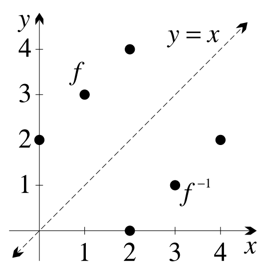
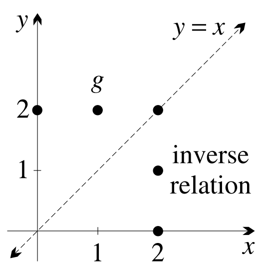
Answers to Exercises

Chapter One

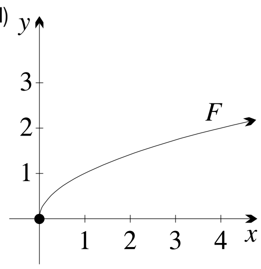
Exercise **1A** (Page 5)

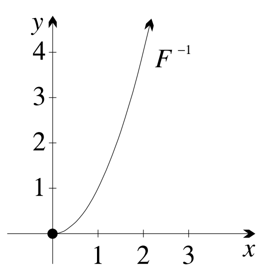
1(a) The inverse of f is $\{(2,0),(3,1),(4,2)\}$.
The inverse of g is $\{(2,0),(2,1),(2,2)\}$.
(c) For f it is, for g it isn't.



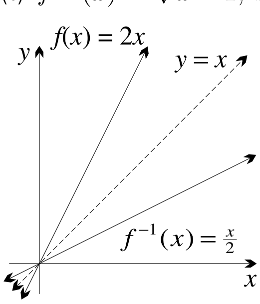


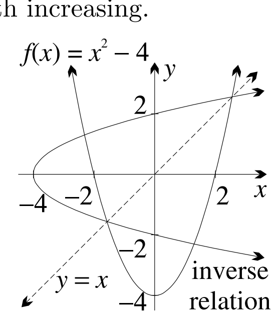
2(a) $3 \le y \le 5$ **(b)** domain: $3 \le x \le 5$, range: $0 \le y \le 2$ **(c)** $f^{-1}(x) = x - 3$
3(a) $0 \le y \le 2$ **(b)** domain: $0 \le x \le 2$, range: $0 \le y \le 4$ **(c)** $F^{-1}(x) = x^2$
(d)



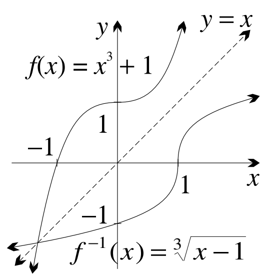


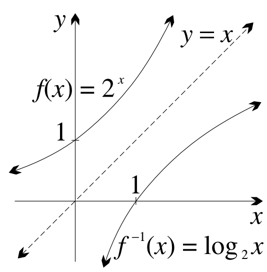
4(a) $f^{-1}(x) = \frac{1}{2}x$, both increasing.
(b) $f^{-1}(x) = \sqrt[3]{x-1}$, both increasing.



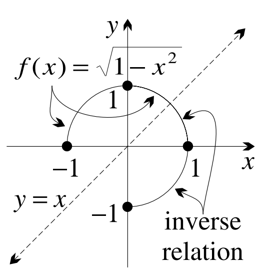


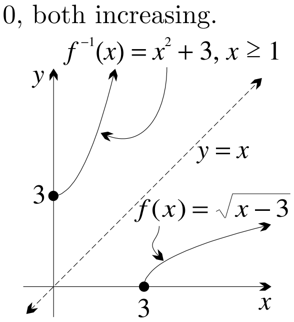
(c) The inverse is not a function, f is neither increasing nor decreasing. **(d)** The inverse is not a function, f is neither increasing nor decreasing.



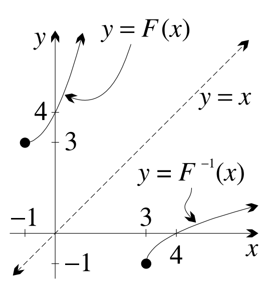


(e) $f^{-1}(x) = \log_2 x$, both increasing.
(f) $f^{-1}(x) = x^2 + 3$, $x \ge 0$, both increasing.



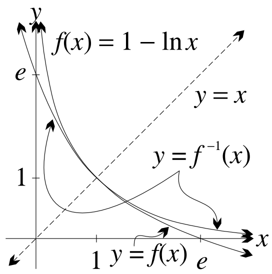


5(a) Both x. **(b)** They are inverse functions.
6(a) $g^{-1}(x) = \sqrt{x}$, domain: $x \ge 0$, range: $y \ge 0$ **(b)** $g^{-1}(x) = -\sqrt{x-2}$, domain: $x \ge 2$, range: $y \le 0$ **(c)** $g^{-1}(x) = \sqrt{4 - x^2}$, $-2 \le x \le 0$, domain: $-2 \le x \le 0$, range: $0 \le y \le 2$
7(a) $3x^2$ **(b)** $\frac{1}{3}(y+1)^{-\frac{2}{3}}$
8 $\dfrac{dy}{dx} = \frac{1}{2\sqrt{x}}$, $\dfrac{dx}{dy} = 2y$
9(b) $F^{-1}(x)$
$= -1 + \sqrt{x-3}$,
domain: $x \ge 3$,
range: $y \ge -1$

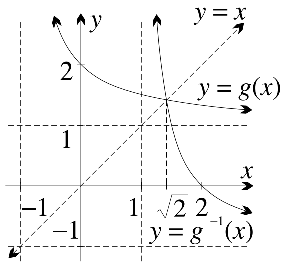


ISBN: 978-1-107-61604-2 Cambridge University Press

10(a) $x = e$
(b) Reflect $y = \ln x$ in the x-axis, then shift it one unit up.
(d) $f^{-1}(x) = e^{1-x}$, domain: all real x, range: $y > 0$
(e) Both are decreasing.

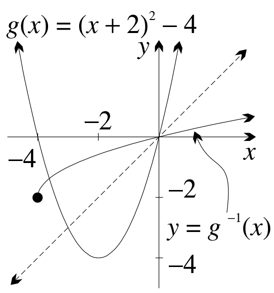


11(b) $g^{-1}(x) = \frac{2-x}{x-1}$, for $x > 1$, decreasing **(c)** $x = \sqrt{2}$. It works because the graphs meet on the line of symmetry $y = x$.

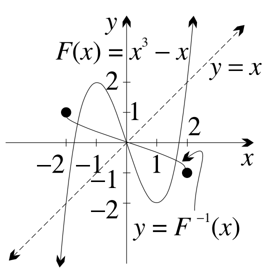


12(a) $y = \sqrt[3]{-x}$ **(b)** $(-1, 1)$, $(0, 0)$ and $(1, -1)$

13(a) Shift two units to the left and four units down.
(b) x-intercepts: -4, 0, vertex: $(-2, -4)$.
(c) $x \geq -2$ **(d)** $x \geq -4$, increasing **(e)** $g^{-1}(x) = -2 + \sqrt{x+4}$
(f) $g(x)$ is neither, $g^{-1}(x)$ is increasing.

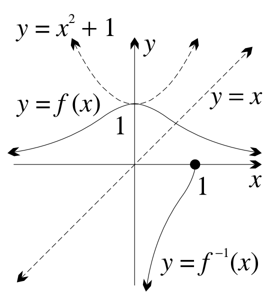


14(b) x-intercepts: 0, $\sqrt{3}$, $-\sqrt{3}$, stationary points: $(-1, 2)$, $(1, -2)$, neither increasing nor decreasing
(c) $-1 \leq x \leq 1$
(d) $-2 \leq x \leq 2$

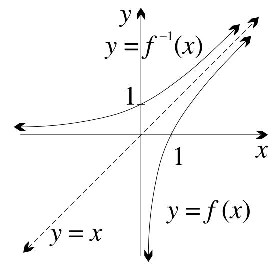


15(a) all real x **(c)** $f'(x) > 0$ for all real x. **(d)** For each value of y, there is only one value of x. That is, the graph of $f(x)$ passes the horizontal line test. $f^{-1}(x) = \log\left(\frac{x}{1-x}\right)$

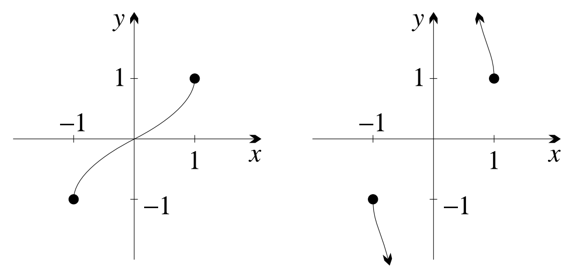
16(a) neither **(b)** $x \leq 0$
(c) $0 < x \leq 1$
(d) $f^{-1}(x) = -\sqrt{\frac{1-x}{x}}$
(e) increasing



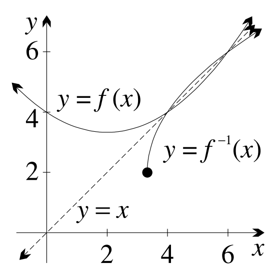
17(b) No. The graph of the inverse is a vertical line, which is not a function.



19(b) From part (a) we see, for example, that $g(\frac{1}{2}) = g(2)$, so the inverse is not a function.
(c)(i) $-1 \leq x \leq 1$ **(iii)** $g^{-1}(x) = \frac{1-\sqrt{1-x^2}}{x}$
(d) domain: $x \leq -1$ or $x \geq 1$, $g^{-1}(x) = \frac{1+\sqrt{1-x^2}}{x}$
(e) Because of the result in part (a).

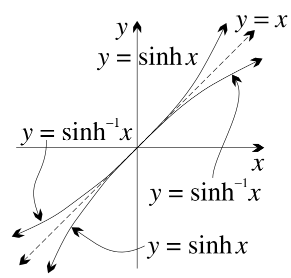


20(a) vertex: $(2, \frac{10}{3})$, y-intercept: 4
(b) $x \geq 2$ **(c)** $x \geq \frac{10}{3}$
(d) The easy way is to solve $y = f(x)$ simultaneously with $y = x$. They intersect at $(4, 4)$ and $(6, 6)$. **(e)** $4 - N$



21(b) functions whose domain is $x = 0$ alone

22(a) all real x **(b)** 0
(d) $\frac{1}{2}(e^x + e^{-x})$, which is positive for all real x.
(e) $y = \frac{1}{2}e^x$ **(f)** $\sinh x$ is a one-to-one function.
(i) $\frac{1}{\sqrt{1+x^2}}$, $\sinh^{-1} x + C$



24(b)(ii) $\frac{1}{6}$

Exercise 1B (Page 12)

1(a) $1{\cdot}16$ **(b)** $0{\cdot}64$ **(c)** $1{\cdot}32$ **(d)** $1{\cdot}67$ **(e)** $1{\cdot}98$
(f) $2{\cdot}42$

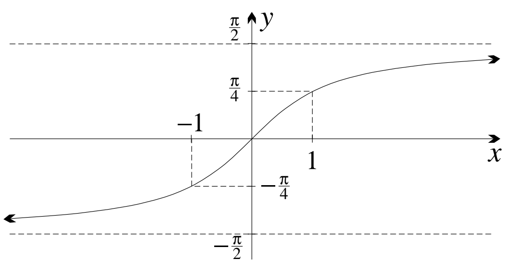
2(a) 0 **(b)** $\frac{\pi}{6}$ **(c)** 0 **(d)** $\frac{\pi}{4}$ **(e)** $-\frac{\pi}{2}$ **(f)** $\frac{\pi}{2}$ **(g)** 0
(h) $-\frac{\pi}{4}$ **(i)** $-\frac{\pi}{3}$ **(j)** $\frac{3\pi}{4}$ **(k)** $-\frac{\pi}{6}$ **(l)** π

ISBN: 978-1-107-61604-2 Cambridge University Press

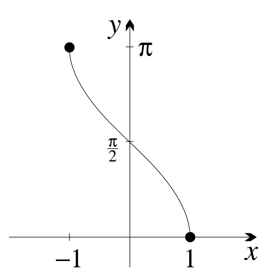
3(a) $1{\cdot}447$ **(b)** $1{\cdot}694$ **(c)** $0{\cdot}730$ **(d)** $-0{\cdot}730$ **(e)** $1{\cdot}373$ **(f)** $-1{\cdot}373$
4(a) $\frac{\pi}{2}$ **(b)** 1 **(c)** 1 **(d)** $\frac{\pi}{6}$ **(e)** $\frac{1}{2}$ **(f)** $\frac{3\pi}{4}$ **(g)** $-\frac{\pi}{6}$ **(h)** 0 **(i)** $\frac{\pi}{3}$
5(a) $-\frac{\pi}{3}$ **(b)** $\frac{\pi}{4}$ **(c)** $-\frac{\pi}{6}$ **(d)** $\frac{3\pi}{4}$ **(e)** $-\frac{\pi}{2}$ **(f)** $\frac{\pi}{3}$
6(a)(i) $\frac{4}{5}$ **(ii)** $\frac{5}{12}$ **(iii)** $\frac{1}{3}\sqrt{5}$ **(iv)** $\frac{8}{17}$ **(v)** $\frac{3}{10}\sqrt{10}$ **(vi)** $-\frac{1}{3}\sqrt{7}$
7(a) $\frac{56}{65}$ **(b)** $\frac{10\sqrt{3}-\sqrt{5}}{20}$ **(c)** 1 **(d)** $\frac{56}{33}$
8(a) $-\frac{7}{9}$ **(b)** $\frac{12}{49}\sqrt{13}$ **(c)** $\frac{4}{3}$
9(b) $\frac{\pi}{4}$
13(a) 2 is outside the range of the inverse sine function, which is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. **(b)** It is because the sine curve is symmetrical about $x = \frac{\pi}{2}$. **(c)** $\pi - 2$
14(a) $\cot\theta$ **(d)** $-\frac{\pi}{2}$
15(a) $\frac{3x}{1-2x^2}$ **(b)** $x = \frac{1}{2}$ (note that $x \ne -1$)
16(a) $x = \frac{1}{3}$ **(b)** $x = \frac{1}{3}$ or 1
20 $x = -\frac{3}{2}$ or $\frac{1}{3}$
23(b) $0 < \tan^{-1}\left(\frac{1}{1+x^2}\right) \le \frac{\pi}{4}$, $0 \le \tan^{-1}\left(\frac{x^2}{1+x^2}\right) < \frac{\pi}{4}$ **(d)** $\frac{\pi}{4} \le y \le \tan^{-1}\frac{4}{3}$

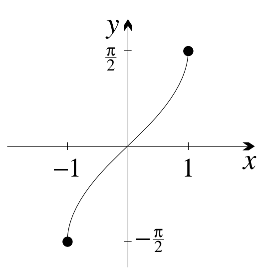
Exercise **1C** (Page 17)

1(a) domain: all real x, range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$, odd

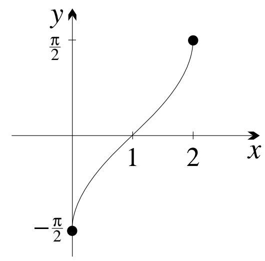


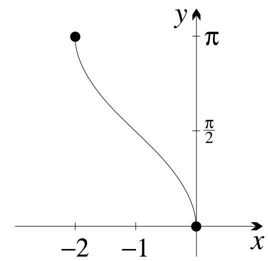
(b) domain: $-1 \le x \le 1$, range: $0 \le y \le \pi$, neither
(c) domain: $-1 \le x \le 1$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd



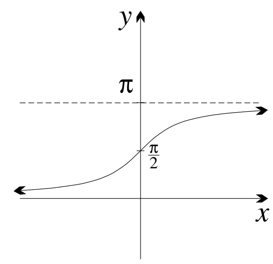


2(a) domain: $0 \le x \le 2$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, neither **(b)** domain: $-2 \le x \le 0$, range: $0 \le y \le \pi$, neither

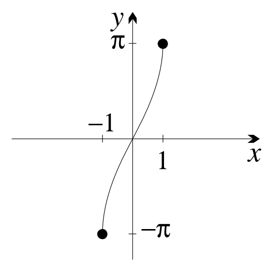


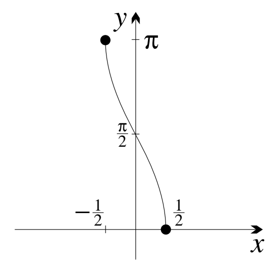


(c) domain: all real x, range: $0 < y < \pi$, neither

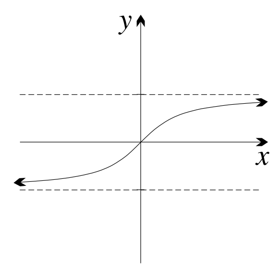


3(a) domain: $-1 \le x \le 1$, range: $-\pi \le y \le \pi$, odd **(b)** domain: $-\frac{1}{2} \le x \le \frac{1}{2}$, range: $0 \le y \le \pi$, neither

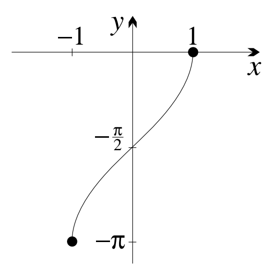


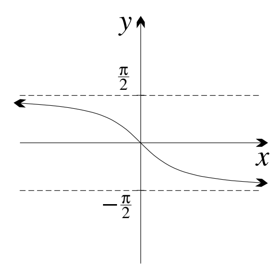


(c) domain: all real x, range: $-\frac{\pi}{4} < y < \frac{\pi}{4}$, odd



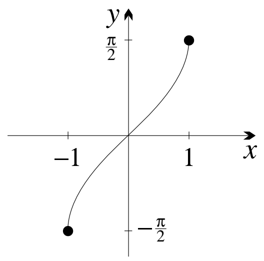
4(a) domain: $-1 \le x \le 1$, range: $-\pi \le y \le 0$, neither
(b) domain: all real x, range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$, odd



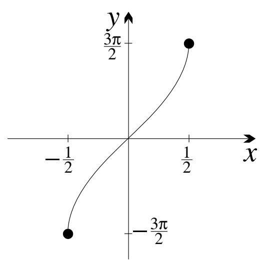


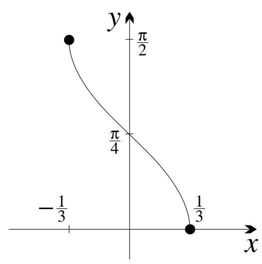
(c) domain: $-1 \le x \le 1$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd

ISBN: 978-1-107-61604-2

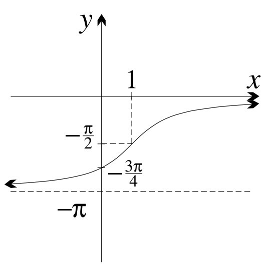


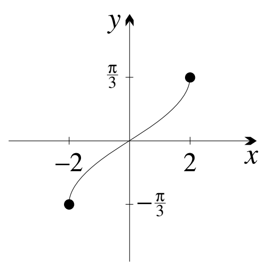
5(a) domain: $-\frac{1}{2} \le x \le \frac{1}{2}$, range: $-\frac{3\pi}{2} \le y \le \frac{3\pi}{2}$, odd **(b)** domain: $-\frac{1}{3} \le x \le \frac{1}{3}$, range: $0 \le y \le \frac{\pi}{2}$, neither



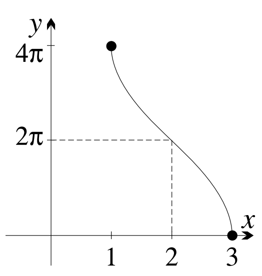


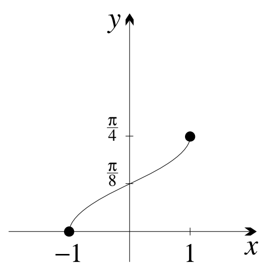
(c) domain: all real x, range: $-\pi < y < 0$, neither
(d) domain: $-2 \le x \le 2$, range: $-\frac{\pi}{3} \le y \le \frac{\pi}{3}$, odd



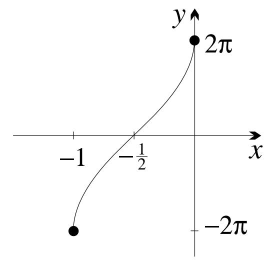


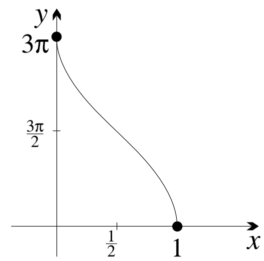
(e) domain: $1 \le x \le 3$, range: $0 \le y \le 4\pi$, neither
(f) domain: $-1 \le x \le 1$, range: $0 \le y \le \frac{\pi}{4}$, neither



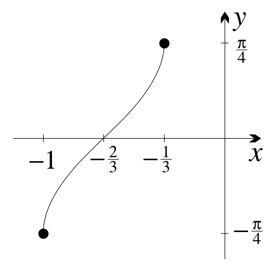


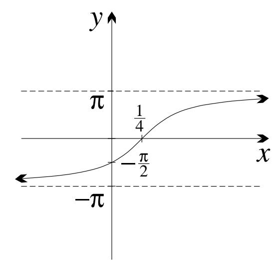
6(a)(i) $-1 \le x \le 0$ **(ii)** $-2\pi \le y \le 2\pi$
(b)(i) domain: $0 \le x \le 1$, range: $0 \le y \le 3\pi$



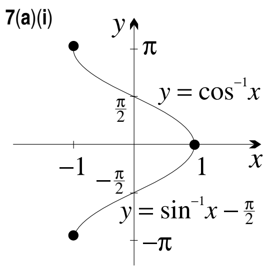


(ii) domain: $-1 \le x \le -\frac{1}{3}$, range: $-\frac{\pi}{4} \le y \le \frac{\pi}{4}$
(iii) domain: all real x, range: $-\pi < y < \pi$

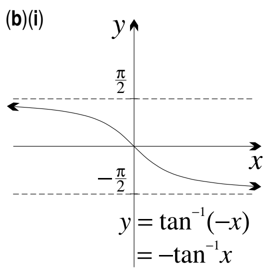




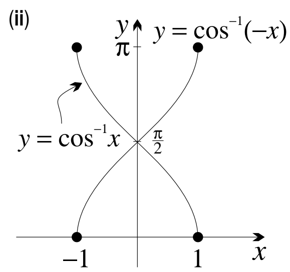
7(a)(i)



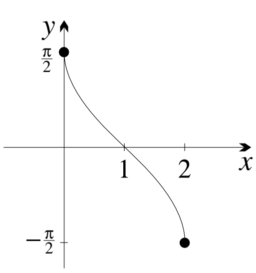
(b)(i)



(ii)

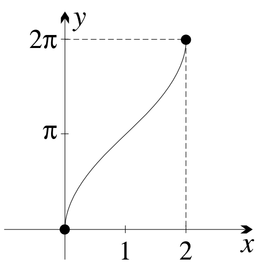


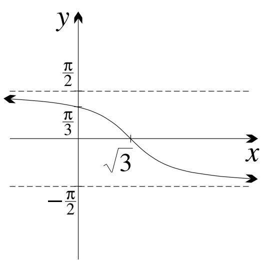
8(a) domain: $0 \le x \le 2$, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
(b) $y = \frac{\pi}{2}, 0, -\frac{\pi}{2}$ **(c)** $x = \frac{1}{2}$



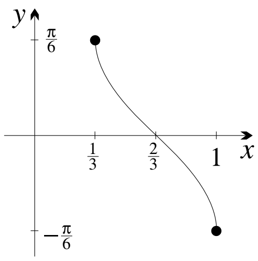
9(a) domain: $0 \le x \le 2$, range: $0 \le y \le 2\pi$
(b) domain: all real x, range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$

ISBN: 978-1-107-61604-2 Cambridge University Press

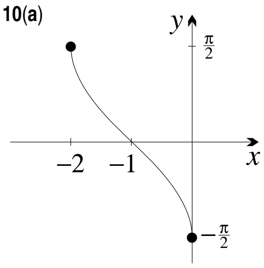




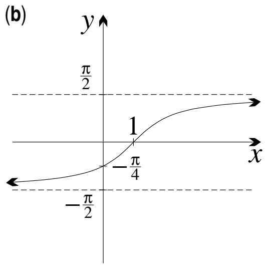
(c) domain: $\frac{1}{3} \le x \le 1$, range: $-\frac{\pi}{6} \le y \le \frac{\pi}{6}$



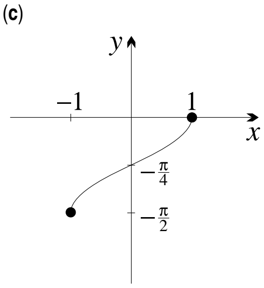
10(a)



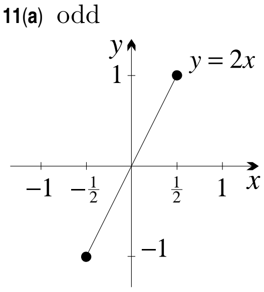
(b)



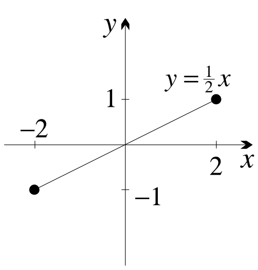
(c)



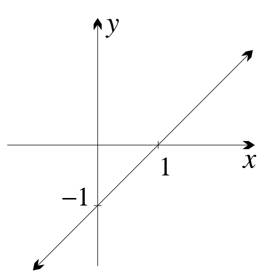
11(a) odd



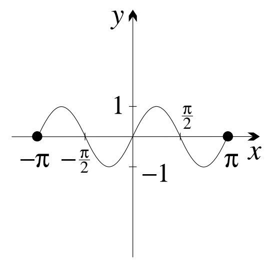
(b) odd



(c) neither

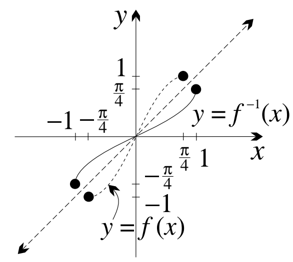


12(a)



(b) $-\frac{\pi}{4} \le x \le \frac{\pi}{4}$

(c)

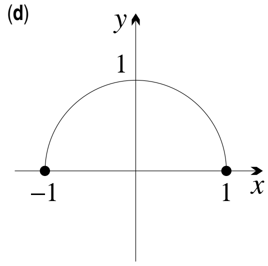


(d) $f^{-1}(x) = \frac{1}{2}\sin^{-1} x$, odd

13(a) $-1 \le x \le 1$, even

(b) $0 \le \cos^{-1} x \le \pi$, so $\sin \cos^{-1} x \ge 0$.

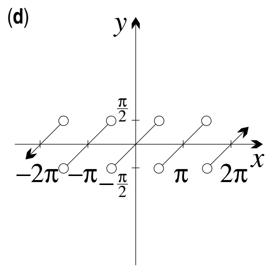
(d)



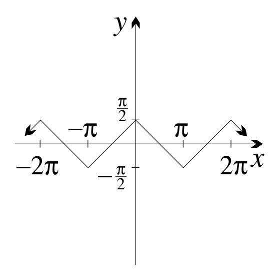
14(a) domain: all real x, $x \ne \frac{(2n+1)\pi}{2}$, where n is an integer, range: $-\frac{\pi}{2} < y < \frac{\pi}{2}$, odd

(b) x **(c)** π

(d)



15 $\sin^{-1} \cos x$
$= \frac{\pi}{2} - \cos^{-1} \cos x$,
so we reflect in the x-axis and then shift $\frac{\pi}{2}$ units up.
It's even.

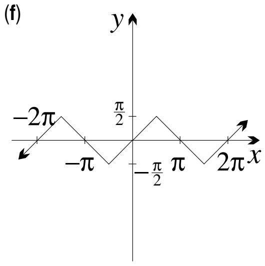


16(a) domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, period: 2π, odd

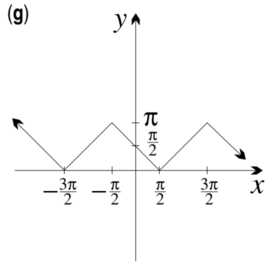
(b) x

(e) $\cos^{-1} \sin x = \frac{\pi}{2} - \sin^{-1} \sin x$, so we reflect in the x-axis and then shift $\frac{\pi}{2}$ units up.

(f)

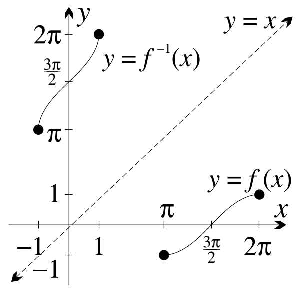


(g)

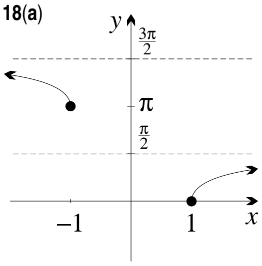


ISBN: 978-1-107-61604-2 Cambridge University Press

17(b) $\pi \le x \le 2\pi$
(e) $f^{-1}(x)$
$= 2\pi - \cos^{-1} x$



18(a)

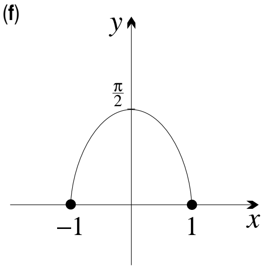


(b)(i) $\frac{\pi}{3}$ **(ii)** $\frac{4\pi}{3}$

Exercise 1D (Page 22)

2(a) $\frac{-1}{\sqrt{1-x^2}}$ **(b)** $\frac{1}{1+x^2}$ **(c)** $\frac{2}{\sqrt{1-4x^2}}$ **(d)** $\frac{3}{1+9x^2}$
(e) $\frac{-5}{\sqrt{1-25x^2}}$ **(f)** $\frac{-1}{\sqrt{1-x^2}}$ **(g)** $\frac{2x}{\sqrt{1-x^4}}$ **(h)** $\frac{3x^2}{1+x^6}$
(i) $\frac{1}{x^2+4x+5}$ **(j)** $\frac{1}{\sqrt{2x-x^2}}$ **(k)** $\sin^{-1} x + \frac{x}{\sqrt{1-x^2}}$
(l) $2x\tan^{-1} x + 1$ **(m)** $\frac{1}{\sqrt{25-x^2}}$
(n) $\frac{4}{16+x^2}$ **(o)** $\frac{-1}{2\sqrt{x-x^2}}$ **(p)** $\frac{1}{2\sqrt{x}(1+x)}$ **(q)** $\frac{-1}{1+x^2}$
3(a) 2 **(b)** 2 **(c)** 1 **(d)** -1
4(a) Tangent is $y = -6x + \pi$, normal is $y = \frac{1}{6}x + \pi$.
(b) Tangent is $y = \frac{1}{\sqrt{2}}x + \frac{\pi}{4} - 1$,
normal is $y = -\sqrt{2}\,x + \frac{\pi}{4} + 2$.
5(b) $\frac{\pi}{2}$
6(a) π **(b)** $\frac{\pi}{2}$
7(b) concave up
9(a) $\cos^{-1} x$ **(b)** $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$ **(c)** $\frac{2}{\sqrt{7+12x-4x^2}}$
(d) $\frac{1}{x^2-2x+2}$ **(e)** $\frac{e^x}{\sqrt{1-e^{2x}}}$ **(f)** $\frac{1}{2\sqrt{1-x^2}\,\sin^{-1} x}$
(g) $\frac{1}{2x\sqrt{\log x(1-\log x)}}$ **(h)** $\frac{\sin^{-1}\sqrt{1-x}}{2\sqrt{x}} - \frac{1}{2\sqrt{1-x}}$
(i) $\frac{1}{1+x^2}$
11(a) $-1 \le x \le 1$, even
(b) The y-axis, since the function is even.
(c) $\frac{-2x}{\sqrt{1-x^4}}$
(e) The tangents at $x = 1$ and $x = -1$ are vertical.

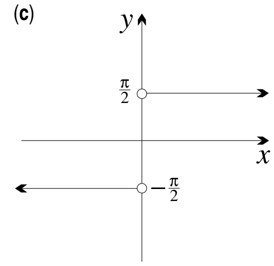
(f)



13(c) $\frac{1}{45}$ rad/s

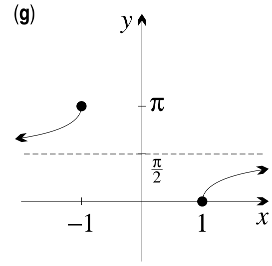
14(a) $x \ne 0$, odd

(c)

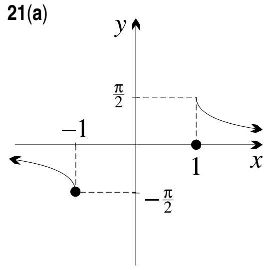


15(a) $-\sqrt{t}$ **(b)** $\frac{t}{2}$
16(a) $x \ge 1$ or $x \le -1$
(c) They are undefined.
(d) When $x > 1$,
$f'(x) = \frac{1}{x\sqrt{x^2-1}}$,
and when $x < -1$,
$f'(x) = \frac{-1}{x\sqrt{x^2-1}}$.
(e) $f'(x) > 0$ for $x > 1$ and for $x < -1$. **(f)(i)** $\frac{\pi}{2}$
(ii) $\frac{\pi}{2}$

(g)

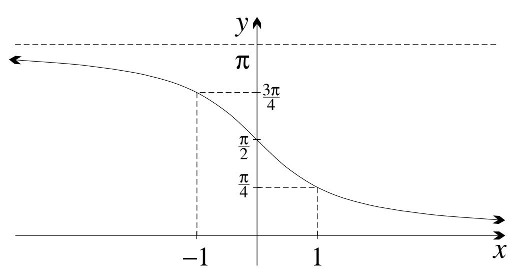


17(a) domain: all real x, range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, odd
(c) No, since $\frac{0}{0}$ is undefined. **(d)** $f'(x) = 1$ when $\cos x > 0$, and $f'(x) = -1$ when $\cos x < 0$.
18(a) $-1 \le x \le 1$ **(c)** $g(x) = \frac{\pi}{2}$ for $0 \le x \le 1$.
19 $\tan^{-1} \dfrac{x+2}{1-2x}$ is $\tan^{-1} x + \tan^{-1} 2$ for $x < \frac{1}{2}$, and is $\tan^{-1} x + \tan^{-1} 2 - \pi$ for $x > \frac{1}{2}$.
20(a) -1 **(b)** $-2\sqrt{1-x^2y^2} - \frac{y}{x}$ **(c)** $\frac{x+y}{x-y}$
21(a)



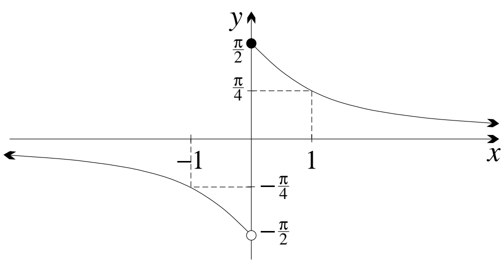
domain:
$x \ge 1$ or $x \le -1$,
range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$,
odd

22(a)(i) domain: all real x, range: $0 < y < \pi$, point symmetry about $(0, \frac{\pi}{2})$



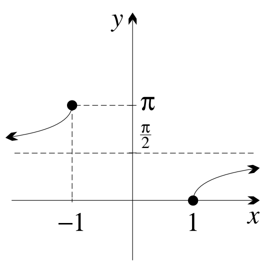
(ii) domain: all real x, range: $-\frac{\pi}{2} < y \le \frac{\pi}{2}$, odd, except for the value at $x = 0$

ISBN: 978-1-107-61604-2 Cambridge University Press



(c) It is only true for the second function. **(d)** The first produces a continuous function with a natural symmetry. The second has the same range (apart from $y = \frac{\pi}{2}$ and $y = 0$) as $\tan^{-1} x$, but the value at $x = 0$ disturbs the symmetry.

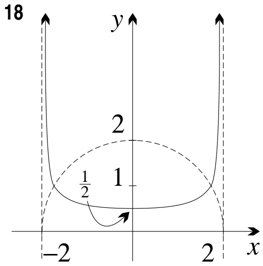
23(c) $\frac{x}{(x^2-1)\sqrt{x^2-2}}$
(d) domain: $x \ge 1$ or $x \le -1$, range: $0 \le y \le \pi$ excluding $y = \frac{\pi}{2}$, point symmetry about $(0, \frac{\pi}{2})$



Exercise **1E** (Page 28)

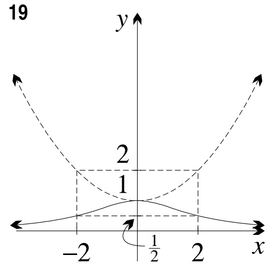
2(a) $\cos^{-1} x + C$ **(b)** $\sin^{-1} \frac{x}{2} + C$ **(c)** $\frac{1}{3}\tan^{-1} \frac{x}{3} + C$
(d) $\sin^{-1} \frac{3x}{2} + C$ **(e)** $\frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + C$
(f) $\cos^{-1} \frac{x}{\sqrt{5}} + C$
3(a) $\frac{\pi}{2}$ **(b)** $\frac{\pi}{8}$ **(c)** $\frac{\pi}{4}$ **(d)** $\frac{\pi}{12}$ **(e)** $\frac{\pi}{6}$ **(f)** $\frac{5\pi}{12}$
4(a) $y = \sin^{-1} x + \pi$ **(b)** $y = \tan^{-1} \frac{x}{4} + \frac{\pi}{4}$
5(a) $\frac{\pi}{3}$ **(b)** $\frac{\pi}{4}$
6(a) $\frac{1}{2}\sin^{-1} 2x + C$ **(b)** $\frac{1}{4}\tan^{-1} 4x + C$
(c) $\frac{1}{\sqrt{2}}\cos^{-1} \sqrt{2}x + C$ **(d)** $\frac{1}{3}\sin^{-1} \frac{3x}{2} + C$
(e) $\frac{1}{15}\tan^{-1} \frac{3x}{5} + C$ **(f)** $\frac{1}{2}\cos^{-1} \frac{2x}{\sqrt{3}} + C$
7(a) $\frac{\pi}{18}$ **(b)** $\frac{\pi}{12}$ **(c)** $\frac{2\pi}{9}\sqrt{3}$ **(d)** $\frac{5\pi}{24}$ **(e)** $\frac{\pi}{12}\sqrt{3}$
(f) $\frac{\pi}{120}\sqrt{10}$
9(c) $(\frac{\pi}{12} + \frac{1}{2}\sqrt{3} - 1)$ unit^2
10(b) $(1 - \frac{1}{2}\sqrt{3})$ unit^2
11(b) $\frac{\pi}{2}$
12(a) $\frac{6x^2}{4+x^6}$ **(b)** $\frac{1}{6}\tan^{-1} \frac{x^3}{2} + C$
13(a) $\frac{\pi^2}{4\sqrt{7}}$ unit^3 **(b)** $\frac{\pi^2}{8}$ unit^3
14(b) $\tan^{-1}(x+3) + C$
15(a) $\tan^{-1} x + \frac{x}{1+x^2}$ **(b)** $\frac{\pi}{4} - \frac{1}{2}\ln 2$
16(a) 0 **(b)** 0 **(c)** $\frac{3\pi}{4}$ **(d)** 0 **(e)** 0 **(f)** 18π
17(a)(i) 0 **(b)(i)** $f(0) = 0$ and $f'(x) < 0$ for $x > 0$.
(ii) $\frac{\pi-2}{8}$

18



(c) domain: $-2 \le x \le 2$, range: $y \ge \frac{1}{2}$, even
(d) $\frac{\pi}{3}$ unit^2
(e) π unit^2

19

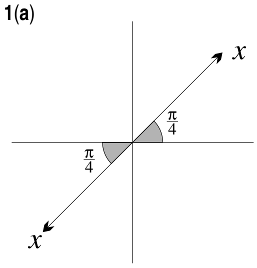


(a) The y-axis, since it's an even function.
(b) domain: all real x, range: $0 < y \le 1$
(d) 0 **(e)** π unit^2
(f) $4\tan^{-1} \frac{a}{2}$ unit^2
(g) 2π unit^2

21(b) $0{\cdot}153$ unit^2
22(a) $\frac{8011}{10\,200}$ **(b)** $I = \frac{\pi}{4}$, four decimal places
24(a) $2\tan^{-1}\sqrt{x} + C$ **(b)** $\tan^{-1} e - \frac{\pi}{4}$
25(g) $\pi \doteqdot 3{\cdot}092$, error $\doteqdot 0{\cdot}050$
26(a) $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 + \cdots + \tan^{-1} n$
(b) $x\tan^{-1} x - \frac{1}{2}\ln(1+x^2)$

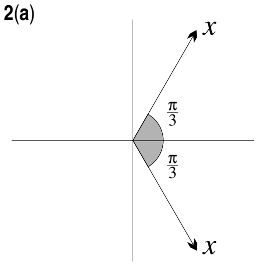
Exercise **1F** (Page 35)

1(a)



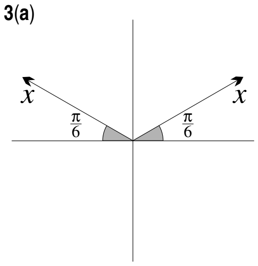
(b) $\frac{\pi}{4}$, $\frac{5\pi}{4}$, $\frac{9\pi}{4}$, $\frac{13\pi}{4}$, $\frac{17\pi}{4}$ or $\frac{21\pi}{4}$
(c) $-\frac{3\pi}{4}$, $-\frac{7\pi}{4}$, $-\frac{11\pi}{4}$, $-\frac{15\pi}{4}$, $-\frac{19\pi}{4}$ or $-\frac{23\pi}{4}$
(d) $x = n\pi + \frac{\pi}{4}$, where $n \in \mathbf{Z}$.

