YEAR 11

CAMBRIDGE Mathematics

3 Unit Extension 1 Enhanced

BILL PENDER

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Preface

This textbook has been written for students in Years 11 and 12 taking the course previously known as '3 Unit Mathematics', but renamed in the new HSC as two courses, 'Mathematics' (previously called '2 Unit Mathematics') and 'Mathematics, Extension 1'. The book develops the content at the level required for the 2 and 3 Unit HSC examinations. There are two volumes — the present volume is roughly intended for Year 11, and the second for Year 12. Schools will, however, differ in their choices of order of topics and in their rates of progress.

Although these Syllabuses have not been rewritten for the new HSC, there has been a gradual shift of emphasis in recent examination papers.

- The interdependence of the course content has been emphasised.
- Graphs have been used much more freely in argument.
- Structured problem solving has been expanded.
- There has been more stress on explanation and proof.

This text addresses these new emphases, and the exercises contain a wide variety of different types of questions.

There is an abundance of questions in each exercise — too many for any one student — carefully grouped in three graded sets, so that with proper selection the book can be used at all levels of ability. In particular, those who subsequently drop to 2 Units of Mathematics, and those who in Year 12 take 4 Units of Mathematics, will both find an appropriate level of challenge. We have written a separate book, also in two volumes, for the 2 Unit 'Mathematics' course alone.

We would like to thank our colleagues at Sydney Grammar School and Newington College for their invaluable help in advising us and commenting on the successive drafts, and for their patience in the face of some difficulties in earlier drafts. We would also like to thank the Head Masters of Sydney Grammar School and Newington College for their encouragement of this project, and Peter Cribb and the team at Cambridge University Press, Melbourne, for their support and help in discussions. Finally, our thanks go to our families for encouraging us, despite the distractions it has caused to family life.

Preface to the enhanced version

To provide students with practice for the new objective response (multiple choice) questions to be included in HSC examinations, online self-marking quizzes have been provided for each chapter, on Cambridge GO (access details can be found in the following pages). In addition, an interactive textbook version is available through the same website.

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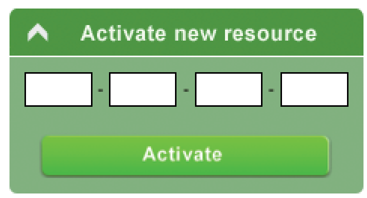

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How to Use This Book

This book has been written so that it is suitable for the full range of 3 Unit students, whatever their abilities and ambitions. The book covers the 2 Unit and 3 Unit content without distinction, because 3 Unit students need to study the 2 Unit content in more depth than is possible in a 2 Unit text. Nevertheless, students who subsequently move to the 2 Unit course should find plenty of work here at a level appropriate for them.

The Exercises: No-one should try to do all the questions! We have written long exercises so that everyone will find enough questions of a suitable standard — each student will need to select from them, and there should be plenty left for revision. The book provides a great variety of questions, and representatives of all types should be selected.

Each chapter is divided into a number of sections. Each of these sections has its own substantial exercise, subdivided into three groups of questions:

FOUNDATION: These questions are intended to drill the new content of the section at a reasonably straightforward level. There is little point in proceeding without mastery of this group.

DEVELOPMENT: This group is usually the longest. It contains more substantial questions, questions requiring proof or explanation, problems where the new content can be applied, and problems involving content from other sections and chapters to put the new ideas in a wider context. Later questions here can be very demanding, and Groups 1 and 2 should be sufficient to meet the demands of all but exceptionally difficult problems in 3 Unit HSC papers.

EXTENSION: These questions are quite hard. Some are algebraically challenging, some establish a general result beyond the theory of the course, some make difficult connections between topics or give an alternative approach, some deal with logical problems unsuitable for the text of a 3 Unit book. Students taking the 4 Unit course should attempt some of these.

The Theory and the Worked Exercises: The theory has been developed with as much rigour as is appropriate at school, even for those taking the 4 Unit course. This leaves students and their teachers free to choose how thoroughly the theory is presented in a particular class. It can often be helpful to learn a method first and then return to the details of the proof and explanation when the point of it all has become clear.

