# THE UNIVERSITY OF SYDNEY

## FACULTIES OF ARTS, EDUCATION & SOCIAL WORK, ENGINEERING AND SCIENCE

# PHYS1902 – PHYSICS 1B (ADVANCED)

## NOVEMBER 2006

### Time allowed: THREE Hours

**MARKS FOR QUESTIONS ARE AS INDICATED**
**TOTAL: 90 marks**

## INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

## DATA

| | | | |
|---|---|---|---|
| Density of water | $\rho$ | = | $1.00 \times 10^{3}$ kg.m$^{-3}$ |
| Density of air | $\rho$ | = | $1.20$ kg.m$^{-3}$ |
| Atmospheric pressure | 1 atm | = | $1.01 \times 10^{5}$ Pa |
| Magnitude of local gravitational field | $g$ | = | $9.81$ m.s$^{-2}$ |
| Avogadro constant | $N_A$ | = | $6.022 \times 10^{23}$ mol$^{-1}$ |
| Permittivity of free space | $\varepsilon_0$ | = | $8.854 \times 10^{-12}$ F.m$^{-1}$ |
| Permeability of free space | $\mu_0$ | = | $4\pi \times 10^{-7}$ T.m.A$^{-1}$ |
| Elementary charge | $e$ | = | $1.602 \times 10^{-19}$ C |
| Speed of light in vacuum | $c$ | = | $2.998 \times 10^{8}$ m.s$^{-1}$ |
| Planck constant | $h$ | = | $6.626 \times 10^{-34}$ J.s |
| Rest mass of an electron | $m_e$ | = | $9.110 \times 10^{-31}$ kg |
| Rest mass of a neutron | $m_n$ | = | $1.675 \times 10^{-27}$ kg |
| Rest mass of a proton | $m_p$ | = | $1.673 \times 10^{-27}$ kg |
| Rest mass of a hydrogen atom | $m_H$ | = | $1.674 \times 10^{-27}$ kg |
| Boltzmann constant | $k$ | = | $1.381 \times 10^{-23}$ J.K$^{-1}$ |
| Atomic mass unit | u | = | $1.661 \times 10^{-27}$ kg |
| Rydberg constant | R | = | $1.097 \times 10^{7}$ m$^{-1}$ |

# SECTION A

## Question 1

A director wants to make a movie to include action on the Moon. She decides to simulate lunar gravity on Earth by using helium-filled balloons to provide buoyancy.

(a) Draw a force diagram showing the desired set-up, including all forces on the actor and the resultant force.

(b) Derive an expression for the lift provided by a helium-filled balloon with volume *V*.

(c) The mass of the actor in his space-suit is 120 kg. The Moon's gravity is 1/6 that of Earth. What is the radius of the (spherical) balloon which needs to be attached to the actor to simulate lunar gravity?

Take the density of helium ($\rho_{He}$) to be 0.16 kg.m$^{-3}$ and the density of air ($\rho_{air}$) as 1.2 kg.m$^{-3}$

**(5 marks)**

## Question 2

(a) Explain why long-distance power lines are operated at a very high voltage.

(b) If the power lines were replaced with superconductors that have zero resistance, would this affect the reason(s) for choosing a high voltage?

Justify your answers with appropriate equations.

**(5 marks)**

## Question 3

A mass spectrometer uses electric and magnetic fields to measure the masses of charged ions.

(a) First, positive ions are created, all of the same charge, but with a distribution of speeds.

Using only the Lorentz force law, explain how electric and magnetic fields can be used as a *velocity selector* to pick out specific speeds of the ions. Draw a diagram to support your explanation.

(b) Next, with all the ions possessing the same speed, a magnetic field is used to separate and measure different masses. The ions travel in circular motion, the radius of which depends on their masses.

Again, using only the Lorentz force law, express the mass of an ion in terms of its velocity $v$, the radius of its motion $R$, its charge $q$, and the magnetic field $B$.

**(5 marks)**

**Question 4**

Consider the Ampere-Maxwell Law: $\oint \mathbf{B}.d\mathbf{l} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ where $\mu_0$ and $\varepsilon_0$ are constants of nature.

