Assignment 3

1. Assuming all the random variables are discrete, show the following statements are true.

(a) (2 pts) $X_1 \perp\!\!\!\perp (X_2, X_3) | X_4$, then $X_1 \perp\!\!\!\perp X_2 | X_4$.

(b) (2 pts) Suppose that for (X_1, X_2, X_3), the joint PMF $p(x_1, x_2, x_3) > 0$ for any set of possible values (x_1, x_2, x_3). Then $X_1 \perp\!\!\!\perp X_2 | X_3$ together with $X_1 \perp\!\!\!\perp X_3 | X_2$ imply $X_1 \perp\!\!\!\perp (X_2, X_3)$.

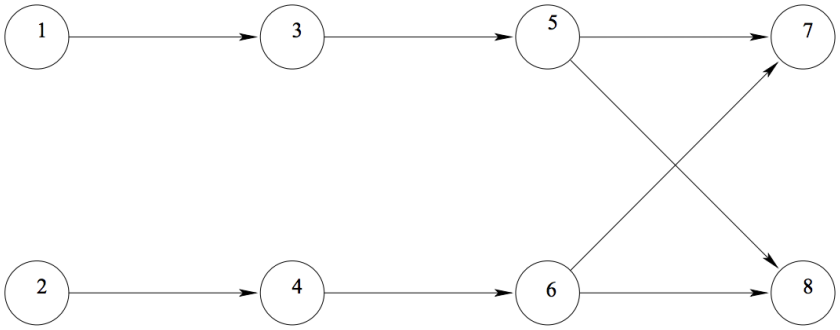
2. For $i = 1, 2, 3$, let X_i be an indicator variable for the event that a coin toss comes up heads (which occurs with probability q). That is, $X_i = 1$ with probability q and $X_i = 0$ with probability $1 - q$. Assuming that the X_i's are independent, define $X_4 = X_1 \oplus X_2$, $X_5 = X_2 \oplus X_3$, where $\oplus$ denotes addition in modulo two arithmetic.

(a) (2 pt) Compute the conditional PMF of (X_2, X_3) given $X_5 = 0$ and $X_5 = 1$ respectively.

(b) (2 pts) Draw a directed graphical model (including the graph and conditional probability tables) for these five random variables. List at least five conditional independence relations implied by the graph.

(c) (2 pts) Under what conditions on q do we have $X_3 \perp\!\!\!\perp X_5$ and $X_1 \perp\!\!\!\perp X_4$? Is either of these marginal independence assertions implied by the graph in (b)?

3. Consider the following directed graph.



(a) (1 pt) What is the corresponding moral graph?

(b) (2 pts) Sketch the sequence of graphs obtained by running the graph elimination algorithm on the moral graph following the ordering

(i) $\{8, 7, 2, 4, 6, 5, 3, 1\}$

(ii) $\{8, 5, 6, 7, 4, 3, 2, 1\}$

(c) (2 pts) Suppose we want to calculate $p(x_1|x_8)$, which of the orderings listed in (b) is preferable? Why?