

# Differences Between Two Groups



# Statistical Golems

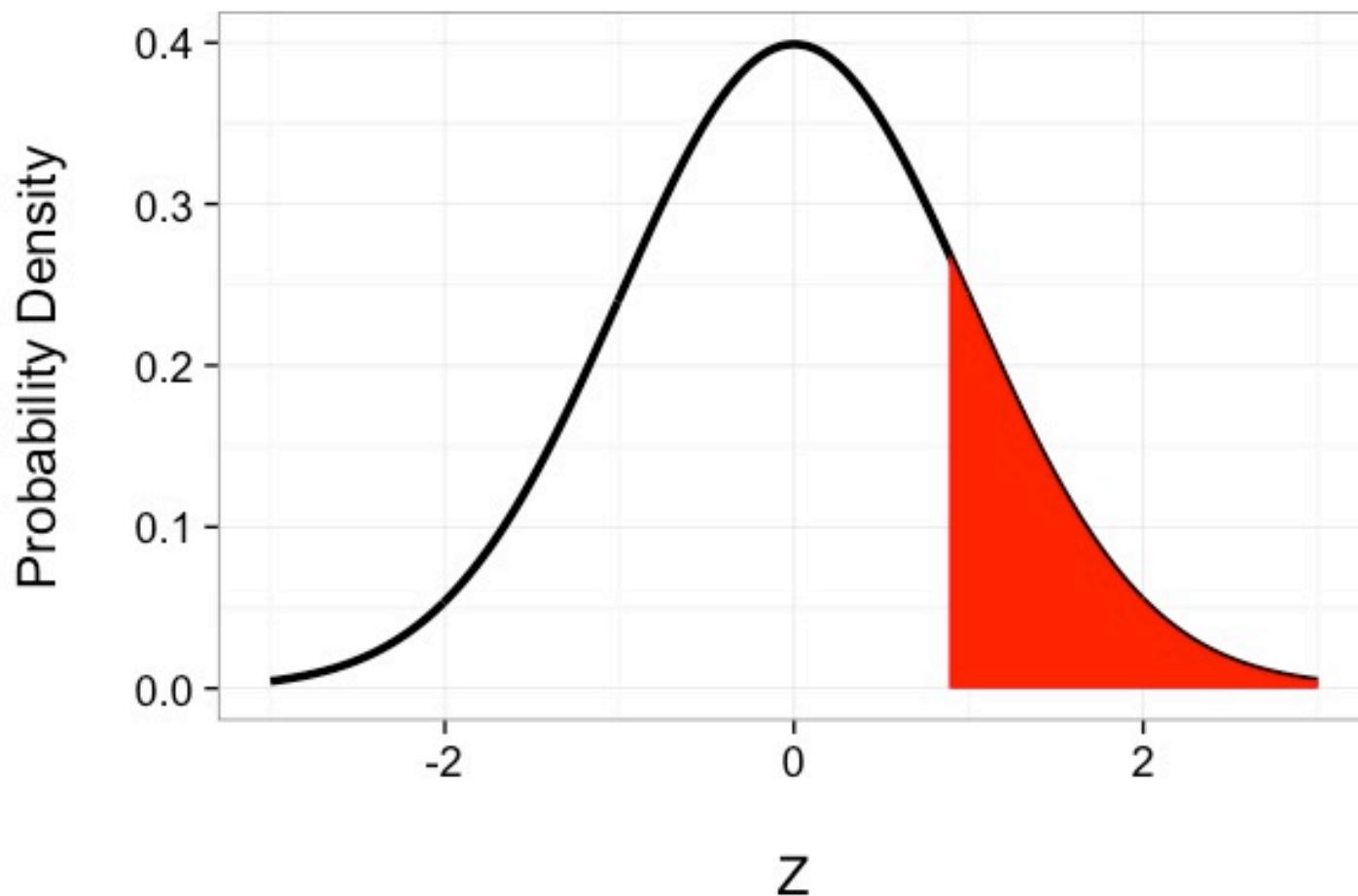
(sensu Richard McElreath)



ravenscar45

# This is a Golem

- What is my data generating process?
- What is my error generating process?



