The University of Sydney
School of Mathematics and Statistics

Sample Quiz 2

MATH1903: Integral Calculus and Modelling (Advanced) Semester 1, 2017

Lecturers: Daniel Daners and David Easdown

Family Name: .....

Other Names: .....

SID: ..... **Day:** .....

Time: ..... **Room:** .....

Signature: .....

Please write your final answers, and only your final answers, in the answer boxes.

Please note:

(a) You have 40 minutes to complete the quiz.

(b) Questions are worth the indicated number of marks. The total number of possible marks is 20.

(c) Answers will only be marked if they are in the answer boxes and supported by working where relevant.

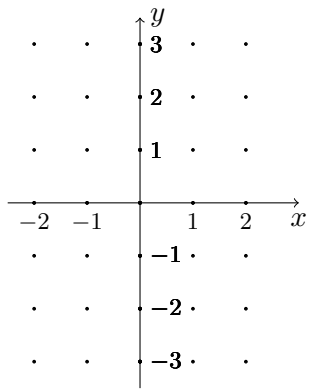
(d) Partial marks may be awarded for the working.

(e) Non-programmable, non-graphics calculators **are** permitted.

(f) There is space for working. No additional paper is allowed.

(Note: In this sample quiz, unlike the real quiz, there is no space for working is provided)

1. Sketch the direction field of the differential equation $y' = y(y-2)$ in the region below. *2 Marks*



2. Find the general solution of the differential equation $y' = y^2 \sin x$. *2 Marks*

Answer Q2:

3. It is given that $y = \dfrac{Cx}{x-C}$ is the general solution of a differential equation. *2 Marks*
Determine the constant C for the particular solution satisfying the initial condition $y(2) = 3$.

Answer Q3:

4. Find the particular solution of the differential equation $z' = (2x+1)z$ with $z(0) = 4$. *2 Marks*

Answer Q4:

5. Suppose that y satisfies the differential equation $\dfrac{dy}{dx} = 3(y+2x)^2$. Find the differential equation for $v = y + 2x$. *2 Marks*

Answer Q5:

6. Find an integrating factor for the differential equation $y' - \dfrac{1+x}{x}y = q(x)$. *2 Marks*

Answer Q6:

7. Find the general solution of $\dfrac{dy}{dt} = -3t\cos^2 y$. *2 Marks*

Answer Q7:

8. A molecule of substance A can combine with two molecules of substance B to form a molecule of substance X, in a reaction which is denoted $A + 2B \to X$. According to the Law of Mass Action, the rate of formation of X is proportional to the product of the amounts of A and B present. A test-tube initially contains amounts a_0 and b_0 of substances A and B, respectively, but none of substance X. The amount of substance X at time t is $x(t)$ and k is a positive constant. *2 Marks*

Which differential equation models the amount of substance X?

(a) $\dfrac{dx}{dt} = k(a_0 - x)(2b_0 - x)$

(b) $\dfrac{dx}{dt} = k(a_0 - x)(b_0 - 2x)$

(c) $\dfrac{dx}{dt} = k(2a_0 - x)(b_0 - x)$

(d) $\dfrac{dx}{dt} = k(a_0 - 2x)(b_0 - x)$

Answer Q8:

9. Find the general solution of $y' - 2xy = 3x$. *2 Marks*

Answer Q9:

10. Find the solution of the differential equation $\dfrac{du}{dx} = \dfrac{1+u^2}{2xu}$ with initial condition $u(1) = 2$. *2 Marks*

Answer Q10: