THE UNIVERSITY OF SYDNEY

PHYS1902 – PHYSICS 1B (ADVANCED)

Solutions

NOVEMBER 2010

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

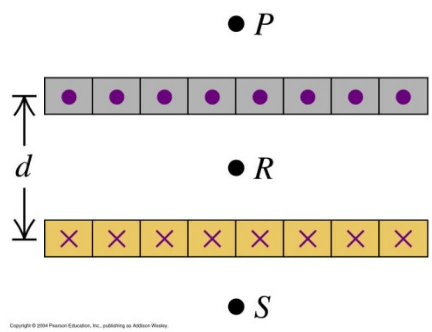
DATA

Density of water	ρ	=	1.00×10^{3} kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^{5} Pa
Magnitude of local gravitational field	g	=	9.80 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^{8} m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg
Rydberg constant	R	=	1.097×10^{7} m^{-1}

ADV_Q01

Semester 2, Y2010

Question 1
Long, straight conductors with square cross section, each carrying current I directed out of the plane of the page, are laid side by side to form an infinite current sheet. A second infinite current sheet is a distance d below the first and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length.



Use Ampère's law to calculate the magnitude and direction of the magnetic field at:

(a) Point P (above the upper sheet as in the diagram above);

b) Point R (midway between the two sheets);

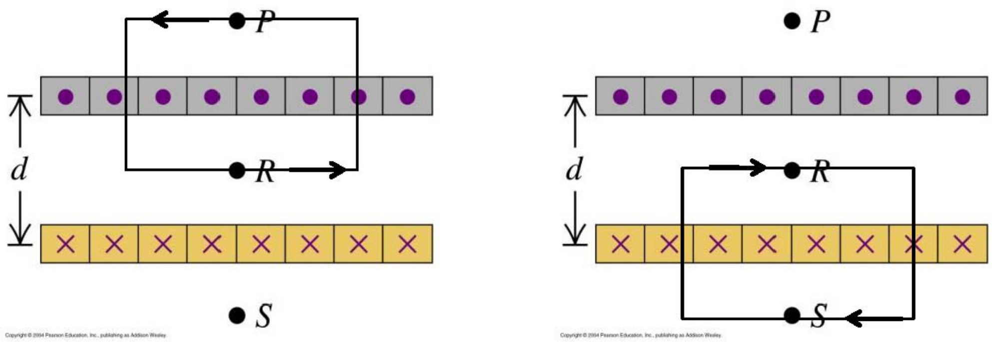
(c) Point S (below the lower sheet).

(5 marks)

Solution

The most direct method is to use Ampère's law for a single sheet, and then use the superposition principle to obtain the net magnetic field from both sheets.

Using Ampère's law, we use rectangular paths in the plane of the page, with the top and bottom portions of the rectangle parallel to the sheets, and the sides perpendicular to the sheets. Choose the width to be some length L.



For a single sheet, a rectangle gives a line integral of the magnetic field of $2BL$ for a length L, enclosing a current nIL. The magnetic field for a single sheet with current coming out of the page is thus $B = \mu_0 nI/2$ to the left above the sheet, and the same but to the right below the sheet. The directions are opposite for a sheet with current going into the page.

(2 marks)

We can obtain the answer at each point by adding the contributions due to each sheet:

(a) $B = 0$ at point P. **(1 mark)**

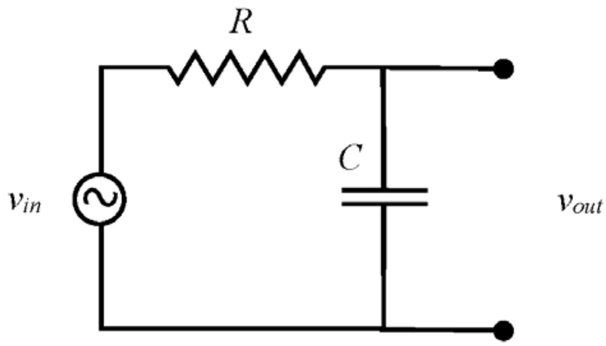
(b) $B = \mu_0 nI$ to the right at point R. **(1 mark)**

(c) $B = 0$ at point S. **(1 mark)**

TEC_Q02=ADV_Q02

Semester 2, Y2010

Question 2



In the circuit shown in the diagram above the resistor has a value of $R = 10.0\,\mathrm{k}\Omega$ and the capacitor has a value of $C = 100\,\mathrm{nF}$.

(a) Briefly describe in words the relationship between v_{out} and v_{in} as a function of frequency.

(b) Explain why the circuit shows this behaviour.

(c) At what frequency f_1 (in Hz) does the reactance of the capacitor C have a value of $10.0\,\mathrm{k}\Omega$?

(d) At this frequency f_1, what is the value of ratio of v_{out} / v_{in}?

(5 marks)

Solution

(a)

The circuit is a low pass filter and high frequencies are attenuated. At low frequencies v_{out}/v_{in} is close to one and at high frequencies v_{out}/v_{in} approaches zero.

(1 mark)

(b)

The circuit acts as a voltage divider. At low frequencies the impedance of the capacitor $\left(X_C = \dfrac{1}{\omega C}\right)$ is large compared with the resistance R and most of the voltage appears across the capacitor. At high frequencies the impedance of the capacitor $\left(X_C = \dfrac{1}{\omega C}\right)$ is small compared with the resistance R and most of the voltage appears across the resistance with little across the capacitor.

(1 mark)

(c)

$$X_C = \frac{1}{\omega C}$$

If $X_C = 10.0 \times 10^3\ \Omega$ then $\omega = \dfrac{1}{(10.0\times 10^3)(100\times 10^{-9})} = 1000\ \text{rad.s}^{-1}$.

The frequency is given by: $f = \dfrac{\omega}{2\pi} = \dfrac{1000}{2\pi} = 159\,\text{Hz}$.

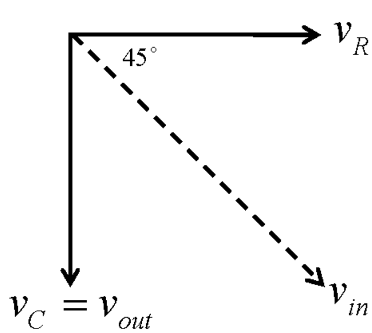
(2 marks)

(d)

At this frequency

$$v_{out}/v_{in} = \frac{1}{\sqrt{2}} = 0.707.$$

The voltages across the resistor and the capacitor are equal in magnitude but are 90° out of phase and are related by a vector sum as shown in the diagram below.