# What Drives My Golem?



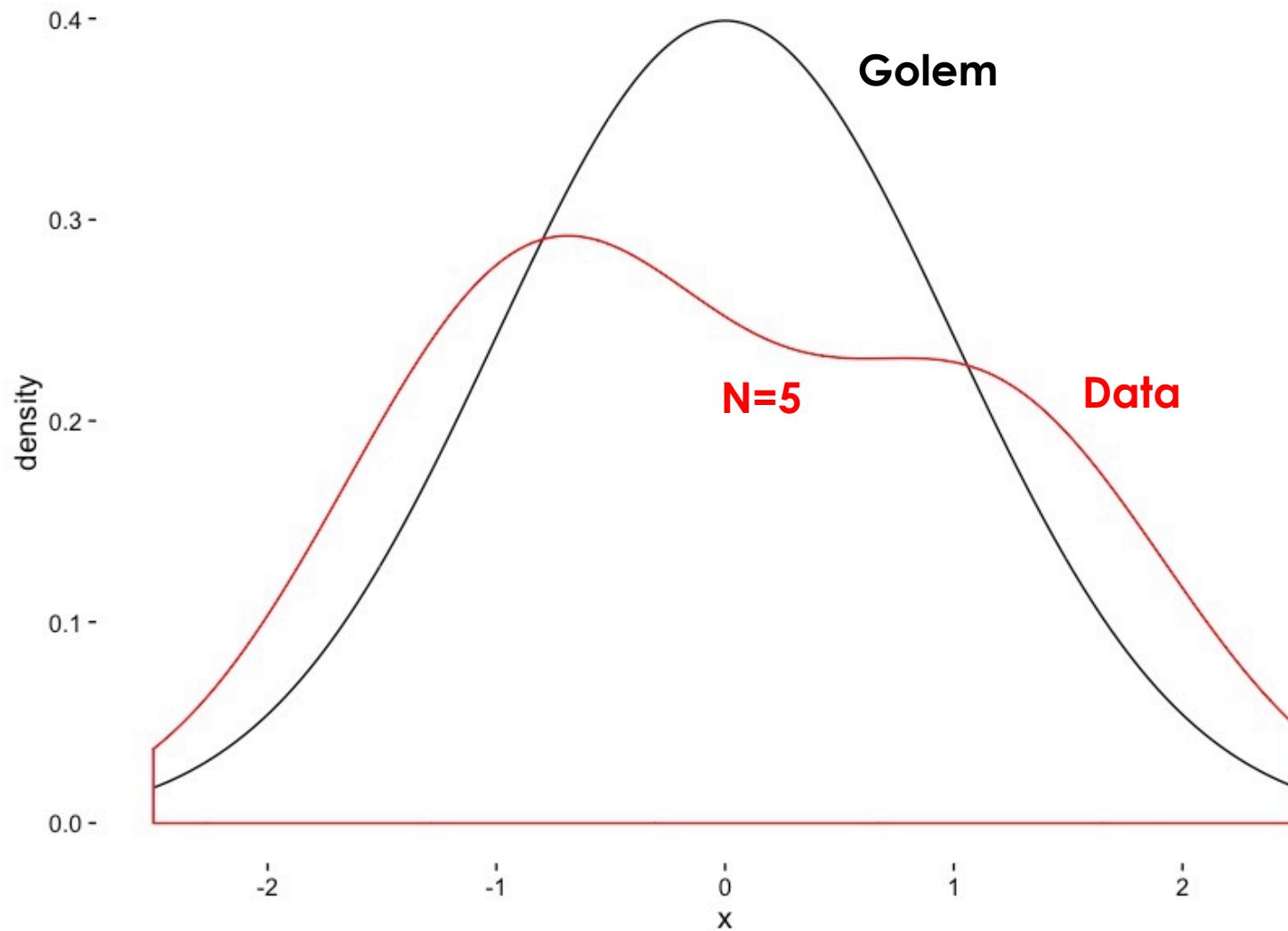
$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

Drawn from Data    Assumption about Population

Assumption about Population                                  Drawn from Data

The diagram illustrates the components of a z-score formula. The term  $\bar{x} - \mu$  is labeled "Drawn from Data" with red arrows pointing to it from both sides. The term  $\sigma / \sqrt{n}$  is labeled "Assumption about Population" with red arrows pointing to it from both sides. The entire formula is centered, with the mean  $\mu$  positioned above the center of the denominator and the standard deviation  $\sigma$  and sample size  $n$  below it.

