Chapter 24: Probability using tree diagrams and Venn diagrams

Key words

- Possible outcomes
- Sample space
- Independent events
- Mutually exclusive events
- Combined events
- Favourable combinations

In this chapter you will learn how to:

- use tree diagrams and Venn diagrams to show all possible outcomes of combined events
- EXTENDED: calculate the probability of simple combined events using tree diagrams.

The probability of getting heads when you toss one coin is 0.5. But what is the probability of getting heads only when you toss two, three or 15 coins at the same time?

In chapter 8 you used probability space diagrams to represent the sample space and all possible outcomes of an event. On these diagrams, outcomes are represented by points on a grid. When events are combined, you can think of them taking place in different stages. For example, when you toss two coins, you can look at the outcomes for the first coin, then the outcomes for the second coin. In experiments like this, it is convenient to use a tree diagram to list the outcomes of each stage in a clear and systematic way.

In this chapter you are going to learn how to use tree diagrams to represent outcomes of simple combined events. You are also going to learn how to use tree diagrams to calculate the probability of different outcomes.

RECAP

You should already be familiar with the following probability work:

Possibility diagrams (Chapter 8)

Grids and tables can be used as possibility diagrams to show all the possible outcomes for combined events.

This table shows the sample space for rolling a die and tossing a coin.

Coin \ Die	1	2	3	4	5	6
Heads	H1	H2	H3	H4	H5	H6
Tails	T1	T2	T3	T4	T5	T6

Calculating the probability of combined events (Chapter 8)

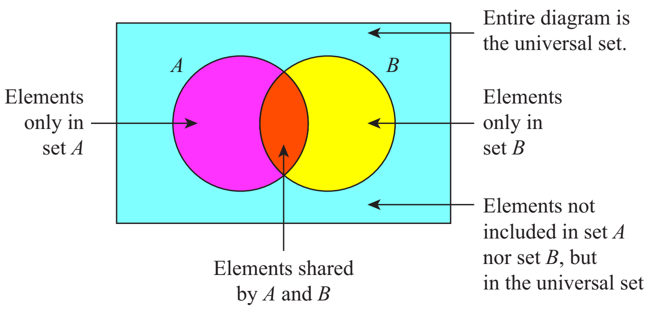
If A and B are independent events then:

P(A and B) = P(A) × P(B)

If A and B are mutually exclusive events then:

P(A or B) = P(A) + P(B)

Venn diagrams (Chapter 9)



Key

ℰ Universal set (sample space)

n(A) Number of elements in set A

$A \cup B$ Union of set A and set B

$A \cap B$ Intersection of set A and set B

24.1 Using tree diagrams to show outcomes

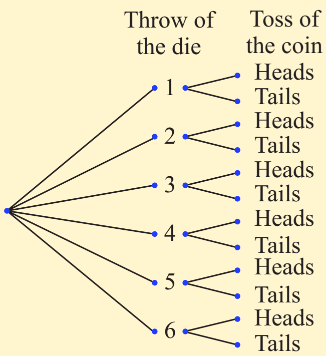
REWIND

For **independent** events
P(A and then B) = P(A) × P(B).

For **mutually exclusive** events
P(A or B) = P(A) + P(B).

Read through chapter 8 again if you have forgotten this. ◀

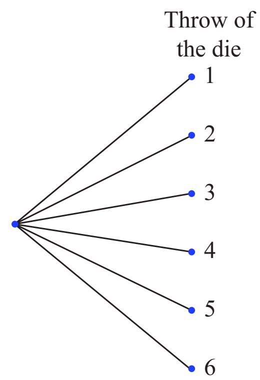
The possible outcomes for the **combined events** of throwing a dice and tossing a coin at the same time:



A tree diagram is a branching diagram that shows all the **possible outcomes** (**sample space**) of one or more events.

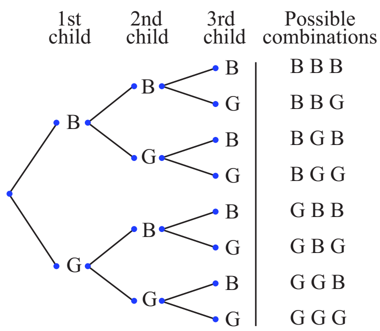
To draw a tree diagram:

- make a dot to represent the first event
- draw branches from the dot to show all possible outcomes of that event only
- write the outcomes at the end of each branch
- draw a dot at the end of each branch to represent the next event
- draw branches from this point to show all possible outcomes of that event
- write the outcomes at the end of the branches.



Worked example 1

When a woman has a child, she can have a girl or a boy. Draw a tree diagram to show the possible outcomes for the first three children born to a couple. Use B for boys and G for girls.