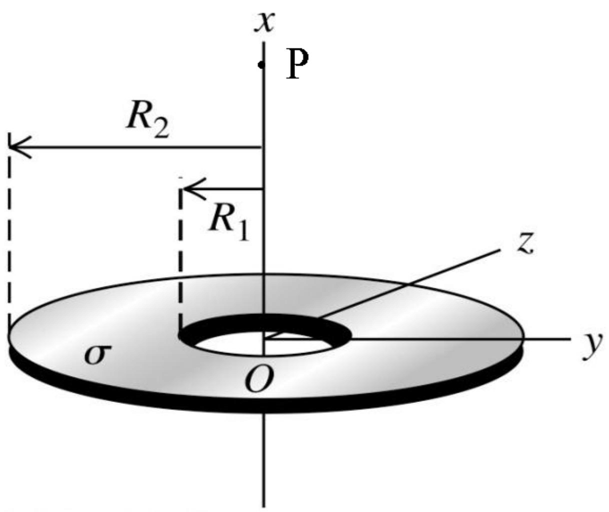


(1 mark, no marks for $v_{out}/v_{in} = 1/2$.)

ADV_Q03

Semester 2, Y2010

Question 3
A thin disk with a circular hole at its centre, called an annulus, has inner radius R_1 and outer radius R_2. The disk has a uniform positive surface charge density σ on its surface.



(a) Determine the total electric charge on the annulus.

(b) Calculate the magnitude and direction of the electric field at a point *P* a distance *x* along the *x*-axis from the centre of the annulus.
Hint: you may need the following integral:

$$\int \frac{r\,dr}{(a^2+r^2)^{3/2}} = -\frac{1}{\sqrt{a^2+r^2}}$$

(5 marks)

Solution

(a)
The total surface area of the annulus is

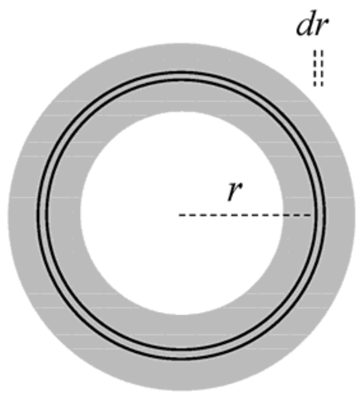
$$A = \pi\left(R_2^{\ 2} - R_1^{\ 2}\right).$$

The total charge is therefore

$$Q = \sigma A = \pi\,\sigma\left(R_2^{\ 2} - R_1^{\ 2}\right).$$

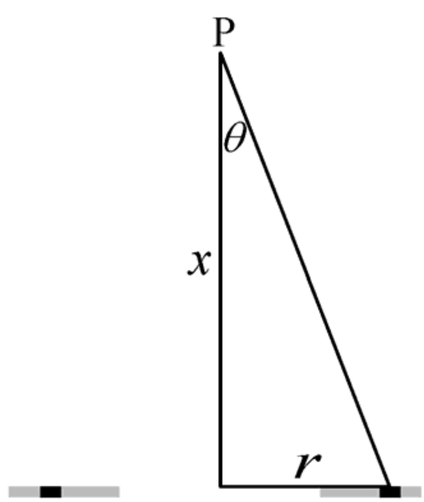
(1 mark)

(b)
Consider a ring of radius r and thickness dr on the annulus as shown on the diagram below (viewed from above the annulus in the yz plane.).



The area of this ring is $2\pi r\,dr$ and so its charge is $\sigma\,2\pi r\,dr$.

All charge on this ring is the same distance $\sqrt{x^2 + r^2}$ from the point P.



By symmetry, all components of the electric field at point P that are in the yz plane will cancel (as for every bit of charge on one side of the ring, the bit of charge on the opposite side will contribute the opposite yz component). Only the x-component of the electric field will contribute.

(2 marks)

By Coulomb's law, the electric field in the x direction arising from this ring is given by

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r}{x^2+r^2}\cos\theta$$

$$dE_x = \frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r}{x^2+r^2}\frac{x}{\sqrt{x^2+r^2}}.$$

(1 mark)

Integrating from the inner to the outer radius gives

$$E_x = \int_{R_1}^{R_2}\frac{1}{4\pi\varepsilon_0}\frac{\sigma 2\pi r}{x^2+r^2}\frac{x}{\sqrt{x^2+r^2}}\,dr$$

$$= \frac{\sigma x}{2\varepsilon_0}\int_{R_1}^{R_2}\frac{r}{\left(x^2+r^2\right)^{3/2}}\,dr$$

This gives

$$E_x = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^{\,2}}} - \frac{1}{\sqrt{x^2+R_2^{\,2}}}\right)$$

(1 mark)

ADV_Q04

Semester 2, Y2010

Question 4

(a) Two identical drops of water will merge if they come into contact. Explain why this happens in terms of energy.

(b) Astronauts in the pressurized environment on the International Space Station find that surface tension effects are very important for the way that liquids behave. Is buoyancy also important for liquids in this environment? Briefly explain your answer.

(5 marks)

Solution

(a)

The surface energy density of a drop of water is equal to the surface tension. So the surface energy of a drop of water with radius r is the surface area multiplied by the surface energy density, i.e.

$$E_1 = 4\pi r_1^2 T$$

where T is the surface tension of the water. Surface tension T is generated by forces between the molecules in the surface of the water.

(1 mark)

Thus the total surface energy for two drops of water is

$$E_{tot} = 2E_1 = 8\pi r_1^2 T$$

When they merge they form a single drop with a volume

$$V_2 = 2V_1 = 2\frac{4}{3}\pi r_1^3 .$$

The radius r_2 of the merged drop is therefore given by

$$r_2 = 2^{1/3} r_1 .$$

This new drop has surface energy

$$E_2 = 4\pi r_2^2 T = 4\pi\left(2^{1/3} r_1\right)^2 = 2^{2/3} E_1 \sim 1.59 E_1 < 2E_1 .$$

Thus the energy of the merged drop is less than the energy of the two separate drops. When the drops come into contact the system will seek the lowest energy state and the drops will merge.

