RESONANT TUNNELING

1.Aim

The aim of this experiment is:

- to ***numerically*** study the transmission coefficient, ***T***, of a general double barrier in nano-heterostructures. Symmetric double barrier case.
- to calculate the cases of asymmetric double barrier:
    1. unequal barriers
    2. external bias

2.Overview and theory.

The transmission probability of an electron through a double-barrier potential (see Fig. 1) becomes unity when its energy is equal to that of the quantum states in the well (see Figs. 2 and 3). Away from this resonant condition, that probability drops exponentially. This phenomenon, called resonant tunneling, was predicted in the 1960´s and subsequently observed in GaAlAs-GaAs-GaAlAs heterostructures. The same phenomenon has been shown in similar structures for the transmission of both light and heavy holes.

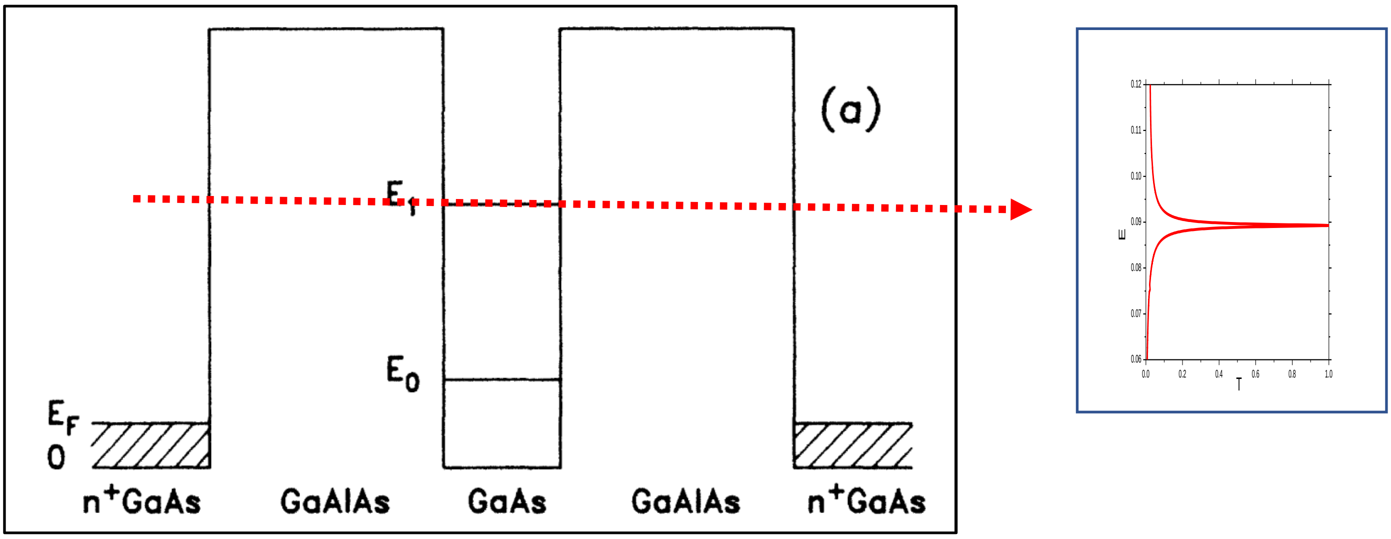


Fig.1

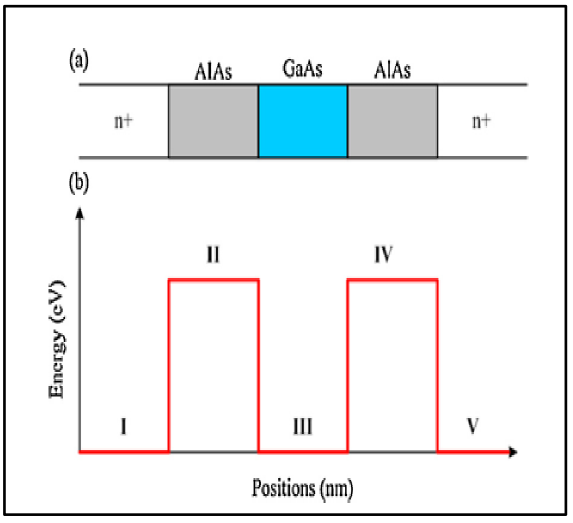


Fig. 2

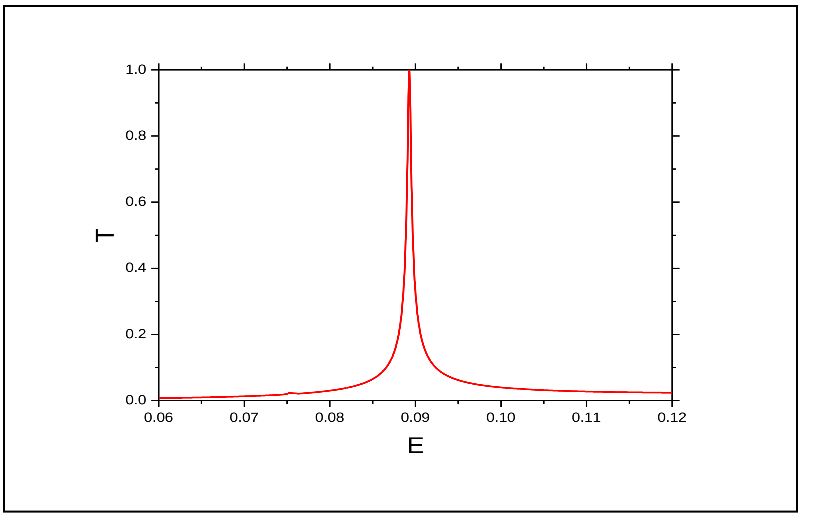


Fig. 3

The simplest of all one-dimensional problems are square barriers and square wells in one dimension. The Schrodinger equation will have the simple form:

$$\left(\frac{p^2}{2m}+V\right)\varphi = E\varphi \quad \textbf{[1]}$$

where V is a constant in a given region. The general solution of the above equation has the well-known form:

$$\varphi(x) = ae^{ikx} + be^{-ikx} \quad \textbf{[2]}$$

$$k = \sqrt{\frac{2m(E-V(x))}{\hbar^2}} \quad \textbf{[3]}$$

Thus, when E - V> 0 the wave functions are plane waves.

But when E - V < 0 the wave function has the form (see Fig. 4)

$$\varphi(x) = ae^{-\chi x} + be^{\chi x} \quad \textbf{[4]}$$

$$\chi = \sqrt{\frac{2m(V(x)-E)}{\hbar^2}} \quad \textbf{[5]}$$

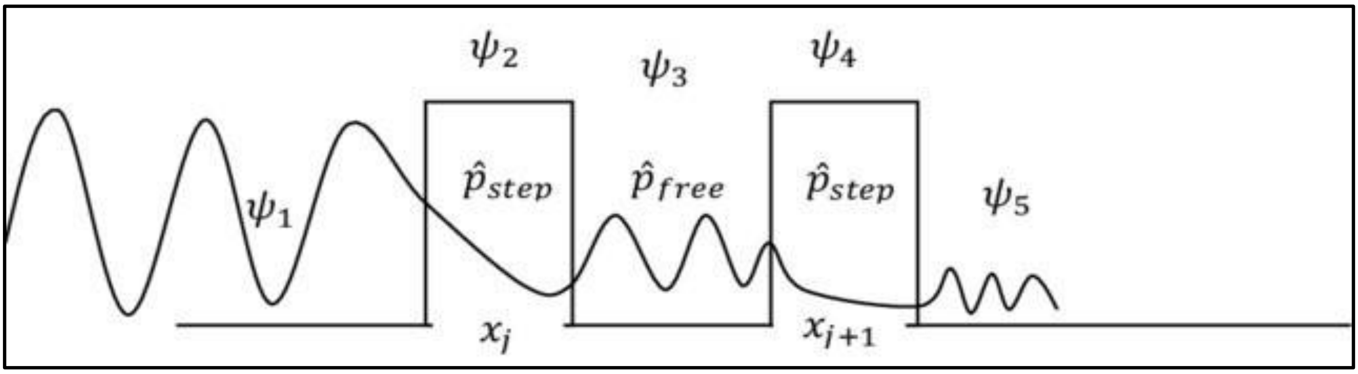


Fig. 4

The wave functions are now exponentially growing and decaying waves characteristic of barrier penetration problems. In the square barrier and "square well" problem, *V* changes abruptly from one constant value to another. The overall wave function is then constructed out of pieces by matching the wave function and the wave function derivative at the discontinuities of *V*:

$$\psi_j\big|_{x=x_{j+1}} = \psi_{j+1}\big|_{x=x_{j+1}}$$

and

$$\left.\frac{d\psi_j}{dx}\right|_{x=x_{j+1}} = \left.\frac{d\psi_{j+1}}{dx}\right|_{x=x_{j+1}} \qquad \textbf{[6]}$$

