THE UNIVERSITY OF SYDNEY SCHOOL OF MATHEMATICS AND STATISTICS

MATH1903/1907
Integral Calculus and Modelling (Advanced)

November 2010 Lecturers: H Dullin,	J Parkinson
TIME ALLOWED: One and a half hours	
Family Name:	,
Other Names:	
SID: Seat Number:	
This examination has two sections: Multiple Choice and Extended Answer.	Marker's use
The Multiple Choice Section is worth 35% of the total examination; there are 20 questions; the questions are of equal value; all questions may be attempted.	
Answers to the Multiple Choice questions must be entered on the Multiple Choice Answer Sheet.	
The Extended Answer Section is worth 65% of the total examination; there are 4 questions; the questions are of equal value; all questions may be attempted; working must be shown.	
Approved non-programmable calculators may be used. There is a table of integrals after the last question in this booklet.	
THE QUESTION PAPER MUST NOT BE REMOVED FROM THE EXAMINATION ROOM.	

Extended Answer Section

There are four questions in this section, each with a number of parts. Write your answers in the space provided below each part. There is extra space at the end of the paper.

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. (a) Let D be the region of the pla and the curve $y = \cosh x$.	ne bounded by the x -axis, the y -axis, the line $x = 1$,
(i) Compute the area of D .	2
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տնասարերում չու ենկան, եմ հանդեսի անգականում է գունասարեն Հայդ որոշատ ոն բանականին գունագի արևականության անգակա	
(ii) Compute the volume of	the solid obtained by rotating D about the y -axis
	the solid obtained by rotating D about the y -axis
	the solid obtained by rotating D about the y -axis

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(b) Let $I(x) = \int_0^x \sqrt{1+t^3} dt$. Calculate the integral

 $\int_0^1 x I(x) \, dx.$

Note: The constant I(1) will appear in your answer.

Note: The constant $I(1)$ will appear in your answer.

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141111	(c) Let $s_n = \sqrt{1} + \sqrt{2} + \sqrt{3} + \dots + \sqrt{n}$.
3	(i) Let P be the partition of $[0, n]$ into n subintervals of length 1. Use the corresponding upper and lower Riemann sums for the integral $\int_0^n \sqrt{x} dx$ to find
	upper and lower bounds for s_n , such that the bounds differ by at most \sqrt{n} .
1	(ii) Hence, or otherwise, calculate the limit $\lim_{n\to\infty} \frac{s_n}{n^{3/2}}$.

2. (a) (i)	Use a comparison test to show that $\int_0^\infty \frac{e^x}{7 + 2\cosh(2x)} dx$ converges.	2
(ii)	Using an appropriate substitution, or otherwise, calculate the integral $\int_0^1 \frac{xe^{\sqrt{1+x^2}}}{\sqrt{1+x^2}} dx.$	2
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(b) (i) For integers $m,n\geq 0$ let $I_{m,n}=\int_0^1 x^m(\ln x)^n\,dx$. Show that for $n\geq 1$, $I_{m,n}=-\frac{n}{m+1}I_{m,n-1},$

and hence compute $I_{m,n}$.

[You may use the fact that $\lim_{x\to 0^+} x^{\alpha} (\ln x)^{\beta} = 0$ for all $\alpha > 0$ and $\beta \ge 0$.]

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(ii) Hence show that

 $\int_0^1 x^{-x} \, dx = \sum_{k=1}^\infty n^{-n}.$

You may assume that any reasonable series manipulations are valid.		ations are valid.	
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3. (a) Find the general solution of the differential equation	
	$\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = 3e^{-2x}$
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(b) F:	ind the	general	solution	of
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$$\frac{dy}{dx} = \frac{2x+1}{x^2+x+1}(1-y),$$

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and show that every sol	ution converges	to the equilibriur	n solution $y = 1$	for $x \to \infty$.
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(c) Consider the differential equation of the form

$$\frac{dy}{dx} - e^{-x-y} + 1 = 0.$$

Introduce a new dependent variable u given by u=x+y, and hence find the general solution of the original equation.

<b>4.</b> (a)	A spherical raindrop evaporates at a rate proportional to its surface area, retaining the spherical shape. Derive a differential equation for the radius $r(t)$ of the raindrop and solve it for a raindrop with initial radius $r_0$ to show that $r(t) = r_0 - \alpha t$
	for a constant $\alpha > 0$ . [Note that the volume of a sphere of radius $r$ is $V = 4\pi r^3/3$ , and that the surface area is $A = 4\pi r^2$ , and assume that the density of water is 1.]
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(b) The evaporating raindrop is falling towards the ground. For this type of problem with time-dependent mass the appropriate form of Newton's second law states that the rate of change of the product of mass m with velocity v is equal to the force. The force is given by mg (with positive direction down), where g is the constant gravitational acceleration, with an additional air friction force proportional to the area πr^2 times the velocity. The friction force opposes the velocity. Show that the differential equation for the velocity v of the falling raindrop can be written as

$$\frac{dv}{dt} - \frac{k\alpha}{r(t)}v = g$$

for some constant k.

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(c)	Find the particular solution of the different which initially the raindrop is at rest.	erential equation for the falling raindrop for Assume that $k \neq -1$.
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(d) Assume that $k = -2$. completely evaporated.	Compute the	e distance	the drop	falls from	rest until	it is
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Table of Standard Integrals

1.
$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$
 $(n \neq -1)$ 9. $\int \sec^2 x \, dx = \tan x + C$

$$9. \int \sec^2 x \, dx = \tan x + C$$

$$2. \int \frac{dx}{x} = \ln|x| + C$$

$$10. \int \csc^2 x \, dx = -\cot x + C$$

$$3. \int e^x dx = e^x + C$$

11.
$$\int \sec x \, dx = \ln \left| \sec x + \tan x \right| + C$$

$$4. \int \sin x \, dx = -\cos x + C$$

12.
$$\int \csc x \, dx = \ln \left| \csc x - \cot x \right| + C$$

$$5. \int \cos x \, dx = \sin x + C$$

$$13. \int \sinh x \, dx = \cosh x + C$$

6.
$$\int \tan x \, dx = -\ln|\cos x| + C$$
 14.
$$\int \cosh x \, dx = \sinh x + C$$

14.
$$\int \cosh x \, dx = \sinh x + C$$

$$7. \int \cot x \, dx = \ln |\sin x| + C$$

15.
$$\int \tanh x \, dx = \ln \cosh x + C$$

8.
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

8.
$$\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$
 16. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1} \left(\frac{x}{a} \right) + C$ ($|x| < a$)

17.
$$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2 + a^2}\right) + C'$$

18.
$$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C = \ln\left(x + \sqrt{x^2 - a^2}\right) + C' \quad (x > a)$$

19.
$$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C \quad (|x| \neq a)$$

End of Extended Answer Section