Chapter 21: Ratio, rate and proportion

Key words

- Ratio
- Scale
- Rate
- Speed
- Direct proportion
- Unitary method
- Ratio method
- Inverse proportion

In this chapter you will learn how to:

- record relationships using ratio notation
- find one quantity when the other is given
- divide amounts in a given ratio
- make sense of scales on maps, models and plans
- read and interpret rates
- calculate average speed
- EXTENDED: solve problems using distance–time and speed–time graphs
- understand what is meant by direct and inverse proportion
- solve problems involving proportionate amounts
- EXTENDED: use algebra to express direct and inverse proportion
- EXTENDED: increase and decrease amounts by a given ratio.

This is an architectural model of the Louvre Abu Dhabi Museum on Saadiyat Island; the man in the photo is the architect, Jean Nouvel. The model is approximately a 1:90 version of the real building, in other words, the real building will be 90 times larger than the model.

A ratio is a comparison of amounts in a particular order. The amounts are expressed in the same units and are called the terms of the ratio. A ratio is usually written in the form $a:b$. Actual measurements are not given in a ratio, what is important is the proportion of the amounts. Ratio is used when working with scale on maps, models and plans.

A rate is a comparison of two different quantities. Speed is a rate which compares distance travelled to the time taken. Other examples of rates in daily life are cost per kilogram of foods, beats per minute in medicine, runs per over in cricket, kilometres per litre of petrol and exchange rates of foreign currencies.

RECAP

You should already be familiar with the following concepts from work on ratio, scale, rate and proportion:

Ratio (Chapter 5; Year 9 Mathematics)

A ratio compares amounts in a specific order, for example 2:5 or 1:2:3.

You can write ratios in the form of $a:b$ or as fractions $\frac{a}{b}$.

As ratios can be expressed as fractions, you can simplify them, find equivalent ratios and calculate with ratios in the same way as you did with fractions.

$$\frac{8}{10} = \frac{4}{5} \quad \text{and} \quad 8:10 = 4:5$$

Scale and ratio (Chapter 15)

Scale is a ratio. It can be expressed as: length on diagram:length in real world.

For example 1:500 means that one unit on the diagram represents 500 of the same units in the real world.

Rate (Year 9 Mathematics)

A rate is a comparison between two quantities measured in different units.

Average speed is a very common rate. It compares distance travelled (km) and time (hrs) to get a rate in km/hr.

Proportion (Year 9 Mathematics)

When two quantities are in direct proportion, they increase or decrease at the same rate.

The graph of a directly proportional relationship is a straight line that passes through the origin. Conversion graphs show directly proportional relationships.

21.1 Working with ratio

When you speak out a ratio, you use the word 'to' so, 5:2 is said as '5 to 2'.

A **ratio** is a numerical comparison of two amounts. The order in which you write the amounts is very important. For example, if there is one teacher for every 25 students in a school, then the ratio of teachers to students is 1:25.

Ratios can also be written as fractions. A ratio of 1:25 can be written as $\frac{1}{25}$ and a ratio of 5:3 can be written as $\frac{5}{3}$.

When you write two quantities as a ratio, you must make sure they are both in the same units before you start. For example, the ratio of 20 c to \$1 is not 20:1, it is 20:100 because there are 100 cents in a dollar.

Writing ratios in simplest form

Ratios are in their simplest form when you write them using the smallest whole numbers possible. The ratio of 20:100 above is not in its simplest form. You can simplify ratios in the same way that you simplified fractions. $\frac{20}{100} = \frac{2}{10} = \frac{1}{5}$ so 20:100 = 1:5.

REWIND

If you have forgotten how to simplify fractions, look again at chapter 5. ◀

LINK

Ratios are particularly important when considering the components of various food items. What proportion is water? Salt? Sugar? This all helps us to understand the impact on the human body.

Worked example 1

Sanjita mixes eight litres of white paint with three litres of red paint to get pink paint. What is the ratio of:

a red paint to white paint?
b white paint to the total amount of paint in the mixture?
c red paint to the total amount of paint in the mixture?

a 3 litres to 8 litres = 3:8

b 8 litres white out of a total of 11 litres so 8:11 is the required ratio

c 3 litres red out of a total of 11 litres so 3:11 is the required ratio

Worked example 2

To make concrete, you mix cement, sand and gravel in the ratio 1:2:4.

a What is ratio of cement to gravel?
b What is the ratio of sand to gravel?
c What is the ratio of gravel to the total amount of concrete?
d What fraction of the concrete is cement?

a Cement to gravel is 1:4

b Sand to gravel is 2:4 = 1:2

c Concrete = 1 + 2 + 4 = 7 parts ∴ gravel is 4 parts out of 7, so 4:7 is the required ratio

d Concrete = 1 + 2 + 4 = 7 ∴ cement = $\frac{1}{7}$

Exercise 21.1

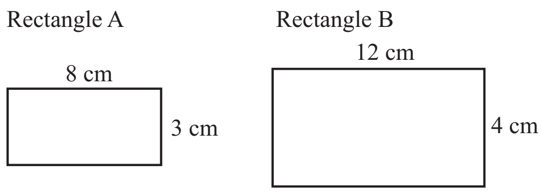
1 Write each of the following as a ratio.

a Nine men to nine women.
b One litre to five litres.
c 25 minutes to 3 minutes.
d 18 seconds out of every minute.
e 15c out of every \$1.
f two millimetres out of every centimetre.

2 A packet of sweets contains 12 red and five yellow sweets. What is the ratio of:

a red to yellow sweets
b yellow to red sweets?

3 Look at these two rectangles.



Express the following relationships as ratios:

a length of rectangle A to length of rectangle B
b width of rectangle A to width of rectangle B
c perimeter of rectangle A to perimeter of rectangle B
d area of rectangle A to area of rectangle B.

4 The table gives the mean life expectancy (in years) of some African animals.

Animal	Life expectancy (years)
Tortoise	120
Parrot	50
Elephant	35
Gorilla	30
Lion	15
Giraffe	10

Find the ratio of the life expectancies of:

a giraffe to tortoise
b lion to gorilla
c lion to tortoise
d elephant to gorilla
e parrot to lion
f parrot to tortoise.

5 What is the ratio of:

a a millimetre to a centimetre
b a centimetre to a metre
c a metre to a centimetre
d a gram to a kilogram
e a litre to a millilitre
f a minute to an hour?

6 Express the following as ratios in their simplest form.

a 25 litres to 50 litres
b 25c to \$2.00
c 75 cm to 2 m
d 600 g to five kilograms
e 15 mm to a metre
f 2.5 g to 50 g
g 4 cm to 25 mm
h 400 ml to 3 ℓ

REWIND

It may be useful to revise common factors from chapter 1. ◀

Equivalent ratios

Equivalent ratios are basically the same as equivalent fractions. If you multiply or divide the terms of the ratio by the same number (except 0) you get an equivalent ratio.

Worked example 3

For each of the following ratios find the missing value:

a 1:4 = x:20
b 4:9 = 24:y.

Method 1: multiplying by a common factor.

a	1:4 = x:20 1:4 = 5:20	20 ÷ 5 = 4, so 20 = 4 × 5 1 must be multiplied by 5 as well.
b	4:9 = 24:y 4:9 = 24:54	24 ÷ 4 = 6, so 4 × 6 = 24 9 must be multiplied by 6 as well.

Method 2: cross multiplying fractions.

a	$\frac{1}{4} = \frac{x}{20}$ $x = \frac{1 \times 20}{4}$ $x = 5$	Write the ratios as fractions. Solve the equation by multiplying both sides by 20.
b	$\frac{4}{9} = \frac{24}{y}$ $\frac{9}{4} = \frac{y}{24}$ $\frac{9 \times 24}{4} = y$ $\frac{216}{4} = y$ $y = 54$	Write each ratio as a fraction. Take the reciprocal of the fractions (turn them upside down) to get y at the top and make the equation easier to solve. Solve the equation by multiplying both sides by 24.

REWIND

You learned about the reciprocals of fractions in chapter 5. ◀

Equivalent ratios are useful when you need to solve problems involving a missing amount.

Exercise 21.2

1 Find the unknown values in the following equivalent ratios. Use whichever method you find easiest.

a $2:3 = 6:x$	**b** $6:5 = y:20$	**c** $12:8 = 3:y$	**d** $27:x = 9:2$
e $3:8 = 66:x$	**f** $1:5 = 13:y$	**g** $x:25 = 7:5$	**h** $40:9 = 800:y$
i $3:7 = 600:y$	**j** $2:7 = 30:x$	**k** $1.5:x = 6:5$	**l** $\frac{1}{6}:\frac{1}{3} = 2:y$

2 Use the equation (cross multiplying fractions) method to find the unknown values in the following equivalent ratios.

a $x:20 = 3:4$	**b** $12:21 = x:14$	**c** $2:5 = 8:y$	**d** $3:5 = x:4$
e $1:10 = x:6$	**f** $8:13 = 2:y$	**g** $4:5 = x:7$	**h** $5:4 = 9:y$

3 Say whether these statements are true or false. If a statement is false, explain why it is false.

a The ratio 1:6 is the same as the ratio 6:1.
b The ratio 1:6 is equivalent to 3:18.
c The ratio 20:15 can be expressed as 3:4.
d If the ratio of a mother's age to her daughter's age is 8:1, the daughter will be nine when her mother is 48 years old.
e If Mr Smith's wages are $\frac{5}{8}$ of Mr Jones' wages, then the ratio of their wages is 20:32.

