Stochastic Calculus

1. In the framework of Black and Scholes model find the price at the time 0 of an asset which gives you the payoff $\max\{\ln(S_t), 0\}$ at the time t. Here S_t is the price of the underlying asset.

2. Let's consider the following system of stochastic differential equations

$$\begin{cases} dX_t = aX_t dt - Y_t dW_t \\ dY_t = aY_t dt + X_t dW_t \end{cases}$$

with initial conditions $X_0 = x_0$ and $Y_0 = 0$

   (a) Find the solution of the form $X_t = f(t)\cos W_t$ and $Y_t = g(t)\sin W_t$;

   (b) Plot the time path of $D_t = X_t^2 + Y_t^2$.