Report 7 - Stochastic methods for finance

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Abstract

Firms and regulators in the financial industry commonly employ VaR (Value at Risk) as a tool to assess the quantity of assets required to mitigate potential losses. In this report we construct a balanced portfolio with two assets: Google and Meta and study different methods for evaluating the Value at Risk: Parametric Var, Var with EWMA volatility, Monte carlo Var, Historical simulation and the Historical Var.

1 Introduction

Value at Risk (VaR) is an attempt to provide a single number summarizing the total risk in a portfolio of financial assets. It has become widely used by corporate treasurers and fund managers as well as by financial institutions. Bank regulators have traditionally used VaR in determining the capital a bank is required to keep for the risks it is bearing. When using the value-at-risk measure, an analyst is interested in making a statement of the following form:

I am X percent certain there will not be a loss of more than V dollars in the next N days.

The variable V is the VaR of the portfolio. It is a function of two parameters: the time horizon (N days) and the confidence level (X%). It is the loss level over N days that has a probability of only (100-X)% of being exceeded. Bank regulators require banks to calculate VaR for market risk with N = 10 and X = 99.

When N days is the time horizon and X% is the confidence level, VaR is the loss corresponding to the (100-X)th percentile of the distribution of the gain in the value of the portfolio over the next N days. VaR is an attractive measure because it is easy to understand. In essence, it asks the simple question How bad can things get? This is the question all senior managers want answered. They are very comfortable with the idea of compressing all the Greek letters for all the market variables underlying a portfolio into a single number.

2 Methods & Results

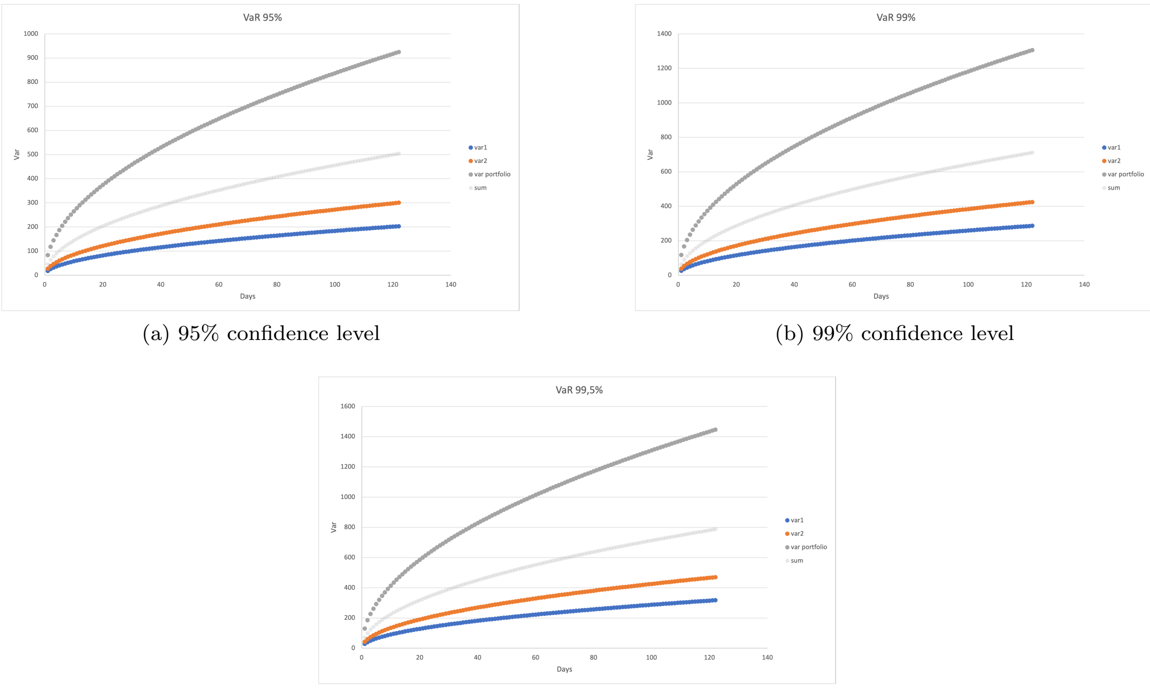
First we build a balanced portfolio of two assets, we have chosen Google and META. The total value of the portfolio is 1000$. We use Yahoo Finance to retrive historical data of the past six months and then compute the portfolio returns for each days. To create this type of portfolio we adjust our investment daily based on their performance. For example, if the first asset gains 1% and the second asset loses 1%, we sell some of the first asset to invest in the second asset, so that we maintain an equal investment in both assets every day. This approach helps us maintain a consistent balance in our portfolio.

