THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Practice for Quiz 2

MATH1903: Integral Calculus and Modelling (Advanced)   Semester 1, 2012

Lecturers: Daniel Daners and James Parkinson

Solutions

Family Name: ....

Other Names: ....

SID: .... Day: ....

Time: .... Room: ....

Signature: ....

Please write your final answers, and only your final answers, in the answer boxes.

Please note:

(a) There are 10 questions, each worth 1 mark. Half marks will **not** be awarded.

(b) Working will **not** be marked. Marks will be awarded on the basis of answers only.

(c) Answers will only be marked if they are in the answer boxes on the answer sheet.

(d) Non-programmable, non-graphics calculators **are** permitted.

(e) There is some space for working. No other paper is allowed.

(f) You have 40 minutes to complete the quiz.

1. Sketch the direction field of the differential equation $y' = y(y-2)$ in the region below.

Answer Q1:
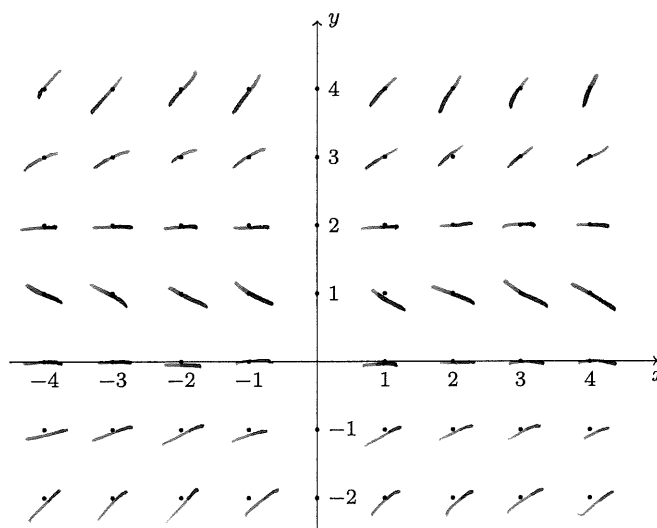
2. Find the general solution of the differential equation $y' = -2y^2$.

Answer Q2:

$$y = \frac{1}{2x + C}$$

3. Find the particular solution of the differential equation $u' = \dfrac{u}{x^2}$ with $u(-1) = 2$. Indicate for which x it is defined.

Answer Q3:

$$u = 2e^{-\frac{1}{x} - 1}, \quad x < 0$$

4. Find an integrating factor for the differential equation $y' = y \tan x + e^x$

Answer Q4:

$$\cos x$$

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5. Suppose that $x(t)$ satisfies the differential equation $x' + x^3 = 2x$. Find the differential equation for $v = \dfrac{1}{x^2}$.

Answer Q5:

$$v' + 4v - 2 = 0$$

6. Find the general solution of the differential equation $y' - 2y = x$.

Answer Q6:

$$y = -\frac{x}{2} - \frac{1}{4} + Ce^{-2x}$$

7. Compute the partial fraction decomposition of $\dfrac{1}{(x+2)(x-5)}$.

Answer Q7:

$$\frac{1}{7}\left(\frac{1}{x-5} - \frac{1}{x+2}\right)$$

8. Find the particular solution of $xy' + y = \sin x$ with $y(\pi) = 2$.

Answer Q8:

$$y = \frac{1}{x}\left(2\pi - \cos x - 1\right)$$

9. A population P of animals grows proportionally to the size of the population. A number of animals is removed at a constant rate N. Write down a differential equation modelling the size of the population as a function of time.

Answer Q9:

$$P' = kP - N \qquad (k > 0 \text{ const})$$

10. Compute the general solution of $x^2 y' + 2y = y^2 + 1$.

Answer Q10:

$$y = 1 + \frac{x}{1 + Cx}$$

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