The University of Sydney

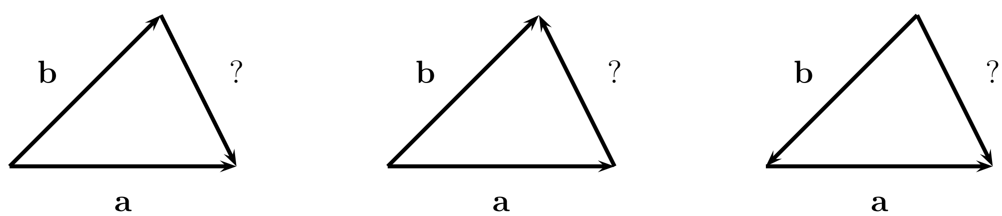
MATH1902 Linear Algebra (Advanced)

Semester 1 | **Board Tutorial for Week 1** | 2017

8. If $|\mathbf{v}| = 2$, find $|\mathbf{u}|$ in each of the following cases.

(i) $\mathbf{u} = 3\mathbf{v}$ (ii) $\mathbf{u} = \frac{1}{2}\mathbf{v}$ (iii) $\mathbf{u} = -3\mathbf{v}$ (iv) $\mathbf{v} = 3\mathbf{u}$

9. In each diagram below, find the unknown vector in terms of $\mathbf{a}$ and $\mathbf{b}$.

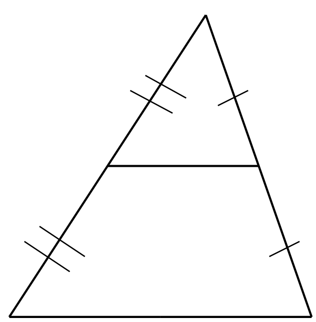


10. Solve for $\mathbf{x}$ in terms of $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ in each case.

(i) $\mathbf{v} + \mathbf{x} = \mathbf{u} - \mathbf{w}$ (ii) $\mathbf{v} - \mathbf{x} = \mathbf{w} - \mathbf{u}$ (iii) $2\mathbf{v} + \mathbf{x} = 2\mathbf{w} - 2\mathbf{u} - \mathbf{x}$

11. A balloon experiences two forces, a buoyancy force of 8 newtons vertically upwards and a wind force of 6 newtons acting horizontally to the right. Calculate the magnitude and direction of the resultant force.

12. Prove, using vectors, that the line segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length of the third side.



13. Explain the associative law for addition of vectors. Explain the triangle inequality and determine when equality occurs.

14. Look at the corner of the room. Each wall is a plane and two planes meet in a line. Follow the line upwards towards the ceiling. Where it meets the ceiling is the point of intersection of three planes. Find the point (x, y, z) of intersection of the following three planes:

$$\begin{aligned} 2x + 3y + 4z &= -4 \\ 5x + 5y + 6z &= -3 \\ 3x + y + 2z &= -1 \end{aligned}$$

15.* Find the line obtained by reflecting the line $ax + by = c$ in the line $y = x + k$.

PTO

Important Ideas and Useful Facts:

(i) A *geometric vector* $\mathbf{v}$ is a directed line segment in space, described by its length $|\mathbf{v}|$ and direction. Two vectors are *equal* if they have the same magnitude and direction, regardless of their position in space.

(ii) A *scalar* λ is a real number. The *scalar multiple* $\lambda\,\mathbf{v}$ has length $|\lambda||\mathbf{v}|$ and the same direction as $\mathbf{v}$ if λ is positive, and opposite direction if λ is negative.

(iii) If P and Q are points in space then $\overrightarrow{PQ}$ denotes the vector pointing from P to Q. The *position vector* of the point P is the vector $\overrightarrow{OP}$ where O denotes the origin in space.

(iv) A *parallelogram* is a quadrilateral such that two opposite sides are parallel and have the same length (which implies that the other two opposite sides are also parallel and have the same length).

(v) Parallelogram Law of Vector Addition: The *vector sum* $\mathbf{v}+\mathbf{w}$ is represented by the diagonal of the parallelogram formed using sides $\mathbf{v}$ and $\mathbf{w}$.

(vi) Commutative Law of Addition: $\mathbf{v}+\mathbf{w}=\mathbf{w}+\mathbf{v}$.

(vii) Associative Law of Addition: $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$.

(viii) Triangle Inequality: $|\mathbf{v}+\mathbf{w}|\leq|\mathbf{v}|+|\mathbf{w}|$.

(ix) The *zero vector* $\mathbf{0}$ has zero length and points in every direction. For every vector $\mathbf{v}$, $\mathbf{0}+\mathbf{v}=\mathbf{v}$ and $0\mathbf{v}=\mathbf{0}$.

(x) The *negative* of $\mathbf{v}$ is $-\mathbf{v}=(-1)\mathbf{v}$ with the same length as $\mathbf{v}$, but pointing in the opposite direction. If P and Q are points then $\overrightarrow{QP}=-\overrightarrow{PQ}$.

(xi) The *vector difference* $\mathbf{v}-\mathbf{w}$ equals $\mathbf{v}+(-\mathbf{w})$ and has the property that $\mathbf{w}+(\mathbf{v}-\mathbf{w})=\mathbf{v}$.

(xii) If $\mathbf{v}$ and $\mathbf{w}$ are vectors and λ and μ are scalars, then

$$\lambda(\mu\mathbf{v})=(\lambda\mu)\mathbf{v}\ ,\quad \lambda(\mathbf{v}+\mathbf{w})=\lambda\mathbf{v}+\lambda\mathbf{w}\ ,\quad (\lambda+\mu)\mathbf{v}=\lambda\mathbf{v}+\mu\mathbf{v}\ ,$$

$$-(-\mathbf{v})=\mathbf{v}\ ,\quad \mathbf{v}-\mathbf{v}=\mathbf{0}\ ,\quad 1\mathbf{v}=\mathbf{v}\ ,\quad (-\lambda)\mathbf{v}=-(\lambda\mathbf{v})\ .$$