Chapter 13: Understanding measurement

Key words

- Metric
- Lower bound
- Upper bound
- Imperial
- Conversion
- Exchange rate

In this chapter you will learn how to:

- convert between units in the metric system
- find lower and upper bounds of numbers that have been quoted to a given accuracy
- EXTENDED: Solve problems involving upper and lower bounds
- use conversion graphs to change units from one measuring system to another
- use exchange rates to convert currencies.

Weather systems are governed by complex sets of rules. The mathematics that describes these rules can be highly sensitive to small changes or inaccuracies in the available numerical data. We need to understand how accurate our predictions may or may not be.

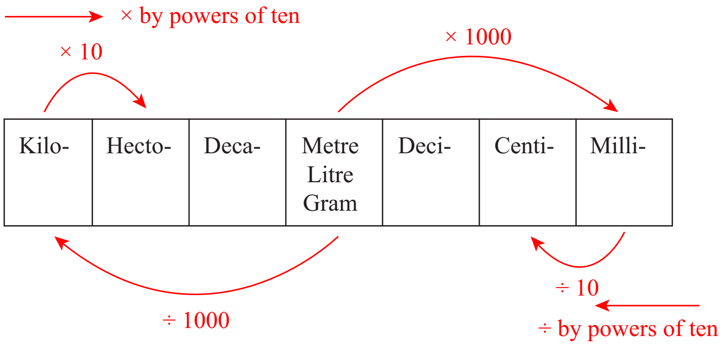
The penalties for driving an overloaded vehicle can be expensive, as well as dangerous for the driver and other road users. If a driver is carrying crates that have a rounded mass value, he needs to know what the maximum mass could be before he sets off and, if necessary, put his truck onto a weighbridge as a precaution against fines and, worse, an accident.

RECAP

You should already familiar with the following measurement work:

Converting units of measurement (Stage 9 Mathematics)

To convert between basic units you need to multiply or divide by powers of ten.



Area is measured in square units so conversion factors are also squared.

Volume is measured in cubic units so conversion factors are also cubed.

For example:

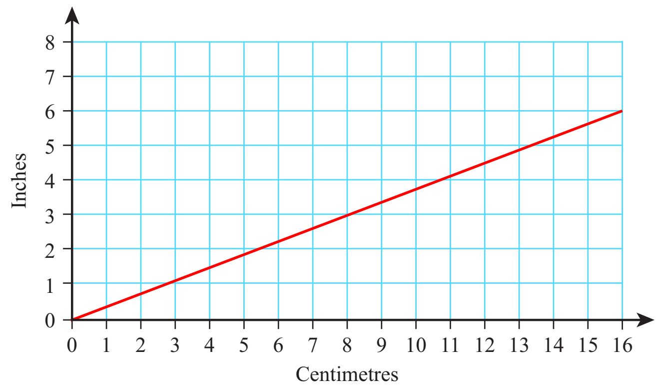
Basic units	cm → mm	$\times 10$
Square units	$cm^2 \rightarrow mm^2$	$\times 10^2$
Cubic units	$cm^3 \rightarrow mm^3$	$\times 10^3$

Units of time can be converted as long as you use the correct conversion factors.

For example 12 minutes = 12×60 seconds = 720 seconds.

Money amounts are decimal. In general one main unit = 100 smaller units.

A conversion graph can be used to convert one set of measurements to another.



13.1 Understanding units

REWIND

You encountered these units in chapter 7 when working with perimeters, areas and volumes. ◀

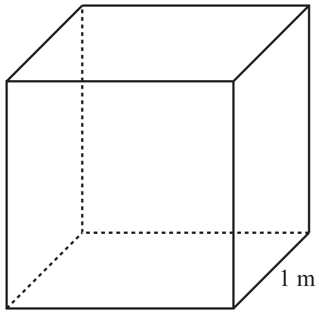
LINK

Physicists need to understand how units relate to one another. The way in which we express masses, speeds, temperatures and a vast array of other quantities can depend on the units used.

Vishal has a 1 m × 1 m × 1 m box and has collected a large number of 1 cm × 1 cm × 1 cm building blocks. He is very tidy and decides to stack all of the cubes neatly into the box.

Try to picture a 1 m × 1 m × 1 m box:

The lengths of each side will be 1 m = 100 cm. The total number of 1 cm × 1 cm × 1 cm cubes that will fit inside will be 100 × 100 × 100 = 1 000 000.



The main point of this example is that if you change the units with which a quantity is measured, the actual numerical values can be wildly different.

Here, you have seen that one cubic metre is equivalent to one million cubic centimetres!

Centimetres and metres are examples of **metric** measurements, and the table below shows some important conversions. You should work through the table and make sure that you understand *why* each of the conversions is as it is.

Measure	Units used	Equivalent to . . .
Length – how long (or tall) something is.	Millimetres (mm) Centimetres (cm) Metres (m) Kilometres (km)	10 mm = 1 cm 100 cm = 1 m 1000 m = 1 km 1 km = 1 000 000 mm
Mass – the amount of material in an object, (sometimes incorrectly called weight).	Milligrams (mg) Grams (g) Kilograms (kg) Tonnes (t)	1000 mg = 1 g 1000 g = 1 kg 1000 kg = 1 t 1 t = 1 000 000 g
Capacity – the inside volume of a container; how much it can hold.	Millilitres (ml) Centilitres (cl) Litres (ℓ)	10 ml = 1 cl 100 cl = 1 ℓ 1 ℓ = 1000 ml
Area – the amount of space taken up by a flat (two-dimensional) shape, always measured in square units.	Square millimetre (mm^2) Square centimetre (cm^2) Square metre (m^2) Square kilometre (km^2) Hectare (ha)	100 mm^2 = 1 cm^2 10 000 cm^2 = 1 m^2 1 000 000 m^2 = 1 km^2 1 km^2 = 100 ha 1 ha = 10 000 m^2
Volume – the amount of space taken up by a three-dimensional object, always measured in cubic units (or their equivalent liquid measurements, e.g. ml).	Cubic millimetre (mm^3) Cubic centimetre (cm^3) Cubic metre (m^3) Millilitre (mℓ)	1000 mm^3 = 1 cm^3 1 000 000 cm^3 = 1 m^3 1 m^3 = 1000 ℓ 1 cm^3 = 1 mℓ

Capacity is measured in terms of what something *can* contain, not how much it *does* contain. A jug can have a capacity of 1 litre but only contain 500 ml. In the latter case, you would refer to the volume of the liquid in the container.

