PHYS1902 Advanced Physics
Exam Solutions Semester 2, 2004.

Question One

At the level of compensation, pressure equal everywhere. Consider 2 points, one below the mountain (A) and one under just the continent (B).

(a)

$P_0 =$ atmosphere pressure

$d =$ depth of root

$y =$ depth from main bottom level of crust to level of compnsation

$\rho_1 =$ density of crust $= 2900\, kg\, m^{-3}$

$\rho_2 =$ density of mantle $= 4700\, kg\, m^{-3}$

$$P_A = P_0 + \rho_1 g(8 + 35 + d) + \rho_2 g\,(y - d)$$
$$P_B = P_0 + \rho_1 g\,(35) + \rho_2\, gy$$

(1 mark)

$$P_A = P_B$$
$$\therefore P_o + \rho_1 g\,(8 + 35 + d) + \rho_2 g\,(y - d) = P_0 + \rho_1 g(35) + \rho_2 gy$$
$$\rho_1(8 + d) - \rho_2 d = 0$$
$$d = \frac{\rho_1 8}{\rho_2 - \rho_1} = \frac{(2900)(8)}{(4700 - 2900)} = 13\, km$$

(calculation 1 mark)

(result 1 mark)

(b) Assume atmosphere pressure is the same at the bottom and top of the mountain $\left(\rho_0 \approx 1.2 kg\, m^{-3}\right)$

(1 mark)

Pressure at top of mountain $= P_0 - \rho_0 gh = 10^5 - (1)(9.8)(8000) = 0.2 \times 10^5$

Compare to pressure at bottom of continent crust, $P = P_0 + \rho_1 gh$

$$P = 10^5 + (2900)(9.8)(35000)$$
$$\approx 10^5 + 9.9 \times 10^8 \approx 10^9$$

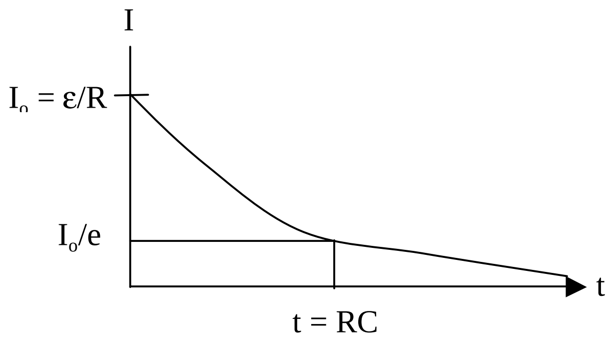
(1mark)

(Total 5 marks)

Question Two

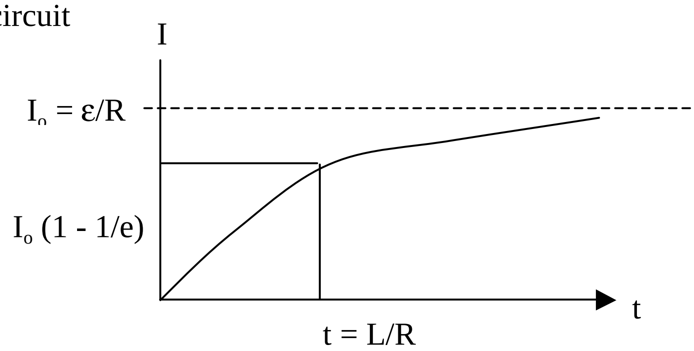
(a)
(graphs really need to go a bit further to show steady state behaviour)

RC circuit



general shape of the graph ***(1 mark)***
correct axes ***(1/2 mark)***

RL circuit



general shape of the graph ***(1 mark)***
correct axes ***(1/2 mark)***

(b) Initial and final current for RC ***(1 mark)***
Initial and final current for RL ***(1 mark)***

(Total 5 marks)

Question Three

Field due to B is R $\Rightarrow$ L

(1 mark)

Field due to B must act to oppose the <u>change</u> in field due to A (not just the field itself)

(1 mark)

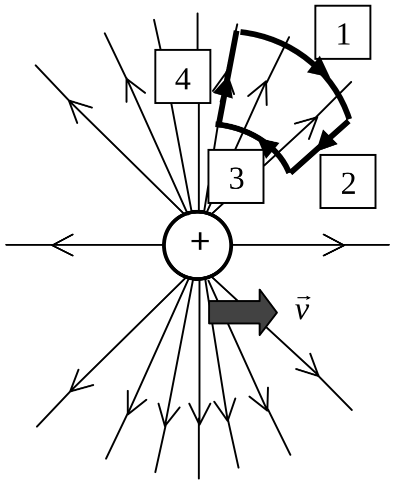
$\Rightarrow$ change in A's field must be to *reduce* the R$\Rightarrow$ L component

- Field due to A is L $\Rightarrow$R
- So a reduction in R $\Rightarrow$ L component
  $\Rightarrow$ A's increase with time.

