

Software implementations allowing new approaches toward data analysis, modeling and curation of biological knowledge for Systems Medicine

Doctoral thesis

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John Zobolas, May 2021

Abstract

Cancer is one the most prevalent human diseases. The scientific community has devoted considerable efforts to understand the mechanisms behind this disease and search for treatments that promise a better quality of life for patients. To accomplish this goal, Biology and Medicine have joined forces with Computer Sciences, using the power of Computational modeling, Mathematics, Machine Learning and Statistics. This interdisciplinary effort to address the cancer problem, constitutes the basis upon which this thesis was formed. We present several contributions to this effort, consisting of software, data analyses and mathematical investigations, which have enabled the more efficient curation of biological knowledge, the use of computer models to prioritize drug treatments for cancer and the derivation of molecular mechanistic insights from the simulation results.

In order to build a computational model of a biological system such as a cancer cell, we first need a way to describe the structure of such a system. A common network-based approach provides an elegant representation of such a structure, where molecular entities such as proteins and genes are connected to each other via causal interactions, which in turn determine cellular behavior and the functional properties of the cell as an integrated system of individual components. These interactions form the Prior Knowledge Network (PKN), which serves as the basic building block for most computational biological models. Nonetheless, several challenges exist, even at this early stage of the modeling process.

The first problem is that biological information by its very nature is largely complex, and therefore its formalization to a structured, computable form for use in modeling applications, demands extra attention. The translation of scientific knowledge from publications into such a computable form is achieved with the use of specialized software tools and is the main responsibility of biocurators. In order to help biocurators be more efficient in their annotation tasks, we proposed the Visual Syntax Method (VSM) as an alternative approach for general-purpose knowledge formalization. In particular, we implemented a user interface component (VSM-box) that enables curators to annotate any type of information, no matter its complexity, and translate it into an intuitive, flexible sentence-like format. This software was used to build a prototype curation interface (CausalBuilder) for the annotation of molecular causal interactions, which constitute the cornerstone of a model's PKN.

The second problem concerns the availability and ease of access of causal molecular interaction data for modeling or other scientific endeavors. A standard format for the representation of such signaling information was developed (CausalTAB), and we supported the export of the causality statements from CausalBuilder's interface to this format. But there exist several other molecular interaction databases that could update their data to fit the new CausalTAB standard. PSICQUIC is a web-service platform that was initially built so that users can conveniently fetch in a standard way molecular interaction data from different sources. We extended PSICQUIC to incorporate the new CausalTAB format, so that causality-enriched information generated by our curation prototype tool or from other data providers could be shared through a common channel.

A third major problem arose during the design process of the VSM-box and its application, CausalBuilder. Behind the scenes, the curator interface has to communicate with a large number of diverse biological data resources, each with its own online API service that provides access to the respective data. In order to present to the user the available terms that pertain to a specific annotation of interest, a uniform way to query all these resources was needed. This prompted us to build the Unified Biological Dictionaries (UBDs), a software suite that provides a unified gateway for life science data, helping users retrieve the right query terms. In addition, curators sometimes come across new knowledge that is not yet available through the standard authoritative resources. To address this related problem, we connected UBDs with PubDictionaries, an online resource of simple dictionaries, allowing curators to publicly create and share ad-hoc terms, and further use them as annotations in VSM-based applications.

After the signaling information has been curated and the causal interactions assembled to form the PKN, we then need to specify the mathematical equations of the cancer cell model. This allows us to describe and analyze its dynamical behavior subject to external stimuli, such as drug perturbations. The modeling approaches can in general range from qualitative to quantitative and in this work we focused on Boolean modeling, where signaling components are assigned either an active or inactive state. An automated computational pipeline was developed to produce an ensemble of Boolean models from a PKN, calibrated to a specific cancer cell signaling phenotype. These models are then analyzed to suggest possible synergistic drug combinations and the results are compared with experimental findings, where all possible combinations are tested in a high-throughput screen setup. We demonstrated that our pipeline could prioritize specific drug combinations, reducing the number of drugs that need to be tested in experiments, before a viable treatment is found for a patient. Moreover, several analyses indicated that our models can be used to derive mechanistic insights about the diseased model and generate novel biological hypotheses. Lastly, we showed the significance of the PKN quality, where even small modifications to the cancer signaling network could severely affect our pipeline's drug prediction performance.

To exploit the range of parameterizations present in the Boolean models produced by our pipeline, we devised several strategies to split and compare the different models in a dedicated R package (emba). This supplementary effort allowed us to find potential biomarkers, which are nodes whose state is decisive for the global behavior of the models and can indicate parts of the PKN that are responsible for a drug combination to be synergistic. Additionally, we noticed particular patterns in the way specific equations always correspond to specific signaling states in our models, so we more deeply investigated the influence of the choice of parameterization on the output behavior of these nodes. This led us to propose a list of Boolean function metrics that can assist modelers in choosing more appropriate equations, meaning those that are consistent with the regulatory information present in the PKN and whose expected output better matches experimental observations. Finally, results from a study of diverse Boolean functions indicated that these also exhibit diverse output behaviors, with some being highly biased towards specific Boolean outcomes while others depending more on the ratio between positive and negative regulators, as these are derived from the two distinct types of causal interactions present in the model's PKN.

Paper list

Primary

Papers that I am first author:

- **Paper 1:** *UniBioDicts: Unified access to Biological Dictionaries*
Zobolas, J., Touré, V., Kuiper, M., & Vercruysse, S. [[1](#)]

SV and VT identified the need for software to connect with the resources listed in MI2CAST used in the CausalBuilder tool. JZ implemented the software and wrote the manuscript. All co-authors revised and provided inputs to the manuscript.

- **Paper 2:** *Linking PubDictionaries with UniBioDicts to support Community Curation*
Zobolas, J., Kim, J.-D., Kuiper, M., & Vercruysse, S. [[2](#)]

MK and SV developed the original idea for this project. JZ wrote the application for a BioHackathon project and invited JDK to collaborate. JZ implemented the client side, JDK the server side. JZ wrote the manuscript. All co-authors revised and provided inputs to the manuscript.

- **Paper 3:** *emba: R package for analysis and visualization of biomarkers in Boolean model ensembles*
Zobolas, J., Kuiper, M., & Flobak, Å. [[3](#)]

AF developed the idea of this project. JZ wrote the software and the manuscript. All co-authors revised and provided inputs to the manuscript.

- **Paper 4:** *Fine tuning a logical model of cancer cells to predict drug synergies: combining manual curation and automated parameterization*¹

Flobak, Å., **Zobolas J.**, Vazquez M., Steigedal T., Thommesen L., Grislingås A., Niederdorfer B., Folkesson E., Kuiper M.

AF designed the project, developed initial software and executed experiments. JZ extended the software, ran all simulations, produced and analyzed results. AF and MK added biological interpretation and wrote the manuscript. JZ provided various inputs and text to the manuscript. TS, LT and AG helped AF with experiments. MV developed prototype software. BN and EF did curation work and performed experiments.

- **Paper 5:** *Boolean function metrics can assist modelers to check and choose logical rules*²
Zobolas, J., Monteiro, P. T., Kuiper, M., & Flobak, Å. [4]

JZ designed this project and wrote the manuscript. PTM and AF provided feedback and ideas to better shape the content of the manuscript. All co-authors revised and provided inputs to the manuscript.

Additional

In the following papers I have contributed to the underlying software and manuscript text:

1. *VSM-box: General-purpose Interface for Biocuration and Knowledge Representation*
Vercruysse, S., **Zobolas, J.**, Touré, V., Andersen, M. K., & Kuiper, M. [5]
2. *CausalBuilder: Bringing the MI2CAST Causal Interaction Annotation Standard to the Curator*
Touré, V., **Zobolas, J.**, Kuiper, M., & Vercruysse, S. [6]
3. *CausalTAB: the PSI-MITAB 2.8 updated format for signalling data representation and dissemination*
Perfetto, L., Acencio, M. L., Bradley, G., Cesareni, G., Del Toro, N., Fazekas, D., Hermjakob, H., Korcsmaros, T., Kuiper, M., Lægreid, A., Lo Surdo, P., Lovering, R. C., Orchard, S., Porras, P., Thomas, P. D., Touré, V., **Zobolas, J.**, & Licata, L. [7]

¹(Manuscript) Shared first co-authorship, to be submitted to the Molecular Systems Biology Journal

²(Preprint) To be submitted to the Journal of Theoretical Biology

Abbreviations

abmlog	All possible Boolean Models Link Operator Generator
AGS	Gastric Adenocarcinoma (cell line)
API	Application Programming Interface
AUC	Area Under the Curve
BioPax	Biological Pathway Exchange
CASCADE	CAnCER Signaling CAusality DatabasE
CNA	Copy Number Alterations
COVID-19	COrona VIRus Disease 2019
drabme	Drug Response Analysis to Boolean Model Ensembles
emba	Ensemble (Boolean) Model Biomarker Analysis
gitsbe	Generic Interactions To Specific Boolean Equations
GO	Gene Ontology
GREEKC	Gene Regulation Ensemble Effort for the Knowledge Commons
HUPO-PSI	HUman Proteome Organization - Proteomics Standards Initiative
IMEx	The International Molecular Exchange Consortium
MI2CAST	Minimum Information about a Molecular Interaction CAusal STatement
MIQL	Molecular Interactions Query Language
MITAB	Molecular Interaction TABular format
ODE	Ordinary Differential Equation
PDE	Partial Differential Equation
PKN	Prior Knowledge Network
PPI	Protein-Protein Interaction
PRC	Precision Recall Curve
PSICQUIC	Proteomics Standard Initiative Common QUery InterfaCe
ROC	Receiver Operating Characteristic
SBGN	Systems Biology Graphical Notation
TF-TG	Transcription Factor - Target Gene
UBDs (UniBioDicts)	Unified Biological Dictionaries

UMAP	Uniform Manifold Approximation and Projection (for dimension reduction)
VSM	Visual Syntax Method
XML	Extensible Markup Language

Summary

One for all

Scientific and technological progress has been the foundation for some of the most astounding achievements of humankind. In the last century in particular, discoveries were made that contributed to the sustainable development of the economy and society, affecting our lives in an unprecedented manner and making possible what was considered impossible. The invention of the digital computer and the Internet for example, revolutionized the access, dissemination and analysis of information [8,9]. We have been to the Moon, a breakthrough that has opened up the possibilities of space exploration and interstellar travel. The industrial revolution of the latest century has enabled us to design machines for every conceivable need. Human well-being has become significantly better: compare a middle class household and the appliances within, with one from 60 years ago. With a higher standard of living and the ongoing efforts to alleviate hunger, poverty and inequality on a global scale, people have started caring more about the planet, paving the way for sustainable economic and environmental growth [10]. Due to advancements in Biology and Medicine, the application of public health interventions such as vaccinations and hygiene measures has become common practice, causing a rapid increase in the global life expectancy during the last century [11]. The genome-editing technology CRISPR [12] has enabled the discovery of new therapeutic solutions for a variety of genetic diseases and has been beneficially used in several agriculture and plant biotechnology applications [13]. The list of achievements is truly endless and all the data points to the fact that the world is getting better [14].

There are three factors that have made technological progress possible. Firstly, every human innovation is based on basic scientific research, without which the development of new technologies would have been impossible. Secondly, society is developing new contracts with science [15], where researchers can only be trusted to continue their work (and get funding for it), if they tackle real-world problems and produce knowledge characterized by a fully transparent and participative spirit. Practically, this means that better communication skills are a necessity for today's scientists and that their research should have translational potential to deliver on society's expectations. But solving these real-world problems is incredibly hard, and so, they cannot be addressed by applying

knowledge from specific fields only, e.g. either from the Computer or the Biological Sciences alone. This brings us to the third factor: in order for science to deliver on its promises to society, collaboration across fields of science is the only way forward.

Medicine, from research to develop new therapies up to delivering the actual product or services to the patients, constitutes the perfect example that encompasses all three reasons that have enabled progress to transpire in its domain. It first starts with a real-life problem: people get sick. The existence of diseases is a societal problem and a hard one at that, since people usually lack the necessary knowledge or the means to deal with it on their own. They have in fact exchanged some of their freedom to have a place in society, and ensure that they receive proper treatment when needed (along with other forms of security, access to free education, etc.). To manage such a complex problem, society provides healthcare services, which have significantly increased across the world in recent years [16]. For most people, the single most applied healthcare interaction is the use of drugs, prescribed by medical doctors. Drugs are the translational product of the pharmaceutical industry, which is the result of basic interdisciplinary research. Medical doctors alone wouldn't be able to find the cause and understand the mechanisms behind many of the diseases that exist today. This knowledge has been the culmination of years of scientific knowledge, built atop collaboration across fields, from Medicine and Biology to Computer Science and Engineering.

So, only by using every possible method and knowledge at our disposal and by working together, we can achieve the solution to complex problems such as human diseases. When these conditions are met, societal challenges can be addressed and science stands as one unified body for the good and progress of all mankind. The field of Systems Medicine has been the direct embodiment of this notion, promising improved prevention, prognosis, diagnosis and treatment of patients via an integrative, interdisciplinary approach [17].

Dots and lines

One of the simplest ways to conceptualize complex systems, either man-made or existing freely in nature, is using the notion of a *network* or *graph*. The idea is that any system is composed of individual entities or components of interest (nodes) and these components interact with each other in various, usually non-obvious ways (links). These two properties, namely having some objects to study, and relationships between these objects, form the basis for the conceptualization of a network (Figure 1). From a cognitive point of view, the conceptualization of a network manifests as a visual representation in our brain, consisting of a bunch of dots (nodes) connected with numerous lines (links) [18]. Such a projection is usually close to what people instinctively draw on paper when they attempt to describe their knowledge about a system and its inner workings (thereby “connecting the dots”). Simple schematics that are abstractly similar to dots and lines, along with further contextual information (e.g. node labels, coloring, directed links, etc.), seem to be able to capture and render information derived from our thought processes, in a unique and comprehensible way.

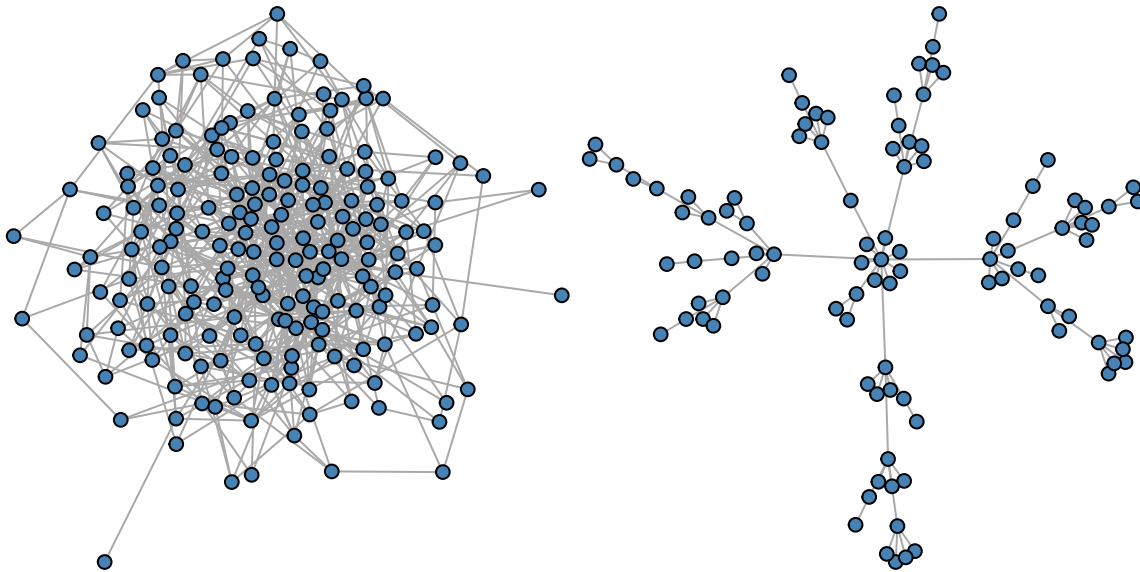


Figure 1: Two examples of networks, composed of dots and lines. The left network is a random graph based on the Erdős–Rényi model [19] and the one on the right is created using the preferential attachment principle that characterizes scale-free networks with hub nodes, such as the World Wide Web [20].

Since studying complex systems falls into the domain of science’s responsibilities, and graphs seem to be an intuitive way of representing such systems, the emergence of a new field called network science was inevitable [21]. Its purpose is to establish a unified set of tools and methods to study the properties of any type of network that emerges across disparate fields. A variety of software tools for network visualization and analysis have been released throughout the years, ranging from generic-purpose [22–25], to tools more suitable for studying biological [26–29] or social networks [30,31]. The use of such tools enables the discovery of fundamental laws that characterize the function of systems represented by networks. In addition, it allows us to study in detail the

networks' systemic structure and derive key principles that drive their evolution and emergent behavior. Anthropological research for example uses network theory to study people and their relationships, and explain emergent complex phenomena such as human behavior. Neuroscience uses network analysis methods to detect anomalies in diseased human brains [32]. The impact of online social networks is studied to understand and predict future personal and profit-oriented communication (online marketing) [33]. Epidemiologists use graph-based methods to model the spread of diseases like COVID-19, predict the future course of outbreaks and evaluate strategies to control epidemics [34]. Molecular biologists study intra- and intercellular signaling networks to understand the mechanisms behind biological processes and investigate the causes of network dysregulation, often leading to the emergence of particular disease phenotypes. Such network-based approaches have significant clinical applications since they have the potential to assist in the discovery of new disease genes and modules, and the identification of drug targets and biomarkers for complex diseases [35].

