PHYS1001 Physics 1 (Regular) Formula Sheet

Vectors

$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

$A = \left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$

$R_x = A_x + B_x,\ R_y = A_y + B_y,\ R_z = A_z + B_z$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi = \left|\vec{\mathbf{A}}\right| \left|\vec{\mathbf{B}}\right| \cos\phi$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$

$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}, \quad C = AB\sin\phi$

$C_x = A_yB_z - A_zB_y, \quad C_y = A_zB_x - A_zB_z,$

$C_z = A_xB_y - A_yB_x$

Kinematics

$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$$\vec{\mathbf{v}}_{\text{av}} = \frac{\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1}{t_2 - t_1} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$\vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

$$\vec{\mathbf{a}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}$$

Simple motions

Constant acceleration in one direction:

$v = v_0 + at$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$v^2 = v_0^2 + 2a(x - x_0)$

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t$$

Projectile motion:

$x = (v_0\cos\alpha_0)t$

$$y = (v_0\sin\alpha_0)t - \frac{1}{2}gt^2$$

$v = v_0\cos\alpha_0$

$v_y = v_0\sin\alpha_0 - gt$

Uniform circular motion:

$$a_{\text{rad}} = \frac{v^2}{R} = \omega^2R = \frac{4\pi^2R}{T^2}$$

Force and Momentum

$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}}, \quad \vec{\mathbf{F}}_{\text{A on B}} = -\vec{\mathbf{F}}_{\text{B on A}}$

$w = mg$

$f_k = \mu_k n, \quad f_s \leqslant \mu_s n$

$$\frac{d\vec{\mathbf{P}}}{dt} = \sum\vec{\mathbf{F}}_{\text{ext}}$$

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \int_{t_1}^{t_2}\sum\vec{\mathbf{F}}\,dt, \qquad M = \sum_i m_i$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}, \quad \sum\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}, \quad \sum\vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{cm}}$$

$\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3 + \ldots = M\vec{\mathbf{v}}_{\text{cm}}$

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{\sum_i m_i\vec{\mathbf{r}}_i}{M} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + m_3\vec{\mathbf{r}}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

Work and Energy

$K = \frac{1}{2}mv^2, \quad U_{\text{grav}} = mgy, \quad U_{\text{el}} = \frac{1}{2}kx^2$

$F = -kx$

$W_{\text{tot}} = K_2 - K_1 = \Delta K$

$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = Fs\cos\phi$

$W = \int_{P_1}^{P_2} F\cos\phi \, dl = \int_{P_1}^{P_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$

$P_{\text{av}} = \frac{\Delta W}{\Delta t}$

$P = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

$W_{\text{el}} = -\Delta U_{\text{el}}, \qquad W_{\text{grav}} = -\Delta U_{\text{grav}}$

$E = K + U$

$\Delta E = W_{\text{other}}$

Periodic Motion

$\omega = 2\pi f = \frac{2\pi}{T}, \quad f = \frac{\omega}{2\pi} = \frac{1}{T}$

$\omega = \sqrt{\frac{k}{m}}, \quad \omega = \sqrt{\frac{\kappa}{I}}$

$\omega = \sqrt{\frac{g}{L}}, \quad \omega = \sqrt{\frac{mgd}{I}}$

$F_x = -kx$

$x = A\cos(\omega t + \phi)$

$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant}$

$x = Ae^{-(b/2m)t}\cos\omega' t, \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$

$b_{\text{critical}} = 2\sqrt{k\,m}$

$A = \frac{F_{\text{max}}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$

Rotational Motion

$\omega_z = \frac{d\theta}{dt}, \qquad v = r\omega_z$

$\alpha_z = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$

$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r, \quad a_{\text{tan}} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha_z$

$I_{\text{P}} = I_{\text{cm}} + Md^2, \quad v_{\text{cm}} = R\omega$

$I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \sum_i m_i r_i^2$

$\tau = rF\sin\theta, \quad \vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$

$\sum \tau_z = I\alpha_z, \quad \sum \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}$

$K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega_z^2, \quad P = \tau_z\omega_z$

$W = \int_{\theta_1}^{\theta_2} \tau_z \, d\theta, \quad W_{\text{tot}} = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$

$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}}$ (particle)

$\vec{\mathbf{L}} = I\vec{\boldsymbol{\omega}}$ (rigid body)