2.1 Parametric VaR

The first method for evaluating the VaR is by the parametric VaR. This method assumes that the underlying distribution is Normal and from the daily returns of the stocks and the portfolio is possible to calculate the parametric VaR using the quantiles. This is done by calculating the daily volatilities (σ) and using them in the parametric VaR formula where the 1 day-VaR is given by:

$$1day - VaR = \sigma\sqrt{T}VK \tag{1}$$

Where V is the initial value of the portfolio and K is the quantile corresponding to confidence level. By following this procedure is possible to calculate the VaR for every horizon from 1 to 120 days. Obtaining the graphs in fig. 1. It is useful also to check the additivity (non additivity) of the VaR: we compute the VaR separately for the two assets and we compare the sum with the global one.



(a) 95% confidence level

(b) 99% confidence level

(c) 99,5% confidence level

Figure 1: Parametric VaR computed for different confidence levels. The light grey line corresponds to the sum of the two assets. Dark grey is the portfolio one.

From the pictures it is evident the non additivity of the VaR.

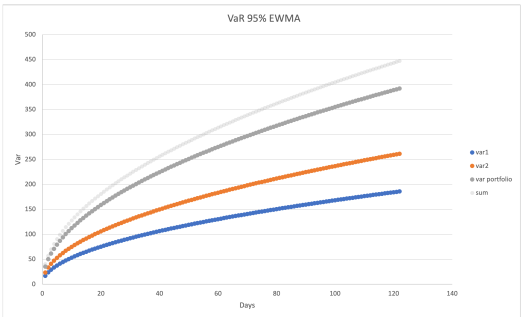
2.2 EWMA volatility

Another method avoids to give the same weights to the volatility σ and gives more importance to closer timeframe volatility. In this case to compute the volatility of the portfolio we use the

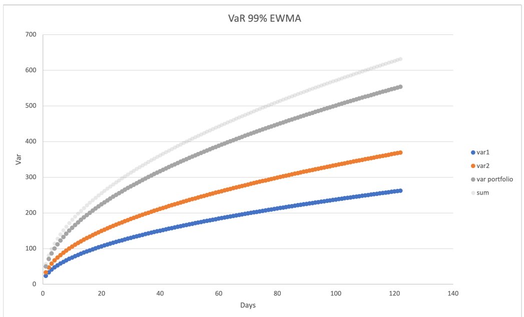
Exponentially Weighted Moving Average method:

$$\sigma^2_{(EWMA)} = \lambda \sigma^2_{n-1} + (1-\lambda) u^2_{n-1} \tag{2}$$

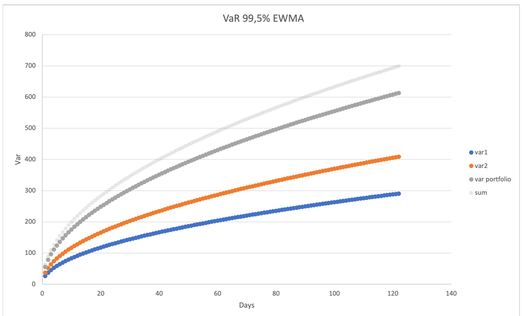
where u^2_{n-1} are the squared daily returns at time n-1 and λ is a weight, chosen optimally as 0,94. From this procedure is possible to obtain an estimation of the VaR for the assets and the portfolio shown in fig.3. It is possible to notice again from the graph that we have sub-additivity for the VaR.



