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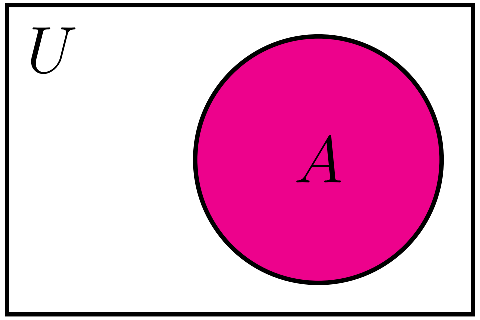

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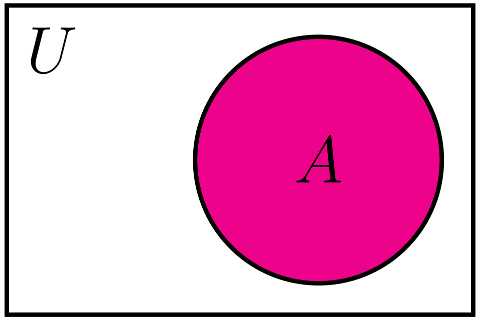

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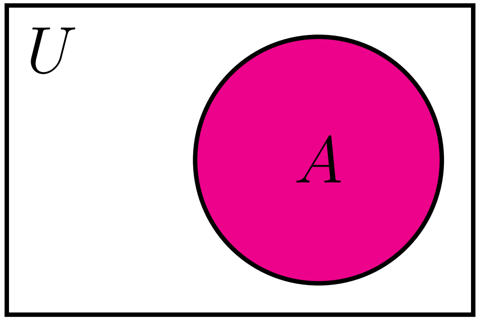

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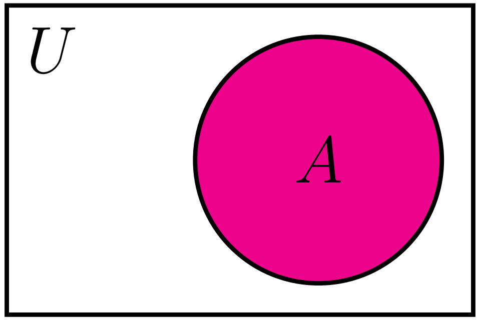

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  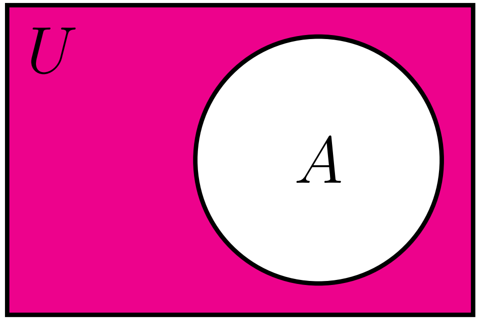

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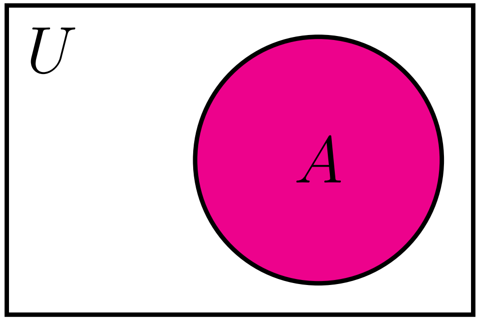

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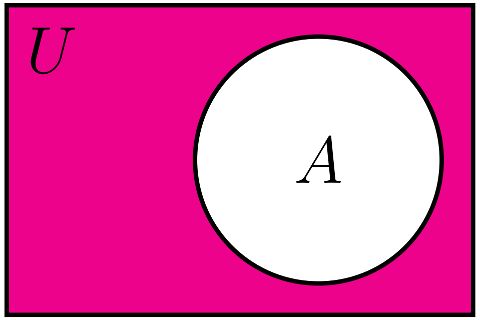

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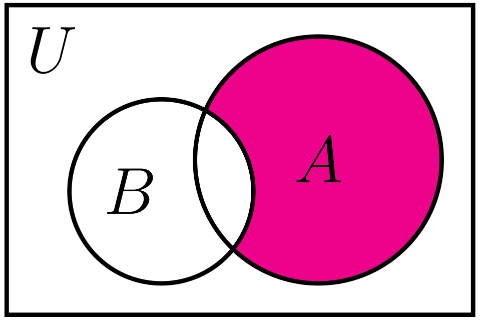

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  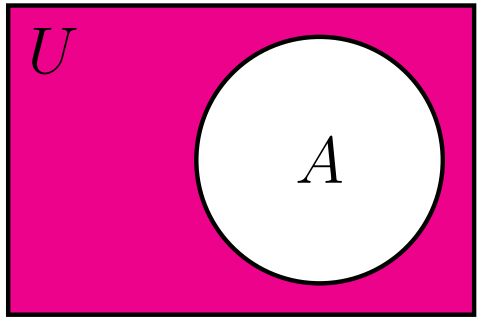

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  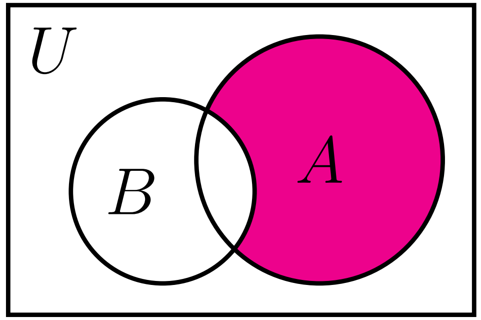

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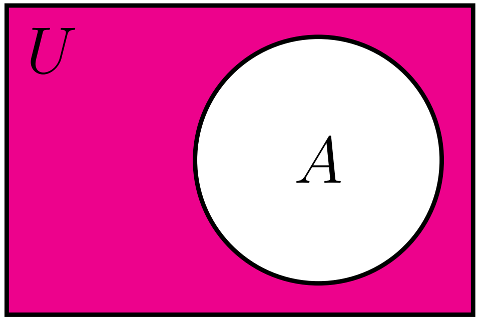

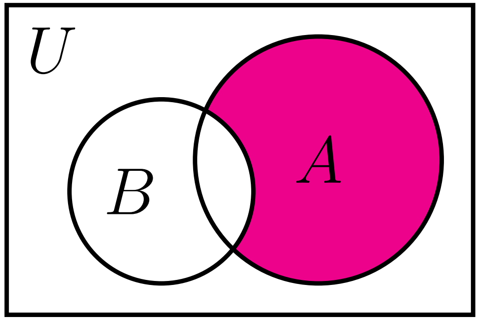

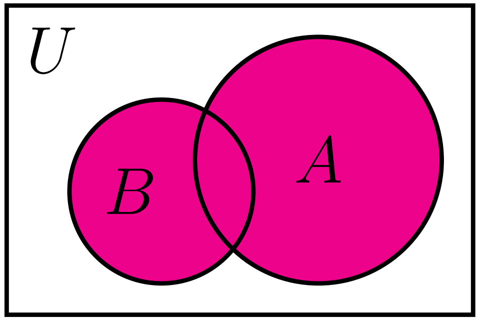

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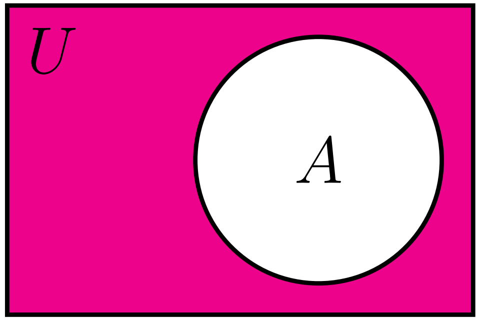

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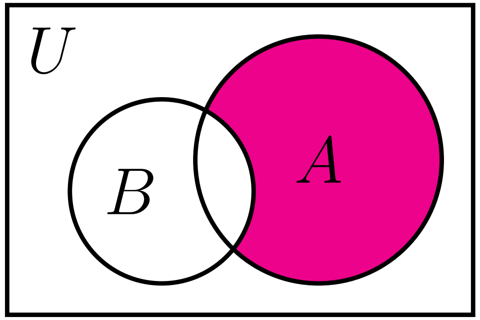

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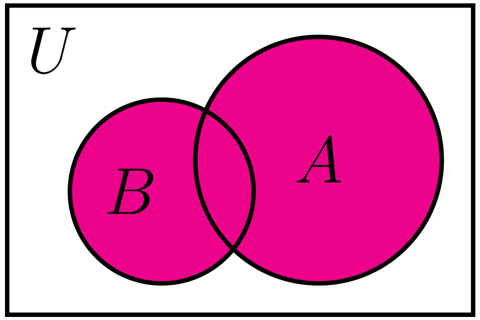

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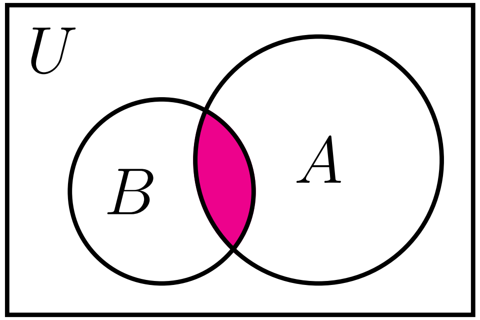

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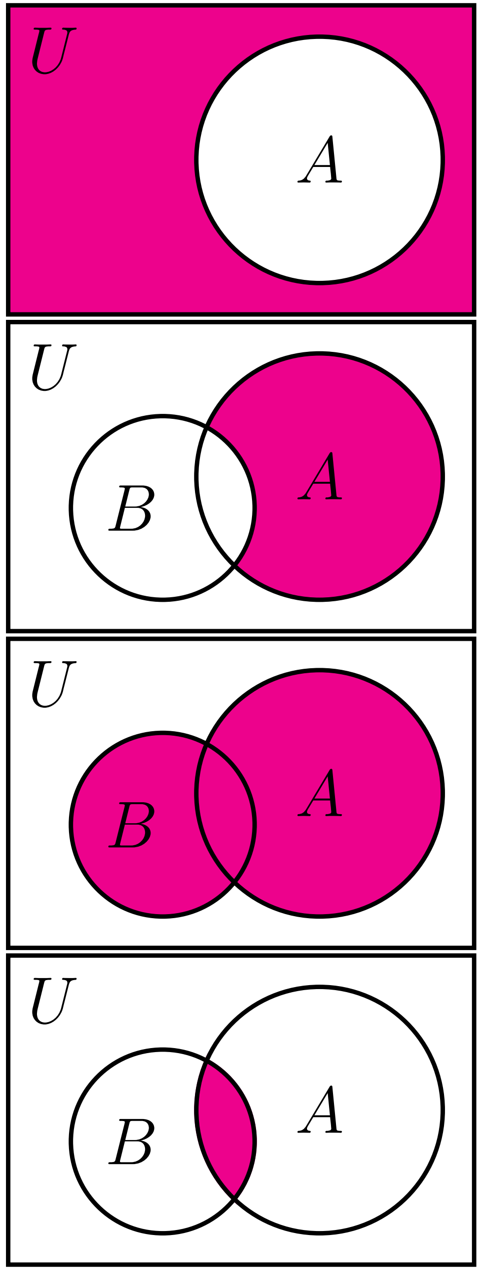

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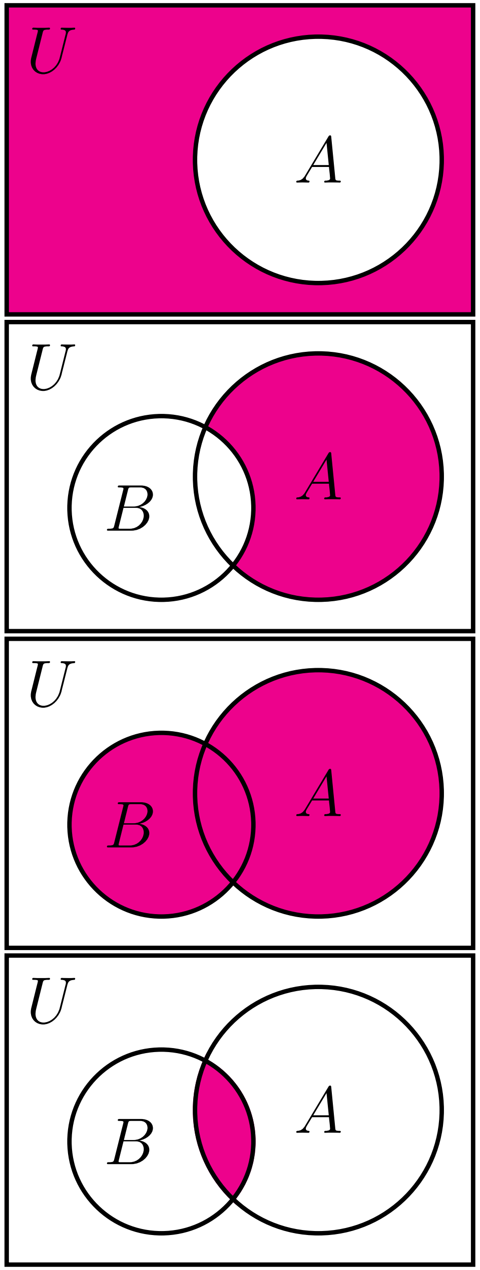

- Two sets A and B are *disjoint* if $A \cap B = \varnothing$.
- The Inclusion-Exclusion Principle: $|A \cup B| = |A| + |B| - |A \cap B|$.

