Answers to Exercises

Chapter One

Exercise **1A** (Page 2)

1(a) $8x+4y$ **(b)** $-3a^2+4a$ **(c)** $-5x^2-12x-3$
(d) $9a-3b-5c$
2(a) $6x$ **(b)** $20a$ **(c)** $5ab+bc$
(d) $2x^3-5x^2y+2xy^2+3y^3$
3(a) $2x$ **(b)** $4x$ **(c)** $-6a$ **(d)** $-4b$
4(a) $2x^2-2x+4$ **(b)** $3a-5b-4c$
(c) $-3a+2b-2c+2d$ **(d)** $2ab-2bc+2cd$
5(a) $2x^2-2x$ **(b)** $6x^2y+2y^3$ **(c)** a^3-c^3-abc
(d) $4x^4-5x^3-2x^2-x+2$
6(a) $10a$ **(b)** $-18x$ **(c)** $-3a^2$ **(d)** $6a^3b$ **(e)** $-8x^5$
(f) $-6p^3q^4$
7(a) $6a^5b^6$ **(b)** $-24a^4b^8$ **(c)** $9a^6$ **(d)** $-8a^{12}b^3$
8(a) 18 **(b)** 2
9(a) 59 **(b)** 40
10(a) 5 **(b)** $-7x^2$ **(c)** $12a$ **(d)** $-3x^3y^4z$
11(a) -2 **(b)** $3x$ **(c)** xy **(d)** $-a^4$ **(e)** $-7ab^3$
(f) $5ab^2c^6$
12(a) $3a^2$ **(b)** $5c^4$ **(c)** a^2bc^6
13(a) $2x^5$ **(b)** $9xy^5$ **(c)** b^4 **(d)** $2a^3$
14 $-x^3+3x^2+7x-8$
15 $-b+11c$
16 $8d-14c-2b$
17 $-18x^{25}y^{22}$
18(a) $0\le x\le 2$ **(b)** $x\le -\sqrt{3}$ or $0\le x\le \sqrt{3}$

Exercise **1B** (Page 4)

1(a) $4a+8b$ **(b)** x^2-7x **(c)** $-3x+6y$ **(d)** $-a^2-4a$
(e) $5a+15b-10c$ **(f)** $-6x+9y-15z$
(g) $-2x^4+4x^3+6x^2-2x$ **(h)** $6x^3y^2-15x^4y$
(i) $-2a^4b^4+4a^5b^2$
2(a) $x+4$ **(b)** $-8a-3b+5c$
(c) $-x^5-25x^4+10x^3+13x^2-6x$
(d) $-12x^5y^5+14x^4y^6-13x^3y^8$
3(a) x^2+5x+6 **(b)** $2a^2+13a+15$ **(c)** x^2-2x-8
(d) $2b^2-13b+21$ **(e)** $12x^2+17x-40$
(f) $30-71x+42x^2$
5(a) $x^2-2xy+y^2$ **(b)** a^2+6a+9 **(c)** $n^2-10n+25$
(d) c^2-4 **(e)** $4a^2+4a+1$ **(f)** $9p^2-12p+4$
(g) $9x^2-16y^2$ **(h)** $16y^2-40xy+25x^2$
6(a) a^2-4b^2 **(b)** $10-17x-20x^2$ **(c)** $16x^2+56x+49$
(d) $x^4-x^2y-12y^2$ **(e)** a^2-ac-b^2+bc **(f)** $27x^3+1$
7(a) $t^2+2+\frac{1}{t^2}$ **(b)** $t^2-2+\frac{1}{t^2}$ **(c)** $t^2-\frac{1}{t^2}$
8(a) $a^2-b^2-c^2+2bc$ **(b)** x^2-2x+3
9(a) $11x-3$ **(b)** $-4b+8c-8$
10(a) $10\,404$ **(b)** $998\,001$ **(c)** $39\,991$
11(a) $2ab-b^2$ **(b)** $2x+3$ **(c)** $18-6a$ **(d)** $4pq$
(e) x^2+2x-1 **(f)** a^2-2a-6
12 $7x^2+16ax+4a^2$
13(a) $x^3-6x^2+12x-8$ **(b)** $x^2+y^2+z^2$
(c) $x^2-y^2-z^2+2yz$ **(d)** $a^3+b^3+c^3-3abc$

Exercise **1C** (Page 6)

1(a) $a(x-y)$ **(b)** $x(x+3)$ **(c)** $3a(a-2b)$
(d) $6x(2x+3)$ **(e)** $2a^3(3+a+2a^2)$
(f) $7xy(x^2-2xy+3y)$
2(a) $(x-y)(a+b)$ **(b)** $(a+b)(a+c)$ **(c)** $(x-3)(x-y)$
(d) $(2a-b)(x-y)$ **(e)** $(b+c)(a-1)$
(f) $(x-3)(2x^2-a)$
3(a) $(x-3)(x+3)$ **(b)** $(1-a)(1+a)$
(c) $(2x-y)(2x+y)$ **(d)** $(5x-4)(5x+4)$
(e) $(1-7k)(1+7k)$ **(f)** $(9ab-8)(9ab+8)$
4(a) $(x+3)(x+5)$ **(b)** $(x-1)(x-3)$ **(c)** $(a+4)(a-2)$
(d) $(y-7)(y+4)$ **(e)** $(c-3)(c-9)$ **(f)** $(p+12)(p-3)$
(g) $(u-20)(u+4)$ **(h)** $(x-3)(x-17)$
(i) $(t+25)(t-2)$ **(j)** $(x-15)(x+6)$
(k) $(x-2y)(x-3y)$ **(l)** $(x+2y)(x+4y)$
(m) $(a-3b)(a+2b)$ **(n)** $(p+8q)(p-5q)$
(o) $(c-11d)(c-13d)$
5(a) $(2x+1)(x+2)$ **(b)** $(3x+2)(x+2)$
(c) $(3x-1)(2x-3)$ **(d)** $(3x-1)(x+5)$

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(e) $(3x-4)(3x+2)$ (f) $(2x-3)(3x+1)$
(g) $(3x-1)(2x-1)$ (h) $(3x-5)(x+6)$
(i) $(3x-4)(4x+3)$ (j) $(12x-5)(x+3)$
(k) $(4x-5)(6x-5)$ (l) $(2x-y)(x+y)$
(m) $(2a-b)(2a-3b)$ (n) $(3p+4q)(2p-q)$
(o) $(9u+4v)(2u-3v)$
6(a) $3(a-2)(a+2)$ (b) $(x-y)(x+y)(x^2+y^2)$
(c) $x(x-1)(x+1)$ (d) $5(x+2)(x-3)$
(e) $y(5-y)(5+y)$ (f) $(2-a)(2+a)(4+a^2)$
(g) $2(2x-3)(x+5)$ (h) $x(x-1)(x-7)$
(i) $(x-2)(x+2)(x^2+1)$ (j) $(x-1)(x+1)(a-2)$
(k) $m(4m-n)(4m+n)$ (l) $a(x-5a)(x+4a)$
7(a) $(9-x)(8+x)$ (b) $(a-b-c)(a-b+c)$
(c) $a(a-4b)(a-6b)$ (d) $(a-b)(a+b-1)$
(e) $(x-4)(x+4)(x^2+16)$
(f) $(2p-q-r)(2p+q+r)$ (g) $x^2(3x-2)(2x+1)$
(h) $(c+1)(a^2-b)$ (i) $9(x+5)(x-1)$
(j) $(2x-1)(2x+1)(x-3)(x+3)$
(k) $(xy-16)(xy+3)$ (l) $x(x-y-z)(x-y+z)$
(m) $(4-5x)(5+4x)$ (n) $(2x-1)(2x+1)(x-3)$
(o) $(2x-3y)(6x+5y)$ (p) $(x+a-b)(x+a+b)$
(q) $9(x-7)(x+5)$ (r) $(x^2-x-1)(x^2+x+1)$
(s) $x(5x-9y)(2x+y)$
(t) $(x+2y-a+b)(x+2y+a-b)$ (u) $4xy$
8(a) $(a+b)(a+b^2)$ (b) $(a+c)(b-d)$ (c) $4ab(a-b)^2$
(d) $(2x^2+3y^2)(2x-3y)(x+y)$
(e) $(x-y)(x+y)^3$
(f) $(a-b-c)(a+b+c)(a-b+c)(a+b-c)$
(g) $(x^2+y^2)(a^2+b^2+c^2)$
(h) $(x+ay)(x-by)$ (i) $(a^2-ab+b^2)(a^2+ab+b^2)$
(j) $(a^2-2ab+2b^2)(a^2+2ab+2b^2)$

Exercise 1D (Page 8)

1(a) $\frac{1}{2}$ (b) $\frac{1}{a}$ (c) $\frac{x}{3y}$ (d) $\frac{3}{a}$ (e) $\frac{4y}{5xz}$ (f) $\frac{w}{u^2v}$
2(a) 1 (b) $\frac{1}{2}$ (c) $\frac{3}{x}$ (d) $\frac{b}{2}$ (e) $\frac{3x}{2y}$ (f) 1 (g) $\frac{1}{2a}$
(h) $\frac{2c}{3b}$ (i) $6a^2$ (j) $\frac{5c}{2a}$ (k) $\frac{x^2}{yz}$ (l) $\frac{1}{2a^2}$
3(a) $\frac{7x}{10}$ (b) $\frac{a}{6}$ (c) $\frac{3x-2y}{24}$ (d) $\frac{13a}{6}$ (e) $\frac{b}{15}$ (f) $-\frac{xy}{45}$
(g) $\frac{3}{2x}$ (h) $\frac{25}{12x}$ (i) $\frac{b-a}{ab}$ (j) $\frac{x^2+1}{x}$ (k) $\frac{a^2+b}{a}$ (l) $\frac{x-2}{2x^2}$
4(a) $\frac{5x+7}{6}$ (b) $\frac{-x-17}{10}$ (c) $\frac{9x+26}{12}$ (d) $\frac{12x+3}{5}$
(e) $\frac{2x-16}{15x}$ (f) $\frac{1}{x(x+1)}$ (g) $\frac{2x}{x^2-1}$ (h) $\frac{5x-13}{(x-2)(x-3)}$
(i) $\frac{-10}{(x+3)(x-2)}$ (j) $\frac{x^2+y^2}{x^2-y^2}$ (k) $\frac{ax-bx}{(x+a)(x+b)}$ (l) $\frac{2x}{x^2-1}$
5(a) $\frac{1}{x+y}$ (b) $\frac{3}{2b}$ (c) $\frac{x}{x-2}$ (d) $\frac{a+3}{a+4}$ (e) $\dfrac{x+y}{x-y}$
(f) $\frac{x+5}{x+4}$ (g) $\frac{c+d}{a}$ (h) $\frac{y-5}{2y+1}$ (i) $\frac{3a+2b}{3x+2y}$
6(a) $\frac{3x}{2(x-1)}$ (b) a (c) $\frac{c+2}{c+4}$ (d) x (e) $\frac{3x-1}{a+b}$ (f) $\frac{x-7}{3(x+3)}$
7(a) $\frac{2}{x^2-1}$ (b) $\frac{2x}{(x-2)^2(x+2)}$ (c) $\frac{3x}{x^2-y^2}$
(d) $\frac{x+1}{(x-2)(x+3)(x+4)}$ (e) $\frac{bx}{a(a-b)(a+b)}$
(f) $\frac{x}{(x-1)(x-2)(x-3)}$
8(a) -1 (b) $-u-v$ (c) $3-x$ (d) $\frac{2}{a-b}$ (e) 1
(f) $\frac{-1}{2x+y}$
9(a) $\frac{1}{3}$ (b) $\frac{7}{13}$ (c) $\frac{3}{11}$ (d) $\frac{1}{5}$ (e) $\frac{1}{x+2}$ (f) $\frac{t^2-1}{t^2+1}$
(g) $\frac{ab}{a+b}$ (h) $\frac{x^2+y^2}{x^2-y^2}$ (i) $\frac{x^2}{2x+1}$ (j) $\frac{x-1}{x-3}$
11(a) $x+y$ (b) $\frac{2x+3}{3x-1}$ (c) $\frac{a-b+c}{ab}$ (d) $\frac{x^4-y^4}{x^2y^2}$ (e) $\frac{16x}{x^2-16}$
(f) $\frac{4}{x+2y}$ (g) $\frac{-13x}{(x+1)(x+2)(x+3)}$ (h) $\frac{2}{(x+1)^2(x-1)}$
12(a) $x^2+2+\frac{1}{x^2}$ (b) 7
13(a) 0 (b) 3 (c) $\frac{3n-m}{2}$ (d) $\frac{1}{x}$

Exercise 1E (Page 11)

3(a) $a^3+3a^2b+3ab^2+b^3$ (b) $x^3-3x^2y+3xy^2-y^3$
(c) b^3-3b^2+3b-1 (d) $p^3+6p^2+12p+8$
(e) $1-3c+3c^2-c^3$ (f) $t^3-9t^2+27t-27$
(g) $8x^3+60x^2y+150xy^2+125y^3$
(h) $27a^3-108a^2b+144ab^2-64b^3$
4(a) $(x+y)(x^2-xy+y^2)$ (b) $(a-b)(a^2+ab+b^2)$
(c) $(y+1)(y^2-y+1)$ (d) $(g-1)(g^2+g+1)$
(e) $(b-2)(b^2+2b+4)$ (f) $(2c+1)(4c^2-2c+1)$
(g) $(3-t)(9+3t+t^2)$ (h) $(5+a)(25-5a+a^2)$
(i) $(3h-1)(9h^2+3h+1)$
(j) $(u-4v)(u^2+4uv+16v^2)$
(k) $(abc+10)(a^2b^2c^2-10abc+100)$
(l) $(6x+5y)(36x^2-30xy+25y^2)$
5(a) $2(x+2)(x^2-2x+4)$ (b) $a(a-b)(a^2+ab+b^2)$
(c) $3(2t+3)(4t^2-6t+9)$
(d) $y(x-5)(x^2+5x+25)$
(e) $2(5p-6q)(25p^2+30pq+36q^2)$
(f) $x(3x+10y)(9x^2-30xy+100y^2)$
(g) $5(xy-1)(x^2y^2+xy+1)$
(h) $x^3(x+y)(x^2-xy+y^2)$
6(a) $\frac{x^2+x+1}{x+1}$ (b) $\frac{a-5}{a^2-2a+4}$ (c) $\frac{a^2-a+1}{2a^2}$ (d) $\frac{1}{x}$
7(a) $\frac{12a+12}{a^3-8}$ (b) $\frac{x^2}{x^3-1}$ (c) $\frac{x^2-3x+8}{(x-4)(x+2)(x^2-2x+4)}$
(d) $\frac{3a^2-ab}{a^3+b^3}$
8(a) $(a+b)(a^2-ab+b^2+1)$
(b) $(x-2)(x+2)(x^2-2x+4)(x^2+2x+4)$
(c) $(2a-3)(a+2)(a^2-2a+4)$ (d) $2y(3x^2+y^2)$
(e) $(s-t)(s^2+st+t^2+s+t)$ (f) $2t(t^2+12)$
(g) $9(a-b)(a^2-ab+b^2)$
(h) $(x-2)(x+1)(x^2+2x+4)(x^2-x+1)$
(i) $(u+1)(u^2+1)(u^4-u^2+1)$
(j) $(1-x)(1+x+x^2)(2+3x^3)$
(k) $(x-1)(x+1)(x+2)(x^2+1)(x^2-2x+4)$
(l) $(a+1)(a^2+a+1)(a^2-a+1)$
9(a) $\frac{6}{a-3}$ (b) 1 (c) $\frac{2}{a}$ (d) $\frac{x}{x^3-27}$ (e) $\frac{3}{x^3-1}$ (f) $\frac{1}{(1-x)^2}$

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10 $(A+B)^4 = A^4+4A^3B+6A^2B^2+4AB^3+B^4$, $(A-B)^4 = A^4-4A^3B+6A^2B^2-4AB^3+B^4$, $A^4+B^4=(A^2-\sqrt{2}AB+B^2)(A^2+\sqrt{2}AB+B^2)$, $A^4-B^4=(A-B)(A+B)(A^2+B^2)$

11(a) $x(x^2+1)(x^2-\sqrt{3}x+1)(x^2+\sqrt{3}x+1)$

(b) $(x-y)(x+y)(x^2+y^2)(x^2+xy+y^2)$ $(x^2-xy+y^2)(x^2-\sqrt{3}xy+y^2)(x^2+\sqrt{3}xy+y^2)$

12 13

13 $8x^3$

15 $a+b$

16 $1+a-a^3$

Exercise 1F (Page 13)

1(a) $x=10$ **(b)** $x>\frac{2}{3}$ **(c)** $a=-5$ **(d)** $x\ge 4$ **(e)** $x=-1$ **(f)** $y=50$ **(g)** $t<0$ **(h)** $x=-16$

2(a) $x=9$ **(b)** $x\ge -5$ **(c)** $x>-4$ **(d)** $x=-7$ **(e)** $a\ge -\frac{7}{5}$ **(f)** $t<-30$ **(g)** $y=-\frac{16}{7}$ **(h)** $u\le 48$

3(a) $x<4$ **(b)** $x=-11$ **(c)** $a>-\frac{1}{2}$ **(d)** $y\ge 2$ **(e)** $x\le \frac{7}{9}$ **(f)** $x=-\frac{3}{5}$ **(g)** $x<\frac{23}{6}$ **(h)** $x=-\frac{2}{5}$

(i) There are no solutions.

(j) All real numbers are solutions. **(k)** $x\le\frac{19}{6}$

(l) $x=\frac{3}{14}$ **(m)** $x>-1$ **(n)** $x=\frac{17}{6}$

4(a) $x=4$ **(b)** $a=8$ **(c)** $y<16$ **(d)** $x=\frac{1}{3}$ **(e)** $a=\frac{2}{5}$ **(f)** $y=\frac{3}{2}$ **(g)** $x\ge -8$ **(h)** $a\ge 1\frac{7}{8}$ **(i)** $x>\frac{1}{4}$ **(j)** $a=-\frac{5}{4}$ **(k)** $t=\frac{3}{5}$ **(l)** $c<\frac{9}{2}$ **(m)** $a=-1$ **(n)** $x=\frac{1}{5}$ **(o)** $x=\frac{7}{17}$ **(p)** $t=-\frac{26}{27}$

5(a) $x\ge 15$ **(b)** $a>-15$ **(c)** $x=\frac{9}{2}$ **(d)** $x=\frac{1}{6}$ **(e)** $x=\frac{1}{2}$ **(f)** $x>6$ **(g)** $x>20$ **(h)** $x=-\frac{23}{2}$ **(i)** $x=-\frac{7}{3}$ **(j)** $x=\frac{5}{6}$ **(k)** $a>-11$ **(l)** $x\le 2$ **(m)** $x>\frac{34}{57}$ **(n)** $x=-\frac{7}{3}$ **(o)** $x=-\frac{5}{2}$ **(p)** $x\le -\frac{43}{69}$

6(a) $a=3$ **(b)** $s=16$ **(c)** $v=\frac{2}{3}$ **(d)** $\ell=21$ **(e)** $C=35$ **(f)** $c=-\frac{14}{5}$

7(a) $x=1,\,2,\,3$ **(b)** $x=-5,\,-4,\,-3,\,-2,\,-1$ **(c)** $x>-4$ **(d)** $x\le 2$ **(e)** $x=2,\,3,\,4,\,5,\,6$ **(f)** $x=-3,\,-2,\,-1,\,0,\,1,\,2$ **(g)** $0<x\le 5$ **(h)** $1\le x\le 4$

8(a) -4 **(b)** 7 **(c)** 36 **(d)** 80 litres **(e)** 15 min **(f)** 16 **(g)** 30 km/h **(h)** 5 hours

9(a) $b=\frac{a+d}{c}$ **(b)** $n=\frac{t-a+d}{d}$ **(c)** $r=\frac{p-qt}{t}$ **(d)** $v=\frac{3}{u-1}$ **(e)** $a=-\frac{2b}{3}$ **(f)** $g=\frac{2fh}{5f-h}$ **(g)** $y=\frac{2x}{1-x}$ **(h)** $b=\frac{4a+5}{a-1}$ **(i)** $d=\frac{5c-7}{3c+2}$ **(j)** $v=\frac{1+u-w-uw}{1-u}$

10(a) $x=\frac{14}{5}$ **(b)** $a=4$

11(b) $x=6$

Exercise 1G (Page 16)

1(a) $x=3$ or -3 **(b)** $a=2$ or -2 **(c)** $t=1$ or -1 **(d)** $x=\frac{3}{2}$ or $-\frac{3}{2}$ **(e)** $x=\frac{1}{2}$ or $-\frac{1}{2}$ **(f)** $y=\frac{4}{5}$ or $-\frac{4}{5}$

2(a) $x=0$ or 5 **(b)** $c=0$ or -2 **(c)** $t=0$ or 1 **(d)** $a=0$ or 3 **(e)** $b=0$ or $\frac{1}{2}$ **(f)** $u=0$ or $-\frac{1}{3}$ **(g)** $y=0$ or $\frac{2}{3}$ **(h)** $u=0$ or $-\frac{12}{5}$

3(a) $x=1$ or 2 **(b)** $x=-4$ or -2 **(c)** $a=-5$ or 3 **(d)** $y=-5$ or 1 **(e)** $p=-2$ or 3 **(f)** $a=-11$ or 12 **(g)** $c=3$ or 6 **(h)** $t=-2$ or 10 **(i)** $u=-8$ or 7 **(j)** $h=-25$ or -2 **(k)** $k=-4$ or 15 **(l)** $\alpha=-22$ or 2

4(a) $a=\frac{1}{3}$ or 2 **(b)** $x=-5$ or $-\frac{1}{2}$ **(c)** $b=-\frac{2}{3}$ or 2 **(d)** $y=-4$ or $\frac{3}{2}$ **(e)** $x=\frac{1}{5}$ or 5 **(f)** $t=\frac{3}{4}$ or 3 **(g)** $t=-\frac{5}{2}$ or 3 **(h)** $u=-\frac{4}{5}$ or $\frac{1}{2}$ **(i)** $x=\frac{3}{5}$ **(j)** $x=-\frac{2}{3}$ or $-\frac{3}{2}$ **(k)** $b=-\frac{3}{2}$ or $-\frac{1}{6}$ **(l)** $k=-\frac{8}{3}$ or $\frac{1}{2}$

5(a) $x=\frac{1}{2}(1+\sqrt{5})\doteqdot 1{\cdot}618$ or $x=\frac{1}{2}(1-\sqrt{5})\doteqdot -0{\cdot}6180$ **(b)** $x=\frac{1}{2}(-1+\sqrt{13})\doteqdot 1{\cdot}303$ or $x=\frac{1}{2}(-1-\sqrt{13})\doteqdot -2{\cdot}303$ **(c)** $a=3$ or 4 **(d)** $u=-1+\sqrt{3}\doteqdot 0{\cdot}7321$ or $u=-1-\sqrt{3}\doteqdot -2{\cdot}732$ **(e)** $c=3+\sqrt{7}\doteqdot 5{\cdot}646$ or $c=3-\sqrt{7}\doteqdot 0{\cdot}3542$ **(f)** $x=-\frac{1}{2}$ **(g)** $a=\frac{1}{2}(2+\sqrt{2})\doteqdot 1{\cdot}707$ or $a=\frac{1}{2}(2-\sqrt{2})\doteqdot 0{\cdot}2929$ **(h)** $x=-3$ or $\frac{2}{5}$ **(i)** $b=\frac{1}{4}(-3+\sqrt{17})\doteqdot 0{\cdot}2808$ or $b=\frac{1}{4}(-3-\sqrt{17})\doteqdot -1{\cdot}781$ **(j)** $c=\frac{1}{3}(2+\sqrt{13})\doteqdot 1{\cdot}869$ or $c=\frac{1}{3}(2-\sqrt{13})\doteqdot -0{\cdot}5352$ **(k)** $t=\frac{1}{4}(1+\sqrt{5})\doteqdot 0{\cdot}8090$ or $t=\frac{1}{4}(1-\sqrt{5})\doteqdot -0{\cdot}3090$ **(l)** no solutions

6(a) $x=-1$ or 2 **(b)** $a=2$ or 5 **(c)** $y=\frac{1}{2}$ or 4 **(d)** $b=-\frac{2}{5}$ or $\frac{2}{3}$ **(e)** $k=-1$ or 3 **(f)** $u=\frac{4}{3}$ or 4

7(a) $x=1+\sqrt{2}$ or $x=1-\sqrt{2}$ **(b)** $a=2+\sqrt{3}$ or $a=2-\sqrt{3}$ **(c)** $a=1+\sqrt{5}$ or $a=1-\sqrt{5}$ **(d)** $m=\frac{1}{5}(2+\sqrt{14})$ or $m=\frac{1}{5}(2-\sqrt{14})$ **(e)** $y=1+\sqrt{6}$ or $y=1-\sqrt{6}$ **(f)** $k=\frac{1}{4}(-5+\sqrt{73})$ or $k=\frac{1}{4}(-5-\sqrt{73})$

8(a) $p=\frac{1}{2}$ or 1 **(b)** $x=-3$ or 5 **(c)** $n=5$

9(a) $a=2b$ or $a=3b$ **(b)** $a=-2b$ or $a=\frac{b}{3}$

10(a) $y=2x$ or $y=-2x$ **(b)** $y=\frac{x}{11}$ or $y=-\frac{x}{2}$

11(a) $x=15$ **(b)** 7 **(c)** 6 and 9 **(d)** 4 cm **(e)** $\frac{2}{5}$ or $\frac{-9}{-6}$ **(f)** 3 cm **(g)** 2 hours, 4 hours **(h)** 55 km/h and 60 km/h

12(a) $a=-\frac{7}{3}$ or 3 **(b)** $k=-4$ or 15

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(c) $t = 2\sqrt{3}$ or $-\sqrt{3}$
(d) $m = \frac{1}{3}(1+\sqrt{2}\,)$ or $m = \frac{1}{3}(1-\sqrt{2}\,)$
13(a) $x = 2c$ or $x = \frac{11c}{14}$ **(b)** $x = a$ or $x = \frac{ab}{a-2b}$

Exercise 1H (Page 20)

1(a) $x = 2,\ y = 4$ **(b)** $x = -1,\ y = 3$
(c) $x = 2,\ y = 2$ **(d)** $x = 9,\ y = 1$
(e) $x = 3,\ y = 4$ **(f)** $x = 4,\ y = -1$
(g) $x = 5,\ y = 3\frac{3}{5}$ **(h)** $x = 13,\ y = 7$
2(a) $x = -1,\ y = 3$ **(b)** $x = 5,\ y = 2$
(c) $x = -4,\ y = 3$ **(d)** $x = 2,\ y = -6$
(e) $x = 1,\ y = 2$ **(f)** $x = 16,\ y = -24$
(g) $x = 1,\ y = 6$ **(h)** $x = 5,\ y = -2$
(i) $x = 5,\ y = 6$ **(j)** $x = 7,\ y = 5$
(k) $x = \frac{1}{2},\ y = \frac{3}{2}$ **(l)** $x = 5,\ y = 8$
3(a) $x = 1,\ y = 1$ or $x = -2,\ y = 4$
(b) $x = 2,\ y = 1$ or $x = 4,\ y = 5$
(c) $x = 0,\ y = 0$ or $x = 1,\ y = 3$
(d) $x = -2,\ y = -7$ or $x = 3,\ y = -2$
(e) $x = -3,\ y = -5$ or $x = 5,\ y = 3$
(f) $x = 1,\ y = 6$ or $x = 2,\ y = 3$
(g) $x = 5,\ y = 3$ or $x = 5,\ y = -3$ or $x = -5,\ y = 3$ or $x = -5,\ y = -3$
(h) $x = 9,\ y = 6$ or $x = 9,\ y = -6$ or $x = -9,\ y = 6$ or $x = -9,\ y = -6$
4(a) Each apple cost 40 cents, each orange cost 60 cents. **(b)** 44 adults, 22 children
(c) The man is 36, the son is 12.
(d) 189 for, 168 against
(e) $\frac{2}{15}$ **(f)** 9 \$20 notes, 14 \$10 notes
(g) 5 km/h, 3 km/h **(h)** 72
5(a) $x = 12,\ y = 20$ **(b)** $x = 3,\ y = 2$
6(a) $x = 6,\ y = 3,\ z = 1$ **(b)** $x = 2,\ y = -1,\ z = 3$
(c) $a = 3,\ b = -2,\ c = 2$ **(d)** $p = -1,\ q = 2,\ r = 5$
(e) $x = 5,\ y = -3,\ z = -4$
(f) $u = -2,\ v = 6,\ w = 1$
7(a) $x = 5,\ y = 10$ or $x = 10,\ y = 5$
(b) $x = -8,\ y = -11$ or $x = 11,\ y = 8$
(c) $x = \frac{1}{2},\ y = 4$ or $x = 2,\ y = 1$
(d) $x = 4,\ y = 5$ or $x = 5,\ y = 4$
(e) $x = 1,\ y = 2$ or $x = \frac{3}{2},\ y = \frac{7}{4}$
(f) $x = 2,\ y = 5$ or $x = \frac{10}{3},\ y = 3$
8(a) $x = 1,\ y = \frac{5}{4}$
(b) $x = 2,\ y = 4$ or $x = -2,\ y = -4$ or $x = \frac{4}{3},\ y = 6$ or $x = -\frac{4}{3},\ y = -6$
9(b) $x = 1,\ y = -2$ or $x = -1,\ y = 2$ or $x = \frac{7}{3},\ y = \frac{2}{3}$ or $x = -\frac{7}{3},\ y = -\frac{2}{3}$

Exercise 1I (Page 22)

1(a) 1 **(b)** 9 **(c)** 25 **(d)** 81 **(e)** $\frac{9}{4}$ **(f)** $\frac{1}{4}$ **(g)** $\frac{25}{4}$
(h) $\frac{81}{4}$
2(a) $(x+2)^2$ **(b)** $(y+1)^2$ **(c)** $(p+7)^2$ **(d)** $(m-6)^2$
(e) $(t-8)^2$ **(f)** $(20-u)^2$ **(g)** $(x+10y)^2$ **(h)** $(ab-12)^2$
3(a) $x^2+6x+9 = (x+3)^2$ **(b)** $y^2+8y+16 = (y+4)^2$
(c) $a^2 - 20a + 100 = (a-10)^2$
(d) $b^2 - 100b + 2500 = (b-50)^2$
(e) $u^2 + u + \frac{1}{4} = (u + \frac{1}{2})^2$
(f) $t^2 - 7t + \frac{49}{4} = (t - \frac{7}{2})^2$
(g) $m^2 + 50m + 625 = (m+25)^2$
(h) $c^2 - 13c + \frac{169}{4} = (c - \frac{13}{2})^2$
4(a) $x = -1$ or 3 **(b)** $x = 0$ or 6 **(c)** $a = -4$ or -2
(d) $y = -5$ or 2 **(e)** $b = -2$ or 7
(f) $x = -2 + \sqrt{3}$ or $x = -2 - \sqrt{3}$
(g) $x = 5 + \sqrt{5}$ or $x = 5 - \sqrt{5}$
(h) no solution for y
(i) $a = \frac{1}{2}(-7 + \sqrt{21}\,)$ or $a = \frac{1}{2}(-7 - \sqrt{21}\,)$
5(a) $p^2 - 2pq + q^2 = (p-q)^2$
(b) $a^2 + 4ab + 4b^2 = (a+2b)^2$
(c) $x^2 - 6xy + 9y^2 = (x-3y)^2$
(d) $c^2 + 40cd + 400d^2 = (c+20d)^2$
(e) $u^2 - uv + \frac{1}{4}v^2 = (u - \frac{1}{2}v)^2$
(f) $m^2 + 11mn + \frac{121}{4}n^2 = (m + \frac{11}{2}n)^2$
6(a) $x = 2$ or 3 **(b)** $x = \frac{1}{2}(2+\sqrt{6}\,)$ or $x = \frac{1}{2}(2-\sqrt{6}\,)$
(c) no solution for x
(d) $x = \frac{1}{2}(-4 + \sqrt{10}\,)$ or $x = \frac{1}{2}(-4 - \sqrt{10}\,)$
(e) $x = -\frac{3}{2}$ or $\frac{1}{2}$
(f) $x = \frac{1}{4}(1 + \sqrt{5}\,)$ or $x = \frac{1}{4}(1 - \sqrt{5}\,)$
(g) $x = -\frac{1}{3}$ or 3 **(h)** $x = -3$ or $\frac{5}{2}$
(i) $x = \frac{1}{2}(5 + \sqrt{11}\,)$ or $x = \frac{1}{2}(5 - \sqrt{11}\,)$
7(b) $a = 3,\ b = 4$ and $c = 25$
(d) $A = -5,\ B = 6$ and $C = 8$
8(a) $x^3 + 12x^2 + 48x + 64 = (x+4)^3$
(b) $u = x + 4,\ u^3 - 18u + 12 = 0$

Exercise 1J (Page 27)

1(a) infinite **(b)** finite, 10 members
(c) finite, 0 members **(d)** infinite
(e) finite, 18 members **(f)** infinite
(g) finite, 6 members **(h)** finite, 14 members
2(a) false **(b)** true **(c)** true **(d)** false **(e)** true
(f) false
3(a) false **(b)** true **(c)** true **(d)** true **(e)** false
4(a) true **(b)** true
5(a) $\emptyset$, { a } **(b)** $\emptyset$, { a }, { b }, { a, b } **(c)** $\emptyset$, { a }, { b }, { c }, { a, b }, { a, c }, { b, c }, { a, b, c }

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(d) $\emptyset$

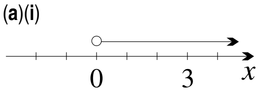
6(a) $\{\,\mathrm{m, n}\,\}, \{\,\mathrm{m}\,\}$ **(b)** $\{\,2, 4, 6, 8\,\}, \{\,4, 6\,\}$
(c) $\{\,1, 2, 3, 4, 5, 6, 7, 8, 9\,\}, \{\,4, 9\,\}$
(d) $\{\,\mathrm{c, o, m, p, u, t, e, r, s, f, w, a}\,\}, \{\,\mathrm{o, t, e, r}\,\}$
(e) $\{\,1, 2, 3, 5, 7, 9, 11\,\}, \{\,3, 5, 7, 11\,\}$

7(a) students who study both Japanese and History **(b)** students who study either Japanese or History or both

8(a) Q **(b)** P

9(a) $\{\,2, 4, 5, 7, 9, 10\,\}$ **(b)** $\{\,1, 2, 5, 8, 9\,\}$
(c) $\{\,1, 2, 4, 5, 7, 8, 9, 10\,\}$ **(d)** $\{\,2, 5, 9\,\}$
(e) $\{\,2, 5, 9\,\}$ **(f)** $\{\,1, 2, 4, 5, 7, 8, 9, 10\,\}$

10(a) III **(b)** I **(c)** I **(d)** II **(e)** IV

11(a)(i)


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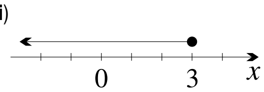

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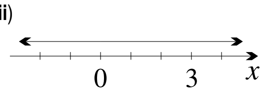

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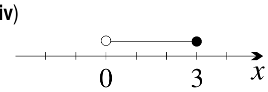
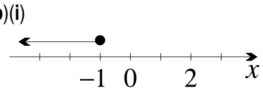


(b)(i)


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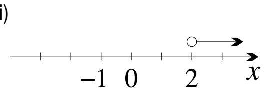

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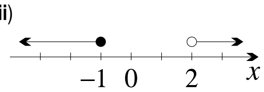

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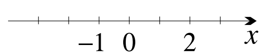
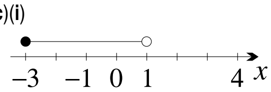


(c)(i)


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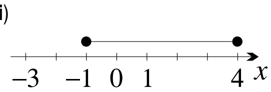

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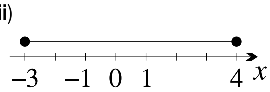

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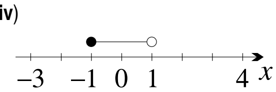
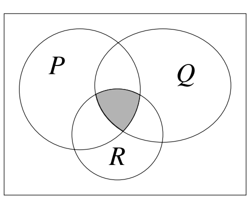
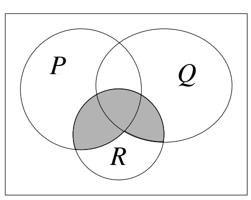
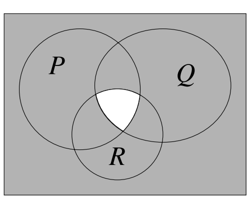


12(a) $|A \cap B|$ is subtracted so that it is not counted twice. **(b)** 5 **(c)** LHS $= 7$, RHS $= 5 + 6 - 4 = 7$

13(a) 10 **(b)** 22 **(c)** 12

14(a)


(b)


(c)


15 4

16(a) Every subset of a four member set can become a subset of a five member set in two ways — leaving it alone, and adding the fifth member.
(b) An n-member set has 2^n subsets.

17 $2^8 = 256$

18 'The set whose only member is the empty set is not equal to the empty set because the empty set is a member of the set whose only member is the empty set.' It is true.

19 It is true.

20 $A \cup B$

21 If $A \in A$, then $A \notin A$. If $A \notin A$, then $A \in A$. Hence A is not well-defined.

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Chapter Two

Exercise **2A** (Page 33)

1(a) 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97 **(b)** 151, 157, 163, 167, 173, 179, 181, 191, 193, 197, 199

2(a) $2^3 \times 3$ **(b)** $2^2 \times 3 \times 5$ **(c)** $2^3 \times 3^2$ **(d)** $2 \times 3^2 \times 7$ **(e)** $2^3 \times 13$ **(f)** $3^3 \times 5$ **(g)** $3^3 \times 7$ **(h)** $2 \times 3 \times 7^2$ **(i)** $3^2 \times 5 \times 7$ **(j)** 5×11^2

3(a) $8,\ \frac{9}{8}$ **(b)** $6,\ \frac{14}{15}$ **(c)** $26,\ \frac{3}{4}$ **(d)** $16,\ \frac{7}{9}$ **(e)** $24,\ \frac{7}{9}$ **(f)** $21,\ \frac{14}{15}$

4(a) $24,\ \frac{5}{24}$ **(b)** $90,\ \frac{13}{90}$ **(c)** $72,\ \frac{1}{72}$ **(d)** $210,\ \frac{173}{105}$ **(e)** $216,\ \frac{5}{72}$ **(f)** $780,\ \frac{401}{780}$

5(a) $0{\cdot}625$ **(b)** $0{\cdot}\dot{6}$ **(c)** $0{\cdot}4375$ **(d)** $0{\cdot}\dot{5}$ **(e)** $0{\cdot}15$ **(f)** $0{\cdot}58\dot{3}$ **(g)** $4{\cdot}64$ **(h)** $5{\cdot}\dot{3}\dot{6}$ **(i)** $2{\cdot}875$ **(j)** $2{\cdot}8\dot{3}$

6(a) $\frac{3}{20}$ **(b)** $\frac{7}{9}$ **(c)** $\frac{27}{250}$ **(d)** $\frac{2}{11}$ **(e)** $\frac{78}{25}$ **(f)** $\frac{60}{11}$ **(g)** $\frac{5}{3}$ **(h)** $\frac{40}{33}$ **(i)** $\frac{19}{90}$ **(j)** $\frac{98}{15}$

7(a) $1{\cdot}8\dot{3}$ **(b)** $1{\cdot}08\dot{3}$ **(c)** $0{\cdot}4\dot{6}$ **(d)** $0{\cdot}\dot{4}3\dot{2}$ **(e)** $0{\cdot}\dot{0}7\dot{4}$ **(f)** $0{\cdot}541\dot{6}$ **(g)** $3{\cdot}\dot{1}4285\dot{7}$ **(h)** $1{\cdot}2\dot{1}4285\dot{7}$ **(i)** $2{\cdot}\dot{0}7692\dot{3}$ **(j)** $1{\cdot}\dot{2}3809\dot{5}$

8(a) $\frac{25}{33}$ **(b)** $\frac{28}{27}$ **(c)** $\frac{169}{37}$ **(d)** $\frac{44}{101}$ **(e)** $\frac{2}{1}$ **(f)** $\frac{5}{2}$ **(g)** $\frac{137}{90}$ **(h)** $\frac{129}{55}$ **(i)** $\frac{257}{36}$ **(j)** $\frac{5}{44}$

9 The digits of each cycle are in the same order, but start at a different place in the cycle.

10(a) $2^8,\ 2^4 = 16$ **(b)** $2^6 \times 3^2,\ 2^3 \times 3 = 24$ **(c)** $5^2 \times 7^2,\ 5 \times 7 = 35$ **(d)** $2^4 \times 11^2,\ 2^2 \times 11 = 44$

11(a) $\text{HCF} = 2^2 \times 3^2 \times 11,\ \ \text{LCM} = 2^3 \times 3^3 \times 11$
(b) $\text{HCF} = 7 \times 13,\ \ \text{LCM} = 2^4 \times 13^2 \times 7$
(c) $\text{HCF} = 3^2 \times 7^2,\ \ \text{LCM} = 2 \times 3^3 \times 7^3$
(d) $\text{HCF} = 2 \times 5^2 \times 7,\ \ \text{LCM} = 2^5 \times 3^2 \times 5^2 \times 7$

12(a) the primes $< \sqrt{250}$, namely 2, 3, 5, 7, 11, 13 **(b)** It is prime since $22 > \sqrt{457}$.
(c) $247 = 13 \times 19$, $329 = 7 \times 47$, $451 = 11 \times 41$, 503 is prime, 727 is prime, $1001 = 7 \times 11 \times 13$.

14(a) $1 + 2 + 4 + 7 + 14 = 28$

15(c) $3{\cdot}000\,300\,03 \neq 3$, showing that some fractions are not stored exactly. (The number you obtain may vary depending on the calculator used.)

Exercise **2B** (Page 37)

2(a) rationals: $4\frac{1}{2} = \frac{9}{2}$, $5 = \frac{5}{1}$, $-5\frac{3}{4} = \frac{-23}{4}$, $0 = \frac{0}{1}$, $\sqrt{4} = \frac{2}{1}$ **(b)** rationals: $\sqrt[3]{27} = \frac{3}{1}$, $\sqrt{\frac{4}{9}} = \frac{2}{3}$, $-3 = \frac{-3}{1}$, $16^{\frac{1}{2}} = \frac{4}{1}$

3 $\dfrac{ad + bc}{2bd}$, which is in the form $\dfrac{p}{q}$, where p and q are integers.

8(b) $\sqrt{11}$

11(c) $\pi \doteqdot \frac{333}{106}$, with error less than 10^{-4}.

12 $1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \frac{1}{2 + \cdots}}}}$, $1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \frac{1}{2 + \cdots}}}}$, $2 + \cfrac{1}{4 + \cfrac{1}{4 + \cfrac{1}{4 + \frac{1}{4 + \cdots}}}}$

13 $\pi \doteqdot 3{\cdot}11$

14(a) Clearly $\frac{p+1}{n} > a$. $\frac{p+1}{n} = \frac{p}{n} + \frac{1}{n} < a + b - a = b$ **(b)** $n = 63\,293$, $p = 2000$ **(c)** $\frac{2001}{63\,293}$

Exercise **2C** (Page 40)

1 The graph is steeper there.

2(a) 4 **(b)** 9 **(c)** 6 **(d)** $2\sqrt{3}$ **(e)** $3\sqrt{3}$ **(f)** $2\sqrt{5}$ **(g)** $x\sqrt{6}$ **(h)** $2y\sqrt{2}$ **(i)** 22 **(j)** $5x$ **(k)** $6\sqrt{2}$ **(l)** $y\sqrt{y}$ **(m)** 4 **(n)** 7 **(o)** $2x$ **(p)** $y\sqrt[3]{4}$

3(a) $\sqrt{6}$ **(b)** $2\sqrt{3}$ **(c)** $3\sqrt{5}$ **(d)** 5 **(e)** $6\sqrt{15}$ **(f)** 84 **(g)** $15\sqrt{3}$ **(h)** 12 **(i)** $6\sqrt{15}$ **(j)** $20\sqrt{21}$ **(k)** $36\sqrt{6}$ **(l)** $420\sqrt{3}$ **(m)** $6\pi\sqrt{2}$ **(n)** $2a^2\pi\sqrt{\pi}$ **(o)** $336x^2\sqrt{33}$

4(a) $\sqrt{20}$ **(b)** $\sqrt{27}$ **(c)** $\sqrt{72}$ **(d)** $\sqrt{150}$ **(e)** $\sqrt{48}$ **(f)** $\sqrt{32}$ **(g)** $\sqrt{567}$ **(h)** $\sqrt{68}$ **(i)** $\sqrt{275x^2}$ **(j)** $\sqrt{216\pi^2}$ **(k)** $\sqrt{117y^3}$ **(l)** $\sqrt{864a^4}$

5(a) $\frac{3}{2}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{1}{5}\sqrt{7}$ **(d)** $\frac{5}{2}$ **(e)** $\frac{7}{3}$ **(f)** $\frac{1}{4}\sqrt[3]{7}$ **(g)** $\frac{3}{2}$ **(h)** $\frac{4}{3}$

6(a) $\sqrt{2}$ **(b)** $\sqrt{6}$ **(c)** $\sqrt{5}$ **(d)** $2\sqrt{3}$ **(e)** $\frac{1}{2}$ **(f)** $\frac{2}{5}$ **(g)** $\frac{2}{3}$ **(h)** $\frac{6}{7}$ **(i)** $2\sqrt{2}$ **(j)** $\frac{1}{5}\sqrt{7}$ **(k)** $\frac{1}{3}\sqrt{11}$ **(l)** $\frac{1}{2}\sqrt{7}$

7(a) $2\sqrt{2} \doteqdot 2{\cdot}82$ **(b)** $2\sqrt{3} \doteqdot 3{\cdot}46$ **(c)** $2\sqrt{5} \doteqdot 4{\cdot}48$ **(d)** $3\sqrt{2} \doteqdot 4{\cdot}23$ **(e)** $3\sqrt{3} \doteqdot 5{\cdot}19$ **(f)** $3\sqrt{5} \doteqdot 6{\cdot}72$ **(g)** $5\sqrt{2} \doteqdot 7{\cdot}05$ **(h)** $5\sqrt{3} \doteqdot 8{\cdot}65$

8 If $a = 3$ and $b = 4$, then LHS $= 5$, but RHS $= 7$. If one of a or b is zero, then they are equal.

9(a) $2\sqrt{2}$ **(b)** $20\sqrt{3}$ **(c)** $3\sqrt{7}$ **(d)** $\sqrt{6}$

10(a) $2\sqrt{3} - 1$ **(b)** $-\sqrt{6}$ **(c)** 0 **(d)** $2\sqrt{10}$ **(e)** $2\sqrt{5}$ **(f)** $4\sqrt{3} - 5\sqrt{2}$ **(g)** $3\sqrt{6} + 6\sqrt{2}$ **(h)** $3\sqrt{3} - \sqrt{13}$ **(i)** $6\sqrt{2} - 2\sqrt{7}$

11(a) $a = 192$ **(b)** $x = 275$ **(c)** $y = 15$ **(d)** $m = 24$

12(a) $\sqrt{6} + 3$ **(b)** $5 + 5\sqrt{3}$ **(c)** $6\sqrt{3} - 12$ **(d)** $3\sqrt{21} - 7\sqrt{2}$ **(e)** $a + \sqrt{ab}$ **(f)** $4\sqrt{a} - 4a$ **(g)** $\sqrt{x^2 + 2x} + x$ **(h)** $x - 1 + \sqrt{x^2 - 1}$

13(a) $\sqrt{15} + \sqrt{6} - \sqrt{10} - 2$ **(b)** $\sqrt{10} + \sqrt{15} + \sqrt{2} + \sqrt{3}$ **(c)** $\sqrt{6} - \sqrt{3} - \sqrt{2} + 1$ **(d)** $3\sqrt{2} + 2\sqrt{3} - \sqrt{6} - 2$ **(e)** $26 + 6\sqrt{6}$ **(f)** $19 + \sqrt{7}$ **(g)** $4\sqrt{5} - 2\sqrt{15} + 2\sqrt{3} - 3$ **(h)** $6\sqrt{3} - 3\sqrt{10} - \sqrt{30} + 5$

14(a) $2\sqrt{2} + 3$ **(b)** 2 **(c)** -4 **(d)** $4 - 2\sqrt{3}$ **(e)** $5 + 2\sqrt{6}$ **(f)** -2 **(g)** -1 **(h)** $14 - 8\sqrt{3}$ **(i)** $4a + 1 - 4\sqrt{a}$ **(j)** $a + 6 + 4\sqrt{a + 2}$ **(k)** $x - 2$

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(l) $2x - 1 + 2\sqrt{(x+1)(x-2)}$ **(m)** $\frac{3}{2} + \frac{1}{2}\sqrt{5}$
(n) $\frac{3}{2} - \frac{1}{2}\sqrt{5}$
15(a) 2 **(b)** $2\sqrt{3}$ **(c)** $\frac{5\sqrt{3}}{2}$ **(d)** 3 **(e)** 5 **(f)** 5 **(g)** $\sqrt{2}$
(h) $\frac{5\sqrt{7}}{9}$
16(a) 3 **(b)** $\sqrt{15}$ **(c)** 4 **(d)** $2\sqrt{15}$
17(a) $-\frac{b}{a}$ **(b)** $\frac{c}{a}$
18(a) $xy\sqrt{y}$ **(b)** x^2y^3 **(c)** $x+3$ **(d)** $(x+1)\sqrt{x}$
(e) $x(x+1)y^2$ **(f)** $x(x+1)$

Exercise **2D** (Page 43)

1(a) $\frac{\sqrt{3}}{3}$ **(b)** $\frac{2\sqrt{7}}{7}$ **(c)** $\frac{5\sqrt{11}}{11}$ **(d)** $\sqrt{5}$ **(e)** $\frac{\sqrt{2}}{2}$ **(f)** $\frac{\sqrt{6}}{2}$
(g) $\frac{\sqrt{15}}{3}$ **(h)** $\frac{7\sqrt{10}}{5}$
2(a) $4,\ 1$ **(b)** $6,\ 4$ **(c)** $2,\ -1$ **(d)** $10,\ 1$ **(e)** $2,\ -11$
(f) $-2,\ -6$
3(a) $1+\sqrt{2}$ **(b)** $-1-\sqrt{2}$ **(c)** $\frac{3(\sqrt{5}+1)}{4}$ **(d)** $-\frac{3(\sqrt{5}+1)}{4}$
(e) $\sqrt{3}-\sqrt{2}$ **(f)** $\frac{\sqrt{5}-\sqrt{3}}{2}$ **(g)** $\frac{4+\sqrt{7}}{3}$ **(h)** $\frac{3(\sqrt{11}+\sqrt{6})}{5}$
4(a) $\frac{\sqrt{2}}{3}$ **(b)** $\frac{\sqrt{5}}{2}$ **(c)** $\sqrt{6}$ **(d)** $\frac{2\sqrt{3}}{3}$ **(e)** $\frac{\sqrt{21}}{7}$ **(f)** $\frac{\sqrt{14}}{35}$
(g) $\frac{\sqrt{17}}{2}$ **(h)** $\frac{6\sqrt{11}}{11}$
5(a) $2\sqrt{2}+\sqrt{5}$ **(b)** $3\sqrt{2}+4$ **(c)** $2(\sqrt{5}+\sqrt{3})$
(d) $\frac{3\sqrt{15}-9}{2}$ **(e)** $\frac{28-10\sqrt{7}}{3}$ **(f)** $2\sqrt{2}-3$
(g) $1+\sqrt{3}$ **(h)** $\frac{9\sqrt{2}+\sqrt{21}-3\sqrt{42}-7}{47}$ **(i)** $2-\sqrt{3}$
(j) $4+\sqrt{15}$ **(k)** $\frac{3\sqrt{x}-6}{x-4}$ **(l)** $\frac{q+\sqrt{p}}{q^2-p}$
(m) $\sqrt{x+1}-\sqrt{x-1}$ **(n)** $\frac{x+y-2\sqrt{xy}}{x-y}$ **(o)** $\frac{a+b+2\sqrt{ab}}{a-b}$
(p) $\frac{\sqrt{2}(\sqrt{5}+1)}{2}$ **(q)** $\sqrt{21}-\sqrt{15}$ **(r)** $\sqrt{x^2+2x}-x$
6(a) 3 **(b)** 1
7(a) $\sqrt{3}$ **(b)** $\frac{\sqrt{5}}{4}$ **(c)** $\frac{-2}{17}$
8 $\frac{\sqrt{x+h}-\sqrt{x}}{h}$
9(a) $2\sqrt{2}$ **(b)** 4 **(c)** 4 **(d)** $\frac{2}{x-1}$
10(b) $2\sqrt{5}$ **(c)** 18 **(d)(i)** 6 **(ii)** 14
11(a) $2\sqrt{3}$ **(b)** $3\sqrt{11}$ **(c)** $3+2\sqrt{2}$ **(d)** $7-2\sqrt{6}$
12(a) $\frac{1}{12}(2\sqrt{3}+3\sqrt{2}-\sqrt{30})$ **(b)** $\frac{1}{2}\sqrt[3]{4}$
(c) $\sqrt[3]{4}+\sqrt[3]{2}+1$
14(a) $8{\cdot}33$ **(b)** $8{\cdot}12$

Exercise **2E** (Page 46)

1(a) $a=7,\ b=-2$ **(b)** $a=2,\ b=3$
(c) $a=-7,\ b=-4$ **(d)** $a=3,\ b=-2$
(e) $a=\frac{5}{7},\ b=\frac{1}{2}$ **(f)** $a=\frac{2}{3},\ b=3$
2(a) $a=2,\ b=18$ **(b)** $a=-1,\ b=2$
(c) $a=3,\ b=-2$ **(d)** $a=5,\ b=20$
(e) $a=\frac{4}{3},\ b=\frac{1}{2}$ **(f)** $a=-\frac{1}{5},\ b=-\frac{3}{4}$
3(a) $x=7,\ y=28$ **(b)** $x=4,\ y=12$
(c) $x=39,\ y=12$ **(d)** $x=11,\ y=5$
(e) $x=9,\ y=6$ **(f)** $x=14,\ y=180$
4(a) $x=0,\ y=-3,\ z=2$
(b) $x=20,\ y=10,\ z=3$
(c) $x=-7,\ y=1,\ z=10$
(d) $x=20,\ y=-10,\ z=3$
5(a) $a=2,\ b=1$ **(b)** $a=-2,\ b=1$
(c) $a=\frac{1}{2},\ b=\frac{1}{2}$ **(d)** $a=-\frac{1}{2},\ b=\frac{1}{2}$
(e) $a=3,\ b=2$ **(f)** $a=\frac{9}{5},\ b=\frac{8}{15}$
6(a) $x=2,\ y=3$ **(b)** $x=\frac{5}{2},\ y=8$
(c) $x=\frac{9}{2},\ y=\frac{3}{2}$ **(d)** $x=12,\ y=-6$
(e) $x=1,\ y=\frac{1}{2}$ **(f)** $x=\frac{1}{3},\ y=\frac{1}{2}$
(g) $x=-2$ and $y=-5$, or $x=5$ and $y=2$
(h) $x=\frac{5}{2}$ and $y=\frac{1}{2}$, or $x=\frac{1}{2}$ and $y=\frac{5}{2}$
7(a) $a=1,\ b=1$ **(b)** $a=2,\ b=-1$
(c) $a=1,\ b=-2$ **(d)** $a=\frac{3}{2},\ b=1$
8(a) $3-\sqrt{6}$
(b)(i) $5-\sqrt{3}$ **(ii)** $7+\sqrt{17}$ **(iii)** $-\frac{1}{2}+\frac{1}{\sqrt{3}}=-\frac{1}{2}+\frac{\sqrt{3}}{3}$

Exercise **2F** (Page 50)

1 a, b, d, e, g, h, j, l
2(a) domain: all real numbers, range: $y \ge -1$
(b) domain: all real numbers, range: $y > -1$
(c) domain: all real numbers, range: all reals
(d) domain: all real numbers, range: $y = 2$
(e) domain: all real numbers, range: $y < 2$
(f) domain: $x \ge -1$, range: all real numbers
(g) domain: $x \ne 0$, range: $y \ne 0$
(h) domain: all real numbers, range: all reals
(i) domain: $0 \le x \le 3$, range: $-3 \le y \le 3$
(j) domain: $x < 4$, range: $y > 0$
(k) domain: all reals, range: $y \le -1, y > 1$
(l) domain: all real numbers, range: $y < 1$
3(a) 2 **(b)** 0 **(c)** a^2-2 **(d)** a^2-2 **(e)** a^2+4a+2
(f) x^2-2x-1 **(g)** $-1\frac{3}{4}$ **(h)** $9t^2+12t+2$ **(i)** t^4-2
(j) $t^2+\frac{1}{t^2}$
4(a) 0 **(b)** -1 **(c)** 8 **(d)** 0 **(e)** t^2-2t **(f)** t^2+2t
(g) w^2-4w+3 **(h)** w^2-2w-1 **(i)** w^2-1
(j) $x^2-2x=g(x)$
5(a) $-3,\ -2,\ -1,\ 0,\ 1,\ 0,\ -1$ **(b)** $3,\ 0,\ 0,\ 1,\ 4$
6(a) all real numbers **(b)** $x \ne 3$ **(c)** $x \ge 0$ **(d)** $x > 0$
(e) $x \le 2$ **(f)** $x < 2$
8(a) h^3-h+1 **(b)** $-h^3+h+1$ **(c)** h^2-1 **(d)** h^2-1
(e) $\frac{13}{16}$ **(f)** $\frac{3}{4}$
9(a) $1,\ 3,\ 6,\ 10,\ 15,\ 21$ **(b)** $1,\ 2,\ 2,\ 3,\ 2,\ 4$
(c) $1,\ 3,\ 4,\ 7,\ 6,\ 12$
10(a) $-3 \le x \le 3$ **(b)** $x \le -2$ or $x \ge 2$
(c) $x > 0$ **(d)** $x \ne -1$ **(e)** $x \ne 3$ and $x \ne 2$
(f) $x \ne 3$ and $x \ne -3$
11(a) 64 **(b)** 28 **(c)** $(x+3)^2$ **(d)** x^2+3 **(e)** 64
(f) 12 **(g)** 2^{3x} **(h)** 3×2^x

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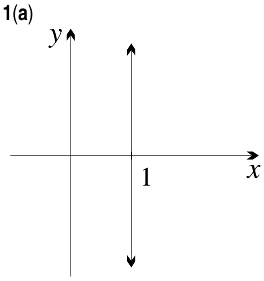
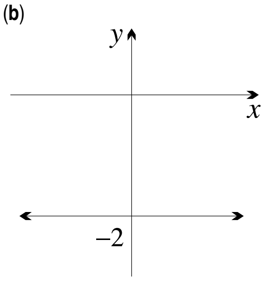
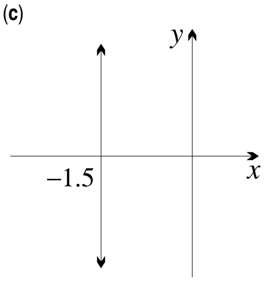
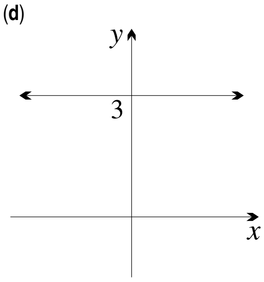
13(a) all real values of a and b
(b) all real values of a and b **(c)** no solutions
(d) no solutions **(e)** $a = 0$ and b is any real number, or $b = 0$ and a is any real number.
14 It approaches 2·72.
15(b) $\Big(s(x)\Big)^2 = \frac{1}{2}\Big(c(2x) - 1\Big)$

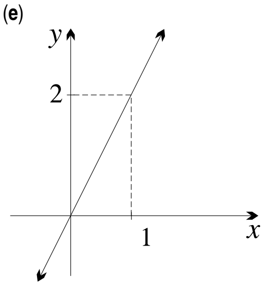
Exercise **2G** (Page 56)

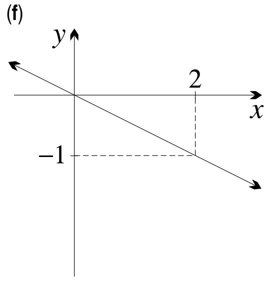
1(a)


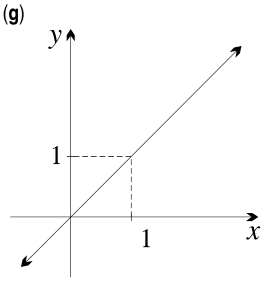
(b)


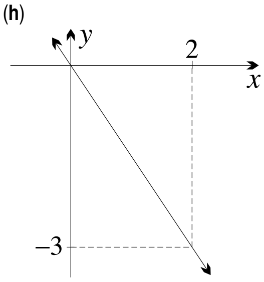
(c)


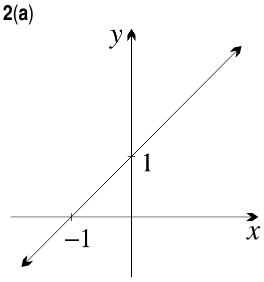
(d)


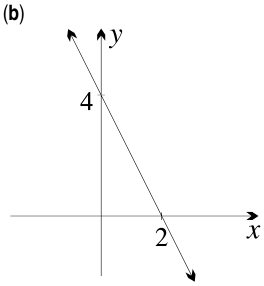
(e)


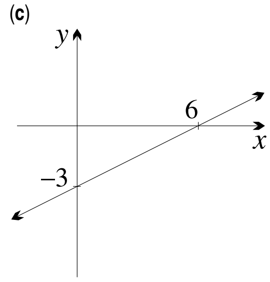
(f)


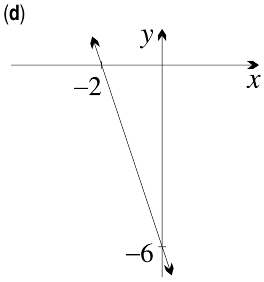
(g)


(h)


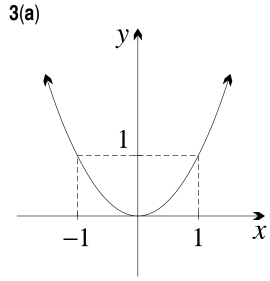
2(a)


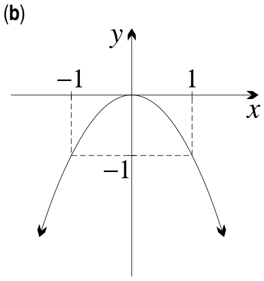
(b)


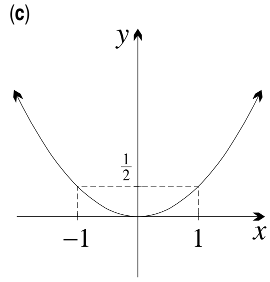
(c)


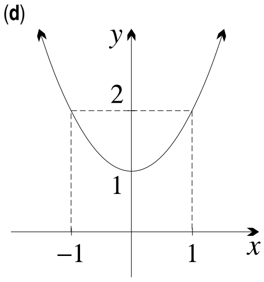
(d)


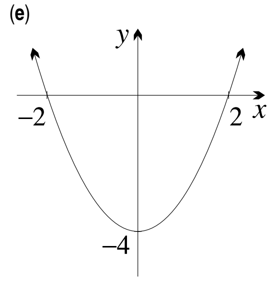
(e) intercepts: $(1, 0)$ and $(0, 1)$
(f) intercepts: $(-1, 0)$ and $(0, 2)$
(g) intercepts: $(3, 0)$ and $(0, -1)$
(h) intercepts: $(4, 0)$ and $(0, -2)$
(i) intercepts: $(6, 0)$ and $(0, -4)$
(j) intercepts: $(-6, 0)$ and $(0, -1\frac{1}{2})$
(k) intercepts: $(2, 0)$ and $(0, 5)$
(l) intercepts: $(3, 0)$ and $(0, -7\frac{1}{2})$

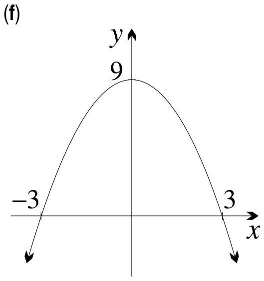
3(a)


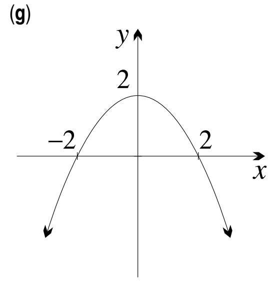
(b)


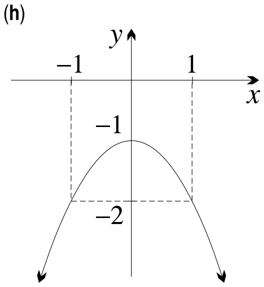
(c)


(d)


(e)


(f)


(g)


(h)


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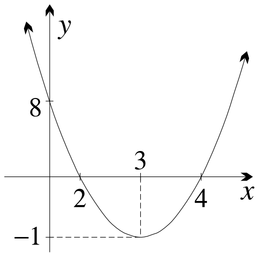
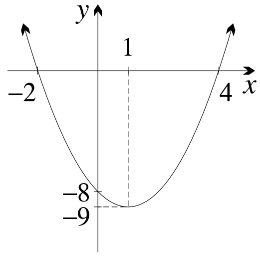
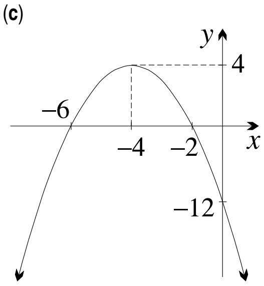
4(a)



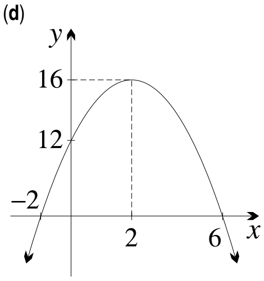
(b)



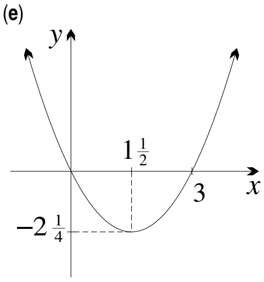
(c)



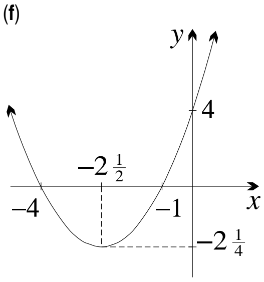
(d)



(e)



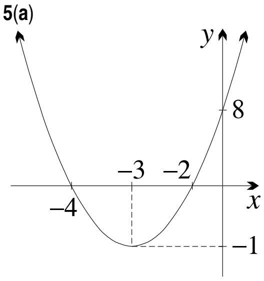
(f)



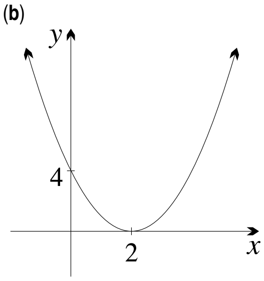
The ranges are:

(a) $y \geq -1$ **(b)** $y \geq -9$ **(c)** $y \leq 4$ **(d)** $y \leq 16$
(e) $y \geq -2\frac{1}{4}$ **(f)** $y \geq -2\frac{1}{4}$

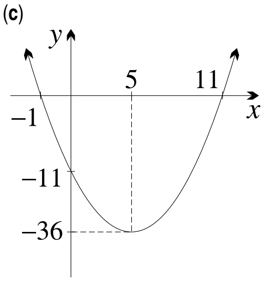
5(a)



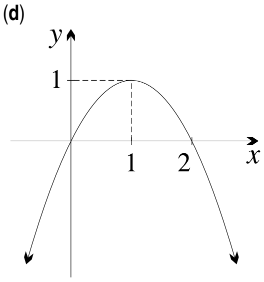
(b)



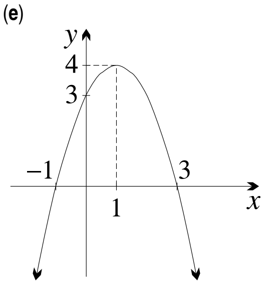
(c)



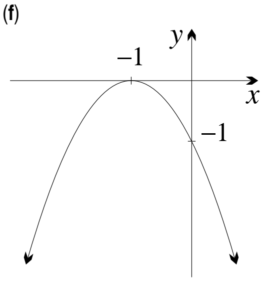
(d)



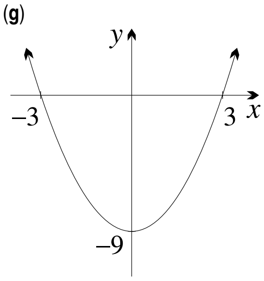
(e)



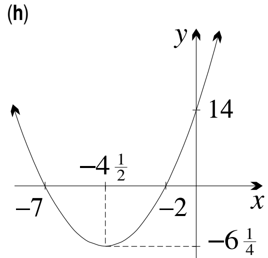
(f)



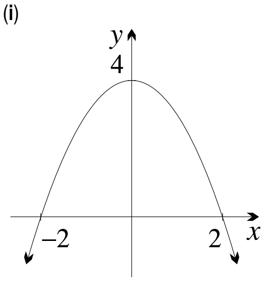
(g)



(h)

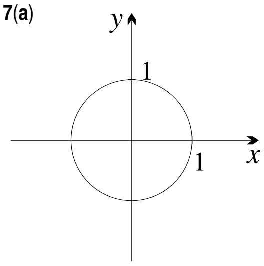


(i)

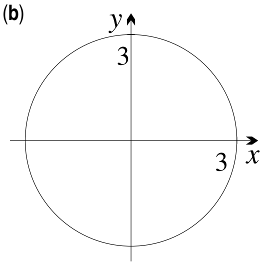


6 The curves are all sketched in the notes of Section 2G.

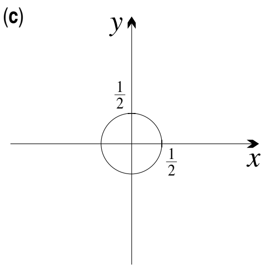
7(a)



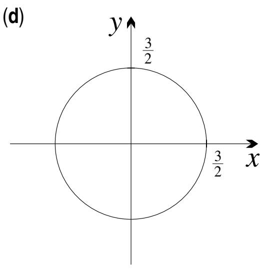
(b)



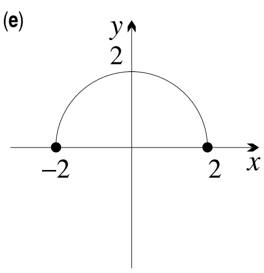
(c)



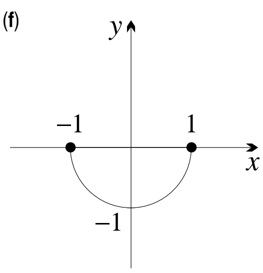
(d)



(e)



(f)



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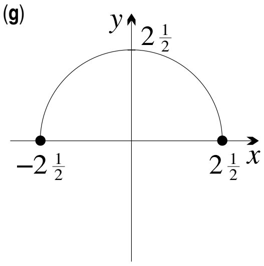
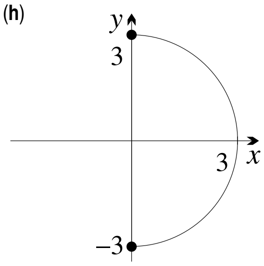
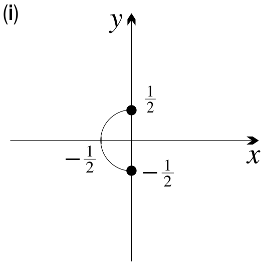
(g)



(h)



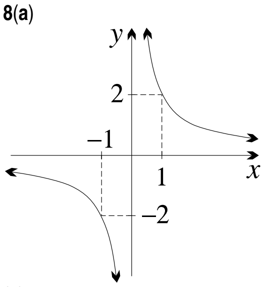
(i)



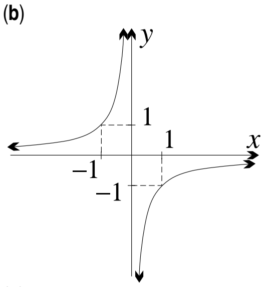
The domains and ranges are respectively:

(a) $-1 \le x \le 1$, $-1 \le y \le 1$
(b) $-3 \le x \le 3$, $-3 \le y \le 3$
(c) $-\frac{1}{2} \le x \le \frac{1}{2}$, $-\frac{1}{2} \le y \le \frac{1}{2}$
(d) $-\frac{3}{2} \le x \le \frac{3}{2}$, $-\frac{3}{2} \le y \le \frac{3}{2}$
(e) $-2 \le x \le 2$, $0 \le y \le 2$
(f) $-1 \le x \le 1$, $-1 \le y \le 0$
(g) $-2\frac{1}{2} \le x \le 2\frac{1}{2}$, $0 \le y \le 2\frac{1}{2}$
(h) $0 \le x \le 3$, $-3 \le y \le 3$
(i) $-\frac{1}{2} \le x \le 0$, $-\frac{1}{2} \le y \le \frac{1}{2}$

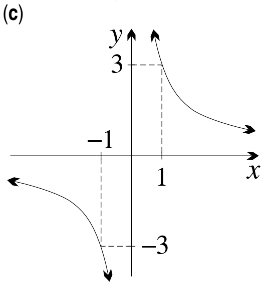
8(a)



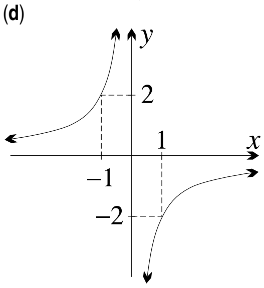
(b)



(c)

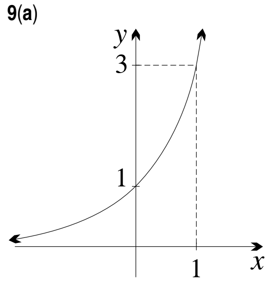


(d)

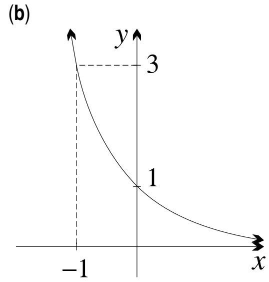


Each domain is $x \neq 0$, each range is $y \neq 0$.

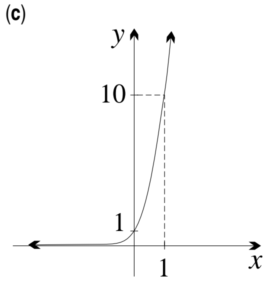
9(a)



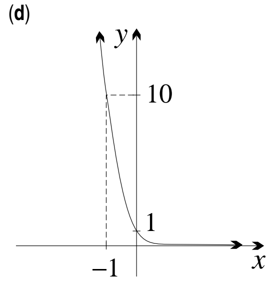
(b)



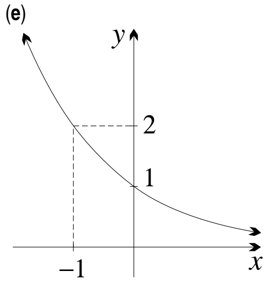
(c)



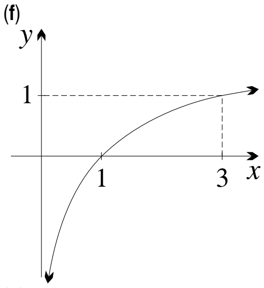
(d)



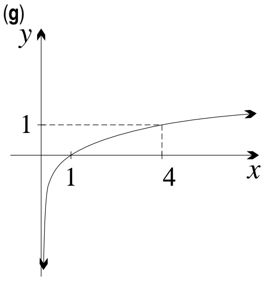
(e)



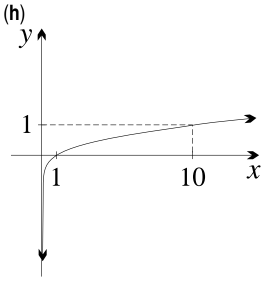
(f)



(g)

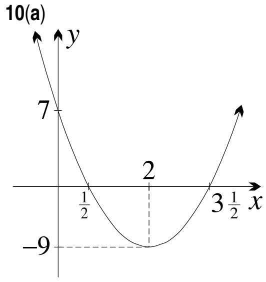


(h)

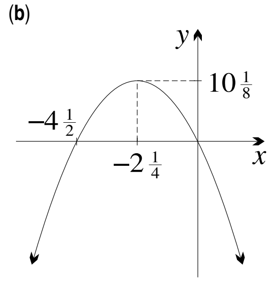


For parts (a)–(e), the domain is all real x, and the range is $y > 0$. For parts (f)–(h), the domain is $x > 0$, and the range is all real y.

