THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Tutorial for Week 11

MATH1903: Integral Calculus and Modelling (Advanced) — Semester 1, 2012

Web Page: http://www.maths.usyd.edu.au/u/UG/JM/MATH1903/
Lecturers: Daniel Daners and James Parkinson

Questions marked with * are more difficult questions.

Material covered

(1) First order linear and separable differential equations

(2) Substitution of variables in differential equations

(3) homogeneous second order differential equations with constant coefficients

Outcomes

After completing this tutorial you should

(1) should consolidate the ability to solve first order linear and separable differential equations by the appropriate methods.

(2) be able to solve homogeneous second order differential equations in case of two distinct solutions of the auxiliary equation.

Questions to do before the tutorial

1. Classify each of the following differential equations as separable or linear, and find its general solution.

(a) $\dfrac{dy}{dx} = \dfrac{2xy^2}{1+x^2}$ (b) $\dfrac{dy}{dx} = x + \dfrac{y}{x}$

2. Find the general solutions of the following homogeneous second-order linear differential equations. Then compute the particular solution satisfying the initial conditions $y(0) = \dot{y}(0) = -1$.

(a) $\dfrac{d^2y}{dt^2} - \dfrac{dy}{dt} - 6y = 0$ (b) $\dfrac{d^2y}{dt^2} + 16y = 0$

Questions to complete during the tutorial

No questions due to quiz

Extra questions for further practice

3. Find the general solutions of the following homogeneous second-order linear differential equations. Then compute the particular solution satisfying the initial conditions $y(0) = \dot{y}(0) = -1$.

(a) $\dfrac{d^2y}{dt^2} + 6\dfrac{dy}{dt} + 13y = 0.$

4. (a) Classify the differential equation $(x^2+1)\dfrac{dy}{dx} + x = xy^2$ as separable or linear, and find its general solution. Then determine the particular solution for which $y = 0$ when $x = 1$.

(b) Use the substitution $v = xw$ to find the general solution of the differential equation

$$2xv\frac{dv}{dx} = 3v^2 - 4x^2.$$

5. (a) Show that a differential equation of the form $y' = f(y/x)$ becomes separable when transformed into a differential equation for $v = y/x$.

(b) Show that the differential equation $\dfrac{dy}{dx} = \dfrac{y^3}{x(x^2+y^2)}$ is of the form $y' = f(y/x)$ and hence solve it by transforming it into a separable equation.

6. Find the general solution of $\dfrac{dy}{dx} = \dfrac{1}{x+y}$.

7. Solve the following differential equations by the transformations indicated:

(a) $3x\dfrac{dy}{dx} + y + x^2y^4 = 0$ (a Bernoulli equation, let $w = \dfrac{1}{y^3}$).

(b) $\dfrac{dy}{dx} + xy^2 + \dfrac{3}{4x^3} = 0$ (a Riccati equation, let $y = \dfrac{1}{2x^2} + \dfrac{1}{w}$).

*(c) $x\dfrac{dy}{dx} - y = \dfrac{1}{4}\left(\dfrac{dy}{dx}\right)^4$ (a Clairaut equation, differentiate both sides).

8. Classify the following differential equations and find the general solution of each.

(a) $\dfrac{dy}{dx} = \dfrac{5-2y}{1+x^2}$ (b) $\dfrac{dy}{dx} = \dfrac{5-2xy}{1+x^2}$ (c) $\dfrac{d^2y}{dt^2} + 3\dfrac{dy}{dt} - 4y = 0$

9. Solve the following differential equations by making suitable substitutions:

(a) $\dfrac{dy}{dx} = \dfrac{x+y}{x+y+2}$ (b) $\dfrac{dy}{dx} = \dfrac{1}{(x+2y)^2+1}$