Stochastic calculus syllabus

Boris Demeshev

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Office: Shabolovka 26, 2112
Office Hours: by appointment
Web: bdemeshev.github.io/sc401
E-mail: bdemeshev@hse.ru

Course Description

Stochastic calculus is used in financial engineering. We will cover the minimum of required math: sigma-algebras, conditional expectations, martingales, Wiener process, stochastic integration. The goal of this course is the Black and Scholes model and option pricing using martingale approach.

The big problem is that stochastic calculus is very hard from a mathematical viewpoint. This course is aimed at students with no measure theory background. We will formulate all the required theorems mostly without proofs. The only prerequisite for this course is probability theory: students should know how to calculate expectations, probabilities and conditional probabilities in discrete and continuous cases.

This course is elective.

Learning objectives and outcomes

Successful student will

- Understand the following mathematical concepts with their properties:
  - sigma-algebra
  - expectation with respect to sigma algebra
  - martingale
  - Wiener process
  - Ito's stochastic integral
- Be able to formulate and apply in simple context the following theorems:
  - Ito's lemma
  - Girsanov's theorem
- Understand the Black and Scholes model:
  - price simple European options using martingale approach
  - price exotic European options using simulations in open source software like R, python or julia

Reading List

Required

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media.

Optional

Shreve, Steven (2012). *Stochastic calculus for finance I: the binomial asset pricing model*. Springer Science and Business Media.

Shreve, Steven E (2004). *Stochastic calculus for finance II: Continuous-time models*. Vol. 11. Springer Science and Business Media.

Steele, J Michael (2012). *Stochastic calculus and financial applications*. Vol. 45. Springer Science and Business Media.

Course Plan

Sigma-algebras

Motivation, formal definition, properties. Random variables measurable with respect to given sigma-algebras.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 1.

Conditional expectation

Motivation, formal definition, properties. Geometry of conditional expectation. Conditional variance. Expectation conditional on another random variable.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 2.

Martingales

Stochastic processes in discrete time, filtration. Martingales: motivation, formal definition properties. Examples of martingales and non-martingales.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 3.

Wiener process

Stochastic processes in continuous time. Two definitions of Wiener process (with and without sigma-algebras). Transformations.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 6.3.

Ito's integral

Motivation, formal definition, properties. Exersices on Ito's isometry.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 7.1 — 7.3.

Ito's lemma and Girsanov theorem

Ito's process. Univariate and multivariate versions of Ito's lemma. Girsanov theorem: motivation and formal statement.

Brzezniak, Zdzislaw and Tomasz Zastawniak (2000). *Basic stochastic processes: a course through exercises*. Springer Science and Business Media. Chap. 7.4 — 7.5.

Black and Scholes model

Assumptions of the model. Pricing European options using martingale approach. Pricing European options with simulations.

Shreve, Steven E (2004). *Stochastic calculus for finance II: Continuous-time models*. Vol. 11. Springer Science and Business Media. Chap. 5.2.

Grading System

- **50%** of the grade will be determined by a hometask.
- **50%** of the grade will be determined by a final exam.

  Showing up is 80 percent of life – Woody Allen, via Marshall Brickman

Methods of Instruction

The stochastic calculus is very taugh from a mathematical viewpoint. Most students are not ready for the math required. So the accent is done on problem solving. We will solve a lot of problems during classes. That is the only way to do math.

Special Equipment and Software Support

We will price some options using free open source software (R or python or julia). Students are supposed to bring their own laptops with installed software during this session. For this last class the projector will be used.