PHYS 1902
Electromagnetism: 1

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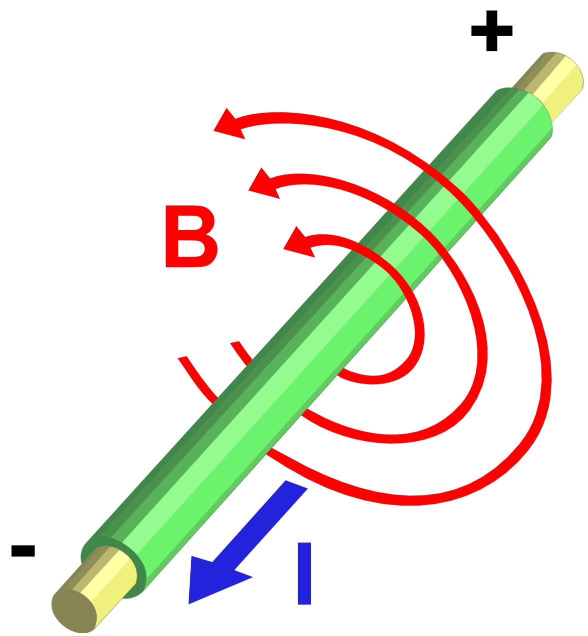


SPOILER WARNING!

Electromagnetism
- The Whole Thing

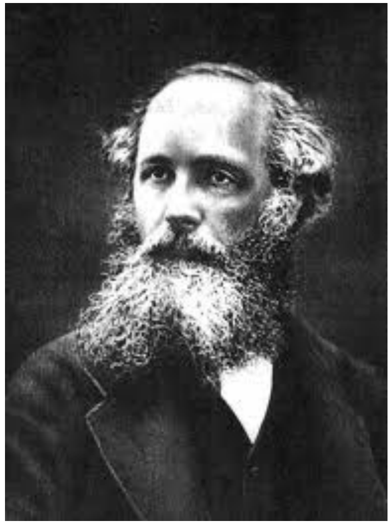
Maxwell's equations:

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

$$\oint \vec{B} \cdot d\vec{A} = 0$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi_B}{dt}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc} + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt}$$



Lorentz Force: $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$

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EM Module

By the end of this module, you should be able to:

- describe electric and magnetic fields and their effects on charges and currents, understanding each field as an alternative way of describing the corresponding force.
- relate each of Maxwell's equations to the properties of the fields it describes
- understand the ways that fields are created and use Maxwell's equations to calculate the fields produced
- describe the relationships between electric and magnetic fields and appreciate that light is a consequence of this relationship

Where to get help

- Textbook, especially broblems
- Workshop tutorials
- Duty Tutor

Note!

A lot of work will be on the board, & slides may not be in step with lecture. Also, examples are only worked out on the board!

Chapter 21
Electric Charge and Electric Field

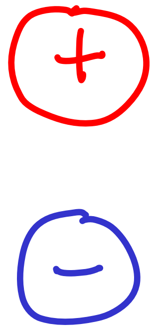
Electric Charge

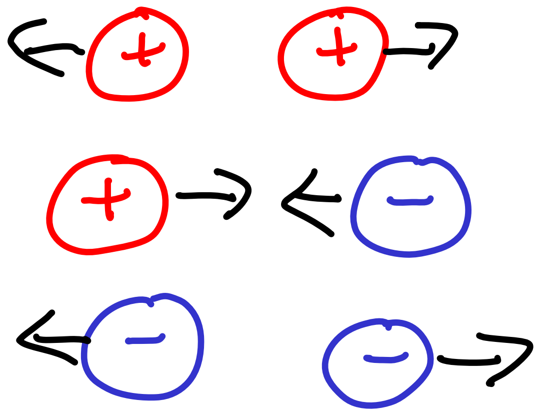
Some things "have" charge just like they "have" mass

Unlike mass, charge can come in two types:

Positive

Negative





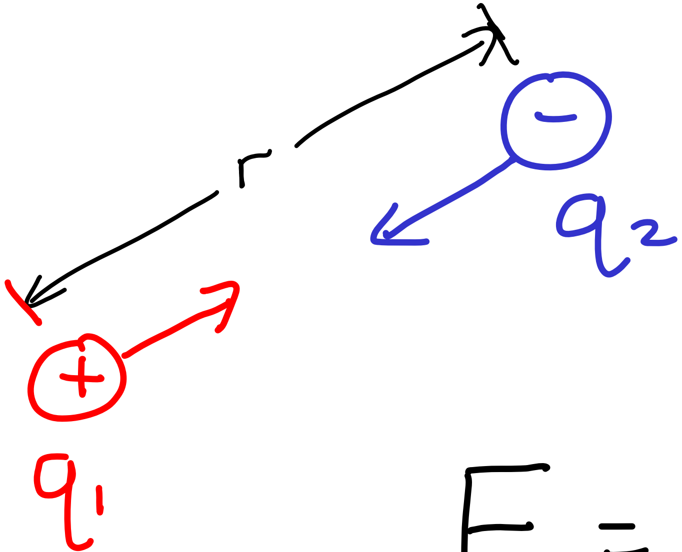
Opposite signs attract, like signs repel

SI Unit of charge is the Coulomb (C)

Coulomb's Law

Charles-Augustin de Coulomb

(from Wikipedia)



$$F = \frac{1}{4\pi\epsilon_0}\frac{|q_1 q_2|}{r^2}$$

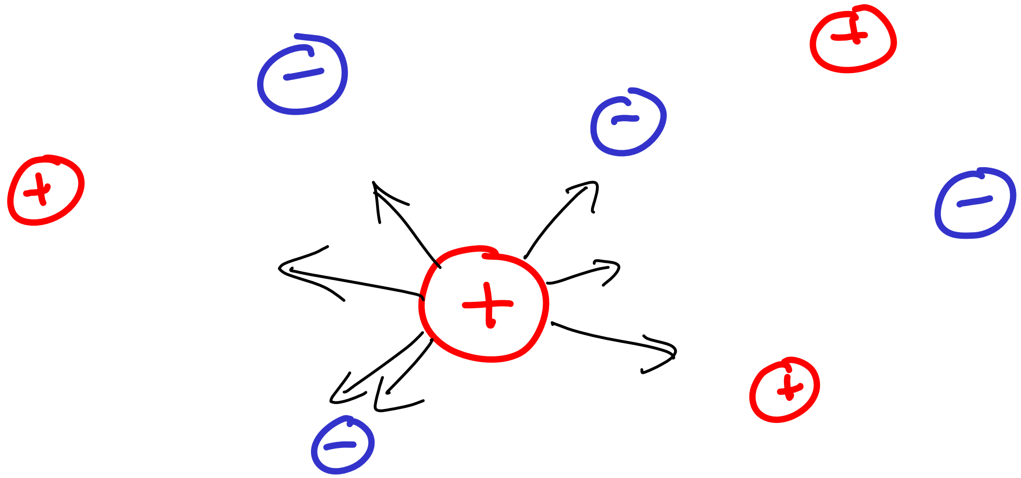
$\epsilon_0 = 8.854 \times 10^{-12}\ C^2/m^2 N$

(Permittivity of Free Space)

Remember:

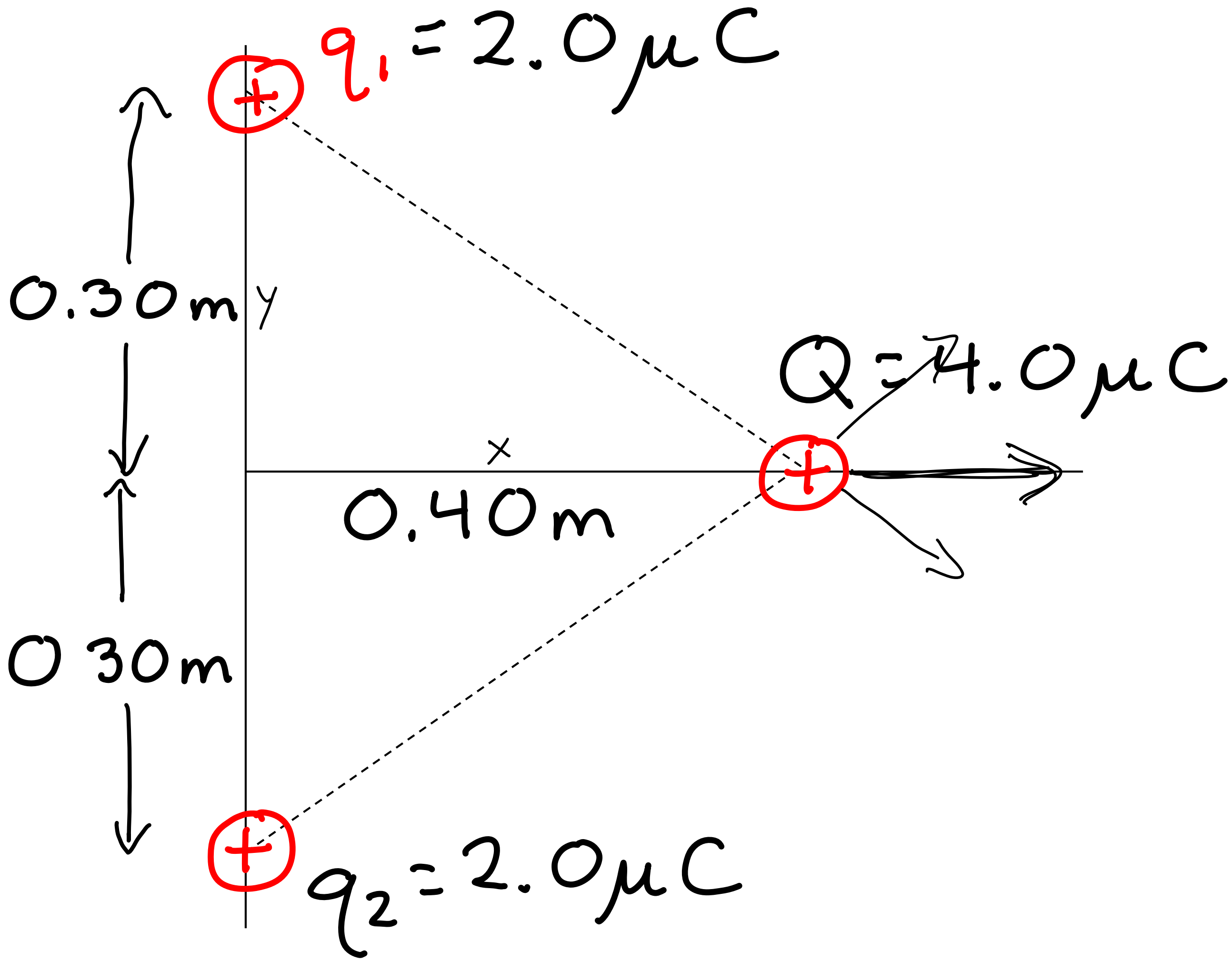
- force is a *vector*
- direction is determined by the *sign* of the charge

Superposition Principle

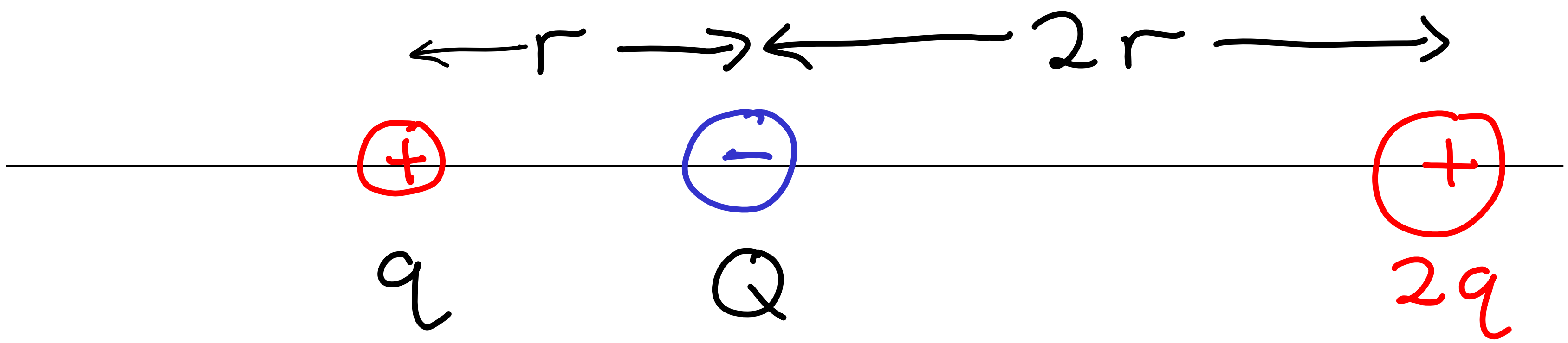


$$\vec{F}_{total} = \sum_i \vec{F}_i$$

A Simple Problem



Another Simple Problem



In which direction is the net force on the charge Q?

The Electric Field

Defined as:

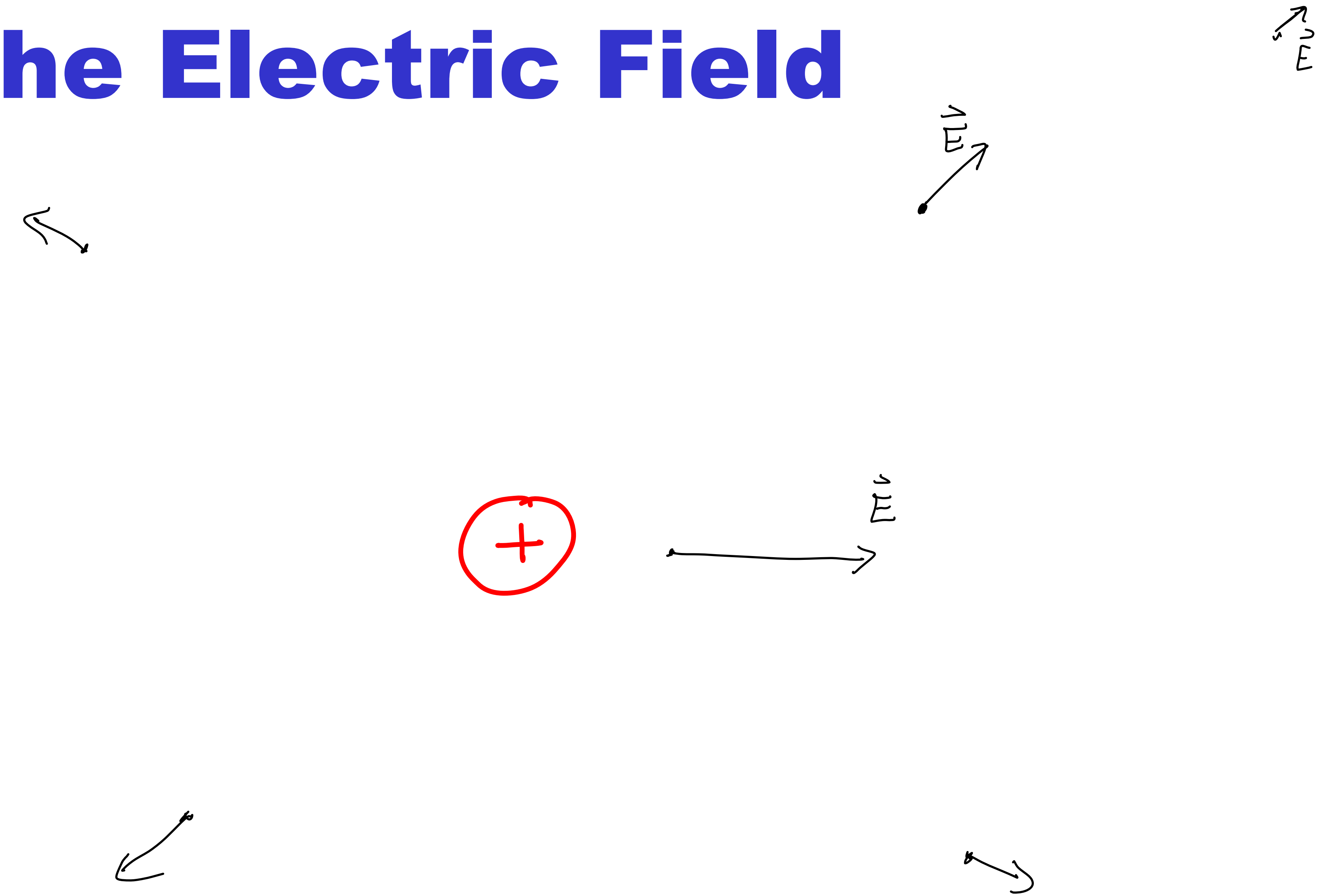
$$\vec{E} = \frac{\vec{F}_o}{q_o}$$

Of a point charge:

$$\vec{E} = \frac{1}{4\pi\epsilon_o}\frac{q}{r^2}\hat{r}$$

Just like forces, there is a superposition principle for the electric field

The Electric Field



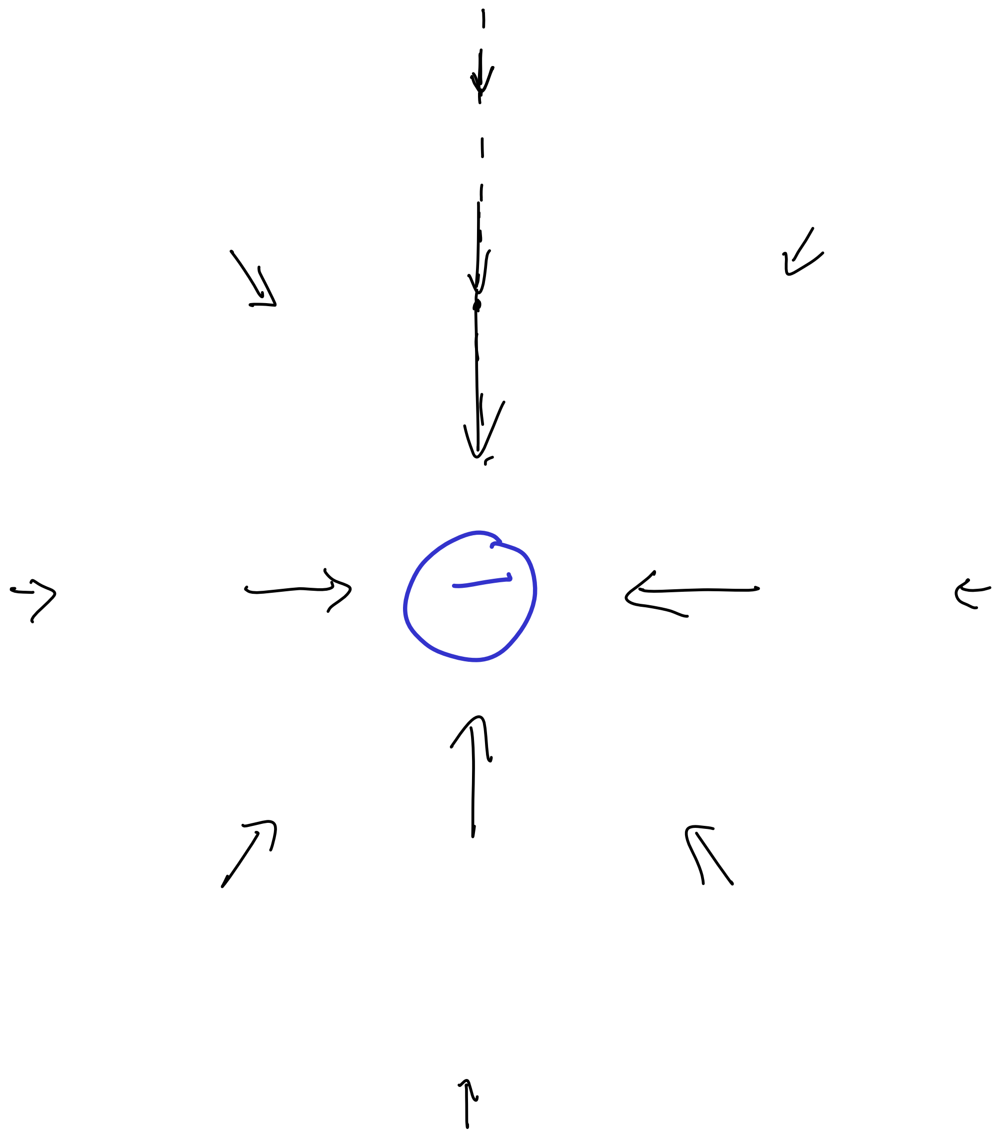
The electric field is a "vector field"

The Electric Field

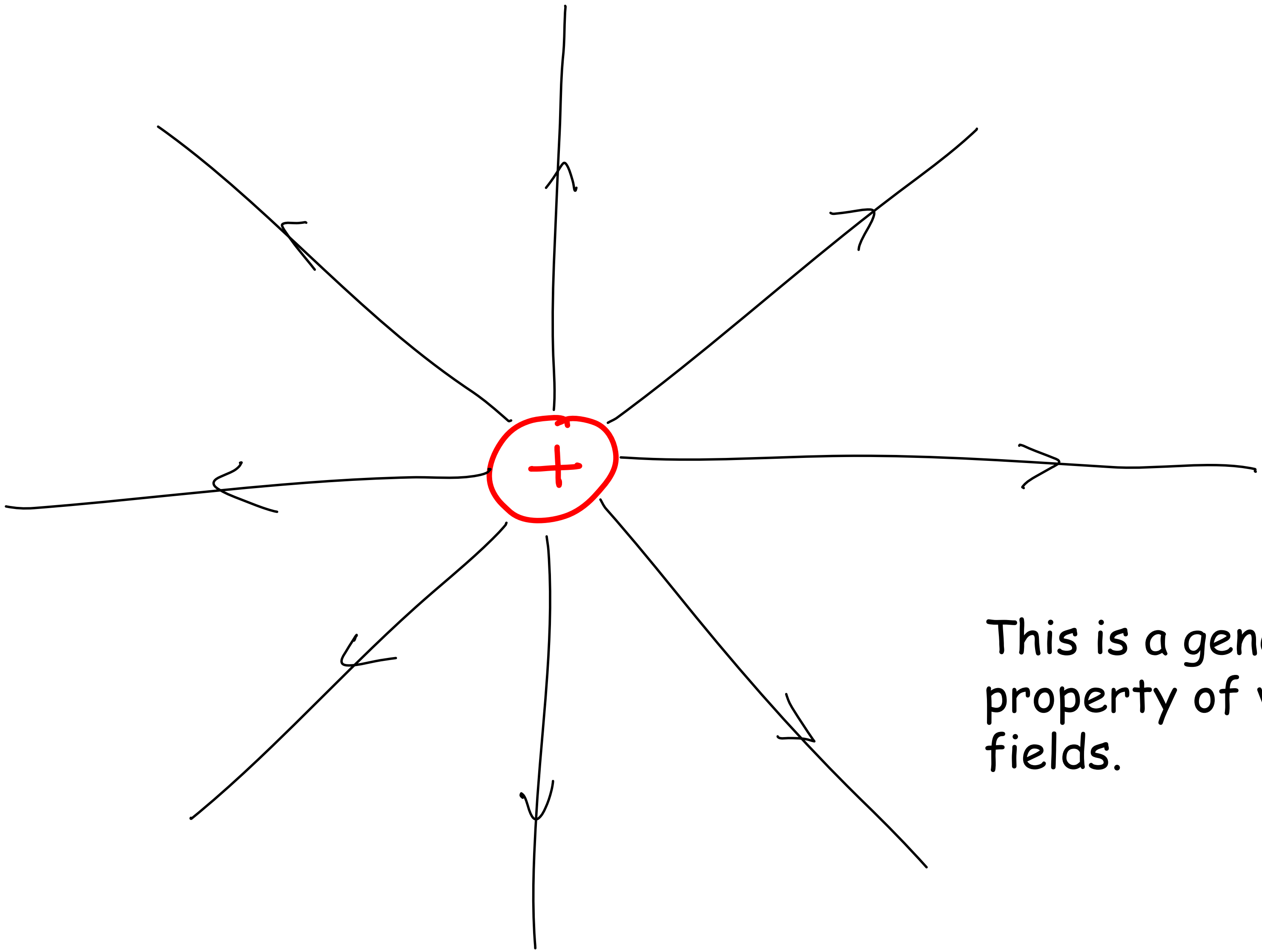
The "test charge" is taken to be positive.

