The University of Sydney

MATH1902 Linear Algebra (Advanced)

Semester 1	**Longer Solutions to Selected Exercises for Week 6**	2017

1. Making each expression equal to 0 produces the point $P(2,-1,4)$, and equal to 1 produces the point $Q(5,1,5)$. A vector in the direction of $\mathcal{L}_1$ is $\overrightarrow{PQ} = 3\mathbf{i}+2\mathbf{j}+\mathbf{k}$.

2. Setting t equal to 0 produces the point $R(8,3,6)$, and equal to 1 produces the point $Q(5,1,5)$. A vector in the direction of $\mathcal{L}_2$ is $\overrightarrow{RQ} = -3\mathbf{i}-2\mathbf{j}-\mathbf{k}$. The lines are the same because they have a point in common and have vectors that point in the same or opposite directions.

4. Setting $z=0$, the equation becomes $4x-3y=-7$, which is clearly satisfied by $x=-1$ and $y=1$, producing the point $(-1,1,0)$. Taking the coefficients of x, y and z as coefficients of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ respectively produces a normal vector $4\mathbf{i}-3\mathbf{j}+6\mathbf{k}$.

6. (iii) Because $\overrightarrow{PQ}\times\overrightarrow{PR} = -40\mathbf{i}-16\mathbf{j}+24\mathbf{k} = -8(5\mathbf{i}+2\mathbf{j}-3\mathbf{k})$, we may take $5\mathbf{i}+2\mathbf{j}-3\mathbf{k}$ as the normal to the plane, with equation $5x+2y-3z=d$, where we find d by substituting any point, say P, to get $d=5+4-9=0$, yielding finally the equation $5x+2y-3z=0$.

7. Observe that

$$\overrightarrow{PQ} = -6\mathbf{i}-15\mathbf{j}+13\mathbf{k} \qquad \text{and} \qquad \overrightarrow{PR} = 8\mathbf{i}-10\mathbf{j}+11\mathbf{k}$$

so that

$$\overrightarrow{PQ}\times\overrightarrow{PR} \;=\; -35\mathbf{i}+170\mathbf{j}+180\mathbf{k} \;=\; 5(-7\mathbf{i}+34\mathbf{j}+36\mathbf{k})\ .$$

Taking as normal vector $\mathbf{n} = -7\mathbf{i}+34\mathbf{j}+36\mathbf{k}$, and using the coordinates of Q, we get the Cartesian equation of the plane containing P, Q, R to be

$$-7x+34y+36z \;=\; 34(-8)+36(11) \;=\; 124\ .$$

9. (i) Observe that $\overrightarrow{PR} = 5\mathbf{i}-4\mathbf{j}-2\mathbf{k}$, so the line $\mathcal{L}_1$ has parametric vector equation

$$\mathbf{r} = \mathbf{i}+\mathbf{j}+\mathbf{k}+t(5\mathbf{i}-4\mathbf{j}-2\mathbf{k})\ ,$$

parametic scalar equations

$$\left.\begin{array}{rcl} x &=& 1+5t \\ y &=& 1-4t \\ z &=& 1-2t \end{array}\right\} t\in\mathbb{R}\,,$$

and Cartesian equations

$$\frac{x-1}{5} = \frac{y-1}{-4} = \frac{z-1}{-2}\ .$$

Observe that $\overrightarrow{QS} = -3\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}$, so the line $\mathcal{L}_2$ has parametric vector equation

$$\mathbf{r} = 5\mathbf{i} - 5\mathbf{j} - 3\mathbf{k} + s(-3\mathbf{i} + 8\mathbf{j} + 6\mathbf{k}) \ ,$$

parametic scalar equations

$$\left.\begin{array}{rcc} x &=& 5 - 3s \\ y &=& -5 + 8s \\ z &=& -3 + 6s \end{array}\right\} s \in \mathbb{R} \, ,$$

and Cartesian equations

$$\frac{x-5}{-3} = \frac{y+5}{8} = \frac{z+3}{6} \ .$$

(ii) To find the intersection point we set

$$1 + 5t = 5 - 3s \, , \quad 1 - 4t = -5 + 8s \, , \quad 1 - 2t = -3 + 6s \, ,$$

and solve for s and t, yielding $s = t = 1/2$ and therefore, from either set of equations, the intersection point $T(7/2, -1, 0)$.

