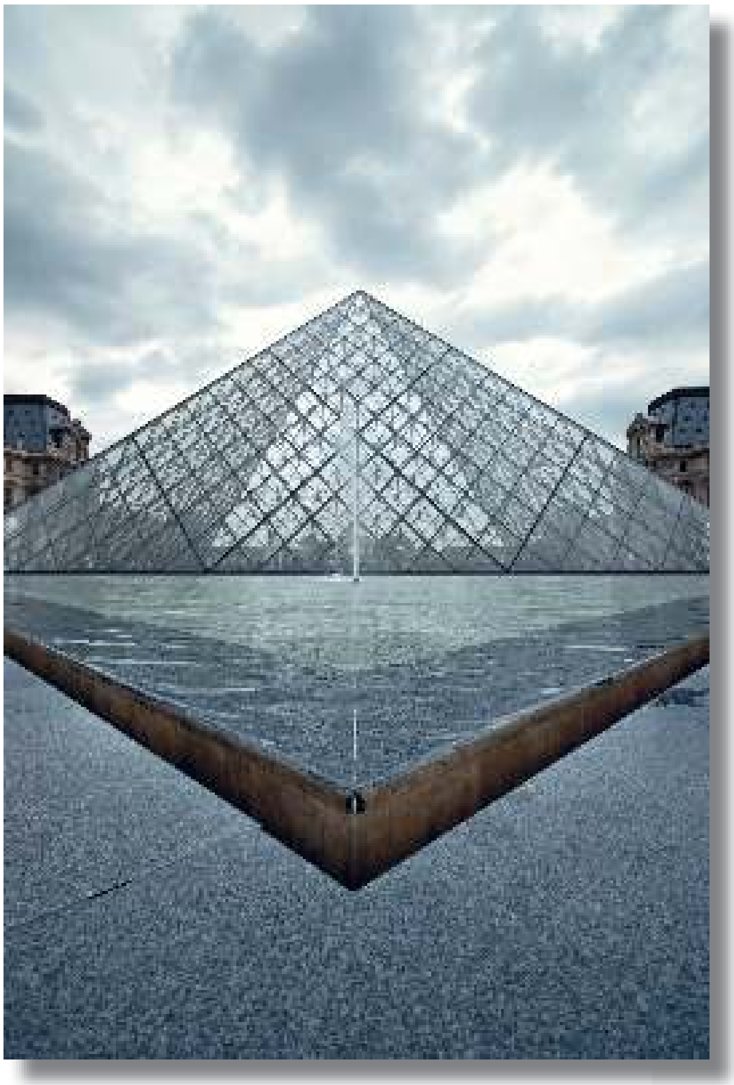
Chapter 7: Perimeter, area and volume

Key words

- Perimeter
- Area
- Irrational number
- Sector
- Arc
- Semi-circle
- Solid
- Net
- Vertices
- Face
- Surface area
- Volume
- Apex
- Slant height

In this chapter you will learn how to:

- calculate areas and perimeters of two-dimensional shapes
- calculate areas and perimeters of shapes that can be separated into two or more simpler polygons
- calculate areas and circumferences of circles
- calculate perimeters and areas of circular sectors
- understand nets for three-dimensional solids
- calculate volumes and surface areas of solids
- calculate volumes and surface area of pyramids, cones and spheres.



The glass pyramid at the entrance to the Louvre Art Gallery in Paris. Reaching to a height of 20.6 m, it is a beautiful example of a three-dimensional object. A smaller pyramid – suspended upside down – acts as a skylight in an underground mall in front of the museum.

When runners begin a race around a track they do not start in the same place because their routes are not the same length. Being able to calculate the perimeters of the various lanes allows the officials to stagger the start so that each runner covers the same distance.

A can of paint will state how much area it should cover, so being able to calculate the areas of walls and doors is very useful to make sure you buy the correct size can.

How much water do you use when you take a bath instead of a shower? As more households are metered for their water, being able to work out the volume used will help to control the budget.

RECAP

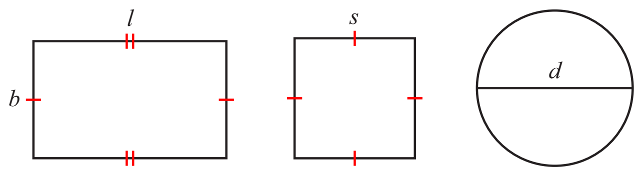
You should already be familiar with the following perimeter, area and volume work:

Perimeter

Perimeter is the measured or calculated length of the boundary of a shape.

The perimeter of a circle is its circumference.

You can add the lengths of sides or use a formula to calculate perimeter.



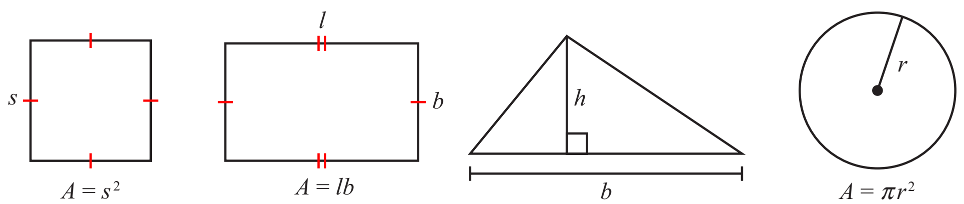
$P = 2(l + b)$ $P = 4s$ $C = \pi d$

Area

The area of a region is the amount of space it occupies. Area is measured in square units.

The surface area of a solid is the sum of the areas of its faces.

The area of basic shapes is calculated using a formula.



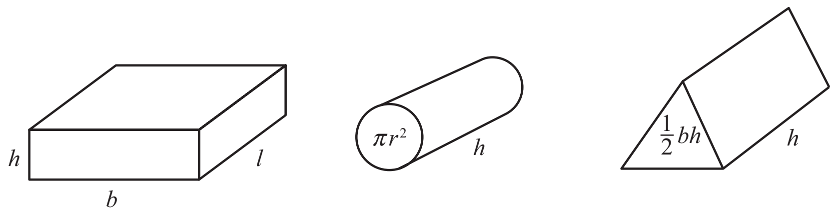
$A = s^2$ $A = lb$ $A = \frac{1}{2}bh$ $A = \pi r^2$

Volume

The volume of a solid is the amount of space it occupies.

Volume is measured in cubic units.

The volume of cuboids and prisms can be calculated using a formula.



$V = l\,b\,h$ $V =$ Area of cross section × height

7.1 Perimeter and area in two dimensions

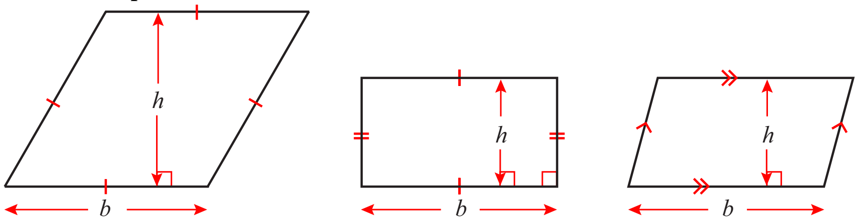
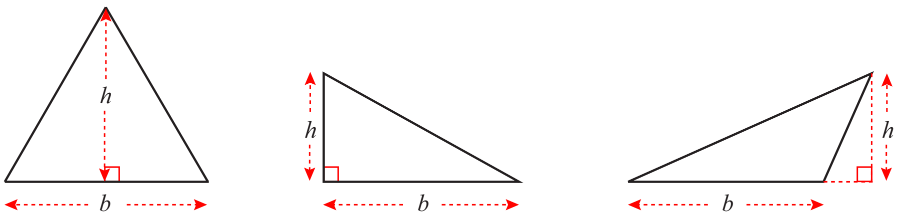
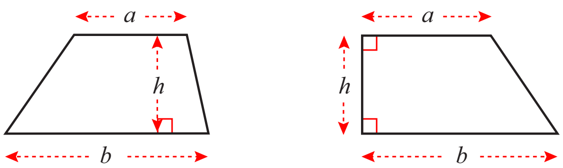
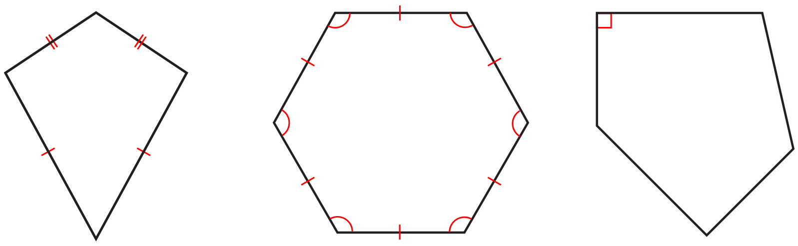
LINK

When geographers study coastlines it is sometimes very handy to know the length of the coastline. If studying an island, then the length of the coastline is the same as the perimeter of the island.

Polygons

A polygon is a flat (two-dimensional) shape with three or more straight **sides**. The **perimeter** of a polygon is the sum of the lengths of its sides. The perimeter measures the total distance around the outside of the polygon.

The **area** of a polygon measures how much space is contained inside it.

Two-dimensional shapes	**Formula for area**
Quadrilaterals with parallel sides  rhombus rectangle parallelogram	Area = bh
Triangles  	Area = $\frac{1}{2}bh$ or $\frac{bh}{2}$
Trapezium  	Area = $\frac{1}{2}(a+b)h$ or $\frac{(a+b)h}{2}$
Here are some examples of other two-dimensional shapes.  kite regular hexagon irregular pentagon	It is possible to find areas of other polygons such as those on the left by dividing the shape into other shapes such as triangles and quadrilaterals.

Tip

You should always give units for a final answer if it is appropriate to do so. It can, however, be confusing if you include units throughout your working.

