PHYS 1901 – Physics 1A (Advanced) Mechanics module

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THE UNIVERSITY OF SYDNEY

Dynamics of Rotational Motion

Chapter

10

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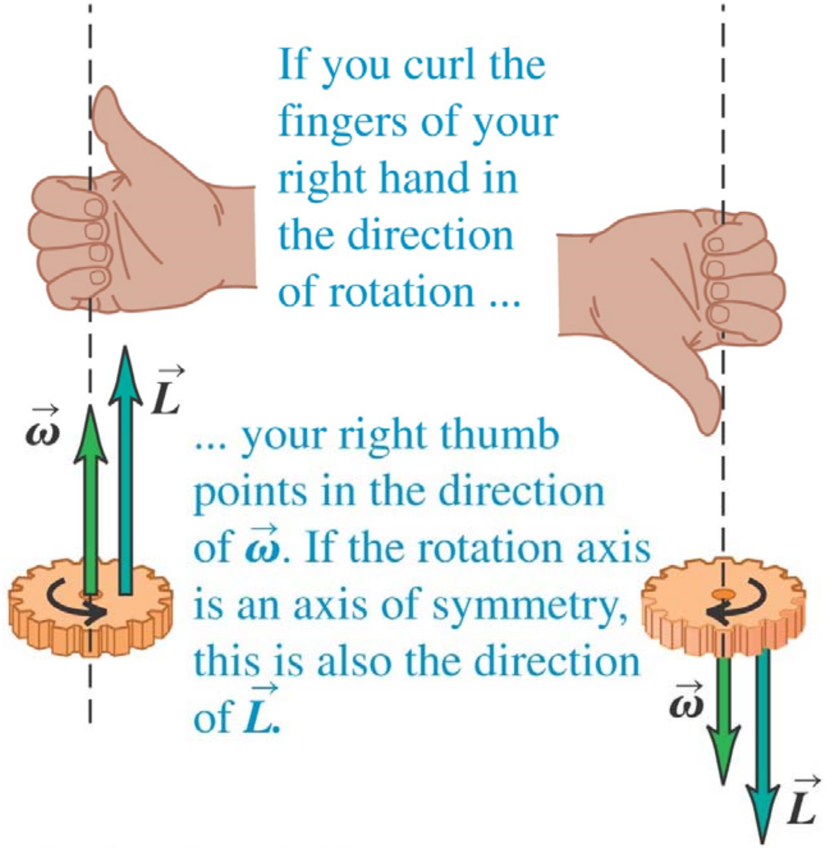
Angular momentum

In linear dynamics, complex interaction (collisions) can be examined using the conservation of momentum.

In rotational dynamics, the concept of angular momentum similarly eases complex interactions.

