Chapter 11: Pythagoras' theorem and similar shapes

Key words

- Right angle
- Hypotenuse
- Similar
- Corresponding sides
- Corresponding angles
- Scale factor of lengths
- Scale factor of volumes
- Scale factor of areas
- Congruent
- Included side
- Included angle

In this chapter you will learn how to:

- use Pythagoras' theorem to find unknown sides of a right-angled triangles
- learn how to use Pythagoras' theorem to solve problems
- decide whether or not triangles are mathematically similar
- use properties of similar triangles to solve problems
- find unknown lengths in similar figures
- use the relationship between sides and areas of similar figures to find missing values

EXTENDED

- recognise similar solids
- calculate the volume and surface area of similar solids

- decide whether or not shapes are congruent.

EXTENDED

- use the basic conditions for congruency in triangles

One man – Pythagoras of Samos – is usually credited with the discovery of the Pythagorean theorem, but there is evidence to suggest that an entire group of religious mathematicians would have been involved.

Right-angled triangles appear in many real-life situations, including architecture, engineering and nature. Many modern buildings have their sections manufactured off-site and so it is important that builders are able to accurately position the foundations on to which the parts will sit so that all the pieces will fit smoothly together.

Many properties of right-angled triangles were first used in ancient times and the study of these properties remains one of the most significant and important areas of Mathematics.

RECAP

You should already be familiar with the following number and shape work:

Squares and square roots (Chapter 1)

To square a number, multiply it by itself. $7^2 = 7 \times 7 = 49$.

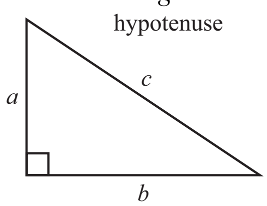
You can also use the square function on your calculator x^2.

To find the square root of a number use the square root function on your calculator $\sqrt{\ }$ $\sqrt{121} = 11$.

Pythagoras’ theorem (Stage 9 Mathematics)

Pythagoras’ theorem states that in a right-angled triangle, the square of the hypotenuse is equal to the sum of the two shorter sides.

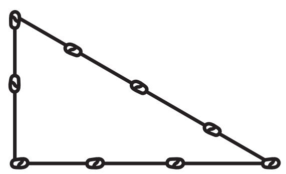
For this triangle:



$a^2 + b^2 = c^2$

The hypotenuse is the longest side and it is always opposite the right angle.

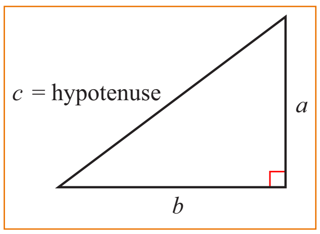
11.1 Pythagoras’ theorem



Centuries before the theorem of right-angled triangles was credited to Pythagoras, the Egyptians knew that if they tied knots in a rope at regular intervals, as in the diagram on the left, then they would produce a perfect **right angle**.

In some situations you may be given a right-angled triangle and then asked to calculate the length of an unknown side. You can do this by using Pythagoras’ theorem if you know the lengths of the other two sides.

Learning the rules



Pythagoras’ theorem describes the relationship between the sides of a right-angled triangle.

The longest side – the side that does not touch the right angle – is known as the **hypotenuse**.

For this triangle, Pythagoras’ theorem states that: $\boxed{a^2 + b^2 = c^2}$

In words this means that the square on the hypotenuse is equal to the sum of the squares on the other two sides. Notice that the square of the hypotenuse is the subject of the equation. This should help you to remember where to place each number.

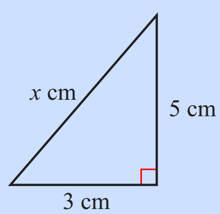
Tip

You will be expected to remember Pythagoras' theorem.

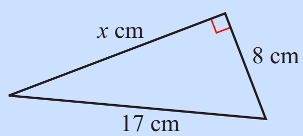
Worked example 1

Find the value of x in each of the following triangles, giving your answer to one decimal place.

a



b



a	$a^2 + b^2 = c^2$ $3^2 + 5^2 = x^2$ $9 + 25 = x^2$ $\Rightarrow x^2 = 34$ $x = \sqrt{34} = 5.8309\ldots$ $\approx 5.8\text{ cm (1 dp)}$	Notice that the final answer needs to be rounded.
b	$a^2 + b^2 = c^2$ $8^2 + x^2 = 17^2$ $64 + x^2 = 289$ $x^2 = 289 - 64$ $x^2 = 225$ $x = \sqrt{225} = 15\text{ cm (1 dp)}$	Notice that a shorter side needs to be found so, after writing the Pythagoras formula in the usual way, the formula has to be rearranged to make x^2 the subject.

Checking for right-angled triangles

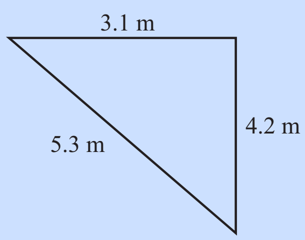
You can also use the theorem to determine if a triangle is right-angled or not. Substitute the values of a, b and c of the triangle into the formula and check to see if it fits. If $a^2 + b^2$ does not equal c^2 then it is *not* a right-angled triangle.

Notice here the theorem is written as $c^2 = a^2 + b^2$; you will see it written like this or like $a^2 + b^2 = c^2$ in different places but it means the same thing.

The symbol '≠' means 'does not equal'.

Worked example 2

Use Pythagoras' theorem to decide whether or not the triangle shown below is right-angled.



Check to see if Pythagoras' theorem is satisfied:

$c^2 = a^2 + b^2$

$3.1^2 + 4.2^2 = 27.25$

$5.3^2 = 28.09 \neq 27.25$

Pythagoras' theorem is not satisfied, so the triangle is not right-angled.

Exercise 11.1

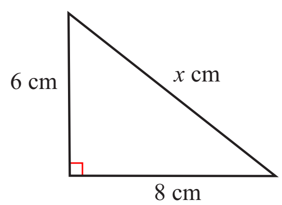
For all the questions in this exercise, give your final answer correct to three significant figures where appropriate.

REWIND

You will notice that some of your answers need to be rounded. Many of the square roots you need to take produce irrational numbers. These were mentioned in chapter 9. ◀

1 Find the length of the hypotenuse in each of the following triangles.

a



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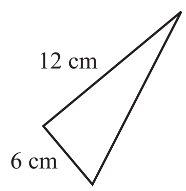

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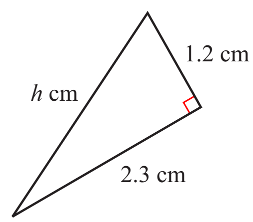

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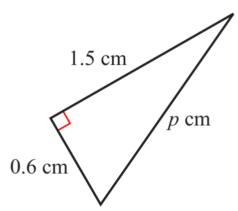

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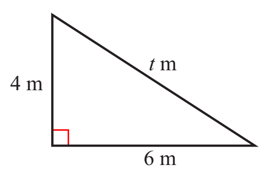
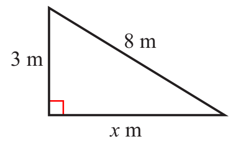


2 Find the values of the unknown lengths in each of the following triangles.

a



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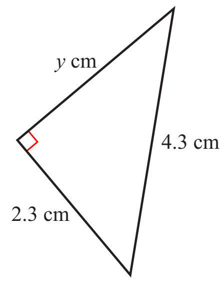

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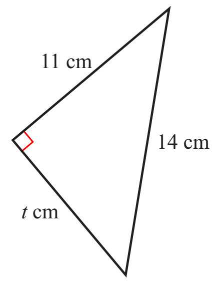

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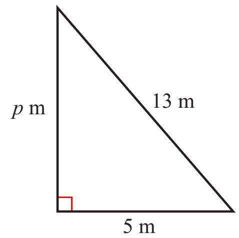

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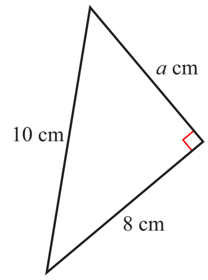
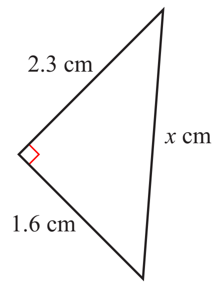


3 Find the values of the unknown lengths in each of the following triangles.

a



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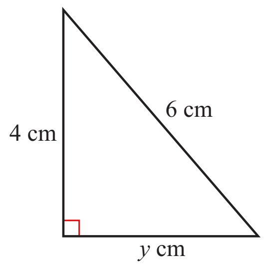

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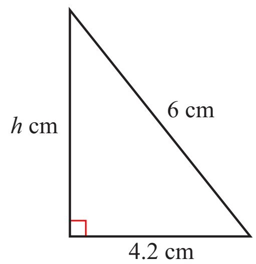

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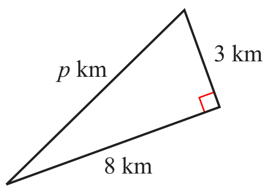

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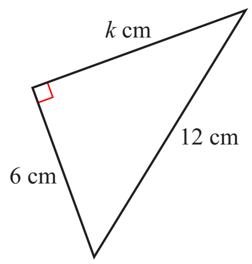

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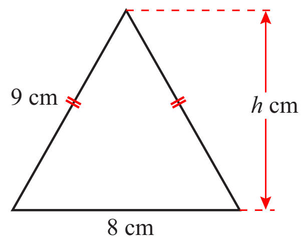

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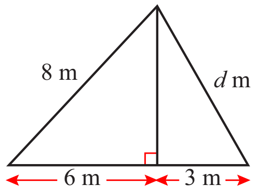