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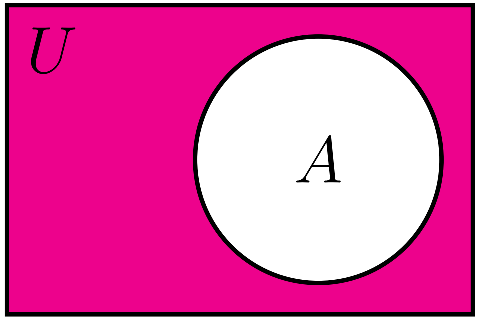

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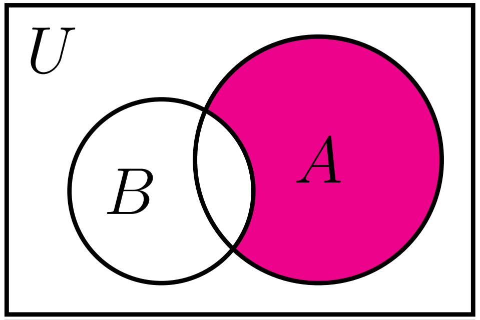

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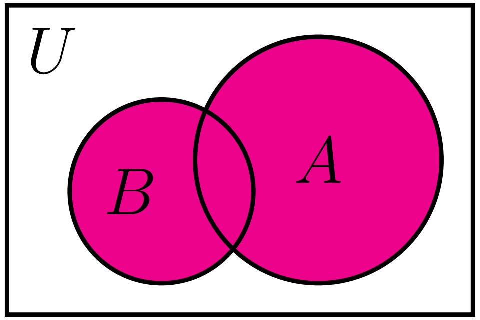

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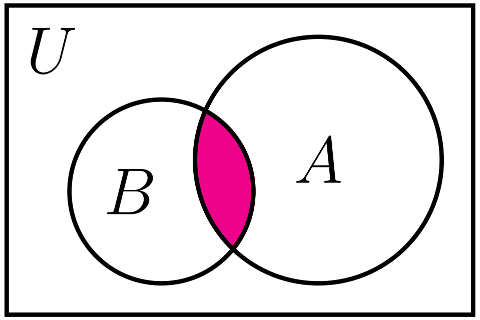

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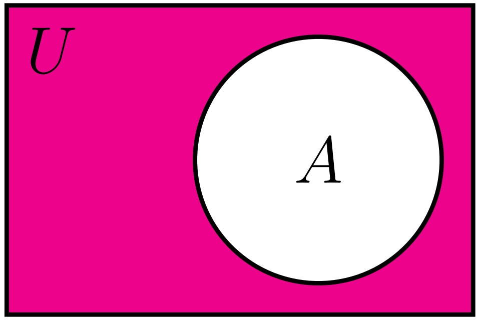

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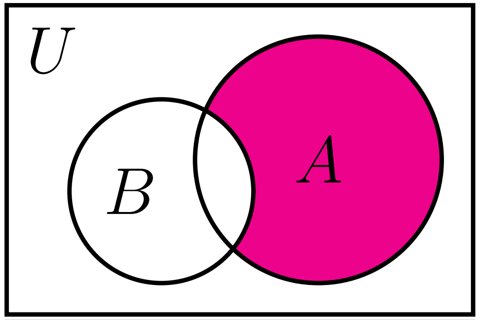

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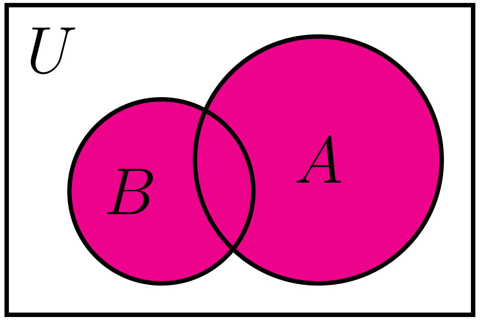

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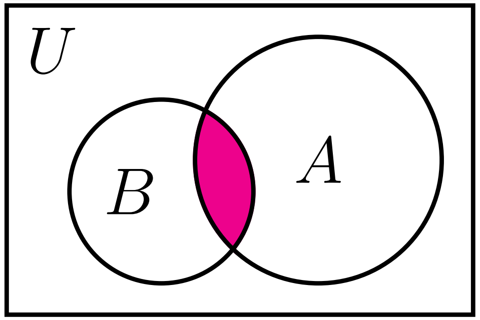

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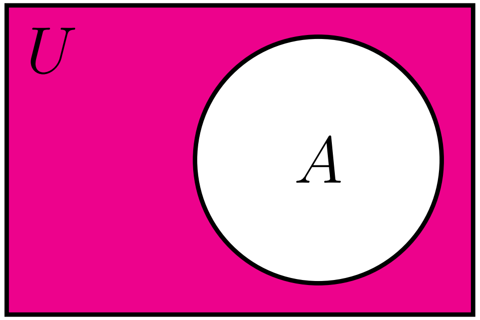

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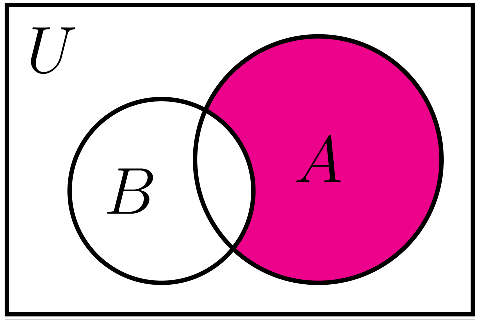

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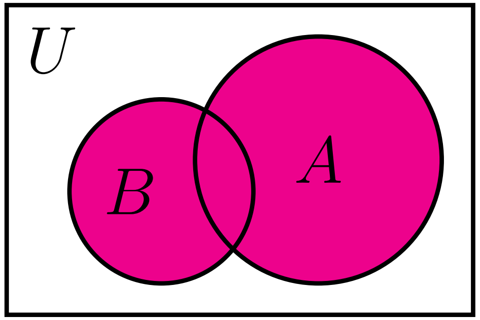

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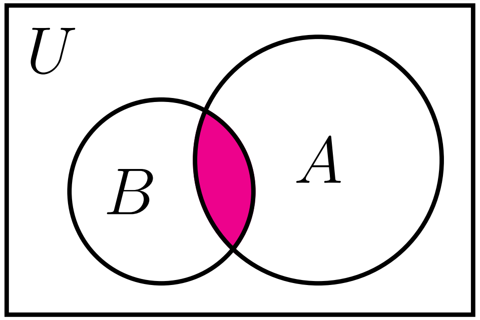

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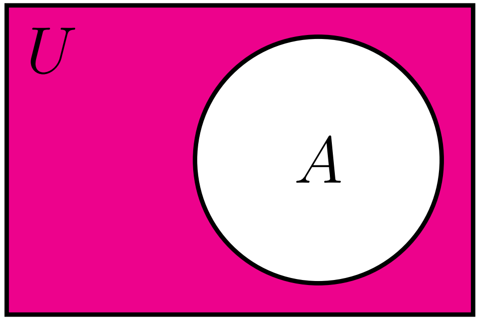

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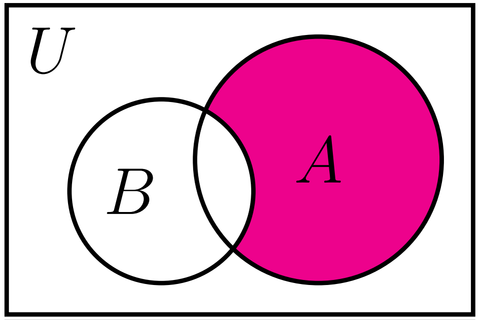

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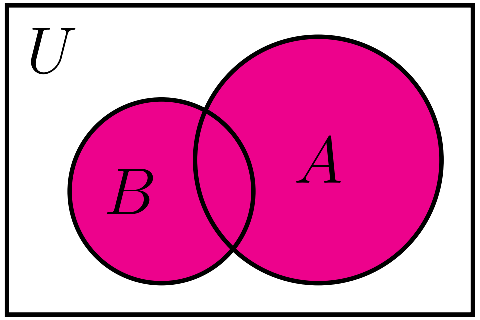

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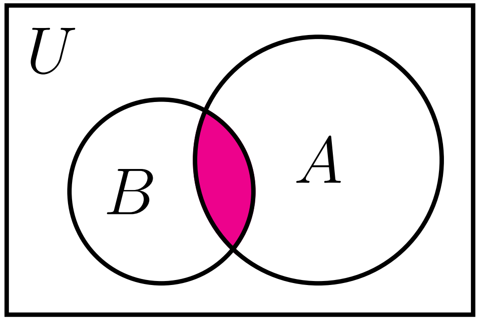

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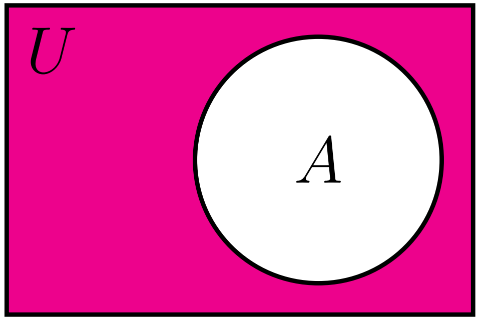

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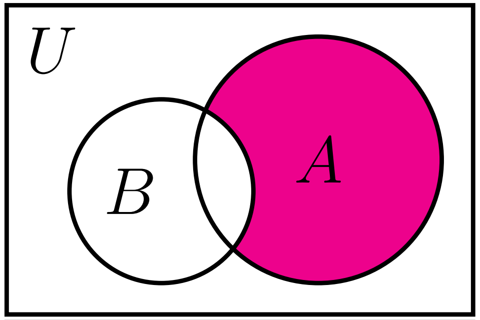

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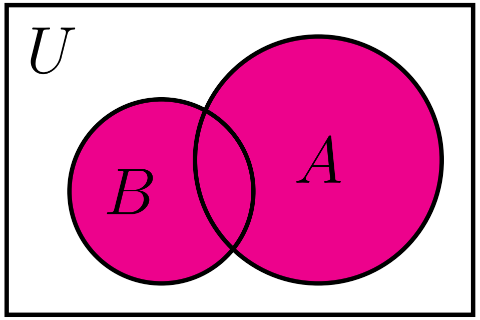

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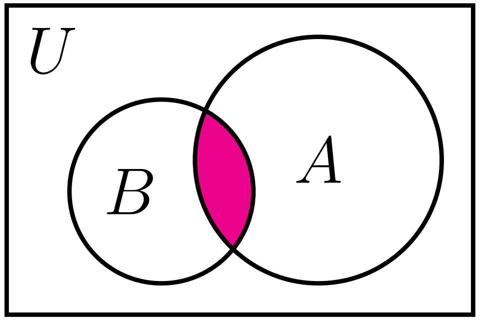

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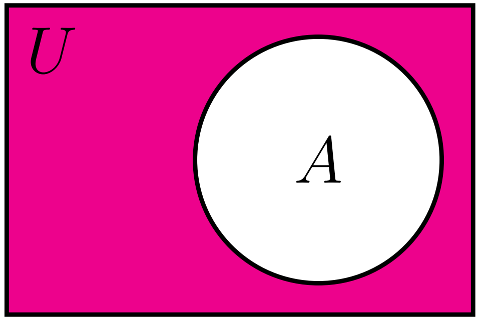

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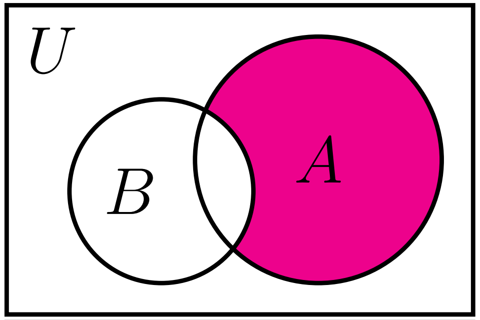

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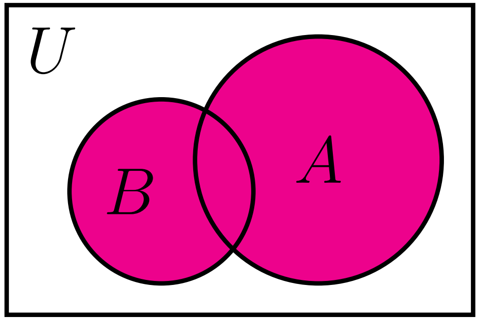

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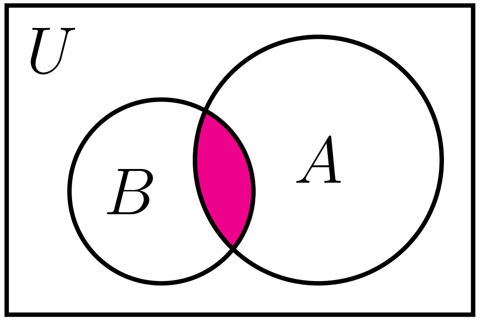

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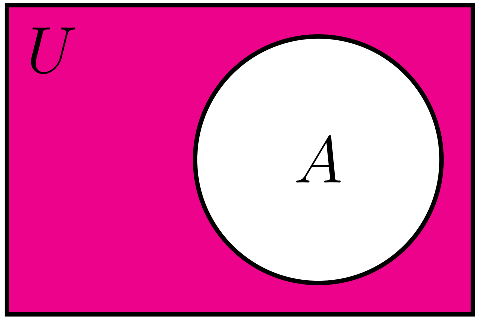

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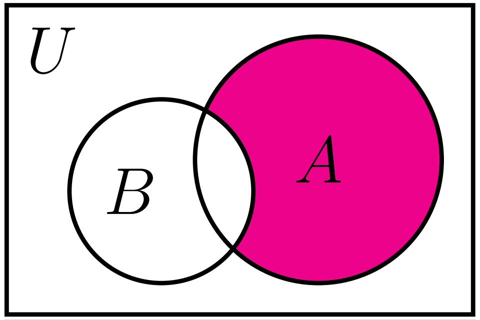

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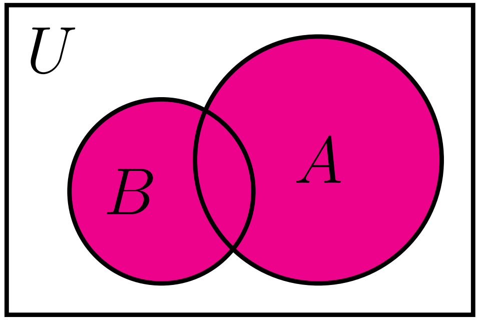

