

If we observe a free falling ball started in perfectly horizontal direction, we can separate its motion to two components according to the two axes. The horizontal motion has zero acceleration, thus the horizontal, x coordinate of the ball can be calculated by the quite simple

$$x = v_{0,x}t$$

at every given t time point. Let upward be the positive vertical direction, thus gravity force (and the acceleration it causes) would be negative. To calculate the vertical, y coordinate, denote the original height by y_0 from where the ball has been dropped. Its vertical motion from the beginning to the first bounce can be described by

$$y = y_0 + \frac{g}{2}t^2$$

(note that it had zero vertical velocity at the beginning). The first bounce happens when this first reaches zero, thus $y = y_0 + \frac{g}{2}t^2 = 0$, i.e.

$$t_1 = \sqrt{\frac{2y_0}{-g}}$$

velocity of the ball then:

$$v_1 = gt_1 = g\sqrt{\frac{2y_0}{-g}} = -\sqrt{2y_0(-g)}.$$

Suppose it loses p' fraction of its kinetic energy upon the impact, then v_1 speed before the impact and v'_1 after it are connected by

$$\begin{aligned}\frac{1}{2}m(v'_1)^2 &= p\frac{1}{2}mv_1^2 \\ (v'_1)^2 &= pv_1^2 \\ v'_1 &= -\sqrt{p}v_1\end{aligned}$$

(using $p := 1 - p'$ substitution here). It moves upwards with v'_1 speed after the bounce, its vertical coordinate is given by

$$y = v'_1t + \frac{g}{2}t^2$$

until the second bounce. This will reach zero again at t_2 :

$$t_2 = t_1 + \frac{2v'_1}{-g}.$$

At this time, its speed will again be v'_1 (because this part of the motion is symmetric to the apex), its direction being downwards of course, exactly as v_1 . Thus

$$v_2 = -v'_1 = \sqrt{p}v_1$$

applies to the v_2 velocity of the ball before it bounces back the **second** time.

v_2 and v'_2 are in the same relation as v_1 and v'_1 . Then the **third** round comes:

$$v_3 = \sqrt{p}v_2 = \sqrt{p^2}v_1$$

$$t_3 = t_1 + t_2 + \frac{2v'_3}{-g} = \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g}(v'_2 + v'_3)$$

Briefly, the collision speeds:

$$\begin{aligned} v_i &= \sqrt{p^{i-1}}v_1 \quad (i \geq 2) \\ v_1 &= -\sqrt{2y_0(-g)} \end{aligned}$$

bounce-back speeds:

$$v'_i = -v_{i+1} = -\sqrt{p^i}v_1 \quad (i \geq 1)$$

impact times:

$$t_i = \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{i-1} v'_j \quad (i \geq 1)$$

the function that gives the y coordinate between t_i és t_{i+1} ($i \geq 1$):

$$y = v'_i t + \frac{g}{2} t^2$$

that is

$$y(t) = \begin{cases} y_0 + \frac{g}{2} t_*^2 & \text{if } 0 \leq t < \sqrt{\frac{2y_0}{-g}} \\ v'_1 t_* + \frac{g}{2} t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} \leq t < \sqrt{\frac{2y_0}{-g}} + \frac{2v'_1}{-g} \\ v'_2 t_* + \frac{g}{2} t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} + \frac{2v'_1}{-g} \leq t < \sqrt{\frac{2y_0}{-g}} + \frac{2v'_1}{-g} + \frac{2v'_2}{-g} \\ \dots & \\ v'_i t_* + \frac{g}{2} t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{i-1} v'_j \leq t < \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{(i+1)-1} v'_j \end{cases}$$

where t_* is the relative distance of t from the start of current section,

$$t_* = t - \left(\sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{i-1} v'_j \right).$$

Using y_0 , p , and g only:

$$y(t) = \begin{cases} y_0 + \frac{g}{2}t_*^2 & \text{if } 0 \leq t < \sqrt{\frac{2y_0}{-g}} \\ \sqrt{p \cdot 2y_0(-g)}t_* + \frac{g}{2}t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} \leq t < \sqrt{\frac{2y_0}{-g}} + \frac{2\sqrt{p \cdot 2y_0(-g)}}{-g} \\ \sqrt{p^2 \cdot 2y_0(-g)}t_* + \frac{g}{2}t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} + \frac{2\sqrt{p \cdot 2y_0(-g)}}{-g} \leq t < \\ & \sqrt{\frac{2y_0}{-g}} + \frac{2\sqrt{p \cdot 2y_0(-g)}}{-g} + \frac{2\sqrt{p^2 \cdot 2y_0(-g)}}{-g} \\ \dots & \\ \sqrt{p^i \cdot 2y_0(-g)}t_* + \frac{g}{2}t_*^2 & \text{if } \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{i-1} \sqrt{p^j \cdot 2y_0(-g)} \leq t < \\ & \sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{(i+1)-1} \sqrt{p^j \cdot 2y_0(-g)} \end{cases}$$

and $t_* = t - \left(\sqrt{\frac{2y_0}{-g}} + \frac{2}{-g} \sum_{j=1}^{i-1} \sqrt{p^j \cdot 2y_0(-g)} \right)$.