The example on page 257 shows how these conversions can be used.

Worked example 1

Express:

a 5 km in metres **b** 3.2 cm in mm **c** 2 000 000 cm² in m².

a 1 km = 1000 m
So, 5 km = 5 × 1000 m = 5000 m

b 1 cm = 10 mm
So, 3.2 cm = 3.2 × 10 = 32 mm

c $1\text{ m}^2 = 100\text{ cm} \times 100\text{ cm} = 10\,000\text{ cm}^2$
So, $2\,000\,000\text{ cm}^2 = \dfrac{2\,000\,000}{10\,000} = 200\text{ m}^2$

Exercise 13.1

1 Express each quantity in the unit given in brackets.

a 4 kg (g) **b** 5 km (m) **c** 35 mm (cm)
d 81 mm (cm) **e** 7.3 g (mg) **f** 5760 kg (t)
g 2.1 m (cm) **h** 2 t (kg) **i** 140 cm (m)
j 2024 g (kg) **k** 121 mg (g) **l** 23 m (mm)
m 3 cm 5 mm (mm) **n** 8 km 36 m (m) **o** 9 g 77 mg (g)

2 Arrange in ascending order of size.

3.22 m, $3\frac{2}{9}$ m, 32.4 cm

3 Write the following volumes in order, starting with the smallest.

$\frac{1}{2}$ litre, 780 ml, 125 ml, 0.65 litres

4 How many 5 ml spoonfuls can be obtained from a bottle that contains 0.3 litres of medicine?

5 Express each quantity in the units given in brackets.

a 14.23 m (mm, km) **b** 19.06 g (mg, t) **c** $2\frac{3}{4}$ litres (ml, cl)
d 4 m² (mm², ha) **e** 13 cm² (mm², ha) **f** 10 cm³ (mm³, m³)

REWIND

You will need to remind yourself how to calculate volumes of three-dimensional shapes in chapter 7. You also need to remember what you learned about standard form in chapter 5. ◄

6 A cube has sides of length 3 m. Find the volume of the cube in:
a m³ **b** cm³ **c** mm³ (give your answer in standard form).

7 The average radius of the Earth is 6378 km. Find the volume of the Earth, using each of the following units. Give your answers in standard form to 3 significant figures. The volume of a sphere $= \frac{4}{3}\pi r^3$

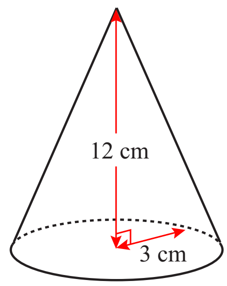
a km³
b m³
c mm³

You were given the formula for the volume of a cone in chapter 7: Volume $= \frac{1}{3}\pi r^2 h$.

8 The dimensions of the cone shown in the diagram are given in cm. Calculate the volume of the cone in:

a cm^3
b mm^3
c km^3

Give your answers in standard form to 3 significant figures.



Applying your skills

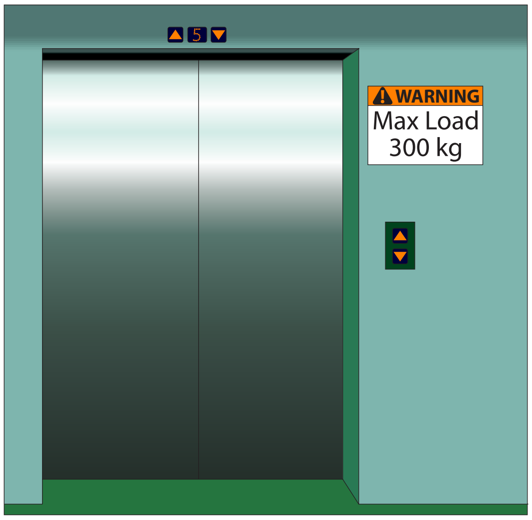
LINK

Scale and measurement is an important element of map skills. If you study geography you need to understand how to convert between units and work with different scales.

9 Miss Molly has a jar that holds 200 grams of flour.

a How many 30 gram measures can she get from the jar?
b How much flour will be left over?

10 This is a lift in an office building.



The lift won't start if it holds more than 300 kg.

a Tomas (105 kg), Shaz (65 kg), Sindi (55 kg), Rashied (80 kg) and Mandy (70 kg) are waiting for the lift. Can they all ride together?
b Mandy says she will use the stairs. Can the others go safely into the lift?
c Rashied says he will wait for the lift to come down again. Can the other four go together in the lift?

13.2 Time

Always read time problems carefully and show the steps you take to solve them. This is one context where working backwards is often a useful strategy.

You have already learned how to tell the time and you should know how to read and write time using the 12-hour and 24-hour system.

The clock dial on the left shows you the times from 1 to 12 (a.m. and p.m. times). The inner dial shows what the times after 12 p.m. are in the 24-hour system.

Always remember that time is written in hours and minutes and that there are 60 minutes in an hour. This is very important when calculating time – if you put 1.5 hours into your calculator, it will assume the number is decimal and work with parts of 100, not parts of 60. So, you need to treat minutes and hours separately.

