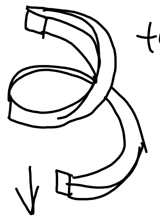
Lecture 2: - the maths of SHM
- the energy of SHM

Homework: ~~H/W~~ - how does a spring work?



twisting the metal

Apr 12-1:59 PM

Equations of SHM

we were told that SHM involves sinusoidal motion

use Newton's 2nd Law

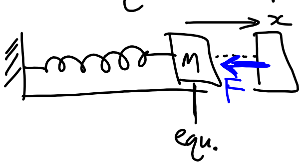
$$F = ma$$

$$\boxed{F = m\frac{d^2x}{dt^2}}$$

forces produce accelerations

Apr 12-2:10 PM

SHM happens when restoring force is linearly proportional to displacement from equilibrium posn.



$$F \propto -x$$

$$\boxed{F = -kx}$$

displacement ~~produces~~ leads to a force

Apr 12-2:15 PM

putting these together
displacement leads to acceleration

$$m\frac{d^2x}{dt^2} = -kx \qquad (14.4)$$

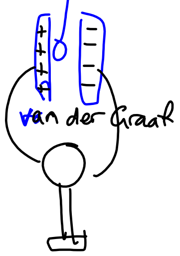
this is a differential equation (D.E.)

Goal: find $x(t)$

My method: guess the solution and substitute into D.E.

Apr 12-2:18 PM

H/W (optional)



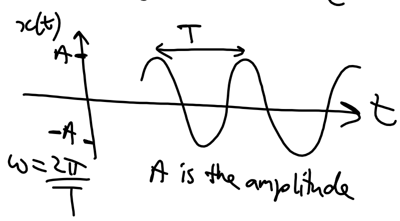
a) is this SHM?

b) what is eqn for ω? (optional)

Apr 12-2:38 PM

Our soln. for the SHM D.E.:

$$x(t) = A\cos(\omega t + \phi)$$



Apr 12-2:40 PM

we derived $\omega = \sqrt{\frac{k}{m}}$

this tells us period depends on k & m
but not on amplitude (as promised)

$\omega = \frac{2\pi}{T}$ so $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$

Others to write the solution:

Apr 12-2:43 PM

I claim most general soln to our DE
is $x(t) = A\cos(\omega t + \phi)$

any sinusoid
can also be written:

- $x(t) = C\cos(\omega t) + S\sin(\omega t)$
(sometimes used)

Apr 12-2:49 PM

- $x(t) = \mathrm{Re}\left[A e^{i(\omega t + \phi)}\right]$

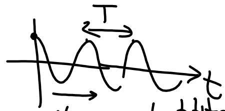
because $e^{i\theta} = \cos\theta + i\sin$..
useful! (Euler)

we'll use this to solve more difficult DEs

Apr 12-2:51 PM

For our DE (14-4), we guess the solution
$x(t) = \cos(\text{something})$

more general guess:
$x(t) = A\cos(\text{something})$
$= A\cos(\omega t + \phi)$ (14.13)



Now substitute (14.13) into (14.4)

Apr 12-2:23 PM

we have $x(t) = A\cos(\omega t + \phi)$

$\Rightarrow \frac{dx}{dt} = -A\omega\sin(\omega t + \phi)$

$\frac{d^2x}{dt^2} = -A\omega^2\cos(\omega t + \phi)$

sub. into DE.

$m\frac{d^2x}{dt^2} = -kx$

LHS $= -A\omega^2 m\cos(\omega t + \phi)$

Apr 12-2:28 PM

LHS $= -m\omega^2 x(t)$
RHS $= -k\,x(t)$

Yes, LHS=RHS provided
$m\omega^2 = k$

$\Rightarrow \omega^2 = k/m \Rightarrow \omega = \sqrt{\frac{k}{m}}$

strength of restoring force

mass of object

angular freq
$\omega = 2\pi f$
$= 2\pi/T$

Apr 12-2:30 PM

Apr 12-2:34 PM