Chapter 5: Fractions and standard form

Key words

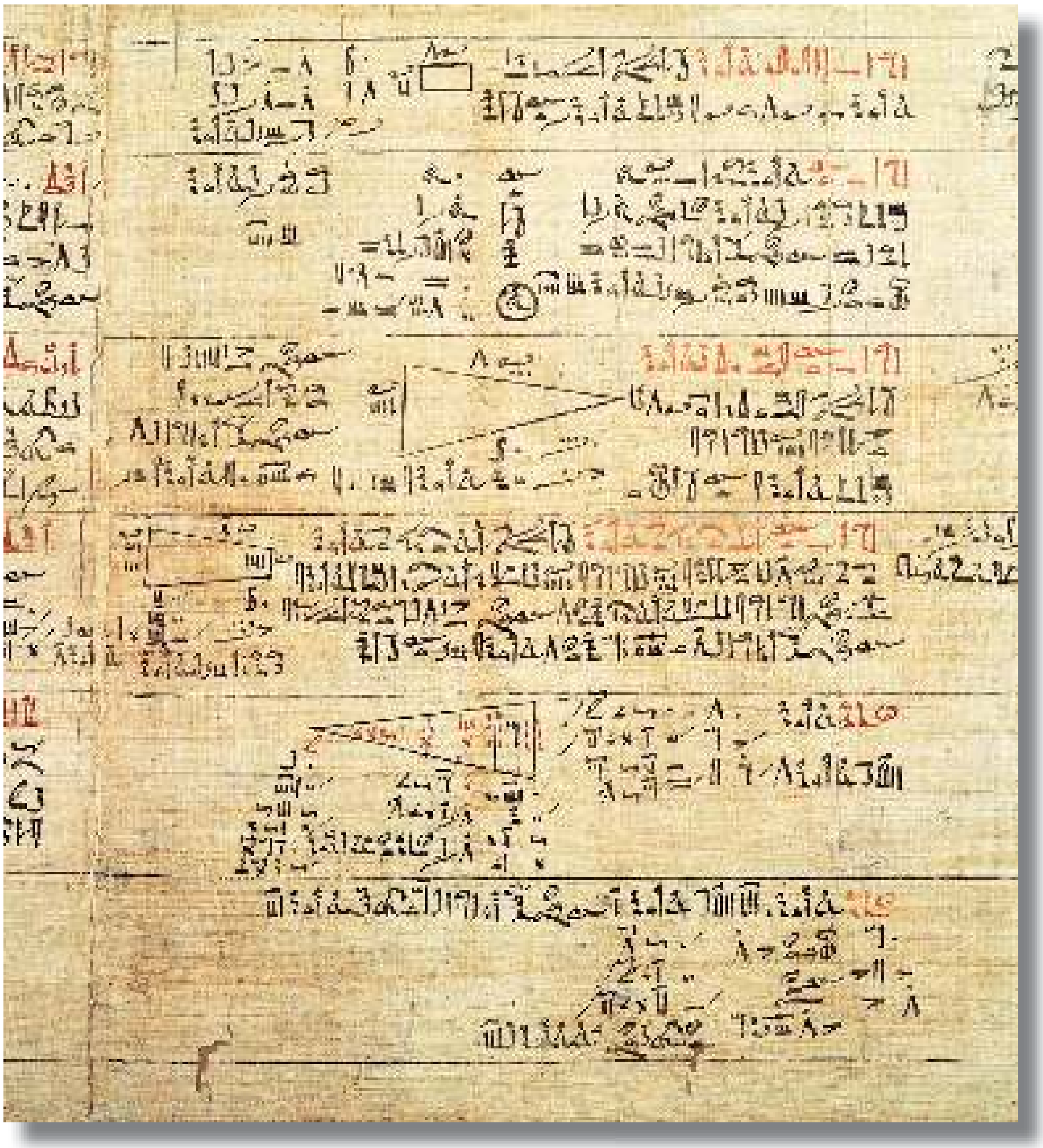
- Fraction
- Vulgar fraction
- Numerator
- Denominator
- Equivalent fraction
- Simplest form
- Lowest terms
- Mixed number
- Common denominator
- Reciprocal
- Percentage
- Percentage increase
- Percentage decrease
- Reverse percentage
- Standard form
- Estimate

In this chapter you will learn how to:

- find equivalent fractions
- simplify fractions
- add, subtract, multiply and divide fractions and mixed numbers
- find fractions of numbers
- find one number as a percentage of another
- find a percentage of a number
- calculate percentage increases and decreases
- increase and decrease by a given percentage

EXTENDED

- handle reverse percentages (undoing increases and decreases)
- work with standard form
- make estimations without a calculator.



The Rhind Mathematical Papyrus is one of the earliest examples of a mathematical document. It is thought to have been written sometime between 1600 and 1700 BC by an Egyptian scribe called Ahmes, though it may be a copy of an older document. The first section of it is devoted to work with fractions.

Fractions are not only useful for improving your arithmetic skills. You use them, on an almost daily basis, often without realising it. How far can you travel on half a tank of petrol? If your share of a pizza is two-thirds will you still be hungry? If three-fifths of your journey is complete how far do you still have to travel? A hairdresser needs to mix her dyes by the correct amount and a nurse needs the correct dilution of a drug for a patient.

RECAP

You should already be familiar with the following fractions work:

Equivalent fractions

Find equivalent fractions by multiplying or dividing the numerator and denominator by the same number.

$\frac{1}{2} \times \frac{4}{4} = \frac{4}{8}$ $\qquad \frac{1}{2}$ and $\frac{4}{8}$ are equivalent

$\frac{40}{50} \div \frac{10}{10} = \frac{4}{5}$ $\qquad \frac{40}{50}$ and $\frac{4}{5}$ are equivalent

To simplify a fraction you divide the numerator and denominator by the same number.

$\frac{18}{40} = \frac{18 \div 2}{40 \div 2} = \frac{9}{20}$

Mixed numbers

Convert between mixed numbers and improper fractions:

$3\frac{4}{7} = \frac{(3 \times 7) + 4}{7} = \frac{25}{7}$

Number of data in that group, not individual values.

Calculating with fractions

To add or subtract fractions make sure they have the same denominators.

$\frac{7}{8} + \frac{1}{3} = \frac{21+8}{24} = \frac{29}{24} = 1\frac{5}{24}$

Class intervals are equal and should not overlap.

To multiply fractions, multiply numerators by numerators and denominators by denominators. Write the answer in simplest form.

Multiply to find a fraction of an amount. The word 'of' means multiply.

$\frac{3}{8} \times \frac{3}{4} = \frac{9}{32}$

$$\begin{aligned} \frac{3}{8} \text{ of } 12 &= \frac{3}{8} \times \frac{12}{1} \\ &= \frac{36}{8} \\ &= 4\frac{1}{2} \end{aligned}$$

To divide by a fraction you multiply by its reciprocal.

$12 \div \frac{1}{3} = 12 \times \frac{3}{1} = 36 \qquad \frac{2}{5} \div \frac{1}{2} = \frac{2}{5} \times \frac{2}{1} = \frac{4}{5}$

Percentages

The symbol % means per cent or per hundred.

Percentages can be written as fractions and decimals.

