THE UNIVERSITY OF SYDNEY

PHYS1902 – PHYSICS 1B (ADVANCED)

NOVEMBER 2008

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

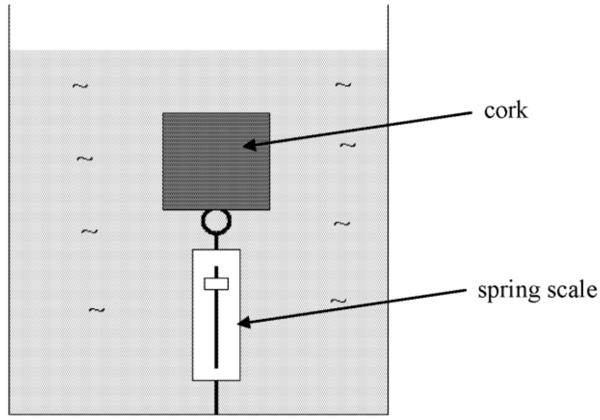
- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

DATA

Density of water	ρ	=	1.00×10^{3} kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^{5} Pa
Magnitude of local gravitational field	g	=	9.81 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^{8} m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg
Rydberg constant	R	=	1.097×10^{7} m^{-1}

SECTION A

Question 1



A large piece of cork weighs 0.285 N in air. When held submerged under water by a spring scale as shown, the scale reads 0.855 N.

(a) Draw a free-body diagram for the cork.

(b) What is the density of the cork?

(c) Now the water is replaced by oil and the scale reads 0.600 N. What is the density of the oil?

(5 marks)

Question 2

A flashlight battery's emf of 1.5 V remains relatively constant over time, but its internal resistance increases substantially with age.

(a) If an (ideal) voltmeter is used to test a very old battery on its own (i.e. not in a circuit), what would be the reading? Explain your answer.

(b) How can you test the 'freshness' of a battery? Describe a testing method and explain why it will reveal if the battery is 'fresh'.

(5 marks)

Question 3

The wires in a household lamp power cord are typically 3.0 mm apart centre to centre, and carry equal currents in opposite directions.

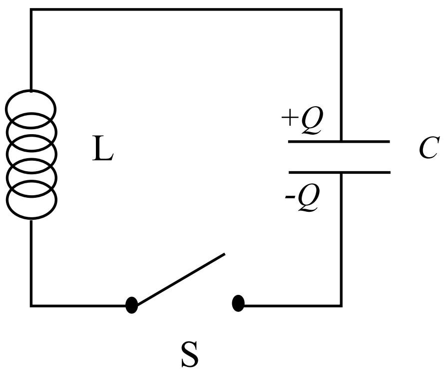
(a) If the cord carries current to a 100 W light bulb connected across a 240 V potential difference, what force per metre does each wire of the cord exert on the other? (Note that for AC power, the power, voltage and force will all be average values.)

(b) Is the force attractive or repulsive?

(c) Is this force large enough that it should be considered in the design of the lamp cord? Explain your answer.

(5 marks)

Question 4



A capacitor C, an inductor L and a switch S are connected in series. The capacitor is initially charged with a charge Q. Assume all connecting wires have zero resistance.

(a) What happens to the charge when the switch is closed? Explain why this occurs.

(b) Sketch a graph of current in the circuit versus time. Label the axes with appropriate values of C, L, and Q.

(c) Explain what happens to the energy initially stored in the capacitor.

(5 marks)

Question 5

Light with frequency ν is incident on a metal with a work function $\phi = 1.0$ eV. Electrons are observed to be ejected from the metal, up to a maximum kinetic energy K_{max} described by

$$K_{max} = h\nu - \phi.$$

(a) What is the maximum kinetic energy of photo-electrons ejected when the light has frequency $\nu = 5.0 \times 10^{14}$ Hz?

(b) What is the maximum kinetic energy of ejected electrons when the metal is illuminated with light with frequency $\nu = 2.0 \times 10^{14}$ Hz?

