Chapter 19: Symmetry

Key words

- Symmetry
- Line symmetry
- Rotational symmetry
- Symmetrical
- Order of rotational symmetry
- Centre of rotation
- Plane symmetry
- Axis of symmetry
- Tangent
- Perpendicular bisector
- Chord
- Equidistant
- Subtend
- Arc
- Cyclic quadrilateral
- Alternate segment

In this chapter you will learn how to:

- identify line symmetry of two-dimensional shapes
- find the order of rotational symmetry of two-dimensional shapes
- recognise and use symmetrical properties of triangles, quadrilaterals and circles

EXTENDED

- recognise symmetry properties of prisms and pyramids
- apply symmetry properties of circles to solve problems

The front of this museum in Ho Chi Minh City in Vietnam, is symmetrical. If you draw a vertical line through the centre of the building (from the centre red flag), the left side will be a mirror image of the right side.

The front of the building in the photograph is symmetrical. One half of the front of the building is the mirror image of the other. The line dividing the building into two halves is called the mirror line, or line of symmetry.

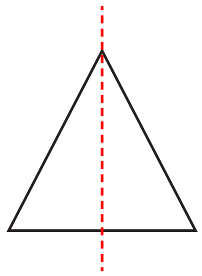
Shapes or objects that can be divided into two or more parts which are identical in shape and size are said to be symmetrical. Symmetry is found in both two-dimensional shapes and three-dimensional objects. In this chapter you are going to learn more about symmetry about a line, and turning, or rotational symmetry, in both two-dimensional shapes and three-dimensional objects.

RECAP

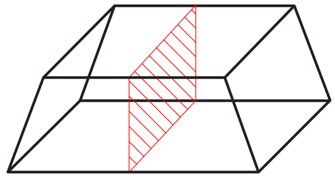
You should already be familiar with the following work on shapes, solids and symmetry:

Symmetry

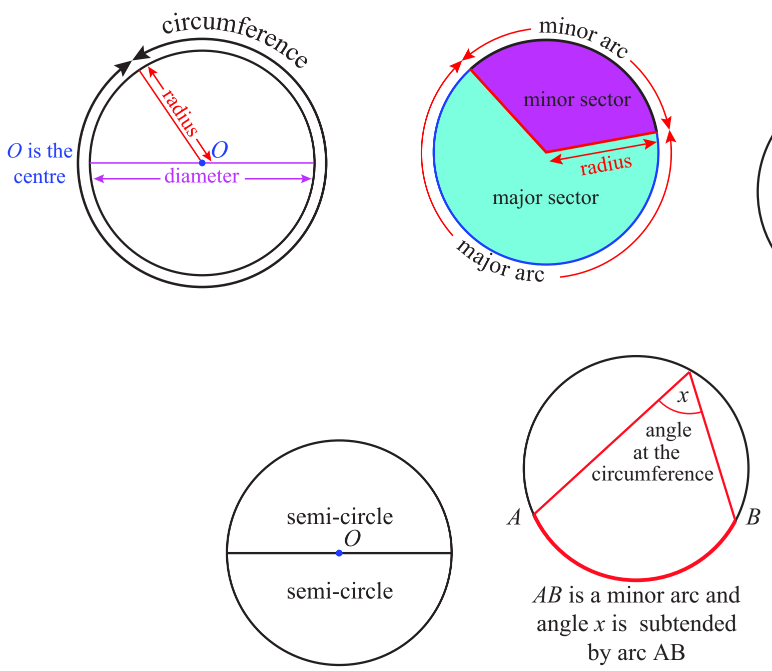
2D shapes are symmetrical if they can be divided into two identical halves by a straight line. Each half is the mirror image of the other.



3D solids are symmetrical if they can be 'cut' into two identical parts by a plane. The two parts are mirror images of each other.



Parts of a circle (Chapter 3)



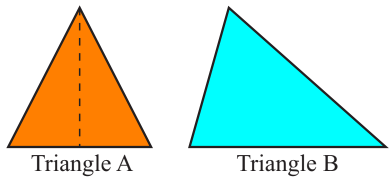
19.1 Symmetry in two dimensions

There are two types of **symmetry** in two-dimensional shapes:

- **Line symmetry**
- **Rotational symmetry**

Line symmetry

If a shape can be folded so that one half fits exactly over the other half, it has line symmetry (also called reflection symmetry).

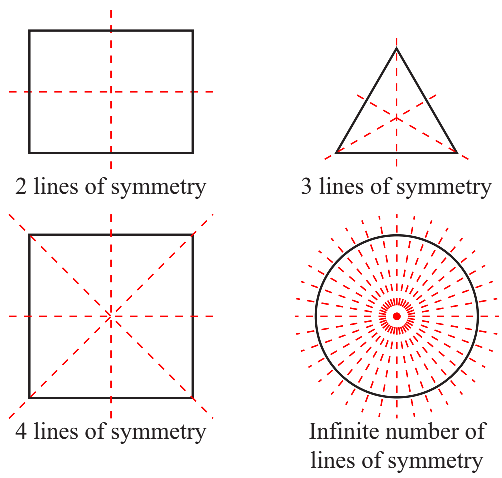


Triangle A is symmetrical. The dotted line divides it into two identical parts. Triangle B is not symmetrical. You cannot draw a line which will divide it into two identical halves.

> **LINK**
> Symmetry is very important when understanding the construction of crystals in chemistry.

If you place a mirror on the dividing line on shape A, the view in the mirror will be that of the whole triangle. The line is called the line of symmetry or mirror line of the shape.

Shapes can have more than one line of symmetry:

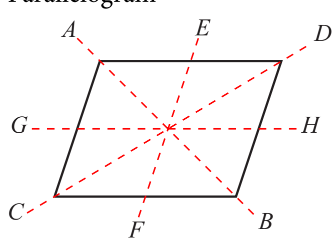


Exercise 19.1

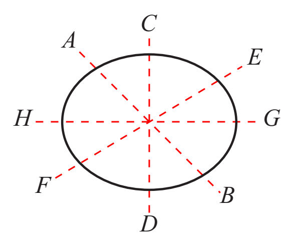
1 Which of the broken lines in these figures are lines of symmetry? Check with a small mirror or trace and fold the shape if you are not sure.

> Another name for an 'oval' is an 'ellipse'.

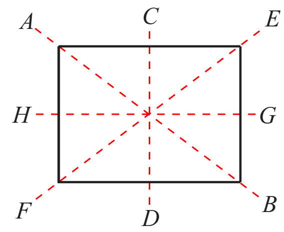
a Parallelogram



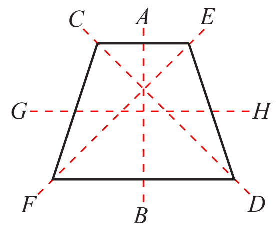
b Oval



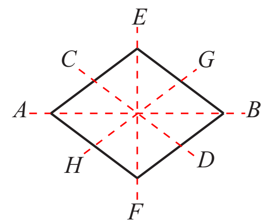
c Rectangle



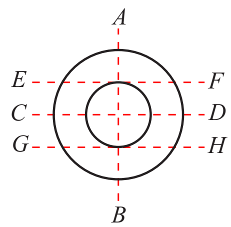
d Isosceles trapezium



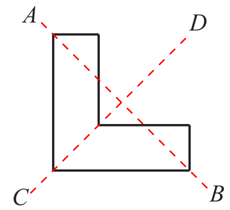
e Rhombus



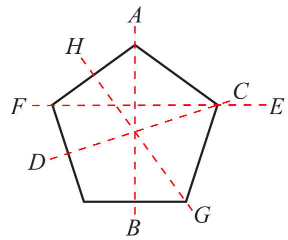
f Torus



g L-shape



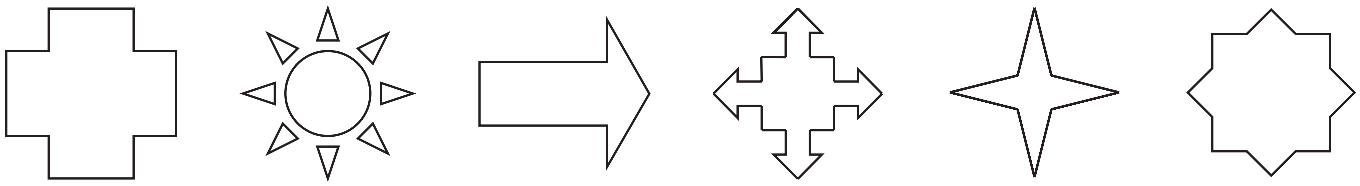
h Regular pentagon



2 Sketch the following polygons and investigate to see how many lines of symmetry each one has. Copy and complete the table to summarise your results.

