Report 4 - Stochastic methods for finance

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Abstract

In this report i study the Greeks's behaviour in respect to their parameters, in particular stock price, maturity time, volatility and dividend rate for the Black&Scholes model.

1 Introduction

The Option Greek formulae express the change in the option price with respect to a parameter change taking as fixed all the other inputs. One significant use of Greek measures is to calibrate risk exposure. A market-making financial institution with a portfolio of options, for instance, would want a snap shot of its exposure to asset price, interest rates, dividend fluctuations. It would try to establish impacts of volatility and time decay. In the formulae below, the Greeks merely evaluate change to only one input at a time. In reality, we might expect a conflagration of changes in interest rates and stock prices etc. Each Greek letter measures a different dimension to the risk in an option position and the aim of a trader is to manage the Greeks so that all risks are acceptable.

2 Methods

I calculated the Greeks using VBA script [4] with fixed values for S, K, t, σ, r and q=0%. In particular S=60,70,..,140, K=100, R=1%, σ = 10%, 20%, 60% and t=0.1,0.2,..1,2,3,4,5 years. Then i further study the relationship between q and the greeks by setting q=20% for stressing them and keeping fixed values above. The results obtained can be visualized by plotting the Greekss value in function of T and S for each one of them and for each volatility obtaining the graphs in section [3]. But first i explain what are the greeks in the following section.

2.1 The Greeks

Delta Δ

Δ measures the rate of change of the theoretical option value with respect to changes in the underlying asset's price. Delta is the first derivative of the value V of the option with respect to the underlying instrument's price S.

$$\Delta = \frac{\delta V}{\delta S} \tag{1}$$

Gamma Γ

Gamma Γ , measures the rate of change in the delta with respect to changes in the underlying price. Gamma is the second derivative of the value function with respect to the underlying price.

$$\Delta = \frac{\delta^2 V}{\delta S^2} \tag{2}$$

Theta Θ

Theta Θ , measures the sensitivity of the value of the derivative to the Option time.

$$\Theta = \frac{\delta V}{\delta t} \tag{3}$$

Vega ν

ν measures sensitivity to volatility. Vega is the derivative of the option value with respect to the volatility of the underlying asset.

$$\nu = \frac{\delta V}{\delta \sigma} \tag{4}$$

Rho ρ

Rho, ρ, measures sensitivity to the interest rate: it is the derivative of the option value with respect to the risk-free interest rate.

$$\rho = \frac{\delta V}{\delta r} \tag{5}$$

3 Results

The first set of plots (from 1 to 5) are done considering q=0% and varying the stock price S and the time T for fixed volatility σ=10%,20%,60%. The second set of figures (from 6 to 10) are done comparing q=0% and q=20% and the other parameters are the same as above in order to do an analysis on the impact of the dividend rate q.

