Let $\boxed{x_1 \quad x_2 \quad x_3 \quad \ldots\ldots \quad x_N}$ be a "population"

SINGLE RANDOM DRAW

X

The distribution of X is given by

x	x_1	x_2	$\ldots\ldots$	x_N
$P(X=x)$	$\frac{1}{N}$	$\frac{1}{N}$	$\ldots\ldots$	$\frac{1}{N}$.

$$E(X) = \sum_x x P(X=x) = \sum_{i=1}^{N} x_i \cdot \frac{1}{N} = \bar{x}$$

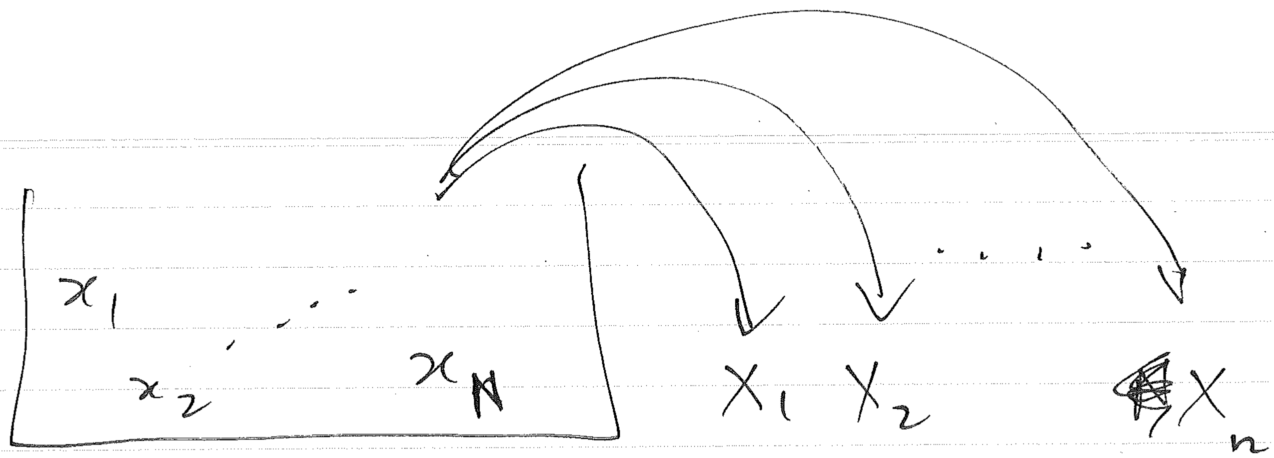
= average of no.s in box.

$$Var(X) = \sum_x (x-\mu)^2 P(X=x) \quad \text{where } \mu = \bar{x} = E(X)$$

$$= E\left[(X-\mu)^2\right]$$

$$= \sum_{i=1}^{N} (x_i - \bar{x})^2 \frac{1}{N} = \frac{1}{N} \sum_{i=1}^{N} (x_i - \bar{x})^2$$

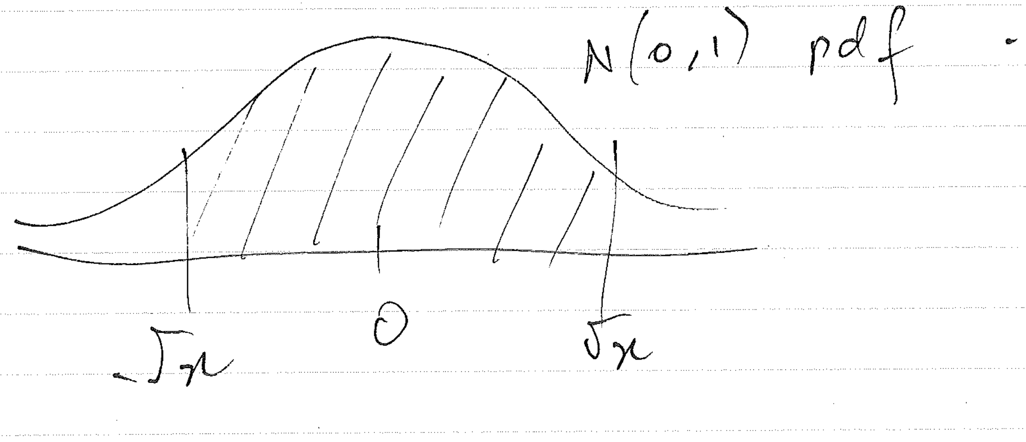
"Population" variance of numbers in the box.