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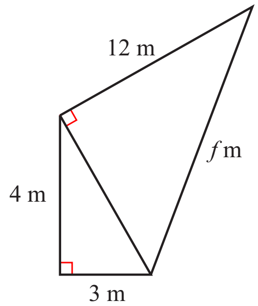
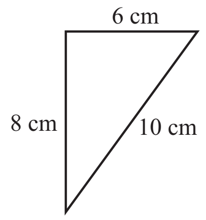


4 Use Pythagoras' theorem to help you decide which of the following triangles are right-angled.

a



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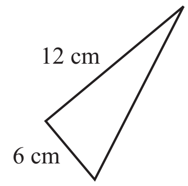

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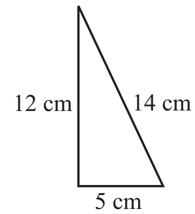

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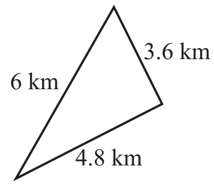

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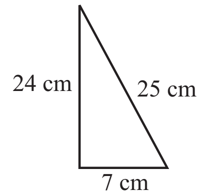
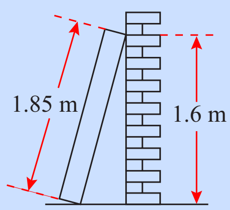


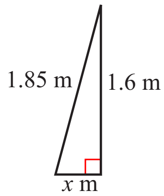
Applications of Pythagoras' theorem

This section looks at how Pythagoras' theorem can be used to solve real-life problems. In each case look carefully for right-angled triangles and draw them separately to make the working clear.

Worked example 3



The diagram shows a bookcase that has fallen against a wall. If the bookcase is 1.85 m tall, and it now touches the wall at a point 1.6 m above the ground, calculate the distance of the foot of the bookcase from the wall. Give your answer to 2 decimal places.



Apply Pythagoras' theorem:

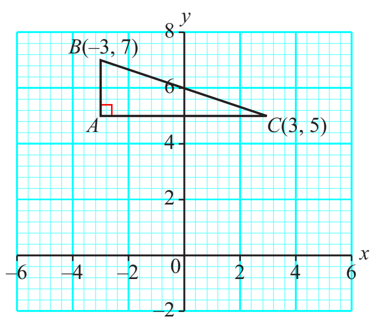
$$a^2 + b^2 = c^2$$
$$x^2 + 1.6^2 = 1.85^2$$
$$x^2 = 1.85^2 - 1.6^2$$
$$= 3.4225 - 2.56$$
$$= 0.8625$$
$$x = \sqrt{0.8625} = 0.93\,\text{m (2dp)}$$

Think what triangle the situation would make and then draw it. Label each side and substitute the correct sides into the formula.

It is usually useful to draw the triangle that you are going to use as part of your working.

Worked example 4

Find the distance between the points $A(3, 5)$ and $B(-3, 7)$.



$AB = 7 - 5 = 2$ units — Difference between y-co-ordinates.

$AC = 3 - -3 = 6$ units — Difference between x-co-ordinates.

$$BC^2 = 2^2 + 6^2$$
$$= 4 + 36$$
$$= 40$$

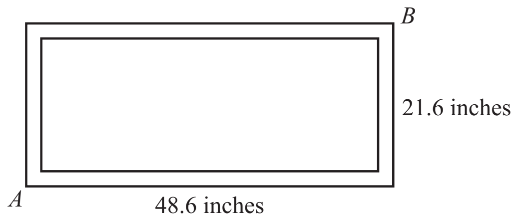
So $BC = \sqrt{40}$

$= 6.32$ units (3sf) — Apply Pythagoras' theorem.

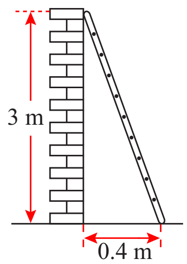
It can be helpful to draw diagrams when you are given co-ordinates.

Exercise 11.2

You generally won't be told to use Pythagoras' theorem to solve problems. Always check for right-angled triangles in the context of the problem to see if you can use the theorem to solve it.

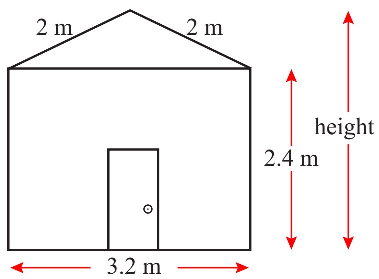
1 

The size of a television screen is its longest diagonal. The diagram shows the length and breadth of a television set. Find the distance AB.

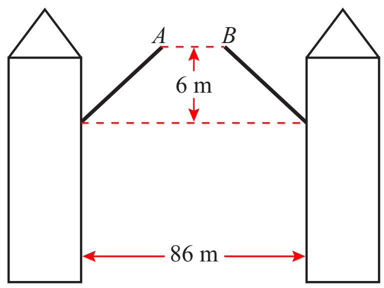
2 

The diagram shows a ladder that is leaning against a wall. Find the length of the ladder.

3 Sarah stands at the corner of a rectangular field. If the field measures 180 m by 210 m, how far would Sarah need to walk to reach the opposite corner in a straight line?

4 

The diagram shows the side view of a shed. Calculate the height of the shed.

5 

The diagram shows a bridge that can be lifted to allow ships to pass below. What is the distance AB when the bridge is lifted to the position shown in the diagram?
(Note that the bridge divides exactly in half when it lifts open.)

6 Find the distance between the points A and B with co-ordinates:

a	$A(3, 2)$	$B(5, 7)$
b	$A(5, 8)$	$B(6, 11)$
c	$A(-3, 1)$	$B(4, 8)$
d	$A(-2, -3)$	$B(-7, 6)$

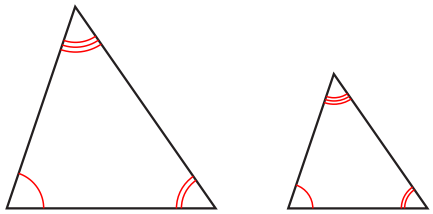
7 The diagonals of a square are 15 cm. Find the perimeter of the square.

11.2 Understanding similar triangles

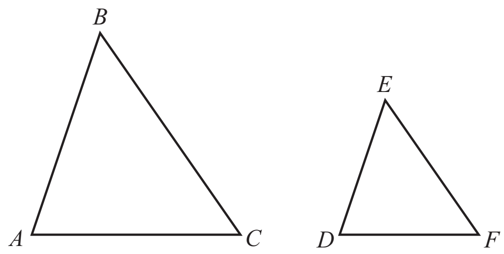
Two mathematically **similar** objects have exactly the same shape and proportions, but may be different in size.

When one of the shapes is enlarged to produce the second shape, each part of the original will *correspond* to a particular part of the new shape. For triangles, **corresponding sides** join the same angles.

All of the following are true for similar triangles:

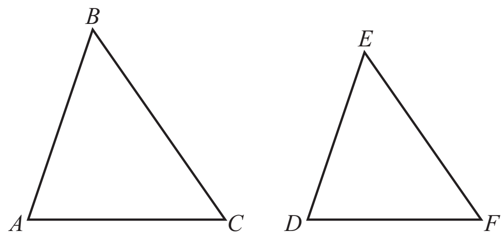


Corresponding angles are equal.



'Internal' ratios of sides are the same for both triangles. For example:

$$\frac{AB}{BC}=\frac{DE}{EF}$$



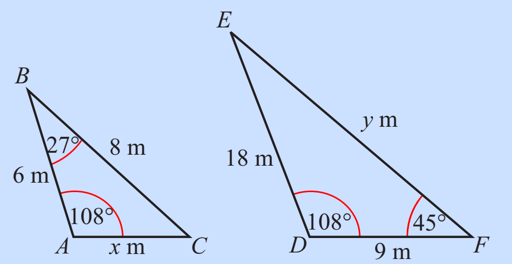
Ratios of corresponding sides are equal:

$$\frac{AB}{DE}=\frac{BC}{EF}=\frac{AC}{DF}$$

If any of these things are true about two triangles, then all of them will be true for both triangles.

Worked example 5

Explain why the two triangles shown in the diagram are similar and work out *x* and *y*.



Angle $ACB = 180° - 27° - 108° = 45°$

Angle $FED = 180° - 45° - 108° = 27°$

So both triangles have exactly the same three angles and are, therefore, similar.

Since the triangles are similar: $\frac{DE}{AB}=\frac{EF}{BC}=\frac{DF}{AC}$

So : $\frac{y}{8}=\frac{18}{6}=3 \Rightarrow y = 24\,\text{m}$

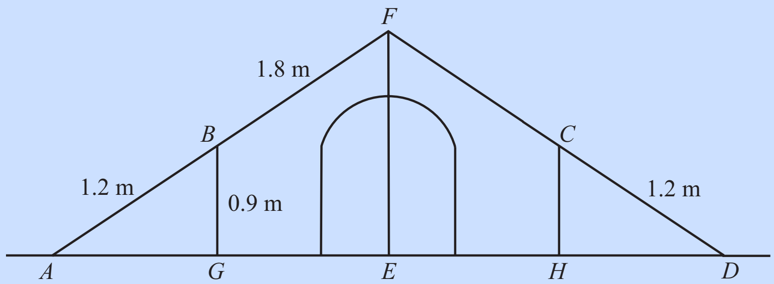
and: $\frac{9}{x}=\frac{18}{6}=3 \Rightarrow x = 3\,\text{m}$

REWIND

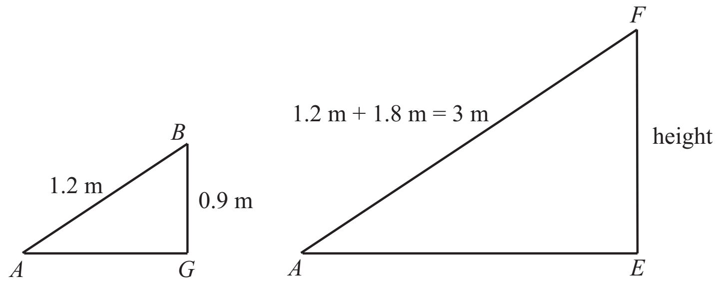
You learned in chapter 2 that the angle sum in a triangle is always 180°. ◀

Worked example 6

The diagram shows a tent that has been attached to the ground using ropes *AB* and *CD*. *ABF* and *DCF* are straight lines. Find the height of the tent.



