PHYS 1901 – Physics 1A (Advanced)
Mechanics module

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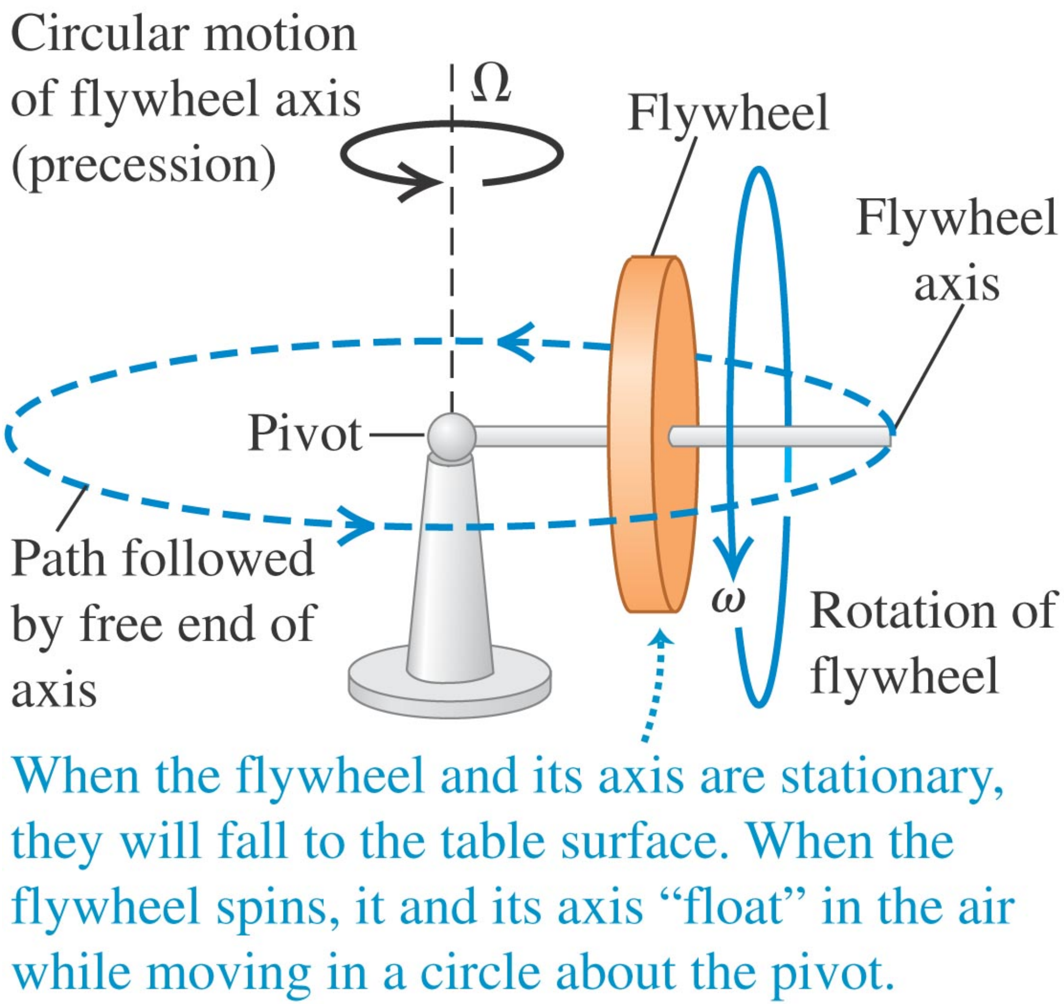
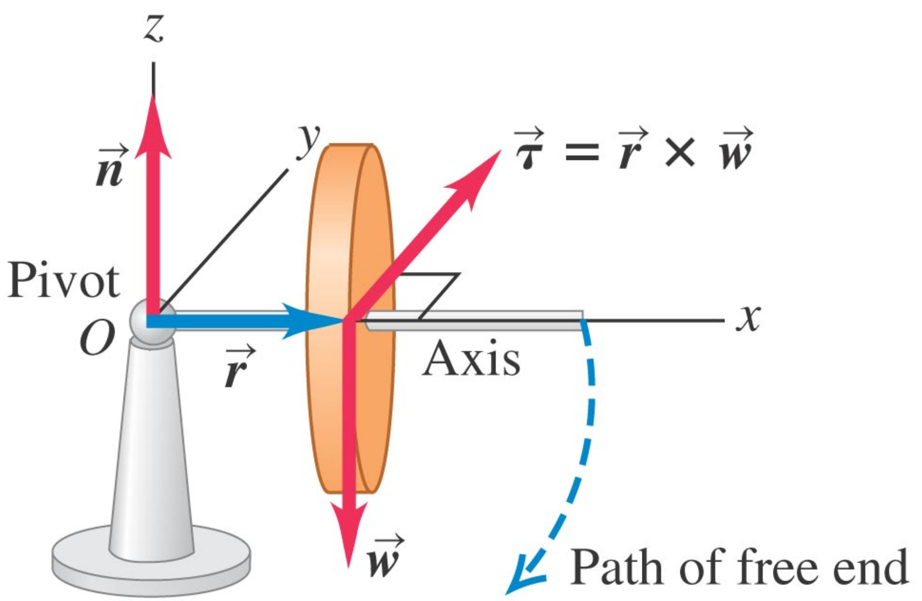
Gyroscope



