Chapter 3: Lines, angles and shapes

Key words

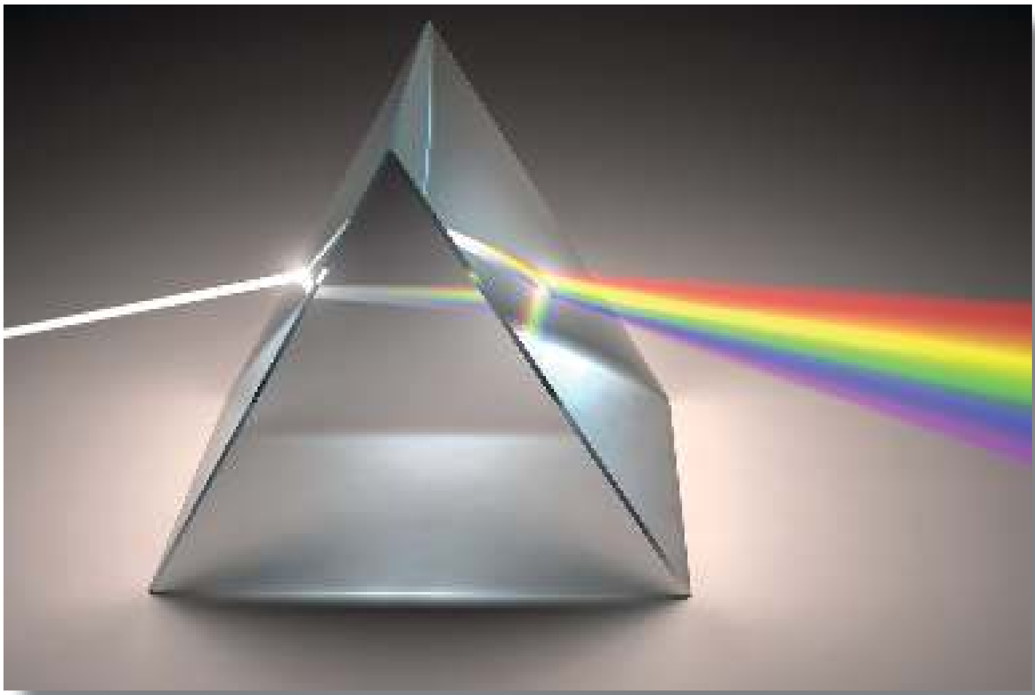
- Line
- Parallel
- Angle
- Perpendicular
- Acute
- Right
- Obtuse
- Reflex
- Vertically opposite
- Corresponding
- Alternate
- Co-interior
- Triangle
- Quadrilateral
- Polygon
- Circle

In this chapter you will learn how to:

- use the correct terms to talk about points, lines, angles and shapes
- classify, measure and construct angles
- calculate unknown angles using angle relationships
- talk about the properties of triangles, quadrilaterals, circles and polygons.
- use instruments to construct triangles.

EXTENDED

- calculate unknown angles in irregular polygons



In this photo white light is bent by a prism and separated into the different colours of the spectrum. When scientists study the properties of light they use the mathematics of lines and angles.

Geometry is one of the oldest known areas of mathematics. Farmers in Ancient Egypt knew about lines and angles and they used them to mark out fields after floods. Builders in Egypt and Mesopotamia used knowledge of angles and shapes to build huge temples and pyramids.

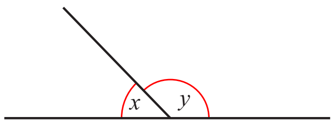
Today geometry is used in construction, surveying and architecture to plan and build roads, bridges, houses and office blocks. We also use lines and angles to find our way on maps and in the software of GPS devices. Artists use them to get the correct perspective in drawings, opticians use them to make spectacle lenses and even snooker players use them to work out how to hit the ball.

RECAP

You should already be familiar with the following geometry work:

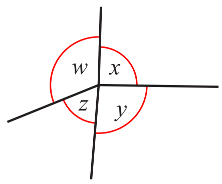
Basic angle facts and relationships

Angles on a line



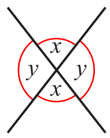
$x + y = 180°$

Angles round a point



$w + x + y + z = 360°$

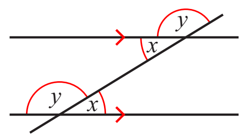
Vertically opposite angles



$x = x$ and $y = y$

$2x + 2y = 360°$

Parallel lines and associated angles



$x = x$ alternate
$y = y$ corresponding
$x + y = 180°$ co-interior

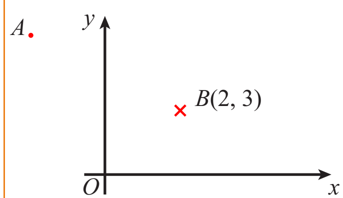
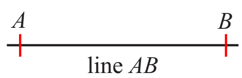
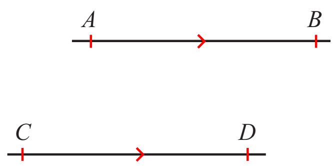
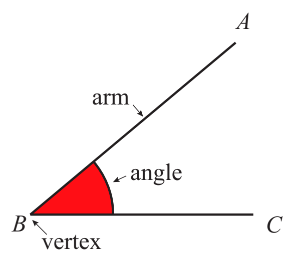
3.1 Lines and angles

Mathematicians use specific terms and definitions to talk about geometrical figures. You are expected to know what the terms mean and you should be able to use them correctly in your own work.

FAST FORWARD

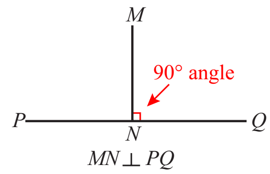
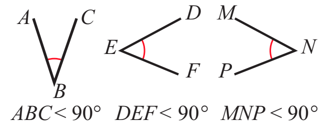
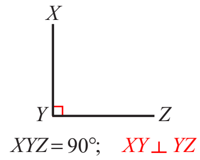
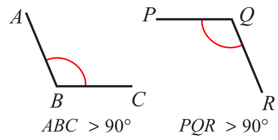
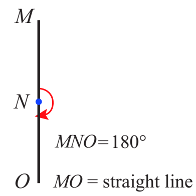
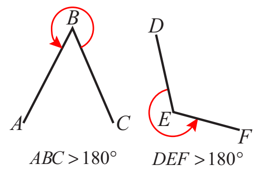
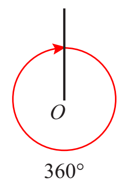
You will use these terms throughout the course but especially in chapter 14, where you learn how to solve simultaneous linear equations graphically. ▶

Terms used to talk about lines and angles

Term	What it means	Examples
Point	A point is shown on paper using a dot (·) or a cross (×). Most often you will use the word 'point' to talk about where two lines meet. You will also talk about points on a grid (positions) and name these using ordered pairs of co-ordinates (x, y). Points are normally named using capital letters.	
Line	A line is a straight (one-dimensional) figure that extends to infinity in both directions. Normally though, the word 'line' is used to talk about the shortest distance between two points. Lines are named using starting point and end point letters.	 line AB
Parallel	A pair of lines that are the same distance apart all along their length are parallel. The symbol $\|\|$ (or $/\!/$) is used for parallel lines, e.g. $AB \| CD$. Lines that are parallel are marked on diagrams with arrows.	 $AB \| CD$
Angle	When two lines meet at a point, they form an angle. The meeting point is called the vertex of the angle and the two lines are called the arms of the angle. Angles are named using three letters: the letter at the end of one arm, the letter at the vertex and the letter at the end of the other arm. The letter in the middle of an angle name always indicates the vertex.	 Angle ABC

LINK

Polygons and circles appear almost everywhere, including sport and music. Think about the symbols drawn on a football pitch or the shapes of musical instruments, for example.

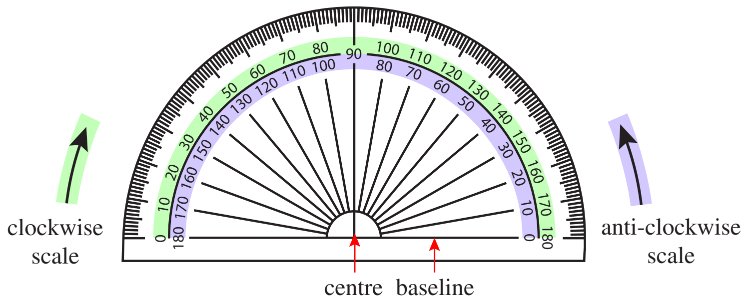
Term	What it means	Examples
Perpendicular	When two lines meet at right angles they are perpendicular to each other. The symbol ⊥ is used to show that lines are perpendicular, e.g. $MN \perp PQ$.	
Acute angle	An acute angle is > 0° but < 90°.	
Right angle	A right angle is an angle of exactly 90°. A square in the corner is usually used to represent 90°. A right angle is formed between perpendicular lines.	
Obtuse angle	An obtuse angle is > 90° but < 180°.	
Straight angle	A straight angle is an angle of 180°. A line is considered to be a straight angle.	
Reflex angle	A reflex angle is an angle that is > 180° but < 360°.	
Revolution	A revolution is a complete turn; an angle of exactly 360°.	

