

# MATH1081 Discrete Mathematics

UNSW 2019T1

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*Welcome !*

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## Formalities

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Please email me if you have any questions or comments!

My office: Room RC-5111, Red Centre Building; just drop by!

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## Overview

TOPIC 1: SETS, FUNCTIONS AND SEQUENCES

TOPIC 2: INTEGERS, MODULAR ARITHMETIC AND RELATIONS

TOPIC 3: LOGIC AND PROOFS

TOPIC 4: ENUMERATION AND PROBABILITY

TOPIC 5: GRAPHS

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- $\mathbb{C}$  - the set of *complex numbers*, which includes all real numbers as well as numbers like  $\sqrt{-1}$ .

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● The *power set*  $P(S)$  of a set  $S$  is the set of all subsets of  $S$ .

★ For any set  $S$ , we have  $\emptyset \subseteq S$  and  $S \subseteq S$ .

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● A set  $S$  is a *subset* of a set  $T$  if each element of  $S$  is also an element of  $T$ .

★  $S = T$  if and only if  $S \subseteq T$  and  $T \subseteq S$ .

● A set  $S$  is a *proper subset* of a set  $T$  if  $S$  is a subset of  $T$  and  $S \neq T$ .

★  $\emptyset$  is a proper subset of any non-empty set.

★ Any non-empty set is an improper subset of itself.

● The *power set*  $P(S)$  of a set  $S$  is the set of all subsets of  $S$ .

★ For any set  $S$ , we have  $\emptyset \subseteq S$  and  $S \subseteq S$ .

★ For any set  $S$ , we have  $\emptyset \in P(S)$  and  $S \in P(S)$ .

● The number of subsets of  $S$  is  $|P(S)| = 2^{|S|}$ . (Why?)

**Example.**  $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$ ,  $\{\frac{1}{2}, \pi\} \not\subseteq \mathbb{N}$

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