PHYS1902 Physics 1B (Advanced)
Exam Solutions – Semester 2, 2003

Question 1

(a)(i) Height supported by atmospheric pressure very large for low density liquid like water (not practical)

(1 mark)

(ii) because mercury has cohesive forces greater than adhesive forces, doesn't wet glass container.
Convex meniscus – easier to measure accurately.

(1 mark)

(b) Body floats therefore weight = buoyancy force
In water buoyancy force $F_B = \rho A h_w g$
In unknown fluid, $F_B = \rho A h g$
therefore $\rho A h g = \rho_w A h_w g$

$$\rho = \rho_w \frac{h_w}{h}$$

(1 mark)

(c) Buoyant force $F = \rho A h g$ for unknown liquid
Different value in water $F_w = \rho_w A h g$
Now need to use spring balance to weigh cylinder (mass m) in air and when submerged.

(1 mark)

Difference in weight = buoyant force

$$\Delta m g = \rho A h g$$

$$\rho = \frac{\Delta m}{A h}$$

Ah = volume of water displaced
= volume of submerged object
Thus

$$\rho = \Delta m \frac{\rho_{cylinder}}{m}$$

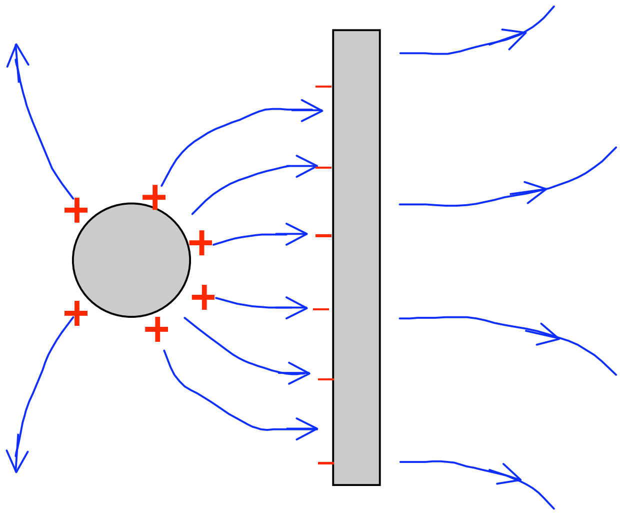
(further explanation - 1 mark)
(Total 5 marks)

Specific objectives tested:

Buoyancy – Archimedes
Pressure changing with depth
Capillary action

Question 2

(a)



(Note: Charge distribution is not required)

- Outward field from sphere
- Bunching of field on side near plate
- Plausible field to right of plate
- Always perpendicular to surfaces

(1/2 mark for each point; total 2 marks)

(b) None.
Electric potential non-zero for all points.
(Note: Some may realise that inside metal is an equipotential, but say it is zero when it is not.)
Must do work to come in from ∞ to any of the points.

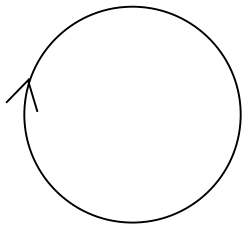
(A,C non-zero 1 mark)
(B also non-zero 1 mark)
(reason 1 mark)
(Total 5 marks)

Specific objectives tested:

General; #1
Specific; ch 22 #2,#7,#8, ch 23 #4, ch 24 #3.

Question 3

(a) Suppose there is an ***E*** line that forms a closed loop:



Then define a path to follow this loop exactly and calculate the path integral $\int \mathbf{E} \bullet d\mathbf{l}$

Since $\mathbf{E} \parallel d\mathbf{l}$ at every point along path, the result must be non-zero, which contradicts the equation.

(3 marks)

(b) Symmetry might lead us to suspect that a changing ***E*** produces a ***B*** (since the reverse is true).
While a capacitor is changing Ampere's Law implies a ***B*** around the core, but not in the region between the plates

Continuity suggests there should be a ***B*** there. Maxwell introduced is
$i_D = E_0 \dfrac{d\Phi_E}{dt}$ to reflect this: a changing ***E*** produces ***B***.

