A

MATH1903 — Lecture 4 — Fri 11/8/2017

Applications of Riemann sums

$$\underbrace{\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\,\Delta x_i}_{} = \underbrace{\int_a^b f(x)\,dx}_{}$$

Riemann sum, arises naturally in many contexts

Fund. Thm of Calc. allows this to be evaluated (in principle) by antidifferentiation

pp 53-73 of notes

- volumes of revolution
  - disc method
  - shell method
- length of a curve
- work
- surface areas

subtle connection with length of a curve

<u>Warning</u>: limiting processes are <u>subtle</u>!

<u>Freshman's Dream</u>: $(x+y)^n = x^n + y^n$?? ✗

Special case $n = \frac{1}{2}$:

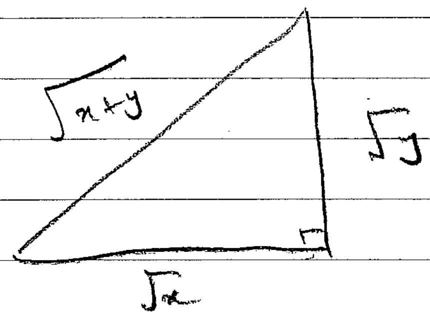
$$\sqrt{x+y} = \sqrt{x} + \sqrt{y} \quad ?? \quad ✗$$

e.g. $\sqrt{13} = \sqrt{9+4} = \sqrt{9} + \sqrt{4} = 3 + 2 = 5$!!

<u>"Proof" using limits</u> (argument puzzled Lebesgue):

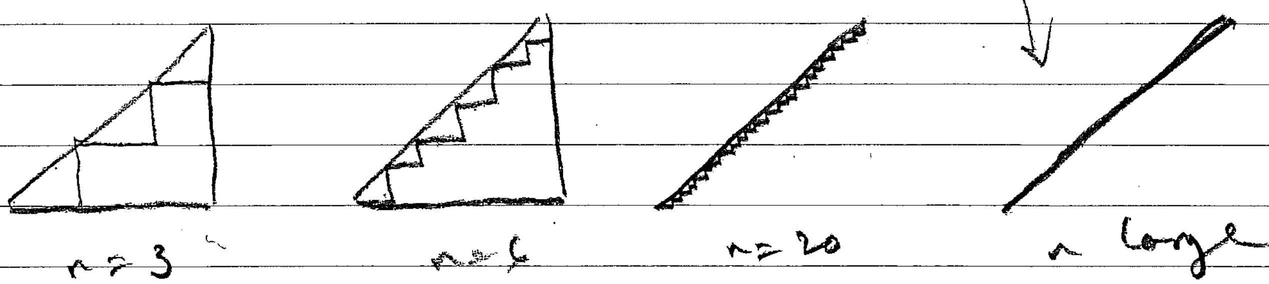
✓✓

correct by Pythagoras



proof by optical illusion

Now approximate the hypotenuse by smaller & smaller "steps":



$$\sqrt{x+y} = \text{length of hypotenuse}$$
$$= \lim_{n\to\infty} (\text{sum of horizontal steps}) + (\text{sum of vertical steps})$$
$$= \sqrt{x} + \sqrt{y} \,. \quad !!$$

(c)

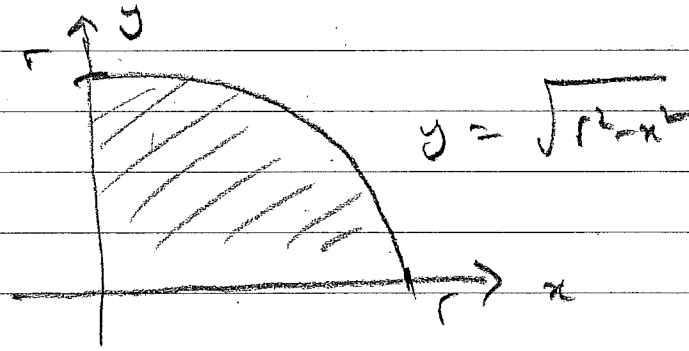
pp 53 – 63 : volumes of revolution

- disc method

- shell method

<u>Example</u> (disc method) :

$x^2+y^2=r^2$



rotate about x-axis to get $\frac{1}{2}$ sphere.

$$V = \int_0^r \pi y^2 \, dx = \pi \int_0^r (r^2-x^2) \, dx$$

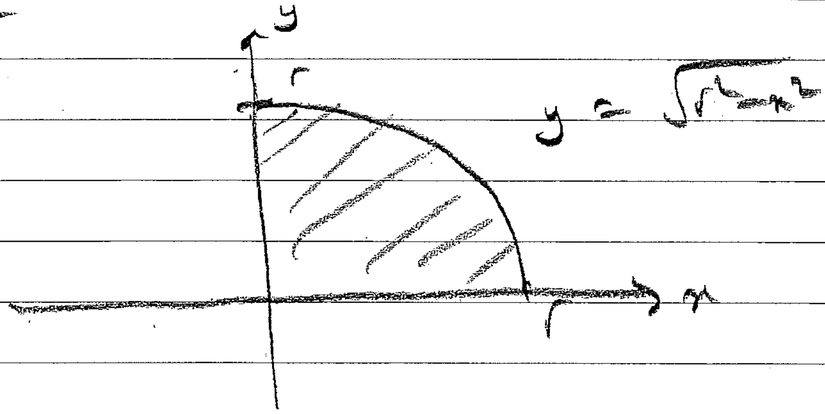
$$= \pi \left[r^2x - \frac{x^3}{3} \right]_0^r$$

$$= \pi \left(r^3 - \frac{r^3}{3} - 0 \right)$$

$$= \frac{2\pi r^3}{3} .$$

Hence $\boxed{\text{volume of sphere} = \frac{4\pi r^3}{3}}$

Example (shell method):



rotate about y-axis to get $\frac{1}{2}$ sphere:

$$V = \int_0^r 2\pi x y \, dx = 2\pi \int_0^r x\sqrt{r^2-x^2}\, dx$$

$$= -\pi \int_{r^2}^{0} u^{1/2}\, du$$

$$= \pi \int_0^{r^2} u^{1/2}\, du$$

$$= \pi \left[\frac{2}{3} u^{3/2}\right]_0^{r^2}$$

$$= \pi \left(\frac{2}{3}\right)\left(r^2\right)^{3/2}$$

$$= \frac{2}{3}\pi r^3$$

Put $u = r^2 - x^2$,

so $\frac{du}{dx} = -2x$,

so $du = -2x\,dx$,

so $x\,dx = -\frac{1}{2}\,du$

as before