

PHYS 1901 – Physics 1A (Advanced) Mechanics module



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9

Dynamics of Rotational Motion

Chapter

10

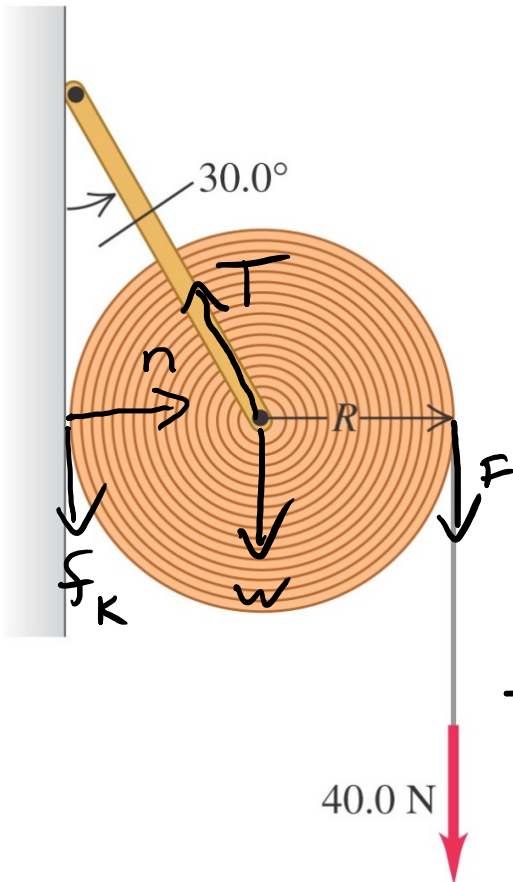
Problem 10.63

Large roll of paper, mass 16.0 kg, radius $R = 18.0$ cm.

Coefficient of kinetic friction with wall: $\mu_k = 0.25$

a) What is the force that the rod exerts on the paper as it unrolls?

b) What is the angular acceleration of the roll?



a) $\vec{F}_{\text{net}} = 0$

$$\sum F_{\text{vert}} = 0 \Rightarrow T \cos 30^\circ = W + F + \mu_k n$$

$$\sum F_{\text{hor}} = 0 \Rightarrow T \sin 30^\circ = n$$

Eliminate n

$$T \cos 30^\circ = W + F + \mu_k T \sin 30^\circ$$

$$T = \frac{W + F}{\cos 30^\circ - \mu_k \sin 30^\circ}$$

$$= 266 \text{ N}$$

b) $\vec{\tau}_{\text{net}} = I \vec{\alpha}$
What is the net torque?