Moments of inertia

Thin rod, axis through centre:$I = \frac{1}{12}ML^2$

Thin rod, axis through one end:$I = \frac{1}{3}ML^2$

Rectangular plate, axis through centre:$I = \frac{1}{12}M(a^2 + b^2)$

Thin rectangular plate, axis along edge:$I = \frac{1}{3}Ma^2$

Hollow cylinder:$I = \frac{1}{2}M(R_1^2 + R_2^2)$

Solid cylinder:$I = \frac{1}{2}MR^2$

Thin-walled hollow cylinder:$I = MR^2$

Solid sphere:$I = \frac{2}{5}MR^2$

Thin-walled hollow sphere:$I = \frac{2}{3}MR^2$

A QUICK GUIDE TO SIGNIFICANT FIGURES

This material comes from the Appendix to the Junior Physics lab manual: Experimental Analysis. Please read that Appendix carefully for more details.

All scientific quantities must be written with the appropriate number of significant figures to indicate uncertainties in the value.

- You can indicate uncertainty **explicitly** using $\pm$ followed by a number. The uncertainty should be written with one *significant figure* (sometimes two figures if the first is 1) and the answer is rounded to have the same number of *decimal places*.

  e.g. a mass measurement with a mean value of $m = 0.679$ kg and a standard error of the mean (SEM) of 0.028 kg would be written as
  $$m = (0.68 \pm 0.03) \text{ kg}$$
  If the SEM is 0.014 kg, you could write the answer as
  $$m = (0.679 \pm 0.014) \text{ kg}$$
  since rounding the SEM to 0.01 kg understates the uncertainty by 40%.

- You can indicate uncertainty **implicitly** by using significant figures. The number is written so that only the last digit is uncertain.

  e.g. in the first example, our mass measurement would be written $m = 0.68$ kg

The number of *significant figures* in a quantity is the number of digits that convey meaning. In counting the number of significant figures, you ignore

- All leading zeros
  e.g. 0.00252 has three significant figures
- Trailing zeros where there is no decimal point
  e.g. 1200 has *two* significant figures (result is uncertain to ± 100), but 1200.0 has *five* significant figures (result is uncertain to ± 0.1). Trailing zeros after the decimal point are significant; thus 0.0120 has *three* significant figures.

The *clearest* way to indicate the number of significant figures is to use scientific notation (powers of ten). The above results can be stated as 1.2×10^3, 1.2000×10^3, and 1.20×10^{-2}, which makes it explicit how many significant figures each number has.

When combining quantities, if you *have* explicit uncertainties, you combine them to determine the number of significant figures in the result.

If you *don't* have explicit uncertainties, but are combining values with different numbers of significant figures, the following rules apply:

1. When **adding** or **subtracting**, the answer should contain the **same** number of **decimal places** as the measurement with the **fewest decimal places**.
2. When **multiplying** or **dividing**, the answer should contain the **same** number of **significant figures** as the measurement with the **fewest significant figures**.

So

- the difference between two measured lengths $l = 0.9570$ m (4 decimal places) and $l = 0.84$ m (2 decimal places) is $\Delta l = 0.12$ m (to 2 decimal places).
- the speed of an object which travels a distance $x = 1.3$ m (2 significant figures) in time $t = 22.0$ s (3 significant figures) is $v = x/t = 0.059 \text{ m.s}^{-1}$ (2 significant figures).

For more information about combining uncertainties, see the table on page A.10 of the *Experimental Analysis* Appendix to the Junior Physics lab manual.

COMBINING UNCERTAINTIES

Rules for combining uncertainties: x and y represent two measured quantities, and the result is u. The uncertainties in the respective quantities are represented by Δx, Δy and Δz.

Calculation of experimental result u	**Uncertainty in** u
Multiplying by a constant a (with negligible uncertainty): $u = ax$	$\Delta u = a\Delta x$
Adding/subtracting $u = x + y$ $u = x - y$	$\Delta u = \Delta x + \Delta y$
Multiplying/dividing $u = xy$ $u = x/y$	$\frac{\Delta u}{u} = \frac{\Delta x}{x} + \frac{\Delta y}{y}$
Raising to a power: $u = x^n$	$\frac{\Delta u}{u} = n\frac{\Delta x}{x}$