PHYS 1902 Electromagnetism: 3

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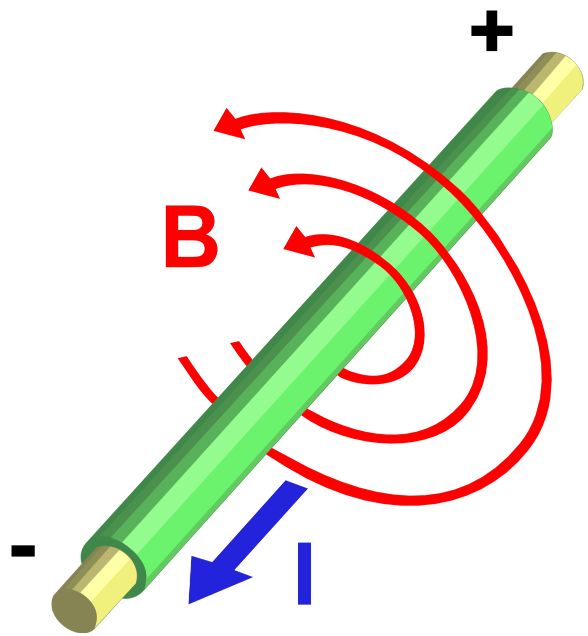
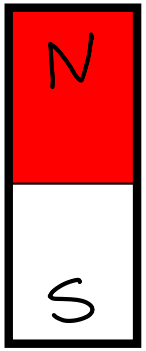


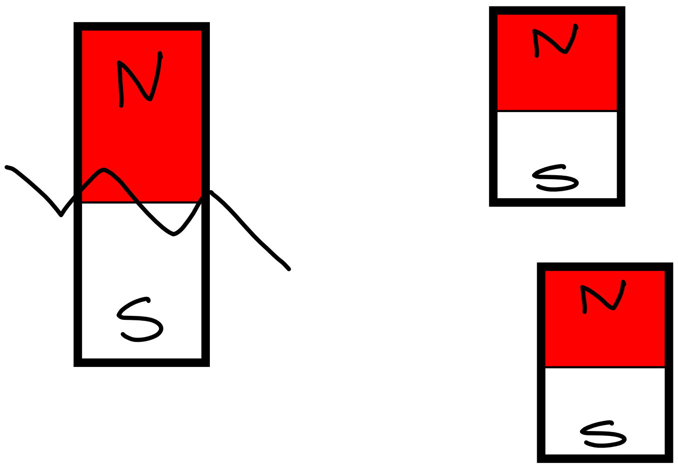
Chapter 27
Magnetic Field and Magnetic Forces

Magnets



Magnets have two poles

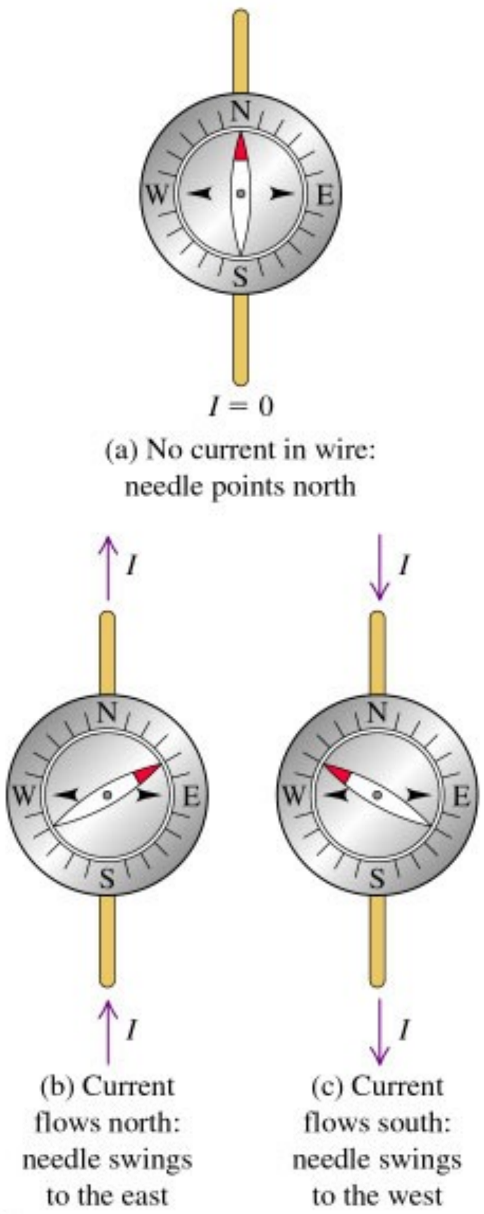
- historically, north (N) and south (S)



Break a magnet, and each piece has two poles

No one has ever observed a magnet with just one pole – *a magnetic monopole*

Magnets and Moving Charges

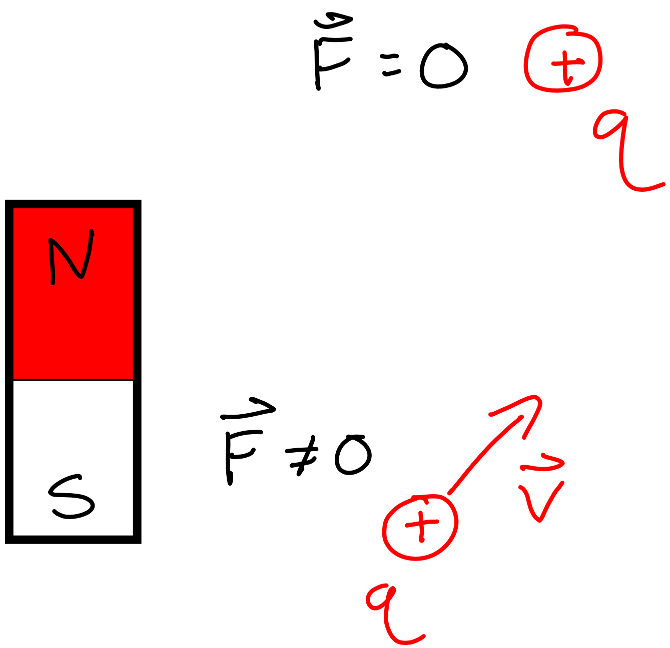


A magnet on a free pivot will align itself to north in the Earth's *magnetic field*

Moving charge (a current I) causes the magnet to deflect, i.e., through a torque

Geraint F. Lewis

Magnetic Force



A *stationary* charge does not feel a force due to a (stationary) magnet

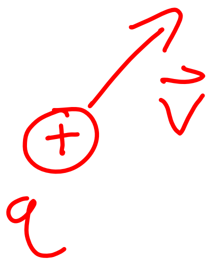
A moving charge does feel a magnetic force

Magnetism is about moving charges

We'll introduce the *magnetic field* in two steps:

1. Define the field through a moving *test charge*
2. Define the magnetic field produced by a magnet or a collection of moving charges (a current)

Magnetic Field



Test charge q
with velocity $\vec{v}$

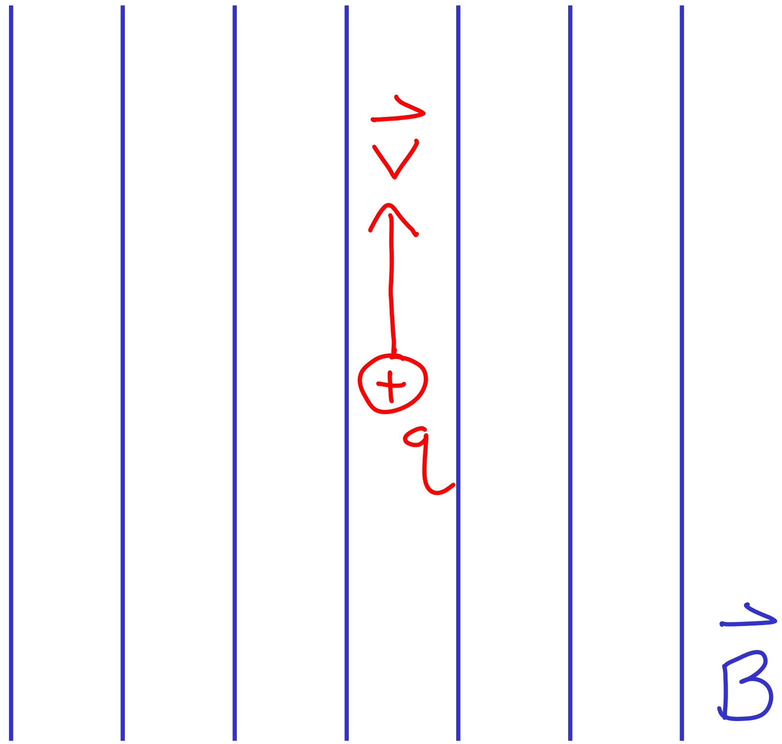
We find empirically that:

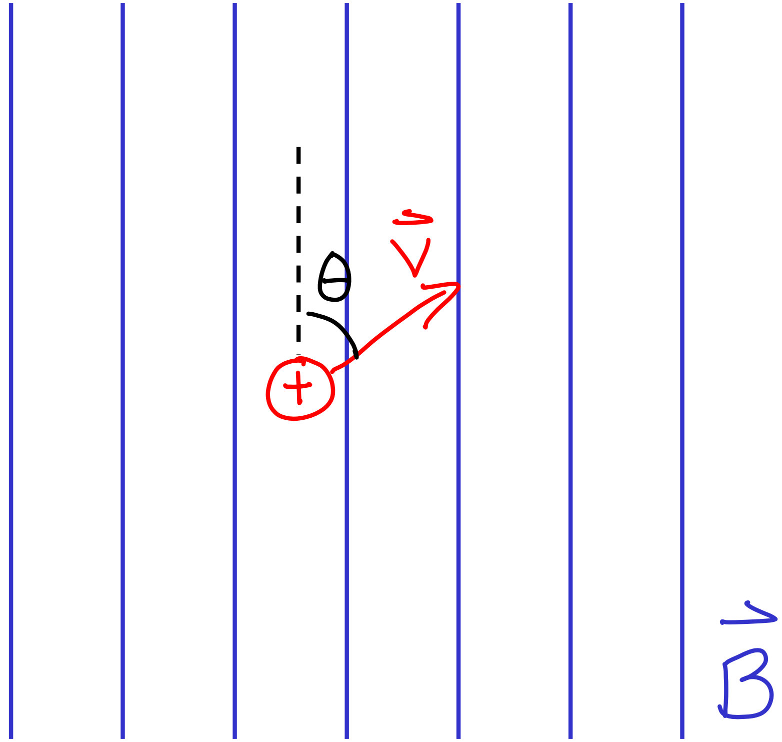
1. the *magnitude* of the force is proportional to
   a) the test charge |q|
   b) the speed |v|
2. the *direction* of the force is always perpendicular to the velocity

Definition of magnetic field:	$\vec{F} = q\vec{v} \times \vec{B}$

Units of B: Tesla $\qquad$ 1 T = 1 N/A·m

Uniform Magnetic Field





Moving parallel to $\vec{B}$:

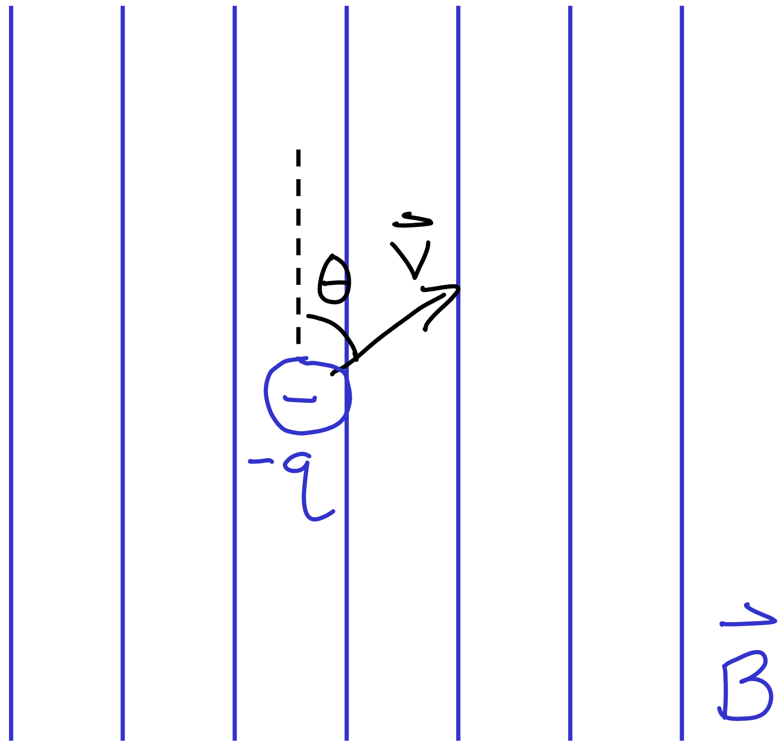
$$\vec{F} = q\,\vec{v} \times \vec{B} = 0$$

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

$$|\vec{v} \times \vec{B}| = |v|\,|B| \sin\theta$$

$$\vec{F} = q\,|v|\,|B| \sin\theta \quad \text{out of board}$$

Uniform Magnetic Field



$\vec{v} \times \vec{B}$ out of board (as befor)

$\vec{F} = -q\vec{v} \times \vec{B}$ into board

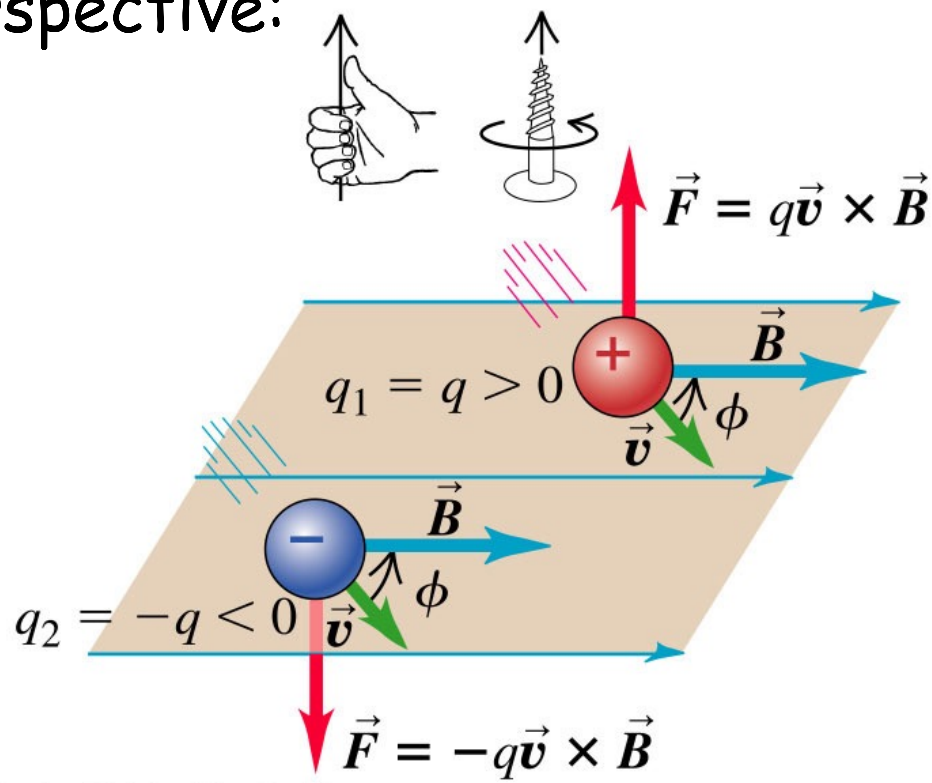
Force is directed *into* the screen/page

- perpendicular to *both* velocity and magnetic field

- opposite direction of positive charge

Uniform Magnetic Field

Another perspective:



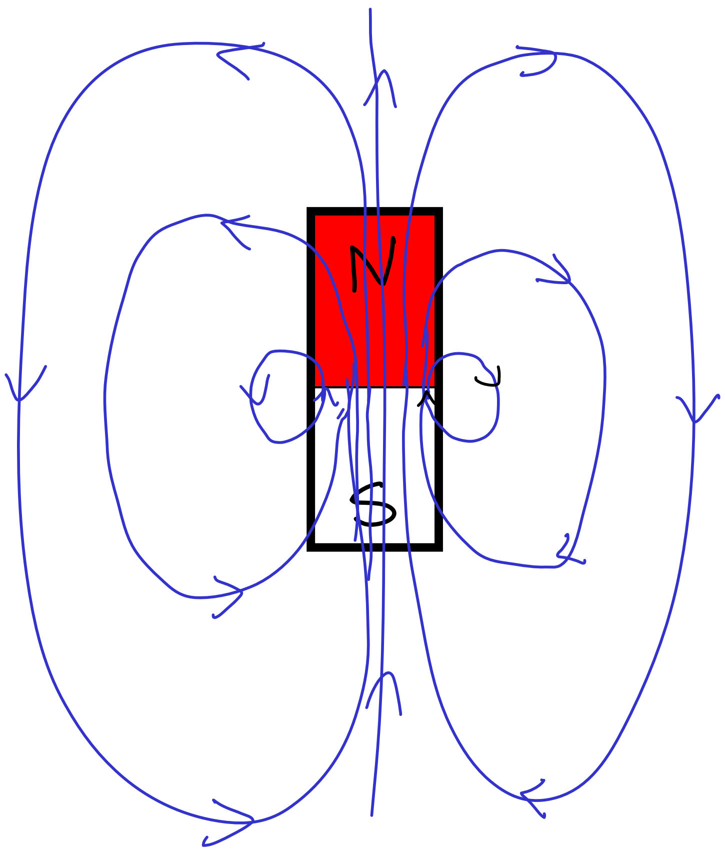
If there is an electric field as well, we have a combined force law:

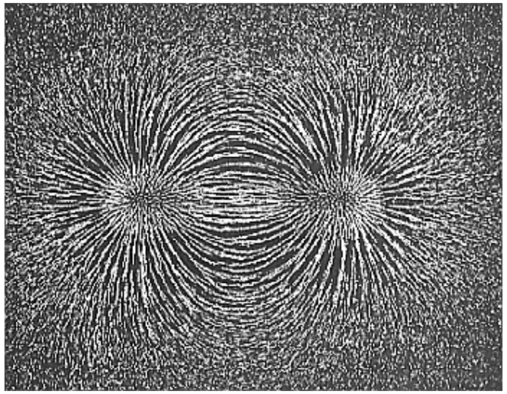
Lorentz force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

Magnetic Field Lines

With moving test charges, we can define the magnetic field everywhere in space: a vector field

Like the electric field, a useful way to visualise the magnetic field is with magnetic field lines



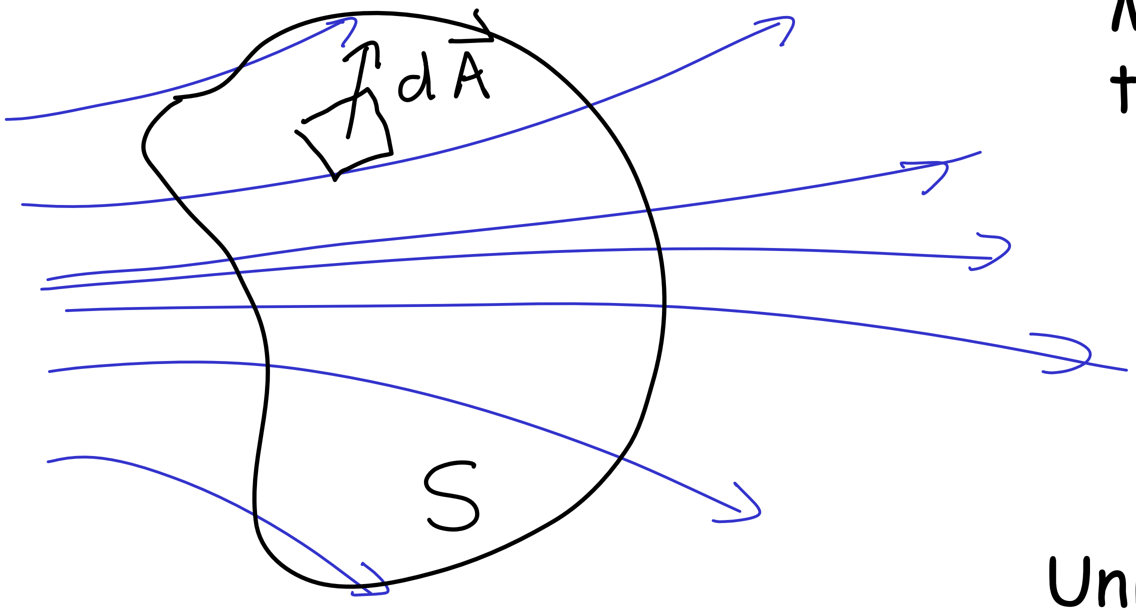


Warning!!!
Magnetic field lines are not lines of force

$\vec{B} \perp \vec{F}$

Note: We are not yet calculating the magnetic field. (We don't have a Coulomb's Law for B.) We're observing properties of magnetic fields that can be verified by moving test charges.