Draw a dot for the first born child.
Draw and label two branches, one B and one G.
Repeat this at the end of each branch for the second and third child.

Note: this example assumes that a boy or a girl is equally likely at each stage, although this may not be the case in many families.

Exercise 24.1

1 Sandra has a bag containing three coloured counters: red, blue and green.

- **a** Draw a tree diagram to show the possible outcomes when one counter is drawn from the bag at random, then returned to the bag before another counter is drawn at random.
- **b** How many possible outcomes are there for the two draws?
- **c** How many outcomes produce two counters the same colour?
- **d** How many outcomes contain at least one blue counter?
- **e** How many outcomes do not contain the blue counter?

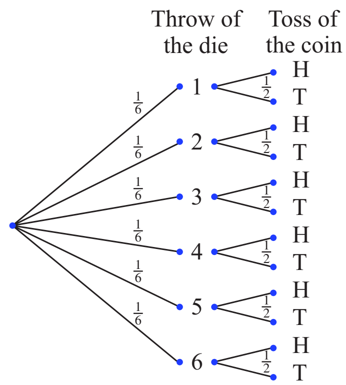
2 Four cards marked A, B, C and D are in a container. A card is drawn, the letter noted, and then it is replaced. Another card is then drawn and the letter noted to make a two-letter combination.

- **a** Draw a tree diagram to show the sample space in this experiment.
- **b** How many outcomes are in the sample space?
- **c** What is the probability of getting the letter combination BD?

24.2 Calculating probability from tree diagrams

Here is the tree diagram showing possible outcomes for throwing a dice and tossing a coin at the same time (H is used for head and T is used for tail).

This is the same diagram as in the previous section but now the probability of each outcome is written at the side of each branch.



LINK

One particular kind of study of tiny particles is called *quantum mechanics*. This looks at the probability of finding particles in particular places at particular times.

The probability of combined events on a tree diagram

To find the probability of one particular combination of outcomes:

- multiply the probabilities on consecutive branches, for example the probability of throwing a 5 and getting an H is $\frac{1}{6}\times\frac{1}{2}=\frac{1}{12}$.

REWIND

You learned in chapter 8 that mutually exclusive events cannot both happen together.

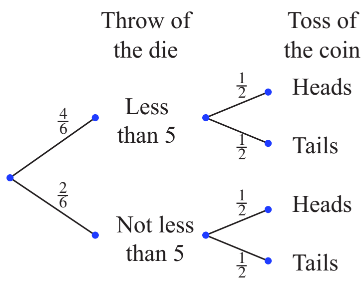
To find the probability when there is more than one **favourable combination** or when the events are mutually exclusive:

- multiply the probabilities on consecutive branches
- add the probabilities (of each favourable combination) obtained by multiplication, for example, throwing 1 or 2 and getting an H is $\left(\frac{1}{6}\times\frac{1}{2}\right)+\left(\frac{1}{6}\times\frac{1}{2}\right)=\frac{1}{12}+\frac{1}{12}=\frac{2}{12}=\frac{1}{6}$

When you are interested only in specific probabilities, you can draw a tree diagram that only shows the favourable outcomes. For example, if you wanted to find the probability of getting a number < 5 and H in the above experiment, you might draw a tree diagram like this one:

There are four numbers on a die less than 5. As they are equally likely to occur, the probability of scoring < 5 is $\frac{4}{6}$.

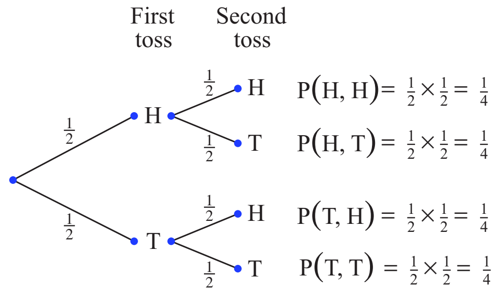
$$P(<5 \text{ and H})=\frac{4}{6}\times\frac{1}{2}=\frac{1}{3}$$



Worked example 2

Two coins are tossed together. Draw a tree diagram to find the probability of getting:

a two tails
b one head and one tail.



a $P(TT) = P(T \text{ on 1st toss}) \times P(T \text{ on 2nd toss})$

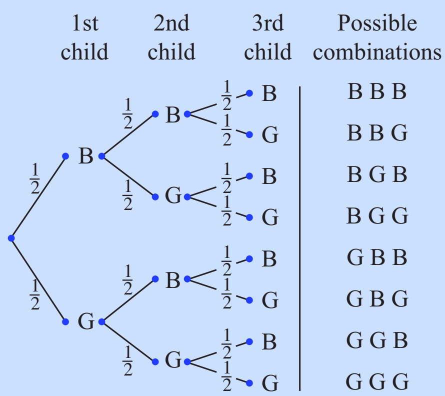
$$= \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

b $P(HT \text{ or } TH) = P(HT) + P(TH)$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$$

Worked example 3

This tree diagram shows all possible combinations of boys and girls in a family of three children.