Worked example 2

Sara and John left home at 2.15 p.m. Sara returned at 2.50 p.m. and John returned at 3.05 p.m. How long was each person away from home?

Sara: 2 hours 50 minutes – 2 hours 15 minutes = 0 h 35 min 2 hours – 2 hours = 0 hours 50 min – 15 min = 35 min Sara was away for 35 minutes.	Think about how many hours you have, and how many minutes. 2.50 p.m. is the same as 2 hours and 50 minutes after 12 p.m., and 2.15 p.m. is the same as 2 hours and 15 minutes after 12 p.m. Subtract the hours separately from the minutes.
John: 3 h 5 min = 2 h 65 min 2 h 65 min – 2 h 15 min = 0 h 50 min John was away for 50 minutes.	3.05 p.m. is the same as 2 hours and 65 minutes after 12 p.m.; do a subtraction like before. Note both times are p.m. See the next example for when one time is a.m. and the other p.m.

You cannot subtract 15 min from 5 min in the context of time (you can't have negative minute) so carry one hour over to the minutes so that 3 h 5min becomes 2 h 65 mins.

Worked example 3

A train leaves at 05.35 and arrives at 18.20. How long is the journey?

18.20 is equivalent to 17 hours and 80 minutes after 12 a.m.	Again, 20–35 is not meaningful in the context of time, so carry one hour over to give 17 h 80 min.
17h – 5h = 12h 80 min – 35 min = 45 min	Now you can subtract the earlier time from the later time as before (hours and minutes separately).
The journey took 12 hours and 45 minutes.	Then add the hours and minutes together.

Again note that you cannot simply do 18.20 – 05.35 because this calculation would not take into account that with time you work in jumps of 60 not 100.

The methods in examples 1–3 are best used when you are dealing with time within the same day. But what happens when the time difference goes over one day?

Worked example 4

How much time passes from 19.35 on Monday to 03.55 on Tuesday?

19.35 to 24.00 is one part and 00.00 to 03.55 the next day is the other part.	The easiest way to tackle this problem is to divide the time into parts.
Part one: 19.35 to 24.00. 24 h = 23 h 60 min (past 12 a.m.) 23 h 60 min – 19 h 35 min = 4 h 25 min	00–35 is not meaningful in time, so carry one hour over so that 24.00 becomes 23 h 60 min. Then do the subtraction as before (hours and minutes separately).

Part two: 0.00 to 03.55 3h 55 min – 0 h 0 min = 3 h 55 min	03.55 is 3 hours and 55 minutes past 12 a.m. (or 0.00) so this is simply a difference of + 3 hours and 55 minutes
4 h 25 min + 3 h 55 min = 7 h 80 min 80 min = 1 h 20 min 7 h 0 min + 1 h 20 min = 8 h 20 min 8 hours and 20 min passes.	Add the result of the two parts together. Change the 80 minutes into hours and minutes. Add together.

Exercise 13.2 *Applying your skills*

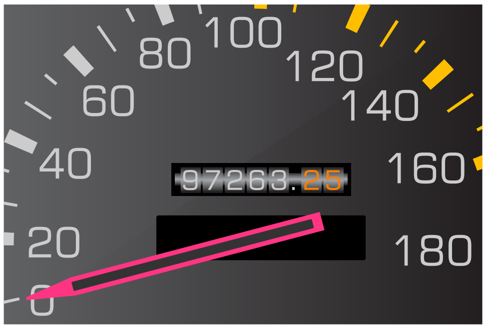
When dealing with time problems, consider what is being asked and what operations you will need to do to answer the question.

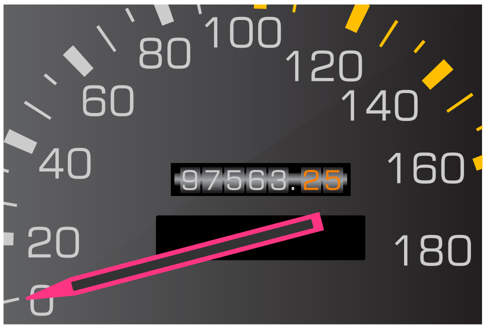
1 Gary started a marathon race at 9.25 a.m. He finished at 1.04 p.m. How long did he take? Give your answer in hours and minutes.

2 Nick has a satellite TV decoder that shows time in 24-hour time. He wants to program the machine to record some programmes. Write down the timer settings for starting and finishing each recording.

a 10.30 p.m. to 11.30 p.m.
b 9.15 a.m. to 10.45 a.m.
c 7.45 p.m. to 9.10 p.m.

3 Yasmin's car odometer shows distance travelled in kilometres. The odometer dial showed these two readings before and after a journey:





a How far did she travel?

b The journey took $2\frac{1}{2}$ hours. What was her average speed in km/h?

4 Yvette records three songs onto her MP4 player. The time each of them lasts is three minutes 26 seconds, three minutes 19 seconds and two minutes 58 seconds. She leaves a gap of two seconds between each of the songs. How long will it take to play the recording?

5 A journey started at 17:30 hours on Friday, 7 February, and finished 57 hours later. Write down the time, day and date when the journey finished.

6 Samuel works in a bookshop. This is his time sheet for the week.

Day	Mon	Tues	Wed	Thurs	Fri
Start	8:20	8:20	8:20	8:22	8:21
Lunch	12:00	12:00	12:30	12:00	12:30
Back	12:45	12:45	1:15	12:45	1:15
End	5:00	5:00	4:30	5:00	5:30
Total time worked					

a Complete the bottom row of the time sheet.
b How many hours in total did Samuel work this week?
c Samuel is paid $5.65 per hour. Calculate how much he earned this week.

