

# Efficient high-dimensional receptive field estimation with a flexible spline basis

Ziwei Huang, Philipp Berens

Institute for Ophthalmic Research, University of Tübingen, Germany



## Introduction

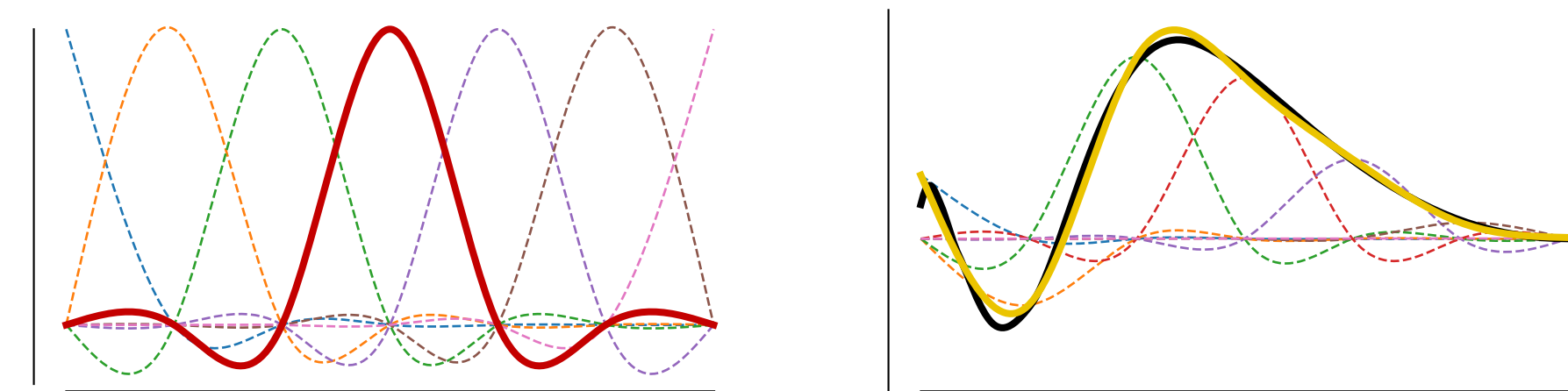
Spatio-temporal receptive field (STRF) models are frequently used to approximate the computation implemented by a sensory neuron. Typically, such STRFs are assumed to be smooth and sparse.

Current empirical Bayes estimation approaches such as automatic relevance determination (ARD), automatic smoothness determination (ASD), automatic locality determination (ALD) and others encode such prior knowledge into a prior covariance matrix, whose hyperparameters are learned from the data, and thus provide STRF estimates with the desired properties even with little or noisy data [1, 2]. However, empirical Bayes methods are not computationally efficient in high-dimensional settings, as often encountered in sensory neuroscience.

Here we pursue an alternative approach and encode prior knowledge for estimation of STRFs by choosing a set of basis function with the desired properties [3].

## Flexible basis functions

We use a *natural cubic spline basis*, which is known as the smoothest possible interpolant [4]. We find that this method provides a good basis for high-dimensional STRFs.



**Example spline basis in 1D.** **Left:** 7 natural cubic spline basis functions, with the 3rd basis highlighted. **Right:** 1D RF fitted by spline-based maximum likelihood (Black line: simulated RF; Yellow line: fitted RF):

