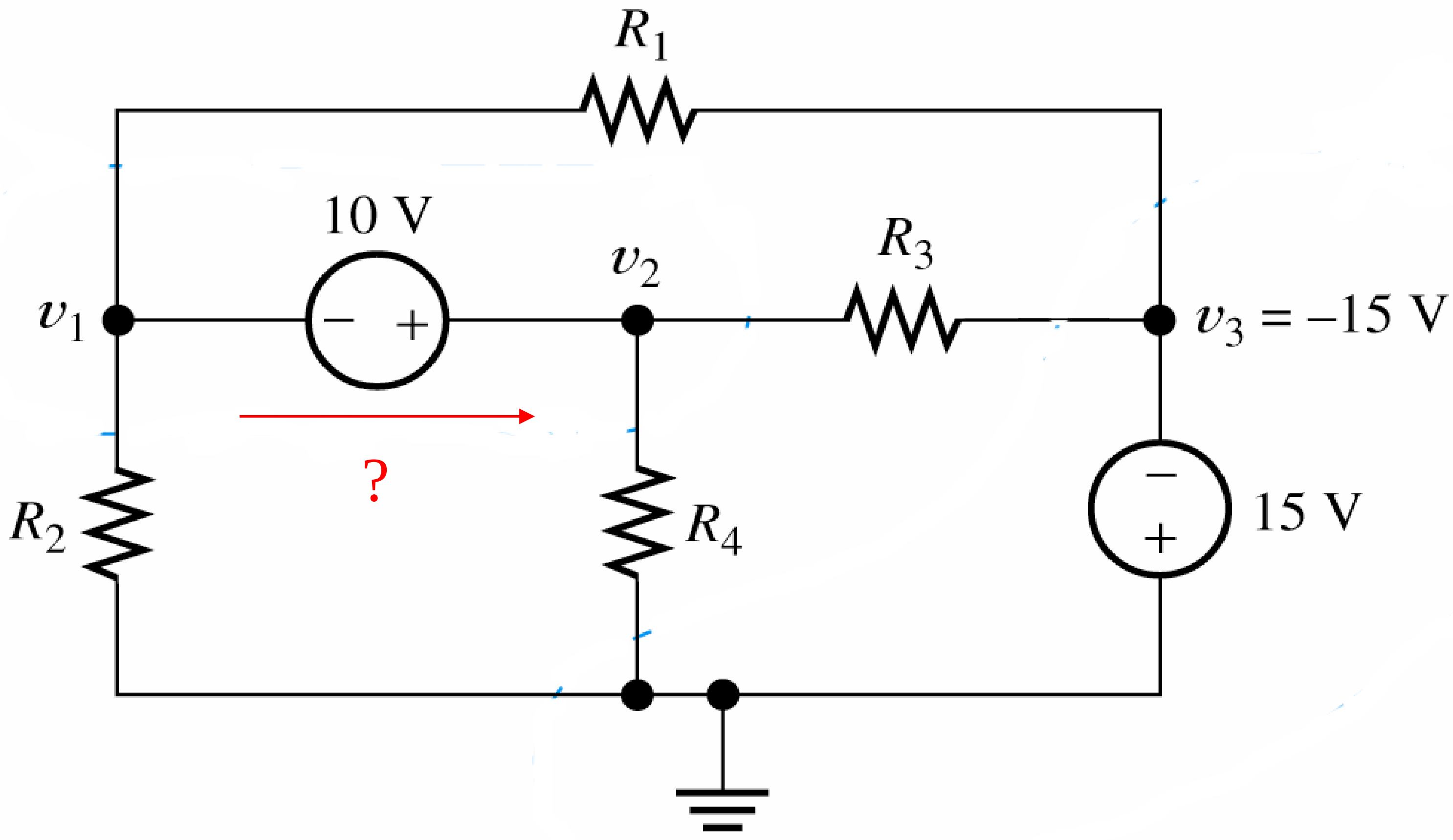
解聯立方程式求 voltages

$$0.7v_1 - 0.2v_2 = 6$$

$$-0.2v_1 + 0.5v_2 = 1$$

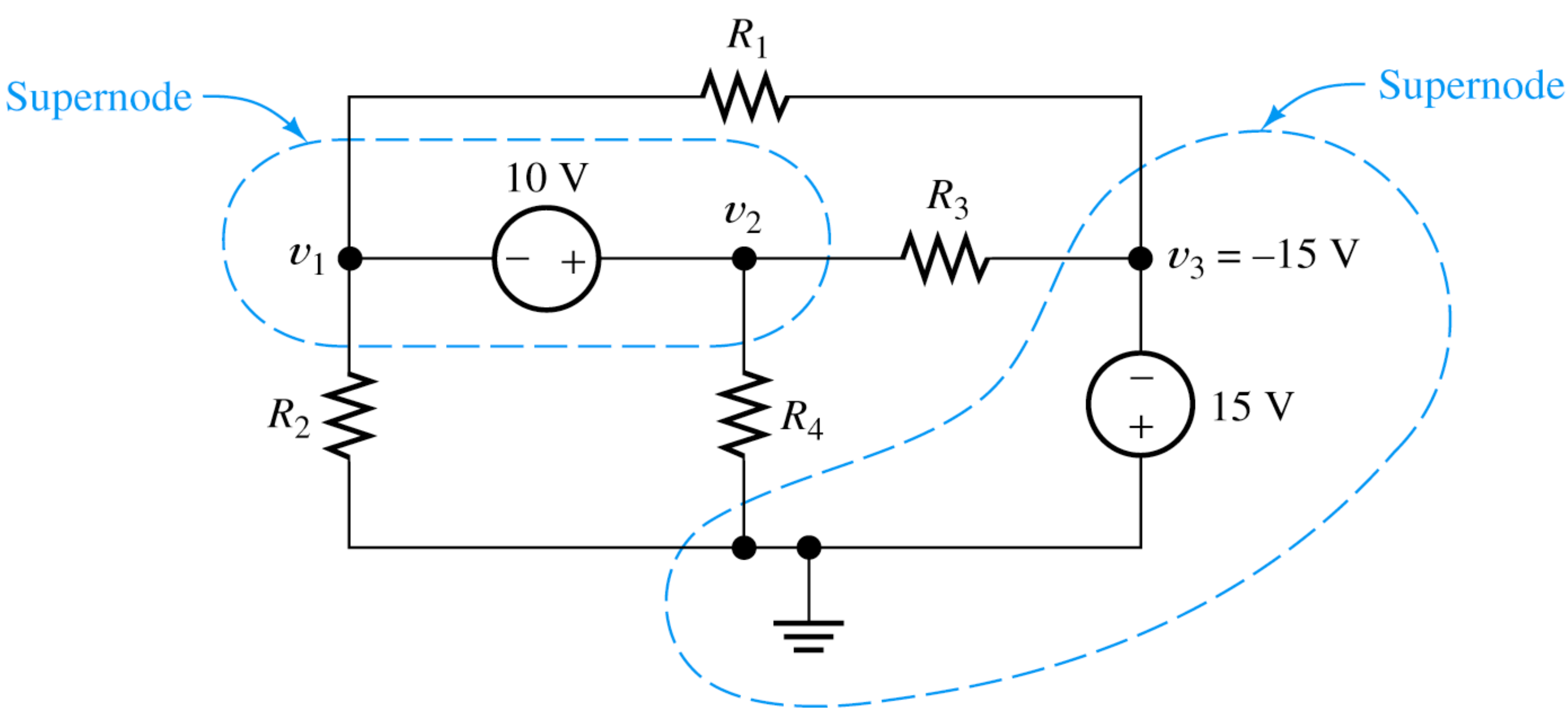
Circuits with Voltage Sources

- 因為電壓源與 V_1, V_2相連，所以我們無法寫出只含節點電壓的電流方程式。



Circuits with Voltage Sources

- 當分枝處於兩非參考節點之間且包含一個電壓源時，即可使用超節點技術(supernode)。
- 將包含電壓源的節點含括為一supernode(超節點)。

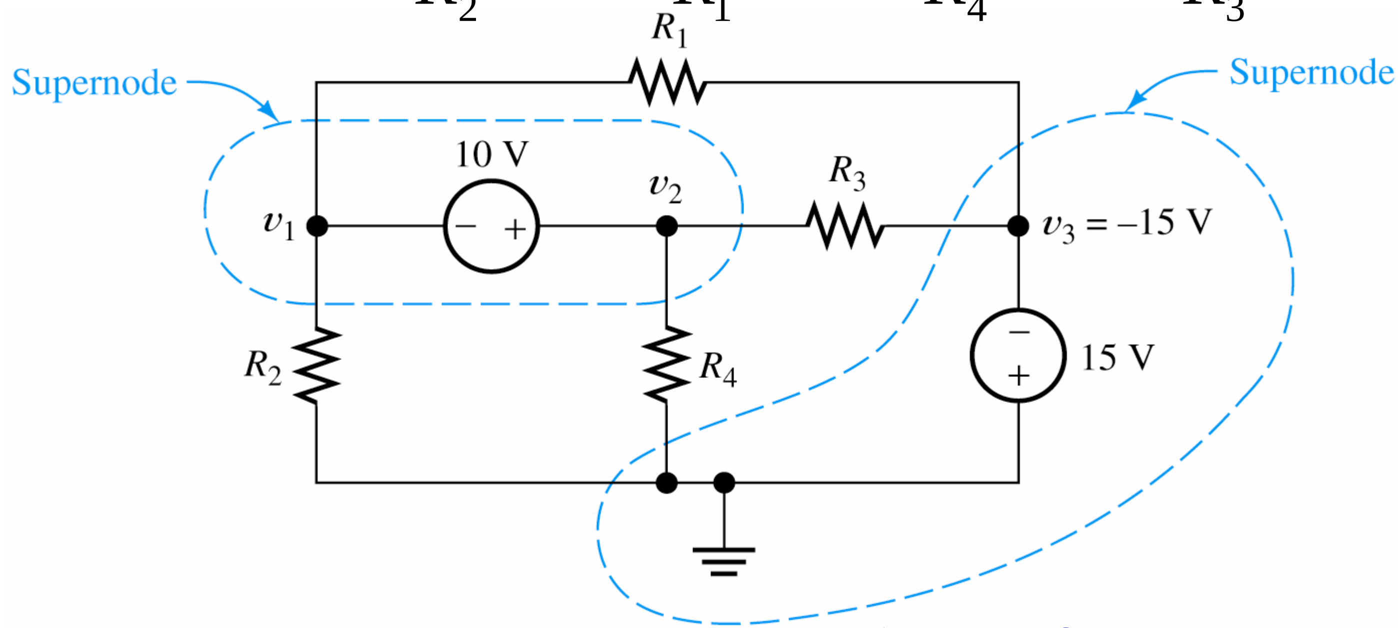


Circuits with Voltage Sources

- 流入(流出)supernode (封閉表面, closed surface)的淨電流(net current)為0.

KCL: Supernode 包含10V voltage source

$$\frac{v_1}{R_2}+\frac{v_1-(-15)}{R_1}+\frac{v_2}{R_4}+\frac{v_2-(-15)}{R_3}=0$$



Note: We obtain dependent equations (相依) if we use all of the nodes in a network to write KCL equations. (KCL S1 與 KCL S2 相依)

Circuits with Voltage Sources

- 將電壓源連結的節點寫出KVL以獲得另外的獨立方程式。

KVL: $$-v_1 - 10 + v_2 = 0$$

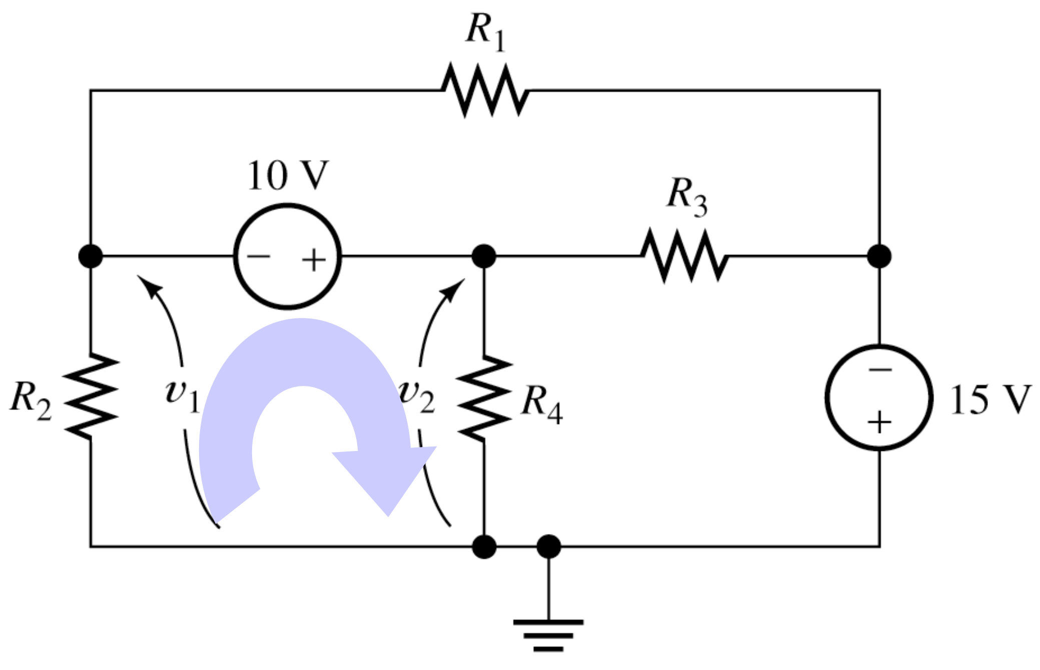
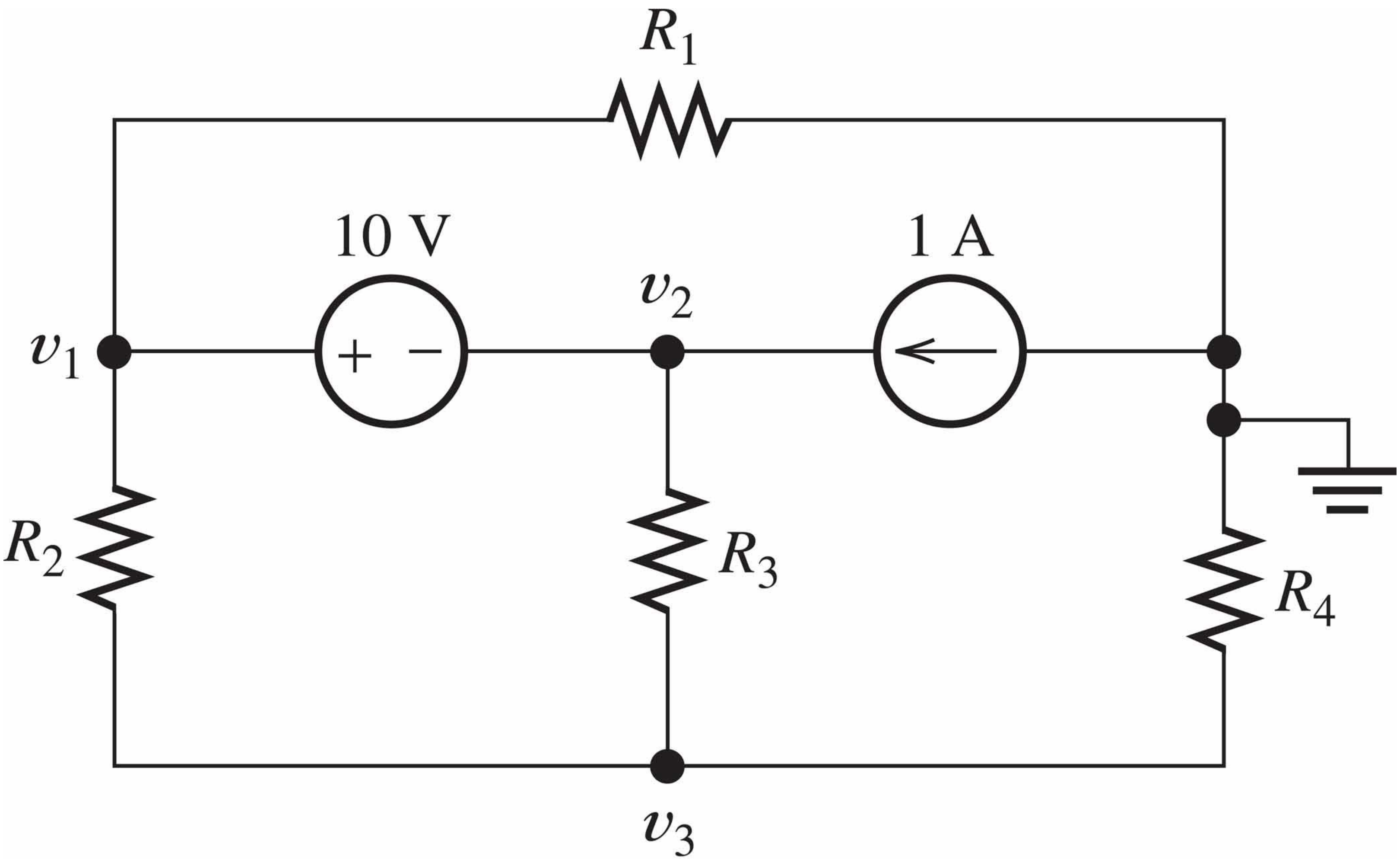
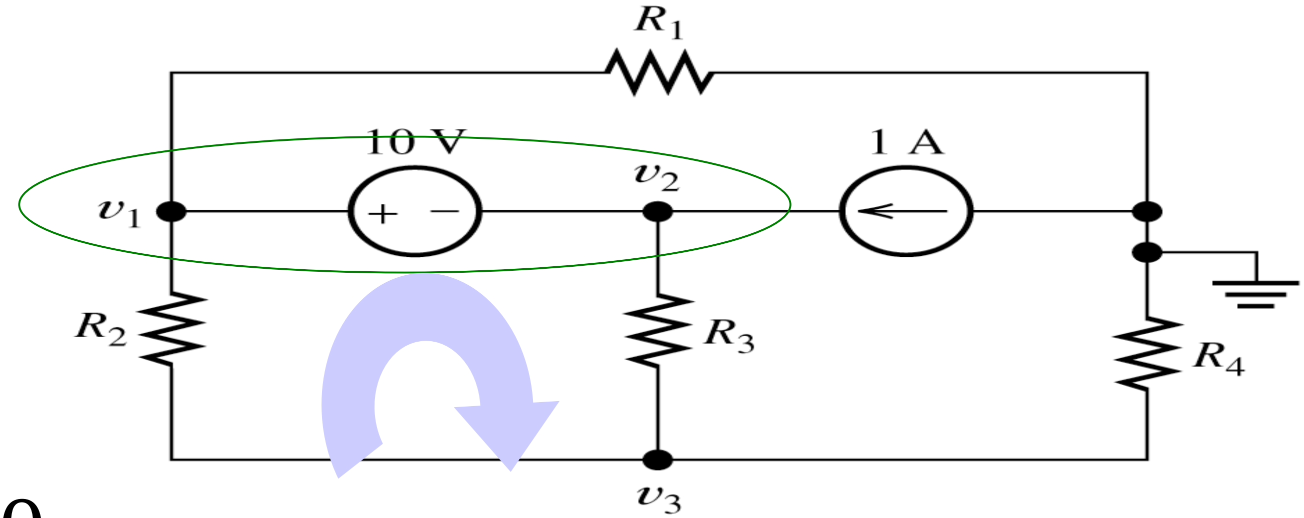


Figure 2.25 Node voltages v_1 and v_2 and the 10-V source form a closed loop to which KVL can be applied. (This is the same circuit as that of Figure 2.24.)