Electric field "points" away from positive charges.

Electric field "points" towards negative charges.

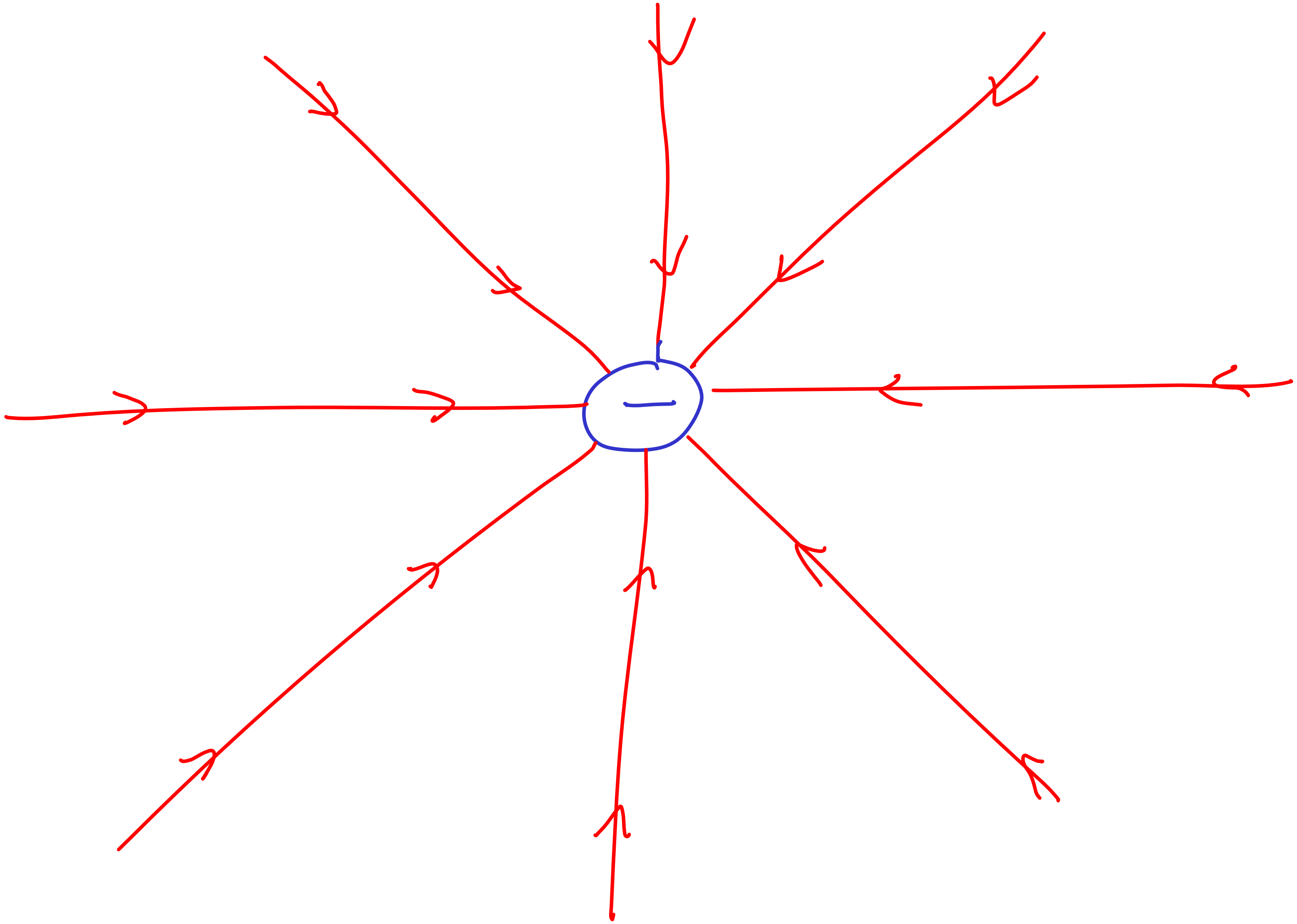


Electric Field Lines

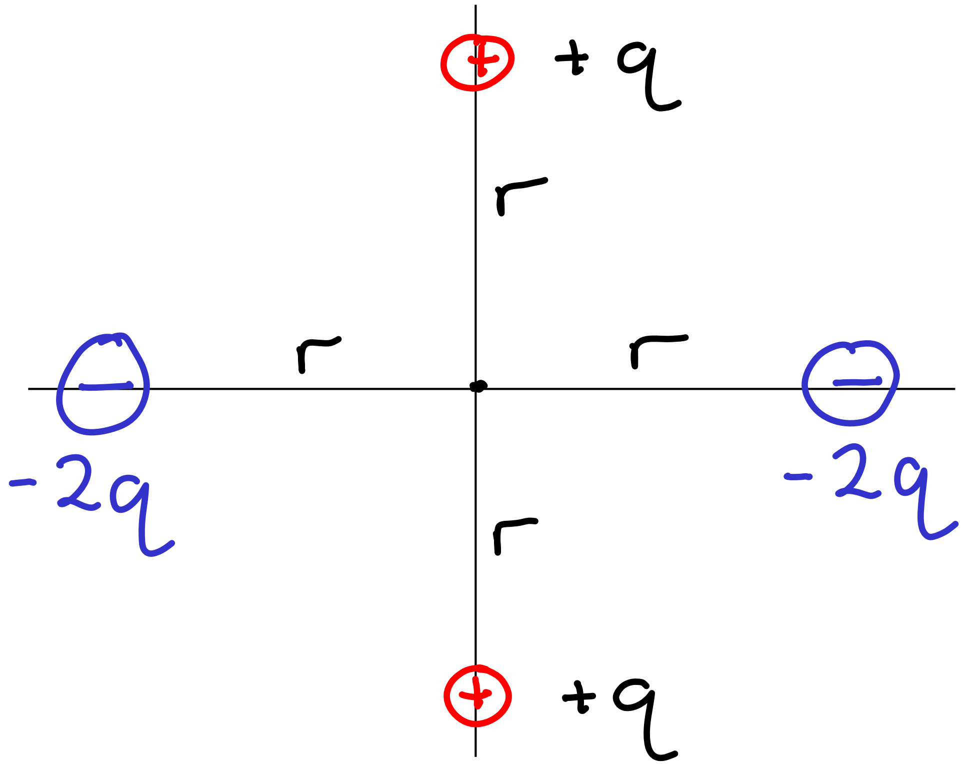


This is a general property of vector fields.

Electric Field Lines



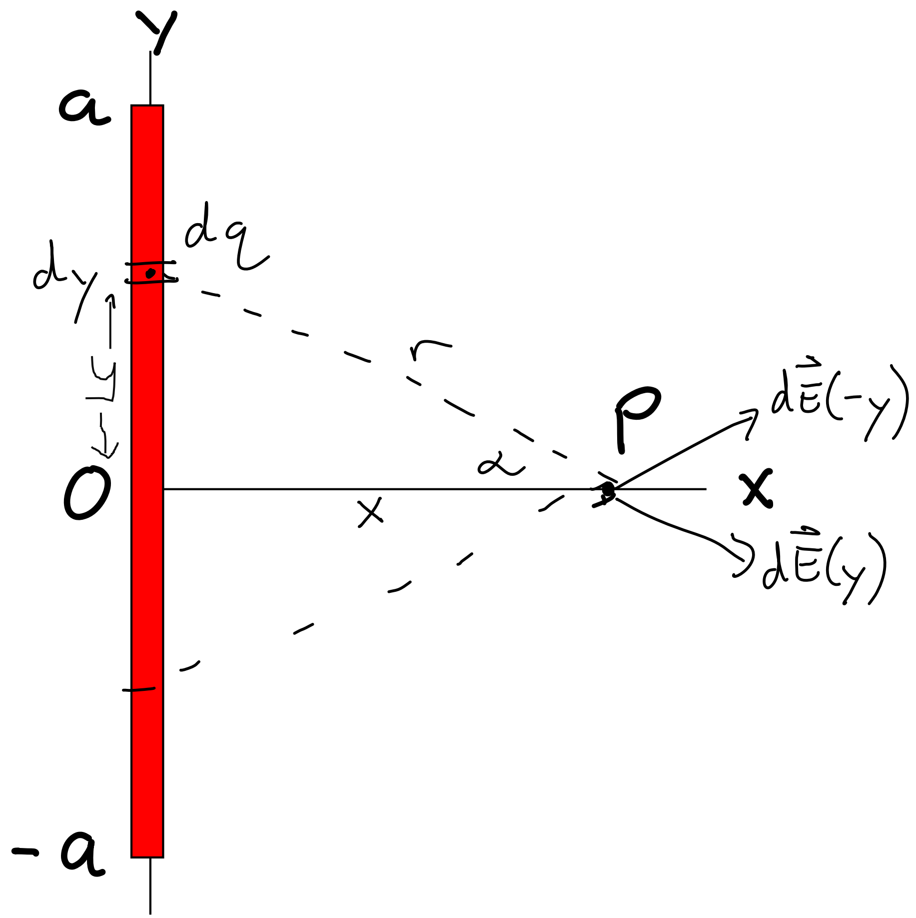
A Problem



What is the electric field at the origin?
What do the field lines look like?

Example 21.11

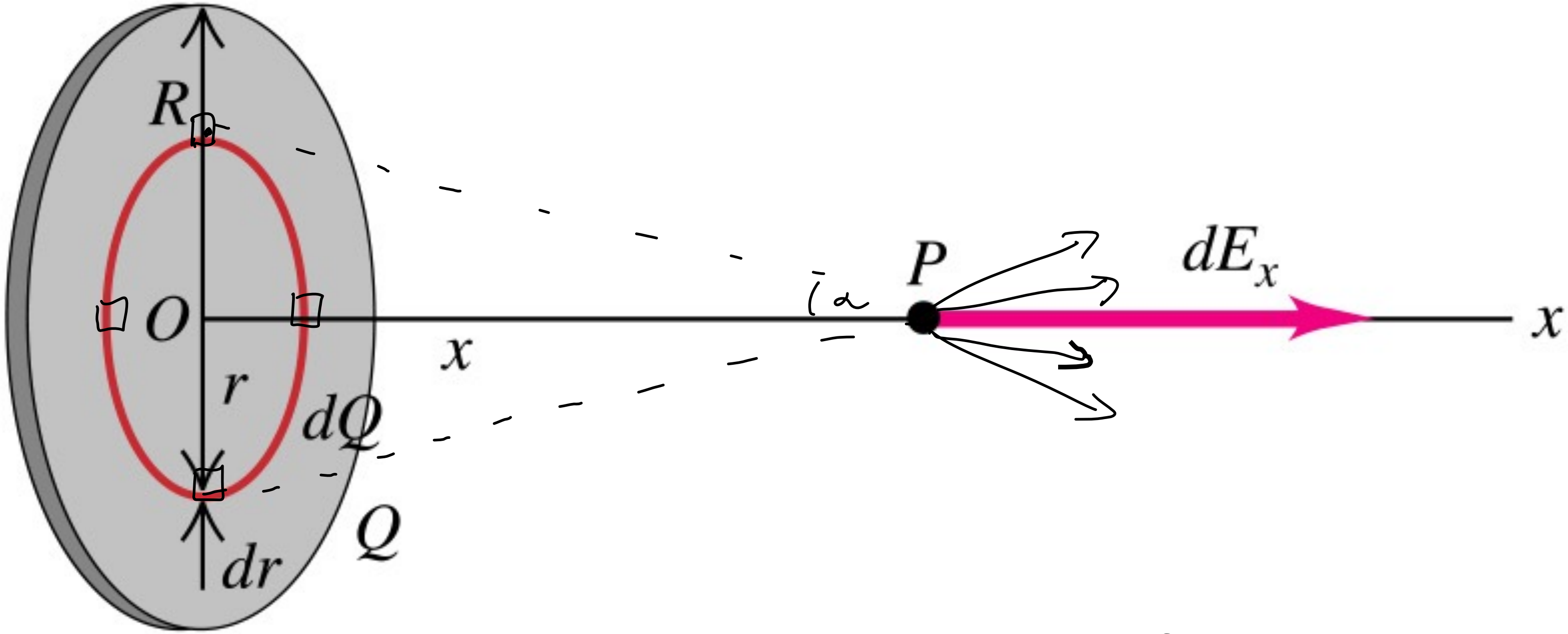
Charged rod
- length 2a
- charge Q



What is the electric field at point P?

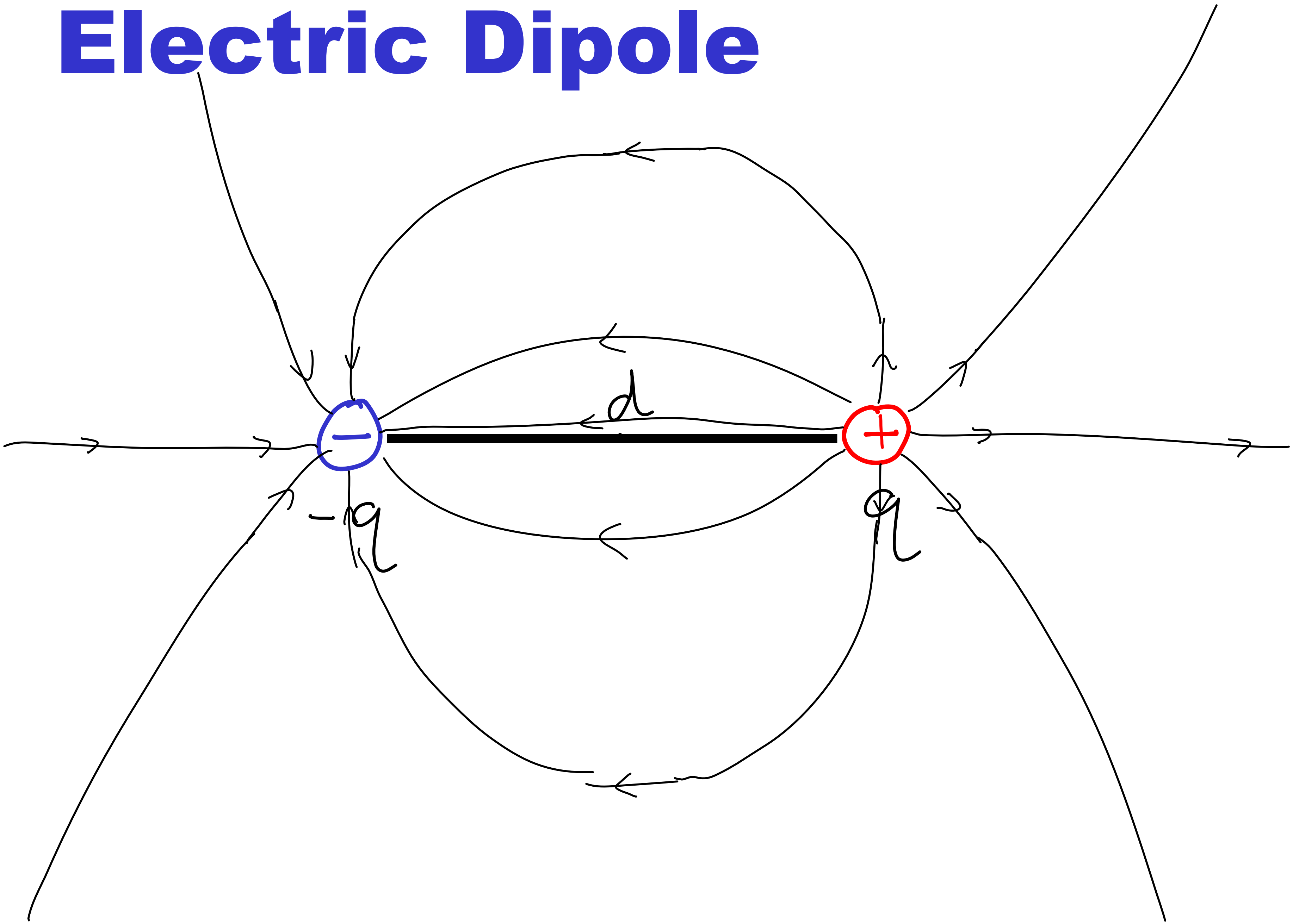
Example 21.12

What is the electric field at point P?



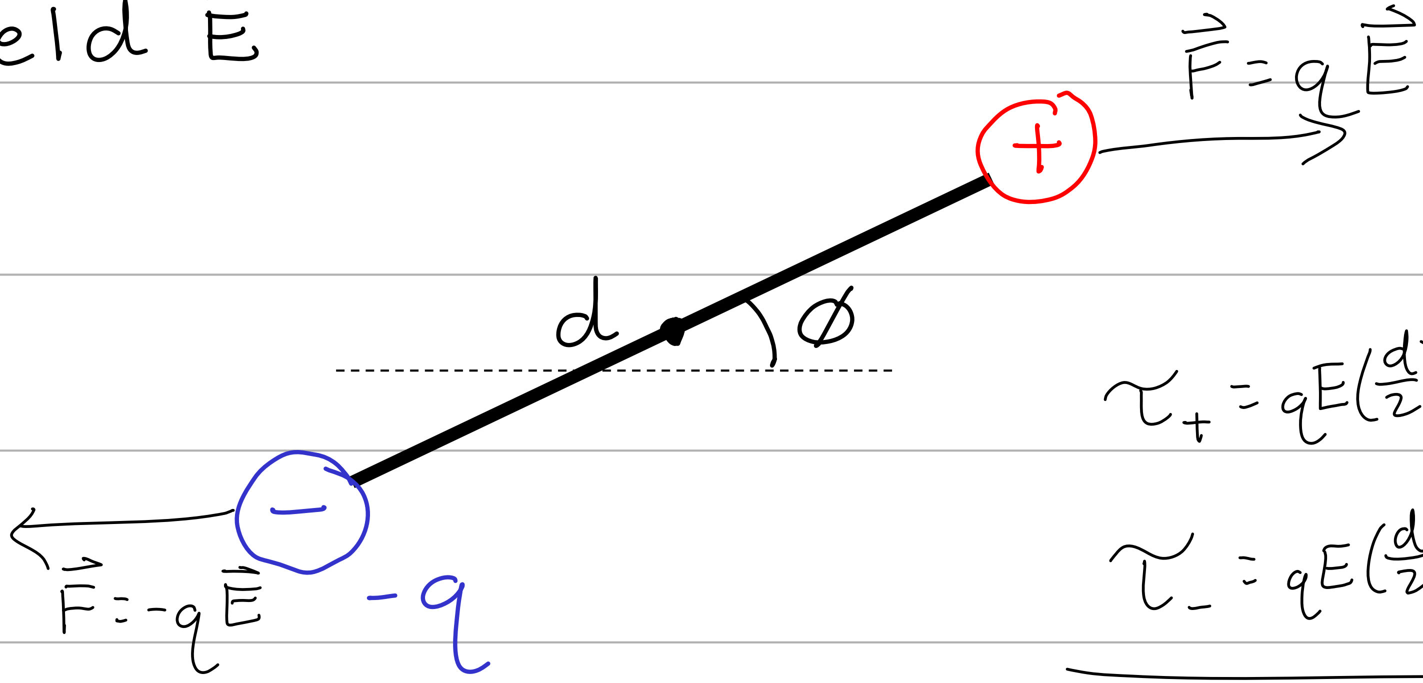
$$E_x = \int dE_x$$

Electric Dipole



Torque on an Electric Dipole

Uniform electric field $\vec{E}$



$$\tau_+ = qE\left(\frac{d}{2}\right)\sin\phi$$

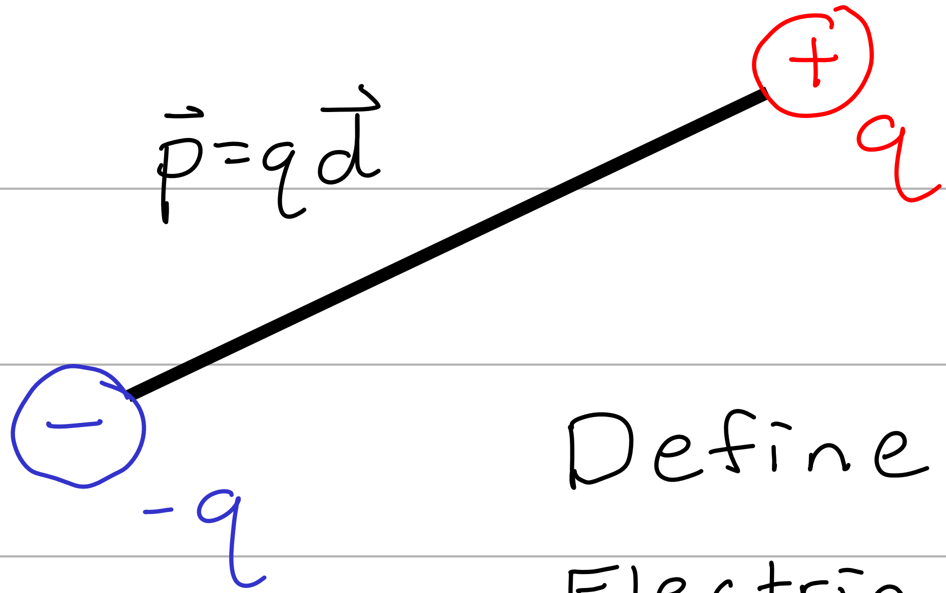
$$\tau_- = qE\left(\frac{d}{2}\right)\sin\phi$$

