# Chapter 10: Straight lines and quadratic equations

**Key words**

- Equation of a line
- Gradient
- $y$-intercept
- Constant
- $x$-intercept
- Line segment
- Midpoint
- Expand
- Constant term
- Quadratic expression
- Factorisation
- Difference between two squares
- Quadratic equation

**In this chapter you will learn how to:**

- construct a table of values and plot points to draw graphs
- find the gradient of a straight line graph
- recognise and determine the equation of a line
- determine the equation of a line parallel to a given line

EXTENDED

- calculate the gradient of a line using co-ordinates of points on the line
- find the gradient of parallel and perpendicular lines
- find the length of a line segment and the co-ordinates of its midpoint

- expand products of algebraic expressions

EXTENDED

- factorise quadratic expressions
- solve quadratic equations by factorisation

**Geoff wishes he had paid more attention when his teacher talked about negative and positive gradients and rates of change.**

On 4 October 1957, the first artificial satellite, Sputnik, was launched. This satellite orbited the Earth but many satellites that do experiments to study the upper atmosphere fly on short, sub-orbital flights. The flight path can be described with a quadratic equation, so scientists know where the rocket will be when it deploys its parachute and so they know where to recover the instruments. The same equation can be used to describe any thrown projectile including a baseball!

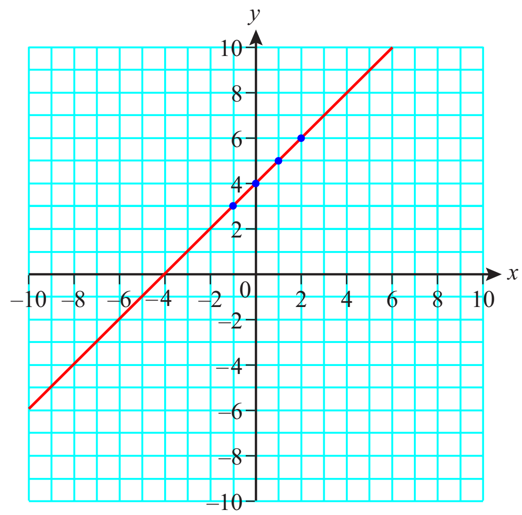
## RECAP

You should already be familiar with the following algebra and graph work:

**Table of values and straight line graphs (Stage 9 Mathematics)**

A table of values gives a set of ordered pairs $(x, y)$ that you can use to plot graphs on a coordinate grid.

| $x$ | –1 | 0 | 1 | 2 |
|---|---|---|---|---|
| $y$ | 3 | 4 | 5 | 6 |




(–1, 3), (0, 4), (1, 5) and (2, 6) are all points on the graph.
Plot them and draw a line through them.

**Equations in the form of $y = mx + c$ (Year 9 Mathematics)**

The standard equation of a straight line graph is $y = mx + c$

- $m$ is the gradient (or steepness) of the graph
- $c$ is the point where the graph crosses the $y$-axis (the $y$-intercept)

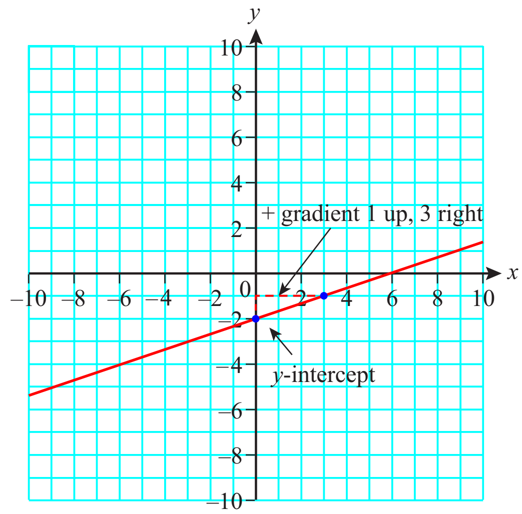
**Gradient of a straight line (Year 9 Mathematics)**

$$\text{Gradient } (m) = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y\text{-values}}{\text{change in } x\text{-values}}$$

**Drawing a straight line graph (Year 9 Mathematics)**

You can use the equation of a graph to find the gradient and $y$-intercept and use these to draw the graph.

For example $y = \frac{1}{3}x - 2$




**Expand expressions to remove brackets (Chapter 2)**

To expand $3x(2x - 4)$ you multiply the term outside the bracket by each term inside the first bracket

$$3x\,(2x - 4) = 3x \times 2x - 3x \times 4$$
$$= 6x^2 - 12x$$

# 10.1 Straight lines

## Using equations to plot lines

Mr Keele owns a boat hire company. If Mr Keele makes a flat charge of \$40 and then another \$15 per hour of hire, you can find a formula for the total cost \$$y$ after a hire time of $x$ hours.

Total cost = flat charge + total charge for all hours

$y = 40 + 15 \times x$

or (rearranging)

$y = 15x + 40$

REWIND

You will recognise that the formulae used to describe $n^{th}$ terms in chapter 9 are very similar to the equations used in this chapter.

Now think about the total cost for a range of different hire times:

one hour: cost = $15 \times 1 + 40 = \$55$

two hours: cost = $15 \times 2 + 40 = \$70$

three hours: cost = $15 \times 3 + 40 = \$85$

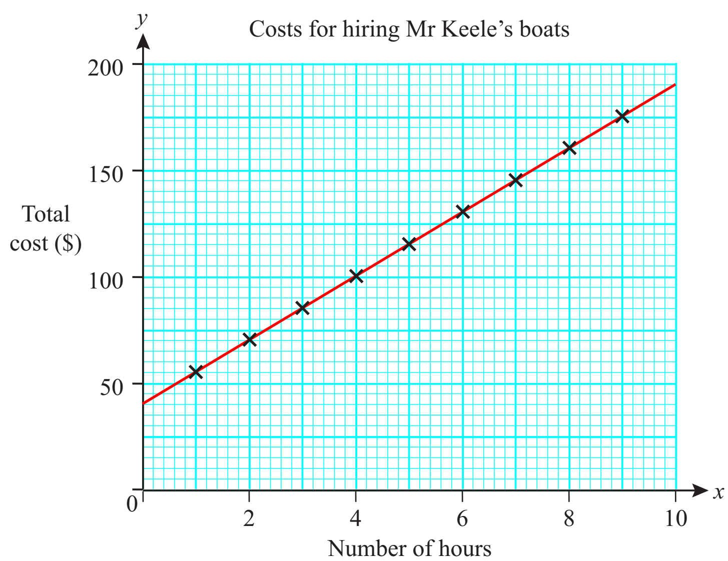
and so on.

If you put these values into a table (with some more added) you can then plot a graph of the total cost against the number of hire hours:

| **Number of hours ($x$)** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| **Total cost ($y$)** | 55 | 70 | 85 | 100 | 115 | 130 | 145 | 160 | 175 |

LINK

Equations of motion, in physics, often include terms that are squared. To solve some problems relating to physical problems, therefore, physicists often need to solve quadratic equations.




The graph shows the total cost of the boat hire (plotted on the vertical axis) against the number of hire hours (on the horizontal axis). Notice that the points all lie on a straight line.

The formula $y = 15x + 40$ tells you how the $y$ co-ordinates of all points on the line are related to the $x$ co-ordinates. This formula is called an **equation of the line**.

The following worked examples show you how some more lines can be drawn from given equations.

## Worked example 1

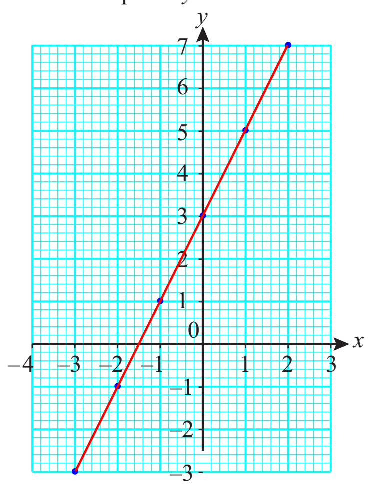
A straight line has equation $y = 2x + 3$. Construct a table of values for $x$ and $y$ and draw the line on a labelled pair of axes. Use integer values of $x$ from –3 to 2.

Substituting the values –3, –2, –1, 0, 1 and 2 into the equation gives the values in the following table:

| $x$ | –3 | –2 | –1 | 0 | 1 | 2 |
|---|---|---|---|---|---|---|
| $y$ | –3 | –1 | 1 | 3 | 5 | 7 |

Notice that the $y$-values range from –3 to 7, so your $y$-axis should allow for this.

Graph of $y = 2x + 3$



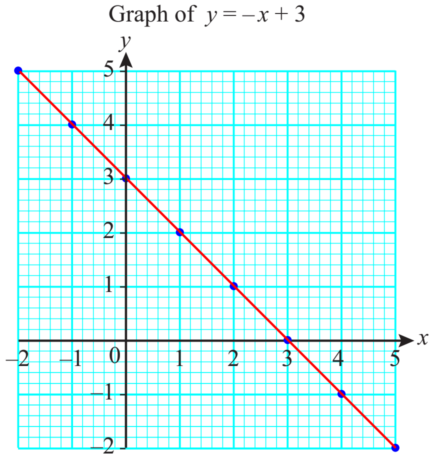

## Worked example 2

Draw the line with equation $y = -x + 3$ for $x$-values between –2 and 5 inclusive.

The table for this line would be:

| $x$ | –2 | –1 | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 5 | 4 | 3 | 2 | 1 | 0 | –1 | –2 |

Graph of $y = -x + 3$



To draw a graph from its equation:

- draw up a table of values and fill in the $x$ and $y$ co-ordinates of at least three points (although you may be given more)
- draw up and label your set of axes for the range of $y$-values you have worked out
- plot each point on the number plane
- draw a straight line to join the points (use a ruler).