Exercise 2.13 Write a set of independent equations for the node voltage in Fig. 2.27.





KVL:

$$-v_1 + 10 + v_2 = 0$$

KCL: Supernode enclosing 10-V source

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1$$

KCL for node 3

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0$$

KCL at reference node

$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1$$

3 variables 4 equations?

KVL:

$$-v_1 + 10 + v_2 = 0 \quad (1)$$

KCL: Supernode enclosing 10-V source

$$\frac{v_1}{R_1} + \frac{v_1 - v_3}{R_2} + \frac{v_2 - v_3}{R_3} = 1 \quad (2)$$

KCL for node 3

$$\frac{v_3 - v_1}{R_2} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} = 0 \quad (3)$$

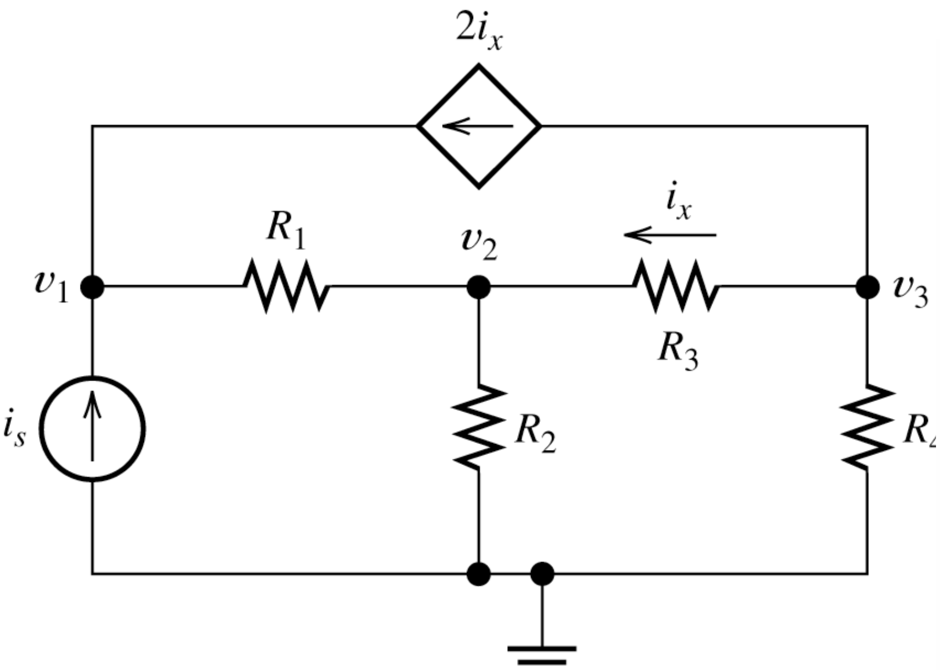
KCL at reference node

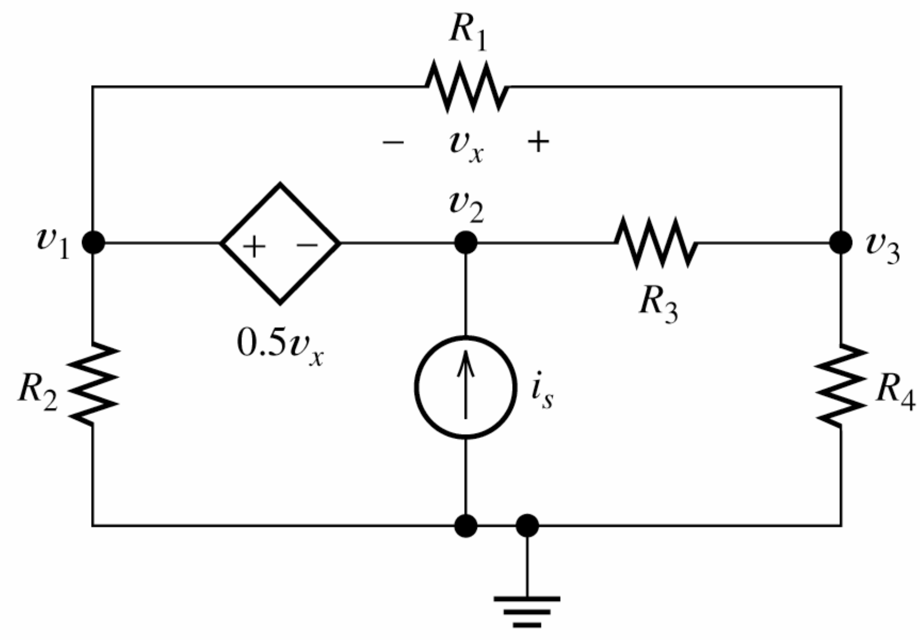
$$\frac{v_1}{R_1} + \frac{v_3}{R_4} = 1 \quad (4)$$

(2) +(3) =(4) (2) (3) and (4) are dependent

⇨ (1) must be included with any two of the three KCL equations for independence.

Node-Voltage Analysis with a Dependent Source

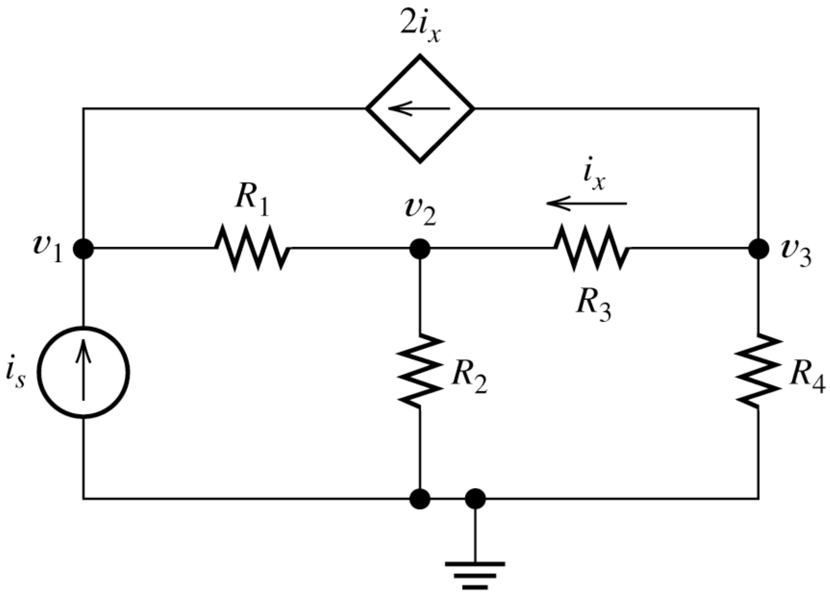




Node-Voltage Analysis with a Dependent Source

- 首先寫出各個節點的 KCL equations，包含controlled source，將其視為一般的source 。

Example 2.10



KCL Node 1:

$$\frac{v_1 - v_2}{R_1} = i_s + 2i_x$$

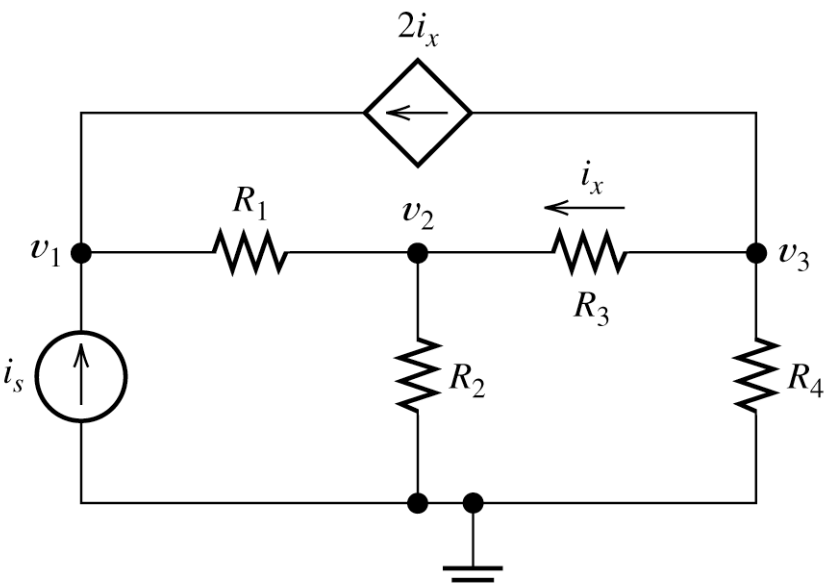
KCL Node 2:

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

KCL Node 3:

$$\frac{v_3 - v_2}{R_3} + \frac{v_3}{R_4} + 2i_x = 0$$

- 接著將controlling variable i_x 以 node voltages 形式表示。



$$i_x = \frac{v_3 - v_2}{R_3}$$

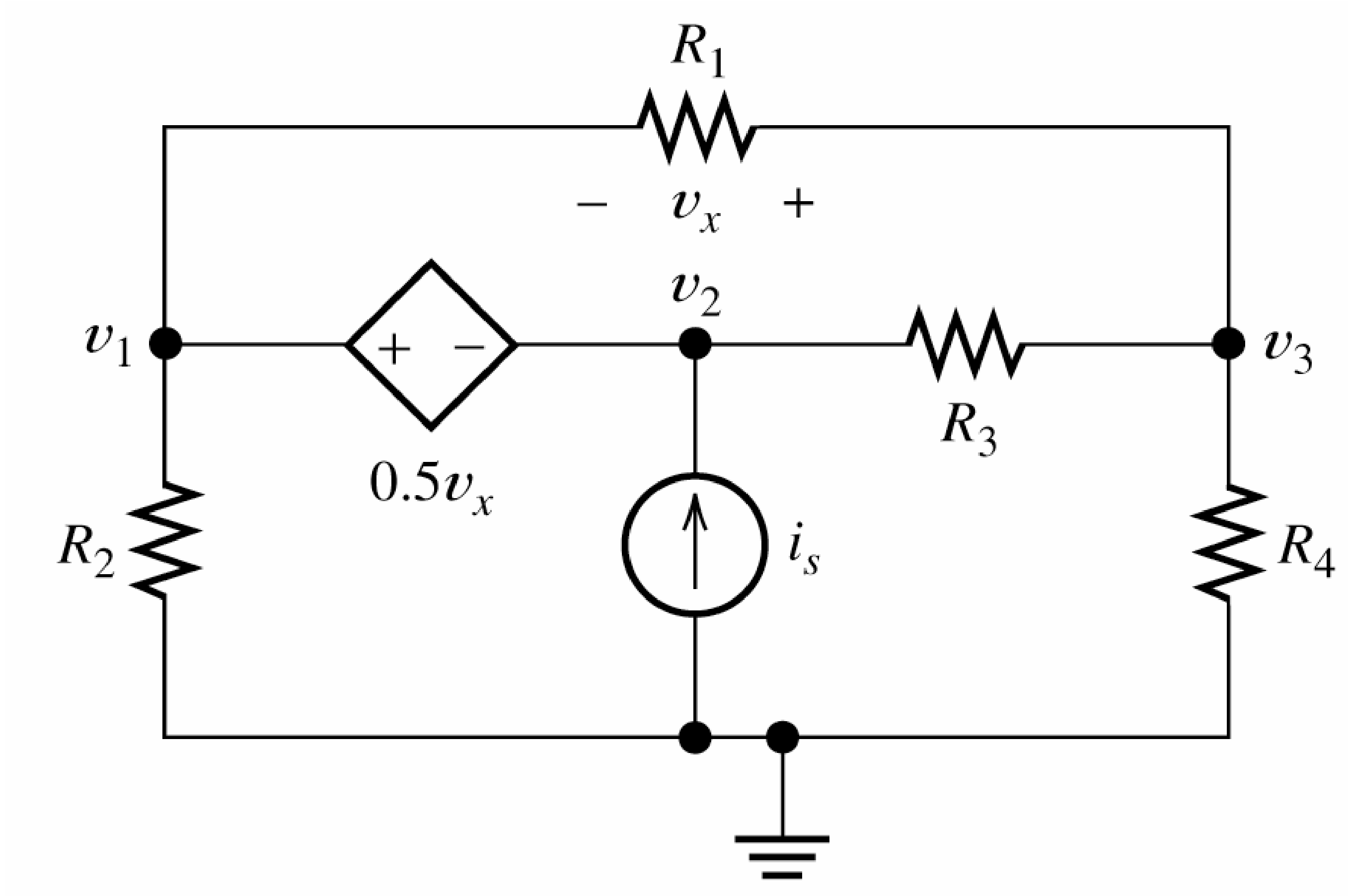
帶入原來方程式

$$\frac{v_1 - v_2}{R_1} = i_s + 2\frac{v_3 - v_2}{R_3}$$

$$\frac{v_2 - v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_2 - v_3}{R_3} = 0$$

$$\frac{v_3 - v_2}{R_3} + \frac{v_4}{R_4} + 2\frac{v_3 - v_2}{R_3} = 0$$

Example 2.11 Node-Voltage Analysis with a Dependent Source



Example 2.10 Node-Voltage Analysis with a Dependent Source

KVL: $-v_1 + 0.5v_x + v_2 = 0$

KCL: Supernode enclosing controlled source

$$\frac{v_1}{R_2} + \frac{v_1 - v_3}{R_1} + \frac{v_2 - v_3}{R_3} = i_s$$

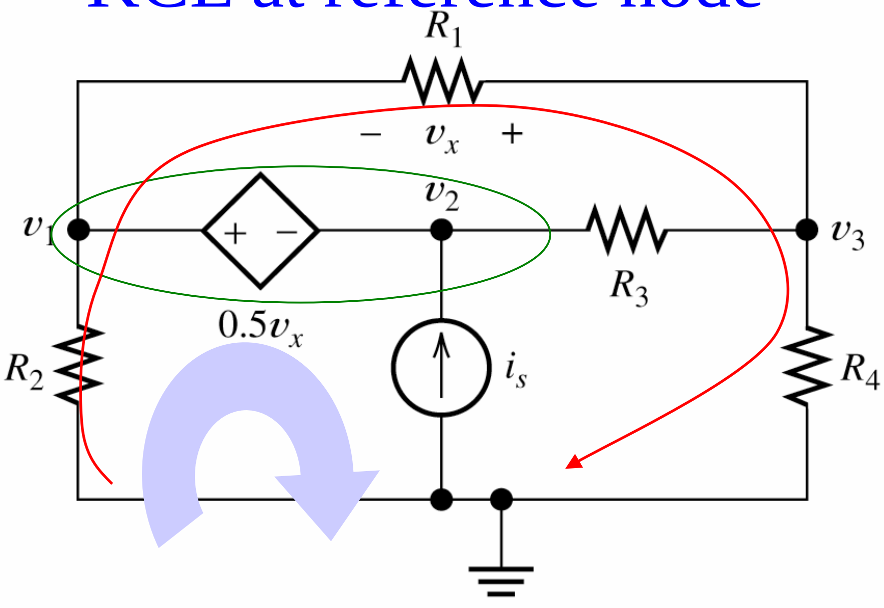
KCL for node 3

$$\frac{v_3}{R_4} + \frac{v_3 - v_2}{R_3} + \frac{v_3 - v_1}{R_1} = 0$$

KCL at reference node

$$\frac{v_1}{R_2} + \frac{v_3}{R_4} = i_s$$

KCLs are dependent



KVL: $-v_1 - v_x + v_3 = 0$

$$v_1 = 0.5(v_3 - v_1) + v_2$$

integrated with any two of the three KCL equations for independence.

Node-Voltage Analysis

1. Select a reference node and assign variables for the unknown node voltages. If the reference node is chosen at one end of an independent voltage source, one node voltage is known at the start, and fewer need to be computed.