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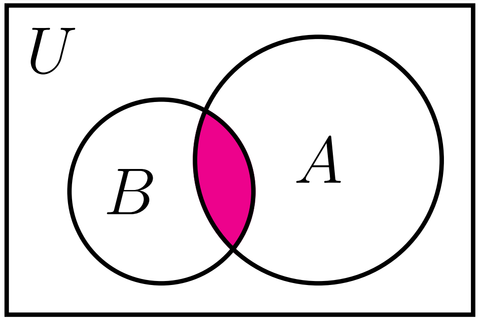
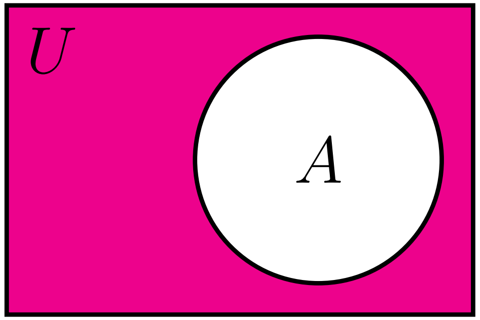


Example. Set $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{1, 3, 5\}$, and $B = \{1, 2\}$. Then

$$A^c = \{2, 4, 6\} \qquad A \cap B = \{1\} \qquad A \cup B = \{1, 2, 3, 5\}$$

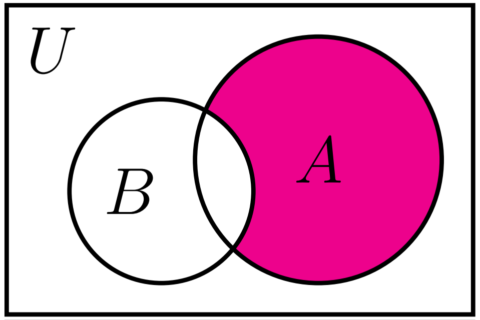
- *complement* (^{c}, $\overline{}$) - “not”

$$\boxed{A^c = \overline{A} = U \setminus A = \{x \in U \,|\, x \notin A\}}$$



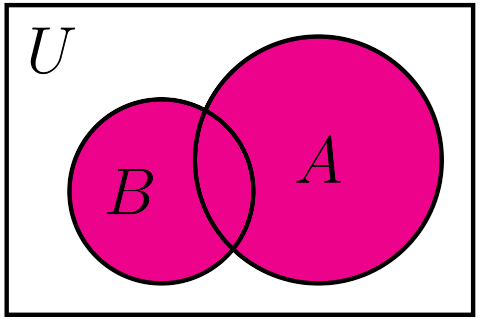
- *difference* ($-$, $\setminus$) - “but not”

$$\boxed{A - B = A \setminus B = \{x \in U \,|\, x \in A \text{ and } x \notin B\}}$$



- *union* ($\cup$) - “or”

$$\boxed{A \cup B = \{x \in U \,|\, x \in A \text{ or } x \in B\}}$$



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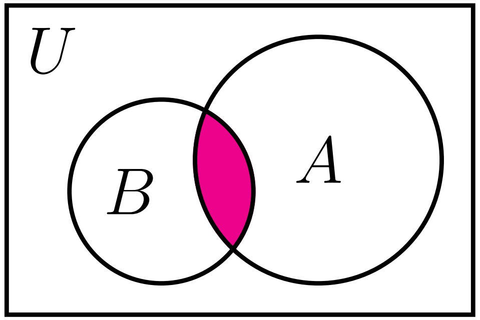

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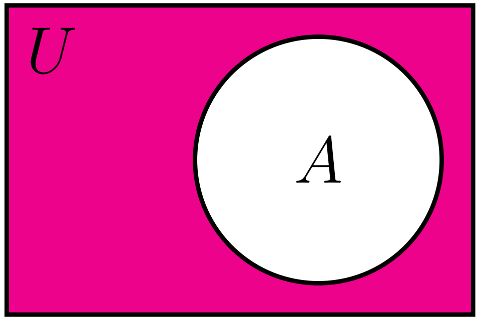

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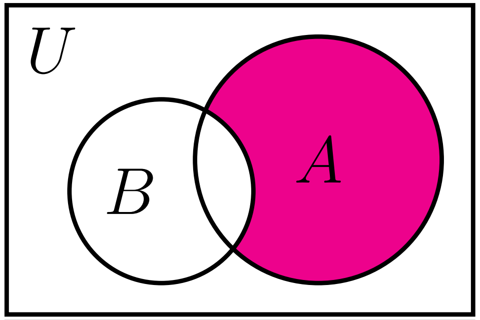

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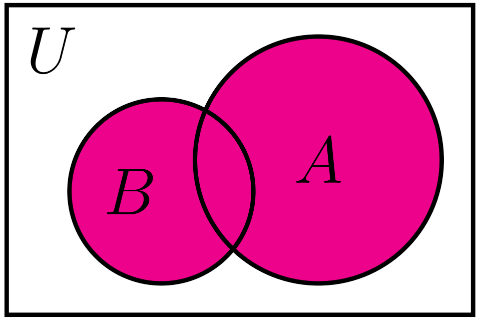

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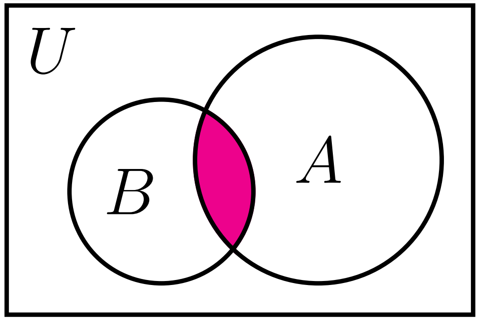

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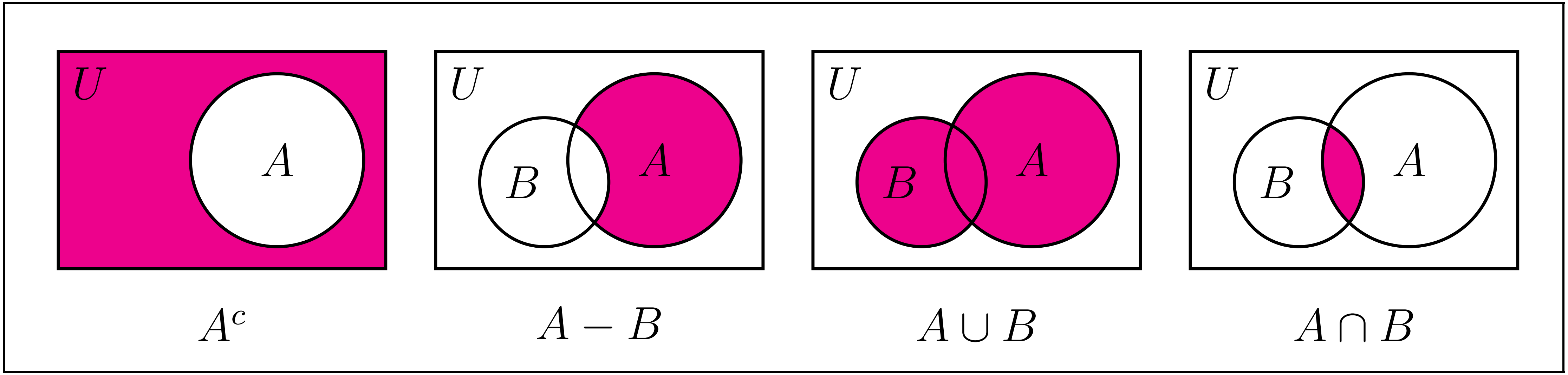
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A^c
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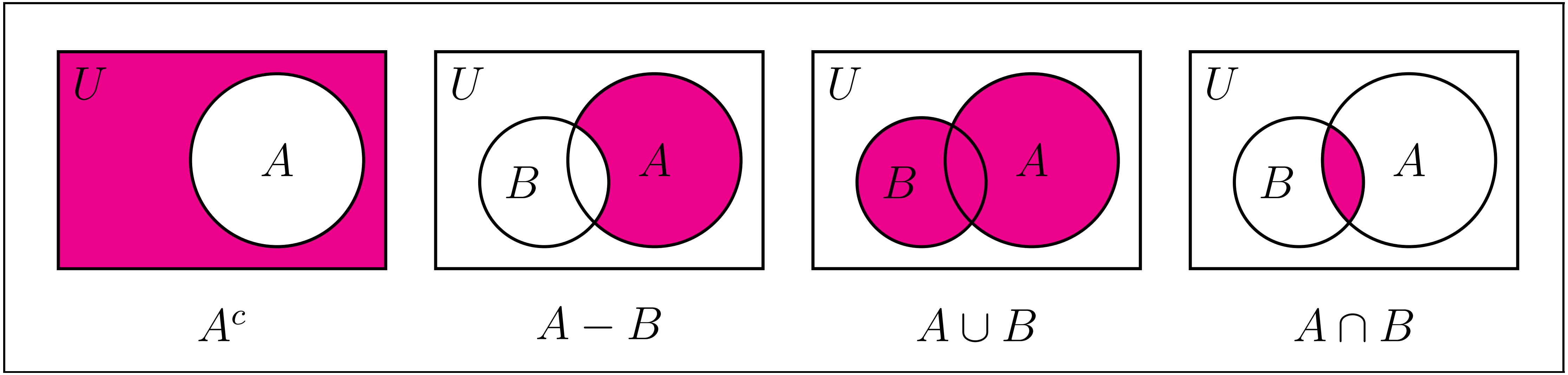

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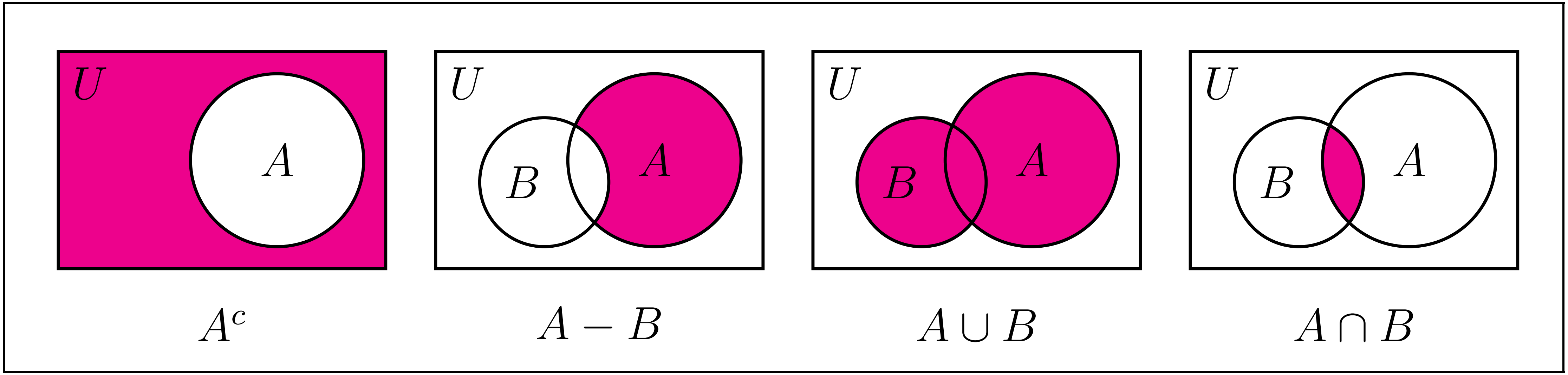

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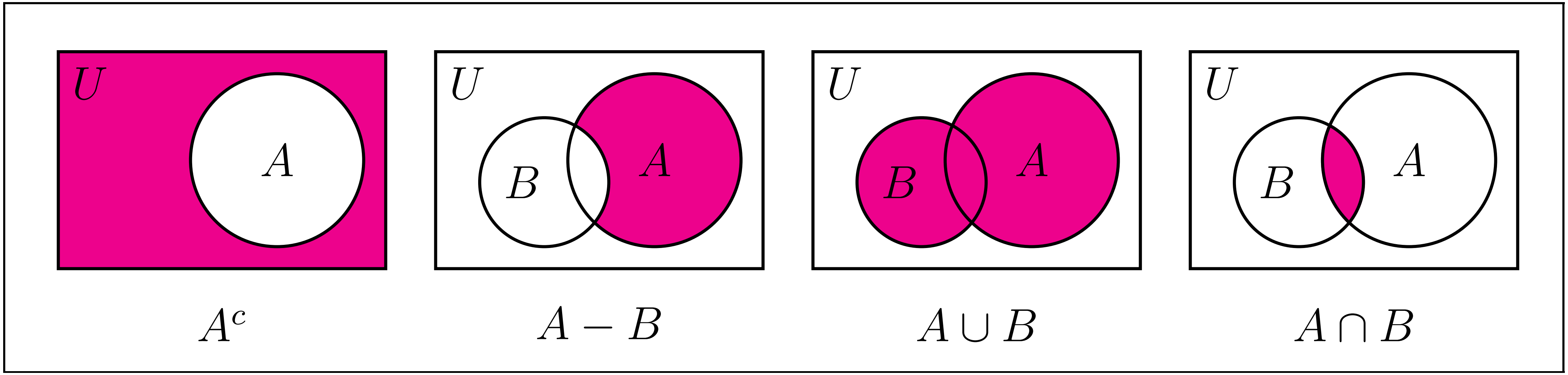

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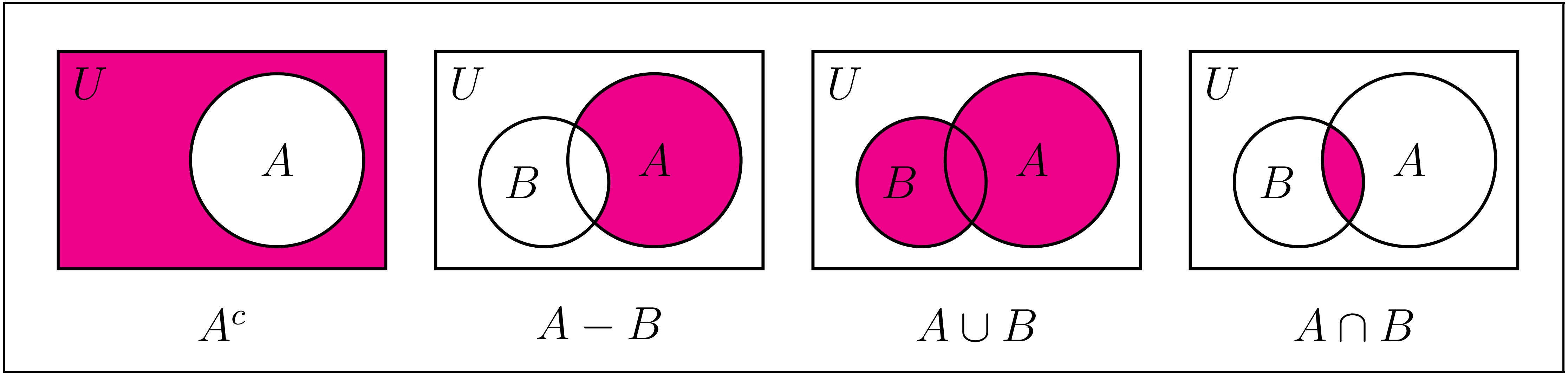

