Estimation of nested and zero-inflated ordered probit models

David Dale
Yandex
Moscow, Russia
dale.david@yandex.ru

Andrei Sirchenko
Higher School of Economics
Moscow, Russia
andrei.sirchenko@gmail.com

Abstract

We introduce three new STATA commands, `nop`, `ziop2` and `ziop3`, for the estimation of a three-part nested ordered probit model, the two-part zero-inflated ordered probit models of Harris and Zhao (2007, *Journal of Econometrics* 141: 1073–1099) and Brooks, Harris and Spencer (2012, *Economics Letters* 117: 683–686), and a three-part zero-inflated ordered probit model for ordinal outcomes, with both exogenous and endogenous switching. The three-part models allow the probabilities of positive, neutral (zero) and negative outcomes to be generated by distinct processes. The zero-inflated models address the preponderance of zero responses and allow the zeros to emerge in two or three latent regimes. We provide the postestimation commands to compute the predicted choice probabilities and their standard errors, the predicted outcomes, the expected values of dependent variable, the marginal effects on the probabilities, the classification tables, the Brier probability score (Brier 1950, *Monthly Weather Review* 78: 1–3), the ranked probability score (Epstein 1969, *Journal of Applied Meteorology* 8: 985–987), and the adjusted noise-to-signal ratios (Kaminsky and Reinhart 1999, *American Economic Review* 89: 473–500), and to perform model comparison using the Vuong test (Vuong 1989, *Econometrica* 57: 307–333) with the corrections based on Akaike and Schwarz information criteria. We investigate the finite-sample performance of proposed maximum likelihood estimators by Monte Carlo simulations, discuss the relations among the models, and illustrate them with an empirical application to the U.S. federal funds rate target.

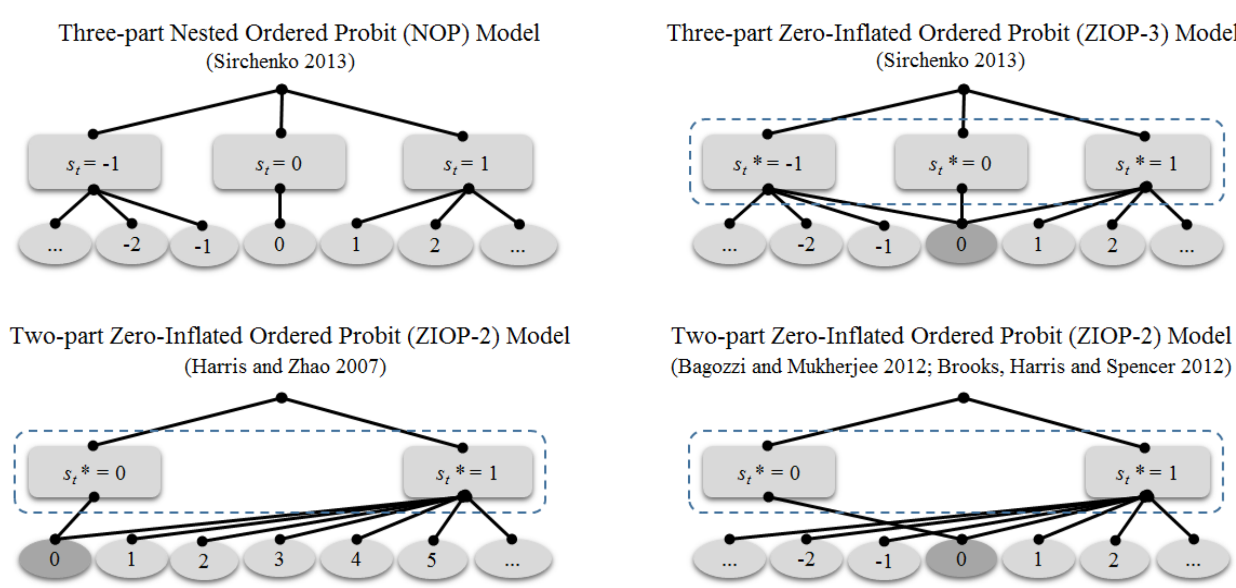
Keywords: ordinal outcomes, zero inflation, nested ordered probit, zero-inflated ordered probit, endogenous switching, Vuong test, nop, ziop2, ziop3, federal funds rate target.

1 Introduction

We introduce the STATA commands, `nop`, `ziop2` and `ziop3`, which estimate the two-level nested and zero-inflated ordered probit (OP) models for ordinal outcomes, including the zero- and middle-inflated models of Harris and Zhao (2007), Bagozzi and Mukherjee (2012), Brooks, Harris and Spencer (2012) and Sirchenko (2013). The rationale behind the two-level nested decision process is standard in the discrete-choice modeling when the set of alternatives faced by a decision-maker can be partitioned into subsets (or nests) with similar alternatives correlated due to the common unobserved factors. A choice among the nests and a choice among the alternatives within each nest can be driven by different sets of observed and unobserved factors (and common factors can have different weights).

In the case of unordered categorical data, in which choices can be grouped into the nests of similar options, the nested logit model is a popular method. The nested models for ordinal data are rare although the rationale behind them is similar: choosing among a negative response (decrease), a neutral response (no change) or a positive response (increase) is quite different from choosing the magnitude of negative response; and choosing the magnitude of negative response can be driven by quite different determinants than choosing the magnitude of positive response. This leads to three implicit decisions: an upper-level regime decision — a choice among the nests, and two lower-level outcome decisions — the choices of the magnitude of negative and positive responses (see the top left panel of Figure 1).

Figure 1. Decision trees of nested and zero-inflated ordered probit models



Notes: Decisionmakers are not assumed to choose sequentially. The tree diagrams simply represent a nesting structure of the system of OP models.

Furthermore, it would be reasonable for the zero (no-change) alternative to be in three nests: its own one, one with negative responses and one with positive responses; so some zeros can be driven by similar factors as negative or positive responses. This leads to a three-part cross-nested model with the nests overlapping at a zero response; hence, the probability of zeros is 'inflated'. Since the regime decision is not observable, the zeros are observationally equivalent — it is never known to which of the three nests the observed zero belongs. While several types of models with overlapping nests for unordered categorical responses are developed (Vovsha 1997; Wen and Koppelman 2001), the cross-nested models for ordinal outcomes are very scarce.[1]

The prevalence of status quo, neutral or zero outcomes is observed in many fields, including economics, sociology, technometrics, psychology and biology. The heterogeneity of zeros is widely recognized — see Winkelmann (2008) and Greene and Hensher (2010) for a review. Studies identify different types of zeros such as: no visits to doctor due to good health, iatrophobia, or medical costs; no illness due to strong immunity or lack of infection; no children due to infertility or choice. In the studies of survey responses using an odd-point Likert-type scale, where the respondents must indicate the negative, neutral or positive attitude or opinion, the heterogeneity of indifferent responses (a true neutral option versus an undecided, or ambivalent, or uninformed one, commonly reported as neutral) is also well-recognized and sometimes labeled as the middle category endorsement or inflation (Bagozzi and Mukhetjee 2012; Hernández, Drasgow and Gonzáles-Romá 2004; Kulas and Stachowski 2009).

The two-part zero-inflated models, developed to address the unobserved heterogeneity of zeros, combines a binary choice model for the probability of crossing the hurdle (to participate or not to participate; to consume or not to consume) with a count or ordered-choice model for nonnegative outcomes above the hurdle: the two parts are estimated jointly, and the zero observations can emerge in both parts. The two-part zero-inflated models include the zero-inflated Poisson (Lambert 1992), negative binomial (Greene 1994), binomial (Hall 2002) and generalized Poisson (Famoye and Singh 2003) models for count outcomes, and the zero-inflated OP model (Harris and Zhao 2007) and zero-inflated proportional odds model (Kelley and Anderson 2008) for non-negative ordinal responses.[2]

The model of Harris and Zhao (2007) is suitable for explaining decisions such as the levels of consumption, when the upper hurdle is naturally binary (to consume or not to consume), the responses are non-negative and the inflated zeros are situated at one end of the ordered scale (see the bottom left panel of Figure 1). Bagozzi and Mukherjee (2012) and Brooks, Harris and Spencer (2012) modified the model of Harris and Zhao (2007) and developed the middle-inflated OP model for an ordinal outcome, which ranges from negative to positive responses, and where an abundant outcome is situated in the middle of the choice spectrum (see the bottom right panel of Figure 1).

The three-part cross-nested zero-inflated OP model (see the top right panel of Figure 1) introduced in Sirchenko (2013) is a natural generalization of the models of Harris and Zhao (2007), Bagozzi and Mukherjee (2012) and Brooks, Harris and Spencer (2012). A trichotomous regime decision is more realistic and flexible than a binary decision (change or no change) if applied to ordinal data with negative, zero and positive values.

[1] Small (1987) introduced an ordered-choice model with overlapping nests, which contain two adjacent choices.

[2] The zero-inflated models, estimation of which is currently implemented in STATA, include: the zero-inflated Poisson model (the zip command), the negative binomial model (the zinb command), and the binomial model (the zib command) and the beta-binomial model (the zibbin command) developed by Hardin and Hilbe (2014).