2(a)



(b) $\frac{\pi}{3}$, $\frac{5\pi}{3}$, $\frac{7\pi}{3}$, $\frac{11\pi}{3}$, $\frac{13\pi}{3}$ or $\frac{17\pi}{3}$
(c) $-\frac{\pi}{3}$, $-\frac{5\pi}{3}$, $-\frac{7\pi}{3}$, $-\frac{11\pi}{3}$, $-\frac{13\pi}{3}$ or $-\frac{17\pi}{3}$
(d) $x = 2n\pi + \frac{\pi}{3}$ or $2n\pi - \frac{\pi}{3}$, where $n \in \mathbf{Z}$.

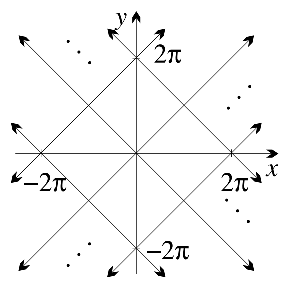
3(a)



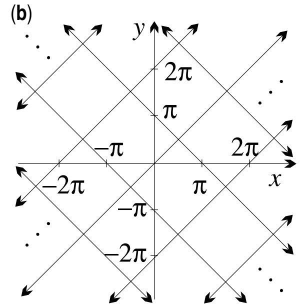
(b) $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{13\pi}{6}$, $\frac{17\pi}{6}$, $\frac{25\pi}{6}$ or $\frac{29\pi}{6}$
(c) $-\frac{7\pi}{6}$, $-\frac{11\pi}{6}$, $-\frac{19\pi}{6}$, $-\frac{23\pi}{6}$, $-\frac{31\pi}{6}$ or $-\frac{35\pi}{6}$
(d) $x = \frac{\pi}{6} + 2n\pi$ or $\frac{5\pi}{6} + 2n\pi$, where $n \in \mathbf{Z}$. [Alternatively, $x = m\pi + (-1)^m\frac{\pi}{6}$, where $m \in \mathbf{Z}$.]

ISBN: 978-1-107-61604-2 Cambridge University Press

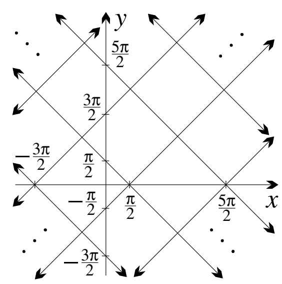
4(a) $x = n\pi + \frac{\pi}{3},\ n \in \mathbf{Z}$ **(b)** $x = 2n\pi \pm \frac{\pi}{4},\ n \in \mathbf{Z}$ **(c)** $x = 2n\pi + \frac{\pi}{3}$ or $x = 2n\pi + \frac{2\pi}{3},\ n \in \mathbf{Z}$. [Alternatively, $x = m\pi + (-1)^m \frac{\pi}{3},\ m \in \mathbf{Z}$.]
(d) $x = n\pi - \frac{\pi}{4},\ n \in \mathbf{Z}$ **(e)** $x = 2n\pi \pm \frac{2\pi}{3},\ n \in \mathbf{Z}$
(f) $x = 2n\pi - \frac{\pi}{6}$ or $x = 2n\pi + \frac{7\pi}{6}$. [Alternatively, $x = m\pi - (-1)^m \frac{\pi}{6} = m\pi + (-1)^{m+1}\frac{\pi}{6},\ m \in \mathbf{Z}$.]
5(a) $\theta = 2n\pi \pm \frac{\pi}{6},\ n \in \mathbf{Z}$ **(b)** $\theta = n\pi + \frac{\pi}{4},\ n \in \mathbf{Z}$
(c) $\theta = 2n\pi + \frac{\pi}{5}$ or $\theta = 2n\pi + \frac{4\pi}{5}$. [Alternatively, $\theta = m\pi + (-1)^m \frac{\pi}{5},\ m \in \mathbf{Z}$.]
(d) $\theta = 2n\pi + \frac{4\pi}{3}$ or $\theta = 2n\pi - \frac{\pi}{3}$. [Alternatively, $\theta = m\pi + (-1)^m \frac{4\pi}{3},\ m \in \mathbf{Z}$.]
(e) $\theta = n\pi - \frac{\pi}{3},\ n \in \mathbf{Z}$ **(f)** $\theta = 2n\pi \pm \frac{5\pi}{6},\ n \in \mathbf{Z}$
6(a) $x = n\pi,\ n \in \mathbf{Z}$ **(b)** $x = 2n\pi,\ n \in \mathbf{Z}$
(c) $x = n\pi,\ n \in \mathbf{Z}$ **(d)** $x = 2n\pi + \frac{\pi}{2},\ n \in \mathbf{Z}$
(e) $x = 2n\pi + \frac{\pi}{2},\ n \in \mathbf{Z}$ **(f)** $x = 2n\pi - \frac{\pi}{2},\ n \in \mathbf{Z}$
7(a)(i) $x = n\pi,\ n \in \mathbf{Z}$ **(ii)** $x = -\pi,\ 0$ or π
(b)(i) $x = \frac{\pi}{2} + 4n\pi$ or $x = \frac{3\pi}{2} + 4n\pi$. [Alternatively, $x = 2m\pi + (-1)^m \frac{\pi}{2},\ m \in \mathbf{Z}$.]
(ii) $x = \frac{\pi}{2}$ **(c)(i)** $x = \frac{n\pi}{3} + \frac{\pi}{18},\ n \in \mathbf{Z}$
(ii) $x = -\frac{17\pi}{18},\ -\frac{11\pi}{18},\ -\frac{5\pi}{18},\ \frac{\pi}{18},\ \frac{7\pi}{18}$ or $\frac{13\pi}{18}$
(d)(i) $x = n\pi + \frac{\pi}{4},\ n \in \mathbf{Z}$ **(ii)** $x = -\frac{3\pi}{4}$ or $\frac{\pi}{4}$
(e)(i) $x = 2n\pi + \frac{7\pi}{12}$ or $2n\pi - \frac{11\pi}{12},\ n \in \mathbf{Z}$
(ii) $x = -\frac{11\pi}{12}$ or $\frac{7\pi}{12}$ **(f)(i)** $x = \frac{n\pi}{2} - \frac{\pi}{12},\ n \in \mathbf{Z}$
(ii) $x = -\frac{7\pi}{12},\ -\frac{\pi}{12},\ \frac{5\pi}{12}$ or $\frac{11\pi}{12}$
(g)(i) $x = n\pi \pm \frac{\pi}{10},\ n \in \mathbf{Z}$
(ii) $x = -\frac{9\pi}{10},\ -\frac{\pi}{10},\ \frac{\pi}{10}$ or $\frac{9\pi}{10}$
(h)(i) $x = \frac{\pi}{6} + \frac{2}{3}n\pi$. [Alternatively, $x = \frac{m\pi}{3} + (-1)^m \frac{\pi}{6},\ m \in \mathbf{Z}$.]
(ii) $x = -\frac{\pi}{2},\ \frac{\pi}{6}$ or $\frac{5\pi}{6}$ **(i)(i)** $x = \frac{n\pi}{4} + \frac{\pi}{12},\ n \in \mathbf{Z}$
(ii) $x = -\frac{11\pi}{12},\ -\frac{2\pi}{3},\ -\frac{5\pi}{12},\ -\frac{\pi}{6},\ \frac{\pi}{12},\ \frac{\pi}{3},\ \frac{7\pi}{12}$ or $\frac{5\pi}{6}$
(j)(i) $x = n\pi + \frac{3\pi}{8},\ n \in \mathbf{Z}$ **(ii)** $x = -\frac{5\pi}{8}$ or $\frac{3\pi}{8}$
(k)(i) $x = 2n\pi + \frac{5\pi}{7}$ or $2n\pi - \frac{3\pi}{7},\ n \in \mathbf{Z}$
(ii) $x = -\frac{3\pi}{7}$ or $\frac{5\pi}{7}$
(l)(i) $x = -\frac{\pi}{5} + n\pi$ or $x = \frac{2\pi}{5} + n\pi$. [Alternatively, if m is even, $x = \frac{m\pi}{2} - \frac{\pi}{5}$; if m is odd, $x = \frac{m\pi}{2} - \frac{\pi}{10}$.] **(ii)** $x = -\frac{3\pi}{5},\ -\frac{\pi}{5},\ \frac{2\pi}{5}$ or $\frac{4\pi}{5}$
8(a)(i) $\theta = n\pi$ or $\theta = \frac{3\pi}{2} + 2n\pi,\ n \in \mathbf{Z}$. [Alternatively, $\theta = m\pi$ or $m\pi + (-1)^{m+1}\frac{\pi}{2},\ m \in \mathbf{Z}$.]
(ii) $\theta = -\pi,\ -\frac{\pi}{2},\ 0$ or π **(b)(i)** $\theta = n\pi + \frac{\pi}{2}$ or $\theta = \frac{\pi}{6} + 2n\pi$ or $\theta = \frac{5\pi}{6} + 2n\pi,\ n \in \mathbf{Z}$. [Alternatively, $\theta = 2m\pi \pm \frac{\pi}{2}$ or $m\pi + (-1)^m \frac{\pi}{6},\ m \in \mathbf{Z}$.]
(ii) $\theta = -\frac{\pi}{2},\ \frac{\pi}{6},\ \frac{\pi}{2}$ or $\frac{5\pi}{6}$ **(c)(i)** $\theta = n\pi$ or $n\pi + \frac{\pi}{3}$, $n \in \mathbf{Z}$ **(ii)** $\theta = -\pi,\ -\frac{2\pi}{3},\ 0,\ \frac{\pi}{3}$ or π
(d)(i) $\theta = 2n\pi \pm \pi$ or $2n\pi \pm \frac{2\pi}{3},\ n \in \mathbf{Z}$
(ii) $\theta = -\pi,\ -\frac{2\pi}{3},\ \frac{2\pi}{3}$ or π **(e)(i)** $\theta = \frac{n\pi}{2} - \frac{\pi}{6},\ n \in \mathbf{Z}$
(ii) $\theta = -\frac{2\pi}{3},\ -\frac{\pi}{6},\ \frac{\pi}{3}$ or $\frac{5\pi}{6}$
(f)(i) $\theta = \frac{n\pi}{2}$ or $\frac{n\pi}{2} + \frac{\pi}{8},\ n \in \mathbf{Z}$
(ii) $\theta = -\pi,\ -\frac{7\pi}{8},\ -\frac{\pi}{2},\ -\frac{3\pi}{8},\ 0,\ \frac{\pi}{8},\ \frac{\pi}{2},\ \frac{5\pi}{8}$ or π
9(c) $0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3}$ or 2π
10(b) $0,\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \pi,\ \frac{5\pi}{4},\ \frac{7\pi}{4}$ or 2π
11(c) $\frac{\pi}{8},\ \frac{5\pi}{8},\ \frac{3\pi}{4},\ \frac{9\pi}{8},\ \frac{13\pi}{8}$ or $\frac{7\pi}{4}$
12(a) $x = 0,\ \frac{\pi}{6},\ \frac{\pi}{2},\ \frac{5\pi}{6}$ or π
(b) $x = 0,\ \frac{\pi}{3},\ \frac{\pi}{2},\ \frac{2\pi}{3}$ or π
(c) $x = \frac{\pi}{12},\ \frac{\pi}{8},\ \frac{5\pi}{12},\ \frac{5\pi}{8}$ or $\frac{3\pi}{4}$
(d) $x = \frac{\pi}{12},\ \frac{3\pi}{8},\ \frac{5\pi}{12},\ \frac{3\pi}{4}$ or $\frac{7\pi}{8}$
13(a)



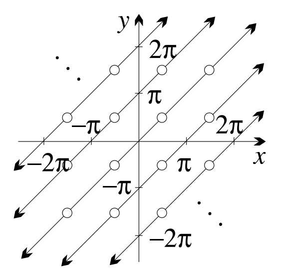
(b)



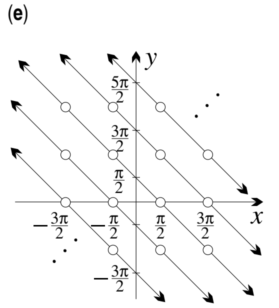
(c)



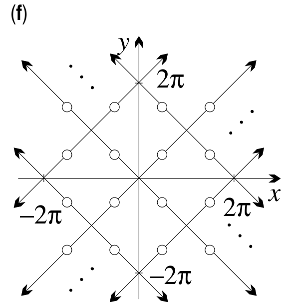
(d)



(e)



(f)



in x-axis: (a), (c), (f); in y-axis: (a), (f); in $y = x$: (a), (b), (d), (f)

ISBN: 978-1-107-61604-2 Cambridge University Press

Chapter Two

Exercise 2A (Page 40)

1(a) $\cos 2\theta$ **(b)** $\sin 40°$ **(c)** $\tan 50°$
(d) $\cos 70°$ **(e)** $\sin 6\alpha$ **(f)** $\frac{1}{\tan\theta} = \cot\theta$
2(a) $\sin 4\theta$ **(b)** $\cos x$ **(c)** $\cos 6\alpha$
(d) $\tan 70°$ **(e)** $\cos 50°$ **(f)** $\tan 8x$
3(a) $\frac{4}{5}$ **(b)** $\frac{7}{25}$ **(c)** $-\frac{16}{65}$ **(d)** $\frac{120}{169}$ **(e)** $\frac{24}{7}$ **(f)** $\frac{33}{56}$
5(a) $\sin 30° = \frac{1}{2}$ **(b)** $\cos 30° = \frac{1}{2}\sqrt{3}$
(c) $\tan 135° = -1$ **(d)** $\cos 45° = \frac{1}{2}\sqrt{2}$
(e) $\frac{1}{2}\sin\frac{\pi}{6} = \frac{1}{4}$ **(f)** $\sin\frac{2\pi}{3} = \frac{1}{2}\sqrt{3}$
(g) $\cos\frac{7\pi}{6} = -\frac{1}{2}\sqrt{3}$ **(h)** $\tan\frac{4\pi}{3} = \sqrt{3}$ **(i)** $\frac{1}{4}\sqrt{2}$
(j) -1
6(a) $\frac{1}{2}\sin\theta$ **(b)** $\sin^2 x$ **(c)** $\cos^2 2x$ **(d)** $2\sin^2 3\theta$
(e) $\cos 20°$ **(f)** $2\cos^2\frac{\alpha}{2}$ **(g)** $\sin 5x$ **(h)** $\frac{1}{4}\sin^2 2\alpha$
7(a) $\frac{3}{\sqrt{10}}$ **(b)** $\frac{1}{\sqrt{10}}$ **(c)** $\frac{1}{3}$
9(a) $-\frac{4}{7}\sqrt{2}$ **(b)** $-\frac{9}{10}$
12(a) $\frac{1}{3}\sqrt{5}$ **(b)** $-\frac{1}{9}$ **(c)** $\frac{4}{9}\sqrt{5}$ **(d)** $\frac{7}{27}\sqrt{5}$ **(e)** $-\frac{8}{81}\sqrt{5}$
(f) $-\frac{79}{81}$ **(g)** $-\frac{7}{22}\sqrt{5}$ **(h)** $\frac{8}{79}\sqrt{5}$ **(i)** $\frac{1}{6}\sqrt{30}$ **(j)** $\frac{1}{5}\sqrt{5}$
15(a) $y = 2x^2 - 8x + 7$ **(b)** $y = \frac{2x-2}{2x-x^2}$ **(c)** $y = \frac{4-x^2}{4+x^2}$
(d) $9y^2 = 16x^2(9-x^2)$
16(a) $\frac{1}{2}\sqrt{2}$
18(a) $a^4 = 2a^2 - b^2$ **(b)** $a^2 + b^2 = 2(c+1)$
19(d) $\frac{\sqrt{5}-1}{4}$
20(b) $$\sin\frac{90°}{2^n} = \frac{1}{2}\underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{\cdots+\sqrt{2}}}}}}_{n \text{ terms}}.$$

Let $$T_n = 2^n\underbrace{\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{\cdots+\sqrt{2}}}}}}_{n \text{ terms}}.$$

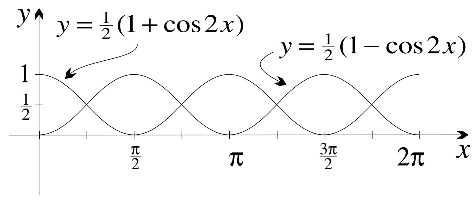
Since $\dfrac{\sin\theta}{\theta} \to 1$ as $\theta \to 0$, $T_n \to \pi$ as $n \to \infty$.
$T_4 \doteqdot 3.1365$, $T_8 \doteqdot 3{\cdot}141\,573$

Exercise 2B (Page 44)

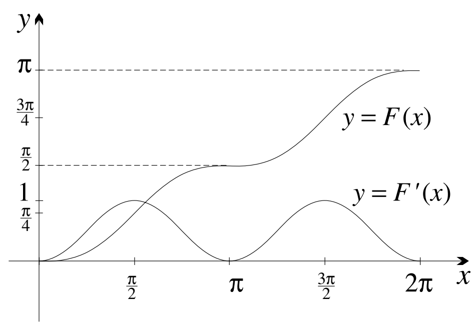
1(a) $\frac{2t}{1+t^2}$ **(b)** $\frac{1-t^2}{1+t^2}$ **(c)** $\frac{2t}{1-t^2}$ **(d)** $\frac{1+t^2}{1-t^2}$ **(e)** $\frac{2t^2}{1+t^2}$
(f) t
2(a) $\frac{1-t^2}{1+t^2}$ **(b)** $\frac{(1-t)^2}{1+t^2}$ **(c)** $\frac{1+t}{1-t}$
3(a) $\tan 20°$ **(b)** $\sin 20°$ **(c)** $\cos 20°$ **(d)** $\sin 4x$
(e) $\tan 4x$ **(f)** $\cos 4x$
4(a) $\tan 30° = \frac{1}{3}\sqrt{3}$ **(b)** $\sin 30° = \frac{1}{2}$
(c) $\cos 150° = -\frac{1}{2}\sqrt{3}$ **(d)** $\sin 225° = -\frac{1}{2}\sqrt{2}$
(e) $\cos\frac{3\pi}{4} = -\frac{1}{2}\sqrt{2}$ **(f)** $\tan\frac{11\pi}{6} = -\frac{1}{3}\sqrt{3}$
6(b)(ii) $\sqrt{2} - 1 = \tan 22\frac{1}{2}°$,
since $\tan 45° = \tan 225° = 1$.
8(a) $-\frac{3}{4}$ **(b)** $-\frac{3}{5}$ **(c)** $\frac{4}{5}$ **(d)** $3 + \sqrt{10}$
9(a)(i) $\cos\theta = 2\cos^2\frac{1}{2}\theta - 1$
(b)(i) $\sin\theta = 2\sin\frac{1}{2}\theta\cos\frac{1}{2}\theta$

Exercise 2C (Page 47)

1(a) $\frac{56}{65}$ **(b)** $\frac{33}{65}$
2(a) $\frac{a}{c}$ **(c)** $\frac{a+b}{c}$
3(a) $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$ **(b)** $\tan\theta = \frac{h}{30}$, $\tan 2\theta = \frac{h}{10}$
4(a) $\frac{a}{x}$, $\frac{b}{x}$ **(d)** The expression under the square root in (c) is not positive unless $b > 2a$.
5(a) $\frac{\pi}{2}$ **(b)** $\frac{1}{8}(\pi+2)$ **(c)** $\frac{1}{12}(\pi-3)$ **(d)** $\frac{1}{32}(\pi+2\sqrt{2})$
(e) $\frac{1}{24}(4\pi+9)$ **(f)** $\frac{1}{24}(2\pi - 3\sqrt{3})$
6(b)



7 $\cos x\sin x = \frac{1}{2}\sin 2x$, and $-\frac{1}{2} \le \frac{1}{2}\sin 2x \le \frac{1}{2}$.
8(b) $\dfrac{\sin\theta}{\sin\phi}$
12(b) $\cos^4 x = \frac{3}{8} + \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$
(c)(i) $\frac{3\pi}{8}$ **(ii)** $\frac{1}{32}(3\pi+8)$
16(b)(i) $x = 0, \pi$ or 2π **(ii)** $0 < x < \pi$ or $\pi < x < 2\pi$
(iii) no values of x
(c) It is because $-\frac{1}{4} \le \frac{1}{4}\sin 2x \le \frac{1}{4}$.



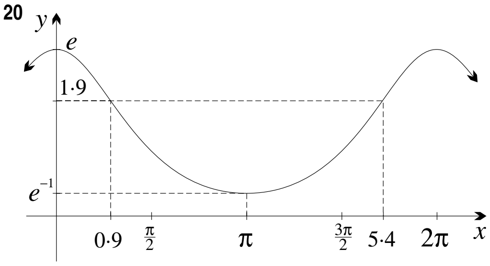
(d) $(\frac{\pi}{2}, \frac{\pi}{4})$ and $(\frac{3\pi}{2}, \frac{3\pi}{4})$ are points of inflexion, while $(0,0)$, $(\pi, \frac{\pi}{2})$ and $(2\pi, \pi)$ are stationary (or horizontal) points of inflexion.
(f)(i) $k = 3\pi$ **(ii)** $k = n\pi$, where n is an integer.

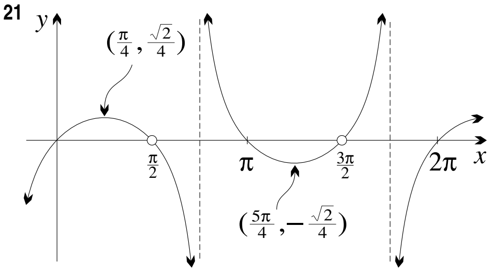
Exercise 2D (Page 53)

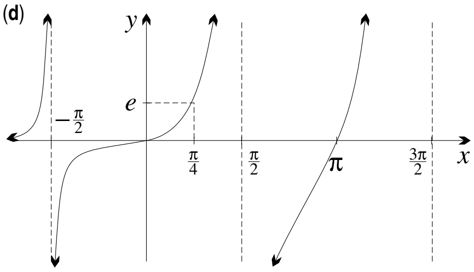
1(a) $x = \frac{\pi}{4}$ or $\frac{3\pi}{4}$ **(b)** $x = \frac{2\pi}{3}$ or, $\frac{4\pi}{3}$
(c) $x = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ **(d)** $x = \frac{\pi}{2}$ or $\frac{3\pi}{2}$
(e) $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}$ or $\frac{11\pi}{6}$ **(f)** $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}$ or $\frac{7\pi}{4}$
2(a) $\alpha = 30°, 120°, 210°$ or $300°$ **(b)** $\alpha = 0°, 180°$ or $360°$ **(c)** $\alpha = 10°, 50°, 130°, 170°, 250°$ or $290°$
(d) $\alpha = 45°, 105°, 165°, 225°, 285°$ or $345°$
3(a) $\theta = \frac{\pi}{2}$ or $\frac{5\pi}{6}$ **(b)** $\theta = \frac{7\pi}{12}$ or $\frac{11\pi}{12}$

ISBN: 978-1-107-61604-2

(c) $\theta = \frac{3\pi}{8},\ \frac{5\pi}{8},\ \frac{11\pi}{8}$ or $\frac{13\pi}{8}$
(d) $\theta = \frac{\pi}{4},\ \frac{7\pi}{12},\ \frac{5\pi}{4}$ or $\frac{19\pi}{12}$
4(a) $x = \frac{\pi}{3}$ or $\frac{4\pi}{3}$ **(b)** $x = \frac{\pi}{6},\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{11\pi}{6}$
(c) $x = \frac{\pi}{12},\ \frac{5\pi}{12},\ \frac{7\pi}{12},\ \frac{11\pi}{12},\ \frac{13\pi}{12},\ \frac{17\pi}{12},\ \frac{19\pi}{12}$ or $\frac{23\pi}{12}$
(d) $x = \frac{2\pi}{3}$ or $\frac{4\pi}{3}$
5(a) $\alpha = 0°, 90°, 180°$ or $360°$ **(b)** $\alpha = 60°$ or $300°$ **(c)** $\alpha = 45°,\ 90°,\ 225°$ or $270°$ **(d)** $\alpha = 75°58'$, $135°$, $255°58'$ or $315°$ **(e)** $\alpha = 90°, 210°$ or $330°$ **(f)** $\alpha = 0°, 60°, 300°$ or $360°$ **(g)** $\alpha = 63°26', 135°$, $243°26'$ or $315°$ **(h)** $\alpha = 45°$ or $225°$ **(i)** $\alpha = 15°$, $75°, 105°, 165°, 195°, 255°, 285°$ or $345°$
(j) $\alpha = 180°$ or $240°$
6(a) $\theta = \frac{\pi}{3}$ or $\frac{4\pi}{3}$ **(b)** $\theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ **(c)** $\theta = \frac{\pi}{9},\ \frac{5\pi}{9}$, $\frac{7\pi}{9},\ \frac{11\pi}{9},\ \frac{13\pi}{9}$ or $\frac{17\pi}{9}$ **(d)** $\theta = 0,\ \frac{\pi}{2},\ \pi,\ \frac{3\pi}{2}$ or 2π
7(a) $x = 0,\ \frac{\pi}{3},\ \pi,\ \frac{5\pi}{3}$ or 2π **(b)** $x = \frac{\pi}{6},\ \frac{5\pi}{6}$ or $\frac{3\pi}{2}$
(c) $x = 0,\ \frac{\pi}{3},\ \frac{2\pi}{3},\ \pi,\ \frac{4\pi}{3},\ \frac{5\pi}{3}$ or 2π
(d) $x = 0,\ \frac{\pi}{4},\ \frac{3\pi}{4},\ \pi,\ \frac{5\pi}{4},\ \frac{7\pi}{4}$ or 2π
8(a) $0 < x < \pi$ **(b)** $0 < x < \frac{\pi}{2}$ or $\pi < x < \frac{3\pi}{2}$
(c) $\frac{\pi}{3} \le x \le \frac{5\pi}{3}$ **(d)** $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$ or $\frac{7\pi}{6} \le x \le \frac{11\pi}{6}$
(e) $\frac{7\pi}{12} \le x \le \frac{23\pi}{12}$ **(f)** $\frac{\pi}{8} \le x < \frac{\pi}{4}$ or $\frac{5\pi}{8} \le x < \frac{3\pi}{4}$ or $\frac{9\pi}{8} \le x < \frac{5\pi}{4}$ or $\frac{13\pi}{8} \le x < \frac{7\pi}{4}$
9(a) $A = 120°$ or $240°$ **(b)** $A = 0°,\ 60°,\ 300°$ or $360°$ **(c)** $A = 45°,\ 161°34',\ 225°$ or $341°34'$ **(d)** $A = 30°,\ 60°,\ 210°$ or $240°$ **(e)** $A = 60°$, $90°,\ 120°,\ 240°,\ 270°$ or $300°$ **(f)** $A = 45°, 60°$, $120°, 135°, 225°, 240°, 300°$ or $315°$ **(g)** $A = 0°$, $180°,\ 210°,\ 330°$ or $360°$ **(h)** $A = 60°$ or $300°$ **(i)** $A = 71°34', 135°, 251°34'$ or $315°$ **(j)** $A = 45°$, $60°, 120°, 135°, 225°, 240°, 300°$ or $315°$
10(a) $\theta = 90°, 194°29', 270°$ or $345°31'$
(b) $\theta = 60°,\ 120°,\ 240°$ or $300°$ **(c)** $\theta = 60°$ or $300°$ **(d)** $\theta = 22°30',\ 67°30',\ 112°30',\ 157°30'$, $202°30', 247°30', 292°30'$ or $337°30'$
(e) $\theta = 41°49', 138°11', 210°$ or $330°$
(f) $\theta = 54°44', 125°16', 234°44'$ or $305°16'$
(g) $\theta = 106°16'$ **(h)** $\theta = 0°,\ 60°,\ 300°$ or $360°$
(i) $\theta = 30°, 90°, 150°, 210°$ or $330°$
(j) $\theta = 45°, 63°26', 225°$ or $243°26'$
11(b) $x = \frac{\pi}{6},\ \frac{\pi}{4},\ \frac{7\pi}{6}$ or $\frac{5\pi}{4}$.
(c) $x = \frac{\pi}{3},\ \frac{3\pi}{4},\ \frac{4\pi}{3}$ or $\frac{7\pi}{4}$
12(a) $x = 2n\pi + \frac{2\pi}{3}$ or $x = 2n\pi - \frac{2\pi}{3}$ or $2n\pi$, where $n \in \mathbf{Z}$. **(b)** $x = \frac{n\pi}{2}$ or $2n\pi + \frac{\pi}{6}$ or $2n\pi - \frac{\pi}{6}$, where $n \in \mathbf{Z}$. **(c)** $x = n\pi$ or $n\pi + (-1)^n\frac{\pi}{6}$, where $n \in \mathbf{Z}$.
(d) $x = n\pi - \frac{\pi}{4}$, where $n \in \mathbf{Z}$.
13(b) $x = 0,\ \frac{2\pi}{5},\ \frac{4\pi}{5},\ \frac{6\pi}{5},\ \frac{8\pi}{5}$ or 2π
14(a) $x = \frac{\pi}{3},\ \frac{2\pi}{3},\ \frac{4\pi}{3}$ or $\frac{5\pi}{3}$
(b) $x = 0,\ \frac{\pi}{4},\ \pi,\ \frac{5\pi}{4}$ or 2π

(c) $x = \frac{\pi}{4},\ \frac{7\pi}{12},\ \frac{5\pi}{4}$ or $\frac{19\pi}{12}$ **(d)** $x = \frac{\pi}{2},\ \frac{7\pi}{6},\ \frac{3\pi}{2}$ or $\frac{11\pi}{6}$
(e) $x = \frac{\pi}{3}, \pi$ or $\frac{5\pi}{3}$ **(f)** $x = 0,\ \frac{3\pi}{4},\ \pi,\ \frac{7\pi}{4}$ or 2π
15(b) $\theta = \frac{\pi}{12}$ or $\frac{5\pi}{12}$
16(b) $x = 36°, 108°, 252°$ or $324°$
17(b) $\theta = 0,\ \frac{\pi}{8},\ \frac{\pi}{2},\ \frac{5\pi}{8}$ or π
18(a) $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ or $\frac{5\pi}{4} \le x \le \frac{7\pi}{4}$ **(b)** $0 < x < \frac{\pi}{4}$ or $\pi < x < \frac{5\pi}{4}$ **(c)** $\frac{\pi}{4} \le x \le \frac{3\pi}{4}$ or $\frac{5\pi}{4} \le x \le \frac{7\pi}{4}$
(d) $\frac{\pi}{3} \le x \le \frac{5\pi}{3}$ **(e)** $\pi \le x \le \frac{7\pi}{6}$ or $\frac{11\pi}{6} \le x \le 2\pi$
(f) $\frac{\pi}{3} \le x < \frac{\pi}{2}$ or $\frac{\pi}{2} < x \le \pi$ or $\frac{4\pi}{3} \le x < \frac{3\pi}{2}$ or $\frac{3\pi}{2} < x \le 2\pi$
19(a) $k = \frac{n\pi}{2}$, where n is an integer.
(b) $\frac{(2n-1)\pi}{2} < k < n\pi$, where n is an integer.
20


21


Because tan $\frac{\pi}{2}$ and tan $\frac{3\pi}{2}$ are undefined, and the function values do not approach ∞ or $-\infty$ as x approaches $\frac{\pi}{2}$ or $\frac{3\pi}{2}$.
22(b) 0 and π. The respective gradients are 1 and e^{π}.
(d)


23 $\sin 18° = \cos 72° = \frac{1}{4}(-1 + \sqrt{5})$,
$\sin 36° = \cos 54° = \frac{1}{4}\sqrt{10 - 2\sqrt{5}}$,
$\sin 54° = \cos 36° = \frac{1}{4}(1 + \sqrt{5})$,
$\sin 72° = \cos 18° = \frac{1}{4}\sqrt{10 + 2\sqrt{5}}$,

ISBN: 978-1-107-61604-2 Cambridge University Press

$\tan 18^\circ = \frac{1}{5}\sqrt{25 - 10\sqrt{5}}$, $\tan 36^\circ = \sqrt{5 - 2\sqrt{5}}$, $\tan 54^\circ = \frac{1}{5}\sqrt{25 + 10\sqrt{5}}$, $\tan 72^\circ = \sqrt{5 + 2\sqrt{5}}$

24(b) $\theta = 160^\circ 55'$ or $289^\circ 5'$

25(c) $x \doteqdot -2{\cdot}571$, $-1{\cdot}368$ or $3{\cdot}939$

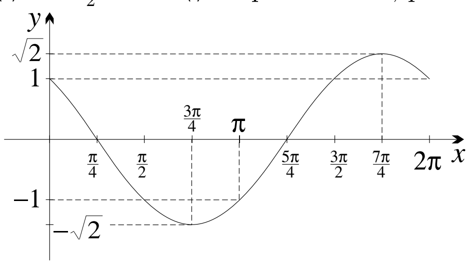
26(e) $x = \tan\frac{\pi}{10}$, $-\tan\frac{\pi}{10}$, $\tan\frac{3\pi}{10}$ or $-\tan\frac{3\pi}{10}$

Exercise **2E** (Page 60)

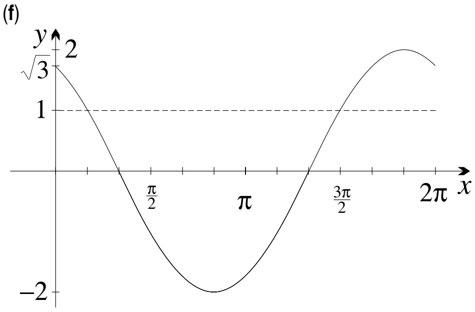
1(a) $R = 2$, $\alpha = \frac{\pi}{3}$ **(b)** $R = 3\sqrt{2}$, $\alpha = \frac{\pi}{4}$

2(a) $R = 13$, $\alpha \doteqdot 22^\circ 37'$ **(b)** $R = 2\sqrt{5}$, $\alpha \doteqdot 63^\circ 26'$

3(b) $A = \sqrt{2}$ **(c)** $\alpha = \frac{\pi}{4}$ **(d)** Maximum is $\sqrt{2}$, when $x = \frac{7\pi}{4}$. Minimum is $-\sqrt{2}$, when $x = \frac{3\pi}{4}$.
(e) $x = \frac{\pi}{2}$ or π **(f)** amplitude: $\sqrt{2}$, period: 2π



5(b) $B = 2$ **(c)** $\theta = \frac{\pi}{6}$ **(d)** Maximum is 2, when $x = -\frac{\pi}{6}$. Minimum is -2, when $x = \frac{5\pi}{6}$.
(e) $x = \frac{\pi}{6}$, $\frac{3\pi}{2}$
(f)



6(c) $x \doteqdot 126^\circ 52'$

7(b) $x = 90^\circ$ or $x \doteqdot 323^\circ 8'$

8(b) $x = 270^\circ$ or $x \doteqdot 306^\circ 52'$

9(a) $3\sin(x + \tan^{-1}\frac{2}{\sqrt{5}})$
(b) $x = 180^\circ$ or $x \doteqdot 276^\circ 23'$

10(a) $x \doteqdot 77^\circ 39'$ or $344^\circ 17'$
(b) $x \doteqdot 103^\circ 29'$ or $156^\circ 8'$
(c) $x \doteqdot 30^\circ 41'$ or $297^\circ 26'$
(d) $x \doteqdot 112^\circ 37'$ or $323^\circ 8'$

11(c) $x = 0$, $\frac{3\pi}{2}$, 2π

12(b) $x = 0$, $\frac{2\pi}{3}$, 2π

13(b) $x = 90^\circ$ or $x \doteqdot 298^\circ 4'$

14(b) $x = 180^\circ$ or $x \doteqdot 67^\circ 23'$

16(a) $x = 90^\circ$ or $x \doteqdot 12^\circ 41'$
(b) $x \doteqdot 36^\circ 52'$ or $241^\circ 56'$
(c) $x \doteqdot 49^\circ 48'$ or $197^\circ 35'$
(d) $x = 180^\circ$ or $x \doteqdot 280^\circ 23'$

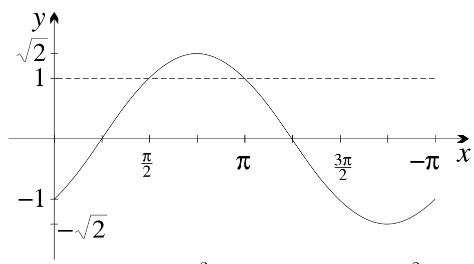
17(a) $A = 2$, $\alpha = \frac{5\pi}{6}$ **(b)** $A = 5\sqrt{2}$, $\alpha = \frac{5\pi}{4}$

18(a) $A = \sqrt{41}$, $\alpha \doteqdot 321^\circ 20'$
(b) $A = 5\sqrt{5}$, $\alpha \doteqdot 259^\circ 42'$

19(a)(i) $2\cos(x + \frac{11\pi}{6})$ **(ii)** $x = \frac{\pi}{2}$ or $\frac{11\pi}{6}$
(b)(i) $\sqrt{2}\sin(x + \frac{3\pi}{4})$ **(ii)** $x = 0$ or $\frac{3\pi}{2}$
(c)(i) $2\sin(x + \frac{5\pi}{3})$ **(ii)** $x = \frac{\pi}{6}$ or $\frac{3\pi}{2}$
(d)(i) $\sqrt{2}\cos(x - \frac{5\pi}{4})$ **(ii)** $x = \pi$ or $\frac{3\pi}{2}$

20(a)(i) $\sqrt{5}\sin(x + 116^\circ 34')$
(ii) $x = 270^\circ$ or $x \doteqdot 36^\circ 52'$
(b)(i) $5\cos(x - 3{\cdot}7851)$ **(ii)** $x \doteqdot 2{\cdot}63$ or $4{\cdot}94$

21(a)(ii) **(iii)** $\frac{\pi}{2} < x < \pi$ **(b)(i)** $\frac{\pi}{2} \le x \le \frac{11\pi}{6}$



(ii) $0 < x < \frac{\pi}{6}$ or $\frac{3\pi}{2} < x < 2\pi$ **(iii)** $\frac{2\pi}{3} < x < \pi$ or $\frac{5\pi}{3} < x < 2\pi$ **(iv)** $0 \le x \le \frac{\pi}{12}$ or $\frac{17\pi}{12} \le x \le 2\pi$

22(a) $x = \frac{7\pi}{12}$, $\frac{11\pi}{12}$ **(b)** $x = \frac{\pi}{3}$, $\frac{4\pi}{3}$
(c) $x = 0$, $\frac{\pi}{8}$, $\frac{\pi}{2}$, $\frac{5\pi}{8}$, π, $\frac{9\pi}{8}$, $\frac{3\pi}{2}$, $\frac{13\pi}{8}$, 2π

23(a) $x \doteqdot 313^\circ 36'$ **(b)** $x \doteqdot 79^\circ 6'$ or $218^\circ 59'$

24(b) $x \doteqdot 36^\circ 52'$

25 $\theta = 0$, $\frac{3\pi}{4}$, $\frac{3\pi}{2}$ or $\frac{7\pi}{4}$

26(b) $x = n\pi + \frac{\pi}{6}$ or $n\pi - \frac{\pi}{12}$, $n \in \mathbf{Z}$

27(b) $\sin x + \sqrt{3}\cos x = 2\sin(x - \frac{5\pi}{3})$ or $2\cos(x - \frac{\pi}{6})$ or $2\cos(x + \frac{11\pi}{6})$
(c) $\cos x - \sin x = \sqrt{2}\cos(x - \frac{7\pi}{4})$ or $\sqrt{2}\sin(x + \frac{3\pi}{4})$ or $\sqrt{2}\sin(x - \frac{5\pi}{4})$

30(b) $x = 2n\pi \pm \cos^{-1}\frac{c}{r} + \theta$, $n \in \mathbf{Z}$
(d)(i) $\angle MOP = \theta$ **(iii)** $\angle MOQ$ is obtained from $2n\pi + \cos^{-1}\frac{c}{r} + \theta$, while $\angle MOQ'$ is obtained from $2n\pi - \cos^{-1}\frac{c}{r} + \theta$. **(e)** $ON > OP$

Exercise **2F** (Page 65)

1(b)(i) $\cos 50^\circ + \cos 20^\circ$ **(ii)** $\sin 80^\circ - \sin 16^\circ$
(iii) $\sin 4\alpha + \sin 2\alpha$ **(iv)** $\cos 2y - \cos 2x$

2(b)(i) $2\cos 14^\circ \cos 2^\circ$ **(ii)** $2\cos 38^\circ \sin 18^\circ$
(iii) $2\sin 5x \cos x$ **(iv)** $-2\sin 2x \sin 3y$

3(a)(ii) $x = 0$, $\frac{\pi}{2}$ or π **(b)** $x = \frac{\pi}{4}$, $\frac{\pi}{2}$ or $\frac{3\pi}{4}$

4(a)(ii) $-\frac{1}{4}\cos 4x - \frac{1}{2}\cos 2x + C$
(b) $\frac{1}{4}\sin 4x + \frac{1}{2}\sin 2x + C$

6(a) $\frac{1}{24}(3\sqrt{3} - 4)$ **(b)** $\frac{1}{48}(3 - 2\sqrt{2})$

7(b)(i) 0 **(ii)** 30π **(iii)** 30π

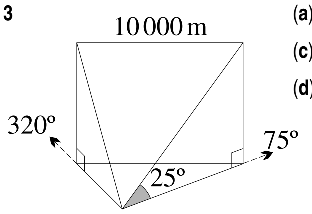
9(a) $2\sin 2x \cos x$ **(b)** $x = 0$, $\frac{\pi}{2}$, $\frac{2\pi}{3}$, π, $\frac{4\pi}{3}$ or $\frac{3\pi}{2}$

ISBN: 978-1-107-61604-2 Cambridge University Press

10(a) $x = \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}$ or $\frac{5\pi}{6}$
(b) $x = 0, \frac{\pi}{5}, \frac{3\pi}{5}, \frac{2\pi}{3}$ or π
(c) $x = \frac{\pi}{8}, \frac{\pi}{3}, \frac{3\pi}{8}, \frac{5\pi}{8}$ or $\frac{7\pi}{8}$
(d) $x = 0, \frac{2\pi}{5}, \frac{\pi}{2}$ or $\frac{4\pi}{5}$ **(e)** $x = 0, \frac{2\pi}{5}, \frac{\pi}{2}$ or $\frac{4\pi}{5}$
(f) $x = 0, \frac{\pi}{14}, \frac{3\pi}{14}, \frac{5\pi}{14}, \frac{\pi}{2}, \frac{9\pi}{14}, \frac{11\pi}{14}, \frac{13\pi}{14}$ or π
11(a) $x = \frac{\pi}{12}, \frac{3\pi}{8}, \frac{5\pi}{12}, \frac{3\pi}{4}$ or $\frac{7\pi}{8}$
(b) $x = \frac{2n\pi}{5} + \frac{\pi}{10}$ or $2n\pi + \frac{\pi}{2}$, where $n \in \mathbf{Z}$.
13(d) No. For example, substitute the values $n = 1$ and $\lambda = 1{\cdot}1$.

Exercise **2G** (Page 70)

1(a) $56°19'$ **(b)** $8{\cdot}8\,\text{cm}$ **(c)** $27°7'$
2(a) $39°52'$, $35°33'$ **(b)** 72 metres
3(b) 110 metres **(c)** $14°$
4(a) x **(g)** $35°16'$
5(b) $35°16'$
6(a) $63°26'$ **(b)** $54°44'$ **(c)** $53°8'$
7 $54°44'$
9(d) 5040 metres
12(c) $67°23'$
13



(a) 10 000 metres
(c) 3941 metres
(d) $54°$
14(a)(ii) $\cos^2\alpha + \cos^2\beta = 1$, where $\alpha + \beta = 90°$.
(b)(ii) $\sin^2\theta + \sin^2\phi = 1$, where $\theta + \phi = 90°$.
15 $16°16'$

Exercise **2H** (Page 75)

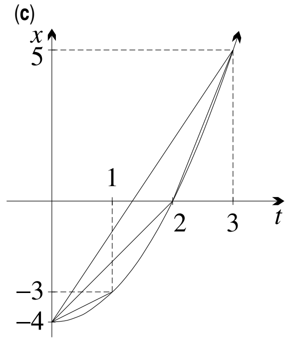
1(a) $h\cot 55°$
(b) It is the angle between south and east.
(d) 114 metres
2(b) 13 metres
3(a) $x\cot 27°$
4(c) 129 metres
5(a) $AT = h\operatorname{cosec} 55°$, $BT = h\operatorname{cosec} 40°$ **(b)** $90°$
(d) 52 metres
6(b) $PL = h\cot 9°$, $QL = h\cot 12°$
8(a) $y = h\cot\beta$
9(b)(i) $\sqrt{3}\,h,\ h,\ h$
(ii) $\cos\alpha = \frac{100}{h}$ or $\frac{80\,000 - h^2}{400h}$, $h = 200$ metres
10(a) $BD = \sqrt{3}\,h$, $CD = h$
11(a) $AC = 2\sqrt{a^2+b^2}$, $AM = \sqrt{a^2+b^2}$, $AT = \sqrt{a^2+b^2+h^2}$ **(b)** $\cos\alpha = \frac{-a^2+b^2+h^2}{a^2+b^2+h^2}$, $\cos\beta = \frac{a^2-b^2+h^2}{a^2+b^2+h^2}$, $\cos\theta = \frac{-a^2-b^2+h^2}{a^2+b^2+h^2}$
12(c) 17 metres
14(b) 535 metres
15(a) $PC = h$, $PD = \frac{1}{3}h\sqrt{3}$ **(c)** $305°$
16(b) $13°41'$
17(b)(i) The foot of the tower is equidistant from P, Q and R, the distance being $h\cot 30°$.

ISBN: 978-1-107-61604-2 Cambridge University Press

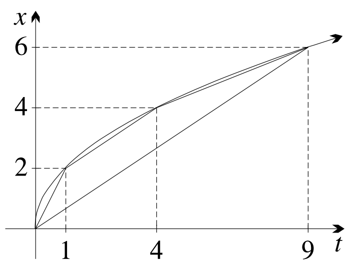
Chapter Three

Exercise 3A (Page 82)

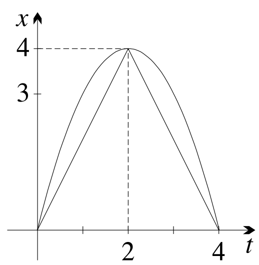
1(a) $x = -4,\ -3,\ 0,\ 5$ **(b)(i)** $1\,\text{m/s}$ **(ii)** $2\,\text{m/s}$ **(iii)** $3\,\text{m/s}$ **(iv)** $5\,\text{m/s}$ **(c)**



2(a) $t = 0,\ 1,\ 4,\ 9,\ 16$ **(b)(i)** $2\,\text{cm/s}$ **(ii)** $\frac{2}{3}\,\text{cm/s}$ **(iii)** $\frac{2}{5}\,\text{cm/s}$ **(iv)** $\frac{2}{3}\,\text{cm/s}$ **(c)** They are parallel.

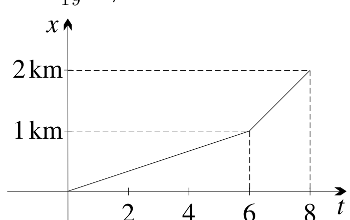


3(a) $x = 0,\ 3,\ 4,\ 3,\ 0$ **(b)(i)** $2\,\text{m/s}$ **(ii)** $-2\,\text{m/s}$ **(iii)** $0\,\text{m/s}$ **(d)** The total distance travelled is 8 metres. All three average speeds are $2\,\text{m/s}$.

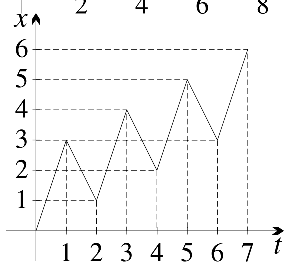


4(a)(i) $-1\,\text{m/s}$ **(ii)** $4\,\text{m/s}$ **(iii)** $-2\,\text{m/s}$ **(b)** 40 metres, $1\frac{1}{3}\,\text{m/s}$ **(c)** 0 metres, $0\,\text{m/s}$ **(d)** $2\frac{2}{19}\,\text{m/s}$

5(a)(i) 6 minutes **(ii)** 2 minutes **(c)** $15\,\text{km/hr}$ **(d)** $20\,\text{km/hr}$



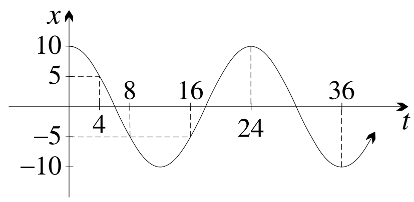
6(b) 7 hours **(c)** $2\frac{4}{7}\,\text{m/hr}$ **(d)** those between 1 and 2 metres high or between 4 and 5 metres high



7(a)(i) once **(ii)** three times **(iii)** twice **(b)(i)** when $t = 4$ and when $t = 14$ **(ii)** when $0 \le t < 4$ and when $4 < t < 14$ **(c)** It rises 2 metres, at $t = 8$. **(d)** It sinks 1 metre, at $t = 17$. **(e)** As $t \to \infty$, $x \to 0$, meaning that eventually it ends up at the surface. **(f)(i)** $-1\,\text{m/s}$ **(ii)** $\frac{1}{2}\,\text{m/s}$ **(iii)** $-\frac{1}{3}\,\text{m/s}$ **(g)(i)** 4 metres **(ii)** 6 metres **(iii)** 9 metres **(iv)** 10 metres **(h)(i)** $1\,\text{m/s}$ **(ii)** $\frac{3}{4}\,\text{m/s}$ **(iii)** $\frac{9}{17}\,\text{m/s}$

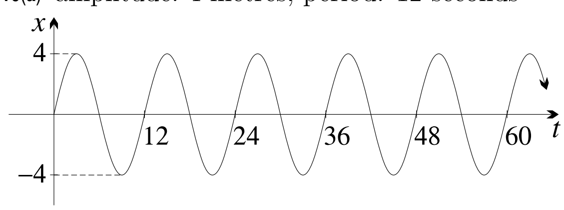
8(b) $t = 4,\ t = 20$ **(c)** $8 < t < 16$ **(d)** $12\,\text{cm},\ \frac{3}{4}\,\text{cm/s}$ **(e)(i)** $t = \frac{8}{\pi}\sin^{-1}\frac{1}{3} \doteqdot 0{\cdot}865$, and $t = 8 - \frac{8}{\pi}\sin^{-1}\frac{1}{3} \doteqdot 7{\cdot}13$ **(ii)** $0{\cdot}865$ seconds and $7{\cdot}13$ seconds, 4 metres, $0{\cdot}638\,\text{m/s}$

9 amplitude: 10 metres, period: 24 seconds



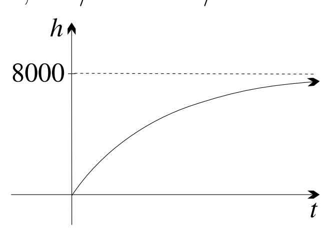
(c) The maximum is 20 metres, when $t = 12,\ 36$ and 60. **(d)** 100 metres, $1\frac{2}{3}\,\text{m/s}$ **(e)** It is at $x = 0$ when $t = 6,\ 18,\ 30,\ 42$ and 54. **(f)** $10,\ 5,\ -5,\ -10,\ -5,\ 5,\ 10$ **(g)** $-1\frac{1}{4}\,\text{m/s},\ -2\frac{1}{2}\,\text{m/s},\ -1\frac{1}{4}\,\text{m/s}$ **(h)** $x = -5$ when $t = 8$ or $t = 16$, $x < -5$ when $8 < t < 16$.

10(a) amplitude: 4 metres, period: 12 seconds

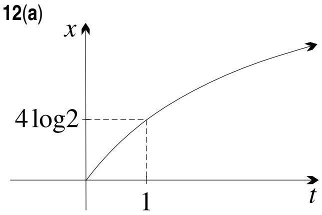


(b) 10 times **(c)** $t = 3,\ 15,\ 27,\ 39,\ 51$ **(d)** It travels 16 metres with average speed $1\frac{1}{3}\,\text{m/s}$. **(e)** $x = 0$, $x = 2$ and $x = 4$, $2\,\text{m/s}$ and $1\,\text{m/s}$

11(a) When $t = 0$, $h = 0$. As $t \to \infty$, $h \to 8000$. **(b)** 0, 3610, 5590, 6678 **(d)** $361\,\text{m/min}$, $198\,\text{m/min}$, $109\,\text{m/min}$ **(f)** 77 minutes



12(a)

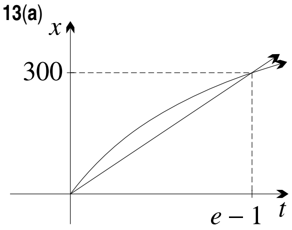


(b) When $t = 2$, $x = 4\log 3 \doteqdot 4{\cdot}394$, the average speed is $2\log 3 \doteqdot 2{\cdot}197\,\text{m/s}$,

ISBN: 978-1-107-61604-2 Cambridge University Press

$\angle AOP = 2\log 3 \doteqdot 2{\cdot}197$ radians,
$AP^2 = 8\Big(1-\cos(2\log 3)\Big)$, $AP \doteqdot 3{\cdot}562$ metres.
(c) $\angle AOB = 2\log(t+1)$. The train is at A when $t = e^{\pi}-1 \doteqdot 22$, when $t = e^{2\pi}-1 \doteqdot 534$, and when $t = e^{3\pi}-1 \doteqdot 12\,391$.
(d) Since $2\log(t+1) \to \infty$ as $t \to \infty$, the train will return to A infinitely many times.

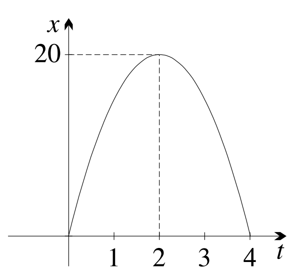
13(a)

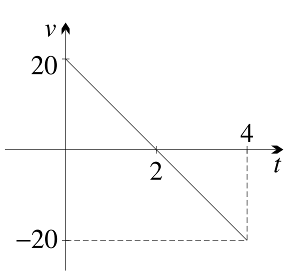


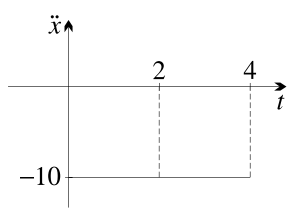
(c) The maximum distance is
$300\log(e-1) - \dfrac{300(e-2)}{e-1} \doteqdot 37$ metres
when $t = e-2 \doteqdot 43''$.

Exercise **3B** (Page 89)

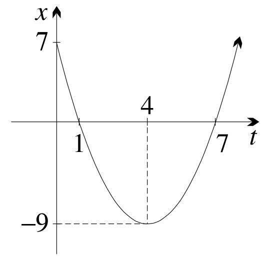
1(a) $v = 2t-8$, $\ddot{x} = 2$, which is constant.
(b) $x = -15$ metres, $v = 2\,\text{m/s}$, $\ddot{x} = 2\,\text{m/s}^2$
(c) When $t = 4$, $v = 0$ and $x = -16$.
2(a) $v = 3t^2-12t-1$, $\ddot{x} = 6t-12$ **(b)** When $t = 0$, $x = 2\,\text{cm}$, $|v| = 1\,\text{cm/s}$, $\ddot{x} = -12\,\text{cm/s}^2$. **(c)(i)** left $(x = -28)$ **(ii)** left $(v = -10)$ **(iii)** right $(\ddot{x} = 6)$
(d) When $t = 2$, $\ddot{x} = 0$ and $|v| = 13\,\text{cm/s}$.
3 $v = 2\pi\cos\pi t$, $\ddot{x} = -2\pi^2\sin\pi t$ **(a)** When $t = 1$, $x = 0$, $v = -2\pi$ and $\ddot{x} = 0$. **(b)(i)** right $(v = \pi)$ **(ii)** left $(\ddot{x} = -\pi^2\sqrt{3})$
4 $v = -4e^{-4t}$, $x = 16e^{-4t}$ **(a)** e^{-4t} is positive for all t, so v is always negative, and $\ddot{x}$ is always positive. **(b)(i)** $x = 1$ **(ii)** $x = 0$ **(c)(i)** $v = -4$, $\ddot{x} = 16$ **(ii)** $v = 0$, $\ddot{x} = 0$
5(a) $x = 5t(4-t)$
$v = 20-10t$
$\ddot{x} = -10$

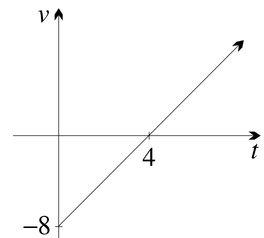


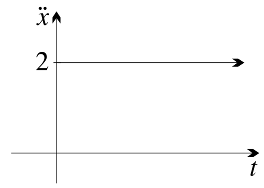




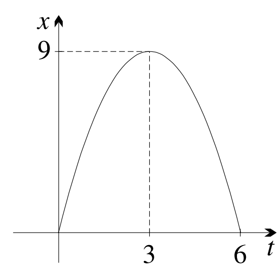
(b) It returns at $t = 4$; both speeds are $20\,\text{m/s}$.
(c) 20 metres after 2 seconds
(d) $-10\,\text{m/s}^2$. Although the ball is stationary, its velocity is changing, meaning that its acceleration is nonzero. **(e)** After 1 second, when $v = 10\,\text{m/s}$, and after 3 seconds, when $v = -10\,\text{m/s}$.
6(a) $x = (t-7)(t-1)$
$v = 2(t-4)$
$\ddot{x} = 2$

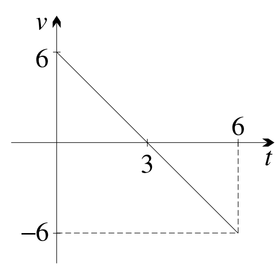






(b)(i) $t = 1$ and $t = 7$ **(ii)** $t = 4$ **(c)(i)** 7 metres when $t = 0$ **(ii)** 9 metres when $t = 4$ **(iii)** 27 metres when $t = 10$ **(d)** $-1\,\text{m/s}$, $t = 3\frac{1}{2}$, $x = -8\frac{3}{4}$
(e) 25 metres, $3\frac{4}{7}\,\text{m/s}$
7(a) $x = t(6-t)$, $v = 2(3-t)$, $\ddot{x} = -2$

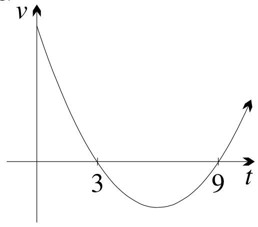


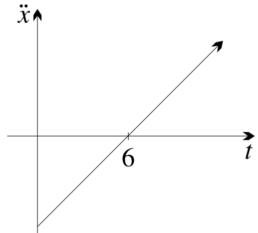


(b)(i) When $t = 2$, it is moving upwards and accelerating downwards. **(ii)** When $t = 4$, it is moving downwards and accelerating downwards.
(c) $v = 0$ when $t = 3$. It is stationary for zero time, 9 metres up the plane, and is accelerating downwards at $2\,\text{m/s}^2$.
(d) $4\,\text{m/s}$. When $v = 4$, $t = 1$ and $x = 5$.
(e) All three average speeds are $3\,\text{m/s}$.
8(a) 8 metres when $t = 3$ **(b)(i)** when $t = 3$ and $t = 9$ **(ii)** when $0 \le t < 3$ and when $t > 9$ **(iii)** when $3 < t < 9$ **(c)** $t = 9$, $v = 0$, accelerating forwards **(d)** $t = 6$, $x = 4$, moving backwards
(e) $0 \le t < 6$
(f)(i) $t \doteqdot 4,\ 12$ **(ii)** $t \doteqdot 10$ **(iii)** $t \doteqdot 4,\ 8,\ 10$

ISBN: 978-1-107-61604-2 Cambridge University Press

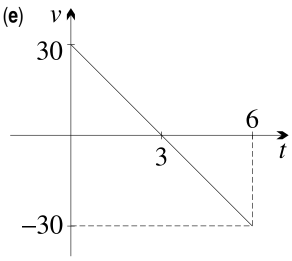
(g)





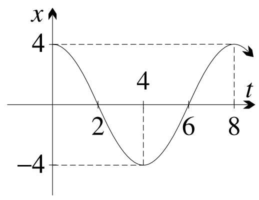
9(a) 45 metres, 3 seconds, 15 m/s **(b)** 30 m/s, 20, 10, 0, −10, −20, −30 **(c)** 0 seconds **(d)** The acceleration was always negative.

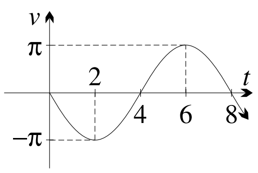
(e)

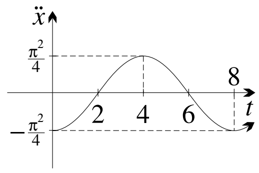


The velocity was decreasing at a constant rate of 10 m/s every second.

10(a) $x = 4\cos\frac{\pi}{4}t$
$v = -\pi\sin\frac{\pi}{4}t$
$\ddot{x} = -\frac{1}{4}\pi^2\cos\frac{\pi}{4}t$

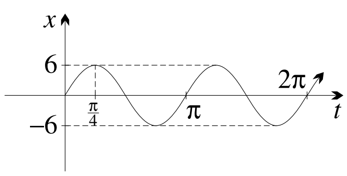


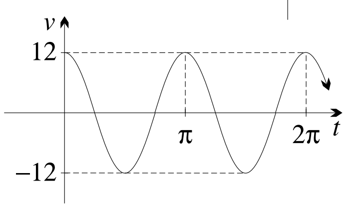


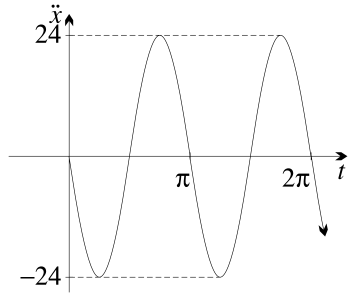


(b) maximum displacement: $x = 4$ when $t = 0$ and $t = 8$, maximum velocity: π m/s when $t = 6$, maximum acceleration: $\frac{1}{4}\pi^2$ m/s² when $t = 4$
(c) 40 metres, 2 m/s **(d)(i)** after $1\frac{1}{3}$ and $6\frac{2}{3}$ seconds **(ii)** $1\frac{1}{3} < t < 6\frac{2}{3}$ **(e)(i)** after $4\frac{2}{3}$ and $7\frac{1}{3}$ seconds **(ii)** $4\frac{2}{3} < t < 7\frac{1}{3}$

11(a) $x = 6\sin 2t$
$v = 12\cos 2t$
$\ddot{x} = -24\sin 2t$

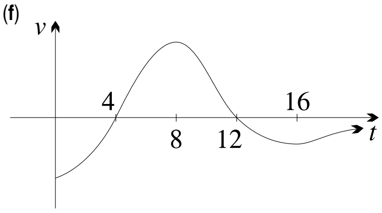


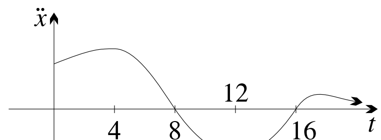




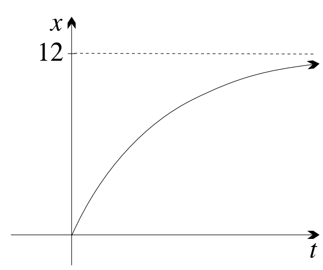
(b) $\ddot{x} = -4x$ **(c)(i)** $x = 0$ when $t = 0$, $\frac{\pi}{2}$ or π.
(ii) $v = 0$ when $t = \frac{\pi}{4}$ or $\frac{3\pi}{4}$. **(iii)** same as (i)
(d)(i) $x < 0$ when $\frac{\pi}{2} < t < \pi$.
(ii) $v < 0$ when $\frac{\pi}{4} < t < \frac{3\pi}{4}$.
(iii) $\ddot{x} < 0$ when $0 < t < \frac{\pi}{2}$. **(e)(i)** $t = \frac{\pi}{12}$ **(ii)** $t = \frac{\pi}{6}$

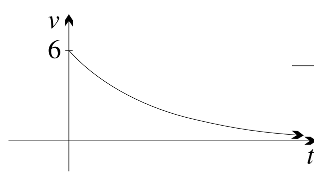
12(a)(i) $0 \le t < 8$ **(ii)** $0 \le t < 4$ and $t > 12$
(iii) roughly $8 < t < 16$ **(b)** roughly $t = 8$
(c)(i) $t \doteqdot 5$, 11, 13 **(ii)** $t \doteqdot 13$, 20
(iii) $t \doteqdot 5$, 11, 13, 20 **(d)** twice **(e)** 17 units
(f)

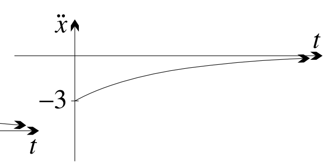




13(a) $x = 12 - 12e^{-0\cdot5t}$
$v = 6e^{-0\cdot5t}$
$\ddot{x} = -3e^{-0\cdot5t}$







(b)(i) downwards (Downwards is positive here.)
(ii) upwards **(c)** The velocity and acceleration tend to zero, and the position tends to 12 metres below ground level. **(d)** $x = 6$ when $e^{-0\cdot5t} = \frac{1}{2}$, that is, $t = 2\log 2$. The speed then is 3 m/min

ISBN: 978-1-107-61604-2 Cambridge University Press

(half the initial speed of $6\,\text{m/min}$) and the acceleration is $-1\frac{1}{2}\,\text{m/min}^2$ (half the initial acceleration of $-3\,\text{m/min}^2$). **(e)** 19 minutes

14(a) The displacement of M is the average of x_A and x_B, $v_M = 2e^{-t}(t^2 - 3t + 1)$, M returns to O after 1 second, when M is moving left at a speed of $2/e$. **(b)** The particle is furthest right at $t = \frac{1}{2}(3 - \sqrt{5}\,)$, and furthest left at $t = \frac{1}{2}(3 + \sqrt{5}\,)$. **(c)** They all move towards O. **(d)** $t = \frac{1}{2}(1 + \sqrt{5}\,)$

15(a) $0 \le x \le 2r$ **(b)(i)** $dx/d\theta = \dfrac{2r \sin\theta}{\sqrt{5 - 4\cos\theta}}$. M is travelling upwards when $0 < \theta < \pi$.
(ii) M is travelling downwards when $\pi < \theta < 2\pi$.
(c) The speed is maximum when $\theta = \frac{\pi}{3}$ (when $\dfrac{dx}{d\theta} = r$) and when $\theta = \frac{5\pi}{3}$ (when $\dfrac{dx}{d\theta} = -r$).
(d) When $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$, $\angle APC$ is a right angle, so AP is a tangent to the circle. At these places, P is moving directly towards A or directly away from A, and so the distance AP is changing at the maximum rate. Again because AP is a tangent, $dx/d\theta$ at these points must equal the rate of change of arc length with respect to θ, which is r or $-r$ when $\theta = \frac{\pi}{3}$ or $\frac{5\pi}{3}$ respectively.

16 $\sin\alpha = 2/g \doteqdot 0{\cdot}204\,08$, $\alpha \doteqdot 11^\circ 47'$

Exercise 3C (Page 96)

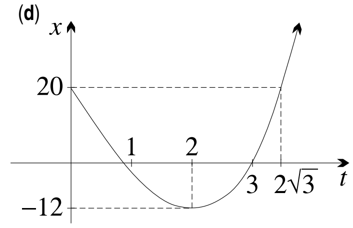
1(a) $v = -4t$, $x = -2t^2$ **(b)** $v = 3t^2$, $x = t^3$
(c) $v = 2e^{\frac{1}{2}t} - 2$, $x = 4e^{\frac{1}{2}t} - 2t - 4$
(d) $v = -\frac{1}{3}e^{-3t} + \frac{1}{3}$, $x = \frac{1}{9}e^{-3t} + \frac{1}{3}t - \frac{1}{9}$
(e) $v = -4\cos 2t + 4$, $x = -2\sin 2t + 4t$
(f) $v = \frac{1}{\pi}\sin \pi t$, $x = -\frac{1}{\pi^2}\cos \pi t + \frac{1}{\pi^2}$
(g) $v = \frac{2}{3}t^{\frac{3}{2}}$, $x = \frac{4}{15}t^{\frac{5}{2}}$
(h) $v = -12(t+1)^{-1} + 12$, $x = -12\log(t+1) + 12t$

2(a) $\ddot{x} = 0$, $x = -4t - 2$ **(b)** $\ddot{x} = 6$, $x = 3t^2 - 2$
(c) $\ddot{x} = \frac{1}{2}e^{\frac{1}{2}t}$, $x = 2e^{\frac{1}{2}t} - 4$
(d) $\ddot{x} = -3e^{-3t}$, $x = -\frac{1}{3}e^{-3t} - 1\frac{2}{3}$
(e) $\ddot{x} = 16\cos 2t$, $x = -4\cos 2t + 2$
(f) $\ddot{x} = -\pi \sin \pi t$, $x = \frac{1}{\pi}\sin \pi t - 2$
(g) $\ddot{x} = \frac{1}{2}t^{-\frac{1}{2}}$, $x = \frac{2}{3}t^{\frac{3}{2}} - 2$
(h) $\ddot{x} = -24(t+1)^{-3}$, $x = -12(t+1)^{-1} + 10$

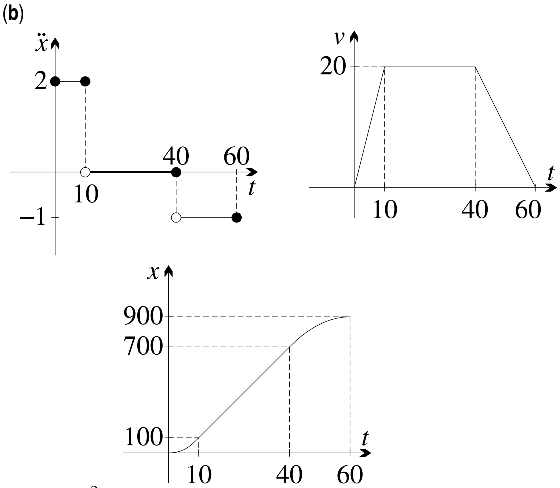
3(a) $v = 10t$, $x = 5t^2$ **(b)(i)** 4 seconds **(ii)** $40\,\text{m/s}$
(c) After 2 seconds, it has fallen 20 metres, and its speed is $20\,\text{m/s}$. **(d)** It is halfway down after $2\sqrt{2}$ seconds, and its speed then is $20\sqrt{2}\,\text{m/s}$.

4(a) $\ddot{x} = -10$, $v = -10t - 25$, $x = -5t^2 - 25t + 120$
(i) 3 seconds **(ii)** $55\,\text{m/s}$ **(b)** $\ddot{x} = 10$, $v = 10t + 25$, $x = 5t^2 + 25t$. Put $5t^2 + 25t = 120$.

5(a) $v = 6t^2 - 24$, $x = 2t^3 - 24t + 20$
(b) $t = 2\sqrt{3}$, $|v| = 48\,\text{m/s}$
(c) $x = -12$ when $t = 2$.
(d)



6(a) $20\,\text{m/s}$, 900 metres
(b)



7(a)(i) $\displaystyle\int_1^2 \frac{4}{t+1}\,dt = 4(\log 3 - \log 2)$
(ii) $\displaystyle\int_1^2 \frac{4}{\log(t+1)}\,dt \doteqdot 4{\cdot}54$
(b)(i) $\displaystyle\int_1^2 \sin \pi t\,dt = -\tfrac{2}{\pi}$
(ii) $\displaystyle\int_1^2 t\sin \pi t\,dt = -\tfrac{1}{4}(1 + 2\sqrt{2}\,) \doteqdot -0{\cdot}957$

8(a) $\ddot{x} = 6t$, $v = 3t^2 - 9$
(b) $x = t^3 - 9t + C_1$, 3 seconds

10(a) $v = -5 + 20e^{-2t}$, $x = -5t + 10 - 10e^{-2t}$, $t = \log 2$ seconds **(b)** It rises $7\frac{1}{2} - 5\log 2$ metres, when acceleration is $10\,\text{m/s}^2$ downwards.
(c) The velocity approaches a limit of $5\,\text{m/s}$ downwards, called the *terminal velocity*.

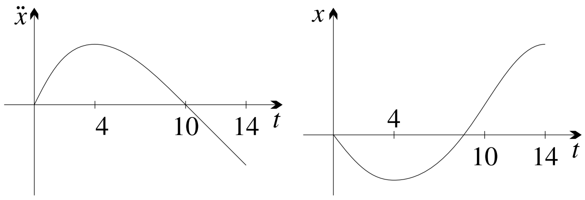
11 $e - 1$ seconds, $v = 1/e$, $\ddot{x} = -1/e^2$.
The velocity and acceleration approach zero, but the particle moves to infinity.

12(a) $x = t^2(t-6)^2$, after 6 seconds, $0\,\text{cm/s}$
(b) $162\,\text{cm}$, $27\,\text{cm/s}$ **(c)** $\ddot{x} = 12(t^2 - 6t + 6)$, $24\sqrt{3}\,\text{cm/s}$ after $3 - \sqrt{3}$ and $3 + \sqrt{3}$ seconds.
(d) The graphs of x, v and $\ddot{x}$ are all unchanged by reflection in $t = 3$, but the mouse would be running backwards!

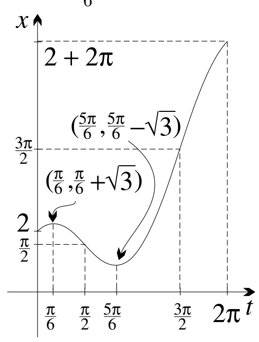
13(a) $4 < t < 14$ **(b)** $0 < t < 10$ **(c)** $t = 14$
(d) $t = 4$ **(e)** $t \doteqdot 8$

ISBN: 978-1-107-61604-2 Cambridge University Press

(f)



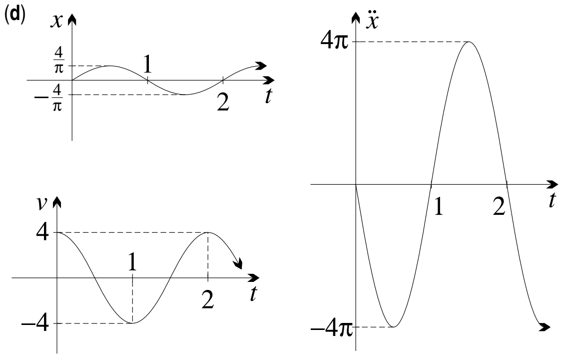
14(a) $\ddot{x} = -4$, $x = 16t - 2t^2 + C$ **(b)** $x = C$ after 8 seconds, when the speed is $16\,\text{cm/s}$. **(c)** $v = 0$ when $t = 4$. Maximum distance right is $32\,\text{cm}$ when $t = 4$, maximum distance left is $40\,\text{cm}$ when $t = 10$. The acceleration is $-4\,\text{cm/s}^2$ at all times.
(d) $104\,\text{cm}$, $10{\cdot}4\,\text{cm/s}$
15(a) $v = 1 - 2\sin t$, $x = t + 2\cos t$
(b) $\frac{\pi}{2} < t < \frac{3\pi}{2}$ **(c)** $t = \frac{\pi}{6}$ when $x = \frac{\pi}{6} + \sqrt{3}$, and $\frac{5\pi}{6}$ when $x = \frac{5\pi}{6} - \sqrt{3}$, $\frac{\pi}{6} < t < \frac{5\pi}{6}$.
(d) $3\,\text{m/s}$ when $t = \frac{3\pi}{2}$ and $x = \frac{3\pi}{2}$, $-1\,\text{m/s}$ when $t = \frac{\pi}{2}$ and $x = \frac{\pi}{2}$.
(e) 2π metres, $1\,\text{m/s}$
(f) $4\sqrt{3} + \frac{2\pi}{3}$ metres, $\frac{1}{3} + \frac{2}{\pi}\sqrt{3}\,\text{m/s}$



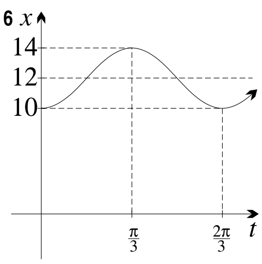
16(a) $x_1 = 2 + 6t + t^2$, $x_2 = 1 + 4t - t^2$, $D = 1 + 2t + 2t^2$ **(b)** D is never zero, the minimum distance is 1 metre at $t = 0$ (t cannot be negative).
(c) $v_M = 5\,\text{m/s}$, $12\frac{1}{2}$ metres
17(a) Thomas, by $15\,\text{m/s}$ **(b)** $x_T = 20\log(t+1)$, $x_H = 5t$ **(c)** during the 10th second, $3\frac{2}{11}\,\text{m/s}$
(d) after 3 seconds, by 13 metres
18(a) For $V \geq 30\,\text{m/s}$, they collide after $180/V$ seconds, $\dfrac{180}{V^2}(V^2 - 900)$ metres above the valley floor.
(b) $V = 30\sqrt{2}\,\text{m/s}$, $3\sqrt{2}$ seconds
19(a) $v = 5(e^{-2t} - 1)$, $x = \frac{5}{2}(1 - e^{-2t}) - 5t$
(b) The speed gradually increases with limit $5\,\text{m/s}$ (the terminal velocity).

Exercise **3D** (Page 105)

1(a) $v = 4\cos\pi t$, $\ddot{x} = -4\pi\sin\pi t$
(b) $a = \frac{4}{\pi}$, $T = 2$ seconds, centre at O
(c) $4\,\text{m/s}$, $4\pi\,\text{m/s}$, $\frac{4}{\pi}$ metres
(d)



(e) $t = 1$ (when $v = -4\,\text{m/s}$) and $t = 2$ (when $v = 4\,\text{m/s}$) **(f)** $t = \frac{1}{2}$ (when $\ddot{x} = -4\pi\,\text{m/s}^2$) and $t = 1\frac{1}{2}$ (when $\ddot{x} = 4\pi\,\text{m/s}^2$)
2(a) $n = \frac{\pi}{2}$ and $a = 12$, so $x = 12\cos\frac{\pi}{2}t$.
(b) $v = -6\pi\sin\frac{\pi}{2}t$, $\ddot{x} = -3\pi^2\cos\frac{\pi}{2}t$ **(c)** 2 seconds
3(a) $x = 4\sin 3t$ **(b)** $x = 2\sin 6t$ **(c)** $x = \frac{3}{2}\cos 8t$
(d) $x = 16\cos\frac{3}{4}t$
4(a) $x = 2\sin 2t$, $v = 4\cos 2t$, $-2 \leq x \leq 2$
(b) $x = 6\sin\frac{2}{3}t$, $v = 4\cos\frac{2}{3}t$, 3π seconds
5(a) $v = bn\cos nt - cn\sin nt$, $\ddot{x} = -bn^2\sin nt - cn^2\cos nt = -n^2 x$
(b) $c = 3$ and $b = 0$, so $x = 3\cos nt$.
(c) $x = 5\cos 2\pi t$, $\frac{1}{4}$ s
6

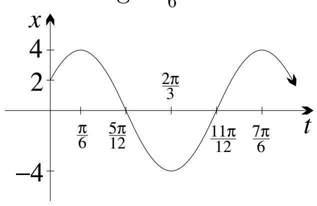


(a) $v = 6\sin 3t$, $\ddot{x} = 18\cos 3t$
(b) $a = 2$, $T = \frac{2\pi}{3}$ seconds, centre $x = 12$
(c) $10 \leq x \leq 14$, $\frac{\pi}{3}$ seconds
(d) $t = \frac{2\pi}{3}$ and $t = \frac{4\pi}{3}$. At both times, $|v| = 0$ and $\ddot{x} = 18\,\text{cm/s}^2$. **(e)** $t = \frac{\pi}{6}$ and $t = \frac{\pi}{2}$. At both times, $|v| = 6\,\text{cm/s}$ and $\ddot{x} = 0\,\text{cm/s}^2$.
7(a) amplitude: 6, period: π, initial phase: $\frac{\pi}{2}$
(b) $\dot{x} = 12\cos(2t + \frac{\pi}{2})$, $\ddot{x} = -24\sin(2t + \frac{\pi}{2})$, $\ddot{x} = -4x$, so $n = 2$.
(c) $t = \frac{\pi}{4}$ when $v = -12$, $t = \frac{3\pi}{4}$ when $v = 12$
(d) $t = \frac{3\pi}{4}$ and $t = \frac{7\pi}{4}$, when $x = 0$
(e) $t = \pi$ and $t = 2\pi$, when $v = 0$ and $\ddot{x} = -24$
8(a)(i) Use expansions of $\sin(\alpha + \beta)$ and $\cos(\alpha - \beta)$.
(ii) The graph of $x = \sin t$ shifted left $\frac{\pi}{2}$ is $x = \cos t$. The graph of $x = \cos t$ shifted right $\frac{\pi}{2}$ is $x = \sin t$.
(b)(i) $\sin(t - \frac{\pi}{2}) = -\cos t$, $\cos(t + \frac{\pi}{2}) = -\sin t$
(ii) The graph of $x = \sin t$ shifted right $\frac{\pi}{2}$ is $x = -\cos t$. The graph of $x = \cos t$ shifted left $\frac{\pi}{2}$ is $x = -\sin t$.

