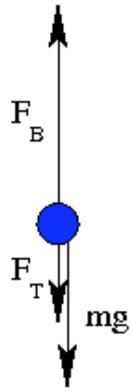
PHYS1902 Advanced Exam Solutions
Semester 2, 2007

SECTION A

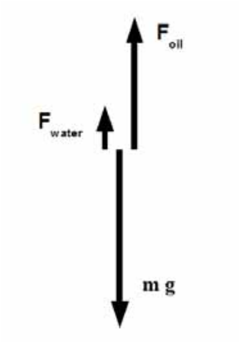
Question 1

a) Free-body Diagram A for block:



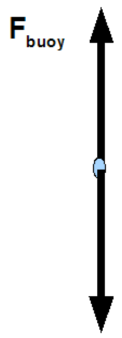
Where, F_B and F_T are the forces on the bottom and top faces of the block due to the pressure: $F_B = p_B A$ and $F_T = p_T A$.

Alternate Diagram B:



where F_{oil} is the buoyancy force exerted by the part of the block that is immersed in oil and F_{water} is the buoyancy force exerted by the part of the block immersed in water. See below for an estimate of the relative magnitudes of the buoyancy forces.

Alternate Diagram C:



Where F_{buoy} is the buoyancy force exerted by the block (oil and water combined).

(1 mark for any of the 3 diagrams; 1/2 mark for forces separated as in diagrams A and B; 1/2 mark for sensible relative magnitudes of the forces separated as in Diagrams A & B). Diagram C hence scores only 1 mark.

b) Buoyancy force is always upwards, and arises because of the difference in pressure between the upper and lower faces of the block. The pressure increases with depth in the liquid, so the resultant force is always upwards.

(1/2 mark for direction; 1/2 mark for reason)

c) The block must be denser than the oil, because it has sunk below the bottom of the oil. Since only a small fraction of the block is submerged in the water, it must be substantially less dense than water. The density of the block in the diagram can be estimated as 840 kg m^{-3} but this is not needed in the answer.

(1/2 mark for "denser than oil and less dense than water");
(1/2 mark for "substantially less dense than water" or "a little more dense than oil")

d) The position of the block would not change. The buoyancy force depends on the weight of fluid displaced; if g changes this weight changes, but so does the weight of the block, so there is no relative change in position. Another way to look at this is that as g increases the pressure at top and bottom both increase by the same amount, so the position of the block does not change.

(1 mark)

Mathematical Treatment of Problem

For a block of cross sectional area A, density ρ, height H, and vertical distance x immersed in the water we can calculate: the buoyancy force from the section of the block immersed in water; and the buoyancy force from the section of the block immersed in oil. The sum of these two (vertical upwards) forces is equal to the weight force (vertically downwards).

$$\rho_{water}\, A\, x\, g \quad + \quad \rho_{oil}\, A\left(H - x\right) g \quad = \quad \rho\, A\, H\, g$$

$$\Rightarrow \rho_{water}\, x \quad + \quad \rho_{oil}\, \left(H - x\right) \quad = \quad \rho\, H$$

We can make an estimate of the relative size of these forces by putting $x = 0.2\,H$, roughly estimated from the diagram.

