Chapter 1: Reviewing number concepts

Key words

- Natural number
- Integer
- Prime number
- Symbol
- Multiple
- Factor
- Composite numbers
- Prime factor
- Square
- Square root
- Cube
- Directed numbers
- BODMAS

In this chapter you will learn how to:

- identify and classify different types of numbers
- find common factors and common multiples of numbers
- write numbers as products of their prime factors
- calculate squares, square roots, cubes and cube roots of numbers
- work with integers used in real-life situations
- revise the basic rules for operating with numbers
- perform basic calculations using mental methods and with a calculator.

This statue is a replica of a 22 000-year-old bone found in the Congo. The real bone is only 10 cm long and it is carved with groups of notches that represent numbers. One column lists the prime numbers from 10 to 20. It is one of the earliest examples of a number system using tallies.

Our modern number system is called the Hindu-Arabic system because it was developed by Hindus and spread by Arab traders who brought it with them when they moved to different places in the world. The Hindu-Arabic system is decimal. This means it uses place value based on powers of ten. Any number at all, including decimals and fractions, can be written using place value and the digits from 0 to 9.

RECAP

You should already be familiar with most of the concepts in this chapter. This chapter will help you to revise the concepts and check that you remember them.

1.1 Different types of numbers

Make sure you know the correct mathematical words for the types of numbers in the table.

Number	Definition	Example
Natural number	Any whole number from 1 to infinity, sometimes called 'counting numbers'. 0 is not included.	1, 2, 3, 4, 5, . . .
Odd number	A whole number that cannot be divided exactly by 2.	1, 3, 5, 7, . . .
Even number	A whole number that can be divided exactly by 2.	2, 4, 6, 8, . . .
Integer	Any of the negative and positive whole numbers, including zero.	. . . −3, −2, −1, 0, 1, 2, 3, . . .
Prime number	A whole number greater than 1 which has only two factors: the number itself and 1.	2, 3, 5, 7, 11, . . .
Square number	The product obtained when an integer is multiplied by itself.	1, 4, 9, 16, . . .
Fraction	A number representing parts of a whole number, can be written as a common (vulgar) fraction in the form of $\frac{a}{b}$ or as a decimal using the decimal point.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{8}, \frac{13}{3}, 2\frac{1}{2}$ 0.5, 0.2, 0.08, 1.7

FAST FORWARD

You will learn about the difference between rational and irrational numbers in chapter 9.

Find the 'product' means 'multiply'. So, the product of 3 and 4 is 12, i.e. 3 × 4 = 12.

Exercise 1.1

FAST FORWARD

You will learn much more about sets in chapter 9. For now, just think of a set as a list of numbers or other items that are often placed inside curly brackets.

1 Here is a set of numbers: $\{-4, -1, 0, \frac{1}{2}, 0.75, 3, 4, 6, 11, 16, 19, 25\}$

List the numbers from this set that are:

a natural numbers
b even numbers
c odd numbers
d integers
e negative integers
f fractions
g square numbers
h prime numbers
i neither square nor prime.

2 List:

a the next four odd numbers after 107
b four consecutive even numbers between 2008 and 2030
c all odd numbers between 993 and 1007
d the first five square numbers
e four decimal fractions that are smaller than 0.5
f four vulgar fractions that are greater than $\frac{1}{2}$ but smaller than $\frac{3}{4}$.

3 State whether the following will be odd or even:

a the sum of two odd numbers
b the sum of two even numbers
c the sum of an odd and an even number
d the square of an odd number
e the square of an even number
f an odd number multiplied by an even number.

Remember that a 'sum' is the result of an addition. The term is often used for *any* calculation in early mathematics but its meaning is very specific at this level.

Being able to communicate information accurately is a key skill for problem solving. Think about what you are being asked to do in this task and how best to present your answers.

Applying your skills

4 There are many other types of numbers. Find out what these numbers are and give an example of each.

a Perfect numbers.
b Palindromic numbers.
c Narcissistic numbers. (In other words, numbers that love themselves!)

Using symbols to link numbers

Mathematicians use numbers and **symbols** to write mathematical information in the shortest, clearest way possible.

You have used the operation symbols +, −, × and ÷ since you started school. Now you will also use the symbols given in the margin below to write mathematical statements.

Exercise 1.2

- $=$ is equal to
- $\neq$ is not equal to
- $\approx$ is approximately equal to
- $<$ is less than
- $\leqslant$ is less than or equal to
- $>$ is greater than
- $\geqslant$ is greater than or equal to
- $\therefore$ therefore
- $\sqrt{\ }$ the square root of

1 Rewrite each of these statements using mathematical symbols.

a 19 is less than 45
b 12 plus 18 is equal to 30
c 0.5 is equal to $\frac{1}{2}$
d 0.8 is not equal to 8.0
e −34 is less than 2 times −16
f therefore the number x equals the square root of 72
g a number (x) is less than or equal to negative 45
h π is approximately equal to 3.14
i 5.1 is greater than 5.01
j the sum of 3 and 4 is not equal to the product of 3 and 4
k the difference between 12 and −12 is greater than 12
l the sum of −12 and −24 is less than 0
m the product of 12 and a number (x) is approximately −40

Remember that the 'difference' between two numbers is the result of a subtraction. The order of the subtraction matters.

2 Say whether these mathematical statements are true or false.

a $0.599 > 6.0$
b $5 \times 1999 \approx 10\,000$
c $8.1 = 8\frac{1}{10}$
d $6.2 + 4.3 = 4.3 + 6.2$
e $20 \times 9 \geqslant 21 \times 8$
f $6.0 = 6$
g $-12 > -4$
h $19.9 \leqslant 20$
i $1000 > 199 \times 5$
j $\sqrt{16} = 4$
k $35 \times 5 \times 2 \neq 350$
l $20 \div 4 = 5 \div 20$
m $20 - 4 \neq 4 - 20$
n $20 \times 4 \neq 4 \times 20$

3 Work with a partner.

a Look at the symbols used on the keys of your calculator. Say what each one means in words.
b List any symbols that you do not know. Try to find out what each one means.

1.2 Multiples and factors

You can think of the multiples of a number as the 'times table' for that number. For example, the multiples of 3 are $3 \times 1 = \mathbf{3}$, $3 \times 2 = \mathbf{6}$, $3 \times 3 = \mathbf{9}$ and so on.

Multiples

A **multiple** of a number is found when you multiply that number by a positive integer. The first multiple of any number is the number itself (the number multiplied by 1).

Worked example 1

a What are the first three multiples of 12?
b Is 300 a multiple of 12?

a	12, 24, 36	To find these multiply 12 by 1, 2 and then 3. $12 \times 1 = 12$ $12 \times 2 = 24$ $12 \times 3 = 36$
b	Yes, 300 is a multiple of 12.	To find out, divide 300 by 12. If it goes exactly, then 300 is a multiple of 12. $300 \div 12 = 25$

Exercise 1.3

1 List the first five multiples of:

a 2 **b** 3 **c** 5 **d** 8
e 9 **f** 10 **g** 12 **h** 100

2 Use a calculator to find and list the first ten multiples of:

a 29 **b** 44 **c** 75 **d** 114
e 299 **f** 350 **g** 1012 **h** 9123

3 List:

a the multiples of 4 between 29 and 53
b the multiples of 50 less than 400
c the multiples of 100 between 4000 and 5000.

4 Here are five numbers: 576, 396, 354, 792, 1164. Which of these are multiples of 12?

5 Which of the following numbers are not multiples of 27?
a 324 **b** 783 **c** 816 **d** 837 **e** 1116

The lowest common multiple (LCM)

The lowest common multiple of two or more numbers is the smallest number that is a multiple of all the given numbers.