The main formulae, methods, definitions and results have been boxed and numbered consecutively through each chapter. They provide a summary only, and represent an absolute minimum of what should be known. The worked examples have been chosen to illustrate the new methods introduced in the section, and should be sufficient preparation for the questions of the following exercise.

The Order of the Topics: We have presented the topics in the order we have found most satisfactory in our own teaching. There are, however, many effective orderings of the topics, and the book allows all the flexibility needed in the many different situations that apply in different schools (apart from the few questions that provide links between topics).

The time needed for the algebra in Chapter One will depend on students' experiences in Years 9 and 10. The same applies to other topics in the early chapters — trigonometry, quadratic functions, coordinate geometry and particularly curve sketching. The Study Notes at the start of each chapter make further specific remarks about each topic.

We have left Euclidean geometry and polynomials until Year 12 for two reasons. First, we believe as much calculus as possible should be developed in Year 11, ideally including the logarithmic and exponential functions and the trigonometric functions. These are the fundamental ideas in the course, and it is best if Year 12 is used then to consolidate and extend them (and students subsequently taking the 4 Unit course particularly need this material early). Secondly, the Years 9 and 10 Advanced Course already develops much of the work on polynomials and Euclidean geometry in Options recommended for those proceeding to 3 Unit, so that revisiting them in Year 12 with the extensions and far greater sophistication required seems an ideal arrangement.

The Structure of the Course: Recent examination papers have included longer questions combining ideas from different topics, thus making clear the strong interconnections amongst the various topics. Calculus is the backbone of the course, and the two processes of differentiation and integration, inverses of each other, dominate most of the topics. We have introduced both processes using geometrical ideas, basing differentiation on tangents and integration on areas, but the subsequent discussions, applications and exercises give many other ways of understanding them. For example, questions about rates are prominent from an early stage.

Besides linear functions, three groups of functions dominate the course:

THE QUADRATIC FUNCTIONS: These functions are known from earlier years. They are algebraic representations of the parabola, and arise naturally in situations where areas are being considered or where a constant acceleration is being applied. They can be studied without calculus, but calculus provides an alternative and sometimes quicker approach.

THE EXPONENTIAL AND LOGARITHMIC FUNCTIONS: Calculus is essential for the study of these functions. We have chosen to introduce the logarithmic function first, using definite integrals of the reciprocal function $y = 1/x$. This approach is more satisfying because it makes clear the relationship between these functions and the rectangular hyperbola $y = 1/x$, and because it gives a clear picture of the new number e. It is also more rigorous. Later, however, one can never overemphasise the fundamental property that the exponential function with base e is its own derivative — this is the reason why these functions are essential for the study of natural growth and decay, and therefore occur in almost every application of mathematics.

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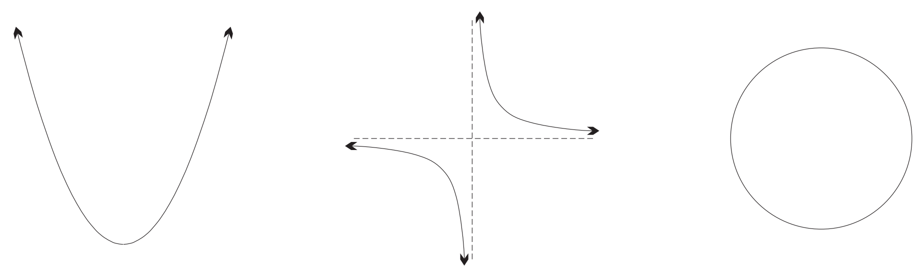
Arithmetic and geometric sequences arise naturally throughout the course. They are the values, respectively, of linear and exponential functions at integers, and these interrelationships need to be developed, particularly in the context of applications to finance.

The Trigonometric Functions: Again, calculus is essential for the study of these functions, whose definition, like the associated definition of π, is based on the circle. The graphs of the sine and cosine functions are waves, and they are essential for the study of all periodic phenomena — hence the detailed study of simple harmonic motion in Year 12.