2 Models

2.1 Notation and assumptions

The observed dependent variable y_t, $t = 1, 2, ..., T$ is assumed to take on a finite number of ordinal values j coded as $\{-J^-, ..., -1, 0, 1, ..., J^+\}$, where a potentially heterogeneous (and typically predominant) response is coded as zero. The latent unobserved (or only partially observed) variables are denoted by $*$. Each model assumes an ordered-choice regime decision and the ordered-choice outcome decisions conditional on the regime. The regime decision is allowed to be correlated with each outcome decision. We denote by $\mathbf{x}_t$, $\mathbf{x}_t^-$, $\mathbf{x}_t^+$ and $\mathbf{z}_t$ the t^{th} rows of the observed data matrices (which in addition to predetermined explanatory variables may also include the lags of y_t); by $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}$ and $\boldsymbol{\gamma}$ the vectors of unknown slope parameters; by $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$ and $\boldsymbol{\mu}$ the vectors of unknown threshold parameters; by ρ, ρ^- and ρ^+ the vectors of unknown correlation coefficients; by ε_t, ε_t^-, ε_t^+ and ν_t the error terms that are independently and identically distributed (iid) across t with normal cumulative distribution function (CDF) Φ, the zero mean and the variances σ^2, σ_-^2, σ_+^2 and σ_ν^2, respectively; and by $\Phi_2(g_1;g_2;\sigma_1^2;\sigma_2^2;\rho)$ the CDF of the bivariate normal distribution of the two random variables g_1 and g_2 with the zero means, the variances σ_1^2 and σ_2^2 and the correlation coefficient ρ:

$$\Phi_2(g_1;g_2;\sigma_1^2;\sigma_2^2;\rho) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \int_{-\infty}^{g_1}\int_{-\infty}^{g_2} \exp\left(-\frac{u^2/\sigma_1^2 - 2\rho uw/\sigma_1\sigma_2 + w^2/\sigma_2^2}{2(1-\rho^2)}\right) dudw.$$

2.2 Three-part nested ordered probit (NOP) model

Despite a wide-spread use of nested logit models for unordered categorical responses, we are aware of only one example of the nested ordered probit model in the literature (Sirchenko 2013). The two-level NOP model can be described as

$$\begin{array}{ll}
\text{Upper-level decision:} & r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t = \begin{cases} 1 & \text{if} \quad \mu_2 < r_t^*, \\ 0 & \text{if} \quad \mu_1 < r_t^* \le \mu_2, \\ -1 & \text{if} \quad r_t^* \le \mu_1. \end{cases} \\
\text{Lower-level decisions:} & y_t^{-*} = \mathbf{x}_t^-\boldsymbol{\beta}^- + \varepsilon_t^-, \quad y_t^{+*} = \mathbf{x}_t^+\boldsymbol{\beta}^+ + \varepsilon_t^+, \\
& y_t = \begin{cases} j(j>0) & \text{if} \quad s_t = 1 \text{ and } \alpha_{j-1}^+ < y_t^{+*} \le \alpha_j^+, \\ 0 & \text{if} \quad s_t = 0, \\ j(j<0) & \text{if} \quad s_t = -1 \quad \text{and } \alpha_j^- < y_t^{-*} \le \alpha_{j+1}^-, \end{cases} \\
& \text{where } -\infty = \alpha_0^+ \le \alpha_1^+ \le ... \le \alpha_{J^+}^+ = \infty \\
& \text{and } -\infty = \alpha_{-J^-}^- \le \alpha_{-J^-+1}^- \le ... \le \alpha_0^- = \infty. \\
\text{Correlation among decisions:} & \begin{bmatrix} \nu_t \\ \varepsilon_t^i \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho^i\sigma_\nu\sigma_i \\ \rho^i\sigma_\nu\sigma_i & \sigma_i^2 \end{bmatrix} \right), \ i \in \{-,+\}.
\end{array}$$

The probabilities of the outcome j in the NOP model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+) = I_{j<0} \Pr(r_t^* \leq \mu_1 \text{ and } \alpha_j^- < y_t^{-*} \leq \alpha_{j+1}^-|\mathbf{z}_t, \mathbf{x}_t^-) \\
&+I_{j=0} \Pr(\mu_1 < r_t^* \leq \mu_2|\mathbf{z}_t) + I_{j>0} \Pr(\mu_2 < r_t^* \text{ and } \alpha_{j-1}^+ < y_t^{+*} \leq \alpha_j^+|\mathbf{z}_t, \mathbf{x}_t^+) \\
&= I_{j<0} \Pr(\nu_t \leq \mu_1 - \mathbf{z}_t\boldsymbol{\gamma} \text{ and } \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^- < \varepsilon_t^- \leq \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-) \\
&+I_{j=0} \Pr(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \leq \mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) \\
&+I_{j>0} \Pr(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+ < \varepsilon_t^+ \leq \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+) \\
&= I_{j<0}[\Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-) - \Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)] \\
&+I_{j>0}[\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_\nu^2; \sigma_+^2; -\rho^+) \\
&-\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_\nu^2; \sigma_+^2; -\rho^+)],
\end{aligned} \tag{1}$$

where $I_{j<0}$ is an indicator function such that $I_{j<0} = 1$ if $j < 0$, and $I_{j<0} = 0$ if $j \geq 0$ (analogously for $I_{j=0}$ and $I_{j>0}$).

In the case of exogenous switching (when $\rho^- = \rho^+ = 0$), the probabilities of the outcome j in the NOP can be computed as

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+, \rho^- = \rho^+ = 0) \\
&= I_{j<0}\Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)[\Phi(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-; \sigma_-^2) - \Phi(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-; \sigma_-^2)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma})] \\
&+I_{j>0}[1 - \Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_+^2) - \Phi(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+; \sigma_+^2)].
\end{aligned}$$

In the case of two or three outcome choices the NOP model degenerates to the conventional single-equation OP model.

2.3 Two-part zero-inflated ordered probit (ZIOP-2) model

The ZIOP-2 model, which represents the two part zero-inflated OP models of Bagozzi and Mukherjee (2012) and Brooks, Harris and Spencer (2012), can be described by the following system

$$\begin{array}{ll}
\text{Regime decision:} & r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t^* = \begin{cases} 1 & \text{if} \quad \mu < r_t^*, \\ 0 & \text{if} \quad r_t^* \leq \mu. \end{cases} \\
\text{Outcome decision:} & y_t^* = \mathbf{x}_t\boldsymbol{\beta} + \varepsilon_t, \\
& y_t = \begin{cases} j & \text{if} \quad s_t^* = 1 \text{ and } \alpha_{j-1} < y_t^* \leq \alpha_j, \\ 0 & \text{if} \quad s_t^* = 0, \end{cases} \\
& \text{where } -\infty = \alpha_{-J^- -1} \leq \alpha_{-J^-} \leq ... \leq \alpha_{J^+} = \infty. \\
\begin{array}{l}\text{Correlation among} \\ \text{decisions:}\end{array} & \begin{bmatrix} \nu_t \\ \varepsilon_t \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho\sigma_\nu\sigma \\ \rho\sigma_\nu\sigma & \sigma^2 \end{bmatrix} \right).
\end{array}$$

The probabilities of the outcome j in the ZIOP-2 model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t) = I_{j=0} \Pr(r_t^* \leq \mu|\mathbf{z}_t) + I_{j\geq 0} \Pr(\mu < r_t^* \text{ and } \alpha_{j-1} < y_t^* \leq \alpha_j|\mathbf{z}_t, \mathbf{x}_t) \\
&= I_{j=0} \Pr(\nu_t \leq \mu - \mathbf{z}_t\boldsymbol{\gamma}) + I_{j\geq 0} \Pr(\mu - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta} < \varepsilon_t \leq \alpha_j - \mathbf{x}_t\boldsymbol{\beta}) \\
&= I_{j=0}\Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) + \Phi_2(-\mu + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j - \mathbf{x}_t\boldsymbol{\beta}; \sigma_\nu^2; \sigma^2; -\rho) \\
&-\Phi_2(-\mu + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta}; \sigma_\nu^2; \sigma^2; -\rho).
\end{aligned} \tag{2}$$

In the case of exogenous switching (when $\rho = 0$), the probabilities of the outcome j in the ZIOP-2 model can be computed as

$$\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t, \rho = 0) = I_{j=0}\Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)$$
$$+[1 - \Phi(\mu - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j - \mathbf{x}_t\boldsymbol{\beta};\sigma^2) - \Phi(\alpha_{j-1} - \mathbf{x}_t\boldsymbol{\beta};\sigma^2)].$$

If $y_t \geq 0$ for $\forall t$, the ZIOP-2 model becomes the model of Harris and Zhao (2007).

2.4 Three-part zero-inflated ordered probit (ZIOP-3) model

The ZIOP-3 model developed by Sirchenko (2013) is a three-part generalization of the ZIOP-2 model, and can be described by the following system

$$\begin{array}{ll}
\text{Regime decision:} & r_t^* = \mathbf{z}_t\boldsymbol{\gamma} + \nu_t, \quad s_t^* = \begin{cases} 1 & \text{if} \quad \mu_2 < r_t^*, \\ 0 & \text{if} \quad \mu_1 < r_t^* \leq \mu_2, \\ -1 & \text{if} \quad r_t^* \leq \mu_1. \end{cases} \\
\text{Outcome decisions:} & y_t^{-*} = \mathbf{x}_t^-\boldsymbol{\beta}^- + \varepsilon_t^-, \quad y_t^{+*} = \mathbf{x}_t^+\boldsymbol{\beta}^+ + \varepsilon_t^+, \\
& y_t = \begin{cases} j (j \geq 0) & \text{if} \quad s_t^* = 1 \quad \text{and } \alpha_{j-1}^+ < y_t^{+*} \leq \alpha_j^+, \\ 0 & \text{if} \quad s_t^* = 0, \\ j(j \leq 0) & \text{if} \quad s_t^* = -1 \text{ and } \alpha_j^- < y_t^{-*} \leq \alpha_{j+1}^-, \end{cases} \\
& \text{where } -\infty = \alpha_{-1}^+ \leq \alpha_0^+ \leq ... \leq \alpha_{J^+}^+ = \infty \\
& \text{and } -\infty = \alpha_{-J^-}^- \leq \alpha_{-J^-+1}^- \leq ... \leq \alpha_1^- = \infty. \\
\text{Correlation among decisions:} & \begin{bmatrix} \nu_t \\ \varepsilon_t^i \end{bmatrix} \overset{iid}{\sim} \mathcal{N}\left(\begin{matrix} 0 \\ 0 \end{matrix}, \begin{bmatrix} \sigma_\nu^2 & \rho^i\sigma_\nu\sigma_i \\ \rho^i\sigma_\nu\sigma_i & \sigma_i^2 \end{bmatrix} \right), i \in \{-, +\}.
\end{array}$$

The probabilities of the outcome j in the ZIOP-3 model are given by

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+) = I_{j\leq 0}\Pr(r_t^* \leq \mu_1 \text{ and } \alpha_j^- < y_t^{-*} \leq \alpha_{j+1}^-|\mathbf{z}_t, \mathbf{x}_t^-) \\
&+I_{j=0}\Pr(\mu_1 < r_t^* \leq \mu_2|\mathbf{z}_t) + I_{j\geq 0}\Pr(\mu_2 < r_t^* \text{ and } \alpha_{j-1}^+ < y_t^{+*} \leq \alpha_j^+|\mathbf{z}_t, \mathbf{x}_t^+) \\
&= I_{j\leq 0}\Pr(\nu_t \leq \mu_1 - \mathbf{z}_t\boldsymbol{\gamma} \text{ and } \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^- < \varepsilon_t^- \leq \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-) \\
&+I_{j=0}\Pr(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \leq \mu_2 - \mathbf{z}_t\boldsymbol{\gamma}) \\
&+I_{j\geq 0}\Pr(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma} < \nu_t \text{ and } \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+ < \varepsilon_t^+ \leq \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+) \\
&= I_{j\leq 0}[\Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-) - \Phi_2(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_\nu^2; \sigma_-^2; \rho^-)] \\
&+I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)] \\
&+I_{j\geq 0}[\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_\nu^2;\sigma_+^2; -\rho^+) \\
&-\Phi_2(-\mu_2 + \mathbf{z}_t\boldsymbol{\gamma}; \alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_\nu^2;\sigma_+^2; -\rho^+)],
\end{aligned} \tag{3}$$

where $I_{j\leq 0}$ is an indicator function such that $I_{j\leq 0} = 1$ if $j \leq 0$, and $I_{j\leq 0} = 0$ if $j > 0$ (analogously for $I_{j\geq 0}$).

In the case of exogenous switching (when $\rho^- = \rho^+ = 0$), the probabilities of the outcome j in the ZIOP-3 model can be computed as

$$\begin{aligned}
&\Pr(y_t = j|\mathbf{z}_t, \mathbf{x}_t^-, \mathbf{x}_t^+, \rho^- = \rho^+ = 0) = I_{j\leq 0}\Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\boldsymbol{\sigma_\nu^2})[\Phi(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_-^2) \\
&-\Phi(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\sigma_-^2)] + I_{j=0}[\Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2) - \Phi(\mu_1 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)] \\
&+I_{j\geq 0}[1 - \Phi(\mu_2 - \mathbf{z}_t\boldsymbol{\gamma};\sigma_\nu^2)][\Phi(\alpha_j^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_+^2) - \Phi(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;\sigma_+^2)].
\end{aligned}$$

The inflated outcome does not have to be in the *very* middle of the ordered choices. If it is located at the *end* of the ordered scale, i.e. if $y_t \geq 0$ for $\forall t$, the ZIOP-3 model reduces to the ZIOP-2 model of Harris and Zhao (2007).

2.5 Maximum likelihood (ML) estimation

The probabilities in each OP equation can be consistently estimated under fairly general conditions by an asymptotically normal ML estimator (Basu and de Jong 2007). The simultaneous estimation of the OP equations in the NOP, ZIOP-2 and ZIOP-3 models can be also performed using an ML estimator of the vector of the parameters $\boldsymbol{\theta}$ that solves

$$\max_{\boldsymbol{\theta}\epsilon\boldsymbol{\Theta}} \sum_{t=1}^{T} \sum_{j=-J^-}^{J^+} I_{tj} \ln[\Pr(y_t = j|\mathbf{x}_t^{all}, \boldsymbol{\theta})], \tag{4}$$

where I_{tj} is an indicator function such that $I_{tj} = 1$ if $y_t = j$ and $I_{tj} = 0$ otherwise; $\boldsymbol{\theta}$ includes $\boldsymbol{\gamma}$, $\boldsymbol{\mu}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$, ρ^- and ρ^+ for the NOP and ZIOP-3 models, and $\boldsymbol{\gamma}$, μ, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and ρ for the ZIOP-2 model; Θ is a parameters' space; $\mathbf{x}_t^{all}$ is a vector that contains the values of all covariates in the model; and $\Pr(y_t = j|\mathbf{x}_t^{all}, \boldsymbol{\theta})$ are the probabilities from either (1) or (2) or (3). The asymptotic standard errors of $\widehat{\boldsymbol{\theta}}$ can be computed from the Hessian matrix.

The intercept components of $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$ and $\boldsymbol{\gamma}$ are identified up to scale and location, that is only jointly with the corresponding threshold parameters $\boldsymbol{\alpha}$, $\boldsymbol{\alpha}^-$, $\boldsymbol{\alpha}^+$ and $\boldsymbol{\mu}$ and variances σ^2, σ_-^2, σ_+^2, and σ_ν^2. As is common in the identification of discrete choice models, the variances σ^2, σ_-^2, σ_+^2, and σ_ν^2 are fixed to one, and the intercept components of $\boldsymbol{\beta}$, $\boldsymbol{\beta}^-$, $\boldsymbol{\beta}^+$ and $\boldsymbol{\gamma}$ are fixed to zero. The probabilities in (1), (2) and (3) are invariant to these (arbitrary) identifying assumptions: up to scale and location, we can identify all parameters in $\boldsymbol{\theta}$ because of the nonlinearity of OP equations, i.e. via the functional form (Heckman 1978; Wilde 2000). However, since the normal CDF is approximately linear in the middle of its support, the simultaneous estimation of two or three equations may experience a weak identification problem if regime and outcome equations contain the same set of covariates. To enhance the precision of parameter estimates we may impose exclusion restrictions on the specification of covariates in each equation.

The three regimes (nests) in the NOP model are fully observable, contrary to the latent (only partially observed) regimes in the ZIOP-2 and ZIOP-3 models. The likelihood function of the NOP model — again in contrast with the ZIOP-2 and ZIOP-3 models — is separable with respect to the parameters in the three equations. Thus, solving (4) for the NOP model is equivalent to maximizing separately the likelihoods of the three OP models representing the upper- and lower-level decisions.[3]

2.6 Marginal effects (ME)

The marginal effect of a continuous covariate k (the k^{th} element of $\mathbf{x}_t^{all}$) on the probability of each discrete outcome j are computed for the ZIOP-3 model as

[3]The data matrices in the lower-level decisions should be truncated to contain only those rows of $\mathbf{x}_t^-$ or $\mathbf{x}_t^+$ for which $y_t < 0$ or $y_t > 0$, respectively.

$$
\begin{aligned}
\mathrm{ME}_{k,j,t} &= \frac{\partial \Pr(y_t=j|\boldsymbol{\theta})}{\partial \mathbf{x}_{t,k}^{all}} = I_{j\le 0}\left\{\left[\Phi\left(\frac{\mu_1-\mathbf{z}_t\boldsymbol{\gamma}-\rho^-(\alpha_j^- -\mathbf{x}_t^-\boldsymbol{\beta}^-)}{\sqrt{1-(\rho^-)^2}}\right) f(\alpha_j^- - \mathbf{x}_t^-\boldsymbol{\beta}^-)\right.\right.\\
&-\Phi\left(\frac{\mu_1-\mathbf{z}_t\boldsymbol{\gamma}-\rho^-(\alpha_{j+1}^- -\mathbf{x}_t^-\boldsymbol{\beta}^-)}{\sqrt{1-(\rho^-)^2}}\right) \left. f(\alpha_{j+1}^- - \mathbf{x}_t^-\boldsymbol{\beta}^-)\right] \boldsymbol{\beta}_k^{-all}\\
&-\left.\left[\Phi\left(\frac{\alpha_{j+1}^- -\mathbf{x}_t^-\boldsymbol{\beta}^- -\rho^-(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})}{\sqrt{1-(\rho^-)^2}}\right) - \Phi\left(\frac{\alpha_{j}^- -\mathbf{x}_t^-\boldsymbol{\beta}^- -\rho^-(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})}{\sqrt{1-(\rho^-)^2}}\right)\right] f(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})\boldsymbol{\gamma}_k^{all}\right\}\\
&-I_{j=0}[f(\mu_2-\mathbf{z}_t\boldsymbol{\gamma})-f(\mu_1-\mathbf{z}_t\boldsymbol{\gamma})]\boldsymbol{\gamma}_k^{all}\\
&+I_{j\ge 0}\left\{\left[\Phi\left(\frac{\mathbf{z}_t\boldsymbol{\gamma}-\mu_2+\rho^+(\alpha_{j-1}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+)}{\sqrt{1-(\rho^+)^2}}\right) f(\alpha_{j-1}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+)\right.\right.\\
&-\Phi\left(\frac{\mathbf{z}_t\boldsymbol{\gamma}-\mu_2+\rho^+(\alpha_{j}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+)}{\sqrt{1-(\rho^+)^2}}\right) \left. f(\alpha_{j}^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+)\right] \boldsymbol{\beta}_k^{+all}\\
&+\left.\left[\Phi\left(\frac{\alpha_{j}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+ +\rho^+(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)}{\sqrt{1-(\rho^+)^2}}\right) - \Phi\left(\frac{\alpha_{j-1}^+ -\mathbf{x}_t^+\boldsymbol{\beta}^+ +\rho^+(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)}{\sqrt{1-(\rho^+)^2}}\right)\right] f(\mathbf{z}_t\boldsymbol{\gamma}-\mu_2)\boldsymbol{\gamma}_k^{all}\right\},
\end{aligned}
$$