Shape	Number of lines of symmetry
Square	
Rectangle	
Equilateral triangle	
Isosceles triangle	
Scalene triangle	
Kite	
Parallelogram	
Rhombus	
Regular pentagon	
Regular hexagon	
Regular octagon	

3 Copy these figures and draw in all possible lines of symmetry.



FAST FORWARD

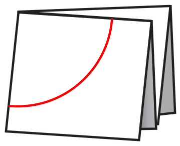
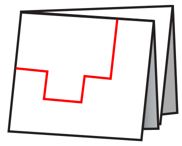
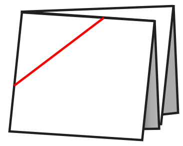
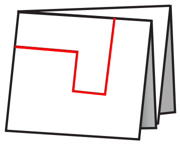
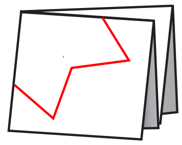
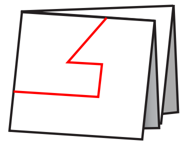
You will deal with line symmetry on the Cartesian plane when you deal with reflections about a line in chapter 23. ▶

Think carefully about how the paper is folded. The diagram shows that it's folded into four.

Applying your skills

4 The children in a primary school class make shapes for a class pattern by cutting out a design drawn on the corner of a folded piece of paper.

a Draw the shapes that will be produced by each of these cut outs.
b Show the lines of symmetry on each shape by means of dotted lines.

i

ii

iii

iv

v

vi


5 Find and draw the badges of five different makes of motor vehicle. Indicate the lines of symmetry on each drawing.

FAST FORWARD

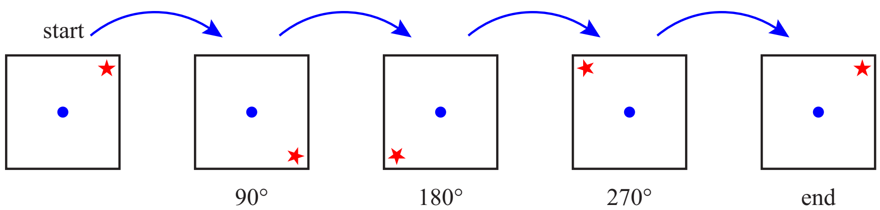
You will deal with rotational symmetry on the Cartesian plane when you deal with rotations in chapter 23. ▶

Rotational symmetry

A rotation is a turn. If you rotate a shape through 360°, keeping its centre point in a fixed position, and it fits onto itself exactly at various positions during the turn, then it has rotational symmetry. The number of times it fits onto itself during a full revolution is its **order of rotational symmetry**.

The diagram shows how a square fits onto itself four times when it is turned through 360°. The dot in the centre of the square is the **centre of rotation**. This is the point around which it is turning. The star shows the position of one corner of the square as it turns.

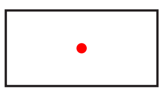
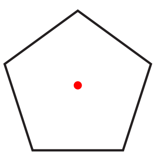
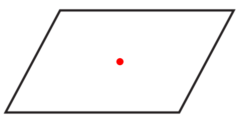
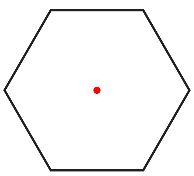
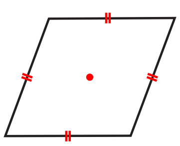
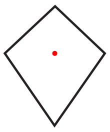
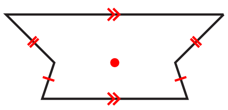
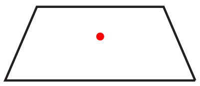
If you have to turn the shape through a full 360° before its fits onto itself then it does *not* have rotational symmetry. Be careful though, because every shape will fit back onto itself after a whole revolution.



The square fits exactly onto itself four times in a rotation, when it has turned through 90°, 180°, 270° and 360°, so its order of rotational symmetry is 4. Remember it has to turn 360° to get back to its original position.

Exercise 19.2

1 State the order of rotational symmetry of each of the following polygons. The dot represents the centre of rotation in each.

a

b

c

d

e

f

g

h


2 This table shows how many lines of symmetry there are in six regular polygons.

Regular polygon	Lines of symmetry	Order of rotational symmetry
Triangle	3	
Quadrilateral	4	
Pentagon	5	
Hexagon	6	
Octagon	8	
Decagon	10	

a Draw each shape and investigate its rotational symmetry. Sometimes it helps to physically turn the shape to see how many times it fits onto itself in a revolution.
b Copy and complete the table by filling in the order of rotational symmetry for each polygon.
c Describe how line symmetry and rotational symmetry are related in regular polygons.
d What pattern can you see relating line symmetry, order of rotational symmetry and number of sides in a regular polygon?

Applying your skills

3 Refer back to the motor vehicle badges you drew in Exercise 19.1. For each one, state its order of rotational symmetry.

4 Using a computer, print out the capital letters of the alphabet. (You can choose whichever font you like). Which letters have:

a only one line of symmetry?
b two or more lines of symmetry?
c rotational symmetry of order 2 or more?

5 Alloy rims for tyres are very popular on modern cars. Find and draw five alloy rim designs that you like. For each one, state its order of rotational symmetry.

19.2 Symmetry in three dimensions

E

REWIND

Three-dimensional figures (solids) were covered in chapter 7.

There are two types of symmetry in three-dimensional shapes:

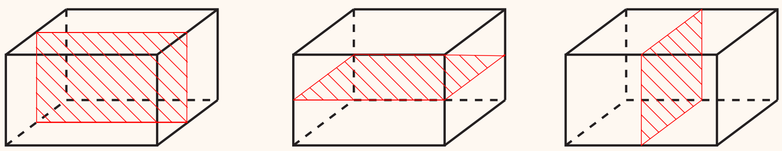
- **Plane symmetry**
- Rotational symmetry

Plane symmetry

A plane of symmetry in a three-dimensional solid is similar to a line of symmetry in a two-dimensional shape.

A plane is a flat surface. If you can cut a solid in half so that each half is the mirror image of the other, then the solid has plane symmetry.

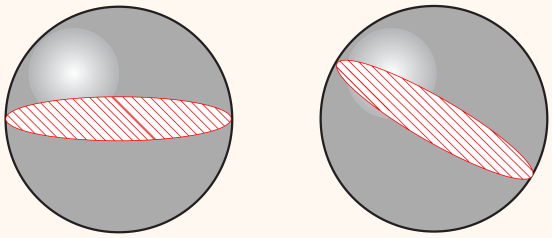
This diagram of a cuboid shows that it can be cut three different ways to make two identical halves. The shaded area on each diagram represents the plane of symmetry (this is where you would cut it).



There are three planes of symmetry in a rectangular cuboid.

E

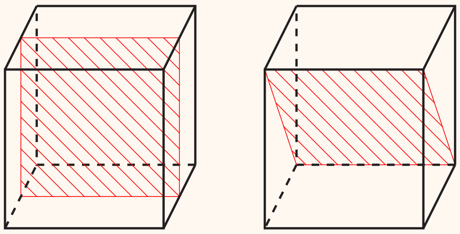
This diagram shows two possible cuts through a sphere that produce two identical halves.



A sphere has an infinite number of planes of symmetry. It is symmetrical about any plane that passes through its centre.

Exercise 19.3

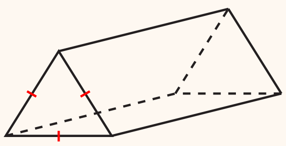
1 Here are two of the planes of symmetry in a cube:



A cube has nine possible planes of symmetry. Make sketches to show the other seven planes.