The formula for the area of a triangle can be written in different ways:

$\frac{1}{2} \times b \times h = \frac{bh}{2}$

OR $= \left(\frac{1}{2}b\right) \times h$

OR $= b \times \left(\frac{1}{2}h\right)$

Choose the way that works best for you, but make sure you write it down as part of your method.

You do not usually have to redraw the separate shapes, but you might find it helpful.

REWIND

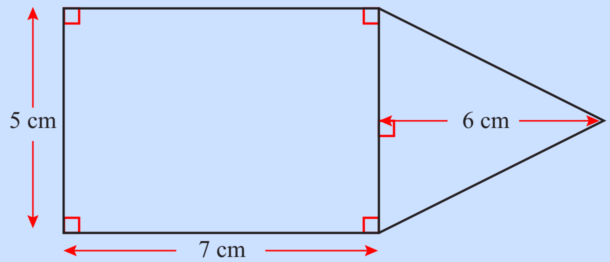
At this point you may need to remind yourself of the work you did on rearrangment of formulae in chapter 6. ◀

Units of area

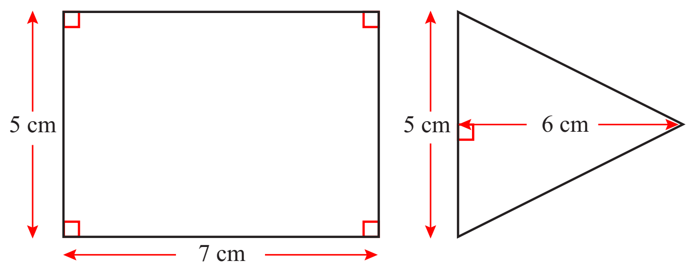
If the dimensions of your shape are given in cm, then the units of area are square centimetres and this is written cm². For metres, m² is used and for kilometres, km² is used and so on. Area is always given in square units.

Worked example 1

a Calculate the area of the shape shown in the diagram.



This shape can be divided into two simple polygons: a rectangle and a triangle. Work out the area of each shape and then add them together.



rectangle triangle

Area of rectangle $= bh = 7 \times 5 = 35\,\text{cm}^2$ (substitute values in place of *b* and *h*)

Area of triangle $= \frac{1}{2}bh = \frac{1}{2} \times 5 \times 6 = \frac{1}{2} \times 30 = 15\,\text{cm}^2$

Total area $= 35 + 15 = 50\,\text{cm}^2$

b The area of a triangle is 40 cm². If the base of the triangle is 5 cm, find the height.

$A = \frac{1}{2} \times b \times h$	Use the formula for the area of a triangle.
$40 = \frac{1}{2} \times 5 \times h$	Substitute all values that you know.
$\Rightarrow 40 \times 2 = 5 \times h$	
$\Rightarrow h = \frac{40 \times 2}{5} = \frac{80}{5} = 16\,\text{cm}$	Rearrange the formula to make *h* the subject.

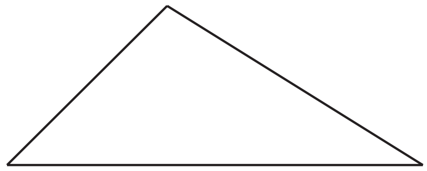
Exercise 7.1

LINK

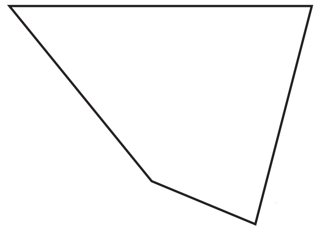
Agricultural science involves work with perimeter, area and rates. For example, fertiliser application rates are often given in kilograms per hectare (an area of 10 000 m^2). Applying too little or too much fertiliser can have serious implications for crops and food production.

1 By measuring the lengths of each side and adding them together, find the perimeter of each of the following shapes.

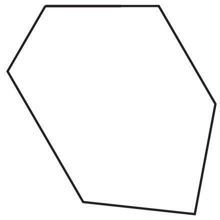
a



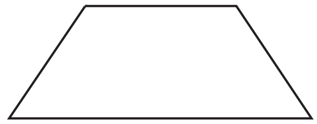
b



c

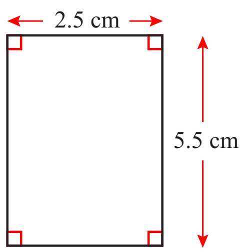


d



2 Calculate the perimeter of each of the following shapes.

a



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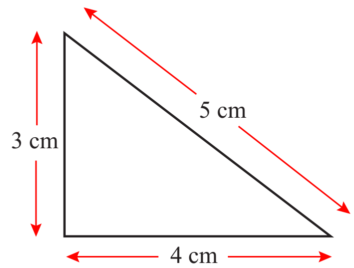

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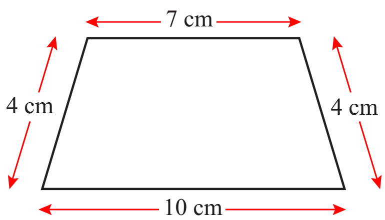

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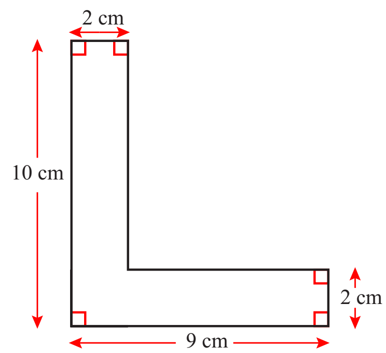

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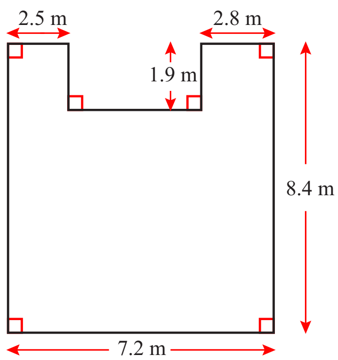

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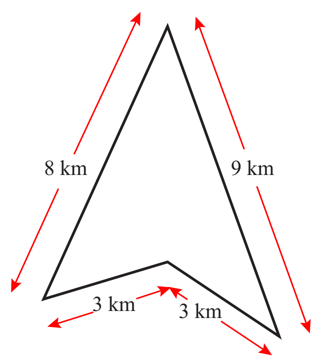
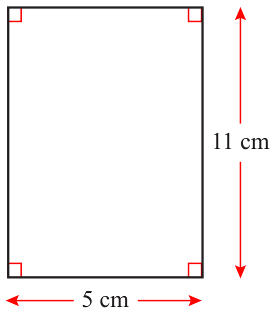


3 Calculate the area of each of the following shapes.

a



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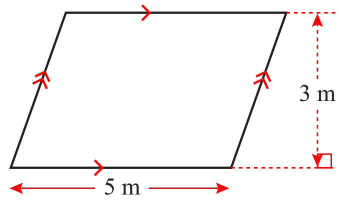

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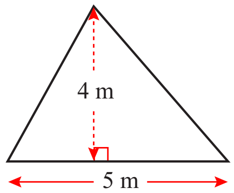

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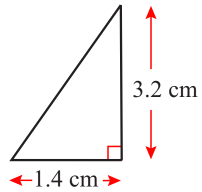

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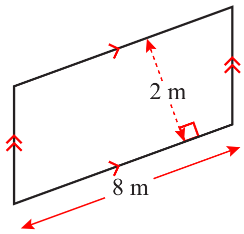

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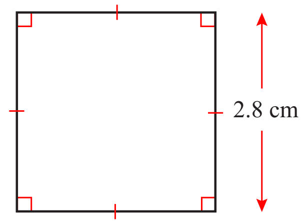

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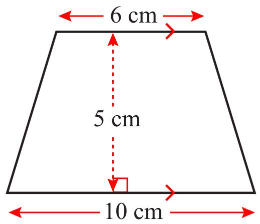
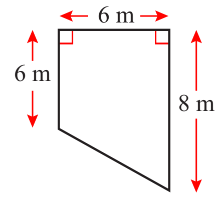
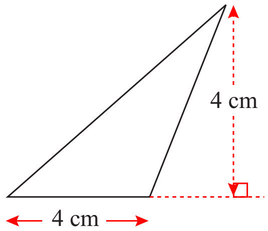


h



i



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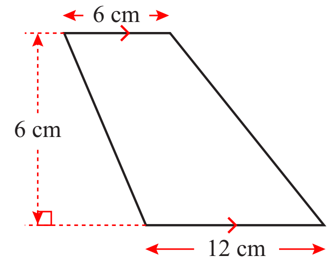
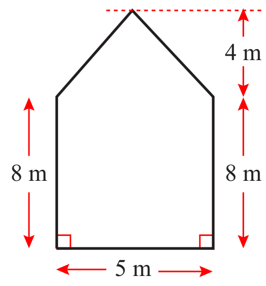


Draw the simpler shapes separately and then calculate the individual areas, as in worked example 1.