$45\% = \frac{45}{100} = \frac{9}{20}$

$45\% = 45 \div 100 = 0.45$

Calculating percentages

To find a percentage of an amount:

use fractions and cancel $\qquad$ or $\qquad$ use decimals $\qquad$ or $\qquad$ use a calculator

25% of 60 =

$\frac{\cancel{25}^{1}}{\cancel{100}_{4}} \times \frac{\cancel{60}^{15}}{1} = 15 \qquad 0.25 \times 60 = 15 \qquad \boxed{2}\ \boxed{5}\ \boxed{\%}\ \boxed{\times}\ \boxed{6}\ \boxed{0}\ \boxed{=}\ 15$

5.1 Equivalent fractions

REWIND

Before reading this next section you should remind yourself about Highest Common Factors (HCFs) in chapter 1. ◀

A **fraction** is part of a whole number.

Common fractions (also called **vulgar fractions**) are written in the form $\frac{a}{b}$. The number on the top, a, can be any number and is called the **numerator**. The number on the bottom, b, can be any number except 0 and is called the **denominator**. The numerator and the denominator are separated by a horizontal line.

If you multiply or divide both the numerator and the denominator by the same number, the new fraction still represents the same amount of the whole as the original fraction. The new fraction is known as an **equivalent fraction**.

For example, $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$ and $\frac{25}{35} = \frac{25 \div 5}{35 \div 5} = \frac{5}{7}$.

REWIND

You have come across simplifying in chapter 2 in the context of algebra. ◀

Notice in the second example that the original fraction $\left(\frac{25}{35}\right)$ has been divided to smaller terms and that as 5 and 7 have no common factor other than 1, the fraction cannot be divided any further. The fraction is now expressed in its **simplest form** (sometimes called the **lowest terms**). So, simplifying a fraction means expressing it using the lowest possible terms.

LINK

Percentages are particularly important when we deal with money. How often have you been in a shop where the signs tell you that prices are reduced by 10%? Have you considered a bank account and how money is added? The study of financial ideas forms the greater part of economics.

Worked example 1

Express each of the following in the simplest form possible.

a $\frac{3}{15}$ **b** $\frac{16}{24}$ **c** $\frac{21}{28}$ **d** $\frac{5}{8}$

a $\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}$

b $\frac{16}{24} = \frac{16 \div 8}{24 \div 8} = \frac{2}{3}$

c $\frac{21}{28} = \frac{21 \div 7}{28 \div 7} = \frac{3}{4}$

d $\frac{5}{8}$ is already in its simplest form (5 and 8 have no common factors other than 1).

Notice that in each case you divide the numerator and the denominator by the HCF of both.

Worked example 2

Which two of $\frac{5}{6}, \frac{20}{25}$ and $\frac{15}{18}$ are equivalent fractions?

Simplify each of the other fractions: $\frac{5}{6}$ is already in its simplest form.

$\frac{20}{25} = \frac{20 \div 5}{25 \div 5} = \frac{4}{5}$

$\frac{15}{18} = \frac{15 \div 3}{18 \div 3} = \frac{5}{6}$

So $\frac{5}{6}$ and $\frac{15}{18}$ are equivalent.

You could have written:

$\frac{\cancel{15}^{5}}{\cancel{18}_{6}} = \frac{5}{6}$

This is called *cancelling* and is a shorter way of showing what you have done.

Exercise 5.1

1 By multiplying or dividing both the numerator and denominator by the same number, find three equivalent fractions for each of the following.

a $\frac{5}{9}$ **b** $\frac{3}{7}$ **c** $\frac{12}{18}$ **d** $\frac{18}{36}$ **e** $\frac{110}{128}$

2 Express each of the following fractions in its simplest form.

a $\frac{7}{21}$ **b** $\frac{3}{9}$ **c** $\frac{9}{12}$ **d** $\frac{15}{25}$ **e** $\frac{500}{2500}$ **f** $\frac{24}{36}$ **g** $\frac{108}{360}$

5.2 Operations on fractions

Multiplying fractions

When multiplying two or more fractions together you can simply multiply the numerators and then multiply the denominators. Sometimes you will then need to simplify your answer. It can be faster to cancel the fractions before you multiply.

Worked example 3

Calculate:

a $\frac{3}{4}\times\frac{2}{7}$ **b** $\frac{5}{7}\times 3$ **c** $\frac{3}{8}$ of $4\frac{1}{2}$

a $\frac{3}{4}\times\frac{2}{7}=\frac{3\times 2}{4\times 7}=\frac{6}{28}=\frac{3}{14}$

Multiply the numerators to get the new numerator value. Then do the same with the denominators. Then express the fraction in its simplest form.

Notice that you can also cancel before multiplying:

$\frac{3}{\cancel{4}_{2}}\times\frac{\cancel{2}^{1}}{7}=\frac{3\times 1}{2\times 7}=\frac{3}{14}$

Divide the denominator of the first fraction, and the numerator of the second fraction, by two.

b $\frac{5}{7}\times 3=\frac{5\times 3}{7\times 1}=\frac{15}{7}$

15 and 7 do not have a common factor other than 1 and so cannot be simplified.

c $\frac{3}{8}$ of $4\frac{1}{2}$

Here, you have a **mixed number** ($4\frac{1}{2}$). This needs to be changed to an improper fraction (sometimes called a top heavy fraction), which is a fraction where the numerator is larger than the denominator. This allows you to complete the multiplication.

$\frac{3}{8}\times 4\frac{1}{2}=\frac{3}{8}\times\frac{9}{2}=\frac{27}{16}$ Notice that the word 'of' is replaced with the $\times$ sign.

To multiply a fraction by an integer you only multiply the numerator by the integer. For example, $\frac{5}{7}\times 3=\frac{5\times 3}{7}=\frac{15}{7}$.

To change a mixed number to a vulgar fraction, multiply the whole number part (in this case 4) by the denominator and add it to the numerator. So: $4\frac{1}{2}=\frac{4\times 2+1}{2}=\frac{9}{2}$

Exercise 5.2

Evaluate each of the following.

1 **a** $\frac{2}{3}\times\frac{5}{9}$ **b** $\frac{1}{2}\times\frac{3}{7}$ **c** $\frac{1}{4}\times\frac{8}{9}$ **d** $\frac{2}{7}\times\frac{14}{16}$

2 **a** $\frac{50}{128}\times\frac{256}{500}$ **b** $1\frac{1}{3}\times\frac{2}{7}$ **c** $2\frac{2}{7}\times\frac{7}{8}$ **d** $\frac{4}{5}$ of $3\frac{2}{7}$

e $1\frac{1}{3}$ of 24 **f** $5\frac{1}{2}\times 7\frac{1}{4}$ **g** $8\frac{8}{9}\times 20\frac{1}{4}$ **h** $7\frac{2}{3}\times 10\frac{1}{2}$

REWIND

You will need to use the lowest common multiple (LCM) in this section. You met this in chapter 1.

Adding and subtracting fractions

You can only add or subtract fractions that are the same type. In other words, they must have the same denominator. This is called a **common denominator**. You must use what you know about equivalent fractions to help you make sure fractions have a common denominator.

The following worked example shows how you can use the LCM of both denominators as the common denominator.

