LSA University of Michigan

ASTRO201: Introduction to Astrophysics
Homework 1

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1. a) Using the formula for the difference in apparent and absolute magnitudes,

$$\begin{aligned} m_V - M_V &= 5 \times \log_{10} d_{pc} - 5 \\ m_V &= 5 \times \log_{10} d_{pc} - 5 + M_V \\ &= 5 \times \log_{10} \left(4.84 \times 10^{-6}\right) - 5 + 4.83 \\ &= -26.75 \\ \therefore m_V &= -27. \end{aligned}$$

Thus, the apparent magnitude of the Sun in the V band is -27.

b) Again, using the formula for the difference in apparent and absolute magnitudes, and with the apparent magnitude in the V band set to $+6$,

$$\begin{aligned} m_V - M_V &= 5 \times \log_{10} d_{pc} - 5 \\ \frac{m_V - M_V + 5}{5} &= \log_{10} d_{pc} \\ \therefore \log d_{pc} &= \frac{m_V - M_V + 5}{5} \\ &= \frac{6 - 4.83 + 5}{5} \\ &= 1.234 \\ \therefore d_{pc} &= 10^{1.234} \\ &= 17.14 \\ \therefore d_{pc} &= 17. \end{aligned}$$

Thus, for the Sun to have an apparent magnitude of $+6$, it must be at a distance of 17 parsecs, or 5.3×10^{19} cm.

2. a) Using the formula for optical depth, with $n = 110$/cm, $l = 5 \times 10^9$ cm, and $\sigma_\lambda = 7 \times 10^{-25}$ cm^2,

$$\begin{aligned} \tau_\lambda &= nl\sigma_\lambda \\ \therefore \tau_\lambda &= 110 \times 5 \times 10^9 \times 7 \times 10^{-25} \\ &= 3.85 \times 10^{-13} \\ \therefore \tau_\lambda &= 3.9 \times 10^{-13}. \end{aligned}$$

Thus, the optical depth of the Sun's corona is 3.9×10^{-13}.

b) Using the formula for radiative transport, in the case of absorption,

$$\begin{aligned} I &= I_0 e^{-\tau} \\ &= I_0 e^{-3.85 \times 10^{-13}} \\ &= I_0. \end{aligned}$$

Clearly, the solar radiation passes through the Sun's corona without attenuation, as $I = I_0$, and so is attenuated 0%.

3. a) Using the formula for energy flux and luminosity, where $R = D$,

$$\begin{aligned} F &= \frac{L}{4\pi D^2} \\ &= \frac{4\pi R^2}{4\pi D^2}\sigma_{\Sigma B}T^4 \\ &= \frac{4\pi R^2}{4\pi R^2}\sigma_{\Sigma B}T^4 \\ &= \sigma_{\Sigma B}T^4 \\ &= 5.7 \times 10^{-5} \times (5770)^4 \\ &= 6.32 \times 10^{10} \\ \therefore F &= 6.3 \times 10^{10}. \end{aligned}$$

Thus, the surface brightness of the Sun is 6.3×10^{10} ergs/cm^2/s.

b) Again, using the formula for energy flux and luminosity, where the radius of the Sun is 696340 km, and the distance to the Sun is 149.6×10^6 km,

$$\begin{aligned} F &= \frac{L}{4\pi D^2} \\ &= \frac{4\pi R^2}{4\pi D^2}\sigma_{\Sigma B}T^4 \\ &= \frac{696340^2}{(149.6 \times 10^6)^2} \times 5.7 \times 10^{-5} \times (5770)^4 \\ &= 1.37 \times 10^6 \\ \therefore F &= 1.4 \times 10^6. \end{aligned}$$

Thus, the flux energy from the Sun received at Earth is 1.4×10^6 ergs/cm^2/s.

c) The textbook quotes the temperature of a sunspot to be 3800 K. Using the formula relating peak wavelength and temperature,

$$\begin{aligned} \lambda_{\text{max}}T &= 2.90 \times 10^6 \\ \therefore \lambda_{\text{max}} &= \frac{2.90 \times 10^6}{T} \\ &= \frac{2.90 \times 10^6}{3800} \\ &= 763.16 \\ \therefore \lambda_{\text{max}} &= 760. \end{aligned}$$

Thus, the peak wavelength of a sunspot is 760 nm.

d) Using the formula for intensity of a black body, where temperature is 5770 K for the Sun, 3800 K for the sunspot, and wavelength is 550 nm, the intensities of the Sun and sunspot at 550 nm are

$$
\begin{aligned}
I_\lambda(\lambda, T) &= \left(\frac{2hc^2}{\lambda^5}\right)\frac{1}{e^{hc/\lambda kT}-1} \\
\therefore I_\lambda(550, 5770) &= \left(\frac{2\times 6.6262\times 10^{-27}\times\left(2.9979\times 10^{17}\right)^2}{550^5}\right) \\
&\quad\times\frac{1}{e^{6.6262\times10^{-27}\times2.9979\times10^{17}/550\times1.3806\times10^{-16}\times5770}-1} \\
&= 2.56\times 10^{-7}, \\
\therefore I_\lambda(550, 5770) &= 2.6\times 10^{-7}, \\
\therefore I_\lambda(550, 3800) &= \left(\frac{2\times 6.6262\times 10^{-27}\times\left(2.9979\times 10^{17}\right)^2}{550^5}\right) \\
&\quad\times\frac{1}{e^{6.6262\times10^{-27}\times2.9979\times10^{17}/550\times1.3806\times10^{-16}\times3800}-1} \\
&= 2.4\times 10^{-8}.
\end{aligned}
$$

Thus, the ratio of intensity of the sunspot to the Sun is $1:11$.

4. a) As Eta Carinae is significantly hotter than the Sun, it will appear bluer than the Sun.

b) Using the formula relating peak wavelength and temperature,

$$
\begin{aligned}
\lambda_{\text{max}}T &= 2.90\times 10^6 \\
\therefore \lambda_{\text{max}} &= \frac{2.90\times 10^6}{T} \\
&= \frac{2.90\times 10^6}{38000} \\
&= 76.32 \\
\therefore \lambda_{\text{max}} &= 76
\end{aligned}
$$

Thus, the peak wavelength of Eta Carinae is 76 nm, which is in the ultraviolet range on the spectrum.