Applying your skills

4 An alloy is a mixture of metals. Most of the gold used in jewellery is an alloy of pure gold and other metals which are added to make the gold harder. Pure gold is 24 carats (ct), so 18 carat gold is an alloy of gold and other metals in the ratio 18:6. In other words, $\frac{18}{24}$ parts pure gold and $\frac{6}{24}$ other metals.

a A jeweller makes an 18 ct gold alloy using three grams of pure gold. What mass of other metals does she add?
b An 18 ct gold chain contains four grams of pure gold. How much other metal does it contain?
c What is the ratio of gold to other metals in 14 ct gold?
d What is the ratio of gold to other metals in 9 ct gold?

5 An alloy of 9 ct gold contains gold, copper zinc and silver in the ratio 9:12.5:2.5.

a Express this ratio in simplest form.
b How much silver would you need if your alloy contained six grams of pure gold?
c How much copper zinc would you need to make a 9 ct alloy using three grams of pure gold?

6 An epoxy glue comes in two tubes (red and black), which have to be mixed in the ratio 1:4.

a If Petrus measures 5 ml from the red tube, how much does he need to measure from the black tube?
b How much would you need from the red tube if you used 10 ml from the black tube?

7 A brand of pet food contains meat and cereal in the ratio 2:9. During one shift, the factory making the pet food used 3500 kg of meat. What mass of cereal did they use?

Dividing a quantity in a given ratio

Ratios can be used to divide or share quantities. There are two ways of solving problems like these.

- Method 1: find the value of one part. This is the **unitary method**.

  1 Add the values in the ratio to find the total number of parts involved.

  2 Divide the quantity by the total number of parts to find the quantity per part (the value of one part).

  3 Multiply the values in the ratio by the quantity per part to find the value of each part.

- Method 2: express the shares as fractions. This is the **ratio method**.
  1. Add the values in the ratio to find the total number of parts involved.
  2. Express each part of the ratio as a fraction of the total parts.
  3. Multiply the quantity by the fraction to find the value of each part.

Worked example 4

Share \$24 between Jess and Anne in the ratio 3:5.

Method 1

$3 + 5 = 8$ $24 \div 8 = 3$ Jess gets \$9, Anne gets \$15.	There are 8 parts in the ratio. This is the value of 1 part. Jess gets 3 parts:$3 \times 3 = 9$ Anne gets 5 parts:$5 \times 3 = 15$

Method 2

$3 + 5 = 8$ Jess gets $\frac{3}{8}$ of \$24 $= \frac{3}{8} \times 24 = \$9$ Anne gets $\frac{5}{8}$ of \$24 $= \frac{5}{8} \times 24 = \$15$.	There are 8 parts in the ratio. Express each part as a fraction of the total parts and multiply by the quantity.

Exercise 21.3

1 Divide:

a 200 in the ratio 1:4	**b** 1500 in the ratio 4:1
c 50 in the ratio 3:7	**d** 60 in the ratio 3:12
e 600 in the ratio 3:9	**f** 38 in the ratio 11:8
g 300 in the ratio 11:4	**h** 2300 in the ratio 1:2:7.

2 Fruit concentrate is mixed with water in the ratio of 1:3 to make a fruit drink. How much concentrate would you need to make 1.2 litres of fruit drink?

3 Josh has 45 marbles. He shares them with his friend Ahmed in the ratio 3:2. How many marbles does each boy get?

4 \$200 is to be shared amongst Annie, Andrew and Amina in the ratio 3:4:5. How much will each child receive?

5 A line 16 cm long is divided in the ratio 3:5. How long is each section?

The capital letters N:P:K on fertiliser bags are the chemical symbols for the elements. The ratio of chemicals is always given on packs of fertiliser.

6 A bag of N:P:K fertiliser contains nitrogen, phosphorus and potassium in the ratio 2:3:3. Work out the mass of each ingredient if the bags have the following total masses:

a one kilogram **b** five kilograms **c** 20 kilograms **d** 25 kilograms.

7 The lengths of the sides of a triangle are in the ratio 4:5:3. Work out the length of each side if the triangle has a perimeter of 5.4 metres.

8 A rectangle has a perimeter of 120 cm. The ratio of its length to its breadth is 5:3. Sketch the rectangle and indicate what the lengths of each side would be.

9 In a group of 3200 elderly people, the ratio of men to women is 3:5. Calculate how many men there are in the group.

10 **a** Show that the ratio *area:circumference* for any circle is r:2, where r is the radius of the circle

b Find the ratio *volume:surface* area for a sphere of radius r, giving your answer as simply as possible

21.2 Ratio and scale

REWIND

Scale drawings were discussed in more detail in chapter 15. ◀

Scale drawings (maps and plans) and models such as the one of the Louvre Museum in Abu Dhabi (on page 453), are the same shape as the real objects but they are generally smaller.

Scale is a ratio. It can be expressed as 'length on drawing:real length'.

The scale of a map, plan or model is usually given as a ratio in the form of 1:n. For example, the architects who designed the new Louvre building for Abu Dhabi made a 6 m wide scale model of the domed roof using aluminium rods to test how light would enter the dome. The scale of this model is 1:33.

A scale of 1:33 means that a unit of measurement on the model must be multiplied by 33 to get the length (in the same units) of the real building. So, if the diameter of the dome in the model was 6 m, then the diameter of the real dome will be 6 m × 33 = 198 m.

Expressing a ratio in the form of 1: *n*

Some scale drawings, such as diagrams of cells in Biology, are larger than the real items they show. In an enlargement the scale is given in the form of n:1 (where $n > 1$).

All ratio scales must be expressed in the form of 1:n or n:1.

To change a ratio so that one part = 1, you need to divide both parts by the number that you want expressed as 1.

Worked example 5

Express 5:1000 in the form of 1: n

$5:1000$ $= \frac{5}{5} : \frac{1000}{5}$ $= 1:200$	Divide both sides by 5, i.e. the number that you want expressed as 1.

Worked example 6

Express 4 mm:50 cm as a ratio scale.

4 mm:50 cm $= 4\text{mm} : 500\text{mm}$	Express the amounts in the same units first.
$= 4:500$ $= \frac{4}{4} : \frac{500}{4}$ $= 1:125$	Divide by 4 to express in the form 1 : n.

Worked example 7

The form of 1 : n or n : 1 does not always the give a ratio with whole number parts.

Write 22 : 4 in the form of n : 1.

22:4 $= \frac{22}{4} : \frac{4}{4}$	Divide both sides by 4, i.e. the number that you want expressed as 1.
$= 5.5:1$	In this form you may get a decimal answer on one side.

Solving scale problems

There are two main types of problems involving scale:

- calculating the real lengths of objects from a scaled diagram or model,
  Real length = diagram length × scale
- calculating how long an object on the diagram will be if you are given the scale,
  Diagram length = real length ÷ scale

Worked example 8

The scale of a map is 1:25 000.

a What is real distance between two points that are 5 cm apart on the diagram?
b Express the real distance in kilometres.

a	Distance on map = 5 cm Scale = 1:25 000 ∴ Real distance = 5 cm × 25 000 = 125 000 cm The real distance is 125 000 cm.	Multiply the map distance by the scale. The units in your answer will the same units as the map distance units.
b	1 km = 100 000 cm 125 000 cm ÷ 100 000 = 1.25 km	From part (a) you know the real distance = 120 000 cm. You know that 1 km = 100 000 cm. So convert the real distance to km.

Worked example 9

A dam wall is 480 m long. How many centimetres long would it be on a map with a scale of 1:12 000?

Real length = 480 m
Scale = 1:12 000
Map length = real length ÷ scale
= 480 ÷ 12 000
= 0.04 m
1 m = 100 cm
0.04 × 100 = 4 cm
The dam wall would be 4 cm long on the map.

Exercise 21.4

1 Write each of the following scales as a ratio in the form

i 1:n **ii** n:1

a 1 cm to 2 m **b** 2 cm to 5 m **c** 4 cm to 1 km
d 5 cm to 10 km **e** 3.5 cm to 1 m **f** 9 mm to 150 km.

2 A scaled diagram of a shopping centre is drawn at a scale of 1:400. Find the real distance in metres of the following lengths measured on the diagram.

a 1 cm **b** 15 mm **c** 3.5 cm **d** 12 cm.

