PHYS1901 Physics 1 (Advanced) Formula Sheet

Vectors

$\vec{\mathbf{A}} = A_x\hat{\mathbf{i}} + A_y\hat{\mathbf{j}} + A_z\hat{\mathbf{k}}$

$A = \left|\vec{\mathbf{A}}\right| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} = \vec{\mathbf{B}} + \vec{\mathbf{A}}$

$R_x = A_x + B_x,\ R_y = A_y + B_y,\ R_z = A_z + B_z$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB\cos\phi = \left|\vec{\mathbf{A}}\right|\left|\vec{\mathbf{B}}\right|\cos\phi$

$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_xB_x + A_yB_y + A_zB_z$

$\vec{\mathbf{C}} = \vec{\mathbf{A}} \times \vec{\mathbf{B}}, \quad C = AB\sin\phi$

$C_x = A_yB_z - A_zB_y, \quad C_y = A_zB_x - A_zB_z,$

$C_z = A_xB_y - A_yB_x$

Kinematics

$\vec{\mathbf{r}} = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$

$$\vec{\mathbf{v}}_{\text{av}} = \frac{\vec{\mathbf{r}}_2 - \vec{\mathbf{r}}_1}{t_2 - t_1} = \frac{\Delta\vec{\mathbf{r}}}{\Delta t}$$

$$\vec{\mathbf{v}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{r}}}{\Delta t} = \frac{d\vec{\mathbf{r}}}{dt}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt}$$

$$\vec{\mathbf{a}}_{\text{av}} = \frac{\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1}{t_2 - t_1} = \frac{\Delta\vec{\mathbf{v}}}{\Delta t}$$

$$\vec{\mathbf{a}} = \lim_{\Delta t\to 0}\frac{\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{d\vec{\mathbf{v}}}{dt}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}$$

Mechanics

$\sum\vec{\mathbf{F}} = m\vec{\mathbf{a}},\ w = mg,\ \vec{\mathbf{F}}_{\text{A on B}} = -\vec{\mathbf{F}}_{\text{B on A}}$

$f_k = \mu_k n, \quad f_s \leqslant \mu_s n$

$$\vec{\mathbf{J}} = \vec{\mathbf{p}}_2 - \vec{\mathbf{p}}_1 = \int_{t_1}^{t_2}\sum\vec{\mathbf{F}}\,dt, \qquad M = \sum_i m_i$$

$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}, \quad \sum\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}, \quad \sum\vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{cm}}$$

$\vec{\mathbf{P}} = m_1\vec{\mathbf{v}}_1 + m_2\vec{\mathbf{v}}_2 + m_3\vec{\mathbf{v}}_3 + \ldots = M\vec{\mathbf{v}}_{\text{cm}}$

$$\vec{\mathbf{r}}_{\text{cm}} = \frac{\sum_i m_i\vec{\mathbf{r}}_i}{\sum_i m_i} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2 + m_3\vec{\mathbf{r}}_3 + \ldots}{m_1 + m_2 + m_3 + \ldots}$$

Simple motions

Constant acceleration in one direction:

$v = v_0 + at$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$v^2 = v_0^2 + 2a(x - x_0)$

$$x - x_0 = \left(\frac{v_0 + v}{2}\right)t$$

Projectile motion:

$x = (v_0\cos\alpha_0)t$

$$y = (v_0\sin\alpha_0)t - \frac{1}{2}gt^2$$

$v = v_0\cos\alpha_0$

$v_y = v_0\sin\alpha_0 - gt$

Uniform circular motion:

$$a_{\text{rad}} = \frac{v^2}{R} = \omega^2 R = \frac{4\pi^2 R}{T^2}$$

Work and Energy

$$W = \vec{\mathbf{F}} \cdot \vec{\mathbf{s}} = Fs\cos\phi$$

$$K = \frac{1}{2}mv^2, \quad U = mgy$$

$$W_{\text{tot}} = K_2 - K_1 = \Delta K$$

$$W = \int_{x_1}^{x_2} F_x\,dx$$

$$W = \int_{P_1}^{P_2} F\cos\phi\,dl = \int_{P_1}^{P_2} F_{\parallel}\,dl = \int_{P_1}^{P_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{l}}$$