Circular motion of flywheel axis (precession)
Ω
Flywheel
Flywheel axis
Pivot
Path followed by free end of axis
ω
Rotation of flywheel
When the flywheel and its axis are stationary, they will fall to the table surface. When the flywheel spins, it and its axis "float" in the air while moving in a circle about the pivot.

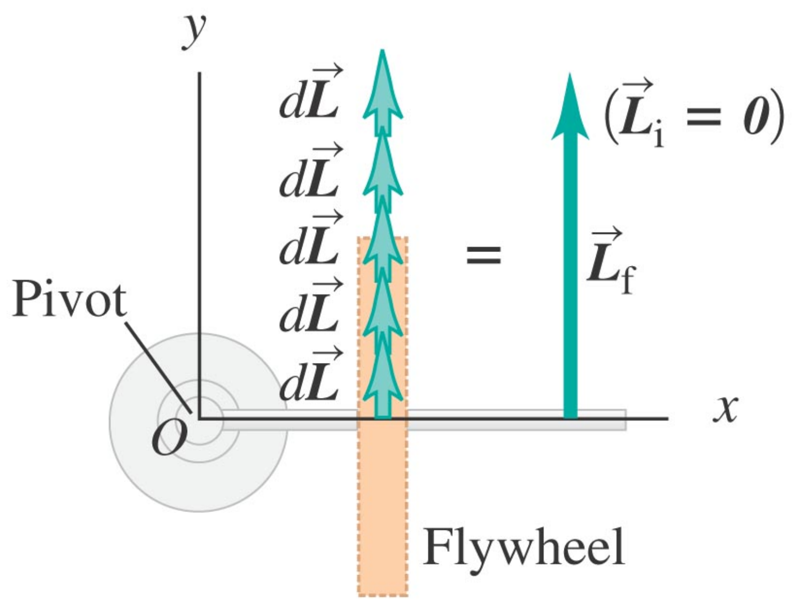
Gyroscope: not rotating

(a) Nonrotating flywheel falls



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

(b) View from above as flywheel falls

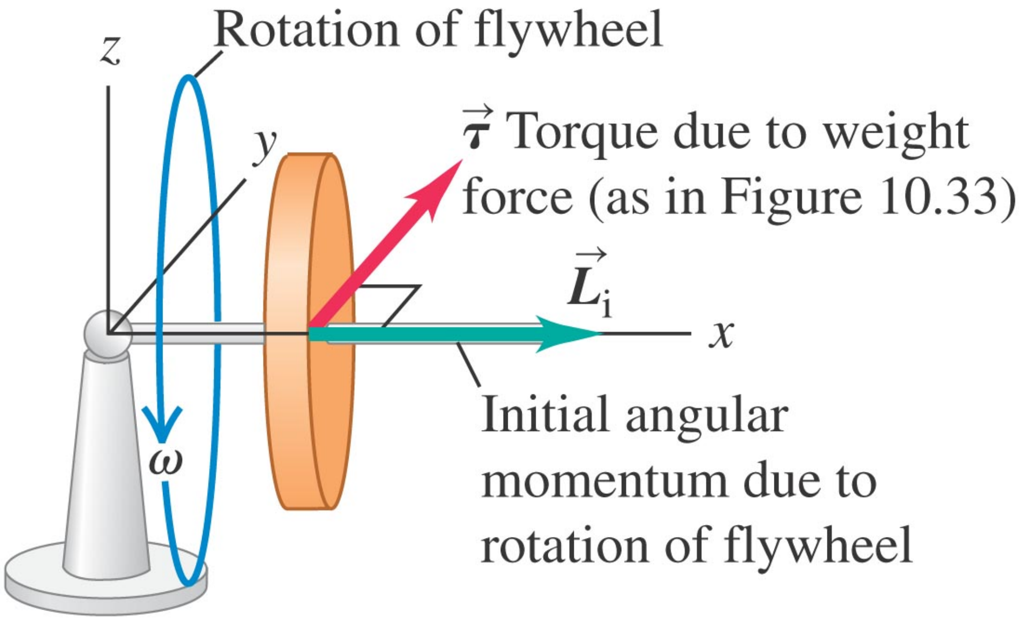


In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec{L}$. The *direction* of $\vec{L}$ stays constant.

