Extended Answer Section

There are **three** *questions in this section, each with a number of parts. Write your answers in the space provided below each part. If you need more space there are extra pages at the end of the examination paper.*

1. The parametric vector form of the line $\mathcal{L}_1$ is given as $\mathbf{r}_1 = \mathbf{u}_1 + r\mathbf{v}_1$ $(r \in \mathbb{R})$ where $\mathbf{u}_1$ is the position vector of $P_1 = (1, 1, -3)$ and $\mathbf{v}_1 = \overrightarrow{P_1Q_1}$ where $Q_1 = (3, 3, -2)$. The parametric vector form of the line $\mathcal{L}_2$ is given as $\mathbf{r}_2 = \mathbf{u}_2 + s\mathbf{v}_2$ $(s \in \mathbb{R})$ where $\mathbf{u}_2$ is the position vector of $P_2 = (-2, 0, 2)$ and $\mathbf{v}_2 = -\mathbf{j} - \mathbf{k}$.

(a) [**2 marks**] Give the parametric scalar equations of $\mathcal{L}_1$.

(b) [**2 marks**] Find a *unit* vector $\hat{\mathbf{n}}$ that is perpendicular to both $\mathcal{L}_1$ and $\mathcal{L}_2$.

Question 1 continues on the next page

(c) [**4 marks**] The shortest distance between two lines is the length of a vector that connects the two lines and is perpendicular to both lines. For $\mathcal{L}_1$ and $\mathcal{L}_2$ this is expressed in the vector equation $\mathbf{r}_1 + t\hat{\mathbf{n}} = \mathbf{r}_2$ where $t \in \mathbb{R}$ is a parameter. The corresponding system of linear equations is

$$\begin{aligned} 2r - \frac{1}{3}t &= -3 \\ 2r + s + \frac{2}{3}t &= -1 \\ r + s - \frac{2}{3}t &= 5\,. \end{aligned}$$

Solve this system of linear equations.

Question 1 continues on the next page

(d) [**2 marks**] Hence find the shortest distance between $\mathcal{L}_1$ and $\mathcal{L}_2$ and find the point Q on $\mathcal{L}_1$ that is closest to $\mathcal{L}_2$.

2. (a) Let $A = \begin{bmatrix} 3 & 1 & 0 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

(i) [**3 marks**] Find the inverse A^{-1}.

Question 2 continues on the next page

(*ii*) [**2 marks**] Hence find the 3×3 lower triangular matrix X such that $AX = B$.

(*iii*) [**1 mark**] Find $\det(A)$. (*Hint: This may be done directly, but also follows easily from a correct answer to part (ii).*)

Question 2 continues on the next page

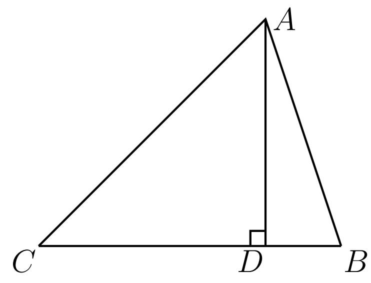
(b) Consider the matrix $C = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 1 & -4 \\ 1 & 0 & -1 \end{bmatrix}$.

(i) [**2 marks**] Find all eigenvalues of C.

Question 2 continues on the next page

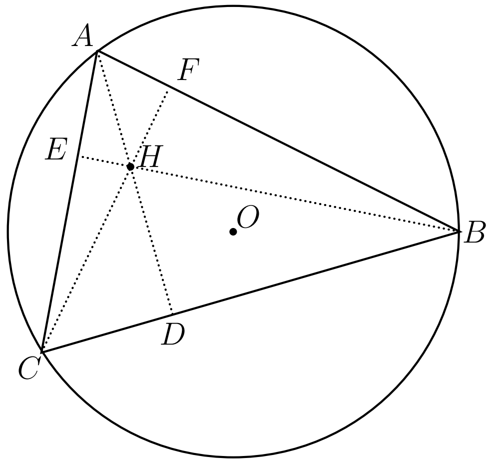
(ii) [**2 marks**] Find the eigenspace of C corresponding to eigenvalue 1.

3. Recall the definition of an *altitude* of a triangle ABC:



It is the line joining a vertex A to the opposite side at D so that $\overrightarrow{AD}$ and $\overrightarrow{BC}$ are orthogonal.

Let O be the centre of a circumscribing circle around a triangle ABC, and let H be the point of intersection of the altitudes AD, BE and CF:



Let $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$, and $\overrightarrow{OC} = \mathbf{c}$.

(a) [**2 marks**] Write the vectors $\overrightarrow{AB}$ and $\overrightarrow{BC}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.

Question 3 continues on the next page

(b) [**2 marks**] Show that $\mathbf{b}+\mathbf{c}$ and $\mathbf{b}-\mathbf{c}$ are orthogonal.

(c) [**2 marks**] Hence, or otherwise, justify the statement:

A parametric vector equation for the line through A and D is

$$\mathcal{L}_1 : \quad \mathbf{r}_1 = \mathbf{a} + t(\mathbf{b}+\mathbf{c}), \quad t \in \mathbb{R}.$$

Question 3 continues on the next page

(d) [**4 marks**] Express the vector $\overrightarrow{OH}$ in terms of $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$.
(*Hint: use a parametric vector equation for the line through C and F.*)

There are no more questions.
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End of Extended Answer Section

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