MATHS

Chapter 1 : Number System

Putting a value

Q1. To reach Ekapathshala a student has to take 100 steps. He can take steps of 3 or 5 only. How many different steps of 5 can he take, if he must take at least one step of each type?

Sol:
- steps can't be in points
- Steps can't be zero as said at least one step of each

Equation: $3x + 5y = 100$

for $x = 1$, $y = 19.4$ {not applicable}
$x = 2$, $y = 18.8$

x	5	10	15	20	25	30	$\{x+5\}$
y	17	14	11	8	5	2	$\{y-3\}$

Diff. steps of 5 means how many diff. values of y i.e. 6.

Ans = 6

Min. value of $x = 5$
Max. value of $y = 17$
Min. value of $y = 2$
Avg. value of $y = 9.5$ $\left\{\frac{11+8}{2}\right\}$ middle values

Q2. PQR × QS = PQRS
PQR ⇒ 3 digit number
QS ⇒ 2 digit number
PQRS ⇒ 4 digit number
Find Q.

Sol^n. try smallest 2 digit no = 10

PQR × QS = PQRS
1 10
let, 419 × 10 = 4190 verifies
let, 318 × 10 = 3180 "

so QS = 10 is verifying all conditions

∴ Q = 1 Ans = 1

Number of trailing Zeroes → zeroes at last {FACTORIAL}

Q3. What is the number of zeroes at the end of 150!

Sol^n. • no. of trailing zeroes = no. of tens
• one 10 can be formed by two prime no. i.e. 2 × 5.

→ Quotients

• So, no. of 2's $= \frac{150}{2} = \frac{75}{2} = \frac{37}{2} = \frac{18}{2}$

$= \frac{9}{2} = \frac{4}{2} = \frac{2}{2} = 1$

$75 + 37 + 18 + 9 + 4 + 2 + 1 = 146$

no. of 2's = 2^{146}

- Similarly, no. of 5's $= \frac{150}{5} = \frac{30}{5} = \frac{6}{5} = 1$

no. of 5's = 5^{37}

[Applicable on Prime No's only]

1^{st} 5×2
2^{nd} 5×2
⋮
37^{th} 5×2

there is no, 38^{th} 5 so no more 10's can be formed. $\Rightarrow 10^{37}$

$\therefore$ Ans = 37

Q4. What is the greatest power of 7 that will divide 100! completely?

Sol^n. $\frac{100}{7} = \frac{14}{7} = \frac{2}{7}$ $\Rightarrow 16$

$\Rightarrow 7^{16}$

Ans = 16

Qs. (i) Find the power of 2 in 100!

Sol^n- $\frac{100}{2} = \frac{50}{2} = \frac{25}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$

$\Rightarrow 50 + 25 + 12 + 6 + 3 + 1 = 97$

$\Rightarrow 2^{97}$ $\therefore$ Ans = 97

(ii) Find the power of 3 in 100!

Sol^n $\frac{100}{3} = \frac{33}{3} = \frac{11}{3} = \frac{3}{3} = 1$

$\Rightarrow 33 + 11 + 3 + 1 = 48$

$\Rightarrow 3^{48}$ $\therefore$ Ans = 48

(iii) find the power of 6 in 100!

Sol^n.
- 6 is formed with 2×3.
- min. power from both of them is of 3 i.e. 48 or greatest of 2, 3 is 3

$\Rightarrow 6^{48}$ $\therefore$ Ans = 48

(iv) Find the power of 10 in 100!

Sol^n • 10 is formed by 2×5
• Greatest of 2, 5 is 5.
• no. of 5's $= \frac{100}{5} = \frac{20}{5} = 4$

$\Rightarrow 20 + 4 = 24 \qquad \Rightarrow 5^{24}$

Ans: 24

Q6. $167! = 5^P \times Y$. What is the greatest value of P (given Y is not a multiple of 5)?

Sol^n: • no. of 5's. $\Rightarrow \frac{167}{5} = \frac{33}{5} = \frac{6}{5} = 1$

$\Rightarrow 33 + 6 + 1 = 40 \qquad \Rightarrow 5^{40}$

$\therefore$ Ans = 40

Q7. Find the smallest number that should be divided 198!, so that it benhomes odd number?

Sol^n: • To make a no., odd no. we need to take out all the 2's.
• We need to find no. of 2's.

$$\frac{198}{2} = \frac{99}{2} = \frac{49}{2} = \frac{24}{2} = \frac{12}{2} = \frac{6}{2} = \frac{3}{2} = 1$$

$\Rightarrow 99 + 49 + 24 + 12 + 6 + 3 + 1 = 194$

$\therefore$ Ans $= 2^{194}$

Remainder

→ Whatever can be done with a number can also be done with the Remainder.

ex: on no, $\frac{104}{5} = 4$, $\frac{103}{5} = 3 \Rightarrow \frac{207}{5} = 2$

on rem, $\frac{4}{5} = 4$, $\frac{3}{5} = 3 \Rightarrow \frac{7}{5} = 2$

Q8. When 2 numbers are divided by a divisor, 19 and 32 are obtained as remainders. When the sum of these 2 no. is divided by the same divisor, the remainder is 8. Find the divisor.

Solⁿ. Apply on Rem,

$$19 + 32 = \frac{51}{D} = 8$$

$$51 - 43 = 8$$

we subtract 43 then only rem becomes 8.

$\therefore$ **Ans = 43**

Remainder Cycle

{* Rem = 0 means last step }

Q. The Remainder when 2^{8491} is divided by 7 is:

Sol^n Cycle of 7 :

$\frac{2}{7} = 2$] (multiply with itself)

$\frac{4}{7} = 4$ (multiply with 1^st rem)

$\frac{8}{7} = 1$

[Stop when : Repeat
: 1
: 0]

Cycle = 3

$\therefore$

```
      283
  3 ) 8491
      6
      24
      24
      x
        9
        9
        x 1
```

$\therefore$ Rem = 1

Ans = 2 as 1^st step cycle *

Q10. What is the Rem. when 3^{92861} is divided by 5

Sol^n.

$\frac{3}{5} = 3$

$\frac{9}{5} = 4$

$\frac{12}{5} = 2$

$\frac{6}{5} = 1$

Cycle of 5 = 4

$4\overline{)92862}$

$\underline{\quad}$

2

$\therefore$ Ans = 4

as 2^{nd} ~~cycle~~ step

Q11. What is the Rem. when 3^{1986} is divided by 11.

Sol^n.

$\frac{3}{11} = 3$	$\frac{15}{11} = 4$
$\frac{9}{11} = 9$	$\frac{12}{11} = 1$
$\frac{27}{11} = 5$	

Cycle of 11 = 5

$5 \overline{)1986}$

$\underline{1}$

1^{st} step = 3

$\therefore$ Ans = 3

Q12. What is Rem. when 3^{1986} is divided by 19

Sol^n.

$\frac{3}{19} = 3$	$\frac{21}{19} = 2$	$\frac{33}{19} = 14$
$\frac{9}{19} = 9$	$\frac{6}{19} = 6$	$\frac{42}{19} = 4$
$\frac{27}{19} = 8$	$\frac{18}{19} = 18$	$\frac{12}{19} = 12$
$\frac{24}{19} = 5$	$\frac{54}{19} = 16$	$\frac{36}{19} = 17$
$\frac{15}{19} = 15$	$\frac{48}{19} = 10$	$\frac{51}{19} = 13$
$\frac{45}{19} = 7$	$\frac{30}{19} = 11$	$\frac{39}{19} = 1$

Cycle of 19 = 18

$18 \overline{)1986}$

$\underline{6}$

6^{st} step = 7

$\therefore$ Ans = 7

Q13. Remainder when 4^{1985} is divided by 6.

