# MATH 1903 Lectures

Semester 2, 2012

Week 13

Daniel Daners

# Review of differential equations

Classification:

First order equations:

- separable equations
- linear homogeneous (separable)
- linear inhomogeneous → integrating factors

Second order equations:

- linear homogeneous equations (const coeff)
  - solve by means of auxiliary equation (3 cases)
- linear inhomogeneous equations
  → to find a particular solution try a solution of the same form as the inhomogeneity.
  → general sol = one part sol + general sol. of hom. problem.

Systems of two (linear) equations of 2nd order:

- eliminate one variable, by reducing to a 2nd order equation

OR
- look at the system matrix and use the eigenvalues and eigenvectors to construct a solution.

Other aspects we looked at:

- direction fields: for first order equations and first order systems: Solution is a curve tangent to the direction field at any point.
- method of substitutions
- equation for inverse function $\frac{dy}{dx} = f(x,y) \to \frac{dx}{dy} = \frac{1}{f(x,y)}$
- modelling

Second order equations

homogeneous: $ay'' + by' + cy = 0$

auxiliary equation: $a\lambda^2 + b\lambda + c = 0$

3 cases:

- two distinct roots $\lambda_1, \lambda_2$, <u>real</u>

  general solution $y = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$

- pair of complex conjugate roots $\lambda = \mu \pm i\omega$

  general solution $y = e^{\mu x}(A\cos\omega x + B\sin\omega x)$

- one double root $\lambda$

  general solution $y = (A + Bx)e^{\lambda x}$

inhomogeneous: $ay'' + by' + cy = f(t)$

general solution:

$$y(t) = y_h(t) + y_p(t)$$

where

- $y_h(t)$ general solution of homogeneous equation
- $y_p(t)$ one particular solution of inhomogeneous eq.

Finding a particular solution:

(1) $f(t)$ poly. of deg $n$ $\rightsquigarrow$ try polynomial of deg. $n$

(2) $f(t)$ exponential $\rightsquigarrow$ try exponential $\times$ const.

(3) $f(t)$ $\cos\omega t / \sin\omega t$ $\rightsquigarrow$ try $A\cos\omega t + B\sin\omega t$

Bad luck in (2) or (3) if $f(t)$ solves homogeneous equation. In that case try

$$y_p(t) = A\,t\,f(t)$$

multiply by t