Briefly explain, using words rather than equations, what is meant by each of the three components of this equation:

(a) $\oint \mathbf{B}.d\mathbf{l}$

(b) $i$

(c) $\frac{d\Phi_E}{dt}$

(d) Explain briefly what this law tells us about how a magnetic field is produced and about the shape of magnetic field lines.

**(5 marks)**

**Question 5**

(a) Write down and briefly explain the terms in the statement of the Heisenberg uncertainty principle for the position and momentum of a particle moving in the $x$ direction.

(b) Explain why simultaneous knowledge of all three components of the angular momentum of a particle is inconsistent with the uncertainty principle.

(c) Consider a particle with mass $m$ undergoing 1-D motion between high potential barriers at $x = 0$ and $x = L$. Using the uncertainty principle, show that the kinetic energy of the particle must be at least $\hbar^2 /(mL^2)$.

**(5 marks)**

## Question 6

Electrons are incident on a potential energy barrier. Their kinetic energy $K$ is lower than the energy $U$ of the barrier so that, if classical physics was correct, the probability of transmission past the barrier would be zero. In fact, the transmission probability is found to be non-zero.

(a) Explain briefly why this “tunnelling through the barrier” occurs.

The same experiment is done with protons. Again they are incident on a barrier with energy $U$ and have kinetic energy $K$.

(b) Given below are four possible outcomes of this experiment. Using the Correspondence Principle or another argument, decide which one of these is correct. Briefly explain why you think so.

(i) No tunnelling occurs.

(ii) Tunnelling does occur, but the tunnelling probability for protons is lower than for electrons.

(iii) Tunnelling does occur, and the tunnelling probability for protons is the same as for electrons.

(iv) Tunnelling does occur, but the tunnelling probability for protons is higher than for electrons.

**(5 marks)**

# SECTION B
## (Please use a separate booklet for this section)

**Question 7**

An aspirator attached to the end of a garden hose may be used to deliver liquid fertiliser from a reservoir (see the diagram below).

Suppose that the diameter at A, near the garden hose, is 20 mm and at C, where the water exits to the atmosphere, it is 10 mm. The flow rate is $0.5 \times 10^{-3}\ m^3.s^{-1}$ and the gauge pressure at A is 19 kPa.

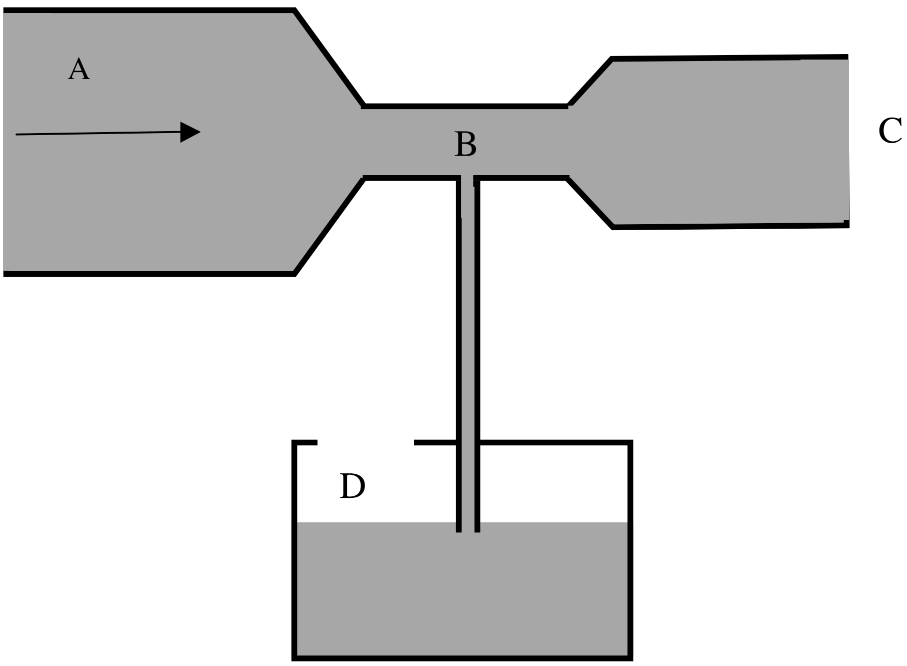
(a) What is the *absolute* pressure (i) at A; and
(ii) at C?