This assumes that the outcomes, boy or girl, are equally likely. A family with three children is chosen at random. Find the probability that:

a at least one child is a girl
b two of the children are girls
c the oldest and youngest children are the same gender.

a

$$P(\text{at least one girl}) = 7\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)$$
$$= 7 \times \frac{1}{8}$$
$$= \frac{7}{8}$$

All outcomes except BBB have at least one girl. This makes 7 outcomes. As each outcome is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ you can simply multiply this by 7.

b

$$P(\text{two are girls}) = \frac{3}{8}$$

There are three out of eight outcomes where there are two girls.

c

$$P(\text{oldest and youngest same gender}) = \frac{4}{8} = \frac{1}{2}$$

GGG, GBG, BGB and BBB all have the oldest and youngest of the same gender. The probability for each combination is $\frac{1}{8}$, so you can simply add the favourable combinations to give $\frac{4}{8}$.

Exercise 24.2

1 An unbiased coin is tossed twice. Draw a tree diagram to show the outcomes and use it to find the probability of the two tosses giving the same results.

2 A bag contains eight blue marbles and two red marbles. Two marbles are drawn at random. The first marble is replaced before the second is drawn.

a Draw a tree diagram to show all possible outcomes.
b What is the probability of getting:

i two red marbles
ii one red marble and one blue marble
iii two blue marbles?

LINK

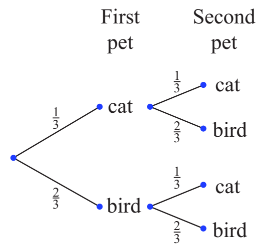
Probability has huge implications in health and medicine. The probability that tests for different diseases are accurate is very high, but it is seldom 100% and an incorrect test result can have really serious implications.

3 A bag contains 12 beads. Five are red and the rest are white. Two beads are drawn at random. The first bead is replaced before the second is drawn.

a Represent the possible outcomes on a tree diagram.
b Find the probability that:

i both beads are red
ii both beads are white.

4 Harold wants to buy two new pets; he will buy them a week apart. He prefers birds to cats, but only slightly, and decides that so long as he buys them as chicks and/or kittens, it doesn't matter what combination he gets. The tree diagram below represents what combination of two pets he might buy.

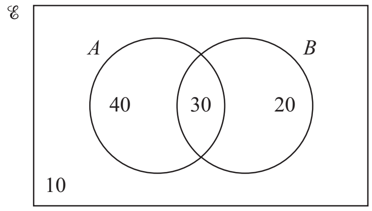


a How many possible combinations of pet could be buy?
b What is the probability that he buys a cat and a bird?
c What is the probability that he buys two cats?
d Based on the probabilities above, what combination is he most likely to buy?

24.3 Calculating probability from Venn diagrams

You used Venn diagrams to show the relationships between sets in Chapter 9. Now you are going to use Venn diagrams to solve probability problems.

This Venn diagram shows the results of a survey in which people were asked whether they watch programme A or programme B. Study the diagram and read the information to see how to determine probabilities from a Venn diagram.



$\mathscr{E}$ means the universal set. In probability this is the sample space, or possible outcomes. In this example the sample space is $40 + 30 + 20 + 10 = 100$ people.

n(A) means the number of elements in set A. P(A) means the probability that an element is in set A. You can write this as the number of elements in set A as a fraction of the sample space.

$n(A) = 40 + 30 = 70$

Remember the elements in the intersection are included in set A.

$P(A) = \frac{70}{100} = \frac{7}{10}$ or 0.7

$A \cap B$ is the intersection of sets A and B, the elements shared by A and B. The probability of two events happening can be written as P(A and B). This is the same as $P(A \cap B)$, the probability that an element is in both set A and set B. The word 'and' is a clue that the probability is found in the intersection of the sets.

$n(A \cap B) = 30$, so $P(A \cap B) = \frac{30}{100} = \frac{3}{10}$ or 0.3

$A \cup B$ is the union of sets A and B, the elements in both sets, with none repeated. The probability of either event A happening or event B happening can be written as P(A or B). This is the same as $P(A \cup B)$, the probability that an element is found either in set A or in set B. The word 'or' is a clue that the probability is found in the union of the sets.

$n(A \cup B) = 40 + 30 + 20 = 90$, so $P(A \cup B) = \frac{90}{100} = \frac{9}{10}$ or 0.9

$1 - P(A \cup B)$ is the same as $P(A \cup B)'$. The complement of $A \cup B$.