$$\tau = qEd\sin\phi$$

Torque on an Electric Dipole

Uniform electric field $\vec{E}$

$\vec{p} = q\vec{d}$



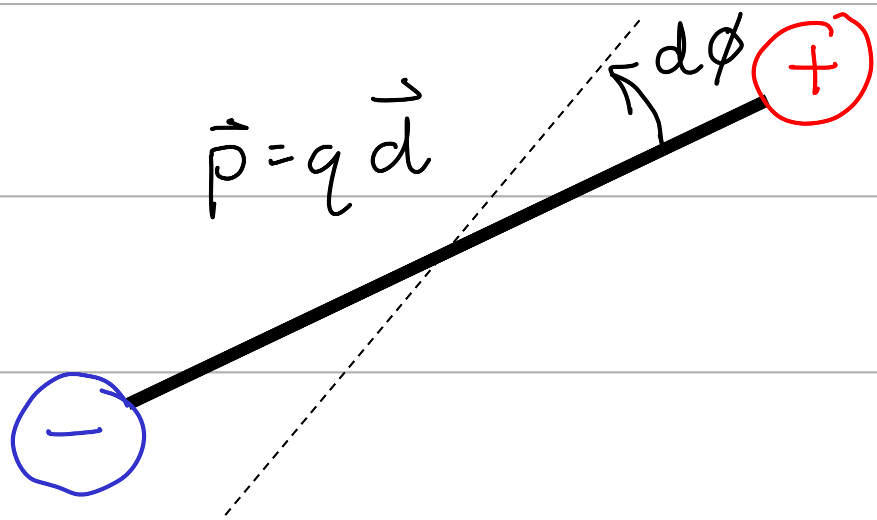
Define $\vec{p} = q\vec{d}$

Electric dipole moment

$$\boxed{\vec{\tau} = \vec{p} \times \vec{E}}$$

Potential Energy for a Dipole

Uniform electric field $\vec{E}$



$$dW = \tau \, d\phi$$
$$= -pE\sin\phi \, d\phi$$

$$U(\phi) = -pE\cos\phi$$
$$= -\vec{p}\cdot\vec{E}$$

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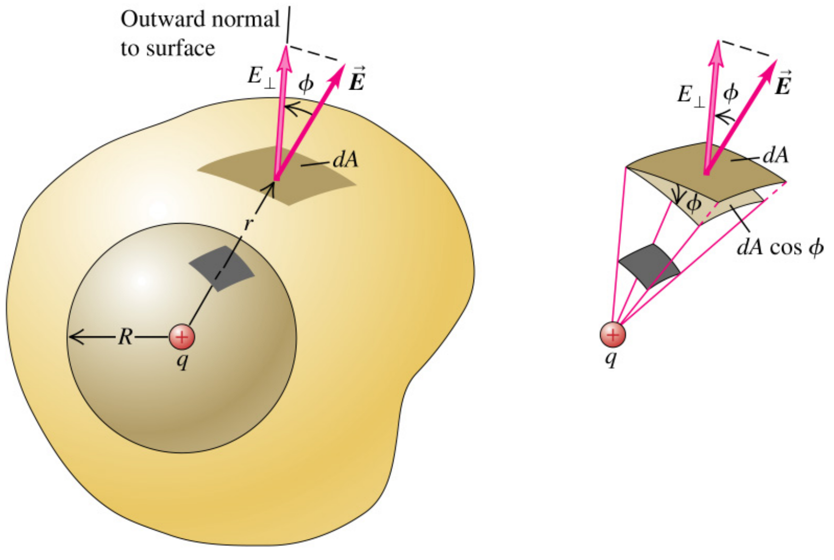
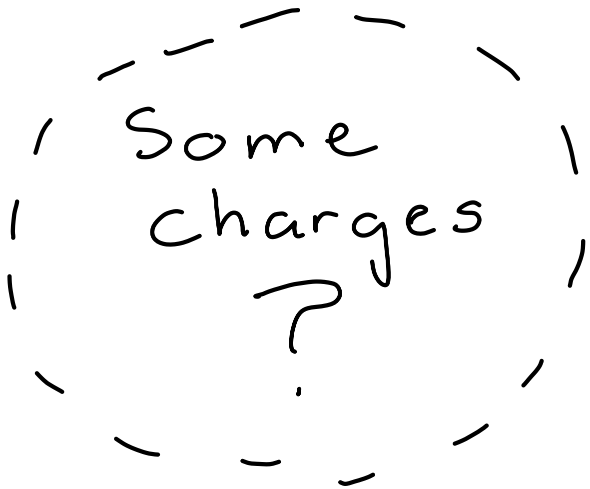
Chapter 22
Gauss's Law

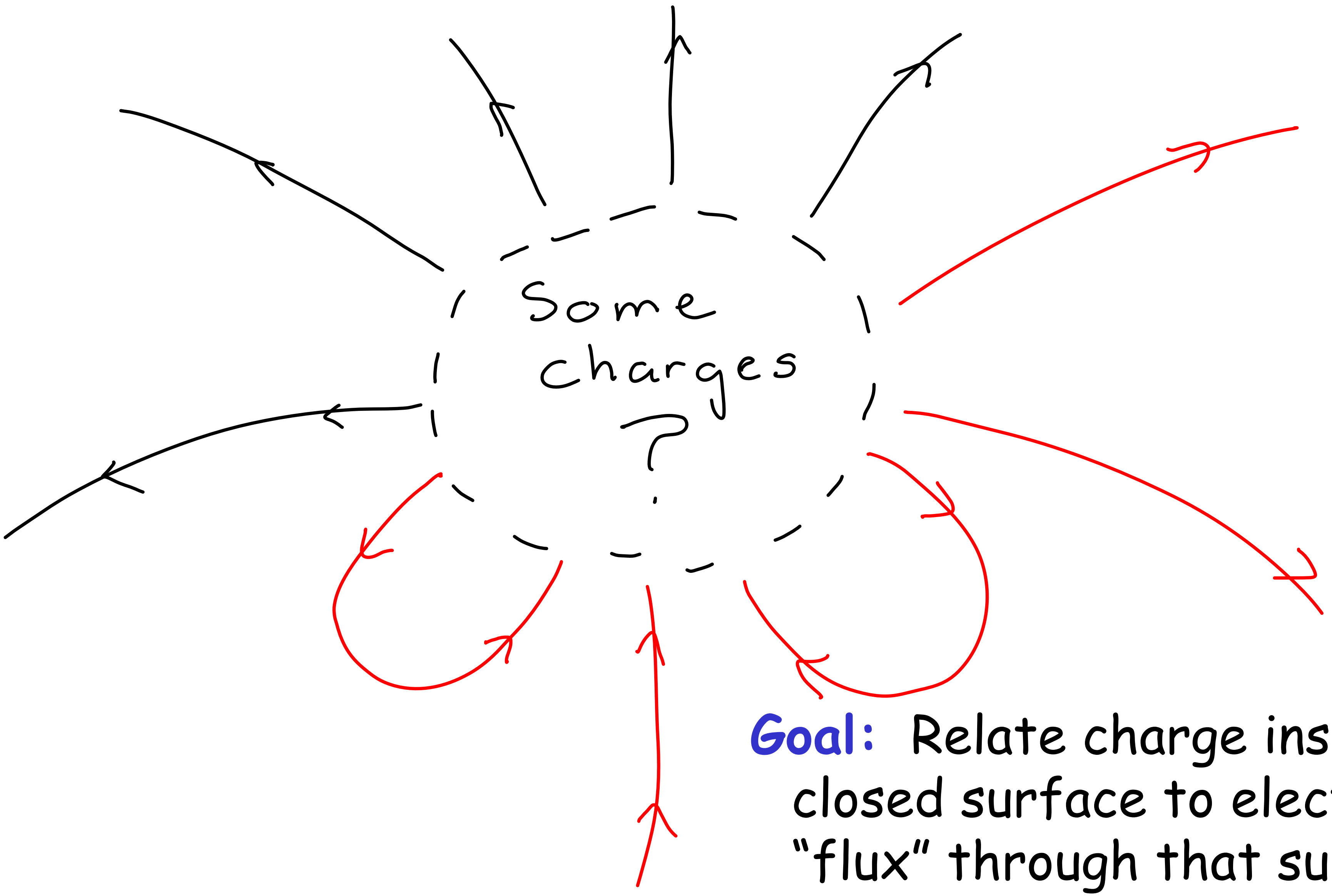


(a) (b)

Charge in a Volume



Charge and Electric Flux



Goal: Relate charge inside a closed surface to electric "flux" through that surface

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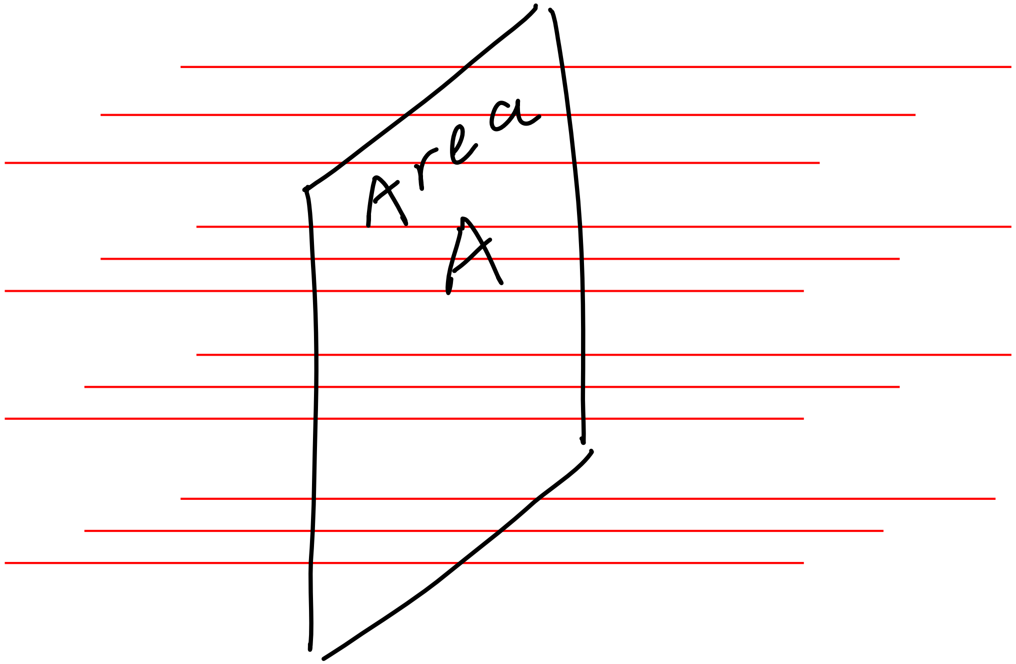
Electric Flux



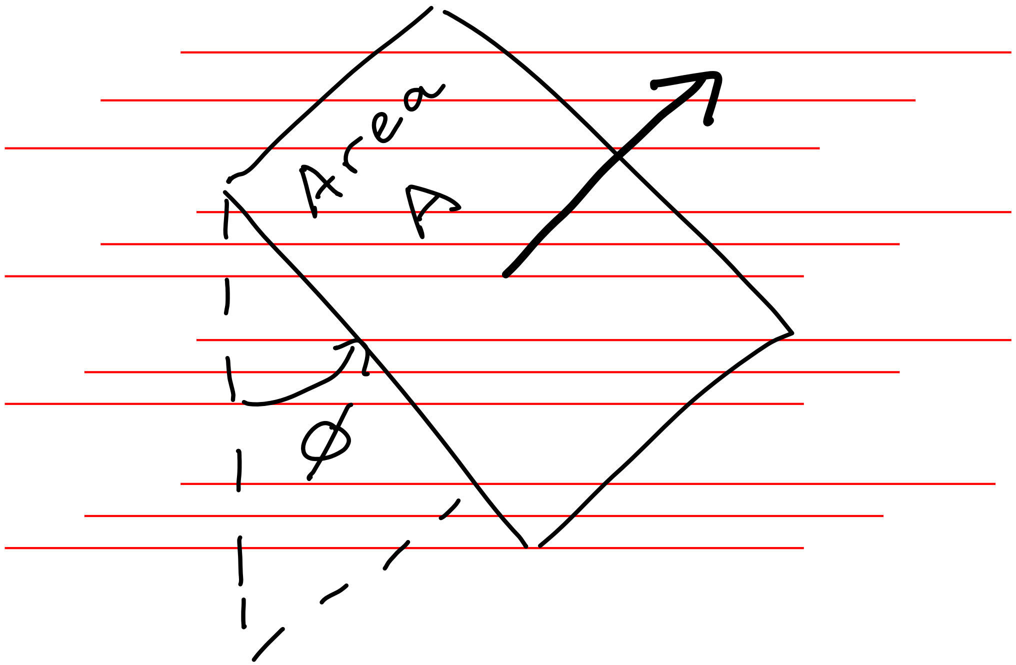
Uniform $\vec{E}$

Electric flux through A:

$$\Phi_E = EA$$

for a uniform perpendicular surface

Electric Flux



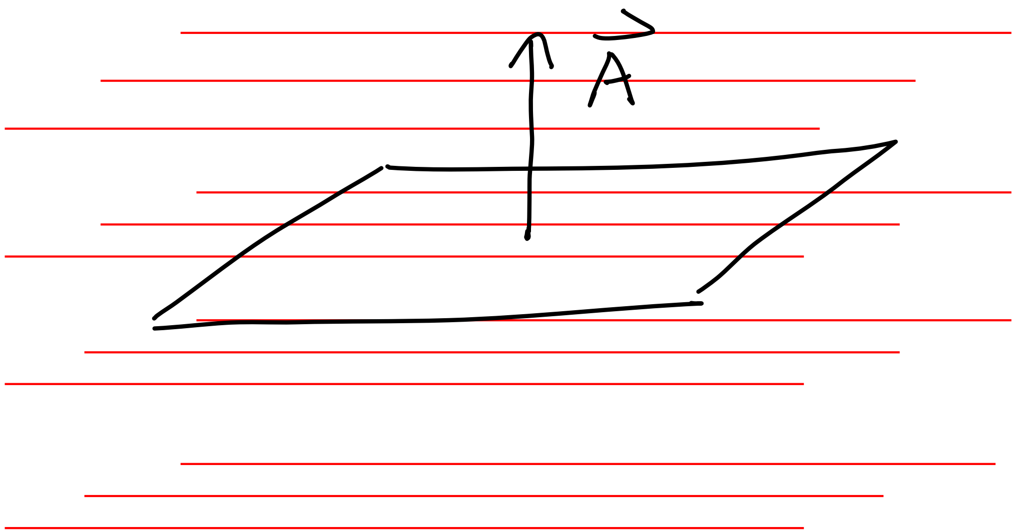
Uniform
$\vec{E}$

Electric flux through A:

$$\Phi_E = EA\cos\phi$$
$$= \vec{E}\cdot\vec{A}$$

for a uniform surface

Electric Flux

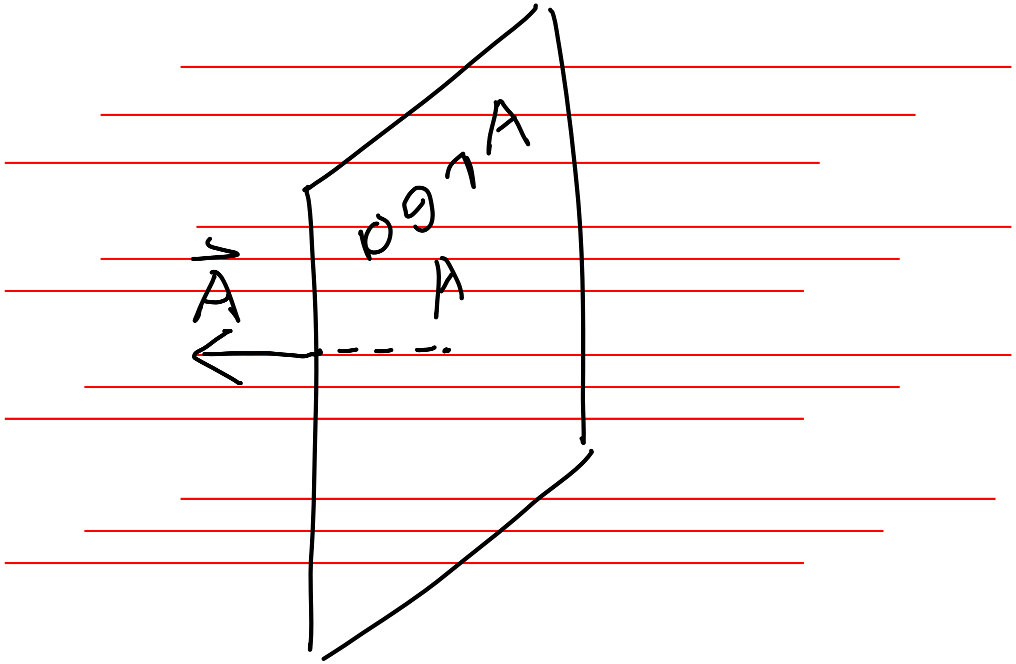


Uniform $\vec{E}$

Electric flux through A :

$$\Phi_E = EA\cos 90^\circ$$
$$= 0$$

Surfaces are Oriented



Uniform $\vec{E}$

Electric flux through A:

$$\Phi_E = \vec{E} \cdot \vec{A}$$
$$= -EA$$

can be negative

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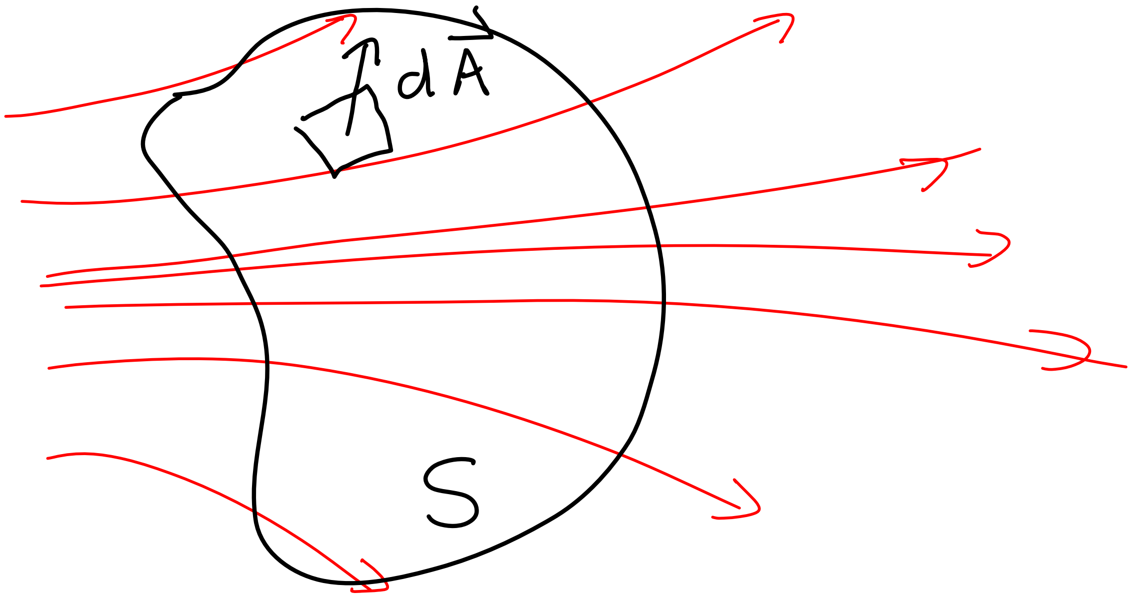
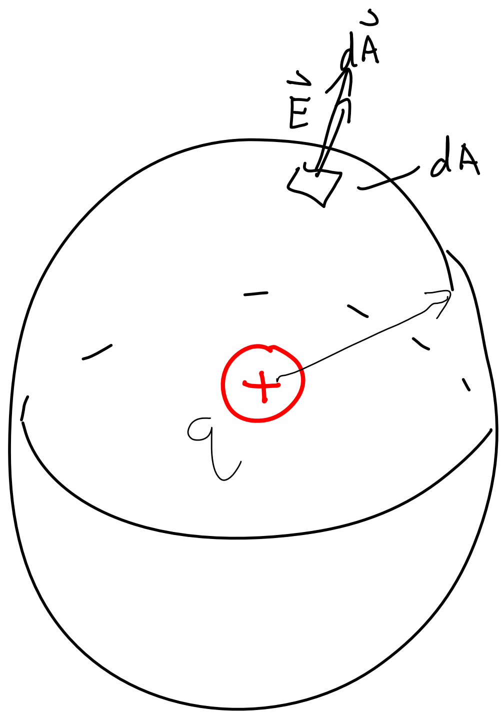
Electric Flux



Electric flux through any surface S :

$$\Phi_E = \int_S \vec{E} \cdot d\vec{A}$$

Electric Flux-Point Charge



Consider a "closed" spherical surface centred on a point charge, q.

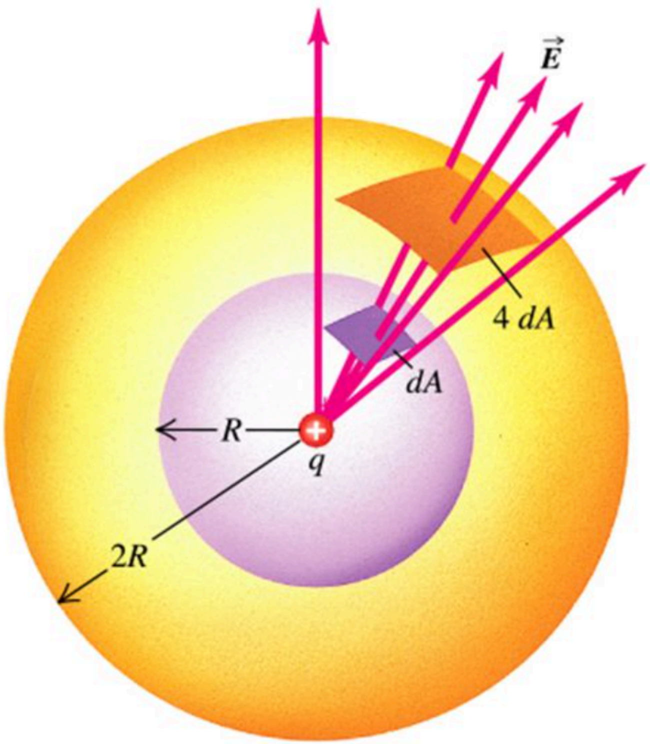
At each point on the sphere, the electric field points radially outwards (spherical symmetry).

At each point on the sphere, the "normal" to the surface is also radial. Choose the normal pointing "towards infinity".

Each infinitesimal vector area, dA, also points radially outwards.

So, what is the electric flux through the surface of the sphere?

Electric Flux-Point Charge



Let's add a second sphere, at double the radius of the first.

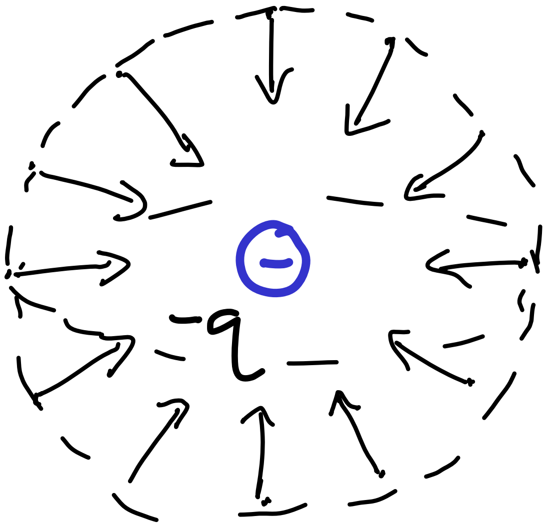
The relative size of the area of this new sphere is 4 times that of the old.

The magnitude of the electric field through the larger sphere is $\frac{1}{4}$ of the smaller sphere.

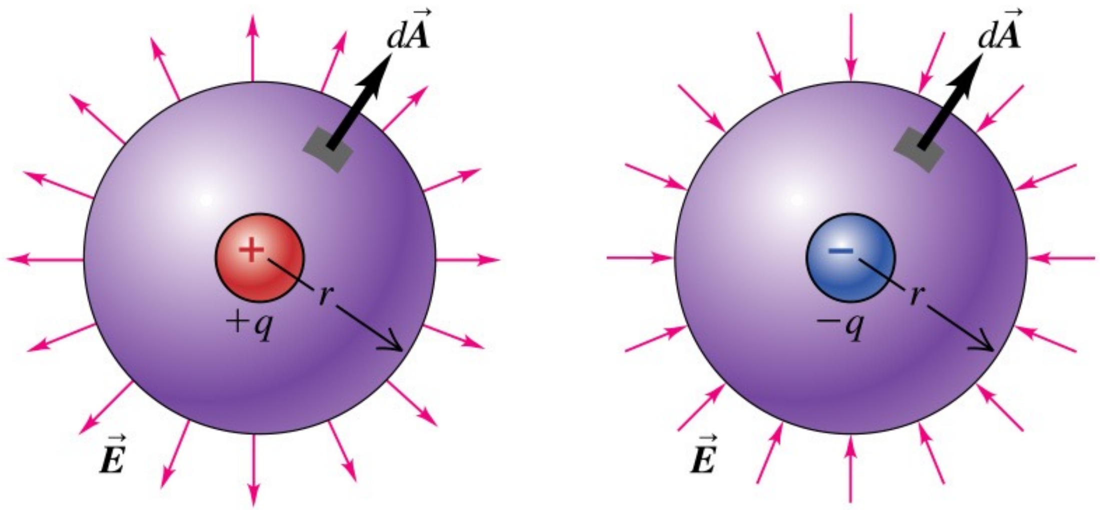
So, the electric flux through the larger sphere is the same as that through the smaller sphere.

$$\Phi_E = \oint \vec{E} \,.\, \overrightarrow{dA} = \frac{q}{\epsilon_0}$$

Electric Flux-Point Charge



$$\Phi_E = \frac{-q}{\epsilon_0}$$



(a) Gaussian surface around positive charge: positive (outward) flux

(b) Gaussian surface around negative charge: negative (inward) flux

Gauss's Law

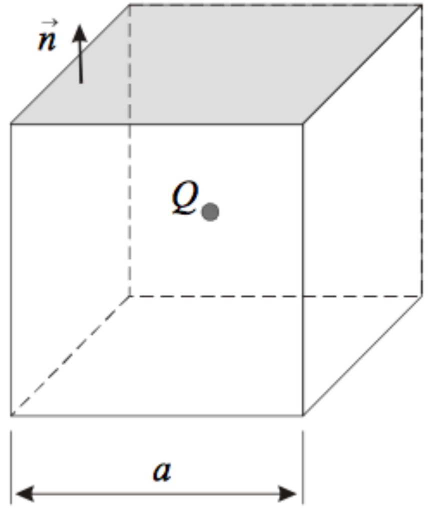
The total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by ε_0.

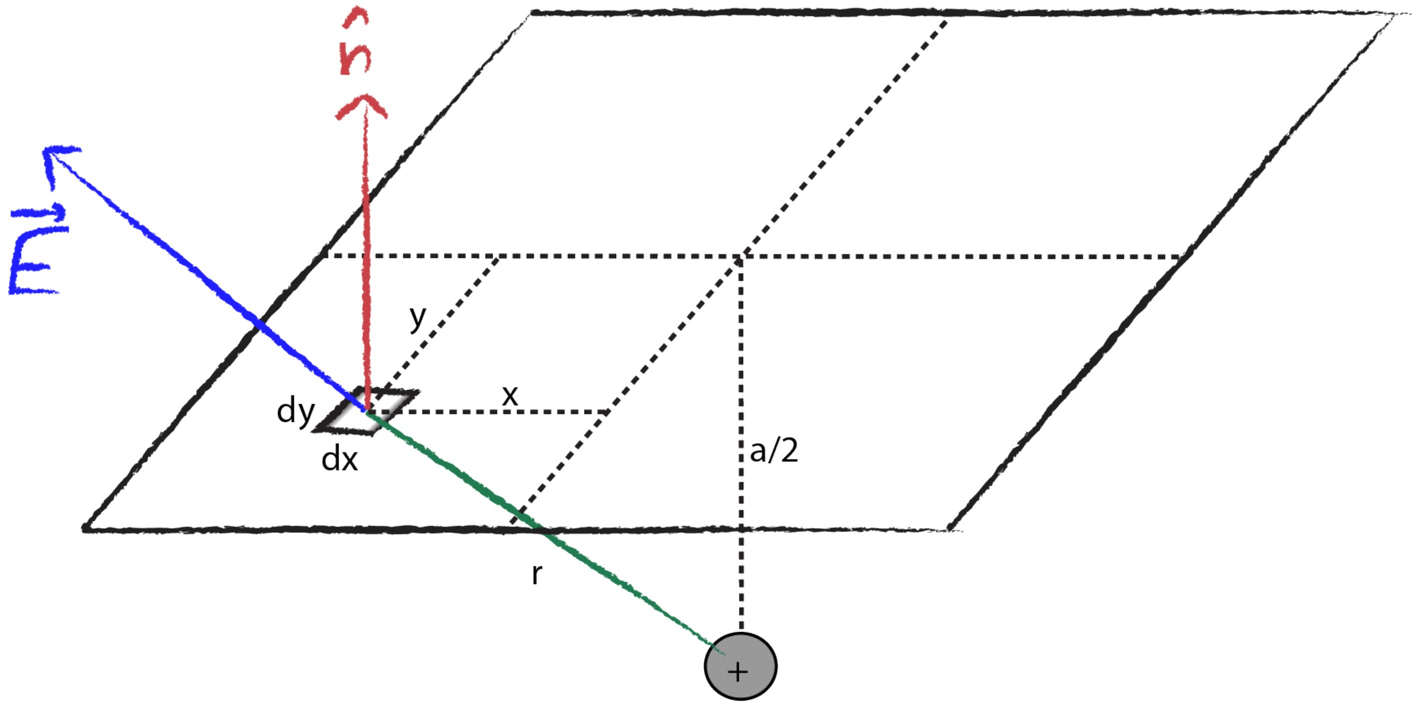
$$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\varepsilon_0}}$$

closed

1st Maxwell's equations = equivalent to Coulomb's law

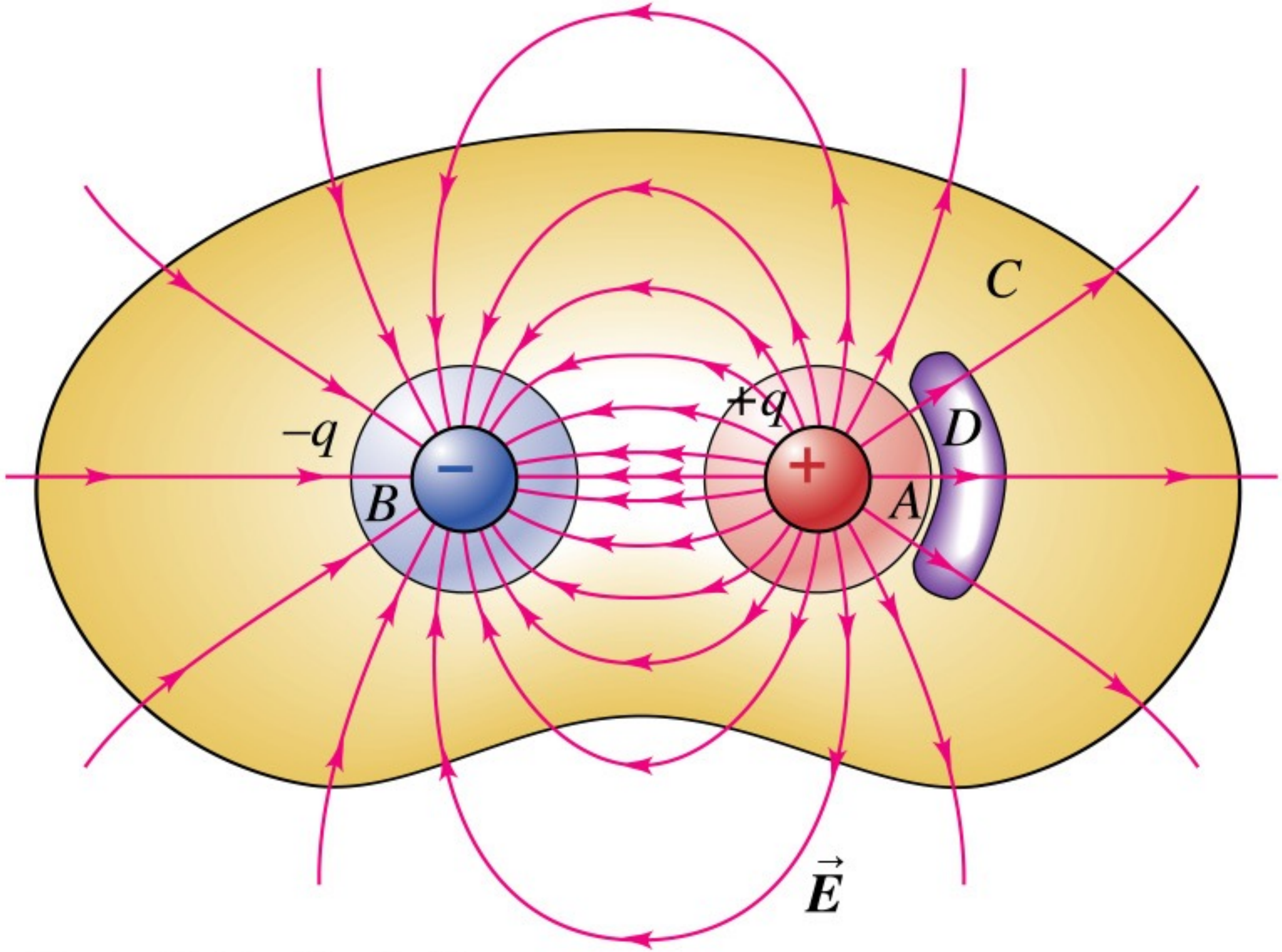
What about other shapes?