LINK

Builders, designers, architects, engineers, artists and even jewellers use shape, space and measure as they work and many of these careers use computer packages to plan and design various items. Most design work starts in 2-D on paper or on screen and moves to 3-D for the final representation. You need a good understanding of lines, angles, shape and space to use Computer-Aided Design (CAD) packages.

Measuring and drawing angles

The size of an angle is the amount of turn from one arm of the angle to the other. Angle sizes are measured in degrees (°) from 0 to 360 using a protractor.



A 180° protractor has two scales. You need to choose the correct one when you measure an angle.

Always take time to measure angles carefully. If you need to make calculations using your measured angles, a careless error can lead to several wrong answers.

If the arm of the angle does not extend up to the scale, lengthen the arm past the scale. The length of the arms of the angle does not affect the size of the angle.

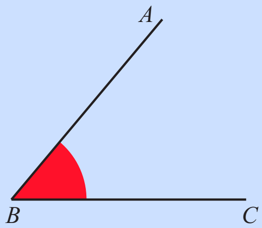
Measuring angles < 180°

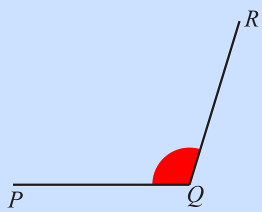
Put the centre of the protractor on the vertex of the angle. Align the baseline so it lies on top of one arm of the angle.

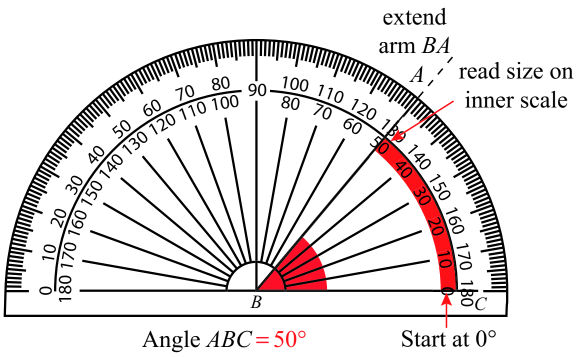
Using the scale that starts with 0° to read off the size of the angle, move round the scale to the point where it crosses the other arm of the angle.

Worked example 1

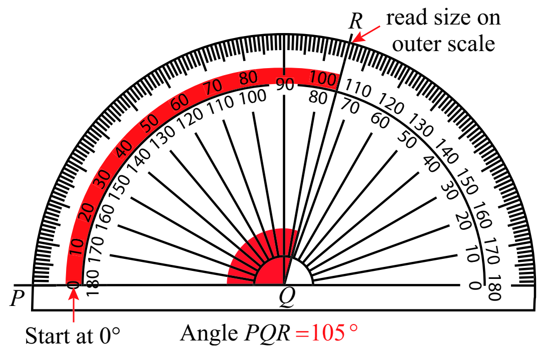
Measure angles $A\hat{B}C$ and $P\hat{Q}R$.







Place the centre of the protractor at *B* and align the baseline so it sits on arm *BC*. Extend arm *BA* so that it reaches past the scale. Read the inner scale. Angle *ABC* = 50°

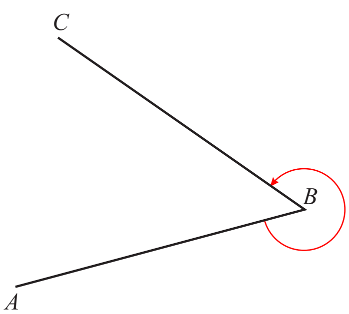


Put the centre of the protractor at *Q* and the baseline along *QP*. Start at 0° and read the outer scale. Angle *PQR* = 105°

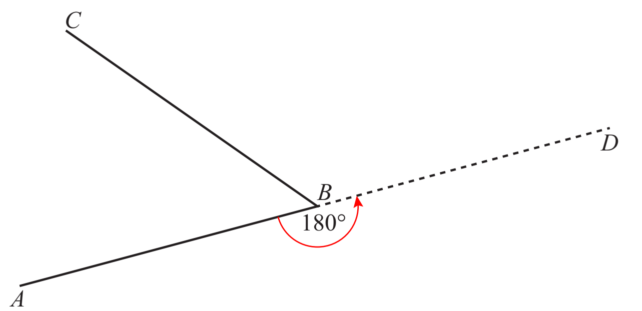
Measuring angles > 180°

Here are two different methods for measuring a reflex angle with a 180° protractor. You should use the method that you find easier to use. Suppose you had to measure the angle *ABC*:

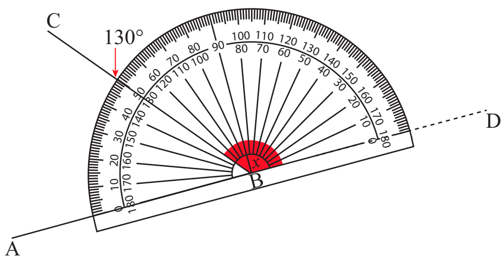
Method 1: Extend one arm of the angle to form a straight line (180° angle) and then measure the 'extra bit'. Add the 'extra bit' to 180° to get the total size.



Angle *ABC* is >180°.



Extend *AB* to point *D*. You know the angle of a straight line is 180°. So *ABD* = 180°.



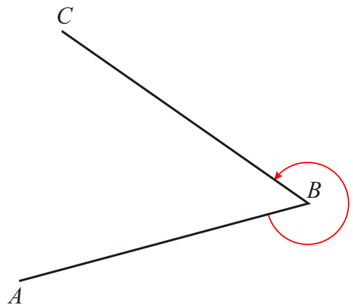
Use the protractor to measure the other piece of the angle *DBC* (marked *x*).

Add this to 180° to find angle *ABC*.

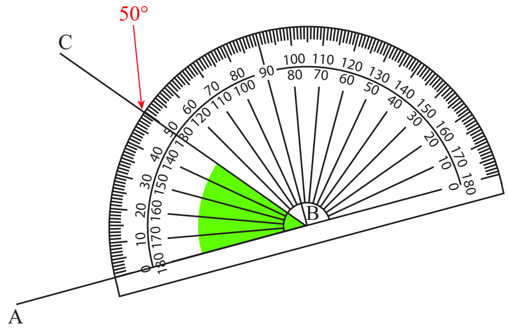
$180° + 130° = 310°$

$\therefore\ ABC = 310°$

Method 2: Measure the inner (non-reflex) angle and subtract it from 360° to get the size of the reflex angle.



You can see that the angle *ABC* is almost 360°.

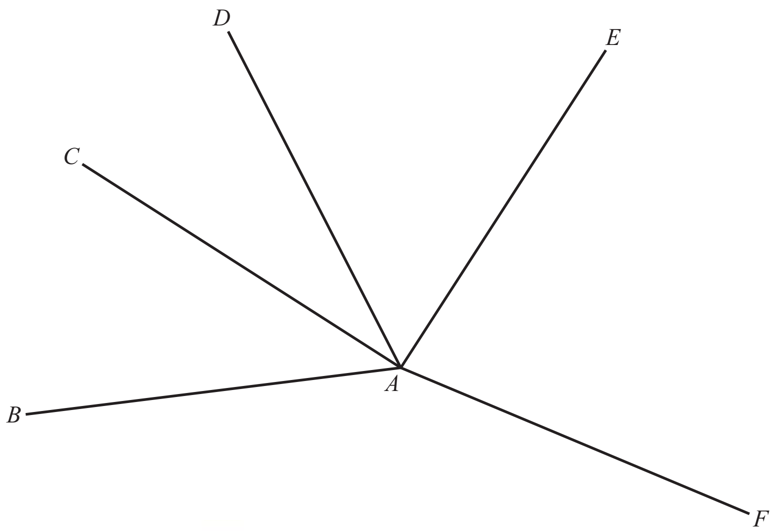


Measure the size of the angle that is < 180° (non-reflex) and subtract from 360°.

$360° - 50° = 310°$

$\therefore\ ABC = 310°$

Exercise 3.1



1 For each angle listed:

i *BAC*	**ii** *BAD*	**iii** *BAE*
iv *CAD*	**v** *CAF*	**vi** *CAE*
vii *DAB*	**viii** *DAE*	**ix** *DAF*

a state what type of angle it is (acute, right or obtuse)
b estimate its size in degrees
c use a protractor to measure the actual size of each angle to the nearest degree.
d What is the size of reflex angle *DAB*?

Applying your skills

2 Some protractors, like the one shown on the left, are circular.

a How is this different from the 180° protractor?
b Write instructions to teach someone how to use a circular protractor to measure the size of an obtuse angle.
c How would you measure a reflex angle with a circular protractor?

Drawing angles

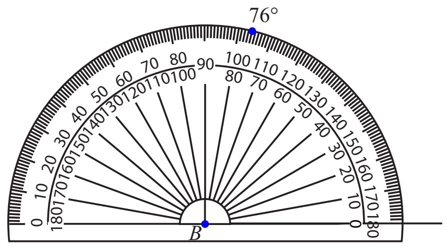
It is fairly easy to draw an angle of a given size if you have a ruler, a protractor and a sharp pencil. Work through this example to remind yourself how to draw angles < 180° and > 180°.