This procedure is applied to calculate the transmission coefficient of a double barrier. Generally speaking, as one might expect, the tunneling attenuation factor of two barriers is approximately the product of the attenuation factors for each. But, for a resonating condition in the intermediate well the electron may pass through both barriers without any attenuation with a transmission coefficient equal to 1 (see Figs. 1, 2 and 3).

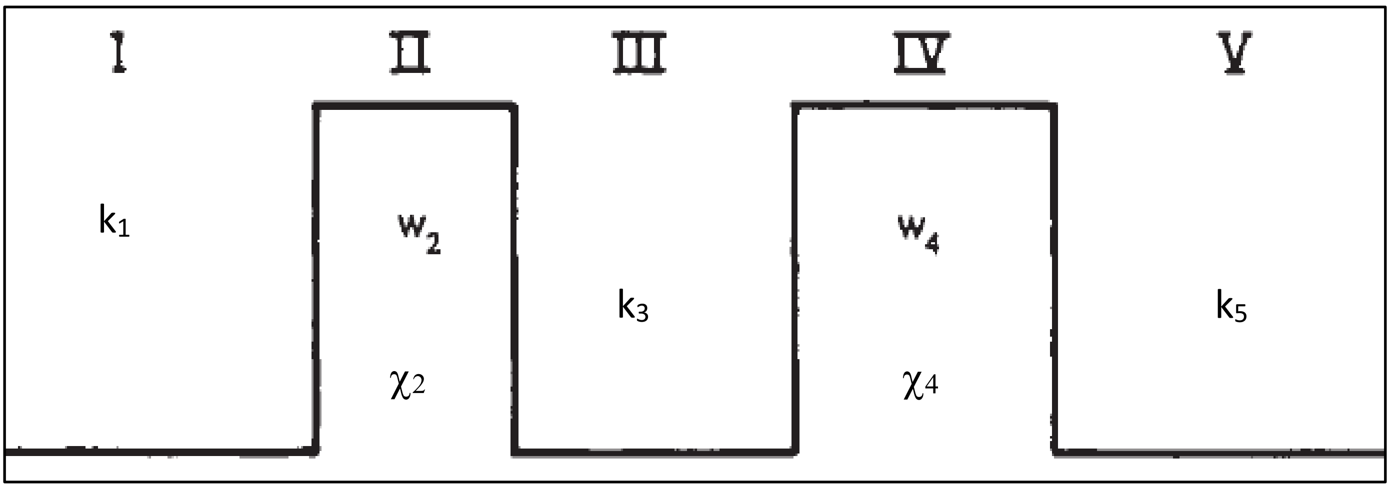
If we assume a wave incident from the left and transmitted on the right, it is possible to obtain analytically an exact expression for the transmission coefficient, ***T***:

$$\boxed{T = \frac{\dfrac{2^8 k_1 \chi_2 k_3 \chi_4 k_5}{|K|^2}}{(k_1^2 + \chi_2^2)(\chi_2^2 + k_3^2)(k_3^2 + \chi_4^2)(\chi_4^2 + k_5^2)}} \tag{7}$$

$$\begin{aligned}
K = {} & \exp\left(\chi_2 w_2 + \chi_4 w_4\right) \times \\
& [\exp i(-\varphi_1 + \varphi_2 + \varphi_3 + \varphi_4 + \varphi_5) - \exp i(\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4 + \varphi_5)] \\
& + \exp\left(\chi_2 w_2 - \chi_4 w_4\right) \\
& \times [-\exp i(-\varphi_1 + \varphi_2 + \varphi_3 - \varphi_4 - \varphi_5) \\
& + \exp i(\varphi_1 + \varphi_2 - \varphi_3 + \varphi_4 - \varphi_5)] \\
& + \exp\left(-\chi_2 w_2 + \chi_4 w_4\right) \\
& \times [-\exp i(-\varphi_1 - \varphi_2 - \varphi_3 + \varphi_4 + \varphi_5) \\
& + \exp i(\varphi_1 - \varphi_2 + \varphi_3 - \varphi_4 + \varphi_5)] \\
& + \exp\left(-\chi_2 w_2 - \chi_4 w_4\right) \\
& \times [\exp\ i(-\varphi_1 - \varphi_2 - \varphi_3 - \varphi_4 - \varphi_5) \\
& - \exp i(\varphi_1 - \varphi_2 + \varphi_3 + \varphi_4 - \varphi_5)]
\end{aligned}$$