(c) If the intensity of the incident light is doubled, does the maximum kinetic energy of ejected electrons change? Does the number of ejected electrons change? Explain each of your answers.

(d) Briefly explain how these results are incompatible with classical physics.

(5 marks)

Question 6

A particle is described at time $t = 0$ by the wave packet

$$\psi(x) = \int_{-\infty}^{+\infty} A(k)\exp(i\,k\,x)\,dk$$

with

$$A(k) = \begin{cases} A_0 / k_0 & \text{for } 0 \le k \le k_0 \\ 0 & \text{for other } k \text{ values.} \end{cases}$$

(a) Show that the wave packet satisfies

$$|\psi(x)|^2 = |A_0|^2 \frac{\sin^2\left(k_0 x / 2\right)}{\left(k_0 x / 2\right)^2}.$$

(b) The uncertainty Δx in the position of the particle at time $t = 0$ is defined by the smallest positive value of x for which the function $|\psi(x)|^2$ obtained in (a) is zero. Show that this is given by

$$\Delta x = \frac{2\pi}{k_0}.$$

(c) The uncertainty in the momentum Δp_x of the particle at time $t = 0$ is given by

$$\Delta p_x = \hbar k_0 .$$

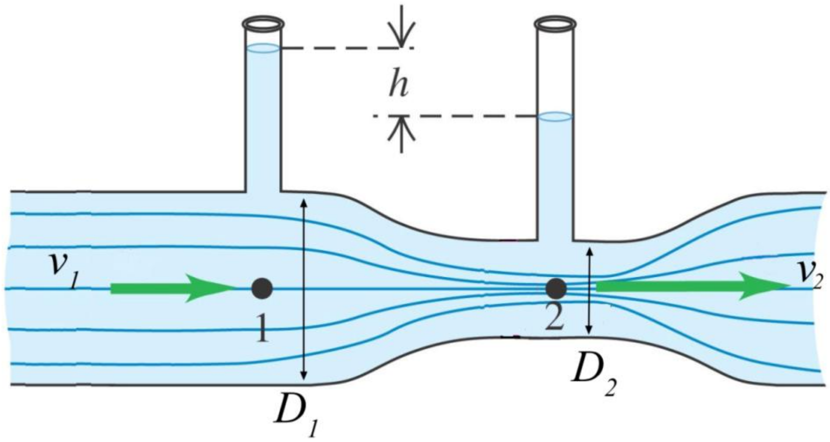
Show that this uncertainty in momentum, together with the uncertainty in position obtained in (b), satisfy the Heisenberg uncertainty principle.

(5 marks)

SECTION B
(Please use a separate booklet for this section)

Question 7

A Venturi meter (used to measure flow rate in pipes) is installed in a horizontal water pipe, as shown in the diagram. The pipe has circular cross-section, with diameter D_1 in the first segment and D_2 in the second segment, with $D_2 < D_1$. The density of water is ρ. The volume flow rate of the water in the pipe is R (measured in $m^3.s^{-1}$).



(a) Express the speed of flow v_1 in the first section of pipe in terms of R and D_1.

(b) What is the speed of flow v_2 in the second section of pipe?

(c) Using the result in (a) find an expression for the pressure difference between point 1 and point 2.

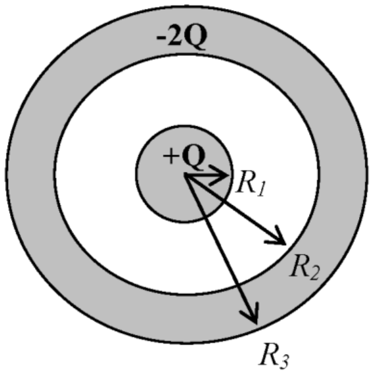
(d) What is the difference h in the water level in the two tubes expressed in terms of R, D_1, and D_2?