Worked example 2

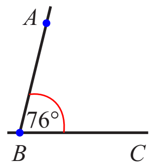
Draw **a** *angle ABC* = 76° and **b** *angle XYZ* = 195°.

a

Use a ruler to draw a line to represent one arm of the angle, make sure the line extends beyond the protractor.
Mark the vertex (*B*).

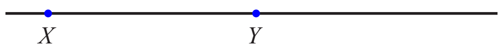


Place your protractor on the line with the centre at the vertex.
Measure the size of the angle you wish to draw and mark a small point.

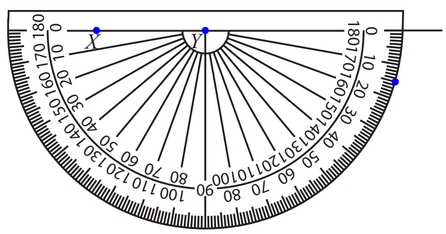


Remove the protractor and use a ruler to draw a line from the vertex through the point.
Label the angle correctly.

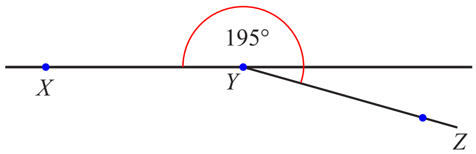
b



For a reflex angle, draw a line as in (a) but mark one arm (*X*) as well as the vertex (*Y*). The arm should extend beyond the vertex to create a 180° angle.



Calculate the size of the rest the angle: 195° − 180° = 15°.
Measure and mark the 15° angle (on either side of the 180° line).



Remove the protractor and use a ruler to draw a line from the vertex through the third point.
Label the angle correctly.

To draw a reflex angle, you could also work out the size of the inner angle and simply draw that. 360° − 195° = 165°. If you do this, remember to mark the reflex angle on your sketch and not the inner angle!

Exercise 3.2

Use a ruler and a protractor to accurately draw the following angles:

a $ABC = 80°$	**b** $PQR = 30°$	**c** $XYZ = 135°$
d $EFG = 90°$	**e** $KLM = 210°$	**f** $JKL = 355°$

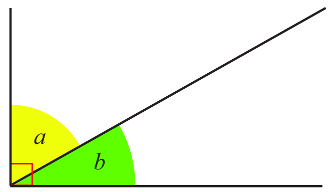
Angle relationships

Make sure you know the following angle facts:

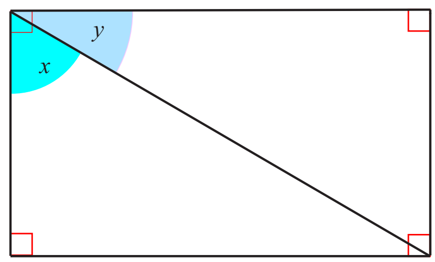
Complementary angles

Angles in a right angle add up to 90°.
When the sum of two angles is 90° those two angles are complementary angles.

> **Tip**
>
> In general terms:
>
> for complementary angles, if one angle is x°, the other must be $90° - x°$ and vice versa.
>
> For supplementary angles, if one angle is x°, the other must be $180° - x°$ and vice versa.



$a + b = 90°$

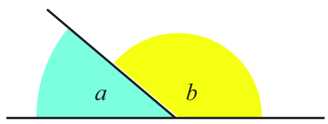


$x + y = 90°$

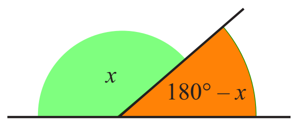
Supplementary angles

Angles on a straight line add up to 180°.

When the sum of two angles is 180° those two angles are supplementary angles.



$a + b = 180°$

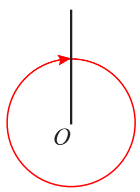


$x + (180° - x) = 180°$

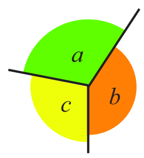
Angles round a point

Angles at a point make a complete revolution.

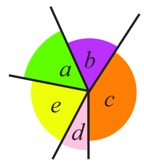
The sum of the angles at a point is 360°.



360°



$a + b + c = 360°$



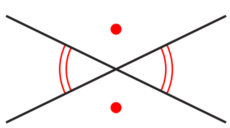
$a + b + c + d + e = 360°$

Vertically opposite angles

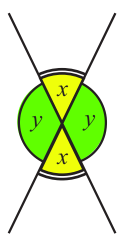
> The adjacent angle pairs in vertically opposite angles form pairs of supplementary angles because they are also angles on a straight line.

When two lines intersect, two pairs of **vertically opposite** angles are formed.

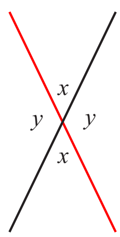
Vertically opposite angles are equal in size.



Two pairs of vertically opposite angles.



The angles marked x are equal to each other. The angles marked y are also equal to each other.



$x + y = 180°$

Using angle relationships to find unknown angles

The relationships between angles can be used to work out the size of unknown angles. Follow these easy steps:

- identify the relationship
- make an equation
- give reasons for statements
- solve the equation to find the unknown value.

In geometry problems you need to present your reasoning in a logical and structured way.

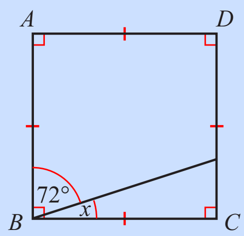
You will usually be expected to give reasons when you are finding the size of an unknown angle. To do this, state the relationship that you used to find the unknown angle after your statements. You can use these abbreviations to give reasons:

- comp angles
- supp angles
- angles on line
- angles round point

Worked example 3

Find the size of the angle marked x in each of these figures. Give reasons.

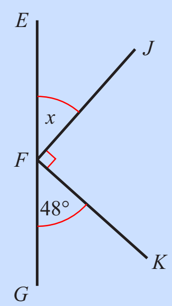
a



a $72° + x = 90°$
(angle $ABC = 90°$, comp angles)
$x = 90° - 72°$
$x = 18°$

You are told that angle ABC is a right angle, so you know that $72°$ and x are complementary angles. This means that $72° + x = 90°$, so you can rearrange to make x the subject.

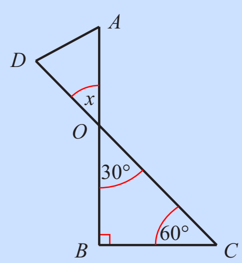
b



b $48° + 90° + x = 180°$
(angles on line)
$x = 180° - 90° - 48°$
$x = 42°$

You can see that $48°$, the right angle and x are angles on a straight line. Angles on a straight line add up to $180°$. So you can rearrange to make x the subject.

c



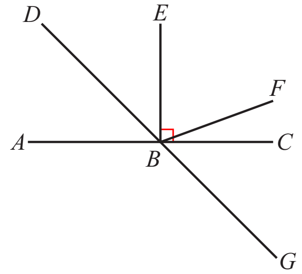
c $x = 30°$
(vertically opposite angles)

You know that when two lines intersect, the resulting vertically opposite angles are equal. x and $30°$ are vertically opposite, so $x = 30°$.

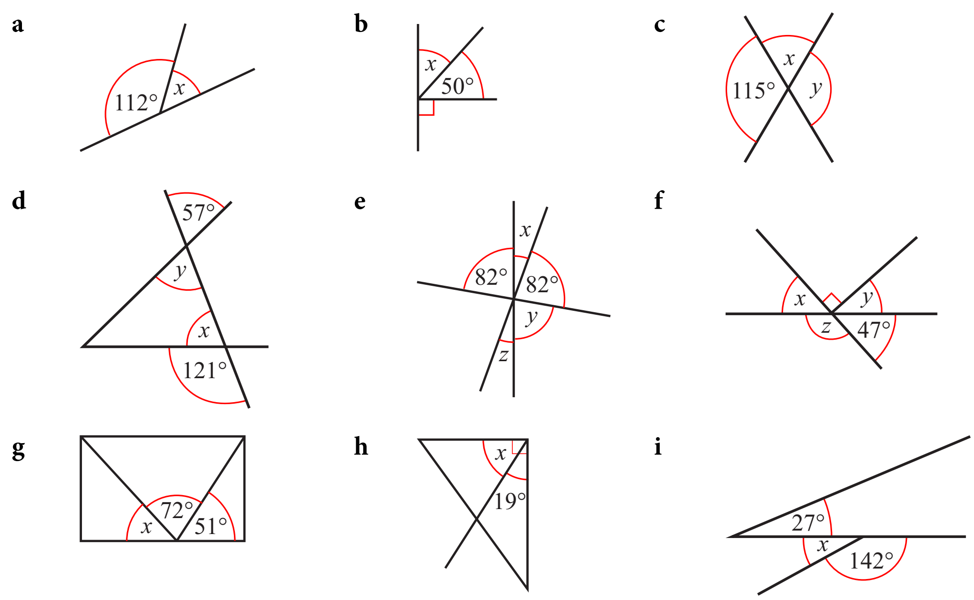
Exercise 3.3

1 In the following diagram, name:

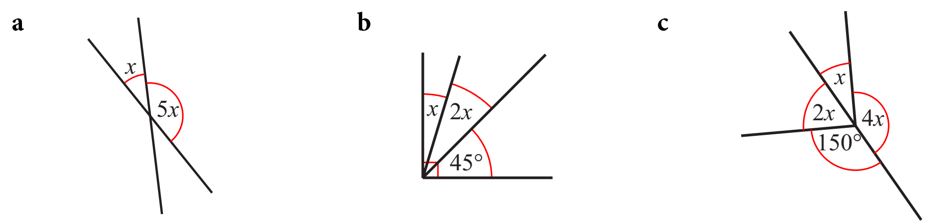
a a pair of complementary angles
b a pair of equal angles
c a pair of supplementary angles
d the angles on line DG
e the complement of angle EBF
f the supplement of angle EBC.



