# Fractals

## Cantor Set

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A fractal is a curve or geometric figure that is bounded, continuous and nowhere differentiable in some region $R$.

# Examples

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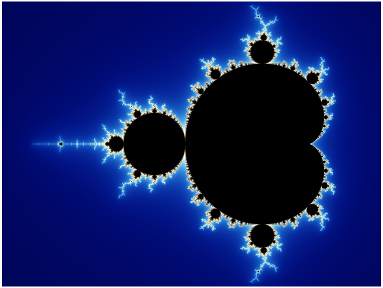

Figure: Mandlebrot Set

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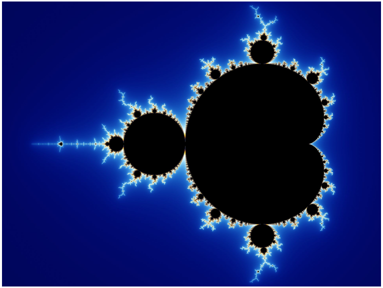

Figure: Mandlebrot Set

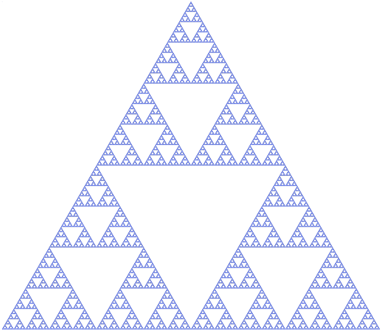

Figure: Serpinski Triangle

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1. Start with a curve, or image.
2. Apply a sequence of transformations that are repeatable to the curve, or image.
3. Repeat ad infinitum.

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**Proof.**

Consider the end points of the closed intervals at each step. These points remain, proving the Cantor Set is non-empty.
Consider now the length of the open interval removed at step $n$. The length of such an interval is $3^n$. At step $n$, $2^{n-1}$ intervals are removed, and so the length of intervals removed is,

$$\sum_{n=1}^{\infty} \frac{2^{n-1}}{3^n} = \frac{1}{3} \sum_{k=0}^{\infty} \frac{2^k}{3^k} = \frac{1}{3} \frac{1}{1-\frac{2}{3}} = 1$$

□

# Cantor Set cont.

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The self-similarity dimension of a fractal is given by

$$\frac{\log N}{\log \frac{1}{r}}$$

where $N$ is the number of self-similar copies, and $r$ is the scaling factor of the self-similar copies.

# Cantor Set cont.

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## Property

The topological dimension of the Cantor Set is 0, as it contains no intervals of non-zero length. The self-similarity dimension of the Cantor Set is $\dfrac{\log 2}{\log 3}$. Thus, the Cantor Set is a fractal.

# Consequences

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- Countably infinite sets and their cardinality.
- Cardinality arithmetic.
- Modelling of nature and natural phenomena.
- Computer graphics and simulations.

# References

Cirstea, F. (2017). Fractals.
Cirstea, F. (2017). Cantor Set.