4 The following shapes can all be divided into simpler shapes. In each case find the total area.

a



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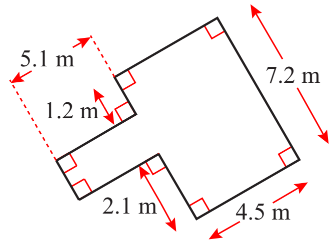

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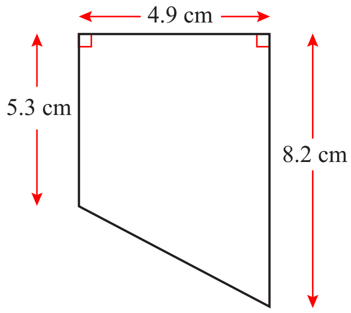

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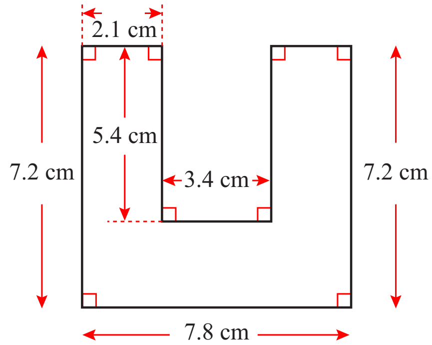

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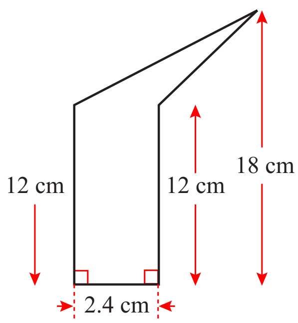

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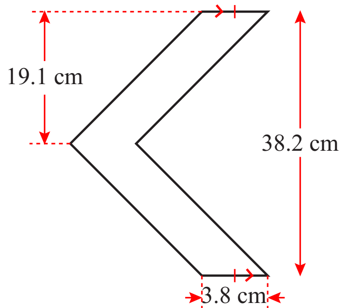

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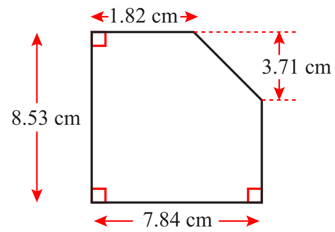
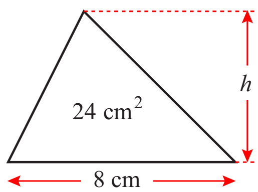


Write down the formula for the area in each case. Substitute into the formula the values that you already know and then rearrange it to find the unknown quantity.

5 For each of the following shapes you are given the area and one other measurement. Find the unknown length in each case.

a



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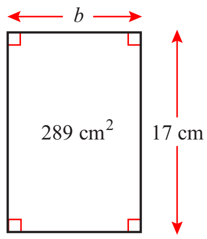

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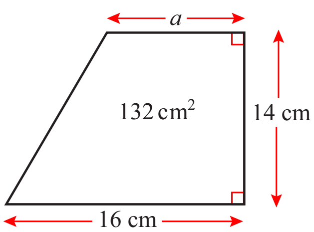

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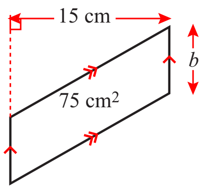

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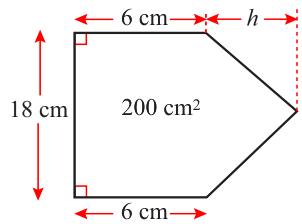
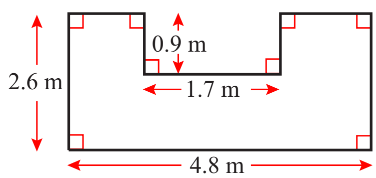


6 How many 20 cm by 30 cm rectangular tiles would you need to tile the outdoor area shown below?



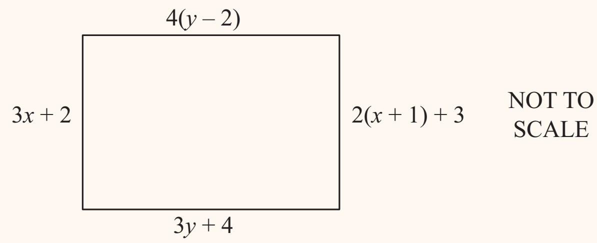
7 Sanjay has a square mirror measuring 10 cm by 10 cm. Silvie has a square mirror which covers twice the area of Sanjay's mirror. Determine the dimensions of Silvie's mirror correct to 2 decimal places.

8 For each of the following, draw rough sketches and give the dimensions:

a two rectangles with the same perimeter but different areas
b two rectangles with the same area but different perimeters
c two parallelograms with the same perimeter but different areas
d two parallelograms with the same area but different perimeters.

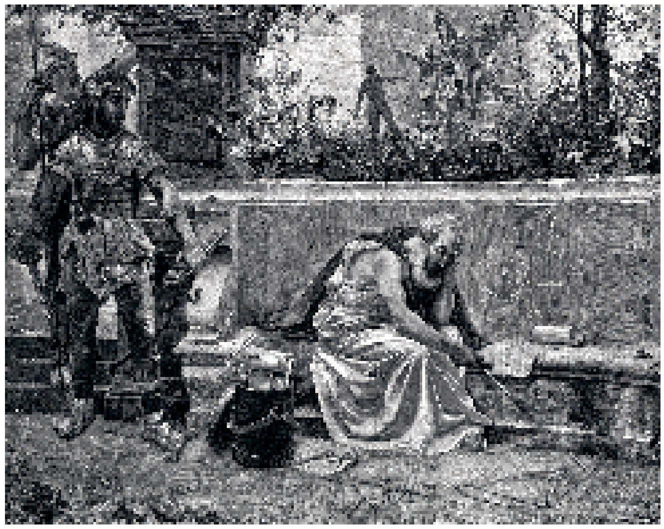
You will need to use some of the algebra from chapter 6.

9 E



Find the area and perimeter of the rectangle shown in the diagram above.

Circles

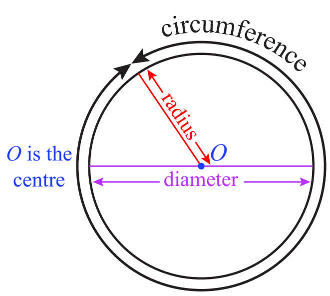


Archimedes worked out the formula for the area of a circle by inscribing and circumscribing polygons with increasing numbers of sides.

'Inscribing' here means to draw a circle inside a polygon so that it just touches every edge. 'Circumscribing' means to draw a circle outside a polygon that touches every vertex.

REWIND

You learned the names of the parts of a circle in chapter 3. The diagram below is a reminder of some of the parts. The diameter is the line that passes through a circle and splits it into two equal halves. ◀



The circle seems to appear everywhere in our everyday lives. Whether driving a car, running on a race track or playing basketball, this is one of a number of shapes that are absolutely essential to us.

Finding the circumference of a circle

Circumference is the word used to identify the perimeter of a circle. Note that the diameter = 2 × radius ($2r$). The Ancient Greeks knew that they could find the circumference of a circle by multiplying the diameter by a particular number. This number is now known as 'π' (which is the Greek letter 'p'), pronounced 'pi' (like apple *pie*). π is equal to 3.141592654. . .

The circumference of a circle can be found using a number of formulae that all mean the same thing:

$$\text{Circumference} = \pi \times \text{diameter}$$
$$= \pi d \qquad \text{(where } d = \text{diameter)}$$
$$= 2\pi r \qquad \text{(where } r = \text{radius)}$$

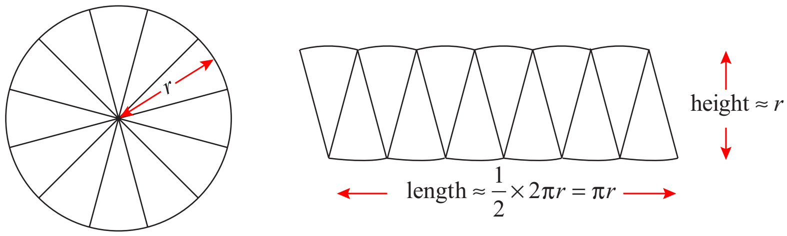
FAST FORWARD

π is an example of an **irrational number**. The properties of irrational numbers will be discussed later in chapter 9. ▶

Finding the area of a circle

There is a simple formula for calculating the area of a circle. Here is a method that shows how the formula can be worked out:

Consider the circle shown in the diagram below. It has been divided into 12 equal parts and these have been rearranged to give the diagram on the right.



Because the parts of the circle are narrow, the shape almost forms a rectangle with height equal to the radius of the circle and the length equal to half of the circumference.

Now, the formula for the area of a rectangle is Area = bh so,

$$\text{Area of a circle} \approx \frac{1}{2} \times 2\pi r \times r \qquad \text{(Using the values of } b \text{ and } h \text{ shown above)}$$
$$= \pi r^2 \qquad \text{(Simplify)}$$

If you try this yourself with a greater number of even narrower parts inside a circle, you will notice that the right-hand diagram will look even more like a rectangle.

This indicates (but does not prove) that the area of a circle is given by: $A = \pi r^2$.

You will now look at some examples so that you can see how to apply these formulae.

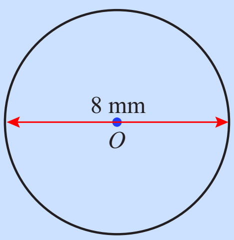
REWIND

BODMAS in chapter 1 tells you to calculate the square of the radius before multiplying by π.

Worked example 2

For each of the following circles calculate the circumference and the area. Give each answer to 3 significant figures.

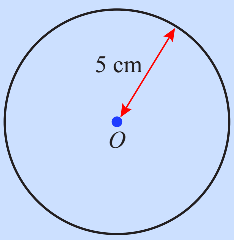
a



a Circumference = $\pi \times$ diameter
$= \pi \times 8$
$= 25.1327...$
≈ 25.1 mm

Area = $\pi \times r^2$
$r = \frac{d}{2}$
$= \pi \times 4^2$
$= \pi \times 16$
$= 50.265...$
$\approx 50.3\text{ mm}^2$

b



b Circumference = $\pi \times$ diameter
$= \pi \times 10 \quad (d = 2 \times r)$
$= 31.415...$
≈ 31.4 cm

Area = $\pi \times r^2$
$= \pi \times 5^2$
$= \pi \times 25$
$= 78.539...$
$\approx 78.5\text{ cm}^2$

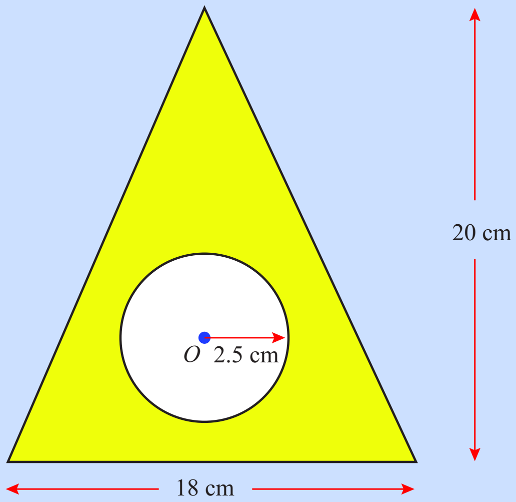
Note that in (a), the diameter is given and in (b) only the radius is given. Make sure that you look carefully at which measurement you are given.