Before drawing your axes, always check that you know the range of $y$-values that you need to use.

## Exercise 10.1

**1** Make a table for $x$-values from $-3$ to 3 for each of the following equations.
Plot the co-ordinates on separate pairs of axes and draw the lines.

**a** $y = 3x + 2$ **b** $y = x + 2$ **c** $y = 2x - 1$ **d** $y = 5x - 4$
**e** $y = -2x + 1$ **f** $y = -x - 2$ **g** $y = 6 - x$ **h** $y = 3x + \frac{1}{2}$
**i** $y = \frac{1}{2}x + 1$ **j** $y = 4x$ **k** $y = -3$ **l** $y = -1 - x$
**m** $x + y = 4$ **n** $x - y = 2$ **o** $y = x$ **p** $y = -x$

**2** Plot the lines $y = 2x$, $y = 2x + 1$, $y = 2x - 3$ and $y = 2x + 2$ on the same pair of axes.
Use $x$-values from $-3$ to 3. What do you notice about the lines that you have drawn?

**3** For each of the following equations, draw up a table of $x$-values for $-3$, 0 and 3.
Complete the table of values and plot the graphs on the same set of axes.

**a** $y = x + 2$ **b** $y = -x + 2$ **c** $y = x - 2$ **d** $y = -x - 2$

**4** Use your graphs from question **3** above to answer these questions.

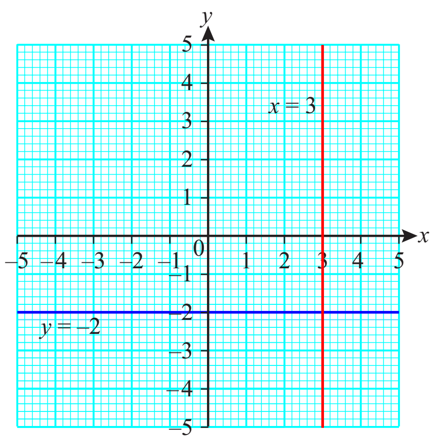
**a** Where do the graphs cut the $x$-axis?
**b** Which graphs slope up to the right?
**c** Which graphs slope down to the right?
**d** Which graphs cut the $y$-axis at (0, 2)?
**e** Which graphs cut the $y$-axis at (0, $-2$)?
**f** Does the point (3, 3) lie on any of the graphs? If so, which?
**g** Which graphs are parallel to each other?
**h** Compare the equations of graphs that are parallel to each other. How are they similar? How are they different?

## Gradient

The **gradient** of a line tells you how steep the line is. For every one unit moved to the right, the gradient will tell you how much the line moves up (or down). When graphs are parallel to each other, they have the same gradient.

### *Vertical and horizontal lines*

Look at the two lines shown in the following diagram:



Every point on the vertical line has $x$ co-ordinate = 3. So the equation of the line is simply $x = 3$.

Every point on the horizontal line has $y$ co-ordinate = –2. So the equation of this line is $y = -2$.

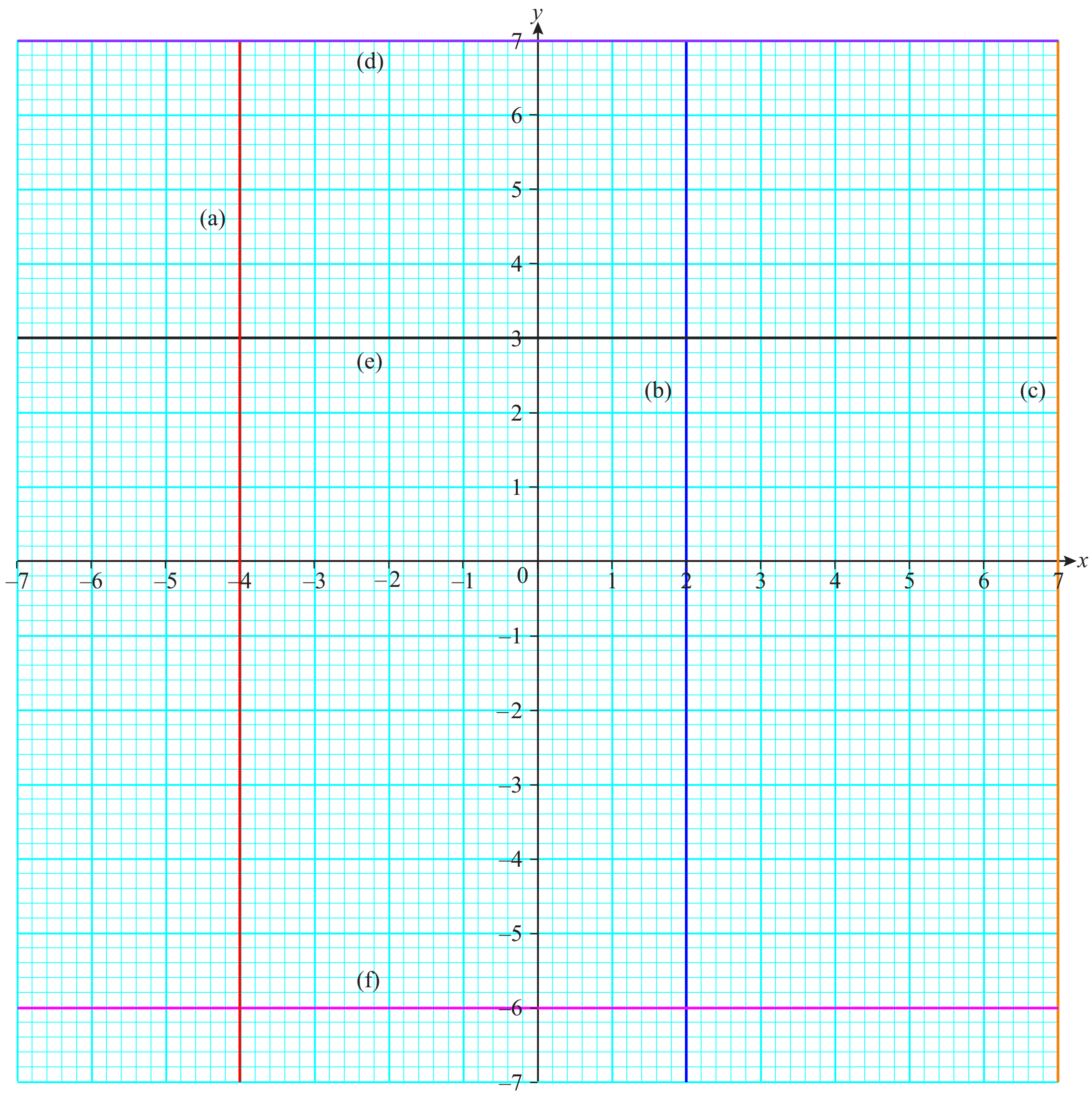
All vertical lines are of the form: $x$ = a number.

All horizontal lines are of the form: $y$ = a number.

The gradient of a horizontal line is zero (it does not move up or down when you move to the right).

## Exercise 10.2

**1** Write down the equation of each line shown in the diagram.




**2** Draw the following graphs on the same set of axes without plotting points or drawing up a table of values.

**a** $y = 3$ **b** $x = 3$ **c** $y = -1$ **d** $x = -1$

**e** $y = -3$ **f** $y = 4$ **g** $x = \frac{1}{2}$ **h** $x = \frac{-7}{2}$

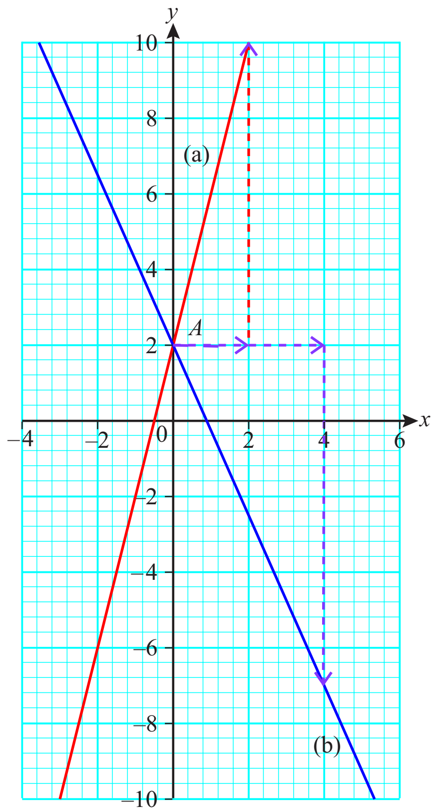
**i** a graph parallel to the $x$-axis which cuts the $y$-axis at (0, 4)

**j** a graph parallel to the $y$-axis which goes through the point (–2, 0)

FAST FORWARD

You will deal with gradient as a rate of change when you work with kinematic graphs in Chapter 21.

### *Lines that are neither vertical nor horizontal*




The diagram shows two different lines. If you take a point $A$ on the line and then move *to the right* then, on graph (a) you need to move up to return to the line, and on graph (b) you need to move down.

The gradient of a line measures how steep the line is and is calculated by dividing the change in the $y$ co-ordinate by the change in the $x$ co-ordinate:

$$\boxed{\text{gradient} = \frac{y\text{-change}}{x\text{-increase}}}$$

Another good way of remembering the gradient formula is gradient $= \frac{\text{'rise'}}{\text{'run'}}$. The 'run' must always be to the right (increase $x$).

