

## Assignment 1

MATH1906: Mathematics Special Studies Program A

Semester 1, 2017

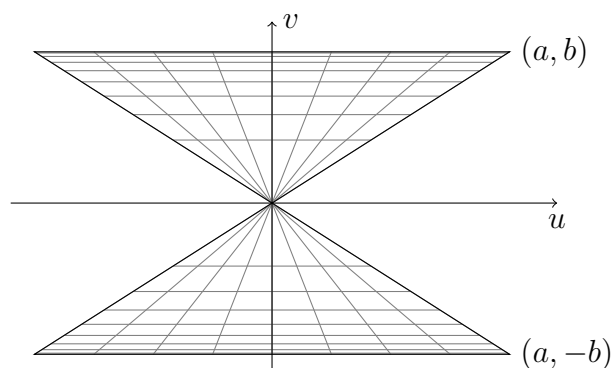
Web Page: <http://sydney.edu.au/science/math/su/UG/JM/MATH1906/>

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This assignment is due by **23:59 on Thursday 13th April**. A scanned or typeset copy of your answers must be upload in the Learning Management System (Blackboard) at <https://elearning.sydney.edu.au>, where it will be passed through the text matching service Turnitin. It should include your name and SID.

The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

1. Design an equal area map in the shape of two triangles. The slopes of the lines bounding the map left and right are  $\pm b/a$ . Given  $b > 0$ , the coordinate  $a > 0$  is chosen so that the total area is that of the surface area of the sphere with radius one.



Represent the meridians as equally spaced straight lines through the origin and the parallels as horizontal lines as shown above.

- (a) Find a differential equation for the spacing of the parallels. To do so look at a small strip between the latitudes  $\theta$  and  $\theta + \Delta\theta$  and write down its approximate area. Then look at the approximate area of the corresponding narrow strip between  $v(\theta)$  and  $v(\theta + \Delta\theta)$  within one of the triangles. Equating the two and letting  $\Delta\theta \rightarrow 0$ , show that

$$b^2 \cos \theta = 2v \frac{dv}{d\theta}.$$

- (b) Solve the differential equation derived in the previous part, making sure the solution also works for  $\theta$  negative.
- (c) Write down the map coordinates in the form  $u = u(\varphi, \theta)$  and  $v = v(\theta)$ , where  $\varphi$  is longitude and  $\theta$  is latitude.