THE UNIVERSITY OF SYDNEY

FACULTIES OF ARTS, EDUCATION, ENGINEERING AND SCIENCE

PHYS1902 – PHYSICS 1B (ADVANCED)

NOVEMBER 2004

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

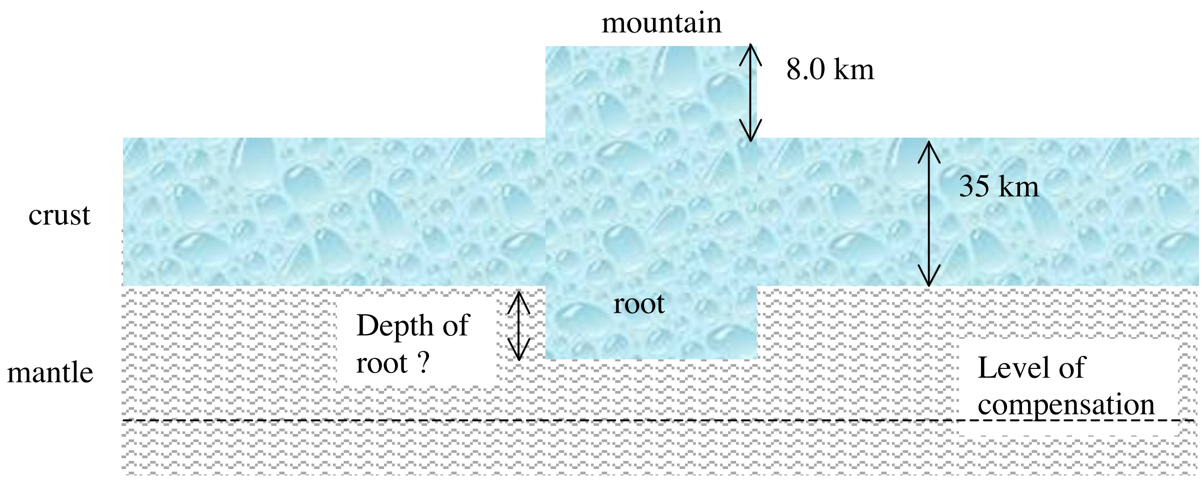
DATA

Density of water	ρ	=	1.00×10^{3} kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^{5} Pa
Magnitude of local gravitational field	g	=	9.81 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^{8} m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg

SECTION A
(Please use a separate booklet for this section)

Question 1

In a simple geological model, the Earth can be represented as a thin crust of continents and mountains (density 2900 $kg.m^{-3}$) floating on a denser mantle of material, which behaves as a fluid and has a density 4700 $kg.m^{-3}$. At some horizontal level in the mantle (called the *level of compensation*), the pressure has a constant value, regardless of what is located in the crust above. This model requires that mountains have roots to enable such a level of equal pressure to exist, as shown in the diagram.



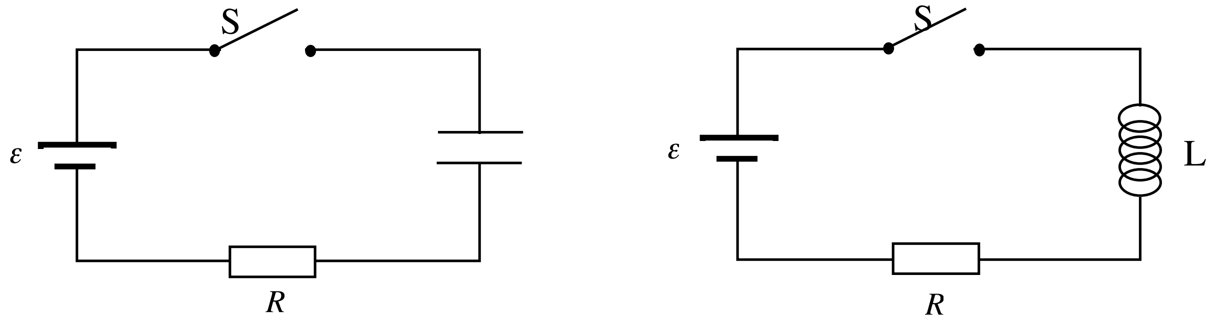
(a) For a crust of depth 35 km and a mountain of height 8.0 km, how deep must the root of the mountain be?

(b) What assumption is made about atmospheric pressure over the various parts of the continent? Is this reasonable? Justify your answer.

(5 marks)

Question 2

Two different circuits are shown below. They include a switch S, a source of emf generating a potential difference $\mathcal{E}$, a resistor of value R, a capacitor of value C which is initially uncharged, and an inductor of value L. The switch S is closed at time $t = 0$ and each circuit then responds, eventually reaching a steady state.