Magnetic Flux



Magnetic flux through a surface S:

$$\Phi_B = \int_S \vec{B} \cdot d\vec{A}$$

$$= \int_S B_\perp \, dA$$

Units: Weber

1 Wb = 1 T m^2

Identical in definition to electric flux

Quantifies the number of magnetic field lines flowing out of a surface

Is there a Gauss's Law for magnetism?

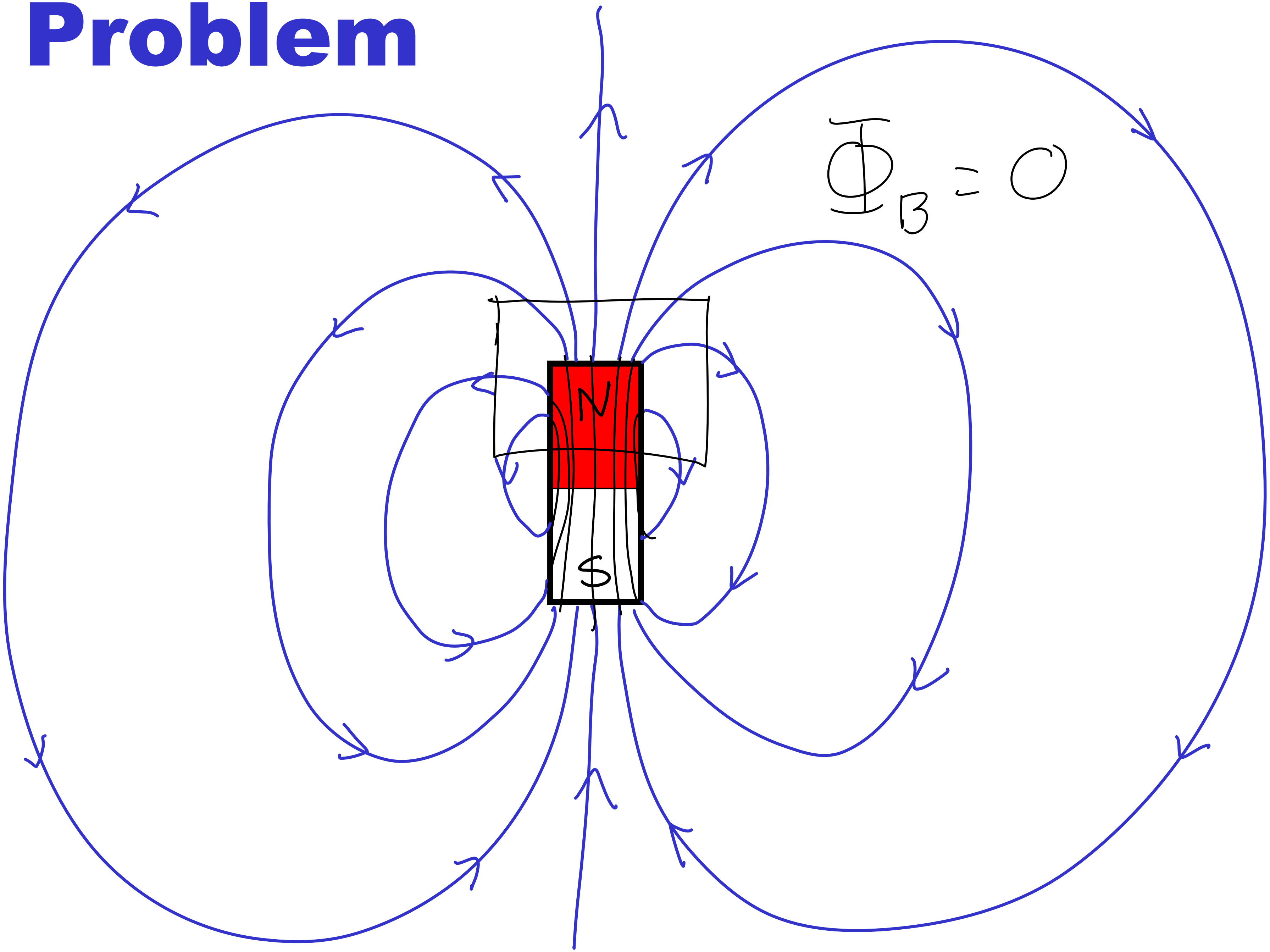
Gauss's Law for Magnetism

The total magnetic flux through a closed surface is zero

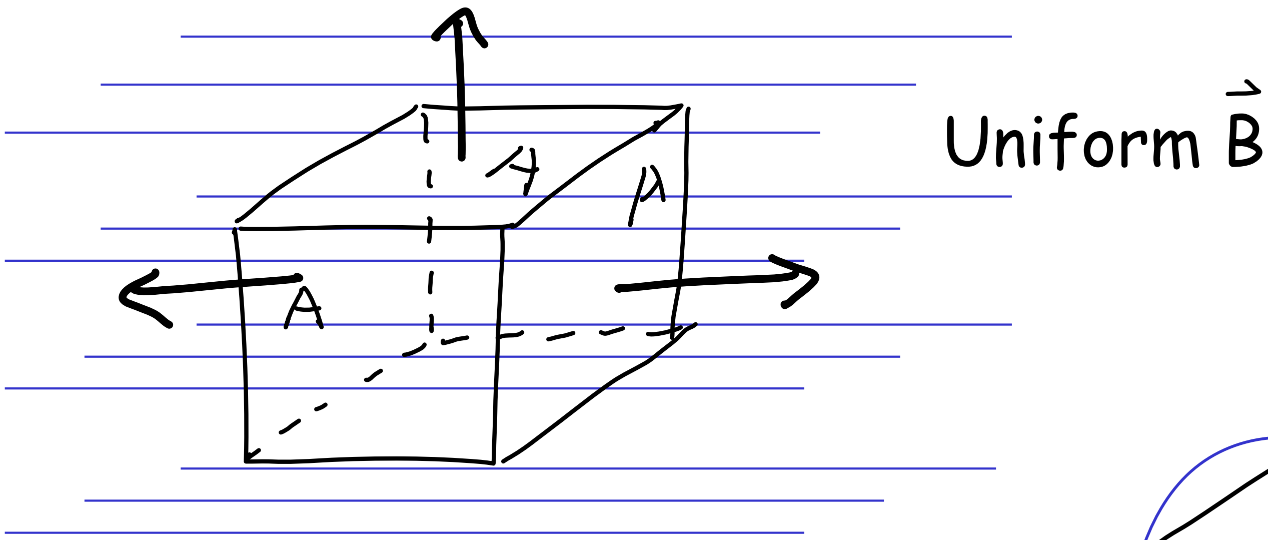
$$\boxed{\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0}$$

1. For the electric field, we derived Gauss's Law from Coulomb's Law, which itself was suggested by experiment. The magnetic version is suggested by experiment and its analogy to the electric version.
2. No magnetic charge, i.e., no magnetic monopoles
3. Magnetic field lines never begin or end – they only form closed loops

Problem



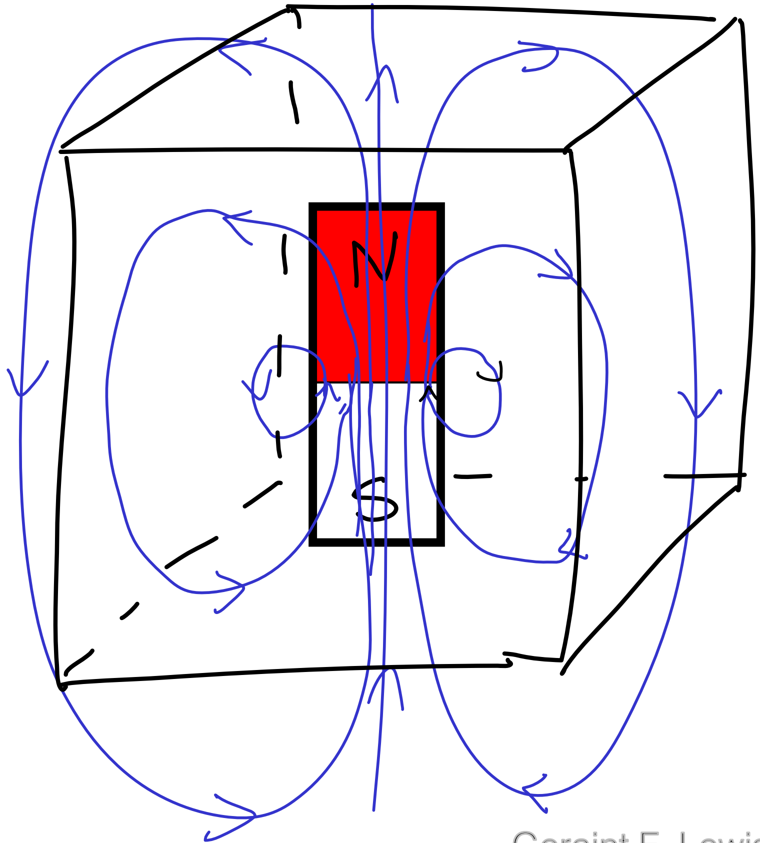
Magnetic Flux through a Box



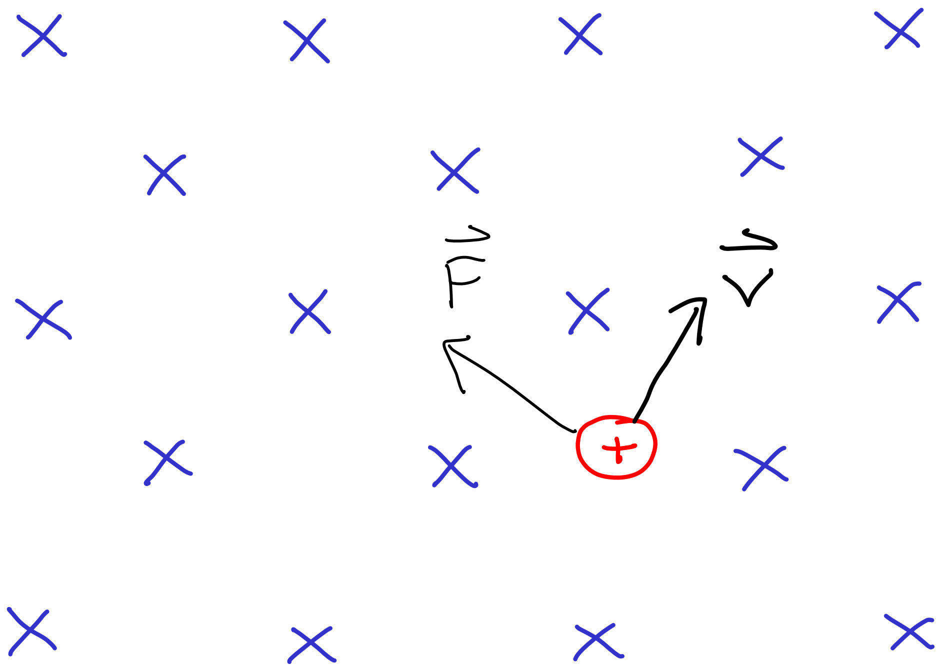
$$\Phi_B = 0$$

for <u>any</u> closed surface

(even one containing magnets)