Tip

Your calculator should have a $\boxed{\pi}$ button. If it does not, use the approximation 3.142, but make sure you write this in your working. Make sure you record the final calculator answer before rounding and then state what level of accuracy you rounded to.

Worked example 3

Calculate the area of the shaded region in the diagram.



Shaded area = area of triangle − area of circle.

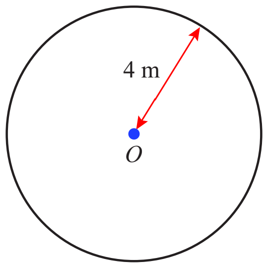
$$\text{Area} = \frac{1}{2}bh - \pi r^2$$
$$= \frac{1}{2} \times 18 \times 20 - \pi \times 2.5^2$$
$$= 160.365\ldots$$
$$\simeq 160\,\text{cm}^2$$

Substitute in values of *b*, *h* and *r*.
Round the answer. In this case it has been rounded to 3 significant figures.

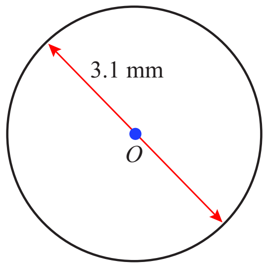
Exercise 7.2

1 Calculate the area and circumference in each of the following.

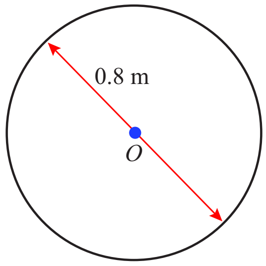
a



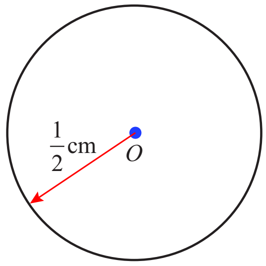
b



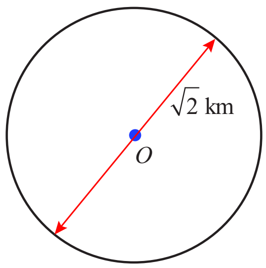
c



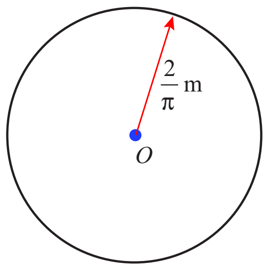
d



e



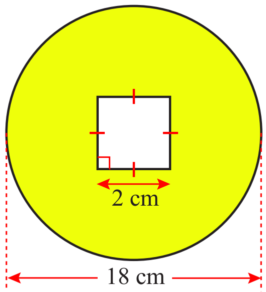
f



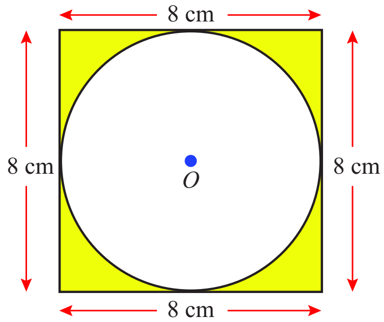
In some cases you may find it helpful to find a decimal value for the radius and diameter before going any further, though you can enter exact values easily on most modern calculators. If you know how to do so, then this is a good way to avoid the introduction of rounding errors.

2 Calculate the area of the shaded region in each case.

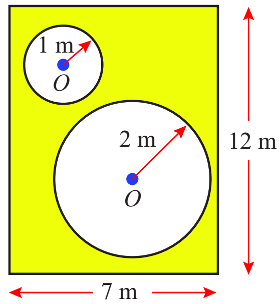
a



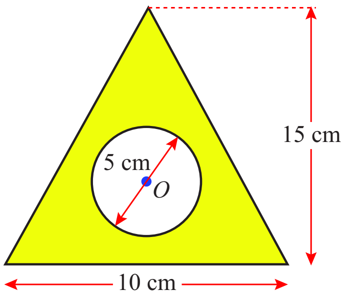
b



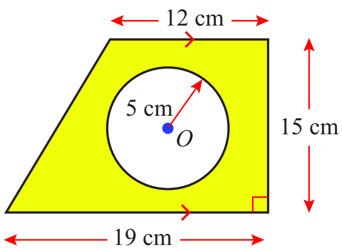
c



d



e



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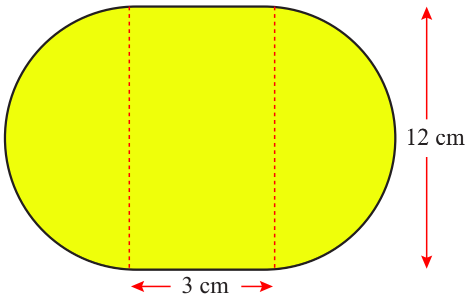
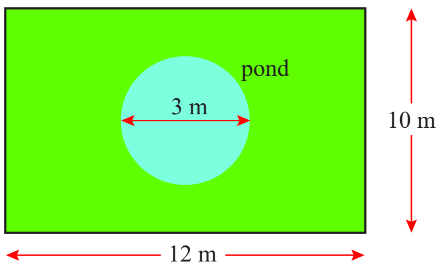


Applying your skills

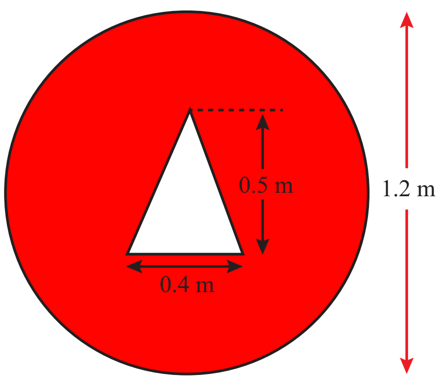
This is a good example of a problem in which you need to carry out a series of calculations to get to the answer. Set your work out in clear steps to show how you get to the solution.

3



The diagram shows a plan for a rectangular garden with a circular pond. The part of the garden not covered by the pond is to be covered by grass. One bag of grass seed covers five square metres of lawn. Calculate the number of bags of seed needed for the work to be done.

4



The diagram shows a road sign. If the triangle is to be painted white and the rest of the sign will be painted red, calculate the area covered by each colour to 1 decimal place.

5 Sixteen identical circles are to be cut from a square sheet of fabric whose sides are 0.4 m long. Find the area of the leftover fabric (to 2dp) if the circles are made as large as possible.

6 Anna and her friend usually order a large pizza to share. The large pizza has a diameter of 24 cm. This week they want to eat different things on their pizzas, so they decide to order two small pizzas. The small pizza has a diameter of 12 cm. They want to know if there is the same amount of pizza in two small pizzas as in one large. Work out the answer.

Exact answers as multiples of π

Pi is an irrational number so it has no exact decimal or fractional value. For this reason, calculations in which you give a rounded answer or work with an approximate value of pi are not exact answers.

If you are asked to give an exact answer in any calculation that uses pi it means you have to give the answer in terms of pi. In other words, your answer will be a multiple of pi and the π symbol should be in the answer.

If the circumference or area of a circle is given in terms of π, you can work out the length of the diameter or radius by dividing by pi.

For example, if $C = 5\pi$ cm the diameter is 5 cm and the radius is 2.5 cm (half the diameter).

Similarly, if $A = 25\pi\ \text{cm}^2$ then $r^2 = 25$ and $r = \sqrt{25} = 5$ cm.

Worked example 4

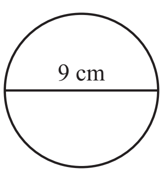
For each calculation, give your answer as a multiple of π.

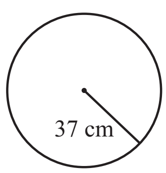
a Find the circumference of a circle with a diameter of 12 cm.
b What is the exact circumference of a circle of radius 4 mm?
c Determine the area of a circle with a diameter of 10 m.
d What is the radius of a circle of circumference 2.8π cm?

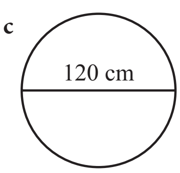
a	$C = \pi d$ $C = 12\pi$ cm	Multiply the diameter by 12 and remember to write the units.
b	$C = \pi d$ $C = 2 \times 4 \times \pi = 8\pi$ mm	Remember the diameter is $2 \times$ the radius.
c	$A = \pi r^2$ $r = 5$ m, so $A = \pi \times 5^2$ $A = 25\pi$ m	
d	$C = \pi d$ So, $d = \frac{C}{\pi}$ $\frac{2.8\pi}{\pi} = 2.8$ cm $r = 1.4$ cm	Divide the diameter by 2 to find the radius.

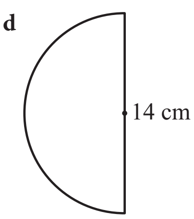
Exercise 7.3

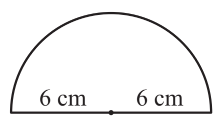
1 Find the circumference and area of each shape. Give each answer as a multiple of π.

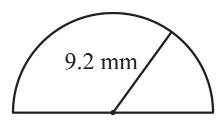
a


b


c


d


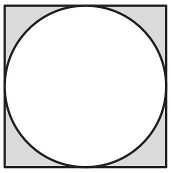
e


f


2 For each of the following, give the answer as a multiple of π.

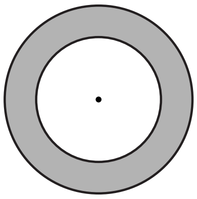
a Calculate the circumference of a circle of diameter 10 cm.
b A circle has a radius of 7 mm. What is its circumference?
c What is the area of a circle of diameter 1.9 cm?
d The radius of a semi-circle is 3 cm. What is the area of the semi-circle?

