Chapter 15: Scale drawings, bearings and trigonometry

Key words

- Scale drawing
- Bearing
- Hypotenuse
- Adjacent
- Opposite
- Tangent ratio
- Inverse function
- Sine ratio
- Cosine ratio
- Sine rule
- Cosine rule
- Projection

In this chapter you will learn how to:

- make scale drawings
- interpret scale drawings
- calculate bearings
- calculate sine, cosine and tangent ratios for right-angled triangles
- use sine, cosine and tangent ratios to calculate the lengths of sides and angles of right-angled triangles

EXTENDED

- solve trigonometric equations finding all the solutions between 0° and 360°
- apply the sine and cosine rules to calculate unknown sides and angles in triangles that are not right-angled
- calculate the area of a triangle that is not right-angled using the sine ratio
- use the sine, cosine and tangent ratios, together with Pythagoras' theorem in three-dimensions.

A full understanding of how waves strengthen or destroy one another can help to save countless lives. Such an understanding begins with the study of trigonometry.

To 'get your bearings' is to find out the direction you need to move from where you are. Sat Navs and the GPS can take a lot of the effort out of finding where you are going but their software uses the basic mathematical principles of calculating angles.

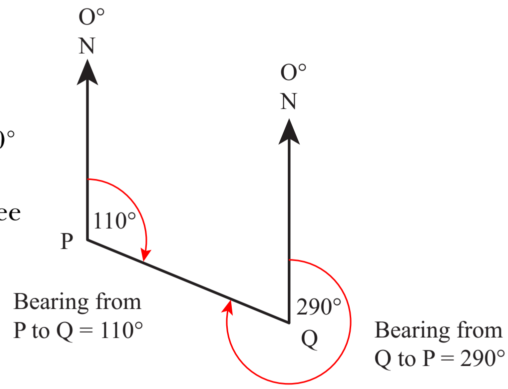
RECAP

You should already be familiar with the following work on bearings and scale drawings:

Bearings (Year 9 Mathematics, Chapter 3)

Bearings are measured from north (0°) in a clockwise direction to 360° (which is the same bearing as north).

You measure bearings using your protractor and write them using three digits, so you would write a bearing of 88 degrees as 088°.



Scale diagrams (Year 9 Mathematics)

Accurately reduced (or enlarged) diagrams are called scale diagrams.

The scale by which the diagram is reduced (or enlarged) can be given as a fraction or a ratio.

For example, $\frac{1}{200}$ or 1 : 200.

The scale factor tells you how much smaller (or larger) the diagram is than the reality it represents.

15.1 Scale drawings

Later in this chapter you will be learning how to use the trigonometric ratios to accurately calculate missing angles and sides. For this you will need the use of a calculator. Missing lengths and angles can also be found using **scale drawings**; although this is less accurate, it is still valid. For scale drawings you will need a ruler, a protractor and a sharp pencil.

Sometimes you have to draw a diagram to represent something that is much bigger than you can fit on the paper or so small that it would be very difficult to make out any detail. Examples include a plan of a building, a map of a country or the design of a microchip. These accurate diagrams are called a scale drawings.

The lines in the scale drawing are all the same fraction of the lines they represent in reality. This fraction is called the *scale* of the drawing.

The scale of a diagram, or a map, may be given as a fraction or a ratio such as $\frac{1}{50000}$ or 1 : 50 000.

A scale of $\frac{1}{50000}$ means that every line in the diagram has a length which is $\frac{1}{50000}$ of the length of the line that it represents in real life. Hence, 1 cm in the diagram represents 50 000 cm in real life. In other words, 1 cm represents 500 m or 2 cm represents 1 km.

REWIND

Some of the construction skills from chapter 3 will be useful for scale drawings. ◀

LINK

Scale drawings are often used to plan the production of items in design technology subjects. Many problems involving the fitting together of different shapes can be solved by using a good quality scale drawing. Maps in geography are also scale drawings and enable us to represent the real world in a diagram of manageable size.

LINK

Draughtspeople, architects and designers all draw accurate scaled diagrams of buildings and other items using pencils, rulers and compasses.

Worked example 1

A rectangular field is 100 m long and 45 m wide. A scale drawing of the field is made with a scale of 1 cm to 10 m. What are the length and width of the field in the drawing?

10 m is represented by 1 cm

$\therefore$ 100 m is represented by $(100 \div 10)$ cm = 10 cm
and 45 m is represented by $(45 \div 10)$ cm = 4.5 cm

So, the dimensions on the drawing are: length = 10 cm and width = 4.5 cm.

Exercise 15.1

1. On the plan of a house, the living room is 3.4 cm long and 2.6 cm wide. The scale of the plan is 1 cm to 2 m. Calculate the actual length and width of the room.

2. The actual distance between two villages is 12 km. Calculate the distance between the villages on a map whose scale is:

   a 1 cm to 4 km **b** 1 cm to 5 km.

3. A car ramp is 28 m long and makes an angle of 15° with the horizontal. A scale drawing is to be made of the ramp using a scale of 1 cm to 5 m.

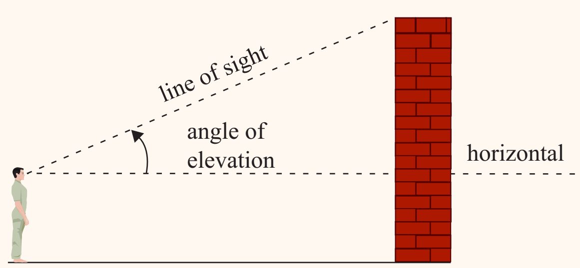
   a How long will the ramp be on the drawing?
   b What angle will the ramp make with the horizontal on the drawing?

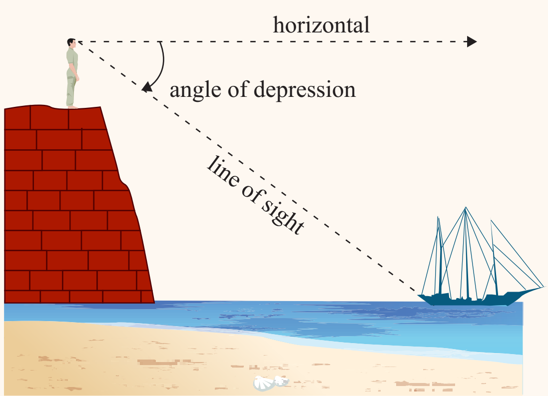
E

Angles of elevation are *always* measured from the *horizontal*.

Angle of elevation and angle of depression

Scale drawing questions often involve the observation of objects that are higher than you or lower than you, for example, the top of a building, an aeroplane or a ship in a harbour. In these cases, the angle of elevation or depression is the angle between the horizontal and the line of sight of the object.





Drawing a scale diagram

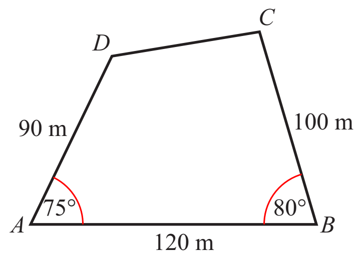
Here are some clues and tips for drawing diagrams to scale:

- Draw a rough sketch, showing all details as given in the question.
- If you are told to use a particular scale you must use it! If you are not given a scale try to choose one that will make your diagram fit neatly onto a page.
- Make a clean, tidy and accurate scale drawing using appropriate geometrical instruments. Show on it the given lengths and angles. Write the scale next to the drawing.
- Measure lengths and angles in the drawing to find the answers to the problem. Remember to change the lengths to *full size* using the scale. Remember that the full size angles are the same as the angles in the scale drawing.

A scale drawing is *similar* to the real object, so the sides are in proportion and corresponding angles are equal.

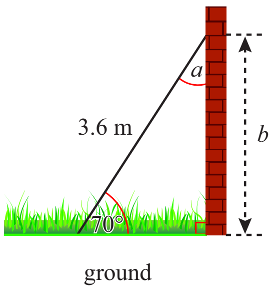
Exercise 15.2

1 The diagram is a rough sketch of a field *ABCD*.



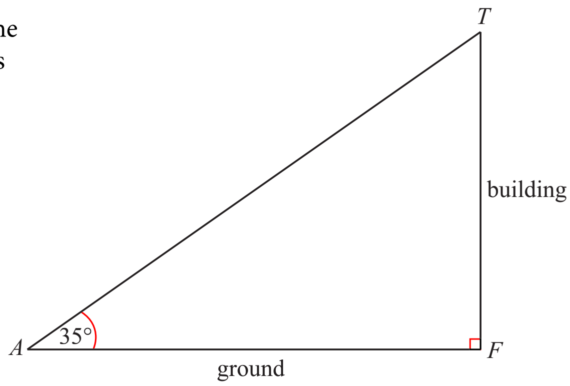
- **a** Using a scale of 1 cm to 20 m, make an accurate scale drawing of the field.
- **b** Find the sizes of $B\hat{C}D$ and $A\hat{D}C$ at the corners of the field.
- **c** Find the length of the side *CD* of the field.

2 A ladder of length 3.6 m stands on horizontal ground and leans against a vertical wall at an angle of 70° to the horizontal (see diagram).



- **a** What is the size of the angle that the ladder makes with the wall (*a*)?
- **b** Draw a scale drawing using a scale of 1 cm to 50 cm, to find how far the ladder reaches up the wall (*b*).

3 The *accurate* scale diagram represents the vertical wall *TF* of a building that stands on horizontal ground. It is drawn to a scale of 1 cm to 8 m.



- **a** Find the height of the building.
- **b** Find the distance from the point *A* to the foot (*F*) of the building.
- **c** Find the angle of elevation of the top (*T*) of the building from the point *A*.

15.2 Bearings

FAST FORWARD

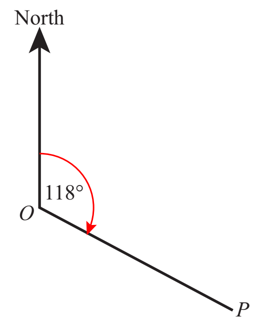
One degree of bearing does not seem like a lot but it can represent a huge distance in the real world. This is why you will need to use the trigonometry you will learn later in this chapter to calculate angles accurately. ▶

You have now used scale drawings to find distances between objects and to measure angles. When you want to move from one position to another, you not only need to know how far you have to travel but you need to know the direction. One way of describing directions is the **bearing**. This description is used around the world.

The angle 118°, shown in the diagram, is measured clockwise from the north direction. Such an angle is called a bearing.

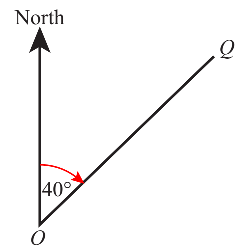
All bearings are measured clockwise from the north direction.

Here the bearing of P from O is 118°.



If the angle is less than 100° you still use three figures so that it is clear that you mean to use a bearing.

Here the bearing of Q from O is 040°.

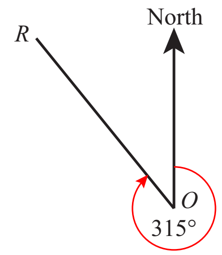


REWIND

You saw in chapter 3 that a reflex angle is >180° but < 360°. ◀

Since you *always* measure clockwise from north it is possible for your bearing to be a reflex angle.

Here the bearing of R from O is 315°.



You may sometimes need to use angle properties from previous chapters to solve bearings problems.

Always make sure that you draw a clear diagram and mark all north lines clearly.

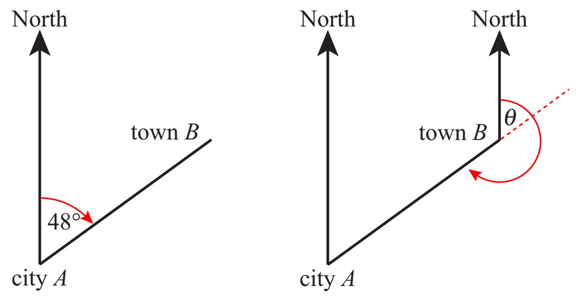
REWIND

You should remind yourself how to deal with alternate and corresponding angles from chapter 3. ◀

This is an example of a 'back' bearing. If you know the bearing of point X from point Y then, to find the bearing to return to point Y from point X, you add 180° to the original bearing (or subtract 180° if adding would give a value greater than 360°).

Worked example 2

The bearing of town B from city A is 048°. What is the bearing of city A from town B?



In the second diagram, the two north lines are parallel. Hence angle $\theta = 48°$ (using the properties of corresponding angles).

The bearing of city A from town B = 48°+180° = 228°.

Notice that the difference between the two bearings (48° and 228°) is 180°.

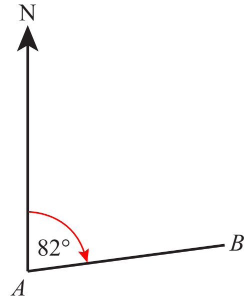
Exercise 15.3

1 Give the three-figure bearing corresponding to:

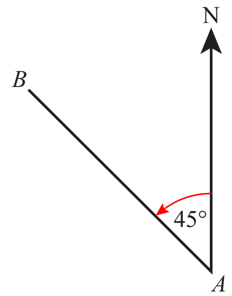
a west **b** south-east **c** north-east

2 Write down the three figure bearings of A from B for each of the following:

a



b



3 Use the map of Southern Africa to find the three-figure bearing of:

a Johannesburg from Windhoek
b Johannesburg from Cape Town
c Cape Town from Johannesburg
d Lusaka from Cape Town
e Kimberley from Durban.

4 Townsville is 140 km west and 45 km north of Beeton. Using a scale drawing with a scale of 1 cm to 20 km, find:

a the bearing of Beeton from Townsville
b the bearing of Townsville from Beeton
c the direct distance from Beeton to Townsville.

5 Village Q is 7 km from village P on a bearing of 060°. Village R is 5 km from village P on a bearing of 315°. Using a scale drawing with a scale of 1 cm to 1 km, find:

a the direct distance from village Q to village R
b the bearing of village Q from village R.

15.3 Understanding the tangent, cosine and sine ratios

Trigonometry is the use of the ratios of the sides of right-angled triangles. The techniques covered in the following sections will help you to make much more precise calculations with bearings.

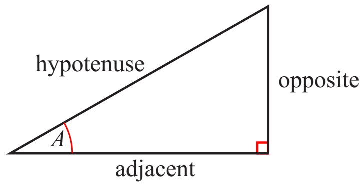
Throughout the remainder of this chapter you must make sure that your calculator is set in degrees mode. A small letter '*D*' will usually be displayed. If this is not the case, or if your calculator displays a '*G*' or an '*R*', then please consult your calculator manual.

REWIND

The hypotenuse was introduced with the work on Pythagoras' theorem in chapter 11. ◀

Labelling the sides of a right-angled triangle

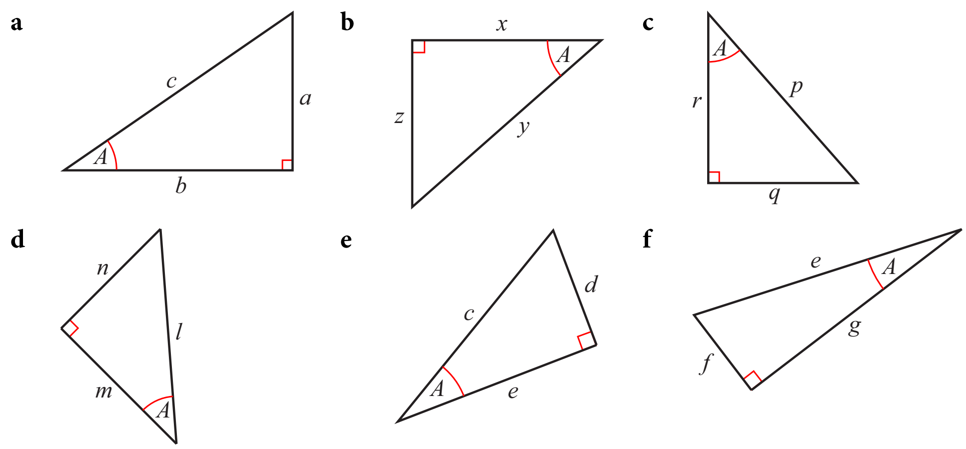
You will have already learned that the longest side of a right-angled triangle is called the **hypotenuse**. If you take one of the two non right-angles in the triangle for reference then you can also 'name' the two shorter sides:



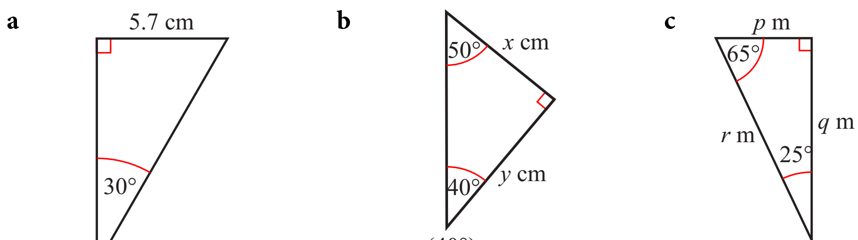
Notice that the **adjacent** is the side of the triangle that touches the angle A, but is not the hypotenuse. The third side does not meet with angle A at all and is known as the **opposite.** Throughout the remainder of the chapter, opp(A) will be used to mean the length of the opposite side, and adj(A) to mean the length of the adjacent. The hypotenuse does not depend upon the position of angle A, so is just written as 'hypotenuse' (or hyp).