$$\varphi_1 = k_3 w_3; \quad \varphi_2 = Atan\frac{\chi_2}{k_1}; \quad \varphi_3 = Atan\frac{\chi_2}{k_3}\ ; \quad \varphi_4 = Atan\frac{\chi_4}{k_3}; \quad \varphi_5 = Atan\frac{\chi_4}{k_5}$$

Fig. 5



The expression of T above corresponds to the structure depicted in Fig. 5

2.1. Asymmetric double barrier.

The simplest example of an asymmetric double barrier es the case where the barriers are different. Another more elaborated case is the one where an external bias is applied between left and right regions (see Figs. 6 and 7). For an asymmetric double barrier, the resonant transmission is still very important but, on the other hand, it is no longer equal to one.

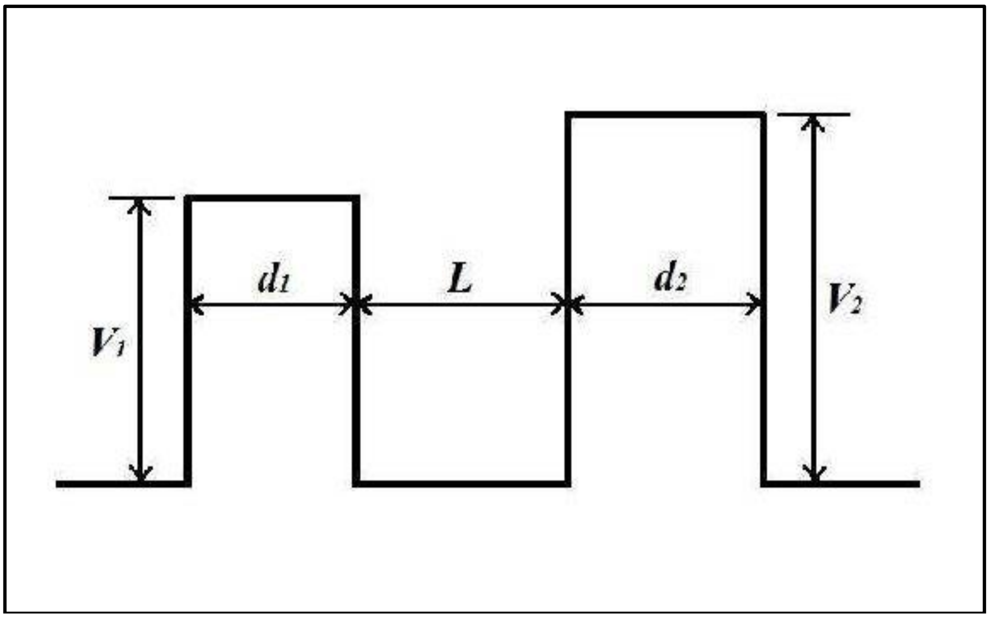


Fig. 6

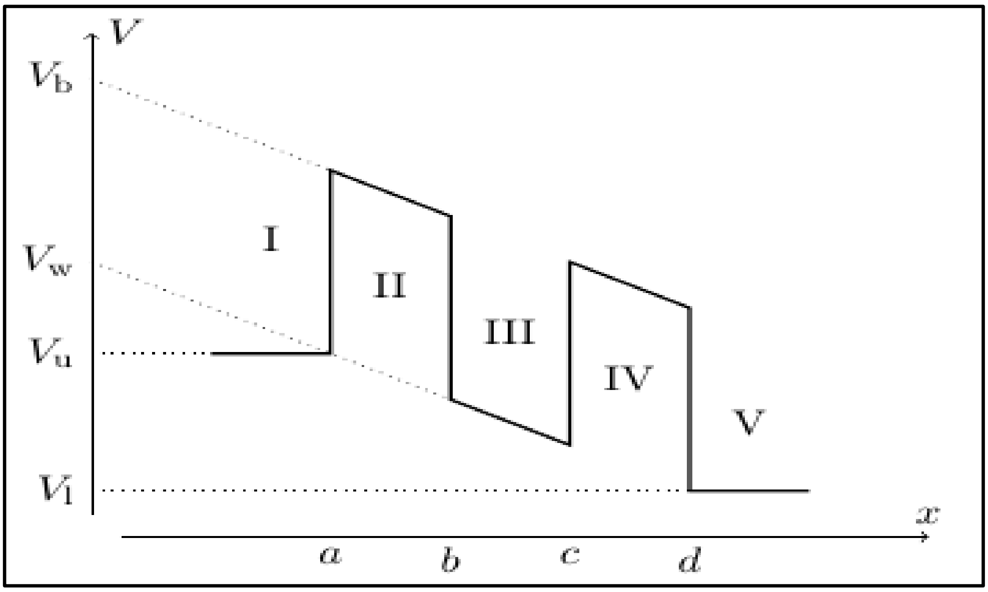


Fig. 7

3. To learn more

Applied Quantum Mechanics. Second Edition. A. F. J. Levi.
Cambridge University Press 2006.