(b) What is the flow speed (i) at A; and
(ii) at C?

(c) What is the pressure at D?

(d) Explain how this device delivers fertiliser from the reservoir to the garden.

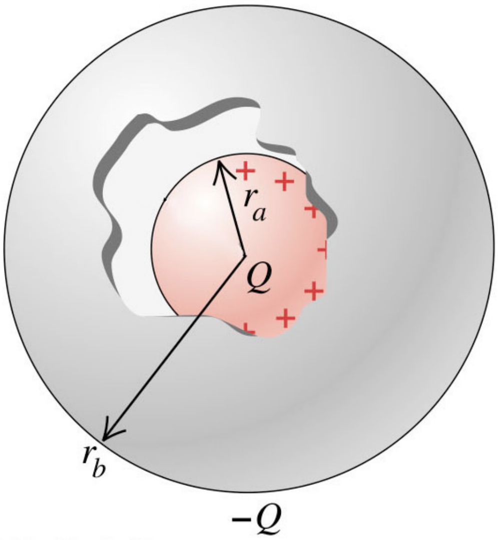
(e) What diameter of the constriction at B will achieve an absolute pressure of 10 kPa in the constricted area at B?




**(10 marks)**

## Question 8

A *conducting* sphere of radius $r_a$ with charge +Q is placed at the origin. It is surrounded by a thin conducting shell of radius $r_b$ with charge –Q.




(a) Find the electric field $E(r)$ at a point a distance $r$ from the origin, when the point is

(i) inside the sphere ( $r < r_a$)

(ii) between the sphere and the shell ($r_a < r < r_b$)

(iii) outside the shell ($r_b < r$)

(b) Find the corresponding electric potential $V(r)$ at these points – i.e.

(i) inside the sphere ( $r < r_a$)

(ii) between the sphere and the shell ($r_a < r < r_b$)

(iii) outside the shell ($r_b < r$)

(assume $V(r) \to 0$ as $r \to \infty$)

(c) Show that the capacitance of the capacitor formed by the sphere and shell can be written as

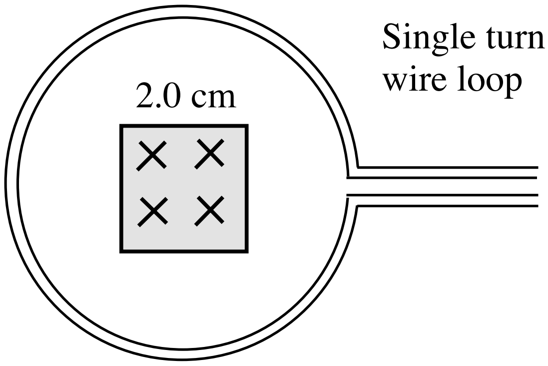
$$C = 4\pi\varepsilon_\circ \left( \frac{r_a r_b}{r_b - r_a} \right)$$

(d) What is the potential energy stored in this capacitor?

**(10 marks)**

## Question 9

A uniform but time-varying magnetic field $B$ exists in the 2.0 cm x 2.0 cm square region within a loop, and is zero outside of this region. The field is perpendicular to the plane of the loop (as shown in the diagram at an instant when $B$ is directed into the page).




(a) At time $t$, $B = B_0\cos(\omega t)$ where $B_0$ = 0.50 T and $\omega$ = 9.42 rad.s$^{-1}$. What is the amplitude of the emf $E$ induced in the wire loop?