Exercise 15.4

1 For each of the following triangles write down the letters that correspond to the length of the hypotenuse and the values of opp(A) and adj(A).



2 In each case copy and complete the statement written underneath the triangle.



a opp(30°) = ... cm

b opp(40°) = ... cm
adj(50°) = ... cm

c ....(65°) = q m
.......(25°) = p m
.... = r m

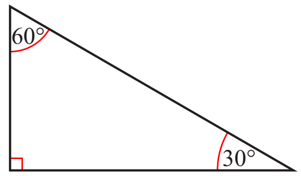
Investigation

REWIND

You will need to use the skills you learned for constructing accurate drawings of triangles in chapter 3. ◀

You will now explore the relationship between the opposite, adjacent and hypotenuse and the angles in a right-angled triangle.

For this investigation you will need to draw four *different* scale copies of the diagram opposite. The right angle and 30° angle must be drawn as accurately as possible, and all four triangles should be of different sizes. Follow the instructions listed on the next page.



1 Label the opp(30°), adj(30°) and hypotenuse clearly.
2 Measure the length of opp(30°) and write it down.
3 Measure the length of the adj(30°) and write it down.
4 Calculate $\frac{\text{opp}(30°)}{\text{adj}(30°)}$ in each case.
5 What do you notice about your answers?
6 Ask a friend to draw some triangles (with the same angles) and make the same calculations. What do you notice?
7 Now repeat the investigation using a triangle with different angles. Record any observations that you make.

REWIND

Look back at the work on calculating gradients in chapter 10 and compare it with the tangent ratio. What connection do you notice?

Tangent ratio

It turns out that $\frac{\text{opp}(A)}{\text{adj}(A)}$ is constant for any given angle A. $\frac{\text{opp}(A)}{\text{adj}(A)}$ depends on the angle only, and not the actual size of the triangle. The ratio $\frac{\text{opp}(A)}{\text{adj}(A)}$ is called the **tangent ratio** and you write:

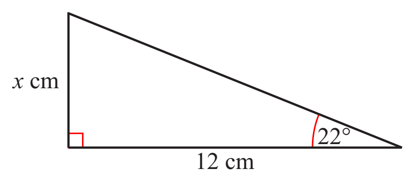
$$\tan A = \frac{\text{opp}(A)}{\text{adj}(A)}$$

Your calculator can work out the tangent ratio for any given angle and you can use this to help work out the lengths of unknown sides of a right-angled triangle.

For example, if you wanted to find the tangent of the angle 22° you enter:

Notice that the answer has many decimal places. When using this value you must make sure that you don't round your answers too soon.

Now, consider the right-angled triangle shown below.



You can find the *unknown side*, x cm, by writing down what you know about the tangent ratio:

$$\tan 22° = \frac{\text{opp}(22°)}{\text{adj}(22°)} = \frac{x}{12}$$

$$\Rightarrow x = 12\tan(22°)$$

$$\therefore \quad x = 4.848314...$$

$$x \approx 4.8\,\text{cm}\ (1\text{dp})$$

Worked example 3

Calculate the value of:

a $\tan 40°$ **b** $\tan 15.4°$

a tan 4 0 =

tan 40

0.8390996312

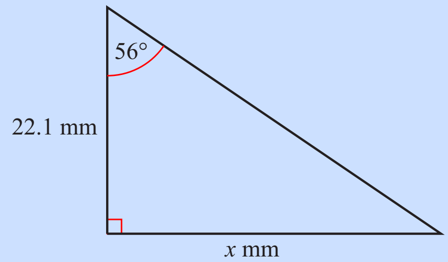
b tan 1 5 . 4 =

tan (15.4)

0.2754458909

Worked example 4

Find the value of x in the diagram. Give your answer to the nearest mm.



$\text{Opp}(56°) = x$

$\text{Adj } 56° = 22.1 \text{ mm}$

$$\tan 56° = \frac{x}{22.1}$$

$$\Rightarrow x = 22.1 \tan(56°)$$

$$= 32.76459\ldots$$

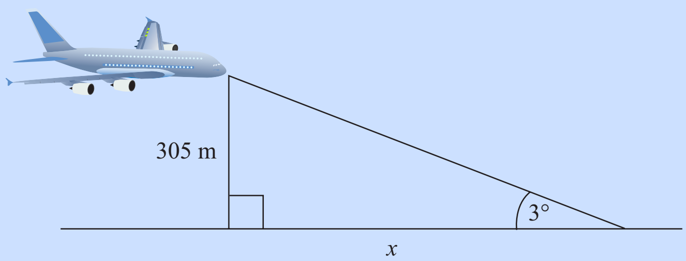
$$\approx 33 \text{ mm (nearest mm)}$$

REWIND

Remind yourself how to deal with equations that involve fractions from chapter 6. ◄

Worked example 5

The angle of approach of an airliner should be 3°. If a plane is 305 metres above the ground, how far should it be from the airfield?



$$\tan 3° = \frac{305}{x}$$

$$\Rightarrow x \tan 3° = 305$$

$$\Rightarrow x = \frac{305}{\tan 3°}$$

$$= 5819.74\ldots$$

$$\approx 5820 \text{ (nearest metre)}$$

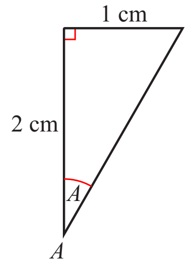
Exercise 15.5

1 Calculate the value of these tangent ratios, giving your answers to 3 significant figures where necessary.

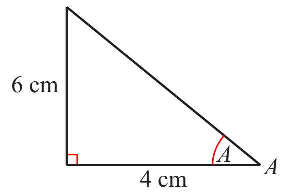
a tan 35°	**b** tan 46°	**c** tan 18°	**d** tan 45°
e tan 15.6°	**f** tan 17.9°	**g** tan 0.5°	**h** tan 0°

2 For each of the following triangles find the required tangent ratio as a fraction in the lowest terms.

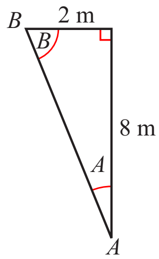
a tan A =



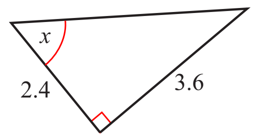
b tan A =



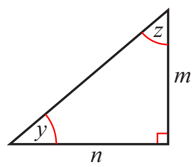
c tan A =
tan B =



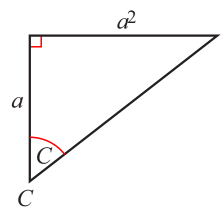
d tan x =



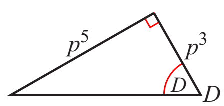
e tan z =
tan y =



f tan C =

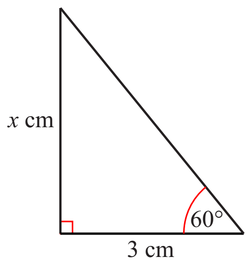


g tan D =

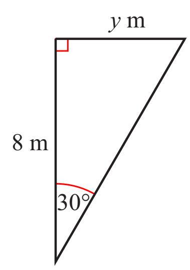


3 Find the length of the lettered side in each case. Give your answers to 3 significant figures where necessary.

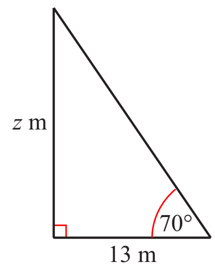
a



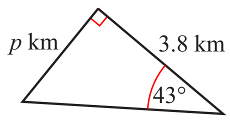
b



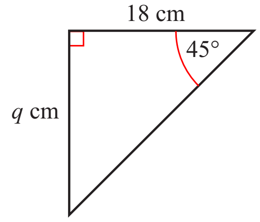
c



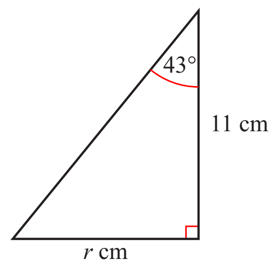
d



e

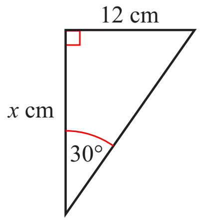


f

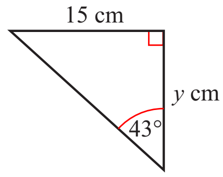


4 Calculate the lettered length in each case. In some cases you are expected to calculate the length of the adjacent. Make sure that you are careful when substituting lengths into the tangent ratio formula.

a



b



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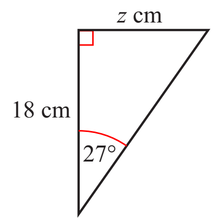
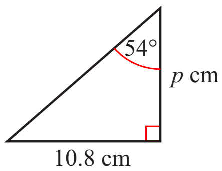


d



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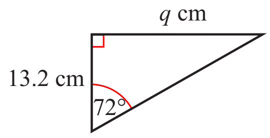
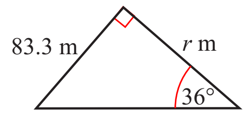
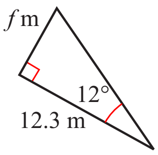


f



g



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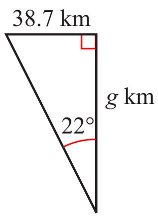
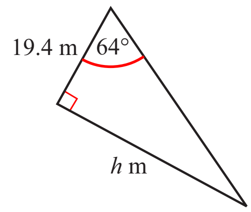
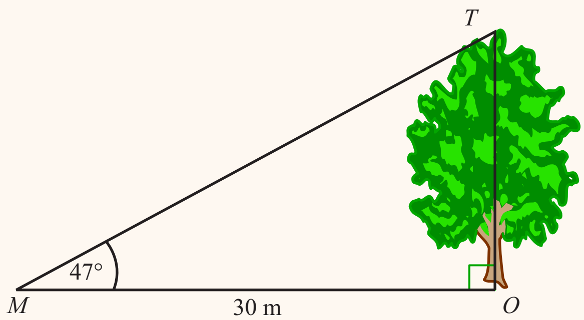


i



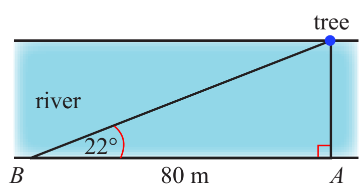
5



a Use your calculator to find the value of $\tan 47°$ correct to 4 decimal places.

b The diagram shows a vertical tree, OT, whose base, O, is 30 m horizontally from point M. The angle of elevation of T from M is 47°. Calculate the height of the tree.

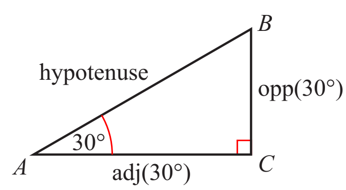
6 Melek wants to estimate the width of a river which has parallel banks. He starts at point A on one bank directly opposite a tree on the other bank. He walks 80 m along the bank to point B and then looks back at the tree. He finds that the line between B and the tree makes an angle of 22° with the bank. Calculate the width of the river.



7 The right-angled ΔABC has $B\hat{A}C = 30°$. Taking the length of BC to be one unit:

a work out the length of AC

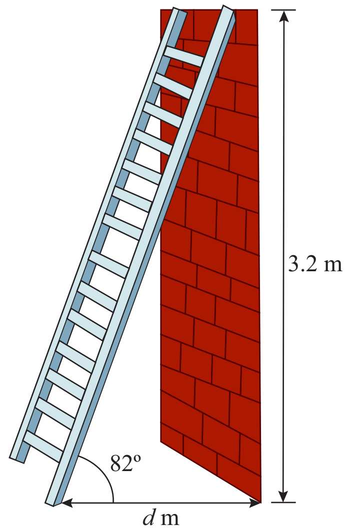
b use Pythagoras' theorem to obtain the length of AB.



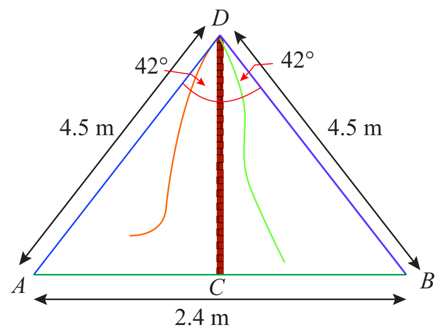
You need to be able to work out whether a problem can be solved using trigonometry or whether you can use Pythagoras' theorem.

Look at these two problems carefully. Can you see why you can't use Pythagoras' theorem to solve 8, but you could use it to solve 9?

8 The diagram shows a ladder that leans against a brick wall. If the angle between the ladder and the floor is 82°, and the ladder reaches 3.2 m up the wall, find the distance d m of the foot of the ladder from the bottom of the wall. Give your answer to the nearest cm.



9 Adi and Sarah are taking part in a Maypole dance. Adi claims that the pole is 4 metres tall but Sarah thinks he is wrong. Adi and Sarah each pull a piece of maypole ribbon tight and pin it to the ground. The points A and B represent where the ribbon was pinned to the ground; Adi and Sarah measure this distance as 2.4 m. Use the diagram to decide if Adi is right or not.



FAST FORWARD

'Functions' are dealt with more thoroughly in chapter 22.

Calculating angles

Your calculator can also 'work backwards' to find the *unknown angle* associated with a particular tangent ratio. You use the **inverse** tangent **function** $\boxed{\tan^{-1}}$ on the calculator. Generally this function uses the same key as the tangent ratio, but is placed above. If this is the case you will need to use $\boxed{\text{2ndF}}$ or $\boxed{\text{shift}}$ before you press the tan button.

Worked example 6

Find the acute angle with the tangents below, correct to 1 decimal place:

a 0.1234 **b** 5 **c** 2.765

a $\boxed{\text{shift}}\ \boxed{\tan}\ \boxed{0}\ \boxed{.}\ \boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{=}$

tan⁻¹ 0.1234 → 7.034735756

So the angle is 7.0°

b $\boxed{\text{shift}}\ \boxed{\tan}\ \boxed{5}\ \boxed{=}$

$\tan^{-1}$ 5 → 78.69006753

So the angle is 78.7°

c $\boxed{\text{shift}}\ \boxed{\tan}\ \boxed{2}\ \boxed{.}\ \boxed{7}\ \boxed{6}\ \boxed{5}\ \boxed{=}$

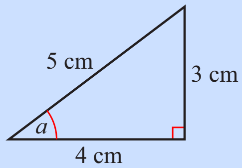
$\tan^{-1}$ 2.765 → 70.11678432

So the angle is 70.1°

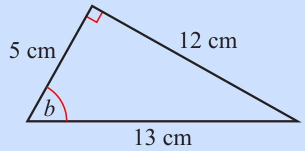
Worked example 7

Calculate, correct to 1 decimal place, the lettered angles.

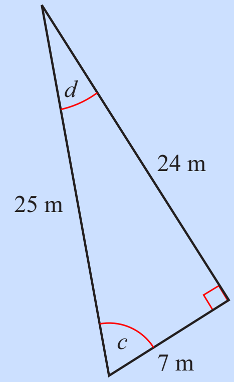
a



b



c



a

$\tan a = \dfrac{\text{opp}(a)}{\text{adj}(a)} = \dfrac{3}{4} = 0.75$

$a = \tan^{-1}(0.75)$
$\quad = 36.869897\ldots$
$a = 36.9°$ (1dp)

b

$\tan b = \dfrac{\text{opp}(b)}{\text{adj}(b)} = \dfrac{12}{5} = 2.4$

$b = \tan^{-1}(2.4)$
$\quad = 67.380135\ldots$
$b = 67.4°$ (1dp)

c

$\tan c = \dfrac{\text{opp}(c)}{\text{adj}(c)} = \dfrac{24}{7}$

$c = \tan^{-1}\left(\dfrac{24}{7}\right)$
$\quad = 73.739795\ldots$
$c = 73.7°$ (1dp)

To find the angle d, you could use the fact that the angle sum in a triangle is 180°. This gives $d = 180° - (90° + 73.7°) = 16.3°$.

