# Estimation of nested and zero-inflated ordered probit models

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**Abstract**

We introduce three new STATA commands, `nop`, `ziop2` and `ziop3`, for the estimation of a three-part nested ordered probit model, the two-part zero-inflated ordered probit models of Harris and Zhao (2007, *Journal of Econometrics* 141: 1073–1099) and Brooks, Harris and Spencer (2012, *Economics Letters* 117: 683–686), and a three-part zero-inflated ordered probit model for ordinal outcomes, with both exogenous and endogenous switching. The three-part models allow the probabilities of positive, neutral (zero) and negative outcomes to be generated by distinct processes. The zero-inflated models address a preponderance of zeros and allow them to emerge in different latent regimes. We provide postestimation commands to compute probabilistic predictions and various measures of their accuracy, to access the goodness of fit, and to perform model comparison using the Vuong test (Vuong 1989, *Econometrica* 57: 307–333) with the corrections based on the Akaike and Schwarz information criteria. We investigate the finite-sample performance of the maximum likelihood estimators by Monte Carlo simulations, discuss the relations among the models, and illustrate the new commands with an empirical application to the U.S. federal funds rate target.

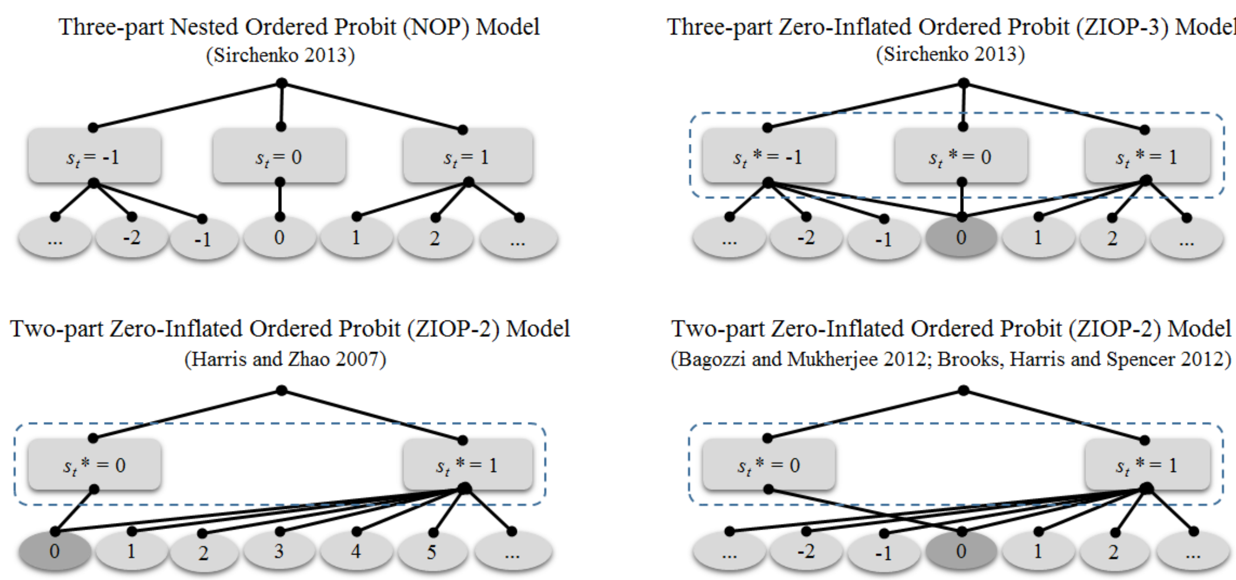
**Keywords:** ordinal outcomes, zero inflation, nested ordered probit, zero-inflated ordered probit, endogenous switching, Vuong test, nop, ziop2, ziop3, federal funds rate target.

# 1 Introduction

We introduce the STATA commands, `nop`, `ziop2` and `ziop3`, which estimate two-level nested and zero-inflated ordered probit (OP) models for ordinal outcomes, including the zero- and middle-inflated OP models of Harris and Zhao (2007), Bagozzi and Mukherjee (2012), Brooks, Harris and Spencer (2012) and Sirchenko (2013). The rationale behind the two-level nested decision process is standard in discrete-choice modeling when the set of alternatives faced by a decision-maker can be partitioned into subsets (or nests) with similar alternatives correlated due to common unobserved factors. The choice among the nests and the choice among the alternatives within each nest can be driven by different sets of observed and unobserved factors (and common factors can have different weights).

In unordered categorical data, in which choices can be grouped into the nests of similar options, the nested logit model is a popular method. Nested models for ordinal data are rare although the rationale behind them is similar: choosing among a negative response (decrease), a neutral response (no change) or a positive response (increase) is quite different from choosing the magnitude of a negative or positive response; and choosing the magnitude of a negative response can be driven by quite different determinants than choosing the magnitude of a positive response. This leads to three implicit decisions: an upper-level regime decision — a choice among the nests, and two lower-level outcome decisions — the choices of the magnitude of the negative and positive responses (see the top left panel of Figure 1).

Figure 1. Decision trees of nested and zero-inflated ordered probit models




Notes: Decisionmakers are not assumed to choose sequentially. The tree diagrams simply represent a nesting structure of the system of OP models.

Furthermore, it would be reasonable for the zero (no-change) alternative to be in three nests: its own, one with the negative responses, and one with the positive responses; hence, some zeros can be driven by similar factors as the negative or positive responses. This leads to a three-part cross-nested model with the nests overlapping at the zero response; hence, the probability of zeros is "inflated". Since the regime decision is not observable, the zeros are observationally equivalent — it is never known to which of the three nests the observed zero belongs. Several types of models with overlapping nests for unordered categorical responses have been developed (Vovsha 1997; Wen and Koppelman 2001); cross-nested models for ordinal outcomes are rare (Small 1987).

The prevalence of status quo, neutral or zero outcomes is observed in many fields, including economics, sociology, technometrics, psychology and biology. The heterogeneity of zeros is widely recognized — see Winkelmann (2008) and Greene and Hensher (2010) for a review. Studies identify different types of zeros such as: no visits to a doctor due to good health, iatrophobia, or medical costs; no illness due to strong immunity or lack of infection; no children due to infertility or choice. In the studies of survey responses using an odd-point Likert-type scale, where the respondents must indicate a negative, neutral or positive attitude or opinion, the heterogeneity of indifferent responses (a true neutral option versus an undecided, or ambivalent, or uninformed one, commonly reported as neutral) is also well-recognized and sometimes labeled as the middle category endorsement or inflation (Bagozzi and Mukhetjee 2012; Hernández, Drasgow and Gonzáles-Romá 2004; Kulas and Stachowski 2009).

Two-part zero-inflated models, developed to address the unobserved heterogeneity of zeros, combines a binary choice model for the probability of crossing the hurdle (to participate or not to participate; to consume or not to consume) with a count or ordered-choice model for non-negative outcomes above the hurdle: the two parts are estimated jointly, and zero observations can emerge in both parts. The two-part zero-inflated models include the zero-inflated Poisson (Lambert 1992), negative binomial (Greene 1994), binomial (Hall 2002) and generalized Poisson (Famoye and Singh 2003) models for count outcomes, and the zero-inflated OP model (Harris and Zhao 2007) and zero-inflated proportional odds model (Kelley and Anderson 2008) for non-negative ordinal responses.[1]

The model of Harris and Zhao (2007) is suitable for explaining decisions such as the levels of consumption, when the upper hurdle is naturally binary (to consume or not to consume), the responses are non-negative and the inflated zeros are situated at one end of the ordered scale (see the bottom left panel of Figure 1). Bagozzi and Mukherjee (2012) and Brooks, Harris and Spencer (2012) modified the model of Harris and Zhao (2007) and developed the middle-inflated OP model for an ordinal outcome, which ranges from negative to positive responses, and where an abundant outcome is situated in the middle of the choice spectrum (see the bottom right panel of Figure 1).

The three-part zero-inflated OP model (see the top right panel of Figure 1) introduced in Sirchenko (2013) is a natural generalization of the models of Harris and Zhao (2007), Bagozzi and Mukherjee (2012) and Brooks, Harris and Spencer (2012). A trichotomous regime decision is more realistic and flexible than a binary decision (change or no change) if applied to ordinal data with negative, zero and positive values.

[1] The zero-inflated models, estimation of which is currently implemented in STATA, include: the zero-inflated Poisson model (the zip command), the negative binomial model (the zinb command), and the binomial model (the zib command) and the beta-binomial model (the zibbin command) developed by Hardin and Hilbe (2014).