$$\rho \quad = \quad \rho_{water}\,\frac{x}{H} + \rho_{oil}\,\frac{H - x}{H}$$

$$1000\,(0.2) \quad + \quad 800\,(0.8) \quad = \quad \rho$$

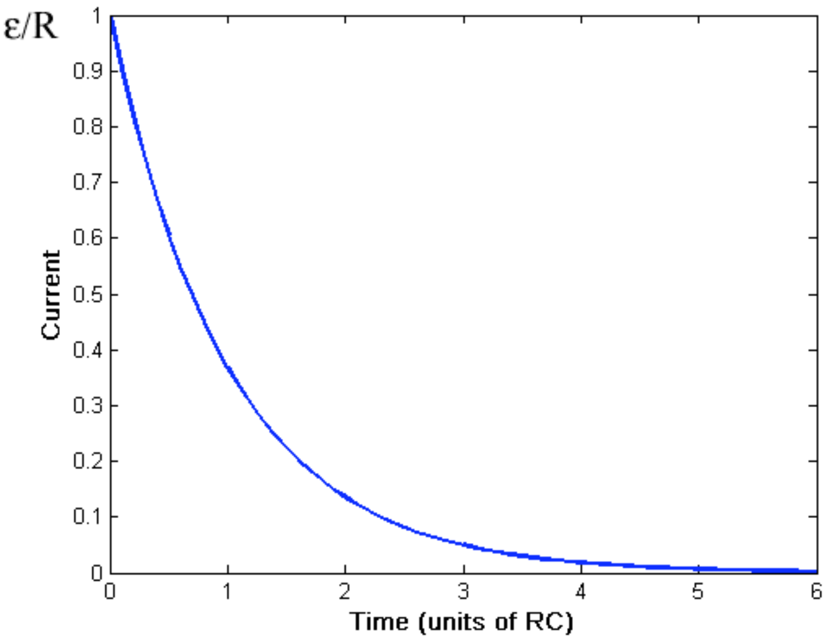
$$200 \quad + \quad 640 \quad = \quad 840$$

$$0.24 \quad : \quad 0.76 \quad : \quad 1.00$$

Question 2

a)

$$I = \frac{\varepsilon}{R} e^{-t/(RC)}$$



(2 marks, deduct ½ mark for missing labels, deduce ½ mark for missing approx scales)

b) Maximum charge is given by $Q_{\max} = C\varepsilon$.
Charge as a function of time is given by:

$$Q = Q_{\max} [1 - e^{-t/(RC)}].$$

$$Q = \frac{Q_{\max}}{2}$$

$$\Rightarrow 1 - e^{-t/(RC)} = 0.5$$

$$\Rightarrow e^{-t/(RC)} = 0.5$$

$$\frac{t}{RC} = \log_e(2)$$

$$t = RC \log_e(2) = 0.69RC$$

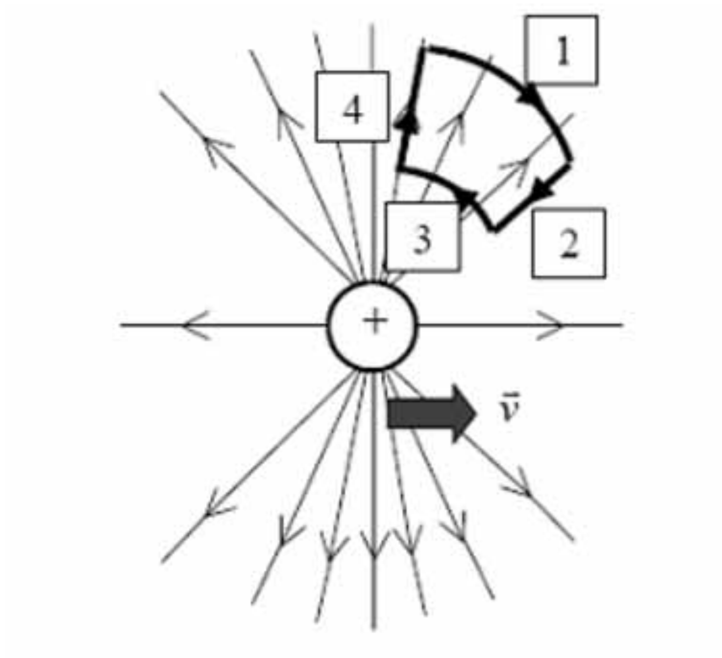
(1 mark for approach; 1 mark for the answer)

c)
Reduce R and you will decrease the time taken to charge the capacitor.

(1 mark = -1/2 if say reduce C and or -1/2 if say change ε)

(Total 5 marks)

Question 3



a)

Take $\oint \boldsymbol{E} \cdot d\boldsymbol{l} = 0$ around a path like that shown in the above diagram. Sides 1 and 3 contribute zero because $\boldsymbol{E}$ is perpendicular to $d\boldsymbol{l}$. Sides 2 and 4 have opposite sign but <u>not</u> the same magnitude. Hence $\oint \boldsymbol{E} \cdot d\boldsymbol{l} \neq 0$.