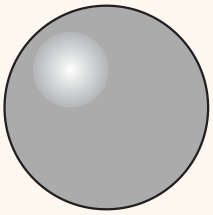
2 How many planes of symmetry does each of the following solids have?

a

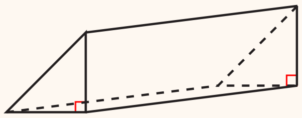


b

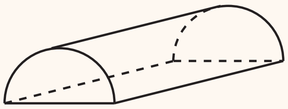
c



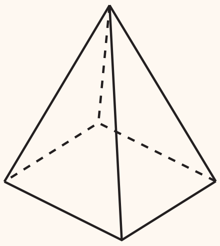
d



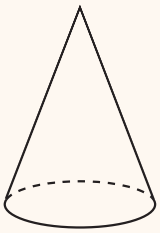
e



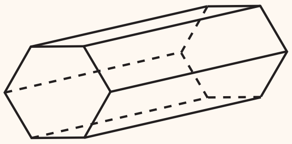
f



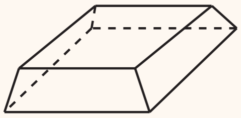
g



h



i

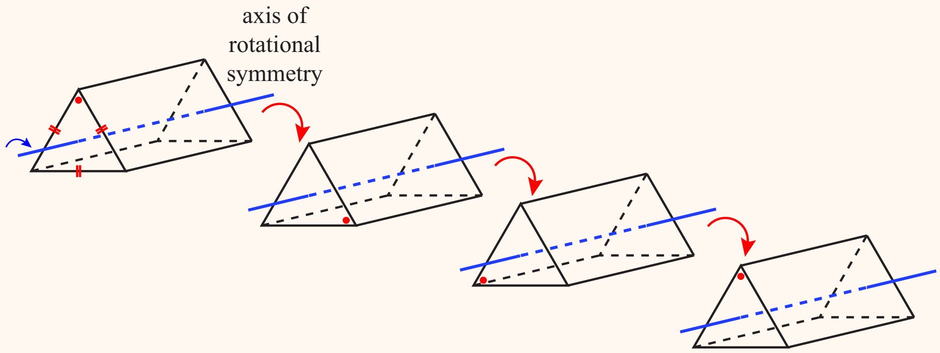


E

Rotational symmetry

Imagine a rod through a solid shape. The rod forms an axis for the shape to turn around. If you rotate the shape around the axis and it looks the same at different points on its rotation, then the shape has rotational symmetry. The rod is then the **axis of symmetry**.

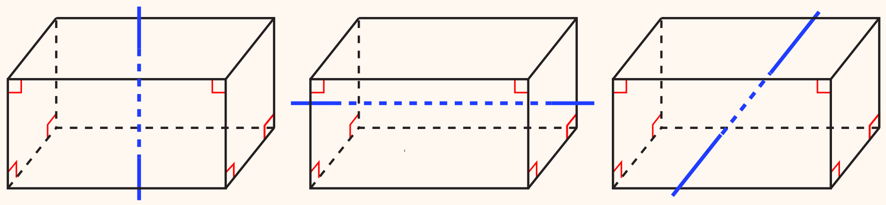
This triangular prism has rotational symmetry of order 3 around the given axis.



The triangular prism looks the same in three positions during a rotation, as it turns for rotation about 120°, 240° and 360°. The dot shows the position of one of the vertices during the turn.

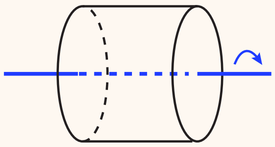
Exercise 19.4

1 This diagram shows three possible axes of symmetry through a cuboid. For each one, state the order of rotational symmetry clockwise through 360°.

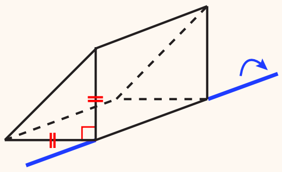


2 For each solid shown, determine the order of rotational symmetry for rotation about the given axis.

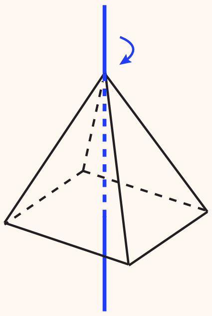
a



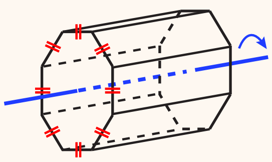
b



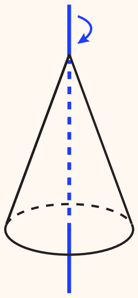
c



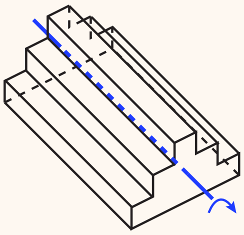
d



e



f



19.3 Symmetry properties of circles

REWIND

You were introduced to chords in chapter 3. ◀

A circle has line symmetry about any diameter and it has rotational symmetry about its centre. From these facts a number of results can be deduced:

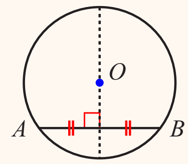
1. The perpendicular bisector of a chord passes through the centre.
2. Equal chords are equidistant from the centre, and chords equidistant from the centre are equal in length.
3. Two **tangents** drawn to a circle from the same point outside the circle are equal in length.

1. The perpendicular bisector of a chord passes through the centre

Tip

You will be expected to state all symmetry and angle properties 'formally'. Learn the statements as they appear throughout the coming pages and be prepared to write them out when you answer questions.

The **perpendicular bisector** of **chord** AB is the locus of points **equidistant** from A and B.



But centre O is equidistant from A and B (OA and OB are radii of circle with centre O).

$\therefore$ O must be on the perpendicular bisector of AB.

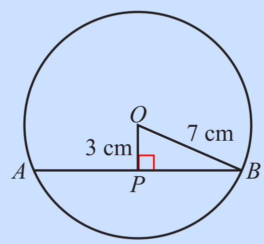
This result can be expressed in other ways:

- The perpendicular from the centre of a circle to a chord meets the chord at its mid-point.
- The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord.

You can use this fact to find the lengths of chords and the lengths of sides of right-angled triangles drawn between the centre and the chord.

Worked example 1

Chord AB is drawn in a circle with a radius of 7 cm. If the chord is 3 cm from the centre of the circle, find the length of the chord correct to 2 decimal places.



REWIND

Pythagoras' theorem was introduced in chapter 11. ◀

$PB^2 = OB^2 - OP^2$ (Rearrange Pythagoras' theorem)

$= 7^2 - 3^2$

$= 49 - 9$

$= 40$

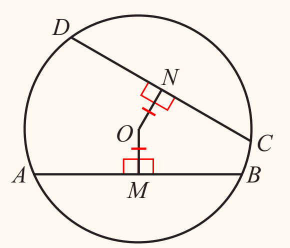
$\therefore PB = \sqrt{40}$

Chord $AB = 2 \times \sqrt{40} = 12.65$ cm

2. Equal chords are equidistant from the centre and chords equidistant from the centre are equal in length

When the distance from a point to a line is asked for, it is always the perpendicular distance which is expected. This is the shortest distance from the point to the line.

If chords AB and CD are the same length, then $OM = ON$, and vice versa.



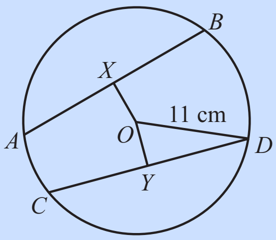
Think about how you could you prove $\Delta OAM \equiv \Delta ODN$; remember OA and OD are radii of the circle.

This is true because triangle OAM is congruent to triangle ODN and because the circle has rotational symmetry about its centre, O.

E

Worked example 2

O is the centre of the circle, radius 11 cm.
AB and *CD* are chords, *AB* = 14 cm.
If *OX* = *OY*, find the length of *OY* correct to 2 decimal places.