# 2 Models

## 2.1 Notation and assumptions

The observed dependent variable $y_t$, $t = 1, 2, ..., T$ is assumed to take on a finite number of ordinal values $j$ coded as $\{-J^-, ..., -1, 0, 1, ..., J^+\}$, where a potentially heterogeneous (and typically predominant) response is coded as zero. The latent unobserved (or only partially observed) variables are denoted by "$*$". Each model assumes an ordered-choice regime decision and ordered-choice outcome decisions conditional on the regime. The regime decision can be correlated with each outcome decision. We denote: by $\mathbf{x}_t$, $\mathbf{x}_t^-$, $\mathbf{x}_t^+$ and $\mathbf{z}_t$ the $t^{\text{th}}$ rows of the observed data matrices (which in addition to the predetermined explanatory variables may also include the lags of $y_t$); by $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$ and $\boldsymbol{\gamma}$ the vectors of slope parameters; by $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$ and $\boldsymbol{\mu}$ the vectors of threshold parameters; by $\rho$, $\rho^-$ and $\rho^+$ the vectors of correlation coefficients; by $\varepsilon_t$, $\varepsilon_t^-$, $\varepsilon_t^+$ and $\nu_t$ the error terms that are independently and identically distributed ($iid$) across $t$ with normal cumulative distribution function (CDF) $\Phi$, the zero means and the variances $\sigma^2$, $\sigma_-^2$, $\sigma_+^2$ and $\sigma_\nu^2$, respectively; and by $\Phi_2(g_1;g_2;\sigma_1^2;\sigma_2^2;\rho)$ the CDF of the bivariate normal distribution of the two random variables $g_1$ and $g_2$ with the zero means, the variances $\sigma_1^2$ and $\sigma_2^2$ and the correlation coefficient $\rho$:

$$\Phi_2(g_1;g_2;\sigma_1^2;\sigma_2^2;\rho) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_{-\infty}^{g_1}\int_{-\infty}^{g_2} \exp\left(-\frac{u^2/\sigma_1^2 - 2\rho uw/\sigma_1\sigma_2 + w^2/\sigma_2^2}{2(1-\rho^2)}\right) du dw.$$

## 2.2 Three-part nested ordered probit (NOP) model

Despite the wide-spread use of nested logit models for unordered categorical responses, we are aware of only one example of the nested ordered probit model in the literature (Sirchenko 2013). The two-level NOP model can be described as

$$\text{Upper-level decision:} \quad r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t = \begin{cases} 1 & \text{if} \quad \mu_2 < r_t^*, \\ 0 & \text{if} \quad \mu_1 < r_t^* \le \mu_2, \\ -1 & \text{if} \quad r_t^* \le \mu_1. \end{cases}$$

$$\text{Lower-level decisions:} \quad y_t^{-*} = \mathbf{x}_t^-\boldsymbol{\beta}^- + \varepsilon_t^-, \quad y_t^{+*} = \mathbf{x}_t^+\boldsymbol{\beta}^+ + \varepsilon_t^+,$$

$$y_t = \begin{cases} j(j>0) & \text{if} \quad s_t = 1 \text{ and } \alpha_{j-1}^+ < y_t^{+*} \le \alpha_j^+, \\ 0 & \text{if} \quad s_t = 0, \\ j(j<0) & \text{if} \quad s_t = -1 \quad \text{and } \alpha_j^- < y_t^{-*} \le \alpha_{j+1}^-, \end{cases}$$

where $-\infty = \alpha_0^+ \le \alpha_1^+ \le ... \le \alpha_{J^+}^+ = \infty$
and $-\infty = \alpha_{-J^-}^- \le \alpha_{-J^-+1}^- \le ... \le \alpha_0^- = \infty$.

$$\text{Correlation among decisions:} \quad \begin{bmatrix} \nu_t \\ \varepsilon_t^i \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho^i\sigma_\nu\sigma_i \\ \rho^i\sigma_\nu\sigma_i & \sigma_i^2 \end{bmatrix}\right), \ i \in \{-,+\}.$$

The probabilities of the outcome $j$ in the NOP model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+) = I_{j<0}\Pr(r_t^* \le \mu_1 \text{ and } \alpha_j^- < y_t^{-*} \le \alpha_{j+1}^-|\mathbf{z}_t, \mathbf{x}_t^-) \\
&+I_{j=0}\Pr(\mu_1 < r_t^* \le \mu_2|\mathbf{z}_t) + I_{j>0}\Pr(\mu_2 < r_t^* \text{ and } \alpha_{j-1}^+ < y_t^{+*} \le \alpha_j^+|\mathbf{z}_t, \mathbf{x}_t^+) \\
&= I_{j<0}\Pr(\nu_t \le \mu_1 - \mathbf{z}_t\boldsymbol{\gamma} \text{ and } \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^- < \varepsilon_t^- \le \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-) \\
&+I_{j=0}\Pr(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \le \mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) \\
&+I_{j>0}\Pr(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+ < \varepsilon_t^+ \le \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+) \\
&= I_{j<0}[\Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-) - \Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)] \\
&+I_{j>0}[\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_\nu^2; \sigma_+^2; -\rho^+) \\
&-\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_\nu^2; \sigma_+^2; -\rho^+)],
\end{aligned} \tag{1}$$

where $I_{j<0}$ is an indicator function such that $I_{j<0} = 1$ if $j < 0$, and $I_{j<0} = 0$ if $j \ge 0$ (analogously for $I_{j=0}$ and $I_{j>0}$).

In the case of exogenous switching (when $\rho^- = \rho^+ = 0$), the probabilities of the outcome $j$ in the NOP can be computed as

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+, \rho^- = \rho^+ = 0) \\
&= I_{j<0}\Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)[\Phi(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-; \sigma_-^2) - \Phi(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-; \sigma_-^2)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma})] \\
&+I_{j>0}[1 - \Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_+^2) - \Phi(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_+^2)].
\end{aligned}$$

In the case of two or three outcome choices the NOP model degenerates to the conventional single-equation OP model.

## 2.3 Two-part zero-inflated ordered probit (ZIOP-2) model

The ZIOP-2 model, which represents the zero-inflated OP model of Brooks, Harris and Spencer (2012) and the middle-inflated OP model of Bagozzi and Mukherjee (2012), can be described by the following system

Regime decision: $\quad r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t^* = \begin{cases} 1 & \text{if} \quad \mu < r_t^*, \\ 0 & \text{if} \quad r_t^* \le \mu. \end{cases}$

Outcome decision: $\quad y_t^* = \mathbf{x}_t\boldsymbol{\beta} + \varepsilon_t,$

$$y_t = \begin{cases} j & \text{if} \quad s_t^* = 1 \text{ and } \alpha_{j-1} < y_t^* \le \alpha_j, \\ 0 & \text{if} \quad s_t^* = 0, \end{cases}$$

where $-\infty = \alpha_{-J^- -1} \le \alpha_{-J^-} \le ... \le \alpha_{J^+} = \infty$.

Correlation among decisions: $\quad \begin{bmatrix} \nu_t \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho\sigma_\nu\sigma \\ \rho\sigma_\nu\sigma & \sigma^2 \end{bmatrix} \right).$

The probabilities of the outcome $j$ in the ZIOP-2 model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t) = I_{j=0}\Pr(r_t^* \le \mu|\mathbf{z}_t) + \Pr(\mu < r_t^* \text{ and } \alpha_{j-1} < y_t^* \le \alpha_j|\mathbf{z}_t, \mathbf{x}_t) \\
&= I_{j=0}\Pr(\nu_t \le \mu - \mathbf{z}_t\boldsymbol{\gamma}) + \Pr(\mu - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta} < \varepsilon_t \le \alpha_j - \mathbf{x}_t\boldsymbol{\beta}) \\
&= I_{j=0}\Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) + \Phi_2(-\mu + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j - \mathbf{x}_t\boldsymbol{\beta}; \sigma_\nu^2; \sigma^2; -\rho) \\
&-\Phi_2(-\mu + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta}; \sigma_\nu^2; \sigma^2; -\rho).
\end{aligned} \tag{2}$$

In the case of exogenous switching (when $\rho = 0$), these probabilities can be computed as

$$\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t, \rho = 0) = I_{j=0}\Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)$$
$$+[1 - \Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j - \mathbf{x}_t\boldsymbol{\beta};\sigma^2) - \Phi(\alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta};\sigma^2)].$$

If $y_t \geq 0$ for $\forall t$, the ZIOP-2 model becomes the model of Harris and Zhao (2007).

## 2.4 Three-part zero-inflated ordered probit (ZIOP-3) model

The ZIOP-3 model developed by Sirchenko (2013) is a three-part generalization of the ZIOP-2 model, and can be described by the following system

Regime decision: $$r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t^* = \begin{cases} 1 & \text{if} \quad \mu_2 < r_t^*, \\ 0 & \text{if} \quad \mu_1 < r_t^* \leq \mu_2, \\ -1 & \text{if} \quad r_t^* \leq \mu_1. \end{cases}$$

Outcome decisions: $$y_t^{-*} = \mathbf{x}_t^-\boldsymbol{\beta}^- + \varepsilon_t^-, \quad y_t^{+*} = \mathbf{x}_t^+\boldsymbol{\beta}^+ + \varepsilon_t^+,$$
$$y_t = \begin{cases} j(j \geq 0) & \text{if} \quad s_t^* = 1 \quad \text{and } \alpha_{j-1}^+ < y_t^{+*} \leq \alpha_j^+, \\ 0 & \text{if} \quad s_t^* = 0, \\ j(j \leq 0) & \text{if} \quad s_t^* = -1 \text{ and } \alpha_j^- < y_t^{-*} \leq \alpha_{j+1}^-, \end{cases}$$
$$\text{where } -\infty = \alpha_{-1}^+ \leq \alpha_0^+ \leq ... \leq \alpha_{J^+}^+ = \infty$$
$$\text{and } -\infty = \alpha_{-J^-}^- \leq \alpha_{-J^-+1}^- \leq ... \leq \alpha_1^- = \infty.$$

Correlation among decisions: $$\begin{bmatrix} \nu_t \\ \varepsilon_t^i \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left( \begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho^i\sigma_\nu\sigma_i \\ \rho^i\sigma_\nu\sigma_i & \sigma_i^2 \end{bmatrix} \right), \ i \in \{-,+\}.$$