$$\vec{L} = I\vec{\omega}$$



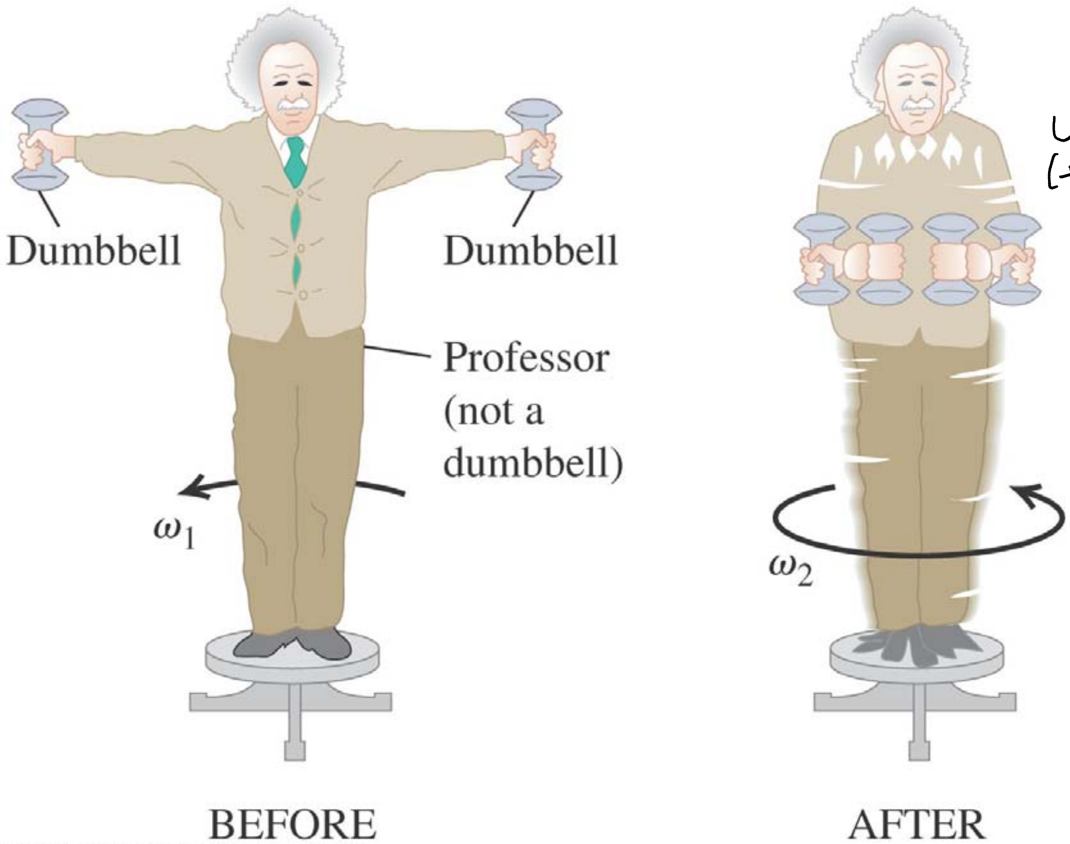
Angular momentum

In linear dynamics: $\frac{d\vec{p}}{dt} = \vec{F}_{net}$

In rotational dynamics: $\frac{d\vec{L}}{dt} = \vec{\tau}_{net}$

Hence, the net torque is equal to the rate of change of angular momentum. Hence, if there is no net torque, angular momentum is conserved.

Angular momentum



up (+ve)

$$\vec{L} = I\vec{\omega}$$

$$L = I\omega$$

constant (no net torque) dec. inc.