(iii) Observe that the coordinates of T are the averages of the corresponding coordinates for P and R, and also for Q and S, so T must be the midpoint of both line segments PR and QS. This is not surprising because the figure $PQRS$ is a parallelogram, and the diagonals of a parallelogram bisect each other (from earlier exercises).

10. Setting $z = 0$ and solving $x + y = 2$ and $x - y = 0$ simultaneously yields the point $P(1, 1, 0)$ on $\mathcal{L}$. Normals to the planes are $\mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$ respectively, and

$$\big(\mathbf{i} + \mathbf{j} + \mathbf{k}\big) \times \big(\mathbf{i} - \mathbf{j} + 3\mathbf{k}\big) \ = \ 4\mathbf{i} - 2\mathbf{j} - 2\mathbf{k} \ = \ 2(2\mathbf{i} - \mathbf{j} - \mathbf{k}) \ .$$

so that a vector pointing in the direction of $\mathcal{L}$ may be taken to be $2\mathbf{i} - \mathbf{j} - \mathbf{k}$. Hence parametric scalar equations for $\mathcal{L}$ are

$$\left.\begin{array}{rcc} x &=& 1 + 2t \\ y &=& 1 - t \\ z &=& -t \end{array}\right\} \quad t \in \mathbb{R} \ ,$$

and Cartesian equations for $\mathcal{L}$ are

$$\frac{x-1}{2} \ = \ \frac{y-1}{-1} \ = \ \frac{z}{-1} \ .$$

11. The angle between two planes may be defined as the angle of rotation along the line of intersection that takes one plane to coincide with the other, usually chosen to be between zero and one hundred and eighty degrees. This is precisely the angle between the normal vectors to the planes. The planes here have normal vectors $\mathbf{v} = \mathbf{i} + \mathbf{j} + \mathbf{k}$ and $\mathbf{w} = \mathbf{i} - 2\mathbf{j} - \mathbf{k}$ respectively, so the cosine of the angle between them is

$$\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{v}||\mathbf{w}|} \ = \ \frac{1 - 2 - 1}{\sqrt{3}\sqrt{6}} \ = \ \frac{-2}{3\sqrt{2}} \ = \ -\sqrt{2}/3 \ .$$

12. Observe that

$$\overrightarrow{OR} = \lambda\overrightarrow{OP} + \mu\overrightarrow{OQ} = \lambda\overrightarrow{OP} + (1-\lambda)\overrightarrow{OQ} = \overrightarrow{OQ} + \lambda(\overrightarrow{OP} + \overrightarrow{QO}) = \overrightarrow{OQ} + \lambda\overrightarrow{QP},$$

which is precisely the parametric vector equation (using point notation for vectors and λ for the parameter) for the line passing through Q pointing in the direction of QP. Hence R must vary over the line that passes through P and Q.

(i) To move along the segment joining P to Q, the scalar λ must be nonnegative and not exceed 1, that is, $0 \leq \lambda \leq 1$.

(ii) To move beyond P on the side away from Q, the scalar λ exceeds 1, that is, $\lambda > 1$.

(iii) To lie beyond Q on the side away from P, the scalar λ is negative, that is, $\lambda < 0$.

(iv) There are two ways in which R could be twice as far from P as it is from Q. If R lies between P and Q then the vector $\lambda\overrightarrow{QP}$ must be one third of the length of $\overrightarrow{QP}$, that is, $\lambda = 1/3$. To lie beyond Q on the side away from P the vector $\lambda\overrightarrow{QP}$ must be the negative of $\overrightarrow{QP}$, that is, $\lambda = -1$. (There is no solution for R beyond P on the side away from Q, because the distance to P would then be less than the distance to Q.)