(correct reasoning – 2 marks)
(correct answer – 1 mark)

(total 5 marks)

Question Four



(a) Take $\oint \mathbf{E} \bullet d\mathbf{l}$ around a path (like that shown):

(1 mark)

Sides 1 and 3 contribute zero $(\mathbf{E} \perp d\mathbf{l})$
Sides 2 and 4 have opposite sign, but <u>not</u> the same magnitude, hence $\oint \mathbf{E} \bullet d\mathbf{l} \neq 0$.

(2 marks)

(b)

(i) The flux of **B** through <u>any</u> imaging surface is zero.

(1 mark)

(ii) Line of **B** can never start or end at a point, so monopoles cannot exist if this equation is correct.

(1 mark)

(1 mark)

(total 5 marks)

Question Five

(a) Each image is made up entirely of small points of light.

(1 mark)

Only the element of the detector the electron strikes lights up, suggesting the electrons are behaving like classical small localised particles.

(1 mark)

(b) The rightmost image shows distinct stripes (fringes).

(1 mark)

A plane wave passing through two slits will produce an interference pattern, as observed in the rightmost image. Thus the electrons are behaving like ordinary waves.

(1 marks)

(c) Quantum mechanics treats electrons as probability waves. They exhibit particle properties (randomness) and wave properties (interference).

(1 mark)

(Total 5 marks)

Question Six

(a) The wavelength is inversely proportional to momentum.

The momentum is $mv = m\sqrt{2K/m}$. Hence the wavelength is inversely proportional to $\sqrt{m}$.

The rank of increasing wavelength goes alpha, neutron, electron.

(2 mark)

(b) velocity is $v = h/m\lambda$

using $m_{He} \approx 6.6 \times 10^{-27}$ kg (they will need to estimate this) and $\lambda = 0.01$ m gives $v = 1.0 \times 10^{-5}$ ms^{-1}

(2 mark)

(c) Exclusion principle allows only two electrons to enter the 1s state since there are only two available distinct states. The next lowest available state is 2s, used in Lithium.

(1 mark)

(Total 5 marks)

Question Seven

(a) By the equation of continuity, the volume flow rate is constant, so

$$A_1 v_1 = A_2 v_2 = R$$

where A is the cross-sectional area of the pipe and v the speed of flow. Hence

$$v_1 = \frac{R}{A_1} = \frac{4R}{\pi D_1^2}$$

(2 marks)

(b) Similarly,

$$v_2 = \frac{R}{A_2} = \frac{4R}{\pi D_2^2}$$

(1mark)

(c) Apply Bernoulli's equation to find the pressures P_1 and P_2 at the centre of the pipe in the large and small segments. They are at the same height so $y_1 = y_2 = 0$. Hence from Bernoulli's equation,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

so

$$\Delta P = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

(2 marks)

(d) By Pascal's law, the pressure P in each tube is equal to the pressure exerted by the column of water. The tops are open to the air, so the pressure at the surface is atmospheric pressure P_A.

$$P_1 = P_A + \rho g y_1 \text{ and } P_2 = P_A + \rho g y_2$$

Hence

$$\Delta h = y_1 - y_2 = \frac{P_1 - P_2}{\rho g} = \frac{v_2^2 - v_1^2}{2g}$$

$$= \frac{8R^2}{\pi^2 g}\left(\frac{1}{D_2^4} - \frac{1}{D_1^4}\right).$$

(3 marks)

(e) With a viscous fluid we can write the energy equation for positions 1 and 2 within the pipe as

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2 + \Delta$$

where Δ (> 0) is a tem representing the energy loss due to viscosity.
Continuity still holds so

$$A_1 v_1 = A_2 v_2$$

or

$$v_1 = \frac{A_2 v_2}{A_1}$$

Hence the pressure drop is

$$\begin{aligned} p_1 - p_2 &= \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \Delta \\ &= \frac{1}{2}\rho v_2^2\left(1 - \frac{A_2^2}{A_1^2}\right) + \Delta \end{aligned}$$

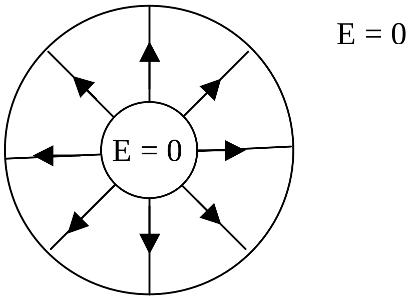
The first term on the right is the pressure drop due to Bernoulli's laws in a *non*-viscous fluid. So, since $\Delta > 0$, $p_1 - p_2$ is greater than in the *non*-viscous fluid.