Solⁿ. $\frac{4}{6} = 4$

$\frac{16}{6} = 4$

Cycle = 1

$\frac{16}{6} = 4$

Always coming 4. $\therefore$ Ans = 4

Q14. Remainder when $3^{185!}$ is divided by 5.

Solⁿ. $\frac{3}{5} = 3$

$\frac{9}{5} = 4$

$\frac{12}{5} = 2$

$\frac{6}{5} = 1$

Cycle = 4

- $\frac{185 \times 184 \times 183 \cdots \times 4 \times 3 \times 2 \times 1}{4}$
- $\frac{4m}{4}$ multiple of 4
- so rem becomes 0
- i.e. go to last step

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$\therefore$ Ans = 1

Fermat Rule

- $\dfrac{a^{even}}{a+1} = 1$

- $\dfrac{a^{odd}}{a+1} = a$

- $\dfrac{(a+1)^{any}}{a} = 1$

Q15. (i) $\dfrac{3^{5^{6^{8^{\cdots^{10001}}}}}}{4}$

Sol^n: $\dfrac{3^{5^{n}}}{4}$ • 5 is odd and odd^{any} = odd

$\therefore$ power is odd

$\dfrac{a^{odd}}{a+1} = a$

$\therefore$ Ans = 3

(ii) $\dfrac{3^{4^{5^{6^{\cdots^{10001}}}}}}{4}$

Sol^n: • 4 is even and $even^{any}$ = even

• $\therefore$ power is even

$$\frac{a^{even}}{a+1} = 1$$

$\therefore$ Ans = 1

(iii) $\dfrac{4^{2^{4^{6^{\cdots^{200023}}}}}}{3}$

Solⁿ $\dfrac{(a+1)^{an\ 4}}{a} = 1$ $\therefore$ Ans = 1

(iv) $\dfrac{7^{11^{12^{13^{14}}}}}{6}$

Solⁿ. $\therefore$ Ans = 1 (3rd pt.)

(v) $\dfrac{5^{12^{13^{14^{15}}}}}{6}$

Ans. $\therefore$ Ans = 1 (1st point)

(vi) $\dfrac{5^{13^{14^{16}}}}{6}$

Solⁿ. Ans = 5 (2nd point)

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Unit Digit Cycle

→ • A number ending with 0 will always end with 0
• ——— 1 ——— 1
• ——— 5 ——— 5
• ——— 6 ——— 6

→ • Whereas the remaining digits 2 have cycle of 4
• ——— 3 ———
• ——— 4 ———
• ——— 7 ———
• ——— 8 ———
• ——— 9 ———

$2^1 = 2$	$2^5 = 32$	$2^9 = 512$
$2^2 = 4$	$2^6 = 64$	$2^{10} = 1024$
$2^3 = 8$	$2^7 = 128$	$2^{11} = 2048$
$2^4 = 16$	$2^8 = 256$	$2^{12} = 4096$

{repeats after 4}

Q16. (ii) $(2013)^{782}$ find Unit Digit

Soln. • 3 has cycle of 4
• $4 \overline{)782}$ (195

$$\begin{array}{r} 782 \\ \underline{4} \\ 38 \\ \underline{36} \\ 22 \\ \underline{20} \\ 2 \end{array}$$

- Rem = 2
- $3^{Rem} = 3^2 = 9$

$\therefore$ **Ans = 9**

(ii) $(21467)^{9885}$

Sol^n^

- 7 has cycle of 4
- 4) 9885 (2471

```
4 ) 9885 ( 2471
    8
    --
    18
    16
    --
     28
     28
     --
     x5
      4
     --
      1
```

- Rem = 1
- $7^1 = 7$

$\therefore$ **Ans = 7**

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(iii) $(21648)^{844}$

Sol^n • 8 has cycle of 4

• $4 \overline{)844} (211$
$\overline{\quad}$
$\overline{x}$

• Rem = 0
• last step i.e. 4

• $8^4 \Rightarrow 8^2 \times 8^2$
$\Rightarrow 64 \times 64$
$\Rightarrow 1\boxed{6}$

$\therefore$ Ans = 6

Q17. (i) Find unit digit of $11^{11} \times 12^{12} \times 13^{13} \times 14^{14}$

Sol^n. • $11^{11} = 1$

• $12^{12} = 4 \overline{)12} ($ $\Rightarrow$ Rem = 0 $\Rightarrow 2^4 = 6$
$\overline{x}$

• $13^{13} = 4 \overline{)13} ($ $\Rightarrow$ Rem = 1 $\Rightarrow 3^1 = 3$
$\overline{1}$

• $14^{14} = 4\overline{)14}(3 \Rightarrow \text{Rem} = 2 \Rightarrow 4^2 = 8$

$\underline{2}$

$\Rightarrow \underbrace{1 \times 6 \times 3}_{18} \times 6 = 48$

$\therefore$ **Ans = 8**

• even	×	even	=	odd	• $5 \times e = 0$
• odd	×	even	=	even	
• odd	×	odd	=	odd	• $5 \times o = 5$

(ii) $11^{11} \times 12^{12} \times 13^{13} \times 14^{14} \times 15^{15} \text{ -------- } 99^{99}$

Sol^n. $11^{11} \times 12^{12}$ = odd × even = even

$15^{15} \rightarrow 5 \times \text{even } (2) = 10$

$\therefore$ **Ans = 0**

(iii) $11^{11} \times 13^{13} \times 15^{15} \text{ ------ } 99^{99}$

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Solⁿ. $11^{11} \times 13^{15}$ = odd × odd = odd

$15^{15} \rightarrow 5 \times$ odd = $\underline{5}$

$0 \times 0 \times 5 \times 0 \times 0 \longrightarrow 5 \times 0$

∴ Ans = 5

NUMBER SYSTEM SHEET

1. What is the unit digit of the product of
207 · 781 · 39 · 94

Solⁿ. 7 × 1 × 9 × 4

7 63 12 ⟹ 2

∴ Ans = 2

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2. What will come in place of unit digit in the value of $(7)^{35} \cdot (3)^{71} \cdot (11)^{55}$?

Solⁿ. Cycle of 7 = 4 ⟹ 4) 35 (8
32
3

⟹ 7^3

Cycle of 3 = 4 ⟹ 4) 71 (17
68
3

$\Rightarrow 3^3$

Cycle of 1 = 1 $\Rightarrow 1^1$

$\Rightarrow 7^3 \times 3^3 \times 1$

$\Rightarrow (49 \times 7)\ (27)\ (1)$

$\Rightarrow 3 \times 7 \times 1 = 21 = 1$

$\therefore$ <u>Ans = 1</u>

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3. Find the number of zeroes at the end of
$1 \times 2 \times 3 \times 4 \times 5 \ldots 99 \times 100$?

