Easy problems!

1. The semi-annual y_t is modelled by $ETS(AAA)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; 16) \\ s_t = s_{t-2} + 0.1u_t \\ b_t = b_{t-1} + 0.2u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + 0.3u_t \\ y_t = \ell_{t-1} + b_{t-1} + s_{t-2} + u_t \end{cases}$$

   Given that $s_{100} = 2$, $s_{99} = -1.9$, $b_{100} = 0.5$, $\ell_{100} = 4$ find 95% predictive interval for y_{101} and y_{102}.

2. Consider a stationary solution of the equation $y_t = 2 + y_{t-1} - 0.25y_{t-2} + u_t$, where u_t is a white noise process.
   (a) Calculate $\mathrm{E}(y_t)$, the first two values of autocorrelation function.
   (b) Assuming normality and independence of u_t, $u_t \sim \mathcal{N}(0; 16)$, $u_{100} = -1$, $y_{100} = 5$, calculate short-term 95% predictive interval for y_{101} and long-term 95% predictive interval for y_{100+h} where $h \to \infty$.

3. Consider six observations $y = (10, 20, 40, 100, 110, 150)$. Construct regression tree to predict y_t using y_{t-1} as unique predictor. The value of TSS is used to split a node. Growing of tree stops when each node contains no more than 2 observations. To split two values use the mean between them.
   (a) Draw your tree!
   (b) What is the forecast of this tree for the next observation?

A little bit harder!

4. Consider the $ETS(AAN)$ process:

$$\begin{cases} u_t \sim \mathcal{N}(0; \sigma^2) \\ b_t = b_{t-1} + \beta u_t \\ \ell_t = \ell_{t-1} + b_{t-1} + \alpha u_t \\ y_t = \ell_{t-1} + b_{t-1} + u_t \end{cases}$$

   with initial constants b_0 and ℓ_0.

   Find $\mathrm{Cov}(y_t, y_s)$. Is the $ETS(AAN)$ stationary?

5. Check whether the sum of two independent $ETS(ANN)$ processes is a $ETS(ANN)$ process.

6. Consider $MA(1)$ process $y_t = u_t + 2u_{t-1}$ where (u_t) is a white noise with variance σ.
   (a) Find $\mathrm{Cov}(y_t, y_s)$ for all t and s.
   (b) Find best linear prediction for y_t using y_{t-1} and y_{t-2}.

   I will give you a hint for the point b. You need to find α_1 and α_2 in $y_t = \alpha_1 y_{t-1} + \alpha_2 y_{t-2} + w_t$ such that prediction error w_t is uncorrelated with predictors y_{t-1} and y_{t-2}.