Worked example 4

Write each of the following as a single fraction in its simplest form.

a $\frac{1}{2}+\frac{1}{4}$ **b** $\frac{3}{4}+\frac{5}{6}$ **c** $2\frac{3}{4}-1\frac{5}{7}$

a	$\frac{1}{2}+\frac{1}{4}$ $=\frac{2}{4}+\frac{1}{4}$ $=\frac{3}{4}$	Find the common denominator. The LCM of 2 and 4 is 4. Use this as the common denominator and find the equivalent fractions. Then add the numerators.
b	$\frac{3}{4}+\frac{5}{6}$ $=\frac{9}{12}+\frac{10}{12}$ $=\frac{19}{12}$ $=1\frac{7}{12}$	Find the common denominator. The LCM of 4 and 6 is 12. Use this as the common denominator and find the equivalent fractions. Add the numerators. Change an improper fraction to a mixed number.
c	$2\frac{3}{4}-1\frac{5}{7}$ $=\frac{11}{4}-\frac{12}{7}$ $=\frac{77}{28}-\frac{48}{28}$ $=\frac{77-48}{28}$ $=\frac{29}{28}$ $=1\frac{1}{28}$	Change mixed numbers to improper fractions to make them easier to handle. The LCM of 4 and 7 is 28, so this is the common denominator. Find the equivalent fractions. Subtract one numerator from the other. Change an improper fraction to a mixed number.

Notice that, once you have a common denominator, you only add the numerators. *Never* add the denominators!

You will sometimes find that two fractions added together can result in an improper fraction (sometimes called a top-heavy fraction). Usually you will re-write this as a mixed number.

The same rules apply for subtracting fractions as adding them.

Egyptian fractions

An Egyptian fraction is the sum of any number of *different* fractions (different denominators) each with numerator one. For example $\frac{1}{2}+\frac{1}{3}$ is the Egyptian fraction that represents $\frac{5}{6}$. Ancient Egyptians used to represent single fractions in this way but in modern times we tend to prefer the single fraction that results from finding a common denominator.

Tip

Egyptian fractions are a good example of manipulating fractions but they are not in the syllabus.

Exercise 5.3

Evaluate the following.

1 **a** $\frac{1}{3}+\frac{1}{3}$ **b** $\frac{3}{7}+\frac{2}{7}$ **c** $\frac{5}{8}-\frac{3}{8}$ **d** $\frac{5}{9}+\frac{8}{9}$

e $\frac{1}{6}+\frac{1}{5}$ **f** $\frac{2}{3}-\frac{5}{8}$ **g** $2\frac{5}{8}-1\frac{3}{4}$ **h** $5\frac{1}{8}-3\frac{1}{16}$

2 **a** $4-\frac{2}{3}$ **b** $6+\frac{5}{11}$ **c** $11+7\frac{1}{4}$ **d** $11-7\frac{1}{4}$

e $3\frac{1}{2}-4\frac{1}{3}$ **f** $5\frac{1}{4}+3\frac{1}{16}+4\frac{3}{8}$ **g** $5\frac{1}{8}-3\frac{1}{16}+4\frac{3}{4}$ **h** $1\frac{1}{3}+2\frac{2}{5}-1\frac{1}{4}$

i $\frac{3}{7}+\frac{2}{3}\times\frac{14}{8}$ **j** $3\frac{1}{2}-2\frac{1}{4}\times\frac{4}{3}$ **k** $3\frac{1}{6}-1\frac{1}{2}+7\frac{3}{4}$ **l** $2\frac{1}{4}-3\frac{1}{3}+4\frac{1}{5}$

Remember to use BODMAS here.

3 Find Egyptian fractions for each of the following.

a $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{5}{8}$ **d** $\frac{3}{16}$

Think which two fractions with a numerator of 1 might have an LCM equal to the denominator given.

FAST FORWARD

The multiplication, division, addition and subtraction of fractions will be revisited in chapter 14 when *algebraic* fractions are considered.

Dividing fractions

Before describing how to divide two fractions, the **reciprocal** needs to be introduced. The reciprocal of any fraction can be obtained by swapping the numerator and the denominator.

So, the reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$ and the reciprocal of $\frac{7}{2}$ is $\frac{2}{7}$.

Also the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or just 2 and the reciprocal of 5 is $\frac{1}{5}$.

If any fraction is multiplied by its reciprocal then the result is always 1. For example:

$\frac{1}{3}\times\frac{3}{1}=1$, $\frac{3}{8}\times\frac{8}{3}=1$ and $\frac{a}{b}\times\frac{b}{a}=1$

To divide one fraction by another fraction, you simply multiply the first fraction by the reciprocal of the second.

Look at the example below:

$$\frac{a}{b}\div\frac{c}{d}=\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}$$

Now multiply both the numerator and denominator by bd and cancel:

$$\frac{a}{b}\div\frac{c}{d}=\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)}=\frac{\left(\frac{a}{\cancel{b}}\right)\times\cancel{b}d}{\left(\frac{c}{\cancel{d}}\right)\times b\cancel{d}}$$

$$=\frac{ad}{bc}=\frac{a}{b}\times\frac{d}{c}$$

Worked example 5

Evaluate each of the following.

a $\frac{3}{4}\div\frac{1}{2}$ **b** $1\frac{3}{4}\div2\frac{1}{3}$ **c** $\frac{5}{8}\div2$ **d** $\frac{6}{7}\div3$

a

$\frac{3}{4}\div\frac{1}{2}=\frac{3}{\cancelto{2}{4}}\times\frac{\cancelto{1}{2}}{1}=\frac{3}{2}=1\frac{1}{2}$

Multiply by the reciprocal of $\frac{1}{2}$. Use the rules you have learned about multiplying fractions.

b	$1\frac{3}{4}\div 2\frac{1}{3}=\frac{7}{4}\div\frac{7}{3}$ $=\frac{\cancel{7}^{1}}{4}\times\frac{3}{\cancel{7}_{1}}$ $=\frac{3}{4}$	Convert the mixed fractions to improper fractions. Multiply by the reciprocal of $\frac{7}{3}$.
c	$\frac{5}{8}\div 2=\frac{5}{8}\div\frac{2}{1}$ $=\frac{5}{8}\times\frac{1}{2}$ $=\frac{5}{16}$	Write 2 as an improper fraction. Multiply by the reciprocal of $\frac{2}{1}$.
d	$\frac{6}{7}\div 3=\frac{\cancel{6}^{2}}{7}\times\frac{1}{\cancel{3}_{1}}$ $=\frac{2}{7}$	

To divide a fraction by an integer you can either just multiply the denominator by the integer, *or* divide the numerator by the same integer.

Exercise 5.4

Evaluate each of the following.

1 $\frac{1}{7}\div\frac{1}{3}$ **2** $\frac{2}{5}\div\frac{3}{7}$ **3** $\frac{4}{9}\div 7$ **4** $\frac{10}{11}\div 5$

5 $4\frac{1}{5}\div\frac{1}{7}$ **6** $3\frac{1}{5}\div 5\frac{2}{3}$ **7** $7\frac{7}{8}\div 5\frac{1}{12}$ **8** $3\frac{1}{4}\div 3\frac{1}{2}$

9 Evaluate

a $\left(2\frac{1}{3}-1\frac{2}{5}\right)\div 1\frac{1}{3}$ **b** $2\frac{1}{3}-1\frac{2}{5}\div 1\frac{1}{3}$

Fractions with decimals

Sometimes you will find that either the numerator or the denominator, or even both, are *not* whole numbers! To express these fractions in their simplest forms you need to

- make sure both the numerator and denominator are converted to an integer by finding an equivalent fraction
- check that the equivalent fraction has been simplified.

