Chapter 17: Managing money

Key words

- Earnings
- Wages
- Salary
- Commission
- Gross income
- Deductions
- Net income
- Tax threshold
- Interest
- Simple interest
- Interest rate
- Principal
- Compound interest
- Cost price
- Selling price
- Profit
- Loss
- Discount

In this chapter you will learn how to:

- calculate earnings (wages and salaries) in different situations
- use and manipulate a formula to calculate simple interest payable and due on a range of loans and investments
- solve problems related to simple and compound interest
- apply what you already know about percentages to work out discounts, profit and loss in everyday contexts
- use a calculator effectively to perform financial calculations
- read and interpret financial data provided in tables and charts.

Knowing how to work well with money is an important skill that you will use again and again throughout your life.

During your life so far, you will have solved problems relating to money on a daily basis. You will continue to do this as you get older but the problems you have to solve may become more complicated as you start earning and spending money, borrowing money and saving money.

In this chapter you will apply some of the maths skills you have already learned to solve real world problems. You will use your calculator to find the answers quickly and efficiently.

You should already be familiar with the following number and formula work:

Fractions and percentages (Chapter 5)

You can convert percentages to equivalent fractions or decimals.

$$65\% \quad = \quad 0.65 \quad = \quad \frac{65}{100} = \frac{13}{20}$$

You can increase or decrease quantities by a percentage.

To increase $40 by 5% multiply by 100% + 5% = 105%

$$\frac{105}{100} \times \$40 = \$42$$

To decrease $45 by 10%, multiply by 100% − 10% = 90%

$$\frac{90}{100} \times \$45 = \$40.50$$

Formulae (Chapter 6)

You can use a formula to calculate a value.

For example, simple interest $I = \dfrac{Prt}{100}$

where P = amount invested r = rate of interest (percentage) t = time period

If, $P = \$75$, $r = 3\%$ and $t = 5$ years

$$I = \frac{(75 \times 3 \times 5)}{100} = \$11.25$$

You can use inverse operations to change the subject of a formula.

$$P = \frac{100I}{rt} \qquad\qquad r = \frac{100I}{Pt} \qquad\qquad t = \frac{100I}{Pr}$$

17.1 Earning money

When you are employed you earn money (get paid) for the work you do. **Earnings** can be worked out in different ways. Make sure you understand these terms:

- **Wages** – pay based on a fixed number of hours worked, usually paid weekly. Extra hours of work are called overtime and these are paid at a higher rate.
- **Salary** – pay based on a fixed yearly amount, usually paid monthly. Overtime may be paid, or workers may be given time off in exchange.
- **Piece work** – pay based on the number of items produced.
- **Commission** – pay is based on a percentage of sales made; sometimes a low wage, called a retainer, is paid as well as commission.

Worked example 1

Emmanuel makes beaded necklaces for a curio stand. He is paid in South African rand at a rate of R14.50 per completed necklace. He is able to supply 55 necklaces per week. Calculate his weekly income.

Income $= 55 \times 14.50$ $= $ R797.50	Multiply items produced by the rate paid.

Worked example 2

Sanjay works as a sales representative for a company that sells mobile phones in the United Arab Emirates. He is paid a retainer of 800 dirhams (Dhs) per week plus a commission of 4.5% of all sales.

a How much would he earn in a week if he made no sales?
b How much would he earn if he sold four phones at Dhs3299 each in a week?

a	Dhs800	If he made no sales, he would earn no commission, only his retainer.
b	Commission $= 4.5\%$ of (3299×4) $= 0.045 \times 13\,196$ $= 593.82$ Earnings $=$ retainer $+$ commission $=$ Dhs800 $+$ Dhs593.82 $=$ Dhs1393.82	Calculate 4.5% of the total sales Sanjay made. Add this to the retainer of Dhs 800.

Worked example 3

Josh's hourly rate of pay is $12.50. He is paid 'time-and-a-half' for work after hours and on Saturdays and 'double-time' for Sundays and Public Holidays.

One week he worked 5.5 hours on Saturday and 3 hours on Sunday. How much overtime pay would he earn?

Saturday overtime $= 1.5 \times \$12.50 \times 5.5$ $= \$103.13$	(time-and-a-half $= 1.5 \times$ normal time)
Sunday overtime $= 2 \times \$12.50 \times 3$ $= \$75$	(double-time $= 2 \times$ normal time)
Total overtime $= \$103.13 + \$75.00$ $= \$178.13$	

Exercise 17.1   *Applying your skills*

1   A waiter earns $8.25 per hour. How much would she earn for a six-hour shift?

2   How much would a receptionist earn for working a 35-hour week if her rate of pay is $9.50 per hour?

3   Calculate the hourly rate for each of the following:

   a $67.50 for five hours
   b $245.10 for a 38-hour week
   c $126.23 for 13.5 hours
   d $394.88 for five $6\frac{1}{2}$-hour shifts
   e $71.82 for working five hours and 15 minutes.

4   A truck driver is paid $15.45 per tonne of wood pulp delivered to a factory in Malaysia. If he delivers 135 tonnes to the factory, how much will he earn?

5   A team of workers in a factory is paid $23.25 per pallet of goods produced. If a team of five workers produces 102 pallets in a shift, how much will each person in the team have earned that shift?