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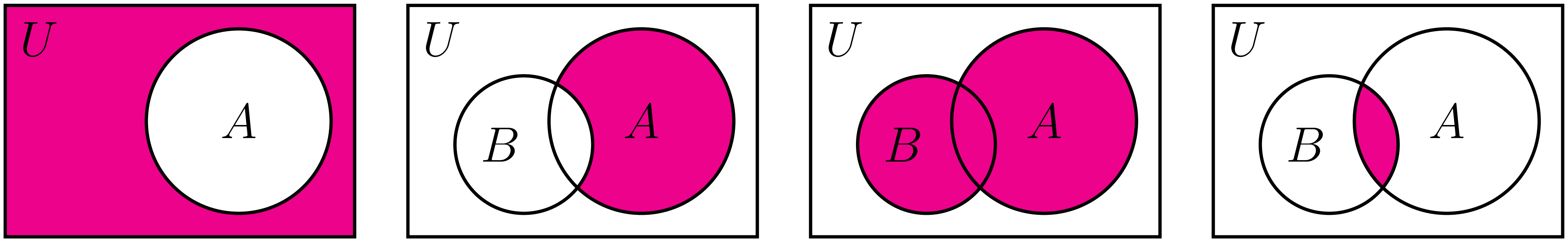

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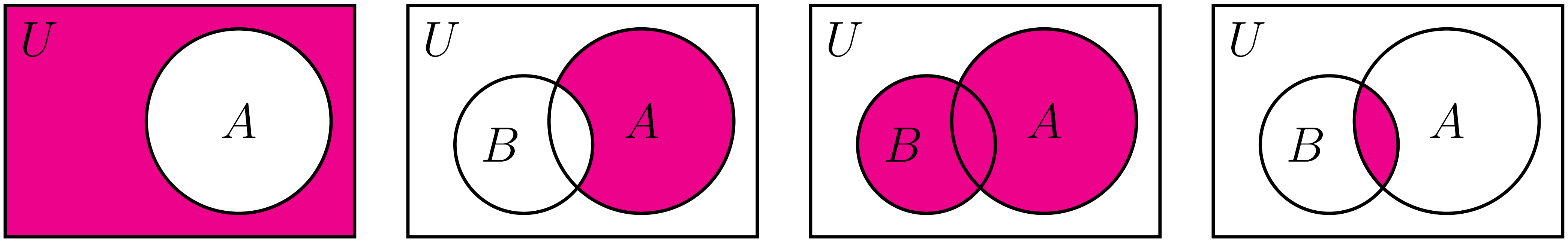

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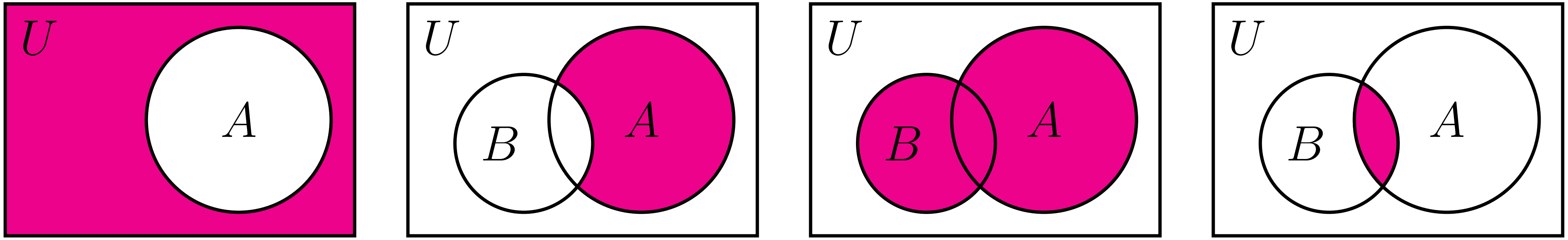

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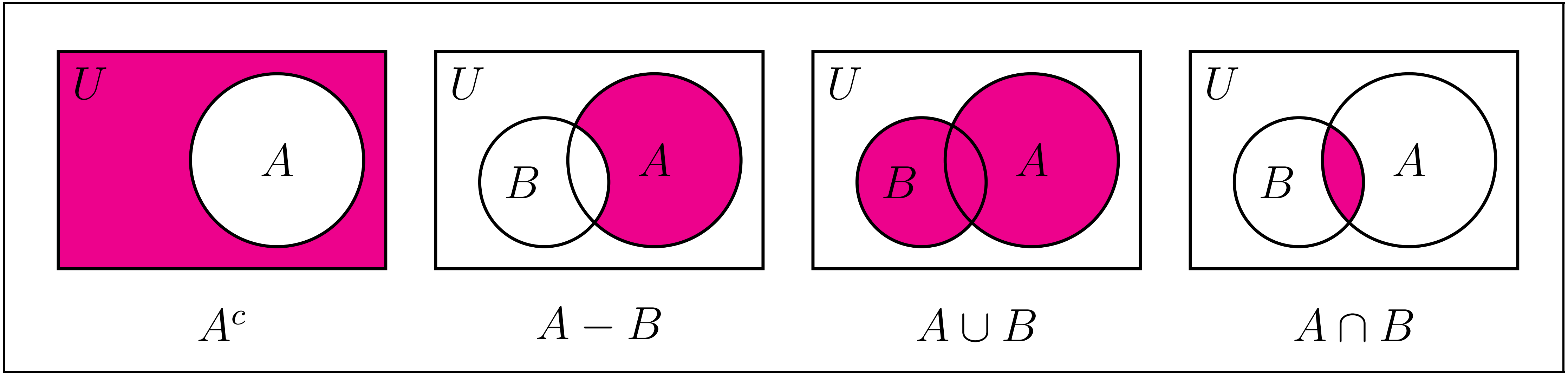

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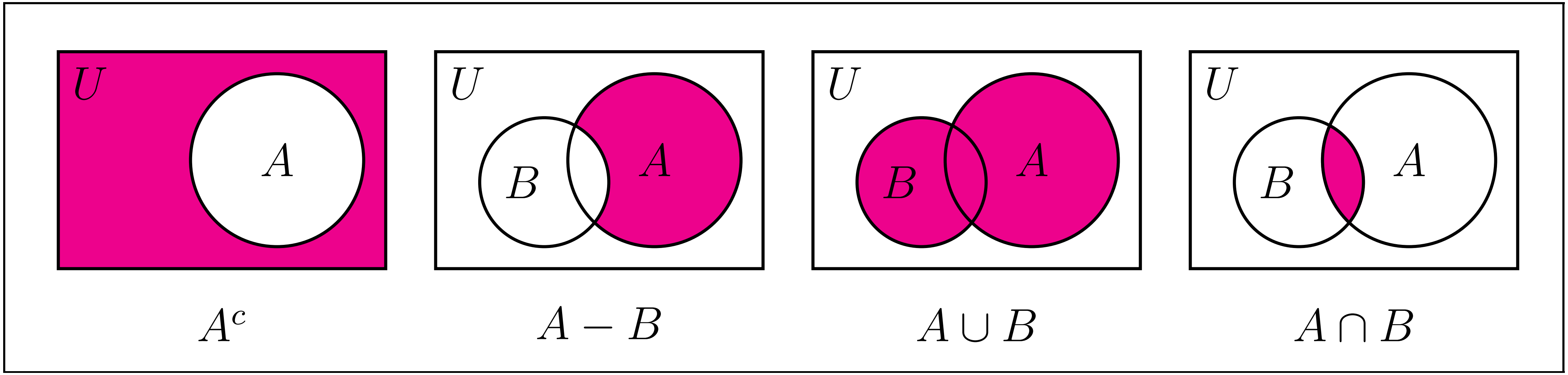

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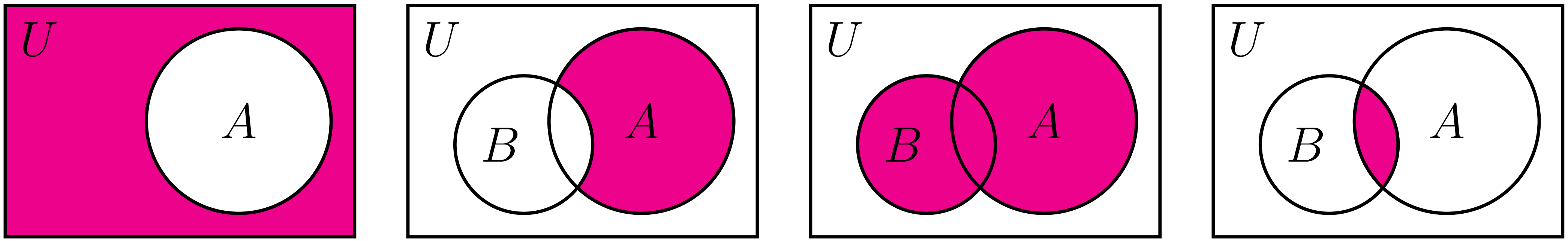

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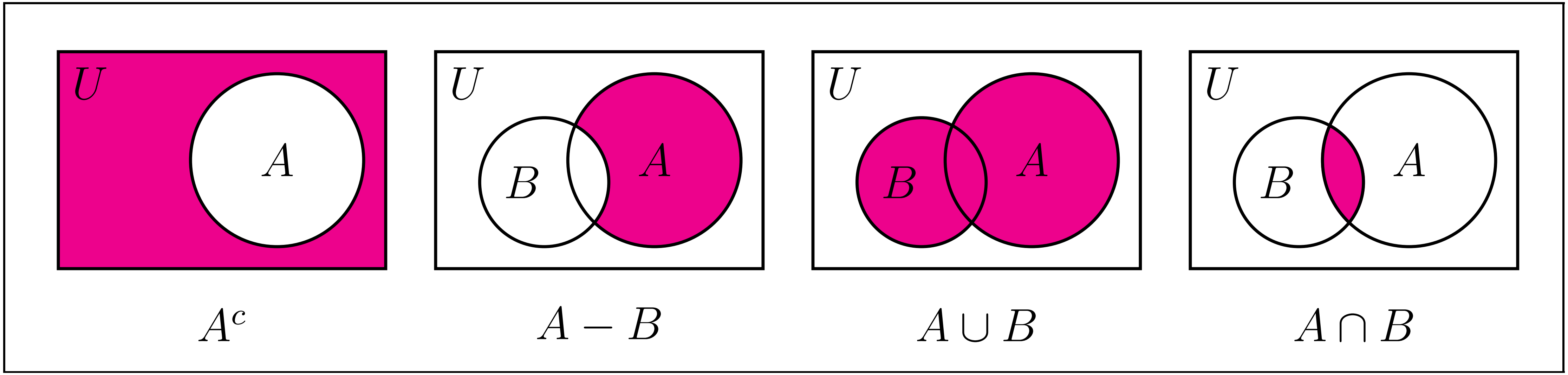

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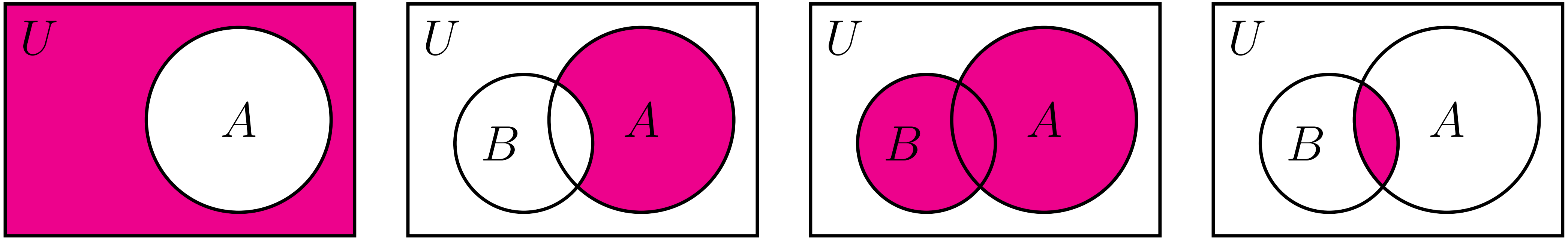

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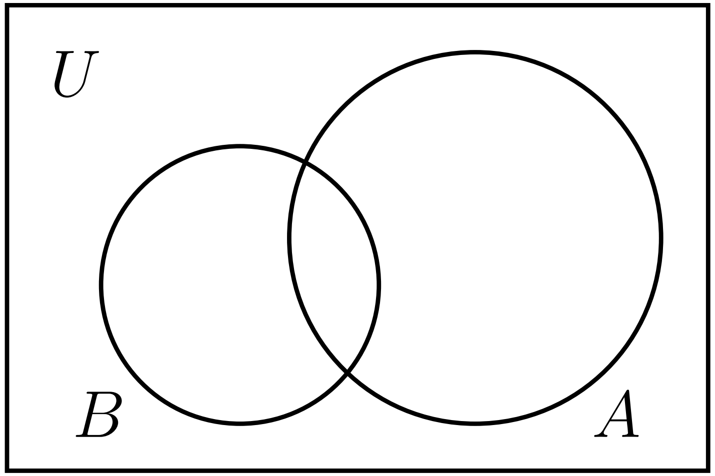

