THE UNIVERSITY OF SYDNEY

PHYS1902 – PHYSICS 1B (ADVANCED)

NOVEMBER 2011

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

DATA

Density of water	ρ	=	1.00×10^{3} kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^{5} Pa
Magnitude of local gravitational field	g	=	9.80 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^{8} m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg
Rydberg constant	R_H	=	1.097×10^{7} m^{-1}

SECTION A
(Please use a separate booklet for each section)

Question 1

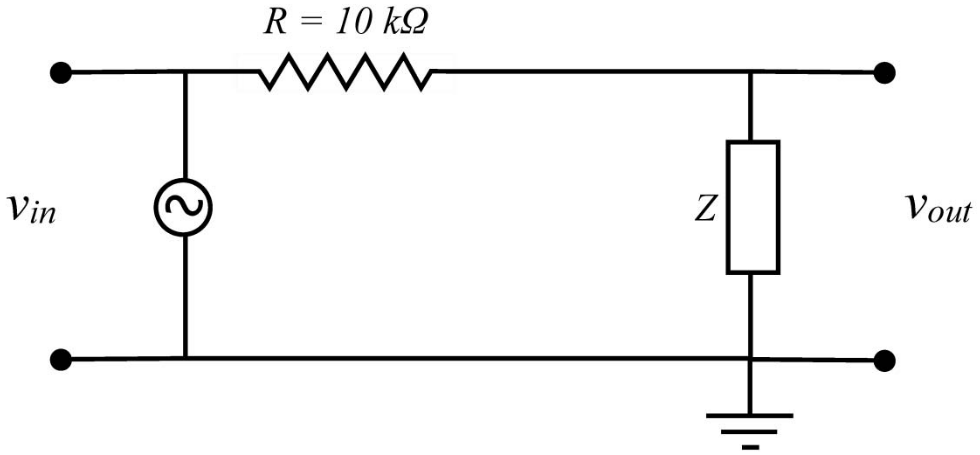
A mass spectrometer uses electric and magnetic fields to measure the masses of ions.

(a) First, positive ions are created, but with a distribution of speeds. Using only the Lorentz force law, explain how electric and magnetic fields can be used as a *velocity selector* to pick out specific speeds of the ions. Draw a diagram to support your explanation.

(b) Next, with all the ions possessing the same speed, a magnetic field is used to separate and measure different masses. The ions move on circular arcs, the radius of which depends on their masses. Again, using only the Lorentz force law, express the mass of an ion in terms of its velocity v, the radius of its motion R, its charge q, and the magnetic field B.

(5 marks)

Question 2



A simple voltage divider is shown in the circuit above, in which one component, R, is a $10\,\mathrm{k\Omega}$ resistor and the other, Z, is an unknown impedance (either an ideal capacitor or inductor).

(a) Derive an expression for v_{out}/v_{in} in terms of R and Z.

(b) A measurement at a frequency of $10\,\mathrm{kHz}$ gives $|v_{out}|/|v_{in}| = 0.5$. Explain how a further measurement, at say $1\,\mathrm{kHz}$, would allow you to determine whether Z is a capacitor or inductor.

(c) Based on the measurement in part (b) show that at $10\,\mathrm{kHz}$ the reactance, X, of Z is given by

$$X = \frac{R}{\sqrt{3}}.$$

(5 marks)

Question 3

A magnet dropped through a hollow copper pipe is observed to fall very slowly. By considering what happens as the magnet moves past a fixed point P on the pipe, carefully explain this observation.