2. Write network equations. First, use KCL to write current equations for nodes and supernodes. Write as many current equations as you can without using all of the nodes. Then if you do not have enough equations because of voltage sources connected between nodes, use KVL to write additional equations.

3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the node voltages. Substitute into the network equations, and obtain equations having only the node voltages as unknowns.

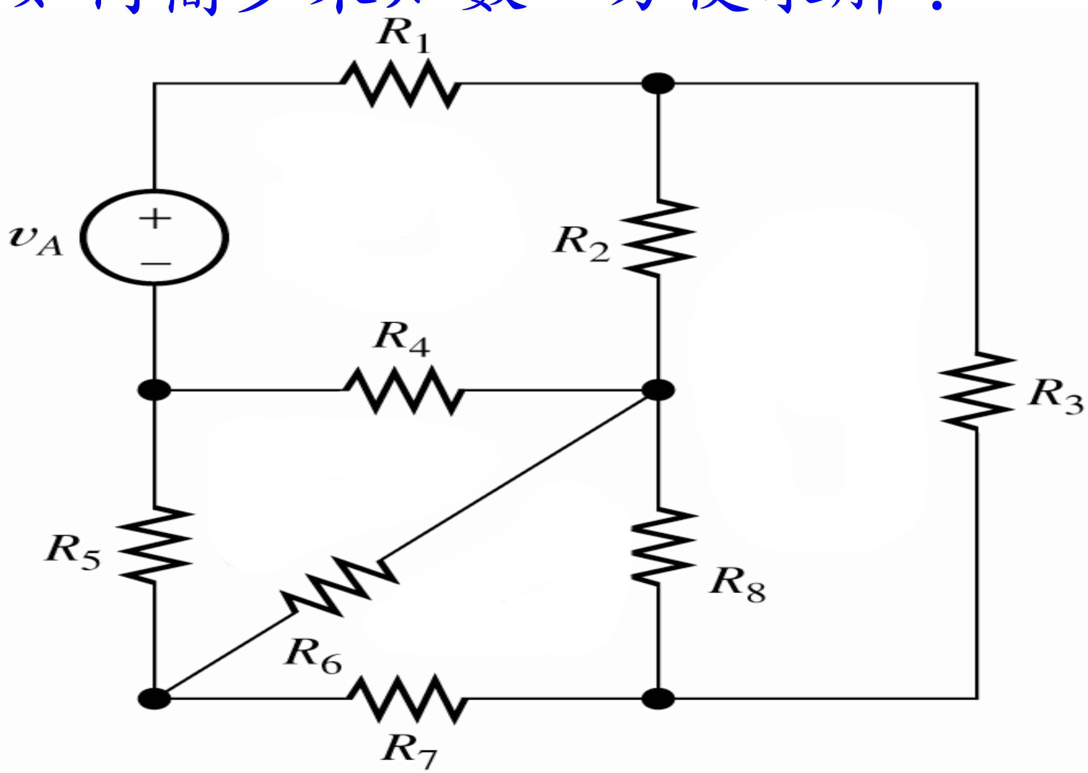
4. Put the equations into standard form and solve for the node voltages.

5. Use the values found for the node voltages to calculate any other currents or voltages of interest.

2.5 Mesh Current Analysis (網目電流分析法)

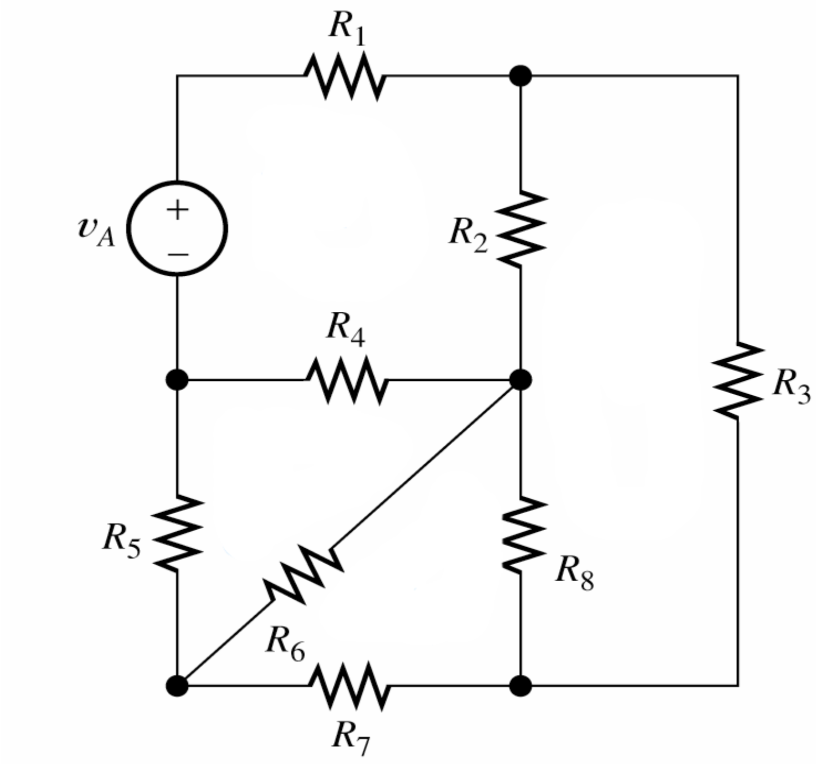
$i_1, i_2, \cdots, i_8$ 待求

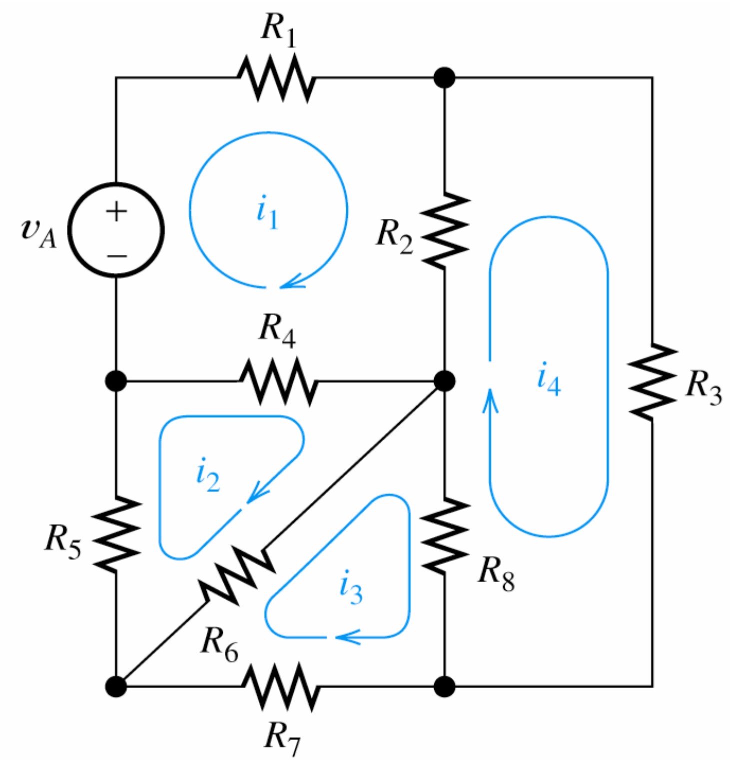
如何簡少未知數，方便求解？



Mesh Current Analysis

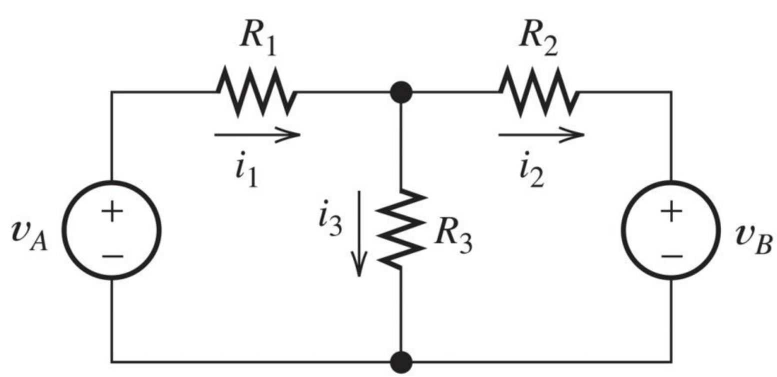
$i_1, i_2, \cdots, i_8$ $\Rightarrow$ i_1, i_2, i_3, i_4



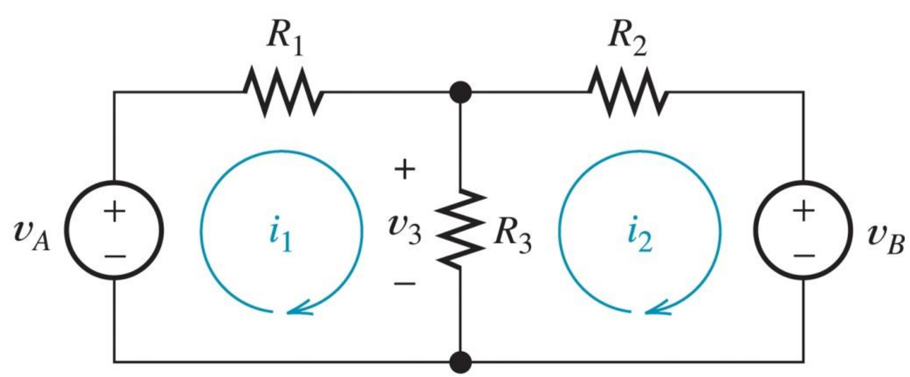


Mesh Current Analysis

- 利用 KVL 環繞網目(mesh)，方程式中的未知數是電流，不論網目中電流方向如何預設，只要 KVL 及歐姆定律正確使用即可。
- 一般進行網目分析時網目電流均取順時針方向。
- 多個網路電流(mesh currents) 都流過一個電路元件時，我們設通過此元件的電流為這些網路電流的和(注意方向與正負)。

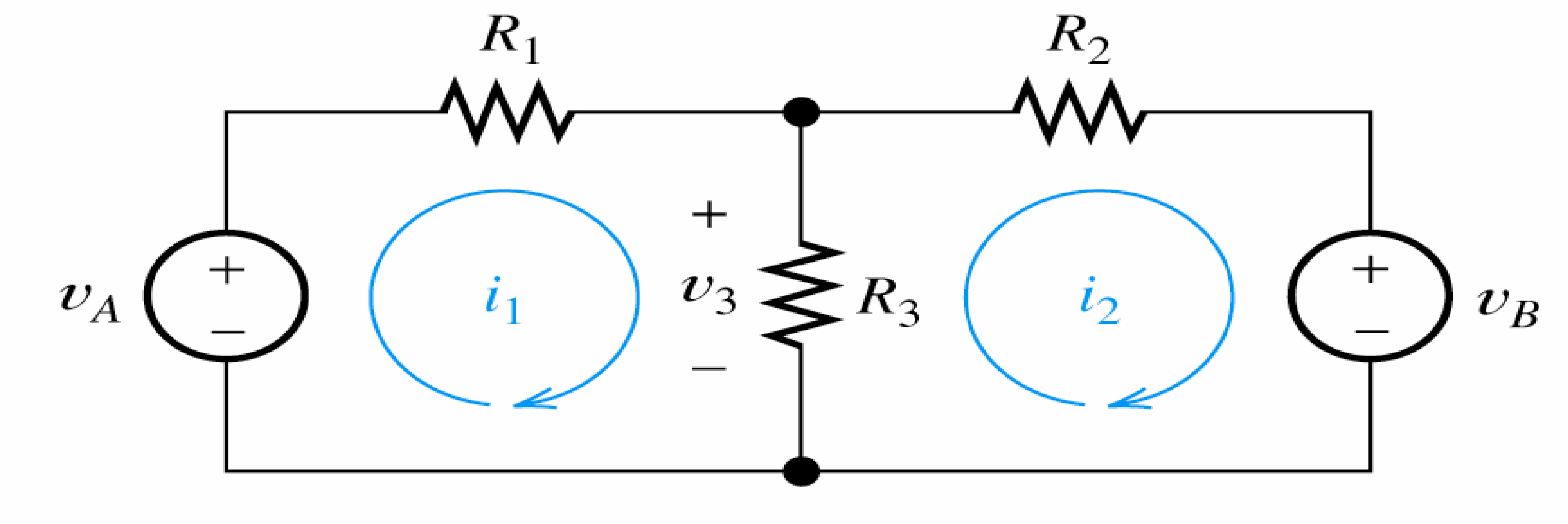


(a) Circuit with branch currents



(b) Circuit with mesh currents

Mesh Current Analysis



KVL for mesh 1

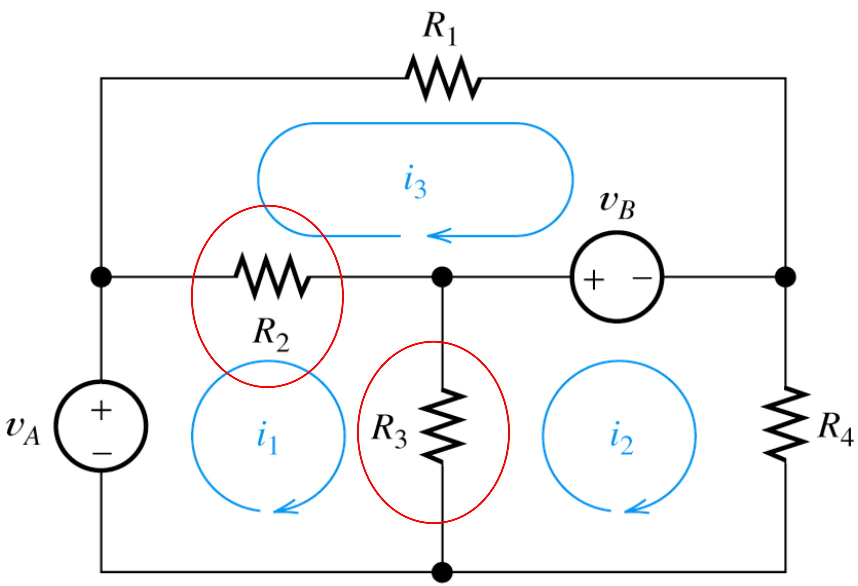
$$-v_A + R_1 i_1 + v_3 = -v_A + R_1 i_1 + R_3(i_1 - i_2) = 0$$

KVL for mesh 2

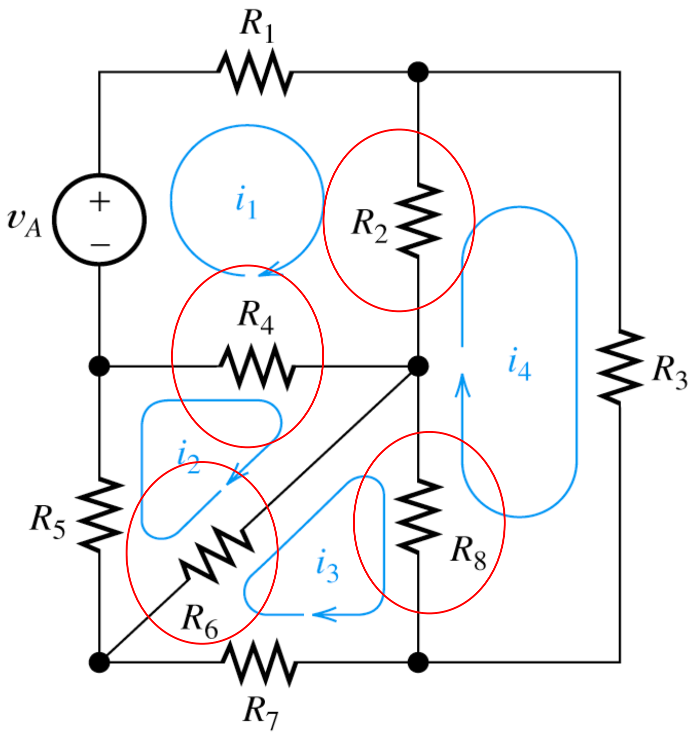
$$v_B + R_2 i_2 + R_3(i_2 - i_1) = 0$$

Choosing the Mesh Currents

- 通常設網目電流為順時針方向(clockwise)。
- 要特別注意有多個網路電流流過的元件(the elements that several mesh currents flow through)。

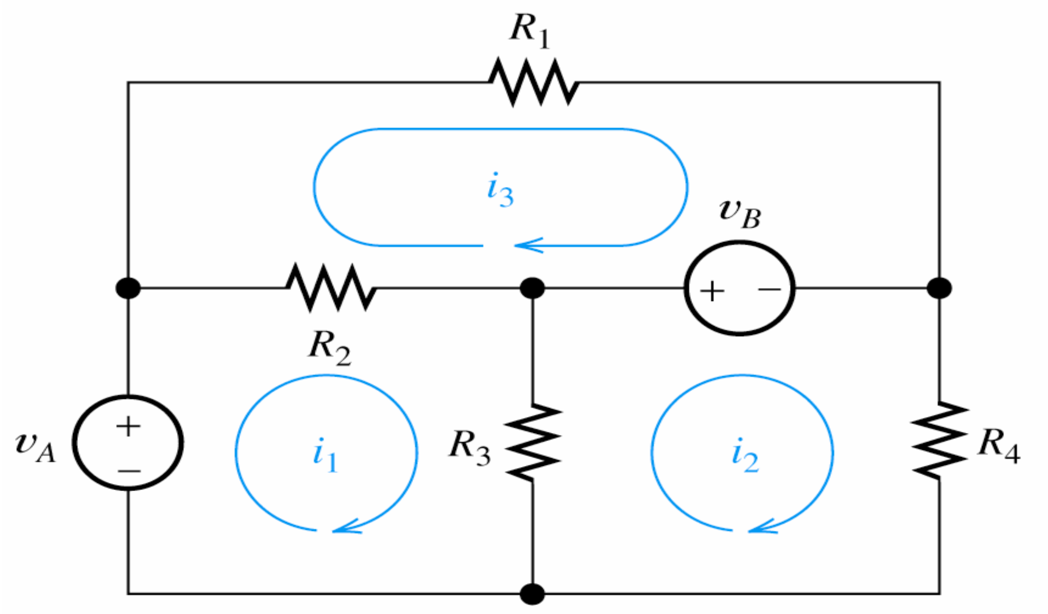


(a)