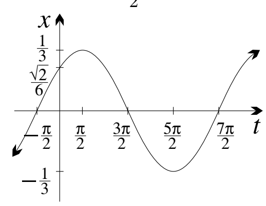
ISBN: 978-1-107-61604-2 Cambridge University Press

9(a) $x = 120 \sin \frac{\pi}{12} t,\ v = 10\pi \cos \frac{\pi}{12} t,\ 10\pi \text{ m/s}$
(b)(i) $\frac{12}{\pi} \sin^{-1} \frac{1}{4} \doteqdot 0{\cdot}9652$ seconds
(ii) $12 + \frac{12}{\pi} \sin^{-1} \frac{1}{4} \doteqdot 12{\cdot}97$ seconds **(c)** 4 seconds and 8 seconds
10(a) $x = 4 \cos 4t,\ v = -16 \sin 4t$ **(b)(i)** $\frac{\pi}{12}$ s **(ii)** $\frac{\pi}{6}$ s
(c) $\frac{\pi}{24}$ seconds and $\frac{5\pi}{24}$ seconds
11 $x = \frac{1}{2} - \frac{1}{2} \cos 2t,\ x_0 = \frac{1}{2},\ \frac{1}{2},\ 0 \le x \le 1,\ \pi$
12(a) $x = 2 - \cos 4t$ **(b)** $x_0 = 2$, 1 cm, $1 \le x \le 3$, $\frac{\pi}{2}$ s **(c)** 4 cm/s when $t = \frac{\pi}{8}$
13 $v = bn \cos nt - cn \sin nt$
(a) $x = 6 \sin \frac{1}{2} t + 6 \cos \frac{1}{2} t,\ \frac{3\pi}{2}$ s and $\frac{7\pi}{2}$ s
(b) $x = \frac{9}{\pi} \sin \frac{\pi}{3} t - 2 \cos \frac{\pi}{3} t$, about $0{\cdot}582$ s and $3{\cdot}582$ s

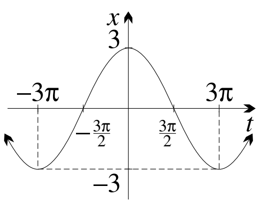
14(a) 4 times, $a = 4$, $T = \pi$, $x = 4 \cos 2t$ shifted right $\frac{\pi}{6}$



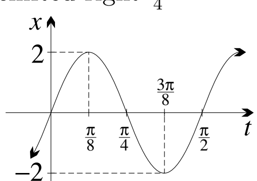
(b) once, $a = \frac{1}{3}$, $T = 4\pi$, $x = \frac{1}{3} \sin \frac{1}{2} t$ shifted left $\frac{\pi}{2}$



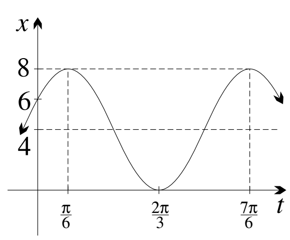
(c) once, $a = 3$, $T = 6\pi$, $x = -3 \cos \frac{1}{3} t$ shifted left 3π



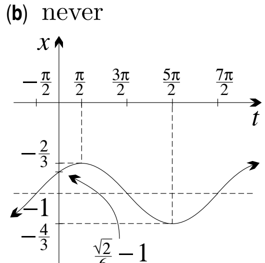
(d) 9 times, $a = 2$, $T = \frac{\pi}{2}$, $x = -2 \sin 4t$ shifted right $\frac{\pi}{4}$



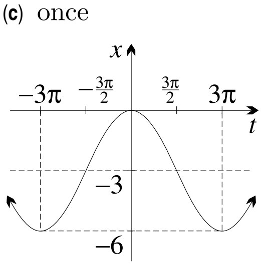
15(a) twice



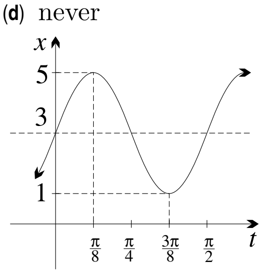
(b) never



(c) once



(d) never



16 $v = an \cos(nt + \alpha)$ **(a)** $n = \frac{\pi}{3},\ \alpha = 0,\ a = \frac{15}{\pi}$
(b) $n = \frac{2}{3},\ \alpha = \frac{3\pi}{2},\ a = 5$ **(c)** $n = 1,\ \alpha = \frac{3\pi}{4},\ a = \sqrt{2}$
17 $v = -2a \sin(2t - \varepsilon)$ **(a)** $\varepsilon = \frac{\pi}{2},\ a = 3$
(b) $\varepsilon = \frac{5\pi}{3},\ a = 2$
18 $a = \frac{32}{\pi}\sqrt{2},\ \alpha = \frac{\pi}{4}$
19 $a = 5,\ \alpha \doteqdot 2{\cdot}248$
20(a) $x = 4 \sin 4t + 3 \cos 4t$ **(b)** $x = 5 \cos(4t - \varepsilon)$, where $\varepsilon = \tan^{-1} \frac{4}{3}$ **(c)** 5 m, 20 m/s
(d) $t = \frac{\pi}{4} - \frac{1}{4} \tan^{-1} \frac{3}{4},\ t = \frac{\pi}{8} + \frac{1}{4} \tan^{-1} \frac{4}{3}$
21(a) $x = 24 \sin \frac{1}{4} t + 4 \cos \frac{1}{4} t,\ t = 8 \tan^{-1} 6 \doteqdot 11{\cdot}2$
(b) $x = 4\sqrt{37}\, \cos(\frac{1}{4} t - \alpha)$, where $\alpha = \tan^{-1} 6$
22(a) $x = \frac{2}{77} t(18 - t),\ 2,\ \frac{162}{77}$
(b) $x = -4 \sin \frac{\pi}{6} t,\ 2,\ 4$
23(a) 10:00 am **(b)** 7:33 am **(c)** 12:27 pm
24 11:45 am to 8:15 pm
25(a) $x = 2\sqrt{5}\, \cos(3\pi t - \varepsilon)$, where $\varepsilon = \pi - \tan^{-1} \frac{1}{2} \doteqdot 2{\cdot}678$. **(b)(i)** $0{\cdot}195$ seconds **(ii)** $0{\cdot}287$ seconds
26 $x = \dfrac{v(0)}{n} \sin nt + x(0) \cos nt$,
$\frac{1}{n}\sqrt{n^2 x(0)^2 + v(0)^2}$
28(a) $\sin nt + \sin(nt + \alpha) \equiv 2 \cos \frac{1}{2}\alpha \sin(nt + \frac{1}{2}\alpha)$

Exercise **3E** (Page 112)

1(a) $t = \frac{1}{6}(x - 1),\ x = 6t + 1$
(b) $t = \frac{1}{18}(1 - x^3),\ x = (1 - 18t)^{\frac{1}{3}}$
(c) $t = \frac{1}{2} \log(2x - 1),\ x = \frac{1}{2}(e^{2t} + 1)$
(d) $t = \frac{1}{6}(x^{-1} - 1),\ x = (1 + 6t)^{-1}$
(e) $t = \frac{1}{12}(x^{-2} - 1),\ x = (12t + 1)^{-\frac{1}{2}}$
(f) $t = \frac{1}{2}(e^{2x} - e^2),\ x = \frac{1}{2} \log(2t + e^2)$
(g) $t = \tan^{-1} x - \frac{\pi}{4},\ x = \tan(t + \frac{\pi}{4})$
(h) $t = \tan x - \tan 1,\ x = \tan^{-1}(t + \tan 1)$
2(a) $\ddot{x} = 0$ **(b)** $\ddot{x} = -72x^{-5}$ **(c)** $\ddot{x} = 2(2x - 1)$
(d) $\ddot{x} = 72x^3$ **(e)** $\ddot{x} = 108x^5$ **(f)** $\ddot{x} = -2e^{-4x}$
(g) $\ddot{x} = 2x(1 + x^2)$ **(h)** $\ddot{x} = -2 \cos^3 x \sin x$
3(a) $v^2 = 4x^3$ **(b)** $v^2 = 2(1 - e^{-x})$
(c) $v^2 = 12x$ **(d)** $v^2 = \log(2x + 1)$
(e) $v^2 = \frac{1}{3}(1 - \cos 6x)$ **(f)** $v^2 = \tan^{-1} \frac{1}{2} x$
4(a) 100 m/s **(b)** Downwards is positive, so while the stone is falling, its velocity is positive.
(c) $x = 5t^2$, 10 seconds
5(a) 20 metres **(b)** Upwards is positive, so while the stone is rising, its velocity is positive.
(c) $t = \frac{1}{10}(20 - \sqrt{400 - 20x}\,),\ x = 20t - 5t^2$, 2 seconds

ISBN: 978-1-107-61604-2 Cambridge University Press

7(a) $v^2 = 2(4 - x^2)$, $\sqrt{6}\,\text{cm/s}$. It starts at $x = 2$, so on the first occasion it reaches $x = 1$, it must be moving backwards.
(b) $-2 \le x \le 2$, $2\sqrt{2}\,\text{cm/s}$
8(a) $t = \frac{1}{8}x^3$, $x = 2t^{\frac{1}{3}}$ **(b)** $v = \frac{2}{3}t^{-\frac{2}{3}}$, $\ddot{x} = -\frac{4}{9}t^{-\frac{5}{3}}$
(c) $\ddot{x} = -\frac{128}{9}x^{-5}$
9(b) $v^2 = 2x\log x$ **(c)** $\ddot{x}$ is initially positive, and the particle moves off in the positive direction; thus x remains greater than 1, $\ddot{x}$ remains positive, and the particle continues moving in that direction. $v = 2e$.
10(a) $v^2 = \frac{1}{3}\tan^{-1}\frac{1}{6}x$. The acceleration is always positive, and the velocity is initially zero, so for $t > 0$ the velocity is always positive.
(b)(i) $\sqrt{\frac{\pi}{12}}$ **(ii)** $\sqrt{\frac{\pi}{6}}$
11(a) $\ddot{x} = -\frac{5}{2}\,\text{m/s}$, $v^2 = 10\,000 - 5x$
(b)(i) $v = 50\sqrt{2}\,\text{m/s}$ **(ii)** $x = 1500$ metres
(c) The plane is still moving forwards while it is braking. **(d)** $x = 100t - \frac{5}{4}t^2$, 40 seconds
12(a) $v^2 = 6 - 2e^{-x}$. The acceleration is always positive, and the velocity is initially 2. Hence the velocity is always increasing with minimum 2. The particle continues to accelerate, but with limiting velocity $\sqrt{6}$.
(b) $v^2 = V^2 + 2(1 - e^{-x})$, $V = -\sqrt{2(e-1)}$. The particle has limiting velocity $\sqrt{2e}$.
13(a) The velocity $\cos^2 2x$ can never be negative, so the particle can never be moving backwards, so it can never return to anywhere it has already been. **(b)** $x = \frac{1}{2}\tan^{-1}2t$, $x \to \frac{\pi}{4}$, $v = \dfrac{1}{1+4t^2}$
(c) $t = \frac{1}{2}$, $v = \frac{1}{2}$, $\ddot{x} = -1$
14(a) $\ddot{x} = -12$ **(b)** $x = 3(1 - e^{-2t})$ **(c)** As $t \to \infty$, the particle moves to the limiting position $x = 3$.
15(a) The velocity is positive everywhere, so it can never be on the negative side of its initial position. $\ddot{x} = 2x^3 e^{-2x^2}(1 - x^2)$, maximum $1/e$ at $x = 1$
(b) 1·695
16(a) $v^2 = e^{-x}$ **(b)** v is initially positive, and is never zero. $x = 2\log\frac{1}{2}(t+2)$. As $t \to \infty$, $x \to \infty$ (slowly) and $v \to 0$.
17(a) $v^2 = 2(x-5)(x-4)$. v^2 cannot be negative. **(b)** $x = 6$ ($x = -5$ is impossible, because the particle can never pass through the origin). The particle moves forwards with increasing velocity.
18 $x_{\text{A}} = \frac{\pi}{16} - \frac{1}{4}\tan^{-1}\frac{1}{4}x$, they take $\frac{\pi}{16}$ seconds, B is released from $x = 2\sqrt{2}$, $x_{\text{B}} = \frac{\pi}{16} - \frac{1}{4}\sin^{-1}\frac{1}{4}x$.
19(a) $v^2 = 2x^3$ **(b)** Initially, v is negative. Since $v^2 = 2x^3$, v can only be zero at the origin. But since $\ddot{x} = 3x^2$ the acceleration at the origin would also be zero. Hence if the particle ever arrived at the origin, it would then be permanently at rest. Thus the velocity can never change from negative to positive, and must always be negative or zero. $x = \dfrac{2}{(t+\sqrt{2})^2}$. The particle starts at $x = 1$ and moves backwards towards the origin, its speed having limit zero, and position having limit the origin.
20(a) $v^2 = 49 - (x-7)^2$, $0 \le x \le 14$, maximum speed 7 at $x = 7$ **(b)** $\ddot{x} = 7 - x$, $4 \le x \le 10$
21(b) $v = 0$, $\ddot{x} = -15$. It moves off in the negative direction. **(c)** $6\sqrt{2}$ at the origin. It oscillates between $x = 3$ and $x = -2$.
22(a) $v^2 = 10^{14} + 2k(1 - \cos\pi x)$. Since $1 - \cos\pi x$ is never negative, v^2 never drops below the square of its initial value. The velocity is minimum at $x = 0, 2, 4, \ldots$ and maximum at $x = 1, 3, 5, \ldots$.
(b) $k = \frac{3}{4} \times 10^{14}$, $\frac{3}{4}\pi \times 10^{14}\,\text{m/s}^2$ at $x = \frac{1}{2}$, $\frac{5}{2}$, $\frac{9}{2}, \ldots$
23 $x = 2^{\frac{1}{10}t+1}$, $x = 8\sqrt{2}$
25(b) $v^2 = V^2 + 2gR^2\,(1/x - 1/R)$, $H = 2gR^2/(2gR - V^2)$ **(c)** $11{\cdot}2\,\text{km/s}$
26(a) $x = 150 - \dfrac{50\log v}{\log 10}$, $x = 150$ metres
(b) $t = \dfrac{1000}{99}\left(\dfrac{1}{v} - \dfrac{1}{1000}\right)$, $t = 10\frac{1}{11}$ s
27(a) $v = 500 - 5x$, $x = 100\left(1 - e^{-5t}\right)$
(b) The bullet moves to a limiting position of $x = 100$ as the velocity decreases to zero.

Exercise **3F** (Page 120)

1(a) $v = -6\sin 2t$, $\ddot{x} = -12\cos 2t$, $v^2 = 4(9 - x^2)$, $\ddot{x} = -4x$ **(b)** $2\sqrt{5}\,\text{m/s}$ or $-2\sqrt{5}\,\text{m/s}$, $-8\,\text{m/s}^2$
2(a) $v^2 = 9(25 - x^2$ **(b)** $v = 12\,\text{m/s}$ or $v = -12\,\text{m/s}$, $\ddot{x} = -27\,\text{m/s}^2$ **(c)** $15\,\text{m/s}$, $\frac{2\pi}{3}$ seconds
3(a) $v^2 = 16(36 - x^2$ **(b)** $6\,\text{cm}$, $\frac{\pi}{2}$ seconds
(c) $|v| = 16\sqrt{2}\,\text{cm/s}$, $\ddot{x} = -32\,\text{cm/s}^2$
4(a) $v^2 = 4(36 - x^2)$, π seconds, $12\dot{\text{m}}/\text{s}$
(b)(i) $x = 6\cos 2t$ **(ii)** $x = -6\cos 2t$ **(iii)** $x = 6\sin 2t$
(iv) $x = -6\sin 2t$
5(a) $32\,\text{cm/min}$ **(b)** $8\,\text{cm}$
6(a) $x = \sin 4t$, $v = 4\cos 4t$, $a = 1$ metre

ISBN: 978-1-107-61604-2 Cambridge University Press

(b) $\frac{4\pi}{3}$ m/s, $\frac{8\pi^2}{9}$ m/s^2
7(a) $\frac{5\pi}{2}$ cm/s, $\frac{5\pi^2}{8}$ cm/s^2
(b) 2π cm/s, $\frac{3\pi^2}{8}$ or $-\frac{3\pi^2}{8}$ cm/s^2
8 $5\sqrt{2}$ m/s, $3\sqrt{2}$ m/s
9(a) $a = 4$, $2\sqrt{7}$ or $-2\sqrt{7}$ m/s
(b) 4 m/s, 6 or -6 m/s^2
10(a) $v^2 = \frac{225}{4}(4 - x^2)$, $v = 10\sqrt{2}$ or $-10\sqrt{2}$ m/s, $\ddot{x} = -37\frac{1}{2}$ m/s^2
(b) $v^2 = -\frac{5}{3}x^2 + \frac{1280}{3}$, amplitude 16
11(b)(i) When $x = 0$, $|v| = an$.
(ii) When $x = \frac{1}{2}a$, $|v| = \frac{1}{2}\sqrt{3}\,an$ and $\ddot{x} = -\frac{1}{2}an^2$.
13 15 cm/s
14(a) $\ddot{x} = -9(x - 1)$, centre: $x = 1$, period: $\frac{2\pi}{3}$, amplitude: 2 **(b)(i)** $\ddot{x} = -16(x - 2)$, centre: $x = 2$, period: $\frac{\pi}{2}$, amplitude: 3 **(ii)** $\ddot{x} = -9(x - 6)$, centre: $x = 6$, period: $\frac{2\pi}{3}$, amplitude: 4
(iii) $\ddot{x} = -2(x + 2)$, centre: $x = -2$, period: $\pi\sqrt{2}$, amplitude: 1 **(iv)** $\ddot{x} = -3(x + \frac{5}{3})$, centre: $x = -\frac{5}{3}$, period: $2\pi/\sqrt{3}$, amplitude: $2\frac{1}{3}$
15(a)(i) $\ddot{x} = 50\cos 10t = 100(\frac{1}{2} - x)$
(ii) $\ddot{x} = 50(1 - 2\sin^2 5t) = 100(\frac{1}{2} - x)$
(b) centre: $x = \frac{1}{2}$, range: $0 \le x \le 1$, period: $\frac{\pi}{5}$ minutes, $t = \frac{\pi}{5}$
16(a) centre: $x = 7$. Since the amplitude is 7, the extremes of motion are $x = 0$ and $x = 14$, and the particle is stationary there.
(b) $v^2 = 9\Big(49 - (x - 7)^2\Big)$, 21 cm/s
(c) Although the particle is stationary for an instant, its acceleration at that time is positive (it is actually 63 m/s^2), and so the speed immediately changes and the particle moves away.
17(a) $\ddot{x} = -9x$ **(b)** period: $\frac{2\pi}{3}$, amplitude: $2\sqrt{13}$, maximum speed $6\sqrt{13}$, $|\ddot{x}| = 9\sqrt{13}$
18(a) $x = 3$, $\frac{\pi}{2}$ **(b)** $x = 3 + 2\sin(4t + \frac{\pi}{3})$
(c) $t = \frac{\pi}{12}(3n - 1)$, where n is a positive integer, $|v| = 8$
19(a) $\ddot{x} = -4(x - 10)$, centre: $x = 10$, period: π, amplitude: 10
(b) $\frac{3\pi}{4} - \frac{1}{2}\tan^{-1}\frac{3}{4}$ ($= \pi - \tan^{-1} 2 \doteqdot 2{\cdot}034$)
20(b) $\dot{x} = -16\pi\sin 2\pi t$, $\dot{y} = 16\pi\cos 2\pi t$, $\ddot{x} = -32\pi^2\cos 2\pi t$, $\ddot{y} = -32\pi^2\sin 2\pi t$
(c)(i) $\frac{\pi}{6}$ or $\frac{7\pi}{6}$ **(ii)** $\frac{\pi}{3}$ or $\frac{4\pi}{3}$ **(iii)** $\frac{3\pi}{4}$ or $\frac{7\pi}{4}$
21(b) $a = \sqrt{\dfrac{{v_2}^2{x_1}^2 - {v_1}^2{x_2}^2}{{v_1}^2 - {v_2}^2}}$
(c) 5 cm, π seconds, 10 cm/s
22 $v = \frac{1}{2}V\sqrt{3}$ or $v = -\frac{1}{2}V\sqrt{3}$, $x = \frac{1}{2}a\sqrt{3}$ or $x = -\frac{1}{2}a\sqrt{3}$
23(b) When $\alpha = \pi$, $A = 3$ and $x = 3\sin t$. When $\alpha = 0$, $A = 1$ and $x = -\sin t$.
(c) twice **(d)** When $\alpha = \frac{\pi}{3}$, $x = \sqrt{3}\cos t$. When $\alpha = \frac{5\pi}{3}$, $x = -\sqrt{3}\cos t$.

Exercise **3G** (Page 127)

1(a) $\dot{x} = 6\sqrt{3}$, $\dot{y} = 6$ **(b)** $\dot{x} = 4\sqrt{2}$, $\dot{y} = -4\sqrt{2}$
(c) $\dot{x} = 12$, $\dot{y} = 16$
2(a) $v = 6\sqrt{2}$, $\theta = 45°$ **(b)** $v = 14$, $\theta = -60°$
(c) $v = \sqrt{74}$, $\theta = \tan^{-1}\frac{7}{5}$ [$\doteqdot 54°28'$]
3(a) Initially, $\dot{x} = \dot{y} = 10$ **(b)** $\dot{x} = 10$, $x = 10t$, $\dot{y} = -10t + 10$, $y = -5t^2 + 10t$
(c) 5 metres, 1 second **(d)** 20 metres, 2 seconds
(e) $\dot{x} = 10$, $x = 5$, $\dot{y} = 5$ and $y = 3{\cdot}75$
(i) 6·25 metres **(ii)** $5\sqrt{5} \doteqdot 11$ m/s, $\tan^{-1}\frac{1}{2} \doteqdot 27°$
4(a) Initially, $\dot{x} = \sqrt{5}$ and $\dot{y} = 2\sqrt{5}$. **(b)** $\dot{x} = \sqrt{5}$, $x = t\sqrt{5}$, $\dot{y} = -10t + 2\sqrt{5}$, $y = -5t^2 + 2t\sqrt{5}$
(d) 1 metre **(e)** 2 metres **(f)** $\dot{x} = \sqrt{5}$, $\dot{y} = -2\sqrt{5}$, $v = 5$ m/s, $\theta = -\tan^{-1} 2$ **(g)** $y = 2x - x^2$
5(a) $x = 20t$, $y = -5t^2 + 20\sqrt{3}t$ **(b)** $4\sqrt{3}$ seconds, $80\sqrt{3}$ metres **(c)** $2\sqrt{3}$ seconds, 60 metres
(d) It is false. The horizontal range would not have changed, although the flight time would have been 4 seconds and the maximum height would have been 20 metres.
6(a) $x = 10\sqrt{3}t$, $y = -5t^2 + 10t$
(b) 5 s, $50\sqrt{3}$ metres **(c)** 80 metres
(d) 44 m/s, 67° **(e)** $y = -\frac{1}{60}x^2 + \frac{1}{\sqrt{3}}x$
7(a) 101 m/s **(b)** $x = 101t$, $y = -5t^2$ **(d)** 149 m/s, $\tan\theta = \frac{20}{101}\sqrt{30}$, $\theta \doteqdot 47°19'$ **(e)** 1·106 km
8(c) $V = 36$, $\theta \doteqdot 41°49'$ **(d)** 129 metres
9(a) $\dot{x} = V\cos\alpha$, $x = Vt\cos\alpha$, $\dot{y} = -gt + V\sin\alpha$, $y = -\frac{1}{2}gt^2 + Vt\sin\alpha$
(b)(i) $H = \dfrac{V^2}{2g}\sin^2\alpha$, $\dfrac{V}{g}\sin\alpha$ seconds **(ii)** $\dfrac{V^2}{2g}$ when $\alpha = 90°$, half this value when $\alpha = 45°$.
(c)(i) $R = \dfrac{v^2}{g}\sin 2\alpha$, $T = \dfrac{2V}{g}\sin\alpha$ **(ii)** V^2/g when $\alpha = 45°$, half this value when $\sin 2\alpha = \frac{1}{2}$, that is, $\alpha = 15°$ or $75°$.
10(c) 50 metres
11(d) $60°15'$ or $72°54'$
12(b) 0·36 s **(c)** 12° **(d)** No, it lands 4·72 metres in front of him.
13(b)(ii) 16 metres **(iii)** 112°
14(d) 15 metres **(e)** 10 m/s, $63°26'$
15(c) $T = 4$, $\theta = 30°$

ISBN: 978-1-107-61604-2 Cambridge University Press

16(b)(ii) Yes. The vertical components of their initial velocities are equal, and they are both subject to the same force (gravity) acting in the vertical direction.
18(b)(iv) 52°

Exercise 3H (Page 134)

1(a) 40 metres **(b)** 10 metres **(c)** $\alpha = 45^\circ$
(d) When $x = 10$, $y = 7\frac{1}{2}$, so the ball goes under A.
(e) $V = 20\,\text{m/s}$
2(a) $\dot{x} = 5\sqrt{2}$, $x = 5t\sqrt{2}$, $\dot{y} = -10t + 5\sqrt{2}$, $y = -5t^2 + 5t\sqrt{2}$ **(b)** range: 10 metres, maximum height: 2·5 metres when $x = 5$ **(c)(i)** 1·6 metres **(ii)** $y' = -\frac{1}{5}x + 1$, $\theta = -\tan^{-1}\frac{3}{5}$ **(d)(i)** $x = 3$ **(ii)** $\theta = \tan^{-1}\frac{2}{5}$
3(b) $R = 21{\cdot}6$ metres, $H = 4{\cdot}05$ metres
(c) $\tan^{-1}\frac{3}{4}$ **(d)** $15\,\text{m/s}$
(e) $t = 0{\cdot}8$, when $x = 9{\cdot}6$, and $t = 1$, when $x = 12$
4(a) $\dot{x} = 200$, $\dot{y} = 0$ **(b)** $\dot{x} = 200$, $x = 200t$, $\dot{y} = -10t$, $y = -5t^2$, $y = -\frac{1}{8000}x^2$ **(c)** 600 metres
(d) $8^\circ 32'$
5(c)(i) $\alpha = 15^\circ$ or 75°
(ii) It will if $\alpha = 75^\circ$, but not if $\alpha = 15^\circ$.
6(b) $62^\circ 22'$ or $37^\circ 5'$
7(b) range: 38·4 metres, height: 12·8 metres
(c)(ii) 33·3 metres
8 $\alpha = 45^\circ$, $V = 4\sqrt{5}\,\text{m/s}$, $\frac{2}{5}\sqrt{10}$ seconds.
The ground is the latus rectum of the parabola.
9(a) $x = \frac{1}{2}Vt\sqrt{2}$, $y = -\frac{1}{2}gt^2 + \frac{1}{2}Vt\sqrt{2}$
10(c)(ii) $R = 18$ metres
11(c) For $0^\circ < \alpha < 45^\circ$, $0 < \tan\alpha < 1$. Hence if α_1 and α_2 are both less than 45°, then the two roots of the quadratic both lie between 0 and 1. But the product of these roots is greater than 1, so α_1 and α_2 cannot both be less than 45°.
13(b) $\frac{1}{2}(1 + \sqrt{2})$ **(c)** $\frac{1}{2}(1 - \sqrt{2})$
(d) The two speeds are equal.
14 $(x - k\sin 2\alpha)^2 = -4k\cos^2\alpha(y - k\sin^2\alpha)$,
vertex: $(k\sin 2\alpha, k\sin^2\alpha)$,
focus: $(k\sin 2\alpha, -k\cos 2\alpha)$, directrix: $y = k$
(a) k is the maximum height when the projectile is fired vertically upwards.
(b) focus: the circle $x^2 + y^2 = k^2$ with centre O and radius k, vertex: the ellipse $\dfrac{x^2}{k^2} + \dfrac{(y - \frac{1}{2}k)^2}{(\frac{1}{2}k)^2} = 1$
(c) The focus is $\left(k\cos(2\alpha - \frac{\pi}{2}), k\sin(2\alpha - \frac{\pi}{2})\right)$.
This is a general property of parabolas.

Chapter Four

Exercise 4A (Page 141)

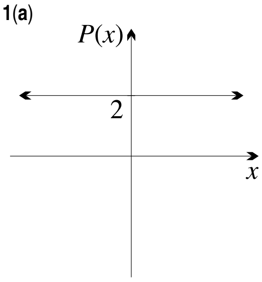
1(a) yes **(b)** no **(c)** no **(d)** no **(e)** yes **(f)** no
(g) yes **(h)** yes **(i)** no **(j)** yes **(k)** yes **(l)** no
2(a) 3, 4, $4x^3$, -11, not monic
(b) 3, -6, $-6x^3$, 10, not monic
(c) 0, 2, 2, 2, not monic **(d)** 12, 1, x^{12}, 0, monic
(e) 3, 1, x^3, 0, monic **(f)** 5, -1, $-x^5$, 0, not monic
(g) no degree, no leading coefficient, no leading term, 0, not monic
(h) 2, -3, $-3x^2$, 0, not monic
(i) 6, -4, $-4x^6$, -5, not monic
3(a) $x^2 + 2x + 3$ **(b)** $x^2 + 2x + 3$ **(c)** $-x^2 + 8x + 1$
(d) $x^2 - 8x - 1$ **(e)** $5x^3 - 13x^2 - x + 2$
(f) $5x^3 - 13x^2 - x + 2$
5(a) $2\left(x^2 - \frac{3}{2}x + 2\right)$ **(b)** $3\left(x^3 - 2x^2 - \frac{5}{3}x + \frac{1}{3}\right)$
(c) $-2\left(x^5 - \frac{7}{2}x^4 + 2x - \frac{11}{2}\right)$ **(d)** $\frac{2}{3}\left(x^3 - 6x + 24\right)$
6(a) $x(x - 10)(x + 2)$, 0, 10, -2
(b) $x^2(2x + 1)(x - 1)$, 0, 1, $-\frac{1}{2}$
(c) $(x - 3)(x + 3)(x^2 + 4)$, 3, -3
(d) $(x - 2)(x^2 + 2x + 4)$, 2
(e) $(x - 3)(x + 3)(x^2 + 9)$, 3, -3
(f) $(x - 1)(x + 1)(x^2 - x + 1)(x^2 + x + 1)$, 1, -1
7(a)(i) $p + q$ **(ii)** the maximum of p and q
(b) $P(x)Q(x)$ still has degree $p + p = 2p$, but $P(x) + Q(x)$ may have degree less than p (if the leading terms cancel out), or it could be the zero polynomial. **(c)** $x^2 + 2$ and $-x^2 + 3$. Do not choose two opposite polynomials, like $x^2 + 1$ and $-x^2 - 1$, because their sum is the zero polynomial, which does not have a degree.
8 $x + 1$
9(a) $a = 3$, $b = -4$ and $c = 1$ **(b)** $a = 2$ and $b = 3$
(c) $a = 1$, $b = 2$ and $c = 1$
(d) $a = 1$, $b = 2$ and $c = -1$
10(a) $a = 4$, $b \neq \frac{2}{3}$ and c arbitrary **(b)** $a = 4$, $b = \frac{2}{3}$ and c arbitrary **(c)** $a = 5$, b and c arbitrary
(d) $a = 4$, $b = \frac{2}{3}$ and $c = \frac{1}{5}$
12(c) A polynomial is even if and only if the coefficients of the odd powers of x are zero. A polynomial is odd if and only if the coefficients of the even powers of x are zero.
13(a) True. If $P(x)$ is even, then the terms are of the form $a_n x^{2n}$, where $n \geq 0$ is an integer. Therefore $P'(x)$ has terms of the form $2na_n x^{2n-1}$, so all powers of x will be odd.

ISBN: 978-1-107-61604-2 Cambridge University Press

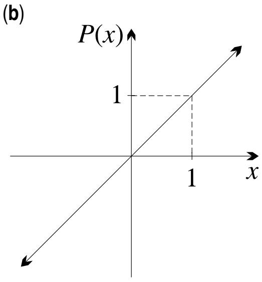
(b) False. For example, $Q(x) = x^3 + 1$ is not odd but $Q'(x) = 3x^2$ is even.
(c) True. If $R(x)$ is odd, then the terms are of the form $a_n x^{2n+1}$, where $n \geq 0$ is an integer. Therefore $R'(x)$ has terms of the form $(2n+1)a_n x^{2n}$, so all powers of x will be even.
(d) True. As $S'(x)$ is odd, it has no constant term, and all powers of x are odd. Therefore all the terms in $S(x)$ will have even powers.
14(a) They are both nonzero constants.
(b) If $f(x)$ were a polynomial, $\dfrac{1}{f(x)}$ would not be a polynomial.

Exercise **4B** (Page 145)

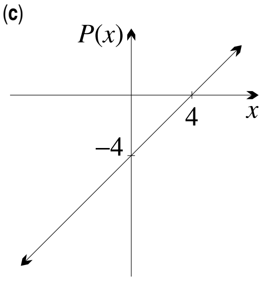
1(a)



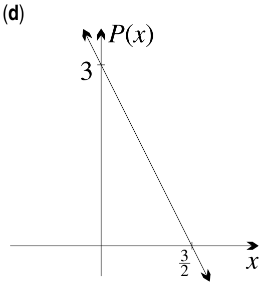
(b)



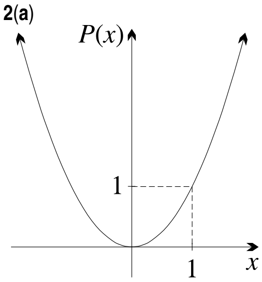
(c)



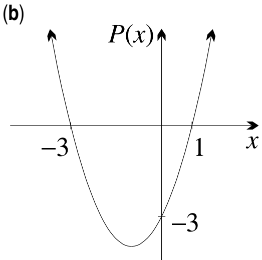
(d)



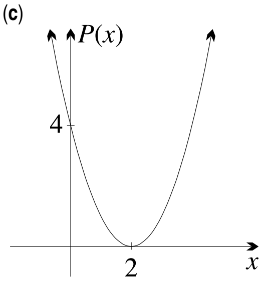
2(a)



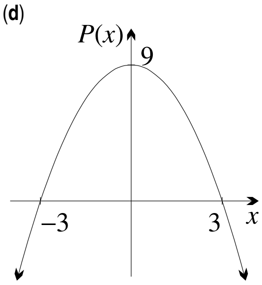
(b)



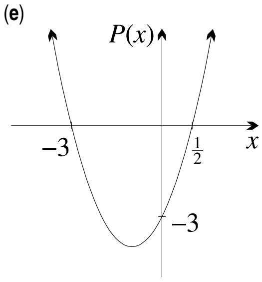
(c)



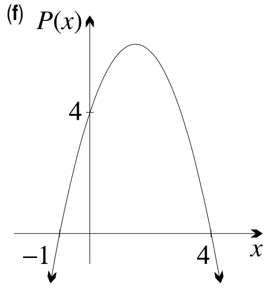
(d)



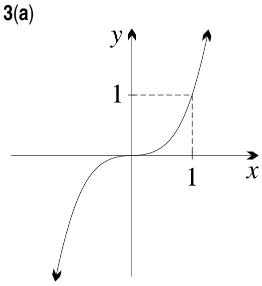
(e)



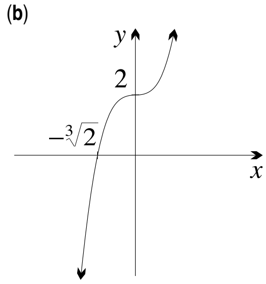
(f)



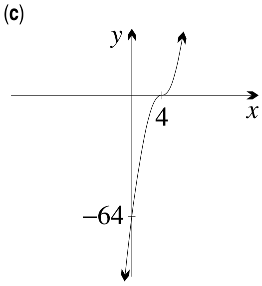
3(a)



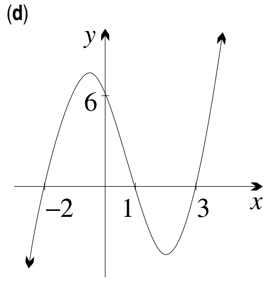
(b)



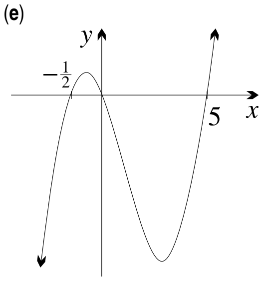
(c)



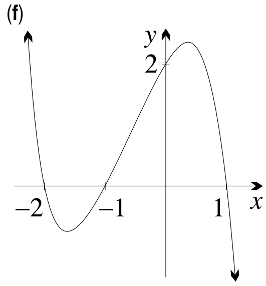
(d)



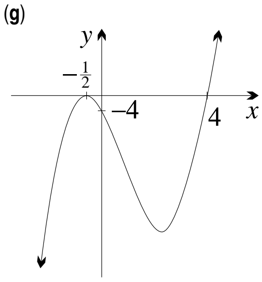
(e)



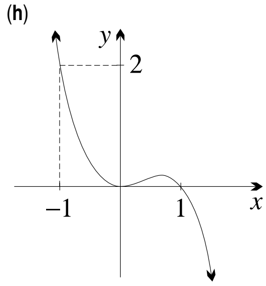
(f)



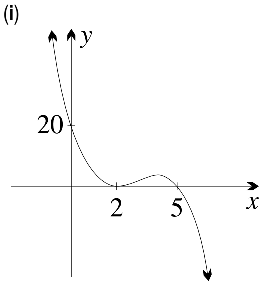
(g)



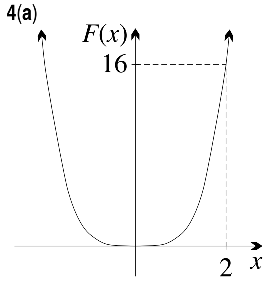
(h)



(i)

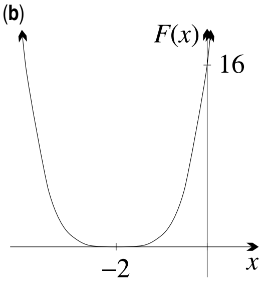


4(a)

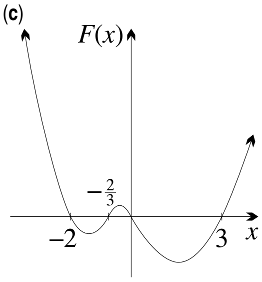


ISBN: 978-1-107-61604-2 Cambridge University Press

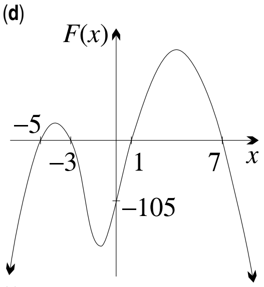
(b)



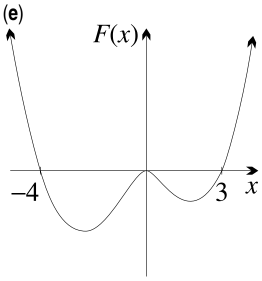
(c)



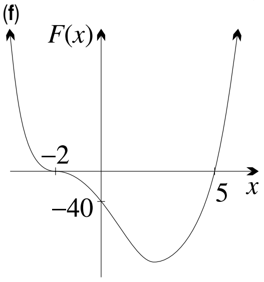
(d)



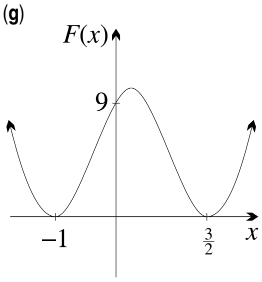
(e)



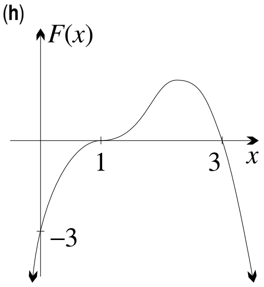
(f)



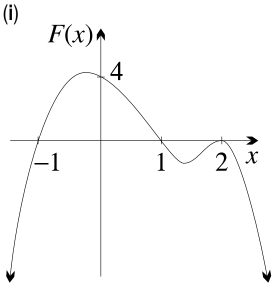
(g)



(h)

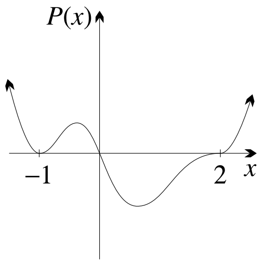


(i)

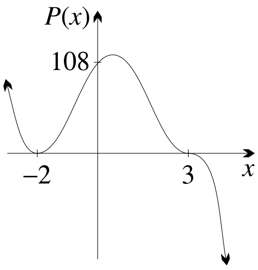


5(a) There are two zeroes, one between 0 and 1, and one between 2 and 3. **(b)** There are three zeroes, one between -2 and -1, one between -1 and 0, and one between 1 and 2.