Alternatively, you could use the tangent ratio again but with the opp and adj re-assigned to match this angle:

$\tan d = \dfrac{\text{opp}(d)}{\text{adj}(d)} = \dfrac{7}{24}$

$d = \tan^{-1}\left(\dfrac{7}{24}\right)$
$\quad = 16.260204\ldots$
$d = 16.3°$ (1dp)

Exercise 15.6

1 Find, correct to 1 decimal place, the acute angle that has the tangent ratio:

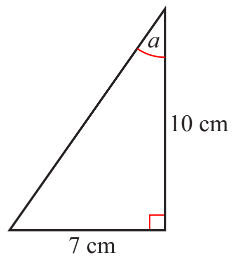
a 0.85 **b** 1.2345 **c** 3.56 **d** 10.

2 Find, correct to the nearest degree, the acute angle that has the tangent ratio:

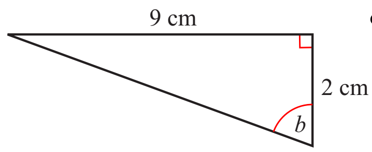
a $\frac{2}{5}$ **b** $\frac{7}{9}$ **c** $\frac{25}{32}$ **d** $2\frac{3}{4}$

3 Find, correct to 1 decimal place, the lettered angles in these diagrams.

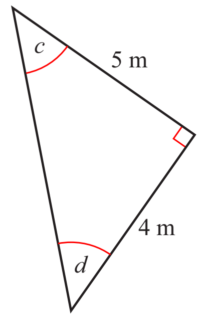
a



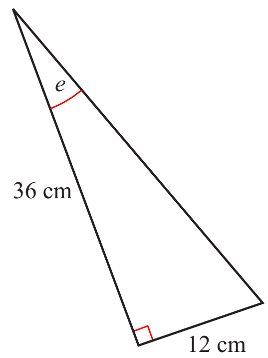
b



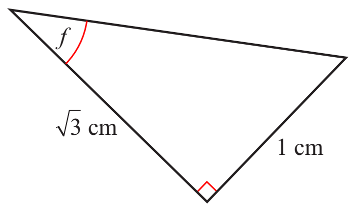
c



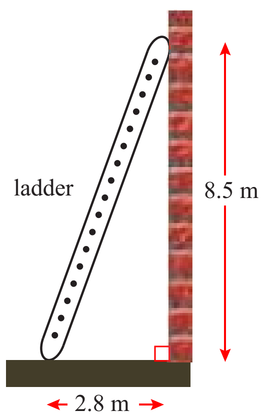
d



e

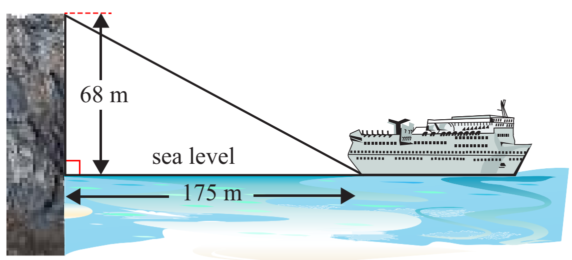


4 A ladder stands on horizontal ground and leans against a vertical wall. The foot of the ladder is 2.8 m from the base of the wall and the ladder reaches 8.5 m up the wall. Calculate the angle the ladder makes with the ground.



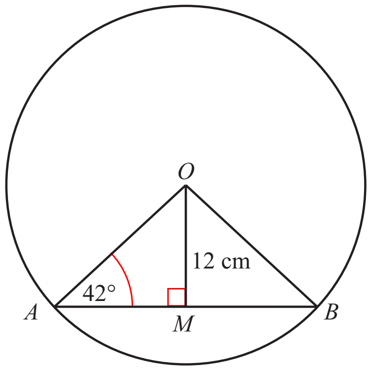
Draw a clear diagram.

5 The top of a vertical cliff is 68 m above sea level. A ship is 175 m from the foot of the cliff. Calculate the angle of elevation of the top of the cliff from the ship.



6 O is the centre of a circle with $OM = 12$ cm.

a Calculate AM.
b Calculate AB.



7 The right-angled triangle ABC has hypotenuse $AC = 7$ cm and side $BC = 3$ cm. Calculate the length AB and hence the angle ACB.

Sine and cosine ratios

You will have noticed that the tangent ratio only makes use of the opposite and adjacent sides. What happens if you need to use the hypotenuse? In fact, there are three possible pairs of sides that you could include in a ratio:

- opposite and adjacent (already used with the tangent ratio)
- opposite and hypotenuse
- or adjacent and hypotenuse.

This means that you need two more ratios:

the **sine ratio** is written as $\sin(A) = \dfrac{\text{opp}(A)}{\text{hyp}}$

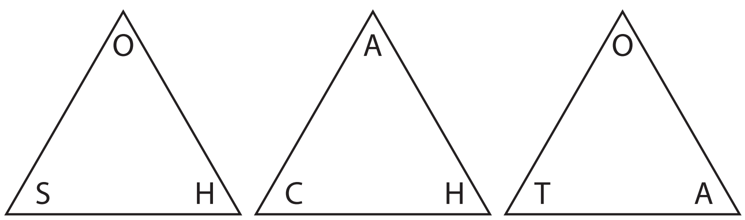
the **cosine ratio** is written as $\cos(A) = \dfrac{\text{adj}(A)}{\text{hyp}}$.

The abbreviation 'cos' is pronounced 'coz' and the abbreviation 'sin' is pronounced 'sine' or 'sign'.

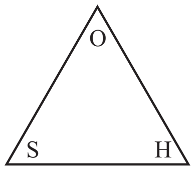
As with the tangent ratio, you can use the sin and cos keys on your calculator to find the sine and cosine ratios associated with given angles. You can also use the shift sin or $\sin^{-1}$ and shift cos or $\cos^{-1}$ 'inverse' functions to find angles.

Before looking at some worked examples you should note that with three possible ratios you need to know how to pick the right one! 'SOHCAHTOA' might help you to remember:

S = sine
O = opposite
H = hypotenuse
C = cosine
A = adjacent
T = tan



The word 'SOHCAHTOA' has been divided into three triangles of letters, each representing one of the three trigonometric ratios. The first letter in each trio tells you which ratio it represents (sine, cosine or tangent), the second letter (at the top) tells you which side's length goes on the top of the ratio, and the third letter tells you which side's length goes on the bottom.



$\Rightarrow S = \frac{O}{H}$

For example, if a problem involves the opposite and hypotenuse you simply need to find the triangle of letters that includes 'O' and 'H': SOH.

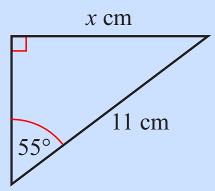
The 'S' tells you that it is the sine ratio, the 'O' at the top of the triangle sits on top of the fraction, and the lower 'H' sits on the bottom.

The use of 'SOHCAHTOA' is shown clearly in the following worked examples. The tangent ratio has been included again in these examples to help show you how to decide which ratio should be used.

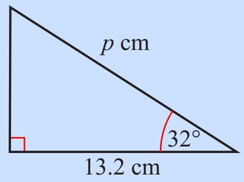
Worked example 8

Find the length of the sides lettered in each of the following diagrams.

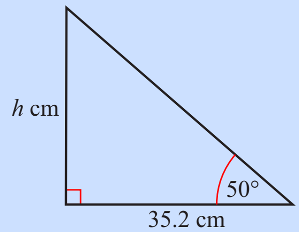
a



b



c



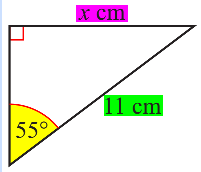
a

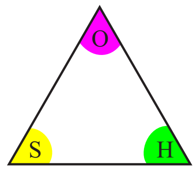
opp(55°) $= x$
hyp $= 11$ cm

So $\sin 55° = \frac{\text{opp}(55°)}{\text{hyp}} = \frac{x}{11}$

$\Rightarrow x = 11 \sin 55°$
$\Rightarrow x = 9.0$ cm (to 1dp)

Identify the sides that you are going to consider clearly:





$\left(S = \frac{O}{H}\right)$

b

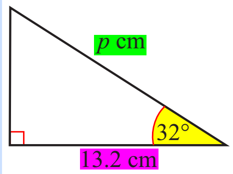
adj(32°) $= 13.2$ cm
hyp $= p$ cm

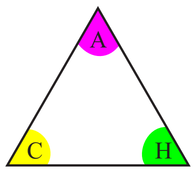
So $\cos 32° = \frac{\text{adj}(32°)}{\text{hyp}} = \frac{13.2}{p}$

$\Rightarrow p \cos 32° = 13.2$

$\Rightarrow p = \frac{13.2}{\cos 32°}$

$\Rightarrow p = 15.6$ cm (to 1dp)





$\left(C = \frac{A}{H}\right)$

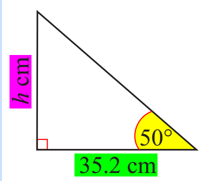
c

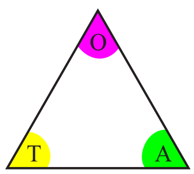
opp(50°) $= h$ cm

adj(50°) $= 35.2$ cm

So $\tan 50° = \frac{\text{opp}(50°)}{\text{adj}(50°)} = \frac{h}{35.2}$

$\Rightarrow h = 35.2 \tan 50°$
$\Rightarrow h = 41.9$ cm (to 1dp)



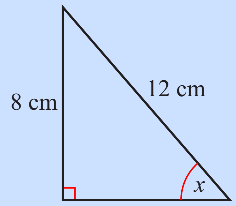


$\left(T = \frac{O}{A}\right)$

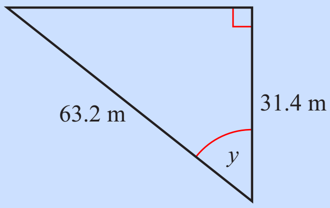
Worked example 9

Find the size of the lettered angles in each of the following diagrams.

a



b



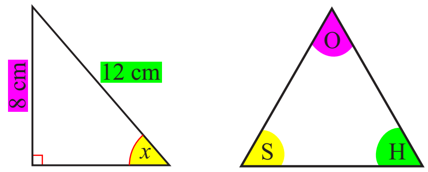
a opp(x) = 8 cm
hyp = 12 cm

So $\sin x = \dfrac{\text{opp}(x)}{\text{hyp}} = \dfrac{8}{12}$

$\Rightarrow x = \sin^{-1}\left(\dfrac{8}{12}\right)$

$\Rightarrow x = 41.8°$ (1 dp)

Once again, clearly identify the sides and ratio to be used:

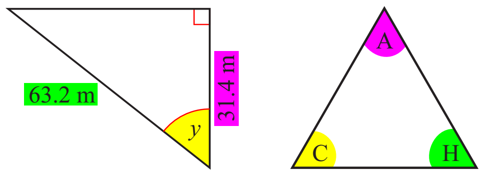


b adj(y) = 31.4 m
hyp = 63.2 m

So $\cos y = \dfrac{\text{adj}(y)}{\text{hyp}} = \dfrac{31.4}{63.2}$

$\Rightarrow y = \cos^{-1}\left(\dfrac{31.4}{63.2}\right)$

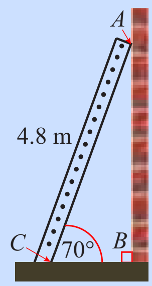
$\Rightarrow y = 60.2°$ (1 dp)



Worked example 10

A ladder 4.8 m long leans against a vertical wall with its foot on horizontal ground. The ladder makes an angle of 70° with the ground.

a How far up the wall does the ladder reach?
b How far is the foot of the ladder from the wall?

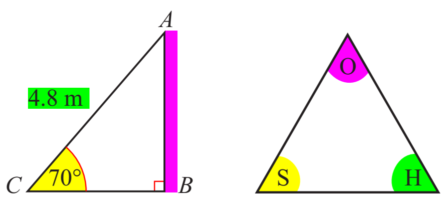


a In the diagram, AC is the hypotenuse of the right-angled ΔABC.
AB is the distance that the ladder reaches up the wall.

opp(70°) = AB
hyp = 4.8 m

So $\sin 70° = \dfrac{\text{opp}(70°)}{\text{hyp}} = \dfrac{AB}{4.8}$

$\Rightarrow AB = 4.8 \sin 70°$
$\Rightarrow AB = 4.5$ m (1 dp)
So the ladder reaches 4.5 m up the wall.

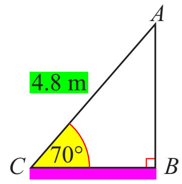


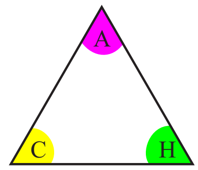
b The distance of the foot of the ladder from the wall is BC.

adj(70°) = BC
hyp = 4.8 m

So $\cos 70° = \dfrac{\text{adj}(70°)}{\text{hyp}} = \dfrac{BC}{4.8}$

$\Rightarrow BC = 4.8 \cos 70°$
$\Rightarrow BC = 1.64$ m (2dp)
The foot of the ladder is 1.64 m from the wall.



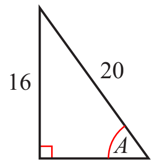


Exercise 15.7

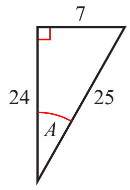
1 For each of the following triangles write down the value of:

i $\sin A$ **ii** $\cos A$ **iii** $\tan A$

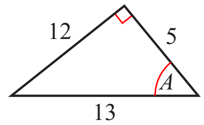
a



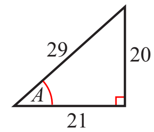
b



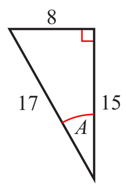
c



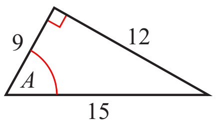
d



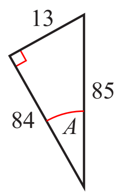
e



f



g



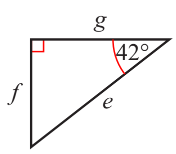
Remember to check that your calculator is in degrees mode. There should be a small '*D*' on the screen.

2 Use your calculator to find the value of each of the following. Give your answers to 4 decimal places.

a sin 5°	**b** cos 5°	**c** sin 30°	**d** cos 30°
e sin 60°	**f** cos 60°	**g** sin 85°	**h** cos 85°

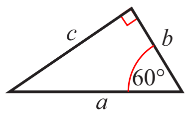
3 For each of the following triangles, use the letters of the sides to write down the given trigonometric ratio.

a



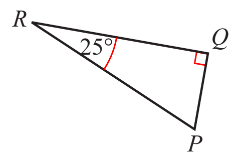
cos 42°

b



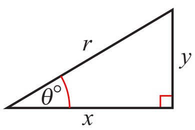
sin 60°

c



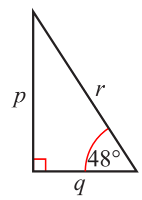
cos 25°

d



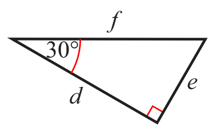
sin θ°

e



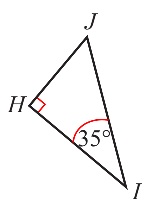
cos 48°

f



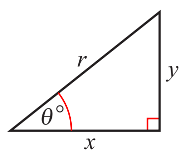
sin 30°

g



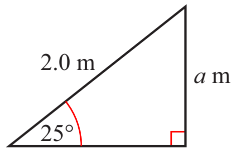
cos 35°

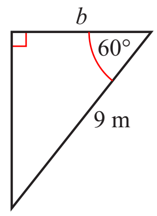
h

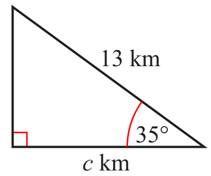


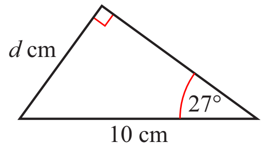
cos θ°

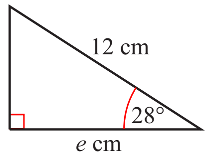
4 For each of the following triangles find the length of the unknown, lettered side. (Again, some questions that require the tangent ratio have been included. If you use SOHCAHTOA carefully you should spot these quickly!)