# Is this a Good Golem for Realistic Sample Sizes?





Chemist & Statistician

**WILLIAM SEALY  
GOSSET**

1876-1937  
Chief Brewer

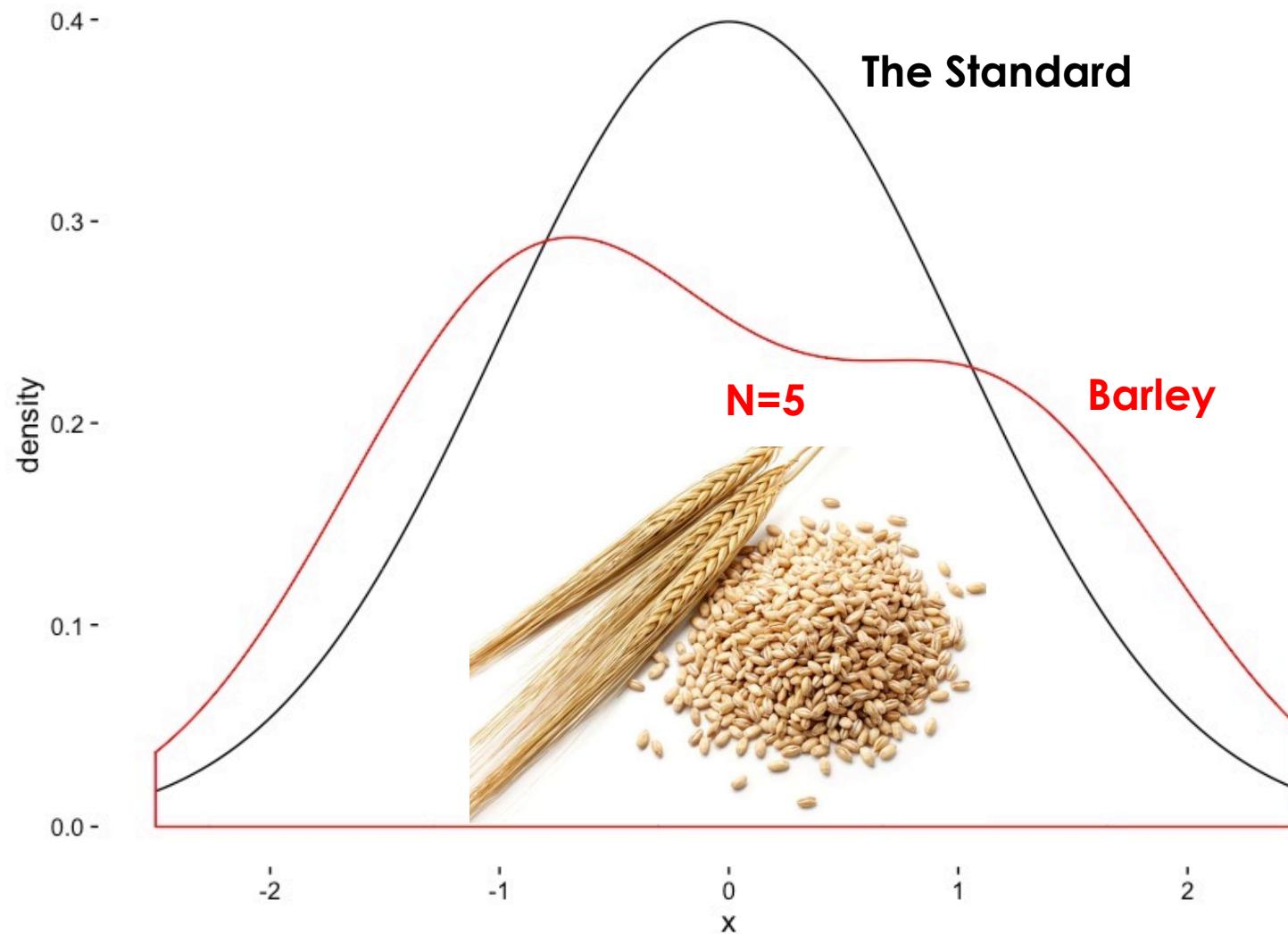
*Student 't' test*

# **INGREDIENTS**

**To understand what makes GUINNESS special,  
you have to start with the raw ingredients.**

Water, barley, hops and yeast: four natural ingredients, carefully selected to ensure that they are of the highest quality. Each ingredient is special in its own right but when mixed together according to our secret recipe, the result is simply extraordinary.

# Does This Farm Produce Barley at the Right Moisture?



# What Does My Golem Know?

$$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$$

Diagram illustrating the components of the t-statistic:

- $\bar{x}$  (sample mean) is labeled "Drawn from Data".
- $\mu$  (population mean) is labeled "Assumption about Population".
- $s$  (sample standard deviation) and  $\sqrt{n}$  (square root of sample size) are both labeled "Drawn from Data".