3 A circle of circumference 12π cm is precision cut from a metal square as shown.

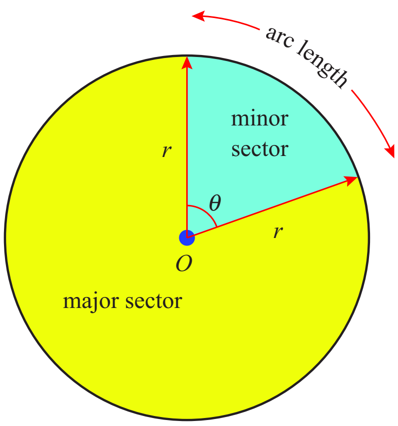


a What is the length of each side of the square?
b What area of metal is left once the circle has been cut from it? Give your answer in terms of π.

4 The diagram shows two concentric circles. The inner circle has a circumference of 14π mm. The outer circle has a radius of 9 mm. Determine the exact area of the shaded portion.



Arcs and sectors



The diagram shows a circle with two radii (plural of radius) drawn from the centre.

The region contained in-between the two radii is known as a **sector**. Notice that there is a *major* sector and a *minor* sector.

A section of the circumference is known as an **arc**.

The Greek letter θ represents the angle *subtended* at the centre.

Notice that the minor sector is a fraction of the full circle. It is $\frac{\theta}{360}$ of the circle.

Area of a circle is πr^2. The sector is $\frac{\theta}{360}$ of a circle, so replace 'of' with '×' to give:

Sector area $= \frac{\theta}{360} \times \pi r^2$

Circumference of a circle is $2\pi r$. If the sector is $\frac{\theta}{360}$ of a circle, then the length of the arc of a sector is $\frac{\theta}{360}$ of the circumference. So;

Arc length $= \frac{\theta}{360} \times 2\pi r$

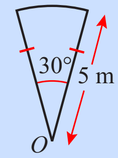
Make sure that you remember the following two special cases:

- If $\theta = 90°$ then you have a quarter of a circle. This is known as a *quadrant*.
- If $\theta = 180°$ then you have a half of a circle. This is known as a **semi-circle**.

Worked example 5

Find the area and perimeter of shapes **a** and **b**, and the area of shape **c**.
Give your answer to 3 significant figures.

a

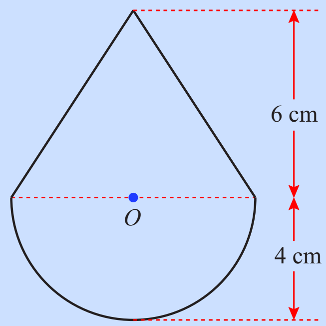


$$\textbf{Area} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{30}{360} \times \pi \times 5^2$$
$$= 6.544\ldots$$
$$\approx 6.54\,\text{m}^2$$

$$\textbf{Perimeter} = \frac{\theta}{360} \times 2\pi r + 2r$$
$$= \frac{30}{360} \times 2 \times \pi \times 5 + 2 \times 5$$
$$= 12.617\ldots$$
$$\approx 12.6\,\text{m}$$

Note that for the perimeter you need to add 5 m twice. This happens because you need to include the two straight edges.

b



Total area = area of triangle + area of a semi-circle.

$$\text{Area} = \frac{1}{2}bh + \frac{1}{2}\pi r^2 \quad \text{(Semi-circle is half of a circle so divide circle area by 2).}$$
$$= \frac{1}{2} \times 8 \times 6 + \frac{1}{2}\pi \times 4^2$$
$$= 49.132\ldots$$
$$\approx 49.1\,\text{cm}^2$$

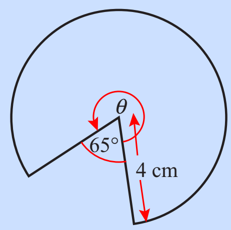
You should have spotted that you do not have enough information to calculate the perimeter of the top part of the shape using the rules you have learned so far.

FAST FORWARD

You will be able to find the perimeter of this third shape after completing the work on Pythagoras' theorem in chapter 11.

Note that the base of the triangle is the diameter of the circle.

c



$$\textbf{Area} = \frac{\theta}{360} \times \pi r^2$$
$$= \frac{360 - 65}{360} \times \pi \times 4^2$$
$$= \frac{295}{360} \times \pi \times 16$$
$$= 41.189\ldots$$
$$\approx 41.2\,\text{cm}^2$$

$$\text{Perimeter} = \frac{\theta}{360} \times 2\pi r + 2r$$
$$= \frac{295}{360} \times 2 \times \pi \times 4 + 2 \times 4$$
$$= 28.594\ldots$$
$$\approx 28.6\,\text{cm}$$

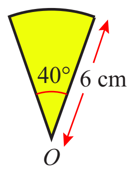
Note that the size of θ has not been given. You need to calculate it ($\theta = 360 - 65$).

E

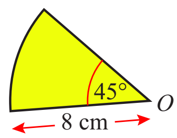
Exercise 7.4

1 For each of the following shapes find the area *and* perimeter.

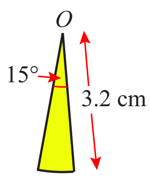
a



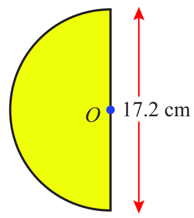
b



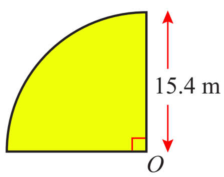
c



d



e



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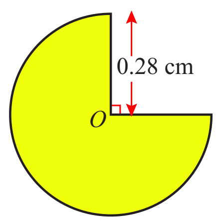
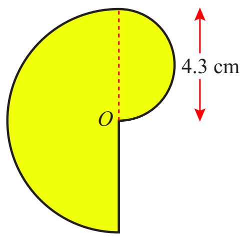
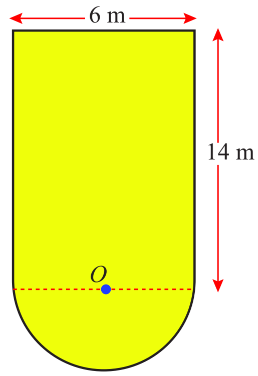


To find the perimeter you need the arc length, so calculate that separately.

g

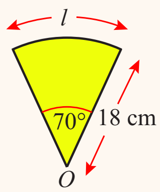


h

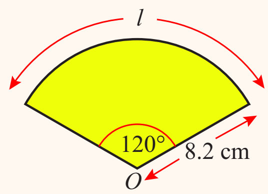


2 Find the area of the coloured region and find the arc length l in each of the following.

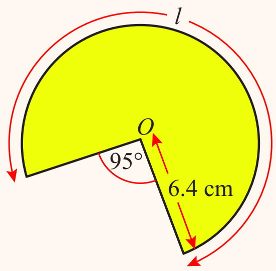
a



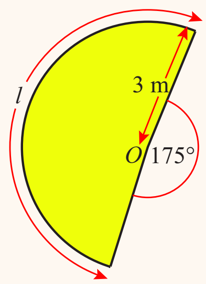
b



c

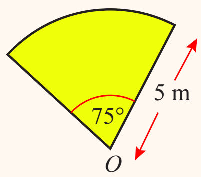


d

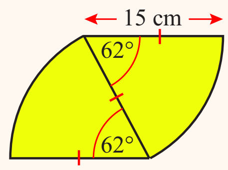


Find the area and perimeter of the following:

e



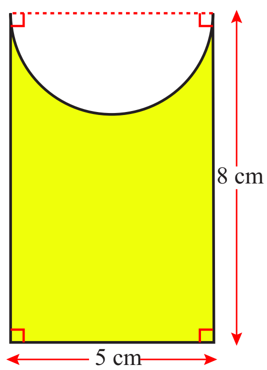
f



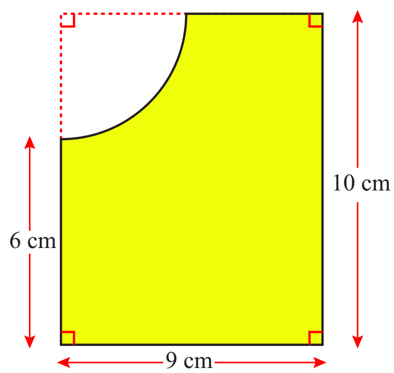
Q2, part b is suitable for Core learners.