where f is the probability density function of the standard normal distribution, and $\boldsymbol{\gamma}_k^{all}$, $\boldsymbol{\beta}_k^{-all}$ and $\boldsymbol{\beta}_k^{+all}$ are the coefficients on the k^{th} covariate in $\mathbf{x}_t^{all}$ in the regime equation, outcome equation conditional on $s_t^*=1$ and outcome equation conditional on $s_t^*=-1$, respectively ($\boldsymbol{\gamma}_k^{all}$, $\boldsymbol{\beta}_k^{-all}$ or $\boldsymbol{\beta}_k^{+all}$ is zero if the k^{th} covariate in $\mathbf{x}_t^{all}$ is not included into the corresponding equation). For a discrete-valued covariate, the ME can be computed as the change in the probabilities when this covariate changes by one increment and all other covariates are fixed.

The MEs for the NOP model are computed by replacing in the above formula $I_{j\ge 0}$ with $I_{j>0}$ and $I_{j\le 0}$ with $I_{j<0}$.

The MEs for the ZIOP-2 model are computed as

$$
\begin{aligned}
\mathrm{ME}_{k,j,t} &= \frac{\partial \Pr(y_t=j|\boldsymbol{\theta})}{\partial \mathbf{x}_{t,k}^{all}} = -I_{j=0}[f(\mu-\mathbf{z}_t\boldsymbol{\gamma})]\boldsymbol{\gamma}_k^{all}\\
&+\left[\Phi\left(\frac{\mathbf{z}_t\boldsymbol{\gamma}-\mu+\rho(\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta})}{\sqrt{1-\rho^2}}\right) f(\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta}) - \Phi\left(\frac{\mathbf{z}_t\boldsymbol{\gamma}-\mu+\rho(\alpha_{j}-\mathbf{x}_t\boldsymbol{\beta})}{\sqrt{1-\rho^2}}\right) f(\alpha_{j}-\mathbf{x}_t\boldsymbol{\beta})\right]\boldsymbol{\beta}_k^{all}\\
&+\left[\Phi\left(\frac{\alpha_j-\mathbf{x}_t\boldsymbol{\beta}+\rho(\mathbf{z}_t\boldsymbol{\gamma}-\mu)}{\sqrt{1-\rho^2}}\right) - \Phi\left(\frac{\alpha_{j-1}-\mathbf{x}_t\boldsymbol{\beta}+\rho(\mathbf{z}_t\boldsymbol{\gamma}-\mu)}{\sqrt{1-\rho^2}}\right)\right] f(\mathbf{z}_t\boldsymbol{\gamma}-\mu)\boldsymbol{\gamma}_k^{all},
\end{aligned}
$$

where $\boldsymbol{\beta}_k^{all}$ is the coefficient on the k^{th} covariate in $\mathbf{x}_t^{all}$ in the outcome equation ($\boldsymbol{\beta}_k^{all}$ is zero if the k^{th} covariate in $\mathbf{x}_t^{all}$ is not included into the outcome equation).

The asymptotic standard errors of the MEs are computed using the Delta method as the square roots of the diagonal elements of

$$
\widehat{Var(\widehat{\mathbf{ME}}_{k,j,t})} = \nabla_\theta \widehat{\mathbf{ME}}_{k,j,t} \widehat{Var(\widehat{\boldsymbol{\theta}})} \nabla_\theta \widehat{\mathbf{ME}}_{k,j,t})'.
$$

2.7 Relations among the models and their comparison

We discuss now the choice of a formal statistical test to compare the NOP, ZIOP-2, ZIOP-3 and conventional OP models. The choice depends on whether the models are nested in each other.

The exogenous-switching version of each model is nested in its endogenous-switching version as its uncorrelated special case; their comparison can be performed using any classical lilkelihood-based test for nested hypotheses, such as the likelihood ratio (LR) test.

The OP is not nested either in the NOP or ZIOP-3 model. We can compare the OP model with them using a likelihood-based test for non-nested models, such as the Vuong test

(Vuong 1989). The OP model is however nested in the ZIOP-2 model. The latter reduces to the former if $\mu \rightarrow -\infty$; hence, $\Pr(y_t = 0|\mathbf{x}_t, s_t^* = 1) \rightarrow 0$. Therefore, the Vuong test for non-nested hypothesis may not be used to compare the OP and ZIOP-2 model. For nested hypothesis, the Voung test reduces to the LR test, which can be performed for the comparison of the OP and ZIOP-2 models. However, the critical values of the classical LR test are invalid in this case since some standard regularity conditions of the classical LR test fail to hold. In particular, the value of μ in the null hypothesis is not an interior point of the parameter space; hence, the asymptotic distribution of the LR statistics is not standard. Instead, one may use the simulated adjusted critical values, which can be computed following an algorithm provided in Andrews (2001).[4]

The NOP model is nested in the ZIOP-3 model. The latter becomes the former if $\alpha_{-1}^{-} \rightarrow \infty$ and $\alpha_1^{+} \rightarrow -\infty$; therefore, $\Pr(y_t = 0|\mathbf{x}_t^{+}, s_t^* = 1) \rightarrow 0$ and $\Pr(y_t = 0|\mathbf{x}_t^{-}, s_t^* = -1) \rightarrow 0$. The values of α_{-1}^{-} and α_1^{+} in the null hypothesis are not the interior points of the parameter space; thus, the asymptotic distribution of the LR statistics is not standard. The comparison of the NOP and ZIOP-3 models can also be performed using the LR test with the simulated adjusted critical values following Andrews (2001).

Generally, the ZIOP-2 model is not a special case of the ZIOP-3 model, and vice versa. We can compare them using the Vuong test. A special case when the ZIOP-3 model nests the ZIOP-2 model emerges under some restrictions on the parameters as explained below. In this case, the selection between the ZIOP-3 and ZIOP-2 models can be performed using any classical likelihood-based test for nested hypotheses.

The special case emerges if y_t takes on only three discrete values $j \in \{-1, 0, 1\}$, the regressors in $\mathbf{x}_t^{-}$ and $\mathbf{x}_t^{+}$ in the outcome equations of the ZIOP-3 model contain all regressors in the ZIOP-2 regime equation (denoted below by $\mathbf{z}_{2t}$ with the parameter vector $\boldsymbol{\gamma}_2$), and the regressors in the regime equation of the ZIOP-3 model (denoted below by $\mathbf{z}_{3t}$ with the parameter vector $\boldsymbol{\gamma}_3$) include all regressors in the $\mathbf{x}_t$ in the ZIOP-2 outcome equation. According to (2) the probabilities of the outcome j in the ZIOP-2 model are given by

$$\begin{aligned}
&\Pr(y_t = -1|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho);\\
&\Pr(y_t = 0|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi(\mu - \mathbf{z}_{2t}\boldsymbol{\gamma}_2) + \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_0 - \mathbf{x}_t\boldsymbol{\beta}; -\rho)\\
&-\Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho) = 1 - \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; -\alpha_0 + \mathbf{x}_t\boldsymbol{\beta}; \rho)\\
&-\Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_{-1} - \mathbf{x}_t\boldsymbol{\beta}; -\rho);\\
&\Pr(y_t = 1|\mathbf{z}_{2t}, \mathbf{x}_t) = \Phi(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}) - \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; \alpha_0 - \mathbf{x}_t\boldsymbol{\beta}; -\rho)\\
&= \Phi_2(-\mu + \mathbf{z}_{2t}\boldsymbol{\gamma}_2; -\alpha_0 + \mathbf{x}_t\boldsymbol{\beta}; \rho),
\end{aligned} \tag{5}$$

since $\Phi_2(x; y; \rho) = \Phi(x) - \Phi_2(x; -y; -\rho)$.

