Royal Institute of Technology
School of Engineering Sciences

Flight Mechanics
Project Work Part

Nikolaos Koukis

Personnummer: 930727T073
Group: D

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1 Abstract

This is the report for the performance analysis project in the *Flight Mechanics* course offered by the School of Engineering Sciences at KTH. The report was written using the *Vim* Editor and the XƎLATEXtypesetting system. For the coding part the *Matlab* technical computing language was used.

2 Introduction

As the jet era started, Sweden foresaw the need for a jet fighter that could intercept bombers at high altitude and also successfully engage fighters. Although other interceptors such as the US Air Force's F-104 Starfighter were being conceived during the same period, Saab's "Draken" would have to undertake a combat role unique to Sweden. Other demanding requirements were the capability to operate from reinforced public roads used as part of wartime airbases, and for refuelling/rearming to be carried out in no more than ten minutes, by conscripts with minimal training. **In September 1949, the Swedish Defence Material Administration issued a request for a fighter/interceptor aircraft, and work began at Saab the same year**

Regarding the aerodynamic design of the J35 Draken the two major options were swept wings and delta wings [1]. The question was quickly resolved by the initial studies which had called for the exploration of a swept wing configuration. In short order it was determined tha in consideration of all other parameteres placed upon the design, the swept wing's aerodynamic drag at high Mach numbers was too high, and its configuration requirements dictated that the fuselage have insufficient volume for equipment, fuel and armament. The Delta wing on the other hand showed great promise following initial tunnel tests. The pure delta soon was ruled out, however, as it suffered from center of gravity and center of pressure anomalies that were difficult to alleviate. A derivative, however, often referred to as *the double delta*, proved much more flexible. In general the double delta was found to offer the attributes of:

- reduced frontal area while permitting optimal wing area
- More favorable wing sweep angles on the center wing section
- Center of gravity and center of pressure being closer to each other
- More favorable area distribution
- Low supersonic drag
- Favorable low speed drag
- Strong and stiff fail safe structure
- Being able to place the air intakese farther forward

3 Performance Characteristics of J35 Draken

3.1 Static Perfomance

3.2 Excess Thrust

Figure 3.1 shows the Excess Thrust with regards to the Mach number is presented:

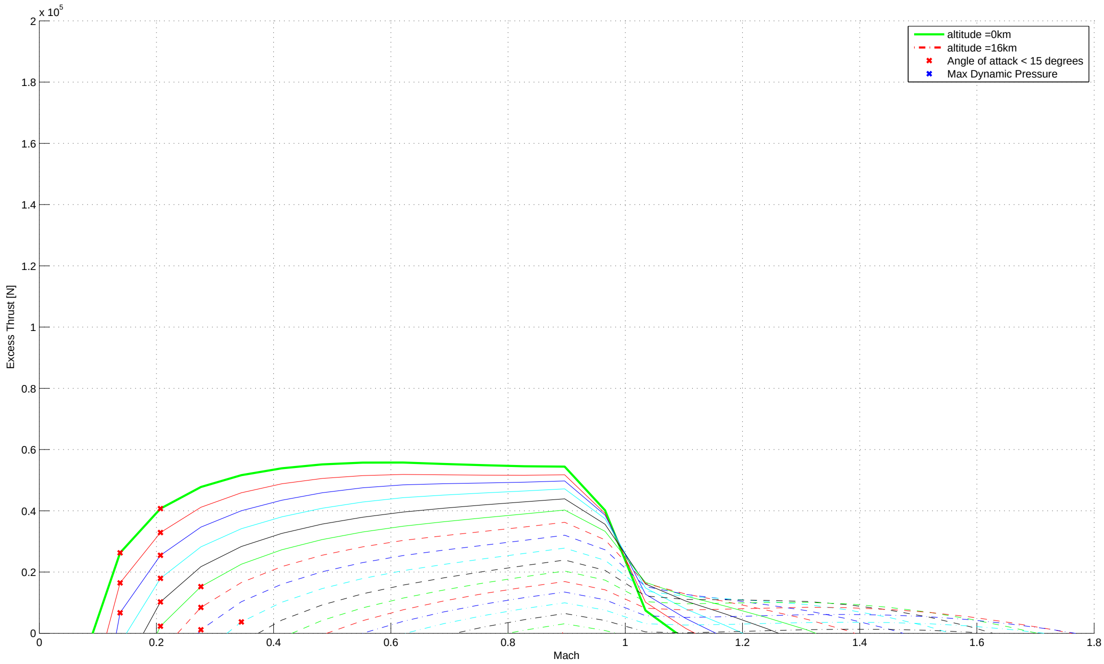


Figure 3.1: Excess Thrust Graph

Using the graph we can now determine the envelope limits corresponding to the dynamic pressure and the maximum angle of attack. These limits are shown in the diagram as red and blue Xs. We should note however that the dynamic pressure limits are not visible in 3.1, when presenting only the part of the diagram above the zero horizontal line, so a supplementary graph (3.2) is given:

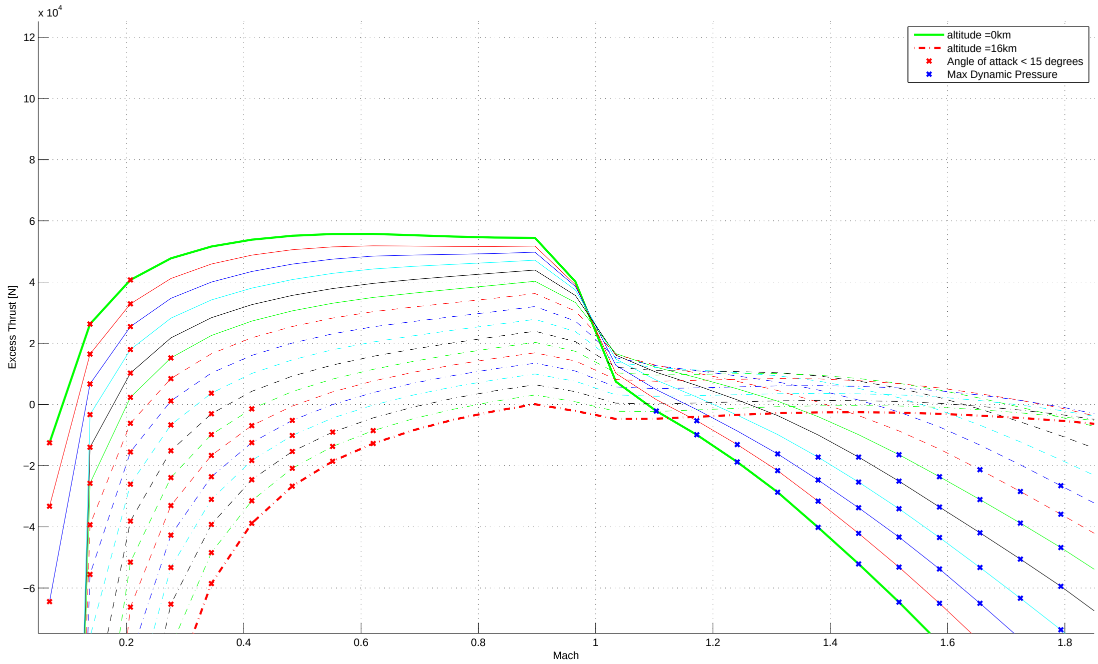


Figure 3.2: Excess Thrust Graph - Dynamic pressure limits Visible

For the calculation of the Excess Thrust the TexSep function was used. Refer to the TexSep.m module for more information on the implementation

3.3 SEP Graph

Using the code provided with the report the Specific Excess Power (SEP) Graph can be plotted:

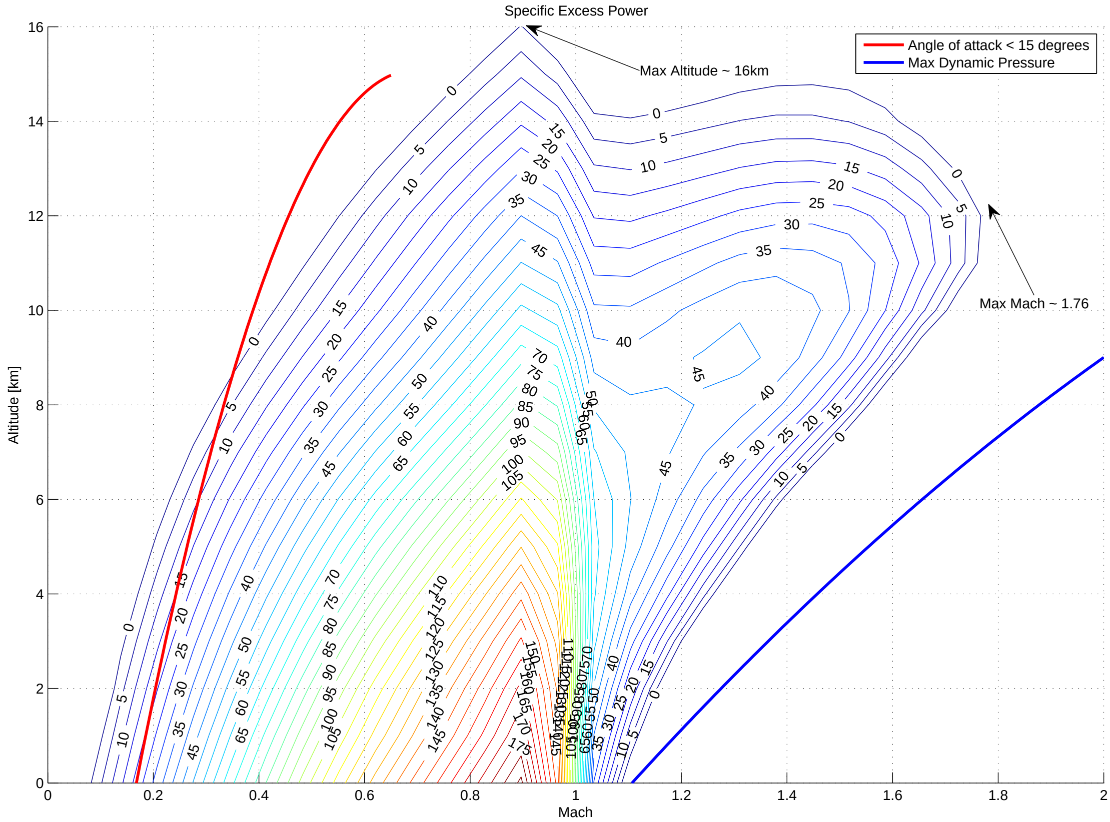


Figure 3.3: SEP Graph

Graph 3.3 shows the contours of same Excess Power with regards to the altitude of the airplane as well as the Mach number thus the velocity of the airplane.

Using the graph we can also determine the envelope limits corresponding to the maximum angle of attack (15°) and the maximum dynamic pressure corresponding to $1350^{km}/h$ at sea level which is $8.6133e + 04Pa$.

3.4 Maximum altitude - Maximum Mach Number

The maximum altitude and the maximum Mach Number at which the aircraft can fly level can be determined using 3.3:

- Max. Altitude: 16 km
- Max Mach: 1.76

3.5 Minimum time to climb

The current task is to find using trial and error methods, the minimum time to reach certain goals. In order to investigate the situation more thoroughly in each try we plot the total energy lines as well as plots for the fuel consumption the speed of the airplane the height and the gamma angle.

3.6 Computing $\gamma(t)$ - minimum time for Mach = 1.5 & h = 1.1km

??

The feasible solution to the above challenge is shown in 3.4. 3.5 is also presented to show the changes of the different properties during the time of flight