3 A map has a scale of 1:50 000. How long would each of these real lengths be on the map?

a 60 m **b** 15 km **c** 120 km **d** 75.5 km.

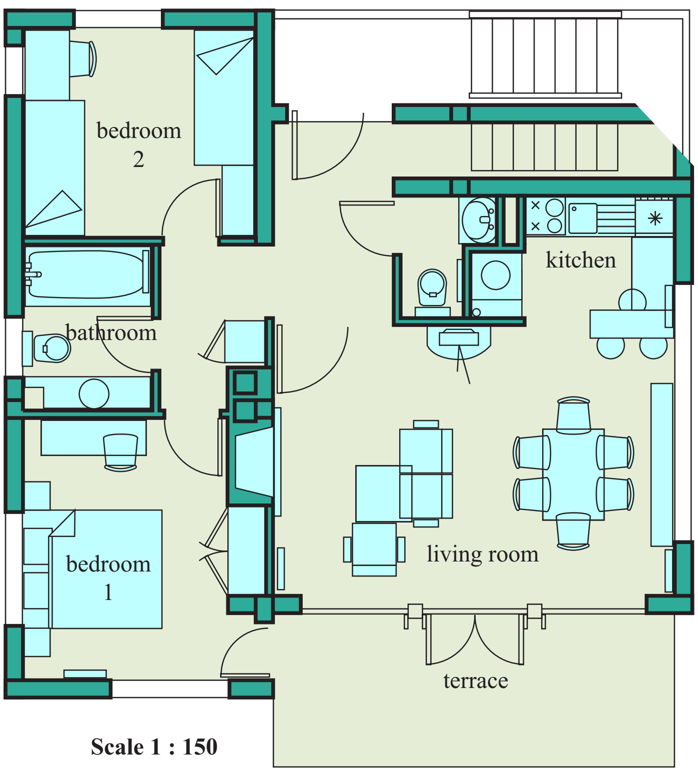
4 A rectangular hall is 20 m long and 50 m wide. Draw scaled diagrams of this hall using a scale of:

a 1:200 **b** 4 mm to 1 m.

Applying your skills

5 Use this map to find the straight line distance between:

a New Delhi and Bangalore **b** Mumbai and Kolkata **c** Srinagar and Nagpur.



6 The scale used for this floor plan of a house is 1:150.

a What is real distance in metres is represented by 1 cm on the plan?

b Calculate the real length in metres of:

i the length of living room

ii the breadth of the living room

iii the length of the bath

iv the breadth of the terrace.

Remember to measure from inside wall to inside wall.

c What is the real area of:

i bedroom 1 **ii** bedroom 2 **iii** the terrace?

Work out the actual size of *each* length before you calculate the area.

d Calculate how much floor space there is in the bathroom (in m^2). (Include the toilet in the floor space.)

e Calculate the cost of tiling the bathroom floor if the tiles cost \$25.99 per square metre and the tiler charges \$15.25 per square metre for laying the tiles.

21.3 Rates

A **rate** is a comparison of two different quantities. In a rate, the quantity of one thing is usually given in relation to one unit of the other thing. For example, 750 ml per bottle or 60 km/h. The units of *both* quantities must be given in a rate.

Rates can be simplified just like ratios. They can also be expressed in the form of 1:n. You solve rate problems in the same ways that you solve ratio and proportion problems.

Worked example 10

492 people live in an area of 12 km². Express this as a rate in its simplest terms.

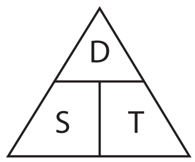
492 people in 12 km²	
$= \frac{492}{12}$ people per km²	Divide by 12 to get a rate per unit.
= 41 people/km²	Don't forget to write the units.

Average speed

A rate like km/h may sometimes be written kph (the 'p' stands for 'per').

Average **speed** (km/h) is one of the most commonly used rates. You need to be able to work with speed, distance and time quantities to solve problems.

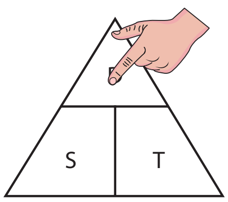
Use the Distance–Time–Speed triangle (shown here on the right) when you have to solve problems related to distance, time or speed.



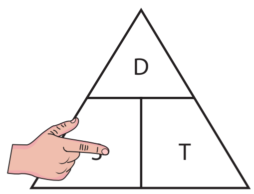
LINK

Nurses and other medical support staff work with ratio and rates when they calculate medicine doses, convert between units of measurement and set the patient's drips to supply the correct amount of fluid per hour.

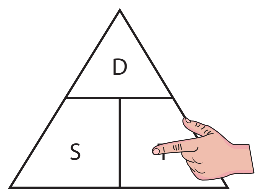
If you cover the letter of the quantity you need to find, then the remaining letters in the triangle give you the calculation you need to do (a multiplication or a division). For example:



$D = S \times T$



$S = \frac{D}{T}$



$T = \frac{D}{S}$

Worked example 11

A bus travels 210 km in three hours, what is its average speed in km/h?

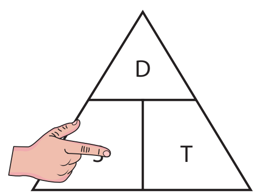
Speed $= \frac{D}{T}$

Distance = 210 km, Time = 3h,

$\therefore$ speed $= \frac{210}{3}$

$= 70$ km/h

Its average speed is 70 km/h.

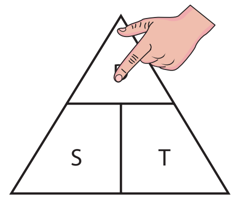


When you see the phrases 'how fast', 'how far' or 'for how long' you know you are dealing with a speed, distance or time problem.

Worked example 12

I walk at 4.5 km/h. How far can I walk in $2\frac{1}{2}$ hours at the same speed?

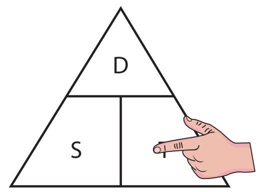
Distance = S × T
Speed = 4.5 km/h, T = 2.5 h,
∴ distance = 4.5 × 2.5 = 11.25 km
I can walk 11.25 km.



Worked example 13

How long would it take to cover 200 km at a speed of 80 km/h?

Time = $\frac{D}{S}$
Distance = 200 km, Speed = 80 km/h,
∴ time = $\frac{200}{80}$ = 2.5
It would take $2\frac{1}{2}$ hours.



Exercise 21.5 *Applying your skills*

1 Express each of these relationships as a rate in simplest form.

- **a** 12 kg for $5.
- **b** 120 litres for 1000 km.
- **c** $315 for three nights.
- **d** 5 km in 20 min.
- **e** 135 students for five teachers.
- **f** 15 hours spent per 5 holes dug.

2 A quarry produces 1200 t of crushed stone per hour. How much stone could it supply in:

- **a** an eight-hour shift
- **b** five shifts

3 Water leaks from a pipe at a rate of 5 ℓ/h. How much water will leak from the pipe in:

- **a** a day
- **b** a week?

4 A machine fills containers at a rate of 135 containers per minute. How long would it take to fill 1000 containers at the same rate?

5 Riku walks at 4.25 km/h. How far will he walk in three hours?

6 How far will a train travelling at 230 km/h travel in:

- **a** $3\frac{1}{2}$ hours
- **b** 20 minutes?

7 A plane flies at an average speed of 750 km/h. How far will it fly in:

- **a** 25 minutes
- **b** four hours?

8 A train left Cairo at 9 p.m. and travelled the 880 km to Aswan, arriving at 5 a.m. What was its average speed?

9 A runner completes a 42 km marathon in two hours 15 minutes. What was her average speed?

10 In August 2009, Usain Bolt of Jamaica set a world record by running 100 m in 9.58 seconds.

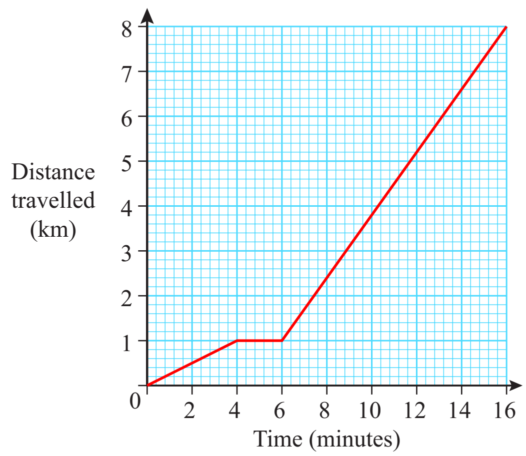
a Translate this speed into km/h.

b How long would it take him to run 420 m if he could run it at this speed?

21.4 Kinematic graphs

Distance–time graphs

Graphs that show the connection between the distance an object has travelled and the time taken to travel that distance are called distance–time graphs or travel graphs. On such graphs, time is normally shown along the horizontal axis and distance on the vertical. The graphs normally start at the origin because at the beginning no time has elapsed (passed) and no distance has been covered.