Reading timetables

Most travel timetables are in the form of tables with columns representing journeys. The 24-hour system is used to give the times.

Here is an example:

	SX	D	D	D	MO	D	SX
Anytown	06:30	07:45	12:00	16:30	17:15	18:00	20:30
Beecity	06:50	08:05	12:25	16:50	17:35	18:25	20:50
Ceeville	07:25	08:40	13:15	17:25	18:15	19:05	21:25

D – daily including Sundays, SX – daily except Saturdays, MO – Mondays only

Make sure you can see that each column represents a journey. For example, the first column shows a bus leaving at 06:30 every day except Saturday (six times per week). It arrives at the next town, Beecity, at 06:50 and then goes on to Ceeville, where it arrives at 07:25.

Exercise 13.3 *Applying your skills*

1 The timetable for evening trains between Mitchell's Plain and Cape Town is shown below.

Mitchell's Plain	18:29	19:02	19:32	20:02	21:04
Nyanga	18:40	19:13	19:43	20:13	21:15
Pinelands	19:01	19:31	20:01	20:31	21:33
Cape Town	19:17	19:47	20:17	20:47	21:49

a Shaheeda wants to catch a train at Mitchell's Plain and get to Pinelands by 8.45 p.m. What is the time of the latest train she should catch?
b Calculate the time the 19:02 train from Mitchell's Plain takes to travel to Cape Town.
c Thabo arrives at Nyanga station at 6.50 p.m. How long will he have to wait for the next train to Cape Town?

2 The timetable for a bus service between Aville and Darby is shown below.

Aville	10:30	10:50	and	18:50
Beeston	11:05	11:25	every	19:25
Crossway	11:19	11:39	20 minutes	19:39
Darby	11:37	11:57	until	19:57

a How many minutes does a bus take to travel from Aville to Darby?
b Write down the timetable for the first bus on this service to leave Aville after the 10:50 bus.
c Ambrose arrives at Beeston bus station at 2.15 p.m. What is the time of the next bus to Darby?

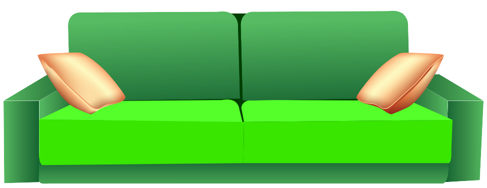
3 The tides for a two-week period are shown on this tide table.

February	High tide		Low tide	
	Morning	Afternoon	Morning	Afternoon
1 Wednesday	1213	--	0518	1800
2 Thursday	0017	1257	0614	1849
3 Friday	0109	1332	0700	1930
4 Saturday	0152	1404	0740	2004
5 Sunday	0229	1434	0815	2038
6 Monday	0303	1505	0848	2111
7 Tuesday	0336	1537	0922	2143
8 Wednesday	0411	1610	0957	2215
9 Thursday	0448	1644	1030	2245
10 Friday	0528	1718	1104	2316
11 Saturday	0614	1757	1140	2354
12 Sunday	0706	1845	1222	--
13 Monday	0808	1948	0041	1315
14 Tuesday	0917	2111	0141	1425

a What is the earliest high tide in this period?
b How long is it between high tides on day two?
c How long is it between the first high tide and the first low tide on day seven?
d Mike likes to go surfing an hour before high tide.
  i At what time would this be on Sunday 5 February?
  ii Explain why it would unlikely to be at 01:29.
e Sandra owns a fishing boat.
  i She cannot go out in the mornings if the low tide occurs between 5 a.m. and 9 a.m. On which days did this happen?
  ii Sandra takes her boat out in the afternoons if high tide is between 11 a.m. and 2.30 p.m. On which days could she go out in the afternoons?

13.3 Upper and lower bounds

Raeman has ordered a sofa and wants to work out whether or not it will fit through his door. He has measured both the door (47 cm) and the sofa (46.9 cm) and concludes that the sofa should fit with 1 mm to spare. Unfortunately, the sofa arrives and doesn't fit. What went wrong?

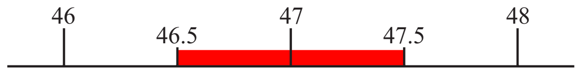


Looking again at the value 47 cm, Raeman realises that he rounded the measurement to the nearest cm. A new, more accurate measurement reveals that the door frame is, in fact, closer to 46.7 cm wide. Raeman also realises that he has rounded the sofa measurement to the nearest mm. He measures it again and finds that the actual value is closer to 46.95 cm, which is 2.5 mm wider than the door!

Finding the greatest and least possible values of a rounded measurement

Consider again, the width of Raeman's door. If 47 cm has been rounded to the nearest cm it can be useful to work out the greatest and least possible values of the *actual* measurement.

If you place the measurement of 47 cm on a number line, then you can see much more clearly what the range of possible values will be:



Notice at the upper end, that the range of possible values stops at 47.5 cm. If you round 47.5 cm to the nearest cm you get the answer 48 cm. Although 47.5 cm does not round to 47 (to the nearest cm), it is still used as the upper value. But, you should understand that the *true* value of the width could be anything up to *but not including* 47.5 cm. The lowest possible value of the door width is called the **lower bound**. Similarly, the largest possible value is called the **upper bound.**

Letting w represent the width of the sofa, the range of possible measurements can be expressed as:

$$46.5 \leqslant w < 47.5$$

This shows that the true value of w lies between 46.5 (including 46.5) and 47.5 (not including 47.5).

Worked example 5

Find the upper and lower bounds of the following, taking into account the level of rounding shown in each case.

a 10 cm, to the nearest cm

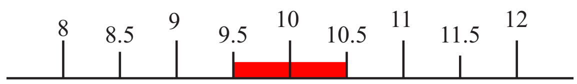
b 22.5, to 1 decimal place

c 128 000, to 3 significant figures.

