# Report 2 - Stochastic methods for finance

Roben Bhatti - 2091187 - Physics of data

March 31, 2023

**Abstract**

In this report i provide accurate implied dividend and interest rate term-structures from Verizon option prices using linear regression with the call-put parity relationship. The result i got are satisfactory taking into account that i have chosen an American option.

## 1 Introduction

Verizon Communications Inc., commonly known as Verizon, is an American multinational telecommunications company headquartered in New York City. The company was formed in 2000 when Bell Atlantic merged with GTE, creating one of the largest telecommunications companies in the world.

Verizon is primarily focused on providing wireless telecommunications services, as well as broadband and fiber-optic internet services to both residential and commercial customers. The company operates in 150 countries worldwide and is considered one of the largest wireless carriers in the United States.

Verizon is a publicly traded company, with its shares listed on the New York Stock Exchange (NYSE) under the ticker symbol VZ. The largest shareholders of the company include institutional investors such as The Vanguard Group, BlackRock Inc., and State Street Corporation.

Over the years, Verizon has been involved in several controversies. In 2013, the company was accused of violating net neutrality rules by throttling the bandwidth of some customers who were using certain applications. In 2014, the Federal Communications Commission (FCC) fined Verizon $7.4 million for failing to notify customers of their right to opt-out of the company's use of their personal information for marketing purposes.

In 2016, Verizon was embroiled in a controversy when it was revealed that the company had provided customer data to the National Security Agency (NSA) without a warrant. The company was also criticized for its handling of a labor dispute with its workers in 2016, which resulted in a strike that lasted for several weeks.

Despite these controversies, Verizon remains a major player in the telecommunications industry and continues to expand its services to new markets and customers around the world.

## 2 Methods

**Put-Call parity**

The put-call parity is a concept that applies only to European options because American options can be exercised at any time before the expiration date.

Put-call parity states that simultaneously holding a short European put and long European call of the same class will deliver the same return as holding one forward contract on the same underlying asset, with the same expiration, and a forward price equal to the option's strike price.

If the prices of the put and call options diverge so that this relationship does not hold, an **arbitrage opportunity** exists. This means that sophisticated traders can theoretically earn a risk-free profit. Such opportunities are uncommon and short-lived in liquid markets.

In general the payoff will be exactly $S - K$. This payoff can be turned into a constant known guaranteed value on the expiration date if we additionally sell the stock. Selling a stock today at $S_0$ will give a payoff of $S_0 - S$ at expiration. Combining this with the options payoff of $S - K$ will result in a total payoff of $S_0 - K$, and this is independent of what the stock price S will be at expiration.

This constant guaranteed value forces a relationship on todays call and put price. Buying a call for a price C and selling a put for a price P will cost $C - P$, and will always have a constant value of $S_0 - K$ on the expiration date. This leads to a parity relation:

$$C - P = S_0 - K \tag{1}$$

The call-put parity formula has two extra elements. The first has to do with interest rates because we are talking about future payoffs. The second has to do with selling the stock. Having a long or short stock position means that we might have to include dividend payment effects (if the stock gives a dividend somewhere before the expiration date). For European options on dividend-paying stocks, the put-call parity formula is given by:

$$P - C = Ke^{-rt} - S_0e^{-qt} \tag{2}$$

Where C, P are today's call and put prices, K the strike, t the time to expiration in years, $r$ the continuously compounded interest rate, $S_0$ the current stock price, and q the continuously compounded dividend yield.

It is possible to compute the **discount factor** as function of maturity:

$$(S - K_1)^+ - (K_1 - S)^+ = S - K_1 \qquad - (S - K_2)^+ + (K_2 - S)^+ = K_2 - S \tag{3}$$

and combining them:

$$C(K1) - C(K2) + P(K2) - P(K1) = (K2 - K1)D(0, T) \tag{4}$$

where $D(0, T)$ is the discount factor and the other quantities are found in the market. Knowing the discount factor it is possible to retrive the implied dividend as function of

maturity using call-put parity and using ATM prices and forward contracts as:

$$Call - Put = D(0,T)(Se^{rT} - Se^{\delta T} - K) \tag{5}$$

Another efficient and elegant way to estimate an implied interest rate is through least squarer's regression.

Indeed the put call relationship can be seen as linear equation in function of K:

$$P - C = Ke^{-rt} - S_0e^{-qt} \qquad y = Ka + b \tag{6}$$

I have chosen to take one share of Verizon Communications Inc. (VZ) at the price of $S = S0 = 38,29\$$ ( this is the price of one share in date 30/03/2023, 5:40PM CET) and several call/put options, for T=1,3,6 and 15 months since 12 months maturity was not available.

# 3 Results

The computations are done in date 30 March 2023 so the data is taken at that date. First i fix maturity T to 1 month and retrive from Yahoo Finance all the available Put and Call options. I compute discount factor with eq.4 and the results are plotted in the figure 1 along with other maturities. Then i compute the linear regression in order to get the implied dividend q and interest rate r eq. 6 in figure 2 for T=1 month. Finally i compute r and q for the other timeframes and get the term structure in figure 3 and 4.
The Discount factors computed are mostly satisfactory since in most cases they are $< 1$ even if we are using an american option that is subject to early exercise. The same conclusions can be said for the dividend yields and the interest rates (mostly non negative), indeed we get values very close to zero except for maturity 1 and 3 months; although they are not compatible with the actual ones.

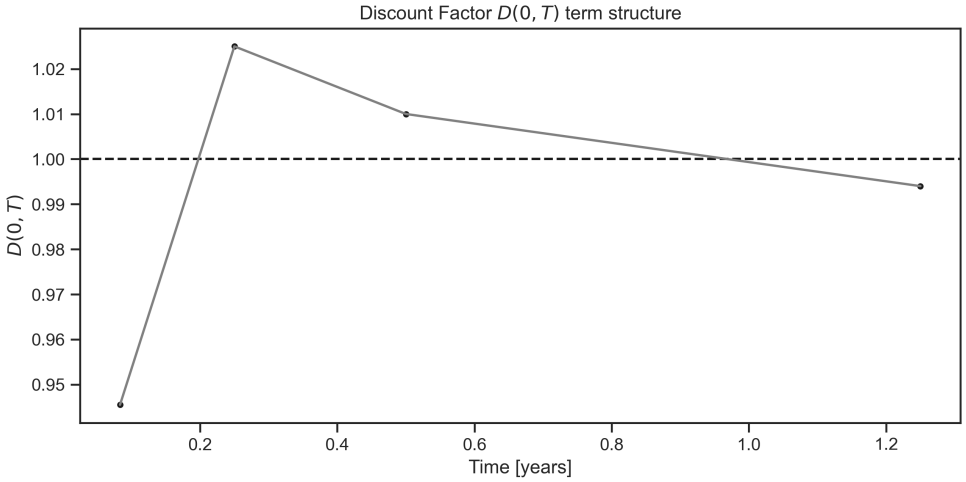


Figure 1: Discount factor computed with eq 1.