Sol^n:
- This is 100!
- Divide by 5 $\Rightarrow \frac{100}{5} = \frac{20}{5} = \underline{4}$

$\Rightarrow 20 + 4 = 24$

$\therefore$ <u>Ans = 24</u>

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* 4. Find the number of zeroes at the end of
~~10 × 20 × 30 × ---- ×~~ $2 \times 4 \times 6 \times \ldots 98 \times 100$

Sol^n: $2 \times 4 \times 6 \times 8 \times \ldots 100$

- There are 50 terms and we take out 2, 50 times

$\Rightarrow 2^{50} \{1 \times 2 \times 3 \times 4 \text{ ---- } 50\}$

$\Rightarrow 2^{50} (50!)$

$\Rightarrow \frac{50}{5} = \frac{10}{5} = 2 \quad \Rightarrow 10 + 2 = 12$

$\therefore$ Ans = 12

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5. Find the number of zeroes at the end of $10 \times 20 \times 30 \times \text{-----} \times 2000$

Sol^n: $10 \times 20 \times 30 \text{ ---- } \times 2000$

$\Rightarrow 10^{200} (1 \times 2 \times 3 \text{ ---- } \times 200)$

$\Rightarrow 10^{200} (200!)$

$\Rightarrow \frac{200}{5} = \frac{40}{5} = \frac{8}{5} = 1 \Rightarrow 40 + 8 + 1 \Rightarrow 49$

No. of zeroes $\Rightarrow 200 + 49 = 249$

$\therefore$ Ans = 249

FACTORS

Q.18. (i) Number of factors of 720. find

Sol^n.

2	720
2	360
2	180
2	90
3	45
3	15
5	5
	1

$720 = 2^{4} \times 3^{2} \times 5^{1}$

$\Rightarrow$ N.F. $= (4+1)(2+1)(1+1)$

$= 5 \times 3 \times 2$

$= 30$

- Add 1 to the powers and multiply.

$\therefore$ Ans $= 30$

(ii) Number of factors of 540

Sol^n.

2	540
2	270
3	135
3	45
3	15
5	5
	1

$540 = 2^{2} \times 3^{3} \times 5^{1}$

N.F $= (2+1)(3+1)(1+1)$

$= 3 \times 4 \times 2$

$= 24$

$\therefore$ Ans $= 24$

NO. OF PRIME FACTORS

- Add the powers

Q19. Find no. of prime factors of 720

Soln. $720 = 2^4 \times 3^2 \times 5^1$

$N.P.F. = 4 + 2 + 1 = 7$

$\therefore$ Ans = 7

NO. OF ODD FACTORS

- Take the power of odd no.
- Add 1 to them and multiply

Q20. Find no. of odd factors in 720

Soln. $720 = 2^4 \times 3^{2} \times 5^{1}$

$N.O.F. = (2+1)(1+1)$
$= 3 \times 2$
$= 6$

$\therefore$ Ans = 6

NO. OF EVEN FACTORS

$\boxed{\text{N.E.F.} = \text{Total factor} - \text{odd factor}}$

Q21. Find no. of even factors of 720

Soln N.E.F = T.F − O.F.
= 30 − 6
= 24

∴ **Ans = 24**

Sum of FACTORS

Q22. Find sum of factors of 720

Soln. $720 = 2^4 \times 3^2 \times 5^1$

$$\text{Sum} = [2^0 + 2^1 + 2^2 + 2^3 + 2^4] \times [3^0 + 3^1 + 3^2] \times [5^0 + 5^1]$$

$$= [1 + 2 + 4 + 8 + 16][1 + 3 + 9][1 + 5]$$

$$= (31)(13)(6)$$

$$= 2418$$

∴ **Ans = 2418**

Q23. Find sum of factors of 540.

Sol^n. $540 = 2^2 \times 3^3 \times 5^1$

$$\begin{aligned} \text{Sum} &= (2^0 + 2^1 + 2^2) \times (3^0 + 3^1 + 3^2 + 3^3) \times (5^0 + 5^1) \\ &= (1 + 2 + 4) \times (1 + 3 + 9 + 27) \times (1 + 5) \\ &= 7 \times 40 \times 6 \\ &= 1680 \end{aligned}$$

$\therefore$ Ans = 1680

AVERAGE OF FACTORS

$$\boxed{\text{avg} = \frac{\text{sum of factors}}{\text{no. of factors}}}$$

Q24. Find average of factors of 720.

Sol^n. Sum of factors of 720 = 2418
No. of factors of 720 = 30

$$\text{Avg} = \frac{2418}{30} = 80.6$$

$\therefore$ Ans = 80.6

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PRODUCT OF FACTORS

$$\boxed{(\text{Number})^{\frac{\text{no. of factors}}{2}}}$$ ← power

Q25. Find product of factors of 720.

Solⁿ. $(720)^{30/2} = (720)^{15}$

$\therefore$ **Ans** $= (720)^{15}$

NUMBER SYSTEM SHEET

6. Find the no. of factor of 100.

Solⁿ.

2	100
2	50
5	25
5	5
	1

$100 = 2^2 \times 5^2$

$N.F. = (2+1)(2+1)$
$= 3 \times 3$
$= 9$

$\therefore$ **Ans** $= 9$

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7. Find the number of factor of 80.

Sol:

2	80
2	40
2	20
2	10
5	5
	1

$80 = 2^4 \times 5^1$

N.F. $= (4+1)(1+1)$
$= 5 \times 2$
$= 10$

$\therefore$ Ans $= 10$

8. Find the sum of factor of 100

Sol: $100 = 2^2 \times 5^2$

N.F. $= (2+1)(2+1) = 3 \times 3 = 9$ (no need)

Sum $= [2^0 + 2^1 + 2^2] \times [5^0 + 5^1 + 5^2]$
$= (1+2+4)(1+5+25)$
$= 7 \times 31$
$= 217$

$\therefore$ Ans $= 217$

9. Find the sum of factos of 50

Sol: $50 = 2^1 \times 5^2$

N.F. $= (1+1)(2+1) = 2 \times 3 = 6$

2	50
5	25
5	5
	1

Sum $= (2^0+2^1)(5^0+5^1+5^2)$
$= (1+2)(1+5+25)$
$= 3 \times 31$
$= 93$

$\therefore$ Ans $= 93$

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10. Find the average of 60.

Solⁿ. $60 = 2^2 \times 3^1 \times 5^1$

2	60
2	30
3	15
5	5
	1

N.F. $= (2+1)(1+1)(1+1)$
$= 3 \times 2 \times 2 = 12$

Sum $= (2^0+2^1+2^2)(3^0+3^1)(5^0+5^1)$
$= (1+2+4)(1+3)(1+5)$
$= 7 \times 4 \times 6$
$= 168$

Avg $= \frac{168}{12} = 14$

$\therefore$ Ans $= 14$

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11. Find the product of factor of 100

Sol^n. $100 = 2^2 \times 5^2$

$N.F. = (2+1)(2+1) = 3 \times 3 = 9$

prod. of factor $= (100)^{9/2} = (10^2)^{9/2}$

$= (10)^9$

$\therefore$ **Ans $= (10)^9$**

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14. What will be the remainder when 17^{200} is divided by 18.

Sol^n. $\dfrac{17^{200}}{18} = \dfrac{a^{even}}{a+1} = 1$ (By fermet rule)

$\therefore$ **Ans $= 1$**

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15. What will be the remainder when $(67^{67} + 67)$ is divided by 68?

Sol^n. $\dfrac{67^{67} + 67}{68} = \dfrac{67^{67}}{68} + \dfrac{67}{68}$ {Find rem so that jo ~~...~~ num k sath vo rem k sath}

$\dfrac{a^{odd}}{a+1} = a \Rightarrow \dfrac{67 + 67}{68} = \dfrac{134}{68}$

$= 66$

$\therefore$ Ans $= 66$

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16. Which of the following numbers will completely divide $(49^{15} - 1)$?

