ASTRO201: Introduction to Astrophysics
Homework 2

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1. a) Using the formula for the energy of level n,

$$\begin{aligned}
E_n &= \frac{-13.6}{n^2} \\
E_1 &= -13.6 \text{ eV}, \\
E_2 &= -3.4 \text{ eV}, \\
E_3 &= -1.5 \text{ eV}, \\
E_4 &= -0.85 \text{ eV}, \\
E_5 &= -0.54 \text{ eV}.
\end{aligned}$$

b) Using the previous part, the energies of the first 3 transitions are

$$\begin{aligned}
E_2 - E_3 &= -1.9 \text{ eV}, \\
E_2 - E_4 &= -2.55 \\
&= -2.6 \text{ eV}, \\
E_2 - E_5 &= -2.856 \\
&= -2.9 \text{ eV}.
\end{aligned}$$

c) Rearranging the formula relating energy and wavelength, with $E_m > E_n$,

$$\begin{aligned}
E_m - E_n &= \frac{hc}{\lambda} \\
\therefore \lambda &= \frac{hc}{E_m - E_n}. \\
\lambda &= \frac{hc}{E_3 - E_2} \\
&= \frac{4.135667696 \times 10^{-15} \times 3 \times 10^{17}}{1.9} \\
\therefore \lambda &= 653 \text{ nm}, \\
\lambda &= \frac{hc}{E_4 - E_2} \\
&= \frac{4.135667696 \times 10^{-15} \times 3 \times 10^{17}}{2.55} \\
&= 486.549 \\
\therefore \lambda &= 490 \text{ nm}, \\
\lambda &= \frac{hc}{E_5 - E_2} \\
&= \frac{4.135667696 \times 10^{-15} \times 3 \times 10^{17}}{2.856} \\
&= 434.4188756 \\
\therefore \lambda &= 430 \text{ nm}.
\end{aligned}$$

d) These Balmer series transitions fall under the Visible Light part of the spectrum.

2. a) From the previous part, the $n = 4$ energy level has an energy of -0.85 eV, and so a hydrogen atom with its electron in the $n = 4$ level requires 0.85 eV to ionise the atom.

   b) Using the formula relating temperature and energy of particles,

$$\begin{aligned}
\kappa T &= h\nu \\
&= E_m - E_n \\
\therefore T &= \frac{E_m - E_n}{\kappa} \\
T &= \frac{E_\infty - E_4}{\kappa} \\
&= \frac{0 - (-0.85)}{8.617333262145 \times 10^{-5}} \\
&= 9863.84 \\
\therefore T &= 9900\ \mathsf{K}
\end{aligned}$$

3. a) Note the conversions $100\ \mathsf{AU} = 1.496 \times 10^{13}$ m, and $745\ \mathsf{yrs} = 2.349 \times 10^{10}$ s. Using the formula for the sum of masses in an edge-on binary system,

$$\begin{aligned}
\frac{4\pi^2 R^3}{G} &= (m_1 + m_2)P^2 \\
\therefore m_1 + m_2 &= \frac{4\pi^2 R^3}{G \times P^2} \\
&= \frac{4\pi^2 \left(1.496 \times 10^{13}\right)^3}{6.674 \times 10^{-11} \times (2.349 \times 10^{10})^2} \\
&= 3.589238201 \times 10^30\ \mathsf{kg} \\
&= 1.804544093\ \mathsf{M}_\odot \\
\therefore m_1 + m_2 &= 1.8\ \mathsf{M}_\odot
\end{aligned}$$

   b) As we have been given the maximum Doppler shift of a 500 nm spectral line, this occurs when the angle of either star relative to the centre of their orbit is 0. Thus, the formula used is

$$\begin{aligned}
\frac{\Delta\lambda}{\lambda} &= \frac{v}{c}\cos(0) \\
\therefore v &= \frac{c\Delta\lambda}{\lambda}, \\
v_1 &= \frac{c\Delta\lambda}{\lambda} \\
&= \frac{3 \times 10^{10} \times 4.05 \times 10^{-10}}{500 \times 10^{-7}} \\
&= 243000\ \mathsf{cm/s} \\
&= 0.5123\ \mathsf{AU/yr} \\
\therefore v_1 &= 0.51\ \mathsf{AU/yr},
\end{aligned}$$

$$
\begin{aligned}
v_2 &= \frac{c\Delta\lambda}{\lambda} \\
&= \frac{3 \times 10^{10} \times 2.59 \times 10^{-10}}{500 \times 10^{-7}} \\
&= 155400 \text{ cm/s} \\
&= 0.3276 \text{ AU/yr} \\
\therefore v_2 &= 0.33 \text{ AU/yr},
\end{aligned}
$$

c) Combining the formulas relating each star's mass, radius and velocity in an edge-on binary system,

$$
\begin{aligned}
m_1 r_1 &= m_2 r_2 \\
\frac{r_1}{r_2} &= \frac{v_1}{v_2} \\
\therefore \frac{m_2}{m_1} &= \frac{v_1}{v_2} \\
&= \frac{0.5123}{0.3276} \\
&= 1.563797314 \\
\therefore \frac{m_2}{m_1} &= 1.6.
\end{aligned}
$$

Thus, the ratio of Star 1's mass to Star 2's mass is $1 : 1.6$.

d) Using the two equations found in the previous parts, and solving simultaneously,

$$
\begin{aligned}
m_1 + m_2 &= 1.804544093 \ldots (A) \\
\frac{m_2}{m_1} &= 1.563797314 \ldots (B) \\
(B) &\Rightarrow \; m_2 = 1.563797314 m_1 \ldots (C) \\
(C) \to (B) &\Rightarrow \; 2.563797314 m_1 = 1.804544093 \\
\therefore m_1 &= 0.703855988 \\
&= 0.70 \text{ M}_\odot \\
\therefore m_2 &= 1.100688105 \\
&= 1.1 \text{ M}_\odot
\end{aligned}
$$

4. a) Converting the apparent magnitudes to luminosities, we have,

$$
\begin{aligned}
\frac{L_{binary}}{L_0} &= \frac{L_{+2.4}}{L_0} + \frac{L_{+5.2}}{L_0} \\
&= 10^{-2.4/2.5} + 10^{-5.2/2.5} \\
&= 0.109647819 + 0.008317637 \\
&= 0.117965456 \\
m_{binary} &= -2.5 \log \left(\frac{L_{binary}}{L_0} \right) \\
&= -2.5 \log 0.117965456 \\
&= 2.320612873 \\
\therefore m_{binary} &= 2.3
\end{aligned}
$$

b) For Star A, using the Appendix, we have an apparent magnitude of $+5.20$, and an absolute magnitude of $+7.4$. Thus,

$$
\begin{aligned}
m_V - M_V &= -2.2 \\
\therefore 5 \log_{10} d_{pc} - 5 &= -2.2 \\
\log_{10} d_{pc} &= 0.56 \\
d_{pc} &= 10^{0.56} \\
&= 3.63 \\
\therefore d_{pc} &= 3.6 \text{ parsecs.}
\end{aligned}
$$

For Star B, using the Appendix, we have an apparent magnitude of $+2.4$, and an absolute magnitude of $+4.6$. Thus,

$$
\begin{aligned}
m_V - M_V &= -2.2 \\
\therefore 5 \log_{10} d_{pc} - 5 &= -2.2 \\
\log_{10} d_{pc} &= 0.56 \\
d_{pc} &= 10^{0.56} \\
&= 3.63 \\
\therefore d_{pc} &= 3.6 \text{ parsecs.}
\end{aligned}
$$

Thus, both stars are 3.6 parsecs away, and so likely are in a binary system.