PHYS 1901 – Physics 1A (Advanced) Mechanics module

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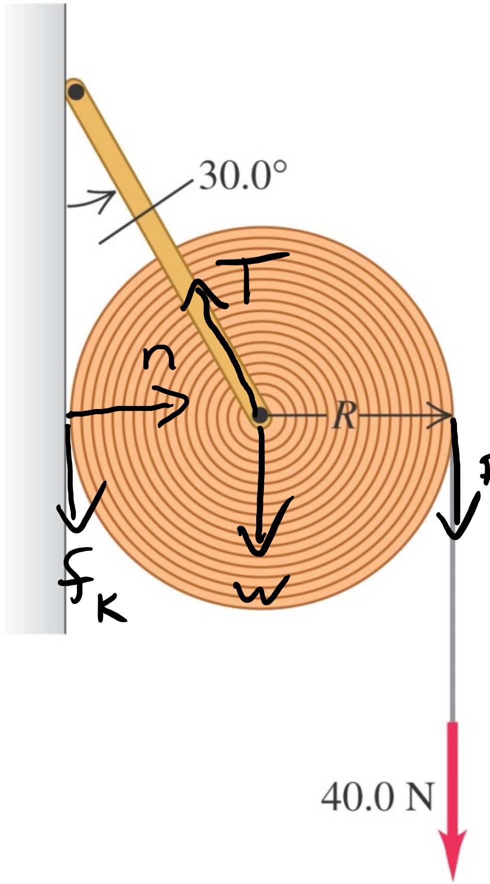
Dynamics of Rotational Motion

Chapter

10

Problem 10.63



Large roll of paper, mass 16.0 kg, radius R= 18.0 cm.

Coefficient of kinetic friction with wall: $\mu_k = 0.25$

a) What is the force that the rod exerts on the paper as it unrolls?

b) What is the angular acceleration of the roll?

a) $\vec{F}_{net} = 0$

$\Sigma F_{vert} = 0 \Rightarrow T\cos 30° = w + F + \mu_k n$

$\Sigma F_{hor} = 0 \Rightarrow T \sin 30° = n$

Eliminate n

$T\cos 30° = w + F + \mu_k T \sin 30°$

$$T = \frac{w + F}{\cos 30° - \mu_k \sin 30°}$$

$= 266 \text{ N}$

b) $\vec{\tau}_{net} = I\vec{\alpha}$

What is the net torque?