When a question contains words like 'is not' or 'neither' it is a clue that you are looking for the complement of a set. For example, 'What is the probability that a person watches neither of the two programmes?' In the Venn diagram this is everything outside of $A \cup B$. So, P(neither A nor B) is $1 - P(A \cup B)$.

Worked example 4

In a survey, 25 people were asked to say if they liked fruit and if they liked vegetables.

15 people said they liked vegetables and 18 said they liked fruit.

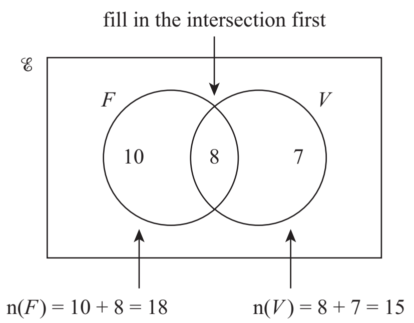
Assuming that everyone surveyed liked fruit or vegetables or both, draw a Venn diagram and use it to work out the probability that a person chosen at random from this group will like both fruit and vegetables.

Start by defining the sets and writing the information in set language. Use letters to make it quicker and easier to refer to the sets.

$\mathscr{E}$ = {number of people surveyed}, $n(\mathscr{E}) = 25$
F = {people who like fruit}, so $n(F) = 18$
V = {people who like vegetables}, so $n(V) = 15$

$n(F) + n(V) = 18 + 15 = 33$
But the total number of people surveyed was only 25.
$33 - 25 = 8$, so 8 people must have said they liked both fruit and vegetables.
$n(F \cap V) = 8$
Once you've defined the sets, you can draw the diagram.

You don't know the names of the people surveyed, so you have to work with the number of people in each set.



Once you've drawn the diagram, calculate the probability.

In words:

$$\text{P(Person likes both)} = \frac{\text{number of people who like both}}{\text{number of people surveyed}}$$

$$= \frac{8}{25} = 0.32$$

In set language:

$$\text{P(F and V)} = \frac{n(F \cap V)}{n(\mathscr{E})}$$

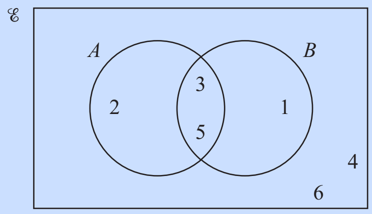
$$= \frac{8}{25} = 0.32$$

Worked example 5

The Venn diagram shows the possible outcomes when a six-sided dice is rolled.

Set A = {prime numbers} and Set B = {Odd numbers}.

Use the diagram to find the probability that a number is either odd or prime.



The events are not mutually exclusive so you need to consider the intersection of the two sets and subtract it so that you don't repeat any events. So:

P(A or B) = P(A) + P(B) − P(A and B) $\text{P(A)} = \frac{3}{6}$	Elements in A as fraction of total number of elements
$\text{P(B)} = \frac{3}{6}$	Elements in B as fraction of total number of elements
$\text{P(A and B)} = \frac{2}{6}$	Elements in the intersection of A and B (found in both)

$\text{P(A or B)} = \frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$

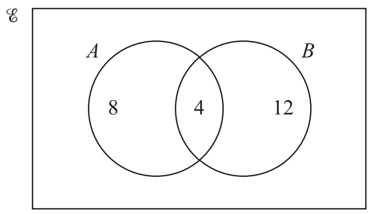
This is the same as $\text{P}(A \cup B)$ which can be calculated as $\frac{1}{6} + \frac{2}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$

REWIND

In chapter 9 you saw that numbers in Venn diagrams can represent either the *elements* in a set or the *number of elements* in a set.

Exercise 24.3

1 Use the Venn diagram to determine the following probabilities.



- **a** P(A)
- **b** P(B)
- **c** P(A and B)
- **d** P(not B)
- **e** P(A or B)

2 Naresh owns 20 T-shirts. Six are long-sleeved and four are black. Only one of the long-sleeved T-shirts is black. Draw a Venn diagram and use it to find the following probabilities:

- **a** P(T-shirt is not black)
- **b** P(T-shirt is not black and long-sleeved)
- **c** P(T-shirt is neither black nor long-sleeved).

3 Twenty students walked into a classroom. Of these students, 13 were wearing headphones and 15 were typing messages on their phones. Four students were not wearing headphones nor typing messages on their phones.

- **a** Draw a Venn diagram to show this information.
- **b** What is the probability that a student was both wearing headphones and typing on their phone when they walked into class?