Motion of Charge Particles in a Magnetic Field



$\vec{B}$ uniform
directed into page

$\vec{v} \perp \vec{B}$

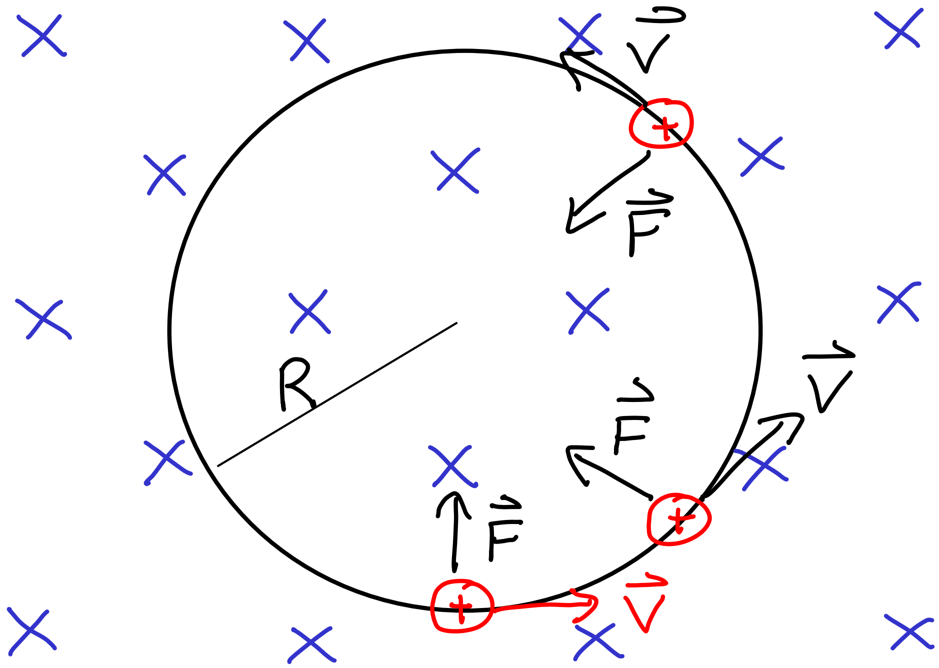
$\vec{F} = q\vec{v} \times \vec{B}$

Recall that a force does work if it acts over a distance

But the motion is always perpendicular to the magnetic force

The magnetic force does no work on a moving charge

Motion of Charge Particles in a Magnetic Field



$\vec{B}$ uniform directed into page

Speed $v = |\vec{v}|$ constant

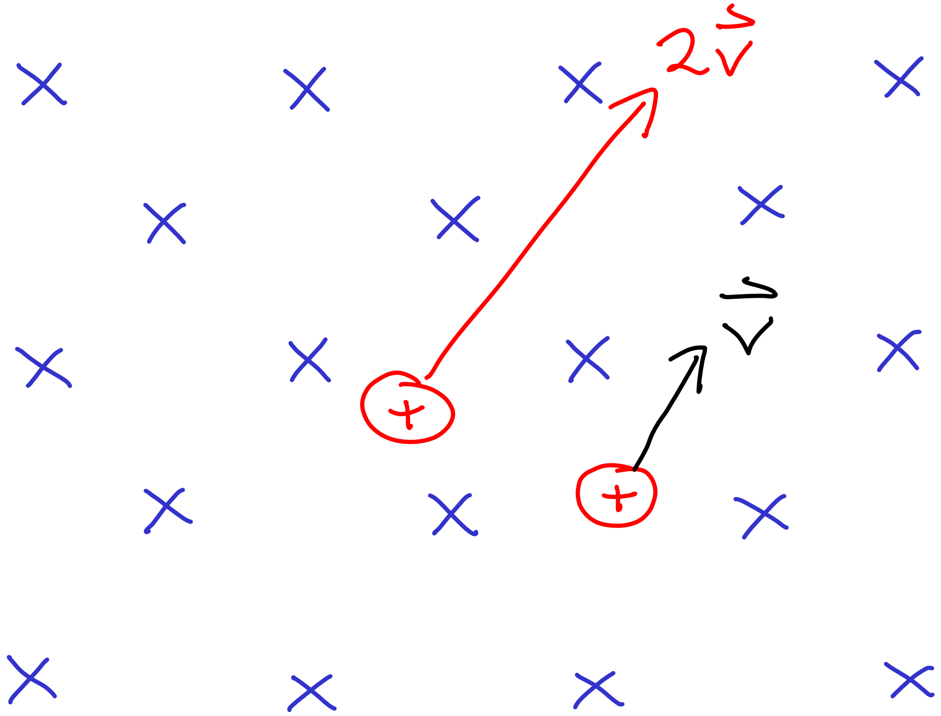
$F = |q| v B$

Newton's 2nd Law with $a = v^2/R$

$|q| v B = m v^2 / R$

Radius of a circular orbit in a magnetic field: $R = \dfrac{mv}{|q| B}$

Motion of Charge Particles in a Magnetic Field



$\vec{B}$ uniform directed into page

$$R = \frac{mv}{qB}$$

$$v\,T = 2\pi R = 2\pi \frac{mv}{qB}$$

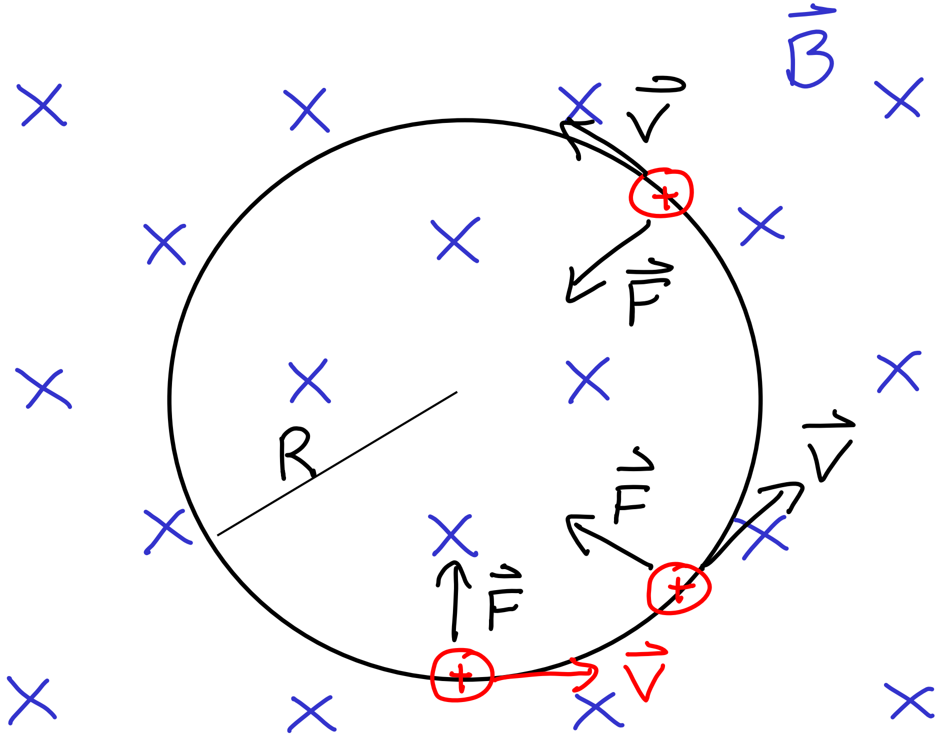
$$T = \frac{2\pi m}{qB}$$

Problem: two identical particles in the same uniform

One has twice the speed of the other

1. How do the radii of their orbits compare? × 2
2. How do the periods of their orbits compare? Same

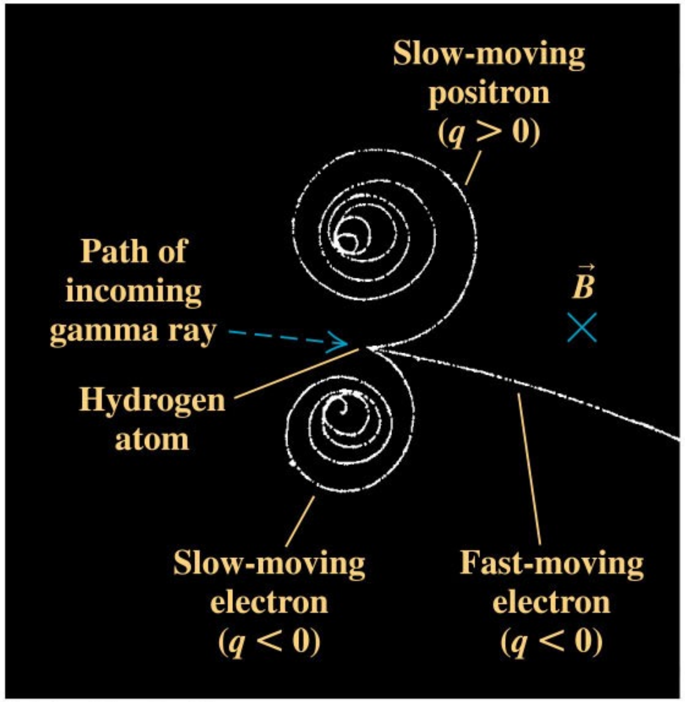
Cyclotron frequency



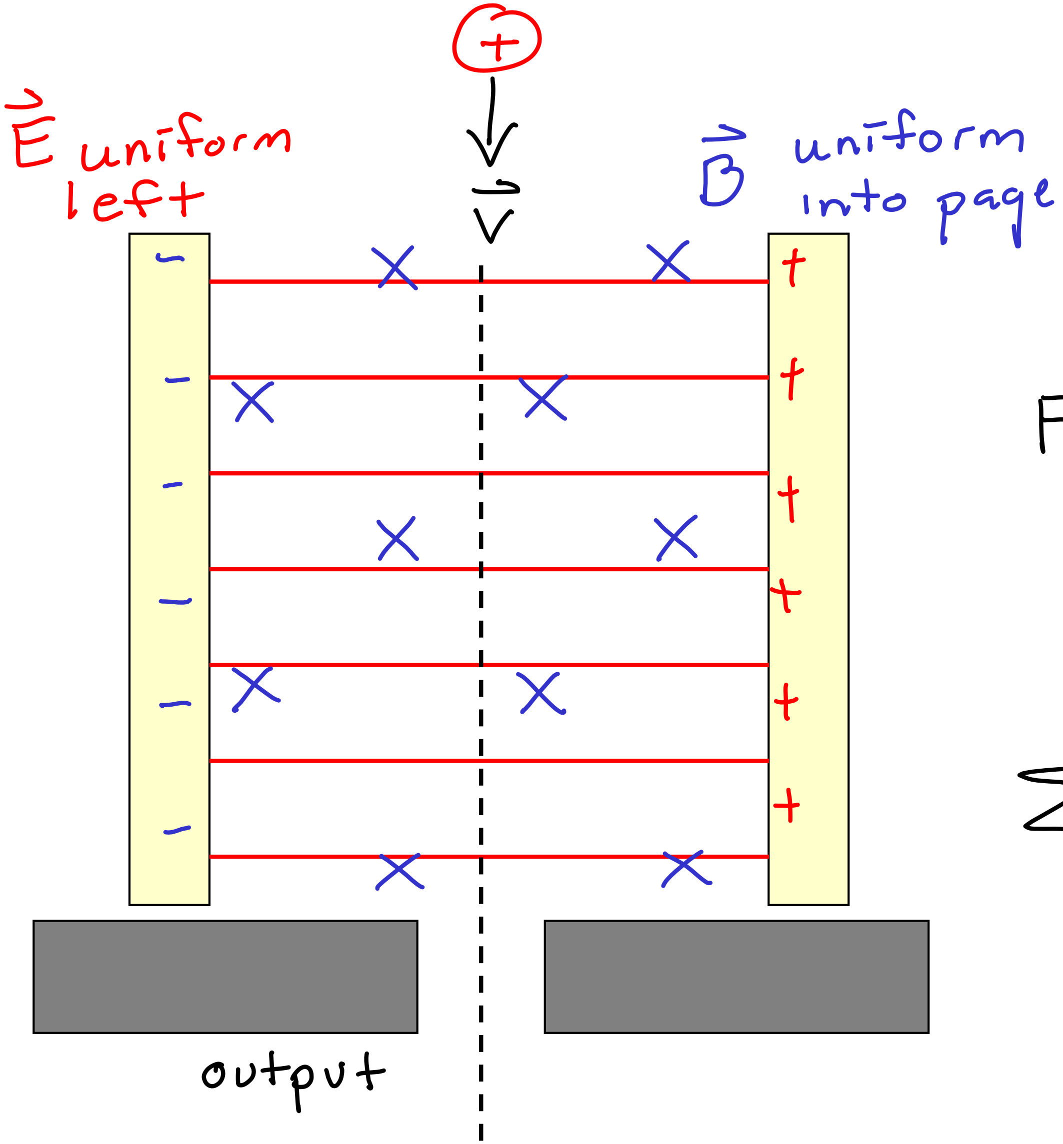
What is the angular frequency of this motion?

$$\omega = \frac{v}{R} = \frac{|q| B}{m}$$

Independent of radius and of speed



Velocity Selector



Moving charges are created at the top, but their velocities are not all the same

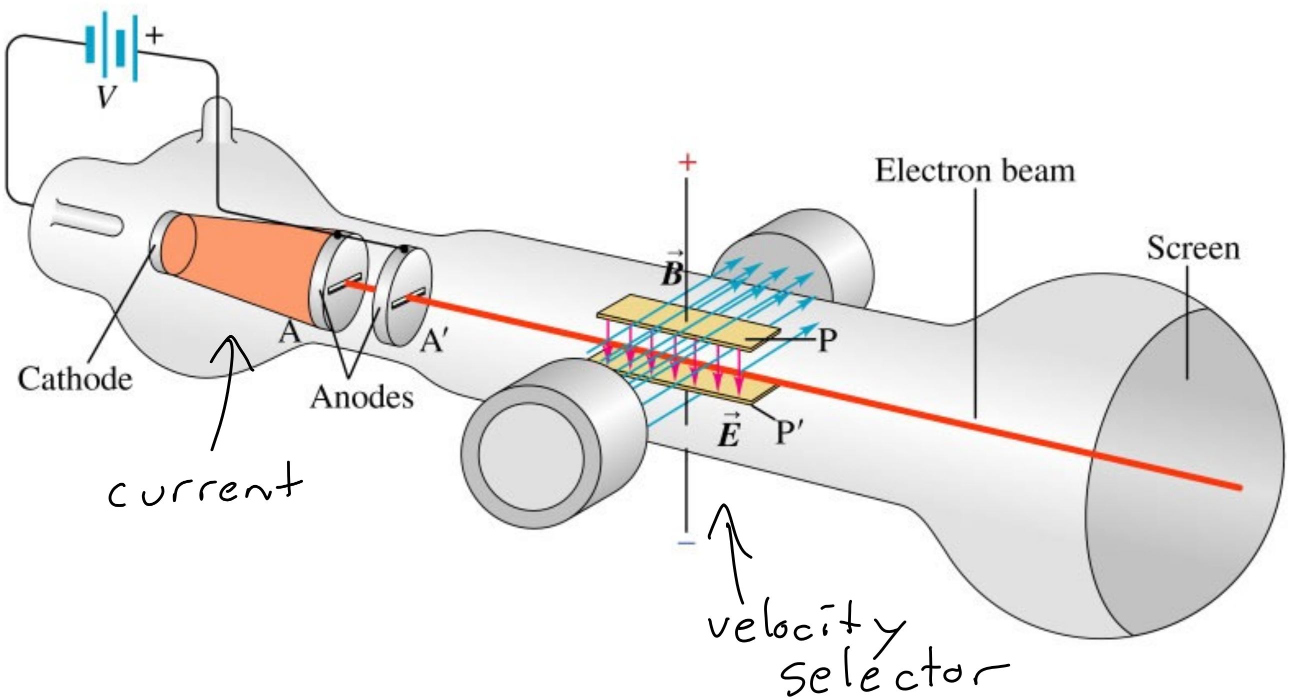
$F_e = qE$ $\quad$ $F_m = qvB$

$\vec{v}$

$\sum F = 0 \Rightarrow v = \dfrac{E}{B}$

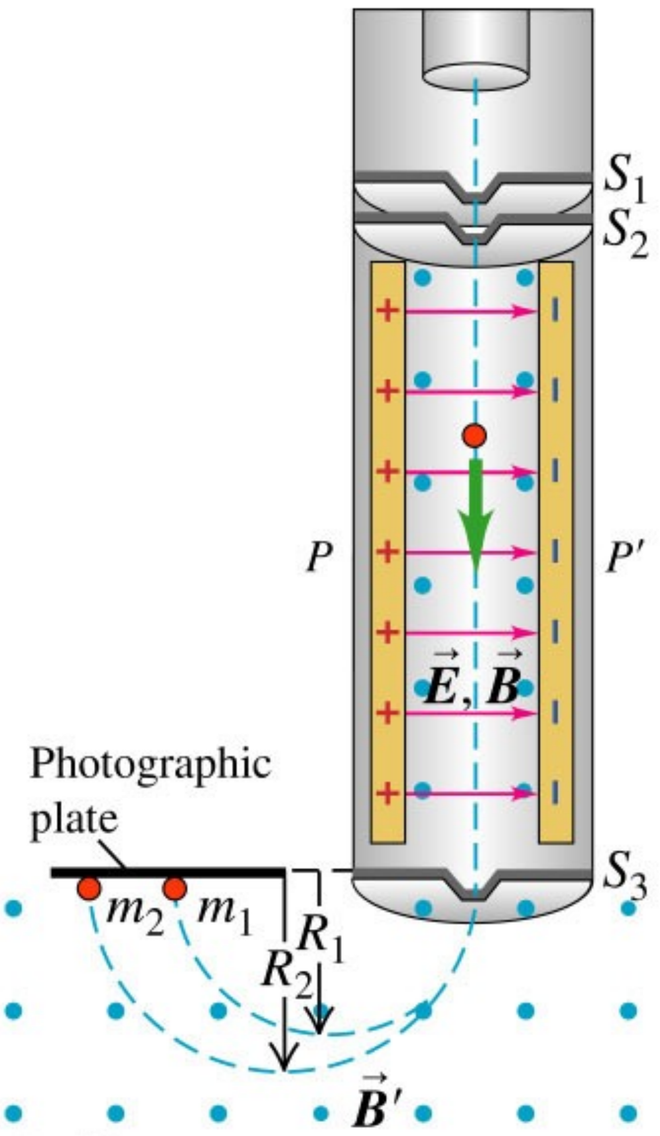
Only charges with this velocity pass through undeflected

Thompson's experiment



How do we use a velocity selector to measure the charge to mass ratio of the electron?

Mass Spectrometer



$\vec{B}$ uniform
directed into page

- Give all particles the same charge (ionize)
- Use velocity selector

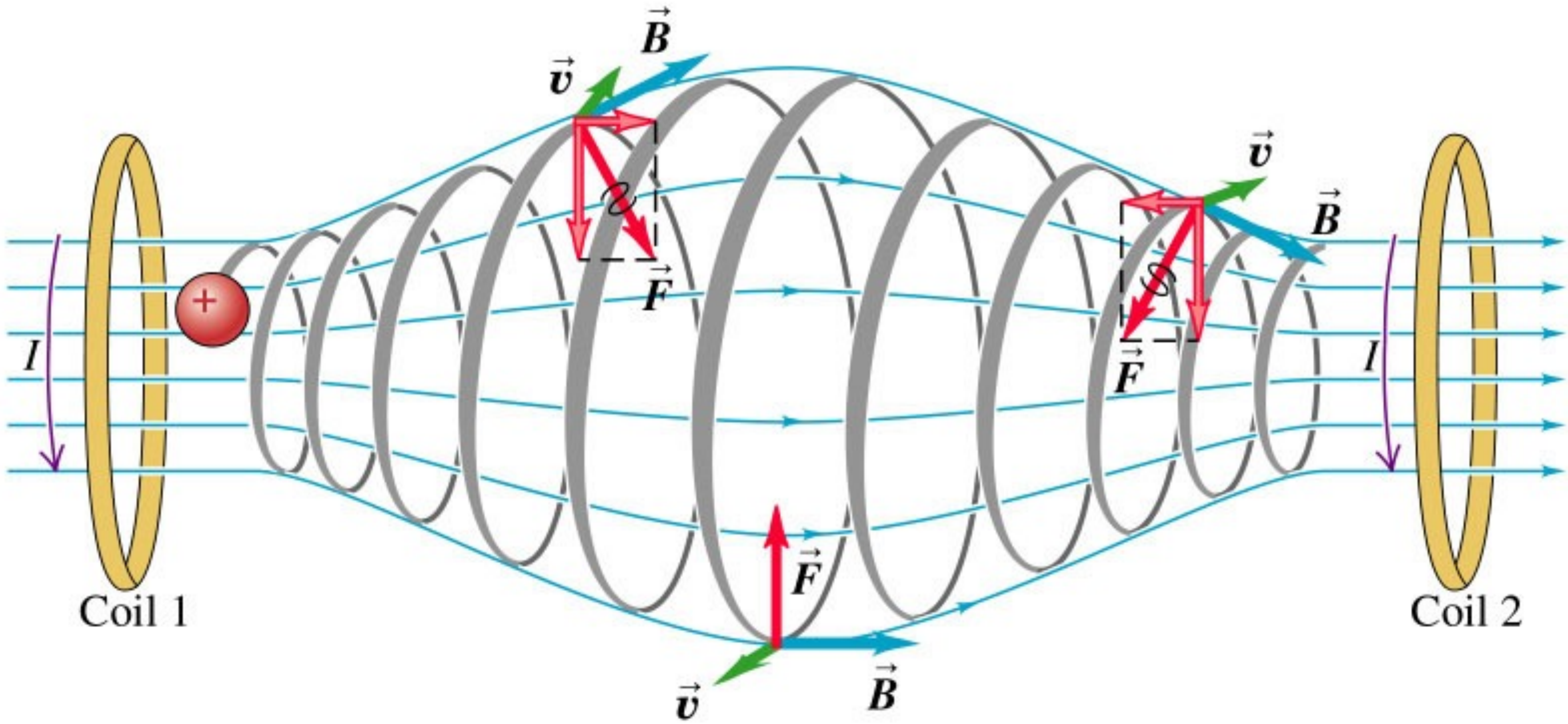
$$R = \frac{mv}{|q|\,B}$$

velocity selector

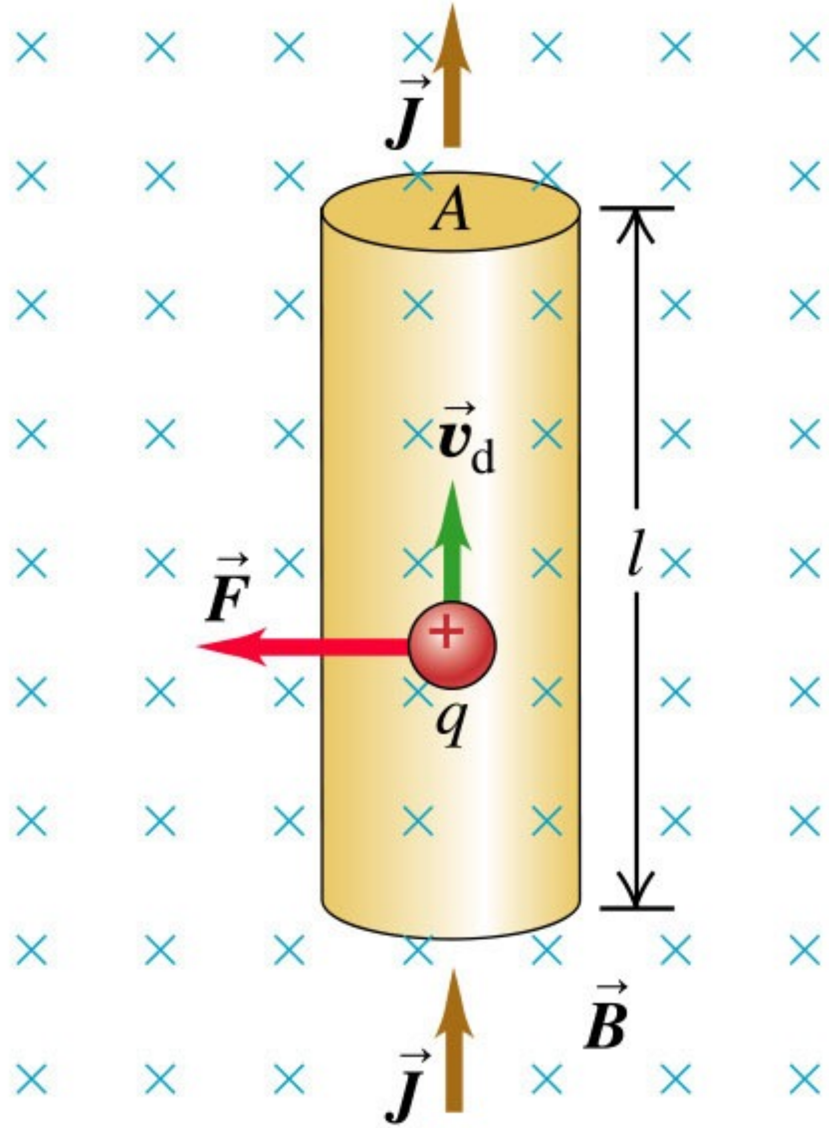
charge of ions

fixed known field

Magnetic Bottle



Magnetic Force on a Current-Carrying Conductor



$\vec{B}$ uniform

directed into page

$$\vec{F} = q\vec{v} \times \vec{B}$$

What is the force on the conductor segment?