$\vec{\tau}_{net} = \vec{\tau}_F + \vec{\tau}_{f_k}$

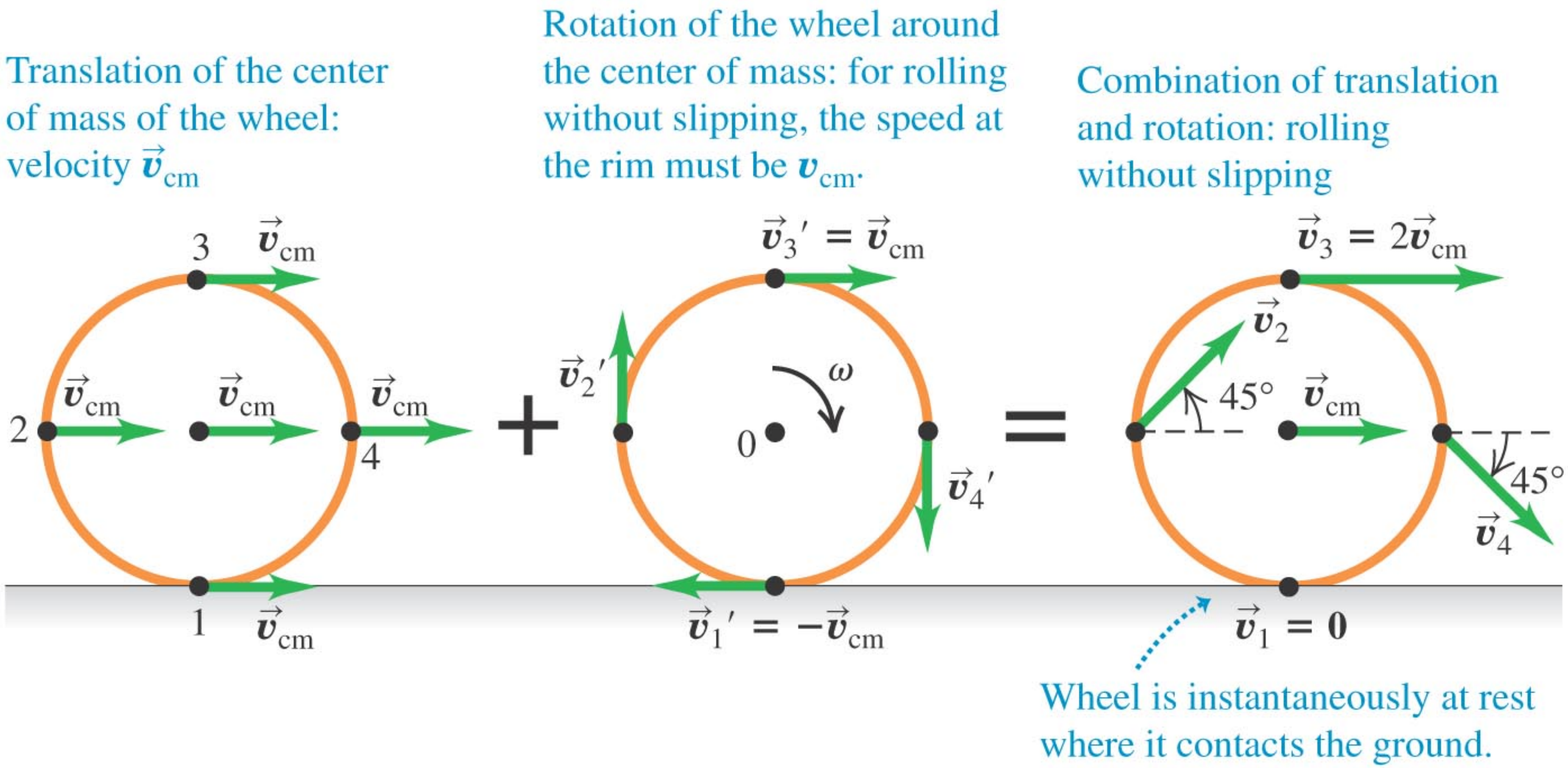
$= RF - Rf_k$

$= 1.2 \text{ N·m}$ into board

$I = \frac{1}{2} MR^2$

$\vec{\alpha} = 4.68 \text{ rad/s}^2$ into board

Rolling without slipping



For a rolling wheel which does not slide, then the distance it travels is related to how much it turns.

$$V_{cm} = R\,\omega$$

Rolling without slipping

The total kinetic energy is

$$K = K_{linear} + K_{rot}$$
$$= \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \omega^2$$

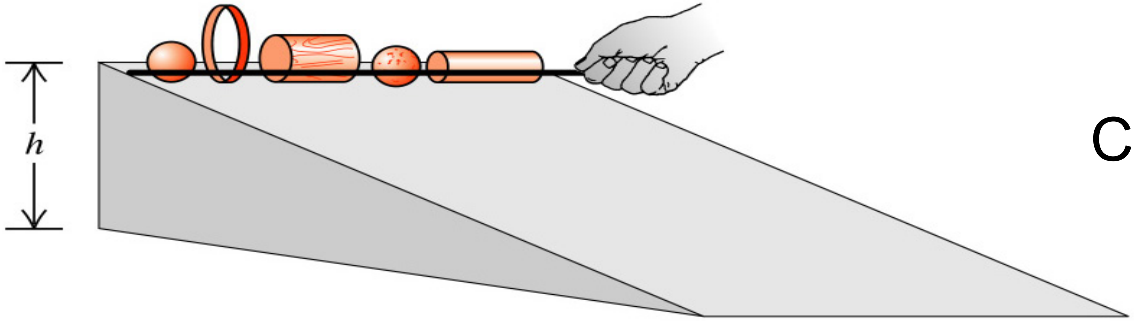
Rolling without slipping $v_{cm} = R\omega$

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} I \left(v_{cm}/R \right)^2$$

Generally $I = cmR^2$ for some constant $c \leq 1$

$$K = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} c m v_{cm}^2$$
$$= \frac{1}{2}(1+c) m v_{cm}^2$$

Rolling without slipping



Conservation of energy

$\Delta K = W_g = -\Delta U_g = mgh$

Starting from rest

$K_f = \frac{1}{2}(1+c)mv_{cm}^2 = mgh$

$\Leftarrow$

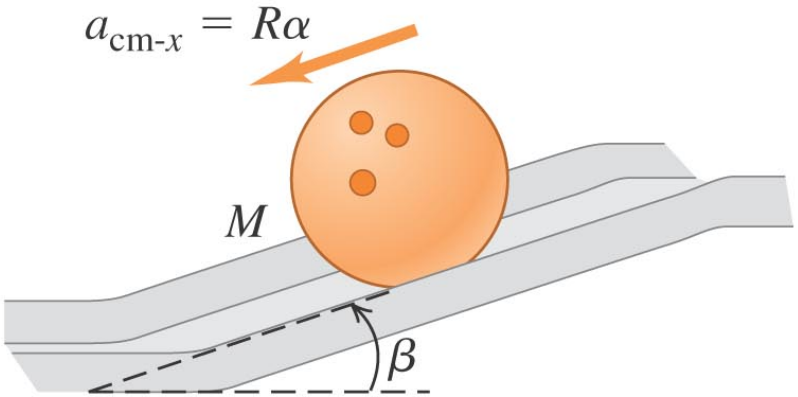
$V_{cm} = \sqrt{\frac{2gh}{(1+c)}}$

- ☐ Independent of mass & size
- ☐ Any sphere beats any hoop!

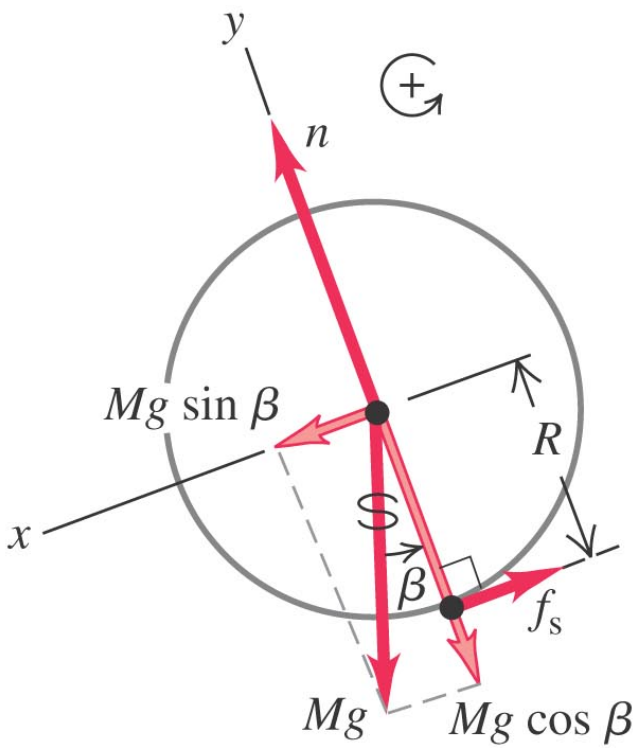
What is the source of torque?

Rolling without slipping

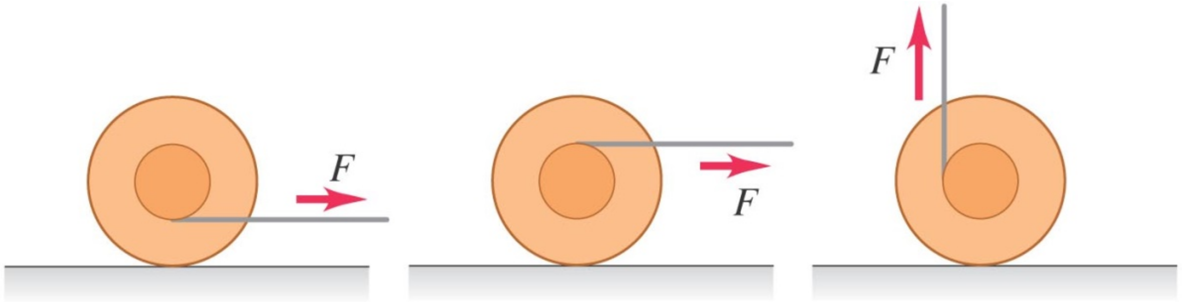
(a) The bowling ball



(b) Free-body diagram for the bowling ball



Problem 10.71



Three yo-yo initially at rest

A string is pulled as shown

There is sufficient friction so that the yo-yos roll without slipping.

In which direction will each yo-yo rotate?