The probabilities of the outcome $j$ in the ZIOP-3 model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+) = I_{j\leq 0}\Pr(r_t^* \leq \mu_1 \text{ and } \alpha_j^- < y_t^{-*} \leq \alpha_{j+1}^-|\mathbf{z}_t, \mathbf{x}_t^-) \\
&+I_{j=0}\Pr(\mu_1 < r_t^* \leq \mu_2|\mathbf{z}_t) + I_{j\geq 0}\Pr(\mu_2 < r_t^* \text{ and } \alpha_{j-1}^+ < y_t^{+*} \leq \alpha_j^+|\mathbf{z}_t, \mathbf{x}_t^+) \\
&= I_{j\leq 0}\Pr(\nu_t \leq \mu_1 - \mathbf{z}_t\boldsymbol{\gamma} \text{ and } \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^- < \varepsilon_t^- \leq \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-) \\
&+I_{j=0}\Pr(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \leq \mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) \\
&+I_{j\geq 0}\Pr(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+ < \varepsilon_t^+ \leq \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+) \\
&= I_{j\leq 0}[\Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-) - \Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)] \\
&+I_{j\geq 0}[\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_\nu^2;\sigma_+^2; -\rho^+) \\
&-\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_\nu^2;\sigma_+^2; -\rho^+)],
\end{aligned} \tag{3}$$

where $I_{j\leq 0}$ is an indicator function such that $I_{j\leq 0} = 1$ if $j \leq 0$, and $I_{j\leq 0} = 0$ if $j > 0$ (analogously for $I_{j\geq 0}$).

In the case of exogenous switching (when $\rho^- = \rho^+ = 0$), these probabilities can be computed as

$$\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+, \rho^- = \rho^+ = 0) = I_{j\leq 0}\Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\boldsymbol{\sigma_\nu^2})[\Phi(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_-^2)$$
$$-\Phi(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_-^2)] + I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)]$$
$$+I_{j\geq 0}[1 - \Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_+^2) - \Phi(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_+^2)].$$

The inflated outcome does not have to be in the *very* middle of the ordered choices. If it is located at the *end* of the ordered scale, i.e. if $y_t \geq 0$ for $\forall t$, the ZIOP-3 model reduces to the ZIOP-2 model of Harris and Zhao (2007).

## 2.5 Maximum likelihood (ML) estimation

The probabilities in each OP equation can be consistently estimated under fairly general conditions by an asymptotically normal ML estimator (Basu and de Jong 2007). The simultaneous estimation of the OP equations in the NOP, ZIOP-2 and ZIOP-3 models can be also performed using an ML estimator of the vector of the parameters $\boldsymbol{\theta}$ that solves

$$\max_{\boldsymbol{\theta}\epsilon\boldsymbol{\Theta}} \sum_{t=1}^{T} \sum_{j=-J^-}^{J^+} I_{tj} \ln[\Pr(y_t = j|\mathbf{x}_t^{all}, \boldsymbol{\theta})], \tag{4}$$

where $I_{tj}$ is an indicator function such that $I_{tj} = 1$ if $y_t = j$ and $I_{tj} = 0$ otherwise; $\boldsymbol{\theta}$ includes $\boldsymbol{\gamma}$, $\boldsymbol{\mu}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$, $\rho^-$ and $\rho^+$ for the NOP and ZIOP-3 models, and $\boldsymbol{\gamma}$, $\mu$, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and $\rho$ for the ZIOP-2 model; $\Theta$ is a parameter space; $\mathbf{x}_t^{all}$ is a vector that contains the values of all independent variables in the model; and $\Pr(y_t = j|\mathbf{x}_t^{all}, \boldsymbol{\theta})$ are the probabilities from either (1) or (2) or (3). The asymptotic standard errors of $\widehat{\boldsymbol{\theta}}$ can be computed from the Hessian matrix.

The intercept components of $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$ and $\boldsymbol{\gamma}$ are identified up to scale and location, that is, only jointly with the corresponding threshold parameters $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$ and $\boldsymbol{\mu}$ and variances $\sigma^2$, $\sigma_-^2$, $\sigma_+^2$, and $\sigma_\nu^2$. As is common in the identification of discrete-choice models, the variances $\sigma^2$, $\sigma_-^2$, $\sigma_+^2$, and $\sigma_\nu^2$ are fixed to one, and the intercept components of $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$ and $\boldsymbol{\gamma}$ are fixed to zero. The probabilities in (1), (2) and (3) are invariant to these (arbitrary) identifying assumptions: up to scale and location, we can identify all parameters in $\boldsymbol{\theta}$ because of the non-linearity of OP equations, i.e. via the functional form (Heckman 1978; Wilde 2000). However, since the normal CDF is approximately linear in the middle of its support, the simultaneous estimation of two or three equations may experience a weak identification problem if the regime and outcome equations contain the same set of independent variables. To enhance the precision of parameter estimates we may impose exclusion restrictions on the specification of the independent variables in each equation.

The three regimes (nests) in the NOP model are fully observable, contrary to the latent (only partially observed) regimes in the ZIOP-2 and ZIOP-3 models. The likelihood function of the NOP model — again in contrast with the ZIOP-2 and ZIOP-3 models — is separable with respect to the parameters in the three equations. Thus, solving (4) for the NOP model is equivalent to maximizing separately the likelihoods of the three OP models representing the upper- and lower-level decisions.[2]

## 2.6 Marginal effects (ME)

The marginal effects of a continuous independent variable $k$ (the $k^{\text{th}}$ element of $\mathbf{x}_t^{all}$) on the probability of each discrete outcome $j$ are computed for the ZIOP-3 model as

[2] The data matrices in the lower-level decisions should be truncated to contain only those rows of $\mathbf{x}_t^-$ or $\mathbf{x}_t^+$ for which $y_t < 0$ or $y_t > 0$, respectively.

$$
\begin{aligned}
\mathrm{ME}_{k,j,t} &= \tfrac{\partial \Pr(y_t=j|\boldsymbol{\theta})}{\partial \mathbf{x}_{t,k}^{all}} = I_{j\le 0}\Bigg\{\Bigg[\Phi\left(\tfrac{\mu_1-\mathbf{z}_t\boldsymbol{\gamma}-\rho^-(\alpha_j^- -\mathbf{x}_t^-\boldsymbol{\beta}^-)}{\sqrt{1-(\rho^-)^2}}\right) f(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-) \\
&-\Phi\left(\tfrac{\mu_1-\mathbf{z}_t\boldsymbol{\gamma}-\rho^-(\alpha_{j+1}^- -\mathbf{x}_t^-\boldsymbol{\beta}^-)}{\sqrt{1-(\rho^-)^2}}\right) f(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-)\Bigg]\boldsymbol{\beta}_k^{-all} \\
&-\Bigg[\Phi\left(\tfrac{\alpha_{j+1}^- -\mathbf{x}_t^-\boldsymbol{\beta}^- -\rho^-(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})}{\sqrt{1-(\rho^-)^2}}\right) - \Phi\left(\tfrac{\alpha_j^- -\mathbf{x}_t^-\boldsymbol{\beta}^- -\rho^-(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})}{\sqrt{1-(\rho^-)^2}}\right)\Bigg] f(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})\boldsymbol{\gamma}_k^{all}\Bigg\} \\
&-I_{j=0}[f(\mu_2-\mathbf{z}_t\boldsymbol{\gamma})-f(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})]\boldsymbol{\gamma}_k^{all} \\
&+I_{j\ge 0}\Bigg\{\Bigg[\Phi\left(\tfrac{\mathbf{z}_t\boldsymbol{\gamma}-\mu_2+\rho^+(\alpha_{j-1}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+)}{\sqrt{1-(\rho^+)^2}}\right) f(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+) \\
&-\Phi\left(\tfrac{\mathbf{z}_t\boldsymbol{\gamma}-\mu_2+\rho^+(\alpha_j^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+)}{\sqrt{1-(\rho^+)^2}}\right) f(\alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+)\Bigg]\boldsymbol{\beta}_k^{+all} \\
&+\Bigg[\Phi\left(\tfrac{\alpha_j^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+ +\rho^+(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)}{\sqrt{1-(\rho^+)^2}}\right) - \Phi\left(\tfrac{\alpha_{j-1}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+ +\rho^+(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)}{\sqrt{1-(\rho^+)^2}}\right)\Bigg] f(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)\boldsymbol{\gamma}_k^{all}\Bigg\},
\end{aligned}
$$

where $f$ is the probability density function of the standard normal distribution, and $\boldsymbol{\gamma}_k^{all}$, $\boldsymbol{\beta}_k^{-all}$ and $\boldsymbol{\beta}_k^{+all}$ are the coefficients on the $k^{\text{th}}$ independent variable in $\mathbf{x}_t^{all}$ in the regime equation, the outcome equation conditional on $s_t^* = 1$ and the outcome equation conditional on $s_t^* = -1$, respectively ($\boldsymbol{\gamma}_k^{all}$, $\boldsymbol{\beta}_k^{-all}$ or $\boldsymbol{\beta}_k^{+all}$ is zero if the $k^{\text{th}}$ independent variable in $\mathbf{x}_t^{all}$ is not included into the corresponding equation). For a discrete-valued independent variable, the ME can be computed as the change in the probabilities when this independent variable changes by one increment and all other independent variables are fixed.

