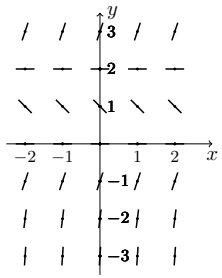
THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Sample Quiz 2

MATH1903: Integral Calculus and Modelling (Advanced) Semester 1, 2017

1. Sketch the direction field of the differential equation $y' = y(y-2)$ in the region below. *2 Marks*



2. Find the general solution of the differential equation $y' = y^2 \sin x$. *2 Marks*

Answer Q2: $y = \dfrac{1}{C + \cos x}$ or $y = \dfrac{1}{-C + \cos x}$

3. It is given that $y = \dfrac{Cx}{x - C}$ is the general solution of a differential equation. *2 Marks*

Determine the constant C for the particular solution satisfying the initial condition $y(2) = 3$.

Answer Q3: $C = \dfrac{6}{5}$

4. Find the particular solution of the differential equation $z' = (2x+1)z$ with $z(0) = 4$. *2 Marks*

Answer Q4: $z = 4e^{x^2+x}$

5. Suppose that y satisfies the differential equation $\dfrac{dy}{dx} = 3(y+2x)^2$. Find the differential equation for $v = y + 2x$. *2 Marks*

Answer Q5: $\dfrac{dv}{dx} = 3v^2 + 2$

6. Find an integrating factor for the differential equation $y' - \dfrac{1+x}{x}y = q(x)$. *2 Marks*

Answer Q6: $\dfrac{1}{x}e^{-x}$

7. Find the general solution of $\dfrac{dy}{dt} = -3t\cos^2 y$. *2 Marks*

Answer Q7: $y = \tan^{-1}\left(C - \dfrac{3t^2}{2}\right)$

8. A molecule of substance A can combine with two molecules of substance B to form a molecule of substance X, in a reaction which is denoted $A + 2B \to X$. According to the Law of Mass Action, the rate of formation of X is proportional to the product of the amounts of A and B present. A test-tube initially contains amounts a_0 and b_0 of substances A and B, respectively, but none of substance X. The amount of substance X at time t is $x(t)$ and k is a positive constant. *2 Marks*

Which differential equation models the amount of substance X?

(a) $\dfrac{dx}{dt} = k(a_0 - x)(2b_0 - x)$

(b) $\dfrac{dx}{dt} = k(a_0 - x)(b_0 - 2x)$

(c) $\dfrac{dx}{dt} = k(2a_0 - x)(b_0 - x)$

(d) $\dfrac{dx}{dt} = k(a_0 - 2x)(b_0 - x)$

Answer Q8: (b)

9. Find the general solution of $y' - 2xy = 3x$. *2 Marks*

Answer Q9: $-\dfrac{3}{2} + Ce^{x^2}$

10. Find the solution of the differential equation $\dfrac{du}{dx} = \dfrac{1+u^2}{2xu}$ with initial condition $u(1) = 2$. *2 Marks*

Answer Q10: $u = \sqrt{5x-1}$