10(a)



(b)



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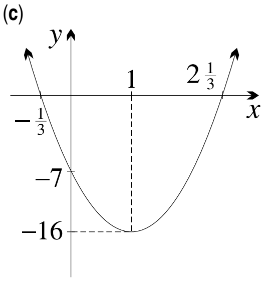
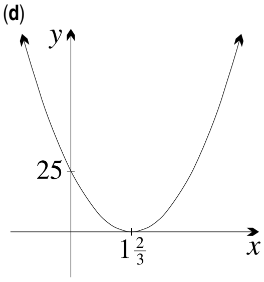
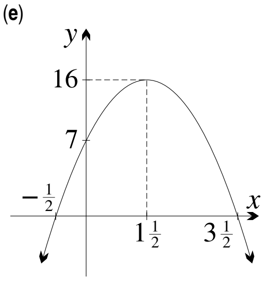
(c)



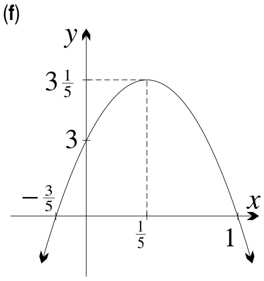
(d)



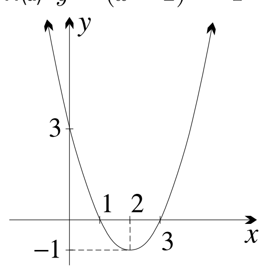
(e)



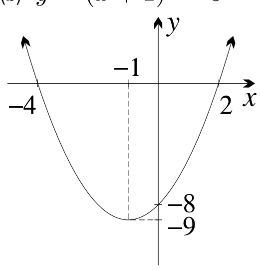
(f)



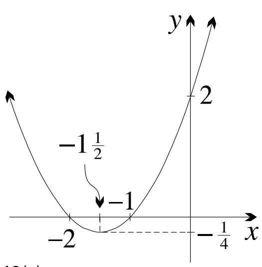
11(a) $y = (x-2)^2 - 1$



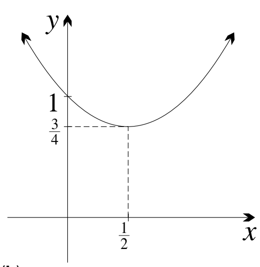
(b) $y = (x+1)^2 - 9$



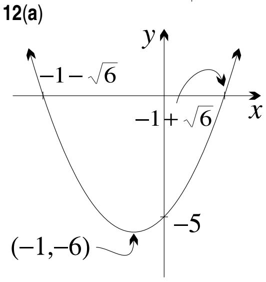
(c) $y = (x + 1\frac{1}{2})^2 - \frac{1}{4}$



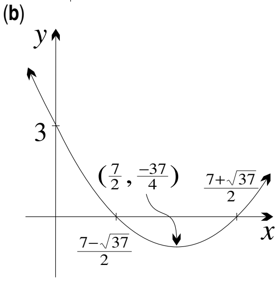
(d) $y = (x - \frac{1}{2})^2 + \frac{3}{4}$



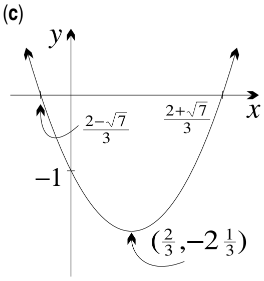
12(a)



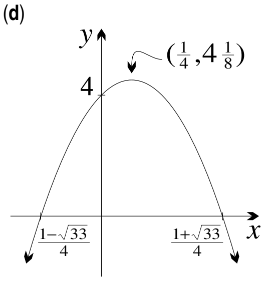
(b)



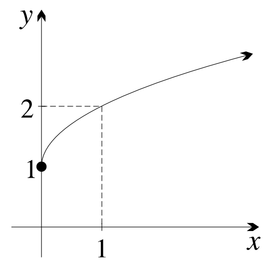
(c)



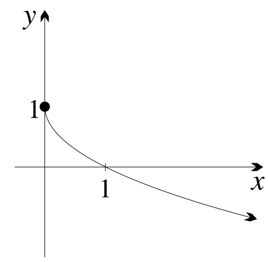
(d)



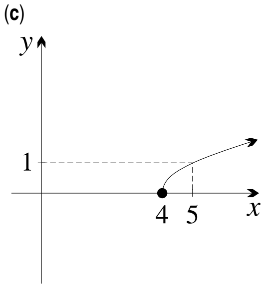
13(a)



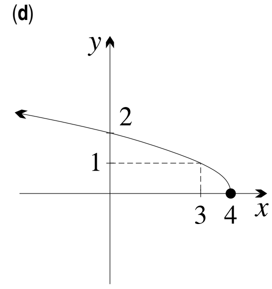
(b)



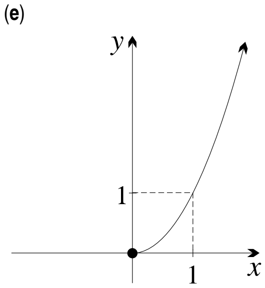
(c)



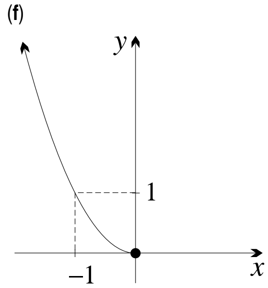
(d)



(e)



(f)



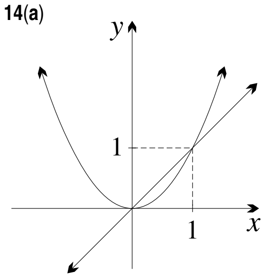
The domains and ranges are respectively:

(a) $x \geq 0,\ y \geq 1$ **(b)** $x \geq 0,\ y \leq 1$

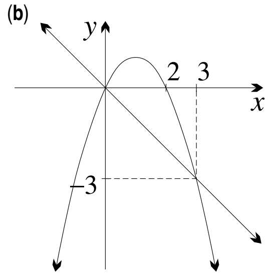
(c) $x \geq 4,\ y \geq 0$ **(d)** $x \leq 4,\ y \geq 0$

(e) $x \geq 0,\ y \geq 0$ **(f)** $x \leq 0,\ y \geq 0$

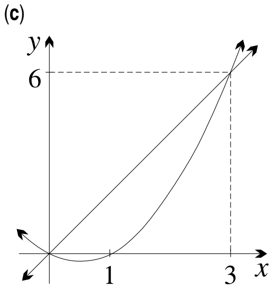
14(a)



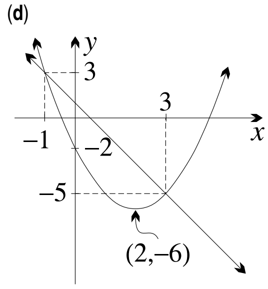
(b)



(c)



(d)



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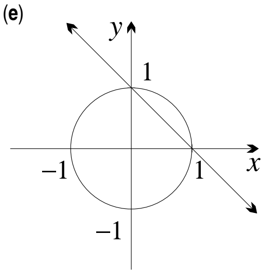
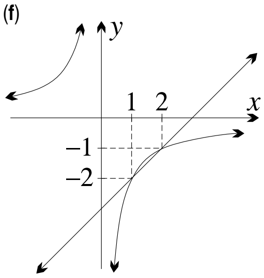
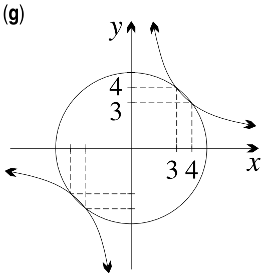
(e)



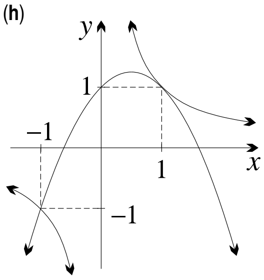
(f)



(g)



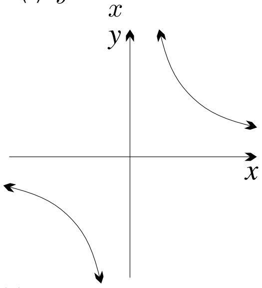
(h)

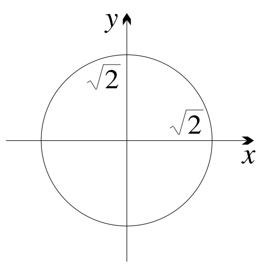


intersection points: **(a)** $(0,0)$, $(1,1)$ **(b)** $(0,0)$, $(3,-3)$ **(c)** $(0,0)$, $(3,6)$ **(d)** $(-1,3)$, $(3,-5)$
(e) $(1,0)$, $(0,1)$ **(f)** $(1,-2)$, $(2,-1)$
(g) $(-4,-3)$, $(-3,-4)$, $(3,4)$, $(4,3)$
(h) $(-1,-1)$, $(1,1)$

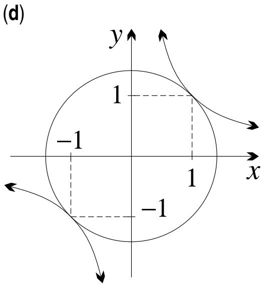
15(a) $r=\sqrt{5}$, $(2,1)$, $(1,2)$, $(-1,2)$, $(-2,1)$, $(-2,-1)$, $(-1,-2)$, $(1,-2)$, $(2,-1)$
(b) $r=\sqrt{2}$, $(1,-1)$, $(-1,-1)$
(c) $r=\sqrt{10}$, $(3,1)$, $(1,3)$, $(1,-3)$, $(3,-1)$
(d) $r=\sqrt{17}$, $(4,1)$, $(1,4)$, $(-1,4)$, $(-4,1)$, $(-4,-1)$, $(-1,-4)$, $(1,-4)$, $(4,-1)$

16(a) $y=\dfrac{1}{x}$ **(b)** $x^2+y^2=2$

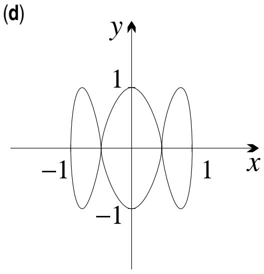




(d)

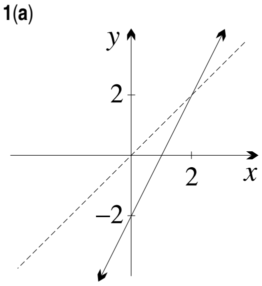


17(a) $(0,2\sqrt{\lambda^2-\alpha^2})$ **(b)** $r=\lambda$
18(a) $P(\frac{2}{b},\frac{b}{2})$ **(c)** 2 units^2
19(a) $a=\frac{1}{4}$, $b=\frac{3}{4}$, $c=1$ **(b)** $\sqrt{2} \doteqdot \frac{23}{16}$, $\frac{1}{\sqrt{2}} \doteqdot \frac{11}{16}$
20(a) $y=(4x^2-1)\sqrt{1-x^2}$ or $y=-(4x^2-1)\sqrt{1-x^2}$
(b) $-1 \le x \le 1$
(c) $(-1,0)$, $(-\frac{1}{2},0)$, $(\frac{1}{2},0)$, $(1,0)$, $(0,-1)$, $(0,1)$
(d)

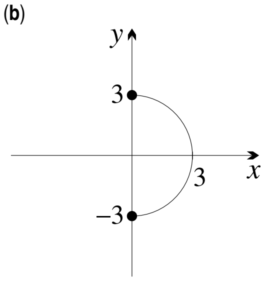


Exercise **2H** (Page 62)

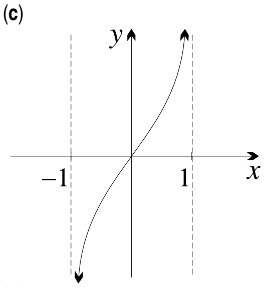
1(a)



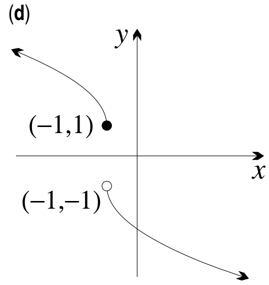
(b)



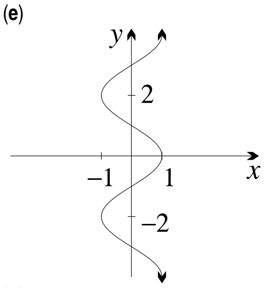
(c)



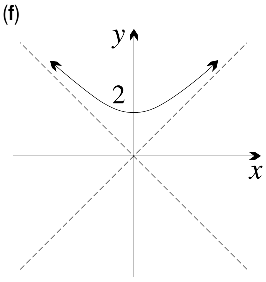
(d)



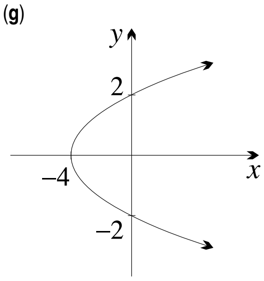
(e)



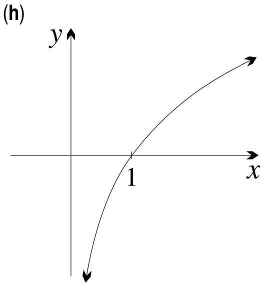
(f)



(g)



(h)



2 Original is a function: all except (f)
Inverse is a function: (a), (c), (d), (f), (h)

3(a) $y=\dfrac{x+2}{3}$ **(b)** $y=2x-2$ **(c)** $y=6-2x$
(d) $y-x+1=0$ **(e)** $2y+5x-10=0$ **(f)** $x=2$

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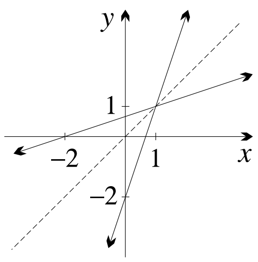
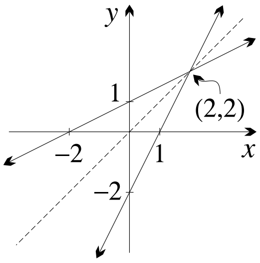
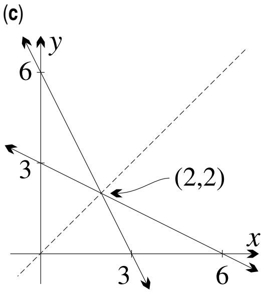
4(a)



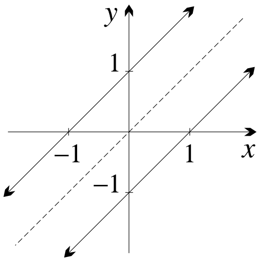
(b)



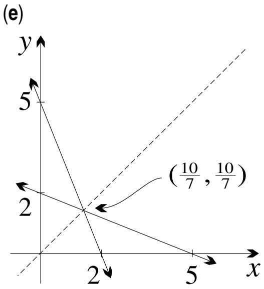
(c)



(d)

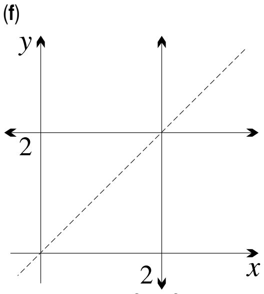


(e)



$(\frac{10}{7}, \frac{10}{7})$

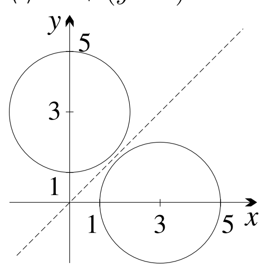
(f)



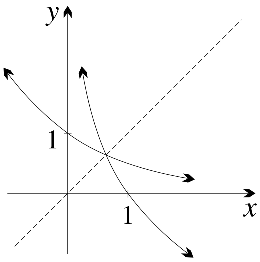
5(a) $y = \frac{1}{x-1}$ **(b)** $y = \frac{1}{x} - 1$ **(c)** $y = \frac{2x+2}{x-1}$
(d) $y = \frac{2x}{3-x}$

6 Each inverse is identical to the original function. Therefore the graph must be symmetric about the line $y = x$.

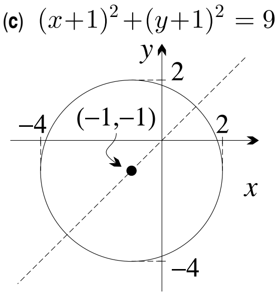
8(a) $x^2 + (y-3)^2 = 4$



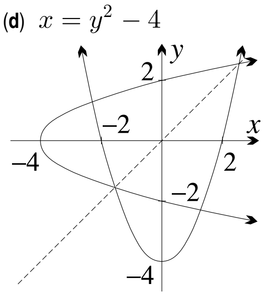
(b) $y = -\log_2 x$



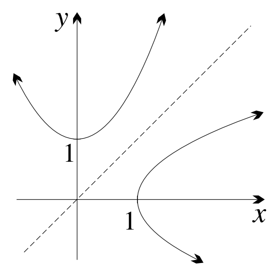
(c) $(x+1)^2+(y+1)^2 = 9$



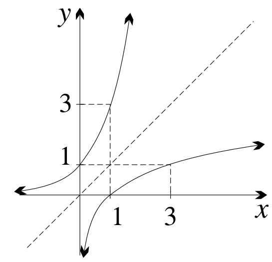
(d) $x = y^2 - 4$



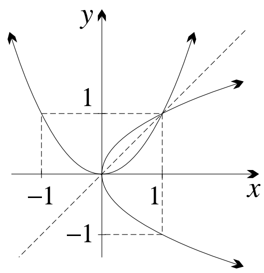
(e) $x = y^2 + 1$



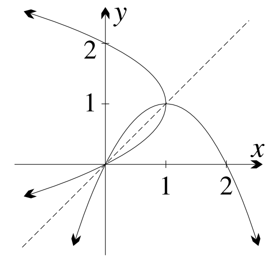
(f) $y = 3^x$



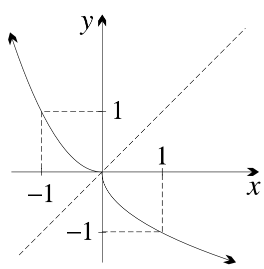
9(a) $x = y^2$



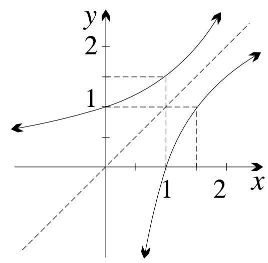
(b) $x = 2y - y^2$



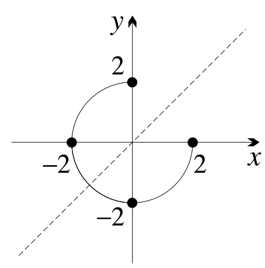
(c) $y = x^2$, where $x \le 0$



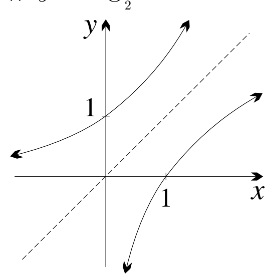
(d) $y = \log_2 x$



(e) $x = -\sqrt{4 - y^2}$



(f) $y = \log_{\frac{3}{2}} x$



10(a) It fails the horizontal line test, for example $f(1) = f(-1) = 1$, so the inverse is not a function.
(b) $f^{-1}(x) = x^2$, where $x \ge 0$.
(c) It fails the horizontal line test, for example $f(1) = f(-1) = 1$, so the inverse is not a function.
(d) $f^{-1}(x) = (x-1)^{\frac{1}{3}}$
(e) It fails the horizontal line test, for example $f(1) = f(-1) = 8$, so the inverse is not a function.
(f) $f^{-1}(x) = \sqrt{9 - x}$
(g) It fails the horizontal line test, for example $f(1) = f(-1) = \frac{1}{3}$, so the inverse is not a function.
(h) $f^{-1}(x) = \dfrac{1 - 3x}{1 + x}$
(i) $f^{-1}(x) = -\sqrt{x}$ **(j)** $f^{-1}(x) = 1 + \sqrt{1 + x}$

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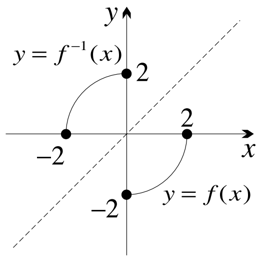
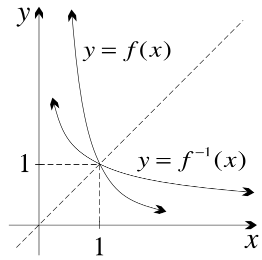
(k) $f^{-1}(x) = 1 - \sqrt{1+x}$ **(l)** $f^{-1}(x) = \dfrac{x+1}{x-1}$

12(b) The inverse of the first, $x = -y^2$, is not a function. The second is a natural restriction of the domain of the first in order that its inverse $y = \sqrt{-x}$ is a function.

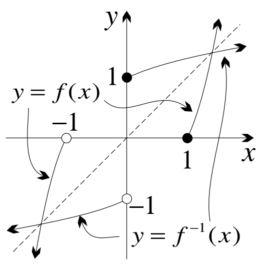
13(a) $0 \le x \le 2$



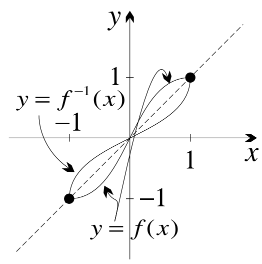
(b) $x > 0$



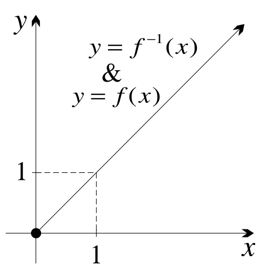
(c) $x < -1$ or $x \ge 1$



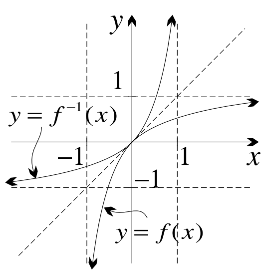
(d) $-1^\circ \le x^\circ \le 1^\circ$



(e) $x \ge 0$



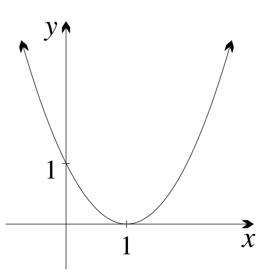
(f) $-1^\circ < x^\circ < 1^\circ$



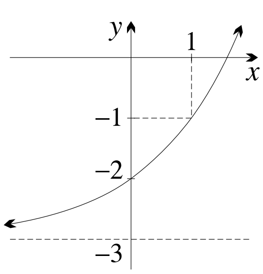
14 $\log_3(x^2) \ne 2\log_3(x)$ if $x < 0$. Instead we must write $\log_3(x^2) = 2\log_3(\sqrt{x^2})$, and neither of these functions has an inverse that is a function.

Exercise **2I** (Page 66)

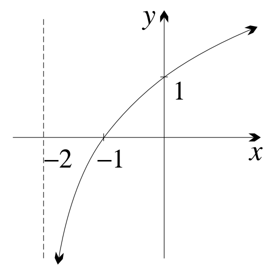
1(a) $y = (x-1)^2$



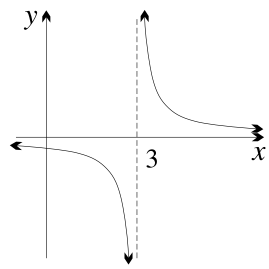
(b) $y = 2^x - 3$



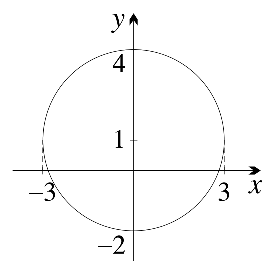
(c) $y = \log_2(x+2)$



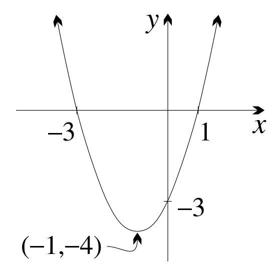
(d) $y = \frac{1}{x-3}$



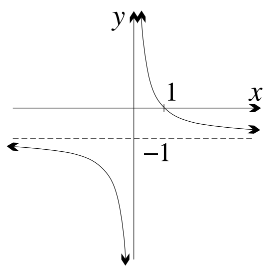
(e) $x^2 + (y-1)^2 = 9$



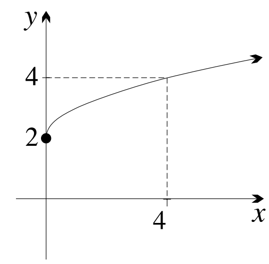
(f) $y = (x+1)^2 - 4$



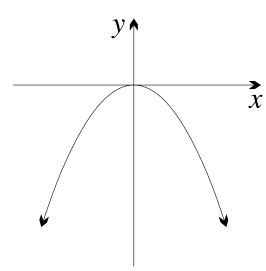
(g) $x(y+1) = 1$



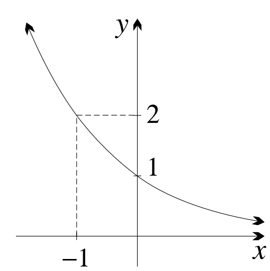
(h) $y = \sqrt{x} + 2$



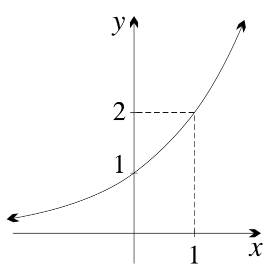
2(a) $y = -x^2$



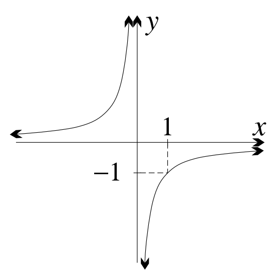
(b) $y = 2^{-x}$



(c) $y = 2^x$



(d) $y = -\frac{1}{x}$



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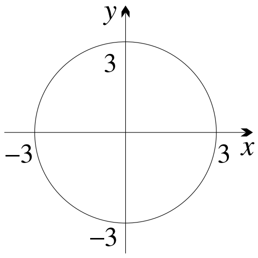
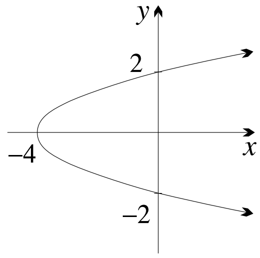
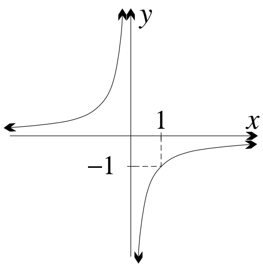
(e) $x^2 + y^2 = 9$



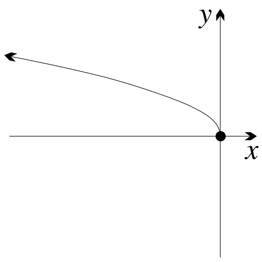
(f) $x = y^2 - 4$



(g) $-xy = 1$

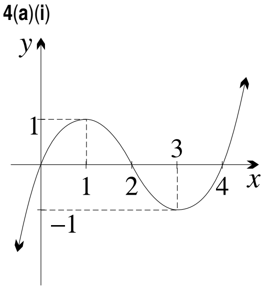


(h) $y = \sqrt{-x}$

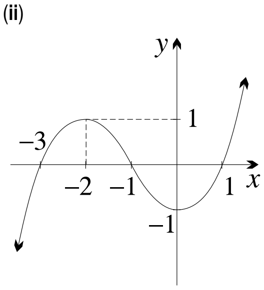


3(a) $r = 2,\ (-1, 0)$ (b) $r = 1,\ (1, 2)$
(c) $r = 3,\ (1, 2)$ (d) $r = 5,\ (-3, 4)$
(e) $r = 3,\ (5, -4)$ (f) $r = 6,\ (-7, 1)$

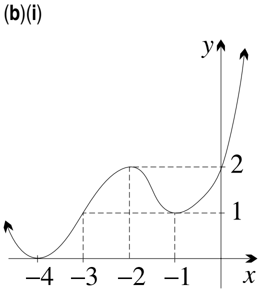
4(a)(i)



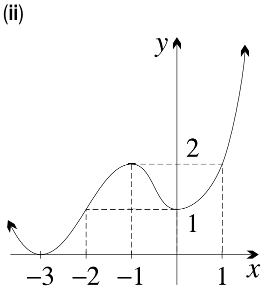
(ii)



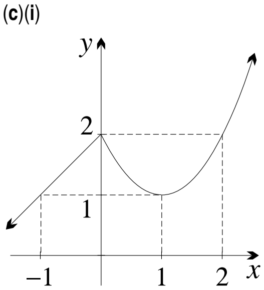
(b)(i)



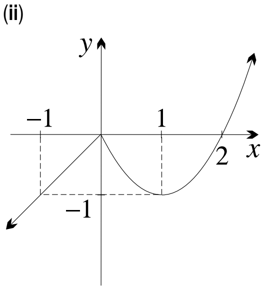
(ii)



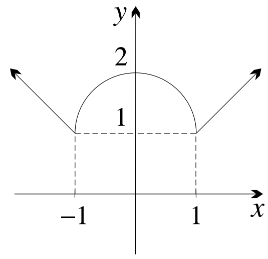
(c)(i)



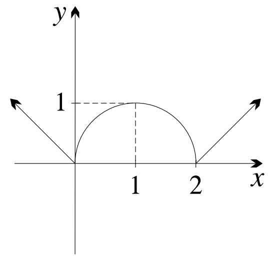
(ii)



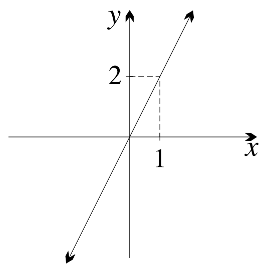
(d)(i)



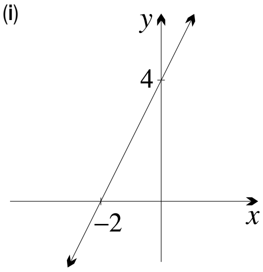
(ii)



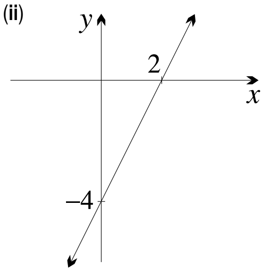
5(a) From $y = 2x$: (i) shift up 4 (or left 2)
(ii) shift down 4 (or right 2)
(iii) reflect in y-axis and shift up 4



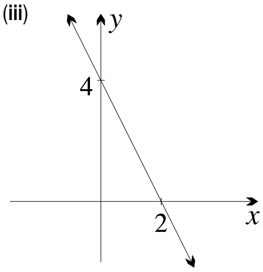
(i)



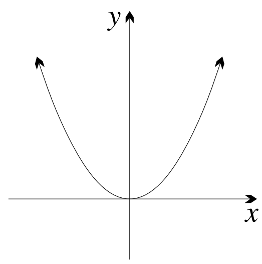
(ii)



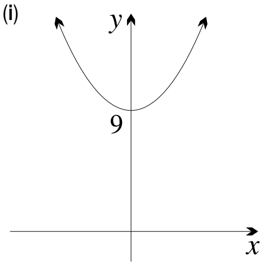
(iii)



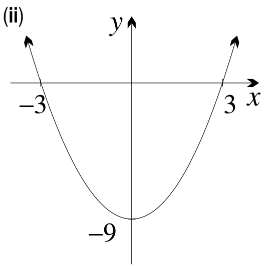
(b) From $y = x^2$: (i) shift 9 down (ii) shift 9 up
(iii) shift 3 right



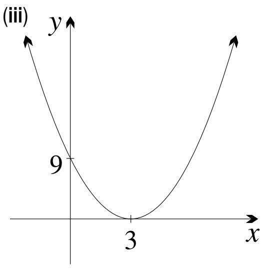
(i)



(ii)



(iii)



(c) From $y = x^2$: (i) shift 1 left
(ii) shift 1 left and reflect in x-axis
(iii) shift 1 left, reflect in x-axis and shift up 2

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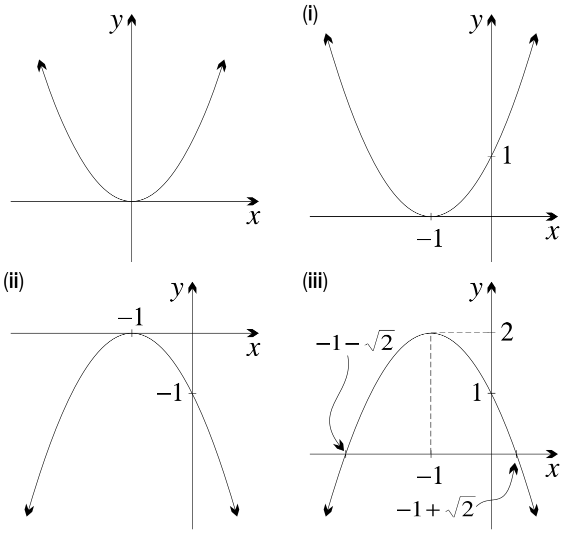
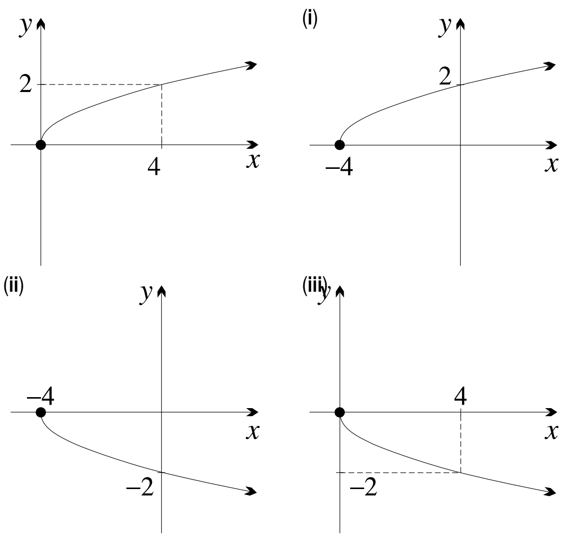
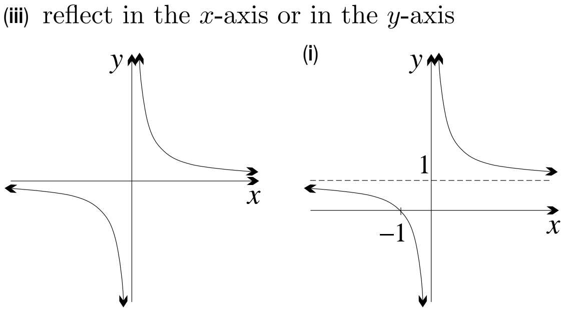
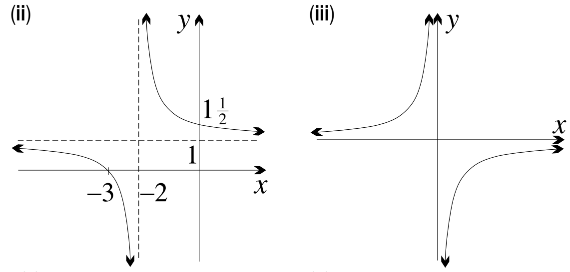


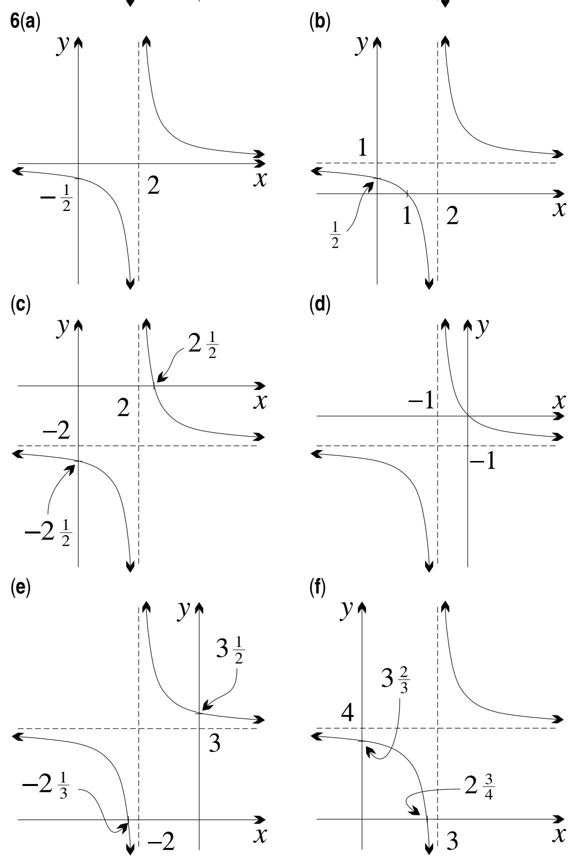
(d) From $y = \sqrt{x}$: **(i)** shift 4 left
(ii) shift 4 left and reflect in x-axis
(iii) reflect in x-axis

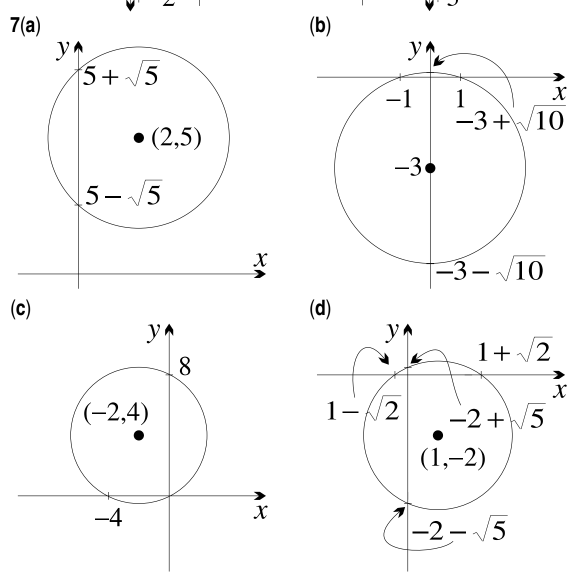


(e) From $y = \dfrac{1}{x}$: **(i)** shift up 1
(ii) shift up 1, left 2
(iii) reflect in the x-axis or in the y-axis









(a) $(x-2)^2 + (y-5)^2 = 9$, $r = 3$, centre: $(2,5)$
$x^2 + (y+3)^2 = 10$, $r = \sqrt{10}$, centre: $(0,-3)$

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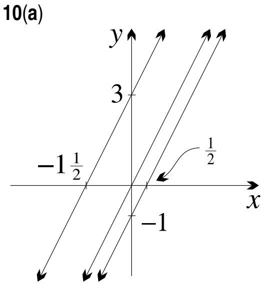
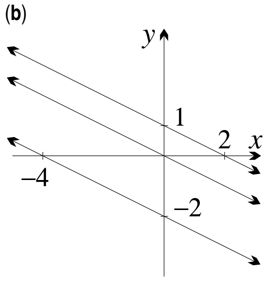
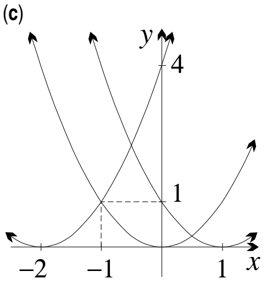
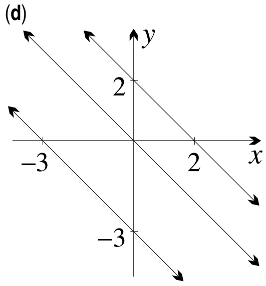
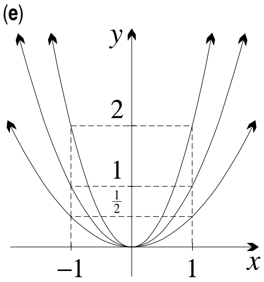
$(x+2)^2+(y-4)^2=20$, $r=2\sqrt{5}$, centre: $(-2,4)$
$(x-1)^2+(y+2)^2=6$, $r=\sqrt{6}$, centre: $(1,-2)$
8(a) $(x+1)^2+(y-2)^2=25$ **(b)** On the line $y=x$, so $\left(\frac{1+\sqrt{41}}{2}, \frac{1+\sqrt{41}}{2}\right)$ and $\left(\frac{1-\sqrt{41}}{2}, \frac{1-\sqrt{41}}{2}\right)$
9(a) The parabola $y=x^2$ shifted left 2, down 1. $y+1=(x+2)^2$
(b) The hyperbola $xy=1$ shifted right 2, down 1. $y+1=\frac{1}{x-2}$
(c) The exponential $y=2^x$ reflected in the x-axis, shifted 1 up. $y=1-2^x$
(d) The parabola $y=x^2$ reflected in $y=0$, then shifted 3 right and 1 up. $y=-(x-3)^2+1$

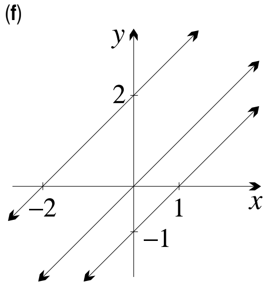
10(a)


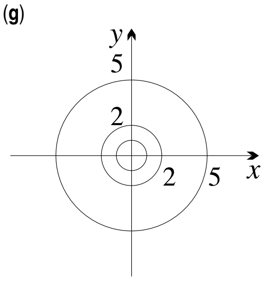
(b)


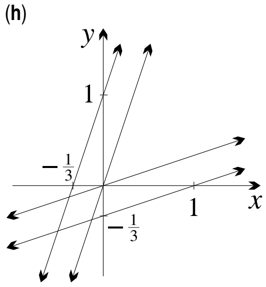
(c)


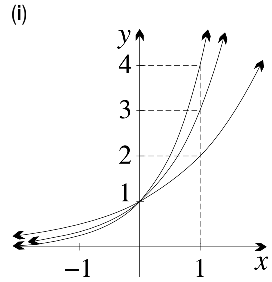
(d)


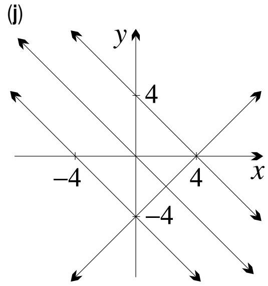
(e)


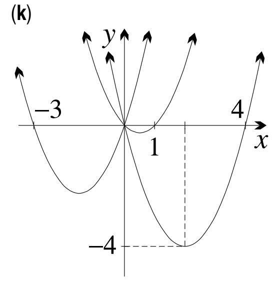
(f)


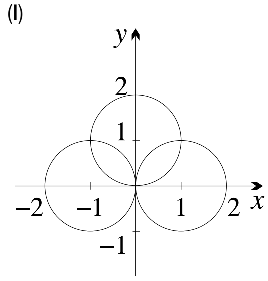
(g)


(h)


(i)


(j)


(k)


(l)


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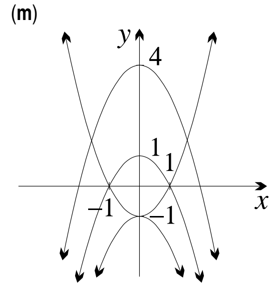

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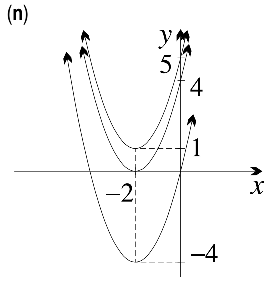

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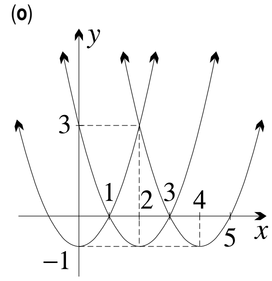

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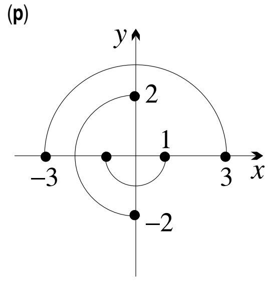

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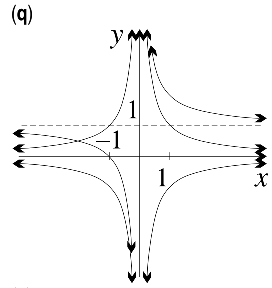
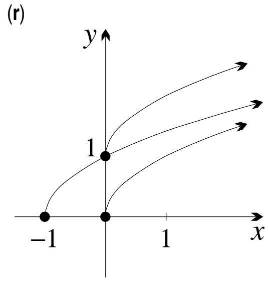


(r)


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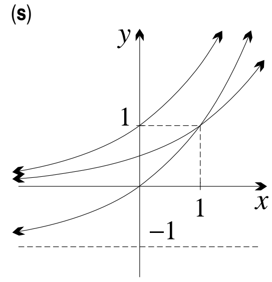

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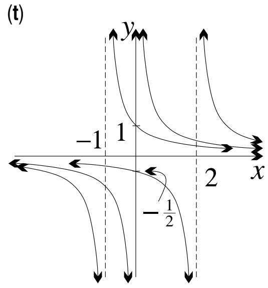

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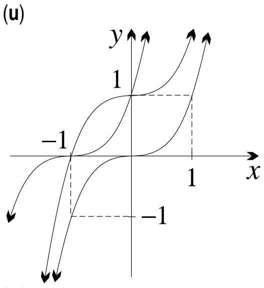
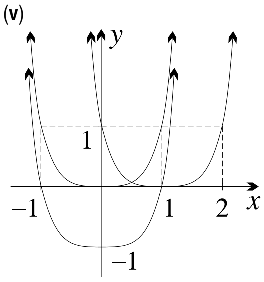
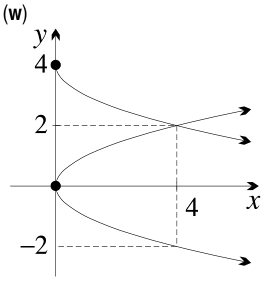
(u)



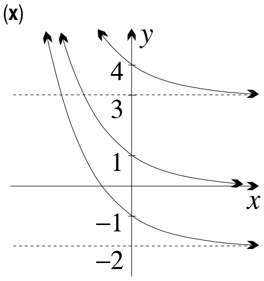
(v)



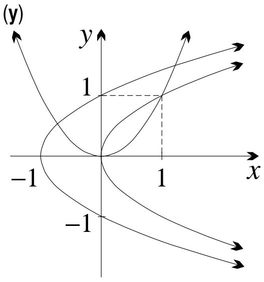
(w)



(x)



(y)



11(a) $x + 2y - 2 = 0$ **(b)** $x + 2y - 2 = 0$

(c) Both shifts yield the same result.

12 $y - y_1 = m(x - x_1)$ is the line $y = mx$ shifted right by x_1 and up by y_1.

13(a) $y - a = f(x)$, $-y - a = f(x)$, $-y = f(x)$, $y = f(x)$

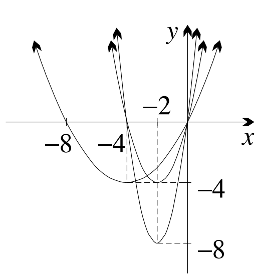
(b) $y = f(x - a)$, $-y = f(x - a)$, $-y = f(x - 2a)$, $y = f(x - 2a)$

(c) $x = f(y)$, $-x = f(y)$, $-y = f(x)$, $y = f(x)$

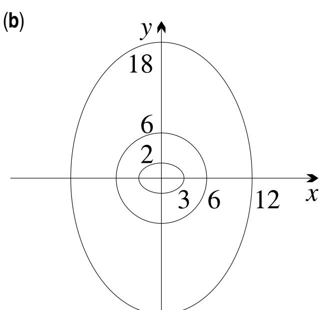
(d) $-x = f(y)$, $-y = f(-x)$, $x = f(-y)$, $y = f(x)$

Exercise **2J** (Page 71)

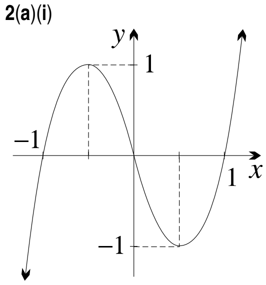
1(a)



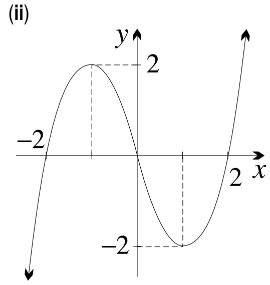
(b)



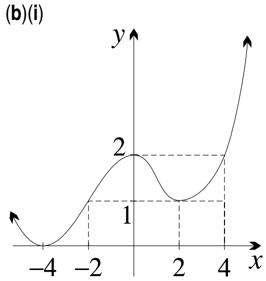
2(a)(i)



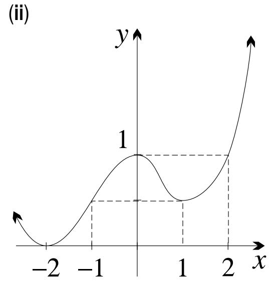
(ii)



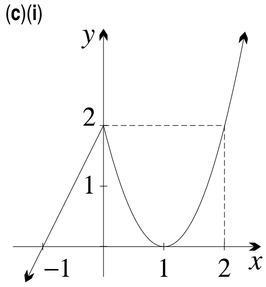
(b)(i)



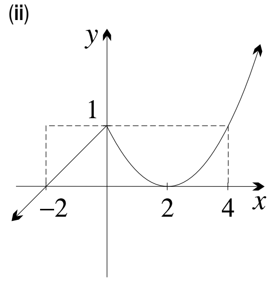
(ii)



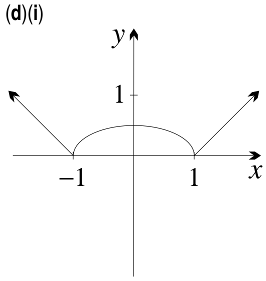
(c)(i)



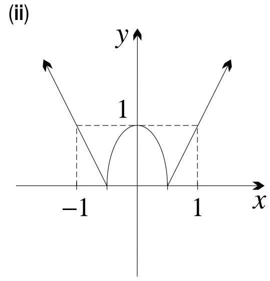
(ii)



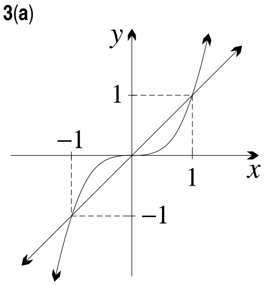
(d)(i)

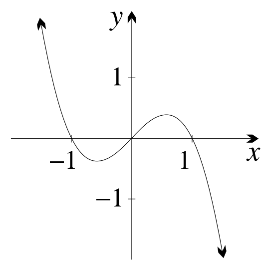


(ii)

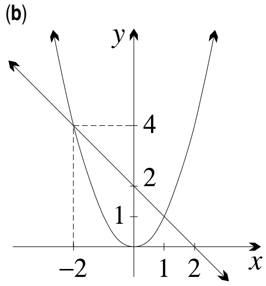


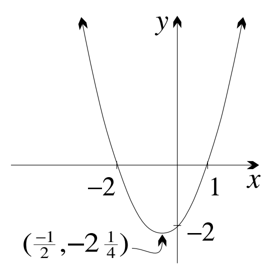
3(a)





(b)





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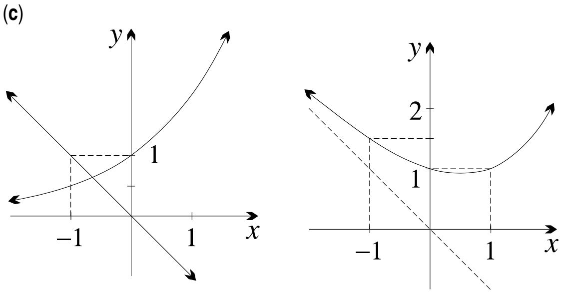
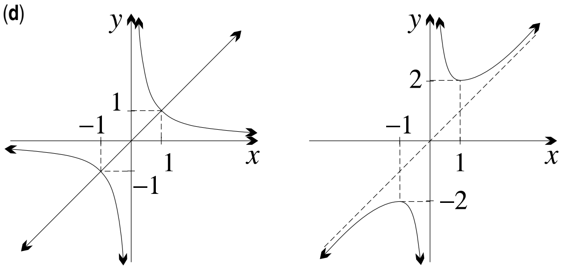
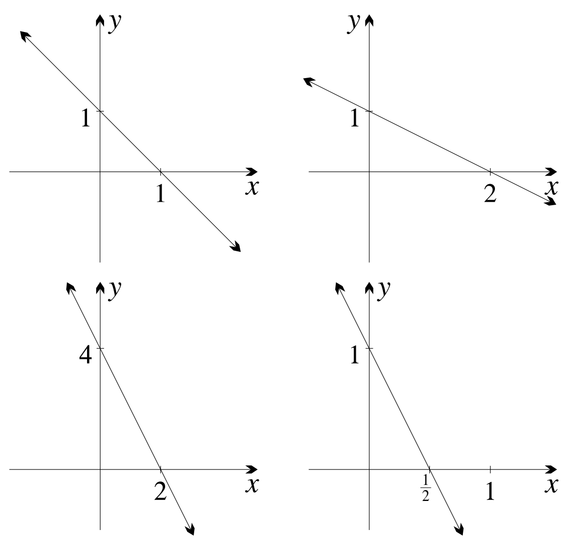
(c)



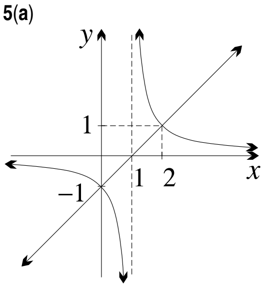
(d)



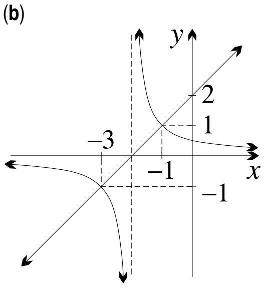
4(a) stretch horizontally by factor 2
(b) stretch horizontally by factor 2, vertically by factor 4
(c) stretch horizontally by factor $\frac{1}{2}$



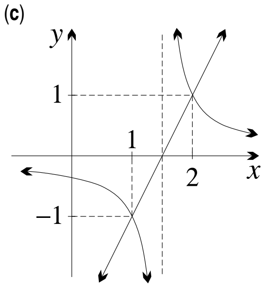
5(a)

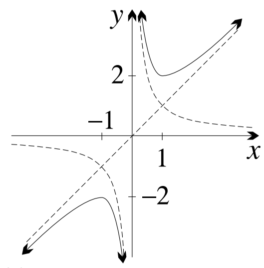


(b)

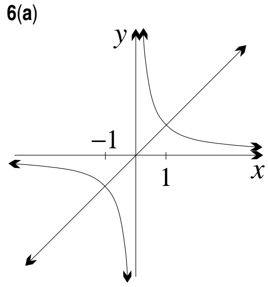


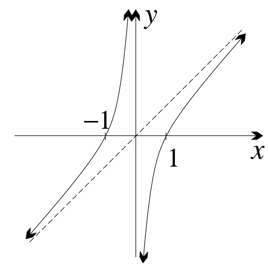
(c)





6(a)





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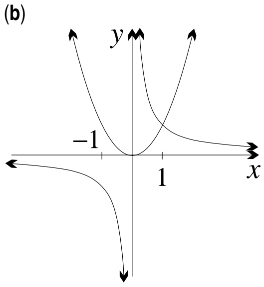
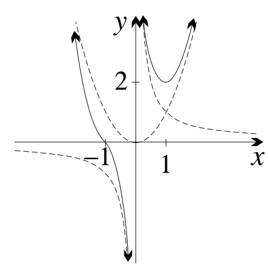
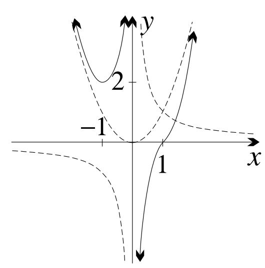
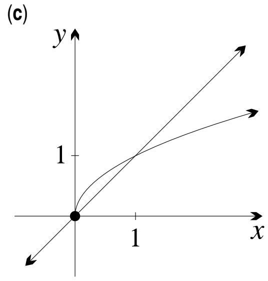
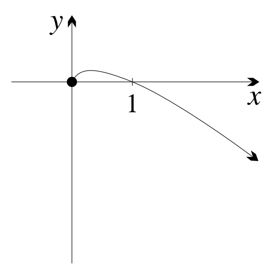




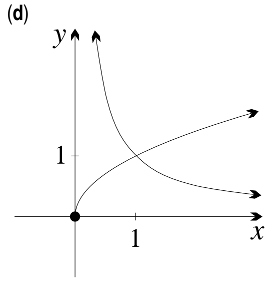


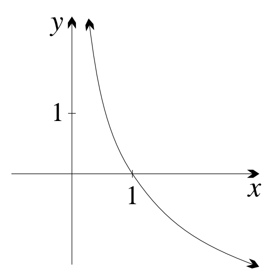
(c)



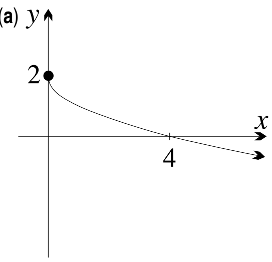


(d)





7(a)



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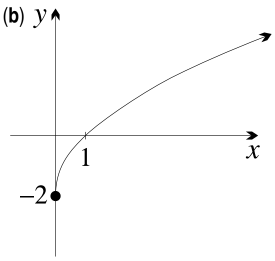
(b)



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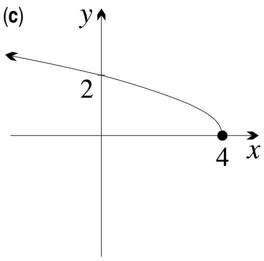
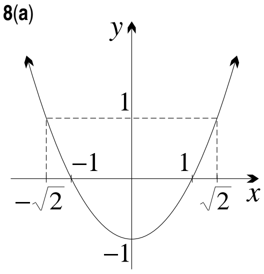
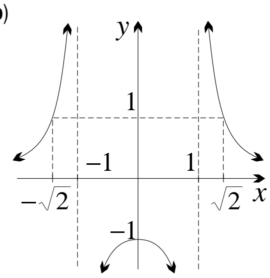


(a) reflect in the x-axis and translate up 2
(b) stretch vertically by a factor of 2, and translate down 2
(c) reflect in y-axis and translate right 4, or translate left 4 and reflect in y-axis

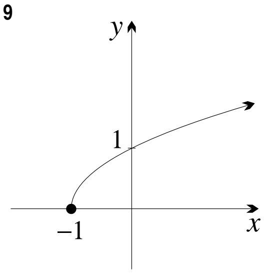
8(a)

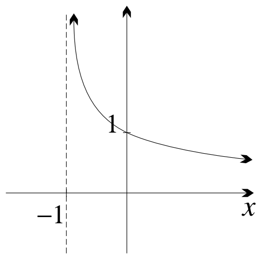


(b)

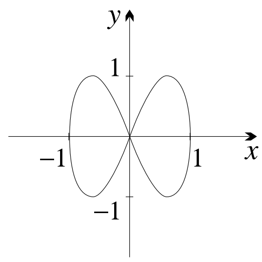


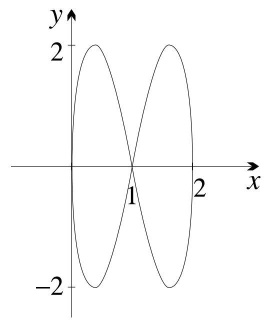
9





10(a)(i) stretch vertically by factor 2, $\frac{y}{2} = 2^x$, or translate left by 1, $y = 2^{(x+1)}$
(ii) stretch along both axes by k, $\dfrac{y}{k} = \dfrac{1}{\frac{x}{k}}$, or stretch horizontally by k^2, $y = \dfrac{1}{\frac{x}{k^2}}$
(iii) reciprocal, $y = \frac{1}{3^x}$, or reflect in the y-axis, $y = 3^{-x}$
11 stretch horizontally by factor $\sqrt{3}$ and vertically by factor $3\sqrt{3}$
12(c) $(-1, 0)$, $(0, 0)$, $(1, 0)$ **(d)** a figure eight
(e) $\Big((x-1)^2 + (\frac{y}{2})^2\Big)^2 = (x-1)^2 - (\frac{y}{2})^2$





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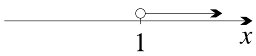
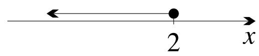
Chapter Three

Exercise **3A** (Page 76)

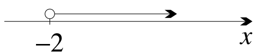
1(a) $x > 1$



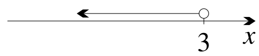
(b) $x \le 2$



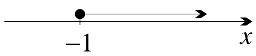
(c) $x > -2$



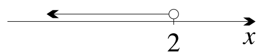
(d) $x < 3$



(e) $x \ge -1$



(f) $x < 2$



(g) $x < 2$

(h) $x \ge 3$

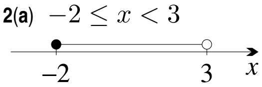
(i) $x \ge 3$

(j) $x \le -2$

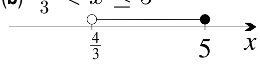
(k) $x > 2$

(l) $x \le -2$

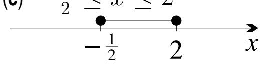
2(a) $-2 \le x < 3$



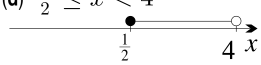
(b) $\frac{4}{3} < x \le 5$



(c) $-\frac{1}{2} \le x \le 2$

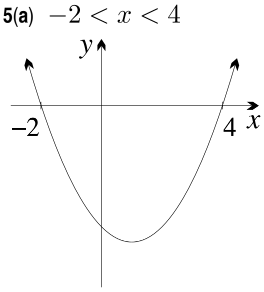


(d) $\frac{1}{2} \le x < 4$

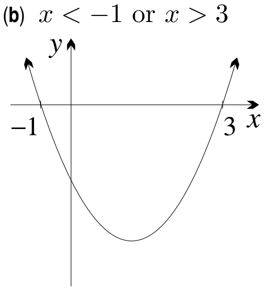


3(a) $x > 4$ **(b)** $x \le 2$ **(c)** $x < 2$ **(d)** $x \le -1$
(e) $-2 \le x < 1$ **(f)** $-6 \le x \le 15$
4(a) $0 < x < 4$ **(b)** $x \le -1$ or $x \ge 3$
(c) $x \le 0$ or $x \ge 2$

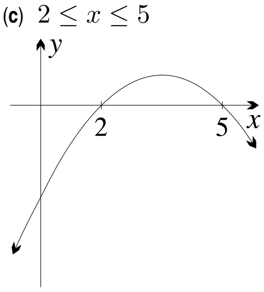
5(a) $-2 < x < 4$



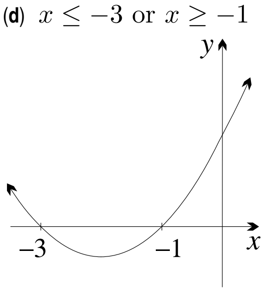
(b) $x < -1$ or $x > 3$



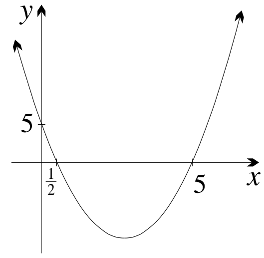
(c) $2 \le x \le 5$



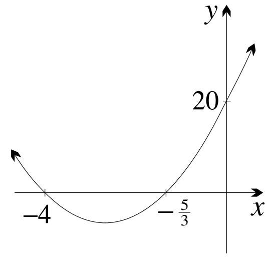
(d) $x \le -3$ or $x \ge -1$



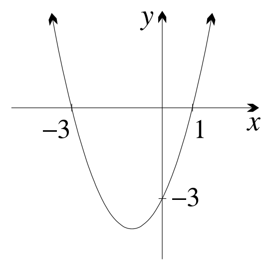
(e) $x < \frac{1}{2}$ or $x > 5$



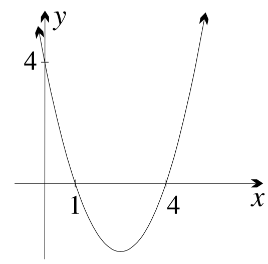
(f) $-4 \le x \le -\frac{5}{3}$



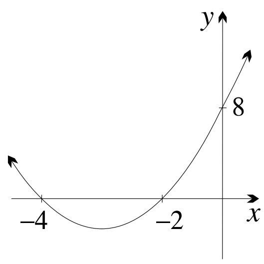
6(a) $-3 < x < 1$



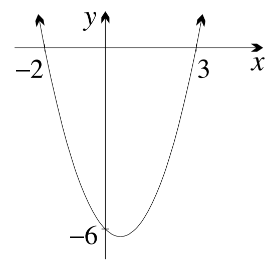
(b) $x \le 1$ or $x \ge 4$



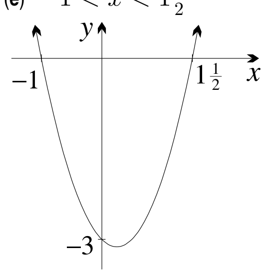
(c) $x < -4$ or $x > -2$



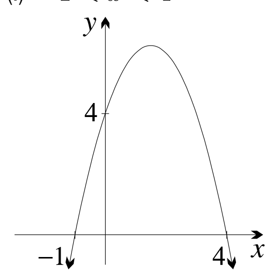
(d) $-2 \le x \le 3$



(e) $-1 < x < 1\frac{1}{2}$



(f) $-1 < x < 4$



7(a) $-1 \le x \le 1$ **(b)** $x < 0$ or $x > 3$
(c) $x \le -12$ or $x \ge 12$
(d) $x < 0$ or $x > 0$ (or simply $x \ne 0$)
(e) $x = 3$ **(f)** $1 \le x \le 3$
8(a) $x < 0$ or $x \ge \frac{1}{2}$ **(b)** $3 < x < 5$
(c) $-4 < x \le -2\frac{1}{2}$ **(d)** $x < \frac{3}{2}$ or $x > 4$
(e) $1 < x < 3$ **(f)** $\frac{5}{3} < x \le 3$
9 The curve is always above the line.
10(a) false: $x = 0$ **(b)** false: $x = \frac{1}{2}$
(c) true **(d)** false: $x = \frac{1}{2}$ or $x = -2$
(e) false: $x = -1$ **(f)** true **(g)** false: $x = -1$
(h) true
13(a) $\frac{1}{2} < x \le 3$ **(b)** $-3 < x < -2$
(c) $x < 1$ or $x \ge 3$ **(d)** $x < -\frac{1}{7}$ or $x > 2$

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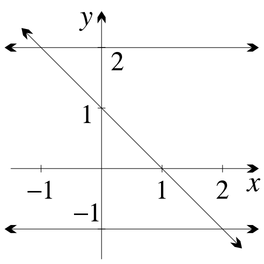
14 The two lines are parallel and thus the first is always below the second.

15(a)



(b) $-1 \le x < 2$. The solution to the inequation is where the diagonal line lies between the horizontal lines.

16 $5x - 4 < 7 - \frac{1}{2}x$, with solution $x < 2$

17(a) $x \ge 3$ **(b)** $0 < x \le 3$ **(c)** $-4 \le x \le 4$ **(d)** $x < -4$ **(e)** $0 < x < 8$ **(f)** $\frac{1}{25} \le x \le 625$

18(a) true **(b)** false: $a = -2$, $b = -1$ **(c)** true **(d)** false: $a = -1$, $b = 1$ **(e)** true **(f)** false: $a = 1$, $b = 2$

19(a) $-4 \le 4t < 12$ **(b)** $-3 < -t \le 1$ **(c)** $6 \le t + 7 < 10$ **(d)** $-3 \le 2t - 1 < 5$ **(e)** $0 \le \frac{1}{2}(t+1) < 2$ **(f)** $-2 \le \frac{1}{2}(3t-1) < 4$ **(g)** $\frac{1}{2} \le 2^t < 8$ **(h)** $0 \le \sqrt{t+1} < 2$

20(a) $7 < x^2 + 3 < 19$ **(b)** $3 \le x^2 + 3 \le 12$

21(b)(i) Either $x^2 > xy > y^2$, or $x^2 - y^2 = (x+y)(x-y) > 0$ so $x^2 > y^2$, or otherwise. **(ii)** $n > 0$

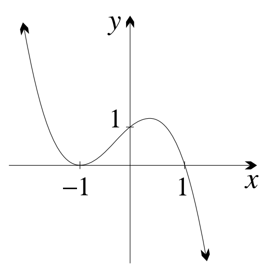
22(a) Put $x = \sqrt{a}$ and $y = \frac{1}{\sqrt{a}}$. **(b)** Put $x = \sqrt{a}$ and $y = \sqrt{b}$.

23 $x^2 + xy + y^2 = \frac{1}{2}(x^2 + y^2) + \frac{1}{2}(x+y)^2$ or otherwise.

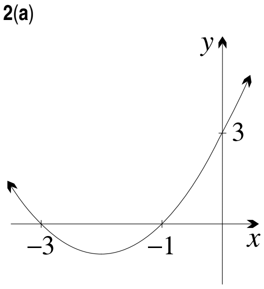
25(a) $2(a^2 + b^2 + c^2 - ab - bc - ac)$ **(b)** $2(a^3 + b^3 + c^3 - 3abc)$

Exercise **3B** (Page 81)

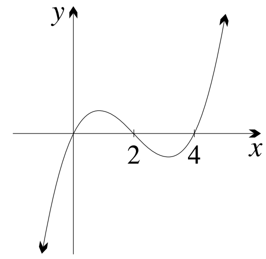
1



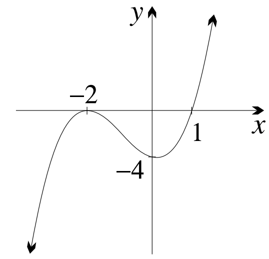
2(a)



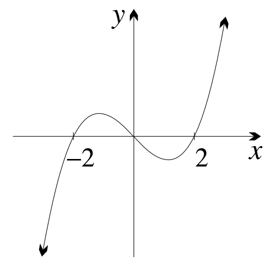
(b)



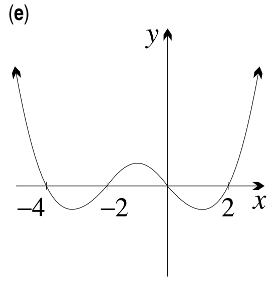
(c)



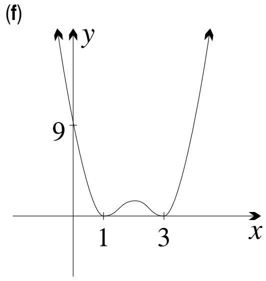
(d)



(e)

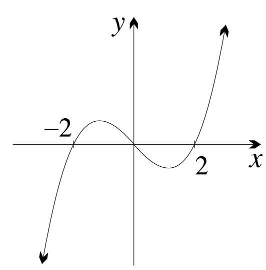


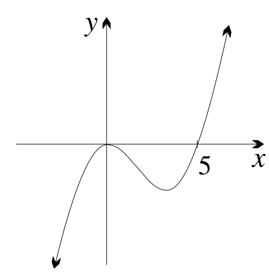
(f)



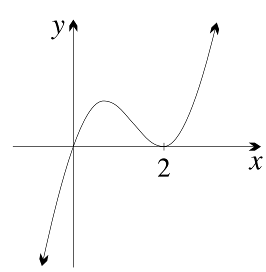
3(a) $x \le 0$ or $1 \le x \le 2$ **(b)** $-2 < x < 0$ or $2 < x < 4$ **(c)** $0 < x < 3$ or $x > 3$ **(d)** $x = 0$ or $x \ge 4$ **(e)** $x = -3$ or $x = 3$ **(f)** $x = -2$ or $x \ge 0$

4(a) $f(x) = x(x-2)(x+2)$ **(b)** $f(x) = x^2(x-5)$





(c) $f(x) = x(x-2)^2$



5(a) $-2 < x < 0$ or $x > 2$ **(b)** $x < 0$ or $0 < x < 5$ **(c)** $x \le 0$ or $x = 2$

6(a) $x < 1$ or $3 < x < 5$ **(b)** $x \ne 1$ and $x \ne 3$ (alternatively, $x < 1$ or $1 < x < 3$ or $x > 3$) **(c)** $-2 < x \le 4$ **(d)** $-3 < x < 0$ or $x > 3$ **(e)** $-3 < x < -1$ **(f)** $x < 0$ or $0 < x < 5$

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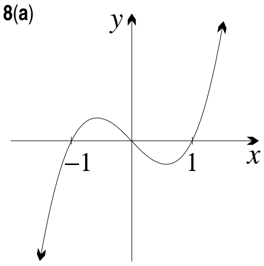
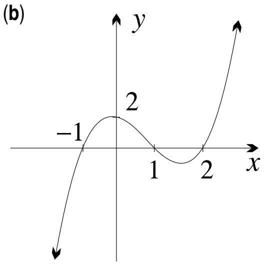
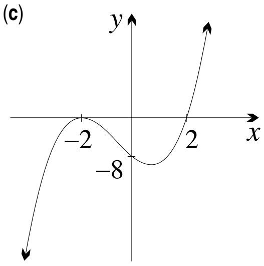
(g) $x \le 0$ or $x \ge 5$ **(h)** $-2 \le x < 0$ or $x \ge 2$
(i) $x < -3$ or $0 < x \le 2$
7(a) $y = x(x+1)(x-1),\ x = -1, 0$ or 1
(b) $y = (x-2)(x-1)(x+1),\ x = -1, 1$ or 2
(c) $y = (x+2)^2(x-2),\ x = -2$ or 2

8(a)


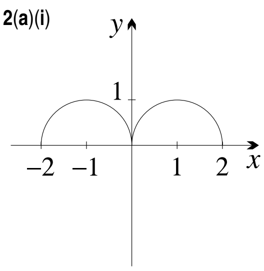
(b)


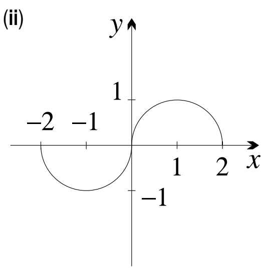
(c)


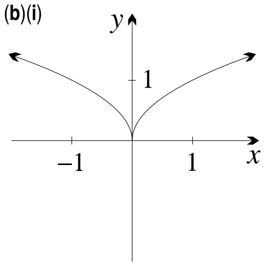
9(a) zero for $x = 0$, undefined at $x = 3$, positive for $x < 0$ or $x > 3$, negative for $0 < x < 3$
(b) zero for $x = 4$, undefined at $x = -2$, positive for $x < -2$ or $x > 4$, negative for $-2 < x < 4$
(c) zero for $x = -3$, undefined at $x = -1$, positive for $x < -3$ or $x > -1$, negative for $-3 < x < -1$
10(a) $x \le -4$ or $-3 < x \le 1$
(b) $-2 < x < -1\frac{1}{2}$ or $x > \frac{1}{2}$
(c) $-\frac{1}{2} \le x < 1\frac{1}{2}$ or $x \ge 2\frac{1}{2}$

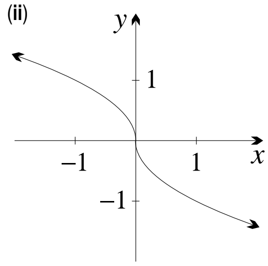
Exercise **3C** (Page 84)

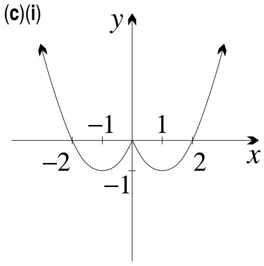
1(a) $x \ne -1$ **(b)** $x \ne \frac{3}{2}$ **(c)** all real numbers
(d) all real numbers **(e)** $x \ge 0$ **(f)** $x \ge 1$
(g) $x \le 7$ **(h)** $x \ge -4$

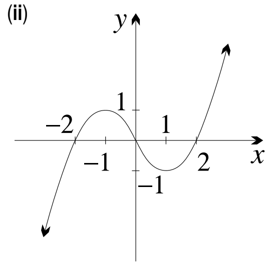
2(a)(i)


(ii)


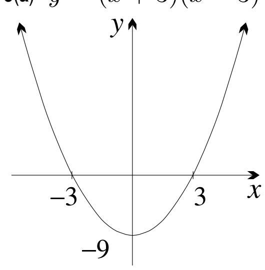
(b)(i)


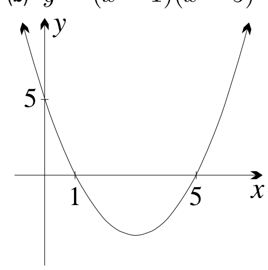
(ii)


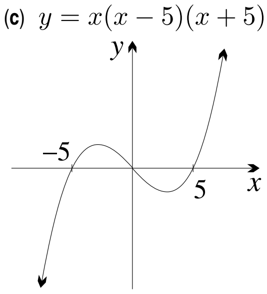
(c)(i)


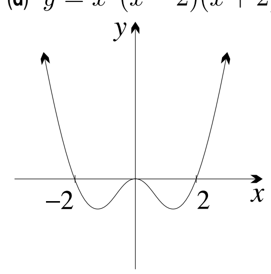
(ii)


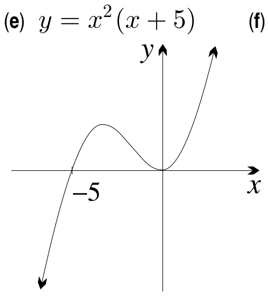
4(a) even **(b)** neither **(c)** odd **(d)** even
(e) neither **(f)** odd **(g)** odd **(h)** neither
If a function is a sum of multiples of odd powers of x, then it is odd. If it is a sum of multiples of even powers, then it is even. If the sum involves even and odd powers, then it is neither.

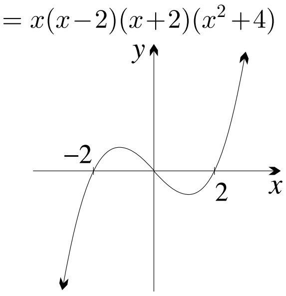
5(a) $y = (x+3)(x-3)$


(b) $y = (x-1)(x-5)$


(c) $y = x(x-5)(x+5)$


(d) $y = x^2(x-2)(x+2)$


(e) $y = x^2(x+5)$


(f) $y = x(x-2)(x+2)(x^2+4)$


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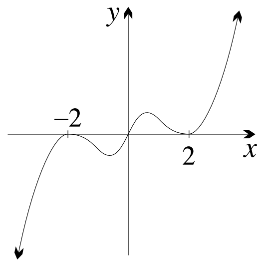
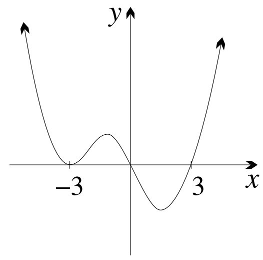
(g) $y = x(x-2)^2(x+2)^2$



(h) $y = x(x-3)(x+3)^2$

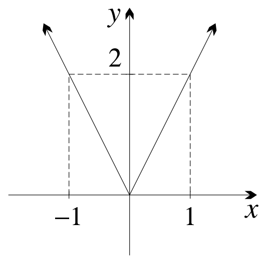


6(a) $x \neq 1$ **(b)** $x \neq 4$ **(c)** $x \neq -1$ **(d)** $x \neq 2$
(e) $x > -4$ **(f)** $x > 1$ **(g)** all real x **(h)** $x \neq 3$
7(a) $x \leq -2$ or $x \geq 2$ **(b)** $x < -2$ or $x > 2$
8(a) $-2 \leq x \leq 2$ **(b)** $-2 < x < 2$ **(c)** $-5 \leq x \leq 5$
(d) $-5 < x < 5$ **(e)** $x \leq -2$ or $x \geq 2$
(f) $x < -2$ or $x > 2$
9(a) even **(b)** even **(c)** odd **(d)** neither
10(a)(i) even **(ii)** even **(iii)** odd **(b)(i)** even **(ii)** odd
(iii) in general, neither
11(a) Suppose $f(0) = c$. Then since $f(x)$ is odd, $f(0) = -f(0) = -c$. So $c = -c$, and hence $c = 0$.
(b) It is not defined at the origin (it is 1 for $x > 0$, and -1 for $x < 0$).
12(a) Let $y = f^{-1}(-x)$. Then $-x = f(y)$, from which it follows $x = f(-y)$ since f is odd. Hence $-y = f^{-1}(x)$, and thus $f^{-1}(-x) = -f^{-1}(x)$ as required. **(b)** The graph will fail the horizontal line test unless it is a single point on the y-axis.
13(b)(i) $g(x) = 1 + x^2$ and $h(x) = -2x$
(ii) $g(x) = \frac{2^x + 2^{-x}}{2}$ and $h(x) = \frac{2^x - 2^{-x}}{2}$
(c) In the first, $g(x)$ and $h(x)$ are not defined for all x in the natural domain of $f(x)$, specifically at $x = -1$. In the second, $x = 0$ is the only place at which $g(x)$ and $h(x)$ are defined.