13. The configuration $\mathbf{S}$ is a sphere in space, centred at the origin, with radius r. The tangent plane to $\mathbf{S}$ at P has normal $\overrightarrow{OP} = x_0\mathbf{i} + y_0\mathbf{j} + z_0\mathbf{k}$ with equation

$$x_0x + y_0y + z_0z = x_0x_0 + y_0y_0 + z_0z_0 = x_0^2 + y_0^2 + z_0^2 = r^2 .$$

14. The triangle PQR is right-angled, with the right angle at R, so both the angles at P and Q must be acute (to add up to ninety degrees). In particular the cosine of the angle at P is positive, so $\overrightarrow{PQ} \cdot \overrightarrow{PR}$ must be positive.

15. The point $Q(0,3,0)$ lies on $\mathcal{P}$, and a normal vector is $\mathbf{n} = 4\mathbf{i} + 2\mathbf{j} - \mathbf{k}$. The distance from P to $\mathcal{P}$ is

$$\left|\overrightarrow{PQ} \cdot \widehat{\mathbf{n}}\right| = \frac{\left|(-3\mathbf{i} + 3\mathbf{j} + \mathbf{k}) \cdot (4\mathbf{i} + 2\mathbf{j} - \mathbf{k})\right|}{\sqrt{16+4+1}} = \frac{|-7|}{\sqrt{21}} = \frac{7}{\sqrt{21}} = \frac{\sqrt{21}}{3} .$$

Denote the closest point of $\mathcal{P}$ to P by R. Observe that $\overrightarrow{PQ} \cdot \mathbf{n} < 0$, so that

$$\overrightarrow{PR} = -\frac{\sqrt{21}}{3}\,\widehat{\mathbf{n}} = -\frac{1}{3}(4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) .$$

Hence

$$\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR} = 3\mathbf{i} - \mathbf{k} - \frac{1}{3}(4\mathbf{i} + 2\mathbf{j} - \mathbf{k}) = \frac{5}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k} ,$$

so that

$$R = (5/3, -2/3, -2/3) .$$

16. The line contains $(1,0,-2)$ and has direction $3\mathbf{i} - 4\mathbf{j} + \mathbf{k}$, so has parametric equations

$$\left.\begin{array}{rcr} x &=& 1+3t \\ y &=& -4t \\ z &=& -2+t \end{array}\right\} \quad t \in \mathbb{R} ,$$

yielding Cartesian equations

$$\frac{x-1}{3} = \frac{y}{-4} = z+2\,.$$

17. The line points in the direction of $\mathbf{v} = 2\mathbf{i}+3\mathbf{j}+4\mathbf{k}$ and the plane has normal $\mathbf{n} = 4\mathbf{i}+4\mathbf{j}-5\mathbf{k}$, so it is sufficient to check that $\mathbf{v}\cdot\mathbf{n} = 0$, which is indeed the case:

$$\mathbf{v}\cdot\mathbf{n} \;=\; 2(4)+3(4)+4(-5) \;=\; 8+12-20 \;=\; 0\,.$$

18. The plane contains $P(1,1,1)$ and $Q(4,-3,1)$ and has a parallel vector in the direction of the line which is

$$\mathbf{v} = -2\mathbf{i}+\mathbf{j}+3\mathbf{k}\,.$$

A normal to the plane therefore is

$$\overrightarrow{PQ}\times\mathbf{v} \;=\; (3\mathbf{i}-4\mathbf{j})\times(-2\mathbf{i}+\mathbf{j}+3\mathbf{k}) \;=\; -12\mathbf{i}-9\mathbf{j}-5\mathbf{k}\,.$$

Hence the Cartesian equation is

$$12x+9y+5z \;=\; 12+9+5 \;=\; 26\,.$$

19. The line $\mathcal{L}_1$ contains the point $(1,2,10)$ and points in the direction of the vector $\mathbf{v} = 4\mathbf{i}+3\mathbf{j}+5\mathbf{k}$. The point $(1,2,10)$ also lies on the line $\mathcal{L}_2$ (setting $t=-2$), and $\mathcal{L}_2$ points in the direction of $-\mathbf{v}$, so the lines must coincide.