Consider triangles *ABG* and *AEF*:



Angle *BAG* = *FAE* — Common to both triangles.

Angle *AGB* = *AEF* = 90°

Angle *ABG* = *AFE* — *BG* and *FE* are both vertical, hence parallel lines. Angles correspond.

Therefore triangle *ABG* is similar to triangle *AEF*.

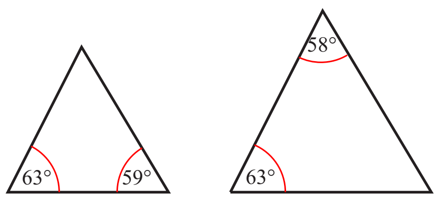
So: $\frac{\text{height}}{0.9} = \frac{3}{1.2} \Rightarrow \text{height} = \frac{0.9 \times 3}{1.2} = 2.25\,\text{m}$

Exercise 11.3

Always look for corresponding sides (sides that join the same angles).

1 For each of the following decide whether or not the triangles are similar in shape. Each decision should be explained fully.

a



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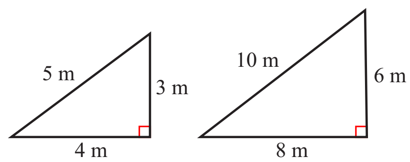

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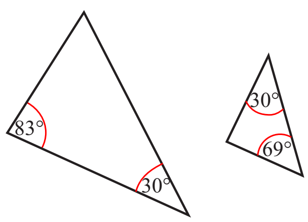

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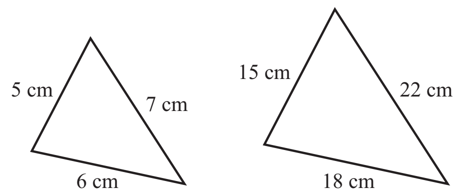

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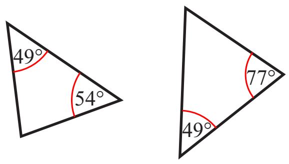

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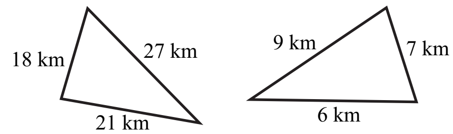

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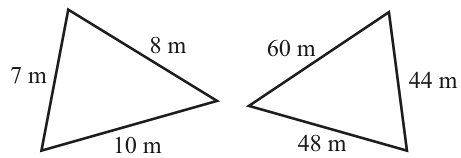

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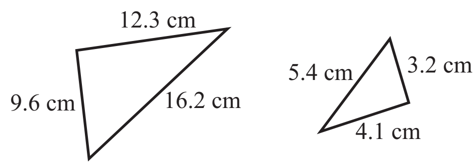

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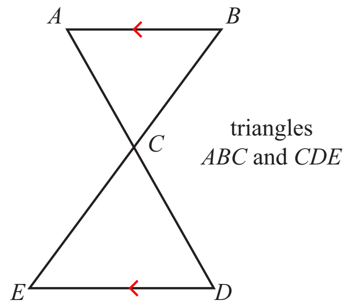

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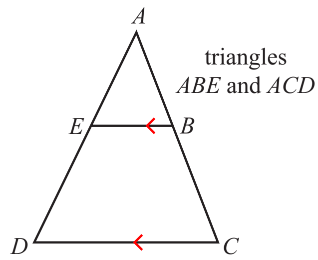
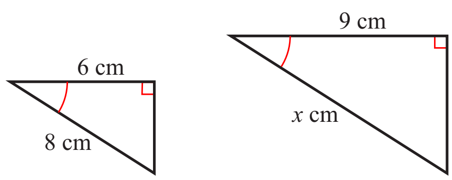


2 The pairs of triangles in this question are similar. Calculate the unknown (lettered) length in each case.

a



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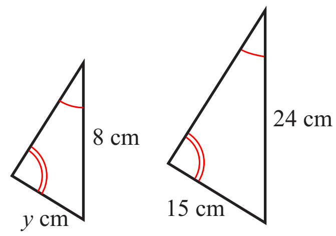

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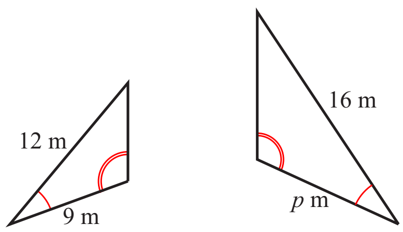

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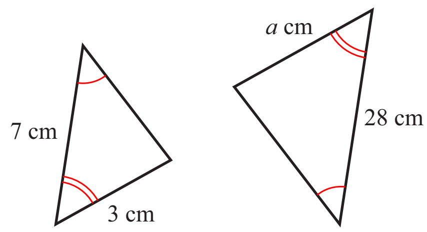
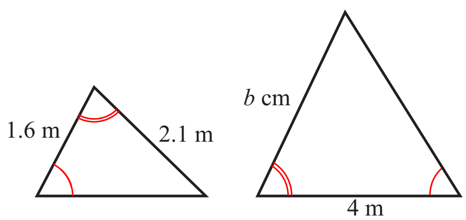


e



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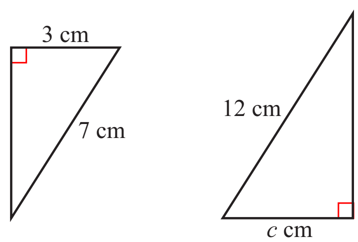
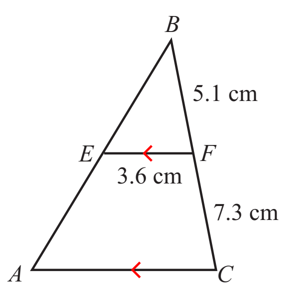
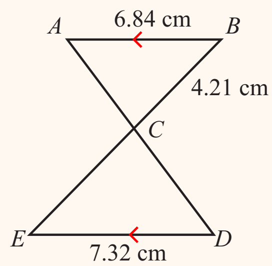


3 The diagram shows triangle ABC. If AC is parallel to EF, find the length of AC.

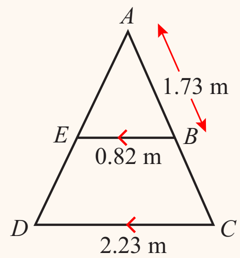


E

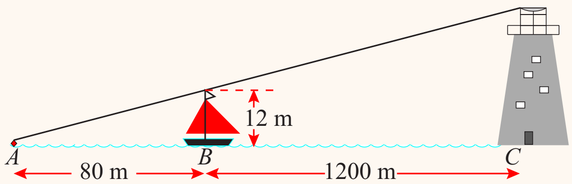
4 In the diagram AB is parallel to DE. Explain why triangle ABC is mathematically similar to triangle CDE and find the length of CE.



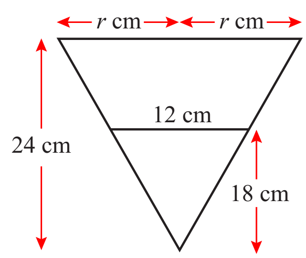
5 The diagram shows a part of a children's climbing frame. Find the length of BC.



6 Swimmer A and boat B, shown in the diagram, are 80 m apart, and boat B is 1200 m from the lighthouse C. The height of the boat is 12 m and the swimmer can just see the top of the lighthouse at the top of the boat's mast when his head lies at sea level. What is the height of the lighthouse?

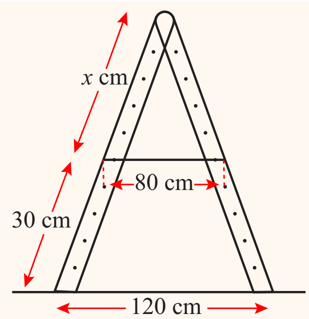


7 The diagram shows a circular cone that has been filled to a depth of 18 cm. Find the radius r of the top of the cone.



E

8 The diagram shows a step ladder that is held in place by an 80 cm piece of wire. Find x.



11. 3 Understanding similar shapes

In the previous section you worked with similar triangles, but any shapes can be similar. A shape is similar if the ratio of corresponding sides is equal and the corresponding angles are equal. Similar shapes are therefore identical in shape, but they differ in size.

You can use the ratio of corresponding sides to find unknown sides of similar shapes just as you did with similar triangles.

LINK

When trying to understand how molecules fit together, chemists will need to have a very strong understanding of shape and space.

Worked example 7

Ahmed has two rectangular flags. One measures 1000 mm by 500 mm, the other measures 500 mm by 350 mm. Are the flags similar in shape?

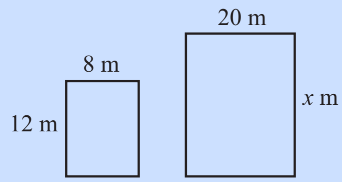
$\frac{1000}{500} = 2$ and $\frac{500}{350} = 1.43$ (Work out the ratio of corresponding sides.)