$$\vec{\tau}_{\text{net}} = \vec{\tau}_F + \vec{\tau}_{f_k}$$

$$= RF - Rf_k$$

$$= 1.2 \text{ N}\cdot\text{m} \text{ into board}$$

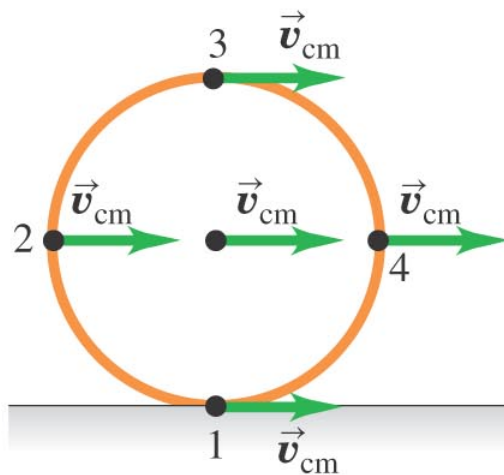
$$I = \frac{1}{2} MR^2$$

$$\vec{\alpha} = 4.68 \text{ rad/s}^2 \text{ into board}$$

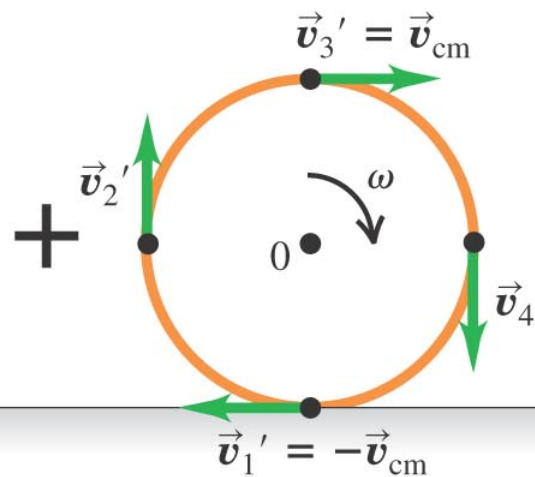


Rolling without slipping

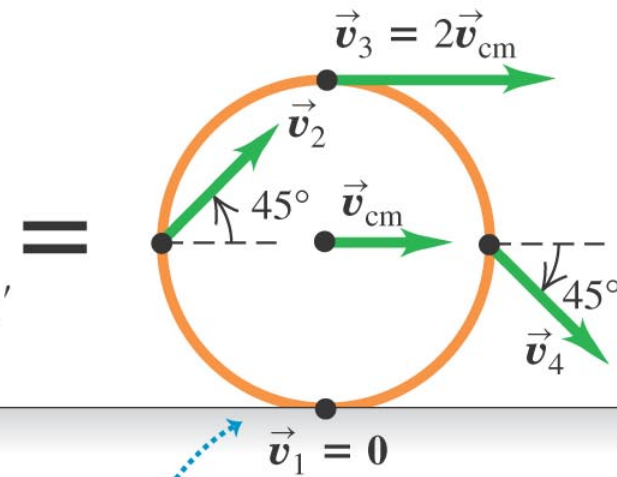
Translation of the center of mass of the wheel: velocity \vec{v}_{cm}



Rotation of the wheel around the center of mass: for rolling without slipping, the speed at the rim must be v_{cm} .



Combination of translation and rotation: rolling without slipping



Wheel is instantaneously at rest where it contacts the ground.

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For a rolling wheel which does not slide, then the distance it travels is related to how much it turns.

$$v_{cm} = R \omega$$

Rolling without slipping

The total kinetic energy is

$$K = K_{\text{linear}} + K_{\text{rot}} \\ = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I \omega^2$$

Rolling without slipping $v_{\text{cm}} = R\omega$

$$K = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} I (\omega_{\text{cm}}/R)^2$$

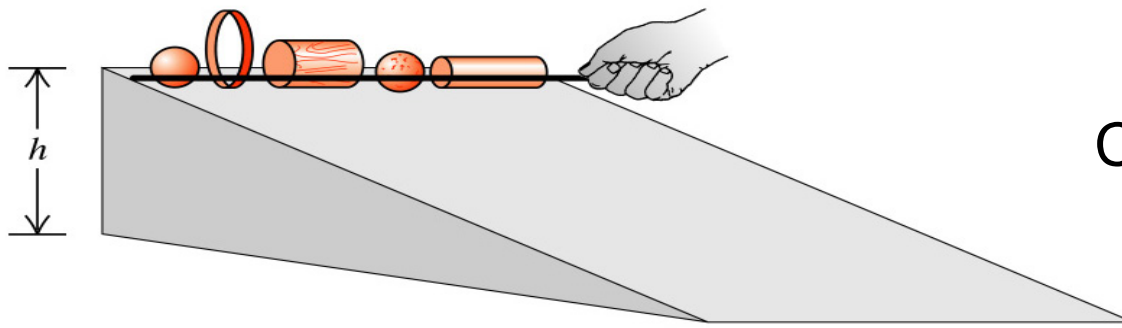
Generally $I = c m R^2$ for some constant $c \leq 1$

$$K = \frac{1}{2} m v_{\text{cm}}^2 + \frac{1}{2} c m v_{\text{cm}}^2 \\ = \frac{1}{2} (1 + c) m v_{\text{cm}}^2$$