2 In each diagram, find the value of the angles marked with a letter.



3 Find the value of x in each of the following figures.



4 Two angles are supplementary. The first angle is twice the size of the second. What are their sizes?

5 One of the angles formed when two lines intersect is 127°. What are the sizes of the other three angles?

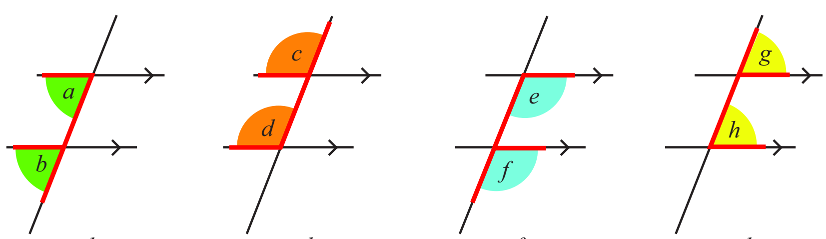
> **Tip**
> Although ‘*F*’, ‘*Z*’ and ‘*C*’ shapes help you to remember these properties, you must use the terms ‘corresponding’, ‘alternate’ and ‘co-interior’ to describe them when you answer a question.

Angles and parallel lines

When two parallel lines are cut by a third line (the transversal) eight angles are formed. These angles form pairs which are related to each other in specific ways.

Corresponding angles (‘*F*’-shape)

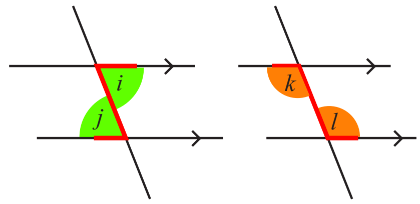
When two parallel lines are cut by a transversal four pairs of **corresponding** angles are formed. Corresponding angles are equal to each other.



$a = b$ $c = d$ $e = f$ $g = h$

Alternate angles ('Z'-shape)

When two parallel lines are cut by a transversal two pairs of **alternate** angles are formed. Alternate angles are equal to each other.



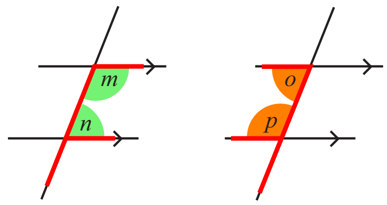
$i = j$ $\qquad$ $k = l$

Co-interior angles will only be equal if the transversal is perpendicular to the parallel lines (when they will both be 90°).

'Co-' means together. Co-interior angles are found together on the same side of the transversal.

Co-interior angles ('C'-shape)

When two parallel lines are cut by a transversal two pairs of **co-interior** angles are formed. Co-interior angles are supplementary (together they add up to 180°).



$m + n = 180°$ $\qquad$ $o + p = 180°$

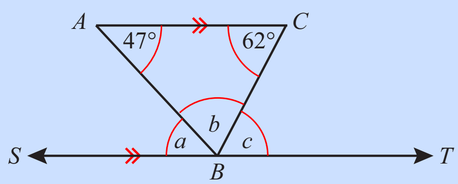
FAST FORWARD

You will use the angle relationships in this section again when you deal with triangles, quadrilaterals, polygons and circles.

These angle relationships around parallel lines, combined with the other angle relationships from earlier in the chapter, are very useful for solving unknown angles in geometry.

Worked example 4

Find the size of angles a, b and c in this figure.

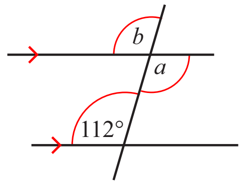


$a = 47°$ (CAB alt SBA) $c = 62°$ (ACB alt CBT) $a + b + c = 180°$ (s on line) $\therefore\ b = 180° - 47° - 62°$ $b = 71°$	CAB and SBA are alternate angles and therefore are equal in size. ACB and CBT are alternate angles and so equal in size. Angles on a straight line = 180°. You know the values of a and c, so can use these to find b.

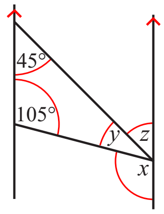
Exercise 3.4

1 Calculate the size of all angles marked with variables in the following diagrams. Give reasons.

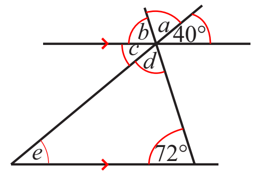
a



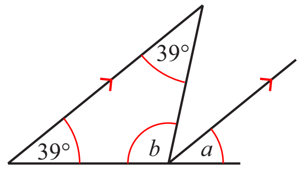
b



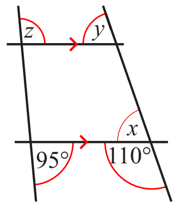
c



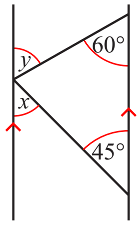
d



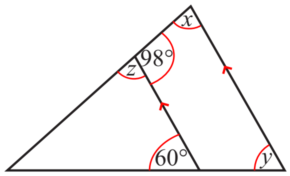
e



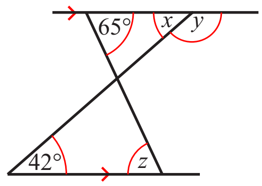
f



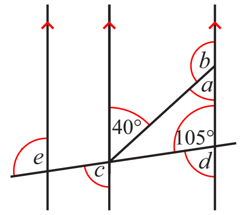
g



h

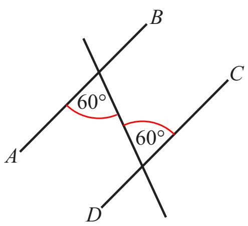


i

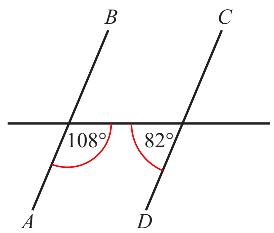


2 Decide whether $AB \| DC$ in each of these examples. Give a reason for your answer.

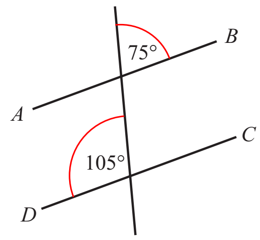
a



b



c



3.2 Triangles

Plane means flat. Plane shapes are flat or two-dimensional shapes.

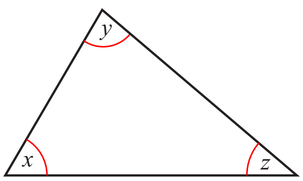
A **triangle** is a plane shape with three sides and three angles.

Triangles are classified according to the lengths of their sides and the sizes of their angles (or both).

FAST FORWARD

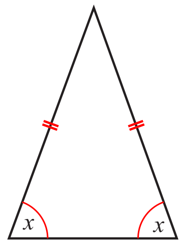
You will need these properties in chapter 11 on Pythagoras' theorem and similar triangles, and in chapter 15 for trigonometry. ▶

Scalene triangle



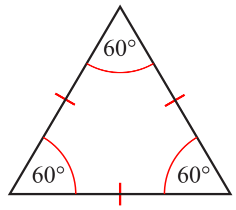
Scalene triangles have no sides of equal length and no angles that are of equal sizes.

Isosceles triangle



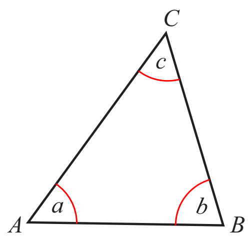
Isosceles triangles have two sides of equal length. The angles at the bases of the equal sides are equal in size.

Equilateral triangle

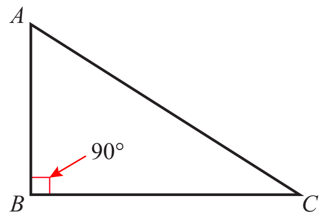


Equilateral triangles have three equal sides and three equal angles (each being 60°).

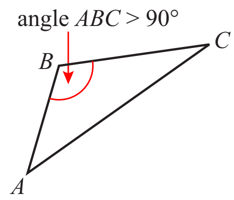
Other triangles



Acute-angled triangles have three angles each < 90°.



Right-angled triangles have one angle = 90°.



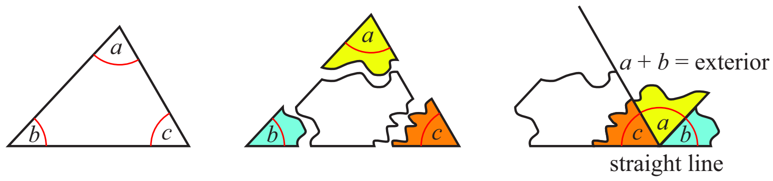
Obtuse-angled triangles have one angle > 90°.