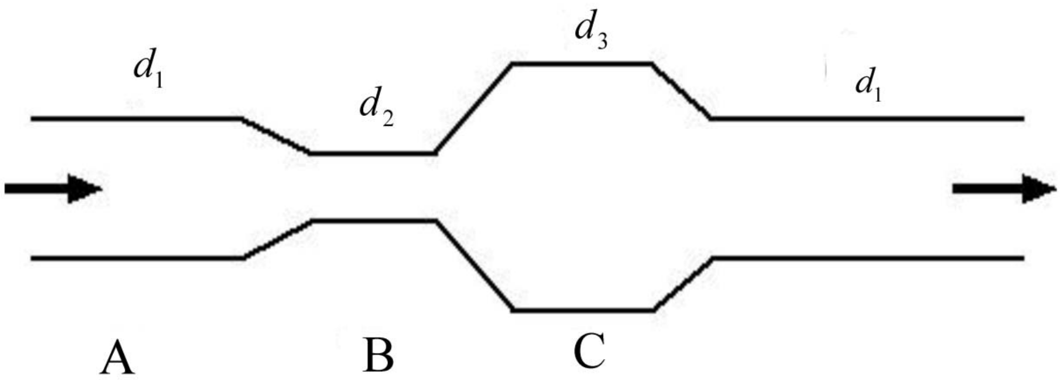
Assume the magnet falls without spinning, with its north pole downwards.

Your explanation should include a diagram and be given in terms of physical principles but without using equations. Your answer should be no more than one page long, excluding the diagram.

(5 marks)

Question 4

A non-viscous, incompressible fluid flows through a horizontal pipe of circular cross-section. The initial diameter of the pipe is d_1, the pipe then shrinks to a diameter $d_2 < d_1$, expands to a diameter $d_3 > d_1$, and then returns to its initial diameter d_1 before exiting to the atmosphere.

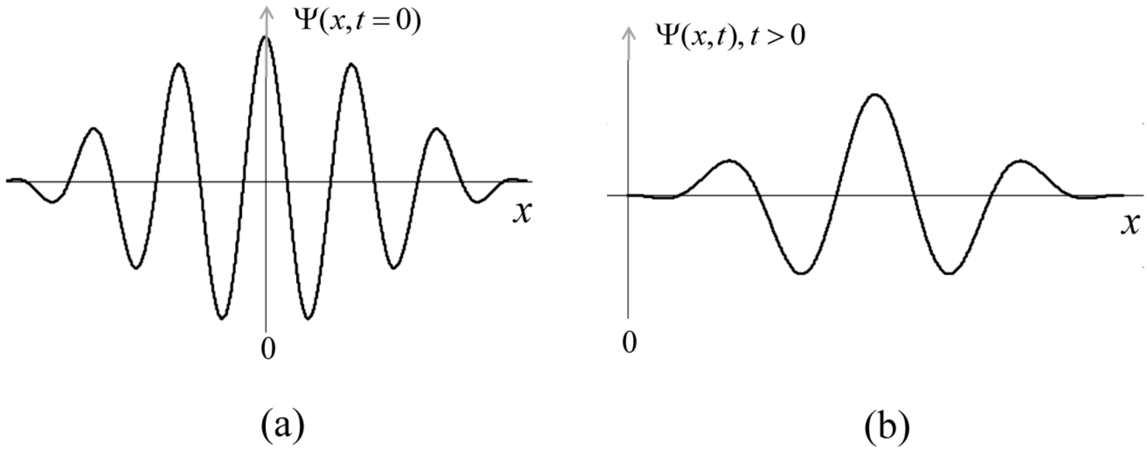


(a) A thief wants to drill a small hole so some fluid leaks out of the pipe before the exit. Will he be successful if he drills the hole at Point A, Point B, or Point C? Briefly explain your answer.

(b) Suppose the pipe contains alcohol, with density $800\,\text{kg.m}^{-3}$ and viscosity $\eta = 2.0\times10^{-3}\,\text{Pa.s}$, and that the exit pipe has a diameter $d_1 = 0.10\,\text{m}$. If the volume flow rate is $8.0\times10^{-4}\,\text{m}^3.\text{s}^{-1}$, will the flow through the last section of pipe be turbulent?

(5 marks)

Question 5



The wave function for an electron free to move in the x direction at time $t = 0$ is shown in panel (a) of the figure above. The wave function for the electron at a later time t is shown in panel (b) of the figure above.

(a) Does this electron have a uniquely defined position at time $t = 0$? Briefly explain.

(b) Does this electron have a uniquely defined momentum at time $t = 0$? Briefly explain.

(c) Does this particle have a uniquely defined energy at time $t = 0$? Briefly explain.