Exercise 3D (Page 89)

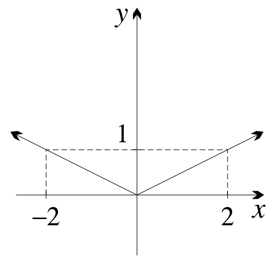
1(a) For $|x-2|$: 3, 2, 1, 0, 1.
For $|x| - 2$: −1, −2, −1, 0, 1.
(b) The first is $y = |x|$ shifted right 2 units, the second is $y = |x|$ shifted down 2 units.
2(a) 5 **(b)** 3 **(c)** 7 **(d)** 3 **(e)** 3 **(f)** 3 **(g)** 16 **(h)** −3
3(a) $x = 3$ or -3 **(b)** $x = 5$ or -5
(c) $x = 10$ or -4 **(d)** $x = 5$ or -7
(e) $x = 6$ or -5 **(f)** $x = 2$ or $-3\frac{1}{3}$
(g) $x = \frac{7}{5}$ or $-\frac{11}{5}$ **(h)** $x = 2$ or $-\frac{8}{7}$
4(a) false: $x = 0$ **(b)** true **(c)** true
(d) false: $x = -2$ **(e)** true **(f)** true
(g) false: $x = -2$ **(h)** true

5(a)



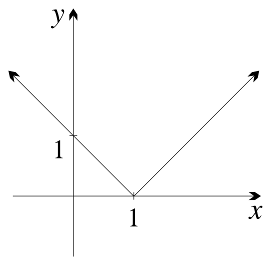
$y = \begin{cases} 2x, & \text{for } x \geq 0, \\ -2x, & \text{for } x < 0. \end{cases}$

(b)



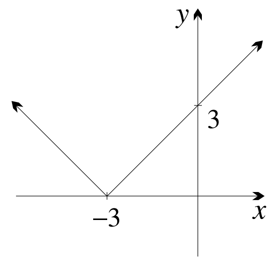
$y = \begin{cases} \frac{1}{2}x, & \text{for } x \geq 0, \\ -\frac{1}{2}x, & \text{for } x < 0. \end{cases}$

(c)



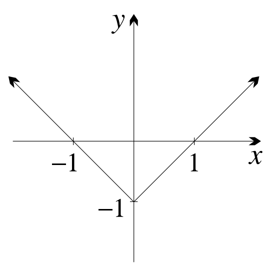
$y = \begin{cases} x-1, & \text{for } x \geq 1, \\ 1-x, & \text{for } x < 1. \end{cases}$

(d)



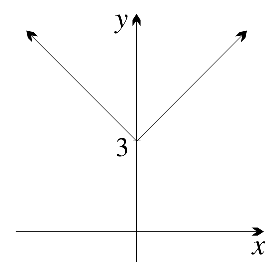
$y = \begin{cases} x+3, & \text{for } x \geq -3, \\ -x-3, & \text{for } x < -3. \end{cases}$

(e)



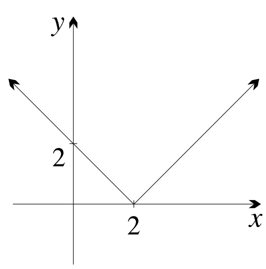
$y = \begin{cases} x-1, & \text{for } x \geq 0, \\ -x-1, & \text{for } x < 0. \end{cases}$

(f)



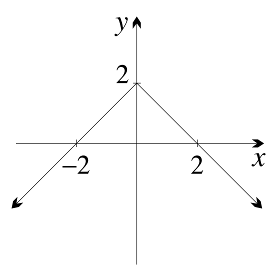
$y = \begin{cases} x+3, & \text{for } x \geq 0, \\ 3-x, & \text{for } x < 0. \end{cases}$

(g)



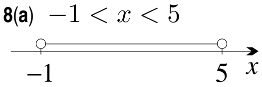
$y = \begin{cases} x-2, & \text{for } x \geq 2, \\ 2-x, & \text{for } x < 2. \end{cases}$

(h)

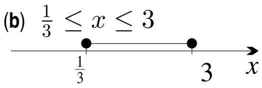


$y = \begin{cases} 2-x, & \text{for } x \geq 0, \\ 2+x, & \text{for } x < 0. \end{cases}$

6 An absolute value can not be negative.
7(a) even **(b)** neither **(c)** odd **(d)** even
8(a) $-1 < x < 5$



(b) $\frac{1}{3} \leq x \leq 3$



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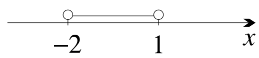
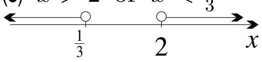
(c) $x \ge 9$ or $x \le 5$

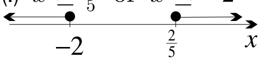
(d) $-2 < x < 1$



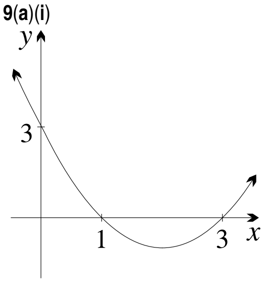
(e) $x > 2$ or $x < \frac{1}{3}$



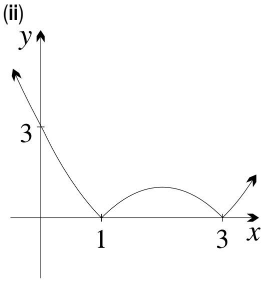
(f) $x \ge \frac{2}{5}$ or $x \le -2$



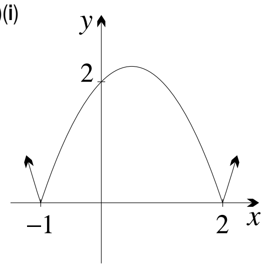
9(a)(i)



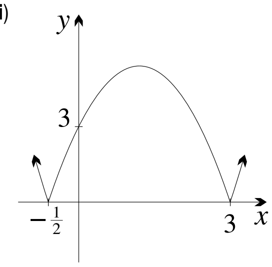
(ii)



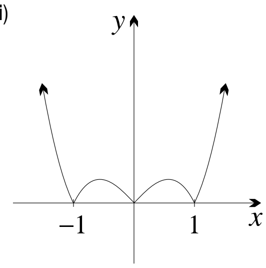
(b)(i)



(ii)



(iii)

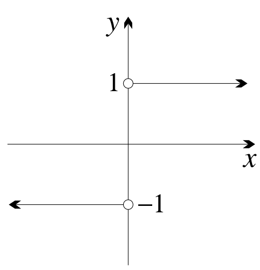


10(a) The first holds when x is positive, the second when x is negative.

(b)(i) $-2 < x < 2$ or $-10 < x < -6$

(ii) $3 \le x < 4\frac{1}{2}$ or $\frac{1}{2} < x \le 2$

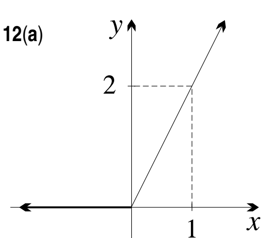
11



(a) y is undefined for $x = 0$.

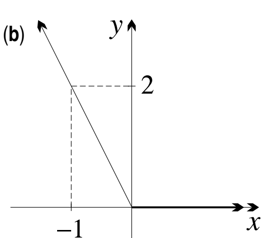
(c) $y = 1$, for $x > 0$, and $y = -1$, for $x < 0$.

12(a)



$y = 2x$, for $x \ge 0$,
$y = 0$, for $x < 0$.

(b)



$y = 0$, for $x \ge 0$,
$y = -2x$, for $x < 0$.

13(a) false: $x = 2$ and $y = -2$ **(b)** true

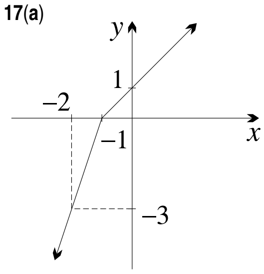
(c) false: $x = 2$ and $y = -2$ **(d)** true **(e)** true

(f) false: $x = -2$

14(a) $x \ne 1$

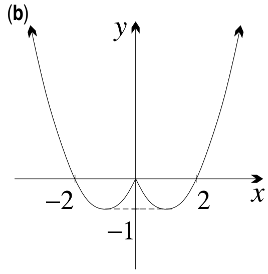
(b) $y = \frac{1}{x-1}$, for $x > 1$, and $y = \frac{1}{1-x}$, for $x < 1$.

17(a)



$$y = \begin{cases} x+1, & \text{for } x \ge -1, \\ 3(x+1), & \text{for } x < -1. \end{cases}$$

(b)

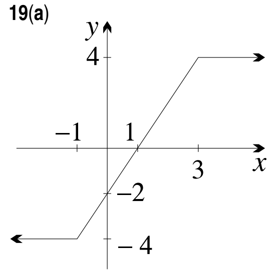


$$y = \begin{cases} x^2 - 2x, & \text{for } x \ge 0, \\ x^2 + 2x, & \text{for } x < 0. \end{cases}$$

18(a) An absolute value must be positive.

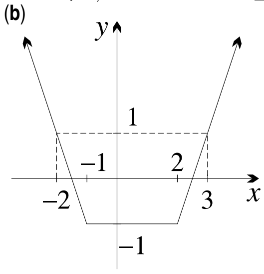
(b) $x > 1$

19(a)



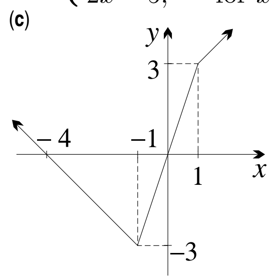
$$y = \begin{cases} -4, & \text{for } x < -1, \\ 2x - 2, & \text{for } -1 \le x < 3, \\ 4, & \text{for } x \ge 3. \end{cases}$$

(b)



$$y = \begin{cases} -2x - 3, & \text{for } x < -1, \\ -1, & \text{for } -1 \le x < 2, \\ 2x - 5, & \text{for } x \ge 2. \end{cases}$$

(c)



$$y = \begin{cases} -x - 4, & \text{for } x < -1, \\ 3x, & \text{for } -1 \le x < 1, \\ x + 2, & \text{for } x \ge 1. \end{cases}$$

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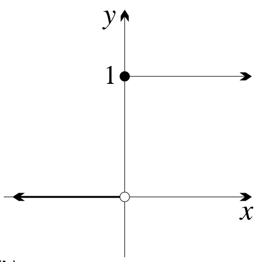
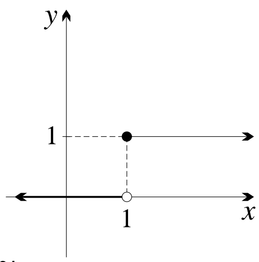
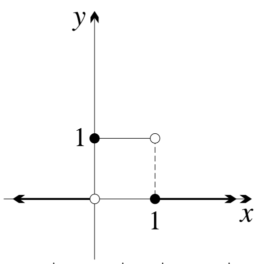
20(a)(i)



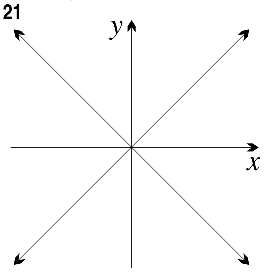
(ii)



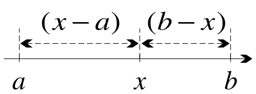
(b)



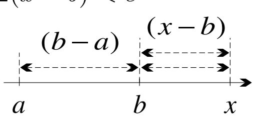
21



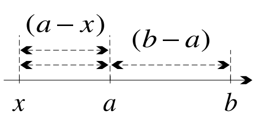
22(a) $|x-a|+|x-b|=(x-a)+(b-x)<c$



(b) $|x-a|+|x-b|=(x-a)+(x-b)=(b-a)+2(x-b)<c$



(c) $|x-a|+|x-b|=(a-x)+(b-x)=(b-a)+2(a-x)<c$

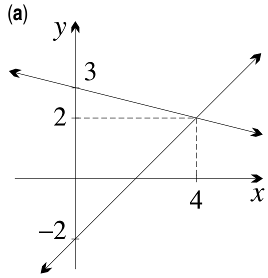


(d) The result follows directly from parts (a), (b) and (c). **(e)** $-3<x<7$

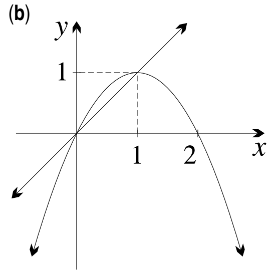
Exercise 3E (Page 93)

1(a) $\sqrt{2} \doteqdot 1{\cdot}4$, $\sqrt{3} \doteqdot 1{\cdot}7$ **(b)** $y=2$ and $y=3$
(c) $x=-1$ or $x=2$ **(d)** $x<-1$ or $x>2$
(e) $x=-2$ or $x=1$, $-2\le x\le 1$
(f) $x \doteqdot 1{\cdot}62$ or $x \doteqdot -0{\cdot}62$
(g)(i) Draw $y=-x$, $x=0$ or $x=-1$.
(ii) Draw $y=x+\frac{1}{2}$, $x \doteqdot 1{\cdot}37$ or $x \doteqdot -0{\cdot}37$.
(iii) Draw $y=\frac{1}{2}x+\frac{1}{2}$, $x=1$ or $x=-\frac{1}{2}$.
2(a) $x\le -3$ **(b)** $0\le x\le 2$ **(c)** $x=1$
3(a) $x<-2$ or $x>1$ **(b)** $0\le x\le 1$
(c) $-1<x<0$ or $x>1$
4(a) $(4,2)$, $x-2=3-\frac{1}{4}x$ **(b)** $(0,0)$ and $(1,1)$, $x=2x-x^2$ **(c)** $(-1,-2)$ and $(2,1)$, $\frac{2}{x}=x-1$
(d) $(-1,-1)$ and $(0,0)$ and $(1,1)$ $x^3=x$

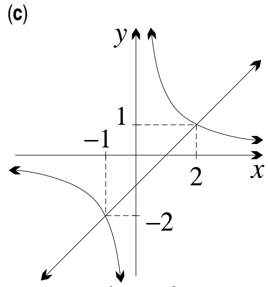
(a)



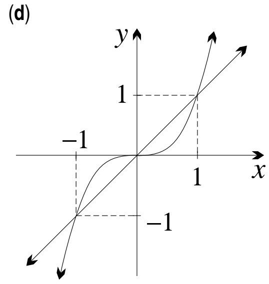
(b)



(c)

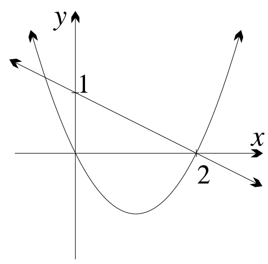


(d)

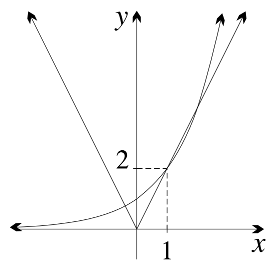


5(a) $x\ge 4$ **(b)** $0<x<1$ **(c)** $x<-1$ or $0<x<2$
(d) $-1<x<0$ or $x>1$

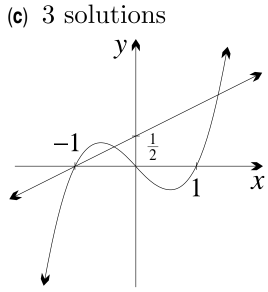
6(a) 2 solutions



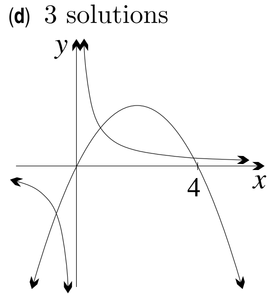
(b) 3 solutions



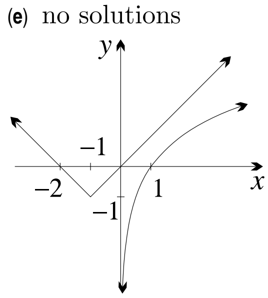
(c) 3 solutions



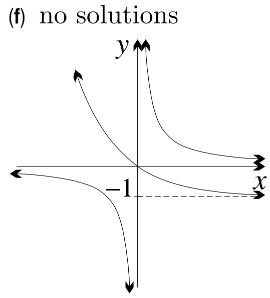
(d) 3 solutions



(e) no solutions



(f) no solutions



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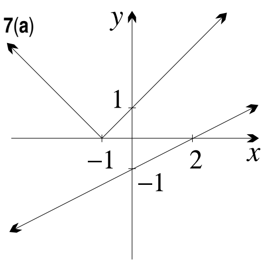
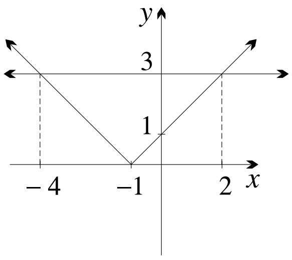
7(a)

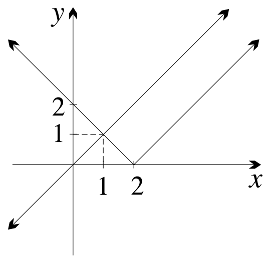


(b) The graph of $y = |x+1|$ is always above the graph of $y = \frac{1}{2}x - 1$.

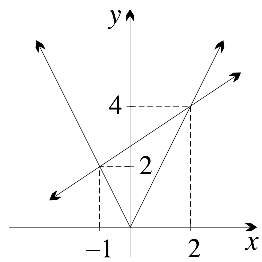
8(a) $(-4,3)$, $(2,3)$



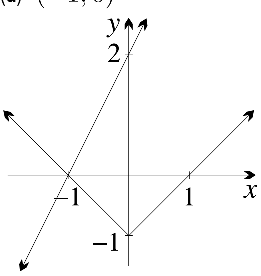
(b) $(1,1)$



(c) $(-1,2)$, $(2,4)$



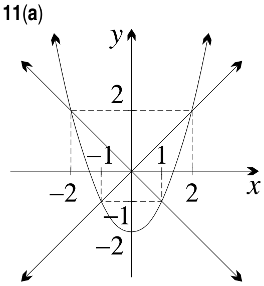
(d) $(-1,0)$



9(a) $-4 \le x \le 2$ **(b)** $x < 1$ **(c)** $x \le -1$ or $x \ge 2$ **(d)** $x < -1$

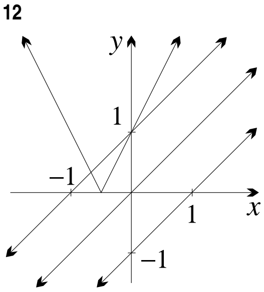
10(b) The right-hand branch is $y = x$, which gives solution $x = 3$, and the left-hand branch is $y = -x$, which gives solution $x = -3$.
(c) $-3 \le x \le 3$

11(a)



(b) $x = 2$ or -2
(c) $x < -2$ or $x > 2$

12



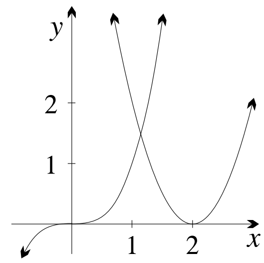
(c) $c > \frac{1}{2}$

13(b) $b^2 < \frac{9}{2}$

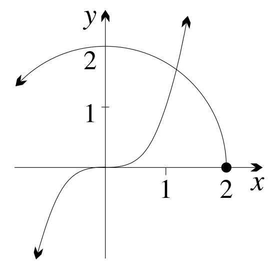
14(a) 2
(b) The solutions are not integers.
(c) $x = \frac{1}{11}$ or $\frac{7}{3}$

15(a) $x \le 2\frac{1}{2}$ **(b)** $x \le -4$ or $x \ge 0$
(c) $-2 < x < \frac{2}{3}$ **(d)** $x < -4$ or $x > \frac{2}{3}$

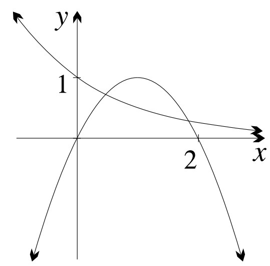
16(a) $x \doteqdot 1{\cdot}1$



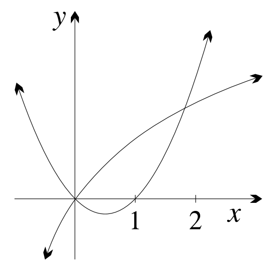
(b) $x \doteqdot 1{\cdot}2$



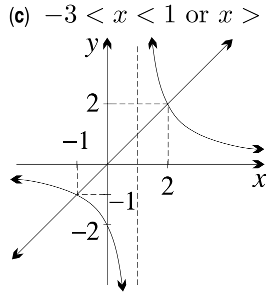
(c) $x \doteqdot 0{\cdot}5$ or $x \doteqdot 1{\cdot}9$

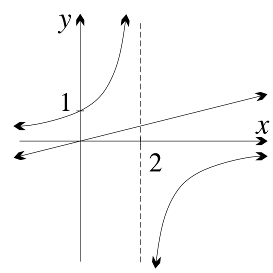


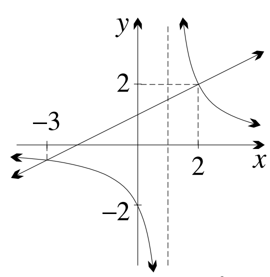
(d) $x = 0$ or $x \doteqdot 1{\cdot}8$



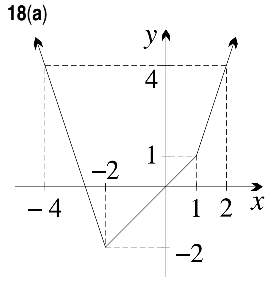
17(a) $-1 \le x < 1$ or $x \ge 2$ **(b)** $x < 2$
(c) $-3 < x < 1$ or $x > 2$







18(a)

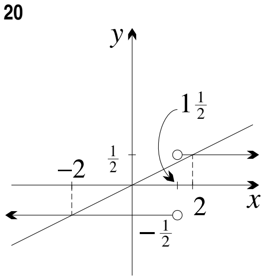


$$y = \begin{cases} -3x - 8, & \text{for } x < -2, \\ x, & \text{for } -2 \le x < 1, \\ 3x - 2, & \text{for } x \ge 1. \end{cases}$$

(b) $-3\frac{1}{3} \le x \le -2\frac{1}{3}$ or $-1 \le x \le 1\frac{1}{3}$

19(b) $b < m$ **(c)** $-p \le m \le p$ and $b < -\frac{qm}{p}$

20 $x \le -2$ or $1\frac{1}{2} < x \le 2$



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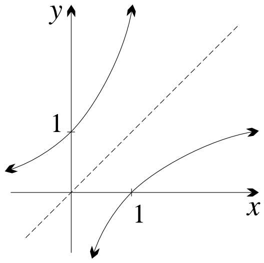
21(a)(i)



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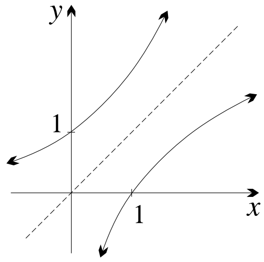

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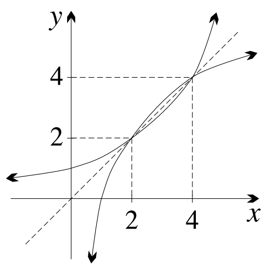
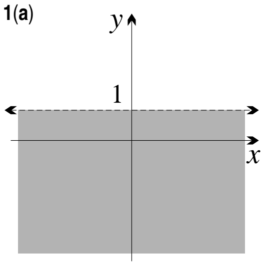


(b) 0, 1 or 2

Exercise **3F** (Page 98)

1(a)



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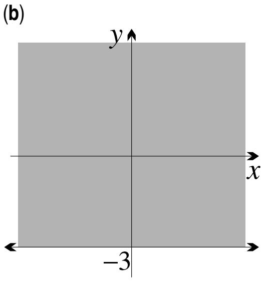
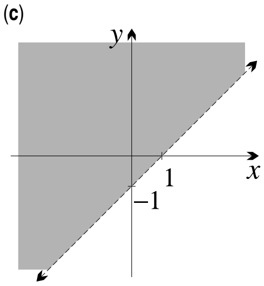


(c)



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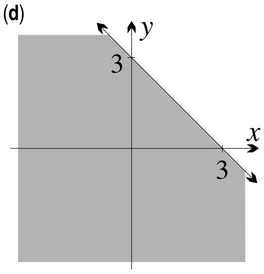

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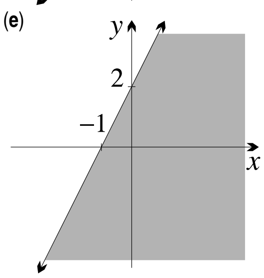

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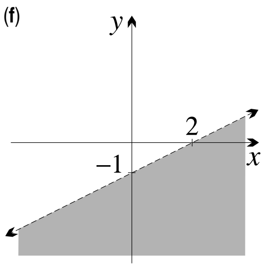

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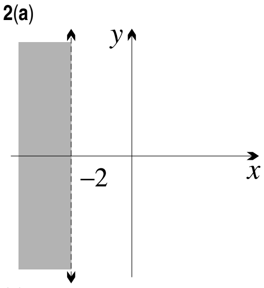

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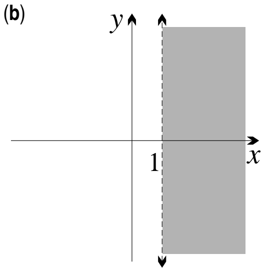

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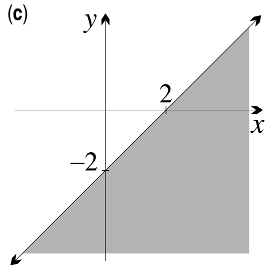

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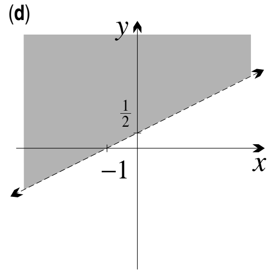

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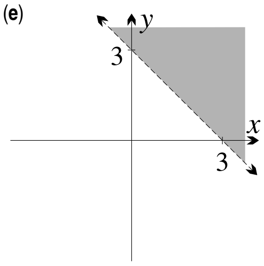

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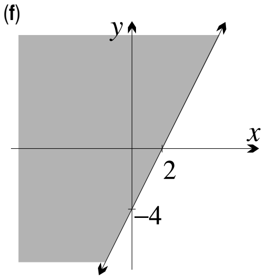

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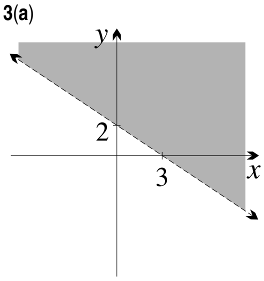

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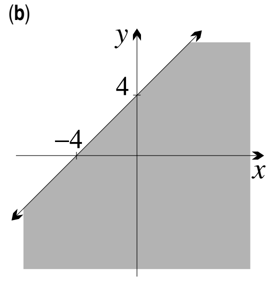

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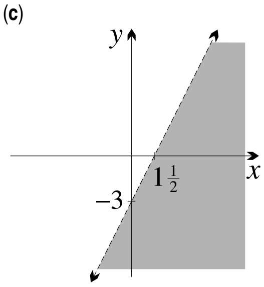

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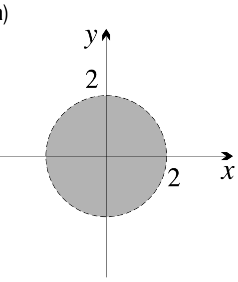

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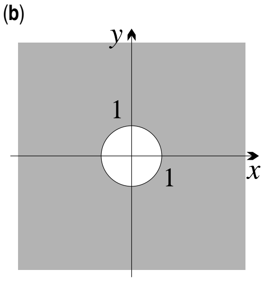

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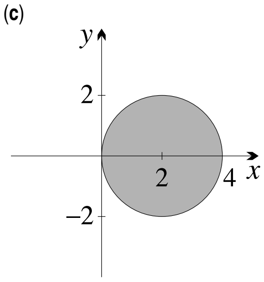

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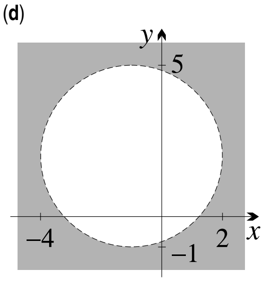
(d)



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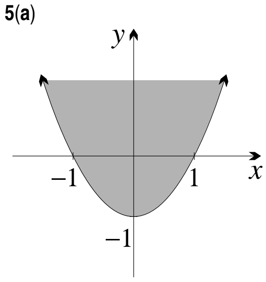

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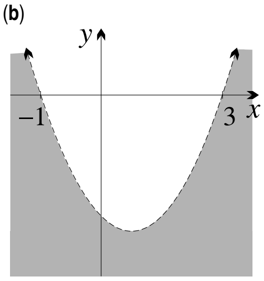

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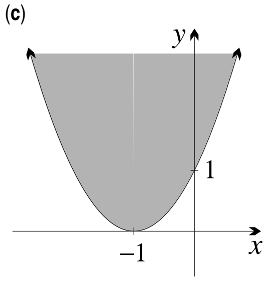

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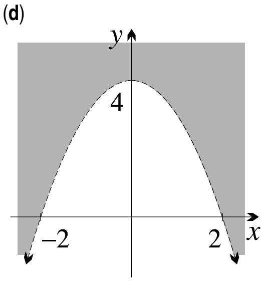

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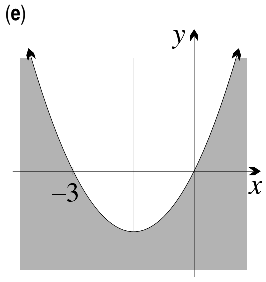

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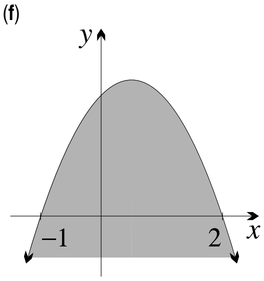

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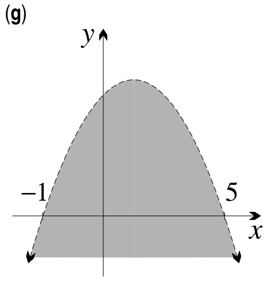

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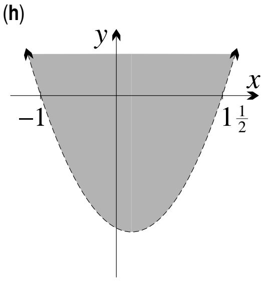

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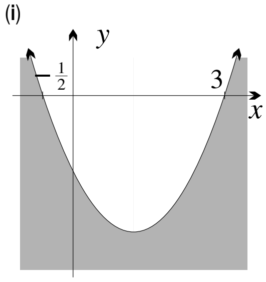

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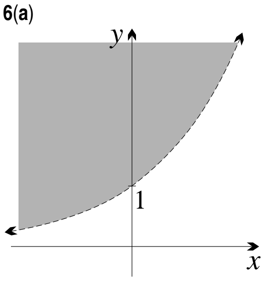

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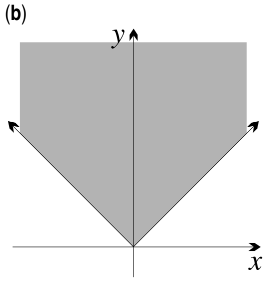

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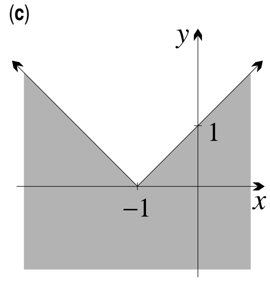

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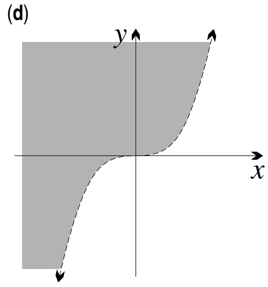

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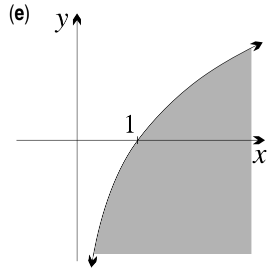

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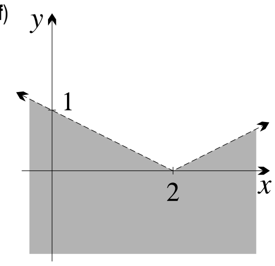
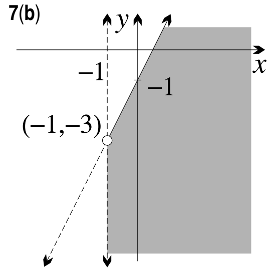


7(b)



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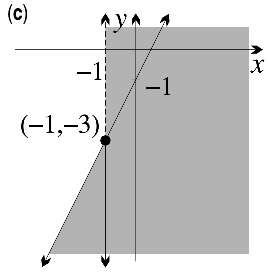
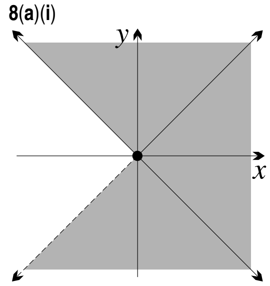


8(a)(i)



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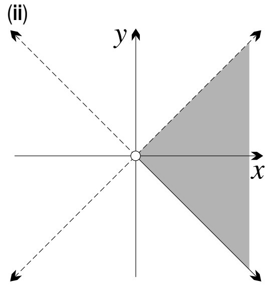
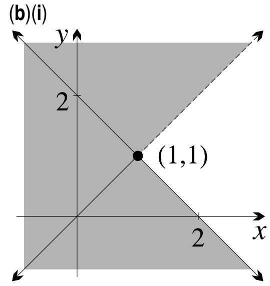
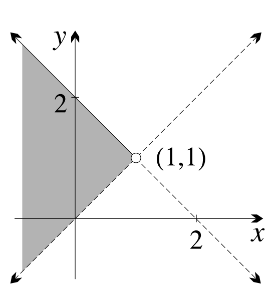


(b)(i)





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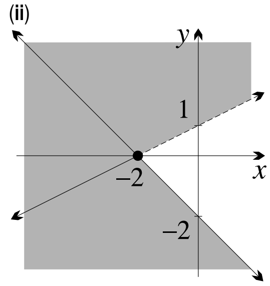

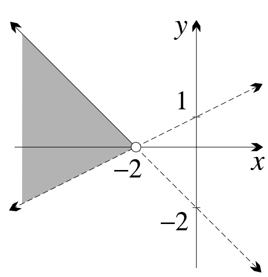

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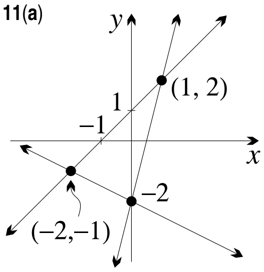
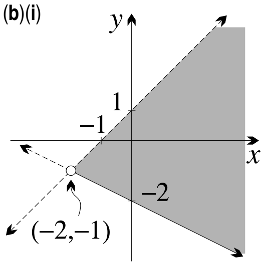
9(a) $x \geq 0$ and $y \geq 0$ **(b)** $x \leq 0$ and $y \geq 0$
(c) $x \leq 0$ and $y \leq 0$ **(d)** $x \geq 0$ and $y \leq 0$
(e) $x \geq 0$ or $y \geq 0$ **(f)** $x \geq 0$ or $y \leq 0$

10(a) $y < x$ and $y \leq 2 - x$
(b) $y \leq -\frac{1}{2}x - 1$ or $y \geq 2 - 2x$
(c) $y < x + 2$ or $y > 4x - 1$

11(a)



(b)(i)



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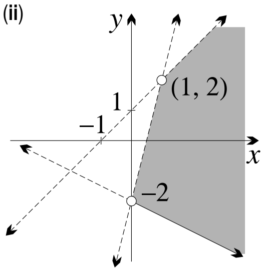

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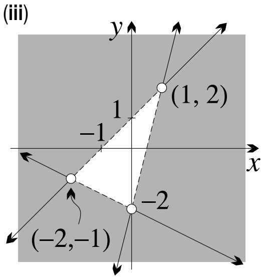
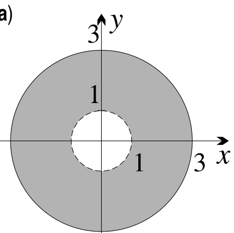
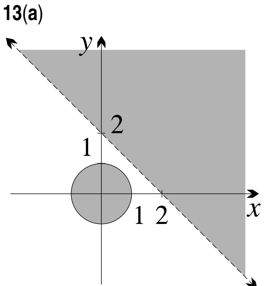


12(a)



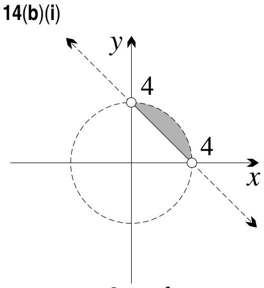
(b) whole plane

13(a)

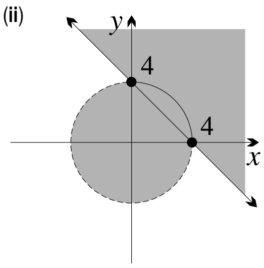


(b) no intersection

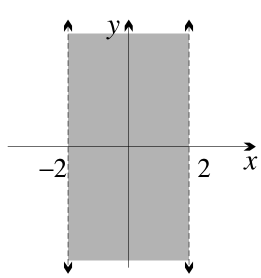
14(b)(i)



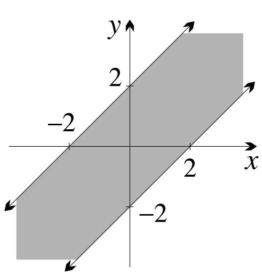
(ii)



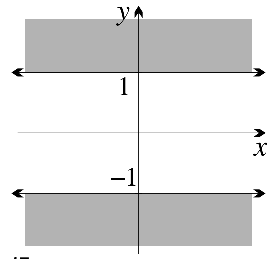
15(a) $x < 2$ and $x > -2$



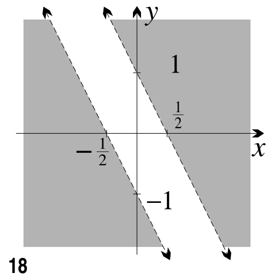
(b) $x - y \leq 2$ and $x - y \geq -2$



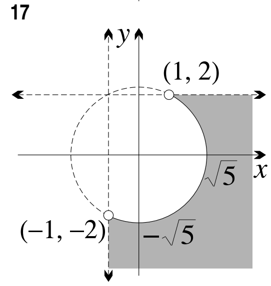
16(a) $y \geq 1$ or $y \leq -1$



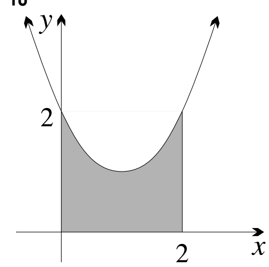
(b) $y + 2x > 1$ or $y + 2x < -1$



17

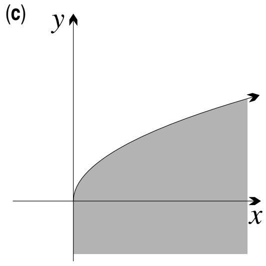


18



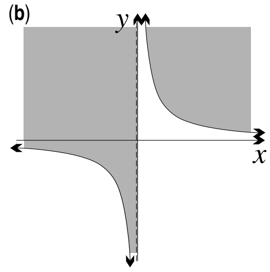
19(b) The curve is undefined for $x < 0$.

(c)

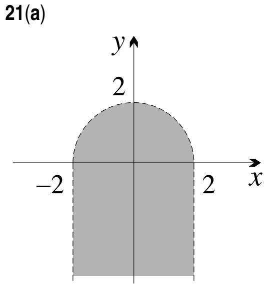


20(a) The curve is undefined when $x = 0$.

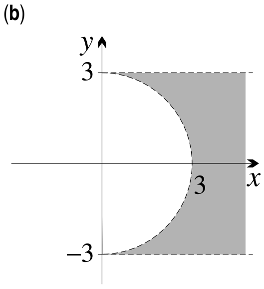
(b)



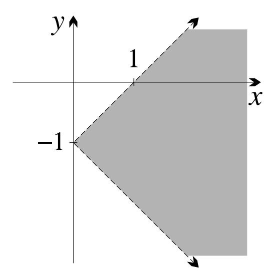
21(a)



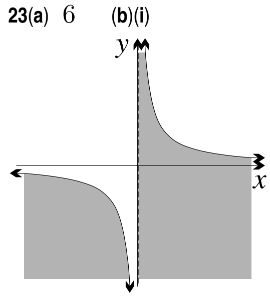
(b)



22



23(a) 6 **(b)(i)**



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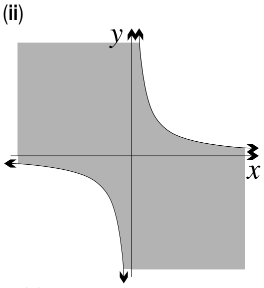
(ii)



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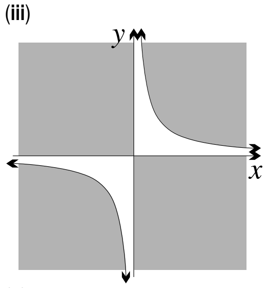
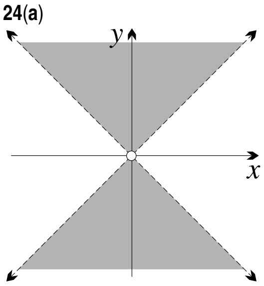


24(a)



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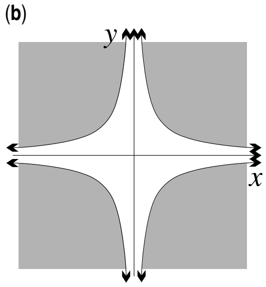
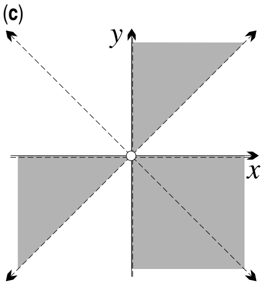


(c)



25(a) A region is connected if every pair of points within the region can be joined by a curve that lies within the region.

(b) A region is not convex if there exist two points in the region which may be joined by a straight line that goes outside of the region.

Exercise 3G (Page 105)

1(a) $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

(b) $f(x) \to 1$ as $x \to \infty$ and $x \to -\infty$

(c) $f(x) \to -2$ as $x \to \infty$ and $x \to -\infty$

(d) $f(x) \to \frac{1}{2}$ as $x \to \infty$ and $x \to -\infty$

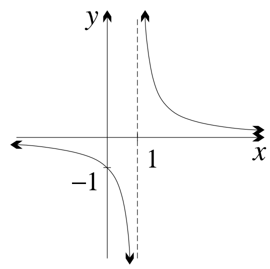
(e) $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

(f) $f(x) \to 0$ as $x \to \infty$ and $x \to -\infty$

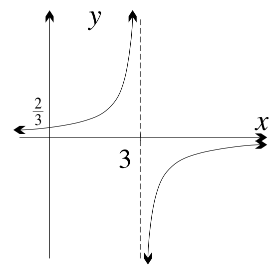
2(a) domain: $x \neq 1$, vertical asymptote: $x = 1$, $y \to \infty$ as $x \to 1^+$, and $y \to -\infty$ as $x \to 1^-$

(b) domain: $x \neq 3$, vertical asymptote: $x = 3$, $y \to -\infty$ as $x \to 3^+$, and $y \to \infty$ as $x \to 3^-$

(a)



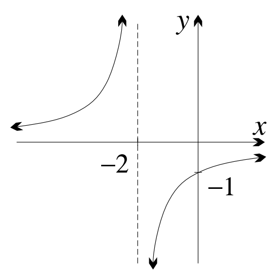
(b)



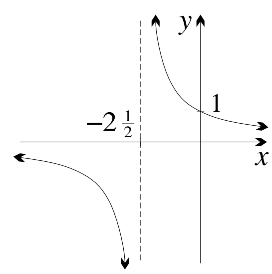
(c) domain: $x \neq -2$, vertical asymptote: $x = -2$, $y \to -\infty$ as $x \to -2^+$, and $y \to \infty$ as $x \to -2^-$

(d) domain: $x \neq -2\frac{1}{2}$, vertical asymptote: $x = -2\frac{1}{2}$, $y \to \infty$ as $x \to -2\frac{1}{2}^+$, and $y \to -\infty$ as $x \to -2\frac{1}{2}^-$

(c)



(d)

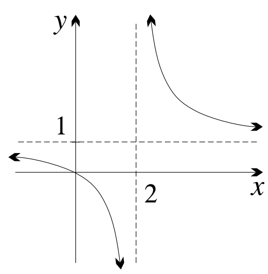


3(a) $x \neq 2$

(b) $x = 0$ and $y = 0$

(c) $y \to 1$ as $x \to \infty$ and as $x \to -\infty$.

(d) $x = 2$ is a vertical asymptote, $y \to \infty$ as $x \to 2^+$, $y \to -\infty$ as $x \to 2^-$.

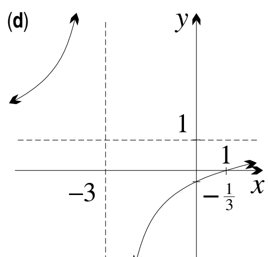


4(a) $x = -3$

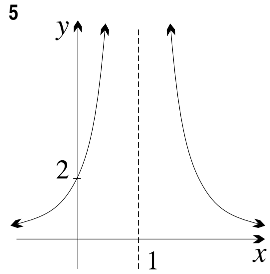
(b) $x = 1$ and $y = -\frac{1}{3}$

(c) $y \to 1$ as $x \to \infty$ and as $x \to -\infty$, $y \to -\infty$ as $x \to -3^+$, $y \to \infty$ as $x \to -3^-$.

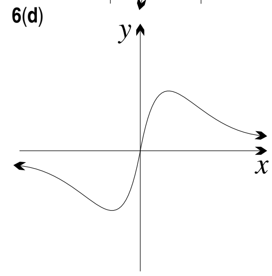
(d)



5



6(d)



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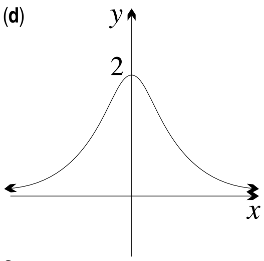
7(a) even

(b) $y = 2$

(d)



8(a) $x = 1$, $x = 3$ and $y = 2$

(b) $x = \frac{1}{3}$, $x = -\frac{1}{3}$ and $y = \frac{4}{9}$

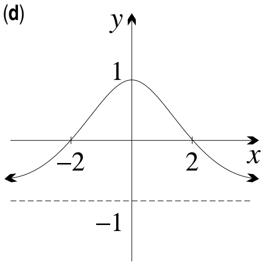
(c) $x = -1$, $x = -4$ and $y = 1$

(d) $x = -5$, $x = 2$ and $y = 0$

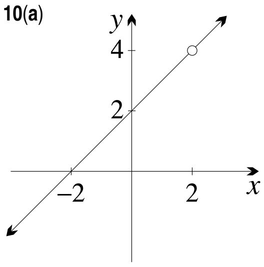
9(b) $x = -2, 2$ and $y = 1$

(c) $y = -1$

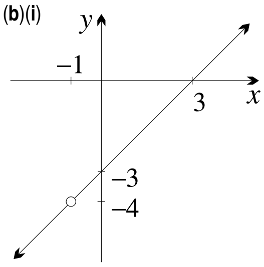
(d)



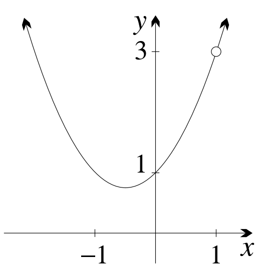
10(a)



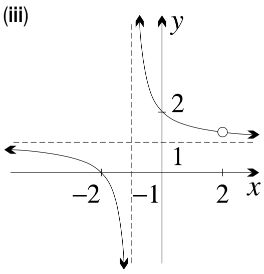
(b)(i)



(ii)



(iii)



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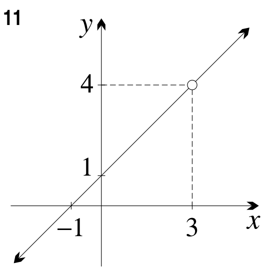
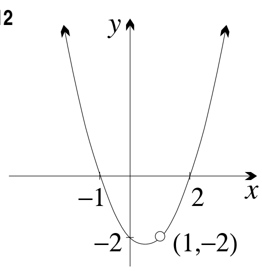
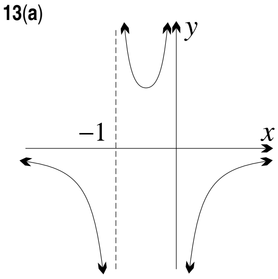


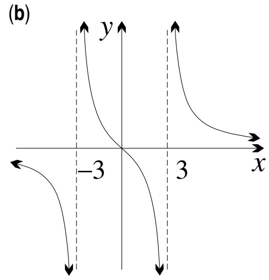
12



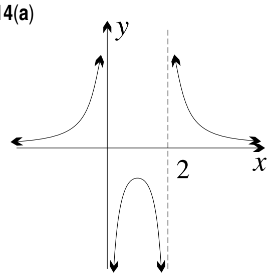
13(a)



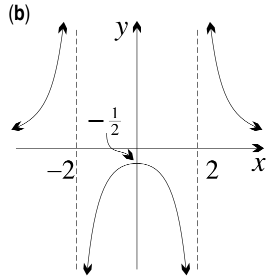
(b)



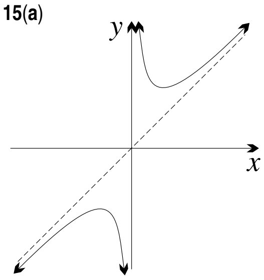
14(a)



(b)



15(a)



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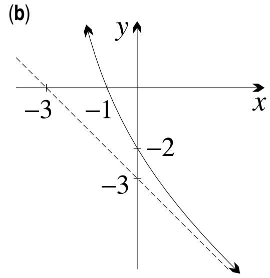
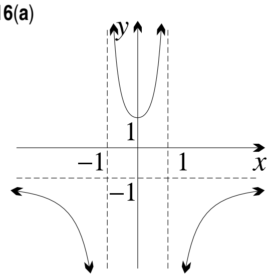
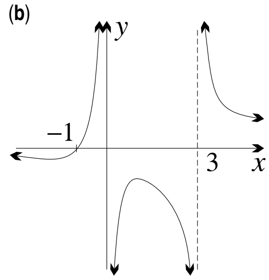


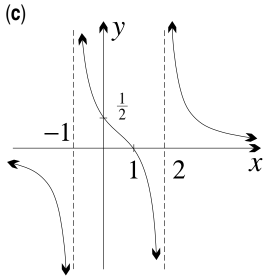
16(a)



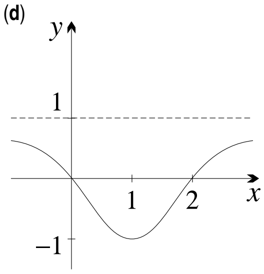
(b)



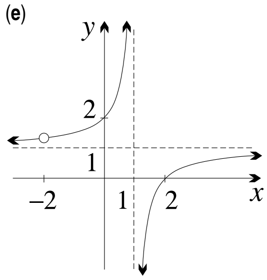
(c)



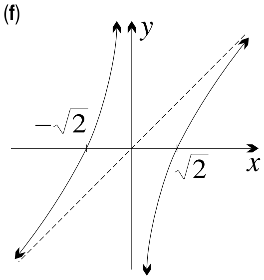
(d)



(e)



(f)



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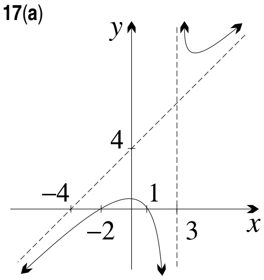
17(a)



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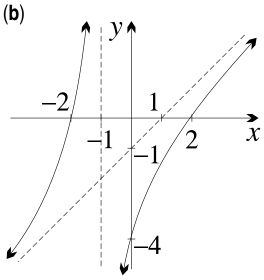
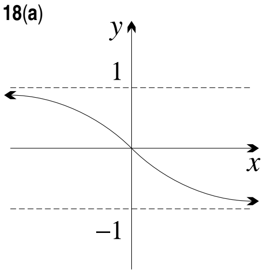
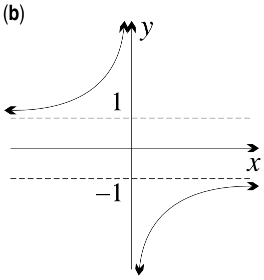


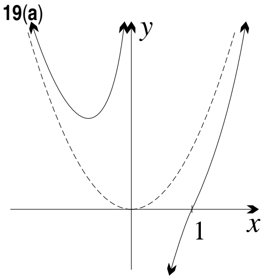
18(a)



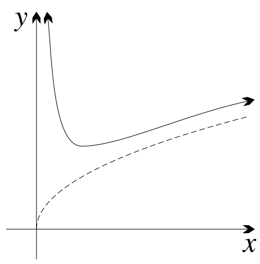
(b)



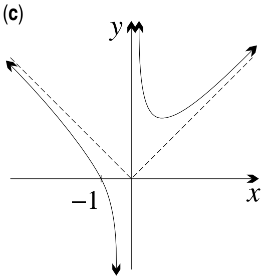
19(a)



(b)



(c)



Chapter Four

Exercise **4A** (Page 111)

1(a) $0{\cdot}4067$ **(b)** $0{\cdot}4848$ **(c)** $0{\cdot}7002$ **(d)** $0{\cdot}8443$ **(e)** $4{\cdot}9894$ **(f)** $0{\cdot}9571$ **(g)** $2{\cdot}9238$ **(h)** $1{\cdot}4945$ **(i)** $0{\cdot}6745$ **(j)** $1{\cdot}8418$ **(k)** $2{\cdot}6372$ **(l)** $1{\cdot}0119$

2(a) 76° **(b)** 27° **(c)** 39° **(d)** 71° **(e)** 10° **(f)** 21°

3(a) $41^\circ 25'$ **(b)** $16^\circ 42'$ **(c)** $46^\circ 29'$ **(d)** $77^\circ 3'$ **(e)** $40^\circ 32'$ **(f)** $75^\circ 24'$

4(a) $\frac{12}{13}$ **(b)** $\frac{5}{12}$ **(c)** $\frac{13}{12}$ **(d)** $\frac{5}{12}$ **(e)** $\frac{13}{12}$ **(f)** $\frac{13}{5}$

5(a) 6 and 17 **(b)(i)** $\frac{15}{17}$ **(ii)** $\frac{4}{5}$ **(iii)** $\frac{3}{4}$ **(iv)** $\frac{17}{8}$ **(v)** $\frac{5}{3}$ **(vi)** $\frac{15}{8}$

6(a) $\frac{\sqrt{3}}{2}$ **(b)** $\frac{1}{\sqrt{3}}$ **(c)** $\frac{1}{\sqrt{2}}$ **(d)** 2 **(e)** $\sqrt{2}$ **(f)** $\sqrt{3}$

7(a) 1 **(b)** $\frac{1}{2}$ **(c)** 4 **(d)** 1

8(a) $x \doteqdot 4{\cdot}4$ **(b)** $a \doteqdot 10{\cdot}4$ **(c)** $h \doteqdot 19{\cdot}0$, $j \doteqdot 16{\cdot}2$ **(d)** $k \doteqdot 17{\cdot}4$, $\ell \doteqdot 12{\cdot}6$

9(a) $\alpha \doteqdot 58^\circ 24'$, $\beta \doteqdot 31^\circ 36'$
(b) $x \doteqdot 31^\circ 47'$, $y \doteqdot 58^\circ 13'$
(c) $\theta \doteqdot 57^\circ 16'$, $\phi \doteqdot 32^\circ 44'$
(d) $\alpha \doteqdot 54^\circ 19'$, $\beta \doteqdot 35^\circ 41'$

10(a) $0{\cdot}61$ **(b)** $2{\cdot}86$ **(c)** $0{\cdot}26$ **(d)** $0{\cdot}31$ **(e)** $1{\cdot}09$ **(f)** $3{\cdot}65$

11(b) 3 **(c)(i)** $\frac{1}{3}\sqrt{5}$, $\frac{2}{3}$

12(a)(i) $\frac{1}{2}\sqrt{22}$ **(ii)** $\frac{3}{2}\sqrt{2}$

14(a) $71^\circ 34'$ **(b)** $21{\cdot}98$ **(c)** $\alpha = 54^\circ 19$, $\beta = 35^\circ 41'$

15(a) $b \doteqdot 8{\cdot}452$ **(b)** $\ell \doteqdot 8{\cdot}476$
(c) $s \doteqdot 10{\cdot}534$, $h \doteqdot 17{\cdot}001$
(d) $a \doteqdot 16{\cdot}314$, $b \doteqdot 7{\cdot}607$

16 73°

17 11°

18(a) $\angle PQR = 20^\circ + 70^\circ = 90^\circ$ (using alternate angles on parallel lines and the fact that due west is 270°). **(b)** $110^\circ + 39^\circ = 149^\circ$

19(a) $5{\cdot}1\,\text{cm}$ **(b)** $16\,\text{cm}$ **(c)** $PQ = 18\sin 40^\circ$, $63^\circ 25'$

20(a) $69^\circ 5'$, $69^\circ 5'$ and $41^\circ 51'$ **(b)** $0{\cdot}4838$ **(c)** $60^\circ 31'$ **(d)** $3{\cdot}172$ **(e)** $64^\circ 1'$ and $115^\circ 59'$ **(f)** $0{\cdot}2217$

21(b) $16\sqrt{3}\,\text{cm}^2$

23 457 metres

24 $1{\cdot}58$ nautical miles

25(a) $y = x\tan 39^\circ$ and $y + 7 = x\tan 64^\circ$

26(a) 108°

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Exercise 4B (Page 115)

2(b)(i) $BD = a\cos B$
3(a) $\angle QPR = 90° - \theta$, so $\angle RPS = \theta$.
(b) $\frac{h}{a}$ and $\frac{b}{h}$
5(a) $\angle OTP = 90°$ (radius $\perp$ tangent) and $\angle OTA = 90° - \theta$ (angle sum of $\triangle OTA$), so $\angle ATP = \theta$.
6 $\frac{1}{2}q$, $\frac{1}{2}p\sqrt{3}$ and $\frac{1}{2}(q + p\sqrt{3})$
8(a) If $\angle RBQ = \alpha$, then $\angle RQB = 90° - \alpha$ (angle sum of $\triangle BQR$) and so $\angle RQP = \alpha$ (complementary angles). Therefore $\angle QPR = 90° - \alpha$ (angle sum of $\triangle PQR$) and so $\angle QPC = \alpha$ (complementary angles). Thus $\angle RBQ = \angle RQP = \angle QPC$.
12(a) If $OA = OB = x$ and $OP = y$, then $AP - PB = (x+y) - (x-y) = 2y = 2 \times OP$.
13(a)(i) $\angle NSO = 90° - \alpha$ (angle sum of $\triangle NOS$), so $\angle QSR = 90° - \alpha$ (vertically opposite) and so $\angle RPQ = \alpha$ (angle sum of $\triangle PQS$). **(ii)** $NR = MQ$ (opposite sides of rectangle $MNRQ$). So $NP = NR + RP = MQ + RP$.

Exercise 4C (Page 119)

3(a) $-320°$ **(b)** $-250°$ **(c)** $-170°$ **(d)** $-70°$ **(e)** $-300°$ **(f)** $-220°$
4(a) $310°$ **(b)** $230°$ **(c)** $110°$ **(d)** $10°$ **(e)** $280°$ **(f)** $170°$
5(a) $70°, 430°, -290°, -650°$
(b) $100°, 460°, -260°, -620°$
(c) $140°, 500°, -220°, -580°$
(d) $200°, 560°, -160°, -520°$
(e) $240°, 600°, -120°, -480°$
(f) $340°, 700°, -20°, -380°$
6(a) $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\operatorname{cosec}\theta = \frac{5}{4}$, $\sec\theta = \frac{5}{3}$, $\cot\theta = \frac{3}{4}$
(b) $\sin\theta = \frac{3}{5}$, $\cos\theta = -\frac{4}{5}$, $\tan\theta = -\frac{3}{4}$, $\operatorname{cosec}\theta = \frac{5}{3}$, $\sec\theta = -\frac{5}{4}$, $\cot\theta = -\frac{4}{3}$
(c) $\sin\theta = -\frac{2}{5}\sqrt{5}$, $\cos\theta = -\frac{1}{5}\sqrt{5}$, $\tan\theta = 2$, $\operatorname{cosec}\theta = -\frac{1}{2}\sqrt{5}$, $\sec\theta = -\sqrt{5}$, $\cot\theta = \frac{1}{2}$
(d) $\sin\theta = -\frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = -\frac{5}{12}$, $\operatorname{cosec}\theta = -\frac{13}{5}$, $\sec\theta = \frac{13}{12}$, $\cot\theta = -\frac{12}{5}$
8(a)(i) $0{\cdot}5$ **(ii)** -0·5 **(iii)** $0{\cdot}95$ **(iv)** $0{\cdot}95$ **(v)** $0{\cdot}59$ **(vi)** $0{\cdot}81$ **(vii)** $-0{\cdot}89$ **(viii)** $0{\cdot}45$ **(ix)** $-0{\cdot}81$ **(x)** $0{\cdot}59$
(b)(i) $30°, 150°$ **(ii)** $120°, 240°$ **(iii)** $64°, 116°$ **(iv)** $53°, 307°$ **(v)** $53°, 127°$ **(vi)** $143°, 217°$ **(vii)** $204°, 336°$ **(viii)** $107°, 253°$
(c) $45°, 225°$
10(b) A circle of radius r_0.
(c) The curve is called a *rectangular hyperbola*, and is the same curve as the curve $y = 1/x$ in the two-dimensional coordinate plane.

Exercise 4D (Page 125)

1(a) $+$ **(b)** $+$ **(c)** $-$ **(d)** $-$ **(e)** $-$ **(f)** $-$ **(g)** $-$ **(h)** $+$ **(i)** $+$ **(j)** $+$ **(k)** $+$ **(l)** $-$ **(m)** $+$ **(n)** $-$ **(o)** $-$ **(p)** $+$
2(a) $36°$ **(b)** $30°$ **(c)** $50°$ **(d)** $20°$ **(e)** $60°$ **(f)** $30°$ **(g)** $60°$ **(h)** $70°$ **(i)** $40°$ **(j)** $60°$
3(a) $-\tan 50°$ **(b)** $\cos 50°$ **(c)** $-\sin 40°$ **(d)** $\cot 80°$ **(e)** $-\sec 10°$ **(f)** $-\operatorname{cosec} 40°$ **(g)** $-\cos 5°$ **(h)** $\operatorname{cosec} 55°$ **(i)** $-\tan 40°$ **(j)** $-\sin 85°$ **(k)** $\sec 80°$ **(l)** $\cot 20°$
4(a) 1 **(b)** -1 **(c)** 0 **(d)** 0 **(e)** undefined **(f)** 1 **(g)** -1 **(h)** 0 **(i)** 1 **(j)** 0 **(k)** undefined **(l)** undefined
5(a) $-\frac{1}{\sqrt{2}}$ **(b)** $\frac{\sqrt{3}}{2}$ **(c)** 1 **(d)** $\frac{\sqrt{3}}{2}$ **(e)** $\sqrt{3}$ **(f)** $-\frac{2}{\sqrt{3}}$ **(g)** -2 **(h)** $\frac{1}{\sqrt{2}}$ **(i)** -2 **(j)** $-\frac{1}{\sqrt{3}}$ **(k)** $-\frac{2}{\sqrt{3}}$ **(l)** $-\frac{1}{\sqrt{2}}$ **(m)** $\frac{1}{2}$ **(n)** $\sqrt{3}$ **(o)** $-\sqrt{2}$ **(p)** $-\frac{2}{\sqrt{3}}$
6(a) $0{\cdot}42$ **(b)** $-0{\cdot}91$ **(c)** $0{\cdot}91$ **(d)** $-0{\cdot}42$ **(e)** $0{\cdot}49$ **(f)** $0{\cdot}49$
7(a) $-0{\cdot}70$ **(b)** $-1{\cdot}22$ **(c)** $-0{\cdot}70$ **(d)** $-0{\cdot}52$ **(e)** $1{\cdot}92$ **(f)** $-0{\cdot}52$
8(a) 1 **(b)** $-\frac{7}{2}$ **(c)** $\frac{3}{4}$
10(a) $(2, 2\sqrt{3})$ **(b)** $(-\sqrt{3}, 1)$ **(c)** $(1, -1)$ **(d)** $(-5, -5\sqrt{3})$
11(a) $53°8'$ **(b)** $138°11'$ **(c)** $300°$ **(d)** $213°41'$
13(a) $-\sin A$ **(b)** $\cos A$ **(c)** $-\tan A$ **(d)** $\sec A$ **(e)** $\sin A$ **(f)** $-\sin A$ **(g)** $-\cos A$ **(h)** $\tan A$ **(i)** $-\sec A$ **(j)** $-\operatorname{cosec} A$ **(k)** $-\cot A$ **(l)** $\sec A$
14(a) $y = \sin\theta$ and $y = \cos\theta$ have range $-1 \le y \le 1$, $y = \tan\theta$ and $y = \cot\theta$ have range $\mathbf{R}$, $y = \sec\theta$ and $y = \operatorname{cosec}\theta$ have range $y \ge 1$ or $y \le -1$.
(b) $\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ and $\sec\theta$ have period $360°$; $\tan\theta$ and $\cot\theta$ have period $180°$.
(c) $\sin\theta$, $\operatorname{cosec}\theta$, $\tan\theta$ and $\cot\theta$ are odd; $\cos\theta$ and $\sec\theta$ are even.
(d) The graphs have point symmetry about every θ-intercept, and about every point where an asymptote crosses the θ-axis.
(e) $\sin\theta$, $\cos\theta$, $\operatorname{cosec}\theta$ and $\sec\theta$ have line symmetry in every vertical line through a maximum or minimum; $\tan\theta$ and $\cot\theta$ have no axes of symmetry.
15(a) $\cos\theta$ **(b)** $\cos\theta$ **(c)** $-\sin\theta$ **(d)** $-\cos\theta$ **(e)** $-\tan\theta$ **(f)** $-\operatorname{cosec}\theta$

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16(a) -1 **(b)** $\tan\alpha$ **(c)** $-\cot\alpha$ **(d)** $-\cos\alpha$

Exercise 4E (Page 128)

1(a) $\sin\theta = \frac{4}{5}$, $\tan\theta = \frac{4}{3}$
(b) $\sin\theta = \frac{5}{13}$, $\sec\theta = -\frac{13}{12}$
2(a) $\cos\alpha = \frac{15}{17}$ or $-\frac{15}{17}$, $\cot\alpha = \frac{15}{8}$ or $-\frac{15}{8}$
(b) $\tan x = -\frac{1}{3}\sqrt{7}$, $\operatorname{cosec} x = \frac{4}{7}\sqrt{7}$
3(a) $\cos\beta = -\frac{3}{13}\sqrt{13}$ **(b)** $\cot\alpha = -\frac{1}{2}\sqrt{21}$
(c) $\operatorname{cosec}\theta = \frac{1}{2}\sqrt{5}$ or $-\frac{1}{2}\sqrt{5}$ **(d)** $\sec A$ is undefined.
4(a) $\operatorname{cosec} P = -\frac{3}{4}\sqrt{2}$ **(b)** $\tan\theta = 0$
(c) $\sin\alpha = \frac{1}{3}\sqrt{5}$ or $-\frac{1}{3}\sqrt{5}$, $\cot\alpha = \frac{2}{5}\sqrt{5}$ or $-\frac{2}{5}\sqrt{5}$
(d) $\operatorname{cosec} x = \frac{1}{5}\sqrt{34}$ or $-\frac{1}{5}\sqrt{34}$, $\sec x = \frac{1}{3}\sqrt{34}$ or $-\frac{1}{3}\sqrt{34}$
5 $\cos\theta = -\dfrac{\sqrt{q^2-p^2}}{q}$, $\tan\theta = -\dfrac{p}{\sqrt{q^2-p^2}}$
6 $\sin\alpha = \pm\dfrac{k}{\sqrt{1+k^2}}$, $\sec\alpha = \pm\sqrt{1+k^2}$
7(b) $\sin x = \dfrac{2t}{1+t^2}$, $\tan x = \dfrac{2t}{1-t^2}$
8 $\tan(\theta+90^\circ) = \dfrac{\sqrt{1-k^2}}{k}$
9 HINT: $\tan\theta = a - \frac{1}{4a}$ or $\frac{1}{4a} - a$

Exercise 4F (Page 131)

2(a) $\operatorname{cosec}\theta$ **(b)** $\cot\alpha$ **(c)** $\tan\beta$ **(d)** $\cot\phi$
3(a) 1 **(b)** 1 **(c)** 1
6(a) $\cos^2\alpha$ **(b)** $\sin^2\alpha$ **(c)** $\sin A$ **(d)** $\cos A$
7(a) $\cos^2\theta$ **(b)** 1 **(c)** $\tan^2\beta$ **(d)** $\cot^2 A$
8(a) $\cos\theta$ **(b)** $\operatorname{cosec}\alpha$ **(c)** $\cot\beta$ **(d)** $\tan\phi$
9(a) 1 **(b)** $\sin^2\beta$ **(c)** $\sec^2\phi$ **(d)** 1
10(a) $\cos^2\beta$ **(b)** $\operatorname{cosec}^2\phi$ **(c)** $\cot^2 A$ **(d)** -1
14(a) $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ **(b)** $\dfrac{y^2}{b^2} - \dfrac{x^2}{a^2} = 1$
(c) $(x-2)^2 + (y-1)^2 = 1$ **(d)** $x^2 + y^2 = 2$
15(a) 2 **(b)** 0 **(c)** 1 **(d)** 0
17(a) $y - x = 1$ **(b)** $x^2 + 2xy + 2y^2 = 5$
(c) $x^2 y = y + 2$

Exercise 4G (Page 137)

1(a) $\theta = 60^\circ$ or 120° **(b)** $\theta = 45^\circ$ or 225°
(c) $\theta = 135^\circ$ or 225° **(d)** $\theta = 120^\circ$ or 300°
(e) $\theta = 210^\circ$ or 330° **(f)** $\theta = 150^\circ$ or 210°
2(a) $\theta = 90^\circ$ **(b)** $\theta = 180^\circ$ **(c)** $\theta = 90^\circ$ or 270°
(d) $\theta = 0^\circ$ or 360° **(e)** $\theta = 0^\circ$ or 180° or 360°
(f) $\theta = 90^\circ$ or 270°
3(a) $x \doteqdot 65^\circ$ or 295° **(b)** $x \doteqdot 7^\circ$ or 173°
(c) $x \doteqdot 98^\circ$ or 278° **(d)** $x \doteqdot 114^\circ$ or 294°
(e) $x \doteqdot 222^\circ$ or 318° **(f)** $x \doteqdot 80^\circ$ or 280°
4(a) $\alpha \doteqdot 5^\circ 44'$ or $174^\circ 16'$
(b) $\alpha \doteqdot 95^\circ 44'$ or $264^\circ 16'$ **(c)** $\alpha = -45^\circ$ or 135°
(d) $\alpha = 270^\circ$ **(e)** no solutions
(f) $\alpha = 45^\circ$ or 315° **(g)** $\alpha = 90^\circ$ or -90°
(h) $\alpha \doteqdot 243^\circ 26'$ **(i)** $\alpha = 150^\circ$ **(j)** $\alpha = 210^\circ$ or 330°
(k) $\alpha = 60^\circ$ or 300° **(l)** $\alpha \doteqdot 18^\circ 26'$ or $198^\circ 26'$
(m) $\alpha = -360^\circ, -180^\circ, 0^\circ, 180^\circ$ or 360°
(n) $\alpha \doteqdot -16^\circ 42'$ or $163^\circ 18'$
(o) $\alpha \doteqdot 224^\circ 26'$, $315^\circ 34'$, $584^\circ 26'$ or $675^\circ 34'$
(p) $\alpha = 157^\circ 30'$ or $337^\circ 30'$
5(a) $\theta = 0^\circ$, 180° or 360°
(b) $\theta = 30^\circ$, 150°, 210° or 330°
(c) $\theta \doteqdot 72^\circ$, 108°, 252° or 288°
(d) $\theta = 45^\circ$, 135°, 225° or 315°
6(a) $x = 15^\circ$, 75°, 195° or 255°
(b) $x = 67^\circ 30'$, $112^\circ 30'$, $247^\circ 30'$ or $292^\circ 30'$
(c) $x = 20^\circ$, 80°, 140°, 200°, 260° or 320°
(d) no solutions
7(a) $\alpha = 75^\circ$ or 255° **(b)** $\alpha = 210^\circ$ or 270°
(c) $\alpha = 345^\circ$ or 165° **(d)** $\alpha = 285^\circ$ or 45°
8(a) $\theta = 45^\circ$ or 225° **(b)** $\theta = 150^\circ$ or 330°
(c) $\theta = 60^\circ$, 120°, 240° or 300°
(d) $\theta = 45^\circ$, 135°, 225° or 315°
9(a) $\theta = 0^\circ$, 90°, 270° or 360°
(b) $\theta = 30^\circ$, 90°, 210° or 270°
(c) $\theta = 0^\circ$, 60°, 180°, 300° or 360°
(d) $\theta = 135^\circ$ or 315°, or $\theta \doteqdot 63^\circ 26'$ or $243^\circ 26'$
(e) $\theta = 90^\circ$, 210° or 330°
(f) $\theta = 60^\circ$ or 300°, or $\theta \doteqdot 104^\circ 29'$ or $255^\circ 31'$
(g) $\theta \doteqdot 70^\circ 32'$ or $289^\circ 28'$
(h) $\theta \doteqdot 23^\circ 35'$, $156^\circ 25'$, $221^\circ 49'$ or $318^\circ 11'$
(i) $\theta = 0^\circ$, 60°, 120°, 180°, 240°, 300° or 360°
10(a) $x = 60^\circ$, 90°, 270° or 300°
(b) $x = 135^\circ$ or 315°, or $x \doteqdot 71^\circ 34'$ or $251^\circ 34'$
(c) $x = 210^\circ$ or 330°, or $x \doteqdot 14^\circ 29'$ or $165^\circ 31'$
(d) $x \doteqdot 48^\circ 11'$ or $311^\circ 49'$
(e) $x \doteqdot 56^\circ 19'$, $116^\circ 34'$, $236^\circ 19'$ or $296^\circ 34'$
11(a) $\alpha = 90^\circ$, or $\alpha \doteqdot 199^\circ 28'$ or $340^\circ 32'$
(b) $\alpha \doteqdot 63^\circ 26'$, $161^\circ 34'$, $243^\circ 26'$ or $341^\circ 34'$
12(a) $A \doteqdot 48^\circ 11'$ or $311^\circ 49'$
(b) $A \doteqdot 23^\circ 35'$ or $156^\circ 25'$
13(a) $x = 45^\circ$, 180° or 225°
(b) $x = 120^\circ$ or 240°, or $x \doteqdot 19^\circ 28'$ or $160^\circ 32'$
14(a) $x = 71^\circ 34'$ or $251^\circ 34'$
(b) $x \doteqdot 75^\circ 58'$, $116^\circ 34'$, $255^\circ 58'$ or $296^\circ 34'$
(c) $x \doteqdot 21^\circ 48'$, $116^\circ 34'$, $201^\circ 48'$ or $296^\circ 34'$
(d) $x = 0^\circ$, 135°, 180°, 315° or 360°

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15(a) $\theta = 54°$, $126°$, $198°$ or $342°$
(b) $\theta \doteqdot 13°41'$, $121°19'$, $193°41'$ or $301°19'$
(c) $\theta \doteqdot 33°41'$, $63°26'$, $213°41'$ or $243°26'$
(d) $\theta = 45°$ or $225°$, or $\theta \doteqdot 18°26'$ or $198°26'$
(e) $\theta = 30°$, $45°$, $135°$, $150°$, $210°$, $225°$, $315°$ or $330°$ **(f)** $\theta = 120°$, $225°$, $300°$ or $315°$
(g) $\theta \doteqdot 78°28'$, $228°35'$, $281°32'$ or $311°25'$
(h) $\theta = 60°$, $180°$ or $300°$
(i) $\theta = 45°$, $120°$, $225°$ or $300°$
(j) $\theta = 135°$ or $315°$, or $\theta \doteqdot 161°34'$ or $341°34'$

Exercise **4H** (Page 143)

1(a) $1{\cdot}9$ **(b)** $9{\cdot}2$ **(c)** $8{\cdot}9$
2(a) $49°$ **(b)** $53°$ **(c)** $43°$
3(a) $5\,\text{cm}^2$ **(b)** $22\,\text{cm}^2$
4 $42°24', 137°36'$
5(a) $49°46'$ **(b)** $77°53'$ **(c)** $3{\cdot}70\,\text{cm}^2$
6(a) $69°2'$ or $110°58'$ **(b)** $16{\cdot}0$ units or $11{\cdot}0$ units
7 Either $B \doteqdot 62°38', C \doteqdot 77°22', c \doteqdot 11{\cdot}5$ or $B \doteqdot 117°22', C \doteqdot 22°38', c \doteqdot 4{\cdot}6$.
8(b) 28 metres
9(b) 6 cm
10 32
11(a) $3\sqrt{6}$ **(b)** $3\sqrt{2}$ **(c)** $2\sqrt{6}$ **(d)** $6\sqrt{2}$
12(b) $79{\cdot}3$ metres
13 $11{\cdot}0$ cm
14(a) $\angle PJK = \angle PBQ = 20°$ (corresponding angles on parallel lines).
But $\angle PJK = \angle PAJ + \angle APJ$ (exterior angle of triangle). So $\angle APJ = 20° - 5° = 15°$.
(d) 53 metres
17(a) $\angle QSM = 36°$ (angle sum of $\triangle QRS$) and $\angle PSM = 48°$ (angle sum of $\triangle PSM$),
so $\angle PSQ = 48° - 36° = 12°$. $\angle SPQ = 24°$.
So $\angle PQS = 180° - 24° - 12° = 144°$ (angle sum of $\triangle PQS$). **(c)** 473 metres
18(b) 12 km **(c)** 9:52 am
19(a) $\sin \angle BMA = \sin(180° - \theta) = \sin\theta$
20(d) $\frac{\sqrt{3}-1}{2\sqrt{2}}$
21 If the related angle for θ is α and the known angle is β and $\alpha < \beta$, then $\theta = \alpha$ is one solution and $\theta = 180° - \alpha$ is the other possibility.
But $(180° - \alpha) + \beta = 180° + (\beta - \alpha) > 180°$, and so $\theta = 180° - \alpha$ is impossible.
23(a) $8\sqrt{3}$ **(b)** As the triangle varies, the circle remains unchanged, because $\angle A$ is an angle at the circumference standing on the chord BC.
(c) $2 : 1$
24(a) Combine the formulae $D_C = c/\sin C$ and $\Delta = \frac{1}{2}ab\sin C$.
(c) $D_C : D_I = abcs : 4\Delta^2$, $D_C D_I = \dfrac{abc}{s}$
(d) $\dfrac{16\Delta^3}{\pi a^2 b^2 c^2}$, $\dfrac{s^2}{\pi\Delta}$

Exercise **4I** (Page 148)

1(c) $\sqrt{13}$ $14{\cdot}43$ cm $\sqrt{10}$
2(c) $57°$ $101°32'$ $-\frac{1}{5}$
3 $11{\cdot}5$ km
4 167 nautical miles
5 $20°$
6(a) $94°48'$ **(b)** $84°33'$
7(a) $101°38'$ **(b)** $78°22'$
9 $13°10'$
10(a) 19 cm **(b)** $\frac{37}{38}$
11 $\cos A = \frac{3}{4}$, $\cos B = \frac{9}{16}$, $\cos C = \frac{1}{8}$
12(b) 108 km **(c)** $\angle ACB \doteqdot 22°$, bearing $\doteqdot 138°$
13(a) $\angle DAP = \angle DPA = 60°$ (angle sum of isosceles triangle), so $\triangle ADP$ is equilateral.
So $AP = 3$ cm. **(b)** $3\sqrt{7}$ cm
15(a)(ii) $9\sqrt{3}$ units^2
17(a) $D_C = \dfrac{abc}{2\sqrt{s(s-a)(s-b)(s-c)}}$,
$D_I = \dfrac{2\sqrt{(s-a)(s-b)(s-c)}}{\sqrt{s}}$

Exercise **4J** (Page 152)

1(a)(i) $44°25'$ **(ii)** $9{\cdot}8\,\text{cm}^2$ **(b)(i)** $11{\cdot}6$ cm **(ii)** $49°$
2(b) $10{\cdot}61$ metres
3(a) $9{\cdot}85$ metres **(b)** $5{\cdot}30$ metres **(c)** $12{\cdot}52$ metres
4(b) $8{\cdot}7$ nautical miles
5(c) 34 metres
6(a) $34°35'$ **(b)** $\angle PDA = \angle ABP$ (base angles of isosceles $\triangle ABD$) and $\angle ABP = \angle PDC$ (alternate angles on parallel lines), so $\angle PDA = \angle PDC$ and $\angle PDC = \frac{1}{2}\angle ADC$. **(c)** $65°35'$
7(a) $46°59'$ or $133°1'$
(b) $66{\cdot}4$ metres or $52{\cdot}7$ metres
8(a) $\frac{1}{2}\sqrt{37}$ cm **(b)** $25°17'$
9 P_1 by $2{\cdot}5$ min
10(a) 42 km **(b)** $78°$
11(a) $\dfrac{86\sin 60°45'}{\sin 65°45'}$ **(b)** 66 metres
13(a) $CQ = x\tan 48°$, $PD = x\tan 52°$
(b) $QD = x\tan\alpha$ and $CQ - CP = PD - QD$

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(both $= PQ$). **(c)** $42°42'$
15(a) $-\cos\theta$
16 $120°$
21(b) $17°52'$
22(f) As $d \to 0$, the quadrilateral becomes a triangle whose circumcircle remains the original circle.