$$\hat{k}_{SPL,MLE} = S(S^T X^T X S)^{-1} S^T X^T y$$

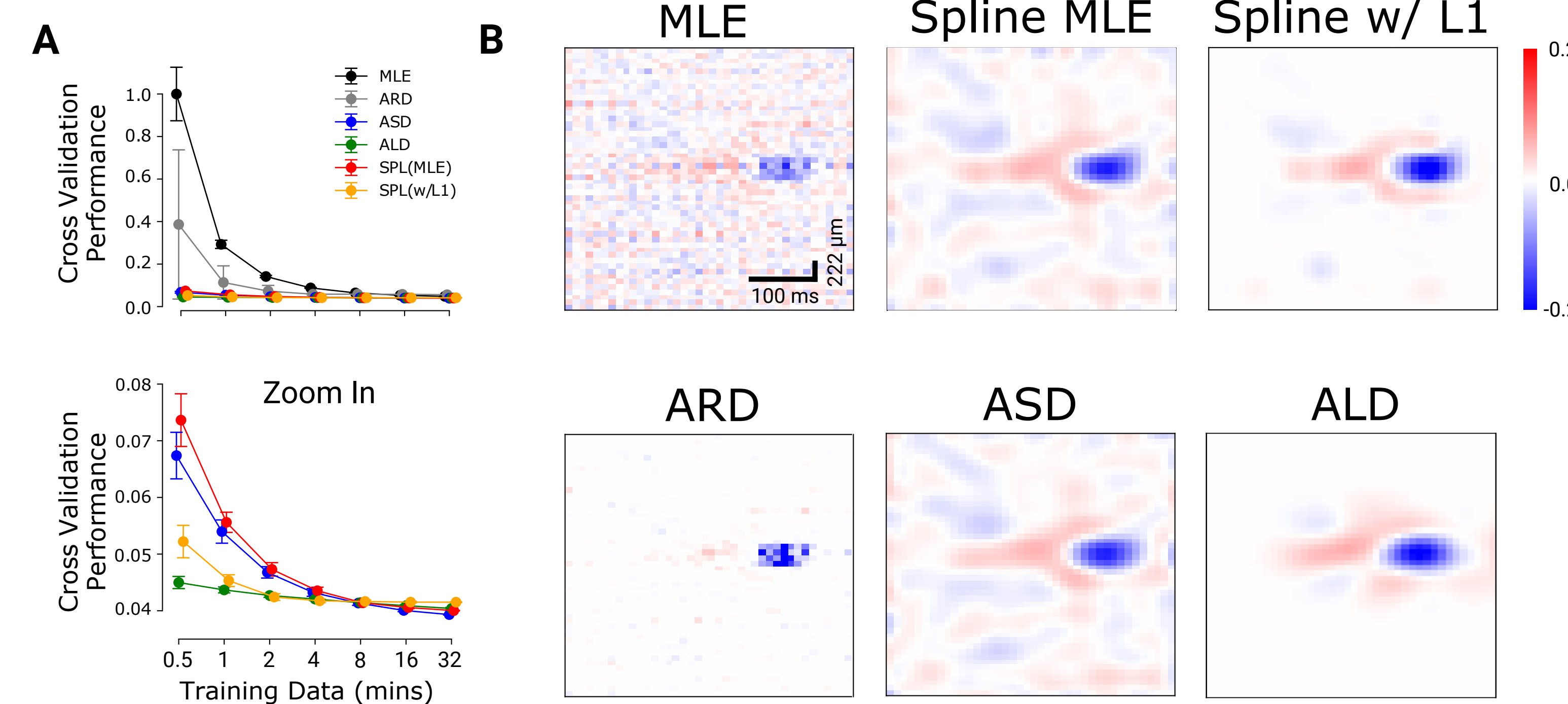
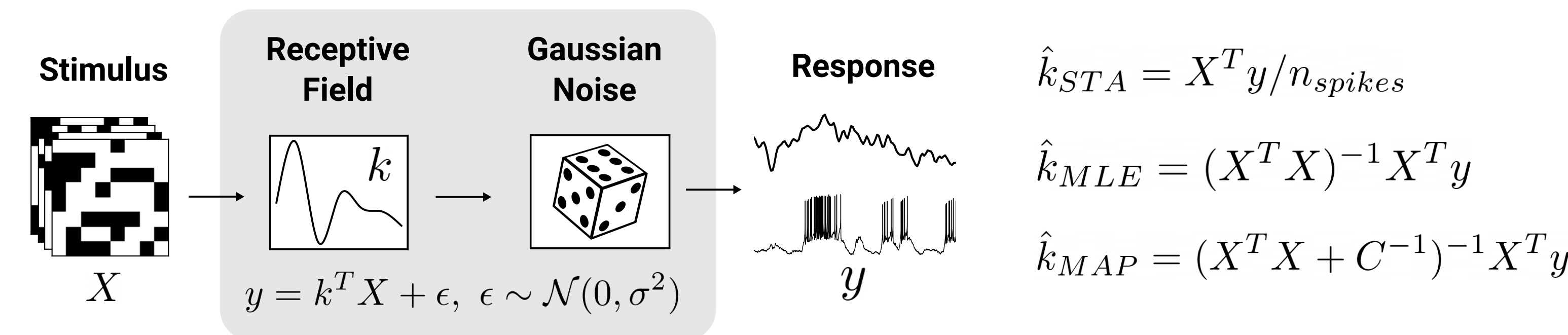
where  $X$  is the stimulus design matrix,  $y$  the response,  $S$  the spline matrix. The spline matrix can be extended to high-dimensions, which is also called *tensor product smooth* [3], by taking the Kronecker product ( $\otimes$ ) of the bases in each dimension:

$$S = S_t \otimes S_x \otimes S_y$$

The computational cost can be reduced from  $O(d^3)$  to  $O(b^3)$ , where  $d$  is the number of STRF coefficients,  $b$  is the number of basis functions.