For graph (a): the $y$-change is 8 and the $x$-increase is 2, so the gradient is $\frac{8}{2} = 4$

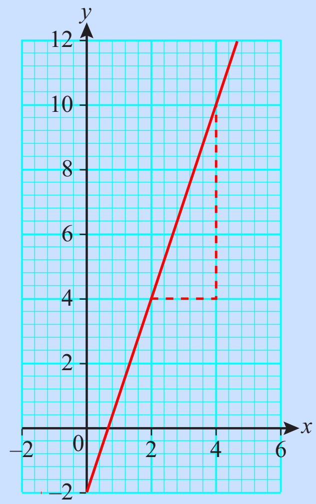
For graph (b): the $y$-change is –9 (negative because you need to move *down* to return to the line) and the $x$-increase is 4, so the gradient is $\frac{-9}{4} = -2.25$.

It is essential that you think about *x-increases* only. Whether the $y$-change is positive or negative tells you what the sign of the gradient will be.

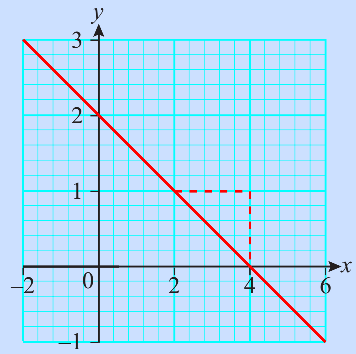
## Worked example 3

Calculate the gradient of each line. Leave your answer as a whole number or fraction in its lowest terms.

**a**




**b**




**a** Notice that the graph passes through the points (2, 4) and (4, 10).

$$\text{gradient} = \frac{y\text{-change}}{x\text{-increase}} = \frac{10-4}{4-2} = \frac{6}{2} = 3$$

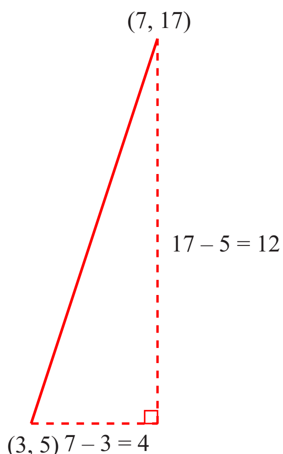
**b** Notice that the graph passes through the points (2, 1) and (4, 0).

$$\text{gradient} = \frac{y\text{-change}}{x\text{-increase}} = \frac{0-1}{4-2} = -\frac{1}{2}$$

E

## Worked example 4

Calculate the gradient of the line that passes through the points (3, 5) and (7, 17).



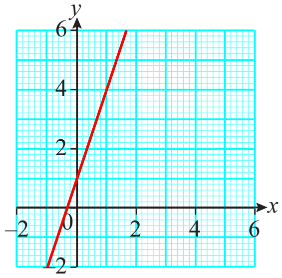

$$\text{gradient} = \frac{y\text{-change}}{x\text{-increase}} = \frac{17-5}{7-3} = \frac{12}{4} = 3$$

Think about where the points would be, in relation to each other, on a pair of axes. You don't need to draw this accurately but the diagram will give you an idea of how it may appear.

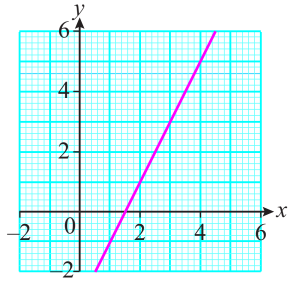
## Exercise 10.3

**1** Calculate the gradient of each line. Leave your answers as a fraction in its lowest terms.

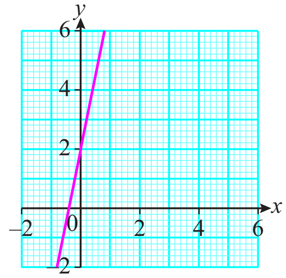
**a**



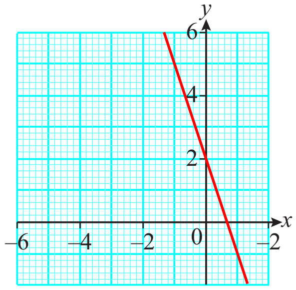
**b**



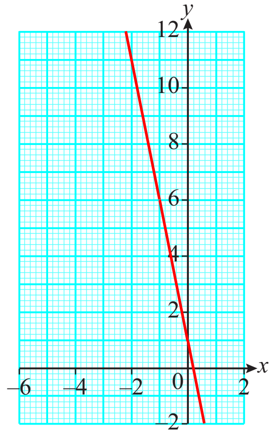
**c**



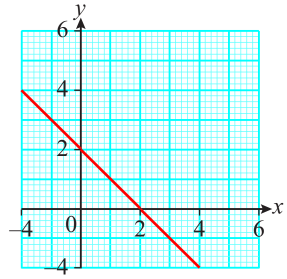
**d**



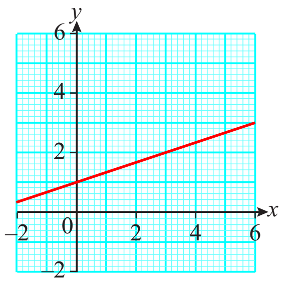
**e**



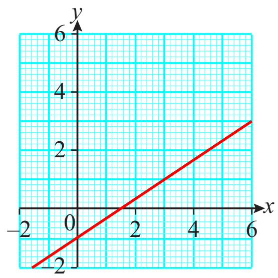
**f**



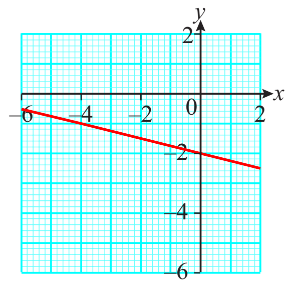
**g**



**h**



**i**



Think carefully about whether you expect the gradient to be positive or negative.

**2** Calculate the gradient of the line that passes through both points in each case. Leave your answer as a whole number or a fraction in its lowest terms.

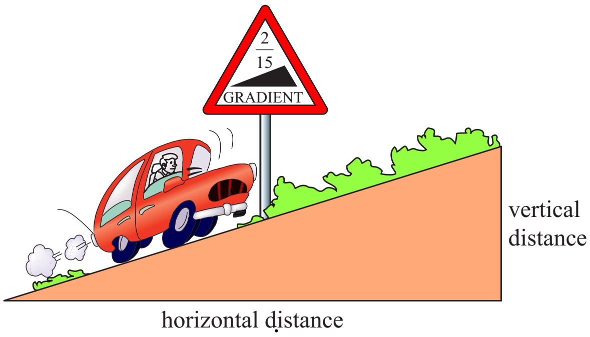
**a** $A$ (1, 2) and $B$ (3, 8)
**b** $A$ (0, 6) and $B$ (3, 9)
**c** $A$ (2, −1) and $B$ (4, 3)
**d** $A$ (3, 2) and $B$ (7, −10)
**e** $A$ (−1, −4) and $B$ (−3, 2)
**f** $A$ (3, −5) and $B$ (7, 12)

E

### *Applying your skills*

Think carefully about the problem and what mathematics you need to do to find the solution.

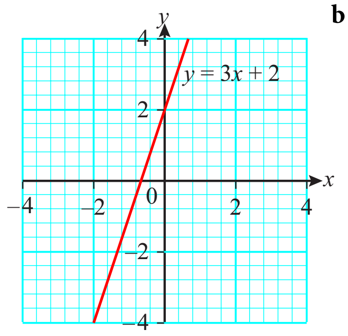
**3** If the car climbs 60 m vertically how far must the car have travelled *horizontally*?



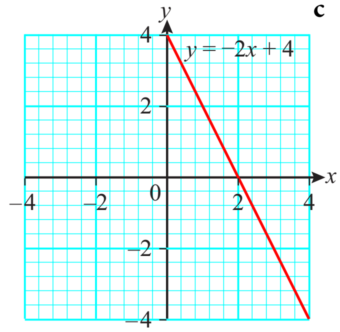
## Finding the equation of a line

Look at the three lines shown below.

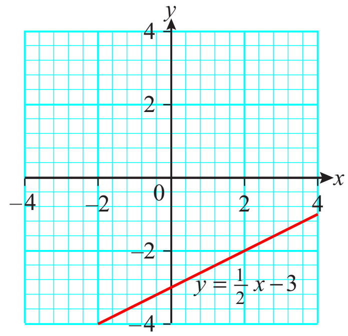
**a**




**b**




**c**




Check for yourself that the lines have the following gradients:

- gradient of line (a) = 3
- gradient of line (b) = −2
- gradient of line (c) = $\frac{1}{2}$

> **REWIND**
> You met the coefficient in chapter 2. ◀

Notice that the gradient of each line is equal to the coefficient of $x$ in the equation and that the point at which the line crosses the $y$-axis (known as the ***y*-intercept**) has a $y$ co-ordinate that is equal to the **constant** term.

In fact this is always true when $y$ is the subject of the equation:

$$y = mx + c$$

- $y$: $y$ is the subject of the equation
- $mx$: gradient
- $c$: $y$-intercept

In summary:

- equations of a straight line graphs can be written in the form of $y = mx + c$
- $c$ (the constant term) tells you where the graph cuts the $y$-axis (the $y$-intercept)
- $m$ (the coefficient of $x$) is the gradient of the graph; a negative value means the graph slopes down the to the right, a positive value means it slopes up to the right. The higher the value of $m$, the steeper the gradient of the graph
- graphs which have the same gradient are parallel to each other (therefore graphs that are parallel have the same gradient).

### Worked example 5

Find the gradient and $y$-intercept of the lines given by each of the following equations.