Similarly, according to (3) the probabilities of the outcome j in the ZIOP-3 model are given by

[4] Analogously, the use of the Vuong test for non-nested fypothesis, which is implemented in the `zip` and `zinb` commands to test for zero inflation in the Poisson and negative binomial model with a binary regime equation, is inappropriate, because these models are actually nested in their two-part zero-inflated extensions (Wilson 2015).

$$
\begin{aligned}
&\Pr(y_t=-1|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\rho^-);\\
&\Pr(y_t=0|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^- - \mathbf{x}_t^-\boldsymbol{\beta}^-;\rho^-)\\
&+\Phi(\mu_2-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)+\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+)\\
&=\Phi_2(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3;-\alpha_0^- + \mathbf{x}_t^-\boldsymbol{\beta}^-;-\rho^-)+\Phi(\mu_2-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)\\
&-\Phi(\mu_1-\mathbf{z}_{3t}\boldsymbol{\gamma}_3)+\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+);\\
&\Pr(y_t=1|\mathbf{z}_{3t},\mathbf{x}_t^-,\mathbf{x}_t^+)=\Phi(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3)-\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;\alpha_0^+ - \mathbf{x}_t^+\boldsymbol{\beta}^+;-\rho^+)\\
&=\Phi_2(-\mu_2+\mathbf{z}_{3t}\boldsymbol{\gamma}_3;-\alpha_0^+ + \mathbf{x}_t^+\boldsymbol{\beta}^+;\rho^+).
\end{aligned} \tag{6}
$$

Suppose the covariates in $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ in the ZIOP-3 outcome equations are identical to the covariates in $\mathbf{z}_{2t}$ in the ZIOP-2 regime equation, the covariates in $\mathbf{z}_{3t}$ in the ZIOP-3 regime equation are identical to the covariates in the $\mathbf{x}_t$ in the ZIOP-2 outcome equation, and the parameters are restricted as follows: $-\boldsymbol{\beta}^-=\boldsymbol{\beta}^+=\boldsymbol{\gamma}_2$, $\boldsymbol{\beta}=\boldsymbol{\gamma}_3$, $\mu_1=\alpha_{-1}$, $\mu_2=\alpha_0$, $-\alpha_0^-=\alpha_0^+=\mu$ and $-\rho^-=\rho^+=\rho$. Then, since $\mathbf{x}_t^-=\mathbf{x}_t^+=\mathbf{z}_{2t}$, $\mathbf{z}_{3t}=\mathbf{x}_t$ and $\Phi(-x)=1-\Phi(x)$, the probabilities for the ZIOP-3 model in (6) can be written as

$$
\begin{aligned}
&\Pr(y_t=-1|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho);\\
&\Pr(y_t=0|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};\mu-\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho)+\Phi(\alpha_0-\mathbf{x}_t\boldsymbol{\beta})-\Phi(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta})\\
&+\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};\mu-\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho)=-\Phi_2(\alpha_{-1}-\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;-\rho)+1\\
&-\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho);\\
&\Pr(y_t=1|\mathbf{x}_t,\mathbf{z}_{2t})=\Phi_2(-\alpha_0+\mathbf{x}_t\boldsymbol{\beta};-\mu+\mathbf{z}_{2t}\boldsymbol{\gamma}_2;\rho),
\end{aligned}
$$

which are identical to the probabilities for the ZIOP-2 model in (5).

Notice that the restrictions $-\boldsymbol{\beta}^-=\boldsymbol{\beta}^+=\boldsymbol{\gamma}_2$ and $-\alpha_0^-=\alpha_0^+=\mu$ impose a sort of symmetry in the ZIOP-3 model, because they imply that the conditional probability of a positive response is equal to the conditional probability of a negative response:

$$
\begin{aligned}
&\Pr(y_t=1|\mathbf{z}_{3t},\mathbf{x}_t^+,s_t^*=1)=1-\Phi(\alpha_0^+-\mathbf{x}_t^+\boldsymbol{\beta}^+)=\\
&=\Phi(-\alpha_0^+ + \mathbf{x}_t^+\boldsymbol{\beta}^+)=\Phi(\alpha_0^- - \mathbf{x}_t^-\boldsymbol{\beta}^-)=\Pr(y_t=-1|\mathbf{z}_t,\mathbf{x}_t^-,s_t^*=-1).
\end{aligned}
$$

In general, if $\mathbf{x}_t^-$ and $\mathbf{x}_t^+$ are not identical to $\mathbf{z}_{2t}$ but contain all covariates in $\mathbf{z}_{2t}$, and if $\mathbf{z}_{3t}$ is not identical to $\mathbf{x}_t$ but contains all covariates in $\mathbf{x}_t$, the ZIOP-2 model is still nested in the ZIOP-3 model with the additional zero restrictions for the coefficients on all extra covariates in $\mathbf{x}_t^-$, $\mathbf{x}_t^+$ and $\mathbf{z}_{3t}$.

3 The nop, ziop2 and ziop3 commands in Stata

The accompanying software includes the command, postestimation and supporting files.

3.1 Syntax

The following commands estimate, respectively, the NOP, ZIOP-2 and ZIOP-3 models for discrete ordinal outcomes:

`nop` *depvar* [*indepvars_reg*] [*if*] [*in*] [, `pos_indepvars(`*varlist*`)`

neg_indepvars(*varlist*) infcat(*choice*) endoswitch cluster(*varname*)
robust nolog vuong initial(*string*)]

ziop2 *depvar* [*indepvars_reg*] [*if*] [*in*] [, indepvars(*varlist*) infcat(*choice*)
endoswitch cluster(*varname*) robust nolog initial(*string*)]

ziop3 *depvar* [*indepvars_reg*] [*if*] [*in*] [, pos_indepvars(*varlist*)
neg_indepvars(*varlist*) infcat(*choice*) endoswitch cluster(*varname*)
robust nolog vuong initial(*string*)]

The dependent variable *depvar* is assumed to take on at least five discrete ordinal values in the NOP model, at least two — in the ZIOP-2 model, and at least three — in the ZIOP-3 model. The set of the covariates *indepvars_reg* in the regime equation may differ from the sets of the covariates pos_indepvars(*varlist*) and neg_indepvars(*varlist*) in the NOP and ZIOP-3 outcome equations conditional on the regimes $s_t = 1$ and $s_t = -1$, respectively, and the set of the covariates indepvars(*varlist*) in the ZIOP-2 outcome equation. The models allow for either exogenous or endogenous switching among the latent regimes. Regime switching is endogenous if the unobserved random term in the regime equation is correlated with the unobserved random term in the outcome equation, and exogenous otherwise.

Options

pos_indepvars(*varlist*) specifies for the NOP and ZIOP-3 models the list of the covariates in the outcome equation conditional on the regime $s_t = 1$ for nonnegative outcomes; by default, it is identical to *indepvars_reg*, the list of the covariates in the regime equation.

neg_indepvars(*varlist*) specifies for the NOP and ZIOP-3 models the list of the covariates in the outcome equation conditional on the regime $s_t = -1$ for nonpositive outcomes; by default, it is identical to *indepvars_reg*, the list of the covariates in the regime equation.

indepvars(*varlist*) specifies for the ZIOP-2 model the list of the covariates in the outcome equation; by default, it is identical to *indepvars_reg*, the list of the covariates in the regime equation.

infcat(*choice*) is a value of the dependent variable in the regime $s_t = 0$ that should be modeled as inflated in the ZIOP-2 and ZIOP-3 models, and as neutral in the NOP model; by default, it equals 0.

endoswitch specifies that endogenous regime switching is to be used instead of the default exogenous switching.

robust specifies that the robust sandwich estimator of variance is to be used; the default estimator is based on the observed information matrix.

cluster(*varname*) specifies a clustering variable for the clustered robust sandwich estimator of variance.

initial(*string*) specifies the space-delimited list *string* of starting values of the parameters, in the following order: $\boldsymbol{\gamma}$, $\boldsymbol{\mu}$, $\boldsymbol{\beta}^+$, $\boldsymbol{\alpha}^+$, $\boldsymbol{\beta}^-$, $\boldsymbol{\alpha}^-$, ρ^- and ρ^+ for the NOP and ZIOP-3 models, and $\boldsymbol{\gamma}$, μ, $\boldsymbol{\beta}$, $\boldsymbol{\alpha}$ and ρ for the ZIOP-2 model.

vuong specifies that the Vuong (1989) test of NOP (or ZIOP-3) model versus the conventional OP one should be performed. The reported Vuong test statistics (the standard one and the two adjusted test statistics with corrections to address the comparison of models with different numbers of parameters based on the Akaike (AIC) and Bayesian

(BIC) information criteria) has a standard normal distribution with large positive values favoring the NOP (or ZIOP-3) model and large negative values favoring the OP model.

`nolog` suppresses the iteration log.

Stored results

The help files include the descriptions of the stored results by the `nop`, `ziop2`, and `ziop3` commands.

3.2 Postestimation commands

The following postestimation commands are available after `nop`, `ziop2` and `ziop3`.

The predict command

`predict` *varname* [*if*] [*in*] [, `zeros regimes output(`*string*`)`]

The `predict` command computes either the predicted probabilities of discrete choices (by default), or the predicted probabilities of the regimes and types of zeros, or the expected values of the dependent variable. `predict` creates $(-J^- + -J^+ + 1)$ new variables under the name with *varname* prefix. The following options are available:

`zeros` indicates that the probabilities of the different types of zeros (the outcomes in the inflated category), conditional on different regimes, must be predicted instead of the probabilities of different discrete choices.

`regimes` indicates that the regime probabilities must be predicted instead of the choice probabilities. This option is ignored if the `zeros` option is used.