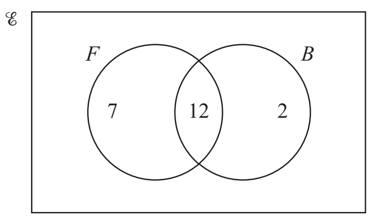
4 In a class of 28 students, 12 take physics, 15 take chemistry and 8 take neither physics nor chemistry.

- **a** Draw a Venn diagram to show the information.
- **b** What is the probability that a student chosen at random from this class:
  - **i** takes physics but not chemistry
  - **ii** takes physics or chemistry
  - **iii** takes physics and chemistry?

5 In a group of 130 students, 56 play the piano and 64 play the violin. 27 of the students play both instruments.

- **a** Draw a Venn diagram to show this information.
- **b** Use your Venn diagram to find the probability that a student chosen at random from this group:
  - **i** plays the violin
  - **ii** plays either the piano or the violin
  - **iii** plays both instruments
  - **iv** plays neither instrument

6 A survey into which cat food is enjoyed by cats is undertaken.
24 cats are tested to see whether they like Fluffy or Bouncer.
Some of the results are shown in the Venn diagram.



$\mathscr{E}$ = {all cats tested}
F = {cats who like Fluffy}
B = {cats who like Bouncer}

- **a** How many cats like both Fluffy and Bouncer?
- **b** How many cats do not like either food?
- **c** Write down the value of $n(F \cup B)$.
- **d** Write down the value of $n(F \cap B)$.
- **e** A tested cat is selected at random. What is the probability that this cat likes Bouncer?
- **f** A cat that likes Fluffy is chosen at random. What is the probability that this cat likes Bouncer?

24.4 Conditional probability

E

When you are using a tree diagram always check whether the probability of an event changes because of the outcome of a previous event. Questions involving conditional probability often contain the instructions 'without replacement' or 'one after the other'.

Conditional probability is used to work out the probability of one event happening when we already know that another has happened.

The information about the first event changes the sample space and affects the calculation.

For two events A and B, P(B given that A has happened) refers to the conditional probability of B happening given that A has already happened.

The way that you work out the conditional probability depends on whether the events are independent or not.

For example, two normal six-sided dice are rolled.

The first dice has already landed on a six. What is the probability of the second dice also being a six?

These two events are independent. The score on the second dice is not affected by the outcome of the first.

If two events A and B are independent, you will find that it is always true that

P(B given that A has happened) = P(B), so in this case, the probability of getting a six with the second dice is $\frac{1}{6}$.

For dependent events, the outcome of the first event affects the probability of the second.

For example, suppose you have an apple, an orange and a banana and you plan to eat only two of the fruits. Once you've eaten the first fruit the options for your second fruit are dependent on what fruit you ate because now you only have two fruits left to choose from. If you eat the apple first you can only choose between the orange and the banana for your second fruit.

The probability of choosing the apple is $\frac{1}{3}$ as there are three fruits to choose from. The probability of choosing the orange or banana given that you've eaten the apple is $\frac{1}{2}$ because there are only two fruits left to choose from.

To find the probability of B given that A has happened, use the rule

$$\text{P(B given that A has happened)} = \frac{\text{P(A and B)}}{\text{P(A)}}.$$

When you deal with Venn diagrams, this rule can be written in set language as

$$\text{P(B given that A has happened)} = \frac{\text{P}(\text{A} \cap \text{B})}{\text{P(A)}}$$

You can use tree diagram and Venn diagrams to help you solve problems involving conditional probability.

E

Worked example 6

There are 21 students in a class, 12 are boys and 9 are girls. The teacher chooses two different students at random to answer questions.

a Draw a tree diagram to represent the situation.

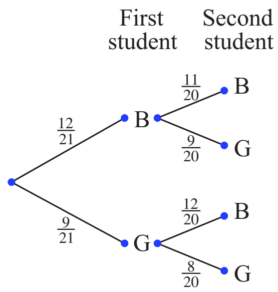
b Find the probability that:

- **i** both students are boys (BB)
- **ii** both students are girls (GG)
- **iii** one student is a girl and the other is a boy.

c The teacher chooses a third student at random. What is the probability that:

- **i** all three students are boys
- **ii** at least one of the students is a girl?

a



Notice that the second set of branches are conditional on the outcomes of the first set. This is an example of conditional probability. The teacher cannot choose the same student twice, so for the second set of branches there are only 20 students to choose from. Notice that if the first student is a boy, there are only 11 boys left for the second student but still 9 girls. If the first student is a girl then there are only 8 girls that could be chosen as the second student, but still 12 boys. In each case the numerator of one branch has changed but the numerators still add up to the value of the denominator (as this has also changed).