Worked example 2

Find the lowest common multiple of 4 and 7.

M_4 = 4, 8, 12, 16, 20, 24, **28**, 32 M_7 = 7, 14, 21, **28**, 35, 42 LCM = 28	List several multiples of 4. (Note: M_4 means multiples of 4.) List several multiples of 7. Find the lowest number that appears in both sets. This is the LCM.

Exercise 1.4

FAST FORWARD

Later in this chapter you will see how prime factors can be used to find LCMs.

1 Find the LCM of:

a 2 and 5 **b** 8 and 10 **c** 6 and 4
d 3 and 9 **e** 35 and 55 **f** 6 and 11
g 2, 4 and 8 **h** 4, 5 and 6 **i** 6, 8 and 9
j 1, 3 and 7 **k** 4, 5 and 8 **l** 3, 4 and 18

2 Is it possible to find the highest common multiple of two or more numbers? Give a reason for your answer.

Factors

A **factor** is a number that divides exactly into another number with no remainder. For example, 2 is a factor of 16 because it goes into 16 exactly 8 times. 1 is a factor of every number. The largest factor of any number is the number itself.

F_{12} means the factors of 12.

To list the factors in numerical order go down the left side and then up the right side of the factor pairs. Remember not to repeat factors.

Worked example 3

Find the factors of:

a 12 **b** 25 **c** 110

a	F_{12} = 1, 2, 3, 4, 6, 12	Find pairs of numbers that multiply to give 12: 1×12 2×6 3×4 Write the factors in numerical order.
b	F_{25} = 1, 5, 25	1×25 5×5 Do not repeat the 5.
c	F_{110} = 1, 2, 5, 10, 11, 22, 55, 110	1×110 2×55 5×22 10×11

Exercise 1.5

1 List all the factors of:

a 4	**b** 5	**c** 8	**d** 11	**e** 18
f 12	**g** 35	**h** 40	**i** 57	**j** 90
k 100	**l** 132	**m** 160	**n** 153	**o** 360

2 Which number in each set is not a factor of the given number?

a 14 {1, 2, 4, 7, 14}
b 15 {1, 3, 5, 15, 45}
c 21 {1, 3, 7, 14, 21}
d 33 {1, 3, 11, 22, 33}
e 42 {3, 6, 7, 8, 14}

FAST FORWARD

Later in this chapter you will learn more about divisibility tests and how to use these to decide whether or not one number is a factor of another.

3 State true or false in each case.

a 3 is a factor of 313	**b** 9 is a factor of 99
c 3 is a factor of 300	**d** 2 is a factor of 300
e 2 is a factor of 122 488	**f** 12 is a factor of 60
g 210 is a factor of 210	**h** 8 is a factor of 420

4 What is the smallest factor and the largest factor of any number?

The highest common factor (HCF)

The highest common factor of two or more numbers is the highest number that is a factor of all the given numbers.

Worked example 4

Find the HCF of 8 and 24.

F_8 = <u>1</u>, <u>2</u>, <u>4</u>, <u>8</u>	List the factors of each number.
F_{24} = <u>1</u>, <u>2</u>, 3, <u>4</u>, 6, <u>8</u>, 12, 24	Underline factors that appear in both sets.
HCF = 8	Pick out the highest underlined factor (HCF).

Exercise 1.6

FAST FORWARD

You will learn how to find HCFs by using prime factors later in the chapter. ▶

1 Find the HCF of each pair of numbers.

- **a** 3 and 6 **b** 24 and 16 **c** 15 and 40 **d** 42 and 70
- **e** 32 and 36 **f** 26 and 36 **g** 22 and 44 **h** 42 and 48

2 Find the HCF of each group of numbers.

- **a** 3, 9 and 15 **b** 36, 63 and 84 **c** 22, 33 and 121

3 Not including the factor provided, find two numbers less than 20 that have:

- **a** an HCF of 2 **b** an HCF of 6

4 What is the HCF of two different prime numbers? Give a reason for your answer.

Applying your skills

Recognising the type of problem helps you to choose the correct mathematical techniques for solving it.

Word problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

5 Simeon has two lengths of rope. One piece is 72 metres long and the other is 90 metres long. He wants to cut both lengths of rope into the longest pieces of equal length possible. How long should the pieces be?

6 Ms Sanchez has 40 canvases and 100 tubes of paint to give to the students in her art group. What is the largest number of students she can have if she gives each student an equal number of canvasses and an equal number of tubes of paint?

7 Indira has 300 blue beads, 750 red beads and 900 silver beads. She threads these beads to make wire bracelets. Each bracelet must have the same number and colour of beads. What is the maximum number of bracelets she can make with these beads?

1.3 Prime numbers

Prime numbers have exactly two factors: one and the number itself.

Composite numbers have more than two factors.

The number 1 has only one factor so it is not prime and it is not composite.

Finding prime numbers

Over 2000 years ago, a Greek mathematician called Eratosthenes made a simple tool for sorting out prime numbers. This tool is called the ‘Sieve of Eratosthenes’ and the figure on page 7 shows how it works for prime numbers up to 100.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~
~~51~~	~~52~~	53	~~54~~	~~55~~	~~56~~	~~57~~	~~58~~	59	~~60~~
61	~~62~~	~~63~~	~~64~~	~~65~~	~~66~~	67	~~68~~	~~69~~	~~70~~
71	~~72~~	73	~~74~~	~~75~~	~~76~~	~~77~~	~~78~~	79	~~80~~
~~81~~	~~82~~	83	~~84~~	~~85~~	~~86~~	~~87~~	~~88~~	89	~~90~~
~~91~~	~~92~~	~~93~~	~~94~~	~~95~~	~~96~~	97	~~98~~	~~99~~	~~100~~

Cross out 1, it is not prime.

Circle 2, then cross out other multiples of 2.

Circle 3, then cross out other multiples of 3.

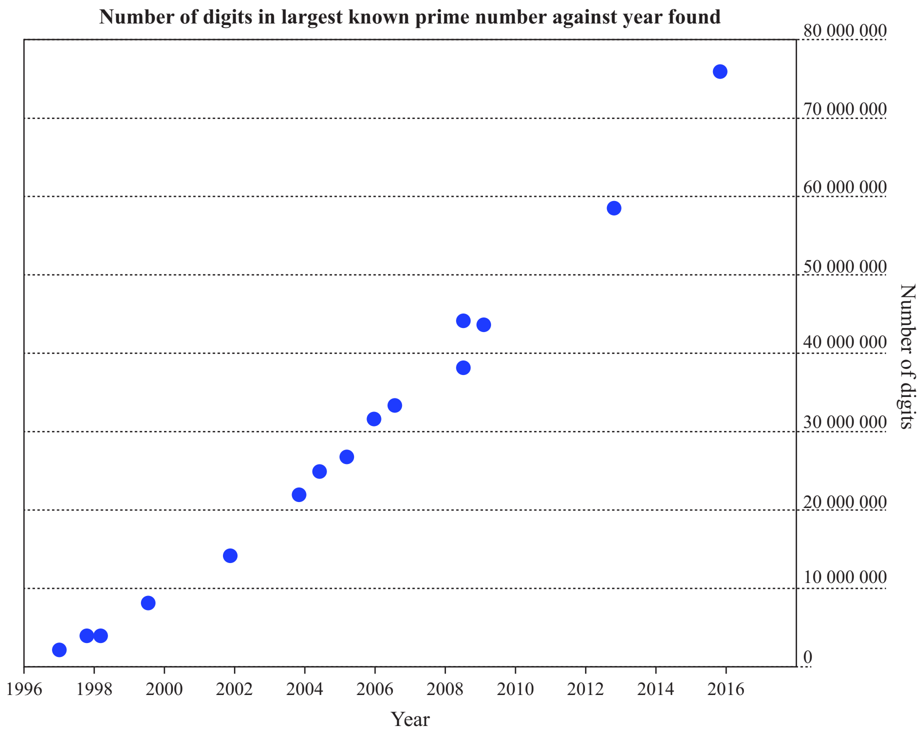
Circle the next available number then cross out all its multiples.