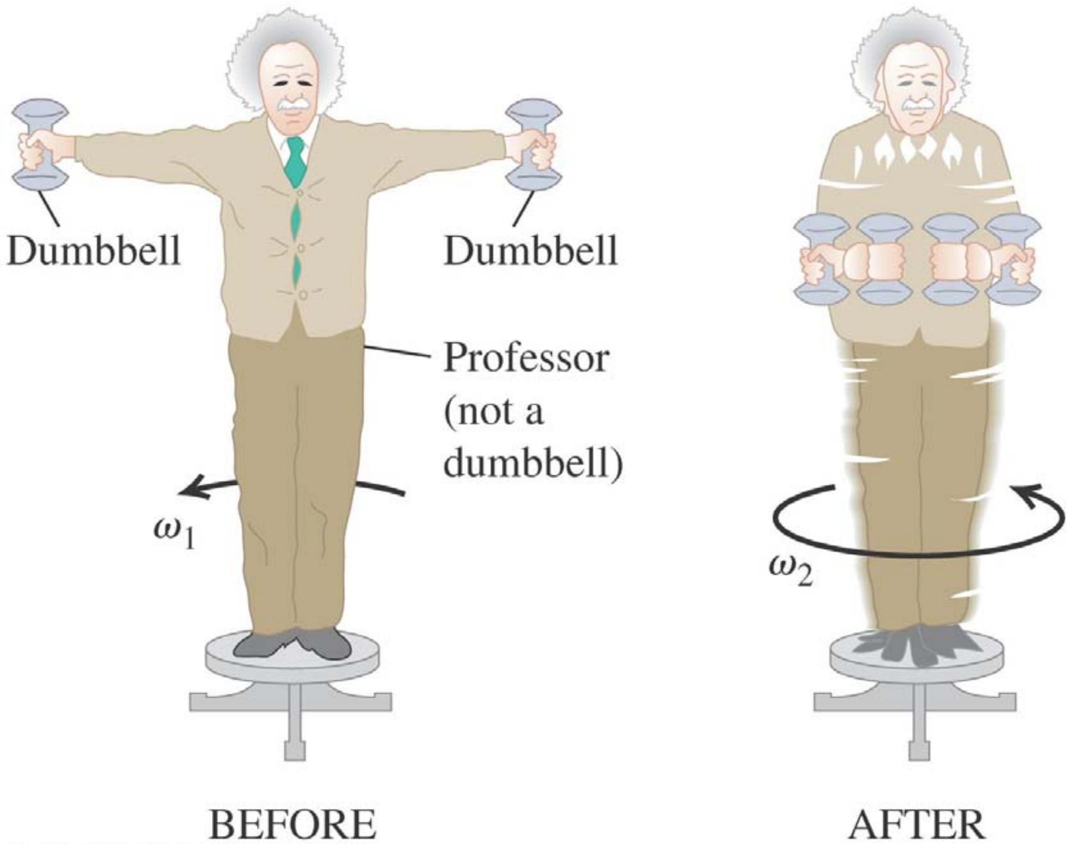
$$K_{rot} = \frac{1}{2}I\omega^2$$

inc.

$$= \frac{1}{2}\underbrace{(I\omega)}_{L \text{ const.}}\omega$$

inc.

Angular momentum



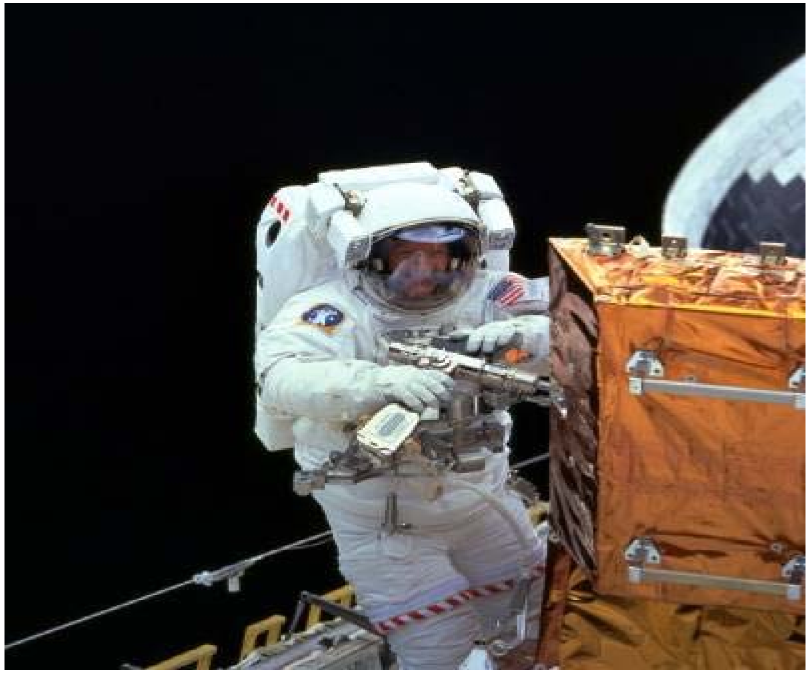
We can change the angular velocity by modifying the moment of inertia.

Angular momentum is conserved, but where has the extra energy come from?

Work done by pulling the dumbbells in

Angular Momentum

Angular Momentum



Angular Momentum

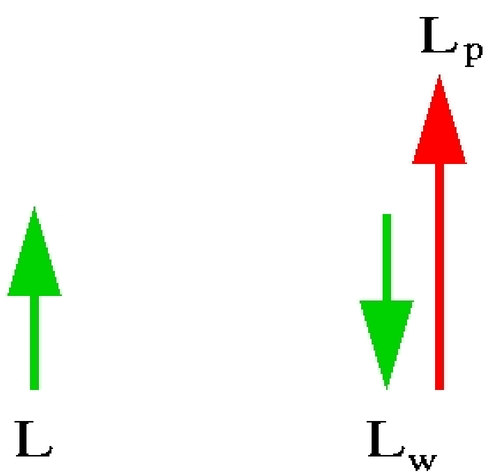
Angular Momentum

http://www.physics.lsa.umich.edu/demolab/demo.asp?id=696

Consider a lecturer on a rotating stool holding a spinning wheel, with the axis of the wheel pointing towards the ceiling.

What happens when the wheel is turned over?

Angular Momentum



$$L = L_p - L_w$$

As with linear momentum, we can use conservation of angular momentum without having to worry about the various (internal) torques in action.