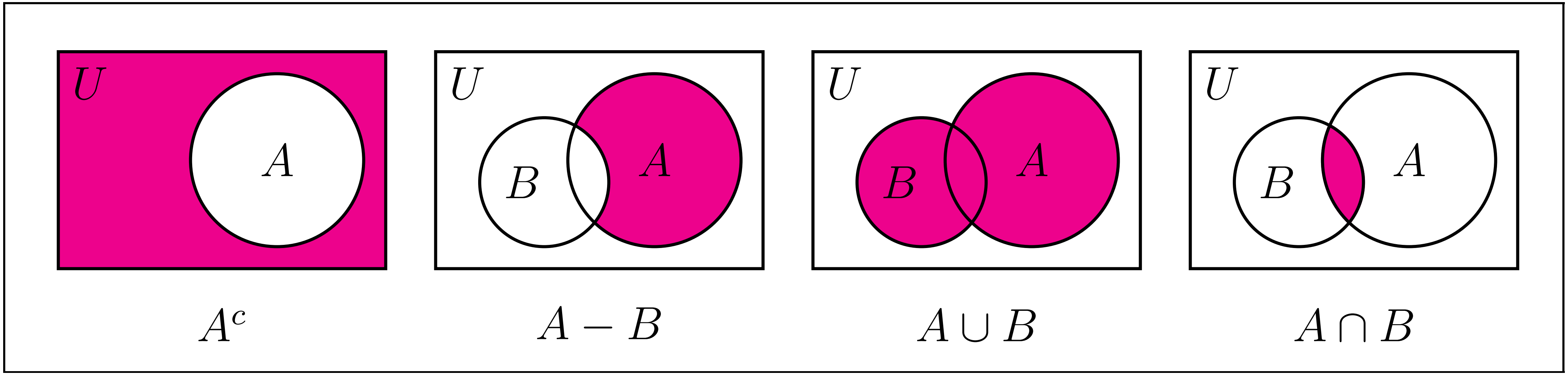

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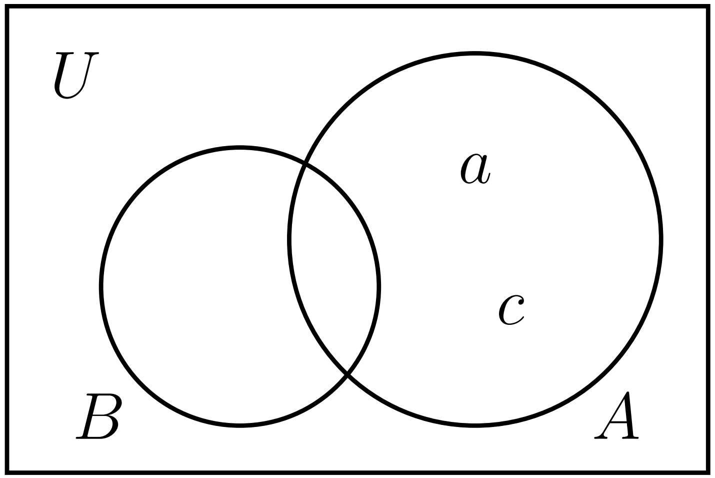

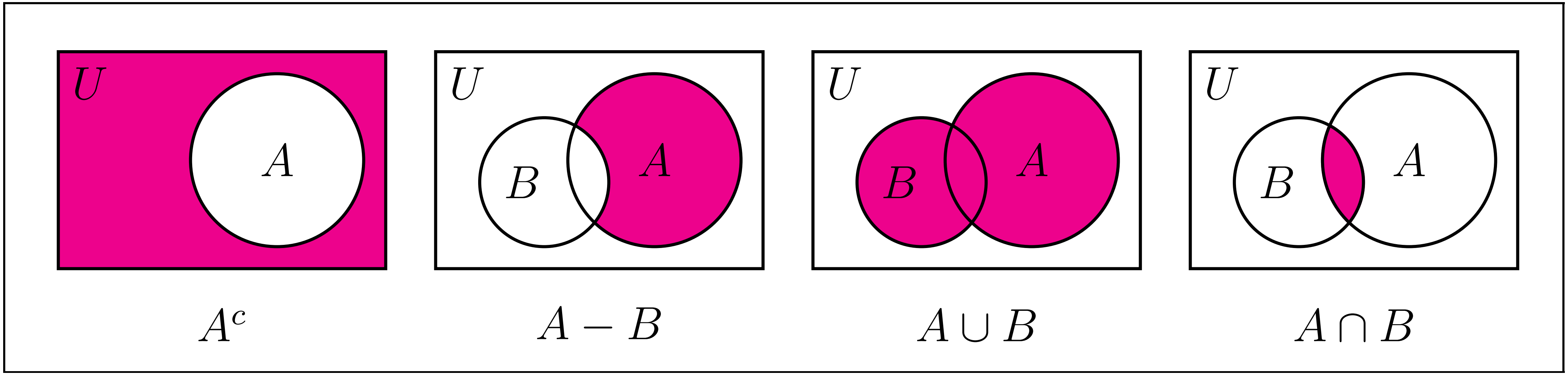

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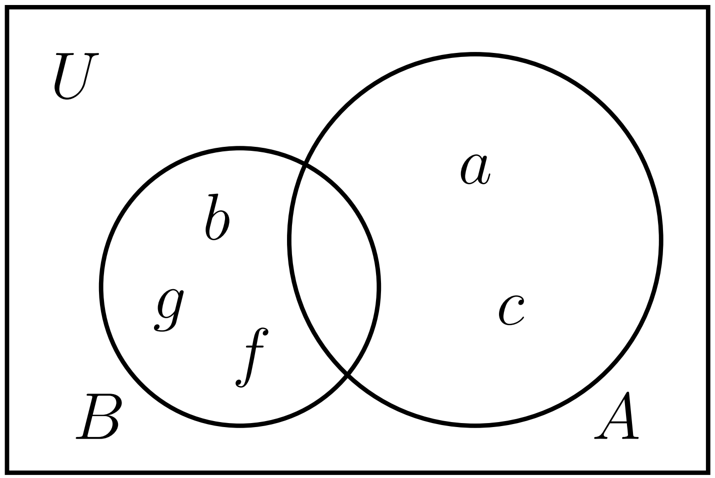

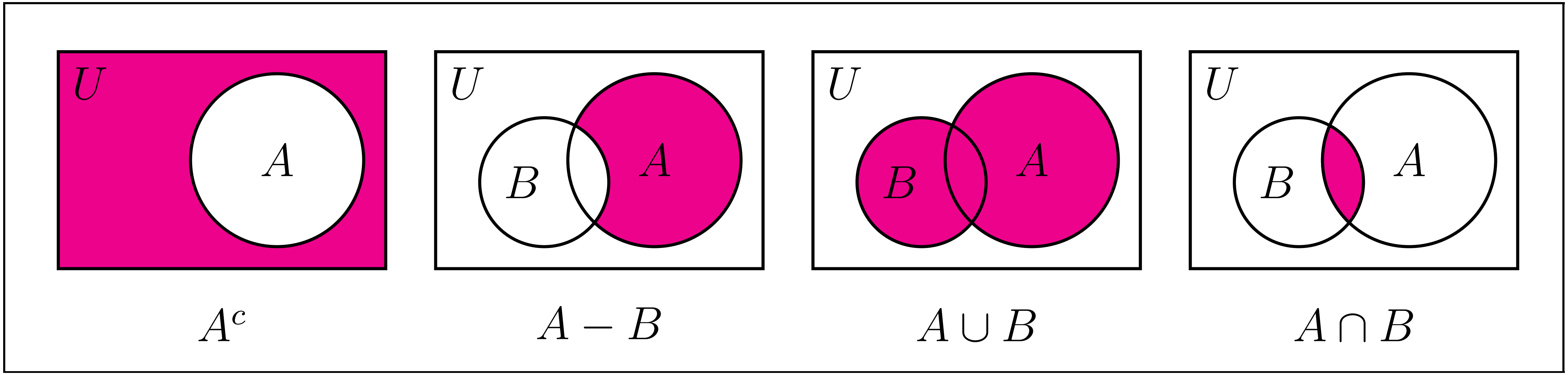

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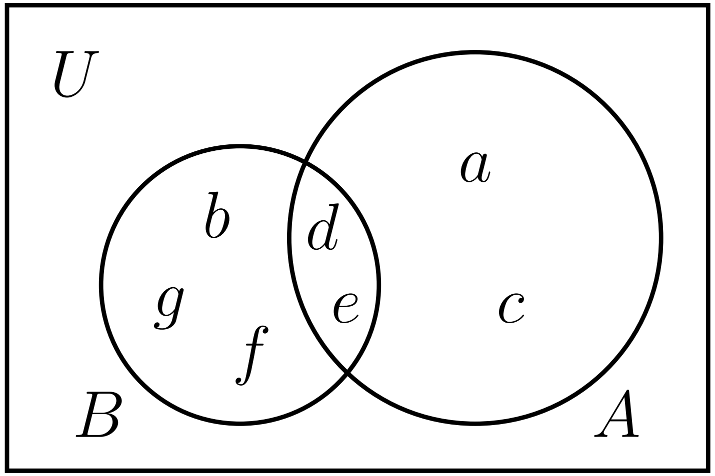

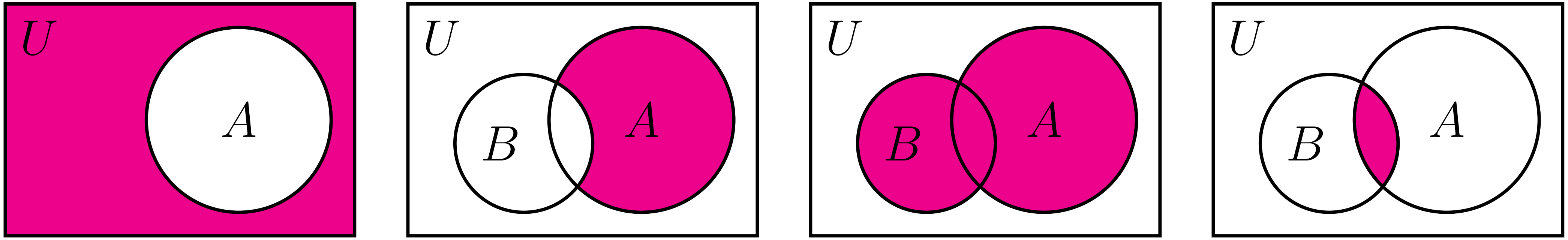

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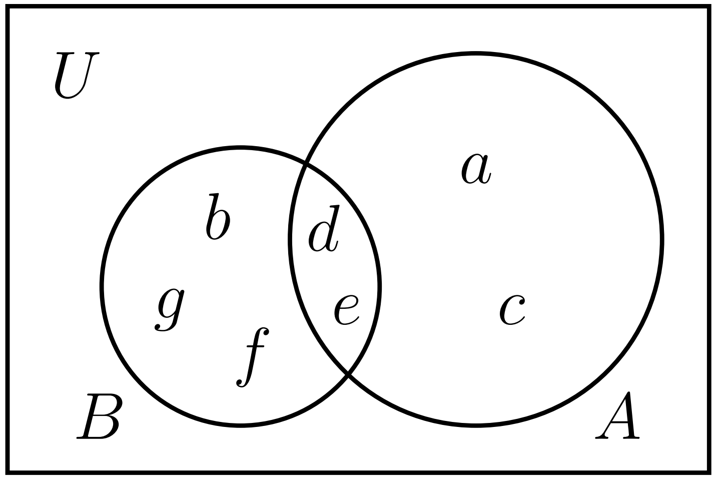

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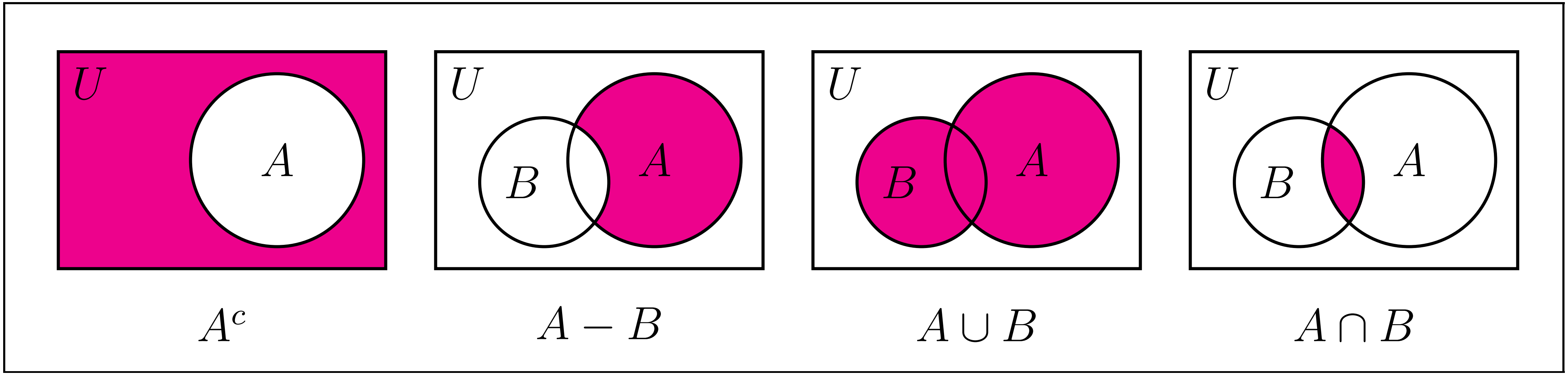

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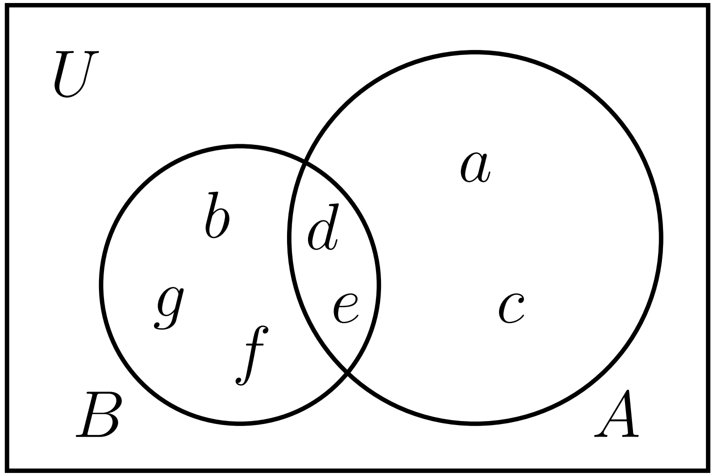

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- 17 can program in C++, Java, and Visual Basic.
- 22 can program in C++ and Java, but not Visual Basic.
- 9 can program in C++ and Visual Basic, but not Java.
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Also, all of the 100 students can program in at least one of these three languages. How many students can program in Java, but not C++ or Visual Basic?

