University of New South Wales

MATH 2901

Higher Theory of Statistics

Assignment 1

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1. (a) If event A is independent of itself, by the definition of independence, the following result holds.

$$\begin{aligned}
\mathbb{P}(A \cap A) &= \mathbb{P}(A)\mathbb{P}(A) \\
&= \big[\mathbb{P}(A)\big]^2 \\
\mathbb{P}(A) &= \big[\mathbb{P}(A)\big]^2 \\
\therefore \big[\mathbb{P}(A)\big]^2 - \mathbb{P}(A) &= 0 \\
\therefore \mathbb{P}(A)\big[\mathbb{P}(A) - 1\big] &= 0 \\
\therefore \mathbb{P}(A) &= 0 \quad \text{OR} \\
\mathbb{P}(A) &= 1
\end{aligned}$$

(b) Suppose that event A has probability $\mathbb{P}(A) = 1$, and the event B has some probability $\mathbb{P}(B)$. Thus the following consequences arise.

$$\begin{aligned}
\mathbb{P}(A \cap B) &= \mathbb{P}(B) \\
\mathbb{P}(A)\mathbb{P}(B) &= 1 \times \mathbb{P}(B) \\
&= \mathbb{P}(B) \\
\therefore \mathbb{P}(A \cap B) &= \mathbb{P}(A)\mathbb{P}(B)
\end{aligned}$$

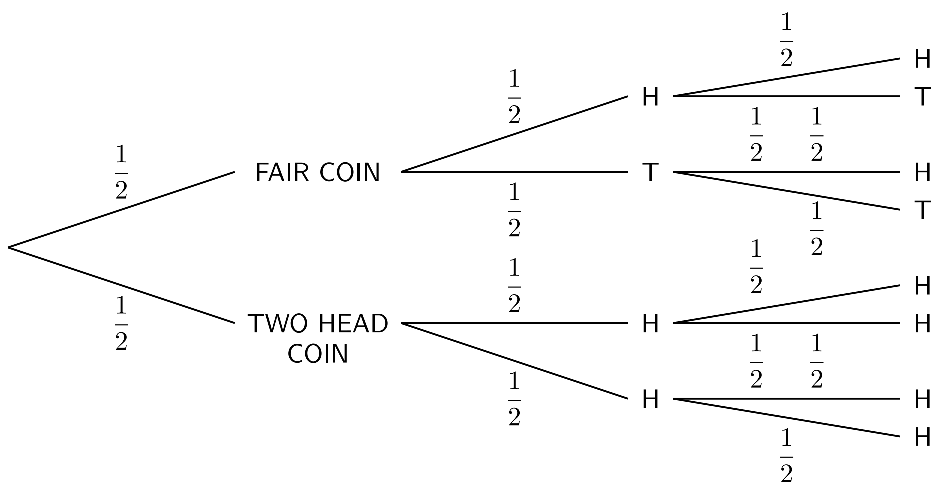
Thus if event A has probability $\mathbb{P}(A) = 1$, events A and B are independent.

Suppose now that event A has probability $\mathbb{P}(A) = 0$, and the event B has some probability $\mathbb{P}(B)$.

$$\begin{aligned}
\mathbb{P}(A \cap B) &= 0 \\
\mathbb{P}(A)\mathbb{P}(B) &= 0 \times \mathbb{P}(B) \\
&= 0 \\
\therefore \mathbb{P}(A \cap B) &= \mathbb{P}(A)\mathbb{P}(B)
\end{aligned}$$

Again, if event A has probability $\mathbb{P}(A) = 0$, events A and B are independent.

2. The following probability tree will be used to answer question 2. Furthermore, the notation FC will denote the Fair Coin.



(a)

$$
\begin{aligned}
\mathbb{P}(FC \mid H) &= \frac{\mathbb{P}(FC \cap H)}{\mathbb{P}(H)} \\
&= \frac{\frac{1}{2} \times \frac{1}{2}}{\left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times 1\right)} \\
&= \frac{\frac{1}{4}}{\frac{1}{4} + \frac{1}{2}} \\
&= \frac{1}{3}
\end{aligned}
$$

Thus the probability of choosing the fair coin given that the coin shows heads after the first flip is $\dfrac{1}{3}$

(b)

$$
\begin{aligned}
\mathbb{P}(FC \mid H, H) &= \frac{\mathbb{P}(FC \cap H, H)}{\mathbb{P}(H, H)} \\
&= \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times 1 \times 1\right)} \\
&= \frac{\frac{1}{8}}{\frac{1}{8} + \frac{1}{2}} \\
&= \frac{1}{5}
\end{aligned}
$$

Thus the probability of choosing the fair coin given that the coin shows heads after the first flip, and heads after the second flip, is $\dfrac{1}{5}$

(c)

$$
\begin{aligned}
\mathbb{P}(FC \mid H, H, T) &= \frac{\mathbb{P}(FC \cap H, H, T)}{\mathbb{P}(H, H, T)} \\
&= \frac{\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}}{\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)} \\
&= 1
\end{aligned}
$$

Thus the probability of choosing the fair coin given that the coin shows heads after the first flip, heads after the second flip, and tails after the third flip, is 1

3.

4.

5. (a)

$$\begin{aligned} F_X(x) &= \int_1^2 f_X(x)dx \\ &= 2\int_1^2 \frac{1}{x^2}dx \\ &= 2\left[\frac{-1}{x}\right]_1^2 \\ &= 2\left[\frac{-1}{2}+1\right] \\ &= 1 \end{aligned}$$

(b)

$$\begin{aligned} \mathbb{E}(X) &= \int_1^2 x f_X(x)dx \\ &= 2\int_1^2 \frac{1}{x}dx \\ &= 2\ln x \Big|_1^2 \\ &= 2[\ln 2 - \ln 1] \\ &= 2\ln 2 \end{aligned}$$

Let M be the location of the median.

$$\begin{aligned} F_X(x) &= \frac{1}{2} \\ \therefore \frac{1}{2} &= \int_1^M f_X(x)dx \\ &= 2\int_1^M \frac{1}{x^2}dx \\ &= 2\left[\frac{-1}{x}\right]_1^M \\ &= 2\left[\frac{-1}{M}+1\right] \\ &= \frac{-2}{M}+2 \\ \therefore \frac{2}{M} &= \frac{3}{2} \\ \therefore M &= \frac{4}{3} \end{aligned}$$