$\frac{1000}{500} \neq \frac{500}{350}$

The ratio of corresponding sides is not equal, therefore the shapes are not similar.

Worked example 8

Given that the two shapes in the diagram are mathematically similar, find the unknown length x.



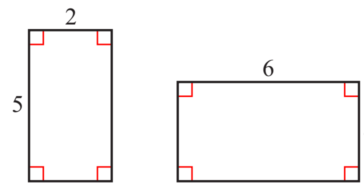
Using the ratios of corresponding sides: $\frac{x}{12} = \frac{20}{8} = 2.5$

$\Rightarrow x = 12 \times 2.5 = 30\text{m}$

Exercise 11.4

1 Establish whether each pair of shapes is similar or not. Show your working.

a



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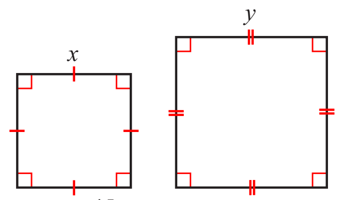

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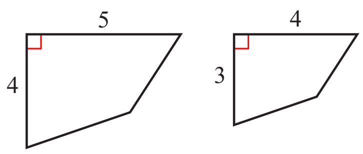

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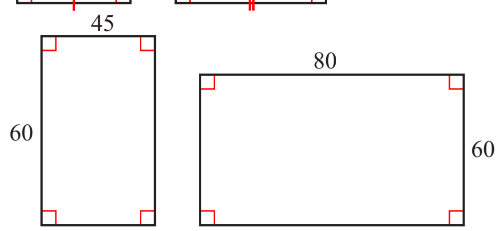

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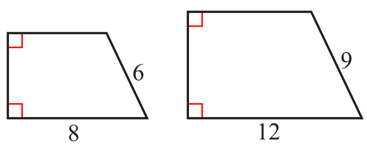

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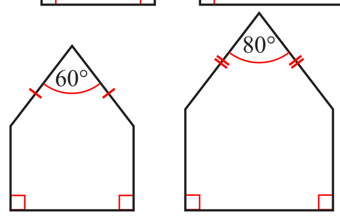
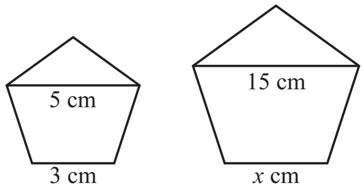


2 In each part of this question the two shapes given are mathematically similar to one another. Calculate the unknown lengths in each case.

a



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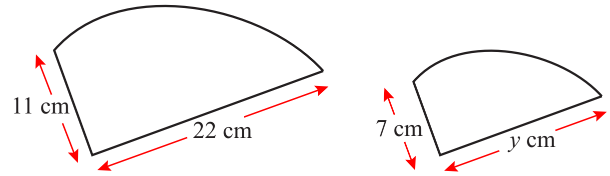

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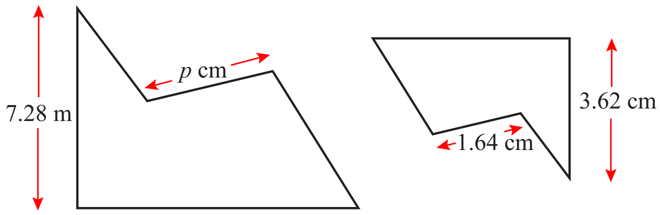

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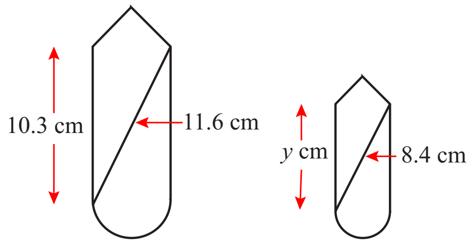

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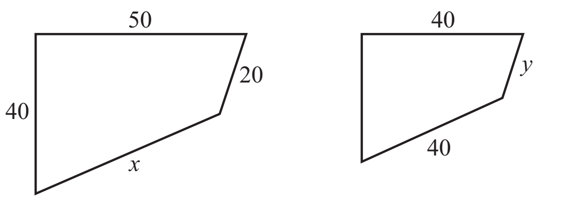

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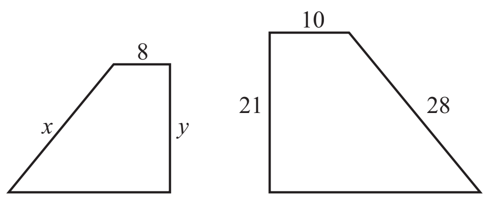

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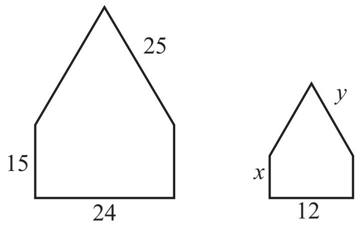

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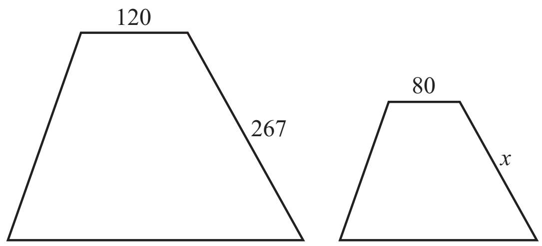
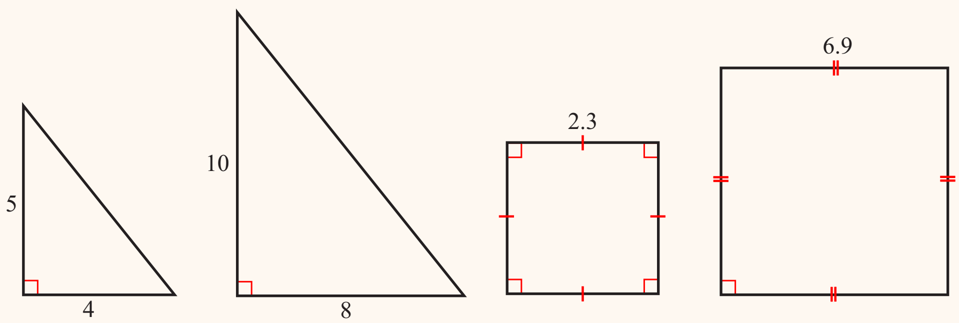


Area of similar shapes

Each pair of shapes below is similar:



Area = 10 Area = 40 Area = 5.29 Area = 47.61

Scale factor $= \frac{10}{5} = 2$ Scale factor $= \frac{6.9}{2.3} = 3$

Area factor $= \frac{40}{10} = 4$ Area factor $= \frac{47.61}{5.29} = 9$

If you look at the diagrams and the dimensions you can see that there is a relationship between the corresponding sides of similar figures and the areas of the figures.

In similar figures where the ratio of corresponding sides is $a : b$, the ratio of areas is $a^2 : b^2$.

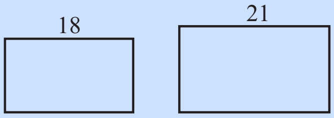
In other words, **scale factor of areas** = (scale factor of lengths)^2

The ratio that compares the measurements of two similar shapes is called the **scale factor**.

E

Worked example 9

These two rectangles are similar. What is the ratio of the smaller area to the larger?



Ratio of sides = 18 : 21

Ratio of areas = $(18)^2 : (21)^2$

$= 324 : 441$

$= 36 : 49$

Worked example 10

Similar rectangles *ABCD* and *MNOP* have lengths in the ratio 3 : 5. If rectangle *ABCD* has area of 900 cm², find the area of *MNOP*.

$$\frac{\text{Area } MNOP}{\text{Area } ABCD} = \frac{5^2}{3^2}$$

$$\frac{\text{Area } MNOP}{900\,\text{cm}^2} = \frac{25}{9}$$

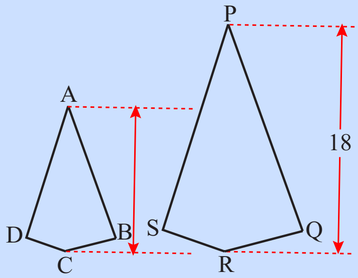
$$\text{Area MNOP} = \frac{25}{9} \times 900$$

$$= 2500\,\text{cm}^2$$

The area of *MNOP* is 2500 cm².

Worked example 11

The shapes below are similar. Given that the area of *ABCD* = 48 cm² and the area of *PQRS* = 108 cm², find the diagonal *AC* in *ABCD*.



Let the length of the diagonal be x cm.

$$\frac{48}{108} = \frac{x^2}{18^2}$$

$$\frac{48}{108} = \frac{x^2}{324}$$

$$\frac{48}{108} \times 324 = x^2$$

$x^2 = 144$

$x = 12$

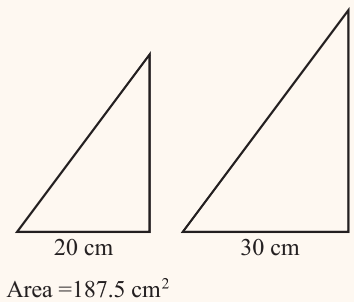
Diagonal *AC* is 12 cm long.