Solⁿ. $49^{15} - 1^{15}$

if $n = odd$, $(a^n - b^n) \div (a - b)$

$a - b = 49 - 1 = 48$

in options, 48 is multiple of 8

$\therefore$ Ans $= 8$

PROPERTIES

- $n = odd$, $a^n - b^n \div a - b$

  $\{ a^3 - b^3 = (a-b)(a^2 + ab + b^2) \}$

- $n = odd$, $a^n + b^n \div a + b$

  $\{ a^3 + b^3 = (a+b)(a^2 - ab + b^2) \}$

27. Which one of the following is the common factor of $(47^{43} + 43^{43})$ and $(47^{47} + 43^{47})$

Solⁿ. $n = \text{odd}$, $a^n + b^n \div a+b$

here. $a = 47$, $b = 43$ or vice-versa

$\Rightarrow a+b = 47+43$

$\therefore$ Ans $= 47 + 43$

PROPERTY

- $n = \text{even}$; $a^n - b^n \div (a+b)(a-b)$

① ② or both

$\{ a^2 - b^2 = (a+b)(a-b) \}$

38. $7^{12} - 4^{12}$ is exactly divisible by which of the following

i) 36 ii) 35 iii) 34 (iv) 33

Solⁿ. div. by $(a+b)(a-b)$

$\Rightarrow (7+4)(7-4)$

$= 11 \times 3$

$= 33$

$\therefore$ Ans $= 33$

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28. Q. Which one of the following is prime no.?
a) 15 b) 31 c) 21 d) 9

Solⁿ. Prime no. :- That is div by 1 and itself

Composite no. :- That is not prime.

To find prime no: follow following steps

- Take square root of ~~nearest~~ greater sq. of that no.
- Then write prime no. till the sq. root.
- Check whether any of those no. divides the no. in question.
- If no, prime else composite

⇒ • 31 so nearest sq. is 36
- $\sqrt{36} = 6$
- ~~no~~. prime no. till 6 = 2, 3, 5
- 2 does not divides 31
  3 ———
  5 ———

∴ 31 is prime

∴ Ans = 31

Q 26. Check 101 is prime or not?

Solⁿ.
- ~~100~~ 121 ⇒ ~~$\sqrt{100}$ = 10~~ ⇒ $\sqrt{121} = 11$
- 2, 3, 5, 7, 11

- 2 → ×, 3 → ×, 5 → ×, 7 → ×, 11 → ×
- They all does not divide 101.

∴ 101 is prime.

∴ **Ans = prime**

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56. If n is a natural no., then $(6n^2 + 6n)$ is always div. by :
a) 6 only c) 12 only b) 6 & 12 both d) by 18 only

Sol^n. Take smallest natural no i.e. 1
by putting 1, $6 + 6 = 12$
ans seems b) 6 & 12 both
by putting 2, $24 + 12 = 36$
still ans is b)

∴ **Ans = b) 6 & 12 both**

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57. If n is natural no., then $(n^3 - n)$ is always divisible by
same options

Sol^n. $n = 1 \Rightarrow 1^3 - 1 = 0$
$n = 2 \Rightarrow 2^3 - 2 = 8 - 2 = 6$

∴ **Ans = a) 6 only**

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58. $(x^n - a^n)$ is completely div. by $(x-a)$ when

a) n natural no   c) n odd no

b) n even no   d) n is prime

Solⁿ. n can be even ass well as odd

$\therefore$ Ans = a)

59. $(x^n - a^n)$ is completely div. by $(x+a)$ when same options

Solⁿ. n can be only even

$\therefore$ Ans = b)

60. $(x^n + a^n)$ is completely div. by $(x+a)$ when same options

Solⁿ. n can be only odd

$\therefore$ Ans = c)

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17 A number when divided by 6 leaves remainder 3. When the sq. of no. is divided by 6, rem is

Soln. A.T.Q. : $\frac{N}{6} = 3$

[जो no. K sath वही Rem K sath]

so, $\frac{N^2}{6} = ?$

we do on Rem i.e. 3

$$\frac{3^2}{6} = \frac{9}{6} = 3$$

$\therefore$ Ans = 3

** Imp. Topic # SUCCESSIVE DIVISION

- 1st stage quotient becomes dividend for next stage.

Q27. 324 is successively divided by 5, 3 and 2 find rem.

Soln

$$5\overline{)324}(64 \qquad 3\overline{)64}(21 \qquad 2\overline{)21}(10$$

$$\begin{array}{ccc} -30 & 6\downarrow & -2\downarrow \\ \hline 24 & 04 & 01 \\ -20 & -3 & -0 \\ \hline 4 & 1 & 1 \end{array}$$

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$$324 \rightarrow \begin{cases} 5 = 4 \\ 3 = 1 \\ 2 = 1 \end{cases}$$

$\therefore$ Ans = 4, 1, 1

Qns. A no. is successively divided by 2, 5, 3 and remainders are 1, 3, 2 resp. find min. such no.

Soln.
- Go in opp. direction [Num = Q × D + R]

Q = 2, 1 | Q = 5, 3 | let Q 3, 2

$2(15Q+13)+1$
$= 30Q+26+1$
$= \boxed{30Q+27}$

$5(3Q+2)+3$
$= 15Q+10+3$
$= \boxed{15Q+13}$

$\boxed{3Q+2}$

- No. is Quotient of prev. no.
- We have to find min no. so put min value of Q in last no. formed i.e. 30 Q + 27

min ⇒ Q = 0 ⇒ 30 × 0 + 27 ⇒ 27

$\therefore$ Ans = 27

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NUMBER SYSTEM SHEET (cont.)

18. A number when divided successively by 4 and 5 leaves rem. 1 and 4 resp. when it is succesively divided by 5 and 4 then resp rem will be

Solⁿ.

4	5
1	4

$\to 5Q+4$

$\to 4(5Q+4)+1 = 20Q+17$

min no., $Q=0$, $\Rightarrow 17$

now, $5\overline{)17}(3$ $\quad 4\overline{)3}(0$

-15 $\quad 0$

2 $\quad 3$

$\therefore$ **Ans = 2, 3**

---

19. A number was divided successively in order by 4, 5 and 6. The rem were resp. 2, 3 and 4. The no. is - a) 214 b) 476 c) 954 d) 1908

Solⁿ.

4	5	6
2	3	4

$\to 6Q+4$

$\to 30Q+20+3 = 30Q+23$

$\to 120Q+92+2 = 120Q+94$

on $Q = 0$, no $= 94$ but it is not in opt.
so $Q = 1$, no $= 120 + 94 = 214$
it is a)

if there is another option e) Cannot determined we will mark this option as on changing Q, no. is changing.

$\therefore$ Ans = a) 214

---

20. Which of the following number will completely divide $(4^{61} + 4^{62} + 4^{63} + 4^{64})$
a) 3 b) 9 c) 11 d) 17

Soln: $4^{61}(1 + 4 + 4^2 + 4^3)$

$= 4^{61}(21 + 64)$

$= 4^{61}(85)$

85 is div. by 17

$\therefore$ Ans = d) 17

24. Which of the following no. will completely divide $5^{51} + 5^{52} + 5^{53}$

a) 11 b) 312 c) 31 d) 32

Sol^n: $5^{51}(1 + 5^2 + 5^2)$

$= 5^{51}(1 + 5 + 25$

$= 5^{51}(31)$

it is div. by 31

$\therefore$ Ans = ~~d) 32~~ c) 31

---

25. If a number is divided by 84 the rem is 37. What will be the rem. if it is divided by 21.

Sol^n: Num = 84 Q + 37

min => Q = 0 , num = 37 ⇒ $\frac{37}{21} = 16$

Q = 1 , num = 84 + 37 = 121 ⇒ $\frac{121}{21} = 16$

OR by Rem. ⇒ $\frac{37}{21} = 16$

$\therefore$ Ans = 16

26. The sum of both digits of a two digit no. is 7. If the digits of the no. are interchanged the no. so formed is greater than the original no. by 27. find original no.

a) 29 b) 25 c) 79 d) 32 e) None

Solⁿ. 1^{st} condition $\rightarrow$ sum = 7

- check from option if sum = 7
- we got b)
- but to verify e) option we check 2^{nd} condn.

