THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Solutions to Tutorial Week 10

MATH1905: Statistics (Advanced) Semester 2, 2017

Web Page: `http://sydney.edu.au/science/maths/MATH1905`
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Please complete any unfinished exercises from week 9.

1. In an effort to compare the durability of two different types of sandpaper, 10 pieces of type A sandpaper were subjected to treatment by a machine which measures abrasive wear. Eleven pieces of type B sandpaper were subjected to the same treatment. Assuming normality and equality of variance, test for equality of mean abrasive wear at the 5% level of significance using the following summary.

Type	Size	Mean	SD
A	10	27.4	2.3
B	11	24.1	3.1

Solution:

- Assumptions: observations sampled independently from two normally distributed populations with common variance: $X_i \sim N(\mu_A, \sigma^2)$ independently of $Y_i \sim N(\mu_B, \sigma^2)$.
- Hypotheses: $H_0 : \mu_A = \mu_B$ against $H_1 : \mu_A \neq \mu_B$.
- Level of significance: $\alpha = 0.05$.
- Whatever be the true values of μ_A and μ_B, the random variable $\dfrac{(\bar{X} - \bar{Y}) - (\mu_A - \mu_B)}{S_p\sqrt{\dfrac{1}{n_A} + \dfrac{1}{n_B}}} \sim t_{19}$.

  To construct a test statistic we "plug in" the hypothesised value $\mu_A - \mu_B = 0$, yielding the test statistic $\dfrac{(\bar{X} - \bar{Y})}{S_p\sqrt{\dfrac{1}{n_A} + \dfrac{1}{n_B}}} \sim t_{19}$ /if H_0 is true.
- Observed test statistic: the pooled variance is $s_p^2 = 7.5637$ so, the test statistic takes the value

$$\tau_{\text{obs}} = \frac{(27.4 - 24.1)}{\sqrt{7.5637}\sqrt{\dfrac{1}{10} + \dfrac{1}{11}}} = 2.746\,,$$

- p-value $= P(|t_{19}| \geq 2.746) = 2P(t_{19} \geq 2.746)$, which we can obtain using R as follows:

```
2*(1-pt(2.746,df=19))
```

```
[1] 0.01284587
```

Hence, we reject H_0 at the 5% level of significance.

2. In the setting of the previous question construct a 95% confidence interval for the (population) mean difference in abrasive wear. Should it be a "one-sided" or "two-sided" confidence interval?

Solution: Since the "scientific question" translates to a two-sided test, the appropriate confidence interval is also two-sided. The form of the interval is "estimate $\pm$ c s.e." where c is an appropriate "multiplier" or "table value". For definiteness we let the population mean difference be $\mu_A - \mu_B$, where μ_A is the population mean abrasive wear for type A (similarly for μ_B).

From the solution to the previous question the estimate is the sample mean difference $\bar{x} - \bar{y} = 3.3$, the standard error of this difference is the denominator of the t-statistic

$$\sqrt{7.5637}\sqrt{\frac{1}{10} + \frac{1}{11}} \approx 1.201657$$

and the "multiplier" or "table value" is that value which cuts off 0.025 in the upper tail of Student's t-distribution with 19 degrees of freedom.

According to R this is

```
qt(.975,19)
```

```
[1] 2.093024
```

This gives the interval $(0.785, 5.815)$.

Note that this interval **does not** include zero, which is consistent with our earlier finding that the two-sided p-value is *less* than 0.05. Note also that a 99% interval would only differ from this by use of a different "multiplier" or "table value" which in this case would cut off 0.005 in the upper tail of the t_{19} distribution; this is (approx.) 2.861, giving the interval $3.3 \pm 2.861 \times 1.201657$, i.e. $(-0.1379407, 6.7379407)$. Note that this **does** include 0 which is also consistent with our finding that the two-sided p-value is *greater* than 0.01.