Evaluate against T Distribution with  $n-1$  Degrees of Freedom

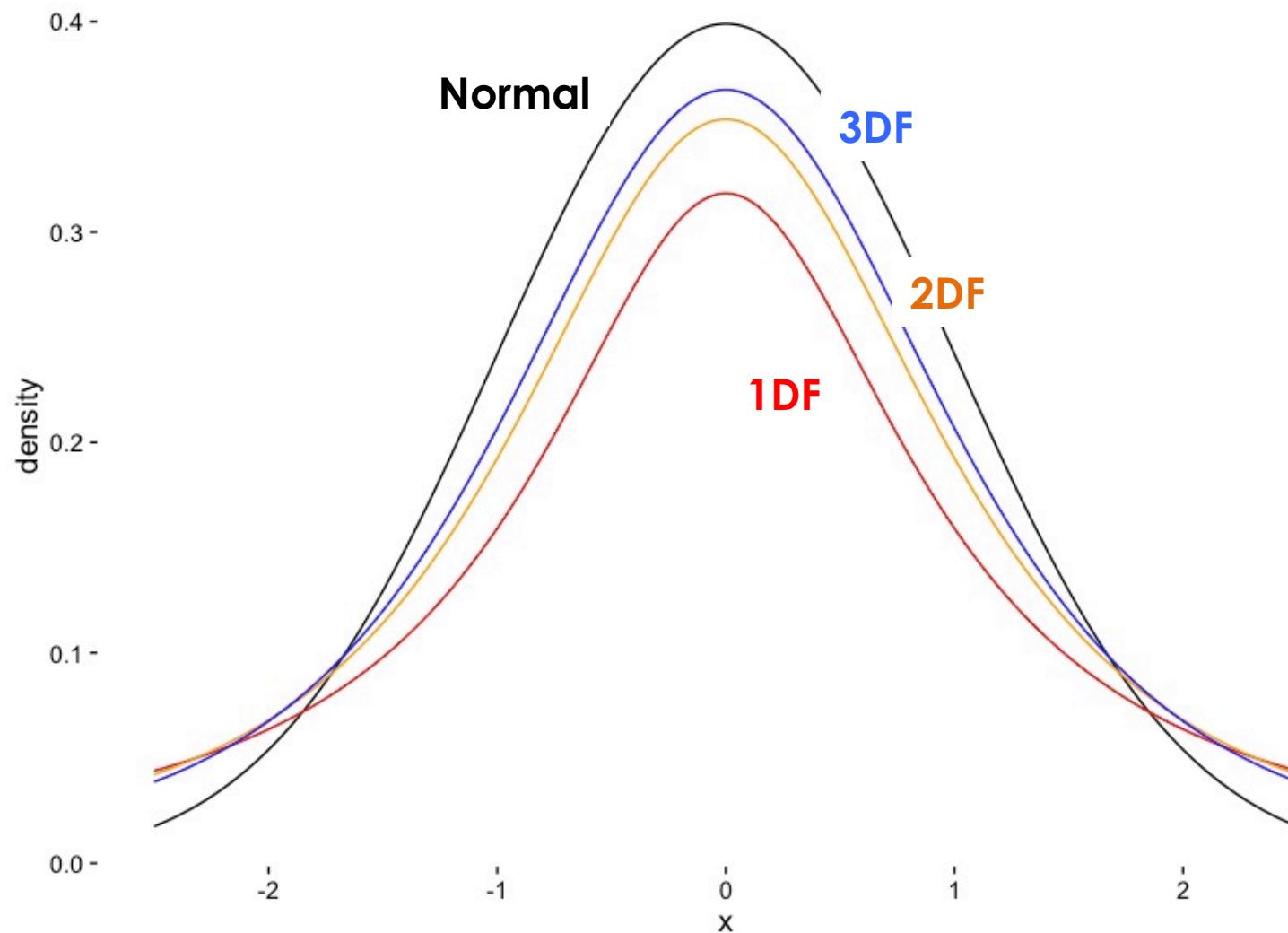
# T Versus N

- A Normal Distribution is defined by a mean and a SD
- A T-Distribution assumes a mean of 0, a SD of 1, but changes shape based on its Degrees of Freedom

# Degrees of What?

- Let's say you **estimate** a mean
- $\text{Mean} = (x_1 + x_2 + x_3)/3$