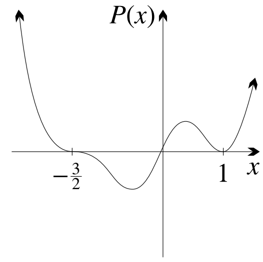
6(a)



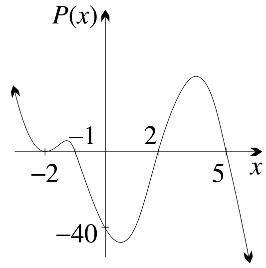
(b)



(c)

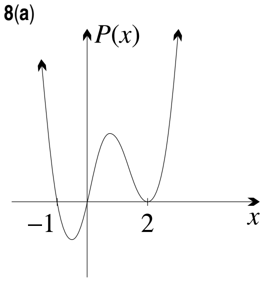


(d)

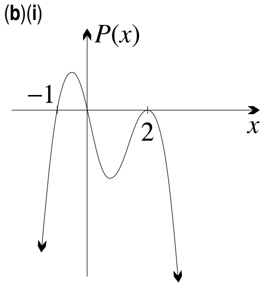


7(a) $x > 2$ or $x < 0$, $x \neq -1$ **(b)** $x \leq 3$
(c) $x \leq -\frac{3}{2}$ or $x \geq 0$ **(d)** $x > 5$ or $-1 < x < 2$

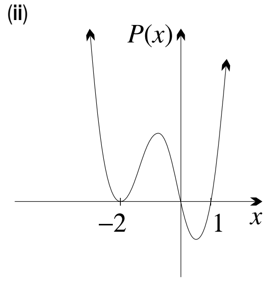
8(a)



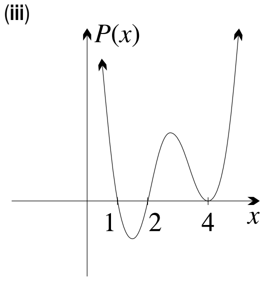
(b)(i)



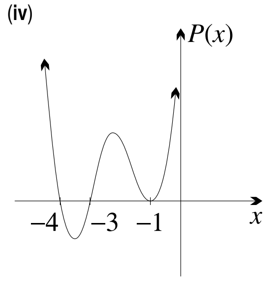
(ii)



(iii)



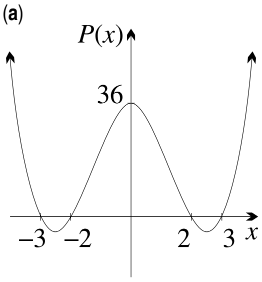
(iv)



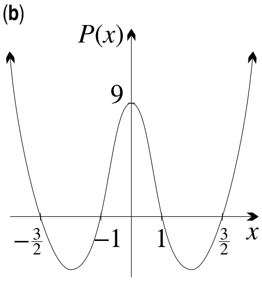
9(a) $P(x) = x^2 - x - 6$ **(b)** $P(x) = x^2 + 4x + 1$
(c) $P(x) = 2x^3 - 6x^2 + 4x$
10(a)(i) $a = c = e = 0$ **(ii)** $b = d = f = 0$
(b) $P(x) = x^4 - 10x^2 + 9$
(c) $P(x) = -3x^5 + 15x^3 - 12x$
11(c) $P(x) = x^3 - 4x$
12(a) $(x+3)(x-3)(x+2)(x-2)$
(b) $(2x-3)(2x+3)(x+1)(x-1)$

ISBN: 978-1-107-61604-2 Cambridge University Press

(a)



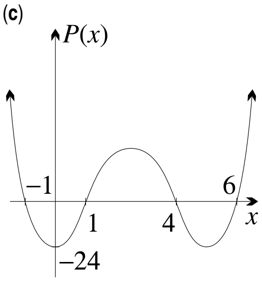
(b)



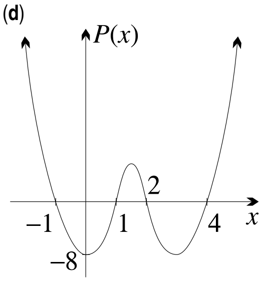
(c) $(x-6)(x+1)(x-4)(x-1)$

(d) $(x-4)(x+1)(x-1)(x-2)$

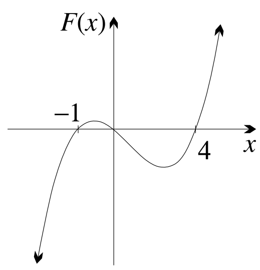
(c)



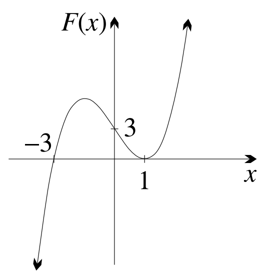
(d)



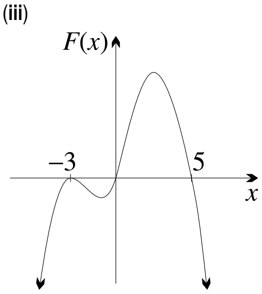
13(a)(i)



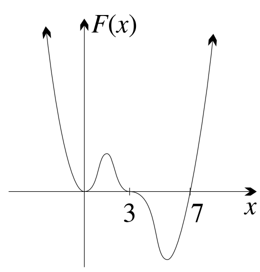
(ii)



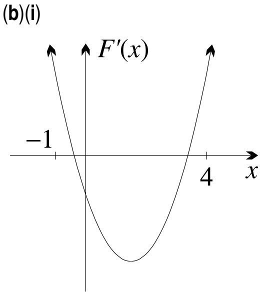
(iii)



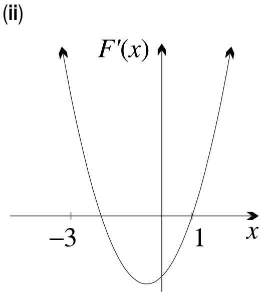
(iv)



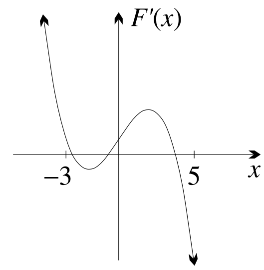
(b)(i)



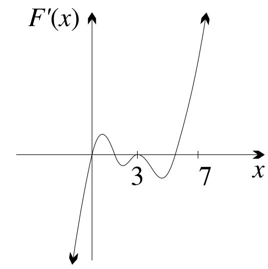
(ii)



(iii)

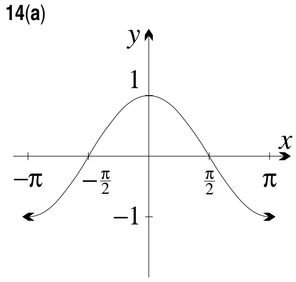


(iv)

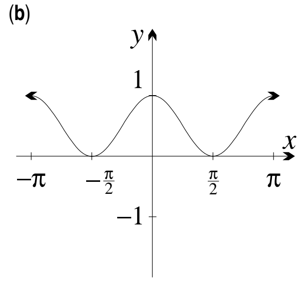


(c) (i) is increasing when $x > 4$ or $-1 < x < 0$ and decreasing when $x < -1$ or $0 < x < 4$. (ii) is increasing when $x > -3$, $x \neq 1$ and decreasing when $x < -3$. (iii) is increasing when $0 < x < 5$ and decreasing when $x > 5$ or $x < 0$, $x \neq -3$. (iv) is increasing when $x < 3$, $x \neq 0$ or $x > 7$ and decreasing when $3 < x < 7$.

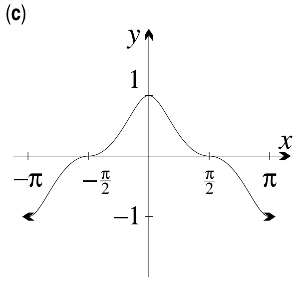
14(a)



(b)



(c)



15(a)(i) $f''(x) = 6ax + 2b$ and $f''(0)$, so $b = 0$. Substituting $x = 0$ gives $d = 0$.

(ii) The curve has point symmetry in the origin.

(b) Translate the curve so that the point of inflexion is at the origin.

(c) We know that the curve has point symmetry in the point of inflexion, so the image of one turning point must be the other one. Now use part (b).

16 The graphs always intersect at $(0, 1)$ and at $(-1, 0)$. If m and n are both even, they also intersect at $(-2, 1)$, and if m and n are both odd, they also intersect at $(-2, -1)$.

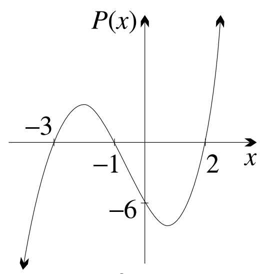
ISBN: 978-1-107-61604-2 Cambridge University Press

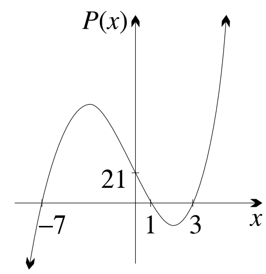
Exercise 4C (Page 149)

1(a) $63 = 5 \times 12 + 3$ **(b)** $125 = 8 \times 15 + 5$
(c) $324 = 11 \times 29 + 5$ **(d)** $1857 = 23 \times 80 + 17$
2(a) $x^2 - 4x + 1 = (x + 1)(x - 5) + 6$
(b) $x^2 - 6x + 5 = (x - 5)(x - 1)$
(c) $x^3 - x^2 - 17x + 24 = (x - 4)(x^2 + 3x - 5) + 4$
(d) $2x^3 - 10x^2 + 15x - 14 = (x - 3)(2x^2 - 4x + 3) - 5$
(e) $4x^3 - 4x^2 + 7x + 14 = (2x + 1)(2x^2 - 3x + 5) + 9$
(f) $x^4 + x^3 - x^2 - 5x - 3 = (x - 1)(x^3 + 2x^2 + x - 4) - 7$
(g) $6x^4 - 5x^3 + 9x^2 - 8x + 2$
$= (2x - 1)(3x^3 - x^2 + 4x - 2)$
(h) $10x^4 - x^3 + 3x^2 - 3x - 2$
$= (5x + 2)(2x^3 - x^2 + x - 1)$
3(a) $\dfrac{x^2 - 4x + 1}{x + 1} = x - 5 + \dfrac{6}{x + 1}$,
$\frac{1}{2}x^2 - 5x + 6\log(x + 1) + C$
(b) $\dfrac{x^2 - 6x + 5}{x - 5} = x - 1$, $\frac{1}{2}x^2 - 5x + C$
(c) $\dfrac{x^3 - x^2 - 17x + 24}{x - 4} = x^2 + 3x - 5 + \dfrac{4}{x - 4}$,
$\frac{1}{3}x^3 + \frac{3}{2}x^2 - 5x + 4\log(x - 4) + C$
(d) $\dfrac{2x^3 - 10x^2 + 15x - 14}{x - 3} = 2x^2 - 4x + 3 - \dfrac{5}{x - 3}$,
$\frac{2}{3}x^3 - 2x^2 + 3x - 5\log(x - 3) + C$
4(a) $x^3 + x^2 - 7x + 6 = (x^2 + 3x - 1)(x - 2) + 4$
(b) $x^3 - 4x^2 - 2x + 3 = (x^2 - 5x + 3)(x + 1)$
(c) $x^4 - 3x^3 + x^2 - 7x + 3$
$= (x^2 - 4x + 2)(x^2 + x + 3) + (3x - 3)$
(d) $2x^5 - 5x^4 + 12x^3 - 10x^2 + 7x + 9$
$= (x^2 - x + 2)(2x^3 - 3x^2 + 5x + 1) + (7 - 2x)$
5(a) 0, 1 or 2 **(b)** $D(x)$ has degree 3 or higher.
6(a) $x^3 - 5x + 3 = (x - 2)(x^2 + 2x - 1) + 1$
(b) $2x^3 + x^2 - 11 = (x + 1)(2x^2 - x + 1) - 12$
(c) $x^3 - 3x^2 + 5x - 4 = (x^2 + 2)(x - 3) + (3x + 2)$,
$\dfrac{x^3 - 3x^2 + 5x - 4}{x^2 + 2} = x - 3 + \dfrac{3x + 2}{x^2 + 2}$,
$\frac{1}{2}x^2 - 3x + \frac{3}{2}\log(x^2 + 2) + \sqrt{2}\tan^{-1}(x/\sqrt{2}) + C$
(d) $2x^4 - 5x^2 + x - 2$
$= (x^2 + 3x - 1)(2x^2 - 6x + 15) + (13 - 50x)$
(e) $2x^3 - 3 = (2x - 4)(x^2 + 2x + 4) + 13$
(f) $x^5 + 3x^4 - 2x^2 - 3$
$= (x^2 + 1)(x^3 + 3x^2 - x - 5) + (x + 2)$,
$\dfrac{x^5 + 3x^4 - 2x^2 - 3}{x^2 + 1} = x^3 + 3x^2 - x - 5 + \dfrac{x + 2}{x^2 + 1}$,
$\frac{1}{4}x^4 + x^3 - \frac{1}{2}x^2 - 5x + \frac{1}{2}\log(x^2 + 1) + 2\tan^{-1}x + C$
7(a) quotient: $\frac{1}{2}x + \frac{7}{4}$, remainder: $\frac{21}{4}$
(b) quotient: $2x^2 + \frac{1}{3}x + \frac{13}{9}$, remainder: $-\frac{5}{9}$
(c) quotient: $\frac{1}{2}x^2 + \frac{1}{4}x + \frac{7}{8}$, remainder: $\frac{29}{8}$
8(a) $P(x) = (x - 3)(x + 1)(x + 4)$
(b) $x > 3$ or $-4 < x < -1$
9(a) $(x - 2)(x + 1)(2x - 1)(x + 3)$
(b) $-3 \le x \le -1$ or $\frac{1}{2} \le x \le 2$
10(a) $30 = 4 \times 7 + 2$, $30 = 7 \times 4 + 2$ **(b)** $D(x)$
11(a) quotient: $x^2 - 3x + 5$, remainder: $12 - 13x$
(b) $a = 8$ and $b = -5$ **(c)** $(x^2 + x - 1)(x^2 - 3x + 5)$
12(a) $x^4 - x^3 + x^2 - x + 1$
$= (x^2 + 4)(x^2 - x - 3) + (3x + 13)$
(b) $a = -4$ and $b = -12$ **(c)** $(x^2 + 4)(x^2 - x - 3)$
13 $a = 41$ and $b = -14$
14(a) $a = 7$ and $b = -32$
(b) $A(x) = \frac{1}{20}(x^3 - 4x^2 - 24)$, $B(x) = -\frac{1}{20}(x - 5)$

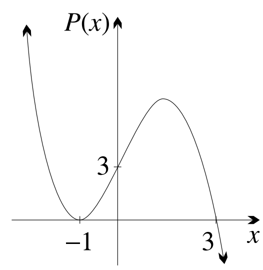
Exercise 4D (Page 153)

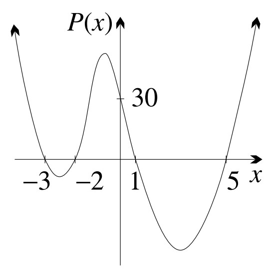
1(a) 3 **(b)** 25 **(c)** -15 **(d)** -3 **(e)** 111 **(f)** -41
2(a) yes **(b)** no **(c)** no **(d)** yes **(e)** no **(f)** yes
3(a) $3\frac{5}{8}$ **(b)** $12\frac{1}{8}$ **(c)** $3\frac{14}{27}$
4(a) $k = 4$ **(b)** $m = -\frac{1}{2}$ **(c)** $p = -14$ **(d)** $a = -1$
5(b) $x \ge 6$ or $-1 \le x \le 3$
6(b) $x \le -2$ or $-\frac{1}{2} \le x \le 3$
7(a) $(x - 2)(x + 1)(x + 3)$ **(b)** $(x - 1)(x - 3)(x + 7)$



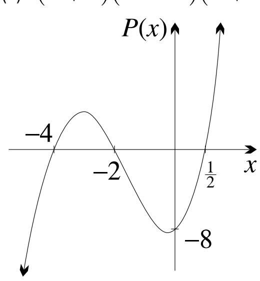


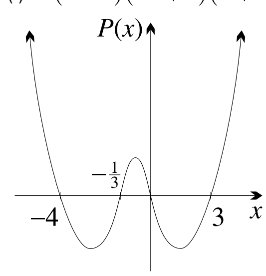
(c) $(x + 1)^2(3 - x)$ **(d)** $(x - 1)(x - 2)(x - 3)(x + 5)$





(e) $(x + 2)(2x - 1)(x + 4)$ **(f)** $x(x - 3)(3x + 1)(x + 4)$





ISBN: 978-1-107-61604-2 Cambridge University Press

8(a) -1, -4 or 2 **(b)** 3 or -2 **(c)** 2, $\frac{1}{2}(-3 \pm \sqrt{13})$ **(d)** 3 **(e)** 2, $-\frac{2}{3}$ or $-\frac{1}{2}$ **(f)** -2, $\frac{1}{4}(-3 \pm \sqrt{17})$

9(a) $(2x-1)(x+3)(x-2)$ **(b)** $(3x+2)(2x+1)(x-1)$

10(a) $P(x) = (x+1)(x-2)^2$, $Q(x) = (x+1)(x-2)(x+3)$ and $R(x) = (x+1)^2(x-2)$ **(b)** $(x+1)(x-2)$ **(c)** $(x+1)^2(x-2)^2(x+3)$

11 $\frac{1}{4}$, 1 and 3

12(a) $a = 4$ and $b = 11$ **(b)** $a = 2$ and $b = -9$ **(c)** $P(x) = -x^3 + 16x$ **(d)** $p = 2$ or $p = 3$

13(a) $3 - x$ **(b)** $3 - 2x$

14(a) $b = 0$, $c = -9$ and $d = 0$ **(c)** $-3 < x < 0$ or $x > 3$

15(b) $t = 1$ or $t = -2$

16(a) $x + 1$ is a factor when n is odd. **(b)** $x + a$ is a factor when n is odd.

17(a) $P(x) = (x-1)(x+3)Q(x) + (2x-1)$ **(b)** 1

18(a) $\frac{1}{2}$ **(b)** $a = -2$ and $b = -7$ **(c)** 8

19(b) $a = -2$

20 $(p^2 + pq + q^2)x - pq(p+q)$

21(a) $(12, 0)$ **(b)** $g(x) = a(x-k)(x-12)^2$ **(c)** 8 **(e)** $a = \frac{2}{27}$, $k = 3$, $g(x) = \frac{2}{27}(x-3)(x-12)^2$ **(f)** $f(x) = \frac{2}{27}(x-3)(x-12)^2 + 4$

23(a) $a^2 + b^2 + c^2 - ab - ac - bc$ **(b)** $(b-c)(c-a)(a-b)(a+b+c)$

Exercise **4E** (Page 159)

1(a) $(x+1)(x-3)(x-4)$ **(b)** $x(x+2)(x-3)(x-1)$ **(c)** $(3x-1)(2x+1)(x-1)$

2(c) $(x-2)(x+3)(x+1)(x-5)$

3(a) $P(x) = (x-1)(x+1)(x-3)(2x+1)$ **(b)** $P(x) = (x-1)(x-2)(x+2)(2x-3)$ **(c)** $P(x) = (2x-5)(3x-2)(x+1)(x-2)$ **(d)** $P(x) = (x-2)(x-3)(3x-1)^2$

4(a) $a = 2$, $b = \frac{1}{3}$ and $c = \frac{5}{2}$ **(b)** $a = -1$, $b = 3$, $c = \frac{1}{2}$ and $d = \frac{5}{4}$

5(a) $a = 3$, $b = -16$ and $c = 27$ **(b)** $a = 2$, $b = -2$, $c = -7$ and $d = -7$ **(c)** $(x+1)^3 - (x+1)^2 - 4(x+1) + 5$ **(d)** $a = 3$, $b = -2$ and $c = 1$

6(a) $P(x) = (x-2)^2(x+5)$ **(b)** $P(x) = (x-1)(x+3)(2x-7)$

8 The maximum of m and n.

9 There must be one turning point between each of the consecutive zeroes.

10 $a = b = c = 0$, $d = k$

11(a) maximum at $(2, 19)$, minimum at $(-2, -13)$, three zeroes **(b)** maximum at $(1, 10)$, minimum at $(-\frac{5}{3}, \frac{14}{27})$, one zero **(c)** maximum at $(3, 26)$, horizontal point of inflexion at $(0, -1)$, two zeroes **(d)** minimum at $(1, 1)$, horizontal point of inflexion at $(0, 2)$, no zeroes

12(a) The curves are tangent at $x = 3$ and cross at $x = -1$. **(b)** The curves are tangent at $x = 2$ and cross at $x = 3$. **(c)** The curves cross at $x = -5$, $x = -2$ and $x = 3$. **(d)** The curves are tangent to one another and cross at $x = 1$ and cross at $x = -2$. **(e)** The curves cross at the origin and cross and are tangent to each other at $x = -1$.

13(b) $a = -5$ and $b = 8$ **(c)** $2(x-2)^2(x-3)^2$

Exercise **4F** (Page 165)

1(a) 4 **(b)** 2 **(c)** 8 **(d)** 2 **(e)** 14 **(f)** 12 **(g)** 6 **(h)** 24 **(i)** $\frac{17}{2}$

2(a) -2 **(b)** -11 **(c)** 12 **(d)** $-\frac{11}{12}$ **(e)** $-\frac{1}{6}$ **(f)** 0 **(g)** -132 **(h)** 26 **(i)** $\frac{13}{72}$ The roots are -1, -4 and 3.

3(a) 5 **(b)** 2 **(c)** 4 **(d)** -3 **(e)** $-\frac{4}{3}$ **(f)** $-\frac{2}{3}$ **(g)** $-\frac{5}{3}$ **(h)** 21

4(a) $x^2 + x - 6 = 0$ **(b)** $x^3 - 7x + 6 = 0$ **(c)** $x^4 + x^3 - 7x^2 - x + 6 = 0$

5(a) 3 **(b)** $-\frac{1}{2}$ **(c)** -3, 1. Hence 1 is a double zero. **(d)** $\frac{2}{3}$, 2

6(a) -1 (triple zero), and 3 **(b)** -4, -1, 2 and 3 **(c)** -1 and 5 (both double zeroes) **(d)** -2 and 2 (No real numbers are solutions of $\alpha + \beta = -1$ and $\alpha\beta = 1$.)

7(a) $-\frac{5}{2}$ **(b)** -2 **(c)** $\frac{41}{4}$ **(d)** $-30\frac{5}{8}$ **(e)** $\frac{1}{2}\sqrt{57}$

8(c) $\alpha^2 - 3\alpha + 5 = 0$, which has negative discriminant. **(d)** once

9(a) $a = 5$, $b = 12$, $(x-3)(x+1)(x-4)$ **(b)** $a = -5$, $b = 8$, $\frac{1}{2}$ is the other zero.

10(a) $a = 3$ and $b = -24$, $(x-3)(x+4)(x+2)$ **(b)** $a = -1$ and $b = 3$, 5, -4, $\sqrt{3}$, $-\sqrt{3}$

13(a) $\frac{3}{2}$, $\frac{3}{2}$ and -1 **(b)** $\frac{1}{3}$, -4 and 4 **(c)** 6, $\frac{1}{2}$ and -4 **(d)** 4, $\frac{1}{2}$ and 2

14(a) $-\frac{1}{3}$, 1 and $\frac{7}{3}$. The inflexion is $(1, 0)$. **(b)** $\frac{1}{4}$, $\frac{1}{2}$ and 1 **(c)** $x = \frac{1}{2}$, -1 or 2

15(a) $a = -12$ and the roots are -2, 2 and -3. **(b)** $a = -5$ and the roots are 4, $\frac{1}{4}$ and -3. **(c)** -1, -2, 2 and 4

18(a)(i) $\frac{1}{2}$ **(ii)** $\frac{1}{2}$ **(iii)** $\frac{1}{2}$ **(iv)** $\frac{1}{4}$

19(c) $\cos\frac{2\pi}{9}$, $\cos\frac{4\pi}{9}$, $\cos\frac{8\pi}{9}$

ISBN: 978-1-107-61604-2 Cambridge University Press

(d)(i) 0 **(ii)** $\frac{1}{8}$ **(iii)** 6 **(iv)** $\frac{3}{2}$
20 12
21 0, because 1 is one of the roots, and so 0 is one of the factors of the expression.
23 $d = \frac{1}{8}b(4c - b^2)$, $e = \frac{1}{64}(4c - b^2)^2$

Exercise **4G** (Page 169)

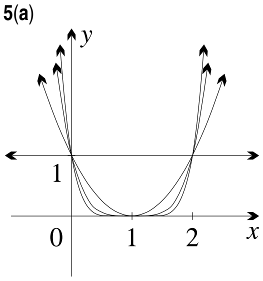
1(b) The equation is $(x - 4)^2 = 0$, so $x = 4$ is a double root, and the line is a tangent at $T(4, -8)$.
2(b)(i) $\alpha+\alpha=4$ **(ii)** $b = -4$.
(iii) $y = -4 - 2x$, $T = (2, -8)$.
3(b) $\alpha + \beta = 4$, $M = (2, 3)$
(d) Since the gradient is 1, rise = run.
$AB = 2\sqrt{10}$.
4(b) The roots are 1, 1 and 3. **(c)** The line is a tangent at $(1, 2)$ because $x = 1$ is a double root of the equation. The curves also intersect at $(3, 0)$.
5(b)(i) $\alpha+\alpha+0 = 5$ **(ii)** $m = -\frac{1}{4}$.
(iii) $y = -\frac{1}{4}x$, $T = (2\frac{1}{2}, -\frac{5}{8})$.
6(b) $\alpha + \beta + 2 = 5$, $M = (1\frac{1}{2}, -\frac{1}{2})$ **(d)** $\sqrt{26}$
7(a) The zeroes of $F'(x)$ satisfy $3x^2 + 2ax + b = 0$.
(b) Since $F''(x) = 6x + 2a$, the inflexion is at $x = -\frac{1}{3}a = \frac{1}{2}(\alpha + \beta)$.
8(a) $y = mx + (m - 7)$
(b) $x^3 - 3x^2 + (4 - m)x + (8 - m) = 0$
(c) The line is a tangent at $x = \alpha$ and meets the curve at $A(-1, -7)$. $\alpha = 2$, $T = (2, 5)$, $m = 4$
9(a) $y = mx - mp + p^3$
(c) $x = -\frac{1}{2}p$, so M lies on $x = -\frac{1}{2}p$.
10(a) $m = 2$, $\alpha = 1$ **(b)** $y + 3 = m(x + 2)$
(c) $y = 2x + 1$
11(a) $y = (x+1)(x-2)(x-5)(x+2)$ **(c)** $\alpha+\beta = 2$, $\alpha^2 + \beta^2 + 4\alpha\beta = -9$, $2\alpha^2\beta + 2\alpha\beta^2 = m - 16$, $\alpha^2\beta^2 = 20 - b$
(d) $m = -10$, $b = -22\frac{1}{4}$, $y = -10x - 22\frac{1}{4}$
12(a) $k = -\frac{1}{4}$, $(\frac{1}{2}\sqrt{2}, \frac{1}{4})$ and $(-\frac{1}{2}\sqrt{2}, \frac{1}{4})$
(b) $c = \frac{1}{2}$, $(\frac{1}{2}\sqrt{2}, \frac{1}{4}\sqrt{2})$ and $(-\frac{1}{2}\sqrt{2}, -\frac{1}{4}\sqrt{2})$
(c) $k = 0$ and $T(0, 0)$, or $k = \frac{4}{27}$ and $(\frac{2}{3}, \frac{8}{27})$
(d) $\alpha = -\frac{1}{2} + \frac{1}{2}\sqrt{5}$
13(a) $y = -\frac{1}{3}x$ **(b)** $x^2 + (y - 1)^2 = 1$. It is the circle with diameter OF. **(c)** $x = 2$ and $y \geq 0$
(d)(i) $y = -mx$ **(ii)** $y = \frac{1}{2}b$ **(e)** $x = -\frac{1}{2}$
14(a) $a = \frac{5}{4}$, $(\frac{1}{2}\sqrt{3}, -\frac{1}{2})$ and $(-\frac{1}{2}\sqrt{3}, -\frac{1}{2})$
(b) Either $a = 1$ and the other points are $(1, 0)$ and $(-1, 0)$, or $a = -1$ and there are no other points. **(c)** $a = \frac{1}{2}$ **(d)** If α is a triple root of $x^3 - 6x^2 - (2+m)x + (1-b) = 0$, then $3\alpha = 6$ and $\alpha = 2$, so the point is $(2, -19)$. **(e)** $y = -16x - 4$, $|\alpha - \beta| = 2\sqrt{6}$, $AB = 2\sqrt{6} \times \sqrt{16^2 + 1} = 2\sqrt{1542}$
15 Let the roots be $\alpha - k$, α and $\alpha + k$.
17(c) $M = \left(\dfrac{\lambda + 2}{2\lambda}, -\dfrac{\lambda + 2}{2\lambda}\right)$, locus: $y = -x$
(d) $\lambda = 2\left(\sqrt{2} + 1\right)$ **(e)** $\lambda < -2\left(\sqrt{2} - 1\right)$ or $\lambda > 2\left(\sqrt{2} + 1\right)$, but $\lambda \neq -1$
18(b) $x = 1/b$

ISBN: 978-1-107-61604-2 Cambridge University Press

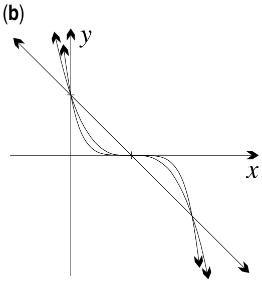
Chapter Five

Exercise **5A** (Page 176)

2(a) $1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$
(b) $1 - 6x + 15x^2 - 20x^3 + 15x^4 - 6x^5 + x^6$
(c) $1 + 9x + 36x^2 + 84x^3 + 126x^4 + 126x^5 + 84x^6 + 36x^7 + 9x^8 + x^9$ **(d)** $1 - 9x + 36x^2 - 84x^3 + 126x^4 - 126x^5 + 84x^6 - 36x^7 + 9x^8 - x^9$ **(e)** $1 + 5c + 10c^2 + 10c^3 + 5c^4 + c^5$ **(f)** $1 + 8y + 24y^2 + 32y^3 + 16y^4$
(g) $1 + \frac{7}{3}x + \frac{7}{3}x^2 + \frac{35}{27}x^3 + \frac{35}{81}x^4 + \frac{7}{81}x^5 + \frac{7}{729}x^6 + \frac{1}{2187}x^7$ **(h)** $1 - 9z + 27z^2 - 27z^3$
(i) $1 - \dfrac{8}{x} + \dfrac{28}{x^2} - \dfrac{56}{x^3} + \dfrac{70}{x^4} - \dfrac{56}{x^5} + \dfrac{28}{x^6} - \dfrac{8}{x^7} + \dfrac{1}{x^8}$
(j) $1 + \dfrac{10}{x} + \dfrac{40}{x^2} + \dfrac{80}{x^3} + \dfrac{80}{x^4} + \dfrac{32}{x^5}$
(k) $1 + \dfrac{5y}{x} + \dfrac{10y^2}{x^2} + \dfrac{10y^3}{x^3} + \dfrac{5y^4}{x^4} + \dfrac{y^5}{x^5}$
(l) $1 + \dfrac{12x}{y} + \dfrac{54x^2}{y^2} + \dfrac{108x^3}{y^3} + \dfrac{81x^4}{y^4}$
4(a)(i) $55x^2$ **(ii)** $165x^8$ **(b)(i)** $-35x^3$ **(ii)** $-21x^5$
(c)(i) $240x^4$ **(ii)** $192x^5$ **(d)(i)** $-\dfrac{12}{x}$ **(ii)** $\dfrac{54}{x^2}$
5(a)



(b)



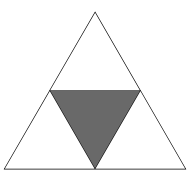
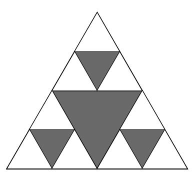
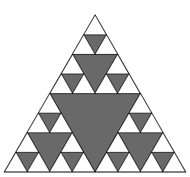
7(a) x^3 **(b)** x^6
8 21
9(a) $a = 76,\ b = 44$ **(b)** $a = 16,\ b = -8$
(c) $a = 433,\ b = 228$ **(d)** $a = 4069,\ b = -2220$
10(a) 152 **(b)** $88\sqrt{3}$
12(a) $1{\cdot}018\,14$ **(b)** $0{\cdot}815\,37$ **(c)** $0{\cdot}032\,00$
13(a)(i) $1 + 4x + 6x^2 + \cdots$ **(ii)** -14
(b)(i) $1 + 10x + 40x^2 + 80x^3 + \cdots$ **(ii)** 40
(c)(i) $1 - 12x + 54x^2 - 108x^3 + \cdots$ **(ii)** -228
14(a) -12 **(b)** 0 **(c)** 380 **(d)** $-\frac{5}{3}$ **(e)** 750 **(f)** -8
15(a) 97 **(b)** $1\frac{10}{27}$
16(a)(i) $15x^2$ **(ii)** $20x^3$ **(iii)** $3 : 4x$
(iv) $135,\ 540,\ 1 : 4$ **(b)(i)** $\dfrac{224}{81x^5}$ **(ii)** $\dfrac{448}{729x^6}$
(iii) $9x : 2$ **(iv)** $\frac{7}{81},\ \frac{7}{729},\ 9 : 1$
17(a) $x = 0$ or $\frac{1}{2}$ **(b)** $x = \frac{5}{2}$ **(c)** $x = 0,\ 1$ or 5
18(a) $k = 5$ **(b)** $k = \frac{1}{2}$ **(c)** $k = -2$
19(a) $\frac{1}{42}\ \mathrm{u}^2$ **(b)** $\frac{1}{630}\ \mathrm{u}^2$ **(c)** $\dfrac{\pi}{20}\ \mathrm{u}^3$
20(b) $\$1124.86$
21 $1{\cdot}0634$
23(a) 3 points, 3 segments, 1 triangle **(b)** 4 points, 6 segments, 4 triangles, 1 quadrilateral
(c) 5 points, 10 segments, 10 triangles, 5 quadrilaterals, 1 pentagon **(d)** 21
24 $(1 + x + y)^0 = 0$, $(1 + x + y)^1 = 1 + x + y$, $(1 + x + y)^2 = 1 + 2x + 2y + 2xy + x^2 + y^2$, $(1 + x + y)^3 = 1 + 3x + 3y + 6xy + 3x^2 + 3y^2 + 3x^2y + 3xy^2 + x^3 + y^3$, $(1+x+y)^4 = 1+4x+4y+12xy+6x^2+6y^2+6x^2y^2+12x^2y+12xy^2+4x^3+4y^3+4x^3y+4xy^3+x^4+y^4$
The coefficients form a triangular pyramid, with 1s on the edges, and each face a copy of Pascal's triangle.

Exercise **5B** (Page 183)

1(a) $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$
(b) $x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$
(c) $r^6 - 6r^5s + 15r^4s^2 - 20r^3s^3 + 15r^2s^4 - 6rs^5 + s^6$
(d) $p^{10} + 10p^9q + 45p^8q^2 + 120p^7q^3 + 210p^6q^4 + 252p^5q^5 + 210p^4q^6 + 120p^3q^7 + 45p^2q^8 + 10pq^9 + q^{10}$
(e) $a^9 - 9a^8b + 36a^7b^2 - 84a^6b^3 + 126a^5b^4 - 126a^4b^5 + 84a^3b^6 - 36a^2b^7 + 9ab^8 - b^9$
(f) $32x^5 + 80x^4y + 80x^3y^2 + 40x^2y^3 + 10xy^4 + y^5$
(g) $p^7 - 14p^6q + 84p^5q^2 - 280p^4q^3 + 560p^3q^4 - 672p^2q^5 + 448pq^6 - 128q^7$
(h) $81x^4 + 216x^3y + 216x^2y^2 + 96xy^3 + 16y^4$
(i) $a^3 - \frac{3}{2}a^2b + \frac{3}{4}ab^2 - \frac{1}{8}b^3$
(j) $\frac{1}{32}r^5 + \frac{5}{48}r^4s + \frac{5}{36}r^3s^2 + \frac{5}{54}r^2s^3 + \frac{5}{162}rs^4 + \frac{1}{243}s^5$
(k) $x^6 + 6x^4 + 15x^2 + 20 + \dfrac{15}{x^2} + \dfrac{6}{x^4} + \dfrac{1}{x^6}$
2(a) $1 + 4x^2 + 6x^4 + 4x^6 + x^8$ **(b)** $1 - 9x^2 + 27x^4 - 27x^6$
(c) $x^{12} + 12x^{10}y^3 + 60x^8y^6 + 160x^6y^9 + 240x^4y^{12} + 192x^2y^{15} + 64y^{18}$
(d) $x^9 - 9x^7 + 36x^5 - 84x^3 + 126x - \dfrac{126}{x} + \dfrac{84}{x^3} - \dfrac{36}{x^5} + \dfrac{9}{x^7} - \dfrac{1}{x^9}$
(e) $x^3\sqrt{x} + 7x^3\sqrt{y} + 21x^2y\sqrt{x} + 35x^2y\sqrt{y} + 35xy^2\sqrt{x} + 21xy^2\sqrt{y} + 7y^3\sqrt{x} + y^3\sqrt{y}$
(f) $\dfrac{32}{x^5} + \dfrac{240}{x^2} + 720x + 1080x^4 + 810x^7 + 243x^{10}$
3(a) ${}^4\mathrm{C}_0 = 1,\ {}^4\mathrm{C}_1 = 4,\ {}^4\mathrm{C}_2 = 6,\ {}^4\mathrm{C}_3 = 4,\ {}^4\mathrm{C}_4 = 1$
(b)(i) 16 **(ii)** 0
4(a) 32 **(b)** 32 **(c)** 20 **(d)** 252
5(a) x^5 **(b)** b^4 **(c)** $8y^3$ **(d)** $64y^6$
6(a)(i) $1024 + 1280x + 640x^2 + 160x^3 + \cdots$ **(ii)** -160
(b)(i) $1 - 12x + 60x^2 - 160x^3 + 240x^4 - \cdots$ **(ii)** 720

ISBN: 978-1-107-61604-2 Cambridge University Press

(c)(i) $2187-5103y+5103y^2-2835y^3+945y^4-\cdots$
(ii) $11\,718$
7(a) $2x^6+30x^4y^2+30x^2y^4+2y^6$
8(a) 540 **(b)** 48 **(c)** -960 **(d)** -8
9(a)(i) $x^3+3x^2h+3xh^2+h^3$ **(ii)** $3x^2h+3xh^2+h^3$
(iii) $3x^2$ **(b)** $5x^4$
10(b) 466 **(c)** $7258\sqrt{2}$ **(d)** 42
11(a) $\frac{7}{2}$ **(b)** $\frac{131}{4}\sqrt{7}$
12 $x^3+y^3+z^3+6xyz+3x^2y+3xy^2+3xz^2+3x^2z+3y^2z+3yz^2$
13(a) $1{\cdot}104\,08$ **(b)** $0{\cdot}903\,92$ **(c)** $51{\cdot}54$
14(a)(i) $\left(x^3+\dfrac{1}{x^3}\right)+3\left(x+\dfrac{1}{x}\right)$
(ii) $\left(x^5+\dfrac{1}{x^5}\right)+5\left(x^3+\dfrac{1}{x^3}\right)+10\left(x+\dfrac{1}{x}\right)$
(iii) $\left(x^7+\dfrac{1}{x^7}\right)+7\left(x^5+\dfrac{1}{x^5}\right)+21\left(x^3+\dfrac{1}{x^3}\right)+35\left(x+\dfrac{1}{x}\right)$ **(b)(i)** 2 **(ii)** 2 **(iii)** 2
15 $a=3$ or $a=-3$
17(a) $x^6+6x^4+15x^2+20+\dfrac{15}{x^2}+\dfrac{6}{x^4}+\dfrac{1}{x^6}$
(b) $A=-6$, $B=9$ and $C=-2$.
18 19
19(a) The limiting figure for this process is called the Sierpinski Gasket. It is one of the classic regular fractals.

(b) Sierpinski's triangle is formed.