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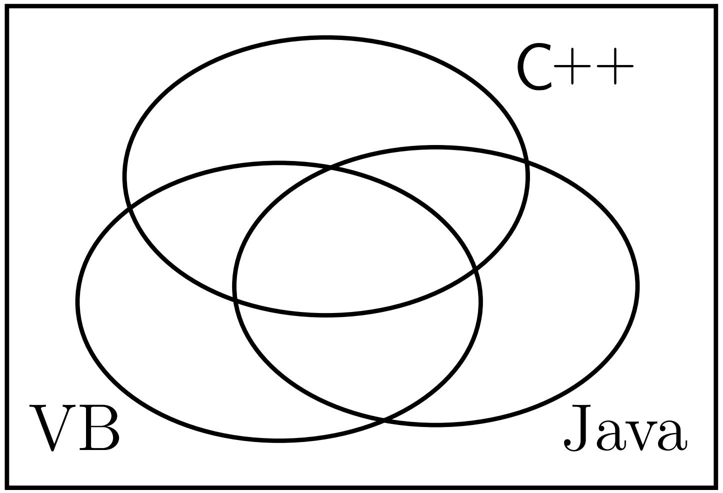

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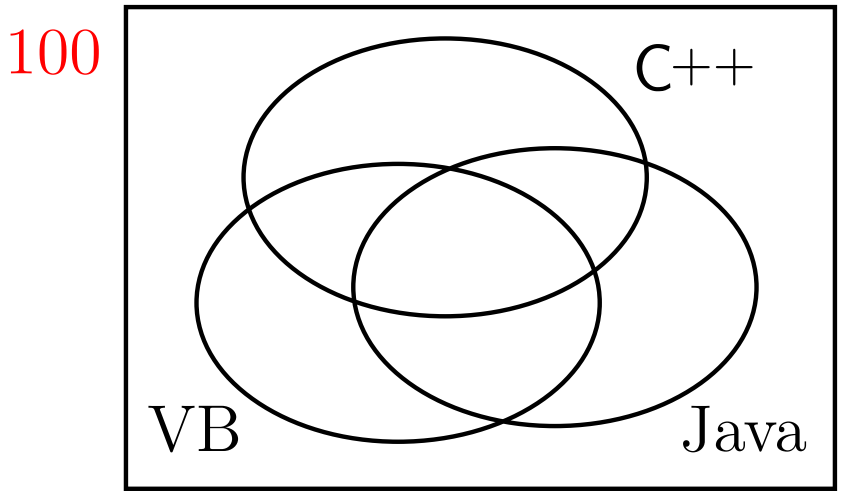

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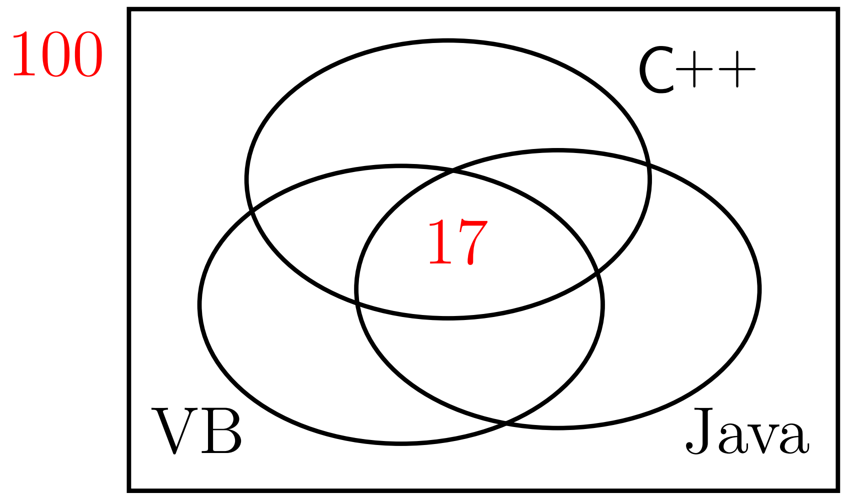

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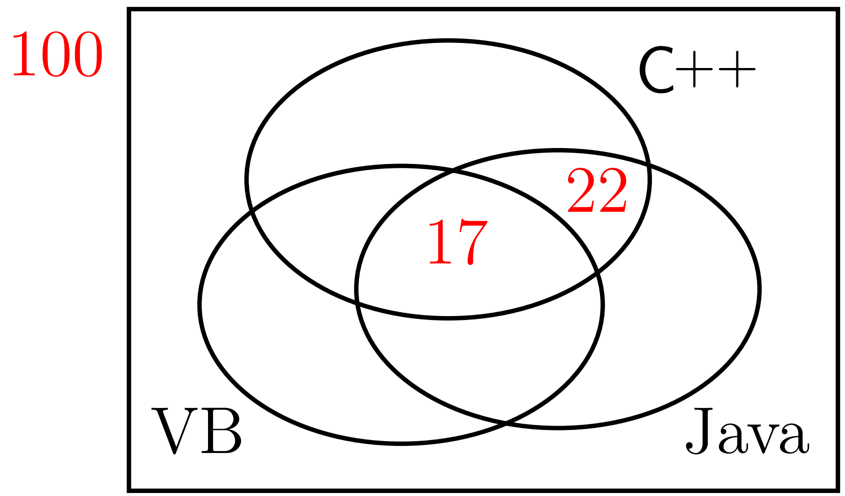

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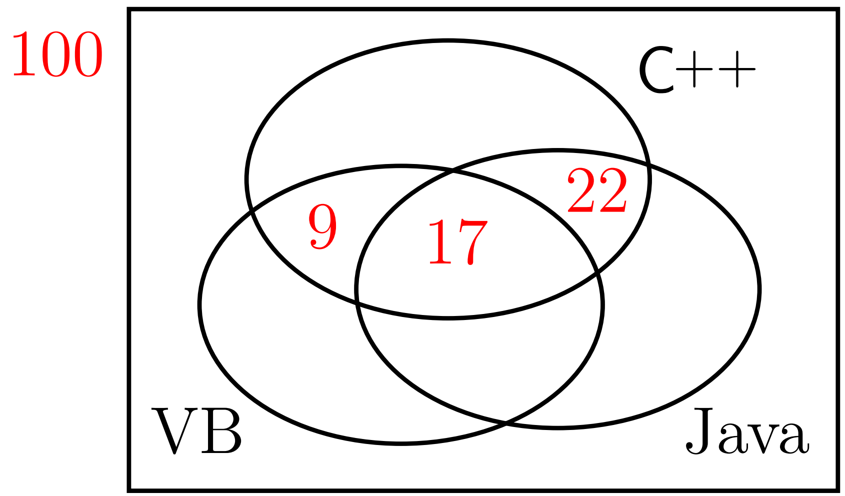

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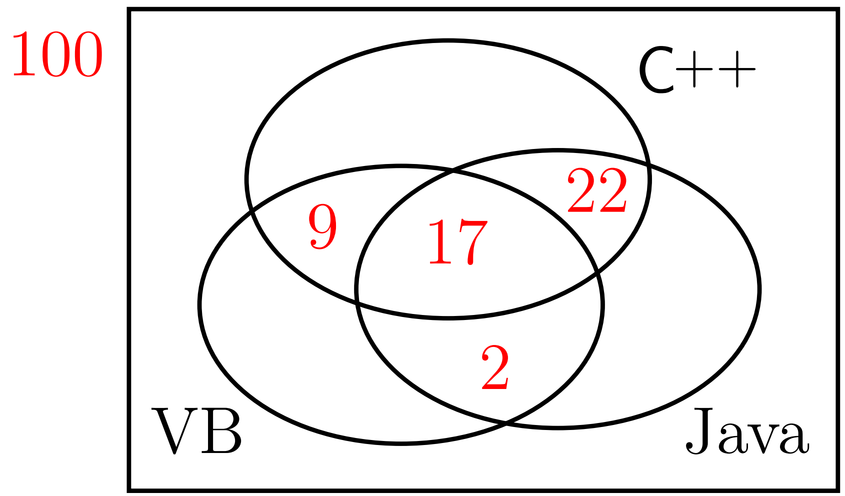

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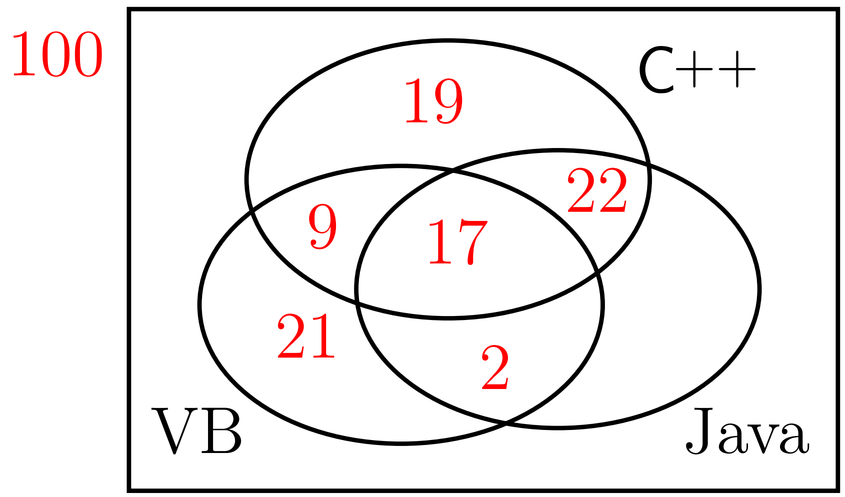

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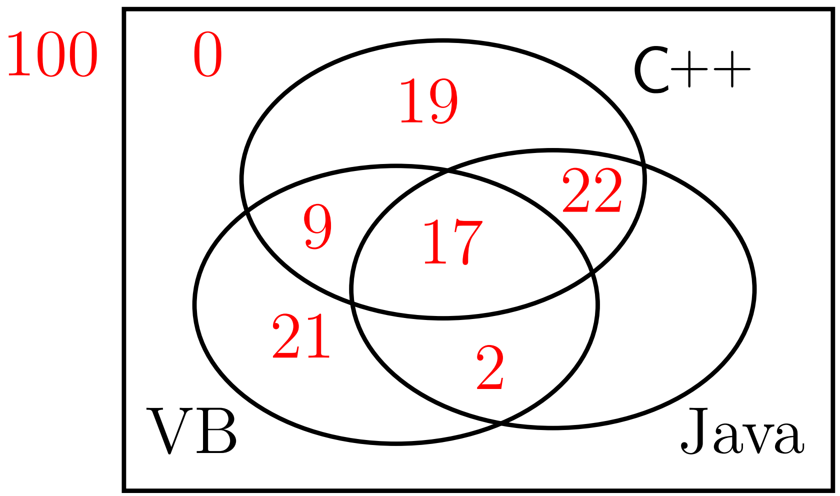

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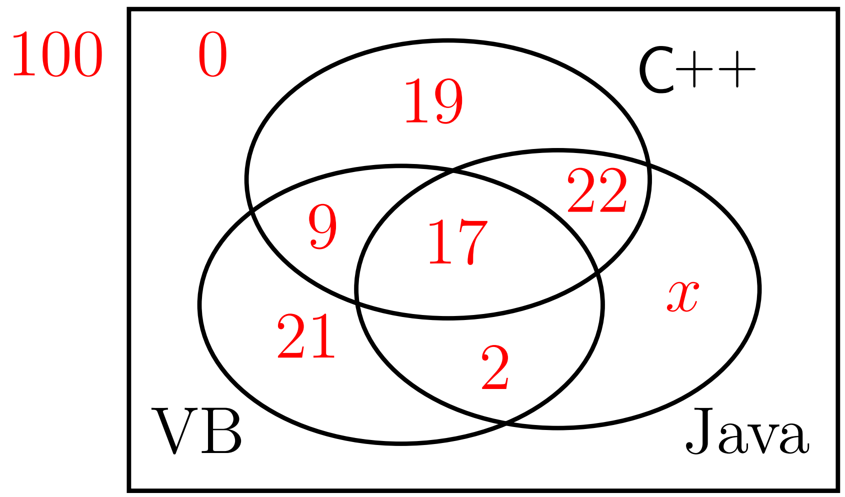

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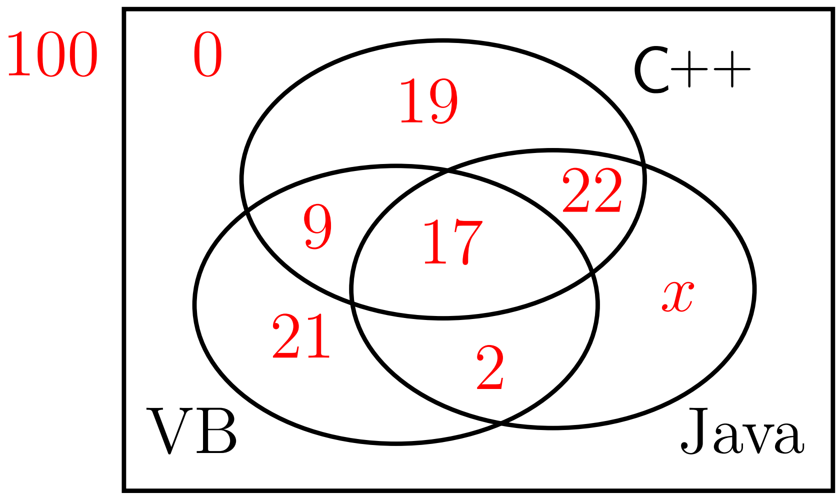

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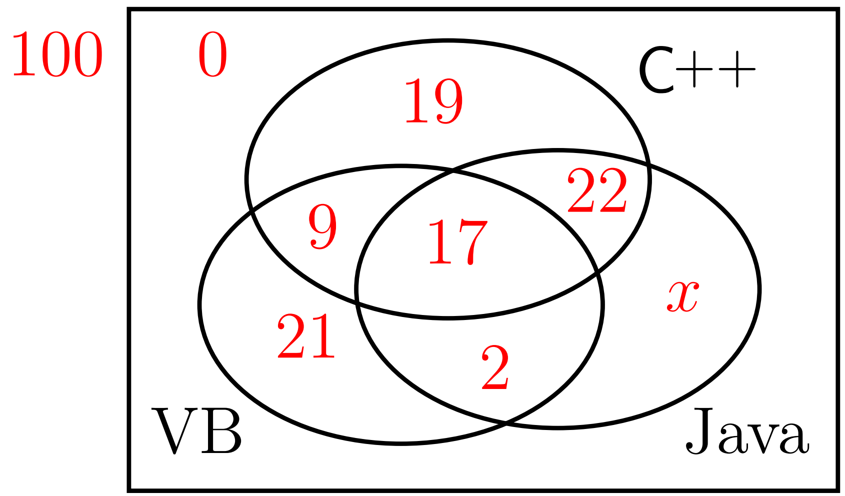

