The University of Sydney
School of Mathematics and Statistics

Tutorial Week 5

MATH1905: Statistics (Advanced) Semester 2, 2017

Web Page: `http://sydney.edu.au/science/maths/MATH1905`
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For a discrete random variable X we have

$$E(X) = \sum_x xP(X = x)\,,\ E(X^2) = \sum_x x^2 P(X = x)\,,\ Var(X) = E(X^2) - [E(X)]^2\,.$$

1. (**Multiple Choice**) For $X \sim B(8, 0.1)$, $P(X \leq 2$ is closest to

(a) 0.9950 (b) 0.8131 (c) 0.7969 (d) 0.9619 (e) 0.6259

2. The following table shows the probability distribution of a random variable X:

x	1	2	3	4	Total
$P(X = x)$	0.35	0.30	0.25	0.10	1.00

Find $E(X)$, $E(1/X)$, $E(X^2)$, $Var(X)$. Verify that $E(1/X) \neq 1/E(X)$.

3. Let X be a discrete random variable with the following incomplete probability distribution table:

x	0	1	2	3	4
$P(X = x)$	0.17	0.36	0.31		0.03

Find

(a) $P(X = 3)$ (b) $E(X)$ (c) $E(X^2)$ (d) $Var(X)$

4. When Mendel crossed a tall strain of pea with a dwarf strain of pea, he found that $\frac{3}{4}$ of the offspring were tall and $\frac{1}{4}$ were dwarf. Suppose five such offspring were selected at random. Let X be the number of 'tall offspring' in this random sample. Find the probability distribution of X by completing the following table:

x	
$P(X = x)$	

Verify that $\sum_x P(X = x) = 1$.

5. Use R to verify the answers to the previous question

6. In a small pond there are 50 fish, 20 of which have been tagged. Seven fish are caught and X represents the number of tagged fish in the catch.

(a) Under what additional conditions is X well-modelled by a binomial random variable? Write a formula for $P(X = x)$ and indicate all possible values x.

(b) Under what additional conditions is X well-modelled by a hypergeometric random variable? Write a formula for $P(X = x)$ and indicate all possible values x.

(c) Find the probability of exactly one tagged fish being in the sample of 7 in each case.

7. (a) Suppose that for some fixed positive integer n, for $x = 1, 2, \ldots, n$, the random variable X has distribution given by $P(X = x) = 1/n$. Find

(i) $E(X)$;

(ii) $Var(X)$.

Hint: you may find the following results useful:

$$\sum_{x=1}^{n} x = \frac{n(n+1)}{2}, \quad \sum_{x=1}^{n} x^2 = \frac{n(n+1)(2n+1)}{6}.$$

(b) If the distribution of X is given by $P(X = x) = C_n x$, $x = 1, 2, 3, \ldots, n$, find C_n.

8. Recall that the expectation of a geometric random variable X with

$$P(X = x) = (1-p)^{x-1} p, \quad \text{for } x = 1, 2, \ldots$$

is given by

$$p\left\{1 + 2(1-p) + 3(1-p)^2 + \cdots\right\}. \tag{$\dagger$}$$

We shall show that in fact $E(X) = 1/p$ by proving that the curly-bracketed expression is $1/p^2$.

(a) For $0 < q < 1$ we know that

$$1 + q + q^2 + \cdots = \frac{1}{1-q}.$$

By differentiating both sides evaluate the infinite sum $1 + 2q + 3q^2 + \cdots$.

(b) Hence show that $E(X) = 1/p$.