York University

MATH 2030 3.0AF (Elementary Probability)
Class Quiz – SOLUTIONS

September 26, 2008

NAME:

STUDENT NUMBER:

You have 20 minutes to complete the quiz. There are 2 pages, containing 2 questions in total. You may bring one letter-sized two-sided formula sheet to the exam. No other books or notes may be used. You may use a calculator. Show all your work, and explain or justify your solutions to the extent possible. You may leave numerical answers unsimplified, including binomial coefficients. Use the back of your page if you run out of room.

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1. Events A and B satisfy

$$P(A) = 0.5 \qquad P(B) = 0.4 \qquad P(A \cap B) = 0.3$$

(a) [5] Find $P(A \cup B)$.
(b) [5] Find $P(A^c \cap B)$.

Solution:

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.4 - 0.3 = 0.6$ by inclusion/exclusion.
(b) $0.4 = P(B) = P(A \cap B) + P(A^c \cap B) = 0.3 + P(A^c \cap B)$, and subtracting, we get $P(A^c \cap B) = 0.4 - 0.3 = 0.1$

Alternatively you could have used a Venn diagram to break $A \cup B$ up into disjoint parts $A^c \cap B$, $A \cap B$, $A \cap B^c$ with probabilities 0.1, 0.3, 0.2 respectively. From this the answer to part (b) is immediate, and the answer to part (a) is the sum, namely $0.1 + 0.3 + 0.2 = 0.6$

2. The game of Scrabble has 100 tiles, each of which is blank or is labeled with a letter. 12 of the tiles have label "E". Each player draws 7 tiles at random to start the game.

(a) [10] Find the probability that the first player draws exactly three "E" tiles.
(b) [5] Indicate what Ω you would use for your model, including how many elements it has.

Solution:

(a) There are $\binom{12}{3}$ ways of choosing the 3 "E" tiles, and $\binom{88}{4}$ ways of choosing the remaining 4 tiles. There are $\binom{100}{7}$ ways of choosing 7 tiles, so the probability is
$$\frac{\binom{12}{3}\binom{88}{4}}{\binom{100}{7}}.$$
(b) The above answer implies that we're using an equally likely outcomes model in which $\#(\Omega) = \binom{100}{7}$. One such Ω is the set of all possible 7-element subsets of the set of 100 tiles.

One alternative to the above would have been to use order in your Ω. For example, you could have used an equally likely outcomes model with Ω consisting of all sequences of 7 tiles chosen from the 100 possibilities. Now we are thinking of drawing tiles one at a time, so it makes a difference what order they are drawn in. Then $\#(\Omega) = (100)_7$. And to work out the probability of 3 "E"s we count as follows: There are $\binom{7}{3}$ ways of choosing the 3 picks that result in an "E". Once those picks are determined, there are $(12)_3$ ways of choosing the 3 "E" tiles, in order. And $(88)_4$ ways of choosing the other tiles, in order. So the probability is
$$\frac{\binom{7}{3}(12)_3(88)_4}{(100)_7}.$$
This is easily seen to be equivalent to the earlier expression.

There are other models you could use as well. For example, to simply count "E"s, a model could be based on $\Omega = \{0, 1, 2, 3, 4, 5, 6, 7\}$. That will not work as an equally likely outcomes model, but if you described how that Ω could be made into a model (ie how you construct the associated probability measure P), this could also become a convincing answer to the question.