2^{nd} condition $\rightarrow$ when inverted $\rightarrow$ original + 27 = new

$\therefore \quad 52 = 25 + 27$

this is also correct

$\therefore$ Ans = b) 25

---

35. If there difference of a number of two digits and a number formed by reversing the digit is 45, ~~which one of the following no. will completely divide N~~. then what is diff of the digits of the original no.

a) 9 b) 81 c) 5 d) 18

Solⁿ. $xy - yx = 45$

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max diff → 90 - 09 ⇒ 81

it can be digits only , so cut b, d

50 - 05 = 45 ✓ (preferably this way)

∴ **Ans = c) 5**

OR (technically)

$xy - yx = 45$

$(10x + y) - (\cancel{y}\ 10y + x) = 45$

$9x - 9y = 45$

$9(x - y) = 45$

$$\boxed{x - y = 5}$$

DIVISIBILITY TESTS

1. DIVISIBLE BY 2 : A no. ending with 0, 2, 4, 6, 8

2. DIVISIBLE BY 3 : Sum of digits ÷ 3

   ex : 123 → 1 + 2 + 3 = 6/3 ✓

3. DIVISIBLE BY 4 : Last two digits ÷ 4

ex: 6<u>24</u> → 24 ÷ 4 ✓

4. <u>DIVISIBLE BY 5</u> : No. ending with 0 or 5

5. <u>DIVISIBLE BY 6</u> : Div. by 2 and 3 <u>both</u>

ex: 126 → 2 ✓
→ 3 ✓ ∴ 6 → ✓

6. <u>DIVISIBLE BY 7</u> :
- Remove unit digit
- Subtract twice of removed unit digit from remaining no.
- Check if div. by 7
- Cont. till its easy to check

ex: 343	ex: 2401	ex: 1234
34 ×2	240 ×2	123
−6	−2	−8
28 ÷ 7 ✓	238	115
	23 ×2	11 ×2
	−16	−10
	7 ÷ 7 ✓	1 ÷ 7 x

7. <u>DIVISIBLE BY 8</u> : Last 3 digits ÷ 8

8. <u>DIVISIBLE BY 9</u> : sum of digits ÷ 9

9. <u>DIVISIBLE BY 10</u> : ending with 0

10. <u>DIVISIBLE BY 11</u> :
- starting from left as +ve put alternate signs till last

- add the digits with that sign
- if sum = 0 or multiple of 11 then yes.

ex: $\overset{+}{1}\overset{-}{2}\overset{+}{3}\overset{-}{4}\overset{+}{3}\overset{-}{2}\overset{+}{1}$
$= 1-2+3-4+3-2+1$
$= 0$
✓

ex: $\overset{+}{3}\overset{-}{2}\overset{+}{4}\overset{-}{4}\overset{+}{3}\overset{-}{2}$
$= 3-2+4-4+3-2$
$= 6-4 \neq 0$
$= 2 \div 11$ ✗

11. DIVISIBLE BY 12 : div. by 4 and 3 both

12. DIVISIBLE BY 17 : • Remove unit digit • multiply by 5 & sub. from remaining
(Like in 7)

23. Which one of the following no. is completely divisible by 99.
a) 3572 b) 13595 c) 913464 d) 114345

Sol: 99 → check for 9 and 11 both

in d) 9 → $1+1+4+3+4+15 = 18$ ✓
11 → $1+4+4-1-3-5$
$= 9-9 = 0$ ✓

Ans = d)

---

24. Which one of the following no. is completely div. by 45.
a) 181560 b) 331145 c) 202860 d) 203350

H.W → except 40 - 52 full sheet.

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Soln. check div. for 9 and 5 both

∵ all are div. by 5

c) · 9 → $2+0+2+8+6+0 = 18$ ✓

∴ ans = c)

---

30. The digit in blank space of the no. $34\overset{*}{\_}7$ so that the no. is div. by 11 will be

blank space

a) 3 b) 6 c) 7 d) 8

Solⁿ for div. by 11 → $3 + x = 4 + 7$

$x = 11 - 3 = 8$

here,

∴ ans = d)

$3 - 4 + x - 7 = 0$ ✓

OR

$3 - 4 + x - 7 = 11$ ✗

$x = 19$

but it is digit.

---

12. How many 3 digit numbers are completely div. by 6?

Solⁿ: max 3 - digit no = 999

$$\begin{array}{r} 166 \\ 6\,\overline{)\,999\,} \\ 6 \\ \hline 39 \\ 36 \\ \hline 39 \\ 36 \\ \hline 3 \end{array}$$

∴ total 166 no. are div. by 6 from 1 to 999

but we need only 3 digit

So we will eliminate 2-digit no. that are div. by 6. i.e. 16

$$6 \overline{)99}(16$$
$$\underline{6}$$
$$39$$
$$\underline{36}$$
$$3$$

1 —— 99 —— 999

$\xleftrightarrow{16}$ 1 to 99, $\xleftrightarrow{166-16}$ 99 to 999

$\xleftrightarrow{166}$ 1 to 999

$\therefore$ $166 - 16 = 150$

$\therefore$ **Ans = 150**

---

13. How many 3 digit no's are divisible by 3 and 4.

Solⁿ. L.C.M of 3 & 4 = 12

$$12 \overline{)999}(83$$
$$\underline{96}$$
$$39$$
$$\underline{36}$$
$$3$$

$$12 \overline{)99}(8$$
$$\underline{96}$$
$$3$$

$\Rightarrow$ $83 - 8 = 75$

$\therefore$ **Ans = 75**

---

27. Find the largest number of 5 digits which is div. by 7

a) 99999 b) 99960 c) 99994 d) 10013

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Solⁿ. Div. by 17 → • Remove unit digit
• Multiply unit digit by 5
• Subtract that prod. from remaining no.

$$99960 \rightarrow \begin{array}{r} 9996 \\ -0 \\ \hline 999\boxed{6} \\ \times 5 \end{array} \rightarrow \begin{array}{r} 999 \\ -30 \\ \hline 96\boxed{9} \\ \times 5 \end{array}$$

~~999~~

$$\begin{array}{r} 96 \\ -45 \\ \hline 51 \end{array}$$

$51 \div 7$ ✓ ←

∴ **Ans = b) 99960**

---

29. Which is the greatest out of the following no.
a) $(2+2+2)^2$ b) $[(2+2+2)^2]^2$
c) $(2+2+2)^3$ d) 4^3

Solⁿ.
a) $\Rightarrow 6^2 = 36$
b) $\Rightarrow (6^2)^2 = (36)^2$
c) $\Rightarrow 6^3 = 216$
d) $\Rightarrow 4^3 = 64$

greatest is $(36)^2$

∴ **Ans = b)**

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31. If $a^* b = a^2 + b^2$ then $-3^* 5$ is equal to

Sol^n. $-3^* 5 = (-3)^2 + (5)^2$

$= 9 + 25$

$= 34$

$\therefore$ Ans = 34

---

32. If the sum of a number of two digits and a number formed by reversing the digit is 99. What is the sum of digits of the original no?

a) 9 b) 81 c) 118 d) 18

Sol^n:

$99 - 18 = 81$ i.e. rev

& $1 + 8 = 9$

OR

$(10x + y) + (10y + x) = 99$

$11x + 11y = 99$

$11(x+y) = 99$

$x + y = \frac{99}{11} = 9$

$\therefore$ Ans = b) 9

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33. If there sum of a number of two digits and a number formed by reversing the digits is N. Which one of the following number will completely divide N.