The MEs for the NOP model are computed by replacing $I_{j\ge 0}$ in the above formula with $I_{j>0}$ and $I_{j\le 0}$ with $I_{j<0}$.

The MEs for the ZIOP-2 model are computed as

$$
\begin{aligned}
\mathrm{ME}_{k,j,t} &= \tfrac{\partial \Pr(y_t=j|\boldsymbol{\theta})}{\partial \mathbf{x}_{t,k}^{all}} = -I_{j=0}[f(\mu-\mathbf{z}_t\boldsymbol{\gamma})]\boldsymbol{\gamma}_k^{all} \\
&+\Bigg[\Phi\left(\tfrac{\mathbf{z}_t\boldsymbol{\gamma}-\mu+\rho(\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta})}{\sqrt{1-\rho^2}}\right) f(\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta}) - \Phi\left(\tfrac{\mathbf{z}_t\boldsymbol{\gamma}-\mu+\rho(\alpha_j-\mathbf{x}_t\boldsymbol{\beta})}{\sqrt{1-\rho^2}}\right) f(\alpha_j-\mathbf{x}_t\boldsymbol{\beta})\Bigg]\boldsymbol{\beta}_k^{all} \\
&+\Bigg[\Phi\left(\tfrac{\alpha_j-\mathbf{x}_t\boldsymbol{\beta}+\rho(\mathbf{z}_t\boldsymbol{\gamma}-\mu)}{\sqrt{1-\rho^2}}\right) - \Phi\left(\tfrac{\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta}+\rho(\mathbf{z}_t\boldsymbol{\gamma}-\mu)}{\sqrt{1-\rho^2}}\right)\Bigg] f(\mathbf{z}_t\boldsymbol{\gamma}-\mu)\boldsymbol{\gamma}_k^{all},
\end{aligned}
$$

where $\boldsymbol{\beta}_k^{all}$ is the coefficient on the $k^{\text{th}}$ independent variable in $\mathbf{x}_t^{all}$ in the outcome equation ($\boldsymbol{\beta}_k^{all}$ is zero if the $k^{\text{th}}$ independent variable in $\mathbf{x}_t^{all}$ is not included into the outcome equation).

The asymptotic standard errors of the MEs are computed using the Delta method as the square roots of the diagonal elements of

$$
\widehat{Var}(\widehat{\mathbf{ME}}_{k,j,t}) = \nabla_\theta \widehat{\mathbf{ME}}_{k,j,t}\widehat{Var}(\widehat{\boldsymbol{\theta}})\nabla_\theta \widehat{\mathbf{ME}}_{k,j,t}'.
$$

## 2.7 Relations among the models and their comparison

We now discuss the choice of a formal statistical test to compare the NOP, ZIOP-2, ZIOP-3 and conventional OP models. The choice depends on whether the models are nested in each other.

The exogenous-switching version of each model is nested in its endogenous-switching version as its uncorrelated special case; their comparison can be performed using any classical likelihood-based test for nested hypotheses, such as the likelihood ratio (LR) test.

The OP is not nested either in the NOP or ZIOP-3 model. We can compare the OP model with them using a likelihood-based test for non-nested models, such as the Vuong test (Vuong 1989). The OP model is however nested in the ZIOP-2 model. The latter reduces to the former if $\mu \to -\infty$; hence, $\Pr(y_t = 0|\mathbf{x}_t, s_t^* = 1) \to 0$. Therefore, the Vuong test for non-nested hypothesis cannot be used to compare the OP and ZIOP-2 model: for nested hypothesis, the Voung test reduces to the LR test. However, the critical values of the classical LR test are invalid in this case since some of the standard regularity conditions of the classical LR test fail to hold (Andrews 2001; Andrews and Cheng 2012). In particular, the value of $\mu$ in the null hypothesis is not an interior point of the parameter space; hence, the asymptotic distribution of the LR statistics is not standard.[3]

The NOP model is nested in the ZIOP-3 model. The latter becomes the former if $\alpha_{-1}^- \to \infty$ and $\alpha_1^+ \to -\infty$; therefore, $\Pr(y_t = 0|\mathbf{x}_t^+, s_t^* = 1) \to 0$ and $\Pr(y_t = 0|\mathbf{x}_t^-, s_t^* = -1) \to 0$. The values of $\alpha_{-1}^-$ and $\alpha_1^+$ in the null hypothesis are not the interior points of the parameter space; thus, the asymptotic distribution of the LR statistics is not standard. The comparison of the NOP and ZIOP-3 models can also be performed using the LR test with simulated adjusted critical values (Andrews 2001; Andrews and Cheng 2012).

Generally, the ZIOP-2 model is not a special case of the ZIOP-3 model, and vice versa. We can compare them using the Vuong test. A special case when the ZIOP-3 model nests the ZIOP-2 model emerges under certain restrictions on the parameters as explained below. In this case, the selection between the ZIOP-3 and ZIOP-2 models can be performed using any classical likelihood-based test for nested hypotheses such as the LR test.

The special case emerges if $y_t$ takes on only three discrete values $j \in \{-1, 0, 1\}$, the regressors in $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ in the outcome equations of the ZIOP-3 model contain all the regressors in the ZIOP-2 regime equation (denoted below by $\mathbf{z}_{2t}$ with the parameter vector $\boldsymbol{\gamma}_2$), and the regressors in the regime equation of the ZIOP-3 model (denoted below by $\mathbf{z}_{3t}$ with the parameter vector $\boldsymbol{\gamma}_3$) include all the regressors in the $\mathbf{x}_t$ in the ZIOP-2 outcome equation. According to (2) the probabilities of the outcome $j$ in the ZIOP-2 model are given by

$$\begin{aligned}
&\Pr(y_t = -1|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho);\\
&\Pr(y_t = 0|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi(\mu - \mathbf{z}_{2t}\boldsymbol{\gamma}_2) + \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_0 - \mathbf{x}_t\boldsymbol{\beta}; -\rho)\\
&-\Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho) = 1 - \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; -\alpha_0 + \mathbf{x}_t\boldsymbol{\beta}; \rho)\\
&-\Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho);\\
&\Pr(y_t = 1|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}) - \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_0 - \mathbf{x}_t\boldsymbol{\beta}; -\rho)\\
&= \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; -\alpha_0 + \mathbf{x}_t\boldsymbol{\beta}; \rho),
\end{aligned} \tag{5}$$

since $\Phi_2(x; y; \rho) = \Phi(x) - \Phi_2(x; -y; -\rho)$.

Similarly, according to (3) the probabilities of the outcome $j$ in the ZIOP-3 model are given by

[3] Analogously, the use of the Vuong test for non-nested hypotheses to test for zero inflation in a Poisson or negative binomial model with a binary regime equation is inappropriate too, because these models are actually nested in their two-part zero-inflated extensions (Wilson 2015).

$$\Pr(y_t=-1|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^- -\mathbf{x}_t^-\boldsymbol{\beta}^-;\rho^-);$$

$$\begin{aligned}&\Pr(y_t=0|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^- -\mathbf{x}_t^-\boldsymbol{\beta}^-;\rho^-)\\&+\Phi(\mu_2-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)+\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+-\mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+)\\&=\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;-\alpha_0^-+\mathbf{x}_t^-\boldsymbol{\beta}^-;-\rho^-)+\Phi(\mu_2-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)\\&-\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)+\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+-\mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+);\end{aligned}$$

$$\begin{aligned}&\Pr(y_t=1|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+-\mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+)\\&=\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;-\alpha_0^++\mathbf{x}_t^+\boldsymbol{\beta}^+;\rho^+).\end{aligned}$$

Suppose the regressors in $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ in the ZIOP-3 outcome equations are identical to the regressors in $\mathbf{z}_{2t}$ in the ZIOP-2 regime equation, the regressors in $\mathbf{z}_{3t}$ in the ZIOP-3 regime equation are identical to the regressors in the $\mathbf{x}_t$ in the ZIOP-2 outcome equation, and the parameters are restricted as follows: $-\boldsymbol{\beta}^-=\boldsymbol{\beta}^+=\boldsymbol{\gamma}_2$, $\boldsymbol{\beta}=\boldsymbol{\gamma}_3$, $\mu_1=\alpha_{-1}$, $\mu_2=\alpha_0$, $-\alpha_0^-=\alpha_0^+=\mu$ and $-\rho^-=\rho^+=\rho$. Then, since $\mathbf{x}_t^-=\mathbf{x}_t^+=\mathbf{z}_{2t}$, $\mathbf{z}_{3t}=\mathbf{x}_t$ and $\Phi(-x)=1-\Phi(x)$, the probabilities for the ZIOP-3 model can be written as

$$\Pr(y_t=-1|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho);$$

$$\begin{aligned}&\Pr(y_t=0|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};\mu-\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho)+\Phi(\alpha_0-\mathbf{x}_t\boldsymbol{\beta})-\Phi(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta})\\&+\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};\mu-\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho)=-\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho)+1\\&-\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho);\end{aligned}$$

$$\Pr(y_t=1|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho),$$

which are identical to the probabilities for the ZIOP-2 model in (5).