3 For each of the following find the area and perimeter of the coloured region.

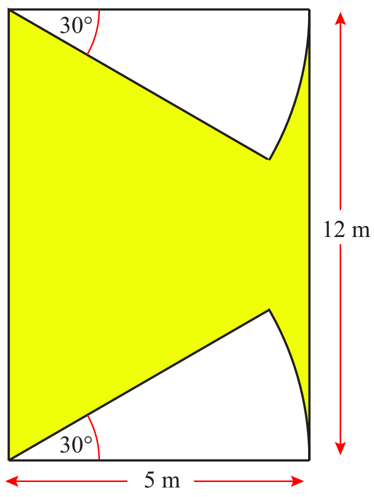
a



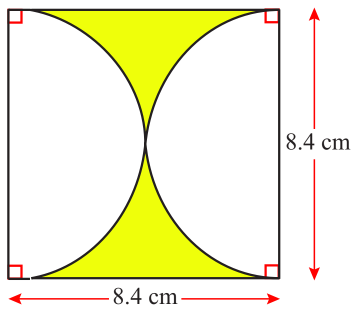
b



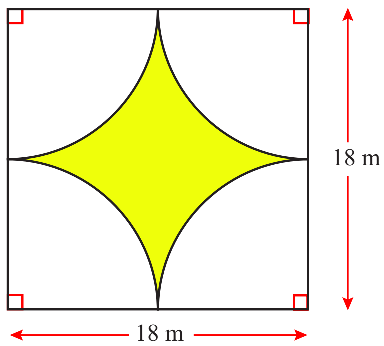
c



d

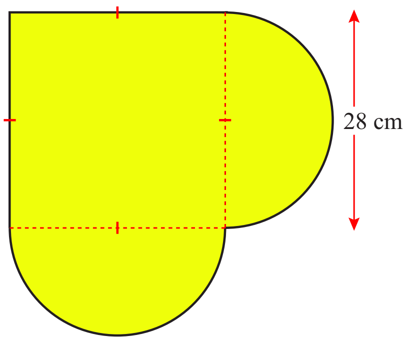


e

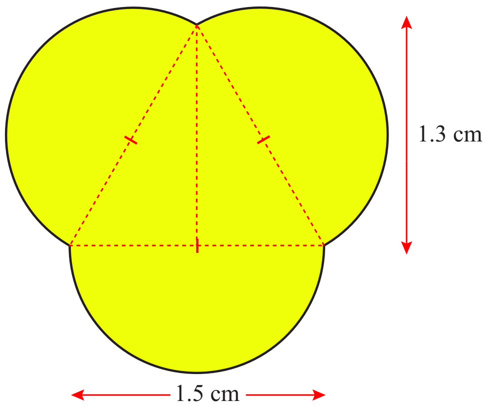


4 Each of the following shapes can be split into simpler shapes.
In each case find the perimeter and area.

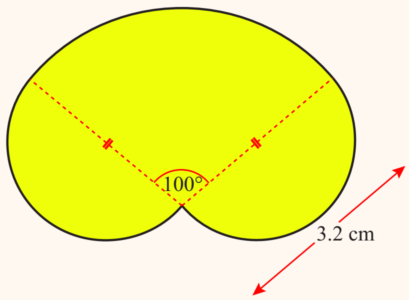
a



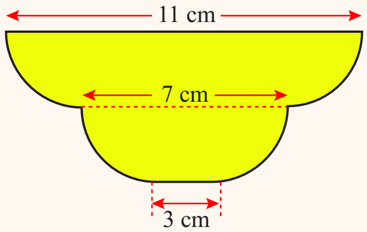
b



c



d



E

7.2 Three-dimensional objects

We now move into three dimensions but will use many of the formulae for two-dimensional shapes in our calculations. A three-dimensional object is called a **solid**.

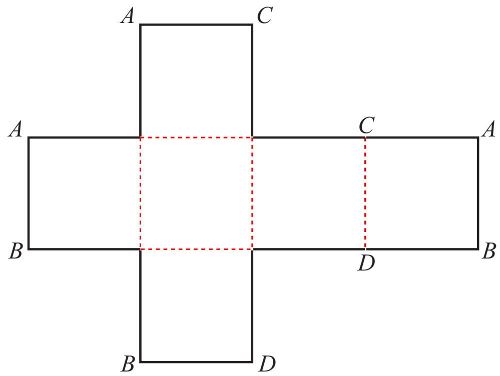
Nets of solids

A **net** is a two-dimensional shape that can be drawn, cut out and folded to form a three-dimensional solid.

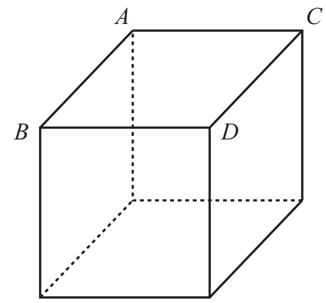
Tip

You might be asked to count the number of vertices (corners), edges and faces that a solid has.

The following shape is the net of a solid that you should be quite familiar with.



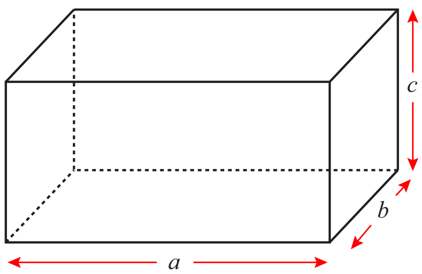
If you fold along the dotted lines and join the points with the same letters then you will form this cube:



You should try this yourself and look carefully at which edges (sides) and which **vertices** (the points or corners) join up.

Exercise 7.5

1 The diagram shows a cuboid. Draw a net for the cuboid.

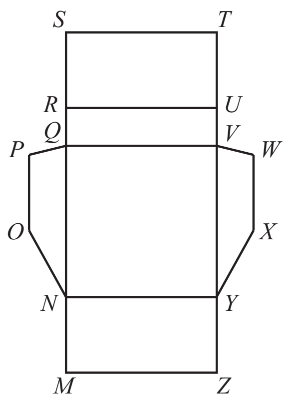


2 The diagram shows the net of a solid.

a Describe the solid in as much detail as you can.

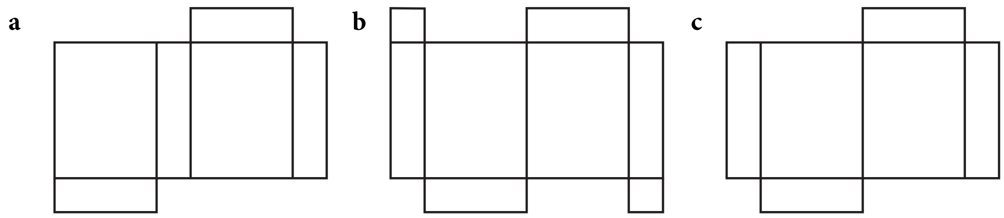
b Which two points will join with point M when the net is folded?

c Which edges are certainly equal in length to PQ?

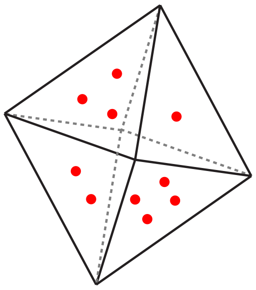


If you can't visualise the solution to problems like this one, you can build models to help you.

3 A teacher asked her class to draw the net of a cuboid cereal box. These are the diagrams that three students drew. Which of them is correct?



4 How could you make a cardboard model of this octahedral dice? Draw labelled sketches to show your solution.



7.3 Surface areas and volumes of solids

It can be helpful to draw the net of a solid when trying to find its surface area.

The flat, two-dimensional surfaces on the outside of a solid are called **faces**. The area of each face can be found using the techniques from earlier in this chapter. The total area of the faces will give us the **surface area** of the solid.

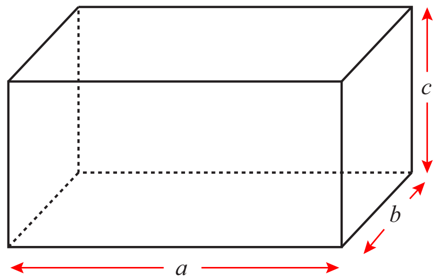
The **volume** is the amount of space contained inside the solid. If the units given are cm, then the volume is measured in cubic centimetres (cm^3) and so on.

Some well known formulae for surface area and volume are shown below.

Cuboids

A cuboid has six rectangular faces, 12 edges and eight vertices.

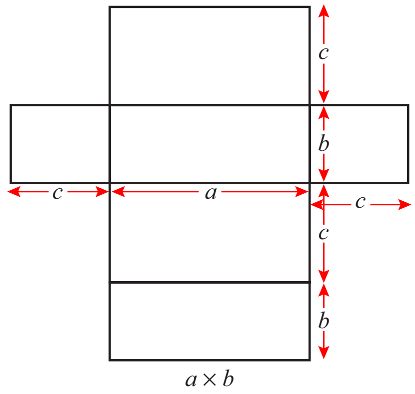
If the length, breadth and height of the cuboid are a, b and c (respectively) then the surface area can be found by thinking about the areas of each rectangular face.



Notice that the surface area is exactly the same as the area of the cuboid's net.

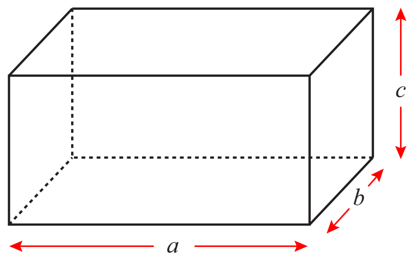
Surface area of cuboid = $2(ab + ac + bc)$

Volume of cuboid = $a \times b \times c$



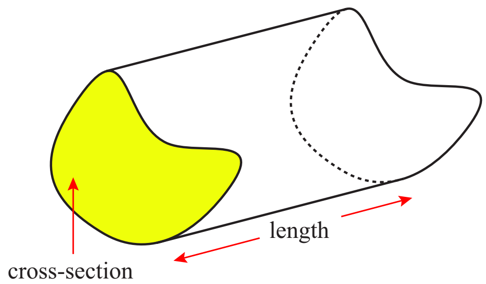
The volume of a cuboid is its length × breadth × height.

So, volume of cuboid = $a \times b \times c$.

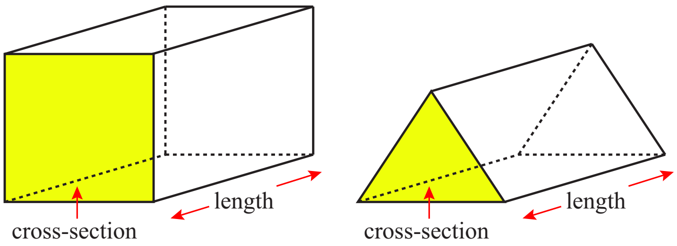


Prisms

A prism is a solid whose cross-section is the same all along its length. (A cross-section is the surface formed when you cut parallel to a face.)



The cuboid is a special case of a prism with a rectangular cross-section. A triangular prism has a triangular cross-section.