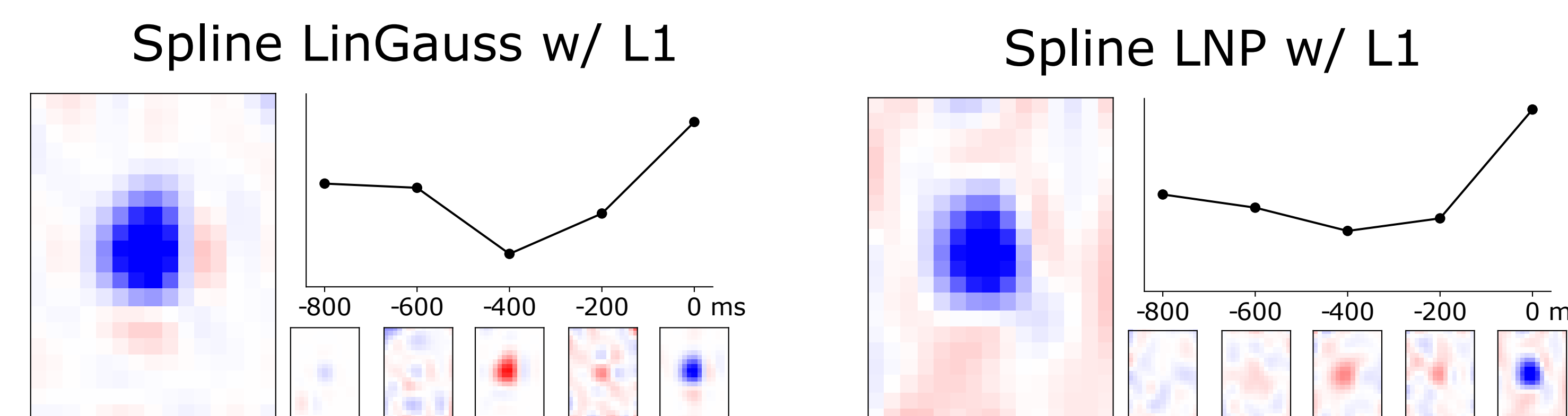
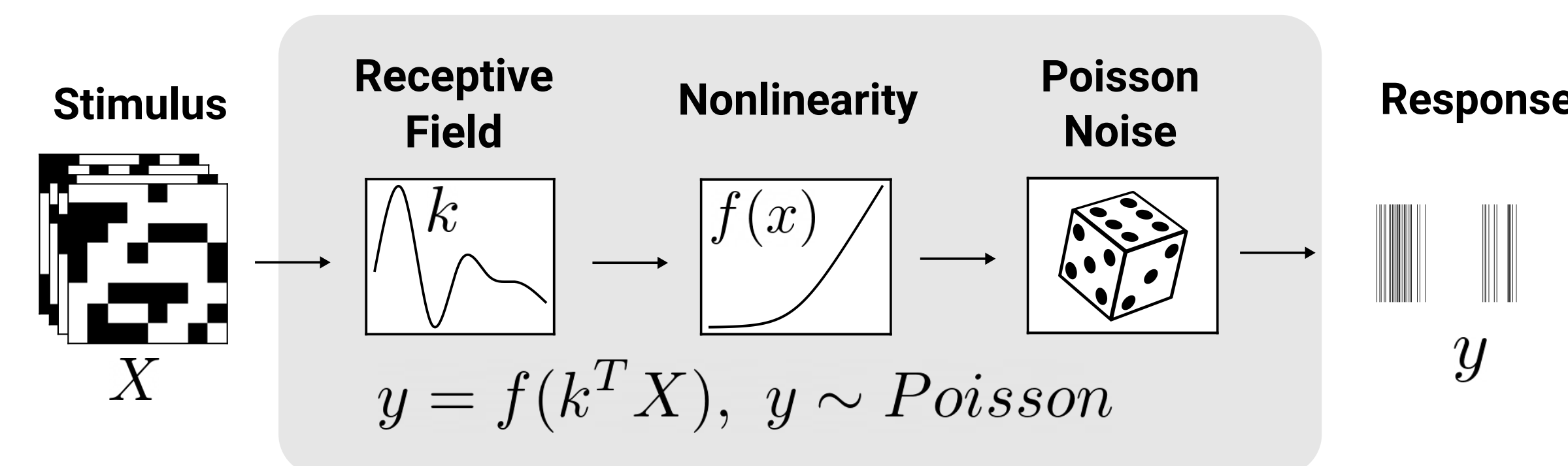
## Single Filter models

### Linear-Gaussian models



**A.** Performance comparison of evidence optimization methods and spline-based methods. **B.** RFs fitted by different methods with 5 minutes of recording (data from [5]).