Gyroscope: rotating

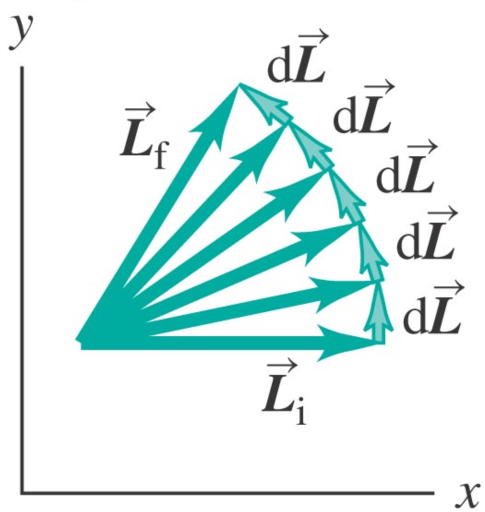
(a) Rotating flywheel

When the flywheel is rotating, the system starts with an angular momentum $\vec{L}_i$ parallel to the flywheel's axis of rotation.



(b) View from above

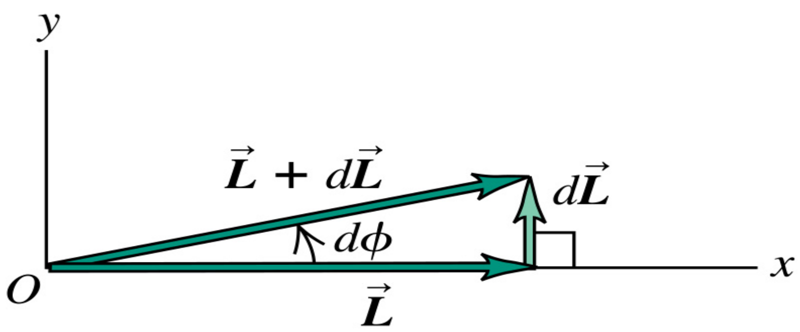
Now the effect of the torque is to cause the angular momentum to precess around the pivot. The gyroscope circles around its pivot without falling.



Precession

Over a small time interval, there is a change in angular momentum given by

As dL is perpendicular to L, only the direction of L changes, not magnitude.



Gravitation

Chapter

12

(Note: we are not covering Chapter 11 in this module)