The work presented in this thesis is heavily based on this network medicine paradigm, with causal molecular interaction networks as the main object of study. Our primary focus is on protein-protein interaction (PPI) networks, with proteins as nodes and their physical contacts and interactions as links, and gene regulatory networks, represented for example by directed regulatory relationships between transcription factors and genes (TF-TG networks). These types of networks demonstrate a system of signal transduction pathways connected by crosstalk and embedded in feedback loops, forming what is known as the *Prior Knowledge Network* (PKN). The causality property of the PKN stems from the fact that the network links are directed (i.e. protein X affects protein Y) and signed (Y is inhibited or activated as a result). It's exactly this causality information that allows the investigation of behaviors from a systems perspective. Such networks form the basis for the study and computational modeling of cancer, which is another subject of investigation in this thesis. In the subsequent chapters, we will discuss how we addressed problems related to the formalization, access and public sharing of the knowledge encoded in the PKN.

Knowledge from a stack of papers

Biological Dictionaries aid in the curation of complex knowledge

Sharing causal interactions with PSICQUIC

Biological modeling: a Prelude

Clean Code

Links to software, documentation and data analyses

GitHub organizations

- UniBioDicts: <https://github.com/UniBioDicts/>
- VSM: <https://github.com/vsm>
- DrugLogics: <https://github.com/druglogics>
- PSICQUIC: <https://github.com/PSICQUIC>

Documentation

- DrugLogics software: <https://druglogics.github.io/druglogics-doc/>
- VSM technology and related projects: <https://vsm.github.io/>
- PSICQUIC: <https://psicquic.github.io/>

DrugLogics software modules

- [gitsbe](#): A Java module that defines Boolean models compliant with observed behavior (e.g. steady state or perturbation data) using an automated, model parameterization genetic algorithm
- [drabme](#): A Java module that performs a drug perturbation response analysis to the Boolean model ensembles generated by gitsbe
- [druglogics-synergy](#): A Java module to execute serially gitsbe and then drabme
- [abmlog](#): A Java-based generator of all possible logical models with AND/OR-NOT link operators in their respective Boolean equations
- [druglogics-roc](#): R Shiny web app to visualize the ROC and PR prediction performance of drabme's ensemble-wise predictions
- [emba](#): R package for analysis and visualization of biomarkers in Boolean model ensembles

R community packages

- [usefun](#): various useful R functions
- [rtemps](#): templates for reproducible data analyses with R [36]

Miscellaneous data analyses and repositories

All the following repositories have been authored exclusively by myself. Each repository has a README .md file with a brief description of the analysis and a link to online documentation and results (in the form of R Markdown documents or R GitBooks).

- [CASCADE](#): repository of the CAncer Signaling CAusality DatabasE developed by the Drug-Logics group and its subsequent versions
- [ags-paper](#): simulation results and data analyses related to the AGS paper manuscript
- [sintef-obs-synergies](#): synergy assessment of the Flobak et al. (2019) [37] drug combination dataset using rbbt [38] and the CImbinator tool [39]
- [brf-bias](#): data analyses related to truth density bias in standardized Boolean regulatory functions (results are part of the Boolean function metrics paper)
- [gitsbe-model-analysis](#): several analyses using Boolean model ensemble datasets generated via gitsbe and the emba R package to analyze them
- [bool-param-maps](#): visualization of model parameterization and node importance using UMAP and random forests on the CASCADE 1.0 Boolean model dataset generated by abmlog

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Papers

PAPER 1

Databases and ontologies

UniBioDicts: Unified access to Biological Dictionaries

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Abstract

Summary: We present a set of software packages that provide uniform access to diverse biological vocabulary resources that are instrumental for current biocuration efforts and tools. The Unified Biological Dictionaries (UniBioDicts or UBDs) provide a single query-interface for accessing the online API services of leading biological data providers. Given a search string, UBDs return a list of matching term, identifier and metadata units from databases (e.g. UniProt), controlled vocabularies (e.g. PSI-MI) and ontologies (e.g. GO, via BioPortal). This functionality can be connected to input fields (user-interface components) that offer autocomplete lookup for these dictionaries. UBDs create a unified gateway for accessing life science concepts, helping curators find annotation terms across resources (based on descriptive metadata and unambiguous identifiers), and helping data users search and retrieve the right query terms.

Availability and implementation: The UBDs are available through npm and the code is available in the GitHub organisation UniBioDicts (<https://github.com/UniBioDicts>) under the Affero GPL license.

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Supplementary information: [Supplementary data](#) are available at *Bioinformatics* online.

1 Motivation

The plethora of ontology terms and biological entity identifiers (IDs) provides a vast resource for use in annotations (by curators) and in database queries (by life scientists and computers), but specifying and finding them requires extensive navigation through an intimidating number of web resources and look-up forms. A universal way to perform a comprehensive search of life science databases, ontologies and vocabularies, supported by an autocomplete function that allows users to choose from a list of candidate terms with defining metadata, will greatly streamline this process. In addition, it will help to eliminate errors that stem from typing these terms manually without autocomplete support or options for semantic input checking. Furthermore, a unified lookup utility makes terms from diverse vocabularies easy to place together into context-rich annotations. The Visual Syntax Method (VSM) for example (Vercruysse and Kuiper, 2020), a technology that allows the flexible annotation of virtually any type of contextual information, can take advantage of unified access to such a large diversity of terms, e.g. in applications like causalBuilder (Touré et al., 2020). For these reasons, we set out to create a software suite that maps many of the diverse resources to a single data access and representation form.

2 Implementation

Each UBD module is an interface to an online server that provides ontology or controlled vocabulary data. A single dictionary module may provide access to one or several apparent ‘sub-dictionaries’; e.g.

the BioPortal UBD presents each of its many combined biological-domain ontologies as a distinct sub-dictionary. When a UBD receives a request for data, it makes a custom request to the associated server’s API, and translates received data back into the format specified by the [generic dictionary interface](#).

2.1 Main methods and data-types

Each UBD module offers the following methods to access a resource’s data, along with options for filtering, sorting and paging of results:

1. `getDictInfos`: returns a list of `dictInfo` objects which each hold information about one sub-dictionary of the data resource.
2. `getEntries`: returns entry objects. Each entry represents all relevant information about a specific biological concept. It is the combination of a computer-processable ID, at least one human-friendly term (a word or word sequence), and various metadata. The combined metadata makes it possible to inform curators of what a concept represents and how its meaning differs from others. For example, the UniProt UBD returns the ‘tp53’ concept via the standard properties: `id` (a URI, Uniform Resource ID: ‘<https://www.uniprot.org/uniprot/P04637>’), `terms` (a list: ‘P53_HUMAN’, ‘Cellular tumor antigen p53’, etc., with recommended name first and synonyms next), `descr` (text description of the protein), `dictID` (URI for the resource: ‘<https://www.uniprot.org/>’); and an extra set of `z` sub-properties for data

specific to UniProt: *z.species* ('Homo Sapiens'), *z.genes* ('TP53', 'P53'), etc.

3. `getEntryMatchesForString`: returns match objects. Each match combines one term-string (which may be a synonym, for one or several entries) with a specific entry that it represents. For example, querying the UniProt dictionary for 'tumor antigen p53' returns among others the above entry object for 'tp53', augmented with the property *str* ('P53_HUMAN').

For each UBD, these 'get-' methods have been harmonized with the associated resource's available search and returned data. This is detailed in each UBD's Readme on GitHub.

2.2 Additional features

1. Several UBDs are **optimized for curator use**: a match object's *descr* and *str* are tweaked so that an autocomplete list can present available concepts in a way that is helpful in biocuration tasks. For example, when the Ensembl UBD queries its server for 'tp53', it receives several gene concepts with the same name and description, but different species and gene-synonyms. So to provide a more informative description, the last three are combined into an optimized *descr*.
2. Identifiers (*id*, *dictID*) are formed as **unambiguous, browsable URIs**. This supports giving users clickable access to details about a returned concept to verify if it conveys the desired semantics for their annotation (McMurry et al., 2017).
3. UBDs *entry* objects are **extensible**. Any extra information offered by a resource's API can be added in the *entry.z* object, where it can later be used to customize or augment what an autocomplete shows to the user.

For further discussion on implementation and the expected impact of UBDs in the biocuration world, see [Supplementary File S1](#).

3 Results

3.1 Implemented UBDs

Current UBDs map and unify the following biological resources and their respective APIs:

- BioPortal (Whetzel et al., 2011), the largest repository of biomedical ontologies, using the [BioPortal REST API](#)
- PubMed MEDLINE database of biomedical literature, using the Entrez programming utilities (Sayers, 2010)
- Noctua Entity Ontology, using their Solr Web service
- UniProt (The UniProt Consortium, 2019), using their [REST API](#)
- Ensembl (Zerbino et al., 2018)
- Ensembl Genomes (Howe et al., 2020)
- RNACentral (The RNACentral Consortium, 2018)
- Complex Portal (Meldal et al., 2019)

The last four UBDs each process a different data domain from the EBI Search API (Madeira et al., 2019). In addition, we provide a [package](#) that can combine several UBDs into one virtual dictionary, enabling the querying of multiple UBDs through one access point (see [demo example](#) where a [vsm-box](#) tool's autocomplete is linked to UBDs).

3.2 Potential users

1. **Research software engineers** who use UBDs as a meta-API. They can programmatically access multiple resources in a uniform

way and avoid dealing with disparate APIs that all have different documentation, specifications and data formats.

2. **Software developers** who build a project-specific curation tool. They can create input fields that offer autocomplete lookup in any set of UBDs and present matching terms and IDs in a selection panel. This is easily achieved by linking any dictionary to our reusable [autocomplete web-component](#). UBDs can also be linked to a [vsm-box](#) (Vercruysse et al., 2020) to build curation applications, like causalBuilder.
3. **Biocurators** who use the above curation tools to find the terms they need. Autocomplete-based annotation allows biocurators to curate papers more quickly, conveniently and precisely, without having to copy text and IDs from elsewhere (Ward et al., 2012).

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Author's comment: In this supplementary file, we discuss what UBDs can offer to the biocuration and systems biology world, and the problems that we faced and had to overcome towards this goal.

Discussion

Although many of the leading resources provide individual support for finding appropriate identifiers, terms and definitions for biological entities and concepts, an overarching function that spans all resources is not yet available. Such a utility, providing real-time access to terminology from diverse biological subdomains through a unified interface, enables the development of tools that build upon the collective information residing in these disparate domains. A unified access to the wealth of descriptive information forms an essential enabling part of computational, semantic systems biology. Continuing in this spirit, we have recently started building another [UBD](#) that connects with PubDictionaries (Kim et al. 2019), and we invite future collaborators to join our [UniBioDicts](#) GitHub organisation and help build a growing collection of client APIs serving biological dictionaries. The currently developed packages cover a diverse range of web-services, API-technologies and associated data-types, providing concrete examples that facilitate the development of additional UBDs, or for that matter, any other domain dictionaries that may need to access online databases or ontologies for curation.

In the process of building the UBDs, we had to consult with at least one developer from each data or API resource, in order to clarify, refine, and simplify both their and our documentation and specification details, which subsequently led to a better design of the software. For example, individual APIs return error objects in different ways, which prompted us to harmonize our error handling specification across all UBDs. In order to deliver robust software that will benefit its users and optimize software development efforts in the future, face-to-face discussions coupled with extensive Q&A email correspondence proved to be essential (Prlić and Procter 2012). Finally, we wish to emphasize the importance that proper documentation has in a healthy software development practice (Karimzadeh and Hoffman 2018), and its vital role in achieving our aforementioned goal.

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PAPER 2

Linking PubDictionaries with UniBioDicts to support Community Curation

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Abstract

One of the many challenges that biocurators face, is the continuous evolution of ontologies and controlled vocabularies and their lack of coverage of biological concepts. To help biocurators annotate new information that cannot yet be covered with terms from authoritative resources, we produced an update of **PubDictionaries**: a resource of publicly editable, simple-structured dictionaries, accessible through a dedicated REST API. PubDictionaries was equipped with both an enhanced API and a new software client that connects it to the **Unified Biological Dictionaries** (UBDs) uniform data exchange format. This client enables efficient search and retrieval of *ad hoc* created terms, and easy integration with tools that further support the curator's specific annotation tasks. A demo that combines the Visual Syntax Method (VSM) interface for general-purpose knowledge formalization, with this new PubDictionaries-powered UBD client, shows it is now easy to incorporate the user-created PubDictionaries terminologies into biocuration tools.

Introduction

The curation of biological information is met with several challenges today. The constant refactoring of ontologies, nomenclature and identifiers, as well as the discovery of new types of information and new uses thereof, makes the life of knowledge curators difficult, especially in the highly diverse domain of biology. For example, expert curators who rely on specialized software tools in the annotation process might come across a new concept or feature that does not exist within an ontology or vocabulary that their annotation tool connects to. Similar difficulties are faced by non-expert curators. Some biologists want to create a project-specific knowledge resource in a biological niche that is only minimally or not yet covered by existing controlled vocabularies. Under time pressure of project milestones, these biologists may not immediately have the resources to organize a multilateral effort to standardize the terminology in their field. Instead they typically resort to creating 'private' vocabularies within their projects, that may later serve as a first step toward larger-scale coordination (Hartmann et al., 2019).

In cases where there is an abundance of resources for controlled vocabularies, ontologies and identifiers, it may still be challenging to coordinate access to these many necessary resources for dedicated annotation endeavours. Alternatively, in cases where no proper controlled vocabulary would exist, the results from all the work that goes into creating new vocabularies will remain largely isolated from general use, if no term sharing mechanism is available. Existing tools would then also be unable to access these newly created terms and for example serve them to curators in an autocomplete panel, to make their task easier and less error-prone.

These problems are addressed by two complementary initiatives. On the one hand, **Unified Biological Dictionaries (UniBioDicts or UBDs)** (Zobolas, Touré, Kuiper, & Vercruysse, 2020) is a set of software packages that offers a unified, single access point to biological terminology resources, and is available to be plugged into any curation platform currently in operation (for example the general-purpose curation interface 'vsm-box' (Vercruysse et al., 2020), based on VSM (Vercruysse & Kuiper, 2020), is out-of-the-box compatible with UBDs). On the other hand, **PubDictionaries** (Kim et al., 2019) is a public repository of dictionaries where users can create and immediately share their own dictionaries based on a simple data format consisting of a term and an identifier.

During the ELIXIR BioHackathon 2020, we updated and improved the PubDictionaries API as well as developed a new UBD client that directly communicates with that API. These implementation efforts constitute a significant step towards the unification of some of the most important data resources across all biological domains. The addition of PubDictionaries to the list of UniBioDicts-interoperable resources now provides a uniform way to search and autocomplete terms from all these community-created dictionaries as well. Such functionality enables the easier integration of PubDictionaries in any curation tool that may have a need for their terms and for such a term suggestion feature.

Results

In the following two subsections, we briefly summarize the results of the hackathon efforts, grouped into two categories:

- Updating the PubDictionaries REST API
- Creating a new UBD client library to access the above API

PubDictionaries REST API

PubDictionaries is a public repository of dictionaries, where each dictionary is a collection of **labels** (human-friendly **terms**) and **identifiers** (unambiguous **IDs**, used by computers). Each label + ID pair is called an **entry**. Multiple entries can have the same ID (for synonymous labels) and the same label can occur in multiple entries (for ambiguous ones). Users can create their own dictionaries and add entries to them via the web-interface. The dictionaries can be used to annotate any piece of text via the PubAnnotation ecosystem (Kim et al., 2019) or to simply lookup terms, and both these services are supported by a RESTful API (Kim, 2020). All the API responses are structured as JSON objects. Prior to the BioHackathon event, the REST service provided the following main endpoints:

1. **find_ids**: given some specific terms and dictionary names, this endpoint returns the corresponding IDs that approximately match the terms in these dictionaries. Example: https://pubdictionaries.org/find_ids.json?labels=TP53&dictionaries=human-UniProt
2. **prefix_completion/substring_completion**: given a term (or partial term string), these endpoints search for prefix (respectively substring) matches in a specified dictionary and return the corresponding entries. Note that only a first page of results was returned with at most 15 entries, prior to the hackathon efforts. Example: https://pubdictionaries.org/dictionaries/human-UniProt/prefix_completion?term=p53
3. **text_annotation**: given a piece of text and dictionary names, this endpoint returns the result of annotation to the text using the dictionaries. The annotation is performed based on computation of string similarity between dictionary entries and expressions in the text. Example: [https://pubdictionaries.org/text_annotation.json?dictionary=human-UniProt&text=The%20tumor%20suppressor%20p53%20\(TP53\)%20is%20the%20most%20frequently%20mutated%20human%20gene](https://pubdictionaries.org/text_annotation.json?dictionary=human-UniProt&text=The%20tumor%20suppressor%20p53%20(TP53)%20is%20the%20most%20frequently%20mutated%20human%20gene)

The following REST Service endpoints were added during the BioHackathon:

1. `dictionaries` endpoint: returns information about a specific dictionary, such as its id, name, a text description, the number of entries it has, etc. Example: <https://pubdictionaries.org/dictionaries/human-UniProt.json>, where `human-UniProt` can be any existing dictionary name.
2. `entries` endpoint: returns all entries of a specific dictionary, paginated and sorted by label. Example: https://pubdictionaries.org/dictionaries/human-UniProt/entries.json?page=1&per_page=15 Note that the users (software developers) can explicitly configure how the result should be paginated, i.e. how many entries of a dictionary should be included in one 'result-page', and what page they want to get results back from.
3. `find_terms` endpoint: this is the complement of the `find_ids` endpoint in the sense that it returns a list of terms and dictionary names that match the given IDs. The result is first sorted by ID and then by dictionary name. If no dictionary name is given to this endpoint, then it searches for the given IDs in all dictionaries. Example: https://pubdictionaries.org/find_terms.json?dictionaries=&ids=https://www.uniprot.org/uniprot/P04637
4. `mixed_completion` endpoint: a combined and updated version of the `prefix_completion` and `substring_completion` endpoints. For a given term (or partial term string) and a specified dictionary it returns a list of entries, putting the prefix completions in the top half and the substring completions in the bottom half, while pruning any possible common entries. In addition, this endpoint supports pagination which is a direct result of extending the prefix and substring endpoints to support this feature as well. Example: https://pubdictionaries.org/dictionaries/human-UniProt/mixed_completion?term=p53&page=2&per_page=3

Additional work on the PubDictionaries server-side included the support of create (via the HTTP POST method) and delete operations of a specific dictionary, given certified user credentials.