6   An estate agent is paid a retainer of $150 per week plus a commission on sales. The rate of commission is 2.5% on sales up to $150 000 and 1.75% on amounts above that. How much would she earn in a week if she sold a house for $220 000 and an apartment for $125 000?

7   Here is the time sheet for five workers in a factory. Calculate each person's income for the week if their standard rate of pay is $8.40 per hour.

Worker	Normal hours worked	Hours overtime at time-and-a-half	Hours overtime at double-time
Annie	35	2	0
Bonnie	25	3	4
Connie	30	1.5	1.75
Donny	40	0	4
Elizabeth	20	3.75	2

8   The media in South Africa published a list of the annual earnings in Rands (R) of ten prominent CEOs in 2016. Here is the list:

Name	Annual salary (R million)
Bernard Fornas	87.9
Hendrik du Toit	86.1
Richard Lepeu	85.1
Mark Cutifani	66.9
David Hathorn	66.8
Nicandro Durante	59.5
David Constable	51.9
Glyn Lewis	51.5
Whitey Basson	49.9
Alan Clark	49.7

a Calculate each person's earnings per month.
b Assuming a tax rate of 35%, work out how much tax each person would pay on these earnings.
c How much would they earn per month after tax is deducted?
d If the average working week is 40 hours long and each person took three weeks leave during the year, what did the highest and lowest earning person earn on average per working hour (before tax)?

Deductions from earnings

Gross income (earnings) refers to the total amount a person earns.

> Gross earnings, deductions and net income are normally shown on a payment advice (slip) which is given to each worker when they get paid.

Deductions, such as income tax, pension contributions, unemployment and health insurance and union dues are often taken from the gross earnings before the person is paid. The amount that is left over after deductions is called the **net income**.

Net income = Gross income − deductions

Exercise 17.2

1 For each person shown in the table:

a calculate their net income
b calculate what percentage their net income is of their gross income. Give your answers to the nearest whole percent.

Employee	Gross weekly earnings ($)	Tax ($)	Other deductions ($)
B Willis	675.90	235.45	123.45
M Freeman	456.50	245.20	52.41
J Malkovich	1289.00	527.45	204.35
H Mirren	908.45	402.12	123.20
M Parker	853.30	399.10	90.56

2 Use the gross weekly earnings to work out:

a the mean weekly earnings
b the median weekly earnings
c the range of earnings.

Applying your skills

3 Study the following two pay advice slips. For each worker, calculate:

a the difference between gross and net income
b the percentage of gross income that each takes home as net income.

<table>
<tr><td colspan="2" align="center">Poovan's Plastics Pty Ltd
PAYMENT ADVICE</td></tr>
</table>

EMPLOYEE DETAILS	SEPTEMBER
M Badru Employee no: MBN 0987	Income tax no. 0987654321A Bank details Big Bucks Bank Account no. 9876598

EARNINGS

Details	Taxable Amount	Payable Amount	Description	Amount
Salary Medical Car allowance	12 876.98 650.50 1 234.99	12 876.98 0.00 0.00	Unemployment Insurance Fund (UIF) First aid course fees Group life insurance Union membership PAYE	89.35 9.65 132.90 32.00 3 690.62
	14 762.47	12 876.98		3 954.52
			NET PAY:	**8 922.46**

Nehru–Kapoor Network Services	Employee name: B Singh Job title: Clerk	ID number: 630907000000
Hours/Days Normal hours 84.00 O/time @ 1.5 hours 11.00	**Earnings** Wages 1402.80 Overtime @ 1.5 275.55	**Deductions** Income tax 118.22 UIF 18.94 Pension fund 105.21 Loan 474.00 Sickpay 8.42
Deduct tools × 2 Deduct cellphone × 2		
Year-to-date Taxable 22 881.40 Benefits 0.00 Tax paid 509.30		
Current period Company Contributions 358.12	TOTAL EARNINGS 1678.35	TOTAL DEDUCTIONS 724.79 NET PAY 953.56

If you earn less than a certain amount each year you don't have to pay income tax. This amount is called the **tax threshold**.

In some Islamic countries, tax is not deducted from earnings. Instead people pay a portion of their earnings as a religious obligation (Zakat).

Getting information from tax tables

In most countries, employers have to take taxes from earnings and pay them over to the government's tax authority. The tax authority publishes a table of tax rates every year so that employers can work out how much tax to deduct. Here is a portion of a tax table:

TAXABLE INCOME (in $)	RATES OF TAX
0 – 132 000	18% of each $1
132 001 – 210 000	$23 760 + 25% of the amount above $132 000
210 001 – 290 000	$43 260 + 30% of the amount above $210 000
290 001 – 410 000	$67 260 + 35% of the amount above $290 000
410 001 – 525 000	$109 260 + 38% of the amount above $410 000
525 001 and above	$152 960 + 40% of the amount above $525 000

Worked example 4

Mr Smith's taxable income is $153 772.00 p.a. How much tax must he pay

a per year?   **b** per month?

a To work out the yearly tax, find his tax bracket on the table. His income is in row two because it is between $132 001 and $210 000.

He has to pay $23 760 + 25% of his earnings above $132 000.

$153 772 − $132 000 = $21 772
25% of $21 772 = $5443

Tax payable = $23 760 + $5443 = $29 203 per year

b $29 203 ÷ 12 = $2433.58 | To find the monthly tax, divide the total from part (a) by 12.

