Physics Department
Quantum Laboratory

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1.Aim

The aim of this experiment is:

- to numerically calculate the wavefunction and probability density of a Landau state and study the convergence with classical mechanics at high Landau levels indexes.

2.Overview and theory.

2.1. MOTION IN A QUANTIZING MAGNETIC FIELD

We consider the motion of an electron orbiting around a constant or stationary magnetic field. Classically, the electron is pulled into a circular orbit such that the magnetic field is directed normal to the plane of the orbit, and the motion arises from the Lorentz force acting on the electron:

$$\vec{F} = -e\left(\vec{v} \times \vec{B}\right) \qquad [1]$$

The electrons execute circular trajectories in the plane normal to magnetic field at constant angular frequency: $w_c = \left|\frac{eB}{m}\right|$ known as the cyclotron orbit. The radius of the orbit, the cyclotron radius is given by: $R_c = \frac{v}{w_c} = \frac{\sqrt{2mE}}{eB}$ where v is the constant magnitude of velocity and *E* is the kinetic energy.
Here, we want to examine the quantization of these orbits. The simplest algebra is in the Landau gauge where there is only one component of the vector potential: $\vec{A} = (0, Bx, 0)$. Thus, in the absence of any external potential, other than the vector potential giving rise to the magnetic field, the stationary Schrödinger's equation can be written as:

$$\left\{\frac{1}{2m}\left[-\hbar^2\frac{\partial^2}{\partial x^2} + \left(-i\hbar\frac{\partial}{\partial y} + eBx\right)^2\right]\right\}\psi(r) = E\psi(r) \qquad [2]$$

Expanding the inner bracket:

$$\left[-\frac{\hbar^2}{2m}\nabla^2 - \frac{ie\hbar Bx}{m} + \left(-i\hbar\frac{\partial}{\partial y} + eBx\right)^2\right]\psi(r) = E\psi(r) \qquad [3]$$

The vector potential does not depend on y, what suggests that the wave function should be a product of a plane wave depending on y and another kind of function that depends on x: $\psi(x)e^{iky}$.

Substituting this function on the Schrödinger's equation we get to:

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}mw_c^2\left(x + \frac{\hbar k}{eB}\right)^2\right]\psi(x) = \psi(x) \qquad [4]$$

Where *k* is the y component of the electron momentum. This is the Schrödinger's equation for a one-dimensional quantum harmonic oscillator. The cyclotron frequency $w_c = \left|\frac{eB}{m}\right|$ appears as in the classical case, but the center of the oscillation is displaced by $x_0 = \frac{\hbar k}{eB}$

Thus, the energy levels that are called ***Landau levels****,* are given by:

$$E = \hbar w_c\left(n + \frac{1}{2}\right) \qquad [5]$$

The corresponding wave function is:

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega_c}{\hbar\pi}\right)^{1/4} \mathrm{e}^{-m\omega_c(x-x_0)^2/2\hbar} H_n\left(\sqrt{\frac{m\omega_c}{\hbar}}(x - x_0)\right)$$

[6]