$$\vec{F}_{wire} = (\#\text{ charges})(\vec{F}_{charge})$$

\# of charges per unit volume

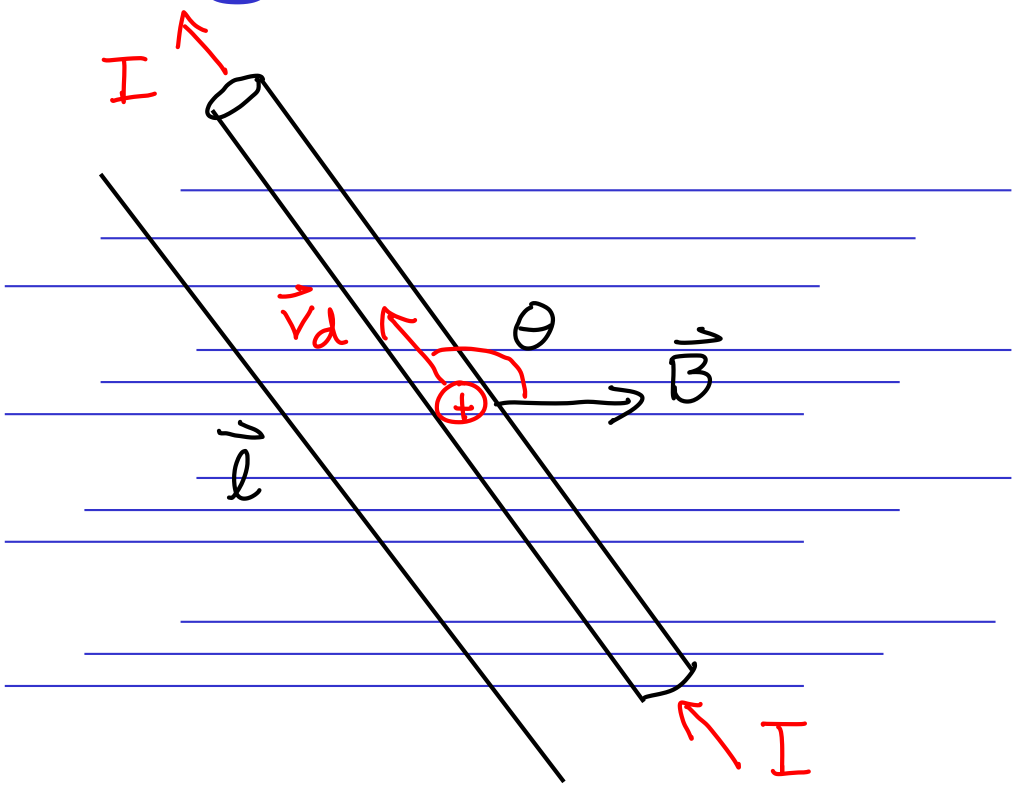
$$= (n A \ell)(q v_d B) \text{ left}$$

$$= (n q v_d)(A \ell B)$$

$$= J A \ell B$$

$$= I \ell B \quad \text{left}$$

Magnetic Force on a Wire



Uniform $\vec{B}$

$$\vec{F}_1 = q\,\vec{v}_d \times \vec{B}$$

into page

On wire:

$$\vec{F} = I\,\vec{\ell} \times \vec{B}$$

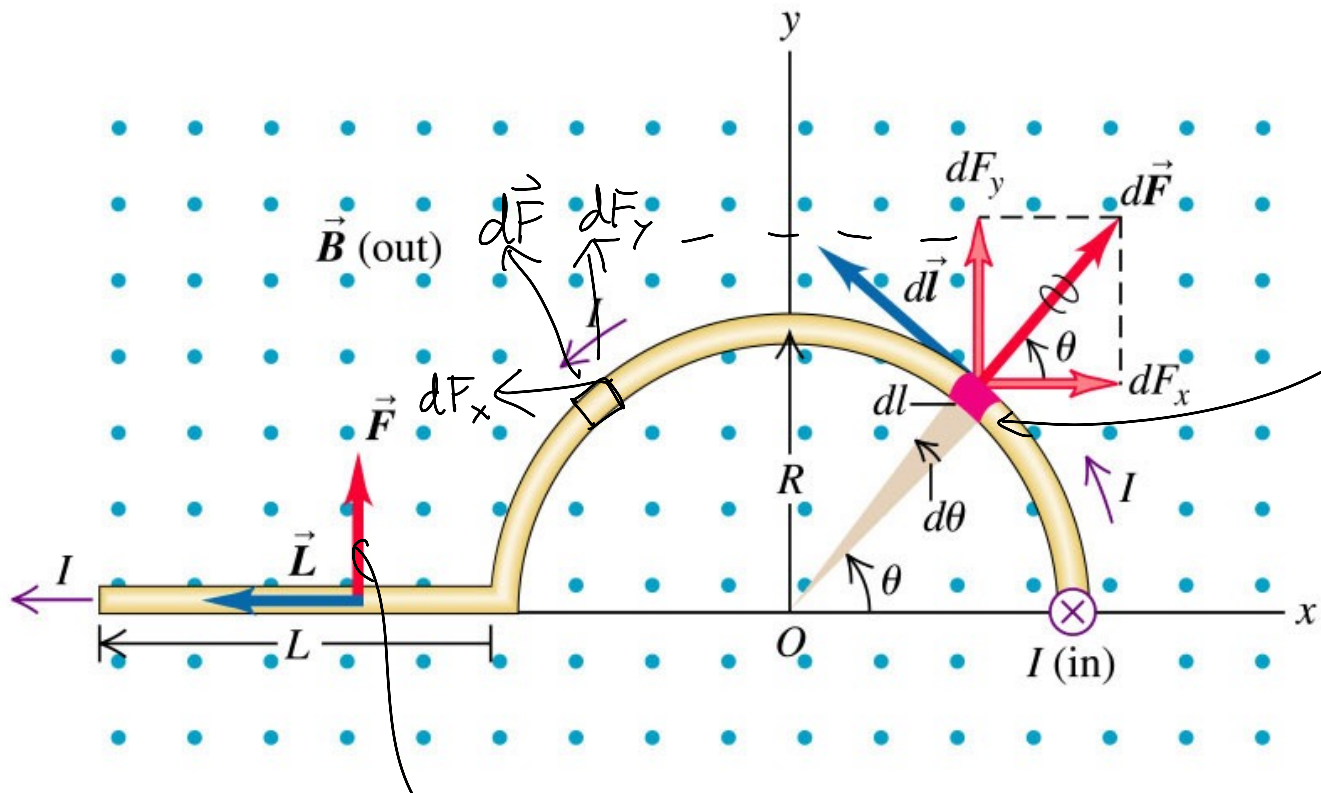
On infinitesimal section of wire:

$$d\vec{F} = I\,d\vec{\ell} \times \vec{B}$$

Magnetic force is always:

1. orthogonal to $\vec{B}$
2. orthogonal to direction of I

Magnetic Force on a Curved Conductor



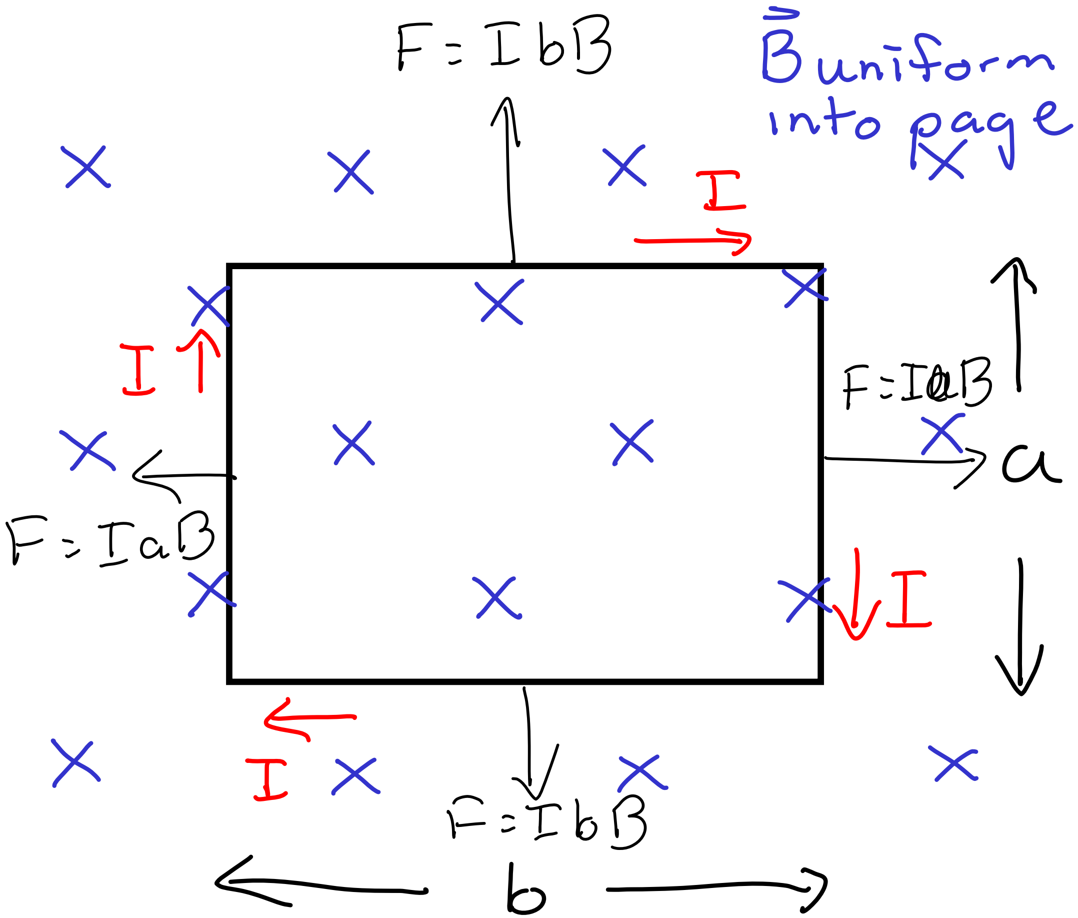
$d\vec{F} = I\, d\vec{l} \times \vec{B}$

$\vec{F} = I \vec{l} \times \vec{B}$

$= I l B$ (up)

Force on a Current Loop

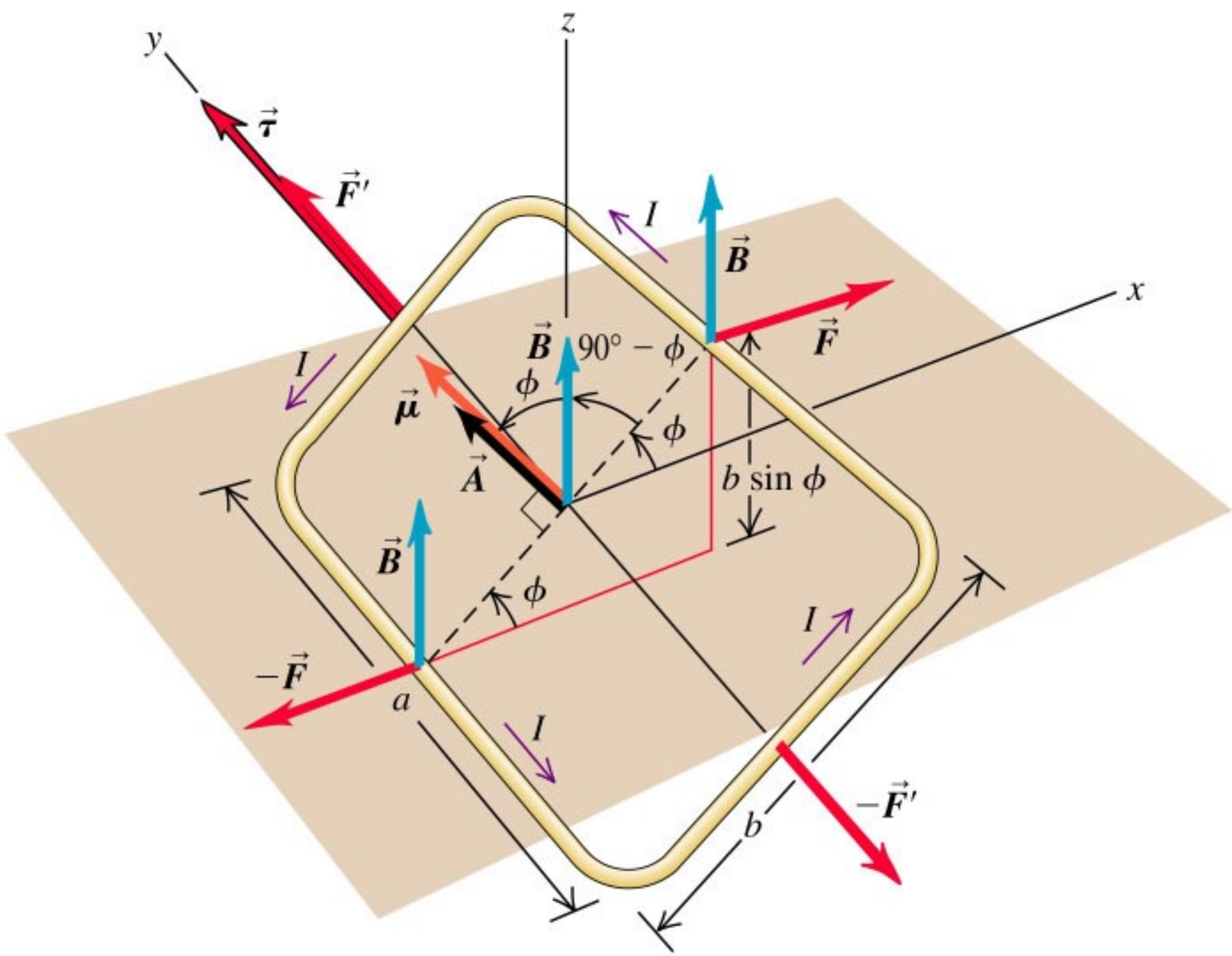
Current usually makes a closed loop...



Find the magnitude and direction of the force on each segment.

What is the net force on the loop?

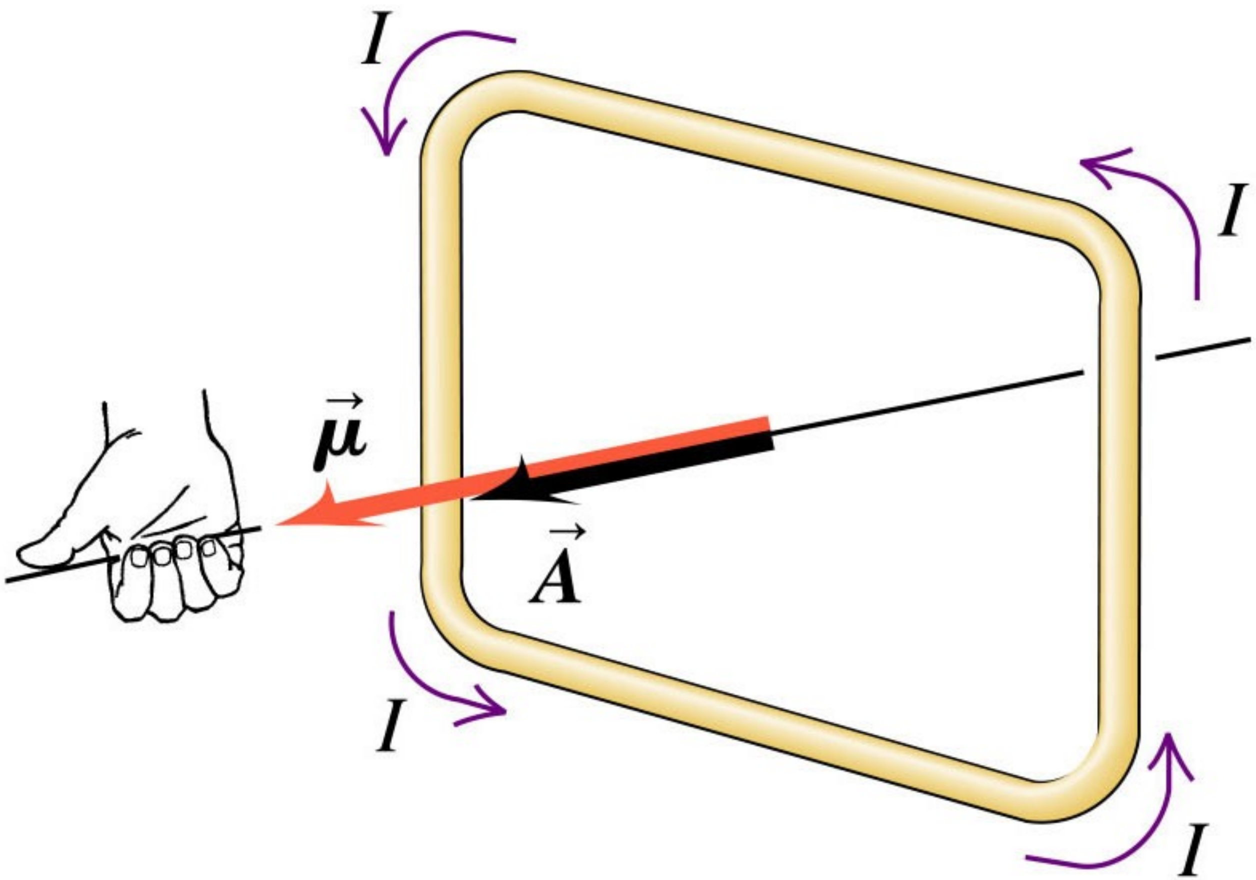
Force on a Current Loop



(a)