(5 marks)

Question 6

In quantum physics the orbital angular momentum $\vec{\boldsymbol{L}} = \vec{\boldsymbol{r}} \times \vec{\boldsymbol{p}}$ of an electron, where $\vec{\boldsymbol{r}}$ is the electron position and $\vec{\boldsymbol{p}}$ is its momentum, is quantised. The magnitude L of the momentum and a component L_z of the momentum with respect to a chosen axis can only take the values

$$L = \sqrt{\ell(\ell+1)}\,\hbar \quad \text{with} \quad \ell = 0,1,2,\ldots,n-1,$$

and

$$L_z = m_\ell \hbar \quad \text{with} \quad m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell,$$

where n is the principal quantum number, ℓ is the orbital quantum number, and m_ℓ is the magnetic quantum number.

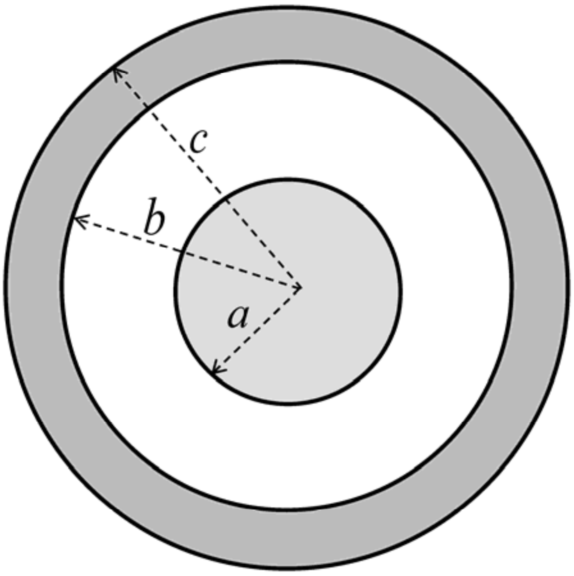
(a) The Bohr model for the hydrogen atom always has a non-zero orbital angular momentum. Does the quantum model for the hydrogen atom, in its ground state, have a non-zero orbital angular momentum? Briefly explain your answer in terms of the allowed values of L and L_z.

(b) The allowed values of L and L_z require that $|L_z| < L$. Explain why this inequality is required for consistency with the Heisenberg Uncertainty Principle.

(5 marks)

SECTION B
(Please use a separate booklet for this section)

Question 7



A long coaxial cable consists of an inner cylindrical conductor with radius a and an outer coaxial cylinder with inner radius b and outer radius c. The outer cylinder is insulated and has no net charge. The inner cylinder has a uniform positive charge per unit length λ.

(a) In a cross-sectional diagram, draw where the charges (positive and negative) will reside on the conductors.

(b) Calculate the magnitude and direction of the electric field for a point between the cylinders at a distance r from the axis $(a < r < b)$.

(c) Calculate the magnitude and direction of the electric field for a point outside the outer cylinder, at a distance r from the axis $(r > c)$.

(d) In a cross-sectional diagram, sketch the electric field lines in all regions.

