The University of Sydney

MATH1902 Linear Algebra (Advanced)

Semester 1 **Board Tutorial for Week 2** 2017

6. Given points $A(4,-1,5)$ and $B(6,-1,-2)$ in space, find

(i) the position vectors of A and B in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$;

(ii) the displacement vector $\overrightarrow{AB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$;

(iii) the unit vector pointing from A towards B;

(iv) the unit vector pointing from B towards A.

7. Let $\mathbf{v} = 2\mathbf{i} - 6\mathbf{j} + 9\mathbf{k}$ and $\mathbf{w} = 4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}$. Find

(i) $-\mathbf{v}$ (ii) $\mathbf{w} - \mathbf{v}$ (iii) $2\mathbf{v}$ (iv) $3\mathbf{w}$ (v) $2\mathbf{v} - 3\mathbf{w}$

(vi) $|\mathbf{v}|$ (vii) $|\mathbf{w}|$ (viii) $\widehat{\mathbf{v}}$ (ix) $\widehat{\mathbf{w}}$ (x) $|\mathbf{v} + \mathbf{w}|$

8. Let $ABCDEF$ be a regular hexagon. True or false:

(i) $\overrightarrow{AC} = \overrightarrow{FD}$ (ii) $\overrightarrow{AC} = \overrightarrow{DF}$ (iii) $\overrightarrow{AC} = \overrightarrow{BD}$ (iv) $|\overrightarrow{AC}| = |\overrightarrow{BD}|$

(v) $|\overrightarrow{AC}| = |\overrightarrow{AD}|$ (vi) The line segments AD and BE bisect each other.

9. Consider the following points in space:

$$O(0,0,0)\,,\;\; P(1,1,1)\,,\;\; Q(-1,-1,0)\,,\;\; R(0,1,2)\,,\;\; S(2,3,3)\,.$$

(i) Find the Cartesian forms of

$$\overrightarrow{OP}\,,\; \overrightarrow{OQ}\,,\; \overrightarrow{OR}\,,\; \overrightarrow{OS}\,,\; \overrightarrow{PQ}\,,\; \overrightarrow{QP}\,,\; \overrightarrow{QR}\,,\; \overrightarrow{RS}\,,\; \overrightarrow{SP}\,.$$

(ii) Verify that the figure $PQRS$ is a rhombus, that is, a parallelogram in which side lengths are equal. How can you tell that it is not a square?

10. Let $\mathbf{i}$ and $\mathbf{j}$ denote displacements of 1 km east and north respectively. An aeroplane travels 300km southeast and then 150 km in the direction 30° west of north. Find

(i) the above displacements of the aeroplane and their vector sum in terms of the unit vectors $\mathbf{i}$ and $\mathbf{j}$;

(ii) the final distance (to the nearest km) and direction (to nearest degree, south of east) of the aeroplane from the starting position.

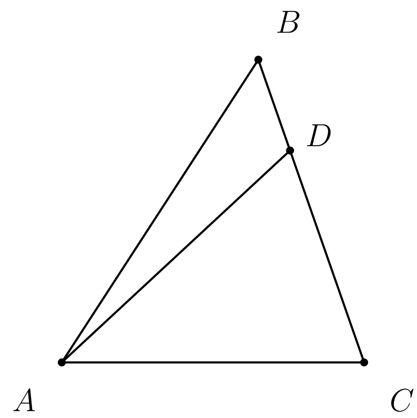
11. Suppose that $\mathbf{v}$ and $\mathbf{w}$ are non-zero vectors which are not parallel (so are linearly independent) and the following vector equation holds for some scalars α and β:

$$\mathbf{v} + \alpha(\mathbf{w} - \mathbf{v}) \;=\; \beta\big(\mathbf{v} + \frac{1}{2}\mathbf{w}\big)\,.$$

Find α and β.

12.* Prove that any three geometric vectors in the plane are linearly dependent.

13.* Let D be the point which divides the side BC of the triangle ABC in the ratio $\alpha : \beta$.



Carefully prove that $\overrightarrow{AD} = \dfrac{\beta\,\overrightarrow{AB} + \alpha\,\overrightarrow{AC}}{\alpha+\beta}$. Is there any natural interpretation if one of α or β is negative? What if both are negative?

14.* Show that $\mathrm{span}(\mathbf{i}, \mathbf{i}+\mathbf{j}, \mathbf{i}+\mathbf{j}+\mathbf{k})$ is the set of all vectors in $\mathbb{R}^3$.
Show that $\mathrm{span}(\mathbf{j}+2\mathbf{k}, -\mathbf{i}+3\mathbf{k}, -2\mathbf{i}-3\mathbf{j})$ does not contain the vector $3\mathbf{i}-2\mathbf{j}+\mathbf{k}$.

If you are done with all these have a look at **19.** in the exercise sheet for this week.