$$P_{\text{av}} = \frac{\Delta W}{\Delta t}$$

$$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

$$W_{\text{el}} = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2 = -\Delta Uel$$

$$W_{\text{grav}} = mgy_1 - mgy_2 = -\Delta Ugrav$$

$$E = K + U$$

$$F_x(x) = -\frac{dU_x}{dx}$$

$$\vec{\mathbf{F}} = -\left(\frac{\partial U}{\partial x}\hat{\mathbf{i}} + \frac{\partial U}{\partial y}\hat{\mathbf{j}} + \frac{\partial U}{\partial z}\hat{\mathbf{k}}\right)$$

Periodic Motion

$$\omega = 2\pi f = \frac{2\pi}{T}, \quad f = \frac{\omega}{2\pi} = \frac{1}{T}$$

$$\omega = \sqrt{\frac{k}{m}}, \quad \omega = \sqrt{\frac{\kappa}{I}}$$

$$\omega = \sqrt{\frac{g}{L}}, \quad \omega = \sqrt{\frac{mgd}{I}}$$

$$F_x = -kx, \quad a_x = \frac{F_x}{m}$$

$$x = A\cos(\omega t + \phi)$$

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2 = \text{constant}$$

$$x = Ae^{-(b/2m)t}\cos\omega' t, \quad \omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$b_{\text{critical}} = 2\sqrt{k\,m}$$

$$A = \frac{F_{\text{max}}}{\sqrt{(k - m\omega_d^2)^2 + b^2\omega_d^2}}$$

Rotational Motion

$$\omega_z = \lim_{x \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$$

$$\alpha_z = \lim_{x \to 0} \frac{\Delta\omega_z}{\Delta t} = \frac{d\omega_z}{dt} = \frac{d^2\theta}{dt^2}$$

$$a_{\text{rad}} = \frac{v^2}{r} = \omega^2 r, \quad a_{\text{tan}} = \frac{dv}{dt} = r\frac{d\omega}{dt} = r\alpha_z$$

$$v = r\omega_z, \quad I_{\text{P}} = I_{\text{cm}} + Md^2, \quad v_{\text{cm}} = R\omega$$

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots = \sum_i m_i r_i^2$$

$$\tau = F\,l = rF\sin\theta, \quad \vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}$$

$$\sum \tau_z = I\alpha_z, \quad \sum \vec{\boldsymbol{\tau}} = \frac{d\vec{\mathbf{L}}}{dt}$$

$$K = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega_z^2, \quad P = \tau_z\omega_z$$

$$W = \int_{\theta_1}^{\theta_2} \tau_z\,d\theta, \quad W_{\text{tot}} = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2$$

$$\vec{\mathbf{L}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}} = \vec{\mathbf{r}} \times m\vec{\mathbf{v}} \quad \text{(particle)}$$

$$\vec{\mathbf{L}} = I\vec{\boldsymbol{\omega}} \quad \text{(rigid body)}$$

$$I = MR^2 \quad \text{(thin hollow cylinder)}$$

$$I = \frac{1}{2}MR^2 \quad \text{(solid cylinder)}$$

$$I = \frac{2}{5}MR^2 \quad \text{(solid sphere)}$$

Newtonian Gravitation

$$F_g = \frac{Gm_1 m_2}{r^2}, \quad g = \frac{Gm_E}{R_E^2}$$

$$w = F_g = \frac{Gm_E m}{R_E^2}, \quad v = \sqrt{\frac{Gm_E}{r}}$$

$$U = -\frac{Gm_E m}{r}, \quad R_S = \frac{2GM}{c^2}$$

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{Gm_E}}$$

Thermal physics

$\Delta L = \alpha L_0 \Delta T, \quad \Delta V = \beta V_0 \Delta T$

$Q = mc\Delta T, \quad Q = nC\Delta T, \quad Q = \pm mL$

$pV = nRT = NkT \qquad N = nN_A$

$C_V = \frac{3}{2}R$ (ideal monatomic gas)