If you get confused when dealing with upper and lower bounds, draw a number line to help you.

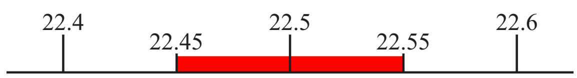
a Show 10 cm on a number line with the two nearest whole number values.

The real value will be closest to 10 cm if it lies between the lower bound of 9.5 cm and the upper bound of 10.5 cm.



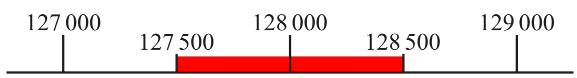
b Look at 22.5 on a number line.

The real value will be closest to 22.5 if it lies between the lower bound of 22.45 and the upper bound of 22.55.



c 128 000 is shown on a number line.

128 000 lies between the lower bound of 127 500 and the upper bound of 128 500.



Exercise 13.4

1 Each of the following numbers is given to the nearest whole number. Find the lower and upper bounds of the numbers.

a 12 **b** 8 **c** 100 **d** 9 **e** 72 **f** 127

2 Each of the following numbers is correct to 1 decimal place. Write down the lower and upper bounds of the numbers.

a 2.7 **b** 34.4 **c** 5.0 **d** 1.1 **e** −2.3 **f** −7.2

3 Each of the numbers below has been rounded to the degree of accuracy shown in the brackets. Find the upper and lower bounds in each case.

a 132 (nearest whole number)
b 300 (nearest one hundred)
c 405 (nearest five)
d 15 million (nearest million)
e 32.3 (1dp)
f 26.7 (1dp)
g 0.5 (1dp)
h 12.34 (2dp)
i 132 (3sf)
j 0.134 (3sf)

Applying your skills

4 Anne estimates that the mass of a lion is 300 kg. Her estimate is correct to the nearest 100 kg. Between what limits does the actual mass of the lion lie?

5 In a race, Nomatyala ran 100 m in 15.3 seconds. The distance is correct to the nearest metre and the time is correct to one decimal place. Write down the lower and upper bounds of:

a the actual distance Nomatyala ran **b** the actual time taken.

6 The length of a piece of thread is 4.5 m to the nearest 10 cm. The actual length of the thread is L cm. Find the range of possible values for L, giving you answer in the form $\ldots \leqslant L < \ldots$

E

Problem solving with upper and lower bounds

Some calculations make use of more than one rounded value. Careful use of the upper and lower bounds of each value, will give correct upper and lower bounds for the calculated answer.

Worked example 6

If $a = 3.6$ (to 1dp) and $b = 14$ (to the nearest whole number), find the upper and lower bounds for each of the following:

a $a + b$ **b** ab **c** $b - a$ **d** $\frac{a}{b}$ **e** $\frac{a+b}{a}$

Firstly, find the upper and lower bounds of a and b:

$3.55 \leqslant a < 3.65$ and $13.5 \leqslant b < 14.5$

a

$$\begin{aligned}\text{Upper bound for } (a+b) &= \text{upper bound of } a + \text{ upper bound of } b\\ &= 3.65 + 14.5\\ &= 18.15\end{aligned}$$

$$\begin{aligned}\text{Lower bound for } (a+b) &= \text{lower bound for } a + \text{ lower bound for } b\\ &= 3.55 + 13.5\\ &= 17.05\end{aligned}$$

This can be written as: $17.05 \leqslant (a + b) < 18.15$

b

$$\begin{aligned}\text{Upper bound for } ab &= \text{upper bound for } a \times \text{upper bound for } b\\ &= 3.65 \times 14.5\\ &= 52.925\end{aligned}$$

$$\begin{aligned}\text{Lower bound for } ab &= \text{lower bound for } a \times \text{lower bound for } b\\ &= 3.55 \times 13.5\\ &= 47.925\end{aligned}$$

This can be written as: $47.925 \leqslant ab < 52.925$

c

Think carefully about $b - a$. To find the upper bound you need to subtract as small a number as possible from the largest possible number. So:

$$\begin{aligned}\text{Upper bound for } (b-a) &= \text{upper bound for } b \quad \text{lower bound for } a\\ &= 14.5 - 3.55\\ &= 10.95\end{aligned}$$

Similarly, for the lower bound:

$$\begin{aligned}\text{Lower bound } (b-a) &= \text{lower bound for } b \quad \text{upper bound for } a\\ &= 13.5 - 3.65\\ &= 9.85\end{aligned}$$

This can be written as: $9.85 \leqslant (b - a) < 10.95$

d

To find the upper bound of $\frac{a}{b}$ you need to divide the largest possible value of a by the smallest possible value of b:

$$\text{Upper bound} = \frac{\text{upper bound for } a}{\text{lower bound for } b} = \frac{3.65}{13.5} = 0.2703\ldots = 0.270\ (3\text{sf})$$

$$\text{Lower bound} = \frac{\text{lower bound for } a}{\text{upper bound for } b} = \frac{3.55}{14.5} = 0.2448\ldots = 0.245\ (3\text{sf})$$

This can be written as: $0.245 \leqslant \frac{a}{b} < 0.270$

e

$$\text{Upper bound of } = \frac{a+b}{a} = \frac{\text{upper bound of } a+b}{\text{lower bound of } a} = \frac{18.15}{3.55} = 5.1126\ldots = 5.11\ (3\text{sf})$$

$$\text{Lower bound of } = \frac{a+b}{a} = \frac{\text{lower bound of } a+b}{\text{upper bound of } a} = \frac{17.05}{3.65} = 4.6712\ldots = 4.67\ (3\text{sf})$$