Thus the three basic functions of the course — x^2, e^x and $\sin x$ — and the related numbers e and π are developed from the three most basic degree 2 curves — the parabola, the rectangular hyperbola and the circle. In this way, everything in the course, whether in calculus, geometry, trigonometry, coordinate geometry or algebra, is easily related to everything else.



The geometry of the circle is mostly studied using Euclidean methods, and the highly structured arguments used here contrast with the algebraic arguments used in the coordinate geometry approach to the parabola. In the 4 Unit course, the geometry of the rectangular hyperbola is given special consideration in the context of a coordinate geometry treatment of general conics.

Polynomials are a generalisation of quadratics, and move the course a little beyond the degree 2 phenomena described above. The particular case of the binomial theorem then becomes the bridge from elementary probability using tree diagrams to the binomial distribution with all its practical applications. Unfortunately the power series that link polynomials with the exponential and trigonometric functions are too sophisticated for a school course. Projective geometry and calculus with complex numbers are even further removed, so it is not really possible to explain that exponential and trigonometric functions are the same thing, although there are many clues.

Algebra, Graphs and Language: One of the chief purposes of the course, stressed in recent examinations, is to encourage arguments that relate a curve to its equation. Being able to predict the behaviour of a curve given only its equation is a constant concern of the exercises. Conversely, the behaviour of a graph can often be used to solve an algebraic problem. We have drawn as many sketches in the book as space allowed, but as a matter of routine, students should draw diagrams for almost every problem they attempt. It is because sketches can so easily be drawn that this type of mathematics is so satisfactory for study at school.

This course is intended to develop simultaneously algebraic agility, geometric intuition, and rigorous language and logic. Ideally then, any solution should

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display elegant and error-free algebra, diagrams to display the situation, and clarity of language and logic in argument.

Theory and Applications: Elegance of argument and perfection of structure are fundamental in mathematics. We have kept to these values as far as is reasonable in the development of the theory and in the exercises. The application of mathematics to the world around us is an equally fundamental, and we have given many examples of the usefulness of everything in the course. Calculus is particularly suitable for presenting this double view of mathematics.

We would therefore urge the reader sometimes to pay attention to the details of argument in proofs and to the abstract structures and their interrelationships, and at other times to become involved in the interpretation provided by the applications.

Limits, Continuity and the Real Numbers: This is a first course in calculus, geometrically and intuitively developed. It is not a course in analysis, and any attempt to provide a rigorous treatment of limits, continuity or the real numbers would be quite inappropriate. We believe that the limits required in this course present little difficulty to intuitive understanding — really little more is needed than $\lim\limits_{x\to\infty} 1/x = 0$ and the occasional use of the sandwich principle in proofs. Characterising the tangent as the limit of the secant is a dramatic new idea, clearly marking the beginning of calculus, and quite accessible. Continuity and differentiability need only occasional attention, given the well-behaved functions that occur in the course. The real numbers are defined geometrically as points on the number line, and provided that intuitive ideas about lines are accepted, everything needed about them can be justified from this definition. In particular, the intermediate value theorem, which states that a continuous function can only change sign at a zero, is taken to be obvious.

These unavoidable gaps concern only very subtle issues of ‘foundations’, and we are fortunate that everything else in the course can be developed rigorously so that students are given that characteristic mathematical experience of certainty and total understanding. This is the great contribution that mathematics brings to all our education.

Technology: There is much discussion, but little agreement yet, about what role technology should play in the mathematics classroom and which calculators or software may be effective. This is a time for experimentation and diversity. We have therefore given only a few specific recommendations about technology, but we encourage such investigation, and to this version we have added some optional technology resources that can be accessed via the Cambridge GO website. The graphs of functions are at the centre of the course, and the more experience and intuitive understanding students have, the better able they are to interpret the mathematics correctly. A warning here is appropriate — any machine drawing of a curve should be accompanied by a clear understanding of why such a curve arises from the particular equation or situation.

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The mathematician's patterns, like the painter's or the poet's,
must be beautiful. The ideas, like the colours or the words,
must fit together in a harmonious way. Beauty is the first test.

— The English mathematician G. H. Hardy (1877–1947)

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