(a) Draw two graphs to indicate how the current changes with time in each current. The time axis of your graph should start at $t = 0$ and extend over an interval of time sufficient to show the steady state behaviour.

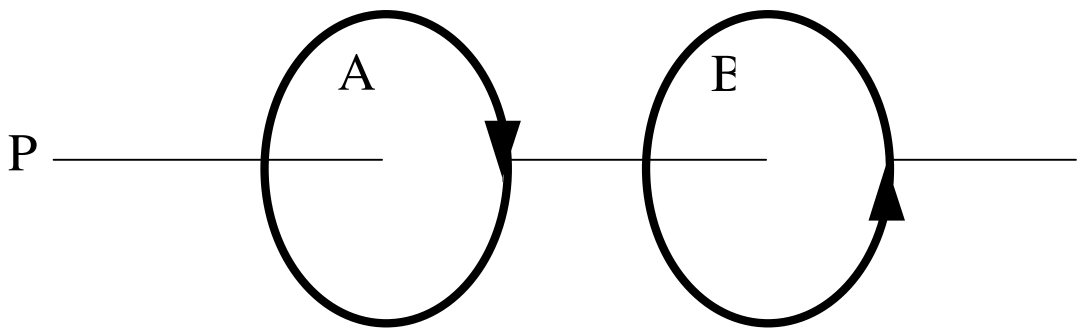
(b) On each of your graphs, indicate clearly values of initial and final current. Where possible express these values in terms of $\mathcal{E}$, R, C and L.

(5 marks)

Question 3

The two circular loops in the diagram below have their planes parallel to each other. As viewed from point P, the current in loop A is clockwise, and the current is anti-clockwise in loop B. The current in loop B is induced by the current in loop A.

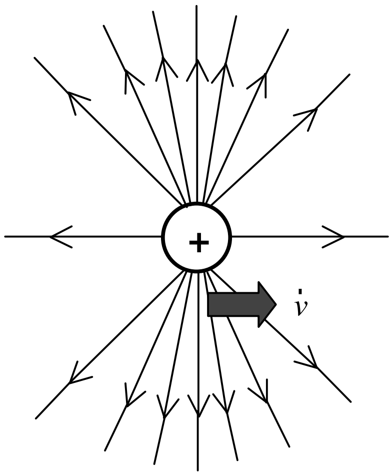
How does the current in loop A change with time? Carefully explain your reasoning.



(5 marks)

Question 4

(a) It turns out that the electric field from a single charge that is moving at high speed is not radially symmetric, as shown in the figure below.



Show that this field is not conservative, i.e. it does not satisfy $\oint \vec{E}.d\vec{l} = 0$

(b) Consider the equation $\oint \vec{B}.d\vec{A} = 0$, which is one of Maxwell's equations.

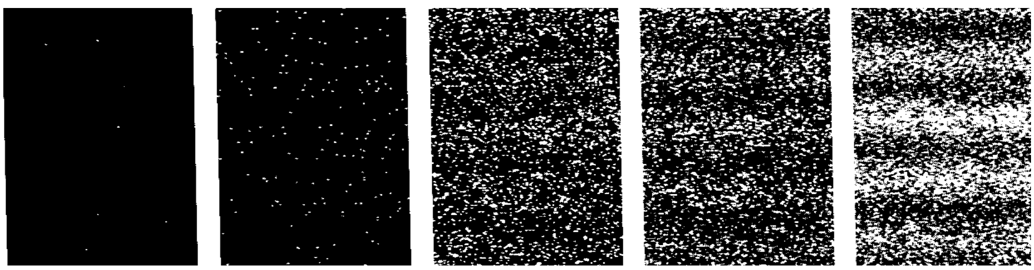
(i) Briefly explain what this equation tells us about the flux of a magnetic field.

(ii) Briefly explain why the discovery of magnetic monopoles would violate this equation.

(5 marks)

Question 5

A beam of electrons is directed through two narrowly spaced horizontal slits. The emerging beam falls on a detector array. Images from the detector array are shown below, with exposure time increasing to the right.



(a) What feature in these images shows that the electrons in the beam are behaving like ordinary classical particles (for example, tiny balls)? Explain why this feature illustrates this behaviour.

(b) What feature in these images show that the electrons are behaving like ordinary waves (for example, waves on the surface of water)? Explain why this feature illustrates this behaviour.

(c) How do physicists describe electrons in order to account for both the observations you have just described?