20. The point $Q(1,1,-4)$ lies on $\mathcal{L}$, which has direction $\mathbf{v} = \mathbf{i}+3\mathbf{j}-\mathbf{k}$. Hence the distance from P to $\mathcal{L}$ is

$$\frac{\left|\overrightarrow{PQ}\times\mathbf{v}\right|}{|\mathbf{v}|} \;=\; \frac{\left|(-\mathbf{i}-5\mathbf{k})\times(\mathbf{i}+3\mathbf{j}-\mathbf{k})\right|}{\sqrt{11}} \;=\; \frac{\left|15\mathbf{i}-6\mathbf{j}-3\mathbf{k}\right|}{\sqrt{11}} \;=\; \frac{\sqrt{270}}{\sqrt{11}}\,.$$

Let R be the closest point on $\mathcal{L}$ to P, so $\left|\overrightarrow{PR}\right| = \dfrac{\sqrt{270}}{\sqrt{11}}$. By Pythagoras,

$$\left|\overrightarrow{QR}\right| \;=\; \sqrt{\left|\overrightarrow{PQ}\right|^2-\left|\overrightarrow{PR}\right|^2} \;=\; \sqrt{26-\frac{270}{11}} \;=\; \frac{4}{\sqrt{11}}\,.$$

Note that

$$\overrightarrow{PQ}\cdot\mathbf{v} \;=\; 4>0\,,$$

so

$$\begin{aligned}\overrightarrow{OR} \;&=\; \overrightarrow{OQ}+\overrightarrow{QR} \;=\; \overrightarrow{OQ}-\frac{4}{\sqrt{11}}\,\widehat{\mathbf{v}}\\[2ex] &=\; \mathbf{i}+\mathbf{j}-4\mathbf{k}-\frac{4}{11}(\mathbf{i}+3\mathbf{j}-\mathbf{k})\\[2ex] &=\; \frac{1}{11}\big(7\mathbf{i}-\mathbf{j}-40\mathbf{k}\big)\,.\end{aligned}$$

Hence $R = (7/11,-1/11,-40/11)\,.$

21. To find the shortest distance d between two parallel lines $\mathcal{L}_1$ and $\mathcal{L}_2$ in space, pointing in the direction of a nonzero vector $\mathbf{v}$, choose points P on $\mathcal{L}_1$ and Q on $\mathcal{L}_2$, and compute the length of the component of $\overrightarrow{PQ}$ orthogonal to $\mathbf{v}$, that is,

$$d \;=\; \left| \overrightarrow{PQ} - \frac{\overrightarrow{PQ}\cdot\mathbf{v}}{|\mathbf{v}|^2}\mathbf{v} \right| .$$

In this case we may take $\mathbf{v} = \mathbf{i} + 4\mathbf{j} - 3\mathbf{k}$, $P = (1,2,3)$, $Q = (-1,3,-1)$ and then the formula becomes

$$\begin{aligned} d \;&=\; \left| -2\mathbf{i} + \mathbf{j} - 4\mathbf{k} - \frac{(-2\mathbf{i} + \mathbf{j} - 4\mathbf{k})\cdot(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k})}{26}(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) \right| \\ &=\; \left| -2\mathbf{i} + \mathbf{j} - 4\mathbf{k} - \frac{14}{26}(\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}) \right| \;=\; \frac{1}{13}|-33\mathbf{i} - 15\mathbf{j} - 31\mathbf{k}| \\ &=\; \frac{1}{13}\sqrt{33^2 + 15^2 + 31^2} \;=\; \sqrt{2275}/13\,. \end{aligned}$$

Alternatively, one can use the formula for the distance from a point to a line, using any point at all on one line and finding its distance to the other line. Here

$$d \;=\; \frac{\left|\overrightarrow{PQ}\times\mathbf{v}\right|}{|\mathbf{v}|} \;=\; \frac{\left|13\mathbf{i} - 10\mathbf{j} - 9\mathbf{k}\right|}{\sqrt{1+16+9}} \;=\; \frac{\sqrt{169+100+81}}{\sqrt{26}} \;=\; \frac{\sqrt{350}}{\sqrt{26}} \;=\; \sqrt{2275}/13$$

as before.