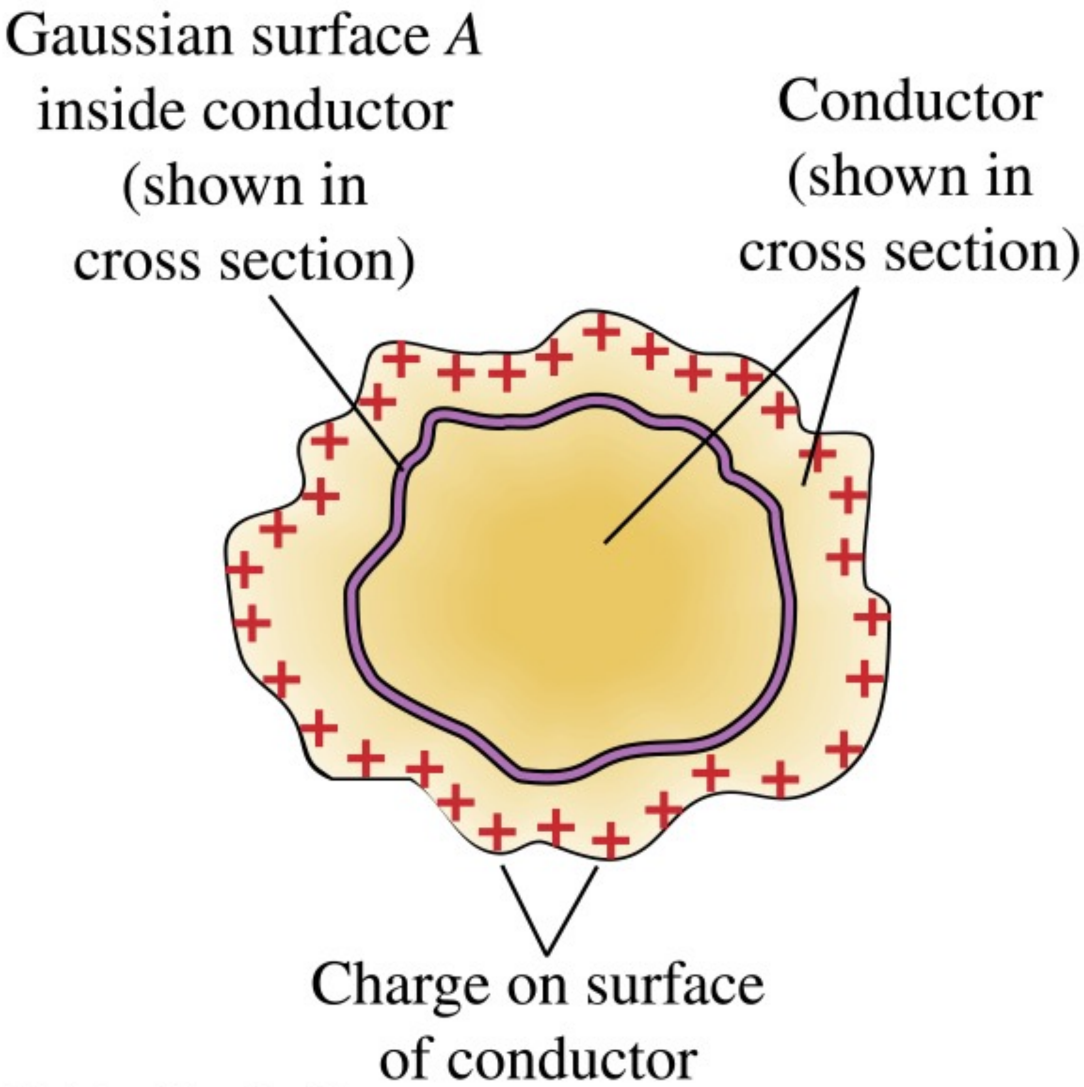


$$r^2 = x^2 + y^2 + ({}^{a}/_{2})^2$$

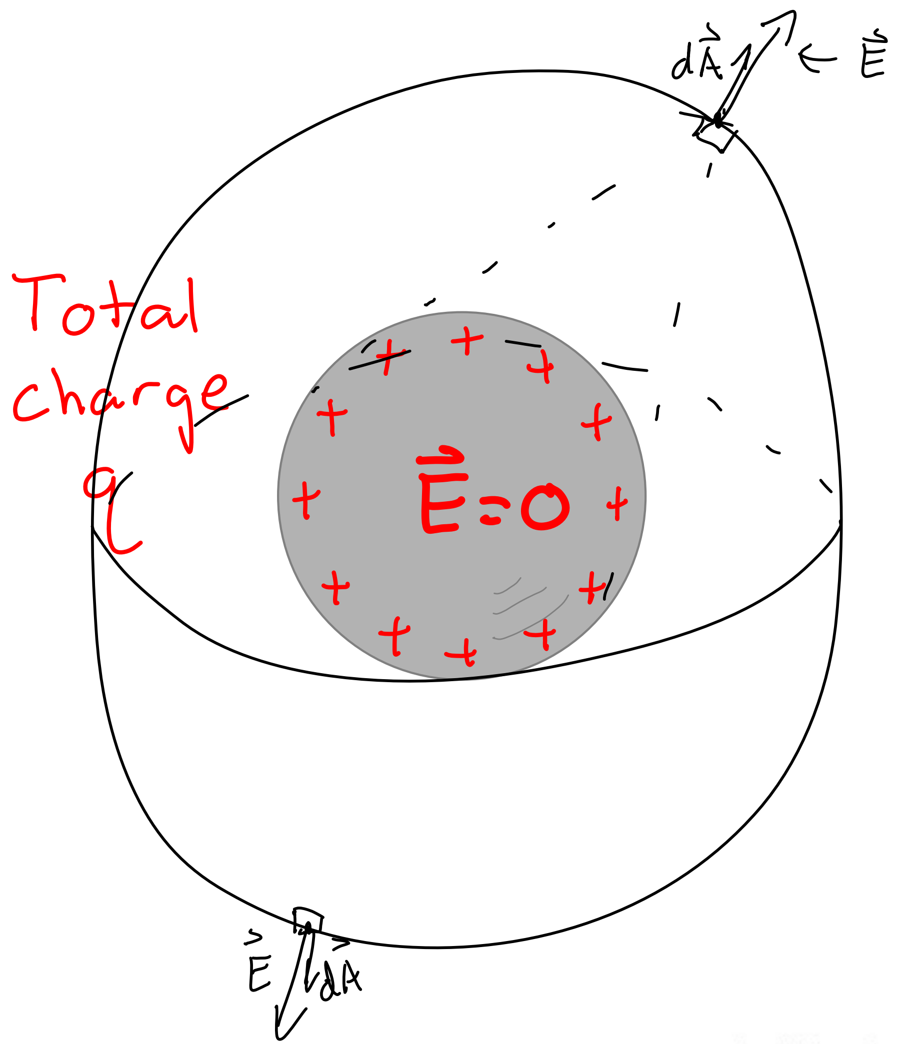
Choices of Surfaces

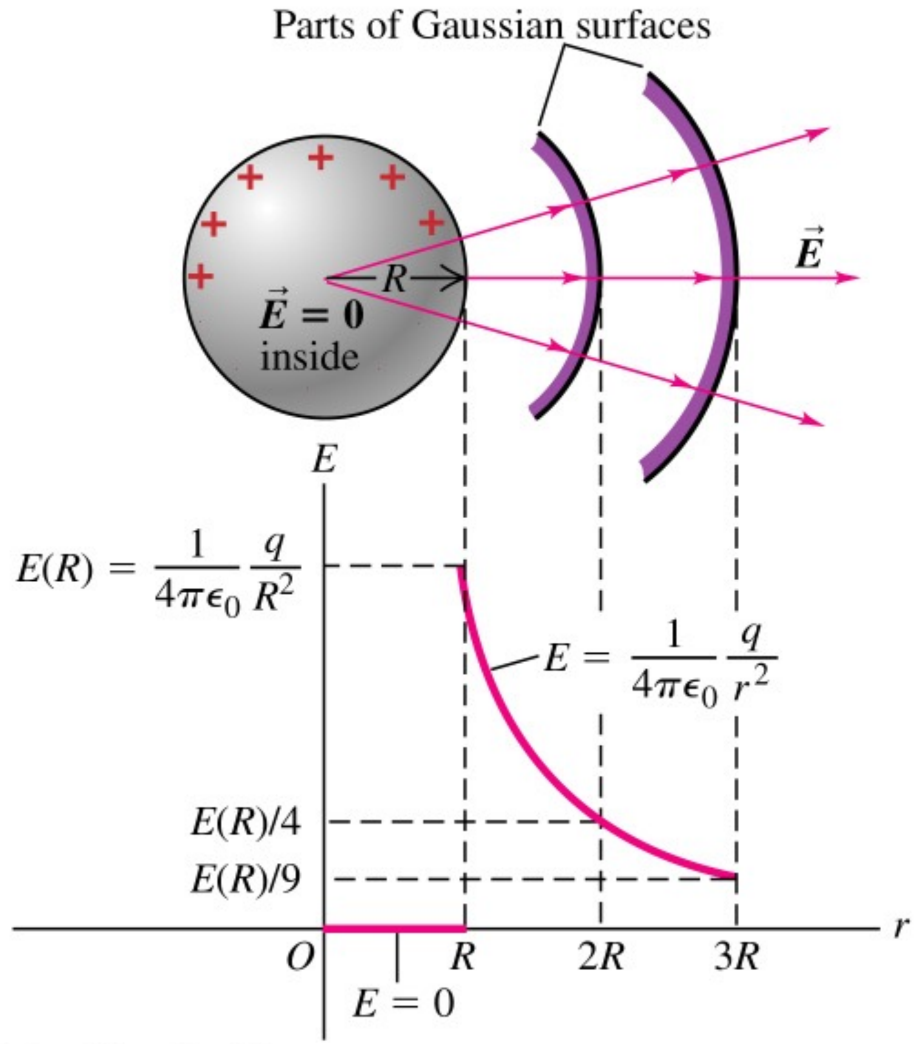


Application: Conductors

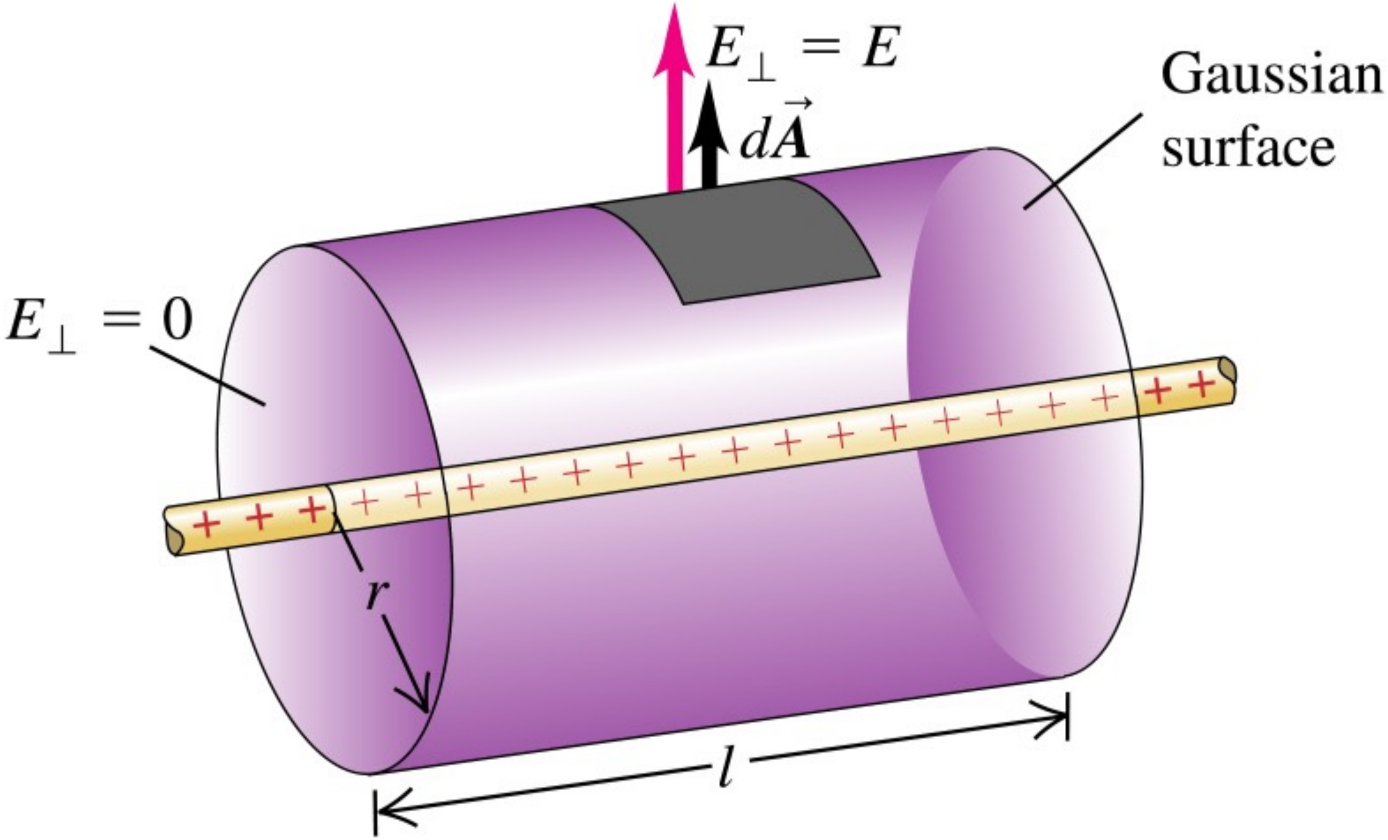


Conducting Sphere: 22.5

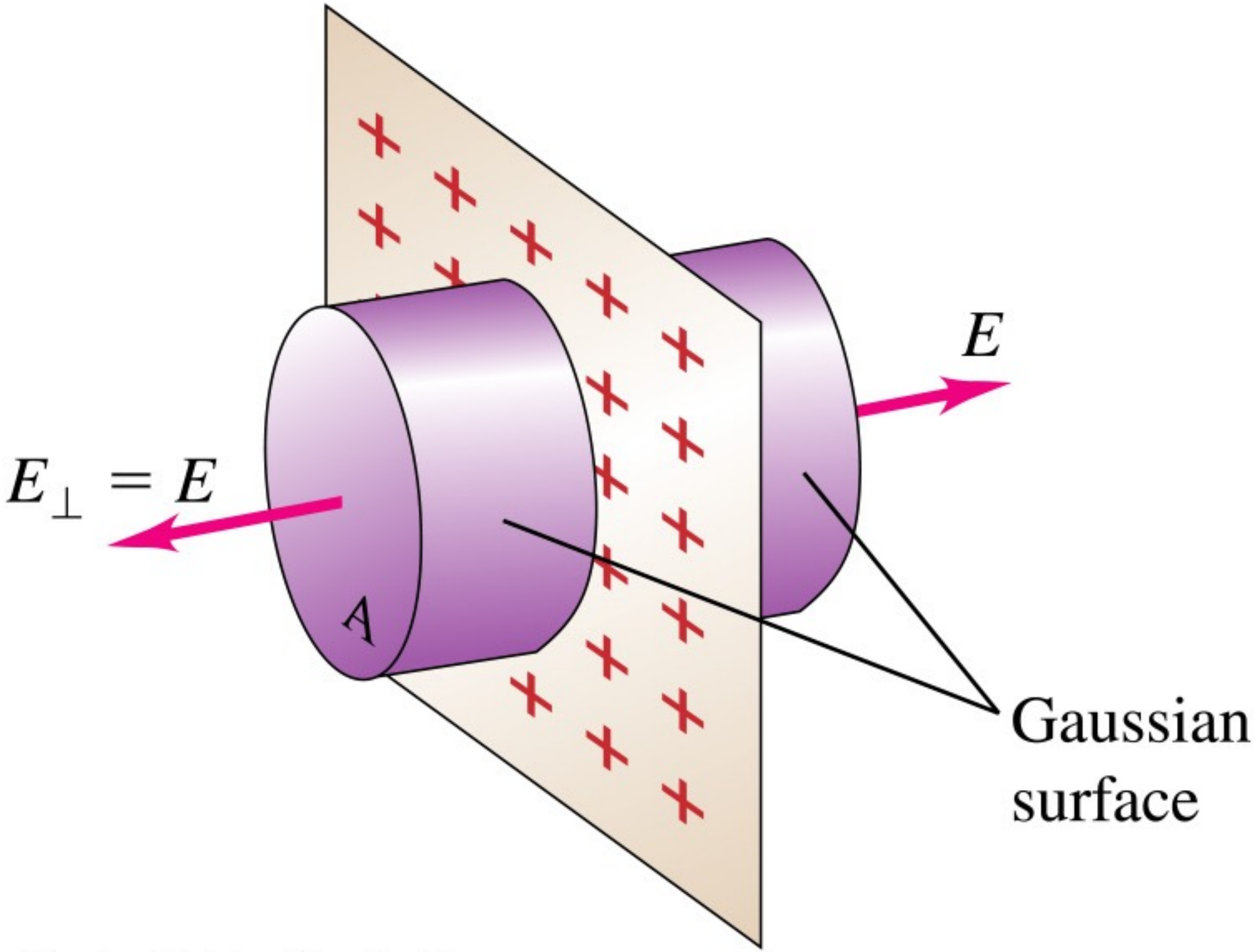




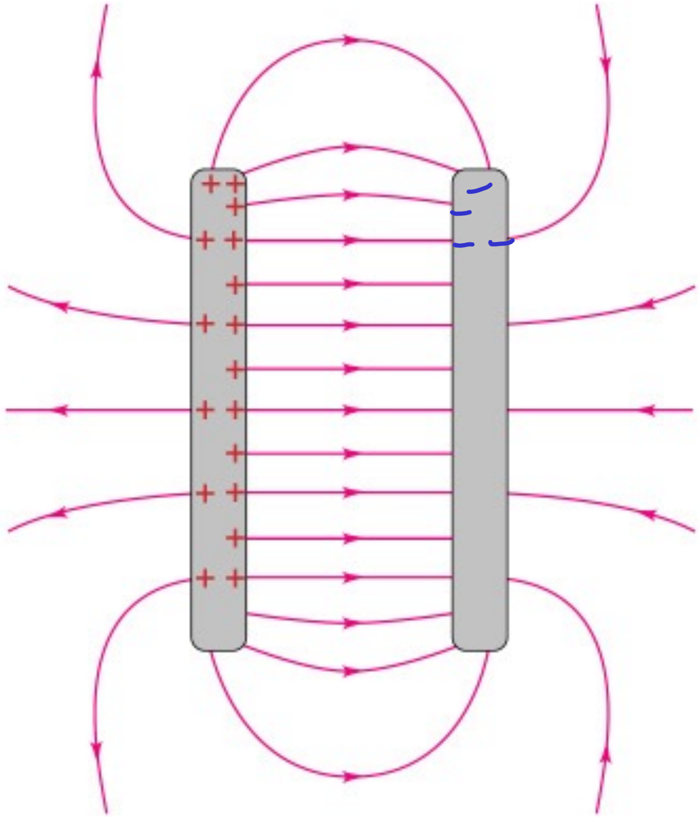
Infinite Line of Charge: 22.6



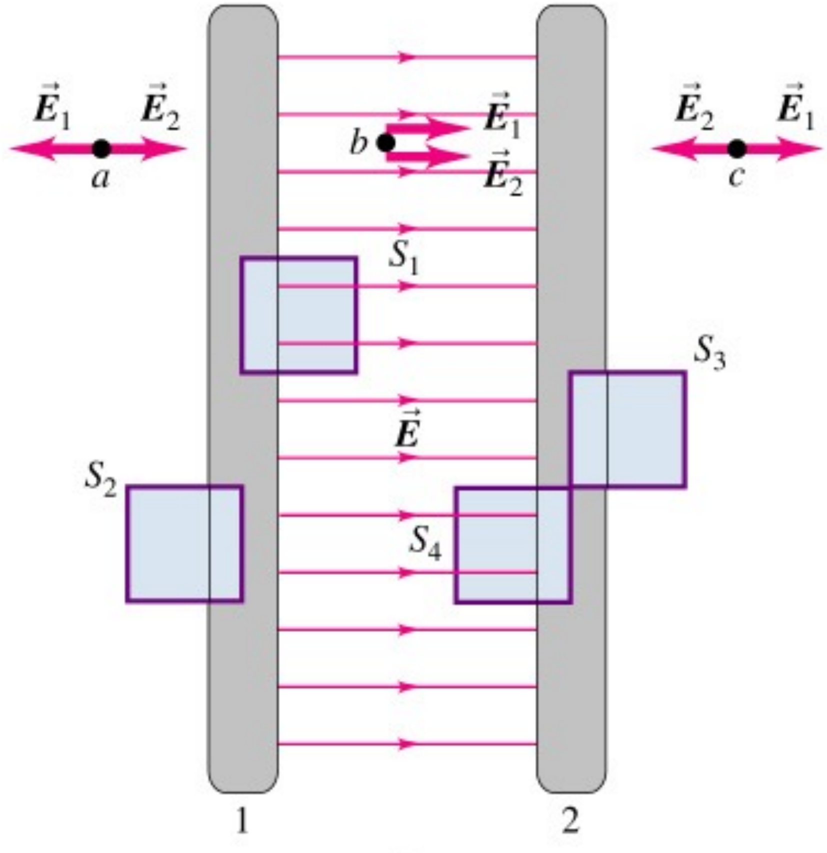
Infinite Plane Sheet: 22.7



Parallel Plates: 22.8



(a)

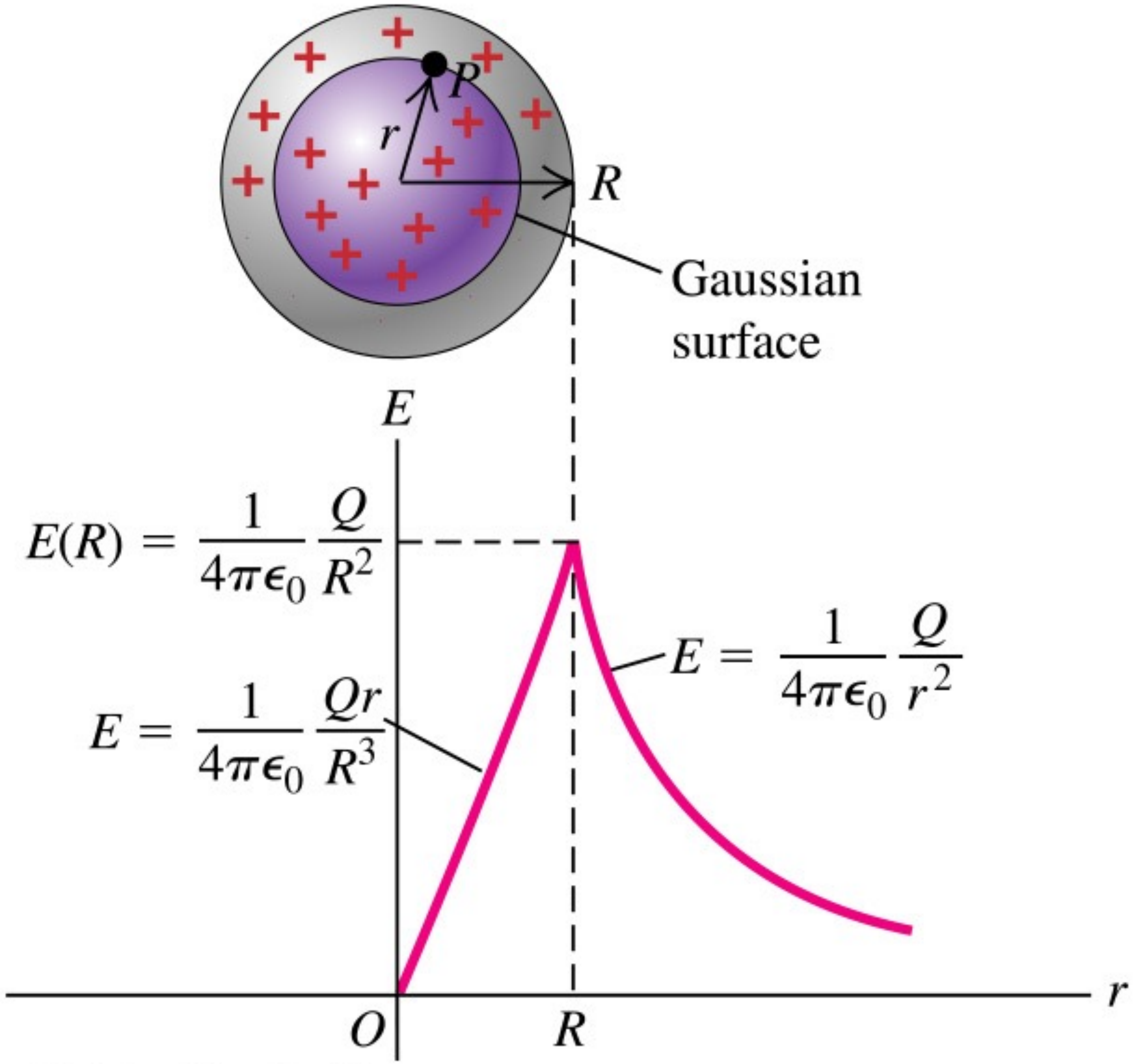


(b)

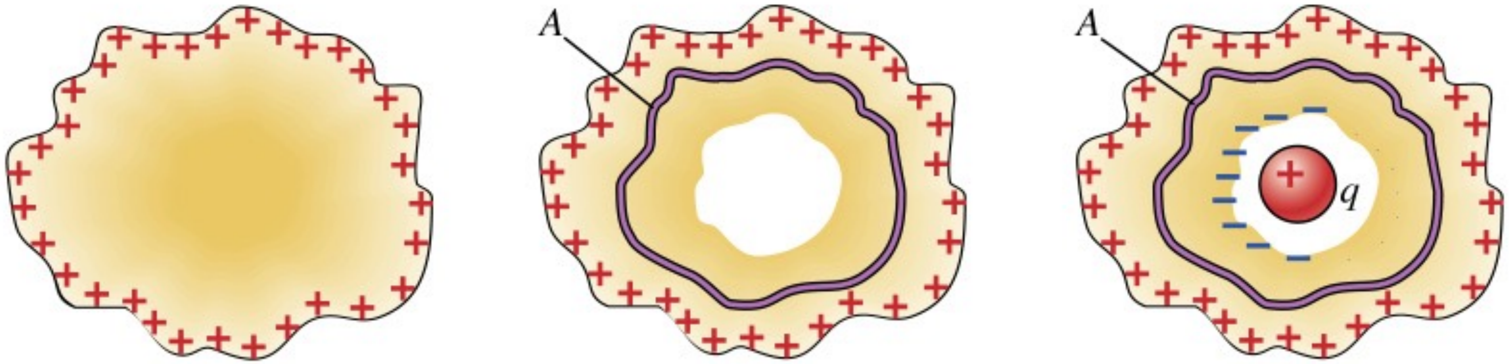
Finite Charged Plates

Infinite Charged Plates

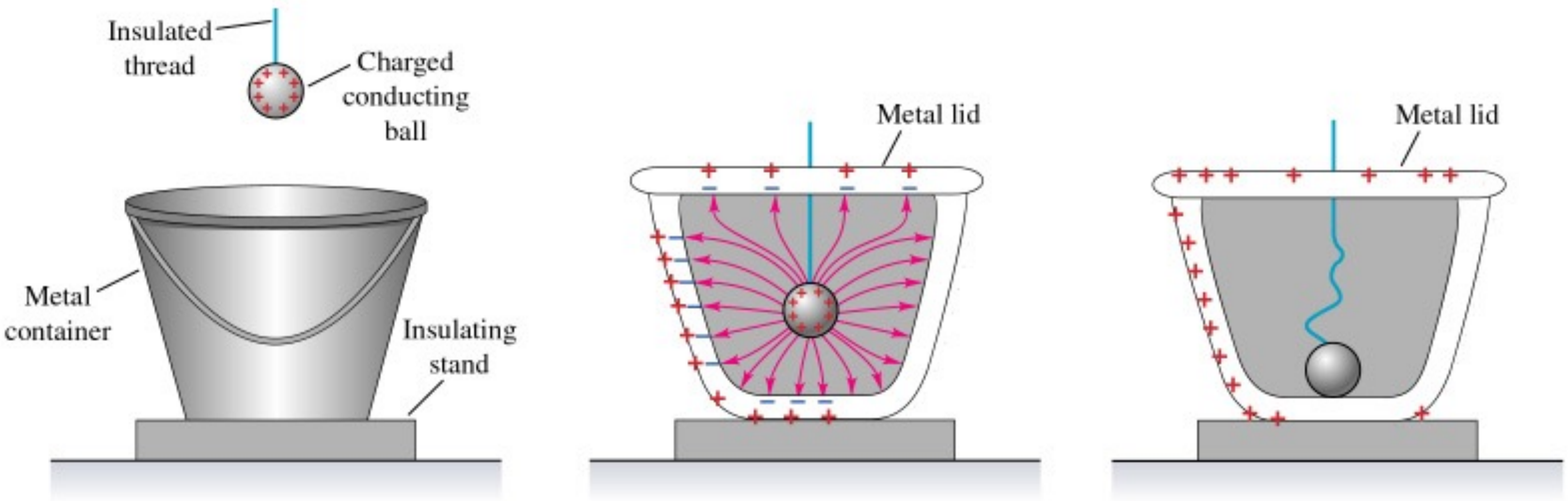
Uniformly Charged Sphere: 22.9



Application: Conductors



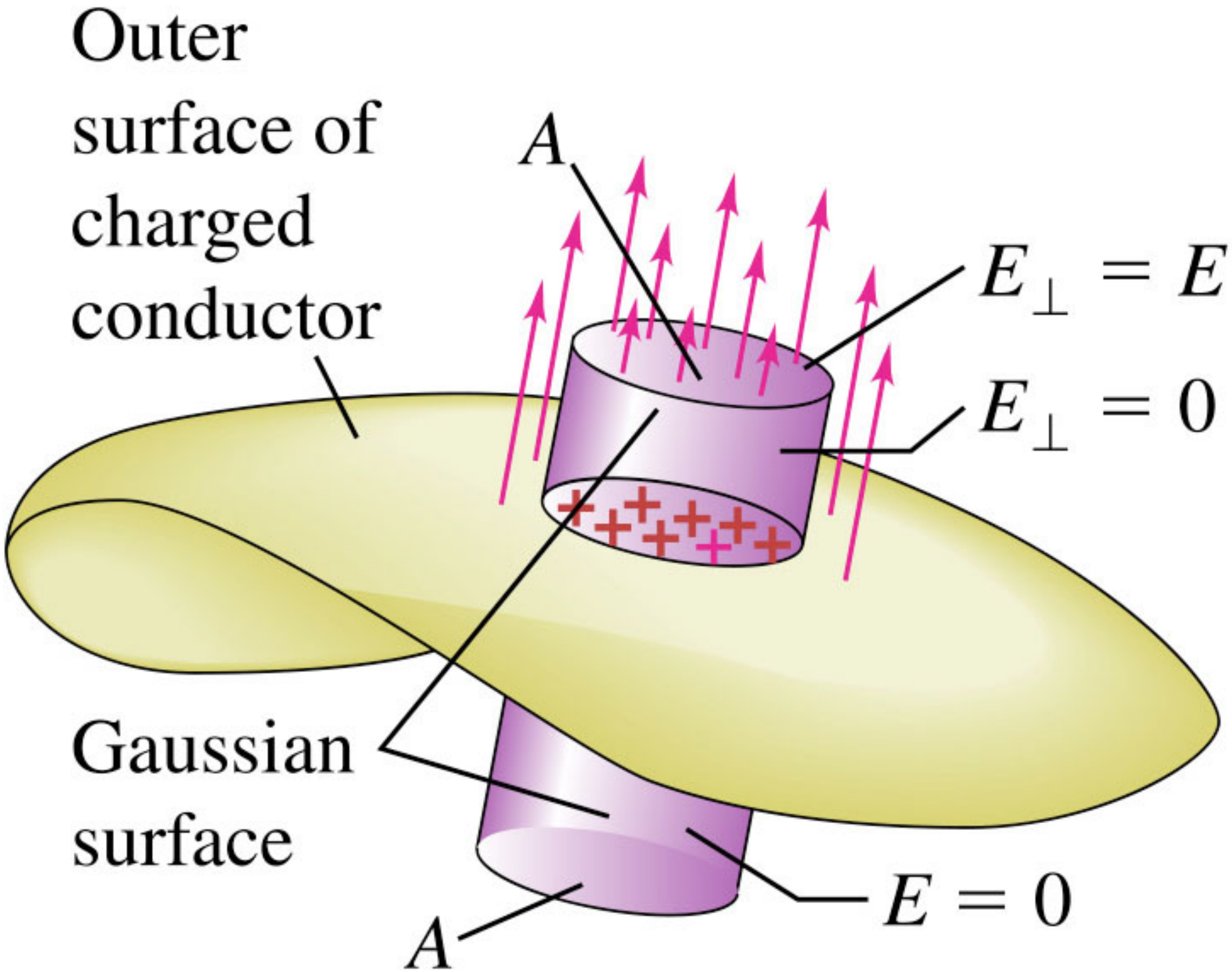
(a) (b) (c)



(a) (b) (c)