`output(`*string*`)` specifies different types of predictions. The possible values of *string* are: *choice* for reporting the predicted outcome (the choice with the largest predicted probability); *mean* for reporting the expected value of the dependent variable computed as $\sum_i i \Pr(y_t = i)$; and *cum* for predicting the cumulative choice probabilities: $\Pr(y_t <= -J^-)$, $\Pr(y_t <= -J^- + 1)$, ..., $\Pr(y_t <= J^+)$. If *string* is not specified, the usual choice probabilities $\Pr(y_t = -J^-)$, $\Pr(y_t = -J^- + 1)$, ..., $\Pr(y_t = J^+)$ are predicted and stores into the new variables with *varname* prefix.

The ziopprobabilities command

`ziopprobabilities [, at(`*string*`) zeros regimes]`

This command shows the predicted probabilities, estimated for the specified values of the covariates, along with the standard errors. The options `zeros` and `regimes` are specified as in `predict`. The option `at()` is specified as follows:

`at(`*string*`)` specifies for which values of the covariates to estimate the predictions. If `at(`*string*`)` is used (*string* is a list of *varname* = *value* expressions, separated by commas), the predictions are estimated for these values and displayed without saving to the dataset. If some covariate names are not specified, their mean values are taken instead. If `at()` is not used, by default the predictions are estimated for the covariate mean values.

The ziopcontrasts command

`ziopcontrasts [, at(`*string*`) to(`*string*`) zeros regime]`

This command shows the differences in the predicted probabilities, estimated first for the values of the covariates in `at()`and then in `to(`*string*`)`, along with the standard errors. The options `zeros`, `regimes` and `at()` are specified as in `ziopprobabilities`. The options `to()` is specified analogously to `at()`.

The ziopmargins command

`ziopmargins [, at(`*string*`) zeros regimes]`

This command prints the marginal effects of each covariate on the choice probabilities, estimated for the specified values of the covariates, along with the standard errors. The options `zeros`, `regimes` and `at()` are specified as in `ziopprobabilities`.

The ziopclassification command

`ziopclassification`

This command prints: the classification (contingency) table; the percentage of correct predictions; the two strictly proper scoring rules: the probability, or Brier, score (Brier 1950) and the ranked probability score (Epstein 1969); and the adjusted noise-to-signal ratios (Kaminsky and Reinhart 1999).

The classification table displays the predicted choices (ones with the highest predicted probability) in rows, the actual choices in columns, and the number of (mis)classifications in each cell.

The Brier probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=J^-}^{J^+}[\Pr(y_t = j) - I_{jt}]^2$, where indicator $I_{jt}=1$ if $y_t = j$ and $I_{jt}=0$ otherwise. The ranked probability score is computed as $\frac{1}{T}\sum_{t=1}^{T}\sum_{j=J^-}^{J^+}[Q_{jt} - D_{jt}]^2$, where $Q_{jt} = \sum_{i=J^-}^{j}\Pr(y_t = i)$ and $D_{jt} = \sum_{i=J^-}^{j} I_{it}$. The better the prediction, the smaller both score values. Both scores have a minimum value of zero when all actual outcomes are predicted with a unit probability.

The adjusted noise-to-signal ratios are defined as follows. Let A denote the event that the outcome was predicted and occurred; let B denote the event that the outcome was predicted but did not occur; let C denote the event that the outcome was not predicted but occurred; and let D denote the event that the outcome was not predicted and did not occur. The desirable outcomes fall into categories A and D, while the noisy ones fall into categories B and C. A perfect prediction has no entries in B and C, while a noisy prediction has many entries in B and C, but few in A and D. The adjusted noise-to-signal ratio is defined for each choice as $[B/(B{+}D)]/[A/(A{+}C)]$.

The ziopvuong command

`ziopvuong` *modelspec*$_1$ *modelspec*$_2$

This command performs the non-nested Vuong test (Vuong 1989) which compares the closeness of two models to the true data distribution using the difference of the pointwise log likelihoods of the two models. Arguments *modelspec*$_1$ and *modelspec*$_2$ are the names under which the estimation results were saved using the `estimates store` command. Any model that stores the vector `e(ll_obs)` of observation-wise log-likelihood technically can be used to perform the test. The command provides three z-scores, the Vuong test statistics: the standard one and the two adjusted test statistics with corrections to address the comparison of models with different numbers of parameters based on the AIC and BIC. They can be used to test the hypothesis that one of the models explains the data better than the other. A significant positive Vuong test statistic indicates preference for the first model, while a significant negative value of the z-score indicates preference for the second model. Non significant z-score implies no preference for either model.

4 Monte Carlo simulations

We conducted extensive Monte Carlo experiments to illustrate the finite sample performance of the ML estimators of each model.

4.1 Monte Carlo design

We simulated six processes generated by the NOP, ZIOP-2 and ZIOP-3 models, each of them with both exogenous and endogenous switching. The repeated samples with 200, 500 and 1,000 observations were independently generated and then estimated by the true model. The number of replications was 10,000 in each experiment.

Three covariates $\mathbf{w_1}$, $\mathbf{w_2}$ and $\mathbf{w_3}$ were drawn in each replication as $\mathbf{w_1} \overset{iid}{\sim} \mathcal{N}(0,1) + 2$, $\mathbf{w_2} \overset{iid}{\sim} \mathcal{N}(0,1)$, and $\mathbf{w_3} = -1$ if $\mathbf{u} \leq 0.3$, 0 if $0.3 < \mathbf{u} \leq 0.7$, or 1 if $\mathbf{u} > 0.7$, where $\mathbf{u} \overset{iid}{\sim} \mathcal{U}[0,1]$. The repeated samples were generated for the NOP and ZIOP-3 models with $\mathbf{Z} = (\mathbf{w_1}, \mathbf{w_2})$, $\mathbf{X}^- = (\mathbf{w_1}, \mathbf{w_3})$, $\mathbf{X}^+ = (\mathbf{w_2}, \mathbf{w_3})$, and for the ZIOP-2 model with $\mathbf{Z} = (\mathbf{w}_1, \mathbf{w}_3)$, $\mathbf{X} = (\mathbf{w}_2, \mathbf{w}_3)$. The dependent variable y was generated with five values: -2, -1, 0, 1 and 2. The parameters were calibrated to yield on average the following frequencies of the above outcomes: 7%, 14%, 58%, 14% and 7%, respectively. To avoid the divergence of ML estimates due to the problem of complete separation (perfect prediction), which could happen if the actual number of observations in any outcome category is very low, the samples with any outcome category frequency lower than 6% were re-generated. The variances of the errors in all equations were fixed to one. The true values of all other parameters in the simulations are shown in Table A1 in Appendix. The starting values for slope and threshold parameters were obtained using the independent OP estimations of each equation. The starting values for ρ, ρ^- and ρ^+ were obtained by maximizing the likelihood functions of the endogenous-switching models holding the other parameters fixed at their estimates in the corresponding exogenous-switching model. The values of the choice probabilities, which depend on the values of the regressors, are computed at the population medians of the covariates.

4.2 Monte Carlo results

Table 1 reports the measures of accuracy for the estimates of the choice probabilities. The results for the estimates of the parameters and MEs are qualitatively and quantitatively similar.

Table 1. Monte Carlo results: The accuracy of ML estimators

Sample size	True and estimated model:	NOP ($\rho^-=\rho^+=0$)	NOP	ZIOP-2 ($\rho=0$)	ZIOP-2	ZIOP-3 ($\rho^-=\rho^+=0$)	ZIOP-3
	The accuracy of the estimates of choice probabilities						
200	Bias, %	2.3	1.5	4.4	5.1	3.3	3.1
500		1.1	0.9	2.3	3.0	1.6	1.5
1000		0.4	0.4	1.3	1.7	0.8	1.0
200	RMSE, ×100	2.4	2.6	2.8	2.9	2.7	2.9
500		1.5	1.6	1.7	1.8	1.6	1.8
1000		1.1	1.1	1.2	1.2	1.1	1.3
200	Coverage rate (at 95% level), %	94.4	94.4	95.3	95.3	95.1	94.8
500		95.4	95.2	95.6	95.6	95.9	95.7
1000		95.5	95.5	95.7	95.7	95.6	95.6
200	Bias of standard error estimates, %	4.2	4.2	6.9	6.4	5.5	15.1
500		3.9	4.6	6.9	6.1	5.3	16.6
1000		2.6	3.4	5.7	5.9	3.7	13.9

Notes: Bias – the absolute difference between the estimated and true values, divided by the true value; RMSE – the absolute root mean square error of the estimates; Coverage rate – the percentage of times the estimated asymptotic 95% confidence intervals cover the true values; Bias of standard error estimates – the absolute difference between the average of the estimated asymptotic standard errors of the estimates and the standard deviation of the estimates in all replications. The above measures are averaged across five outcome categories.

The simulations show that the ML estimators are consistent and reliable even in samples with only 200 observations: the biases of choice probability estimates are smaller than five percent and the asymptotic coverage rates differ from the nominal 0.95 level by less than one percent. For each model, the bias and RMSE decrease as sample size increases. The RMSE decreases in most cases faster than the asymptotic rate $\sqrt{n}$. This may be caused by a small number of large deviations in parameter estimation in small samples. For all models and sample sizes, the bias and RMSE are expectedly slightly higher for a more complex endogenous-switching version. The standard error estimates on average correspond to the actual standard errors; however, large deviations make standard error estimates biased in small samples, but do not move the coverage rates from the nominal level by more than one percent even with only 200 observations. The accuracy in the NOP models is expectedly higher than in the zero-inflated OP models.

