THE UNIVERSITY OF SYDNEY

MATH1905 STATISTICS ADVANCED

Semester 2 **Tutorial Week 4** 2012

1. Write down the 3 axioms of Probability without looking in your notes.

2. Two unbiased dice are thrown. Write out the 36 outcomes to which equal probability are assigned and use counting methods to find the probability that the total showing

(a) exceeds 9 (b) is at most 5 (c) is odd.

3. A "fair" die has two of its faces white, one red and three green. It is thrown three times. Use probability rules to find the probability that

(a) a white face is uppermost at each throw (Hint: The throws are independent)

(b) the same colour is uppermost at each throw (Hint: Express this outcome as the *union* of three mutually exclusive events)

4. An electronic assembly consists of two subsystems A and B, say. From previous testing procedures the following probabilities are assumed to be known:

$\mathrm{P}(A \text{ fails}) = 0.20$, $\mathrm{P}(A \text{ and } B \text{ fail}) = 0.15$, $\mathrm{P}(B \text{ fails and } A \text{ does not fail}) = 0.15$.

Evaluate the following probabilities.

(a) $\mathrm{P}(A \text{ fails} \mid B \text{ has failed})$.

(b) $\mathrm{P}(A \text{ fails and } B \text{ does not fail})$.

5. (**Multiple choice**) A six-sided die is loaded in such a way that an even number is twice as likely to occur as an odd number. The probability that a number (strictly) less than 4 occurs in a single toss of the die is:

(a) 1/9 (b) 4/9 (c) 7/9 (d) 11/9 (e) none of these.

6. (**Multiple choice**) In Q5, let A be the event that an even number turns up and let B be the event that a number is divisible by 3 occurs. $\mathrm{P}(A \text{ or } B)$ and $\mathrm{P}(A \text{ and } B)$ respectively are:

(a) 1/9 and 2/9 (b) 1 and 0 (c) 7/9 and 5/9 (d) 7/9 and 2/9 (e) none of these.

7. Four students are to be selected at random to participate in a survey of Senior Statistics students. The Senior Statistics class consists of 12 females and 16 males.

(a) What is the probability of selecting all males?

(b) What is the probability of selecting at least one female?

8. Read the help page of the R `sample` function by typing `help(sample)` at the prompt. Use the R function `sample` to simulate the following experiments:

(a) Draw 6 balls from an urn which contains 6 numbered balls with replacement

(b) Draw 6 balls from an urn which contains 6 numbered balls without replacement

(c) Draw 7 balls from an urn which contains 52 numbered balls without replacement

(d) Throw a (fair) six faced die once

(e) Throw a (fair) six faced die 6 times

Assignment 1 for MATH1905 STATISTICS (due on Tuesday, 2nd October, in week 9) will consist of selected questions from the Problem Sheets for weeks 1, 2, 3, 4, 5, 6, 7, 8.

1. A doctor has prescribed a skin cream for 10 of his patients. Unknown to him the treatment was successful in exactly 8 of the 10 cases. He asks his secretary to contact any 5 of the 10 patients as a follow-up. Assuming that the selection is made randomly, use counting methods to find the probability that

(a) exactly 3 of those contacted had a successful result with the cream;

(b) at least 4 of those contacted had a successful result.

2. Let (E) be an experiment whose outcome is not predictable. Explain in words what is the sample space associated to (E)? In each of the following scenarios, write down the sample space.

(1) A fair coin is thrown twice.
(2) A car passes through a sequence of 3 lights. At each light the car stops 's' or continues 'c'.
(3) Rolling two dice once (each die has 6 faces).
(4) Measuring height of a single person. Recording heights in a class of 28 students.
(5) Recording birth dates in a class room with 18 students (disregard leap years).
(6) Recording the time until an electronic component fails.

3. (Simpson's paradox) A yellow box contains 5 pink and 6 green balls. A blue box contains 3 pink and 4 green balls. You can choose a box and a ball within that box (at random). To win a price you must get a pink ball. Which box should you choose from? Same question with a yellow box with 6 pink and 3 green balls and a blue box with 9 pink and 5 green balls. In the last game, the contents of the second yellow box is added to the contents of the first yellow box and the contents of the second blue box is added to the first blue box. Which box should you choose from the yellow box combo or the blue box combo ?

4. Use the R function `sample` to simulate the following experiments

(a) throw a fair coin 100 times (1=head, 0=tail)

(b) throw a coin 100 times where P(Head) = 3/10

(c) throw a coin 100 times where P(Head) = 9/10

(d) throw a 6 faced-die 100 times, the die is loaded in such a way that it is twice more likely to get an even number than an odd number.

(e) Toss a fair coin 50 times. Note: you can sample from a vector whose components are letters: `c('H','T')`

(f) Pick 6 of 54 lottery numbers (without replacement)

(g) Pick 5 `cards=paste(rep(c('A',2:10,'J','Q','K'),4),c('H','D','S','C'))`

(h) Roll 2 dice 5 times. (Hint: `dice=as.vector(outer(1:6,1:6,paste))`)