Notice that the restrictions $-\boldsymbol{\beta}^-=\boldsymbol{\beta}^+=\boldsymbol{\gamma}_2$ and $-\alpha_0^-=\alpha_0^+=\mu$ impose a sort of symmetry in the ZIOP-3 model, because they imply that the conditional probability of a positive response is equal to the conditional probability of a negative response:

$$\begin{aligned}&\Pr(y_t=1|\mathbf{z}_{3t},\mathbf{x}_t^+,s_t^*=1)=1-\Phi(\alpha_0^+-\mathbf{x}_t^+\boldsymbol{\beta}^+)=\\&=\Phi(-\alpha_0^++\mathbf{x}_t^+\boldsymbol{\beta}^+)=\Phi(\alpha_0^- -\mathbf{x}_t^-\boldsymbol{\beta}^-)=\Pr(y_t=-1|\mathbf{z}_t,\mathbf{x}_t^-,s_t^*=-1).\end{aligned}$$

In general, if $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ are not identical to $\mathbf{z}_{2t}$ but contain all the regressors in $\mathbf{z}_{2t}$, and if $\mathbf{z}_{3t}$ is not identical to $\mathbf{x}_t$ but contains all the regressors in $\mathbf{x}_t$, the ZIOP-2 model is still nested in the ZIOP-3 model with the additional zero restrictions for the coefficients on all the extra regressors in $\mathbf{x}_t^-$, $\mathbf{x}_t^+$ and $\mathbf{z}_{3t}$.

## 3 The nop, ziop2 and ziop3 commands in Stata

The accompanying software includes the three new commands, the postestimation commands and the supporting help files.

## 3.1 Syntax

The following commands estimate, respectively, the NOP, ZIOP-2 and ZIOP-3 models for discrete ordinal outcomes:

`nop` *depvar* [*indepvars*] [*if*] [*in*] [, `posindepvars(`*varlist*`)` `negindepvars(`*varlist*`)` `infcat(`*choice*`)` `endoswitch` `robust` `cluster(`*varname*`)` `nolog` `initial(`*string*`)vuong`]

`ziop2` *depvar* [*indepvars*] [*if*] [*in*] [, `outindepvars(`*varlist*`)` `infcat(`*choice*`)` `endoswitch` `robust` `cluster(`*varname*`)` `nolog` `initial(`*string*`)`]

`ziop3` *depvar* [*indepvars*] [*if*] [*in*] [, `posindepvars(`*varlist*`)` `negindepvars(`*varlist*`)` `infcat(`*choice*`)` `endoswitch` `robust` `cluster(`*varname*`)` `nolog` `initial(`*string*`)` `vuong`]

An ordinal dependent variable *depvar* is assumed to take on at least five discrete ordinal values in the NOP model, at least two in the ZIOP-2 model, and at least three in the ZIOP-3 model. A list of the independent variables in the regime equation *indepvars* may be different from the lists of the independent variables in the outcome equations.

### Options

`posindepvars(`*varlist*`)` specifies a list of the independent variables in the outcome equation, conditional on the regime $s_t^* = 1$ for non-negative outcomes in the NOP and ZIOP-3 models; by default, it is identical to *indepvars*, the list of the independent variables in the regime equation.

`negindepvars(`*varlist*`)` specifies a list of the independent variables in the outcome equation, conditional on the regime $s_t^* = -1$ for non-positive outcomes in the NOP and ZIOP-3 models; by default, it is identical to *indepvars*, the list of the independent variables in the regime equation.

`outindepvars(`*varlist*`)` specifies a list of the independent variables in the outcome equation of the ZIOP-2 model; by default, it is identical to *indepvars*, the list of the independent variables in the regime equation.

`infcat(`*choice*`)` is the value of the dependent variable in the regime $s_t^* = 0$ that should be modeled as inflated in the ZIOP-2 and ZIOP-3 models, and as neutral in the NOP model; by default, *choice* equals 0.

`endoswitch` specifies that endogenous regime switching is to be used instead of default exogenous switching. Regime switching is endogenous if the unobserved random term in the regime equation is correlated with the unobserved random terms in the outcome equations, and exogenous otherwise.

`robust` specifies that a robust sandwich estimator of variance is to be used; the default estimator is based on the observed information matrix.

`cluster(`*varname*`)` specifies a clustering variable for the clustered robust sandwich estimator of variance.

`initial(`*string*`)` specifies a space-delimited list *string* of the starting values of the parameters in the following order: $\boldsymbol{\gamma}$, $\boldsymbol{\mu}$, $\boldsymbol{\beta}^{+}$, $\boldsymbol{\alpha}^{+}$, $\boldsymbol{\beta}^{-}$, $\boldsymbol{\alpha}^{-}$, $\rho^{-}$ and $\rho^{+}$ for the NOP and ZIOP-3 models, and $\boldsymbol{\gamma}$, $\mu$, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and $\rho$ for the ZIOP-2 model.

`vuong` specifies that the Vuong test of the NOP (or ZIOP-3) model versus the conventional OP model should be performed. The reported Vuong test statistics (the standard one and the two adjusted test statistics with corrections to address the comparison of models with different numbers of parameters based on the Akaike (AIC) and Bayesian (BIC) information criteria) have a standard normal distribution with large positive values favoring the NOP (or ZIOP-3) model and large negative values favoring the OP model.

`nolog` suppresses the iteration log and preliminary results.

### Stored results

The descriptions of the stored results can be found in the help files.

## 3.2 Postestimation commands

The following postestimation commands are available after `nop`, `ziop2` and `ziop3`:

### The predict command

`predict` *newvar* [*if*] [*in*] [, `zeros regimes output(`*string*`)`]

This command computes the predicted probabilities of the discrete choices (by default), the regimes and the types of zeros conditional on the regime, and the predicted outcomes and the expected values of the dependent variable for all observed values of the independent variables in the sample. The command creates $(J^- + J^+ + 1)$ new variables under the names with a *newvar* prefix. The following options are available:

`regimes` indicates that the probabilities of the regimes $s_t \in \{-1, 0, 1\}$ must be predicted instead of the choice probabilities. This option is ignored if the `zeros` option is used.

`zeros` indicates that the probabilities of the different types of zeros (the outcomes in the inflated category `infcat(`*choice*`)` in the ZIOP-2 and ZIOP-3 models), conditional on different regimes, must be predicted instead of the choice probabilities.

`output(`*string*`)` specifies the different types of predictions. The possible values of *string* are: *choice* for reporting the predicted outcome (the choice with the largest predicted probability); *mean* for reporting the expected value of the dependent variable computed as $\sum_i i \Pr(y_t = i)$; and *cum* for predicting the cumulative choice probabilities: $\Pr(y_t <= -J^-)$, $\Pr(y_t <= -J^- + 1)$, ..., $\Pr(y_t <= J^+)$. If *string* is not specified, the usual choice probabilities $\Pr(y_t = -J^-)$, $\Pr(y_t = -J^- + 1)$, ..., $\Pr(y_t = J^+)$ are predicted and saved into the new variables with the *newvar* prefix.

### The ziopprobabilities command

`ziopprobabilities [, at(`*string*`) zeros regimes]`

This command shows the predicted probabilities estimated at the specified values of the independent variables along with the standard errors. The options `zeros` and `regimes` are specified as in `predict`. The option `at`() is specified as follows:

`at(`*string*`)` specifies for which values of the independent variables to estimate the predictions. If `at(`*string*`)` is used (*string* is a list of *varname* = *value* expressions, separated by commas), the predictions are estimated at these values and displayed without saving to the dataset. If some independent variable names are not specified, their median values are taken instead. If `at()` is not used, by default the predictions are estimated at the median values of the independent variables.

### The ziopcontrasts command

`ziopcontrasts [, at(`*string*`) to(`*string*`) zeros regimes]`

This command shows the differences in the predicted probabilities, estimated first at the values of the independent variables in `at()` and then at the values in `to()`, along with the standard errors. The options `zeros`, `regimes` and `at()` are specified as in `ziopprobabilities`. The options `to()` is specified analogously to `at()`.

### The ziopmargins command

`ziopmargins [, at(`*string*`) zeros regimes]`

This command shows the marginal effects of each independent variable on the predicted probabilities estimated at the specified values of the independent variables along with the standard errors. The options `zeros`, `regimes` and `at()` are specified as in `ziopprobabilities`.

### The ziopclassification command

`ziopclassification [`*if*`] [`*in*`]`

This command shows the classification table (or confusion matrix); the percentage of correct predictions; the two strictly proper scores — the probability, or Brier, score (Brier 1950) and the ranked probability score (Epstein 1969); the precisions, the hit rates (or recalls) and the adjusted noise-to-signal ratios (Kaminsky and Reinhart 1999).

The classification table reports the predicted choices (the ones with the highest predicted probability) in columns, the actual choices in rows, and the number of (mis)classifications in each cell.

The Brier probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=-J^-}^{J^+}[\Pr(y_t = j) - I_{jt}]^2$, where indicator $I_{jt} = 1$ if $y_t = j$ and $I_{jt} = 0$ otherwise. The ranked probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=-J^-}^{J^+}[Q_{jt} - D_{jt}]^2$, where $Q_{it} = \sum_{i=-J^-}^{j}\Pr(y_t = i)$ and $D_{it} = \sum_{i=-J^-}^{j} I_{jt}$. The better the prediction, the smaller both score values. Both scores have a minimum value of zero when all the actual outcomes are predicted with a unit probability.

