THE UNIVERSITY OF SYDNEY
MATH1905 Statistics Advanced

Semester 2 **Tutorial Week 04 – Solutions** 2012

1. (1) $\mathrm{P}(A) \geq 0$, for any event A;
(2) $\mathrm{P}(\Omega) = 1$;
(3) If $A_1, ..., A_n$ are mutually exclusive events then $\mathrm{P}(A_1 \cup ... \cup A_n) = \mathrm{P}(A_1) + ... + \mathrm{P}(A_n)$.

2. (a) $6/36 = 1/6$ (b) $10/36 = 5/18$ (c) $18/36 = 1/2$

3. (a) $(2/6) \times (2/6) \times (2/6) = 1/27$

(b) $\mathrm{P}(\text{same colour}) = \mathrm{P}(3W \text{ or } 3R \text{ or } 3G) \overset{\text{mut. excl.}}{=} \mathrm{P}(3W) + \mathrm{P}(3R) + \mathrm{P}(3G)$
$\overset{\text{indep.}}{=} \frac{1}{3}.\frac{1}{3}.\frac{1}{3} + \frac{1}{6}.\frac{1}{6}.\frac{1}{6} + \frac{1}{2}.\frac{1}{2}.\frac{1}{2} = \frac{1}{6}.$

4. (a) $\mathrm{P}(A \text{ fails } | B \text{ has failed}) = \dfrac{\mathrm{P}(A \text{ and } B \text{ fail})}{\mathrm{P}(B \text{ fails})}$
$\mathrm{P}(B \text{ fails}) = \mathrm{P}(A \text{ and } B \text{ fail}) + \mathrm{P}(A \text{ does not fail } \cap B \text{ fails}) = 0.15 + 0.15 = 0.3$
and so $\mathrm{P}(A \text{ fails } | B \text{ has failed}) = \dfrac{0.15}{0.3} = 1/2.$

(b) $\mathrm{P}(B \text{ does not fail } \cap A \text{ fails}) = \mathrm{P}(A \text{ fails}) - \mathrm{P}(A \text{ and } B \text{ fail}) = 0.2 - 0.15 = 0.05$

5. Answer (b), since

$$\mathrm{P}(X<4) = \mathrm{P}(X=1) + \mathrm{P}(X=2) + \mathrm{P}(X=3) = \tfrac{1}{9} + \tfrac{2}{9} + \tfrac{1}{9} = \tfrac{4}{9}.$$

6. Answer (d), since

$A = \{2, 4, 6\}; \quad B = \{3, 6\}.$
A or B is $A \cup B = \{2, 3, 4, 6\}$ and $\mathrm{P}(A \cup B) = \frac{7}{9}$.
A and B is $A \cap B = \{6\}$ and $\mathrm{P}(A \cap B) = \frac{2}{9}$.

7. (a) Number of ways of selecting 4 students among 28: $\binom{28}{4} = \dfrac{28 \times 27 \times 26 \times 25}{4 \times 3 \times 2 \times 1} = 20,475.$

Number of ways of selecting 4 male students among 16: $\binom{16}{4} = \dfrac{16.15.14.13}{4.3.2.1} = 1,820.$

Hence, $\mathrm{P}(\text{all male}) = 1,820/20,475 = 0.089 \quad (3dp)$.

(b) $\mathrm{P}(\text{at least one female}) = 1 - 1,820/20,475 = 0.911 \quad (3dp)$

8. (a) `sample(1:6,6,replace=T)`

(b) `sample(1:6,6,replace=F)`

(c) `sample(1:52,7,replace=F)`

(d) `sample(1:6,1,replace=T)`

(e) `sample(1:6,6,replace=T)`