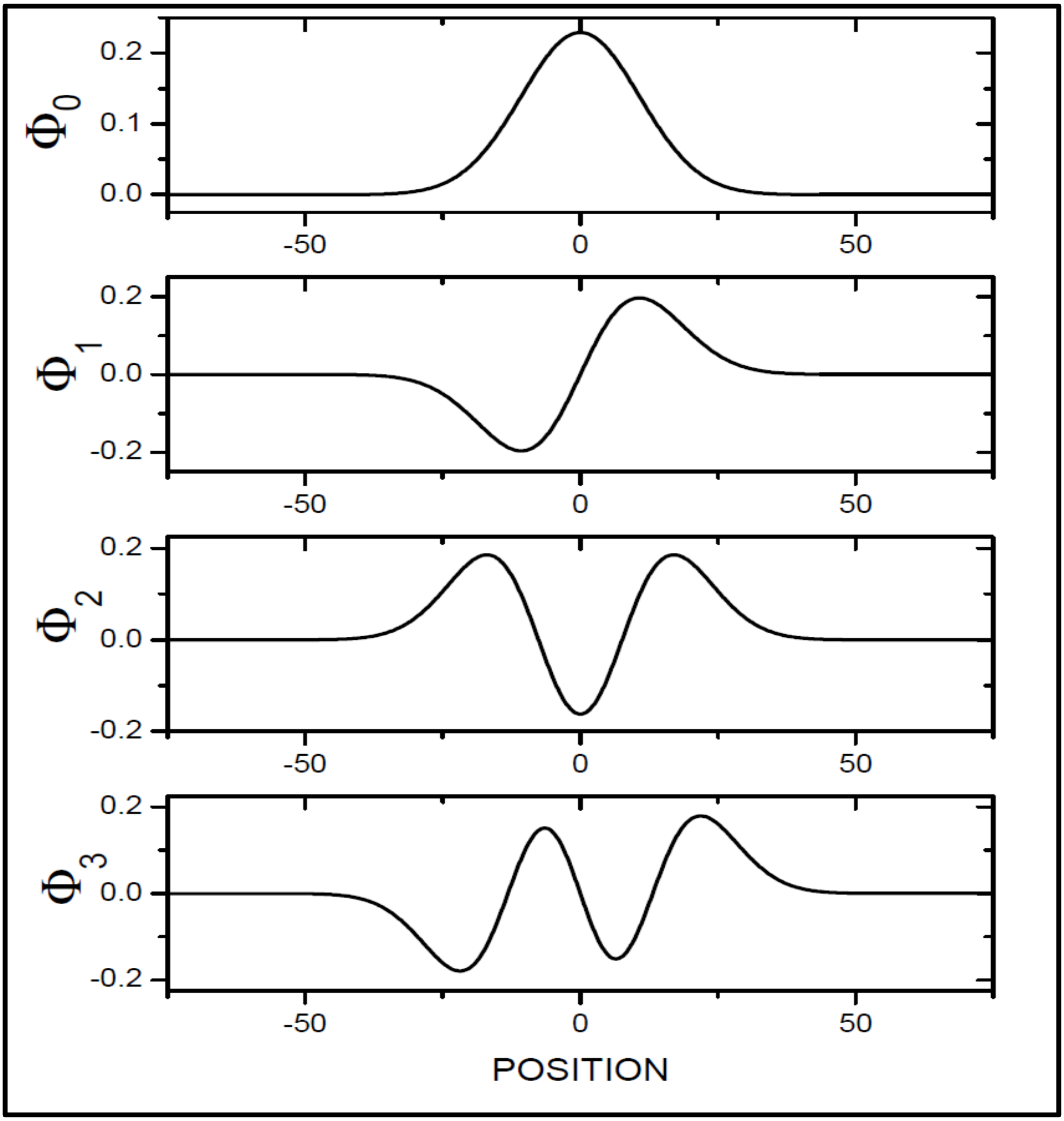
where *n* the Landau level index *n = 1, 2, 3, 4…..* and H_n are Hermite polynomials. These solutions are, of course, just the normal harmonic oscillator wave functions shifted by the offset x_0.

The characteristic length of this problem is called *magnetic length*, l_B and is given by:

$$l_B = \sqrt{\frac{\hbar}{mw_c}} = \sqrt{\frac{\hbar}{eB}}$$

[7]

Fig. 1 Graphical representation of the wave function of the first four Landau levels vs position.