(b)

Example 2.12



KVL for mesh 1

$$R_2(i_1 - i_3) + R_3(i_1 - i_2) - v_A = 0$$

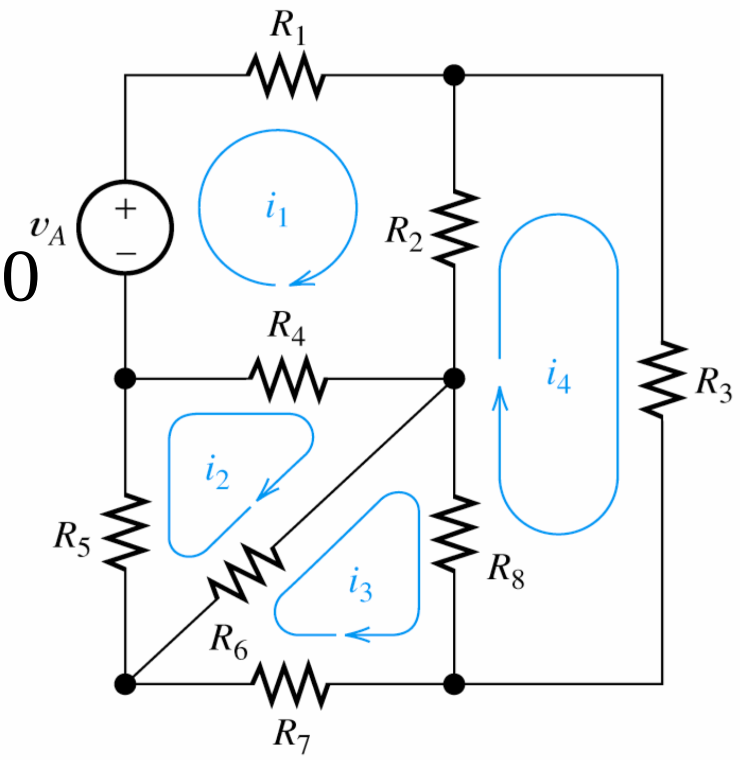
KVL for mesh 2

$$R_3(i_2 - i_1) + R_4 i_2 + v_B = 0$$

KVL for mesh 3

$$R_2(i_3 - i_1) + R_1 i_3 - v_B = 0$$

Exercise 2.18



KVL for mesh 1

$$R_1 i_1 + R_2(i_1 - i_4) + R_4(i_1 - i_2) - v_A = 0$$

KVL for mesh 2

$$R_5 i_2 + R_4(i_2 - i_1) + R_6(i_2 - i_3) = 0$$

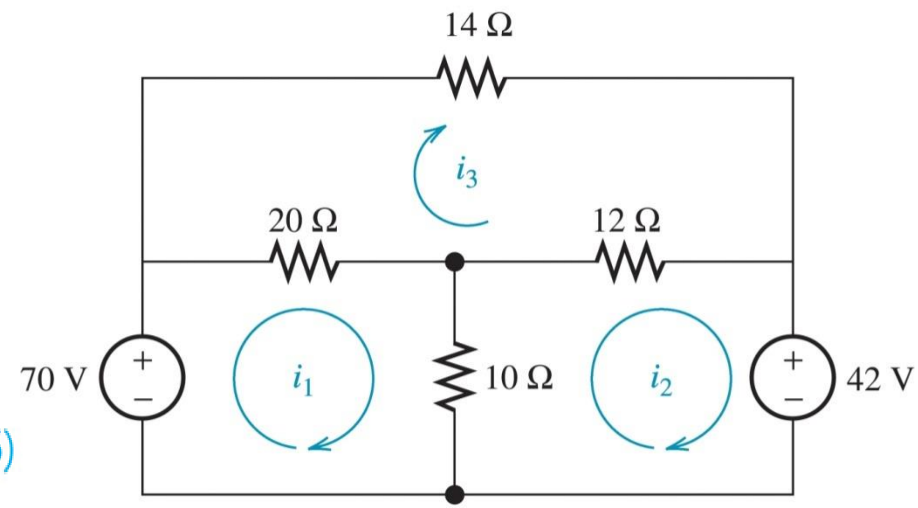
KVL for mesh 3

$$R_7 i_3 + R_6(i_3 - i_2) + R_8(i_3 - i_4) = 0$$

KVL for mesh 4

$$R_3 i_4 + R_2(i_4 - i_1) + R_8(i_4 - i_3) = 0$$

Example 2.13



For mesh 1

$$20(i_1 - i_3) + 10(i_1 - i_2) - 70 = 0 \tag{2.56}$$

For meshes 2 and 3, we have:

$$10(i_2 - i_1) + 12(i_2 - i_3) + 42 = 0 \tag{2.57}$$

$$20(i_3 - i_1) + 14i_3 + 12(i_3 - i_2) = 0 \tag{2.58}$$

Putting the equations into standard form, we have:

$$30i_1 - 10i_2 - 20i_3 = 70 \tag{2.59}$$

$$-10i_1 + 22i_2 - 12i_3 = -42 \tag{2.60}$$

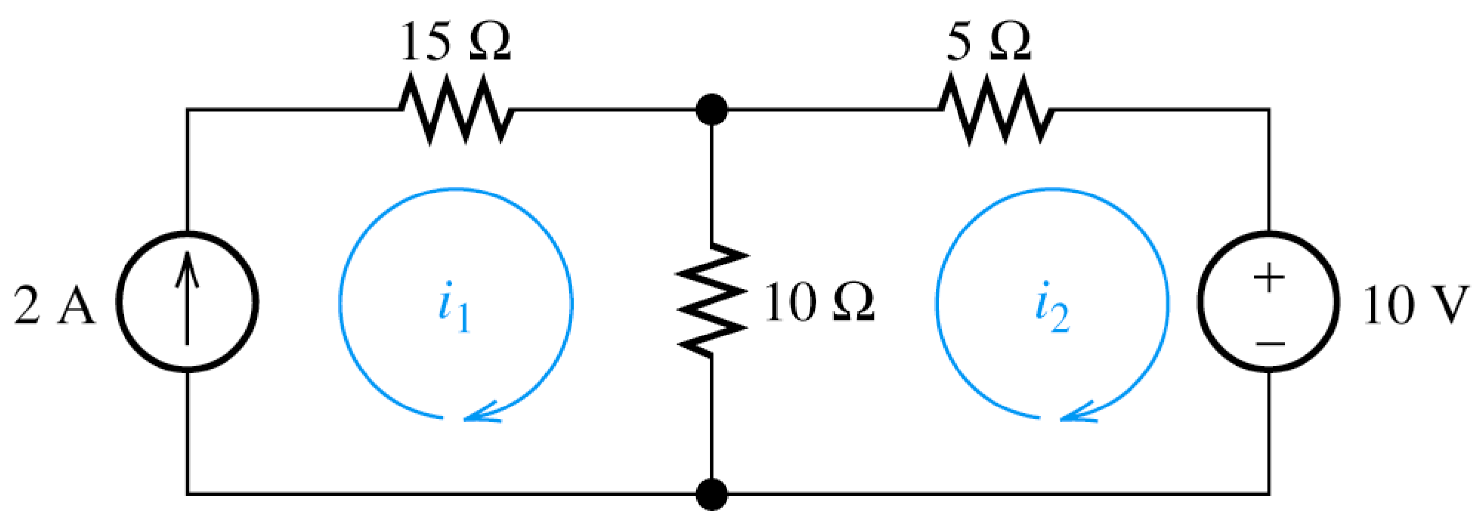
$$-20i_1 - 12i_2 + 46i_3 = 0 \tag{2.61}$$

In matrix form, the equations become:

$$\begin{bmatrix} 30 & -10 & -20 \\ -10 & 22 & -12 \\ -20 & -12 & 46 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 70 \\ -42 \\ 0 \end{bmatrix}$$

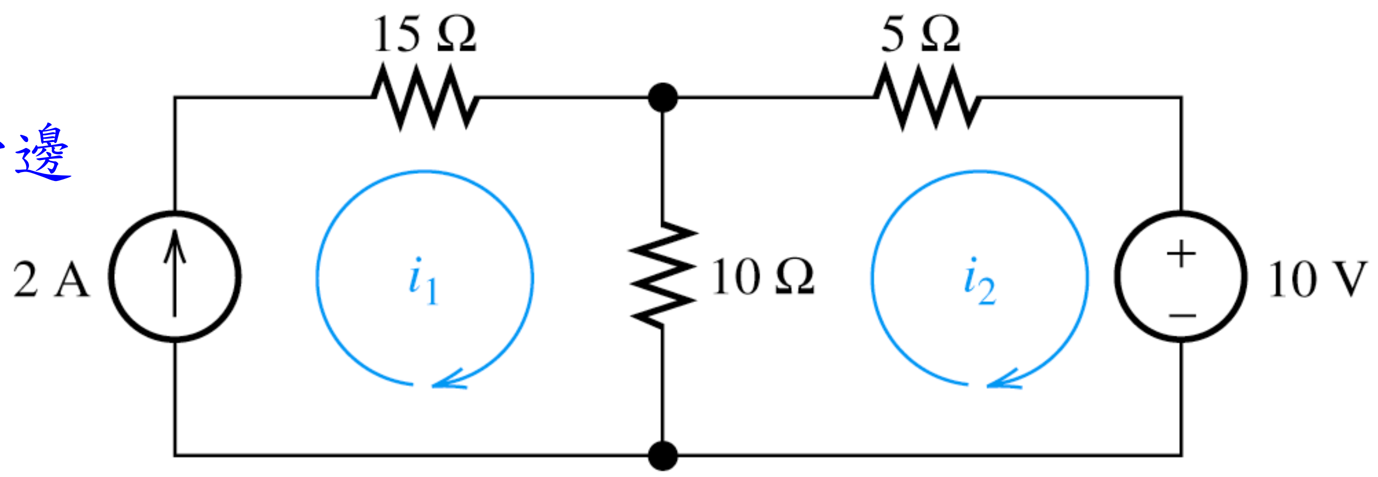
Mesh Currents in Circuits Containing Current Sources

- 要利用KVL 時電流源上的電壓為何？
- 將電流源上的電壓設為0 是常犯的錯誤。



Mesh Currents in Circuits Containing Current Sources

電流源在旁邊

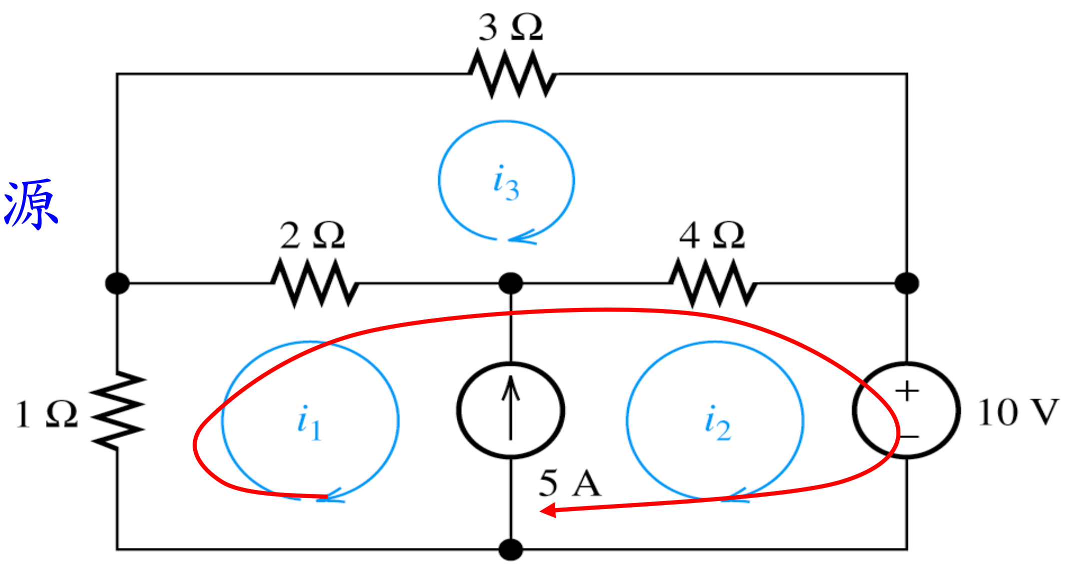


$$i_1 = 2\text{A}$$

KVL for mesh 2 $$10(i_2 - i_1) + 5i_2 + 10 = 0$$

Supermesh

兩個mesh 都包含電流源



KVL for mesh 1?　　KVL for mesh 2?

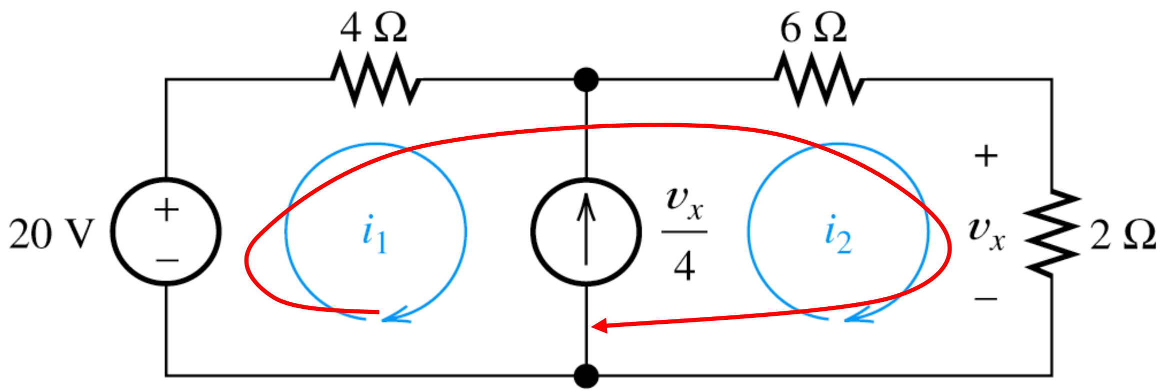
結合 meshes 1 and 2 為一個 **supermesh (超網目)**.

KVL for supermesh $$i_1 + 2(i_1 - i_3) + 4(i_2 - i_3) + 10 = 0$$

KVL for mesh 3 $$3i_3 + 4(i_3 - i_2) + 2(i_3 - i_1) = 0$$

Additional Equation $$i_2 - i_1 = 5$$

Circuits with Controlled Sources



KVL for supermesh $$-20+4i_1+6i_2+2i_2=0$$

Source current $$\frac{v_x}{4}=i_2-i_1$$

Controlling voltage $$v_x=2i_2$$

$$\frac{i_2}{2}=i_2-i_1$$

Mesh-Current Analysis

1. Define the mesh currents. Select a clockwise direction for each of the mesh currents.

2. Write network equations.
First, use KVL to write voltage equations for meshes. Express current sources in terms of the mesh currents. Finally, if a current source is common to two meshes, write a KVL equation for the supermesh.

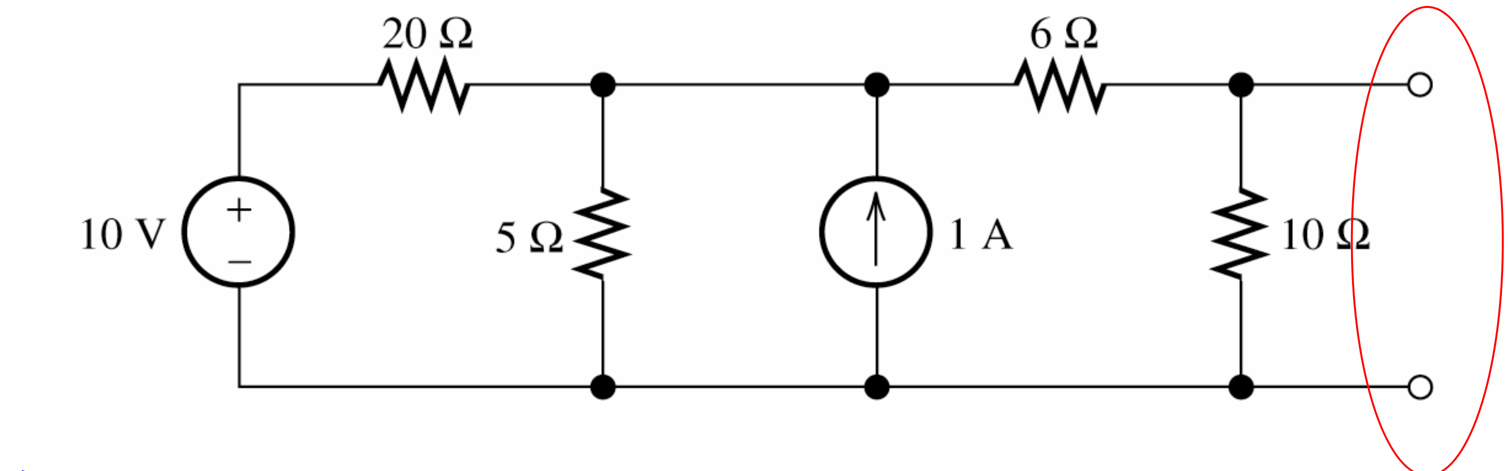
3. If the circuit contains dependent sources, find expressions for the controlling variables in terms of the mesh currents.

