The University of Sydney
School of Mathematics and Statistics

Assignment 2

MATH1902: Linear Algebra (Advanced) — Semester 1, 2017

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1902/
Lecturer: Holger Dullin

This assignment is due by **11:59pm Monday 8th May**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard) at https://elearning.sydney.edu.au. Please submit only a single PDF document (scan or convert other formats). It should include

- your name and SID,
- your tutorial time, day, room and Tutor's name.

Printed/typed solutions are acceptable. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission. The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 5% of your final assessment for this course.

Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master.

The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts of all questions correctly, with clear accurate explanations and all relevant diagrams and working. There are at most only minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on at least 1 question and good progress on the remaining question, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making good progress on 1 question and some progress on the second, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making substantial progress on only 1 question and some progress on another question.
4	D	Some attempt, with substantial progress made on only 1 question.
2	E	No substantial progress made on any of the 2 questions.
0	F	No credit awarded.

The *rank* of a matrix M is the number of pivots in its (reduced) row echelon form. We write $\operatorname{rank} M$ for the rank of M. When we talk about the dimension of the solution of a system of linear equations we mean the number of parameters in the general solution.

1. Consider the system of equations $A\mathbf{x} = \mathbf{b}$ where

$$A = \begin{bmatrix} 1 & 2 & -1 & -2 \\ -2 & 1 & 2 & -1 \\ -1 & -2 & 1 & 2 \\ 2 & -1 & -2 & x \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} c \\ d \\ e \\ f \end{bmatrix}$$

(a) Solve the system of equations, carefully treating the various cases depending on the values of the parameters x, c, d, e, f.

(b) Define $M = [A|\mathbf{b}]$, the augmented matrix. For each of the cases found in part (a) state what is $\operatorname{rank} A$, $\operatorname{rank} M$, and (if consistent) the dimension of the solution set. (Note that when computing the rank of M you should ignore the vertical bar.)

2. In the unit square the distance of two corners from a diagonal is $1/\sqrt{2}$. This question is about similar distances in the unit cube in $\mathbb{R}^3$ and the unit (hyper) cube in $\mathbb{R}^4$.

In $\mathbb{R}^3$ you may work with the basic unit vectors $\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$, or alternatively fixing these basic vectors you may work with column vectors that have the components as entries, e.g. the vector

$$\mathbf{v} = a\mathbf{i} + b\mathbf{j} + c\mathbf{k} \quad \text{may be written as} \quad \mathbf{v} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Vectors in $\mathbb{R}^4$ are similarly represented as column vectors with 4 entries.

The unit cube in dimension n has corners with each coordinate either 0 or 1, and hence there are 2^n corners. By the main diagonal of the unit cube we mean a diagonal with the largest possible length, e.g. the one from the origin to the corner which has all coordinates equal to 1.

(a) For the unit cube in $\mathbb{R}^3$ find the distances of corners to the main diagonal. Here you may use our usual formula involving the cross product for the distance between a point and a line.

(b) Given a line $\mathbf{r}_0 + t\mathbf{v}, t \in \mathbb{R}$, and a point P, show that the point R on the line that is closest to P is given by

$$\overrightarrow{OR} = \overrightarrow{OQ} + \frac{\overrightarrow{QP} \cdot \mathbf{v}}{|\mathbf{v}|^2}\mathbf{v}$$

where Q is an arbitrary point on the line.

(c) For the unit cube in $\mathbb{R}^4$ find the distances of corners to the main diagonal. Since the cross product is only defined in $\mathbb{R}^3$ the formula mentioned in part (a) cannot be used here. Instead use the vector projection formula from part (b) to find the distance.