EE 703: Digital Message Transmission (Autumn 2022)
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Assignment 1: 20 points | Date: August 12, 2022

1. [5 points] Let $\hat{s}_p(t) = s_p(t) * \frac{1}{\pi t}$ be the Hilbert transform of a passband signal $s_p(t)$. Show that $\langle s_p, \hat{s}_p \rangle = 0$.

2. [5 points] Suppose we define the complex envelope of a passband signal $s_p(t)$ centered at $\pm f_c$ as

$$S(f) = S_p(f + f_c)u(f + f_c)$$

where $S_p(f)$ is the Fourier transform of $s_p(t)$. Derive the following with explanations for each step.

(a) $s_p(t)$ in terms of $s(t)$

(b) $s_p(t)$ in terms of $s_c(t)$ and $s_s(t)$ (the in-phase and quadrature components of $s(t)$)

(c) $s(t)$ in terms of $s_p(t)$

(d) $S_p(f)$ in terms of $S(f)$

(e) The relationship between $\|s\|^2$ and $\|s_p\|^2$.

3. [5 points] Consider the passband signals $s_1(t) = \sqrt{2}\cos(2\pi f_1 t)$ and $s_2(t) = \sqrt{2}\cos(2\pi f_2 t)$ where $f_1 \neq f_2$. Calculate the complex baseband representations of these signals for $f_c = f_1$.

4. [5 points] Suppose $x_p(t)$ and $y_p(t)$ are passband signals. Let $z_p(t) = x_p(t) * y_p(t)$ also be a passband signal. Show that the complex envelopes of these signals satisfy the relation

$$y(t) = \frac{1}{\sqrt{2}}\, x(t) * y(t).$$

Here $x(t), y(t), z(t)$ are the complex envelopes of $x_p(t), y_p(t), z_p(t)$ respectively.