Stochastic calculus part

Here W_t always denotes the standard Wiener process.

1. (10 points) Find $\mathbb{E}(W_6^2|W_4)$ and $\mathbb{P}(W_6 > 0|W_4)$.

   Hint: the answer may contain the normal cumulative distribution function.

2. (10 points) I wait until Wiener process will hit the level 5 or the level -3, that is up to the moment $\tau = \min\{t|W_t = 5 \cup W_t = -3\}$.

   Find $\mathbb{P}(W_\tau = 5)$ and $\mathbb{E}(\tau)$.

   Hint: you may use the martingales W_t and $W_t^2 - t$.

3. (10 points) The process X_t is defined as

$$X_t = \begin{cases} 1, & t \in [0; 1) \\ -2, & t \in [1; 2) \\ W_1, & t \in [2; \infty) \end{cases}$$

   (a) Explicitely find $Z_t = \int_0^t X_u dW_u$ for every $t > 0$.
   (b) Find $\mathbb{E}(Z_t)$ and $\mathbb{V}\mathrm{ar}(Z_t)$.

4. (10 points) Find the price of the «Asset-or-nothing» call option at time $t = 0$ in the framework of Black and Scholes model. The risk-free interest rate is equal to r. The volatility of the share is equal to σ. The current share price is S_0. The «Asset-or-nothing» call option pays you at fixed time T the sum S_T if S_T is higher than the strike price K or nothing otherwise.

   Hint: the correct answer will contain the normal cumulative distribution function $F()$.