(1 mark for suitable path; 1 mark for evaluating E along the sections of the path; 1 mark for summation and conclusion)

b)

(i) $\oint \boldsymbol{B} \cdot d\boldsymbol{A} = 0$. This means that the flux of B through any imaginary closed surface is zero.

(1 mark = must say 'closed' surface -1/2 if not)

(ii) Lines of B can never start or end at a point so monopoles can not exist if this equation is correct. Conversely if a magnetic monopole existed then it would be possible to set up a surface enclosing the monopole for which

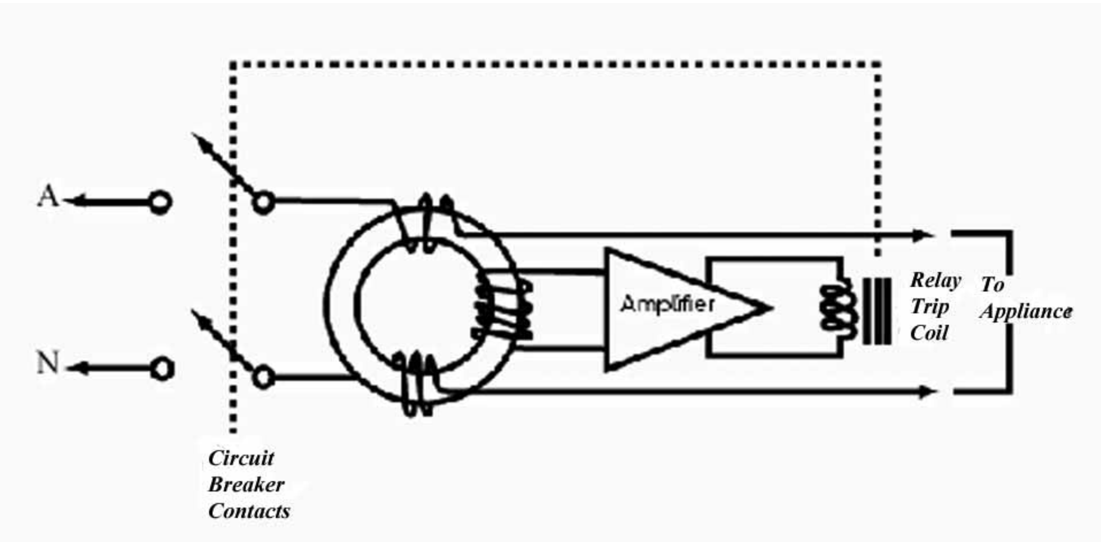
$\oint \boldsymbol{B} \cdot d\boldsymbol{A} \neq 0$.

(1 mark)

(Total 5 marks)

Question 4 (Common Question TEC Q.4=ADV Q.4)

Preamble - Under normal conditions of operation the current in the active and neutral lines should be the same. In an electrocution a person may come into contact with the active lead so that the current in the neutral lead will be less than the current in the active lead - a situation which is referred to as earth leakage. The diagram shows the earth leakage circuit breaker, which detects this imbalance and immediately cuts of the electrical supply.



The active and neutral leads are wound in opposite senses on a ring of ferromagnetic material (usually referred to as a 'core'). The function of the ferromagnetic core is to guide the magnetic flux from each winding through the third winding.

(2 marks)

When the currents in the active and neutral leads are equal the fluxes due to each cancel and there is no net flux through the third winding. If however there is leakage to earth from the active lead the fluxes will not cancel and there will be a net flux through the third coil.