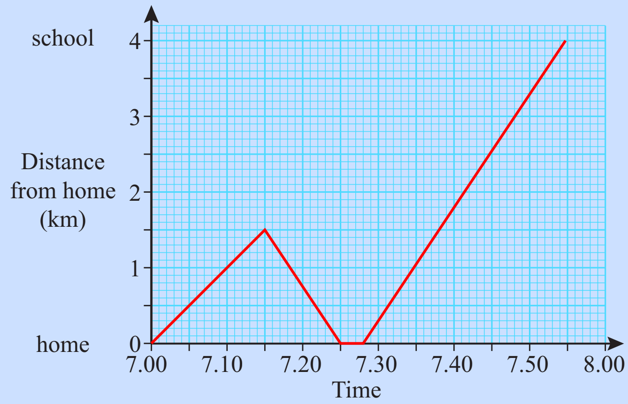
Look at the graph. Can you see that it shows the following journey:

- a cycle for 4 minutes from home to a bus stop 1 km away
- a 2 minute wait for the bus
- 7 km journey on the bus that takes 10 minutes.

The line of the graph remains horizontal while the person is not moving (waiting for the bus) because no distance is being travelled at this time. The steeper the line, the faster the person is travelling.

Worked example 14

Ashraf's school is 4 km from home and it takes him 40 minutes to walk to school. One morning he leaves at 7 a.m. After 15 minutes, he realises he has left his boots at home, so he runs back in 10 minutes. It takes him three minutes to find the boots. He runs at the same speed to school. The graph shows his journey.



a How far had he walked before he remembered his boots?
b What happens to the graph as he returns home?
c What does the horizontal line on the graph represent?
d How fast did he run in m/min to get back home?

a Ashraf walked 1.5 km before he remembered his boots.

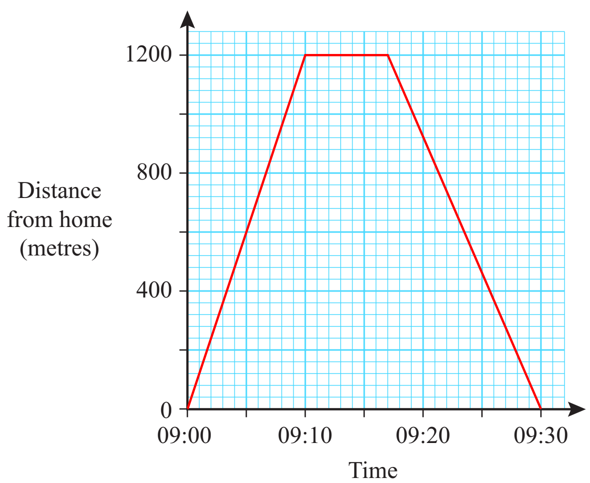
b The graph slopes downwards (back towards 0 km) as he goes home.

c The horizontal part of the graph corresponds with the three minutes at home.

d He runs 1.5 km in 10 minutes, an average speed 150 m/min.

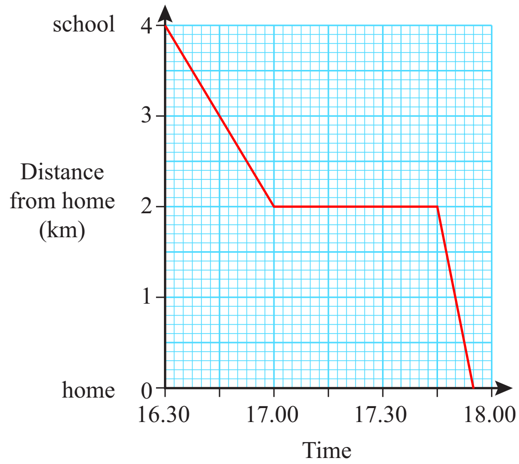
Exercise 21.6 *Applying your skills*

1 This distance–time graph represents Monica's journey from home to a supermarket and back again.



a How far was Monica from home at 09:06 hours?
b How many minutes did she spend at the supermarket?
c At what times was Monica 800 m from home?
d On which part of the journey did Monica travel faster, going to the supermarket or returning home?

2 Omar left school at 16.30. On his way home, he stopped at a friend's house before going home on his bicycle. The graph shows this information.



a How long did he stay at his friend's house?
b At what time did Omar arrive home?
c Omar's brother left school at 16.45 and walked home using the same route as Omar. If he walked at 4 km per hour, work out at what time the brother passed Omar's friend's house.

3 A swimming pool is 25 m long. Jasmine swims from one end to the other in 20 seconds. She rests for 10 seconds and then swims back to the starting point. It takes her 30 seconds to swim the second length.

a Draw a distance–time graph for Jasmine's swim.
b How far was Jasmine from her starting point after 12 seconds?
c How far was Jasmine from her starting point after 54 seconds?

Speed in distance–time graphs

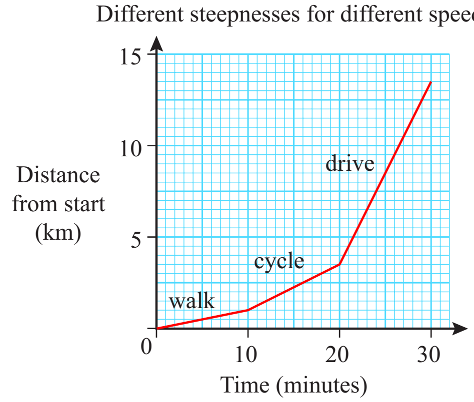
The steepness gradient of a graph gives an indication of speed.

- A straight line graph indicates a constant speed.
- The steeper the graph, the greater the speed.
- An upward slope and a downward slope represent movement in opposite directions.

The distance–time graph shown is for a person who walks, cycles and then drives for three equal periods of time. For each period, speed is given by the formula:

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

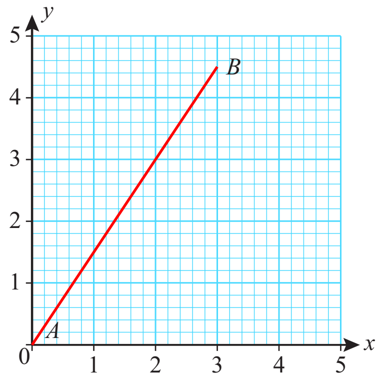
Different steepnesses for different speeds



Remember speed is a rate of change. A straight line from the origin shows a constant rate of change and you can use the formula $\frac{\text{change in } y}{\text{change in } x}$ to find the rate of change. If the gradient changes along the graph, the speed will be different for different time periods.

In mathematics, it is necessary to be more precise about what is meant by the steepness of a line. In the diagram, the steepness of line AB is measured by: $\dfrac{\text{increase in } y \text{ co-ordinate}}{\text{increase in } x \text{ co-ordinate}}$

This is the same as $\dfrac{\text{rise}}{\text{run}}$ or gradient of a straight line graph.



REWIND

You learned how to calculate the gradient of a straight line graph in chapter 10. ◀

REWIND

The gradient of a line is:

- positive if the line slopes up from left to right,
- negative if the line slopes down from left to right.

See chapter 10 if you need a reminder. ◀

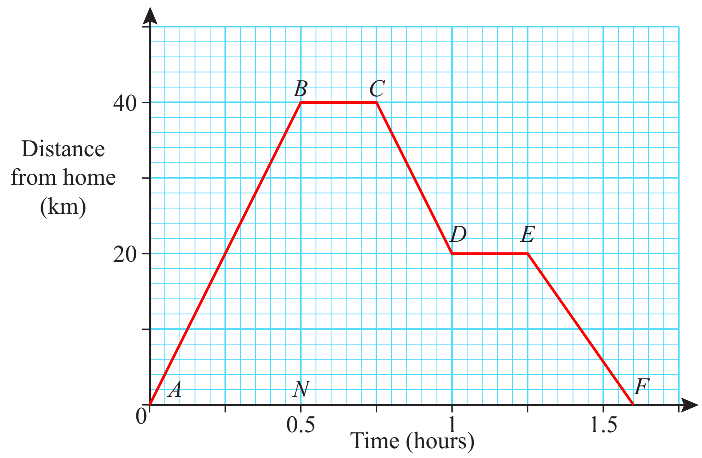
For a distance–time graph, a positive gradient indicates the object is moving in the direction of y, increasing, a zero gradient indicates the object is not moving and a negative gradient indicates the object is moving in the direction of y, decreasing.

For a distance–time graph:

$$\frac{\text{change in } y \text{ co-ordinate (distance)}}{\text{change in } x \text{ co-ordinate (time)}} = \frac{\text{time travelled}}{\text{time taken}} = \text{speed}$$

Thus, the gradient of the graph gives us the speed of the object and its direction of motion. This is known as the *velocity* of the object.