Repeat until all the numbers in the table are either circled or crossed out.

The circled numbers are the primes.

You should try to memorise which numbers between 1 and 100 are prime.

Other mathematicians over the years have developed ways of finding larger and larger prime numbers. Until 1955, the largest known prime number had less than 1000 digits. Since the 1970s and the invention of more and more powerful computers, more and more prime numbers have been found. The graph below shows the number of digits in the largest known primes since 1996.



Source: https://www.mersenne.org/primes/

Today anyone can join the Great Internet Mersenne Prime Search. This project links thousands of home computers to search continuously for larger and larger prime numbers while the computer processors have spare capacity.

Exercise 1.7

FAST FORWARD

A good knowledge of primes can help when factorising quadratics in chapter 10.

1 Which is the only even prime number?

2 How many odd prime numbers are there less than 50?

3 **a** List the composite numbers greater than four, but less than 30.

b Try to write each composite number on your list as the sum of two prime numbers. For example: $6 = 3 + 3$ and $8 = 3 + 5$.

> **Tip**
> Whilst super-prime numbers are interesting, they are not on the syllabus.

4 Twin primes are pairs of prime numbers that differ by two. List the twin prime pairs up to 100.

5 Is 149 a prime number? Explain how you decided.

6 Super-prime numbers are prime numbers that stay prime each time you remove a digit (starting with the units). So, 59 is a super-prime because when you remove 9 you are left with 5, which is also prime. 239 is also a super-prime because when you remove 9 you are left with 23 which is prime, and when you remove 3 you are left with 2 which is prime.
   a Find two three-digit super-prime numbers less than 400.
   b Can you find a four-digit super-prime number less than 3000?
   c Sondra's telephone number is the prime number 987-6413. Is her phone number a super-prime?

Prime factors

Prime factors are the factors of a number that are also prime numbers.

Every composite whole number can be broken down and written as the product of its prime factors. You can do this using tree diagrams or using division. Both methods are shown in worked example 5.

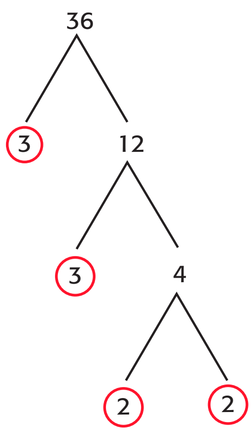
> Remember a product is the answer to a multiplication. So if you write a number as the product of its prime factors you are writing it using multiplication signs like this: $12 = 2 \times 2 \times 3$.

Worked example 5

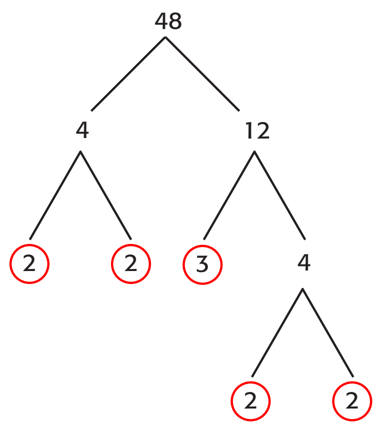
Write the following numbers as the product of prime factors.

a 36 **b** 48

Using a factor tree



$36 = 2 \times 2 \times 3 \times 3$



$48 = 2 \times 2 \times 2 \times 2 \times 3$

Write the number as two factors.

If a factor is a prime number, circle it.

If a factor is a composite number, split it into two factors.

Keep splitting until you end up with two primes.

Write the primes in ascending order with × signs.

Using division

2	36
2	18
3	9
3	3
	1

$36 = 2 \times 2 \times 3 \times 3$

2	48
2	24
2	12
2	6
3	3
	1

$48 = 2 \times 2 \times 2 \times 2 \times 3$

Divide by the smallest prime number that will go into the number exactly.
Continue dividing, using the smallest prime number that will go into your new answer each time.
Stop when you reach 1.
Write the prime factors in ascending order with × signs.

> Prime numbers only have two factors: 1 and the number itself. As 1 is not a prime number, do not include it when expressing a number as a product of prime factors.

> Choose the method that works best for you and stick to it. Always show your method when using prime factors.

Exercise 1.8

When you write your number as a product of primes, group all occurrences of the same prime number together.

1 Express the following numbers as the product of prime factors.

a 30	**b** 24	**c** 100	**d** 225	**e** 360
f 504	**g** 650	**h** 1125	**i** 756	**j** 9240

Using prime factors to find the HCF and LCM

When you are working with larger numbers you can determine the HCF or LCM by expressing each number as a product of its prime factors.

FAST FORWARD

You can also use prime factors to find the square and cube roots of numbers if you don't have a calculator. You will deal with this in more detail later in this chapter. ▶

Worked example 6

Find the HCF of 168 and 180.

$168 = \underline{2} \times \underline{2} \times 2 \times \underline{3} \times 7$ $180 = \underline{2} \times \underline{2} \times \underline{3} \times 3 \times 5$ $2 \times 2 \times 3 = 12$ HCF = 12	First express each number as a product of prime factors. Use tree diagrams or division to do this. Underline the factors *common* to both numbers. Multiply these out to find the HCF.

Worked example 7

Find the LCM of 72 and 120.

$72 = \underline{2 \times 2 \times 2} \times \underline{3 \times 3}$ $120 = 2 \times 2 \times 2 \times 3 \times \underline{5}$ $2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$ LCM = 360	First express each number as a product of prime factors. Use tree diagrams or division to do this. Underline the *largest* set of multiples of each factor. List these and multiply them out to find the LCM.

Exercise 1.9

1 Find the HCF of these numbers by means of prime factors.

a 48 and 108	**b** 120 and 216	**c** 72 and 90	**d** 52 and 78
e 100 and 125	**f** 154 and 88	**g** 546 and 624	**h** 95 and 120

2 Use prime factorisation to determine the LCM of:

a 54 and 60	**b** 54 and 72	**c** 60 and 72	**d** 48 and 60
e 120 and 180	**f** 95 and 150	**g** 54 and 90	**h** 90 and 120

3 Determine both the HCF and LCM of the following numbers.

a 72 and 108	**b** 25 and 200	**c** 95 and 120	**d** 84 and 60

You won't be told to use the HCF or LCM to solve a problem, you will need to recognise that word problems involving LCM usually include repeating events. You may be asked how many items you need to 'have enough' or when something will happen again at the same time.

Applying your skills

4 A radio station runs a phone-in competition for listeners. Every 30th caller gets a free airtime voucher and every 120th caller gets a free mobile phone. How many listeners must phone in before one receives both an airtime voucher *and* a free phone?

5 Lee runs round a track in 12 minutes. James runs round the same track in 18 minutes. If they start in the same place, at the same time, how many minutes will pass before they both cross the start line together again?

Divisibility tests to find factors easily

Sometimes you want to know if a smaller number will divide into a larger one with no remainder. In other words, is the larger number divisible by the smaller one?

> **Tip**
> Divisibility tests are not part of the syllabus. They are just useful to know when you work with factors and prime numbers.