(2 marks)

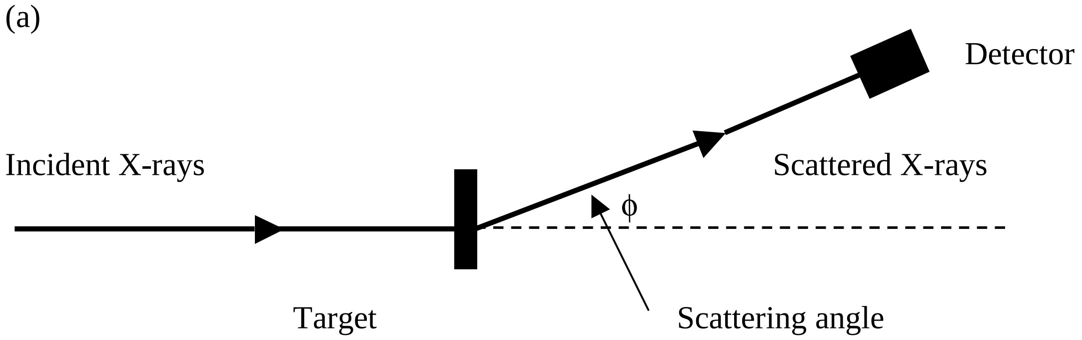
As the flux will be alternating (at a frequency of 50 Hz) it will induce an emf in the third coil. With the aid of further electronics this signal can be used to trigger a circuit breaker which disconnects the mains supply.

(1 mark)

(total 5 marks)

Question 5 (Common Question TEC Q.5 = ADV Q.5)

(a)



(2 marks)

(b) Electromagnetic radiation sometimes behaves like a beam of particles called photons, each with energy $h\nu$ and momentum $h\nu / c$.

(1 mark)

(c) The other wavelength is very close to the incident wavelength to the point of it not being distinguishable from the incident wavelength.

(1 mark)

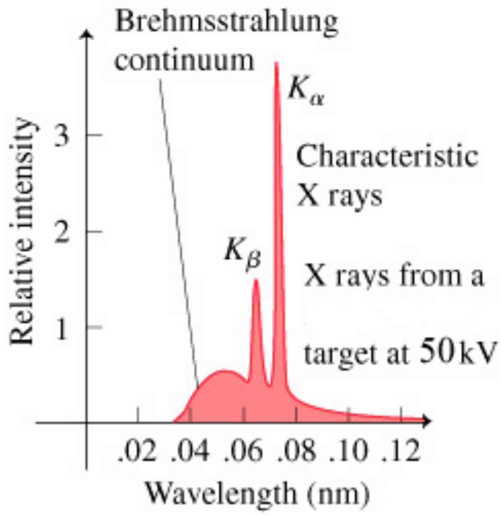
The radiation is due to Compton scattering from the tightly bound electrons whose effective mass is that of the whole atom. The Compton shift is too small to be detected.

(1 mark)

(total 5 marks)

Question 6

a)



The wide, continuous, range of X-rays produced when high-energy electrons are suddenly brought to rest by colliding with a target are called *bremsstrahlung* X rays. These are emitted as the electrons rapidly slow down when they hit the target. **(1 mark)**

The target material affects the characteristic X-rays emitted, not the bremsstrahlung X rays. The characteristic X-rays are emitted when the incoming electrons "dislodge" inner shell electrons in the target material. Electrons from higher energy levels in the target then "fall" into these lower energy level vacancies, emitting photons of specific wavelengths, typically of X-ray energy. **(1 mark)**

b)

By energy conservation, the shortest wavelength X rays are formed when *all* of the electrical energy carried by the electrons is transferred to an X ray photon. The more energy the electrons are given by the accelerating voltage, the shorter the minimum wavelength of the X rays produced.

The incident electron energy is: eV and this is equal to the energy of the outgoing photon

$hf = \frac{hc}{\lambda}$. Hence

$$\frac{hc}{\lambda} = eV \Rightarrow \lambda = \frac{hc}{eV}$$

$$= \frac{(6.63\times10^{-34})(3.00\times10^{8})}{(1.60\times10^{-19})(50\times10^{3})}$$

(1 mark for approach; 1 mark for correct answer)

$$= 2.49\times10^{-11}\text{ m}$$

0.025 nm

c)
The X-rays are emitted when the incoming electrons dislodge *K*-shell, or innermost shell, electrons in the target. The K_α line is formed when a K-shell vacancy is filled by an $n = 2$ shell electron, the K_β when it is filled with an $n = 3$ shell electron, etc.