Exercise 17.3   *Applying your skills*

1 Use the tax table above to work out the annual tax payable and the monthly tax deductions for each of the following taxable incomes.

    a $98 000   **b** $120 000   **c** $129 000   **d** $135 000   **e** $178 000

2 Use the tax table below to answer the questions that follow.

Single person (no dependants)	
Taxable income	Income tax payable
$0–$8375	10% of the amount over $0
$8375–$34 000	$837.50 plus 15% of the amount over $8375
$34 000–$82 400	$4681.25 plus 25% of the amount over $34 000
$82 400–$171 850	$16 781.25 plus 28% of the amount over $82 400
$171 850–$373 650	$41 827.25 plus 33% of the amount over $171 850
$373 650+	$108 421.25 plus 35% of the amount over $373 650

a Li-Gon has a taxable income of $40 000 for this tax year. He tells his friends that he is in the 25% tax bracket.
   i   Is this correct?
   ii Does it mean that he pays $10 000 in income tax? Explain why or why not.
   iii When Li-Gon checks his tax return, he finds that he only has to pay $6181.25 income tax. Show how this amount is calculated by the revenue services.

b How much tax would a person earning $250 000 pay in this tax year?
c Cecelia earned $30 000 in taxable income in this year. Her employer deducts $320.25 income tax per month from her salary.
   i   Will Cecelia have to pay in additional tax at the end of the tax year or will she be due for a tax refund as a result of overpaying?
   ii How much is the amount due in (i) above?

3 Income tax is one form of direct taxation. Carry out some research of your own to find out about each type of tax below, who pays this tax, how it is paid, and the rate/s at which it is charged.

   a Value-Added-Tax
   b General sales tax
   c Customs and Excise duties
   d Capital Gains Tax
   e Estate duties

17.2 Borrowing and investing money

When you borrow money or you buy things on credit, you are normally charged **interest** for the use of the money. Similarly, when you save or invest money, you are paid interest by the bank or financial institution in return for allowing them to keep and use your money.

Simple interest

Simple interest is a fixed percentage of the original amount borrowed or invested. In other words, if you borrow $100 at an **interest rate** of 5% per year, you will be charged $5 interest for every year of the loan.

Simple interest involves adding the interest amount to the original amount at regular intervals. The formula used to calculate simple interest is:

$$I = \frac{PRT}{100} \text{, where:}$$

P = the **principal**, which is the original amount borrowed or saved

R = the interest rate

T = the time (in years)

> **LINK**
>
> Business studies students will need to understand how the quantity of money in an account changes through the application of interest.

> In Islam, interest (*riba*) is forbidden so Islamic banks do not charge interest on loans or pay interest on investments. Instead, Islamic banks charge a fee for services which is fixed at the beginning of the transaction (*murabaha*). For investments, the bank and its clients share any profits or losses incurred over a given period in proportion to their investment (*musharaka*). Many banks in Islamic countries have the responsibility of collecting *Zakat* on behalf of the government. *Zakat* is a religious tax which all Muslims are obliged to pay. It is usually calculated at about 2.5% of personal wealth.

> Per annum means each year or annually. It is often abbreviated to p.a.

Worked example 5

$500 is invested at 10% per annum simple interest. How much interest is earned in three years?

10% of $500 = $\frac{10}{100} \times 500 = \$50$	The interest rate is 10% per annum.
The interest every year is $50. So after three years, the interest is: $3 \times \$50 = \$150$	Multiply by the number of years.

Worked example 6

Sam invested $400 at 15% per annum for three years. How much money did he have at the end of the period?

At the end of the period he would have P + I (the principal plus the interest paid).

$I = \dfrac{PRT}{100}$ and P = 400, so:

$P + I = 400 + \dfrac{(400 \times 15 \times 3)}{100}$

$\qquad = 400 + 180$

$\qquad = \$580$

Worked example 7

How long will it take for $250 invested at the rate of 8% per annum simple interest to amount to $310?

Amount = principal + interest
Interest = amount − principal
∴ Interest = $310 − $250 = $60

$\text{Rate} = 8\% \text{ per annum} = \dfrac{8}{100} \times 250 = \$20$

So the interest per year is $20.
Total interest (60) ÷ annual interest (20) = 3
So it will take three years for $250 to amount to $310 at the rate of 8% per annum simple interest.

Worked example 8

Calculate the rate of simple interest if a principal of $250 amounts to $400 in three years.

Interest paid = $400 − $250 = $150

$I = \dfrac{PRT}{100}$

$100I = PRT$

$R = \dfrac{100I}{PT} = \dfrac{100 \times 150}{250 \times 3} = 20$

So, the interest rate = 20%

Change the subject of the formula to R to find the rate.