Worked example 6

Simplify each of the following fractions.

a $\frac{0.1}{3}$ **b** $\frac{1.3}{2.4}$ **c** $\frac{36}{0.12}$

a	$\frac{0.1}{3}=\frac{0.1\times 10}{3\times 10}=\frac{1}{30}$	Multiply 0.1 by 10 to convert 0.1 to an integer. To make sure the fraction is equivalent, you need to do the same to the numerator and the denominator, so multiply 3 by 10 as well.
b	$\frac{1.3}{2.4}=\frac{1.3\times 10}{2.4\times 10}=\frac{13}{24}$	Multiply both the numerator and denominator by 10 to get integers. 13 and 24 do not have a HCF other than 1 so cannot be simplified.
c	$\frac{36}{0.12}=\frac{36\times 100}{0.12\times 100}=\frac{3600}{12}=300$	Multiply 0.12 by 100 to produce an integer. Remember to also multiply the numerator by 100, so the fraction is equivalent. The final fraction can be simplified by cancelling.

Exercise 5.5

Remember that any fraction that contains a decimal in either its numerator or denominator will not be considered to be simplified.

What fraction can be used to represent 0.3?

Simplify each of the following fractions.

1 $\frac{0.3}{12}$ **2** $\frac{0.4}{0.5}$ **3** $\frac{6}{0.7}$ **4** $\frac{0.7}{0.14}$

5 $\frac{36}{1.5}$ **6** $0.3\times\frac{5}{12}$ **7** $0.4\times\frac{1.5}{1.6}$ **8** $\frac{2.8}{0.7}\times\frac{1.44}{0.6}$

Further calculations with fractions

You can use fractions to help you solve problems.

Remember for example, that $\frac{2}{3}=2\times\frac{1}{3}$ and that, although this may seem trivial, this simple fact can help you to solve problems easily.

Worked example 7

Suppose that $\frac{2}{5}$ of the students in a school are girls. If the school has 600 students, how many girls are there?

Remember in worked example 3, you saw that 'of' is replaced by ×.

$\frac{2}{5}$ of $600=\frac{2}{5}\times600=\frac{2}{\cancel{5}_{1}}\times\frac{\cancel{600}^{120}}{1}=2\times120=240$ girls

Worked example 8

Now imagine that $\frac{2}{5}$ of the students in another school are boys, and that there are 360 boys. How many students are there in the whole school?

$\frac{2}{5}$ of the total is 360, so $\frac{1}{5}$ of the total must be half of this, 180. This means that $\frac{5}{5}$ of the population, that is all of it, is 5 × 180 = 900 students in total.

Exercise 5.6

1 $\frac{3}{4}$ of the people at an auction bought an item. If there are 120 people at the auction, how many bought something?

2 An essay contains 420 sentences. 80 of these sentences contain typing errors. What fraction (given in its simplest form) of the sentences contain errors?

3 28 is $\frac{2}{7}$ of which number?

4 If $\frac{3}{5}$ of the people in a theatre buy a snack during the interval, and of those who buy a snack $\frac{5}{7}$ buy ice cream, what fraction of the people in the theatre buy ice cream?

5 Andrew, Bashir and Candy are trying to save money for a birthday party. If Andrew saves $\frac{1}{4}$ of the total needed, Bashir saves $\frac{2}{5}$ and Candy saves $\frac{1}{10}$, what fraction of the cost of the party is left to pay?

E

6 Joseph needs $6\frac{1}{2}$ cups of cooked rice for a recipe of Nasi Goreng. If 2 cups of uncooked rice with $2\frac{1}{2}$ cups of water make $4\frac{1}{3}$ cups of cooked rice, how many cups of uncooked rice does Joseph need for his recipe ? How much water should he add ?

5.3 Percentages

A **percentage** is a fraction with a denominator of 100. The symbol used to represent percentage is %.

To find 40% of 25, you simply need to find $\frac{40}{100}$ of 25. Using what you know about multiplying fractions:

$$\frac{40}{100}\times 25 = \frac{\cancel{40}^{\,2}}{{}_{5}\cancel{100}}\times\frac{25}{1}$$

$$= \frac{2}{{}_{1}\cancel{5}}\times\frac{\cancel{25}^{\,5}}{1}$$

$$= \frac{2}{1}\times\frac{5}{1} = 10$$

$\therefore$ 40% of 25 = 10

Equivalent forms

A percentage can be converted into a decimal by dividing by 100 (notice that the digits move two places to the right). So, $45\% = \frac{45}{100} = 0.45$ and $3.1\% = \frac{3.1}{100} = 0.031$.

A decimal can be converted to a percentage by multiplying by 100 (notice that the digits move two places to the left). So, $0.65 = \frac{65}{100} = 65\%$ and $0.7 \times 100 = 70\%$.

Converting percentages to vulgar fractions (and vice versa) involves a few more stages.

Worked example 9

Convert each of the following percentages to fractions in their simplest form.

a 25% **b** 30% **c** 3.5%

a	$25\% = \frac{25}{100} = \frac{1}{4}$	Write as a fraction with a denominator of 100, then simplify.
b	$30\% = \frac{30}{100} = \frac{3}{10}$	Write as a fraction with a denominator of 100, then simplify.
c	$3.5\% = \frac{3.5}{100} = \frac{35}{1000} = \frac{7}{200}$	Write as a fraction with a denominator of 100, then simplify.

Remember that a fraction that contains a decimal is not in its simplest form.

Worked example 10

Convert each of the following fractions into percentages.

a $\frac{1}{20}$ **b** $\frac{1}{8}$

a	$\frac{1}{20} = \frac{1\times 5}{20\times 5} = \frac{5}{100} = 5\%$	Find the equivalent fraction with a denominator of 100. (Remember to do the same thing to both the numerator and denominator). $\left(\frac{5}{100} = 0.05,\ 0.05\times 100 = 5\%\right)$

b $\frac{1}{8} = \frac{1 \times 12.5}{8 \times 12.5}$

$= \frac{12.5}{100} = 12.5\%$

Find the equivalent fraction with a denominator of 100. (Remember to do the same thing to both the numerator and denominator).

$\left(\frac{12.5}{100} = 0.125,\ 0.125 \times 100 = 12.5\%\right)$

Although it is not always easy to find an equivalent fraction with a denominator of 100, any fraction can be converted into a percentage by multiplying by 100 and cancelling.

Worked example 11

Convert the following fractions into percentages:

a $\frac{3}{40}$ **b** $\frac{8}{15}$

a $\frac{3}{40} \times \frac{100}{1} = \frac{30}{4} = \frac{15}{2} = 7.5,$

so $\frac{3}{40} = 7.5\%$

b $\frac{8}{15} \times \frac{100}{1} = \frac{160}{3} = 53.3$ (1 d.p.),

so $\frac{8}{15} = 53.3\%$ (1 d.p.)

Exercise 5.7

Later in the chapter you will see that a percentage can be greater than 100.

1 Convert each of the following percentages into fractions in their simplest form.

a 70% **b** 75% **c** 20% **d** 36% **e** 15% **f** 2.5%

g 215% **h** 132% **i** 117.5% **j** 108.4% **k** 0.25% **l** 0.002%

2 Express the following fractions as percentages.

a $\frac{3}{5}$ **b** $\frac{7}{25}$ **c** $\frac{17}{20}$ **d** $\frac{3}{10}$ **e** $\frac{8}{200}$ **f** $\frac{5}{12}$

Finding one number as a percentage of another

To write one number as a percentage of another number, you start by writing the first number as a fraction of the second number then multiply by 100.

Worked example 12

a Express 16 as a percentage of 48.

$\frac{16}{48} = \frac{16}{48} \times 100 = 33.3\%$ (1 d.p.)