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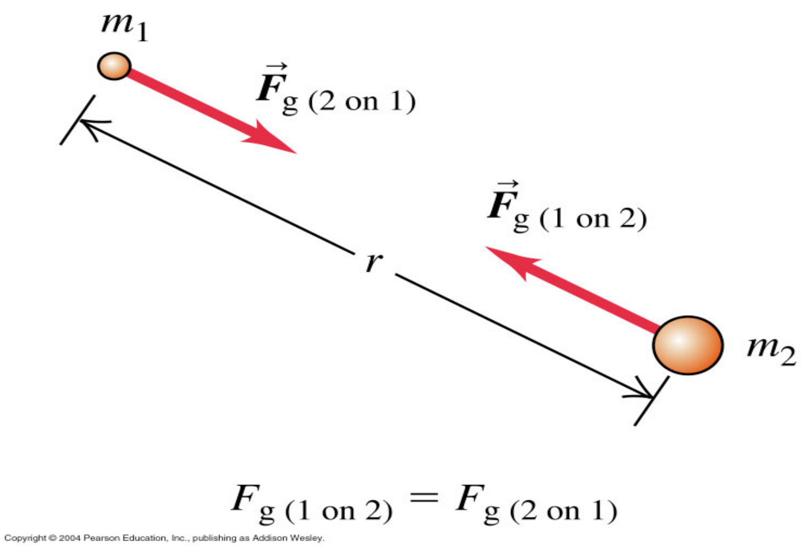
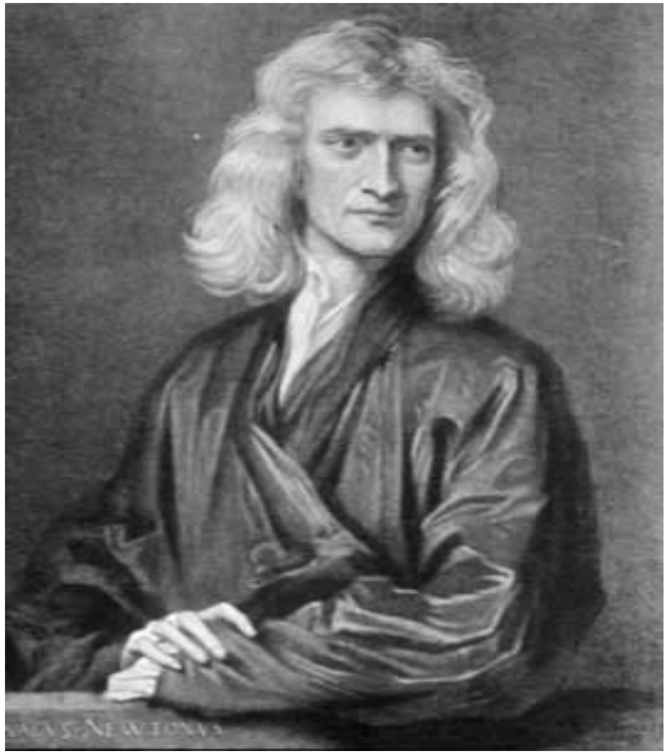
Gravitation





$$F_G = G\frac{m_1 m_2}{r^2}$$

for point masses

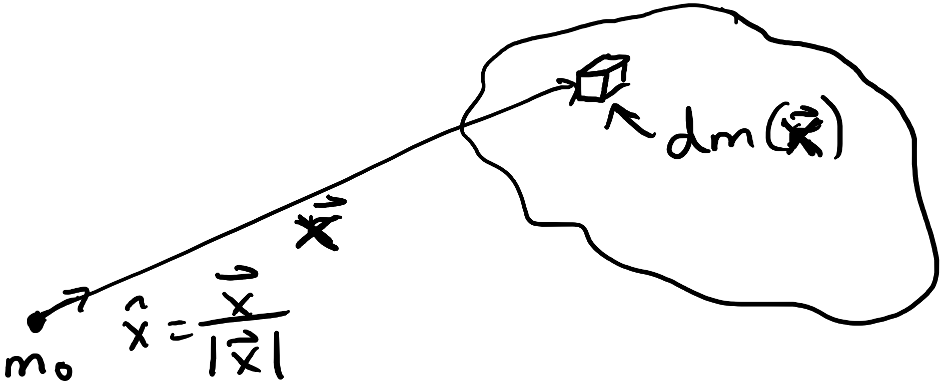
$$G = 6.67 \times 10^{-11} N\ m^2\ kg^{-2}$$

Gravitation

Newton realised that using his gravitational formula was actually pretty tricky.

If we have a randomly shaped object, what is the force it produces on a test (small) mass locate near by?

$$\vec{F}_G = \int_{\text{entire object}} \frac{G m_0 \, dm(\vec{x})}{|\vec{x}|^2} \hat{x}$$