The precision, the hit rate (or recall) and the adjusted noise-to-signal ratios are defined as follows. Let *TP* denote a true positive event, that is, the outcome was predicted and occurred; let *FP* denote a false positive event, that is, the outcome was predicted but did not occur; let *FN* denote a false positive event, that is, the outcome was not predicted but did occur; and let *TN* denote a true negative event, that is, the outcome was not predicted and did not occur. The desirable outcomes fall into categories *TP* and *TN*, while the noisy ones fall into categories *FP* and *FN*. A perfect prediction has no entries in *FP* and *FN*, while a noisy prediction has many entries in *FP* and *FN*, but few in *TP* and *TN*. The precision is defined for each choice as $TP/(TP+FP)$, the recall — as $TP/(TP+FN)$, and the adjusted noise-to-signal ratio — as $[FP/(FP+TN)]/[TP/(TP+FN)]$.

**The ziopvuong command**

`ziopvuong` *modelspec*$_1$ *modelspec*$_2$

This command performs the Vuong test for non-nested hypotheses, which compares the closeness of two models to the true data distribution using the differences in the pointwise log likelihoods of the two models. The arguments *modelspec*$_1$ and *modelspec*$_2$ are the names under which the estimation results are saved using the `estimates store` command. Any model that stores the vector `e(ll_obs)` of observation-wise log-likelihood can technically be used to perform the test. The command provides the three Vuong test statistics ($z$-scores): the standard one and two adjusted ones with corrections to address the comparison of models with different numbers of parameters based on AIC and BIC. They can be used to test the hypothesis that one of the models explains the data better than the other. A significant positive $z$-score indicates a preference for the first model, while a significant negative value of the $z$-score indicates a preference for the second model. An insignificant $z$-score implies no preference for either model.

# 4 Monte Carlo simulations

We conducted extensive Monte Carlo experiments to illustrate the finite sample performance of the ML estimators of each model.

## 4.1 Monte Carlo design

We simulated six processes generated by the NOP, ZIOP-2 and ZIOP-3 models, each of them with both exogenous and endogenous switching. Repeated samples with 200, 500 and 1,000 observations were independently generated and then estimated by the true model. There were 10,000 replications in each experiment.

Three independent variables $\mathbf{w_1}$, $\mathbf{w_2}$ and $\mathbf{w_3}$ were drawn in each replication as $\mathbf{w_1} \overset{iid}{\sim} \mathcal{N}(0,1)+2$, $\mathbf{w_2} \overset{iid}{\sim} \mathcal{N}(0,1)$, and $\mathbf{w_3} = -1$ if $\mathbf{u} \leq 0.3$, 0 if $0.3 < \mathbf{u} \leq 0.7$, or 1 if $\mathbf{u} > 0.7$, where $\mathbf{u} \overset{iid}{\sim} \mathcal{U}[0,1]$. The repeated samples were generated for the NOP and ZIOP-3 models with $\mathbf{Z} = (\mathbf{w_1}, \mathbf{w_2})$, $\mathbf{X}^- = (\mathbf{w_1}, \mathbf{w_3})$, $\mathbf{X}^+ = (\mathbf{w_2}, \mathbf{w_3})$, and for the ZIOP-2 model with $\mathbf{Z} = (\mathbf{w}_1, \mathbf{w}_3)$, $\mathbf{X} = (\mathbf{w}_2, \mathbf{w}_3)$. The dependent variable $y$ was generated with five values: -2, -1, 0, 1 and 2. The parameters were calibrated to yield, on average, the following frequencies of the above outcomes: 7%, 14%, 58%, 14% and 7%, respectively. To avoid the divergence of ML estimates due to the problem of complete separation (perfect prediction), which could happen if the actual number of observations in any outcome category is very low, the samples with any outcome category frequency lower than 6% were re-generated. The variances of the error terms in all equations were fixed to one. The true values of all other parameters in the simulations are shown in Table A1 in Appendix. The starting values for the slope and threshold parameters were obtained using the independent OP estimations of each equation. The starting values for $\rho$, $\rho^-$ and $\rho^+$ were obtained by maximizing the likelihood functions of the endogenous-switching models holding the other parameters fixed at their estimates in the corresponding exogenous-switching model. The values of the choice probabilities, which depend on the values of the regressors, are computed at the population medians of the regressors.

## 4.2 Monte Carlo results

Table 1 reports the measures of the accuracy for the estimates of the choice probabilities. The results for the estimates of the parameters and MEs are qualitatively and quantitatively similar. The simulations show that the ML estimators are consistent and reliable even in samples with only 200 observations: the biases of choice probability estimates are smaller than five percent and the asymptotic coverage rates differ from the nominal 0.95 level by less than one percent. For each model, the bias and RMSE decrease as the sample size increases. The RMSE decreases, in most cases, faster than the asymptotic rate $\sqrt{n}$. This may be caused by a small number of large deviations in the parameter estimation in small samples. For all models and sample sizes, the bias and RMSE are, as expected, slightly higher for a more complex endogenous-switching version. The standard error estimates, on average, correspond to the actual standard errors; however, large deviations make standard error estimates biased in small samples, but do not move the coverage rates from the nominal level by more than one percent even with only 200 observations. The accuracy in the NOP models is, as expected, higher than in the zero-inflated OP models.

Table 1. Monte Carlo results: The accuracy of ML estimators

| Sample size | True and estimated model: | NOP ($\rho^-=\rho^+=0$) | NOP | ZIOP-2 ($\rho=0$) | ZIOP-2 | ZIOP-3 ($\rho^-=\rho^+=0$) | ZIOP-3 |
|---|---|---|---|---|---|---|---|
| | The accuracy of the estimates of choice probabilities | | | | | | |
| 200 | Bias, % | 2.3 | 1.5 | 4.4 | 5.1 | 3.3 | 3.1 |
| 500 | | 1.1 | 0.9 | 2.3 | 3.0 | 1.6 | 1.5 |
| 1000 | | 0.4 | 0.4 | 1.3 | 1.7 | 0.8 | 1.0 |
| 200 | RMSE, ×100 | 2.4 | 2.6 | 2.8 | 2.9 | 2.7 | 2.9 |
| 500 | | 1.5 | 1.6 | 1.7 | 1.8 | 1.6 | 1.8 |
| 1000 | | 1.1 | 1.1 | 1.2 | 1.2 | 1.1 | 1.3 |
| 200 | Coverage rate (at 95% level), % | 94.4 | 94.4 | 95.3 | 95.3 | 95.1 | 94.8 |
| 500 | | 95.4 | 95.2 | 95.6 | 95.6 | 95.9 | 95.7 |
| 1000 | | 95.5 | 95.5 | 95.7 | 95.7 | 95.6 | 95.6 |
| 200 | Bias of standard error estimates, % | 4.2 | 4.2 | 6.9 | 6.4 | 5.5 | 15.1 |
| 500 | | 3.9 | 4.6 | 6.9 | 6.1 | 5.3 | 16.6 |
| 1000 | | 2.6 | 3.4 | 5.7 | 5.9 | 3.7 | 13.9 |

Notes: Bias – the absolute difference between the estimated and true values, divided by the true value; RMSE – the absolute root mean square error of the estimates; Coverage rate – the percentage of times the estimated asymptotic 95% confidence intervals cover the true values; Bias of standard error estimates – the absolute difference between the average of the estimated asymptotic standard errors of the estimates and the standard deviation of the estimates in all replications. The above measures are averaged across five outcome categories.

# 5 Examples

The new commands are applied to a real-world time-series sample of all decisions of the U.S. Federal Open Market Committee (FOMC) on the federal funds rate target made at scheduled and unscheduled meetings during the 9/1987 – 9/2008 period.

The dependent variable, the change to the rate target, is classified into five ordered categories: "-0.5" (a cut of 0.5% or more), "-0.25" (a cut less than 0.5% but more than 0.0625%), "0" (no change or change by no more than 0.0625%), "0.25" (a hike more than 0.0625% but less than 0.5%) and "0.5" (a hike of 0.5% or more). The FOMC decisions are aligned with the real-time values of the explanatory variables as they were truly available to the public on the previous day before each FOMC meeting. The explanatory variables include: `spread` (the difference between the one-year treasury constant maturity rate and the effective federal funds rate, five-business-day moving average; data source: ALFRED[4]); `pb` (the trichotomous indicator that we constructed from the "policy bias" statements at the previous FOMC meeting: it equals 1 if the statement was asymmetric toward tightening, 0 if the statement was symmetric, and -1 if the statement was asymmetric toward easing; data source: FOMC statements and minutes[5]); `houst` (the Greenbook projection for the current quarter of the total number of new privately owned housing units started; data source: RTDSM[6]); `gdp` (the Greenbook projection for the current quarter of quarterly growth in the nominal gross domestic (before 1992: national) product, annualized percentage points; data source: RTDSM).

