Stochastic Calculus

1. Two stochastic processes are defined by the system of SDE:

$$\begin{cases} dX_t = (2 + 5t + X_t)dt + 3dW_t \\ dY_t = 4X_t dt + 8Y_t dW_t \end{cases}$$

   (a) Calculate $d(X_t^2 Y_t)$.

   (b) Is $X_t^2 Y_t$ a martingale?

2. Consider the following stochastic differential equation

$$dX_t = -0.5e^{-2X_t}dt + e^{-X_t}dW_t,$$

   with deterministic initial value X_0.

   (a) Is X_t a martingale?

   (b) Using the substitution $Y_t = f(X_t)$ solve this differential equation.

   Hint: try to find a function f such that the term before dt cancels out.