Since $OX = OY$, chords are equidistant so, $AB = CD = 14$ cm
$CY = YD = 7$ cm (OY is perpendicular bisector of CD)
$\angle OYD = 90°$

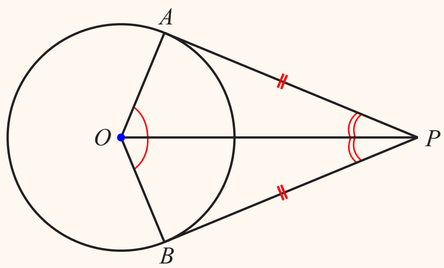
$$\begin{aligned} OY^2 &= OD^2 - YD^2 \text{ (Pythagoras)} \\ &= 11^2 - 7^2 \\ &= 121 - 49 \\ &= 72 \end{aligned}$$

$\therefore\ OY = \sqrt{72} = 8.49$ cm

3. Two tangents drawn to a circle from the same point outside the circle are equal in length

A and *B* are the points of contact of the tangents drawn from *P*.

The result is *PA* = *PB*.



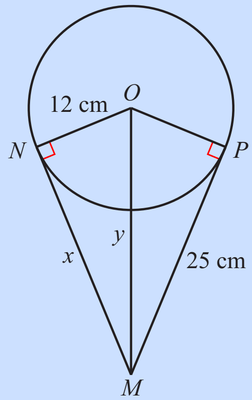
In addition:

- the tangents **subtend** equal angles at the centre (i.e. angle *POA* = angle *POB*)
- the line joining the centre to the point where the tangents meet bisects the angle between the tangents. (i.e. angle *OPA* = angle *OPB*)

This is true because the figure is symmetrical about the line *OP*. It can also be shown by proving that ΔOAP is congruent to ΔOBP. You need to use the 'tangent perpendicular to radius' property for this.

Worked example 3

Find the length of *x* and *y* in this diagram correct to 2 decimal places where appropriate.



$NM = PM = 25$ cm (equal tangents)
$\therefore\ x = 25$ cm

$$\begin{aligned} y^2 &= x^2 + NO^2 \text{ (Pythagoras)} \\ &= 25^2 + 12^2 \\ &= 625 + 144 \\ &= 769 \end{aligned}$$

$y = \sqrt{769} = 27.73$ cm

E

Worked example 4

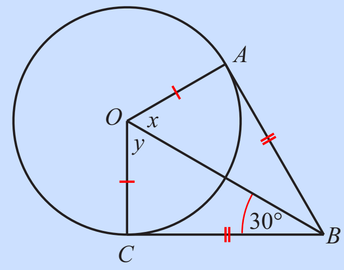
Find the size of angles x and y in this diagram.

Angle $OCB = 90°$ (OC perpendicular to tangent CB)

$\therefore y = 180° - 90° - 30°$ (angle sum of triangle)

$y = 60°$

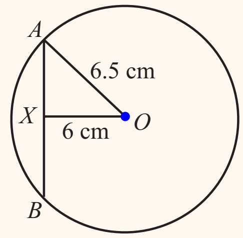
$y = x = 60°$ (tangents subtend equal angles at centre)



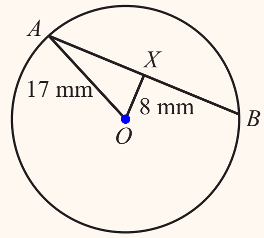
Exercise 19.5

1 Calculate the length of the chord AB in each of the following circles. (In each case, X is the mid-point of AB.)

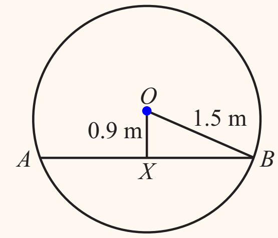
a



b



c

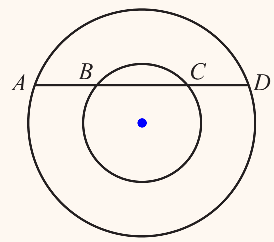


2 P is a point inside a circle whose centre is O. Describe how to construct the chord that has P as its mid-point

Concentric circles are circles with different radii but the same centre.

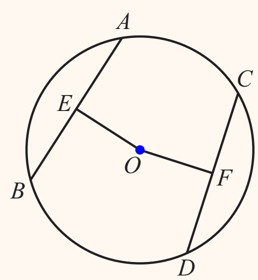
3 A straight line cuts two concentric circles at A, B, C and D (in that order).

Prove that $AB = CD$.



4 Apply what you have learned about circle properties to calculate the diameter of each circle. Give approximate answers to 3 significant figures. Show all your working and give reasons for any deductions.

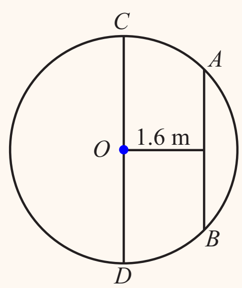
a



$AB = CD = 11.4$ cm

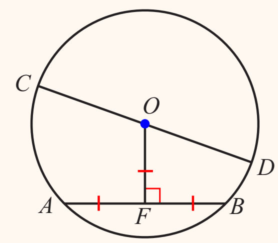
$OF = 6.5$ cm

b



$AB = 2.8$ m

c

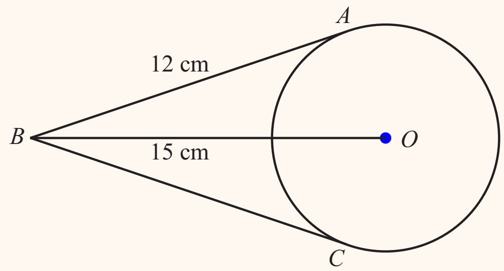


$AB = 22$ mm

5 A circle with a radius of 8.4 cm has a chord 5 cm from its centre. Calculate the length of the chord correct to 2 decimal places.

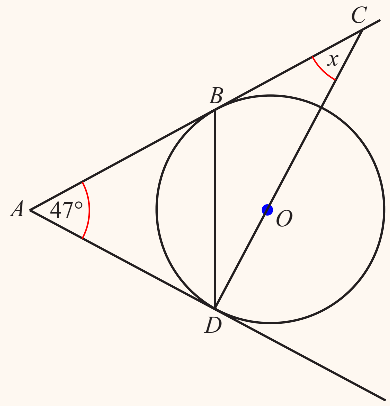
6 In this diagram, find the length of AO and the area of quadrilateral $AOCB$.

BA and BC are tangents to the circle.



7 In the diagram, AB and AD are tangents to the circle. ABC is a straight line.

Calculate the size of angle x.



19.4 Angle relationships in circles

Circles have many useful angle properties that can be used to solve problems.

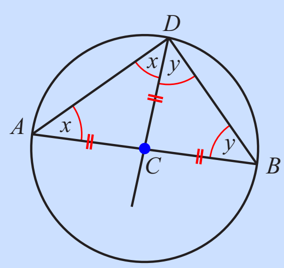
You will now explore more of these properties. The following examples and theorems will help you to solve problems involving angles and circles.

The angle in a semi-circle is a right angle (90°)

Read through the worked example to see how to work out the size of an angle in a semi-circle.

Worked example 5

AB is the diameter of a circle. C is the centre. D is any point on the circumference. Remember that all radii of a circle are equal. Work out the size of angle ADB.



$AC = CB = CD$ (radii of circle)

$\therefore \Delta ACD$ and ΔBCD are isosceles.

$\therefore$ angle CAD = angle $ADC = x$

and angle CDB = angle $DBC = y$

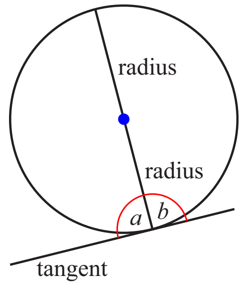
But $2x + 2y = 180°$ (sum angles ΔABD)

$\therefore x + y = 90°$

so angle $ADB = 90°$

The angle between the tangent and radius is 90°

Look at the diagram carefully. You already know that the diameter divides the circle evenly into two equal parts.