These simple divisibility tests are useful for working this out:

A number is exactly divisible by:

2 if it ends with 0, 2, 4, 6 or 8 (in other words is even)
3 if the sum of its digits is a multiple of 3 (can be divided by 3)
4 if the last two digits can be divided by 4
5 if it ends with 0 or 5
6 if it is divisible by both 2 and 3
8 if the last three digits are divisible by 8
9 if the sum of the digits is a multiple of 9 (can be divided by 9)
10 if the number ends in 0.

There is no simple test for divisibility by 7, although multiples of 7 do have some interesting properties that you can investigate on the internet.

Exercise 1.10

23	65	92	10	104	70	500	21	64	798	1223

1 Look at the box of numbers above. Which of these numbers are:

a divisible by 5? **b** divisible by 8? **c** divisible by 3?

2 Say whether the following are true or false.

a 625 is divisible by 5
b 88 is divisible by 3
c 640 is divisible by 6
d 346 is divisible by 4
e 476 is divisible by 8
f 2340 is divisible by 9
g 2890 is divisible by 6
h 4562 is divisible by 3
i 40 090 is divisible by 5
j 123 456 is divisible by 9

3 Can $34.07 be divided equally among:

a two people? **b** three people? **c** nine people?

4 A stadium has 202 008 seats. Can these be divided equally into:

a five blocks? **b** six blocks? **c** nine blocks?

5 **a** If a number is divisible by 12, what other numbers must it be divisible by?
b If a number is divisible by 36, what other numbers must it be divisible by?
c How could you test if a number is divisible by 12, 15 or 24?

6 Jacqueline and Sophia stand facing one another. At exactly the same moment both girls start to turn steadily on the spot.
It takes Jacqueline 3 seconds to complete one full turn, whilst Sophia takes 4 seconds to make on full turn.
How many times will Jacqueline have turned when the girls are next facing each other?

E

1.4 Powers and roots

> REWIND
> In section 1.1 you learned that the product obtained when an integer is multiplied by itself is a square number. ◀

Square numbers and square roots

A number is squared when it is multiplied by itself. For example, the **square** of 5 is $5 \times 5 = 25$. The symbol for squared is ^{2}. So, 5×5 can also be written as 5^2.

The **square root** of a number is the number that was multiplied by itself to get the square number. The symbol for square root is $\sqrt{\ }$. You know that $25 = 5^2$, so $\sqrt{25} = 5$.

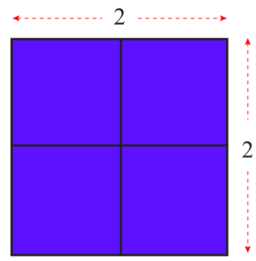
Cube numbers and cube roots

A number is cubed when it is multiplied by itself and then multiplied by itself again. For example, the **cube** of 2 is $2 \times 2 \times 2 = 8$. The symbol for cubed is ^{3}. So $2 \times 2 \times 2$ can also be written as 2^3.

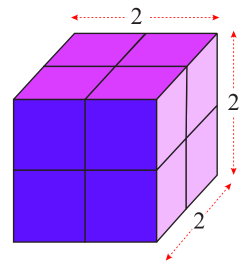
LINK

Fractional powers and roots are used in many different financial calculations involving investments, insurance policies and economic decisions.

The cube root of a number is the number that was multiplied by itself to get the cube number. The symbol for cube root is $\sqrt[3]{\ }$. You know that $8 = 2^3$, so $\sqrt[3]{8} = 2$.



a) Square numbers can be arranged to form a square shape. This is 2^2.



b) Cube numbers can be arranged to form a solid cube shape. This is 2^3.

Finding powers and roots

You can use your calculator to square or cube numbers quickly using the $\boxed{x^2}$ and $\boxed{x^3}$ keys or the $\boxed{x^{\square}}$ key. Use the $\boxed{\sqrt{\ }}$ or $\boxed{\sqrt[3]{\ }}$ keys to find the roots. If you don't have a calculator, you can use the product of prime factors method to find square and cube roots of numbers. Both methods are shown in the worked examples below.

Not all calculators have exactly the same buttons. $\boxed{x^{\square}}$ $\boxed{x^y}$ and $\boxed{\wedge}$ all mean the same thing on different calculators.

Worked example 8

Use your calculator to find:

a 13^2 **b** 5^3 **c** $\sqrt{324}$ **d** $\sqrt[3]{512}$

a	$13^2 = 169$	Enter $\boxed{1}$ $\boxed{3}$ $\boxed{x^2}$ $\boxed{=}$
b	$5^3 = 125$	Enter $\boxed{5}$ $\boxed{x^3}$ $\boxed{=}$. If you do not have a $\boxed{x^3}$ button then enter $\boxed{5}$ $\boxed{x^{\square}}$ $\boxed{3}$ $\boxed{=}$; for this key you have to enter the power.
c	$\sqrt{324} = 18$	Enter $\boxed{\sqrt{\ }}$ $\boxed{3}$ $\boxed{2}$ $\boxed{4}$ $\boxed{=}$
d	$\sqrt[3]{512} = 8$	Enter $\boxed{\sqrt[3]{\ }}$ $\boxed{5}$ $\boxed{1}$ $\boxed{2}$ $\boxed{=}$

Worked example 9

If you do not have a calculator, you can write the integer as a product of primes and group the prime factors into pairs or threes. Look again at parts (c) and (d) of worked example 8:

c $\sqrt{324}$ **d** $\sqrt[3]{512}$

c	$324 = \underbrace{2\times 2}_{2} \times \underbrace{3\times 3}_{3} \times \underbrace{3\times 3}_{3}$ $2\times 3\times 3 = 18$ $\sqrt{324} = 18$	Group the factors into pairs, and write down the square root of each pair. Multiply the roots together to give you the square root of 324.
d	$512 = \underbrace{2\times 2\times 2}_{2} \times \underbrace{2\times 2\times 2}_{2} \times \underbrace{2\times 2\times 2}_{2}$ $2\times 2\times 2 = 8$ $\sqrt[3]{512} = 8$	Group the factors into threes, and write the cube root of each threesome. Multiply together to get the cube root of 512.

Other powers and roots

Make sure that you know which key is used for each function on your calculator and that you know how to use it. On some calculators these keys might be second functions.

You've seen that square numbers are all raised to the power of 2 (5 squared $= 5 \times 5 = 5^2$) and that cube numbers are all raised to the power of 3 (5 cubed $= 5 \times 5 \times 5 = 5^3$). You can raise a number to any power. For example, $5 \times 5 \times 5 \times 5 = 5^4$. This is read as 5 to the power of 4. The same principle applies to finding roots of numbers.

$5^2 = 25 \qquad \sqrt{25} = 5$

$5^3 = 125 \qquad \sqrt[3]{125} = 5$

$5^4 = 625 \qquad \sqrt[4]{625} = 5$

FAST FORWARD

You will work with higher powers and roots again when you deal with indices in chapter 2, standard form in chapter 5 and rates of growth and decay in chapters 17 and 18.

You can use your calculator to perform operations using any roots or squares.

The $\boxed{y^x}$ key calculates any power.

So, to find 7^5, you would enter 7 $\boxed{y^x}$ 5 and get a result of 16 807.

The $\boxed{\sqrt[x]{\ }}$ key calculates any root.

So, to find $4\sqrt{81}$, you would enter 4 $\boxed{\sqrt[x]{\ }}$ 81 and get a result of 3.

Exercise 1.11

1 Calculate:

a 3^2	**b** 7^2	**c** 11^2	**d** 12^2	**e** 21^2
f 19^2	**g** 32^2	**h** 100^2	**i** 14^2	**j** 68^2

2 Calculate:

a 1^3	**b** 3^3	**c** 4^3	**d** 6^3	**e** 9^3
f 10^3	**g** 100^3	**h** 18^3	**i** 30^3	**j** 200^3

Learn the squares of all integers between 1 and 20 inclusive. You will need to recognise these quickly. Spotting a pattern of square numbers can help you solve problems in different contexts.

