PHYS 1901 – Physics 1A (Advanced) Mechanics module

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Chapter

8

Momentum, Impulse, and Collisions

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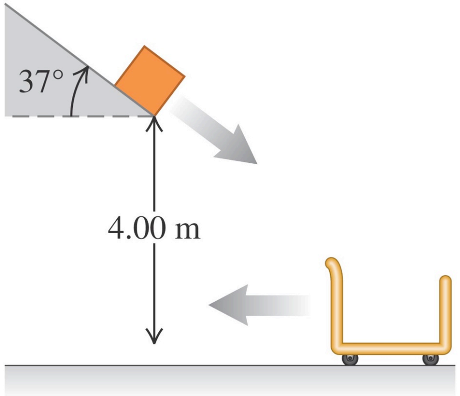
Problem 8.87



An open cart of mass 50.0 kg is rolling to the left at a speed of 5.00 m/s. Ignore friction between the cart and floor.

A 15.0-kg package leaves a chute at 3.00 m/s and falls into the cart.

a) What is the speed of the package just before landing in the cart?

b) What is the final speed of the cart?

a) $W_G = \Delta K$

$mgh = K_f - K_i$

$= \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$v_f = 9.35\ m/s$

b) $\vec{p}_{cart,i} = m_c \vec{v}_{c,i} = -250\ kg\ m/s$ (left)

$\vec{p}_{package,i} = m_p \vec{v}_{p,i}$

$p_{p,i,x} = (15\,kg)(3.00\,m/s)\cos 37°$

$= 35.9\ kg\ m/s$ (right)

$p_{f,x} = -214.1\ kg\ m/s$ (left)

$v_{f,x} = \frac{p_{f,x}}{m_c + m_p} = -3.29\ m/s$ (left)

Types of collisions

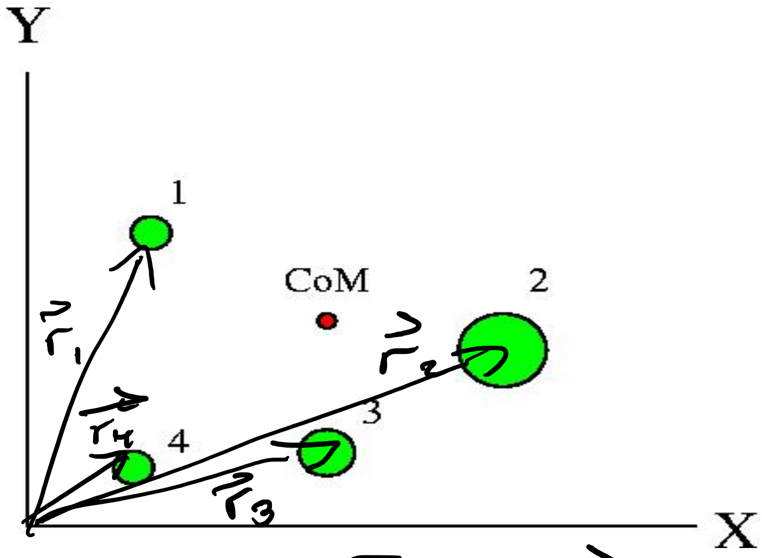
Momentum is conserved in all collisions

But we can define two kinds of collision

- **Elastic:** Both kinetic energy and momentum are conserved
- **Inelastic:** Only momentum is conserved

Where does the energy go?

Centre of Mass



For the collection of objects (pool balls, cars, planets etc) we can define the **centre of mass**.

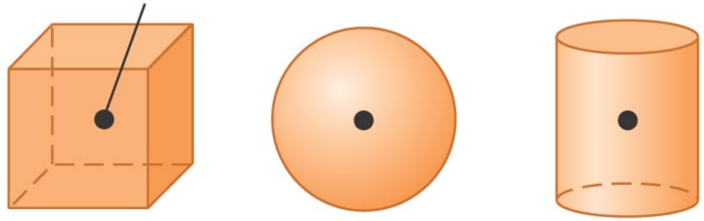
This is weighted average position of all the individual masses.

$$\vec{r}_{CM} = \frac{\sum_i m_i \vec{r}_i}{\sum_i m_i} = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2 + m_3 \vec{r}_3 + m_4 \vec{r}_4}{m_1 + m_2 + m_3 + m_4}$$