Here is another example:



The travel graph represents a car journey. The horizontal sections have zero gradient (so the car was stationary during these times).

For section AB, the gradient is positive.

$$\text{Gradient} = \frac{NB}{AN} = \frac{40}{0.5} = 80\,\text{km/h}$$

For section CD, the gradient is negative.

$$\text{Gradient} = \frac{20-40}{0.25} = -80\,\text{km/h}$$

$\therefore$ the velocity was 80 km/h in the direction towards home.

For section EF the gradient is negative.

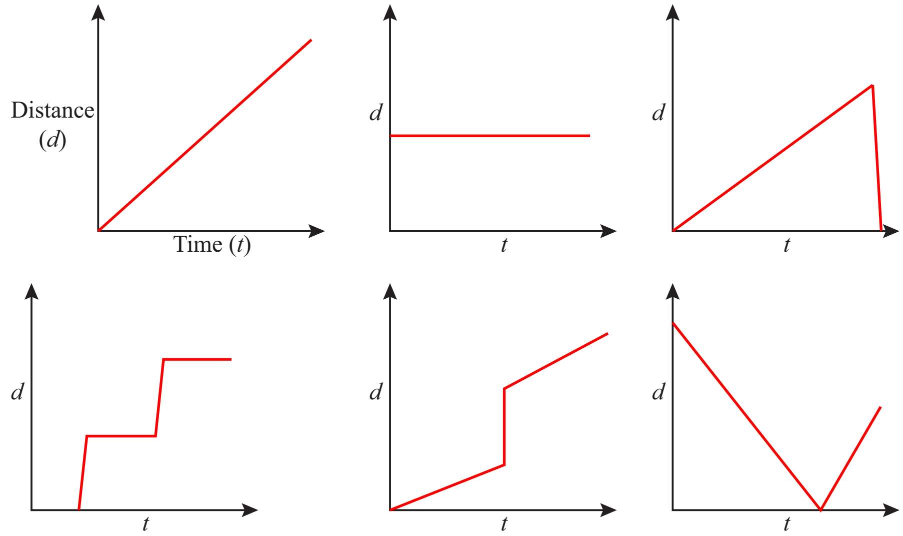
$$\text{Gradient} = \frac{0-20}{0.35} = -57.1\,\text{km/h}$$

$\therefore$ the velocity was 57.1 km/h in the direction towards home.

Exercise 21.7

1 **a** Clearly describe what is happening in each of the distance–time graphs below.

b Suggest a possible real life situation that would result in each graph.



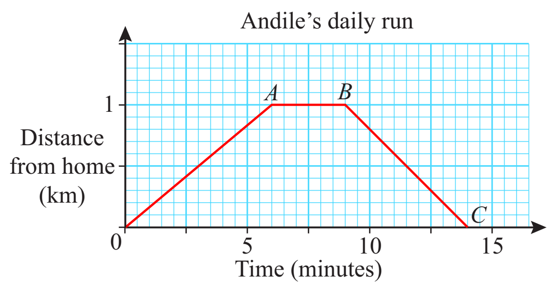
2 The graph shows Andile's daily run.

a For how many minutes does he run before taking a rest?

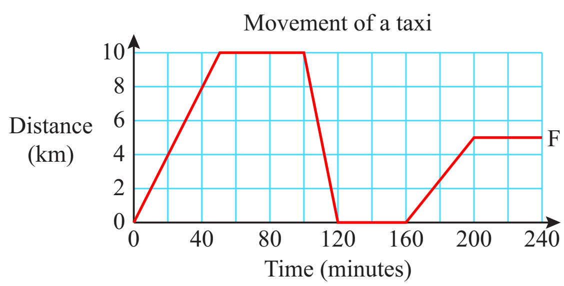
b Calculate the speed in km/h at which he runs before taking a rest.

c For how many minutes does he rest?

d Calculate the speed in m/s at which he runs back home.



3 This graph shows the movement of a taxi in city traffic during a 4-hour period.



a Clearly and concisely describe the taxi's journey.

b For how many minutes was the taxi waiting for passengers in this period? How can you tell this?

c What was the total distance travelled?

d Calculate the taxi's average speed during:

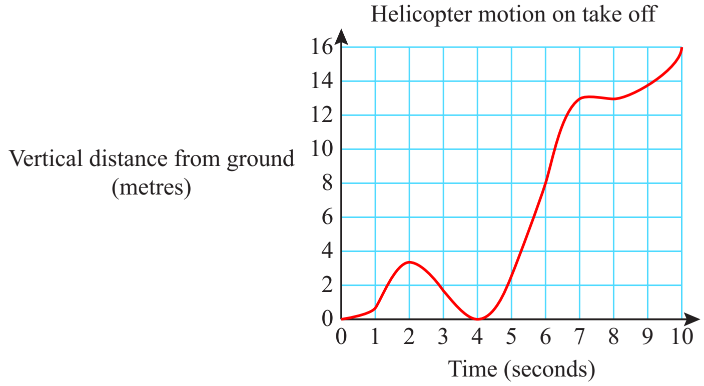
i the first 20 minutes

ii the first hour

iii from 160 to 210 minutes

iv for the full period of the graph.

4 This is a real distance–time graph showing distance from the ground against time for a helicopter as it takes off and flies away from an airport.

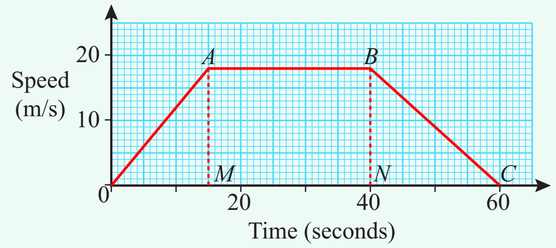


a Make up five mathematical questions that can be answered from the graph.
b Exchange questions with another student and try to answer each other's questions.

Speed–time graphs

In certain cases, the speed (or velocity) of an object may change. An increase in speed is called *acceleration*; a decrease in speed is called *deceleration*. A speed–time graph shows speed (rather than distance) on the vertical axis.

This graph shows a train journey between two stations.



- The train starts at zero speed.
- The speed increases steadily reaching 18 m/s after 15 seconds.
- The train travels at a constant speed (horizontal section) of 18 m/s for 25 seconds.
- The train then slows down at a steady rate till it stops.
- The entire journey took 60 seconds.

Look at the first part of the journey again.

The speed increased by 18 m/s in 15 seconds.

$\frac{18\text{ m/s}}{15\text{ seconds}}$ is the gradient of the line representing the first part of the journey.

This is a rate of 1.2 m/s every second. This is the rate of acceleration. It is written as 1.2 m/s^2 (or m/s/s).

For a speed–time graph, the gradient = acceleration.

A positive gradient (acceleration) is an increase in speed.

A negative gradient (deceleration) is a decrease in speed.

You may sometimes see m/s written as m s^{-1} and m/s^2 as m s^{-2}.

Distance travelled in a speed–time graph

You already know that distance = speed × time. On a speed–time graph, this is represented by the area of shapes under the sections of graph. You can use the graph to work out the distance travelled.

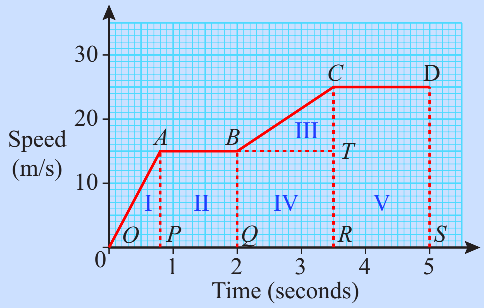
REWIND

You worked with distance, time and speed earlier in this chapter. Refer back if you have forgotten the formulae.

E

Worked example 15

This speed–time graph represents the motion of a particle over a period of five seconds.



a During which periods of time was the particle accelerating?
b Calculate the particle's acceleration 3 seconds after the start.
c Calculate the distance travelled by the particle in the 5 seconds.

a The particle was accelerating in the period 0 to 0.8 seconds (section *OA*) and in the period 2 to 3.5 seconds (section *BC*).

b Acceleration $= \dfrac{\text{speed}}{\text{time}}$

The acceleration was constant in the period 2 to 3.5 seconds, so the acceleration 3 seconds after the start is:

$$\frac{25-15\,(\text{m/s})}{3.5-2\,(\text{s})} = \frac{10}{1.5} = 6.7\,\text{m/s}^2$$

c Distance travelled = area under graph

$$\begin{aligned} &= \text{area I} + \text{area II} + \text{area III} + \text{area IV} + \text{area V} \\ &= \tfrac{1}{2}(0.8 \times 15) + (1.2 \times 15) + \tfrac{1}{2}(1.5 \times 10) + (1.5 \times 15) + (1.5 \times 25) \\ &= 6 + 18 + 7.5 + 22.5 + 37.5 \\ &= 91.5 \end{aligned}$$

The distance travelled is 91.5 m.

