PHYS 1901 – Physics 1A (Advanced)
Mechanics module

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Gravitation

Chapter

12

(Note: we are not covering Chapter 11 in this module)

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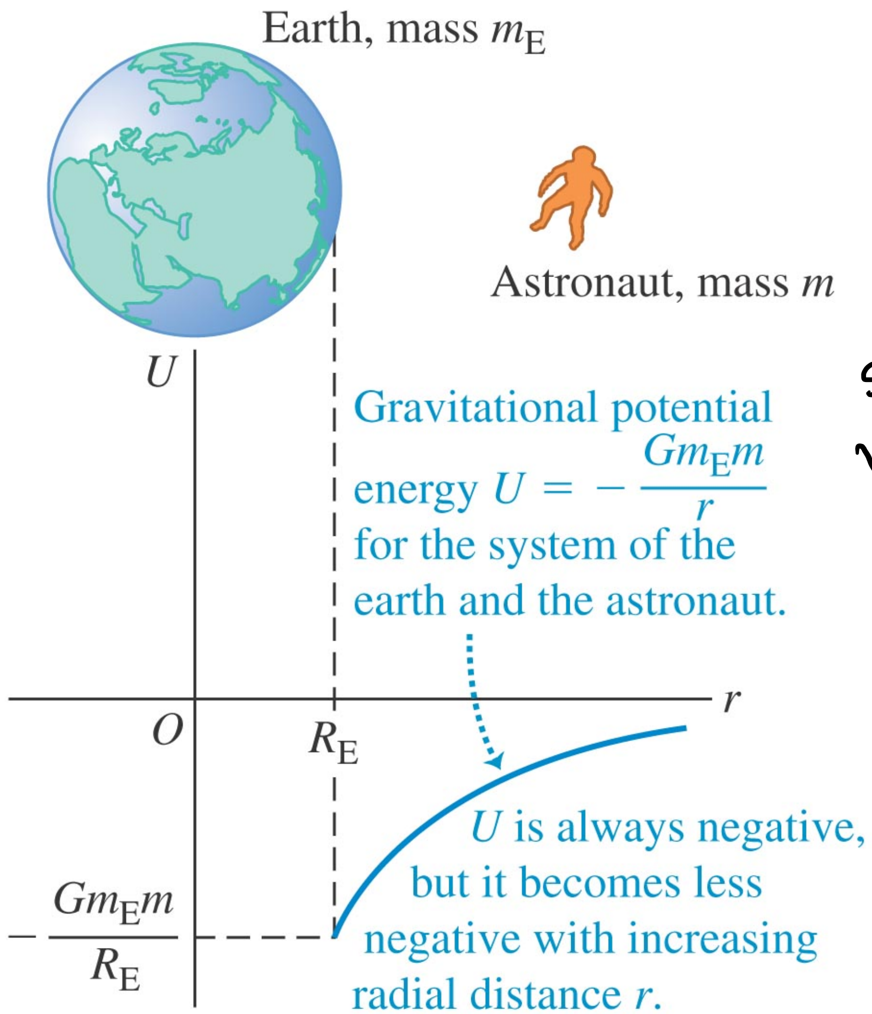
Gravitational potential energy



Imagine bringing two mass from far apart to close together. There is a change in the gravitational potential given by:

$$W_G = -\Delta U_G \leftarrow$$ what is this?