We start by estimating the conventional OP model using the `oprobit` command:

```
. oprobit rate_change spread pb houst gdp, nolog

Ordered probit regression                       Number of obs   =        210
                                                LR chi2(4)      =     214.54
                                                Prob > chi2     =     0.0000
Log likelihood = -159.56242                     Pseudo R2       =     0.4020

------------------------------------------------------------------------------
 rate_change |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      spread |   1.574232   .1870759     8.41   0.000      1.20757    1.940894
          pb |   .9262378   .1479364     6.26   0.000     .6362877    1.216188
       houst |   1.373179   .3459397     3.97   0.000     .6951499    2.051209
         gdp |   .2390714   .0571926     4.18   0.000     .1269761    .3511668
-------------+----------------------------------------------------------------
       /cut1 |   .4656819   .5382091                     -.5891885    1.520552
       /cut2 |     1.8382   .5339707                      .7916362    2.884763
       /cut3 |   4.835985   .6359847                      3.589478    6.082492
       /cut4 |   6.331172   .6875922                      4.983516    7.678828
------------------------------------------------------------------------------

. estat ic

Akaike´s information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |    Obs    ll(null)   ll(model)     df          AIC         BIC
-------------+---------------------------------------------------------------
           . |    210   -266.8308   -159.5624      8     335.1248    361.9017
-----------------------------------------------------------------------------
```

[4] ALFRED (ArchivaL Federal Reserve Economic Data) is available at https://alfred.stlouisfed.org/.
[5] https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm.
[6] RTDSM (Real-Time Data Set for Macroeconomists) is available at https://www.philadelphiafed.org.

We now allow the negative, zero and positive changes to the rate target to be generated by different processes, and estimate the three-part NOP model. The `nop` command yields the following results:

```
. nop rate_change spread pb houst gdp, neg(spread gdp) pos(spread pb) inf(0) nolog vuong
Nested ordered probit regression
Regime switching:        exogenous
Number of observations =       210
Log likelihood         = -150.9638
McFadden pseudo R2     =    0.4342
LR chi2( 8)            =  231.7341
Prob > chi2            =    0.0000
AIC                    =  325.9276
BIC                    =  366.0929
------------------------------------------------------------------------------
         rate_change |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
--------------------+---------------------------------------------------------
Regime equation      |
              spread |   1.579634   .2195074     7.20   0.000     1.149407     2.00986
                  pb |   .8769436   .1582913     5.54   0.000     .5666983    1.187189
               houst |   2.303497   .4324382     5.33   0.000     1.455934     3.15106
                 gdp |   .2742909   .0696122     3.94   0.000     .1378535    .4107283
               /cut1 |   3.299825   .6832466     4.83   0.000     1.960686    4.638963
               /cut2 |   6.496983   .8339921     7.79   0.000     4.862389    8.131578
--------------------+---------------------------------------------------------
Outcome equation (+) |
              spread |   1.627788   .6748859     2.41   0.016     .3050354     2.95054
                  pb |   2.255519   .8805447     2.56   0.010     .5296829    3.981355
               /cut1 |    3.13416   .9511016     3.30   0.001     1.270035    4.998285
--------------------+---------------------------------------------------------
Outcome equation (-) |
              spread |   .9489572   .3821965     2.48   0.013     .1998659    1.698049
                 gdp |   .1339181   .1006124     1.33   0.183    -.0632785    .3311147
               /cut1 |  -.4720761   .4202012    -1.12   0.261    -1.295655     .351503
------------------------------------------------------------------------------
Vuong test versus ordered probit:
  Mean difference in log likelihood                     0.0409
  Standard deviation of difference in log likelihood    0.2626
  Number of observations                                   210
  Vuong test statistic                           z =    2.2600
  P-Value                                     Pr>z =    0.0119
     with AIC (Akaike) correction                z =    1.2087
  P-Value                                     Pr>z =    0.1134
     with BIC (Schwarz) correction               z =   -0.5508
  P-Value                                     Pr>z =    0.7091
```

The NOP model provides a substantial improvement of the likelihood, and is preferred to the standard OP model according to AIC and the Vuong test (the $p$-value is 0.01). However, the Vuong tests with the corrections based on AIC and BIC are indifferent between the two models. Endogenous switching does not significantly improve the likelihood of the NOP model (the log likelihood with endogenous switching is -150.2, the $p$-value of the LR test of the null of exogenous switching is 0.48), the correlation coefficients $\rho^{-}$and $\rho^{+}$ are not significant, and both AIC and BIC favor the NOP model with exogenous switching.

Next we allow for an inflation of zero outcomes and estimate the three-part ZIOP-3 model. The `ziop3` command with exogenous switching yields the following results:

```
. ziop3 rate_change spread pb houst gdp, neg(spread gdp) pos(spread pb) inf(0) nolog vuong
 (output omitted)
Zero-inflated ordered probit regression
Zero inflation:          three regimes
Regime switching:        exogenous
Number of observations =       210
Log likelihood         = -139.5529
McFadden pseudo R2     =    0.4770
LR chi2(10)            =  254.5558
Prob > chi2            =    0.0000
AIC                    =  307.1058
BIC                    =  353.9653
------------------------------------------------------------------------------------
         rate_change |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
---------------------+--------------------------------------------------------------
Regime equation      |
              spread |   2.106257    .364262     5.78   0.000     1.392317    2.820198
                  pb |   1.628486   .3356997     4.85   0.000     .9705269    2.286446
               houst |   5.311379   .9913486     5.36   0.000     3.368372    7.254387
                 gdp |   .3809606   .1085468     3.51   0.000     .1682127    .5937084
               /cut1 |   9.103481   1.772781     5.14   0.000     5.628894    12.57807
               /cut2 |    12.3481   1.952013     6.33   0.000     8.522227    16.17398
---------------------+--------------------------------------------------------------
Outcome equation (+) |
              spread |   1.809669   .7282205     2.49   0.013     .3823831    3.236955
                  pb |   2.620109   .9836793     2.66   0.008     .6921334    4.548085
               /cut1 |  -1.481781   1.015198    -1.46   0.144    -3.471532    .5079697
               /cut2 |   3.509078   1.070858     3.28   0.001     1.410236    5.607921
---------------------+--------------------------------------------------------------
Outcome equation (-) |
              spread |   1.072859   .2690323     3.99   0.000     .5455655    1.600153
                 gdp |    .177697   .0742318     2.39   0.017     .0322055    .3231886
               /cut1 |  -.6373707   .3361142    -1.90   0.058    -1.296142     .021401
               /cut2 |   .7569744   .3460019     2.19   0.029     .0788232    1.435126
------------------------------------------------------------------------------------
Vuong test versus ordered probit:
  Mean difference in log likelihood                    0.0953
  Standard deviation of difference in log likelihood   0.3851
  Number of observations                                  210
  Vuong test statistic                         z =     3.5853
  P-Value                                   Pr>z =     0.0002
     with AIC (Akaike) correction              z =     2.5102
  P-Value                                   Pr>z =     0.0060
     with BIC (Schwarz) correction             z =     0.7110
  P-Value                                   Pr>z =     0.2385
```

The empirical evidence in favor of zero inflation is convincing: with only two extra parameters, the ZIOP-3 model has a much higher likelihood than the NOP model (-139.6 vs. -151.0), and is clearly preferred by both AIC and BIC to the NOP and OP models. The Vuong tests for zero inflation (the standard one and one with the correction based on AIC) favor the ZIOP-3 model over the OP model at the 0.001 and 0.01 level, respectively. Endogenous switching does not significantly improve the likelihood of the ZIOP-3 model either (the $p$-value of the LR test of exogenous switching is 0.30, and both AIC and BIC prefer the exogenous switching).

In contrast, the likelihood of the two-part ZIOP-2 model is even lower than that of the NOP model. According both to AIC and BIC, the ZIOP-2 model is inferior to all the above models, including the OP one. The `ziop2` command yields the following results:

```
. ziop2 rate_change spread pb houst gdp, out(spread pb houst gdp ) infcat(0) nolog
Zero-inflated ordered probit regression
Zero inflation:          two regimes
Regime switching:        exogenous
Number of observations =       210
Log likelihood         = -154.3563
McFadden pseudo R2     =    0.4215
LR chi2( 9)            =  224.9490
Prob > chi2            =    0.0000
AIC                    =  334.7126
BIC                    =  378.2250
------------------------------------------------------------------------------
     rate_change |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-----------------+------------------------------------------------------------
Regime equation  |
          spread |  -.5718098   .4932372    -1.16   0.246    -1.538537    .3949173
              pb |   2.220756   1.124943     1.97   0.048      .015908    4.425605
           houst |   .4317792   .9262931     0.47   0.641    -1.383722     2.24728
             gdp |  -.3039409   .1561281    -1.95   0.052    -.6099462    .0020645
           /cut1 |  -3.269292   2.104548    -1.55   0.120    -7.394131    .8555464
-----------------+------------------------------------------------------------
Outcome equation |
          spread |   1.920514   .2407834     7.98   0.000     1.448587    2.392441
              pb |    1.21367   .1982338     6.12   0.000     .8251391    1.602201
           houst |   1.637904   .3932584     4.16   0.000     .8671315    2.408676
             gdp |   .2358575   .0628755     3.75   0.000     .1126239    .3590911
           /cut1 |   .5651226   .5985828     0.94   0.345    -.6080782    1.738323
           /cut2 |   2.422641   .6270021     3.86   0.000     1.193739    3.651542
           /cut3 |   5.397053   .7416277     7.28   0.000      3.94349    6.850617
           /cut4 |   7.039527   .8100945     8.69   0.000     5.451771    8.627283
------------------------------------------------------------------------------
```

The Vuong tests prefer the ZIOP-3 model to the ZIOP-2 model at the 0.01 significance level using the standard test statistic, and at the 0.02 and 0.03 levels using the corrected statistics based, respectively, on AIC and BIC:

```
. quietly ziop3 rate_change pb spread houst gdp, neg(spread gdp ) pos(pb spread) inf(0)
 (output omitted)

 . est store ziop3_model

. quietly ziop2 rate_change spread pb houst gdp, out(spread pb houst gdp) inf(0)

. est store ziop2_model

. ziopvuong ziop3_model ziop2_model
Vuong non-nested test for ziop3_model vs ziop2_model
  Mean difference in log likelihood                   0.0705
  Standard deviation of difference in log likelihood    0.4235
  Number of observations                                 210
  Vuong test statistic                        z =     2.4119
  P-Value                                  Pr>z =     0.0079
     with AIC (Akaike) correction             z =     2.2490
  P-Value                                  Pr>z =     0.0123
     with BIC (Schwarz) correction            z =     1.9763
  P-Value                                  Pr>z =     0.0241
```

Now we report the selected output of the postestimation commands, performed for the ZIOP-3 model.

