The University of Sydney
School of Mathematics and Statistics

Assignment 3

MATH1906: Mathematics Special Studies Program A — Semester 1, 2012

Web Page: http://www.maths.usyd.edu.au/u/UG/JM/MATH1906/
Lecturer: Anne Thomas

> Due on **Friday, June 8** by **16:30** in Carslaw **Room 615**
> (Slide under the door when locked).
>
> Late assignments are not accepted without *prior arrangement* well before the deadline!
>
> You must attach the signed cover-sheet to the front of your assignment (see over)!

1. Given a subset X of the complex plane $\mathbb{C}$, recall that the notation $f : X \to X$ means that f is a function defined on X so that $f(X) \subseteq X$. If $f : X \to X$ then we say that f is *distance-preserving* if for all $z_1, z_2 \in X$,

$$|f(z_1) - f(z_2)| = |z_1 - z_2|.$$

(a) Show that for all $X \subseteq \mathbb{C}$, any function $f : X \to X$ which is distance-preserving must be injective. *1 Mark*

(b) Denote by $S(a, r)$ the circle in $\mathbb{C}$ with centre a and radius $r > 0$. Let b be a point on $S(a, r)$. Show that: *2 Marks*

(i) If w is any point on $S(a, r)$ then $0 \leq |w - b| \leq 2r$.

(ii) If $0 < d < 2r$ then there are exactly 2 points on $S(a, r)$ at distance d from b, and if $d = 2r$ there is a unique point on $S(a, r)$ at distance d from b.

(c) Suppose that $f : \mathbb{C} \to \mathbb{C}$ is a distance-preserving function. Fix $w \in \mathbb{C}$ and let $r = |f(0) - w|$. *3 Marks*

(i) Show that the points $f(r + i0) = f(r)$, $f(-r)$ and w all lie on the circle $S(f(0), r)$.

(ii) Let $d = |f(r) - w|$. Use (a), (b) and (c)(i) to show that there is a $z \in \mathbb{C}$ such that $f(z) = w$. Conclude that any distance-preserving function $f : \mathbb{C} \to \mathbb{C}$ is a bijection.

(d) Let $\mathcal{U} = \{z \in \mathbb{C} \mid \mathrm{Re}(z) > 0\}$ be the *upper half-plane*. Give an example of a distance-preserving function $f : \mathcal{U} \to \mathcal{U}$ which is not a bijection. *1 Mark*

2. Sketch a tiling pattern that you find around the university, and a tiling pattern that you find off-campus (and say where you found them). On your sketches, indicate any kaleidescopes, gyration points and miracles, and hence determine the signature of each pattern. *3 Marks*

You will not obtain full marks if either of your examples has the same signature as the brick pattern in our classroom.

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Assignment Cover Sheet

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Family Name ....

Given Names .... **SID** ....

Some collaboration between students on assignments is encouraged, since it can be a real aid to understanding. Thus it is legitimate for students to discuss assignment questions at a general level, provided everybody involved makes some contribution. However, students should produce their own individual written solution. Copying someone else's work is plagiarism, and is unacceptable. The University may impose severe penalties in cases where plagiarism is detected.

I certify that:

- I have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure* at http://www.maths.usyd.edu.au/u/UG/Plagiarism.pdf.
- this assignment is all my own work, and that no part of this assignment has been copied from another person.
- I have not allowed my work to be copied by another person.

Signature .... **Date** ....

This part to be completed by the marker:

Grand total out of 10 ....