Lastly, the error handling was harmonized across all REST URL endpoints. In particular, when a user searches for a non-existent dictionary name, the PubDictionaries server returns a proper HTTP response status code, 400 (Bad Request), together with a JSON-formatted description as follows: `{ "message": "Unknown dictionary: <name>." }`. For example, all the following URL links return such a response object:

- <https://pubdictionaries.org/dictionaries/non-existent-dictionary-name.json>
- https://pubdictionaries.org/find_terms.json?dictionaries=non-existent-dictionary-name&ids=id1,id2
- <https://pubdictionaries.org/dictionaries/non-existent-dictionary-name/entries.json>

UBD Client for PubDictionaries

UBDs are a set of software packages that provide a unified query-interface for accessing the online API services of key biological vocabulary-data providers (Zobolas et al., 2020). The main feature of UBDs is their string-search functionality, which returns for a given label (or partial label) a list of matching term, identifier and metadata units from databases (e.g. UniProt (The UniProt Consortium, 2019)), controlled vocabularies (e.g. PSI-MI), and ontologies (e.g. Gene Ontology, via BioPortal (Whetzel et al., 2011)). This feature makes UBDs ideal for enabling autocomplete support in user-interface components that serve terms to curators from disparate resources, thus allowing the more efficient annotation of information.

Our work in the ELIXIR BioHackathon 2020 included the creation of a new UBD client ([vsm-pubdictionaries](https://github.com/elixir-europe/vsm-pubdictionaries)) that utilizes the updated PubDictionaries API in order to solve a long-standing problem in the biocurator community: how can *ad hoc*, project-specific terms and new information be effectively annotated with, and served via a curation platform, without the need to first negotiate the storage, update and reconciliation of that information with a third party, e.g. a database or ontology provider? Our client software addresses this problem

by presenting a mediator solution that can easily be plugged into current curation applications and serve *ad hoc* terms from PubDictionaries' public curator-created dictionaries.

Regarding the software client code, we wrote [extensive documentation](#) to delineate the mapping between the terms and IDs from PubDictionaries and the unified UBD format and how this is achieved via the updated PubDictionaries REST API endpoints, all in accordance with the UBDs' shared [dictionary interface specification](#). We also enabled continuous integration support via [GitHub Actions](#) and wrote extensive tests ([code coverage](#) is at 95%), so as to deliver more reliable, fault-tolerant and easy-to-extend software. Moreover, the documentation includes two examples; one showcasing the search term functionality via Node.js and one indicating how to use the client library in a web-based environment, with an HTML file. Finally, the [demo example](#) (see Figure 1) that was presented during the last report session of the BioHackathon, demonstrates a simple use-case where a few public dictionaries were created and their terms served in a vsm-box curation interface (Vercruysse et al., 2020). Thus we show how straightforward the annotation of new information can be by means of the autocomplete functionality of the provided curation tool, and how this new knowledge can be connected with semantically aware annotations.

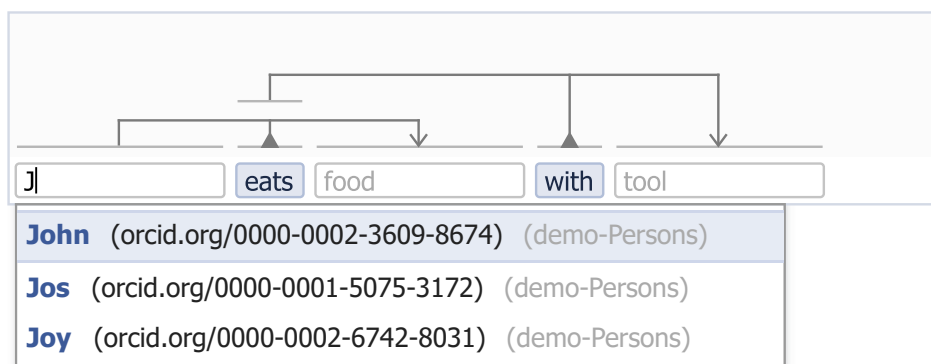


Figure 1: Demo example that uses a 'vsm-box' curation interface component, pre-filled with a VSM-template. An autocomplete panel appears while the user enters terms linked to identifiers. These term+ID pairs come from demo-dictionaries that we created at PubDictionaries, and are fetched through the new REST API-client described in this paper. Placeholders like 'food' and 'tool' indicate the kinds of dictionaries that specific fields of the template are connected with. On top, VSM-connectors formalize the structure and meaning of this knowledge unit.

Discussion

The advantages of a software package that connects with and queries any dictionary created in the PubDictionaries web-interface are multiple. This novel software enables annotation tools to use a common language and interface to link to information that is not yet available in standard databases. Note that the process of integrating new terms into standard resources can be time-consuming, so supporting communities of curators to create and utilize new terms that are at least publicly shared, in PubDictionaries, is a helpful first measure to tackle the problem of missing terms during ongoing curation work. Our software is a step towards achieving that goal, since it positions the community-manageable PubDictionaries into the mainstream of controlled vocabularies (CVs) and ontology resources. It fills the niche of new and *ad hoc* CVs that in turn may prompt new dedicated efforts to mature these CVs for consensus and expert maintenance.

Propelled by an ELIXIR BioHackathon event, our work underpins the goals of several of ELIXIR's activities, or so-called 'Platforms'. In the ELIXIR Data Platform, the drive to use, re-use and value life science data takes precedence. Our efforts exemplify how to achieve this by providing a scalable solution for curation platforms, especially ones that include support for annotation efforts that involve new information types. Furthermore, our main objective matches the goals of the ELIXIR Interoperability Platform: we offer a way to publicly-access and integrate new curated knowledge in a unified form, which enables new knowledge to be used by humans and machines alike, and to build knowledge systems that will aid future endeavors in understanding biological processes.

Future Work

Our future work includes updates on the PubDictionaries API to support the addition, update and deletion of dictionary entries, which is a functionality currently only available in the web-interface of PubDictionaries. Consequently, a further update on the UBD client will provide the necessary backbone to help build user interfaces, where curators would not even need to log in to the PubDictionaries website to create new dictionaries, and add, update or delete entries, but rather would be able to do that from within their own in-house curation tool. This functionality is currently not offered by any other biological data provider. Lastly, we expect that these updates, coupled with the search-string functionality provided by the PubDictionaries UBD client, will contribute in efforts to significantly increase the autonomy of biocurators and their potential for creating shareable annotations.

Links to software and documentation

- PubDictionaries API documentation: <https://docs.pubdictionaries.org>
- PubDictionaries source code: <https://github.com/pubannotation/pubdictionaries>
- UBD Client for PubDictionaries: <https://github.com/UniBioDicts/vsm-pubdictionaries>
- Demo example with vsm-box curation interface: https://github.com/UniBioDicts/vsm-pubdictionaries/blob/master/test/test_vsm_box_pubdictionaries.html

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PAPER 3

emba: R package for analysis and visualization of biomarkers in boolean model ensembles

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Introduction

Computational modeling of cellular systems has been one of the most powerful tools used to build interpretable knowledge of biological processes and help identify molecular mechanisms that drive diseases such as cancer (Aldridge, Burke, Lauffenburger, & Sorger, 2006). In particular, the use of logical modeling has proven to be a substantially useful approach, since it allows the easy construction, simulation and analysis of predictive models, capable of providing a qualitative and insightful view on the extremely complex landscape of biological systems (Abou-Jaoudé et al., 2016; Morris, Saez-Rodriguez, Sorger, & Lauffenburger, 2010; Wang, Saadatpour, & Albert, 2012). Such mechanistic models, with the systematic integration of prior knowledge and experimental data, have been extensively used to better understand what drives deregulation of signal transduction, the outcome of which is the manifestation of diseases (Traynard, Tobalina, Eduati, Calzone, & Saez-Rodriguez, 2017). Furthermore, their explanatory power has been used to provide insights into a drug's mode of action, investigate the mechanisms of resistance to drugs (Eduati et al., 2017) and suggest new therapeutic combination candidates, among others (Flobak et al., 2015).

One of the major challenges in systems medicine, has been the identification of scientifically validated, predictive biomarkers that correlate with patient response to given therapies. The analysis of biological predictive markers of pharmacologic response can not only further our understanding of the systemic processes involved in diseases but can also help to classify patients into groups with similar responses to specific therapeutic interventions, advancing personalized medicine (Senft, Leiserson, Rupp, & Ronai, 2017). In addition, the identification of biomarkers in tumor cells (e.g. mutations) has enabled the discovery of drug targets which are utilized in combinatorial molecular-targeted therapies - a strategy which aims to treat specific patient subgroups and has shown larger overall survival rates and reduced side-effects than monotherapy (Al-Lazikani, Banerji, & Workman, 2012). Despite the huge advancements towards drug combination therapy, genetic heterogeneity, drug resistance and drug combination synergy mechanisms still pose fundamental challenges to clinicians, modelers and lab researchers.

To help bridge the model simulation results with the (clinical) laboratory observations, several optimization methods have been used, such as model calibration, parameter estimation and sensitivity analysis. These methods also allow us to determine which model parameters have the biggest influence in the overall behaviour of the system (Aldridge et al., 2006). For example, in Fröhlich et al. (2018), a computational framework that allowed for the efficient parameterization and contextualization of a large-scale cancer signaling network, was used to predict combination treatment outcome from single drug data. This model was calibrated to fit and accurately describe specific cell-line experimental data, while enabling the identification

of biomarkers of drug sensitivity as well as molecular mechanisms that affect drug resistance. Furthermore, in Dorier et al. (2016), a network optimization approach which topologically parameterized boolean models according to a genetic algorithm was used, in order to best match the experimentally observed behaviour. This method resulted in an ensemble of boolean models which can be used to simulate response under drug perturbations in order to assess the underlying mechanisms and to generate new testable hypotheses. Such an aggregation of best-fit models (wisdom of the crowds) has been shown to be quite robust and effective for model prediction performance (Marbach et al., 2012).

Statement of need

There is a plethora of software tools devoted to the qualitative modeling and analysis of biological networks. The Consortium for the development of Logical Models and Tools (CoLoMoTo) is a community effort which aims to standardize the representation of logical networks and provide a common repository of methods and tools to analyze these networks (Naldi et al., 2015). Furthermore, to facilitate the access to several software logical modeling tools and enable reproducible computational workflows, the CoLoMoTo Interactive Notebook was introduced as a unified computational framework (Aurélien Naldi, Hernandez, Levy, et al., 2018). The incorporated tools are accessed via a common programming interface (though originally implemented in different programming languages e.g. Java, Python, C++ and R) and offer a collection of features like accessing online model repositories (Helikar et al., 2012), model editing (Aurélien Naldi, Hernandez, Abou-Jaoudé, et al., 2018), dynamical analysis (finding attractors, stochastic simulations, reachability properties, model-checking techniques) (Klarner, Streck, Siebert, & Sahinalp, 2016; Müssel, Hopfensitz, & Kestler, 2010; Aurélien Naldi, 2018; Paulevé, 2017; Stoll et al., 2017) and model parameterization/optimization to fit perturbation signaling data (Gjerga et al., 2020; Terfve et al., 2012). Despite the diverse and multi-purpose logical modeling tools that exist, there is still a lack of data analysis-oriented software that assists with the discovery of predictive biomarkers in ensembles of parameterized boolean networks that have been subject to drug combination perturbations.

The *emba* R package aims to fill that gap and provide a first implementation of such a novel software. Initially, it was designed as a complementary software tool, to help the analysis of the parameterized boolean model ensembles which were produced by modules from the DrugLogics NTNU software pipeline (see respective documentation (Zobolas, 2020a)). Later, we generalized most of the functions in the package and modularized them to package-essential (that form the core of the *emba* package) and various general-purpose yet useful functions (that are now part of the dependency package *usefun* (Zobolas, 2020b)).

Summary

The main functionality of the *emba* R package is to find *performance* and *synergy* biomarkers. Performance biomarkers are nodes in the input boolean networks whose activity state and/or model parameterization affects the predictive performance of those models. The prediction performance can be assessed via the number of true positive predictions or the Matthews correlation coefficient score which is more robust to class imbalances (Chicco & Jurman, 2020). On the other hand, synergy biomarkers are nodes which provide hints for the mechanisms behind the complex process of synergy manifestation in drug combination datasets.

For more information, see our “Get started guide” and the reference manual in the package website (Zobolas, 2020c). Several analyses using the *emba* R package are available in a separate repository (Zobolas, 2020d). Future developments will include the implementation of a method for the identification of *topology* biomarkers, where we will be able to assess

which interactions in the network are important for the manifestation of synergies in specific cell-contexts.

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PAPER 4

BOOLEAN FUNCTION METRICS CAN ASSIST MODELERS TO CHECK AND CHOOSE LOGICAL RULES

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Abstract

Computational models of biological processes provide one of the most powerful methods for a detailed analysis of the mechanisms that drive the behavior of complex systems. Logic-based modeling has enhanced our understanding and interpretation of those systems. Defining rules that determine how the output activity of biological entities is regulated by their respective inputs has proven to be challenging. Partly this is because of the inherent noise in data that allows multiple model parameterizations to fit the experimental observations, but some of it is also due to the fact that models become increasingly larger, making the use of automated tools to assemble the underlying rules indispensable.

We present several Boolean function metrics that provide modelers with the appropriate framework to analyze the impact of a particular model parameterization. We demonstrate the link between a semantic characterization of a Boolean function and its consistency with the model's underlying regulatory structure. We further define the properties that outline such consistency and show that several of the Boolean functions under study violate them, questioning their biological plausibility and subsequent use. We also illustrate that regulatory functions can have major differences with regard to their asymptotic output behavior, with some of them being biased towards specific Boolean outcomes when others are dependent on the ratio between activating and inhibitory regulators.

Application results show that in a specific signaling cancer network, the function bias can be used to guide the choice of logical operators for a model that matches data observations. Moreover, graph analysis indicates that the standardized Boolean function bias becomes more prominent with increasing numbers of regulators, confirming the fact that rule specification can effectively determine regulatory outcome despite the complex dynamics of biological networks.

Keywords Boolean regulatory networks · Boolean functions · Truth Density · Bias · Complexity

1 Introduction

The understanding of biological processes has been greatly stimulated by systems biology approaches [1, 2, 3]. The integration of mathematical models with the underlying biological knowledge and empirical observations can help us observe emergent systems properties, test new hypotheses, enhance the interpretability of the studied systems and guide innovations in areas such as medicine and drug discovery [4]. While multiple mathematical modeling frameworks exist, the scarcity of experimental data and the challenges posed by the development of quantitative large-scale biological networks, has favoured the simplicity and intuitiveness of more qualitative approaches, such as logic-based modeling [5].

At the heart of the mathematical representation of molecular biological networks lies the concept of regulation. Regulation of activity, typically by changing the modification state, location or concentration of a biological entity, is a process which can be expressed by a mathematical function that combines the various regulatory inputs that affect the target, with a logic that describes how these regulators are integrated. In Boolean logic-based modeling, the regulatory inputs are entities which can be expressed in two states: active (1) or inactive (0). These entities are combined with logical rules to derive the *Boolean regulatory function* (BRF) of the target entity. For every possible regulatory input (combination of 0 and 1's) the BRF will produce the end regulatory product, which is the activity of the target (0 or 1).

The construction of a Boolean computational model starts with the assembly of information from literature and experimental observations, in the form of a Prior Knowledge Network (PKN), i.e. a list of network entities and their causal interactions (positive or negative) [6, 7]. The use of a PKN for accurate representation of biological reality and subsequent analysis and simulation requires the definition of the model formalism. This is one of the most important steps in dynamical modeling since it directly translates to the choice of BRFs, i.e. the logical rules that together with the regulators define the activity state of each network target [8]. There have been several approaches related to the choice of BRFs, from using a standardized format [9], to automatically generating all possible BRFs compatible with the PKN and calibrating the rules in order to fit perturbation data [10, 11, 12]. State-of-the-art approaches involve the automated construction of large-scale logical networks by inferring the logical rules from the topology and semantics of molecular interaction maps [13].

Regardless of how a logical model is constructed, it has been shown in practice that expert curation, i.e. the manual fine-tuning of the logical rules to fit experimental data, can result in highly predictive models [14, 15], yet this is not trivially obtained with automatically constructed networks [16]. Because of the large function space complemented with a sparsity of observations and inherent noise in existing data, there is a wide range of plausible BRFs. Thus, it is crucial to properly define function characteristics that can guide the modeler to a more informed function choice. Our work is focused on explicating some of these metrics and using them to show for example which BRFs can be discarded due to biological inconsistencies with the underlying regulatory topology and which are biased towards specific Boolean outcomes.

The paper is structured as follows: Section 2 provides a list of notations and definitions to be used later in the text. In Section 3, we discuss the benefits of using the equivalent disjunctive normal form of a Boolean function to delineate its biological interpretability. In Section 4, we provide a set of properties that characterize the Boolean functions that are consistent with a given regulatory topology and show that several functions under study violate them. In Section 5, we present the truth density metric as a means to evaluate if a Boolean function is biased or balanced with increasing number of regulators. We also discuss the asymptotic properties of different functions relating to the ratio between activators and inhibitors. Lastly, in Section 6, we present evidence that the standardized Boolean functions are indeed biased and show how modelers can exploit such information for their own benefit. The results are demonstrated in Boolean models derived from a cancer signaling network as well as from scale-free topologies that are applicable to most biological networks. We close the paper with some discussion points in Section 7 and directions for future research in Section 8.