Exercise **5C** (Page 187)

1(a) 6 **(b)** 5040 **(c)** $3\,628\,800$ **(d)** 1 **(e)** 1 **(f)** $15\,120$
(g) 15 **(h)** 6720 **(i)** 45 **(j)** 220 **(k)** 70 **(l)** 2520
(m) 5005 **(n)** $13\,860$
2(a) $6x^5$ **(b)** $30x^4$ **(c)** $120x^3$ **(d)** $360x^2$ **(e)** $720x$
(f) 720 **(g)** 0
3(a) n **(b)** $n!$ **(c)** 1 **(d)** $n(n+1)$ **(e)** $(n+1)(n+2)$
(f) $\dfrac{1}{n(n-1)}$ **(g)** $\dfrac{n-2}{n}$ **(h)** $\dfrac{(n-1)!}{n+1}$
4(a) $7\times 7!$ **(b)** $n\times n!$ **(c)** $57\times 6!$
(d) $(n^2+n+1)\times(n-1)!$ **(e)** $9^2\times 7!$
(f) $(n+1)^2\times(n-1)!$
5(a) $\dfrac{1+n}{n!}$ **(b)** $\dfrac{n}{(n+1)!}$ **(c)** $\dfrac{1-n-n^2}{(n+1)!}$
6(a)(i) nx^{n-1} **(ii)** $n(n-1)x^{n-2}$ **(iii)** $n!$
(iv) $n(n-1)(n-2)\cdots(n-k+1)x^{n-k}$
$=\dfrac{n!}{(n-k)!}\,x^{n-k}$
(b)(i) $-1!\,x^{-2}$ **(ii)** $2!\,x^{-3}$ **(iii)** $-5!\,x^{-6}$
(iv) $(-1)^n\,n!\,x^{-(n+1)}$
7(b) $(n+1)!-1$
9(a)(i) 2^8 **(ii)** 10^2 **(b)(i)** 2^{97} **(ii)** 5^{24} **(iii)** 7^{16} **(iv)** 13^7
11(a) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{8}$, $\frac{1}{30}$, $\frac{1}{144}$ **(b)** $\frac{1}{2}$, $\frac{5}{6}$, $\frac{23}{24}$, $\frac{119}{120}$, $\frac{719}{720}$
(c) $\displaystyle\sum_{k=1}^{n}\frac{k}{(k+1)!}=1-\frac{1}{(n+1)!}$. The limit is 1.
(d) The sequence can be written as $\left(\dfrac{1}{1!}-\dfrac{1}{2!}\right)+\left(\dfrac{1}{2!}-\dfrac{1}{3!}\right)+\left(\dfrac{1}{3!}-\dfrac{1}{4!}\right)+\cdots+\left(\dfrac{1}{n!}-\dfrac{1}{(n+1)!}\right)$.
12(a) $2^{15}\times 15!$ **(b)** $\dfrac{30!}{2^{15}\times 15!}$ **(c)** $\dfrac{2^{30}\times(15!)^2}{30!}$
13(a)(i) $1+x+x^2+x^3+\cdots$ **(ii)** It is an infinite GP, so for $-1<x<1$ it converges to $\dfrac{1}{1-x}$, as expected.
(b) $-x-\frac{1}{2}x^2-\frac{1}{3}x^3-\cdots$ **(c)(i)** $x-\dfrac{x^3}{3!}+\dfrac{x^5}{5!}-\dfrac{x^7}{7!}+\cdots$
(ii) $1+x+\dfrac{x^2}{2!}+\dfrac{x^3}{3!}+\cdots$
(iii) $x+x^2+\left(\dfrac{1}{2!}-\dfrac{1}{3!}\right)x^3+\left(\dfrac{1}{4!}-\dfrac{1}{2!\,3!}+\dfrac{1}{5!}\right)x^5$
14 $0{\cdot}14\%$

Exercise **5D** (Page 193)

1(a) 10 **(b)** 210 **(c)** 20 **(d)** 1287 **(e)** 792 **(f)** 1
(g) 9 **(h)** 11 **(i)** 35 **(j)** 10 **(k)** 14 **(l)** 132
2(a)(i) 56 **(ii)** 35 **(b)** 5
3(b)(i) 6 **(ii)** 10 **(iii)** 30 **(iv)** $n=4$ or $n=8$
4(a)(i) $672x^2$ **(ii)** $280x^4$ **(b)(i)** $\frac{1001}{16}x^9y^5$
(ii) $\frac{1001}{256}x^5y^9$ **(c)(i)** $-\frac{33}{1024}x^{10}y^2$ **(ii)** $\frac{168\,399}{16}x^5y^{12}$
(d)(i) $-1140a^3b^{\frac{17}{2}}$ **(ii)** $190a^2b^9$
5(a)(i) 1 **(ii)** n **(iii)** $\frac{1}{2}n(n-1)$ **(iv)** $\frac{1}{6}n(n-1)(n-2)$
(b)(i) 16 **(ii)** 9 **(iii)** 4 **(iv)** 6 **(v)** 4 **(vi)** 7
6(a) $5x^2:39$ **(b)** $5:2$ **(c)** $18\,304:1$
7(b)(i) 126 **(ii)** 36 **(iii)** 84
8(b)(i) ${}^{10}\mathrm{C}_4\,2^6\,3^4=2^7\times 3^5\times 5\times 7$
(ii) ${}^{10}\mathrm{C}_7\,2^3\,3^7=2^6\times 3^8\times 5$
(iii) ${}^{10}\mathrm{C}_6\,2^4\,3^6=2^5\times 3^7\times 5\times 7$
9(b)(i) ${}^{15}\mathrm{C}_2\,5^2\,2^{-13}$ **(ii)** $-{}^{15}\mathrm{C}_7\,5^7\,2^{-8}$
(iii) ${}^{15}\mathrm{C}_{10}\,5^{10}\,2^{-5}$

ISBN: 978-1-107-61604-2 Cambridge University Press

10(a) ${}^{8}\mathrm{C}_4 \times 3^4 = 5670$ **(b)** $-{}^{12}\mathrm{C}_9 \times 2^3 = -1760$
(c) ${}^{10}\mathrm{C}_2 \times 5^2 \times 2^8 = 288\,000$
(d) ${}^{6}\mathrm{C}_4 \times a^2 \times (\frac{1}{2})^4 = \frac{15}{16}a^2$
11(a) -672 **(b)** $\frac{969}{2}$ **(c)** $-112\,266$ **(d)** $21\,875$ **(e)** $\frac{40}{49}$
(f) $-{}^{19}\mathrm{C}_9\,(\frac{3}{5})^{10}$
12(a)(i) 3640 **(ii)** 140 **(b)(i)** -385 **(ii)** 66
(c)(i) $-2\,379\,520$ **(ii)** $10\,920$ **(d)(i)** -1241
(ii) $161\,838$
13(a)(i) $-1\,959\,552x^5y^5$ **(ii)** $924x^3y^2$
(iii) $-\frac{9724}{390\,625}x^9y^{18}$ **(iv)** $\dfrac{160x^3}{27y^3}$
(b)(i) $90a^3b^2,\ 270a^2b^3$ **(ii)** $-\frac{77}{2592}a^6b^5,\ \frac{77}{3888}a^5b^6$
(iii) $6435a^{\frac{8}{3}}b^{\frac{7}{2}},\ 6435a^{\frac{7}{3}}b^4$ **(iv)** $\dfrac{63b^4}{8a^5},\ -\dfrac{63b^5}{16a^4}$
14(a) $x = \frac{11}{2}$ **(b)** $x = -\frac{7}{3}$
15(a) 6 **(b)** 45 **(c)** 84
16(a) $a = -24,\ b = 158$ **(b)** $n = 13,\ 286$
17(a) $a = 2$ and $n = 14$ **(b)** $a = -\frac{1}{3}$ and $n = 10$
18(a) $n = 14$ **(b)** $n = 13$ **(c)** $n = 9$
20(a) $0{\cdot}877\,52$ **(b)** $1{\cdot}1157$ **(c)** $0{\cdot}985\,10$
21(a) $n = 10$ **(b)** $n = 7$ **(c)(ii)** $n = 14$ or $n = 7$
22 $1 - 4x + 10x^2 - 16x^3 + 19x^4 - \cdots$
23 ${}^{3n}\mathrm{C}_n\ (= {}^{3n}\mathrm{C}_{2n})$
24(a) ${}^{12}\mathrm{C}_r\,(-1)^r\,a^{12-r}\,b^r\,x^r$ **(b)** $\frac{5}{8}$
26(a) ${}^{n}\mathrm{C}_0\,x^n + {}^{n}\mathrm{C}_1\,x^{n-1}h + {}^{n}\mathrm{C}_2\,x^{n-2}h^2 + \cdots + {}^{n}\mathrm{C}_n\,h^n$ **(b)** nx^{n-1}
28(a) $\dfrac{9-9n}{2}$ **(b)** $-\frac{9}{2}n(n-1)(2n-1)$
29(b) $1001,\ 2002,\ 3003$
30(b) 12
31(c) If you add any column downwards from the top to any point, then the sum is diagonally below and to the right.
32(a) $\dfrac{n-r+1}{r}$ **(b)** $\dfrac{n(n+1)}{2}$
33(b)(i) $1-x+x^2-x^3+\cdots$ **(ii)** $1+2x+3x^2+4x^3+\cdots$
(iii) $1 - 2x + 3x^2 - 4x^3 + \cdots$
(iv) $1 + \frac{1}{2}x - \frac{1}{8}x^2 + \frac{3}{48}x^3 - \cdots$

Exercise **5E** (Page 199)

1(a) $t_{k+1} = {}^{12}\mathrm{C}_{k+1}\,2^{11-k}\,3^{k+1},\ t_k = {}^{12}\mathrm{C}_k\,2^{12-k}\,3^k$
(c) ${}^{12}\mathrm{C}_7\,2^5\,3^7$
2(a) $c_{k+1} = {}^{25}\mathrm{C}_{k+1}\,7^{24-k}\,3^{k+1},\ c_k = {}^{25}\mathrm{C}_k\,7^{25-k}\,3^k$
(c) ${}^{25}\mathrm{C}_7\,7^{18}\,3^7$
3(a) $T_{k+1} = {}^{13}\mathrm{C}_{k+1}\,3^{12-k}\,4^{k+1}x^{k+1}$,
$T_k = {}^{13}\mathrm{C}_k\,3^{13-k}\,4^kx^k$ **(c)** ${}^{13}\mathrm{C}_5\,3^8\,2^5$
4(a) $T_{k+1} = {}^{21}\mathrm{C}_{k+1}\,5^{k+1}\,x^{k+1},\ T_k = {}^{21}\mathrm{C}_k\,5^k\,x^k$
(c) ${}^{21}\mathrm{C}_{16}\,3^{16}$
5(a)(i) $t_{k+1} = {}^{15}\mathrm{C}_{k+1}\,5^{14-k}\,2^{k+1}$,
$t_k = {}^{15}\mathrm{C}_k\,5^{15-k}\,2^k$ **(iii)** ${}^{15}\mathrm{C}_4\,5^{11}\,2^4$
(b)(i) $T_{k+1} = {}^{15}\mathrm{C}_{k+1}\,5^{14-k}\,2^{k+1}x^{k+1}$,
$T_k = {}^{15}\mathrm{C}_k\,5^{15-k}\,2^kx^k$ **(iii)** ${}^{15}\mathrm{C}_6\,5^{15}\,(\frac{2}{3})^6$
6(a)(i) ${}^{11}\mathrm{C}_9\,4^9$ **(ii)** $T_8 = {}^{11}\mathrm{C}_8(\frac{8}{3})^8$
(b)(i) ${}^{9}\mathrm{C}_2\,2^3\,3^2$ **(ii)** $T_1 = T_2 = 1152$
(c)(i) ${}^{12}\mathrm{C}_8\,3^4\,5^8$ **(ii)** $T_6 = {}^{12}\mathrm{C}_6\,10^6$
(d)(i) ${}^{11}\mathrm{C}_6\,5^5\,6^6$, **(ii)** $T_5 = {}^{11}\mathrm{C}_5\,5^6\,4^5$
7(a)(i) ${}^{9}\mathrm{C}_8\,7^8$ **(ii)** $T_8 = {}^{9}\mathrm{C}_8\,(\frac{14}{3})^8$
(b)(i) $-{}^{14}\mathrm{C}_3\,7^{11}\,2^3$ **(ii)** $T_2 = {}^{14}\mathrm{C}_2\,7^{12}\,(\frac{4}{3})^2$
(c)(i) ${}^{12}\mathrm{C}_8\,2^8$ **(ii)** $T_{11} = -{}^{12}\mathrm{C}_{11} \times 6^{11}$
(d)(i) $-{}^{15}\mathrm{C}_5\,2^{10}$ **(ii)** $T_{11} = -{}^{15}\mathrm{C}_{11}\,4^4\,3^7$
8(a)(i) ${}^{10}\mathrm{C}_6\,2^4\,3^6$ **(ii)** ${}^{10}\mathrm{C}_3\,9^7\,2^{-4}$ **(b)(i)** ${}^{12}\mathrm{C}_7\,2^5\,3^7$
(ii) Two terms have the greatest value, which is $10\,264\,320$.
9 The equal terms are ${}^{14}\mathrm{C}_6\,(\frac{2}{3})^6$ and ${}^{14}\mathrm{C}_5\,(\frac{2}{3})^5$.
10(a) $T_{r+1} = {}^{n}\mathrm{C}_{r+1}\,x^{n-r-1}\,y^{r+1}$,
$T_r = {}^{n}\mathrm{C}_r\,x^{n-r}\,y^r$ **(c)** $T_5 = \frac{63}{256}$
11(a) $n = 17,\ r = 2$ **(b)** ${}^{17}\mathrm{C}_8 = {}^{17}\mathrm{C}_9$
12 $r = 8$
13 $\theta = 22^\circ$
16 2

Exercise **5F** (Page 204)

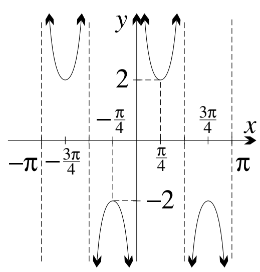
1(a) The five numbers on the row indexed by $n = 4$ have a sum of $2^4 = 16$.
(b)(i) The sum of the first, third and fifth terms on the row equals the sum of the second and fourth terms.
(ii) The sum of the first, third and fifth terms on the row is half the sum of the whole row.
(c)(i) $4(1+x)^3 = {}^{4}\mathrm{C}_1 + 2\,{}^{4}\mathrm{C}_2\,x + 3\,{}^{4}\mathrm{C}_3\,x^2 + 4\,{}^{4}\mathrm{C}_4\,x^3$
6(b) $\left({}^{2n}\mathrm{C}_0\right)^2 - \left({}^{2n}\mathrm{C}_1\right)^2 + \left({}^{2n}\mathrm{C}_2\right)^2 - \cdots + \left({}^{2n}\mathrm{C}_{2n}\right)^2 = (-1)^n\,{}^{2n}\mathrm{C}_n$
7(a) $4 \le r \le n$ **(b)** $r \le p$ and $r \le q$
9(a) ${}^{3n}\mathrm{C}_{n+r}$
10(a) $\dfrac{5^{n+1}-1}{n+1}$
12(a) $k \times {}^{5}\mathrm{C}_k = 5 \times {}^{4}\mathrm{C}_{k-1}$, and more generally, $k \times {}^{n}\mathrm{C}_k = n \times {}^{n-1}\mathrm{C}_{k-1}$. **(b)** $\dfrac{{}^{5}\mathrm{C}_k}{k+1} = \dfrac{{}^{6}\mathrm{C}_{k+1}}{6}$, and more generally, $\dfrac{{}^{n}\mathrm{C}_k}{k+1} = \dfrac{{}^{n+1}\mathrm{C}_{k+1}}{n+1}$.
15(d) $\frac{8}{15}$
20(c) ${}^{7}\mathrm{C}_1,\ {}^{7}\mathrm{C}_2,\ {}^{7}\mathrm{C}_3$, and ${}^{7}\mathrm{C}_4,\ {}^{7}\mathrm{C}_5,\ {}^{7}\mathrm{C}_6$; ${}^{14}\mathrm{C}_4,\ {}^{14}\mathrm{C}_5,\ {}^{14}\mathrm{C}_6$, and ${}^{14}\mathrm{C}_8,\ {}^{14}\mathrm{C}_9,\ {}^{14}\mathrm{C}_{10}$; ${}^{23}\mathrm{C}_8,\ {}^{23}\mathrm{C}_9,\ {}^{23}\mathrm{C}_{10}$, and ${}^{23}\mathrm{C}_{13},\ {}^{23}\mathrm{C}_{14},\ {}^{23}\mathrm{C}_{15}$

ISBN: 978-1-107-61604-2 Cambridge University Press

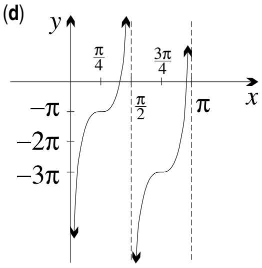
Chapter Six

Exercise **6A** (Page 211)

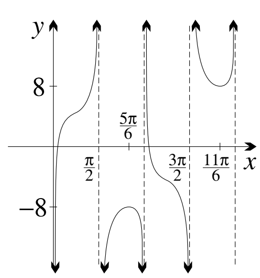
1(a) $\sec x \tan x$ **(b)** $-\operatorname{cosec} x \cot x$ **(c)** $-\operatorname{cosec}^2 x$
(d) $-3 \operatorname{cosec} 3x \cot 3x$ **(e)** $\operatorname{cosec}^2(1-x)$
(f) $5 \sec(5x-2) \tan(5x-2)$
2(a) $4\sqrt{3}$ **(b)** $-\frac{8}{3}$
3(a) $12x + 2y = \pi + 2$ **(b)** $\sqrt{2}\,x + y = \sqrt{2}(1 + \frac{\pi}{4})$
(c) $3\sqrt{3}\,x - 2y = \pi\sqrt{3} - 5$ **(d)** $y = -1$
4(a) $-\operatorname{cosec}^2 x e^{\cot x}$ **(b)** $\tan x$
(c) $\operatorname{cosec} x(1 - x \cot x)$ **(d)** $-2 \cot x \operatorname{cosec}^2 x$
(e) $4 \sec^4 x \tan x$ **(f)** $-\operatorname{cosec} x \sec x$
(g) $2e^{2x} \sec 2x(1 + \tan 2x)$ **(h)** $\frac{-2\operatorname{cosec}^2 x(x \cot x + 1)}{x^3}$
5(a) $-\frac{\pi}{2}$, 0 and $\frac{\pi}{2}$ **(b)** odd **(c)** $y > 0$ in quadrants 1 and 3, $y < 0$ in quadrants 2 and 4. **(e)** minimum turning points at $(-\frac{3\pi}{4}, 2)$ and $(\frac{\pi}{4}, 2)$, maximum turning points at $(-\frac{\pi}{4}, -2)$ and $(\frac{3\pi}{4}, -2)$
(f)



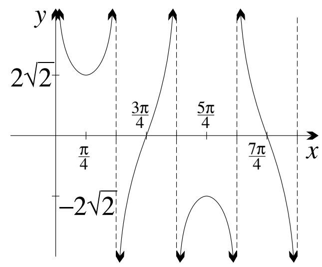
8(b) $\frac{\pi}{2} < x < \frac{3\pi}{2}$
9(b) $x = \frac{\pi}{2}$ **(c)** $(\frac{\pi}{4}, -\pi)$ and $(\frac{3\pi}{4}, -3\pi)$ are horizontal points of inflexion. **(d)**



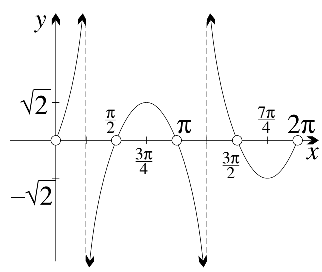
10(a) $x = \frac{\pi}{2}$, π, $\frac{3\pi}{2}$
(c) $(\frac{5\pi}{6}, -8)$ is a maximum turning point, $(\frac{11\pi}{6}, 8)$ is a minimum turning point. **(d)** $y \to \infty$ as $x \to \frac{\pi}{2}^-$, $x \to \pi^+$, $x \to \frac{3\pi}{2}^+$, $x \to 2\pi^-$ and $y \to -\infty$ as $x \to 0^+$, $x \to \frac{\pi}{2}^+$, $x \to \pi^-$, $x \to \frac{3\pi}{2}^-$.
(e)



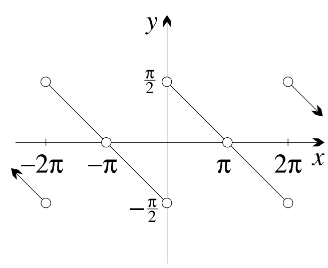
11



12(c)(i) 0 **(ii)** π
13(a) $\frac{1}{x^2} \operatorname{cosec}^2 \frac{1}{x}$ **(b)** $\frac{\tan x}{\log(\sec x)}$
(c) $\frac{3 \sec 3x}{\tan 3x - \sec 3x}$
14(b) $3\sqrt{2}\,x - 2y = 6$
16(b) $\theta = \frac{\pi}{6}$
18(a) $\frac{\operatorname{cosec} x \cot x}{\operatorname{cosec}^2 y}$ **(b)** $\frac{y - \sec(x+y)\tan(x+y)}{\sec(x+y)\tan(x+y) - x}$
19



20



Exercise **6B** (Page 216)

1(a) $\frac{1}{2}\sin 2x + C$ **(b)** $-\frac{1}{2}\cos 2x + C$ **(c)** $\frac{1}{2}\tan 2x + C$
(d) $-\frac{1}{2}\cot 2x + C$ **(e)** $\frac{1}{2}\sec 2x + C$
(f) $-\frac{1}{2}\operatorname{cosec} 2x + C$
2(a) $3\sin\frac{1}{3}x + C$ **(b)** $2\cos\frac{1}{2}(1-x) + C$
(c) $-\frac{1}{3}\tan(4-3x) + C$ **(d)** $-\frac{5}{2}\cot\frac{1}{5}(2x+3) + C$
(e) $\frac{1}{a}\sec(ax+b) + C$ **(f)** $\frac{1}{b}\operatorname{cosec}(a-bx) + C$
3(a) $(2-\sqrt{2})$ units^2 **(b)** $\frac{1}{6}\sqrt{3}$ units^2
(c) $3(2-\sqrt{2})$ units^2 **(d)** $\frac{1}{2}\log 2$ units^2
4(a) $\ln 2$ **(b)** $\frac{1}{6}\ln 2$ **(c)** $\ln(\sqrt{2}+1)$ **(d)** $\frac{1}{2}\ln 3$
5 $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ **(a)** $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$
(b) $\frac{1}{2}x - \frac{1}{8}\sin 4x + C$ **(c)** $\frac{1}{2}x - \sin\frac{1}{2}x + C$ **(d)** $\frac{\pi}{6}$
6 $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ **(a)** $\frac{1}{2}x + \frac{1}{4}\sin 2x + C$
(b) $\frac{1}{2}x + \frac{1}{24}\sin 12x + C$ **(c)** $\frac{1}{2}x + \frac{1}{2}\sin x + C$ **(d)** $\frac{\pi}{8}$
7(a)(i) $\frac{1}{2}\tan 2x - x + C$ **(ii)** $-2\cot\frac{1}{2}x - x + C$
(b)(i) $\sqrt{3} - 1 - \frac{\pi}{12}$ **(ii)** $\frac{1}{4}\sqrt{3} - \frac{\pi}{12}$
8(a) $\frac{1}{8}(14 - \pi)$ **(b)** $\frac{1}{2}(3 + \sqrt{3})$
9(a) $\frac{\pi}{2}(\sqrt{3} - 1)$ units^3 **(b)** $\frac{\pi}{2}(4 - \pi)$ units^3

ISBN: 978-1-107-61604-2 Cambridge University Press

(c) $\frac{\pi}{4}$ units^3 **(d)** $\pi \ln 2$ units^3
(e) $\frac{1}{2}(4-\pi)$ units^3 **(f)** $\pi\sqrt{3}$ units^3
10(a) $\frac{1}{4}\sin^4 x + C$ **(b)** $-\frac{1}{5}\cot^5 x + C$
(c) $\frac{1}{7}\sec^7 x + C$ **(d)** $\frac{2}{7}$ **(e)** $\frac{1}{3}$ **(f)** $\frac{2}{3}(4-\sqrt{2})$
11(a) $\sec x^2 + C$ **(b)** $-\ln(1+\cot x) + C$
(c) $\ln\sin e^x + C$ **(d)** $\frac{1}{2}e^{\sec 2x} + C$
12(a) $\frac{1}{4}\sqrt{3}$ **(b)** $1+\sqrt{2}-\sqrt{3}$ **(c)** $\frac{1}{2}+\frac{1}{3}\sqrt{3}$ **(d)** $\ln\frac{3}{2}$
13(a) $(\pi+2)$ units^2, $\frac{\pi}{2}(3\pi+8)$ units^3
(b) $\frac{1}{2}(\sqrt{3}+1)$ units^2, $\frac{\pi}{12}(4\pi+9)$ units^3
(c) $\frac{1}{4}\sqrt{2}$ units^2, $\frac{\pi}{32}(\pi+2)$ units^3
(d) $\frac{1}{2}\ln 3$ units^2, $\frac{2\pi}{\sqrt{3}}$ units^3
(e) $\frac{2\pi}{3}+\ln(7+4\sqrt{3})$ units^2,
$\frac{\pi}{3}\left(2\pi+6\sqrt{3}+6\ln(7+4\sqrt{3})\right)$ units^3
14(a) $\frac{1}{6}(\pi\sqrt{3}+6\ln 2)$ **(b)** $\sqrt{3}-1$ **(c)** $\frac{58}{15}$
(d) $\frac{1}{2}\sqrt{2}+\frac{1}{2}\ln(\sqrt{2}+1)$ **(e)** $\frac{2}{3}(3\sqrt{3}-2)$ **(f)** $\frac{3\pi}{16}$
15 $\frac{1}{2}$
17(b) $\frac{32}{3}\sqrt{2}+\frac{15}{2}\ln(\sqrt{2}+1)$

Exercise 6C (Page 220)

1(c) $\frac{1}{4}(1+x^2)^4 + C$
2(a) $\frac{1}{4}(2x+3)^4 + C$ **(b)** $\frac{1}{5}(1+x^3)^5 + C$
(c) $\frac{-1}{1+x^2} + C$ **(d)** $2\sqrt{3x-5} + C$
(e) $\frac{1}{4}\sin^4 x + C$ **(f)** $\log_e(1+x^4) + C$
3(c) $-\sqrt{1-x^2} + C$
4(a) $\frac{1}{24}(x^4+1)^6 + C$ **(b)** $\frac{2}{9}(x^3-1)^{\frac{3}{2}} + C$ **(c)** $\frac{1}{3}e^{x^3} + C$
(d) $\frac{-1}{(1+\sqrt{x})^2} + C$ **(e)** $\frac{1}{6}\tan^3 2x + C$ **(f)** $-e^{\frac{1}{x}} + C$
5(a) $\frac{65}{12}$ **(b)** $\sqrt{2}-1$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{24}$ **(e)** 2 **(f)** $\frac{1}{2}(e^2-1)$
(g) $\frac{1}{10}$ **(h)** $\frac{\pi^4}{64}$ **(i)** 3 **(j)** $\frac{1}{2}\ln 3$
6(a) $\frac{\pi}{12}$ units^2 **(b)** $\frac{\pi^2}{6}$ units^3
7(a) $\log_e\frac{3}{2}$ **(b)** $\frac{\pi}{4}$ **(c)** $\frac{2}{3}$ **(d)** $\frac{4}{3}$
8(a) $\sqrt{1+e^{2x}} + C$ **(b)** $\ln(\ln x) + C$
(c) $-\ln(\ln\cos x) + C$ **(d)** $\frac{1}{4}\tan^4 x + \frac{1}{6}\tan^6 x + C$
9(a) $y = \frac{1}{2}\tan^{-1} e^{2x}$ **(b)** $y = \sin^{-1}\frac{x}{2} + \frac{x}{2} + \frac{1}{2}$
10(b)(i) $\frac{2}{\ln 2}$ **(ii)** $\frac{1}{5}(4\sqrt{2}-1)$
11(a) $\ln 2$ **(b)** $\frac{e}{e+1}$
12 $\tan^{-1}\sqrt{x-1} + C$
13(a) $(3-2\ln 2)$ units^2 **(b)** $\frac{\pi}{4}(15-16\ln 2)$ units^3
14(a) $2\sin^{-1}\sqrt{x} + C_1$ **(b)** $\sin^{-1}(2x-1) + C_2$

Exercise 6D (Page 223)

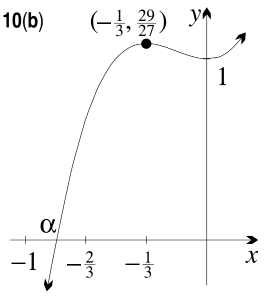
1(c) $\frac{1}{7}(x-1)^7 + \frac{1}{6}(x-1)^6 + C$
2(a) $\frac{2}{3}(x-1)^{\frac{3}{2}} + 2(x-1)^{\frac{1}{2}} + C$ **(b)** $\ln(x-1) - \frac{1}{x-1} + C$
3(c) $\frac{2}{5}(x+1)^{\frac{5}{2}} - \frac{2}{3}(x+1)^{\frac{3}{2}} + C$
4(a) $\frac{2}{7}(x+1)^{\frac{7}{2}} - \frac{4}{5}(x+1)^{\frac{5}{2}} + \frac{2}{3}(x+1)^{\frac{3}{2}} + C$
(b) $\frac{4}{3}(x+1)^{\frac{3}{2}} + 2(x+1)^{\frac{1}{2}} + C$
5(a) $(x+2) - 4\ln(x+2) + C$
(b) $\frac{1}{3}(2x-1)^{\frac{3}{2}} + 2(2x-1)^{\frac{1}{2}} + C$
(c) $\frac{3}{40}(4x-5)^{\frac{5}{2}} + \frac{5}{8}(4x-5)^{\frac{3}{2}} + C$
(d) $2(1+\sqrt{x}) - 2\ln(1+\sqrt{x}) + C$
6(a) $\frac{49}{20}$ **(b)** $2\ln 2 - \frac{1}{2}$ **(c)** $\frac{8}{9}$ **(d)** $\frac{1}{9}$ **(e)** $\frac{128}{15}$ **(f)** $\frac{4}{3}$
(g) $4 - 6\ln\frac{5}{3}$ **(h)** $\frac{2517}{40}$
7(a) $\sin^{-1}\frac{x+2}{3} + C$ **(b)(i)** $\frac{1}{\sqrt{3}}\tan^{-1}\frac{x+1}{\sqrt{3}} + C$
(ii) $\sin^{-1}\frac{x+1}{\sqrt{5}} + C$ **(iii)** $\frac{\pi}{6}$ **(iv)** $\frac{\pi}{16}$
8(b)(i) $\frac{1}{3}\tan^{-1}\frac{x}{3} + C$ **(ii)** $\cos^{-1}\frac{x}{\sqrt{3}} + C$
(iii) $\frac{1}{2}\sin^{-1} 2x + C$ **(iv)** $\frac{1}{4}\tan^{-1} 4x + C$ **(v)** $\frac{\pi}{6}$ **(vi)** $\frac{\pi}{24}$
9(b)(i) $\frac{x}{4\sqrt{4+x^2}} + C$ **(ii)** $\frac{\pi}{12} - \frac{\sqrt{3}}{8}$ **(iii)** π
(iv) $-\frac{\sqrt{25-x^2}}{25x} + C$ **(v)** $-\frac{\sqrt{9+x^2}}{9x} + C$ **(vi)** $\frac{\sqrt{3}}{8}$
10(b) $\frac{\pi}{8}(\pi+2)$ units^3
11 $y = \sqrt{x^2-9} - 3\tan^{-1}\frac{\sqrt{x^2-9}}{3}$
12 $\frac{1}{3}(6\sqrt{3}-7\sqrt{2})$ units^2
14(a) $\ln(\sec\theta + \tan\theta) + C$
15(b) $\frac{8}{3}$

Exercise 6E (Page 230)

1(b) $P(\frac{5}{2}) = \frac{1}{4}$ **(c)** The root is closer to $2\frac{1}{2}$.
2(a)(ii) The root is between $\frac{3}{4}$ and 1. **(b)(ii)** $1{\cdot}2$
3(a)(ii) $0{\cdot}9$ **(b)(ii)** $2{\cdot}2$ **(c)(ii)** $1{\cdot}1$
4(a) $2{\cdot}3$ **(c)** $2{\cdot}236\,067\,98$
5(a) $x_1 = 3{\cdot}1$, $x_5 = 3{\cdot}105\,482\,62$
(b) $x_1 = 1{\cdot}9$, $x_5 = 1{\cdot}903\,813\,69$
(c) $x_1 = 1{\cdot}9$, $x_5 = 1{\cdot}895\,494\,27$
6(a) $2{\cdot}42$, $2{\cdot}414\,213\,56$ **(b)** $0{\cdot}84$, $0{\cdot}843\,734\,28$
(c) $1{\cdot}21$, $1{\cdot}203\,947\,57$ **(d)** $0{\cdot}85$, $0{\cdot}850\,651\,21$
(e) $2{\cdot}22$, $2{\cdot}219\,107\,15$ **(f)** $1{\cdot}14$, $1{\cdot}141\,890\,55$
7(b) The root is between $2{\cdot}5$ and $2{\cdot}625$. **(c)** no
8(a) $3{\cdot}61$, $3{\cdot}605\,551\,28$
(b) $3{\cdot}27$, $3{\cdot}271\,066\,31$
(c) $2{\cdot}75$, $2{\cdot}752\,525\,92$
9 $3{\cdot}162\,277\,66$

ISBN: 978-1-107-61604-2 Cambridge University Press

10(b)

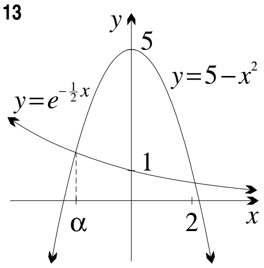


(c) 4 **(d)** The curve has negative gradient at $x = -\frac{1}{4}$, so the tangent at $x = -\frac{1}{4}$ will cut the x-axis further away from α.

11 No. The tangent at $x = a$ has positive gradient, and so will cut the x-axis further away from r.

12(a) A **(b)** A **(c)** A **(d)** D **(e)** C **(f)** B **(g)** B **(h)** A **(i)** D **(j)** C **(k)** C **(l)** C

13



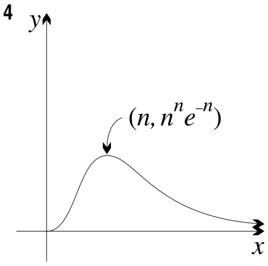
14(a)(ii) It is an AP, with $a = d = \frac{1}{k}$.
(b) $x_{n+1} = x_n(1 + \frac{1}{k})$, and so the sequence is a GP with $a = r = 1 + \frac{1}{k}$. **(c)** e^{-kx} is steeper, and so approaches zero more quickly as $x \to \infty$.

16(e) $x_1 = \cot\theta_1 = \cot\left(\dfrac{m\pi}{2^n - 1}\right)$, where m and n are integers with $n \ge 1$ and $0 \le m \le n$.

Exercise **6F** (Page 235)

1(b) $y'' = -\frac{1}{4}x^{-\frac{3}{2}}$, which is negative for $x > 0$.
2(a)(i) $\frac{\sqrt{3}}{4}$ square units **(ii)** $\frac{3\sqrt{3}}{2}$ square units
(b)(i) $\frac{1}{\sqrt{3}}$ square units **(ii)** $2\sqrt{3}$ square units
3(b)(i) e^{-1} **(ii)** $\frac{1}{2}(1 + e^{-1})$
4(a) $y = x$ **(b)** $y = \frac{2x}{\pi}$ **(c)** $y = \frac{3x}{\pi}$
5(a)(i) $\frac{1}{2}r^2 \sin x$ **(ii)** $\frac{1}{2}r^2 x$ **(iii)** $\frac{1}{2}r^2 \tan x$
6(b) $\frac{1}{3}$
7(a) $-1 < x < 1$ **(b)** $f'(x) = \frac{2}{1-x^2}$
8(b) $M(1, \frac{8}{9})$, $N(2, \frac{4}{9})$
(c) The area of $ABDC$ is $\frac{3}{4}$ square units and the area of $MNDC$ is $\frac{2}{3}$ square units.
9(e) zero, one, two, three, four, five
10(a) $y'' = -\frac{1}{x^2}$, which is negative for all $x > 0$.
(c) $\left(\frac{a+2b}{3}, \frac{\ln a + 2\ln b}{3}\right)$
11(a) $x = \frac{\pi}{4}$
12 $\frac{1}{2}$
13(a) $\frac{1}{6}$

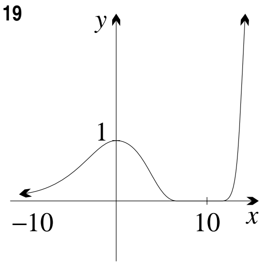
14



(c) For $x > n$, $f(x) < f(n)$, that is, $x^n e^{-x} < n^n e^{-n}$.

15(b) $\frac{4900}{3321}$ **(c)** $\frac{1}{2}(\frac{3}{2} - \frac{1}{n} - \frac{1}{n+1})$, limiting sum is $\frac{3}{4}$.
16(b) 3
17(b) $f(x)$ is stationary at $x = 1$ and increasing for all other positive values of x. Also, $f(0) = 0$. Hence the graph of $f(x)$ lies completely above the x-axis for $x > 0$.
18(c) From part (b), $\sqrt[n]{n}$ is not an integer. Therefore $\sqrt[n]{n}$ is not rational.

19



(b) $(0, 1)$ is a maximum turning point, $(10, 0)$ is a minimum turning point.
(c) As $x \to \infty$, $y \to \infty$, and as $x \to -\infty$, $y \to 0$.

20(b)(i) $x > 1 + \sqrt{2}$ or $x < 1 - \sqrt{2}$ **(c)(i)** $a^n - b^n$
26(c) when $x_1 = x_2 = x_3 = \cdots = x_n$
27(a) $y''''' = u'''''v + 5u''''v' + 10u'''v'' + 10u''v''' + 5u'v'''' + v'''''$ **(b)** $(x^2 - 9x + 16)e^{-x}$
(c) $y^{(n)} = \sum_{k=0}^{n} {}^n\mathrm{C}_k \, u^{(n-k)} v^{(k)}$

ISBN: 978-1-107-61604-2 Cambridge University Press

Chapter Seven

Exercise 7A (Page 245)

1(a) $300\,500$ **(b)** 125 **(c)(i)** $d = -3$ **(ii)** $T_{35} = -2$
(iii) $S_n = \frac{1}{2}n(203 - 3n)$
2(a)(i) $\frac{3}{2}$ **(ii)** $26\,375$ **(iii)** $|r| = \frac{3}{2} > 1$
(b)(i) $\frac{1}{3}$ **(ii)** $|r| = \frac{1}{3} < 1$, $S_\infty = 27$
3(a) $48 000, $390 000 **(b)** the 7th year
4(a) $62 053, $503 116 **(b)** the 13th year
5(a) $25 000, $27 500, $30 000, $d = \$2500$
(b) $20 000, $23 000, $26 450, $r = 1{\cdot}15$ **(c)** $2727
6(a) the 18th year **(b)** the 19th year
7(a) SC50: 50%, SC75: 25%, SC90: 10% **(c)** 4
(d) at least 7 **8(a)** 2000 **(b)** 900 **(c)** 10 years
9(a) 18 times **(b)** 1089 **(c)** Monday
10(a) 6 metres, 36 metres, 66 metres **(b)** $30n - 24$
(c) 6 **(d)** 486 metres
11(a) $T_n = 3 \times (\frac{2}{3})^{n-1}$ **(b)** 4·5 metres **(c)(ii)** 16
12 4 units
13(a) $D = 3200$ **(b)** $D = 3800$ **(c)** the 15th year
(d) $S_{13} = \$546\,000$, $S_{14} = 602\,000$
14(a) the 10th year **(b)** the 7th year
15(a) $(\frac{1}{2})^{\frac{1}{4}}$ **(b)** $S_\infty = \dfrac{F}{1 - (\frac{1}{2})^{\frac{1}{4}}} \doteqdot 6{\cdot}29F$
16(a)(i) $x \neq n\pi$, where $n \in \mathbf{Z}$, $S_\infty = \operatorname{cosec}^2 x$
(ii) $x \neq \frac{\pi}{2} + n\pi$, where $n \in \mathbf{Z}$, $S_\infty = \sec^2 x$
(b)(i) $\dfrac{1+t^2}{(1+t)^2}$ **(ii)** $\dfrac{1+t^2}{(1-t)^2}$
17(b) at $x = 16$ **(c)(i)** at $x = 18$, halfway between the original positions **(ii)** 36 metres, the original distance between the bulldozers
18(a) 125 metres **(b)** 118·75 metres
(d) $a = 118{\cdot}75$, $d = -6{\cdot}25$, $\ell = 6{\cdot}25$ and $n = 19$
(e) $2 \times S_{19} + 125 = 20 \times 125$, which is $2\frac{1}{2}$ km.
19(a) $\frac{1}{2}\cos\theta\sin\theta$ **(b)** $\sin^2\theta$ **20(a)** $\frac{1}{\sqrt{n}}$
(d) No — the spiral keeps turning without bound.

Exercise 7B (Page 251)

1(a) $6050 **(b)** $25 600 **(c)** 11 **(d)** 5·5%
2(a) $59 750 **(b)** $13 250
3(a) Howard — his is $21 350, and hers is $21 320.
(b) Juno — hers is now $21 360.67.
4(a) $16 830.62 **(b)** $8000 **(c)(ii)** 3 years **(d)** 7·0%
5(a) $1120 **(b)** $1123.60 **(c)** $1125.51 **(d)** $1126.83
(e) $1127.34 **(f)** $1127.47 The values in the previous parts are converging towards $1000 \times e^{0.12}$
6 $101 608.52 **7** $A_n = P(1 + 0{\cdot}12\,n)$ **(a)** 9 years
(b) 17 years **(c)** 25 years **(d)** 75 years
8 $A_n = P \times 1{\cdot}12^n$ **(a)** 7 years **(b)** 10 years
(c) 13 years **(d)** 21 years **9(a)** $40 988 **(b)** $42 000
10(a) $12 209.97 **(b)** 4·4% per annum
11 8 years and 9 months **12** $1 110 000 **13** Sid
14(a)(i) $A_n = P(1+r)^n$ **(ii)** $n = \frac{\log 2}{\log(1+r)}$
(b)(i) $B_n = P(1 + Rn)$ **(ii)** $R = \frac{1}{n}((1+r)^n - 1)$
15 If an amount P is invested for one year at an interest rate r per annum, compounded n times per year, then the final amount at the end of one year is $A = P\left(1 + \frac{r}{n}\right)^n$. Hence as the number of compoundings increases, the final amount A converges to the limit Pe^r, and does not increase without bound, as one might possibly expect.
16(a) $A_n = P + PRn$ **(c)** P is the principal, PRn is the simple interest and $\displaystyle\sum_{k=2}^{n} {}^n\mathrm{C}_k\, R^k$ is the result of compound interest over and above simple interest.
17(a) $\displaystyle\sum_{k=0}^{n} {}^n\mathrm{C}_k\, PR^k$ **(b)** ${}^n\mathrm{C}_k$ is the number of ways of choosing k years from the n years of the investment. PR^k is the contribution to the interest for each of those sets of k years. In particular, PRn, the simple interest, is the contribution to the total interest of applying the interest rate to all possible combinations of one year. **(c)** In the life of a loan, more interest is earned from this term than from any other.