- If you know the mean,  $x_1$ , and  $x_2$ , you can calculate  $x_3$
- How much unique information is there in calculating a parameter?

# T Distribution Versus Normal



# Comparing Paired Groups

$H_0$ : Difference = 0

$$t = \frac{\bar{x}_d}{s / \sqrt{n}}$$

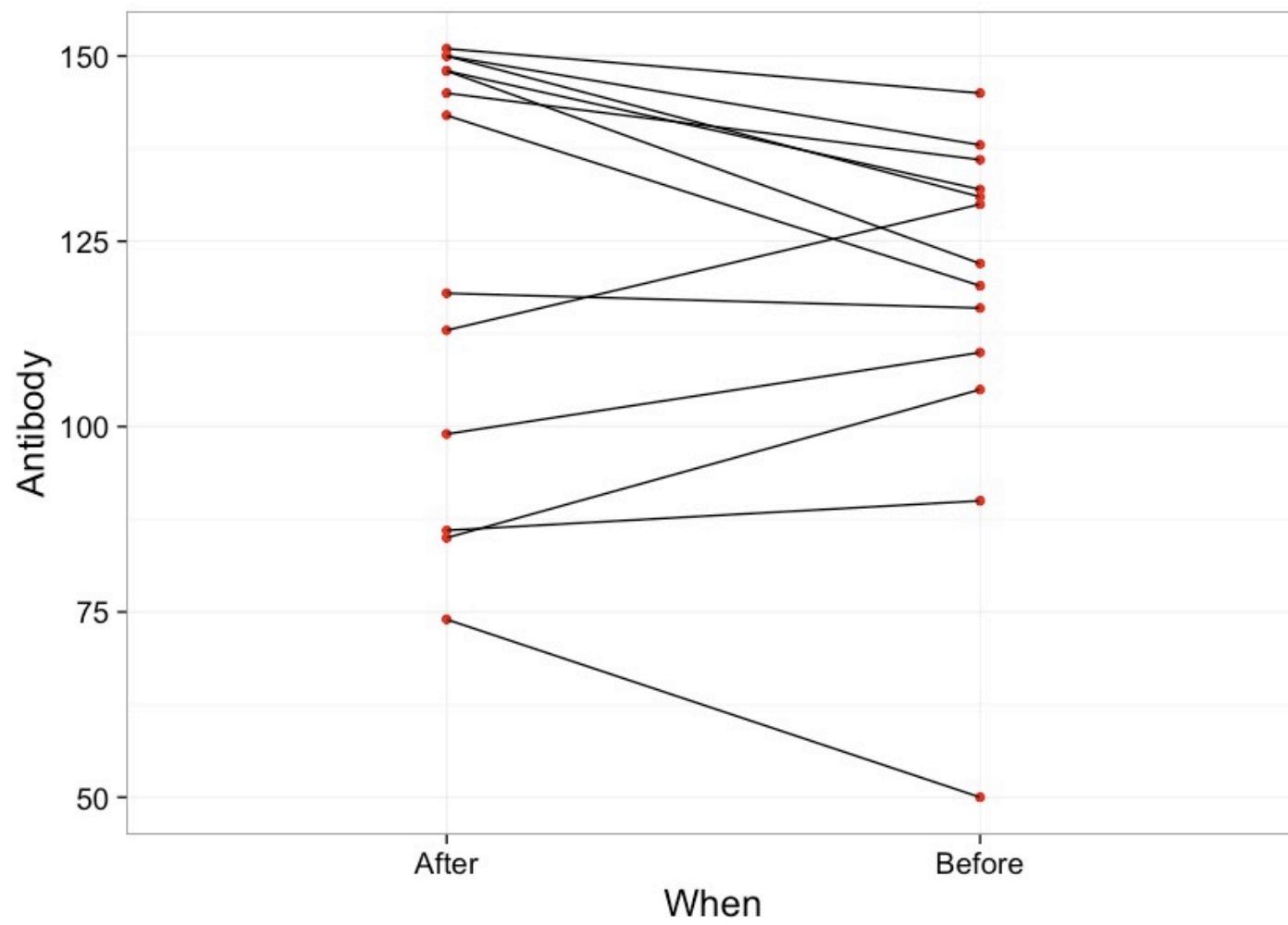
Drawn from Data  Drawn from Data  Drawn from Data 