(a) 95% confidence level



(b) 99% confidence level



(c) 99,5% confidence level

Figure 2: Parametric VaR computed with EWMA volatility for different confidence levels. The light grey line corresponds to the sum of the two assets. Dark grey is the portfolio one.

The VaR for the sum of the two assets is always bigger than the VaR of the global portfolio proving the portfolio strategy.

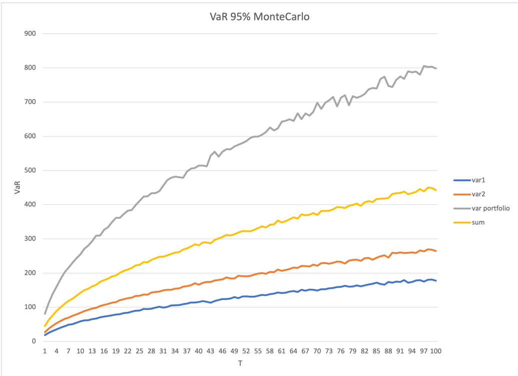
2.3 Monte Carlo simulation

To calculate VaR using M.C. simulation we:

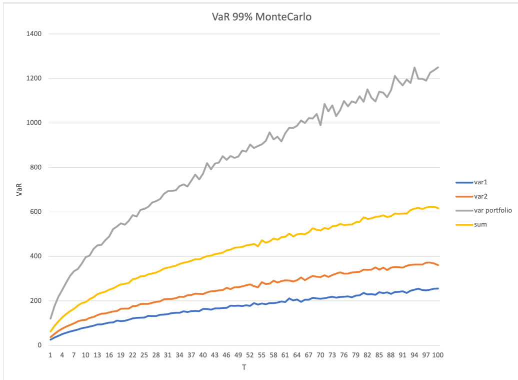
- Value portfolio today;
- Sample once from the multivariate distributions of the Δx_i
- Use the Δx_i to determine market variables at end of one day;
- Revalue the portfolio at the end of day;

- Calculate ΔP ;
- Repeat many times to build up a probability distribution for ΔP ;
- VaR is the appropriate fractile of the distribution times square root of N

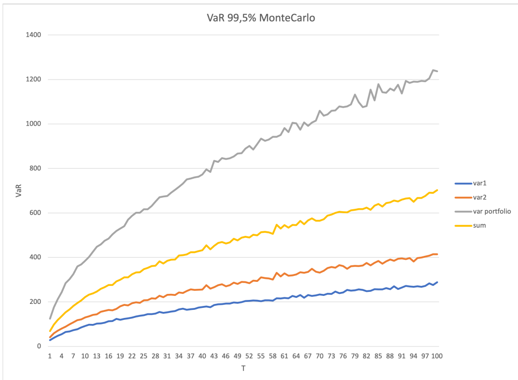
In practice we will use a VBA script 3 and simulate a large number of trajectories for the portfolio value over the considered days, for each day compute the VaR as the difference between the initial value of the portfolio and the percentile related to the confidence level considered. The interest rate was set to zero.



(a) 95% confidence level



(b) 99% confidence level



(c) 99,5% confidence level

Figure 3: MC VaR computed for different confidence levels. The yellow line corresponds to the sum of the two assets. Grey is the portfolio one.

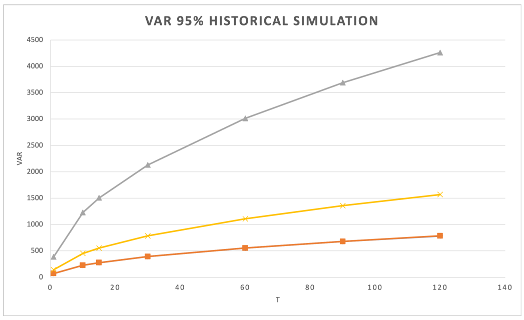
Also in this case there is no additivity.