Sol^n.

$$(10x + y) + (10y + x) = N$$
$$11x + 11y = N$$
$$11(x+y) = N$$
$$x+y = \frac{N}{11}$$

N is always div. by 11

$\therefore$ **Ans = 11**

---

34 If there diff. of a no. of two digits and a number formed by reversing the digits is N, which will completely divide N

a) 9 b) 81 c) 11 d) 18

Sol^n.

$$(10x + y) - (10y + x) = N$$
$$10x + y - 10y - x = N$$
$$9x - 9y = N$$
$$9(x-y) = N$$
$$x-y = \frac{N}{9}$$

N is alway div. by 9

$\therefore$ **Ans = 9**

---

36. A number being successively divided by 9, 11 and 13 leaves rem. 8, 9 and 8 resp. If order of divisors is reversed then rem. will be.

Sol^n.

		(Rem)
9	N	
11	A	→ 8
13	B	→ 9
	1	→ 8

$N = 9 \times 240 + 8 = 2168$
$A = 11 \times 21 + 9 = 240$
$B = 13 \times 1 + 8 = 21$

now, $N = 2168$
rev. divisors → 13, 11, 9

$\cancel{\frac{2168}{13} = 166 \to \frac{166}{11} = 15 \to \frac{15}{9} = 6}$

13	2168	
11	166	→ 10
9	15	→ 1
	1	→ 6

∴ **Ans = 10, 1, 6**

---

37. A four digit no. is formed by repeating 2-digit no. such as 2525, 3232 etc. Any no. ~~is fo~~ of this form is always div by

a) 11 b) 7 c) 13 d) 101

Solⁿ. $xy\,xy$

$y \to y$

$x \to 10x$

$y \to 100y$

$x \to 1000x$

$\Rightarrow 1000x + 100y + 10x + y$

$= 1010x + 101y$

$= 101(10x + y)$

It is multiple of 101 $\therefore$ it is always div. by 101

$\therefore$ **Ans = 101**

---

39. Find the sum of first fifty numbers

Solⁿ. $\rightarrow 1 + 2 + 3 + \cdots\cdots + 50$

$= \boxed{\dfrac{n(n+1)}{2}}$ Sum of 1st n Natural No.

$= \dfrac{50 \times (50+1)}{2}$

$= 25 \times 51$

$= 1275$

$\therefore$ **Ans = 1275**

53. Find the no. of prime factors of $6^{20} \cdot 11^{11} \cdot 21^{21}$?

Solⁿ:

$6^{20} \rightarrow (3 \times 2)^{20} \rightarrow 3^{20} \times 2^{20}$

$11^{11} \rightarrow 11^{11}$

$21^{21} \rightarrow (3 \times 7)^{4} \rightarrow 3^{21} \times 7^{21}$

$\rightarrow 3^{20} \times 2^{20} \times 11^{11} \times 3^{21} \times 7^{21}$

no. of prime factors $=$ sum of powers

$= 20 + 20 + 11 + 21 + 21$

$= 40 + 11 + 42$

$= 93$

$\therefore$ Ans $= 93$

---

54. Find the no. of prime factors of $14^{14} \cdot 15^{15}$.

Solⁿ:

$14^{14} \rightarrow (2 \times 7)^{14} = 2^{14} \times 7^{14}$

$15^{15} \rightarrow (3 \times 5)^{15} = 3^{15} \times 5^{15}$

$\rightarrow 2^{14} \times 7^{14} \times 3^{15} \times 5^{15}$

no. of prime factors $= 14 + 14 + 15 + 15$

$= 28 + 30$

$= 58$

$\therefore$ Ans $= 58$

---

55. What will be the remainder when $(27^{27} + 17^{27})$ is divided by 11?

Solⁿ:

$$\boxed{\frac{x^n + a^n}{x+a} = Rem = 0 \quad \text{if } n = \text{odd}}$$

$$\frac{27^{27} + 17^{27}}{27+17} = 0$$

$$27 + 17 = 44 \longrightarrow 4 \times 11$$

It is completely div. by 11

$\therefore$ Rem = 0

$\therefore$ Ans = 0

H.C.F. and L.C.M.

- factor → that divides a no.
- H.C.F → highest factor that divides a no.
  ↓ common

Ex:

48, 80

2	48
2	24
2	12
2	6
3	3
	1

2	80
2	40
2	20
2	10
5	5
	1

$48 = 2^4 \times 3^1$

$80 = 2^4 \times 5$

$\Rightarrow$ H.C.F $= 2^4 = 16$

LONG DIVISION :-

- Divide greater no. with smaller no.
- then continue by making divisor as dividend.
- H.C.F. = divisor that makes rem = 0.

Q29. 84, 68

$68 \overline{)84}$ 1
$\underline{68}$
$16 \overline{)68}$ 4
$\underline{64}$
$4 \overline{)16}$ 4
$\underline{16}$
$\underline{0}$

∴ **Ans = 4**

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Q30. 65, 85

Sol^n.

$$
\begin{array}{r|l}
65 & 85 \quad (1 \\
 & 65 \\
\hline
20 & 65 \quad (3 \\
 & 60 \\
\hline
5 & 20 \quad (4 \\
 & 20 \\
\hline
 & 0
\end{array}
$$

Ans = 5

Q31. 96, 72

$$
\begin{array}{r|l}
72 & 96 \quad (1 \\
 & 72 \\
\hline
24 & 72 \quad (3 \\
 & 72 \\
\hline
 & 0
\end{array}
$$

∴ Ans = 24

Q32. Find H.C.F of 42, 63 and 140

Sol^n.

- a, b, c

$\underbrace{a, b}$, c

↓

H.C.F , c

↓

H.C.F.

$$
\begin{array}{l}
63 \overline{)140}\ (2 \\
\quad 126 \\
\quad \overline{14)\ 63}\ (4 \\
\qquad 56 \\
\qquad \overline{7)\ 14}\ (2 \\
\qquad\quad 14 \\
\qquad\quad \overline{0}
\end{array}
$$

63, 140

now 7, 42

$$
\begin{array}{l}
7 \overline{)42}\ (6 \\
\quad 42 \\
\quad \overline{0}
\end{array}
$$

$\therefore$

Q33. Find the H.C.F. of $a^2 b^4 c^6$, $b^3 c^8 a^4$ and $a^8 b^6 c^2$.

a) $a^4 b^4 c^4$ b) $a^2 b^2 c^2$ c) $a^2 b^3 c^2$ d) $a^2 b^3 c^3$

Sol.ⁿ =

$a^2 b^4 c^6$

$a^4 b^3 c^8$

$a^8 b^6 c^2$

$$\left.\begin{array}{l} a \rightarrow a^2 \\ b \rightarrow b^3 \\ c \rightarrow c^2 \end{array}\right\} \text{Common}$$

$\therefore$ **Ans = c) $a^2 b^3 c^2$**

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Q34. Find H.C.F. of 0.63, 1.05 and 2.1
a) 0.21 b) 0.021 c) 21 d) 2.1

Sol^n-

$0.63 \rightarrow \frac{63}{100}$

$1.05 \rightarrow \frac{105}{100}$

$2.1 \rightarrow \frac{21}{10}$

$$\boxed{\text{H.C.F. of fraction} = \frac{\text{H.C.F. of num}}{\text{L.C.M of den.}}}$$

- H.C.F. of num = 63, 105, 21
  = 21

- L.C.M of den = 100, 100, 10
  = 100

H.C.F $\Rightarrow \frac{21}{100} = 0.21$ Ans = 0.21

L.C.M.