**a** $y = 3x + 4$ **b** $y = 5 - 3x$ **c** $y = \frac{1}{2}x + 9$ **d** $x + y = 8$

**e** $3x + 2y = 6$

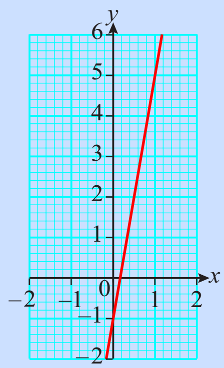
| | | |
|---|---|---|
| **a** | $y = 3x + 4$<br>Gradient = 3<br>$y$-intercept = 4 | The coefficient of $x$ is 3.<br>The constant term is 4. |
| **b** | $y = 5 - 3x$<br>Gradient = −3<br>$y$-intercept = 5 | Re-write the equation as $y = -3x + 5$.<br>The coefficient of $x$ is −3.<br>The constant term is 5. |

| | | |
|---|---|---|
| **c** | $y = \frac{1}{2}x + 9$<br>Gradient $= \frac{1}{2}$<br>$y$-intercept $= 9$ | The gradient can be a fraction. |
| **d** | $x + y = 8$<br>Gradient $= -1$<br>$y$-intercept $= 8$ | Subtracting $x$ from both sides, so that $y$ is the subject, gives $y = -x + 8$. |
| **e** | $3x + 2y = 6$<br>Gradient $= \frac{-3}{2}$<br>$y$-intercept $= 3$ | Make $y$ the subject of the equation.<br>$3x + 2y = 6$<br>$2y = -3x + 6$<br>$y = \frac{-3}{2}x + \frac{6}{2}$<br>$y = \frac{-3}{2}x + 3$ |

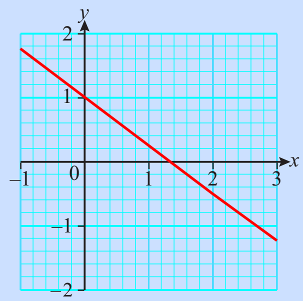
## Worked example 6

Find the equation of each line shown in the diagrams.

**a**




**b**




| | | |
|---|---|---|
| **a** | Gradient $= 6$ and the $y$-intercept $= -1$<br>So the equation is $y = 6x - 1$ | Gradient $= \frac{6}{1} = 6$<br>Graph crosses $y$-axis at $-1$ |
| **b** | Gradient $= \frac{-3}{4}$ and the $y$-intercept $= 1$<br>So the equation is $y = -\frac{3}{4}x + 1$ | Gradient $= \frac{-1.5}{2} = \frac{-3}{4}$<br>Graph crosses $y$-axis at 1. |

You should always label your axes $x$ and $y$ when drawing graphs – even when they are sketches.

## Exercise 10.4

Look carefully at your sketches for answers 1(d) and 1(g). If you draw them onto the same axes you will see that they are parallel. These lines have the same gradient but they cut the $y$-axis at different places. If two or more lines are parallel, they will have the same gradient.

**1** Find the gradient and $y$-intercept of the lines with the following equations. Sketch the graph in each case, taking care to show where the graph cuts the $y$-axis.

**a** $y = 4x - 5$ **b** $y = 2x + 3$ **c** $y = -3x - 2$ **d** $y = -x + 3$

**e** $y = \frac{1}{3}x + 2$ **f** $y = 6 - \frac{1}{4}x$ **g** $x + y = 4$ **h** $x + 2y = 4$

**i** $x + \frac{y}{2} = 3$ **j** $x = 4y - 2$ **k** $x = \frac{y}{4} + 2$ **l** $2x - 3y = -9$

**2** Rearrange each equation so that it is in the form $y = mx + c$ and then find the gradient and $y$-intercept of each graph.

**a** $2y = x - 4$ **b** $2x + y - 1 = 0$ **c** $x = \frac{y}{2} - 2$ **d** $2x - y - 5 = 0$

**e** $2x - y + 5 = 0$ **f** $x + 3y - 6 = 0$ **g** $4y = 12x - 8$ **h** $4x + y = 2$

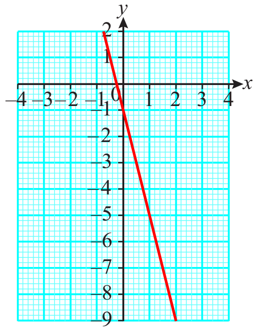
**i** $\frac{y}{2} = x + 2$ **j** $\frac{y}{3} = 2x - 4$ **k** $\frac{x}{2} - 4y = 12$ **l** $\frac{-y}{3} = 4x - 2$

**3** Find the equation (in the form of $y = mx + c$) of a line which has:

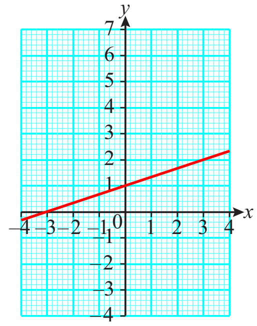
**a** a gradient of 2 and a $y$-intercept of 3
**b** a gradient of $-3$ and a $y$-intercept of $-2$
**c** a gradient of 3 and a $y$-intercept of $-1$
**d** a gradient of $-\frac{3}{2}$ and a $y$-intercept at $(0, -0.5)$
**e** a $y$-intercept of 2 and a gradient of $-\frac{3}{4}$
**f** a $y$-intercept of $-3$ and a gradient of $\frac{4}{8}$
**g** a $y$-intercept of $-0.75$ and a gradient of 0.75
**h** a $y$-intercept of $-2$ and a gradient of 0
**i** a gradient of 0 and a $y$-intercept of 4

**4** Find an equation for each line.

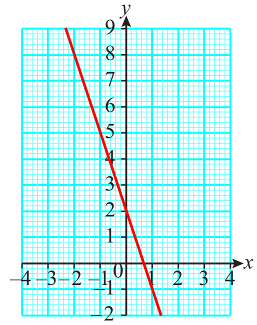
**a**



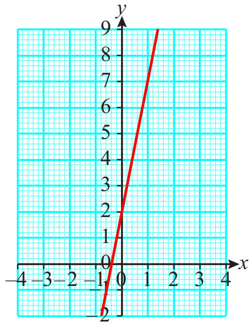

**b**



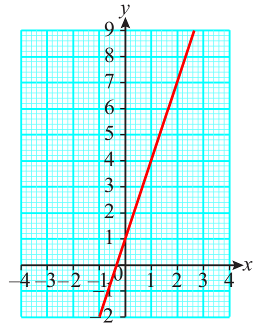

**c**



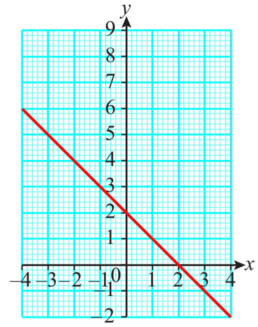

**d**



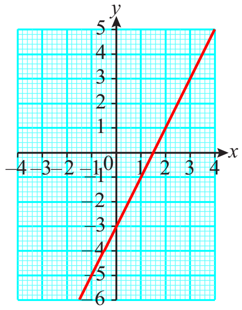

**e**



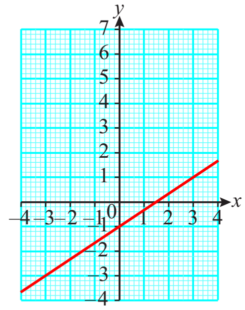

**f**



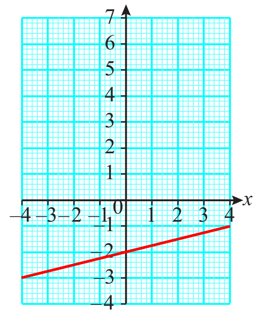

**g**




**h**




**i**



E

**5** Find the equation of the line which passes through both points in each case.

**a** $A$ (2, 3) and $B$ (4, 11) **b** $A$ (4, 5) and $B$ (8, –7)
**c** $A$ (–1, –3) and $B$ (4, 6) **d** $A$ (3, –5) and $B$ (7, 12)

**6** Write down the equation of a line that is parallel to:

**a** $y = -3x$ **b** $y = 2x - 3$ **c** $y = \frac{x}{2} + 4$
**d** $y = -x - 2$ **e** $x = 8$ **f** $y = -6$

**7** Which of the following lines are parallel to $y = \frac{1}{2}x$?

**a** $y = \frac{1}{2}x + 1$ **b** $y = 2x$ **c** $y + 1 = \frac{1}{2}x$ **d** $2y + x = -6$ **e** $y = 2x - 4$

**8** Find the equation of a line parallel to $y = 2x + 4$ which:

**a** has a $y$-intercept of –2
**b** passes through the origin
**c** passes through the point (0, –4)
**d** has a $y$-intercept of $\frac{1}{2}$

**9** A graph has the equation $3y - 2x = 9$.

**a** Write down the equation of one other graph that is parallel to this one.
**b** Write down the equation of one other graph that crosses the $y$-axis at the same point as this one.
**c** Write down the equation of a line that passes through the $y$-axis at the same point as this one and which is parallel to the $x$-axis.

If the product of the gradients of two lines is equal to –1, it follows that the lines are perpendicular to each other.

E

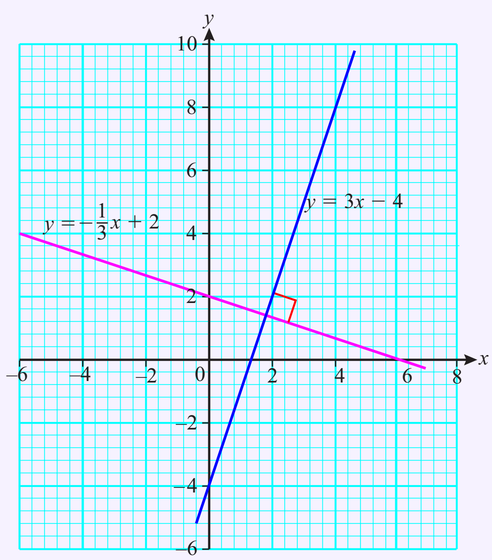
## Parallel and perpendicular lines

You have already seen that parallel lines have the same gradient and that lines with the same gradient are parallel.