Faraday's Bucket

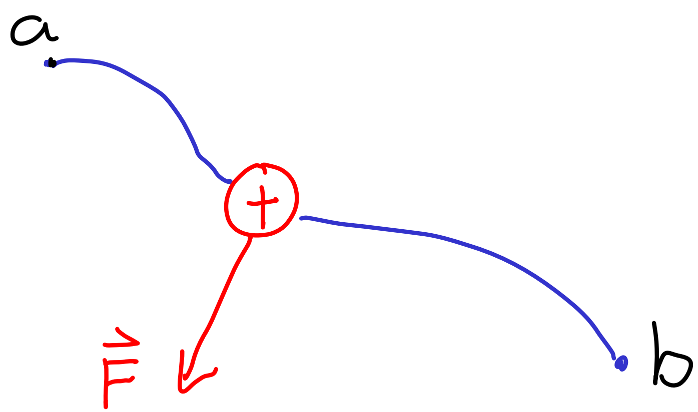
Surface of a Conductor



Chapter 23
Electric Potential

Electric Potential Energy

Electric force $\vec{F}$



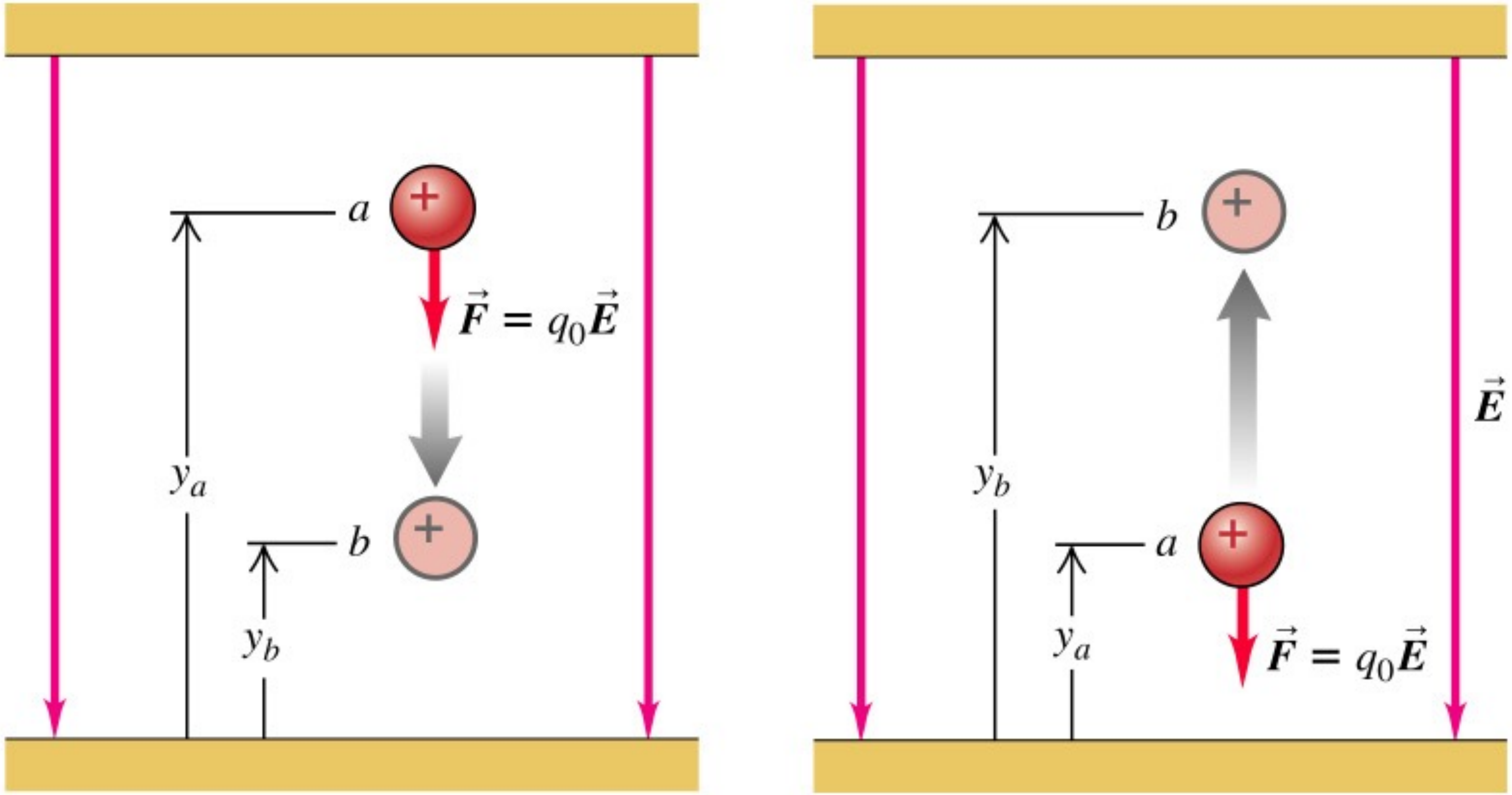
$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{\ell}$$

Electric force is conservative

$$W_{a\to b} = U_a - U_b = -(U_b - U_a) = -\Delta U$$

(work done by a conservative force)

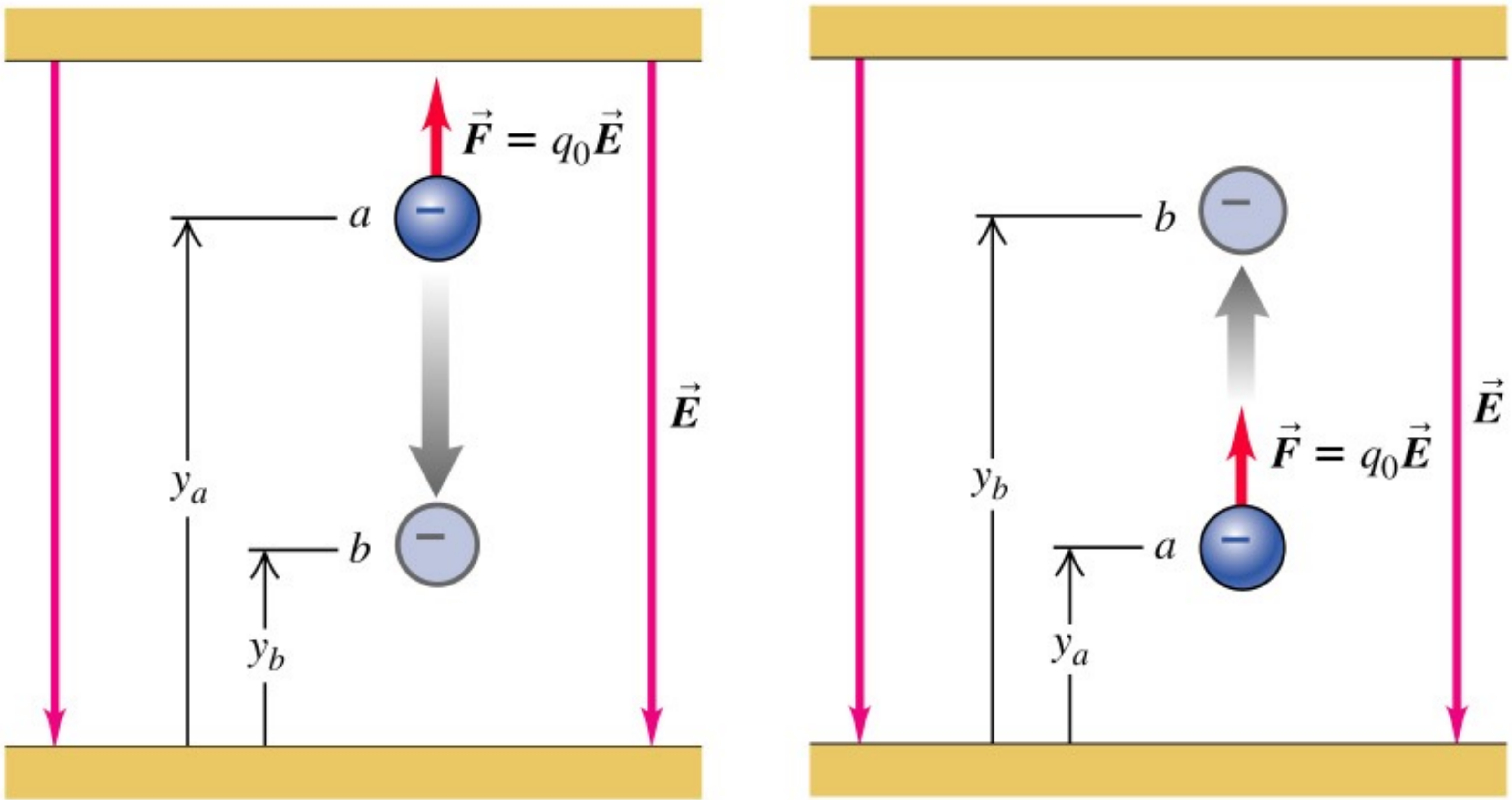
Electric Potential Energy in a Uniform Field



(a) Positive charge moves in direction of $\vec{E}$: field does positive work on charge, potential energy U decreases

(b) Positive charge moves in direction opposite $\vec{E}$: field does negative work on charge, potential energy U increases

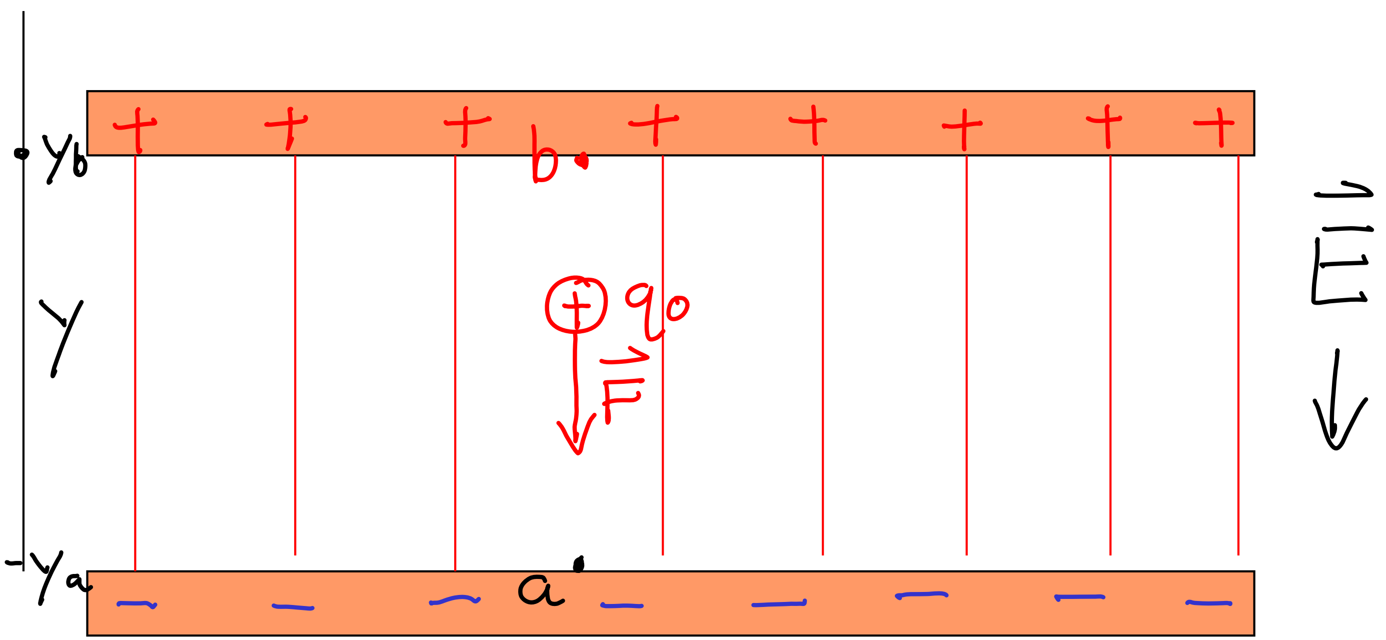
Electric Potential Energy in a Uniform Field



(a) Negative charge moves in direction of $\vec{E}$: field does negative work on charge, potential energy U increases

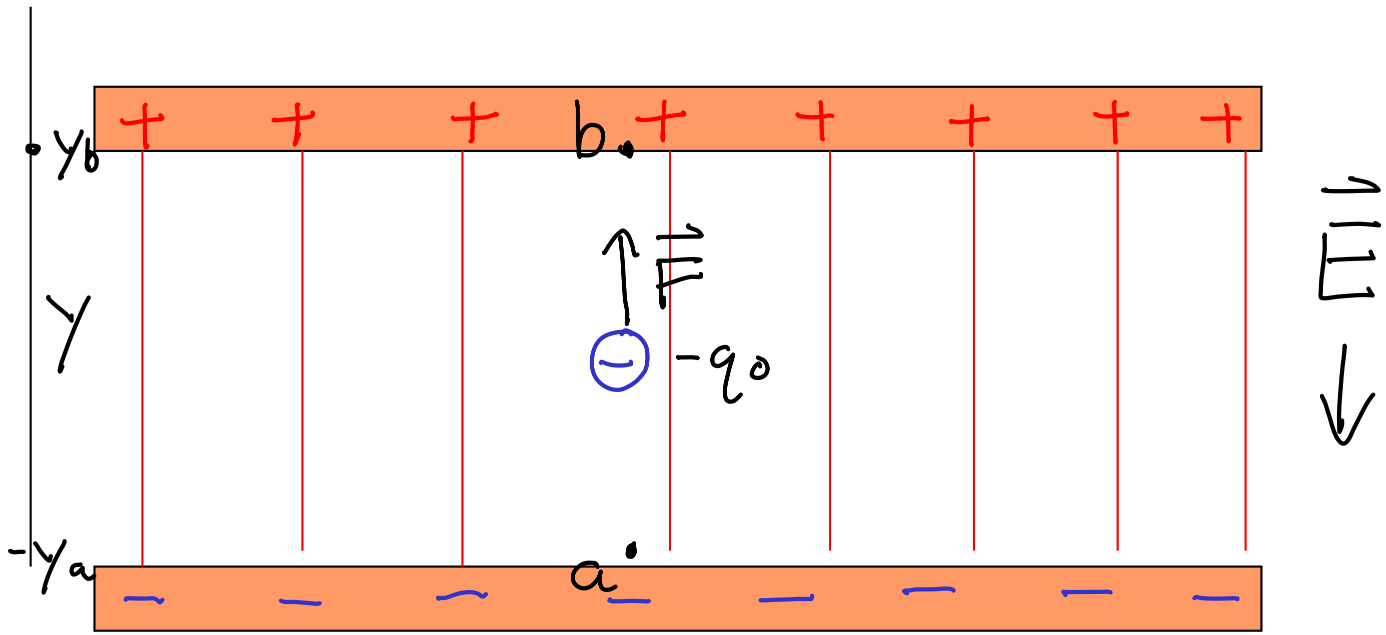
(b) Negative charge moves in direction opposite $\vec{E}$: field does positive work on charge, potential energy U decreases

Electric Potential Energy in a Uniform Field

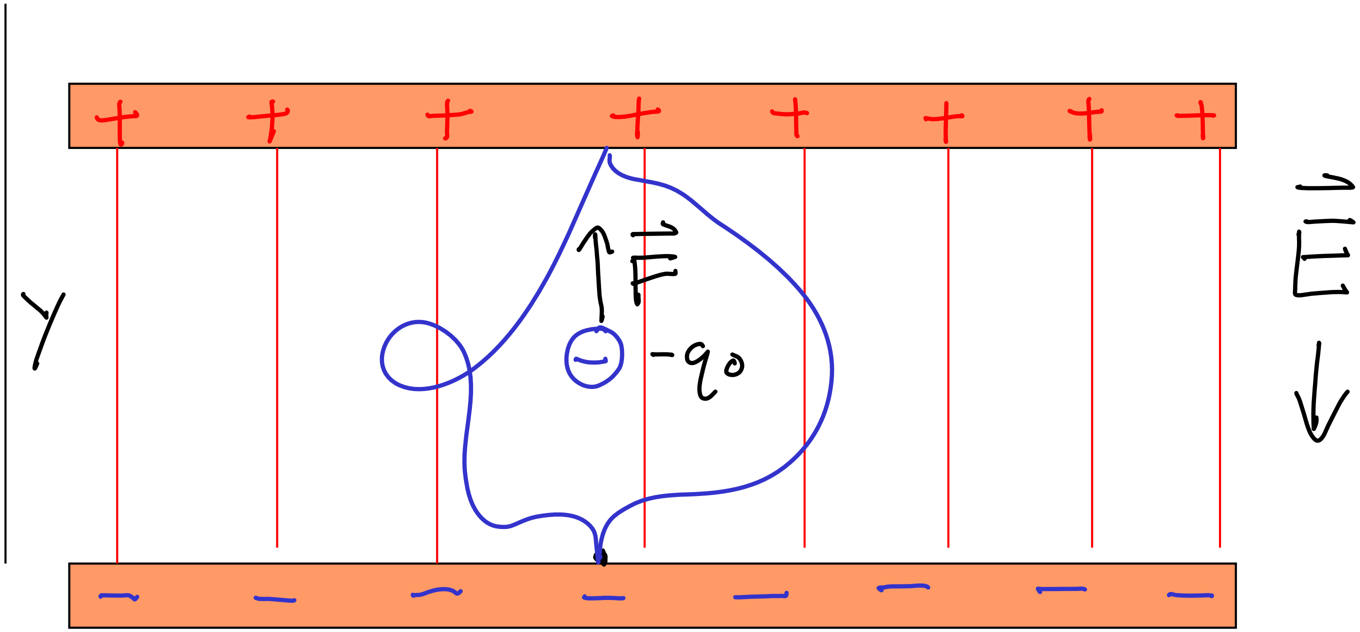


$$U_a - U_b = W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$$

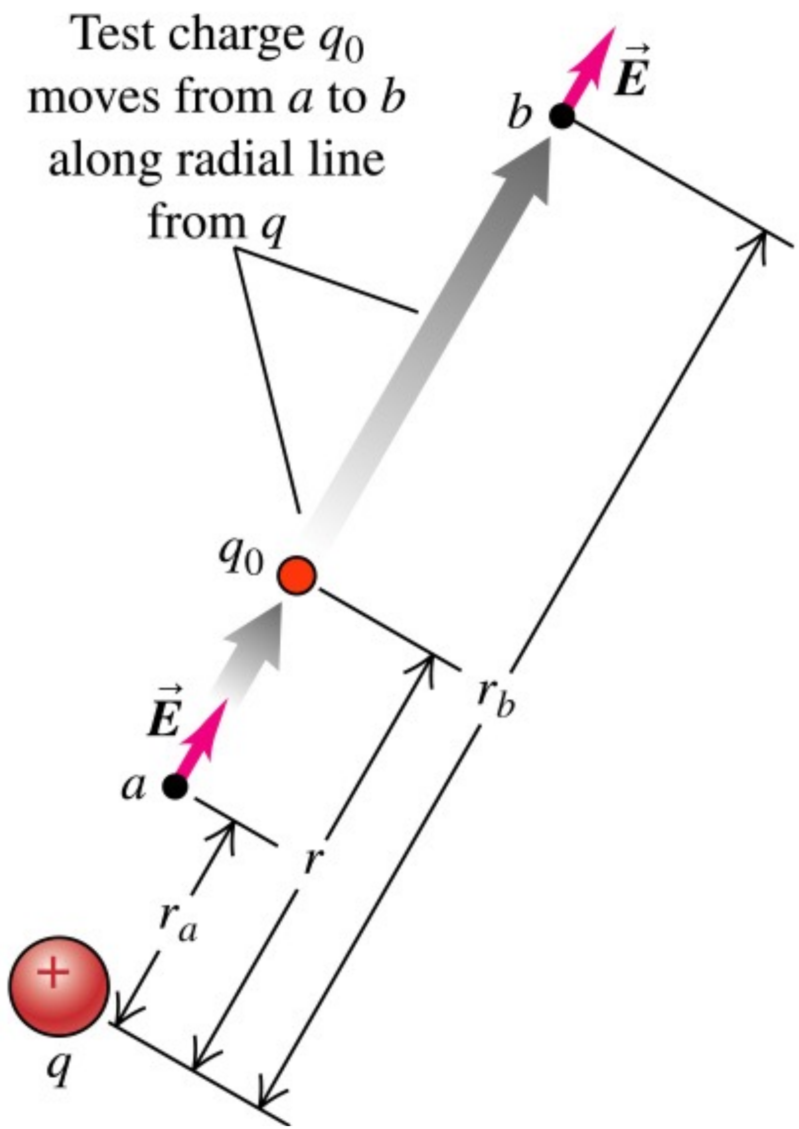
Electric Potential Energy in a Uniform Field



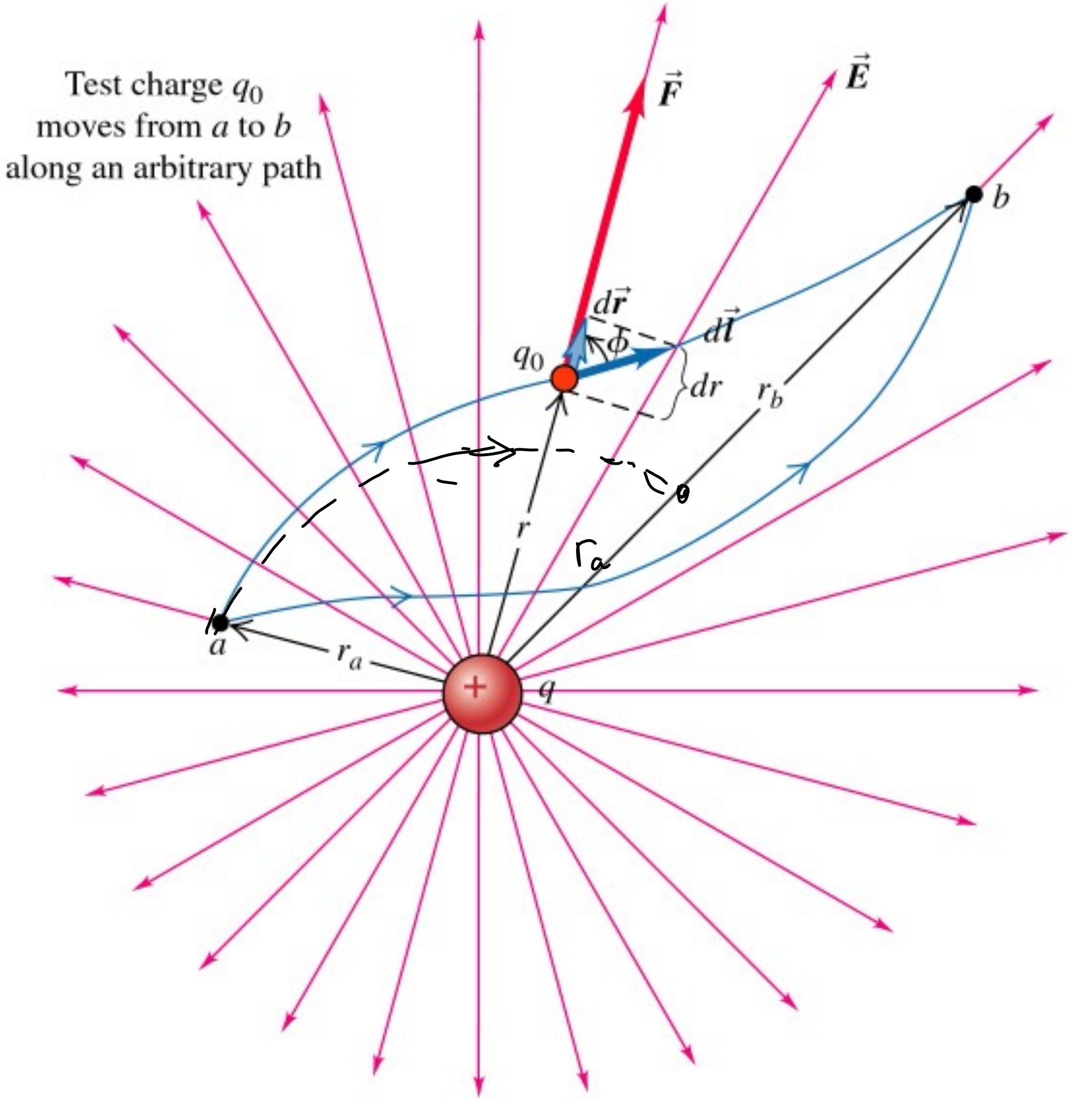
Electric Potential Energy in a Uniform Field



