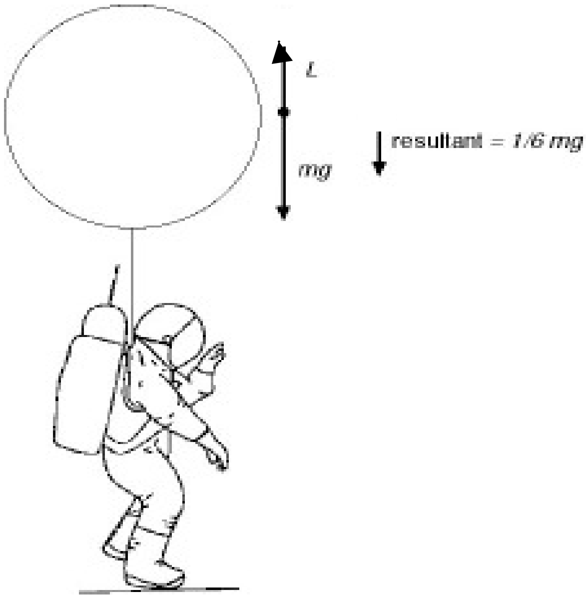
PHYS1902 Advanced Exam Solutions
Semester 2, 2006

SECTION A

Question 1

(a)



(1 mark)

(b) Buoyancy = weight of fluid displaced

Lift Force = buoyancy – weight of helium

$$= Vg\rho_{air} - Vg\rho_{He}$$

$$= Vg(\rho_{air} - \rho_{He})$$

(2 marks)

(c) From the diagram,

$$mg - L = \frac{1}{6}mg$$

so,

$$L = \frac{5}{6}mg$$

From (b),

$$L = Vg(\rho_{air} - \rho_{He})$$

so,

$$V = \frac{5m}{6(\rho_{air} - \rho_{He})}$$

$$= \frac{5 \times 120}{\left(6 \times \left(1.2 - 0.16\right)\right)}$$

$$= 96.2\text{m}^3$$

$$= \frac{4}{3}\pi r^3$$

so,

$$r = 2.84 \text{ m}$$

(2 marks)
(Total 5 marks)

Question 2

(a) High voltage is chosen to minimize the power lost to the resistance of the wires.

(1 mark)

Justification: the lines must deliver a power P to the destination, which is fixed. The voltage and current are then fixed by the equation $P = VI$ = constant.

(1 mark)

The current I is the same throughout the circuit. The power loss due to a line with resistance R is given by $P_{loss} = I^2 R$. To minimize the power loss, the current is chosen to be very low (and thus the voltage very high).

(1 mark)

(b) Lines with zero resistance will have no power loss.

(1 mark)

Thus, any voltage can be chosen to give $P = VI$ = constant.

(1 mark)

(Total 5 marks)

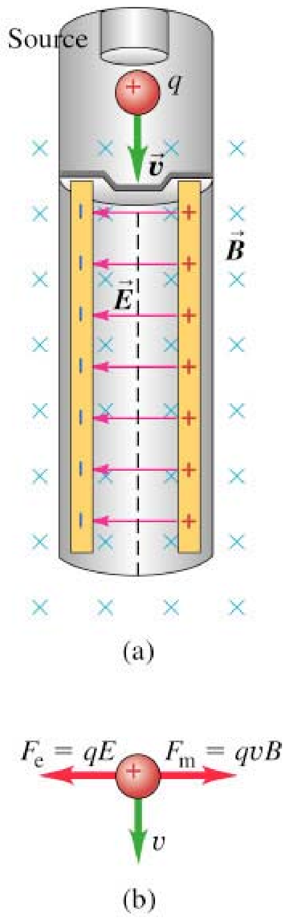
Question 3

(a) An electric field is created perpendicular to the direction of the ions, and a magnetic field perpendicular to both of these directions following the right-hand rule. A positive charge feels an electric force qE (to the left in the picture), and a magnetic force qvB (to the right).

(1 mark)

These forces will balance if v = E/B, regardless of the charge. Only ions with v = E/B will pass through the velocity selector.

(1 mark)



(1 mark for diagram)

(b) In a magnetic field B, a particle with charge q and velocity v experiences a force F=qvB perpendicular to its direction of motion. This force is then the centripetal force F = mv^2/R.

