THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 1

MATH1901/1906: Differential Calculus (Advanced) Semester 1, 2017

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1901/
Lecturer: Daniel Daners

This assignment is due by **23:59 Thursday 23rd March 2017**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard) at https://elearning.sydney.edu.au.

Please submit a single PDF document (scan or convert other formats). It should include your name and SID; your tutorial time, day, room and Tutor's name. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission. After submitting you can go back and view your submission to check it.

The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 2.5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making substantial progress on only one out of the two questions.
4	D	Some attempt, with substantial progress made on only 2 of the 4 parts parts of Question 1 or on Question 2.
2	E	No substantial progress made on any part or question.
0	F	No credit awarded.

The first question is revision/practice using assumed knowledge from your HSC mathematics on manipulating algebraic expressions, inequalities and a proof by induction.

1. (a) Show that for every integer $n \geq 1$ we have

$$x^{n+1} - (n+1)x + n = (x-1)\big[1 + x + x^2 + \cdots + x^n - (n+1)\big]$$

for all $x \in \mathbb{R}$ (no induction required).

(b) Hence show that

$$x^{n+1} - (n+1)x + n \geq 0 \tag{1}$$

for all $x \geq 0$.

(c) Let $x_1, x_2, x_3, \ldots$ be a sequence of positive real numbers. For every $n \geq 1$ we consider the averages

$$a_n := \frac{x_1 + x_2 + \cdots + x_n}{n}.$$

By setting $x = \dfrac{a_{n+1}}{a_n}$ in (1), show that for $n \geq 1$

$$a_{n+1}^{n+1} \geq a_n^n x_{n+1}$$

(d) Let x_n and a_n be as in the previous part. Using mathematical induction by n, show that

$$a_n^n = \left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right)^n \geq x_1 x_2 \ldots x_n.$$

for all $n \geq 1$.

The second question is about mapping properties of functions of a complex variable.

2. Consider the map $f(z) = \dfrac{z+i}{z-i}$ of the complex variable $z \neq i$. Show that the image of the real axis under f lies on a circle centred at the origin of the complex plane. Which points from the circle are missing in the image?