Assignment 1 solutions. (Chapter 1 of Pitman.)

Exercises 1.1.

3. (a) $1/n^2$. (b) $(n-1)/n^2$. (c) The probability that the two numbers are different is $1-1/n$, so the probability that the second number is bigger is half of that, $(1-1/n)/2$. (d) $1/[n(n-1)]$; $(n-1)/[n(n-1)] = 1/n$; by symmetry, 1/2.

5. (a) $52{\cdot}51$. (b) $(4{\cdot}51)/(52{\cdot}51) = 1/13$. (c) $(4{\cdot}3+48{\cdot}4)/(52{\cdot}51) = 1/13$. (d) $(4 \cdot 3)/(52 \cdot 51) = 1/221$. (e) $1-(48 \cdot 47)/(52 \cdot 51) = 33/221$.

7. (a) $1+1, 1+2, 2+1, 2+2$ only; 4/36 or 1/9. (b) $1+1, 1+2, 1+3, 2+1, 2+2, 2+3, 3+1, 3+2, 3+3$ only; 9/36 or 1/4. (c) $9/36-4/36 = 5/36$.

9. $1/(1+10) = 1/11$. $1/(1+5) = 1/6$.

Exercises 1.2.

2. 1/100 would be correct if the bookmake were offering a fair bet, which is unlikely. So we should expect less that 1/100.

Exercises 1.3.

5. See Solutions, page 533

6. 1 with probability 1/10; 2 w.p. 2/10; 4 w.p. 3/10; 6 w.p. 2/10; 7 w.p. 1/10; 8 w.p. 1/10.

9. (a) $P(F \cup G) = P(F) + P(G) - P(FG) = 0.7 + 0.6 - 0.4 = 0.9$. (b) $P(F \cup G \cup H) = P(F) + P(G) + P(H) - P(FG) - P(FH) - P(GH) + P(FGH) = 0.7 + 0.6 + .05 - 0.4 - 0.3 - 0.2 + 0.1 = 1$. (c) $P(F^cG^cH) = P(H) - P(FH) - P(GH) + P(FGH) = 0.5 - 0.3 - 0.2 + 0.1 = 0.1$.

10. (a) $P(AB) + P(AC) + P(BC) - 3P(ABC)$. (b) $P(A) + P(B) + P(C) - 2P(AB) - 2P(AC) - 2P(BC) + 3P(ABC)$. (c) $1 - P(A \cup B \cup C) = 1 - P(A) - P(B) - P(C) + P(AB) + P(AC) + P(BC) - P(ABC)$.

14. Follows from $P(A \cup B) = P(A) + P(B) - P(AB)$ and $P(A \cup B) \leq 1$.