Exercise 7C (Page 255)

1(a)(i) $M \times 1{\cdot}065^n$ **(ii)** $M \times 1{\cdot}065^{n-1}$
(iii) $M \times 1{\cdot}065$
(iv) $A_n = 1{\cdot}065M + 1{\cdot}065^2 M + \cdots + 1{\cdot}065^n M$
(c) $188 146 and $75 000 **(d)(i)** $\frac{300\,000}{188\,146} \times M \doteqdot \$4784$
(ii) $1784
2(a) M, $1{\cdot}04M$, $1{\cdot}04^{n-1}M$ **(c)** $893 342
3(a) $200 000 **(b)** $67 275 **(c)** $630 025
4(a) $360 **(b)** $970.27
5(a) $31 680 **(b)** $394 772 **(c)** $1 398 905
6(a) $67 168.92 **(b)** $154 640.32
7 $3086 **8(a)** $25 718.41
(b) $25 718.41 + $23 182.17 = $48 900.58
9(a) $286 593 **(b)(i)** $107 355 **(ii)** $152 165
10(a) $424 195.23 **(b)** $431 235.13
11 $55 586.38
12(c) $A_2 = 1{\cdot}01\,M + 1{\cdot}01^2\,M$,
$A_3 = 1{\cdot}01\,M + 1{\cdot}01^2\,M + 1{\cdot}01^3\,M$,
$A_n = 1{\cdot}01\,M + 1{\cdot}01^2\,M + \cdots + 1{\cdot}01^n\,M$

ISBN: 978-1-107-61604-2 Cambridge University Press

(e) \$4350.76 **(f)** \$363.70

13(b) $A_2 = 1{\cdot}002 \times 100 + 1{\cdot}002^2 \times 100$,
$A_3 = 1{\cdot}002 \times 100 + 1{\cdot}002^2 \times 100 + 1{\cdot}002^3 \times 100$,
$A_n = 1{\cdot}002 \times 100 + 1{\cdot}002^2 \times 100 + \cdots + 1{\cdot}002^n \times 100$
(d) about 549 weeks

14(b) exponential **(c)** \$10 436 **(d)(i)** They are the same. **(ii)** Superannuation is an approximation for the area under the graph of compound interest.

15(a) The function FV calculates the value just after the last premium has been paid, not at the end of that year.

Exercise 7D (Page 259)

1(a)(i) $P \times 1{\cdot}015^n$ **(ii)** $M \times 1{\cdot}015^{n-1}$
(iii) $M \times 1{\cdot}015^{n-2}$ and M **(iv)** $A_n = P \times 1{\cdot}015^n - (M + 1{\cdot}015M + 1{\cdot}015^2M + \cdots + 1{\cdot}015^{n-1}M)$
(c) 0 **(d)** $M = \dfrac{P \times 1{\cdot}015^n \times 0{\cdot}015}{1{\cdot}015^n - 1}$ **(e)** \$254

2(a) $A_n = P \times 1{\cdot}006^n - (M + 1{\cdot}006M + 1{\cdot}006^2M + \cdots + 1{\cdot}006^{n-1}M)$
(c) \$162 498, which is more than half.
(d) $-$\$16 881 **(f)** 8 months

3(a) The loan is repaid in 25 years. **(b)** \$1226.64
(c) \$367 993 **(d)** \$187 993 and 4·2%

4 \$345

5(a) \$4202 **(b)** $A_{10} = \$6.65$ **(c)** Each instalment is approximately 48 cents short because of rounding.

6 \$216 511

7 It will take 57 months, but the final payment will be lower than usual.

8(a) \$160 131.55 **(b)** \$1633.21 < \$1650, so the couple can afford the loan.

9 \$44 131.77

10(a) \$2915.90 **(b)** \$84.10

11(b) zero balance after 20 years **(c)** \$2054.25

12(c) $A_2 = 1{\cdot}005^2\,P - M - 1{\cdot}005\,M$,
$A_3 = 1{\cdot}005^3\,P - M - 1{\cdot}005\,M - 1{\cdot}005^2\,M$,
$A_n = 1{\cdot}005^n\,P - M - 1{\cdot}005\,M - \cdots - 1{\cdot}005^{n-1}\,M$
(e) \$1074.65 **(f)** \$34 489.78

13(b) $A_2 = 1{\cdot}008^2\,P - M - 1{\cdot}008\,M$,
$A_3 = 1{\cdot}008^3\,P - M - 1{\cdot}008\,M - 1{\cdot}008^2\,M$,
$A_n = 1{\cdot}008^n\,P - M - 1{\cdot}008\,M - \cdots - 1{\cdot}008^{n-1}\,M$
(d) \$136 262
(e) $n = \log_{1\cdot008} \dfrac{125M}{125M - P}$, 202 months

14(a) \$542 969.89 **(b)** \$285 151.16

15(b) 10·5% per annum

16(a) \$839 343 **(b)** \$6478

Exercise 7E (Page 264)

1(b) $1\,\text{m}^2/\text{s}$ **(c)** 7 metres **(d)** $9\,\text{m}^2$

2(a) $A = \frac{1}{2}\ell^2$ **(c)(i)** $5\,\text{m}^2/\text{s}$ **(ii)** $3\,\text{m}^2/\text{s}$
(d) 34 metres

3(a) $15{\cdot}1\,\text{m}^3/\text{s}$ **(b)** $30{\cdot}2\,\text{m}^2/\text{s}$

4(b) $\dfrac{2}{9\pi}$ cm/s **(c)** $\dfrac{10}{\sqrt{\pi}}$ cm, $\dfrac{4000}{3\sqrt{\pi}}\,\text{cm}^3$

5(a) $90\,000\pi\,\text{mm}^3/\text{min}$ **(b)** the rate is constant at 6π mm/min.

6(b) $\frac{1}{6}$ cm/s

7(b) 5 degrees per second

8(a) $V = \frac{4}{3}\pi h^3$ **(b)** $\frac{1}{32\pi}$ m/s

9 2 degrees per minute

11(a) $-2\sqrt{1 - x^2}$
(b) -2 m/s — as the point crosses the y-axis it is travelling horizontally at a speed of 2 m/s.

12(a) $\dfrac{2CV^2}{L^2}\,\text{m/s}^2$
(c) As L decreases, the speed passing the truck increases, so the driver should wait as long as possible before beginning to accelerate. A similar result is obtained if the distance between car and truck is increased. Optimally, the driver should allow both L to decrease and C to increase.
(d) 950 metres

13(b) This is just two applications of the chain rule.
(d) 6

14(c) $x = h = 50(\sqrt{3} + 1)$ metres **(d)** 200 km/h

Exercise 7F (Page 268)

1(a) 25 minutes **(b)** $V = 5(t^2 - 50t) + 3145$
(c) 3145 litres

2(a) $P = 6{\cdot}8 - 2\log(t + 1)$
(b) approximately 29 days

3(a) $-2\,\text{m}^3/\text{s}$ **(b)** 20 s **(c)** $V = 520 - 2t + \frac{1}{20}t^2$
(d) $20\,\text{m}^3$ **(e)** 2 minutes and 20 seconds

4(a) no **(b)** $x = \frac{5}{2}(1 - e^{-0\cdot4t})$ **(c)** $t \doteqdot 1{\cdot}28$
(d) $x = \frac{5}{2}$

5(a) 0 **(b)** 250 m/s
(c) $x = 1450 - 250(5e^{-0\cdot2t} + t)$

6(a) $I = 18\,000 - 5t + \frac{48}{\pi}\sin\frac{\pi}{12}t$
(b) $\dfrac{dI}{dt}$ has a maximum of -1, so it is always negative. **(c)** There will be 3600 tonnes left.

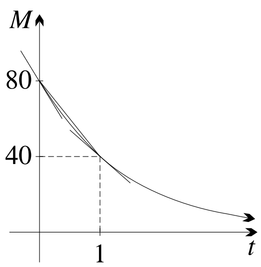
7(a) $\theta = \tan^{-1} t + \frac{\pi}{4}$
(b) $t = \tan(\theta - \frac{\pi}{4})$
(c) As $t \to \infty$, $\tan^{-1} t \to \frac{\pi}{2}$, and so $\theta \to \frac{3\pi}{4}$.

ISBN: 978-1-107-61604-2

8(a) $1200\,\text{m}^3$ per month at the beginning of July **(b)** $W = 0{\cdot}7t - \frac{3}{\pi}\sin\frac{\pi}{6}t$
9(b) $r = k(t-12)$ **(c)** $k = -\frac{1}{48}$
10(a) $V = \frac{1}{3}\pi r^3$ **(b)** $\dfrac{dr}{dt} = \dfrac{1}{2\pi r^2}$
(c) $t = \frac{2\pi}{3}(r^3 - 1000)$ **(d)** 25 minutes 25 seconds
11(a) $V = \frac{\pi}{3}(128 - 48h + h^3)$ **(b)(i)** $A = \pi(16 - h^2)$
(iii) 1 hour 20 minutes

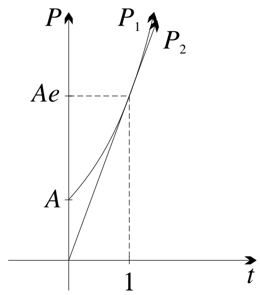
Exercise **7G** (Page 273)

1(b) 1350 **(c)** 135 per hour **(d)** 23 hours
2(c) $6{\cdot}30$ grams, $1{\cdot}46$ grams per minute
(d) 6 minutes 58 seconds **(e)** $20\,\text{g}$, $20e^{-k} \doteqdot 15{\cdot}87\,\text{g}$, $20e^{-2k} \doteqdot 12{\cdot}60\,\text{g}$, $20e^{-3k} = 10\,\text{g}$, $r = e^{-k} = 2^{\frac{1}{3}} \doteqdot 0{\cdot}7937$
3(b) $-\frac{1}{5}\log\frac{7}{10}$ **(c)** 10 290 **(d)** At $t \doteqdot 8{\cdot}8$, that is, some time in the fourth year from now.
4(b) 30 **(c)(i)** 26 **(ii)** $\frac{1}{5}\log\frac{15}{13}$ (or $-\frac{1}{5}\log\frac{13}{15}$)
5(a) $80\,\text{g}$, $40\,\text{g}$, $20\,\text{g}$, $10\,\text{g}$ **(e)**

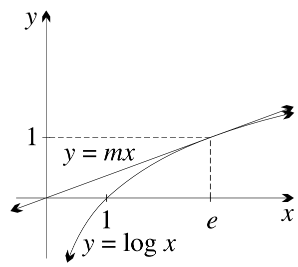


(b) $40\,\text{g}$, $20\,\text{g}$, $10\,\text{g}$.
During each hour, the average mass loss is 50%.
(c) $M_0 = 80$, $k = \log 2 \doteqdot 0{\cdot}693$
(d) $55{\cdot}45\,\text{g/hr}$, $27{\cdot}73\,\text{g/hr}$, $13{\cdot}86\,\text{g/hr}$, $6{\cdot}93\,\text{g/hr}$
6(a) $C = C_0 \times 1{\cdot}01^t$ **(i)** $1{\cdot}01^{12} - 1 \doteqdot 12{\cdot}68\%$
(ii) $\log_{1\cdot 01} 2 \doteqdot 69{\cdot}66$ months **(b)** $k = \log 1{\cdot}01$
(i) $e^{12k} - 1 \doteqdot 12{\cdot}68\%$ **(ii)** $\frac{1}{k}\log 2 \doteqdot 69{\cdot}66$ months
7(b) $L = \frac{1}{2}$
8(c) 25 **(d)** $\frac{k}{A} = \frac{1}{2}\log\frac{5}{3}$ (or $-\frac{1}{2}\log\frac{3}{5}$)
(e) 6 hours 18 minutes
9(b) $C_0 = 20\,000$, $k = \frac{1}{5}\log\frac{9}{8} \doteqdot 0{\cdot}024$
(c) 64 946 ppm **(d)(i)** 330 metres from the cylinder
(ii) If it had been rounded down, then the concentration would be above the safe level.
10(a) $y(3) = A_0\,e^{3k} = A_0(e^k)^3$ and we know that $e^k = \frac{3}{4}$. **(b)** $y(3) = \frac{27}{64}A_0$
11(a)(ii) $k = \frac{1}{12}\log\frac{122}{105}$ **(b)(ii)** $\ell = \frac{1}{12}\log\frac{217}{100}$
(c) At $t = \dfrac{\log\frac{525}{100}}{\ell - k} \doteqdot 31{\cdot}85$, that is, in the 32nd month.
(d) $\ell C = \ell \times 100 \times e^{32\ell} \doteqdot 51$ cents per month
12(a) $\dfrac{dV}{dt} = -0{\cdot}15V$ **(b)** $V = 12\,000\,e^{-0\cdot 15t}$
(c) \$10 328.50, a decrease of about $13{\cdot}9\%$
(d) \$1549.27 per year **(e)** At $t \doteqdot 15{\cdot}4$, that is, during the 16th year.

13(a) $A = 5000 \times 1{\cdot}07^t$ **(c)** $A = 5000 \times e^{t\log 1\cdot 07}$
(e) $A = \$7503.65$ **(f)** \$507.69 per year
14(a)(ii)



(b)(i)



(c) In part (a), changing the base is equivalent to stretching the graph horizontally. Since both curve and straight line are equally stretched, the straight line will still pass through the origin. The same is true in part (b) except that the stretch is vertical.
(d) The graph in part (b) is just a reflection in the line $y = x$ of the graph in part (a).
15(a) $B = \dfrac{2{N_0}^2}{N_c}$ and $C = \left(\dfrac{N_0}{N_c}\right)^2$ **(b)** $\dfrac{B}{C} = 2N_c$

Exercise **7H** (Page 279)

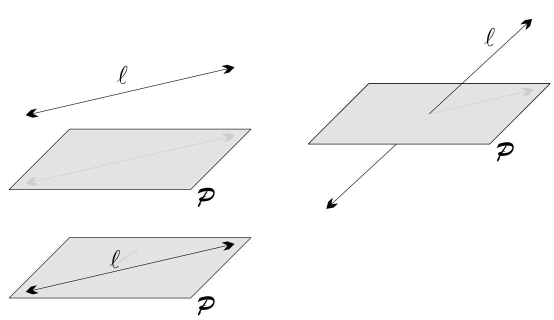
1(a)(ii) 12 000, $P \to \infty$ as $t \to \infty$
(b)(ii) 12 000, $P \to 10\,000$ as $t \to \infty$
(c)(ii) 8000, $P \to 10\,000$ as $t \to \infty$
2(b) $A = 1000$, $k = \frac{1}{3}\log 6$ **(c)** 67 420 bugs
(d) $10{\cdot}4$ weeks
3(b) $B = 970\,000$, $k = -\frac{1}{10}\log\frac{47}{97} = \frac{1}{10}\log\frac{97}{47}$
(c) 158 000 flies **(d)** 73 days
4(b) $T_e = 20$, $A = 70$ **(c)** $k = \frac{1}{6}\log\frac{7}{3}$ [Alternatively, $k = -\frac{1}{6}\log\frac{3}{7}$.] **(d)** 13 minutes 47 seconds
5(a) $A = 34$ **(b)** $\frac{1}{45}\log 2$ (or $-\frac{1}{45}\log\frac{1}{2}$) **(c)** $16{\cdot}5°\text{C}$
6(a) $1 - e^{-\frac{1}{16}t}$ is always positive for $t > 0$. The body is falling. **(b)** It is the acceleration of the body. **(c)** $-160\,\text{m/s}$ **(d)** $16\log\frac{8}{7} \doteqdot 2{\cdot}14\,\text{s}$
7(b)(i) 15 cm **(ii)** 15 is the average **(iii)** 15
(c) $\frac{1}{5}\log\frac{5}{3}$
8(b) $-\frac{V}{R}$ **(c)** $I \to \frac{V}{R}$ **(d)** $4{\cdot}62 \times 10^{-4}\,\text{s}$
9(b) $M \to a$ as $t \to \infty$ **(c)** $k = \frac{1}{120}\log_e 100$
(d) 2 minutes 45 seconds
10(a) $2w\,\text{g/min}$ **(b)** $\dfrac{Q}{1000}\,\text{g/L}$ **(c)** $\dfrac{Qw}{1000}\,\text{g/min}$
(f) -2000 **(g)** $Q \to 2000$
(h) $w = \frac{1000}{345}\log 2 \doteqdot 2\,\text{L/min}$
12(b) $A = 1000$, $I = 9000$ and $k = \frac{1}{7}\log 3$
(c) 36 000
13 The man's coffee is cooler.

ISBN: 978-1-107-61604-2 Cambridge University Press

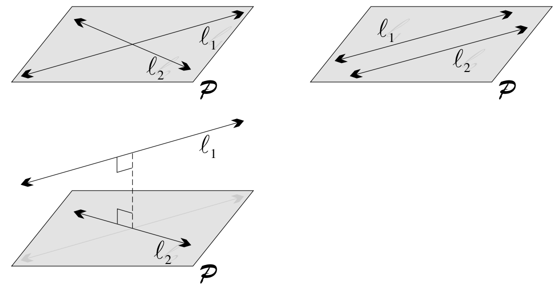
Chapter Eight

Exercise 8A (Page 288)

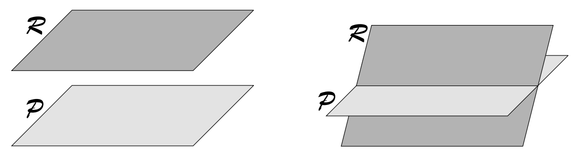
1(a) 70° **(b)** 45° **(c)** 60° **(d)** 50° **(e)** 22°
(f) $\alpha = 153^\circ$, $\beta = 27^\circ$ **(g)** 34°
(h) $\alpha = 70^\circ$, $\beta = 70^\circ$
2(a) 35° **(b)** 43° **(c)** 60° **(d)** $\alpha = 130^\circ$, $\beta = 50^\circ$
(e) $\alpha = 123^\circ$, $\beta = 123^\circ$ **(f)** 60°
(g) $\alpha = 65^\circ$, $\beta = 65^\circ$ **(h)** $\alpha = 90^\circ$, $\beta = 90^\circ$
3(a) equal alternate angles **(b)** equal corresponding angles **(c)** supplementary co-interior angles
(d) supplementary co-interior angles
5(a) $\alpha = 52^\circ$, $\beta = 38^\circ$ **(b)** $\alpha = 30^\circ$, $\beta = 60^\circ$
(c) 24° **(d)** 36° **(e)** $\alpha = 15^\circ$, $\beta = 105^\circ$, $\gamma = 60^\circ$, $\delta = 105^\circ$ **(f)** 24° **(g)** 15° **(h)** 22°
6(a) $\alpha = 75^\circ$, $\beta = 105^\circ$ **(b)** $\alpha = 252^\circ$, $\beta = 72^\circ$
(c) 32° **(d)** 62° **(e)** 60° **(f)** 135° **(g)** 48° **(h)** 35°
10(a) $\theta = 58^\circ$ **(b)** $\theta = 37^\circ$, $\phi = 15^\circ$ **(c)** $\theta = 10^\circ$
(d) $\theta = 12^\circ$, $\phi = 41^\circ$
11(a) $\angle DOB$ and $\angle COE$ are straight angles; $\angle BOC$ and $\angle DOE$ are vertically opposite angles, and so are $\angle BOE$ and $\angle COD$.
(b) $GA \parallel BD$ (alternate angles are equal)
(c) $\angle BOE = 90^\circ$
12(a) $\alpha = 60^\circ$ **(b)** $\alpha = 90^\circ$ **(c)** $\alpha = 105^\circ$
14 $\angle FBE = \angle FBD + \angle DBE$
$= \frac{1}{2}(\angle ABD + \angle BDC) = \frac{1}{2}180^\circ = 90^\circ$
16 $\angle FBE = \frac{k}{k+\ell} \times 180^\circ$
17(a) two walls and the ceiling of a room
(b) three pages of a book
(c) the floors of a multi-storey building
(d) the sides of a simple tent and the ground
(e) the floor, ceiling and one wall of a room
(f) a curtain rod in front of a window pane
(g) the corner post of a soccer field
18(a) The line is parallel to the plane, or intersects with it at a point, or lies in the plane.



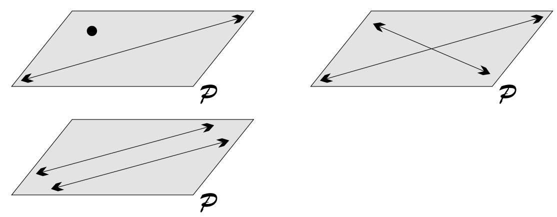
(b) The lines lie in one plane and intersect, or lie in one plane and are parallel, or are skew.



(c) The two planes are parallel, or intersect.



19(a) a point and a line, or two intersecting lines, or two parallel lines



(b) AB and CD, BC and AD, CA and BD

Exercise 8B (Page 295)

1(a) 55° **(b)** 55° **(c)** 52° **(d)** 70° **(e)** 60° **(f)** 30°
(g) 18° **(h)** 20°
2(a) 108° **(b)** 129° **(c)** 24° **(d)** 74°
3(a) 99° **(b)** 138° **(c)** 65° **(d)** 56° **(e)** 60° **(f)** 80°
(g) 36° **(h)** 24°
5(a)(i) 108° **(ii)** 72° **(b)(i)** 120° **(ii)** 60° **(c)(i)** 135°
(ii) 45° **(d)(i)** 140° **(ii)** 40° **(e)(i)** 144° **(ii)** 36°
(f)(i) 150° **(ii)** 30°
6(a)(i) 8 **(ii)** 10 **(iii)** 45 **(iv)** 180 **(b)(i)** 5 **(ii)** 9
(iii) 20 **(iv)** 720
(c) Solving for n does not give an integer value.
(d) Solving for n does not give an integer value.
8(a) $\alpha = 59^\circ$, $\beta = 108^\circ$ **(b)** $\alpha = 45^\circ$, $\beta = 60^\circ$
(c) $\alpha = 76^\circ$, $\beta = 106^\circ$ **(d)** $\alpha = 110^\circ$, $\beta = 50^\circ$
(e) $\alpha = 106^\circ$, $\beta = 40^\circ$ **(f)** $\alpha = 51^\circ$, $\beta = 43^\circ$
(g) $\alpha = 104^\circ$, $\beta = 48^\circ$ **(h)** $\alpha = 87^\circ$
9(a) 50° **(b)** $\theta = 35^\circ$ $\phi = 40^\circ$ **(c)** $\theta = 40^\circ$ $\phi = 50^\circ$
(d) $\theta = 108^\circ$ $\phi = 144^\circ$
10(a) 23° **(b)** 17° **(c)** 22° **(d)** 31° **(e)** 44° **(f)** 38°
(g) 60° **(h)** 45°

ISBN: 978-1-107-61604-2 Cambridge University Press

11(a) $140°$ **(b)** $68°$ **(c)** $47°$ **(d)** $50°$

16(a) $\frac{n-2}{2}$ **(b)(i)** No, because $n = 3\frac{1}{3}$, which is not an integer. **(ii)** Yes, $n = 9$ and it is a nonagon.

17(a) $360°$ **(b)** They are the same.

19(a) $\theta > 60°$ **(b)** $\theta < 120°$

20(a) $\frac{720}{n}$ **(b)** $m = \frac{2n}{n-4}$ **(c)** $n = 5$ gives a pentagon and decagon, $n = 6$ gives a hexagon with itself, $n = 8$ gives an octagon and a square, $n = 12$ gives a dodecagon and an equilateral triangle.

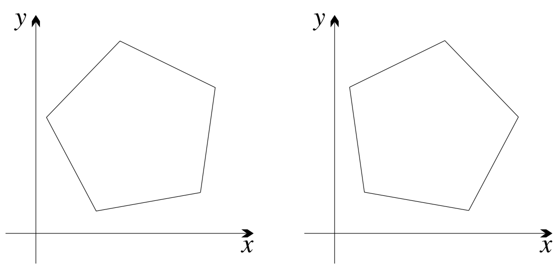
(d) For $n = 5$, $2\cos 36° \doteqdot 1.62$. For $n = 6$, 1. For $n = 8$, $\frac{1}{\sqrt{2}}$. For $n = 12$, $\frac{1}{\sqrt{3}}$.

24(a) $\dfrac{(\cos\alpha - 1) + \sqrt{2(1-\cos\alpha)}}{(\cos\alpha + 1)}$, where $\alpha = \dfrac{(n-2)180°}{n}$. **(b)(i)** $\frac{1}{3}$ **(ii)** $\sqrt{2} - 1$

25(a) exterior angle of triangle theorem

(b) One angle is obtuse. **(c)** Nothing. $m_1m_2m_3$ may be positive, negative, zero, or even undefined if one line is vertical.

26 If n is even, then the product must be positive because opposite sides are parallel. If n is odd, then the product could be positive or negative, depending on its orientation.



27(a) 1 **(b)** 0 **(c)** 2

Exercise 8C (Page 304)

1(a) $\triangle ABC \equiv \triangle RQP$ (AAS)

(b) $\triangle ABC \equiv \triangle CDA$ (SSS)

(c) $\triangle ABC \equiv \triangle CDE$ (RHS)

(d) $\triangle PQR \equiv \triangle GEF$ (SAS)

2(a) $\triangle ABC \equiv \triangle DEF$ (AAS), $x = 4$

(b) $\triangle GHI \equiv \triangle LKJ$ (RHS), $x = 20$

(c) $\triangle QRS \equiv \triangle UTV$ (SAS), $x = \sqrt{61}$

(d) $\triangle MLN \equiv \triangle MPN$ (AAS), $x = 12$

3(a) $\triangle ABC \equiv \triangle FDE$ (SSS), $\theta = 67°$

(b) $\triangle XYZ \equiv \triangle XVW$ (SAS), $\theta = 86°$

(c) $\triangle ABC \equiv \triangle BAD$ (SSS), $\theta = 49°$

(d) $\triangle PQR \equiv \triangle HIG$ (RHS), $\theta = 71°$

4(a) $\theta = 64°$ **(b)** $\theta = 69°$ **(c)** $\theta = 36°$ **(d)** $\theta = 84°$ **(e)** $\theta = 64°$ **(f)** $\theta = 90°$ **(g)** $\theta = 45°$ **(h)** $\theta = 120°$

5(a) The diagram does not show a pair of equal sides. The correct reason is AAS.

(b) The diagram does not show a pair of equal hypotenuses. The correct reason is SAS.

6(a) $\triangle AXB \equiv \triangle CXD$ (SAS)

(b) $\triangle ABD \equiv \triangle CBD$ (SSS)

(c) $\triangle ABC \equiv \triangle ADC$ (RHS)

(d) $\triangle ABF \equiv \triangle DEC$ (AAS)

7 This is the spurious ASS test — the angles are not the included angles.

8 In both cases, two sides are given but not the included angle.

9(a) It has reflection symmetry in its altitude.

(b) It has reflection symmetry in each of its altitudes. It has rotational symmetry of $120°$ and $240°$ about the point where the altitudes are concurrent.

10(a)(ii) AM **(b)(i)** Each altitude is an axis of symmetry. **(ii)** There is $60°$ rotational symmetry about the point where the three altitudes meet.

12 $AB = AC = BC$ by construction

13(a) SSS

16(b) SAS

17(b) SAS

18(a) SSS **(b)** The base angles are equal.

(c) $CX = AC - AX = BD - BX = DX$

(d) equal alternate angles

19(a)(i) $66°$ **(ii)** $24°$ **(b)(i)** $\angle A = \angle C = 2\alpha$

(ii) $\triangle ABC$ is equilateral, and in triangle ABE, $\angle A = \angle E = 30°$.

20(a) $\angle BXY = \alpha + \beta = \angle BYX$, hence $\triangle BXY$ is isosceles (base angles equal).

(b)(i) $\triangle ADX \equiv \triangle CDX$ (AAS), hence $AD = CD$. **(ii)** $\triangle CDB \equiv \triangle ADB$ (SAS)

21(a) exterior angle of $\triangle ACD$ **(b)** $180° - (2\alpha + \beta)$

(c) $\angle EDB = \frac{1}{2}\beta$

22(a) Two equal radii form two sides of each triangle. **(b)** SSS **(c)** AAS or SAS **(d)** matching sides and matching angles, $\triangle AMO \equiv \triangle BMO$

23(a) $AB = BC$ (given), $\angle CAB = 36°$ **(b)** SAS

(c) $36°$ **(d)** area $= x^2 \sin 108° + \frac{1}{4}x^2 \tan 72°$

24(a) SAS **(b)** AAS

(c) $\triangle OQM \equiv \triangle OSM$ (SAS or SSS)

25(a) SAS **(b)** SAS

26(a) exterior angle of $\triangle ABP$

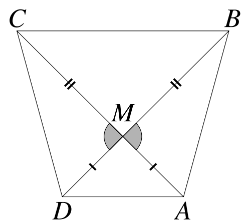
(b) base angles of isosceles $\triangle PBC$

ISBN: 978-1-107-61604-2 Cambridge University Press

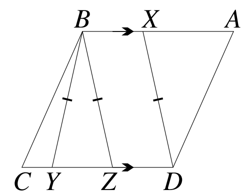
28 $\triangle BDM$ is an isosceles right-angled triangle with the right angle at M.

Exercise **8D** (Page 311)

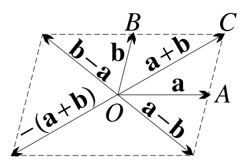
1(a) $\alpha = 115°$, $\beta = 72°$ **(b)** $\alpha = 128°$, $\beta = 52°$ **(c)** $\alpha = 90°$, $\beta = 102°$ **(d)** $\alpha = 47°$, $\beta = 133°$
2(a) $\alpha = 27°$, $\beta = 99°$ **(b)** $\alpha = 41°$, $\beta = 57°$ **(c)** $\alpha = 40°$, $\beta = 100°$ **(d)** $\alpha = 30°$, $\beta = 150°$
3 Test for a parallelogram: two opposite sides are equal and parallel.
4 Test for a parallelogram: diagonals bisect each other.
5 No. It could be a trapezium with a pair of equal but non-parallel sides.
6(a) $180°$ about the intersection of the diagonals **(b)** A trapezium with equal non-parallel sides has reflection symmetry.
7 $\sin(180° - \theta) = \sin\theta$, that is, the sine of an angle and its supplement are equal.
8(a) Co-interior angles are supplementary.
(b)(i) AAS **(c)(i)** AAS
9(a)(i) angle sum of a quadrilateral **(ii)** Co-interior angles are supplementary. **(b)(i)** SSS **(iii)** Test for a parallelogram: opposite angles are equal. **(c)(i)** SAS **(ii)** Test for a parallelogram: opposite sides are equal. **(d)(i)** SAS
10(a) SAS **(b)** matching angles, $\triangle BAD \equiv \triangle ABC$ **(d)** Co-interior angles are supplementary.
11(a) Properties of a parallelogram: opposite angles are equal. **(b)** Properties of a parallelogram: opposite sides are equal. **(c)** SAS
(d) A quadrilateral with equal opposite sides is a parallelogram.
12(a) SAS **(b)** SAS **(c)** Test for a parallelogram: opposite sides are equal. Alternatively, use the equality of alternate angles to prove that the opposite sides are parallel.
13(a) AAS **(b)** Z is the midpoint of AC, and this is where BD meets AC.
14 In the first question, $AD = CB$ (opposite sides of parallelogram $ABCD$), and hence $AY = CX$. Thus the sides AY and CX are equal and parallel, and so $AYCX$ is a parallelogram.
In the second question, the diagonals BD and AC of the parallelogram $ABCD$ bisect each other. Hence the intervals BD and PQ bisect each other, and so $BPDQ$ is also a parallelogram.
In the third question, $AXCY$ is a parallelogram because the sides AX and CY are equal and parallel. Hence XY meets AC at the midpoint of AC, which is where BD meets AC.
16 In both parts, $DABF$ is a parallelogram, so $\angle F = \alpha$ (opposite angles are equal) and $FB = DA$ (opposite sides are equal). Also, $\angle ABC = \angle BCF$ (alternate angles, $DF \parallel AB$).
(a)(ii) $\triangle BFC$ is isosceles, so $\angle BCF = \alpha$.
(b)(ii) $\angle BCF = \angle F = \alpha$, hence $\triangle BCF$ is isosceles.
17(b) $\triangle AMD$ is isosceles.



18(b) a trapezium **(c)** $DX > DA$



19



20(a) Test for a parallelogram: one pair of opposite sides are equal and parallel. **(b)** AAS **(c)** equal corresponding angles **(d)** Repeated use of the result in part (c) yields $PQ \parallel SR$ and $QR \parallel PS$. Hence $PQRS$ is a parallelogram by definition.

Exercise **8E** (Page 316)

1(a) $45°$ **(b)** $76°$ **(c)** $15°$ **(d)** $9°$
2(a) $\alpha = 15°$, $\phi = 105°$
3(a)(i) A rectangle has horizontal and vertical reflection symmetries. It has rotation symmetry of $180°$ about the intersection of its diagonals.
(ii) A rhombus has reflection symmetries in each diagonal. It has rotation symmetry of $180°$ about the intersection of its diagonals.
(iii) A square has horizontal and vertical reflection symmetries, as well as those in each diagonal. It has rotation symmetries of $90°$, $180°$ and $270°$ about the intersection of its diagonals. **(b)** A circle has reflection symmetry in every diagonal,

ISBN: 978-1-107-61604-2

and rotation symmetry of every number of degrees about the centre.

4(a) The diagonals bisect each other at right angles. **(b)** The diagonals are equal and bisect each other. **(c)** By parts (a) and (b), $ABCD$ is both a rectangle and a rhombus.

5(a) Test for a rhombus: all sides are equal.

(b) Property of a rhombus: diagonals bisect vertex angles.

6(a) base angles of isosceles triangle ABD

(b) alternate angles, $AB \parallel DC$ **(d)** angle sum of triangles

7(a) Since opposite sides are equal, it is a parallelogram and it has a pair of equal adjacent sides. **(b)(i)** Test for a parallelogram: the diagonals bisect each other. **(ii)** SAS **(c)(i)** half the angle sum of a quadrilateral **(iii)** Test for a parallelogram: opposite angles are equal. **(iv)** The base angles of $\triangle ABD$ are equal.

8(b)(ii) SAS

9(b)(i) Test for a parallelogram: diagonals bisect each other.

(ii) base angles of isosceles triangle ABM

(iii) base angles of isosceles triangle BCM

10(a)(i) SAS **(b)(i)** SAS

11(a) Test for a rhombus: all sides are equal.

(b) Properties of a rhombus: diagonals bisect each other at right angles.

12(a) Test for a rhombus: all sides are equal.

(b) Properties of a rhombus: diagonals bisect each other at right angles.

13(a) Test for a rhombus: all sides are equal.

14(a) Test for a rhombus: all sides are equal.

(b) Properties of a rhombus: diagonals bisect each other at right angles.

15(a)(i) SAS **(ii)** Test for a rhombus: all sides are equal. **(b)(i)** Opposite sides are parallel by construction. **(ii)** Test for a rhombus: diagonals bisect vertex angles.

16(a) AAS **(b)** By definition: $PQRS$ is a parallelogram with a pair of adjacent sides equal.

17(a) SAS

18(a) Property of a rhombus: diagonals bisect vertex angles. **(b)** SAS

(c) alternate angles, $AD \parallel BC$ **(d)** 90°

19(a) alternate angles, $BC \parallel AR$ **(b)** The diagonals of a rectangle are equal and bisect each other.

20(a)(i) SSS **(b)(i)** SAS

Exercise 8F (Page 322)

1(a) 33 **(b)** 50 **(c)** 28 **(d)** 72

2(a) $A = 36,\ P = 24$ **(b)** $A = 18,\ P = 12\sqrt{2}$ **(c)** $A = 60,\ P = 32$ **(d)** $A = 48,\ P = 28$

3(a) $A = 54,\ P = 18 + 4\sqrt{13}$, diagonals: $\sqrt{205}$ and $\sqrt{61}$ **(b)** $A = 264,\ P = 72$, diagonals: 30 and $4\sqrt{37}$ **(c)** $A = 120,\ P = 52$, diagonals: 24 and 10 **(d)** $A = 600,\ P = 100$

NOTE: The second diagonal, which is 30, is most easily obtained from the area of the rhombus.

4(a) A square is a rhombus, so the result follows from the area formula for a rhombus. **(b)** $s^2 = ab$, so $s = \sqrt{ab}$. **(c)** Since $C = 90^\circ$, $\sin C = 1$, so the trigonometric formula becomes $A = \frac{1}{2}ab$, and a and b are the base and the perpendicular height.

6(a) Both triangles have the same base and altitude — the distance between the parallel lines. **(b)** $\triangle BCX = \triangle ABC - \triangle ABX$
$= \triangle ABD - \triangle ABX = \triangle ADX$

7(a) sums of equal areas **(b)** Triangles with the same base and area have the same altitude.

8 Any two adjacent triangles have the same height and equal bases. They will all be congruent when the parallelogram is also a rhombus.

9(a) Properties of a parallelogram: the diagonals bisect each other. **(b)** $a^2 : b^2$

(c) When the parallelogram is a rectangle.

10(a) $2x^2 - 2x + 1$ **(b)** $\frac{1}{2}\,\text{m}^2$ when $x = \frac{1}{2}$ metre.

11(a) $20\,\text{m}^2$ **(b)** $76\,\text{m}^2$

12(a)(i) $\frac{\sqrt{3}}{4}$ **(ii)** $\frac{3\sqrt{3}}{2}$ **(b)(i)** $\frac{1}{\sqrt{3}}$ **(ii)** $2\sqrt{3}$

(c) The area of the inscribed hexagon is smaller than the circle, which is in turn smaller than the escribed hexagon.

13(a) AAS **(b)** $AG = \frac{7}{4}$ **(c)** $37\frac{1}{2}$

14(a) the area of the two triangles formed by one diagonal **(b)** $a^2 + b^2 - 2ab\cos\theta,\ a^2 + b^2 + 2ab\cos\theta$

(c) area of annulus $= \pi ab|\cos\theta|$

15(a) In the trapezium $DBRC$, the areas of $\triangle CPR$ and $\triangle DBP$ are equal, proven in question 15 of Section 8D. The result then follows. **(b)** $\frac{1}{5}$

16(a) The fat rhombus has angles of 72° and 108°. The thin rhombus has angles of 36° and 144°.

17(a) AQ dissects $\triangle OQX$ into two triangles of equal area, hence AQ bisects the base OX.

(c) $\frac{\pi}{6\sqrt{2-\sqrt{2}}}$ **(d)** $\frac{\pi}{12}(\sqrt{2}+1)$

ISBN: 978-1-107-61604-2 Cambridge University Press

(e) $AR = \frac{1}{12}\left(\pi(\sqrt{2}+1)-6\right)$,
$RX = \frac{1}{12}(12-\pi(\sqrt{2}+1))$ **(g)** $162°48'$

Exercise **8G** (Page 326)

1 a, c, d
2(a) $c = 13$ **(b)** $c = \sqrt{41}$ **(c)** $a = 5\sqrt{7}$
(d) $b = 2\sqrt{10}$
3 The cosine rule is $c^2 = a^2 + b^2 - 2ab\cos C$, but here $C = 90°$ and $\cos 90° = 0$, so the third term disappears, giving Pythagoras' theorem.
4(a) $29'49''$ **(b)** $20'7''$
5(a) $a^2 = s^2 - b^2$ **(i)** $108\,\text{cm}^2$ **(ii)** $40\sqrt{14}\,\text{cm}^2$
(b) This is an equilateral triangle with $a = b\sqrt{3}$ and area $= b^2\sqrt{3}$.
6(a)(i) $17\,\text{cm}$ **(ii)** $56°9'$, $123°51'$ **(b)** $4\sqrt{11}\,\text{cm}$
(c)(i) $10\,\text{cm}$ **(ii)** $5\sqrt{5}\,\text{cm}$
7(b) $x = 3$ or 4, so the diagonals are 6 cm and 8 cm.
8(a)(ii) 10 cm when $t = 3$, and 17 cm when $t = 4$.
(b)(i) 3, 4, 5 **(ii)** 5, 12, 13 **(iii)** 7, 24, 25
(iv) 33, 56, 65
9(b)(i) c^2 **(ii)** $(b-a)^2$ **(iii)** Each is $\frac{1}{2}ab$.
10(a) $p^4 + p^2q^2$, $q^4 + p^2q^2$
11(a) $\angle PRS = 15°$ **(b)** $RS = 2$, $QS = \sqrt{3}$
12(b) $x^2 + y^2 = 25$, $(x+10)^2 + y^2 = 169$
(c) $x = \frac{11}{5}$, $\cos\alpha = \frac{61}{65}$
13(a) 4
19(a) $a^2 + b^2$, $b^2 + c^2$, $c^2 + d^2$, $a^2 + d^2$
21(a) $(c+x)^2 + h^2 = a^2$, $(c-x)^2 + h^2 = b^2$,
$h^2 + x^2 = d^2$
23(c) There are two possible configurations:
$\theta = 77°$ for P outside and $\theta = 166°$ for P inside.
25(b) The possible remainders are 0, 1 and 4.
A simple addition table for the LHS shows that there are only six cases, and in each case one of the integers has remainder 0.

