Lec 8 : — wave eq'n
     — standing waves

Wave
Eq'n
(P.D.E.)
$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$y(x,t)$ is disp. of medium from equilib.
(↑ speed)

May 3-2:07 PM

---

y

Claim: any wave with small amplitude
     obeys wave eq'n.
       (interest: look eqn (∗))
One soln is sinusoidal travelling wave
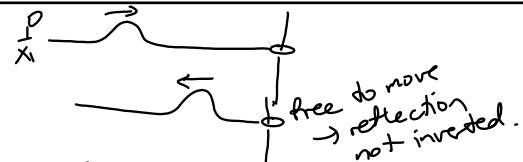$$y(x,t) = A\cos(kx - \omega t)$$
Note $v = \frac{\omega}{k}$

May 3-2:10 PM

---

we derived for stretched rope
     and showed $v = \sqrt{\frac{E}{\mu}}$

Reflection

fixed end
→ reflection inverted

May 3-2:12 PM

---

free to move
→ reflection
not inverted.

Superposition
Principle of Superposition
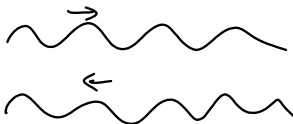   two waves in medium, total displ. is
     just sum of two displ.
True provided small amplitudes. $y_{tot} = y_1 + y_2$
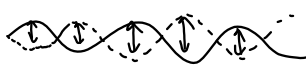
May 3-2:17 PM

— suppose two sinusoidal waves in
     opposite directions
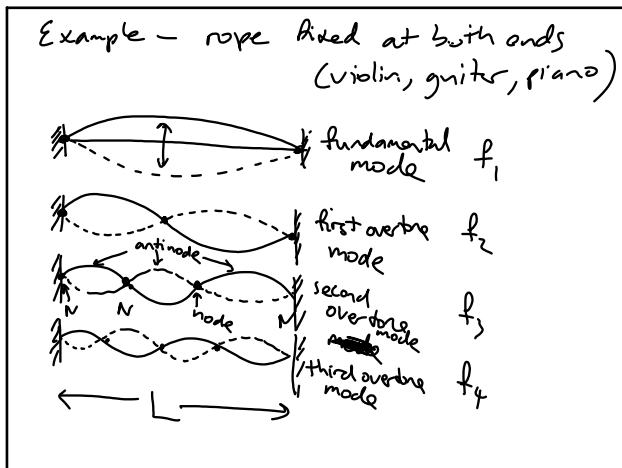
add:

Result
(see movie)

May 3-2:22 PM

---

so mathematically, can think of
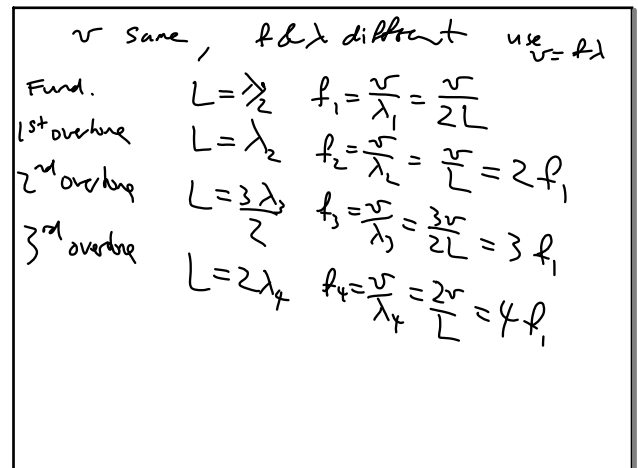standing wave as sum of oppositely
travelling sinusoidal waves.
In practice we achieve using reflection

Another way to think of standing wave
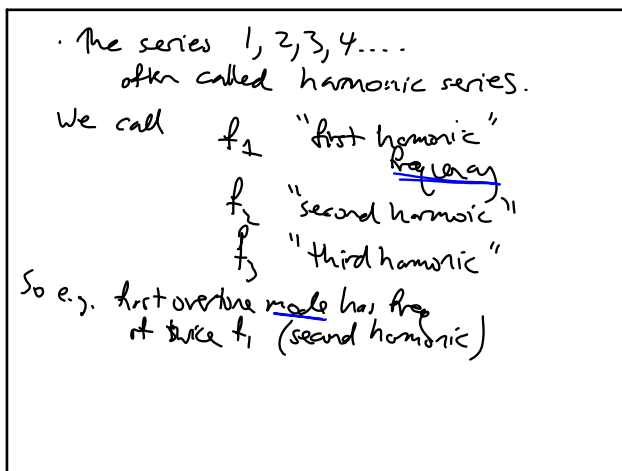is an oscillating system
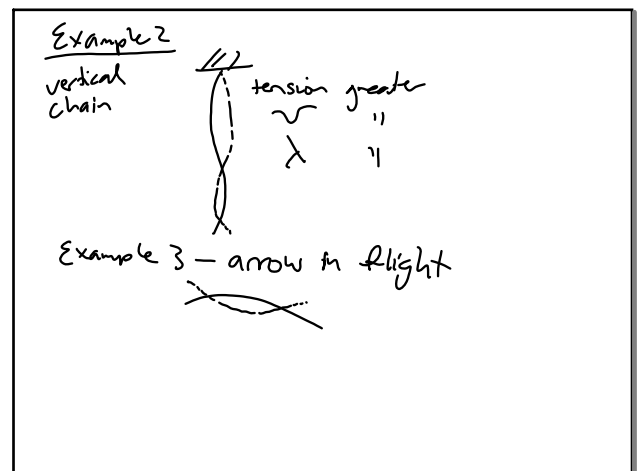   each part oscillating SHM.

May 3-2:24 PM

1

Example — rope fixed at both ends
(violin, guitar, piano)

fundamental mode f_1

first overtone mode f_2

antinode

second overtone mode f_3

N N node N

third overtone mode f_4

$\longleftarrow L \longrightarrow$

May 3-2:28 PM

---

v same, f & λ different   use $v = f\lambda$

Fund. $L = \dfrac{\lambda}{2}$ $f_1 = \dfrac{v}{\lambda_1} = \dfrac{v}{2L}$

1st overtone $L = \lambda_2$ $f_2 = \dfrac{v}{\lambda_2} = \dfrac{v}{L} = 2f_1$

2nd overtone $L = \dfrac{3\lambda_3}{2}$ $f_3 = \dfrac{v}{\lambda_3} = \dfrac{3v}{2L} = 3f_1$

3rd overtone $L = 2\lambda_4$ $f_4 = \dfrac{v}{\lambda_4} = \dfrac{2v}{L} = 4f_1$

May 3-2:34 PM

· The series $1, 2, 3, 4 \ldots$
often called harmonic series.

We call f_1 "first harmonic"
       frequency

        f_2 "second harmonic"

        f_3 "third harmonic"

So e.g. first overtone mode has freq
at twice f_1 (second harmonic)

May 3-2:38 PM

Example 2
vertical chain

tension greater

v "

λ "

Example 3 — arrow in flight

May 3-2:49 PM