(1 mark)

Equating these expressions and solving for m yields

m = qRB/v

(1 mark)

(Total 5 marks)

Question 4

(a) This is the line integral of the magnetic field around a closed imaginary path.

(1 mark)

(b) i = current encircled by the path

(1 mark)

(c') $\frac{d\phi_E}{dt}$ = rate of chage of flux of electric field through path

(1 mark)

(d) Law means B is produced in circles *around* a current and also around a changing electric field.

Note: B lines always form closed loops – true, but not from this equation.

Must say *around i or* $\frac{d\phi_E}{dt}$

(2 marks)

(Total 5 marks)

Question 5

(a) The Heisenberg uncertainty principle may be written

$$\Delta p_x \Delta x \geq \hbar$$

where Δp_x is the uncertainty in p_x, the x – component of momentum, and Δx is the uncertainty in th position x. Also $\hbar = \frac{h}{(2\pi)}$, where h is Planck's constant.

(1 mark)

(b) If the angular momentum is known precisely, then we can write $\boldsymbol{L} = \boldsymbol{L}_z \hat{\boldsymbol{z}}$, where the z axis is chosen to be aligned with the angular momentum.

However, this implies that the motion is in a plane perpendicular to $\hat{z}$, say the x-y plane (choosing the origin to lie in the plane)

Hence, $\Delta p_z = 0$, but then the uncertainty principle implies $\Delta z = \infty$, i.e. the motion isn't in a plane, which is a contradiction.

(This or similar argument 2 marks)

(c) The kinetic energy of the particle is $K = \frac{p_x^2}{2m}$ (assuming non-relativistic motion).

So the uncertainty in energy is $\Delta K = \frac{2 p_x \Delta p_x}{2m}$.

The momentum must be at least as large as the uncertainty in the momentum, so

$$\Delta K \geq \frac{(\Delta p_x)^2}{m}.$$

From the uncertainty principle $\Delta p \, \Delta x \geq \hbar$, which implies $\Delta K \geq \frac{\hbar^2}{m(\Delta x)^2}$. Assuming $\Delta x \approx L$ we have (approximately) $\Delta K \geq \frac{\hbar^2}{(mL^2)}$, hence $K \approx \frac{\hbar^2}{(mL^2)}$

(This or similar argument 2 marks)

(Total 5 marks)

Question 6

(a) The wave function of the electron extends through the barrier and to the other side if the barrier is of a finite width. Since the modulus squared of the wave function is directly proportional to the probability of finding the particle at a certain point, then if the wave function is non-zero on the other side of the barrier, then there will be a finite probability of the particle being found on the other side, i.e. tunnelling.

(2 marks)

(b) The correct choice is (ii). Transmission coefficient formula is given by

$$T \propto e^{-2\kappa L}$$

where L is the barrier width, and $\kappa = \frac{\sqrt{2m(U-K)}}{\hbar}$, where m is the mass of the particle. From this it can be seen that the greater the mass, the less the transmission coefficient.

Note: However, this formula WAS NOT presented to the Technological class in lectures (although it's in the book), but may have been presented to the Advanced students.

The Technological class has seen that the wave function decays exponentially through barrier (graphically – no equation). They were also told about the correspondence principle. So for a very large mass, the particle must behave classically – that is, no tunneling through the barrier. Since the proton is more massive than the electron, one would expect the tunneling probability to be less.

(3 marks)

(Total 5 marks)

(Section B)

Question 7

diameter of A = 20 mm = 0.020 m r_A=0.010 m area of circle A = $\pi\ r^2$

Cross-sectional area $A_A = \pi(0.01)^2 = 3.14 \times 10^{-4}\,\mathrm{m}^2$

diameter of C = 10 mm = 0.010 m $r_C = 0.005$ m

Cross-sectional area $A_C = \pi(0.05)^2 = 7.85 \times 10^{-5}\,\mathrm{m}^2$

$P_C = P_{atm} = 1.01\times 10^5$ Pa $P_A = 19$ kPa + $P_{atm} = 1.20 \times 10^5$ Pa