- multiple → that is in table
- L.C.M → least common multiple i.e. in table of both

Q35. Find the H.C.F of $2^2 3^3 5^2$, $2^3 3^2 5^2 7$ and $2^4 3^4 5 7^2 11$ is :

Sol^n:
$2^2 \; 3^3 \; 5^2$
$2^3 \; 3^2 \; 5^2 \; 7$
$2^4 \; 3^4 \; 5 \; 7^2 \; 11$

$\rightarrow 2^2 \; 3^2 \; 5$ $\therefore$ Ans $= 2^2 3^2 5$

Q36. Find H.C.F of $\frac{2}{3}, \frac{8}{9}, \frac{64}{81}$ and $\frac{10}{27}$

Sol^n: H.C.F of num $= 2, 8, 64, 10$
$= 2$

~~8, 64~~ $\rightarrow$

L.C.M. of den. $= 3, 9, 81, 27$

Shortcut for L.C.M

- In given no. is there any multiple of 3, if any, $\Rightarrow$ cut

$\cancel{3}, \cancel{9}, 81, \cancel{27}$

$\therefore$ Ans $= \frac{2}{81}$

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LCM

Q37 Find the L.C.M of 24, 36 and 40

Soln.

2	24, 36, 40
2	12, 18, 20
2	6, 9, 10
3	3, 9, 5
3	1, 3, 5
5	1, 1, 5
	1, 1, 1

L.C.M $\rightarrow 2^3 \times 3^2 \times 5 \rightarrow 8 \times 9 \times 5$

$\rightarrow 72 \times 5$

$= 72 \times \frac{10}{2}$

$= 36 \times 10 = 360$

$\therefore$ Ans = 360

Q38. Find the L.C.M. of 3, 2.7 and 0.09

Soln.

$\frac{3}{1}, \frac{27}{10}, \frac{9}{100}$

H.C.F of num $= 3, 27, 9$

$= 3, 3^3, 3^2$

$= 3$

L.C.F of den $= 1, 10, 100$

$= 100$

$$\boxed{\text{L.C.M. of fraction} = \frac{\text{L.C.M of num}}{\text{H.C.F of den}}}$$

L.C.M. of num $= 3, 27, 9 \rightarrow \cancel{3}, 27, \cancel{9}$
$= 27$

H.C.F of den $= 1, 10, 100$ → common no. that divides all.
$= 1$

$\therefore$ Ans $= 27$

Q39. Find L.C.M of $\frac{1}{3}, \frac{5}{6}, \frac{2}{9}$ and $\frac{4}{27}$?

Sol^n: L.C.M of num $= 1, 5, 2, 4$
$= 20$

H.C.F of den $= 3, 6, 9, 27$
$= 3$

$\therefore$ Ans $= \frac{20}{3}$

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HCF	L.C.M
• greatest	• least
• largest	• lowest
• max	• smallest
• biggest	• minimum
• highest	• simultaneous occurrence
• distribution into groups	

Q40. The greatest no. that exactly divides 147, 168, ~~120~~ 210 and 315 is

Sol^n. H.C.F of 147, 168

$$147\overline{)168}(1$$
$$\underline{147}$$
$$21\overline{)147}(7$$
$$\underline{147}$$
$$0$$

H.C.F of 210 and 315

$$210\overline{)315}(1$$
$$\underline{210}$$
$$105\overline{)210}(2$$
$$\underline{210}$$
$$0$$

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now H.C.F of 21 and 105

$$\begin{array}{r|l} & 5 \\ 21 & 105 \\ & 105 \\ \hline & 0 \end{array}$$

$\therefore$ Ans = 21

Q41.

Q42. Find the greatest possible length of a scale that can be used to measure exactly the following length of cloth 3m, 5m 10 cm, and 12m 90 cm.

Soln.

3m $\longrightarrow$ 300

5m 10 cm $\longrightarrow$ 510

12 m 90 cm $\longrightarrow$ 1290

Q44. Find min. possible length of scale to measure exactly the following length, 64 cm, 80 cm and 96 cm.

Solⁿ.

* Q45. Traffic lights at three diff. points are changing respectively at 24, 48 and 72 sec. If all the three are changed together at 9:10:24 hours then when will next changes take place together.

Solⁿ. changed together ⟶ simultaneously
∴ LCM

$\cancel{24}$, 48, 72

L.C.M.

$$\begin{array}{r|l} 12 & 48, 72 \\ 2 & 4, 6 \\ & 2, 3 \end{array} \rightarrow 12 \times 2 \times 2 \times 3$$

If no common → 48 × 3 or 72 × 2
→ 144 sec.
→ 2 min 24 sec.

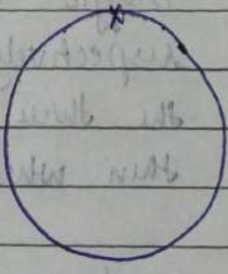
add in time

Ans = 9 : 12 : 48

Q46. A, B and C start at the same time in the same direction to run around a circular stadium. A completes one around in 252 sec., B in 308 sec. and C in 198 sec., all starting at same point. After what time will they meet again at starting point.

Soln: same point → L.C.M.

252, 308, 198



2	252, 308, 198
2	126, 154, 99
	63

Q47. A, B and C start at same time in same direction to run around a circular stadium of length 12 Km and speed 3km/h, 4 km/h and 6 km/h resp. After what time will they meet again at starting point?

Sol^n.

$$\text{time} = \frac{\text{dist}}{\text{speed}}$$

$$\text{L.C.M} \rightarrow \left[\frac{12}{3}, \frac{12}{4}, \frac{12}{6}\right]$$

L.C.M. of num = 12

H.C.F. of den = 1

$\therefore$ Ans = 12

Q48. The HCF of two numbers is 11 and L.CM is 7700. If one of the number is 275, then the other is :

a) 279 b) 283 c) 308 d) 318

Sol^n

$$\boxed{\text{Prod. of 2 no.} = \text{H.C.F} \times \text{L.CM}}$$

$$275 \times x = 11 \times 7700$$

$$x = \frac{11 \times 7700}{275}$$

$$= 308$$

$\therefore$ Ans = 308

Q49. The L.C.M of two no. is 495 and their HCF is 5. If sum of the number is 100 then their diff is a) 10 b) 46 c) 70 d) 90

Sol^n.

$$x + y = 100$$
$$y = 100 - x$$

$$x(100 - x) = 495 \times 5$$
$$x^2 - 100x + 2475 = 0$$
$$\boxed{x = 55, 45}$$

$$diff = 55 - 45 = 10$$

OR

let a) → 10 → diff

$$x + y = 100$$
$$x - y = 10$$

$$\left.\begin{array}{l} x = 55 \\ y = 45 \end{array}\right\}$$ find L.CM i.e 495

∴ Ans = 10

Q50. Product of two co-prime no. is 117. Their L.CM should be.

Sol: • co-prime no. → H.C.F = 1

$H.CF \times L.CM = Prod.$

$1 \times L = 117$

$L = 117$

∴ Ans = 117

Q51. The L.C.M of three diff. no. is 120. Which of the following cannot be their H.Cf.

a) 8 b) 12 c) 24 d) 35

Sol:

• LCM is always divisible by H.Cf.