This can be written as: $4.67 \leqslant \frac{a+b}{a} < 5.11$

Exercise 13.5

1 You are given that:

$a = 5.6$ (to 1dp) $\quad b = 24.1$ (to 1dp) $\quad c = 145$ (to 3sf) $\quad d = 0.34$ (to 2dp)

Calculate the upper and lower bounds for each of the following to 3 significant figures:

a a^2 **b** b^3 **c** cd^3 **d** $a^2 + b^2$ **e** $\frac{c}{b^2}$

f $\frac{ab}{cd}$ **g** $\frac{c}{a} - \frac{b}{d}$ **h** $\frac{a}{d} \div \frac{c}{b}$ **i** $dc + \sqrt{\frac{a}{b}}$ **j** $dc - \sqrt{\frac{a}{b}}$

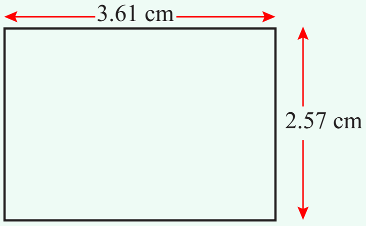
Applying your skills

2 Jonathan and Priya want to fit a new washing machine in their kitchen. The width of a washing machine is 79 cm to the nearest cm. To fit in the machine, they have to make a space by removing cabinets. They want the space to be as small as possible.

a What is the smallest space into which the washing machine can fit?
b What is the largest space they might need for it to fit?

3 12 kg of sugar are removed from a container holding 50 kg. Each measurement is correct to the nearest kilogram. Find the lower and upper bounds of the mass of sugar left in the container.

4 The dimensions of a rectangle are 3.61 cm and 2.57 cm, each correct to 3 significant figures.



a Write down the upper and lower bounds for each dimension.
b Find the upper and lower bounds of the area of the rectangle.
c Write down the upper and lower bounds of the area correct to 3 significant figures.

REWIND
Look back at chapter 7 to remind yourself about calculating areas.

5 The mean radius of the Earth is 6378 km, to the nearest km. Assume that the Earth is a sphere. Find upper and lower bounds for:

a the surface area of the Earth in km^2
b the volume of the Earth in km^3.

6 A cup holds 200 ml to the nearest ml, and a large container holds 86 litres to the nearest litre. What is the largest possible number of cupfuls of water needed to fill the container? What is the smallest possible number of cupfuls?

REWIND
Gradient was covered in chapter 10.

7 A straight road slopes steadily upwards. If the road rises 8 m (to the nearest metre) over a horizontal distance of 120 m (given to the nearest 10 m), what is the maximum possible gradient of the road? What is the minimum possible gradient? Give your answers to 3 significant figures.

REWIND

Remind yourself about Pythagoras' theorem from chapter 11. ◀

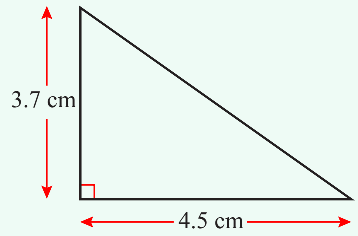
E

8 The two short sides of a right-angled triangle are 3.7 cm (to nearest mm) and 4.5 cm (to nearest mm). Calculate upper and lower bounds for:

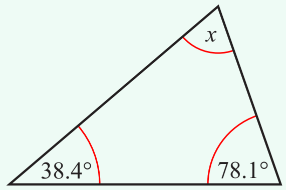
a the area of the triangle

b the length of the hypotenuse.

Give your answers to the nearest mm.



9 The angles in a triangle are $x°$, 38.4° (to 1 d.p.) and 78.1° (to 1 d.p.). Calculate upper and lower bounds for x.



10 Quantity x is 45 to the nearest integer. Quantity y is 98 to the nearest integer. Calculate upper and lower bounds for x as a percentage of y to 1 decimal place.

11 The following five masses are given to 3 significant figures.

138 kg   94.5 kg   1090 kg   345 kg   0.354 kg

Calculate upper and lower bounds for the *mean* of these masses.

12 Gemma is throwing a biased die. The probability that she throws a five is 0.245 to 3 decimal places. If Gemma throws the die exactly 480 times, calculate upper and lower bounds for the number of fives Gemma *expects* to throw. Give your answer to 2 decimal places.

13 A cuboid of height, h, has a square base of side length, a.

a In an experiment, a and h are measured as 4 cm and 11 cm respectively, each measured to the nearest cm.
What are the minimum and maximum possible values of the volume in cm^3?

b In another experiment, the volume of the block is found to be 350 cm^3, measured to the nearest 50 cm^3, and its height is measured as 13.5 cm, to the nearest 0.5 cm.

i What is the maximum and minimum possible values of the length a, in centimetres?

ii How many significant figures should be used to give a reliable answer for the value of a?

13.4 Conversion graphs

Generally speaking, the imperial equivalents of common metric units are shown below:

metric	imperial
mm/cm	inches
metres	feet/yards
kilometres	miles

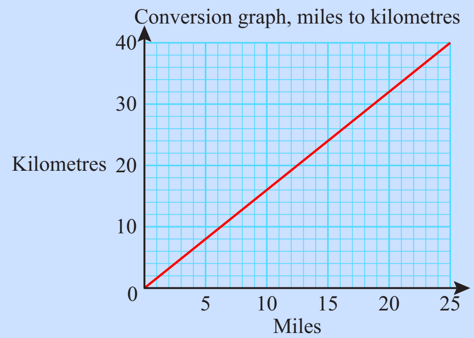
So far in this chapter, you have seen that it is possible to convert between different units in the metric system. Another widely used measuring system is the **imperial** system. Sometimes you might need to convert a measurement from metric to imperial, or the other way around. Similarly different countries use different currencies: dollars, yen, pounds, euros. When trading, it is important to accurately convert between them.

Conversion graphs can be used when you need to convert from one measurement to another. For example from miles (imperial) to kilometres (metric) or from dollars to pounds (or any other currency!).

Worked example 7

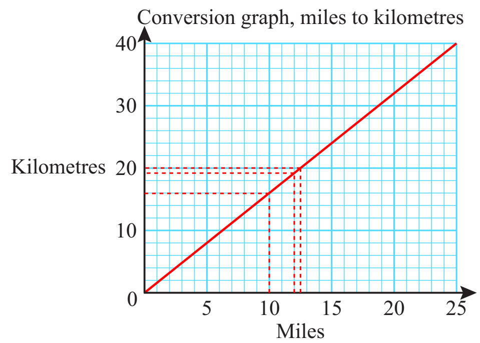
8 km is approximately equal to five miles. If you travel no distance in kilometres then you also travel no distance in miles. These two points of reference enable you to draw a graph for converting between the two measurements.

If the line is extended far enough you can read higher values. Notice, for example, that the line now passes through the point with co-ordinates (25, 40), meaning that 25 miles is approximately 40 km.



Check for yourself that you can see that the following are true:

10 miles is roughly 16 km
12 miles is roughly 19 km
20 km is roughly 12.5 miles, and so on.

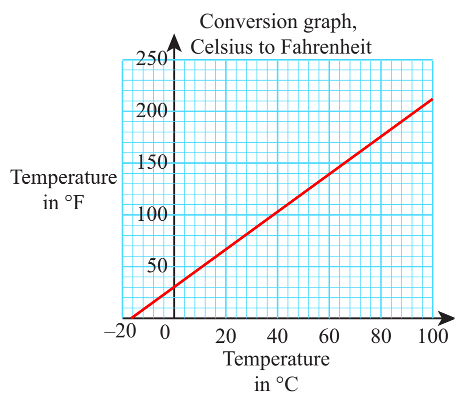


Exercise 13.6 *Applying your skills*

1 The graph shows the relationship between temperature in degrees Celsius (°C) and degrees Fahrenheit (°F).

Use the graph to convert:

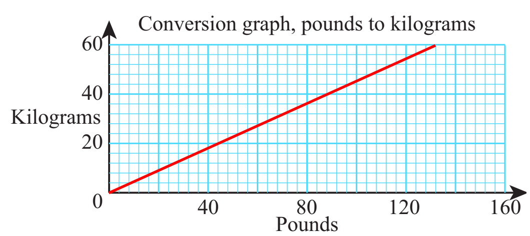
a 60 °C to °F
b 16 °C to °F
c 0 °F to °C
d 100 °F to °C.



The unit symbol for the imperial mass, pounds, is lb.

2 The graph is a conversion graph for kilograms and pounds. Use the graph to answer the questions below.

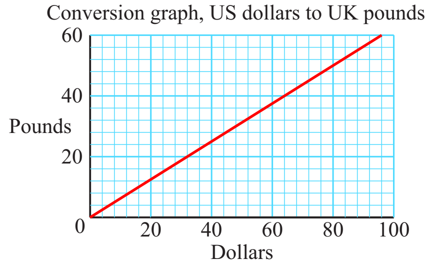
a What does one small square on the horizontal axis represent?
b What does one small square on the vertical axis represent?
c Change 80 pounds to kilograms
d The minimum mass to qualify as an amateur lightweight boxer is 57 kg. What is this in pounds?



e Which of the following conversions are incorrect? What should they be?

i 30 kg = 66 pounds **ii** 18 pounds = 40 kg
iii 60 pounds = 37 kg **iv** 20 pounds = 9 kg

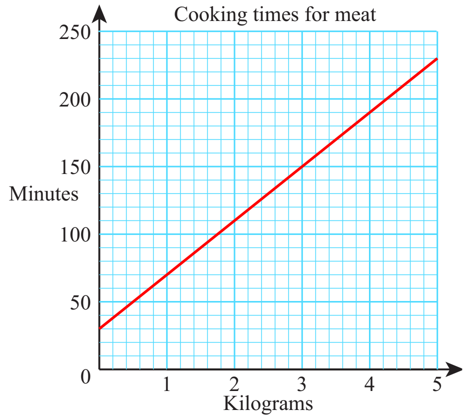
3 The graph shows the conversion between UK pounds (£) and US dollars ($), as shown on a particular website in February, 2011.



Use the graph to convert:

a £25 to $
b £52 to $
c $80 to £
d $65 to £.

4 The cooking time (in minutes) for a joint of meat (in kilograms) can be calculated by multiplying the mass of the joint by 40 and then adding 30 minutes. The graph shows the cooking time for different masses of meat.



Use the graph to answer the following questions.

a If a joint of this meat has a mass of 3.4 kg, approximately how long should it be cooked?
b If a joint of meat is to be cooked for 220 minutes, approximately how much is its mass?
c By calculating the mass of a piece of meat that takes only 25 minutes to cook, explain carefully why it is not possible to use this graph for every possible joint of meat.

ft is the abbreviation for the imperial unit foot (plural, feet). One foot is a little over 30 cm.

5 You are told that Mount Everest is approximately 29 000 ft high, and that this measurement is approximately 8850 m.

a Draw a conversion graph for feet and metres on graph paper.
b You are now told that Mount Snowdon is approximately 1085 m high. What is this measurement in feet? Use your graph to help you.
c A tunnel in the French Alps is 3400 feet long. Approximately what is the measurement in metres?

Mount Everest.