3 Find a value of x to make each of these statements true.

a $x \times x = 25$	**b** $x \times x \times x = 8$	**c** $x \times x = 121$
d $x \times x \times x = 729$	**e** $x \times x = 324$	**f** $x \times x = 400$
g $x \times x \times x = 8000$	**h** $x \times x = 225$	**i** $x \times x \times x = 1$
j $\sqrt{x} = 9$	**k** $\sqrt{1} = x$	**l** $\sqrt{x} = 81$
m $\sqrt[3]{x} = 2$	**n** $\sqrt[3]{x} = 1$	**o** $\sqrt[3]{64} = x$

4 Use a calculator to find the following roots.

a $\sqrt{9}$	**b** $\sqrt{64}$	**c** $\sqrt{1}$	**d** $\sqrt{4}$	**e** $\sqrt{100}$
f $\sqrt{0}$	**g** $\sqrt{81}$	**h** $\sqrt{400}$	**i** $\sqrt{1296}$	**j** $\sqrt{1764}$
k $\sqrt[3]{8}$	**l** $\sqrt[3]{1}$	**m** $\sqrt[3]{27}$	**n** $\sqrt[3]{64}$	**o** $\sqrt[3]{1000}$
p $\sqrt[3]{216}$	**q** $\sqrt[3]{512}$	**r** $\sqrt[3]{729}$	**s** $\sqrt[3]{1728}$	**t** $\sqrt[3]{5832}$

5 Use the product of prime factors given below to find the square root of each number. Show your working.

a $324 = 2 \times 2 \times 3 \times 3 \times 3 \times 3$	**b** $225 = 3 \times 3 \times 5 \times 5$
c $784 = 2 \times 2 \times 2 \times 2 \times 7 \times 7$	**d** $2025 = 3 \times 3 \times 3 \times 3 \times 5 \times 5$
e $19\,600 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 7 \times 7$	**f** $250\,000 = 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$

6 Use the product of prime factors to find the cube root of each number. Show your working.

a $27 = 3 \times 3 \times 3$	**b** $729 = 3 \times 3 \times 3 \times 3 \times 3 \times 3$
c $2197 = 13 \times 13 \times 13$	**d** $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

e $15\,625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

f $32\,768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Brackets act as grouping symbols. Work out any calculations inside brackets before doing the calculations outside the brackets.

Root signs work in the same way as a bracket. If you have $\sqrt{25+9}$, you must add 25 and 9 before finding the root.

7 Calculate:

a $(\sqrt{25})^2$	**b** $(\sqrt{49})^2$	**c** $(\sqrt[3]{64})^3$	**d** $(\sqrt[3]{32})^3$
e $\sqrt{9}+\sqrt{16}$	**f** $\sqrt{9+16}$	**g** $\sqrt{36}+\sqrt{64}$	**h** $\sqrt{36+64}$
i $\sqrt{100-36}$	**j** $\sqrt{100}-\sqrt{36}$	**k** $\sqrt{25}\times\sqrt{4}$	**l** $\sqrt{25\times 4}$
m $\sqrt{9\times 4}$	**n** $\sqrt{9}\times\sqrt{4}$	**o** $\sqrt{\dfrac{36}{4}}$	**p** $\dfrac{\sqrt{36}}{4}$

8 Find the length of the edge of a cube with a volume of:

a $1000\,\text{cm}^3$ **b** $19\,683\,\text{cm}^3$ **c** $68\,921\,\text{mm}^3$ **d** $64\,000\,\text{cm}^3$

9 If the symbol * means 'add the square of the first number to the cube of the second number', calculate:

a 2 * 3	**b** 3 * 2	**c** 1 * 4	**d** 4 * 1	**e** 2 * 4
f 4 * 2	**g** 1 * 9	**h** 9 * 1	**i** 5 * 2	**j** 2 * 5

10 Evaluate.

a $2^4\times 2^3$	**b** $3^5\times\sqrt[6]{64}$	**c** $3^4+\sqrt[4]{256}$
d $2^4\times\sqrt[5]{7776}$	**e** $\sqrt[4]{625}\times 2^6$	**f** $8^4\div(\sqrt[5]{32})^3$

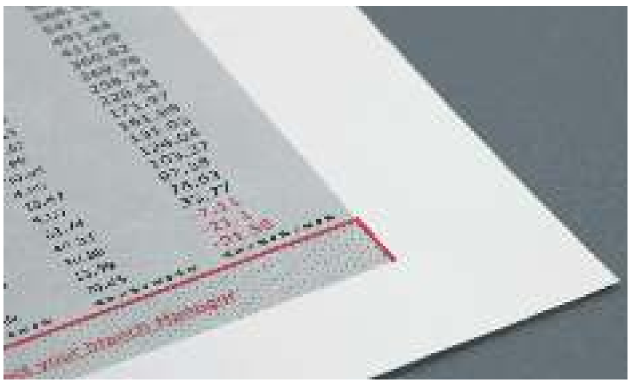
11 Which is greater and by how much?

a $8^0\times 4^4$ or $2^4\times 3^4$ **b** $\sqrt[4]{625}\times 3^6$ or $\sqrt[6]{729}\times 4^4$

1.5 Working with directed numbers

Once a direction is chosen to be positive, the opposite direction is taken to be negative. So:

- if up is positive, down is negative
- if right is positive, left is negative
- if north is positive, south is negative
- if above 0 is positive, below 0 is negative.



A negative sign is used to indicate that values are less than zero. For example, on a thermometer, on a bank statement or in an elevator.

When you use numbers to represent real-life situations like temperatures, altitude, depth below sea level, profit or loss and directions (on a grid), you sometimes need to use the negative sign to indicate the direction of the number. For example, a temperature of three degrees below zero can be shown as $-3\,°\text{C}$. Numbers like these, which have direction, are called **directed numbers**. So if a point 25 m above sea level is at +25 m, then a point 25 m below sea level is at −25 m.

Exercise 1.12

1 Express each of these situations using a directed number.

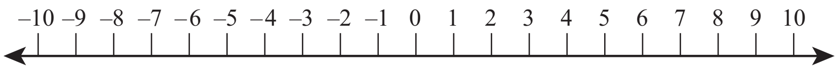
a a profit of \$100	**b** 25 km below sea level
c a drop of 10 marks	**d** a gain of 2 kg
e a loss of 1.5 kg	**f** 8000 m above sea level
g a temperature of 10 °C below zero	**h** a fall of 24 m
i a debt of \$2000	**j** an increase of \$250
k a time two hours behind GMT	**l** a height of 400 m
m a bank balance of \$450.00	

FAST FORWARD

You will use similar number lines when solving linear inequalities in chapter 14. ▶

Comparing and ordering directed numbers

In mathematics, directed numbers are also known as integers. You can represent the set of integers on a number line like this:



The further to the right a number is on the number line, the greater its value.

Exercise 1.13

It is important that you understand how to work with directed numbers early in your IGCSE course. Many topics depend upon them!