$Q_A = 0.5 \times 10^{-3}\,\mathrm{m}^3.\mathrm{s}^{-1}$

(a) absolute pressure = gauge pressure + atmospheric pressure

$$P_A = 1.20\times 10^5 \text{ Pa} \qquad P_C = 1.01\times 10^5 \text{ Pa}$$

(2 marks)

(b) Equation of continuity

$$Q_A = Q_C = A_A\, v_A = A_C\, v_C$$

$$v_A = \frac{Q}{A_A} \quad = \quad \frac{0.5 \times 10^{-3}}{3.14 \times 10^{-4}} \quad = \quad 1.6\ \mathrm{m.s}^{-1}$$

$$v_A = \frac{Q}{A_C} \quad = \quad \frac{0.5 \times 10^{-3}}{7.85 \times 10^{-5}} \quad = \quad 6.4\ \mathrm{m.s}^{-1}$$

(2 marks)

(c) $P_D = P_{atm} = 1.013 \times 10^5$ Pa since D is open to the atmosphere.

(1 mark)

(d) $P_D > P_B \Rightarrow$ liquid flow from reservoir to pipe at B and then to the garden

(1 mark)

(e) Bernoulli's Principle: A and B at same height

$$\frac{1}{2}\rho {v_A}^2 + P_A = \frac{1}{2}\rho {v_B}^2 + P_B$$

(Use of equation - 1 mark)

$$v_B = \sqrt{\frac{2(P_A - P_B)}{\rho} + v_A{}^2}$$

Assume ρ = $10^3\,kg.m^{-3}$

$$P_A - P_B = (120 - 10)\ \text{kPa} = 9\times10^{3}\ \text{Pa}$$

$$v_B = \sqrt{\frac{(2)(110\times10^3)}{10^3} + 1.6}$$

$$= 15\,\text{m.s}^{-1}$$

$$A_B = \frac{Q}{v_B}$$

$$= \frac{0.5\times10^{-3}}{15}$$

$$= 3.3\times10^{-5}\,\text{m}^2$$

$$r_B = \sqrt{\frac{A_B}{\pi}}$$

$$= \sqrt{\frac{3.3\times10^{-5}}{\pi}}$$

$$= 0.003\ \text{m}$$

diameter of B = 0.006 m = 6 mm

(radius acceptable if explicitly stated)

(Working - 2 marks)

(1 mark)

(Total 10 marks)

Question 8

(a) $\int \mathbf{E}.d\mathbf{A} = \frac{Q}{\varepsilon_o}$

Choosing a spherical Gaussian surface then, over that surface, E is a constant E is parallel to dl,

so,

(i) $E(r)\int dA = 0$ for $r < R$

(method 1 mark)

$E(r) = 0$

(results – 1 mark)

(ii) $E(r)\int dA = \frac{Q}{\varepsilon_o}$ for $r \geq R$

$$E(r)\left(4\pi r^2\right) = \frac{Q}{\varepsilon_o}$$

$$E(r) = \frac{Q}{4\pi\varepsilon_o r^2}$$

(method 1 mark)

(results – 1 mark)

(b) $v_r - v_\infty = \int_r^\infty \mathbf{E}.d\mathbf{r}$

$$= \frac{Q}{4\pi\varepsilon_o}\int_r^\infty \frac{1}{r^2}$$

$$= \frac{Q}{4\pi\varepsilon_o}\left[\frac{-1}{r}\right]_r^\infty$$

$$= \frac{Q}{4\pi\varepsilon_o r}$$

so,

(i) $V = \frac{Q}{4\pi\varepsilon_o R}$ for $r < R$

(ii) $V = \frac{Q}{4\pi\varepsilon_o r}$ for $r \geq R$

(method 1 mark)

(results – 1 mark)

(c) $V_a - V_b = \dfrac{Q}{4\pi\varepsilon_o r_a} - \dfrac{Q}{4\pi\varepsilon_o r_b}$

$$= \frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{r_a} - \frac{1}{r_b}\right)$$

$$= \frac{Q}{4\pi\varepsilon_o}\left(\frac{r_b - r_a}{r_a r_b}\right)$$

so C $= \dfrac{Q}{(v_a - v_b)}$

$$= 4\pi\varepsilon_o\left(\frac{r_a r_b}{r_b - r_a}\right)$$

(2 marks)

