PHYS 1901 Physics Advanced
Tutorial 5: Mechanics

A. Qualitative Questions:

1. ***Merry-go-round***

Imagine standing on the edge of a merry-go-round that is rotating clockwise.

a. Are you more likely to slip off the merry-go-round near the middle or near the edge?

b. If you walk in towards the centre of the merry-go-round from the edge, what will happen to the angular momentum and angular velocity of the system (you and the merry-go-round)?

You stay at the edge and walk in the direction of rotation of the merry-go-round.

c. What will happen to the angular momentum of the system (you plus the merry-go-round)?

d. What will happen to your angular momentum and angular velocity?

e. What will happen to the angular momentum and angular velocity of the merry-go-round?

f. How would it be different if you walked in the opposite direction?

2. ***Kepler' laws***

Kepler's empirical laws state that: 1. Each planet moves in an elliptical orbit with the sun at one focus of the ellipse, 2. A line from sun to a given planet sweeps out equal areas in equal times, 3. The periods of the planets are proportional to 3/2 power of the major axis lengths of their orbits.

a. Draw a diagram of the sun-earth system, and indicate the velocity vector of the earth at several points on its orbit. At what points does the earth have the greatest and smallest speeds?

b. Show that the quantity in question in the second law is proportional to the angular momentum. Hence the second law is a statement of the conservation of angular momentum.

c. Newton showed that the third law can be stated mare precisely as $T = 2\pi a^{3/2}/(Gm_s)$, where T is the period, $2a$ is the length of the major axis, G is the gravitational constant and m_s is the sun's mass. Comet Halley moves in an elongated elliptical orbit around the sun with the min and max distances 8.75×10^7 and 5.26×10^9 km. Find the period of the orbit.

B. Demonstration Questions:

1. ***A loaded race***

Spheres and cylinders of various diameters and an inclined plane are available on the demonstration table.

Do all the cylinders roll down with same speed?

Do all the spheres roll down with same speed?

Try to explain why some of them roll down faster than the others.

2. ***The rotating stool***

Sit on the stool and start rotating with equal weights held in your hands. Start with the hands stretched and slowly bring your hands towards your chest. What do you observe? Explain.

Try dropping the weights with your arms outstretched without moving anything else. What change, if any, is there in your angular speed? Is angular momentum conserved?

3. ***Falling cats***

The diagram on display shows how a cat can rotate itself around so that it always lands on its feet.

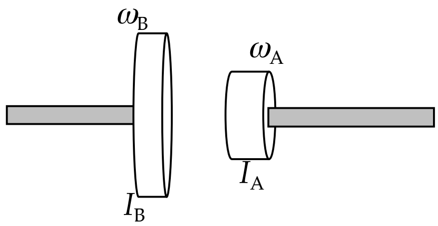
Sit on the rotating stool and see if you can turn yourself around in the same way that cats do.

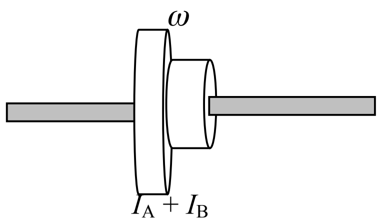
How is it possible to do this without violating the law of conservation of angular momentum?

C. Quantitative Questions:

1. *Two Rotating Discs*

The figure shows two disks, one an engine flywheel, the other a clutch plate attached to a transmission shaft. Their moments of inertia are I_A and I_B, and they are initially rotating with constant angular velocities ω_A and ω_B, respectively. The disks are pushed together with forces acting along the axis, so as to keep their faces parallel. The disks rub against each other and eventually reach a common final angular velocity, ω.





a. Derive an expression for ω.

Suppose the flywheel A has a mass of 2.0 kg, a radius of 0.20 m, and an initial angular velocity of 50 rad.s^{-1} and the clutch plate B has a mass of 4.0 kg, a radius of 0.10 m, and an initial angular velocity of 200 rad.s^{-1}.

b. Find the common angular velocity ω after the disks are pushed into contact.

c. Calculate the initial and final kinetic energies of the system. Why isn't kinetic energy conserved in this process?

2. *Collapsing Stars*

There are two competing forces in stars, which roughly balance for most of a star's lifetime. One is the *gravitational force* that pulls the atoms and subatomic particles in towards each other. The other is an outward *pressure* due to the thermal energy of the particles. When a star runs low on nuclear fuel and nuclear processes begin to slow down, the star's core cools and the force of gravity overcomes thermal pressures: the star begins to collapse under gravity.

a. As the star collapses, what happens to its moment of inertia?

The surface of our Sun rotates at about one revolution per month (although, being made of gas it can rotate more slowly at the equator than at the poles). It has a mass of 2.0×10^{30} kg and a radius of 1.4 million kilometres.

b. Assuming that the whole Sun rotates at one revolution per month, what is the angular momentum of the Sun?

c. Old collapsed stars, called neutron stars, can rotate at speeds of up to 800 revolutions per second. By how much would our Sun have to collapse to spin at this speed?

[Note: You can approximate the Sun as a solid sphere, which has a moment of inertia of 2/5 (MR^2)]

Extra Question: *(challenging)*

A rod of length l and mass M is rotating on a frictionless table about a vertical axis through its centre. An ant of mass m crawls along the rod at a constant speed v. At time $t = 0$ the ant is at the centre of the rod, which is rotating at angular speed ω_o.

a. Find the angular speed $\omega(t)$ as a function of time for $0 \leq t \leq \frac{1}{2}\, l \,/\, v$.

b. Find the total kinetic energy of the system as a function of time for $0 \leq t \leq \frac{1}{2}\, l \,/\, v$.

c. The kinetic energy is not conserved. Find the additional source or sink of energy.

d. Suppose that the motion of the ant is reversed. Show that the sign of the rate of change of kinetic energy also reverses. (Note: your answer to **c.** should also make sense in this case.)