REWIND

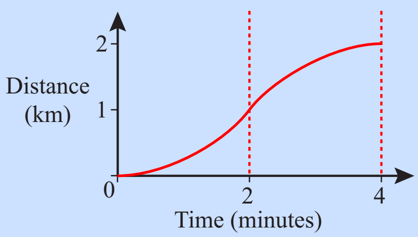
Apply the formulae for area of shapes that you learned in chapter 7 to find the area of the shapes under the graphs.

Units are important

When calculating acceleration and distance travelled from a speed–time graph, it is essential that the unit of speed on the vertical axis involves the same unit of time as on the horizontal axis. In the example above the speed unit was metres per second and the horizontal axis was graduated in seconds. These units are compatible.

If the units of time are not the same, it is essential that the unit on one of the axes is converted to a compatible unit.

Worked example 16

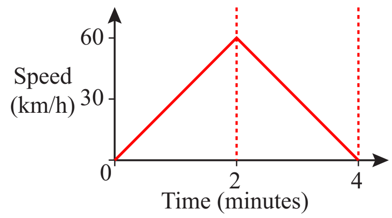


The diagram shown above is the distance–time graph for a short car journey. The greatest speed reached was 60 km/h. The acceleration in the first two minutes and the deceleration in the last two minutes are constant.

a Draw the speed–time graph of this journey.
b Calculate the average speed, in km/h, for the journey.

E

a



Since the acceleration and deceleration are both constant, the speed–time graph consists of straight lines. The greatest speed is 60 km/h.

b

$$\text{Average speed} = \frac{2\,\text{km}}{(4\ \text{minutes})}$$

Units of speed are more commonly m/s or km/h so the units need changing.

$$= \frac{2 \times 15}{60\ (\text{minutes})}$$

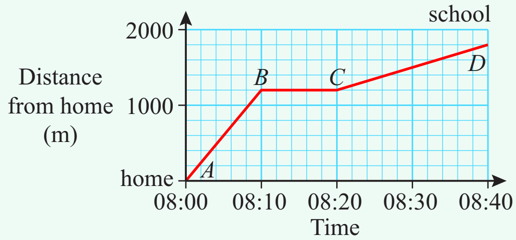
Multiply top and bottom by 15 to get 60 minutes (= one hour) on the bottom.

$= 30$ km/h

Calculate to give answer in km/h.

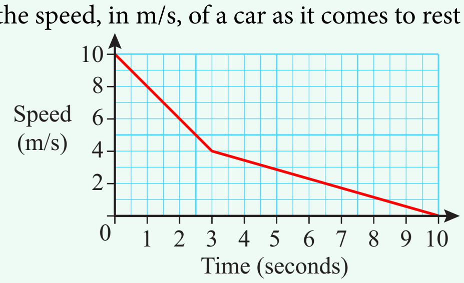
Exercise 21.8

1 The distance–time graph represents Ibrahim's journey from home to school one morning.



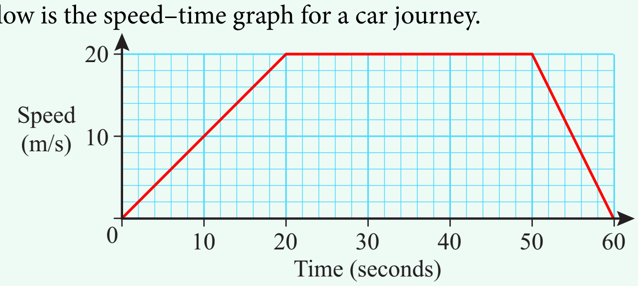
a How far was Ibrahim from home at 08:30 hours?
b How fast, in m/s, was Ibrahim travelling during the first 10 minutes?
c Describe the stage of Ibrahim's journey represented by the line *BC*.
d How fast, in m/s, was Ibrahim travelling during the last 20 minutes?

2 The graph below shows the speed, in m/s, of a car as it comes to rest from a speed of 10 m/s.



a Calculate the rate at which the car is slowing down during the first three seconds.
b Calculate the distance travelled during the 10 second period shown on the graph.
c Calculate the average speed of the car for this 10 second period.

3 The diagram below is the speed–time graph for a car journey.



a Calculate the acceleration during the first 20 seconds of the journey.
b Calculate the distance travelled in the last 10 seconds of the journey.
c Calculate the average speed for the whole journey.

21.5 Proportion

In mathematics, proportion is an equation or relationship between two ratios. In general, $a : b = c : d$.

Quantities are said to increase or decrease in proportion if multiplying (or dividing) one quantity by a value results in multiplying (or dividing) the other quantity by the same value. In other words, there is a constant ratio between the corresponding elements of two sets.

LINK

Home economists, chefs and food technologists use proportional reasoning to mix ingredients, convert between units or work out the cost of a dish.

Direct proportion

When two quantities are in **direct proportion** they increase or decrease at the same rate. In other words, the ratio of the quantities is equivalent. If there is an increase or decrease in one quantity, the other will increase or decrease in the same proportion.

Here are some examples of quantities that are in direct proportion:

Speed (km/h)	0	45	60	75	90	120
Distance covered in an hour (km)	0	45	60	75	90	120

Distance = speed × time, so the faster you drive in a set time, the further you will travel in that time.

Number of items	0	1	2	3	4
Mass (kg)	0	2	4	6	8

If one item has a mass of 2 kg, then two of the same item will have a mass of 4 kg and so on. The more you have of the same item, the greater the mass will be.

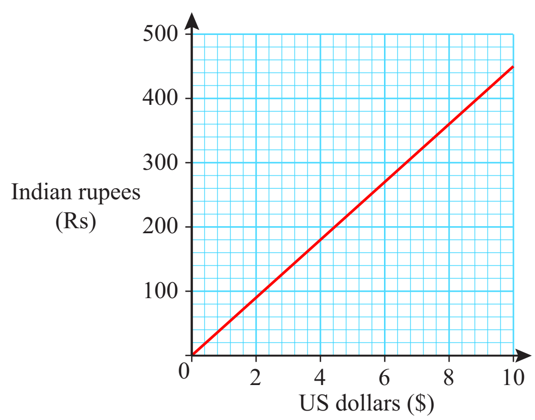
Number of hours worked	0	1	2	3
Amount earned ($)	0	12	24	36

The more hours you work, the more you earn.

Graphs of directly proportional relationships

If you graph a directly proportional relationship you will get a straight line that passes through the origin.

Of course, the converse (opposite) of this is also true. When a graph is a straight line that passes through the origin, one quantity is directly proportional to the other.



This graph shows the amount of Indian rupees you would get for different amounts of US dollars at an exchange rate of US$1:Rs 45. Exchange rates are a good example of quantities that are in direct proportion.

Exercise 21.9

1 Which of these could be examples of direct proportion?

a The length of a side of a square and its area.
b The ages and heights of students.
c The amount of money collected in a sponsored walk if you are paid 5c per kilometre.
d The time it takes to cover different distances at the same speed.
e The heights of objects and the lengths of their shadows.
f The amount of petrol used to travel different distances.
g The number of chickens you could feed with 20 kg of feed.
h The height of the tree and the number of years since it was planted.
i The area of the sector of a circle and the angle at the centre.

The unitary method

The unitary method is useful for solving a range of problems to do with proportion. In this method, you find the value of one unit of the quantity. For example, the price of one cupcake or the amount of rupees you would get for one dollar.

REWIND
You have already used the unitary method to solve ratio problems earlier in this chapter.

Worked example 17

Five bottles of perfume cost \$200. What would 11 bottles cost?

5 bottles cost \$200
1 bottle costs \$200 ÷ 5 = \$40
11 bottles cost 11 × \$40 = \$440

The same problem can be solved using the ratio method.

REWIND
You have already used the ratio method earlier in this chapter.

Worked example 18

Using the problem from worked example 17, let x be the cost of 11 bottles.

$\frac{5}{200} = \frac{11}{x}$	Write out each part as a fraction.
$\frac{200}{5} = \frac{x}{11}$	Take the reciprocal both ratios (tum them upside down) to make it easier to solve the equation.
$\frac{200 \times 11}{5} = x$ $x = \$440$	

Exercise 21.10

1 Four soft drinks cost \$9. How much would you pay for three?

2 A car travels 30 km in 40 minutes. How long would it take to travel 45 km at the same speed?

3 If a clock gains 20 seconds in four days, how much does it gain in two weeks?

4 Six identical drums of oil weigh 90 kg in total. How much would 11.5 drums weigh?

5 An athlete runs 4.5 kilometres in 15 minutes. How far could he run in 35 minutes at the same speed?

Applying your skills

Familiarise yourself with the language used in word problems involving proportion. This will help you recognise proportion problems in other contexts.