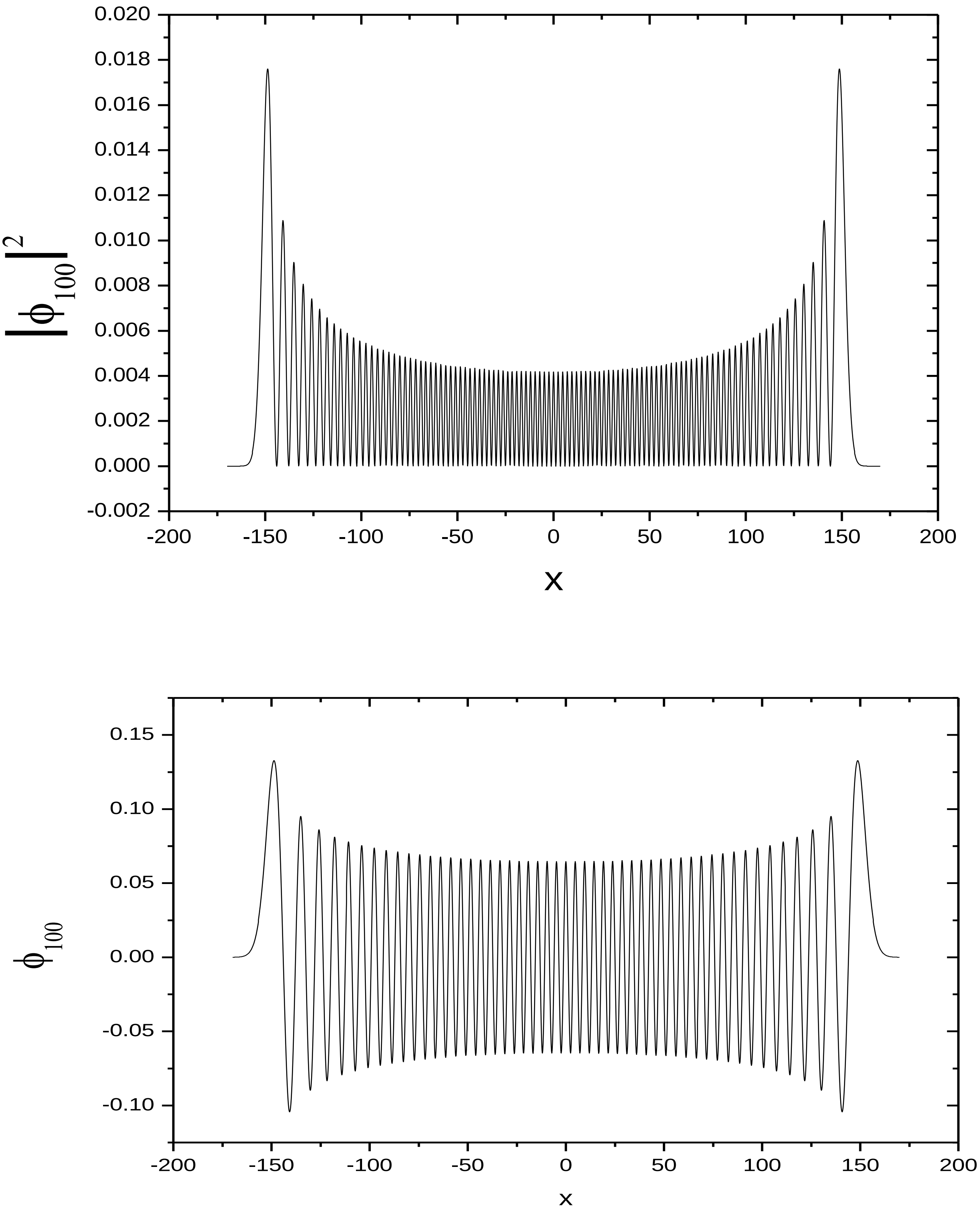


Fig. 2. Graphical representation of the wave function and probability density of the Landau level of index 100.

3. To learn more

Applied Quantum Mechanics. First Edition. A. F. J. Levi.
Cambridge University Press 2006.

Quantum Mechanics. An Introduction for Device Physicists and Electrical Engineers. Third Edition. David Ferry.
CRC Press 2001.

4.Procedure.

4.1. Landau level wave functions and probability density (square of the wave function).

1. Write a computer program with the software of your choice that plots the n-th Landau state and probability density (wave function squared) vs position (see Figs. 1 and 2 as an example). Neglect, for simplicity, the offset x_0.

   Knowing the two starting wave functions ϕ_0 and ϕ_1 of a quantum harmonic oscillator, the n-th wave function can be generated by using the recursive expression:

$$\phi_n(\xi) = \sqrt{\frac{2}{n}}\left(\xi\phi_{n-1}(\xi) - \sqrt{\frac{n-1}{2}}\,\phi_{n-2}(\xi)\right) \qquad [8]$$

$$\xi = \left(\frac{mw_c}{\hbar}\right) x \qquad [9]$$

Look up the two first wave functions of a quantum harmonic oscillator and adapt them to a Landau level scenario. Use Eq. 8 as a recurrence relation and Eq. 9 to generate the wave function and plot the n=18, n=50 and n=100 Landau states wave functions and wave function squared (probability density) and comment about the convergence between classical and quantum mechanics.

(Hint: Pages 319-320. **Applied Quantum Mechanics**. *First Edition*. A. F. J. Levi. Cambridge University Press 2006.)