THE UNIVERSITY OF SYDNEY

MATH1901 DIFFERENTIAL CALCULUS (ADVANCED)

Semester 1	**Tutorial Week 11**	2012

1. (*This question is a preparatory question and should be attempted before the tutorial. Answers are provided at the end of the sheet – please check your work.*)

Compute the partial derivatives $f_x(x,y)$, $f_y(x,y)$ of the following functions $f(x,y)$.

(a) xy^3 (b) $\sin(2x+3y)$ (c) $\ln(x+\sqrt{x^2+y^2})$

Questions for the tutorial

2. Find the limit, if it exists, or show that the limit does not exist.

(a) $\displaystyle\lim_{(x,y)\to(2,3)}(x^2y^2-2xy^5+3y)$ (b) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2y^3+x^3y^2-5}{2-xy}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x-y}{x^2+y^2}$ (d) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^3+xy^2}{x^2+y^2}$

3. Consider the function

$$f(x,y)=\frac{\sin(x^2+y^2)}{x^2+y^2},\ \text{defined for } (x,y)\neq(0,0).$$

Is it possible to define $f(0,0)$ so that f is continuous at $(0,0)$?

4. Decide whether the limits exist.

(a) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{xy^2}{x^2+y^4}$ (b) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{xy^2}{x^2+y^2}\sin\frac{1}{x^2+y^4}$

(c) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2-y^2}{x^2+y^2}$ (d) $\displaystyle\lim_{(x,y)\to(0,0)}\frac{x^2-y^2}{\sqrt{x^2+y^2}}$

5. Define $f:\mathbb{R}^2\to\mathbb{R}$ as follows:

$$f(x,y)=\begin{cases}1 & \text{if } x=y\neq 0,\\ 0 & \text{otherwise.}\end{cases}$$

Show that f is not continuous at $(0,0)$ but both f_x and f_y exist at $(0,0)$.

6. Verify that the functions given by the following formulas are solutions of the *Laplace equation* $f_{xx}+f_{yy}=0$.

(a) x^2-y^2 (b) $2xy$ (c) $e^x\cos y$ (d) $e^x\sin y$

7. Suppose that f is a diffentiable function of one variable. Show that if $z=f\left(\dfrac{x}{y}\right)$, then

$$x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=0.$$

8. Find the equation of the tangent plane to the surface $z = e^x \ln y$ at $(3, 1, 0)$.

9. Find the single point at which the tangent plane to the surface $z = x^2 + 2xy + 2y^2 - 6x + 8y$ is horizontal.

Extra Question

10. Use the ϵ, δ definition of the limit of a function of two variables to show that

$$\lim_{(x,y)\to(1,2)} x^2 + y = 3.$$

Solution to Question 1

(a) $f_x = y^3,\ f_y = 3xy^2$

(b) $f_x = 2\cos(2x + 3y),\ f_y = 3\cos(2x + 3y)$

(c) $f_x = \dfrac{1 + x(x^2 + y^2)^{-1/2}}{x + \sqrt{x^2 + y^2}} = \dfrac{1}{\sqrt{x^2 + y^2}},\ f_y = \dfrac{y}{(x + \sqrt{x^2 + y^2})\sqrt{x^2 + y^2}}$