22. The vector $\overrightarrow{Q_2Q_1}$ is perpendicular to both lines (for otherwise we could shift one or both of the points slightly to shorten the length of the vector), so

$$\overrightarrow{Q_2Q_1}\cdot\mathbf{v}_1 \;=\; \overrightarrow{Q_2Q_1}\cdot\mathbf{v}_2 \;=\; 0\,.$$

Hence $\overrightarrow{Q_2Q_1}$ is normal to the plane $\mathcal{P}$ that contains the point P_1 and is parallel to $\mathbf{v}_1$ and $\mathbf{v}_2$. When we shift the vector $\overrightarrow{Q_2Q_1}$ parallel to itself in space so that it coincides with the vector $\overrightarrow{P_2R}$ positioned in space with tail at P_2, then the point R lies in the plane $\mathcal{P}$ (because to translate Q_2 to P_2 along the line $\mathcal{L}_2$ we are moving parallel to $\mathbf{v}_2$). Since the vector $\overrightarrow{P_2R} = \overrightarrow{Q_2Q_1}$ is normal to $\mathcal{P}$, the triangle P_1P_2R is right-angled, with the right angle at R. The length of $\overrightarrow{Q_2Q_1}$ then becomes the distance from P_2 to $\mathcal{P}$, and the formula

$$d \;=\; |\overrightarrow{Q_1Q_2}| \;=\; \frac{\left|\overrightarrow{P_1P_2}\cdot(\mathbf{v}_1\times\mathbf{v}_2)\right|}{\left|\mathbf{v}_1\times\mathbf{v}_2\right|}$$

is immediate from Useful Fact (ix). A method for finding Q_1 and Q_2 would be to parametrise the lines in terms of parameters s and t respectively and minimize the function

$$f(s,t) \;=\; |\overrightarrow{P_1(s)P_2(t)}|^2$$

as $P_1(s)$ varies over $\mathcal{L}_1$ and $P_2(t)$ over $\mathcal{L}_2$, using the methods of calculus of two variables. (The minimum of $f(s,t)$ will occur when the partial derivatives are zero.) In the case in question, we can parametrise $\mathcal{L}_1$ as

$$\left.\begin{aligned} x &= 2 \\ y &= s \\ z &= 1-s \end{aligned}\right\} \quad s\in\mathbb{R}\,,$$

and $\mathcal{L}_2$ as

$$\left.\begin{array}{rcr} x &=& -1+t \\ y &=& 3+4t \\ z &=& -1-3t \end{array}\right\} \quad s \in \mathbb{R}\,.$$

Then form the function

$$\begin{aligned} f(s,t) &= |(3-t)\mathbf{i} + (s-3-4t)\mathbf{j} + (2-s+3t)\mathbf{k}|^2 \\ &= (3-t)^2 + (s-3-4t)^2 + (2-s+3t)^2\,. \end{aligned}$$

This will be minimised when

$$0 \;=\; \partial f/\partial s \;=\; 2(s-3-4t) - 2(2-s+3t) \;=\; -10+4s-14t$$

and

$$0 \;=\; \partial f/\partial t \;=\; -2(3-t) - 8(s-3-4t) + 6(2-s+3t) \;=\; 30-14s+52t\,.$$

Solving simultaneously yields $s = 25/3$ and $t = 5/3$, and we get the points

$$Q_1 \;=\; (2, 25/3, -22/3)\,, \quad Q_2 \;=\; (2/3, 29/3, -6)\,.$$

In particular $d = |\overrightarrow{Q_1Q_2}| = |-4/3\,\mathbf{i} + 4/3\,\mathbf{j} + 4/3\,\mathbf{k}| = 4\sqrt{3}/3 = 4/\sqrt{3}$.

