Assignment 3 solutions. (Chapters 1 and 2 of Pitman.)

Exercises 1.6.

4. (a) $(1/20)(9/20)(1/20) = 9/8000$.

(b) $(1/20)(9/20)(19/20) + (1/20)(11/20)(1/20) + (19/20)(9/20)(1/20) = 353/8000$.

(c) In (a), $(3/20)(1/20)(3/20) = 9/8000$.

In (b), $(3/20)(1/20)(17/20)+(3/20)(19/20)(3/20)+(17/20)(1/20)(3/20) = 273/8000$. More near misses will keep the player playing longer.

7. (a) (1 or 2) and 3: $(p_1 + p_2 - p_1p_2)p_3$.

(b) [(1 or 2) and 3] or 4: $(p_1 + p_2 - p_1p_2)p_3 + p_4 - (p_1 + p_2 - p_1p_2)p_3p_4$.

Review Exercises

7. (a) $(20/50)(19/49)(18/48)(17/47) = 116280/5527200 = 0.02104$.

(b) $1 - (20/50)(19/49)(18/48)(17/47) = 0.97896$.

(c) $(20/50)(19/49)(18/48)(30/47) + (20/50)(19/49)(30/48)(18/47) + (20/50)(30/49)(19/48)(18/47) + (30/50)(20/49)(19/48)(18/47) = 205200/5527200 = 0.03713$.

(d) 3 or 4 in favor: $4(30)(29)(28)(20)/5527200+(30)(29)(28)(27)/5527200 = 2606520/5527200 = 0.4716$.

8. (a) $(39/52)(38/51)(37/50)(36/49)(13/48) = 25662312/311875200 = 0.08228$.

(b) $1 - (39/52)(38/51)(37/50)(36/49)(35/48) = 69090840/311875200 = 0.2215$.

(c) $(39/52)(38/51)(13/50)/[1-(39/52)(38/51)(37/50)(36/49)(35/48)] = (19266/132600)/(69090840/311875200) = 0.6559$.

Exercises 2.1.

1. (a) $\binom{7}{4} = 35$.

(b) $\binom{7}{4}(1/6)^4(5/6)^3 = (35)(125)/279936 = 0.01563$.

3. $P(A) = \binom{5}{2}(1/6)^2(5/6)^3$. $P(B) = 1-\binom{5}{0}(1/6)^0(5/6)^5-\binom{5}{1}(1/6)^1(5/6)^4$. $P(C) = \binom{5}{0}(1/6)^0(5/6)^5+\binom{5}{1}(1/6)^1(5/6)^4+\binom{5}{2}(1/6)^2(5/6)^3$. $P(D) = \binom{5}{3}(1/2)^3(1/2)^2$. $P(E) = \binom{5}{3}(1/2)^3(1/2)^2 + \binom{5}{4}(1/2)^4(1/2)^1 + \binom{5}{5}(1/2)^5(1/2)^0$.

7. $p = (1-6/36)/2 = 15/36$. $P(\text{at least 4 wins}) = \binom{5}{4}(15/36)^4(21/36)^1 + \binom{5}{5}(15/36)^5(21/36)^0$.

9. (a) $\lfloor np + p\rfloor = \lfloor(325 + 1)/38\rfloor = \lfloor 8.5789\rfloor = 8$. $P(7)/P(6) = (325 - 6)(1/38)/[7(37/38)]$ and $P(8)/P(7) = (325 - 7)(1/38)/[8(37/38)]$, so

$$P(8) = P(6)\frac{319}{7 \cdot 37}\frac{318}{8 \cdot 37} = 0.1387.$$

Compare with $P(8) = \binom{325}{8}(1/38)^8(37/38)^{317} = 0.1387$, which is hard to compute, even with a calculator.

(b)

$$P(10) = P(8)\frac{317}{9 \cdot 37}\,\frac{316}{10 \cdot 37} = 0.1128.$$

(c) $P(10, 326) = P(10, 325)(37/38) + P(9, 325)(1/38) = (0.1128)(37/38) + (0.1321)(1/38) = 0.1133$.

13. (a) No.

(b) $P(Bb) = P(bb) = 1/2$, so $1/2$ is the answer.

(c) $P(BB) = 1/4$, $P(Bb) = 1/2$, and $P(bb) = 1/4$, so the answer is $3/4$.

(d) The prior probability is $2/3$ (that is, $(1/2)/[(1/4) + (1/2)]$). But we have additional information, that her child with a blue-eyed man has brown eyes. By Bayes's rule, answer is $(2/3)(1/2)/[(2/3)(1/2) + (1/3)(1)] = 1/2$.