Report 1 - Stochastic methods for finance

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Abstract

In this report i provide the price of a Alphabet Inc. (GOOG) *call* option with maturity T=3 months and T=6 months by using the binomial model. Then i compare it with the market quote.

1 Introduction

Alphabet Inc. is a multinational conglomerate company that specializes in internet-related services and products. It was created in 2015 as a result of a restructuring of Google, which became a subsidiary of the new company. Alphabet's headquarters are located in Mountain View, California.

- **History**: Alphabet Inc. was created by Google co-founders Larry Page and Sergey Brin in 2015, as a way to separate Google's core internet businesses from its more experimental ventures. At the time of its creation, Alphabet was made up of Google and several smaller subsidiaries, including Google X, Calico, and Nest. Larry Page served as CEO of Alphabet until 2019, when he stepped down and was replaced by Sundar Pichai, who was already CEO of Google.

- **businesses**: Alphabet's main business is Google, which includes search, advertising, and various other online services such as Gmail, Google Maps, and Google Drive. Alphabet also includes several other subsidiaries, such as YouTube, Waymo (a self-driving car company), and Verily (a healthcare company). Other Alphabet subsidiaries include X (formerly known as Google X), which is responsible for developing "moonshot" projects like self-driving cars and internet-beaming balloons, and Google Ventures and CapitalG, which are Alphabet's venture capital arms.

- **Leadership**: Alphabet's current CEO is Sundar Pichai, who took over the position from Larry Page in 2019. Pichai has been with Google since 2004, and prior to becoming CEO of Alphabet, he was CEO of Google. Other notable executives at Alphabet include Ruth Porat, who serves as CFO, and Kent Walker, who is Google's senior vice president for global affairs.

- **Finances**: Alphabet is one of the largest companies in the world, with a market capitalization of over \$1.5 trillion as of March 2023. In 2021, the company reported revenues of \$182.5 billion and net income of \$59.3 billion. Alphabet's primary source of revenue is advertising, which accounted for over 80% of its 2021 revenue.

- **Investments and Acquisitions**: Alphabet has made several high-profile investments and acquisitions in recent years. In 2018, the company acquired HTC's smartphone team for \$1.1 billion, which helped bolster its hardware division. Alphabet has also invested heavily in artificial intelligence and machine learning, with acquisitions of companies like DeepMind and Kaggle. In addition, Alphabet has invested in several companies focused on alternative energy, such as wind power company Makani Power and geothermal energy company Dandelion Energy.

- **lawsuits and controversies**: Alphabet and its subsidiaries have faced a number of antitrust investigations and lawsuits. In 2020, the US Department of Justice filed a lawsuit against Google, alleging that the company used its dominant market position in search and advertising to stifle competition. Alphabet has also faced antitrust investigations in Europe and other countries.
Alphabet and Google have faced a number of controversies related to data privacy. In 2019, Google was fined \$57 million by the French government for violating the EU's General Data Protection Regulation (GDPR). The company has also faced criticism for its data collection practices and for failing to adequately protect user data.

2 Methods

I have chosen to take one share of Alphabet Inc. at the price of $S = S0 = 92,59\$$ (this is the price of one share in date 13/03/2023, 2:14PM CET) and two call options at the money (ATM), for T=3 months and for T=6 months.

Maturity	Contract Name	Strike K (\$)	Last Price (\$)	Bid (\$)	Ask (\$)	Mid price (\$)
3 months	GOOG230616C00092500	92.50	7.05	7.35	7.50	7.425
6 months	GOOG230915C00092500	92.50	9.65	10.15	10.90	10.525

Table 1: Source: `https://finance.yahoo.com`

I downloaded the historical data of GOOG shares from Yahoo Finance. The value adj close corresponds to S_t for each day so i compute the daily returns:

$$return_t = \frac{S_t - S_{t-1}}{S_{t-1}} \tag{1}$$

Then i compute the daily volatility from the standard deviation of the returns σ_{daily} and the annual one σ_{yearly} related to each maturity:

$$\sigma_{yearly} = \sigma_{daily}\sqrt{252} \tag{2}$$

where 252 is the number of working days in a year for the financial market.
I compute the parameters u,d of the binomial model:

$$u, d = e^{\pm \sigma_{daily}\sqrt{252T}} \tag{3}$$

The value of the payoff is given by:

$$\begin{pmatrix} f^u = (S \cdot u - K)^+ \\ f^d = (S \cdot d - K)^+ \end{pmatrix} \tag{4}$$

Where S is the market value and K is the strike price.
Then i compute the capitalisation factor r and simple discounting $\frac{1}{r}$ using simple compounding with the interest rate R taken from https://www.global-rates.com/en/:

$$r = 1 + R \cdot T \tag{5}$$

Finally the risk probability weight q:

$$q = \frac{r - d}{u - d} \tag{6}$$

and the call option price p:

$$p = \frac{1}{r}(f^u \cdot q + (1 - q) \cdot f^d) \tag{7}$$

In this case there are no dividends so the process is finished.

3 Results

The computations are done in date 13 March 2023 so the data is limited to that date. The results of the computation are showed in the following table:

Maturity	σ_{daily}	σ_{daily}	u	d	$f^u(\$)$	$f^d(\$)$	r (%)	q	p (\$)
3 months	0.0245	0.389	1.214	0.823	19.972	0	1.012	0.484	9.554
6 months	0.0253	0.402	1.328	0.752	30.55	0	1.027	0.451	13.619

Table 2: calculations

Recalling the contracts chosen in table 1 we get that:

Maturity	p ($)	Mid price	comparison (%)
3 months	9.554	7.425	77.7
6 months	13.619	10.525	77.3

Table 3: comparison of the results between yahoo finance and the calculations from the binomial tree.

Comparing the price computed here with the one given by yahoo it is clear that there is a big difference between them using both maturities. In particular for T=3 months we got a 77.7% difference and for T=6 months we got a 77.3% difference. It is expected because the binomial tree is a very simple discrete time model applied on a more complex continuous time world.