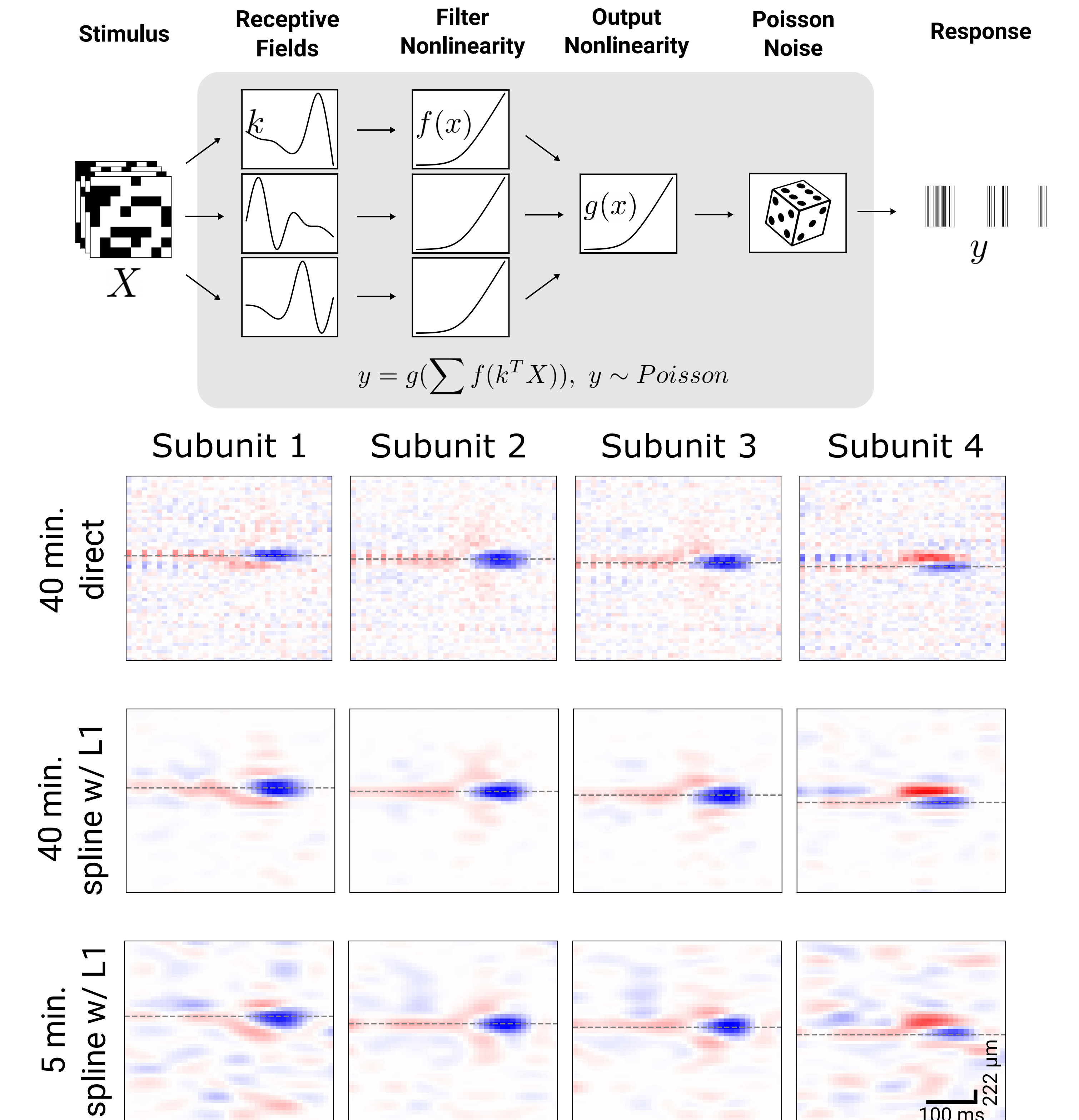
### Linear-Nonlinear-Poisson model



3D STRFs estimated from 5 minutes of simultaneous recording of somatic  $\text{Ca}^{2+}$  (left) and voltage (right) signal (data from [6]).

## Multifilters (subunits) models

### Hierarchical linear-nonlinear model



The resulting subunits of the spline-based LNLN are already well-separated and smooth using only 5 minutes of data, while traditional estimation methods require much more data to achieve similar results.

## Conclusion

Here we showed that *natural cubic spline* is a good basis for high-dimensional STRFs, and can be easily incorporated into a wide range of existing RF estimation methods.

## Python Package: RFest

We provide a Python toolbox to efficiently use all implemented methods. See:

<https://github.com/berenslab/RFest>



## Reference

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- [2] Park, M. & Pillow, J. W. (2011) *PLoS Comput. Biol.*
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- [4] Wood, S. N. (2017) *Generalized additive models: an introduction with R*.
- [5] Maheswaranathan, N., et al. (2018) *PLoS Comput. Biol.*
- [6] Ran, Y., et al. (2020). *bioRxiv*. 753335

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