(2 marks)

(b)

No, buoyancy effects are not important in zero gravity. Buoyancy arises because of a pressure gradient in the liquid, which arises on Earth because of gravity. In zero gravity, there are no pressure gradients in the liquid and hence there is no buoyancy.

(1 mark for answer; 1 mark for reasoning)

ADV_Q05

Semester 2, Y2010

Question 5

An electron free to move in one direction and subject to no forces is described by the wave function:

$$\Psi\left(x,t\right) = A\exp[i(kx - \omega t)],$$

where E is the energy of the electron, m is the mass of the electron, $k = \sqrt{2mE}/\hbar$ and $\omega = E/\hbar$.

(a) Does this electron have a uniquely defined momentum? Briefly explain.

(b) Does this electron have a uniquely defined position? Briefly explain.

(c) Briefly explain how your answers to (a) and (b) are consistent with the Heisenberg Uncertainty Principle.

(5 marks)

Solution

(a)

For a specified E the electron has the unique momentum

$$p = \hbar k = \sqrt{2mE}\ .$$

(1½ marks)

(b)

The position of the particle is completely unknown: $|\Psi(x,t)|^2 = |A|^2$, a constant, implying that the particle is equally likely to be anywhere, because $|\Psi|^2$ is the probability per unit distance of the electron being at position x at time t.

(1½ marks)

(c)

The Heisenberg Uncertainty Principle is $\Delta p \, \Delta x \geq \hbar$ in one dimension, where Δp is the momentum uncertainty and Δx is the position uncertainty. This electron corresponds to the limiting case $\Delta p = 0$, $\Delta x \to \infty$, i.e. the momentum is exactly known and the position is completely unknown.

(2 marks)

ADV_Q06

Semester 2, Y2010

Question 6
The Planck law for the intensity distribution of radiation from a blackbody is written:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(\exp\left(\frac{hc}{\lambda kT} \right) - 1 \right)}$$

where h is Planck's constant, c is the speed of light, k is Boltzmann's constant, T is the absolute temperature of the blackbody, and λ is wavelength.

The Rayleigh radiation law is expressed as:

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4}.$$

(a) Sketch both laws as a function of wavelength at a given temperature (your plot only needs to be qualitatively correct).

(b) Show that the Planck law reduces to the Rayleigh law at very large wavelengths $\left(\lambda >> \frac{hc}{kT} \right)$.

Hint: You may need the Taylor series expansion of the exponential function which is expressed as:

$$\exp(z) = 1 + z + \frac{z^2}{2!} + \ldots .$$

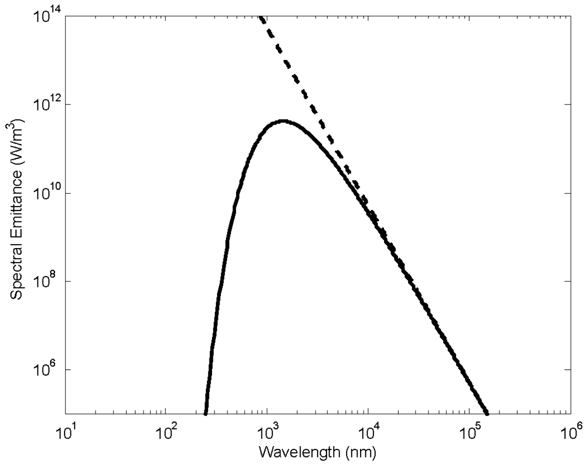
(c) Planck's constant does not appear in the Rayleigh law, but does appear in the Planck law. Briefly explain why, in terms of the physical origin of blackbody radiation.

(5 marks)

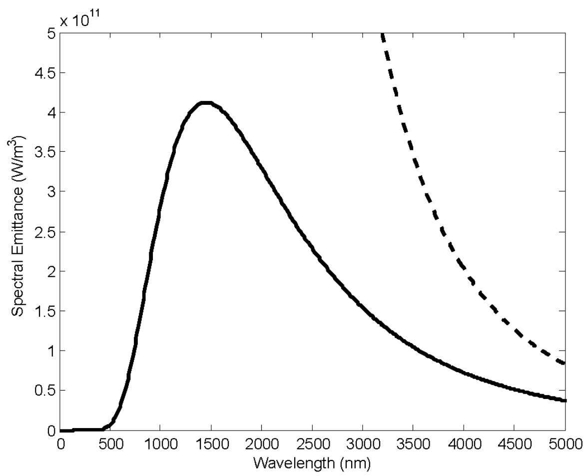
Solution

(a)

In figure below the solid line is the Planck law and the dashed line is the Rayleigh law.



or more likely as a linear plot



Only the general details of the above diagram need to be shown.

(2 marks)

(b)

Planck law is given by:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)}.$$

If $\lambda >> \frac{hc}{kT}$

then we can write

$$\exp\left(\frac{hc}{\lambda kT}\right) \approx 1 + \frac{hc}{\lambda kT}$$

and

$$\exp\left(\frac{hc}{\lambda kT}\right) - 1 \approx \frac{hc}{\lambda kT}.$$

The Planck law then becomes

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5 \left(\frac{hc}{\lambda kT}\right)}$$

$$= \frac{2\pi ckT}{\lambda^4}$$

which is the Rayleigh law.

(2 marks)

(c)

The Rayleigh law is based on classical physics whereas the Planck law takes into account quantum physics. At very long wavelengths and correspondingly small photon energies quantum effects are not important and the two laws agree.

Planck derived the law by postulating that the energies of the oscillators in a blackbody producing the radiation are quantized such that the energy is an integer multiple of Planck's constant times the frequency. This quantization is a quantum characteristic with no classical counterpart.