6 Mount Rubakumar, on the planet Ktorides is 1800 *Squidges* high. This measurement is equivalent to 3450 *Splooges*.

a Draw a conversion graph for *Squidges* and *Splooges*.
b If Mount Otsuki, also on planet Ktorides, is 1200 *Splooges* high, what is this measurement in *Squidges*?
c There are, in fact, 80 *Ploggs* in a *Splooge*. If Mount Adil on planet Ktorides is 1456 *Squidges* high, what is the measurement in *Ploggs* ?

13.5 More money

You have used graphs to convert from one currency to another. However, if you know the **exchange rate**, then you can make conversions without a graph.

Working with money is the same as working with decimal fractions, because most money amounts are given as decimals. Remember though, that when you work with money you need to include the units ($ or cents) in your answers.

REWIND

Before trying this section it will be useful to remind yourself about working with fractions from chapter 5. ◀

Foreign currency

The money a country uses is called its currency. Each country has its own currency and most currencies work on a decimal system (100 small units are equal to one main unit). The following table shows you the currency units of a few different countries.

Country	Main unit	Smaller unit
USA	Dollar ($)	= 100 cents
Japan	Yen (¥)	= 100 sen
UK	Pound (£)	= 100 pence
Germany	Euro (€)	= 100 cents
India	Rupee (₹)	= 100 paise

Worked example 8

Convert £50 into Botswana pula, given that £1 = 9.83 pula.

£1 = 9.83 pula
£50 = 9.83 pula × 50 = 491.50 pula

Worked example 9

Convert 803 pesos into British pounds given that £1 = 146 pesos.

146 pesos = £1

So 1 peso = £$\frac{1}{146}$

803 pesos = £$\frac{1}{146} \times 803$ = £5.50

Exercise 13.7

Applying your skills

1 Find the cost of eight apples at 50c each, three oranges at 35c each and 5 kg of bananas at $2.69 per kilogram.

2 How much would you pay for: 240 textbooks at $15.40 each, 100 pens at $1.25 each and 30 dozen erasers at 95c each?

3 If 1 Bahraini dinar = £2.13, convert 4000 dinar to pounds.

4 If US $1 = £0.7802, how many dollars can you buy with £300?

R is the symbol for Rands.

5 An American tourist visits South Africa with $3000. The exchange rate when she arrives is $1 = 12.90. She changes all her dollars into rands and then spends R900 per day for seven days. She changes the rands she has left back into dollars at a rate of $1 = R12.93. How much does she get in dollars?

Summary

Do you know the following?

- There are several measuring systems, the most widely used being metric and imperial.
- Every measurement quoted to a given accuracy will have both a lower bound and an upper bound. The actual value of a measurement is greater than or equal to the lower bound, but strictly less than the upper bound.
- You can draw a graph to help convert between different systems of units.
- Countries use different currencies and you can convert between them if you know the exchange rate.

Are you able to …?

- convert between various metric units
- calculate upper and lower bounds for numbers rounded to a specified degree of accuracy
- calculate upper and lower bounds when more than one rounded number is used in a problem E
- draw a conversion graph
- use a conversion graph to convert between different units
- convert between currencies when given the exchange rate.

Examination practice

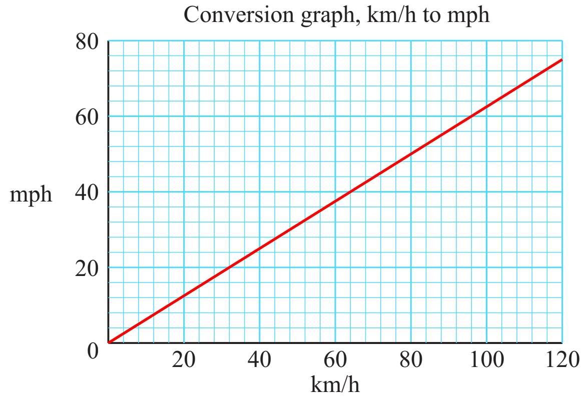
Exam-style questions

1 A cuboid has dimensions 14.5 cm, 13.2 cm and 21.3 cm. These dimensions are all given to 1 decimal place. Calculate the upper and lower bounds for the volume of the cuboid in:

a cm^3 **b** mm^3

Give your answers in standard form.

2 The graph shows the relationship between speeds in mph and km/h.



Use the graph to estimate:

a the speed, in km/h, of a car travelling at 65 mph
b the speed, in mph, of a train travelling at 110 km/h.

3 You are given that $a = 6.54$ (to 3 significant figures) and $b = 123$ (to 3 significant figures). Calculate upper and lower bounds for each of the following, give your answers to 3 significant figures:

a $a + b$ **b** ab **c** $\frac{a}{b}$ **d** $b - \frac{1}{a}$

Past paper questions

1 A carton contains 250 ml of juice, correct to the nearest millilitre.

Complete the statement about the amount of juice, j ml, in the carton.

$\ldots\ldots \leqslant j < \ldots\ldots$ [2]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q11 October/November 2012]

2 George and his friend Jane buy copies of the same book on the internet.

George pays \$16.95 and Jane pays £11.99 on a day when the exchange rate is \$1 = £0.626.
Calculate, in dollars, how much more Jane pays. [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q6 May/June 2013]

3 Joe measures the side of a square correct to 1 decimal place.

He calculates the **upper** bound for the area of the square as 37.8225 cm^2.

Work out Joe's measurement for the side of the square. [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q8 May/June 2013]

4 The length, l metres, of a football pitch is 96 m, correct to the nearest metre.

Complete the statement about the length of this football pitch.

$\ldots\ldots \leqslant j < \ldots\ldots$ [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q6 October/November 2014]

E

5 The base of a triangle is 9 cm correct to the nearest cm.

The area of this triangle is $40\,cm^2$ correct to the nearest $5\,cm^2$.

Calculate the upper bound for the perpendicular height of this triangle. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q13 May/June 2016]