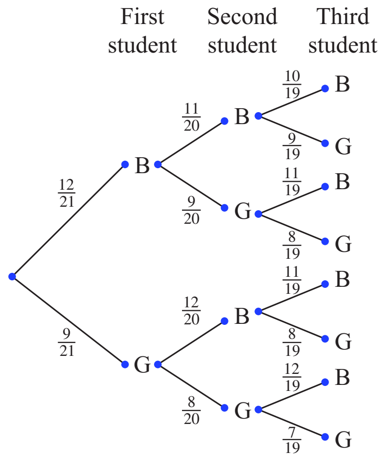
b

i $P(BB) = \frac{12}{21} \times \frac{11}{20} = \frac{11}{35}$

ii $P(GG) = \frac{9}{21} \times \frac{8}{20} = \frac{6}{35}$

iii $P(BG) + P(GB) = \frac{12}{21} \times \frac{9}{20} + \frac{9}{21} \times \frac{12}{20} = \frac{18}{35}$ The boy and girl can be chosen in either order.

c



You may find it helpful to add a third set of branches to the diagram but, if you can see the pattern of the probabilities on the branches, you can just show the arithmetic as follows:

i $P(BBB) = \frac{12}{21} \times \frac{11}{20} \times \frac{10}{19} = \frac{22}{133}$

ii $P(\text{at least one G}) = 1 - P(\text{all boys})$

$= 1 - \frac{22}{133} = \frac{111}{133}$

(Sometimes it is faster to work out the probabilities that you don't want and subtract the result from 1.)

E

Worked example 7

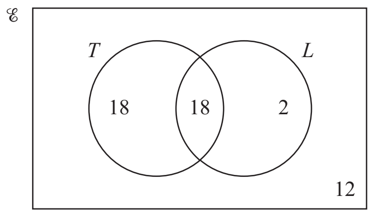
In a group of 50 students, 36 students work on tablet computers, 20 work on laptops and 12 work on neither of these.

A student is chosen at random. What is the probability that this student

a works on a tablet and a laptop computer.
b works on at least one type of computer.
c works on a tablet given that he or she works on a laptop.
d doesn't work on a laptop, given that he or she works on a tablet.

Start by identifying the sets and drawing a Venn diagram.

T = {students who work on tablets}
n(T) = 36
L = {students who work on laptops}
n(L) = 20
50 − 12 = 38, so there are 38 students in T and L combined.
36 + 20 = 56, but there are only 38 students in T and L combined
58 − 38 = 18, so 18 students work on both (T ∩ L)



a $\text{P(works on both)} = \text{P}(T \cap L) = \frac{18}{50} = \frac{9}{25}$

b $\text{P(works on at least one)} = 1 - \text{P(works on neither)} = 1 - \frac{12}{50} = \frac{38}{50} = \frac{19}{25}$

c $\text{P(T given that L has happened)} = \frac{\text{P(L and T)}}{\text{P(L)}} = \frac{n(L \cap T)}{n(L)}$

$= \frac{18}{20} = \frac{9}{10}$

d $\text{P(Not L given T has happened)} = \frac{\text{P(L}' \text{ and T)}}{\text{P(T)}} = \frac{18}{36} = \frac{1}{2}$

Tip

P(T given that L has happened) is dependent on students already using a laptop, so the probability is calculated using the total number of students who use a laptop not the total number of students. n(T and L) is the number of students in the intersection of the two sets.

Exercise 24.4

A set of 52 playing cards contains 13 each of hearts, diamonds, clubs and spades. Diamonds and hearts are red. Clubs and spades are black. There are no jokers in a 52 card pack.

1 A card is randomly selected from a pack of 52 playing cards, and its suit is recorded. The card is not replaced. Then a second card is chosen.

a Draw a tree diagram to represent this situation.
b Use the tree diagram to find the probability that:

i both cards are hearts
ii both cards are clubs
iii the first card is red and the second card is black.

2 Mohammed has four scrabble tiles with the letters A, B, C and D on them. He draws a letter at random and places it on the table, then he draws a second letter and a third, placing them down next to the previously drawn letter.

a Draw a tree diagram to show the possible outcomes.
b What is the probability that the letters he has drawn spell the words:

i CAD ii BAD iii DAD?

c What is the probability that he won't draw the letter B?
d What is Mohammed's chance of drawing the letters in alphabetical order?

3 In a group of 25 people, 15 like coffee (C), 17 like tea (T) and 2 people like neither.Using an appropriate sample space diagram, calculate the probability that a person will:

a like coffee
b like coffee given that he or she likes tea

4 100 teenagers went on a computer camp. 80 of them learned coding and 42 learned animation techniques. Each student did at least one of these activities.

a Draw a Venn diagram to show how many teenagers did both activities.
b A teenager is randomly selected. Find the probability that he or she:

i learned coding but not animation techniques.
ii learned animation techniques given that he or she learned coding.