Torque $= I a b B \sin\phi = I A B \sin\phi = \vec{\mu} \times \vec{B}$

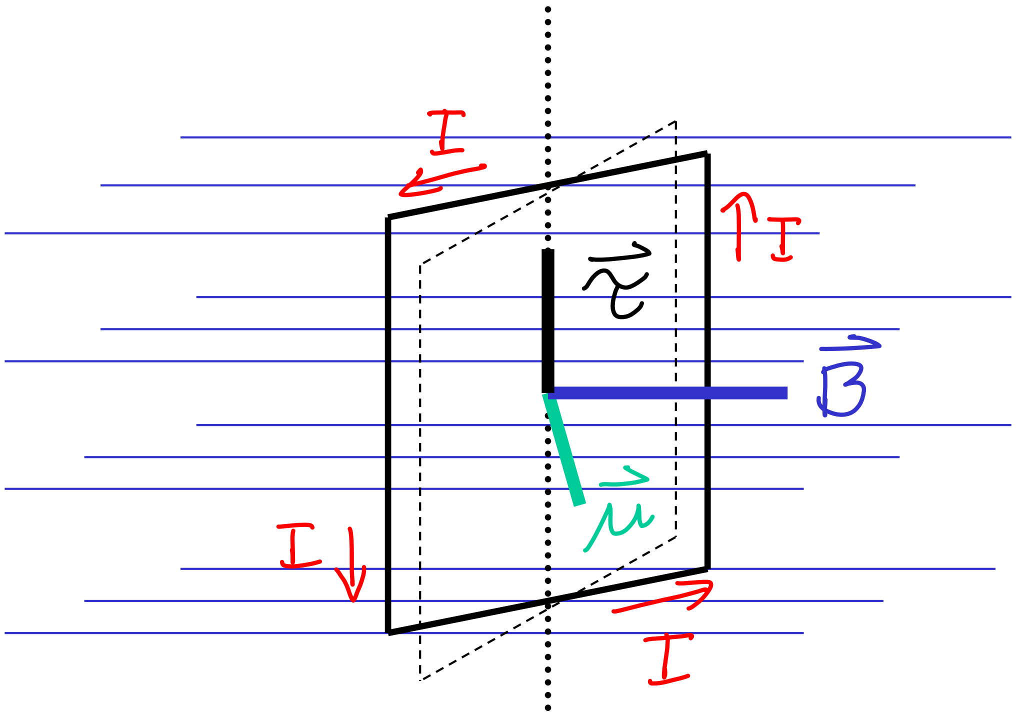
Magnetic Moment



$\mu = I A$

Right hand rule gives it a direction

Torque on a Current Loop



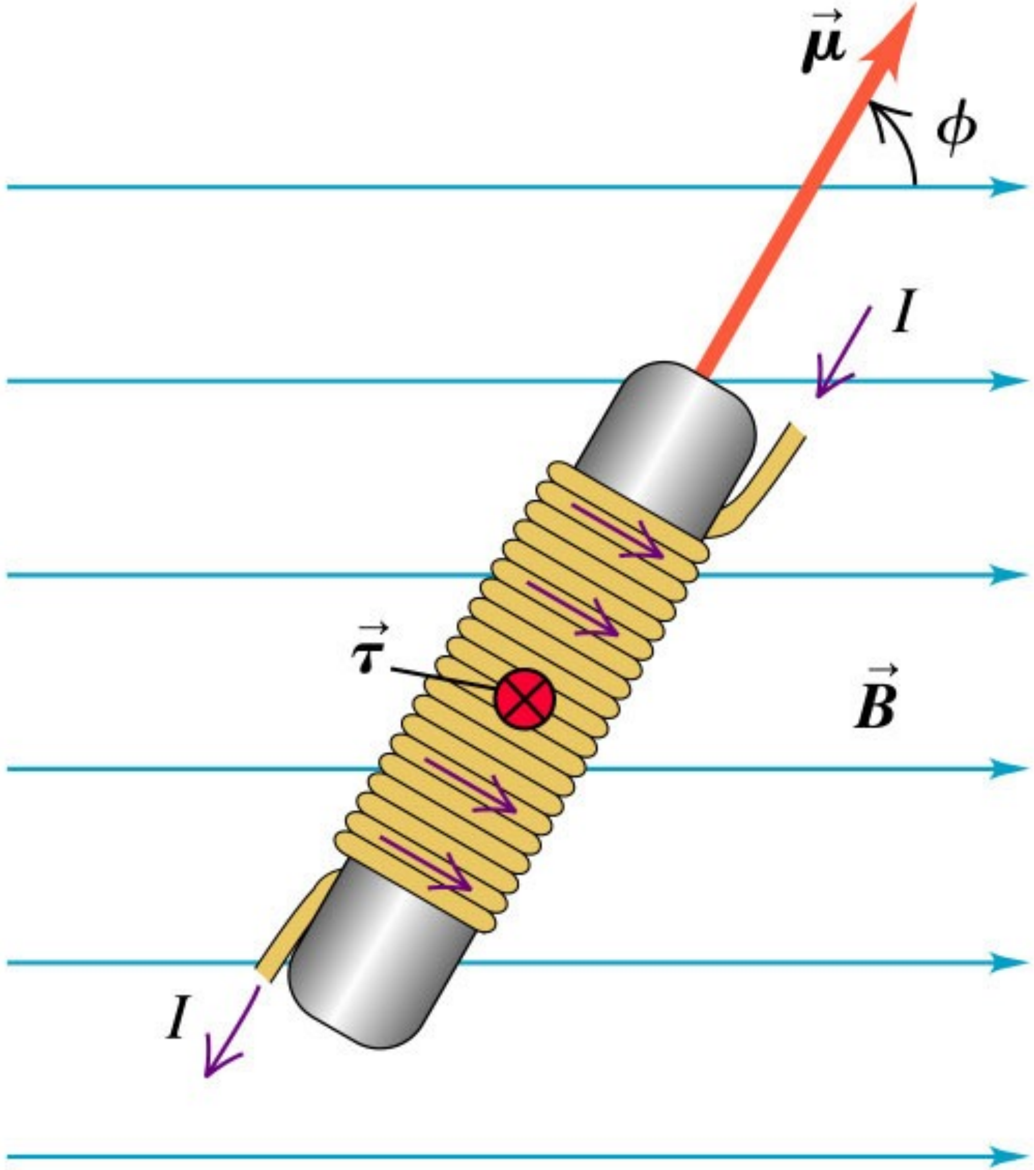
Uniform $\vec{B}$

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Vector torque on a current loop in a magnetic field

$U = -\vec{\boldsymbol{\mu}} \cdot \vec{\boldsymbol{B}} = -\mu B \cos\phi$ (potential energy for a magnetic dipole)

Torque on a Coil

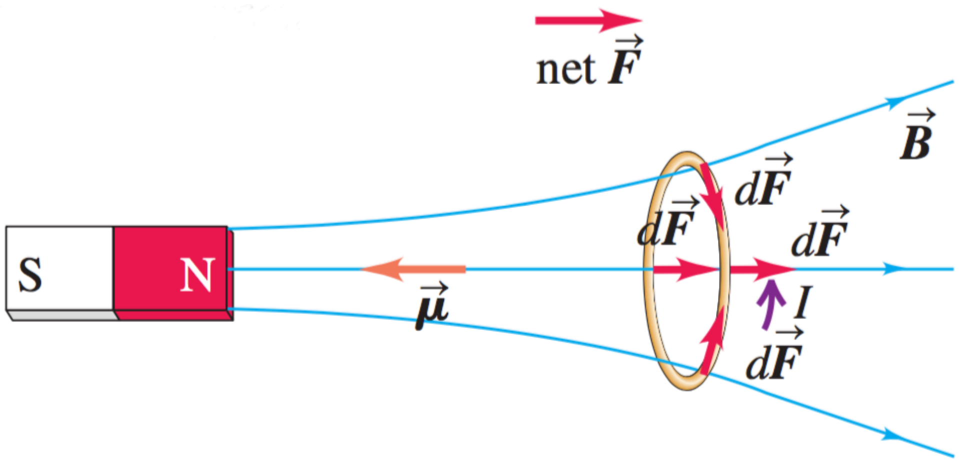


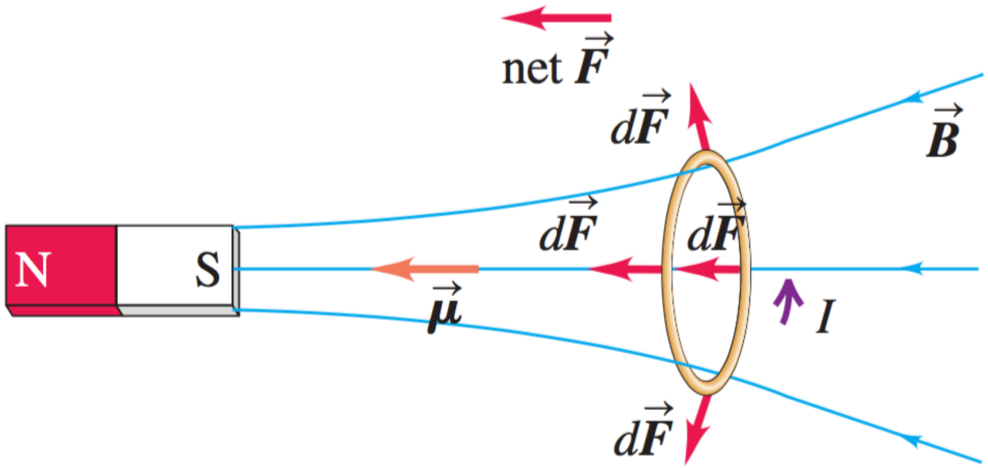
$$\vec{\mu} = N \vec{\mu}_1$$

# of turns

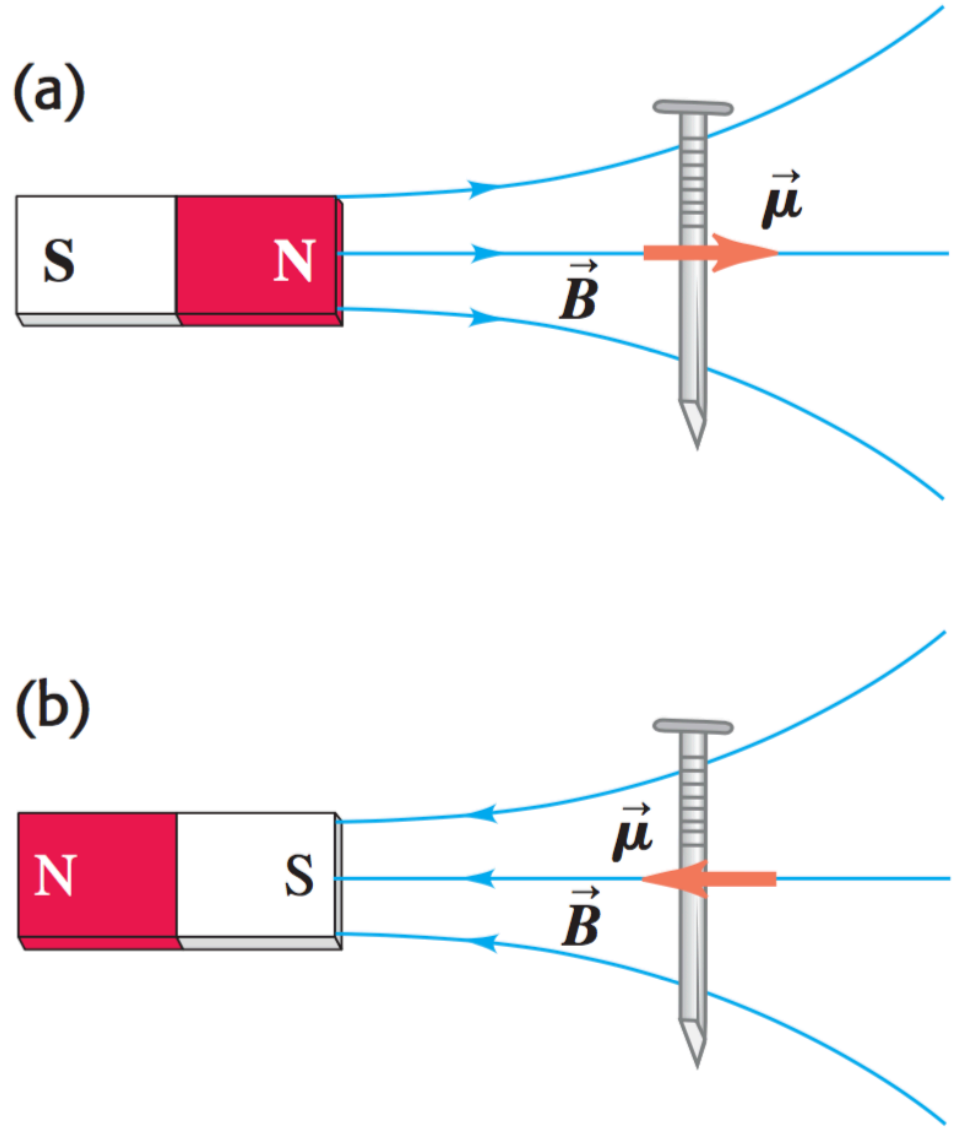
magnetic moment of one turn

Non-Uniform Magnetic Field





Non-Uniform Magnetic Field



Chapter 28
Sources of Magnetic Field

What produces a magnetic field?

All charges feel a force due to an electric field

- all charges *create* an electric field

Moving charges feel a force due to a magnetic field

- moving charges *create* a magnetic field

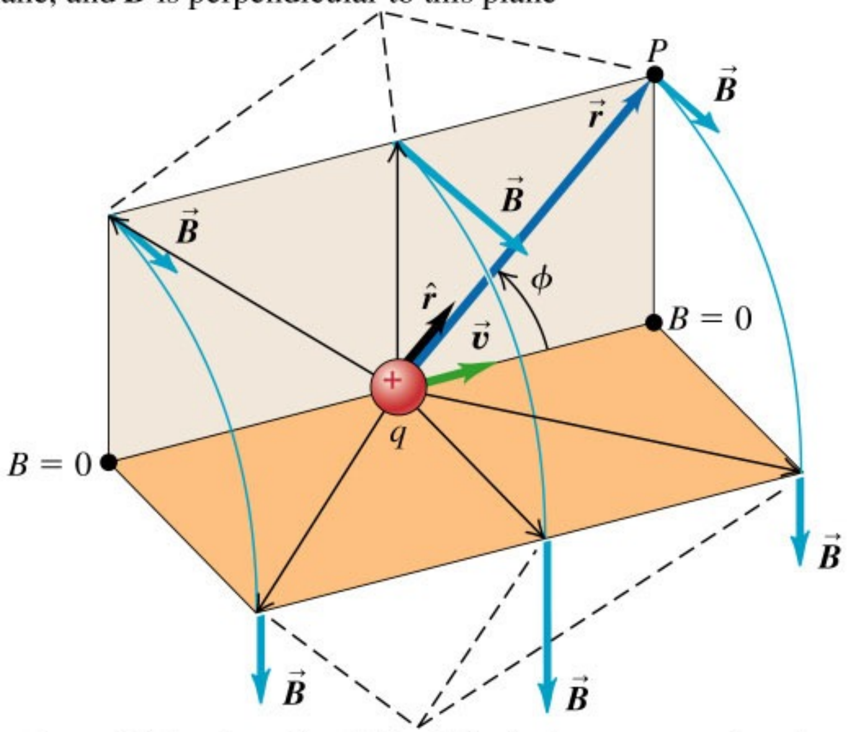
$\vec{B} = 0$

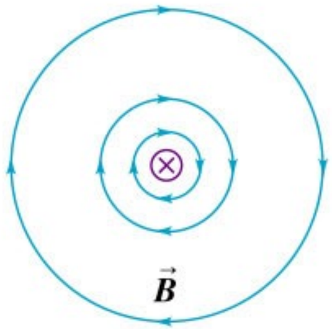
everywhere

+

Charge q
Stationary

Magnetic Field of a Moving Charge



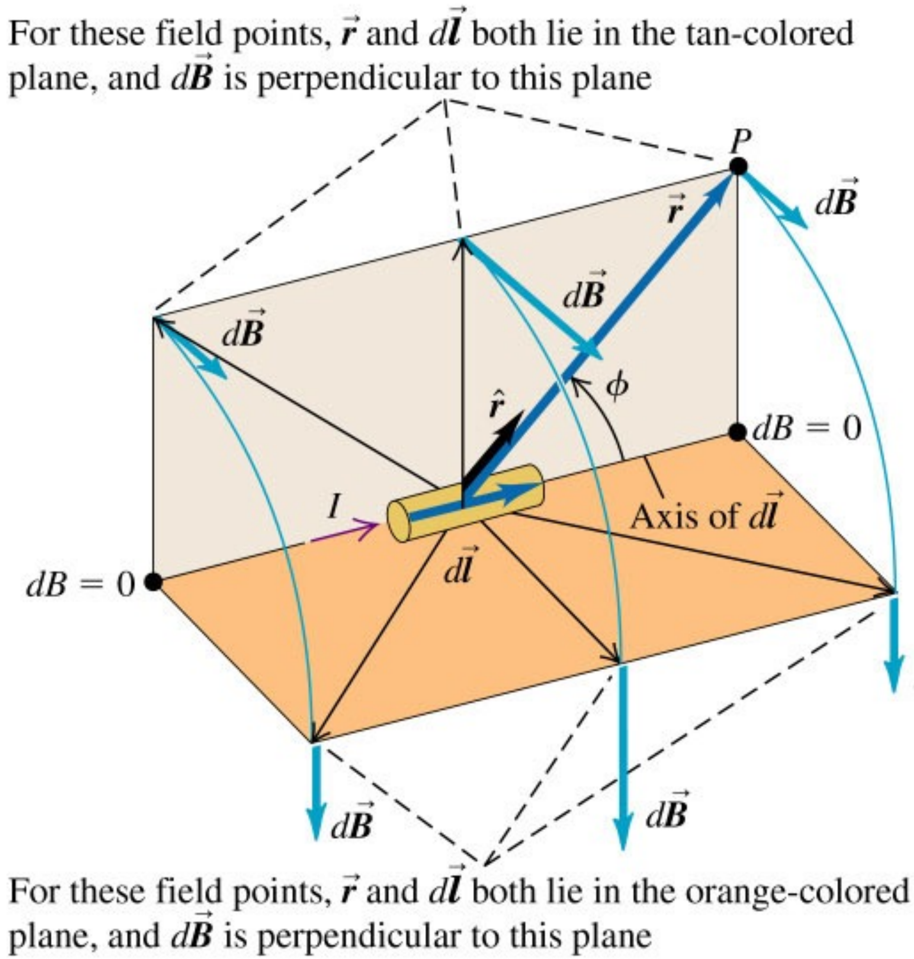


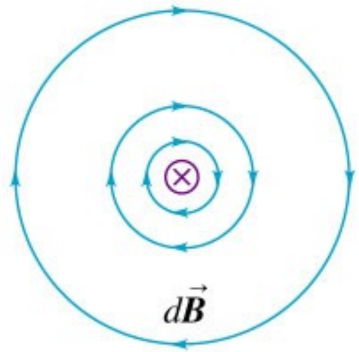
Use right hand rule

Magnetic field of a moving charge $$B = \frac{\mu_0}{4\pi} \frac{|q| v \sin\phi}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7} \ T \cdot m / A$$

Magnetic Field of a Current





(b)

Magnetic field of a current element

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{l} \times \hat{r}}{r^2}$$

Who is moving?

$$F_{\mu\nu} = \eta_{\mu\alpha} F^{\alpha\beta} \eta_{\beta\nu} = \begin{bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & -B_z & B_y \\ -E_y/c & B_z & 0 & -B_x \\ -E_z/c & -B_y & B_x & 0 \end{bmatrix}.$$

Electromagnetic Tensor

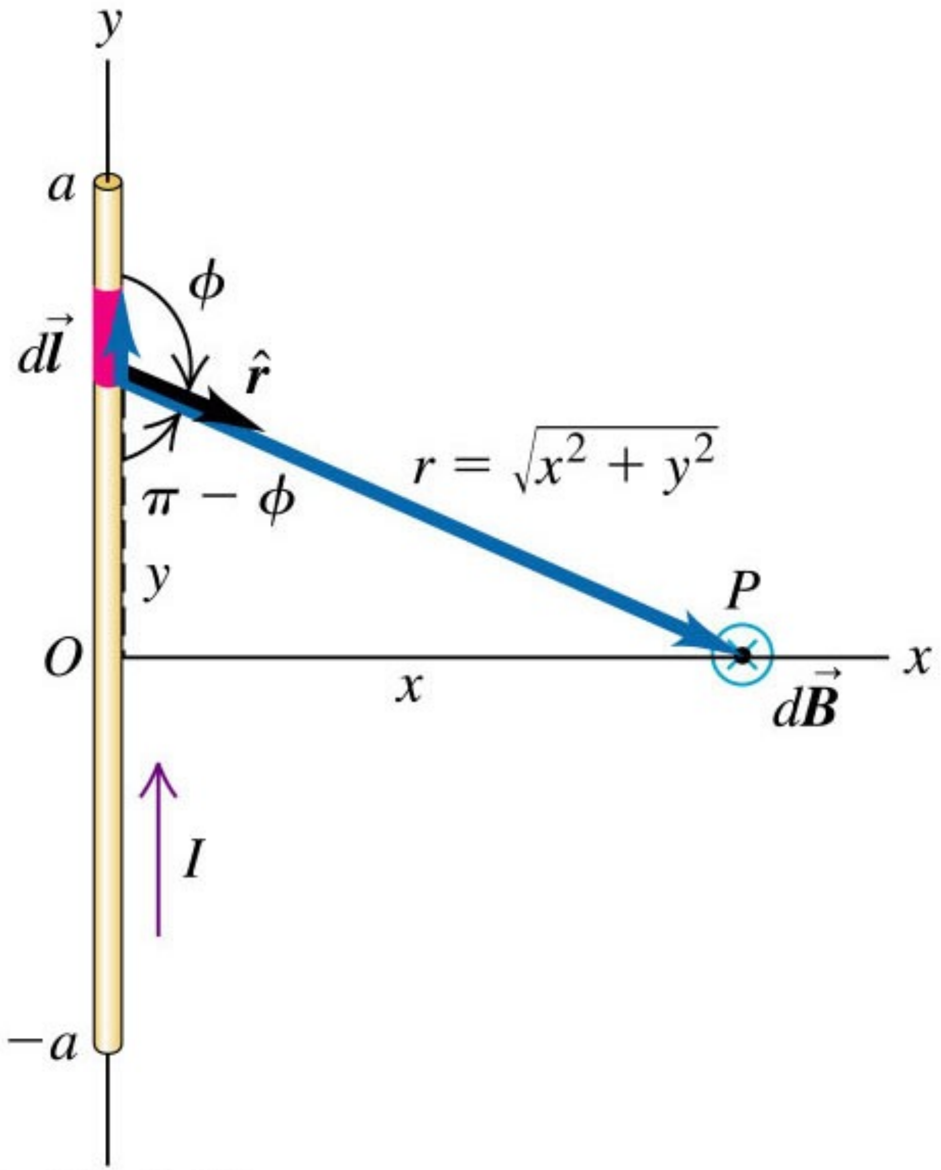
$+$

q

$$\begin{aligned} t' &= \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}} \\ x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\ y' &= y \\ z' &= z \end{aligned}$$

