Practice for Quiz 2

MATH1903: Integral Calculus and Modelling (Advanced)   Semester 2, 2015

Lecturers: Florica-Corina Cîrstea and Anne Thomas

Quiz 2 will be held during your tutorial in week 11 (beginning 12 October).
The quiz questions will be based on material covered during lectures in weeks
7–9, which corresponds to material covered in tutorials in weeks 8–10. They
will also draw on knowledge of integration techniques from earlier weeks.

Family Name: ....

Other Names: ....

SID: .... **Day:** ....

Time: .... **Room:** ....

**Signature:** ....

Please write your final answers, and only your final answers, in the answer boxes.

Please note:

(a) There are 10 questions, each worth 1 mark. Half marks will **not** be awarded.

(b) Working will **not** be marked. Marks will be awarded on the basis of answers only.

(c) Answers will only be marked if they are in the answer boxes on the answer sheet.

(d) Non-programmable, non-graphics calculators **are** permitted.

(e) There is some space for working. No other paper is allowed.

(f) You have 40 minutes to complete the quiz.

1. Sketch the direction field of the differential equation $y' = y(y-2)$ in the region below.

Answer Q1:
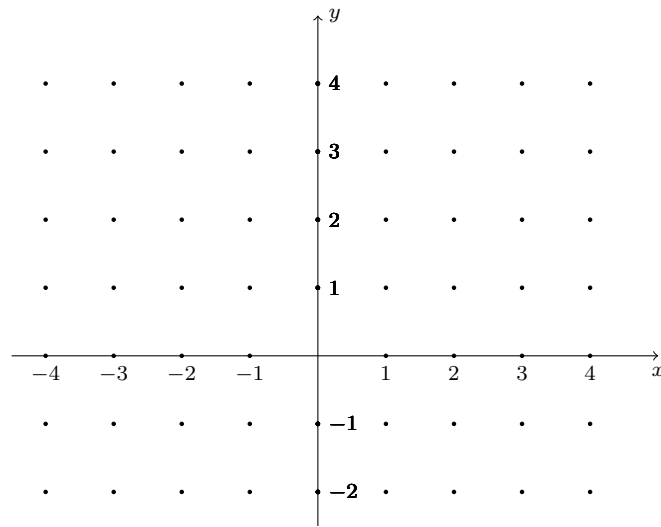
2. Find the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{3y-1}{x}$.

Answer Q2:

3. Find an integrating factor for the linear differential equation $e^x (y' + y\sin x) = 2$.

Answer Q3:

4. Find the particular solution $y = y(t)$ of the differential equation $\dfrac{dt}{dy} = \dfrac{5t}{t^2 + t - 6}$ for $0 < t < 2$, which satisfies the condition $y(1) = \dfrac{1}{5}$.

Answer Q4:

This page is intentionally blank, and may be used for working.

5. Suppose that $y(x)$ is a solution of the differential equation $\dfrac{dy}{dx} = \dfrac{x^2 + xy}{xy + y^2}$. We make the transformation $v = \dfrac{y}{x}$. Find a differential equation for $v = v(x)$.

Answer Q5:

6. Find the equilibrium solution of the equation $\dfrac{dy}{dx} = (y+1)\ln y$.

Answer Q6:

7. Determine whether the equilibrium solution in the above question is stable or unstable.

Answer Q7:

8. Find the particular solution of $\dfrac{dy}{dx} + y\cos x = 2xe^{-\sin x}$ with $y(\pi) = 0$.

Answer Q8:

9. Determine the general solution $x = x(t)$ of the differential equation $\dfrac{dx}{dt} = \dfrac{5t}{t^2 + t - 6}$, where $t > 2$.

Answer Q9:

10. The number N of bacteria in a culture grew according to the Malthusian law given by $\dfrac{dN}{dt} = kN$, where k denotes the constant of proportionality. The value of N was initially 82 and increased to 375 in 2 hours. Find the value of k correct to two significant figures.

Answer Q10: