THE UNIVERSITY OF SYDNEY
MATH1905 Statistics Advanced

Semester 2	**Sample Quiz 1 – Week 7**	2012

Time allowed 40 minutes

For the multiple choice questions please circle the letter corresponding to your answer. For the other questions write your answer in pen in the box provided on the answer sheet.

This sample quiz shows a range of possible problem topics that will be asked in the quiz. However, the various versions of the quiz will differ. For more sample questions check tutorials up and including week 6.

1. Find the median of the 6 observations : 1 2 3 4 5 6
[illegible]

2. How many observations of the following sample of size $n = 9$ lie in the interval $[Q_0, Q_1]$: $1, 2, \ldots, 9$
[illegible]

3. A correlation coefficient of $r = 0.6$ is reported for a sample of pairs (x_i, y_i). Without any further information this implies:
(a) the least squares straight line can explain 36% of the variability of the y values
(b) the least squares straight line can explain 60% of the variability of the y values
(c) the points (x_i, y_i) lie on a straight line of slope 0.6
(d) the points (x_i, y_i) are scattered about a straight line of slope 0.6
(e) with a probability of 60% the relationship between x and y is best described with a straight line.

4. For a sample $y_1, y_2, \ldots, y_{10}$ we have $\sum_{i=1}^{10} y_i = 55$ and $\sum_{i=1}^{10} y_i^2 = 385$. To 2 d.p., the sample variance s_y^2 is
[illegible]

5. An electronic system consists of two subsystems A and B, say. From previous testing the following probabilities are known: P(A fails) = 0.40, P(A and B fails) = 0.2, P(B fails and A does not fail) = 0.3. The probability that B fails is
[illegible]

6. The random variable X is described by the following probabilities.

i	-1	4
$P(X = i) = p_i$	0.8	0.2

What is the expected value of X?
[illegible]

7. What is $P(|X - 1.5| = 2.5)$ for the random variable X in Question 6?
[illegible]

8. The random variable X has probability generating function

$$\pi(s) = (1 - p + ps)^n.$$

The expected value of X is

$\mathrm{E}(X) =$ [illegible]

9. (1 mark) Let $\pi(s)$ be the probability generating function of X. Then $\mathrm{E}(X(X-1))$ equals
(a) $\pi''(1)$

10. The probabilities $\mathrm{P}(X = k)$ of $X \sim \mathcal{P}(\lambda)$ are:

$\mathrm{P}(X = k) =$ [illegible]

11. The following table shows the data for age (in years), x and height (in cm) y of 3 children:

Age, x	2	3	4
Height, y	80	93	97

Enter the data in R and calculate the value of the correlation coefficient:

Correlation = [illegible]

12. For the data in Question 11, the slope of the regression line of y explained by x is

b = [illegible]

13. If $X \sim \mathcal{N}(2, 9)$, use tables or R to find $\mathrm{P}(2X < 6)$

Probability = [illegible]

14. The success probability of passing an exam is 0.8. Use R to find the probability that all 8 students who sit the exam pass. (Assume equal success probability for each student and independence of outcomes).

Probability = [illegible]

15. The function $F(x) = \begin{cases} 0, & x < c \\ (x - c), & c \le x \le c + 1 \\ 1, & \text{otherwise} \end{cases}$ is a cumulative distribution function

[illegible]

FORMULA SHEET FOR MATH1905 STATISTICS

- **Calculation formulae**:
  - *For a sample $x_1, x_2, \ldots, x_n$*

Sample mean $\bar{x}$	$\frac{1}{n}\sum_{i=1}^{n} x_i$
Sample variance s^2	$\frac{1}{n-1}\left[\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2\right] = \frac{1}{n-1}S_{xx}.$

  - *For paired observations $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$*

S_{xy}	$\sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$
S_{xx}	$\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2$
S_{yy}	$\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2$
r	$\frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

For the regression line $y = a + bx$:	
b	$\frac{S_{xy}}{S_{xx}}$
a	$\bar{y} - b\bar{x}$

- **Some probability results:**

For any two events A and B	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ and $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B\|A)$
If A and B are mutually exclusive (m.e.)	$\mathrm{P}(A \cap B) = 0$ and $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
If A and B are independent	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

- If $X \sim \mathcal{B}(n, p)$, then :
  $\mathrm{P}(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$, $i = 0, \ldots, n$, $\quad \mathrm{E}(X) = np \quad$ and $\quad \mathrm{Var}(X) = np(1-p)$

- **Some test statistics** and sampling distributions under appropriate assumptions and hypotheses:

$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$
$\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0, 1)$
$\frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{n-1}$

$\frac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \sim t_{n_x+n_y-2}$, where $S_p^2 = [(n_x - 1)S_x^2 + (n_y - 1)S_y^2]/(n_x + n_y - 2)$
$\sum_i \frac{(O_i - E_i)^2}{E_i} \sim \chi^2_\nu$, for appropriate ν

TABLE 1. **Some values of the standard normal distribution**: $\Phi(x) = F(z) = \mathrm{P}(Z \leq z)$, where $Z \sim \mathcal{N}(0,1)$. The point tabulated is $1 - p$, where $\mathrm{P}(Z \leq z) = 1 - p$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

TABLE 2. **Quantiles of the $\mathcal{N}(0,1)$ distribution**: Some percentage points of the standard normal. The point tabulated is z, where $\mathrm{P}(Z > z) = p$, where $Z \sim \mathcal{N}(0,1)$.

				p				
0.25	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001
0.674	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090