4. Put the equations into standard form. Solve for the mesh currents by use of determinants or other means.

5. Use the values found for the mesh currents to calculate any other currents or voltages of interest.

2.6 Thévenin Equivalent Circuits (戴維寧等效電路)

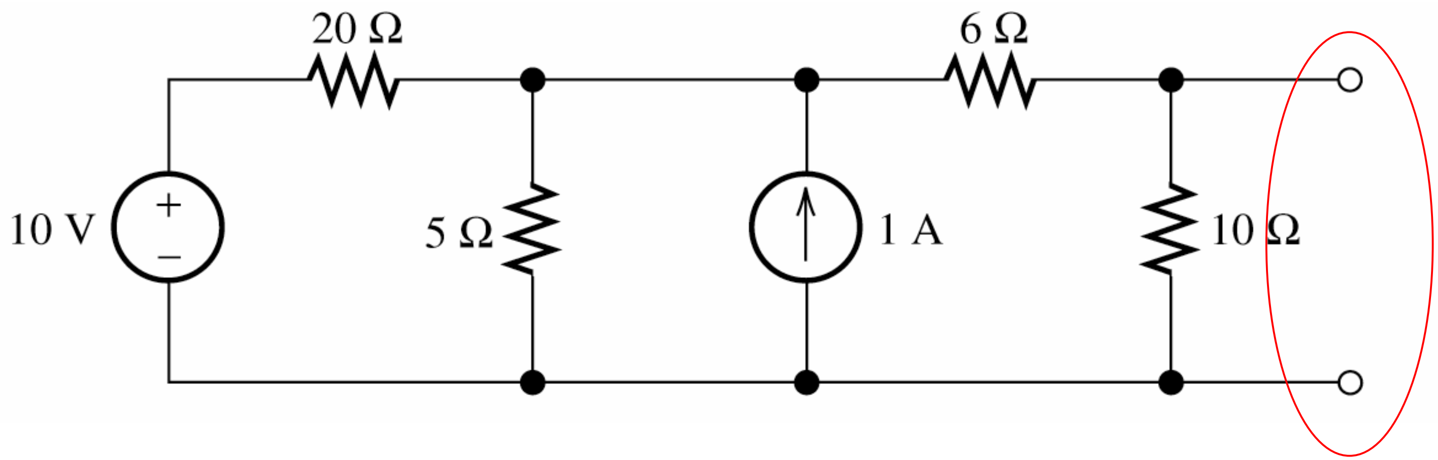
- 兩端點電路(two-terminal circuit): 一個電路只有兩點可以與其他電路連接。



每連接一個不同元件(負載)是否要將此複雜電路重新分析一遍？

有沒有取巧的方法？

ANS: 等效電路



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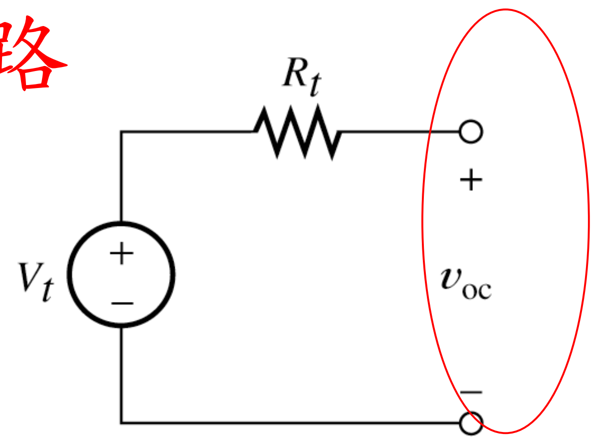
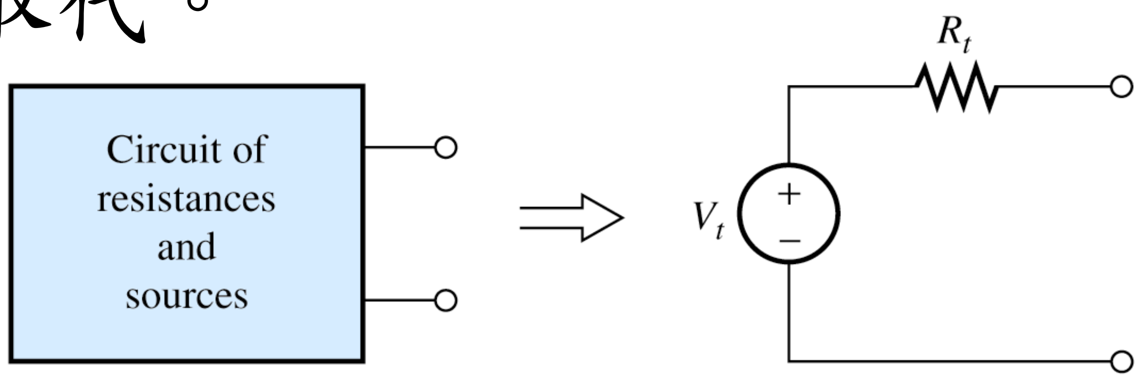
戴維寧等效電路



Thévenin Equivalent Circuits (戴維寧等效電路)

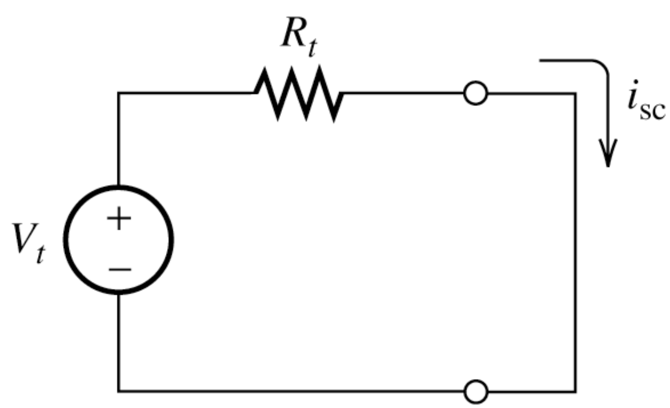
- 一個包含電阻與source的兩端點電路可藉由一包含一獨立電壓源串接一電阻的等效電路可取代。



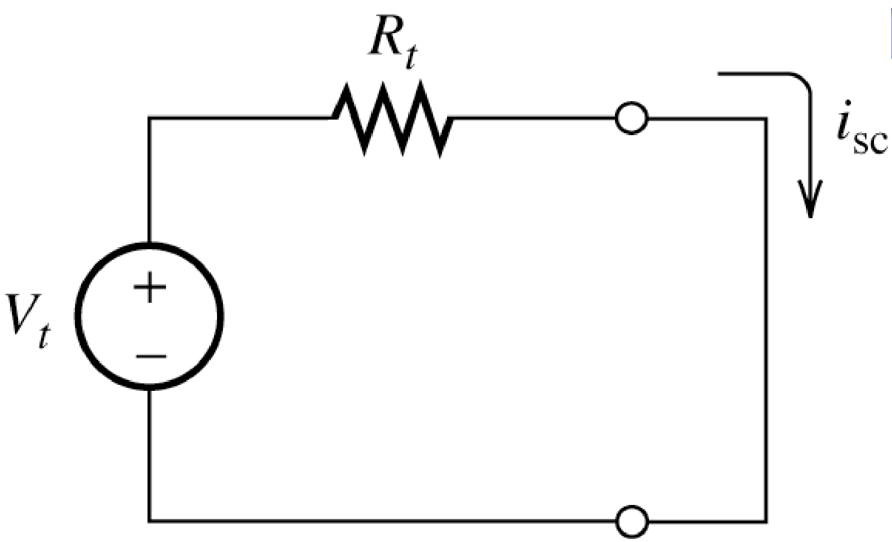
- V_t 等於原來電路的斷路(open circuit)電壓 V_{oc}。(KVL)

- 將戴維寧等效電路短路(short circuit)，則流過此電路的電流為

$$i_{sc} = \frac{V_t}{R_t}$$



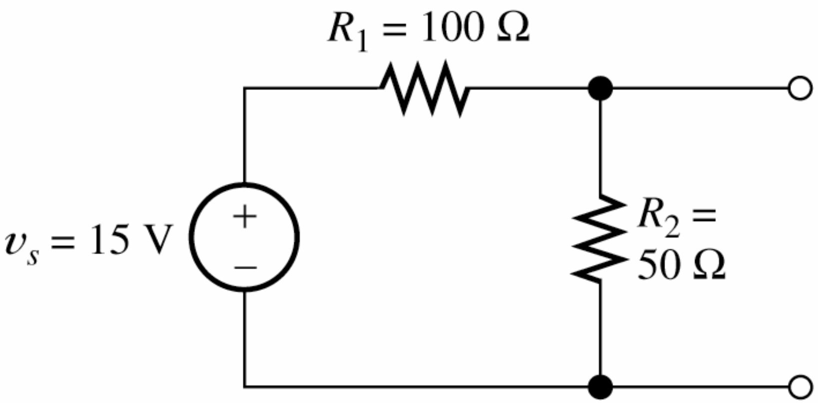
Thévenin Equivalent Circuits



戴維寧電阻(Thévenin resistance)

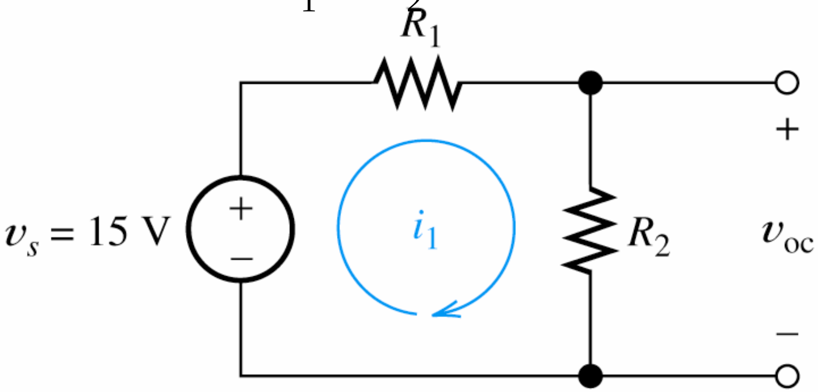
$$R_t = \frac{v_t}{i_{sc}} = \frac{v_{oc}}{i_{sc}}$$

Example 2.16



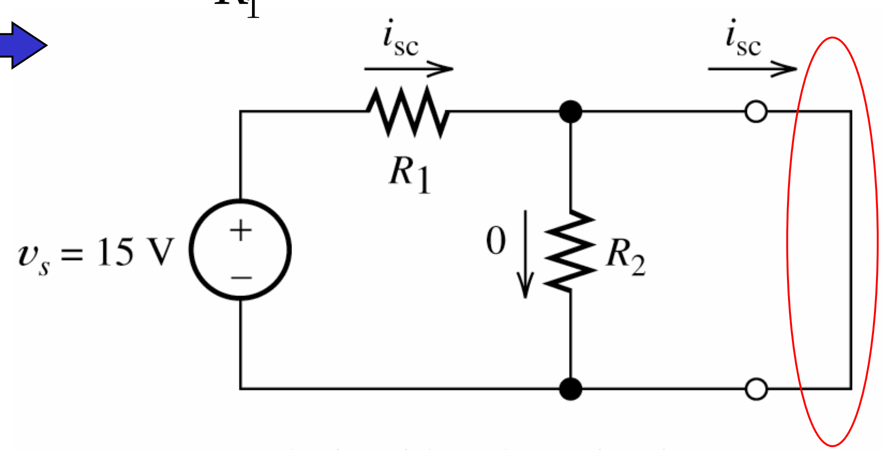
(a) Original circuit

$$v_{oc} = 15 \cdot \frac{R_2}{R_1 + R_2} = 5\text{ V}$$



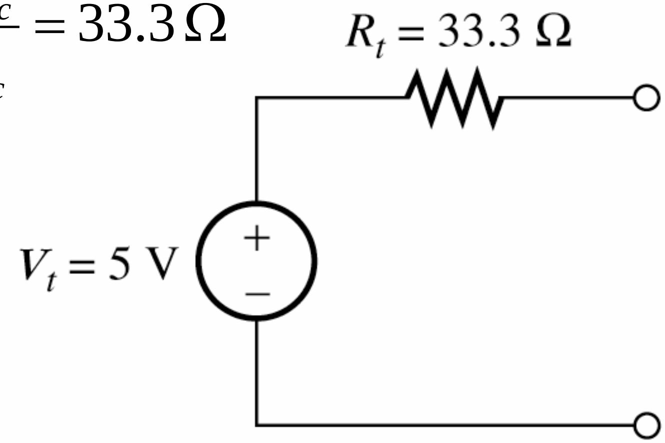
(b) Analysis with an open circuit

$$i_{sc} = \frac{v_s}{R_1} = 0.15\text{ A}$$



(c) Analysis with a short circuit

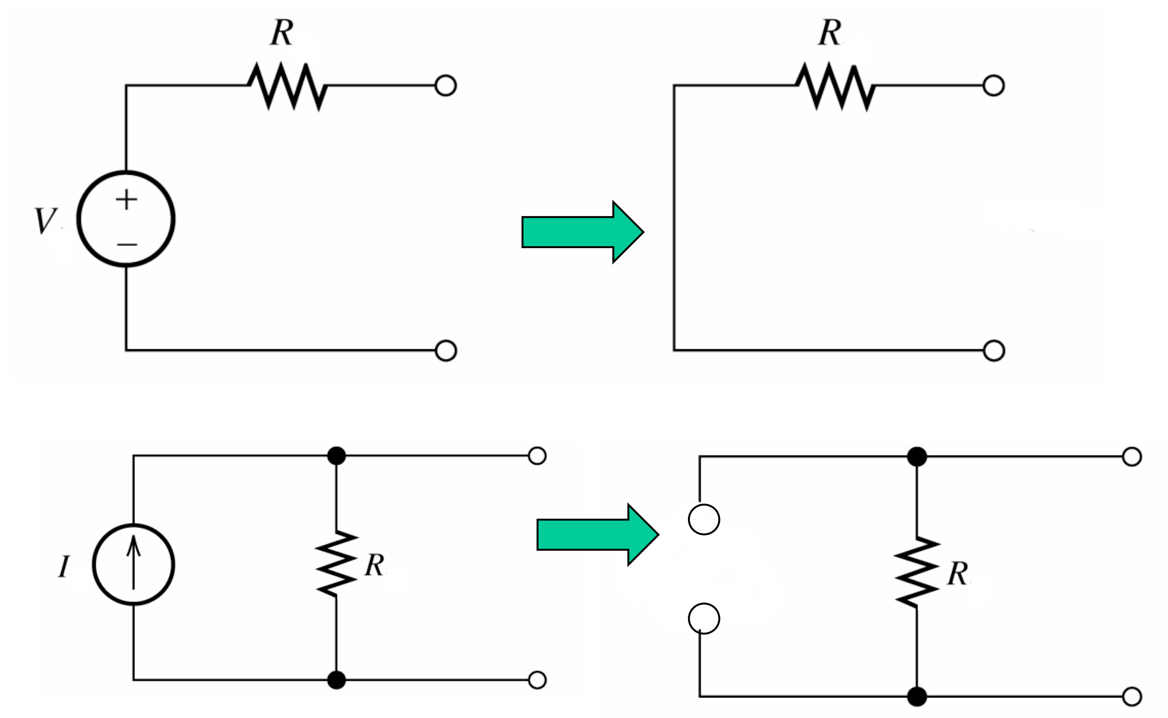
$$R_t = \frac{v_{oc}}{i_{sc}} = 33.3\,\Omega$$



(d) Thévenin equivalent

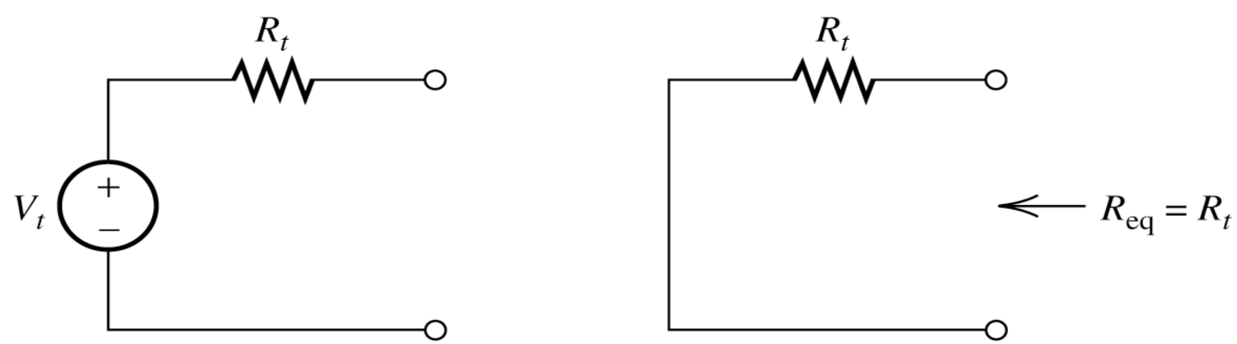
Finding the Thévenin Resistance Directly

When zeroing a voltage source, it becomes a short circuit. When zeroing a current source, it becomes an open circuit.