So $a = b$

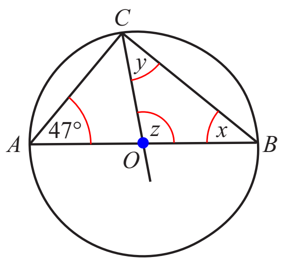
and $a + b = 180°$ (angles on a straight line)

$\therefore a = b = 90°$

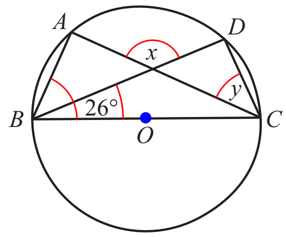
Exercise 19.6

1 Calculate the size of the lettered angles in each diagram. Show your working and give reasons for any deductions.

a

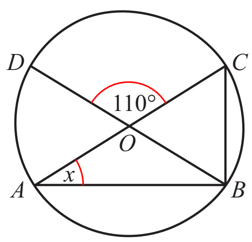


b

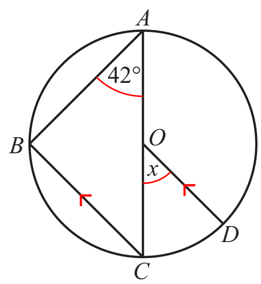


angle $ABC = 60°$

c

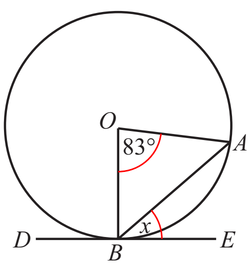


d

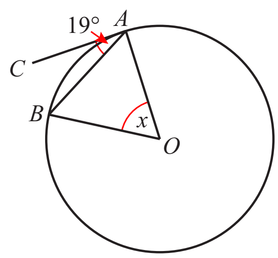


2 Calculate the value of x in each diagram. Show your working and give reasons for any deductions.

a

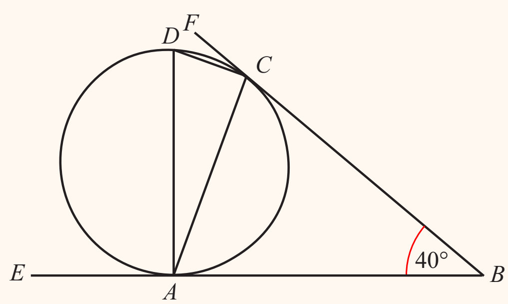


b



3 In the diagram, BCF and BAE are the tangents to the circle at C and A respectively.

AD is a diameter and angle $ABC = 40°$.



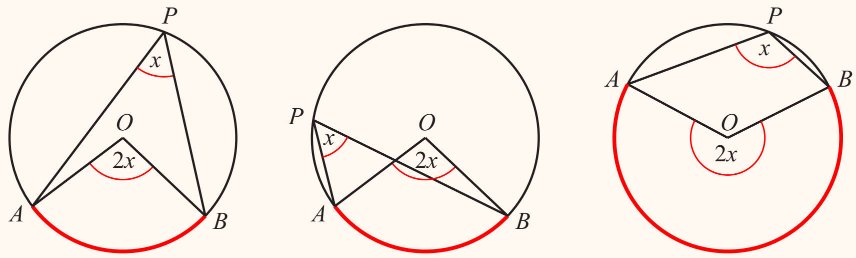
E

a Explain why ΔABC is isosceles.
b Calculate the size of:

i angle CAB
ii angle DAC
iii angle ADC.

Further circle theorems

The angle at the centre of a circle is twice the angle at the circumference

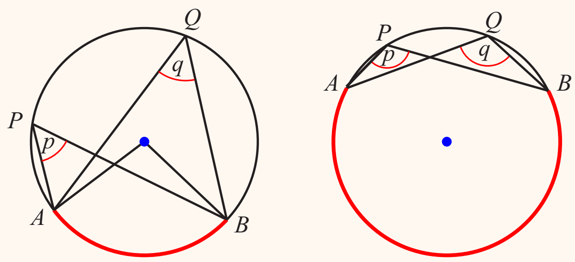


AB is an **arc** of a circle with centre O. P is a point on the circumference, but not on the arc AB. The angle at the centre theorem states that

angle $AOB = 2 \times$ angle APB

As you saw before, this is also true when AB is a semi-circular arc. The angle at the centre theorem states that the angle in a semi-circle is 90°. This is because, in this case, angle AOB is a straight line (180°).

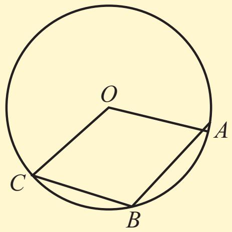
Angles in the same segment are equal



In these two diagrams, $p = q$. Each of the angles p and q is half the angle subtended by the arc AB at the centre of the circle.

You may sometimes see 'cyclic quadrilateral' written as 'cyclical quadrilateral'.

A common error is to see the following as a cyclic quadrilateral:



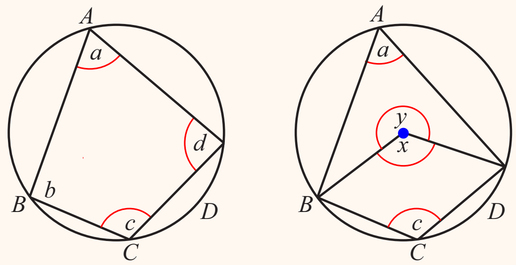
You must check that *all four* vertices sit on the circumference of the circle.

E

The opposite angles of a cyclic quadrilateral add up to 180°

A **cyclic quadrilateral** is one that has all four vertices touching the circumference of a circle.

Look at the diagram and follow the working to see why this is the case.



$x = 2a$ (angle at centre theorem, minor arc BD)

$y = 2c$ (angle at centre theorem, major arc BD)

$\therefore x + y = 2a + 2c$

But $x + y = 360°$ (angles around a point)

$\therefore a + c = 180°$ (opposite angles sum of a cyclic quadrilateral)

By a similar argument:

$b + d = 180°$

Each exterior angle of a cyclic quadrilateral is equal to the interior angle opposite to it

The worked example shows why this is the case.

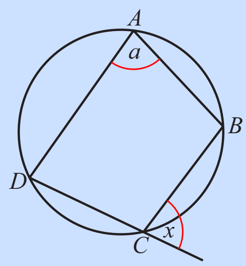
Worked example 6

Prove that $x = a$.

x + angle $BCD = 180°$ (angles on a straight line)

a + angle $BCD = 180°$ (opposite interior angles of a cyclic quadrilateral)

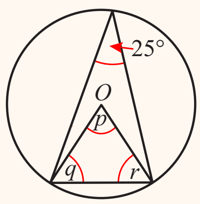
$x = a$



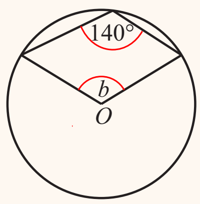
Exercise 19.7

1 Find the size of each lettered angle in these sketches.
When it is marked, O is the centre of the circle.

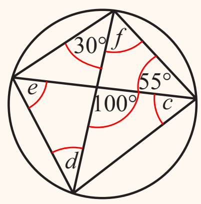
a



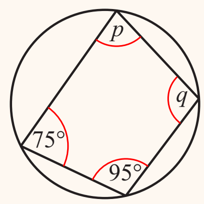
b



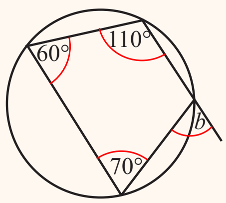
c



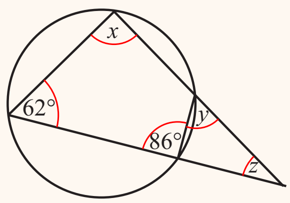
d



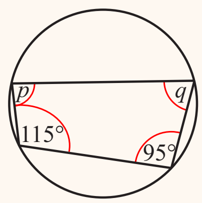
e



f

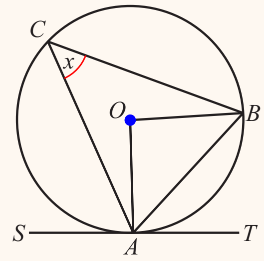


g



E

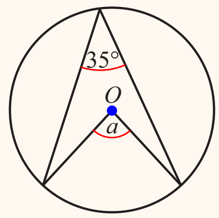
2 In the diagram SAT is the tangent to the circle at point A. The points B and C lie on the circle and O is the centre of the circle. If angle $ACB = x$, express, in terms of x, the size of:



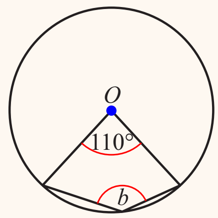
a angle AOB
b angle OAB
c angle BAT.