(b) U $= \dfrac{1}{2}\dfrac{Q^2}{c}$

$$= \frac{Q^2}{2}\frac{1}{4\pi\varepsilon_0}\left(\frac{r_b - r_a}{r_a r_b}\right)$$

(2 marks)

(Total 10 marks)

Question 9

(a) The magnetic flux through the loop will be $\Phi = \Phi_0 \cos(\omega t)$, with

$\Phi_0 = B_0 A = (0.50 \text{ T})(0.02 \text{ m})^2 = 2.0 \times 10^{-4}$ V s

The induced emf obeys Faraday's law

$E = - d\Phi/dt = \omega \Phi_0 \sin(\omega t) = (1.88 \times 10^{-3} \text{ V}) \sin(\omega t)$

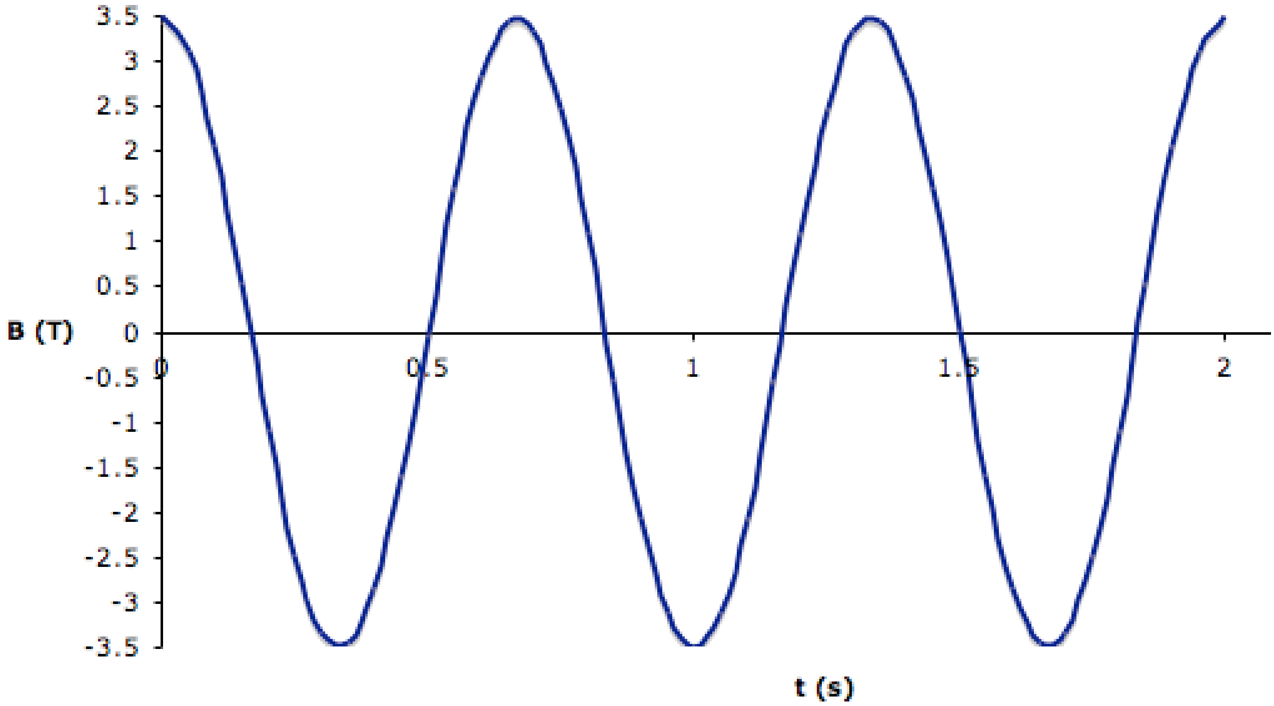
(2 marks)

