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THE UNIVERSITY OF SYDNEY

School of Mathematics and Statistics

MATH1901 DIFFERENTIAL CALCULUS (ADV)

SAMPLE QUIZ 2

Family Name : . . . . . .

Other Names : . . . . . .

Day/Time/Room : . . . . . .

Signature : . . . . . .

Time allowed: 40 minutes. Write your answers in the boxes provided. Each answer box (except Q9) is worth 1 mark. Non-programmable calculators may be used. Use the blank spaces provided for rough working, if necessary. Rough working will not be marked. Use a pen, not a pencil. All pages must be handed in at the end of the quiz. Please switch off mobile phones.

The actual quiz will resemble this sample quiz, but not necessarily cover the same topics.

1. Write the Taylor polynomial of order two for $\tan(2x^2)$ about $x = 0$. (You should be able to answer this question without taking any derivatives.)

Answer . . . . . $T_2(x) =$

2. What is the right derivative of the function, $f(x) = 3|x| + 4x^{4/3}$, at $x = 0$.

Answer . . . . . $f'_+(0) =$

3. What is the limit, $L = \lim_{x\to\infty} x\, e^{-\sqrt{x}}$?

Answer . . . . . $L =$

4. Use a suitable Taylor polynomial for e^x to evaluate the 30th derivative of $f(x) = \exp(x^{10})$ at $x = 0$. (Express your answer in terms of factorials.)

Answer . . . . . $f^{(30)}(0) =$

5. Let $f(x) = x^{3/5}\cos x$. If $f'(0)$ exists, write its value in the box. If the graph of $y = f(x)$ has a vertical tangent or cusp at $x = 0$, write "vertical tangent" or "cusp" as appropriate in the box. If $f'(0)$ does not exist for any other reason, write "does not exist" in the box.

Answer . . . . . $f'(0) =$

6. In the case of the functions $f(x) = x + \sin x$ and $g(x) = 2x + \cos x$, l'Hôpital's rule fails to evaluate the limit $\lim_{x\to\infty} f(x)/g(x)$ because

(a) the limit is not of $0/0$ type or ∞/∞ type;
(b) the limit of $f(x)/g(x)$ as $x \to \infty$ does not exist;
(c) the limit of $f'(x)/g'(x)$ as $x \to \infty$ does not exist;
(d) there are zeros in the denominator as the limit is approached;
(e) none of the above, because l'Hôpital's rule gives the correct limit.

Answer (a), (b), (c), (d) or (e) . . . . .

7. The function $f(x) = a\sqrt{x} - bx$ is continuous on $[0, 1]$ and differentiable on $(0, 1)$. Find all points $c \in (0, 1)$ that satisfy the Mean Value Theorem (as it is usually stated) for $f(x)$ on $[0, 1]$.

Answer(s) . . . . . $c =$

8. Find the critical points of the function, $f(x) = \begin{cases} \sin 2x, & x \geq 0 \\ \sinh 2x, & x < 0, \end{cases}$ on the interval $(-1, 1)$.

Answer(s) . . . . . $x =$

9. [*2 marks*] Sketch the natural domain $D \subseteq \mathbf{R}^2$ of the function, $f : D \to \mathbf{R}$, $(x, y) \mapsto \ln(4 - x^2 - y^2)$.

Answer . . . . .

Answers:

1. $T_2(x) = 2x^2$ **2**. $f'_+(0) = 3$ **3**. $L = 0$ **4**. $f^{(30)}(0) = 30!/3!$

5. $f'(0) =$ "vertical tangent" **6**. (c) **7**. $c = 1/4$ only **8**. $x = \pi/4$ only

9. [*2 marks*] Interior of circle (dashed), radius 2