Report 3 - Stochastic methods for finance

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Abstract

In this report i compare the price of a Call Option computed with three different methods: Binomial Model, Black&Scholes formula (BS) and the Leisen-Reimer Model (LR). I show the smoother convergence of the LR Model with respect to the Binomial model.

1 Introduction

The Binomial Model is a simple toy model but very powerful. Indeed it is a well known result that for large number of steps this model converges to the Black&Scholes formula. However this convergence has periodic oscillations around the B&S value. A solution to this problem was proposed by Leisen and Reimer in 1995 and it is based on the fact that the underlying price tree is centered around the option's strike price at expiration, unlike the other model which center the tree around the initial underlying price.

2 Methods

Leisen-Reimer Model

Leisen and Reimer developed a method in which the parameters u, d and p of the binomial tree can be altered in order to get better convergence behavior. Instead of choosing the parameters p, u and d to get convergence to the normal distribution Leisen-Reimer suggest to use inversion formulae reverting the standard method. They use normal approximations to determine the binomial distribution $B(n,p)$. In particular, they suggest the following three inversion formulae to replace p (probability of an up move) by p(d). The Camp-Paulson-Inversion formula, Peizer-Pratt-Inversion formula 1 or the reported below Peizer-Pratt-Inversion formula 2.

Peizer-Pratt-Inversion formula 2 (n = 2j + 1)

$$p(z) = \frac{1}{2} + sign(z)\frac{1}{2}\sqrt{1 - exp\left[-\left(\frac{z}{n+\frac{1}{3}+\frac{1}{10(n+1)}}\right)^2\left(n+\frac{1}{6}\right)\right]} \tag{1}$$

With a = n - j, b = j + 1 and z as input values for the standard normal distribution one uses in the Black-Scholes formula.

Then the model parameters are defined by:

$$u = e^{(r_d - r_f)\Delta t}\frac{p(d_+)}{d_-} \tag{2}$$

$$d = \frac{e^{(r_d - r_f)\Delta t} - p(d_-)u}{1 - p(d_-)} \tag{3}$$

$$d_\pm = \frac{\ln \frac{S_0}{K} + (r_d - r_f \pm \frac{1}{2}\sigma)T}{\sigma T} \tag{4}$$

Using this method, Leisen and Reimer observe much better convergence behavior. This model, Binomial model and also the B&S formula are implemented in Excel using VBA. The scripts used for these computations are reported in the Appendix section.

3 Results

The parameter chosen for the computation are reported in table 1 for various n steps. In the figure 1 it is clear that the binomial model oscillates around the B&S value:

$$Price_{B\&S} = 8,678\ \$ \tag{5}$$

Stock price S	Strike price K	Risk-free interest rate r	Maturity time T	Volatilty
100	100	0,01	6 months	30%

Table 1

However in figure 2 it is evident that the Leisen-Reimer Model converges smoothly and faster than the binomial one. In conclusion as n increases both the binomial model and the Leisen-Reimer model converge but the latter does it more precisely and fastly.

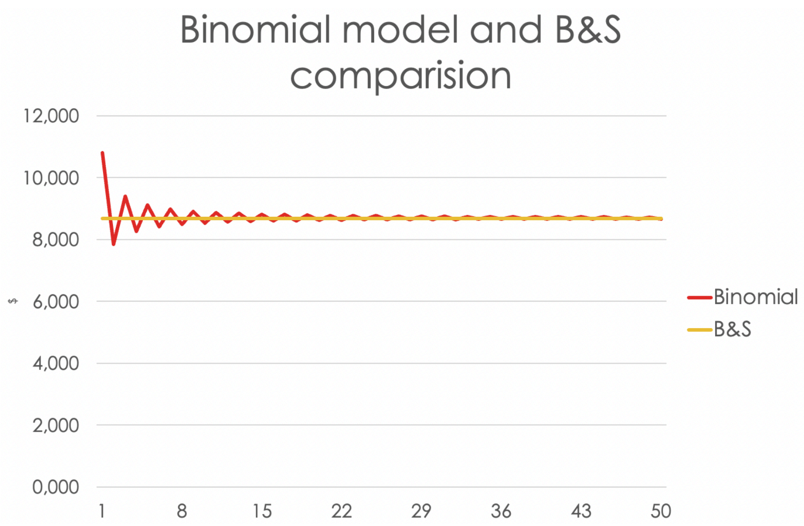

Figure 1

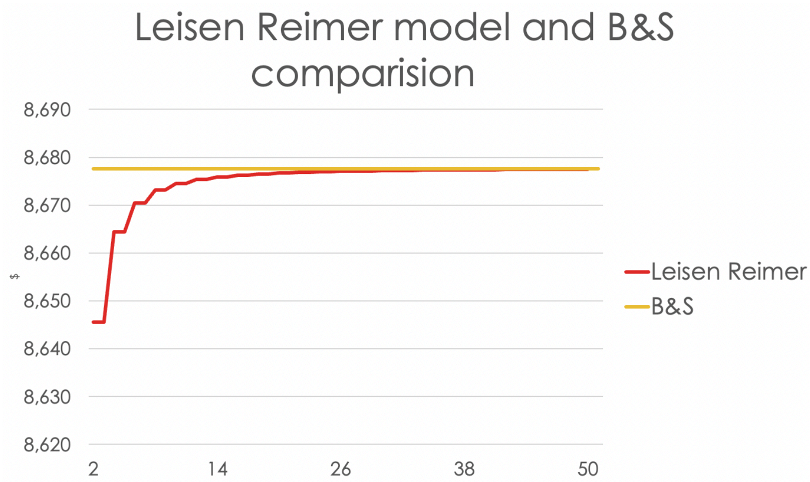

Figure 2

Appendix

VBA codes are taken from these links:

- Binomial Model: `http://www.anthony-vba.kefra.com/vba/vba7.htm`
- B&S formula: `http://www.anthony-vba.kefra.com/vba/vba6.htm`
- Leisen-Reimer: `https://sites.google.com/view/vinegarhill-financelabs/`