As the atomic number of the target increases, the binding energy of the inner electrons increases as $(Z-1)^2$ because more protons are present in the nucleus. With a larger binding energy, it takes more energy to remove these electrons. Consequently, more energy is released when these electron vacancies are filled. If a photon carries away this energy (hf) then $f \sim (Z-1)^2$

(1 mark for plausible explanation)

(Total 5 marks)

Section B

Question 7

a)
Apply Bernoulli's equation between the fluid at the surface of the container (point A) and the fluid leaving the siphon (point B). These two points are on the same streamline, so Bernoulli's equation applies.

$$p_A + \frac{1}{2}\rho v_A{}^2 + \rho g\, y_A \quad = \quad p_B + \frac{1}{2}\rho v_B{}^2 + \rho g\, y_B$$

Both point A and B are at atmospheric pressure and so $p_A = p_B$. Taking $y_B = 0$ at the end of the siphon and assuming $v_A = 0$, we have

$$p_A + \; 0 + \rho g\, d \quad = \quad p_A + \frac{1}{2}\rho v_B{}^2 + 0$$

$$\Rightarrow \frac{1}{2}\rho v_B{}^2 = \rho g\, d$$

$$\Rightarrow v_B = \sqrt{2\, g\, d}$$

which is is the velocity of the fluid as it leaves the siphon.

(1 mark for correct approach (Bernoulli's equation); 1 mark for correct assumptions in solving equation; 1 mark for correct solution.)

b)
To find the pressure at the top of the siphon, apply Bernoulli's equation again between the top of the tube (point X) and the exit (point B).

$$p_X + \frac{1}{2}\rho v_X{}^2 + \rho g\, y_X \quad = \quad p_B + \frac{1}{2}\rho v_B{}^2 + \rho g\, y_B$$

Since the tube has constant cross-section, the flow-rate through the tube is constant velocity at X is equal to the velocity at B ($v_B = v_X$). The pressure at the exit point B is just atmospheric pressure p_0. Also, again take $y_B = 0$. This means that $y_X = d + h$. Hence

$$p_X + \frac{1}{2}\rho v_B{}^2 + \rho g\,(d+h) \quad = \quad p_0 + \frac{1}{2}\rho v_B{}^2 + 0$$

$$\Rightarrow p_X = p_0 - \; \rho g\,(d+h)$$

(1 mark for correct assumptions in solving Bernoulli's equation; 1 mark for correct solution.)

(c) The greatest height h occurs when $p_X = 0$ so

$$0 = p_0 - \; \rho g\,(d+h)$$

$$\Rightarrow p_0 = \rho g\,(d+h)$$

$$\Rightarrow h = \frac{p_0}{\rho g} - d$$

This has a maximum when $d = 0$ in which case

$$h = \frac{p_0}{\rho g}$$

(1 mark for approach; 1 mark for correct answer). It is not clear from the question that the value of d can also be varied so mark as correct even if the d=0 step is not taken.

(d)

$$h = \frac{p_0}{\rho\, g} = \frac{1.01\times10^5}{(13.6\times10^3)(9.8)} = 0.76 \text{ m}$$

(1 mark) Mark as correct if d is left in equation i.e. $0.76 - d$ **m**

e) On Venus,

$$h = \frac{p_0}{\rho\, g} = \frac{(9.3\times10^6)}{(13.6\times10^3)(8.9)} = 76.8 \text{ m}$$

(1 mark) Mark as correct if d is left in equation

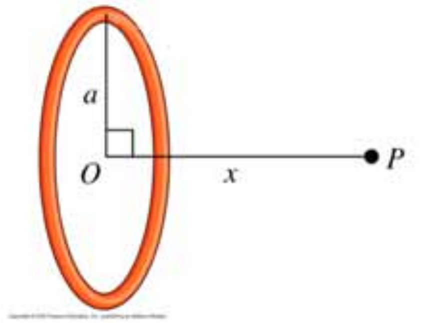
On Mars

$$h = \frac{p_0}{\rho\, g} = \frac{(0.8\times10^3)}{(13.6\times10^3)(3.7)} = 0.016 \text{ m}$$