1 Copy the numbers and fill in < or > to make a true statement.

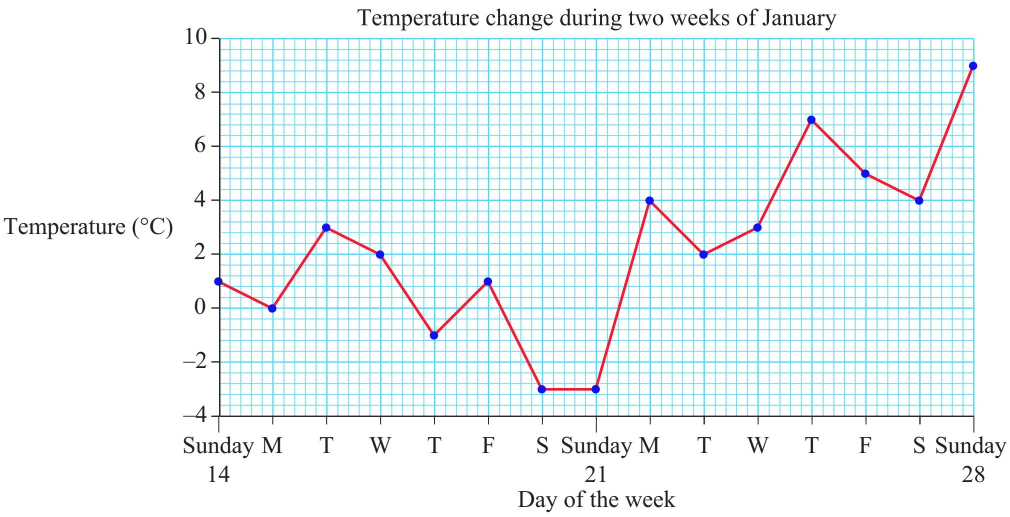
a 2 □ 8 **b** 4 □ 9 **c** 12 □ 3 **d** 6 □ –4 **e** –7 □ 4
f –2 □ 4 **g** –2 □ –11 **h** –12 □ –20 **i** –8 □ 0 **j** –2 □ 2
k –12 □ –4 **l** –32 □ –3 **m** 0 □ –3 **n** –3 □ 11 **o** 12 □ –89

2 Arrange each set of numbers in ascending order.

a –8, 7, 10, –1, –12 **b** 4, –3, –4, –10, 9, –8
c –11, –5, –7, 7, 0, –12 **d** –94, –50, –83, –90, 0

Applying your skills

3 Study the temperature graph carefully.



a What was the temperature on Sunday 14 January?
b By how much did the temperature drop from Sunday 14 to Monday 15?
c What was the lowest temperature recorded?
d What is the difference between the highest and lowest temperatures?
e On Monday 29 January the temperature changed by –12 degrees. What was the temperature on that day?

The difference between the highest and lowest temperature is also called the *range* of temperatures.

4 Matt has a bank balance of \$45.50. He deposits \$15.00 and then withdraws \$32.00. What is his new balance?

5 Mr Singh's bank account is \$420 overdrawn.

a Express this as a directed number.
b How much money will he need to deposit to get his account to have a balance of \$500?
c He deposits \$200. What will his new balance be?

6 A diver 27 m below the surface of the water rises 16 m. At what depth is she then?

7 On a cold day in New York, the temperature at 6 a.m. was –5 °C. By noon, the temperature had risen to 8 °C. By 7 p.m. the temperature had dropped by 11 °C from its value at noon. What was the temperature at 7 p.m.?

8 Local time in Abu Dhabi is four hours ahead of Greenwich Mean Time. Local time in Rio de Janeiro is three hours behind Greenwich Mean Time.

a If it is 4 p.m. at Greenwich, what time is it in Abu Dhabi?
b If it is 3 a.m. in Greenwich, what time is it in Rio de Janiero?
c If it is 3 p.m. in Rio de Janeiro, what time is it in Abu Dhabi?
d If it is 8 a.m. in Abu Dhabi, what time is it in Rio de Janeiro?

1.6 Order of operations

At this level of mathematics you are expected to do more complicated calculations involving more than one operation (+, –, × and ÷). When you are carrying out more complicated calculations you have to follow a sequence of rules so that there is no confusion about what operations you should do first. The rules governing the order of operations are:

- complete operations in grouping symbols first
- do division and multiplication next, working from left to right
- do addition and subtractions last, working from left to right.

Many people use the letters **BODMAS** to remember the order of operations. The letters stand for:

Brackets

Of (Sometimes, 'I' for 'indices' is used instead of 'O' for 'of')

Divide **M**ultiply

Add **S**ubtract

BODMAS indicates that indices (powers) are considered after brackets but before all other operations.

Grouping symbols

The most common grouping symbols in mathematics are brackets. Here are some examples of the different kinds of brackets used in mathematics:

$$(4 + 9) \times (10 \div 2)$$

$$[2(4 + 9) - 4(3) - 12]$$

$$\{2 - [4(2 - 7) - 4(3 + 8)] - 2 \times 8\}$$

When you have more than one set of brackets in a calculation, you work out the innermost set first.

Other symbols used to group operations are:

- fraction bars, e.g. $\frac{5-12}{3-8}$
- root signs, such as square roots and cube roots, e.g. $\sqrt{9+16}$
- powers, e.g. 5^2 or 4^3

Worked example 10

Simplify:

a $7 \times (3 + 4)$ **b** $(10 - 4) \times (4 + 9)$ **c** $45 - [20 \times (4 - 3)]$

a $7 \times 7 = 49$

b $6 \times 13 = 78$

c $45 - [20 \times 1] = 45 - 20$
$= 25$

Worked example 11

Calculate:

a $3 + 8^2$ **b** $\dfrac{4 + 28}{17 - 9}$ **c** $\sqrt{36 \div 4} + \sqrt{100 - 36}$

a
$3 + (8 \times 8)$
$= 3 + 64$
$= 67$

b
$(4 + 28) \div (17 - 9)$
$= 32 \div 8$
$= 4$

c
$\sqrt{36 \div 4} + \sqrt{100 - 36}$
$= \sqrt{9} + \sqrt{64}$
$= 3 + 8$
$= 11$

Exercise 1.14

1 Calculate. Show the steps in your working.

a $(4 + 7) \times 3$ **b** $(20 - 4) \div 4$ **c** $50 \div (20 + 5)$ **d** $6 \times (2 + 9)$
e $(4 + 7) \times 4$ **f** $(100 - 40) \times 3$ **g** $16 + (25 \div 5)$ **h** $19 - (12 + 2)$
i $40 \div (12 - 4)$ **j** $100 \div (4 + 16)$ **k** $121 \div (33 \div 3)$ **l** $15 \times (15 - 15)$

2 Calculate:

a $(4 + 8) \times (16 - 7)$ **b** $(12 - 4) \times (6 + 3)$ **c** $(9 + 4) - (4 + 6)$
d $(33 + 17) \div (10 - 5)$ **e** $(4 \times 2) + (8 \times 3)$ **f** $(9 \times 7) \div (27 - 20)$
g $(105 - 85) \div (16 \div 4)$ **h** $(12 + 13) \div 5^2$ **i** $(56 - 6^2) \times (4 + 3)$

A bracket 'type' is always twinned with another bracket of the same type/shape. This helps mathematicians to understand the order of calculations even more easily.