Finding the Thévenin Resistance Directly

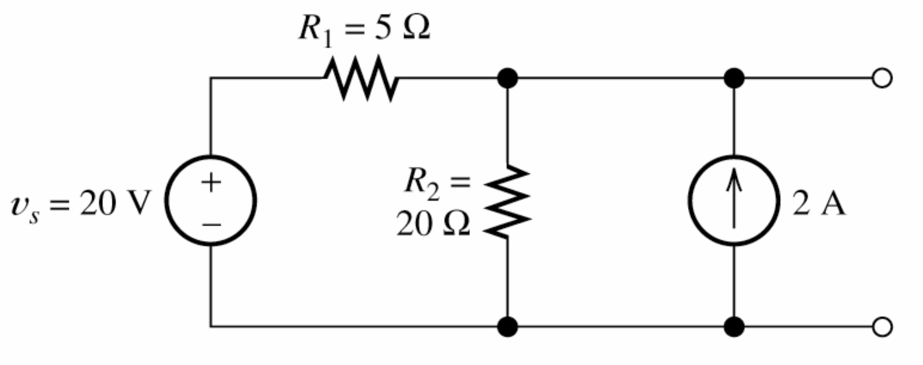
We can find the Thévenin resistance by zeroing the sources in the original network and then computing the resistance between the terminals.



(a) Thévenin equivalent

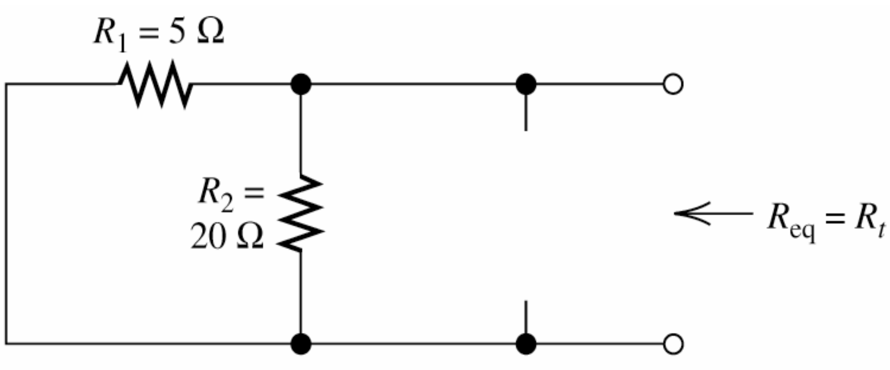
(b) Thévenin equivalent with its source zeroed

Example 2.17



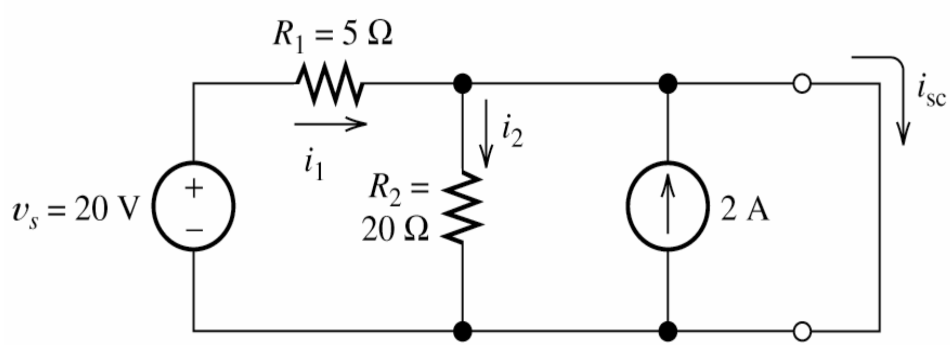
(a) Original circuit

$$R_{eq} = R_t = \frac{R_1 R_2}{R_1 + R_2} = 4\Omega$$



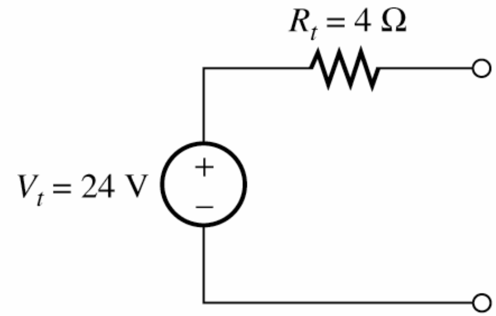
(b) Circuit with sources zeroed

$$i_{sc} = \frac{v_s}{R_1} + 2 = 6 \text{ A}$$



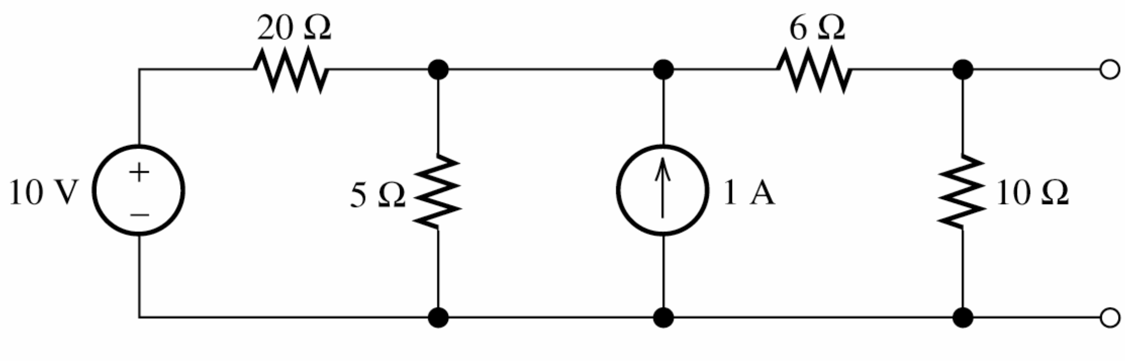
(c) Circuit with a short circuit

$$v_t = R_t i_{sc} = 24 \text{ V}$$

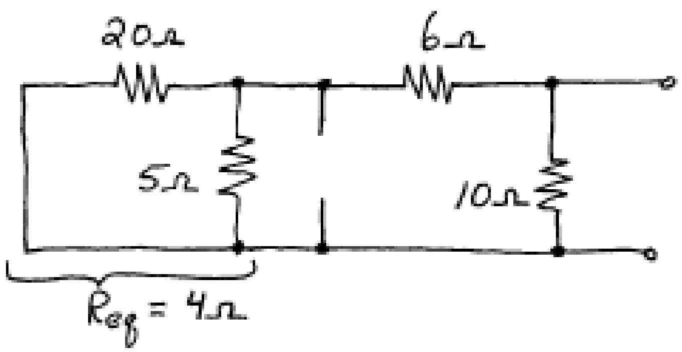


(d) Thévenin equivalent circuit

Exercise 2.28 Find R_t by zeroing the sources



(c)



$$R_t = \frac{1}{\frac{1}{10} + \frac{1}{6 + \frac{1}{(1/5 + 1/20)}}} = 5\,\Omega$$

Note: we can not find the Thévenin resistance by zeroing the dependent source.

a. Determine the open-circuit voltage $V_t = v_{oc}$.

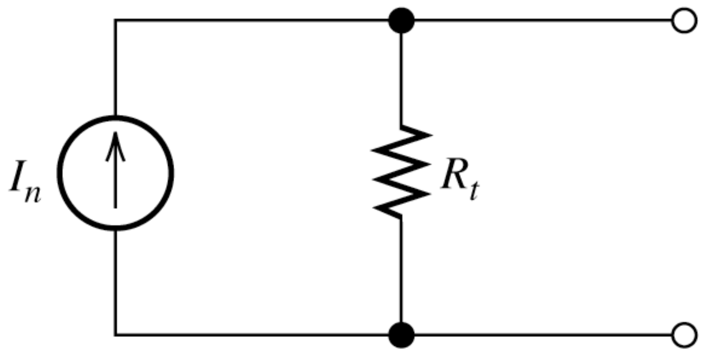
b. Determine the short-circuit current i_{sc}.

c. Thévenin resistance R_t

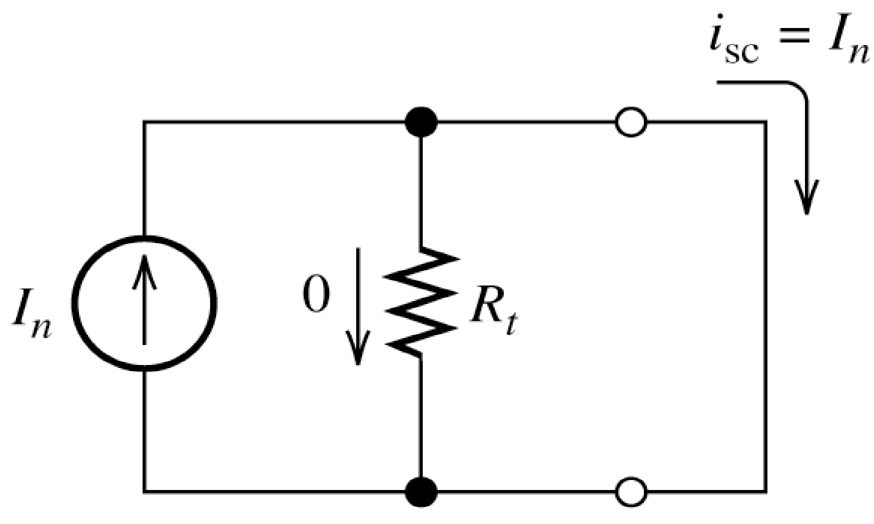
$$R_t = \frac{v_{\mathrm{t}}}{i_{\mathrm{sc}}} = \frac{v_{\mathrm{oc}}}{i_{\mathrm{sc}}}$$

Norton Equivalent Circuits (諾頓等效電路)

- 一個包含電阻與source的兩端點電路可藉由一包含一獨立電流源並聯一電阻的等效電路取代。



•將諾頓等效電路短路(short circuit)，則流過等效電阻的電流為0, 短路電流為諾頓等效電流。



The Norton equivalent circuit with a short circuit across its terminals.

Thévenin/Norton-Equivalent-Circuit Analysis

1. Perform two of these:

a. Determine the open-circuit voltage V_t **=** v_{oc}.

b. Determine the short-circuit current I_n **=** i_{sc}.

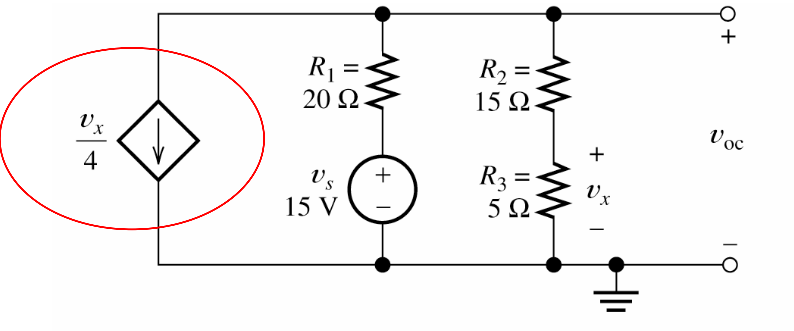
c. Zero the sources and find the Thévenin resistance R_t looking back into the terminals
(if NO dependent source is in the circuit).

2. Use the equation $V_t = R_t I_n$ to compute the remaining value.

3. The Thévenin equivalent consists of a voltage source V_t in series with R_t .

4. The Norton equivalent consists of a current source I_n in parallel with R_t .

Example 2.19 Norton Equivalent Circuit



(a) Original circuit under open-circuit conditions

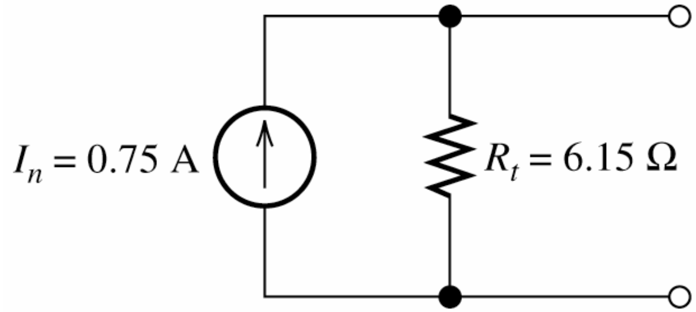
KCL $\dfrac{v_{oc}}{R_2 + R_3} + \dfrac{v_{oc} - v_s}{R_1} + \dfrac{v_x}{4} = 0$

分壓定律 (voltage-divider principle)

$$v_x = \frac{R_3}{R_2 + R_3} v_{oc} = 0.25 v_{oc}$$

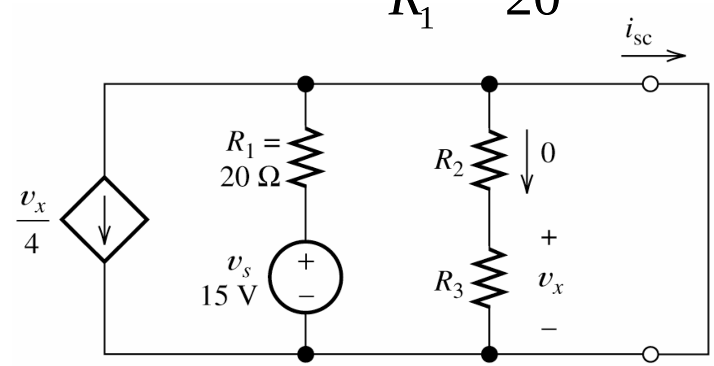
$$v_{oc} = 4.62 \text{ V}$$

$$R_t = \frac{v_{oc}}{i_{sc}} = \frac{4.62}{0.75} = 6.15\,\Omega$$



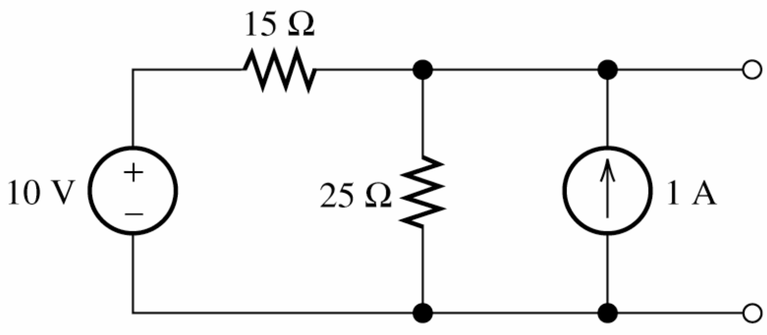
(c) Norton equivalent circuit

$$I_n = i_{sc} = \frac{v_s}{R_1} = \frac{15}{20} = 0.75 \text{ A}$$



(b) Circuit with a short circuit

Exercise 2.29 Norton Equivalent Circuit



(a)

$$R_t = \frac{15 \cdot 25}{15 + 25} = 9.375\,\Omega$$

$$I_n = i_{sc} = 1 + \frac{10}{15} = 1.67\text{ A}$$

Source Transformations

•Source Transformations: 一我們可以將一個電壓源串聯一電阻的電路由諾頓等效電路取代(或反之)。

•Source Transformation 為外部等效(external equivalence)非內部等效。

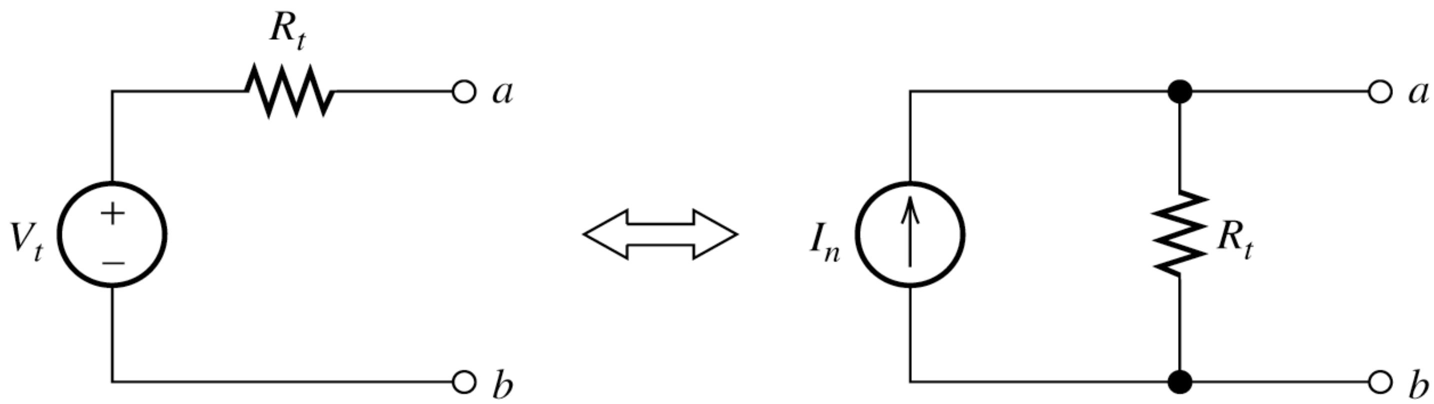
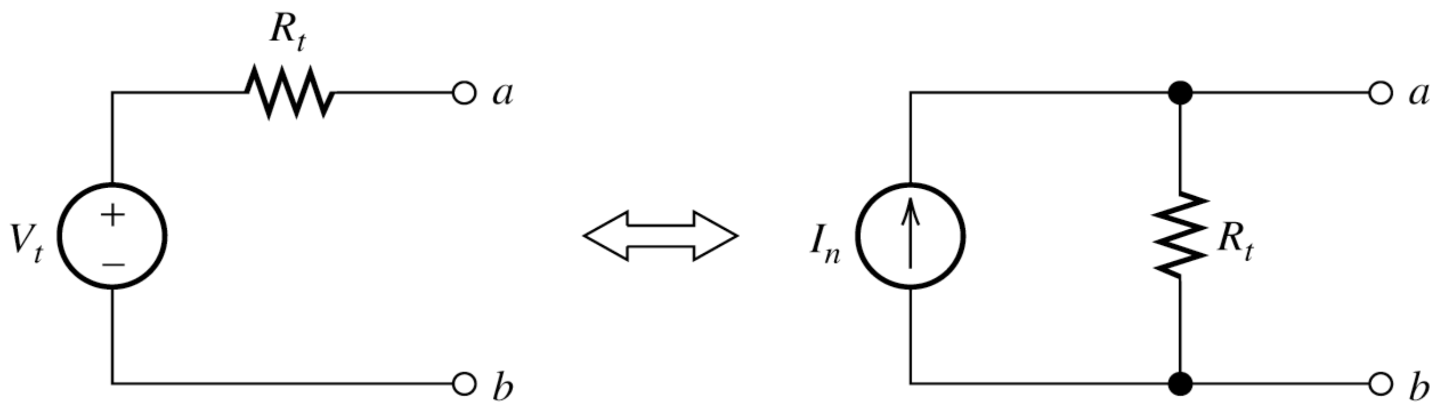


Figure 2.53 A voltage source in series with a resistance is externally equivalent to a current source in parallel with the resistance, provided that $I_n = V_t / R_t$.

