

Semester 2	Tutorial Week 06 – Solutions	2012
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1. (a) (i) 0.9192 (ii) 0.3821 (iii) 0.5284
(iv) 0.05.
- (b) (i) $z = 1.28$ (ii) $z = -1.645$ (iii) $z = 1.645$
2. (a) 0.3085 (b) 0.8413 (c) 0.4649
(d) 0.4502 (e) 0.8664
3. $X \sim \mathcal{N}(15.4, 0.48^2)$
(a) $P(X > 16) = 0.1057$ (b) $P(X \leq 14.2) = 0.0062$ (c) $P(15 < X < 15.8) = 0.5934$
4. Let X denote intraocular pressure, $X \sim \mathcal{N}(16, 4^2)$.
(a) $P(12 \leq X \leq 20) = 0.6826$ i.e. about 68% of unaffected adults in this range.
(b) We want c such that $P(X > c) = 0.01$: In R `qnorm(1-0.01,16,4)` gives $c = 25.31$. Pressure above 25.32mm Hg are considered abnormally high.
5. (a) $1 = \int_0^\infty Cxe^{-x}dx = C[-xe^{-x} - e^{-x}]_0^\infty = C$. Thus $C = 1$.
- (b) (i) $E(X) = \int_0^\infty x^2e^{-x}dx = [x^2(-e^{-x})]_0^\infty + 2 \int_0^\infty xe^{-x}dx = 2$.
(ii) $E(X^2) = \int_0^\infty x^3e^{-x}dx = [x^3(-e^{-x})]_0^\infty + 3 \int_0^\infty x^2e^{-x}dx = 6$.
(iii) $\text{var}(x) = E(X^2) - (E(X))^2 = 2$.
- (c) $P(X > 6) = P(X - 2 > 4) \leq P(|X - 2| > 2\sqrt{2}\sigma)$ as $\sigma^2 = 2$. By Chebyshev's inequality:
$$P(|X - 2| > 2\sqrt{2}\sigma) \leq \frac{1}{(2\sqrt{2})^2} = 1/8$$
- (d) $P(X > 6) = \int_6^\infty xe^{-x}dx = [-xe^{-x} - e^{-x}]_6^\infty = 7e^{-6} = 0.0174$