Exercise 11.5

E

1 In each part of this question, the two figures are similar. The area of one figure is given. Find the area of the other.

a



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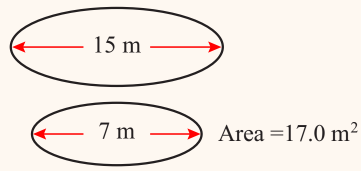

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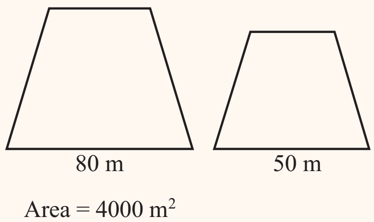

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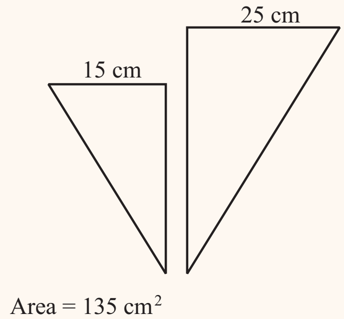
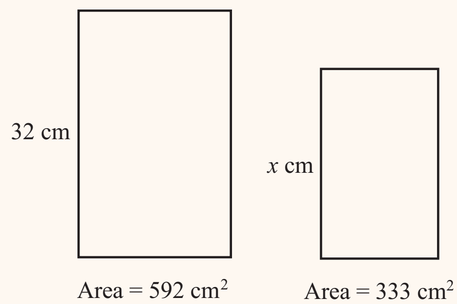


2 In each part of this question the areas of the two similar figures are given. Find the length of the side marked x in each.

a



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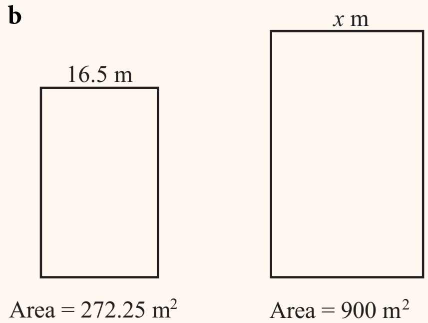

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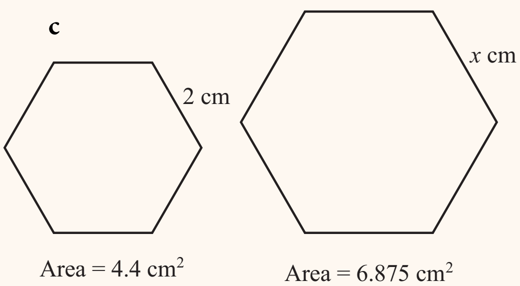

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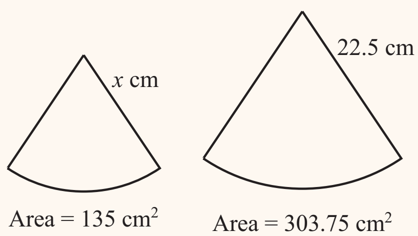


3 Clarissa is making a pattern using a cut out regular pentagon. How will the area of the pentagon be affected if she:

a doubles the lengths of the sides?
b trebles the lengths of the sides?
c halves the lengths of the sides?

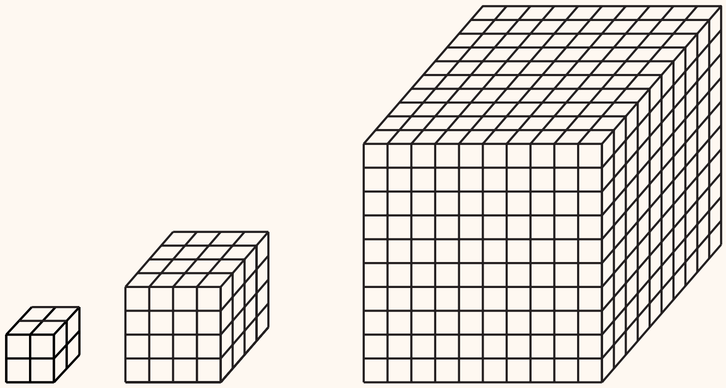
4 If the areas of two similar quadrilaterals are in the ratio 64 : 9, what is the ratio of matching sides?

Similar solids

Three-dimensional shapes (solids) can also be similar.

Similar solids have the same shape, their corresponding angles are equal and all corresponding linear measures (edges, diameters, radii, heights and slant heights) are in the same ratio. As with similar two-dimensional shapes, the ratio that compares the measurements on the two shapes is called the scale factor.

Volume and surface area of similar solids



The following table shows the side length and volume of each of the cubes above.

× 5

× 2

Length of side (units)	2	4	10
Volume (units^3)	$2\times2\times2=8$	$4\times4\times4=64$	$10\times10\times10=1000$

× 8

× 125

Notice that when the side length is multiplied by 2 the volume is multiplied by $2^3 = 8$

Here, the **scale factor of lengths** is 2 and the **scale factor of volumes** is 2^3.

Also, when the side length is multiplied by 5 the volume is multiplied by $5^3 = 125$.

This time the scale factor of lengths is 5 and the scale factor of volumes is 5^3.

In fact this pattern follows in the general case:

$$\boxed{\text{scale factor of volumes} = (\text{scale factor of lengths})^3}$$

By considering the surface areas of the cubes you will also be able to see that the rule from page 221 is still true:

$$\boxed{\text{scale factor of areas} = (\text{scale factor of lengths})^2}$$

In summary, if two solids (A and B) are similar:

- the ratio of their volumes is equal to the cube of the ratio of corresponding linear measures (edges, diameter, radii, heights and slant heights). In other words: Volume A ÷ Volume $B = \left(\frac{a}{b}\right)^3$
- the ratio of their surface areas is equal to the square of the ratio of corresponding linear measures. In other words: Surface area A ÷ Surface area $B = \left(\frac{a}{b}\right)^2$

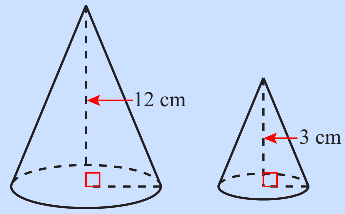
The following worked examples show how these scale factors can be used.

Sometimes you are given the scale factor of areas or volumes rather than starting with the scale factor of lengths. Use square roots or cube roots to get back to the scale factor of lengths as your starting point.

E

Worked example 12

The cones shown in the diagram are mathematically similar. If the smaller cone has a volume of 40 cm³ find the volume of the larger cone.



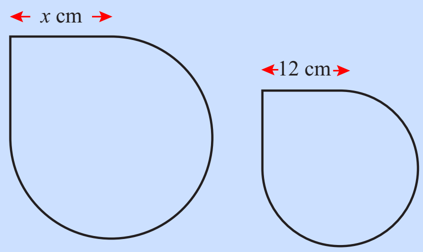
Scale factor of lengths $= \frac{12}{3} = 4$

$\Rightarrow$ Scale factor of volumes $= 4^3 = 64$

So the volume of the larger cone $= 64 \times 40 = 2560\,\text{cm}^3$

Worked example 13

The two shapes shown in the diagram are mathematically similar. If the area of the larger shape is $216\,\text{cm}^2$, and the area of the smaller shape is $24\,\text{cm}^2$, find the length x in the diagram.



Scale factor of areas $= \frac{216}{24} = 9$

$\Rightarrow$ (Scale factor of lengths)$^2 = 9$

$\Rightarrow$ Scale factor of lengths $= \sqrt{9} = 3$

So: $x = 3 \times 12 = 36$ cm

Worked example 14

A shipping crate has a volume of 2000 cm³. If the dimensions of the crate are doubled, what will its new volume be?

$$\frac{\text{Original volume}}{\text{New volume}} = \left(\frac{\text{original dimensions}}{\text{new dimensions}}\right)^3$$

$$\frac{2000}{\text{New volume}} = \left(\frac{1}{2}\right)^3$$

$$\frac{2000}{\text{New volume}} = \frac{1}{8}$$

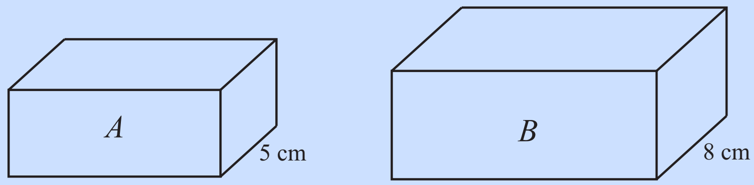
New volume $= 2000 \times 8$

New volume $= 16\,000\,\text{cm}^3$

E

Worked example 15

The two cuboids A and B are similar. The larger has a surface area of 608 cm². What is the surface area of the smaller?



$$\frac{\text{Surface area } A}{\text{Surface area } B} = \left(\frac{\text{width } A}{\text{width } B}\right)^2$$

$$\frac{\text{Surface area } A}{608} = \left(\frac{5}{8}\right)^2$$

$$\frac{\text{Surface area } A}{608} = \frac{25}{64}$$

$$\text{Surface area } A = \frac{25}{64} \times 608$$

Surface area A = 237.5 cm²

Cuboid A has a surface area of 237.5 cm²

Exercise 11.6

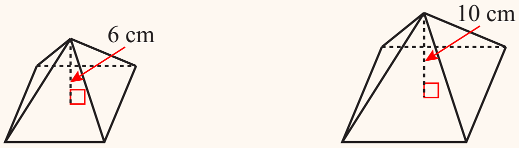
1 Copy and complete the statement.

When the dimensions of a solid are multiplied by k, the surface area is multiplied by __ and the volume is multiplied by __.

2 Two similar cubes A and B have sides of 20 cm and 5 cm respectively.

- **a** What is the scale factor of A to B?
- **b** What is the ratio of their surface areas?
- **c** What is the ratio of their volumes?

3 Pyramid A and pyramid B are similar. Find the surface area of pyramid A.



Surface area = 600 cm²

4 Yu has two similar cylindrical metal rods. The smaller rod has a diameter of 4 cm and a surface area of 110 cm². The larger rod has a diameter of 5 cm. Find the surface area of the larger rod.