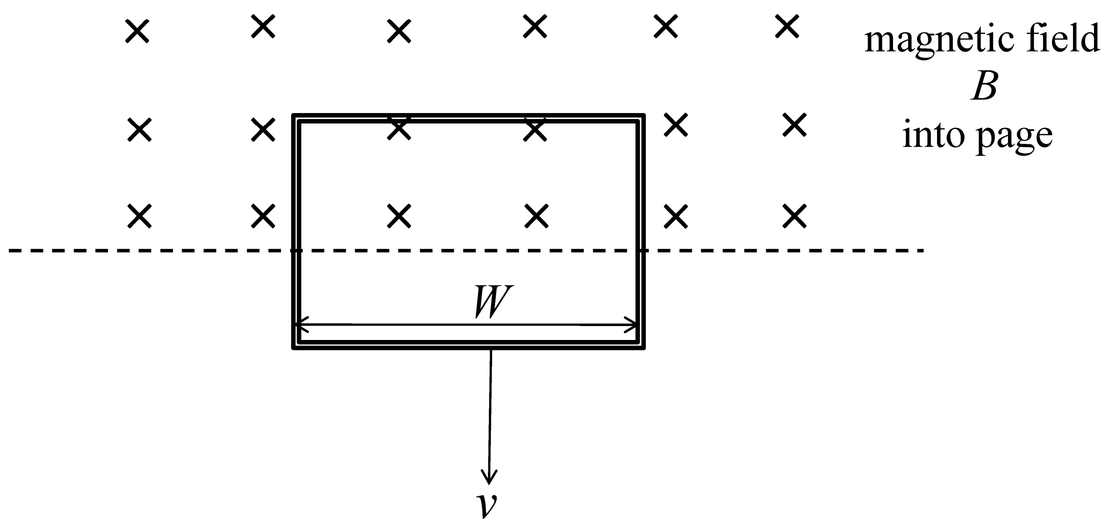
Some damage to the coaxial cable allowed a conducting strip to connect the inner cylindrical conductor to the inside of the outer cylindrical conductor.

(e) In a cross-sectional diagram, draw where the charges (positive and negative) will now reside.

(f) In a cross-sectional diagram, sketch the electric field lines for this new configuration in all regions.

(10 marks)

Question 8



A rectangular conducting loop of wire has width W and mass m. It is initially at rest in a region of uniform magnetic field B directed into the page. The loop is oriented perpendicular to the direction of the field. The loop is allowed to fall out of the region at uniform speed v as shown in the figure above.

(a) Show that the emf induced in the loop as it falls out of the region of the magnetic field is given by

$$\varepsilon = BWv.$$

(b) Derive an expression for the magnitude of the induced current flowing in the loop if it has a resistance of R?

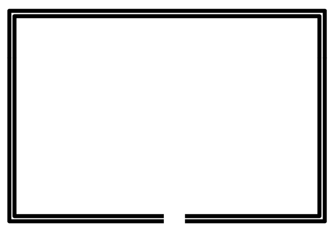
(c) What is the direction of the current flow in the loop? Briefly explain the reason for your answer.

(d) Find an expression for the force exerted by the magnetic field on the loop. What is the direction of the force?

(e) Show that the constant speed v of the loop falling out of the region of magnetic field is given by

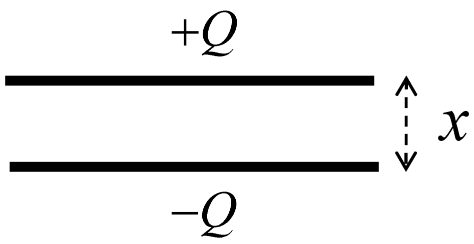
$$v = \frac{m\,g\,R}{W^2 B^2}.$$

(f) If the loop had been cut as shown in the diagram below, describe what its motion would be as it falls out of the region of the magnetic field.



(10 marks)

Question 9



A parallel-plate capacitor with plate area A and separation x has charges $+Q$ and $-Q$ on its plates. The capacitor is disconnected from the voltage source that placed this charge, so that the charge on each plate now remains fixed.

(a) Show that the total energy stored in this capacitor is given by:

$$U = \frac{Q^2 x}{2\varepsilon_0 A}.$$

(b) The plates are pulled apart an additional distance dx. Obtain an expression for the change in the stored energy.

(c) The change in stored energy must equal the work $dW = F\,dx$ done in pulling the plates apart, where F is the force with which the plates attract each other. Find an expression for F.

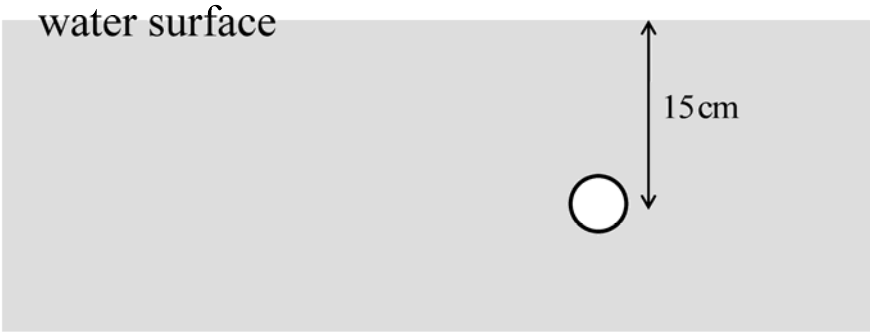
(d) Determine the strength of the electric field E between the two plates, in terms of the plate area A, the separation x, the charge Q, and fundamental constants.