6 To make 12 muffins, you need:

240 g flour
48 g sultanas
60 g margarine
74 ml milk
24 g sugar
12 g salt

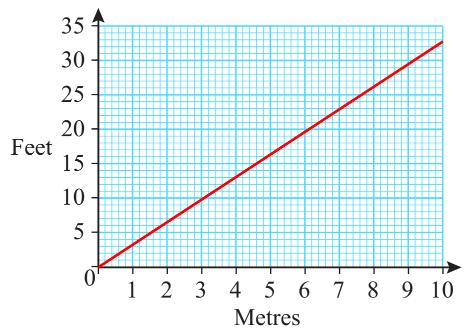
a How much of each ingredient would you need to make 16 muffins?
b Express the amount of flour to margarine in this recipe as a ratio.

7 A vendor sells frozen yoghurt in 250 g and 100 g tubs. It costs \$1.75 for 250 g and 80 cents for 100 g. Which is the better buy?

8 A car used 45 litres of fuel to travel 495 km.

a How far could the car travel on 50 ℓ of fuel at the same rate?
b How much fuel would the car use to travel 190 km at the same rate?

9 This graph shows the directly proportional relationship between lengths in metres (metric) and lengths in feet (imperial).



Feet is the plural of (one) foot.

a Use the graph to estimate how many feet there are in four metres.
b Given that 1 m = 3.28 feet and one foot = 0.305 m, calculate how many feet there are in 4 metres.
c Which is longer:

i four metres or 12 feet **ii** 20 feet or 6.5 metres?

d Mr Bokomo has a length of fabric 9 m long.

i What is its length to the nearest foot?
ii He cuts and sells 1.5 m to Mrs Johannes and 3 ft to Mr Moosa. How much is left in metres?

e A driveway was 18 feet long. It was resurfaced and extended to be one metre longer than previously. How long is the newly resurfaced driveway in metres?

Inverse proportion

In **inverse proportion**, one quantity decreases in the same proportion as the other quantity increases. For example, if work is to be done it can be done in less time if more people help. As the number of people who are working increases, the time it takes to get the job done decreases.

The graph of an inversely proportional relationship is a hyperbolic curve and not a straight line.

You can solve problems involving inverse proportion using either the ratio method or the unitary method.

Worked example 19

A person travelling at 30 km/h takes 24 minutes to get home from work.
How long would it take him if he travelled at 36 km/h?

30 km/h takes 24 minutes
So, 1 km/h would take 30×24 minutes
$\therefore$ at 36 km/h it would take $\frac{30 \times 24}{36}$
$= 20$ minutes.

Worked example 20

A woman working six hours per day can complete a job in four days.
How many hours per day would she need to work to complete the job in three days?

4 days takes 6 hours per day
$\therefore$ 1 day would take 4×6 hours $= 24$ hours per day
3 days would take $\frac{24}{3} = 8$ hours per day.

Exercise 21.11

1 A hurricane disaster centre has a certain amount of clean water. The length of time the water will last depends on the number of people who come to the centre. Calculate the missing values in this table.

No. of people	120	150	200	300	400
Days the water will last	40	32			

2 It takes six people 12 days to paint a building. Work out how long it would take at the same rate using:

a 9 people **b** 36 people

3 Sanjay has a 50 m long piece of rope. How many pieces can he cut it into if the length of each piece is:

a 50 cm **b** 200 cm **c** 625 cm?

d He cuts the rope into 20 equal lengths. What is the length of each piece?

4 An airbus usually flies from Mumbai to London in 11 hours at an average speed of 920 km/h. During some bad weather, the trip took 14 hours. What was the average speed of the plane on that flight?

5 A journey takes three hours when you travel at 60 km/h. How long would the same journey take at a speed of 50 km/h?

21.6 Direct and inverse proportion in algebraic terms

E

Direct proportion

If the values of two variables are always in the same ratio, the variables are said to be in direct proportion. If the variables are P and Q, you write this as $P \propto Q$.

This is read as P is *directly* proportional to Q.

$P \propto Q$ means that $\frac{P}{Q}$ is constant. That is, $P = kQ$ where k is a constant.

Tip

You might be asked to find a proportion equation and then find unknowns by solving. Recap how to solve equations from chapter 6.

E

If the constant is 2, then $P = 2Q$. This means that whatever P is, Q will be double that.

You can also write this as $\frac{P}{Q} = 2$.

Inverse proportion

If the product of the value of two variables is constant, the variables are said to be inversely proportional. If the variables are P and Q, you say $PQ = k$, where k is a constant. This means that P is inversely proportional to Q.

$PQ = k$ can also be written as $P = \frac{k}{Q}$.

So P is inversely proportional to Q can be written as $P \propto \frac{1}{Q}$.

The relationship between quantities is usually described in words and you will need to add the $\propto$ symbol in your working, as in these examples. The word 'direct' is not always used but if quantities are in inverse (or indirect) proportion this will be stated. Sometimes you will see, '*a* varies with *t*', instead of '*a* is in proportion to *t*.'

Worked example 21

y is directly proportional to x^3 and when $x = 2$, $y = 32$.

a Write this relationship as an equation.
b Find the value of y when $x = 5$.

a $y \propto x^3$ which means (as an equation) $y = kx^3$.
When $x = 2$, $x^3 = 2^3 = 8$
$\therefore\ 32 = 8k$
$\therefore\ k = \frac{32}{8} = 4$ and $y = 4x^3$

b If $x = 5$ then, $y = 4x^3 = 4 \times 5^3 = 500$

Worked example 22

F is inversely proportional to d^2 and when $d = 3$, $F = 12$.
Find the value of F when $d = 4$.

$F \propto \frac{1}{d^2}$ which means (as an equation) $F = \frac{k}{d^2}$.

When $d = 3$ and $F = 12$, $12 = \frac{k}{3^2} = \frac{k}{9}$

$\therefore\ k = 12 \times 9 = 108$ So, $F = \frac{108}{d^2}$

If $d = 4$ then, $F = \frac{108}{d^2} = \frac{108}{4^2} = 6.75$

Worked example 23

Some corresponding values of the variables p and q are shown in the table. Are p and q directly proportional?

p	2.8	7	11.2	16.8
q	2	5	8	12

E

Compare each pair in turn:

$\frac{2.8}{2} = 1.4$ $\quad$ $\frac{7}{5} = 1.4$ $\quad$ $\frac{11.2}{8} = 1.4$ $\quad$ $\frac{16.8}{12} = 1.4$

All the values are the same.
So, the values are directly proportional, $p = 1.4q$.

Worked example 24

x	3	4	5	6
y	12			

Copy and complete this table of values for:

a $y \propto x$

b $y \propto \frac{1}{x}$

a $y = kx$ — Write the relationship as an equation

$12 = 3k$ — Solve the equation for k

$\therefore\ k = 4$ and $y = 4x$. — Substitute the value of k into original equation

x	3	4	5	6
y	12	16	20	24

b $y \propto \frac{1}{x}$ means $xy = k$ — Write the relationship as an equation

$\therefore\ k = 3 \times 12 = 36$ and $y = \frac{36}{x}$. — Use a value of x and corresponding value of y, to solve the equation for k

x	3	4	5	6
y	12	9	7.2	6

Worked example 25

The speed of water in a river is determined by a water-pressure gauge.
The speed (v m/s) is directly proportional to the square root of the height (h cm) reached by the liquid in the gauge. Given that $h = 36$ when $v = 8$, calculate the value of v when $h = 18$.

$v \propto \sqrt{h}$ means that $v = k\sqrt{h}$ where k is constant.
When $v = 8$, $h = 36$ and so $8 = k\sqrt{36} = 6k$.

It follows that $k = \frac{4}{3}$ and the formula connecting v and h is $v = \frac{4\sqrt{h}}{3}$.

When $h = 18$, $v = \frac{4\sqrt{18}}{3} = 5.66$ (to 3 s.f.)

Exercise 21.12

E

1 For each of the following, y is inversely proportional to x. Write an equation expressing y in terms of x if:

a $y = 0.225$ when $x = 20$
b $y = 12.5$ when $x = 5$
c $y = 5$ when $x = 0.4$
d $y = 0.4$ when $x = 0.7$
e $y = 0.6$ when $x = 8$

Sometimes you may see 'varies as' written instead of 'is directly proportional to'. Similarly, if $y \propto \frac{1}{x}$, then 'y varies inversely as x'.

2 y is inversely proportional to x^3. If $y = 80$ when $x = 4$, find:

a the constant of proportionality
b the value of y when $x = 8$
c the value y when $x = 6$
d the value of x when $y = 24$

3 Given that y is inversely proportional to x^2. Complete the table.

x	0.1	0.25	0.5	
y			1	64

4 Given that y is inversely proportional to $\sqrt{x}$. Complete the table.

x	25	100		
y	10		26	50

5 x and y are known to be proportional to each other. When $x = 20$, $y = 50$. Find k, if:

a $y \propto x$
b $y \propto \dfrac{1}{x}$
c $y \propto x^2$

6 A is directly proportional to r^2 and when $r = 3$, $A = 36$. Find the value of A when $r = 10$.

7 l is inversely proportional to d^3. When $d = 2$, $l = 100$. Find the value of l when $d = 5$.