Perpendicular lines meet at right angles. The product of the gradients is –1.

So, $m_1 \times m_2 = -1$, where $m$ is the gradient of each line.

The sketch shows two perpendicular graphs.



E

$y = -\frac{1}{3}x + 2$ has a gradient of $-\frac{1}{3}$

$y = 3x - 4$ has a gradient of 3

The product of the gradients is $-\frac{1}{3} \times 3 = -1$.

## Worked example 7

Given that $y = \frac{2}{3}x + 2$, determine the equation of the straight line that is:

a perpendicular to this line and which passes through the origin

b perpendicular to this line and which passes through the point (–3, 1).

a

$y = mx + c$

$m = -\frac{3}{2}$ The gradient is the negative reciprocal of $\frac{2}{3}$

$c = 0$

The equation of the line is $y = -\frac{3}{2}x$.

b

| | |
|---|---|
| $y = -\frac{3}{2}x + c$ | Using $m = -\frac{3}{2}$ from part (a) above. |
| $x = -3$ and $y = 1$ | |
| $1 = -\frac{3}{2}(-3) + c$ | Substitute the values of $x$ and $y$ for the given point to solve for c. |
| $1 = \frac{9}{2} + c$ | |
| $c = -3\frac{1}{2}$ | |
| $y = -\frac{3}{2}x - 3\frac{1}{2}$ | |

## Exercise 10.5

**1** A line perpendicular to $y = \frac{x}{5} + 3$ passes through (1, 3). What is the equation of the line?

**2** Show that the line through the points $A(6, 0)$ and $B(0, 12)$ is:

**a** perpendicular to the line through $P(8, 10)$ and $Q(4, 8)$

**b** perpendicular to the line through $M(-4, -8)$ and $N(-1, -\frac{13}{2})$

**3** Given $A(0, 0)$ and $B(1, 3)$, find the equation of the line perpendicular to $AB$ with a $y$-intercept of 5.

**4** Find the equation of the following lines:

**a** perpendicular to $2x - y - 1 = 0$ and passing through $(2, -\frac{1}{2})$

**b** perpendicular to $2x + 2y = 5$ and passing through (1, –2)

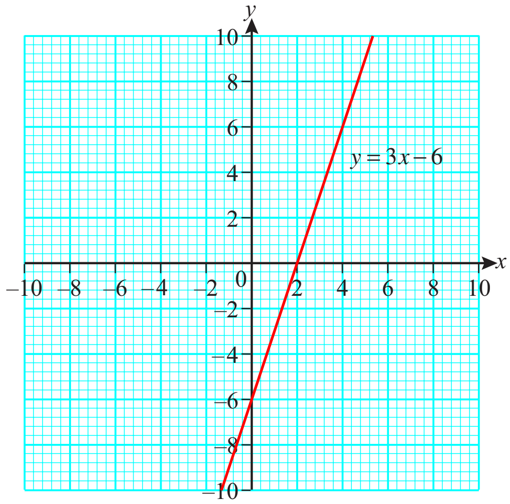
**5** Line $A$ joins the points (6, 0) and (0, 12) and Line $B$ joins the points (8, 10) and (4, 8). Determine the gradient of each line and state whether $A$ is perpendicular to $B$.

**6** Line $MN$ joins points (7, 4) and (2, 5). Find the equation of $AB$, the perpendicular bisector of $MN$.

**7** Show that points $A(-3, 6)$, $B(-12, -4)$ and $C(8, -5)$ could not be the vertices of a rectangle $ABCD$.

### *Intersection with the x-axis*

So far only the $y$-intercept has been found, either from the graph or from the equation. There is, of course, an **$x$-intercept** too. The following sketch shows the line with equation $y = 3x - 6$.




Notice that the line crosses the $x$-axis at the point where $x = 2$ and, importantly, $y = 0$. In fact, all points on the $x$-axis have $y$ co-ordinate = 0. If you substitute $y = 0$ into the equation of the line:

**FAST FORWARD**
You will need to understand this method when solving simultaneous equations in chapter 14. ▶

| | |
|---|---|
| $y = 3x - 6$ | |
| $0 = 3x - 6$ | (putting $y = 0$) |
| $3x = 6$ | (add 6 to both sides) |
| $x = 2$ | (dividing both sides by 3) |

this is exactly the answer that you found from the graph.

You can also find the $y$-intercept by putting $x = 0$. The following worked examples show calculations for finding both the $x$- and $y$-intercepts.

## Worked example 8

Find the $x$- and $y$-intercepts for each of the following lines. Sketch the graph in each case.

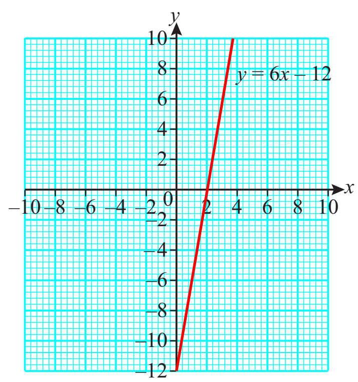
**a** $y = 6x - 12$ **b** $y = -x + 3$ **c** $2x + 5y = 20$

**a** $y = 6x - 12$

$x = 0 \Rightarrow y = -12$

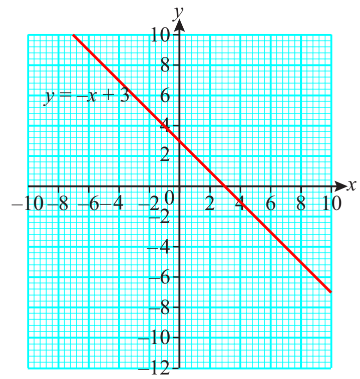
$y = 0 \Rightarrow 6x - 12 = 0$

$\Rightarrow x = 2$



**b** $y = -x + 3$

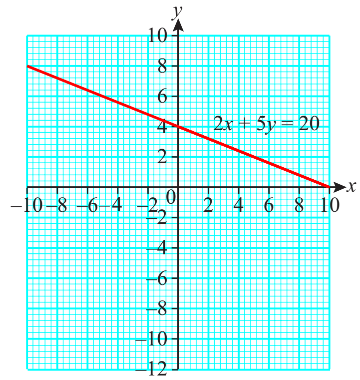
$x = 0 \Rightarrow y = 3$
$y = 0 \Rightarrow -x + 3 = 0$
$\Rightarrow x = 3$




**c** $2x + 5y = 20$

$x = 0 \Rightarrow 5y = 20$
$\Rightarrow y = 4$

$y = 0 \Rightarrow 2x = 20$
$\Rightarrow x = 10$




## Exercise 10.6

**1** Find the $x$- and $y$-intercepts for each of the following lines. Sketch the graph in each case.

**a** $y = -5x + 10$ **b** $y = \frac{x}{3} - 1$ **c** $y = -3x + 6$ **d** $y = 4x + 2$

**e** $y = 3x + 1$ **f** $y = -x + 2$ **g** $y = 2x - 3$ **h** $y = \frac{2x}{3} - 1$

**i** $y = \frac{x}{4} - 2$ **j** $y = \frac{2x}{5} + 1$ **k** $-2 + y = \frac{x}{4}$ **l** $\frac{-y}{3} = 4x - 2$

**2** For each equation, find $c$, if the given point lies on the graph.

**a** $y = 3x + c$ $(1, 5)$ **b** $y = 6x + c$ $(1, 2)$

**c** $y = -2x + c$ $(-3, -3)$ **d** $y = \frac{3}{4}x + c$ $(4, -5)$

**e** $y = \frac{1}{2}x + c$ $(-2, 3)$ **f** $y = c - \frac{1}{2}x$ $(-4, 5)$

**g** $y = c + 4x$ $(-1, -6)$ **h** $\frac{2}{3}x + c = y$ $(3, 4)$

E

## Finding the length of a straight line segment

Although lines are infinitely long, usually just a part of a line is considered. Any section of a line joining two points is called a **line segment**.

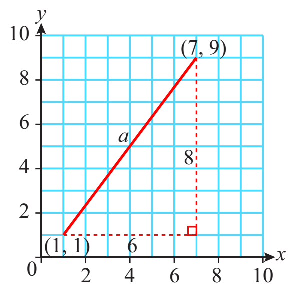
If you know the co-ordinates of the end points of a line segment you can use Pythagoras' theorem to calculate the length of the line segment.

**FAST FORWARD**

Pythagoras' theorem is covered in more detail in chapter 11. Remember though, that in any right-angled triangle the square on the hypotenuse is equal to the sum of the squares on the other two sides. We write this as $a^2 + b^2 = c^2$.

### Worked example 9

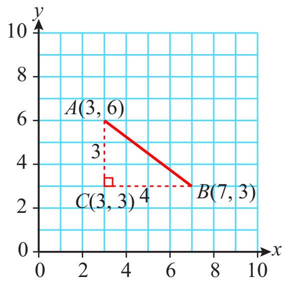
Find the distance between the points (1, 1) and (7, 9)




| | |
|---|---|
| $a^2 = 8^2 + 6^2$ | $a^2 = b^2 + c^2$ (Pythagoras' theorem) |
| $a^2 = 64 + 36$ | Work out each expression. |
| $a^2 = 100$ | Undo the square by taking the square root of both sides. |
| $\therefore a = \sqrt{100}$ | |
| $a = 10$ units | |

### Worked example 10

Given that $A(3, 6)$ and $B(7, 3)$, find the length of $AB$.