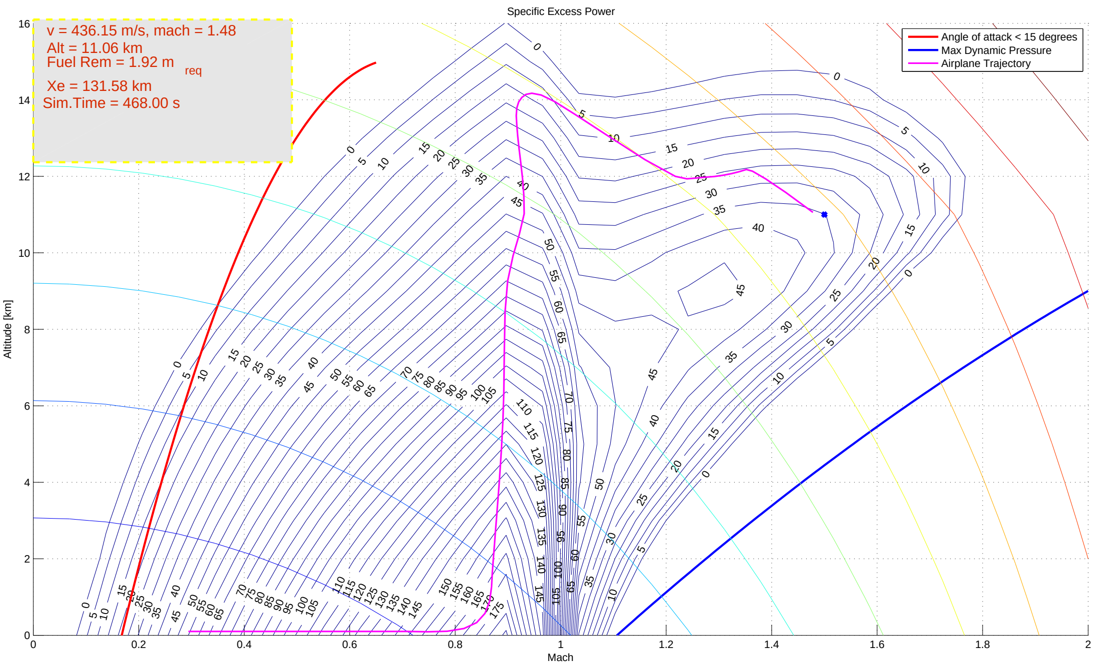


Figure 3.4: Trajector of the airplane to reach M = 1.5 & h = 11km

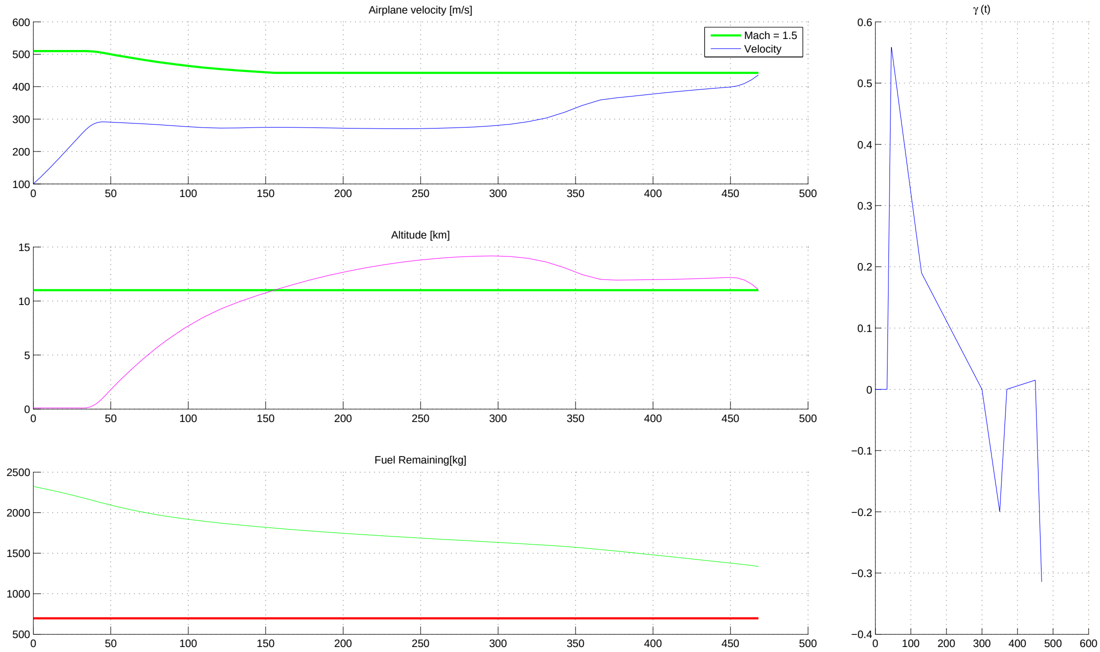


Figure 3.5: Velocity, height fuel consumption and gamma angle during the flight

The strategy of the climb is first to climb to higher altitude than needed and then dive into the supersonic area "with the same slope as an energy line" in order to lose the least amount of energy during the manuever. As we see the goal is reached at 468 seconds which is acceptable both in terms of fuel consumption and afterburner time of use.

3.7 Trajectory for maximum Mach number in minimum time

In order to reach the maximum Mach number given by the SEP graph we use the same tactic as in **??**. The diagram 3.6 corresponding to the flight are also given below:

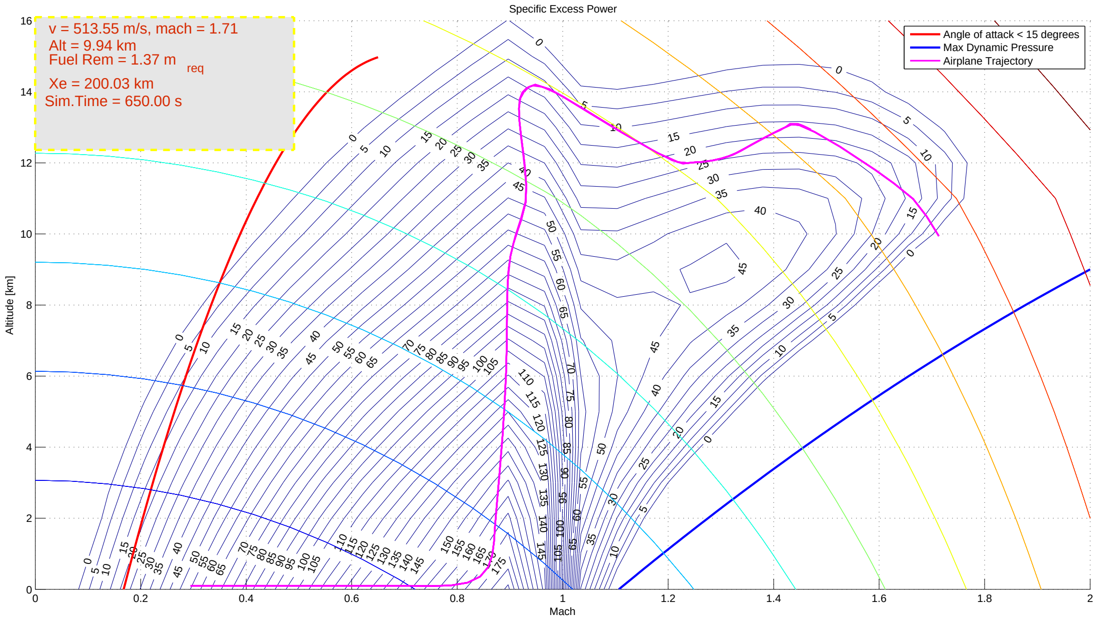


Figure 3.6: Trajector of the airplane to reach M = 1.76

3.8 Trajectory for maximum altitude in minimum time

The diagram for reaching the maximum altitude is given in **??**. It is visible that the trajectory is not the best possible as the aircraft reaches a maximum of 15.4 km instead of the actual goal which is 16km.

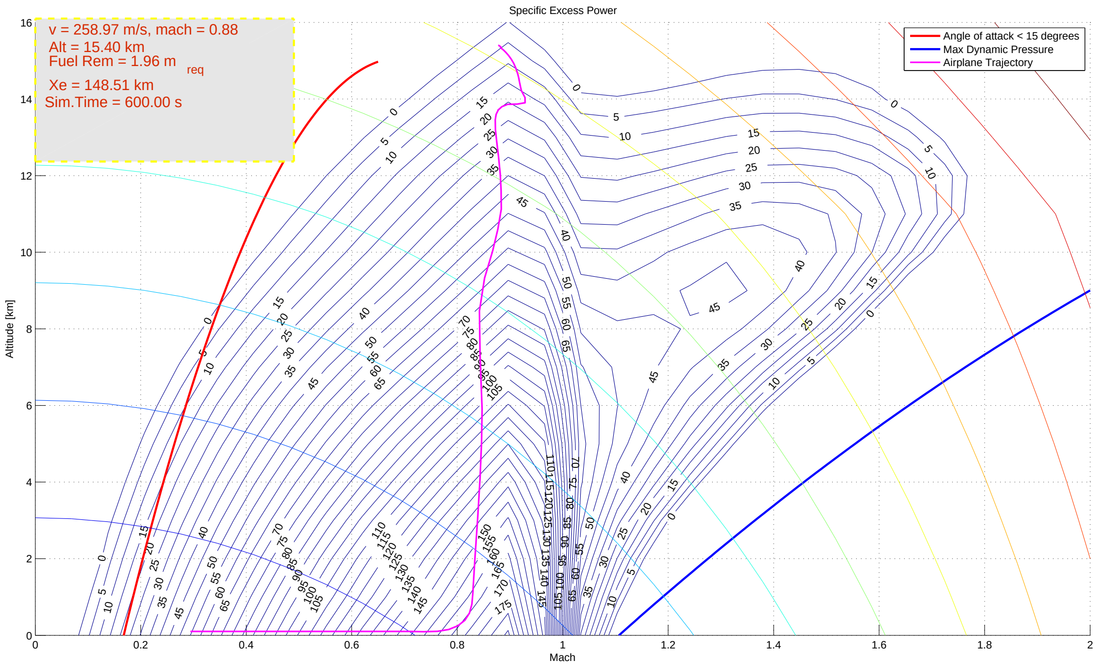


Figure 3.7: Trajector of the airplane to reach h = 16km

4 Derivation of C_l rolling moment coefficient

In this part of the project the rolling moment coefficient of the J35 Draken is to be calculated. For the simulation, the model of fig.4.1 was used:

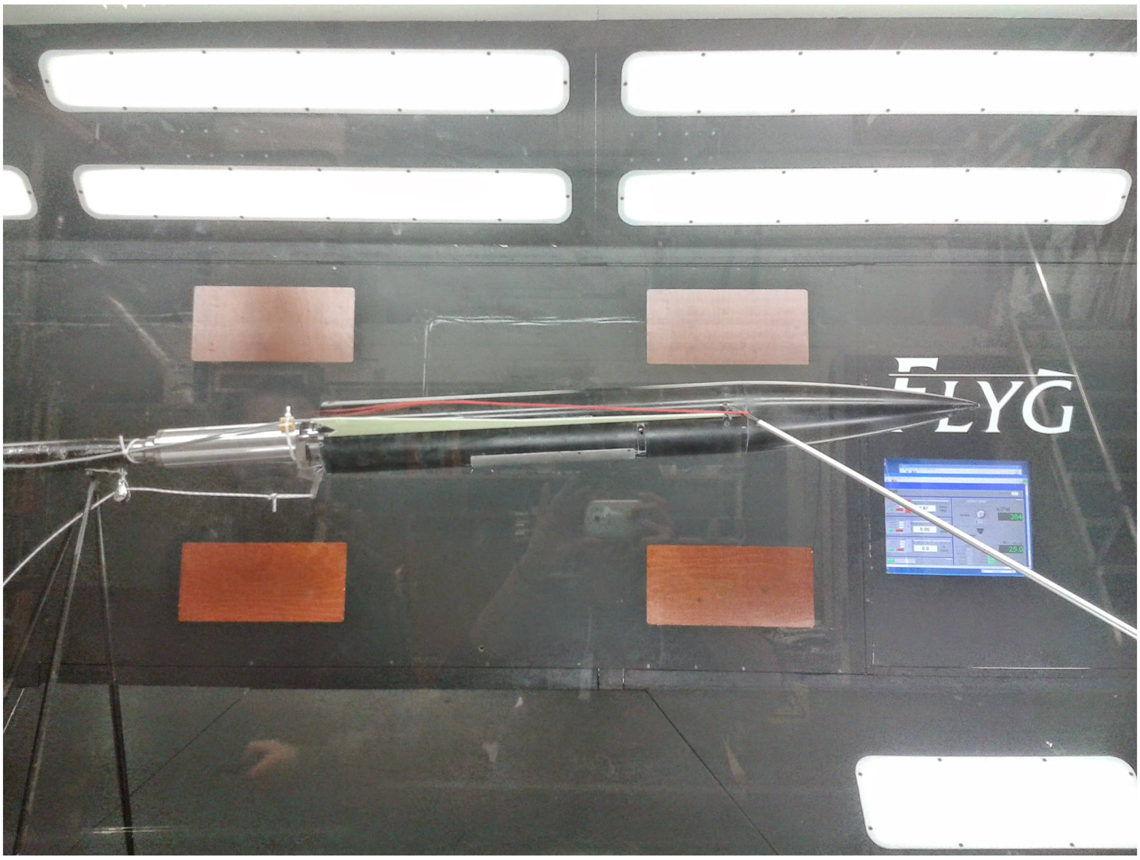


Figure 4.1: Picture of model during wind tunnel test

4.1 Simulation Considerations

The following can be stated when comparing the j35 Draken aircraft with the wind tunnel model used(fig.4.1

- Apparently the major difference is the *absence of the fin*. This fact eventually caused considerable divergence between the values computed during the simulation and the data extracted from Draken data diagramms provided.
- The wind tunnel model scale was 1:14.7 except for the bodywidth and nose-length.

4.2 Mathematical Modelling

In order to proceed with the analysis and the computation of $C_{lp}, C_{l\beta} parameters$, a valid mathematical model needs to be derived first. If I implement the Newton's second law for the rolling movement I can derive equation 4.1.

$$\Sigma T = I_{xx}\ddot{\phi} \Rightarrow L - C\dot{\phi} - k\phi = I_{xx}\ddot{\phi}, \tag{4.1}$$

where $C\dot{\phi}$ is the moment exerted due to the (mechanical) damping and $k\phi$ is the moment due to the (mechanical) spring stiffness of the device holding the airplane during the simulation. For the calculation of the rolling moment the following equations can be used:

$$L = C_l q b S \tag{4.2}$$

$$C_l = C_{lp}\frac{pb}{2u} + C_{l\beta}\beta \tag{4.3}$$

$$p = \dot{\phi}$$

$$q = \frac{1}{2}\rho v^2$$

A sufficient model of calculating C_l - the rolling moment coefficient - is given in equation 4.3. According to this, C_l is a function of C_{lp} the *damping-in-roll* coefficient and $C_{l\beta}$ the dihedral effect. C_{lp} expresses the resistance of the airplane to rolling [2], while $C_{l\beta}$ expresses the change in rolling moment coefficient per degree of change in the sideslip angle β. A sufficient relation to calculating β can be the following:

$$\beta = \alpha\phi \tag{4.4}$$

If I now substitute the expression of rolling moment 4.2 into the initial equation 4.1, and move all the $\phi, \dot{\phi}$ terms to the other side of the equation I can end up with the following second order differential equation:

$$I_{xx}\ddot{\phi} + \dot{\phi}\big(C_{mech} - \frac{C_{lp}Sb^2q}{2u}\big) + \big(k - C_{l\beta}aqbS\big)\phi = 0 \tag{4.5}$$

The analytical solution of 4.5 for the general case of complex roots (which is what I expect due to the oscillatory behavior of the model movement) is the following:

$$\phi = c_1 e^{\alpha' x} cos(\beta' x) + c_2 e^{\alpha' x} sin(\beta' x) \tag{4.6}$$

$$\alpha' = -\frac{\beta}{2\alpha}$$

$$\beta' = \frac{\sqrt{4ac - b^2}}{2a} \tag{4.7}$$

where α, β, c are the coefficients of 4.5 respectively:

$$\alpha = I_{xx}$$

$$\beta = C_{mech} - \frac{C_{lp}Sb^2q}{2u}$$

$$c = k - C_{l\beta}aqbS$$

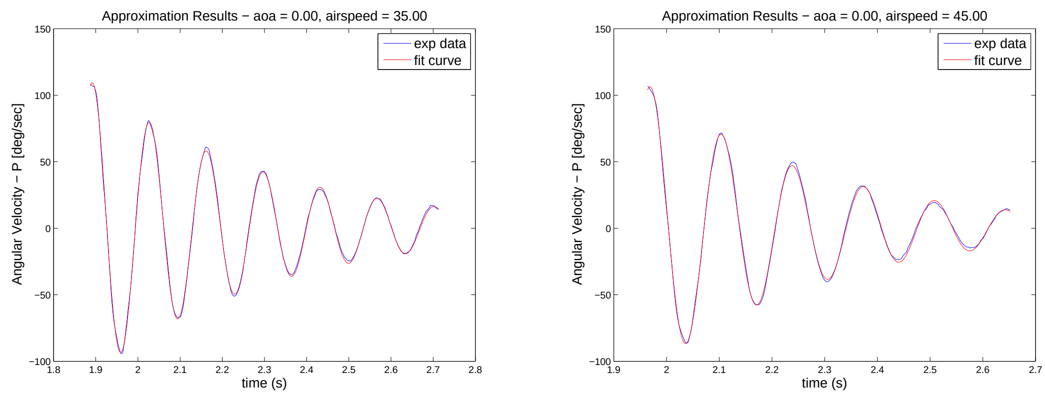
In order to get the analytical solution of the roll rate $\dot{\phi}$ I take the derivative of 4.6:

$$
\begin{aligned}
\dot{\phi} &= \big(-C_1\beta' sin(\beta' x) + C_2\beta' cos(\beta' x)\big)e^{\alpha' x} = \\
&= \beta'\big(-C_1 + C_2\big)e^{\alpha'}x\big(sin(\beta' x) + cos(\beta' x)\big) = \\
&\Rightarrow \big\{A = \beta'\big(-C_1 + C_2\big) \quad \big\} \Rightarrow \\
\dot{\phi} &= Ae^{\alpha' x}\big(sin(\beta' x) + cos(\beta' x)\big) = \\
&\Rightarrow \big\{asin(\theta) + bcos(\theta) = Rsin\big(\theta \pm \alpha\big)\big\} \Rightarrow \\
\dot{\phi} &= Ae^{\alpha' x}\big(sin(\beta' x + \Theta\big) \qquad (4.8)
\end{aligned}
$$

4.3 Derivation of *damping-in-roll* derivative C_{lp}

In order to derive the C_{lp} parameter, I first need to extract information out of the experimental data gathered. The oscillation of the rolling motion can be modelled as a decaying sinusoidal function of time. Therefore I can fit an exponential sinusoidal function of the form 4.8 in the data and estimate each parameter by using the least squares method.

The data fit procedure uses the Fourier transform to extract the basic frequency as well as the least squares method. Figures 4.2a,4.2b show some examples of the data fits that demonstrate the efficiency of the approximation algorithm used.