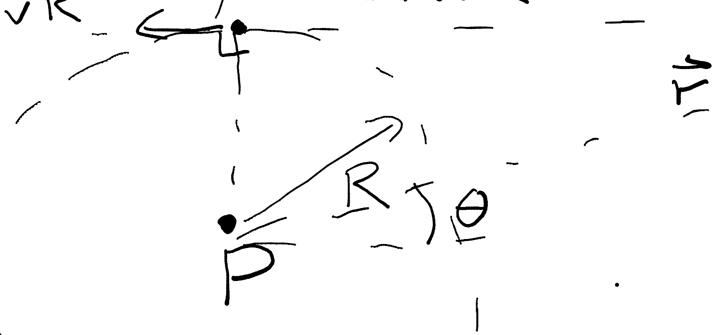
External torques will change the value of the total angular momentum.

Linear and Angular Momentum

(In a circle $L = I\omega = (mR^2)(v/R) = mvR$

$L = R(mv)\sin 90^\circ = mvR$

What is the angular momentum of an object moving along a straight line?



$$L = \left(\frac{R}{\sin\theta}\right)(mv)\sin\theta = mvR$$

$$\vec{L} = \vec{r} \times \vec{p}$$

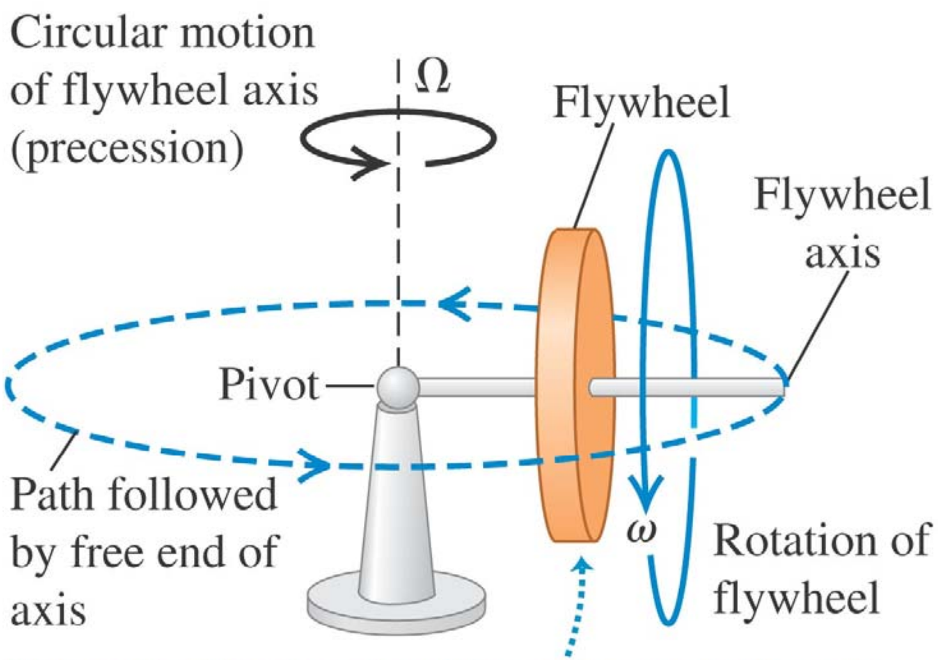
vector from P to the object

linear momentum

Objects moving linearly have **constant** angular momentum.

Rotational mechanics is linear mechanics in a different coordinate system.