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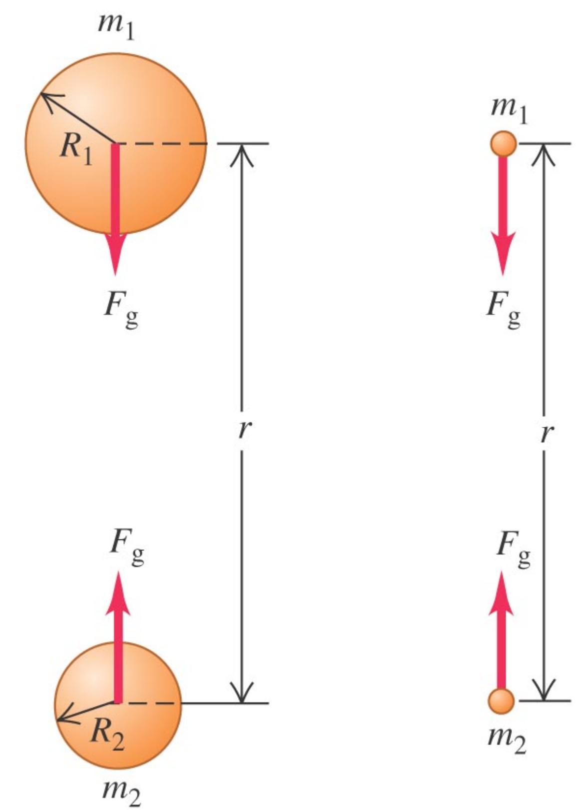
Gravitation

(a) The gravitational force between two spherically symmetric masses m_1 and m_2 ...

(b) ... is the same as if we concentrated all the mass of each sphere at the sphere's center.



- Newton discovered that a spherical body has special properties. If you are outside the body, the gravitational force was the same as if all the mass were concentrated at the centre of the body
- Newton realised he could treat planets as basically being points!
- What about **inside** a spherical shell?

$$\vec{F}_{inside} = 0$$

Weight

At the surface of the Earth

$$w = \frac{G M_E m}{R_E^2} = mg \Rightarrow g = \frac{G M_E}{R_E^2}$$

We know R_E, g and G, so can calculate M_E

$$M_E = 5.98 \times 10^{24} \text{ kg}$$

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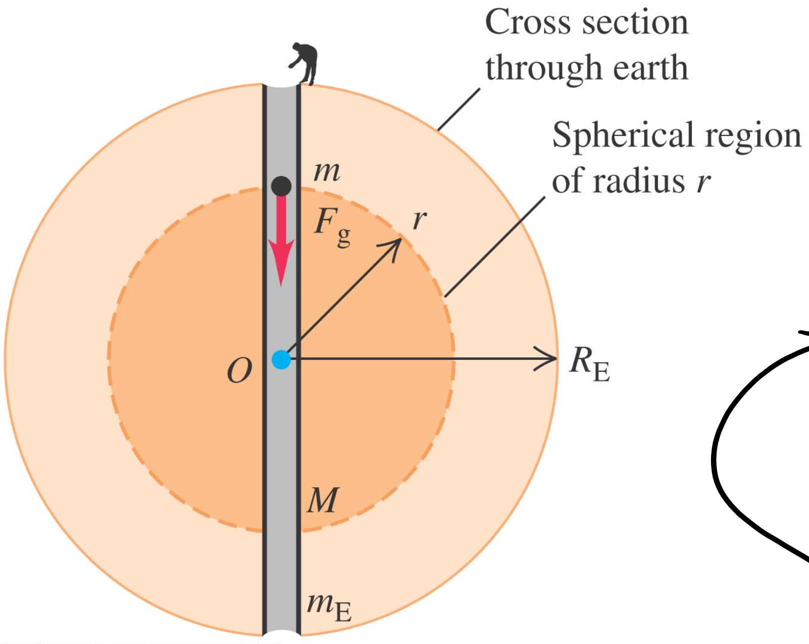
Falling through the earth



Assume a mass is dropped down a tunnel in a uniform density Earth. What is its equation of motion? How long does it take to return?

$M_E(r)$ is the mass within a sphere of radius $r < R_E$

$$\frac{M_E}{V_E} = \frac{M_E(r)}{V_E(r)} \Rightarrow \frac{M_E(r)}{M_E} = \frac{r^3}{R_E^3}$$

$$F_G(r) = \frac{G M_E(r)\, m}{r^2}$$

$$= \frac{G M_E (r/R_E)^3 m}{r^2}$$

$$F_G(r) = \frac{G M_E m}{R_E^3} \cdot r$$

$$ma = \frac{G M_E m}{R_E^3} \cdot r$$

Hooke's law with spring constant $k = \frac{G M_E m}{R_E^3}$