PHYS 1901 – Physics 1A (Advanced)
Mechanics module

Prof Stephen Bartlett
School of Physics

THE UNIVERSITY OF SYDNEY

Chapter

9

Rotation of Rigid Bodies

Rotational motion

So far we have examined linear motion;

- Newton’s laws
- Energy conservation
- Momentum

Rotational motion seems quite different, but is actually familiar.

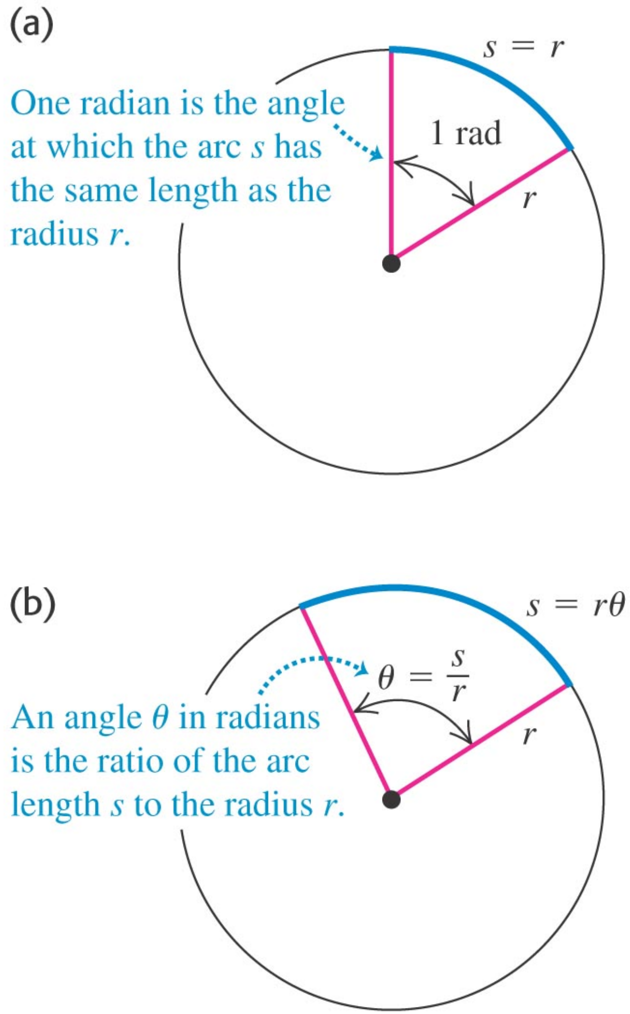
Remember: We are looking at rotation in fixed coordinates, not rotating coordinate systems.

Angular variables

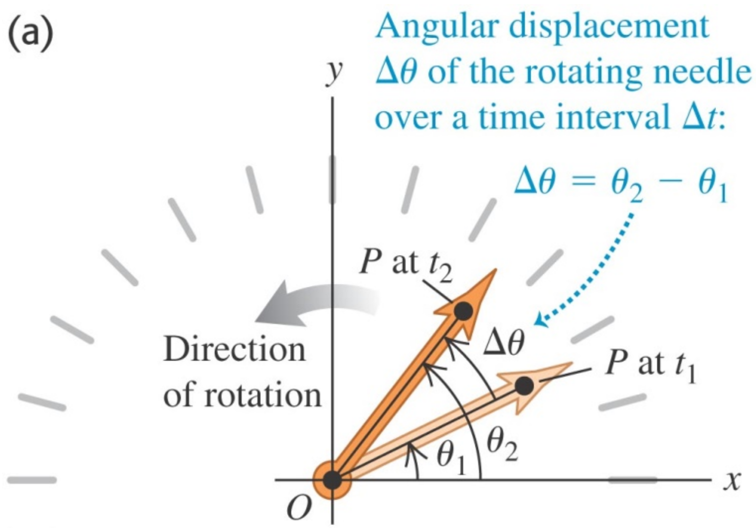
For a circle of radius *r*, an angular displacement of θ corresponds to an arc length of

