THE UNIVERSITY OF SYDNEY

MATH1905 STATISTICS ADVANCED

Semester 2 **Tutorial Week 11** 2012

1. The performance in kilometres/litre (km/l) of a particular model of car tested by machine is 7 km/l and the distribution appears normal. Company engineers have redesigned the carburettor in an effort to improve the performance and have equipped a sample of 36 cars with this new carburettor. When tested the average performance of the sample was 7.6 km/l, with a sample standard deviation of 1.5 km/l. The sample has no 'outliers'. What are the null (H_0) and alternative (H_1) hypotheses to be tested using the sample?

(a) $H_0 : \mu = 7,\ H_1 : \mu \neq 7,$

(b) $H_0 : \mu < 7,\ H_1 : \mu = 7,$

(c) $H_0 : \mu > 7,\ H_1 : \mu \leq 7,$

(d) $H_0 : \mu = 7,\ H_1 : \mu > 7.$

2. In the preceding problem the appropriate P-value of the test is

(a) 0.6554,

(b) 0.0082,

(c) between 0.01 and 0.025,

(d) none of the these.

3. Suppose 10 pieces of type sandpaper were subjected to treatment by a machine which measures abrasive wear. The sample mean abrasive wear was calculated to be 27.4 with a sample standard deviation of 2.3. Construct a 95% confidence interval for the mean abrasive wear of type A sandpaper. Would the 90% confidence interval be longer or shorter?

4. A sample of 80 observations from a population **known** to have a standard deviation of 12, gave a sample average of $\overline{x} = 6.55$.

(a) Provide a 99% confidence interval for the population mean.

(b) What sample size would be necessary to produce a 99% confidence interval of length 4?

5. Of 30 donors arriving at the Blood Bank in one hour, the number found to have Type O blood was 13. Construct an approximate 95% confidence interval for the proportion of donors who have Type O blood. What assumptions are necessary for the above calculation?

6. A study is made to estimate the proportion p of people in a large city who favour having their water fluoridated. A pilot sample of size 50 residents contained exactly 32 in favour of the proposal.

(a) Provide an approximate 95% confidence interval for p based on the pilot survey.

(b) It is desired to estimate the true proportion to within 0.01 of the true value. Estimate n, the sample size needed for a 95% confidence interval of this precision. (Use a method which will definitely not underestimate the sample size needed.)

(c) A sample of 10,000 contained exactly 5,140 in favour. Check that the conservative 95% confidence interval is as short as required.

1. Below are the weights (in grams) of malignant tumours removed from the abdomen of 21 patients. Calculate the mean and sample standard deviation of the weights and produce a 95% confidence interval for the average weight of such tumours assuming that the tumour weight follows a normal distribution.

1050	650	790	875	790	1460	1215
870	780	900	820	980	695	860
1105	850	910	800	1080	720	830

2. Let $\overline{X}$ denote the mean of a random sample of size n from $\mathcal{N}(\mu, 10^2)$.

(a) Find n such that the probability is approximately 0.954 that the random interval $(\overline{X} - \frac{1}{2}, \overline{X} + \frac{1}{2})$ covers μ.

(b) How would your answer in (a) change when the variance σ^2 was unknown but instead the sample variance $s^2 = 100$ was given, i.e. do you need at least n or at most n observations?