THE UNIVERSITY OF SYDNEY

PHYS1902 – PHYSICS 1B (ADVANCED)

NOVEMBER 2010

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

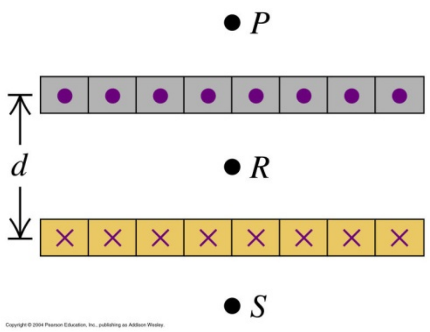
DATA

Density of water	ρ	=	1.00×10^3 kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^5 Pa
Magnitude of local gravitational field	g	=	9.80 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^8 m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg
Rydberg constant	R	=	1.097×10^7 m^{-1}

SECTION A
(Please use a separate booklet for each section)

Question 1

Long, straight conductors with square cross section, each carrying current I directed out of the plane of the page, are laid side by side to form an infinite current sheet. A second infinite current sheet is a distance d below the first and is parallel to it. The second sheet carries current into the plane of the page. Each sheet has n conductors per unit length.



Use Ampere's law to calculate the magnitude and direction of the magnetic field at:

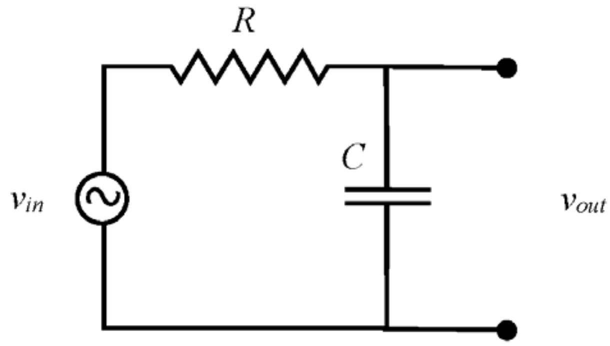
(a) Point *P* (above the upper sheet as in the diagram above);

b) Point *R* (midway between the two sheets);

(c) Point *S* (below the lower sheet).

(5 marks)

Question 2



In the circuit shown in the diagram above the resistor has a value of $R = 10.0\,\mathrm{k\Omega}$ and the capacitor a value of $C = 100\,\mathrm{nF}$.

(a) Briefly describe in words the relationship between v_{out} and v_{in} as a function of frequency.

(b) Explain why the circuit shows this behaviour.

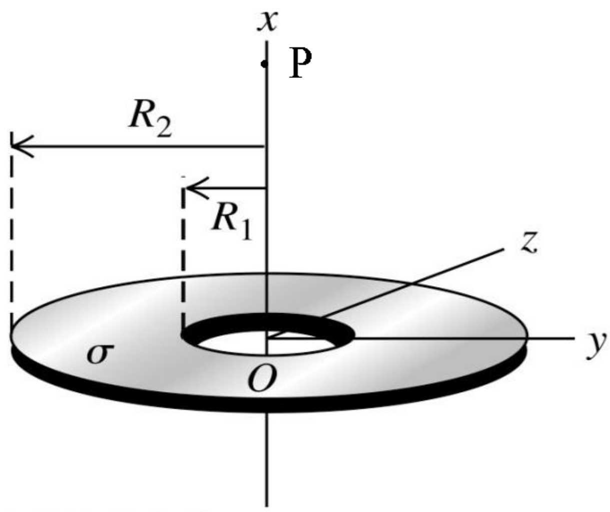
(c) At what frequency f_1 (in Hz) does the reactance of the capacitor C have a value of $10.0\,\mathrm{k\Omega}$?

(d) At this frequency f_1, what is the value of ratio of v_{out} / v_{in}?

(5 marks)

Question 3

A thin disk with a circular hole at its centre, called an annulus, has inner radius R_1 and outer radius R_2. The disk has a uniform positive surface charge density σ on its surface.



(a) Determine the total electric charge on the annulus.

(b) Calculate the magnitude and direction of the electric field at a point *P* a distance *x* along the *x*-axis from the centre of the annulus.
Hint: you may need the following integral:

$$\int \frac{r\,dr}{(a^2+r^2)^{3/2}} = -\frac{1}{\sqrt{a^2+r^2}}$$

(5 marks)

Question 4

(a) Two identical drops of water will merge if they come into contact. Explain why this happens in terms of energy.