(1 mark) Mark as correct if d is left in equation

Question 8 (Common Question TEC Q.8=ADV Q.8)

a)



Split the ring up into charge elements dq. The potential at a distance r from a charge element dq is then

$$V' = \frac{1}{4\pi\varepsilon_o}\frac{dq}{r}$$

(1 mark)

From the diagram, the distance of the point P on the axis of the ring from a charge element dq of the ring is given by

$$r = \sqrt{x^2 + a^2}$$

(1 mark)

Using this expression, the net potential is then given by summing all the charge element dq to yield the total charge q

(using or stating this approach - 1 mark)

i.e.

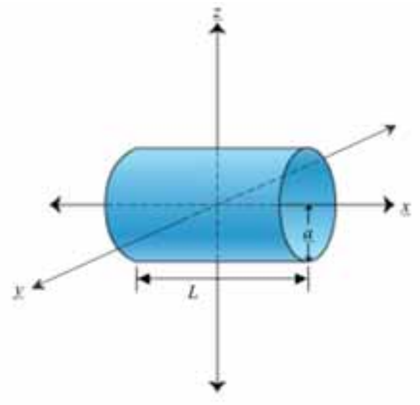
$$\begin{aligned} V &= \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{r} \\ &= \frac{1}{4\pi\varepsilon_o}\int\frac{dq}{\sqrt{x^2 + a^2}} \\ &= \frac{1}{4\pi\varepsilon_o}\frac{1}{\sqrt{x^2 + a^2}}\int dq \end{aligned}$$

Hence the result,

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{\sqrt{x^2 + a^2}}$$

(valid derivation - 2 marks)

b)



The cylinder can be divided into a series of rings of thickness *dx*, each with a potential given by the result in (a). The charge on each ring is given by

$$dq = Q\frac{dx}{L}$$

(1 mark)

So the potential due to one ring is then

$$V' = \frac{Q\,dx}{4\pi\varepsilon_o L}\frac{1}{\sqrt{x^2+a^2}}$$

(1 mark)

Using this expression, the net potential is then given by summing all the rings of width *dx* to yield the total cylinder

(using or stating this approach - 1 mark)

i.e.

$$V = \frac{Q}{4\pi\varepsilon_o L}\int\frac{dx}{\sqrt{x^2+a^2}}$$

Using the integral given $\int\frac{da}{\sqrt{a^2+b^2}} = \ln\left(\frac{a}{b}+\sqrt{1+\frac{a^2}{b^2}}\right)$ and setting the limits of integration as 0 to L/2 and then doubling the result yields

$$V = \frac{Q}{4\pi\varepsilon_o L}\left[\ln\left(\frac{x}{a}+\sqrt{1+\frac{x^2}{a^2}}\right)\right]_0^{L/2}\times 2$$

$$= \frac{Q}{2\pi\varepsilon_o L}\left[\ln\left(\frac{L}{2a}+\sqrt{1+\frac{L^2}{4a^2}}\right)-\ln\left(0+\sqrt{1+0}\right)\right]$$

$$= \frac{Q}{2\pi\varepsilon_o L}\ln\left(\frac{L}{2a}+\sqrt{1+\frac{L^2}{4a^2}}\right)$$

(valid derivation - 2 marks)

(total 10 marks)

Question 9

a)
If the energy of a light photon is larger than the work function of the cathode, electrons are ejected from the cathode. Depending on the anode voltage these photoelectrons are collected by the anode and appear as a current flow between cathode and anode.

(2 marks)

b)
The kinetic energy of electrons ejected from the cathode is equal to the difference between the photon energy and the work function of the cathode:

$KE = h\,f - \phi$

If the anode voltage is positive the electrons are accelerated towards the anode producing a current. But if the anode voltage is negative the energy of the electrons is reduced. At a critical negative voltage V_0, the electrons can only just reach the anode.

(2 marks)

At this critical value

$eV_0 = KE = h\,f - \phi$

$\Rightarrow V_0 = \frac{h}{e} f - \frac{\phi}{e}$

where e is the electronic charge and h is Planck's constant.