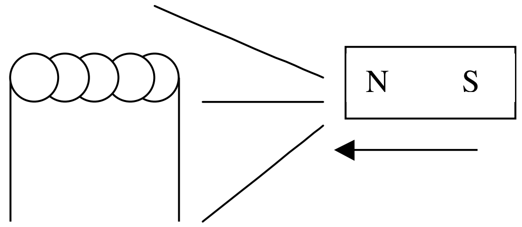
(2 marks)

(Total 5 marks)

Question 4

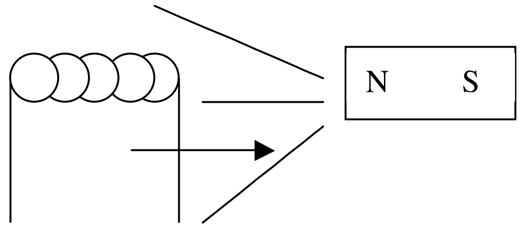
(Is this marking scheme workable??)

Solenoid frame:



Changing |**B**| induces |**E**| **in circles**, which drives current around the solenoid loops.

Magnet Frame:



Change in wire moving in |**B**|, feel a $q\mathbf{v}\times\mathbf{B}$ force that drives them in circles.

(basics idea of each – 2 marks each)
(extra detail – 1 mark

(Total 5 marks)

Question 5

(a) The energy of the photon is

$$E = h\nu = hc/\lambda = 6.63\times10^{-34}\ \text{J.s}\times 3.00\times10^{8}\ \text{m.s}^{-1}/700\times10^{-9}\ \text{m}$$
$$= 2.84\times10^{-19}\ \text{J} = 1.78\ \text{eV}$$

(2 marks)

Since $h\nu = K_{max} + \Phi$ where Φ is the work function of the metal,
$\Phi = 1.78\ \text{eV} - 0.78\ \text{eV} = 1.00\ \text{eV}$.

(1 mark)

(b) The minimum kinetic energy of the electrons is 0 eV.

(1 mark)

An electron below the surface of the metal loses energy as it tries to escape due to interactions with other particles in the metal. The work function is the *minimum* energy it can lose. Some electrons lose all their energy and leave the surface with none.

(1 mark)

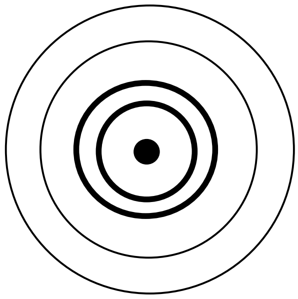
(Total 5 marks)

Module objectives:

- Calculate the energy and momentum of a photon in terms of its wavelength or frequency.
- Convert between electron volts and joules.
- Describe the photoelectric effect and its properties.
- Explain and use Einstein's theory for the photoelectric effect.

Question 6

(a)



*Concentric circles sufficient - details **not** required*

(1 mark)

(b) The electrons are diffracted like waves from the metal (the foil consists of very small randomly oriented aluminium crystals) producing circles where constructive interference occurs.

(1mark)

(c) The pattern would be the same.
It indicates the probability that individual electrons will strike particular points on the photographic plate. It is not due to interactions between electrons!
It reveals that individual electrons exhibit wave-like properties under certain conditions.
The corresponding wavelength is the de Broglie wavelength.

(1/2 mark per point. Total 2 marks)

(d) The kinetic energy of the electrons will double and hence their momentum would increase by a factor $\sqrt{2}$, the de Broglie wavelength of the electrons and hence the radii of the circles in the pattern would decrease by this factor.

(radii decrease 1/2 mark)
(extra detail 1/2 mark)
(Total 5 Marks)
(round up))

Module objectives:

- Explain and use the de Broglie relation between momentum and wavelength.
- Describe the Davisson and Germer and the Thomson diffraction experiments and their physical significance.
- Describe the two-slit interface experiment that reveals both the wave and the particle nature of the particles.

Question 7

(a)(i) To calculate volume flow rate as liquid exits, use Bernoulli;
Take surface of tank as position A, reference bottom of tank on axis of horizontal pipes.