Evaluate against T Distribution with  $n-1$  Degrees of Freedom  
 $N$  is the sample size per group

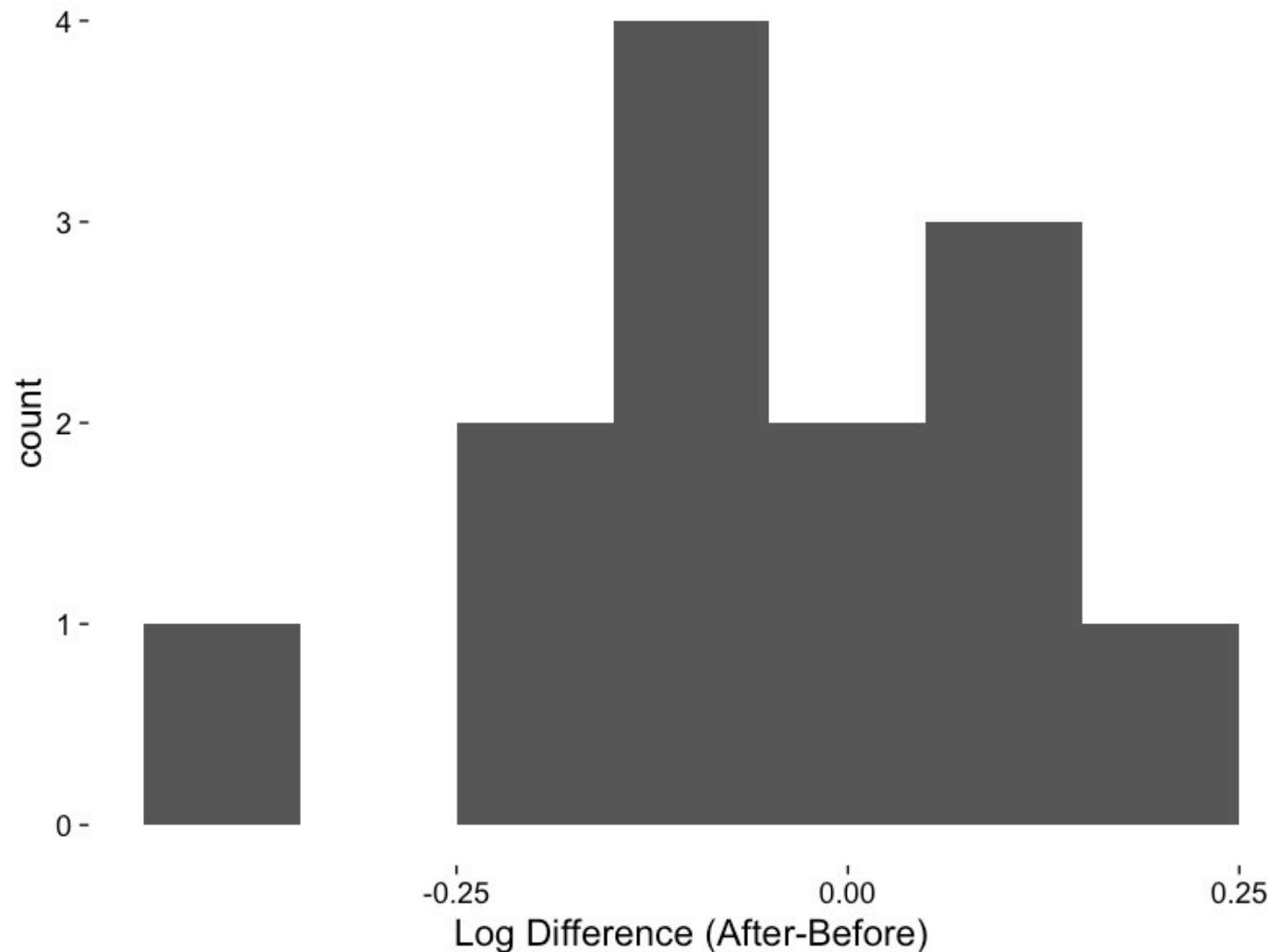
# Testosterone and Birds