2 Background

2.1 Boolean regulatory functions

Boolean regulatory functions (BRFs) are Boolean functions used in the context of biological networks and modeling. A mathematical description of such a function associates the activity output of a target biological entity with the Boolean input values of n variables (the *regulators*), such that $f_{BRF} : \{0, 1\}^n \rightarrow \{0, 1\}$. Thus, the target's output state is binary, i.e. either 0 (*False*, denoting an inactive or inhibited state) or 1 (*True*, indicating an active state).

One intuitive representation of a Boolean function is its *truth table*, which is a list of all possible Boolean input configurations of the n regulators along with their associated function output. Since every regulator can be assigned two possible values (0 and 1), the total number of input configurations (i.e. rows) in a truth table is 2^n . For example, a Boolean function $f(x_1, x_2, x_3)$ with 3 regulators has a total of $2^3 = 8$ rows in its corresponding truth table, starting from the input configuration (0, 0, 0) and ending with (1, 1, 1) (Table 1).

The total number of BRFs with n regulators is 2^{2^n} since for each of the 2^n input configurations (i.e. rows of the truth table) there can be two possible function outcomes (0 or 1). For example, with 3 regulators and a total of 8 rows in the truth table, that would be a total of $2^8 = 256$ functions, three of which are shown in Table 1.

2.2 Disjunctive normal form

The most frequently used form of a Boolean function is its analytical expression, where variables are connected with logical operators such as AND (\wedge), OR (\vee), NOT (\neg), XOR (\oplus), etc. and the output of the function is calculated using basic Boolean algebra. In Table 1 for example, we provide the analytical forms for the functions f_1 and f_2 . Note that there can be multiple analytical forms that essentially compute the same function, e.g. another form of the f_1 function is $f'_1 = (\neg x_1 \wedge x_2 \wedge \neg x_3) \vee (x_1 \wedge \neg x_2 \wedge \neg x_3) \vee (x_1 \wedge x_2 \wedge \neg x_3)$.

Truth Table			Boolean functions		
x_1	x_2	x_3	$f_1 = (x_1 \wedge \neg x_3) \vee (x_2 \wedge \neg x_3)$	$f_2 = x_1 \vee (\neg x_2 \wedge \neg x_3)$	$f_3 = 1$
0	0	0	0	1	1
0	0	1	0	0	1
0	1	0	1	0	1
0	1	1	0	0	1
1	0	0	1	1	1
1	0	1	0	1	1
1	1	0	1	1	1
1	1	1	0	1	1

Table 1: Truth table of three Boolean functions with three input variables x_1, x_2 and x_3 . Functions f_1 and f_2 are expressed in disjunctive normal form (DNF) with the minimum possible number of terms. f_3 is a tautology.

This brings us to the notion of a general form which could be used to define useful metrics common to all Boolean functions (e.g. complexity), as well as the need to provide minimal forms based on specific criteria. For example, a more compact function form enhances readability, which can be seen by comparing f_1 with f'_1 .

Every Boolean function can be represented in a *disjunctive normal form* (DNF), requiring only AND (\wedge), OR (\vee) and NOT (\neg) operators as building blocks. In such a representation, *literals*, which are variables (e.g. positive literal x) or their logical negations (e.g. negative literal NOT x), are connected by AND's, producing *terms*, which are then in turn connected by OR's [17]. For example, every function in Table 1 is expressed in DNF, while the Boolean expressions $\neg(x_1 \vee x_2)$ and $\neg(x_1 \wedge x_2) \vee x_3$ are not. Note that a Boolean function can have multiple DNF formulations.

2.3 Link operator functions

We consider the class of BRFs that partitions the input regulators to two sets: the set of positive regulators (*activators*) and the set of negative regulators (*inhibitors*). Let f be such a Boolean function $f_{BRF}(x, y) : \{0, 1\}^n \rightarrow \{0, 1\}$, with $m \geq 1$ activators $x = \{x_i\}_{i=1}^m$ and $k \geq 1$ inhibitors $y = \{y_j\}_{j=1}^k$, that is a total of $n = m + k$ regulators. The *link operator* BRFs have an analytical formula which places the two distinct types of regulators in two separate expressions, while connecting them with a special logical operator that we call a *link operator*. An example of such a function that has been used extensively in the logical modeling literature is the standardized BRF formula with the “AND-NOT” link operator [9]:

$$f_{AND-NOT}(x, y) = \left(\bigvee_{i=1}^m x_i \right) \wedge \neg \left(\bigvee_{j=1}^k y_j \right) \quad (1)$$

A variation of the above function is the “OR-NOT” link operator function:

$$f_{OR-NOT}(x, y) = \left(\bigvee_{i=1}^m x_i \right) \vee \neg \left(\bigvee_{j=1}^k y_j \right) \quad (2)$$

Note that the presence of the link operator is what forces the condition $m, k \geq 1$ (at least one regulator in each category). For the rest of this work, we will not consider BRFs with only one type of regulator, since these can be represented by simple logical functions without loss of biological consistency. Following the notation introduced in Mendoza et al. [9], in the case of only positive regulators, the presence of at least one activator makes the target active, i.e. $f(x) = \bigvee_{i=1}^m x_i$. In the case of only inhibitory regulators, the presence of at least one inhibitor is sufficient to make the target inactive, i.e. $f(y) = \neg \bigvee_{j=1}^k y_j = \bigwedge_{j=1}^k \neg y_j$.

Borrowing notation from circuit theory, we will also use other link operators like the “NAND”, “NOR”, “XNOR” gates, with or without the “NOT” symbol in front. Note that the logical operator used to connect the same type of regulators (e.g. the activators) is usually OR, but other operators could be used as well.

Another link operator function that we will consider in this work is the “Pairs” function:

$$f_{Pairs}(x, y) = \bigvee_{\forall(i,j)}^{m,k} (x_i \wedge \neg y_j) \quad (3)$$

The intuition behind the name is derived from the fact that the function will return *True* if there is at least one pair of regulators consisting of a present activator and an absent inhibitor. For a formulation of the “Pairs” function that is consistent with the link operator terminology as defined above, see (Eq. 9).

2.4 Threshold functions

Threshold functions are a special type of Boolean functions, the output of which depends on the condition that the sum of (possibly weighted) activities of the input regulators surpasses a given *threshold* value [18, 19].

In this work we will consider two simple threshold functions, which both output *True* when the number of present activators is larger than the number of present inhibitors. As such, the activities of the positive and negative regulators are combined in an *additive* manner, with their respective assigned weights set to ± 1 and the threshold parameter to 0, formulating thus a *majority rule* which defines the value of the function [20, 21]. These functions differ with regards to their output when there is balance between the activities of the positive and negative regulators: the first outputs 1 (the activators “win”) while the second outputs 0 (the inhibitors “win”):

$$f_{Act-win}(x, y) = \begin{cases} 1, & \sum_{i=1}^m x_i \geq \sum_{j=1}^k y_j \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

$$f_{Inh-win}(x, y) = \begin{cases} 1, & \sum_{i=1}^m x_i > \sum_{j=1}^k y_j \\ 0, & \text{otherwise} \end{cases} \quad (5)$$

3 Disjunctive Normal Form unmasks biological interpretation

3.1 Interpretability issues in Boolean modeling

Two main features make Boolean modeling attractive to users. First, transforming conditions for the activation or inhibition of a target biological entity to Boolean equations is a relatively easy task using a qualitative, logic-based modeling formalism. Second, the reverse is also true, i.e. Boolean equations can be more interpretable and closer to a simplified description of biological reality that “makes sense” than the use of other kinds of formalisms (e.g. kinetic modeling). For example, consider the simple case of a target entity, which is regulated by one positive regulator x_1 and one negative regulator y_1 . The use of the “AND-NOT” link operator function in this case (Eq. 1) is very easy to understand and interpret since the formula directly connects to the underlying biology. Thus, the mathematical formulation is simply written as $f_{AND-NOT} = x_1 \text{ AND NOT } y_1$, while the modeler reads “the target becomes active when x_1 (the activator) is present and y_1 (the inhibitor) absent”.

Issues start arising when considering the *interpretability* of such Boolean expressions in cases where a larger number of regulators act on a target, e.g. in a more complex scenario with three positive (x_1, x_2, x_3) and three negative (y_1, y_2, y_3) regulators, the mathematical formulation expressing the target’s activity output can be easily written using the link operator function form, as $f_{AND-NOT} = (x_1 \text{ OR } x_2 \text{ OR } x_3) \text{ AND NOT } (y_1 \text{ OR } y_2 \text{ OR } y_3)$. A modeler could read this as “the target becomes active when at least one activator is present, and all of its inhibitory regulators are absent”, but a precise semantic description that explicates the conditions under which the target gets activated, can in general be difficult to assess. A similar issue arises when reflecting on the use of a different link operator instead of the standard “AND-NOT” or even of an entirely different regulatory function, for which the biological interpretation might be difficult to derive from the expression itself.

BRF (standard form)	BRF (CDNF)	Biological Interpretation	Consistent	Complexity
$(x_1 \text{ OR } x_2) \text{ NOR } (y_1 \text{ OR } y_2)$	NOT x_1 AND NOT x_2 AND NOT y_1 AND NOT y_2	Absence of all regulators	NO	1 (always)
$(x_1 \text{ OR } x_2) \text{ NAND } (y_1 \text{ OR } y_2)$	(NOT x_1 AND NOT x_2) OR (NOT y_1 AND NOT y_2)	Absence of all activators or absence of all inhibitors	NO	2 (always)
$(x_1 \text{ OR } x_2) \text{ AND NOT } (y_1 \text{ OR } y_2)$ “AND-NOT” (Eq. 1)	$(x_1 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$ OR $(x_2 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$	Presence of at least one activator and absence of all inhibitors	YES	2 (m)
$(x_1 \text{ OR } x_2) \text{ NOR NOT } (y_1 \text{ OR } y_2)$	$(y_1 \text{ AND NOT } x_1 \text{ AND NOT } x_2)$ OR $(y_2 \text{ AND NOT } x_1 \text{ AND NOT } x_2)$	Presence of at least one inhibitor and absence of all activators	NO	2 (k)
$(x_1 \text{ OR } x_2) \text{ OR NOT } (y_1 \text{ OR } y_2)$ “OR-NOT” (Eq. 2)	$x_1 \text{ OR } x_2 \text{ OR } (NOT y_1 \text{ AND NOT } y_2)$	Presence of any activator or absence of all inhibitors	YES	3 ($m + 1$)
$(x_1 \text{ OR } x_2) \text{ NAND NOT } (y_1 \text{ OR } y_2)$	$y_1 \text{ OR } y_2 \text{ OR } (NOT x_1 \text{ AND NOT } x_2)$	Presence of any inhibitor or absence of all activators	NO	3 ($k + 1$)
$(x_1 \text{ OR } x_2) \text{ XOR } (y_1 \text{ OR } y_2)$	$(x_1 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$ OR $(x_2 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$ OR $(NOT x_1 \text{ AND NOT } x_2 \text{ AND } y_1)$ OR $(NOT x_1 \text{ AND NOT } x_2 \text{ AND } y_2)$	Presence of at least one activator and absence of all inhibitors or presence of at least one inhibitor and absence of all activators	NO	4 ($m + k$)
$(x_1 \text{ OR } x_2) \text{ AND } (NOT y_1 \text{ OR NOT } y_2)$ “Pairs” (Eq. 3)	$(x_1 \text{ AND NOT } y_1)$ OR $(x_1 \text{ AND NOT } y_2)$ OR $(x_2 \text{ AND NOT } y_1)$ OR $(x_2 \text{ AND NOT } y_2)$	Presence of at least one activator and absence of at least one inhibitor	YES	4 ($m \times k$)
$(x_1 \text{ OR } x_2) \text{ XNOR } (y_1 \text{ OR } y_2)$	$(x_1 \text{ AND } y_1)$ OR $(x_1 \text{ AND } y_2)$ OR $(x_2 \text{ AND } y_1)$ OR $(x_2 \text{ AND } y_2)$ OR $(NOT x_1 \text{ AND NOT } x_2 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$	Presence of at least one activator and inhibitor pair or absence of all regulators	NO	5 ($m \times k + 1$)
<i>True</i> when $x_1 + x_2 > y_1 + y_2$ “Inh-win” (Eq. 5)	$(x_1 \text{ AND } x_2 \text{ AND NOT } y_1)$ OR $(x_1 \text{ AND } x_2 \text{ AND NOT } y_2)$ OR $(x_1 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$ OR $(x_2 \text{ AND NOT } y_1 \text{ AND NOT } y_2)$	Number of present activators is larger than the number of present inhibitors	YES	4
<i>True</i> when $x_1 + x_2 \geq y_1 + y_2$ “Act-win” (Eq. 4)	$(x_1 \text{ AND } x_2)$ OR $(x_1 \text{ AND NOT } y_1)$ OR $(x_1 \text{ AND NOT } y_2)$ OR $(x_2 \text{ AND NOT } y_1)$ OR $(x_2 \text{ AND NOT } y_2)$	Number of present activators is larger than or equal to the number of present inhibitors	YES	5

Table 2: Several Boolean regulatory functions with four regulators ($m = 2$ positive $\{x_1, x_2\}$, $k = 2$ negative $\{y_1, y_2\}$) and some metrics are presented. The two first columns provide two different function forms: a standard one, i.e. either the link operator form distinguishing activating and inhibiting regulators or a simple description in the case of the threshold functions, and the CDNF which is a special case of DNF (Section 4.1). The “Biological Interpretation” states in words the conditions that make a BRF become *True*, and is explicitly translated from the terms in the corresponding CDNF. The “Consistent” column states if the functions satisfy the properties 1-3 from Section 4.1 (YES, green-colored) or there are inconsistencies with the underlying regulatory structure (NO, red-colored), i.e. if an activator (resp. inhibitor) appears as a negative (resp. positive) literal in the corresponding CDNF. The functions are sorted according to an increasing complexity metric (“Complexity” column), which is the number of terms in each respective, minimum-length CDNF expression. In parentheses we provide the generalized formula for the number of CDNF terms of the link operator functions with m activators and k inhibitors.

3.2 DNF links to biological semantics

We argue here that the DNF is the most adequate function form to help us address the aforementioned issues. Every Boolean regulatory function expressed in DNF, has a biological characterization that is directly derived from the formula itself: each term in the DNF is an activation condition, i.e. a list of regulators, some present (the positive literals) and some absent (the negative literals), which, when combined, make the target (output of the function) active. Further merging of all the conditions using OR-semantics into a description of how the regulators influence the target’s output, facilitates the biological interpretation of any Boolean regulatory function.

In Table 2, we show a list of BRFs with two positive and two negative regulators. Most of the BRFs presented have a different link operator separating the activators from the inhibitors. Using the functions standard expressions (1st column) makes it very hard to derive a meaningful biological characterization as expressed in the 3rd column of Table 2. For example, defining a meaningful description of the “NOR” or “NAND-NOT” equations using only their standard expression, is a very difficult task. In contrast, by using the equivalent DNFs (2nd column) we can make an explicit, “1-1” correspondence between mathematical formulation and biological interpretation and use it to compare the different functions’ meanings. Thus, by expressing the “NAND-NOT” equation in DNF, we can precisely identify the conditions that make the outcome of the function *True* and translate these into a meaningful description such as “Presence of any inhibitor or absence of all activators”. Consequently, we are led to a generalized and independent of the number of regulators description of this link operator function. Such a description is intuitive to human interpretation and reasoning, in terms of the function’s applicability, e.g. in comparing the “AND-NOT” and “NAND-NOT” biological interpretations, we see that the first is semantically plausible while the second completely contradicts the underlying biology.

4 Characterizing consistent regulatory functions

4.1 The 3 consistency properties

As observed in Table 2, not only can the DNF be used to uncover the biological interpretation of any BRF and subsequently help determine its plausibility, but it also provides a means to compare the different function meanings. Still, we need a more refined, technical description that is able to express the implausibility of the “NAND-NOT” or “NOR” cases directly from their mathematical formulas, and which would be applicable to every BRF. We define the *consistency* attribute of a BRF to describe its compliance with the underlying regulatory network structure.

The first step in making a Boolean model is to build a graph (PKN), assembling the regulatory entities of interest from various databases or the scientific literature, and use causality information to connect them through their regulatory action on other entities. As such, a network structure can be defined, in which entities can regulate (either positively or negatively) some of the other entities. Using such a simple network-driven formalization, we define a set of three properties that describe the set of all the *consistent Boolean regulatory functions*, i.e. the functions that comply with the underlying regulatory structure. So, for a consistent BRF, the following propositions are satisfied [22]:

1. Its regulators can be partitioned into two disjoint sets: the set of *activators* (positive regulators, enhance target’s activity) and the set of *inhibitors* (negative regulators, suppress target’s activity). This stems from the fact that every interaction in the PKN has a fixed sign (either positive or negative). As such, there are no dual regulations, i.e. a regulator cannot activate and inhibit a target at the same time. This property essentially makes the set of consistent BRFs a subset of the *monotone* Boolean functions [17].

2. All regulators are *essential*: for every regulatory input, inverting their values, will also, in at least one configuration of states of other regulators, change the output of the function. This means that all regulators are indispensable for deriving the target’s activity output.
3. A consistent BRF can be represented in a unique *complete* DNF (CDNF) which is also known as Blake’s Canonical Form [23]. This is a consequence of property (1), since monotone Boolean functions expressed in any DNF, can be further simplified by removing redundant literals, resulting in the equivalent unique CDNF expression [17]. This property is really important since it allows us to identify which regulatory entities are activators and which are inhibitors from the corresponding CDNF expression of a consistent BRF: an activator will always appear as a positive literal, whereas an inhibitor will always appear as a negative literal.