$\frac{16}{48} = \frac{1}{3} \times 100 = 33.3\%$ (1 d.p.)

First, write 16 as a fraction of 48, then multiply by 100.

This may be easier if you write the fraction in its simplest form first.

b Express 15 as a percentage of 75.

$\frac{15}{75} \times 100$

$= \frac{1}{5} \times 100 = 20\%$

Write 15 as a fraction of 75, then simplify and multiply by 100. You know that 100 divided by 5 is 20, so you don't need a calculator.

c Express 18 as a percentage of 23.

You need to calculate $\frac{18}{23} \times 100$, but this is not easy using basic fractions because you cannot simplify it further, and 23 does not divide neatly into 100. Fortunately, you can use your calculator. Simply type:

1 8 ÷ 2 3 × 1 0 0 = 78.26% (2 d.p.)

Exercise 5.8

Where appropriate, give your answer to 3 significant figures.

1 Express 14 as a percentage of 35.

2 Express 3.5 as a percentage of 14.

3 Express 17 as a percentage of 63.

4 36 people live in a block of flats. 28 of these people jog around the park each morning. What percentage of the people living in the block of flats go jogging around the park?

5 Jack scores $\frac{19}{24}$ in a test. What percentage of the marks did Jack get?

6 Express 1.3 as a percentage of 5.2.

7 Express 0.13 as a percentage of 520.

Percentage increases and decreases

Suppose the cost of a book increases from \$12 to \$15. The actual increase is \$3. As a fraction of the original value, the increase is $\frac{3}{12} = \frac{1}{4}$. This is the fractional change and you can write this fraction as 25%. In this example, the value of the book has increased by 25% of the original value. This is called the **percentage increase**. If the value had reduced (for example if something was on sale in a shop) then it would have been a **percentage decrease**.

Note carefully: whenever increases or decreases are stated as percentages, they are stated as percentages of the *original* value.

Worked example 13

The value of a house increases from \$120 000 to \$124 800 between August and December. What percentage increase is this?

$\$124\,800 - \$120\,000 = \$4800$	First calculate the increase.
$\%\,\text{increase} = \frac{\text{increase}}{\text{original}} \times 100\%$	Write the increase as a fraction of the original and multiply by 100.
$= \frac{4800}{120\,000} \times 100\% = 4\%$	Then do the calculation (either in your head or using a calculator).

Exercise 5.9

Applying your skills

Where appropriate, give your answer to the nearest whole percent.

1 Over a five-year period, the population of the state of Louisiana in the United States of America decreased from 4 468 976 to 4 287 768. Find the percentage decrease in the population of Louisiana in this period.

2 Sunil bought 38 CDs one year and 46 the next year. Find the percentage increase.

3 A theatre has enough seats for 450 audience members. After renovation it is expected that this number will increase to 480. Find the percentage increase.

4 Sally works in an electrical component factory. On Monday she makes a total of 363 components but on Tuesday she makes 432. Calculate the percentage increase.

5 Inter Polation Airlines carried a total of 383 402 passengers one year and 287 431 the following year. Calculate the percentage decrease in passengers carried by the airline.

6 A liquid evaporates steadily. In one hour the mass of liquid in a laboratory container decreases from 0.32 kg to 0.18 kg. Calculate the percentage decrease.

Increasing and decreasing by a given percentage

If you know what percentage you want to increase or decrease an amount by, you can find the actual increase or decrease by finding a percentage of the original. If you want to know the new value you either add the increase to or subtract the decrease from the original value.

Remember that you are always considering a percentage of the *original* value.

Worked example 14

Increase 56 by: **a** 10% **b** 15% **c** 4%

a

$10\% \text{ of } 56 = \frac{10}{100} \times 56$

$= \frac{1}{10} \times 56 = 5.6$

$56 + 5.6 = 61.6$

First of all, you need to calculate 10% of 56 to work out the size of the increase.

To increase the original by 10% you need to add this to 56.

If you don't need to know the actual increase but just the final value, you can use this method:

If you consider the original to be 100% then adding 10% to this will give 110% of the original. So multiply 56 by $\frac{110}{100}$, which gives 61.6.

b

$\frac{115}{100} \times 56 = 64.4$

A 15% increase will lead to 115% of the original.

c

$\frac{104}{100} \times 56 = 58.24$

A 4% increase will lead to 104% of the original.

Worked example 15

In a sale all items are reduced by 15%. If the normal selling price for a bicycle is \$120 calculate the sale price.

$100 - 15 = 85$

$\frac{85}{100} \times \$120 = \$102$

Note that reducing a number by 15% leaves you with 85% of the original. So you simply find 85% of the original value.

Exercise 5.10

1 Increase 40 by:

a 10% **b** 15% **c** 25% **d** 5% **e** 4%

2 Increase 53 by:

a 50% **b** 84% **c** 13.6% **d** 112% **e** $\frac{1}{2}\%$

3 Decrease 124 by:

a 10% **b** 15% **c** 30% **d** 4% **e** 7%

4 Decrease 36.2 by:

a 90% **b** 35.4% **c** 0.3% **d** 100% **e** $\frac{1}{2}$%

Applying your skills

5 Shajeen usually works 30 hours per week but decides that he needs to increase this by 10% to be sure that he can save enough for a holiday. How many hours per week will Shajeen need to work?

6 12% sales tax is applied to all items of clothing sold in a certain shop. If a T-shirt is advertised for $12 (before tax) what will be the cost of the T-shirt once tax is added?

7 The Oyler Theatre steps up its advertising campaign and manages to increase its audiences by 23% during the year. If 21 300 people watched plays at the Oyler Theatre during the previous year, how many people watched plays in the year of the campaign?

8 The population of Trigville was 153 000 at the end of a year. Following a flood, 17% of the residents of Trigville moved away. What was the population of Trigville after the flood?

9 Anthea decides that she is watching too much television. If Anthea watched 12 hours of television in one week and then decreased this by 12% the next week, how much time did Anthea spend watching television in the second week? Give your answer in hours and minutes to the nearest minute.

E

Reverse percentages

Sometimes you are given the value or amount of an item *after* a percentage increase or decrease has been applied to it and you need to know what the original value was. To solve this type of **reverse percentage** question it is important to remember that you are always dealing with percentages of the *original* values. The method used in worked example 14 (b) and (c) is used to help us solve these type of problems.

Worked example 16

A store is holding a sale in which every item is reduced by 10%. A jacket in this sale is sold for $108. How can you find the original price of the Jacket?

$\frac{90}{100} \times x = 108$ $x = \frac{100}{90} \times 108$ original price = $120.	If an item is reduced by 10%, the new cost is 90% of the original (100–10). If x is the original value of the jacket then you can write a formula using the new price. Notice that when the $\times \frac{90}{100}$ was moved to the other side of the = sign it became its reciprocal, $\frac{100}{90}$.

Important: Undoing a 10% decrease is *not* the same as increasing the reduced value by 10%. If you *increase* the sale price of $108 by 10% you will get $\frac{110}{100} \times \$108 = \$118.80$ which is a different *(and incorrect)* answer.

Exercise 5.11

E

1 If 20% of an amount is 35, what is 100%?

2 If 35% of an amount is 127, what is 100%?

3 245 is 12.5% of an amount. What is the total amount?

4 The table gives the sale price and the % by which the price was reduced for a number of items. Copy the table, then complete it by calculating the original prices.