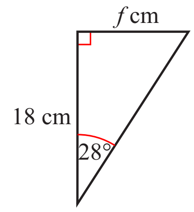
a


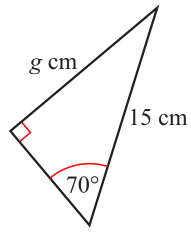
b


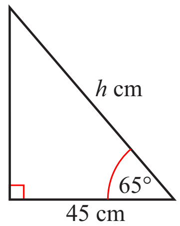
c


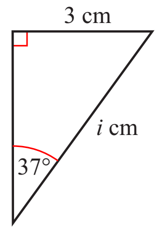
d


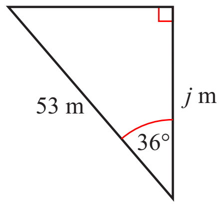
e


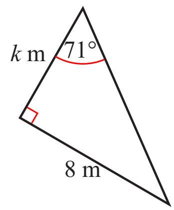
f


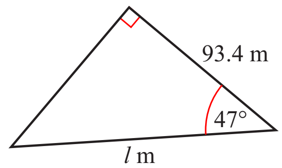
g


h


i


j


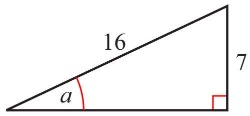
k


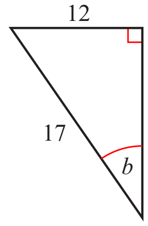
l


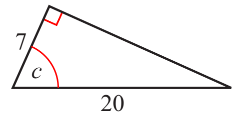
5 Use your calculator to find, correct to 1 decimal place:

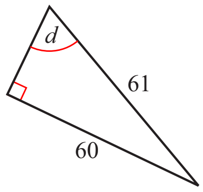
a an acute angle whose sine is 0.99
b an acute angle whose cosine is 0.5432
c an acute angle whose sine is $\frac{3}{8}$
d an acute angle whose cosine is $\frac{\sqrt{3}}{2}$.

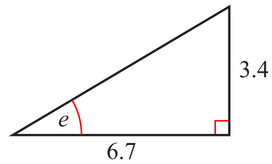
6 Find, to 1 decimal place, the lettered angle in each of the following triangles.

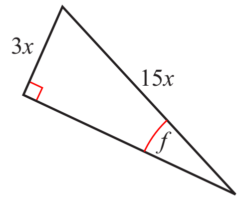
a


b


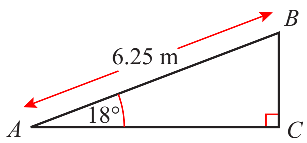
c


d


e


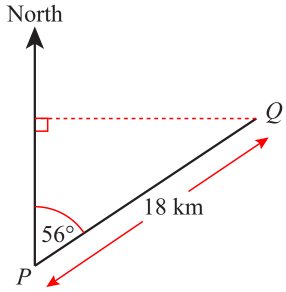
f


7 The diagram shows a ramp, AB, which makes an angle of 18° with the horizontal. The ramp is 6.25 m long. Calculate the difference in height between A and B (this is the length BC in the diagram).



8 Village Q is 18 km from village P, on a bearing of 056°.

a Calculate the distance Q is north of P.
b Calculate the distance Q is east of P.

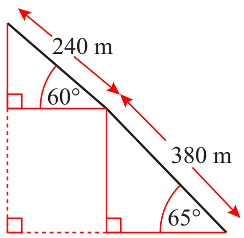


9 A 15 m beam is resting against a wall. The base of the beam forms an angle of 70° with the ground.

a At what height is the top of the beam touching the wall?
b How far is the base of the beam from the wall?

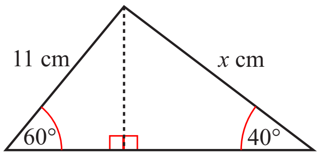
10 A mountain climber walks 380 m along a slope that is inclined at 65° to the horizontal, and then a further 240 m along a slope inclined at 60° to the horizontal.

Calculate the total vertical distance through which the climber travels.

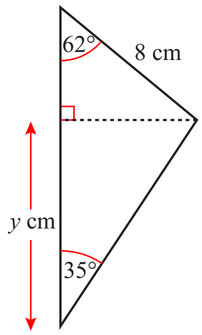


11 Calculate the unknown, lettered side(s) in each of the following shapes. Give your answers to 2 decimal places where necessary.

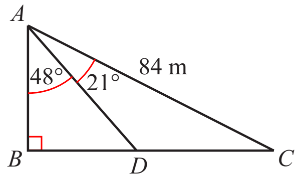
a



b

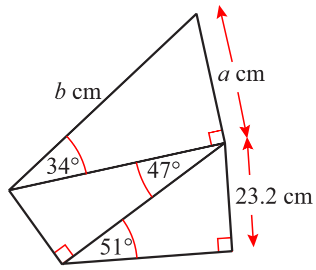


c



Find length AD.

d



12 For each of the following angles calculate:

i $\tan x$ **ii** $\dfrac{\sin x}{\cos x}$

a $x = 30°$ **b** $x = 48°$ **c** $x = 120°$ **d** $x = 194°$

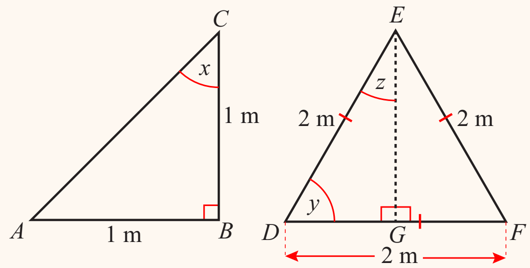
What do you notice?

13 Calculate:

a $(\sin 30°)^2 + (\cos 30°)^2$
b $(\sin 48°)^2 + (\cos 48°)^2$.
c Choose another angle and repeat the calculation.
d What do you notice?

E

14 The diagrams show a right-angled isosceles triangle and an equilateral triangle.



a Write down the value of angle ACB.
b Use Pythagoras' theorem to calculate length AC, leaving your answer in exact form.
c Copy triangle ABC, including your answers to **(a)** and **(b)**.
d Use your diagram to find the exact values of sin 45°, cos 45° and tan 45°.
e Write down the value of angle y.
f Write down the value of angle z.
g Use Pythagoras' theorem to calculate the length EG, leaving your answer in exact form.
h Copy the diagram, including your answers to **(e)**, **(f)** and **(g)**.
i Use your diagram to find the exact values of sin 30°, cos 30°, tan 30°, sin 60°, cos 60°, tan 60°.
j Copy and complete the table below, using your answers from previous parts of this question.

Angle x	$\sin x$	$\cos x$	$\tan x$
30°			
60°			
45°			

Exact form

You have already met irrational numbers in chapter 9. An example was $\sqrt{2}$. This is called a 'surd'. Written like this it is exact, but if you put it through a calculator and then use a rounded value, your answer is an approximation. The same is true for recurring decimals like $\frac{2}{3}$.

When a question asks for an answer in exact form it intends you to leave any surds in root form and any recurring decimals as fractions. So, $5\sqrt{2}$ is exact but putting it through a calculator and writing 7.07 (2 d.p.) is not. Similarly $\frac{2}{3}$ is exact but 0.67 (2 d.p.) is not.

15.4 Solving problems using trigonometry

REWIND

It will be useful to remind yourself about general angle properties of triangles from chapter 3. ◀

You may need to make use of more than one trigonometric ratio when solving problems that involve right-angled triangles. To make it clear which ratio to use and when, you should follow these guidelines.

- If the question does not include a diagram, draw one. Make it clear and large.
- Draw any triangles that you are going to use separately and clearly label angles and sides.

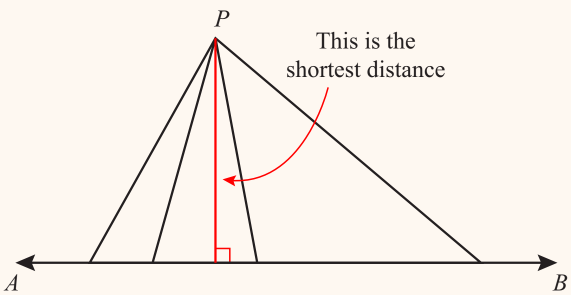
- Identify any right-angled triangles that may be useful to you.
- Identify any sides or angles that you already know.
- Write down which of the opposite, adjacent and hypotenuse are going to be used, and then use SOHCAHTOA to help you decide which ratio to use.
- Write down the ratio and solve, either for an angle or a side.
- If you need to use a side or angle that you have calculated for another part of a question, try hard to use the unrounded value that you have in your calculator memory. This will help to avoid rounding errors later on.

Calculating distances

E

In mathematics, when you are asked to calculate the distance from a point to a line you are expected to find the shortest distance between the point and the line. This distance is equal to the length of a perpendicular from the point to the line.

In this diagram, the distance from P to the line AB is 5 units.

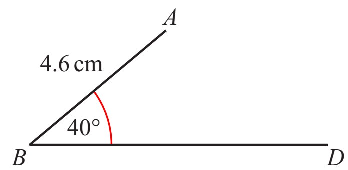


Any other line from the point to the line creates a right-angled triangle and the line itself becomes the hypotenuse of that triangle. Any hypotenuse must be longer than the perpendicular line, so, all the other distances from P to the line are longer than 5 units.

If you are given the equation of the line ($y = mx + c$) and the coordinates of the point (x, y), you can use what you know about coordinate geometry and simultaneous equations to work out the distance.

- Determine the equation of the line perpendicular to the given line that goes through (x, y). (Remember that if the gradient of one line is a/b, then the gradient of a line perpendicular to it is $-b/a$.)
- Solve the two equations simultaneously to find the point of intersection.
- Calculate the distance between the point of intersection and the given point.

There are different ways of working out the distance between a point and a line. The method you choose will depend on the information you are given. For example, if you were asked to find the distance between point A and line BD given the information on the diagram below, you could draw in the perpendicular and use trigonometry to find the lengths of the other sides of the triangle.



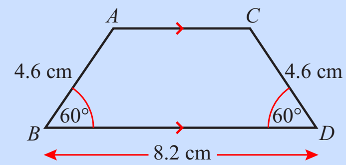
FAST FORWARD

You will use the distance between a point and a line again when you deal with circles in chapter 19.

The following worked example shows you how trigonometry is used to find the distance between point A and the line BD and then how to use it to solve the problem given. It also shows you how the solution to a trigonometry problem could be set out.

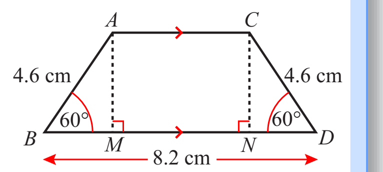
Worked example 11

The diagram shows an isosceles trapezium *ABDC*.
Calculate the area of the trapezium.



The area of a trapezium = (mean of the parallel sides) × (perpendicular distance between them)

In this diagram, perpendiculars have been added to form right-angled triangles so that trigonometry can be used.



$AC = MN$ and you can find the length of MN if you calculate the lengths of BM and ND.

In $\triangle ABM$, $\sin 60° = \frac{\text{opp}(60°)}{\text{hyp}} = \frac{AM}{4.6}$ and $\cos 60° = \frac{\text{adj}(60°)}{\text{hyp}} = \frac{BM}{4.6}$

Hence, $AM = 4.6 \times \sin 60°$ and $BM = 4.6 \times \cos 60°$
$AM = 3.983716\ldots$ cm and $BM = 2.3$ cm

By symmetry, $ND = BM = 2.3$ cm
and $\therefore\ MN = 8.2 - (2.3 + 2.3) = 3.6$ cm

Hence, $AC = 3.6$ cm and $AM = CN = 3.983716\ldots$ cm

$$\text{The area of } ABDC = \left(\frac{AC + BD}{2}\right) \times AM$$
$$= \left(\frac{3.6 + 8.2}{2}\right) \times 3.983716\ldots \text{cm}^2$$
$$= 23.503929\ldots \text{cm}^2$$

Area of *ABDC* = 23.5 cm^2 (to 3sf)

LINK

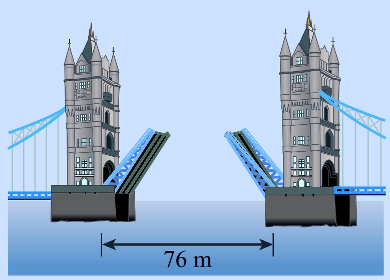
Trigonometry is used to work out lengths and angles when they can't really be measured. For example in navigation, surveying, engineering, construction and even the placement of satellites and receivers.

Tip

Give your answer to 3 significant figures if no degree of accuracy is specified.

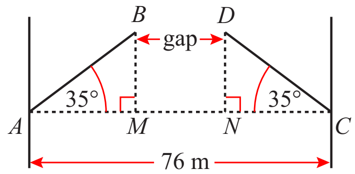
Worked example 12

The span between the towers of Tower Bridge in London is 76 m. When the arms of the bridge are raised to an angle of 35°, how wide is the gap between their ends?



Drawing a clear, labelled sketch can help you work out what mathematics you need to do to solve the problem.

Here is a simplified labelled drawing of the bridge, showing the two halves raised to 35°.



The gap = $BD = MN$ and $MN = AC - (AM + NC)$.

The right-angled triangles ABM and CDN are congruent, so $AM = NC$. When the two halves are lowered, they must meet in the middle.

$\therefore AB = CD = \frac{76}{2} = 38\,\text{m}$

In ΔABM, $\cos 35° = \frac{\text{adj}(35°)}{\text{hyp}} = \frac{AM}{38}$

$AM = 38 \times \cos 35°$
$= 31.1277\ldots\text{m}$

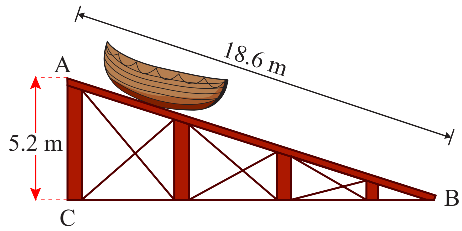
$\therefore MN = 76 - (31.1277\ldots + 31.1277\ldots)$
$= 13.744\ldots\text{m}$

The gap $BD = 13.7\,\text{m}$ (to 3sf)

Exercise 15.8 *Applying your skills*

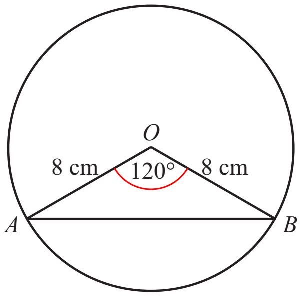
1 The diagram represents a ramp *AB* for a lifeboat. *AC* is vertical and *CB* is horizontal.

a Calculate the size of angle *ABC* correct to 1 decimal place.
b Calculate the length of *BC* correct to 3 significant figures.



2 *AB* is a chord of a circle, centre *O*, radius 8 cm. Angle $AOB = 120°$.

Calculate the length of *AB*.



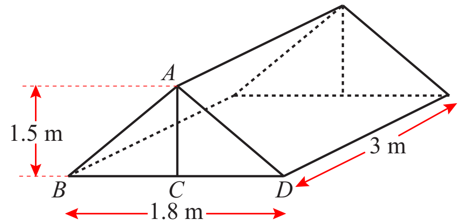
REWIND
Look at chapter 7 and remind yourself about the formula for the volume of a prism.

3 The diagram represents a tent in the shape of a triangular prism. The front of the tent, *ABD*, is an isosceles triangle with $AB = AD$.

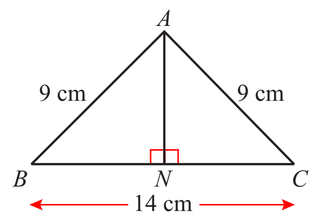
The width, *BD*, is 1.8 m and the supporting pole *AC* is perpendicular to *BD* and 1.5 m high. The tent is 3 m long.

Calculate:

a the angle between *AB* and *BD*
b the length of *AB*
c the capacity inside the tent (i.e. the volume).



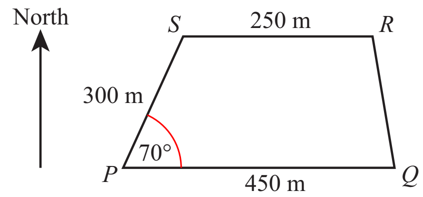
4 Calculate the angles of an isosceles triangle that has sides of length 9 cm, 9 cm and 14 cm.



5 The sketch represents a field *PQRS* on level ground.

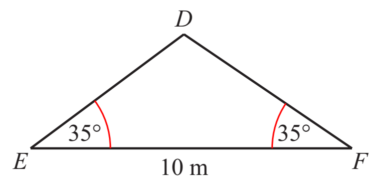
The sides *PQ* and *SR* run due east.

a Write down the bearing of *S* from *P*.
b Calculate the perpendicular (shortest) distance between *SR* and *PQ*.
c Calculate, in square metres, the area of the field *PQRS*.

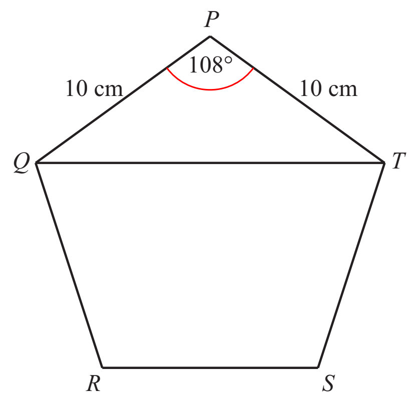


6 In the isosceles triangle *DEF*, $E = F = 35°$ and side $EF = 10$ m.

a Calculate the perpendicular (shortest) distance from *D* to *EF*.
b Calculate the length of the side *DE*.