The three angles inside a triangle are called interior angles.

If you extend a side of a triangle you make another angle outside the triangle. Angles outside the triangle are called exterior angles.

Angle properties of triangles

Look at the diagram below carefully to see two important angle properties of triangles.



The diagram shows two things:

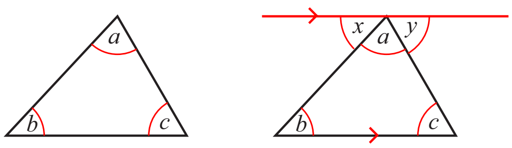
- The three interior angles of a triangle add up to 180°.
- Two interior angles of a triangle are equal to the opposite exterior angle.

You don't need to know these proofs, but you do need to remember the rules associated with them.

If you try this yourself with any triangle you will get the same results. But why is this so? Mathematicians cannot just show things to be true, they have to prove them using mathematical principles. Read through the following two simple proofs that use the properties of angles you already know, to show that angles in a triangle will always add up to 180° and that the exterior angle will always equal the sum of the opposite interior angles.

Angles in a triangle add up to 180°

To prove this you have to draw a line parallel to one side of the triangle.



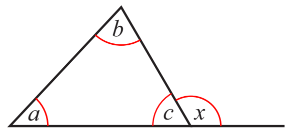
$x + a + y = 180°$ (angles on a line)

but:

$b = x$ and $c = y$ (alternate angles are equal)

so $a + b + c = 180°$

The exterior angle is equal to the sum of the opposite interior angles



$$c + x = 180° \text{ (angles on a line)}$$
$$\text{so, } c = 180° - x$$
$$a + b + c = 180° \text{ (angle sum of triangle)}$$
$$c = 180° - (a + b)$$
$$\text{so, } 180° - (a + b) = 180° - x$$
$$\text{hence, } a + b = x$$

These two properties allow us to find the missing angles in triangles and other diagrams involving triangles.

FAST FORWARD

Some of the algebraic processes used here are examples of the solutions to linear equations. You've done this before, but it is covered in more detail in chapter 14. ▶

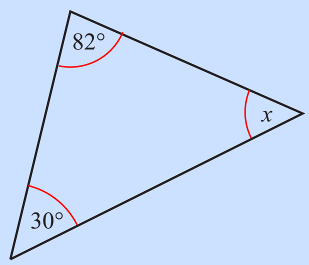
FAST FORWARD

Many questions on trigonometry require you to make calculations like these before you can move on to solve the problem. ▶

Worked example 5

Find the value of the unknown angles in each triangle. Give reasons for your answers.

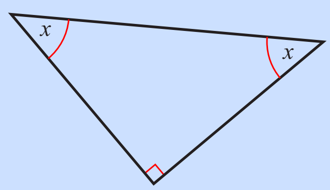
a



a

$82° + 30° + x = 180°$	(angle sum of triangle)
$x = 180° - 82° - 30°$	
$x = 68°$	

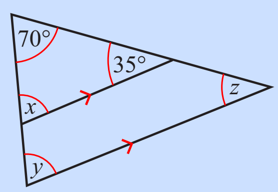
b



b

$2x + 90° = 180°$	(angle sum of triangle)
$2x = 180° - 90°$	
$2x = 90°$	
$x = 45°$	

c



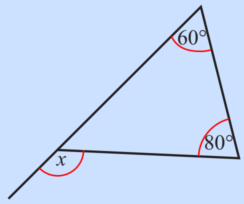
c

$70° + 35° + x = 180°$	(angle sum of triangle)
$x = 180° - 70° - 35°$	
$x = 75°$	
$y = 75°$	(corresponding angles)
$70° + y + z = 180°$	
$70° + 75° + z = 180°$	(angle sum of triangle)
$z = 180° - 75° - 70°$	
$z = 35°$	
or $z = 35°$	(corresponding angles)

Worked example 6

Find the size of angles x, y and z.

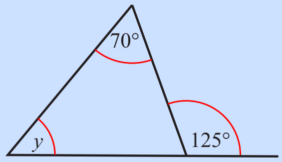
a



a $x = 60° + 80°$ (exterior angle of triangle)

$x = 140°$

b

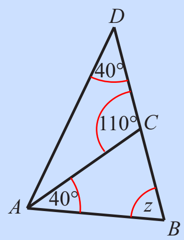


b $y + 70° = 125°$ (exterior angle of triangle)

$y = 125° - 70°$

$y = 55°$

c



c $40° + z = 110°$ (exterior angle triangle ABC)

$z = 110° - 40°$

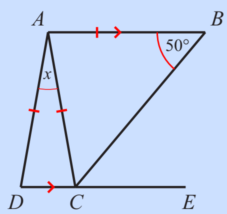
$z = 70°$

The exterior angle of one triangle may be inside another triangle as in worked example 6, part (c).

The examples above are fairly simple so you can see which rule applies. In most cases, you will be expected to apply these rules to find angles in more complicated diagrams. You will need to work out what the angle relationships are and combine them to find the solution.

Worked example 7

Find the size of angle x.



Angle $ACB = 50°$	(base angles isos triangle ABC)
$\therefore CAB = 180° - 50° - 50°$ $CAB = 80°$	(angle sum triangle ABC)
Angle $ACD = 50°$	(alt angles)
$\therefore ADC = 80°$ $\therefore x = 180° - 80° - 80°$ $x = 20°$	(base angles isos triangle ADC) (angle sum triangle ADC)

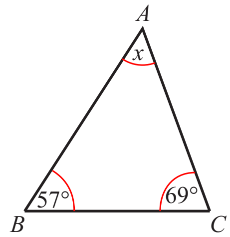
REWIND

An isosceles triangle has two sides and two angles equal, so if you know that the triangle is isosceles you can mark the two angles at the bases of the equal sides as equal.

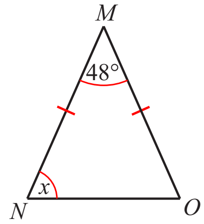
Exercise 3.5

1 Find the size of the marked angles. Give reasons.

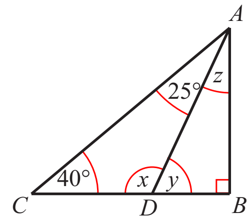
a



b

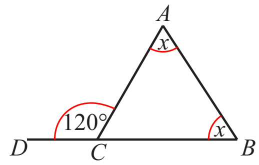


c

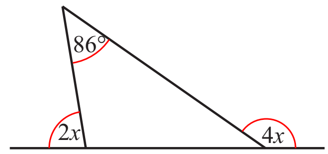


2 Calculate the value of x in each case. Give reasons.

a

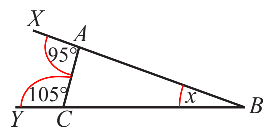


b

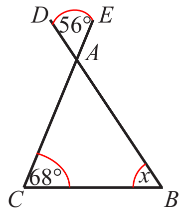


3 What is the size of the angle marked x in these figures? Show all steps and give reasons.

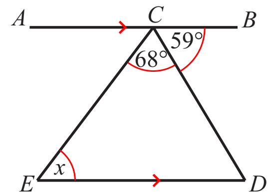
a



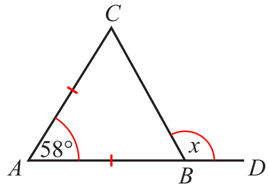
b



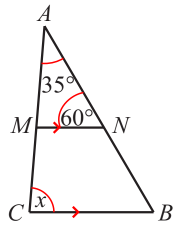
c



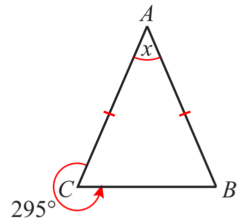
d



e



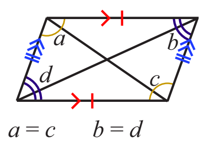
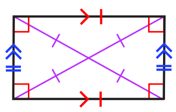
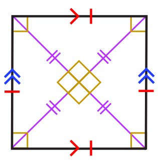
f

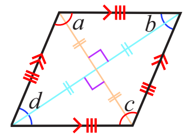
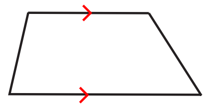
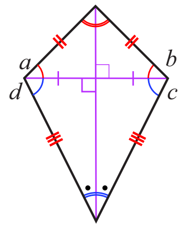


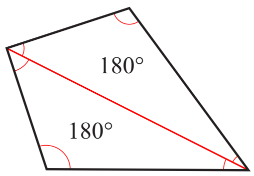
3.3 Quadrilaterals

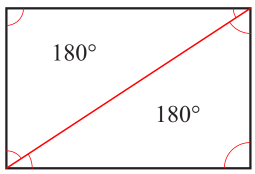
Quadrilaterals are plane shapes with four sides and four interior angles. Quadrilaterals are given special names according to their properties.

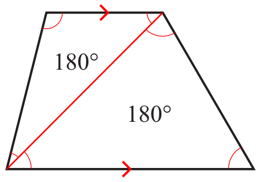
Some of these shapes are actually 'special cases' of others. For example, a square is also a rectangle because opposite sides are equal and parallel and all angles are 90°. Similarly, any rhombus is also a parallelogram. In both of these examples the converse is not true! A rectangle is not also a square. Which other special cases can you think of?