The surface area of a prism is found by working out the area of each face and adding the areas together. There are two ends with area equal to the cross-sectional area. The remaining sides are all the same length, so their area is equal to the perimeter of the cross-section multiplied by the length:

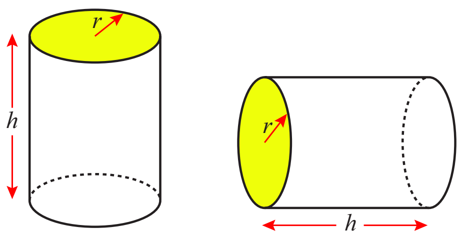
surface area of a prism = 2 × area of cross-section + perimeter of cross-section × length

The volume of a prism is found by working out the area of the cross-section and multiplying this by the length.

volume of a prism = area of cross-section × length

Cylinders

A cylinder is another special case of a prism. It is a prism with a circular cross-section.



Tip

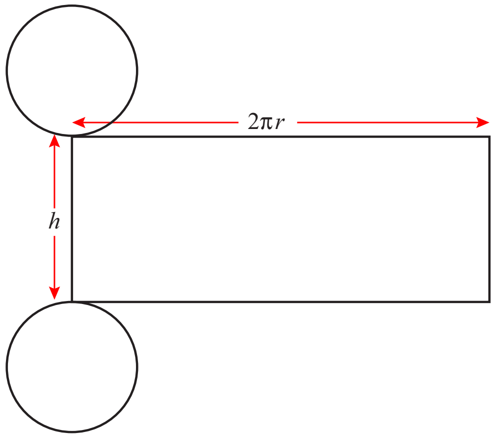
You may be asked to give exact answers to surface area and volume calculations where pi is part of the formula. If so, give your answer as a multiple of π.

A cylinder can be 'unwrapped' to produce its net. The surface consists of two circular faces and a curved face that can be flattened to make a rectangle.

Curved surface area of a cylinder = $2\pi rh$

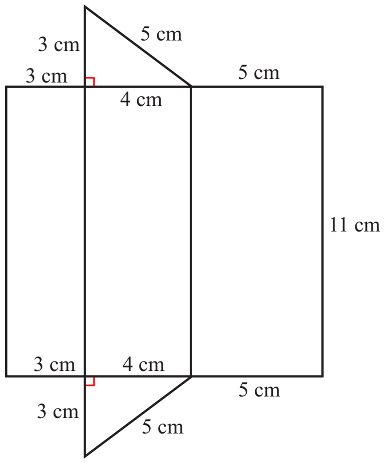
and

Volume = $\pi r^2 h$.



Exercise 7.6

1 Find the volume and surface area of the solid with the net shown in the diagram.



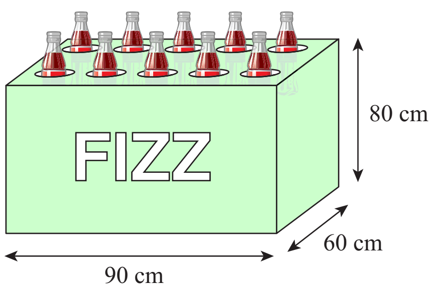
2 Find **(i)** the volume and **(ii)** the surface area of the cuboids with the following dimensions:

a length = 5 cm, breadth = 8 cm, height = 18 cm

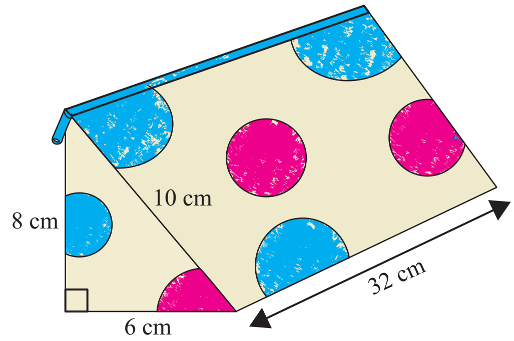
b length = 1.2 mm, breadth = 2.4 mm, height = 4.8 mm

Applying your skills

3 The diagram shows a bottle crate. Find the volume of the crate.



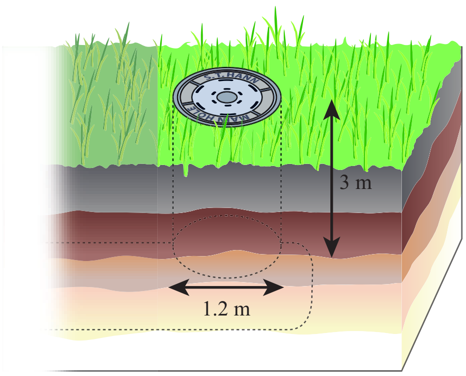
4 The diagram shows a pencil case in the shape of a triangular prism.



Calculate:

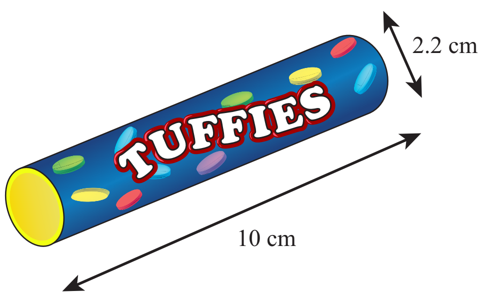
a the volume and **b** the surface area of the pencil case.

5 The diagram shows a cylindrical drain. Calculate the volume of the drain.

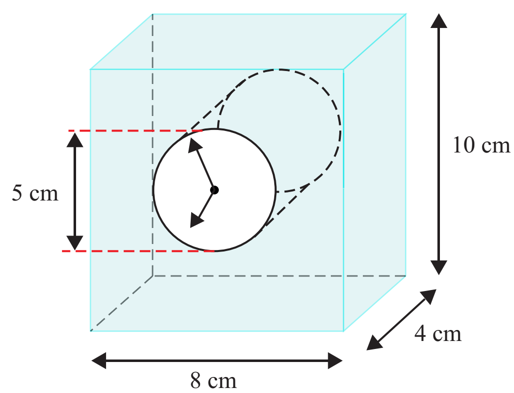


Don't forget to include the circular faces.

6 The diagram shows a tube containing chocolate sweets. Calculate the *total* surface area of the tube.



7 The diagram shows the solid glass case for a clock. The case is a cuboid with a cylinder removed (to fit the clock mechanism). Calculate the volume of glass required to make the clock case.



8 A storage company has a rectangular storage area 20 m long, 8 m wide and 2.8 m high.

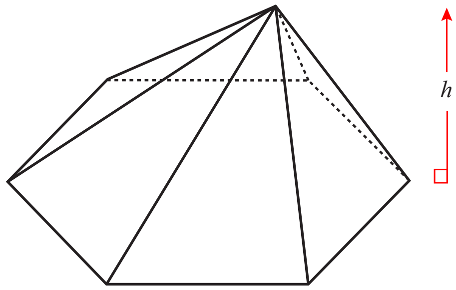
a Find the volume of the storage area.
b How many cardboard boxes of dimensions 1 m × 0.5 m × 2.5 m can fit into this storage area?
c What is the surface area of each cardboard box?

9 Vuyo is moving to Brazil for his new job. He has hired a shipping container of dimensions 3 m × 4 m × 4 m to move his belongings.

a Calculate the volume of the container.
b He is provided with crates to fit the dimensions of the container. He needs to move eight of these crates, each with a volume of 5 m^3. Will they fit into one container?

Pyramids

A pyramid is a solid with a polygon-shaped base and triangular faces that meet at a point called the **apex**.



If you find the area of the base and the area of each of the triangles, then you can add these up to find the total surface area of the pyramid.

The perpendicular height is the shortest distance from the base to the apex.

The volume can be found by using the following formula:

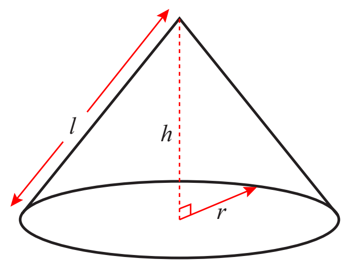
$$\text{Volume} = \frac{1}{3} \times \text{base area} \times \text{perpendicular height}$$

Cones

FAST FORWARD

The slant height can be calculated by using Pythagoras' theorem, which you will meet in chapter 11.

A cone is a special pyramid with a circular base. The length l is known as the **slant height**. h is the perpendicular height.



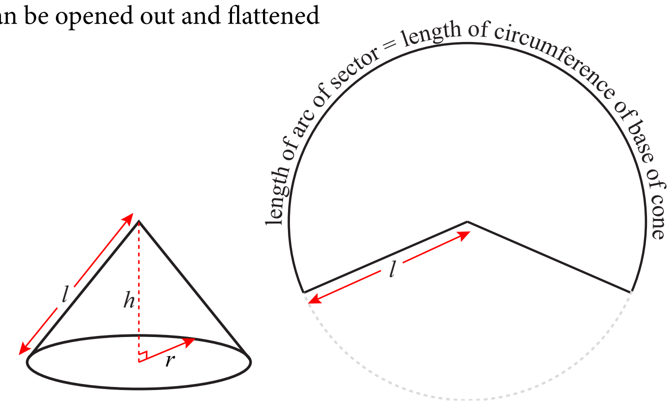
If you are asked for the total surface area of a cone, you must work out the area of the circular base and add it to the curved surface area.

The curved surface of the cone can be opened out and flattened to form a sector of a circle.

Curved surface area = πrl

and

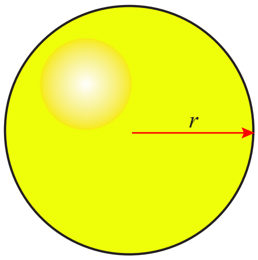
$$\text{Volume} = \frac{1}{3}\pi r^2 h$$



Remember, if you are asked for an exact answer you must give the answer as a multiple of π and you cannot use approximate values in the calculation.

Spheres

The diagram shows a sphere with radius r.