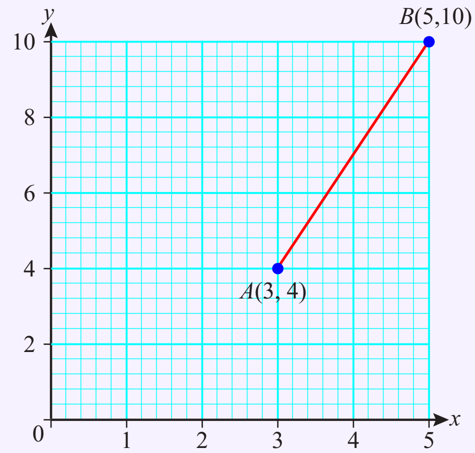

| | |
|---|---|
| $AB^2 = AC^2 + CB^2$ | $a^2 = b^2 + c^2$ (Pythagoras' theorem) |
| $AB^2 = 3^2 + 4^2$ | Work out each expression. |
| $= 9 + 16$ | |
| $= 25$ | |
| $\therefore AB = \sqrt{25} = 5$ units | |

E

## Midpoints

It is possible to find the co-ordinates of the **midpoint** of the line segment (i.e. the point that is exactly halfway between the two original points).

Consider the following line segment and the points $A(3, 4)$ and $B(5, 10)$.




FAST FORWARD

In chapter 12 you will learn about the *mean* of two or more numbers. The midpoint uses the mean of the *x* co-ordinates and the mean of the *y* co-ordinates.

If you add both $x$ co-ordinates and then divide by two you get $\frac{(3+5)}{2} = \frac{8}{2} = 4$.

If you add both $y$ co-ordinates and then divide by two you get $\frac{(4+10)}{2} = \frac{14}{2} = 7$.

This gives a new point with co-ordinates (4, 7). This point is exactly half way between $A$ and $B$.

## Exercise 10.7

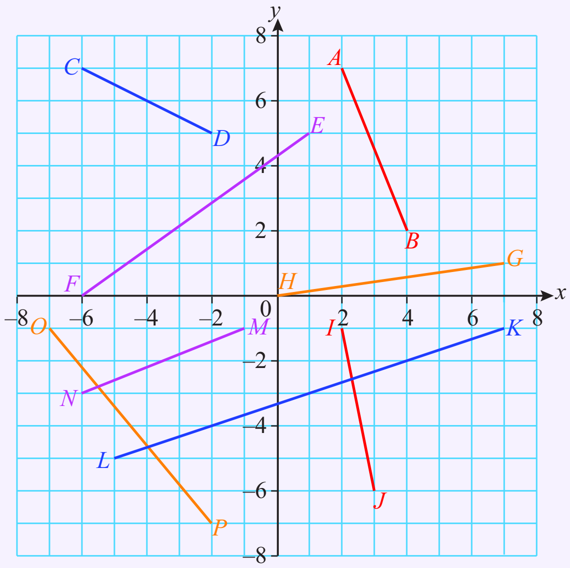
REWIND

Check that you remember how to deal with negative numbers when adding.

**1** Find the length and the co-ordinates of the midpoint of the line segment joining each pair of points.

**a** (3, 6) and (9, 12) **b** (4, 10) and (2, 6) **c** (8, 3) and (4, 7)
**d** (5, 8) and (4, 11) **e** (4, 7) and (1, 3) **f** (12, 3) and (11, 4)
**g** (−1, 2) and (3, 5) **h** (4, −1) and (5, 5) **i** (−2, −4) and (−3, 7)

**2** Use the graph to find the length and the midpoint of each line segment.



3 Find the distance from the origin to point (–3, –5).

4 Which of the points $A(5, 6)$ or $B(5, 3)$ is closer to point $C(-3, 2)$?

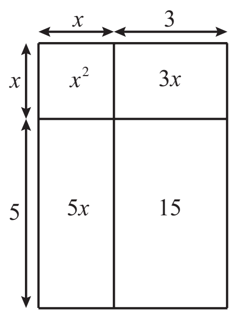
5 Which is further from the origin, $A(4, 2)$ or $B(-3, -4)$?

6 Triangle $ABC$ has its vertices at points $A(0, 0)$, $B(4, -5)$ and $C(-3, -3)$. Find the length of each side.

7 The midpoint of the line segment joining $(10, a)$ and $(4, 3)$ is $(7, 5)$. What is the value of $a$?

8 The midpoint of line segment $DE$ is $(-4, 3)$. If point $D$ has the co-ordinates $(-2, 8)$, what are the co-ordinates of $E$?

## 10.2 Quadratic (and other) expressions



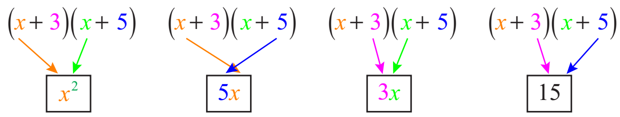

The diagram shows a rectangle of length $(x + 3)$ cm and width $(x + 5)$ cm that has been divided into smaller rectangles.

The area of the whole rectangle is equal to the sum of the smaller areas, so the area of whole rectangle = $(x + 3) \times (x + 5)$.

The sum of smaller rectangle areas: $x^2 + 3x + 5x + 15 = x^2 + 8x + 15$.

This means that $(x+3)\times(x+5) = x^2 + 8x + 15$ and this is true for *all* values of $x$.

Notice what happens if you multiply every term in the second bracket by every term in the first:




Notice that the four terms in boxes are exactly the same as the four smaller areas that were calculated before.

Another way to show this calculation is to use a grid:

| | $x$ | 3 |
|---|---|---|
| $x$ | $x^2$ | $3x$ |
| 5 | $5x$ | 15 |

You will notice that this is almost the same as the areas method above but it can also be used when the constants are negative, as you will see in the worked examples shortly.

When you remove the brackets and re-write the algebraic expression you are **expanding** or **multiplying out** the brackets. The resulting algebraic expression contains an $x^2$ term, an $x$ term and a **constant term**. This is called a **quadratic expression**.

The following worked example shows these two methods and a third method for expanding pairs of brackets. You should try each method when working through the next exercise and decide which you find easiest, though you will begin to notice that they are all, in fact, the same.

## Worked example 11

Expand and simplify:

**a** $(x+2)(x+9)$ **b** $(x-7)(x+6)$ **c** $(2x-1)(x+9)$

**a**

$(x+2)(x+9)$

$x^2+9x+2x+18$
$= x^2+11x+18$

In this version of the method you will notice that the arrows have not been included and the multiplication 'arcs' have been arranged so that they are symmetrical and easier to remember.

**b**

| | $x$ | $-7$ |
|---|---|---|
| $x$ | $x^2$ | $-7x$ |
| $+6$ | $6x$ | $-42$ |

$x^2-7x+6x-42$
$= x^2-x-42$

The grid method with a negative value.

**c**

$(2x-1)(x+9)$

**F**irsts: $2x \times x = 2x^2$
**O**utsides: $2x \times 9 = 18x$
**I**nsides: $-1 \times x = -x$
**L**asts: $-1 \times 9 = -9$

$2x^2+18x-x-9$
$= x^2+17x-9$

A third method that you can remember using the mnemonic '**FOIL**' which stands for **F**irst, **O**utside, **I**nside, **L**ast. This means that you multiply the first term in each bracket together then the 'outside' pair together (i.e. the first term and last term), the 'inside' pair together (i.e. the second term and third term) and the 'last' pair together (i.e. the second term in each bracket).

You need to choose which method works best for you but ensure that you show all the appropriate stages of working clearly.

**LINK**

Quadratic expressions and formulae are useful for modelling situations that involve movement, including acceleration, stopping distances, velocity and distance travelled (displacement). These situations are studied in Physics but they also have real life applications in situations such as road or plane accident investigations.

E

### *The product of more than two sets of brackets*

You can multiply in steps to expand three (or more) sets of brackets. Your answer might contain terms with powers of 3 (cubic expressions).

## Worked example 12

Expand and simplify $(3x+2)(2x+1)(x-1)$

| | |
|---|---|
| $(3x+2)(2x+1)(x-1)$ | Expand the first two brackets. |
| $= (6x^2+4x+3x+2)(x-1)$ | Collect like terms. |
| $= (6x^2+7x+2)(x-1)$ | |
| $= 6x^3+7x^2+2x-6x^2-7x-2$ | Multiply each term in the first bracket by each term in the second. |
| $= 6x^3+x^2-5x-2$ | Collect like terms to simplify. |

## Exercise 10.8

REWIND

You will need to remember how to multiply fractions. This was covered in chapter 5.

**1** Expand and simplify each of the following.

**a** $(x+3)(x+1)$ **b** $(x+6)(x+4)$ **c** $(x+9)(x+10)$
**d** $(x+3)(x+12)$ **e** $(x+1)(x+1)$ **f** $(x+5)(x+4)$
**g** $(x+4)(x-7)$ **h** $(x-3)(x+8)$ **i** $(x-1)(x+1)$
**j** $(x-9)(x+8)$ **k** $(x-6)(x-7)$ **l** $(x-13)(x+4)$
**m** $(y+3)(y-14)$ **n** $(z+8)(z-8)$ **o** $(t+17)(t-4)$
**p** $(h-3)(h-3)$ **q** $(g-\frac{1}{2})(g+4)$ **r** $(d+\frac{3}{2})(d-\frac{3}{4})$

**2** Find the following products.

**a** $(4-x)(3-x)$ **b** $(3-2x)(1+3x)$ **c** $(3m-7)(2m-1)$
**d** $(2x+1)(3-4x)$ **e** $(4a-2b)(2a+b)$ **f** $(2m-n)(-3n-4m)$
**g** $\left(x+\frac{1}{2}\right)\left(x+\frac{1}{4}\right)$ **h** $\left(2x+\frac{1}{3}\right)\left(x-\frac{1}{2}\right)$ **i** $\left(2x^2-4y\right)\left(y-x^2\right)$
**j** $(7-9b)(4b+6)$ **k** $(x+y)\left(2y^2-4x^3\right)$ **l** $(3x-3)(5+2x)$

**3** Expand and simplify each of the following.

**a** $(2x+3)(x+3)$ **b** $(3y+7)(y+1)$ **c** $(7z+1)(z+2)$
**d** $(t+5)(4t-3)$ **e** $(2w-7)(w-8)$ **f** $(4g-1)(4g+1)$
**g** $(8x-1)(9x+4)$ **h** $(20c-3)(18c-4)$ **i** $(2m-4)(3-m)$

REWIND

Refer to chapter 2 to remind you how to multiply different powers of the same number together.