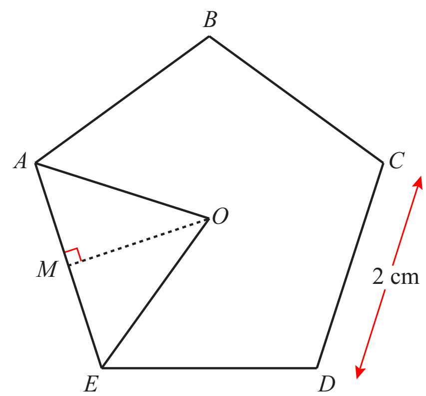
7 Find the length of a diagonal (*QT*) of a regular pentagon that has sides of length 10 cm. Give your answer to the nearest whole number.



REWIND
Areas of two-dimensional shapes were covered in chapter 7. ◀

8 The diagram shows a regular pentagon with side 2 cm. *O* is the centre of the pentagon.

a Find angle *AOE*.
b Find angle *AOM*.
c Use trigonometry on triangle *AOM* to find the length *OM*.
d Find the area of triangle *AOM*.
e Find the area of the pentagon.



9 Using a similar method to that described in question 8 find the area of a regular octagon with side 4 cm.

10 Find the area of a regular pentagon with side $2a$ metres.

11 Find the area of a regular polygon with n sides, each of length $2a$ metres.

15.5 Sines, cosines and tangents of angles more than 90°

E

FAST FORWARD

$\sin\theta$, $\cos\theta$ and $\tan\theta$ are actually functions. Functions take input values and give you a numerical output. For example, if you use your calculator to find $\sin 30°$ you will get the answer $\frac{1}{2}$. Your calculator takes the input 30°, finds the sine of this angle and gives you the output $\frac{1}{2}$. You will learn more about functions in Chapter 22. ▶

You have now seen that we can find the sine, cosine or tangent of an angle in a triangle by using your calculator.

It is possible to find sines, cosines and tangents of angles of any size.

Investigation

Use a calculator to find each of the following.

sin 30° and sin 150°

sin 10° and sin 170°

sin 60° and sin 120°

sin 5° and sin 175°

What did you notice? What is the relationship between the two angles in each pair.

Now do the same for these pairs.

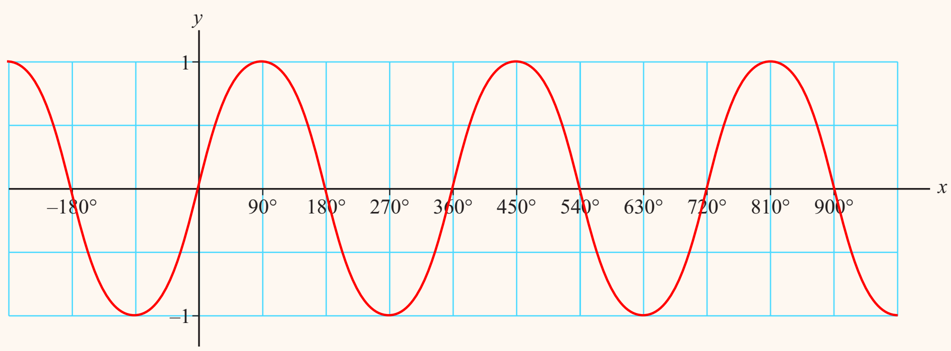
cos 30° and cos 330°	tan 30° and tan 210°
cos 60° and cos 300°	tan 60° and tan 240°
cos 50° and cos 310°	tan 15° and tan 195°
cos 15° and cos 345°	tan 100 and tan 280°

The pattern is different for each of sine, cosine and tangent.

You will now explore the graphs of $y = \sin\theta$, $y = \cos\theta$ and $y = \tan\theta$ to see why.

The graph of $y = \sin\theta$

If you plot several values of $\sin\theta$ against θ you will get the following:



The graph repeats itself every 360° in both the positive and negative directions.

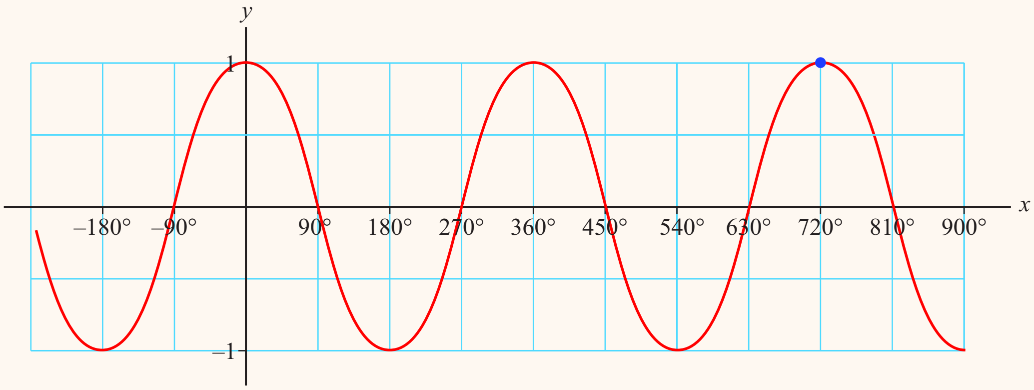
Notice that the section of the graph between 0 and 180° has reflection symmetry, with the line of reflection being $\theta = 90°$. This means that $\sin\theta = \sin(180° - \theta)$, exactly as you should have seen in the investigation above.

It is also very important to notice that the value of $\sin\theta$ is never larger than 1 nor smaller than −1.

E

The graph of $y = \cos\theta$

If you plot several values of $\cos\theta$ against θ you will get a similar shape, but the line of symmetry is in a different place:



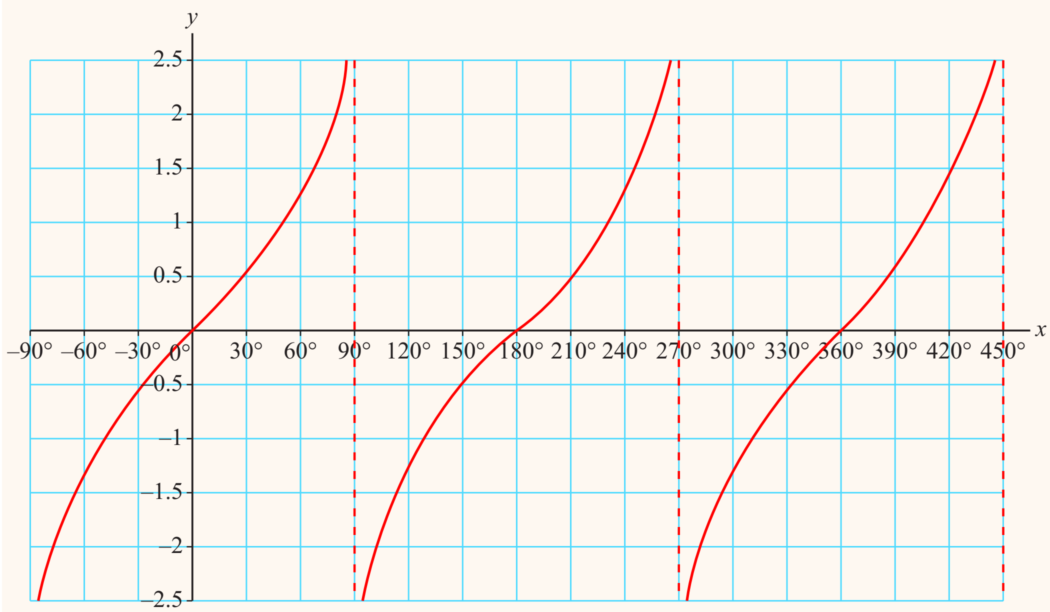
The graph repeats itself every 360° in both the positive and negative directions.

Here the graph is symmetrical from 0 to 360°, with the reflection line at $\theta = 180°$. This means that $\cos\theta = \cos(360° - \theta)$, again you should have seen this in the investigation above. By experimenting with some angles you will also see that $\cos\theta = -\cos(180° - \theta)$

It is also very important to notice that the value of $\cos\theta$ is never larger than 1 nor smaller than –1.

The graph of $y = \tan\theta$

Finally, if you plot values of $\tan\theta$ against θ you get this graph:



FAST FORWARD

A line that is approached by a graph in this way is known as an ***asymptote***. You will learn more about asymptotes in Chapter 18.

The vertical dotted lines are approached by the graph, but it never touches nor crosses them.

Notice that this graph has no reflection symmetry, but it does repeat every 180°. This means that $\tan\theta = \tan(180° + \theta)$.

Note that, unlike $\sin\theta$ and $\cos\theta$, $\tan\theta$ is not restricted to being less than 1 or greater than –1.

The shapes of these three graphs means that equations involving sine, cosine or tangent will have multiple solutions. The following examples show you how you can find these. In each case the question is done using a sketch graph to help.

E

Worked example 13

Which acute angle has the same sine as 120°?

$\sin(180° - \theta) = \sin\theta$ But in this case, $\theta = 120°$
$180° - \theta = 120$
$180° - 120° = \theta$
$60° = \theta$
$\sin 60° = \sin 120°$

Worked example 14

Express each of the following in terms of another angle between 0° and 180°.

a $\cos 100°$ **b** $-\cos 35°$

a $\cos(180° - \theta) = -\cos\theta$ In this case $\theta = 100°$
$\cos 100° = -\cos(180° - 100°) = -\cos 80°$

b $-\cos\theta = \cos(180° - \theta)$
$-\cos 35° = \cos 145°$

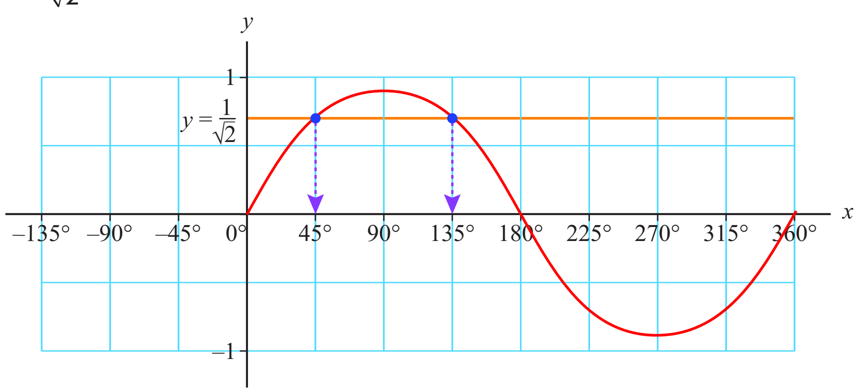
Worked example 15

Solve the following equations, giving all possible solutions in the range 0 to 360 degrees.

a $\sin\theta = \frac{1}{\sqrt{2}}$ **b** $\tan\theta = 3$ **c** $\cos x = -\frac{1}{2}$

a Use your calculator to find one solution: $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45°$

Now mark $\theta = 45$ degrees on a sketch of the graph $y = \sin\theta$ and draw the line $y = \frac{1}{\sqrt{2}}$ like this:



Using the symmetry of the graph, you can see that there is another solution at $\theta = 135°$.
Use your calculator to check that $\sin 135° = \frac{1}{\sqrt{2}}$.

Notice that $135° = 180° - 45°$. You can use this rule, but drawing a sketch graph always makes it easier to understand *why* there is a second solution.

As well as θ, other variables can be used to represent the angle. In part c of the example, x is used. You work in exactly the same way whatever variable is used.

This only needs to be a sketch and doesn't need to be accurate. It should be just enough for you to see how the symmetry can help.

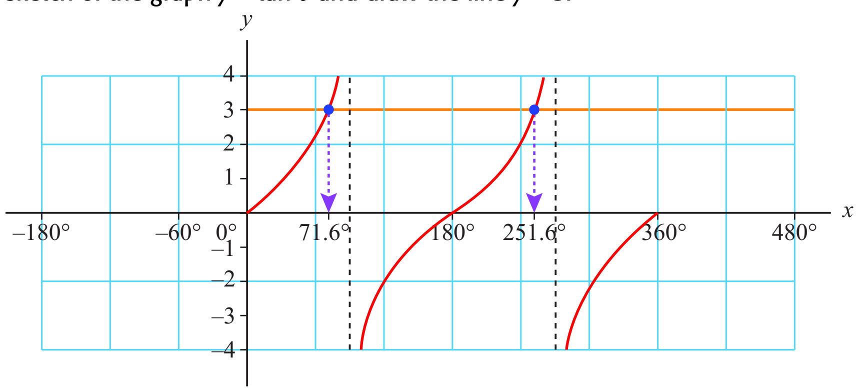
Tip

Note that you only need to sketch the part of the graph for 0 to 360°, because you are only looking for solutions between these two values of θ.

There are more solutions, but in the range 0 to 360° the line $y = \frac{1}{\sqrt{2}}$ only meets the graph $y = \sin\theta$ twice.

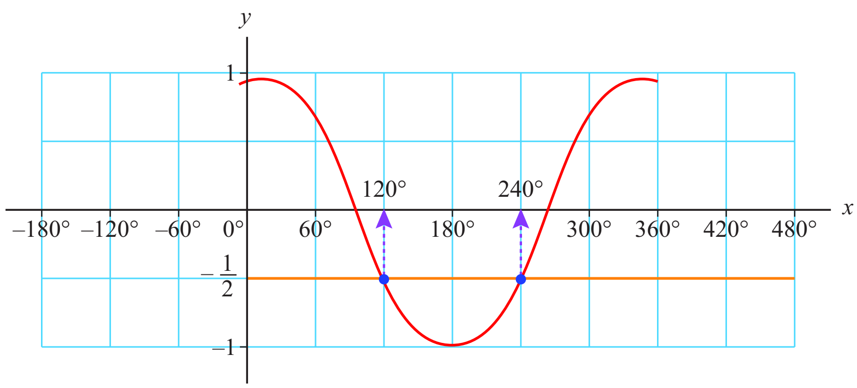
E

b Use your calculator to find one solution: $\tan^{-1}3 = 71.6°$. As before mark this on a sketch of the graph $y = \tan\theta$ and draw the line $y = 3$.



You can see that the second solution is 180° + 71.6° = 251.6°.
More solutions can be found by adding 180 degrees over and over, but these will all be larger than 360 degrees, so they are not in the range that you want.

c Use your calculator to find one solution: $\cos^{-1}\left(-\frac{1}{2}\right) = 120°$.
Draw a sketch and mark the values:



You can now see that the second solution will be at 360° – 120° = 240°.

Exercise 15.9

1 Express each of the following in terms of the same trig ratio of another angle between 0° and 180°.

a cos 120° **b** sin 35° **c** cos 136° **d** sin 170° **e** cos 88°
f –cos 140° **g** sin 121° **h** sin 99° **i** –cos 45° **j** –cos 150°

2 Solve each of the following equations, giving all solutions between 0 and 360 degrees.

a $\sin\theta = \frac{1}{2}$ **b** $\sin\theta = 1$ **c** $\cos\theta = \frac{\sqrt{2}}{2}$

d $\tan\theta = 5$ **e** $\cos\theta = -\frac{\sqrt{3}}{2}$ **f** $\sin\theta = -0.2$

g $\cos\theta = -\frac{1}{3}$ **h** $\tan\theta = \sqrt{3}$ **i** $\tan\theta = -4$

E

3 For each of the following find the smallest positive value of x for which

a $\sin x = \sin 135°$ **b** $\cos x = \cos 120°$ **c** $\tan x = \tan 235°$
d $\cos x = \cos(-45°)$ **e** $\sin x = \sin 270°$ **f** $\tan x = \tan 840°$
g $\sin(x - 30°) = \sin 240°$ **h** $\cos(2x) = \cos(540°)$ **i** $\tan\left(\frac{x}{6}\right) = \tan(-476°)$

4 Solve, giving all solutions between 0 and 360 degrees:

$$(\sin x)^2 = \frac{1}{4}$$

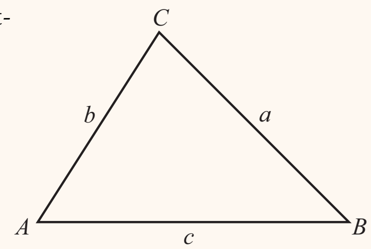
Write $\cos x = y$ and try to factorise

5 Solve, giving all solutions between 0 and 360 degrees:

$$8(\cos x)^2 - 10\cos x + 3 = 0$$

15.6 The sine and cosine rules

The sine and cosine ratios are not only useful for right-angled triangles. To understand the following rules you must first look at the standard way of labelling the angles and sides of a triangle. Look at the triangle shown in the diagram.