5 Examples

The real-data application analyzes a time-series sample of all decisions of the U.S. Federal Open Market Committee (FOMC) on the federal funds rate target made at the scheduled and unscheduled meetings during the 9/1987 – 9/2008 period.

The dependent variable, the change to the rate target, is classified into five ordered categories: "-0.5" (a cut of 0.5% or more), "-0.25" (a cut less than 0.5% but more than 0.0625%), "0" (no change or change by not more than 0.0625%), "0.25" (a hike more than

0.0625% but less than 0.5%) and "0.5" (a hike of 0.5% or more). The FOMC decisions are aligned with the real-time values of the explanatory variables as they were truly available to the public on the previous day before each FOMC meeting. The explanatory variables include: `spread` (the difference between the one-year treasury constant maturity rate and the effective federal funds rate, five-business-day moving average, data source: ALFRED); `pb` (the trichotomous indicator that we constructed from the 'policy bias' statements at the previous FOMC meeting: it is equal to 1 if the statement was asymmetric toward tightening, 0 if the statement was symmetric, and -1 if the statement was asymmetric toward easing; data source: FOMC statements and minutes[5]); `houst` (the Greenbook projection for the current quarter of the total number of new privately owned housing units started, data source: RTDSM[6]); `gdp` (the Greenbook projection for the current quarter of quarterly growth in the nominal gross domestic (before 1992: national) product, annualized percentage points, data source: RTDSM).

We start by estimating the conventional OP model using the `oprobit` command:

```
. oprobit rate_change spread pb houst gdp, nolog

Ordered probit regression                       Number of obs     =        210
                                                LR chi2(4)        =     214.54
                                                Prob > chi2       =     0.0000
Log likelihood = -159.56242                     Pseudo R2         =     0.4020

------------------------------------------------------------------------------
 rate_change |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      spread |   1.574232   .1870759     8.41   0.000      1.20757    1.940894
          pb |   .9262378   .1479364     6.26   0.000     .6362877    1.216188
       houst |   1.373179   .3459397     3.97   0.000     .6951499    2.051209
         gdp |   .2390714   .0571926     4.18   0.000     .1269761    .3511668
-------------+----------------------------------------------------------------
       /cut1 |   .4656819   .5382091                     -.5891885    1.520552
       /cut2 |     1.8382   .5339707                      .7916362    2.884763
       /cut3 |   4.835985   .6359847                      3.589478    6.082492
       /cut4 |   6.331172   .6875922                      4.983516    7.678828
------------------------------------------------------------------------------

. estat ic

Akaike's information criterion and Bayesian information criterion

-----------------------------------------------------------------------------
       Model |        Obs  ll(null)  ll(model)      df         AIC        BIC
-------------+---------------------------------------------------------------
           . |        210 -266.8308  -159.5624       8    335.1248   361.9017
-----------------------------------------------------------------------------
```

We now allow the negative, zero and positive changes to the rate target to be generated by different processes, and estimate the three-part NOP model. The `nop` command yields the following results:

[5] https://www.federalreserve.gov/monetarypolicy/fomc_historical.htm.

[6] RTDSM (Real-Time Data Set for Macroeconomists) is available at https://www.philadelphiafed.org.

```
. nop rate_change spread pb houst gdp, xn(spread gdp) xp(spread pb) infcat(0) endoswitch nolog
  Three-part nested ordered probit model with endogenous switching
  Number of observations = 210
  Log likelihood = -150.2325
  AIC            = 328.465
  BIC            = 375.3246
```

rate_change	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Regime equation						
spread	1.626627	.2240853	7.26	0.000	1.187428	2.065826
pb	.8733051	.1596507	5.47	0.000	.5603955	1.186215
houst	2.370716	.4309395	5.50	0.000	1.52609	3.215342
gdp	.2583803	.072696	3.55	0.000	.1158988	.4008617
/cut1	3.31599	.6781702	4.89	0.000	1.986801	4.645179
/cut2	6.52286	.8276111	7.88	0.000	4.900772	8.144948
Outcome equation (+)						
spread	1.738494	.6426342	2.71	0.007	.4789541	2.998034
pb	2.241514	.8656279	2.59	0.010	.5449148	3.938114
/cut1	3.418268	.92534	3.69	0.000	1.604635	5.231901
Outcome equation (-)						
spread	1.220589	.4027208	3.03	0.002	.4312708	2.009907
gdp	.2099337	.1125002	1.87	0.062	-.0105625	.43043
/cut1	-.6292352	.4181413	-1.50	0.132	-1.448777	.1903067
Correlation coefficients						
rho(+)	.4950234	.7379739	0.67	0.502	-.9513788	1.941426
rho(-)	.5371233	.4628359	1.16	0.246	-.3700184	1.444265

The NOP model provides a substantial improvement of the likelihood, and is preferred to the standard OP model according to the AIC (the BIC favors the OP model). The endogenous switching does not significantly change the likelihood (the log likelihood with exogenous switching is -151.0, the p-value of the LR test of the null of exogenous switching is 0.48), the correlation coefficients $\rho^-$and ρ^+ are not significant, and both information criteria favor the NOP model with exogenous switching (the AIC is 325.9, the BIC is 366.1).

Next we allow for an inflation of zero outcomes and estimate the three-part ZIOP-3 model. The `ziop3` command with exogenous switching yields the following results:

```
. ziop3 rate_change spread pb houst gdp, xn(spread gdp) xp(spread pb) infcat(0) nolog
```

(output omitted)

```
Three-part zero-inflated ordered probit model with exogenous switching
Number of observations = 210
Log likelihood = -139.5529
AIC            = 307.1058
BIC            = 353.9653
```

rate_change	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Regime equation						
spread	2.106257	.364262	5.78	0.000	1.392317	2.820198
pb	1.628486	.3356997	4.85	0.000	.9705269	2.286446
houst	5.311379	.9913486	5.36	0.000	3.368372	7.254387
gdp	.3809606	.1085468	3.51	0.000	.1682127	.5937084
/cut1	9.103481	1.772781	5.14	0.000	5.628894	12.57807
/cut2	12.3481	1.952013	6.33	0.000	8.522227	16.17398
Outcome equation (+)						
spread	1.809669	.7282205	2.49	0.013	.3823831	3.236955
pb	2.620109	.9836793	2.66	0.008	.6921334	4.548085
/cut1	-1.481781	1.015198	-1.46	0.144	-3.471532	.5079697
/cut2	3.509078	1.070858	3.28	0.001	1.410236	5.607921
Outcome equation (-)						
spread	1.072859	.2690323	3.99	0.000	.5455655	1.600153
gdp	.177697	.0742318	2.39	0.017	.0322055	.3231886
/cut1	-.6373707	.3361142	-1.90	0.058	-1.296142	.021401
/cut2	.7569744	.3460019	2.19	0.029	.0788232	1.435126

The empirical evidence in favor of zero inflation is convincing: with only two extra parameters, the ZIOP-3 model has much higher likelihood than the NOP model (-139.6 vs. -151.0), and is clearly preferred by both the AIC and the BIC to the NOP and OP models. The endogenous switching does not significantly improve the likelihood of the ZIOP-3 model either (the p-value of the LR test of exogenous switching is 0.30, and both the AIC and BIC prefer the exogenous switching), though one of the correlation coefficients is significant at the 0.05 level:

```
. ziop3 rate_change spread pb houst gdp, xn(spread gdp) xp(spread pb) infcat(0) endoswitch
```

(output omitted)

```
Three-part zero-inflated ordered probit model with endogenous switching
Number of observations = 210
Log likelihood = -138.3437
AIC            = 308.6873
BIC            = 362.2411
```

(output omitted)

In contrast, the likelihood of the two-part ZIOP-2 model is even lower than that of the NOP model. According both to the AIC and BIC, the ZIOP-2 model is inferior to all the above models, icluding the OP one. The `ziop2` command yields the following results:

```
. ziop2 rate_change spread pb houst gdp, x(spread pb houst gdp ) infcat(0) nolog
  Two-part zero-inflated ordered probit model with exogenous switching
  Number of observations = 210
  Log likelihood = -154.3563
  AIC            = 334.7126
  BIC            = 378.225
```

rate_change	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
Regime equation						
spread	-.5718098	.4932372	-1.16	0.246	-1.538537	.3949173
pb	2.220756	1.124943	1.97	0.048	.015908	4.425605
houst	.4317792	.9262931	0.47	0.641	-1.383722	2.24728
gdp	-.3039409	.1561281	-1.95	0.052	-.6099462	.0020645
/cut1	-3.269292	2.104548	-1.55	0.120	-7.394131	.8555464
Outcome equation						
spread	1.920514	.2407834	7.98	0.000	1.448587	2.392441
pb	1.21367	.1982338	6.12	0.000	.8251391	1.602201
houst	1.637904	.3932584	4.16	0.000	.8671315	2.408676
gdp	.2358575	.0628755	3.75	0.000	.1126239	.3590911
/cut1	.5651226	.5985828	0.94	0.345	-.6080782	1.738323
/cut2	2.422641	.6270021	3.86	0.000	1.193739	3.651542
/cut3	5.397053	.7416277	7.28	0.000	3.94349	6.850617
/cut4	7.039527	.8100945	8.69	0.000	5.451771	8.627283

The Vuong test prefers the ZIOP-3 model to the ZIOP-2 model at the 0.01 significance level using the standard critical values, and at the 0.03 level using the corrections based on the AIC and BIC:

```
. quietly ziop3 rate_change pb spread houst gdp, xn(spread gdp )xp(pb spread) infcat(0)
```

