

Assignment 2

MATH1905: Statistics (Advanced)

Semester 2, 2017

Web Page: <http://sydney.edu.au/science/math/MATH1905>

Lecturer: Michael Stewart

This assignment is due by **5pm Tuesday 3rd October 2017**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard) at <https://elearning.sydney.edu.au>. Please submit only a PDF document. It should include your name and SID; your tutorial time, day, room and Tutor's name. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission, and then to check your submission receipt. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts of all questions correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on both questions, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making good progress on 1 question and moderate progress on the other, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making moderate progress on both questions.
4	D	Some attempt, with moderate progress made on only 1 question.
2	E	Some attempt, with minimal progress made on only 1 question.
0	F	No credit awarded.

You may use R to check your answers, but please provide a complete handwritten solution, scan it to PDF and then submit it through Turnitin. **Please do not include any R output/graphics in your assignment solution.**

You may use R in question 3 to obtain the required probability but please *write out the solution* (including the R command(s) you used) by hand.

1. Suppose $X \sim \text{Poisson}(\lambda)$ independently of $Y \sim \text{Poisson}(\mu)$. Define also $T = X + Y$.
 - (a) Determine the *joint distribution* of X and T , that is
 - (i) clearly state what pairs (x, t) are possible “values” of the random ordered pair (X, T) ;
 - (ii) for each such possible pair determine

$$P(X = x, T = t) = P\{(X = x) \cap (T = t)\} .$$

- (b) Using probability generating functions or otherwise, derive the distribution of T .
 - (c) For each possible value t that T can take, the conditional probability

$$P(X = x|T = t)$$

defines a full distribution for X , known as the *conditional distribution of X given $T = t$* .

- (i) Specify what possible values x have positive probability under this conditional distribution.
 - (ii) Show that this conditional distribution is a binomial distribution.
2. (a) Verify the following computing formula for covariance: if random variables X and Y satisfy $E(X) = \mu_X$ and $E(Y) = \mu_Y$ then

$$\text{Cov}(X, Y) = E[(X - \mu_X)(Y - \mu_Y)] = E(XY) - \mu_X \mu_Y .$$

- (b) Suppose that X has distribution given by

x	0	1	2	3	4	5	6
$P(X = x)$	0.1	0.05	0.3	0.1	0.3	0.05	0.1

and that $Y = |X - 3|$. Determine

- (i) $E(X)$;
 - (ii) the distribution of Y ;
 - (iii) $E(Y)$;
 - (iv) $E(XY)$;
 - (v) $\text{Cov}(X, Y)$.
 - (c) Are X and Y independent? Explain.

3. Suppose S is the sum of 20 independent rolls of a fair 6-sided die¹.
 - (a) Determine $E(S)$ and $\text{Var}(S)$.
 - (b) Compute a normal approximation with continuity correction to $P(S \leq 55)$ (to 4 decimal places). You may use R to obtain the normal probability but please write out *by hand* both the command you used as well as the answer.
 - (c) The exact probability in the previous part *can* be determined using a computer. It is 0.0285 (to 4 decimal places). Using this, determine the *relative error* (as a percentage) in the normal approximation obtained in the previous part, that is the absolute error divided by the thing being approximated.

¹The die has 6 sides, numbered 1, 2, 3, 4, 5 and 6 and the sum refers to the sum of the numbers showing “uppermost” after each roll.