(2 marks)

(Total 10 marks)

Question Eight

(a)



(1 mark)

$$\mathrm{E}(\mathrm{r}) = \begin{cases} 0 & r < r_a \\ \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2} & r_a < r < r_b \\ 0 & r > r_b \end{cases}$$

(3 marks)

(b) $$V_{ab} = \int_{r_a}^{r_b} \mathrm{E\,dr} = \frac{Q}{4\pi\varepsilon_0}\int_{r_a}^{r_b}\frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(2 marks)

(c) $$C = \frac{Q}{V_{ab}} = 4\pi\varepsilon_0 \frac{r_a r_b}{r_b - r_a} \quad \left[\text{full marks if say } C = \frac{\mathrm{Q}}{\text{answer from (b)}}\right]$$

(2 marks)

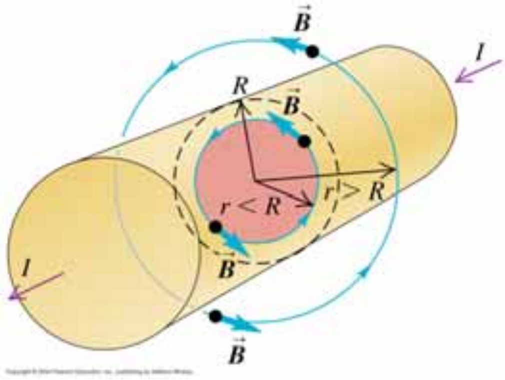
(d) $$U = \tfrac{1}{2} CV^2 = \frac{Q^2}{2C} = \frac{Q^2}{2}\frac{1}{4\pi\varepsilon_0}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

(2 marks)

(Total 10 marks)

Question Nine

(a) From symmetry arguments, the magnetic field is in circles. ***(1 mark)***
Apply Ampere's Law using a circular path of radius *r* (as illustrated) ***(1 mark)***



We use the fact the B has constant magnitude along the path and is parallel to the path. ***(1 mark)***

Thus Amperes law gives $B(2\pi\, r) = u_0 i$

And thus $B(r > R) = \dfrac{u_0 i}{2\pi\, r}$

(1 mark)

(b) Similar argument but with $i\,(r) = \dfrac{r^2}{R^2} i$ inside the cylinder ***(1 mark)***

yields $B(r < R) = \dfrac{u_0 ir}{2\pi\, R^2}$

(1 mark)

(c) $B(r < R) = \dfrac{u_0 ir}{2\pi\, R^2}$ and $u = \dfrac{B^2}{2\mu_o}$

so at a point within the field the energy density is $du = \dfrac{\mu_o i^2 r^2}{8\pi^2 R^4}$

Volume of thin cylindrical shell of radius *r* is $dV = (2\pi r)drL$

Then the energy with a volume element is

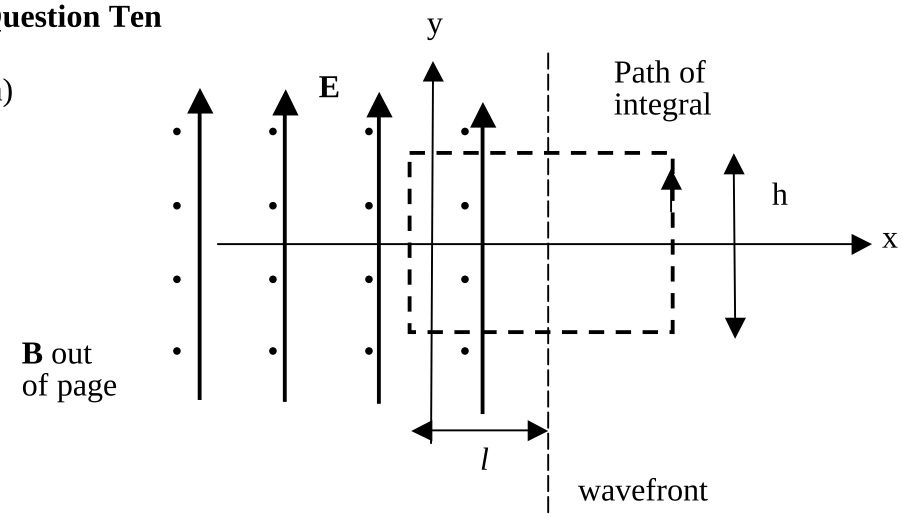
$$dU = dudV$$
$$= (2\pi rLdr)\left(\frac{\mu_o i^2 r^2}{8\pi^2 R^4}\right)$$
$$= \frac{\mu_o i^2 L}{4\pi R^4} r^3 dr$$

Integrate over the cylinder to get $U = \dfrac{\mu_o i^2 L}{16\pi}$

(procedure 3 marks)
(result 1 mark)
(Total 10 marks)

Question Ten

(a)