3 Find the size of each lettered angle in these sketches.
When it is marked, O is the centre of the circle.

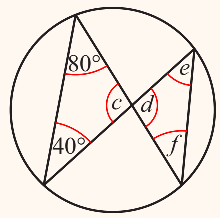
a



b

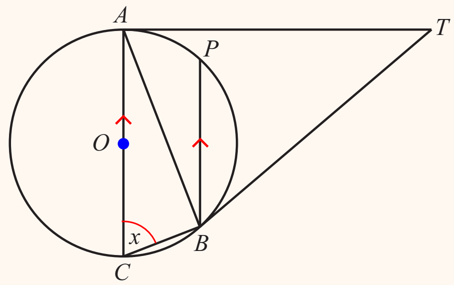


c



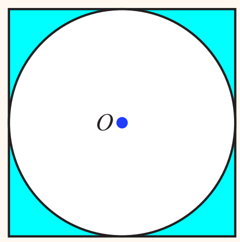
4 In the diagram, TA and TB are the tangents from T to the circle whose centre is O. AC is a diameter of the circle and angle $ACB = x$.

a Find angle CAB in terms of x.
b Find angle ATB in terms of x.
c The point P on the circumference of the circle is such that BP is parallel to CA. Express angle PBT in terms of x.



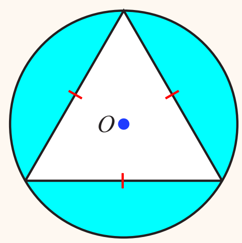
Applying your skills

5 The diagram shows a circular disc cut out of a square of silver sheet metal plate. The circle has a radius of 15 mm. O is the centre of the circle.



a Calculate the length of the sides and hence the area of the uncut metal square.
b What area of metal is left over once the circle has been cut from the square?

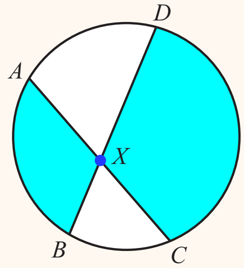
6 Mahindra makes badges by sticking an equilateral triangle onto a circular disc as shown. If the triangle has sides of 15 cm, find the diameter of the circular disc.



7 The diagram shows two chords, AC and BD, drawn in a circle. The chords intersect at the point X.

a Use angle properties to show that triangle ABX is similar to triangle CDX.
b Use the fact that the two triangles are similar to show that:

$$AX \times CX = BX \times DX$$

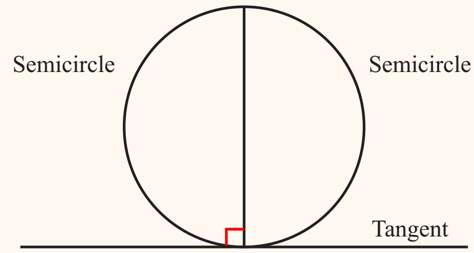


This is called the intersecting chords theorem.

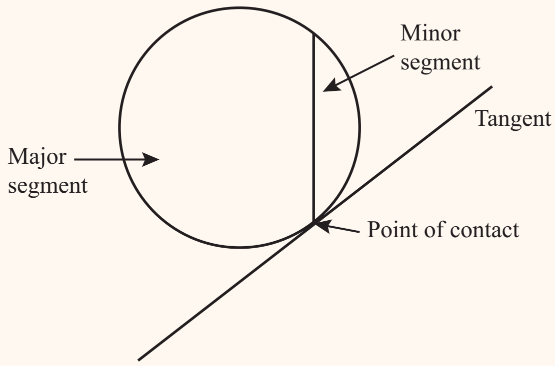
E

Alternate segment theorem

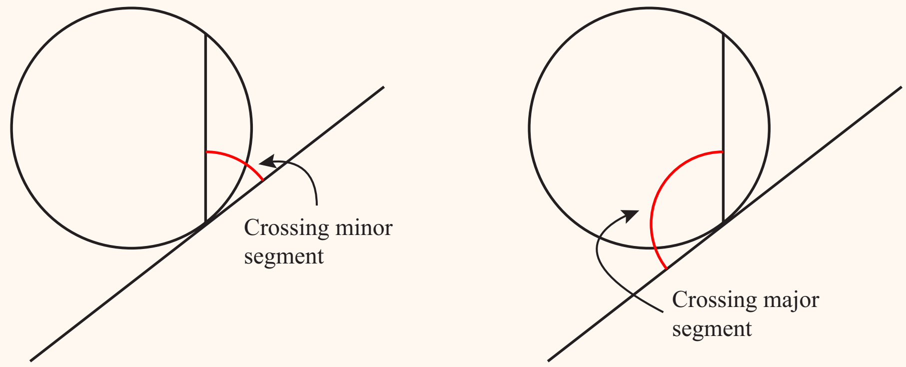
You already know that where a tangent and diameter of a circle meet they form a right-angle. The diameter will divide the circle into two semicircles.



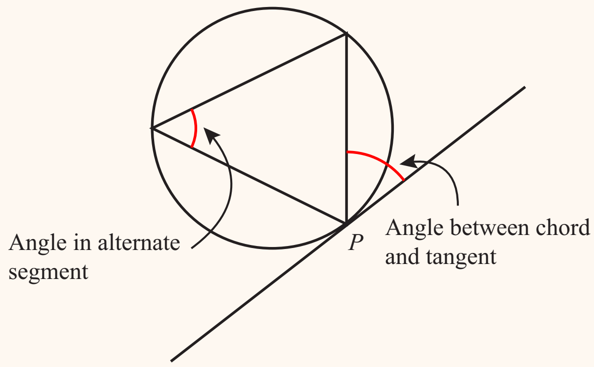
You also know that a diameter of any circle is a chord that passes through the centre of the circle. When a chord meets a tangent but does *not* pass through the centre, the circle is divided into a major segment and a minor segment. The point at which the tangent and chord meet is called the point of contact.



You can drawe two possible angles between the tangent and this chord. One crosses the major segment and the other crosses the minor segment.



The segment that the angle does *not* cross is called the **alternate segment**. Draw an angle in the alternate segment.

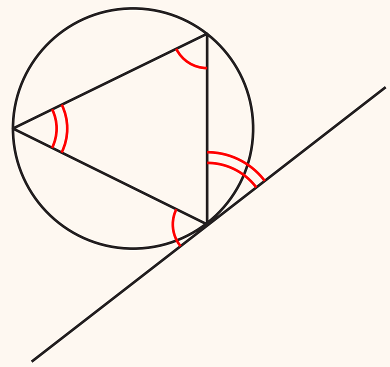


E

Investigation

- Draw 3 large circles
- Draw a tangent to each circle, taking care to be as accurate as possible – your line should only touch the circle once
- Draw a chord so that it meets the tangent
- Draw an angle between the tangent and chord. You can choose either angle
- Work out which segment is the alternate segment and draw an angle in it
- Measure, as accurately as you can, both the angle in the angle in the alternate segment and the angle between the tangent and the chord.
- What do you notice?

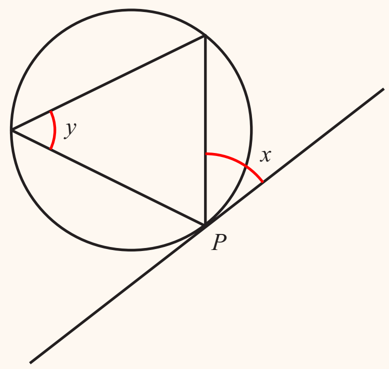
The **alternate segment theorem** states that the angle between the tangent and chord is always equal to the angle in the alternate segment. The diagram shows which angles are equal to which:



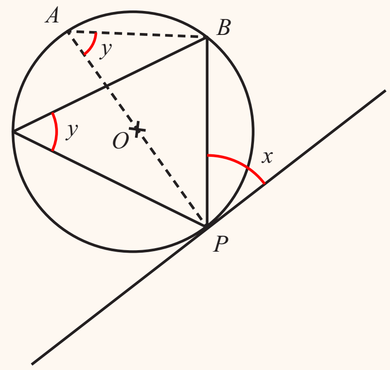
Proof of the Alternate Segment Theorem

This is an interesting proof because it will use some of the theorems that you already know.