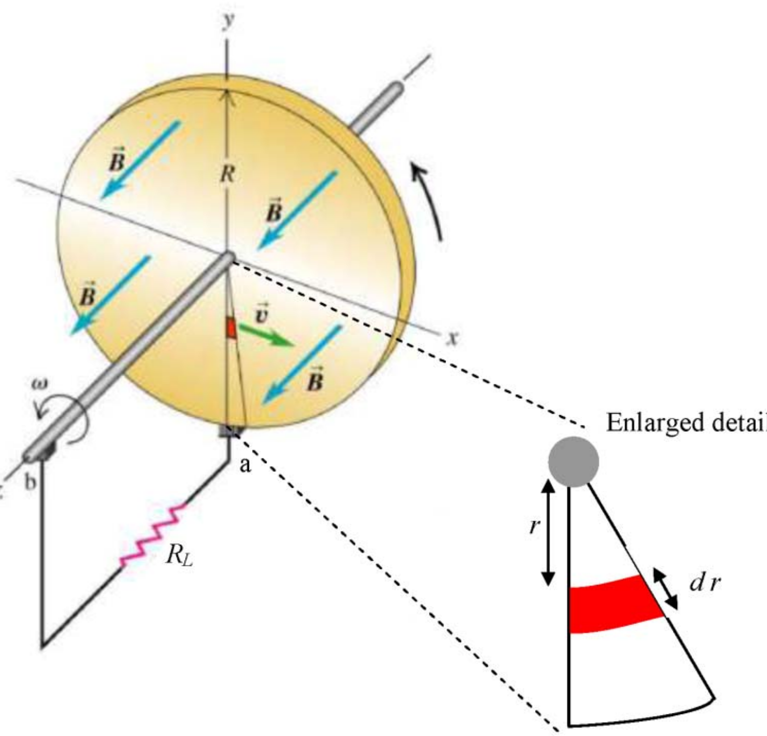
(1 mark)

ADV_Q07

Semester 2, Y2010

Question 7

A conducting disk with radius R, as shown below, lies in the xy-plane and rotates with a constant angular velocity ω about the z-axis. The disk is in a uniform, constant magnetic field $\vec{B}$ parallel to the z-axis. Sliding contacts at points **a** and **b** in the diagram below allow current through the resistor R_L.



(a) Consider a small radial segment of radial extent dr located at radius r from the axis of rotation at the centre of the disk. Calculate the contribution $d\mathcal{E}$ of this segment to the total induced emf.

(b) Integrate over the radius to find the total induced emf $\mathcal{E}$ between the axis and the outer edge of the disk.

(c) Use Lenz's law to identify the direction of current through the resistor R_L, i.e. does the current flow from point **a** to **b**, or **b** to **a**?

(d) Write an expression for the power required to maintain the constant angular velocity of the disk.

(10 marks)

Solution
(a)
Motional emf is given by:

$$\varepsilon = \oint \vec{v} \times \vec{B} \bullet d\vec{l}$$

As $\vec{v}$ and $\vec{B}$ are at right angles, the magnitude is given by:

$$\left|\vec{v} \times \vec{B}\right| = v\,B = r\,\omega\,B\,,$$

where ω is the angular velocity of the disk.

(1 mark)

For this small segment, with velocity $\vec{v}$, the vector $\vec{v} \times \vec{B}$ is directed radially outwards.

The induced emf $d\varepsilon$ of this segment tends to make current flow radially outward, and is given by

$$d\varepsilon = \left(\vec{v} \times \vec{B}\right) \bullet d\vec{l} = \omega\,r\,B\,dr$$

(**1 mark correct approach; 1 mark correct answer; 1 mark correct direction)**

(b) Integrating from 0 to R:

$$\varepsilon = \int_0^R \omega\,r\,B\,dr = \frac{1}{2}\omega\,B\,R^2$$

(**1 mark approach; 1 mark correct answer)**

(c) Current will flow from point **a** to **b**.

(1 mark)

(d) The power dissipated in the resistor is

$$P = \frac{\varepsilon^2}{R_L} = \frac{\omega^2\,B^2\,R^4}{4\,R_L}$$

(2 marks)

This equals the power required to maintain the constant angular velocity of the disk.

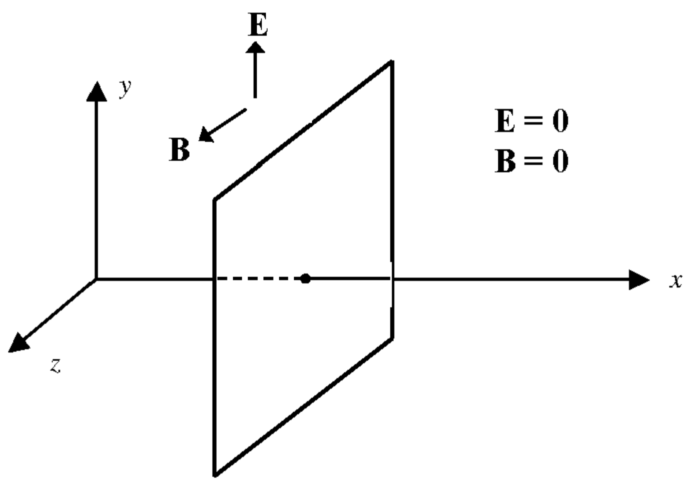
(1 mark)

ADV_Q08

Semester 2, Y2010

Question 8

Imagine that we are far from any charges and currents, and space is divided into two regions by a plane perpendicular to the x axis. We call this boundary plane the *wavefront*, which we take to be moving in the $+x$ direction with speed v. To the right of the wavefront there are no electric or magnetic fields. To its left, there is a uniform electric field in the $+y$ direction and a uniform magnetic field in the $+z$ direction.



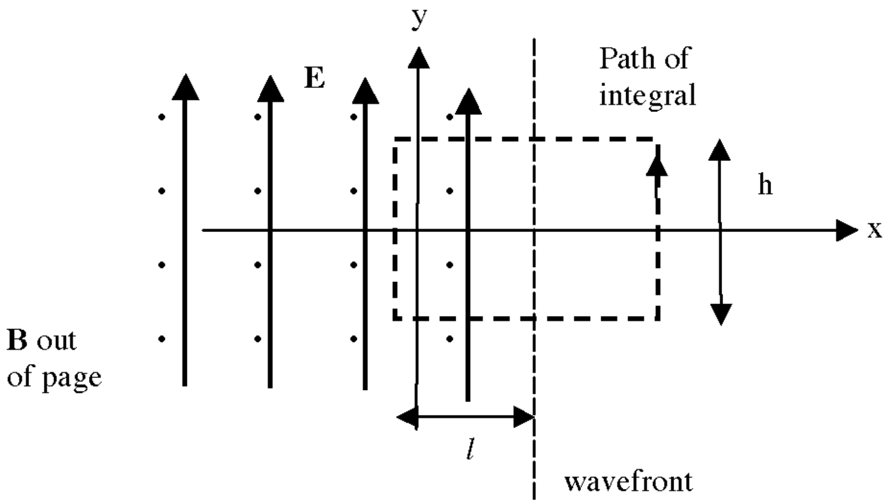
(a) Show that this situation satisfies Faraday's Law, provided the magnitudes of the fields satisfy $E = vB$.
(*Hint:* consider a path integral around a rectangle in the $x-y$ plane.)