Apply $\oint \mathbf{E} \bullet d\mathbf{l} = -\frac{d\Phi_B}{dt}$

In this equation: $LHS = -Eh$

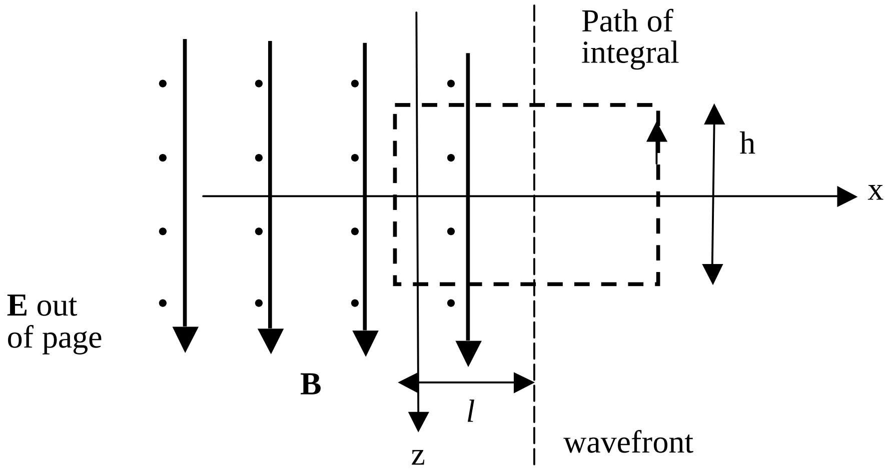
$RHS = -\frac{d}{dt}(B \times \text{area})$

$= -\frac{d}{dt} l\, h\, B = -hB\frac{dl}{dt} = -hvB$

$\Rightarrow E = vB$

(4 marks)

(b)



Apply $\oint B \bullet d\mathbf{l} = \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$

In this equation: $LHS = Bh$

$RHS = \mu_0 \varepsilon_0 \frac{d}{dt}(E \times \text{area})$

$= \mu_0 \varepsilon_0 \frac{d}{dt}(E\, l\, h) = \mu_0 \varepsilon_0\ Ehv \Rightarrow B = \varepsilon_0 \mu_0\, v\, E$

$\Rightarrow B = \varepsilon_0 \mu_0\, v\, E$

(4 marks)

(c) $\frac{E}{B} = v$ and $\frac{\mathrm{E}}{\mathrm{B}} = \frac{1}{\varepsilon_0 \mu_0 v} \Rightarrow v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

(2 marks)

(Total 10 marks)

Question Eleven

(a) (i) Since in classical physics energy is not quantised, as the duration of the pulse tends to zero, the energy available to any electron in the metal tends to zero, and will reach a point where it is insufficient to overcome the work function. No photocurrent would be seen for short pulses.

(2 marks)

(ii) in quantum physics, the very first photon incident on the metal can cause ejection and so a current is observed for even the shortest pulses.

(2 marks)

(b) The energy in the photon is $\dfrac{hc}{\lambda} = E + \phi$

0.5 eV is $0.5 \times 1.6 \times 10^{-19}$ J

the photon energy is $6.63 \times 10^{-34} \times 3 \times 10^{8}/700 \times 10^{-9}$ J

The work function W is the difference ≈ 1.3 eV

(2 marks)

(c) Increasing wavelength means decreasing energy per photon. With total energy constant this implies an increasing number of photons.

(2 marks)

(i) The classical case will have the number of electrons ejected should remain approx constant since the total energy available is constant.

(1 mark)

(ii) In quantum physics, a threshold wavelength is reached where no electrons are ejected.

(1 mark)

(Total 10 marks)

Question Twelve

(a) constant inside, zero outside

(2 marks)

(b) $-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = E\psi(x)$

try A cos kx + B sin kx where $k=\frac{1}{\hbar}\sqrt{2mE}$

differentiating *wr*t *x*, get –A*ksinkx*+B*kcoskx* and again, -A*k*2*coskx*-B*k*2*sinkx* which gives E(A cos*kx* +B *sinkx*) as required for a solution

(2 marks)

(c) outside the well, the SE is, where

$k = \frac{1}{\hbar}\sqrt{2m(V-E)}$,the Schrodinger equation is

$\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} = (V-E)\psi(x)$,

try as a solution $Ce^{-kx}+De^{kx}$ where C and D are constants.

Differentiating twice we get $k2Ce^{-kx}+Dk2e^{kx} = k^2\psi = \frac{1}{\hbar^2}(2m(V-E))\psi$

Hence the LHS is $(V\text{-}E)\psi$ =RHS

In order to preserve the finite integral of the wavefunction over all space we need the solution that exponentially decays from the edge of the well. For x<0 this is the solution with the positive power of *x*.

(4 marks)

(d) Sketch of the probability showing smooth matching of the probability across the edge of the well, with decreasing probability near the edge of the well and exponential decay outside the well.

(2 marks)

(Total 10 marks)