Chapter Five

Exercise **5A** (Page 159)

1(a) 5 **(b)** 13 **(c)** 10 **(d)** $\sqrt{8} = 2\sqrt{2}$ **(e)** $\sqrt{80} = 4\sqrt{5}$
(f) 13
2(a) $(3, 2\frac{1}{2})$ **(b)** $(\frac{1}{2}, 1)$ **(c)** $(-1, 1)$ **(d)** $(4, 5)$
(e) $(0, 1)$ **(f)** $(2\frac{1}{2}, -6)$
3(a)(i) $(3, 3)$ **(ii)** $(5, 4)$ **(iii)** $(9, 6)$ **(iv)** $(9, 6)$
(b)(i) $(0, \frac{1}{2})$ **(ii)** $(2, -\frac{1}{2})$ **(iii)** $(-3, 2)$ **(iv)** $(5, -2)$
(c)(i) $(-1, 1)$ **(ii)** $(3, -1)$ **(iii)** $(11, -5)$ **(iv)** $(37, -18)$
(d)(i) $(-6, 3)$ **(ii)** $(-4, -1)$ **(iii)** $(-10, 11)$
(iv) $(-8\frac{1}{2}, 8)$
4(a) $2 : 3$ **(b)** $3 : 2$ **(c)** $3 : -1$ (or $-3 : 1$)
(d) $1 : -3$ (or $-1 : 3$) **(e)** $4 : -1$ (or $-4 : 1$)
(f) $1 : -4$ (or $-1 : 4$)
5(a)(i) $2 : -5$ **(ii)** $5 : -3$ **(b)(i)** $3 : -5$ **(ii)** $5 : -2$
(c)(i) $3 : -2$ **(ii)** $2 : 1$ **(d)(i)** $1 : 2$ **(ii)** $2 : -3$
(e)(i) $4 : -3$ **(ii)** $3 : 1$ **(f)(i)** $1 : 3$ **(ii)** $3 : -4$
6(a) $(13, 7)$ **(b)** $(-7, -3)$ **(c)** $(-1, 0)$ **(d)** $(15, 8)$
7 $AB = BC = \sqrt{10}$ and $CD = DA = 5$.
Such a quadrilateral is sometimes called a *kite*.
8(a) $XY = 2\sqrt{13}$, $YZ = 2\sqrt{13}$, $ZX = 2\sqrt{26}$, so $XY^2 + YZ^2 = 104 = ZX^2$ **(b)** 26 square units
9(a) ABC is an equilateral triangle.
(b) PQR is a right triangle.
(c) DEF is none of these.
(d) XYZ is an isosceles triangle.
10(a) $(0, -4)$, $-2, -3)$, $(-4, -2)$
(b) $(0, 6)$, $(2, 9)$, $(4, 12)$, $(6, 15)$
11(a) Both midpoints are $M(2\frac{1}{2}, 2\frac{1}{2})$. It must be a parallelogram, since its diagonals bisect each other. **(b)** $AB = AD = \sqrt{5}$. $ABCD$ is a rhombus, since it is a parallelogram with a pair of adjacent sides equal.
12 $\sqrt{17}$, $2\sqrt{17}$, $2\pi\sqrt{17}$, 17π
13(a) $(x-5)^2 + (y+2)^2 = 45$
(b) $(x+2)^2 + (y-2)^2 = 74$
14(a) $S(-5, -2)$ **(b)(i)** $P = (-1, -17)$
(ii) $P = (7, -7)$ **(c)** $B = (0, 7)$ **(d)** $R = (12, -9)$
15(a) Check the results using the distance formula — there are eight such points.
(b) $y = 4$ or 10 **(c)** $a = 1 + \sqrt{2}$ or $1 - \sqrt{2}$
16(a) $P(1, -2\frac{1}{2})$ $(0, -1)$
17(a) $7 : 2$ **(b)(i)** $-3 : 5$ **(ii)** $2 : 3$ **(iii)** $-5 : 2$
18(a) $AB : BM = -2 : 1$ **(b)** $AB : BM = -4 : 3$
(c) $AB : BM = -11 : 4$ **(d)** $AB : BM = 1 : 1$
(e) $AB : BM = -2 : 3$ **(f)** $AB : BM = 1 : 3$

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19(a)(i) $2:1$ **(ii)** $1:3$
20(a) $(-11,5)$ and $(17,-7)$ **(b)** $R(\frac{1}{4},-3)$
21(a) $P(\frac{-1+2k}{1+k},\frac{4-2k}{1+k})$ **(b)** $k=2$, $P(1,0)$
22(a) $(-2,1)$ **(b)** $M=(4\frac{1}{2},1\frac{1}{2})$
23(a) $Q(-\frac{2}{t},\frac{1}{t^2})$
(b)(i) $S(\frac{a}{r^2},\frac{b}{r^2})$ **(ii)** $S(\frac{a}{a^2+b^2},\frac{b}{a^2+b^2})$
24(a) $x=\frac{3}{2}a$, a vertical straight line through the midpoint of AB. **(b)** $(x-4a)^2+y^2=(2a)^2$, a circle with centre $(4a,0)$ and radius $2a$.
25(a) Q **(b)(i)** $k>0$ **(ii)** $k<-1$ **(iii)** $-1<k<0$ **(iv)** As $k\to(-1)^+$, M moves infinitely far along the ray QP. As $k\to(-1)^-$, M moves infinitely far along the ray PQ.

Exercise 5B (Page 165)

1(a) $2,-\frac{1}{2}$ **(b)** $-1,1$ **(c)** $\frac{3}{4},-\frac{4}{3}$ **(d)** $-\frac{p}{q},\frac{q}{p}$
2(a) $-1,1$ **(b)** $2,-\frac{1}{2}$ **(c)** $\frac{1}{2},-2$ **(d)** $-\frac{1}{2},2$
(e) $3,-\frac{1}{3}$ **(f)** $-\frac{b}{2a},\frac{2a}{b}$
3(a) $0{\cdot}27$ **(b)** $-1{\cdot}00$ **(c)** $0{\cdot}41$ **(d)** $3{\cdot}08$
4(a)(i) 45°, upwards **(ii)** 120°, downwards
(iii) 76°, upwards **(iv)** 30°, upwards
(b)(i) 45° **(ii)** 30° **(iii)** 14° **(iv)** 60°
5(a) $m=3$, $\alpha\doteqdot72^\circ$ **(b)** $m=-\frac{1}{2}$, $\alpha\doteqdot153^\circ$
(c) $m=-\frac{3}{4}$, $\alpha\doteqdot143^\circ$ **(d)** $m=\frac{2}{3}$, $\alpha\doteqdot34^\circ$
(e) $m=\frac{4}{5}$, $\alpha\doteqdot39^\circ$ **(f)** $m=-\frac{5}{2}$, $\alpha\doteqdot112^\circ$
6 Check your answers by substitution into the gradient formula.
7(a) $3{\cdot}73$ **(b)** $1{\cdot}00$ **(c)** $2{\cdot}41$ **(d)** $0{\cdot}32$
8(a) non-collinear **(b)** collinear with gradient $\frac{2}{3}$
9(a) $m_{AB}=\frac{1}{2}$, $m_{BC}=-2$ and $m_{AC}=0$, so $AB\perp BC$.
(b)(i) $m_{PQ}=4$, $m_{QR}=-\frac{1}{4}$ and $m_{PR}=-\frac{5}{3}$, so $PQ\perp QR$. Area $=8\frac{1}{2}$ units^2
(ii) $m_{XY}=\frac{7}{3}$, $m_{YZ}=\frac{2}{5}$ and $m_{XZ}=-\frac{5}{2}$, so $XZ\perp YZ$. Area $=14\frac{1}{2}$ units^2
10(d) square
11 In each case, show that each pair of opposite sides is parallel. **(a)** Show also that two adjacent sides are equal. **(b)** Show also that two adjacent sides are perpendicular. **(c)** Show that it is both a rhombus and a rectangle.
12(a) -5 **(b)** 5
13 $\lambda=-\frac{1}{2}$
14 $k=2$ or -1
15(a) OA has gradient $\frac{1}{2}$, and OB has gradient 2. Their product is $\frac{1}{2}\times(-2)=-1$, thus they are perpendicular. **(b)** $OA=AB=\sqrt{5}$ **(c)** $D(\frac{3}{2},-\frac{1}{2})$
(d) $C(1,-2)$ **(e)** square
16(b) $WZ=5$, $XY=10$. It is a trapezium, but not a parallelogram.
17(a) $P=(2,-1)$, $Q=(-1,4)$, $R=(-3,2)$, $S=(0,-3)$
(b) $m_{PQ}=m_{RS}=-\frac{5}{3}$ and $m_{PS}=m_{QR}=1$
18(a) $P=(2,3)$, $Q=(3,5)$
(b) $m_{PQ}=m_{BC}=2$ and $PQ=\sqrt{5}$
19 $\frac{1}{2}(p+q)$
20 $x^2+(y-1)^2=5^2$, a circle with centre $(0,1)$ and radius 5.
21(a) $\dfrac{y^2}{x(x-4)}=-1$ (products of gradients is -1), and $(x-4)^2+y^2=1$
(b) $Q(\frac{15}{4},\frac{1}{4}\sqrt{15})$ or $Q(\frac{15}{4},-\frac{1}{4}\sqrt{15})$
23(b) They are collinear if and only if $\Delta=0$, that is $a_1b_2+a_2b_3+a_3b_1=a_2b_1+a_3b_2+a_1b_3$.
24(a) $x=\frac{4p}{1-p^2}$ **(b)** $x=p-\frac{1}{p}$

Exercise 5C (Page 169)

1(a) not on the line **(b)** on the line **(c)** on the line
2 Check your answer by substitution.
3(a) $x=1$, $y=2$ **(b)** $x=-1$, $y=1$
(c) $x=3$, $y=-4$ **(d)** $x=5$, $y=1$
(e) $x=-2$, $y=-3$ **(f)** $x=-4$, $y=1$
4(a) $m=4$, $b=-2$ **(b)** $m=\frac{1}{5}$, $b=-3$
(c) $m=-1$, $b=2$
5(a) $x-y+3=0$ **(b)** $2x+y-5=0$
(c) $x-5y-5=0$ **(d)** $x+2y-6=0$
6(a) $m=1$, $b=3$ **(b)** $m=-1$, $b=2$
(c) $m=2$, $b=-5$ **(d)** $m=\frac{1}{3}$, $b=2$
(e) $m=-\frac{3}{4}$, $b=\frac{5}{4}$ **(f)** $m=1\frac{1}{2}$, $b=-2$
7 The sketches are clear from the intercepts.
(a) $A(-3,0)$, $B(0,3)$ **(b)** $A(2,0)$, $B(0,2)$
(c) $A(2\frac{1}{2},0)$, $B(0,-5)$ **(d)** $A(-6,0)$, $B(0,2)$
(e) $A(1\frac{2}{3},0)$, $B(0,1\frac{1}{4})$ **(f)** $A(1\frac{1}{3},0)$, $B(0,-2)$
8(a) $\alpha=45^\circ$ **(b)** $\alpha=135^\circ$ **(c)** $\alpha\doteqdot63^\circ26'$
(d) $\alpha\doteqdot18^\circ26'$ **(e)** $\alpha\doteqdot143^\circ8'$ **(f)** $\alpha\doteqdot56^\circ19'$
10(a) $y=-2x+3$, $y=\frac{1}{2}x+3$
(b) $y=\frac{5}{2}x+3$, $y=-\frac{2}{5}x+3$
(c) $y=-\frac{3}{4}x+3$, $y=\frac{4}{3}x+3$
11(a) $x-y+3=0$ **(b)** $-\sqrt{3}x+y+1=0$
(c) $x-\sqrt{3}y-2\sqrt{3}=0$ **(d)** $x+y-1=0$

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12 The angles of inclination are about 61° and 119°, so the two lines make acute angles of 61° with the x-axis.

13(a) $k = -\frac{1}{3}$ **(b)** $k = 3$

14(a) $2x - y = -4$ **(b)** $x - y = -3$ **(c)** $5x + y = -3$

15 $(x - a)^2 + (y - x)^2 = a^2$,
where $a = 2 - \sqrt{2}$ or $a = 2 + \sqrt{2}$,
$(x - \sqrt{2}\,)^2 + (y + \sqrt{2}\,)^2 = 2$,
$(x + \sqrt{2}\,)^2 + (y - \sqrt{2}\,)^2 = 2$

16(a) From their gradients, two pairs of lines are parallel and two lines are perpendicular.

(b) The distance between the x-intercepts of one pair of lines must equal the distance between the y-intercepts of the other pair. Thus $k = 2$ or 4.

Exercise **5D** (Page 173)

1(a) $2x-y-1=0$ **(b)** $x+y-4=0$ **(c)** $3x-y+8=0$
(d) $5x+y=0$ **(e)** $x+3y-8=0$ **(f)** $4x+5y+8=0$

2(a) $y = 2x-2$ **(b)** $2x+y-1=0$ **(c)** $x+2y+6=0$
(d) $3y = x + 13$

3(a) $\frac{y}{2} - x = 1$, $2x - y + 2 = 0$
(b) $\frac{x}{2} + \frac{y}{3} = 1$, $3x + 2y - 6 = 0$
(c) $-\frac{x}{4} - y = 1$, $x + 4y + 4 = 0$
(d) $\frac{x}{3} - \frac{y}{3} = 1$, $x - y - 3 = 0$

4(c)(i) No, the first two intersect at $(-4, 7)$, which does not lie on the third.
(ii) They all meet at $(5, 4)$.

5(a) $y = -2x + 5$, $y = \frac{1}{2}x + 6$
(b) $y = 2\frac{1}{2}x - 8\frac{1}{2}$, $y = -\frac{2}{5}x + 4\frac{1}{5}$
(c) $y = -1\frac{1}{3}x + 3$, $y = \frac{3}{4}x + 6\frac{1}{2}$

6(a) $3x + 2y + 1 = 0$ **(b)** $2x - 3y - 8 = 0$

7(a) $x - y - 1 = 0$ **(b)** $\sqrt{3}x + y + \sqrt{3} = 0$
(c) $x - y\sqrt{3} - 4 - 3\sqrt{3} = 0$
(d) $x + \sqrt{3}y + 2 + 5\sqrt{3} = 0$

8 $\ell_1 \parallel \ell_2$, and $\ell_3 \parallel \ell_4$, so there are two pairs of parallel sides. The vertices are
$(-2, -1)$, $(-4, -7)$, $(1, -2)$, $(3, 4)$.

9 $m_{BC} \times m_{AC} = -1$ so $BC \perp AC$,
AB: $y = x - 1$, BC: $y = \frac{1}{2}x + 2$, AC: $y = 2 - 2x$

10(a)(i) $x - 3 = 0$ **(ii)** $y + 1 = 0$
(b) $3x + 2y - 6 = 0$ **(c)(i)** $x - y + 4 = 0$
(ii) $\sqrt{3}x + y - 4 = 0$ **(d)** $x\sqrt{3} + y + 6\sqrt{3} = 0$

11(a) $m_{AC} = \frac{2}{3}$, $\theta \doteqdot 34^\circ$ **(b)** $3y = 2x - 2$
(c) $D(4, 2)$ **(d)** $m_{AC} \times m_{BD} = \frac{2}{3} \times -\frac{3}{2} = -1$, hence they are perpendicular. **(e)** isosceles
(f) area $= \frac{1}{2} \times AC \times BD = \frac{1}{2} \times \sqrt{52} \times \sqrt{52} = 26$
(g) $E(8, -4)$

12(b) $3x - 4y - 12 = 0$ **(c)** OB and AC are vertical and hence parallel, and from their gradients $(\frac{3}{4})$ OA is parallel to BC.
(d) $12\,\text{units}^2$, $AB = 2\sqrt{13}$

13(b) $4y = 3x + 12$ **(c)** $ML = MP = 5$ **(d)** $N(4, 6)$
(f) $x^2 + (y - 3)^2 = 25$

14(a) $(0, 2)$
(d) gradient $= \tan(180^\circ - \theta) = -\tan\theta = -2$ so $2x + y - 6 = 0$ **(e)** $R(3, 0)$, hence area $= 8\,\text{units}^2$.
(f) $QR = 2\sqrt{5}$, $PS = \frac{8}{5}\sqrt{5}$

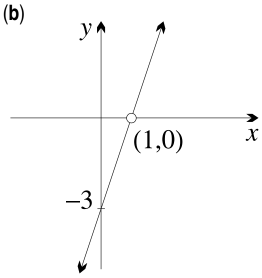
15 $k = 2\frac{1}{2}$

16(a)(i) $\mu = 4$ **(ii)** $\mu = -9$
(b) $\mu \neq 4$ **(c)(i)** $\lambda = 8$ **(ii)** $\lambda = 0$ or 16

17(a) $2x - 3y + k = 0$
(b)(i) $2x - 3y + 2 = 0$ **(ii)** $2x - 3y - 9 = 0$

18(a) $4x - 3y + k = 0$
(b)(i) $4x - 3y - 8 = 0$ **(ii)** $4x - 3y + 11 = 0$

19(a) $x \neq 1$
(b)



20 Stretch horizontally by a factor of a and vertically by a factor of b.

21 $\left(\frac{1}{p+q}, \frac{1}{p+q}\right)$ The two lines are inverse functions of each other, and so are reflections in the line $y = x$.

22 $3x + 4y - 24 = 0$

23 $y - 2 = m(x+1)$ **(a)** $2x - y = -4$ **(b)** $x - y = -3$
(c) $5x + y = -3$

25(a) $bx + ay = 2ab$ **(b)** $bx + 2ay = 3ab$
(c) $\ell bx + kay = (k + \ell)ab$

28(c)(i) 1 **(ii)** $\frac{1}{13}\sqrt{13}$

Exercise **5E** (Page 178)

1(a) $\frac{1}{2}\sqrt{10}$ **(b)** $\frac{4}{\sqrt{5}} = \frac{4}{5}\sqrt{5}$ **(c)** $\frac{5}{\sqrt{20}} = \frac{1}{2}\sqrt{5}$

2(a) 1 **(b)** 2 **(c)** $\sqrt{17}$ **(d)** $\frac{1}{2}\sqrt{10}$
(e) 0 (The point is on the line.) **(f)** $\frac{3}{2}\sqrt{5}$

3(a) D is distant $\frac{3}{10}$. **(b)** C is distant $4\frac{1}{10}$.

4(a) ℓ_3 is distant $\frac{3}{5}\sqrt{10}$. **(b)** ℓ_1 is distant $\frac{17}{13}\sqrt{13}$.

5(a) $\mu = 15$ or -5 **(b)** $\lambda = \frac{1}{2}$ or -1

6(a) $h > -4$ or $h < -6$ **(b)** $-6 \le k \le 4$

7(a) They do not intersect.

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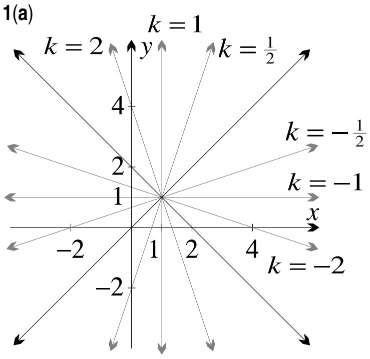
(b) Once, the line is tangent to the circle.
(c) Once, the line is tangent to the circle.
(d) They intersect twice.
8(a) $\frac{7}{\sqrt{10}} = \frac{7}{10}\sqrt{10}$ **(b)** $\frac{10}{\sqrt{17}} = \frac{10}{17}\sqrt{17}$
9(a) $x - 2y - 1 = 0$ **(b)** $2\sqrt{5}$ **(c)** $AB = 3\sqrt{5}$ so the area is 15 square units. **(d)** 10 square units
10(e) AC is common, $AO = AB$ and both triangles are right-angled, thus they are congruent by the RHS test. **(f)** 50 units^2 **(g)** $2\frac{2}{5}$
11(a) centre $(-2, -3)$ and $r = 2$, distance $\frac{4}{\sqrt{5}}$
(b) $\frac{4}{\sqrt{5}}$
12 The distances should differ. Since the distances differ, the lines are not parallel, and must intersect.
13(b) $x + 3y - 4 = 0$ and $3x - y - 2 = 0$
14(a) $y = mx$ **(b)** $\frac{|3m-1|}{\sqrt{m^2+1}}$
(d) $y = \frac{1}{5}(3 + 2\sqrt{6})x$ or $y = \frac{1}{5}(3 - 2\sqrt{6})x$
16(b) $\dfrac{|2q - q^2 - 3|}{\sqrt{5}}$ **(c)** $\frac{2}{\sqrt{5}}$
17(a) $(x - 7)^2 + (y + 1)^2 = 25$ **(b)** $\frac{7m+1}{\sqrt{m^2+1}}$
(c) $m = -\frac{4}{3}$ or $\frac{3}{4}$
(d) $4x + 3y + 6 = 0$ or $3x - 4y + 17 = 0$
19(b) Substitution gives $(p^2+q^2)(r^2-q^2d^2) = p^2r^2$. Rearranging this, $d^2 = \frac{r^2}{p^2+q^2}$.

Exercise **5F** (Page 182)

1(a)



(b) $k = 2$: $3x + y - 4 = 0$, $k = 1$: $x = 1$, $k = \frac{1}{2}$: $3x - y - 2 = 0$
(c) $k = -\frac{1}{2}$: $x - 3y + 2 = 0$, $k = -1$: $y = 1$, $k = -2$: $x + 3y - 4 = 0$
2(a) $x + 2y + 9 + k(2x - y + 3) = 0$
(b) $k = -3$ gives $y = x$.
3(a) $x - 2y - 4 = 0$ **(b)** $2x + y - 3 = 0$ **(c)** $y = x - 3$
4(b) $k = -2$ gives $y = 3$. **(c)** $k = 1$ gives $x = 1$.
(d) $(1, 3)$
5(a) $(1, 1)$ **(c)** $3y + x - 4 = 0$
6(a) $2x - 3y + 6 + k(x + 3y - 15) = 0$ **(b)(i)** $x = 3$
(ii) $4x + 3y - 24 = 0$ **(iii)** $x - 6y + 21 = 0$ **(iv)** $3y = 4x$
7(b)(i) $3x + 4y + 5 = 0$ **(ii)** $3x + 2y + 7 = 0$
(iii) $2y + 5x + 13 = 0$ **(iv)** $x - y + 4 = 0$
8(a) $(4, 1)$ **(b)(i)** $(0, 4)$ **(ii)** $(-3, 7)$
10(a) $\sqrt{10}$ **(b)** $2y + x - 4 = 0$
11(a) $2x - y = 0$
(b) Using $(x + 2y + 10) + k(2x - y) = 0$ yields $k = -1$, hence the line is $3y - x + 10 = 0$.
12(a) $y = 3x - 9$ **(b)** $4y = x + 8$ **(c)** $5y = 4x - 1$
(d) $2x + 3y - 6 = 0$
13(b) $\mu = \frac{3}{2}$ and the circle is $(x - \frac{3}{2})^2 + (y + \frac{3}{2})^2 = \frac{13}{2}$.
14(c) They are all 1 and -1. **(d)** When $\lambda = -1$ the equation reduces the straight line to $x = 0$.
(e) $\lambda = -\frac{1}{9}$, giving $(x - \frac{5}{2})^2 + y^2 = \frac{29}{4}$.
15(b) $k = -\frac{3}{5}$, giving $y = 4(x^2 - 1)$. **(c)** Using $(h+1)y = x^2(h-1) - 2x(2h-1)$ gives $h = 1$ and the result is $y = -x$, which is not a parabola.

Exercise **5G** (Page 185)

1(a)(i) $M = (4, 5)$ **(ii)** $OM = PM = QM = \sqrt{41}$
(iii) OM, PM and QM are three radii of the circle.
(b) $M = (p, q)$, $OM = PM = QM = \sqrt{p^2 + q^2}$
2(a) $PQ^2 = 5$, $RS^2 = 25$, $PS^2 = 17$, $QR^2 = 13$
(b) $PQ^2 = p^2 + q^2$, $RS^2 = r^2 + s^2$, $PS^2 = p^2 + s^2$, $QR^2 = q^2 + r^2$
3(a)(i) $P(2, 0)$, $Q(0, 2)$
(ii) $m_{PQ} = m_{AC} = -1$ and $AC = 4\sqrt{2}$
(b) $P(a + b, c)$, $Q(b, c)$, $m_{PQ} = m_{AC} = 0$ and $PQ = a$
4(a) $\frac{x}{3} + \frac{y}{4} = 1$ and $4y = 3x$, thus $C(\frac{48}{25}, \frac{36}{25})$.
(b) $OA = 3$, $AB = 5$, $OC = \frac{12}{5}$, $BC = \frac{16}{5}$, $AC = \frac{9}{5}$
6(a) $AB = BC = CA = 2a$ **(b)** $AB = AD = 2a$
(c) $BD = 2a\sqrt{3}$
7(a) D is the origin, $AB^2 = (a - b)^2 + c^2$, $BC^2 = (a + b)^2 + c^2$, $CD^2 = a^2$, $BD^2 = b^2 + c^2$.
(b) The sum of the squares on two sides of a triangle equals twice the square on the median to the third side plus twice the square of half the third side.
8 Both midpoints are at the origin.
9 The condition reduces to $x = q$, so that R is vertically in line with Q.
10(a) $P = \left(\frac{1}{2}(a_1 + b_1), \frac{1}{2}(a_2 + b_2)\right)$, $Q = \left(\frac{1}{2}(b_1 + c_1), \frac{1}{2}(b_2 + c_2)\right)$,

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$R = \left(\frac{1}{2}(c_1+d_1), \frac{1}{2}(c_2+d_2)\right)$,
$S = \left(\frac{1}{2}(d_1+a_1), \frac{1}{2}(d_2+a_2)\right)$
(b) The midpoint of both PR and QS is $M\left(\frac{1}{4}(a_1+b_1+c_1+d_1), \frac{1}{4}(a_2+b_2+c_2+d_2)\right)$.
(c) a parallelogram
12(a) $P=(1,4)$, $Q=(-1,0)$ and $R=(3,2)$, BQ: $x-y+1=0$, CR: $y-2=0$, AP: $x=1$
(b) The medians intersect at $(1,2)$.
13(a) The median through B is $3a(y+6b)=(c+3b)(x+6a)$. The median through A is $-3a(y-6b)=(c-3b)(x-6a)$.
(b) The medians intersect at $(0,2c)$.
14(a) perpendicular bisector of AB: $x=0$, of BC: $c(c-y)=(b+a)(x-b+a)$, of AC: $c(c-y)=(b-a)(x-b-a)$
(b) They all meet at $(0, \frac{c^2+b^2-a^2}{c})$.
(c) Any point on the perpendicular bisector of an interval is equidistant from the endpoints of that interval.
15 A suitable choice is $A(0,0)$, $B(2b,0)$ and $C(0,2c)$.
16 C has coordinates $(\frac{ab^2}{a^2+b^2}, \frac{a^2b}{a^2+b^2})$.

Chapter Six

Exercise **6A** (Page 190)

1(a) 1 **(b)** $\frac{1}{5}$ **(c)** $3\frac{1}{2}$ **(d)** $\frac{4}{21}$ **(e)** $\frac{1}{49}$ **(f)** 64 **(g)** $\frac{81}{16}$ **(h)** $\frac{4}{25}$ **(i)** $\frac{27}{1000}$ **(j)** 1
2(a) 5 **(b)** 3 **(c)** 9 **(d)** 4 **(e)** 8 **(f)** 27 **(g)** 81 **(h)** $\frac{125}{8}$ **(i)** $\frac{8}{27}$ **(j)** $\frac{3}{2}$
3(a) 169 **(b)** $\frac{9}{8}$ **(c)** $\frac{1}{2}$ **(d)** 4000 **(e)** $\frac{1}{900}$
4(a) $\frac{7}{x}$ **(b)** $\frac{3}{x^3}$ **(c)** $\frac{x^2}{(y+1)^2}$ **(d)** $\frac{t+3}{s^2}$ **(e)** $\frac{9x^2y^2}{8}$
5(a) $\frac{y}{x^2}$ **(b)** $\frac{21}{x}$ **(c)** $\frac{s^6}{y^9}$ **(d)** $\frac{c^2}{5d^3}$ **(e)** $21m^2$ **(f)** $\frac{b^2}{a}$ **(g)** $\frac{2x}{y^2}$ **(h)** $\frac{p^2}{q^6}$
6(a) -1 **(b)** -2 **(c)** $\frac{1}{3}$ **(d)** $\frac{3}{2}$ **(e)** $\frac{1}{2}$ **(f)** $-\frac{1}{2}$
7(a) $\frac{1}{2}$ **(b)** $\frac{1}{128}$ **(c)** $\frac{25}{4}$ **(d)** $\frac{343}{64}$ **(e)** $\frac{16}{81}$ **(f)** $\frac{125}{8}$ **(g)** $\frac{25}{36}$ **(h)** $\frac{1}{8}$ **(i)** $\frac{2}{3}$ **(j)** $\frac{125}{27}$
8(a) $7^{\frac{1}{3}}$ **(b)** $5^{\frac{3}{4}}$ **(c)** $5^{-1}x^{\frac{4}{3}}$ **(d)** $11^{-\frac{1}{5}}a^{\frac{1}{2}}$ **(e)** $7y^{-\frac{4}{5}}$ **(f)** $x^{\frac{3}{2}}$ **(g)** $x^{\frac{5}{2}}$ **(h)** $2x^{-\frac{1}{2}}$ **(i)** $\frac{1}{8}x^{-1}$ **(j)** $7(3x+2)^{-1}$
9(a) 9 **(b)** 3 **(c)** $\frac{1}{20}$ **(d)** $\frac{3}{10}$
10(a) $x^2+10+\frac{25}{x^2}$ **(b)** $x^4-14+\frac{49}{x^4}$ **(c)** $9x-12+\frac{4}{x}$
11(a) 2^{4n} **(b)** 5^{n-1} **(c)** 3^{6x+1} **(d)** 11^{1-5n} **(e)** $7^{2n-\frac{1}{2}}$ **(f)** 2^{3x-5}
12(a) $x=-\frac{1}{3}$ **(b)** $x=\frac{1}{4}$ **(c)** $-\frac{2}{3}$ **(d)** $x=\frac{5}{12}$ **(e)** $x=-4$ **(f)** $x=-2$
13(a) $b=\frac{1}{343}$ **(b)** $\frac{1}{11}$ **(c)** $x=\frac{1}{81}$
14(a) $x=3$ and $y=4$ **(b)** $x=0$ and $y=-1$ **(c)** $x=-2$ and $y=\frac{1}{2}$
15(a) $\dfrac{b-a}{ab}$ **(b)** $\dfrac{y}{y+1}$ **(c)** $\dfrac{x^2y^2}{y^2-x^2}$ **(d)** $\dfrac{ab}{b-a}$ **(e)** $\dfrac{x^3-y^3}{x^3y^3}$ **(f)** $\dfrac{1}{a+1}$
16(a) 2^{6n} **(b)** 81 **(c)** 2^{3x} **(d)** $2^{2x}\,3^{2x}$ (or 6^{2x}) **(e)** $5^{4n-4}\,2^{4n-5}$ **(f)** $2^x\,3^{1-x}$
17(a) 50×7^n **(b)** 26 **(c)** $124\times 5^{n-3}$ **(d)** 7 **(e)** $7\times 2^{2n-1}$ **(f)** 2^n
18(a) $\frac{3^n}{2}$ **(b)** $\dfrac{1}{3^x}$ **(c)** $-2^n\,3^n$
19(a) $>$ **(b)** $>$ **(c)** $<$ **(d)** $<$ **(e)** $>$ **(f)** $>$
20(a) $1\frac{1}{2}$ **(b)** $4\frac{1}{2}$ **(c)** 5 **(d)** 4 **(e)** 6 **(f)** $7\sqrt{2}$
22(a) $12 < 2^{\frac{11}{3}} < 13$ **(b)** $13 < 2^{\frac{15}{4}} < 14$
23 $\lim_{x\to 0^+} 0^x = 0$ and $\lim_{x\to 0} x^0 = 1$

Exercise **6B** (Page 195)

1(a) $3^x=9$, $x=2$ **(b)** $2^x=16$, $x=4$
(c) $5^x=125$, $x=3$ **(d)** $10^x=\frac{1}{10}$, $x=-1$
(e) $7^x=\frac{1}{49}$, $x=-2$ **(f)** $(\frac{1}{3})^x=\frac{1}{81}$, $x=4$
(g) $5^x=\sqrt{5}$, $x=\frac{1}{2}$ **(h)** $11^x=1/\sqrt{11}$, $x=-\frac{1}{2}$
2 For $y=2^x$: $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{2}$, 1, 2, 4, 8.
For $y=\log_2 x$: -3, -2, -1, 0, 1, 2, 3.

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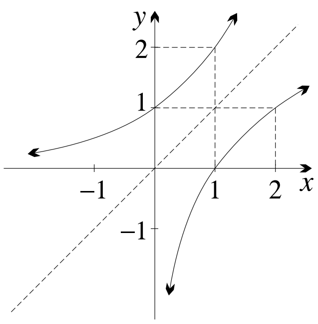

3(a) $x = 4^3 = 64$ **(b)** $x = 13^{-1} = \frac{1}{13}$
(c) $x = 9^{\frac{1}{2}} = 3$ **(d)** $x = 10^{-2} = \frac{1}{100}$
(e) $x = (\frac{1}{16})^{-\frac{1}{4}} = 2$ **(f)** $x = 7^{-\frac{1}{2}} = 1/\sqrt{7}$
(g) $x = 36^{1\cdot 5} = 216$ **(h)** $x = 8^{-\frac{2}{3}} = \frac{1}{4}$
4(a) $x^3 = 27$, $x = 3$ **(b)** $x^{-1} = \frac{1}{7}$, $x = 7$
(c) $x^3 = 1000$, $x = 10$ **(d)** $x^{\frac{1}{2}} = 3$, $x = 9$
(e) $x^{-2} = 25$, $x = \frac{1}{5}$ **(f)** $x^2 = \frac{4}{9}$, $x = \frac{2}{3}$
(g) $x^{\frac{4}{3}} = 16$, $x = 8$ **(h)** $x^{-\frac{1}{2}} = 9$, $x = \frac{1}{81}$
5(a) 1 **(b)** -1 **(c)** 3 **(d)** -2 **(e)** $\frac{1}{2}$ **(f)** $-\frac{1}{2}$
(g) 0 **(h)** $-1\frac{1}{2}$
6(a) 3 and 4, 3·46 **(b)** 3 and 4, 3·01
(c) 2 and 3, 2·21 **(d)** 8 and 9, 8·64
7(a) $2\log_2 3$ **(b)** $1 + 2\log_2 3$ **(c)** $-1 - \log_2 3$
(d) $-1 + \log_2 5$
8(a) $x = \log_2 13 \doteqdot 3{\cdot}700$
(b) $x = 2 + \log_3 20 \doteqdot 4{\cdot}727$
(c) $x > \log_7 1000 \doteqdot 3{\cdot}550$
(d) $x < -1 + \log_2 10 \doteqdot 2{\cdot}322$
(e) $x < \log_5 0{\cdot}04 = -2$
(f) $x = -1 + \log_{\frac{1}{2}} 10 \doteqdot -4{\cdot}322$
(g) $x < \log_{\frac{1}{3}} 100 \doteqdot -4{\cdot}192$
(h) $x > \log_{0\cdot 06} 0{\cdot}001 \doteqdot 2{\cdot}455$
9(a) 33 powers **(b)** 14 powers
10(a) $6x$ **(b)** $-x-y-z$ **(c)** $3y+5$ **(d)** $2x+2z-1$
(e) $y-x$ **(f)** $x+2y-2z-1$ **(g)** $-2z$ **(h)** $3x-y-z-2$
11(a) $-3+\log_2 5$ **(b)** $\log_2 5+\frac{3}{2}\log_2 3$ **(c)** $-\frac{1}{2}-\log_2 3$
(d) $\frac{1}{2} + \frac{3}{2}\log_2 3 - \frac{3}{2}\log_2 5$
12(a) 5 **(b)** 7 **(c)** n **(d)** y
13(a) $3 = 2^{\log_2 3}$ **(b)** $u = 3^{\log_3 u}$ **(c)** $7 = a^{\log_a 7}$
(d) $u = v^{\log_v u}$
14(a) $\frac{1}{2}$ **(b)** 49 **(c)** 15 **(d)** x^n **(e)** $1/x$ **(f)** $x \times 5^x$
(g) x^x **(h)** $x^{1/x}$
15(a) $x + y = xy$ **(b)** $x = 1000y$ **(c)** $x = y^4$
(d) $x^2y^3 = z^4$ **(e)** $2^x = y$ **(f)** $x = yz^n$
(g) $64x^3 = y^2$ **(h)** $(2x+1)^2 = (2x-1)^3$
16(b) $2, -2, -2$
18(a) $SD = \frac{1}{4}(2^{2x} - 2^{-2x})$, $S + D = 2^x$,
$S - D = 2^{-x}$, $S^2 - D^2 = 1$
(b) $x = \log_2\left(S + \sqrt{S^2 - 1}\right)$,
$x = \log_2\left(D + \sqrt{D^2 + 1}\right)$

Exercise **6C** (Page 198)

1(a) 21, 25, 29, 33 **(b)** 24, 48, 96, 192
(c) $-1, -10, -19, -28$ **(d)** $3, 1, \frac{1}{3}, \frac{1}{9}$
(e) $1, -1, 1, -1$ **(f)** 64, 81, 100, 121
(g) $\frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}$ **(h)** $-2, 1, -\frac{1}{2}, \frac{1}{4}$
2(a) 3, 8, 13, 18 **(b)** 5, 25, 125, 625
(c) 1, 8, 27, 64 **(d)** $5, -2, -9, -16$
(e) 12, 36, 108, 324 **(f)** 4, 12, 24, 40
(g) $-1, 2, -3, 4$ **(h)** $-3, 9, -27, 81$
3(a) 5, 17, 29, 41 **(b)** $\frac{3}{4}, \frac{3}{2}, 3, 6$ **(c)** 1, 2, 6, 24
(d) $28, -14, 7, -3\frac{1}{2}$ **(e)** $37, 13, -11, -35$
(f) $2\sqrt{2}, 4, 4\sqrt{2}, 8$ **(g)** 5, 15, 30, 50
(h) $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}$
4(a) $77 = T_{10}$, 349 is not a member, $1577 = T_{260}$.
(b) 63 terms are less than 400, $T_{64} = 401$.
5(a) 60 is not a member, $80 = T_4$, $605 = T_{11}$.
(b) 14 terms are less than 1000, $T_{15} = 1125$.
6(a) 0, 2, 0, 2 **(b)** $-50, 100, -200, 400$
(c) $-36x, 18x, -9x, \frac{9}{2}x$ **(d)** $5a, 3a, a, -a$
(e) $4a, 8a, 16a, 32a$ **(f)** 1, 5, 19, 65
(g) $3, 1, -5, 9$ **(h)** $1, 2\sqrt{2}, 8, 16\sqrt{2}$
(i) $\frac{3}{4}x, 3x, \frac{27}{4}x, 12x$
7(a) $T_n = T_{n-1} + 5$ **(b)** $T_n = 2T_{n-1}$
(c) $T_n = T_{n-1} - 7$ **(d)** $T_n = -T_{n-1}$
8(a) -10 is not a member, $-15 = T_9$.
(b) 106 terms
9(a) $28 = T_7$, $70 = T_{10}$ **(b)** 5 terms
10(a) $1\frac{1}{2} = T_4$, $96 = T_{10}$ **(b)** $T_7 = 12$
11 From Q2: **(a)** $y = 5x-2$ **(b)** $y = 5^x$ **(c)** $y = x^3$
(d) $y = 12 - 7x$ **(e)** $y = 4 \times 3^x$ **(f)** $y = 2x(x+1)$
(g) nothing simple **(h)** nothing simple
12(a) $1, 0, -1, 0$, T_n where n is even.
(b) $0, -1, 0, 1$, T_n where n is odd.
(c) $-1, 1, -1, 1$, no terms are zero.
(d) 0, 0, 0, 0, all terms are zero.
(e) $-1, 0, -1, 0$, T_n where n is even.
(f) 1, 1, 0, 0, the third and fourth term in each group of 4 is zero.
13(a) 11, 15 **(b)** 4, 8
14(a) $\frac{4}{5}$ **(b)** $\frac{n}{n+1}$ **(d)** $\frac{1}{30} = T_5$
15(a) $0{\cdot}9 = T_{10}$, $0{\cdot}99 = T_{100}$ **(b)** $n^2 : (n^2-1)$ **(d)** $\frac{1}{n}$
16(a) The Fibonacci sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, ... The Lucas sequence is 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, ...

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The sum of two odd integers is even, and the sum of an even and an odd is odd.
(b) The first is 2, 4, 6, 10, 16, ..., which is $2F_{n+1}$. The second is 0, 2, 2, 4, 6, ..., which is $2F_{n-1}$.
(c) $\frac{1}{2}+\frac{1}{2}\sqrt{5}$, $\frac{3}{2}+\frac{1}{2}\sqrt{5}$, $\frac{4}{2}+\frac{2}{2}\sqrt{5}$, $\frac{7}{2}+\frac{3}{2}\sqrt{5}$

Exercise **6D** (Page 201)

1(a) $d=3$, $T_n=5+3n$, $T_{10}=35$
(b) $d=-6$, $T_n=27-6n$, $T_{10}=-33$
(c) not an AP (but $T_n=2^{4-n}$, $T_{10}=\frac{1}{64}$)
(d) $d=4$, $T_n=4n-7$, $T_{10}=33$
(e) $d=1\frac{1}{4}$, $T_n=\frac{1}{4}(2+5n$, $T_{10}=13$
(f) $d=-17$, $T_n=29-17n$, $T_{10}=-141$
(g) $d=-\sqrt{2}$, $T_n=5+2\sqrt{2}-n\sqrt{2}$, $T_{10}=5-8\sqrt{2}$
(h) not an AP (but $T_n=n^2$, $T_{10}=100$)
(i) $d=3\frac{1}{2}$, $T_n=\frac{1}{2}(7n-12)=\frac{7}{2}n-6$, $T_{10}=29$
2(a) $d=-3$, $T_n=85-3n$, $T_{25}=10$, $T_{29}=-2$
(b) $d=-8$, $T_n=353-8n$, $T_{25}=153$, $T_{45}=-7$
(c) $d=-\frac{5}{4}$, $T_n=\frac{1}{4}(103-5n)$, $T_{25}=-5\frac{1}{2}$, $T_{21}=-\frac{1}{2}$
3(a) $x=23$, $d=9$ **(b)** $x=-4$, $d=18$
(c) $x=10$, $d=8$ **(d)** $x=-2$, $d=-4$
4(a) cost $=200+300n$ **(b)** cost $=\$4700$
(c) 32 windows
5(a) 2120, 2240, 2360, 2480
(b) $A_n=2000+120n$, $A_{12}=3440$ **(c)** 34 years
6(a) 667 terms **(b)** 44 terms **(c)** 81 terms
7(a) 11, 15, 19, 23, $a=11$, $d=4$
(b) $T_{50}+T_{25}=314$, $T_{50}-T_{25}=100$
(d) $815=T_{202}$ **(e)** $T_{248}=999$, $T_{249}=1003$
(f) $T_{49}=203$, ..., $T_{73}=299$ lie between 200 and 300, making 25 terms.
8(a)(i) $T_{63}=504$ **(ii)** $T_{106}=848$ **(iii)** 44 terms
(b)(i) $T_{91}=1001$, $T_{181}=1991$, 91 terms
(ii) $T_{115}=805$, $T_{285}=1995$, 171 terms
9(a) $d=4$, $a=-1$ **(b)** $d=-9$, $a=60$
(c) $d=3\frac{1}{2}$, $a=-4\frac{1}{2}$ **(d)** $d=2-\sqrt{5}$, $a=7\sqrt{5}-16$
10(a) $T_8=37$ **(b)** $T_6=-2$
11(a) $d=4$, $x=1$ **(b)** $d=6x$, $x=\frac{1}{3}$
(c) $d=-3-3x$, $x=-2$
12(a) $d=\log_3 2$, $T_n=n\log_3 2$
(b) $d=-\log_a 3$, $T_n=\log_a 2+(4-n)\log_a 3$
(c) $d=x+4y$, $T_n=nx+(4n-7)y$
(d) $d=-4+7\sqrt{5}$, $T_n=9-4n+(7n-13)\sqrt{5}$
(e) $d=-1{\cdot}88$, $T_n=3{\cdot}24-1{\cdot}88n$
(f) $d=-\log_a x$, $T_n=\log_a 3+(3-n)\log_a x$
13 The 13 terms $T_{28}=19$, ..., $T_{40}=-17$ have squares less than 200.
14(a) $a=m+b$, $d=m$ **(b)** $f(x)=a+(x-1)d$
15(a) $a=\lambda a_1+\mu a_2$, $d=\lambda d_1+\mu d_2$
(b) $\mathcal{A}(1,0)$ is 1, 1, 1 ..., $\mathcal{A}(0,1)$ is 0, 1, 2 ..., $\mathcal{A}(a,d)=a\mathcal{A}(1,0)+d\mathcal{A}(0,1)$.
(c) $\lambda=\dfrac{ad_2-a_2d}{a_1d_2-a_2d_1}$, $\mu=\dfrac{ad_1-a_1d}{a_2d_1-a_1d_2}$.
Note: $a_1:a_2\neq d_1:d_2$ ensures $a_2d_1-a_1d_2\neq 0$.

Exercise **6E** (Page 205)

1(a) 1, 3, 9, 27, $T_n=3^{n-1}$
(b) 5, -10, 20, -40, $T_n=5\times(-2)^{n-1}$
(c) 18, 6, 2, $\frac{2}{3}$, $T_n=18\times(\frac{1}{3})^{n-1}$
(d) 6, -3, $1\frac{1}{2}$, $-\frac{3}{4}$, $T_n=6\times(-\frac{1}{2})^{n-1}$
(e) 1, $\sqrt{2}$, 2, $2\sqrt{2}$, $T_n=(\sqrt{2})^{n-1}$
(f) -7, 7, -7, 7, $T_n=-7\times(-1)^{n-1}=7\times(-1)^n$
2(a) $r=2$, $T_n=10\times 2^{n-1}$, $T_6=320$
(b) $r=\frac{1}{3}$, $T_n=180\times(\frac{1}{3})^{n-1}$, $T_6=\frac{20}{27}$
(c) not a GP (but $T_n=(n+7)^2$, $T_6=169$)
(d) not a GP (It's an AP with $T_n=20+15n$, $T_6=110$.) **(e)** $r=4$, $T_n=\frac{3}{4}\times 4^{n-1}$, $T_6=768$
(f) $r=\frac{1}{4}$, $T_n=-24\times(\frac{1}{4})^{n-1}$, $T_6=-\frac{3}{128}$
3(a) $r=-1$, $T_n=(-1)^{n-1}$, $T_6=-1$
(b) $r=-2$, $T_n=-2\times(-2)^{n-1}=(-2)^n$, $T_6=64$
(c) $r=-3$, $T_n=-8\times(-3)^{n-1}$, $T_6=1944$
(d) $r=-\frac{1}{2}$, $T_n=60\times(-\frac{1}{2})^{n-1}$, $T_6=-\frac{15}{8}$
(e) $r=-\frac{1}{2}$, $T_n=-1024\times(-\frac{1}{2})^{n-1}$, $T_6=32$
(f) $r=-12$, $T_n=\frac{3}{8}\times(-12)^{n-1}$, $T_6=-2^7\times 3^6$
4(a) $r=2$ **(b)** $r=3$ or -3 **(c)** $r=\frac{1}{9}$ or $-\frac{1}{9}$
(d) $r=-\frac{3}{2}$ **(e)** $r=0{\cdot}1$ or $-0{\cdot}1$
(f) $r=\sqrt{2}$ or $-\sqrt{2}$
5(a) $r=4$, $a=\frac{1}{16}$
(b) $r=3$ and $a=\frac{1}{9}$, or $r=-3$ and $a=-\frac{1}{9}$
(c) $r=\sqrt{2}$ and $a=\frac{3}{2}$, or $r=-\sqrt{2}$ and $a=\frac{3}{2}$
(d) $r=\frac{1}{2}$, $a=128\sqrt{2}$
6(a) $r=\sqrt{2}$, $T_n=\sqrt{6}\times(\sqrt{2})^{n-1}$, $T_6=8\sqrt{3}$
(b) $r=ax^2$, $T_n=a^nx^{2n-1}$, $T_6=a^6x^{11}$
(c) $r=y/x$, $T_n=-x^{2-n}y^{n-2}$, $T_6=-y^4/x^4$
7(a) 50, 100, 200, 400, 800, 1600, $a=50$, $r=2$
(b) $T_{50}\times T_{25}=5^4\times 2^{75}$, $T_{50}\div T_{25}=2^{25}$
(d) $6400=T_8$ **(f)** $T_6=1600$, ..., $T_{11}=51\,200$ lie between 1000 and 100 000, making 6 terms.
8(a) AP: $x=-48$, $d=72$; GP: $x=6$, $r=4$
(b) AP: $x=60$, $d=36$;
GP: $x=48$ and $r=2$, or $x=-48$ and $r=-2$
(c) They can't form an AP. GP: $x=9$, $r=2$
(d) AP: $x=2$, $d=4$;

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GP: $x = 4$ and $r = 3$, or $x = 0$ and $r = -1$
9(a) $T_n = 7 \times 2^{n-1}$, 6 terms
(b) $T_n = 2 \times 7^{n-1}$, 5 terms
(c) $T_n = 5^{n-3}$, 7 terms
10(a) 18 terms **(b)** 7 terms **(c)** 11 terms
11(a) $T_9, \ldots, T_{18}$, 10 terms **(b)** $T_5, \ldots, T_7$, 3 terms
(c) $T_8, \ldots, T_{11}$, 4 terms
12(a) $P \times 1{\cdot}07$, $P \times (1{\cdot}07)^2$, $P \times (1{\cdot}07)^3$
(b) $A_n = P \times (1{\cdot}07)^n$ **(c)** 11 full years to double, 35 full years to increase tenfold.
13(a) $T_n = 98 \times (\frac{1}{7})^{n-1}$, 10 terms
(b) $T_n = 25 \times (\frac{1}{5})^{n-1} = (\frac{1}{5})^{n-3}$, 11 terms
(c) $T_n = (0{\cdot}9)^{n-1}$, 132 terms
14(a) $W_1 = 20\,000 \times 0{\cdot}8$, $W_2 = 20\,000 \times (0{\cdot}8)^2$, $W_3 = 20\,000 \times (0{\cdot}8)^3$, $W_n = 20\,000 \times (0{\cdot}8)^n$
(b) 11 years
15 152 sheets
16(a) $T_n = 2x^n$, $x = 1$ or -1
(b) $T_n = x^{6-2n}$, $x = \frac{1}{3}$ or $-\frac{1}{3}$
(c) $T_n = 2^{-16} \times 2^{4n-4}x = 2^{4n-20}x$, $x = 6$
17(a) $a = 6\frac{1}{4}$ and $b = 2\frac{1}{2}$, or $a = 4$ and $b = -2$
(b) $a = 1, b = 0$ **(c)** $a = 66$ **(d)** $a = \frac{3}{16}$ **(e)** $a = 28$, $d = -1$ **(f)** $a = \frac{1}{3}$ and $r = 3$, or $a = \frac{2}{3}$ and $r = -3$
18(c) $r = 1, \frac{1}{2} + \frac{1}{2}\sqrt{5}$ or $\frac{1}{2} - \frac{1}{2}\sqrt{5}$ **(d)** $1, 2, 4, 8, \ldots$
19(a) $T_n = 2^{8-3n}$
20(a) first term $= 2^a$, ratio $= 2^d$
(b) first term $= \log_2 a$, ratio $= \log_2 r$
(c) No, it can be any positive number except 1.
21(a) $a = kb$, $r = b$ **(b)** $f(x) = ar^{x-1}$
22(a) first term $= aA$, ratio $= rR$
23(a) first term $= {a_1}^{\lambda}{a_2}^{\mu}$, ratio $= {r_1}^{\lambda}{r_2}^{\mu}$
(b) $\mathcal{G}(2,1)$ is $2, 2, 2, 2, \ldots$, $\mathcal{G}(1,2)$ is $1, 2, 4, 8, \ldots$, $\lambda = \log_2 a$, $\mu = \log_2 r$

Exercise **6F** (Page 209)

1(a) 10, 8 or -8 **(b)** $20\frac{1}{2}$, 20 or -20
(c) $-12\frac{1}{2}$, 10 or -10 **(d)** -15, no GM
(e) $3\frac{3}{4}$, 3 or -3 **(f)** $25a^2$, $7a^2$ or $-7a^2$
(g) 0, no GM **(h)** $\frac{1}{2}(a+1)$, $\sqrt{a}$ or $-\sqrt{a}$
(i) 40, 2^5 or -2^5 **(j)** 72, $2^5\sqrt{2}$ or $-2^5\sqrt{2}$
(k) $\frac{1}{2}a^3(a^2+1)$, a^4 or $-a^4$ **(l)** $\dfrac{1+x^6}{2x^3}$, 1 or -1
2(a) $x = 2$ **(b)** $x = -4$ **(c)** $x = \frac{13}{3}$ **(d)** $x = 1$ or 6
3(a) 14, 21, 28, 35 **(b)** 18, 12 **(c)** $36\frac{1}{2}$, 33, $29\frac{1}{2}$, 26, $22\frac{1}{2}$, 19, $15\frac{1}{2}$, 12, $8\frac{1}{2}$ **(d)** $\sqrt{10}$, 10, $10\sqrt{10}$, 100, $100\sqrt{10}$ or $-\sqrt{10}$, 10, $-10\sqrt{10}$, 100, $-100\sqrt{10}$
4(a) $a = 14\frac{1}{4}$, $b = 25\frac{1}{2}$, $c = 36\frac{3}{4}$
(b) $a = 6$, $b = 12$, $c = 24$ or $a = -6$, $b = 12$, $c = -24$
5(a) $\sqrt{5}$, 2 or -2 **(b)** $\frac{3}{8}\sqrt{2}$, $\frac{1}{2}$ or $-\frac{1}{2}$
(c) x, $\sqrt{x^2-y^2}$ or $-\sqrt{x^2-y^2}$
(d) x^2+y^2, x^2-y^2 or y^2-x^2
(e) $\dfrac{x}{x^2-y^2}$, $\dfrac{1}{\sqrt{x^2-y^2}}$ or $\dfrac{-1}{\sqrt{x^2-y^2}}$
(f) $\frac{5}{2}\log_2 3$, $2\log_2 3$ or $-2\log_2 3$
(g) $2\log_2 3$, $\sqrt{3}\log_2 3$ or $-\sqrt{3}\log_2 3$
(h) $5\log_b 2$, $4\log_b 2$ or $-4\log_b 2$
(i) $\frac{1}{4}\sqrt{5}$, $\frac{1}{2}$ or $-\frac{1}{2}$
6(a) $0{\cdot}100\,01$, $0{\cdot}002$ or $-0{\cdot}002$
(b) $0{\cdot}150\,005$, $0{\cdot}100\,01$, $0{\cdot}050\,015$; $0{\cdot}02$, $0{\cdot}002$, $0{\cdot}0002$ or $-0{\cdot}02$, $0{\cdot}002$, $-0{\cdot}0002$
7(a) $\dfrac{x^2+y^2}{2xy}$, 1 **(c)** $x = y$
8(b) The sign of the AM is the sign of the larger in absolute value.
10(b) $(a-b)^2 \geq 0$, so $(a+b)^2 \geq 4ab$, so $a+b \geq 2\sqrt{ab}$. **(c)** When $a = b$.
13(b) $XP = AM$
14(a) $c : a = 5 : 3$ **(b)** $c : a = (1+\sqrt{5}) : 2$
15(b) $T_8/T_1 = (\frac{1}{2})^{\frac{7}{12}} \doteqdot 0{\cdot}6674 \doteqdot \frac{2}{3}$
(c) $T_5/T_1 = (\frac{1}{2})^{\frac{4}{12}} \doteqdot 0{\cdot}7937 \doteqdot \frac{4}{5}$
(d) $T_6/T_1 = (\frac{1}{2})^{\frac{5}{12}} \doteqdot 0{\cdot}7491 \doteqdot \frac{3}{4}$, $T_4/T_1 = (\frac{1}{2})^{\frac{3}{12}} \doteqdot 0{\cdot}8409 \doteqdot \frac{5}{6}$
(e) $T_3/T_1 = (\frac{1}{2})^{\frac{2}{12}} \doteqdot 0{\cdot}8908 \doteqdot \frac{8}{9}$, $T_2/T_1 = (\frac{1}{2})^{\frac{1}{12}} \doteqdot 0{\cdot}9439 \doteqdot \frac{17}{18}$
16(a) $\lambda = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$ **(b)** $\lambda = \frac{3}{2} + \frac{1}{2}\sqrt{5}$ (M to the left of A), or $\lambda = \frac{3}{2} - \frac{1}{2}\sqrt{5}$ (M to the right of B)

Exercise **6G** (Page 212)

1(a) 75 **(b)** 55 **(c)** 10 **(d)** 40 **(e)** 404 **(f)** 0 **(g)** 31 **(h)** 10 **(i)** -10 **(j)** -1 **(k)** 1 **(l)** 80
2(a) $\displaystyle\sum_{n=1}^{40} n^3$ **(b)** $\displaystyle\sum_{n=1}^{40} \frac{1}{n}$ **(c)** $\displaystyle\sum_{n=1}^{20} (n+2)$ **(d)** $\displaystyle\sum_{n=1}^{12} 2^n$
(e) $\displaystyle\sum_{n=1}^{13} 2^{n-1}$ **(f)** $\displaystyle\sum_{n=1}^{k} ar^{n-1}$ **(g)** $\displaystyle\sum_{n=1}^{k} \Big(a+(n-1)d\Big)$
(h) $\displaystyle\sum_{n=1}^{10} (-1)^n n$ **(i)** $\displaystyle\sum_{n=1}^{10} (-1)^{n-1} n$
(j) $\displaystyle\sum_{n=1}^{2k+1} (-1)^{n-1} x^{n-1}$
3(c)(i) $\displaystyle\sum_{n=0}^{6} (3n+1)$ **(ii)** $\displaystyle\sum_{n=2}^{8} (3n-5)$ **(iii)** $\displaystyle\sum_{n=7}^{13} (3n-20)$
5(b) 3
6(a) 125 **(b)** 0 **(c)** 873 **(d)** 56 700

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Exercise 6H (Page 214)

1(a) S_n: 2, 7, 15, 26, 40, 57, 77 (AP with $a = 2$ and $d = 3$) **(b)** T_n: 2, 4, 8, 16, 32, 64, 128 (GP with $a = 2$ and $r = 2$)
2 They are the partial sums of the AP 2, 6, 10, 14, ... For further explanation, see your chemistry teacher.
3(a) 3, 8, 15, 24, 35 **(b)** 3, 5, 7, 9, 11
(c) $T_n = 2n + 1$
4(a) $T_n = 5 - 2n$ **(b)** $T_n = 6n - 8$ **(c)** $T_n = 11 - 10n$
6(a) 2, 8, 26, 80, 242 **(b)** 2, 6, 18, 54, 162
(c) $T_n = 2 \times 3^{n-1}$
7(a) $T_n = 5 \times 2^n$ **(b)** $T_n = 16 \times 5^{n-1}$
(c) $T_n = 3 \times 4^{n-2}$
8(a) $T_n = 6n$, 6, 12, 18
(b) $T_n = n + 1$, 2, 3, 4 **(c)** $T_n = 6 - 2n$, 4, 2, 0
(d) $T_n = 4$, 4, 4, 4 **(e)** $T_n = 3n^2 - 3n + 1$, 1, 7, 19
(f) $T_n = 2 \times 3^{-n}$, $\frac{2}{3}$, $\frac{2}{9}$, $\frac{2}{27}$,
(g) $T_n = -6 \times 7^{-n}$, $-\frac{6}{7}$, $-\frac{6}{49}$, $-\frac{6}{343}$
(h) $T_n = a + (n-1)d$, a, $a + d$, $a + 2d$
(i) $T_n = n^2$, 1, 4, 9 **(j)** $T_n = n^3$, 1, 8, 27
(k) $T_n = ar^{n-1}$, a, ar, ar^2
9(a) $T_1 = 8$, $T_n = 2n + 3$ for $n \ge 2$
(b) $T_1 = -7$, $T_n = 14 \times 3^{n-1}$ for $n \ge 2$
(c) $T_1 = 1$, $T_n = \dfrac{-1}{n(n-1)}$ for $n \ge 2$
(d) $T_n = 3n^2 - n + 1$ for $n \ge 1$
The formula holds for $n = 1$ when $S_0 = 0$.

Exercise 6I (Page 216)

1 185
2(a) 222 **(b)** -630 **(c)** 78 400 **(d)** 0 **(e)** 65 **(f)** 30
3(a) 101 terms, 10 100 **(b)** 13 terms, 650
(c) 11 terms, 275 **(d)** 100 terms, 15 250
(e) 11 terms, 319 **(f)** 10 terms, $61\frac{2}{3}$
4(a) 500 terms, 250 500 **(b)** 2001 terms, 4 002 000
(c) 3160 **(d)** 1440
5(a) $S_n = n(1 + 2n)$ **(b)** $S_n = \frac{1}{2}n(5n - 23)$
(c) $S_n = \frac{1}{4}n(21 - n)$ **(d)** $S_n = \frac{1}{2}n(2 + n\sqrt{2} - 3\sqrt{2})$
6(a) $\frac{1}{2}n(n+1)$ **(b)** n^2 **(c)** $\frac{3}{2}n(n+1)$ **(d)** $100n^2$
7(a) 450 legs. No creatures have the mean number of 5 legs. **(b)** 21 835 years **(c)** \$352 000
8(a) n terms, $\frac{1}{2}nx(n+1)$ **(b)** $60 + 190d$
(c) 21 terms, $21(a - 50)$ **(d)** $40\,400b$
(e) $6(13 + 24\sqrt{2})$ **(f)** 20 terms, $230\sqrt{3}$
9(b)(i) 16 terms **(ii)** more than 16 terms
(c) 5 terms or 11 terms
(d) $n = 18$ or $n = -2$, but n must be a positive integer. **(e)** $n = 4, 5, 6, \ldots, 12$
(f) Solving $S_n > 256$ gives $(n-8)^2 < 0$, which has no solutions.
10(b)(i) $n = 3$ **(ii)** $n = 10$ **(iii)** $n = 40$
(c) 21 or more terms **(d)** Solving $S_n = 50$ gives $n^2 + n - 100 = 0$, which has no integer solutions because $b^2 - 4ac = 401$ is not a square.
11(a) 20 rows, 29 logs on bottom row
(b) $S_n = 5n^2$, 7 seconds
(c) 11 trips, the deposits are 1 km apart.
12(a) 10 terms, $55\log_a 2$ **(b)** 11 terms, 0
(c) 6 terms, $3(4\log_b 3 - \log_b 2)$
(d) $15(\log_x 2 - \log_x 3)$
13(a) $d = 11$ **(b)** $\ell = 22$ **(c)** $a = -7{\cdot}1$ **(d)** $a = -3$
(e) $d = -2$, $a = 11$, $S_{10} = 20$
(f) $a = 9$, $d = -2$, $T_2 = 7$
(g) $d = -3$, $a = 28\frac{1}{2}$, $T_4 = 19\frac{1}{2}$
14(c) $a = -27$, $d = -2$ **(d)** $n = 15$
15(a) $37 + 45 + \cdots + 101 = 621$ **(c)** $n = 11$
(d) 666 667 or more
16(a) $n(43 - n)$, $n = 43$ **(b)(i)** $\frac{3}{2}n(41 - n)$, $n = 41$
(ii) $3n(n + 14)$, $n = 3$ **(iii)** $\frac{1}{4}n(n + 9)$, $n = 6$
17(a)(i) 14 850 **(ii)** 30 000 **(b)** 150 000
(c) $149\,700 + 150\,400 = 300\,100$
(d) 322 multiples, sum is 442 911
18(a) $n = 17$, $a = -32$ **(b)** $n = 11$, $a = 20$
19(a) 300 **(c)** 162
20(a) $\dfrac{n}{n+1}$
(b) $\dfrac{3}{4} - \dfrac{2n+3}{2(n+1)(n+2)}$ and $\dfrac{1}{4} - \dfrac{1}{2(n+1)(n+2)}$

Exercise 6J (Page 220)

1 2186
2 2800 kits, cats, sacks and wives
3(a) 1023, $2^n - 1$ **(b)** -341, $\frac{1}{3}\Big(1 - (-2)^n\Big)$
(c) 242, $3^n - 1$ **(d)** 122, $\frac{1}{2}\Big(1 - (-3)^n\Big)$
(e) $\frac{1023}{64}$, $16\Big(1 - (\frac{1}{2})^n\Big)$ **(f)** $\frac{341}{64}$, $\frac{16}{3}\Big(1 - (-\frac{1}{2})^n\Big)$
(g) $\frac{364}{27}$, $\frac{27}{2}\Big(1 - (\frac{1}{3})^n\Big)$ **(h)** $\frac{182}{27}$, $\frac{27}{4}\Big(1 - (-\frac{1}{3})^n\Big)$
(i) $\frac{1820}{27}$, $\frac{135}{2}\Big(1 - (\frac{1}{3})^n\Big)$ **(j)** $-11\,111$, $\frac{1}{9}(1 - 10^n)$
(k) -9091, $\frac{1}{11}\Big((-10)^n - 1\Big)$ **(l)** $\frac{211}{24}$, $\frac{4}{3}\Big((\frac{3}{2})^n - 1\Big)$
4(a) $5\Big((1{\cdot}2)^n - 1\Big)$, 25·96 **(b)** $20\Big(1 - (0{\cdot}95)^n\Big)$, 8·025
(c) $100\Big((1{\cdot}01)^n - 1\Big)$, 10·46
(d) $100\Big(1 - (0{\cdot}99)^n\Big)$, 9·562

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5(a)(i) 2^{63} **(ii)** $2^{64}-1$ **(b)** $615\,\text{km}^3$

6(a) $S_n = \dfrac{cx(3^n x^n - 1)}{3x-1}$ **(b)** $S_n = \dfrac{x^n - 1}{(x-1)x^{n-1}}$

(c) $S_n = \dfrac{cx\Big(1-(-3x)^n\Big)}{1+3x}$ **(d)** $S_n = \dfrac{y^n - x^n}{(y-x)y^{n-1}}$

7(a) $S_n = \Big((\sqrt{2})^n - 1\Big)\Big(\sqrt{2}+1\Big)$,
$S_{10} = 31\Big(\sqrt{2}+1\Big)$

(b) $S_n = \frac{1}{20}\Big(1-(-\sqrt{5})^n\Big)\Big(\sqrt{5}-1\Big)$,
$S_{10} = -\frac{781}{5}\Big(\sqrt{5}-1\Big)$

8(a)(i) $121\frac{1}{3}$ **(ii)** $9\log_a 3$ **(iii)** $\frac{765}{32}$ **(b)** $\frac{3}{4}+\frac{9}{2}+27 = \frac{129}{4}$

9(b) $n=8$ **(c)** 14 terms **(d)** $S_{14} = 114\,681$

10(a) 41 powers of 3 **(b)** 42 terms

11(a)(i) $0{\cdot}01172$ tonnes **(ii)** $11{\cdot}99$ tonnes

(b) $4{\cdot}9 \times 10^{-3}$ g

(c)(i) $S_n = 10P(1{\cdot}1^1 0 - 1)$ **(ii)** \$56.47

12(a) 34 010 and 26 491 **(c)** $3{\cdot}30$

13(a) $\dfrac{n+1}{n}$ **(b)** $\dfrac{3}{2^n+1}$

14(b) $n=6$ **(c)** $T_{12} = -708\,588$

(d) $S_{13} = 1\,594\,324$

15(a)(i) 2 097 151 **(ii)** $\frac{6560}{4374}$ **(b)** $r=4$ and $n=4$

(c) $n=6$ and $\ell = -1215$

16(a) $r=2$ or $r=-2$ **(c)** $r = 3^{-\frac{1}{2}}$ or $-3^{-\frac{1}{2}}$

17(a) $3\times 3^n + 6\times 2^n - 9$ **(b)** $2\times 2^n + n^2 + 4n - 2$

(c) $a=1$, $d=3$, $b=3$, $S_n = \frac{3}{2}n^2 + \frac{5}{2}n - 6 + 6\times 2^n$

18 112

19 694

Exercise **6K** (Page 225)

1 18, 24, 26, $26\frac{2}{3}$, $26\frac{8}{9}$, $26\frac{26}{27}$, $S_\infty = 27$,
$S_\infty - S_6 = \frac{1}{27}$

2(a) $r=\frac{1}{2}$, $S_\infty = 2$ **(b)** $r=-\frac{1}{2}$, $S_\infty = \frac{2}{3}$

(c) $r=\frac{1}{3}$, $S_\infty = 18$ **(d)** $r=-1$, no limiting sum

(e) $r=\frac{9}{10}$, $S_\infty = 1000$ **(f)** $r=-\frac{1}{5}$, $S_\infty = -\frac{5}{3}$

(g) $r=\frac{1}{5}$, $S_\infty = -\frac{5}{6}$

(h) $r = 1{\cdot}01$, no limiting sum

(i) $r=-0{\cdot}99$, $S_\infty = \frac{100}{199}$

(j) $r = (1{\cdot}01)^{-1}$, $S_\infty = 101$

(k) $r=-\frac{1}{6}$, $S_\infty = \frac{108}{175}$ **(l)** $r=\frac{1}{4}$, $S_\infty = \frac{64}{3}\sqrt{5}$

3(a) $x=\frac{1}{2}$ **(b)** $x=-\frac{2}{3}$ **(c)** $x=-\frac{2}{3}$

4(a) $a=\frac{4}{3}$ **(b)** $a=\frac{8}{3}$ **(c)** $a=\frac{2}{3}$

5(a) $0<x<2$, $\dfrac{1}{2-x}$ **(b)** $-2<x<0$, $-\dfrac{1}{x}$

(c) $\frac{1}{3}<x<1$, $\dfrac{1}{3-3x}$ **(d)** $-1<x<-\frac{1}{3}$, $\dfrac{1}{3x+3}$

6(a) $\frac{7}{6}(7+\sqrt{7})$ **(b)** $4(2-\sqrt{2})$ **(c)** $5(5-2\sqrt{5})$

(d) $r = \frac{1}{3}\sqrt{10} > 1$, so there is no limiting sum.

(e) $\frac{1}{3}\sqrt{3}$ **(f)** $\frac{1}{2}(\sqrt{3}+1)$ **(g)** $2\sqrt{5}+4$

(h) $r>1$, so there is no limiting sum.

7 $\frac{3}{7}$

8(a) The successive down-and-up distances form a GP with $a=15$ and $r=\frac{2}{3}$.

(b) $S_\infty = 45$ metres

9(a) $r=\frac{1}{2}$, $S_\infty = 14$ **(b)** $r=-\frac{4}{25}$, $S_\infty = -\frac{25}{29}$

(c) The first GP has $r=\frac{1}{5}$ and $S_\infty = 5$, the second GP has $r=\frac{1}{4}$ and $S_\infty = 6\frac{2}{3}$, so the total is $11\frac{2}{3}$.

11(a) $r=\frac{4}{5}$ **(b)** $18+6+2+\cdots$ or $9+6+4+\cdots$

(c) $r=\frac{5}{6}$ **(d)(i)** $r=-\frac{1}{2}+\frac{1}{2}\sqrt{5}$ ($r=-\frac{1}{2}-\sqrt{5}<-1$, so it is not a possible solution.)

(ii) $r=\frac{1}{2}$ **(iii)** $r=\frac{1}{2}\sqrt{2}$ or $-\frac{1}{2}\sqrt{2}$ **(e)** $r=2^{-\frac{1}{3}}$

12(b)(i) 96 **(ii)** 32 **(iii)** 64 **(iv)** 32

13(a) $1:10$ **(b)** 45th year

14(a) 66 667 **(b)** $88{\cdot}2\%$ **(c)** 12th month **(d)** 98%

15(b) $r=-3$, which is impossible. **(d)(i)** $S_\infty > 3$

(ii) $S_\infty < -4$ **(iii)** $S_\infty > \frac{1}{2}a$ **(iv)** $S_\infty < \frac{1}{2}a$

16(a) First term is $\dfrac{ar}{1-r}$, ratio is r, it converges to zero because its ratio is between -1 and 1.

(b) $D_n = 3^{3-n}$, $D_5 = \frac{1}{9}$, 16 terms **(c)** 10 terms

17(a) $-\sqrt{2}<x<\sqrt{2}$ and $x\neq 0$, $S_\infty = \dfrac{1}{2-x^2}$

(b) $1<x<\sqrt{3}$ or $-1>x>-\sqrt{3}$, $S_\infty = \dfrac{1}{3-x^2}$

(c) $x>\frac{1}{5}$ or $x<-\frac{1}{5}$, $S_\infty = \dfrac{5x}{5x-1}$

(d) $x>2$ or $x<-2$, $S_\infty = \dfrac{x}{x+2}$

(e) $x\neq 0$, $S_\infty = \dfrac{1+x^2}{x^2}$

(f) $x>4$ or $x<2$, $S_\infty = \dfrac{3-x}{4-x}$

(g) $x\neq 1$ and $x\neq -1$, $S_\infty = \dfrac{x^2+1}{(x-1)^2}$

19(b) $S_n = 4 - (\frac{1}{2})^{n-2} - \dfrac{n}{2^{n-1}}$ **(c)** $S_\infty = 4$

20(b) $S_n = \dfrac{ax}{x-1} + \dfrac{dx}{(x-1)^2}\left(1-(\tfrac{1}{2})^{n-1}\right)$
$-\dfrac{a+(n-1)d}{(x-1)x^{n-1}}$ **(c)** $S_\infty = \dfrac{ax}{x-1} + \dfrac{dx}{(x-1)^2}$

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Exercise 6L (Page 228)

1(a) $0{\cdot}7 + 0{\cdot}07 + 0{\cdot}007 + \cdots = \frac{7}{9}$ **(b)** $\frac{2}{3}$
(c) $0{\cdot}27 + 0{\cdot}0027 + 0{\cdot}000027 + \cdots = \frac{3}{11}$ **(d)** $\frac{26}{33}$
(e) $\frac{5}{11}$ **(f)** $\frac{1}{37}$ **(g)** $\frac{5}{37}$ **(h)** $\frac{5}{27}$
2(a) $12 + (0{\cdot}4 + 0{\cdot}04 + \cdots) = 12\frac{4}{9}$ **(b)** $7\frac{9}{11}$
(c) $8{\cdot}4 + (0{\cdot}06 + 0{\cdot}006 + \cdots) = 8\frac{7}{15}$
(d) $0{\cdot}2 + (0{\cdot}036 + 0{\cdot}000\,36 + \cdots) = \frac{13}{55}$
4(a) $0{\cdot}\dot{9} = 0{\cdot}9 + 0{\cdot}09 + 0{\cdot}009 + \cdots = \frac{0{\cdot}9}{1-0{\cdot}1} = 1$
(b) Zero is the only number that is not negative, but is less than every positive number.
(d) $74 = 73{\cdot}\dot{9}$, $7{\cdot}282 = 7{\cdot}281\dot{9}$
5(a) $\frac{29}{303}$ **(b)** $\frac{25}{101}$ **(c)** $\frac{3}{13}$ **(d)** $\frac{3}{7}$
(e) $0{\cdot}25 + (0{\cdot}0057 + 0{\cdot}000\,057 + \cdots) = \frac{211}{825}$ **(f)** $1\frac{14}{135}$
(g) $\frac{1}{3690}$ **(h)** $7\frac{27}{35}$
7(a) $\frac{1}{4}$, $\frac{13}{16}$, $\frac{27}{64}$ **(b)** $0{\cdot}1$, $0{\cdot}11$, $0{\cdot}011$, $0{\cdot}1011$
(c) $\frac{2}{3}$, $\frac{5}{7}$, $\frac{1}{5}$, 1 **(d)** $0{\cdot}\dot{0}\dot{1}$, $0{\cdot}\dot{1}10\dot{0}$, $0{\cdot}\dot{0}0\dot{1}$
8(a) Notice that $\frac{1}{9} = 0{\cdot}\dot{1}$, $\frac{1}{99} = 0{\cdot}\dot{0}\dot{1}$, $\frac{1}{999} = 0{\cdot}\dot{0}0\dot{1}$, and so on. If the denominator of a fraction can be made a string of nines, then the fraction will be a multiple of one of these recurring decimals.
(b) Periods: 1, 6, 1, 2, 6, 3, 3, 5, 4, 5

Exercise 6M (Page 230)

1(a) $(x-1)(x+1)$ **(b)** $(x-1)(x^2+x+1)$
(c) $(x-1)(x^4+x^3+x^2+x+1)$
(d) $(t-1)(t^6+t^5+t^4+t^3+t^2+t+1)$
(e) $(t+1)(t^2-t+1)$ **(f)** $(t+1)(t^4-t^3+t^2-t+1)$
(g) $(x+1)(x^6-x^5+x^4-x^3+x^2-x+1)$
(h) $(x-5)(x^2+5x+25)$ **(i)** $(x+2)(x^2-2x+4)$
(j) $(x-3)(x^4+3x^3+9x^2+27x+81)$
(k) $(x+5)(x^2-5x+25)$
(l) $(x+y)(x^4-x^3y+x^2y^2-xy^3+y^4)$
(m) $(x+2)(x^4-2x^3+4x^2-8x+16)$
(n) $(2t+1)(16t^4-8t^3+4t^2-2t+1)$
(o) $(1-ax)(1+ax+a^2x^2+a^3x^3+a^4x^4+a^5x^5+a^6x^6)$ **(p)** $(3t+2a)(9t^2-6ta+4a^2)$
2(a) x^2+xy+y^2 **(b)** $x^3+x^2y+xy^2+y^3$ **(c)** x^2-y+1
(d) $16x^4-8x^3y+4x^2y^2-2xy^3+y^4$
(e) $\dfrac{x^6+x^5y+x^4y^2+x^3y^3+x^2y^4+xy^5+y^6}{x^4+x^3y+x^2y^2+xy^3+y^4}$
(f) $\dfrac{x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6}{x^4-x^3y+x^2y^2-xy^3+y^4}$
3(a) $(x-1)(x+1)(x^2+1)$
(b) $(x-1)(x^2+x+1)(x+1)(x^2-x+1)$
(c)(i) $(x^4+a^4)(x^2+a^2)(x+a)(x-a)$
(ii) $(x-1)(x^4+x^3+x^2+x+1)(x+1)(x^4-x^3+x^2-x+1)$
4(a) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$
(b) $(\sqrt{x}-\sqrt{y})(x+\sqrt{xy}+y)$
(c) $(\sqrt{x}+\sqrt{y})(x-\sqrt{xy}+y)$
5(a) $\dfrac{1}{\sqrt{x}+\sqrt{y}}$ **(b)** $\sqrt{x}-\sqrt{y}$ **(c)** $x+\sqrt{xy}+y$
(d) $\dfrac{1}{x-\sqrt{xy}+y}$
6(a) $2n+1$ **(b)** $4n$ **(c)** $3n^2+3n+1$ **(d)** $2(3n^2+1)$
(e) $4an$ **(f)** $35(n+a)(n-a)$
7(a) $u+x$ **(b)** u^2+ux+x^2 **(c)** $u^3+u^2x+ux^2+x^3$
(d) $-\dfrac{1}{ux}$ **(e)** $\dfrac{1}{\sqrt{u}+\sqrt{x}}$ **(f)** $-\dfrac{u+x}{u^2x^2}$
8(b)(i) $(x^2+1)(x^2+x\sqrt{3}+1)(x^2-x\sqrt{3}+1)$
(ii) $(x-1)(x+1)(x^2+1)(x^2+x\sqrt{2}+1)(x^2-x\sqrt{2}+1)$
(iii) $(x-1)(x+1)(x^2+1)(x^2+x+1)(x^2-x+1)(x^2+x\sqrt{3}+1)(x^2-x\sqrt{3}+1)$
9(a) $2^{ab}-1 = (2^a)^b-1$, which factors when $a>1$ and $b>1$. $M_2=3$, $M_3=7$, $M_5=31$, $M_7=127$, $M_{11}=2047=23\times 89$
(b) If b is odd, then $2^{ab}+1=(2^a)^b+1$, which factors. $F_0=2^1+1=3$, $F_1=2^2+1=5$, $F_2=2^4+1=17$, $F_3=2^8+1=257$, $F_4=2^{16}+1=65\,537$, $F_5=2^{32}+1=641\times 6\,700\,417$
(c) The divisors of N less than N are $1, 2, 2^2, \ldots 2^{p-1}$ with sum $2^p-1=M_p$, and M_p, $2M_p$, 2^2M_p, $\ldots 2^{p-2}M_p$ with sum $(2^{p-1}-1)M_p$. The combined sum is N. Some perfect numbers: $6=2\times 3$, $28=2^2\times 7$, $496=2^4\times 31$, $8128=2^6\times 127$
(d) $F_{n+1}-2=(2^n+1)(2^n-1)=F_n(F_n-2)$. The result now follows, since $F_0-2=1$. If $n>m$, then F_m is a divisor of F_n-2, so since F_n and F_m are both odd, they are relatively prime.

Exercise 6N (Page 234)

2 $\dfrac{1}{1\times 2}+\dfrac{1}{2\times 3}+\dfrac{1}{3\times 4}+\cdots=1$,
$\dfrac{1}{1\times 4}+\dfrac{1}{4\times 7}+\dfrac{1}{7\times 10}+\cdots=\frac{1}{3}$,
$\dfrac{1}{1\times 2\times 3}+\dfrac{1}{2\times 3\times 4}+\dfrac{1}{3\times 4\times 5}+\cdots=\frac{1}{4}$
5 n^3-n is divisible by 24, for odd cardinals n.
7 $2^n>2n^3$, for $n\ge 12$.
17(d) $L_n=\left(\dfrac{1+\sqrt{5}}{2}\right)^n+\left(\dfrac{1-\sqrt{5}}{2}\right)^n$

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Chapter Seven

Exercise 7A (Page 240)

1 The graph of $y = f'(x)$ should approximate a line of gradient 2 through the origin; its equation is $f'(x) = 2x$.

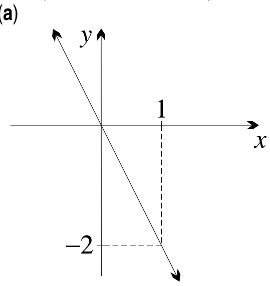
2(a) 2 **(b)** -3 **(c)** $\frac{1}{2}$ **(d)** 0 **(e)** a **(f)** $\frac{2}{3}$ **(g)** $-\frac{5}{4}$ **(h)** $-\frac{10}{3}$ **(i)** 0

3(a) $\frac{7}{2}$ **(b)** 12 **(c)** 0

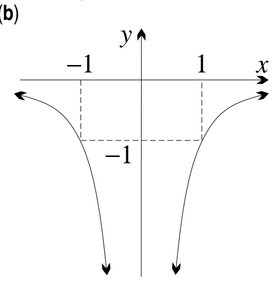
4(a) $-\frac{4}{3}$ **(b)** $-\frac{3}{4}$ **(c)** 0 **(d)** $\frac{4}{3}$ **(e)** $\frac{3}{4}$

5(a) $\dfrac{-x}{\sqrt{1-x^2}}$ **(b)** $\dfrac{x}{\sqrt{1-x^2}}$ **(c)** $\dfrac{-x}{\sqrt{4-x^2}}$

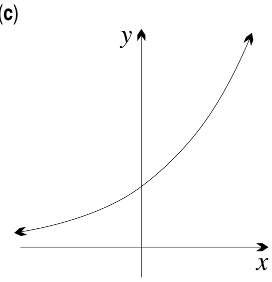
6(a)



(b)



(c)



7(a) $\dfrac{-x}{\sqrt{9-x^2}}$ **(b)** $\dfrac{x}{\sqrt{16-x^2}}$ **(c)** $\dfrac{7-x}{\sqrt{36-(x-7)^2}}$ **(d)** $\dfrac{x-1}{\sqrt{2x-x^2}}$

Exercise 7B (Page 243)

1(a) $2x+h-4$ **(c)** -2 **(d)** 0 at C, 2 at B

2(a) $5,\ 5$ **(b)** $-3,\ -3$ **(c)** $2x+h,\ 2x$ **(d)** $2x+h-4,\ 2x-4$ **(e)** $2x+h+3,\ 2x+3$ **(f)** $4x+2h+3,\ 4x+3$ **(g)** $-8x-4h,\ -8x$ **(h)** $3x^2+3xh+h^2,\ 3x^2$ **(i)** $4x^3+6x^2h+4xh^2+h^3,\ 4x^3$

3(a) $5,\ 11,\ y=5x+1$ **(b)** $-3,\ -2,\ y=4-3x$ **(c)** $4,\ 14,\ y=4x+6$ **(d)** $0,\ -4,\ y=-4$ **(e)** $7,\ 12,\ y=7x-2$ **(f)** $11,\ 14,\ y=11x-8$ **(g)** $-16,\ -7,\ y=-16x+25$ **(h)** $12,\ 8,\ y=12x-16$ **(i)** $32,\ 16,\ y=32x-48$

4(a) none **(b)** none **(c)** $x=0$ **(d)** $x=2$ **(e)** $x=-1\frac{1}{2}$ **(f)** $x=-\frac{3}{4}$ **(g)** $x=0$ **(h)** $x=0$ **(i)** $x=0$

5(a) 5 **(b)** -3 **(c)** $u+x$ **(d)** $u+x-4$ **(e)** $u+x+3$ **(f)** $2u+2x+3$ **(g)** $-4u-4x$ **(h)** u^2+ux+x^2 **(i)** $u^3+u^2x+ux^2+x^3$ (derivatives as before)

6(b)(i) $6,\ 80°32'$ **(ii)** $0,\ 0°$ **(iii)** $1,\ 45°$ **(iv)** $-1,\ 135°$ **(v)** $-4,\ 104°2'$

7(b)(i) $(3,-6)$ **(ii)** $(2,-6)$ **(iii)** $(5,0)$ **(iv)** $(0,0)$ **(v)** $(2\frac{1}{2},-6\frac{1}{4})$

8(b)(i) $x=2$ **(ii)** $x=-2$ **(iii)** $x=2\sqrt{3}$ **(iv)** $x=-2\sqrt{3}$ **(v)** $x=\frac{2}{3}\sqrt{3}$ **(vi)** $x=-\frac{2}{3}\sqrt{3}$ **(vii)** $x=2\tan 37° \doteqdot 1{\cdot}507$

10(b) $f'(0)=-5$, $y=-5x+6$, whose x-intercept is $\frac{6}{5}$. **(c)** At $(2,0)$, $f'(2)=-1$ and angle of inclination is $135°$. At $(3,0)$, $f'(3)=1$ and angle of inclination is $45°$. **(d)** $71°34,\ 108°26'$

11(b)(i) 4 **(ii)** -1 **(iii)** 0 **(iv)** $2{\cdot}01$

13(a) It is the difference-of-squares identity.

14(a) -7 **(b)** 6 **(c)** 1 **(d)** 5 **(e)** 6 or -6

15 The line is a tangent when the two points coincide, that is when $m=2a$, so the gradient of the tangent is twice the x-coordinate.

16 They meet at $x=\frac{1}{2}\left(m+\sqrt{m^2+4b}\right)$ and $x=\frac{1}{2}\left(m-\sqrt{m^2+4b}\right)$. The line is a tangent when these coincide, that is, when $m^2+4b=0$, in which case the tangent at $x=\frac{1}{2}m$ has gradient m, which is twice the x-coordinate.

Exercise 7C (Page 247)

1(a) $7x^6$ **(b)** $45x^4$ **(c)** $2x^5$ **(d)** $6x-5$ **(e)** $4x^3+3x^2+2x+1$ **(f)** $-3-15x^2$ **(g)** $2x^5-2x^3+2x$ **(h)** x^3+x^2+x+1 **(i)** $4ax^3-2bx$ **(j)** $\ell x^{\ell-1}$ **(k)** $3b^2x^{3b-1}$ **(l)** $(5a+1)x^{5a}$

2(a) $0,\ 7$ **(b)** $0,\ 45$ **(c)** $0,\ 2$ **(d)** $-5,\ 1$ **(e)** $1,\ 10$ **(f)** $-3,\ -18$ **(g)** $0,\ 2$ **(h)** $1,\ 4$ **(i)** $0,\ 4a-2b$ **(j)** $0,\ \ell$ **(k)** $0,\ 3b^2$ **(l)** $0,\ 5a+1$

3(a) $3x^2+1$ **(b)** $6x-6x^2-16x^3$ **(c)** $2x+2$ **(d)** $8x$ **(e)** $4x^3+12x$ **(f)** $3x^2-28x+49$ **(g)** $3x^2-10x+3$ **(h)** $2a^2x-10a$

4(a) $-3x^{-2}$ **(b)** $-10x^{-3}$ **(c)** $4x^{-4}$ **(d)** $-4x^{-3}-4x^{-9}$

5(a) $-2/x^3$ **(b)** $-15/x^4$ **(c)** $-2/x^5$ **(d)** $3/x^6$ **(e)** $-c/ax^2$ **(f)** $-6/x^7+8/x^9$ **(g)** $-a/x^2+2b/x^3$ **(h)** $-7n/2x^{n+1}$

6(a) $-\dfrac{3}{x^2}$ **(b)** $-\dfrac{1}{3x^2}$ **(c)** $\dfrac{7}{3x^2}$ **(d)** $-\dfrac{a}{x^2}$

7(a) $\dfrac{3}{2\sqrt{x}}$ **(b)** $\dfrac{5}{\sqrt{x}}$ **(c)** $\dfrac{7}{2\sqrt{x}}$ **(d)** $\dfrac{\sqrt{7}}{2\sqrt{x}}$

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8(a) $1,\ -1$ **(b)** $-1,\ 1$ **(c)** $-6,\ \frac{1}{6}$ **(d)** $1/\sqrt{3},\ -\sqrt{3}$
9(a) $45^\circ,\ 135^\circ$ **(b)** $135^\circ,\ 45^\circ$
(c) about $99^\circ 28',\ 9^\circ 28'$ **(d)** $30^\circ,\ 120^\circ$
10(a) $y=-6x+14,\ x-6y+47=0$ **(b)** $y=4x-21,\ x+4y-18=0$ **(c)** $y=-8x+15,\ x-8y+120=0$
(d) $y=-1,\ x=4$
11 $f'(x)=3x^2$, which is positive for $x\neq 0$ and zero for $x=0$.
12 $2x+y=16,\ A=(8,0),\ B=(0,16)$, $AB=8\sqrt{5}$, $|\triangle OAB|=64$ square units
13(a) $(2,8)$ **(b)** $(2,8)$ and $(-2,40)$ **(c)** $(2a,4a^2)$
(d) $(0,0)$ and $(1,-1)$ and $(-1,-1)$
14(a) $y=-3x+12,\ x-3y+16=0$
(b) $y=-\frac{1}{3}x+4,\ y=3x-16$
15 $y=-2x+5,\ y=2x+5,\ (0,5)$
16 $y=-2x+10,\ x-2y+15=0,\ A=(5,0)$, $B=(-15,0),\ AB=20$,
$|\triangle AKB|=80$ square units
17 $y=3x-2,\ x+3y=4,\ P=(0,-2)$, $Q=(0,1\frac{1}{3}),\ |\triangle QUP|=1\frac{2}{3}$ square units
18 $f'(x)=2ax+b,\ \left(-\dfrac{b}{2a},\ \dfrac{4ac-b^2}{4a}\right)$
19 $f'(9)=14,\ f'(-5)=-14$
20 $f'(x)=3x^2+a,\ x=\sqrt{-a/3}$ and $x=-\sqrt{-a/3},\ a\le 0$ (but no restriction on b)
21(a) 9 **(b)** $-\frac{1}{4}$ **(c)** $a^2,\ -a^2,\ 2a^2$
22 The tangent has gradient $2a-6$, the normal has gradient $\dfrac{1}{6-2a}$. **(a)(i)** 3 **(ii)** 4 **(iii)** $3\frac{1}{4}$
(b)(i) $2\frac{7}{8}$ **(ii)** 1 **(iii)** impossible **(c)** $2\frac{1}{2}$ **(d)** $3-\frac{1}{2}\sqrt{3}$
(e)(i) $3\frac{1}{3}$ **(ii)** $2\frac{1}{4}$
23(a) $y=2ax-a^2,\ U=(\frac{1}{2}a,0),\ V=(0,-a^2)$
(b) $T=(5,25)$ or $(-5,25)$
24(a) $y=2x_0x+9-{x_0}^2$ **(b)** Put the x-intercept equal to 0. $(3,18)$ and $(-3,18)$
25(a) $(\frac{1}{2},2)$ **(b)** $(-2,-\frac{1}{2})$ (a cannot be zero)
(c) $(1,1)$ and $(3,\frac{1}{3})$ **(d)** impossible, as $a\neq 0$
26(a) $2y\sqrt{t}=x+t-2\sqrt{t}$
(c) $t=4,\ x=4y$ ($t=0$ is not allowed, because there is no tangent at the endpoint.)
27 The tangent where $x=t$ is $y=2tx+5-t^2$. To pass through O, $t=\sqrt{5}$ or $t=-\sqrt{5}$.
The tangents are $y=2x\sqrt{5}$ and $y=-2x\sqrt{5}$.
31 The tangents are $y=2ax-a^2$ and $y=2bx-b^2$. They meet at $K=\left(\frac{1}{2}(a+b)\,,\ ab\right)$.
32(a) $y=(2ax_0+b)x-a{x_0}^2+c$. a and c must have the same sign, or $c=0$ (b is arbitrary).
$y=(2\sqrt{ac}+b)x$ and $y=(-2\sqrt{ac}+b)x$
(b) Points of contact: $\left(\sqrt{c/a}\,,\ 2c+b\sqrt{c/a}\right)$ and $\left(-\sqrt{c/a}\,,\ 2c-b\sqrt{c/a}\right)$,
whose midpoint is $(0,2c)$.
(c) $2\sqrt{c^3/a}$ square units

Exercise **7D** (Page 252)

1(a) $4x^3-2x,\ -2$ **(b)** $2ax+b,\ b-2a$
(c) $4x-5,\ -9$ **(d)** $3ax^2-2cx,\ 3a+2c$
(e) $-27/x^4,\ -27$
(f) $6/\sqrt{x}$, undefined
(g) $-a/x^2-2a/x^3,\ a$
(h) $11/2\sqrt{x}$, undefined
2(a) $9x^2-5$ **(b)** $5x^2+\frac{8}{3}x$ **(c)** $2+6x^{-3}$
(d) $a-\dfrac{c}{x^2}+\dfrac{2d}{x^3}$
3(a) $\frac{5}{2}x^{1\frac{1}{2}}$ **(b)** $-\frac{1}{2}x^{-\frac{3}{2}}$ **(c)** $3x^{-\frac{1}{4}}$ **(d)** $-\frac{10}{3}x^{-1\frac{2}{3}}$
(e) $6x^{-1\cdot 6}$
4(a) $18x^{\frac{1}{2}}=18\sqrt{x}$ **(b)** $10x^{\frac{3}{2}}=10x\sqrt{x}$
(c) $-3x^{-\frac{3}{2}}=\dfrac{-3}{x\sqrt{x}}$ **(d)** $-\frac{15}{2}x^{-\frac{5}{2}}=\dfrac{-15}{2x^2\sqrt{x}}$
(e) $3x^{-\frac{4}{5}}$
5(a) $y=-6x,\ y=\frac{1}{6}x$ **(b)** $y=\frac{1}{4}x+1,\ y=-4x+18$
(c) $y=2x+2,\ x+2y+1=0$ **(d)** $y=0,\ x=1$
6(a) $(1,1)$ and $(-1,-1)$ **(b)** $(1,\frac{1}{2})$
(c) $(1,2\frac{2}{3})$ and $(-1,3\frac{1}{3})$ **(d)** none **(e)** $(\frac{1}{4},-\frac{1}{2})$
7(a) $(1,-6\frac{2}{3}),\ (-1,-7\frac{1}{3})$ **(b)** $(-1,\frac{2}{3})$
(c) $(-\frac{1}{2}\sqrt{3},1\frac{3}{4})$ **(d)** $(3,2\sqrt{3})$
8(a) 1 **(b)** $0,\ 3,\ -3$ **(c)** $1,\ -1$
9(b) At $(2,-4)$, $71^\circ 34'$. At $(-3,6)$, $98^\circ 8'$.
10(a) $x=\frac{1}{2}(\tan 22^\circ-3)\doteqdot -1{\cdot}298$
(b) $x^3=\frac{1}{4}\tan 142^\circ 17',\ x\doteqdot -0{\cdot}5782$
(c) This is impossible, because all the tangents to $y=1/x$ have negative gradients.
11(a) $y=(2a-10)x-a^2+9,\ a=3$ and $y=-4x$, or $a=-3$ and $y=-16x$ **(b)** $y=(2a+15)x-a^2+36$, $a=6$ and $y=27x$, or $a=-6$ and $y=3x$
(c) $y=(4a-7)x-2a^2+6,\ a=\sqrt{3}$ and $y=(4\sqrt{3}-7)x$, or $a=-\sqrt{3}$ and $y=(-4\sqrt{3}-7)x$
12(a) $b=7,\ c=0$ **(b)** $b=-2,\ c=-3$
(c) $b=-10,\ c=25$ **(d)** $b=-1,\ c=-2$
(e) $b=-9,\ c=17$ **(f)** $b=-5\frac{2}{3},\ c=7$
13(a) $\frac{15}{2}x\sqrt{x}-3\sqrt{x}$ **(b)** $48x^3$ **(c)** 36
(d) $x^{\pi-1}+x^{\frac{1}{\pi}-1}$ **(e)** $\frac{9}{2}x^{\frac{1}{2}}-x^{-\frac{1}{2}}-2x^{-1\frac{1}{2}}$
(f) $\frac{1}{2}x^{-\frac{1}{2}}-\frac{1}{2}x^{-\frac{3}{2}}$ **(g)** $2x-2x^{-3}$ **(h)** $\frac{3}{2}\sqrt{x}$
(i) $4-4x^{-2}$ **(j)** $4ax^3-4ax^{-5}$

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14(a) $a^2 - 2a$ **(b)** $-a - a^{-1}$ **(c)** $-2\frac{1}{2}$ **(d)** 0 **(e)** $4n^4$
(f) 21 **(g)** -21 **(h)** $-3\frac{1}{2}$ **(i)** $2^n n$

15 $\dfrac{dP}{dx} = 2tx + 3u$, $\dfrac{dP}{du} = 6tu + 3x$, $\dfrac{dP}{dt} = x^2 + 3u^2 + 1$

16(a) 12 metres **(b)** $x = 6$ **(c)** 36 metres
(d) about $85°14'$
(e) The gradients are 12 and -12, so the acute angle with the ground is the same.
(f) about $82°52'$
(g) The gradients are $12 - 2a$ and $2a - 12$. The acute angle with the ground will be the same.
(h) $y = 12x$, $HA = 3$ metres, $HB = 36$ metres

17 At $(1, -3)$ the tangent is $x + y + 2 = 0$, at $(-1, 3)$ the tangent is $x + y - 2 = 0$.

18 The tangent is $y = x$.

19 At $(2, 1)$ the gradient is 2, which is perpendicular to $x + 2y = 4$; at $(-\frac{1}{2}, \frac{9}{4})$ the gradient is -3.

20 $y = 2(a + 1)x - a^2 - 8$, $(1, -5)$, $(3, 7)$

21(c)(i) $y\dfrac{dy}{dx} = \frac{1}{2}a^2$ **(ii)** $y\dfrac{dy}{dx} = na^2x^{2n-1}$

22 $y = 2(\alpha - 3)x + 9 - \alpha^2$, $A = \left(\frac{1}{2}(\alpha + 3), 0\right)$, $B = (0, 9 - \alpha^2)$, $M = \left(\frac{1}{4}(\alpha + 3), \frac{1}{2}(9 - \alpha^2)\right)$, $\alpha = 1$

23(a) $cx + t^2y = 2ct$, $A = (2t, 0)$, $B = (0, 2c/t)$
(b) $2|c|$
(e) $AB = \dfrac{2}{|t|}\sqrt{t^4 + c^2}$,
perpendicular distance $= \dfrac{2|ct|}{\sqrt{t^4 + c^2}}$

25(b) $y = x^2 - 6x$ and $y = \frac{25}{81}x^2 + \frac{2}{9}x$
(c) $y = x^2 - x - 6$

26 The equation of the tangent at $x = t$ is a cubic in t, and every cubic has at least one solution. (Why?)

Exercise **7E** (Page 258)

1(a) $12(3x + 7)^3$ **(b)** $-28(5 - 4x)^6$ **(c)** $8p(px + q)^7$
(d) $24x(x^2 + 1)^{11}$ **(e)** $-64x(7 - x^2)^3$
(f) $9(2x + 3)(x^2 + 3x + 1)^8$
(g) $-18(3x^2 + 1)(x^3 + x + 1)^5$ **(h)** $\dfrac{5}{2\sqrt{5x + 4}}$
(i) $\dfrac{-1}{\sqrt{3 - 2x}}$ **(j)** $\dfrac{7x}{\sqrt{x^2 + 1}}$ **(k)** $\dfrac{-x}{\sqrt{9 - x^2}}$
(l) $\dfrac{b^2x}{\sqrt{a^2 - b^2x^2}}$

2(a) $25(5x - 7)^4$ **(b)** $-21(4 - 3x)^6$
(c) $15(2 - 3x)^{-6}$ **(d)** $4p(q - x)^{-5}$ **(e)** $\dfrac{1}{(2 - x)^2}$
(f) $\dfrac{-5}{(3 + 5x)^2}$ **(g)** $\dfrac{15}{(x + 1)^4}$ **(h)** $\dfrac{1}{2\sqrt{x + 4}}$
(i) $\dfrac{-3}{2\sqrt{4 - 3x}}$ **(j)** $\dfrac{m}{2\sqrt{mx - b}}$ **(k)** $\frac{1}{2}(5 - x)^{-1\frac{1}{2}}$

3(a) $4t$, -4 **(b)** $-1/t^2$, -1 **(c)** b/a, b/a
(d) $\frac{9}{4}t$, $-\frac{9}{4}$

4(a) $6x(x^2 - 1)^2$, $(0, -1)$, $(1, 0)$, $(-1, 0)$
(b) $8(x - 2)(x^2 - 4x)^3$, $(0, 0)$, $(2, 256)$, $(4, 0)$
(c) $10(x + 1)(2x + x^2)^4$, $(0, 0)$, $(-2, 0)$, $(-1, -1)$
(d) $\dfrac{-5}{(5x + 2)^2}$, none **(e)** $-14(x - 5)$, $(5, 24)$
(f) $6(x - 5)^5$, $(5, 4)$ **(g)** $2a(x - h)$, (h, k)
(h) $\dfrac{-1}{\sqrt{3 - 2x}}$, none **(i)** $\dfrac{-2x}{(1 + x^2)^2}$, $(0, 1)$
(j) $\dfrac{x - 1}{\sqrt{x^2 - 2x + 5}}$, $(1, 2)$
(k) $\dfrac{x - 1}{\sqrt{x^2 - 2x}}$, none ($x = 1$ is outside the domain)

5(a) $y = 20x - 19$, $x + 20y = 21$ **(b)** $y = 24x - 16$, $x + 24y = 193$ **(c)** $x + 2y = 2$, $y = 2x - 1\frac{1}{2}$
(d) none ($x = 1$ is outside the domain)

6(a) $2\frac{1}{2}$ and 1 **(b)** 2 and $1\frac{1}{2}$

7(a) $y = \frac{1}{3}x + 15$ **(b)** $y = 3x - 4$

8(a) -5 or -7 **(b)** 4 or 8

9(a) $x + y(b - 4)^2 = 2b - 4$ **(b)(i)** $x + 4y = 0$
(ii) $x + y = 6$

10(a) $\dfrac{11(\sqrt{x} - 3)^{10}}{2\sqrt{x}}$ **(b)** $\dfrac{-3}{4\sqrt{4 - \frac{1}{2}x}}$
(c) $\dfrac{3\sqrt{2}}{(1 - x\sqrt{2}\,)^2}$ **(d)** $\frac{1}{2}(5 - x)^{-1\frac{1}{2}}$
(e) $\frac{1}{2}a^2(1 + ax)^{-1\frac{1}{2}}$ **(f)** $\frac{1}{4}b(c - \frac{1}{2}x)^{-1\frac{1}{2}}$
(g) $-16\left(1 - \dfrac{1}{x^2}\right)\left(x + \dfrac{1}{x}\right)^3$
(h) $6\left(\dfrac{1}{2\sqrt{x}} - \dfrac{1}{2x\sqrt{x}}\right)\left(\sqrt{x} + \dfrac{1}{\sqrt{x}}\right)^5$

11(a) $a = \frac{1}{16}$, $b = 12$ **(b)** $a = \frac{1}{9}$, $b = -10$

13(a) $12x + 5y = 169$, $(\frac{169}{12}, 0)$, $(0, \frac{169}{5})$ **(c)** $\frac{169^2}{120}$
(d) $\frac{13^3}{60} + \frac{169}{12} + \frac{169}{5} = \frac{169}{2}$

14(a) $4x + 3y = 25$, $4x + 5y = 25$, they intersect at $(6\frac{1}{4}, 0)$. **(b)** $\lambda x_0 x + y\sqrt{25 - x_0{}^2} = 25\lambda$, $T = (25/x_0, 0)$, $OM \times OT = 25 = OA^2$

15(a) $P = (7\frac{1}{2}, 3\frac{1}{4})$, $Q = (6\frac{1}{2}, 3\frac{1}{4})$
(b) area $= \frac{1}{2}PQ^2 = \frac{1}{2}$

16(a) At P, $x = h + \frac{1}{2}m$. At Q, $x = h - \frac{1}{2}m$.
(b) $\frac{1}{4}m(m^2 + 1)$

17(b) The vertical distance is $a(\alpha - h)^2$.
(c) $\alpha = \sqrt{h^2 + k/a}$ or $-\sqrt{h^2 + k/a}$

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18(a) $\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dv} \times \dfrac{dv}{dx}$, $\dfrac{x}{(1+\sqrt{1-x^2}\,)^2\sqrt{1-x^2}}$
(b) $\dfrac{dy}{dx} = \dfrac{dy}{du_1} \times \dfrac{du_1}{du_2} \times \cdots \times \dfrac{du_{n-1}}{dx}$

Exercise **7F** (Page 261)

1(a) $2x^2(2x-3)$ **(b)** $4x-9$ **(c)** $4x^3$
2(a) $3(3-2x)^4(1-4x)$, $1\frac{1}{2}$, $\frac{1}{4}$
(b) $x^2(x+1)^3(7x+3)$, 0, -1, $-\frac{3}{7}$
(c) $x^4(1-x)^6(5-12x)$, 0, 1, $\frac{5}{12}$
(d) $(x-2)^2(4x-5)$, 2, $\frac{5}{4}$
(e) $2(x+1)^2(x+2)^3(7x+10)$, -1, -2, $-\frac{10}{7}$
(f) $6(2x-3)^3(2x+3)^4(6x-1)$, $1\frac{1}{2}$, $-1\frac{1}{2}$, $\frac{1}{6}$
3(a) $y=x$, $y=-x$ **(b)** $y=2x-1$, $x+2y=3$
4(a) $(x^2+1)^4(11x^2+1)$ **(b)** $2\pi x^2(1-x^2)^3(3-11x^2)$
(c) $-2(x^2+x+1)^2(7x^2+4x+1)$
(d) $6x(3x^2-2)^3(3x^2+2)^4(27x^2-2)$
5 $10x^3(x^2-10)^2(x^2-4)$, $(0,0)$, $(\sqrt{10},0)$, $(-\sqrt{10},0)$, $(2,-3456)$, $(-2,-3456)$
6(a) $\dfrac{3(3x+2)}{\sqrt{x+1}}$, $-\frac{2}{3}$ **(b)** $\dfrac{4(3x-1)}{\sqrt{1-2x}}$, $\frac{1}{3}$
(c) $\dfrac{10x(5x-2)}{\sqrt{2x-1}}$, 0 and $\frac{2}{5}$
7(a) $-1 \le x \le 1$ **(b)** $\dfrac{1-2x^2}{\sqrt{1-x^2}}$
(c) $\left(\sqrt{\tfrac{1}{2}}, \tfrac{1}{2}\right)$ and $\left(-\sqrt{\tfrac{1}{2}}, -\tfrac{1}{2}\right)$
(d) $y=x$, $y=-x$
8(a) $y' = a(2x-\alpha-\beta)$ **(b)** $y'(\alpha) = a(\alpha-\beta)$
$y'(\beta) = a(\beta-\alpha)$, $M = \left(\frac{1}{2}(\alpha+\beta), -\frac{1}{2}a(\alpha-\beta)^2\right)$
(c) $V = \left(\frac{1}{2}(\alpha+\beta), -\frac{1}{4}a(\alpha-\beta)^2\right)$
9 $f'(x) = (x-a)^{n-1}\Big(n\,q(x) + (x-a)q'(x)\Big)$.
The x-axis is a tangent to the curve at $x=a$.
10 $y' = x^2(1-x)^4(3-8x)$
12(a) $P = \left(\dfrac{r}{r+s}, \dfrac{r^r\, s^s}{(r+s)^{r+s}}\right)$.
(b) When $r=s$, $P = (\frac{1}{2}, 2^{-2r})$.
13 $y' = u'vw + uv'w + uvw'$
(a) $2x^4(x-1)^3(x-2)^2(3x-5)(2x-1)$, 0, 1, 2, $\frac{1}{2}$ and $\frac{5}{3}$
(b) $\dfrac{(x-2)^3(11x^2-x-2)}{\sqrt{2x+1}}$,
2, $\frac{1}{22}(1+\sqrt{89})$, $\frac{1}{22}(1-\sqrt{89})$
14 $y' = u_1{}'u_2\ldots u_n + u_1u_2{}'\ldots u_n + \cdots + u_1u_2\ldots u_n{}'$

Exercise **7G** (Page 263)

1(a) $\dfrac{-2}{(x-1)^2}$, none **(b)** $\dfrac{4}{(x+2)^2}$, none
(c) $\dfrac{-13}{(x+5)^2}$, none **(d)** $\dfrac{x(2-x)}{(1-x)^2}$, 0, 2
(e) $\dfrac{4x}{(x^2+1)^2}$, 0 **(f)** $\dfrac{m^2-b^2}{(bx+m)^2}$, none
(g) $\dfrac{2x(a-b)}{(x^2-b)^2}$, 0 (provided $a \ne b$)
(h) $\dfrac{6nx^{n-1}}{(x^n+3)^2}$, 0 (provided $n > 1$)
2 $\dfrac{-3}{(3x-2)^2}$
3 $\dfrac{20}{(5-2x)^2}$
4(a) $y' = \dfrac{5}{(5-3x)^2}$, $y = 5x-12$, $78^\circ 41'$, $x+5y+8=0$, $168^\circ 41'$
(b) $y' = \dfrac{x^2-2x+4}{(x-1)^2}$, $4x-3y=4$, $53^\circ 8'$, $3x+4y=28$, $143^\circ 8'$
5(a) $\dfrac{1}{2\sqrt{x}(\sqrt{x}+2)^2}$, none **(b)** $\dfrac{x+5}{2(x+1)^{\frac{3}{2}}}$, none ($x=-5$ is outside the domain.)
6(a) $\dfrac{c^2+2c}{(c+1)^2} = -3$, $c = -\frac{1}{2}$ or $-1\frac{1}{2}$
(b) $\dfrac{12k}{(9-k)^2} = 1$, $k=3$ or 27
7(a) $y' = \dfrac{\alpha-\beta}{(x-\beta)^2}$ **(b)** The denominator is positive, being a square, so the sign of y' is the sign of $\alpha-\beta$. **(c)** When $\alpha=\beta$, the curve is the horizontal line $y=1$, and $y'=0$ (except that y is undefined at $x=\beta$).
8(a) $\dfrac{dy}{dx} = \dfrac{-(t+1)^2}{(t-1)^2}$, $T = (\frac{2}{3}, 2)$, $3x-27y+52=0$
(b) $y = \dfrac{x}{2x-1}$, $\dfrac{dy}{dx} = \dfrac{-1}{(2x-1)^2}$, $\frac{1}{9}$
9(a) $f'(x) = \dfrac{-\sqrt{2}}{\sqrt{x}(\sqrt{x}-\sqrt{2})^2}$, $f'(8) = -\frac{1}{4}$ **(b)** 3
10(a) domain: $x \ne -1$, range: $y \ne 1$
(c) $I = (-1,0)$, $G = (1,0)$ **(d)(ii)** Substitute $(c,0)$, then $c+a^2=0$, so $a = \sqrt{-c}$ or $-\sqrt{-c}$. For $-1 < c < 0$, they are both on the right-hand branch. For $c < -1$, they are on different branches.
12(b)(i) 54, $\frac{3}{2}$, $9\sqrt{37}$, $\frac{3}{2}\sqrt{37}$ **(ii)** $\frac{1}{2}$, 8, $\frac{1}{2}\sqrt{17}$, $2\sqrt{17}$

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Exercise **7H** (Page 266)

1 $\dfrac{dy}{dt} = (3x^2+1)\dfrac{dx}{dt}$ **(a)** 65 **(b)** $-\frac{3}{14}$

2(a) $240\pi\,\text{cm}^2/\text{s}$ **(b)** $\frac{1}{12\pi}\,\text{cm/s}$

3(a) $2/\pi\,\text{cm/s}$ **(b)** $5/\pi\,\text{cm}$ **(c)** $50/9\pi\,\text{cm/s}$

4(a) $840\,\text{cm}^2/\text{s},\ 6\sqrt{2}\,\text{cm/s}$

(b) $1200\,\text{cm}^2/\text{s},\ 6\sqrt{2}\,\text{cm/s}$

5(a)(i) $1350\,\text{cm}^3/\text{min},\ 180\,\text{cm}^2/\text{min},\ 6\,\text{cm/min}$

(ii) $600\,\text{cm}^3/\text{min},\ 120\,\text{cm}^2/\text{min},\ 6\,\text{cm/min}$

(b) $10\sqrt{2}\,\text{cm}$

6(a) $\dfrac{dA}{dt} = \frac{1}{2}s\sqrt{3}\,\dfrac{ds}{dt},\ \dfrac{dh}{dt} = \frac{1}{2}\sqrt{3}\,\dfrac{ds}{dt}$

(b) $\frac{9}{5}\sqrt{3}\,\text{cm}^2/\text{s},\ \frac{3}{20}\sqrt{3}\,\text{cm/s}$

7(a)(i) $\frac{1}{24\pi}\,\text{cm/s}$ **(ii)** $1\frac{1}{2}\,\text{cm}^3/\text{s}$ **(b)** $\dfrac{32\,000\pi}{3}\,\text{cm}^3$

8(a) $\frac{1}{24}\,\text{cm/min},\ 83\frac{1}{3}\,\text{cm}^2/\text{min}$

(b) $8\,\text{litre/min},\ 200\,\text{cm}^2/\text{min}$

9(a)(i) $40\,\text{m/s}$ **(ii)** $-160\,\text{m/s}$

(iii) travelling horizontally

(iv) $100\,\text{m/s}$ or $-100\,\text{m/s}$

(b)(i) 1955 metres **(ii)** 1755 metres

(c) $\dfrac{dy}{dx} = 20$, angle is about $87^\circ 8'$

(d) 2 km high, 20 metres away

10 $\dfrac{dV}{dt} = 3\pi h^2\,\dfrac{dh}{dt}$ **(a)(i)** $\frac{1}{160\pi}\,\text{m/min}$

(ii) $\frac{3}{160\pi}\,\text{m/min}$ **(b)** $\dfrac{dA}{dt} = 6\pi h\,\dfrac{dh}{dt},\ \frac{3}{20}\,\text{m}^2/\text{min},$ $\frac{3}{20}\sqrt{3}\,\text{m}^2/\text{min}$ **(c)** $\frac{96\pi}{25}\,\text{m}^3/\text{min}$

11 $\frac{1}{125\pi}\,\text{cm/s},\ \frac{4}{5}\,\text{cm}^2/\text{s},\ \frac{4}{5}\sqrt{2}\,\text{cm}^2/\text{s}$

12(b) $0{\cdot}096\,\text{m}^3/\text{s},\ \frac{3}{125}(\sqrt{17}+1)\,\text{m}^2/\text{s}$

13 $\dfrac{dy}{dt} = \dfrac{-x}{\sqrt{169-x^2}}\,\dfrac{dx}{dt}$ **(a)** $\frac{5}{12}\,\text{cm/s}$ **(b)** $1\frac{1}{5}\,\text{cm/s}$

14(a) $\dfrac{dV}{dt} = \pi h(20-h)\dfrac{dh}{dt},\ \dfrac{1}{6\pi}\,\text{cm/s}$

(b) radius $= \sqrt{2hr-h^2},\ \frac{8}{3}\,\text{cm}^2/\text{s}$

15 $4\sqrt{6}\,\text{cm}^2/\text{s}$

16 $0{\cdot}246\,\text{cm/s}$. Some exact forms are $\dfrac{3}{(27+9\sqrt{3})^{\frac{2}{3}}}$ and $\dfrac{1}{(3+3\sqrt{3}\,)^{\frac{2}{3}}}$.

Exercise **7I** (Page 271)

1(a)(i) $y = \dfrac{1-4/x+3/x^2}{2-7/x+6/x^2},\ y \to \frac{1}{2}$

(ii) $y = \dfrac{2/x-5}{15+11/x},\ y \to -\frac{1}{3}$

(iii) $y = \dfrac{1/x+1/x^2+1/x^3}{1+1/x+1/x^2+1/x^3},\ y \to 0$

(iv) $y = \dfrac{x\sqrt{x}-5/\sqrt{x}}{1/\sqrt{x}+1},\ y \to \infty$

(b)(i) $y \to \frac{1}{2}$ **(ii)** $y \to -\frac{1}{3}$ **(iii)** $y \to 0$

(iv) y is undefined for $x < 0$.

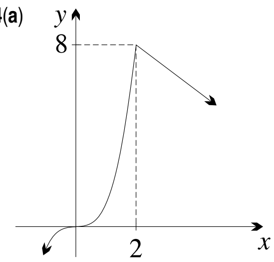
2(a) 4 **(b)** 27 **(c)** 1 **(d)** -6 **(e)** -2 **(f)** 0

3(a) $y \to \frac{1}{2}$ **(b)** $y \to \frac{7}{5}$ **(c)** $y \to -4$

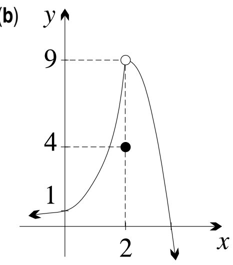
(d) $y \to \infty$ as $x \to -\frac{1}{2}^-$, $y \to -\infty$ as $x \to -\frac{1}{2}^+$

(e) $y \to 0$ **(f)** $y \to -1$ **(g)** $y \to \frac{1}{2}$ **(h)** $y \to 5$

4(a)



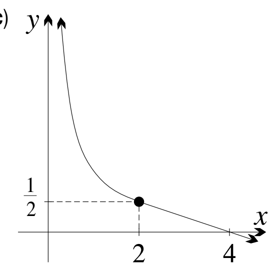
(b)



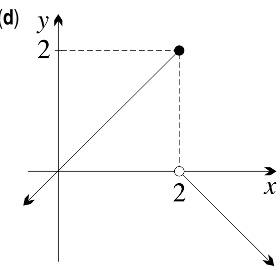
(a) $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = f(2) = 8$, continuous at $x = 2$, domain $= \mathbf{R}$, range: $y \le 8$

(b) $\lim\limits_{x\to 2^-} f(x) = 9$, $\lim\limits_{x\to 2^+} f(x) = 9$, $f(2) = 4$, not continuous at $x = 2$, domain $= \mathbf{R}$, range: $y < 9$

(c)



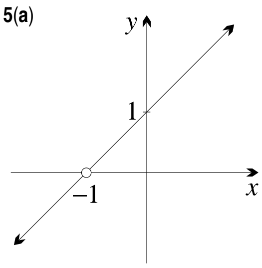
(d)



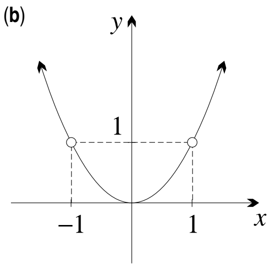
$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^+} f(x) = \frac{1}{2}$, $f(2) = \frac{1}{2}$, continuous at $x = 2$, domain: $x > 0$, range $= \mathbf{R}$

$\lim\limits_{x\to 2^-} f(x) = f(2) = 2$, $\lim\limits_{x\to 2^+} f(x) = 0$, not continuous at $x = 2$, domain $= \mathbf{R}$, range: $y \le 2$

5(a)



(b)



(a) $y = x + 1$ where $x \ne -1$, domain: $x \ne -1$, range: $y \ne 0$

(b) $y = x^2$ where $x \ne -1$ or 1, domain: $x \ne -1$ or 1, range: $y \ge 0,\ y \ne 1$

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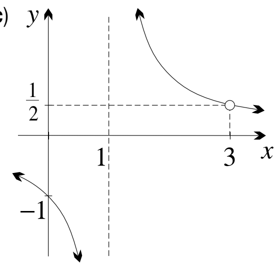
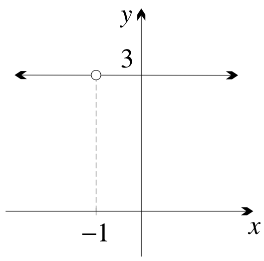
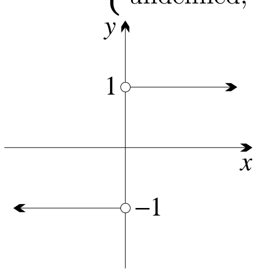
(c)


(d)


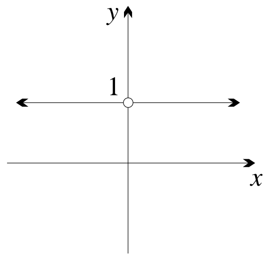
(c) $y = \dfrac{1}{x-1}$ where $x \neq 3$,
domain: $x \neq 1$ or 3, range: $y \neq 0$ or $\frac{1}{2}$
(d) $y = 3$ where $x \neq -1$,
domain: $x \neq -1$, range: $y = 3$
6(a) (gradient of PQ) $= 2x + h - 1 \to 2x - 1$ as $h \to 0$ (b) (gradient of PQ) $= u^3 + u^2x + ux^2 + x^3 - 3 \to 4x^3 - 3$ as $u \to x$
7(a)(i) $2c^2$ (ii) $\frac{4}{3}c$ (iii) $\frac{5}{3}c^2$
(b) $\dfrac{x^n - a^n}{x - a} = x^{n-1} + x^{n-2}a + \cdots + a^{n-1}$,
$\lim\limits_{x \to a} = na^{n-1}$
(c) $\dfrac{u^{2n+1} + 2^{2n+1}}{u + 2} = u^{2n} - u^{2n-1}2 + u^{2n-2}2^2 - \cdots + 2^{2n}$, $\lim\limits_{u \to -2} = (2n+1)\,2^{2n}$
8(a) $a = 5$ (b) $a = -2$
9(a) zeroes: 0, discontinuities: 3 (b) zeroes: 0, discontinuities: 7 and -1 (c) zeroes: none, discontinuities: $180n^\circ$, where $n \in \mathbf{Z}$ (d) zeroes: none, discontinuities: $360n^\circ$, where $n \in \mathbf{Z}$ (e) zeroes: $180n^\circ$, where $n \in \mathbf{Z}$, discontinuities: $90^\circ + 180n^\circ$, where $n \in \mathbf{Z}$, (f) zeroes: 1 and -1, discontinuities: 0, 3 and -3

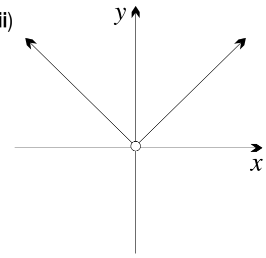
10(a) $y = \begin{cases} 1, & \text{for } x > 0, \\ -1, & \text{for } x < 0, \\ \text{undefined}, & \text{for } x = 0. \end{cases}$



(b)(i) $y = \begin{cases} 1, & \text{for } x \neq 0, \\ \text{undefined}, & \text{for } x = 0. \end{cases}$

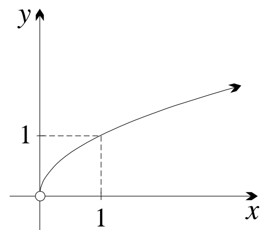
(ii) $y = \begin{cases} |x|, & \text{for } x \neq 0, \\ \text{undefined}, & \text{for } x = 0. \end{cases}$

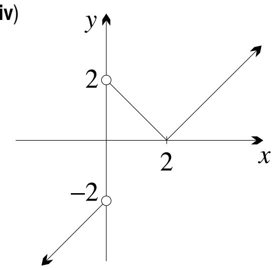
(i)


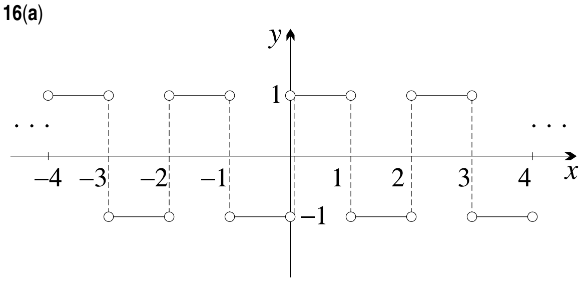
(ii)


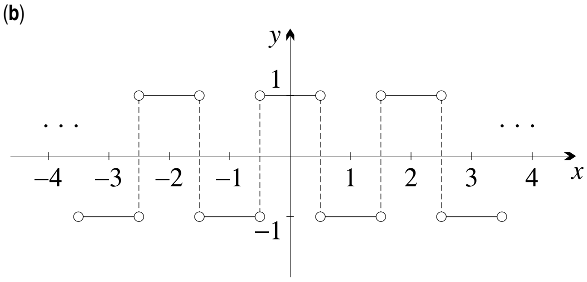
(iii) $y = \begin{cases} \sqrt{x}\,, & \text{for } x > 0, \\ \text{undefined}, & \text{for } x \leq 0. \end{cases}$

(iv) $y = \begin{cases} x - 2, & \text{for } x \geq 2, \\ 2 - x, & \text{for } 0 < x < 2, \\ \text{undefined}, & \text{for } x = 0, \\ x - 2, & \text{for } x < 0. \end{cases}$

(iii)


(iv)


12(a) They are all $\dfrac{1}{2\sqrt{x}}$. (b) All are examples of differentiation by first principles (using the definition of the derivative as a limit).
13 All except (b) are continuous in the closed interval $-1 \leq x \leq 1$.
14(a) zeroes: $135^\circ + 180n^\circ$, where $n \in \mathbf{Z}$,
discontinuities: $45^\circ + 180n^\circ$, where $n \in \mathbf{Z}$
(b) zeroes: $45^\circ + 180n^\circ$, where $n \in \mathbf{Z}$,
discontinuities: $135^\circ + 180n^\circ$, where $n \in \mathbf{Z}$
15(a) $\frac{1}{4}$ (b) $\frac{1}{4}$ (c) $-\frac{1}{9}$ (d) $-\frac{1}{250}$
16(a)


(b)


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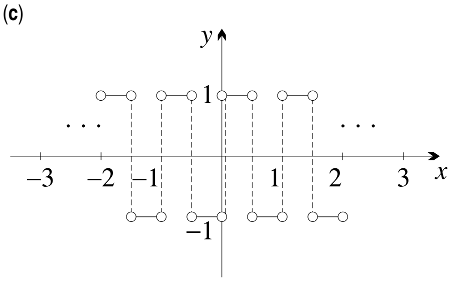
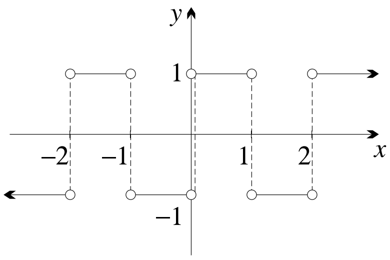
(c)



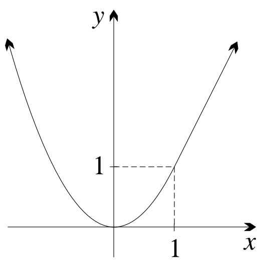
(d)



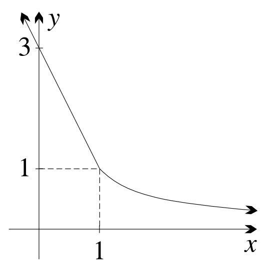
(e) same as (a)

Exercise **7J** (Page 275)

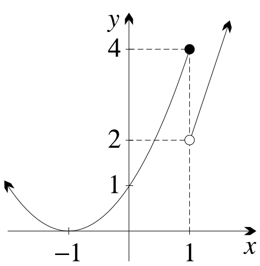
1(a) differentiable at $x = 1$



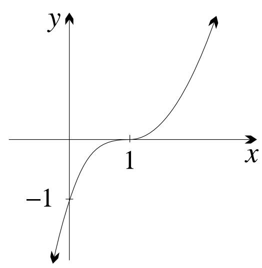
(b) continuous but not differentiable at $x = 1$



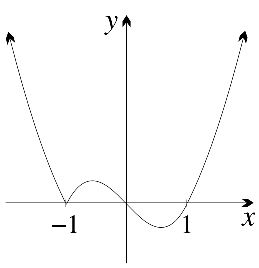
(c) not continuous at $x = 1$



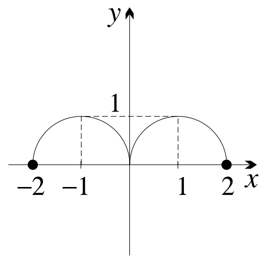
(d) differentiable at $x = 1$



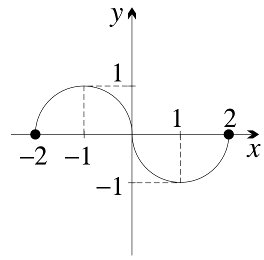
2 continuous but not differentiable at $x = -1$



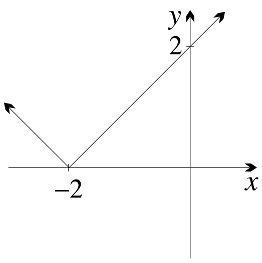
3(a) cusp at $x = 0$



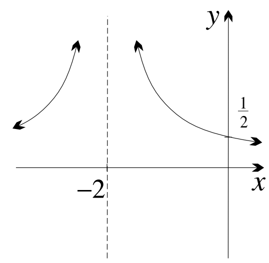
(b) vertical tangent at $x = 0$



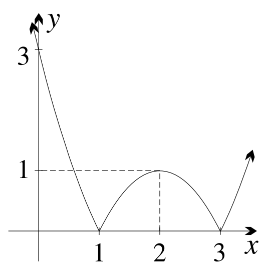
4(a) not differentiable at $x = -2$



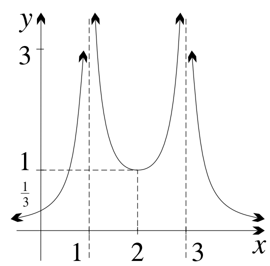
(b) not continuous at $x = -2$



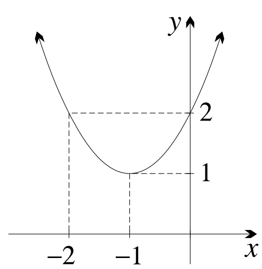
(c) not differentiable at $x = 1$ or 3



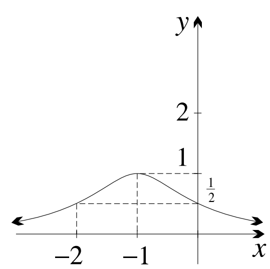
(d) not continuous at $x = 1$ or 3



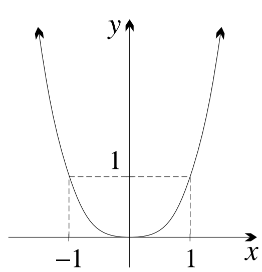
(e) differentiable everywhere



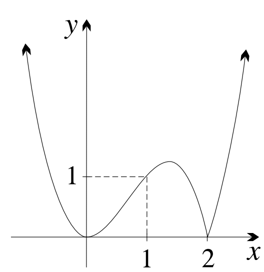
(f) differentiable everywhere



(g) differentiable everywhere



(h) continuous but not differentiable at $x = 2$



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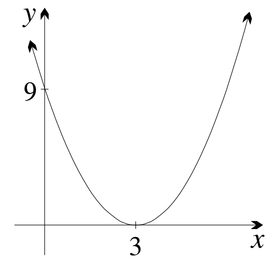
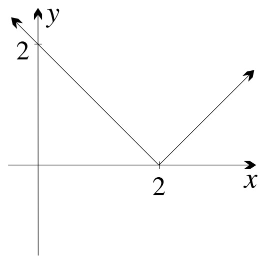
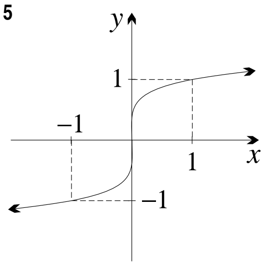
(i) differentiable everywhere

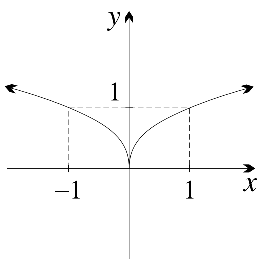


(j) continuous but not differentiable at $x = 2$



5





(a) $f'(x) = \frac{1}{5}x^{-\frac{4}{5}}$, $f'(x) \to \infty$ as $x \to 0^-$ and as $x \to 0^+$, vertical tangent at (0,0).
(b) $f'(x) = \frac{2}{5}x^{-\frac{3}{5}}$, $f'(x) \to -\infty$ as $x \to 0^-$ and $f'(x) \to \infty$ as $x \to 0^+$, cusp at (0,0).
6(a) $3,\ \left(4\frac{1}{2}, -6\frac{3}{4}\right)$ **(b)** $3,\ (1, -8)$
(c) $1,\ \left(\frac{1}{3}\sqrt{3}, \frac{1}{9}\sqrt{3}\right),\ \left(-\frac{1}{3}\sqrt{3}, -\frac{1}{9}\sqrt{3}\right)$
(d) $\frac{1}{3},\ \left(\frac{9}{4}, \frac{3}{2}\right)$ **(e)** $-\frac{1}{2},\ \left(\sqrt{2}, 1/\sqrt{2}\right)$ **(f)** 1. There are none, because all the tangents have negative gradients. **(g)** 0. There are none, because the tangents have gradient 1 for $x > 0$ and gradient -1 for $x < 0$.
(h) $0,\ (0,0)$ **(i)** $\alpha + \beta,\ \left(\frac{1}{2}(\alpha + \beta), \frac{1}{4}(\alpha + \beta)^2\right)$
(j) $-1/\alpha\beta,\ \left(\sqrt{\alpha\beta}, 1/\sqrt{\alpha\beta}\right)$ when α and β are positive, $\left(-\sqrt{\alpha\beta}, -1/\sqrt{\alpha\beta}\right)$ when α and β are negative, impossible when α and β have opposite signs.
7 The tangent is $y = \frac{4}{3}ka^{\frac{1}{3}}x - \frac{1}{3}ka^{\frac{4}{3}}$, $A = (\frac{1}{4}a, 0)$, $B = (0, -\frac{1}{3}ka^{\frac{4}{3}})$, $G = (a, 0)$, $H = (0, ka^{\frac{4}{3}})$, $|OGPH| : |\triangle OAB| = 24 : 1$.
8 The tangent is $y = nka^{n-1}x - (n-1)ka^n$, $A = \left(\dfrac{(n-1)a}{n}, 0\right)$, $B = (0, -(n-1)ka^n)$, $G = (a, 0)$ and $H = (0, ka^n)$.
$|OGPH| : |\triangle OAB| = (n-1)^2 : |2n|$, and the rectangle is bigger when $2 - \sqrt{3} < n < 2 + \sqrt{3}$. B is on the other side from H for $n > 1$ and on the same side for $n < 1$, so for $a > 0$, B is above the origin if and only if $n < 1$.
The number a can be negative provided n is a rational number p/q with odd denominator q, in which case (taking q positive) B is above the origin for $n < 1$ and p even, or for $n > 1$ and p odd.
9(a) q must be odd. **(b)** $p \geq 0$ (When $p = 0$ it is reasonable to take $f(0) = 1$ and ignore the problem of 0^0, because $\lim_{x \to 0} x^0 = 1$; thus when $p = 0$ the function is $y = 1$.) **(c)** no conditions on p and q **(d)** $p \geq 0$ and q is odd. **(e)** $p \geq 0$ ($p = 0$ requires the qualification above.) **(f)** $p \geq q$ and q is odd. **(g)** $0 < p < q$ and q is odd and p is odd. **(h)** $0 < p < q$ and q is odd and p is even.

Exercise **7K** (Page 277)

1(a) $4y^3y'$ **(b)** $y + xy'$ **(c)** $-1 + y' - y - xy'$
(d) $6x + 8yy'$ **(e)** $y(3x^2 + y^2) + xy'(x^2 + 3y^2)$
(f) $\dfrac{y - xy'}{y^2}$ **(g)** $3(x+y)^2(1 + y')$ **(h)** $\dfrac{2(xy' - y)}{(x-y)^2}$
(i) $\dfrac{1 + y'}{2\sqrt{x + y}}$ **(j)** $\dfrac{x + yy'}{\sqrt{x^2 + y^2}}$
2(a) $y' = -\dfrac{x}{y}$ **(b)** $y' = -\dfrac{3x}{2y}$ **(c)** $y' = \dfrac{x}{y}$
(d) $y' = -\dfrac{2x + 3y}{3x + 4y}$ **(e)** $y' = \dfrac{(x-y)^2 + 2x^2}{(x-y)^2 + 2y^2}$
(f) $y' = -\dfrac{2y}{3x}$ **(g)** $y' = -\dfrac{rax^{r-1}}{sby^{s-1}}$ **(h)** $y' = -\dfrac{\sqrt{y}}{\sqrt{x}}$
(i) $y' = \dfrac{y}{x}$
3(a) $y' = -\dfrac{x}{y}$, tangent: $5x - 12y + 169 = 0$, normal: $12x + 5y = 0$. The normal to a circle at any point is a radius, and so must pass through the centre.
(b) $A = \left(-\frac{169}{5}, 0\right)$, $B = \left(0, \frac{169}{12}\right)$
(c) $|\triangle AOB| = \frac{169^2}{120}$ **(ii)** $AB = \frac{13^3}{60}$
4(a) $y' = -\dfrac{y}{x}$, tangent: $3x + 2y = 12$
normal: $2x - 3y + 5 = 0$ **(b)** $P(2,3)$ is the midpoint of the interval joining $(4,0)$ and $(0,6)$.
5(a) $y' = \dfrac{1}{2y}$, tangent: $x - 6y + 9 = 0$,
normal: $6x + y = 57$ **(b)** $(0, 1\frac{1}{2})$ is the midpoint of the interval joining $P(9,3)$ and $(-9,0)$.
6(a) $\dfrac{dy}{dx} = \dfrac{t^2 + 1}{t^2 - 1}$, tangent: $5x - 3y = 8$,
normal: $3x + 5y = 15$ **(b)** $x^2 - y^2 = 4$
7 $x\dfrac{dx}{dt} + y\dfrac{dy}{dt} = 0$
(a) The top is slipping down at $\frac{2}{15}\sqrt{15}$ cm/s.
(b) The bottom is slipping out at $\frac{14}{15}\sqrt{15}$ mm/s.

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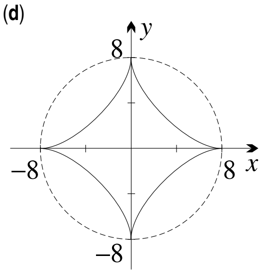
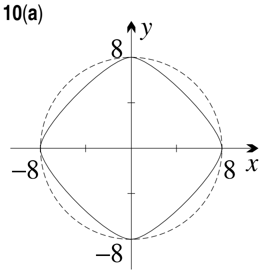
8(a) $S^2 \dfrac{dS}{dt} = 24\pi V \dfrac{dV}{dt}$ or $r \dfrac{dS}{dt} = 2 \dfrac{dV}{dt}$
(b) $10\,\mathrm{cm}^3/\mathrm{s}$

9(a) The symmetries arise because the equation is unchanged when x is replaced by $-x$, or y by $-y$, or x and y are exchanged. **(b)** Neither $x^{\frac{2}{3}}$ nor $y^{\frac{2}{3}}$ can be negative. **(c)** As $x \to 8^-$, either $y \to 0^+$ and $y' \to 0^-$, or $y \to 0^-$ and $y' \to 0^+$.

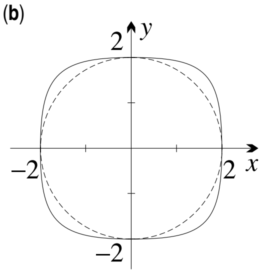
(d)



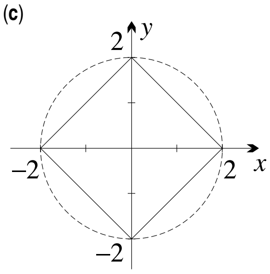
10(a)



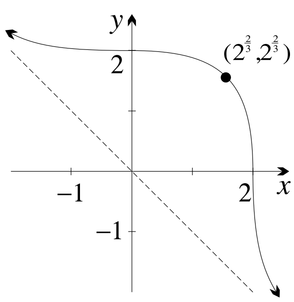
(b)



(c)

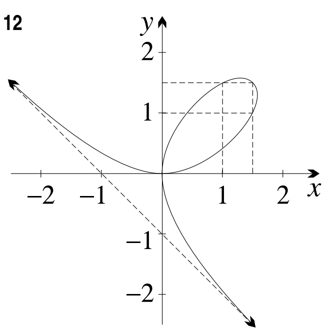


11



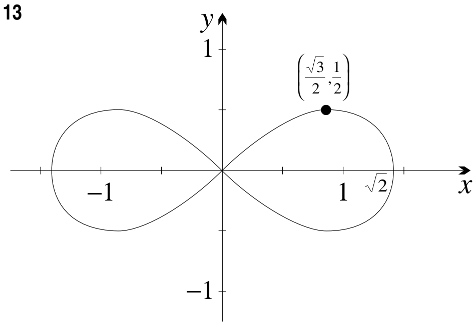
(a) The tangent at $(2,0)$ is $x=2$, the tangent at $(0,2)$ is $y=2$, and the tangent at $(2^{\frac{2}{3}}, 2^{\frac{2}{3}})$ is $x+y=2^{\frac{5}{3}}$.

12



$(y^2-x)y' = y - x^2$, the tangent at $(1\frac{1}{2}, 1\frac{1}{2})$ is $x+y=3$, the tangent at $(2^{\frac{1}{3}}, 2^{\frac{2}{3}})$ is horizontal, the tangent at $(2^{\frac{2}{3}}, 2^{\frac{1}{3}})$ is vertical.

13



$(x^2+y^2)(x+yy') = x - yy'$, the tangents at $(\sqrt{2},0)$ and $(-\sqrt{2},0)$ are vertical, the tangents at $\left(\frac{1}{2}\sqrt{3}, \frac{1}{2}\right)$, $\left(\frac{1}{2}\sqrt{3}, -\frac{1}{2}\right)$, $\left(-\frac{1}{2}\sqrt{3}, \frac{1}{2}\right)$ and $\left(-\frac{1}{2}\sqrt{3}, -\frac{1}{2}\right)$ are horizontal.

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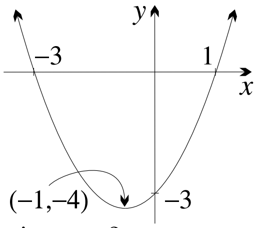
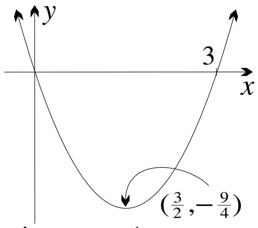
Chapter Eight

Exercise **8A** (Page 283)

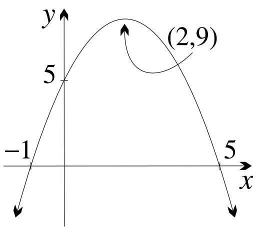
1(a) axis: $x = -1$



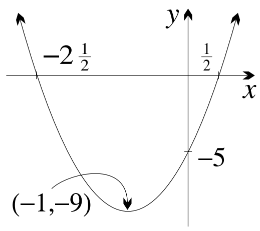
(b) axis: $x = 1\frac{1}{2}$



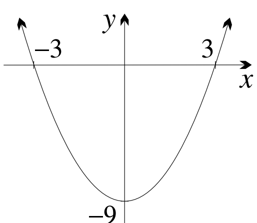
(c) axis: $x = 2$



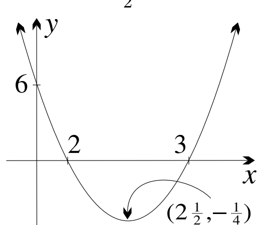
(d) axis: $x = -1$



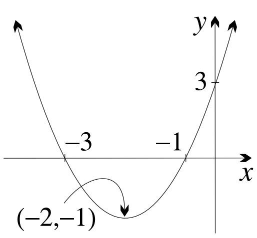
(e) axis: $x = 0$



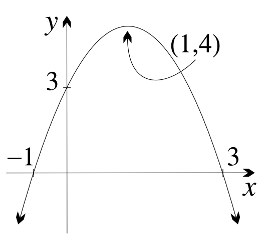
(f) axis: $x = 2\frac{1}{2}$



(g) axis: $x = -2$

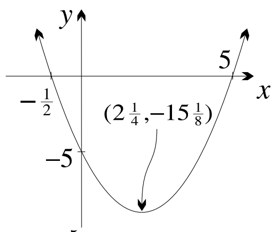


(h) axis: $x = 1$

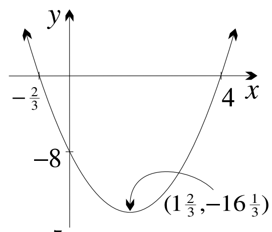


2(a) $x < -3$ or $x > 1$ **(b)** $0 \le x \le 3$
(c) $-1 \le x \le 5$ **(d)** $-\frac{5}{2} < x < \frac{1}{2}$
(e) $x \le -3$ or $x \ge 3$ **(f)** $2 < x < 3$
(g) $-3 \le x \le -1$ **(h)** $-1 < x < 3$
3(a) any positive multiple of $y = (x-3)(x-5)$
(b) any positive multiple of $y = x(x+4)$
(c) any positive multiple of $y = -(x+1)(x-3)$
(d) any positive multiple of $y = -x(x-2)$
4(a) $x = 5$, $y = (x-4)(x-6)$
(b) $x = 5\frac{1}{2}$, $y = (x-3)(x-8)$
(c) $x = 1$, $y = (x+3)(x-5)$
(d) $x = -3\frac{1}{2}$, $y = (x+6)(x+1)$

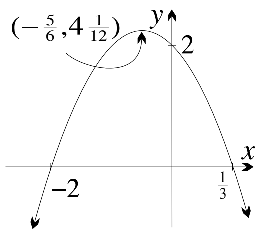
5(a) $x = 2\frac{1}{4}$



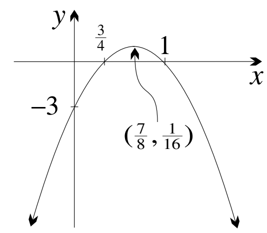
(b) $x = \frac{5}{3}$



(c) $x = -\frac{5}{6}$

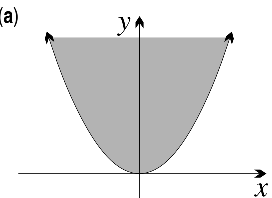


(d) $x = \frac{7}{8}$

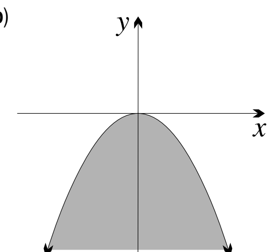


6(a) $y = -(x-2)(x-8)$ **(b)** $y = -2(x-2)(x-8)$
(c) $y = -\frac{3}{16}(x-2)(x-8)$ **(d)** $y = 3(x-2)(x-8)$
(e) $y = \frac{4}{3}(x-2)(x-8)$ **(f)** $y = -\frac{20}{7}(x-2)(x-8)$
7 $y = (x-\alpha)(x-1)$ **(a)** $y = x(x-1)$
(b) $y = (x-1)^2$ **(c)** $y = (x+15)(x-1)$
(d) $y = \frac{1}{2}(2x+3)(x-1)$
8(a)(i) $y \ge -1$ **(ii)** $y \ge 3$ **(iii)** $-1 \le y \le 8$
(b)(i) $y \ge -9$ **(ii)** $y \ge -9$ **(iii)** $-8 \le y \le 27$
(c)(i) $y \le 1$ **(ii)** $y \le -8$ **(iii)** $-8 \le y \le 1$

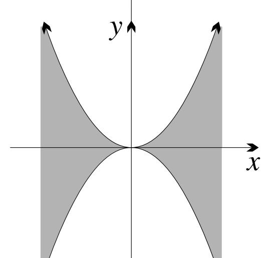
9(a)



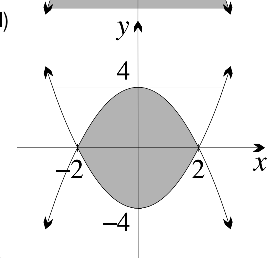
(b)



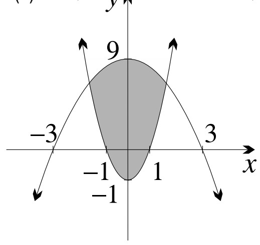
(c)



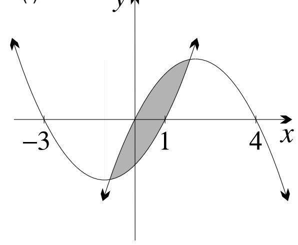
(d)



(e)



(f)



10(a) $y = x(x+3)$ **(b)** $y = -4x(x-2)$ **(c)** $y = \frac{3}{2}x^2$
(d) $y = -\frac{1}{4}x^2$ **(e)** $y = -\frac{2}{5}x(x-5)$
(f) $y = -2x(x+6)$

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11(a) $a = \dfrac{c}{\alpha\beta}$ **(b)** $a = -\dfrac{b}{\alpha+\beta}$

(c) $a = \dfrac{2}{(1-\alpha)(1-\beta)}$

12(a) $y = (x+1)(x-2)$ **(b)** $y = -(x+3)(x-2)$
(c) $y = 3(x+2)(x-4)$ **(d)** $y = -\frac{1}{2}(x-2)(x+2)$

13(a) $-b$, $-c$, $x = -\frac{1}{2}(b+c)$

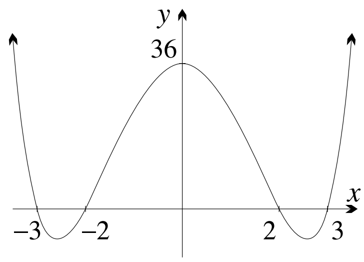
(b) -1, a^2, $x = \frac{1}{2}(a^2-1)$ **(c)** -1, $\dfrac{a+b}{a}$, $x = \dfrac{b}{2a}$

(d) $c+1$, $c-1$, $x = c$

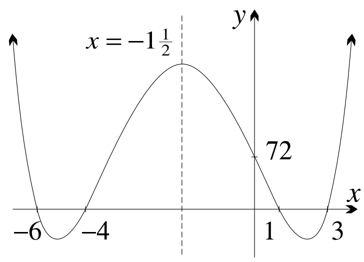
14(a)(ii) $f'(x) = 2(x-3)$. The graph is tangent to the x-axis at $x = 3$.

(b) The graph is tangent to the x-axis at $x = q$.

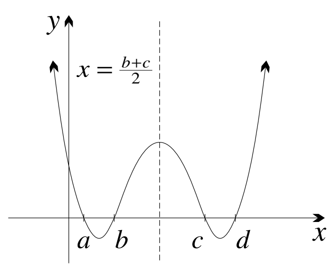
16(a)



17(d)

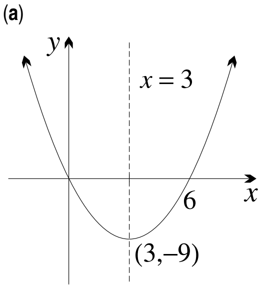


(e)

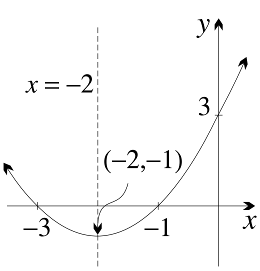


Exercise 8B (Page 287)

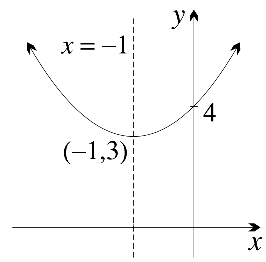
1(a)



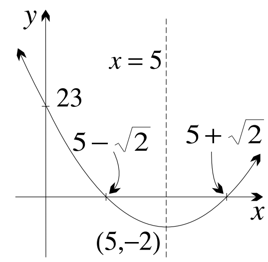
(b)



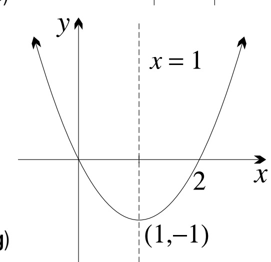
(c)



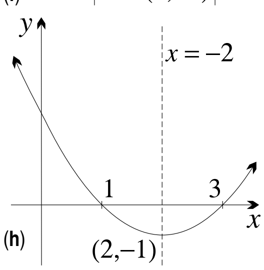
(d)



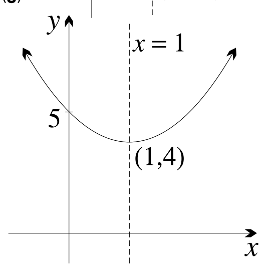
(e)



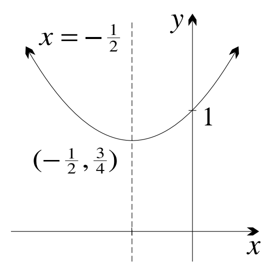
(f)



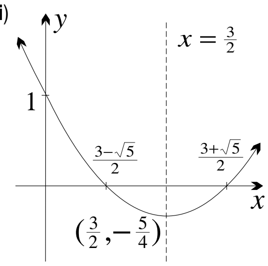
(g)



(h)



(i)



2(a) $y = a(x-1)^2+2$ for any $a > 0$. ($a = 1$ is okay.) **(b)** $y = a(x+2)^2 - 3$. ($a = 1$ is okay — a must be larger than $\frac{3}{4}$.) **(c)** $y = -a(x-2)^2 - 1$ for any $a > 0$. ($a = 1$ is okay.) **(d)** $y = -a(x-3)^2 + 5$. ($a = 1$ is okay — a must be larger than $\frac{5}{9}$.)

3(a) $y = (x-2)^2 + 5$, $x = 2$, $(0, 9)$ **(b)** $y = x^2 - 3$, $x = 0$, $(0, -3)$ **(c)** $y = (x+1)^2 + 7$, $x = -1$, $(0, 8)$ **(d)** $y = (x-3)^2 - 11$, $x = 3$, $(0, -2)$

4 The graph of $y = ax^2+1$ is the graph of $y = ax^2$ shifted up one unit. Put $h = 0$ and $k = 1$ in the formula $y = a(x-h)^2 + k$. **(a)** $y = 2x^2 + 1$ **(b)** $y = -3x^2+1$ **(c)** $y = \frac{1}{3}x^2+1$ **(d)** $y = -\frac{1}{4}x^2+1$

5 Put $h = -4$ and $k = 2$ in the formula $y = a(x-h)^2+k$. **(a)** $y = (x+4)^2+2$ **(b)** $y = 3(x+4)^2+2$ **(c)** $y = -\frac{2}{49}(x+4)^2+2$ **(d)** $y = \frac{7}{8}(x+4)^2+2$ **(e)** $y = -\frac{1}{8}(x+4)^2+2$ **(f)** $y = \frac{18}{25}(x+4)^2+2$

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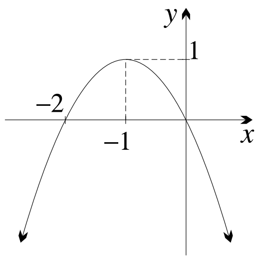
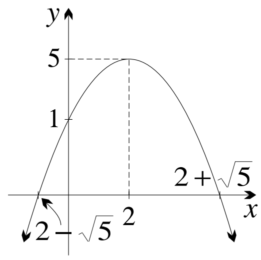
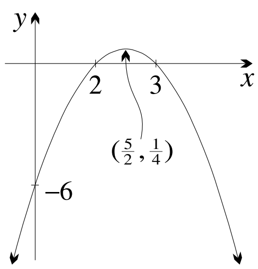
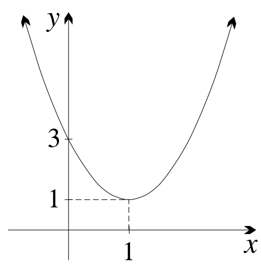
6(a) $y = -(x+1)^2 + 1$ **(b)** $y = -(x-2)^2 + 5$



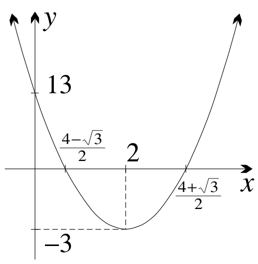


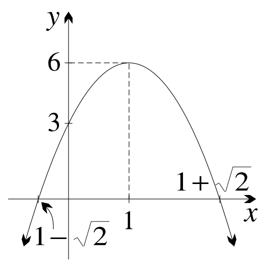
(c) $y = -(x - 2\frac{1}{2})^2 + \frac{1}{4}$ **(d)** $y = 2(x-1)^2 + 1$



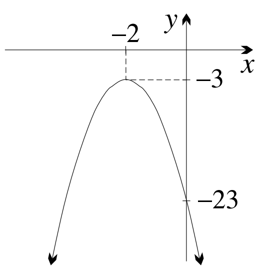


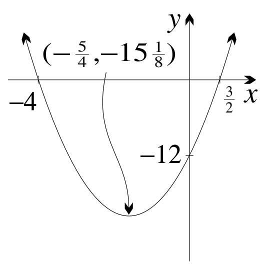
(e) $y = 4(x-2)^2 - 3$ **(f)** $y = -3(x-1)^2 + 6$



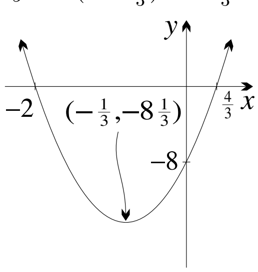


(g) $y = -5(x+2)^2 - 3$ **(h)** $y = 2(x + \frac{5}{4})^2 - 15\frac{1}{8}$





(i) $y = 3(x + \frac{1}{3})^2 - 8\frac{1}{3}$



7(a) $y \geq 2,\ y \geq 6,\ 2 \leq y \leq 6$ **(b)** $y \geq 1,\ y \geq 33,\ 3 \leq y \leq 33$ **(c)** $y \leq 5,\ y \leq 5,\ -11 \leq y \leq 4$

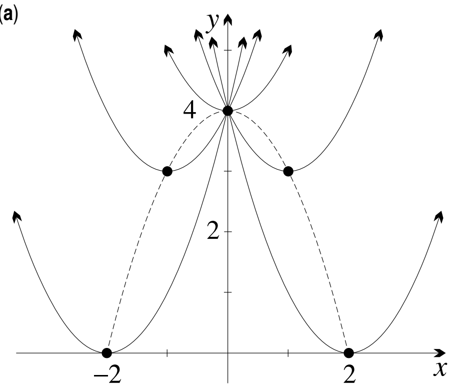
8 $y = (x-3)^2 + c - 9$ **(a)** $c = 9$ **(b)** $c < 9$ **(c)** $c > 9$

9 $y = (x+2)^2 + k$ **(a)** $y = (x+2)^2 - 4$
(b) $y = (x+2)^2 - 48$ **(c)** $y = (x+2)^2 - 9$
(d) $y = (x+2)^2 - 10$ **(e)** $y = (x+2)^2 - 2$
(f) $y = (x+2)^2 + 7$ **(g)** $y = (x+2)^2 - 3$
(h) $y = (x+2)^2 - 2$

10(a) $y = 2(x-1)^2 + 1$ **(b)** $y = -(x-3)^2 + 2$
(c) $y = \frac{1}{2}(x+2)^2 - 4$ **(d)** $y = -3(x+1)^2 + 4$

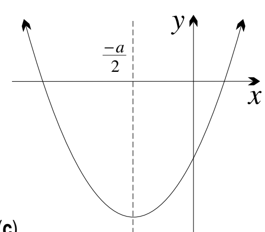
11(a) $-2,\ 3$ **(b)** $2 + \sqrt{3},\ 2 - \sqrt{3}$ **(c)** $-\frac{5}{2},\ 1$
(d) $\frac{3}{2} + \frac{1}{10}\sqrt{5},\ \frac{3}{2} - \frac{1}{10}\sqrt{5}$

12(a)

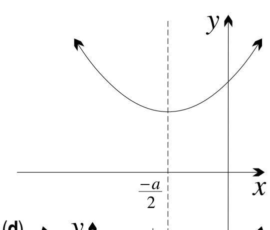


(b) The vertex moves on the parabola $y = 4 - x^2$.