Remember

You can manipulate the formula to find any of the values:

$I = \dfrac{PRT}{100}$

$P = \dfrac{100I}{RT}$

$R = \dfrac{100I}{PT}$

$T = \dfrac{100I}{PR}$

Exercise 17.4

1 For each of the following savings amounts, calculate the simple interest earned.

Principal amount ($)	Interest rate (%)	Time invested
500	1	3 years
650	0.75	$2\frac{1}{2}$ years
1000	1.25	5 years

Principal amount ($)	Interest rate (%)	Time invested
1200	4	$6\frac{3}{4}$ years
875	5.5	3 years
900	6	2 years
699	7.25	3.75 years
1200	8	9 months
150 000	$9\frac{1}{2}$	18 months

2 Calculate how much would have to be repaid in total for the following loans.

Principal amount ($)	Interest rate (%)	Time invested
500	4.5	2 years
650	5	2 years
1000	6	2 years
1200	12	18 months
875	15	18 months
900	15	3 years
699	20	9 months
1200	21.25	8 months
150 000	18	$1\frac{1}{2}$ years

3 $1400 is invested at 4% per annum simple interest. How long will it take for the amount to reach $1624?

4 The simple interest on $600 invested for five years is $210. What is the rate percentage per annum?

Applying your skills

5 If you invest a sum of money at a simple interest rate of 6%, how long will it take for your original amount to treble?

6 Jessica spends $\frac{1}{4}$ of her income from odd jobs on books, $\frac{1}{3}$ on transport and $\frac{1}{6}$ on clothing. The rest she saves.

    a If she saves $8 per month, how much is her income each month?
    b How much does she save in a year at a rate of $8 per month?
    c She deposits one year's savings into an account that pays 8.5% interest for five years.
        i How much interest would she earn?
        ii How much would she have altogether in the end?

7 Mrs MacGregor took a personal loan of ($)8000 over three years. She repaid ($)325 per month in that period.

    a How much did she repay in total?
    b How much interest did she pay in pounds?
    c At what rate was simple interest charged over the three years?

Hire purchase

In HP agreements, the deposit is sometimes called the down-payment. When interest is calculated as a proportion of the amount owed it is called a flat rate of interest. This is the same as simple interest.

Many people cannot afford to pay cash for expensive items like television sets, furniture and cars so they buy them on a system of payment called hire purchase (HP).

On HP you pay a part of the price as a deposit and the remainder in a certain number of weekly or monthly instalments. Interest is charged on outstanding balances. It is useful to be able to work out what interest rate is being charged on HP as it is not always clearly stated.

Worked example 9

The cash price of a car was $20 000. The hire purchase price was $6000 deposit and instalments of $700 per month for two years. How much more than the cash price was the hire purchase price?

Deposit = $6000
One instalment = $700
24 instalments = $700 × 24 = $16 800 (once per month over two years = 24 monthly instalments)
Total HP price = deposit + 24 instalments
= $6000 + $16 800
= $22 800
The hire purchase price was $2800 more than the cash price.

Worked example 10

A man buys a car for $30 000 on hire purchase. A deposit of 20% is paid and interest is paid on the outstanding balance for the period of repayment at the rate of 10% per annum. The balance is paid in 12 equal instalments. How much will each instalment be?

Cash price = $30 000

Deposit of 20% = $\dfrac{20}{100} \times 30\,000 = \$6000$

Outstanding balance = $30 000 − $6000 = $24 000

Interest of 10% = $\dfrac{10}{100} \times 24\,000 = \$2400$

Amount to be paid by instalments = outstanding balance + interest
= $24 000 + $2400
= $26 400

Each instalment = $\dfrac{26\,400}{12} = \$2200$ (divide by total number of instalments)

Exercise 17.5

1 A shopkeeper wants 25% deposit on a bicycle costing $400 and charges 20% interest on the remaining amount. How much is:
   a the deposit   **b** the interest   **c** the total cost of the bicycle?

2 A person pays 30% deposit on a fridge costing $2500 and pays the rest of the money in one year with interest of 20% per year. How much does she pay altogether for the fridge?

3 A student buys a laptop priced at $1850. She pays a 20% deposit and 12 equal monthly instalments. The interest rate is charged at 15% per annum on the outstanding balance.
   a How much is each monthly instalment?
   b What is the total cost of buying the laptop on HP?

4 A large flat screen TV costs $999. Josh agrees to pay $100 deposit and 12 monthly payments of $100.
   a Calculate the total amount of interest Josh will pay.
   b What rate of interest was he charged?

5 A second-hand car is advertised for $15 575 cash or $1600 deposit and 24 monthly payments of $734.70.
   a What is the difference between the cash price and the HP price?
   b What annual rate of interest is paid on the HP plan?

Compound interest

Simple interest is calculated on the original amount saved or borrowed. It is more common, however, to earn or to be charged **compound interest**. With a loan where you are charged compound interest, the interest is added to the amount you owe at regular intervals so the amount you owe increases for the next period. When you invest money for a fixed period, you can earn compound interest. In this case, the interest earned is added to the amount each period and you then earn interest on the amount plus the interest for the next period.

One way of doing compound interest calculations is to view them as a series of simple interest calculations. This method is shown in the following worked example.

When the principal, rate and time are the same, compound interest will be higher than simple interest. The exception is when the interest is only calculated for one period (for example one year), in that case, the compound interest and the simple interest will be the same.

Worked example 11

Priya invests $100 at a rate of 10%, compounded annually. How much money will she have after three years?

Year 1

$$I = \frac{PRT}{100} = \frac{100 \times 10 \times 1}{100} = \$10$$

$P + I = \$100 + \$10 = \$110$

Use the formula for simple interest.