Lorentz Transformations

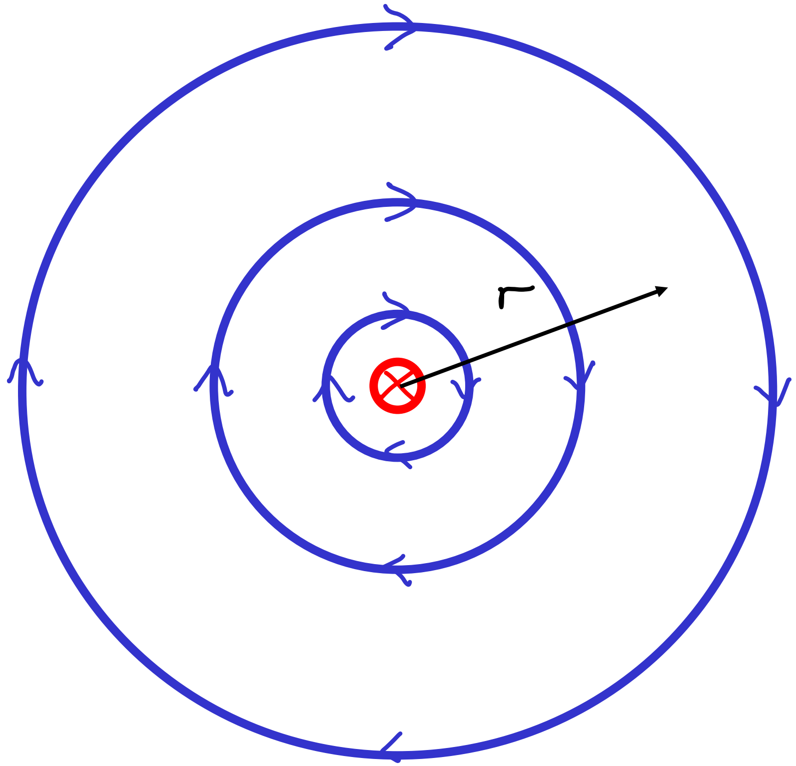
Magnetic Field of a Straight Current-Carrying Conductor



As $a \to \infty$

$$B_z = -\frac{\mu_0 I}{2\pi x}$$

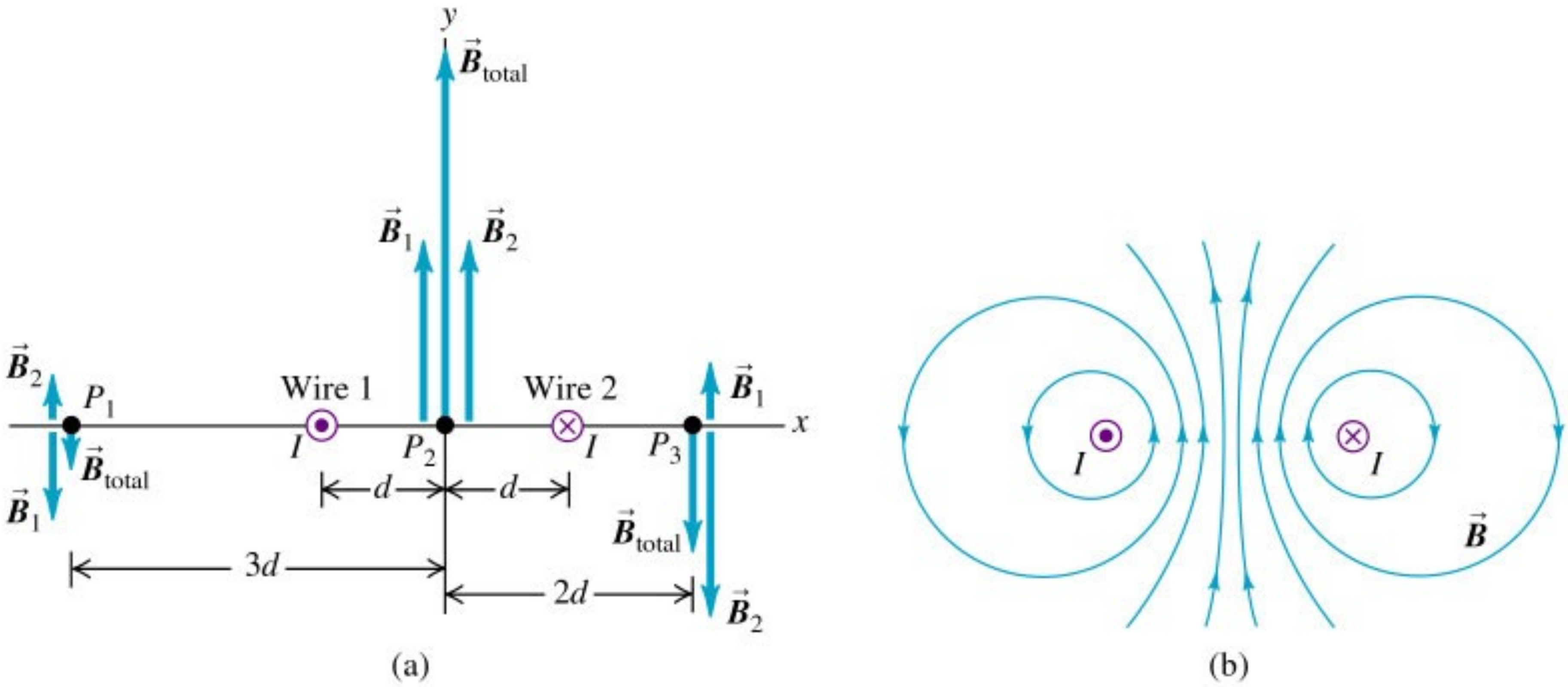
Magnetic Field of a Straight Current-Carrying Conductor



$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic field due to an (infinitely) long, straight, current carrying conductor

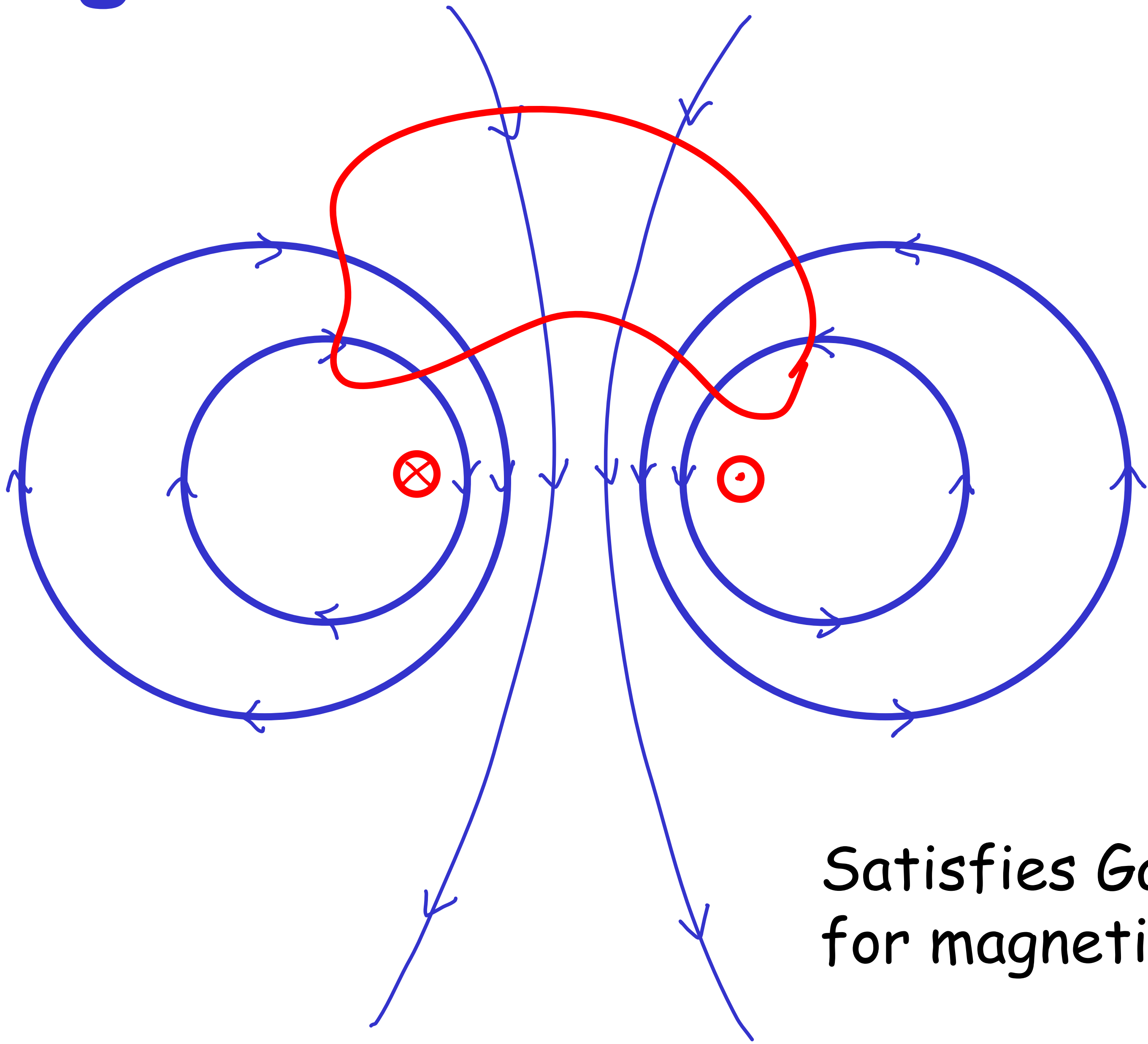
Magnetic Field of a Two Wires



Use the principle of superposition

Magnetic field at each point is given by the vector sum of the fields from each wire

Magnetic Field of a Two Wires



Satisfies Gauss's law for magnetism.

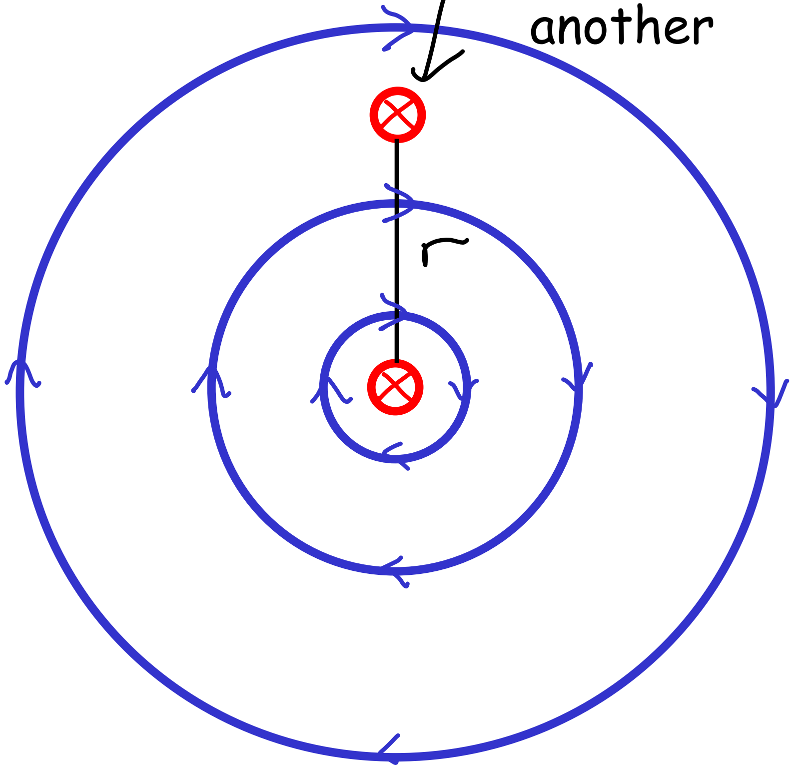
Force Between Parallel Wires

$$B = \frac{\mu_0 I}{2\pi r}$$

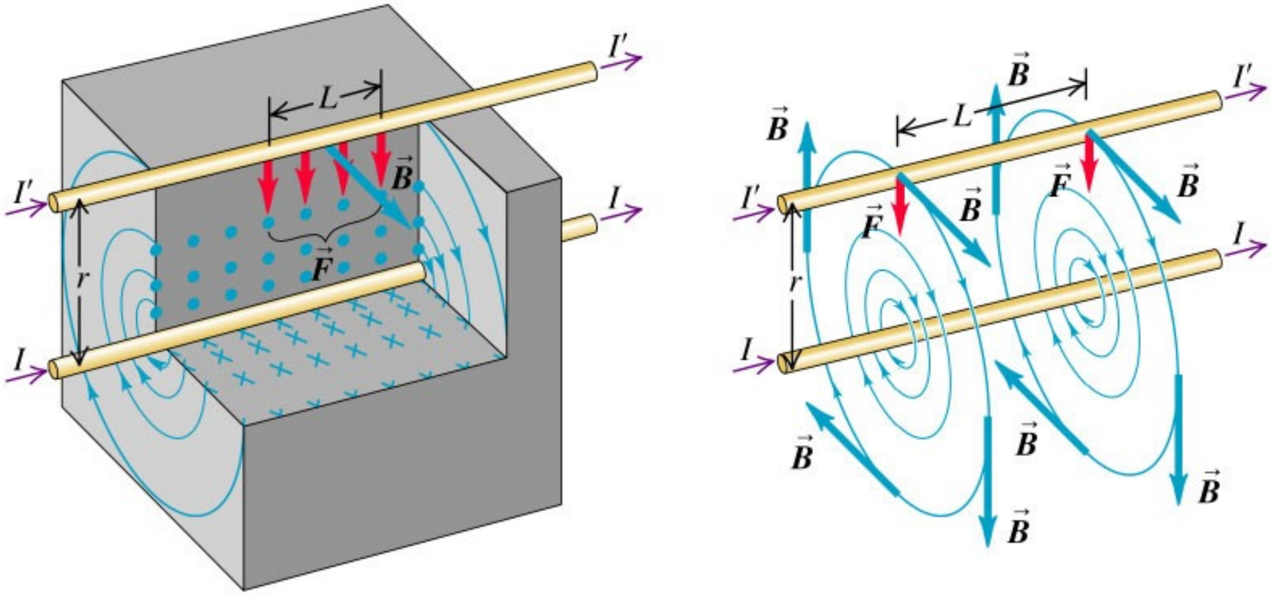
Use $$\vec{F} = I \vec{l} \times \vec{B}$$

Force per unit length $$\frac{F}{l} = \frac{\mu_0 I^2}{2\pi r}$$

place one wire in the field of another



Force Between Parallel Conductor



(a) (b)

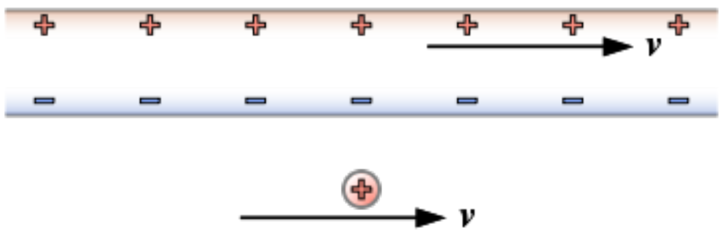
$$\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

force per unit length between two long, parallel current-carrying conductors

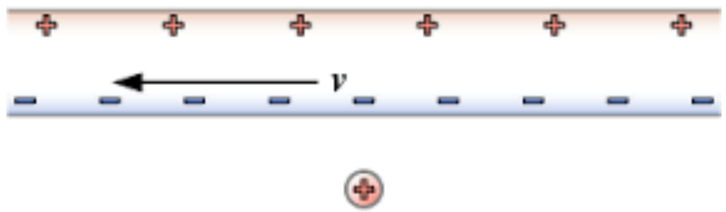
One ampere is that unvarying current that, if present in each of two parallel conductors of infinite length and one meter apart in free space, causes each conductor to experience a force of exactly 2 x 10^{-7} Newtons per meter of length

Explains why $\mu_0 = 4\pi \times 10^{-7}$ T m/A

Who is moving?