We provide an example to delineate the difference between the DNF and CDNF forms and show violations of the consistency properties. In Table 3, we present three Boolean functions, expressing the output of a target regulated by one activator (x_1) and one inhibitor (y_1). The functions f_2 and f_3 are in CDNF whereas f_1 is in normal DNF, since the positive regulator x_1 appears both as a positive and a negative literal (i.e. it acts as a dual regulator, making f_1 inconsistent). Notice that f_1 reduces to f_2 by removing the redundant negative literal ($\neg x_1$) in the term $(\neg x_1 \wedge y_1)$: y_1 “absorbs” the larger term and thus a shorter expression manifests, one that covers more *True* outcomes (i.e. 1’s) in the truth table. In addition, we observe that f_2 is inconsistent, since inhibitor y_1 appears as a positive literal. On the other hand, using a negative literal for inhibitor y_1 and a positive one for activator x_1 , makes f_3 consistent.

Truth Table		Term	Boolean functions		
x_1	y_1	$(\neg x_1 \wedge y_1)$	$f_1 = x_1 \vee (\neg x_1 \wedge y_1)$	$f_2 = x_1 \vee y_1$	$f_3 = x_1 \vee \neg y_1$
0	0	0	0	0	1
0	1	1	1	1	0
1	0	0	1	1	1
1	1	0	1	1	1

Table 3: Truth table of three different Boolean regulatory functions with two input regulators, one positive (x_1) and one negative (y_1). All functions are expressed in DNF. f_1 and f_2 result in the same target Boolean output, with f_2 expressed in CDNF. Activator x_1 regulates the target both positively and negatively in f_1 , making the function non-monotone and thus inconsistent. Inhibitor y_1 is a positive literal in f_2 ’s CDNF, making it inconsistent as well. Function f_3 is consistent since it’s written in CDNF with the activator x_1 and inhibitor y_1 appearing as positive and negative literals respectively.

4.2 Most link operator functions are inconsistent

Examining Table 2, we note that the 2nd column presents not just any DNF expression of the studied Boolean regulatory functions, but precisely the CDNF. Thus we can immediately identify which BRFs violate at least one of the three properties discussed in Section 4.1 and are therefore inconsistent with the regulatory topology (this information is presented in the 4th column, labeled “Consistent”). Two examples of such inconsistencies include the “NAND-NOT” and “XOR” link operator functions, which have terms in their corresponding CDNF in which an activator x_i appears as a negative literal (NOT x_i) and an inhibitor y_j as a positive literal (as itself). In total, from all the BRFs presented in Table 2, only the standardized “AND-NOT”, the “OR-NOT”, the “Pairs” and the two threshold functions respect the underlying regulatory topology, as

can be verified by examining their respective CDNFs. The rest of the link operator functions presented are inconsistent and will not be considered for further analysis in this paper.

5 Truth Density as a measure of expected function output

In this section we present another interesting Boolean function metric, whose properties can be used to add further knowledge about a Boolean function’s behavior. This metric, which we call *truth density*, allows us to project what the regulatory target’s output will most likely be when the number of input regulators changes and investigate how the ratio between activators and inhibitors may affect that output. From a modeler’s perspective, this metric is useful to check if an assigned model parameterization (i.e. use of a specific BRF) can asymptotically predefine the activity state of some targets. Equipped with this knowledge, a modeler can verify the degree of fitness with the observations that such a parameterization allows, and thus discard a specific function in favor of another, if the latter has a truth density value that better matches the outcome observed in the data.

5.1 Truth Density

We define the truth density (TD) of a Boolean function as the fraction of all input configurations in its corresponding truth table that yield a *True* (1) outcome. As such, $TD \in [0, 1]$. This quantity was first introduced in [24] and more recently in [25] under the name of *bias* and was similarly defined as the probability that a Boolean function takes on the value 1. Using the example with the three Boolean functions from Table 1, we have $TD_{f_1} = 3/8 = 0.375$, $TD_{f_2} = 5/8 = 0.625$ and $TD_{f_3} = 8/8 = 1$, where the last function is a tautology, with the maximum possible truth density. Colloquially, we can say that a Boolean function is *biased*, when it’s truth density is close to 0 or 1. Since the size of a truth table grows exponentially with the number of inputs of the Boolean function (n inputs correspond to 2^n rows), the existence of bias conveys the information that most of the input regulatory configurations result in either an activated or inhibited target (bias towards 1 or 0 respectively). On the other hand, we shall say that a Boolean function is *balanced*, if it takes on the values 0 and 1 equally often, or equivalently, it’s truth density is approximately centered around $1/2$ [26].

5.2 Asymptotic truth density results of the consistent regulatory functions

In Appendix A, we present a list of propositions and proofs that provide the exact truth density formulas for the generic forms of the five consistent BRFs we studied in previous sections, namely the “AND-NOT”, “OR-NOT” and “Pairs” link operator functions, and the two threshold functions, “Act-win” and “Inh-win”. A very important element that enables the straightforward derivation of these formulas, is the use of the equivalent DNF expressions in the proofs, especially for the case of the link operator Boolean functions. We also noted that the truth densities of all the aforementioned BRFs depend on two variables: the number of activators and the number of inhibitors (the total number of regulators also appears as a separate variable but it depends on the first two, i.e. it is just their sum). Thus, we logically asked if a BRF’s truth density asymptotically tends towards specific values in the $[0, 1]$ interval (e.g. the function could be biased or balanced), when the number of its input regulators increases or the ratio between activators and inhibitors changes. The results of the asymptotic behavior of the truth density formulas are analytically presented in Appendix B.

The asymptotic analysis of the truth density formulas confirmed the intuitive perception that the link operator “AND-NOT” and “OR-NOT” functions show a characteristically opposite behavior with increasing number of regulators: the standardized “AND-NOT” formula depends only on the number of inhibitors and its output tends towards 0, whereas the “OR-NOT” formula depends only on the number of activators and is biased towards 1. On the other hand, the “Pairs” and threshold functions truth densities don’t have an asymptotic

limit since they depend on both the number of activators and inhibitors. Therefore, we proceeded in clarifying the role of the *activator-to-inhibitor* ratio by investigating three scenarios which explicitly reveal the functions truth density behavior for a significantly large number of regulators:

- A 1 : 1 activator-to-inhibitor ratio, where approximately half of the regulators are activators and half are inhibitors.
- A high activator-to-inhibitor ratio, where all regulators are activators except one inhibitor.
- A low activator-to-inhibitor ratio, where all regulators are inhibitors except one activator.

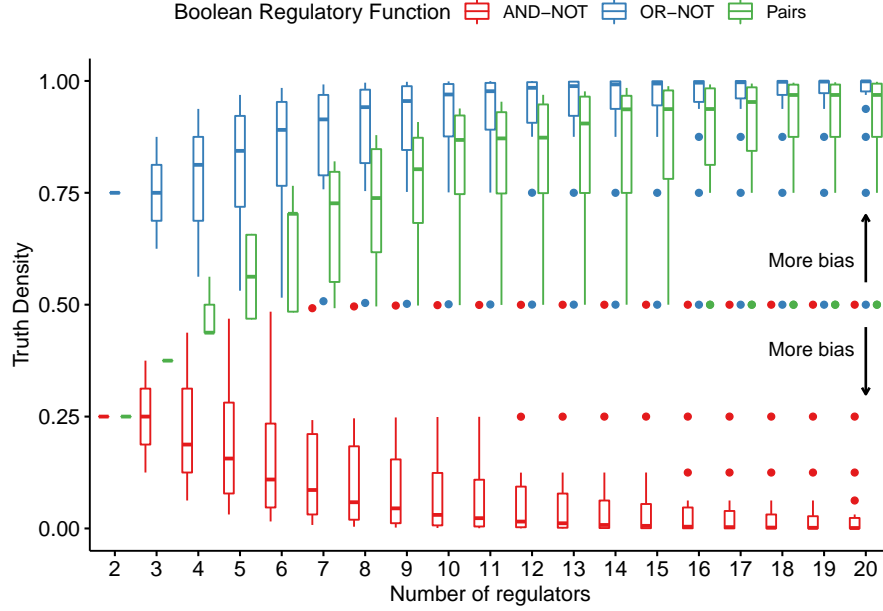
In the 1 : 1 ratio scenario, where there is an equal number of activators and inhibitors, the asymptotic behavior of the “AND-NOT” and “OR-NOT” functions corresponds to absolute inhibition (0) and activation (1) respectively, following the biased behavior shown previously. The “Pairs” function behaves similarly to the “OR-NOT” function and therefore is also biased towards 1. Only the threshold functions show balanced behavior with their truth density value reaching asymptotically $1/2$, since the majority rule does favor neither activators nor inhibitors in this scenario. On the other hand, in the two extremely unbalanced scenarios, where one set of regulators completely outweighs the other, the asymptotic truth density results of the “AND-NOT” and “OR-NOT” functions depend on each respective scenario. Specifically, when the inhibitors dominate over the activators, the “OR-NOT” is balanced and the “AND-NOT” is biased, since the former has been shown to depend exclusively on the number of activators (which is just one in this case) for increasingly more regulators, whereas the latter on the number of inhibitors. Their behavior is reversed when the activators outbalance the inhibitors. In contrast, the “Pairs” function behaves in a balanced manner, having a truth density asymptotically equal to $1/2$ in both these scenarios, since the single minority regulator is paired with every regulator from the dominant group in the respective DNF expression (Eq. 3) and as a result, it significantly influences the function’s output. Lastly, the asymptotic results for the threshold functions follow the larger size regulatory group, being biased towards 0 with significantly more inhibitors and biased towards 1 with significantly more activators.

5.3 Validation of asymptotic behavior

One key issue of immense practical importance for the modeler, which arises when analyzing the asymptotic behavior of the truth density formulas, is the actual number of regulators that effectively make each of the studied functions exhibit the demonstrated behavior. We noticed that most of the truth density formulas (Eq. 11, 12 and 13) are the sum of two to three terms, with only one of them depending exclusively on the number of regulators n . Also, this term is usually $1/2^n$, and can be omitted when considering values larger than $n = 10$ regulators since it’s insignificant ($1/2^{10} \approx 0.001$). This suggests that the limit value of the truth density formulas may be already derived from a much smaller number of regulators than what is implied by the study of asymptotes. Therefore, we need to have a more data-centric view of the results from our previous asymptotics analysis of the different BRFs, one that will enable us to verify the mathematically observed behaviors but also identify an approximate range for the number of regulators where the asymptotics decide the outcome of the studied functions.

We generated the complete truth tables for the five consistent BRFs of Table 2, from 2 up to 20 regulators, accounting for every possible activator-to-inhibitor ratio. For example, for $n = 10$ regulators, every combination of at least one activator and one inhibitor that adds up to 10 (1 activator + 9 inhibitors, 2 activators + 8 inhibitors, etc.) resulted in a different truth table for each considered Boolean function. Subsequently, using the generated truth tables, we could easily calculate the exact truth density value for each function at every considered ratio. The results are shown in Figures 1A and 1B for the link operator and threshold functions, respectively.

A



B

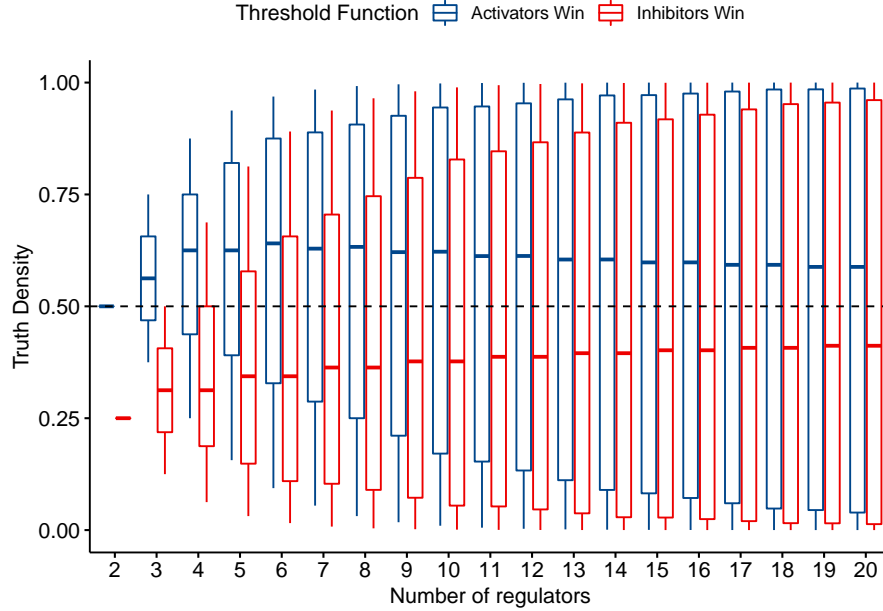


Figure 1: Comparing the truth densities of five different Boolean regulatory functions for different numbers of regulators and activator-to-inhibitor ratios. For each specific number of regulators, every possible combination of at least one activator and one inhibitor that add up to that number, results in a different truth table output with its corresponding truth density value. All such possible configurations up to 20 regulators are shown. (A) The standardized “AND-NOT” function, along with the “OR-NOT” and “Pairs” functions, show an increasingly biased behavior with more regulators. (B) The two threshold functions “Act-win” and “Inh-win” show a more balanced behavior, since they respect the activator-to-inhibitor ratio and thus demonstrate a larger spectrum of possible truth density values even for higher numbers of regulators.

The data in general shows that the different regulatory functions demonstrate quite dissimilar behaviors with regard to their asymptotic outcome. In particular, we recapitulate the findings from the asymptotics analysis, namely the bias of the link operator functions, which is evident even from 7 to 10 input regulators. Interestingly, the “Pairs” function follows asymptotically the behavior of the “OR-NOT” function but is in general less biased. We note that the outliers in Figure 1A with truth density values closer to $1/2$, represent imbalanced activator-to-inhibitor ratio scenarios, i.e. either considerably more activators than inhibitors for the “AND-NOT” function and the reverse for the “OR-NOT” function, or any imbalanced ratio for the “Pairs” function. Lastly, Figure 1B shows that the threshold functions exhibit a more balanced behavior, expressed as a higher spectrum of truth density values for any single number of regulators and with the median truth density asymptotically reaching $1/2$. This result is due to the fact that threshold functions faithfully follow the activator-to-inhibitor ratio, i.e. with more activators the outcome is biased towards 1 whereas with more inhibitors the function outcome tends towards 0.

6 Link operator parameterization determines activity state in biological networks

In this section we investigate if a model’s parameterization can effectively decide the activity state of nodes in biological networks. In more detail, we will use the “AND-NOT” link operator function [9] and its symmetric function “OR-NOT” (Eq. 1 and 2), to build Boolean models from prior causal knowledge and check if their activity state profile as determined by dynamic attractor analysis, shows the biased behavior that we observed in Section 5.

A major motivation for this analysis is the fact that the “AND-NOT” function is extensively used by logical modelers and thus the knowledge of its bias, made possible through the lens of the truth density metric, should be clearly demonstrated in practical use cases, e.g. biological network targets should mostly be in an inhibited state when the “AND-NOT” parameterization is used in their respective Boolean equations and in an active state in the case of the “OR-NOT”. As such, a modeler could make use of the link operator function bias to select the appropriate model parameterization which statistically guarantees an activity state profile that best matches the one supported by experimental evidence.

6.1 From topology to link operator Boolean models

In order to define Boolean models with the “AND-NOT” and “OR-NOT” link operator parameterization forms, we implemented the software *abmlog*, which stands for “**A**ll possible **B**oolean **M**odels **L**ink **O**perator **G**enerator” ([Software and Data Availability](#)). Given a simple interaction (.sif) format file [7], representing a PKN with clearly defined, positive and negative causal interactions, the *abmlog* software outputs all combinatorially possible Boolean models where each link operator equation (deciding the state of a *link operator node*, i.e. one whose Boolean activity state is determined by both positive and negative regulators) will have either the “AND-NOT” or the “OR-NOT” function form. The models are saved in both the widely-used BoolNet (.bnet) [27] format and the gitsbe format [28], with the latter additionally including the attractors of the Boolean model, calculated via the BioLQM Java library [29]. A simple overview of the software is presented in Figure 2.

By default, *abmlog* generates all possible Boolean models with the two link operator parameterizations, the number of which depends on the number of link operator nodes. For example, if a network has 12 nodes with both activating and inhibitory regulators, then a total of $2^{12} = 4096$ Boolean models will be generated. In case the number of all possible Boolean models is very large or space restrictions do not allow the storage of that many models, the software can also be used to generate a random sample of link operator Boolean models from the total parameterization space. In summary, *abmlog* is a useful tool that can generate a large pool of Boolean models for subsequent analyses, each with a unique link operator parameterization.