Type of quadrilateral	Examples	Summary of properties
Parallelogram	 $a = c$ $b = d$	Opposite sides parallel and equal. Opposite angles are equal. Diagonals bisect each other.
Rectangle		Opposite sides parallel and equal. All angles = 90°. Diagonals are equal. Diagonals bisect each other.
Square		All sides equal. All angles = 90°. Diagonals equal. Diagonals bisect each other at 90°. Diagonals bisect angles.

Type of quadrilateral	Examples	Summary of properties
Rhombus	 $a = c$ $b = d$	All sides equal in length. Opposite sides parallel. Opposite angles equal. Diagonals bisect each other at 90°. Diagonals bisect angles.
Trapezium		One pair of sides parallel.
Kite	 $a = b$ $c = d$	Two pairs of adjacent sides equal. One pair of opposite angles is equal. Diagonals intersect at 90°. Diagonals bisect angles.







The angle sum of a quadrilateral

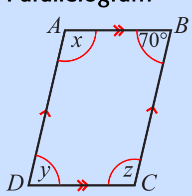
All quadrilaterals can be divided into two triangles by drawing one diagonal. You already know that the angle sum of a triangle is 180°. Therefore, the angle sum of a quadrilateral is 180° + 180° = 360°.

This is an important property and we can use it together with the other properties of quadrilaterals to find the size of unknown angles.

Worked example 8

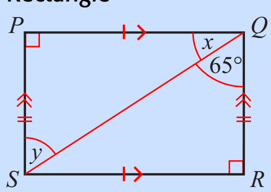
Find the size of the marked angles in each of these figures.

a Parallelogram



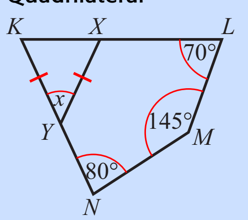
a
$x = 110°$ (co-interior angles)
$y = 70°$ (opposite angles of || gram)
$z = 110°$ (opposite angles of || gram)

b Rectangle



b
$x + 65° = 90°$ (right angle of rectangle)
$\therefore x = 90° - 65°$
$x = 25°$
$y = 65°$ (alt angles)

c Quadrilateral



c
$LKN = 360° - 70° - 145° - 80°$ (angle sum of quad)
$LKN = 65°$
$\therefore KXY = 65°$ (base angles isos triangle)
$\therefore x = 180° - 65° - 65°$ (angle sum triangle KXY)
$x = 50°$

Exercise 3.6

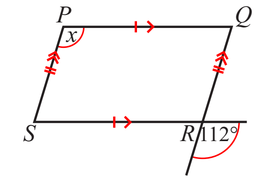
1 A quadrilateral has two diagonals that intersect at right angles.

a What quadrilaterals could it be?

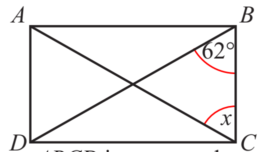
b The diagonals are not equal in length. What quadrilaterals could it NOT be?

2 Find the value of x in each of these figures. Give reasons.

a

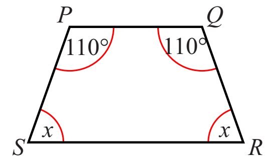


b

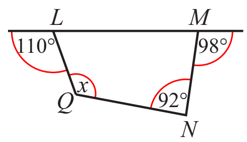


ABCD is a rectangle

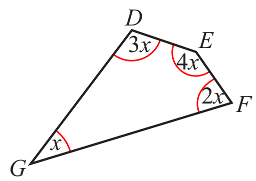
c



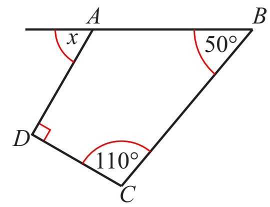
d



e



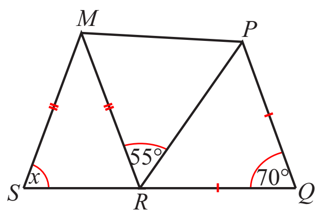
f



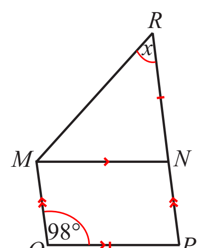
You may need to find some other unknown angles before you can find ***x***. If you do this, write down the size of the angle that you have found and give a reason.

3 Find the value of x in each of these figures. Give reasons.

a

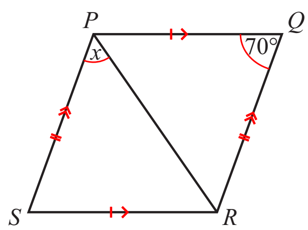


b



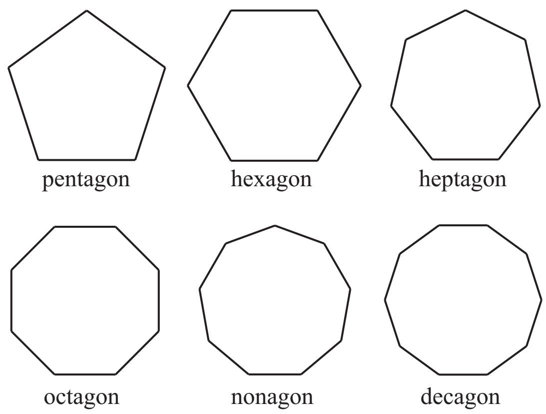
$QP = RN$

c



3.4 Polygons

A **polygon** is a plane shape with three or more straight sides. Triangles are polygons with three sides and quadrilaterals are polygons with four sides. Other polygons can also be named according to the number of sides they have. Make sure you know the names of these polygons:



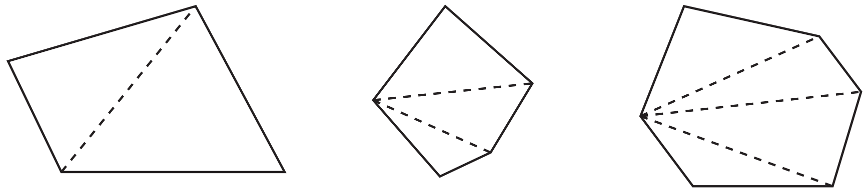
If a polygon has any reflex angles, it is called a concave polygon.

All other polygons are convex polygons.

A polygon with all its sides and all its angles equal is called a regular polygon.

Angle sum of a polygon

By dividing polygons into triangles, we can work out the sum of their interior angles.



Can you see the pattern that is forming here?

The number of triangles you can divide the polygon into is always two less than the number of sides. If the number of sides is n, then the number of triangles in the polygon is $(n - 2)$.

The angle sum of the polygon is $180° \times$ the number of triangles. So for any polygon, the angle sum can be worked out using the formula:

sum of interior angles $= (n - 2) \times 180°$

Worked example 9

Find the angle sum of a decagon and state the size of each interior angle if the decagon is regular.

sum of interior angles $= (n - 2) \times 180°$	Sum of angles
$= (10 - 2) \times 180°$	A decagon has 10 sides, so $n = 10$.
$= 1440°$	
$= \dfrac{1440}{10}$	A regular decagon has 10 equal angles.
$= 144°$	Size of one angle

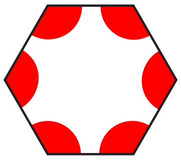
Worked example 10

A polygon has an angle sum of 2340°. How many sides does it have?

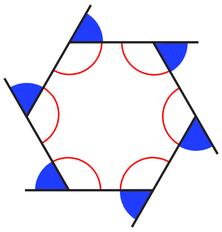
$2340° = (n - 2) \times 180°$	Put values into angle sum formula.
$\dfrac{2340}{180} = n - 2$	
$13 = n - 2$	Rearrange the formula to get n.
$13 + 2 = n$	
$\therefore \quad 15 = n$	So the polygon has 15 sides.

The sum of exterior angles of a convex polygon

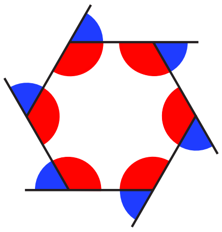
The sum of the exterior angles of a convex polygon is always 360°, no matter how many sides it has. Read carefully through the information about a hexagon that follows, to understand why this is true for every polygon.



A hexagon has six interior angles.
The angle sum of the interior angles $= (n-2)\times 180°$
$= 4\times 180°$
$= 720°$



If you extend each side you make six exterior angles; one next to each interior angle.
Each pair of interior and exterior angles adds up to 180° (angles on line).
There are six vertices, so there are six pairs of interior and exterior angles that add up to 180°.



$$\begin{aligned}
\therefore\ \text{sum of (interior + exterior angles)} &= 180\times 6\\
&= 1080°\\
\text{But, sum of interior angles} &= (n-2)\times 180\\
&= 4\times 180\\
&= 720°\\
\text{So, } 720° + \text{sum of exterior angles} &= 1080\\
\text{sum of exterior angles} &= 1080-720\\
\text{sum of exterior angles} &= 360°
\end{aligned}$$

Tip

You do not have to remember this proof, but you must remember that the sum of the exterior angles of any convex polygon is 360°.