Sale price ($)	% reduction	Original price ($)
52.00	10	
185.00	10	
4700.00	5	
2.90	5	
24.50	12	
10.00	8	
12.50	7	
9.75	15	
199.50	20	
99.00	25	

5 A shop keeper marks up goods by 22% before selling them. The selling price of ten items are given below. For each one, work out the cost price (the price before the mark up).

a \$25.00 **b** \$200.00 **c** \$14.50 **d** \$23.99 **e** \$15.80
f \$45.80 **g** \$29.75 **h** \$129.20 **i** \$0.99 **j** \$0.80

6 Seven students were absent from a class on Monday. This is 17.5% of the class.

a How many students are there in the class in total?
b How many students were present on Monday?

7 A hat shop is holding a 10% sale. If Jack buys a hat for \$18 in the sale, how much did the hat cost before the sale?

8 Nick is training for a swimming race and reduces his weight by 5% over a 3-month period. If Nick now weighs 76 kg how much did he weigh before he started training?

9 The water in a pond evaporates at a rate of 12% per week. If the pond now contains 185 litres of water, approximately how much water was in the pond a week ago?

5.4 Standard form

When numbers are very small, like 0.0000362, or very large, like 358 000 000, calculations can be time consuming and it is easy to miss out some of the zeros. **Standard form** is used to express very small and very large numbers in a compact and efficient way. In standard form, numbers are written as a number multiplied by 10 raised to a given power.

Remember that digits are in place order:

1000s	100s	10s	units		10ths	100ths	1000ths
3	0	0	0	•	0	0	0

Standard form for large numbers

The key to standard form for large numbers is to understand what happens when you multiply by powers of 10. Each time you multiply a number by 10 each digit within the number moves one place order to the left (notice that this *looks* like the decimal point has moved one place to the right).

3.2

$3.2 \times 10 = 32.0$ The digits have moved one place order to the left.

$3.2 \times 10^2 = 3.2 \times 100 = 320.0$ The digits have moved two places.

$3.2 \times 10^3 = 3.2 \times 1000 = 3200.0$ The digits have moved three places.

... and so on. You should see a pattern forming.

Any large number can be expressed in standard form by writing it as a number between 1 and 10 multiplied by a suitable power of 10. To do this write the appropriate number between 1 and 10 first (using the non-zero digits of the original number) and then count the number of places you need to move the first digit to the left. The number of places tells you by what power, 10 should be multiplied.

Worked example 17

Write 320 000 in standard form.

3.2	Start by finding the number between 1 and 10 that has the same digits in the same order as the original number. Here, the extra 4 zero digits can be excluded because they do not change the size of your new number.
5 4 3 2 1 3 2 0 0 0 0.0 3.2	Now compare the position of the first digit in both numbers: '3' has to move 5 place orders to the left to get from the new number to the original number.
$320\,000 = 3.2 \times 10^5$	The first digit, '3', has moved five places. So, you multiply by 10^5.

Calculating using standard form

REWIND

The laws of indices can be found in chapter 2.◄

Once you have converted large numbers into standard form, you can use the index laws to carry out calculations involving multiplication and division.

Worked example 18

Solve and give your answer in standard form.

a $(3\times10^5)\times(2\times10^6)$ **b** $(2\times10^3)\times(8\times10^7)$

c $(2.8\times10^6)\div(1.4\times10^4)$ **d** $(9\times10^6)+(3\times10^8)$

a

$$(3\times10^5)\times(2\times10^6) = (3\times2)\times(10^5\times10^6)$$
$$= 6\times10^{5+6}$$
$$= 6\times10^{11}$$

Simplify by putting like terms together. Use the laws of indices where appropriate.

Write the number in standard form.

You may be asked to convert your answer to an ordinary number. To convert 6×10^{11} into an ordinary number, the '6' needs to move 11 places to the left:

6.0×10^{11}

11 10 9 8 7 6 5 4 3 2 1

= 6 0 0 0 0 0 0 0 0 0 0 0.0

b

$$(2\times10^3)\times(8\times10^7) = (2\times8)\times(10^3\times10^7)$$
$$= 16\times10^{10}$$
$$16\times10^{10} = 1.6\times10\times10^{10}$$
$$= 1.6\times10^{11}$$

The answer 16×10^{10} is numerically correct but it is not in standard form because 16 is not between 1 and 10. You can change it to standard form by thinking of 16 as 1.6×10.

c

$$(2.8\times10^6)\div(1.4\times10^4) = \frac{2.8\times10^6}{1.4\times10^4} = \frac{2.8}{1.4}\times\frac{10^6}{10^4}$$
$$= 2\times10^{6-4}$$
$$= 2\times10^2$$

Simplify by putting like terms together. Use the laws of indices.

Although it is the place order that is changing; it looks like the decimal point moves to the right.

When you solve problems in standard form you need to check your results carefully. Always be sure to check that your final answer is in standard form. Check that all conditions are satisfied. Make sure that the number part is between 1 and 10.

To make it easier to add up the ordinary numbers make sure they are lined up so that the place values match:

$$\begin{array}{r} 300\,000\,000 \\ +\quad 9\,000\,000 \end{array}$$

d $(9 \times 10^6) + (3 \times 10^8)$

When adding or subtracting numbers in standard form it is often easiest to re-write them both as ordinary numbers first, then convert the answer to standard form.

$9 \times 10^6 = \quad 9\,000\,000$

$3 \times 10^8 = 300\,000\,000$

So $(9 \times 10^6) + (3 \times 10^8) = 300\,000\,000 + 9\,000\,000$

$= 309\,000\,000$

$= 3.09 \times 10^8$

Exercise 5.12

When converting standard form back to an ordinary number, the power of 10 tells you how many places the first digit moves to the left (or decimal point moves to the right), not how many zeros there are.

Remember that you can write these as ordinary numbers before adding or subtracting.

1 Write each of the following numbers in standard form.

a 380 **b** 4 200 000 **c** 45 600 000 000 **d** 65 400 000 000 000
e 20 **f** 10 **g** 10.3 **h** 5

2 Write each of the following as an ordinary number.

a 2.4×10^6 **b** 3.1×10^8 **c** 1.05×10^7 **d** 9.9×10^3 **e** 7.1×10^1

3 Simplify each of the following, leaving your answer in standard form.

a $(2 \times 10^{13}) \times (4 \times 10^{17})$ **b** $(1.4 \times 10^8) \times (3 \times 10^4)$ **c** $(1.5 \times 10^{13})^2$
d $(12 \times 10^5) \times (11 \times 10^2)$ **e** $(0.2 \times 10^{17}) \times (0.7 \times 10^{16})$ **f** $(9 \times 10^{17}) \div (3 \times 10^{16})$
g $(8 \times 10^{17}) \div (4 \times 10^{16})$ **h** $(1.5 \times 10^8) \div (5 \times 10^4)$ **i** $(2.4 \times 10^{64}) \div (8 \times 10^{21})$
j $(1.44 \times 10^7) \div (1.2 \times 10^4)$ **k** $\dfrac{(1.7 \times 10^8)}{(3.4 \times 10^5)}$ **l** $(4.9 \times 10^5) \times (3.6 \times 10^9)$

4 Simplify each of the following, leaving your answer in standard form.

a $(3 \times 10^4) + (4 \times 10^3)$ **b** $(4 \times 10^6) - (3 \times 10^5)$ **c** $(2.7 \times 10^3) + (5.6 \times 10^5)$
d $(7.1 \times 10^9) - (4.3 \times 10^7)$ **e** $(5.8 \times 10^9) - (2.7 \times 10^3)$

Standard form for small numbers

You have seen that digits move place order to the left when multiplying by powers of 10. If you *divide* by powers of 10 move the digits in place order to the right and make the number *smaller*.