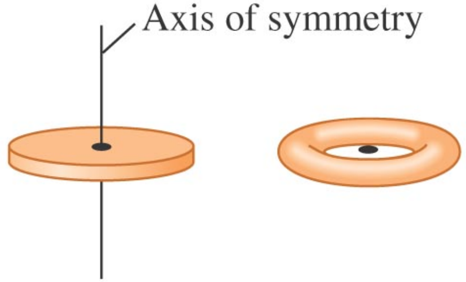
$$M_{total} = \sum_i m_i$$

Note in the continuous limit where we consider a distribution of density rather than point masses;

Centre of Mass



If a homogeneous object has a geometric center, that is where the center of mass is located.



If an object has an axis of symmetry, the center of mass lies along it. As in the case of the donut, the center of mass may not be within the object.

The centre of mass is not a physical thing!

Total momentum and centre of mass

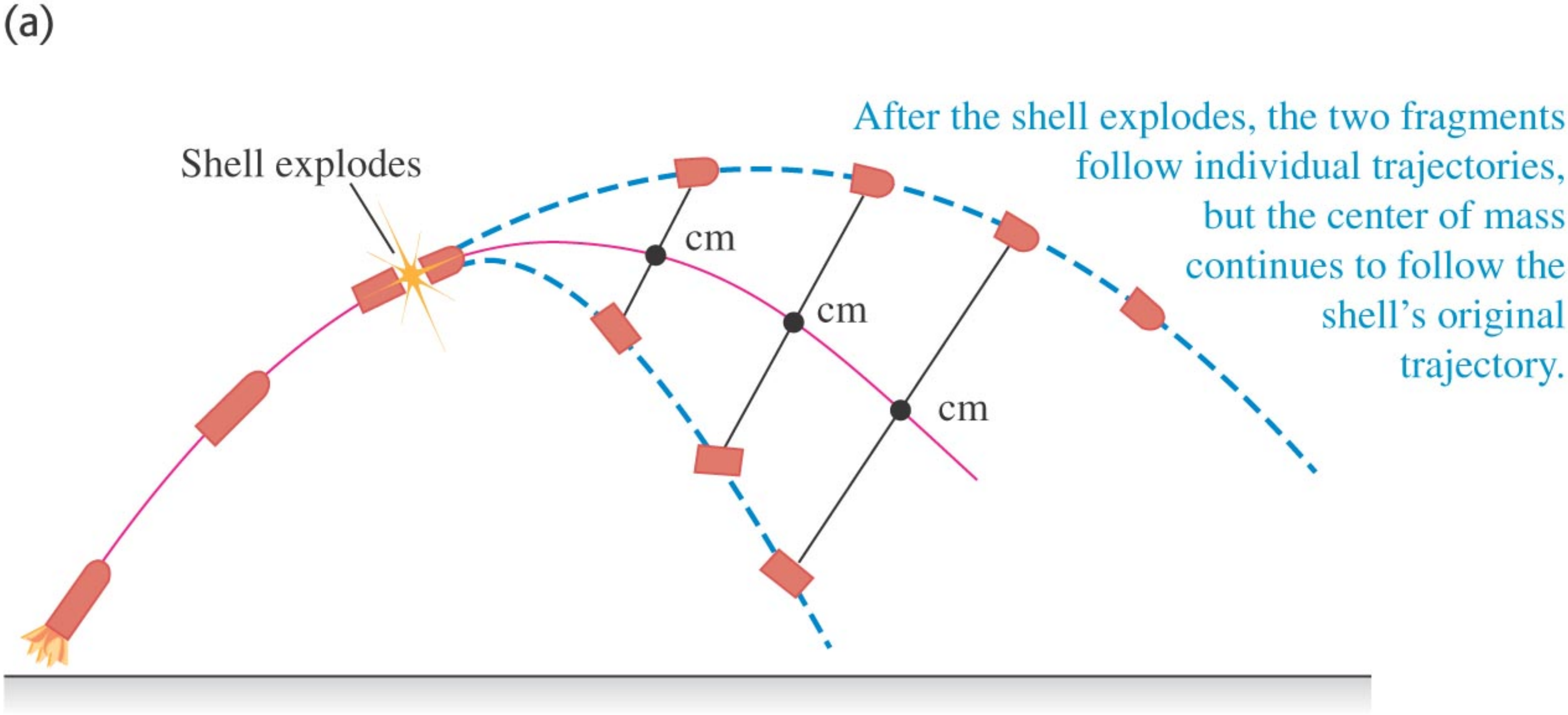
So, the momentum of the centre of mass is equal to the momentum of the entire system. But

