The University of Sydney
School of Mathematics and Statistics

# Practice Questions for Quiz 1

MATH1903: Integral Calculus and Modelling (Advanced) Semester 2, 2012

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Quiz 1 is held in **week 6** (starting 3rd September) in tutorials. You have **40 minutes** to complete the quiz. It covers material up to and including lecture 8 (chapters 1, 2, 3 and 4 in the course notes; plus the appendix on integration techniques). The quiz will have considerably fewer questions than this set of practice questions. There will be answer boxes for you to write your answers in, and **only** your final answers will be marked (1 mark if correct and 0 marks otherwise). **No half marks are awarded**, and so it is important to work carefully and check your answers. Non-programmable calculators are allowed. You will **not** be provided with a table of standard integrals.

*Solutions to these practice questions will be posted later.*

**1.** Let $f(x) = e^x$, and let $P = \{x_0, \ldots, x_n\}$ be the partition of $[0, 1]$ into $n$ equal parts.

(a) Find a closed formula for the corresponding lower Riemann sum $L_n$.

(b) Compute $\lim_{n\to\infty} L_n$.

**2.** Compute the upper Riemann sum of $f(x) = \sin \pi x$ over the interval $[0, 3/2]$ using the partition $P = \{0, 1/6, 1, 5/4, 3/2\}$.

**3.** Compute $\displaystyle\int_0^{2\pi} \mathrm{Si}(x)\,dx$ where $\displaystyle\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}\,dt$.

**4.** Given that $\displaystyle f(x) = x\int_0^{2x} te^{-t}\,dt$, find $f''(1)$.

**5.** Find the derivative of the function $\displaystyle f(x) = \int_{\sin x}^{3+e^x} \sin t\,dt$.

**6.** Given that $\sin(e^x) - \sin(1) = \displaystyle\int_0^x e^t f(t)\,dt$, find $f(x)$.

**7.** Find the value of $x > 0$ which maximises the function $\displaystyle I(x) = \int_0^x (1 - t^2)(t^2 - 16)\,dt$.

**8.** Find the area between the curves $y = \sin \pi x$ and $y = \cos \pi x$ with $1/2 \le x \le 3/2$.

**9.** Compute the volume of the solid obtained by rotating about the $x$-axis the region bounded by the curve $y = \sinh x$, the $x$-axis, and the line $x = 2$.

**10.** Let $D$ be the region bounded by the curve $y = \ln x$, the $x$-axis, and the line $x = e$.

(a) Compute the area of $D$.

(b) Compute the volume of the solid formed by rotating $D$ around the $x$-axis.

(c) Find the volume of the solid formed by rotating $D$ around the $y$-axis.

**11.** Let $D$ be the region bounded by the curve $y = \cosh x$, the $x$-axis, the $y$-axis, and the line $x = 1$.

(a) Calculate the perimeter of $D$.

(b) Calculate the volume of the solid obtained by revolving $D$ around the $x$-axis.

(c) Calculate the volume of the solid obtained by revolving $D$ around the $y$-axis.

(d) Calculate the surface area of the solid obtained by rotating $D$ around the $x$-axis. Remember to include the area of the end caps.

**12.** Compute the volume of the solid obtained by rotating about the $y$-axis the region bounded by the curve $y = e^{-x^2}$, the $x$-axis, the $y$-axis, and the line $x = 1$.

**13.** Find the length of the curve with parametrisation $x(t) = t - \sin t$ and $y(t) = 1 - \cos t$ with $t \in [0, 2\pi]$.

**14.** Compute the surface area of the solid obtained by revolving the part of the graph of $y = \sqrt{x}$ between $x = 0$ and $x = 2$ around the $x$-axis. Remember to include any end caps.

**15.** Decide if the following improper integrals exist or not:

(a) $\displaystyle\int_1^\infty \frac{3 + 2\sin(x^2)}{x}\,dx$ (b) $\displaystyle\int_0^1 \frac{1}{x^2}\sin\left(\frac{1}{x}\right)\,dx$ (c) $\displaystyle\int_0^1 \frac{\cosh x}{\sqrt{x}}\,dx$

**16.** Compute the value of the following improper integrals:

(a) $\displaystyle\int_0^\infty e^{-x}\cos x\,dx.$ (b) $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx.$ (c) $\displaystyle\int_0^1 \frac{x}{\sqrt{1-x}}\,dx.$

**17.** Compute the indefinite integral $\displaystyle\int x^n \ln x\,dx$, where $n \neq -1$.

**18.** Compute the indefinite integral $\displaystyle\int \frac{x^2}{\sqrt{1+x^2}}\,dx$.

**19.** Find a reduction formula for the integral $I_n = \displaystyle\int x^n \cos x\,dx$.

**20.** Calculate the limit $\displaystyle\lim_{x\to 0}\frac{S(x)}{x^3}$, where $S(x) = \displaystyle\int_0^x \sin(t^2)\,dt$.

**21.** Decide if the following improper integrals exist or not (either use the Comparison Test, or make a direct limit calculation). If they exist, try to compute their value (this is not always possible!).

(a) $\displaystyle\int_1^\infty \frac{\ln x}{x^2}\,dx$

(b) $\displaystyle\int_1^\infty \sin(\pi x)\,dx$

(c) $\displaystyle\int_1^\infty \frac{e^{-x}}{\sqrt{x}}\,dx$

(d) $\displaystyle\int_0^\infty \frac{\cosh x}{x^2+1}\,dx$

(e) $\displaystyle\int_{\pi/4}^{\pi/2} \sec^2 x\,dx$

(f) $\displaystyle\int_{-\infty}^0 e^x \cos x\,dx$

(g) $\displaystyle\int_0^1 \sin\left(\frac{1}{x}\right)\,dx$

(h) $\displaystyle\int_0^\infty \frac{\cos x}{x^2+1}\,dx$

(i) $\displaystyle\int_1^\infty \frac{e^{-x}}{\sqrt{x}}\,dx$

(j) $\displaystyle\int_0^\infty x^3 e^{-x}\,dx$

(k) $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx$

(l) $\displaystyle\int_{-1}^1 \frac{1}{\sqrt{1-x^2}}\,dx$

(m) $\displaystyle\int_1^\infty \frac{e^{-x^2}}{\sqrt{x-1}}\,dx$

(n) $\displaystyle\int_0^1 \sin\left(\frac{1}{x^2}\right)\,dx$

(o) $\displaystyle\int_0^\infty \operatorname{erf}(x)\,dx$

(p) $\displaystyle\int_0^\infty \cosh(3x)e^{-4x}\,dx$

(q) $\displaystyle\int_1^2 \frac{1}{\ln x}\,dx$

(r) $\displaystyle\int_2^\infty \frac{\operatorname{Li}(x)}{x^2}\,dx$