$\boxed{\text{Surface area} = 4\pi r^2}$

and

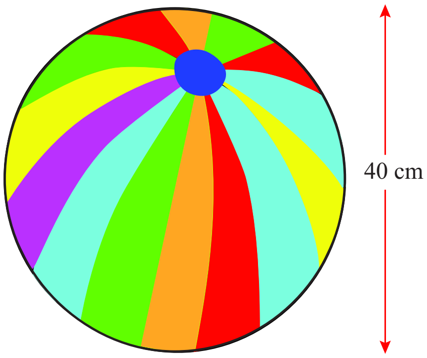
$\boxed{\text{Volume} = \frac{4}{3}\pi r^3}$

Exercise 7.7

1 The diagram shows a beach ball.

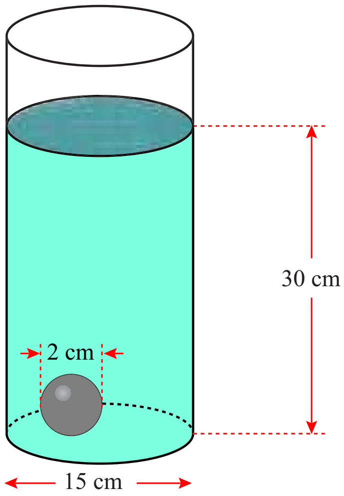
a Find the surface area of the beach ball.

b Find the volume of the beach ball.



The volume of the water is the volume in the cylinder minus the displacement caused by the metal ball. The displacement is equal to the volume of the metal ball.

2 The diagram shows a metal ball bearing that is completely submerged in a cylinder of water. Find the volume of water in the cylinder.



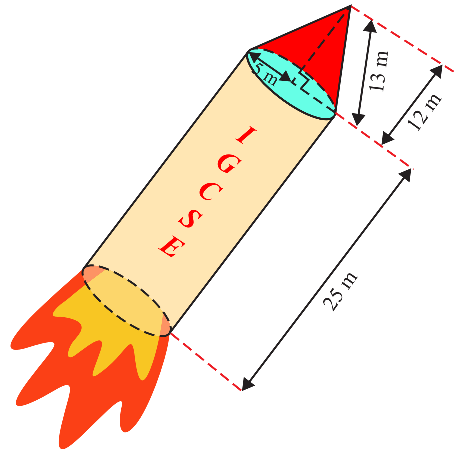
3 The Great Pyramid at Giza has a square base of side 230 m and perpendicular height 146 m.

Find the volume of the Pyramid.

4 The diagram shows a rocket that consists of a cone placed on top of a cylinder.

a Find the surface area of the rocket.

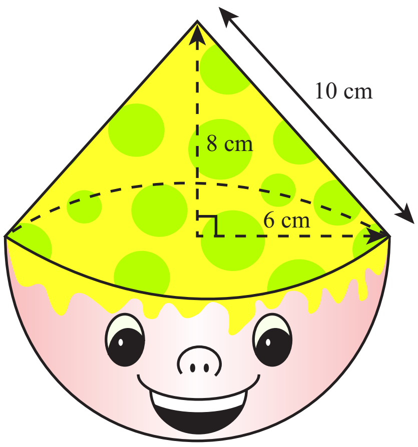
b Find the volume of the rocket.



5 The diagram shows a child's toy that consists of a hemisphere (half of a sphere) and a cone.

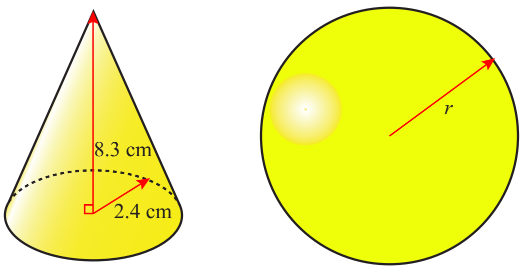
a Find the volume of the toy.

b Find the surface area of the toy.



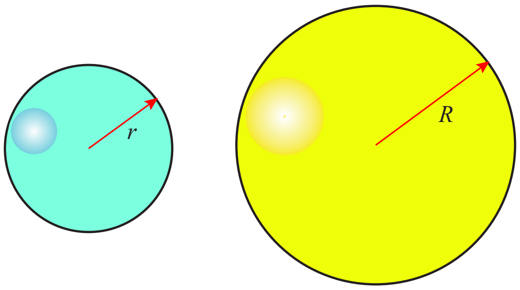
6 The sphere and cone shown in the diagram have the same volume.

Find the radius of the sphere.



7 The volume of the larger sphere (of radius R) is twice the volume of the smaller sphere (of radius r).

Find an equation connecting r to R.



8 A 32 cm long cardboard postage tube has a radius of 2.5 cm.

a What is the exact volume of the tube?
b For posting the tube is sealed at both ends. What is the surface area of the sealed tube?

9 A hollow metal tube is made using a 5 mm metal sheet. The tube is 35 cm long and has an exterior diameter of 10.4 cm.

a Draw a rough sketch of the tube and add its dimensions
b Write down all the calculations you will have to make to find the volume of metal in the tube.
c Calculate the volume of metal in the tube.
d How could you find the total surface area of the outside plus the inside of the tube?

Summary

Do you know the following?

- The perimeter is the distance around the outside of a two-dimensional shape and the area is the space contained within the sides.
- Circumference is the name for the perimeter of a circle.
- If the units of length are given in cm then the units of area are cm^2 and the units of volume are cm^3. This is true for any unit of length.
- A sector of a circle is the region contained in-between two radii of a circle. This splits the circle into a minor sector and a major sector.
- An arc is a section of the circumference.
- Prisms, pyramids, spheres, cubes and cuboids are examples of three-dimensional objects (or solids).
- A net is a two-dimensional shape that can be folded to form a solid.
- The net of a solid can be useful when working out the surface area of the solid.

Are you able to . . . ?

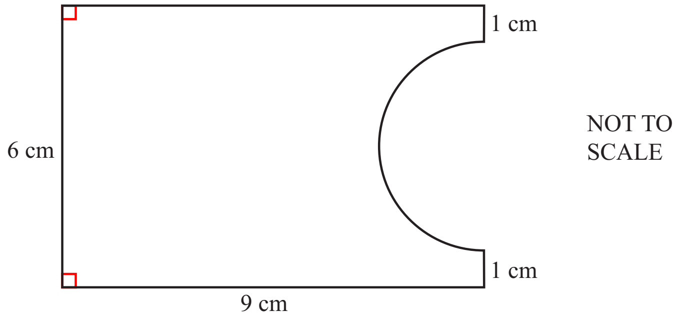
- recognise different two-dimensional shapes and find their areas
- give the units of the area
- calculate the areas of various two-dimensional shapes
- divide a shape into simpler shapes and find the area
- find unknown lengths when some lengths and an area are given
- calculate the area and circumference of a circle
- calculate the perimeter, arc length and area of a sector
- recognise nets of solids
- fold a net correctly to create its solid
- find the volumes and surface areas of a cuboid, prism and cylinder
- find the volumes of solids that can be broken into simpler shapes
- find the volumes and surface areas of a pyramid, cone and sphere.

Examination practice

Exam-style questions

1 A piece of rope is wound around a cylindrical pipe 18 times. If the diameter of the pipe is 600 mm, how long is the rope?

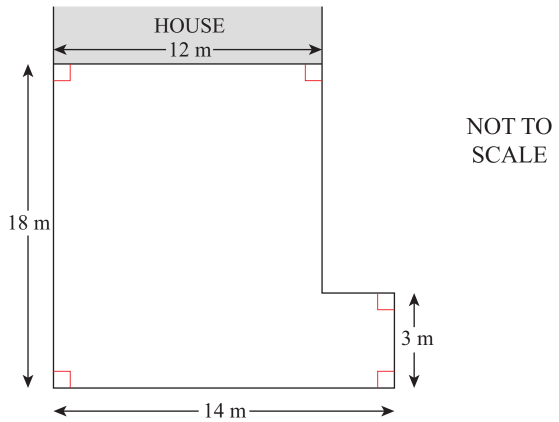
2 Find the perimeter and area of this shape.



3 A cylindrical rainwater tank is 1.5 m tall with a diameter of 1.4 m. What is the maximum volume of rainwater it can hold?

Past paper questions

1 This diagram shows the plan of a driveway to **a** house.

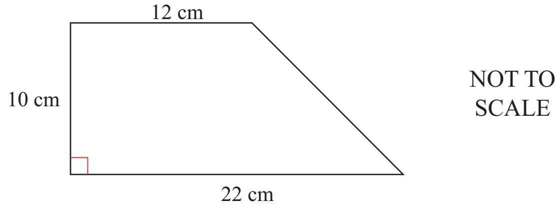


a Work out the perimeter of the driveway. [2]

b The driveway is made from concrete. The concrete is 15 cm thick. Calculate the volume of concrete used for the driveway. Give your answer in cubic metres. [4]

[Cambridge IGCSE Mathematics 0580 Paper 33 Q8 d, e October/November 2012]

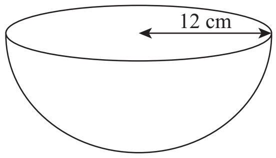
2



Find the area of the trapezium. [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q7 October/November 2013]

3



A **hemisphere** has a radius of 12 cm.
Calculate its volume.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q8 October/November 2013]

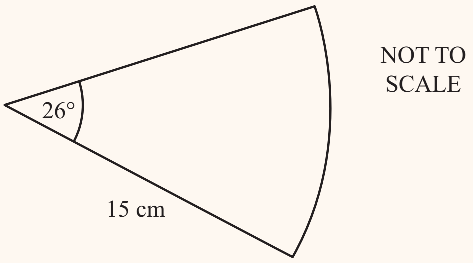
4 Calculate the volume of a hemisphere with radius 5 cm.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q5 October/November 2015]

E

5



The diagram shows a sector of a circle with radius 15 cm.
Calculate the perimeter of this sector. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q16 October/November 2015]