Source Transformations



If nodes a & b are open

$$V_{ab} = V_t$$

$$i_{ab} = 0$$

External equivalence

$$\equiv$$

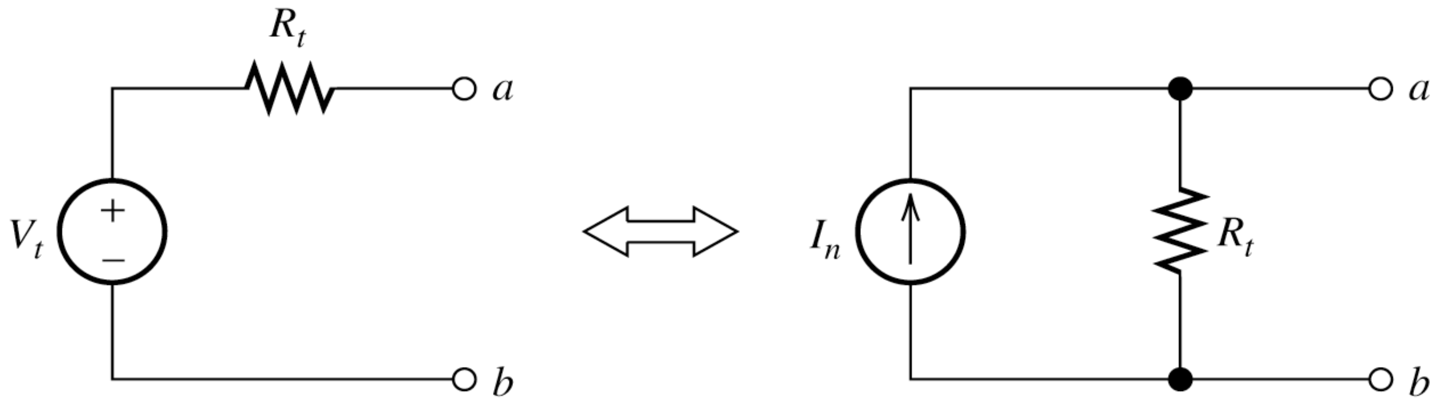
$$I_n = \frac{V_t}{R_t}$$

$$V_{ab} = I_n \cdot R_t = V_t$$

$$i_{ab} = 0$$

- **Source Transformation 要注意電流源、電壓源方向(以維持等效)。**

Source Transformations



If nodes a & b are open

$$i_t = 0$$

Not internal equivalence

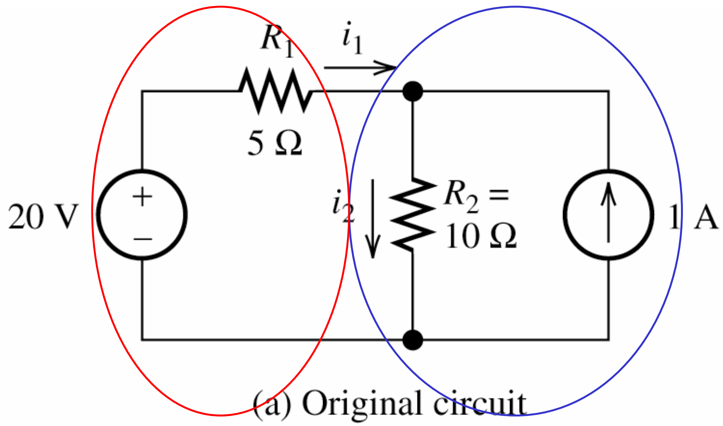
$$i_t = I_n \neq 0$$

Source Transformations

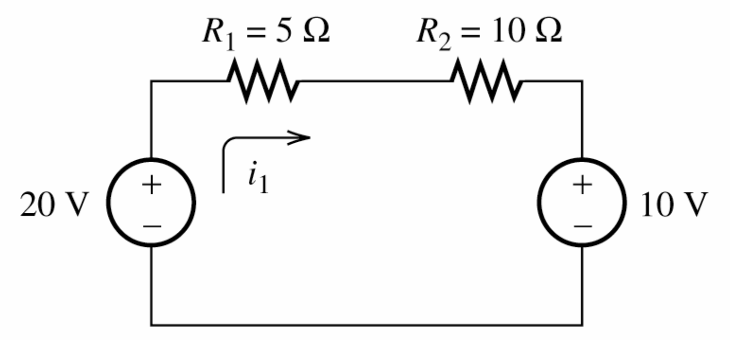
- 如何由外部估測是否有諾頓(Norton)等效?

諾頓等效會持續作工(發熱)
$P=i^2R,\ i\neq 0$

Example 2.20 Source Transformations

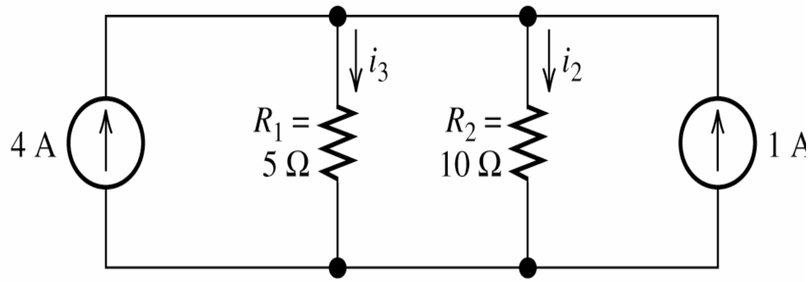


(a) Original circuit



(b) Circuit after transforming the current source into a voltage source

$$i_2 = \frac{R_1}{R_1 + R_2}(4+1) = 1.667\text{A}$$

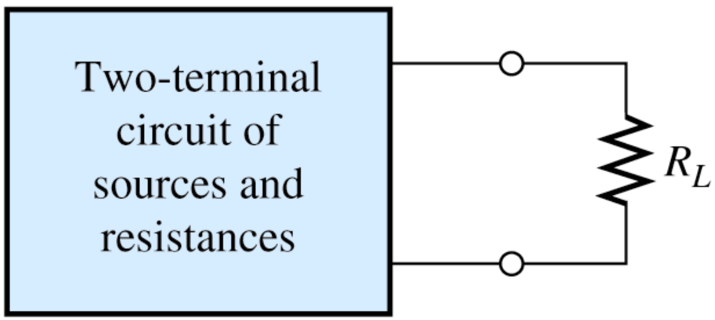


(c) Circuit after transforming the voltage source into a current source

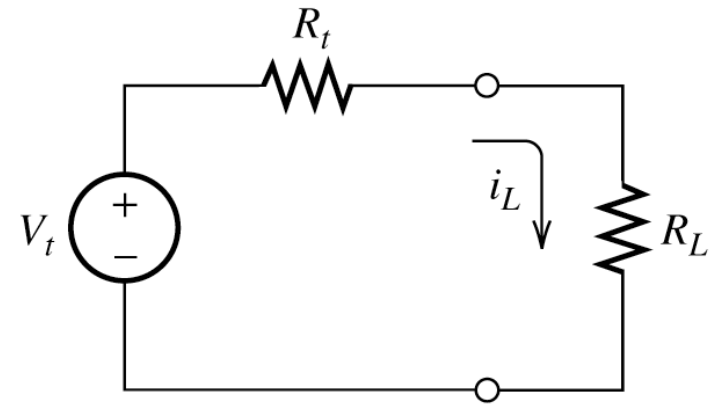
$$i_1 = \frac{20-10}{10+5} = 0.667\text{A}$$

$$i_2 = i_1 + 1 = 1.667\ \text{A}$$

Maximum Power Transfer

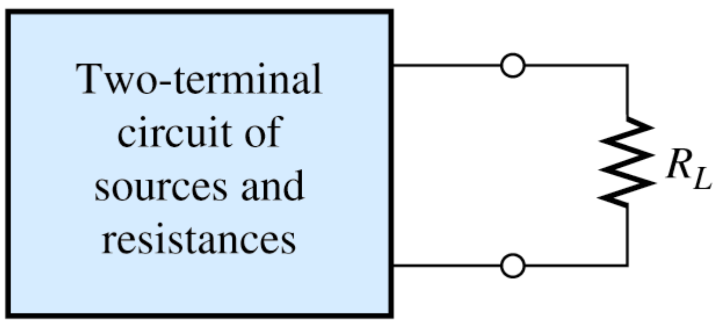


(a) Original circuit with load

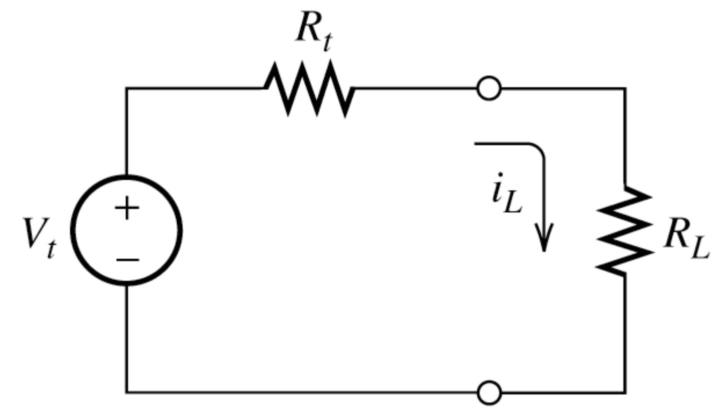


(b) Thévenin equivalent circuit with load

一個兩端電路外接一負載電阻，負載電阻值為何其所消耗功率(maximum power)最大？



(a) Original circuit with load



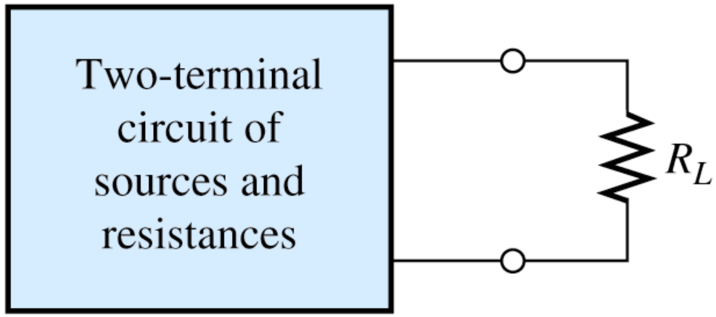
(b) Thévenin equivalent circuit with load

$$i_L = \frac{V_t}{R_t + R_L}$$

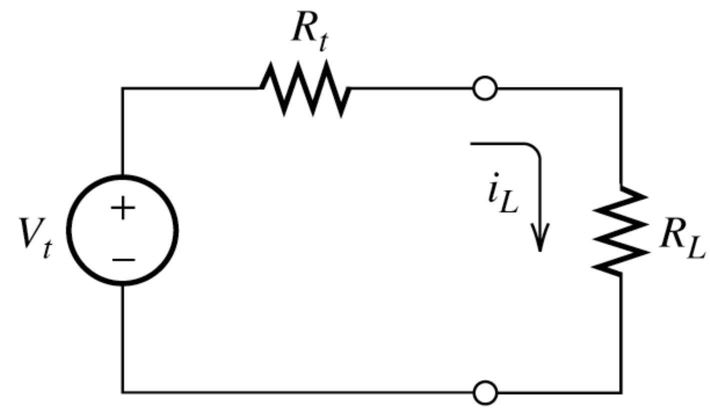
$$p_L = i_L^2 R_L = \frac{V_t^2 R_L}{(R_t + R_L)^2}$$

$$\frac{dp_L}{dR_L} = 0 = \frac{V_t^2}{(R_t + R_L)^2} - \frac{2V_t^2 R_L}{(R_t + R_L)^3} = \frac{V_t^2 (R_t + R_L) - 2V_t^2 R_L}{(R_t + R_L)^3}$$

$$R_t = R_L$$



(a) Original circuit with load

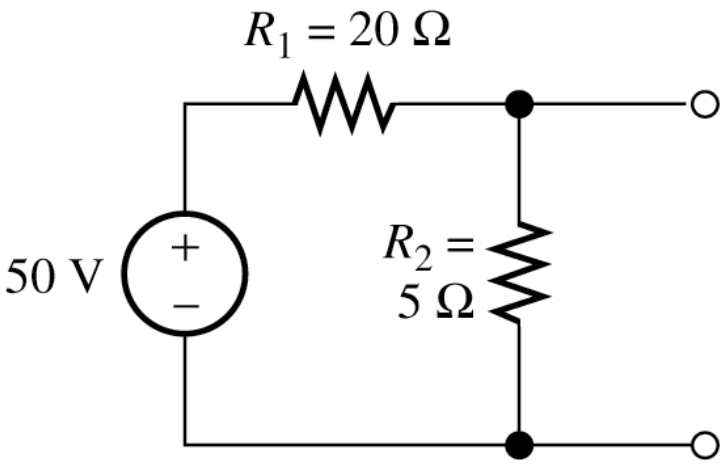


(b) Thévenin equivalent circuit with load

Maximum Power

$$p_{L\max} = i_L^2 R_L = \frac{V_t^{\,2} R_t}{(R_t + R_t)^2} = \frac{V_t^{\,2}}{4R_t}$$

Example 2.21 Find the load resistance for maximum power transformation



$$R_t = \frac{R_1 R_2}{R_1 + R_2} = 4\Omega$$

$$V_t = \frac{R_2}{R_1 + R_2} \cdot 50 = 10\ \text{V}$$

$$R_L = R_t = 4\,\Omega$$

$$p_{L\max} = \frac{V_t^2}{4R_t} = \frac{10^2}{4 \times 4} = 6.25\ \text{W}$$

2.7 SUPERPOSITION PRINCIPLE (疊加原理)

The **superposition principle** states that the total response is the sum of the responses to each of the independent sources acting individually.

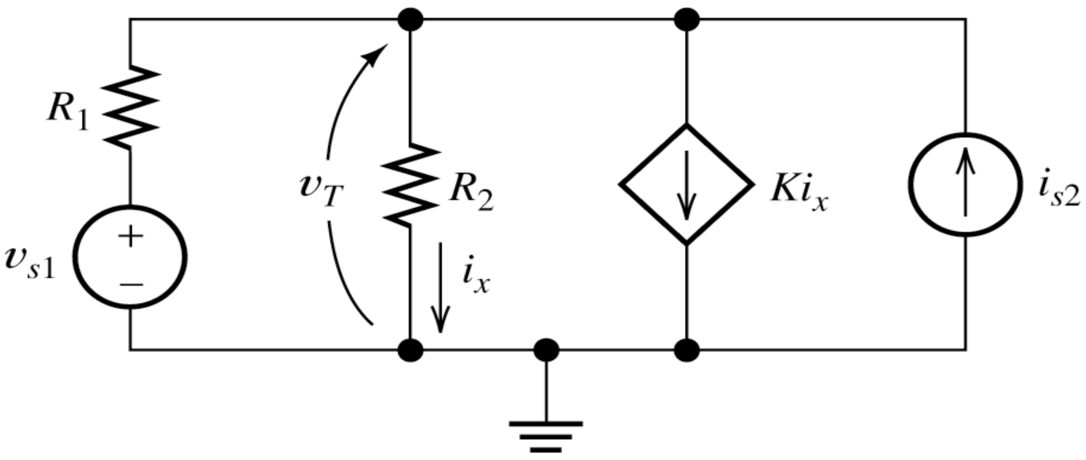
In equation form, this is

$$r_T = r_1 + r_2 + \cdots + r_n$$

2.7 SUPERPOSITION PRINCIPLE (疊加原理)

- 要獲得某一 independent source 所造成的 response，則將其他 independent source zeroing (reduce the source value to zero)。

Suppose the response is the voltage across R_2

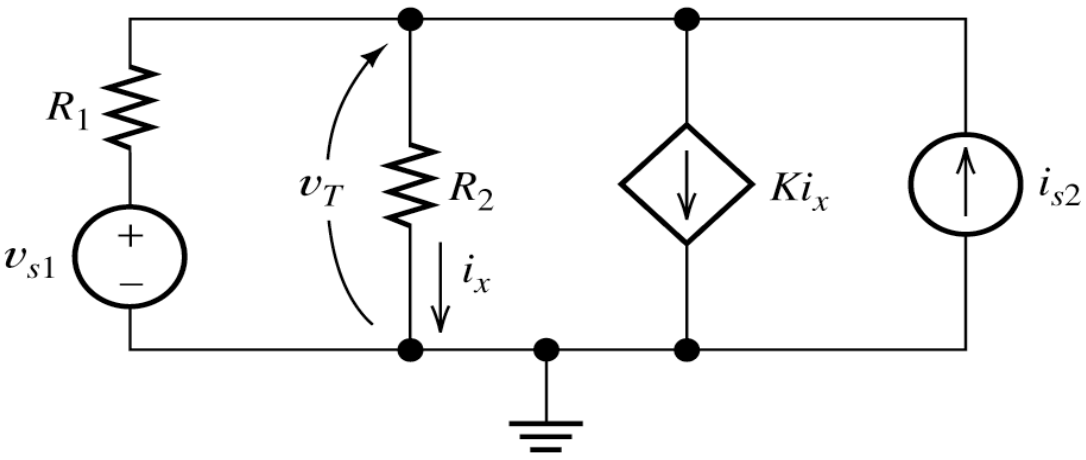


$$\frac{v_T - v_{s1}}{R_1} + \frac{v_T}{R_2} + Ki_x = i_{s2}$$

$$i_x = \frac{v_T}{R_2}$$

$$v_T = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} + \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2}$$

Suppose the response is the voltage across R_2



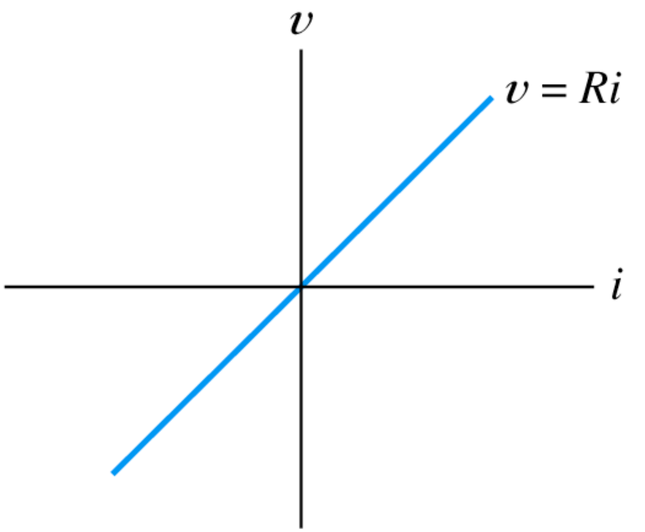
$$v_T = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} + \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2}$$

$i_{s2} = 0$ $\qquad$ $v_{s1} = 0$

$$v_T = v_1 + v_2$$

Linearity

- Ohm's law is a linear equation.
- The controlled source $i_{cs}=Ki_x$ is also a linear equation.
- Superposition principle does not apply to circuits that have element(s) described by nonlinear equation(s).