Electric Potential Energy of Two Charges



Electric Potential Energy of Two Charges

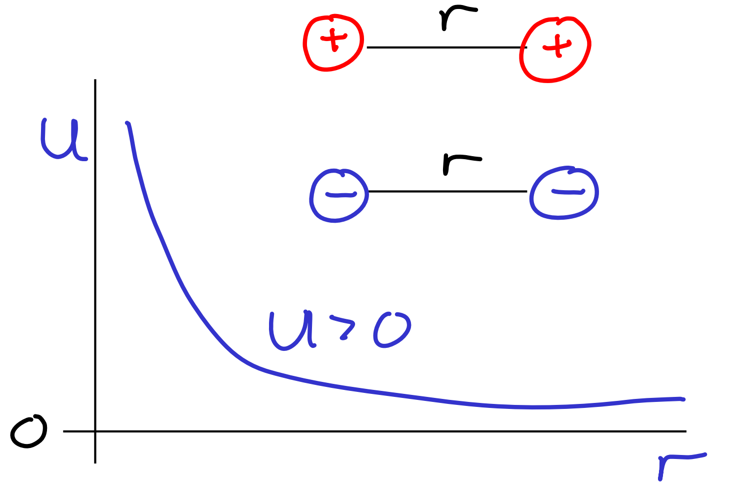


Electric Potential Energy of Two Charges

$$U_a - U_b = \frac{q q_0}{4\pi\epsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

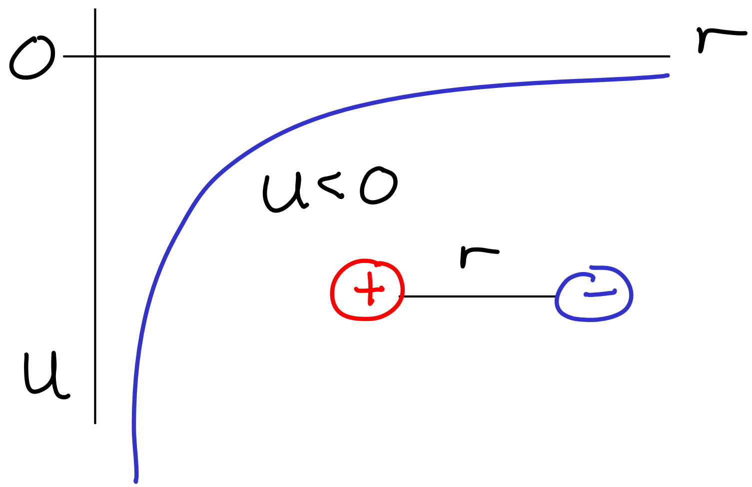
$$U_a = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r_a} + C$$

$$U_b = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r_b} + C$$



$$U = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$$

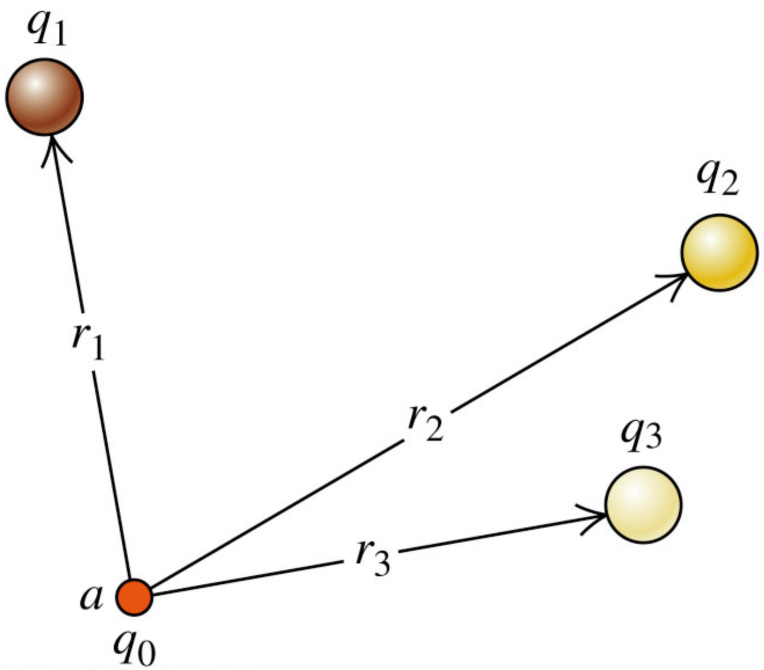
electric potential energy for two point charges q + q_0



Note :- $U = 0$ at $r = \infty$

- works for both signs of charges

Electric Potential Energy of Point Charges



$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

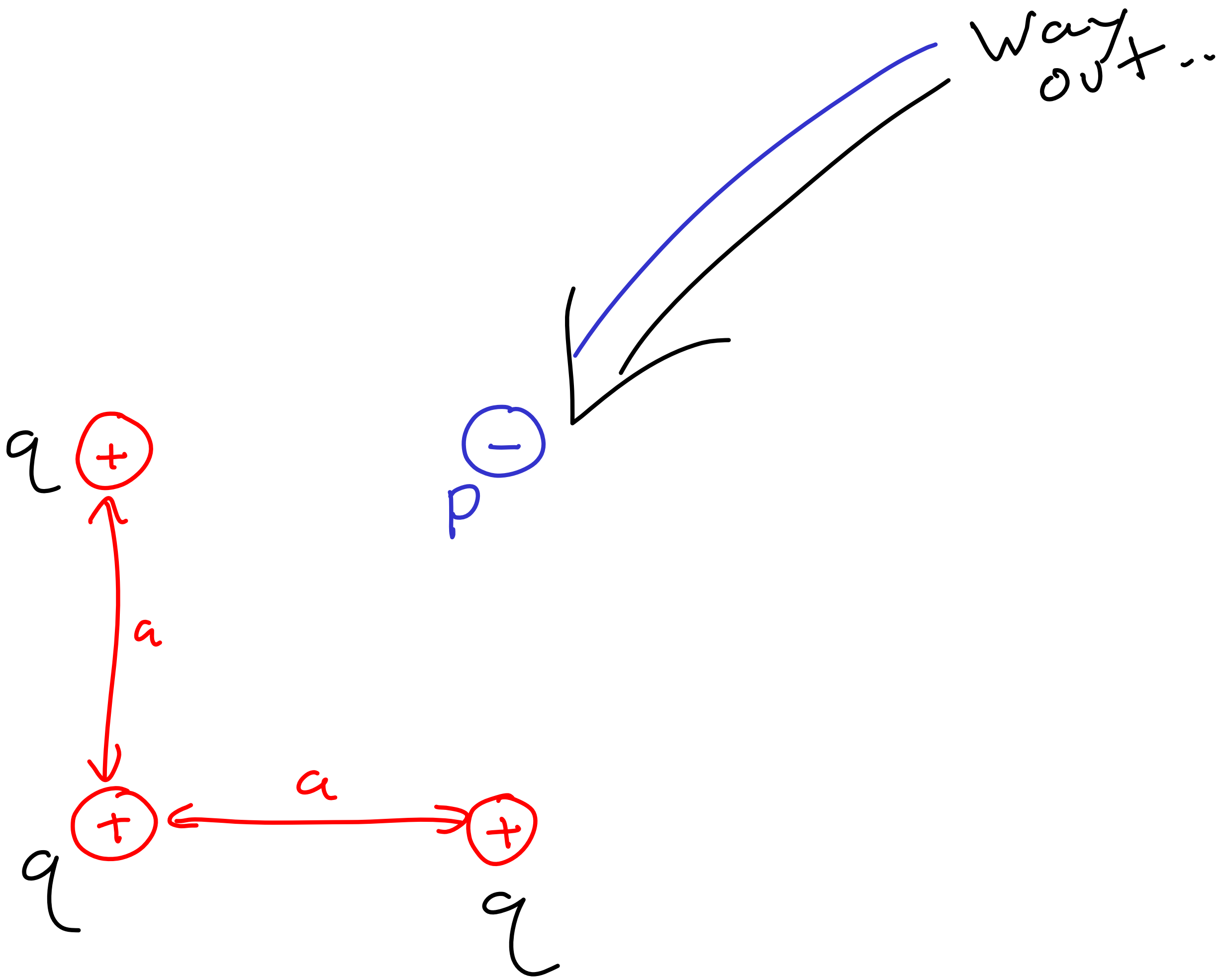
$$= \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

Algebraic sum
(not vector)

Generally:

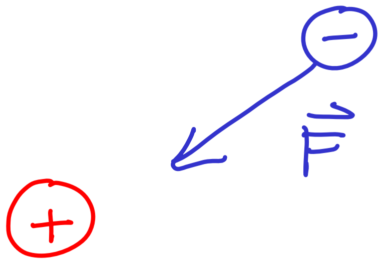
$$U = \frac{q_0}{4\pi\epsilon_0}\int \frac{dq}{r}$$

What happens?



Electric Potential

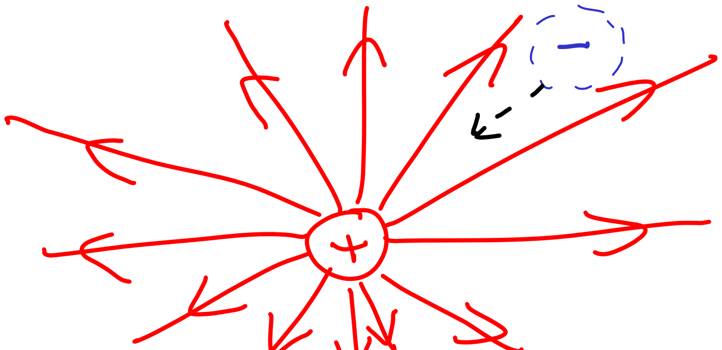
Electric force
must act on a charge



Electric potential energy:
depends on the charge

$$U = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$$

Electric field:
exists independent of test charge



Electric potential

$$V = \frac{U}{q_0} \leftarrow \text{test charge}$$

or

$$U = q_0 V$$

Units of Volts $= \frac{J}{C}$

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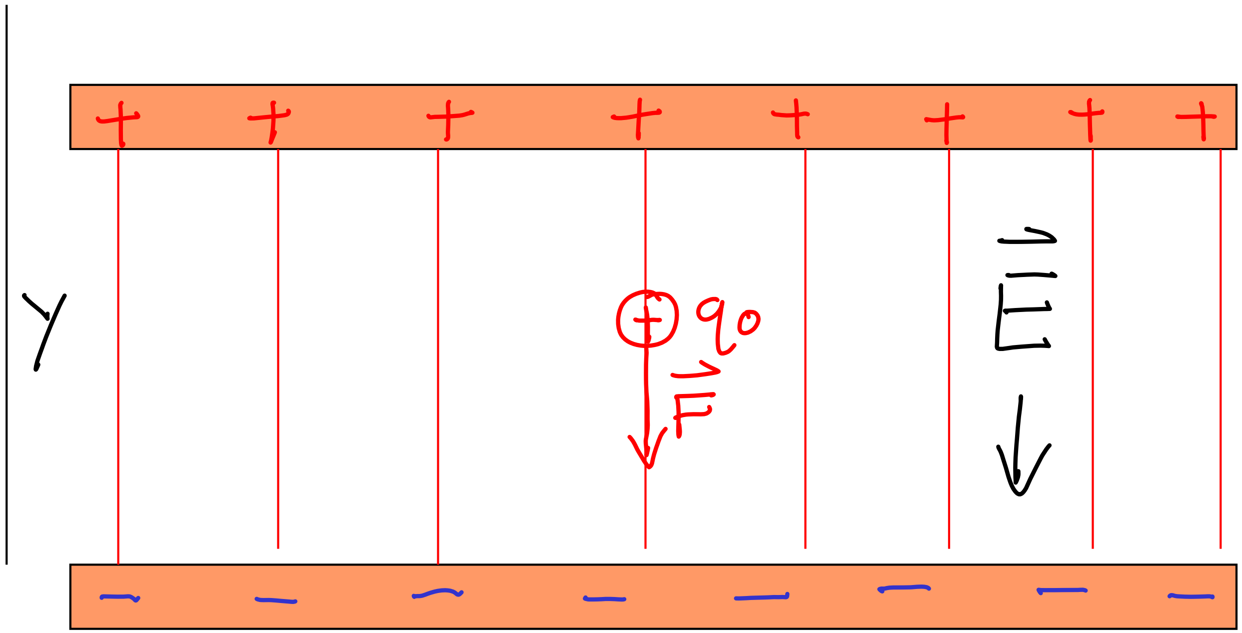
Electric Potential of a Uniform Field

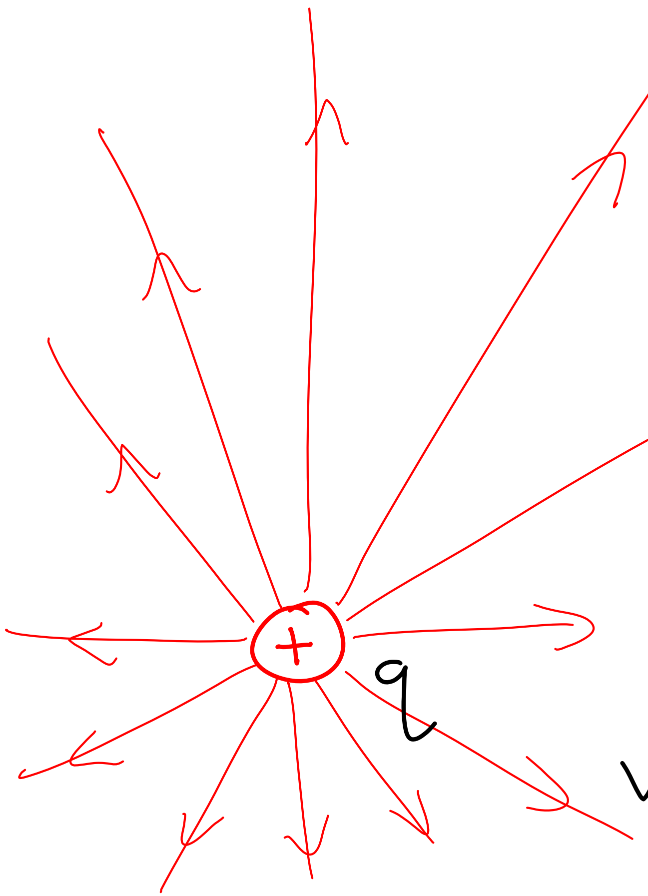


$U(y) = q_0 E y$

$V(y) = E y$

Can speak of the electric potential (voltage) at any point, without reference to a test charge

Electric Potential of a Point Charge

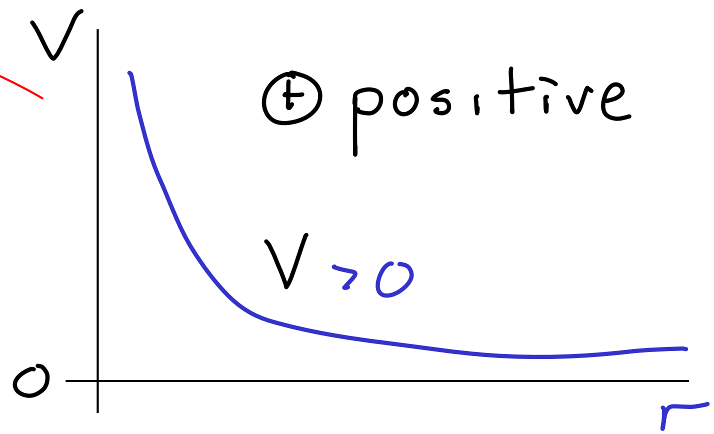


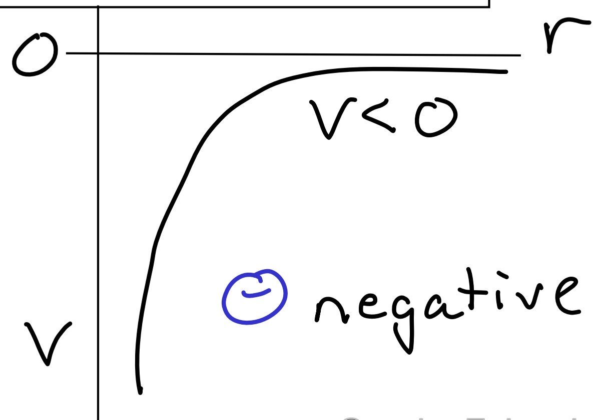
$$U(r) = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r}$$

$$V(r) = \frac{U(r)}{q_0}$$

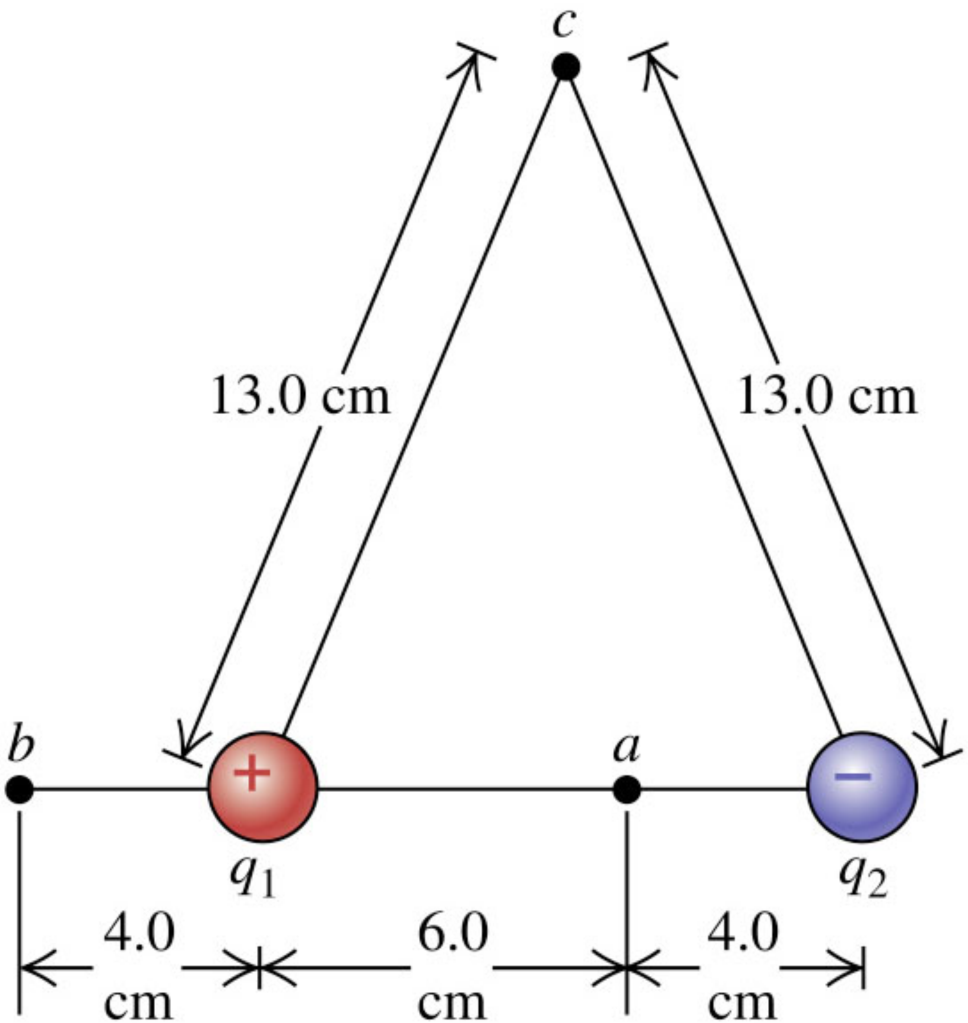
$$\boxed{V(r) = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