3.1 Analysis for σ=10%,20%,60%

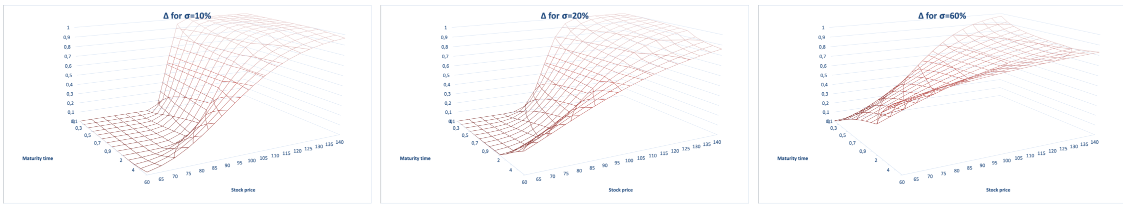

Figure 1: Delta

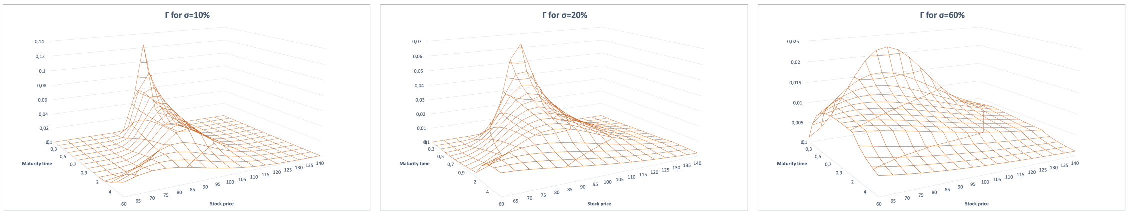

Figure 2: Gamma

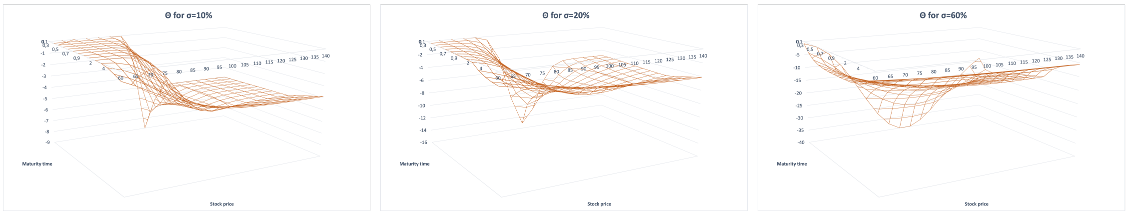

Figure 3: Theta

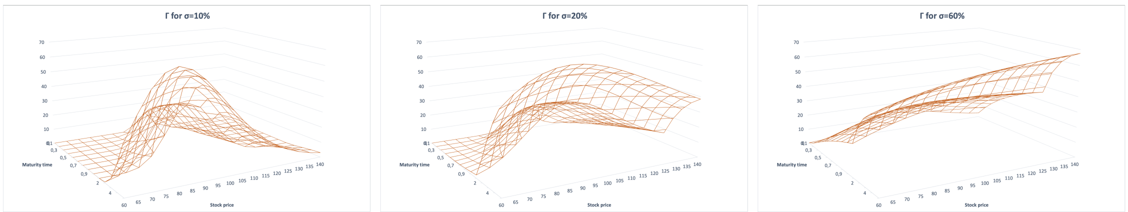

Figure 4: Vega

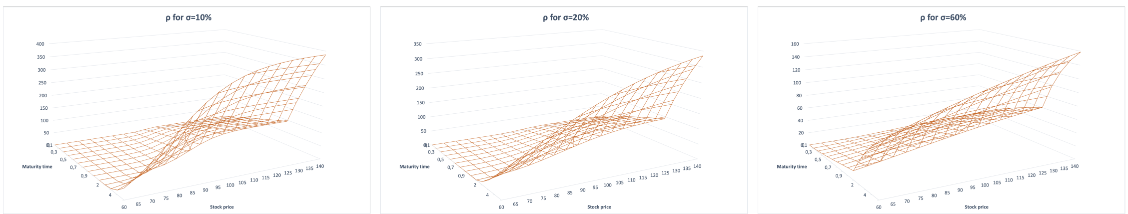

Figure 5: Rho

3.2 Analysis on the dividend rate q

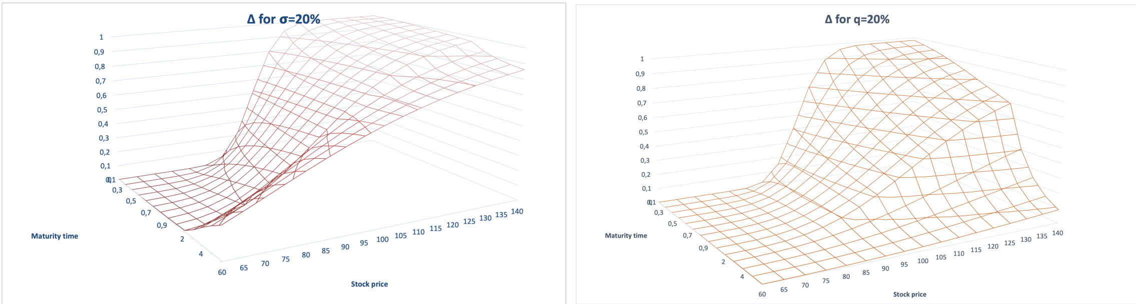

Figure 6: In the left figure delta is computed for q=0% and in the right for q=20%.

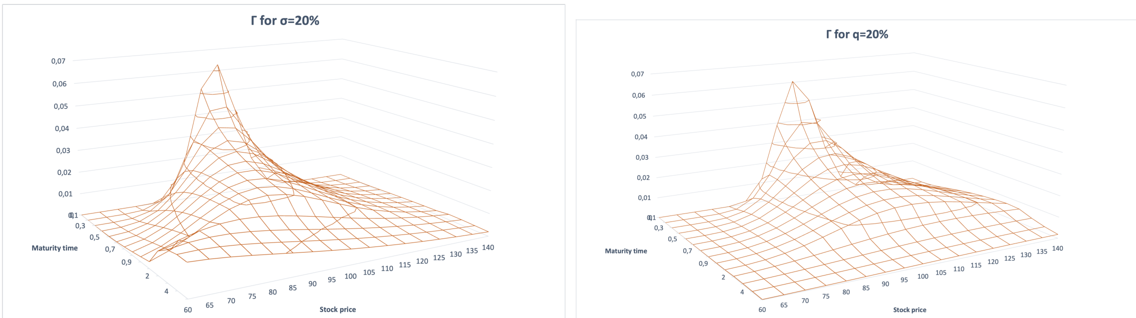

Figure 7: In the left figure Gamma is computed for q=0% and in the right for q=20%.

