Graded home assignment. Deadline: 2018-12-27, before exam.

1. I throw a fair die until first six appears. Let's denote the total number of throws by X and the number of odd integers thrown by Y.

   Find $\mathrm{E}(Y|X)$, $\mathrm{E}(X|Y)$ and $\mathrm{Var}(Y|X)$.

2. Let $X_t = \exp(-\alpha t)\left(1 + \int_0^t \exp(\alpha u)\, dW_u\right)$.

   a) Simplify the expression $X_t + \alpha \int_0^t X_u\, du$.

   б) Find $\mathrm{E}(X_t)$ and $\mathrm{Var}(X_t)$.

3. Today the price of a share is $S_0 = 100$ roubles. Each day the price S_t goes up by one rouble with probability $p \in (0; 1)$ or by two roubles with probability $1 - p$.

   a) Find a number a such that $M_t = a^{S_t}$ is a martingale.

   б) Let τ be the first moment of time when the price will be greater or equal to 200 roubles. Find $\mathrm{E}(\tau)$.

4. In the framework of Black and Scholes model find the price at time $t = 0$ of an asset that pays you at time T the amount

$$X_T = \min\{S_T, S_T^2\}.$$

5. Consider the stochastic differential equation

$$dX_t = 2W_t \exp(-X_t)\, dW_t + 2t \exp(-2X_t)\, dt$$

   Find at least one solution of the form $X_t = h(f(W_t) + g(t))$.