(e) Suppose the volume flow rate through the pipe is $1.0\times10^{-3}\,m^3.s^{-1}$ and the pipe has diameter 0.10 m at point 1. Is the flow through the pipe turbulent? Note that the density of water is 1000 $kg.m^{-3}$ and the viscosity of water is 1.0×10^{-3} Pa.s.

(f) What change could you make to the system so that the flow is exactly on the boundary between turbulent and laminar flow?

(10 marks)

Question 8



A conducting sphere of radius R_1 has a charge of $+Q$. It is surrounded by a larger conducting concentric spherical shell of inner radius R_2 and outer radius R_3. The shell has charge $-2Q$.

(a) With the aid of a diagram describe how charge is arranged on the sphere and on the spherical shell.

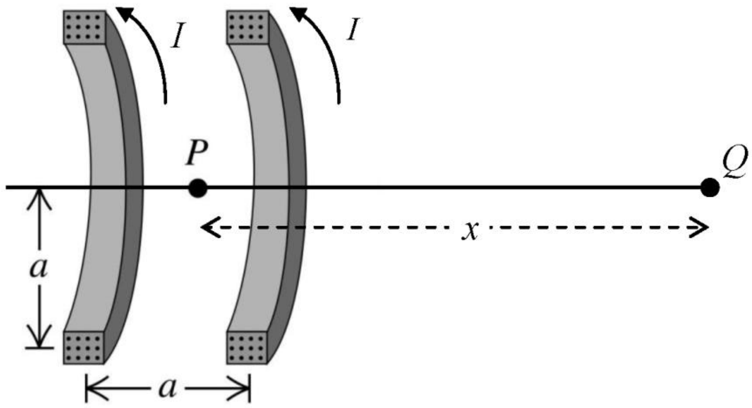
(b) Find expressions for the magnitude of the electric field as a function of the distance r from the centre *and* give its direction:
(i) inside the sphere;
(ii) between the sphere and shell;
(iii) inside the shell;
(iv) outside the shell.
Explain your conclusions.

(c) Plot the *magnitude* of electric field as a function of r. Label the axes on your graph and mark in R_1, R_2 and R_3.

(10 marks)

Question 9

The figure below is a cross-sectional view of two circular coils with radius a, each wound with N turns of wire carrying current I, circulating in the same direction in both coils (as shown in the figure). The coils are separated by a distance a that is equal to their radii. In this configuration, the coils produce a uniform magnetic field in the region between them.



(a) Sketch the magnetic field lines of this configuration.

(b) What is the direction of the magnetic field at point P, which is midway between the coils?

(c) Show that the magnetic field B at a point Q on the axis, a distance x to the right of point P, is given by:

$$B(x) = \frac{\mu_0 N I a^2}{2}\left[\frac{1}{\left[\left(\frac{a}{2}+x\right)^2 + a^2\right]^{3/2}} + \frac{1}{\left[\left(\frac{a}{2}-x\right)^2 + a^2\right]^{3/2}}\right]$$

(d) Obtain an expression for the magnitude of the magnetic field at point P.

(e) What is the value of $\frac{dB}{dx}$ at point P ($x = 0$). Justify your answer with reference to the nature of the function $B(x)$ given above.

(10 marks)

Question 10

The long horizontal straight wire shown in the figures below has constant current *I* directed to the right.

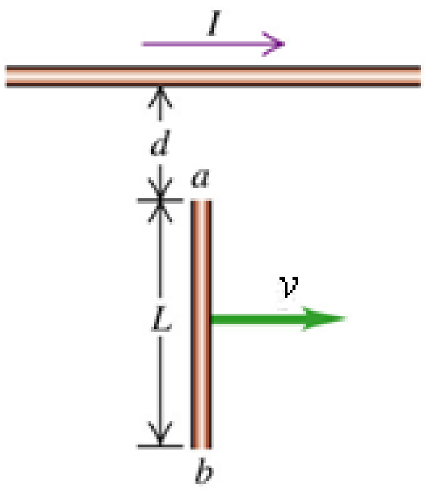


figure (1)