Solve for W_G using F_G

Start at r_1, end at $r_2 < r_1$

$$W_G(r_1 \to r_2) = \int^{r_2} \vec{F}_G \cdot d\vec{s}$$

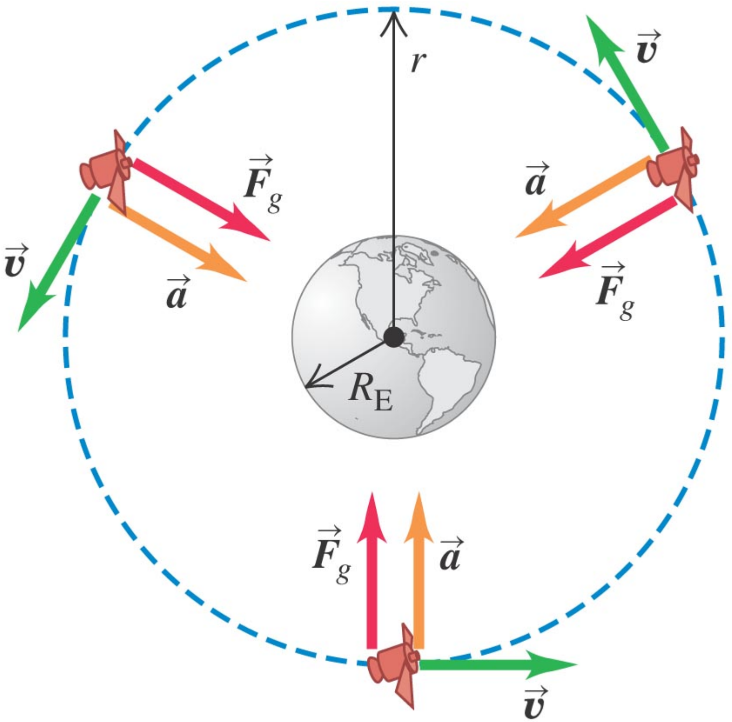
$$= \int_{r_1}^{r_2} \left(-\frac{GMm}{r^2}\right) dr$$

$$= \int_{r_2}^{r_1} \frac{GMm}{r^2}\, dr = -\left[\frac{GMm}{r}\right]_{r_2}^{r_1}$$

$$= \underbrace{-\frac{GMm}{r_1} + \frac{GMm}{r_2}}_{-\Delta U_G} > 0$$ positive if $r_2 < r_1$

$$\boxed{U_G(r) = -\frac{GMm}{r}}$$

Orbits – Motion of satellites



The satellite is in a circular orbit: Its acceleration $\vec{a}$ is always perpendicular to its velocity $\vec{v}$, so its speed v is constant.

Gravity provides a centripetal force. If a satellite has the correct velocity, *v*, it will move in a circular orbit, continually falling towards the Earth, but not getting any closer.

$$F_G = \frac{G M_E m}{r^2}$$

$$F_c = F_G$$

$$\frac{m v^2}{r} = \frac{G M_E m}{r^2}$$

$$\Rightarrow v = \sqrt{\frac{G M_E}{r}}$$

Motion of satellites

The period of the orbit is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\sqrt{GM_E}} r^{3/2}$$

With this, you can calculate the radius of a geostationary orbit (42 000 km).

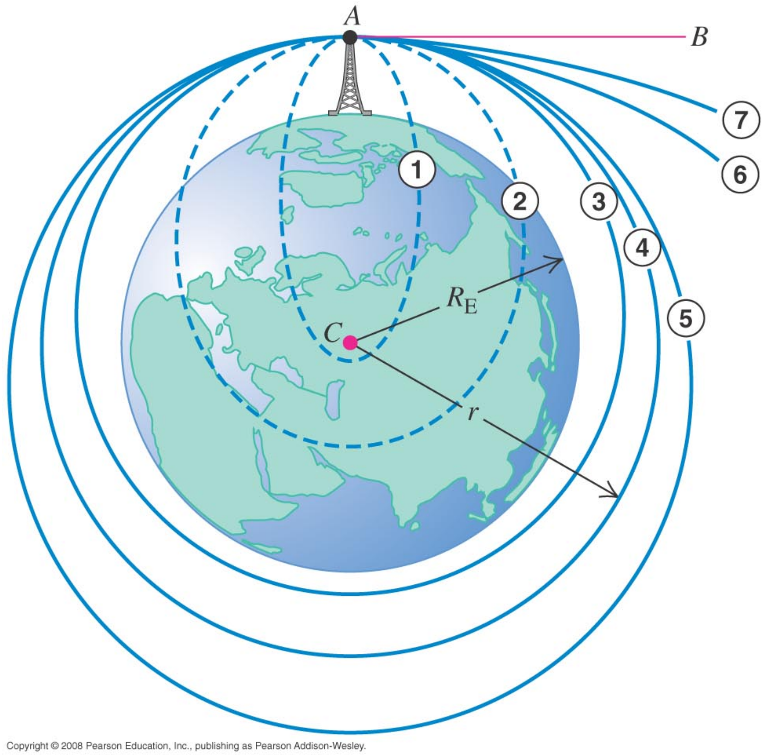
The total energy for any circular orbit is

$$E = K + U_G$$
$$= \frac{1}{2} m v^2 - \frac{GMm}{r}$$
$$= \frac{1}{2} \frac{GM_E m}{r} - \frac{GMm}{r}$$
$$= -\frac{1}{2} \frac{GM_E m}{r}$$

Recall

$$v = \sqrt{\frac{GM_E}{r}}$$

Non-circular orbits



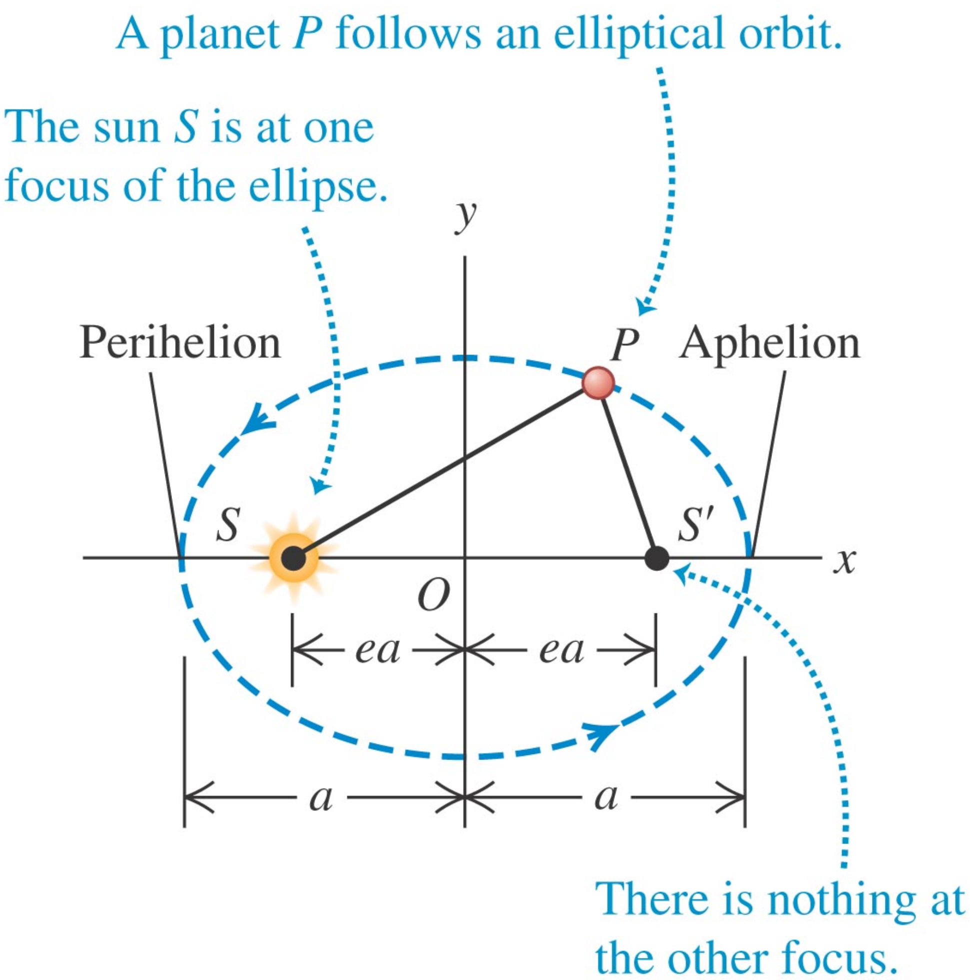
A projectile is launched from A toward B. Trajectories 1 through 7 show the effect of increasing initial speed.

What if the velocity is too small for circular motion?

Gravity is greater than the required centripetal force and the objects radial position changes. By conservation of energy, it speeds up, then being too fast for circular motion.

Newton showed that the resultant motion is elliptical, or if the velocity is much greater than circular, the orbit is unbound and hyperbolic.