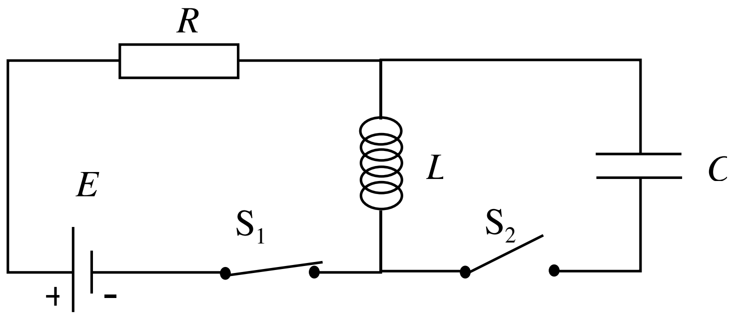
(b) Draw sketches of $B$ vs $t$ and $E$ vs $t$ for the range $t$ = 0 to 2.0 s, labelling all axes clearly with suitable scales and units.

(c) If two turns of the wire are used instead of a single turn as shown, what is the resulting change in the amplitude of the induced emf? Explain your answer.

**(10 marks)**

## Question 10

Consider the following circuit, with an emf $E$ = 10.0 V, a resistor $R$ = 125 Ω, an ideal inductor $L = 10.0$ mH, a capacitor $C$ = 25.0 μF, and two switches $S_1$ (initially closed) and $S_2$ (initially open).




(a) Assuming switch $S_1$ has been closed (as illustrated above) for a long time, calculate the current through the inductor $L$.

(b) The switch $S_1$ is now opened, and simultaneously the switch $S_2$ is closed. Calculate the current in the closed LC circuit immediately after $S_1$ is opened and $S_2$ is closed. Briefly explain why the current has this value.

(c) Calculate the earliest time at which the charge on the capacitor $C$ is at its maximum value.

(d) Calculate the energy stored in the capacitor at this time.

(e) Explain where the energy in this capacitor is stored.

(f) Explain where did this energy originate from, i.e. where was this energy at the time that the LC circuit was first closed?

**(10 marks)**

**Question 11**

The 1-D time-independent Schrödinger equation describing a particle with mass $m$ and energy $E$ subject to a conservative force described by a potential $U(x)$ is

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$$

(a) Briefly explain the physical interpretation of the wave function $\psi(x)$.

(b) Consider a free particle, i.e. a particle subject to no forces, for which the potential $U(x) = 0$. Show that

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}$$

is a solution to the Schrödinger equation in this case. Specify the value of $k$ required.

(c) Consider a potential which is zero between $x = 0$ and $x = L$, and which is infinite at $x = 0$ and $x = L$. The solution for the Schrödinger equation with this potential describes a 'particle in a box'.

(i) State the boundary conditions which should be imposed on the wave function at $x = 0$ and $x = L$.

(ii) Why are these boundary conditions necessary?

(iii) Solve the Schrödinger equation in the region $0 \le x \le L$ to show that the allowed values of energy of the particle are given by

$$E = \frac{h^2 n^2}{8mL^2} \text{ where } n = 1, 2, \ldots.$$

(d) Give a brief qualitative explanation of how the results in (c) part (iii) change if the potential is zero between $x=0$ and $x=L$ and has the constant value $U_0 > E$ everywhere else - i.e. a finite square potential well. (You are not required to solve the Schrödinger equation.)

**(10 marks)**

## Question 12

The spectrum of a blackbody is described by the Planck formula

$$I_\lambda(\lambda) = \frac{2\pi hc^2}{\lambda^5\left[e^{hc/(\lambda kT)} - 1\right]}.$$

(a) On a single diagram, sketch the blackbody spectrum for two temperatures, $T_0$ and $T_1$, where $T_0 < T_1$.

(b) With reference to your diagram, briefly explain what is meant by the Wien displacement law.

(c) Show that when the wavelength $\lambda$ is much greater than $hc/(kT)$, the Planck spectrum takes the form of the Rayleigh-Jeans law:

$$I_\lambda(\lambda) \approx \frac{2\pi ckT}{\lambda^4}.$$

(d) Briefly explain how classical physics was unable to explain the blackbody spectrum.

(e) By making a change in the variable of integration, show that

$$I = \int_0^\infty I_\lambda(\lambda)\,d\lambda$$

is equal to a constant times $T^4$ (the Stefan-Boltzmann law).

Hint: try the change of variable $x = \dfrac{hc}{\lambda kT}$

**(10 marks)**

## This is the end of your questions