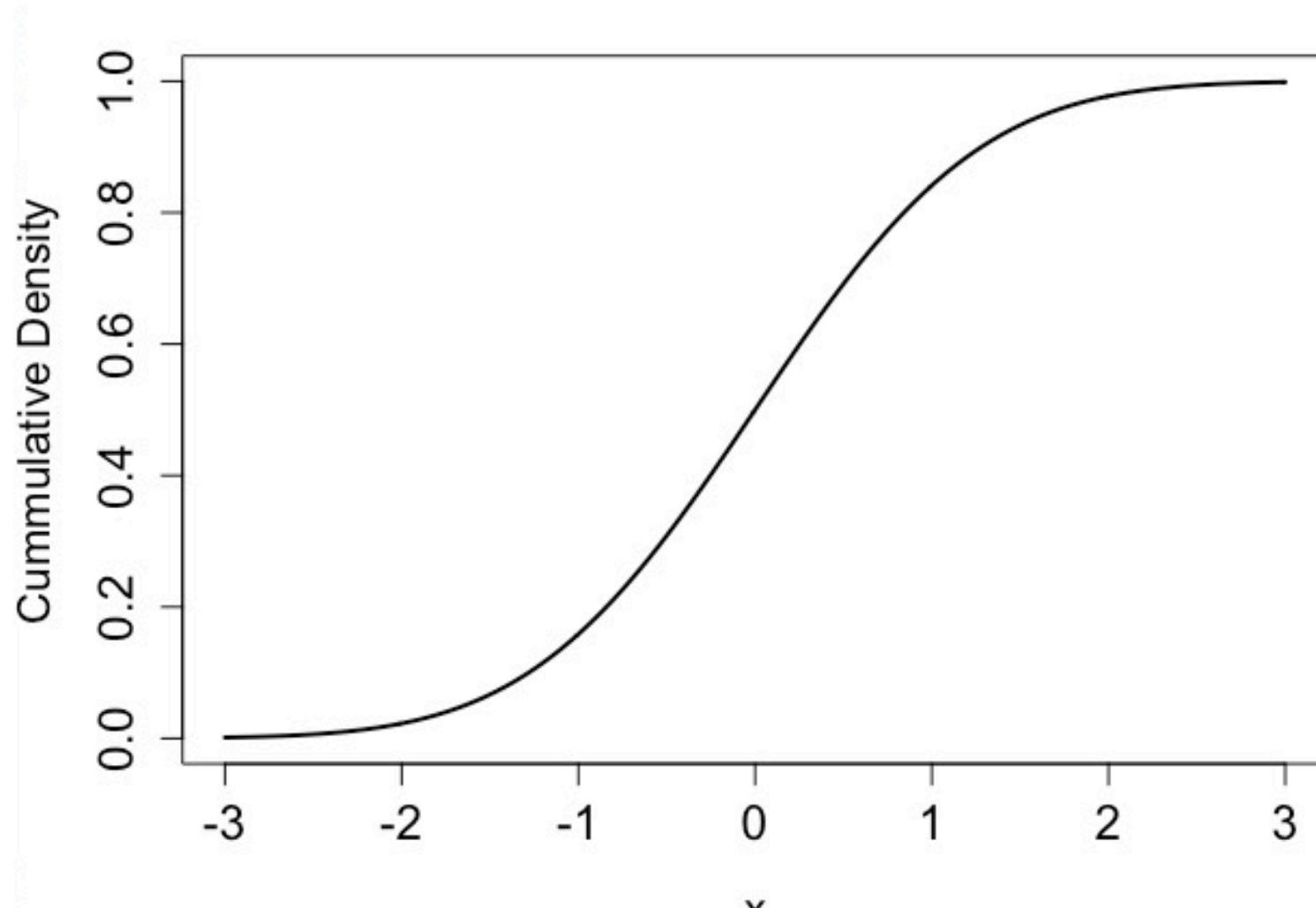
# Differences in Antibody Performance



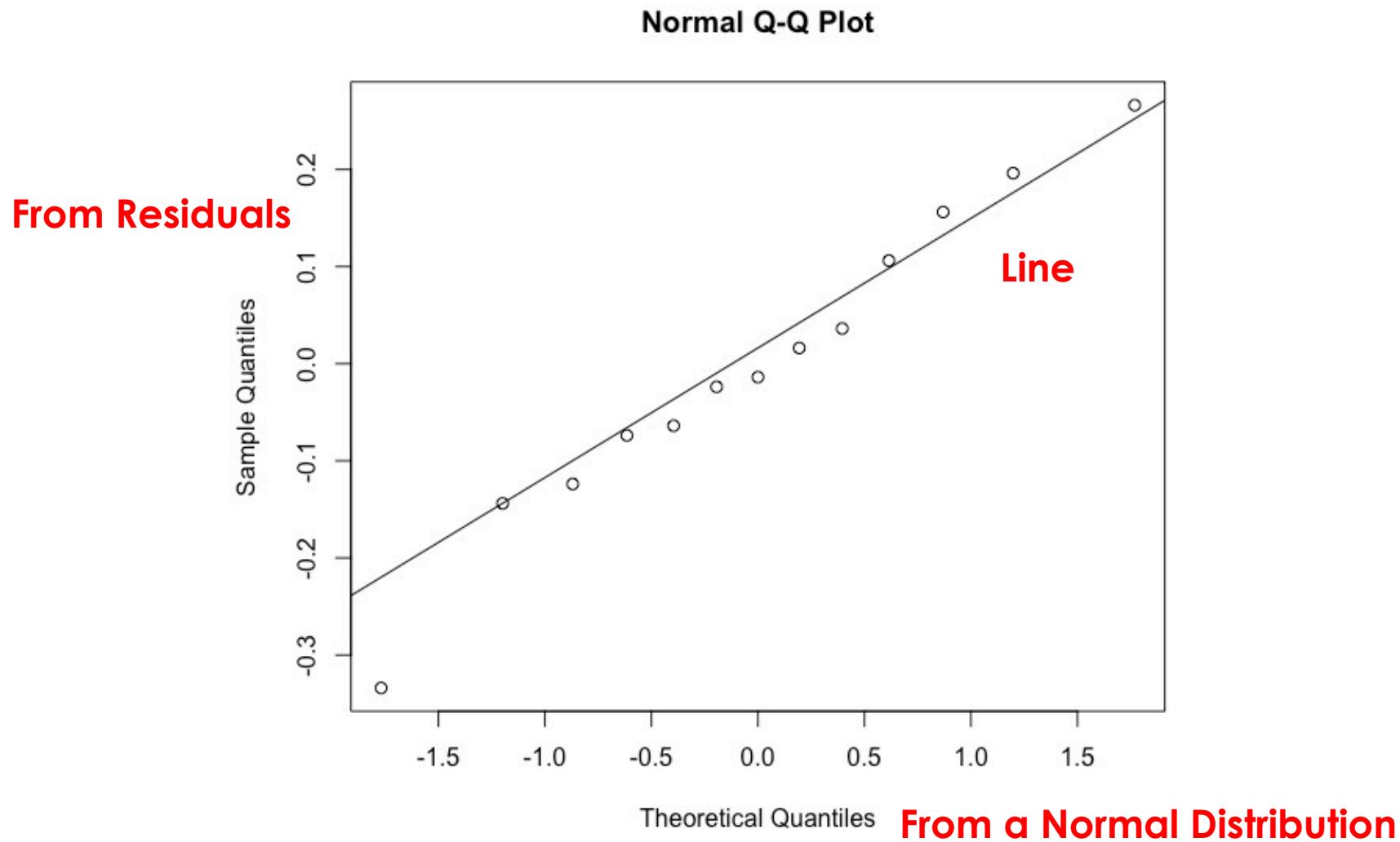
# Is the Log Difference Different from 0?