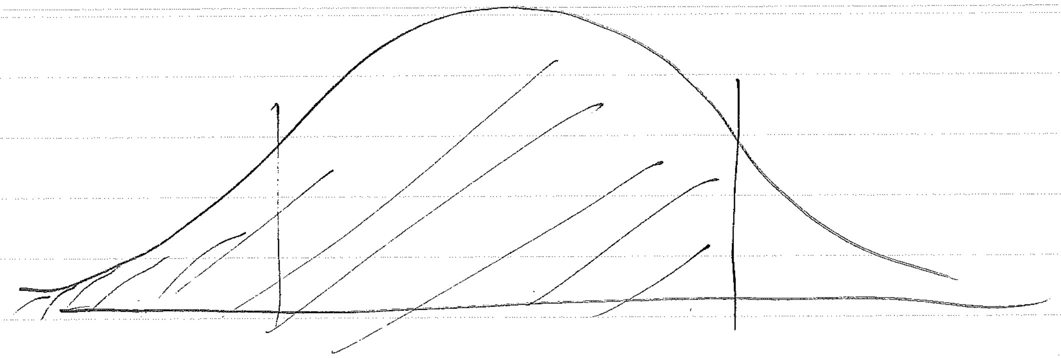


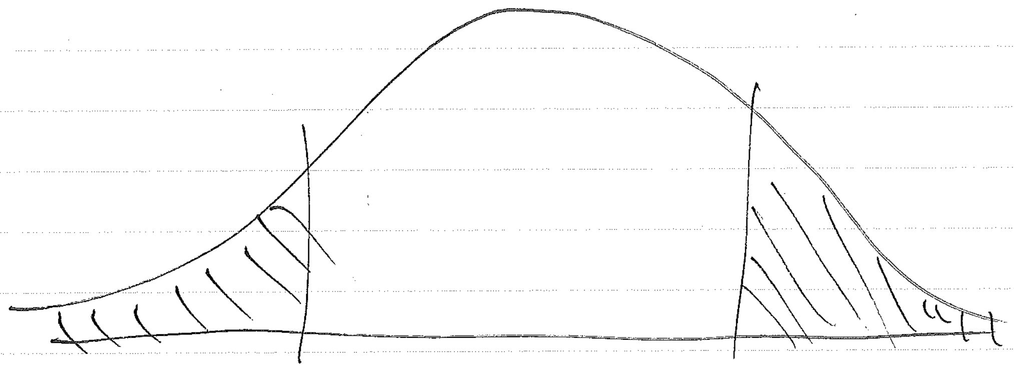
n random draws WITH REPLACEMENT

Then $X_1, X_2, \dots, X_n$ are iid
(independent and identically distributed)
with common expectation $E(X_i) = \bar{x}$
and common variance $Var(X_i) = \frac{1}{N}\sum_{j=1}^{N}(x_j - \bar{x})^2$
the "population variance"

$$\{Z^2 \le x\} = \{-\sqrt{x} \le Z \le \sqrt{x}\}$$







$-\sqrt{x}$ $\qquad\qquad$ $\sqrt{x}$

$$\sum_{i=1}^{n}\left(Z_i-\bar{Z}\right)^2 = \sum_{i=1}^{n}\left(Z_i^2\right) - \frac{\left(\sum_{i=1}^{n} Z_i\right)^2}{n}$$

$\boxed{\text{Fix } \frac{1}{n} \text{ on slide 347}}$

$$\left(1+\frac{t^2}{n-1}\right)^{-n/2} = \underbrace{\left(1+\frac{2}{n-1}\frac{t^2}{2}\right)^{-\frac{(n-1)}{2}}}_{\to e^{-\frac{t^2}{2}}} \left(1+\frac{t^2}{n-1}\right)^{-\frac{1}{2}}$$

$$\left(1+\frac{x}{N}\right)^{N} \to e^{x}$$