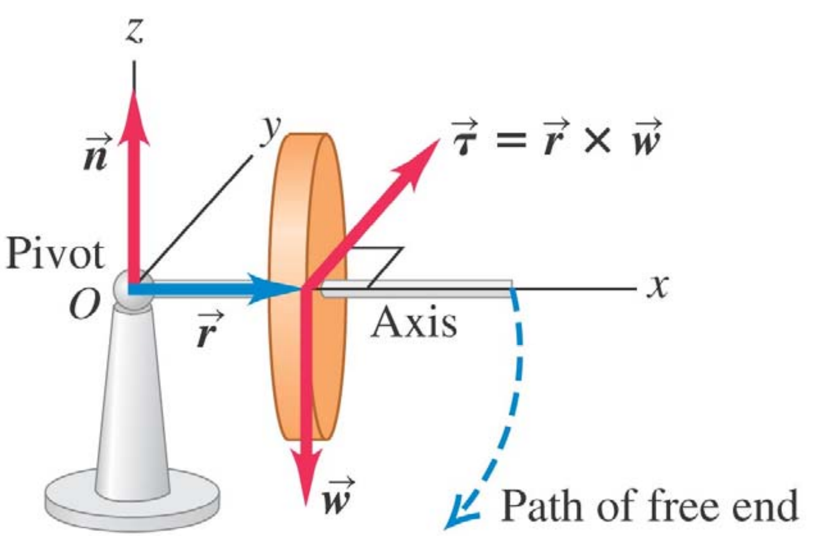
Gyroscope



When the flywheel and its axis are stationary, they will fall to the table surface. When the flywheel spins, it and its axis "float" in the air while moving in a circle about the pivot.

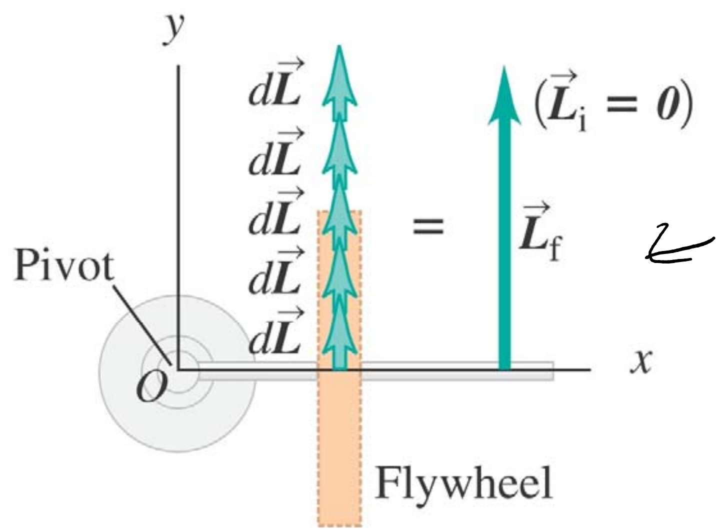
Gyroscope: not rotating

(a) Nonrotating flywheel falls



When the flywheel is not rotating, its weight creates a torque around the pivot, causing it to fall along a circular path until its axis rests on the table surface.