E

**4** Expand and simplify each of the following.

**a** $(3x^2+1)(2x+3)$ **b** $(5x^2-1)(3x^2-3)$ **c** $(3x^2-y)(2x+3y)$

**5** Expand and simplify.

**a** $(5x+2)(3x-3)(x+2)$
**b** $(x-5)(x-5)(x+5)$
**c** $(4x-1)(x+1)(3x-2)$
**d** $(x+4)(2x+4)(2x+4)$
**e** $(2x-3)(3x-2)(2x-1)$
**f** $(3x-2)^2(2x-1)$
**g** $(x+2)^3$
**h** $(2x-2)^3$
**i** $(x^2y^2+x^2)(xy+x)(xy-x)$
**j** $\left(\frac{1}{3}+\frac{x}{2}\right)\left(\frac{1}{9}-\frac{x^2}{4}\right)\left(\frac{1}{3}-\frac{x}{2}\right)$

**6** The volume of a cuboid can be found using the formula $V = lbh$, where $l$ is the length, $b$ is the breadth and $h$ is the height. A cuboid has length $\left(2x+\frac{1}{2}\right)$ m, breadth $(x-2)$ m and height $(x-2)$ m.

**a** Write an expression for the volume of the cuboid in factor form.
**b** Expand the expression.
**c** Determine the volume of the cuboid when $x = 2.2$ m.

E

## Squaring a binomial

$(x + y)^2$ means $(x + y)(x + y)$

To find the product, you can use the method you learned earlier.

$(x + y)(x + y) = x^2 + xy + xy + y^2 = x^2 + 2xy + y^2$

However, if you think about this, you should be able to solve these kinds of expansions by inspection. Look at the answer. Can you see that:

- the first term is the square of the first term ($x^2$)
- the middle term is twice the product of the middle terms ($2xy$)
- the last term is the square of the last term ($y^2$)?

## Exercise 10.9

**1** Find the square of each binomial. Try to do this by inspection first and then check your answers.

**a** $(x-y)^2$ **b** $(a+b)^2$ **c** $(2x+3y)^2$ **d** $(3x-2y)^2$

**e** $(x+2y)^2$ **f** $(y-4x^2)^2$ **g** $(x^2-y^2)^2$ **h** $(2+y^3)^2$

**i** $(-2x-4y^2)^2$ **j** $\left(\frac{1}{2x}-\frac{1}{4y}\right)^2$ **k** $\left(\frac{3x}{4}-\frac{y}{2}\right)^2$ **l** $\left(a+\frac{1}{2}b\right)^2$

**m** $(-ab-c^4)^2$ **n** $(3x^2y-1)^2$ **o** $\left(\frac{2x}{3}+4y\right)^2$ **p** $[-(x-3)]^2$

**2** Simplify.

**a** $(x-2)^2-(x-4)^2$

**b** $(x+2)(x-2)-(3-x)(5+x)$

**c** $(y+2x)^2+(2x-y)(-y+2x)$

**d** $\frac{1}{2}(3x-2)\left(\frac{x}{3}+2\right)$

**e** $3(x+2)(2x+0.6)$

**f** $(\sqrt{2x}-y)(\sqrt{2x}+y)-(4x-y)^2$

**g** $(x+4)(x-5)-2(x-1)^2$

**h** $(2x-y)^2+(x-2y)(x+2y)-(x+4y)^2$

**i** $-2x(x+1)^2-(x-5)(-3x)$

**j** $(3+2x)^2-5(5x+2)$

**3** Evaluate each expression when $x = 4$.

**a** $(x+7)(x-7)-x^2$

**b** $x^2-(x-3)(x+3)$

**c** $(3+2x)^2-(2x+3)(2x-3)$

**d** $(x + 2)^2$

**e** $(x^2+3)(x-4)$

**f** $(2x+3)^2-4(x+1)(2-3x)$

## Factorising quadratic expressions

Look again at the expansion of $(x + 2)(x + 9)$, which gave $x^2+11x+18$:

$$(x+2)(x+9) = x^2 + \overset{2+9=11}{11x} + \underset{2\times 9=18}{18}$$

Here the two numbers add to give the coefficient of $x$ in the final expression and the two numbers multiply to give the constant term.

E

This works whenever there is just one $x$ in each bracket.

## Worked example 13

Expand and simplify: **a** $(x+6)(x+12)$ **b** $(x+4)(x-13)$

| | | |
|---|---|---|
| **a** | $(x+6)(x+12) = x^2 + 18x + 72$ | $6 + 12 = 18$ and $6 \times 12 = 72$ so this gives $18x$ and 72. |
| **b** | $(x+4)(x-13) = x^2 - 9x - 52$ | $4 + -13 = -9$ and $4 \times -13 = -52$ so this gives $-9x$ and $-52$. |

REWIND

$1 \times 72$ and $6 \times 12$ are the factor pairs of 72. You learned about factor pairs in chapter 1. ◀

If you use the method in worked example 11 and work backwards you can see how to put a quadratic expression back into brackets. Note that the coefficient of $x^2$ in the quadratic expression must be 1 for this to work.

Consider the expression $x^2 + 18x + 72$ and suppose that you want to write it in the form $(x+a)(x+b)$.

From the worked example you know that $a + b = 18$ and $a \times b = 72$.

Now $72 = 1 \times 72$ but these two numbers don't add up to give 18.

However, $72 = 6 \times 12$ and $6 + 12 = 18$.

So, $x^2 + 18x + 72 = (x+6)(x+12)$.

The process of putting a quadratic expression *back* into brackets like this is called **factorisation**.

## Worked example 14

Factorise completely:

**a** $x^2 + 7x + 12$ **b** $x^2 - 6x - 16$ **c** $x^2 - 8x + 15$

List the factor pairs of 12.
(If you spot which pair of numbers works straight away then you don't need to write out all the other factor pairs.)

| | | |
|---|---|---|
| **a** | | You need two numbers that multiply to give 12 and add to give 7. |
| | $12 = 1 \times 12$ | These don't add to give 7. |
| | $12 = 2 \times 6$ | These don't add to give 7. |
| | $12 = 3 \times 4$ and $3 + 4 = 7$ | These multiply to give 12 and add to give 7. |
| | So, $x^2 + 7x + 12 = (x+3)(x+4)$ | |
| **b** | $-8 \times 2 = -16$ and $-8 + 2 = -6$<br>So, $x^2 - 6x - 16 = (x-8)(x+2)$. | You need two numbers that multiply to give $-16$ and add to give $-6$. Since they multiply to give a negative answer, one of the numbers must be negative and the other must be positive. (Since they add to give a negative, the larger of the two numbers must be negative.) |
| **c** | $-5 \times -3 = 15$ and $-5 + -3 = -8$<br>So, $x^2 - 8x + 15 = (x-3)(x-5)$. | You need two numbers that multiply to give 15 and add to give $-8$. Since they multiply to give a positive value but add to give a negative then both must be negative. |

## Exercise 10.10

When looking for your pair of integers, think about the factors of the constant term first. Then choose the pair which adds up to the $x$ term in the right way.

**1** Factorise each of the following.

| | | |
|---|---|---|
| **a** $x^2+14x+24$ | **b** $x^2+3x+2$ | **c** $x^2+7x+12$ |
| **d** $x^2+12x+35$ | **e** $x^2+12+27$ | **f** $x^2+7x+6$ |
| **g** $x^2+11x+30$ | **h** $x^2+10x+16$ | **i** $x^2+11x+10$ |
| **j** $x^2+8x+7$ | **k** $x^2+24x+80$ | **l** $x^2+13x+42$ |

**2** Factorise each of the following.

| | | |
|---|---|---|
| **a** $x^2-8x+12$ | **b** $x^2-9x+20$ | **c** $x^2-7x+12$ |
| **d** $x^2-6x+8$ | **e** $x^2-12x+32$ | **f** $x^2-14x+49$ |
| **g** $x^2-8x-20$ | **h** $x^2-7x-18$ | **i** $x^2-4x-32$ |
| **k** $x^2+x-6$ | **l** $x^2+8x-33$ | **m** $x^2+10x-24$ |

**3** Factorise each of the following.

| | | |
|---|---|---|
| **a** $y^2+7y-170$ | **b** $p^2+8p-84$ | **c** $w^2-24w+144$ |
| **d** $t^2+16t-36$ | **e** $v^2+20v+75$ | **f** $x^2-100$ |

### Difference between two squares

The very last question in the previous exercise was a special kind of quadratic. To factorise $x^2-100$ you must notice that $x^2-100=x^2+0x-100$.

Now, proceeding as in worked example 12:

$10\times-10=-100$ and $-10+10=0$ so, $x^2+0x-100=(x-10)(x+10)$.

Now think about a more general case in which you try to factorise $x^2-a^2$. Notice that $x^2-a^2=x^2+0x-a^2$.

Since $a\times-a=-a$ and $a+-a=0$, this leads to: $\boxed{x^2-a^2=(x-a)(x+a)}$.