8 Some corresponding values of p and q are given in the table. Are p and q inversely proportional? Justify your answer.

q	2	5	8	12
p	75	30	20	15

9 An electric current I flows through a resistance R. I is inversely proportional to R and when $R = 3$, $I = 5$. Find the value of I when $R = 0.25$.

10 Corresponding values of s and t are given in the table.

s	2	6	10
t	0.4	10.8	50

Which of the following statements is true?
a $t \propto s$ **b** $t \propto s^2$ **c** $t \propto s^3$.

11 It takes 4 people 10 hours to plaster a section of a building. How long will it take 8 people to do the same job working at the same rate?

12 In an industrial experiment it is found that the force, f, needed to break a concrete beam varies inversely with the length, l, of the beam. If it takes 50 000 newtons to break a concrete beam 2 metres long, how many newtons will it take to break a beam that is 6 metres long?

E

13 A submarine crew discovers that the water temperature (°C) varies inversely with the depth to which they submerge (km). When they were at a depth of 4 km, the water temperature was 6 °C.

a What would the water temperature be at a depth of 12 km?

b To what depth would they need to submerge for the water temperature to be −1 °C?

14 Variable P varies directly as variable m and inversely as variable n. If $P = 24$ when $m = 3$ and $n = 2$, find P when $m = 5$ and $n = 8$.

21.7 Increasing and decreasing amounts by a given ratio

In worked example 17 you found the cost of 11 bottles of perfume having been given the cost of five bottles. This is an example of increasing an amount in a given ratio. You could have been asked to increase \$200 in the ratio 11 : 5.

Worked example 26

Increase \$200 in the ratio 11 : 5

$$\text{New value} : \text{original value} = 11 : 5$$
$$\text{New} : 200 = 11 : 5$$
$$\frac{\text{New}}{200} = \frac{11}{5}$$
$$\text{New value} = \frac{11 \times 200}{5} = \$440$$

Worked example 27

Decrease 45 m in the ratio 2:3

$$\text{New value} : \text{original value} = 2 : 3$$
$$\text{New} : 45 = 2 : 3$$
$$\frac{\text{New}}{45} = \frac{2}{3}$$
$$\text{New value} = \frac{2 \times 45}{3} = 30\,\text{m}$$

Exercise 21.13

1 Increase 40 in the ratio 7:5.

2 Decrease 32 in the ratio 3:4.

3 Increase 84 in the ratio 5:4.

4 Decrease 57 in the ratio 2:3.

5 Nick has a picture of his dog that is 16 cm long and 10 cm wide. If he enlarges the picture in the ratio 5 : 2, what are the new dimensions?

Summary

Do you know the following?

- A ratio is a comparison of two or more quantities in a set order. Ratios can be expressed in the form a:b or $\frac{a}{b}$. Ratios have no units.
- Ratios can be simplified by multiplying or dividing both quantities by the same number. This method produces equivalent ratios.
- Map scales are good examples of ratios in everyday life. The scale of a map is usually given in the form 1:n. This allows you to convert map distances to real distances using the ratio scale.
- A rate is a comparison of two different quantities. Usually a rate gives an amount of one quantity per unit of the other. Rates must include the units of the quantities.
- Speed is one of the most common rates of change. Speed = distance ÷ time.
- Kinematic graphs are used to show relationships between:
  - distance and time (distance–time graph)
  - speed and time (speed–time graph) and to solve problems in these areas.
- The gradient of a distance-time graph shows how the speed changes over time.
- Proportion is a constant ratio between the corresponding elements of two sets.
- When quantities are in direct proportion they increase or decrease at the same rate.
- When quantities are inversely proportional, one increases as the other decreases.
- You can use algebraic expressions to represent direct and indirect (inverse) proportion and to solve problems related to these concepts. The symbol for proportion is $\propto$.
- You can increase and decrease amounts by a given ratio.

E

Are you able to …?

- simplify ratios and find the missing values in equivalent ratios
- divide quantities in a given ratio
- convert measurements on maps, plans and other scale diagrams to real measurements and vice versa
- express relationships between different quantities as rates in their simplest form and solve problems relating to rates
- read and interpret kinematic graphs
  - by calculating average speed
  - by calculating acceleration and deceleration from a graph and finding the distance travelled using the area under a linear speed–time graph E
- solve problems involving direct and indirect proportion
- express direct and inverse proportion in algebraic terms
- solve direct and inverse proportion problems using algebraic methods
- increase and decrease amounts by a given ratio. E

Examination practice

Exam-style questions

1 Sandra and Peter share a packet of 30 marshmallow eggs in the ratio 2:3. How many marshmallow eggs does Sandra receive?

2 Manos and Raja make \$96 selling handcrafts. They share the income in the ratio 7:5. How much does Raja receive?

3 Silvia makes a scale drawing of her bedroom using a scale of 1:25. If one wall on the diagram is 12 cm long, how long is the wall in her room?

4 Mrs James bakes a fruit cake using raisins, currants and dates in the ratio 4:5:3. The total mass of the three ingredients is 4.8 kilograms. Calculate the mass of:

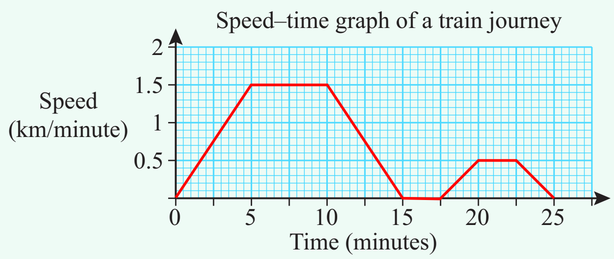
a the raisins
b the dates.

5 During an election, the ratio of female to male voters in a constituency was 3:2. If 2 400 people voted, how many of them were male?

6 A recipe for dough uses three parts wholemeal flour for every four parts of plain flour. What volume of wholemeal flour would you need if you used 12 cups of plain flour?

E

7 The speed–time graph below represents the journey of a train between two stations. The train slowed down and stopped after 15 minutes because of engineering work on the railway line.



a Calculate the greatest speed, in km/h, which the train reached.
b Calculate the deceleration of the train as it approached the place where there was engineering work.
c Calculate the distance the train travelled in the first 15 minutes.
d For how long was the train stopped at the place where there was engineering work?
e What was the speed of the train after 19 minutes?
f Calculate the distance between the two stations.

Past paper questions

1 A map is drawn to a scale of 1 : 1 000 000.
A forest on the map has an area of 4.6 cm^2.
Calculate the actual area of the forest in square kilometres. [2]

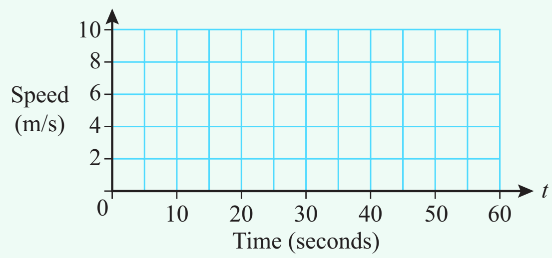
[Cambridge IGCSE Mathematics 0580 Paper 22 Q7 May/June 2016]

2 y is directly proportional to the square of $(x - 1)$.
$y = 63$ when $x = 4$.
Find the value of y when $x = 6$. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q17 October/November 2015]

3 A car passes through a checkpoint at time $t = 0$ seconds, travelling at 8 m/s.
It travels at this speed for 10 seconds.
The car then decelerates at a constant rate until it stops when $t = 55$ seconds.

a On the grid, draw the speed-time graph.

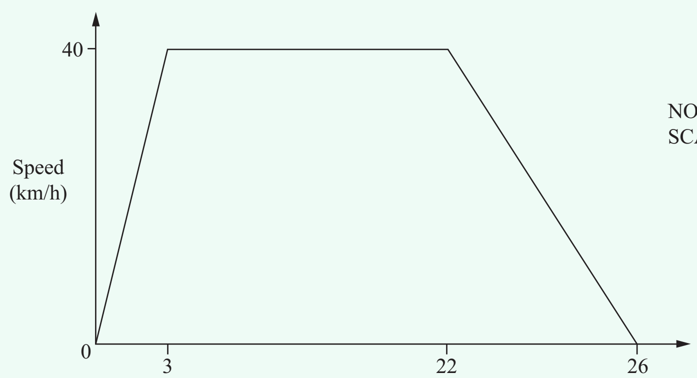


[2]

b Calculate the total distance travelled by the car after passing through the checkpoint. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q20 October/November 2015]

4



The diagram shows the speed-time graph of a train journey between two stations.
The train accelerates for 3 minutes, travels at a constant maximum speed of 40 km/h, then takes 4 minutes to slow to a stop.
Calculate the distance in kilometres between the two stations. [4]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q16 May/June 2013]