(e) Is the force F determined from part (c) equal to QE, where E is the electric field between the plates? Explain why or why not?

(10 marks)

Question 10

A small ball has a density $250\,\text{kg.m}^{-3}$ and volume of $1.0\times10^{-4}\,\text{m}^3$.



(a) When floating on the surface of the water, what fraction of the ball's volume is submerged?

(b) You now push the ball underwater. Draw a free-body diagram of the ball when it is held so that its centre is submerged by 15 cm under the surface of the water.

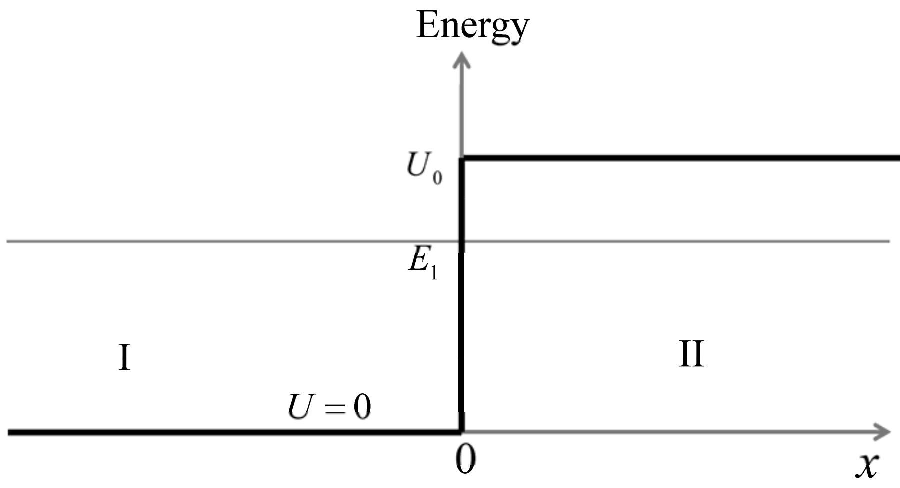
(c) Calculate the work that you do to push the ball to this depth at constant velocity.

(d) You now release the ball. Briefly describe its motion (qualitative only).

(e) Using energy considerations, calculate the maximum height above the surface of the water the ball will reach. Ignore any drag forces.

(10 marks)

Question 11



An electron with energy E_1 is free to move in x direction. It is located in a region of space with the potential energy configuration shown in the figure. In the region $x < 0$ (denoted region I) the potential energy is $U = 0$, and in the region $x > 0$ (denoted region II) the potential energy is $U = U_0 > E_1$. This situation may be interpreted physically as describing an electron incident from the left on a step barrier potential of height U_0. The stationary state of the electron is described by the $1-\mathrm{D}$ time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + U(x)\psi(x) = E\psi(x).$$

(a) The stationary state wave function of the electron in region I may be written

$$\psi_{\mathrm{I}}(x) = A\mathrm{e}^{ikx} + B\mathrm{e}^{-ikx}.$$

Using the Schrödinger equation, determine the value of k.

(b) The stationary state wave function of the electron in region II may be written

$$\psi_{\mathrm{II}}(x) = C\mathrm{e}^{-ax} + D\mathrm{e}^{ax}.$$

Explain why $D = 0$ is required, and using the Schrödinger equation, determine the value of a.