(b) Show that this situation also satisfies the Ampère-Maxwell Law.

(c) Hence derive an expression for the speed of the wavefront.

(10 marks)

Solution
(a)



Apply $\oint \vec{E} \bullet d\vec{l} = -\frac{d\Phi_B}{dt}$

In this equation: $\text{LHS} = -E\,h$

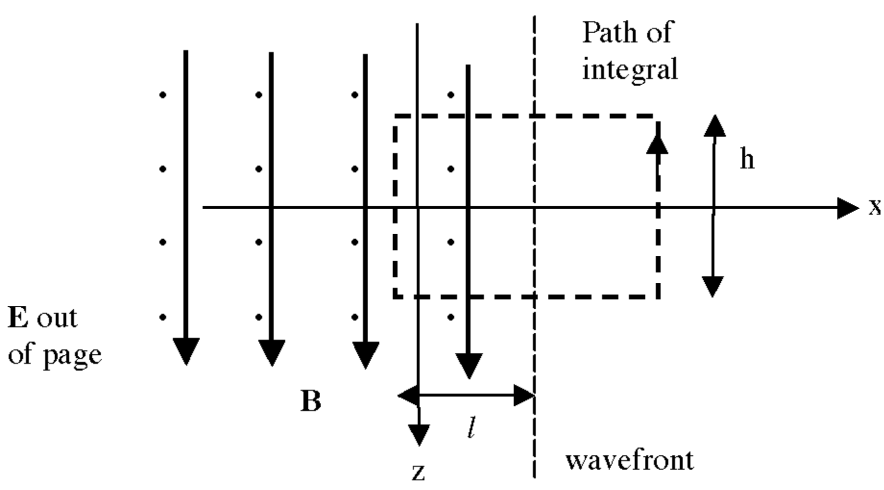
$$\text{RHS} = -\frac{d}{dt}\left(B \times area\right)$$

$$= -\frac{d}{dt} l\,h\,B = -h\,B\frac{dl}{dt}$$

$\Rightarrow E = v\,B$

(4 marks)

(b)



Apply $\oint \vec{B} \bullet d\vec{l} = \mu_0\,\varepsilon_0 \frac{d\Phi_E}{dt}$

In this equation: $\text{LHS} = B\,h$

$$RHS = \mu_0 \, \varepsilon_0 \frac{d}{dt}\left(E \times area\right)$$

$$= \mu_0 \, \varepsilon_0 \frac{d}{dt}\left(E \, l \, h\right) = \mu_0 \, \varepsilon_0 \, E \, h \, v$$

$$\Rightarrow B = \mu_0 \, \varepsilon_0 \, v \, E$$

(4 marks)

(c)

$$\frac{E}{B} = v \quad \text{and} \quad \frac{E}{B} = \frac{1}{\varepsilon_0 \mu_0 v}$$

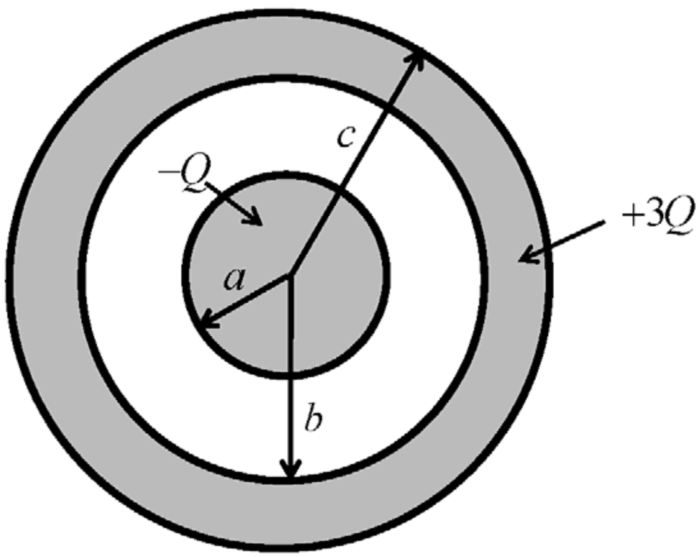
$$\Rightarrow v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

(2 marks)

TEC_Q09=ADV_Q09

Semester 2, Y2010

Question 9
A solid conducting sphere of radius a has an excess charge of $-Q$. It is surrounded by a concentric conducting spherical shell which has a charge of $+3Q$ placed on it. The inner radius of the shell is b, its outer radius c.



(a) Describe how the charges arrange themselves on the sphere and on the shell.

(b) Determine expressions for the electric field, **E** (and indicate its direction with radially outwards taken as positive), as a function of radius r for the following locations:

(i) inside the conducting sphere $(r < a)$;

(ii) between the sphere and the shell $(a < r < b)$;

(iii) inside the spherical shell $(b < r < c)$;

(iv) outside the shell $(c < r)$.

(c) Plot the magnitude of the electric field as a function of radius. Label your graph carefully.

(d) Calculate the potential difference $(V_a - V_b)$ between the sphere and the shell.

(e) Will the potential difference be positive, negative or zero? Justify your answer in one or two sentences.

(10 marks)

Solution

(a)

On the sphere a charge of $-Q$ is distributed uniformly on its surface.

On the inner surface of the shell a charge of $+Q$ is distributed uniformly.

On the outer surface of the shell a charge of $+2Q$ is distributed uniformly.

(1 mark)

(b) Calculate the electric field in different regions.

We construct a spherical Gaussian surface of radius r with its centre at the common centre of the sphere and shell and then use Gauss's law

$$\oint \vec{E} \bullet d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

to work out the electric field. The spherical symmetry means that the electric field is uniform over the Gaussian surface and is radial in direction. So Gauss's law can be written as:

$$E\left(4\pi r^2\right) = \frac{q_{enc}}{\varepsilon_0}$$

$$\Rightarrow E = \frac{q_{enc}}{4\pi \varepsilon_0 r^2}$$

(i) inside the conducting sphere $(r < a)$

The electric field inside a conductor must be zero as all excess charge resides on the surface. There is no enclosed charge.