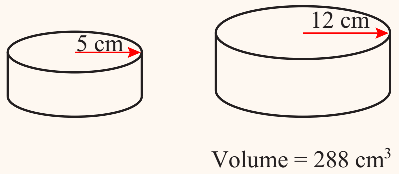
5 Cuboid X and cuboid Y are similar. The scale factor X to Y is $\frac{3}{4}$.

- **a** If a linear measure in cuboid X is 12 mm, what is the length of the corresponding measure on cuboid Y?
- **b** Cuboid X has a surface area of 88.8 cm². What is the surface area of cuboid Y?
- **c** If cuboid X has a volume of 35.1 cm³, what is the volume of cuboid Y?

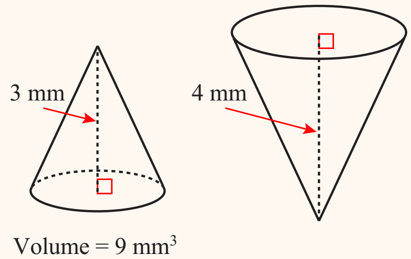
E

6 For each part of this question, the solids are similar. Find the unknown volume.

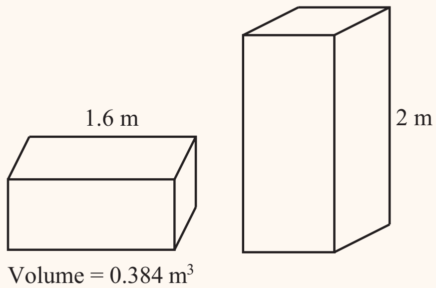
a



b



c



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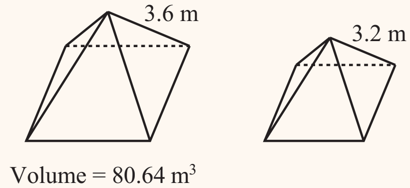
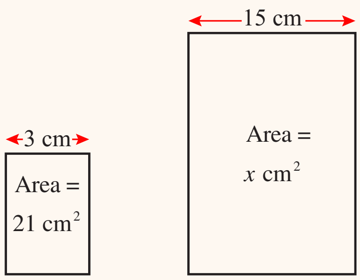
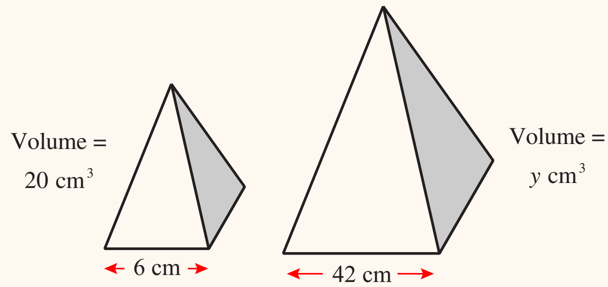


7 Find the unknown quantity for each of the following pairs of similar shapes.

a



b



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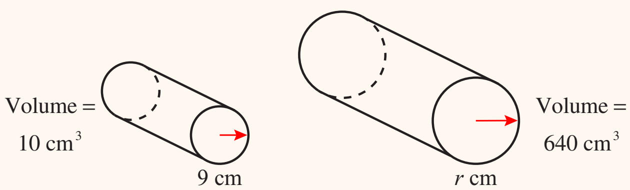

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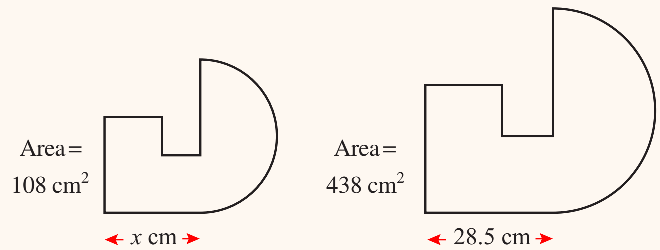


Applying your skills

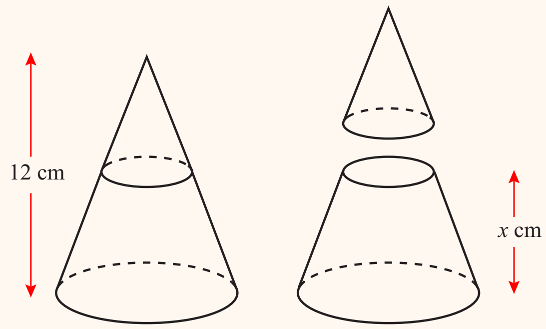
Organised tables and lists are a useful problem-solving strategy. Include headings for rows and/or columns to make sure your table is easy to understand.

8 Karen has this set of three Russian dolls. The largest doll is 13 cm tall, the next one is 2 cm shorter and the third one is 4 cm shorter. Draw up a table to compare the surface area and volume of the three dolls in algebraic terms.

A cone cut in this way produces a smaller cone and a solid called a frustum.

9 A manufacturer is making pairs of paper weights from metal cones that have been cut along a plane parallel to the base. The diagram shows a pair of these weights.

If the volume of the larger (uncut) cone is 128 cm³ and the volume of the smaller cone cut from the top is 42 cm³ find the length x.

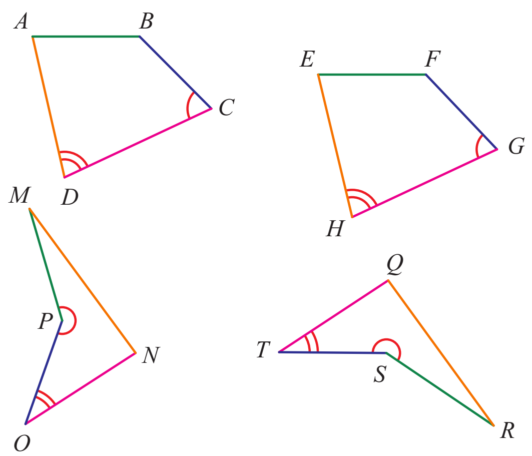


11.4 Understanding congruence

If two shapes are congruent, we can say that:

- corresponding sides are equal in length
- corresponding angles are equal
- the shapes have the same area.

Look at these pairs of congruent shapes. The corresponding sides and angles on each shape are marked using the same colours and symbols.



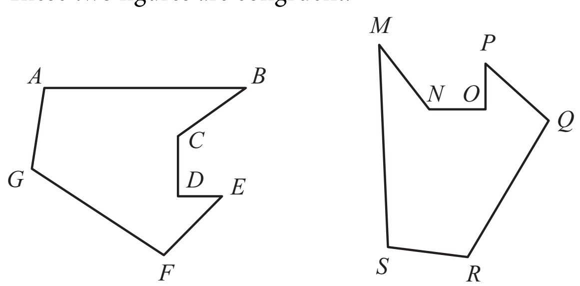
When you make a congruency statement, you name the shape so that corresponding vertices are in the same order.

For the shapes above, we can say that

- *ABCD* is congruent to *EFGH*, and
- *MNOP* is congruent to *RQTS*

Exercise 11.7

1 These two figures are congruent.



a Which side is equal in length to:

i *AB* **ii** *EF* **iii** *MN*

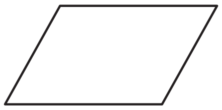
b Which angle corresponds with:

i *BAG* **ii** *PQR* **iii** *DEF*

c Write a congruency statement for the two figures.

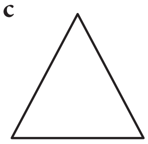
2 Which of the shapes in the box are congruent to each shape given below? Measure sides and angles if you need to.