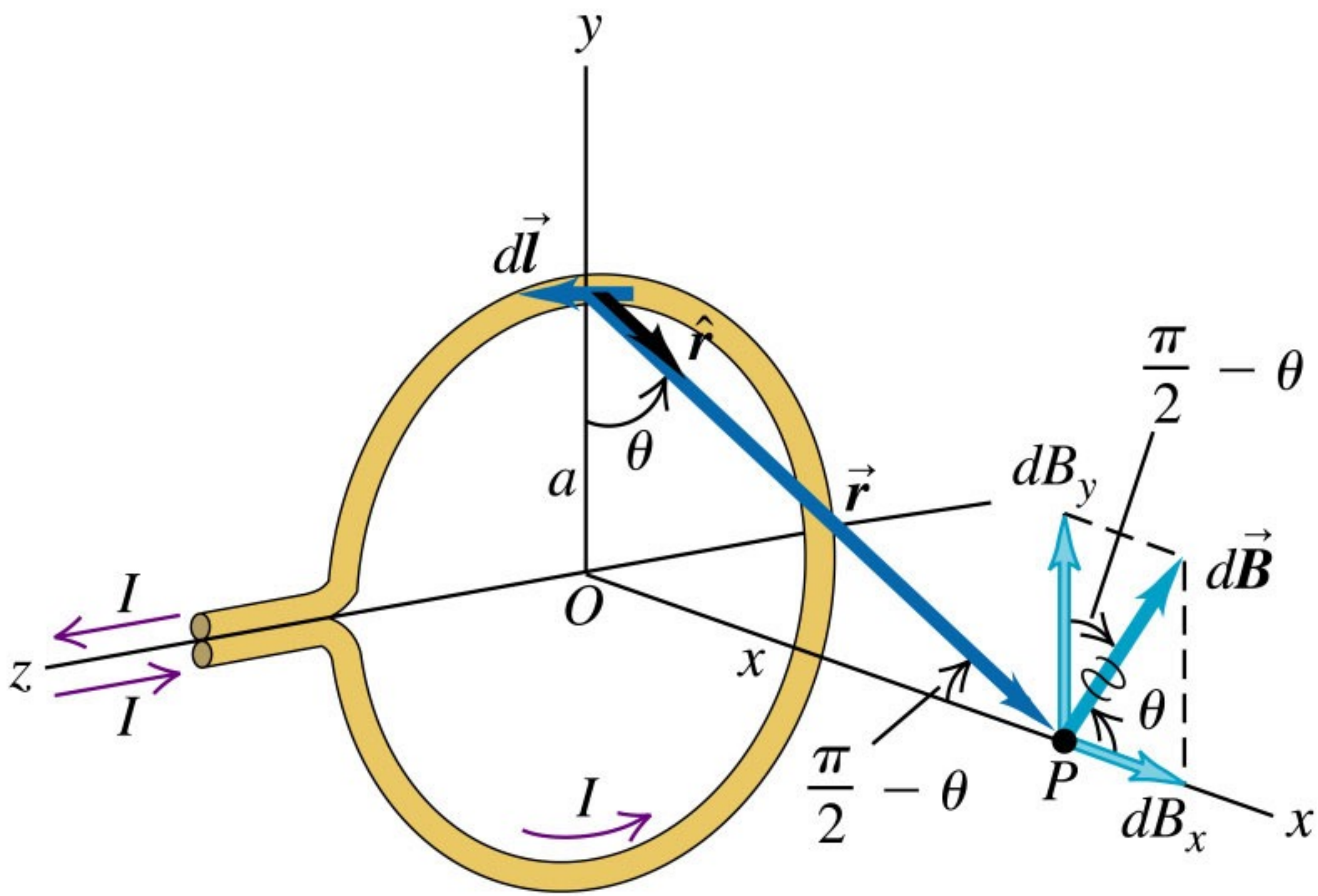


Laboratory Frame



Charge Frame

Magnetic Field of a Circular Current Loop

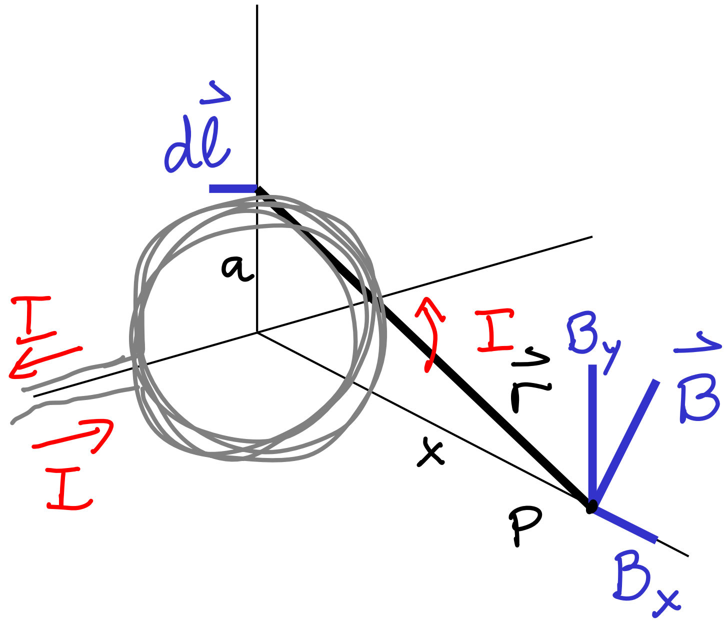


Magnetic field on axis of circular loop:

$$\vec{B} = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

in the +ve x-direction

Magnetic Field of a Circular Current Coil of Loops



Magnetic field on axis of circular loop:

$$\vec{B} = \frac{\mu_0 N I a^2}{2(x^2+a^2)^{3/2}}$$ in the +ve x-direction

$$\vec{B} = \frac{\mu_0 \vec{\mu}}{2\pi(x^2+a^2)^{3/2}}$$

Line Integral of B

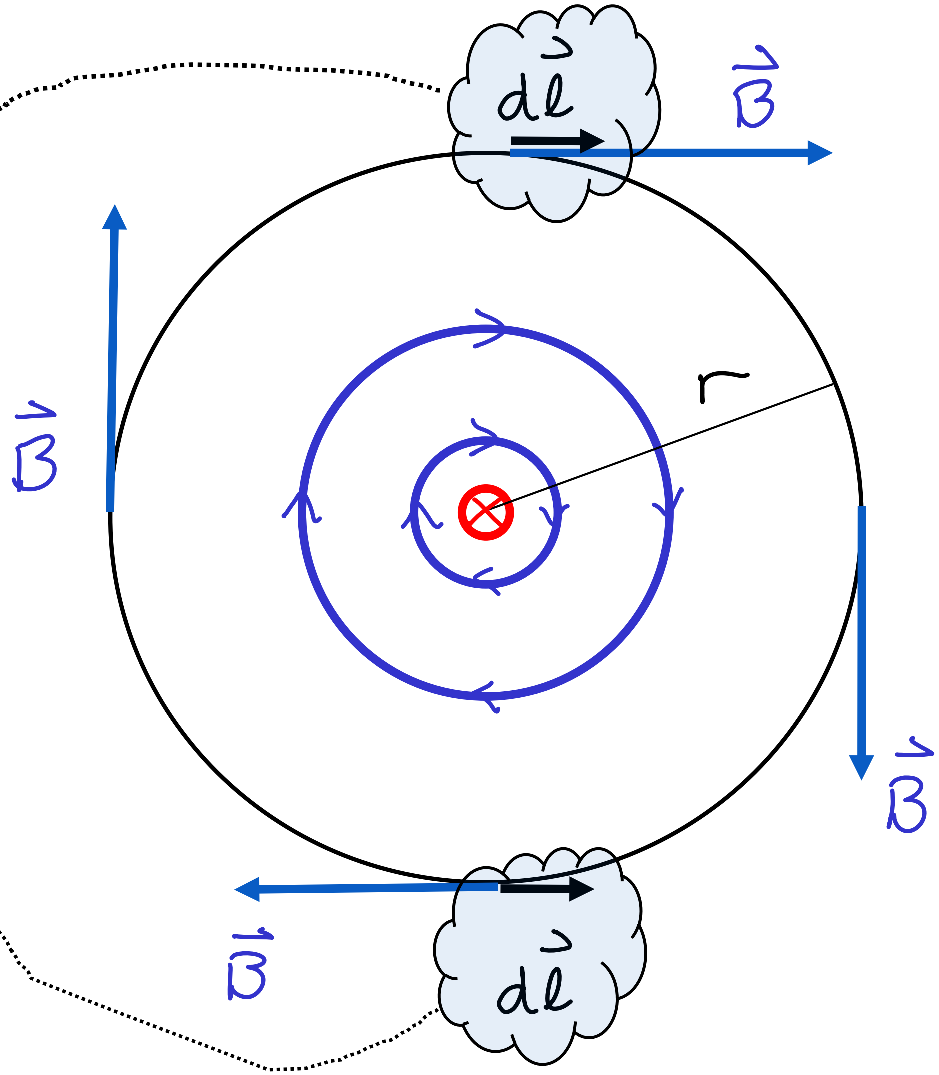
$$\oint \vec{B} \cdot d\vec{l}$$

If B is constant on the path, for a circle:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r$$

Line integral is directed; if we integrate the other way:

$$\oint \vec{B} \cdot d\vec{l} = -B \cdot 2\pi r$$



Line Integral of B

$$\oint \vec{B} \cdot d\vec{l}$$

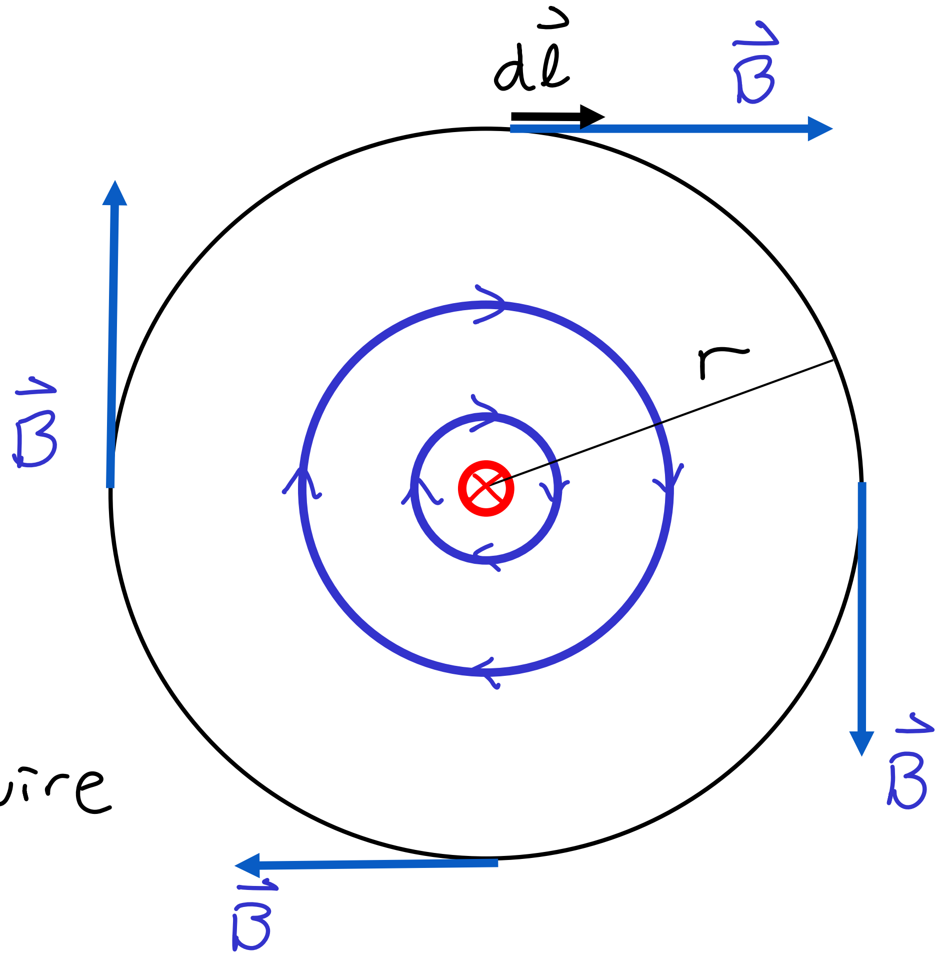
If B is constant on the path, for a circle:

$$\oint \vec{B} \cdot d\vec{l} = B \cdot 2\pi r$$

Recall:

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{for wire}$$

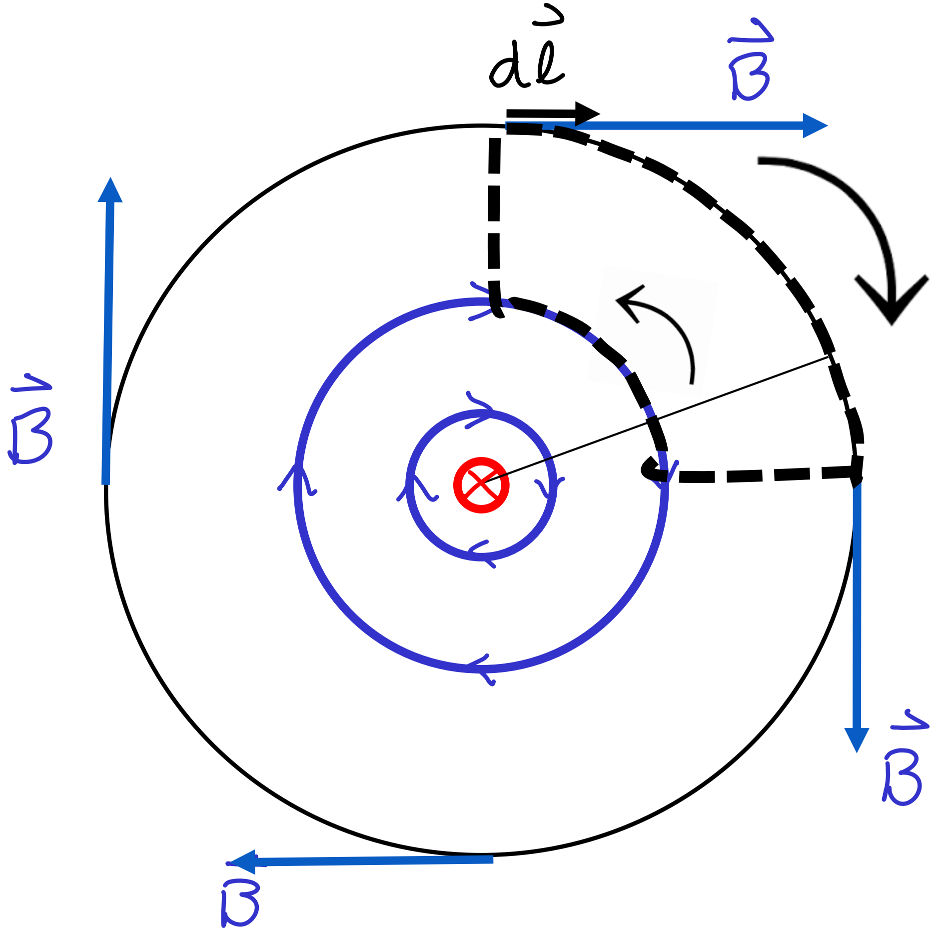
$$\oint \vec{B}\, d\vec{l} = \mu_0 I$$



Line Integral of B

If we integrate around path which does not encompass the current then:

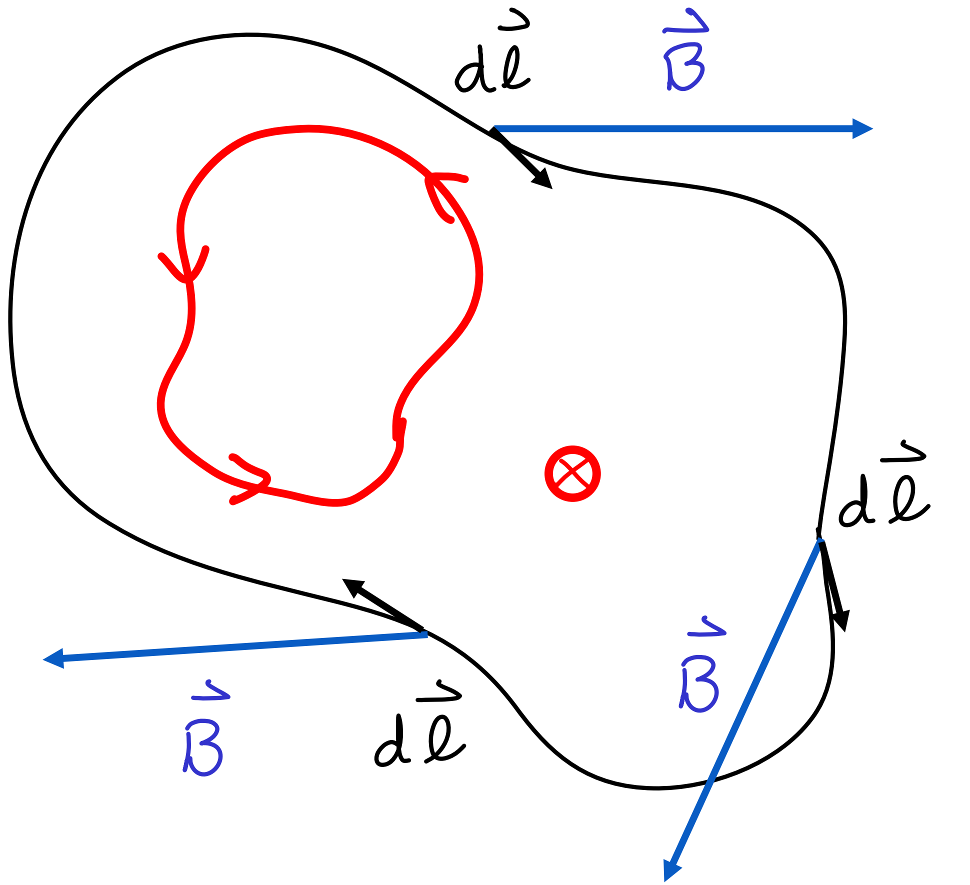
$$\oint \vec{B} \cdot d\vec{l} = 0$$



Ampere's Law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

The line integral of the magnetic field around any path is proportional to the current passing through any area defined by that path



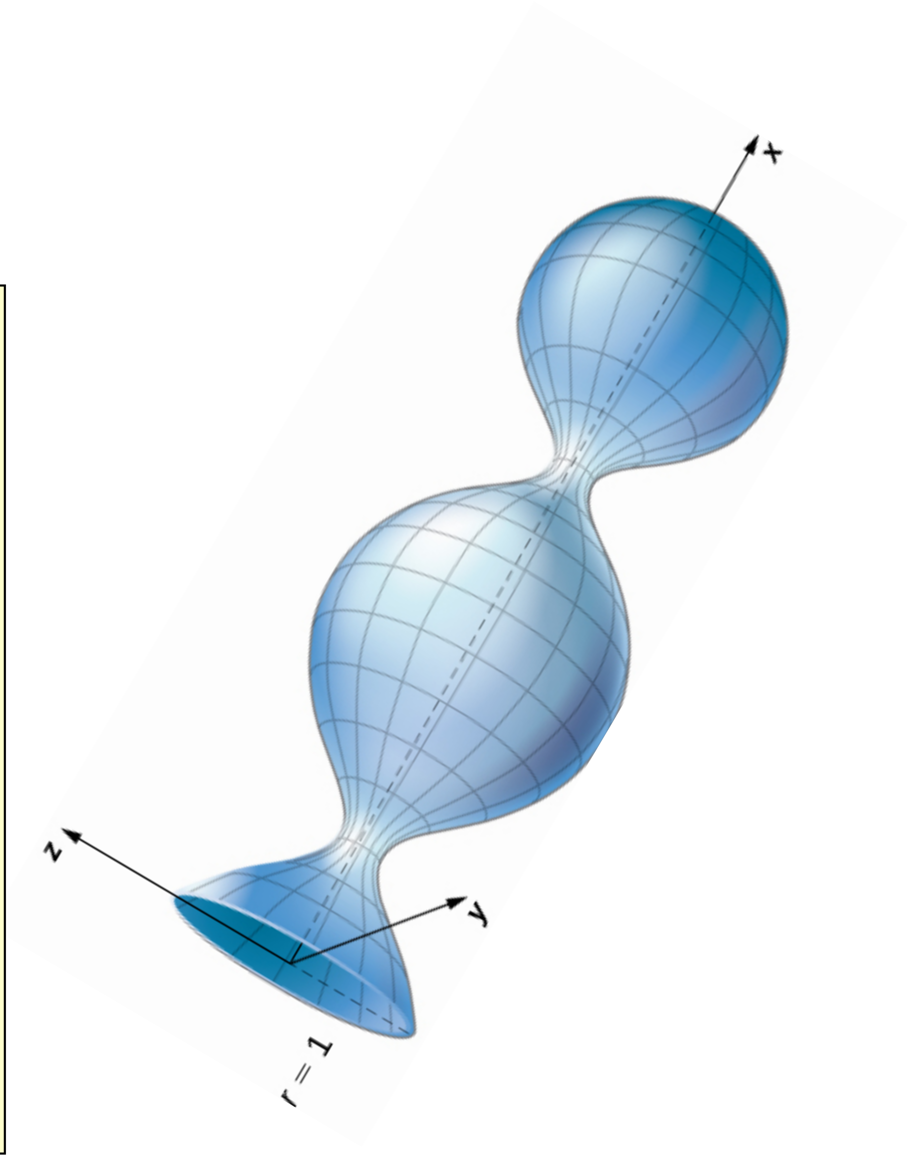
Signs matter!