23. The vector $\mathbf{r}'$ represents the instantaneous rate of change of displacement with respect to time, that is $\mathbf{r}'$ is the velocity vector of the particle. We have, collecting terms, and applying the usual limit laws,

$$\begin{aligned} \mathbf{r}'(t) &= \lim_{\delta\to 0} \frac{\mathbf{r}(t+\delta) - \mathbf{r}(t)}{\delta} \\ &= \lim_{\delta\to 0} \frac{f(t+\delta)\mathbf{i} + g(t+\delta)\mathbf{j} + h(t+\delta)\mathbf{k} - \big(f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}\big)}{\delta} \\ &= \left(\lim_{\delta\to 0} \frac{f(t+\delta) - f(t)}{\delta}\right)\mathbf{i} + \left(\lim_{\delta\to 0} \frac{g(t+\delta) - g(t)}{\delta}\right)\mathbf{j} + \left(\lim_{\delta\to 0} \frac{h(t+\delta) - h(t)}{\delta}\right)\mathbf{k} \\ &= f'(t)\mathbf{i} + g'(t)\mathbf{j} + h'(t)\mathbf{k}\,, \end{aligned}$$

so that $\mathbf{r}'(t)$ is described parametrically by $\left.\begin{array}{l} x = f'(t) \\ y = g'(t) \\ z = h'(t) \end{array}\right\} \quad t \in \mathbb{R}\,.$

24. The partial derivative $\dfrac{\partial f}{\partial x}(x_0, y_0)$ measures the slope of the tangent line $\mathcal{L}_1$ to the curve of intersection of the surface $z = f(x,y)$ with the vertical plane $y = y_0$. Slope is just vertical rise over horizontal run. Hence, since $\mathbf{i}$ is a unit vector in the horizontal x-direction and $\mathbf{k}$ a unit vector in the vertical z-direction, the vector

$$\mathbf{v} \;=\; \mathbf{i} + \frac{\partial f}{\partial x}(x_0, y_0)\mathbf{k}$$

points in space in the direction of $\mathcal{L}_1$. Similarly, the partial derivative $\dfrac{\partial f}{\partial y}(x_0, y_0)$ measures the slope of the tangent line $\mathcal{L}_2$ to the curve of intersection with the vertical plane $x = x_0$. Now, since $\mathbf{j}$ is a unit vector in the horizontal y-direction, the vector

$$\mathbf{w} \;=\; \mathbf{j} + \frac{\partial f}{\partial y}(x_0, y_0)\mathbf{k}$$

points in space in the direction of $\mathcal{L}_2$. Clearly $\mathbf{v}$ and $\mathbf{w}$ are not parallel to each other, but each is parallel to the tangent plane to the surface at (x_0, y_0, z_0) (since the tangent plane contains both tangent lines). Hence a normal to the tangent plane is

$$\begin{aligned}\mathbf{v}\times\mathbf{w} &= \left(\mathbf{i}+\frac{\partial f}{\partial x}(x_0,y_0)\,\mathbf{k}\right)\times\left(\mathbf{j}+\frac{\partial f}{\partial y}(x_0,y_0)\,\mathbf{k}\right)\\ &= -\frac{\partial f}{\partial x}(x_0,y_0)\,\mathbf{i}\ -\frac{\partial f}{\partial y}(x_0,y_0)\,\mathbf{j}+\mathbf{k}\,.\end{aligned}$$

The Cartesian equation is therefore

$$-\frac{\partial f}{\partial x}(x_0,y_0)\,x-\frac{\partial f}{\partial y}(x_0,y_0)\,y+z \;=\; -\frac{\partial f}{\partial x}(x_0,y_0)\,x_0-\frac{\partial f}{\partial y}(x_0,y_0)\,y_0+z_0\,,$$

which rearranges to yield

$$z-z_0 \;=\; \frac{\partial f}{\partial x}(x_0,y_0)\,(x-x_0)+\frac{\partial f}{\partial y}(x_0,y_0)\,(y-y_0)\,.$$