$$\vec{F}_{net} = \frac{d\vec{p}_{tot}}{dt}$$

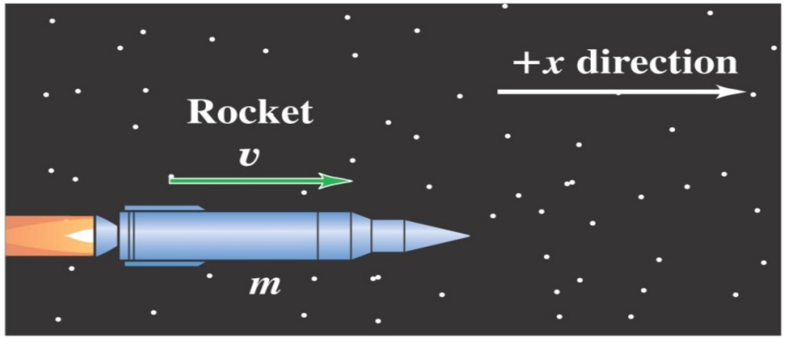
$$\vec{F}_{net} = \sum_{all} \vec{F}_i = \sum_{\substack{all \\ external}} \vec{F}_i + \cancel{\sum_{\substack{all \\ internal}} \vec{F}_i} = 0 \quad = \sum_{ext} \vec{F}_i$$

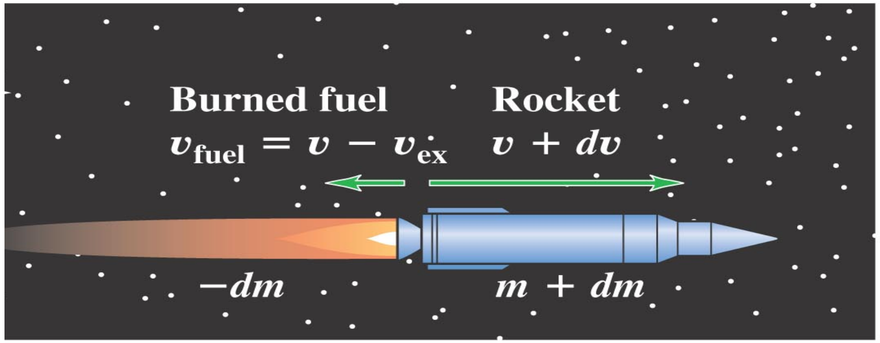
Only external forces can change the momentum of the centre of mass!

Centre of mass



Rocket propulsion





$$P_1 = mv$$

$$P_{2,tot} = (m+dm)(v+dv) + (-dm)(v - v_{ex})$$

$$P_1 = P_2$$

$$mv = (m+dm)(v+dv) + (-dm)(v - v_{ex})$$

$$m\,dv = -dm\,v_{ex} - \cancel{dm\,dv}$$

$$m\frac{dv}{dt} = -\frac{dm}{dt} v_{ex}$$

$$a = -\frac{v_{ex}}{m}\frac{dm}{dt}$$

If v_{ex} constant

$$\int_{v_0}^{v_f} dv = -v_{ex}\int_{m_0}^{m_f}\frac{dm}{m}$$

$$v_f - v_0 = -v_{ex}\ln\frac{m_f}{m_0}$$

$$\boxed{v_f = v_0 + v_{ex}\ln\frac{m_0}{m_f}}$$

Rocket propulsion example

What is a reasonable top speed for a rocket?
Assume a rocket is fired from rest, from deep space
It ejects gas at a relative speed of 2000 m/s
What is the fraction of the initial mass of the rocket that is not fuel, for a top speed of:
(a) (1/1000) c ?
(b) 3000 m/s ?