The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1907: Mathematics Special Studies Program B	Semester 2, 2017

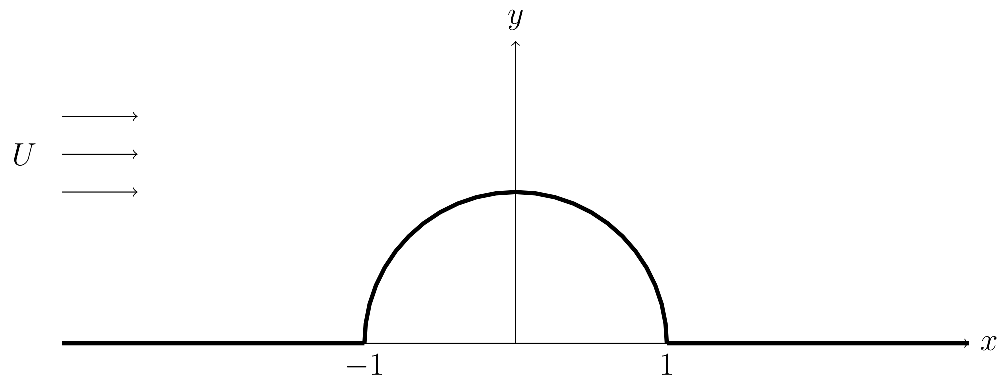
Lecturer: Sharon Stephen

This assignment is due by **5pm Thursday 31st August 2017**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard) at https://elearning.sydney.edu.au. Please submit only a PDF document (scan or convert other formats). It should include your name and SID. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission, and then to check your submission was successful. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 3.33% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	**Grade**	**Criterion**
10	A+	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on at least 1 of the 2 parts and good progress on the remaining part, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making good progress on both parts, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making substantial progress on only 1 of the 2 parts.
4	D	Some attempt, with some progress made on only 1 part.
2	E	No substantial progress made on any of the 2 parts.
0	F	No credit awarded.

1. Consider the two-dimensional, steady, incompressible, inviscid, irrotational fluid flow past the semi-circular cylinder of radius 1 shown below.



(a) (i) Explain why the transformation $\zeta = f(z) = z + \dfrac{1}{z}$ is conformal at all points except $z = 0$ and $z = \pm 1$.

(ii) Show that the transformation $\zeta = \xi + i\eta = z + \dfrac{1}{z}$ maps the boundary of the domain shown above to the ξ axis in the ζ-plane and the region above the boundary to the upper-half of the ζ-plane.

(b) (i) By considering a uniform flow of strength U in the ξ direction in the upper-half ζ-plane, where $\zeta = \xi + i\eta$, and using the results of part (a), find the complex potential, $w(z)$, of the fluid flow in the z-plane.

(ii) Explain why this uniform flow in the ξ-direction in the upper-half ζ-plane corresponds to the uniform flow of strength U in the x-direction around the semi-circle in the figure above.

(iii) Find the streamfunction $\psi(r, \theta)$ for the flow in terms of polar coordinates (r, θ).

(iv) Show that the boundary of the domain is a streamline. Sketch the streamlines for the flow in the z-plane, including the form for large r.

(v) Detemine the stagnation points of the flow.