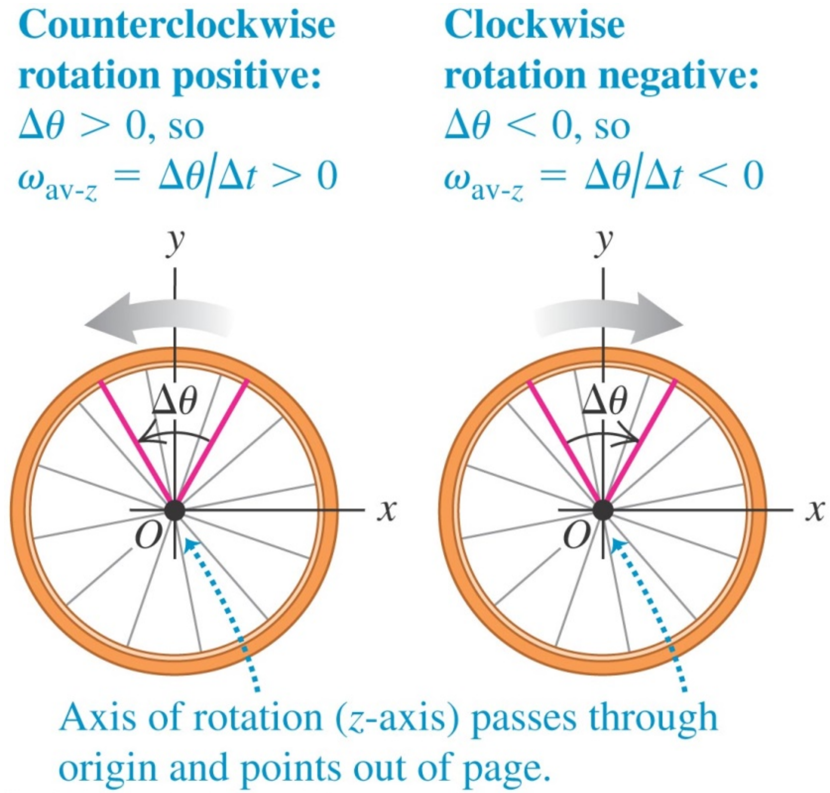
$$s = r\theta$$

(Remember: use radians!)