Ampere's Law

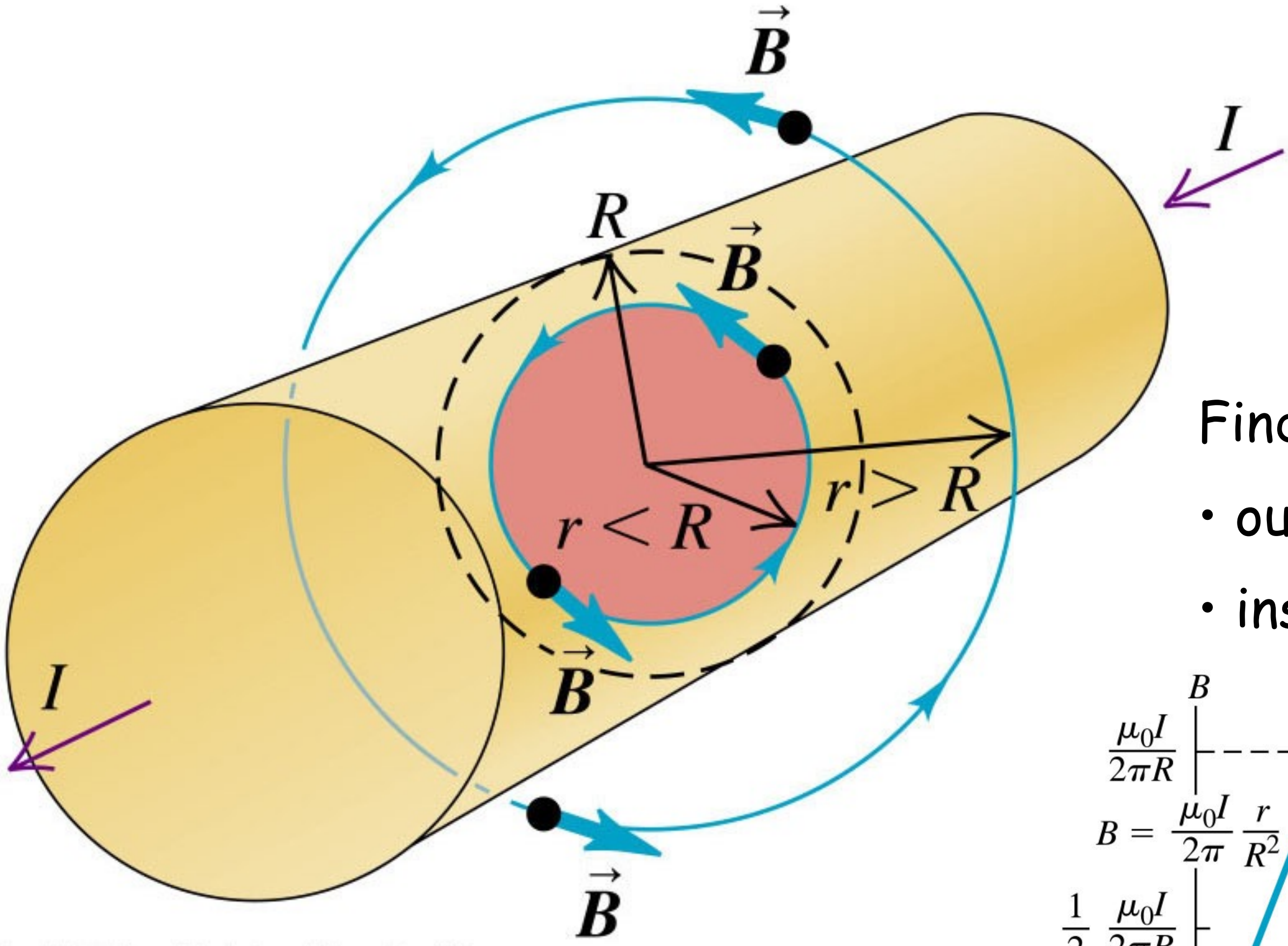
$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$$

The current has to "pierce" any surface bounded by the path.

How can this be? Surely different surfaces will be pierced by different currents?

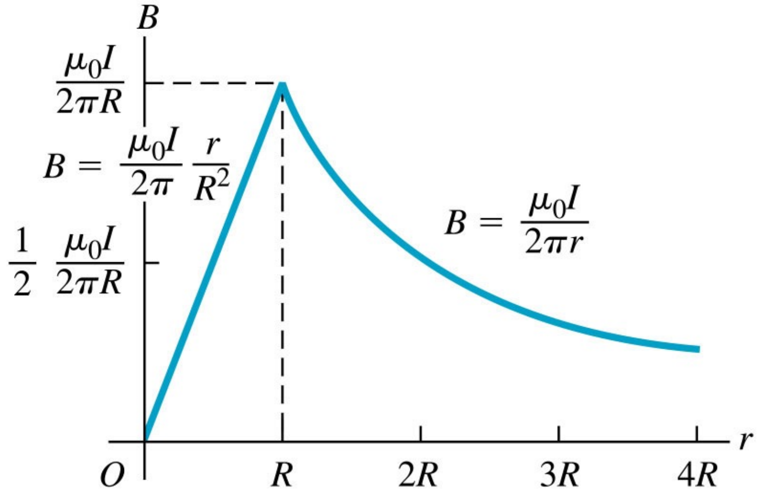


Example 28.9

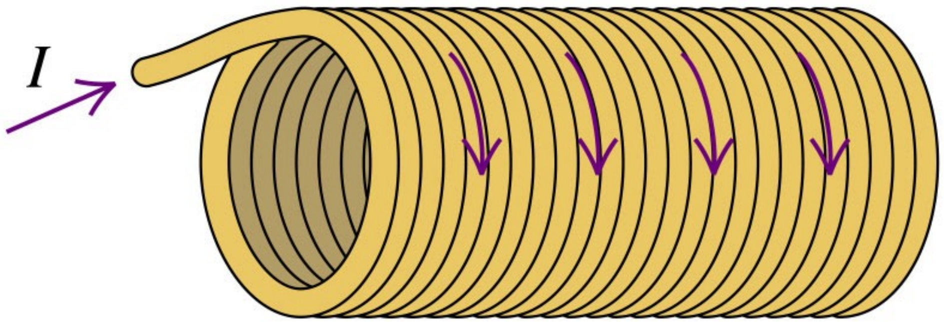


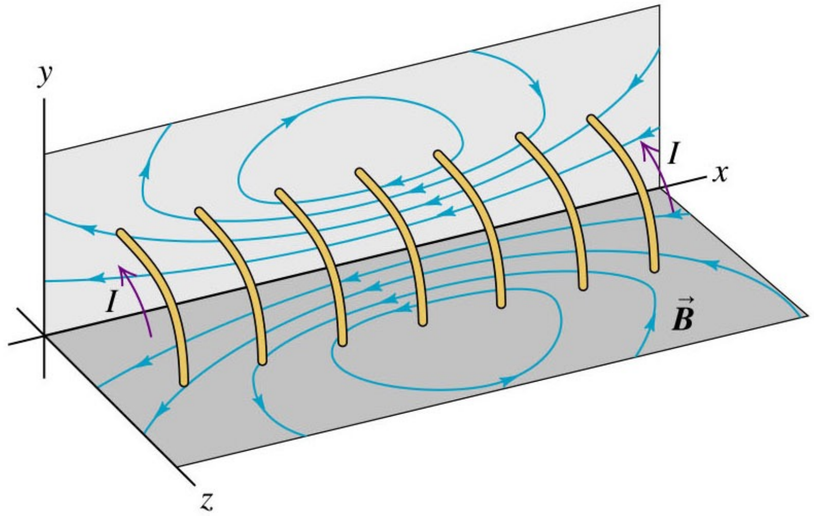
Find the magnetic field:

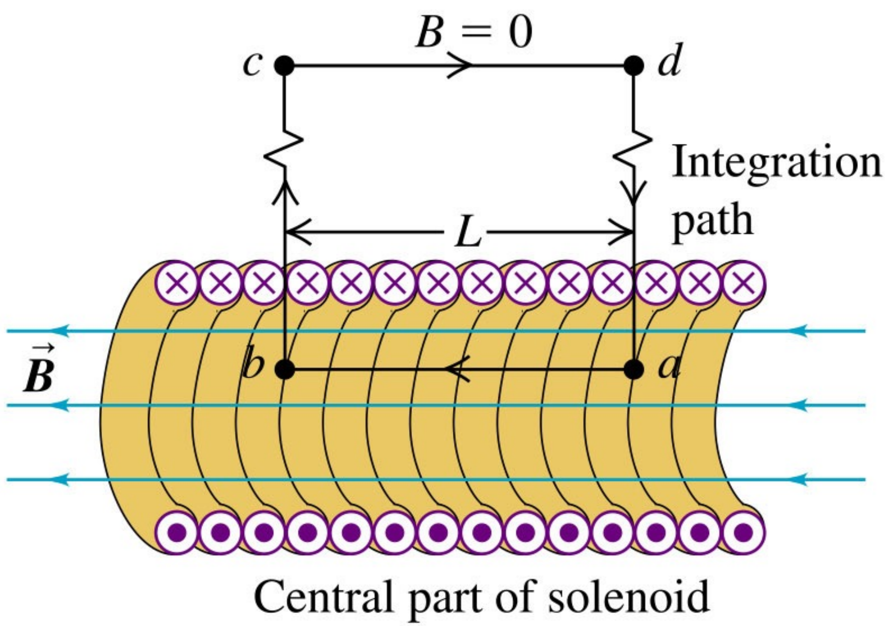
- outside the wire
- inside the wire



Example 28.10 - Solenoid

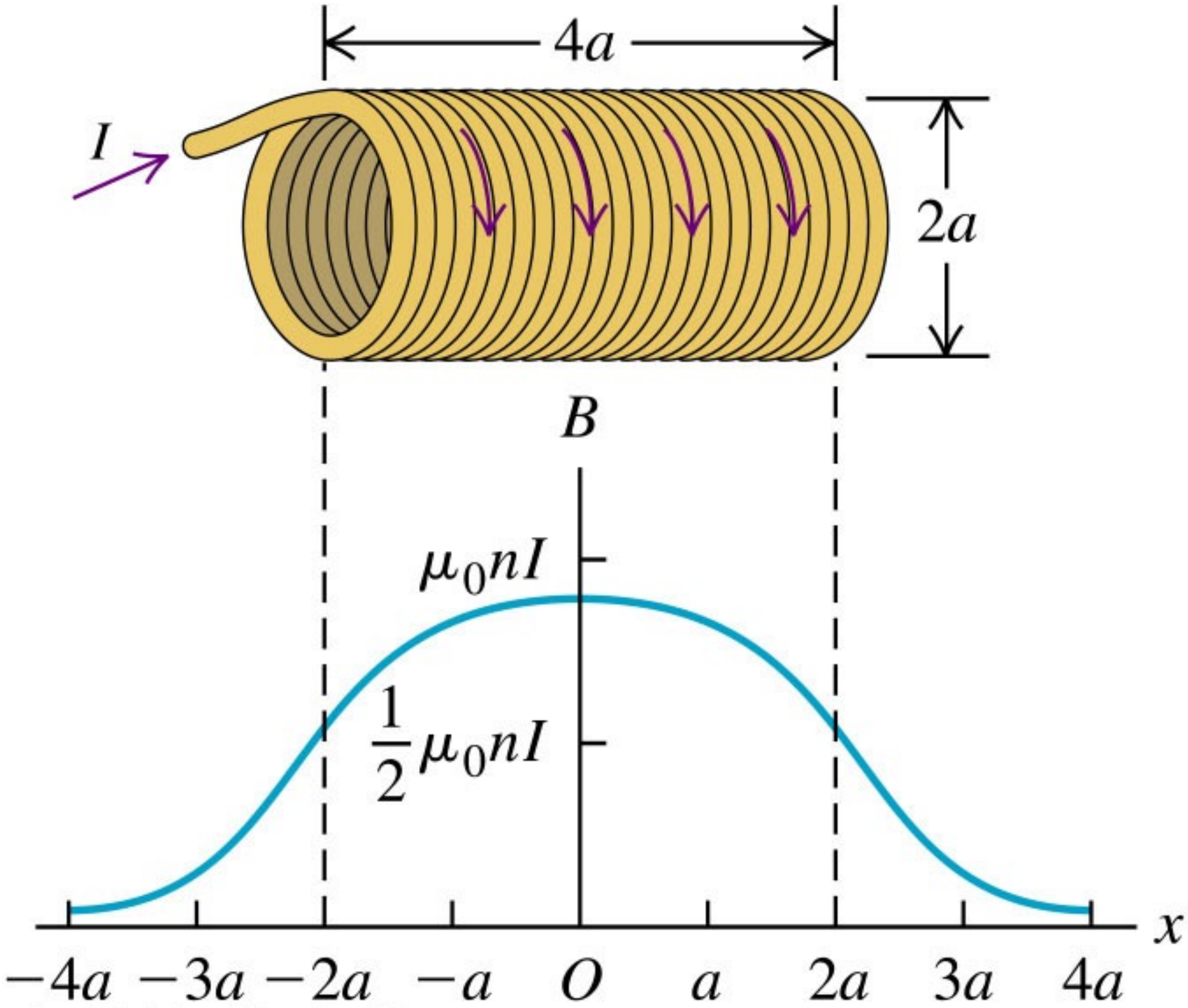




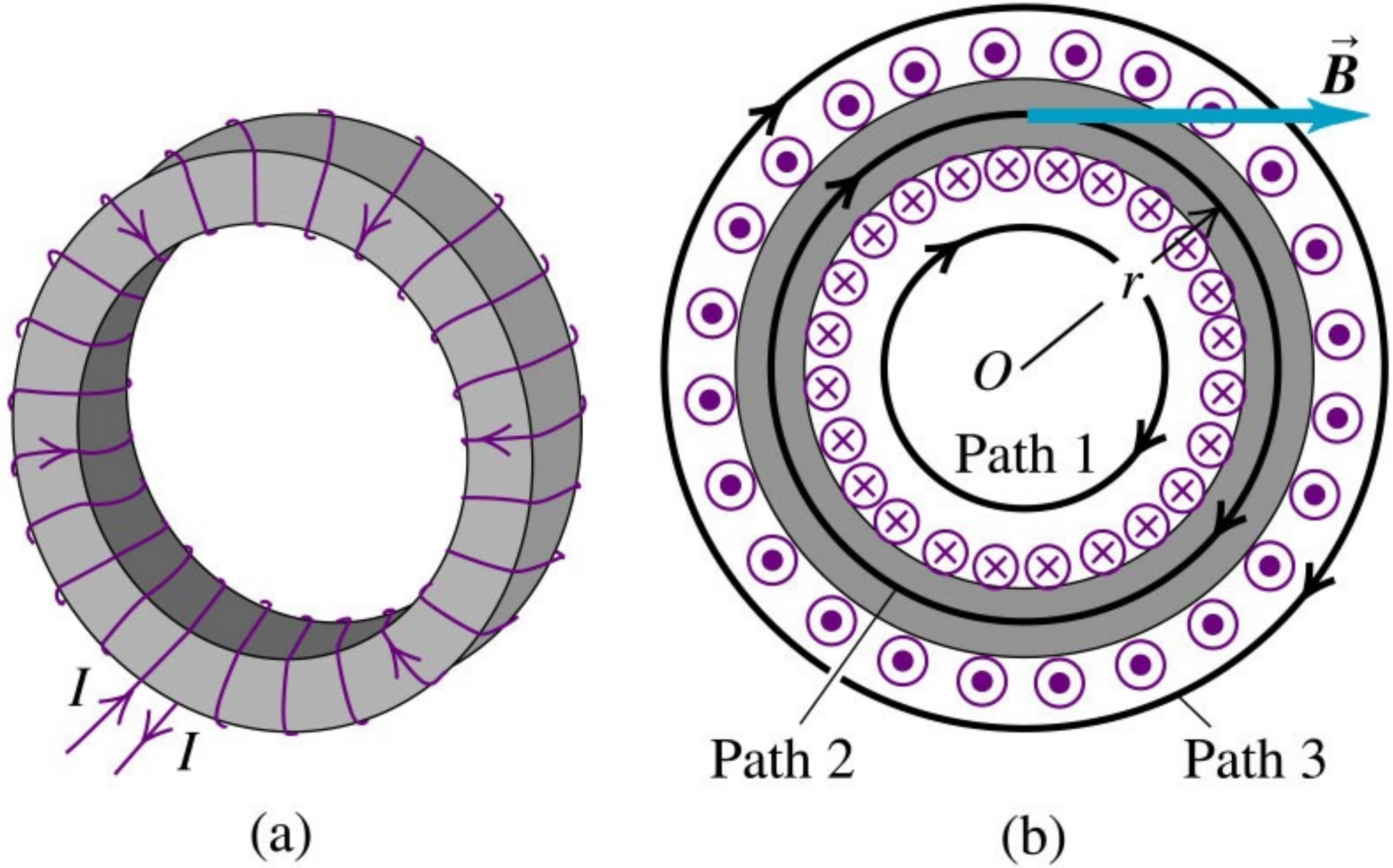


$$B = \mu_0 n I$$

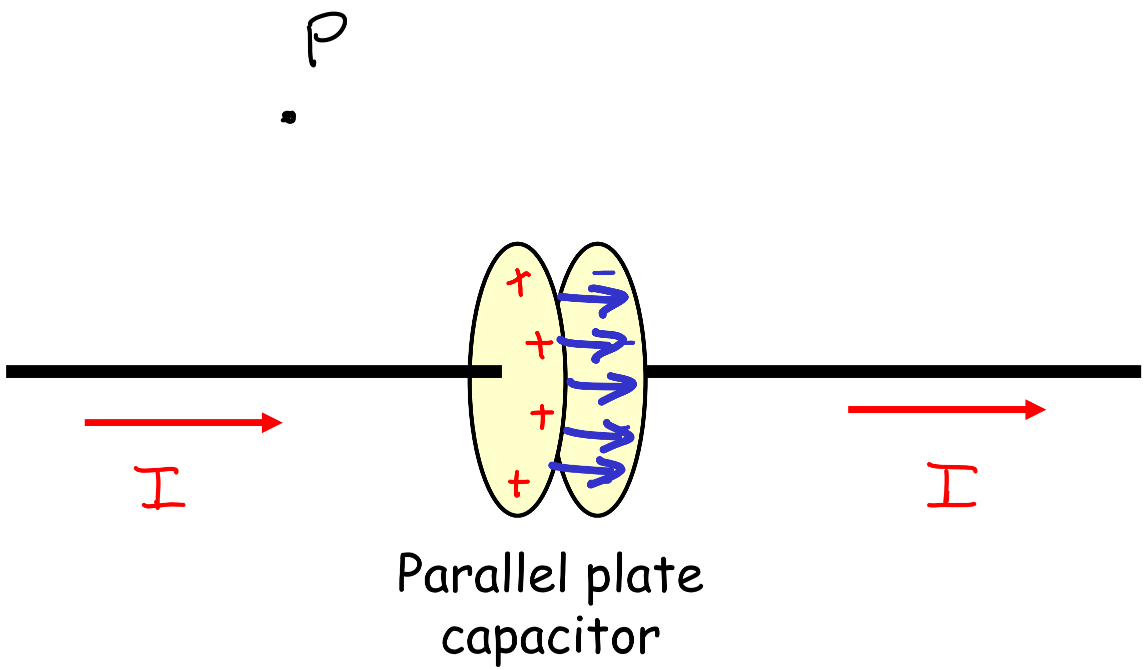
Example 28.10 - Solenoid



Example 28.11 – Toroidal solenoid



Puzzle: Charging capacitor



What is the magnetic field at point P?

Puzzle: Charging capacitor