Year 2

$$I = \frac{PRT}{100} = \frac{110 \times 10 \times 1}{100} = \$11$$

$P + I = 110 + 11 = \$121.00$

P for year two is $110; T is one year as you are only finding the interest for year two.

Year 3

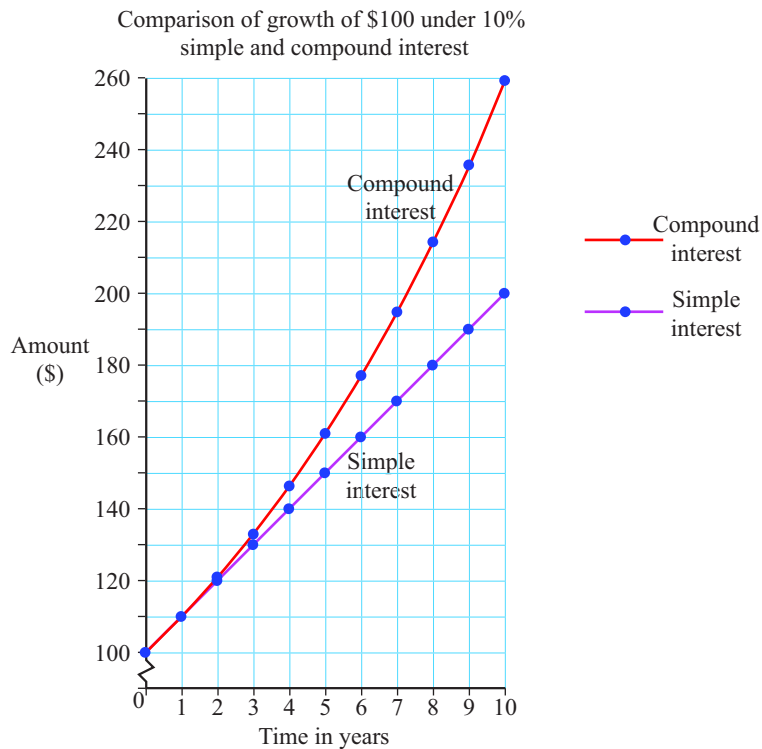$$I = \frac{PRT}{100} = \frac{121 \times 10 \times 1}{100} = \$12.10$$

$P + I = \$133.10$

P for year three is $121; T remains one year.

This table and graph compare the value of two $100 investments. The first is invested at 10% simple interest, the second at 10% compound interest.

Year (T)	Total $ 10% simple interest	Total $ 10% interest compounded annually
1	110	110
2	120	121
3	130	133.10
4	140	146.41
5	150	161.05
6	160	177.16
7	170	194.87
8	180	214.36
9	190	235.79
10	200	259.37

Comparison of growth of $100 under 10%
simple and compound interest

It is clear that choosing a compound interest rate is to the advantage of the investor. Remember though, that the same effect is felt with borrowing – the outstanding debt increases each period as the interest is compounded.

It takes a long time and lots of calculation to work out compound interest as a series of simple interest calculations. But there is a quicker method. Look at the calculations in the third column of the table.

Year (T)	Total $ 10% interest compounded annually	Working using a multiplier
1	110	$100 \times 1.1 = 110$
2	121	$100 \times 1.1 \times 1.1 = 121$
3	133.10	$100 \times 1.1 \times 1.1 \times 1.1 = 133.10$
4	146.41	$100 \times (1.1)^4 = 146.41$
5	161.05	$100 \times (1.1)^5 = 161.05$
6	177.16	$100 \times (1.1)^6 = 177.16$
7	194.87	$100 \times (1.1)^7 = 194.87$
8	214.36	$100 \times (1.1)^8 = 214.36$
9	235.79	$100 \times (1.1)^9 = 235.79$
10	259.37	$100 \times (1.1)^{10} = 259.37$

Multiply the decimal by itself the same number of times as the number of years. For three years it would be $1.1 \times 1.1 \times 1.1$ or $(1.1)^3$ not 1.1×3!

Can you see the rule?

● Add the annual interest rate to 100 to get a percentage increase (subtract for a decrease):
$100\% + 10\% = 110\%$

● Express this as a decimal: $\dfrac{110\%}{100} = 1.1$

● Multiply the principal by a power of the decimal using the number of years as the power. So, for five years: $100 \times (1.1)^5$

You can also insert values into a formula to calculate the value of an investment when it is subject to compound interest.

$$V = P\left(1 + \frac{r}{100}\right)^n, \text{ where}$$

P is the amount invested

r is the percentage rate of interest

n is the number of years of compound interest.

Worked example 12

1 $1500 is invested at 5% p.a. compound interest. What will the investment be worth after 5 years?

$V = P\left(1 + \dfrac{r}{100}\right)^n$ $= 1500\,(1 + 0.05)^5$ $= \$1914.42$	Insert values in the formula and then use your calculator.

2 A sum of money invested for 5 years at a rate of 5% interest, compounded yearly, grows to $2500. What was the initial sum invested?

$V = P\left(1 + \dfrac{r}{100}\right)^n$ So, $P = A\left(1 + \dfrac{r}{100}\right)^n$ $= \dfrac{2500}{(1 + 0.05)^5}$ $= \$1958.82$	Change the subject of the formula to make P the subject.

Exercise 17.6

1 Calculate the total amount owing on a loan of $8000 after two years at an interest rate of 12%:

   a compounded annually   b calculated as a flat rate.