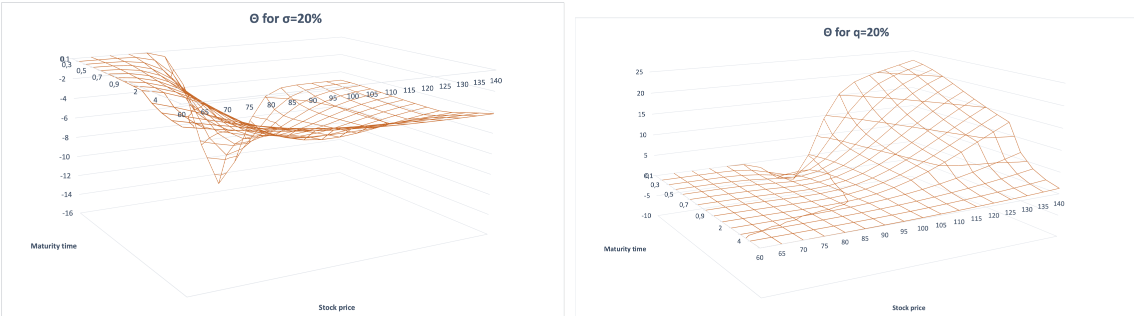

Figure 8: In the left figure Theta is computed for q=0% and in the right for q=20%.

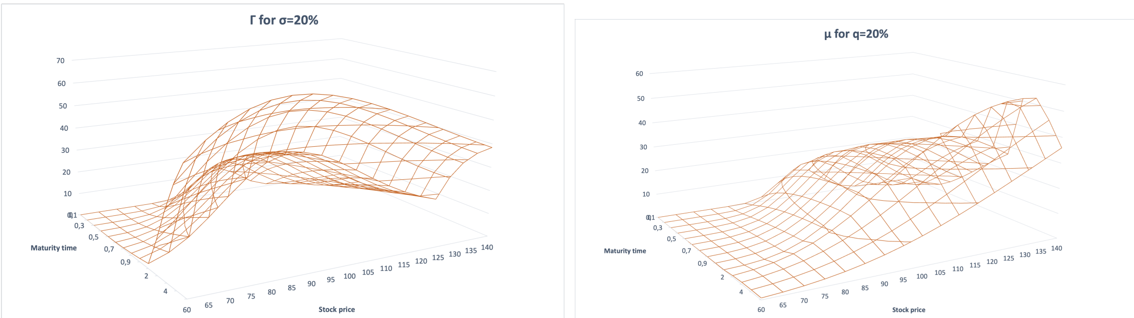

Figure 9: In the left figure Vega is computed for q=0% and in the right for q=20%.

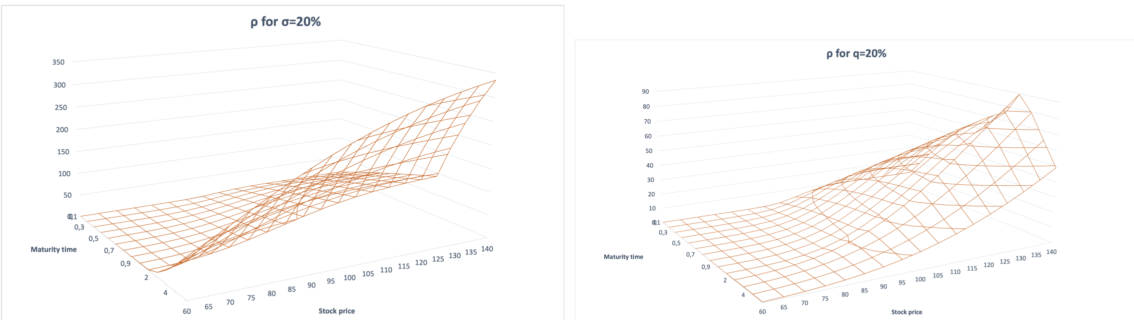

Figure 10: In the left figure Rho is computed for q=0% and in the right for q=20%.

4 Conclusions

A higher volatility values indicate greater instability and thus higher risk levels. In terms of options, this translates to a higher price, in terms of Delta, i.e. the first derivative of the call option price with respect to the price of the underlying: a higher volatility increases the delta for Over the money (OTM) options. The more volatility, the less OTM an option is. A higher volatility decreases the delta for In the money (ITM) options. The more volatility, the less ITM option is. So, more volatile stocks therefore have a less pronounced delta, as we see in the plots. The behaviour of Gamma, Theta and Vega are, on the other hand, similar, with the obvious adjustments. For the three of them, as the volatility increases the surface becomes more and more relaxed and viceversa. As for the dividend rate we see that for high maturity time all greeks tends to go down for every value of S, this is expected since q goes as e^{-qT}.

Appendix

VBA codes are taken from these links:

- Black Scholes Greeks: `https://sites.google.com/view/vinegarhill-financelabs/black-scholes\-merton/black-scholes-greeks`
- used for testing greeks: `https://investexcel.net/black-scholes-greeks-vba/`