(ii) between the sphere and the shell $(a < r < b)$

The enclosed charge is $-Q$ and so the electric field has a value

$$E = -\frac{Q}{4\pi \varepsilon_0 r^2}$$

(iii) inside the spherical shell $(b < r < c)$

The total enclosed charge is again zero and hence the electric field is zero.

$$E = 0$$

(iv) outside the shell $(c < r)$

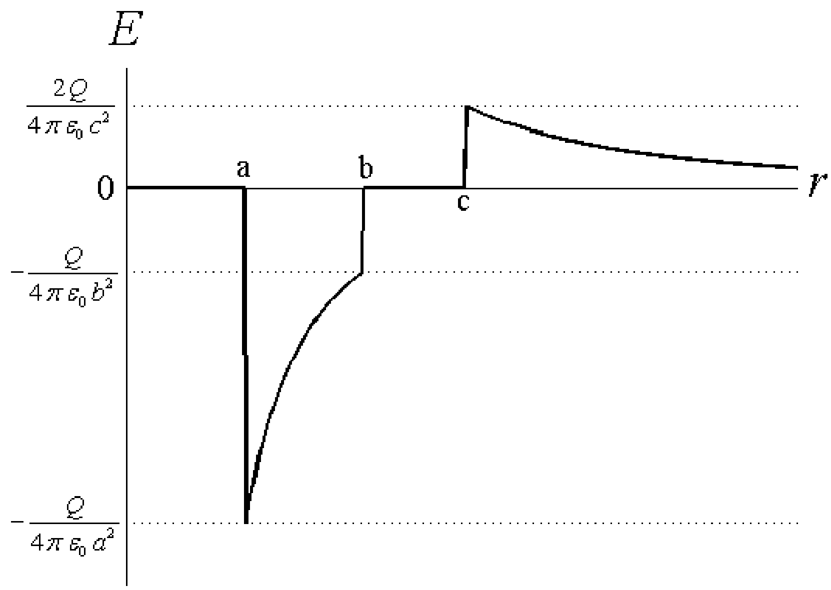
The total enclosed charge is now $+2Q$ and the electric field is given by

$$E = \frac{2Q}{4\pi \varepsilon_0 r^2} = \frac{Q}{2\pi \varepsilon_0 r^2}$$

(1 mark for each answer = 4 marks in total)

(c)

Sketch of the electric field is shown below.



(2 marks, exact shape depends on values of a,b and c)

(d)
Potential difference is

$$V_a - V_b = \int_a^b \vec{E} \bullet d\vec{r}$$

$$= \int_a^b \frac{-Q}{4\pi\varepsilon_0 r^2} dr \quad = \quad -\frac{Q}{4\pi\varepsilon_0} \int_a^b \frac{1}{r^2} dr \quad = \quad -\frac{Q}{4\pi\varepsilon_0} \left[\frac{-1}{r} \right]_a^b$$

$$= \frac{Q}{4\pi\varepsilon_0} \left[\frac{1}{b} - \frac{1}{a} \right]$$

Moving from a to b is against the direction of the E field. The potential increases.

$$\left(b > a \quad \Rightarrow \quad \frac{1}{b} - \frac{1}{a} < 0 \right)$$

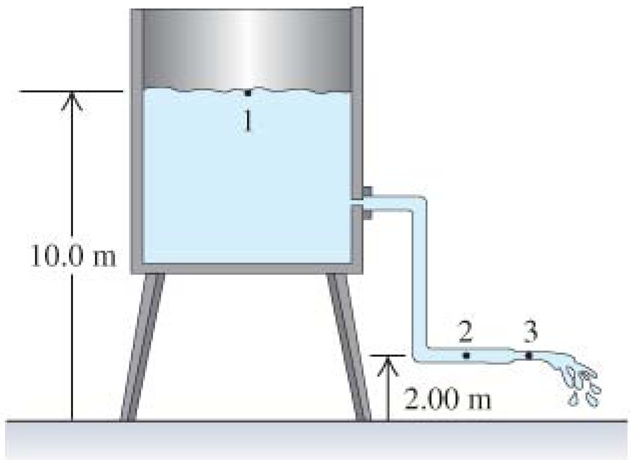
Therefore
$V_a - V_b$ is negative.

(2 marks for answer; 1 mark for direction)

ADV_Q10=TEC_Q10

Semester 2, Y2010

Question 10



Water flows steadily from an open tank as shown in the above figure. The elevation of point 1 inside the tank is $10.0\,\mathrm{m}$, and the elevation of points 2 and 3 is $2.00\,\mathrm{m}$. The cross-sectional area of the pipe at point 2 is $0.0480\,\mathrm{m}^2$; and at point 3 it is $0.0160\,\mathrm{m}^2$. The area of the tank is very large compared with the cross-sectional area of the pipe.

(a) Assuming that Bernoulli's equation applies, what is the velocity of the flow as it discharges from the pipe at point 3?

(b) What is the discharge flow rate at point 3?

(c) What is the velocity of the flow at point 2?

(d) What is the gauge pressure at point 2?

(e) Is the flow turbulent as it discharges from the pipe at point 3? Justify your answer.

(10 marks)

Solution
(a)
Bernoulli's equation for points 1 and 3 is:

$$p_3 + \frac{1}{2}\rho\, v_3{}^2 + \rho\, g\, h_3 = p_1 + \frac{1}{2}\rho\, v_1{}^2 + \rho g\, h_1$$

At point 1 (the top of the tank) we have $h_1 = 10.0\,\text{m}$ and $v_1 = 0\,\text{m.s}^{-1}$ (the cross-sectional area of the tank is large making the flow velocity essentially zero). The tanks is open to the atmosphere and so $p_1 = 1\text{atm}$.

At point 3 ($h_3 = 2.00\,\text{m}$) we have velocity v_3 which is unknown as yet. It is also open to the atmosphere and so $p_3 = 1\text{atm}$.

Bernoulli's equation can then be written as

$$\frac{1}{2}\rho v_3{}^2 + \rho g h_3 = \rho g h_1$$

$$\Rightarrow v_3 = \sqrt{2\,g\,(h_1 - h_3)}$$

Thus $\quad v_3 = \sqrt{2(9.80)(8.0)} \quad = \quad 12.5\,\text{m/s}$.

(3 marks)

(b)
The discharge rate is therefore: $(0.0160)(12.52) = 0.20\,\text{m}^3.\text{s}^{-1}$.

(1 mark)

(c)
Find the velocity at point 2 using the equation of continuity and the previous result.

$$A_2\, v_2 = A_3\, v_3$$

$$\Rightarrow v_2 = v_3\, \frac{A_3}{A_2} = 4.17\,\text{m.s}^{-1}\,.$$

(2 marks)

(d)
Using Bernoulli's equation for points 1 and 2 we have:

$$p_a + \rho\, g\, h_1 = \; p_2 + \rho\, g\, h_2 + \frac{1}{2}\rho\, v_2{}^2$$

where atmospheric pressure p_a cannot be ignored as it is not equal at both points.

This implies that the gauge pressure at point 2 is

$$p_2 - p_a \quad = \rho\, g\,(h_1 - h_2) \; - \; \frac{1}{2}\rho\, v_2{}^2$$

$$= 1000[(9.80)(8.0) - (0.5)(4.17)^2]$$

$$= 6.97\times 10^4 \text{ Pa}$$

(2 marks)

(e)

Reynolds number is given by:

$$R_e = \frac{\rho v L}{\eta}$$

The characteristic length L is taken as the diameter of the pipe at point 3 which is given by:

$$L = \sqrt{\frac{0.016}{\pi}} = 0.071\,\text{m}.$$

Substituting values we have

$$R_e = \frac{(1.0\times10^{3})(12.5)(0.71)}{1.0\times10^{-3}} = 8900\,.$$

(1 mark)

$R_e > 2000$ so the flow is turbulent.

(1 mark)

TEC_Q11=ADV_Q11

Semester 2, Y2010

Question 11

(a) Draw a labelled diagram showing the essential features of the apparatus for a Compton scattering experiment. Indicate the scattering angle on your diagram.

(b) With a detector placed at that scattering angle, briefly describe what is observed.

(c) The Compton shift is given by the equation

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\phi).$$

What is the *maximum* change in wavelength predicted by this equation?

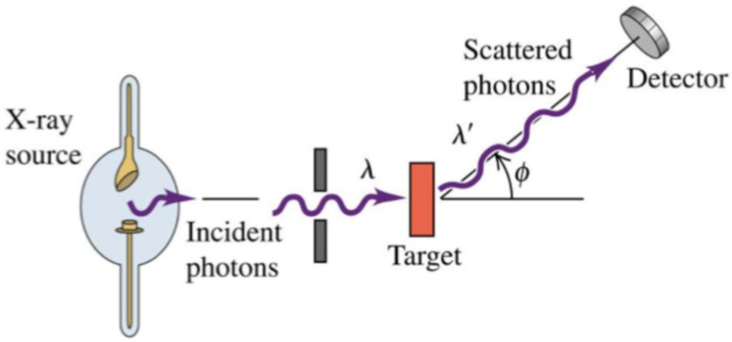
(d) What are the implications of the maximum value identified in part (c) for observing Compton scattering in wavelength bands other than X-rays?

(e) If the incident X-ray photons have a wavelength of $29.0\times10^{-12}\,\mathrm{m}$, what is the corresponding maximum change in energy of the X-ray photons caused by the Compton scattering process? Express your answer in eV.

(10 marks)

Solution

(a)



key features:

- X-ray source
- Target
- Detector
- Scattering angle

(1/2 mark each – total 2 marks)

(b)

The detector sees two peaks – one at the same wavelength as the incident X-rays and one at a longer wavelength (lower energy).

(1 mark)

(c)

Maximum shift occurs when $\phi = 180$ degrees, so the term $(1-\cos\phi) = 2$

(1 mark)

$$\begin{aligned}\lambda' - \lambda &= \frac{h}{m_e c}(1-\cos\phi) = \frac{2h}{m_e c} \quad (\phi = 180^\circ) \\ &= \frac{6.626\times10^{-34}}{(9.10938\times10^{-31})(2.998\times10^{8})}(2) \\ &= 4.855\times10^{-12}\,\text{m} \\ &= 4.855\text{pm}\end{aligned}$$

(1 mark)

(d)

The shift in wavelength is comparable to the wavelength of the X-rays $(10^{-10}\,\text{m})$, but is negligible compared to wavelength for long wavelength (low energy) X-rays and longer wavelengths. It would be dramatic for gamma rays if observed at these wavelengths.

(2 marks)

(e)

Many people will get this wrong – you can't use $\lambda' - \lambda$ directly.

The two wavelengths are 29.0 pm and 33.855 pm.

Then

$$
\begin{aligned}
E' - E &= \frac{hc}{\lambda'} - \frac{hc}{\lambda} \\
&= hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right) \\
&= \left(6.626\times10^{-34}\right)\left(2.998\times10^{8}\right)\left(\frac{1}{33.855\times10^{-12}} - \frac{1}{29.0\times10^{-12}}\right) \\
&= -9.823\times10^{-16}\,\mathrm{J}
\end{aligned}
$$

(correct method, sign irrelevant - 2 marks)

Converting to eV by dividing by 1.602×10^{-19} J/eV the result is a change in energy of $6.13\times10^{3}\,\mathrm{eV} \approx 6.13\,\mathrm{keV}$.

(correct answer in eV - 1 mark)

ADV_Q12

Semester 2, Y2010

Question 12

An electron free to move in one direction and subject to no forces is in a state described by the wave function

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\exp\left[i\left(kx - \omega t\right)\right] + \frac{1}{\sqrt{2}}\exp\left[i\left(k'x - \omega' t\right)\right],$$

where E is the energy of the electron of mass m and where

$$k = \frac{\sqrt{2mE}}{\hbar}, \quad \omega = \frac{E}{\hbar},$$

and

$$k' = \frac{\sqrt{2m\left(E + \Delta E\right)}}{\hbar}, \quad \omega' = \frac{E + \Delta E}{\hbar},$$

with $\Delta E << E$.

(a) Is this state time dependent? Briefly explain.

(b) Show that:

$$k' \approx \left(1 + \frac{\Delta E}{2E}\right)k.$$

You may use the approximation:

$$\sqrt{1+\varepsilon} \approx 1 + \frac{1}{2}\varepsilon,$$

which is valid for $\varepsilon << 1$.

(c) Using the approximation obtained in (b), show that:

$$\left|\Psi(x,t)\right|^2 \approx 1 + \cos\left(Kx - \Omega t\right),$$

with

$$K = \frac{\Delta E}{2E}k \quad \text{and} \quad \Omega = \frac{\Delta \mathrm{E}}{\hbar}.$$

(d) On the same diagram sketch the probability distribution function $\left|\Psi(x,t)\right|^2$ given in part (c) versus position x at times:

(i) $t_0 = 0$;

(ii) $t_{1/4} = \frac{1}{4}\frac{2\pi}{\Omega}$;

(ii) $t_{1/2} = \frac{1}{2}\frac{2\pi}{\Omega}$.

(e) Briefly describe how the probability distribution given in part (c) changes in space and time.

(10 marks)

Solution

(a)
The state does not correspond to a single energy (it is a superposition of states with energies E and $E+\Delta E$) and hence will be time dependent.

(1 mark)

(b)
We have

$$k' = \frac{\sqrt{2m\left(E+\Delta E\right)}}{\hbar}$$
$$= \frac{\sqrt{2\,m\,E}}{\hbar}\left(1+\frac{\Delta E}{E}\right)^{1/2}$$
$$\approx \frac{\sqrt{2\,m\,E}}{\hbar}\left(1+\frac{\Delta E}{2\,E}\right)$$
$$\approx k\left(1+\frac{\Delta E}{2\,E}\right)$$

using the given approximation.

(3 marks)

(c)
We have

$$\left|\Psi\right|^2 = \left|\frac{1}{\sqrt{2}}\exp\left[i\left(kx-\omega t\right)\right] \quad + \quad \frac{1}{\sqrt{2}}\exp\left[i\left(k^{'}x-\omega^{'}t\right)\right]\right|^2$$
$$= \frac{1}{2}+\frac{1}{2}+\frac{1}{2}\exp\left[i\left(k-k^{'}\right)x\right]\exp\left[-i\left(\omega-\omega^{'}\right)t\right]+\frac{1}{2}\exp\left[-i\left(k-k^{'}\right)x\right]\exp\left[i\left(\omega-\omega^{'}\right)t\right]$$

and we have

$$k-k^{'} \approx -\frac{\Delta E}{2E}k \qquad \text{using part (b)}$$

and

$$\omega-\omega^{'} = -\frac{\Delta E}{\hbar}.$$

So

$$\left|\Psi\right|^2 = 1+\frac{1}{2}\exp\left[-i\left(k\frac{\Delta E}{2E}x-\frac{\Delta E}{\hbar}t\right)\right]+\frac{1}{2}\exp\left[i\left(k\frac{\Delta E}{2E}x-\frac{\Delta E}{\hbar}t\right)\right]$$
$$= 1+\frac{1}{2}\cos\left(k\frac{\Delta E}{2E}-\frac{\Delta E}{\hbar}t\right) \quad -\frac{1}{2}\sin\left(k\frac{\Delta E}{2E}-\frac{\Delta E}{\hbar}t\right)$$
$$+\frac{1}{2}\cos\left(k\frac{\Delta E}{2E}-\frac{\Delta E}{\hbar}t\right) \quad +\frac{1}{2}\sin\left(k\frac{\Delta E}{2E}-\frac{\Delta E}{\hbar}t\right)$$

$$= 1 + \cos\left(k\frac{\Delta E}{2E} - \frac{\Delta E}{\hbar} t \right)$$

This has the form of:

$$\left|\Psi(x,t)\right|^2 \approx 1 + \cos\left(K x - \Omega t\right),$$

where

$$K = \frac{\Delta E}{2E} k \quad \text{and} \quad \Omega = \frac{\Delta \mathrm{E}}{\hbar}.$$

(3 marks)

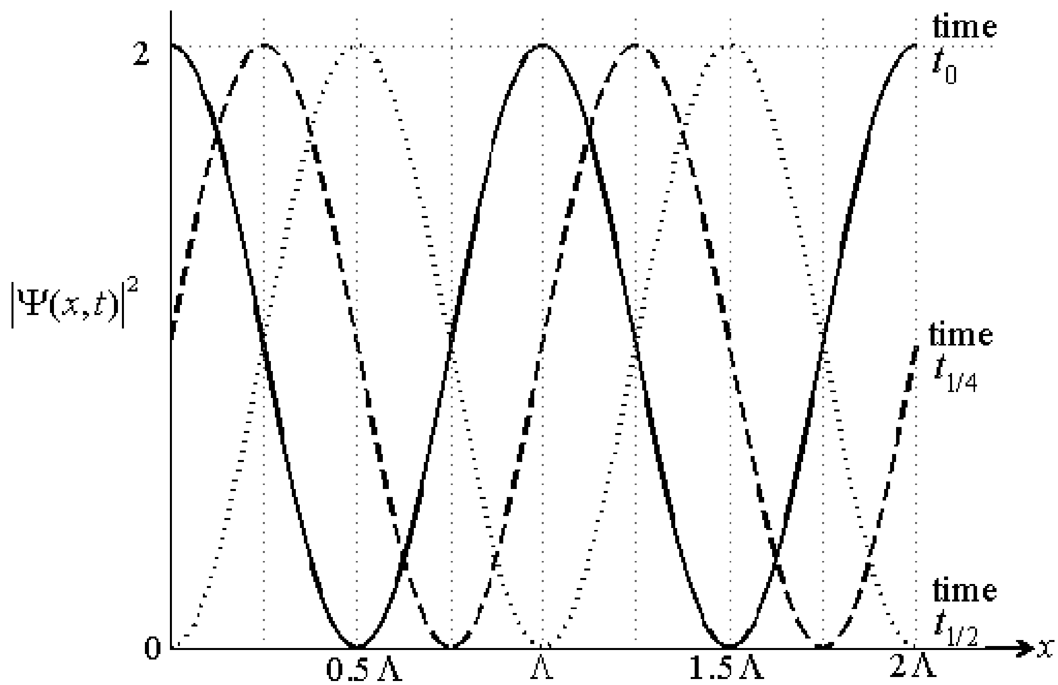
(d)

The probability distribution $\left|\Psi(x,t)\right|^2$ is a cosine form which shifts itself to the right by a distance $\Lambda = \frac{2\pi}{\mathrm{K}}$ in a time $T = \frac{2\pi}{\Omega}$.

We have:

$t_0 = 0$; $t_{1/4} = \frac{1}{4}T$; and $t_{1/2} = \frac{1}{2}T$.

So the distributions are:



(3 marks)