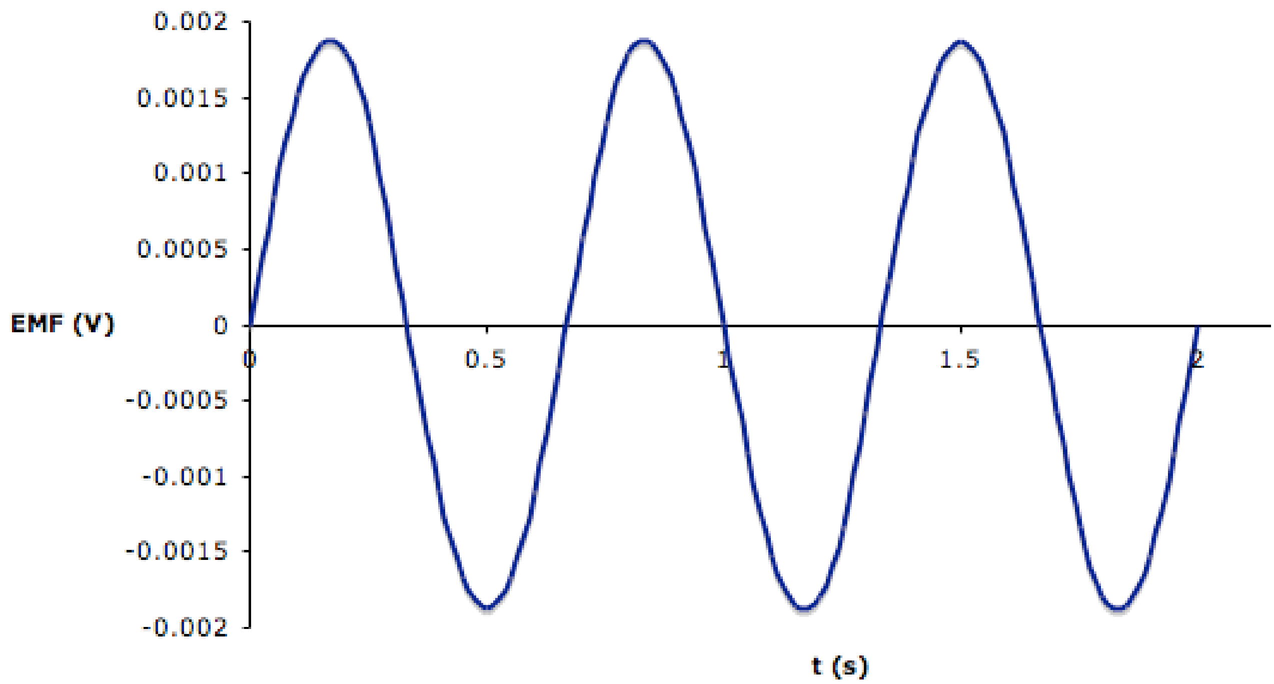
The amplitude of the emf is thus 1.88×10^{-3} V

(1 mark)

(b) The angular frequency is 3π rad. s^{-1} = 1.5 Hz. Thus, 3 full cycles are completed in 2.0 s.

(1 mark)





(1 mark for correct t = 0 value on B plot)

(1 mark for correct t = 0 and maximum values on emf plot)

(1 mark for correct cycles/phases for B plot)

(1 mark for correct cycles/phases for emf plot)

(c) If two turns are used, the amplitude of the induced emf is doubled.

(1 mark)

The reason is that the flux through the loop is doubled.

(1 mark)

(Total 10 marks)

Question 10

(a) In equilibrium, the inductor will (ideally) have zero resistance. The current in the circuit loop, through the inductor, is

$I = V/R = (10.0\ V)/(125\ \Omega) = 0.080\ A$

(1 mark)

(b) Also 0.080 A. The inductor forbids any instantaneous change in the current.

(2 marks)

(c) The angular frequency of oscillation for this LC circuit is

$\omega = (LC)^{-1/2} = ((10.0 \times 10^{-3}\ H) \times (25.0 \times 10^{-6}\ F))^{-1/2} = 2.00 \times 10^{3}\ Hz$

(1 mark)

The charge on the capacitor will first be at a maximum after $\frac{1}{4}$ of a period

(1 mark)

which corresponds to a time of

$T/4 = (0.25)(2\pi)/\omega = 7.85 \times 10^{-4}\ s = 0.785\ ms$

(1 mark)

(d) The initial energy in the inductor is

$\frac{1}{2}\,Li^2 = (10.0 \times 10^{-3}\ H)(\ 0.080\ A)^2/2 = 3.20 \times 10^{-5}\ J$

(1 mark)

(e) At time T/4, this energy is all stored in the capacitor. Thus, the answer is

$E = 3.20 \times 10^{-5}\ J$

(1 mark)

(f) The energy of the capacitor is stored in the electric field between the plates.