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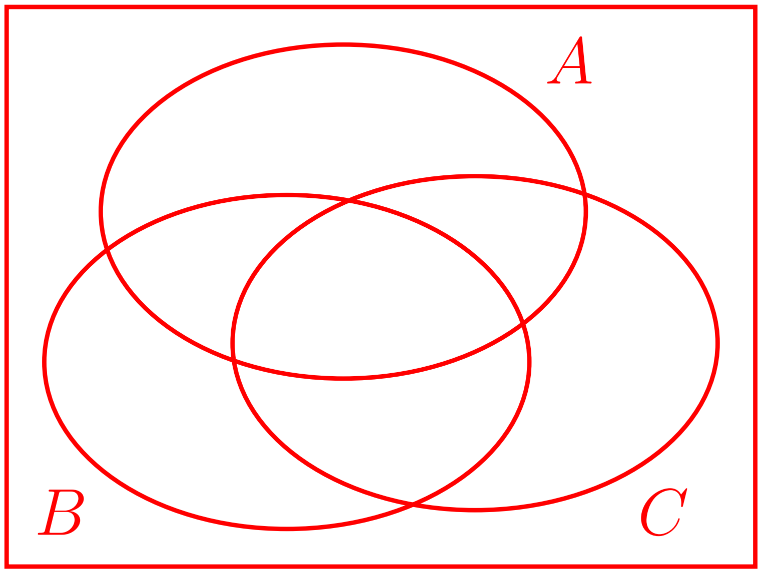

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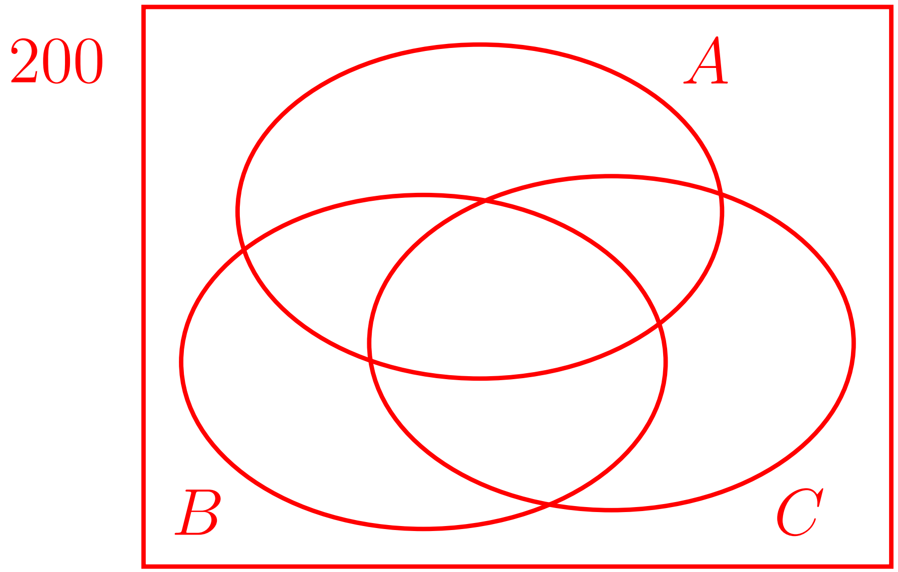

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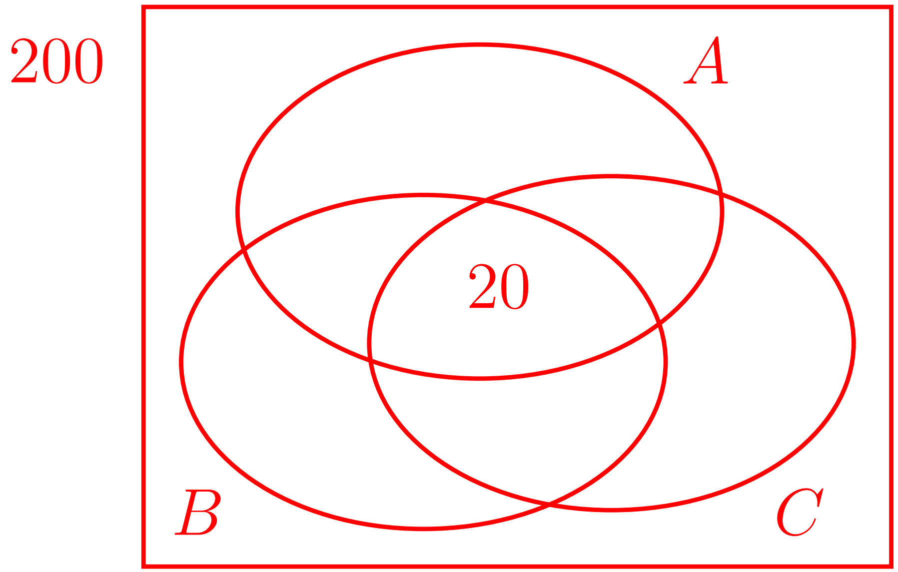

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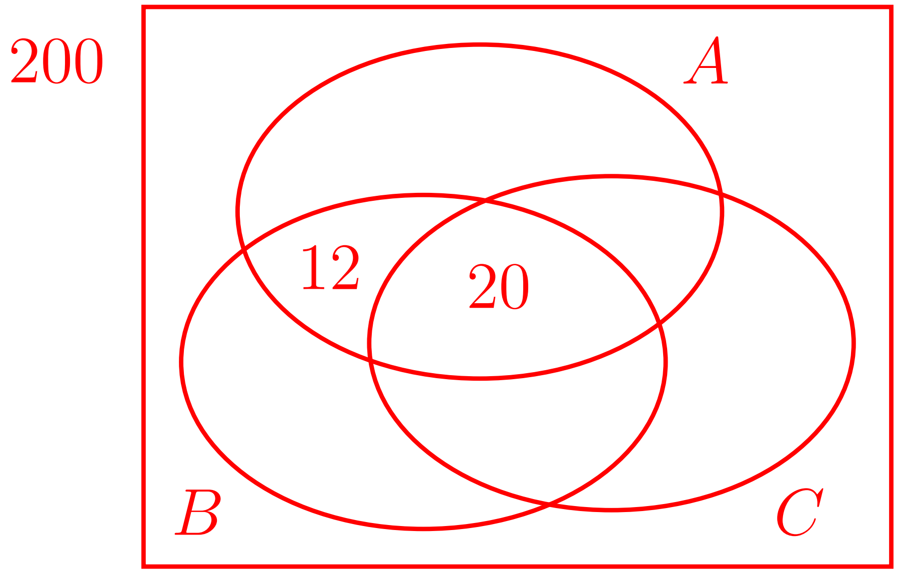

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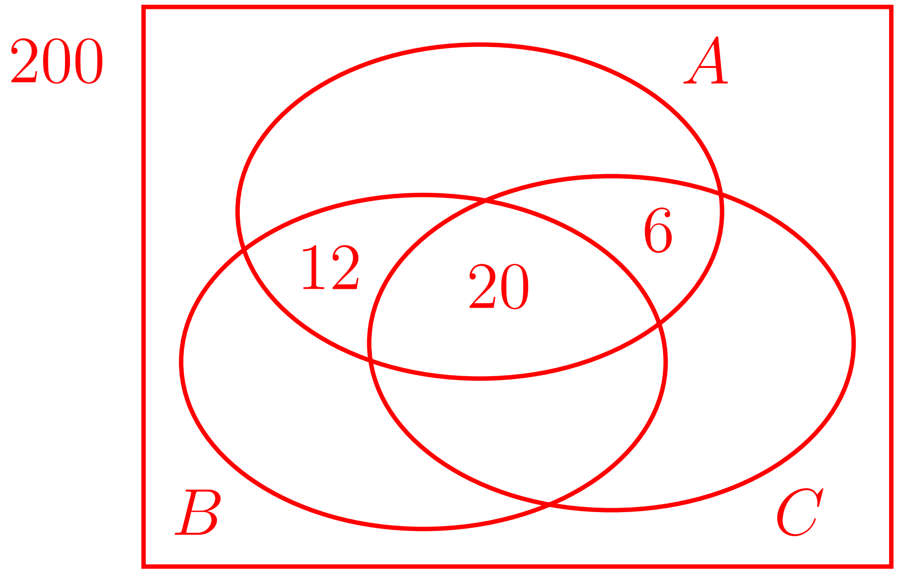

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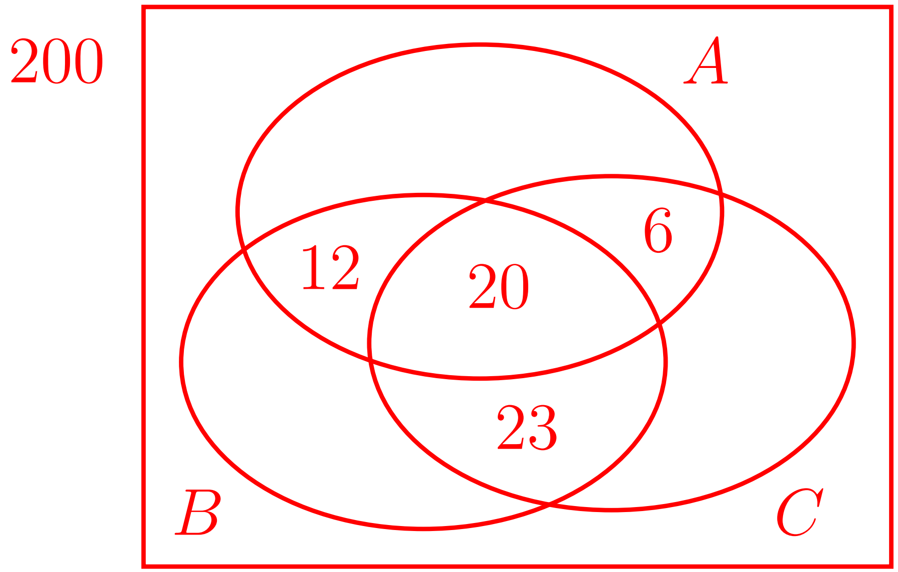

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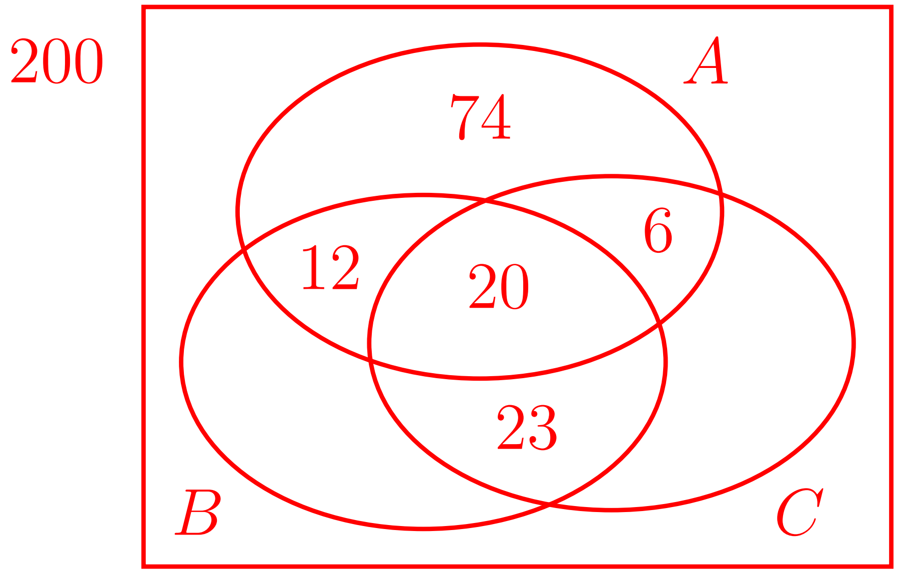

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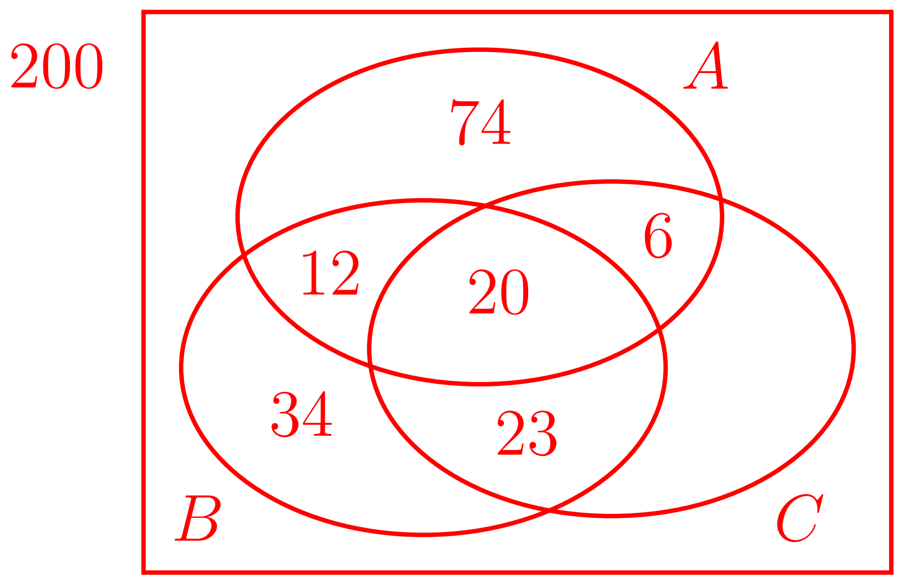