Angular variables





Angular velocity is the change of angle with time

$$\omega = \frac{d\theta}{dt} \quad (\text{rad/s})$$

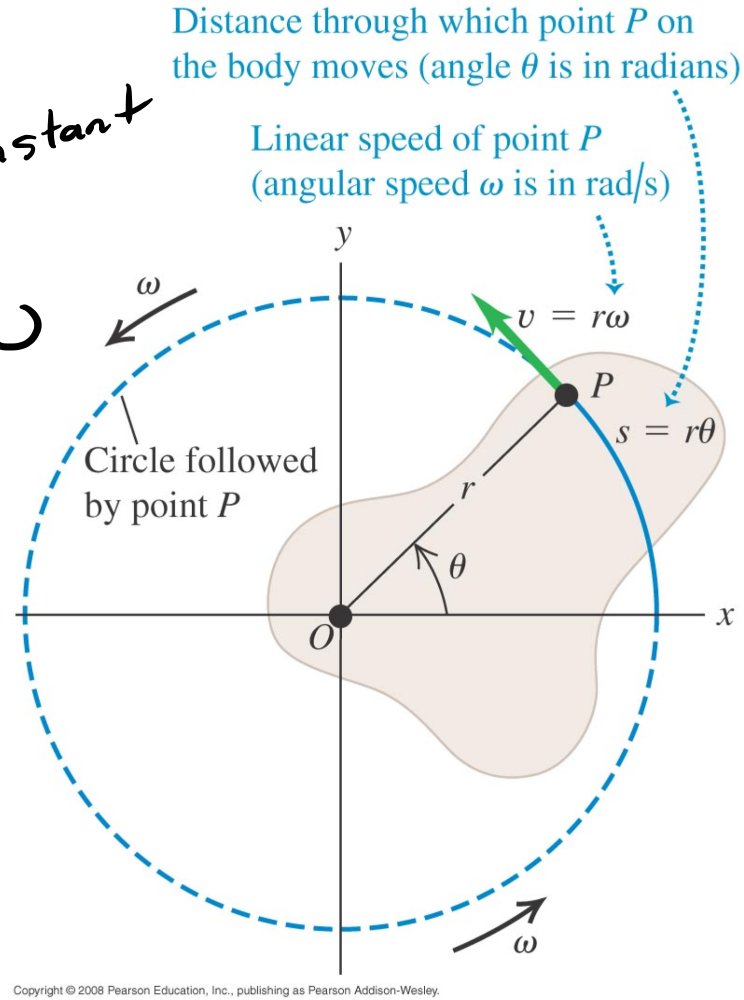
Angular variables

There is a simple relation between angular velocity and speed

if r is constant

$$v = \frac{ds}{dt} = \frac{d(r\theta)}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\boxed{v = r\omega}$$



Angular variables

Angular acceleration is the change of ω with time

$$\alpha = \frac{d\omega}{dt} \quad (\text{rad}/s^2)$$

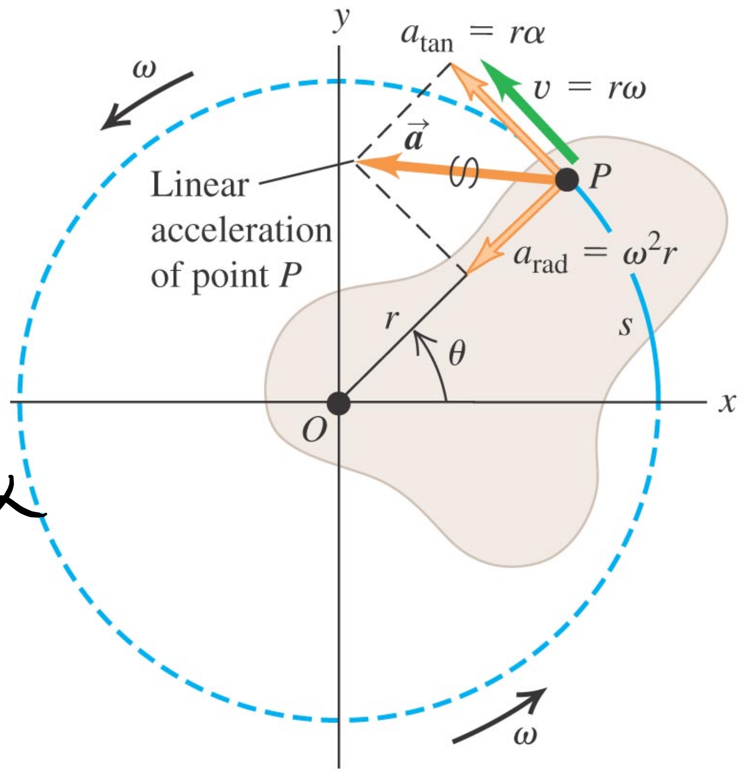
Tangential acceleration is given by

$$a_{tan} = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\frac{d\omega}{dt} = r\alpha$$

$$\boxed{a_{tan} = r\alpha}$$

Radial and tangential acceleration components:

- $a_{rad} = \omega^2 r$ is point P's centripetal acceleration.
- $a_{tan} = r\alpha$ means that P's rotation is speeding up (the body has angular acceleration).



Rotational kinematics

Notice that the form of rotational relations is the same as the linear variables. Hence, we can derive identical kinematic equations:

Linear	*Rotational*
If a is constant	If α is constant
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$s = s_0 + v_0 t + \frac{1}{2}at^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

Net acceleration

Remember, for circular motion, there is always centripetal acceleration

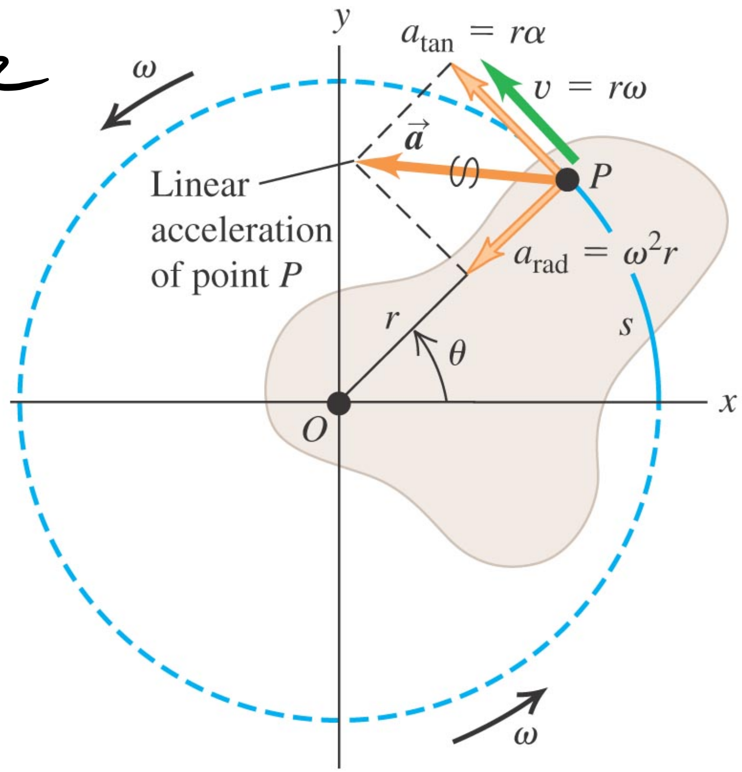
$$a_{rad} = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = r\omega^2$$

The total acceleration is the vector sum of a_{rad} and a_{tan}.

What is the source of a_{rad}?

Radial and tangential acceleration components:

- $a_{rad} = \omega^2 r$ is point P's centripetal acceleration.
- $a_{tan} = r\alpha$ means that P's rotation is speeding up (the body has angular acceleration).



Rotational dynamics

As with rotational kinematics, we will see that the framework is familiar, but we need some new concepts;

Linear	*Rotational*
Mass	Moment of Inertia
Force	Torque

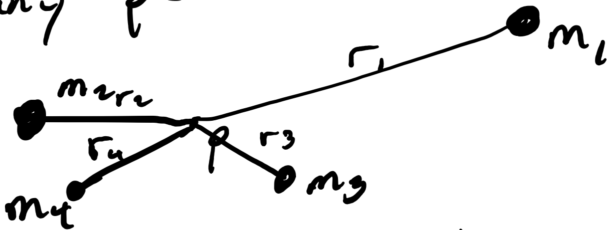
Moment of inertia:

depends on 1) the distribution of the mass
2) location of pivot

Point mass:

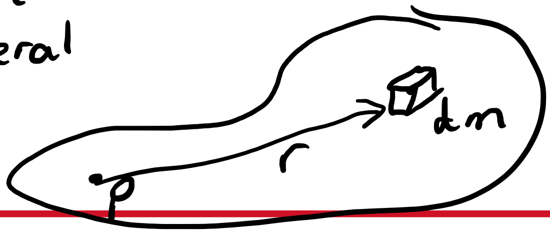
$$I_{\text{point mass}} = mr^2 \quad (\text{units: } kg\, m^2)$$

Many point masses:



$$I_{\text{many point masses}} = \sum_{\text{all masses } i} I_i = \sum_{\text{all masses } i} m_i r_i^2$$

General

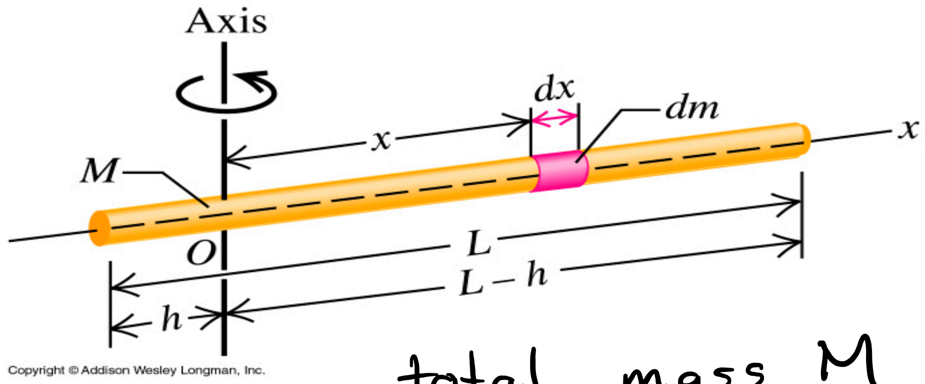


$$I_{\text{extended object}} = \int_{\text{object}} r^2\, dm$$

THE UNIVERSITY OF SYDNEY

Moment of inertia

Calculate the moment of inertia of a rod of mass M, length L, rotating about an axis a distance h from one end.