- 8 divides 120
- 12 ―――
- 24 ―――
- 35 does not divide 120

∴ Ans = d) 35

Eliminating Options

Q52. H.C.F of two no. is 8. Which of following can never be their LCM?

a) 24 b) 48 c) 56 d) 60

Soln. LCM ÷ HCF

- 24 ÷ 8 ✓
- 48 ÷ 8 ✓
- 56 ÷ 8 ✓
- 60 ÷ 8 ✗

∴ Ans = 8

Q53. Sum of two no. is 216 and their HCF is 27. The numbers are :

a) 27, 189 b) 108, 108 c) 200, 16 d) 100, 116

Soln.

- sum is 216 in all
- HCF = 27 of a) 27, 189

$$\begin{array}{r|l} & 7 \\ 27 & 189 \\ & \underline{189} \\ & \underline{0} \end{array}$$

∴ Ans = 27, 189

Q54. The ratio of two no. is 3:4. and their HCF is 4. The numbers are :

a) 9, 12 b) 12, 16 c) 16, 18 d) 20, 24

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Solⁿ: • H.C.F $= 4 \rightarrow$ div. by 4
$\rightarrow$ b, d

• Ratio $= 3:4 \rightarrow$ b) ✓ $3(4) : 4(4)$

$\therefore$ Ans = 12, 16

Q55 The ratio of two no. is 4:5 and their HCF is 2. The LCM is

a) 20 b) 10 c) 40 d) 60

Solⁿ: Ratio $= \frac{4}{5}$, factor $= 2$

$$\frac{4 \times 2}{5 \times 2} = \frac{8}{10}$$

L.C.M of 8, 10 = 40

$\therefore$ Ans = 40

Q56. The ratio of two no. is 2:3 and their LCM is 48. The no. are

a) 16, 24 b) 8, 6 c) 12, 18 d) 12, 24

Solⁿ: a) $\rightarrow 8 \times 2, 8 \times 3$ ✓
c) $\rightarrow 4 \times 2, 4 \times 3$ ✓

L.C.M of 12, 18 = 36

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L.C.M of 16, 24 = 48

∴ Ans = 16, 24

Q57. The smallest no. from which if 17 is subtracted is exactly div. by 2, 4, 3, 5, 6, 8. and 10 is
a) 113 b) 120 c) 127 d) 137

Sol^n. easy options → 127, 137

127 − 7 = 120 → div. by all

OR

smallest = LCM

2	2, 3, 4, 5, 6, 8, 10
2	1, 3, 2, 5, 3, 4, 5
2	1, 3, 1, 5, 3, 2, 5
5	
3	

→ $2^3 \times 5 \times 3$
→ 120

OR

~~2~~, ~~3~~, ~~4~~, ~~5~~, 6, 8; 10

LCM of 6, 8, 10 = 120

- we need to add 7 in L.CM for ans

∴ Ans = 127

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Q58. The smallest no. from which if 8 added is exactly div. by 10, 12, 15 and 20.
a) 60 b) 68 c) 52 d) 38

Sol^n

if we add 8 in 52 → 60
it is only that is div by 10.

OR

$\cancel{10}, 12, 15, 20$ L.CM = 60

$x + 8 = 60$
$x = 52$

∴ Ans = 52

Q59. The smallest no. that will be divisible by 4, 6, 8, 12 and 16 leaving a rem. 2 in each case is
a) 46 b) 50 c) 48 d) 56.

Sol^n. L.CM → $\cancel{4}, \cancel{6}, \cancel{8}, 12, 16$

$$\begin{array}{r|l} 4 & 12, 16 \\ \hline & 3, 4 \end{array}$$

→ 12 × 4 or 16 × 3
→ 48

→ 48 + 2 = 50
• L.CM of no + Rem

∴ Ans = 50

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Q60. Which is smallest no. that can be ~~divisible~~ subtracted from 1936 so that on being divided by 9, 10, 15 the rem is 7 everytime?

a) 93 b) 46 c) 76 d) 39

Solⁿ.

3	9, 10, 15
3	3, 10, 5
5	1, 10, 5
2	1, 2, 1
	1, 1, 1

L.C.M = 90

L.CM (9, 10, 15) + Rem

→

90 + 7 = 97

180 + 7 = 187

270 + 7 = 277

all these can give ans

⋮

→ 90 K + 7

→ 90 (20) + 7 = 1807 → 1936 − 1807

→ 90 × 21 + 7 = 1897 → 1936 − 1897 = 39 ✓

• this diff is not in options

∴ Ans = 39

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Q01. Find greatest number that will divide 187, 233 and 279 so as to leave same rem in each case.

a) 30 b) 36 c) 46 d) 56

Sol^n greatest = HCF if Rem = 0

$$\begin{array}{r|l} 187 & 233 \;(1 \\ & 187 \\ \hline 46 & 187 \;(4 \\ & 184 \\ \hline & 3 \end{array} \qquad \begin{array}{r|l} 233 & 279 \;(1 \\ & 233 \\ \hline 46 & 233 \;(5 \\ & 230 \\ \hline & 0 \end{array}$$

} This is by chance

$$\frac{187}{D} = \frac{233}{D} = \frac{279}{D} = \text{same Rem.}$$

- $233 - 187 = 46$
- $279 - 187 = \cancel{47}\ 92$
- $279 - 233 = \cancel{92}\ 46$

• make pairs of diff.
• Take HCF of diff.

- H.CF of 46, 46, 92 = 46

• Divisor is always a factor of diff. of no.

∴ Ans = 46

Q62. The numbers 1305, 4655 and 6905 divided by a four digit no. N, giving same Rem. The sum of digits of N is
a) 4 b) 5 c) 6 d) 8

Solⁿ.
$4665 - 1305 = 3360$
$6905 - 4665 = 2240$
$6905 - 1305 = 5600$

HCF ~~LCM~~ of $3360, 2240, 5600 = 1120$

sum of digits $= 1+1+2+0 = 4$

$\therefore$ Ans = 4.

Q63. The greatest no. which can divide 110 and 128 leaving the same rem 2 in each case is
a) 8 b) 18 c) 28 d) 38

Solⁿ
$128 - 110 = 18$

H.CF of $18 = 18$

$\therefore$ Ans = 18

OR

$\frac{110}{D}$, $R = 2$

$\frac{128}{D}$, $R = 2$

Rem.

take H.CF of $(110-2)$ & $(128-2)$

H.C.F = 18

Q64. The greatest no. which can divide 122 and 243 leaves a rem 2 and 3 resp.

a) 12 b) 24 c) 30 d) 120

Sol^n.

$120 - 2 = 120$
$243 - 3 = 240$

↓
Rem.

H.C.F of 120, 240 = 120

∴ Ans = 120

★★

Q65. The least no. which when divided by 12, 15 and 16 leaves 7, 10 and 11 as rem. resp is

a) 115 b) 235 c) 247 d) 475

Sol^n.

N		R		
12	-	7	=	5
15	-	10	=	5
16	-	11	=	5

L.CM (n) - Diff. = 240 - 5
= 235

∴ Ans = 235

Q66. ~~The~~ ~~LCM~~ of A no. when divided by 10 leaves a rem. 9, when divided by 9 leaves a rem. of 8, when divided by 8 leaves a rem of 7, so on when divided by 2 leaves a rem of 1. Find no.

a) 31 b) 1029 c) 2519 d) 1679.

Soln.

N	R
10	9
9	8
8	7
7	6
⋮	
2	1

L.CM of (N) − Diff

= 2520 − 1

= 2519

∴ Ans = 2519