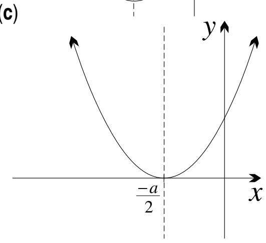
13(a)



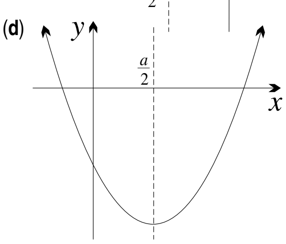
(b)



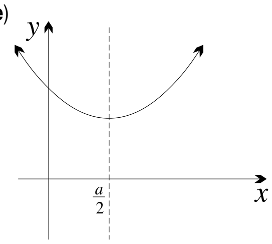
(c)



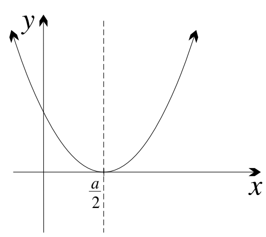
(d)



(e)



(f)



14 vertex $\left(-\dfrac{b}{2a}, \dfrac{4ac - b^2}{4a}\right)$,
zeroes $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, y-intercept c

15 $y = a\Big(x - \frac{1}{2}(\alpha + \beta)\Big)^2 - \frac{1}{4}a(\alpha - \beta)^2$,
vertex $\Big(\frac{1}{2}(\alpha + \beta), -\frac{1}{4}a(\alpha - \beta)^2\Big)$

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16 Tangents drawn at points equidistant from the axis of symmetry have opposite gradients.

17 $y = a(x-h)^2 + k$ **(a)** $a = \dfrac{c-k}{h^2}$

(b) $a = \dfrac{2-k}{(1-h)^2}$ **(c)** $a = -\dfrac{b}{2h}$

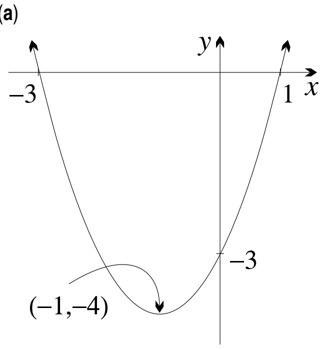
(d) $a = -\dfrac{k}{(\alpha - h)^2}$

18(a) $-d+\sqrt{e}$, $-d-\sqrt{e}$ **(b)** $2\sqrt{e}$

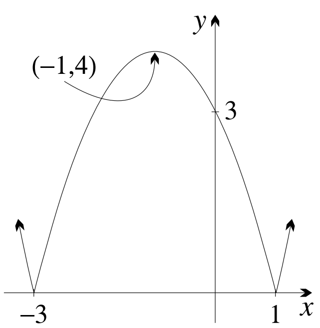
(c) $e = 1$. They have vertex on the line $y = -1$.

19 $h_1 = h_2$, but $k_1 \neq k_2$. The two curves have the same axis of symmetry, but different vertices.

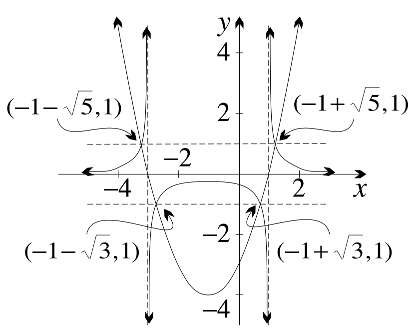
20(a)



(b)



(d)



Exercise 8C (Page 291)

1(a) -1 or -5 **(b)** -2 **(c)** -4 or 6

(d) $1+\sqrt{2}$ or $1-\sqrt{2}$, $2{\cdot}414$ or $-0{\cdot}4142$

(e) $-2+\sqrt{5}$ or $-2-\sqrt{5}$, $0{\cdot}2361$ or $-4{\cdot}236$

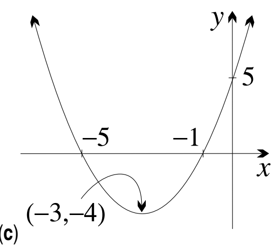
(f) $\frac{1}{2}(-1+\sqrt{3})$ or $\frac{1}{2}(-1-\sqrt{3})$, $-1{\cdot}366$ or $0{\cdot}3660$

(g) $\frac{1}{10}(7+\sqrt{109})$ or $\frac{1}{10}(7-\sqrt{109})$, $0{\cdot}3440$ or $1{\cdot}744$

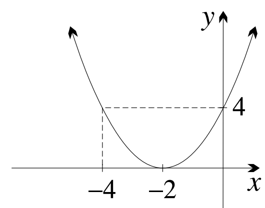
(h) $\frac{1}{8}(3+\sqrt{57})$ or $\frac{1}{8}(3-\sqrt{57})$, $1{\cdot}319$ or $-0{\cdot}5687$

(i) $\frac{3}{2}$ or $-\frac{3}{2}$

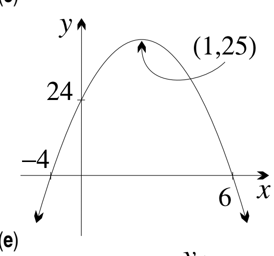
2(a)



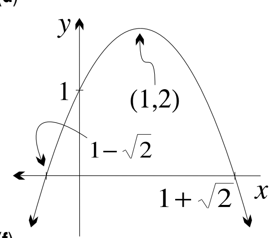
(b)



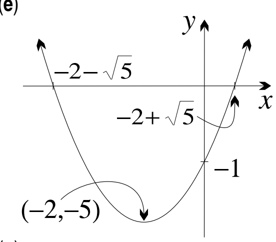
(c)



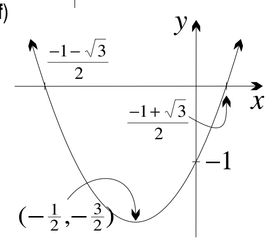
(d)



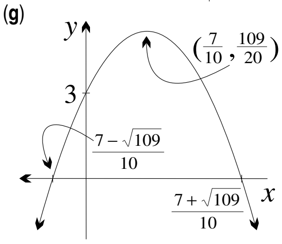
(e)



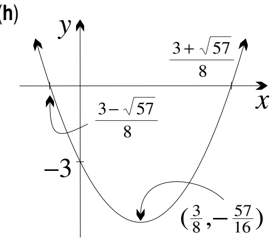
(f)



(g)



(h)



3(a) $-5 < x < -1$ **(b)** $x \neq -2$

(c) $x \le -4$ or $x \ge 6$ **(d)** $1-\sqrt{2} \le x \le 1+\sqrt{2}$

5(a) 16, twice **(b)** 5, twice **(c)** 0, once

(d) -31, no times

6(a) $4, -\frac{2}{3}$ **(b)** $1+\sqrt{5}, 1-\sqrt{5}$ **(c)** $3+2\sqrt{2}, 3-2\sqrt{2}$

(d) $2, -\frac{1}{3}$

7(a) $y = (x-3+\sqrt{5})(x-3-\sqrt{5})$

(b) $y = 3(x+1+\frac{1}{3}\sqrt{3})(x+1-\frac{1}{3}\sqrt{3})$

(c) $y = -(x-\frac{3}{2}-\frac{1}{2}\sqrt{13})(x-\frac{3}{2}+\frac{1}{2}\sqrt{13})$

(d) $y = -2(x+1)(x-\frac{1}{2})$

8(a) $(0,3)$ and $(5,8)$. The line and parabola intersect twice. **(b)** The line intersects the parabola once at $(-1,-9)$, and so it is a tangent to the parabola. **(c)** The line and the parabola do not intersect. **(d)** $(1,2)$ and $(2,1)$. The line and the parabola intersect twice.

9 $\frac{1}{2}p(-1+\sqrt{5})$

10 $f'(x) = 2ax + b$. The axis of symmetry of a quadratic can be found by solving $f'(x) = 0$.

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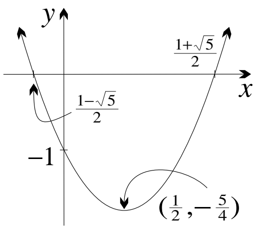
11(a) $x = h + \sqrt{-k}$ or $h - \sqrt{-k}$

12(a) $x = -\frac{1}{2}b$, vertex $\left(-\frac{1}{2}b, \frac{1}{4}(4c - b^2)\right)$

(b) Difference between zeroes is $\sqrt{b^2 - 4c}$.

(c) $b^2 - 4c = 1$

13



Exercise **8D** (Page 293)

1(a) $3, -3, 1$ or -1 **(b)** $2, -2, 5$ or -5

(c) $\sqrt{2}\,, -\sqrt{2}\,, \frac{2}{3}\sqrt{3}$ or $-\frac{2}{3}\sqrt{3}$ **(d)** 1 or 2

(e) 1 or 3 **(f)** $\frac{1}{4}, -\frac{1}{4}, 4$ or -4 **(g)** $3, -3, 4$ or -2

(h) $2 + 2\sqrt{2}$ or $2 - 2\sqrt{2}$ **(i)** 1 or 2 **(j)** 2 or 3

2(a) 30°, 90°or 150° **(b)** 120°, 180°or 240°

(c) 135°or 315° **(d)** 30°, 150°or 270°

3(a) $(1, 3)$ and $(\frac{9}{5}, \frac{13}{5})$ **(b)** $(2, -1)$

(c) $(-2, 3)$ and $(\frac{100}{13}, -\frac{45}{13})$

4(a) $\frac{1}{2}, 2, \frac{1}{2}(-3 + \sqrt{5}\,)$ or $\frac{1}{2}(-3 - \sqrt{5}\,)$

(b) $\frac{1}{2}(-3 + \sqrt{29}\,)$ or $\frac{1}{2}(-3 - \sqrt{29}\,)$

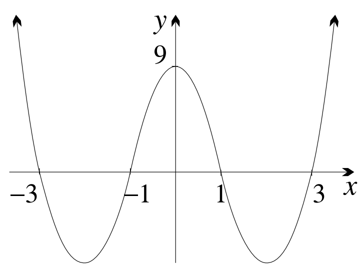
(c) $\frac{1}{2}(-5 + \sqrt{13}\,)$, $\frac{1}{2}(-5 - \sqrt{13}\,)$, $\frac{1}{2}(-5 + \sqrt{17}\,)$ or $\frac{1}{2}(-5 + \sqrt{17}\,)$

5(a) 1 **(b)** 3 **(c)** 4 **(d)** 11

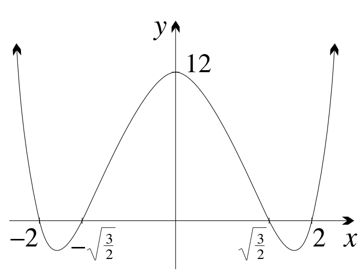
6(a) -1 or 0 **(b)** $5^{-\frac{3}{2}}$ or 125

7(a) $x = \dfrac{pr}{p+q}$, $y = \dfrac{qr}{p+q}$ **(b)** $x = \frac{6}{7}$, $y = \frac{8}{7}$

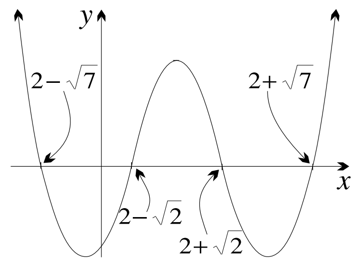
8(a)(i)



(ii)



(iii)



(b)(i) $x \le -3$ or $-1 \le x \le 1$ or $x \ge 3$

(ii) $-2 \le x \le -\frac{1}{2}\sqrt{6}$ or $\frac{1}{2}\sqrt{6} \le x \le 2$

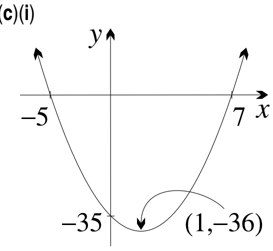
(iii) $x < 2 - \sqrt{7}\,, 2 - \sqrt{2} < x < 2 + \sqrt{2}$ or $x > 2 + \sqrt{7}$

9(a) $1, \frac{1}{2}(3{+}\sqrt{5}\,)$ or $\frac{1}{2}(3{-}\sqrt{5}\,)$ **(b)(i)** $1, \frac{1}{2}(-5{+}\sqrt{21}\,)$ or $\frac{1}{2}(-5 - \sqrt{21}\,)$ **(ii)** $\frac{1}{6}(7 + \sqrt{13}\,)$ or $\frac{1}{6}(7 - \sqrt{13}\,)$

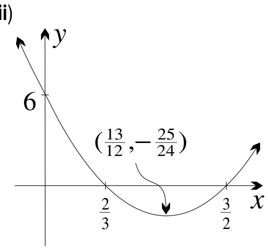
Exercise **8E** (Page 296)

1(a)(i) -4 **(ii)** -9 **(iii)** $-\frac{1}{4}$ **(iv)** $-\frac{49}{8}$ **(b)(i)** 4 **(ii)** 3 **(iii)** -2 **(iv)** $\frac{11}{4}$

(c)(i)

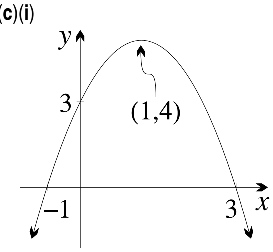


(ii)

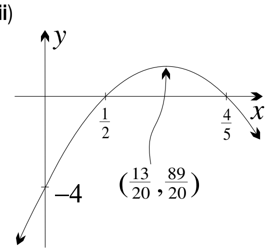


2(a)(i) 9 **(ii)** $\frac{25}{4}$ **(iii)** 9 **(iv)** $\frac{1}{8}$ **(b)(i)** -1 **(ii)** $-\frac{5}{4}$ **(iii)** $\frac{17}{4}$ **(iv)** $\frac{17}{8}$

(c)(i)



(ii)



3(b) $\frac{9}{4}$ when $x = \frac{3}{2}$

4 225 when the numbers are 15 and 15

5(b) 18 when $x = 3$

6 $16\,\text{m}^2$

7 105 metres

8 2 machines, \$7000

9(a) $2x^2 - 64x + 1024$ **(b)** $x = y = 16$

10(b) $15\frac{5}{8}\,\text{cm}^2$

11(a) $-x^2 + 2015x$ **(b)** $1\,015\,056{\cdot}25\,\text{m}^2$

12(a) $2x + 5y = 40$ **(b)** $\frac{1280}{41}$ cm and $\frac{2000}{41}$ cm

13(b) $x = \frac{800}{3}$ and $y = 200$

14(b) 23

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15 $\dfrac{288}{4+\pi}$ cm

16 $\dfrac{m}{4r} \times \dfrac{m}{2(n+r)}$

17 profit$= -\frac{5}{6}x^2 + 15x - 27$, \$40.50

18(a) $x(16-x)$ (c) 4 (d) \$22

19(a) $(\frac{9}{2}, \frac{9}{4})$ and $(0,0)$ (b) $9x - 2x^2$, $\frac{81}{8}$

20 250 metres × $\frac{500}{\pi}$ metres

21(a) $11300 - 14300h + 4525h^2$

(b) approximately 94·8 minutes

22 $(\frac{1}{3}, \frac{1}{3})$ 26(a) $1200\,\text{cm}^2$

Exercise 8F (Page 302)

1(a) irrational, unequal (b) unreal (that is, no roots) (c) rational, equal (that is, a single rational root) (d) rational, unequal (e) rational, unequal (f) unreal

2(a) $\Delta = 4$, two rational roots

(b) $\Delta = -31$, no roots

(c) $\Delta = 0$, one rational root

(d) $\Delta = 32$, two irrational roots

(e) $\Delta = 361 = 19^2$, two rational roots

(f) $\Delta = 36$, two rational roots

3(a) $\Delta = 100 - 4g$, $g = 25$ (b) $\Delta = 16 - 4g$, $g = 4$

(c) $\Delta = 1 - 8g$, $g = \frac{1}{8}$ (d) $\Delta = 44 - 4g$, $g = 11$

(e) $\Delta = g^2 - 4g$, $g = 4$ (If $g = 0$, then $y = 1$ for all x, and so y is never zero.)

(f) $\Delta = 49 - 4g^2$, $g = \frac{7}{2}$ or $-\frac{7}{2}$

(g) $\Delta = 16(g^2 - 6g - 7)$, $g = -1$ or 7

(h) $\Delta = 4(g^2 + 2g - 8)$, $g = -4$ or 2

4(a) $\Delta = 4 - 4k$, $k \le 1$ (b) $\Delta = 64 - 8k$, $k \le 8$

(c) $\Delta = 4 - 12k$, $k \le \frac{1}{3}$ (d) $\Delta = 33 - 16k$, $k \le \frac{33}{16}$

(e) $\Delta = k^2 - 16$, $k \le -4$ or $k \ge 4$

(f) $\Delta = 9k^2 - 36$, $k \le -2$ or $k \ge 2$

(g) $\Delta = k^2 + 12k + 20$, $k \le -10$ or $k \ge -2$

(h) $\Delta = k^2 - 12k$, $k \le 0$ or $k \ge 12$

5(a) $-4 < \ell < 4$ (b) $\ell < -3$ or $\ell > 3$

(c) $-5 < \ell < 3$ (d) no values (e) $0 < \ell < 1$

(f) $\ell < -6$

6(b) $\Delta = 28$. Since $\Delta > 0$, the quadratic equation has two roots.

7(a) They intersect twice.

(b) They do not intersect.

(c) They intersect once.

(d) They intersect twice.

8(a) $\Delta = (m+4)^2$ (b) $\Delta = (m-2)^2$

(c) $\Delta = (2m-n)^2$ (d) $\Delta = (4m-1)^2$

(e) $\Delta = (m-6)^2$ (f) $\Delta = 36m^2$

9(a) $\Delta = \lambda^2 + 4$ (b) $\Delta = 4\lambda^2 + 48$ (c) $\Delta = \lambda^2 + 16$

(d) $\Delta = (\lambda - 1)^2 + 8$

10(a) $m > -\frac{9}{4}$ (b) $m = \frac{39}{8}$ (c) $-1 < m < 2$

(d) $m \le -1$ or $m \ge -\frac{1}{2}$

12(a) -4 (b) $\frac{17}{8}$ (c) -1 or 11

13 $3x - y - 19 = 0$

14 -7 and 5

15 $y = 4x^2 - 4x - 3$

16(a) 2 (b) 1 (c) 0

17 If $ac < 0$, then $\Delta > 0$.

18(a) $(x-4)^2 + y^2 = 4$ (b) $y = mx$

(d) $m = \dfrac{1}{\sqrt{3}}$ or $m = -\dfrac{1}{\sqrt{3}}$

(e) $(3, \sqrt{3})$ or $(3, -\sqrt{3})$

19 $m = \frac{1}{\sqrt{3}}$ or $m = -\frac{1}{\sqrt{3}}$, $P(\sqrt{3}, -1)$ or $P(-\sqrt{3}, -1)$

20 $\frac{4}{3}$ and $-\frac{3}{4}$

21(a) $-\frac{4}{3} < a < 1$ (b) $b \ne -5$ (c) $g = 3$ or $-\frac{13}{29}$

(d) $-1 < k < 14$

22(b) $b^2 = ac$

Exercise 8G (Page 305)

1(a) $x \le 0$ or $x \ge 1$ (b) $-\sqrt{7} < x < \sqrt{7}$ (c) $x \ne 3$

(d) $-1 - \sqrt{34} \le x \le -1 + \sqrt{34}$ (e) $-2 \le x \le \frac{1}{3}$

(f) $\frac{3}{2} \le x \le 5$ (g) $x < -\frac{1}{2}$ or $x > \frac{5}{2}$

(h) There are no solutions.

(i) All real numbers are solutions.

2(a) positive definite (b) indefinite

(c) negative definite (d) indefinite

(e) indefinite (f) positive definite

3(a)(i) $k > \frac{25}{32}$ (ii) $k \le \frac{25}{32}$

(b)(i) $-8 < k < 8$ (ii) $k \le -8$ or $k \ge 8$

(c)(i) $0 < k < 24$ (ii) $k \le 0$ or $k \ge 24$

(d)(i) $2 < k < 5$ (ii) $k \le 2$ or $k \ge 5$

4(a)(i) $-4 < m < 4$ (ii) $m \le -4$ or $m \ge 4$

(b)(i) $m > \frac{9}{8}$ (ii) $m \le \frac{9}{8}$

(c)(i) $-8 < m < 12$ (ii) $m \le -8$ or $m \ge 12$

(d)(i) $0 < m < 2$ (ii) $m \le 0$ or $m \ge 2$

5(a) -4 and 4

(b) 2. (When $\ell = 0$, it is not a quadratic.)

(c) 1. (When $\ell = -\frac{3}{10}$, the expression becomes $-\frac{1}{10}(5x-4)^2$, which is a multiple of a perfect square, but is not itself a perfect square.)

(d) $-\frac{2}{9}$ and 2

6(a) $2 < k < 18$ (b) no values

(c) $k \le 2$ or $k \ge 18$, but $k \ne \frac{5}{3}$

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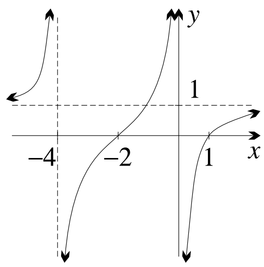
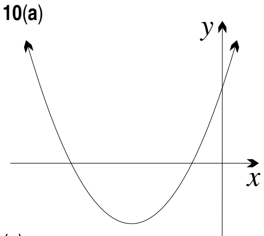
7(a)

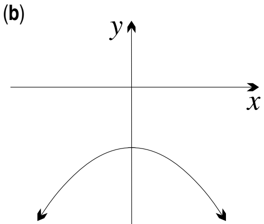


(c) Whatever the value of k, every horizontal line $y = k$ intersects the graph.
8(a) $x \le \frac{1}{2}(5 - \sqrt{21}\,)$ or $x \ge \frac{1}{2}(5 + \sqrt{21}\,)$
(b) no values **(c)** $x \le \frac{1}{2}$ or $x \ge 4$
(d) $0 < x < 2$ **(e)** $-\frac{3}{4} \le x \le 2$
(f) $x < -3$ or $x > 3$
9(a) $a > 0$ and $b^2 < 3ac$
(b) $a < 0$ and $b^2 < 3ac$ **(c)** $b^2 \ge 3ac$

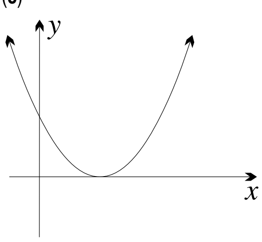
10(a)



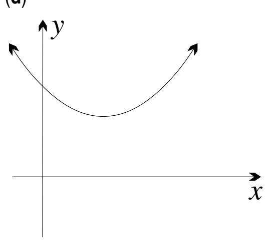
(b)



(c)



(d)



11(a) $b^2 = 3c$ **(b)** $b^2 > 3c$ **(c)** $b^2 < 3c$ **(d)** $c < 0$
(e) $c > 0$ and $b < 0$ (and $b^2 > 3c$)
(f) $c > 0$ and $b > 0$ (and $b^2 > 3c$)
13 $x \le -22$ or $x \ge 2$, $y \le -15 - 12\sqrt{2}$ or $y \ge -15 + 12\sqrt{2}$
14 $(3x - 4y + 1)(x + 2y - 3)$
15 $-\frac{5}{4}$ and $\frac{3}{4}$
16 9. Note that a^2 cannot be negative.
17(b) $(x-1)^2 + (x+3)^2$

Exercise **8H** (Page 309)

1 $\alpha + \beta = -7$, $\alpha\beta = 10$, the roots are -2 and -5.
2(a) $\alpha + \beta = \frac{10}{3}$, $\alpha\beta = 1$, the roots are $\frac{1}{3}$ and 3. **(b)** $\alpha + \beta = -4$, $\alpha\beta = 1$, the roots are $-2 + \sqrt{3}$ and $-2 - \sqrt{3}$. **(c)** $\alpha + \beta = 1$, $\alpha\beta = -1$, the roots are $\frac{1}{2} + \frac{1}{2}\sqrt{5}$ and $\frac{1}{2} - \frac{1}{2}\sqrt{5}$.
3(a) $2,\ 5$ **(b)** $-1,\ -6$ **(c)** $-1,\ 0$ **(d)** $-\frac{3}{2},\ -\frac{1}{2}$
(e) $-1,\ \frac{4}{5}$ **(f)** $\frac{2}{3}\ \ -\frac{4}{3}$ **(g)** $m,\ n$ **(h)** $-q/p,\ -3r/p$
(i) $(a-4)/a,\ -3/a$
4(a) $x^2 - 4x + 3 = 0$ **(b)** $x^2 - 4x - 12 = 0$
(c) $x^2 + 5x + 4 = 0$ **(d)** $4x^2 - 8x + 3 = 0$
(e) $x^2 - 4x + 1 = 0$ **(f)** $x^2 + 2x - 4 = 0$
5(a) 3 **(b)** 2 **(c)** 21 **(d)** 6 **(e)** 20 **(f)** $\frac{3}{2}$ **(g)** 5
(h) $\frac{5}{2}$
6(a) $\frac{5}{2}$ **(b)** $\frac{1}{2}$ **(c)** -1 **(d)** 5 **(e)** $\frac{5}{8}$ **(f)** 10 **(g)** $\frac{21}{4}$
(h) 21
7(b)(i) $3,\ 1,\ 5$ **(ii)** $-5,\ -7,\ 53$ **(iii)** $\frac{7}{3},\ \frac{2}{3},\ 2\frac{7}{9}$
(c) $\sqrt{5},\ \sqrt{53},\ \frac{5}{3}$
9 $\alpha = \dfrac{-b - \sqrt{b^2 - 4ac}}{2a}$, $\beta = \dfrac{-b + \sqrt{b^2 - 4ac}}{2a}$
10 $\frac{5}{6},\ \frac{2}{3}\sqrt{3}$
11(a) $3x^2 + 4x + 28 = 0$ **(b)** $7x^2 + 2x + 3 = 0$
(c) $9x^2 + 18x + 29 = 0$ **(d)** $9x^2 + 38x + 49 = 0$
12(a) $\frac{5}{2}$ **(b)** -4 **(c)** $\frac{7}{2}$ **(d)** $\frac{1}{3}$
13(a) -1 **(b)** 1 **(c)** $\frac{1}{10}$ **(d)** -3
14(a) $ac < 0$ **(b)** If $ca > 0$ the roots have the same sign. If $ab < 0$ they are both positive, if $ab > 0$ they are both negative. **(c)** If $ac < 0$ the roots have opposite signs. If $ab < 0$ the positive root is numerically greater, if $ab > 0$ the negative root is numerically greater.
15 $\frac{305}{27}$
16 8
17 $\frac{1}{3}(2 + 2\sqrt{10}\,)$ or $\frac{1}{3}(2 - 2\sqrt{10}\,)$
20(b) 6 **(c)** $(3, 27)$
21(a) $\left(\frac{1}{2}, \frac{1}{2}\right)$ **(b)** $\left(\frac{5}{2}, \frac{11}{2}\right)$ **(c)** $(-2, -2)$
22 $y = -\frac{1}{3}x$. It is the line through the centre of the circle perpendicular to the given line.
23(b) $\sqrt{14}$
24 $5\sqrt{2}$ units
25 $2 \le m < 3$
26 When $m = 1$, $x = -1$, and when $m = -3$, $x = 1$.
27 $-1,\ \frac{1}{2}(1 + \sqrt{21}\,)$ or $\frac{1}{2}(1 - \sqrt{21}\,)$

Exercise **8I** (Page 313)

2 It is true for all values of x, and hence is an identity.
4(a) $a = 1,\ b = 3,\ c = 3$ **(b)** $a = 1,\ b = 7,\ c = 12$
5(a) $2(x+1)^2 - (x+1) - 7$
(b) $a = 2$, $b = 16$ and $c = 35$
(c) $2(x-2)^2 + 3(x-2) + 1$

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6(a) $(x+1)^2-2(x+1)+1$ **(b)** $(n-4)^2+8(n-4)+16$
(c) $(x+2)^2-4(x+2)+4$
8(a) $a=2$, $b=1$, $c=-\frac{1}{2}$ or $a=2$, $b=-\frac{1}{2}$, $c=1$
(b) $3(m-1)^2-3(m-2)^2+(m-3)^2$
9(a) $7(2x+1)-12(x+1)$ **(b)** $\dfrac{1}{x+1}+\dfrac{1}{x+2}$
(c) $\dfrac{1}{2x}+\dfrac{1}{2(x+2)}$
10(a) $y=x^2-4x$, no
(b) All four points lie on $y=x^2-5x+6$.
11(b) 961 **(d)** n^2
12(a) 1 **(b)** 1

Chapter Nine

Exercise 9A (Page 318)

1(a) $y=-2$ **(b)** $x=-1$ **(c)** $y=2$
(d) $y=3x$ or $y=-3x$ **(e)** $x^2+y^2=9$
(f) $y=x+5$ **(g)** $(x+3)^2+(y-1)^2=9$
2 $(x-3)^2+(y-1)^2=16$
3(a) $6x-4y+15=0$ **(b)** $6x-4y+15=0$. The locus of the point P which moves so that it is equidistant from R and S is the perpendicular bisector of RS.
4(a) $\dfrac{y}{x-4}$, $\dfrac{y}{x+2}$ **(c)** $(x-1)^2+y^2=9$, which is a circle with centre $(1,0)$ and radius 3.
5(a) $2x+y-1=0$, the perpendicular bisector of AB **(b)** $x^2+y^2+2x-6y+5=0$, circle with centre $(-1,3)$ and radius $\sqrt{5}$
(c) $x^2-2x-8y+17=0$, parabola
6 $(x+1)^2=6(y-\frac{3}{2})$, vertex: $(-1,\frac{3}{2})$
7(b) $C(2,1)$, radius $\sqrt{5}$
8 $x^2+y^2=1$, the circle with centre $(0,0)$ and radius 1
9(a) $x^2+y^2=4$ **(b)** $3x^2+3y^2-28x+18y+39=0$
10 $3x^2-y^2+12x+10y-25=0$
11(a) P may satisfy $x-y+12=0$ or $7x+7y-60=0$. **(b)** The gradients are 1 and -1, so the lines are perpendicular.
12 $x^2=-4(y-1)$ and $y>0$
13 $8x-2y+3=0$
14 $10x-15y+18=0$
15(b) $y=\sqrt{3}x+2$, $y=-1$, $y=-\sqrt{3}x+2$
(c) $x^2+y^2=4$ **(d)** circle with centre O and radius 2 — the circumcircle of the triangle
16(b) $x^2+y^2+z^2=a^2$, which is the equation of a sphere with centre $(0,0,0)$ and radius a.
(c) $C(0,0,0)$ and $r=\frac{1}{2}\sqrt{3}$

Exercise 9B (Page 323)

3(b) $x^2=12y$
4(a) $x^2=20y$ **(b)** $x^2=-4y$ **(c)** $y^2=8x$
(d) $y^2=-6x$
5(a) $x^2=-4ay$ **(b)** $y^2=4ax$
6(vi) Only parts (a), (e), (i) and (m) are sketched below. The details of all the parabolas follow these sketches.

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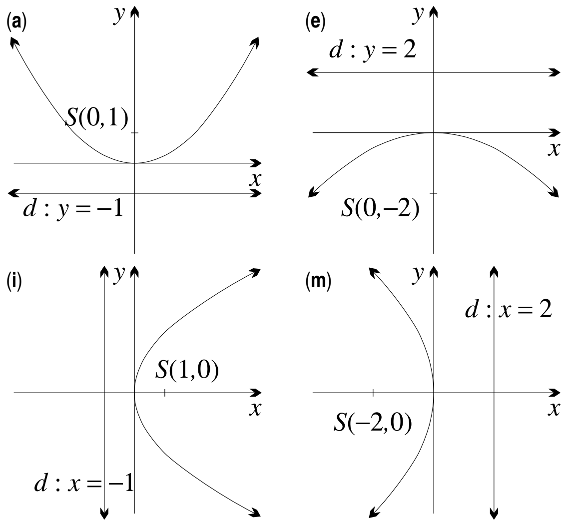
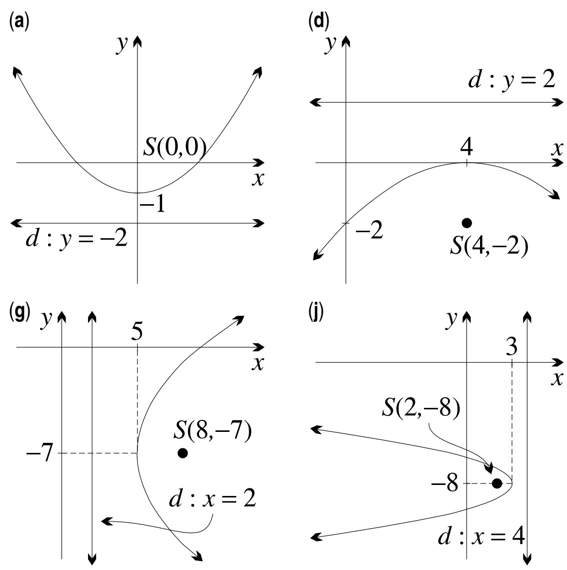


(a) $V(0,0)$, $S(0,1)$, axis: $x=0$, directrix: $y=-1$, $4a=4$
(b) $V(0,0)$, $S(0,2)$, axis: $x=0$, directrix: $y=-2$, $4a=8$
(c) $V(0,0)$, $S(0,\frac{1}{4})$, axis: $x=0$, directrix: $y=-\frac{1}{4}$, $4a=1$
(d) $V(0,0)$, $S(0,\frac{1}{3})$, axis: $x=0$, directrix: $y=-\frac{1}{3}$, $4a=\frac{4}{3}$
(e) $V(0,0)$, $S(0,-2)$, axis: $x=0$, directrix: $y=2$, $4a=8$
(f) $V(0,0)$, $S(0,-3)$, axis: $x=0$, directrix: $y=3$, $4a=12$
(g) $V(0,0)$, $S(0,-\frac{1}{2})$, axis: $x=0$, directrix: $y=\frac{1}{2}$, $4a=2$
(h) $V(0,0)$, $S(0,-0{\cdot}1)$, axis: $x=0$, directrix: $y=0{\cdot}1$, $4a=0{\cdot}4$
(i) $V(0,0)$, $S(1,0)$, axis: $y=0$, directrix: $x=-1$, $4a=4$
(j) $V(0,0)$, $S(\frac{1}{4},0)$, axis: $y=0$, directrix: $x=-\frac{1}{4}$, $4a=1$
(k) $V(0,0)$, $S(\frac{3}{2},0)$, axis: $y=0$, directrix: $x=-\frac{3}{2}$, $4a=6$
(l) $V(0,0)$, $S(\frac{1}{8},0)$, axis: $y=0$, directrix: $x=-\frac{1}{8}$, $4a=\frac{1}{2}$
(m) $V(0,0)$, $S(-2,0)$, axis: $y=0$, directrix: $x=2$, $4a=8$
(n) $V(0,0)$, $S(-3,0)$, axis: $y=0$, directrix: $x=3$, $4a=12$
(o) $V(0,0)$, $S(-\frac{1}{4},0)$, axis: $y=0$, directrix: $x=\frac{1}{4}$, $4a=1$
(p) $V(0,0)$, $S(-0{\cdot}3,0)$, axis: $y=0$, directrix: $x=0{\cdot}3$, $4a=1{\cdot}2$
7 Details rather than sketches are given:
(a) $V(0,0)$, $S(0,\frac{1}{8})$, directrix: $y=-\frac{1}{8}$
(b) $V(0,0)$, $S(0,-1)$, directrix: $y=1$
(c) $V(0,0)$, $S(\frac{1}{9},0)$, directrix: $x=-\frac{1}{9}$
(d) $V(0,0)$, $S(-\frac{5}{2},0)$, directrix: $x=\frac{5}{2}$
8(a) $x^2=20y$ **(b)** $x^2=-12y$ **(c)** $x^2=8y$
(d) $x^2=-2y$ **(e)** $x^2=4y$ **(f)** $x^2=-\frac{1}{2}y$
9(a) $y^2=2x$ **(b)** $y^2=-4x$ **(c)** $y^2=16x$
(d) $y^2=-8x$ **(e)** $y^2=12x$ **(f)** $y^2=-6x$
10(a) $x^2=16y$ **(b)** $x^2=\frac{1}{2}y$ **(c)** $y^2=2x$
(d) $y^2=-x$
11(a) $x^2=8y$ or $x^2=-8y$
(b) $x^2=12y$, $x^2=-12y$, $y^2=12x$ or $y^2=-12x$
(c) $x^2=y$ or $y^2=x$ **(d)** $y^2=2x$ or $y^2=-2x$
12(a) $k=4$ **(b)** $y=-3x^2$
13(a) $x^2+y^2+8x-8y+2xy=0$
(b) $x^2+y^2-24x+24y+2xy=0$
In both cases, the distance from the focus to the origin equals the distance from the directrix to the origin, being $\sqrt{2}$ and $3\sqrt{2}$ respectively.
15(a) $x^2+z^2=12y$ **(b)** $y^2+z^2=4x$
(c) $x^2+y^2=-8z$ **(d)** $x^2+z^2=-6y$
16(a) focus: $(0,0,-2)$, directrix: $z=2$
(b) focus: $(\frac{1}{2},0,0)$, directrix: $x=-\frac{1}{2}$
(c) focus: $(0,-\frac{1}{4},0)$, directrix: $4y-1=0$

Exercise **9C** (Page 326)

1(b) $(x-3)^2=8(y-1)$
2(a) $(x+7)^2=12(y+5)$ **(b)** $(y-2)^2=4(x+1)$
3 Only the graphs of (a), (d), (h) and (j) have been sketched. The details of all graphs are given afterwards.



(a) vertex: $(0,-1)$, focus: $(0,0)$,

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axis: $x = 0$, directrix: $y = -2$
(b) vertex: $(-2, 0)$, focus: $(-2, 1)$,
axis: $x = -2$, directrix: $y = -1$
(c) vertex: $(3, -5)$, focus: $(3, -3)$,
axis: $x = 3$, directrix: $y = -7$
(d) vertex: $(4, 0)$, focus: $(4, -2)$,
axis: $x = 4$, directrix: $y = 2$
(e) vertex: $(0, -3)$, focus: $(0, -3\frac{1}{2})$,
axis: $x = 0$, directrix: $y = -2\frac{1}{2}$
(f) vertex: $(-5, 3)$, focus: $(-5, 2)$,
axis: $x = -5$, directrix: $y = 4$
(g) vertex: $(-2, 0)$, focus: $(-\frac{1}{2}, 0)$,
axis: $y = 0$, directrix: $x = -1\frac{1}{2}$
(h) vertex: $(0, 1)$, focus: $(4, 1)$,
axis: $y = 1$, directrix: $x = -4$
(i) vertex: $(5, -7)$, focus: $(8, -7)$,
axis: $y = -7$, directrix: $x = 2$
(j) vertex: $(3, -8)$, focus: $(2, -8)$,
axis: $y = -8$, directrix: $x = 4$
(k) vertex: $(-6, 0)$, focus: $(-8\frac{1}{2}, 0)$,
axis: $y = 0$, directrix: $x = -3\frac{1}{2}$
(l) vertex: $(0, 3)$, focus: $(-\frac{1}{2}, 3)$,
axis: $y = 3$, directrix: $x = \frac{1}{2}$

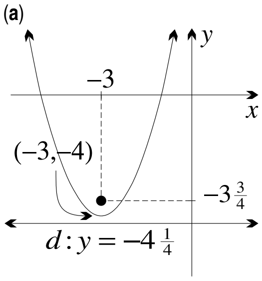
4(a) $(x+2)^2 = 8(y-4)$ **(b)** $(y-1)^2 = 16(x-1)$
(c) $(x-2)^2 = -12(y-2)$ **(d)** $y^2 = -4(x-1)$
(e) $(x+5)^2 = 8(y-2)$ **(f)** $(y+2)^2 = 16(x+7)$
(g) $(x-8)^2 = -12(y+7)$ **(h)** $(y+3)^2 = -8(x+1)$
(i) $(x-6)^2 = 12(y+3)$

5(a) $(x-2)^2 = 8(y+1)$ **(b)** $y^2 = 4(x-1)$
(c) $(x+3)^2 = -8(y-4)$ **(d)** $(y-5)^2 = -12(x-2)$
(e) $(x-3)^2 = 8(y-1)$ **(f)** $(y-2)^2 = 12(x+4)$
(g) $x^2 = -8(y+\frac{3}{2})$ **(h)** $(y+4)^2 = -12(x+1)$
(i) $(x+7)^2 = 2(y+5)$

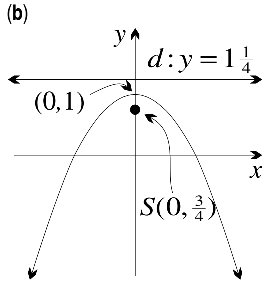
6(a) $x^2 = 8(y-2)$ **(b)** $y^2 = 12(x-3)$
(c) $x^2 = -4(y+1)$ **(d)** $y^2 = -8(x+2)$
(e) $(x-1)^2 = 8(y-5)$ **(f)** $(y+2)^2 = 4(x-2)$
(g) $(x+1)^2 = -2(y-\frac{9}{2})$ **(h)** $(y-\frac{1}{2})^2 = -4(x-4)$
(i) $(x-5)^2 = 10(y+\frac{13}{2})$

7 Only graphs (a) and (b) have been sketched.

(a)



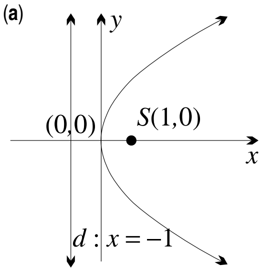
(b)



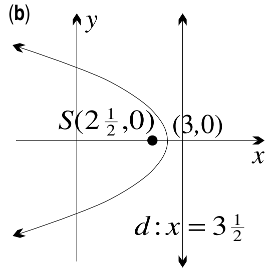
(a) $y + 4 = (x+3)^2$, vertex: $(-3, -4)$,
focus: $(-3, -3\frac{3}{4})$, directrix: $y = -4\frac{1}{4}$
(b) $x^2 = -(y-1)$, vertex: $(0, 1)$,
focus: $(0, \frac{3}{4})$, directrix: $y = \frac{5}{4}$
(c) $(x-6)^2 = 6(y+6)$, vertex: $(6, -6)$,
focus: $(6, -4\frac{1}{2})$, directrix: $y = -7\frac{1}{2}$
(d) $x^2 = 4(y+\frac{1}{2})$, vertex: $(0, -\frac{1}{2})$,
focus: $(0, \frac{1}{2})$, directrix: $y = -\frac{3}{2}$
(e) $(x+3)^2 = y + 25$, vertex: $(-3, -25)$,
focus: $(-3, -24\frac{3}{4})$, directrix: $y = -25\frac{1}{4}$
(f) $(x+4)^2 = 8(y-3)$, vertex: $(-4, 3)$,
focus: $(-4, 5)$, directrix: $y = 1$
(g) $(x-3)^2 = -2(y+1\frac{1}{2})$, vertex: $(3, -1\frac{1}{2})$,
focus: $(3, -2)$, directrix: $y = -1$
(h) $(x-4)^2 = -12(y-1)$, vertex: $(4, 1)$,
focus: $(4, -2)$, directrix: $y = 4$

8 Only graphs (a) and (b) have been sketched.

(a)



(b)



(a) $y^2 = 4x$ vertex: $(0, 0)$,
focus: $(1, 0)$, directrix: $x = -1$
(b) $y^2 = -2(x-3)$ vertex: $(3, 0)$,
focus: $(2\frac{1}{2}, 0)$, directrix: $x = 3\frac{1}{2}$
(c) $y^2 = 6(x-3)$, vertex: $(3, 0)$,
focus: $(4\frac{1}{2}, 0)$, directrix: $x = 1\frac{1}{2}$
(d) $(y-1)^2 = 4(x-1)$, vertex: $(1, 1)$,
focus: $(2, 1)$, directrix: $x = 0$
(e) $(y-2)^2 = 8(x+\frac{1}{2})$, vertex: $(-\frac{1}{2}, 2)$,
focus: $(1\frac{1}{2}, 2)$, directrix: $x = -2\frac{1}{2}$
(f) $(y-3)^2 = 2(x+1)$, vertex: $(-1, 3)$,
focus: $(-\frac{1}{2}, 3)$, directrix: $x = -1\frac{1}{2}$
(g) $(y+2)^2 = -6(x-5)$, vertex: $(5, -2)$,
focus: $(3\frac{1}{2}, -2)$, directrix: $x = 6\frac{1}{2}$
(h) $(y-5)^2 = 12(x+1)$, vertex: $(-1, 5)$,
focus: $(2, 5)$, directrix: $x = -4$

9(a) $y = 2x^2 + 3x - 5$ **(b)** $y = -x^2 - 5x + 1$
(c) $x = y^2 - 4y + 3$ **(d)** $x = -2y^2 + y - 3$

10(a) $(x-1)^2 = 4(y-4)$ **(b)** $(x+2)^2 = -(y-3)$
(c) $(y+2)^2 = 2(x+3)$ **(d)** $(y-5)^2 = -\frac{1}{2}(x-2)$

11(a) $(x-3)^2 = 8(y+1)$ or $(x-3)^2 = -8(y+1)$ or $(y+1)^2 = 8(x-3)$ or $(y+1)^2 = -8(x-3)$

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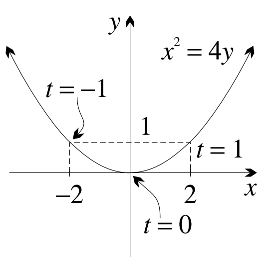
(b) $(y+1)^2 = 8(x+1)$ or $(y+1)^2 = -8(x-3)$
(c) $(x+2)^2 = 4(y-3)$ or $(x+2)^2 = -4(y-5)$
(d) $(y-2)^2 = 6(x-3)$ or $(y-2)^2 = -6(x-3)$
(e) $(x-6)^2 = -20(y-2)$ or $(y+3)^2 = 20(x-1)$
12(a) $(x-3)^2 = y+1$ **(b)** $(y-2)^2 = -\frac{1}{2}(x+4)$
13(a) $x^2+y^2+6x-18y+2xy+33=0$
(b) $9x^2+16y^2-34x-88y-24xy+121=0$
14(a) $(x-1)^2+(y-2)^2 = 8(z-1)$
(b) $(y-2)^2+(z+1)^2 = -4x$
(c) $x^2+(z-3)^2 = -14(y+\frac{3}{2})$
(d) $(x-4)^2+(y+3)^2 = 6(y-\frac{11}{2})$
15(a) $V(1,2,-1)$, $S(1,2,0)$, directrix: $z+2=0$
(b) $V(1,-3,0)$, $S(-1,-3,0)$, directrix: $x=3$

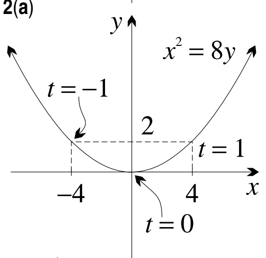
Exercise **9D** (Page 329)

	t	-3	-2	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	2	3
1(a)	x	-6	-4	-2	-1	0	1	2	4	6
	y	9	4	1	$\frac{1}{4}$	0	$\frac{1}{4}$	1	4	9

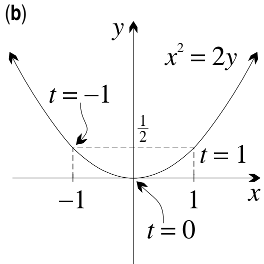
(b) $x^2 = 4y$ **(c)** $(0,0)$, $(0,1)$ **(d)** $t=0$
(e) $(2,1)$, $(-2,1)$, $t=1$ or -1



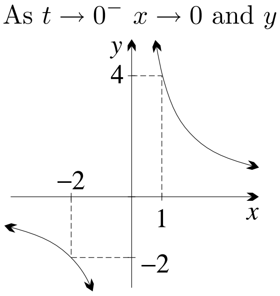
2(a)



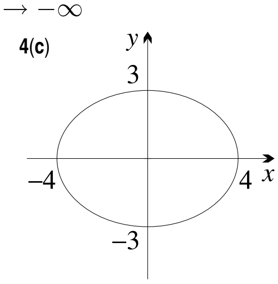
(b)



3(c) As $t \to \infty$, $x \to \infty$ and $y \to 0$.
As $t \to -\infty$, $x \to -\infty$ and $y \to 0$.
As $t \to 0^+$, $x \to 0$ and $y \to \infty$.
As $t \to 0^-$ $x \to 0$ and $y \to -\infty$



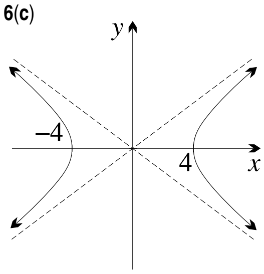
4(c)



$$\frac{x^2}{16}+\frac{y^2}{9}=1$$

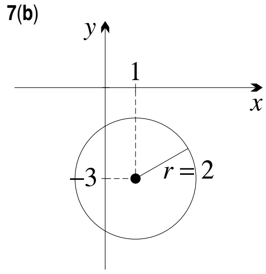
5(a) $2x+y-7=0$ **(b)** $4(y+4)^2-9(x-1)^2=36$
(c) $y=x^2-2$ **(d)** $x^2+y^2=2$

6(c)

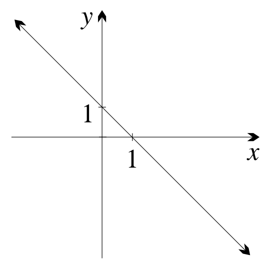


$$\frac{x^2}{16}-\frac{y^2}{9}=1$$

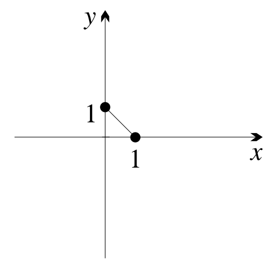
7(b)



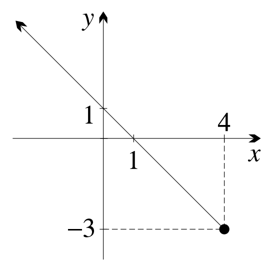
8(a)



(b)



(c)



9(a) $(x-3)^2+(y+2)^2 = r^2$,
circle with centre $(3,-2)$ and radius r
(b) $y = x\tan\theta - (3\tan\theta+2)$,
straight line with gradient $\tan\theta$
10(b) P_1 has parameter $\lambda = 0$. As λ moves from 0 to ∞, P moves from P_1 to P_2. As λ moves from $-\infty$ to -1, P moves from P_2 infinitely far along the line. As λ moves from 0 to -1, P moves from P_1 infinitely far along the line in the other direction. **(c)** $\frac{3}{10}$
(d)(i) $({x_2}^2-4ay_2)\lambda^2 + 2(x_1x_2-2ay_1-2ay_2)\lambda + ({x_1}^2-4ay_1) = 0$
(ii) $(x_1-x_2)(x_1y_2-x_2y_1)+a(y_1-y_2)^2 = 0$

Exercise **9E** (Page 332)

1(a) $x+y-3=0$ **(b)** $4y-5x+8=0$
(c) $3x+2y+2=0$ **(d)** $y-x-2=0$
2(a) $4y-3x-12=0$ **(b)** $x^2=12y$, $(0,3)$
3(d) $(0,a)$ **(f)** $(-\dfrac{4a}{p+q}, -a)$

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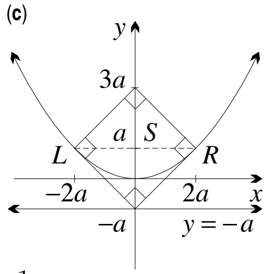
4(a) $P(2ap, ap^2)$, $Q(2aq, aq^2)$, $S(0, a)$
(b) $a(p^2+1)$ **(c)** $a(q^2+1)$
5(a) The x-intercept is $(4a, 0)$.
6(a) $t=-2$ or $t=1$ **(b)** $t=-3$ or $t=1$
8(a) Since $a=2$, the line is $y=\frac{1}{2}\times 4x-(-5)a$, so $p+q=4$ and $pq=-5$. $t=5$ gives $(20, 50)$, and $t=-1$ gives $(-4, 2)$.
(b) The point of contact is $(8, 8)$.
(c) $p+q=4$, $pq=5$ has no solutions.
9 The midpoint lies of $x=k$ if and only if k is the average of $2ap$ and $2aq$.

Exercise 9F (Page 336)

1(a) $y=x-1$ **(b)** $x-2y-1=0$
(c) $6x+2y+9=0$ **(d)** $y=qx-3q^2$
2(a) $x+2y-12=0$ **(b)** $4x-2y+9=0$
(c) $x+my-3m^3-6m=0$
(d) $x+qy-aq^3-2aq=0$
3(a) $y=px-ap^2$ **(b)** $(0, -ap^2)$, $(ap, 0)$ **(c)** $\frac{1}{2}a^2|p|^3$
4(a) $x+py=2ap+ap^3$
(b) $(2ap+ap^3, 0)$, $(0, 2a+ap^2)$ **(c)** $\frac{1}{2}a^2|p|(p^2+2)^2$
5(a) tangents: $y=x-a$, $y=-x-a$; normals: $y=-x+3a$, $y=x+3a$
(b) vertices: $(-2a, a)$, $(2a, a)$, $(0, 3a)$, $(0, -a)$; area $=8a^2$ (half product of the diagonals)
(c)



6(a) $y=tx-t^2$ **(b)** 3 or -1
(c) $y=3x-9$ and $x+y+1=0$
7(a) parameters: 5 and $-\frac{1}{5}$ **(b)** gradients: 5 and $-\frac{1}{5}$; points of contact: $(50, 125)$ and $(-2, \frac{1}{5})$
8(a) $A=(-1, 1)$, $B=(2, 4)$ **(b)** at A, $y=-2x-1$; at B, $y=4x-4$ **(c)** $M=(1\frac{1}{4}, 1)$, $D=(\frac{1}{2}, \frac{1}{4})$, AB and the tangent at D both have gradient 1.
9(a) $t=1$, point of contact: $(6, 3)$
(b) $\sqrt{5}$ **(d)** $k=2$ or -2
10(b) $y+x=3a$, $y-x=3a$
11(b) $(0, 0)$, $(24, 36)$, $(-24, 36)$
14(a) $(0, -ap^2)$ **(b)** $(0, 2a+ap^2)$
15(a) $(2ap, -a)$
16(a) $M=\left(a(p+q), \frac{1}{2}a(p^2+q^2)\right)$, T has parameter $\frac{1}{2}(p+q)$ and $T=\left(a(p+q), \frac{1}{4}a(p+q)^2\right)$, $M=\left(a(p+q), apq\right)$. **(d)** $2:1$
17 When the two points are on the same side of the axis, it is the positive geometric mean, and when they are on opposite sides of the axis, it is the negative geometric mean.
19(b) $\frac{1}{2}a^2|p-q|^3$

Exercise 9G (Page 339)

1(a) $y=x-1$ **(b)** $y=2x-3$ **(c)** $y=-4x+4$
(d) $y=4x-7$
2(a) $x+2y-3=0$ **(b)** $x-3y+33=0$
(c) $x-3y-18=0$ **(d)** $x+7y-21=0$
3(a) $(6, 9)$ **(b)** $(\frac{1}{2}, \frac{23}{4})$
4(a) $x+y=3$, $x+2y=12$ **(b)** $(-6, 9)$
5(b) tangents: $y+x+a=0$, $y-x+a=0$; normals: $x+y=3a$, $x=y-3a$ **(c)** $8a^2$ square units
6(a) $y=\frac{1}{3}x^2+\frac{2}{3}$ **(b)** $2x-3y+1=0$, $3x+2y-5=0$
7(a) $(2, 10)$ and $(-\frac{4}{5}, \frac{8}{5})$
(b) $y=10x-10$ and $y=-4x-\frac{8}{5}$ **(c)** $(\frac{3}{5}, -4)$
9(b) $m=2$ gives $y=2x-2$, $m=-2$ gives $y=-2x-2$.
(c) $y=x-2$ and $y=-x-2$, which are perpendicular because their gradients are 1 and -1.
10(a) $b=-6$ **(b)** $y=-2x-6$ **(c)(i)** $y+2x+4=0$
(ii) $16y-8x+9=0$
11 $a=-\frac{1}{4}$, $(-1, 1)$
12(b) $y=7x-147$
13(b) $y=x+1$ and $y=-2x+4$
14(b) Since $\Delta=3^2+4$ is positive, there are two possible gradients. Since the product of the roots is -1, these gradients are perpendicular.
15(b) $3x-y-27=0$ and $3x+y+27=0$
(c) $y=x-3$ and $x-3y=1$

Exercise 9H (Page 342)

1(a) $y=2$ **(b)** $3x-2y=0$ **(c)** $x+y-1=0$
(d) $2x-y+6=0$
2(a) $y=3$ **(b)** $x+3y=0$ **(c)** $y=x-1$ **(d)** $5x+6y-24=0$
3(a) $x=2y$ **(b)** $(0, 0)$, $(4, 2)$ **(c)** $y=0$, $y=x-2$
4(a) $(0, 2)$
(b) $x-4y+8=0$, $3x-4y+8=0$, $x+y-2=0$
5(a) $y_0y=2a(x+x_0)$ **(b)** The point $(-5, 2)$ lies on the directrix $x=-5$ of the parabola.
6(a) $y=x+1$ **(c)** 4 **(d)** $(2, 3)$
8(a)(i) $x=2y$ **(ii)** $x-4y+4=0$
9(a) $x_0x=2a(y+y_0)$ **(b)** $x^2-2x_0x+4ay_0=0$
(c) $(x_0, \frac{{x_0}^2}{2a}-y_0)$
10(a) $x_0x=2a(y+y_0)$
11(a) $y=\frac{x_0}{2a}x-y_0$

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12(a) $x^2 - 2x - 1 = 0$, sum is 2, product is -1
(b) $y^2 - 6y + 1 = 0$, sum is 6, product is 1
(d) $2\sqrt{10}$ units
13 $10\sqrt{2}$ units
14(c) ${x_0}^2 > 4ay_0$, which is the condition for the point (x_0, y_0) to lie outside the parabola.
15(a) $x_1 x = 2a(y + y_1)$
16 $x_0 = 2am$
18(d) $4x + 5y = 25$, $(0, 5)$ and $(\frac{200}{41}, \frac{45}{41})$
19(a) $xy_0 + x_0 y = 2c^2$
(c) $8x + 2y = 50$, $(5, 5)$ and $(\frac{5}{4}, 20)$

Exercise 9I (Page 346)

1(a) $y = qx - aq^2$ **(e)** $N\left(a(p+q), \frac{1}{4}a(p+q)^2\right)$
2(c) q
3(b) $B(0, at^2)$, $N(0, 2a + at^2)$ **(c)** $BN = 2a$ units
5(a) $T(2at, -a)$ **(e)** A rhombus, because the diagonals bisect each other at right angles.
6(a) $A(2at + at^3, 0)$, $B(0, 2a + at^2)$
7(a) It is a focal chord. **(b)** $y = qx - aq^2$
8(b) $y = tx - at^2$ **(c)** $R(0, -at^2)$
9(a) $A\Big(a(p+q), apq\Big)$
12(a) (x_1, y_1) satisfies $x^2 = 4ay$.
(c) $Q(\frac{2ay_1}{x_1}, 0)$, $R(0, -y_1)$
15(b) $Q\left(-2a(p + \frac{2}{p}), a(p + \frac{2}{p})^2\right)$
16(a) $M = (p, 1 + p^2)$
(b) $P = (2\sqrt{3}, 3)$ or $P = (-2\sqrt{3}, 3)$
18(b) $p^2 + q^2 + 2$
19 $P = (4, 4)$, $Q = (-6, 9)$, $T = (-1, -6)$
22(a) $q = (2\sqrt{2} - 3)p$ or $(-2\sqrt{2} - 3)p$
23(a) $K = (\frac{kx_1 + x_0}{k+1}, \frac{ky_1 + y_0}{k+1})$
(b) $k^2({x_1}^2 - 4ay_1) + ({x_0}^2 - 4ay_0) = 0$

Exercise 9J (Page 352)

1(b) $x = at$, $y = \frac{at^2}{2}$ **(d)** A parabola, axis the y-axis, vertex the origin, focal length $\frac{a}{2}$.
2(b) $T(at, 0)$ **(c)** $M(\frac{3at}{2}, \frac{at^2}{2})$
3(c) The locus of R is $x^2 = 2(y - 2)$.
4(b) $M(\frac{1}{2}at, a + \frac{1}{2}at^2)$
6(a) $Q(2at, -a)$ **(c)** $x^2 = 2a(y - a)$. A parabola, axis the y-axis, vertex $(0, a)$, focal length $\frac{1}{2}a$.
7(a) $y = px + a$ **(c)** $y = \frac{1}{2}a$ **(e)** $x^2 = a(y - a)$
8(a) $N(\frac{1}{2}at^2, \frac{3}{2}at)$
9(d) A parabola, axis the y-axis, vertex $(0, 4a)$, focal length $\frac{1}{2}a$.
10(b) $T(at, 0)$ **(c)** $y = 0$ (the x-axis)
12(a) $y = px - ap^2$ **(b)** $y = px$ **(d)** $y = 2px - 4ap^2$
(f) $2x^2 = 9ay$
13(a) $M\Big(a(p+q), \frac{1}{2}a(p^2 + q^2)\Big)$, $x^2 = 4a(y - \frac{a}{4})$
(b) A parabola, axis the y-axis, vertex $(0, \frac{a}{4})$, focal length a.
14(c) $y = -4a$ **(e)** $x^2 = 16a(y - 6a)$
15(a) $y = px - ap^2$, $px - p^2 y - a = 0$
(b) $y = a$, for $x > 2a$ or $x < -2a$
16(c) $(4(p+q), 4pq)$
17(c) $x^2 = a(y - 3a)$
18(a) $x + py = 2p + p^3$ **(d)** $N(2m, 4m^2 + 3)$
(e) $y = x^2 + 3$
20(c) $x = 3$ and $y < \frac{3}{4}$
(d) The points P and Q coincide, in which case T is not uniquely defined, but the limit of its position coincides with P and Q.
21(c) $x^2 = 8ay$
22(b) $x^2 = 4a(y - 4a)$
(d) 3, since the cubic equation in part (c) has at most three real solutions for t.
23(c) $y = \frac{1}{4}x^2 - x + 2$
24(d) $N(2mk, 2a + k + 4am^2)$
(e) $y = \frac{x}{2m} + 2a(1 + 2m^2)$
26(c) $y = -a$, which is the directrix.

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Chapter Ten

Exercise **10A** (Page 359)

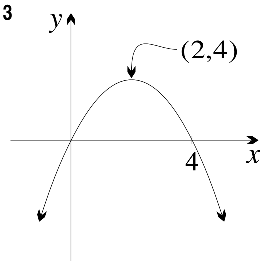
1(a) A, G and I **(b)** C and E **(c)** B, D, F and H

3(a) $4-2x$ **(b)(i)** $x<2$ **(ii)** $x>2$ **(iii)** $x=2$

4(a) $3x^2-6x$ **(b)(i)** $x<0$ or $x>2$ **(ii)** $0<x<2$

(iii) $x=0$ or $x=2$

3



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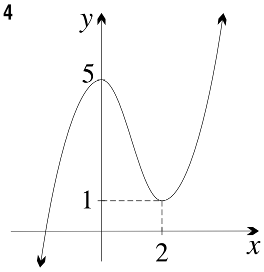


5(a) $\dfrac{3}{x^2}$

(b) The function is not continuous at $x=0$.

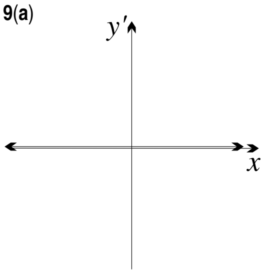
6(a) $x>2$ **(b)** $x<-3$ **(c)** $x>1$ or $x<-1$

(d) $x<0$ or $x>2$

7(a) $x<-1$ or $x>-\frac{1}{3}$ **(b)** $x<-2$ or $0<x<2$

8(a) III **(b)** I **(c)** IV **(d)** II

9(a)



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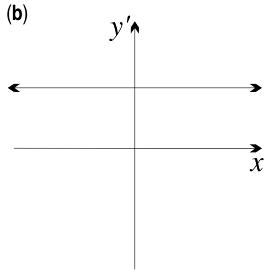

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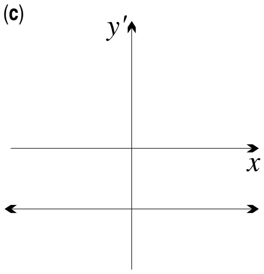

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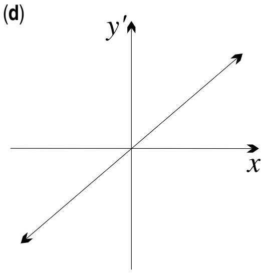

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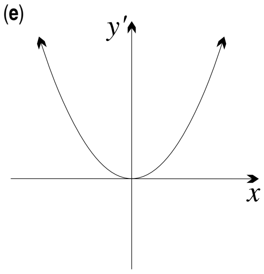

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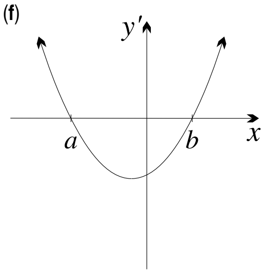

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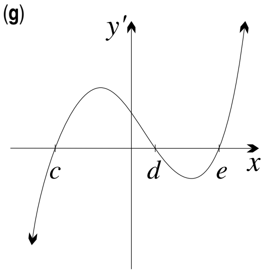

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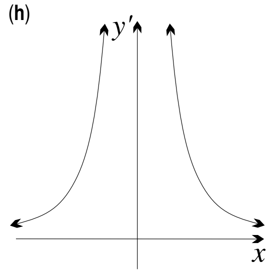


10(a) $-\dfrac{6}{(x-3)^2}$ **(b)** $\dfrac{x^2(x^2+3)}{(x^2+1)^2}$

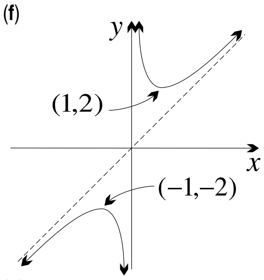
11(a) x^2+2x+5

13(b) 1

14(c)(i) $x<-1$ or $x>1$ **(ii)** $-1<x<1$, $x\neq 0$

(iii) $x=-1$ or $x=1$ **(d)** $2, -2$ **(e)** $x=0$

(f)



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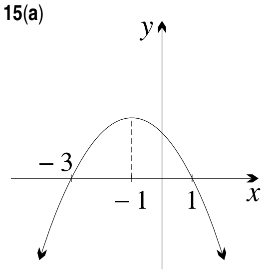

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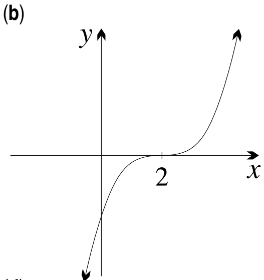

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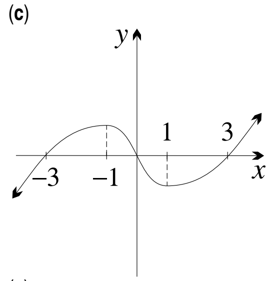

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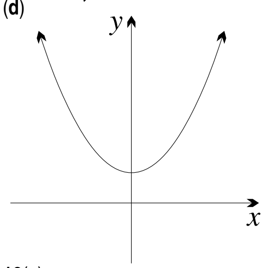

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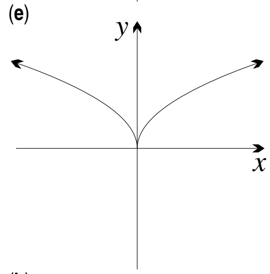

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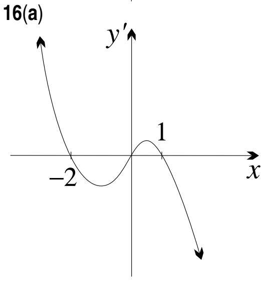

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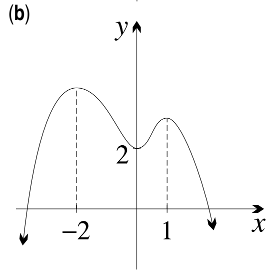

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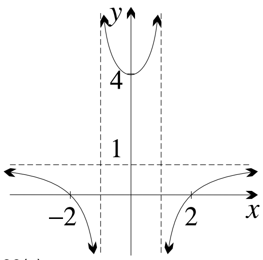
17

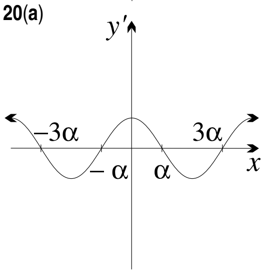


18 $-2 < x < 0$

19(a)(i) $\dfrac{-4x}{(x^2+1)^2}$

(b) $f(x)$ is increasing for $x < 0$ and decreasing for $x > 0$.

20(a)



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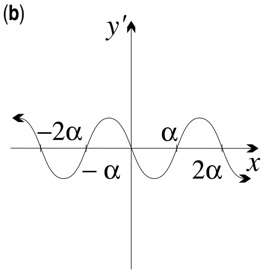

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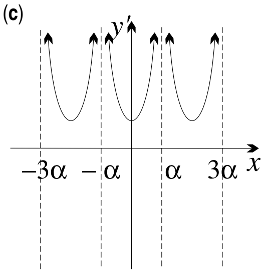

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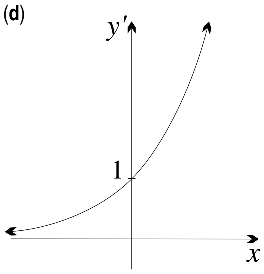
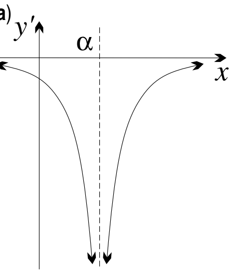


21(a)



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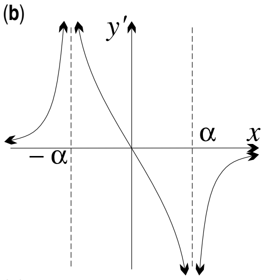

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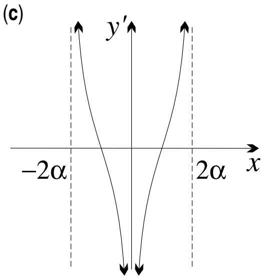

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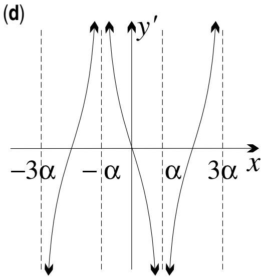
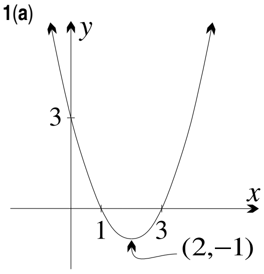


Exercise **10B** (Page 365)

1(a)



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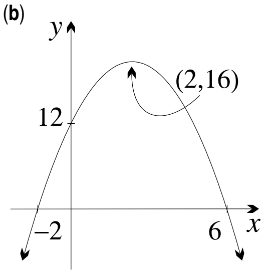

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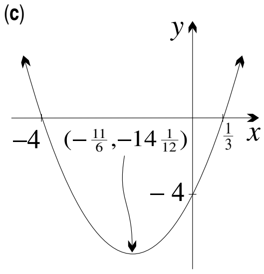

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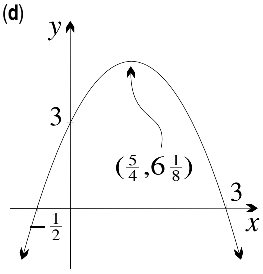

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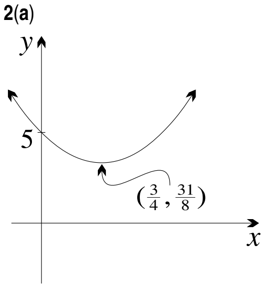

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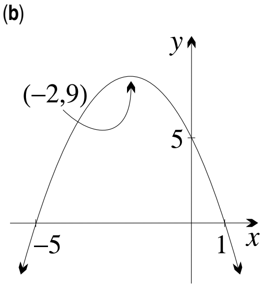

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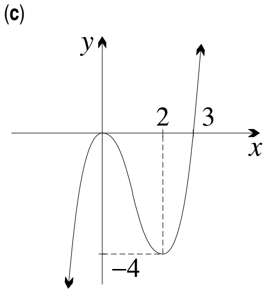

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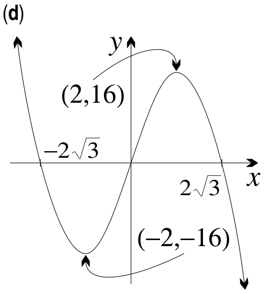

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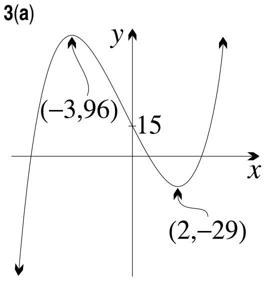

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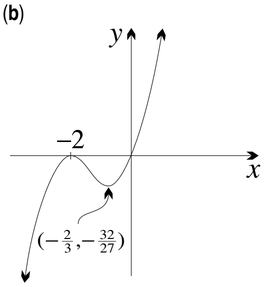
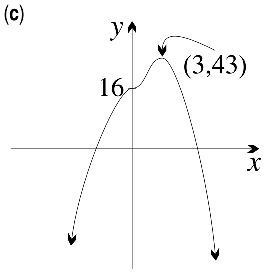
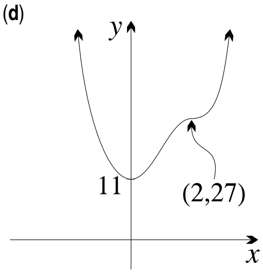


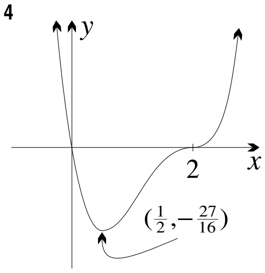
(c)



(d)



4



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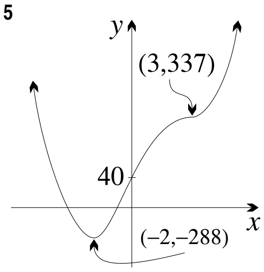

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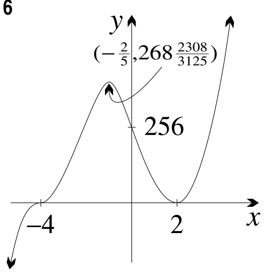
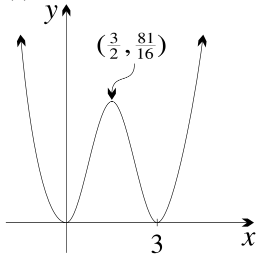
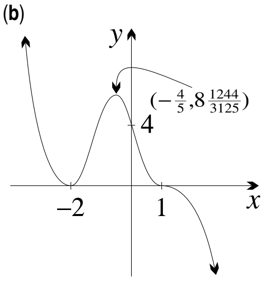
6



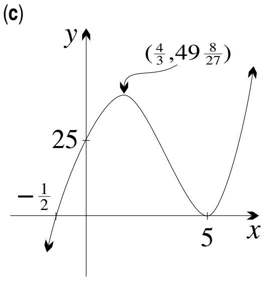
7(a)



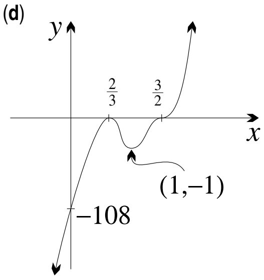
(b)



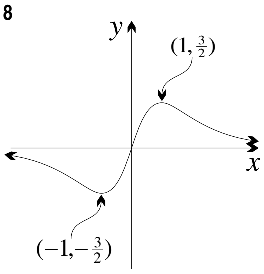
(c)



(d)



8



(d)(i) 0 **(ii)** 1 distinct root **(iii)** 2 **(iv)** 1

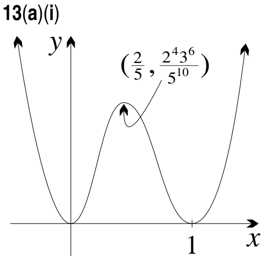
9 -8

10 $a = b = -1$, $c = 6$

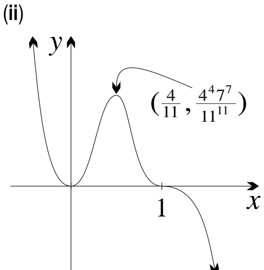
11 $a = -1$, $b = 2$, $c = 0$

12 $a = 2$, $b = 3$, $c = -12\, d = 7$

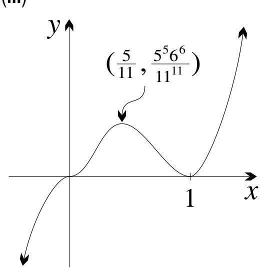
13(a)(i)



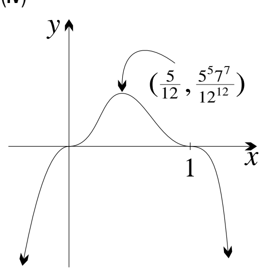
(ii)



(iii)



(iv)



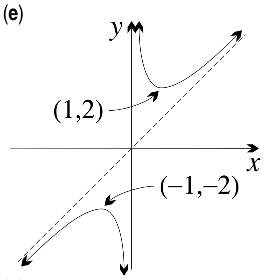
Exercise 10C (Page 369)

1(a) A relative maximum, B relative minimum
(b) C relative minimum **(c)** D horizontal point of inflexion, E relative maximum
(d) F relative minimum, G relative maximum, H relative minimum **(e)** I relative minimum
(f) J horizontal point of inflexion, K relative minimum, L relative maximum

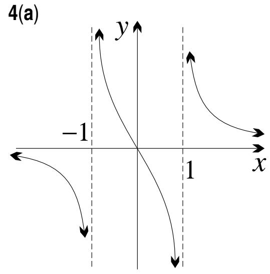
2(a) $x = 1$ turning point
(b) $x = 3$ turning point, $x = -\frac{1}{2}$ turning point
(c) $x = 0$ turning point, $x = 3$ horizontal point of inflexion
(d) $x = -2$ turning point, $x = 4$ turning point
(e) $x = 0$ turning point, $x = 1$ critical value
(f) $x = 0$ horizontal point of inflexion, $x = 1$ critical value
(g) $x = 0$ turning point, $x = 1$ critical value
(h) $x = 0$ horizontal point of inflexion, $x = 1$ critical value **(i)** $x = 0$ critical value
(j) $x = -1$ turning point, $x = 1$ turning point, $x = 0$ critical value
(k) $x = 1$ turning point, $x = 0$ critical value
(l) $x = 2$ turning point, $x = -2$ critical value, $x = 1$ critical value

3(a) $x \neq 0$

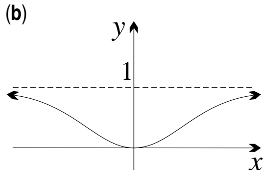
(e)



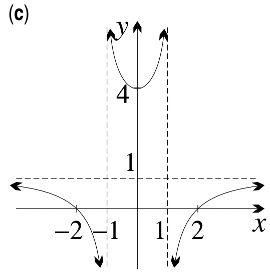
4(a)



(b)



(c)



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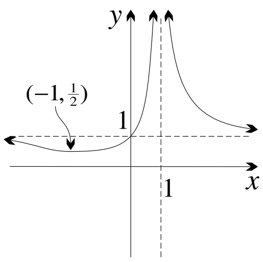
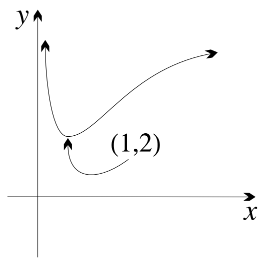
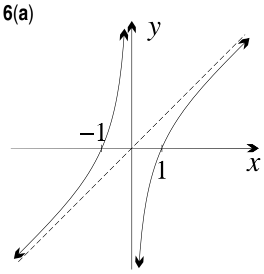
(d)



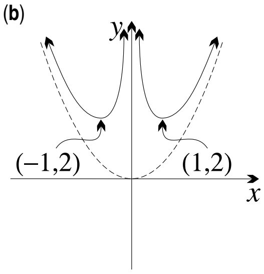
5(a) $x > 0$



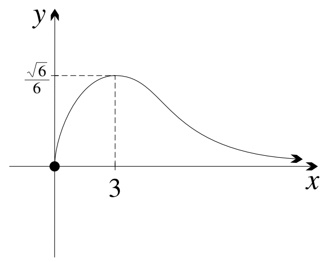
6(a)



(b)

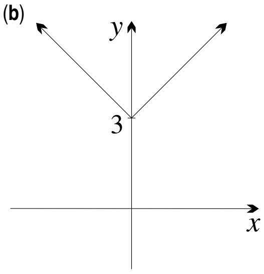


7

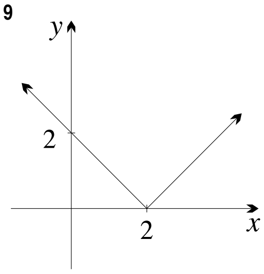


(a) domain: $x \geq 0$.
horizontal asymptotes: $x = 0$
(c) $(3, \frac{1}{6}\sqrt{6})$ is a maximum turning point.
(d) As $x \to 0^+$, $y \to 0$ and $y' \to \infty$, so the curve emerges vertically from the origin. (Notice that $y(0) = 0$, so the origin lies on the curve.)
8(a) When $x < 0$, $dy/dx = -1$.
When $x > 0$, $dy/dx = 1$.