(5 marks)

Question 6

(a) An electron, a neutron and an alpha particle (helium nucleus) all have the same kinetic energy. State the de Broglie hypothesis and rank the particles in order of increasing wavelength.

(b) A beam of helium nuclei travelling in space is slowed down by collisions with photons. What velocity must the helium nuclei have in order for them to have a wavelength of 0.01 m?

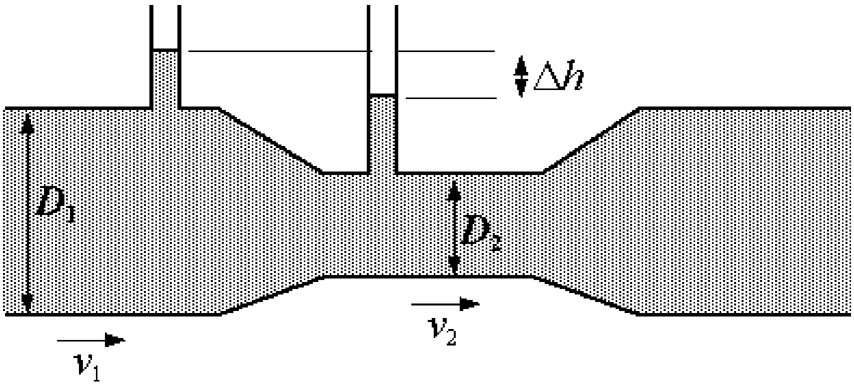
(c) Explain briefly how the exclusion principle forces one of the electrons in an unexcited lithium atom to enter a 2s state.

(5 marks)

SECTION B
(Please use a separate booklet for this section)

Question 7

A Venturi meter is installed in a water main, as shown in the diagram. The pipe has circular cross-section, with diameter D_1 in the first segment and D_2 in the second segment, with $D_2 < D_1$. The density of water is ρ. The volume flow rate of the water in the pipe is R (measured in m^3s^{-1}).



(a) In terms of the variables defined above, what is the speed of flow v_1 in the first section of pipe?

(b) What is the speed of flow v_2 in the second section of pipe?

(c) What is the pressure difference between point 1 and point 2?

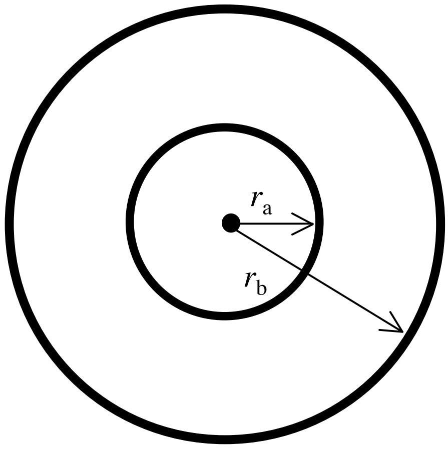
(d) What is the difference in the water level Δh in the two tubes? Express your answer in terms of the dimensional parameters D_1, D_2 and R and any other constants required.

(e) If the water were replaced with a viscous liquid, would the difference in water level in the two tubes Δh be larger or smaller? Why?

(10 marks)

Question 8

Two concentric thin spherical shells are separated by a vacuum. The inner shell has total charge $+Q$ and radius r_a, while the outer shell has total charge $-Q$ and radius r_b. The thickness of the shells can be neglected.



(a) Sketch the electric field (showing its direction) and calculate its magnitude as a function of the distance r from the centre. Be sure to consider the three regimes: $r < r_a$, $r_a < r < r_b$ and $r > r_b$.

(b) Calculate the potential difference between the two shells.

(c) Calculate the capacitance of the two shells.

(d) Calculate the total electrical potential energy stored in the system.

(10 marks)

Question 9

Consider a very long *solid* metal cylinder with radius R. A current i flows along the cylinder and the current density is uniform across the cylinder. Let r be the distance from the axis of the cylinder.

(a) Using Ampere's Law derive an expression for the magnitude of magnetic field outside the cylinder ($r > R$). State all the steps in the argument.

(b) Show that the magnitude of magnetic field inside the cylinder ($r < R$) is

$$B(r) = \frac{\mu_0 \, i \, r}{2\pi R^2}$$

(c) Now consider a segment of the cylinder of length L. Show that the total energy stored in the magnetic field inside this segment of the cylinder is independent of R.