Notice that the sides are labelled with lower case letters and the angles are labelled with upper case letters. The side that is placed opposite angle A is labelled 'a', the side that is placed opposite angle B is labelled 'b' and so on.

The sine rule

For the triangle shown above, the following are true:

$$\frac{\sin A}{a} = \frac{\sin B}{b} \quad \text{and} \quad \frac{\sin A}{a} = \frac{\sin C}{c} \quad \text{and} \quad \frac{\sin B}{b} = \frac{\sin C}{c}$$

These relationships are usually expressed in one go:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Remember, the sine rule is used when you are dealing with pairs of opposite sides and angles.

This is the **sine rule**. This version of the rule, with the sine ratios placed on the tops of the fractions, is normally used to calculate angles.

The formulae can also be turned upside down when you want to calculate lengths:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

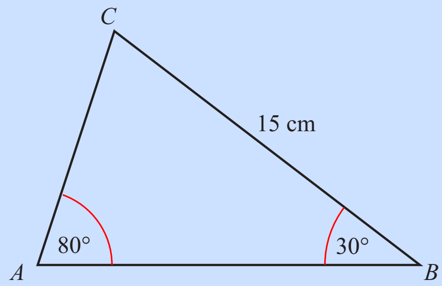
You should remember that this represents *three* possible relationships.

Notice that in each case, both the upper and lower case form of each letter is used. This means that each fraction that you use requires an angle and the length of its opposite side.

Worked example 16

In $\triangle ABC$, $A = 80°$, $B = 30°$ and side $BC = 15$ cm.

Calculate the size of C and the lengths of the sides AB and AC.



To calculate the angle C, use the fact that the sum of the three angles in a triangle is always 180°.

So, $C + 80 + 30 = 180 \Rightarrow C = 180 - 30 - 80 = 70°$

Now think about the side AB. AB is opposite the angle C (forming an 'opposite pair') and side BC is opposite angle A, forming a second 'opposite pair'.

So, write down the version of the sine rule that uses these pairs:

$$\frac{a}{\sin A} = \frac{c}{\sin C} \Rightarrow \frac{BC}{\sin A} = \frac{AB}{\sin C}$$

So, $\dfrac{15}{\sin 80°} = \dfrac{AB}{\sin 70°} \Rightarrow AB = \dfrac{15}{\sin 80°} \times \sin 70° = 14.3\text{ cm (3sf)}$

Similarly:

AC forms an opposite pair with angle B, so once again use the pair BC and angle A:

$$\frac{a}{\sin A} = \frac{b}{\sin B} \Rightarrow \frac{BC}{\sin A} = \frac{AC}{\sin B}$$

So, $\dfrac{15}{\sin 80°} = \dfrac{AC}{\sin 30°} \Rightarrow AC = \dfrac{15}{\sin 80°} \times \sin 30° = 7.62\text{ cm (3sf)}$

The ambiguous case of the sine rule

The special properties of the sine function can lead to more than one possible answer. The following example demonstrates how this may happen.

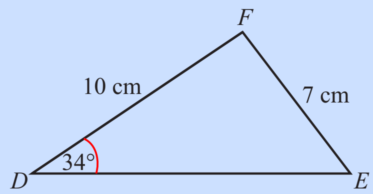
E

Worked example 17

In $\triangle DEF$, $DF = 10$ cm, $EF = 7$ cm and $\hat{D} = 34°$.

Calculate, to the nearest degree, the possible size of:

a angle $\hat{E}$ **b** angle $\hat{F}$.



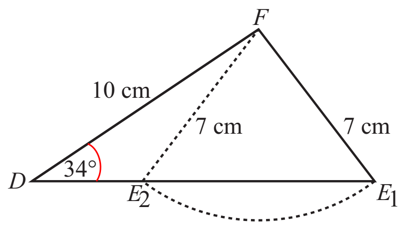
a Angle $\hat{E}$ is opposite a side of length 10 cm. This forms one pair.
Angle $\hat{D}$ is opposite a side of length 7 cm. This forms the second pair.
You are trying to find an angle, so choose the version of the sine rule with the value of sine ratios in the numerators:

$$\frac{\sin 34°}{7} = \frac{\sin E}{10} \Rightarrow \sin E = 10 \times \frac{\sin 34°}{7}$$

So, $\hat{E} = \sin^{-1}\left(10 \times \frac{\sin 34°}{7}\right) = 53.0°$

But there is actually a second angle E such that $\sin E = 10 \times \frac{\sin 34°}{7}$. You can see this if you consider the sine graph. The values of both sin x and cos x repeat every 360°. This property of both functions is called 'periodicity', i.e., both sin x and cos x are *periodic*. The periodicity of the function tells you that the second possible value of $\hat{E}$ is $180 - 53.0 = 127.0°$.

Both of these are possible values of $\hat{E}$ because there are two ways to draw such a triangle.



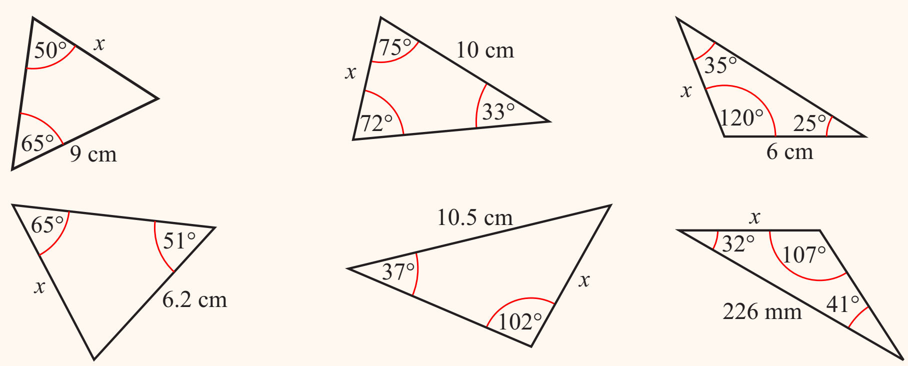
b Of course, the answers to part (a) must lead to two possible answers for part (b).
If $\hat{E} = 127.0°$, then $\hat{F} = 180 - 127 - 34 = 19°$ (shown as E_1 in the diagram).
If $\hat{E} = 53.0°$, then $\hat{F} = 180 - 53 - 34 = 93°$ (shown as E_2 in the diagram).
(If asked for, this would also have led to two possible solutions for the length DE).
You absolutely must take care to check that all possible answers have been calculated. Bear this in mind as you work through the following exercise.

Exercise 15.10

1 Find the value of x in each of the following equations.

a $\frac{x}{\sin 50} = \frac{9}{\sin 38}$ **b** $\frac{x}{\sin 25} = \frac{20}{\sin 100}$ **c** $\frac{20.6}{\sin 50} = \frac{x}{\sin 70}$ **d** $\frac{\sin x}{11.4} = \frac{\sin 63}{16.2}$

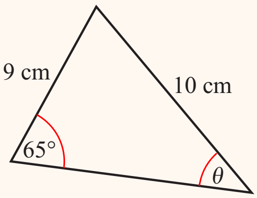
2 Find the length of the side marked x in each of the following triangles.



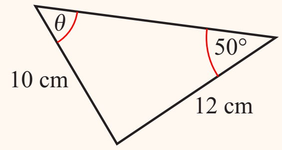
E

3 Find the size of the angle marked θ in the following triangles. Give your answers correct to 1 decimal place.

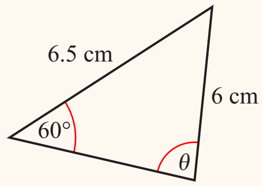
a



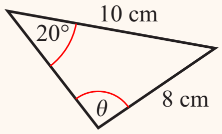
b



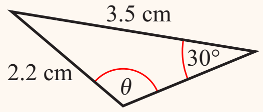
c



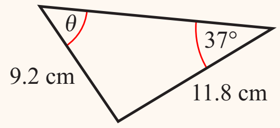
d



e

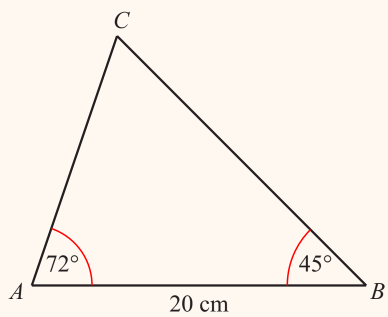


f



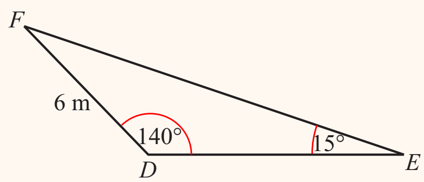
4 In ΔABC, $A = 72°$, $B = 45°$ and side $AB = 20$ cm.

Calculate the size of C and the lengths of the sides AC and BC.

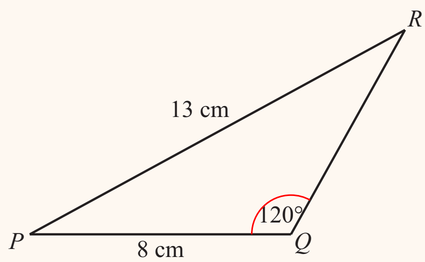


5 In ΔDEF, $D = 140°$, $E = 15°$ and side $DF = 6$ m.

Calculate the size of F and the lengths of the sides DE and EF.

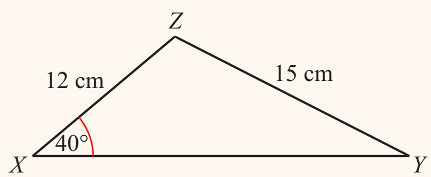


6 In ΔPQR, $Q = 120°$, side $PQ = 8$ cm and side $PR = 13$ cm. Calculate the size of R, the size of P, and the length of side QR.
Give your answers to the nearest whole number.



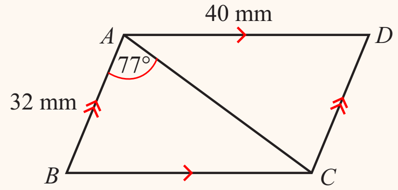
7 In ΔXYZ, $X = 40°$, side $XZ = 12$ cm and side $YZ = 15$ cm.

a Explain why Y must be less than 40°.
b Calculate, correct to 1 decimal place, Y and Z.
c Calculate the length of the side XY.



E

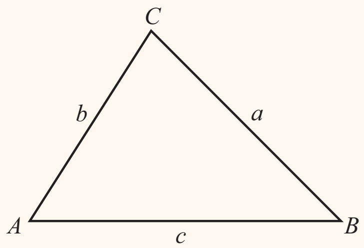
8 $ABCD$ is a parallelogram with $AB = 32$ mm and $AD = 40$ mm and angle $BAC = 77°$.



a Find the size of angle BCA (to the nearest degree)
b Find the size of angle ABC (to the nearest degree)
c Find the length of diagonal AC correct to 2 decimal places.

Cosine rule

For the **cosine rule**, consider a triangle labelled in exactly the same way as that used for the sine rule.



The cosine rule is stated as a single formula:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Notice the all three sides are used in the formula, and just one angle. The side whose square is the subject of the formula is opposite the angle used (hence they have the same letter but in a different case). This form of the cosine rule is used to find unknown sides.

By rearranging the labels of angles (but making sure that opposite sides are still given the lower case version of the same letter for any given angle) the cosine rule can be stated in two more possible ways:

$$b^2 = a^2 + c^2 - 2ac\cos B \qquad \text{or} \qquad c^2 = a^2 + b^2 - 2ab\cos C$$

Remember, if you know all three sides of a triangle, you can use the cosine rule to find any angle.

If you know two sides, and the unknown side is opposite a *known* angle, then the cosine rule can be used to calculate the unknown side.

Notice, also, that you can take any version of the formula to make the cosine ratio the subject.

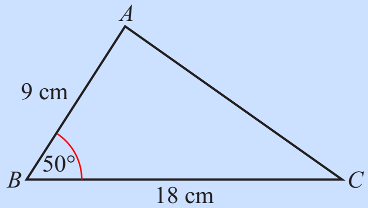
This version can be used to calculate angles:

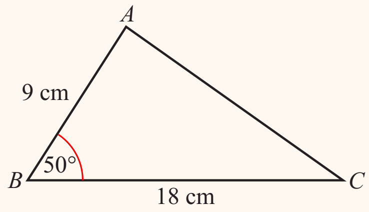
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$\Rightarrow a^2 + 2bc\cos A = b^2 + c^2$$
$$\Rightarrow 2bc\cos A = b^2 + c^2 - a^2$$
$$\Rightarrow \cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Worked example 18

In $\triangle ABC$, $B = 50°$, side $AB = 9$ cm and side $BC = 18$ cm.

Calculate the length of AC.





Notice that $AC = b$ and you know that $B = 50°$.
Use the cosine rule in the form, $b^2 = a^2 + c^2 - 2ac\cos B$

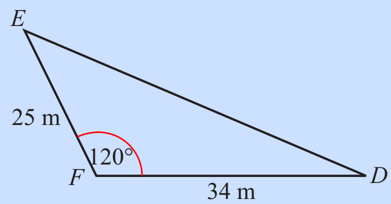
$$\begin{aligned} b^2 &= 9^2 + 18^2 - (2 \times 9 \times 18 \times \cos 50°) \\ &= 81 + 324 - (208.2631\ldots) \\ &= 196.7368\ldots \\ \therefore b &= \sqrt{196.7368\ldots} \\ &= 14.0262\ldots \end{aligned}$$

Length of $AC = 14.0$ cm (to 3 s.f.)

Worked example 19

In $\triangle DEF$, $F = 120°$, side $EF = 25$ m and side $FD = 34$ m.

Calculate the length of side DE.



$DE = f$, so use the cosine rule in the form, $f^2 = d^2 + e^2 - 2de\cos F$.

$$\begin{aligned} f^2 &= 25^2 + 34^2 - (2 \times 25 \times 34 \times \cos 120°) \\ &= 625 + 1156 - (-850) \quad \text{(notice that cos 120° is negative)} \\ &= 625 + 1156 + 850 \\ &= 2631 \\ \therefore f &= \sqrt{2631} \\ &= 51.2932\ldots \end{aligned}$$

Length of $DE = 51.3$ m (to 3 s.f.)

Combining the sine and cosine rules

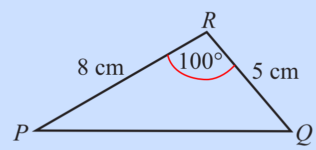
The following worked examples show how you can combine the sine and cosine rules to solve problems.

Worked example 20

Combining the sine and cosine rules

In $\triangle PQR$, $R = 100°$, side $PR = 8$ cm and side $RQ = 5$ cm.

a Calculate the length of side PQ.
b Calculate, correct to the nearest degree, P and Q.



a $PQ = r$, so use the cosine rule in the form, $r^2 = p^2 + q^2 - 2pq\cos R$.

$$\begin{aligned} r^2 &= 5^2 + 8^2 - (2 \times 5 \times 8 \times \cos 100°) \\ &= 25 + 64 - (-13.8918\ldots) \quad \text{(notice that cos 100° is negative)} \\ &= 102.8918\ldots \\ \therefore r &= \sqrt{102.8918\ldots} \\ &= 10.1435\ldots \end{aligned}$$

Length of $PQ = 10.1$ cm (to 3 s.f.)

If you need to use a previously calculated value for a new problem, leave unrounded answers in your calculator to avoid introducing rounding errors.

