Assignment 4 solutions. (Chapter 2 of Pitman.)

Exercises 2.2.

1. $400(1/2) = 200$ and $\sqrt{400(1/2)(1/2)} = 10$.

(a) $\Phi((210 + 1/2 - 200)/10) - \Phi((190 - 1/2 - 200)/10) = \Phi(1.05) - \Phi(-1.05) = 2\Phi(1.05) - 1 = 2(0.8531) - 1 = 0.7062$.

(b) $\Phi((220 + 1/2 - 200)/10) - \Phi((210 - 1/2 - 200)/10) = \Phi(2.05) - \Phi(0.95) = 0.9798 - 0.8289 = 0.1509$.

(c) $\Phi((200 + 1/2 - 200)/10) - \Phi((200 - 1/2 - 200)/10) = \Phi(0.05) - \Phi(-0.05) = 2\Phi(0.05) - 1 = 2(0.5199) - 1 = 0.0398$.

(d) $\Phi((210 + 1/2 - 200)/10) - \Phi((210 - 1/2 - 200)/10) = \Phi(1.05) - \Phi(0.95) = 0.8531 - 0.8289 = 0.0242$.

4. $300(1/3) = 100$ and $\sqrt{300(1/3)(2/3)} = 10\sqrt{2/3}$. $1 - \Phi((121 - 1/2 - 100)/(10\sqrt{2/3})) = 1 - \Phi(2.5107) = 1 - 0.9940 = 0.0060$.

5. It is the probability of 13 or more wins, which is $1 - \Phi((13 - 1/2 - 25(18/38))/\sqrt{25(18/38)(20/38)}) = 1 - \Phi(0.2635) = 1 - 0.6039 = 0.3961$.

9. (a) $n = 324$, $p = 9/10$. We want probability of 301 or more successes. That probability is $1 - \Phi((301 - 1/2 - 324(9/10))/\sqrt{324(9/10)(1/10)}) = 1 - \Phi(1.6481) = 1 - 0.9539 = 0.0461$.

(b) Overbooking becomes more likely.

(c) If people travel in pairs, with each pair have a 90% chance of showing up, then the probability in (a) becomes $1-\Phi((151-1/2-162(9/10))/\sqrt{162(9/10)(1/10)}) = 1 - \Phi(1.2309) = 1 - 0.8909 = 0.1091$.

10. $200(1/2) = 100$ and $\sqrt{200(1/2)(1/2)} = 10\sqrt{2}/2$. $\Phi((100 + 1/2 - 100)/(10\sqrt{2}/2)) - \Phi((100 - 1/2 - 100)/(10\sqrt{2}/2)) = 2\Phi(1/(10\sqrt{2})) - 1 = 2\Phi(0.07071) - 1 = 2(0.5282) - 1 = 0.0564$. This is the probability for one specified student getting exactly 100 heads. For none to get exactly 100 heads, the probability is $(1 - 0.0564)^{30} = 0.1752$.

12. $10000(1/2) = 5000$ and $\sqrt{10000(1/2)(1/2)} = 50$. $2/3 = \Phi((5000 + m+1/2-5000)/50)-\Phi((5000-m-1/2-5000)/50) = 2\Phi((m+1/2)/50)-1$. Solving, we get $\Phi((m+1/2)/50) = 5/6$. Reading the normal table backwards gives $(m + 1/2)/50 = 0.967$ or $m = 47.85$. But m is an integer, so $m = 48$.

13. We want $P(|\hat{p} - p| < 0.01) \geq 0.95$. We know $\Phi(-z, z) = 0.95$ if $z = 1.96$. $\hat{p}$ will belong to the interval $p \pm 1.96\sqrt{p(1-p)/n}$ with probability about 0.95. Hence it will belong to $p \pm 0.98\sqrt{1/n}$ with probability at least 0.95. We want $0.98/\sqrt{n}$ to be about 0.01. So $n = (0.98/0.01)^2 = 98^2 = 9604$.

Exercises 2.4.

5. $n = 52$ and $p = 1/100$ so $\mu = 0.52$. Chance of k wins is approximately Poisson(μ). $k = 0$: $e^{-0.52} = 0.5945$. $k = 1$: $0.52e^{-0.52} = 0.3092$. $k = 2$: $(0.52)^2e^{-0.52}/2 = 0.0804$.

7. (a) $m = \lfloor (25+1)(1/10) \rfloor = 2$.

(b) $P(S=2) = \binom{25}{2}(1/10)^2(9/10)^{23} = 0.2659$.

(c) $25(1/10) = 2.5$ and $\sqrt{25(1/10)(9/10)} = 1.5$. $\Phi((2+1/2-2.5)/1.5) - \Phi((2-1/2-2.5)/1.5) = \Phi(0) - \Phi(-0.6667) = 0.5 - (1-0.7465) = 0.2465$.

(d) $\mu = 2.5$. $(2.5)^2 e^{-2.5}/2 = 0.2565$.

(e) $n = 2500$ and $p = 1/10$. $m = \lfloor (2500+1)(1/10) \rfloor = 250$. The normal approximation is better suited to this case. $np = 250$ and $\sqrt{np(1-p)} = 15$. $\Phi((250+1/2-250)/15) - \Phi((250-1/2-250)/15) = 2\Phi(1/30) - 1 = 2(0.5133) - 1 = 0.0266$.

(f) $n = 2500$ and $p = 1/1000$. $m = \lfloor (2500+1)(1/1000) \rfloor = 2$. The Poisson approximation is better suited to this case. $\mu = 2.5$. Same answer as in (d).