Quantum Mechanics. An Introduction for Device Physicists and Electrical Engineers. Third Edition. David Ferry.
CRC Press 2001.

4.Procedure.

4.1. Transmission of a symmetric double barrier

1. Write a computer program with the software of your choice, to find the transmission coefficient of a symmetric double barrier and transmission resonances (bound states energies of the double barrier well). In the calculation use the expression of T given in page 4, Eq. 7. Use the parameters given in Fig. 8. Use the following masses, $0.067m_e$ for the electron mass m* in left region, well and right region (effective mass of GaAs) and the same mass $0.067m_e$ for the GaAlAs barriers; m_e being the bare electron mass.

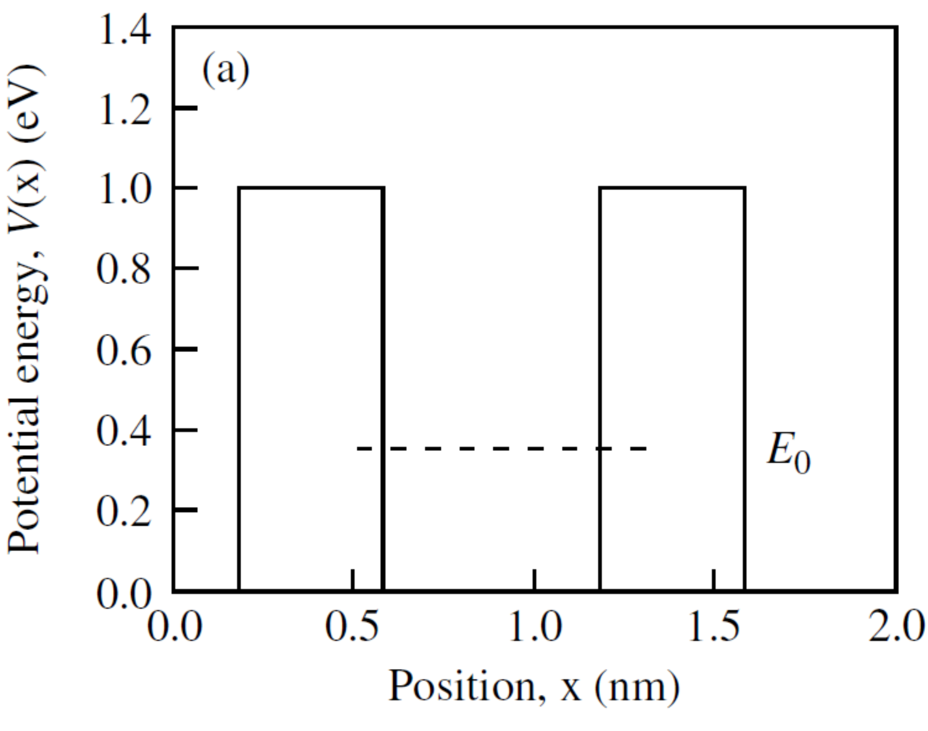


Fig. 8

Parameters of Fig.8: symmetric double rectangular potential barrier with barrier width 0.4 nm, well width 0.6 nm, and barrier energy 1 eV. There is a resonance at energy E_0. This resonance energy is indicated by the broken line.