(output omitted)

```
. est store ziop3_model

. quietly ziop2 rate_change spread pb houst gdp, x(spread pb houst gdp ) infcat(0)

. est store ziop2_model

. ziopvuong ziop3_model ziop2_model
Vuong non-nested test for ziop3_model vs ziop2_model
  Mean difference in log likelihood                  .0704922
  Standard deviation of difference in log likelihood .4235306
  Number of observations                             210
  Vuong test statistic                           z = 2.411939
  P-Value                                     Pr>z = .007934
     with AIC (Akaike) correction                z = 2.249007
  P-Value                                     Pr>z = .012256
     with BIC (Schwarz) correction               z = 1.976332
  P-Value                                     Pr>z = .0240586
```

Now we report the selected output of some postestimation commands for the ZIOP-3 model with exogenous switching, which is preferred by both the AIC and BIC. The `ziopprobabilities` command reports (by default) the choice probabilities and their standard errors, evaluated for the specified values of the covariates:

```
. ziopprobabilities, at (pb=1, spread=0.426, houst=1.6, gdp=6.8)
Evaluated at:

     gdp   houst      pb  spread
     6.8     1.6       1    .426

Predicted probabilities of different outcomes

   Pr(y=-0,5)  Pr(y=-0,25)      Pr(y=0)   Pr(y=0,25)    Pr(y=0,5)
    6.308e-09    .00006312    .19508651    .58811913    .21673124

Standard errors of the probabilities

   Pr(y=-0,5)  Pr(y=-0,25)      Pr(y=0)   Pr(y=0,25)    Pr(y=0,5)
    1.947e-08    .00009072    .05340945    .06898914    .06586911
```

Using the command `predict` with the option `zeros` or `regimes`, we can compute the predicted probabilities of the three latent regimes $s_t \in \{-1, 0, 1\}$ or the probabilities of the three types of zeros conditional on each regime:

```
. predict p_zero, zeros

. predict p_reg, regime

. tabstat p_zero* p_reg*, stat(mean)
```

stats	p_zero_0	p_zero_n	p_zero_p	p_reg_n	p_reg_0	p_reg_p
mean	.3895957	.1453901	.0042672	.4028259	.3895957	.2075784

The `ziopmargins` command reports (by default) the marginal effects of the covariates on the choice probabilities and their standard errors, evaluated for the specified values of the covariates:

The `ziopclassification` command reports several measures of fit such as the classification (contingency) table, the percentage of correct predictions, the Brier probability and the ranked probability scores:

6 Concluding remarks

This article describes the ML estimation of the nested and cross-nested zero-inflated ordered probit models using the new STATA commands `nop`, `ziop2` and `ziop3`. Such models can be applied to a variety of data sets in which the discrete ordinal outcomes can be divided into the groups (nests) of similar choices, for example, the decisions to reduce, or leave unchanged, or increase the choice variable (monetary policy interest rates, rankings, prices, consumption levels), or the negative, or neutral, or positive attitudes to the survey questions. The choice among the nests is driven by an ordered-choice switching mechanism that can be either exogenous or endogenous to the outcome decisions, which are also naturally ordered (large or small increase/decrease; disagree or strongly disagree; etc.). The models allow the probabilities of choices from different nests (e.g., no change and an increase) to be driven by distinct mechanisms. Moreover, the zero-inflated cross-nested models allow the often abundant no-change or neutral outcomes to belong to all nests and be inflated by several different processes. The results of Monte Carlo simulations indicate that the proposed ML estimators are consistent and perform well in small samples.

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References

Andrews, D. W. K. 2001. Testing when a parameter is on the boundary of the maintained hypothesis. *Econometrica* 69 (3): 683–734.

Bagozzi, B. E., and B. Mukherjee. 2012. A mixture model for middle category inflation in ordered survey responses. *Political Analysis* 20: 369–386.

Basu, D., and R. M. de Jong. 2007. Dynamic multinomial ordered choice with an application to the estimation of monetary policy rules. *Studies in Nonlinear Dynamics and Econometrics* 11 (4): 1–35.

Brier, G. W. 1950. Verification of forecasts expressed in terms of probability. *Monthly Weather Review* 78 (1): 1–3.

Brooks, R., M. N. Harris, and C. Spencer. 2012. Inflated ordered outcomes. *Economics Letters* 117 (3): 683–686.

Epstein, E. S. 1969. A scoring system for probability forecasts of ranked categories. *Journal of Applied Meteorology* 8: 985–987.

Famoye, F., and K. P. Singh. 2003. On inflated generalized Poisson regression models. *Advanced Applied Statistics* 3 (2): 145–158.

Greene, W. H. 1994. Accounting for excess zeros and sample selection in Poisson and negative binomial regression models. Working Paper No. 94-10, Department of Economics, Stern School of Business, New York University.

Greene, W. H., and D. A. Hensher. 2010. *Modeling ordered choices: A primer*. Cambridge University Press.

Hardin, J. W., and J. M. Hilbe. 2014. Estimation and testing of binomial and beta-binomial regression models with and without zero inflation. *Stata Journal* 14(2): 292-303.

Harris, M. N., and X. Zhao. 2007. A zero-inflated ordered probit model, with an application to modelling tobacco consumption. *Journal of Econometrics* 141 (2): 1073–1099.

Heckman, J. J. 1978. Dummy endogenous variables in a simultaneous equation system. *Econometrica* 46: 931–959.

Hernández, A., F. Drasgow, and V. Gonzáles-Romá. 2004. Investigating the functioning of a middle category by means of a mixed-measurement model. *Journal of Applied Psychology* 89 (4): 687-699.

Kaminsky, G. L., and C. M. Reinhart. 1999. The twin crises: the causes of banking and balance-of-payments problems. *American Economic Review* 89 (3): 473–500.

Kelley, M. E., and S. J. Anderson. 2008. Zero inflation in ordinal data: incorporating susceptibility to response through the use of a mixture model. *Statistics in Medicine* 27: 3674–3688.

Kulas, J. T., and A. A. Stachowski. 2009. Middle category endorsement in odd-numbered Likert response scales: Associated item characteristics, cognitive demands, and preferred meanings. *Journal of Research in Personality* 43: 489-493.

Lambert, D. 1992. Zero-inflated Poisson regression with an application to defects in manufacturing. *Technometrics* 34 (1): 1–14.

MacKinnon, J. G. 1996. Numerical distribution functions for unit root and cointegration tests. *Journal of Applied Econometrics* 11: 601–618.
Sirchenko, A. 2013. A model for ordinal responses with an application to policy interest rate. National Bank of Poland Working Paper No. 148.
Small, K. 1987. A discrete choice model for ordered alternatives. *Econometrica* 55: 409–424.
Vovsha, P. 1997. Application of cross-nested logit model to mode choice in Tel Aviv, Israel, Metropolitan Area. *Transportation Research Record* 1607: 6–15.
Vuong, Q. 1989. Likelihood ratio tests for model selection and non-nested hypotheses. *Econometrica* 57 (2): 307–333.
Wen, C.-H., and F. Koppelman. 2001. The generalized nested logit model. *Transportation Research B* 35: 627–641.
Wilde, J. 2000. Identification of multiple equation probit models with endogenous dummy regressors. *Economics Letters* 69 (3): 309–312.
Wilson, P. 2015. The misuse of the Vuong test for non-nested models to test for zero-inflation. *Economics Letters* 127: 51–53.
Winkelmann, R. 2008. *Econometric analysis of count data.* 5th edition, Springer.

Appendix

Table A1. Monte Carlo simulations: The true values of parameters

	NOP (exog)	NOP	ZIOP-2 (exog)	ZIOP-2	ZIOP-3 (exog)	ZIOP-3
γ	(0.6, 0.4)'	(0.6, 0.4)'	(0.6, 0.8)'	(0.6, 0.8)'	(0.6, 0.4)'	(0.6, 0.4)'
μ	(0.21, 2.19)'	(0.21, 2.19)'	0.45	0.45	(0.9, 1.5)'	(0.9, 1.5)'
β			(0.5, 0.6)'	(0.5, 0.6)'		
β^-	(0.3, 0.9)'	(0.3, 0.9)'			(0.3, 0.9)'	(0.3, 0.9)'
β^+	(0.2, 0.3)'	(0.2, 0.3)'			(0.2, 0.3)'	(0.2, 0.3)'
α			(-1.45, -0.55, 0.75, 1.65)'	(-1.18,-0.33, 0.9, 1.76)'		
α^-	-0.17	-0.5			(-0.67, 0.36)'	(-0.88, 0.12)'
α^+	0.68	1.3			(0.02, 1.28)'	(0.49, 1.67)'
ρ			0	0.5		
ρ^-	0	0.3			0	0.3
ρ^+	0	0.6			0	0.6

Notes: (exog) – exogenous switching: $\rho = \rho^- = \rho^+ = 0$. The variances σ^2, σ^2_-, σ^2_+, and σ^2_ν are all fixed to one in all models.

To include into the help files:

store the following in `e()`:

Scalars

`e(N)`	number of observations
`e(ll)`	total log-likelihood of the model

Macros

`e(cmd)`	`nop`, `ziop2`, or `ziop3`, respectively
`e(depvar)`	dependent variable of regression

ß

Matrices

`e(b)`	parameters vector
`e(V)`	variance-covariance matrix
`e(ll_obs)`	vector of observation-wise log-likelihood

Functions

`e(sample)`	marks estimation sample