(b) Astronauts in the pressurized environment on the International Space Station find that surface tension effects are very important for the way that liquids behave. Is buoyancy also important for liquids in this environment? Briefly explain your answer.

(5 marks)

Question 5
An electron free to move in one direction and subject to no forces is described by the wave function:

$$\Psi(x,t) = A\exp[i(kx - \omega t)],$$

where E is the energy of the electron, m is the mass of the electron, $k = \sqrt{2mE}/\hbar$ and $\omega = E/\hbar$.

(a) Does this electron have a uniquely defined momentum? Briefly explain.

(b) Does this electron have a uniquely defined position? Briefly explain.

(c) Briefly explain how your answers to (a) and (b) are consistent with the Heisenberg Uncertainty Principle.

(5 marks)

Question 6
The Planck law for the intensity distribution of radiation from a blackbody is written:

$$I(\lambda) = \frac{2\pi hc^2}{\lambda^5\left(\exp\left(\frac{hc}{\lambda kT}\right) - 1\right)}$$

where h is Planck's constant, c is the speed of light, k is Boltzmann's constant, T is the absolute temperature of the blackbody, and λ is wavelength.

The Rayleigh radiation law is expressed as:

$$I(\lambda) = \frac{2\pi ckT}{\lambda^4}.$$

(a) Sketch both laws as a function of wavelength at a given temperature (your plot only needs to be qualitatively correct).

(b) Show that the Planck law reduces to the Rayleigh law at very large wavelengths $\left(\lambda >> \frac{hc}{kT}\right)$.

Hint: You may need the Taylor series expansion of the exponential function which is expressed as:

$$\exp(z) = 1 + z + \frac{z^2}{2!} + \ldots .$$

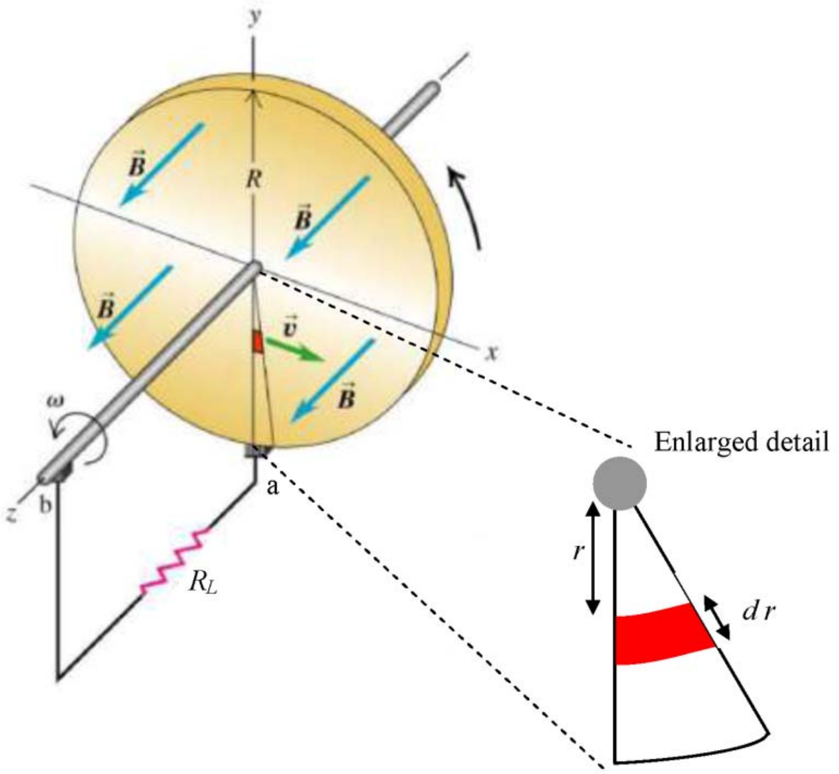
(c) Planck's constant does not appear in the Rayleigh law, but does appear in the Planck law. Briefly explain why, in terms of the physical origin of blackbody radiation.

(5 marks)

SECTION B
(Please use a separate booklet for this section)

Question 7

A conducting disk with radius R, as shown below, lies in the xy-plane and rotates with a constant angular velocity ω about the z-axis. The disk is in a uniform, constant magnetic field $\vec{B}$ parallel to the z-axis. Sliding contacts at points **a** and **b** in the diagram below allow current through the resistor R_L.



(a) Consider a small radial segment of radial extent dr located at radius r from the axis of rotation at the centre of the disk. Calculate the contribution $d\mathcal{E}$ of this segment to the total induced emf.