2 How much would you have to repay on a credit card debt of $3500 after two years if the interest rate is:

   a 19.5% compounded annually?
   b 19.5% compounded half-yearly (the interest rate will be half of 19.5 for half a year)?

3 Calculate the total amount owing on a housing loan of $60 000 after ten years if the interest rate is 4% compounded annually.

4 Jessica bought an apartment in Hong Kong for (US)$320 000 as an investment. If the value of her apartment appreciates at an average rate of 3.5% per annum, what would it be worth in five years' time?

Exponential growth and decay

When a quantity increases (grows) in a fixed proportion (normally a percentage) at regular intervals, the growth is said to be exponential. Similarly, when the quantity decreases (decays) by a fixed percentage over regular periods of time, it is called exponential decay.

Increasing exponential functions produce curved graphs that slope steeply up to the right. Decreasing exponential functions produce curved graphs that slope down steeply to the right.

When financial investments increase or decrease in value at an exponential rate we talk about appreciation (growth) and depreciation. When the number of individuals in a population increase or decrease exponentially over time, we usually talk about growth or decay.

You will deal with exponential curves in more detail in chapter 18. ▶

E

Exponential growth and decay can be expressed using formulae.

For growth: $y = a(1 + r)^n$

For decay: $y = a(1 - r)^n$

Where a is the original value or principal, r is the rate of change expressed as a decimal and n is the number of time periods.

Worked example 13

$100 is invested subject to compound interest at a rate of 8% per annum. Find the value of the investment correct to the nearest cent after a period of 15 years.

Value = $a(1 + r)^n$	Use the formula for exponential growth and substitute the given values.
$= 100(1 + 0.8)^{15}$	
$= 100(1.08)^{15}$	
$= 317.2169114$	

Value of investment is $317.22 (correct to the nearest cent).

Worked example 14

The value of a new computer system depreciates by 30% per year. If it cost $1200 new, what will it be worth in two years' time?

Value = $a(1 - r)^n$	Use the formula for exponential decay and substitute the given values.
$= 1200(1 - 0.3)^2$	
$= 1200(0.7)^2$	
$= 588$	

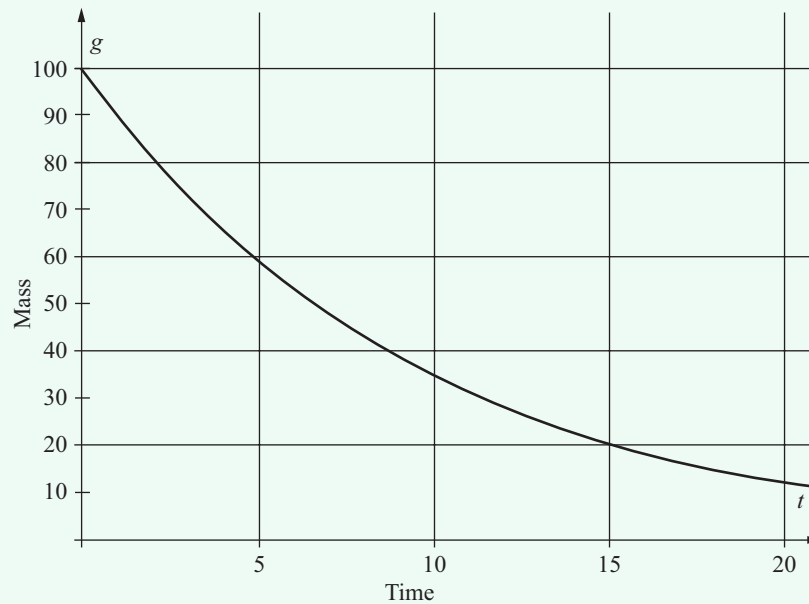Value after two years is $588.

Exercise 17.7

1 The human population of Earth in August 2010 was estimated to be 6.859 billion people. In August 2009, the population grew at a rate of 1.13%. Assuming this growth rate continues, estimate the population of the world in August of:

   a 2015   **b** 2020   **c** 2025.

2 In 2010 there were an estimated 1600 giant pandas in China. Calculate the likely panda population in 2025 if there is:

   a an annual growth in the population of 0.5%
   b an annual decline in the population of 0.5%.

3 A population of microbes in a laboratory doubles every day. At the start of the period, the population is estimated to be 1 000 000 microbes.

   a Copy and complete this table to show the growth in the population.

Time (days)	0	1	2	3	4	5	6	7	8
Total number of microbes (millions)	1	2	4						

   b Draw a graph to show growth in the population over 8 days.
   c Use the graph to determine the microbe population after:
      i 2.5 days   **ii** 3.6 days
   d Use the graph to determine how long it will take the microbe population to reach 20 million.

4 This graph shows how a radio-active substance loses its radioactivity over time.

E

a The half life of the substance is how long it takes to decay to half its original mass. What is the half life of this substance?

b What mass of the substance is left after 20 minutes?

> This is essentially the same as the compound interest formula above.

5 Ms Singh owns a small business. She borrows $18 500 from the bank to finance some new equipment. She repays the loan in full after two years. If the bank charged her compound interest at the rate of 21% per annum, how much did she repay after two years?