$$I = \int r^2\, dm$$

$$= \int_{-h}^{L-h} x^2\, dm$$

$$= \int_{-h}^{L-h} x^2 \left(\frac{M}{L}\right) dx$$

$$= \frac{M}{L}\left[\frac{1}{3}x^3\right]_{-h}^{L-h}$$

$$= \frac{1}{3}\frac{M}{L}\left[(L-h)^3 - (-h)^3\right]$$

$$= \frac{1}{3} M \frac{(L-h)^3 + h^3}{L}$$

total mass M

total length L

mass per unit length $\lambda = \frac{M}{L}$

length dx

mass $\lambda dx = \frac{M}{L} dx$

Middle $h = L/2$

$I_{middle} = \frac{1}{12} M L^2$

End $h = 0$

$I_{end} = \frac{1}{3} M L^2$

Moment of inertia

Luckily, the moment of inertia is *typically;*

$$I = cMR^2$$

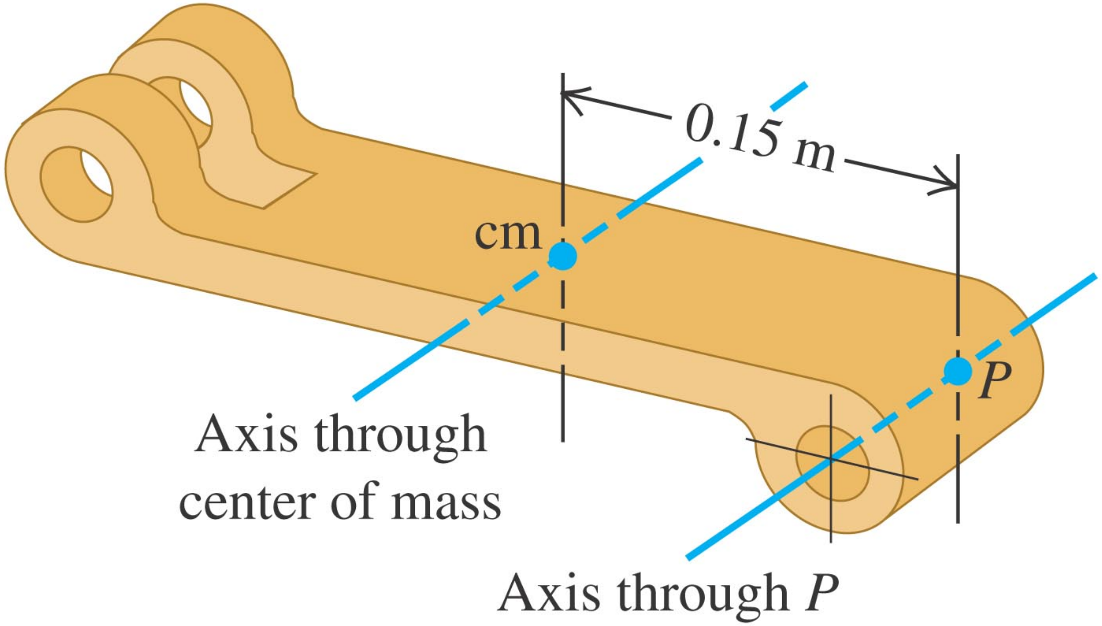
where *c* is a constant and is <1.

Object	I
Solid sphere (on axis)	$2/5\ M\ R^2$
Hollow sphere (on axis)	$2/3\ M\ R^2$
Rod (centre)	$1/12\ M\ L^2$
Rod (end)	$1/3\ M\ L^2$

Parallel axis theorem

If we know the moment of inertia through the centre of mass, the moment of inertia along a parallel axis d is:

$$I_p = I_{c.m.} + Md^2$$



The axis does not have to be through the body!