$v_A = 0 \quad$ (given tank large)
$y_A = h$
$P_A = P_0$

At exit point, position C $v_C = ? \quad P_C = P_0 \quad y_C = 0$

$$P_A + \frac{1}{2}\rho v_A^2 + \rho g y_A = P_C + \frac{1}{2}\rho v_C^2 + \rho g y_C$$

$$P_0 + 0 + \rho gh = P_0 + \frac{1}{2}\rho v_C^2 + 0 \quad \textit{therefore} \quad v_C = \sqrt{2gh}$$

i.e $v_C = \left[(2)(9.81)(0.8)\right]^{\frac{1}{2}} = 3.96 \text{ m.s}^{-1} \approx 4.0\ m.s^{-1}$

Volume flow rate $\dfrac{dV}{dt} = v_C A_C = (3.96)(0.3\times10^{-4})$
$= 1.19\times10^{-4} \text{ m}^3.\text{s}^{-1}$
$\approx 1.2\times10^{-4} \text{ m}^3.\text{s}^{-1}$

(2 marks)

(ii) To calculate speed in larger pipe, position B use continuity equation i.e
$\dfrac{dV}{dt} = \text{constant} \quad v_B A_B = v_C A_C$

$$v_B = v_C \frac{A_C}{A_B} = (3.96)\left(\frac{0.3\times10^{-4}}{10^{-4}}\right)$$
$= 1.2\ m.s^{-1}$

(2 marks)

(iii) Height of liquid in vertical tubes: pressure increases with depth; take height of liquid = x therefore $P_B = P_0 + \rho g x$.
Use Bernoulli between positions A and B.

$P_A = P_0 \quad v_A = 0 \quad y_A = h \qquad P_B = P_0 + \rho g x \quad v_B = 1.19\ m.s^{-1} \quad y_B = 0$

$$P_0 + 0 + \rho gh = P_0 + \rho g x + \frac{1}{2}\rho v_B^2 + 0$$

$$x = h - \frac{v_B^2}{2g} = 0.8 - \frac{(1.19)^2}{(2)(9.81)} = 0.8 - 0.072$$
$= 0.73 \text{ m}$

(2 marks)

(Continued on next page)

(b) Viscous liquid with $\rho = 800\ \text{kg/m}^3, \quad \eta = 0.006, \quad \frac{dV}{dt} = \text{same}.$

(i) To calculate difference in levels, need difference in pressure between 2 tubes; use Poiseiulle's equation.

$$\Delta P = \frac{dV}{dt} \quad \frac{8\eta L}{\pi R^4}$$
$$= \frac{dV}{dt} \quad \frac{8\pi\eta L}{A^2}$$

and area of pipe $A = \pi R^2$

$$A = 10^{-4}\,\text{m}^2 \quad L = 0.2\text{m}$$
$$\frac{dV}{dt} = 1.19 \times 10^{-4}\ \text{m}^3\text{s}^{-1}\ \text{(given by a(i))}$$

Again height difference due to pressure difference $\Delta p = \rho g \Delta x$

Required to find height difference $= \Delta x = \frac{\Delta P}{\rho g} = \left(\frac{dV}{dt}\right) \quad \frac{8\pi\eta L}{\rho g A^2}$

$$\Delta x = \frac{(1.19 \times 10^{-4})(8\pi)(0.006)(0.2)}{(800)(9.81)(10^{-4})^2}$$
$$= 0.046\ \text{m}$$

(2 marks)

(ii) Viscous liquid – energy loss through internal friction; heating of liquid. Bernoulli's principle still applicable, but at any point $\sum$ terms = different value.

(2 marks)

(Total 10 marks)

Question 8

Big Error! Part (b) of question was omitted. Mark this part out of 5 only!!

Observationally: B proportional to q
B proportional to $1/r^2$
B azimuthal around velocity vector via right hand rule

Summarised as

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{|q|\mathbf{v}\times\mathbf{B}}{r^2}$$ (almost the Biot-Savart Law)

Thus:

At point A $\left|\underset{\sim}{B}\right| = \frac{\mu_0}{4\pi}\frac{qv\sin 45^\circ}{r^2}$ out of page

(formula – 1 mark)
(direction – 1 mark)

At point B $\left|\underset{\sim}{B}\right| = 0$

(1 mark)

(reasoning – 2 marks)

(Total 5 marks)

Question 9

(a) Sketch of field: - zero inside inner cylinder and outside outer cylinder
- radially outwards between the cylinders

(1 mark)

Use symmetry and Gauss' Law. Use Cylindrical Gaussian surface
$\Phi = 2\pi r L E(r)$

(method-1 mark)