This can be expressed as a general rule like this:

If I = sum of the interior angles, E = sum of the exterior angles and n = number of sides of the polygon

$$\begin{aligned}
I+E &= 180n\\
E &= 180n - I\\
\text{but } I &= (n-2)\times 180\\
\text{so } E &= 180n-(n-2)\times 180\\
E &= 180n-180n+360\\
E &= 360°
\end{aligned}$$

Exercise 3.7

A regular polygon has all sides equal and all angles equal. An irregular polygon does *not* have all equal sides and angles.

1 Copy and complete this table.

Number of sides in the polygon	5	6	7	8	9	10	12	20
Angle sum of interior angles								

2 Find the size of one interior angle for each of the following regular polygons.

a pentagon **b** hexagon **c** octagon
d decagon **e** dodecagon (12 sides) **f** a 25-sided polygon

3 A regular polygon has 15 sides. Find:

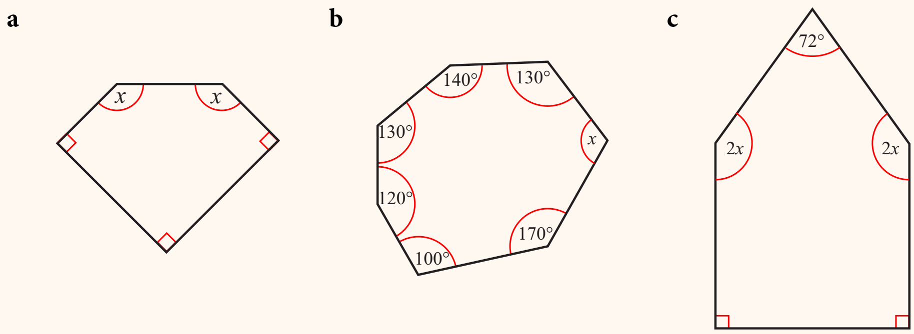
- **a** the sum of the interior angles
- **b** the sum of the exterior angles
- **c** the size of each interior angle
- **d** the size of each exterior angle.

4 A regular polygon has n exterior angles of 15°. How many sides does it have?

The rule for the sum of interior angles, and for the sum of exterior angles is true for both regular and irregular polygons. *But* with irregular polygons, you can't simply divide the sum of the interior angles by the number of sides to find the size of an interior angle: all interior angles may be different.

E

5 Find the value of x in each of these irregular polygons.

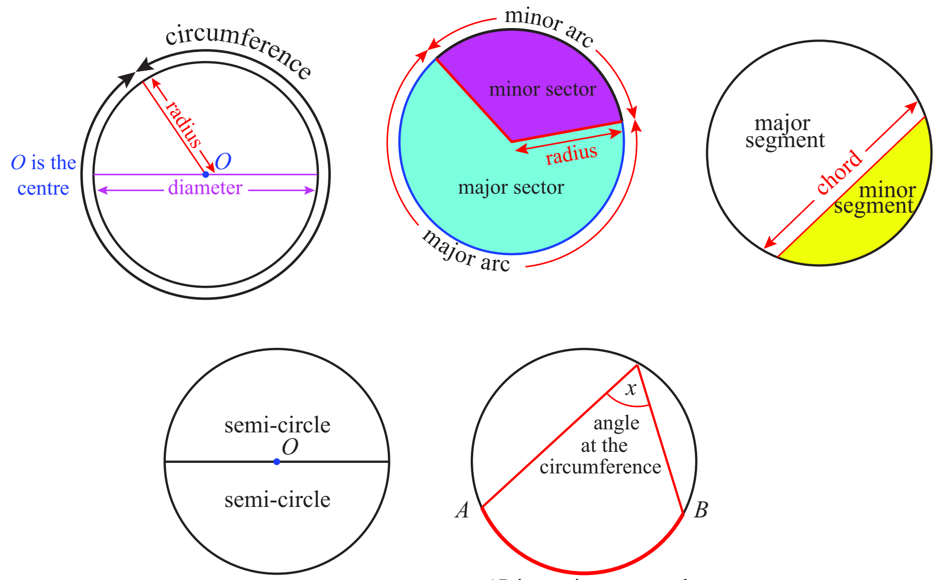


3.5 Circles

In mathematics, a **circle** is defined as a set of points which are all the same distance from a given fixed point. In other words, every point on the outside curved line around a circle is the same distance from the centre of the circle.

There are many mathematical terms used to talk about circles. Study the following diagrams carefully and then work through exercise 3.8 to make sure you know and can use the terms correctly.

Parts of a circle

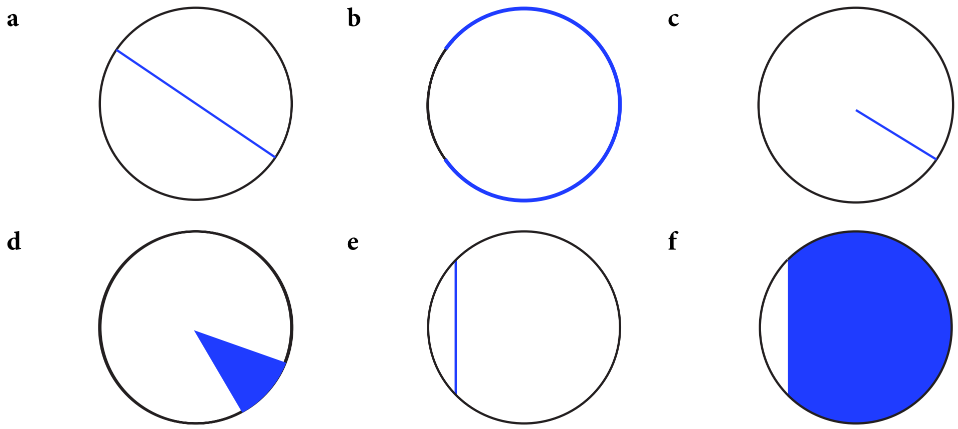


AB is a minor arc and angle x is subtended by arc AB

The angle x is subtended at the circumference. This means that it is the angle formed by two chords passing through the end points of the arc and meeting again at the edge of the circle.

Exercise 3.8

1 Name the features shown in blue on these circles.



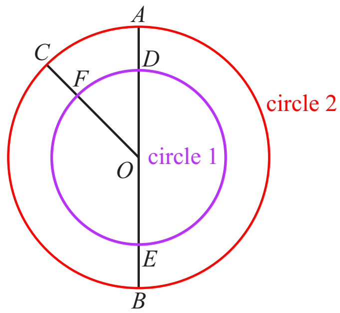
2 Draw four small circles. Use shading to show:

- **a** a semi-circle
- **b** a minor segment
- **c** a tangent to the circle
- **d** angle y subtended by a minor arc MN.

FAST FORWARD

You will learn more about circles and the angle properties in circles when you deal with circle symmetry and circle theorems in chapter 19.

3 Circle 1 and circle 2 have the same centre (O). Use the correct terms or letters to copy and complete each statement.



- **a** OB is a __ of circle 2.
- **b** DE is the __ of circle 1.
- **c** AC is a __ of circle 2.
- **d** __ is a radius of circle 1.
- **e** CAB is a __ of circle 2.
- **f** Angle FOD is the vertex of a __ of circle 1 and circle 2.

3.6 Construction

The photograph shows you the basic equipment that you are expected to use.

It is important that you use a sharp pencil and that your pair of compasses are not loose.

In geometry, constructions are accurate geometrical drawings. You use mathematical instruments to construct geometrical drawings.

Using a ruler and a pair of compasses

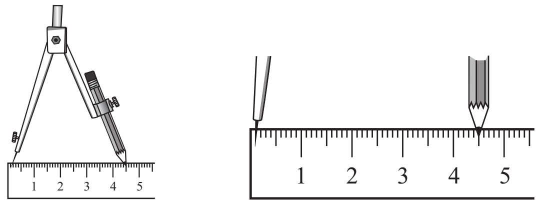
Your ruler (sometimes called a straight-edge) and a pair of compasses are probably your most useful construction tools. You use the ruler to draw straight lines and the pair of compasses to measure and mark lengths, draw circles and bisect angles and lines.

Once you can use a ruler and pair of compasses to measure and draw lines, you can easily construct triangles and other geometric shapes.

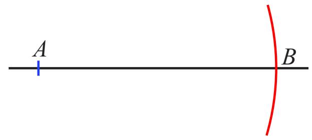
Do you remember how to use a pair of compasses to mark a given length? Here is an example showing you how to construct line AB that is 4.5 cm long. (Diagrams below are NOT TO SCALE.)

- Use a ruler and sharp pencil to draw a straight line that is longer than the length you need. Mark point A on the line with a short vertical dash (or a dot).

- Open your pair of compasses to 4.5 cm by measuring against a ruler.



- Put the point of the pair of compasses on point A. Twist the pair of compasses lightly to draw a short arc on the line at 4.5 cm. Mark this as point B. You have now drawn the line AB at 4.5 cm long.



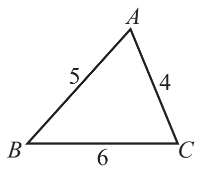
Constructing triangles

You can draw a triangle if you know the length of three sides.