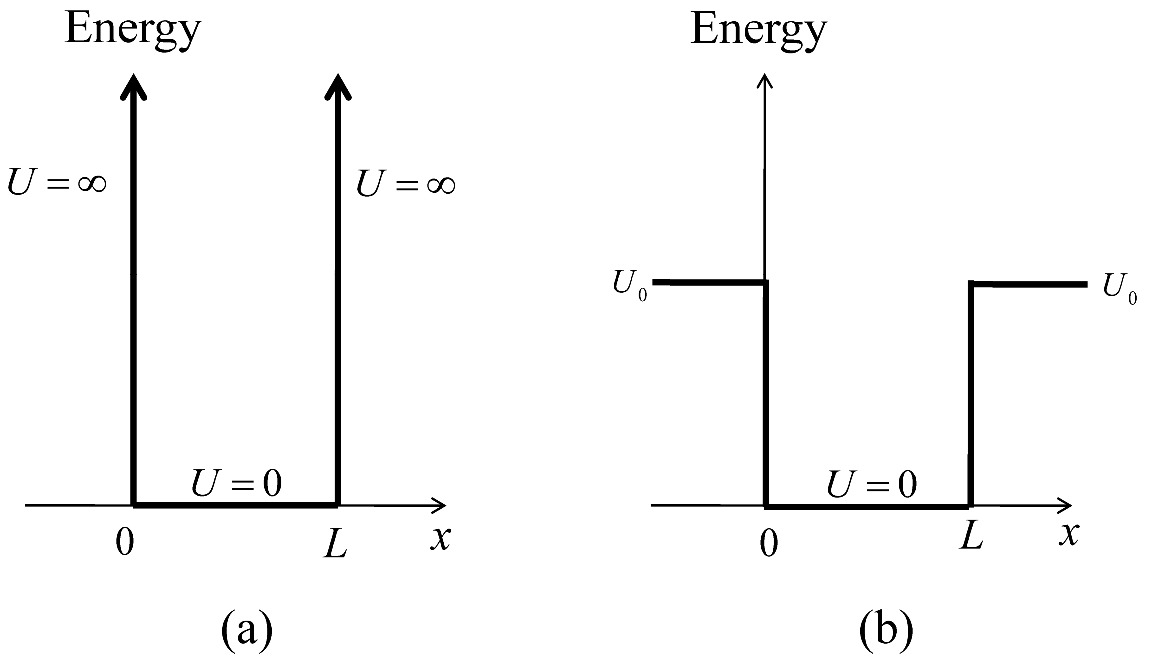
(c) By applying continuity of the wave function and its derivative at $x = 0$, show that

$$B = \frac{a + ik}{-a + ik}A \quad \text{and} \quad C = \frac{2ik}{-a + ik}A.$$

(d) Show that the result in part (c) implies $|B|^2 = |A|^2$. Give a brief physical interpretation for this result.

(10 marks)

Question 12



Consider an electron in an infinite square well extending from $x=0$ to $x=L$ (a "particle in a box"), as shown in panel (a) of the figure above, and also consider an electron in a finite square well extending from $x=0$ to $x=L$, as shown in panel (b) of the figure above. The potential inside each well is $U=0$. The infinite square well has $U=\infty$ in the regions $x<0$ and $x>L$, and the finite square well has $U=U_0$ in these regions.

(a) Sketch the wave functions $\psi_{\text{inf},1}(x)$ and $\psi_{\text{inf},2}(x)$ of the two lowest energy stationary states for the infinite square well shown in panel (a) of the figure above.

(b) Sketch $|\psi_{\text{inf},1}(x)|^2$ and $|\psi_{\text{inf},2}(x)|^2$ for the two states in part (a).

(c) Sketch the wave functions $\psi_{\text{fin},1}(x)$ and $\psi_{\text{fin},2}(x)$ of the two lowest energy stationary states for the finite square well shown in panel (b) of the figure above (assuming two such states exist).

(d) Sketch $|\psi_{\text{fin},1}(x)|^2$ and $|\psi_{\text{fin},2}(x)|^2$ for the two states in part (c).

(e) Compare the energies $E_{\text{inf},1}$ and $E_{\text{fin},1}$ of the lowest energy stationary states for the infinite square well as shown in panel (a) and for the finite square well as shown in panel (b). Briefly explain your answer.

(10 marks)

This is the end of your questions