(1 mark)

c)
For large (positive) V_{anode} essentially all of the photoelectrons are collected by the anode. The current is determined by the number of incident photons which is directly related to the intensity of the incident light. Hence doubling the light intensity also doubles the current.

(2 marks)

d)
For a different frequency f' then we have:

$$V_0' = \frac{h}{e} f' - \frac{\phi}{e} \quad .$$

If $f' = 2\,f$ then

$$V_0' = \frac{h}{e}(2\,f) - \frac{\phi}{e}$$

$$= 2\left(\frac{h}{e} f - \frac{\phi}{e}\right) + \frac{\phi}{e}$$

$$= 2\,V_0 + \frac{\phi}{e}$$

(2 marks)

e)
In classical physics, if the intensity increases electrons should be able to gain more energy increasing the stopping potential V_0. But V_0 is found to *not* depend on intensity.

(1 mark)

(Total 10 marks)

Question 10

a)
Motional emf is given by: $\varepsilon = \oint (\boldsymbol{v}\times\boldsymbol{B})\bullet d\boldsymbol{l}$

As **v** and **B** are at right angles, the magnitude is given by
$|\,\boldsymbol{v}\times\boldsymbol{B}\,| = v\,B = r\,\omega\,B$, where ω is the angular velocity of the disk.

(1 mark)

For this small segment, with velocity **v**, the vector **v** x **B** is directed radially outwards.

The induced emf $d\varepsilon$ of this segment tends to make current flow radially outward, and is given by
$d\varepsilon = (\boldsymbol{v}\times\boldsymbol{B})\bullet\, d\boldsymbol{l} = \omega\, r\, B\, dr$

(1 mark correct approach; 1 mark correct answer; 1 mark correct direction)

b) Integrating from 0 to R:

$$\varepsilon = \int_0^R \omega r B dr = \frac{1}{2}\omega B R^2$$

(1 mark approach; 1 mark correct answer)

c) Current will flow from a to b.

(1 mark)

d) The power dissipated in the resistor is

$$P = \frac{\varepsilon^2}{R_L} = \frac{\omega^2\, B^2\, R^4}{4\,R_L}$$

(2 marks)

This equals the power required to maintain the constant angular velocity of the disk.

(1 mark)

(Total 10 marks)

Question 11 (Common Question TEC Q.10=ADV Q.10)

a)
If the energy of a light photon is larger than the work function of the cathode, electrons are ejected from the cathode. Depending on the anode voltage these photoelectrons are collected by the anode and appear as a current flow between cathode and anode.

(2 marks)

b)
The kinetic energy of electrons ejected from the cathode is equal to the difference between the photon energy and the work function of the cathode:

$KE = h\,f - \phi$

If the anode voltage is positive the electrons are accelerated towards the anode producing a current. But if the anode voltage is negative the energy of the electrons is reduced. At a critical negative voltage V_0, the electrons can only just reach the anode.

(2 marks)

At this critical value

$$eV_0 = KE = h\,f - \phi$$

$$\Rightarrow V_0 = \frac{h}{e} f - \frac{\phi}{e}$$

where e is the electronic charge and h is Planck's constant.

(1 mark)

c)
For large (positive) V_{anode} essentially all of the photoelectrons are collected by the anode. The current is determined by the number of incident photons which is directly related to the intensity of the incident light. Hence doubling the light intensity also doubles the current.

(2 marks)

d)
For a different frequency f' then we have:

$$V_0' = \frac{h}{e} f' - \frac{\phi}{e} \quad .$$

If $f' = 2\,f$ then

$$V_0' = \frac{h}{e}(2\,f) - \frac{\phi}{e}$$

$$= 2\left(\frac{h}{e} f - \frac{\phi}{e}\right) + \frac{\phi}{e}$$

$$= 2\,V_0 + \frac{\phi}{e}$$

(2 marks)

e)
In classical physics, if the intensity increases electrons should be able to gain more energy increasing the stopping potential V_0. But V_0 is found to *not* depend on intensity.

(1 mark)

(Total 10 marks)

Question 12

a)
Observationally it was known that atoms emit light at specific wavelengths producing characteristic line spectra. There was no classical explanation for this, and worse, classical models of atoms consisting of electrons orbiting the nucleus were unstable (the accelerating electrons emit radiation continuously and lose energy). Hence, Bohr made the quantisation assumption, which he postulated defined stable orbits.

(2 marks for reasonable explanation, mentioning problems with classical model and need for an explanation of line spectra)

b)
The hydrogen atom consists of an electron of charge *e* and mass *m* revolving around a much more massive single proton also of charge *e*. The electrostatic force between the electron and the proton is give by:

$$F = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^{\,2}}$$

where r_n is the radius of the nth orbit.

For a particle moving in a circular orbit there will be a centrally directed acceleration given by: $\dfrac{v_n^{\,2}}{r_n}$.

A force $F = \dfrac{m v_n^{\,2}}{r_n}$ is needed to produce this acceleration.

(1 mark for setting up problem i.e. electrostatic force and centripetal acceleration)

For the hydrogen atom, the electrostatic force produces this inward directed acceleration and we can equate the two forces.

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^{\,2}} = \frac{m v_n^{\,2}}{r_n} \qquad (1)$$

$$\Rightarrow r_n = \frac{4\pi\varepsilon_0}{m e^2}(m v_n r_n)^2$$

If we use the expression ($m v_n r_n = \dfrac{n h}{2\pi}$) given in the question for the angular momentum we get:

(1 mark)

$$r_n = \frac{4\pi\varepsilon_0}{m e^2}(m v_n r_n)^2 = \frac{4\pi\varepsilon_0}{m e^2}\frac{n^2 h^2}{4\pi^2}$$

$$= \frac{\varepsilon_0 h^2}{m\pi e^2}\; n^2$$

This is in the form:

$$a_n = n^2 a_0 \qquad \text{where} \qquad a_0 = \frac{\varepsilon_0 h^2}{\pi\, m\, e^2}$$

(1 mark for correct answer)

c)
Electrostatic potential energy is given by:

$$U_n = -\frac{e^2}{4\pi\varepsilon_0}\frac{1}{r_n} = -\frac{e^2}{4\pi\varepsilon_0}\frac{m\pi e^2}{\varepsilon_0 h^2}\frac{1}{n^2}$$

$$= -\frac{m e^4}{4\varepsilon_0^{\ 2} h^2 n^2}$$

(1 mark for potential energy)

The kinetic energy of electron is $\frac{1}{2}m v_n^{\ 2}$.
From Part A equation (1) this equals

$$K_n = \frac{1}{2}m v_n^{\ 2} = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r_n} = \frac{e^2}{8\pi\varepsilon_0}\frac{m\pi e^2}{\varepsilon_0 h^2}\frac{1}{n^2}$$

$$= \frac{m e^4}{8\varepsilon_0^{\ 2} h^2 n^2}$$

(1 mark for Kinetic Energy)

Total energy is given by:

$$E_n = K_n + U_n$$

$$= \frac{m e^4}{8\varepsilon_0^{\ 2} h^2 n^2} + -\frac{m e^4}{4\varepsilon_0^{\ 2} h^2 n^2}$$

$$= -\frac{m e^4}{8\varepsilon_0^{\ 2} h^2 n^2}$$

(1 mark for Total Energy)

d)
This equation shows us how to calculate the value of the Rydberg constant from fundamental physical constants, all of which can be determined independently of the Bohr Model. The result agrees to four significant figures with that derived from measurements of the wavelengths of lines in the hydrogen spectrum.

(1 mark)

e)
The ionisation energy of the hydrogen atom is the energy to remove the electron completely. Ionisation corresponds to a transition from the ground state $n = 1$ to an infinitely large orbit $n = \infty$. Substituting the values into the energy equation in part c gives a value of 13.606 eV compared with the measured value of 13.60 eV. The two values agree to within 0.1%.

(1 mark for reasonable reason)

(Total 10 marks)