6 The value of a car depreciates each year by 8%. A new small car is priced at $11 000. How much will this car be worth in:

a 1 year   **b** 3 years   **c** 8 years   **d** n years?

7 Nils invests his savings in an account that pays 6% interest compounded half yearly. If he puts $2300 into his account and leaves it there for two years, how much money will he have at the end of the period?

8 The total population of a European country is decreasing at a rate of 0.6% per year. In 2014, the population of the country was 7.4 million people.

a What is the population likely to be in 2020 if it decreases at the same rate?

b How long will it take for the population to drop below 7 million people?

9 A colony of bacteria grows by 5% every hour. How long does it take for the colony to double in size?

17.3 Buying and selling

When people trade they buy goods, mark them up (decide on a price) and then sell them.

The price the trader pays for goods is called the **cost price**.

The price the goods are sold at is called the **selling price**.

If the selling price is higher than the cost price, the goods are sold at a **profit**.

If the selling price is lower than the cost price, the goods are sold at a **loss**.

profit = selling price − cost price

loss = cost price − selling price

Percentage profit and loss

Profit and loss are normally calculated as percentages of the cost price.

The following formulae are used to calculate percentage profit or loss:

$$\text{percentage profit} = \frac{\text{actual profit}}{\text{cost price}} \times 100\%$$

$$\text{percentage loss} = \frac{\text{actual loss}}{\text{cost price}} \times 100\%$$

Worked example 15

A shopkeeper buys an article for $500 and sells it for $600. What is the percentage profit?

Profit = selling price − cost price

$\quad\quad = \$600 − \$500$

$\quad\quad = \$100$

Percentage profit $= \dfrac{\text{profit}}{\text{cost}} \times 100\%$

$\quad\quad\quad\quad\quad\quad = \dfrac{\$100}{\$500} \times 100\%$

$\quad\quad\quad\quad\quad\quad = 20\%$

Worked example 16

A person buys a car for $16 000 and sells it for $12 000. Calculate the percentage loss.

Loss = cost price − selling price

$\quad\quad = \$16\,000 − \$12\,000$

$\quad\quad = \$4000$

Percentage loss $= \dfrac{\text{loss}}{\text{cost}} \times 100\%$

$\quad\quad\quad\quad\quad\quad = \dfrac{\$4000}{\$16\,000} \times 100\%$

$\quad\quad\quad\quad\quad\quad = 25\%$

Exercise 17.8

1 Find the actual profit and percentage profit in the following cases (use an appropriate degree of accuracy where needed):

   a cost price $20, selling price $25   **b** cost price $500, selling price $550

   c cost price $1.50, selling price $1.80   **d** cost price 30 cents, selling price 35 cents.

2 Calculate the percentage loss in the following cases (use an appropriate degree of accuracy where needed):

   a cost price $400, selling price $300   **b** cost price 75c, selling price 65c

   c cost price $5.00, selling price $4.75   **d** cost price $6.50, selling price $5.85.

3 A market trader buys 100 oranges for $30. She sells them for 50 cents each.
Calculate the percentage profit or loss she made.

Calculating the selling price, cost price and mark up

The cost price is always 100%. If you add 10% mark up, the selling price will be 110%.

People who sell goods have to decide how much profit they want to make. In other words, they have to decide by how much they will mark up the cost price to make the selling price.

   cost price + % mark up = selling price

Worked example 17

A trader sells her product for $39. If her mark up is 30%, what is the cost price of the product?

Cost price + mark up = selling price
Selling price = 130% of the cost price
So, $39 = 130% × selling price
To find 100%:

$$\frac{39}{130} \times 100 = \$30$$

The cost price was $30

Worked example 18

At a market, a trader makes a profit of $1.08 on an item selling for $6.48. What is his percentage profit?

Cost price + mark up = selling price
Selling price − mark up = cost price
$6.48 − $1.08 = $5.40

$$\text{Percentage profit} = \frac{\text{actual profit}}{\text{cost price}} \times 100$$

$$\frac{1.08}{5.40} \times 100 = 20\%$$

Express the mark up as a percentage of cost price.

Worked example 19

Find the selling price of an article bought for $400 and sold at a loss of 10%.

Cost price = $400
Loss = 10% of $400

$$= \frac{10}{100} \times 400$$
$$= \$40$$

Selling price = cost price − loss
$$= \$400 − \$40$$
$$= \$360$$

Exercise 17.9

1 Find the cost price of each of the following items:
   a selling price $130, profit 20%
   b selling price $320, profit 25%
   c selling price $399, loss 15%
   d selling price $750, loss $33\frac{1}{3}$%.

2 Find the selling price of an article that was bought for $750 and sold at a profit of 12%.

3 Calculate the selling price of a car bought for $3000 and sold at a profit of 7.5%.

4 Hakim bought a computer for $500. Two years later he sold it at a loss of 28%.
   What was his selling price?

5 An article costing $240 is sold at a loss of 8%. Find the selling price.

6 Kwame makes jewellery and sells it to her friends. Her costs to make 10 rings were $377. She wants to sell them and make a 15% profit. What should she charge?

7 Tim sells burgers for $6.50 and makes a profit of $1.43 on each one. What is his percentage profit on cost price?

Applying your skills

8 VAT at a rate of 17% is added *each* time an item is sold on. The original cost of an item is $112.00. The item is sold to a wholesaler, who sells it on to a retailer. The retailer sells it to the public.

   a How much tax will the item have incurred?
   b Express the tax as a percentage of the original price.

