1. Consider the two independent Brownian motions, (W_t) and (V_t). Which of the following processes is Brownian motion:
   (a) $Z_t = \frac{1}{2}W_t + \frac{1}{2}V_t$
   (b) $Q_t = \frac{1}{\sqrt{2}}W_t + \frac{1}{\sqrt{2}}V_t$
2. Let $Y_t = \int_0^t (W_u + u)^2 \, dW_u$. Find $\mathrm{E}(Y_t)$ and $\mathrm{Var}(Y_t)$.
3. James Bond plays in a casino. At every bet he wins one pound with probability 0.5, looses one pound with probability 0.4 or wins nothing with probability 0.1. His initial fortune is $X_0 = 10$ pounds. He stops playing if he goes bankrupt or if he achieves the fortune of 300 pounds (airline ticket price from London to Moscow).
   (a) Find the constant a such that $M_t = a^{X_t}$ is a martingale.
   (b) What is the probability that James Bond will win enough to buy the ticket?
4. Let R_t be the exchange rate at time t. We suppose that $dR_t = \mu R_t dt + \sigma R_t dW_t$. Consider the inverse exchange rate $I_t = 1/R_t$. Find the expression for dI_t. The expression should not contain R_t.
5. It is known that M_t is a martingale. We also know that in short-hand notation $(dM_t)^2 = dt$. What can we say about the process $Y_t = M_t^2 - t + 2017$?
6. Solve the stochastic differential equation
$$dX_t = \frac{-1}{1-t} X_t \, dt + dW_t, \; X_0 = 0 \tag{1}$$
   You may use or not use the following hints:
   (a) The correct answer will contain the integral $\int_0^t \frac{1}{1-u} dW_u$ that cannot be simplified.
   (b) Solve the ordinary differential equation
$$dY_t = \frac{-1}{1-t} Y_t \, dt, \; Y_0 = 1 \tag{2}$$
   (c) Represent X_t as $X_t = Y_t \cdot Z_t$ and find the equation for dZ_t. Find the expression for Z_t.
7. Consider the framework of the Black and Scholes model. Let P be the original probability measure and $\tilde{\mathrm{P}}$ be the risk-neutral probability measure. Provide an example of three events A, B and C such that $\mathrm{P}(A) > \tilde{\mathrm{P}}(A)$, $\mathrm{P}(B) < \tilde{\mathrm{P}}(B)$, $\mathrm{P}(C) = \tilde{\mathrm{P}}(C)$.
8. Consider the framework of the Black and Scholes model. The asset pays you 1 dollar at fixed time T if and only if the price of a share S_T is above the strike-price K. Find the current price X_0 of this asset.

You may decide not to solve any number of problems from this hometask and send me a solution of corresponding number of problems from the old exams collection instead (see `http://bdemeshev.github.io/sc401/`). In this case you should use LaTeX or markdown formats. Please, be polite and avoid sending solutions of already solved problems or problems taken by someone else. You should fill in your choice at `http://goo.gl/8TFhWE`.

The due date for this hometask — 10.01.2017 before exam. If you decide to solve problems from an old exam you should send the solution of these problems by e-mail to boris.demeshev@gmail.com before 09.01.2017 consultation hour. I will check your solutions and add them into the exams collection so they are available for everyone.