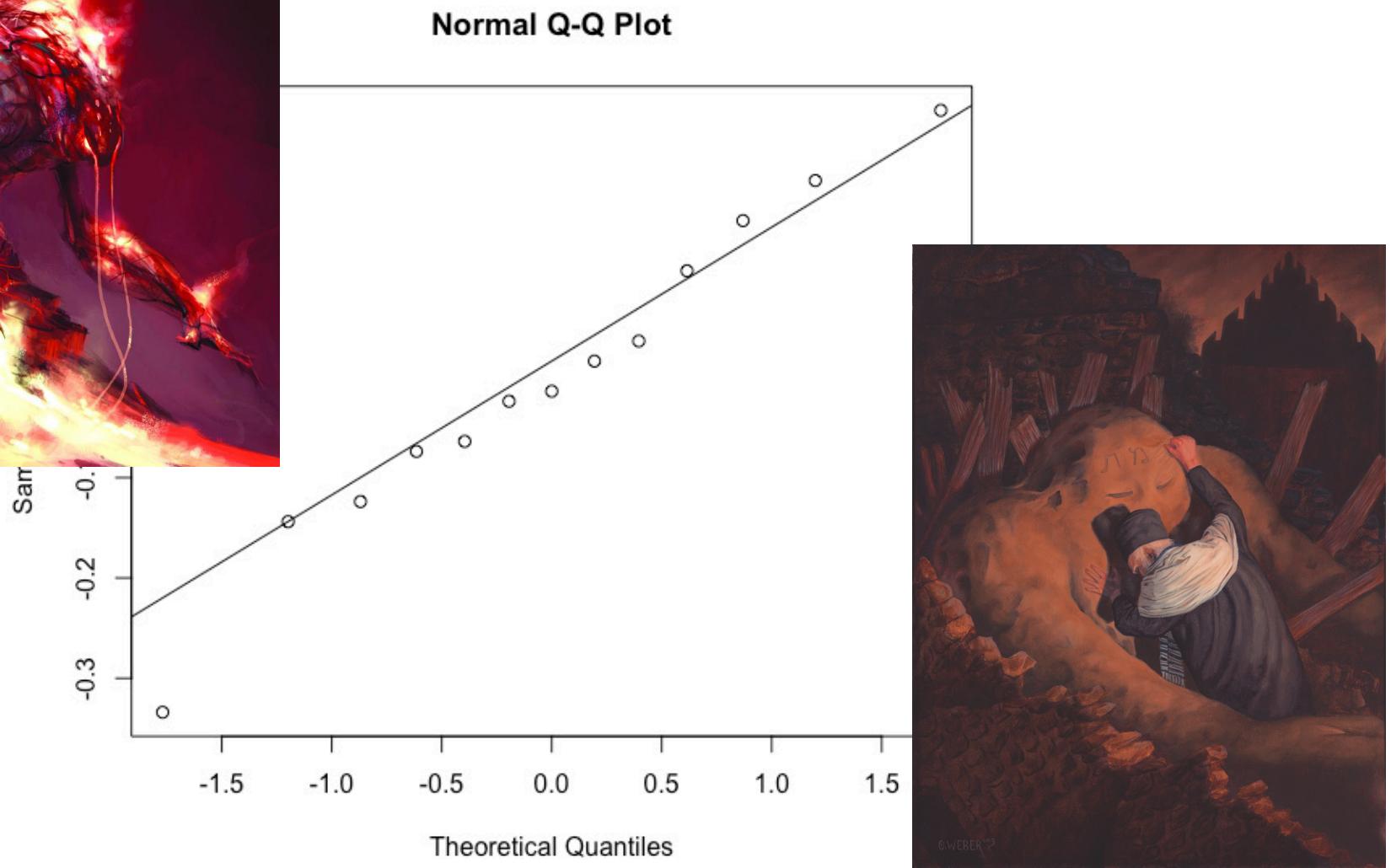
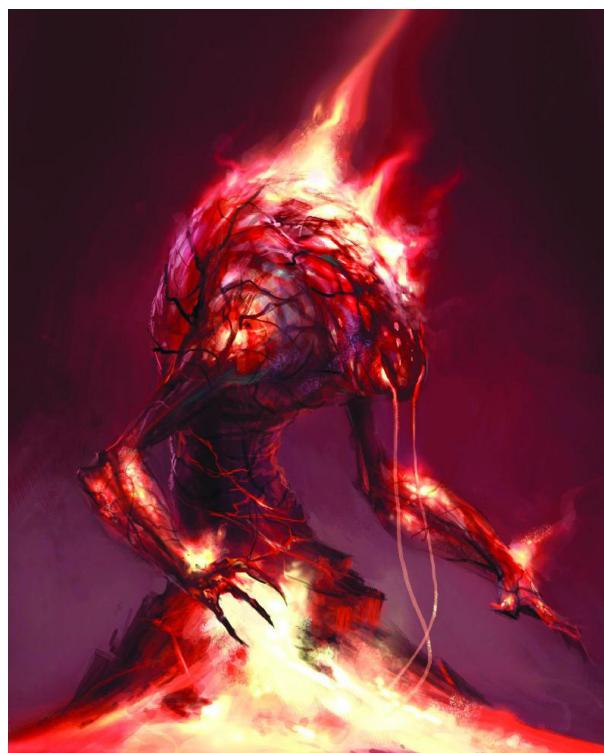
# One Way to Evaluate If Your Golem is Good: Quantiles



# QQ Plot to Evaluate If Residuals are Normal



# QQ Plot to Evaluate If Residuals are Normal



# General Testing Workflow

1. Build a Test
2. Evaluate Assumptions of Test
3. Evaluate Results
4. Visualize Results

# Comparing Groups

$H_0$ : Difference = 0

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_{12} / \sqrt{1/n}}$$

Pooled Sample SD =  $\sqrt{(s_1 + s_2)}$

Evaluate against T Distribution with n-1 Degrees of Freedom

N is the sample size per group

Assumes equal sample size and equal variance of populations

# Troubleshooting Your Golem

1. Unequal Sample Sizes
  - Alternate Formula for Denominator
2. Unequal Population Variances
  - Welch's T-Test (different denominator and DF)
3. Residuals Not Normal
  - Transform
  - Non-Parametric Test
  - Golem with a different error structure