Begin by drawing the circle, chord, tangent, angle and angle in the alternate segment. Call the angles x and y respectively.



You know that any two angles drawn in the same segment are equal. This means that you can draw another angle in the alternate segment, using the diameter as one of the lines forming the angle, and know that it is still equal to y.



You can see that triangle PAB is right-angled, as it is the angle in a semicircle. This means that the angle APB must be $90 - y$.

You also know that the angle between a diameter and a tangent is 90 degrees, so $90 - y + x = 90$. This shows that $x = y$.

E

Worked example 7

Find the unknown angle x in the diagram.

Angles in a triangle add to 180°

$\therefore ABC + BAC + 40 = 180°$

Both AC and BC are tangents to the circle so ΔABC is isosceles.

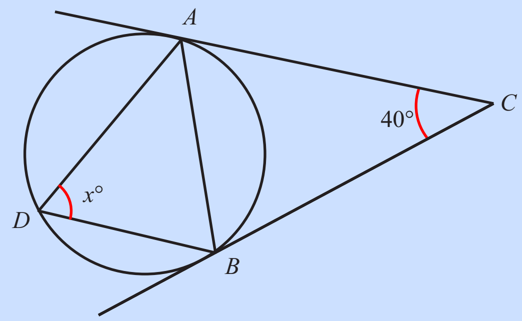
$ABC + BAC + 40 = 180°$

$\therefore ABC = BAC = 70°$

By the alternate segment theorem

Angle ABC = Angle ADB = 70°

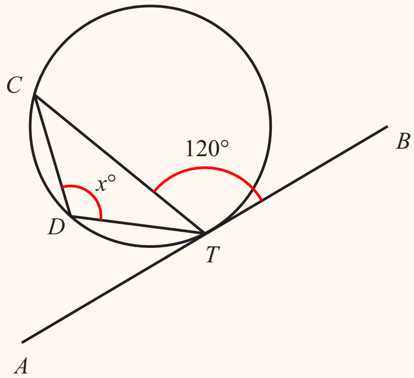
So x = 70°



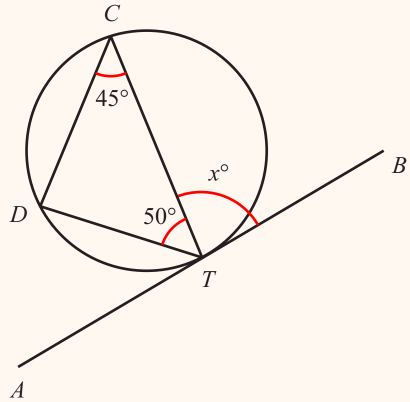
Exercise 19.8

1 Find the angle x in each of the following. Give full reasons for your answers.

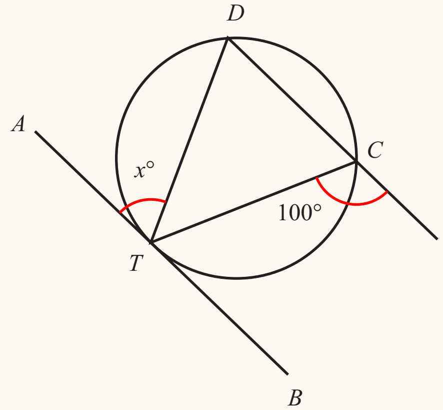
a



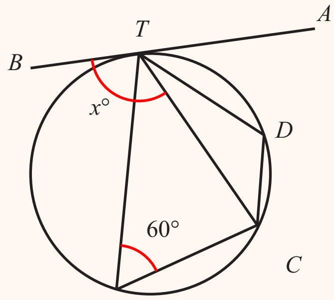
b



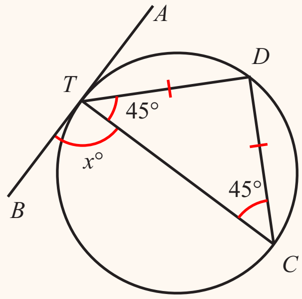
c



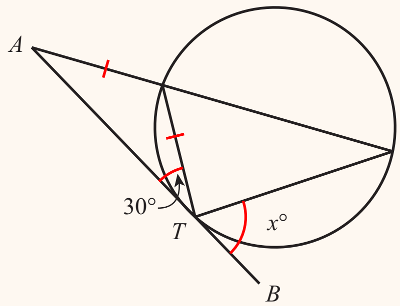
d



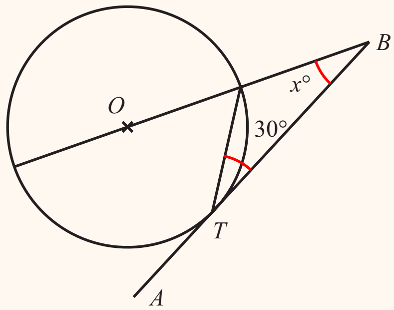
e



f

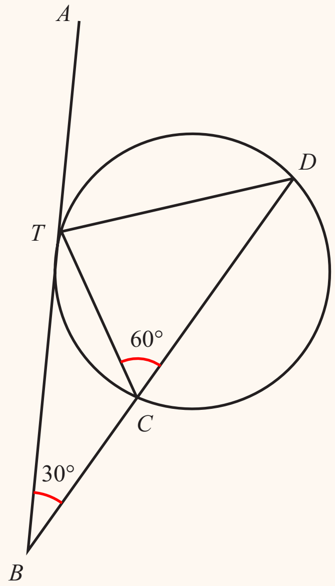


g

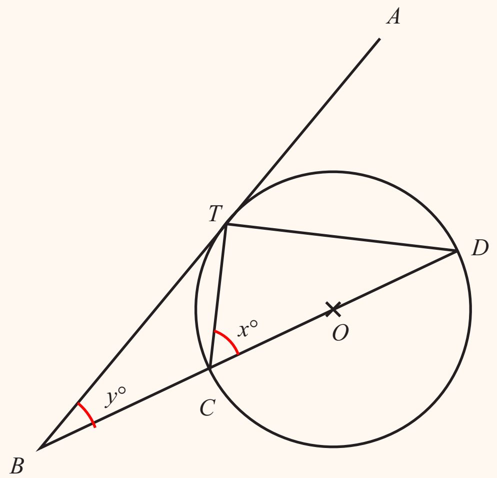


E

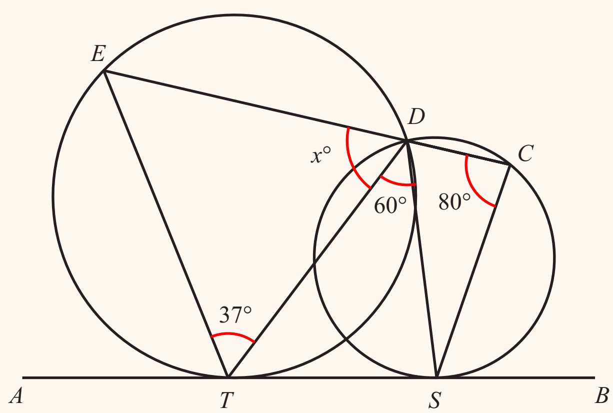
2 Prove that CD is a diameter of the circle.



3 Given that O is the centre of the cirlcle, prove that $2x - y = 90°$



4 Find the unknown angle x.



Summary

Do you know the following?

- If a two-dimensional shape can be folded along a line so that the two halves are mirror images of each other it is said to have line symmetry.
- Shapes can have more than one line of symmetry. The number of lines of symmetry of a regular polygon corresponds with the number of sides the shape has.
- If a shape can be rotated around a point (the centre of rotation) so that it matches itself at least once in a complete revolution it has rotational symmetry.
- The order of rotational symmetry tells you the number of times a shape fits onto itself in one rotation.