a

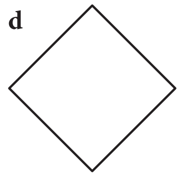


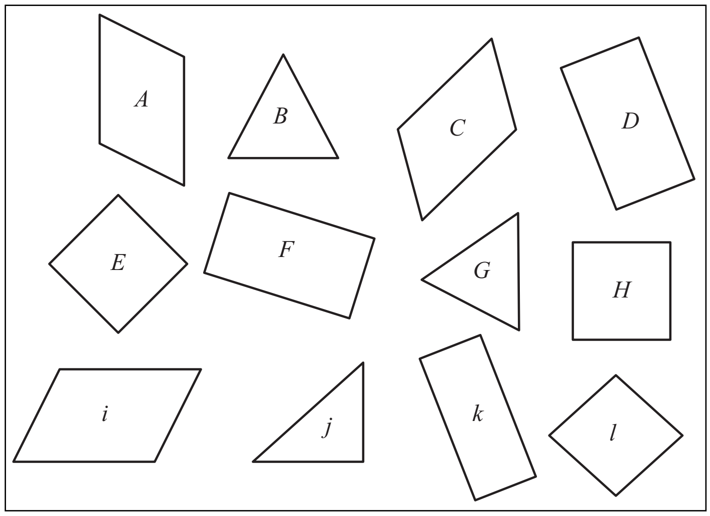
b

c



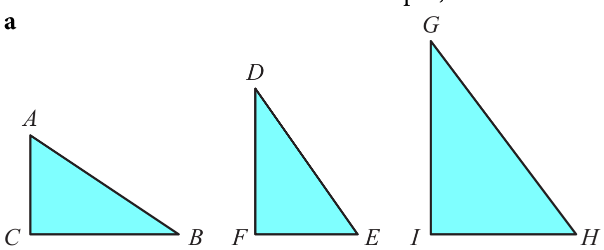
d



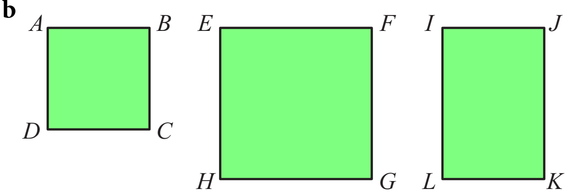


3 For each set of shapes, state whether any shapes are congruent or similar.

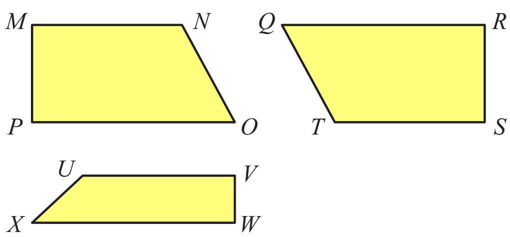
a



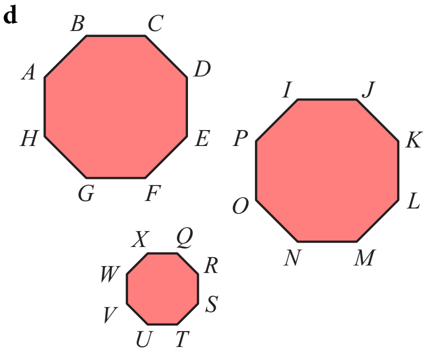
b



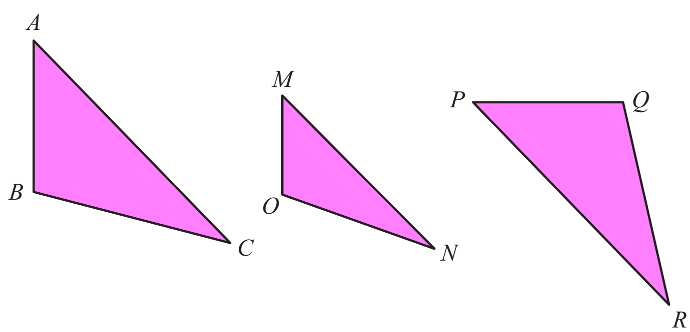
c



d

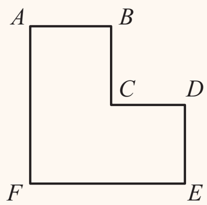


e



E

4 Figure *ABCDEF* has *AB* = *BC* = *CD* = *DE*.

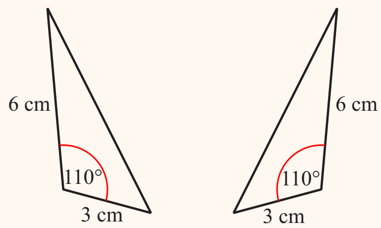


Redraw the figure and show how you could split it into:

a two congruent shapes **b** three congruent shapes
c four congruent shapes

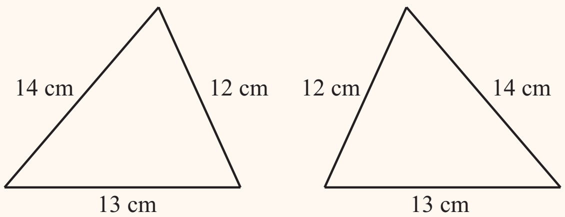
Congruent triangles

Triangles are congruent to each other if the following conditions are true.



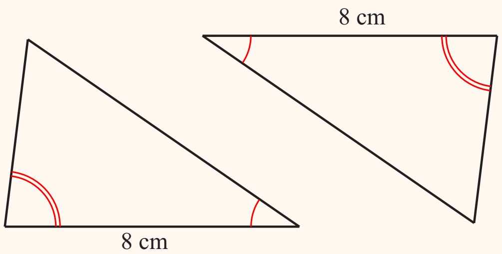
Two sides and the **included angle** (this is the angle that sits between the two given sides) are equal.

This is remembered as SAS – ***S****ide* ***A****ngle* ***S****ide*.



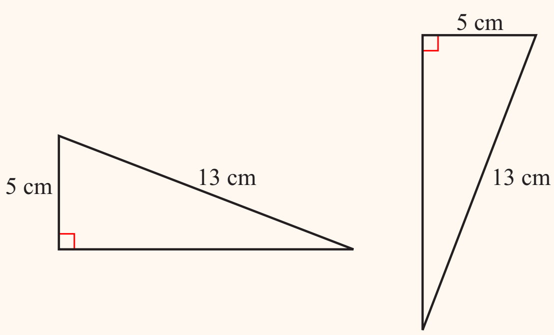
There are three pairs of equal sides.

Remember this as SSS – ***S****ide* ***S****ide* ***S****ide*.



Two angles and the **included side** (the included side is the side that is placed between the two angles) are equal.

Remember this as ASA – ***A****ngle* ***S****ide* ***A****ngle*.



If you have right-angled triangles, the angle does not need to be included for the triangles to be congruent. The triangles must have the same length of hypotenuse and one other side equal.

Remember this as RHS – ***R****ight-angle* ***S****ide* ***H****ypotenuse*.

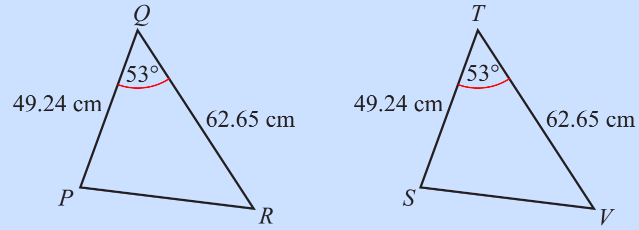
If any one of these conditions is satisfied then you have two congruent triangles.

E

Worked example 16

For each of the following pairs of triangles, show that they are congruent.

a



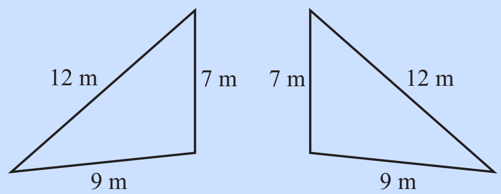
a Length PQ = Length ST

$P\hat{Q}R = S\hat{T}V$

Length QR = Length TV

So the condition is SAS and the triangles are congruent.

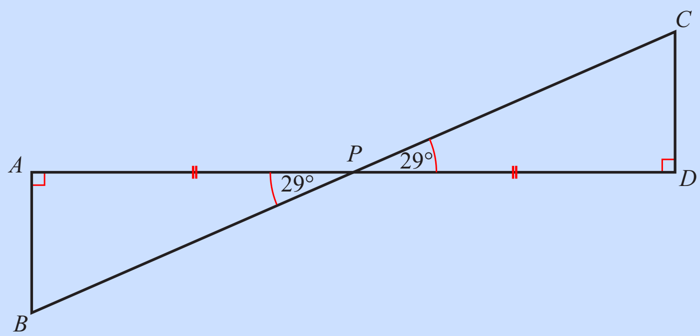
b



b There are three pairs of equal sides.

So the condition is SSS and the triangles are congruent.

c



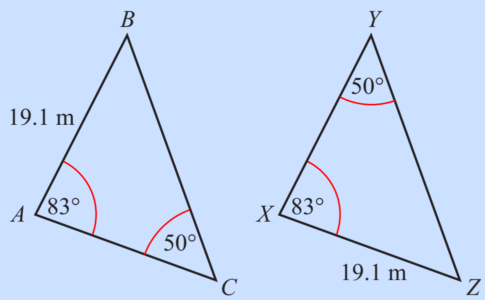
c Angle BAP = CDP (both are right angles)

Angle AP = PD (given on diagram)

Angle APB = CPD (vertically opposite)

So the condition is ASA and the triangles are congruent.

d



d Angle BAC = YXZ = 83°

Angle BCA = ZYX = 50°

Angle ABC = XZY = 47° (angles in a triangle)

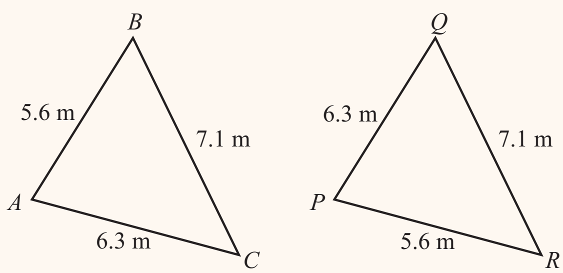
Length AB = Length XZ

So the condition is ASA and the triangles are congruent.

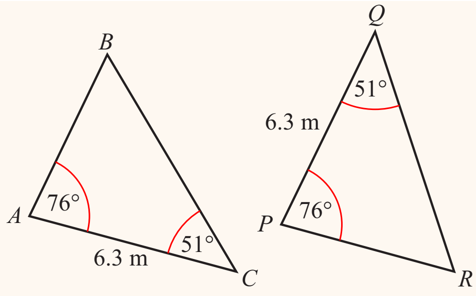
Exercise 11.8

For each question show that the pair of shapes are congruent to one another. Explain each answer carefully and state clearly which of SAS, SSS, ASA or RHS you have used.

