THE UNIVERSITY OF SYDNEY
MATH1905 Statistics Advanced

| Semester 2 | **Tutorial Week 09 – Solutions** | 2012 |
|---|---|---|

**1.** (d)

**2.** (b)

**3.** Test $H_0 : \mu = 175$ against the alternative $H_1 : \mu > 175$. To use the one sample $t$-test you need to assume that the data is obtained by sampling from a normal population.
To check the assumption obtain the boxplot and draw a lineplot. There is no outlier. A histogram does not tell anything because of the small sample size. Anyway, if the sample size is that small it is challenging to find a violation against the normality assumption unless the points are ridiculously scattered.

```
> summary(x)
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  170.0   173.0   178.5   178.0   183.2   184.0
> sd(x)
[1] 5.503246

> t.test(x, mu=175, alt="greater")

One Sample t-test

data:  x
t = 1.5419, df = 7, p-value = 0.0835
alternative hypothesis: true mean is greater than 175
95 percent confidence interval:
 174.3137      Inf
sample estimates:
mean of x
      178
> 1-pnorm(1.5419)
[1] 0.06154895
```

The 2 $P$-values are close to 0.05. The $t$ value will always be slightly larger than the normal value.
Now for the larger data set:

```
> Z = (174.955 - 175)/7*sqrt(200)
> Z
[1] -0.09091373
```

Here the outcome is clearer as the $P$-value is larger than 1/2.

**4.** Test $H_0 : p_+ = 0.5$ against $H_1 : p_+ \neq 0.5$ Observe 60 successes out of 100 trials. Let $X$ denote the number of successes. The $P$-value is

$$\mathrm{P}(|X - 50| \geq 10) = 0.0569.$$

(On R, pbinom(40,100,0.5) + 1-pbinom(59,100,0.5)=2*pbinom(40,100,0.5) )
The approximating normal is $Y \sim \mathcal{N}(50, 25)$ so the normal approximatyion for the $P$-value is

$$\mathrm{P}(|Z| \geq (10 - 0.5)/\sqrt{25}) = \mathrm{P}(|Z| \geq 1.9) = 0.0574.$$

**5.** (a)

```
> qt(0.99,12)
[1] 2.680998

> qt(0.95, 5)
[1] 2.015048

> qt(0.975,25)
[1] 2.059539
```

(b)

```
> 1-pt(2.5,11)
[1] 0.01475319

> 2*(1-pt(2.2,15))
[1] 0.04389558
```

**6.** Let $p$ be the proportion of '+' signs in the sample differences. We wish to test $H_0 : p_+ = 0.5$ against $H_1 : p_+ \neq 0.5$. Let $X$ denote the number of positive differences. If $H_0$ is true then $X \sim \mathcal{B}(12, 0.5)$ as we ignore the 0 term. We observe 9 + signs. The $P$-value for the test is

$$P = 2 \times \mathrm{P}(X \geq 9) = 0.1460.$$

Thus the data are consistent with $H_0$, i.e. with the scales giving the same measurements.