(b)

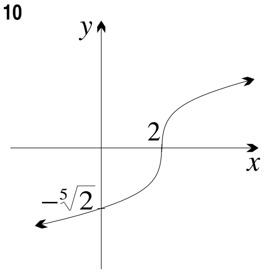


9

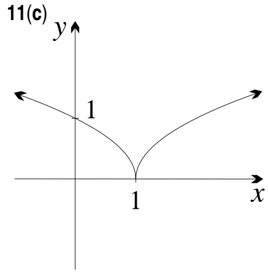


(a) When $x > 2$, $dy/dx = 1$.
When $x < 2$, $dy/dx = -1$ **(b)** $x = 2$ is a critical value, because y' is not defined there.

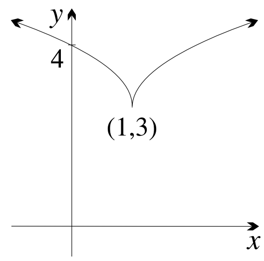
10



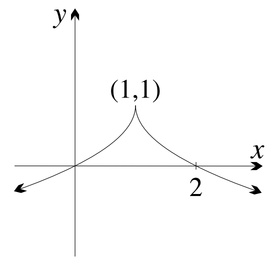
11(c)



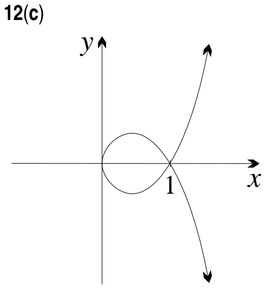
(d)(i)



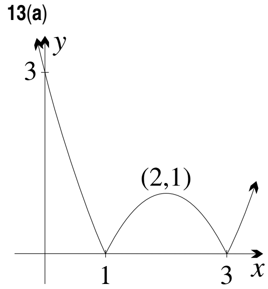
(ii)



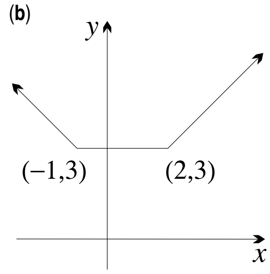
12(c)



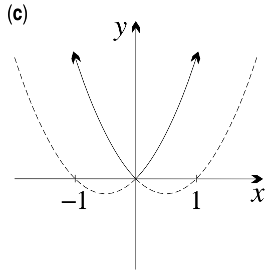
13(a)



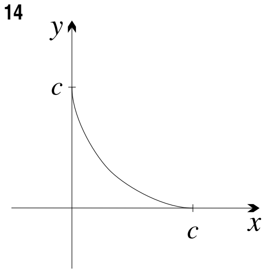
(b)



(c)



14



Exercise **10D** (Page 371)

1(a) $10x^9$, $90x^8$, $720x^7$ **(b)** $15x^4$, $60x^3$, $180x^2$
(c) -3, 0, 0 **(d)** $2x - 3$, 2, 0
(e) $12x^2 - 2x$, $24x - 2$, 24
(f) $0{\cdot}3x^{-0{\cdot}7}$, $-0{\cdot}21x^{-1{\cdot}7}$, $0{\cdot}357x^{-2{\cdot}7}$
(g) $-\dfrac{1}{x^2}$, $\dfrac{2}{x^3}$, $-\dfrac{6}{x^4}$ **(h)** $-\dfrac{2}{x^3}$, $\dfrac{6}{x^4}$, $-\dfrac{24}{x^5}$
(i) $-\dfrac{15}{x^4}$, $\dfrac{60}{x^5}$, $-\dfrac{300}{x^6}$ **(j)** $2x - \dfrac{1}{x^2}$, $2 + \dfrac{2}{x^3}$, $-\dfrac{6}{x^4}$

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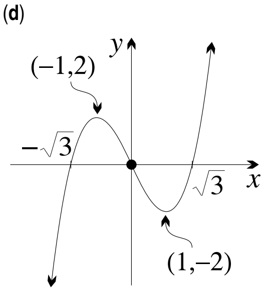
2(a) $2(x+1),\ 2$ **(b)** $9(3x-5)^2,\ 54(3x-5)$
(c) $8(4x-1),\ 32$ **(d)** $-11(8-x)^{10},\ 110(8-x)^9$
3(a) $\dfrac{-1}{(x+2)^2},\ \dfrac{2}{(x+2)^3}$ **(b)** $\dfrac{2}{(3-x)^3},\ \dfrac{6}{(3-x)^4}$
(c) $\dfrac{-15}{(5x+4)^4},\ \dfrac{300}{(5x+4)^5}$
(d) $\dfrac{12}{(4-3x)^3},\ \dfrac{108}{(4-3x)^4}$
4(a) $\dfrac{1}{2\sqrt{x}},\ \dfrac{-1}{4x\sqrt{x}}$ **(b)** $\frac{1}{3}x^{-\frac{2}{3}},\ -\frac{2}{9}x^{-\frac{5}{3}}$
(c) $\frac{3}{2}\sqrt{x},\ \dfrac{3}{4\sqrt{x}}$ **(d)** $-\frac{1}{2}x^{-\frac{3}{2}},\ \frac{3}{4}x^{-\frac{5}{2}}$
(e) $\dfrac{1}{2\sqrt{x+2}},\ \dfrac{-1}{4(x+2)^{\frac{3}{2}}}$ **(f)** $\dfrac{-2}{\sqrt{1-4x}},\ \dfrac{-4}{(1-4x)^{\frac{3}{2}}}$
5(a)(i) $2\frac{13}{16}$ **(ii)** $\frac{3}{8}$
(b)(i) -8 **(ii)** 48 **(iii)** -192 **(iv)** 384
6 $a=-\frac{1}{2},\ b=3,\ c=\frac{5}{2}$
7(a) $\dfrac{1}{(x+1)^2},\ \dfrac{-2}{(x+1)^3}$ **(b)** $\dfrac{7}{(2x+5)^2},\ \dfrac{-28}{(2x+5)^3}$
(c) $\dfrac{1-x^2}{(1+x^2)^2},\ \dfrac{2x(x^2-3)}{(1+x^2)^3}$
8 $(x-1)^3(5x-1),\ 4(x-1)^2(5x-2)$
9(a) $1,-1$ **(b)** $-\frac{1}{3}$ **(c)** $-\frac{4}{3}$
11(a) $nx^{n-1},\ n(n-1)x^{n-2},\ n(n-1)(n-2)x^{n-3}$
(b) $n(n-1)(n-2)\dots 1,\ 0$
12(a) $\frac{2}{3},\ 0$ **(b)** $\dfrac{-1}{2t^2},\ \dfrac{1}{2t^3}$ **(c)** $-\frac{3}{5}t^2,\ \frac{6}{25}t$
(d) $\dfrac{t^2-1}{1+t^2},\ \dfrac{4t^3}{(1+t^2)^3}$ **(e)** $\dfrac{3}{2(t-2)},\ \dfrac{-3}{4(t-2)^3}$
(f) $-\dfrac{(1-t)^2}{1+t)^2},\ \dfrac{2(1-t)^3}{(1+t)^3}$
13 $a=3,\ b=4$
14 $\dfrac{1}{r}$

Exercise **10E** (Page 376)

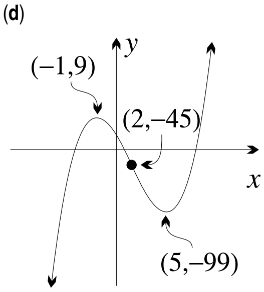
1

Point	A	B	C	D	E	F	G	H	I
y'	0	+	0	−	0	−	0	+	0
y''	+	0	−	0	0	0	+	0	0

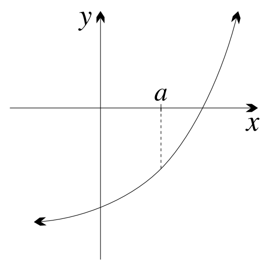
3(a) $f'(x)=3x^2-3$, $f''(x)=6x$
(d)



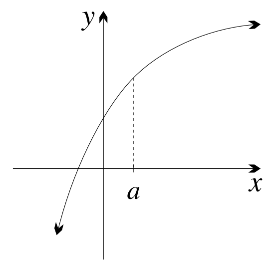
4(a) $f'(x)=3x^2-12x-15$, $f''(x)=6x-12$
(d)



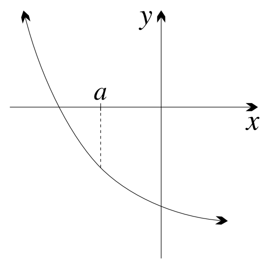
5(a) $x>2$ or $x<-1$ **(b)** $-1<x<2$ **(c)** $x>\frac{1}{2}$
(d) $x<\frac{1}{2}$
6(a) $x=-5$ **(b)** none **(c)** $x=3,\ x=-2$
(d) $x=-2$
7(a)



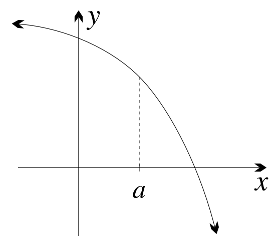
(b)



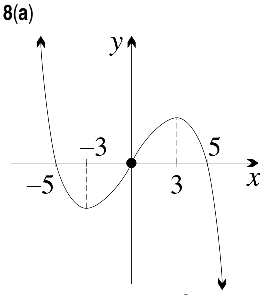
(c)



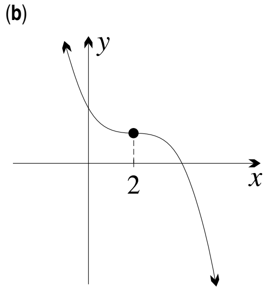
(d)



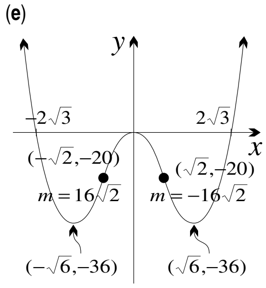
8(a)



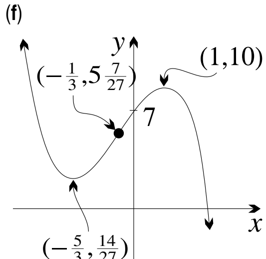
(b)



10(a) $f'(x)=4x^3-24x$, $f''(x)=12x^2-24$
(e)



11(a) $f'(x)=5-2x-3x^2$, $f''(x)=-2-6x$
(e) $\frac{16}{3}$
(f)



12(a) $y'=3x^2+6x-72,\ y''=6x+6$
(d) $75x+y-13=0$
13(b) $f''(x)=g''(x)=0$, no
(c) $f(x)$ has a horizontal point of inflexion,

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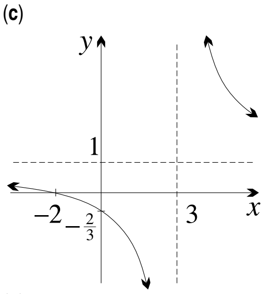
$g(x)$ has a minimum turning point.

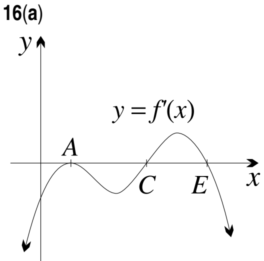
14 $a = 2$, $b = -3$, $c = 0$ and $d = 5$.

15(b) concave up when $x > 3$, concave down when $x < 3$

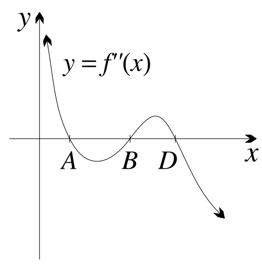
(c)



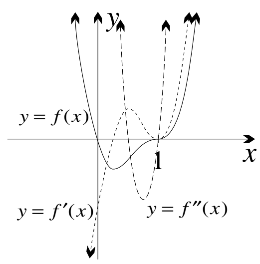
16(a)



(b)

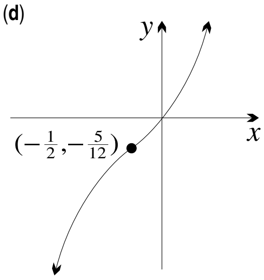


17(b)

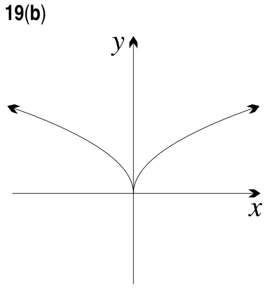


18(a) $f'(x) = x^2 + x + 1$, $f''(x) = 2x + 1$

(d)

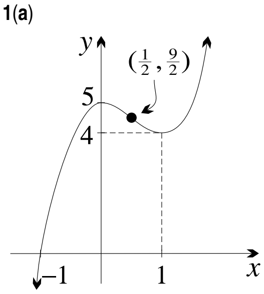


19(b)

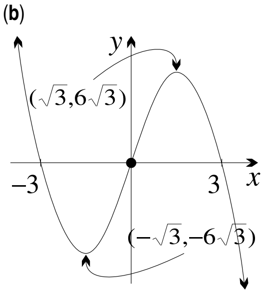


Exercise **10F** (Page 379)

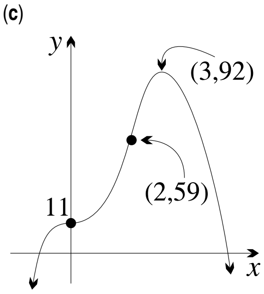
1(a)



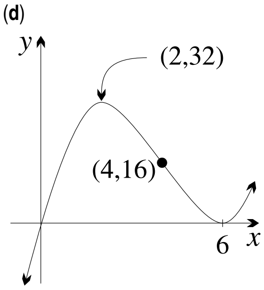
(b)



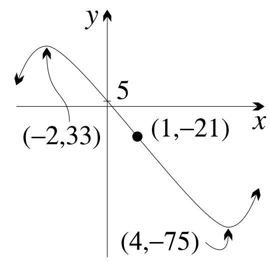
(c)



(d)



(e)



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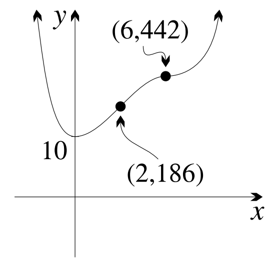

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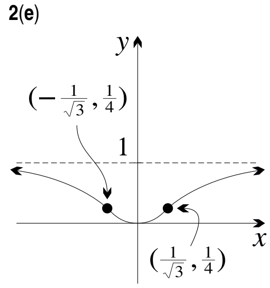

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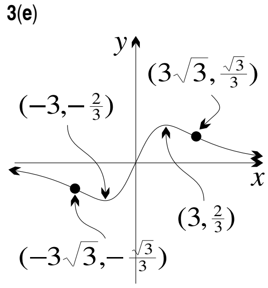

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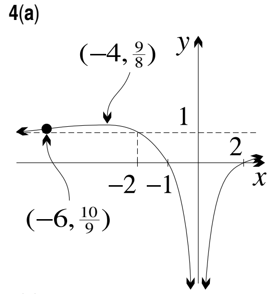

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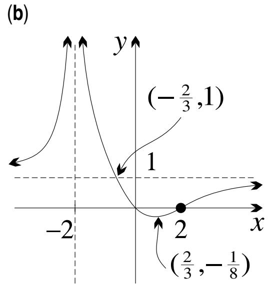

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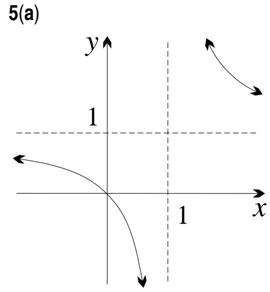

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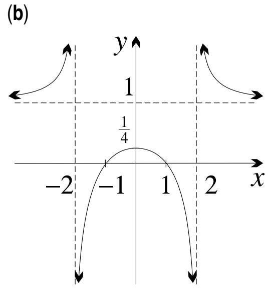

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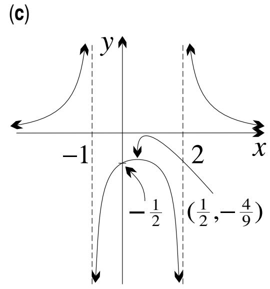

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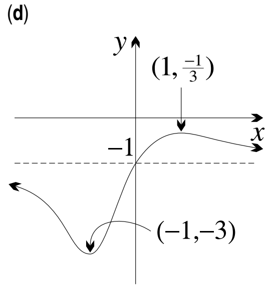

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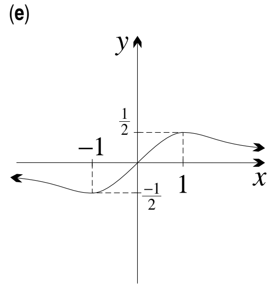

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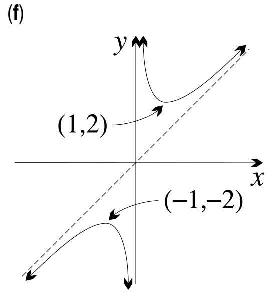

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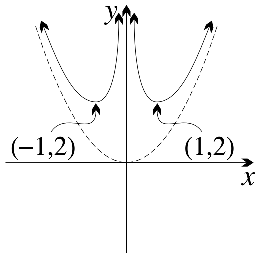
(g)



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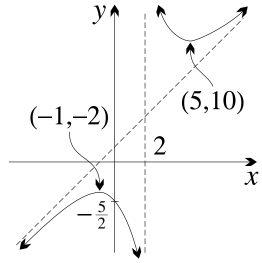

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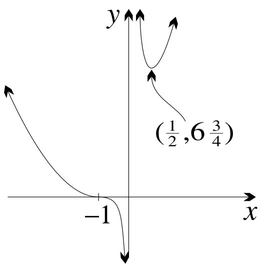

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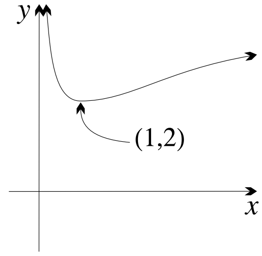

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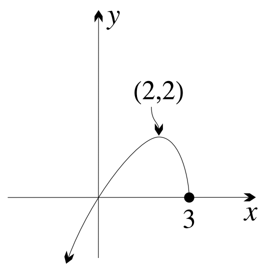

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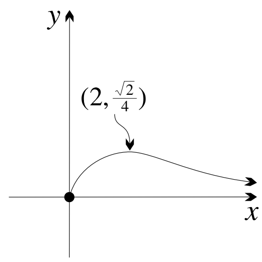

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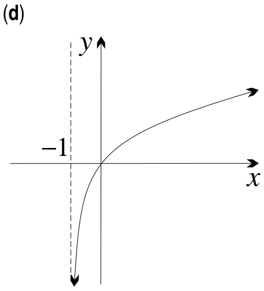
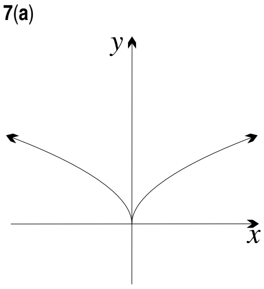


7(a)



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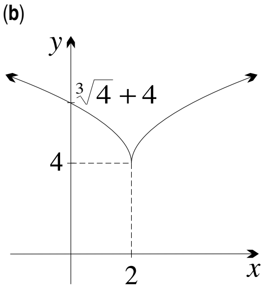

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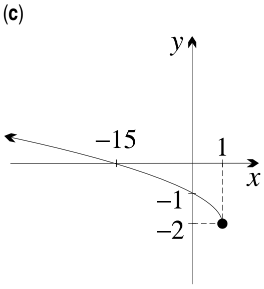

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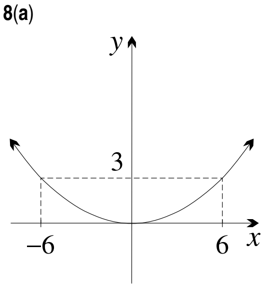

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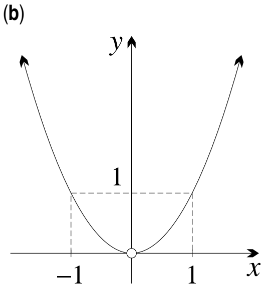

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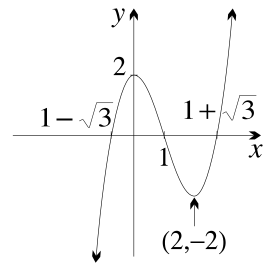
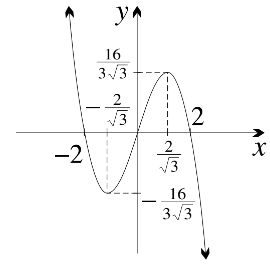
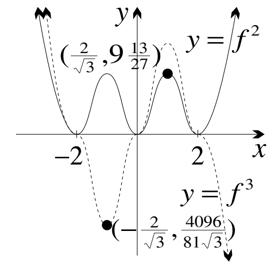


9(a)

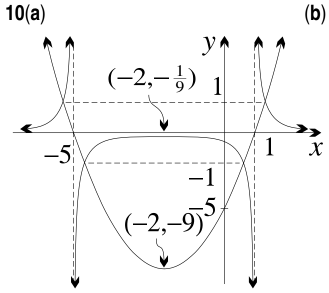


(b) The x-intercepts and the x-coordinates of the stationary points of $y = f(x)$ give the stationary points of $y = (f(x))^2$.

(c)

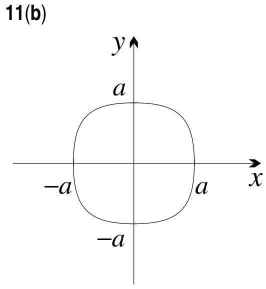


10(a)



(b) $(-2+2\sqrt{2},-1)$, $(-2-2\sqrt{2},-1)$, $(-2+\sqrt{10},1)$, $(-2-\sqrt{10},1)$

11(b)



12(b) $x > \frac{2}{3}$ or $x < 0$

Exercise **10G** (Page 382)

1(a) A relative maximum, B relative minimum **(b)** C absolute maximum, D relative minimum, E relative maximum, F absolute minimum **(c)** G absolute maximum, H horizontal point of inflexion **(d)** I horizontal point of inflexion, J absolute minimum

2(a) $0,\ 4$ **(b)** $2,\ 5$ **(c)** $0,\ 4$ **(d)** $0,\ 5$ **(e)** $0,\ 2\sqrt{2}$ **(f)** $-1,\ -\frac{1}{4}$ **(g)** $1,\ 2$ **(h)** $-1,\ 2$

3(a) $7,\ 7$ **(b)** $\frac{1}{16},\ \frac{1}{9}$ **(c)** $-1,\ 8$ **(d)** $-49,\ 5$ **(e)** $0,\ 4$ **(f)** $0,\ 9$

4(a) global minimum -5, global maximum 20 **(b)** global minimum -5, local maximum 11, global maximum 139

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(c) global minimum 4, global maximum 11
5(a) $1,\ 5$ **(b)** $1,\ \sqrt{5}$ **(c)** $\frac{1}{5}\sqrt{5},\ 1$ **(d)** $-\frac{2}{5}\sqrt{5},\ \frac{1}{2}\sqrt{2}$
7(a) $2,\ 19,\ 181$ **(b)** infinitely many
(c) As $x \to \infty$, $y \to 0$. There is no limit as $x \to 0^+$ — however close one goes towards 0, the function takes every value in the interval $-1 \le y \le 1$.
(d) The global maximum and minimum are 1 and -1 respectively, and they are attained infinitely often.

Exercise **10H** (Page 384)

1(a) $2x^2 - 16x + 64$ **(b)** 4 **(c)** 32
2(a) $11x - 2x^2$ **(b)** $\frac{11}{4}$ **(c)** $\frac{121}{8}$
3(b) $3\frac{1}{8}$ metres
4(b) 5 **(c)** $25\,\text{cm}^2$
5(c) 20 **(d)** $200\,\text{m}^2$
6(a) $R = x(47 - \frac{1}{3}x)$ **(b)** $-\frac{8}{15}x^2 + 32x - 10$ **(c)** 30
7(a) 20 and 20 **(b)** 20 and 20 **(c)** 24 and 16
8(b) $24\,\text{cm}$
9(a) $\dfrac{x}{4},\ \dfrac{10-x}{4}$ **(c)** 5 **(d)** $\frac{25}{8}\,\text{m}^2$
10(c) $\frac{40\,000}{\pi}\,\text{m}^2$
11(c) $4 \times 4 \times 2$
12(c) Each of the 6 rectangles has dimensions $\frac{3}{4} \times \frac{2}{3}$.
13 $27 \times 9 \times 18$
14(b) $80\,\text{km/hr}$ **(c)** $400
15 $12 \times 8 \times 8$
16 $\frac{3}{2}$ sq units
17 $\frac{7}{2}$ units
18 8 units
19(b) $y = -2ax + a^2 + 4$ **(c)** $\frac{2}{3}\sqrt{3}$
20(b) $(\frac{1}{2}, \frac{1}{4})$
21 $64\,\text{cm}^2$
22 width $16\sqrt{3}\,\text{cm}$, depth $16\sqrt{6}\,\text{cm}$
23(b) $\frac{2}{3}a \times \frac{4}{3}a\sqrt{3}$
24 $A(10, 0),\ B(0, 6)$
25 $5\sqrt{2}$ metres
26 $8\,\text{km}$
27 $2(\sqrt{10} + 1) \times 4(\sqrt{10} + 1)$
28(a) $\frac{10}{3}$ **(b)** $\frac{1}{6}(a + b - \sqrt{a^2 + b^2 - ab})$
30 $2\frac{34}{39}$

Exercise **10I** (Page 389)

1(b) $\dfrac{1000}{27\pi}\,\text{m}^3$
2(c) $20\sqrt{10}\pi\,\text{cm}^3$
4(d) $\frac{2}{27}\pi s^3\sqrt{3}$
5(a) $\theta = \dfrac{180(\ell - 2r)}{\pi r}$
8(b) $\dfrac{\pi R^2 h(r - R)}{r}$ **(c)** $\frac{4}{27}\pi r^2 h$
9 $\dfrac{\pi r h}{2}$
11(b) $V = \frac{S}{2}r - \frac{1}{2}\pi r^3$
12(b) $30\,\text{cm} \times 40\,\text{cm}$
13(c) $2\pi R^2$
14(a) $y = \frac{b}{a}\sqrt{a^2 - x^2}$ **(c)** $A = 2ab,\ x = \frac{1}{2}a\sqrt{2}$
17 4:3
18(a) $4\sqrt{3}\,\text{cm}^2$
19 $\frac{1}{2}a^2$. This problem can be done very easily without calculus — the triangle has area $\frac{1}{2}a^2 \sin\theta$, where θ is the apex angle, and $\sin\theta$ is maximum when $\theta = 90^\circ$.
21 $3\sqrt{3}r^2$
22 There is only one such cone. Its height is $48\,\text{cm}$ and its radius is $12\sqrt{2}\,\text{cm}$.
23 $\frac{4}{41}x\sqrt{2}(7 - 2\sqrt{2})$

Exercise **10J** (Page 394)

1(a) $\frac{1}{7}x^7 + C$ **(b)** $\frac{3}{2}x^2 + C$ **(c)** $5x + C$ **(d)** $\frac{1}{2}x^{10} + C$
(e) $3x^7 + C$ **(f)** $\frac{1}{39}x^{13} + C$ **(g)** C
(h) $\frac{2}{3}x^3 + \frac{5}{8}x^8 + C$ **(i)** $x^3 - x^4 - x^5 + C$
(j) $\frac{1}{4}ax^4 + \frac{1}{3}bx^3 + C$ **(k)** $\dfrac{x^{a+1}}{a+1} + C$
(l) $\dfrac{ax^{a+1}}{a+1} + \dfrac{bx^{b+1}}{b+1} + C$
2(a) $\frac{1}{3}x^3 - \frac{3}{2}x^2 + C$ **(b)** $\frac{1}{5}x^5 + \frac{2}{3}x^3 + x + C$
(c) $x^3 + \frac{11}{2}x^2 - 4x + C$ **(d)** $\frac{5}{6}x^6 - x^4 + C$
(e) $\frac{4}{5}x^5 + \frac{4}{3}x^3 + x + C$ **(f)** $\frac{1}{4}a^2x^4 - 2ax^3 + \frac{9}{2}x^2 + C$
3(a) $-\dfrac{1}{x} + C$ **(b)** $-\dfrac{1}{x^2} + C$ **(c)** $-\dfrac{1}{3x} + C$
(d) $\dfrac{2}{15x^3} + C$ **(e)** $-\dfrac{1}{x} + \dfrac{1}{2x^2} + C$ **(f)** $-\dfrac{a}{bx} + C$
(g) $\dfrac{1}{(1-a)x^{a-1}} + C$ **(h)** $x + \dfrac{x^{b-a+1}}{b-a+1} + C$
4(a) $\frac{2}{3}x^{\frac{3}{2}} + C$ **(b)** $2\sqrt{x} + C$ **(c)** $\frac{3}{4}x^{\frac{4}{3}} + C$ **(d)** $\frac{4}{3}\sqrt{x} + C$
(e) $\frac{5}{8}x^{\frac{8}{5}} + C$
5(a) $y = x^2 + 3x + 4$ **(b)** $y = 3x^3 + 4x + 1$
(c) $y = \frac{2}{3}x^{\frac{3}{2}} - 16$ **6** The rule would give the primitive of x^{-1} as $x^0/0$, which is meaningless. This problem is resolved in Chapter 12.

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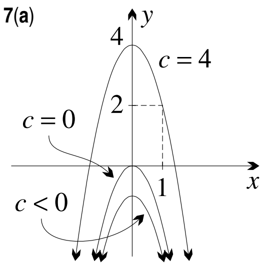
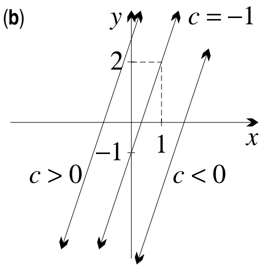
7(a)



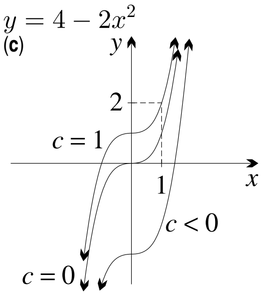
$y = -2x^2 + C,$
$y = 4 - 2x^2$

(b)



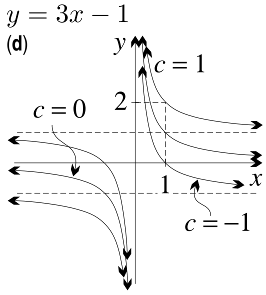
$y = 3x + C,$
$y = 3x - 1$

(c)



$y = x^3 + C,$
$y = x^3 + 1$

(d)



$y = \frac{1}{x} + C,$
$y = \frac{1}{x} + 1$

8(a) $\frac{1}{4}(x+1)^4 + C$ **(b)** $\dfrac{-1}{3(x-2)^3} + C$
(c) $\frac{1}{21}(3x-4)^7 + C$ **(d)** $-\frac{1}{28}(1-7x)^4 + C$
(e) $\dfrac{1}{6a}(ax-b)^6 + C$ **(f)** $\dfrac{2}{81(1-9x)^9} + C$
9(a) $\frac{2}{3}(x+1)^{\frac{3}{2}} + C$ **(b)** $-\frac{2}{3}(1-x)^{\frac{3}{2}} + C$
(c) $\frac{1}{3}(2x-7)^{\frac{3}{2}} + C$ **(d)** $-\frac{2}{3}\sqrt{2-3x} + C$
(e) $\dfrac{2}{3a}(ax+b)^{\frac{3}{2}} + C$ **(f)** $\frac{3}{4}\sqrt{4x-1} + C$
10(a) $\frac{1}{8}(2x+1)^4 - \frac{9}{8}$ **(b)** $y = x^3 + 2x^2 - 5x + 6$
(c) $y = \frac{4}{15}(3-x)^{\frac{5}{2}} + \frac{32}{3}x + 16\frac{2}{3}$
11(a) $\dfrac{x^{a+b+1}}{a+b+1} + C$ **(b)** $\dfrac{x^{a-b+1}}{a-b+1} + C$
(c) $\dfrac{x^{ab+1}}{ab+1} + C$ **(d)** $\dfrac{ax^{b+1}}{b+1} + \dfrac{bx^{a+1}}{a+1} + C$
(e) $\frac{2}{5}x^{\frac{5}{2}} + C$ **(f)** $2\sqrt{x} + \frac{2}{3}x^{\frac{3}{2}} + C$
(g) $\frac{2}{5}x^{\frac{5}{2}} + \frac{2}{3}x^{\frac{3}{2}} + C$ **(h)** $\frac{1}{2}x - \sqrt{x} + C$
12(a) $y = \frac{3}{5}x^5 - \frac{1}{4}x^4 + x$ **(b)** $y = -\frac{1}{4}x^4 + x^3 + 2x - 2$
(c) $y = -\frac{1}{20}(2-5x)^4 + \frac{21}{20}$ **(d)** $y = x^2 - 4x + 4$
13 30 **14(a)** -121 **(b)** $y = -x^3 + 4x^2 + 3$
15 25 seconds
16 \$4.08. Since $p = \frac{10}{t^3} + 4$, and $t > 0$, the price will always exceed \$4 (but by a decreasing amount).
17(a) $x = -\frac{7}{2}$ **(b)** $x = -18\frac{2}{3}$ **18** 3 seconds
19 $f(x) = \dfrac{1}{x} + 1$ for $x > 0$,
and $f(x) = \dfrac{1}{x} + 3$ for $x < 0$

Chapter Eleven

Exercise 11A (Page 401)

1(a) 8 **(b)** 24 **(c)** 8π **(d)** $\frac{25}{4}\pi$ **(e)** 30 **(f)** 15
2(b) When $a = \frac{1}{4}$, the integral is $\frac{1}{192}$.
When $a = \frac{1}{2}$, it is $\frac{1}{24}$. When $a = \frac{3}{5}$, it is $\frac{9}{125}$.
When $a = \frac{4}{5}$, it is $\frac{64}{375}$.
5(a) $\frac{1}{3} + \dfrac{1}{6n^2}$ **(b)** The lines P_0P_1, $P_1P_2 \ldots$ lie above the curve. Therefore the combined area of the trapezia is greater than the area under the curve.

Exercise 11B (Page 406)

1(a) 15 **(b)** $41\frac{2}{3}$ **(c)** 19 **(d)** 62 **(e)** 30 **(f)** 3. The notation $\displaystyle\int_4^7 dx$ means $\displaystyle\int_4^7 1\,dx$, which is $\Big[x\Big]_4^7$.
(g) 4 **(h)** $33\frac{3}{4}$ **(i)** $66\frac{2}{3}$
2(a)(i) $\frac{1}{10}$ **(ii)** $\frac{5}{36}$ **(iii)** $3\frac{3}{4}$ **(b)(i)** $\frac{1}{2}$ **(ii)** $\frac{15}{32}$ **(iii)** 7
3(a) 4 **(b)** $32\frac{2}{3}$ **(c)** $\frac{3}{2}$ **(d)** $13\frac{1}{3}$ **(e)** $\frac{8}{15}$ **(f)** $27\frac{1}{2}$
4(a) $1 + \frac{\pi}{2}$ **(b)** $2\frac{1}{2}$
5(a) $k = 5$ **(b)** $k = \frac{4}{9}$ **(c)** $k = 5$
6 The function is discontinuous at $x = 0$, which lies in the given interval.
7(a) $4x^3 - 3x^2 + x - 1$ **(b)** $-(7-6x)^4$ **(c)(i)** $(a-x)u(x)$
8 The function defined by $f(x) = 0$ for $x \neq \frac{1}{2}$, and $f(\frac{1}{2}) = 1$ satisfies the conditions. This function, however, is not continuous, and so a slight extension of our definition of the definite integral is required.

Exercise 11C (Page 410)

1(a) $\frac{1}{4}\pi$ **(b)** $-\frac{9}{2}\pi$ **(c)** -6 **(d)** -8 **(e)** $\frac{3}{2}$ **(f)** -8
2(a) $3\frac{3}{4}$ **(b)** 0 **(c)** -36 **(d)** 0 **(e)** 12 **(f)** 0
3(a)(i) $\frac{2}{3}$ **(ii)** 2 **(iii)** $\frac{45}{4}$ **(b)(i)** $\frac{14}{3}$ **(ii)** $96\frac{4}{5}$ **(iii)** 4
4(a) $-\frac{2}{3}$ **(b)** $2\frac{2}{3}$ **(c)** $1\frac{19}{32}$ **(d)** $143\frac{3}{4}$ **(e)** $3\frac{1}{3}$ **(f)** $42\frac{3}{4}$
5(a) $\frac{\pi}{2} - \frac{1}{2}$ **(b)** $1 - \frac{\pi}{2}$
6(a) $k = 3$ or $k = -5$ **(b)** $k = 2$ or $k = -\frac{8}{5}$
(c) $k = 1$
7(a) 0 (the interval has zero width).
(b) 0 (the interval has zero width).
(c) 0 (the integrand is odd).
(d) 0 (the integrand is odd).
(e) 0 (the integrand is odd).
(f) 0 (the integrand is odd).
8(a) The function is odd, so the integral is zero.
(b) The integral can be split into the sum of the six integrals. Each odd power is an odd function, and so has integral zero. Each even power is an even

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function, and so its integral is twice the integral over $0 \le x \le \alpha$.

9(a) The curves meet at $(0,0)$ and at $(1,1)$.

(b) In the interval $0 \le x \le 1$, the curve $y = x^2$ is always below the curve $y = \sqrt{x}$.

10(a)(i) 5 **(ii)** $\frac{25}{2}$ **(b)(i)** 25 **(ii)** $17\frac{1}{2}$ **(iii)** $27\frac{1}{2}$

11(a)(i) $\frac{1}{2}$ **(ii)** 18 **(iii)** 8 **(b)(i)** $-\frac{1}{2}$ **(ii)** -18 **(iii)** -8

12(b)(i) 3 **(ii)** 4 **(iii)** $-\frac{7}{3}$ **(iv)** $\frac{10}{3}$ **(v)** $60\frac{2}{3}$ **(vi)** 33

13(a) True, as the function is odd.

(b) True, as $\sin 4x^\circ$ is odd and $\cos 2x^\circ$ is even.

(c) False, as $2^{-x^2} > 0$ for all x.

(d) True, as $2^x < 3^x$ for $0 < x < 1$.

(e) False, as $2^x > 3^x$ for $-1 < x < 0$.

(f) True, as $t^n > t^{n+1}$ for $0 \le t \le 1$ and hence $\dfrac{1}{1+t^n} \le \dfrac{1}{1+t^{n+1}}$.

14(a) The integral is $-\dfrac{1}{N}+1$, which converges to 1 as $N \to \infty$.

(b) The integral is $-1+\dfrac{1}{\varepsilon}$, which diverges to ∞ as $\varepsilon \to 0^+$.

(c) The integral is $2\sqrt{N} - 2$, which diverges to ∞ as $N \to \infty$.

(d) The integral is $2 - 2\sqrt{\varepsilon}$, which converges to 2 as $\varepsilon \to 0^+$.

Exercise **11D** (Page 413)

1(a) $4x+C$ **(b)** x^2+C **(c)** x^3+C **(d)** C **(e)** $\frac{2}{3}x^6+C$

(f) $\frac{5}{7}x^{1\cdot 4} + C$ **(g)** $\frac{1}{2}x^{14} + \frac{1}{3}x^9 + C$

(h) $4x - \frac{3}{2}x^2 + C$ **(i)** $x^3 - 2x^4 + \frac{7}{5}x^5 + C$

(j) $\dfrac{a}{3}x^3 + \dfrac{b}{2}x^2 + C$ **(k)** $\dfrac{1}{a+1}x^{a+1} + C$

(l) $\dfrac{a}{a+1}x^{a+1} + \dfrac{b}{b+1}x^{b+1} + C$

2(a) $-\dfrac{1}{x} + C$ **(b)** $-\dfrac{1}{x^3} + C$ **(c)** $\dfrac{1}{10x^2} + C$

(d) $\dfrac{1}{4x^4} - \dfrac{1}{x} + C$ **(e)** $\dfrac{x^{1-a}}{1-a} + C$ **(f)** $\dfrac{x^{a-b+1}}{a-b+1} + C$

(g) $x^a + C$ **(h)** $x + \dfrac{x^{b-a+1}}{b-a+1} + C$

3(a) $\frac{2}{3}x^{\frac{3}{2}}+C$ **(b)** $\frac{3}{4}x^{\frac{4}{3}}+C$ **(c)** $2\sqrt{x}+C$ **(d)** $\frac{3}{5}x^{\frac{5}{3}}+C$

4(a) $\frac{5}{3}x^3 - \frac{3}{4}x^4 + C$ **(b)** $\frac{4}{3}x^3 + 2x^2 + x + C$

(c) $x - \frac{2}{3}x^3 + \frac{1}{5}x^5 + C$ **(d)** $\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + C$

(e) $\frac{1}{2}x^2 - 4x + C$ **(f)** $2x^2 - \frac{8}{3}x^{\frac{3}{2}} + x + C$

(g) $\frac{1}{2}x^2 - \dfrac{1}{x} + C$ **(h)** $\frac{1}{6}x^3 - \frac{1}{16}x^4 + C$

(i) $\frac{2}{5}x^{\frac{5}{2}} + \frac{4}{3}x^{\frac{3}{2}} + C$

5(a) $\frac{1}{6}(x+1)^6 + C$ **(b)** $\frac{1}{4}(x+2)^4 + C$

(c) $-\frac{1}{5}(4-x)^5 + C$ **(d)** $\frac{1}{15}(3x+1)^5 + C$

(e) $-\frac{5}{4}(1-\dfrac{x}{5})^4 + C$ **(f)** $-\dfrac{1}{2(x+1)^2} + C$

(g) $\frac{1}{11}(2x-1)^{11} + C$ **(h)** $-\dfrac{1}{2(4x+1)^4} + C$

(i) $\dfrac{1}{5-20x} + C$

6(a) $\frac{2}{3}(x+1)^{\frac{3}{2}} + C$ **(b)** $\frac{1}{3}(2x-1)^{\frac{3}{2}} + C$

(c) $-\frac{1}{6}(7-4x)^{\frac{3}{2}} + C$ **(d)** $\frac{3}{16}(4x-1)^{\frac{4}{3}} + C$

(e) $\frac{2}{3}\sqrt{3x+5} + C$

(f) $-\frac{4}{3}(1-\dfrac{x}{2})^{\frac{3}{2}} + C$ **(g)** $2\sqrt{x+1} + 2\sqrt{x+2} + C$

(h) $-\frac{2}{3}(4-x)^{\frac{3}{2}} - 2\sqrt{4-x} + C$ **(i)** $\dfrac{2}{3a}(ax)^{\frac{3}{2}} + \dfrac{2}{a}\sqrt{ax} + C$

7(a) $\frac{242}{5}$ **(b)** 0 **(c)** $121\frac{1}{3}$ **(d)** 1 **(e)** $\frac{1}{3}a^2 + ab + b^2$

(f) $\frac{13}{6}$ **(g)** 2 **(h)** $\frac{45}{4}$ **(i)** 46 **(j)** $6\frac{2}{3}$ **(k)** $\frac{112}{9}$ **(l)** $8\frac{2}{5}$

8(b) $\frac{2}{3}x(1+x)^{\frac{3}{2}} - \frac{4}{15}(1+x)^{\frac{5}{2}} + C$

9 Here is one clue: $\displaystyle\int_{-\infty}^{-1} \frac{dx}{x^2} = \int_1^{\infty} \frac{dx}{x^2} = 1$ (an extension question in the previous exercise explains the meaning of ∞ in the limits of integration).

Exercise **11E** (Page 416)

1(a) $2\,\mathrm{u}^2$ **(b)** $\frac{21}{2}\,\mathrm{u}^2$ **(c)** $\frac{9}{2}\,\mathrm{u}^2$

2(a) $9\,\mathrm{u}^2$ **(b)** $6\frac{2}{3}\,\mathrm{u}^2$ **(c)** $\frac{128}{3}\,\mathrm{u}^2$ **(d)** $6\,\mathrm{u}^2$ **(e)** $\frac{1}{4}\,\mathrm{u}^2$

(f) $57\frac{1}{6}\,\mathrm{u}^2$ **(g)** $36\,\mathrm{u}^2$ **(h)** $60\,\mathrm{u}^2$

3(a) $\frac{9}{2}\,\mathrm{u}^2$ **(b)** $34\frac{2}{3}\,\mathrm{u}^2$ **(c)** $18\,\mathrm{u}^2$ **(d)** $2\,\mathrm{u}^2$

4(a) $\frac{4}{3}\,\mathrm{u}^2$ **(b)** $\frac{27}{2}\,\mathrm{u}^2$ **(c)** $\frac{81}{4}\,\mathrm{u}^2$ **(d)** $46\frac{2}{5}\,\mathrm{u}^2$

5(a) $\frac{9}{2}\,\mathrm{u}^2$ **(b)** $\frac{4}{3}\,\mathrm{u}^2$ **(c)** $\frac{45}{4}\,\mathrm{u}^2$ **(d)** $9\,\mathrm{u}^2$

6 $8\,\mathrm{u}^2$

7(a) $11\frac{2}{3}\,\mathrm{u}^2$ **(b)** $128\frac{1}{2}\,\mathrm{u}^2$ **(c)** $4\,\mathrm{u}^2$ **(d)** $8\frac{1}{2}\,\mathrm{u}^2$

(e) $16\,\mathrm{u}^2$ **(f)** $6\frac{3}{4}\,\mathrm{u}^2$ **(g)** $11\frac{5}{6}\,\mathrm{u}^2$ **(h)** $32\frac{3}{4}\,\mathrm{u}^2$ **(i)** $\frac{17}{3}\,\mathrm{u}^2$

(j) $21\frac{2}{15}\,\mathrm{u}^2$

8(a) $13\,\mathrm{u}^2$ **(b)** $2\frac{1}{2}\,\mathrm{u}^2$ **(c)** $9\frac{1}{3}\,\mathrm{u}^2$ **(d)** $2\,\mathrm{u}^2$

9(a)(i) $64\,\mathrm{u}^2$ **(ii)** $128\,\mathrm{u}^2$ **(iii)** $\frac{12}{5}\sqrt{3}$ **(b)(i)** $50\,\mathrm{u}^2$

(ii) $18\,\mathrm{u}^2$ **(iii)** $\frac{32}{3}\,\mathrm{u}^2$

10(a) $4\,\mathrm{u}^2$ **(b)** $\frac{1024}{15}\,\mathrm{u}^2$ **(c)** $2\sqrt{3}\,\mathrm{u}^2$ **(d)** $\frac{5}{3}\sqrt{5}\,\mathrm{u}^2$

11(a) $(2,0)$, $(0,4\sqrt{2})$, $(0,-4\sqrt{2})$ **(b)** $\frac{16}{3}\sqrt{2}\,\mathrm{u}^2$

13(a) $f(x) = \frac{1}{3}x^3 - x^2 - 3x$, relative maximum at $(-1, \frac{5}{3})$, relative minimum at $(3,-9)$ **(b)** $16\frac{5}{6}\,\mathrm{u}^2$

15(a) $2 : n+1$ **(b)** $1 : n+1$

16(b) $a^2 = \frac{1}{2}(3+\sqrt{5})$, $a^4 = \frac{1}{2}(7+3\sqrt{5})$, $a^5 = \frac{1}{2}(11+5\sqrt{5})$

(c) Areas are $\frac{1}{5}\,\mathrm{u}^2$, $\frac{1}{10}\,\mathrm{u}^2$ and $\frac{1}{10}\,\mathrm{u}^2$.

17(a) $\frac{2}{3}ah^3 + 2ch$

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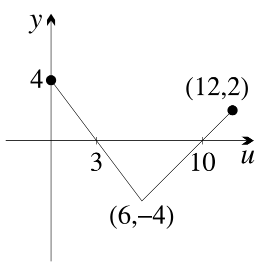
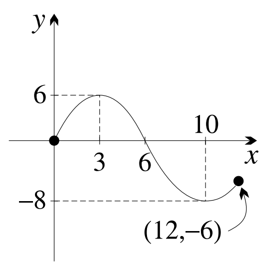
18(a) 

(d) 

(b) maximum at $(3,6)$, minimum at $(10,-8)$
(c) 0,6 **(e)** $24\,\mathrm{u}^2$
19(a) $-\dfrac{1}{n+1}$ **(b)** $\dfrac{1}{n+1}$

Exercise **11F** (Page 421)

1(a) $20\frac{5}{6}\,\mathrm{u}^2$ **(b)** $36\,\mathrm{u}^2$ **(c)** $16\frac{2}{3}\,\mathrm{u}^2$ **(d)** $\frac{9}{4}\,\mathrm{u}^2$ **(e)** $9\frac{1}{3}\,\mathrm{u}^2$
(f) $\frac{1}{12}\,\mathrm{u}^2$ **(g)** $\frac{1}{6}\,\mathrm{u}^2$ **(h)** $\frac{4}{3}\,\mathrm{u}^2$
2(b) $4\frac{1}{2}\,\mathrm{u}^2$
3(a) $(-1,16)$, $(5,4)$ **(b)** $36\,\mathrm{u}^2$
4(a) $20\frac{5}{6}\,\mathrm{u}^2$ **(b)** $57\frac{1}{6}\,\mathrm{u}^2$ **(c)** $\frac{4}{15}\,\mathrm{u}^2$ **(d)** $\frac{1}{32}\,\mathrm{u}^2$
5(b) $36\,\mathrm{u}^2$
6 $5\frac{5}{8}\,\mathrm{u}^2$
7(a) $4\frac{1}{2}\,\mathrm{u}^2$ **(b)** $\frac{1}{2}\,\mathrm{u}^2$ **(c)** $20\frac{5}{6}\,\mathrm{u}^2$ **(d)** $21\frac{1}{3}\,\mathrm{u}^2$
8(b) $\frac{1}{3}\,\mathrm{u}^2$
9 $5\frac{1}{3}\,\mathrm{u}^2$
10(c) $108\,\mathrm{u}^2$
11(a) $4\frac{1}{2}\,\mathrm{u}^2$ **(b)** $\frac{9}{16}\,\mathrm{u}^2$
12(a) $-1<x<1$ or $x>4$ **(b)** $21\frac{1}{12}\,\mathrm{u}^2$
13 $\frac{1}{16}\,\mathrm{u}^2$
14(b) $y=2x-7$ **(c)** $\frac{7}{12}\,\mathrm{u}^2$
15 $1-\dfrac{1}{\sqrt[3]{2}}$

Exercise **11G** (Page 426)

1(b) $81\pi\,\mathrm{u}^3$
2(b) $36\pi\,\mathrm{u}^3$
3(a) $16\pi\,\mathrm{u}^3$ **(b)** $9\pi\,\mathrm{u}^3$ **(c)** $\frac{32}{5}\pi\,\mathrm{u}^3$ **(d)** $6\pi\,\mathrm{u}^3$
(e) $\frac{16}{3}\pi\,\mathrm{u}^3$ **(f)** $\frac{1}{7}\pi\,\mathrm{u}^3$ **(g)** $9\pi\,\mathrm{u}^3$ **(h)** $16\pi\,\mathrm{u}^3$
4(a) $3\pi\,\mathrm{u}^3$ **(b)** $\frac{256}{3}\pi\,\mathrm{u}^3$ **(c)** $618\frac{3}{5}\pi\,\mathrm{u}^3$ **(d)** $\frac{1}{2}\pi\,\mathrm{u}^3$
(e) $85\frac{1}{3}\pi\,\mathrm{u}^3$ **(f)** $\frac{243}{5}\pi\,\mathrm{u}^3$ **(g)** $\frac{16}{15}\pi\,\mathrm{u}^3$ **(h)** $\frac{16}{3}\pi\,\mathrm{u}^3$
5(a) $\frac{296}{3}\pi\,\mathrm{u}^3$ **(b)** $19\frac{5}{6}\pi\,\mathrm{u}^3$ **(c)** $104\frac{1}{6}\pi\,\mathrm{u}^3$ **(d)** $\frac{16}{105}\pi\,\mathrm{u}^3$
6(a) $\frac{1}{3}\pi\,\mathrm{u}^3$ **(b)** $\frac{28}{15}\pi\,\mathrm{u}^3$ **(c)** $\frac{81}{10}\pi\,\mathrm{u}^3$ **(d)** $\frac{1}{2}\pi\,\mathrm{u}^3$
7(b) $\frac{1024}{5}\pi\,\mathrm{u}^3$ **(c)** $256\pi\,\mathrm{u}^3$ **(d)** $128\pi\,\mathrm{u}^3$ **(e)** $128\pi\,\mathrm{u}^3$
8(a) $\frac{32}{5}\pi\,\mathrm{u}^3$, $8\pi\,\mathrm{u}^3$ **(b)** $\frac{50}{3}\pi\,\mathrm{u}^3$, $\frac{5}{3}\pi\,\mathrm{u}^3$
(c) $8\pi\,\mathrm{u}^3$, $\frac{128}{5}\pi\,\mathrm{u}^3$ **(d)** $\frac{24}{5}\pi\,\mathrm{u}^3$, $\frac{1}{2}\pi\,\mathrm{u}^3$
9 $682\frac{2}{3}\pi\,\mathrm{u}^3$
10 $5270\frac{2}{5}\pi\,\mathrm{u}^3$
11 $\frac{9}{2}\pi\,\mathrm{u}^3$
12(b) $\frac{4}{3}\,\mathrm{u}^2$ **(c)(i)** $\frac{6}{5}\pi\,\mathrm{u}^3$ **(ii)** $\frac{24}{5}\pi\,\mathrm{u}^3$
13(b)(i) $\frac{2}{35}\pi\,\mathrm{u}^3$ **(ii)** $\frac{1}{10}\pi\,\mathrm{u}^3$
14(b) $21\frac{1}{3}\pi\,\mathrm{u}^3$
15(a)(ii) $\dfrac{\pi r^2(b^3-a^3)}{3h^2}$
16(a) $\frac{4}{3}ab^2\pi\,\mathrm{u}^3$ **(b)** $2ah^2\pi\,\mathrm{u}^3$
17(a) $2\pi a^3\,\mathrm{u}^3$ **(b)** $\frac{16}{5}\pi a^3\,\mathrm{u}^3$
18(a) $x\le 9$, $y\ge 0$ **(c)** $18\,\mathrm{u}^2$
(d)(i) $\frac{81}{2}\pi\,\mathrm{u}^3$ **(ii)** $129\frac{3}{5}\pi\,\mathrm{u}^3$
19(a) $y=3x$ **(c)(i)** $\frac{15}{7}\pi\,\mathrm{u}^3$ **(ii)** $\frac{2}{5}\pi\,\mathrm{u}^3$
20(b) $72-\frac{9}{2}\pi\,\mathrm{u}^2$ **(c)** $1000\frac{4}{5}\pi\,\mathrm{u}^3$
21(a) relative minimum at $(1,2)$,
relative maximum at $(-1,-2)$ **(c)** $\frac{9}{4}\pi\,\mathrm{u}^3$
22(a) $(0,0)$ and $(1,1)$ **(d)** $\frac{1}{3}\pi\,\mathrm{u}^3$
23(a) maximum turning point at $(\frac{1}{3},\frac{2}{9}\sqrt{3})$
(c) $\frac{8}{15}\,\mathrm{u}^2$ **(d)** $\frac{1}{12}\pi\,\mathrm{u}^3$
24(c) $8\pi\mathrm{u}^3$
25(b) $6\pi^2\,\mathrm{u}^3$

Exercise **11H** (Page 430)

1(a) $8x(x^2+3)^3$
(b)(i) $(x^2+3)^4+C$ **(ii)** $\frac{1}{8}(x^2+3)^4+C$
2(a) $12(x^2+2x)(x^3+3x^2+5)^3$
(b)(i) $(x^3+3x^2+5)^4+C$ **(ii)** $\frac{1}{12}(x^3+3x^2+5)^4+C$
3(a) $-7(2x+1)(5-x^2-x)^6$
(b)(i) $(5-x^2-x)^7+C$ **(ii)** $-\frac{1}{7}(5-x^2-x)^7+C$
4(a) $15x^2(x^3-1)^4$
(b)(i) $(x^3-1)^5+C$ **(ii)** $\frac{1}{5}(x^3-1)^5+C$
5(a) $\dfrac{2x}{\sqrt{2x^2+3}}$
(b)(i) $\sqrt{2x^2+3}+C$ **(ii)** $\frac{1}{2}\sqrt{2x^2+3}+C$
6(a) $\dfrac{3(\sqrt{x}+1)^2}{2\sqrt{x}}$
(b)(i) $(\sqrt{x}+1)^3+C$ **(ii)** $\frac{2}{3}(\sqrt{x}+1)^3+C$
7(a) $\frac{1}{3}(5x^2+3)^3+C$ **(b)** $\frac{1}{4}(x^2+1)^4+C$
(c) $\frac{1}{6}(1+4x^3)^6+C$ **(d)** $\frac{1}{30}(1+3x^2)^5+C$
(e) $\frac{1}{8}(x^2-4x-5)^4+C$ **(f)** $-\frac{1}{32}(1-x^4)^8+C$
(g) $\frac{2}{3}(x^3-1)^{\frac{3}{2}}+C$ **(h)** $\frac{1}{15}(5x^2+1)^{\frac{3}{2}}+C$
(i) $\sqrt{x^2+3}+C$ **(j)** $\frac{1}{4}\sqrt{4x^2+8x+1}+C$
(k) $-\dfrac{1}{4(x^2+5)^2}+C$ **(l)** $\frac{2}{5}(\sqrt{x}-3)^5+C$
(m) $\dfrac{p}{8q}(qx^2-3)^4+C$ **(n)** $\dfrac{r}{15p}(px^3+q)^5+C$
8(a) $\frac{32}{15}$ **(b)** $\frac{3}{64}$ **(c)** $\frac{1}{12}$ **(d)** 936 **(e)** $\dfrac{a^2}{2(a^3+1)}$
(f) $\frac{38}{3}b^3$
9(a) $x\ge 1$ or $x\le -1$ **(b)** $\dfrac{2x^2-1}{\sqrt{x^2-1}}$ **(d)** $\frac{16}{3}\sqrt{2}\,\mathrm{u}^2$

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10(a) horizontal points of inflexion at $(\sqrt{7}, 0)$ and $(-\sqrt{7}, 0)$, relative maximum at $(1, 216)$, relative minimum at $(-1, -216)$ **(b)** $600\frac{1}{4}$ u^2
11(b)(i) $136\frac{8}{15}$ **(ii)** $\frac{16}{105}$

Exercise 11I (Page 432)

2(b) 10 **(c)** $10\frac{2}{3}$, the curve is concave down.
(d) $6\frac{1}{4}\%$
3(b) $10\frac{1}{10}$ **(c)** $y'' = 12x^{-3}$, which is positive in the interval $1 \le x \le 5$, so the curve is concave up.
4(b) 24·7
(c) $24\frac{2}{3}$. $y'' = -\frac{1}{4}x^{-1\frac{1}{2}}$, which is negative in the interval $9 \le x \le 16$, so the curve is concave down.
5(c) $1\frac{27}{260}$
6(a) 0·729 **(b)** 3·388
7(a) 0·7489 **(b)** $\pi \doteqdot 2{\cdot}996$, the estimate is less than the integral, because the curve is concave down.
8 9·2 metres
9 550 m^2
10(a) 38π u^3 **(b)** 36π u^3, $5\frac{5}{9}\%$
11 180π u^3
12(e) 876 400

Exercise 11J (Page 435)

1(b) $\frac{25}{18}$ **(c)** $\frac{73}{90}$ **(d)** $\frac{11}{5}$
2(b) 2·80
3(b) 14·137 **(d)(i)** 12·294 **(ii)** 13·392
4(b) $\frac{32}{3}$ **(c)** $\frac{32}{3}$
5(a) $\frac{7}{15}$ **(b)** $\frac{22}{9}$
6(a) 7·740 **(b)** 0·9376 **(c)** 660
7(a) 0·7709 **(b)** 3·084
8 $6\frac{19}{30}$ metres
9 $613\frac{1}{3}$ m^2
10 115·19 u^3
11(a) maximum turning point at $(\frac{2}{3}, \frac{4}{9}\sqrt{6})$
(c) 1·45 u^2 **(d)** 4·19 u^3 **(e)** $\frac{16}{15}\sqrt{2}$ u^2, $\frac{4}{3}\pi$ u^3
12 4·65 units

Chapter Twelve

Exercise 12A (Page 440)

1(c) reflection in the line $y = x$
(d) For $y = 3^x$, domain: all real numbers, range: $y > 0$. For $y = \log_3 x$, domain: $x > 0$, range: all real numbers.
2(c) reflection in the line $y = x$
(d) For $y = 10^x$, domain: all real numbers, range: $y > 0$. For $y = \log_{10} x$, domain: $x > 0$, range: all real numbers.
3(a) 4 **(b)** -1 **(c)** 2 **(d)** -2 **(e)** -2 **(f)** -3 **(g)** 3 **(h)** -2
4(a) 14 **(b)** 4 **(c)** 6 **(d)** 10 **(e)** $\frac{1}{5}$ **(f)** 2 **(g)** 9 **(h)** 49
5(a) 3 **(b)** 5 **(c)** $-2\frac{1}{2}$ **(d)** 10 **(e)** 3·5 **(f)** -2 **(g)** 2π **(h)** y
6(a) -2 **(b)** -2 **(c)** $\frac{5}{6}$ **(d)** 4·5
7(a) 1 **(b)** 2 **(c)** 2 **(d)** 3
8(a) -3 **(b)** 16 **(c)** -2 **(d)** -3 **(e)** 4 **(f)** 3 **(g)** 9 **(h)** $\frac{13}{7}$ **(i)** 0
9(a) 1·58 **(b)** 3·17 **(c)** 1·72 **(d)** 1·89 **(e)** 1·00 **(f)** $-3{\cdot}97$
10(a) $x = 2$, $y = -1$ **(b)** $x = 2$, $y = -3$
(c) $x = \frac{17}{8}$, $y = \frac{15}{8}$ **(d)** $x = \frac{5}{4}$, $y = \frac{3}{4}$
11(a) $\frac{3}{2}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{5}{3}$ **(d)** $\frac{4}{3}$ **(e)** $-\frac{5}{2}$ **(f)** $\frac{-3}{2}$ **(g)** 4 **(h)** $\frac{6}{5}$
14(b)(i) $\frac{1}{3}$ **(ii)** $\frac{2}{3}$
15(b)(i) $\frac{7}{3}$ **(ii)** $\frac{8}{3}$
(c) $x = 2$, regardless of the value of a.
16(b)(i) $\log_2 3$ **(ii)** $\log_3 2$ **(iii)** $\log_3 5$
17(c) 2·3222
18(a) $x = 3$ **(b)** $x = 3$

Exercise 12B (Page 448)

1(a) $\frac{1}{x}$ **(b)** $\frac{1}{x}$ **(c)** $\frac{2}{x}$ **(d)** $1 + \frac{4}{x}$ **(e)** $\frac{1}{x}$ **(f)** $\frac{1}{x}$ **(g)** $\frac{3}{x}$ **(h)** $12x^2 - \frac{1}{x}$
2(a) $\frac{2}{2x+5}$ **(b)** $\frac{3}{3x-7}$ **(c)** $\frac{2}{3+2x}$ **(d)** $\frac{-1}{4-x}$ **(e)** $\frac{7}{4+7x}$ **(f)** $-\frac{5}{2-5x}$ **(g)** $1+\frac{1}{1-x}$ **(h)** $\frac{-1}{3-x}$ **(i)** $\frac{a}{ax-b}$ **(j)** $\frac{\pi}{\pi x+1}$ **(k)** $\frac{\pi}{\pi x-2}$ **(l)** $\frac{2}{2x-ab}$
3 $e \doteqdot 2{\cdot}7$
4 Check the answers using the calculator.
5(a) e **(b)** $-\frac{1}{e}$ **(c)** 6 **(d)** $\frac{1}{2}$ **(e)** $2e$ **(f)** 0 **(g)** e **(h)** 1 **(i)** 0
6(a) $x = \frac{1}{3}$ **(b)** $x = 3$ or 4
7(a) $\frac{2x}{x^2+1}$ **(b)** $\frac{2x+3}{x^2+3x+2}$ **(c)** $\frac{-2x}{2-x^2}$ **(d)** $\frac{1}{2x+\sqrt{x}}$
8(a) $\frac{2}{x}$ **(b)** $\frac{3}{x}$ **(c)** $\frac{1}{2x}$ **(d)** $\frac{-1}{x}$ **(e)** $\frac{1}{4+2x}$

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(f) $\frac{1}{1+x}+\frac{1}{1-x}$ **(g)** $\frac{1}{3x+3}$ **(h)** $\frac{1}{x}+\frac{1}{2(x+1)}$

9(a) $\frac{1}{x\log 2}$ **(b)** $\frac{1}{x\log 10}$ **(c)** $\frac{1}{x\log 2}$ **(d)** $\frac{5}{x\log 3}$

10(a) $1+\log x$ **(b)** $\frac{2x}{2x+1}+\log(2x+1)$

(c) $\frac{2x+1}{x}+2\log x$ **(d)** $\frac{2+\log x}{2\sqrt{x}}$

11 $x-e^2y+e^2=0$

12 $ex+e^2y+e^2-1=0,\ (\frac{1}{e}-e,0)$

13(a) $\frac{1}{2x-2}-\frac{2x}{x^2+1}$ **(b)** $\frac{2x-2}{x^2-2x}+\frac{1}{2x}$ **(c)** $\log\pi$

14(a) $x(1+2\log x)$ **(b)** $\frac{1-\log x}{x^2}$ **(c)** $\frac{2\log x}{x}$

(d) $\frac{4(\log x)^3}{x}$ **(e)** $\frac{-1}{x(1+\log x)^2}$ **(f)** $\frac{1}{2x\sqrt{\log x}}$

(g) $\frac{8}{x}(2\log x-3)^3$ **(h)** $\frac{-1}{x(\log x)^2}$ **(i)** $\frac{1}{x\log x}$

(j) $\frac{-\log 3}{x(\log x)^2}$ **(k)** $\frac{\log x-1}{(\log x)^2}$

(l) $\log_x 3-\frac{\log 3}{(\log x)^2}=\frac{\log 3(\log x-1)}{(\log x)^2}$

15(a) $(\frac{1}{e},\frac{-1}{e})$ **(b)** $(1,1)$ **(c)** $(\frac{1}{\sqrt{e}},\frac{1}{2e})$

17 $x=\frac{1}{\log 10}$

18(a) $\frac{4x-3}{2x^2-3x}$ **(b)** $\frac{1-4x}{x-1-2x^2}$ **(c)** $\frac{1}{x(1+\log x)}$

(d) $\frac{2+\sqrt{x}}{2x(\sqrt{x}+\log x)}$ **(e)** $\frac{2x+x^2}{2(x-2)}+2(x+1)\log\sqrt{x-2}$

(f) $\frac{3-12x-7x^2}{2x(3+2x-x^2)}$

20(a) $\frac{x(x+5)}{2\sqrt{x-1}(x+2)^2}$ **(b)** $\frac{(x-1)^2(x+2)(x^2-15x-4)}{(x-3)^5}$

(c) $\frac{x(2x^2-x-2)}{(x-1)^{3/2}\sqrt{x+1}}$ **(d)** $\frac{(x-1)(3x^2+6x-1)}{2\sqrt{x}(x+1)^2}$

(e) $\frac{(x+\frac{1}{x})^{\frac{1}{\pi}}(x^2-1)}{\pi x(x^2+1)}$ **(f)** $\frac{3x^2+6x+2}{2\sqrt{x}\sqrt{x+1}\sqrt{x+2}}$

21(a) $x^x(1+\log x)$ **(b)** $2x^{\log x-1}\log x$

(c) $x^{\frac{1}{x}-2}(1-\log x)$

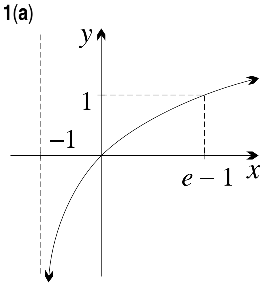
22 $2\frac{28}{39}$

23(d)(i) 2 **(ii)** $2{\cdot}5937$ **(iii)** $2{\cdot}7048$ **(iv)** $2{\cdot}7169$

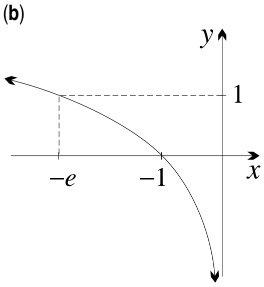
(v) $2{\cdot}7181$

Exercise **12C** (Page 452)

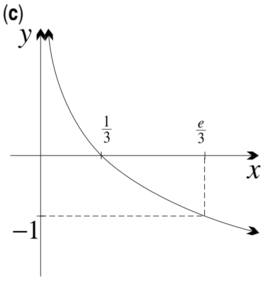
1(a)



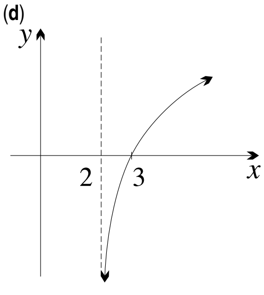
(b)



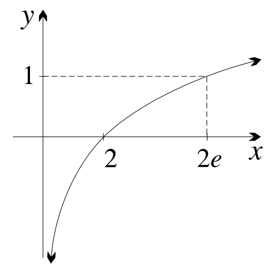
(c)



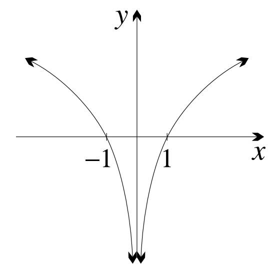
(d)



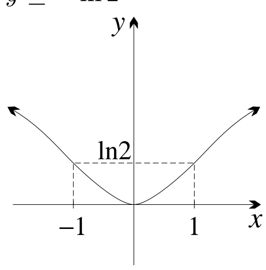
(e)

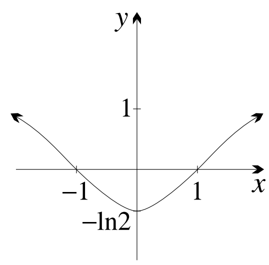


(f)



2(a) all real x **(e)** $y\ge 0$ **(f)** all real x, $y\ge -\ln 2$





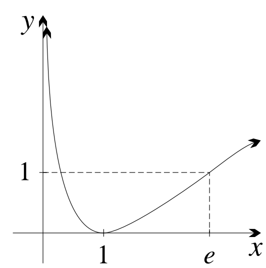
3(a) $x>0$

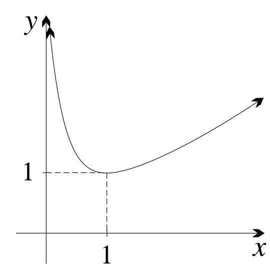
(b) $y'=\frac{2}{x}\log x$, $y''=\frac{2}{x^2}(1-\log x)$

(c) $y\ge 0$

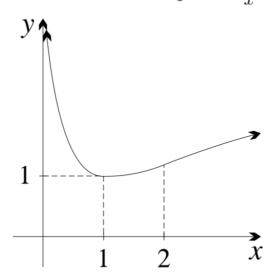
4(a) $y'=1-\frac{1}{x}$, $y''=\frac{1}{x^2}$

(b) $x>0$ **(d)** $y\ge 1$





5(a) $x>0$ **(b)** $y'=\frac{x-1}{x^2}$ and $y''=\frac{2-x}{x^3}$ **(d)** $y\ge 1$



6(a) $x>0$. $y\to 0^+$ as $x\to\infty$ so the x-axis is a horizontal asymptote. $y\to-\infty$ as $x\to 0^+$ so the y-axis is a vertical asymptote.

(b) $y'=\frac{1}{x^2}(1-\log x)$ and $y''=\frac{1}{x^3}(2\log x-3)$

(d) $(e^{\frac{3}{2}},\frac{3}{2}e^{-\frac{3}{2}})$ **(e)** $y\le e^{-1}$

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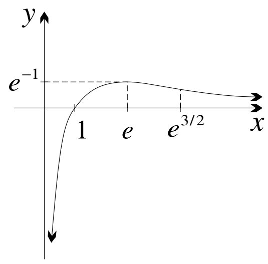
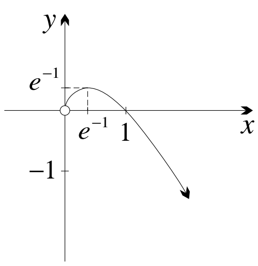
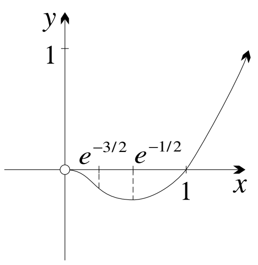


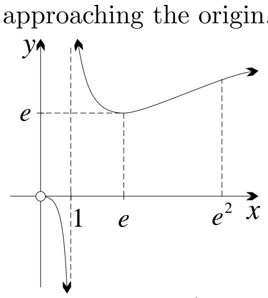
7(a) $y - 2\log c = \frac{2}{c}(x - c)$, $c = e$ **(b)(i)** $c = 1$ or e^2
(ii) e^{-1}
8(b) $\frac{e}{\sqrt{5}}$
9(a) $y = \frac{2\log t}{t}x - 2\log t + (\log t)^2$ **(b)** $\frac{e}{4}$
10(a) $x > 0$
(c) $y \to 0^+$ as $x \to 0^+$, $y' \to \infty$ as $x \to 0^+$, hence the graph becomes vertical approaching the origin. $y \le e^{-1}$.



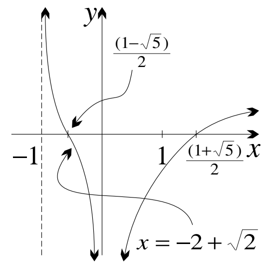
11(a) $x > 0$. Minimum at $(\frac{1}{\sqrt{e}}, -\frac{1}{2e})$ **(c)** $y \to 0$ as $x \to 0^+$, $y' \to 0$ as $x \to 0^+$, hence the graph becomes horizontal approaching the origin.
(d) $y \ge -\frac{1}{2e}$



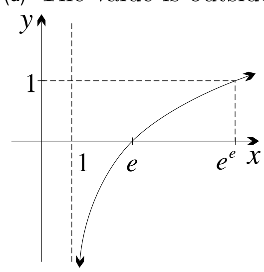
12 $x > 0$, $x \ne 1$, $y < 0$ or $y \ge e$. $x = 1$ is a vertical asymptote and the curve becomes horizontal approaching the origin.



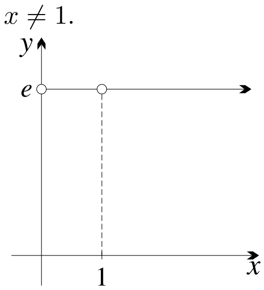
13(a) $x > -1$, $x \ne 0$
(c) $x = -2$ is outside the domain.
(d) one at $x = -2 + \sqrt{2}$



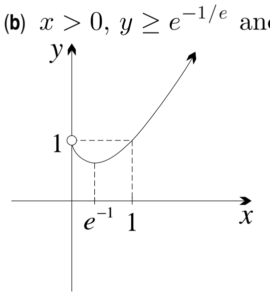
14(a) $x > 1$ **(c)** $y' = \frac{1}{x\log x}$, which can never be zero, $y'' = -\frac{1+\log x}{(x\log x)^2}$
(d) The value is outside the domain.



15(a) $\log a$ **(b)** For the graph of $y = \log x$ a horizontal enlargement of factor $\frac{1}{a}$ is equivalent to a translation of $\log a$ upwards. **(c)** The change to base b stretches the graph vertically by a factor $\frac{1}{\log b}$, otherwise the result is as above.
16(a) $\frac{2c}{1+c^2}$
18 $y = e$ for all x in the domain, which is $x > 0$, $x \ne 1$.



19(a) $y' = x^x(1 + \log x)$
(b) $x > 0$, $y \ge e^{-1/e}$ and $y' = 1$ when $x = 1$.