(a) V = 35 $^m/_s$

(b) V = 45 $^m/_s$

To derive only the value of the C_{lp} parameter I have to take into account *only the data for* $\alpha = 0$, because otherwise $C_{l\beta}$ starts to play a role in the behavior of the oscilation (see eq. 4.5).

In order to extract the maximum amount of data and be as accurate as possible I *exploit all the* $\alpha = 0$ *measurements taken during the laboratory sessions.* [1].

The procedure can be summarised in the next steps:

1. Calculation of C_{mech}, I_{xx}[2] using one of the $\alpha = 0 \& V = 0$ measurements available.

2. Approximation of the n-v [3]curve by using a wide rage of velocities to calculate n-values and by using the least squares method.

[1]The reader is encouraged to refer to stiff.m of the code part for the actual implementation

[2]I know that $I_x x = \sqrt{\frac{k}{\omega^2}}$ where k has already been determined experimentally.

[3]n is the damping coefficient, equivalent to β' of eq. 4.8

3. Calculation of the n-v curve slope from which knowing every other quantity I can extract an estimation of the C_{lp} value.

Using this strategy I can now derive the approximation curve of the n-v points:

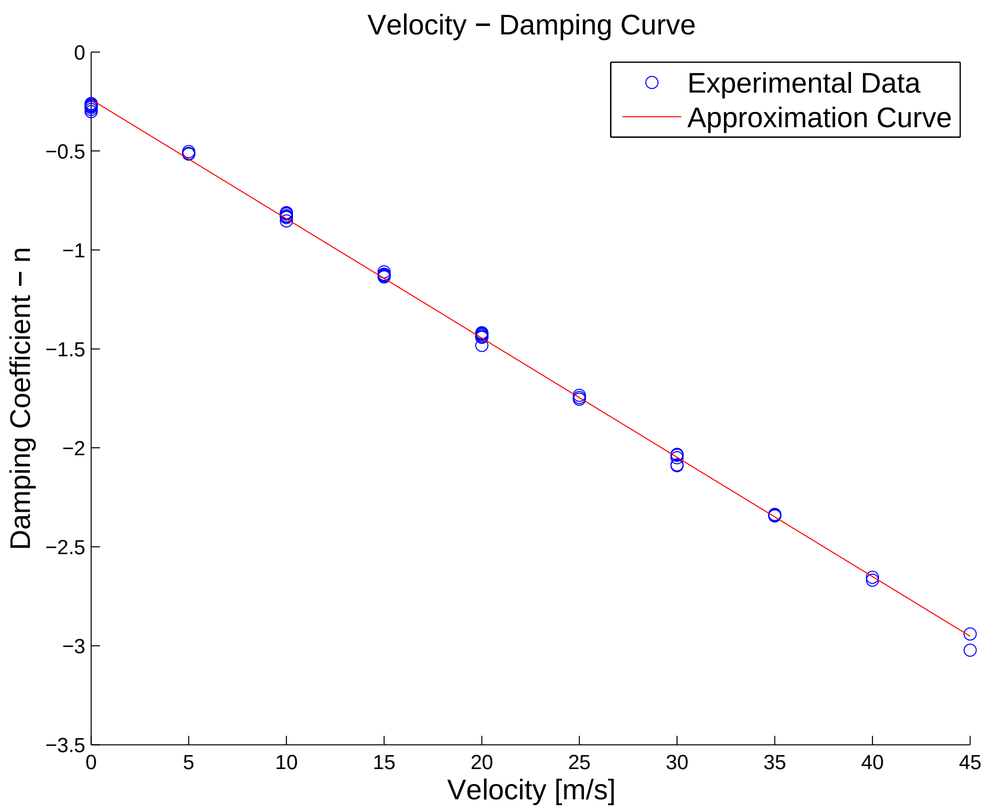


Figure 4.3: Velocity - Damping approximation curve

From the above graph and using the methodology described in the previous steps I can now conclude to the value of C_{lp}:

$$C_{lp} \simeq -0.190 \tag{4.9}$$

Since this value is (strictly) negative and is close to the data given for the real aircraft (for $\alpha = 0$), I can conclude that it is a logical estimation of the damping-in-roll coefficient.

4.4 Derivation of the *Dihedral effect* $C_{l\beta}$

References

[1] Robert Dorr, Rene Francillon, and Jay Miller. *Aerofax Minigraph 12, Saab J35 Draken*. Aerofax Inc., 1987.

[2] Bernanrd Etkin. *Dynamics of Atmospheric Flight*. John Wiley & Sons, Inc, Toronto, Canada, 1972. 00000.