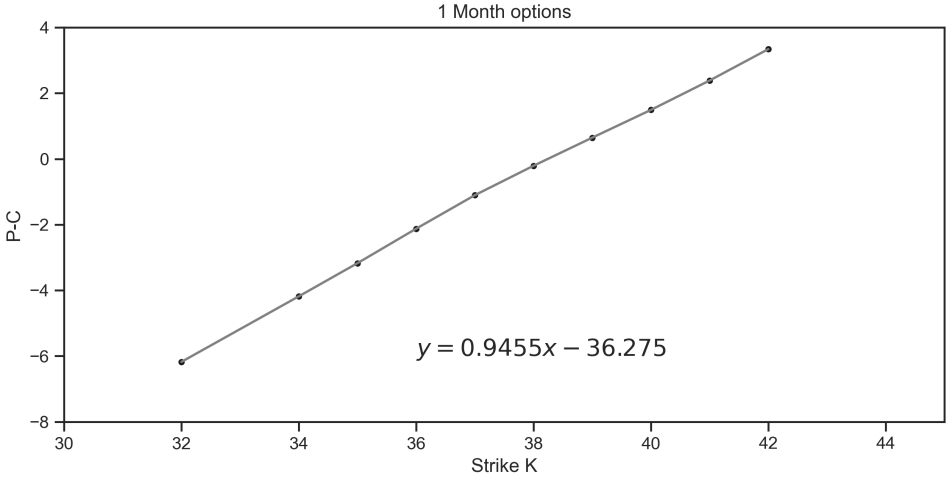


Figure 2: Relative options prices (Put - Call) computed for different strike K for 1 month maturity. A linear regression is computed in order to extrapolate implicit dividend yield and interest rate. Data is taken from Yahoo Finance.

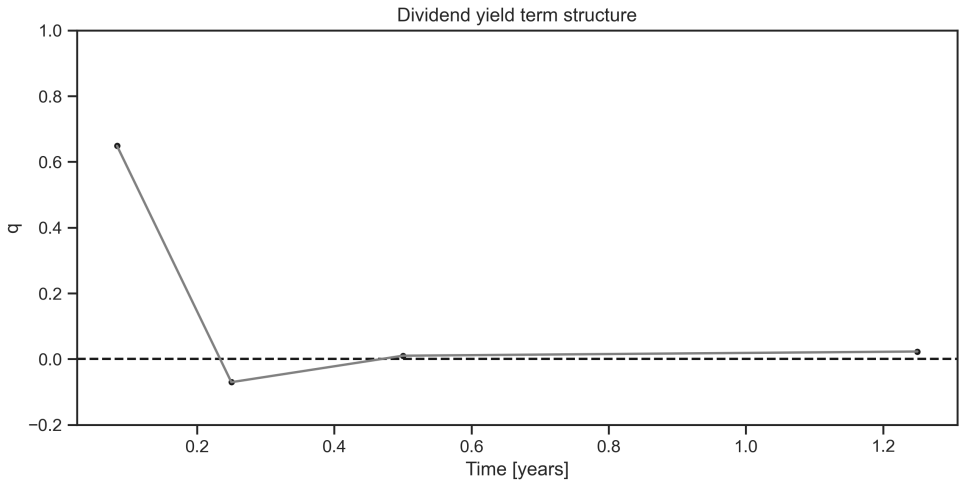


Figure 3: Dividend Yield computed with linear regression for different maturity time.

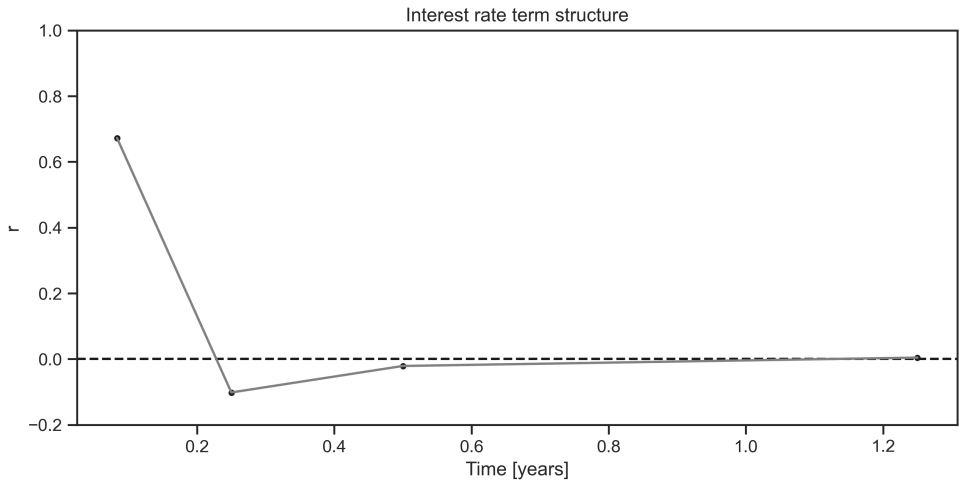


Figure 4: Interest rate computed with linear regression for different maturity time.