Inside, $E = 0$ since $q_{enc} = 0$. Same for outside

Between, $q_{enc} = L\lambda \quad \Rightarrow E = \frac{1}{2\pi\varepsilon_0} \lambda/r \quad a < r < b$ ***(1 mark)***

$$\left.\begin{array}{ll} 0 & r < a \\ 0 & r > b \end{array}\right\}$$ ***(1 mark)***

(b) $V = \int_a^b E dr \quad$ since $E \| dl$

$$= \frac{\lambda}{2\pi\varepsilon_0}\int_a^b \frac{dr}{r}$$

$$= \frac{\lambda}{2\pi\varepsilon_0}\ln\left(b/a\right)$$ ***(2 marks)***

(c) $C = q/V = \frac{\lambda L}{V} \quad \Rightarrow \quad \frac{C}{L} = \frac{2\pi\varepsilon_0}{\ln\left(b/a\right)}$ ***(2 marks)***

(d) $U = \frac{q^2}{2C} = \frac{1}{2} \frac{\ln\left(b/a\right)}{2\pi\varepsilon_0 L} \lambda^2 L^2 = \frac{\ln\left(b/a\right)}{4\pi\varepsilon_0}\lambda^2 L$ ***(2 marks)***

(Total 10 marks)

Module objectives:

Ch 23 point 3
Ch 24 point 4
Ch 25 points 2,4.

Question 10

(a) Internal current $I_0 = 0$ .Current in an inductor cannot change instantaneously.

(2 marks)

(b)

$$V_C + V_L = 0$$

Hence

$$\frac{q}{C} + L\frac{di}{dt} = 0$$

(1 mark)

(c)

Now $i = \frac{dq}{dt}$ so $\frac{di}{dt} = \frac{d^2q}{dt^2}$

Hence $\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0$

(2 marks)

(d)

Substitute $q = Q\cos\omega t$ into the previous DE

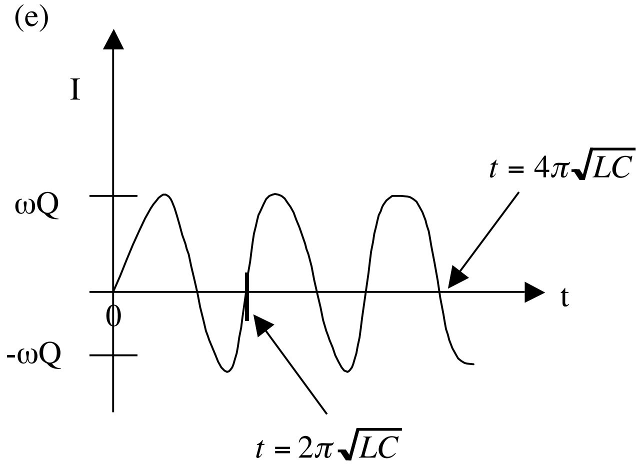
$$-Q\omega^2\cos\omega t + \frac{1}{LC}Q\cos\omega t = 0$$

Hence

$$\omega^2 = \frac{1}{LC} \quad \text{or} \quad \omega = \sqrt{\frac{1}{LC}}$$

(2 marks)

(e)



(2 marks)

Note $\omega = \frac{1}{\sqrt{LC}}$

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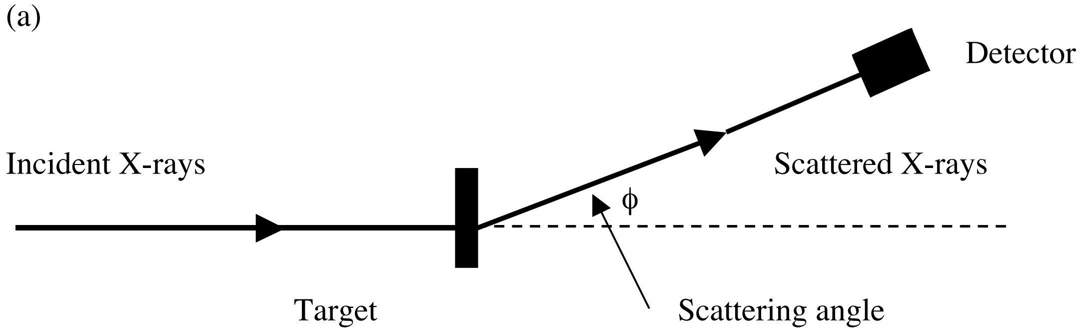
(f) Energy is exchanged between electric field in capacitor and magnetic field of the inductor.

