THE UNIVERSITY OF SYDNEY

MATH1905 STATISTICS ADVANCED

Semester 2	**Tutorial Week 5**	2012

1. If $X \sim \mathcal{B}(10, 0.3)$ use the binomial formula and your calculator to compute $\mathrm{P}(X = 2)$, $\mathrm{P}(X = 1)$, $\mathrm{P}(X = 0)$. Use these values to compute $\mathrm{P}(X \leq 3)$. Check your answer with the `pbinom()` R-function.

2. Each child born to a particular set of parents has probability 0.25 of having blood type O. If these parents have 5 children, what is the probability that exactly two of them have blood type O?

3. (Using R) If $X \sim \mathcal{B}(8, 0.4)$ the value of $\mathrm{P}(X < 6)$ is to 4dp

(a) 0.9502 (b) 0.9915 (c) 0.8936 (d) 0.0498 (e) none of these.

4. A six-sided fair die is thrown 7 times what is the probability that we observe at least three 6's ?

5. If $X \sim \mathcal{P}(0.5)$ use the Poisson distribution formula to compute $\mathrm{P}(X = 2)$, $\mathrm{P}(X = 1)$, $\mathrm{P}(X = 0)$. Use these values to compute $\mathrm{P}(X \leq 2)$. Check your answer with the `ppois()` R-function.

6. Suppose that an office receives telephone calls as a Poisson distribution with mean $\lambda = 0.5$ per min. What is the probability of receiving exactly 1 call during a 1 minute interval? What is the probability of receiving no call during a 1' interval? The number of calls in a 5' interval (also) follows a Poisson distribution with $\lambda = 5 \times 0.5$. What is the probability of receiving no call during a 5 minute interval?

7. Use `ppois()` R-function: if $X \sim \mathcal{P}$ with mean 5 the value of $\mathrm{P}(X > 4)$ is to 4dp

(a) 0.2650 (b) 0.4405 (c) 0.5595 (d) 0.0067 (e) none of these.

8. The following table shows the probability distribution of X, where $p_i = \mathrm{P}(X = i)$.

i	1	2	3	4	Total
p_i	0.35	0.30	0.25	0.10	1.00

Find $\mathrm{E}(X)\,, \mathrm{E}\left(\dfrac{1}{X}\right)\,, \mathrm{E}(X^2)\,, \mathrm{Var}(X)\,.$

9. Let Y be a random variable with a geometric distribution with parameter p. The probability distribution is $\mathrm{P}(Y = i) = q^i p, \quad i = 0, 1, 2, 3, \ldots,$ where $q = 1 - p$.

(a) Determine the probability generating function for Y.

(b) Hence find the expected value of Y.

(c) A six-sided fair die is thrown until a 3 is observed. What is the probability that 6 throws are required? What is the probability that more than 7 throws are required?

10. If X has mean 3 and variance 25 write down the mean and variance of

(a) $Y = 2X$

(b) $Y = X + 2$

(c) $Y = \dfrac{X - 3}{5}$.

11. Roll a fair die 100 times (6 sides). Let X be the number of fives obtained. What is the probability that $X < 21$?

Assignment 1 for MATH1905 STATISTICS (due on Tuesday, 2nd October, in week 9) will consist of selected questions from the Problem Sheets for weeks 1, 2, 3, 4, 5, 6, 7, 8.

1. (a) Let $p_i = \mathrm{P}(X = i) = \dfrac{1}{n}$, $i = 1, 2, 3, \ldots, n$.

(i) Check that $\sum_{i=1}^{n} p_i = 1$.

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

[Hint: $\sum_{i=1}^{n} i = \dfrac{n}{2}(n+1)$, $\sum_{i=1}^{n} i^2 = \dfrac{n}{6}(n+1)(2n+1)$.]

(b) If $p_i = \mathrm{P}(X = i) = C \times i$, $i = 1, 2, 3, \ldots, n$, find C.

2. The random variable X has probability distribution given by

i	0	1	2
p_i	0.25	0.5	0.25

(a) Calculate $\mathrm{E}(X)$ and $\mathrm{Var}(X)$.

(b) Find the probability generating function for X. Use it to verify the results in the previous part.

3. A (fair) six sided die is thrown until it shows a three. What is the probability that only 4 throws are required? What is the probability that more than 4 throws are required?

4. Using R, create a vector `x` of 1000 observations of a poisson distribution with mean 3. Summarise your data `x` with a frequency table. Create a vector `y` of 1000 observations of a $\mathcal{B}(1000, 3/1000)$. Summarise your data (`y`) with a frequency table. Type `par(mfrow=c(1,2))` to set your graphic window with 2 plots. Plot the two ordinate diagrams (one for `x` and one for `y`) side by side. Comment on the plot.