The University of Sydney
MATH1901/06 Differential Calculus (Advanced)

Semester 1 **Assignment 1** 2012

This assignment is due by 4:00pm on **Tuesday, 15 May, 2012**. It should be posted in the locked collection boxes on the verandah of Carslaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue. (NOT the locked collection boxes near the pyramids on Carslaw Level 3, nor the open pigeonholes.) Please do not post your assignment before the due date since the boxes are also used for the collection of assignments in other units. Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. A cover sheet must be signed and attached.

This assignment is worth 10% of the assessment for MATH1901/06. It covers topics on complex numbers, functions, limits, continuity, differentiability and Taylor polynomials from Weeks 1–8 of lectures.

1. [*5 marks*] Find all solutions of the cubic equation,

$$z^3 + 4(i-1)z^2 + (2-7i)z + 5(3+i) \;=\; 0,$$

given that $2+i$ is a root.

2. [*4 marks*] For $\alpha > 0$, define the function,

$$g(x) \;=\; \begin{cases} |x|^\alpha \cos(1/x^2), & x \neq 0, \\ 0, & x = 0. \end{cases}$$

Determine for which values of α, $g(x)$ is (a) differentiable at $x=0$, (b) continuously differentiable at $x=0$.

3. [*4 marks*] Use any method to evaluate the limit, $L = \lim_{x\to 0} (\cosh 2\lambda x)^{1/x^2}$.

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4. In the real domain, the inverse tangent function $\tan^{-1} x$ (also called $\arctan x$) has domain $\mathbf{R}$ and range $(-\pi/2, \pi/2)$. In the complex domain, $\tan^{-1} z$ is a multi-valued function with similar behaviour to $\log z$. Every complex number except $\pm i$ has infinitely many inverse tangents. To simplify matters, restrict attention to the horizontal strip, $-1 < \operatorname{Im} z < 1$, which is not the full domain. [End of preamble.]

In the given strip, show that one of the values of the inverse tangent is

$$\operatorname{Tan}^{-1} z = \frac{\log(1+iz) - \log(1-iz)}{2i},$$

where the logarithms take their principal values. (Use a capital T to distinguish this case from the real-variable inverse tangent.) Your proof needs to include the following three steps:

(a) [*2 marks*] Express $\tan z$ in terms of the exponential function e^{2iz}.

(b) [*3 marks*] Use part (a) to show that $\tan(\operatorname{Tan}^{-1} z) \equiv z$ in the strip.

(c) [*2 marks*] Show directly that, when x is real, $\operatorname{Tan}^{-1} x = \tan^{-1} x$, where the right-hand side is the real-variable inverse tangent. (This result justifies calling $\operatorname{Tan}^{-1} z$ the principal value of the multi-valued function $\tan^{-1} z$.)