(b) View from above as flywheel falls

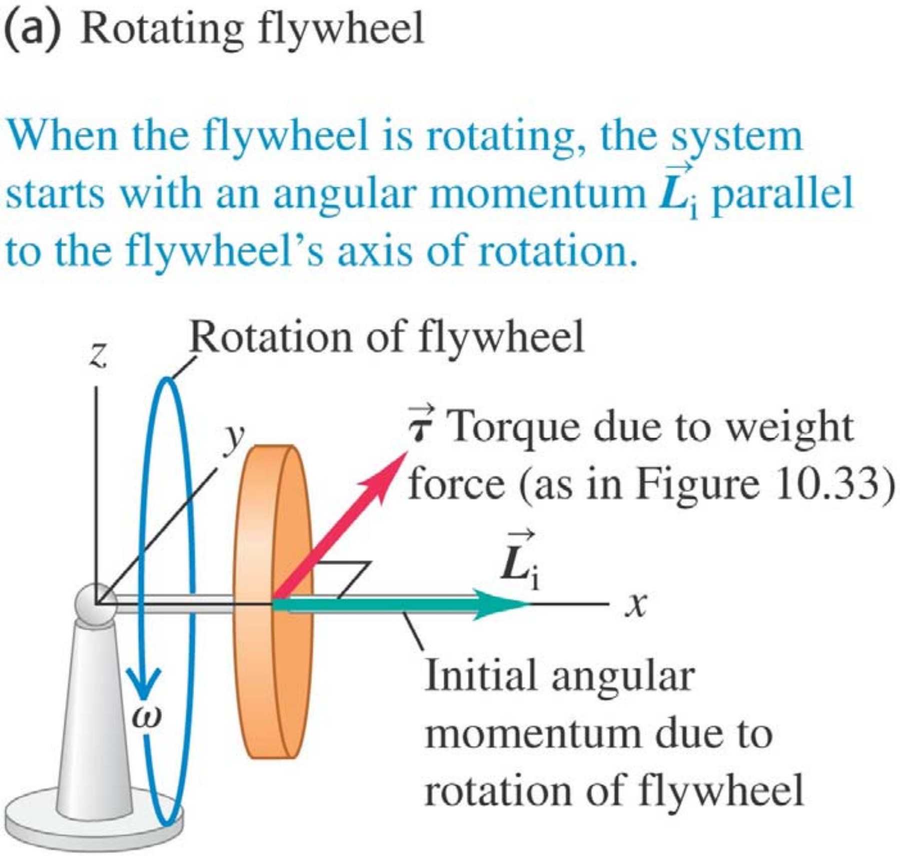


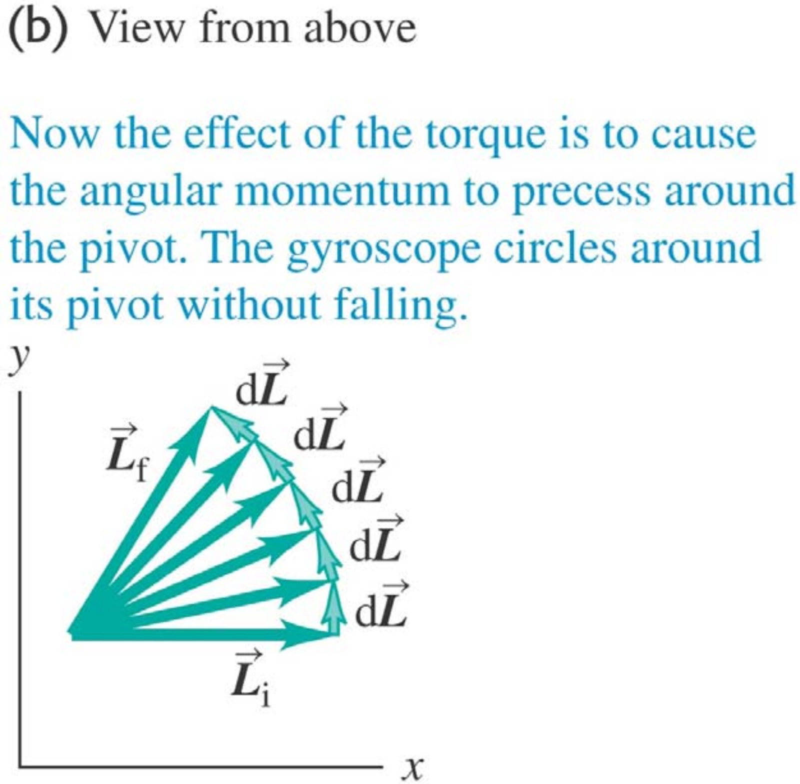
$\frac{d\vec{L}}{dt} = \vec{\tau}$

starting from $\vec{L}_i = 0$

In falling, the flywheel rotates about the pivot and thus acquires an angular momentum $\vec{L}$. The *direction* of $\vec{L}$ stays constant.

Gyroscope: rotating



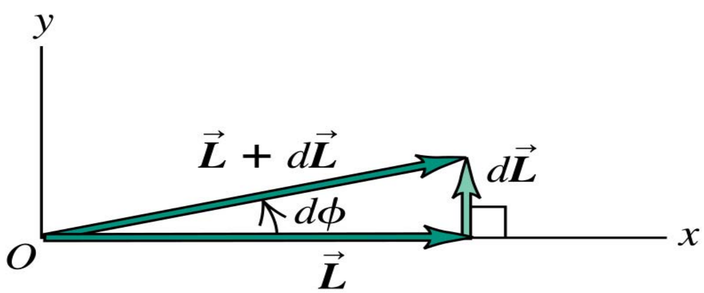


Precession

Over a small time interval, there is a change in angular momentum given by

$$\vec{\tau} = \frac{d\vec{L}}{dt}$$

$$d\vec{L} = \vec{\tau}\, dt$$



As dL is perpendicular to L, only the direction of L changes, not magnitude.

$$\Omega = \frac{d\phi}{dt} = \left|\frac{d\vec{L}}{\vec{L}}\right| / dt$$

$$= \left|\frac{d\vec{L}}{dt}\right| / L$$

$$= \tau / L$$

$$= mgr / I\omega$$