2. Represent in a graph the obtained results for the transmission coefficient and obtain the energy of the possible well resonances that would correspond to the energy of the electron bound states inside the well. See Fig.3 as an example.

4.2. Transmission of an asymmetric double barrier

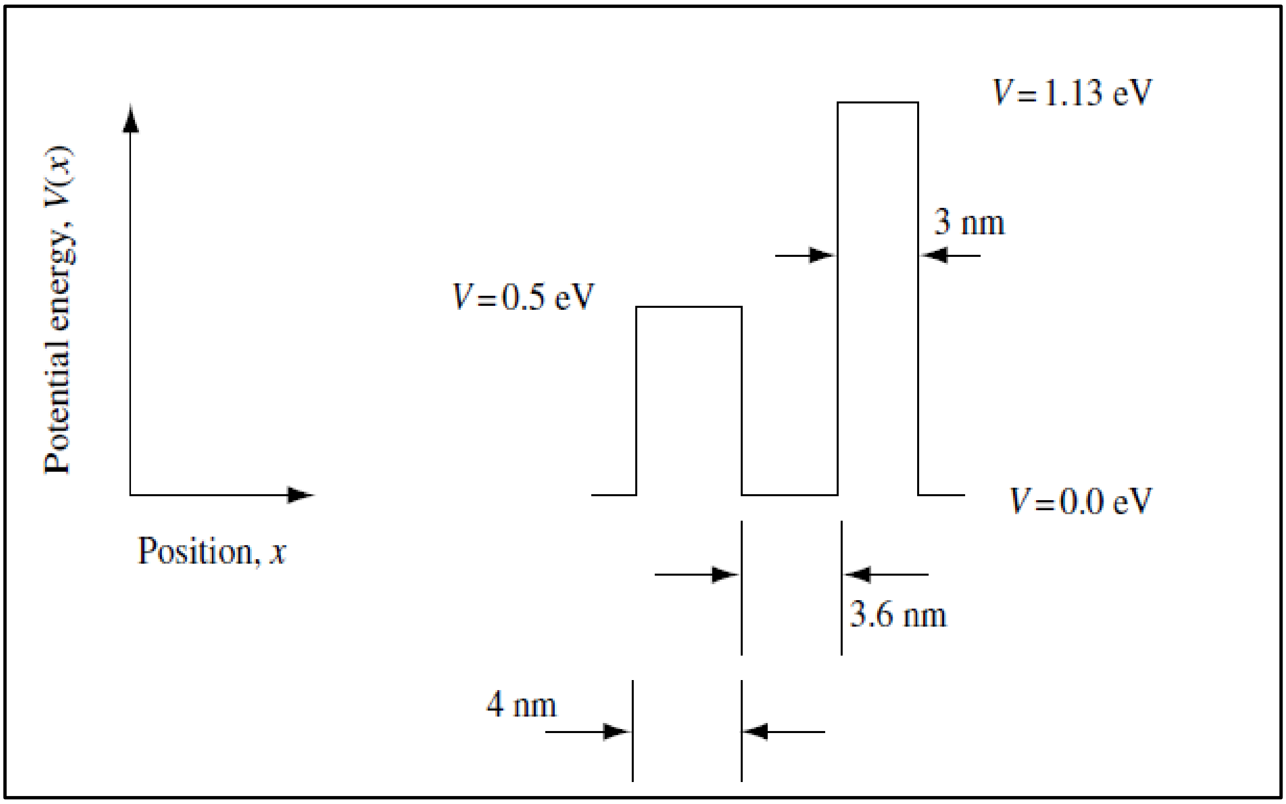


Fig. 9

1. Case (a) Unequal double barrier (see Fig. 9)
   Using the previous computer program, repeat the calculation for the asymmetric double barrier using the same electron masses. Use the parameters presented in Fig. 9. Represent the transmission coefficient vs energy and obtain the resonances.

