The University of Sydney
School of Mathematics and Statistics

Tutorial Week 4

MATH1905: Statistics (Advanced) Semester 2, 2017

Web Page: http://sydney.edu.au/science/maths/MATH1905
Lecturer: Michael Stewart

Be sure to complete questions 7 and 8 from week 3 and discuss any difficulties with your tutor.

1. (**Multiple Choice**) A six-sided die is loaded in such a way that each of the (equally likely) even numbers is twice as likely to occur as each of the (equally likely) odd numbers. The die is thrown (independently) two times. The probability that a total of 5 is observed is:

(a) 1/9 (b) 4/81 (c) 2/9 (d) 2/81 (e) 8/81

2. An unknown number N of animals of a certain species are present in a certain habitat. To try to "estimate" N, 10 animals are captured and tagged. Some time later (to allow the tagged animals to "randomly mix" in the habitat) a further 10 animals are captured and it turns out only 2 of these 10 had tags.

(a) Write the probability of getting 2 tagged animals in the second sample as a function of N.

(b) What is the smallest possible value for N?

3. In a large factory there are four machines M_1, M_2, M_3 and M_4 all producing identical items for computers. The production from all four machines for a day is collected in a large storage bin. The quality control manager has noted the following information on percentages of daily production and defective items produced by each machine:

Machine:	M_1	M_2	M_3	M_4
Production:	10%	20%	30%	40%
Defectives:	0.1%	0.05%	0.50%	0.20%

An inspector selects an item at random from the storage bin in a particular day.

(a) What is the probability that the selected item is defective?

(b) What is the conditional probability that the selected item is from machine M_4 given that it is defective?

4. Suppose A and B are two independent events. Noting that $A = (A \cap B) \cup (A \cap B^c)$ prove (using the third axiom) that A and B^c are also independent.

5. A fair coin is flipped 3 times and a fair 6-sided die is rolled twice, all independently. Let X equal the number of heads obtained in the coin flips *plus* the number obtained in the die rolls. Find $P(X = 10)$.

6. *Does a flush beat a straight?* In the card game of Poker, all possible subsets of size 5 (called "hands") from the 52 standard playing cards[1] are classified into different types. The *more difficult* (i.e. unlikely) the type of hand is to get, the higher it is ranked in the hierarchy of hands.

Two such types of hands are a *flush* (where all cards are of the same suit) and a *straight* (where the numerical values on the cards are in sequence, e.g. 2,3,4,5,6 or 8,9,10,J,Q *irrespective of the suits.*

[1]There are 13 numerical values A,2,3,. . . ,9,10,J,Q,K in each of 4 suits. Note that A can also be "high", that is a straight can be 10,J,Q,K,A.

(a) Count how many possible hands give a flush by enumerating how many ways each of the following steps may be performed:

   (i) choose a suit;

   (ii) choose 5 of the 13 cards in that suit.

(b) Count how many possible hands give a straight by enumerating how many ways each of the following steps may be performed:

   (i) choose a lowest numerical value;

   (ii) for each value, choose 1 of the 4 possible cards of that value.

(c) Does a flush beat a straight?