Applying your skills

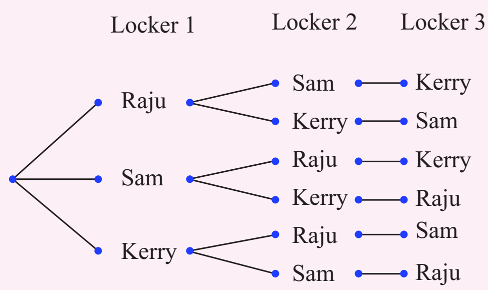
5 Clarissa is having a baby. She knows that the baby will be a girl and she wants her to have a first and a second name. The names she is considering are Olga, Shirley, Karen and Anne.

a Draw a tree diagram to show all possible combinations of names for the baby.
b If Clarissa chooses two names at random, what are the chances that the baby will be called Karen Anne?
c What is the chance of the baby being named Anne Shirley?

6 Sindi, Lee, Marita, Roger, Bongile and Simone are the six members of a school committee. The committee needs to choose a chairperson and a treasurer. One person cannot fill both positions.

a Draw a tree diagram to show how many ways there are of choosing a chairperson and a treasurer.
b If the chairperson and treasurer are chosen at random, what is the probability of choosing Sindi as chairperson and Lee as treasurer?

7 A cleaner accidentally knocked the name labels off three students' lockers. The name labels are Raju, Sam and Kerry. The tree diagram shows the possible ways of replacing the name labels.



a Copy the diagram and write the probabilities next to each branch.
b Are these events conditional or independent? Why?
c How many correct ways are there to match the name labels to the lockers?
d How many possible ways are there for the cleaner to label the lockers?
e If the cleaner randomly stuck the names back onto the lockers, what are his chances of getting the names correct?

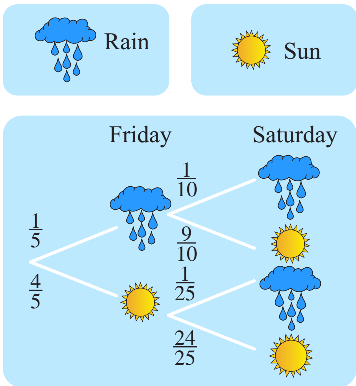
8 In a group of 120 students, 25 are in the sixth form and 15 attend maths tutorials.

Four of the students are sixth formers who attend maths tutorials.

What is the probability that a randomly chosen student who attends maths tutorials will be in the sixth form?

9 A climatologist reports that the probability of rain on Friday is 0.21. If it rains on Friday, there is a 0.83 chance of rain on Saturday, if it doesn't rain on Friday, the chance of rain on Saturday is only 0.3.

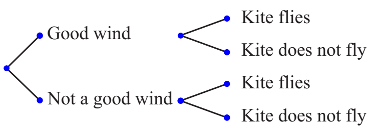
- **a** Draw a tree diagram to represent this situation.
- **b** Use your diagram to work out the probability of rain on:
  - **i** Friday and Saturday
  - **ii** Saturday.



10 Look at this tree diagram sketched by a weather forecaster.

- **a** Give the tree diagram a title.
- **b** What does it tell you about the weather for the next two days in this place?

  (Make sure you include probabilities as part of your answer).



11 Mahmoud enjoys flying his kite. On any given day, the probability that there is a good wind is $\frac{3}{4}$. If there is a good wind, the probability that the kite will fly is $\frac{5}{8}$. If there is not a good wind, the probability that the kite will fly is $\frac{1}{16}$.

- **a** Copy this tree diagram. Write the probabilities next to each branch.
- **b** What is the probability of good wind and the kite flying?
- **c** Find the probability that, whatever the wind, the kite does not fly.
- **d** If the kite flies, the probability that it gets stuck in a tree is $\frac{1}{2}$. Calculate the probability that, whatever the wind, the kite gets stuck in a tree.

Summary

Do you know the following?

- The sample space of an event is all the possible outcomes of the event.
- When an event has two or more stages it is called a combined event.
- Tree diagrams and Venn diagrams are useful for organising the outcomes of different stages in an event. They are particularly useful when there are more than two stages because a probability space diagram can only show outcomes for two events.
- The outcomes are written at the end of branches on a tree diagram. The probability of each outcome is written next to the branches as a fraction or decimal.
- For independent events you find the probability by multiplying the probabilities on each branch of the tree. P(A and then B) = P(A) × P(B).
- When events are mutually exclusive, you need to add the probabilities obtained by multiplication.
- The probability that an event happens, given that another event has already happened is called conditional probability.

Are you able to …?