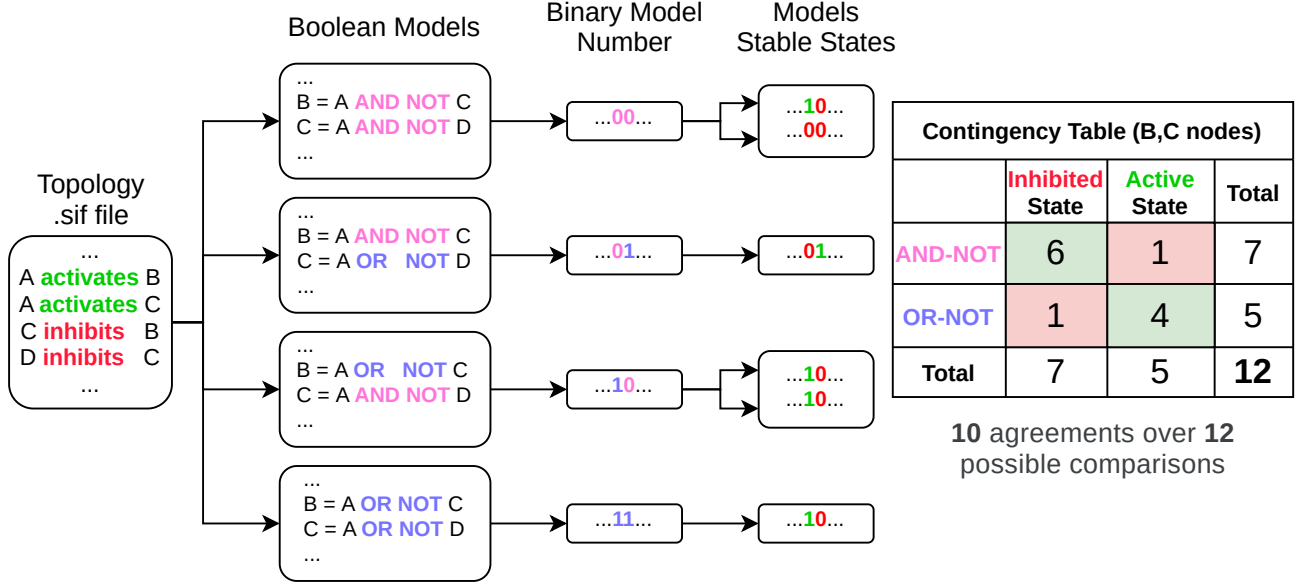


Figure 2: Data-flow overview diagram of the abmlog software and its related contingency table between output model parameterization and stable state activity. A simple interaction file is given as an input to produce a series of Boolean models where equations with both activating and inhibitory regulators have either the “AND-NOT” or the “OR-NOT” formulation. Two link operator equations give rise to a total of $2^2 = 4$ different Boolean models. Each unique parameterization can be represented by a single binary model number, where a “0” corresponds to an equation with the “AND-NOT” link operator and a “1” to an equation with the “OR-NOT”. This representation of parameterization can be directly compared to each of the models’ stable states, which enables the creation of a contingency table for the data pertaining to nodes B and C and the derivation of measures of agreement (see Section 6.2).

6.2 Measuring agreement between parameterization and stable state

In order to quantify the link operator function bias, we use measures of agreement between parameterization and stable state. The idea is that the more biased the link operator parameterization is, the higher the expected agreement will be between a target node’s link operator assignment and its corresponding stable state. For the rest of this work, we shall use two measures of agreement, namely the *percent agreement* and Cohen’s *kappa statistic* [30].

In more detail, using the Boolean model data generated by abmlog, we focus in two categorical variables related to a particular node of interest: its link operator parameterization (“AND-NOT”/“0” or “OR-NOT”/“1”) and its corresponding stable state activity (“inhibition” or “activation”), obtained via attractor analysis. We shall say that these two variables “agree” when a node whose target Boolean equation has the “AND-NOT” link operator (resp. “OR-NOT”) ends up with an inhibited (resp. active) state in the corresponding attractor. In the case of a Boolean model with multiple attractors, each of the stable states is used separately to measure the agreement between the two aforementioned variables, since the activity of a node might change between the different attractors, but its parameterization stays the same.

To define measures of agreement between the two proposed categorical variables, we visualize their interrelation using a contingency table. A total of four data comparison counts can be used to fill in the table’s cells: two where the parameterization and stable state match (i.e. node had the “AND-NOT” link operator form and an inhibited stable state or the “OR-NOT” form and an active state) and two where they differ (i.e. node had the “OR-NOT” form and its state was inhibited, or the “AND-NOT” form and an active state). The

percent agreement is then simply defined as the total number of matches divided by the total number of comparisons and is directly interpreted as the percentage of data that the two variables agree upon. In the example of Figure 2, the corresponding contingency table counts all the matches and mismatches between the link operator assignments for nodes B and C and their corresponding activity state (12 comparisons in total). Since there are only two mismatches, the percent agreement is equal to $10/12 = 0.83$, meaning that in 83% of the presented data, the link operator parameterization dictated function outcome. Naturally, a value of 0 is the absolute minimum score and indicates complete disagreement between the two variables while a perfect agreement score is equal to 1 or 100%.

A more robust statistic that we also apply in the Boolean model data is Cohen’s kappa (κ) coefficient [30]. This statistic is used to measure the extent to which data collectors (raters) assign the same score to the same variable (inter-rater reliability) and takes into account the possibility of agreement occurring by chance. In our case, this can be conceived as one rater that assigns link operator parameterization (“AND-NOT” or “OR-NOT”) and another that assigns stable state activity (“inhibition” or “activation”). Both variables are converted to a binary outcome (0 or 1), allowing the creation of a contingency table and subsequently the calculation of Cohen’s formula for κ . The kappa statistic ranges from -1 to $+1$, where a value of 0 represents the amount of agreement that can be expected from random chance, and a value of 1 (resp. -1) indicates perfect agreement (resp. disagreement) between the raters. In the example contingency table of Figure 2, $\kappa = 0.657$, which is a considerable reduction in the level of congruence compared to the 0.83 percent agreement.

6.3 Truth Density bias in biological networks

6.3.1 Bias guides model parameterization in a cancer signaling network

We used abmlog on a cancer signaling network, consisting of 77 nodes and a total of 149 curated causal interactions that cover a variety of pathways linked to pro-survival and anti-survival cell signaling (e.g. cyclin expression and caspase activation). This PKN, named CASCADE (**C**Ancer **S**ignaling **C**Ausality **D**atabas**E**), was successfully used to build a Boolean model able to predict anti-cancer drug combination effects in gastric cell lines [14]. We used the CASCADE version from the Flobak paper (version CASCADE 1.0), with some node naming changes for compatibility with the newest versions [31]. The number of nodes with both activating and inhibiting regulators in the CASCADE 1.0 topology is 23, while the rest of the nodes have regulators that belong to only one of the two regulatory categories. Thus, using abmlog, we generated all 2^{23} possible Boolean models with the “AND-NOT” and “OR-NOT” link operator parameterizations. The resulting stable state distribution across all produced models is presented in Figure 3A. For our subsequent analysis we will use only the 2802224 Boolean models that had exactly one stable state, as it makes the calculation of agreement between a node’s assigned link operator and its corresponding activity state across all the selected models more straightforward.

The agreement results between link operator parameterization and stable state activity across all the selected CASCADE models are presented in Figures 3B (percent agreement, per node) and 3C (Cohen’s κ , nodes with the same number of regulators are grouped together). The percent agreement results show a high variability across the link operator nodes and range from a minimum of 53% to a perfect agreement (100%). This suggests that for all nodes, across any selected CASCADE 1.0 Boolean model, there is a higher than random probability that the assignment of the “AND-NOT” (resp. “OR-NOT”) link operator formula in the associated Boolean equations will result in the inhibition (resp. activation) of the target nodes. So, even though none of the nodes have more than 5 regulators, we already start seeing signs of the truth density bias in the link operator regulatory functions across a wide range of Boolean models.

When applying Cohen’s κ to evaluate level of agreement, we chose a conservative threshold equal to 0.6, corresponding empirically to a substantial level of agreement [32, 33]. We found that 60% (14 out of 23) of

the nodes have a κ value below the specified threshold. Our conclusion is that biological networks with higher in-degree nodes (i.e. more than 7 – 10 regulators) are needed to properly assess if there is a truly high level of agreement between Boolean parameterization and function state outcome in the case of the link operator regulatory functions, providing thus conclusive proof of their bias (Section 6.3.2).

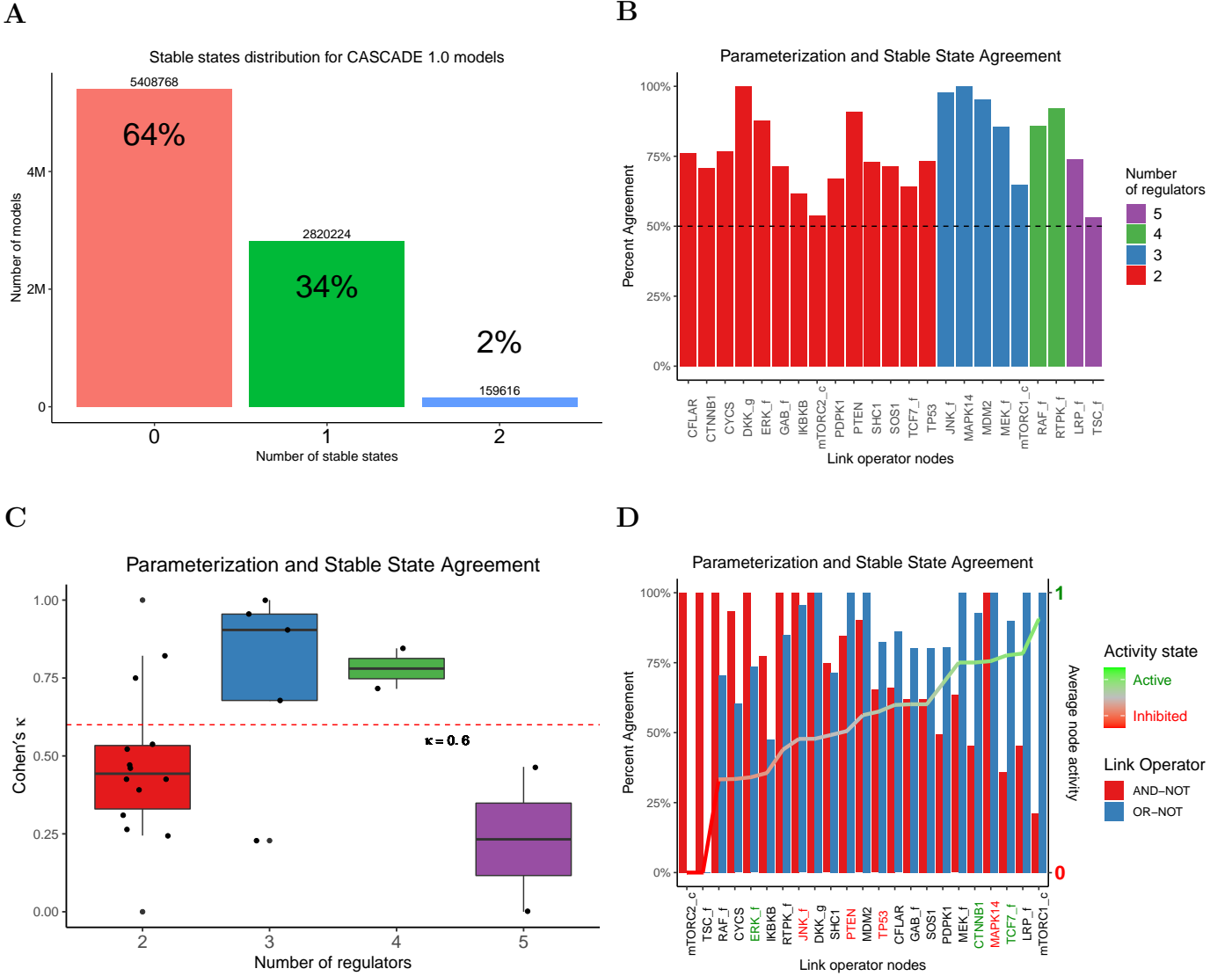


Figure 3: (A) Stable states distribution across all link operator parameterized Boolean models generated by the abmlog software using the CASCADE 1.0 signaling topology. (B) Percent agreement scores between parameterization and activity state across all single stable state CASCADE 1.0 models, for 23 nodes with both inhibiting and activating regulators. Nodes are sorted according to the total number of input regulators. (C) Same as (B), with the difference that the link operator nodes are now grouped into categories based on the total number of input regulators and Cohen's κ is used as an agreement statistic. (D) Same as (B), with the agreement now calculated as the proportion of matches between a node's link operator and its activity state, in the models that had the specific parameterization. The link operator nodes are sorted according to the average activity state across the considered CASCADE models and the colored node labels indicate literature curated activity profiles from Flobak et al. [14]

Regardless of the presence of bias or not, the agreement results can be used to show how experimental data and topological regulatory knowledge (e.g. the activator-to-inhibitor ratio) can be coupled with the truth density metric to guide the choice of regulatory functions. In one example scenario, a modeler asks what the most probable link operator parameterization is among the “AND-NOT” and “OR-NOT” forms that matches available experimental evidence. We used a literature curated activity profile derived for the AGS cell line from [14], to annotate 7 of the link operator nodes in Figure 3D according to their experimentally validated state (activation or inhibition). To clearly identify which of the two parameterizations best fits the observed data, for each node we split the CASCADE models in two model pools, representing the “AND-NOT” and “OR-NOT” node parameterizations, and calculated the proportion of models within each pool whose link operator matched the expected state outcome. For example, in the contingency table of Figure 2, the equivalent calculation would be to divide the number of matches in each row with the corresponding row total sum, resulting in $6/7 = 85.7\%$ of the “AND-NOT” Boolean equations with an inhibited stable state and $4/5 = 80\%$ of the “OR-NOT” equations with an active target node. Moreover, the link operator nodes of Figure 3D are sorted in increasing order by their average stable state activity in the considered CASCADE 1.0 Boolean models. It is evident that nodes with higher average activity in the stable state have a higher agreement with the “OR-NOT” parameterization whereas nodes with lower average activity, a higher agreement with the “AND-NOT” parameterization (0.85 and -0.74 Pearson correlation coefficients with $p_{corr}^{OR-NOT} = 2.6 \times 10^{-7}$ and $p_{corr}^{AND-NOT} = 5 \times 10^{-5}$ respectively, see [Software and Data Availability](#)).

More specifically, we observe that for all experimentally validated nodes, a modeler could a priori set the link operator to the appropriate form and get a stable state activation profile that matches the observations (“AND-NOT” to match an inhibition node profile or “OR-NOT” for an activation profile) with a higher probability than if he was randomly choosing one of the two. For example, the data shows that 90% of the models with an “OR-NOT” Boolean equation for the target family node TCF7_f, had the node as active in their respective stable state. The same is observed for the CTNNB1 (92%) and ERK_f active nodes (74%), as well as for the TP53 (65%) and PTEN (85%) inhibited nodes with the choice of the “AND-NOT” parameterization. Additionally, all the aforementioned nodes have two regulators (one activator and one inhibitor) and using the respective truth density formulas (Eq. 6 and 7) with $n = 2$ and $m = k = 1$, we have that $TD_{AND-NOT} = 0.25$ (closer to 0 or inhibition) and $TD_{OR-NOT} = 0.75$ (closer to 1 or activation), as was also shown in Figure 1A. As such, the nodes observed output matches the statistically expected binary outcomes, showing that even with a low number of regulators, the BRF bias can be used to guide function choice.

In another scenario, a modeler knows that a particular node has a skewed activator-to-inhibitor ratio and wants to exploit such knowledge to make the node conform to a particular activity state of his choice. A nice example from our data is the family node LRP_f, with four activators and one inhibitor. Using the truth density formulas for the two link operator parameterizations (Eq. 6 and 7) with $n = 5$, $m = 4$ and $k = 1$, we have that $TD_{AND-NOT} = 0.47$ and $TD_{OR-NOT} = 0.97$. So, if the modeler wants to have an active LRP_f in the stable state, the “OR-NOT” parameterization should be preferred since the “AND-NOT” has an approximate 50% probability for this to happen from a statistical point of view. These truth density values also match the results from Figure 3D, since only half of the models that use the “AND-NOT” parameterization end up with an inhibited LRP_f in the stable state while all of them have an active LRP_f (100% agreement) in the case where the “OR-NOT” form is used. Also, the average activity of LRP_f across all models is one of the highest in the data, suggesting that imbalanced activator-to-inhibitor ratios could be a direct proxy for predicting regulation outcome. In a similar situation, but at the other range of the activity spectrum, we have the TSC_f family node with one activator and four inhibitors. The truth density values (now using $n = 5$, $m = 1$ and $k = 4$) are $TD_{AND-NOT} = 0.03$ and $TD_{OR-NOT} = 0.53$ respectively. Therefore, the “AND-NOT” parameterization guarantees the inhibition of the TSC_f node (data shows 100% agreement) and it should be a modeler’s first choice if that is the desired outcome. On the other hand, if the activation of TSC_f was a modeler’s preference, then the choice of the “OR-NOT” form would be the most

statistically appropriate according to the truth density metric. We observe though that there was no model having `TSC_f` inhibited in the stable state, indicating that the complex dynamics of the cancer network can also play a significant role in the function outcome. In general, we note that the particular configuration of activating and inhibiting regulators of a target in a specific model instance, can influence the dynamics attributable to the parameterization, causing several results from our analysis to differ from the expected behavior of the Boolean functions studied.

6.3.2 Hub node bias in random scale-free networks

In the previous section we showed that the truth density bias can be used to predict regulatory function outcome in a specific cancer signaling network, but the question still remains open for general biological networks. Also, we found evidence suggesting that Boolean dynamics also plays a significant role in deciding each node’s state in the attractors and in some cases activity state results may contradict what is expected from the use and asymptotic interpretation of the truth density formulas. Therefore, we now proceed to investigate if networks with higher in-degree nodes (i.e. more input regulators) have stable states that can be unquestionably decided a priori by the truth density metric, using the respective *TD* formulas for the “AND-NOT” and “OR-NOT” link operator parameterizations.