25. For each $t\le 0$, the line $\mathcal{L}_{\mathsf{left}}(t)$ contains the point $(t,-10,0)$ and points in the direction of the vector

$$(t-10)\,\mathbf{i}-10\,\mathbf{j}-\sqrt{101}\,\mathbf{k}\,,$$

so has parametric scalar equations

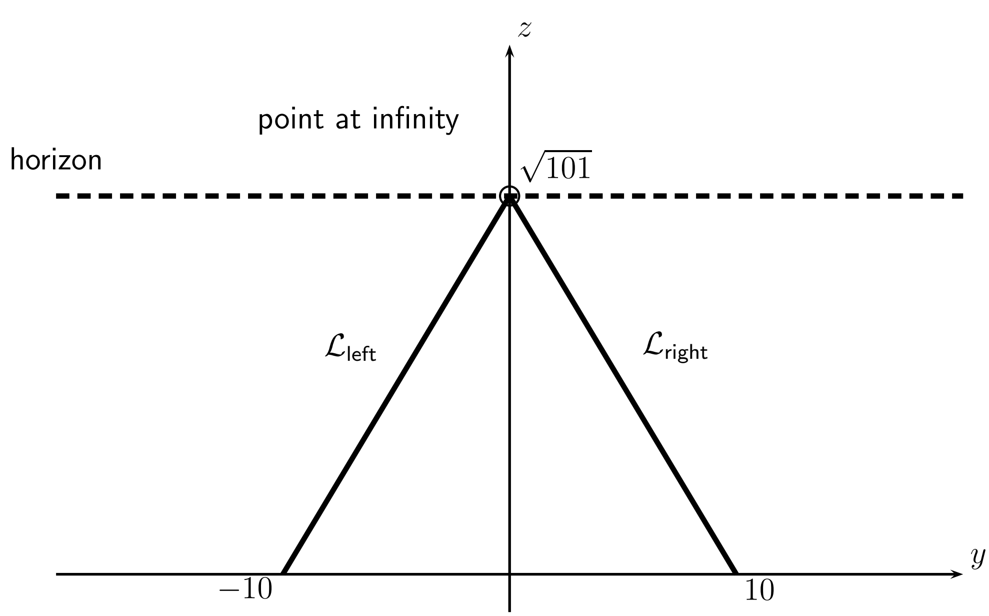
$$\left.\begin{array}{l} x=t+s(t-10)\\ y=-10-10s\\ z=-s\sqrt{101}\end{array}\right\}\qquad s\in\mathbb{R}\,.$$

This line intersects the yz-plane when $x=0$, so that

$$s \;=\; \frac{-t}{t-10}\,,$$

yielding the intersection point

$$(0,-10-10s,-s\sqrt{101}) \;=\; \left(0,\frac{100}{t-10},\frac{t\sqrt{101}}{t-10}\right)\,.$$



These intersection points draw a line on the yz-canvas that satisfies the equation

$$z = \frac{\sqrt{101}}{10} y + \sqrt{101}$$

where $y = \dfrac{100}{t-10}$ varies along the half open interval $[-10, 0)$ as t ranges over the negative half-line $(-\infty, 0]$. Similarly, the intersection points of the lines $\mathcal{L}_{\mathsf{right}}(t)$ with the yz-plane are

$$\left(0, \frac{-100}{t-10}, \frac{t\sqrt{101}}{t-10}\right),$$

and these trace the line

$$z = -\frac{\sqrt{101}}{10} y + \sqrt{101}$$

as $y = \dfrac{-100}{t-10}$ varies along the half open interval $(0, 10]$. The half open intervals $[-10, 0)$ and $(0, 10]$ are disjoint, so the two lines drawn on the canvas do not intersect, even though they are not parallel (having different gradients). Nevertheless, in the limit, as $t \to \infty$, $y \to 0$, and $z \to \sqrt{101}$, so the lines converge to the point $(0, \sqrt{101})$ on the canvas.