(b) Integrate over the radius to find the total induced emf $\mathcal{E}$ between the axis and the outer edge of the disk.

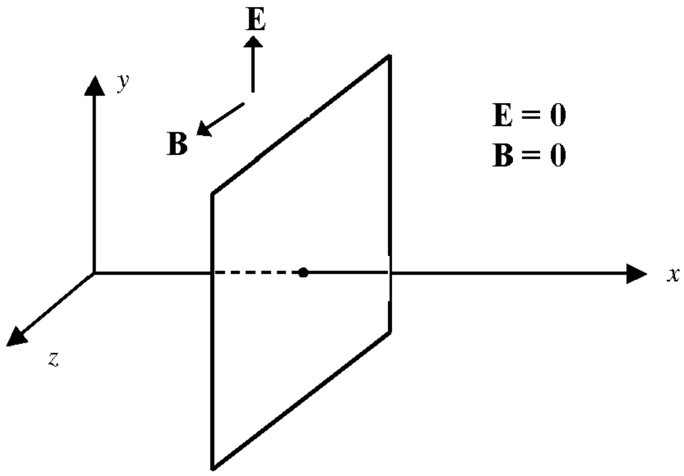
(c) Use Lenz's law to identify the direction of current through the resistor R_L, i.e. does the current flow from point **a** to **b**, or **b** to **a**?

(d) Write an expression for the power required to maintain the constant angular velocity of the disk.

(10 marks)

Question 8

Imagine that we are far from any charges and currents, and space is divided into two regions by a plane perpendicular to the x axis. We call this boundary plane the *wavefront*, which we take to be moving in the $+x$ direction with speed v. To the right of the wavefront there are no electric or magnetic fields. To its left, there is a uniform electric field in the $+y$ direction and a uniform magnetic field in the $+z$ direction.



(a) Show that this situation satisfies Faraday's Law, provided the magnitudes of the fields satisfy $E = vB$.
(*Hint:* consider a path integral around a rectangle in the $x-y$ plane.)

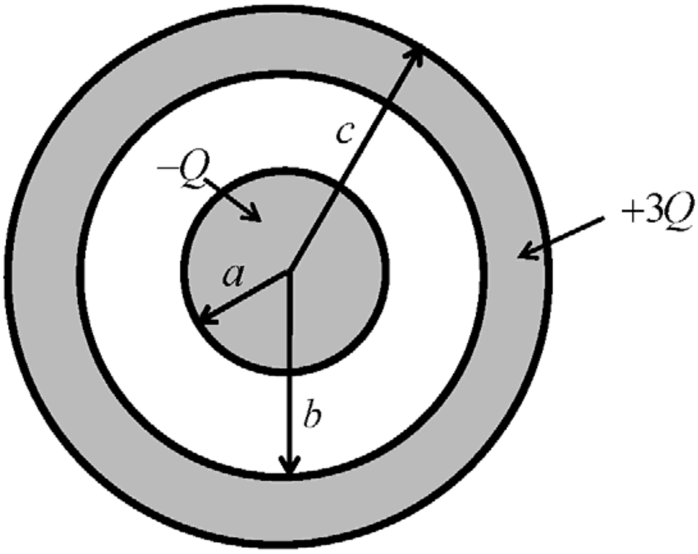
(b) Show that this situation also satisfies the Ampère-Maxwell Law.

(c) Hence derive an expression for the speed of the wavefront.

(10 marks)

Question 9

A solid conducting sphere of radius a has an excess charge of $-Q$. It is surrounded by a concentric conducting spherical shell which has a charge of $+3Q$ placed on it. The inner radius of the shell is b, its outer radius c.



(a) Describe how the charges arrange themselves on the sphere and on the shell.

(b) Determine expressions for the electric field, **E** (and indicate its direction with radially outwards taken as positive), as a function of radius r for the following locations:

- (i) inside the conducting sphere $(r < a)$;
- (ii) between the sphere and the shell $(a < r < b)$;
- (iii) inside the spherical shell $(b < r < c)$;
- (iv) outside the shell $(c < r)$.

