LSA University of Michigan

ASTRO201: Introduction to Astrophysics
Homework 5

Name: Keegan Gyoery UM-ID: 31799451

1. Cygnus X-1 has a mass of 10 solar masses, and its accretion powered luminosity is given to be 5×10^{38} erg/sec.

   a) Using the formula for the Schwarzschild radius, we calculate the radius as

   $$\begin{aligned} r_s &= \frac{2GM}{c^2} \\ &= \frac{2 \times 6.674 \times 10^{-8} \times 10 \times 2 \times 10^{33}}{(3 \times 10^{10})^2} \\ &= 2966222.222. \end{aligned}$$

   Thus, the Schwarzschild radius of Cygnus X-1 is 2.97×10^6 cm.

   b) Using the formula for the Eddington Luminosity, we have

   $$\begin{aligned} L_{\text{edd}} &= 1.3 \times 10^{38} \times \frac{M}{M_\odot} \\ &= 1.3 \times 10^{38} \times \frac{10 \times M_\odot}{M_\odot} \\ &= 1.3 \times 10^{39}. \end{aligned}$$

   Thus, accretion powered luminosity provides 38% of the Eddington Luminosity.

   c) Using the formula for rest-mass energy conversion in a blackhole, with 10% efficiency, we have

   $$\begin{aligned} \Delta E &= 0.1 \times mc^2 \\ \therefore m &= \frac{10 \times E}{c^2} \\ &= \frac{10 \times 5 \times 10^{38}}{(3 \times 10^{10})^2} \\ &= 5.555555556 \times 10^{18}. \end{aligned}$$

   Thus, the mass accretion rate onto Cygnus X-1 is 5.56×10^{18} g/sec, which is 8.6×10^{-8} $\mathrm{M}_\odot$/year.

2. Assume a typical neutron star, with a radius of 10 km and a mass of 1.4 solar masses.

   a) Using the formula for gravitational binding energy for a star with constant mass density, we have

$$\begin{aligned} U &= \frac{3GM^2}{5R} \\ &= \frac{3 \times 6.674 \times 10^{-8} \times \left(1.4 \times 2 \times 10^{33}\right)^2}{5 \times 10^6} \\ &= 3.1394496 \times 10^{53}. \end{aligned}$$

   Thus, the gravitational binding energy of the neutron star is 3.14×10^{53} erg. Using the energy, rest-mass formula, we have

$$\begin{aligned} E &= mc^2 \\ &= 1.4 \times 2 \times 10^{33} \times \left(3 \times 10^{10}\right)^2 \\ &= 2.52 \times 10^{54}. \end{aligned}$$

   Thus, the rest-mass energy of the neutron star is 2.52×10^{54} erg. Clearly, the neutron star's gravitational binding energy is 20.7% of the rest-mass energy of the neutron star.

   b) If the energy of a neutron is 1.5×10^{-5} erg, and the energy released is 3.14×10^{53} , then the number of neutrinos released is 2.09×10^{58}.

   c) Using the equation for dispersion of radiation over a distance, we have

$$\begin{aligned} \#\ \text{neutrinos/m}^2 &= \frac{\#\ \text{neutrinos}}{4\pi R^2} \\ &= \frac{2.09 \times 10^{58}}{4\pi \left(1.543 \times 10^24\right)^2} \\ &= 698561376.8. \end{aligned}$$

   Thus, the number of neutrinos per m^2 at Earth is 6.99×10^8. A 10 m by 10 m box has a surface area of 100 m^2, and so 6.99×10^{10} neutrinos pass through the face of the box.

3. Assuming a star has the same size, that is radius, as the Sun, and the same magnetic field of the Sun, 5 Gauss, using the conservation of magnetic fields, when the star collapses to a neutron star of radius 10 km, its magnetic field becomes

$$\begin{aligned} B_\odot R_\odot^2 &= BR^2 \\ \therefore B &= B_\odot \left(\frac{R_\odot}{R}\right)^2 \\ &= 5 \times \left(\frac{6.955 \times 10^{10}}{10^6}\right)^2 \\ &= 2.41860125 \times 10^{10} \end{aligned}$$

   Thus, the magnetic field of the neutron star after the collapse is 2.42×10^{10} Gauss. The LHC produces magnetic fields of up to 8.36 Tesla, which is equivalent to 8.36×10^4 Gauss, which is approximately 300,000 times smaller.

4. In order for the rotating star to remain intact, the force of gravity of the star must exceed the centrifugal force exerted by rotation. Thus, knowing $w = \frac{2\pi}{P}$, we have the following inequality,

$$
\begin{aligned}
\frac{GMm}{R^2} &> mRw^2 \\
\frac{GM}{R^3} &> w^2 \\
\frac{GM}{R^3} &> \left(\frac{2\pi}{P}\right)^2 \\
\frac{GM}{R^3} &> \frac{4\pi^2}{P^2} \\
\frac{3M}{4\pi R^3} &> \frac{3\pi}{GP^2} \\
\rho &> \frac{3\pi}{P^2}.
\end{aligned}
$$

Clearly, the rotation period of a star translates into a density. If the star has a rotation period of 1 millisecond, or 10^{-3} seconds, the minimum density is

$$
\begin{aligned}
\rho &> \frac{3\pi}{GP^2} \\
\therefore \rho_{\text{min}} &= \frac{3\pi}{6.674 \times 10^{-8} \times (10^{-3})^2} \\
&= 1.412163314 \times 10^{14}
\end{aligned}
$$

Thus, the minimum density of the star is 1.41×10^{14} g/cm^3.