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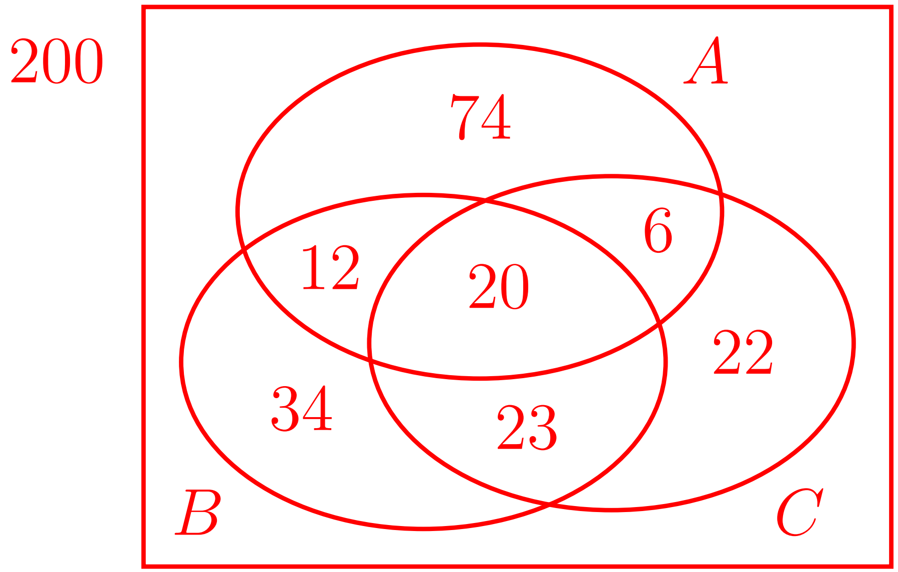

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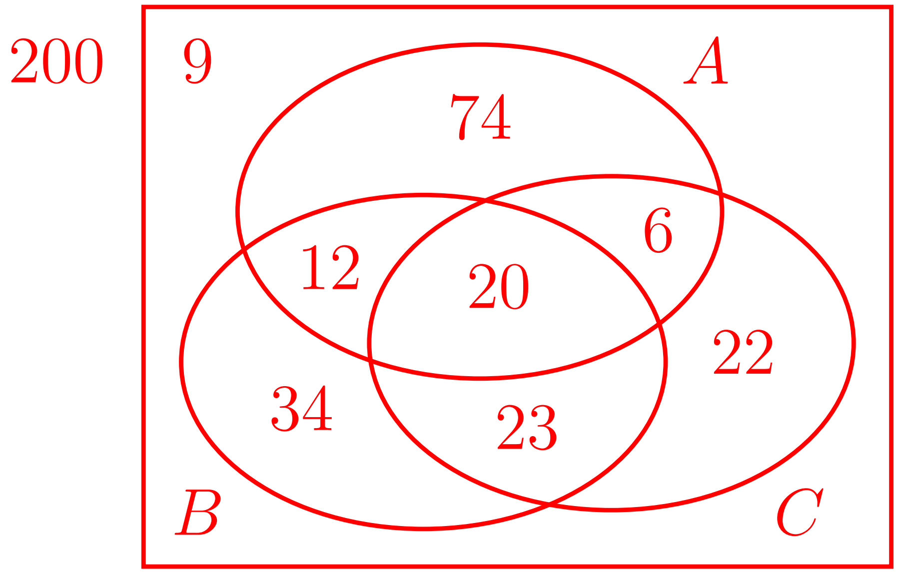

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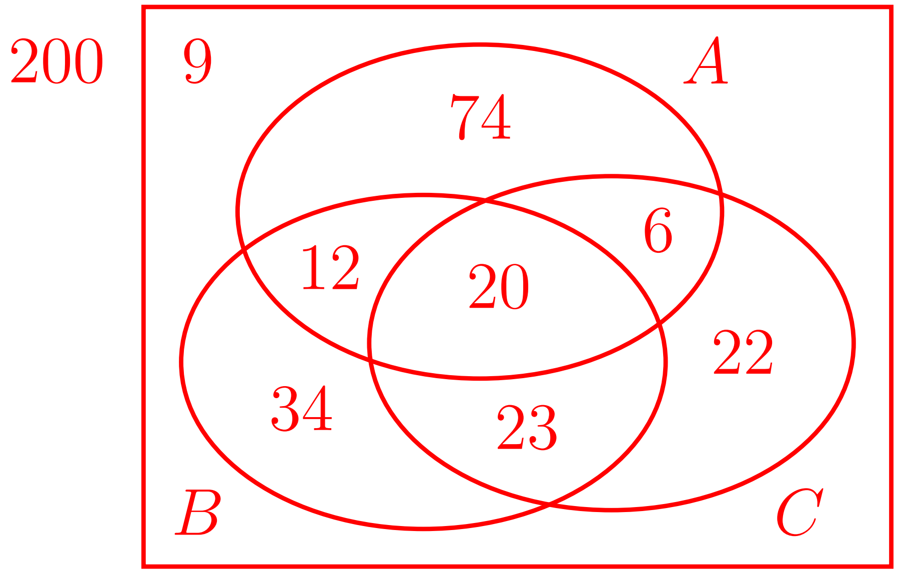

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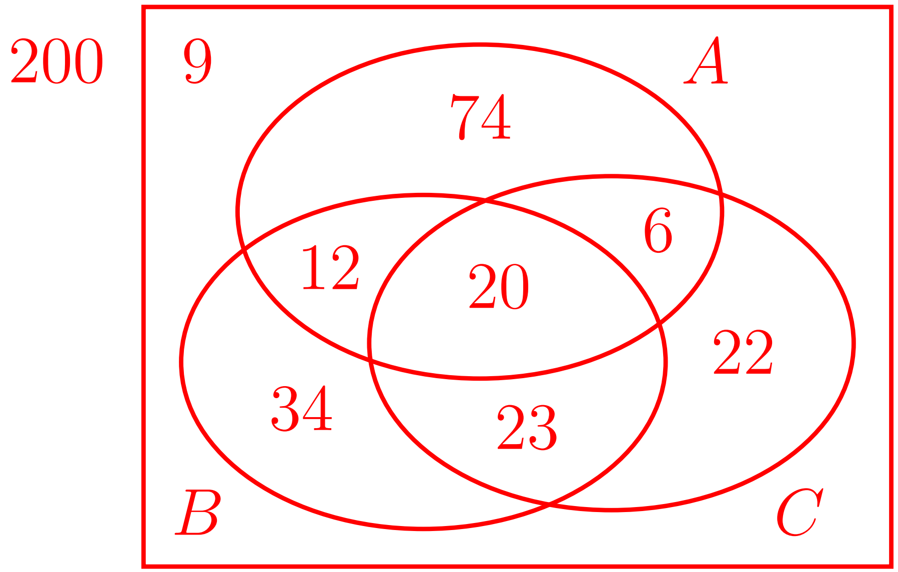

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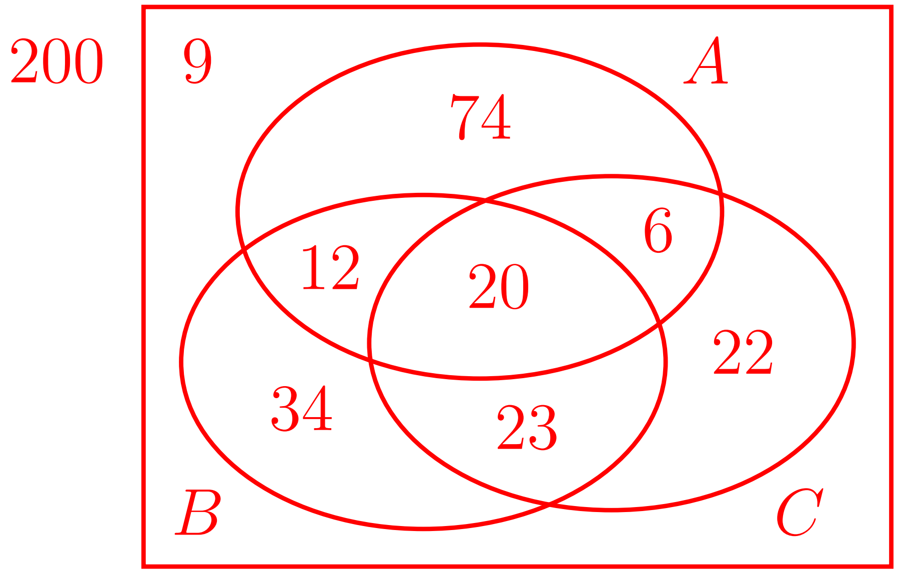

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In both cases, we have $x \in C$, which proves that $A \cup B \subseteq C$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Example. Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

Proof. Suppose that $A \subseteq C$ and $B \subseteq C$ and that $x \in A \cup B$.
Then either $x \in A$ or $x \in B$ (maybe both).
If $x \in A$, then $x \in C$, because $A \subseteq C$.
Likewise, if $x \in B$, then $x \in C$, since $B \subseteq C$.
In both cases, we have $x \in C$, which proves that $A \cup B \subseteq C$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Example. Prove that if $A \subseteq C$ and $B \subseteq C$, then $A \cup B \subseteq C$.

Proof. Suppose that $A \subseteq C$ and $B \subseteq C$ and that $x \in A \cup B$.
Then either $x \in A$ or $x \in B$ (maybe both).
If $x \in A$, then $x \in C$, because $A \subseteq C$.
Likewise, if $x \in B$, then $x \in C$, since $B \subseteq C$.
In both cases, we have $x \in C$, which proves that $A \cup B \subseteq C$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

Proof.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

Proof. Clearly, $A \cap B \subseteq A$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

Proof. Clearly, $A \cap B \subseteq A$.
Also, $A \subseteq A$ and $A \subseteq B$

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

Proof. Clearly, $A \cap B \subseteq A$.
Also, $A \subseteq A$ and $A \subseteq B$,
so $A \subseteq A \cap B$ by the previous exercise.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Proof. Suppose that $A \subseteq B$ and $A \subseteq C$ and that $x \in A$.
Then we have $x \in B$, because $A \subseteq B$.
Also, we have $x \in C$, since $A \subseteq C$.
Hence $x \in B \cap C$.
This proves that $A \subseteq B \cap C$.

Exercise. Prove that if $A \subseteq B$, then $A \cap B = A$.

Proof. Clearly, $A \cap B \subseteq A$.
Also, $A \subseteq A$ and $A \subseteq B$,
so $A \subseteq A \cap B$ by the previous exercise.
Therefore, $A \cap B = A$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Exercise. Prove that if $A \cap B = A$, then $A \cup B = B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Proof. Clearly, $A \cap B \subseteq A$.
Also, $A \subseteq A$ and $A \subseteq B$,
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Exercise. Prove that if $A \cap B = A$, then $A \cup B = B$.

Proof.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Proof. Clearly, $A \cap B \subseteq A$.
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Therefore, $A \cap B = A$.

Exercise. Prove that if $A \cap B = A$, then $A \cup B = B$.

Proof. Suppose that $A \cap B = A$ and let $x \in A \cup B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Proof. Clearly, $A \cap B \subseteq A$.
Also, $A \subseteq A$ and $A \subseteq B$,
so $A \subseteq A \cap B$ by the previous exercise.
Therefore, $A \cap B = A$.

Exercise. Prove that if $A \cap B = A$, then $A \cup B = B$.

Proof. Suppose that $A \cap B = A$ and let $x \in A \cup B$.
Then either $x \in A$ or $x \in B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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Proof. Clearly, $A \cap B \subseteq A$.
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Proof. Suppose that $A \cap B = A$ and let $x \in A \cup B$.
Then either $x \in A$ or $x \in B$.
If $x \in A$, then since $A = A \cap B$, we see that $x \in B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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If $x \in A$, then since $A = A \cap B$, we see that $x \in B$.
Hence, $x \in B$ in both cases, and so $A \cup B \subseteq B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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If $x \in A$, then since $A = A \cap B$, we see that $x \in B$.
Hence, $x \in B$ in both cases, and so $A \cup B \subseteq B$.
Since $B \subseteq A \cup B$, we conclude that $A \cup B \subseteq B$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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If $x \in A$, then since $A = A \cap B$, we see that $x \in B$.
Hence, $x \in B$ in both cases, and so $A \cup B \subseteq B$.
Since $B \subseteq A \cup B$, we conclude that $A \cup B \subseteq B$.

Exercise. Is the statement $A \cap (B \cup C) = (A \cap B) \cup C$ true?
Provide a proof if it is true or give a counter example if it is false.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Prove that if $A \cap B = A$, then $A \cup B = B$.

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Then either $x \in A$ or $x \in B$.
If $x \in A$, then since $A = A \cap B$, we see that $x \in B$.
Hence, $x \in B$ in both cases, and so $A \cup B \subseteq B$.
Since $B \subseteq A \cup B$, we conclude that $A \cup B \subseteq B$.

Exercise. Is the statement $A \cap (B \cup C) = (A \cap B) \cup C$ true?
Provide a proof if it is true or give a counter example if it is false.
It is false.
For example, take $A = \{1\}$, $B = \{1\}$, $C = \{2\}$.
Then $A \cap (B \cup C) = \{1\}$
but $(A \cap B) \cup C = \{1, 2\}$.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

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  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
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It is false.
For example, take $A = \{1\}$, $B = \{1\}$, $C = \{2\}$.
Then $A \cap (B \cup C) = \{1\}$
but $(A \cap B) \cup C = \{1, 2\}$.

Exercise. Is the statement $A - (B - C) = (A - B) - C$ true?
Provide a proof if it is true or give a counter example if it is false.

- Hints for proofs:
  - To prove that $S \subseteq T$, we can assume that $x \in S$ and show that $x \in T$.
  - To prove that $S = T$, we can show that $S \subseteq T$ and $T \subseteq S$.

Exercise. Is the statement $A \cap (B \cup C) = (A \cap B) \cup C$ true?
Provide a proof if it is true or give a counter example if it is false.

It is false.

For example, take $A = \{1\}$, $B = \{1\}$, $C = \{2\}$.

Then $A \cap (B \cup C) = \{1\}$

but $(A \cap B) \cup C = \{1, 2\}$.

Exercise. Is the statement $A - (B - C) = (A - B) - C$ true?
Provide a proof if it is true or give a counter example if it is false.

It is false.

For example, take $A = \{a, b, c\}$, $B = \{b, c\}$, $C = \{c\}$.

Then $A - (B - C) = \{a, c\}$

but $(A - B) - C = \{a\}$.