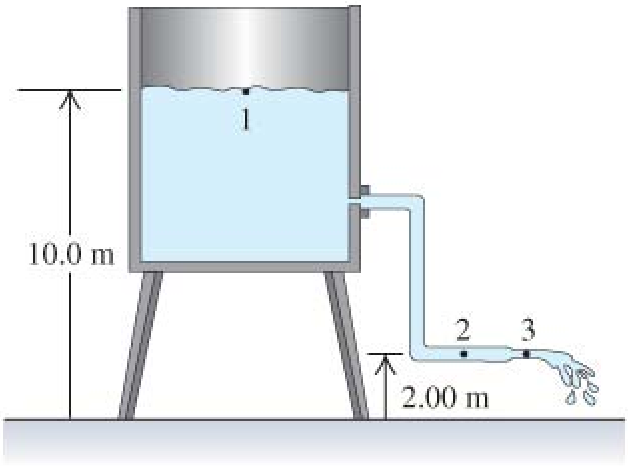
(c) Plot the magnitude of the electric field as a function of radius. Label your graph carefully.

(d) Calculate the potential difference $(V_a - V_b)$ between the sphere and the shell.

(e) Will the potential difference be positive, negative or zero? Justify your answer in one or two sentences.

(10 marks)

Question 10



Water flows steadily from an open tank as shown in the above figure. The elevation of point 1 inside the tank is $10.0\,\text{m}$, and the elevation of points 2 and 3 is $2.00\,\text{m}$. The cross-sectional area of the pipe at point 2 is $0.0480\,\text{m}^2$; and at point 3 it is $0.0160\,\text{m}^2$. The area of the tank is very large compared with the cross-sectional area of the pipe.

(a) Assuming that Bernoulli's equation applies, what is the velocity of the flow as it discharges from the pipe at point 3?

(b) What is the discharge flow rate at point 3?

(c) What is the velocity of the flow at point 2?

(d) What is the gauge pressure at point 2?

(e) Is the flow turbulent as it discharges from the pipe at point 3? Justify your answer. Take the viscosity of water as 1.0×10^{-3} Pa.s.

(10 marks)

Question 11

(a) Draw a labelled diagram showing the essential features of the apparatus for a Compton scattering experiment. Indicate the scattering angle on your diagram.

(b) With a detector placed at that scattering angle, briefly describe what is observed.

(c) The Compton shift is given by the equation

$$\lambda' - \lambda = \frac{h}{m_e c}\left(1 - \cos\phi\right).$$

What is the *maximum* change in wavelength predicted by this equation?

(d) What are the implications of the maximum value identified in part (c) for observing Compton scattering in wavelength bands other than X-rays?

(e) If the incident X-ray photons have a wavelength of $29.0 \times 10^{-12}\,\mathrm{m}$, what is the corresponding maximum change in energy of the X-ray photons caused by the Compton scattering process? Express your answer in eV.

(10 marks)

Question 12

An electron free to move in one direction and subject to no forces is in a state described by the wave function

$$\Psi(x,t) = \frac{1}{\sqrt{2}}\exp\left[i\left(kx - \omega t\right)\right] + \frac{1}{\sqrt{2}}\exp\left[i\left(k'x - \omega' t\right)\right],$$

where E is the energy of the electron of mass m and where

$$k = \frac{\sqrt{2mE}}{\hbar}, \quad \omega = \frac{E}{\hbar},$$

and

$$k' = \frac{\sqrt{2m\left(E + \Delta E\right)}}{\hbar}, \quad \omega' = \frac{E + \Delta E}{\hbar},$$

with $\Delta E << E$.

(a) Is this state time dependent? Briefly explain.

(b) Show that:

$$k' \approx \left(1 + \frac{\Delta E}{2E}\right)k.$$

You may use the approximation:

$$\sqrt{1+\varepsilon} \approx 1 + \frac{1}{2}\varepsilon,$$

which is valid for $\varepsilon << 1$.

(c) Using the approximation obtained in (b), show that:

$$\left|\Psi(x,t)\right|^2 \approx 1 + \cos\left(Kx - \Omega t\right),$$

with

$$K = \frac{\Delta E}{2E}k \quad \text{and} \quad \Omega = \frac{\Delta \mathrm{E}}{\hbar}.$$

(d) On the same diagram sketch the probability distribution function $\left|\Psi(x,t)\right|^2$ given in part (c) versus position x at times:

(i) $t_0 = 0$;

(ii) $t_{1/4} = \frac{1}{4}\frac{2\pi}{\Omega}$;

(ii) $t_{1/2} = \frac{1}{2}\frac{2\pi}{\Omega}$.

(e) Briefly describe how the probability distribution given in part (c) changes in space and time.

(10 marks)

This is the end of your questions