3 Simplify. Remember to work from the innermost grouping symbols to the outermost.

a $4 + [12 - (8 - 5)]$ **b** $6 + [2 - (2 \times 0)]$
c $8 + [60 - (2 + 8)]$ **d** $200 - [(4 + 12) - (6 + 2)]$
e $200 \times \{100 - [4 \times (2 + 8)]\}$ **f** $\{6 + [5 \times (2 + 30)]\} \times 10$
g $[(30 + 12) - (7 + 9)] \times 10$ **h** $6 \times [(20 \div 4) - (6 - 3) + 2]$
i $1000 - [6 \times (4 + 20) - 4 \times (3 + 0)]$

4 Calculate:

a $6 + 72$ **b** $29 - 23$ **c** 8×42
d $20 - 4 \div 2$ **e** $\dfrac{31 - 10}{14 - 7}$ **f** $\dfrac{100 - 40}{5 \times 4}$
g $\sqrt{100 - 36}$ **h** $\sqrt{8 + 8}$ **i** $\sqrt{90 - 9}$

5 Insert brackets into the following calculations to make them true.

a $3 \times 4 + 6 = 30$ **b** $25 - 15 \times 9 = 90$ **c** $40 - 10 \times 3 = 90$
d $14 - 9 \times 2 = 10$ **e** $12 + 3 \div 5 = 3$ **f** $19 - 9 \times 15 = 150$
g $10 + 10 \div 6 - 2 = 5$ **h** $3 + 8 \times 15 - 9 = 66$ **i** $9 - 4 \times 7 + 2 = 45$
j $10 - 4 \times 5 = 30$ **k** $6 \div 3 + 3 \times 5 = 5$ **l** $15 - 6 \div 2 = 12$
m $1 + 4 \times 20 \div 5 = 20$ **n** $8 + 5 - 3 \times 2 = 20$ **o** $36 \div 3 \times 3 - 3 = 6$
p $3 \times 4 - 2 \div 6 = 1$ **q** $40 \div 4 + 1 = 11$ **r** $6 + 2 \times 8 + 2 = 24$

FAST FORWARD

You will apply the order of operation rules to fractions, decimals and algebraic expressions as you progress through the course.

Working in the correct order

Now that you know what to do with grouping symbols, you are going to apply the rules for order of operations to perform calculations with numbers.

Exercise 1.15

1 Simplify. Show the steps in your working.

a $5 \times 10 + 3$ **b** $5 \times (10 + 3)$ **c** $2 + 10 \times 3$
d $(2 + 10) \times 3$ **e** $23 + 7 \times 2$ **f** $6 \times 2 \div (3 + 3)$

g $\frac{15-5}{2\times 5}$ h $(17+1)\div 9+2$ i $\frac{16-4}{4-1}$
j $17+3\times 21$ k $48-(2+3)\times 2$ l $12\times 4-4\times 8$
m $15+30\div 3+6$ n $20-6\div 3+3$ o $10-4\times 2\div 2$

2 Simplify:

a $18-4\times 2-3$ b $14-(21\div 3)$ c $24\div 8\times(6-5)$
d $42\div 6-3-4$ e $5+36\div 6-8$ f $(8+3)\times(30\div 3)\div 11$

3 State whether the following are true or false.

a $(1+4)\times 20+5=1+(4\times 20)+5$ b $6\times(4+2)\times 3>(6\times 4)\div 2\times 3$
c $8+(5-3)\times 2<8+5-(3\times 2)$ d $100+10\div 10>(100+10)\div 10$

4 Place the given numbers in the correct spaces to make a correct number sentence.

a 0, 2, 5, 10 $\square-\square\div\square=\square$
b 9, 11, 13, 18 $\square-\square\div\square=\square$
c 1, 3, 8, 14, 16 $\square\div(\square-\square)-\square=\square$
d 4, 5, 6, 9, 12 $(\square+\square)-(\square-\square)=\square$

In this section you will use your calculator to perform operations in the correct order. However, you will need to remember the *order of operations* rules and apply them throughout the book as you do more complicated examples using your calculator.

Using your calculator

A calculator with algebraic logic will apply the rules for order of operations automatically. So, if you enter $2+3\times 4$, your calculator will do the multiplication first and give you an answer of 14. (Check that your calculator does this!).

When the calculation contains brackets you must enter these to make sure your calculator does the grouped sections first.

Experiment with your calculator by making several calculations with and without brackets. For example: $3\times 2+6$ and $3\times(2+6)$. Do you understand why these are different?

Your calculator might only have one type of bracket (and). If there are two different shaped brackets in the calculation (such as $[4\times(2-3)]$, enter the calculator bracket symbol for each type.

Worked example 12

Use a calculator to find:

a $3+2\times 9$ b $(3+8)\times 4$ c $(3\times 8-4)-(2\times 5+1)$

a	21	Enter	3	+	2	×	9	=			
b	44	Enter	(	3	+	8	)	×	4	=	
c	9	Enter	(	3	×	8	−	4	)	−	
			(	2	×	5	+	1	)	=	

Exercise 1.16

Some calculators have two '–' buttons: − and (–). The first means 'subtract' and is used to subtract one number from another. The second means 'make negative'. Experiment with the buttons and make sure that your calculator is doing what you expect it to do!

1 Use a calculator to find the correct answer.

a $10-4\times 5$ b $12+6\div 7-4$
c $3+4\times 5-10$ d $18\div 3\times 5-3+2$
e $5-3\times 8-6\div 2$ f $7+3\div 4+1$
g $(1+4)\times 20\div 5$ h $36\div 6\times(3-3)$
i $(8+8)-6\times 2$ j $100-30\times(4-3)$
k $24\div(7+5)\times 6$ l $[(60-40)-(53-43)]\times 2$
m $[(12+6)\div 9]\times 4$ n $[100\div(4+16)]\times 3$
o $4\times[25\div(12-7)]$

2 Use your calculator to check whether the following answers are correct. If the answer is incorrect, work out the correct answer.

a $12\times 4+76=124$ b $8+75\times 8=698$
c $12\times 18-4\times 23=124$ d $(16\div 4)\times(7+3\times 4)=76$
e $(82-36)\times(2+6)=16$ f $(3\times 7-4)-(4+6\div 2)=12$

The more effectively you are able to use your calculator, the faster and more accurate your calculations are likely to be.

FAST FORWARD

When you work with indices and standard form in chapter 5, you will need to apply these skills and use your calculator effectively to solve problems involving any powers or roots.

LINK

The idea of 'rounding' runs through all subjects where numerical data is collected. Masses in physics, temperatures in biology, prices in economics: these all need to be recorded sensibly and will be rounded to a degree of accuracy appropriate for the situation.

3 Each * represents a missing operation. Work out what it is.

a 12 * (28 * 24) = 3 **b** 84 * 10 * 8 = 4 **c** 3 * 7(0.7 * 1.3) = 17
d 23 * 11 * 22 * 11 = 11 **e** 40 * 5 * (7 * 5) = 4 **f** 9 * 15 * (3 * 2) = 12

4 Calculate:

a $\dfrac{7\times\sqrt{16}}{2^3+7^2-1}$ **b** $\dfrac{5^2\times\sqrt{4}}{1+6^2-12}$ **c** $\dfrac{2+3^2}{5^2+4\times 10-\sqrt{25}}$

d $\dfrac{6^2-11}{2(17+2\times 4)}$ **e** $\dfrac{3^2-3}{2\times\sqrt{81}}$ **f** $\dfrac{3^2-5+6}{\sqrt{4}\times 5}$

g $\dfrac{36-3\times\sqrt{16}}{15-3^2\div 3}$ **h** $\dfrac{-30+[18\div(3-12)+24]}{5-8-3^2}$

5 Use a calculator to find the answer.

a $\dfrac{0.345}{1.34+4.2\times 7}$ **b** $\dfrac{12.32\times 0.0378}{\sqrt{16}+8.05}$ **c** $\dfrac{\sqrt{16}\times 0.087}{2^2-5.098}$ **d** $\dfrac{19.23\times 0.087}{2.45^2-1.03^2}$

6 Use your calculator to evaluate.

a $\sqrt{64\times 125}$ **b** $\sqrt{2^3\times 3^2\times 6}$ **c** $\sqrt[3]{8^2+19^2}$ **d** $\sqrt{41^2-36^2}$

e $\sqrt{3.2^2-1.17^3}$ **f** $\sqrt[3]{1.45^3-0.13^2}$ **g** $\dfrac{1}{4}\sqrt{\dfrac{1}{4}+\dfrac{1}{4}+\sqrt{\dfrac{1}{4}}}$ **h** $\sqrt[3]{2.75^2-\frac{1}{2}\times 1.7^3}$

1.7 Rounding numbers

In many calculations, particularly with decimals, you will not need to find an exact answer. Instead, you will be asked to give an answer to a stated level of accuracy. For example, you may be asked to give an answer correct to 2 decimal places, or an answer correct to 3 significant figures.