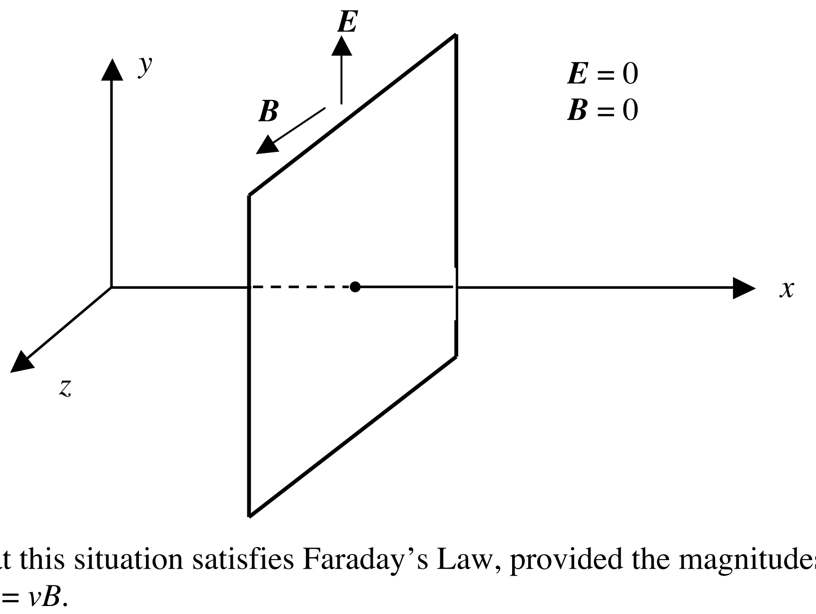
(*Hint:* Remember that energy density of the magnetic field is $u = \dfrac{1}{2\mu_0} B^2$.

First find the energy in a thin cylindrical shell of radius r and thickness dr.)

(10 marks)

Question 10

Imagine that we are far from any charges and currents, and space is divided into two regions by a plane perpendicular to the x axis. We call this boundary plane the *wavefront*, which we take to be moving in the $+x$ direction with speed v. To the right of the wavefront there are no electric or magnetic fields. To its left, there is a uniform electric field in the $+y$ direction and a uniform magnetic field in the $+z$ direction.



(a) Show that this situation satisfies Faraday's Law, provided the magnitudes of the fields satisfy $E = vB$.
(Hint: consider a path integral around a rectangle in the x–y plane.)

(b) Show that this situation also satisfies the Ampere-Maxwell Law.

(c) Hence derive an expression for the speed of the wavefront.

(10 marks)

Question 11

A burst of light of wavelength 700 nm shines on a metallic surface.

(a) What prediction would you make for the dependence of the number of electrons ejected as the duration in time of the burst tends to zero, based on:
   (i) classical physics,
   (ii) quantum physics.

   Carefully justify your answers.

(b) If the maximum energy of any electron ejected is observed to be 0.5 eV, what is the work function of the metal in eV?

(c) What prediction would you make for the dependence on the number of electrons ejected as the wavelength of the light is increased, while keeping the total energy in the burst constant? Justify your answers carefully, based on:
   (i) classical physics,
   (ii) quantum physics.

(10 marks)

Question 12

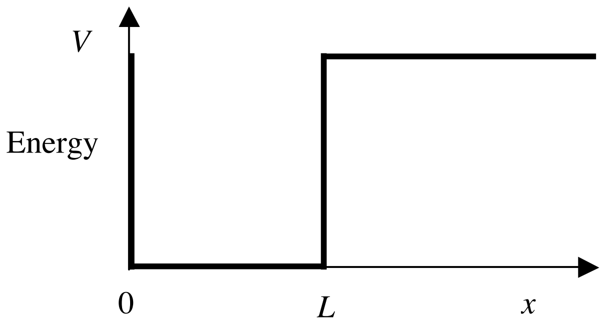
The potential energy as a function of position experienced by a particle is shown in the diagram. The depth of the well is V relative to the zero of energy represented by the bottom of the well. A particle is located in the well and has an energy less than V.

(a) If the particle obeys classical physics, sketch the probability distribution of the particle inside and outside the well.

(b) If instead the particle obeys quantum physics, the probability distribution of the particle is governed by wavefunctions that are solutions to the 1-dimensional Schrodinger equation. Show by substitution that the wavefunction for a particle of energy $E < V$ *inside* the well is a linear combination of *sin kx* and *cos kx* where $k = \frac{1}{\hbar}\sqrt{2mE}$

(c) Write down the Schrodinger equation that applies *outside* the well where $E < V$. Show by substitution that the wavefunction outside the well can be a linear combination of e^{kx} and e^{-kx}.

(d) Sketch a possible solution for the wavefunction of a bound state of the particle inside and outside the well *and* the corresponding probability distribution.



(10 marks)

This is the end of your questions