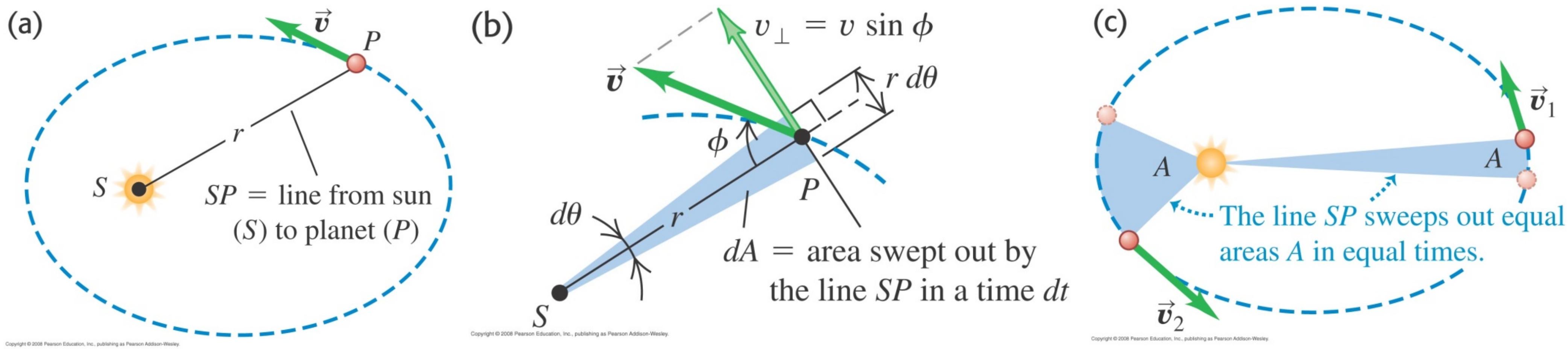
Kepler's first law



Before Newton derived the mathematical form of orbits, Kepler determined three empirical laws

His 1st Law says that orbits in the solar system are elliptical, with the Sun at one focus

Kepler's second law



The 2nd Law says that the area an orbit sweeps out in a fixed time is a constant.

$$\frac{dA}{dt} = \text{constant} = \frac{1}{2} r \cdot \frac{r\,d\theta}{dt}$$

$$= \frac{1}{2} r^2 \omega$$

not const.   not constant

Kepler's second law

Recall that the angular momentum is

$v_\perp = r\omega$

$$\vec{L} = \vec{r} \times \vec{p} \Rightarrow L = r p_\perp = r m v_\perp = m r^2 \omega$$

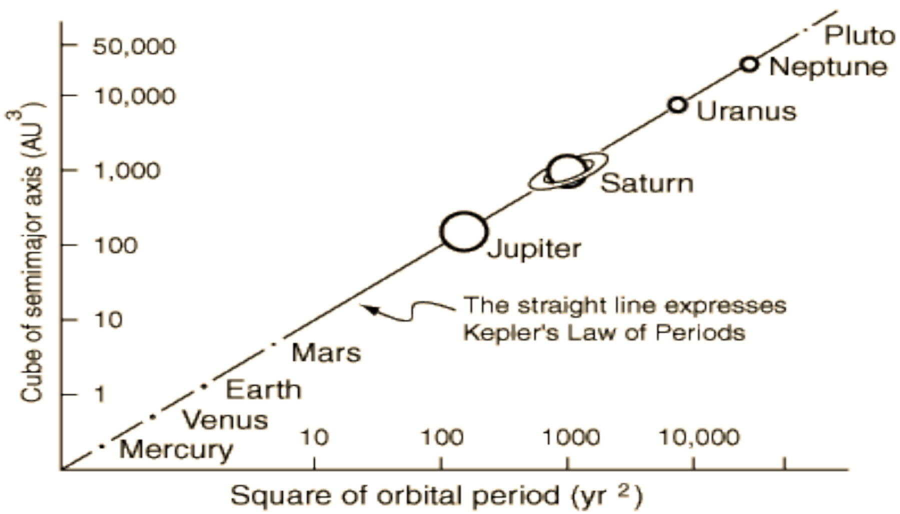
For an elliptical orbit, *r* and ω are continually changing, but *L* remains a constant.

Given this, we see that

$$\frac{dA}{dt} = \frac{1}{2} r^2 \omega = \frac{L}{2m}$$

Kepler's 2nd Law is simply an expression of the conservation of angular momentum.

Kepler's third law



http://hyperphysics.phy-astr.gsu.edu/hbase/kepler.html

Kepler's 3rd Law is relates the period of an elliptical orbit with semi-major axis a

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$

Note that this is not dependent upon the mass of the orbiting object.