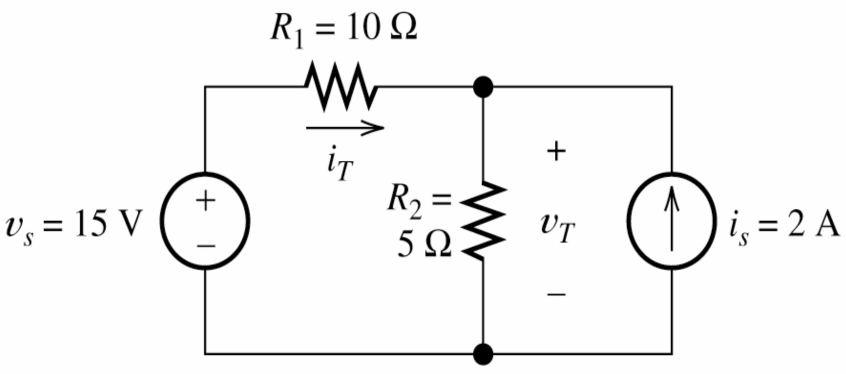
•Dependent source do not contribute a separate term to the total response. We must not zero dependent source in applying superposition。

•However, dependent source affect the contributions of the independent sources.

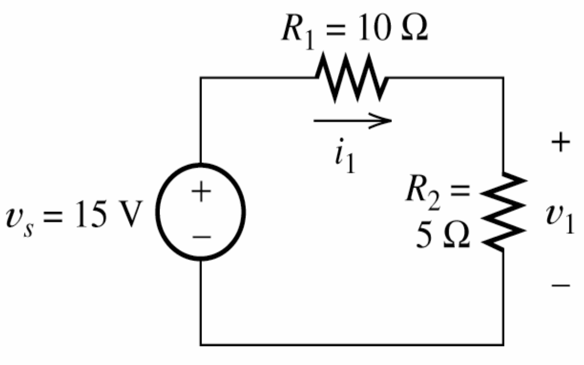
$$v_T = \frac{R_2}{R_1 + R_2 + KR_1} v_{s1} + \frac{R_1 R_2}{R_1 + R_2 + KR_1} i_{s2}$$

Example 2.22 Find v_T

$$V_1 = \frac{R_2}{R_1 + R_2} Vs = \frac{5}{5+10}(15) = 5\text{V}$$



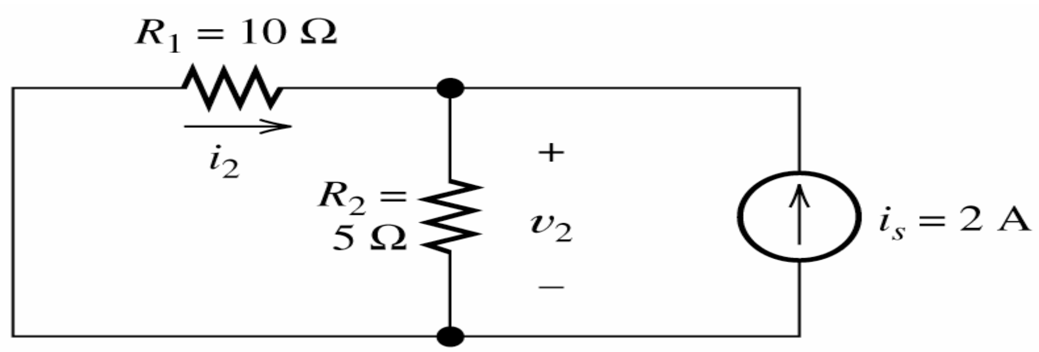
(a) Original circuit



(b) Circuit with only the voltage source active

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{1}{\frac{1}{10} + \frac{1}{5}} = 3.33\Omega$$

$$V_2 = i_s R_{eq} = 2 \times 3.33 = 6.66\text{V}$$

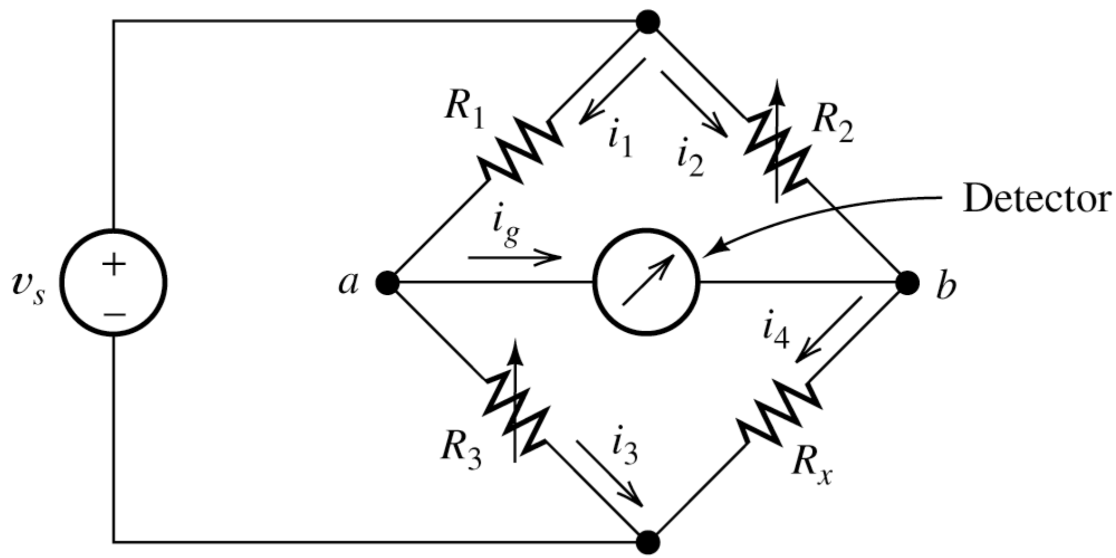


(c) Circuit with only the current source active

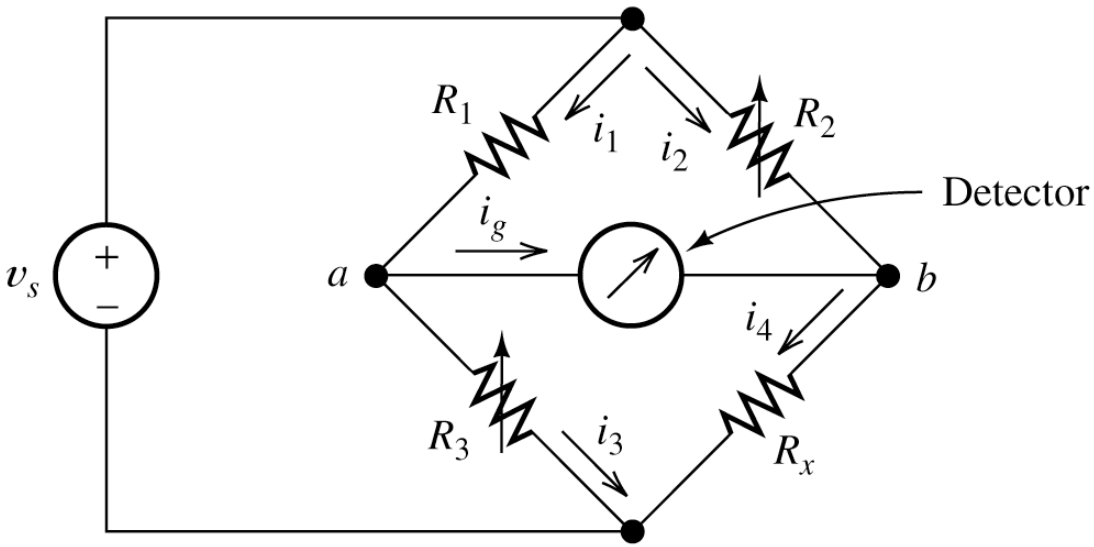
$$V_T = V_1 + V_2 = 5 + 6.66 = 11.66\ \text{V}$$

2.8 Wheatstone Bridge 惠斯登電橋測直流電阻

- 如同天平秤，以三個已知電阻量測一個未知電阻 R_x 。
- 平衡時， $i_g = 0$, v_{ab}=0.



WHEATSTONE BRIDGE



KCL node a

$$i_1 = i_3$$

KCL node b

$$i_2 = i_4$$

KVL upper loop

$$R_1 i_1 + v_{ab} = R_2 i_2$$

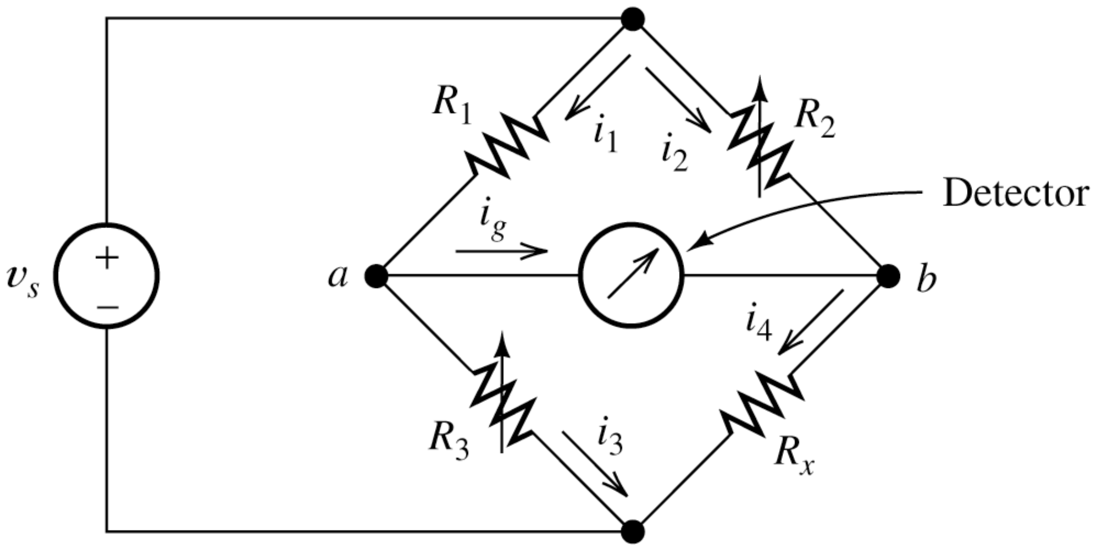
$$R_1 i_1 = R_2 i_2$$

KVL lower loop

$$R_3 i_3 = R_x i_4$$

$$R_3 i_1 = R_x i_2$$

WHEATSTONE BRIDGE

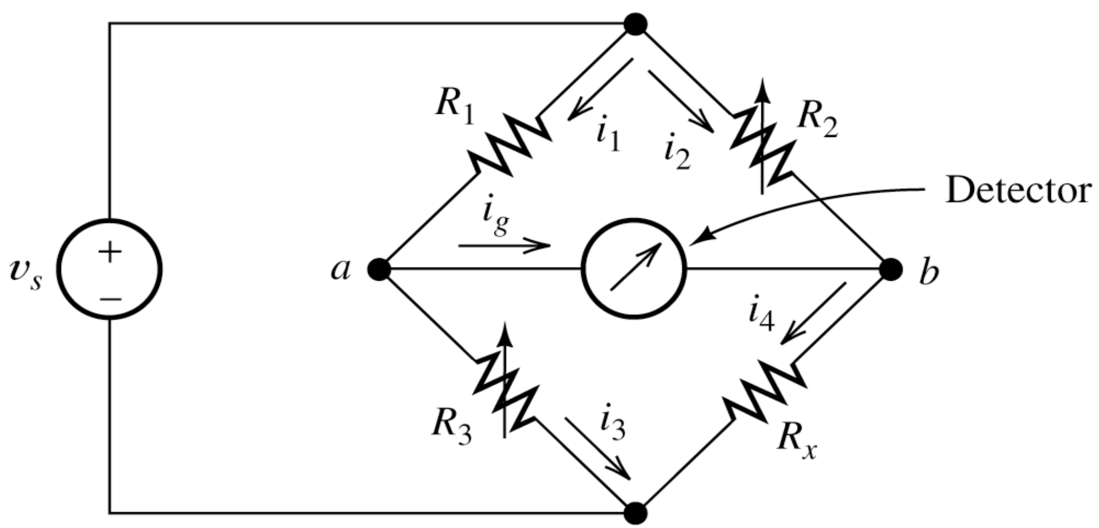


$$\frac{R_3}{R_1} = \frac{R_x}{R_2}$$

$$R_x = \frac{R_2}{R_1} R_3$$

Example 2.23

If R_1=1-kΩ,
R_3= 0~1100-Ω steps
by 1-Ω
R_2: 1k, 10k, 100k or 1MΩ



(a) What is the value of R_x such that the bridge is balanced with R_3=732 Ω, R_2=10k Ω?

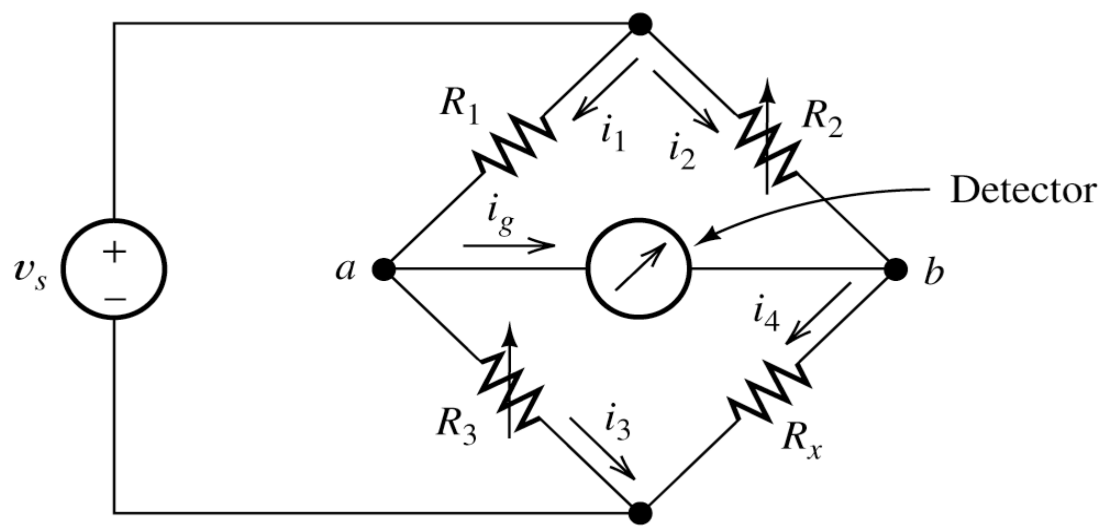
$$R_x = \frac{R_2}{R_1} R_3 = \frac{10k\Omega}{1k\Omega} \times 732\Omega = 7320\Omega$$

(b) What is the largest value of R_x for which bridge is balanced.

$$R_{x\max} = \frac{R_{2\max}}{R_1} R_{3\max} = \frac{1M\Omega}{1k\Omega} \times 1100\Omega = 1.1M\Omega$$

Example 2.22

If R_1=1-kΩ,
R_3= 0~1100-kΩsteps
by 1-Ω
R_2: 1k, 10k, 100k or 1MΩ



(c) Suppose R_2=1M Ω. What is the increment between values of R_x for which the bridge can be precisely balanced?

$$R_{xinc} = \frac{R_2}{R_1} R_{3inc} = \frac{1M\Omega}{1k\Omega} \times 1\Omega = 1k\Omega$$