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Rolling without slipping



Conservation of energy

$$\Delta K = W_g = -\Delta U_g = mgh$$

Starting from rest +

$$v_{cm} = \sqrt{\frac{2gh}{1+c}}$$

$$\Leftarrow K_{\text{f}} = \frac{1}{2}(1+c)mv_{cm}^2 = mgh$$

☐ Independent of mass & size

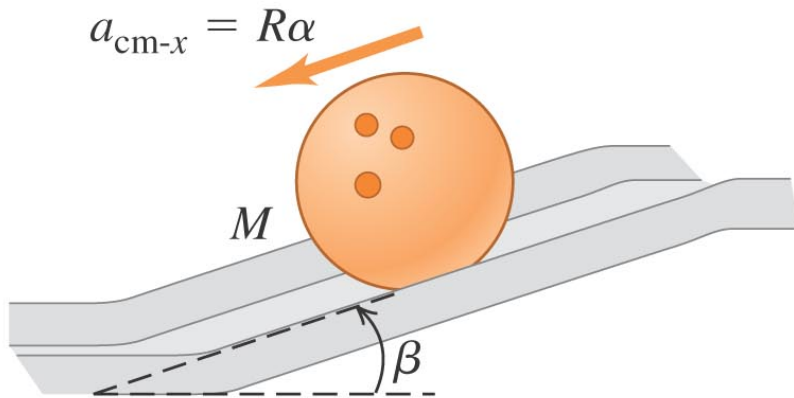
☐ Any sphere beats any hoop!

What is the source of torque?

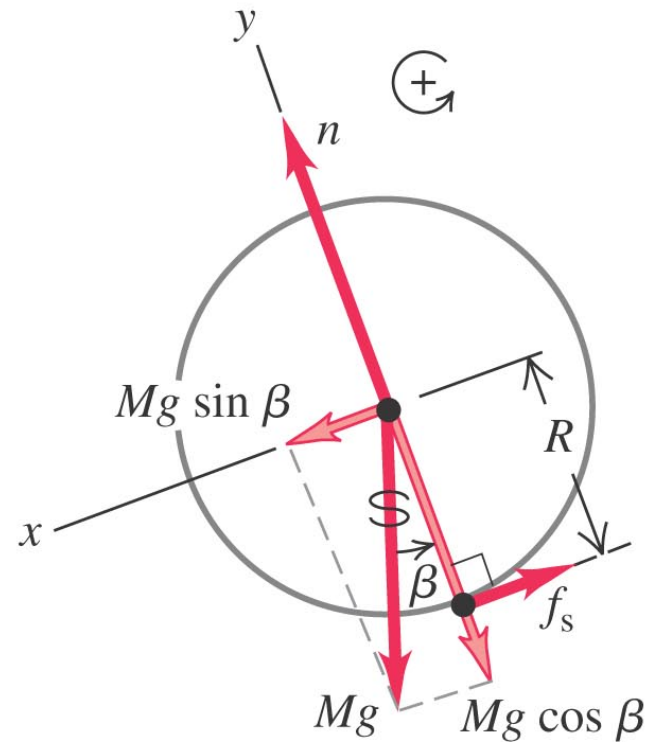


Rolling without slipping

(a) The bowling ball

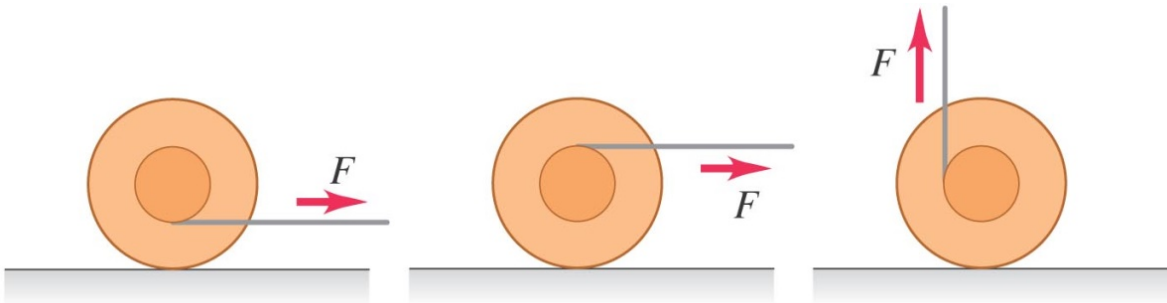


(b) Free-body diagram for the bowling ball





Problem 10.71



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Three yo-yo initially at rest

A string is pulled as shown

There is sufficient friction so that the yo-yos roll without slipping.

In which direction will each yo-yo rotate?