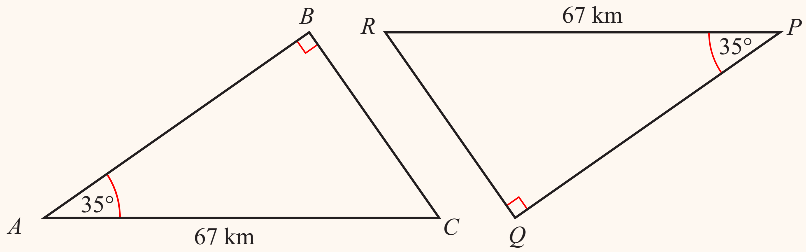
1



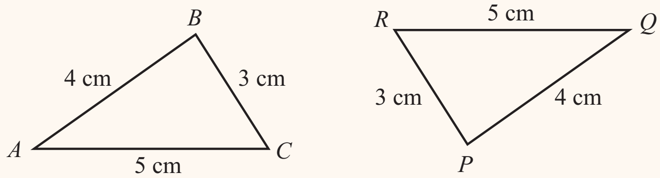
2


B
A
C
76°
51°
6.3 m
Q
P
R
51°
76°
6.3 m

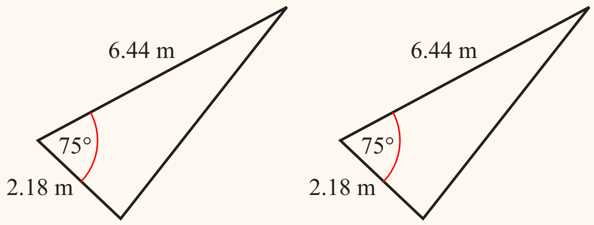
3


B
A
C
35°
67 km
R
P
Q
67 km
35°

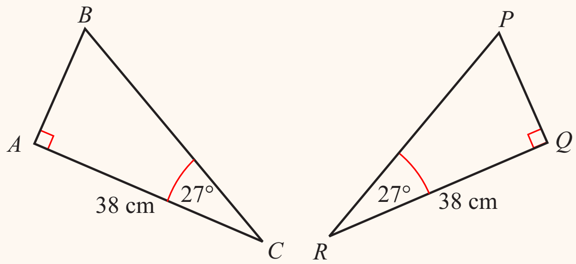
4


B
A
C
4 cm
3 cm
5 cm
R
Q
P
5 cm
3 cm
4 cm

5

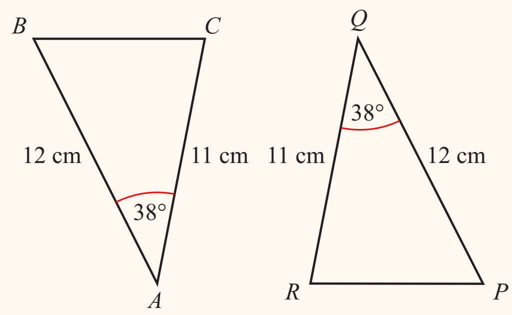

6.44 m
75°
2.18 m
6.44 m
75°
2.18 m

6

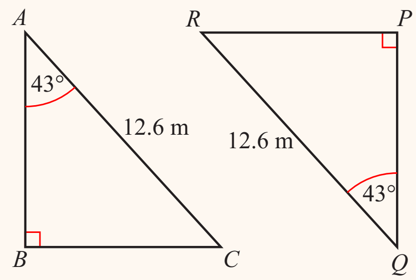

B
A
C
27°
38 cm
P
Q
R
27°
38 cm

E

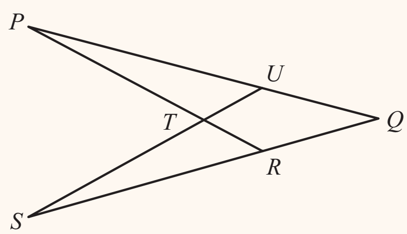
7



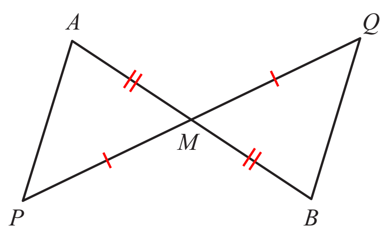
8



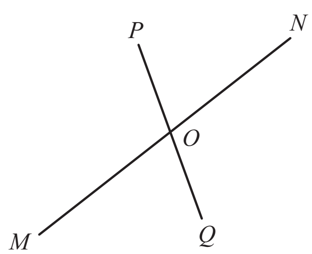
9 In the figure, $PR = SU$ and $RTUQ$ is a kite. Prove that triangle PQR is congruent to triangle SQU.



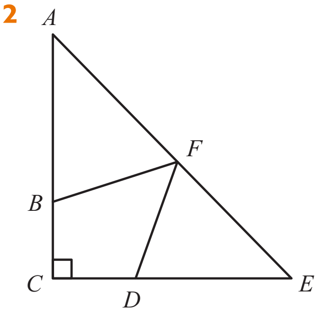
10 In the diagram, $AM = BM$ and $PM = QM$. Prove that AP // QB.



11 Two airstrips PQ and MN bisect each other at O, as shown in the diagram.



Prove that $PM = NQ$

12 

Triangle *FAB* is congruent to triangle *FED*. Prove that *BFDC* is a kite.

Summary

Do you know the following?

- The longest side of a right-angled triangle is called the hypotenuse.
- The square of the hypotenuse is equal to the sum of the squares of the two shorter sides of the triangle.
- Similar shapes have equal corresponding angles and the ratios of corresponding sides are equal.
- If shapes are similar and the lengths of one shape are multiplied by a scale factor of n:
  - then the areas are multiplied by a scale factor of n^2
  - and the volumes are multiplied by a scale factor of n^3. E
- Congruent shapes are exactly equal to each other.
- There are four sets of conditions that can be used to test for congruency in triangles. If one set of conditions is true, the triangles are congruent. E

Are you able to …?

- use Pythagoras' theorem to find an unknown side of a right-angled triangle
- use Pythagoras' theorem to solve real-life problems
- decide whether or not two objects are mathematically similar
- use the fact that two objects are similar to calculate:
  - unknown lengths
  - areas or volumes E
- decide whether or not two shapes are congruent.
- test for congruency in triangles. E

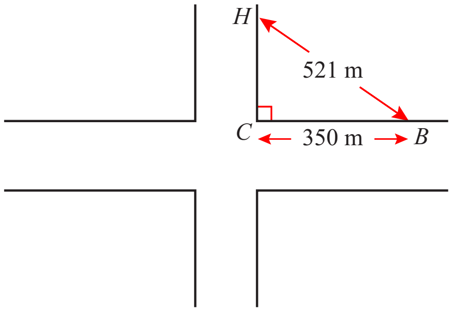
Examination practice

Exam-style questions

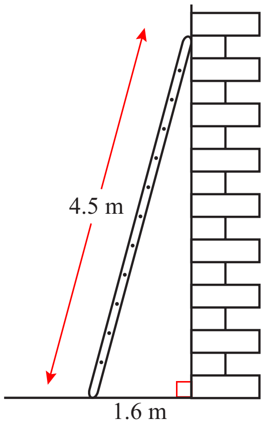
1 Mohamed takes a short cut from his home (H) to the bus stop (B) along a footpath HB.

How much further would it be for Mohamed to walk to the bus stop by going from H to the corner (C) and then from C to B?

Give your answer in metres.

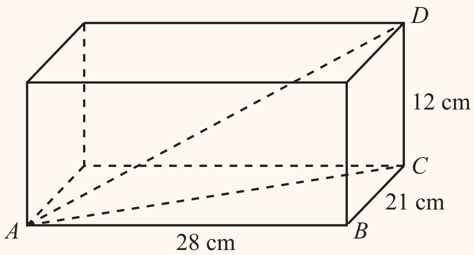


2 A ladder is standing on horizontal ground and rests against a vertical wall. The ladder is 4.5 m long and its foot is 1.6 m from the wall. Calculate how far up the wall the ladder will reach. Give your answer correct to 3 significant figures.



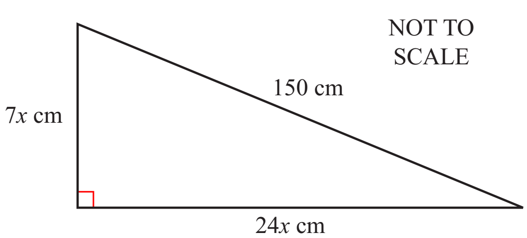
3 A rectangular box has a base with internal dimensions 21 cm by 28 cm, and an internal height of 12 cm. Calculate the length of the longest straight thin rod that will fit:

a on the base of the box
b in the box.



E

4

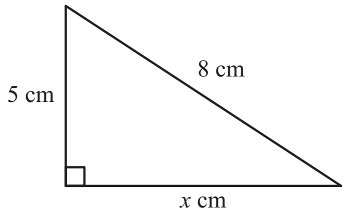


The right-angled triangle in the diagram has sides of length $7x$ cm , $24x$ cm and 150 cm.

a Show that $x^2 = 36$
b Calculate the perimeter of the triangle.

Past paper questions

1

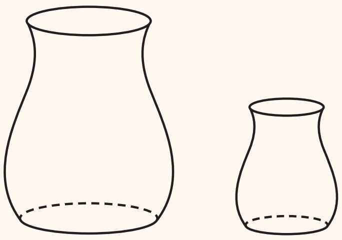


NOT TO SCALE

Calculate the value of x. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q11 October/November 2015]

2 E



NOT TO SCALE

The two containers are mathematically similar in shape.

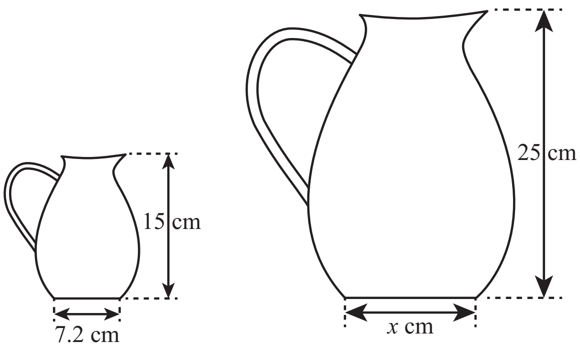
The larger container has a volume of $3456\,cm^3$ and a surface area of $1024\,cm^2$.

The smaller container has a volume of $1458\,cm^3$.

Calculate the surface area of the smaller container. [4]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q18 May/June 2014]

3



NOT TO SCALE

The diagram shows two jugs that are mathematically similar.

Find the value of x. [2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q21 October/November 2015]