E

b Now you know the value of r as well as the value of R, you can make use of the sine rule:

$$\frac{\sin P}{p} = \frac{\sin Q}{q} = \frac{\sin R}{r}$$

$$\frac{\sin P}{5} = \frac{\sin Q}{8} = \frac{\sin 100^\circ}{10.1435\ldots}$$

Using the first and third fractions, $\sin P = \dfrac{5 \times \sin 100^\circ}{10.1435\ldots} = 0.4853\ldots$

R is obtuse so P is acute, and $P = 29.0409\ldots^\circ$
$P = 29^\circ$ (to the nearest degree)
To find Q you can use the angle sum of a triangle $= 180^\circ$:

$Q = 180 - (100 + 29)$
$\therefore\ Q = 51^\circ$ (to the nearest degree)

Worked example 21

a Change the subject of the formula $c^2 = a^2 + b^2 - 2ab\cos C$ to $\cos C$.
b Use your answer to part (a) to find the smallest angle in the triangle which has sides of length 7 m, 8 m and 13 m.

a $c^2 = a^2 + b^2 - 2ab\cos C$

$2ab\cos C = a^2 + b^2 - c^2$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

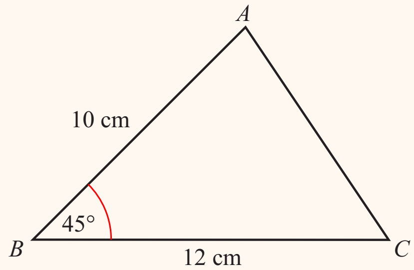
b The smallest angle in a triangle is opposite the shortest side. In the given triangle, the smallest angle is opposite the 7 m side. Let this angle be C.
Then $c = 7$ and
take $a = 8$ and $b = 13$.
Using the result of part (a):

$$\begin{aligned}\cos C &= \frac{8^2 + 13^2 - 7^2}{2 \times 8 \times 13}\\ &= \frac{64 + 169 - 49}{208}\\ &= \frac{184}{208}\\ C &= \cos^{-1}\frac{184}{208}\\ &= 27.7957\ldots^\circ\end{aligned}$$

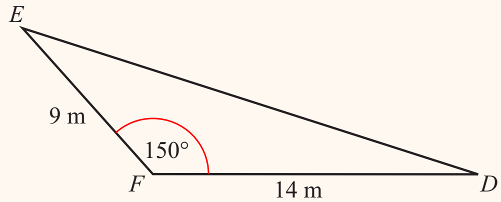
The smallest angle of the triangle $= 27.8^\circ$ (to 1 d.p.)

Exercise 15.11

1 In ΔABC, $B = 45°$, side $AB = 10$ cm and side $BC = 12$ cm. Calculate the length of side AC.

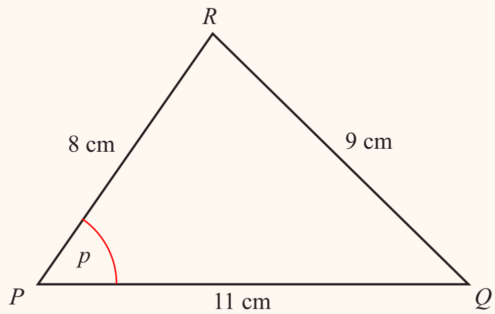


2 In ΔDEF, $F = 150°$, side $EF = 9$ m and side $FD = 14$ m. Calculate the length of side DE.



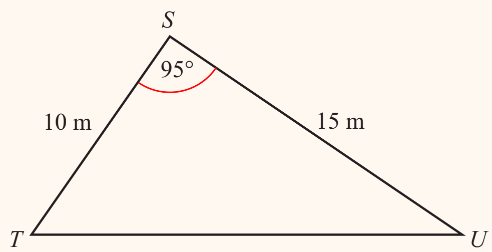
3 In ΔPQR, side $PQ = 11$ cm, side $QR = 9$ cm and side $RP = 8$ cm.

Calculate the size of p correct to 1 decimal place.



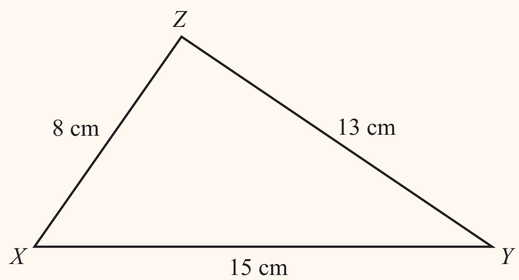
4 In ΔSTU, $S = 95°$, side $ST = 10$ m and side $SU = 15$ m.

a Calculate the length of side TU.
b Calculate U.
c Calculate T.



5 In ΔXYZ, side $XY = 15$ cm, side $YZ = 13$ cm and side $ZX = 8$ cm. Calculate the size of:

a X
b Y
c Z.



REWIND

Look back at the beginning of this chapter and remind yourself about bearings.

6 A boat sails in a straight line from Aardvark Island on a bearing of 060°. When the boat has sailed 8 km it reaches Beaver Island and then turns to sail on a bearing of 150°. The boat remains on this bearing until it reaches Crow Island, 12 km from Beaver Island. On reaching Crow Island the boat's pilot decides to return directly to Aardvark Island.

Calculate:

a The length of the return journey.
b The bearing on which the pilot must steer his boat to return to Aardvark Island.

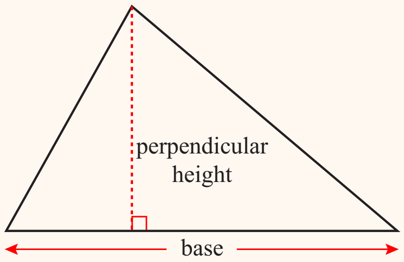
7 Jason stands in the corner of a very large field. He walks, on a bearing of 030°, a distance of d metres. Jason then changes direction and walks twice as far on a new bearing of 120°. At the end of the walk Jason calculates both the distance he must walk and the bearing required to return to his original position. Given that the total distance walked is 120 metres, what answers will Jason get if he is correct?

15.7 Area of a triangle

REWIND

The area of a triangle was first encountered in chapter 7. ◀

You already know that the area of a triangle is given by the following formula:

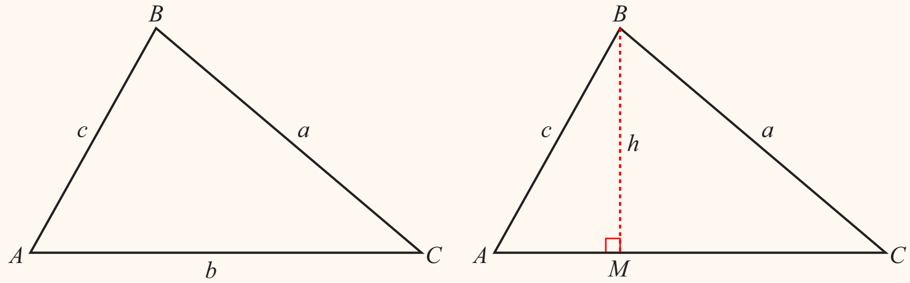


$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

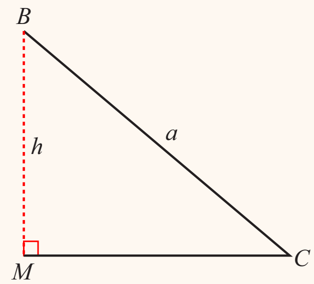
This method can be used if you know both the length of the base and perpendicular height but if you don't have these values you need to use another method.

You can calculate the area of any triangle by using trigonometry.

Look at the triangle ABC shown in the diagram:



The second copy of the triangle is drawn with a perpendicular height that you don't yet know. But if you draw the right-angled triangle BCM separately, you can use basic trigonometry to find the value of h.



Now note that opp(C) = h and the hypotenuse = a.

Using the sine ratio: $\sin C = \frac{h}{a} \Rightarrow h = a \sin C$

This means that you now know the perpendicular height and can use the base length b to calculate the area:

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular height}$$

$$= \frac{1}{2} \times b \times a \sin C$$

$$\text{Area} = \frac{1}{2} ab \sin C$$

In fact you could use any side of the triangle as the base and draw the perpendicular height accordingly. This means that the area can also be calculated with:

$\text{Area} = \frac{1}{2} ac \sin B$ or $\text{Area} = \frac{1}{2} bc \sin A$

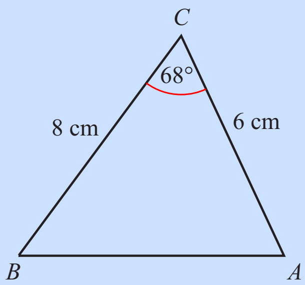
In each case the sides used meet at the angle that has been included.

E

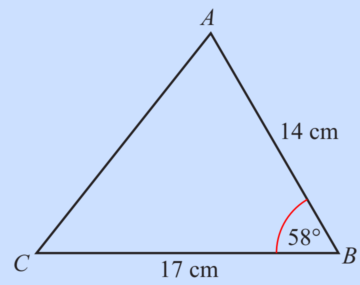
Worked example 22

Calculate the areas of each of the following shapes.

a



b



a

$$\text{Area} = \frac{1}{2}ab\sin C$$
$$= \frac{1}{2} \times 8 \times 6 \times \sin 68°$$
$$= 22.3\,\text{cm}^2 \text{ (to 1 dp)}$$

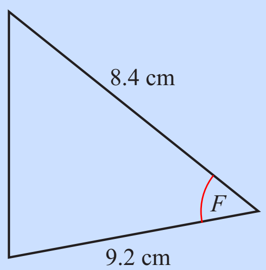
b

$$\text{Area} = \frac{1}{2}ac\sin B$$
$$= \frac{1}{2} \times 17 \times 14 \times \sin 58°$$
$$= 100.9\,\text{cm}^2 \text{ (to 1 dp)}$$

Worked example 23

The diagram shows a triangle with area $20\,\text{cm}^2$.

Calculate the size of angle F.



Notice that the area $= \frac{1}{2} \times 8.4 \times 9.2 \times \sin F = 20$

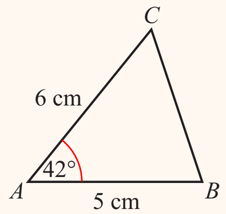
$$\sin F = \frac{2 \times 20}{8.4 \times 9.2}$$
$$\text{So } F = \sin^{-1}\left(\frac{2 \times 20}{8.4 \times 9.2}\right)$$
$$= 31.2° \text{ (to 1 dp)}$$

Exercise 15.12

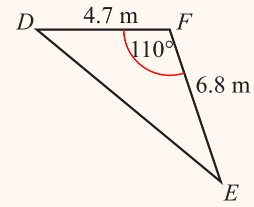
E

1 Find the area of each triangle.

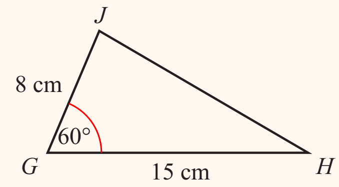
a



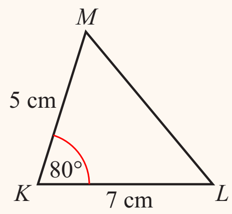
b



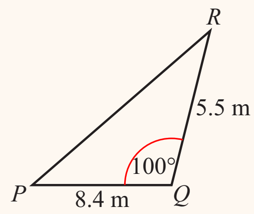
c



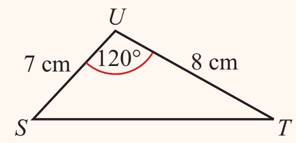
d



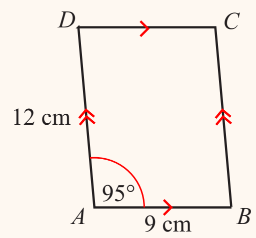
e



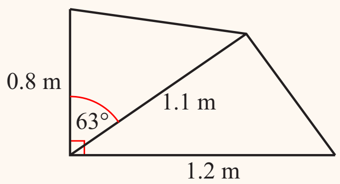
f



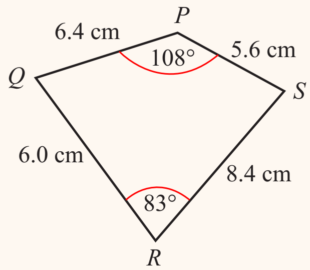
2 Find the area of the parallelogram shown in the diagram.



3 The diagram shows the dimensions of a small herb garden. Find the area of the garden. Give your answer correct to two decimal places.

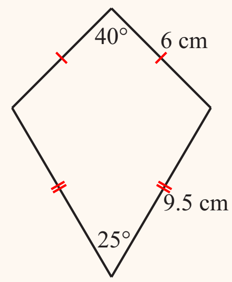


4 Find the area of *PQRS*.

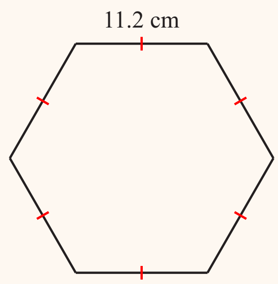


5 Find the area of each polygon. Give your answers to 1 decimal place.

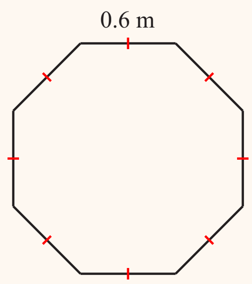
a



b



c



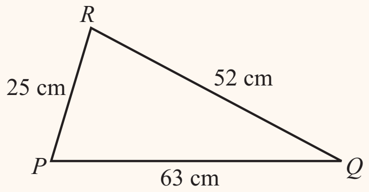
E

6 The diagonals of a parallelogram bisect each other at an angle of 42°. If the diagonals are 26 cm and 20 cm long, find:

a the area of the parallelogram

b the lengths of the sides.

7



The diagrams shows ΔPQR, which has an area of 630 cm^2.

a Use the formula Area $= \frac{1}{2} pr \sin Q$ to find Q correct to 1 decimal place.

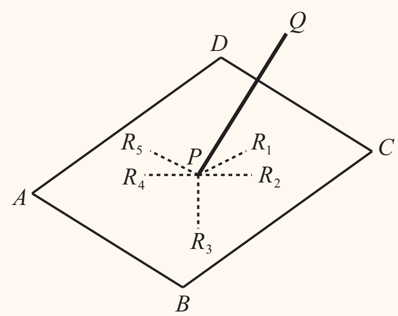
b Find P correct to 1 decimal place.

15.8 Trigonometry in three dimensions

The final part of the trigonometry chapter looks at how to use the ratios in three dimensions. With problems of this kind you must draw and label each triangle as you use it. This will help you to organise your thoughts and keep your solution tidy.

When you work with solids you may need to calculate the angle between an edge, or a diagonal, and one of the faces. This is called the angle between a line and a plane.

Consider a line PQ, which meets a plane $ABCD$ at point P. Through P draw lines PR_1, PR_2, PR_3, … in the plane and consider the angles QPR_1, QPR_2, QPR_3…

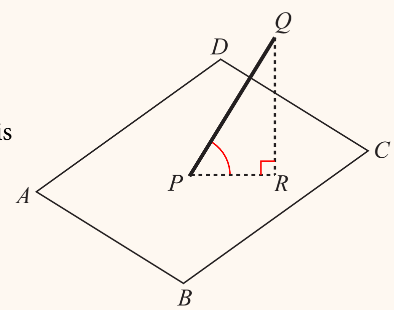


- If PQ is perpendicular to the plane, all these angles will be right angles.
- If PQ is not perpendicular to the plane, these angles will vary in size.

It is the smallest of these angles which is called the angle between the line PQ and the plane $ABCD$.

To identify this angle, do the following:

- From Q draw a perpendicular to the plane. Call the foot of this perpendicular R.
- The angle between the line PQ and the plane is angle QPR.



PR is called the **projection** of PQ on the plane $ABCD$.

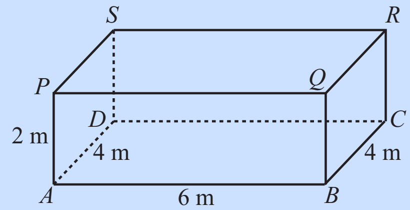
The following worked example shows how a problem in three dimensions might be tackled.

E

It can be helpful to use colour or shading in diagrams involving 3D situations.

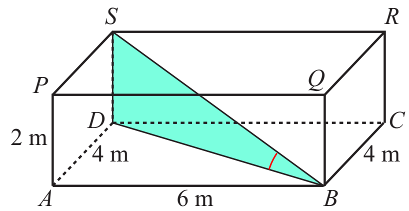
Worked example 24

The diagram represents a room which has the shape of a cuboid. $AB = 6\,\text{m}$, $AD = 4\,\text{m}$, and $AP = 2\,\text{m}$. Calculate the angle between the diagonal BS and the floor $ABCD$.



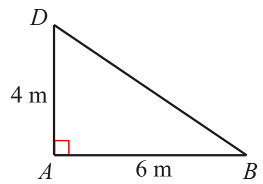
First identify the angle required. B is the point where the diagonal BS meets the plane $ABCD$.

SD is the perpendicular from S to the plane $ABCD$ and so DB is the projection of SB onto the plane.



The angle required is SBD.
You know that ΔSBD has a right angle at D and that $SD = 2\,\text{m}$ (equal to AP).

To find angle SBD, you need to know the length of DB or the length of SB. You can find the length of BD by using Pythagoras' theorem in ΔABD.