Discount

If items are not being sold as quickly as a shop would like or if they want to clear stock as new fashions come out, then goods may be sold at a **discount**. Discount can be treated in the same way as percentage change (loss) as long as you remember that the percentage change is always calculated as a percentage of the original amount.

Worked example 20

During a sale, a shop offers a discount of 15% on jeans originally priced at $75. What is the sale price?

Discount $= 15\%$ of $75

$\qquad = \dfrac{15}{100} \times 75$

$\qquad = \$11.25$

Sale price $=$ original price $-$ discount

$\qquad = \$75 - 11.25$

$\qquad = \$63.75$

You can also work out the price by considering the sale price as a percentage of 100%. $100 - 15 = 85$, so the sale price is 85% of $75:

$\dfrac{85}{100} \times 75 = \$63.75$

Exercise 17.10

1 Copy and complete the following table.

Original price ($)	% discount	Savings ($)	Sale price ($)
89.99	5		
125.99	10		
599.00	12		
22.50	7.5		
65.80	2.5		
10 000.00	23		

2 Calculate the percentage discount given on the following sales. Give your answer rounded to the nearest whole per cent.

Original price ($)	Sale price ($)	% discount
89.99	79.99	
125.99	120.00	
599.00	450.00	
22.50	18.50	
65.80	58.99	
10 000.00	9500.00	

Summary

Do you know the following?

- People in employment earn money for the work they do. This money can be paid as wages, salaries, commission or as a fee per item produced (piece work).
- Gross earnings refers to how much you earn before deductions. Gross earnings – deductions = net earnings. Your net earnings are what you actually receive as payment.
- Companies are obliged by law to deduct tax and certain other amounts from earnings.
- Simple interest is calculated per time period as a fixed percentage of the original amount (the principal). The formula for finding simple interest is $I = \dfrac{PRT}{100}$.
- Compound interest is interest added to the original amount at set intervals. This increases the principal and further interest is compounded. Most interest in real life situations is compounded.
- The formula for calculating compound interest is
$$V = P\left(1 + \frac{r}{100}\right)^n$$
- Hire purchase (HP) is a method of buying goods on credit and paying for them in instalments which include a flat rate of interest added to the original price.
- When goods are sold at a profit they are sold for more than they cost. When they are sold at a loss they are sold for less than they cost. The original price is called the cost price. The price they are sold for is called the selling price. If goods are sold at a profit, selling price – cost price = profit. If they are sold at a loss, cost price – selling price = loss.
- A discount is a reduction in the usual price of an item. A discount of 15% means you pay 15% less than the usual or marked price.

Are you able to …?

- use given information to solve problems related to wages, salaries, commission and piece work
- read information from tables and charts to work out deductions and tax rates
- calculate gross and net earnings given the relevant information
- use the formula to calculate simple interest
- manipulate the simple interest formula to calculate the principal amount, rate of interest and time period of a debt or investment
- solve problems related to HP payments and amounts
- calculate compound interest over a given time period and solve problems related to compound interest
- use exponential growth and decay in relation to finance and population changes   Ⓔ
- calculate the cost price, selling price, percentage profit or loss and actual mark up using given rates and prices
- work out the actual price of a discounted item and calculate the percentage discount given the original and the new price.

Examination practice

Exam-style questions

1 Sayed is paid $8.50 per hour for a standard 36-hour week. He is paid 'time-and-a-half' for all overtime worked. Calculate:

   a his gross weekly earnings if he works $4\frac{3}{4}$ hours overtime
   b the hours overtime worked if he earns $420.75 for the week.

2 Ahmed bought a DVD for $15. He sold it to Barbara, making a 20% loss.

   a How much did Barbara pay for it?
   b Barbara later sold the DVD to Luvuyo. She made a 20% profit.
   How much did Luvuyo pay for it?

3 Last year, Jane's wages were $80 per week. Her wages are now $86 per week. Calculate the percentage increase.

4 What is the simple interest on $160 invested at 7% per year for three years?

5 Senor Vasquez invests $500 in a Government Bond, at 9% simple interest per year. How much will the Bond be worth after three years?

6 Simon's salary has increased by 6% p.a. over the past three years. It is now $35 730.40 p.a.

   a What did he earn per year three years ago?
   b What is his gross monthly salary at the present rate?
   c His deductions each month amount to 22.5% of his gross salary. What is his net pay per month?

7 A new car cost $14 875. Three years later, the insurance company valued it at $10 700. Calculate the percentage reduction in value over the three years.

8 Exercise equipment advertised at $2200 is sold on sale for $1950. What percentage discount is this?

Past paper questions

1 Robert buys a car for $8000.   **E**

   At the end of each year the value of the car has decreased by 10% of its value at the beginning of that year.

   Calculate the value of the car at the end of 7 years.   [2]

   [Cambridge IGCSE Mathematics 0580 Paper 22 Q8 October/November 2015]

2 Anita buys a computer for $391 in a sale.

   The sale price is 15% less than the original price.

   Calculate the original price of the computer.   [3]

   [Cambridge IGCSE Mathematics 0580 Paper 22 Q11 May/June 2014]