Consider the following pattern:

2300

$2300 \div 10 = 230$

$2300 \div 10^2 = 2300 \div 100 = 23$

$2300 \div 10^3 = 2300 \div 1000 = 2.3$

. . . and so on.

The digits move place order to the right (notice that this looks like the decimal point is moving to the left). You saw in chapter 1 that if a direction is taken to be positive, the opposite direction is taken to be negative. Since moving place order to the left raises 10 to the power of a *positive* index, it follows that moving place order to the right raises 10 to the power of a *negative* index.

Also remember from chapter 2 that you can write negative powers to indicate that you divide, and you saw above that with small numbers, you divide by 10 to express the number in standard form.

LINK

Astronomy deals with very large and very small numbers and it would be clumsy and potentially inaccurate to write these out in full every time you needed them. Standard form makes calculations and recording much easier.

Worked example 19

Write each of the following in standard form.

a 0.004 **b** 0.000 000 34 **c** $(2\times10^{-3})\times(3\times10^{-7})$

a

1 2 3
0.0 0 4
4.0
$= 4\times10^{-3}$

Start with a number between 1 and 10, in this case 4.

Compare the position of the first digit: '4' needs to move 3 place orders to the right to get from the new number to the original number. In worked example 17 you saw that moving 5 places to the *left* meant multiplying by 10^5, so it follows that moving 3 places to the *right* means multiply by 10^{-3}.

Notice also that the first non-zero digit in 0.004 is in the 3rd place after the decimal point and that the power of 10 is –3.

Alternatively: you know that you need to divide by 10 three times, so you can change it to a fractional index and then a negative index.

$$0.004 = 4\div10^3$$
$$= 4\times10^{\frac{1}{3}}$$
$$= 4\times10^{-3}.$$

b

$$0.000\,000\,34 = 3.4\div10^7$$
$$= 3.4\times10^{-7}$$

1 2 3 4 5 6 7
0. 0 0 0 0 0 0 3 4 $= 3.4\times10^{-7}$

Notice that the first non-zero digit in 0.000 000 34 is in the 7th place after the decimal point and that the power of 10 is –7.

c

$$(2\times10^{-3})\times(3\times10^{-7})$$
$$=(2\times3)\times(10^{-3}\times10^{-7})$$
$$=6\times10^{-3+-7}$$
$$=6\times10^{-10}$$

Simplify by gathering like terms together.

Use the laws of indices.

Exercise 5.13

When using standard form with negative indices, the power to which 10 is raised tells you the position of the first *non-zero* digit after (to the right of) the decimal point.

For some calculations, you might need to change a term into standard form before you multiply or divide.

Remember that you can write these as ordinary numbers before adding or subtracting.

1 Write each of the following numbers in standard form.

a 0.004 **b** 0.00005 **c** 0.000032 **d** 0.0000000564

2 Write each of the following as an ordinary number.

a 3.6×10^{-4} **b** 1.6×10^{-8} **c** 2.03×10^{-7} **d** 8.8×10^{-3} **e** 7.1×10^{-1}

3 Simplify each of the following, leaving your answer in standard form.

a $(2\times10^{-4})\times(4\times10^{-16})$ **b** $(1.6\times10^{-8})\times(4\times10^{-4})$
c $(1.5\times10^{-6})\times(2.1\times10^{-3})$ **d** $(11\times10^{-5})\times(3\times10^{2})$
e $(9\times10^{17})\div(4.5\times10^{-16})$ **f** $(7\times10^{-21})\div(1\times10^{16})$
g $(4.5\times10^{8})\div(0.9\times10^{-4})$ **h** $(11\times10^{-5})\times(3\times10^{2})\div(2\times10^{-3})$

4 Simplify each of the following, leaving your answer in standard form.

a $(3.1\times10^{-4})+(2.7\times10^{-2})$ **b** $(3.2\times10^{-1})-(3.2\times10^{-2})$
c $(7.01\times10^{3})+(5.6\times10^{-1})$ **d** $(1.44\times10^{-5})-(2.33\times10^{-6})$

Applying your skills

5 Find the number of seconds in a day, giving your answer in standard form.

6 The speed of light is approximately 3×10^8 metres per second. How far will light travel in:

a 10 seconds **b** 20 seconds **c** 102 seconds

7 Data storage (in computers) is measured in gigabytes. One gigabyte is 2^{30} bytes.

a Write 2^{30} in standard form correct to 1 significant figure.
b There are 1024 gigabytes in a terabyte. How many bytes is this? Give your answer in standard form correct to one significant figure.

5.5 Your calculator and standard form

Standard form is also called scientific notation or exponential notation.

On modern scientific calculators you can enter calculations in standard form. Your calculator will also display numbers with too many digits for screen display in standard form.

Keying in standard form calculations

You will need to use the $\boxed{\times 10^x}$ button or the $\boxed{\text{Exp}}$ or $\boxed{\text{EE}}$ button on your calculator. These are known as the exponent keys. All exponent keys work in the same way, so you can follow the example below on your own calculator using whatever key you have and you will get the same result.

When you use the exponent function key of your calculator, you do NOT enter the '$\times$ 10' part of the calculation. The calculator does that part automatically as part of the function.

Worked example 20

Using your calculator, calculate:

a 2.134×10^4 **b** 3.124×10^{-6}

a	2.134×10^4 $= 21\,340$	Press: $\boxed{2}\ \boxed{.}\ \boxed{1}\ \boxed{3}\ \boxed{4}\ \boxed{\times 10^x}\ \boxed{4}\ \boxed{=}$ This is the answer you will get.
b	3.124×10^{-6} $= 0.000003123$	Press: $\boxed{3}\ \boxed{.}\ \boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{\text{Exp}}\ \boxed{-}\ \boxed{6}\ \boxed{=}$ This is the answer you will get.

Making sense of the calculator display

Different calculators work in different ways and you need to understand how your own calculator works. Make sure you know what buttons to use to enter standard form calculations and how to interpret the display and convert your calculator answer into decimal form.

Depending on your calculator, answers in scientific notation will be displayed on a line with an exponent like this:

```
5.98E-06
```

This is 5.98×10^{-06}

or on two lines with the calculation and the answer, like this:

```
6.23E23*4.11
          2.56E24
```

This is 2.56×10^{24}

If you are asked to give your answer in standard form, all you need to do is interpret the display and write the answer correctly. If you are asked to give your answer as an ordinary number (decimal), then you need to apply the rules you already know to write the answer correctly.

Exercise 5.14

1 Enter each of these numbers into your calculator using the correct function key and write down what appears on your calculator display.

a 4.2×10^{12}	**b** 1.8×10^{-5}	**c** 2.7×10^{6}
d 1.34×10^{-2}	**e** 1.87×10^{-9}	**f** 4.23×10^{7}
g 3.102×10^{-4}	**h** 3.098×10^{9}	**i** 2.076×10^{-23}

2 Here are ten calculator displays giving answers in standard form.

i	1.09 05	**ii**	2.876 -06	**iii**	4.012 09	**iv**	1.89 07
v	3.123E13	**vi**	2.876E-04	**vii**	9.02E15	**viii**	8.076E-12
ix	8.124E-11	**x**	5.0234 19				

a Write out each answer in standard form.
b Arrange the ten numbers in order from smallest to largest.