- draw a tree diagram to organise the outcomes for simple combined events
- find the probability of each branch of a tree diagram
- calculate the probability of events using tree diagrams
- draw a Venn diagram to represent sets of information and use it to calculate probabilities
- use tree diagrams and Venn diagrams to determine conditional probability.

Examination practice

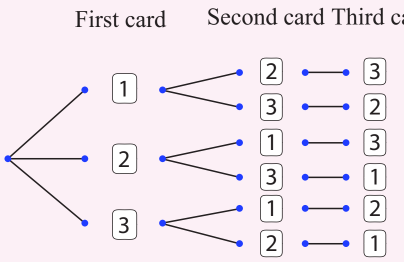
Exam-style questions

1 **a** Draw a tree diagram to show all possible outcomes when two unbiased dice are thrown at the same time.

b Find the probability, as a fraction in its lowest terms, that:

- **i** the two dice will show a total score of eight
- **ii** the two dice will show the same score as each other.

2 The tree diagram shows the possible outcomes when three number cards are placed in a container and then a card is drawn at random three times. Each time a card is drawn, it is placed on the table next to the previous card drawn.



a Copy the diagram and fill in the probabilities on each branch.

b How many three-digit numbers can be formed in this experiment?

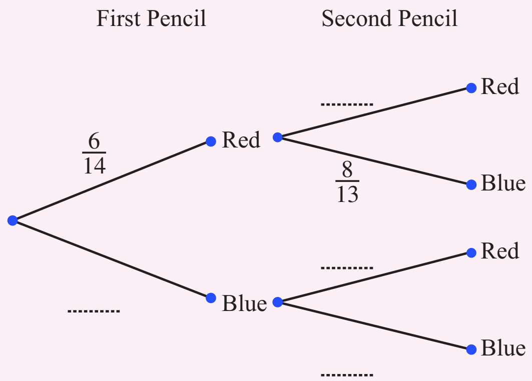
c What is the probability of the three-digit number being:

i 123 **ii** > 200 **iii** even **iv** divisible by three?

Past paper questions

1 A box contains 6 red pencils and 8 blue pencils.
A pencil is chosen at random and not replaced.
A second pencil is then chosen at random.

a Complete the tree diagram.



[2]

b Calculate the probability that

- **i** both pencils are red, [2]
- **ii** at least one of the pencils is red. [3]

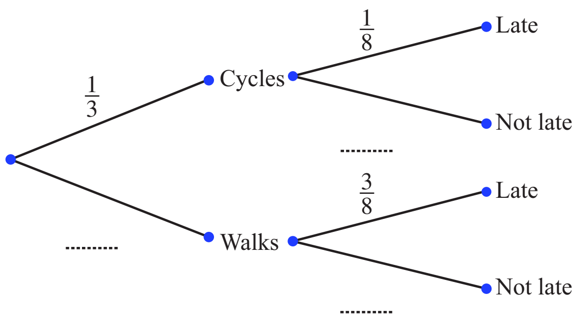
[Cambridge IGCSE Mathematics 0580 Paper 22 Q23 October/November 2015]

2 Dan either walks or cycles to school.
The probability that he cycles to school is $\frac{1}{3}$.

a Write down the probability that Dan walks to school. [1]

b When Dan cycles to school the probability that he is late is $\frac{1}{8}$

When Dan walks to school the probability that he is late is $\frac{3}{8}$
Complete the tree diagram.



[2]

c Calculate the probability that

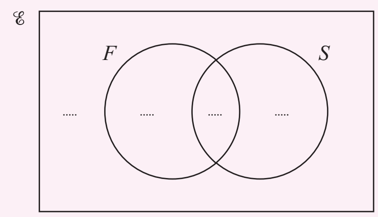
i Dan cycles to school and is late, [2]

ii Dan is not late. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q21 Feb/March 2016]

E

3 a $\mathscr{E}$ = {25 students in a class}
F = {students who study French}
S = {students who study Spanish}
16 students study French and 18 students study Spanish.
2 students study neither of these.



i Complete the Venn diagram to show this information. [2]

ii Find $n(F')$. [1]

iii Find $n(F \cap S)'$. [1]

iv One student is chosen at random.
Find the probability that this student studies both French and Spanish. [1]

v Two students are chosen at random without replacement.
Find the probability that they both study only Spanish. [2]

b In another class the students all study at least one language from French, German and Spanish.
No student studies all three languages.
The set of students who study German is a proper subset of the set of students who study French.
4 students study both French and German.
12 students study Spanish but not French.
9 students study French but not Spanish.
A total of 16 students study French.

i Draw a Venn diagram to represent this information. [4]

ii Find the total number of students in this class. [1]

[Cambridge IGCSE Mathematics 0580 Paper 42 Q9 October/November 2012]