Assignment 2

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March 2021

1 Solve the following questions:

1. In the product Hilbert Space $C^2 \otimes C^2$, the Bell states are given by :

$$\left|\phi^+\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \quad \left|\phi^-\right\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$$
$$\left|\psi^+\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \quad \left|\psi^-\right\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

These form the orthonormal basis for C^4. Here, $\{|0\rangle, |1\rangle\}$ is an arbitrary orthonormal basis in the Hilbert space C^2. Let :

$$|0\rangle = \begin{pmatrix} e^{i\phi}\cos\theta \\ \sin\theta \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} -e^{i\phi}\sin\theta \\ \cos\theta \end{pmatrix}$$

   (a) Find $|\phi^+\rangle, |\phi^-\rangle, |\psi^+\rangle, |\psi^-\rangle$ for this basis

   (b) Find it for the special case of when $\phi = 0$, and $\theta = 0$.

2. Let Alice and Bob share an entangled state $|\psi\rangle = \frac{|00\rangle + n^*|11\rangle}{\sqrt{1+|n|^2}}$ as a resource. Find out the fidelity of teleportation of an unknown quantum state from Alice's side to Bob's side. (Consider Fidelity to be the mod-square of the inner product between the state you desired and state you got)

3. Extend the proof of the Schmidt decomposition to the case where two parties A and B may have state spaces of different dimensionality.

4. Carry out the entanglement swapping at the qubits 2 and 3 of the entangled states, $\rho_{12} = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{|00\rangle + n|11}{\sqrt{1+|n|^2}}$ and $\rho_{34} = |\psi\rangle\langle\psi|$, where $|\psi\rangle = \frac{|00\rangle + m|11}{\sqrt{1+|m|^2}}$, $(0 < |n| < 1, 0 < |m| < 1)$.

5. Find all the two qubit and single qubit reduced density matrices of the state $|\psi\rangle = (1/\sqrt{2})|000\rangle + |111\rangle$.

6. Find out whether projection operators are positive operators or not.