The University of Sydney
School of Mathematics and Statistics

Tutorial for Week 12

MATH1903: Integral Calculus and Modelling (Advanced) — Semester 1, 2012

Web Page: http://www.maths.usyd.edu.au/u/UG/JM/MATH1903/
Lecturers: Daniel Daners and James Parkinson

Material covered

(1) Homogeneous linear second order differential equations with constant coefficients.

(2) Inhomogeneous linear second order differential equations with constant coefficients.

Outcomes

After completing this tutorial you should

(1) be confident in solving homogeneous second order homogeneous and inhomogeneous differential equations in various contexts.

Questions to do before the tutorial

1. Find the general solution of each of the following.

(a) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} - 5y = 0.$ (b) $\dfrac{d^2y}{dt^2} + 9y = 0.$

2. Consider the second-order non-homogeneous differential equation $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + y = x^2.$

(a) Find the general solution of the above differential equation.

(b) Find the particular solution of the above differential equation satisfying the initial conditions $y(0) = y'(0) = 4$.

Questions to complete during the tutorial

3. Find the general solution of each of the following.

(a) $\dfrac{d^2x}{dt^2} - 6\dfrac{dx}{dt} + 9x = 0.$ (b) $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 25y = 0.$

4. Solve the following equations, giving the general solution and then the particular solution $y(x)$ satisfying the given boundary or initial conditions.

(a) $y'' + 4y' + 5y = 0, \quad y(0) = 2,\ y'(0) = 4$ (b) $y'' - 2y' + y = 0, \quad y(2) = 0,\ y'(2) = 1$

5. First find the general solution of each of the following non-homogeneous second-order differential equations, and then the particular solution for the given initial conditions.

(a) $y'' + 3y' + 2y = 6e^t, \quad y(0) = 1,\ y'(0) = 0.$ (b) $y'' + 3y' + 2y = 6e^{-t},\ y(0) = 2,\ y'(0) = 1.$

6. (a) For $\omega \neq 5$, find the general solution of the non-homogeneous differential equation,

$$\frac{d^2y}{dt^2} + 25y = 100 \sin \omega t,$$

and the particular solution subject to the initial conditions $y(0) = 0$ and $\dot{y}(0) = 0$.

(b) For $\omega = 5$, find a particular solution of the differential equation. Then determine the particular solution with $y(0) = 0$ and $\dot{y}(0) = 0$.

(c) Find the corresponding particular solution of the differential equation for $\omega = 5$ by fixing t in the result of part (a) and taking the limit as ω approaches its special value.

Extra questions for further practice

7. Find the general solution of the differential equation

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 0,$$

expressing your answer in real form. What is the particular solution satisfying $y(0) = 1$ and $y(\pi/4) = 2$?

8. Solve the following equations, giving the general solution and then the particular solution $y(x)$ satisfying the given boundary or initial conditions.

(a) $2y'' - 7y' + 5y = 0, \quad y(0) = 1,\ y'(0) = 1$

(b) $y'' + 4y' + 3y = 0, \quad y(-2) = 1,\ y(2) = 1$

(c) $2y'' - 2y' + 5y = 0, \quad y(0) = 0,\ y(2) = 2$

(d) $y'' - 4y' + 4y = 0, \quad y(0) = -2,\ y(1) = 0$

9. Find the particular solution of the differential equation $y'' - 6y' + 9y = e^{3x}$ which satisfies the initial conditions $y(0) = 1$ and $y'(0) = 0$.