(1 mark)

(Total 10 marks)

Question 11

(a)



(2 marks)

(b) Electromagnetic radiation sometimes behaves like a beam of particles called photons, each with energy $h\nu$ and momentum $h\nu/c$.

(1 mark)

(c) X-ray photons with wavelength λ are scattered through an angle ϕ by a stationary electron and become photons with a longer wavelength λ' .

The increase in wavelength depends only on the scattering angle and is independent of λ .

(2 marks)

(d) The collision between the X-ray photon and the electron is elastic, i.e. both total energy and total momentum are conserved.

(1 mark)

(e) The Compton shift cannot be observed with visible light.

This is because the Compton shift is independent of the wavelength of the incident photons. Visible photons have long wavelengths and the spread in wavelengths in a monochromatic beam of visible light is much bigger than the shift.

(2 marks)

(Continued on next page)

(f) The other wavelength is the same as the incident wavelength. ***(1 mark)***

The radiation is due to Compton scattering from the tightly bound electrons whose effective mass is that of the whole atom. The Compton shift is too small to be detected.

(1 mark)

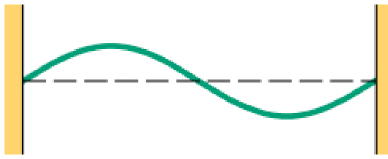
(Total 10 marks)

Module objectives:

- Explain Einstein's hypothesis that light is composed of bundles of energy (light quanta or photons).
- Calculate the energy and momentum of a photon in terms of its wavelength or frequency.
- Describe the Compton experiment.
- Discuss the assumptions in Compton's theory (no derivation required) and the implications of the theory.

Question 12

a.

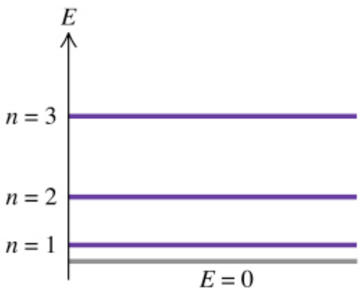


(1 mark)

b. The electron is most likely to be found at a distance 0.25 nm on either side of the centre of the well. The probability of finding the electron at any point is proportional to the square modulus of the wavefunction.

(1 mark)

c.



(1 mark)

d. The energy of the n-th state is

$$E_n = \left(\frac{h^2}{8\,m\,L^2}\right) n^2 = \left(\frac{\left(6.63\times10^{-34}\ \mathrm{J.s}\right)^2}{8\times 9.11\times10^{-31}\ \mathrm{kg}\times\left(1.00\times10^{-9}\ \mathrm{m}\right)^2}\right) n^2$$

$$= 6.03\times10^{-20}\ n^2\ \mathrm{J} = 0.377\ n^2\ \mathrm{eV}\ .$$

The energies of the three lowest states are therefore 0.377 eV, 1.51 eV and 3.39 eV.

(2 marks)

e. The energy difference between the $n = 3$ and the $n = 1$ states is

$$6.03\times10^{-20}\left(3^2 - 1^2\right)\mathrm{J} = 4.82\times10^{-19}\ \mathrm{J}\ .$$

A photon with this energy has wavelength $\lambda = h\,c / E = 4.12\times10^{-7}\ \mathrm{m}$.

(2 marks)

f. The new photons will have energy $2.41\times10^{-19}\ \mathrm{J} = 1.51\ \mathrm{eV}$. This does not correspond to a transition energy from the ground state to a higher energy state and so the photons will not be absorbed.

(1 mark)

g. The ground state energy would increase.

(1 mark)

The uncertainty in the position of the electron decreases as the well is narrowed. The Uncertainty Principle implies that the uncertainty in the momentum of the electron must increase. The average momentum and the energy of the electron also increase.

(1 mark)

Specific objectives tested

- Define the wavefunction and the probability density function for a particle.
- Describe how confinement of a matter wave leads to discrete (i.e. quantised) energy levels.
- Explain and use the equation for the energy levels of a particle in an infinite one-dimensional potential well.
- Describe qualitatively the wavefunctions and probability density functions for a particle in the energy levels of an infinite one-dimensional potential well.
- Explain how emission and absorption line spectra suggest the existence of energy levels in atoms.
  Discuss the relationship between wavepackets and the Uncertainty Principle.