You must remember this special case. This kind of expression is called a **difference between two squares**.

### Worked example 15

Factorise the following using the difference between two squares:

**a** $x^2-49$ **b** $x^2-\frac{1}{4}$ **c** $16y^2-25w^2$

| | | |
|---|---|---|
| **a** | $49=7^2$<br>$x^2-49=x^2-7^2$<br>$=(x-7)(x+7)$ | Use the formula for the difference between two squares: $x^2-a^2=(x-a)(x+a)$.<br>You know that $\sqrt{49}=7$ so you can write 49 as $7^2$. This gives you $a^2$. Substitute $7^2$ into the formula. |
| **b** | $\left(\frac{1}{2}\right)^2=\frac{1}{4}$<br>$x^2-\frac{1}{4}=x^2-\left(\frac{1}{2}\right)^2$<br>$=(x-\frac{1}{2})(x+\frac{1}{2})$ | $\sqrt{\frac{1}{4}}$ is $\frac{1}{2}$ so you can rewrite $\frac{1}{4}$ as $\left(\frac{1}{2}\right)^2$ and substitute it into the formula for the difference between two squares. |

E

| | | |
|---|---|---|
| c | $(4y)^2 = 4y \times 4y = 16y^2$<br>and<br>$(5w)^2 = 5w \times 5w = 25w^2$<br>$16y^2 - 25w^2 = (4y)^2 - (5w)^2$<br>$= (4y - 5w)(4y + 5w)$ | The $\sqrt{16y^2} = (4y)^2$.<br>$25w^2 = (5w)^2$<br>Substitute in $(4y)^2$ and $(5w)^2$. |

## Exercise 10.11

From question (l) you should notice that the numbers given are not square. Try taking a common factor out first.

**1** Factorise each of the following.

| | | | |
|---|---|---|---|
| **a** $x^2 - 36$ | **b** $p^2 - 81$ | **c** $w^2 - 16$ | **d** $q^2 - 9$ |
| **e** $k^2 - 400$ | **f** $t^2 - 121$ | **g** $x^2 - y^2$ | **h** $81h^2 - 16g^2$ |
| **i** $16p^2 - 36q^2$ | **j** $144s^2 - c^2$ | **k** $64h^2 - 49g^2$ | **l** $27x^2 - 48y^2$ |
| **m** $200q^2 - 98p^2$ | **n** $20d^2 - 125e^2$ | **o** $x^4 - y^4$ | **p** $xy^2 - x^3$ |

**2** Factorise and simplify $36^2 - 35^2$ without using a calculator.

**3** Factorise and simplify $(6\frac{1}{4})^2 - (5\frac{3}{4})^2$ without using a calculator.

## Using factors to solve quadratic equations

You can now use the factorisation method to solve some **quadratic equations**. A quadratic equation is an equation of the form $ax^2 + bx + c = 0$. The method is illustrated in the following worked examples.

### Worked example 16

Solve each of the following equations for $x$.

**a** $x^2 - 3x = 0$ **b** $x^2 - 7x + 12 = 0$ **c** $x^2 + 6x - 4 = 12$

**d** $x^2 - 8x + 16 = 0$

**a** Notice that both terms of the left-hand side are multiples of $x$ so you can use common factorisation.

$x^2 - 3x = 0$

$x(x - 3) = 0$

Now the key point:

If two or more quantities multiply to give zero, then at least one of the quantities must be zero.

So either $x = 0$ or $x - 3 = 0 \Rightarrow x = 3$.

Check: $0^2 - 3 \times 0 = 0$ (this works).

$3^2 - 3 \times 3 = 9 - 9 = 0$ (this also works).

In fact both $x = 0$ and $x = 3$ are solutions.

**b** Use the factorisation method of worked example 12 on the left-hand side of the equation.

$x^2 - 7x + 12 = 0$

$(x - 4)(x - 3) = 0$

Therefore either $x - 4 = 0 \Rightarrow x = 4$

or $x - 3 = 0 \Rightarrow x = 3$.

Again, there are two possible values of $x$.

When solving quadratic equations they should be rearranged so that a zero appears on one side, i.e. so that they are in the form $ax^2 + bx + c = 0$

**c** $x^2 + 6x - 4 = 12$

$\Rightarrow x^2 + 6x - 16 = 0$ (subtract 12 from both sides)

Factorising, you get $(x + 8)(x - 2) = 0$

So either $x + 8 = 0 \Rightarrow x = -8$

or $x - 2 = 0 \Rightarrow x = 2$.

**d** Factorising, $x^2 - 8x + 16 = 0$

$(x - 4)(x - 4) = 0$

So either $x - 4 = 0 \Rightarrow x = 4$

or $x - 4 = 0 \Rightarrow x = 4$

Of course these are both the same thing, so the only solution is $x = 4$.

There are still two solutions here, but they are identical.

## Exercise 10.12

**1** Solve the following equations by factorisation.

| | | |
|---|---|---|
| **a** $x^2 - 9x = 0$ | **b** $x^2 + 7x = 0$ | **c** $x^2 - 21x = 0$ |
| **d** $x^2 - 9x + 20 = 0$ | **e** $x^2 + 8x + 7 = 0$ | **f** $x^2 + x - 6 = 0$ |
| **g** $x^2 + 3x + 2 = 0$ | **h** $x^2 + 11x + 10 = 0$ | **i** $x^2 - 7x + 12 = 0$ |
| **j** $x^2 - 8x + 12 = 0$ | **k** $x^2 - 100 = 0$ | **l** $t^2 + 16t - 36 = 0$ |
| **m** $y^2 + 7y - 170 = 0$ | **n** $p^2 + 8p - 84 = 0$ | **o** $w^2 - 24w + 144 = 0$ |

# Summary

### Do you know the following?

- The equation of a line tells you how the $x$- and $y$ co-ordinates are related for all points that sit on the line.
- The gradient of a line is a measure of its steepness.
- The $x$- and $y$-intercepts are where the line crosses the $x$- and $y$-axes respectively.
- The value of $m$ in $y = mx + c$ is the gradient of the line.
- The value of $c$ in $y = mx + c$ is the $y$-intercept.
- The $x$-intercept can be found by substituting $y = 0$ and solving for $x$.
- The $y$-intercept can be found by substituting $x = 0$ and solving for $y$.
- Two lines with the same gradient are parallel.
- The gradients of two perpendicular lines will multiply to give $-1$.
- There is more than one way to expand brackets.
- Some quadratic expressions can be factorised to solve quadratic equations.
- Quadratic equations usually have two solutions, though these solutions may be equal to one another.

### Are you able to …?

- draw a line from its equation by drawing a table and plotting points
- find the gradient, $x$-intercept and $y$-intercept from the equation of a line
- calculate the gradient of a line from its graph
- find the equation of a line if you know its gradient and $y$-intercept
- find the equation of a vertical or horizontal line
- calculate the gradient of a line from the co-ordinates of two points on the line
- find the length of a line segment and the co-ordinates of its midpoint
- expand double brackets
- expand three or more sets of brackets
- factorise a quadratic expression
- factorise an expression that is the difference between two squares
- solve a quadratic equation by factorising.

# Examination practice

## Exam-style questions

**1** Expand and simplify each of the following.

**a** $(x+2)(x+18)$ **b** $(2x+3)(2x-3)$ **c** $(4y^2-3)(3y^2+1)$

**2 a** Factorise each of the following.

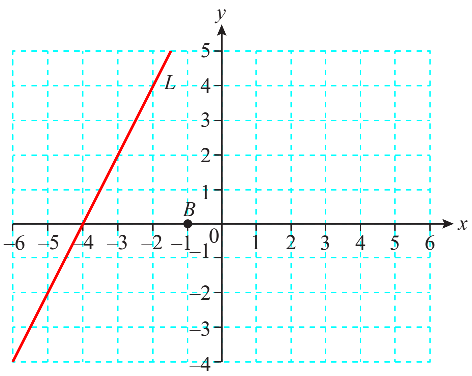
**i** $12x^2-6x$ **ii** $y^2-13y+42$ **iii** $d^2-196$

**b** Solve the following equations.

**i** $12x^2-6x=0$ **ii** $y^2-13y+30=-12$ **iii** $d^2-196=0$

## Past paper questions

**1**



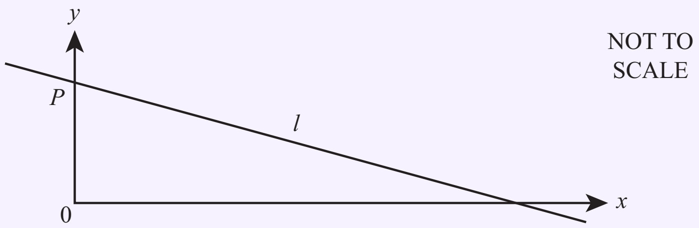

**a** On the grid mark the point (5, 1). Label it $A$. [1]

**b** Write down the co-ordinates of the point $B$. [1]

**c** Find the gradient of the line $L$. [2]

*[Cambridge IGCSE Mathematics 0580 Paper 13 Q19 October/November 2012]*

2




The equation of the line $l$ in the diagram is $y = 5 - x$.

**a** The line cuts the $y$-axis at $P$.
Write down the co-ordinates of $P$. [1]

**b** Write down the gradient of the line $l$. [1]

*[Cambridge IGCSE Mathematics 0580 Paper 22 Q5 May/June 2014]*

3 Factorise
$9w^2 - 100$,

*[Cambridge IGCSE Mathematics 0580 Paper 22 Q15 (a) October/November 2015]*

4 Factorise

$mp + np - 6mq - 6nq$. [2]

*[Cambridge IGCSE Mathematics 0580 Paper 22 Q15(b) October/November 2015]*