Efficient high-dimensional receptive field inference using a flexible spline basis

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Summary

Spatio-temporal receptive field (STRF) models are frequently used to approximate the computation implemented by a sensory neuron. Typically, such STRFs are assumed to be smooth and sparse. Current empirical Bayes estimation approaches such as automatic relevance determination (ARD), automatic smoothnes determination (ASD) and others encode such prior knowledge into a prior covariance matrix, whose hyperparameters are learned from the data, and thus provide STRF estimates with the desired properties even with little or noisy data [1, 2]. However, empirical Bayes methods are not computationally efficient in high-dimensional settings, as often encountered in sensory neuroscience.

Here we pursue an alternative approach and encode prior knowledge for estimation of STRFs by choosing a set of basis function with the desired properties. We use a natural cubic regression spline basis, which is known as the smoothest possible interpolant [3]. We find that this method provides a good basis for high-dimensional STRFs. On spike recordings from retinal ganglion cells, we show that in the linear STRF model with Gaussian noise, spline-based STRFs are as smooth as STRFs estimated by ASD, provide similar prediction performance but are computationally much more efficient. Adding an L1 penalty allows to achieve smoothness and sparseness at the same time, resulting in very good STRF visualizations. In addition, this method allows efficient recovery of 3D STRFs from noisy two-photon calcium recordings from ganglion cell dendrites. Finally, our spline-based method can be naturally extended to hierarchical subunit models (such as the LNLN-Poisson model). The resulting subunits are already well-separated and smooth using only 5 minutes of data, while traditional estimation methods require much more data to achieve similar results.

Additional Detail

Data: We used multi-electrode extracellular recording data from salamander retinal ganglion cells from [4] for the comparison of prediction performance and computation time between different methods (A), estimating 2D STRFs (B) and the LNLN-Poisson model (D). For 3D STRFs (C), we used two-photon calcium imaging data from [5].

Models: RFs in Fig. A-C were computed in the linear Gaussian noise model. The Maximum Likelikehood estimate (MLE) was calculated as $\hat{k}_{MLE} = (X^T X)^{-1} X^T y$, where X the stimulus design matrix, y the response. Maximum a posteriori (MAP) estimators for the empirical Bayes approach were calculated as $\hat{k}_{MAP} = (X^T X + C^{-1})^{-1} X^T y$, where $C_{ii} = \theta_i^{-1}$ for ARD, $C_{ij} = \exp(-\rho - (x_i - x_j)^2/2\delta^2)$ for ASD, and hyperparameters chosen via evidence optimization [see [1] for details]. For spline-based methods, maximum likelihood with cubic splines can be calculated in closed-form: $\hat{k}_{MLE,SPL} = S(S^T X^T X S)^{-1} S^T X^T y$, where S is the spline basis matrix. The size of S depends only on the size of RF and the degree of freedom (df) of the splines [see [3] for details]. df for all dimensions were chosen by cross-validation. For the computation time analysis, we time

the last step of the computation with the aforementioned equations. Cross-validation performance is defined as $e_{xv} = \frac{1}{n}\sum_{j=1}^{n}(y_{test,j} - X_{test,j}\hat{k})^2 - \frac{1}{n}\sum_{j=1}^{n}(y_{test,j} - X_{test,j}\hat{k}_{test})^2$, where $\hat{k}$ is the estimated RF, $\hat{k}_{test}$ is the MLE from the test data. All values were then normalized by dividing by the MLE performance under 30 seconds of data. The subunit model in Fig. D was a hierachical Linear-Nonlinear Poisson model with fixed softplus nonlinearity. We provide a Python toolbox to efficiently use all implemented methods (`https://github.com/berenslab/RFEst/`).

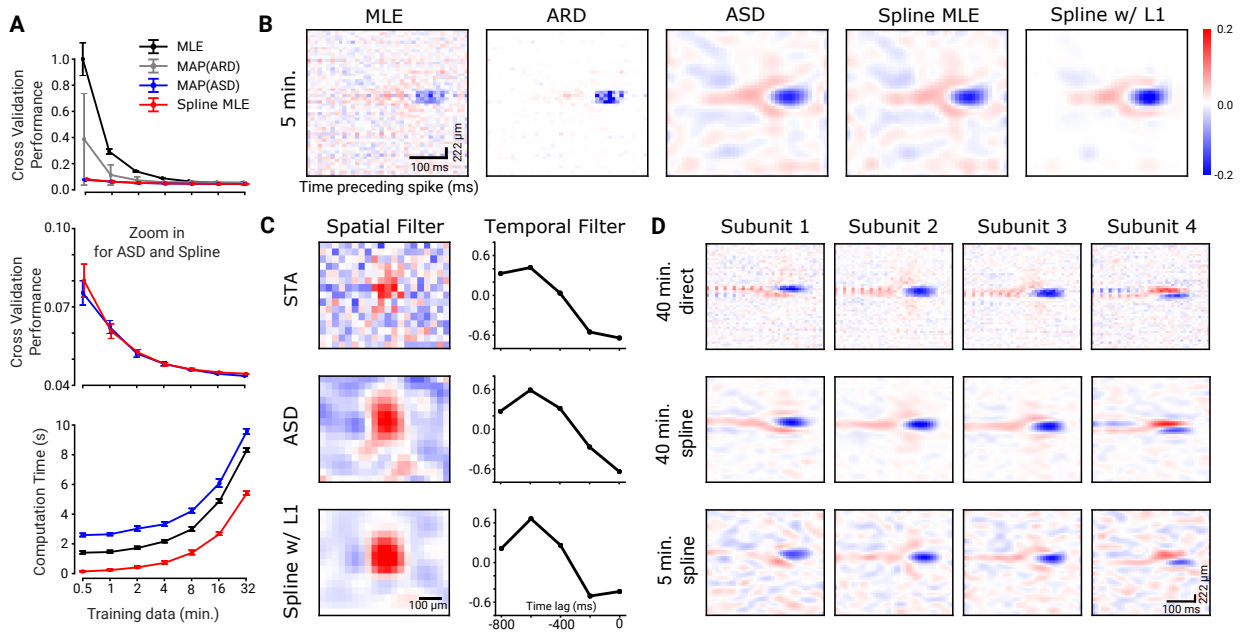

Figure 1: **Model comparison.** **A**. Prediction performance (top, middle) and computation time (bottom) versus different amounts of training data. Spline MLE achieved very similar prediction performance as ASD but was much more efficient on salamander retinal ganglion cell spike data. **B**. STRFs estimated by different methods under a Gaussian noise model using 5 minutes of spike data from salamander retinal ganglion cells. Spline MLE yields STRFs as smooth as ASD. Fitted with L1 penalty, the spline-based STRFs can achieve additional sparseness. **C**. Spatial and temporal components of a 3D STRF (separated by Singular Value Decomposition) estimated using 5 minutes of two-photon calcium recording from mouse retinal ganglion cell dendrites. Here we show spike-triggered average (STA) of the STRFs instead of MLE, because the decorrelation term in MLE rendered the STRF unrecognizable. **D**. LNLN-Poisson model with 4 subunits and fixed softplus nonlinearity. Spline-based subunits are already clearly recognizable with only 5 min of training data.

References

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