20(b) $y' = x^{-2}\, x^{\frac{1}{x}}(1 - \log x)$

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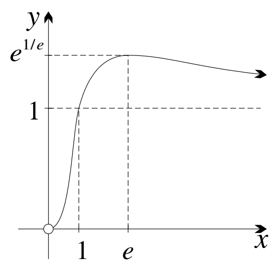
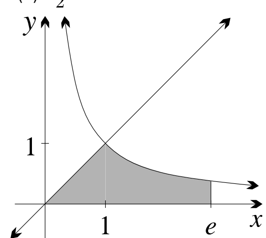


Exercise **12D** (Page 456)

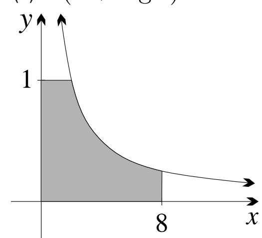
1(a) $\log x + C$ **(b)** $2\log x + C$
(c) $\frac{1}{3}\log x + C$ **(d)** $\frac{1}{5}\log(5x+4) + C$
(e) $\frac{5}{2}\log(3+2x) + C$ **(f)** $\frac{1}{4}\log(4x-1) + C$
(g) $\frac{1}{2}\log(2x-1) + C$ **(h)** $\frac{3}{2}\log(2x+1) + C$
(i) $\log(2x-1) + C$ **(j)** $-\frac{1}{5}\log(3-5x) + C$
(k) $-\frac{2}{7}\log(5-7x) + C$ **(l)** $\frac{e}{\pi}\log(\pi x+1) + C$
(m) $-\frac{1}{e}\log(2-ex) + C$ **(n)** $\frac{\sqrt{2}}{3}\log(3x-\pi) + C$
(o) $-\frac{1}{a}\log(b-ax) + C$ **(p)** $-\frac{a}{c}\log(b-cx) + C$
2(a) 1 **(b)** $1\frac{1}{2}$ **(c)** $\log 5$ **(d)** $\log 3$ **(e)** $\log 2$
(f) $3\log 2$ **(g)** $\frac{2}{3}\log 2$ **(h)** $\frac{3}{2}\log 3$ **(i)** $\frac{1}{2}\log 3$
3(a) $x + \log x + C$ **(b)** $\frac{2}{3}\log x - \frac{1}{3}x + C$
(c) $3x - 2\log x$ **(d)** $\frac{3x^2}{2} + 4\log x + \frac{1}{x} + C$
4(a) $\log(x^2-9) + C$ **(b)** $\log(3x^2+x) + C$
(c) $\log(x^2+x-3) + C$ **(d)** $\log(2+5x-3x^2) + C$
(e) $\frac{1}{2}\log(x^2+6x-1) + C$
(f) $\frac{1}{4}\log(12x-3-2x^2) + C$
5(a) $y = \frac{1}{4}(\log x + 2)$, $x = e^{-2}$
(b) $y = 2\log(x+1) + 1$
(c) $y = \log\left(\frac{x^2+5x+4}{10}\right) + 1$
(d) $y = x + \log x + \frac{1}{2}x^2$
(e) $y = 2\log x + x + C$, $y = 2\log x + x$,
$y(2) = \log 4 + 2$
6(a) $\log(x^3-5) + C$ **(b)** $\log(x^4+x-5) + C$
(c) $\frac{1}{4}\log(x^4-6x^2)+C$ **(d)** $\frac{1}{2}\log(5x^4-7x^2+8)+C$
(e) $\frac{2}{3}\log(\sqrt{x^3}+1)+C$ **(f)** $\frac{1}{3}\log(x^3-2x^{\frac{3}{2}}+1)+C$
7(a) $\log\frac{6}{e^2-1} - 1$ **(b)** $\log 14$ **(c)** $\log\left(\frac{2e^2}{e+1}\right)$
8(a)(i) $x\log x - x + C$ **(ii)** $\frac{\sqrt{e}}{2}$ **(b)** $10 - \frac{9}{\log 10}$ **(c)** $\frac{e^2}{4}$
(d) $2\sqrt{x}(\log x - 2) + C$
9(a) $(n-1)\log a$ **(b)** $\frac{1}{t^2}\log(s+tx) + C$
(c) $\frac{1}{b^2}\log(b+1)$
10(a) $\log\frac{5}{4} - \frac{1}{5}$ **(b)** $\frac{1}{6}\log\left(\frac{x-3}{x+3}\right) + C$
11(a) $\log 9$ **(b)** $\log 2$ **(c)** $\frac{1}{2}$ **(d)** $9\log 2$
12 $\log(x + \sqrt{x^2+1}) + C$
13 $2\log(\sqrt{x}+1) + C$
14(a) They are both $\dfrac{a}{ax+b}$. $-\log 3$
(b) $y = \begin{cases} \log x + 1, & \text{for } x > 0, \\ \log(-x) + 2, & \text{for } x < 0. \end{cases}$
15(d)(i) $\log\frac{3}{2} \doteqdot 0{\cdot}41$ **(ii)** $\log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots$.
(e) $\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots$, $\log\frac{1}{2}$ $\doteqdot -0{\cdot}69$ **(f)** Using $x = \frac{1}{2}$, $\log 3 \doteqdot 1{\cdot}0986$.

Exercise **12E** (Page 459)

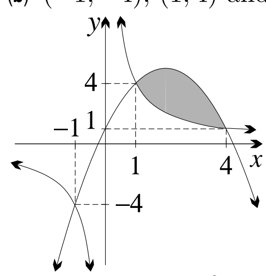
1(a) $1\,\mathrm{u}^2$ **(b)** $2\log 2\,\mathrm{u}^2$
2(a) $(6 - 3\log 3)\,\mathrm{u}^2$ **(b)** $(3\frac{3}{4} - 2\log 4)\,\mathrm{u}^2$ **(c)** $\frac{1}{2}\,\mathrm{u}^2$
3(a) $\frac{1}{2}\log 5 \doteqdot 0{\cdot}805\,\mathrm{u}^2$
4(a) $2\log 2\,\mathrm{u}^2$ **(b)** $(1 - \log 2)\,\mathrm{u}^2$
5(b) $\frac{3}{2}\,\mathrm{u}^2$



6(b) $4(1 + \log 2)\,\mathrm{u}^2$



7(a) $(\frac{1}{3}, 3)$ and $(1, 1)$ **(b)** $(\frac{4}{3} - \log 3)\,\mathrm{u}^2$
8 $\pi\log 6\,\mathrm{u}^3$
9(a) $\pi\log 2\,\mathrm{u}^3$ **(b)** $\pi\log 16\,\mathrm{u}^3$ **(c)** $\pi(\frac{25}{6} + \log 36)\,\mathrm{u}^3$
10(a) $x^3 - 4x^2 - x + 4$
(b) $(-1, -4)$, $(1, 4)$ and $(4, 1)$



(c) $(12 - 4\log 4)\,\mathrm{u}^2$
11 $(\log 4 - \frac{1}{2})\,\mathrm{u}^2$
12(a) $x\log x - x$ **(b)** $e^2\,\mathrm{u}^2$ **(c)** $(e^c - 1)\,\mathrm{u}^2$
13(a) $\pi\left(\frac{15}{2} - \log 4\right)\mathrm{u}^3$ **(b)** $\pi\left(\frac{3}{2} + \log 4\right)\mathrm{u}^3$. There is a difference because $(a-b)^2 \neq a^2 - b^2$.
14 $4\pi(2\log 2 - 1)\,\mathrm{u}^3$
15(a) $M = \log 3$ **(b)** $\overline{x} = \dfrac{2}{\log 3}$
16(a) The upper rectangle has height 2^{-n}, the lower rectangle has height 2^{-n-1}, both rectangles have width $2^{n+1} - 2^n = 2^n$.
17(d) $2{\cdot}715$
18 $\log\left(\frac{25}{16}\right)\mathrm{u}^2$

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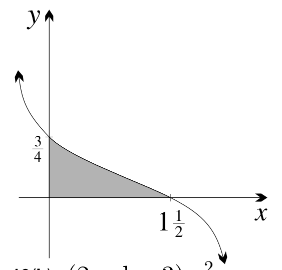
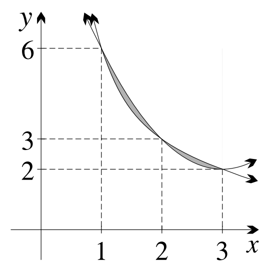


19(b) $(2 - \log 3)\,\mathrm{u}^2$

20(a) Using symmetry, $\displaystyle\int_2^3 \frac{dx}{x} = \log\left(\tfrac{3}{2}\right)\mathrm{u}^2$.

(b)(i) $3\,\mathrm{u}^2$ **(ii)** $\log 3\,\mathrm{u}^2$

21(b) **(c)** $(2 - 6\log\frac{4}{3})\,\mathrm{u}^2$



23 $\frac{\pi(e-1)}{e}\,\mathrm{u}^3$

24 $\pi\left(\frac{(1+c)\log(1+c)-c}{1+c}\right)\mathrm{u}^3$

25(a) $-\sqrt{3}, 0, \sqrt{3}$ **(b)** The curve is concave down.

Chapter Thirteen

Exercise 13A (Page 465)

1(a) $2e^{2x}$ **(b)** $-3e^{-3x}$ **(c)** $-5e^{5x}$ **(d)** $\frac{1}{2}e^{\frac{1}{2}x}$ **(e)** ae^{ax} **(f)** $-ke^{-kx}$ **(g)** $\pi e^{-\pi x}$ **(h)** $\frac{-1}{c}e^{-\frac{1}{c}x}$

2(a) $2e^{2x-1}$ **(b)** $-e^{1-x}$ **(c)** $-9e^{-3x+4}$ **(d)** $e^{\frac{1}{2}x+4}$ **(e)** pe^{px+q} **(f)** $2e^{2x} + 3e^{-3x}$ **(g)** $\frac{e^x + e^{-x}}{2}$ **(h)** $e^{ax} - e^{-bx}$

3(a) $2e^{2x}$ **(b)** $\frac{-3}{e^{3x}}$ **(c)** $\frac{1}{2}e^{\frac{x}{2}}$ **(d)** $\frac{-3}{2}e^{\frac{-3x}{2}}$

4(a) $2xe^{x^2}$ **(b)** $-2xe^{1-x^2}$ **(c)** $2(x+1)e^{x^2+2x}$ **(d)** $(1-2x)e^{6+x-x^2}$ **(e)** $(3x-1)e^{3x^2-2x+1}$ **(f)** $(x+1)e^x$ **(g)** $(1-x)e^{-x}$ **(h)** xe^x **(i)** $(3x+4)e^{3x-4}$ **(j)** $(2x^2+1)e^{x^2+1}$ **(k)** $(x^2+7x)e^x$ **(l)** $(x^3-4x^2+2x)e^{-x}$

6(a) $(2x^2-1)e^{2x-1}$ **(b)** $\frac{-e^x}{1-e^x}$ **(c)** $\frac{e^x+1}{e^x+x}$ **(d)** $e^x\left(\log x + \frac{1}{x}\right)$ **(e)** $\frac{e^x-e^{-x}}{e^x+e^{-x}}$ **(f)** $\frac{(x-1)e^x}{x^2}$ **(g)** $-xe^{-x}$ **(h)** $-\frac{2e^x}{(e^x-1)^2}$ **(i)** $\frac{(x-1)e^x}{(x+1)^3}$ **(j)** $\frac{4}{(e^x+e^{-x})^2}$

7(a) $2^x\log 2$ **(b)** $10^x\log 10$ **(c)** $\pi^x\log\pi$ **(d)** $a^x\log a$ **(e)** $2^{3x-1}3\log 2$ **(f)** $-5^{2-x}\log 5$ **(g)** $a^{x-2}\log a$ **(h)** $a^{bx+c}b\log a$ **(i)** $(x\log 2+1)2^x$ **(j)** $(3x^2-3)3^{x^3-3x}\log 3 = (x^2-1)3^{x^3-3x+1}\log 3$

8(a) 1 **(b)** $\frac{-1}{x^2}$ **(c)** -1 **(d)** 3

9(a) $\frac{Ae}{e-1}$ **(b)** The common ratio is e^d.

10(c)(i) -5 or 2 **(ii)** $-\frac{1+\sqrt{5}}{2}$ or $-\frac{1-\sqrt{5}}{2}$

12(a) $\frac{-1}{x^2}e^{\frac{1}{x}}$ **(b)** $(x+1)e^x$ **(c)** $\frac{\log 3}{x^2}3^{-\frac{1}{x}}$

13(a) $\frac{-1}{\sqrt{2}}$ or $\frac{1}{\sqrt{2}}$ **(b)** $-2-\sqrt{2}$ and $-2+\sqrt{2}$ **(c)** $1-x^2$

15(b) The secant has gradient 1 and the tangent is less steep. The gradient of the tangent is 0·69 to 2 decimal places. **(d)** $\log 2$ **(e)** $y' = 2^x \lim\limits_{h\to 0}\frac{2^h-1}{h} = 2^x\log 2$

17(a) $y = e^{x\log x}$, so $y' = (\log x + 1)x^x$.
(b) $y = e^{(2-x)\log x}$, so $y' = \left(\frac{2}{x} - 1 - \log x\right)x^{2-x}$.
(c) $y = e^{(\log x)^2}$, so $y' = \frac{2x^{\log x}\log x}{x}$.
(d) $y = e^{\log x \times \frac{1}{\log x}}$, so $y' = 0$.

18(a) $\frac{-b+\sqrt{b^2-4ac}}{2a}$ or $\frac{-b-\sqrt{b^2-4ac}}{2a}$ **(b)** when $b^2 - 4ac < 0$

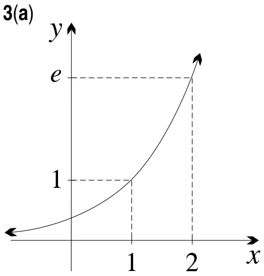
19(b) $f(x)$ simplifies to e^x.

Exercise 13B (Page 469)

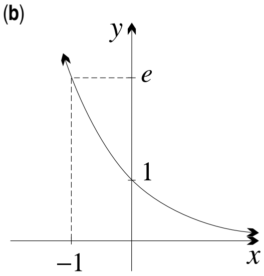
1(c) reflection in the line $y = x$ **(d)** For $y = e^x$, domain: all real numbers, range: $y > 0$. For $y = \log x$, domain: $x > 0$, range: all real numbers.

2(d) The x-intercept is 1 unit left of the point of contact.

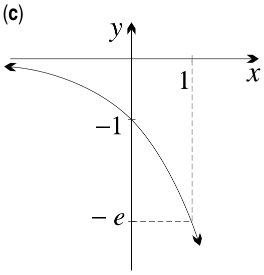
3(a)



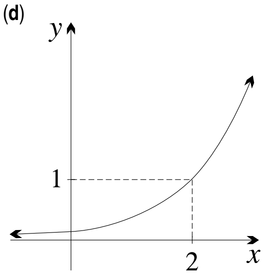
(b)



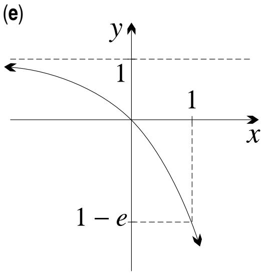
(c)



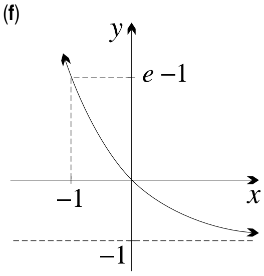
(d)



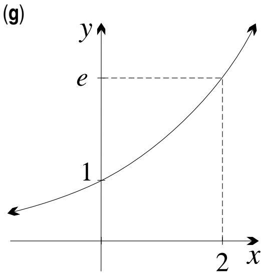
(e)



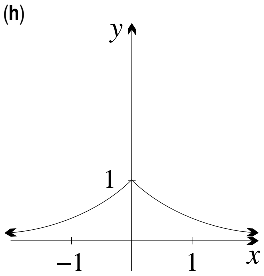
(f)



(g)



(h)

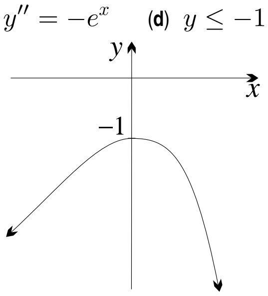


4(a) $y = x + 1$ **(b)** $y = (1 - e)x$

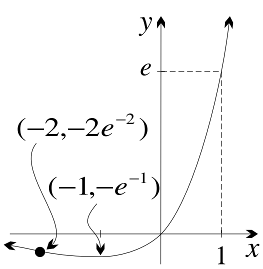
5(a) $x - ey + e^2 + 1 = 0$ **(b)** $x = -e^2 - 1,\ y = e + e^{-1}$

(c) $\frac{1}{2}(e^3 + 2e + e^{-1})$

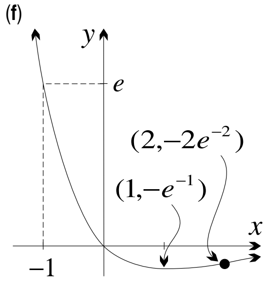
6(a) $y' = 1 - e^x$,
$y'' = -e^x$ **(d)** $y \le -1$



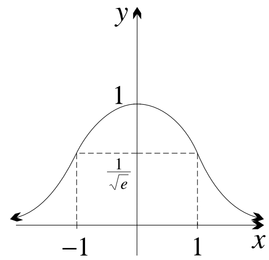
7(e) $y \ge -e^{-1}$



(f)



8(a) $y' = -xe^{-\frac{1}{2}x^2}$,
$y'' = (x^2 - 1)e^{-\frac{1}{2}x^2}$
(d) $0 < y \le 1$



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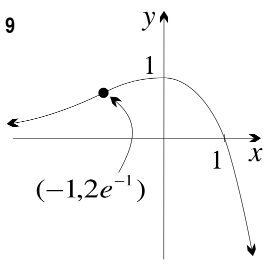


(a) $y' = -x\,e^x$,
$y'' = -(x + 1)e^x$
(c) $y \le 1$

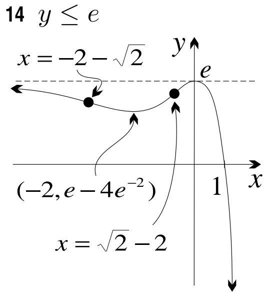
10 -3

11(b) The gradient of $y = a^x$ at $x = 0$ is $\log a$, which is 1 if and only if $a = e$.

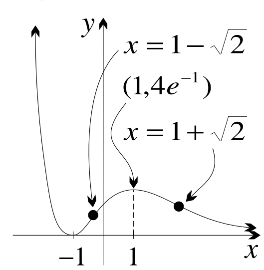
(c) The gradient of $y = Aa^x$ at $x = 0$ is $A \log a$, which is 1 if and only if $a = e^{1/A}$.

13(a) $y = e^t(x - t + 1)$

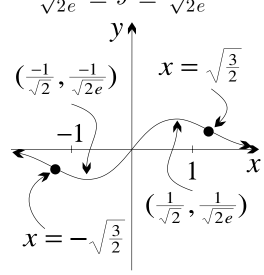
14 $y \le e$



15 $y \ge 0$



16 $\frac{-1}{\sqrt{2e}} \le y \le \frac{1}{\sqrt{2e}}$



17(a) $x - 2te^{-t^2}y + t\left(2e^{-2t^2} - 1\right) = 0$

(b) $t\left(1 - 2e^{-2t^2}\right)$ **(c)** $-\sqrt{\frac{\log 2}{2}}$, 0 and $\sqrt{\frac{\log 2}{2}}$

18(a) $A(p-1, 0)$, $B(p, 0)$, $C(p+q^2, 0)$, $D(0, (1-p)q)$, $E(0, q)$, $F(0, q + \frac{p}{q})$ **(c)(i)** $\frac{q}{2}(q^2 + 1)$ **(ii)** $\frac{p^2}{2}\left(q + \frac{1}{q}\right)$

19 $x = 1$ or $x = -1$

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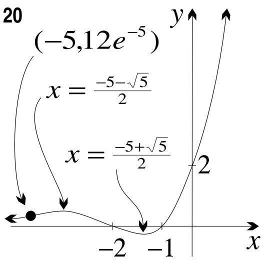
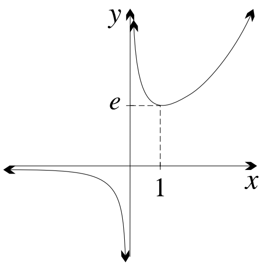
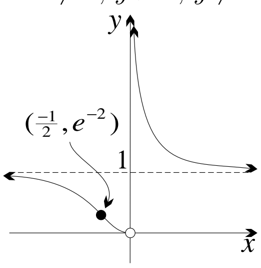


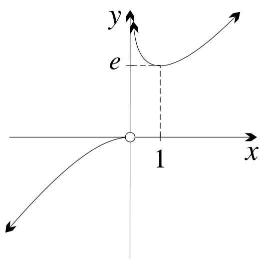
21 $x \neq 0$, $y < 0$ or $y \geq e$



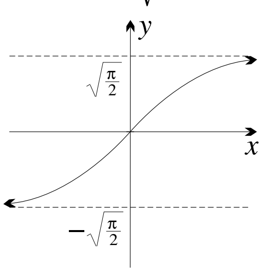
22 $x \neq 0$, $y > 0$, $y \neq 1$



23



24(a) $y = e^{kx}$ and $y = \frac{1}{k}\log x$
(b) Since $y = a^x$ and $y = \log_a x$ are inverse functions, they are symmetric in the line $y = x$. The common tangent must therefore be the line $y = x$, which has gradient 1. NOTE: This assertion is untrue if there is more than 1 intersection point.
(c) $k\,e^{kx} = 1$ and $\frac{1}{kx} = 1$ **(d)** $k = \frac{1}{e}$, $a = e^{\frac{1}{e}}$
26(a) 0 **(b)** $y' = e^{-\frac{1}{2}x^2}$ so $y' > 0$ for all x.
(c) y is an odd function.
(e) $y' \to 0$ as $x \to \infty$. This does not imply an asymptote. For example, $y = \log x$ has gradient $y' = \frac{1}{x}$, so $y' \to 0$ as $x \to \infty$, but $y = \log x$ clearly does not have a horizontal asymptote. **(f)** $-\sqrt{\frac{\pi}{2}} < y < \sqrt{\frac{\pi}{2}}$



Exercise **13C** (Page 473)

1(a)(i) $e - 1 \doteqdot 1\cdot72$ **(ii)** $1 - e^{-1} \doteqdot 0\cdot63$
(iii) $1 - e^{-2} \doteqdot 0\cdot86$ **(iv)** $1 - e^{-3} \doteqdot 0\cdot95$
(d) The total area is exactly 1.
2(a) $\frac{1}{2}e^{2x} + C$ **(b)** $3e^{\frac{1}{3}x} + C$ **(c)** $\frac{1}{4}e^{4x+5} + C$
(d) $\frac{1}{3}e^{1+3x} + C$ **(e)** $\frac{1}{4}e^{4x-2} + C$ **(f)** $e^{x-1} + C$
(g) $2e^{3x+2} + C$ **(h)** $e^{2x-1} + C$ **(i)** $-e^{3-x} + C$
(j) $\frac{-1}{2}e^{7-2x} + C$ **(k)** $\frac{1}{\pi}e^{\pi x-1} + C$
(l) $-\frac{1}{2e}e^{1-ex} + C = -\frac{1}{2}e^{-ex} + C$ **(m)** $\frac{\pi}{3}e^{3x-\sqrt{2}} + C$
(n) $-e^{b-ax} + C$ **(o)** $\frac{a}{b}e^{bx+c} + C$ **(p)** $e^{a-\pi x} + C$
3(a) $e^2 - e$ **(b)** $e - e^{-3}$ **(c)** $e - e^{-1}$ **(d)** $\frac{e^2}{2}(e^2 - 1)$
(e) 1 **(f)** $\frac{1}{12}$ **(g)** $\pi e(e-1)$ **(h)** $\frac{e^{ab}}{b}(e^{ab} - 1)$
4(a) $\frac{-1}{2}e^{-2x} + C$ **(b)** $x - e^{-x} + C$ **(c)** $2e^{\frac{x}{2}} + C$
(d) $2e^{\frac{x}{2}} + \frac{2}{3}e^{\frac{-3x}{2}} + C$
5(a) $\frac{2^x}{\log 2} + C$ **(b)** $\frac{3^x}{\log 3} + C$ **(c)** $\frac{-5^{-x}}{\log 5} + C$ **(d)** $\frac{\pi^x}{\log \pi} + C$
6(a) $y = e^{x-1}$, $y = e^{-1}$
(b) $y = e^2 + 1 - e^{2-x}$, $y = e^2 + 1$
(c) $y = \frac{1-2^{-x}}{\log 2}$ **(d)** $f(x) = e^x + \frac{x}{e} - 1$, $f(0) = 0$
7(a) $e^{x^2+3} + C$ **(b)** $e^{5x^2-2x} + C$ **(c)** $\frac{1}{2}e^{3x^2+4x+1} + C$
(d) $\frac{1}{3}e^{x^3-3x^2} + C$
8(a) $2e^{x^3-2x^2+3x-5} + C$ **(b)** $\log x + \frac{e^{3x}}{3} + C$
(c) $2\sqrt{x} + \frac{1}{2}e^{-x^2} + C$ **(d)** $\frac{1}{2}e^{2x} + 2e^x + x + C$
(e) $\frac{e^{2x}}{2} - 2x - \frac{e^{-2x}}{2} + C$ **(f)** $-e^{\frac{1}{x}} + C$
(g) $\frac{2}{3}e^{x\sqrt{x}} + C$ **(h)** $\frac{x^3}{3} + C$ **(i)** $kx^2 + C$
9(b) $e^2 + 1$
10(a) $e^x - e^{-x}$ **(b)** $\log\left(\frac{e^2+e^{-2}}{2}\right)$
11 $\log(e^x + 1) + C$
12(a) $x^2 + 2\log x + \frac{2^x}{\log 2} + C$ **(b)** $\frac{a^x}{\log a} + \frac{ax^2}{2} + C$
(c) $\frac{3^{x^2+2x}}{\log 3} + C$
13(a) $y' = 2(x - x^3)e^{-x^2}$ **(b)** $\frac{2e^3-5}{2e^4}$
14(b) $(x+1)e^x$
15(a) $y = \frac{2e^{-x\sqrt{x}}+1}{3}$ **(b)** $y = 2 - 3^{-x}$, $y(0) = 1$
17 $\frac{1}{\log 2+1}$
20(b) $1\cdot1276$
(d) $e^{0\cdot5} = \alpha + \sqrt{\alpha^2 - 1}$, $e^{-0\cdot5} = \alpha - \sqrt{\alpha^2 - 1}$

Exercise **13D** (Page 477)

1(a) $1 - e^{-1}$ square units **(b)** $e(e^2 - 1)$ square units
(c) 3 square units **(d)** $3\frac{1}{2}$ square units
2 $3 - e^{-2}$ square units
3(a) $0\cdot8863$ square units **(b)** $0\cdot8362$ square units
4(a) $1 + e^{-2}$ square units
(b) $2\log 2 - 1$ square units **(c)** e^{-1} square units
(d) $3 + e^{-2}$ square units
5 $e - 1$ square units
6 $\displaystyle\int_0^1 e^x - 1 - x\,dx = e - 2\frac{1}{2}$ square units
7 $\displaystyle\pi\int_0^1 (e^x)^2\,dx$, $\frac{\pi}{2}(e^2 - 1)$ cubic units
8(b) $e - 1\frac{1}{3}$ square units

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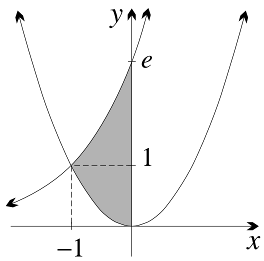
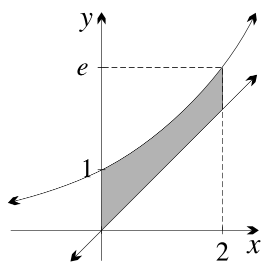
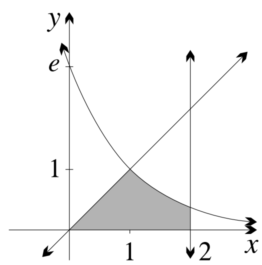


9 $e^2 - 3$ square units **10** $1\frac{1}{2} - e^{-1}$ square units





11 $\pi[\frac{e-e^{-1}}{2} - 1]$ cubic units

12 $\pi[2 + 2e^{-1}(1 - e^{-2}) + \frac{1}{2}e^{-2}(1 - e^{-4})] \doteqdot 8{\cdot}491$ cubic units

13 $\pi(1 - e^{-4}) \doteqdot 3{\cdot}084\,\text{mL}$

14 intercepts $(0, 7)$ and $(3, 0)$ and area $24 - \frac{7}{\log 2}$ square units

15(a) $x = \frac{1}{2}\log\left(\frac{b}{a}\right)$, $y = \sqrt{ab}$ **(b)** $a + b - 2\sqrt{ab}$ square units

16 $\frac{\pi}{2}(8\log 2 - 3)$ cubic units

17 area $= e - \frac{1}{2} - \frac{1}{e}\,\text{m}^2$, so cost = \$29·80.

18(a)(i) $1 - e^{-N}$ **(ii)** 1 **(b)** 1

(c)(i) $\displaystyle\int_0^N 2xe^{-x^2}\,dx = 1 - e^{-N^2}$, thus in the limit as $N \to \infty$ this is just 1.

19(a) $2(e - e^{\sqrt{\delta}})$ **(b)** It approaches $2(e - 1)$.

20(b) $\frac{e^b - 1 - b}{b(e^b - 1)}$ **(c)** $\frac{1}{2}$ square units; the triangle with vertices at the origin, $(1, 0)$ and $(0, 1)$.

21(a) $1 - (1 + N)e^{-N}$ **(b)** 1 **(c)** 2

Exercise **13E** (Page 482)

1(a)(i) 13·6 **(ii)** 1·12 **(iii)** 47·4 **(b)(i)** 1·39 **(ii)** −3·22 **(iii)** 3·58 **(c)(i)** 4 **(ii)** $5\frac{1}{2}$ **(iii)** $\frac{1}{6}$ **(d)(i)** 82·789 **(ii)** 12·345 **(iii)** 4·330

2(a) $y \doteqdot 60{\cdot}26$ **(b)** $t \doteqdot 0{\cdot}6931$ **(c)** $\dfrac{dy}{dt} = -3$

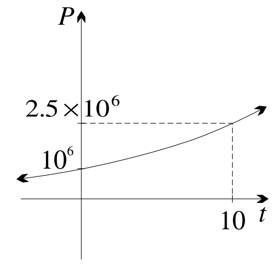
(d) $\dfrac{dy}{dt} = 2e$

3(a) $k = \frac{1}{10}\log\frac{5}{2}$ $\doteqdot 0{\cdot}092$

(b) 8·23 million

(c) during 2000

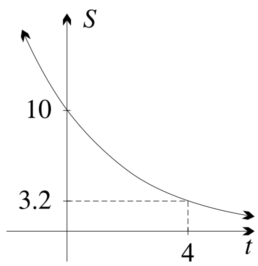
(d) $\dfrac{dP}{dt} = kP$ $\doteqdot 916\,000$



4(a) $k = -\frac{1}{4}\log\frac{3{\cdot}2}{10}$ $\doteqdot 0{\cdot}28$

(b) $t \doteqdot 8{\cdot}08$ hours $\doteqdot 8$ hours 5 minutes

(c) 2 kg/h



5(b) V_0 is the value of V at $t = 0$.

(c) $-\log\frac{7}{10} \doteqdot 0{\cdot}36$ **(d)** $\frac{1}{k}\log 20 \doteqdot 9$ years

6(b) $k = \frac{1}{3}\log 2 \doteqdot 0{\cdot}23$ **(c)** $2{\cdot}52\,\text{cm}^2$

(d) 21 hours 50 minutes

7(b) $h_0 = 100$ **(c)** $k = -\frac{1}{5}\log\frac{2}{5} \doteqdot 0{\cdot}18$ **(d)** 6·4° C

8(a) 72% **(b)** 37% **(c)** 7%

9(a) $k = \frac{\log 2}{1690} \doteqdot 4{\cdot}10 \times 10^{-4}$

(b) $t = \frac{\log 5}{k} \doteqdot 3924$ years

10(b) $k = \frac{2}{3}\log\frac{3}{2} \doteqdot 0{\cdot}27$

(c) 5 min 57 sec, to the nearest second

11(b) $k = \frac{\log 2}{5750} \doteqdot 1{\cdot}21 \times 10^{-4}$

(c) $t = \frac{1}{k}\log\frac{100}{15} \doteqdot 16\,000$ years, to the nearest 1000 years

12(b) 8 more years

13(b) $k = \frac{2\log 2}{3} \doteqdot 0{\cdot}46$ **(c)** 10 hours

14(b) $\mu_1 = 1{\cdot}21 \times 10^{-4}$ **(c)** $\mu_2 = 1{\cdot}16 \times 10^{-4}$

(d) The values of μ differ so the data are inconsistent. **(e)(i)** 625·5 millibars

(ii) 1143·1 millibars **(iii)** 19 205 metres

15(a) 34 minutes **(b)** 2·5%

16(a) $k = \frac{\log 2}{5730} \doteqdot 1{\cdot}21 \times 10^{-4}$, $C_0 = 15{\cdot}3$

(b) 2728 years old **(c)(i)** 15 847 years old

(ii) Further tests should be carried out.

17(a) $p = \frac{1}{10}\log\frac{5}{4} \doteqdot 0{\cdot}022$, $q = \frac{1}{10}\log\frac{6}{5} \doteqdot 0{\cdot}018$

(b) $t = \frac{\log 2}{p+q} \doteqdot 17{\cdot}10$ years, that is during 1997.

18(b) $t = \frac{1}{\lambda}\log(1 + \frac{D}{P})$

(c)(i) $1{\cdot}47 \times 10^{-11}$ **(ii)** 500 million

20(b) $z' = y'' - \lambda y'$ **(d)** $z = A\,e^{\mu t}$

(f) Only the last step would change, with $e^{-\lambda t}(y' - \lambda y) = A$ and hence $y = (At + B)e^{\lambda t}$.

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Chapter Fourteen

Exercise 14A (Page 490)

1(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{\pi}{6}$ **(d)** $\frac{\pi}{3}$ **(e)** $\frac{2\pi}{3}$ **(f)** $\frac{5\pi}{6}$ **(g)** $\frac{3\pi}{4}$ **(h)** $\frac{5\pi}{4}$ **(i)** 2π **(j)** $\frac{5\pi}{3}$ **(k)** $\frac{3\pi}{2}$ **(l)** $\frac{7\pi}{6}$

2(a) $180°$ **(b)** $360°$ **(c)** $720°$ **(d)** $90°$ **(e)** $60°$ **(f)** $45°$ **(g)** $120°$ **(h)** $150°$ **(i)** $135°$ **(j)** $270°$ **(k)** $240°$ **(l)** $315°$ **(m)** $330°$

3(a) $1{\cdot}274$ **(b)** $0{\cdot}244$ **(c)** $2{\cdot}932$ **(d)** $0{\cdot}377$ **(e)** $1{\cdot}663$ **(f)** $3{\cdot}686$

4(a) $114°35'$ **(b)** $17°11'$ **(c)** $82°30'$ **(d)** $7°3'$ **(e)** $183°16'$ **(f)** $323°36'$

5(a) $0{\cdot}91$ **(b)** $-0{\cdot}80$ **(c)** $0{\cdot}07$ **(d)** $1{\cdot}55$ **(e)** $2{\cdot}99$ **(f)** $-0{\cdot}97$

6(a) $\frac{\sqrt{3}}{2}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $-\frac{\sqrt{3}}{2}$ **(d)** $\sqrt{3}$ **(e)** -1 **(f)** $\frac{1}{2}$ **(g)** $-\frac{1}{\sqrt{2}}$ **(h)** $\frac{1}{\sqrt{3}}$

7(a) $x = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ **(b)** $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$ **(c)** $x = \frac{3\pi}{4}$ or $\frac{7\pi}{4}$ **(d)** $x = \frac{\pi}{2}$ **(e)** $x = \frac{\pi}{6}$ or $\frac{11\pi}{6}$ **(f)** $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ **(g)** $x = \pi$ **(h)** $x = \frac{5\pi}{4}$ or $\frac{7\pi}{4}$ **(i)** $x = \frac{\pi}{4}$ or $\frac{5\pi}{4}$

8(a) $\frac{\pi}{9}$ **(b)** $\frac{\pi}{8}$ **(c)** $\frac{\pi}{5}$ **(d)** $\frac{5\pi}{9}$ **(e)** $\frac{5\pi}{8}$ **(f)** $\frac{7\pi}{5}$ **(g)** $\frac{13\pi}{72}$ **(h)** $\frac{263\pi}{720}$

9(a) $15°$ **(b)** $72°$ **(c)** $400°$ **(d)** $247{\cdot}5°$ **(e)** $306°$ **(f)** $276°$

10(a) $\frac{\pi}{3}$ **(b)** $\frac{5\pi}{6}$

11 $\frac{4\pi}{9}$

12(a) $0{\cdot}733$ **(b)** $0{\cdot}349$

13(a) $0{\cdot}283$ **(b)** $0{\cdot}819$

14(a) $0{\cdot}841$, $0{\cdot}997$, $0{\cdot}909$ **(b)** $1{\cdot}0$

15(a) $\sin x$ **(b)** $\cos x$ **(c)** $-\cos x$ **(d)** $-\cos x$ **(e)** $\tan x$ **(f)** $-\tan x$ **(g)** $-\sec x$ **(h)** $\sec x$

16(a) $\frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $-\frac{1}{\sqrt{3}}$ **(d)** $-\frac{1}{\sqrt{3}}$ **(e)** -1 **(f)** $-\sqrt{2}$

17(a) $x = \frac{\pi}{8}$ or $\frac{9\pi}{8}$ **(b)** $x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$ or $\frac{3\pi}{2}$ **(c)** $x = \frac{\pi}{2}, \pi$ or $\frac{3\pi}{2}$ **(d)** $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ or 2π **(e)** $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ **(f)** $x = \frac{\pi}{3}, \frac{3\pi}{4}, \frac{4\pi}{3}$ or $\frac{7\pi}{4}$

18(a) $x = \frac{\pi}{12}$ or $\frac{5\pi}{12}$ **(b)** $x = -\pi, -\frac{\pi}{3}, \frac{\pi}{3}$ or π **(c)** $x = -\frac{\pi}{2}$ or $\frac{\pi}{2}$ **(d)** $x = -\pi$ or $\frac{\pi}{2}$ **(e)** $x = -\frac{3\pi}{4}$ **(f)** $x = -\frac{2\pi}{3}$ or $\frac{\pi}{3}$

19 $6°11'15''$

20 $\frac{169\pi}{1200}$

21 $\frac{2\pi}{5}, \frac{\pi}{2}, \frac{3\pi}{5}, \frac{7\pi}{10}, \frac{4\pi}{5}$

22(a) The solutions of $\sin x = 0$ are $x = k\pi$ where k is an integer. Since π is irrational, $k\pi$ is never an integer when k is an integer. **(b)** $n = 22$ is the first positive integer solution of $|\sin n| < 0{\cdot}01$. Because $\pi \doteqdot \frac{22}{7}$, $\sin 22 \doteqdot \sin 7\pi = 0$.

23 $\theta = \frac{3\pi}{10}, \frac{7\pi}{10}, \frac{11\pi}{10}$ or $\frac{19\pi}{10}$

Exercise 14B (Page 493)

1(a) $12\,\text{cm}$ **(b)** $2\pi\,\text{cm}$

2(a) $32\,\text{cm}^2$ **(b)** $12\pi\,\text{cm}^2$

3 $4\,\text{cm}$

4 $1{\cdot}5$ radians

5(a) $2{\cdot}4\,\text{cm}$ **(b)** $4{\cdot}4\,\text{cm}$

6 $8727\,\text{m}^2$

7(a) $8\pi\,\text{cm}$ **(b)** $16\pi\,\text{cm}^2$

8 $84°$

9 $11{\cdot}6\,\text{cm}$

10(a) $6\pi\,\text{cm}^2$ **(b)** $9\sqrt{3}\,\text{cm}^2$ **(c)** $3(2\pi - 3\sqrt{3})\,\text{cm}^2$

11 $\frac{4}{3}(5\pi - 3)$, $\frac{4}{3}(7\pi + 3)$

12 $15\,\text{cm}^2$

13(a) $4(\pi + 2)\,\text{cm}$ **(b)** $8\pi\,\text{cm}^2$

14(a) $\frac{25\pi}{2}\,\text{cm}^2$ **(b)** $\frac{25(4-\pi)}{2}\,\text{cm}^2$

15(a) $\frac{4\pi}{3}$ **(b)** $5\,\text{cm}$

16(a) $21\,\text{rad/s}$ **(b)** $6\,\text{m}$

17 $\frac{4}{3}(4\pi - 3\sqrt{3})\,\text{cm}^2$

18(a) $1{\cdot}38$ radians **(b)** $10\,\text{cm}^2$

19(c) $3\sqrt{55}\pi\,\text{cm}^3$ **(d)** $24\pi\,\text{cm}^2$

20(a) $\frac{2\pi}{3}\,\text{cm}$ **(b)** $\frac{2\pi}{3}\,\text{cm}^2$ **(c)** $2\pi\,\text{cm}$ **(d)** $\sqrt{3}\,\text{cm}^2$, $2(\pi - \sqrt{3})\,\text{cm}^2$

21(a) $1{\cdot}2661$ radians **(b)** $49{\cdot}2\,\text{cm}$

22(b) $9\,\text{cm}^2$

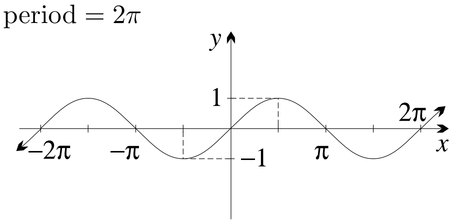
24(a) 1 radian **(b)** Since $\triangle OAC$ is equilateral, chord AC = the radius = arc AB, so B must lie on the minor arc AC. Since $\angle AOC = 60°$ and $\angle AOB = 1$ radian, it follows that 1 radian $< 60°$.

25 $2{\cdot}54\,\text{cm}^2$

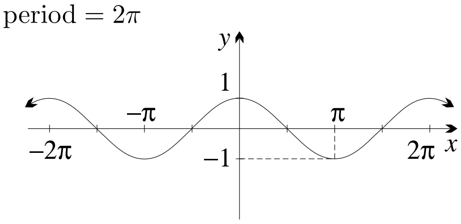
26 36 seconds

Exercise 14C (Page 501)

1(a) period $= 2\pi$



(b) period $= 2\pi$



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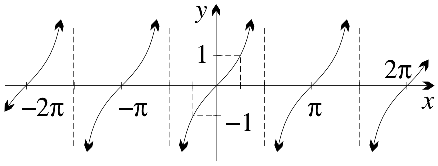
(c) period $= \pi$



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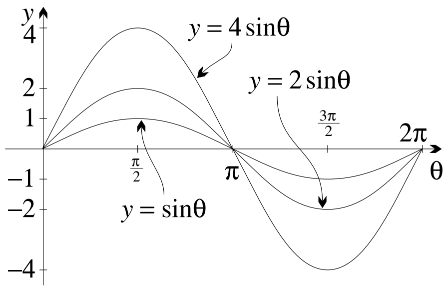

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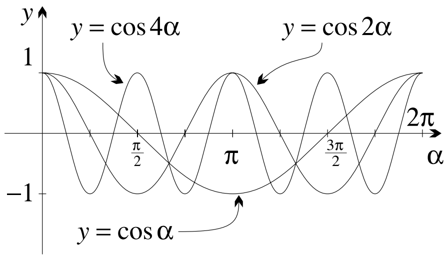

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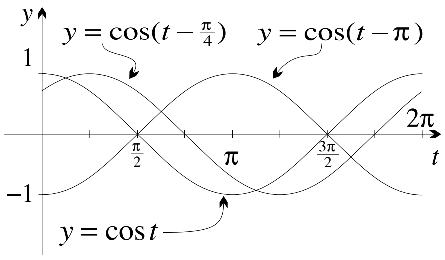
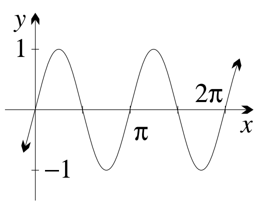
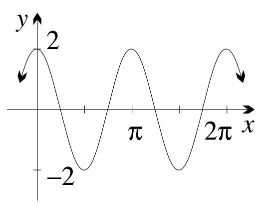


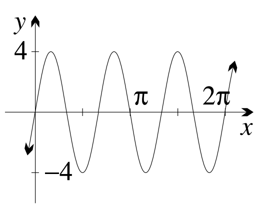
5(a) period $= \pi$, amplitude $= 1$



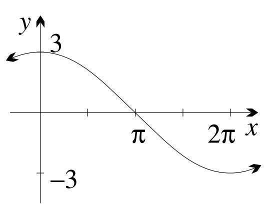
(b) period $= \pi$, amplitude $= 2$



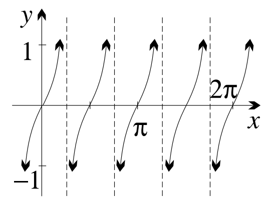
(c) period $= \frac{2\pi}{3}$, amplitude $= 4$



(d) period $= 4\pi$, amplitude $= 3$



(e) period $= \frac{\pi}{2}$, no amplitude



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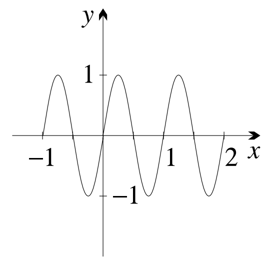

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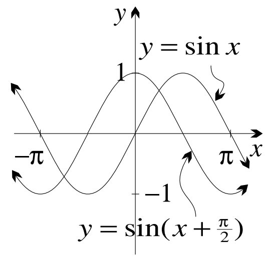
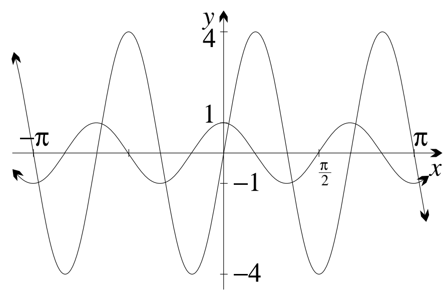


(c) $\sin(x + \frac{\pi}{2}) = \cos x$

8(a) 3 **(b)** 3 solutions, 1 positive solution **(c)** Outside this domain the line is beyond the range of the sine curve.

9 $x \doteqdot 1{\cdot}9$, $x \doteqdot -1{\cdot}9$ or $x = 0$

10



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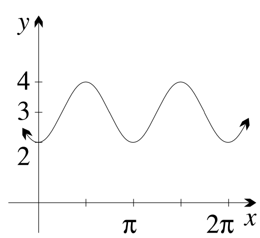
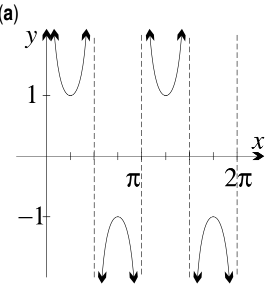
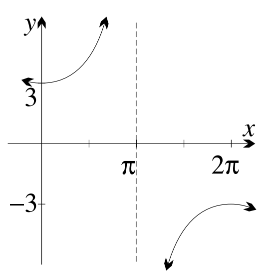


12(a)



(b)



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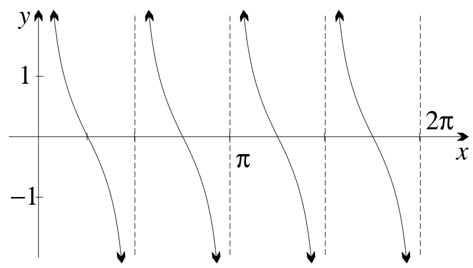
(c)



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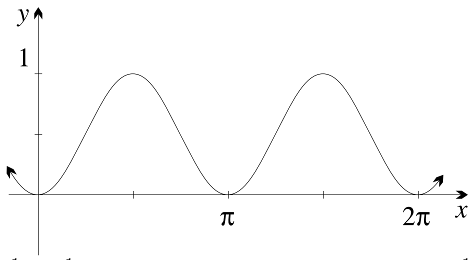
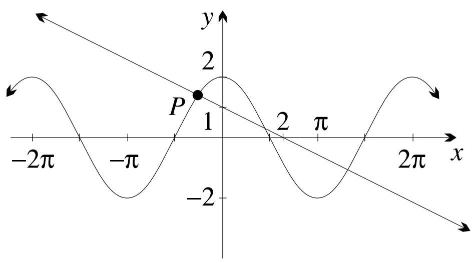


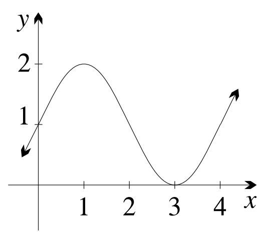
(a) $0, \frac{1}{2}, 1, \frac{1}{2}, 0$ **(c)** period $= \pi$, amplitude $= \frac{1}{2}$

14



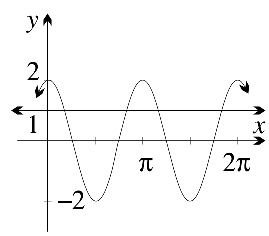
(c) 3 **(d)** P is in the second quadrant.

15



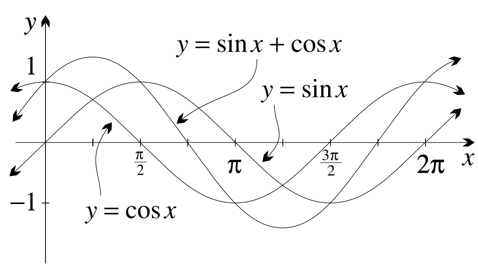
(a) 4 **(c)** the origin
(d) $m > \frac{1}{4}$

16



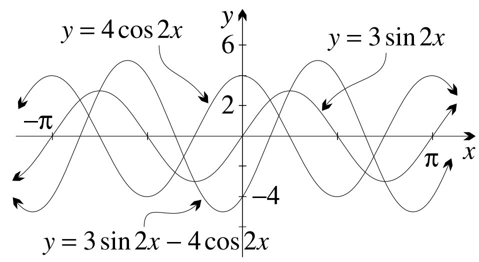
(c) 4 **(d)** $x = \frac{\pi}{6}$
(e) $\frac{\pi}{6} < x < \frac{5\pi}{6}$
or $\frac{7\pi}{6} < x < \frac{11\pi}{6}$

17



(c) 2π **(d)** $1{\cdot}4$

18

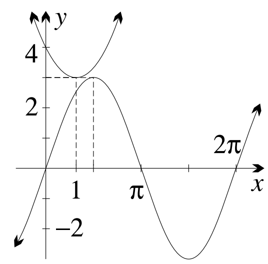


(c) amplitude $= 5$

19(a)(ii) 1 **(iii)** $0 < k < 1$ **(b)(ii)** $1{\cdot}3$
(iii) $\angle AOB = 2\theta \doteqdot 2{\cdot}6$ radians **(c)(ii)** $\ell > 300$

20(a)(ii) $2{\cdot}55$ **(b)** 146° **(c)(ii)** 205°

21



22(a) $x = -2\pi,\ x = -\pi,\ x = 0,\ x = \pi,\ x = 2\pi$
(b) each of its x-intercepts **(c)** translations to the right or left by 2π or by integer multiples of 2π
(d) translation right or left by π **(e)** translation to the right by $\frac{\pi}{2}$ or to the left by $\frac{3\pi}{2}$
(f) $x = \frac{\pi}{4},\ x = -\frac{3\pi}{4}$

23(a) There are none. **(b)** each of its x-intercepts
(c) translations to the right or left by π or by integer multiples of π **(d)** $x = \frac{\pi}{4},\ x = -\frac{\pi}{4}$

24(a) 63 **(b)** 218 (There is more than one answer.)

Exercise **14D** (Page 507)

1(a) $\sin x \cos y - \cos x \sin y$
(b) $\cos 2A \cos 3B - \sin 2A \sin 3B$
(c) $\sin 3\alpha \cos 5\beta + \cos 3\alpha \sin 5\beta$
(d) $\cos\theta \cos\frac{\phi}{2} + \sin\theta \sin\frac{\phi}{2}$ **(e)** $\frac{\tan A + \tan 2B}{1 - \tan A \tan 2B}$
(f) $\frac{\tan 3\alpha - \tan 4\beta}{1 + \tan 3\alpha \tan 4\beta}$

2(a) $\cos(x + y)$ **(b)** $\sin(3\alpha + 2\beta)$ **(c)** $\tan 20^\circ$
(d) $\sin 3A$ **(e)** $\cos 50^\circ$ **(f)** $\tan(\alpha + 10^\circ)$

3(a) $\sin 2x$ **(b)** $\cos 2\theta$ **(c)** $\tan 2\alpha$ **(d)** $\sin 40^\circ$
(e) $\cos 100^\circ$ **(f)** $\tan 140^\circ$ **(g)** $\sin 6\theta$ **(h)** $\cos 4A$
(i) $\tan 8x$

7(b)(i) $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ **(ii)** $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ **(iii)** $2 + \sqrt{3}$

8(a) $\frac{7}{25}$ **(b)** $\frac{1}{9}$ **(c)** $\frac{120}{169}$ **(d)** $\frac{4}{3}$

9(a) 1 **(b)** $\frac{63}{65}$ **(c)** $\frac{\sqrt{5}(1+2\sqrt{3})}{12}$

11 $-\frac{3\sqrt{7}}{8}$

13(a) $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **(b)** $\frac{1-\sqrt{3}}{2\sqrt{2}}$ **(c)** $\sqrt{3} - 2$

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17(b) $\frac{\sqrt{3}}{2}$
19(a) -1 **(c)** $\sqrt{2}+1$
20(b) $\sqrt{2}-1$
21(a) $\sin\alpha\cos\beta+\cos\alpha\sin\beta$

Exercise 14E (Page 511)

1(a) $37°$ **(b)** $41°$ **(c)** $33°$
2(a) $45°$ **(b)** $45°$ **(c)** $45°$ **(d)** $90°$ **(e)** $30°$
(f) The lines are parallel and distinct, and so do not intersect at all.
3 $3°11'$
4 $36°52'$
5(a) $(1,1)$ **(c)** $53°$
6(b) $63°$
7(a) $30°$, $150°$ **(b)** $45°$, $135°$
8 $\angle A = 71°34'$, $\angle B = 56°19'$ and $\angle C = 52°8'$. The sum is $180°1'$. The error is due to rounding.
9(a) $m=-3$ or $\frac{1}{3}$
(b) Let A and B be the points where $y=-3x$ and $y=\frac{1}{3}x$ meet $y=2x-4$. $\triangle AOB$ has two $45°$ angles, so the third angle, $\angle AOB$, is $90°$.
10(b) $3x+2y-5=0, 2x-3y+1=0$
11(a) $(2-\sqrt{3})x-y+\sqrt{3}=0, (2+\sqrt{3})x-y-\sqrt{3}=0$
(b) $x-y+1=0, 7x-y-11=0$
12(b) $75°58'$ at $(0,0)$ and $17°6'$ at $(1,3)$
13(b) 3
14 At $(0,0)$ the curves are perpendicular. At $(1,1)$ and at $(-1,1)$, $\tan\theta=\frac{4}{7}$, so the angle is $29°45'$.
15 $x=0, y=-\sqrt{3}x$

Exercise 14F (Page 515)

1(a) The entries under $5°$ are $0{\cdot}087\,27$, $0{\cdot}087\,16$, $0{\cdot}9987$, $0{\cdot}087\,49$, $1{\cdot}003$, $0{\cdot}9962$
(b) $\sin x < x < \tan x$ **(c)** 1 and 1
(d) $x \le 0{\cdot}0774$ (to 4 decimal places), that is, $x \le 4°26'$
2(a) 1 **(b)** 2 **(c)** $\frac{1}{2}$ **(d)** $\frac{3}{2}$ **(e)** $\frac{5}{3}$ **(f)** 8
3(a) 1 **(b)** $\frac{7}{5}$ **(c)** $\frac{7}{15}$
4(a) 2 **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{6}$
5(a) 2 **(b)** $\frac{3}{2}$ **(c)** $\frac{5}{7}$
6(a) 1 **(b)** $\frac{a}{b}$ **(c)** 0
8(a) $\cos A\cos B-\sin A\sin B$ **(c)** 2
9(a) $\frac{\pi}{90}$ **(b)** $\sin 2° = \sin\frac{\pi}{90} \doteqdot \frac{\pi}{90}$ **(c)** $0{\cdot}0349$
10 87 metres
11 $26'$
14(c) $23°$ **(d)** $4{\cdot}924$ metres
15(a) $\sin(A-B)=\sin A\cos B-\cos A\sin B$ **(d)** 6
16(a) $AB^2=2r^2(1-\cos x)$, arc $AB = rx$
(b) The arc is longer than the chord, so $\cos x$ is larger than the approximation.

Exercise 14G (Page 520)

1 $y=\cos x$
2(a) $\cos x$ **(b)** $-\sin x$ **(c)** $\sec^2 x$ **(d)** $2\cos 2x$
(e) $-2\sin x$ **(f)** $4\sec^2 2x$ **(g)** $2\pi\cos 2\pi x$
(h) $\frac{\pi}{2}\sec^2\frac{\pi}{2}x$ **(i)** $3\cos x-5\sin 5x$
(j) $4\pi\cos\pi x-3\pi\sin\pi x$ **(k)** $-5\sin(5x+4)$
(l) $-21\cos(2-3x)$ **(m)** $-10\sec^2(10-x)$
(n) $-2\sec^2(\frac{\pi}{2}-2\pi x)$ **(o)** $3\cos(\frac{x+1}{2})$
(p) $6\sin(\frac{3-2x}{5})$
3(a) $2x\cos(x^2)$ **(b)** $-3x^2\sin(x^3+1)$ **(c)** $-\frac{1}{x^2}\cos(\frac{1}{x})$
(d) $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$ **(e)** $x\cos x+\sin x$ **(f)** $2x(\cos 2x - x\sin 2x)$ **(g)** $-2\cos x\sin x$ **(h)** $3\sin^2 x\cos x$
(i) $-\frac{\cos x}{(1+\sin x)^2}$ **(j)** $\frac{1}{1+\cos x}$ **(k)** $\frac{-1}{1+\sin x}$
(l) $\frac{-1}{(\cos x+\sin x)^2}$ **(m)** $-\tan x$ **(n)** $\sec^2 x.e^{\tan x}$
4(a) The graphs are reflections of each other in the x-axis. **(b)** The graphs are identical.
5(a) $2\cos 2x.e^{\sin 2x}$ **(b)** $2e^{2x}\cos(e^{2x})$ **(c)** $\frac{2\cos 4x}{\sqrt{\sin 4x}}$
(d) $4\cot 4x$ **(e)** $9\sin 3x(1-\cos 3x)^2$
(f) $2(\cos 2x\sin 4x+2\sin 2x\cos 4x)$
(g) $-15\cos^4 3x\sin 3x$ **(h)** $\frac{20\sin 5x}{(3+4\cos 5x)^2}$
(i) $15\tan^2(5x-4)\sec^2(5x-4)$ **(j)** $\frac{x\cos\sqrt{x^2+1}}{\sqrt{x^2+1}}$
7(a) $\frac{\pi x}{180}$ **(b)(i)** $\frac{\pi}{180}\cos x°$ **(ii)** $\frac{\pi}{180}\sec^2(x°+45°)$
(iii) $-\frac{\pi}{90}\sin 2x°$
10(a) $\log_b P-\log_b Q$
11(b) $\frac{1}{2}\Big((m+n)\cos(m+n)x+(m-n)\cos(m-n)x\Big)$
(c) $\frac{1}{2}(\cos(m+n)x+\cos(m-n)x)$
$= \cos mx\cos nx$,
$-\frac{1}{2}\Big((m+n)\sin(m+n)x+(m-n)\sin(m-n)x\Big)$
12(b) $\sin(\frac{n\pi}{2}+x)$
16(a) $\frac{\cos x}{\sin y}$ **(b)** $\frac{-y\cos x-\sin y}{x\cos y+\sin x}$ **(c)** $\frac{\sin(x-y)+\cos(x+y)}{\sin(x-y)-\cos(x+y)}$
18(b) $\sin 1 \doteqdot 0{\cdot}8415$, $\cos 1 \doteqdot 0{\cdot}5403$

Exercise 14H (Page 525)

1(a) $-\frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** 1 **(d)** -2 **(e)** $\frac{1}{4}$ **(f)** 8
2(a) $y=-2x+\frac{\pi}{2}$ **(b)** $x+y=\frac{\pi}{3}+\frac{\sqrt{3}}{2}$
(c) $y=-\pi x+\pi^2$
3(a) $x-y=\frac{\pi}{4}-\frac{1}{2}$, $x+y=\frac{\pi}{4}+\frac{1}{2}$ **(b)** $\frac{\pi^2-4}{32}$ units^2
4(a) $\frac{\pi}{2}, \frac{3\pi}{2}$ **(b)** $\frac{\pi}{6}, \frac{5\pi}{6}$ **(c)** $\frac{5\pi}{6}, \frac{7\pi}{6}$ **(d)** $\frac{\pi}{2}, \frac{3\pi}{2}$
5 $12\sqrt{3}x-6y=2\sqrt{3}\pi-3, 6x+12\sqrt{3}y=\pi+6\sqrt{3}$
6(a) $y'=-\sin x+\sqrt{3}\cos x$, $y''=-\cos x-\sqrt{3}\sin x$
(b) maximum turning point $(\frac{\pi}{3},2)$,
minimum turning point $(\frac{4\pi}{3},-2)$

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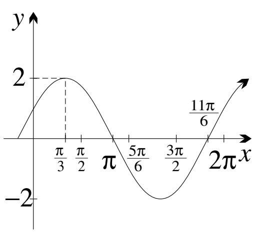
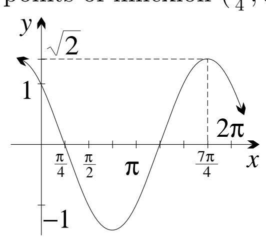
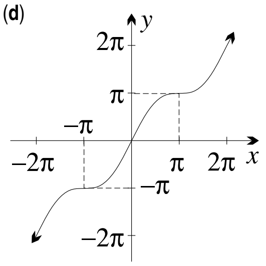
(c) $(\frac{5\pi}{6}, 0)$, $(\frac{11\pi}{6}, 0)$ **(d)**



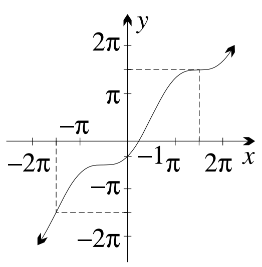
7 $y' = -\sin x - \cos x$, $y'' = -\cos x + \sin x$,
minimum turning point $(\frac{3\pi}{4}, -\sqrt{2})$,
maximum turning point $(\frac{7\pi}{4}, \sqrt{2})$,
points of inflexion $(\frac{\pi}{4}, 0)$, $(\frac{5\pi}{4}, 0)$



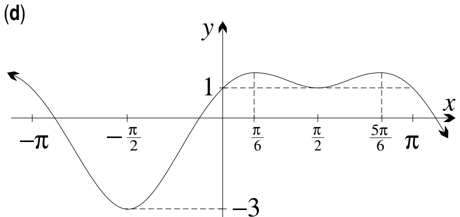
8(a) $y' = 1 + \cos x$, $y'' = -\sin x$ **(b)** $(-\pi, -\pi)$ and (π, π) are horizontal points of inflexion. **(c)** $(0, 0)$
(d)



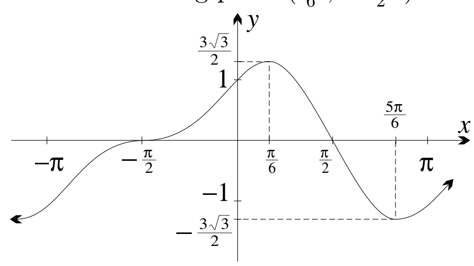
9 $y' = 1 + \sin x$, $y'' = \cos x$,
horizontal points of inflexion $(-\frac{\pi}{2}, -\frac{\pi}{2})$, $(\frac{3\pi}{2}, \frac{3\pi}{2})$,
points of inflexion $(-\frac{3\pi}{2}, -\frac{3\pi}{2})$, $(\frac{\pi}{2}, \frac{\pi}{2})$



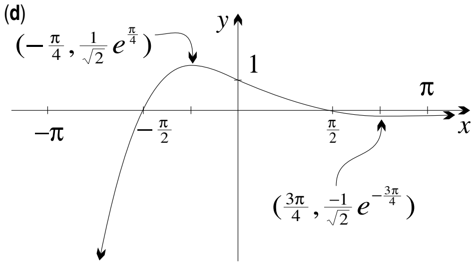
10(a) $y' = 2\cos x - 2\sin 2x$, $y'' = -2\sin x - 4\cos 2x$
(c) maximum turning points $(\frac{\pi}{6}, \frac{3}{2})$ and $(\frac{5\pi}{6}, \frac{3}{2})$,
minimum turning points $(-\frac{\pi}{2}, -3)$ and $(\frac{\pi}{2}, 1)$
(d)



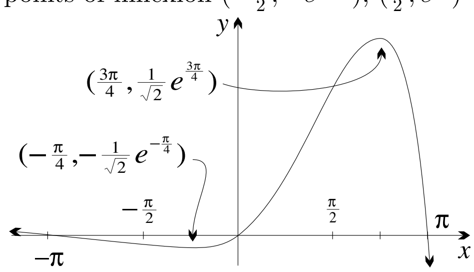
11 horizontal point of inflexion $(-\frac{\pi}{2}, 0)$,
maximum turning point $(\frac{\pi}{6}, \frac{3\sqrt{3}}{2})$,
minimum turning point $(\frac{5\pi}{6}, -\frac{3\sqrt{3}}{2})$



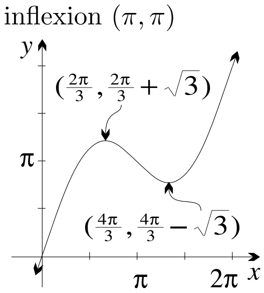
12(a) $y' = -e^{-x}(\cos x + \sin x)$, $y'' = 2e^{-x}\sin x$
(b) minimum turning point $(\frac{3\pi}{4}, -\frac{1}{\sqrt{2}}e^{-\frac{3\pi}{4}})$,
maximum turning point $(-\frac{\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{\pi}{4}})$
(c) $(-\pi, -e^{\pi})$, $(0, 1)$, $(\pi, -e^{-\pi})$
(d)



13 minimum turning point $(-\frac{\pi}{4}, -\frac{1}{\sqrt{2}}e^{-\frac{\pi}{4}})$,
maximum turning point $(\frac{3\pi}{4}, \frac{1}{\sqrt{2}}e^{\frac{3\pi}{4}})$,
points of inflexion $(-\frac{\pi}{2}, -e^{-\frac{\pi}{2}})$, $(\frac{\pi}{2}, e^{\frac{\pi}{2}})$



14(b) $\frac{9\sqrt{2}}{10}\ \text{cm}^2/\text{min}$ **(c)** $\theta = \pi$
19(a) The angle of inclination is $\pi - \alpha$ and so $m = \tan(\pi - \alpha) = -\tan\alpha$.
(b) $P = (\frac{1}{\tan\alpha} + 2, 0), Q = (0, 2\tan\alpha + 1)$
20 minimum $\sqrt{3}$ when $\theta = \frac{\pi}{6}$,
maximum 2 when $\theta = 0$
21(a) maximum turning point $(\frac{2\pi}{3}, \frac{2\pi}{3} + \sqrt{3})$,
minimum turning point $(\frac{4\pi}{3}, \frac{4\pi}{3} - \sqrt{3})$,
inflexion (π, π)



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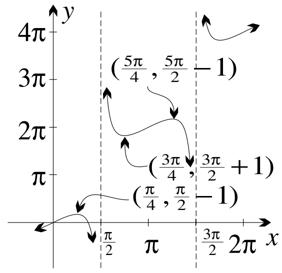
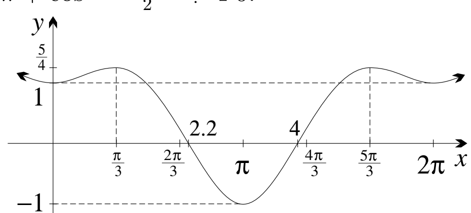
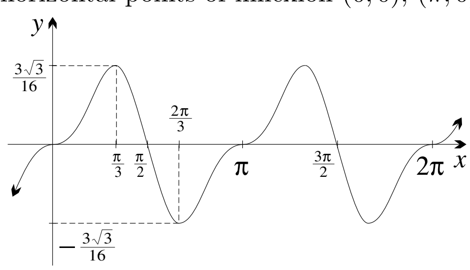
(b) point of inflexion $(\pi, 2\pi)$,
minimum turning points $(\frac{3\pi}{4}, \frac{3\pi}{2}+1)$, $(\frac{7\pi}{4}, \frac{7\pi}{2}+1)$,
maximum turning points $(\frac{\pi}{4}, \frac{\pi}{2}-1)$, $(\frac{5\pi}{4}, \frac{5\pi}{2}-1)$



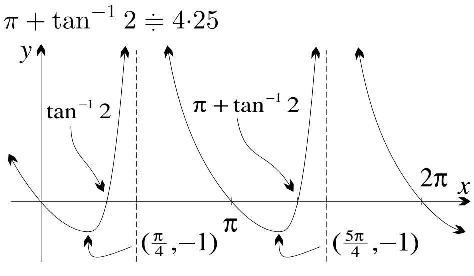
22(a) maximum turning points $(\frac{\pi}{3}, \frac{5}{4})$, $(\frac{5\pi}{3}, \frac{5}{4})$,
minimum turning points $(0, 1)$, $(\pi, -1)$, $(2\pi, 1)$,
x-intercepts $\pi - \cos^{-1}\frac{\sqrt{5}-1}{2} \doteqdot 2{\cdot}2$,
$\pi + \cos^{-1}\frac{\sqrt{5}-1}{2} \doteqdot 4{\cdot}0$.



(b) maximum turning points $(\frac{\pi}{3}, \frac{3\sqrt{3}}{16})$, $(\frac{4\pi}{3}, \frac{3\sqrt{3}}{16})$,
minimum turning points $(\frac{2\pi}{3}, -\frac{3\sqrt{3}}{16})$, $(\frac{5\pi}{3}, -\frac{3\sqrt{3}}{16})$
horizontal points of inflexion $(0, 0)$, $(\pi, 0)$, $(2\pi, 0)$



(c) minimum turning points $(\frac{\pi}{4}, -1)$, $(\frac{5\pi}{4}, -1)$,
vertical asymptotes $x = \frac{\pi}{2}$, $x = \frac{3\pi}{2}$,
x-intercepts 0, π, 2π, $\tan^{-1} 2 \doteqdot 1{\cdot}1$,
$\pi + \tan^{-1} 2 \doteqdot 4{\cdot}25$



23 $(\pi^2 - 8)x - (4\pi - 8)y + (32 - 8\pi) = 0$

24(c) $f'(x) = \frac{x\cos x - \sin x}{x^2}$

25(a) $y' = e^{\lambda x}(\lambda \sin nx + n\cos nx)$

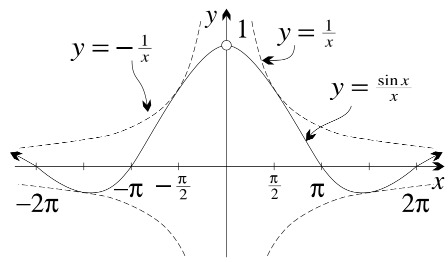
(c)(i) They approach $\frac{k\pi}{n}$ where k is an integer.

(ii) They approach $\frac{(k+\frac{1}{2})\pi}{n}$, where k is an integer.

26(a) Domain: $x \neq 0$, $f(x)$ is even because it is the ratio of two odd functions, the zeroes are $x = n\pi$ where n is an integer, $\lim\limits_{x\to\infty} f(x) = 0$.

(b) $f'(x) = \dfrac{x\cos x - \sin x}{x^2}$, which is zero when $\tan x = x$. **(c)** The graph of $y = x$ crosses the graph of $y = \tan x$ just to the left of $x = \frac{3\pi}{2}$, of $x = \frac{5\pi}{2}$ and of $x = \frac{7\pi}{2}$. Using the calculator, the three turning points of $y = f(x)$ are approximately $(1{\cdot}43\pi, -0{\cdot}217)$, $(2{\cdot}46\pi, 0{\cdot}128)$ and $(3{\cdot}47\pi, -0{\cdot}091)$.

(d) There is an open circle at $(0, 1)$.



Exercise 14I (Page 530)

2(a) $\tan x + C$ **(b)** $\sin(x+2) + C$ **(c)** $-\frac{1}{2}\cos 2x + C$
(d) $3\tan\frac{1}{3}x + C$ **(e)** $\frac{1}{3}\sin(3x-2) + C$
(f) $\frac{1}{5}\cos(7-5x) + C$ **(g)** $-\tan(4-x) + C$
(h) $-3\tan(\frac{1-x}{3}) + C$

3(a) $\frac{1}{2}$ **(b)** $\frac{1}{\sqrt{2}}$ **(c)** $\sqrt{3}$ **(d)** 1 **(e)** $\frac{3}{4}$ **(f)** 2 **(g)** 1
(h) 4

4(a) $2\sin 3x + 8\cos\frac{1}{2}x + C$
(b) $4\tan 2x - 40\sin\frac{1}{4}x - 36\cos\frac{1}{3}x + C$

5(a) $-\cos(ax+b) + C$ **(b)** $\pi\sin\pi x + C$
(c) $\frac{1}{u^2}\tan(v+ux) + C$ **(d)** $\tan ax + C$

6(a) $1 + \tan^2 x = \sec^2 x$, $\tan x - x + C$
(b) $1 - \sin^2 x = \cos^2 x$ $\quad 2\sqrt{3}$

7 The integrand $y = \sec^2 x$ is undefined at $x = \frac{\pi}{2}$, so it is not possible to form the definite integral over the interval $0 \le x \le \pi$. In any case, the result could hardly be 0, since the integrand is a square and can never be negative.

8(a) $\log_e f(x) + C$
(b) $\tan x = \frac{\sin x}{\cos x}$, $\displaystyle\int \tan x = -\ln\cos x + C$
(c) $\cos x e^{\sin x}$, $e - 1$ **(d)** $e^{f(x)} + C$, $e - 1$

9(a) $2x\cos x^2$, $\sin x^2 + C$
(b) $-3x^2\sin x^3$, $-\frac{1}{3}\cos x^3 + C$
(c) $\frac{1}{2\sqrt{x}}\sec^2\sqrt{x}$, $2\tan\sqrt{x} + C$

10(a) 1 **(b)** $\frac{5}{24}$

11(a) $5\sin^4 x\cos x$, $\frac{1}{5}\sin^5 x + C$
(b) $-3\sec^2 x(\tan x)^{-4}$, $-\frac{1}{3}(\tan x)^{-3} + C$

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12(a) $\frac{2}{9}$ **(b)** $\frac{1}{n+1}$ **(c)** 0 **(d)** $\frac{1}{2^{n+1}(n+1)}$ **(e)** $10\frac{1}{8}$
(f) $\frac{1}{n+1}$
13(b)(i) $\frac{1}{4}$ **(ii)** $\frac{1}{4}$ **(iii)** 0 (the integrand is odd)
14(d)(i) $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ **(ii)** $\frac{\pi}{4}$
(e) $\sin^2 2x = \frac{1}{2}(1 - \cos 4x)$, $\frac{\pi}{4}$
(f)(i) $\frac{1}{12}(2\pi + 3\sqrt{3})$ **(ii)** $\frac{1}{8}(\pi - 2\sqrt{2})$
15(a) $\frac{1}{2}\sin e^{2x} + C$ **(b)** $\frac{1}{2}\cos e^{-2x} + C$
(c) $\frac{1}{3}\log_e(3\tan x + 1) + C$
(d) $-\frac{3}{5}\log_e(4 + 5\cos x) + C$
(e) $\tan x - \sin x + C$ **(f)** $\frac{2}{3}$
16 $\sin 2x + 2x\cos 2x$, $\frac{\pi-2}{8}$
17(b) $\frac{4}{3}$
18(b)(i) $\frac{1}{2}\tan^2 x + \log_e(\cos x) + C$
(ii) $\frac{1}{4}\tan^4 x - \frac{1}{2}\tan^2 x - \log_e(\cos x) + C$
19(b)(i) $\frac{6}{5}$ **(ii)** $-\frac{6}{7}$
20(b)(i) $1\frac{1}{5}$ **(ii)** 0 **(c)(i)** $\frac{\sin(m+n)x}{2(m+n)} + \frac{\sin(m-n)x}{2(m-n)} + C$
(ii) $\frac{\sin(m+n)x}{2(m+n)} + C$
21(b) $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, so $\frac{1}{2}\sin^2 x + C$
$= \frac{1}{4} - \frac{1}{4}\cos 2x + C = -\frac{1}{4}\cos 2x + (C + \frac{1}{4})$
$= -\frac{1}{4}\cos 2x + D$, where $D = C + \frac{1}{4}$.
22(a) $A = 5, B = 3$
23(b) $y = (1 - \lambda)e^{\lambda x}\sin x + e^{\lambda x}\cos x$

Exercise **14J** (Page 535)

1(a) $2\,\mathrm{u}^2$ **(b)** $1\,\mathrm{u}^2$
2(a) $(2 - \sqrt{2})\,\mathrm{u}^2$ **(b)** $\frac{1}{4}\,\mathrm{u}^2$
3(a) $2\,\mathrm{u}^2$ **(b)** $\sqrt{2}\,\mathrm{u}^2$ **(c)** $\frac{2}{3}\sqrt{3}\,\mathrm{u}^2$ **(d)** $2\,\mathrm{u}^2$ **(e)** $\frac{1}{2}\,\mathrm{u}^2$
(f) $4\,\mathrm{u}^2$ **(g)** $4\,\mathrm{u}^2$ **(h)** $1\,\mathrm{u}^2$
4(a) $y = \sin x + \cos 2x - 1$ **(b)** $\frac{1}{\pi}$ **(c)** $\frac{1}{2}\sqrt{3}$
(d) $f(x) = -2\cos 3x + x + (1 - \frac{\pi}{2})$
5(a) $\pi\,\mathrm{u}^3$ **(b)** $\frac{\pi}{4}\,\mathrm{u}^3$ **(c)** $\frac{\pi}{4}(\pi + 2)\,\mathrm{u}^3$
6(b) $\frac{4}{\pi}\,\mathrm{u}^2$
7 $3{\cdot}8\,\mathrm{m}^2$
8(b) They are all $4\,\mathrm{u}^2$.
9(b) $\frac{1}{6}(1 + 2\sqrt{2})$
10 $4\,\mathrm{u}^2$
11(a) $\ln 2\,\mathrm{u}^2$ **(b)** $\frac{\pi}{3}(3\sqrt{3} - \pi)\,\mathrm{u}^3$
12(a) $\log\sin x$ **(b)** $\log 2\,\mathrm{u}^2$
(c)(i) The calculation is valid. The regions above and below the x-axis have equal area, so the integral is zero.
(ii) There are asymptotes within the interval at $x = \pi$ and $x = 2\pi$, so the definite integral is not defined, and the calculation is therefore invalid.
13(a) $1\,\mathrm{u}^2$ **(b)** $\frac{\pi^2}{4}\,\mathrm{u}^3$
14(a) $\frac{\sqrt{3}}{2}\,\mathrm{u}^2$ **(b)** $\frac{\pi}{24}(4\pi + 3\sqrt{3})\,\mathrm{u}^3$
16(b) $\frac{1}{2}(3 + \sqrt{3})\,\mathrm{u}^2$
17(c) $\frac{3}{4}\sqrt{3}\,\mathrm{u}^2$
18(b) The curve is below $y = 1$ just as much as it is above $y = 1$, so the area is equal to the area of a rectangle n units long and one unit high.
19(a) 0 **(b)** As $n \to \infty$ the period of the sine curve approaches zero, and so the area approaches zero.
20(b)(i) $2\sqrt{2}\,\mathrm{u}^2$ **(ii)** $\pi^2\,\mathrm{u}^3$
21(b)(i) $(\frac{\pi}{4} - \frac{1}{2}\ln 2)\,\mathrm{u}^2$ **(ii)** $\pi(1 - \ln 2)\,\mathrm{u}^3$
22 $71{\cdot}62\,\mathrm{mL}$
23 12
24(a) We know that $\sin x < x < \tan x$ for $0 < x < \frac{\pi}{2}$. Since $x^2 > 0$, the result follows.
25(b) $\cos x$ and $(1 + \sin x)^2$ are both positive in the given domain, so y' is negative there.
26(a) 0, since the integrand is odd. **(b)** 0, since the integrand is odd. **(c)** 2, since the integrand is even. **(d)** $6\sqrt{3}$, since the integrand is even. **(e)** 6π. The first term is even, the other two are odd. **(f)** $-\frac{2}{3} + \frac{\pi^3}{4}$. The first term is odd, the other two are even.
27(b) $\frac{1}{24}\sqrt{2}(12 - \pi)$ **(c)** $\frac{\pi}{24}(\sqrt{3} + 1)$ and $\frac{\pi}{12}\sqrt{2}$

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