2.4 Historical simulation

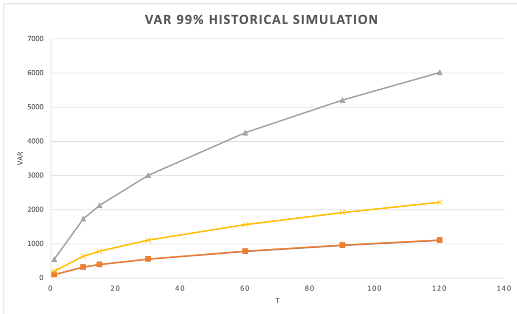
Historical simulation is one popular way of estimating VaR. It involves using past data as a guide to what will happen in the future. Data are collected on movements in the market variables over the most recent 120 days (6 months). This provides 119 alternative scenarios for what can happen between today and tomorrow. Denote the first day for which we have data as Day 0, the second day as Day 1, and so on. Scenario 1 is where the percentage changes in the values of all variables are the same as they were between Day 0 and Day 1, Scenario 2 is where they are the same as between Day 1 and Day 2, and so on. To express the approach algebraically, dene v_i as the value of a market variable on Day i and suppose that today is Day m. The i-th scenario in the historical simulation approach assumes that the value of the market variable tomorrow will be:

$$v_{m+1} = v_m \frac{v_i}{v_{i-1}} \tag{3}$$

We estimate the 1-Day VaR at confidence level X% as the limit of the (1 - X) percentile of the corresponding distribution and we know that from the 1-Day VaR we can obtain the VaR for any horizon. These are plotted in figure 4



(a) 95% confidence level



(b) 99% confidence level

Figure 4: Historical simulation VaR computed for different confidence levels. The yellow line corresponds to the sum of the two assets. Grey is the portfolio one. The single asset ones (orange and blue) overlaps.

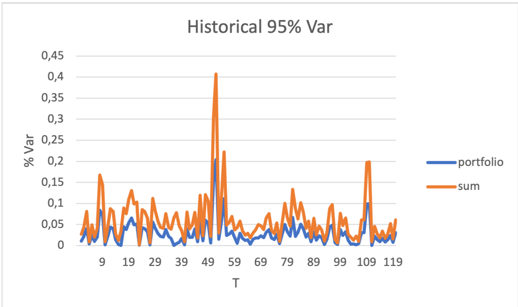
Also in this case there is no additivity.

2.5 Historical VaR

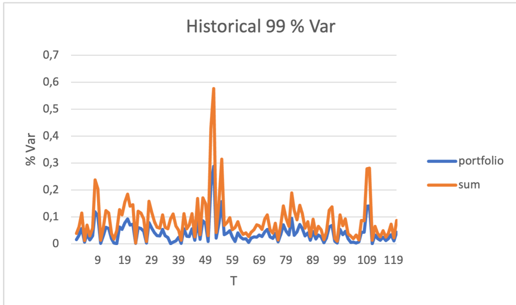
We computed the volatility as the standard deviation between the i-th and i-th+1 returns for each asset and for the portfolio. Than we computed the VaR for each historical return using the formula :

$$VaR = X\sigma_i \tag{4}$$

where X is equal to 2.326,1.645 and 2.576 (confidence levels 99%,95% and 99.5%) and σ_i is the standard deviation between the ith and ith+1 close returns. Plots are reported in the figure 5.



(a) 95% confidence level



(b) 99% confidence level

Figure 5: Historical VaR computed for different confidence levels. The orange line corresponds to the sum of the two assets and blue is the portfolio one.

In this case there is sub additivity for some days, in the most volatile days the VaR for the sum gets very high.

3 Conclusions

In this report we have seen different methods for evaluating the Value at Risk:

- Parametric Var
- Var with EWMA volatility
- Monte carlo Var
- Historical simulation
- Historical Var

In almost all the cases we found out that there is no additivity property. As expected VaR increases with time since the uncertainty about the future grows. In particular, in this cases we observed that investing in the global portfolio reduces the Value at Risk meaning that it is convenient to invest in the global portfolio instead of on the single asset.

Appendix

VBA codes are taken from these links:

- MC VaR: https://investexcel.net/calculate-value-at-risk-with-monte-carlo-simulation/