MATH1081 Discrete Mathematics UNSW 2019T1

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Welcome!

MATH1081 Discrete Mathematics UNSW 2019T1

Formalities

Me: Thomas Britz

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Please email me if you have any questions or comments!

My office: Room RC-5111, Red Centre Building; just drop by!

Check Moodle for course material, information and the Help Forum.

Join the Facebook group to easily find and share information.

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Overview

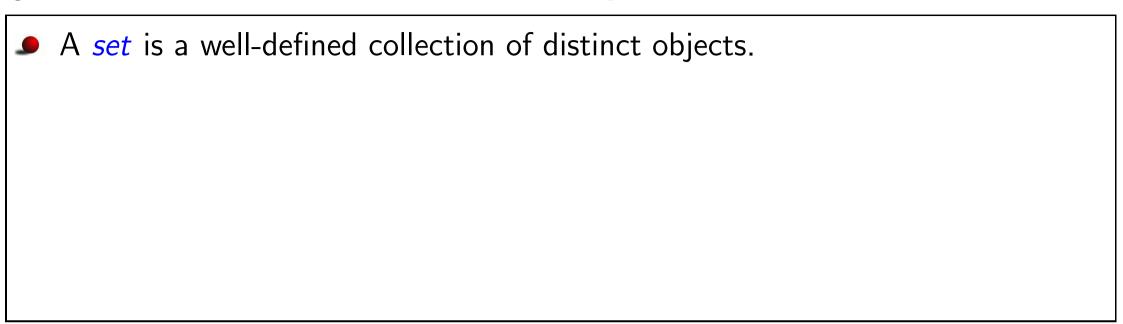
TOPIC 1: SETS, FUNCTIONS AND SEQUENCES

Topic 2: Integers, Modular Arithmetic and Relations

Topic 3: Logic and Proofs

TOPIC 4: ENUMERATION AND PROBABILITY

TOPIC 5: GRAPHS



- A set is a well-defined collection of distinct objects.
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Example. Some commonly-used sets in our number system:

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- the set of complex numbers, which includes all real numbers as well as numbers like $\sqrt{-1}$.

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|A| = 2.

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Exercise. What is the difference between the sets \emptyset , $\{\emptyset\}$, and $\{\emptyset, \{\emptyset\}\}$? \emptyset is the empty set which does not contain anything.

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Example. $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$,

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Example.
$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}, \quad \{\frac{1}{2}, \pi\} \nsubseteq \mathbb{N}$$

$$\{\frac{1}{2},\pi\}\nsubseteq\mathbb{N}$$

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- ullet The power set P(S) of a set S is the set of all subsets of S.
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- The number of subsets of S is $|P(S)| = 2^{|S|}$. (Why?)

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Elements: $0, 1, \text{ and } \{0, 1\}.$

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What are the subsets of A? Find P(A) and |P(A)|.

Elements: $0, 1, \text{ and } \{0, 1\}.$

Subsets: \emptyset , $\{0\}$, $\{1\}$, $\{\{0,1\}\}$, $\{0,1\}$, $\{0,\{0,1\}\}\}$, $\{1,\{0,1\}\}\}$, $\{0,1,\{0,1\}\}\}$.

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- $4. \varnothing \subseteq P(A)$
- $5. \quad 0 \in A$

- $6. \quad 0 \subseteq A$
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