$C_V = \frac{5}{2}R$ (ideal diatomic gas)

$C_V = 3R$ (ideal monatomic solid)

$C_P = C_V + R, \quad \gamma = \frac{C_P}{C_V}$

$v_{\text{rms}} = \sqrt{\frac{3RT}{M}}$

$m_{\text{tot}} = nM = nN_A m$

$e = \frac{W}{Q_H} = 1 + \frac{Q_C}{Q_H} = 1 - \left|\frac{Q_C}{Q_H}\right|$

$e_{\text{Carnot}} = 1 - \frac{T_C}{T_H} = \frac{T_H - T_C}{T_H}$

$K_{\text{Carnot}} = \frac{T_C}{T_H - T_C}$

$e_{\text{Otto}} = 1 - \frac{1}{r^{\gamma-1}}$

$K_{\text{tr}} = \frac{3}{2}nRT, \quad \frac{1}{2}m(v^2)_{\text{av}} = \frac{3}{2}kT$

$W = \int_{V_1}^{V_2} p\,dV, \quad \Delta U = Q - W$

$dU = dQ - dW$ (infinitesimal process)

$K = \frac{|Q_C|}{|W|} = \frac{|Q_C|}{|Q_H| - |Q_C|}$

$\Delta S = \int_1^2 \frac{dQ}{T}$(reversible process), $S = k \ln w$

$H = \frac{dQ}{dt} = kA\frac{T_H - T_C}{L}, \; H_{\text{net}} = Ae\sigma(T^4 - T_s^4)$

$$\begin{aligned} W &= nC_V(T_1 - T_2) \\ &= \frac{C_V}{R}(p_1V_1 - p_2V_2) \quad \text{(adiabatic process, ideal gas)} \\ &= \frac{1}{\gamma - 1}(p_1V_1 - p_2V_2) \end{aligned}$$

$W = nRT \ln\left(\frac{V_2}{V_1}\right)$

Reversible Processes for Ideal Gases:

Adiabatic (no heat transfer):

$Q = 0, \quad pV^\gamma =$ constant

Isochoric (constant volume): $W = 0$

Isobaric (constant pressure): $W = p(V_2 - V_1)$

Isothermal (constant temperature)

Mechanical waves

$v = \lambda f, \quad k = \frac{2\pi}{\lambda}$

$\omega = 2\pi f = vk, \quad v = \sqrt{\frac{F}{\mu}}$

$y(x,t) = A\cos(kx \pm \omega t)$

$y(x,t) = (A_{\text{SW}} \sin kx) \sin \omega t$ (standing wave)

String fixed at both ends:

$f_n = n\frac{v}{2L} = nf_1 \quad (n = 1, 2, 3, \ldots)$

$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

$\beta = (10\text{ dB}) \log \frac{I}{I_0}$

Longitudinal sound waves

$v = \sqrt{\frac{B}{\rho}}$ (fluid), $v = \sqrt{\frac{Y}{\rho}}$ (solid rod)

$v = \sqrt{\frac{\gamma RT}{M}}$ (ideal gas)

$f_n = \frac{nv}{2L} \quad (n = 1, 2, 3, \ldots)$ (open pipe)

$f_n = \frac{nv}{4L} \quad (n = 1, 3, 5, \ldots)$ (stopped pipe)

$f_L = \frac{v + v_L}{v + v_S} f_s, \qquad \sin\alpha = \frac{v}{v_S}$

$f_{\text{beat}} = |f_a - f_b|$