2. Case (b) External bias (see Fig. 10)
   Repeat the calculation but now apply a uniform electric field across the double barrier and single well structure as shown in the Fig. 10. Consider that the right barrier is 3 nm thick, the left one 4 nm thick and the well width is 4 nm. The right-hand edge of the 3 nm-thick barrier is at a potential −0_63eV below the left-hand edge of the 4

nm-thick barrier. Comment on the changes in transmission you observe.

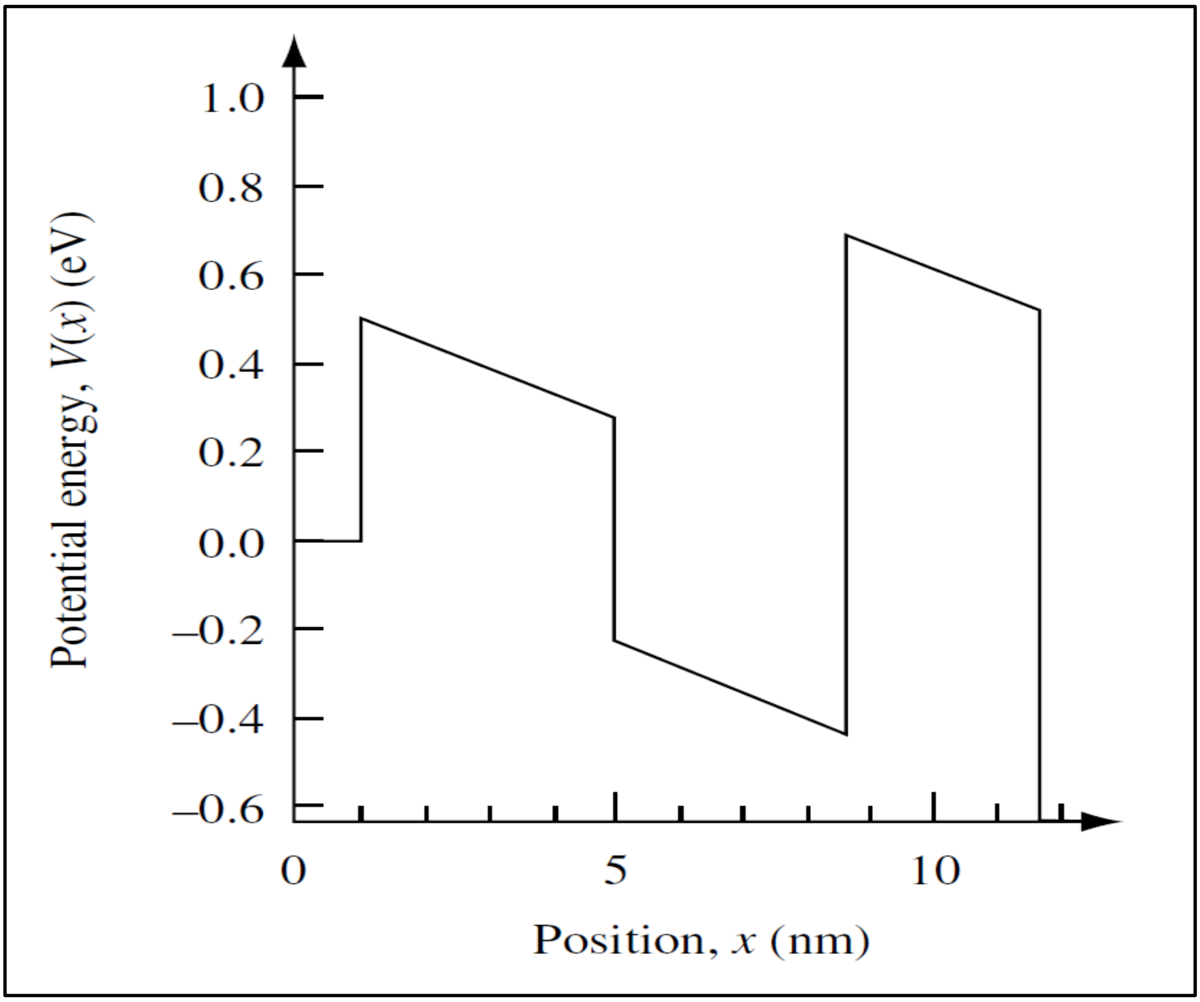


Fig. 10