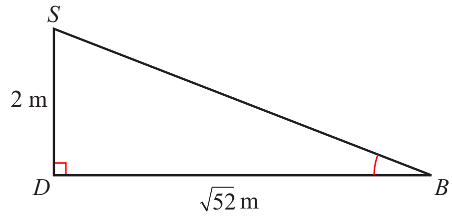
$BD^2 = 6^2 + 4^2 = 36 + 16 = 52$

$BD = \sqrt{52}$

So, using right-angled triangle SBD:

$$\tan B = \frac{\text{opp}(B)}{\text{adj}(B)} = \frac{SD}{BD} = \frac{2}{\sqrt{52}}$$

$$\text{Angle } S\hat{B}D = \tan^{-1}\left(\frac{2}{\sqrt{52}}\right) = 15.5013\ldots$$



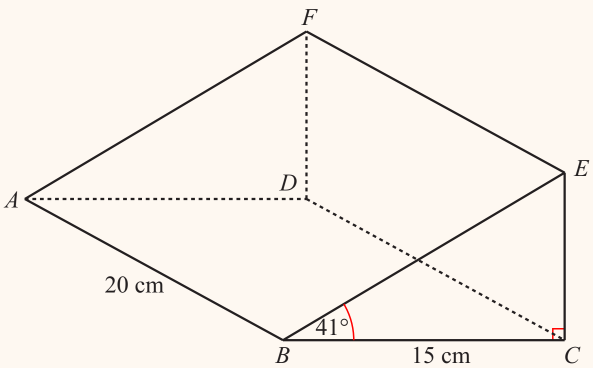
The angle between diagonal BS and the floor $ABCD = 15.5°$ (to 1 d.p.)

Exercise 15.13

1 The diagram represents a triangular prism. The rectangular base, $ABCD$, is horizontal.

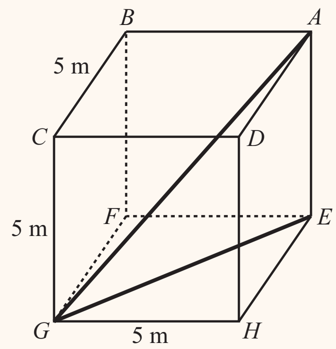
$AB = 20\,\text{cm}$ and $BC = 15\,\text{cm}$.
The cross-section of the prism, BCE, is right-angled at C and angle $EBC = 41°$.

a Calculate the length of AC.
b Calculate the length of EC.
c Calculate the angle which the line AE makes with the horizontal.



E

2 The cube shown in the diagram has sides of 5 m.

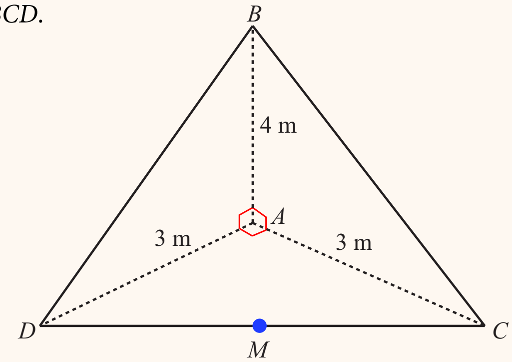


a Use Pythagoras' theorem to calculate the distance *EG*. Leave your answer in exact form.

b Use Pythagoras' theorem to calculate the distance *AG*. Leave your answer in exact form.

c Calculate the angle between the line *AG* and the plane *EFGH*. Give your answer to 1 decimal place.

3 The diagram shows a tetrahedron *ABCD*. *M* is the mid-point of *CD*. *AB* = 4 m, *AC* = 3 m, *AD* = 3 m.



a Calculate angle *ACB*.

b Calculate *BC*.

c Calculate *CD*.

d Calculate the length of *BM*.

e Calculate the angle *BCD*.

4 A cuboid is 14 cm long, 5 cm wide and 3 cm high. Calculate:

a the length of the diagonal on its base

b the length of its longest diagonal

c the angle between the base and the longest diagonal.

5 *ABCD* is a tetrahedral drinks carton. Triangle *ABC* is the base and *B* is a right angle. *D* is vertically above *A*.

Calculate the following in terms of the appropriate lettered side(s):

a the length of *AC*

b length of *DA*

c the length of *DC*

d the size of angle *DAB*

e the size of angle *BDC*

f the size of angle *ADC*.

Summary

Do you know the following?

- A scale drawing is an accurate diagram to represent something that is much bigger, or much smaller.
- An angle of elevation is measured upwards from the horizontal.
- An angle of depression is measured downwards from the horizontal.
- Bearings are measured clockwise from north.
- The ratio of any two lengths in a right-angled triangle depends on the angles in the triangle:
  - $\sin A = \frac{\text{opp}(A)}{\text{hyp}}$.
  - $\cos A = \frac{\text{adj}(A)}{\text{hyp}}$.
  - $\tan A = \frac{\text{opp}(A)}{\text{adj}}$.
- You can use these trigonometric ratios to calculate an unknown angle from two known sides.
- You can use these trigonometric ratios to calculate an unknown side from a known side and a known angle.
- The sine, cosine and tangent function can be extended beyond the angles in triangles.
- The sine, cosine and tangent functions can be used to solve trigonometric equations.
- The sine and cosine rules can be used to calculate unknown sides and angles in triangles that are not right-angled.
- The sine rule is used for calculating an angle from another angle and two sides, or a side from another side and two known angles. The sides and angles must be arranged in opposite pairs.
- The cosine rule is used for calculating an angle from three known sides, or a side from a known angle and two known sides.
- You can calculate the area of a non right-angled triangle by using the sine ratio.

Are you able to…..?

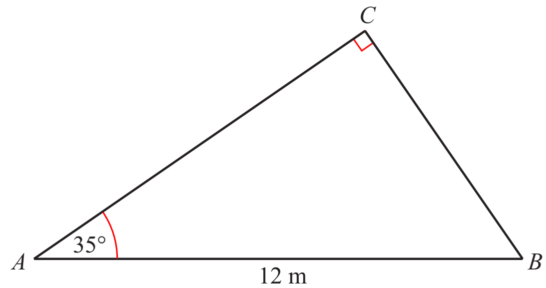
- calculate angles of elevation
- calculate angles of depression
- use trigonometry to calculate bearings
- identify which sides are the opposite, adjacent and hypotenuse
- calculate the sine, cosine and tangent ratio when given lengths in a right-angled triangle
- use the sine, cosine and tangent ratios to find unknown angles and sides
- solve more complex problems by extracting right-angled triangles and combining sine, cosine and tangent ratios
- use the sine and cosine rules to find unknown angles and sides in right-angled triangles
- use the sine, cosine and tangent functions to solve trigonometric equations, finding all the solutions between 0 and 360°
- use sine and cosine rules to find unknown angles and sides in triangles that are not right-angled
- use trigonometry in three dimensions
- find the area of a triangle that is not right-angled.

Examination practice

Exam-style questions

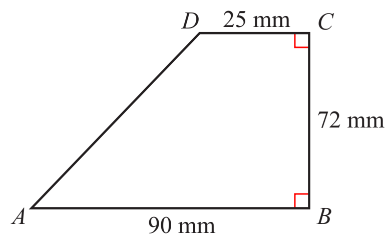
1 The diagram shows the cross-section of the roof of Mr Haziz's house. The house is 12 m wide, angle $CAB = 35°$ and angle $ACB = 90°$.

Calculate the lengths of the two sides of the roof, AC and BC.



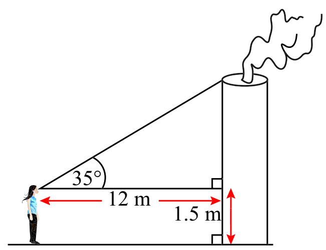
2 The diagram shows a trapezium $ABCD$ in which angles $ABC = BCD = 90°$. $AB = 90$ mm, $BC = 72$ mm and $CD = 25$ mm.

Calculate the angle DAB.

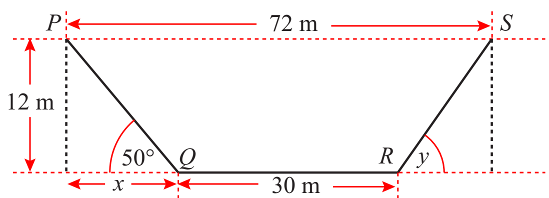


3 A girl, whose eyes are 1.5 m above the ground, stands 12 m away from a tall chimney. She has to raise her eyes 35° upwards from the horizontal to look directly at the top of the chimney.

Calculate the height of the chimney.



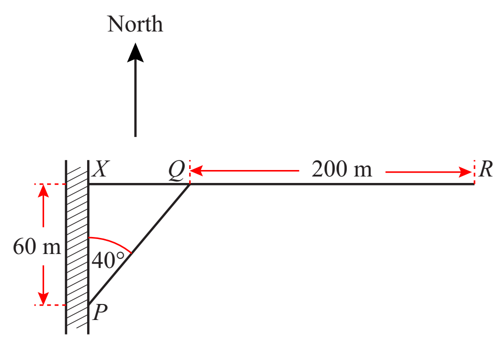
4 The diagram shows the cross-section, $PQRS$, of a cutting made for a road. PS and QR are horizontal. PQ makes an angle of 50° with the horizontal.



a Calculate the horizontal distance between P and Q (marked x in the diagram).

b Calculate the angle which RS makes with the horizontal (marked y in the diagram).

5 A game warden is standing at a point P alongside a road which runs north–south. There is a marker post at the point X, 60 m north of his position. The game warden sees a lion at Q on a bearing of 040° from him and due east of the marker post.



a **i** Show by calculation that the distance, QX, of the lion from the road is 50.3 m, correct to 3 significant figures.

ii Calculate the distance, PQ, of the lion from the game warden.

b Another lion appears at R, 200 m due east of the first one at Q.

i Write down the distance XR.

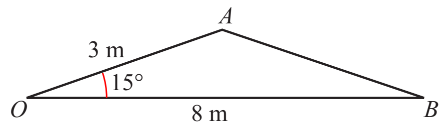
ii Calculate the distance, PR, of the second lion from the game warden.

iii Calculate the bearing of the second lion from the game warden, correct to the nearest degree.

6 In the ΔOAB, angle $AOB = 15°$, $OA = 3$ m and $OB = 8$ m.
Calculate, correct to 2 decimal places:

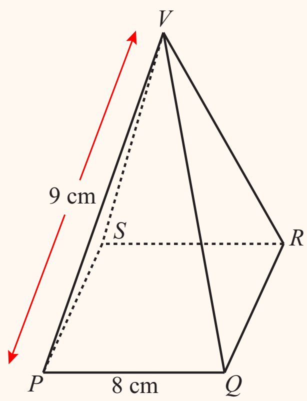
a the length of AB

b the area of ΔOAB.



E

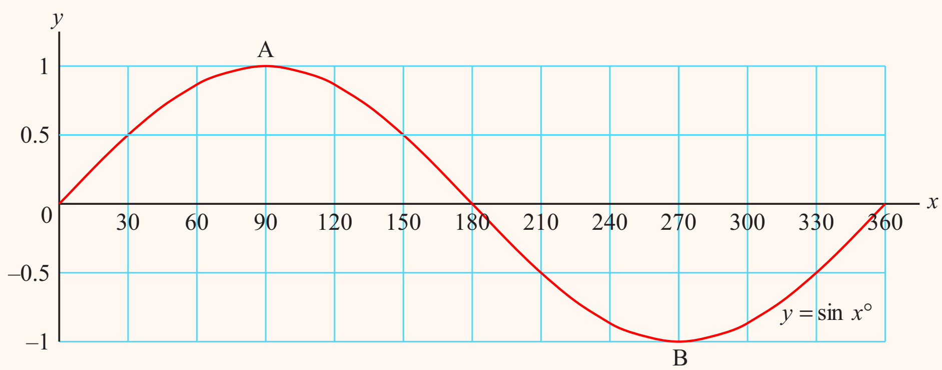
7 A pyramid, $VPQRS$, has a square base, $PQRS$, with sides of length 8 cm. Each sloping edge is 9 cm long.



a Calculate the perpendicular height of the pyramid.

b Calculate the angle the sloping edge VP makes with the base.

8 The diagram shows the graph of $y = \sin x$ for $0 \leqslant x \leqslant 360$.



a Write down the co-ordinates of A, the point on the graph where $x = 90°$.

b Find the value of $\sin 270°$.

c On a copy of the diagram, draw the line $y = -\frac{1}{2}$ for $0 \leqslant x \leqslant 360$.

d How many solutions are there for the equation $\sin x = -\frac{1}{2}$ for $0 \leqslant x \leqslant 360$?

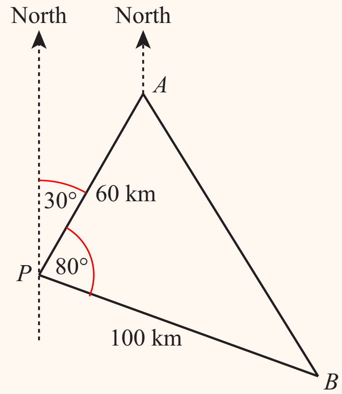
9 Two ships leave port P at the same time. One ship sails 60 km on a bearing of 030° to position A. The other ship sails 100 km on a bearing of 110° to position B.

a Calculate:

- **i** the distance AB
- **ii** $P\hat{A}B$
- **iii** the bearing of B from A.

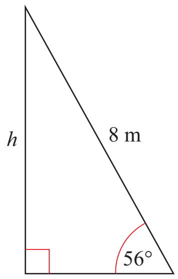
b Both ships took the same time, t hours, to reach their positions. The speed of the *faster* ship was 20 km/h. Write down:

- **i** the value of t
- **ii** the speed of the slower ship.



Past paper questions

1 The diagram shows a ladder of length 8 m leaning against a vertical wall.

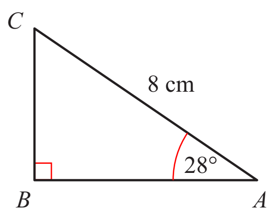


NOT TO SCALE

Use trigonometry to calculate h.
Give your answer correct to 2 significant figures. [3]

[Cambridge IGCSE Mathematics 0580 Paper 11 Q19 October/November 2013]

2



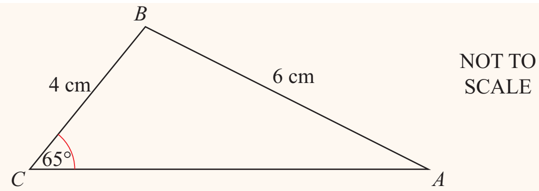
NOT TO SCALE

Calculate the length of AB.

[2]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q4 May/June 2014]

3 In triangle ABC, $AB = 6$ cm, $BC = 4$ cm and angle $BCA = 65°$.



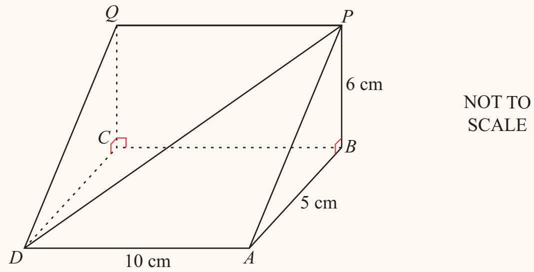
Calculate

a angle CAB, [3]

b the area of triangle ABC. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q21 October/November 2013]

4 The diagram shows a triangular prism.



$ABCD$ is a horizontal rectangle with $DA = 10$ cm and $AB = 5$ cm.

$BCQP$ is a vertical rectangle and $BP = 6$ cm.

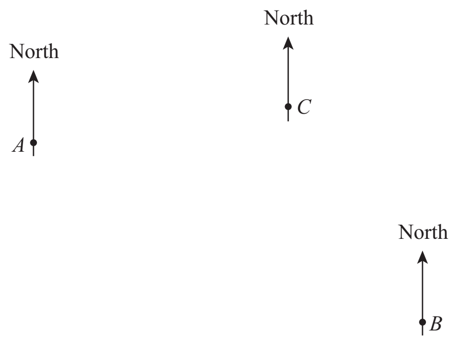
Calculate

a the length of DP, [3]

b the angle between DP and the horizontal rectangle $ABCD$. [3]

[Cambridge IGCSE Mathematics 0580 Paper 23 Q24 October/November 2012]

5 The diagram shows the positions of three towns A, B and C. The scale is 1 cm represents 2 km.



Scale: 1cm = 2 km

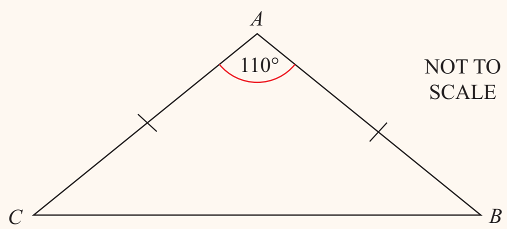
a **i** Find the distance in kilometres from A to B. [2]

ii Town D is 9 km from A on a bearing of 135°. Mark the position of town D on the diagram. [2]

iii Measure the bearing of A from C. [1]

[Cambridge IGCSE Mathematics 0580 Paper 33 Q10 October/November 2012]

6



Triangle ABC is isosceles with $AB = AC$.
Angle $BAC = 110°$ and the area of the triangle is 85 cm^2.
Calculate AC. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q13 October/November 2014]