3 Use your calculator. Give the answers in standard form correct to 5 significant figures.

a 4234^5 **b** $0.0008 \div 9200^3$ **c** $(1.009)^5$
d $123\ 000\ 000 \div 0.00076$ **e** $(97 \times 876)^4$ **f** $(0.0098)^4 \times (0.0032)^3$
g $\frac{8543 \times 9210}{0.000034}$ **h** $\frac{9754}{(0.0005)^4}$

4 Use your calculator to find the answers correct to 4 significant figures.

a $9.27 \times (2.8 \times 10^5)$ **b** $(4.23 \times 10^{-2})^3$ **c** $(3.2 \times 10^7) \div (7.2 \times 10^9)$
d $(3.2 \times 10^{-4})^2$ **e** $231 \times (1.5 \times 10^{-6})$ **f** $(4.3 \times 10^5) + (2.3 \times 10^7)$
g $\sqrt{3.24 \times 10^7}$ **h** $\sqrt[3]{4.2 \times 10^{-8}}$ **i** $\sqrt[3]{4.126 \times 10^{-9}}$

5.6 Estimation

REWIND

For this section you will need to remember how to round an answer to a specified number of significant figures. You covered this in chapter 1.

It is important that you know whether or not an answer that you have obtained is at least roughly as you expected. This section demonstrates how you can produce an approximate answer to a calculation easily.

To **estimate**, the numbers you are using need to be rounded *before* you do the calculation. Although you can use any accuracy, usually the numbers in the calculation are rounded to one significant figure:

$3.9 \times 2.1 \approx 4 \times 2 = 8$

Notice that $3.9 \times 2.1 = 8.19$, so the estimated value of 8 is not too far from the real value!

Worked example 21

Estimate the value of:

a $\frac{4.6 + 3.9}{\sqrt{398}}$ **b** $\sqrt{42.2 - 5.1}$

a

$\frac{4.6+3.9}{\sqrt{398}} \approx \frac{5+4}{\sqrt{400}}$ $= \frac{9}{20} = \frac{4.5}{10} = 0.45$	Round the numbers to 1 significant figure.
Check the estimate: $\frac{4.6+3.9}{\sqrt{398}} = 0.426$ (3sf)	Now if you use a calculator you will find the exact value and see that the estimate was good.

Tip

Note that the '≈' symbol is *only* used at the point where an approximation is made. At other times you should use '=' when two numbers are exactly equal.

b	$\sqrt{42.2-5.1} \approx \sqrt{40-5}$ $= \sqrt{35}$ $\approx \sqrt{36}$ $= 6$	In this question you begin by rounding each value to one significant figure but it is worth noting that you can only easily take the square root of a square number! Round 35 up to 36 to get a square number.

A good starting point for the questions in the following exercise will be to round the numbers to 1 significant figure. Remember that you can sometimes make your calculation even simpler by modifying your numbers again.

Exercise 5.15

1 Estimate the value of each of the following. Show the rounded values that you use.

a $\frac{23.6}{6.3}$ **b** $\frac{4.3}{0.087 \times 3.89}$ **c** $\frac{7.21 \times 0.46}{9.09}$

d $\frac{4.82 \times 6.01}{2.54 + 1.09}$ **e** $\frac{\sqrt{48}}{2.54 + 4.09}$ **f** $(0.45 + 1.89)(6.5 - 1.9)$

g $\frac{23.8 + 20.2}{4.7 + 5.7}$ **h** $\frac{109.6 - 45.1}{19.4 - 13.9}$ **i** $(2.52)^2 \times \sqrt{48.99}$

j $\sqrt{223.8 \times 45.1}$ **k** $\sqrt{9.26} \times \sqrt{99.87}$ **l** $(4.1)^3 \times (1.9)^4$

2 Work out the actual answer for each part of question **1**, using a calculator.

Summary

Do you know the following?

- An equivalent fraction can be found by multiplying or dividing the numerator and denominator by the same number.
- Fractions can be added or subtracted, but you must make sure that you have a common denominator first.
- To multiply two fractions you multiply their numerators and multiply their denominators.
- To divide by a fraction you find its reciprocal and then multiply.
- Percentages are fractions with a denominator of 100.
- Percentage increases and decreases are always percentages of the original value.
- You can use reverse percentages to find the *original* value. E
- Standard form can be used to write very large or very small numbers quickly.
- Estimations can be made by rounding the numbers in a calculation to one significant figure.

Are you able to. . . ?

- find a fraction of a number
- find a percentage of a number
- find one number as a percentage of another number
- calculate a percentage increase or decrease
- find a value before a percentage change E
- do calculations with numbers written in standard form
- find an estimate to a calculation.

Examination practice

Exam-style questions

1 Calculate $\frac{5}{6}\left(\frac{1}{4}+\frac{1}{8}\right)$ giving your answer as a fraction in its lowest terms.

2 93 800 students took an examination.
19% received grade A.
24% received grade B.
31% received grade C.
10% received grade D.
11% received grade E.
The rest received grade U.

a What percentage of the students received grade U?
b What fraction of the students received grade B? Give your answer in its lowest terms.
c How many students received grade A?

3 During one summer there were 27 500 cases of *Salmonella* poisoning in Britain. The next summer there was an increase of 9% in the number of cases. Calculate how many cases there were in the second year.

4 Abdul's height was 160 cm on his 15th birthday. It was 172 cm on his 16th birthday. What was the percentage increase in his height?

Past paper questions

1 Write 0.0000574 in standard form. [1]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q1 May/June 2016]

2 **Do not use a calculator in this question and show all the steps of your working.**
Give each answer as a fraction in its lowest terms.
Work out

a $\frac{3}{4}-\frac{1}{12}$ [2]

b $2\frac{1}{2}\times\frac{4}{25}$ [2]

[Cambridge IGCSE Mathematics 0580 Paper 11 Q21 October/November 2013]

3 Calculate 17.5% of 44 kg. [2]

[Cambridge IGCSE Mathematics 0580 Paper 11 Q10 October/November 2013]

4 **Without using your calculator,** work out
$5\frac{3}{8}-2\frac{1}{5}$.
Give your answer as a fraction in its lowest terms.
You must show all your working. [3]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q17 October/November 2012]

5 Samantha invests \$600 at a rate of 2% per year simple interest.
Calculate the interest Samantha earns in 8 years. [2]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q5 October/November 2012]

6 Show that $\left(\frac{1}{10}\right)^2 + \left(\frac{2}{5}\right)^2 = 0.17$

Write down all the steps in your working. [2]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q6 October/November 2012]

7 Maria pays \$84 rent.
The rent is increased by 5%. Calculate Maria's new rent. [2]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q10 October/November 2012]

8 Huy borrowed \$4500 from a bank at a rate of 5% per year compound **interest**.
He paid back the money and **interest** at the end of 2 years.
How much **interest** did he pay? [3]

[Cambridge IGCSE Mathematics 0580 Paper 13 Q13 May/June 2013]

9 Jasijeet and her brother collect stamps.

When Jasjeet gives her brother 1% of her stamps, she has 2475 stamps left.

Calculate how many stamps Jasjeet had originally [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q14 October/November 2014]

10 **Without using a calculator**, work out $2\frac{5}{8} \times \frac{3}{7}$.

Show all your working and give your answer as a mixed number in its lowest terms. [3]

[Cambridge IGCSE Mathematics 0580 Paper 22 Q14 May/June 2016]