The predicted choice probabilities at the specified values of the independent variables can be estimated using the `ziopprobabilities` command:

```
. ziopprobabilities, at (pb=1, spread=0.426, houst=1.6, gdp=6.8)
  Evaluated at:
   gdp    houst     pb    spread
6.8000   1.6000 1.0000    0.4260

  Predicted probabilities of different outcomes
   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
      0.0000        0.0000     0.1027       0.4908      0.4065

  Standard errors of the probabilities
   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
      0.0000        0.0000     0.0491       0.1173      0.1154
```

The predicted probabilities of the three latent regimes $s_t^* \in \{-1, 0, 1\}$ or the probabilities of the three types of zeros conditional on each regime can be estimated for each sample observation using the command `predict` with the option `zeros` or `regimes`, respectively:

```
. predict p_zero, zeros

. predict p_reg, regimes

. tabstat p_zero* p_reg*, stat(mean)

   stats |  p_zero_0  p_zero_n  p_zero_p   p_reg_n   p_reg_0   p_reg_p
---------+------------------------------------------------------------
    mean |  .3895957  .1453901  .0042672  .4028259  .3895957  .2075784
----------------------------------------------------------------------
```

The average predicted probabilities of the regimes $s_t = -1$, $s_t = 0$ and $s_t = 1$ in the sample are 0.40, 0.39 and 0.21, respectively. However, the average probability of zeros conditional on the regime $s_t = -1$ (0.15) is much higher than on the regime $s_t = 1$ (0.00).

The marginal effects of the independent variables on the choice probabilities at the specified values of the independent variables can be estimated using the `ziopmargins` command:

```
. ziopmargins, at (pb=1, spread=0.426, houst=1.6, gdp=6.8)
  Evaluated at:
   gdp    houst     pb    spread
6.8000   1.6000 1.0000    0.4260

  Marginal effects of all variables on the probabilities of different outcomes
           |   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
-----------+-----------------------------------------------------------------
       gdp |     -0.0000       -0.0000    -0.0682       0.0373      0.0309
     houst |     -0.0000       -0.0000    -0.9503       0.5198      0.4305
        pb |     -0.0000       -0.0000    -0.2914      -0.7720      1.0634
    spread |     -0.0000       -0.0000    -0.3769      -0.4372      0.8140

  Standard errors of marginal effects
           |   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
-----------+-----------------------------------------------------------------
       gdp |      0.0000        0.0000     0.0244       0.0156      0.0143
     houst |      0.0000        0.0000     0.2840       0.1924      0.1799
        pb |      0.0000        0.0000     0.0772       0.4059      0.3890
    spread |      0.0000        0.0000     0.1115       0.3106      0.2971
```

The differences in the predicted choice probabilities (along with the standard errors) at two different values of the independent variables can be estimated using the `ziopcontrasts` command. In particular, this command may be used to compute the MEs of the discrete ordinal independent variables such as `pb` (instead of using the `ziopmargins` command, which computes the derivatives of the probabilities):

```
. ziopcontrasts, at(pb=1, spread=0.426, houst=1.6, gdp=6.8) ///
>                to(pb=0, spread=0.426, houst=1.6, gdp=6.8)
  Evaluated between
           |    gdp    houst     pb    spread
-----------+----------------------------------
      from | 6.8000   1.6000 1.0000    0.4260
        to | 6.8000   1.6000 0.0000    0.4260

  Contrasts of the predicted probabilities of different outcomes
   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
      0.0000        0.0003     0.5427      -0.1376     -0.4054

  Standard errors of the contrasts
   Pr(y=-.5)    Pr(y=-.25)    Pr(y=0)    Pr(y=.25)    Pr(y=.5)
      1.8053        0.9350     0.2971       0.3404      0.7325
```

Finally, the different measures of model fit and the accuracy of the probabilistic predictions can be computed using the `ziopclassification` command:

```
. ziopclassification
Classification table

    Actual |                  Predicted outcomes
  outcomes |       -.5       -.25          0        .25         .5 |     Total
-----------+-------------------------------------------------------+----------
       -.5 |         7          9          2          0          0 |        18
      -.25 |         2         21         12          0          0 |        35
         0 |         1          8        100          5          0 |       114
       .25 |         0          0          9         25          0 |        34
        .5 |         0          0          2          4          3 |         9
-----------+-------------------------------------------------------+----------
     Total |        10         38        125         34          3 |       210

Accuracy (% of correct predictions) =    0.7429
Brier score                         =    0.3731
Ranked probability score            =    0.2160

    Actual |
  outcomes |   Precision    Recall    Adjusted noise-to-signal ratio
-----------+-----------------------------------------------------------
       -.5 |       0.7000    0.3889                          0.0402
      -.25 |       0.5526    0.6000                          0.1619
         0 |       0.8000    0.8772                          0.2969
       .25 |       0.7353    0.7353                          0.0695
        .5 |       1.0000    0.3333                          0.0000
```

As Table 2 reports, the ZIOP-3 model demonstrates the best fit according to all the criteria.

Table 2. Comparison of the alternative models

| Measure of fit | OP | NOP | ZIOP-2 | ZIOP-3 |
|---|---|---|---|---|
| AIC | 335.1 | 325.9 | 334.7 | **307.1** |
| BIC | 361.9 | 366.1 | 378.2 | **354.0** |
| Percentage of correct predictions | 0.66 | 0.70 | 0.70 | **0.74** |
| Brier probability score | 0.42 | 0.40 | 0.41 | **0.37** |
| Ranked probability score | 0.24 | 0.23 | 0.23 | **0.22** |
| Adjusted noise-to-signal ratio for zeros | 0.44 | 0.41 | 0.36 | **0.30** |

Notes: The NOP, ZIOP-2 and ZIOP-3 models are estimated with exogenous switching.

# 6 Concluding remarks

This article describes the ML estimation of the nested and cross-nested zero-inflated ordered probit models using the new STATA commands `nop`, `ziop2` and `ziop3`. Such models can be applied to a variety of data sets in which the discrete ordinal outcomes can be divided into groups (nests) of similar choices, for example, the decisions to reduce, leave unchanged, or increase the choice variable (monetary policy interest rates, rankings, prices, consumption levels), or the negative, neutral, or positive attitudes to survey questions. The choice among the nests is driven by an ordered-choice switching mechanism that can be either exogenous or endogenous to the outcome decisions, which are also naturally ordered (large or small increase/decrease; disagree or strongly disagree; etc.). The models allow the probabilities of choices from different nests (e.g., no change and an increase) to be driven by distinct mechanisms. Moreover, the cross-nested zero-inflated models allow the often abundant no-change or neutral outcomes to belong to all nests and be inflated by several different processes. The results of Monte Carlo simulations indicate that the proposed ML estimators are consistent and perform well in small samples.

# 7 Acknowledgments

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# Appendix

Table A1. Monte Carlo simulations: The true values of parameters

| | NOP (exog) | NOP | ZIOP-2 (exog) | ZIOP-2 | ZIOP-3 (exog) | ZIOP-3 |
|---|---|---|---|---|---|---|
| $\gamma$ | (0.6, 0.4)' | (0.6, 0.4)' | (0.6, 0.8)' | (0.6, 0.8)' | (0.6, 0.4)' | (0.6, 0.4)' |
| $\mu$ | (0.21, 2.19)' | (0.21, 2.19)' | 0.45 | 0.45 | (0.9, 1.5)' | (0.9, 1.5)' |
| $\beta$ | | | (0.5, 0.6)' | (0.5, 0.6)' | | |
| $\beta^-$ | (0.3, 0.9)' | (0.3, 0.9)' | | | (0.3, 0.9)' | (0.3, 0.9)' |
| $\beta^+$ | (0.2, 0.3)' | (0.2, 0.3)' | | | (0.2, 0.3)' | (0.2, 0.3)' |
| $\alpha$ | | | (-1.45, -0.55, 0.75, 1.65)' | (-1.18,-0.33, 0.9, 1.76)' | | |
| $\alpha^-$ | -0.17 | -0.5 | | | (-0.67, 0.36)' | (-0.88, 0.12)' |
| $\alpha^+$ | 0.68 | 1.3 | | | (0.02, 1.28)' | (0.49, 1.67)' |
| $\rho$ | | | 0 | 0.5 | | |
| $\rho^-$ | 0 | 0.3 | | | 0 | 0.3 |
| $\rho^+$ | 0 | 0.6 | | | 0 | 0.6 |

Notes: (exog) – exogenous switching: $\rho = \rho^- = \rho^+ = 0$. The variances $\sigma^2$, $\sigma^2_-$, $\sigma^2_+$, and $\sigma^2_\nu$ are all fixed to one in all models.