To round a number to a given decimal place you look at the value of the digit to the right of the specified place. If it is 5 or greater, you round up; if it less than 5, you round down.

Worked example 13

Round 64.839906 to:

a the nearest whole number **b** 1 decimal place **c** 3 decimal places

a	$6\underline{4}.839906$ $64.\underline{8}39906$ = 65 (to nearest whole number)	4 is in the units place. The next digit is 8, so you will round up to get 5. To the nearest whole number.
b	$64.\underline{8}39906$ $64.8\underline{3}9906$ = 64.8 (1dp)	8 is in the first decimal place. The next digit is 3, so the 8 will remain unchanged. Correct to 1 decimal place.
c	$64.83\underline{9}906$ $64.839\underline{9}06$ = 64.840 (3dp)	9 is in the third decimal place. The next digit is 9, so you need to round up. When you round 9 up, you get 10, so carry one to the previous digit and write 0 in the place of the 9. Correct to 3 decimal places.

The first significant digit of a number is the first *non-zero* digit, when reading from left to right. The next digit is the second significant digit, the next the third significant and so on. All zeros *after* the first significant digit are considered significant.

To round to 3 significant figures, find the third significant digit and look at the value of the digit to the right of it. If it is 5 or greater, add one to the third significant digit and lose all of the other digits to the right. If it is less than 5, leave the third significant digit unchanged and lose all the other digits to the right as before. To round to a different number of significant figures, use the same method but find the appropriate significant digit to start with: the fourth for 4sf, the seventh for 7sf etc. If you are rounding to a whole number, write the appropriate number of zeros after the last significant digit as place holders to keep the number the same size.

Worked example 14

Round:

a 1.076 to 3 significant figures **b** 0.00736 to 1 significant figure

a	1.076 = 1.08 (3sf)	The third significant figure is the 7. The next digit is 6, so round 7 up to get 8. Correct to 3 significant figures.
b	0.00736 = 0.007 (1sf)	The first significant figure is the 7. The next digit is 3, so 7 will not change. Correct to 1 significant figure.

Exercise 1.17

Remember, the first significant digit in a number is the first *non-zero* digit, reading from left to right. Once you have read past the first non-zero digit, all zeros then become significant.

FAST FORWARD

You will use rounding to a given number of decimal places and significant figures in almost all of your work this year. You will also apply these skills to estimate answers. This is dealt with in more detail in chapter 5.

1 Round each number to 2 decimal places.

a 3.185 **b** 0.064 **c** 38.3456 **d** 2.149 **e** 0.999
f 0.0456 **g** 0.005 **h** 41.567 **i** 8.299 **j** 0.4236
k 0.062 **l** 0.009 **m** 3.016 **n** 12.0164 **o** 15.11579

2 Express each number correct to:

i 4 significant figures **ii** 3 significant figures **iii** 1 significant figure

a 4512 **b** 12 305 **c** 65 238 **d** 320.55
e 25.716 **f** 0.000765 **g** 1.0087 **h** 7.34876
i 0.00998 **j** 0.02814 **k** 31.0077 **l** 0.0064735

3 Change $2\frac{5}{9}$ to a decimal using your calculator. Express the answer correct to:

a 3 decimal places **b** 2 decimal places **c** 1 decimal place
d 3 significant figures **e** 2 significant figures **f** 1 significant figure

Summary

Do you know the following?

- Numbers can be classified as natural numbers, integers, prime numbers and square numbers.
- When you multiply an integer by itself you get a square number (x^2). If you multiply it by itself again you get a cube number (x^3).
- The number you multiply to get a square is called the square root and the number you multiply to get a cube is called the cube root. The symbol for square root is $\sqrt{\ }$. The symbol for cube root is $\sqrt[3]{\ }$.
- A multiple is obtained by multiplying a number by a natural number. The LCM of two or more numbers is the lowest multiple found in all the sets of multiples.
- A factor of a number divides into it exactly. The HCF of two or more numbers is the highest factor found in all the sets of factors.
- Prime numbers have only two factors, 1 and the number itself. The number 1 is not a prime number.
- A prime factor is a number that is both a factor and a prime number.
- All natural numbers that are not prime can be expressed as a product of prime factors.
- Integers are also called directed numbers. The sign of an integer (– or +) indicates whether its value is above or below 0.
- Mathematicians apply a standard set of rules to decide the order in which operations must be carried out. Operations in grouping symbols are worked out first, then division and multiplication, then addition and subtraction.

Are you able to . . . ?

- identify natural numbers, integers, square numbers and prime numbers
- find multiples and factors of numbers and identify the LCM and HCF
- write numbers as products of their prime factors using division and factor trees
- calculate squares, square roots, cubes and cube roots of numbers
- work with integers used in real-life situations
- apply the basic rules for operating with numbers
- perform basic calculations using mental methods and with a calculator.

Examination practice

Exam-style questions

1 Here is a set of numbers: {–4, –1, 0, 3, 4, 6, 9, 15, 16, 19, 20}
Which of these numbers are:

a natural numbers? **b** square numbers? **c** negative integers?
d prime numbers? **e** multiples of two? **f** factors of 80?

2 **a** List all the factors of 12. **b** List all the factors of 24. **c** Find the HCF of 12 and 24.

3 Find the HCF of 64 and 144.

4 List the first five multiples of:

a 12 **b** 18 **c** 30 **d** 80

5 Find the LCM of 24 and 36.

6 List all the prime numbers from 0 to 40.

7 **a** Use a factor tree to express 400 as a product of prime factors.
b Use the division method to express 1080 as a product of prime factors.
c Use your answers to find:

i the LCM of 400 and 1080 **ii** the HCF of 400 and 1080
iii $\sqrt{400}$ **iv** whether 1080 is a cube number; how can you tell?

8 Calculate:

a 26^2 **b** 43^3

9 What is the smallest number greater than 100 that is:

a divisible by two? **b** divisible by ten? **c** divisible by four?

10 At noon one day the outside temperature is 4 °C. By midnight the temperature is 8 degrees lower.
What temperature is it at midnight?

11 Simplify:

a $6 \times 2 + 4 \times 5$ **b** $4 \times (100 - 15)$ **c** $(5 + 6) \times 2 + (15 - 3 \times 2) - 6$

12 Add brackets to this statement to make it true.

$7 + 14 \div 4 - 1 \times 2 = 14$

Past paper questions

1 Insert **one pair** of brackets only to make the following statement correct.
$6 + 5 \times 10 - 8 = 16$ [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q1 October/November 2014]

2 Calculate $\dfrac{8.24 + 2.56}{1.26 - 0.72}$ [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q2 October/November 2014]

3 Write 3.5897 correct to 4 significant figures. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q3 May/June 2016]

4

8	9	10	11	12	13	14	15	16

From the list of numbers, write down

a the square numbers, [1]

b a prime factor of 99. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q5 May/June 2016]

5 **a** Write 90 as a product of prime factors. [2]

b Find the lowest common multiple of 90 and 105. [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q15 October/November 2014]