E

- Three-dimensional solids can also have symmetry.
- When a shape can be cut along a plane to form two solid parts that are mirror images of each other then it has plane symmetry.
- If a three-dimensional shape is rotated around an axis and it looks the same at one or more positions during a complete revolution then it has rotational symmetry.
- Circles have symmetry properties.
- The perpendicular bisector of a chord passes through the centre of a circle.
- Equal chords are equidistant from the centre and chords equidistant from the centre are equal in length.
- Two tangents drawn to a circle from a point outside the circle are equal in length.

- The angle in a semi-circle is a right angle.
- The angle between a tangent and the radius of a circle is a right angle.

E

- The angle subtended at the centre of a circle by an arc is twice the angle subtended at the circumference by the arc.
- Angles in the same segment, subtended by the same arc, are equal.
- Opposite angles of a cyclic quadrilateral add up to 180°.
- Each exterior angle of a cyclic quadrilateral is equal to the interior angle opposite to it.
- The alternate segment theorem states that the angle between the tangent and chord is always equal to the angle in the alternate segment.

Are you able to …?

- recognise rotational and line symmetry in two-dimensional shapes
- find the order of symmetry of a two-dimensional shape

E

- recognise rotational and line symmetry in three-dimensional shapes

- use the symmetry properties of polygons and circles to solve problems
- calculate unknown angles in a circle using its angle properties:
  - angle in a semi-circle
  - angle between tangent and radius of a circle

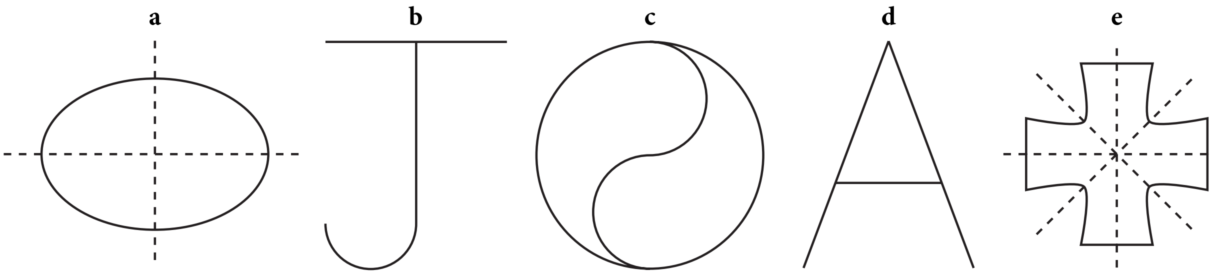
E

  - angle at centre of a circle
  - angles in the same segment
  - angles in opposite segments
  - alternate segment theorem
- use the symmetry properties of circles:
  - equal chords are equidistant from the centre
  - the perpendicular bisector of a chord passes through the centre
  - tangents from an external point are equal

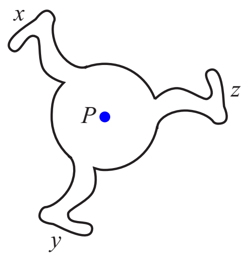
Examination practice

Exam-style questions

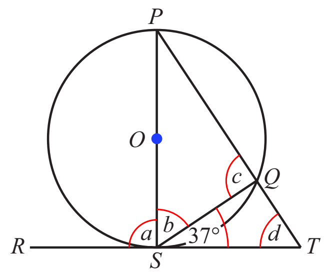
1 Which of the following figures have both line and rotational symmetry?



2 Using P as the centre of rotation, state the order of rotational symmetry in this figure.

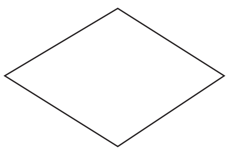


3 RST is a tangent to the circle with centre O. PS is a diameter. Q is a point on the circumference and PQT is a straight line.
Angle $QST = 37°$.
Write down the values of a, b, c and d.

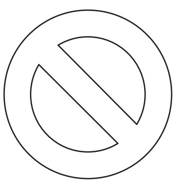


Past paper questions

1 **a** Write down the order of rotational symmetry of this shape. [1]

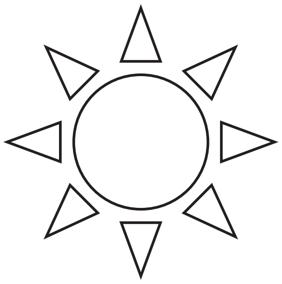


b Draw the lines of symmetry on this shape. [1]



[Cambridge IGCSE Mathematics 0580 Paper 11 Q05 October/November 2013]

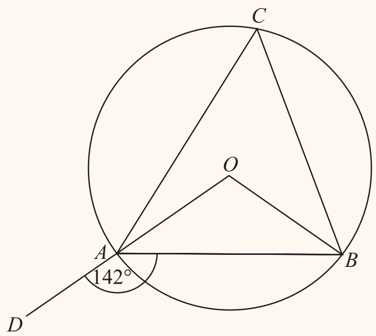
2



Write down the order of rotational symmetry of this shape. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22, Q3, November 2014]

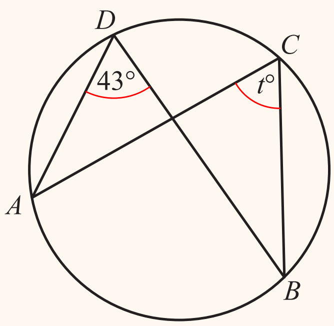
3



A, B and C are points on the circumference of a circle centre O.
OAD is a straight line and angle $DAB = 142°$.
Calculate the size of angle ACB. [3]

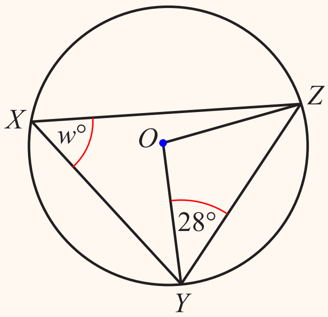
[Cambridge IGCSE Mathematics 0580 Paper 22 Q14 October/November 2013]

4 **a** **i** A, B, C and D lie on the circumference of the circle.



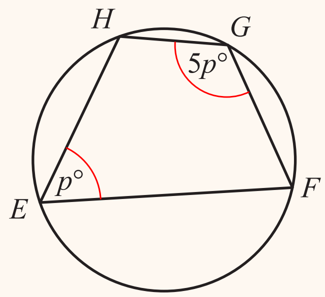
Find the value of t. [1]

ii X, Y and Z lie on the circumference of the circle, centre O.



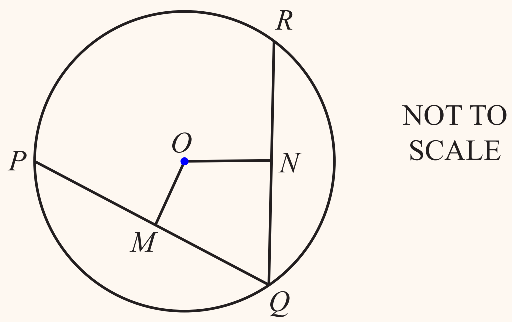
Find the value of w, giving reasons for your answer. [3]

iii E, F, G and H lie on the circumference of the circle.



Find the value of p, giving a reason for your answer. [3]

b



The diagram shows a circle, centre O.
PQ and QR are chords.
OM is the perpendicular from O to PQ.

i Complete the statement.

PM : PQ = ______________ : ______________ [1]

ii ON is the perpendicular from O to QR and $PQ = QR$.

Complete the statements to show that triangle OMQ is congruent to triangle ONQ.

______________ is a common side.

______________ = ______________ because M is the midpoint of PQ and N is the midpoint of RQ.

______________ = ______________ because equal chords are equidistant from ______________ [4]

[Cambridge IGCSE Mathematics 0580 Paper 42, Q6, November 2015]

5 A quadrilateral has rotational symmetry of order 2 and no lines of symmetry.

Write down the mathematical name of this quadrilateral. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22, Q4, June 2016]