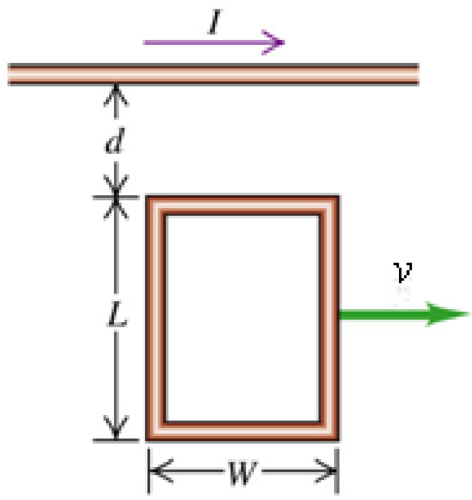


figure (2)

(a) Use Ampère's Law to determine the magnetic field $\vec{B}$ (both magnitude and direction) at a distance *r* below the current carrying wire.

(b) A metal bar with length *L* is moving at a constant velocity *v* to the right in figure (1). Show that the magnitude of the emf, ε, induced in the bar is

$$\varepsilon = \frac{\mu_0 I v}{2\pi} \ln\left(\frac{d+L}{d}\right)$$

(c) Which point, *a* or *b* in figure (1), is at the higher electric potential? Explain your answer.

(d) In figure (2) the wire is replaced by a rectangular wire loop of resistance *R*. What is the magnitude of the current induced in the loop? Explain your answer.

(10 marks)

Question 11

An electron with energy E is incident on a potential step with height $U_0 > E$. The situation may be described by the 1-D time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x),$$

with potential:

$$U(x)=\begin{cases} 0 & \text{for } x<0 \\ U_0 & \text{for } x\geq 0\ . \end{cases}$$

(a) The wave function in the region $x < 0$ is

$$\psi_{\mathrm{I}}(x)=Ae^{ikx}+Be^{-ikx}.$$

Using the Schrödinger equation, determine the value of k.

(b) The wave function in the region $x \geq 0$ is

$$\psi_{\mathrm{II}}(x)=Ce^{-\alpha x}.$$

Using the Schrödinger equation, determine the value of α.

(c) The boundary conditions on the wave function at $x = 0$ are

$$\psi_{\mathrm{I}}(0)=\psi_{\mathrm{II}}(0)$$

and

$$\frac{d\psi_I(0)}{dx}=\frac{d\psi_{II}(0)}{dx}.$$

Applying these boundary conditions, show that

$$A=\frac{1}{2}\frac{k+i\alpha}{k}C$$

and

$$B=\frac{1}{2}\frac{k-i\alpha}{k}C$$

(d) Using the results of (c), show that the wave function may be written

$$\psi(x)=\begin{cases} C\left[\cos kx-\left(\alpha/k\right)\sin kx\right] & \text{for } x<0 \\ Ce^{-\alpha x} & \text{for } x\geq 0\ . \end{cases}$$

(10 marks)

Question 12

The spectrum of a blackbody is described by the Planck formula

$$I_\lambda(\lambda) = \frac{2\pi h c^2}{\lambda^5\left[e^{(hc)/(\lambda kT)} - 1\right]}.$$

(a) On a single diagram, sketch $I_\lambda(\lambda)$ versus λ for two temperatures T_1 and T_2 such that $T_1 < T_2$. State the Wien displacement law and with reference to your diagram explain what it means.

(b) Using $x = (hc)/(\lambda_p kT)$ show that the peak of the spectrum λ_p satisfies the equation

$$5 - x - 5e^{-x} = 0.$$

(c) The solution to the equation in (b) is $x \approx 4.965$. Using this result, derive the value of the constant in the Wien displacement law and determine the peak wavelength corresponding to $T = 6000$ K.

(10 marks)

This is the end of your questions