We study the specific class of scale-free networks [34], based on the hypothesis that most biological networks exhibit that property, i.e. their node degree distribution follows asymptotically a power law $P(k) \sim k^{-\gamma}$, with k the number of regulators and γ the scale-free exponent. We note that the CASCADE 1.0 model also exhibits the scale-free property (see [Software and Data Availability](#)) and there has been evidence in the literature both in favor and against this hypothesis. In particular, earlier studies showed that many complex networks (including metabolic ones) are approximately scale-free [35, 36, 37, 38], whereas more recent efforts demonstrated that not all cellular biological networks may share that property [39], but those that do, exhibit the strongest level of evidence of scale-free structure [40]. Consequently, we shall use scale-free topologies as acceptable substitutes of real biological networks in our analysis.

The methodology is as follows: we start by generating scale-free topology files with a total of 50 nodes each and a maximum in-degree $k_{max} = 50$ [27]. For each network, the number of input regulators per node is drawn from a Riemann Zeta distribution with parameter γ [41]. The choice of regulators for each network node, as well as the type of regulation (positive or negative), is uniformly random. The Zeta distribution allows the creation of in-degree values that far exceed the average connectivity in a network, giving rise to the highest-degree nodes (often called “hubs”), which are the most defining characteristic of the scale-free networks. The value of the scale-free exponent influences the number of hubs and their in-degree distribution. More specifically, we created scale-free networks with $\gamma = 2$ and $\gamma = 2.5$, since most of the studied networks have an exponent between 2 and 3 [41, 42]. Comparing the networks built with the above methodology, we found that those with $\gamma = 2$ have more nodes with both activating and inhibiting regulators and higher degree hubs than networks with $\gamma = 2.5$ (Figures 4A and 4B). These two characteristics suggest that the scale-free networks with $\gamma = 2$ are better suited for use with the abmlog software, since the larger the number of link operator nodes, the more Boolean models can be generated and thus more data comparisons can be made between node parameterization and stable state activity. Additionally, the presence of higher degree hubs is the perfect testbed for the link operator function bias, which manifests especially for nodes with more than 7 – 10 regulators, as we found from our earlier truth density asymptotics analysis (Figure 1A).

Our methodology proceeds with using each of the scale-free topologies with $\gamma = 2$ as input to the abmlog software, and generating ensembles of Boolean models parameterized with every possible mix of the “AND-NOT” and “OR-NOT” regulatory functions along with the calculation of their stable states (as demonstrated in Figure 2). The produced Boolean models had zero, one, or more stable states. Interestingly, we observed that around half of the tested scale-free topologies generated Boolean models with no stable states, no matter which combination of link operators was used to define the model parameterization. Therefore, the randomly

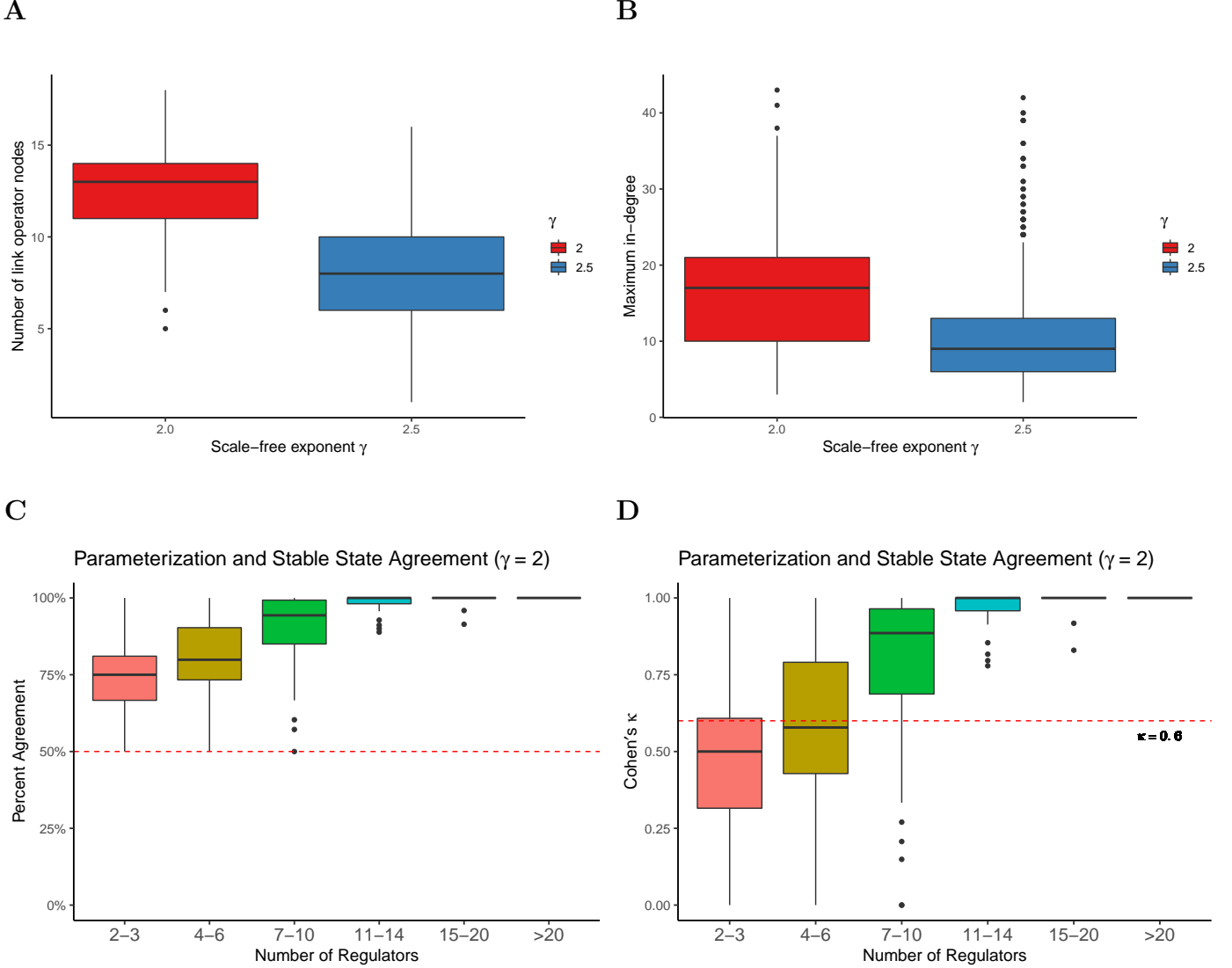


Figure 4: (A)-(B) Network statistics for scale-free topologies with different degree exponents. Every network tested has 50 nodes and a maximum in-degree $k_{max} = 50$. A total of 100 topologies for $\gamma = 2$ and 1000 topologies for $\gamma = 2.5$ are compared. Networks with $\gamma = 2$ have a higher median number of nodes with both activating and inhibiting regulators and higher degree hubs. (C)-(D) Agreement statistics between link operator parameterization and stable state activity. The data is taken from Boolean models generated with the abmlog software, using scale-free topologies with exponent $\gamma = 2$. A total of 757 link operator nodes were compared across multiple link operator parameterization configurations with their corresponding stable states. Nodes are grouped in buckets, where each bucket indicates a different range of input regulators. Both the percent agreement and Cohen's κ show considerable congruence between link operator assignment (“AND-NOT” or “OR-NOT”) and resulting stable state (inhibition or activation respectively) for nodes with more than 10 input regulators.

assigned regulators, regulatory effects, and Zeta distribution in-degree values, may result in networks which do not have stable phenotypes, suggesting that alternative parameterizations might be more suitable in modeling scenarios which specifically examine stable dynamics. Nonetheless, we discarded the models with no stable states and used the rest that had single or multiple attractors in our analysis. Then, for each model

node with both activating and inhibiting regulators, we compared its assigned link operator with the activity state value in the corresponding stable state(s), across all the link operator parameterization spectrum that yielded models with stable phenotypes. The agreement results between parameterization and stable state activity are presented in Figure 4C for the percent agreement and in Figure 4D for Cohen’s kappa statistic.

We observe that both presented statistics show a large variation of agreement for nodes with less than 10 regulators and an increasing agreement with more regulators. This agreement manifests in link operator nodes parameterized with the “AND-NOT” or “OR-NOT” Boolean functions, while at the same time exhibiting inhibited or active states respectively in the associated model attractors. Therefore, we conclude that the considered standardized Boolean regulatory functions are biased and their outcomes can be determined a priori from the choice of the corresponding link operator parameterization, especially for nodes with more than 7 – 10 regulators.

7 Discussion

The specification of mathematical rules that describe the behavior of biological systems is one of the core aspects of computational modeling. It is therefore of considerable value to have a list of metrics that can be used to compare different model parameterizations and make an informed decision with regard to the selection of an appropriate regulatory function that better matches the expected behavior in a specific modeling application.

We specifically discussed two characterizations that can assist modelers in comparing various regulatory functions and select the most plausible ones with regard to the causal interaction-based knowledge at hand. Expressing Boolean functions in DNF makes biological interpretation concrete by explicitly specifying the conditions (presence or absence of the positive and negative regulators, respectively) that make a target active. Expressing the functions in CDNF allows to easily check for compliance with the underlying regulatory topology and subsequently, the rejection of functions that violate such consistency. The difference between these two characterizations lies in the fact that the consistency terminology stems from the mathematical world, while biological interpretability is tightly connected to the world of language semantics and thus closer to the modeler’s point of view. Finally, truth density is an informative measure which can be used to verify if the function parameterization dictates biased Boolean outcomes. It can also be used as a test metric to understand how a function behaves when the number of regulators increases or the balance between the number of activators and inhibitors changes.

Using the truth density metric, we showed the presence of link operator function bias in the hubs of randomly constructed scale-free networks. A potential application of this finding could be to dramatically decrease the time needed to train Boolean models to fit observations via various optimization methods, by pre-assigning the parameterization of link operator nodes with sufficiently many regulators. The pruning of the searchable parameterization space, guided by the truth density metric, can result in more efficient automated methods and can enable the training of larger models against data from numerous resources (e.g. large cell line panels). The hub node bias has also interesting links to the presence of order in biological networks [43]. The dynamics of a Boolean network can exhibit ordered or chaotic behavior. Ordered dynamics is characterized by the presence of less stable states and limit cycle attractors with smaller mean length (number of states in a complex attractor) and transition times (number of steps needed to reach an attractor starting out from an arbitrary configuration) [41]. It is also known that the truth density (probability of target expression) as well as the degree exponent γ (related to network connectivity) can modulate the dynamic transition between the ordered and chaotic phases. Moreover, it has been shown that above the critical value of $\gamma_c \sim 2.47$, ordered behavior in the form of stable state dynamics manifests independently of the truth density, whereas for values closer to $\gamma = 2$, order coincides with the presence of high biased nodes (see Fig. 4 in [41]). Our work confirms this phenomenon, since the use of the link operator parameterization guarantees the presence of biased hubs,

which enable the scale-free networks to exhibit stability and homogeneity in terms of regulatory output, and thus stay in the ordered dynamic regime.

Searching for other function metrics that are applicable to logical modeling, the *sensitivity* of a Boolean function is one of the most relevant [44]. As its name suggests, it measures how sensitive the output of the function is to small changes of its inputs. Sensitivity is tightly linked to the truth density metric, since a highly homogeneous Boolean function (i.e. a biased one), is unlikely to change its value between similar regulatory input configurations and so, its sensitivity is relatively low. To compute the average sensitivity value for an arbitrary Boolean function we need to sum over all the *influences* of the input variables, which essentially represent a way to measure individual variable importance. In the context of regulatory functions, a regulator’s influence is defined as the probability that a random toggle on its activity (from active to inactive and vice-versa) will change the value of the Boolean function [45]. Therefore, by calculating the influence of every regulator, the modeler can gain knowledge of which ones are more important and control the respective function’s outcome. This transition of perspective from the function level to the regulator level might be advantageous in cases where the modeler’s intention is to compare different parameterizations and choose the one for which a particular regulator is labeled as significantly more important than the others, based on the available biological knowledge.

Lastly, an important addition to a universal list of Boolean function metrics for modeling purposes, is the notion of function *complexity*. A recent definition is given by Gherardi et al. [25], where the authors defined it as the number of terms in the shortest possible DNF expression of a given Boolean function, divided by the total number of rows in the corresponding truth table. We presented this information in the last column of Table 2, where the BRFs are sorted from lower to higher complexity (note that the CDNF has the minimum number of terms for every BRF included in the table). One useful observation is that the standardized “AND-NOT” formula [9] is the function with the lowest complexity that is also consistent and thus biologically plausible - all properties that make it a good choice from the modeler’s perspective. Assessing the complexity of the studied regulatory functions using the derived formulas for the minimum number of CDNF terms for any number of activators m and inhibitors k (see last column of Table 2), we comment on the fact that all BRFs have very low complexity since $\mathcal{O}(m \times k) \ll 2^{m+k}$, i.e. the number of function terms does not grow as fast as the number of rows in the corresponding truth table. Same observation has been shown to be true in manually-tuned, experimentally-validated Boolean functions [25], providing us with another confirmation that the consistent functions from Table 2 are good candidates for logic-based modeling approaches.

8 Future work

In this work we make an attempt to address the logical rule specification problem, which can be simply stated as: “Many functions may fit the available observations, which one is the most proper to use?” Of course what is “proper” can be fairly subjective, but the main point is that a careful consideration of the underlying application context (i.e. what output do I expect in a specific scenario of interest) along with a list of metrics that explicate a Boolean function’s behavior and semantics, provides the user with the appropriate framework to decide on the function parameterization that sets the basis for further model analysis and simulation. In that regard, interesting directions for further research include the application of the metrics presented in this work in different published biological models, and the subsequent comparison of different regulatory functions within this framework. Such meta-analyses could potentially indicate regulatory functions that achieve a higher degree of fitness with the observed data or general properties that are common in all Boolean functions used to model biological systems.

An interesting study for example would be to analyze Boolean functions from published biological models that have extreme activator-to-inhibitor ratios. If such imbalanced ratios also result in proportionally skewed Boolean outcomes (i.e. with more activators, the truth density is closer to 1 and the reverse with more inhibitors), suggesting that target outcome follows the majority regulatory groups, then the use of threshold

functions could be a more proper parameterization alternative, as was shown in Figure 1B. Of course, we note that each individual case must be examined with care, since there might be high influence nodes, whose activity defines the target’s output even in the presence of a much larger regulatory group with opposite effects. For example, **CASP3** is a biological entity that, when activated, will almost certainly result in the cell’s death even in the presence of a majority of proliferation-positive regulators at any given time. Subsequently, a more appropriate choice based on the results of this study can be made, either by choosing between the biased functions, which demonstrate a more balanced behavior for such extreme activator-to-inhibitor ratios (e.g. using the “Pairs” or the “AND-NOT” functions which are balanced vs using the “OR-NOT” which would make the target activated most of the time, see **Scenario 2**) or by using refined threshold functions, in which each regulator’s weight will differ in order to match the influence that it has on the target.

There have been only a handful examples of published logical models [46, 47] and research papers [21, 48, 49, 50, 51, 52, 53, 54] that use the threshold modeling framework in biological systems. This is partly due to the lack of tools that make threshold functions accessible to the average user, and the availability of such software in open-source environments such as the CoLoMoTo Interactive Notebook [55]. We believe that the existence of such novel software will enable the construction and configuration of generic Boolean threshold models and provide users of the logical-modeling community and beyond with the necessary toolbox to further study these models. This will enable applications that depend on the dynamical analysis of Boolean threshold models (identification of attractors, reachability properties, formal verification and control) and the use of optimization methods to calibrate the threshold function parameters to best fit the available experimental data, as is done currently with analytical logic-based functions [12].

Software and Data Availability

The *abmlog* software that was used to generate Boolean models with the “AND-NOT” and “OR-NOT” Boolean regulatory functions is available at <https://github.com/druglogics/abmlog> under the MIT License. We used the version 1.6.0 for this analysis, which is also offered as a standalone package at <https://github.com/druglogics/abmlog/packages>.

An extended analysis accompanying the results of this paper is available at <https://druglogics.github.io/brf-bias>. It includes links to the produced model datasets and scripts to reproduce the results and figures of this paper. In particular, the correlation analysis between average node state in the CASCADE 1.0 models and percent agreement per each link operator is available at <https://druglogics.github.io/brf-bias/cascade-1-0-data-analysis.html#node-state-and-percent-agreement-correlation>. The degree distribution of the CASCADE 1.0 topology and other network statistics are examined in <https://druglogics.github.io/brf-bias/cascade-1-0-data-analysis.html#network-properties>.

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A Truth Density formula proofs

For all the following propositions, we consider f to be a Boolean regulatory function $f_{BRF} : \{0, 1\}^n \rightarrow \{0, 1\}$, with a total of n input regulators separated to two distinct sets, the set of $m \geq 1$ activators $x = \{x_i\}_{i=1}^m$ and the set of $k \geq 1$ inhibitors $y = \{y_j\}_{j=1}^k$, such that $n = m + k$.

Proposition 1 (“AND-NOT” Truth Density). *The truth density of the “AND-NOT” link operator function $f_{AND-NOT}(x, y) = (\bigvee_{i=1}^m x_i) \wedge \neg \left(\bigvee_{j=1}^k y_j\right)$, with $m \geq 1$ activators and $k \geq 1$ inhibitors, is given by the formula:*

$$TD_{AND-NOT} = \frac{2^m - 1}{2^n} = \frac{1}{2^k} - \frac{1}{2^n} \quad (6)$$

Proof. Using the distributive property and De Morgan’s law we can express $f_{AND-NOT}$ (Eq. 1) in the equivalent DNF:

$$\begin{aligned} f_{AND-NOT}(x, y) &= \left(\bigvee_{i=1}^m x_i\right) \wedge \neg \left(\bigvee_{j=1}^k y_j\right) \\ &= \bigvee_{i=1}^m \left(x_i \wedge \neg \left(\bigvee_{j=1}^k y_j\right)\right) \\ &= \bigvee_{i=1}^m (x_i \wedge \bigwedge_{j=1}^k \neg y_j) \\ &= \bigvee_{i=1}^m (x_i \wedge \neg y_1 \wedge \dots \wedge \neg y_k) \end{aligned}$$

To calculate $TD_{AND-NOT}$, we need to find the number of rows in $f_{AND-NOT}$ ’s truth table that result in a *True* output result and divide that by the total number of rows, which is 2^n (n input regulators).