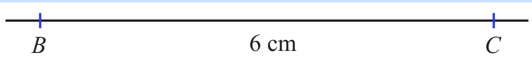
Read through the worked example carefully to see how to construct a triangle given three sides.

Worked example 11

Construct $\triangle ABC$ with $AB = 5$ cm, $BC = 6$ cm and $CA = 4$ cm.

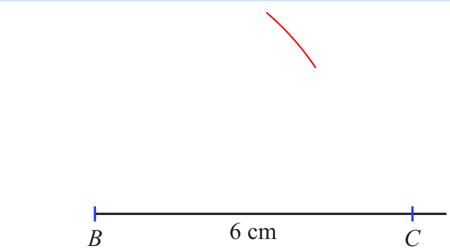


Always start with a rough sketch.



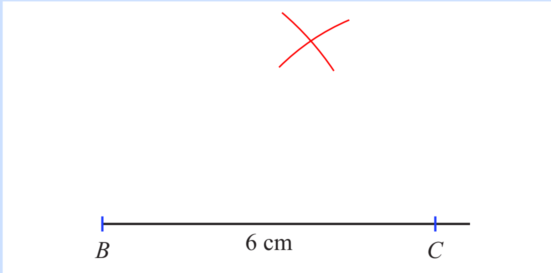
Draw the longest side ($BC = 6$ cm) and label it.

It is a good idea to draw the line longer than you need it and then measure the correct length along it. When constructing a shape, it can help to mark points with a thin line to make it easier to place the point of the pair of compasses.

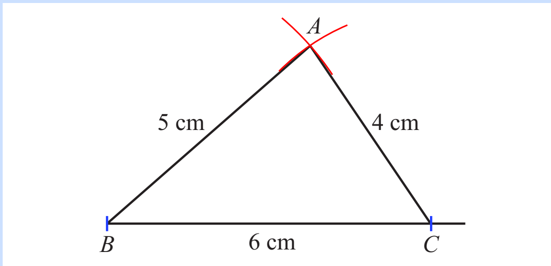


Set your pair of compasses at 5 cm. Place the point on B and draw an arc.

Please note that the diagrams here are NOT TO SCALE but your diagrams ***must*** use the accurate measurements!



Set your pair of compasses at 4 cm. Place the point on *C* and draw an arc.



The point where the arcs cross is *A*. Join *BA* and *CA*.

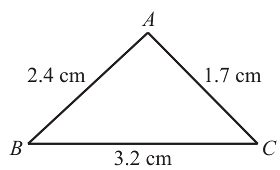
Exercise 3.9

1 Construct these lines.

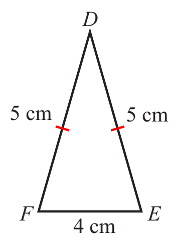
a $AB = 6$ cm **b** $CD = 75$ mm **c** $EF = 5.5$ cm

2 Accurately construct these triangles.

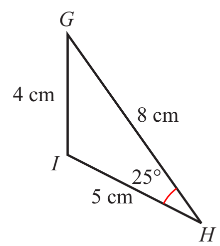
a



b



c



3 Construct these triangles.

a ΔABC with $BC = 8.5$ cm, $AB = 7.2$ cm and $AC = 6.9$ cm.
b ΔXYZ with $YZ = 86$ mm, $XY = 120$ mm and $XZ = 66$ mm.
c Equilateral triangle *DEF* with sides of 6.5 cm.
d Isosceles triangle *PQR* with a base of 4 cm and $PQ = PR = 6.5$ cm.

Exercise 3.10

E

1 Draw a large circle. Draw any two chords in the same circle, but make sure that they are not parallel.

Now construct the perpendicular bisector of each chord. What do you notice about the point at which the perpendicular bisectors meet? Can you explain this?

Summary

Do you know the following?

- A point is position and a line is the shortest distance between two points.
- Parallel lines are equidistant along their length.
- Perpendicular lines meet at right angles.
- Acute angles are $< 90°$, right angles are exactly $90°$, obtuse angles are $> 90°$ but $< 180°$. Straight angles are exactly $180°$. Reflex angles are $> 180°$ but $< 360°$. A complete revolution is $360°$.
- Scalene triangles have no equal sides, isosceles triangles have two equal sides and a pair of equal angles, and equilateral triangles have three equal sides and three equal angles.
- Complementary angles have a sum of $90°$. Supplementary angles have a sum of $180°$.
- Angles on a line have a sum of $180°$.
- Angles round a point have a sum of $360°$.
- Vertically opposite angles are formed when two lines intersect. Vertically opposite angles are equal.
- When a transversal cuts two parallel lines various angle pairs are formed. Corresponding angles are equal. Alternate angles are equal. Co-interior angles are supplementary.
- The angle sum of a triangle is $180°$.
- The exterior angle of a triangle is equal to the sum of the two opposite interior angles.
- Quadrilaterals can be classified as parallelograms, rectangles, squares, rhombuses, trapeziums or kites according to their properties.
- The angle sum of a quadrilateral is $360°$.
- Polygons are many-sided plane shapes. Polygons can be named according to the number of sides they have: e.g. pentagon (5); hexagon (6); octagon (8); and decagon (10).
- Regular polygons have equal sides and equal angles.
- Irregular polygons have unequal sides and unequal angles. E
- The angle sum of a polygon is $(n - 2) \times 180°$, where n is the number of sides.
- The angle sum of exterior angles of any convex polygon is $360°$.

Are you able to … ?

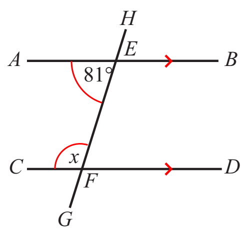
- calculate unknown angles on a line and round a point
- calculate unknown angles using vertically opposite angles and the angle relationships associated with parallel lines
- calculate unknown angles using the angle properties of triangles, quadrilaterals and polygons
- accurately measure and construct lines and angles
- construct a triangle using given measurements

Examination practice

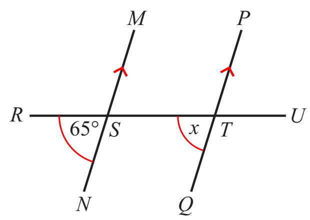
Exam-style questions

1 Find x in each figure. Give reasons.

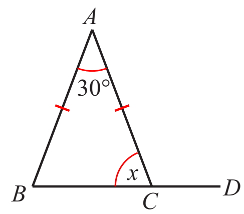
a



b



c



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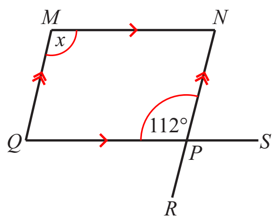
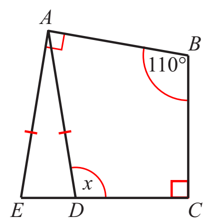
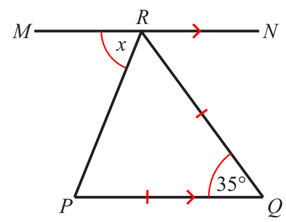


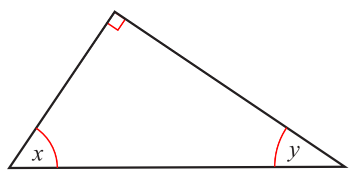
e



f



2 Study the triangle.



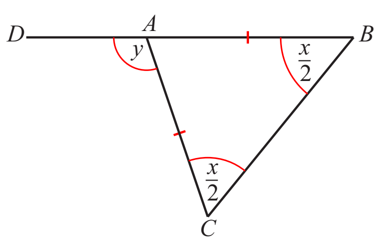
a Explain why $x + y = 90°$.
b Find y if $x = 37°$.

3 What is the sum of interior angles of a regular hexagon?

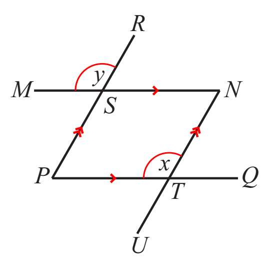
4 **a** What is the sum of exterior angles of a convex polygon with 15 sides?
b What is the size of each exterior angle in this polygon?
c If the polygon is regular, what is the size of each interior angle?

5 Explain why $x = y$ in the following figures.

a



b



6 **a** Measure this line and construct *AB* the same length in your book using a ruler and compasses.

A ____ *B*

b At point *A*, measure and draw angle *BAC*, a 75° angle.

c At point B, measure and draw angle *ABD*, an angle of 125°.

7 **a** Construct triangle *PQR* with sides $PQ = 4.5$ cm, $QR = 5$ cm and $PR = 7$ cm.

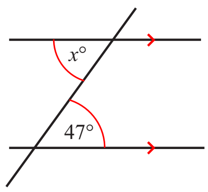
Past paper questions

1 A regular polygon has an interior angle of 172°.

Find the number of sides of this polygon. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q9 May/June 2016]

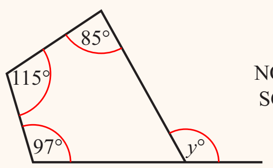
2 **a**



NOT TO SCALE

Find the value of *x*. [1]

b



NOT TO SCALE

Find the value of *y*. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q18 Parts a) and b) February/March 2016]