(1 mark)

(g) The energy was originally stored in the magnetic field through the inductor.

(1 mark)

(Total 10 marks)

Question 11

(a) The physical interpretation of the wave function is that $\left|\psi(x)\right|^2 \Delta x = \psi(x)\psi^*(x)\Delta x$ is the probability of finding the particle between x and $x + \Delta x$, for small Δx.

(This or similar statement – 1 mark)

(b) When $U=0$ the Schrodinger Equation becomes

$$\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi \qquad (1)$$

(1 mark)

Differentiating the given form for the wave function:

$$\psi = A_1 e^{ikx} + A_2 e^{-ikx}$$

$$\frac{d\psi}{dx} = ik\left(A_1 e^{ikx} - A_2 e^{-ikx}\right)$$

$$\frac{d^2\psi}{dx^2} = -k^2\left(A_1 e^{ikx} + A_2 e^{-ikx}\right)$$

$$= -k^2\psi$$

Comparing this with (1), the given from for the wave function is clearly a solution provided

$$k = \frac{\sqrt{2mE}}{\hbar}$$

(2 marks)

(c) (i) The required boundary conditions are $\psi(0) = \psi(L) = 0$

(1 mark)

(ii) This ensures that the term $U(x)\psi(x)$ in the wave function is finite at $x = 0$ and $x = L$

(1 mark)

(iii) In $0 < x < L$ The Schrodinger Equation is $\frac{d^2\psi}{dx^2} = \frac{-2mE}{\hbar^2}\psi$, which has the general solution

$$\psi(x) = A_1 e^{ikx} + A_2 e^{-ikx}, \quad k = \frac{(2mE)^{1/2}}{\hbar}, \text{ as shown in (b).}$$

(1 mark)

The wave function must be continuous, so we require $\psi(0) = \psi(L) = 0$

i.e. $$A_1 + A_2 = 0 \qquad (2)$$

$$A_1 e^{ikL} + A_2 e^{-ikL} = 0 \qquad (3)$$

Using (2) in (3) gives

$$A_1\left(e^{ikL} - e^{-ikL}\right) = 0$$

i.e. $$2iA \sin(kL) = 0$$

$$\Rightarrow \quad kL = n\pi \quad \text{for } n = 0,1,2,3,$$

(2 marks)

The energy of the particle is then given by

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2mL^2}$$

$$= \frac{h^2 n^2}{8mL^2}$$

(1 mark)

(d) For the case of a finite square potential well:

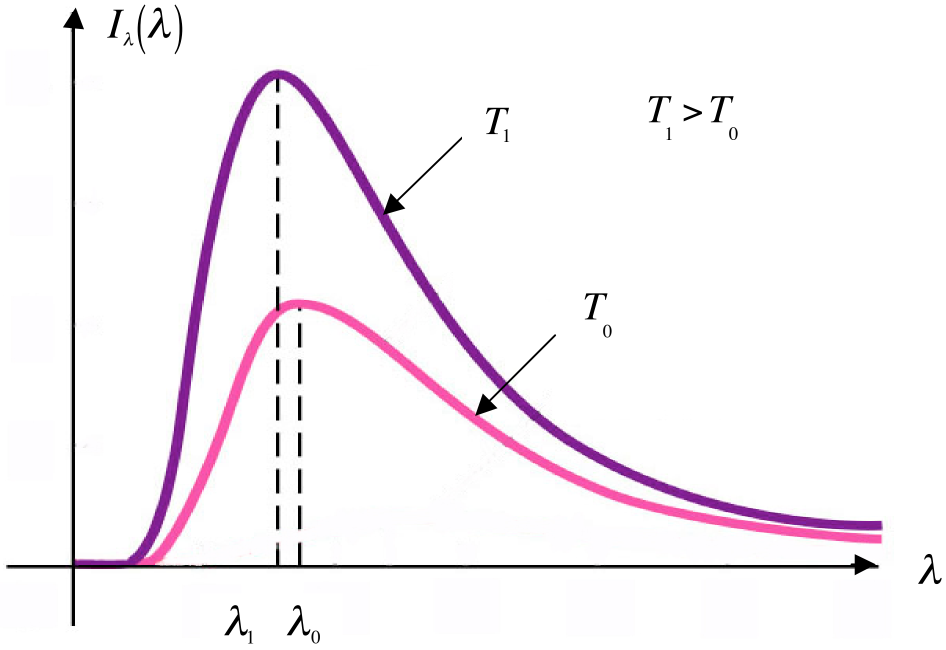
- There are also discrete energies.
- There is always at least one energy, but there may be only one energy for a shallow well.
- The energy levels of the finite well are slightly lower than the corresponding energy levels for the infinitely deep well.
- This may be understood in that the wave function does not go to zero at $x = 0$ and $x = L$ for the finite well, but matches to decaying exponential forms in $x < 0$ and $x > L$.
- Hence, the wavelength of the wave function in $0 < x < L$ is longer than in the infinite well case. This corresponds to a lower energy.

