The University of Sydney
School of Mathematics and Statistics

Assignment 1

MATH1906: Mathematics Special Studies Program A — Semester 1, 2012

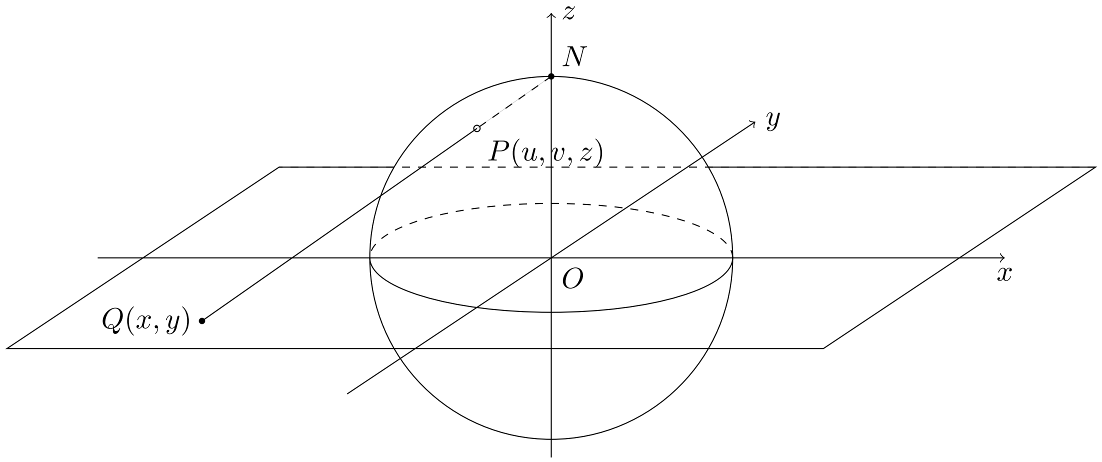
Web Page: http://www.maths.usyd.edu.au/u/UG/JM/MATH1906/
Lecturer: Daniel Daners

Due on **Tuesday, April 24** by **16:00** in Carslaw **Room 715**
(Slide under the door when locked).

Late assignments are not accepted without *prior arrangement* well before the deadline!

You must attach the signed cover-sheet to the front of your assignment!

1. The stereographic projection of a point P on the unit sphere from the north pole N onto the plane given by the equator is the point Q where the line NP meets the plane as shown below.



If $P(u,v,z)$ is a point of longitude φ and latitude θ, then clearly $Q(x,y)$ has polar coordinates of the form

$$x = r(\theta)\cos\varphi \qquad \text{and} \qquad y = r(\theta)\sin\varphi$$

with $r(\theta)$ to be determined.

(a) Using suitable similar triangles, or otherwise, show that *1 Mark*

$$r(\theta) = \frac{\cos\theta}{1-\sin\theta} = \sec\theta + \tan\theta.$$

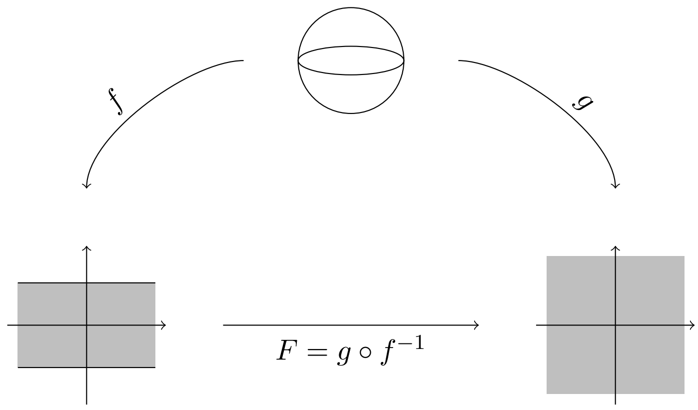
Hint: Look at the cross-section in the plane given by N, Q and O.

(b) Show that $x = \dfrac{u}{1-z}$, $y = \dfrac{v}{1-z}$ and $r^2 = \dfrac{1+z}{1-z}$. *2 Marks*

(c) Show that $u = \dfrac{2x}{r^2+1}$, $v = \dfrac{2y}{r^2+1}$ and $z = \dfrac{r^2-1}{r^2+1}$, where $r^2 = x^2+y^2$. *2 Marks*

(d) A circle on the sphere is the intersection of a plane $au+bv+cz=d$ with the sphere $u^2+v^2+z^2=1$. Use (c) to show that the image of a circle on the sphere under the stereographic projection is a circle or a straight line. *2 Marks*

2. The Mercator projection defines a bijection f from the sphere minus the two poles onto the strip $-\pi < y \leq \pi$ in the plane. The stereographic projection defines a bijection g from the sphere minus one of the poles to the whole plane:



Hence we can define a map $F := g \circ f^{-1}$ from the strip $-\pi < x \leq \pi$ onto the punctured plane $\mathbb{R}^2 \setminus \{0\}$. The complex numbers $\mathbb{C}$ can be interpreted as points in $\mathbb{R}^2$ using an Argand diagram. Hence Mercator and stereographic projection can be written as $\mathbb{C}$-valued functions of φ and θ.

(a) Write down the Mercator projection in the form $f(\varphi, \theta) = x + iy$ so that the meridian $\varphi = 0$ is the real axis (the usual map rotated by $90°$ to the right). *1 Mark*

(b) If g is the stereographic projection from the north pole, show that $F = g^{-1} \circ f$ is given by $F(z) = e^z$. *2 Marks*

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Assignment Cover Sheet

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Family Name ....

Given Names .... **SID** ....

Some collaboration between students on assignments is encouraged, since it can be a real aid to understanding. Thus it is legitimate for students to discuss assignment questions at a general level, provided everybody involved makes some contribution. However, students should produce their own individual written solution. Copying someone else's work is plagiarism, and is unacceptable. The University may impose severe penalties in cases where plagiarism is detected.

I certify that:

- I have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure* at http://www.maths.usyd.edu.au/u/UG/Plagiarism.pdf.
- this assignment is all my own work, and that no part of this assignment has been copied from another person.
- I have not allowed my work to be copied by another person.

Signature .... **Date** ....

This part to be completed by the marker:

Grand total out of 10 ....