Note that $f_{AND-NOT}$, written in its equivalent DNF, has exactly m terms. Each term has a unique *True/False* assignment of regulators that makes it *True*. This happens when the activator of the term is *True* and all of the inhibitors *False*. Since the condition for the inhibitors is the same regardless of the term we are examining and f is expressed in DNF, the *True* outcomes of the function f are defined by all logical assignment combinations of the m activators that have at least one of them being *True* and all inhibitors assigned as *False*. There are a total of 2^m possible *True/False* logical assignments of the m activators (from all *False* to all *True*) and $f_{AND-NOT}$ becomes *True* on all except one of them (i.e. when all activators are *False*), with the corresponding $2^m - 1$ truth table rows having all inhibitors assigned as *False*. Therefore, $TD_{AND-NOT} = (2^m - 1)/2^n$. \square

Proposition 2 (“OR-NOT” Truth Density). *The truth density of the “OR-NOT” link operator function $f_{OR-NOT}(x, y) = (\bigvee_{i=1}^m x_i) \vee \neg \left(\bigvee_{j=1}^k y_j\right)$, with $m \geq 1$ activators and $k \geq 1$ inhibitors, is given by the formula:*

$$TD_{OR-NOT} = \frac{2^n - (2^k - 1)}{2^n} = 1 - \frac{1}{2^m} + \frac{1}{2^n} \quad (7)$$

Proof. Using De Morgan's law we can express f_{OR-NOT} (Eq. 2) in the equivalent DNF:

$$\begin{aligned} f_{OR-NOT}(x, y) &= \left(\bigvee_{i=1}^m x_i \right) \vee \neg \left(\bigvee_{j=1}^k y_j \right) \\ &= \left(\bigvee_{i=1}^m x_i \right) \vee \left(\bigwedge_{j=1}^k \neg y_j \right) \\ &= x_1 \vee x_2 \vee \dots \vee x_m \vee (\neg y_1 \wedge \dots \wedge \neg y_k) \end{aligned}$$

To calculate TD_{OR-NOT} , we find the number of rows of f_{OR-NOT} 's truth table that result in a *False* output (R_{false}), subtract that number from the total number of rows (2^n) to get the rows that result in f being *True*, and then divide by the total number of rows. As such, $TD_{OR-NOT} = (2^n - R_{false})/2^n$.

Note that f_{OR-NOT} , expressed in its equivalent DNF, has exactly $m + 1$ terms. To make f_{OR-NOT} *False*, we assign the m activators as *False* and then we investigate which logical assignments of the inhibitors $\{y_j\}_{j=1}^k$ make the last DNF term also *False*. Out of all the possible 2^k *True/False* logical assignments of the k inhibitors (ranging from all *False* to all *True*) there is only one that does not make the last term of f_{OR-NOT} *False*, which happens specifically when all k inhibitors are *False*. Therefore, $R_{false} = 2^k - 1$ and $TD_{OR-NOT} = (2^n - (2^k - 1))/2^n$. \square

Proposition 3 (“Pairs” Truth Density). *The truth density of the “Pairs” link operator function $f_{Pairs}(x, y) = \bigvee_{\forall(i,j)}^{m,k} (x_i \wedge \neg y_j)$, with $m \geq 1$ activators and $k \geq 1$ inhibitors, is given by the formula:*

$$TD_{Pairs} = \frac{(2^m - 1)(2^k - 1)}{2^n} \quad (8)$$

Proof. Using the distributive property we can express f_{Pairs} (Eq. 3) in its equivalent conjunction normal form (CNF), where two separate clauses are connected with AND's (\wedge) and inside the clauses the literals are connected with OR's (\vee):

$$f_{Pairs}(x, y) = \bigvee_{\forall(i,j)}^{m,k} (x_i \wedge \neg y_j) = \left(\bigvee_{i=1}^m x_i \right) \wedge \left(\bigvee_{j=1}^k \neg y_j \right) \quad (9)$$

To calculate TD_{Pairs} , based on its given CNF, we find the number of rows in its truth table that have at least one *True* activator (R_{act}) and subtract from these the rows in which all inhibitors are *True* (R_{inh}). Therefore, only the rows that have at least one *True* activator and at least one *False* inhibitor will be left, corresponding to the biological interpretation of f_{Pairs} . As such, $TD_{Pairs} = (R_{act} - R_{inh})/2^n$.

R_{act} can be found by subtracting from the total number of rows (2^n), the rows that have all activators as *False*. The number of these rows depends on the number of inhibitors, since for each one of the total possible 2^k *True/False* logical assignments of the k inhibitors (ranging from all *False* to all *True*), there will be a row in the truth table with all activators as *False*. Therefore, $R_{act} = 2^n - 2^k = 2^{m+k} - 2^k = 2^k(2^m - 1)$.

R_{inh} depends on the number of activators, since for each one of the total possible 2^m *True/False* logical assignments of the m activators (ranging from all *False* to all *True*), there will be a row in the truth table with all inhibitors as *True*. Note that we have to exclude one row from this result, which is exactly the row that has all activators as *False* since it's not included in the R_{act} rows. Therefore, $R_{inh} = 2^m - 1$ and $TD_{Pairs} = (R_{act} - R_{inh})/2^n = (2^k(2^m - 1) - (2^m - 1))/2^n$. \square

Proposition 4 (Threshold functions Truth Density). *The truth density of the Boolean threshold functions “Act-win” (Eq. 4) and “Inh-win” (Eq. 5), with $m \geq 1$ activators and $k \geq 1$ inhibitors, is given by the formula:*

$$TD_{thres} = \frac{\sum_{i=1}^m \left[\binom{m}{i} \times \sum_{j=0}^{\min(u,k)} \binom{k}{j} \right]}{2^n} \quad (10)$$

where $u = i$ or $i - 1$, depending on the use of the “Act-win” or “Inh-win” function respectively.

Proof. The truth density formula can be easily derived from the observation that we need to count the number of rows in the respective truth table that have more *True* activators than *True* inhibitors. In the case of the “Act-win” function, we also need to add the rows that have an equal number of *True* regulators in each respective category.

Firstly, we count all the subset input configurations that have up to m activators assigned to *True*. These include the partial *True/False* logical assignments that have either a single *True* activator, a pair of *True* activators, a triplet, etc. This is exactly the term $\sum_{i=1}^m \binom{m}{i}$. Note that each of these activator input configurations is multiplied by a factor of 2^k in the truth table to make *complete* rows, i.e. rows where the activators logical assignments stay unchanged and the inhibitor values range from all *False* to all *True*. Therefore, we need to specify exactly which inhibitor logical assignments are appropriate for each activator subset input configuration. To do that, we multiply the size of each activator subset $\binom{m}{i}$ with the number of configurations that have less *True* inhibitors, i.e. $\sum_{j=0}^{i-1} \binom{k}{j}$.

Let’s consider an example with $m, k > 2$ and set $i = 2$. We find that the number of subsets with 2 *True* activators is $\binom{m}{2}$. Next, we multiply by the number of configurations that have one or no *True* inhibitors, i.e. $\sum_{j=0}^1 \binom{k}{j}$. This results in the number of rows of interest for the “Inh-win” function, i.e. the rows where there are exactly 2 activators assigned to *True* and less than 2 *True* inhibitors. For “Act-win”, we have to multiply up to the *True* inhibitor pairs, i.e. $\sum_{j=0}^2 \binom{k}{j}$. In summation, we count the configurations that have exactly i out of m activators assigned to *True*, and for each one, we multiply by the number of cases that have 0 up to i inhibitors assigned to *True* to find the respective rows, i.e. $\binom{m}{i} \times \sum_{j=0}^i \binom{k}{j}$. Repeating this calculation for every possible subset of i activators (from 1 up to all m of them), and summing the rows up, will result in the numerator of the TD_{thres} formula for the “Act-win” function.

Lastly, note that the *largest* inhibitor configuration subset size that we consider, is the minimum value between the current activator subset size ($u = i$ or $i - 1$, depending on which threshold function we use) and the total number of inhibitors k . Therefore, we take into account the case where the number of inhibitors is less than the activator subset size, i.e. $k < u$. This explains the term $\min(u, k)$ in the truth density formula and concludes the proof. \square

B Truth Density asymptotic behavior

We study the asymptotic behavior of the four truth density formulas (Appendix A) for a large number of regulators ($n \rightarrow \infty$). Note that for the calculations involving the two threshold functions, we will only use the truth density formula corresponding to the “Act-win” function (Eq. 10, with $u = i$), since both functions have similar formulas and therefore, their limiting behavior is analogous. The asymptotics results for each regulatory function are as follows:

1. The “AND-NOT” function truth density (Eq. 6) depends only on the number of inhibitors k :

$$TD_{AND-NOT} = \frac{1}{2^k} - \frac{1}{2^n} \sim \frac{1}{2^k} \quad (11)$$

For large k , it is biased towards 0:

$$TD_{AND-NOT} = \frac{1}{2^k} \xrightarrow{k \rightarrow \infty} 0$$

2. The “OR-NOT” function truth density (Eq. 7) depends only on the number of activators m :

$$TD_{OR-NOT} = 1 - \frac{1}{2^m} + \frac{1}{2^n} \sim 1 - \frac{1}{2^m} \quad (12)$$

For large m , it is biased towards 1:

$$TD_{OR-NOT} = 1 - \frac{1}{2^m} \xrightarrow{m \rightarrow \infty} 1$$

3. The “Pairs” function truth density (Eq. 8) depends on both activators and inhibitors:

$$TD_{Pairs} = \frac{(2^m - 1)(2^k - 1)}{2^n} = \frac{2^n - 2^m - 2^k + 1}{2^n} = 1 - \frac{2^m + 2^k}{2^n} + \frac{1}{2^n} \sim 1 - \frac{1}{2^k} - \frac{1}{2^m} \quad (13)$$

4. The threshold functions truth density (Eq. 10) depends on both m and k variables and does not have a single fixed limit for $n \rightarrow \infty$.

We now focus on the effect of the ratio ($m : k$) between number of activators and inhibitors on the asymptotic truth density values for $n \rightarrow \infty$. We consider the following three scenarios for each of the Boolean functions:

Scenario 1 A 1 : 1 activator-to-inhibitor ratio, where approximately half of the regulators are activators and half are inhibitors, i.e. $m \approx k \approx n/2$ (consider n is even without loss of generality).

1. The “AND-NOT” function truth density is biased towards 0:

$$(\text{Eq. 11}) \Rightarrow TD_{AND-NOT} \sim \frac{1}{2^{n/2}} \xrightarrow{n \rightarrow \infty} 0$$

2. The “OR-NOT” function truth density is biased towards 1:

$$(\text{Eq. 12}) \Rightarrow TD_{OR-NOT} \sim 1 - \frac{1}{2^{n/2}} \xrightarrow{n \rightarrow \infty} 1$$

3. The “Pairs” function truth density is biased towards 1:

$$(\text{Eq. 13}) \Rightarrow TD_{Pairs} \sim 1 - \frac{1}{2^{n/2}} - \frac{1}{2^{n/2}} \xrightarrow{n \rightarrow \infty} 1$$

4. The threshold functions truth density is balanced, meaning its limit asymptotically approaches 1/2.

Proof. We first rewrite the truth density formula substituting $m = k = n/2$:

$$(\text{Eq. 10}) \Rightarrow TD_{thres} = \frac{\sum_{i=1}^{n/2} \left[\binom{n/2}{i} \times \sum_{j=0}^{\min(i, n/2)} \binom{n/2}{j} \right]}{2^n} = \frac{\sum_{i=1}^{n/2} \left[\binom{n/2}{i} \times \sum_{j=0}^i \binom{n/2}{j} \right]}{2^n} = \frac{N}{2^n}$$

Next we simplify N , by using the notation $z = n/2$ and \mathbf{x} as a meta-symbol for $\binom{z}{x}$. For example, $\binom{n/2}{1} = \binom{z}{1} = \mathbf{1}$. N is therefore expressed as:

$$N = \mathbf{1}(\mathbf{0} + \mathbf{1}) + \mathbf{2}(\mathbf{0} + \mathbf{1} + \mathbf{2}) + \dots + \mathbf{z}(\mathbf{0} + \mathbf{1} \dots + \mathbf{z})$$

Using the symmetry of binomial coefficients: $\binom{z}{x} = \binom{z}{z-x} \sim \mathbf{x} = \mathbf{z} - \mathbf{x}$, we can re-write N as:

$$N = (\mathbf{z} - \mathbf{1})[\mathbf{z} + (\mathbf{z} - \mathbf{1})] + (\mathbf{z} - \mathbf{2})[\mathbf{z} + (\mathbf{z} - \mathbf{1}) + (\mathbf{z} - \mathbf{2})] + \dots + \mathbf{0}[\mathbf{z} + \dots + \mathbf{0}]$$

Adding the two expressions for N we have that:

$$2N = [\mathbf{0} + \mathbf{1} \dots + \mathbf{z}]^2 + \mathbf{1}^2 + \mathbf{2}^2 + \dots + (\mathbf{z} - \mathbf{1})^2 = 2^{2z} + \sum_{\mathbf{x}=1}^{z-1} \mathbf{x}^2$$

Substituting back $\binom{z}{x} = \mathbf{x}$ and $i = x$ (change of index) in expression N , we have that the threshold functions truth density is written as:

$$TD_{thres} = \frac{N}{2^{2z}} = \frac{(1/2) \left[2^{2z} + \sum_{i=1}^{z-1} \binom{z}{i}^2 \right]}{2^{2z}}$$

As $n \rightarrow \infty$ (and hence $z \rightarrow \infty$), the term $\sum_{i=1}^{z-1} \binom{z}{i}^2$ does not grow as fast as 2^{2z} - it is smaller by a factor of $\sqrt{\pi z}$ (see answer to Problem 9.18 in [56]), and so it becomes negligible:

$$\lim_{z \rightarrow \infty} TD_{thres} = \lim_{z \rightarrow \infty} \frac{(1/2)2^{2z}}{2^{2z}} = \frac{1}{2}$$

□

Scenario 2 A high activator-to-inhibitor ratio $(n-1 : 1)$, where all regulators are activators except one inhibitor, i.e. $m = n-1, k = 1$.

1. The “AND-NOT” function truth density is balanced:

$$(\text{Eq. 11}) \Rightarrow TD_{AND-NOT} \sim \frac{1}{2^1} = \frac{1}{2}$$

2. The “OR-NOT” function truth density is biased towards 1:

$$(\text{Eq. 12}) \Rightarrow TD_{OR-NOT} \sim 1 - \frac{1}{2^{n-1}} \xrightarrow{n \rightarrow \infty} 1$$

3. The “Pairs” function truth density is balanced:

$$(\text{Eq. 13}) \Rightarrow TD_{Pairs} \sim 1 - \frac{1}{2^1} - \frac{1}{2^{n-1}} \xrightarrow{n \rightarrow \infty} \frac{1}{2}$$

4. The threshold functions truth density is biased towards 1:

$$\begin{aligned} (\text{Eq. 10}) \Rightarrow TD_{thres} &= \frac{\sum_{i=1}^{n-1} \left[\binom{n-1}{i} \times \sum_{j=0}^{\min(i,1)} \binom{1}{j} \right]}{2^n} = \frac{\sum_{i=1}^{n-1} \left[\binom{n-1}{i} \times \sum_{j=0}^1 \binom{1}{j} \right]}{2^n} \\ &= \frac{\sum_{i=1}^{n-1} \binom{n-1}{i} \times 2}{2^n} = \frac{2^{n-1} - 1}{2^{n-1}} = 1 - \frac{1}{2^{n-1}} \xrightarrow{n \rightarrow \infty} 1 \end{aligned}$$

Scenario 3 A low activator-to-inhibitor ratio $(1 : n-1)$, where all regulators are inhibitors except one activator, i.e. $m = 1, k = n-1$.

1. The “AND-NOT” function truth density is biased towards 0:

$$(\text{Eq. 11}) \Rightarrow TD_{AND-NOT} \sim \frac{1}{2^{n-1}} \xrightarrow{n \rightarrow \infty} 0$$

2. The “OR-NOT” function truth density is balanced:

$$(\text{Eq. 12}) \Rightarrow TD_{OR-NOT} \sim 1 - \frac{1}{2^1} = \frac{1}{2}$$

3. The “Pairs” function truth density is balanced:

$$(\text{Eq. 13}) \Rightarrow TD_{Pairs} \sim 1 - \frac{1}{2^{n-1}} - \frac{1}{2^1} \xrightarrow{n \rightarrow \infty} \frac{1}{2}$$

4. The threshold functions truth density is biased towards 0:

$$\begin{aligned} (\text{Eq. 10}) \Rightarrow TD_{thres} &= \frac{\sum_{i=1}^1 \left[\binom{1}{i} \times \sum_{j=0}^{\min(i,n-1)} \binom{n-1}{j} \right]}{2^n} = \frac{\sum_{j=0}^{\min(1,n-1)} \binom{n-1}{j}}{2^n} \\ &= \frac{\sum_{j=0}^1 \binom{n-1}{j}}{2^n} = \frac{1 + (n-1)}{2^n} = \frac{n}{2^n} \xrightarrow[n \rightarrow \infty]{\text{L'Hôpital Rule}} 0 \end{aligned}$$

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