(1/2 mark for each valid point – these or others – Total 2 marks)

(Total 10 marks)

Question 12

(a)



(2 marks)

(b) The peak temperature of the spectrum shifts to shorter wavelengths with increasing temperature. The peaks wavelength λ_0 and λ_1, for temperatures T_0 and T_1 are shown. The product of the location $\lambda_{\max}$ of the maximum and the temperature is a constant, which is "Wien's Displacement Law".

(1 mark)

(c) For $\lambda >> \dfrac{hc}{kT}$ the exponential in the denominator may be expanded as a power series

$$\exp\left(\frac{hc}{\lambda kT}\right) \approx 1 + \frac{hc}{\lambda kT} + \frac{1}{2}\left(\frac{hc}{\lambda kT}\right)^2 + ...$$

Keeping only the first two terms leads to

$$I_\lambda(\lambda) \approx \frac{2\pi hc^2}{\lambda^5} \times \frac{\lambda kT}{hc} = \frac{2\pi ckT}{\lambda^4}$$

(2 marks)

(d) Rayleigh attempted to derive the spectrum for radiation from a hot cavity using classical physics. He assumed the radiation in the cavity may be described as a superposition of EM wave modes, and apportioned an energy kT to each mode (equipartition of energy). The spectrum he derived was the Rayleigh-Jeave law [see (c)], which matches the observed spectrum for long wavelengths but diverges as $\lambda \to 0$ (the "ultraviolet catastrophe").

(1 mark)

Planck managed to derive the correct form for the spectrum, but he was forced to make a number of ad hoc assumptions. In particular he assumed oscillators in the wall of the cavity producing radiation of a given frequency could only have certain fixed energies. His assumptions had no justification in the context of classical physics.

(Other relevant points – 1 mark)

(e) We have

$$I = \int_0^\infty I_\lambda(\lambda)d\lambda$$

$$= 2\pi hc^2 \int_0^\infty \frac{d\lambda}{\lambda^5\left(e^{hc/(\lambda kT)}-1\right)}$$

Setting $x = \dfrac{hc}{\lambda kT}, \quad dx = \dfrac{-hc}{\lambda^2 kT}.d\lambda$

$$\lambda = \frac{-hc}{xkT}$$

so, $dx = \dfrac{-hc}{kT}\,.\,x^2\left(\dfrac{kT}{hc}\right)^2 d\lambda$

$$\frac{dx}{x^2} = -\frac{kT}{hc}d\lambda$$

so: $I = 2\pi hc^2 \dfrac{hc}{kT}.\left(\dfrac{kT}{hc}\right)^5 \displaystyle\int_\circ^\infty \frac{x^5 dx}{x^2\left(e^x-1\right)}$

$$= \frac{2\pi k^4T^4}{h^3c^2}.\int_\circ^\infty \frac{x^3dx}{\left(e^x-1\right)},$$

$$= cT^4$$

where c is a constant.

(3 marks)

(Total 10 marks)