Electric potential of a point charge





Electric Potential of Many Point Charges



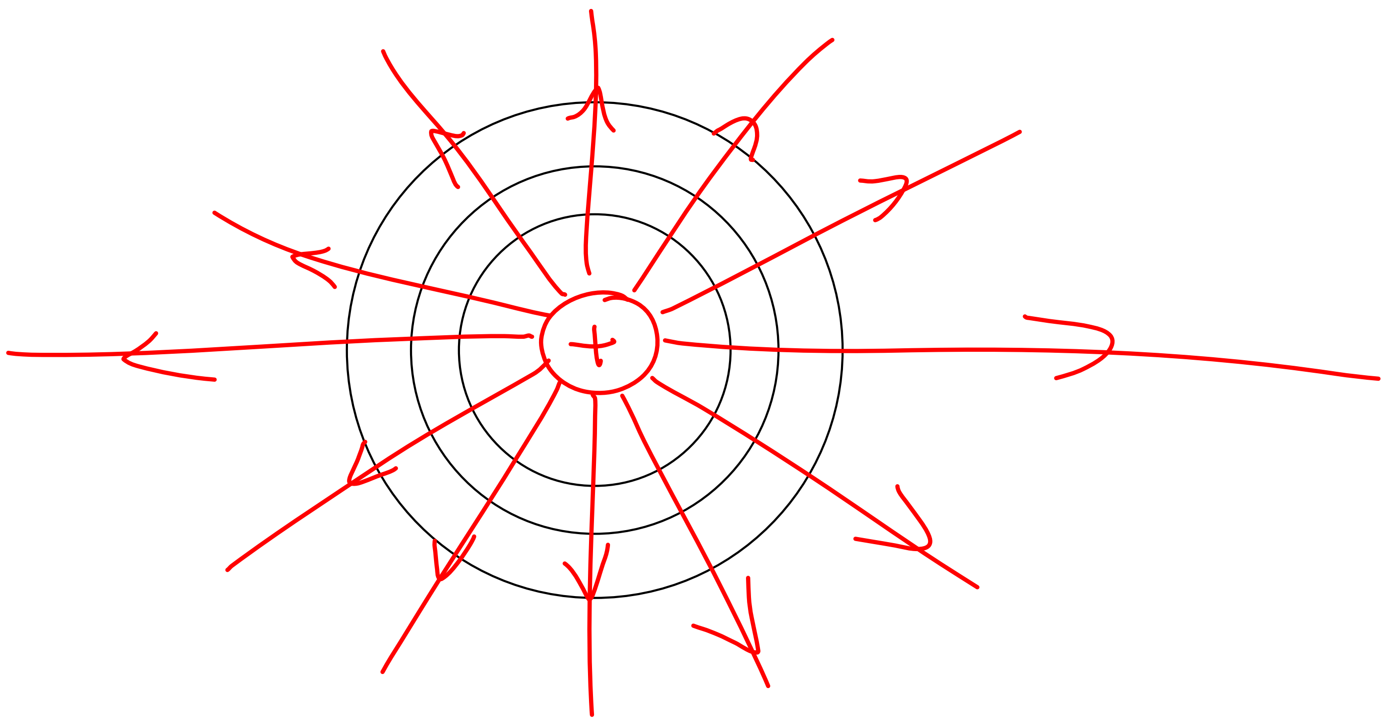
$$V = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i}$$

or $$V = \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r}$$

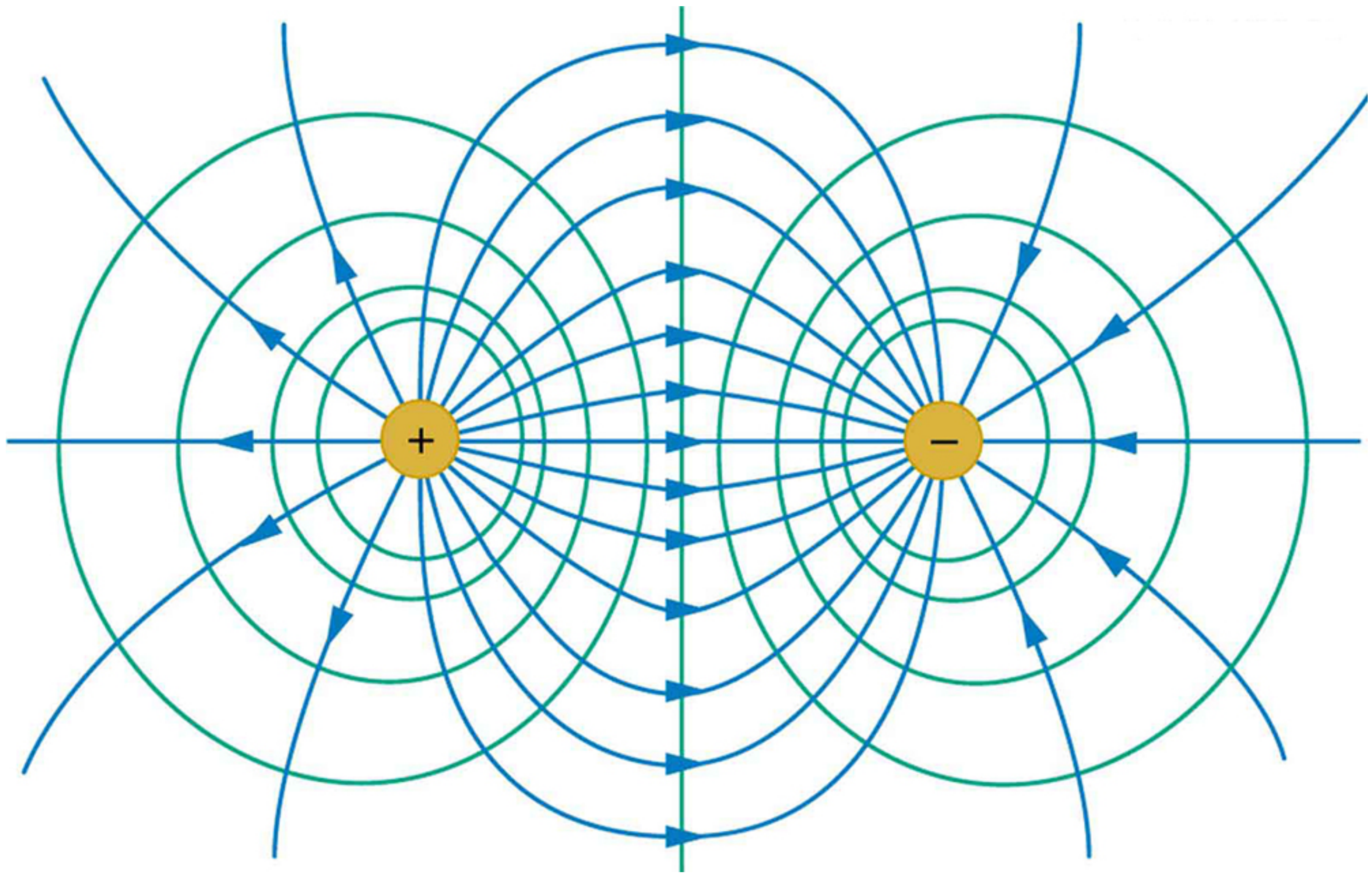
Equipotential surfaces

Visual means of expressing the electric potential

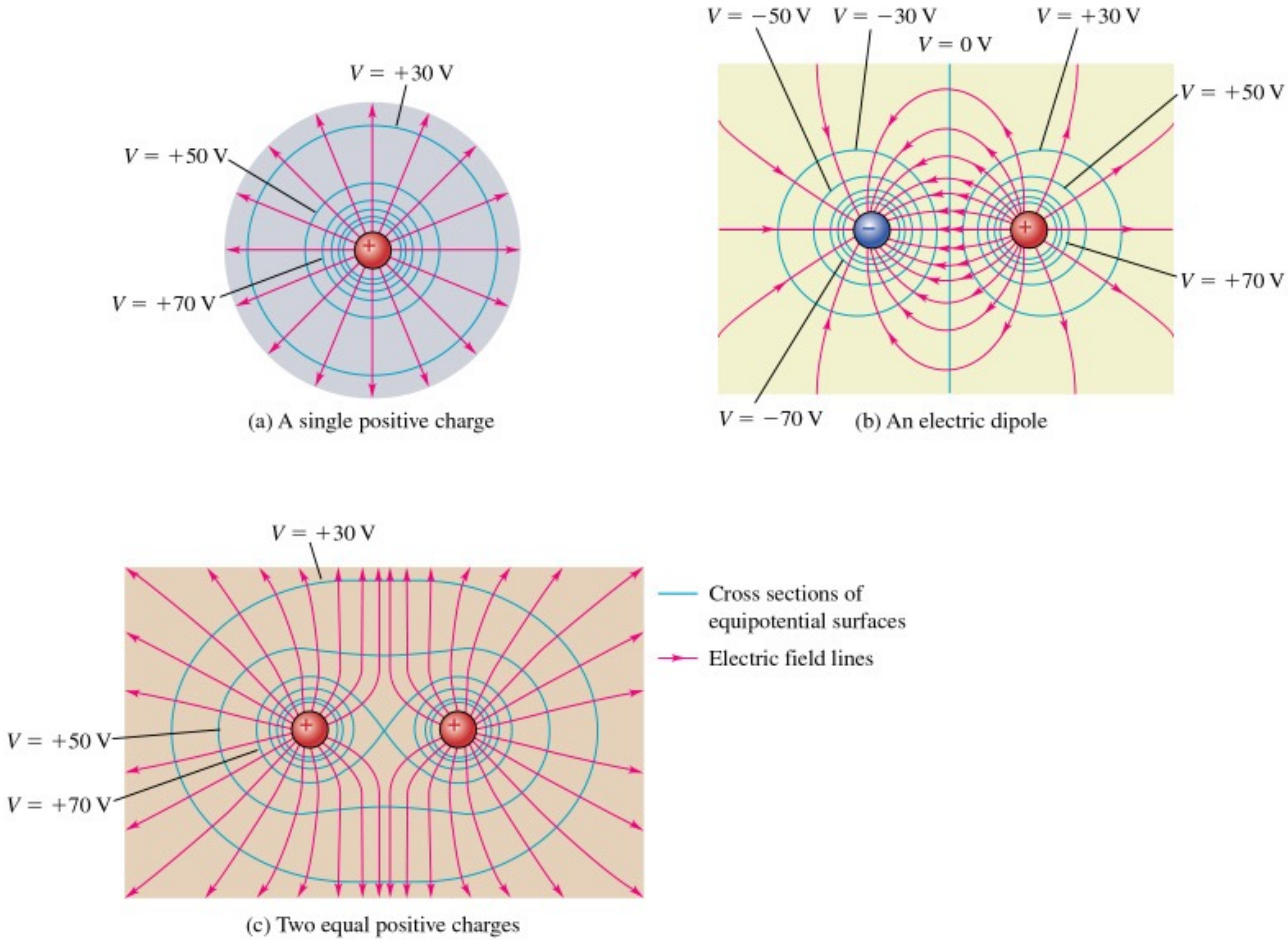
Equipotential surface = Surface of constant electric potential



Equipotential surfaces



Equipotential surfaces



Electric Potential and Electric Field

Work done on a test charge q_0 by $\vec{E}$

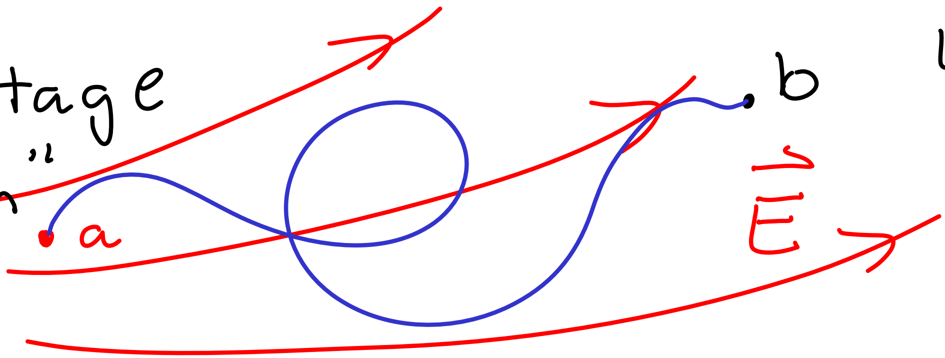
$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{\ell} = \int_a^b q_0 \vec{E}\cdot d\vec{\ell}$$

$$W_{a\to b} = q_0 (V_a - V_b)$$

$$V_a - V_b = \frac{-(U_b - U_a)}{q_0} = \frac{-\Delta U_{ba}}{q_0} = \frac{W_{a\to b}}{q_0}$$

$$\boxed{V_a - V_b = \int_a^b \vec{E}\cdot d\vec{\ell}}$$

High voltage
"upstream"



Low voltage
"downstream"

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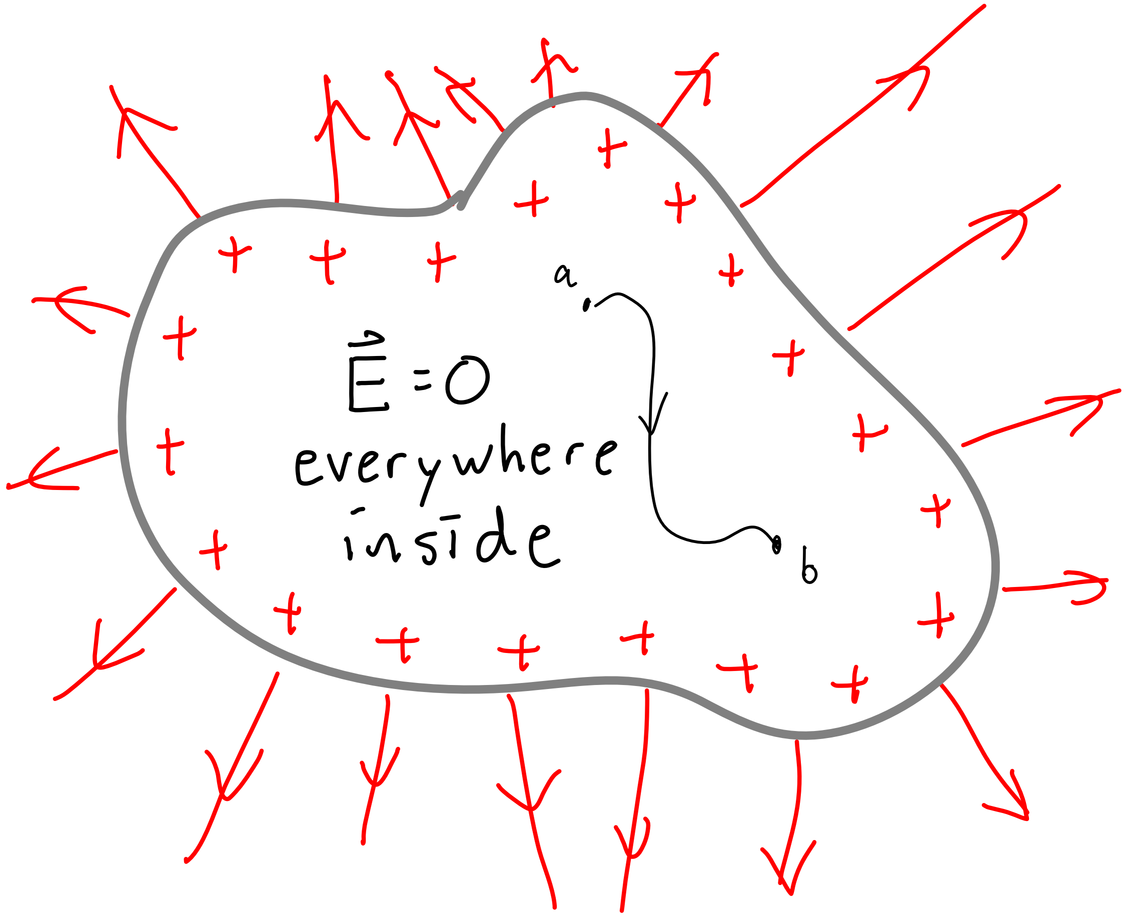
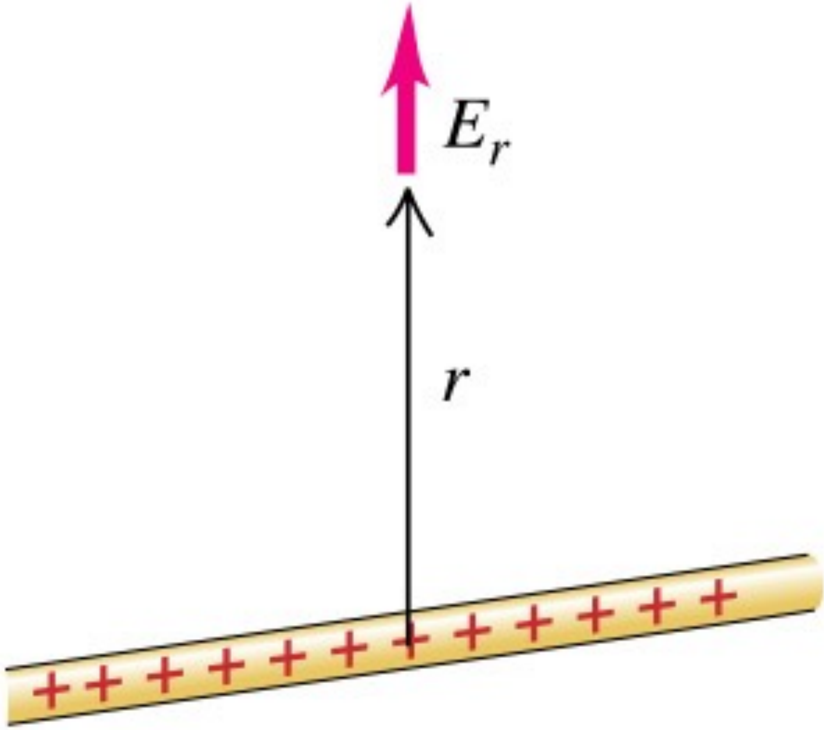
Electric Potential for a Conductor



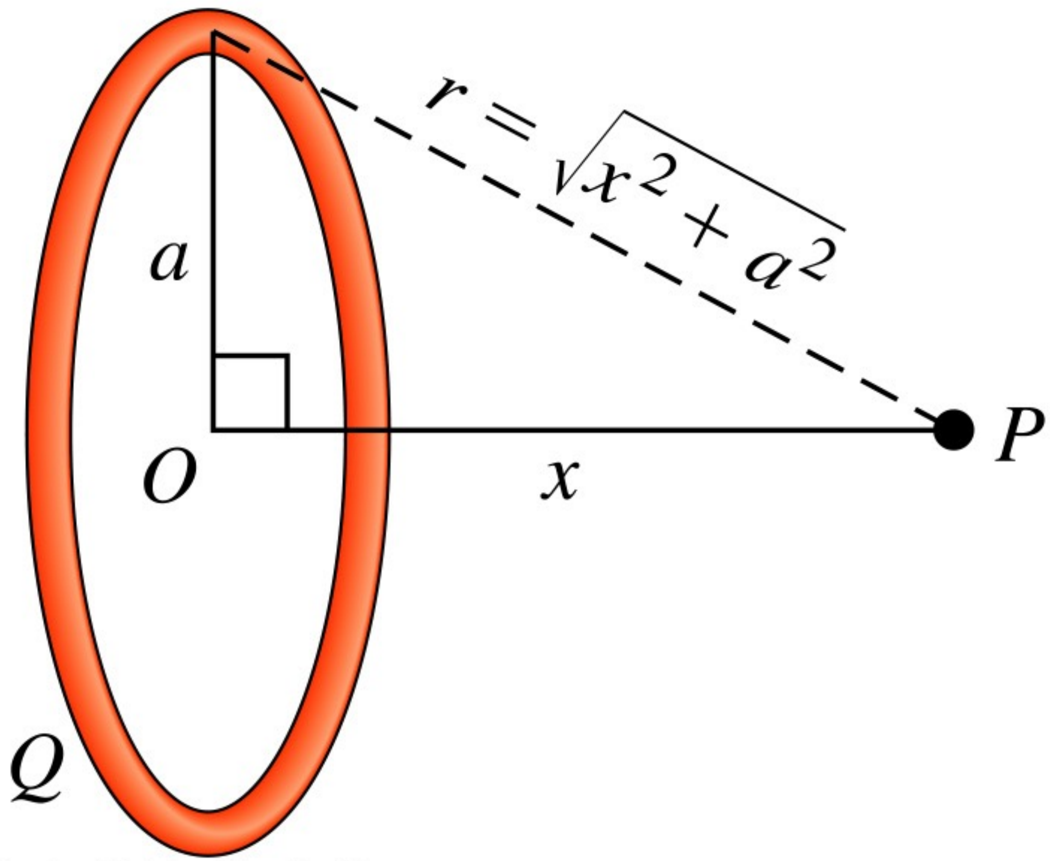
Example 23.10



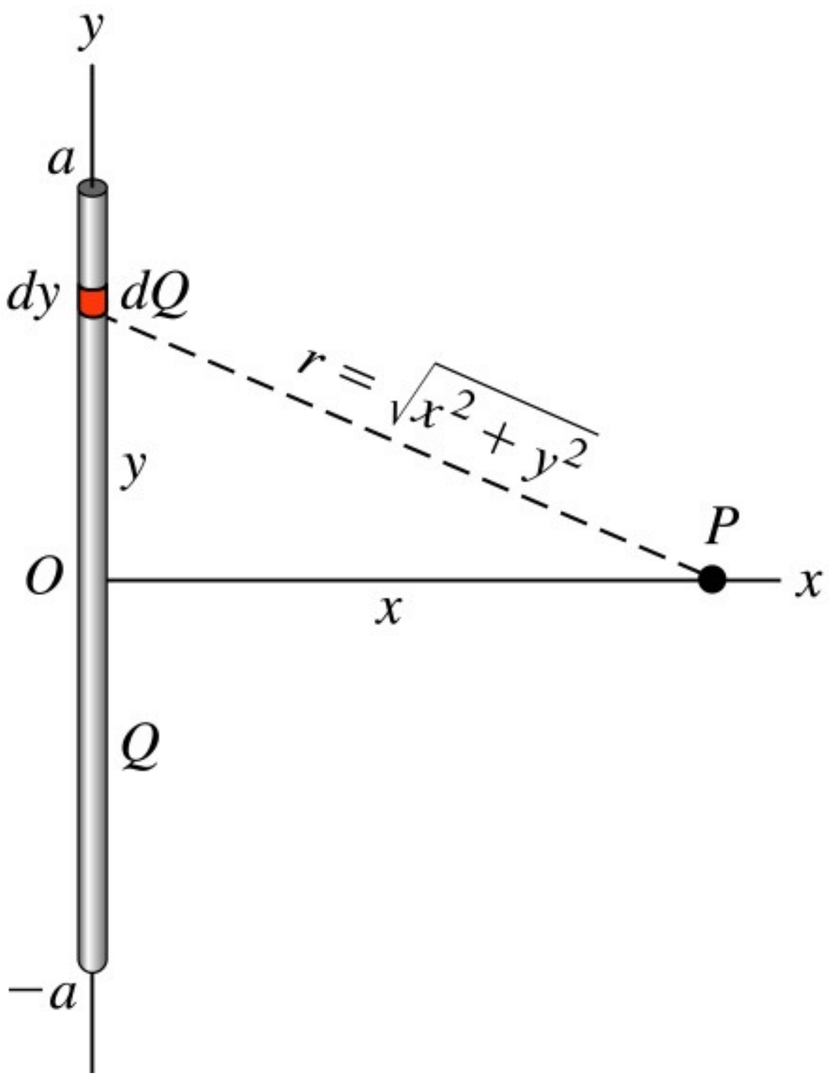
$$E_r = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{\ell}$$

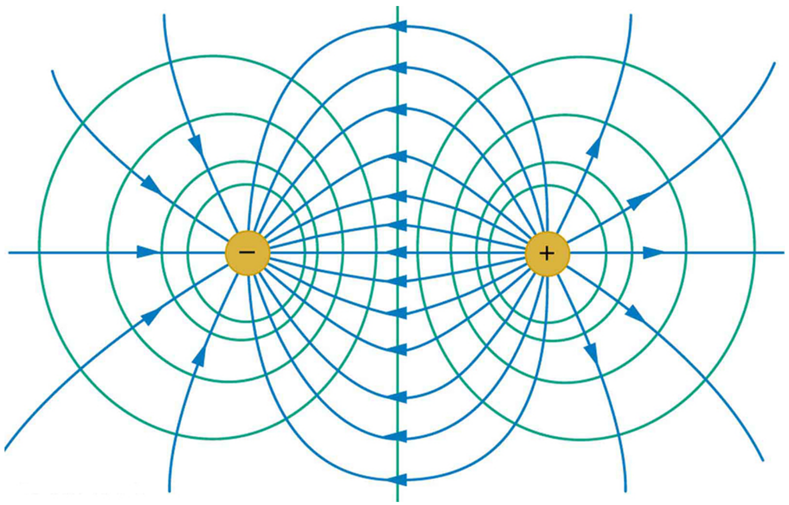
Example 23.11



Example 23.12



Potential Gradients



$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right)$$