Exercise **8H** (Page 333)

1(a) $\triangle ABC \,|||\, \triangle QPR$ (AA)
(b) $\triangle ABC \,|||\, \triangle CAD$ (SSS)
(c) $\triangle ABD \,|||\, \triangle DBC$ (RHS)
(d) $\triangle ABC \,|||\, \triangle ACD$ (SAS)
2(a) $\triangle ABC \,|||\, \triangle DEF$ (AA), $x = 4\frac{4}{5}$
(b) $\triangle GHI \,|||\, \triangle LKJ$ (RHS), $x = 9$
(c) $\triangle QRS \,|||\, \triangle UTV$ (SAS), $x = 61$
(d) $\triangle LMN \,|||\, \triangle LPM$ (AA), $x = 18$
3(a) $\triangle ABC \,|||\, \triangle FDE$ (SSS), $\theta = 67°$
(b) $\triangle XYZ \,|||\, \triangle XVW$ (SAS), $\theta = 86°$
$VW \parallel ZY$ because alternate angles are equal.
(c) $\triangle PQR \,|||\, \triangle PRS$ (SSS), $\theta = 52°$
(d) $\triangle PQR \,|||\, \triangle HIG$ (RHS), $\theta = 71°$
4(a) SAS **(b)** AA **(c)** RHS **(d)** AA
5(a) 64 metres. Use the AA similarity test.
(b) $5\,\text{cm}$, $15\,\text{cm}^2$, $15\,\text{cm}^3$ **(c)** $2 \div 4^{\frac{1}{3}} \doteqdot 1{\cdot}26\,\text{cm}$
6(a) SSS. Alternate angles $\angle BAC$ and $\angle ACD$ are equal. **(b)** AA, $ON = 21$, $PN = 17$ **(c)** SAS, trapezium with $AB \parallel KL$ (alternate angles $\angle BAL$ and $\angle ALK$ are equal) **(d)** AA, $AB = 16$, $FB = 7$
(e) AA, $FQ = 6$, $GQ = 8$, $PQ = 3\sqrt{5}$, $RQ = 4\sqrt{5}$
(f) AA, $RL = 6$
7(a) $\frac{1}{2}ab\sin C$ **(b)** $\frac{1}{2}k^2ab\sin C$
10(a) AA, $AD = 15$, $DC = 20$, $BC = 16$
(b) $AM = 12$, $BM = 16$, $DM = 9$
12(a) $BD = \frac{a^2}{c}$ **(b)** $AD = \frac{b^2}{c}$
13(a) $\sin^2\alpha$ **(b)** $\cos^2\alpha$
14(a) AA, $\frac{c^2}{b}$ **(b)** AA
15(a) Yes, the similarity factor is the ratio of their side lengths. **(b)** No, the ratio of side lengths may differ in the two rectangles. **(c)** No, the ratio of diagonals may differ in the two rhombuses. **(d)** Yes, the similarity factor is the ratio of their side lengths. **(e)** No, the ratio of leg to base may differ in the two triangles. **(f)** Yes, the similarity factor is the ratio of their radii. **(g)** Yes, the similarity factor is the ratio of their focal lengths. **(h)** Yes, the similarity factor is the ratio of their side lengths.
16(a) AA **(b)** $\frac{1}{2}(\sqrt{5}-1)$ **(c)** $\cos 72° = \frac{1}{4}(\sqrt{5}-1)$
17(a) Properties of a parallelogram: opposite sides are equal — used twice. **(b)** SAS
(c) $\triangle QPB \,|||\, \triangle QCA$ (SAS)
18(a)(i) AA **(ii)** matching sides of similar triangles
(iii) Since $ad = bc$, it follows that
$\frac{1}{2}ad\sin\theta = \frac{1}{2}bc\sin\theta$. **(b)(ii)** $\frac{27\sqrt{15}}{4}$
19(a) AA **(b)** $k : \ell$ **(d)** multiples of 10
20(a) AA **(b)** similar triangles in the ratio of $2:1$
(c) Square part (b), then use Pythagoras' theorem and the given data.
21(a) SAS **(d)** The diagonals of a parallelogram bisect each other.
22(a) SAS
23(a) $3:4:5$ **(b)** $\sqrt{\sqrt{5}-1} : \sqrt{2} : \sqrt{\sqrt{5}+1}$
$= 2 : \sqrt{2(\sqrt{5}+1)} : \sqrt{5}+1$

ISBN: 978-1-107-61604-2

24(a) Reflections preserve distances.
(b) SSS **(c)** AA
25(a) SAS **(c)** circumcircle of $\triangle CDO$

Exercise 8I (Page 340)

1(a) $x = 7\frac{1}{2}$ **(b)** $x = 11$ **(c)** $x = 15$, $y = 7$
(d) $x = 5$, $y = 18$, $z = 12$
2(a) $x = 7$ **(b)** $x = 5$ **(c)** $x = 2$ **(d)** $x = 6$
(e) $x = 12\frac{1}{2}$ **(f)** $x = 10\frac{1}{2}$ **(g)** $x = 4$ **(h)** $x = 13$
3(a) $x = 6$, $y = 4\frac{1}{2}$, $z = \frac{2}{3}$
(b) $x = 2\frac{2}{3}$, $y = 4\frac{1}{2}$, $z = 2$
(c) $x = 7\frac{1}{2}$, $y = 15$, $z = 3\frac{1}{2}$
(d) $x = 7$, $y = 6$
4(a) $x = 12$ **(b)** $x = 2$ **(c)** $x = 1$, $y = 2\frac{1}{4}$
(d) $x = 1$, $y = 1\frac{2}{3}$
5(a) $x = 2$ **(b)** $x = 4$ **(c)** $x = 5$ **(d)** $x = 1 + \sqrt{22}$
6(a) $x = 12$, $y = 6\frac{2}{5}$, $z = 9\frac{3}{5}$ **(b)** $1 : 2$
(c) parallelogram, $1 : 2$
7(a) SAS
(b) $PQ \parallel BC$ (corresponding angles are equal)
8(a) AA
9(a) SAS **(b)** Test for a parallelogram: a pair of opposite sides are equal and parallel.
11(a) A line parallel to the base divides the other two sides in the same ratio. Since $AB = AC$, it follows that $DB = EC$. **(b)** SAS
12(a) EG divides two sides of $\triangle DFC$ in half and hence is parallel to AC.
(b) $FC = 2 \times EG = 2 \times EB$
13(b) 6 cm
15(a) The opposite triangles formed by the wires and the poles are similar, with ratio $2 : 3$. Now using horizontal intercepts, the crossover point is 6 metres above the ground. The height is unchanged when the distance apart changes.
(b) The ladders reach 3·2 metres and 1·8 metres respectively up the wall. The opposite triangles formed by the ladders and the walls have similarity ratio $9 : 16$. The crossover point is 1·152 metres above the ground.
16(a) The base angles are equal.
18 Q describes a circle with centre B and radius $\frac{R(k+\ell)}{k}$.

Chapter Nine

Exercise 9A (Page 349)

1(a) $OA = OB$ (radii) **(b)** $OF = OG$ (radii)
(c) All sides are equal, being radii.
(d) They subtend equal angles at the centre.
(e) The diagonals bisect each other.
(f) The diagonals are equal and bisect each other.
2(a) $\alpha = 35^\circ$, $\beta = 10^\circ$, $\gamma = 45^\circ$
(b) $\alpha = 100^\circ$, $\beta = 120^\circ$, $\gamma = 20^\circ$ **(c)** $\alpha = 40^\circ$
(d) $\alpha = 30^\circ$ **(e)** $\alpha = 80^\circ$, $\beta = 40^\circ$
(f) $\alpha = 100^\circ$, $\beta = 27\frac{1}{2}^\circ$ **(g)** $\alpha = 50^\circ$, $\beta = 65^\circ$, both arcs and both chords subtend 100° at the centre. **(h)** $\alpha = \beta = \gamma = 110^\circ$, $\delta = 30^\circ$, both arcs subtend 220° at the centre, both chords subtend 140° at the centre.
3(a) 13 **(b)** $4\sqrt{10}$, $\frac{2}{11}\sqrt{10}$ **(c)** $\sqrt{51}$, $2\sqrt{51}$, 0·7
4(b) The perpendicular bisector of AB is a diameter, so its midpoint is the centre.
6(a) SSS test **(b)** matching angles **(c)** Matching altitudes of congruent triangles are equal.
7(a) SAS test **(b)** matching sides
8(a) RHS test **(b)** RHS test **(c)** Using matching lengths, $AB = AM + MB = XN + NY = XY$.
9 $5\sqrt{15} + 10\sqrt{3}$ or $5\sqrt{15} - 10\sqrt{3}$
10(a) Use exterior angles at Q, then at P. **(c)** The alternate angles $\angle ODA$ and $\angle DAP$ are equal.
11(a) $\triangle OAF \equiv \triangle OBG$ (AAS). Notice that $\triangle OFG$ and $\triangle OAG$ are each isosceles, with equal base angles. **(b)** $\triangle AOF \equiv \triangle BOF$ (AAS)
(c) $PZ = ZQ$, so $FO = OG$ (intercepts).
12(a) $\triangle FGJ \equiv \triangle KJG$ (SSS), so $\angle MGJ = \angle MJG$. **(b)** $\triangle PAB \equiv \triangle QAB$ (SSS), arc BPA = arc BQA.
(c) arc SP = arc SQ, $\triangle PST \equiv \triangle QST$ (SSS), so ST bisects the apex angle of the isosceles $\triangle PSQ$.
13(a) SSS test **(b)** SAS test **(d)** $OAPB$ is a rhombus if and only if the circles have equal radii.
14(a) $\triangle OPA$ and $\triangle OPB$ are both equilateral.
(b) $\sqrt{3} : 1$ **(c)** $4\pi - 3\sqrt{3} : 6\pi$
15(a) $\triangle CAO \equiv \triangle CBO$ (SSS)
(b) $\triangle CAM \equiv \triangle CBM$ (SAS or AAS)
16 $x^2 + h^2 = 1$ and $9x^2 + h^2 = 4$, $x = \frac{1}{4}\sqrt{6}$, $h = \frac{1}{4}\sqrt{10}$ **(a)** $\frac{3}{2}\sqrt{6}$ **(b)** $\frac{1}{4}\sqrt{10}$
17(a) Use the cosine rule. **(b)** Use simple trigonometry. **(c)** $\cos\theta = 1 - 2\sin^2 \frac{1}{2}\theta$

ISBN: 978-1-107-61604-2 Cambridge University Press

18(a) $(2,4)$, $2\sqrt{5}$ **(b)** $(2,5\frac{5}{6})$, $6\frac{1}{6}$ **(c)** $(2,\frac{2}{3}\sqrt{3})$ or $(2,-\frac{2}{3}\sqrt{3})$, $\frac{4}{3}\sqrt{3}$ **(d)** $(2\frac{1}{2},2\frac{1}{2})$, $\frac{5}{2}\sqrt{2}$
19 The perpendicular distance of the chord from the centre remains constant. $\frac{1}{4}(4-\lambda^2):1$
20(a) $\lambda=\frac{3}{2\pi}\sqrt{3}$, $\frac{2}{\pi}\sqrt{2}$, $\frac{3}{\pi}$, $\frac{4}{\pi}\sqrt{2-\sqrt{2}}$
$\mu=\frac{3}{4\pi}\sqrt{3}$, $\frac{2}{\pi}$, $\lambda=\frac{3}{2\pi}\sqrt{3}$, $\frac{2}{\pi}\sqrt{2}$
(b) $\mu=\frac{n}{\pi}\sin\frac{\pi}{n}$, $\mu=\frac{n}{2\pi}\sin\frac{2\pi}{n}$, $\lim\limits_{x\to 0}\frac{\sin x}{x}=1$
(i) $n=82$ **(ii)** $n=41$

Exercise 9B (Page 354)

1(a) $\alpha=90°$, $\beta=55°$ **(b)** $\alpha=80°$, $\beta=40°$ **(c)** $\alpha=55°$, $\beta=35°$ **(d)** $\alpha=40°$, $\beta=20°$ **(e)** $\alpha=\beta=\gamma=22°$ **(f)** $\alpha=90°$, $\beta=40°$, $\gamma=20°$ **(g)** $\alpha=140°$, $\beta=20°$ **(h)** $\alpha=70°$, $\beta=220°$, $\gamma=110°$ **(i)** $\alpha=90°$, $\beta=40°$, $\gamma=60°$, $\delta=30°$ **(j)** $\alpha=70°$, $\beta=20°$ **(k)** $\alpha=18°$, $\beta=36°$, $\gamma=54°$ **(l)** $\alpha=22°$, $\beta=30°$, $\gamma=38°$ **(m)** $\alpha=45°$, $\beta=70°$ **(n)** $\alpha=90°$, $\beta=55°$, $\gamma=35°$ **(o)** $\alpha=110°$, $\beta=140°$, $\gamma=70°$ **(p)** $\alpha=100°$, $\beta=140°$, $\gamma=70°$
2 In all parts, the interval named as diameter subtends a right angle at the other two points on the circle. **(a)** $ACBD$, diameter AB **(b)** $FGHI$, diameter FI **(c)** $OMXN$, diameter OX **(d)** $OXFY$, diameter OF
3 The photographer must stand on the circle with the building as diameter, so that the midpoint of the building is the centre of the circle.
6(a)(i) exterior angles, base angles of isosceles triangles **(b)** If the diagonals of a quadrilateral are equal and bisect each other, then it is a rectangle. **(c)** $OM\perp AB$ because OM bisects the chord AP. $OM\parallel BP$ by intercepts. Hence $\angle P=\angle OMA=90°$ (corresponding angles, $OM\parallel BP$).
7(a)(i) intercepts **(ii)** SAS **(iii)** $AO=PO$ by matching angles. **(b)(i)** It is an angle in a semicircle. **(ii)** The corresponding angles are equal. **(iii)** Parallel lines through the common point B are the same line.
8(a) $\angle B=\alpha$ (opposite angles of parallelogram), reflex $\angle O=360°-\alpha$, reflex $\angle O=2\times\angle B$.
(b) $\alpha=32°$, $\beta=48°$, $\gamma=66°$
(c) $\alpha=56°$, $\beta=34°$, $\angle PAQ=34°$, so $AP\parallel BQ$ (alternate angles are equal).
9(a) $\triangle BCA\equiv\triangle BCP$ (RHS) **(b)** Use intercepts. **(c)** Opposite sides of a parallelogram are equal.
10(a) $\alpha=30°$ (angles on the same arc BC), $\beta=60°$ (base angles of isosceles $\triangle OBC$), $\gamma=30°$ (angle sum of $\triangle MOC$), $\triangle OCM\equiv\triangle BCM$ (AAS)
(b) $\alpha=60°$ (equilateral $\triangle OFG$), $\beta=30°$ (angles on the same arc FG), $\gamma=90°$ ($\triangle OPF\equiv\triangle GPF$, SSS)
(c) $PQRM$ is a parallelogram because the diagonals bisect each other, and is a rhombus because the adjacent sides OP and OR are equal. Hence $\alpha=90°$ (diagonals of a rhombus are perpendicular). Also, $OR=QR$ ($\triangle OMR\equiv\triangle QMR$, SAS), hence $\beta=60°$ (equilateral $\triangle OQR$).
11(a) $\alpha=115°$, $\beta=75°$ **(b)** $\alpha=110°$, $\beta=35°$ **(c)** $\alpha=120°$, $\beta=30°$, $\gamma=60°$ **(d)** $\alpha=40°$, $\beta=70°$, $\gamma=110°$, $\delta=70°$
12(a)(i) Each is an angle in a semicircle.
(ii) $\angle FBG=180°$ **(iii)** $\triangle AFB\equiv\triangle AGB$ (RHS)
(b)(i) SSS **(ii)** Use matching angles.
(iii) angles standing on the same arc AB
(iv) Use angle sums of $\triangle BPQ$ and $\triangle BOZ$.
13(a)(i) The intervals FH and HG subtend right angles at M, the interval GF subtends a right angle at H. **(ii)** Use Pythagoras' theorem.
(b)(i) angles on the same arc BD **(ii)** angle sum of $\triangle MCD$ **(iii)** BD subtends $90°$ at O and M.
14(b) $\frac{\pi}{2}$ **(c)** $\lambda=2+\sqrt{3}$ (when $\lambda>1$) or $\lambda=2-\sqrt{3}$ (when $0<\lambda<1$)
16 The rate of turning of the binoculars is half the angular velocity of the horse.

Exercise 9C (Page 359)

1(a) $\alpha=\beta=25°$ **(b)** $\alpha=90°$, $\beta=110°$ **(c)** $\alpha=15°$, $\beta=100°$ **(d)** $\alpha=25°$, $\beta=65°$ **(e)** $\alpha=40°$, $\beta=85°$ **(f)** $\alpha=142°$, $\beta=95°$ **(g)** $\alpha=60°$, $\beta=70°$ **(h)** $\alpha=100°$, $\beta=110°$, $\gamma=80°$ **(i)** $\alpha=35°$, $\beta=30°$, $\gamma=30°$ **(j)** $\alpha=25°$, $\beta=32°$, $\gamma=57°$ **(k)** $\alpha=50°$, $\beta=130°$, $\gamma=25°$ **(l)** $\alpha=32°$, $\beta=34°$, $\gamma=34°$
2(a) $\alpha=72°$, $\beta=58°$ **(b)** $\alpha=22°$, $\beta=52°$ **(c)** $\alpha=100°$, $\beta=100°$, $\gamma=100°$ **(d)** $\alpha=76°$, $\beta=34°$, $\gamma=76°$ **(e)** $\alpha=64°$, $\beta=36°$, $\gamma=40°$ **(f)** $\alpha=66°$, $\beta=114°$ **(g)** $\alpha=60°$, $\beta=25\frac{5}{7}°$ **(h)** $\alpha=20°$, $\beta=50°$, $\gamma=44°$
3 If two angles are supplementary, then the sines of the two angles are equal.

ISBN: 978-1-107-61604-2 Cambridge University Press

4(a) Using exterior angles of the cyclic quadrilateral, $\angle ECD = \alpha$ and $\angle EDC = \alpha$. Then the corresponding angles $\angle ECD$ and $\angle A$ are equal, and the base angles of $\triangle BCD$ are equal. **(b)** Use angles on the same arcs BD and AC, and alternate angles. Then $\triangle AMB$ and $\triangle CMD$ are isosceles. **(c)** angles on the same arc AB, angles on equal arcs AB and AD **(d)** $\angle C = 60°$ (equilateral $\triangle BCD$), $\angle A = 120°$ (opposite angles of cyclic quadrilateral), $\angle ABC = 30°$ (angle sum of isosceles $\triangle ABD$)
5(a) angles on the same arc DC, angles on the same arc BC **(b)** Use the angle sum of $\triangle BCD$.
6(a) opposite angles of cyclic quadrilateral
(b) angles on the same arc AB, supplements of equal angles $\angle APB$ and $\angle AQB$, angle sum of quadrilateral $PMQN$ **(c)** angles on equal arcs AB and AC, alternate angles are equal **(d)** angles on the same arc PB, angle sums of $\triangle AMP$ and $\triangle QBP$ ($\angle QBP = 90°$, being an angle in a semicircle) **(e)** angles on the same arc BY, angle sums of $\triangle XMB$ and $\triangle AYB$ **(f)** exterior angle of cyclic quadrilateral $BADE$, $\angle DAC = \angle DEC = \angle XEY$
7(a) Exterior angle of cyclic quadrilateral, $\angle C = 180° - \alpha$ and so the co-interior angles are supplementary. **(b)** Both are $90°$(angles in semicircles), they add to a straight angle. **(c)** Both are $90°$(opposite angle of cyclic quadrilateral, and angle in a semicircle), they add to a straight angle.
(d) $\angle FGP = 90° = \angle GFQ$. If the radii are equal, $\triangle FGP \equiv \triangle GFQ$ (RHS). **(e)** Angles on the same arc FA, opposite angles are equal. **(f)** Angles on the same arc AB. Angles on the same arc FP, vertically opposite angles at B, angles on the same arc GQ.
8(a) 10 **(b)** $5\sqrt{3}$ **(c)** $13\frac{1}{3}$ **(d)** $13\frac{1}{2}$
9(a) opposite interior angle of cyclic quadrilateral $ABCD$ **(d)** The bisector of the angle formed by the other two sides is perpendicular to the pair of parallel sides. (What if both pairs of opposite sides are parallel?)
10(a) Each side is an equal chord of the circumcircle, and so subtends the same angle at P. Since $\angle APG = 135°$, each angle is $22\frac{1}{2}° = \frac{1}{8} \times 180°$.
14(a) OC subtends right angles at P and Q, AB subtends right angles at P and Q. **(b)** OQ subtends equal angles at C and P, AQ subtends equal angles at P and B. **(c)** Use angle sums of $\triangle OQC$ and $\triangle ORB$.
16(a) α stands on the fixed arc KL.
(c) $\dfrac{dy}{d\theta} = \dfrac{a}{\sin\alpha}\Big(\cos\theta + \cos(\theta+\alpha)\Big)$ **(d)** $a\sec\frac{1}{2}\alpha$
18(a) angles on the same arc CX
20(a) SAS similarity test **(b)** $\angle AOM = \angle PMO$ (matching angles), hence $AO \parallel MP$ (alternate angles are equal), hence $AO \perp BC$ (corresponding angles, $AO \parallel MP$). **(c)** The same construction can be done from B and the midpoint of AC, and again from C and the midpoint of AB. Hence O lies on all three altitudes.

Exercise **9D** (Page 366)

1(a) Opposite angles are supplementary.
(b) Exterior angle equals opposite interior angle.
(c) Arc AD subtends equal angles at B and C.
2(a) Arc BC subtends equal angles at P and Q.
(b) $\angle A = 40°$, $\alpha = 70°$, exterior angle of $PQCB$ equals opposite interior angle. **(c)** $\angle AJK = \beta$ (corresponding angles, $JK \parallel BC$), $\angle AKJ = \beta$ (base angles of isosceles $\triangle AJK$), exterior angle of $JKCB$ equals opposite interior angle.
(d) $\angle BDE = 180° - \angle A$ (opposite angles of parallelogram, angles on a straight line), $\angle E = \angle BDE$ (base angles of isosceles $\triangle BDE$), opposite angles of $ABEC$ are supplementary.
(e) $\angle AMO = \angle ONA = 90°$ (intervals joining centre to midpoint of chord) **(f)** $\angle A$ and $\angle C$ are supplementary and $\angle BGH = \angle C$. **(g)** $\angle A$ and $\angle Q$ are supplementary and $\angle A = \angle PCD$.
(h) $BP \perp AQ$ (angle in a semicircle)
3(a) BC subtends equal angles at E and D, so $BEDC$ is cyclic. The angle equalities then follow by angles on the same arc ED and exterior angle of the cyclic quadrilateral. **(b)** $\angle BMD = 2\theta$, because the opposite interior angles in $\triangle AMB$ are both θ. Now BD subtends equal angles of 2θ at O and M.
4(a) The opposite angles are supplementary.
(b) Since $\angle A + \angle C = \angle B + \angle D$ and the sum of the angles is $360°$, it follows that $\angle A + \angle C = 180°$.
9(a) intercepts in $\triangle MAB$ and $\triangle MBC$ **(b)** $\angle A + \angle C = 180°$ (opposite angles of cyclic quadrilateral $MABC$), and $\angle QPM = \angle A$ and $\angle QRM = \angle C$ (corresponding angles on parallel lines), so

ISBN: 978-1-107-61604-2

$\angle QPM + \angle QRM = \angle C$. Hence $MPQR$ is cyclic (opposite angles are supplementary).

11(a) $\angle P$ is subtended by the fixed chord AB.
(b) PM subtends right angles at X and Y.
(c) The locus of M is an arc of a circle through AB.

13(a) The circle has radius c. **(b)** Opposite angles are right angles. **(c)** angles on the same arc BG, and exterior opposite angle of cyclic quadrilateral **(d)** angles on the same arc FC **(f)** The point C now lies outside the circle, but $CMFB$ is still cyclic. Being opposite angles of a cyclic quadrilateral, $\angle FBC$ and $\angle FMC$ are now supplementary, so $\tan\angle FBC = -\tan\angle FMC$.

15(a) the midpoint of the diameter AB
(b) The interval XY is a chord of the circle with diameter AB, and subtends a constant angle at A.
(c) a circle with centre the midpoint of AB

16(a) Use the circles $ROPB$, $QOPC$ and $RQPB$.
(b) $\angle RAO = \angle RQO = \angle PQO = \angle PCO = \beta$, $\angle QAO = \angle QRO = \angle PRO = \angle PBO = \gamma$
(c) $\triangle ABC$ and $\triangle PQR$ have the same angle sum $2\alpha + 2\beta + 2\gamma$. **(d)** The arc RQ subtends equal angles at P and L. Since BC is the diameter of the circle $BRQC$, RQ subtends 2α at the midpoint of BC. The interval RQ can only subtend 2α at two points on BC (the locus of such points forms a circle — these points are P and L. **(e)** $NHLP$ is a cyclic quadrilateral. Thus $\angle LHC = 90° + \alpha$, so $\angle HLC = \gamma = \angle OBL$, so $OB \parallel HL$, so H bisects OC.

17(a) When P is inside the square, $\angle APB = 135°$. When P is outside the square, $\angle APB = 45°$. **(b)** Construct the square $PQAB$ adjacent to the square $ABCD$, and construct the circle $PQAB$. Then any angle at the circumference subtended by AB is 45° if the point is outside $ABCD$, and 135° if it is inside. Construct X and Y dividing AB internally and externally respectively in the ratio $1 : 2$, and construct the circle with diameter XY. Then the two points of intersection of the circles are the two positions of P.

Exercise 9E (Page 372)

1(a) $\alpha = 36°$ **(b)** $\alpha = 41°$, $\beta = 49°$ **(c)** $\alpha = 45°$, $\beta = 67\frac{1}{2}°$ **(d)** $\alpha = 55°$ **(e)** $\alpha = 44°$ **(f)** $\alpha = 54°$ **(g)** $\alpha = 50°$ **(h)** $\alpha = 100°$
2(a) $x = 12\,\text{cm}$ **(b)** $x = 6$ **(c)** $x = 4\sqrt{2} - 4$ **(d)** $x = 6$ **(e)** $x = 7$ **(f)** $x = 4$ **(g)** $x = 12$ **(h)** $x = 9$
3(a) $\alpha = 60°$, $\beta = 30°$, $\gamma = 30°$ **(b)** $\alpha = 140°$, $\beta = 80°$ **(c)** $\alpha = 130°$, $\beta = 115°$, $\gamma = 80°$
(d) $\alpha = 100°$, $\beta = 30°$
7(a) $r\sqrt{2}$ **(b)** $2r$
8(c) 28
9(b) $r(\sqrt{3} + 2)$
10(a) $x = 2$ **(b)** $x = 10$ **(c)** $x = 5\frac{1}{3}$
(d) $x = \frac{1}{2}(7 + \sqrt{17})$ or $x = \frac{1}{2}(7 - \sqrt{17})$
14(b) $a = \ell + m$, $b = \dfrac{m^2 + \ell^2}{m - \ell}$, $c = \dfrac{2\ell m}{m - \ell}$
17(a) $2\sqrt{15}\,\text{cm}$ **(b)** $14\sqrt{2}\,\text{cm}$ **(c)** $\sqrt{143}\,\text{cm}$, $3\sqrt{7}\,\text{cm}$
19(a) AAS test
20(a) $TP = 17\frac{1}{7}$, $BP = 12\frac{6}{7}$
(b) $21\sqrt{3} - 36 - \frac{21\pi}{2} + 6\pi\sqrt{3} \doteqdot 0{\cdot}0347\,\text{m}^2$
22(a) $r^2(\sqrt{3} - \frac{\pi}{2})$, $r(\frac{2}{3}\sqrt{3} - 1)$ **(b)** $\frac{2}{3}r(3 + \sqrt{6})$

Exercise 9F (Page 378)

2(a) $\alpha = 70°$, $\beta = 50°$ **(b)** $\alpha = \beta = \gamma = 65°$ **(c)** $\alpha = \beta = \gamma = 60°$ **(d)** $\alpha = \beta = 70°, \gamma = 40°$ **(e)** $\alpha = 68°$, $\beta = 50°$ **(f)** $\alpha = 70°$, $\beta = 55°$ **(g)** $\alpha = \beta = 44°, \gamma = 92°$ **(h)** $\alpha = 50°$, $\beta = 40°$
(i) $\alpha = 55°$, $\beta = 66°, \gamma = 59°$
(j) $\alpha = 50°$, $\beta = 55°, \gamma = 50°$, $\delta = 25°$
(k) $\alpha = \beta = 30°$ **(l)** $\alpha = 85°$, $\beta = \gamma = 25°$
3(a) $\alpha = \theta$, $\beta = \theta$, $\gamma = 180° - 3\theta$
(b) $\alpha = \theta$, $\beta = 180° - 2\theta$, $\gamma = 2\theta$
(c) $\alpha = 90° - \frac{1}{2}\theta$, $\beta = 90° - \frac{1}{2}\theta$
(d) $\alpha = \theta$, $\beta = \theta$, $\gamma = \theta$, $\delta = 180° - 2\theta$
4(a) $\angle A = \alpha$ (alternate segment theorem)
(b) alternate angles, $PQ \parallel AB$, and alternate segment theorem **(c)** alternate segment theorem and base angles of isosceles triangle **(d)** alternate segment theorem and base angles of isosceles triangle
5(a) $\alpha = 40°$, $\beta = 30°$
(b) $\alpha = 65°$, $\beta = 50°$, $\gamma = 25°$
6(a) $\angle OAB = 90° - \alpha$ (radius and tangent)
(b) $\angle OBA = 90° - \alpha$ ($OA = OB$, radii), $\angle AOB = 2\alpha$ (angles in the triangle OAB)
(c) The angle at the centre is twice the angle at the circumference.

ISBN: 978-1-107-61604-2

7(a) $\angle P = \alpha$, using the alternate segment theorem and vertically opposite angles.
(b) Using alternate angles and the alternate segment theorem, $\angle BTY = \angle QTX$. But XTY is a straight line, so by the converse of vertically opposite angles, QTB is a straight line.
8(a)(i) $\angle AST = \beta$ and $\angle BST = \alpha$ (alternate segment theorem) **(ii)** AA similarity test
(iii) matching sides
(b) The argument is similar to that in part (a).
9(a) $\angle CTS = \alpha$ (alternate segment theorem), so $\angle CBS = 180° - \alpha$ (cyclic quadrilateral $CBST$), so $CB \parallel TA$ (co-interior angles are supplementary). **(b)** $\angle BST = \alpha = \angle TBQ$ (alternate segment theorem), making the alternate angles equal.
10(a) $\angle ETA = \beta$ (alternate segment theorem), and $\angle TGA = \alpha + \beta$ (exterior angle of $\triangle TGB$)
(b) $\angle ABQ = 180° - \alpha$ (opposite angles of cyclic quadrilateral $PABQ$), $\angle ATS = \alpha$ (alternate angles, $PQ \parallel ST$) and $\angle ABT = \alpha$ (alternate segment theorem), so $\angle ABQ$ and $\angle ABT$ are supplementary.
12(a) $\angle XSU = \angle SUX = \alpha$, $\angle XTU = \angle XUT = \beta$, using base angles of isosceles triangles and the alternate segment theorem. Hence $\alpha + \beta = 90°$ (angle sum of $\triangle SUT$). **(b)** AS and BT are diameters, because they both subtend right angles at U. They are parallel because of the equal alternate angles $\angle ASB$ and $\angle B$. **(c)** If AB were a tangent to either circle, then it would be parallel to AT, so $ABTS$ would be a rectangle.
14(a) Because the two circles have equal radii, the chord AB subtends equal angles at the circumferences of the two circles. **(b)** $\triangle BPQ$ is isosceles, so $\triangle PMB \equiv \triangle QMB$ (SSS). **(c)** Since AB always subtends a right angle at M, the locus is the circle with diameter AB. **(d)** The proof is the same, except that $\angle APB$ and $\angle AQB$ are now supplementary.

Exercise **9G** (Page 384)

1(a) $x = 9$ **(b)** $x = 2\sqrt{7}$ **(c)** $x = 3$ **(d)** $x = 9\sqrt{2}$
(e) $x = 15$ **(f)** $x = 3$ **(g)** $x = 4$ **(h)** $x = 6$
2(a)(i) The perpendicular to a chord from the centre bisects the chord. **(ii)** $x = 4$ **(iii)** 40 **(b)(i)** The line through the centre bisecting a chord is perpendicular to the chord. **(ii)** $x = 7\frac{1}{4}$ **(iii)** $72\frac{1}{2}$
3(a) tangent and secant from an external point
(b) $SK^2 = KA \times KB = TK^2$
4(b) the SAS similarity test **(c)** matching angles
(d) The interval BC subtends equal angles at A and D.
5(a) the SAS similarity test **(b)** matching angles
(c) The external angle at D equals the internal angle at A.
6(a) The perpendicular from the centre to a chord bisects the chord. **(b)** the intersecting chord theorem **(c)** $a + b$ is the diameter.
(d) The semichord x is less than the radius.
7(a) tangent and secant from A **(b)** Pythagoras' theorem in $\triangle ABT$ **(c)** angle in the semicircle BMT **(d)** M **(e)** T **(f)** secant and tangent from B to new circle **(g)** AA similarity test **(h)** matching sides, $\triangle ATM \parallel\!| \triangle TBM$ **(i)** matching sides, $\triangle ATM \parallel\!| \triangle TBM$
8(a) AA similarity test **(b)** matching sides
(c) $OP = OM + MP$ **(d)** Use Pythagoras' theorem in $\triangle OMT$.
9(a)(i) alternate segment theorem
(ii) $\angle OTF' = \alpha$ $(OT = OF')$, $\angle FTM = \alpha$ because $TM \perp F'F$ and $F'T \perp FT$.
(iii) $F'T \perp FT$ **(b)(i)** $TM \perp F'F$ and $F'T \perp FT$
(ii) $\angle F'TO = \alpha$ $(OT = OF')$, $2\alpha + \angle OTM = 90°$
(iii) TP is perpendicular to the radius OT.
10(a) The result still holds, because $\sin(180° - \theta) = \sin\theta$.
18(a) $\triangle BMA$, $\triangle CMB$, $\triangle CBA$, $\triangle ADC$, $\triangle BAF$
(c) $AG : GB = (r^2 - 1) : r^2$, $r = \sqrt{2}$ **(d)** Yes. Choose $r = \sqrt{2}$. **(e)** No. Were $DFGB$ a circle, then $\angle FDG = \angle FBG$. But $\angle ADG = \angle FBG$, so this is impossible unless G and B coincide.

ISBN: 978-1-107-61604-2

Chapter Ten

Exercise 10A (Page 395)

1(a) $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** 0
2(a) $\frac{1}{6}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{3}$
3(a) $\frac{4}{9}$ **(b)** $\frac{5}{9}$ **(c)** $\frac{11}{18}$
4(a) $\frac{4}{9}$ **(b)** $\frac{5}{9}$ **(c)** $\frac{11}{18}$ **(d)** $\frac{7}{18}$ **(e)** $\frac{1}{3}$ **(f)** $\frac{1}{6}$
5(a) $\frac{1}{26}$ **(b)** $\frac{5}{26}$ **(c)** $\frac{21}{26}$ **(d)** 0 **(e)** $\frac{3}{26}$ **(f)** $\frac{5}{26}$
6(a) $\frac{1}{20}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{2}{5}$ **(f)** $\frac{1}{5}$ **(g)** $\frac{1}{4}$ **(h)** 0 **(i)** 1
7(a) $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{13}$ **(d)** $\frac{1}{52}$ **(e)** $\frac{1}{4}$ **(f)** $\frac{3}{13}$ **(g)** $\frac{1}{2}$ **(h)** $\frac{1}{13}$ **(i)** $\frac{3}{13}$ (counting an ace as a one)
8(a) $\frac{1}{15}$ **(b)** $\frac{7}{150}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{4}{25}$ **(e)** $\frac{1}{75}$ **(f)** $\frac{17}{50}$
9(a) $\frac{1}{5}$ **(b)** $\frac{3}{40}$ **(c)** $\frac{9}{20}$ **(d)** $\frac{7}{100}$ **(e)** $\frac{7}{50}$ **(f)** $\frac{1}{200}$
10(a) $\frac{3}{4}$ **(b)** $\frac{1}{4}$
11 AB, AC, AD, BC, BD, CD **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{3}$ **(d)** $\frac{1}{6}$
12 HH, HT, TH, TT **(a)** $\frac{1}{4}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{4}$
13(a) $\frac{1}{4}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{4}$
14(a) 23, 32, 28, 82, 29, 92, 38, 83, 39, 93, 89, 98 **(b)(i)** $\frac{1}{12}$ **(ii)** $\frac{1}{2}$ **(iii)** $\frac{1}{2}$ **(iv)** $\frac{1}{6}$ **(v)** $\frac{1}{4}$ **(vi)** 0
15(a) The captain is listed first and the vice-captain second. AB, AC, AD, AE, BC, BD, BE, CD, CE, DE, BA, CA, DA, EA, CB, DB, EB, DC, EC, ED **(b)(i)** $\frac{1}{20}$ **(ii)** $\frac{2}{5}$ **(iii)** $\frac{3}{5}$ **(iv)** $\frac{1}{5}$
16(a)(i) $\frac{2}{5}$ **(ii)** $\frac{3}{5}$ **(iii)** $\frac{1}{5}$ **(b)** $\frac{9}{25}$, $\frac{16}{25}$, $\frac{1}{5}$
17 11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66 **(a)** $\frac{1}{6}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{36}$ **(d)** $\frac{1}{6}$ **(e)** $\frac{1}{6}$ **(f)** $\frac{1}{4}$ **(g)** $\frac{11}{36}$ **(h)** $\frac{4}{9}$ **(i)** $\frac{5}{36}$ **(j)** $\frac{1}{6}$
18 187 or 188
19(a) $\frac{227}{300}$ **(b)** Since a probability of $\frac{3}{4}$ would predict about 225 heads and $\frac{7}{9}$ would predict about 233 heads, both these fractions seem consistent with the experiment. Probabilities of $\frac{1}{2}$ and $\frac{5}{8}$ can safely be excluded.
20 HHH, HHT, HTH, HTT, THH, THT, TTH, TTT **(a)** $\frac{1}{8}$ **(b)** $\frac{3}{8}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{2}$ **(e)** $\frac{1}{2}$ **(f)** $\frac{1}{2}$
21(a)(i) $\frac{1}{4}$ **(ii)** $\frac{1}{4}$ **(iii)** $\frac{1}{2}$ **(b)(i)** $\frac{1}{8}$ **(ii)** $\frac{3}{8}$ **(iii)** $\frac{1}{2}$
22(a) $\frac{1}{16}$ **(b)** $\frac{1}{4}$ **(c)** $\frac{11}{16}$ **(d)** $\frac{5}{16}$ **(e)** $\frac{3}{8}$ **(f)** $\frac{5}{16}$
23(a) $\frac{2}{9}$ **(b)** $\frac{\pi}{18}$
24(a) The argument is invalid, because on any one day the two outcomes are not equally likely. The argument really can't be corrected.
(b) The argument is invalid. One team may be significantly better than the other, the game may be played in conditions that suit one particular team, and so on. Even when the teams are evenly matched, the high-scoring nature of this game makes a draw an unlikely event. The three outcomes are not equally likely. The argument really can't be corrected.
(c) The argument is invalid, because we would presume that Peter has some knowledge of the subject, and is therefore more likely to choose one answer than another. The argument would be valid if the questions were answered at random.
(d) The argument is only valid if there are equal numbers of red, white and black beads, otherwise the three outcomes are not equally likely.
(e) This argument is valid. He is as likely to pick the actual loser of the semi-final as he is to pick any of the other three players.
25(a) $\dfrac{1}{2^n}$ **(b)** $1 - 2^{1-n}$
26(b) Throw the needle 1000 times, say, and let S be the number of times it lies across a crack. Then $\pi \doteqdot \dfrac{2000}{S}$. **(c)** $\frac{1}{\pi}$ **(d)** $\frac{2}{3\pi}(6 - 3\sqrt{3} + \pi) \doteqdot 0{\cdot}837$

Exercise 10B (Page 400)

1(a)(i) $A \cup B = \{1, 3, 5, 7\}$, $A \cap B = \{3, 5\}$
(ii) $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A \cap B = \{4, 9\}$
(iii) $A \cup B = \{\text{h, o, b, a, r, t, i, c, e, n}\}$, $A \cap B = \{\text{h, o, b}\}$
(iv) $A \cup B = \{1, 2, 3, 5, 7, 9\}$, $A \cap B = \{3, 5, 7\}$
(b)(i) $\{2, 4, 5, 6, 8, 9\}$ **(ii)** $\{1, 2, 3, 5, 8, 10\}$
(iii) $\{7\}$ **(iv)** $\{1, 2, 3, 4, 5, 6, 8, 9, 10\}$
(v) $\{1, 3, 4, 6, 7, 9, 10\}$ **(vi)** $\{2, 5, 8\}$
2 78%
3(a) $\frac{4}{7}$ **(b)** 32
4(a) $\frac{14}{15}$ **(b)** 8
5(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{3}$ **(d)** 0 **(e)** 1 **(f)** 0 **(g)** $\frac{1}{6}$ **(h)** $\frac{2}{3}$
6(a) $\frac{1}{13}$ **(b)** $\frac{1}{13}$ **(c)** $\frac{2}{13}$ **(d)** 0 **(e)** $\frac{11}{13}$ **(f)** $\frac{1}{2}$ **(g)** $\frac{3}{13}$ **(h)** $\frac{3}{26}$ **(i)** $\frac{8}{13}$ **(j)** $\frac{5}{13}$
7(a) no **(b)(i)** $\frac{1}{2}$ **(ii)** $\frac{2}{3}$ **(iii)** $\frac{1}{3}$ **(iv)** $\frac{5}{6}$
8(a) $\frac{1}{2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{3}{4}$ **(e)** $\frac{1}{4}$ **(f)** $\frac{1}{6}$ **(g)** $\frac{1}{6}$ **(h)** $\frac{1}{36}$ **(i)** $\frac{11}{36}$ **(j)** $\frac{25}{36}$
9(a)(i) $\frac{1}{2}$ **(ii)** $\frac{2}{3}$ **(iii)** $\frac{1}{3}$ **(iv)** $\frac{1}{2}$ **(v)** $\frac{1}{2}$ **(b)(i)** $\frac{3}{5}$ **(ii)** $\frac{4}{5}$ **(iii)** $\frac{3}{5}$ **(iv)** 0 **(v)** 1 **(c)(i)** $\frac{1}{2}$ **(ii)** $\frac{2}{3}$ **(iii)** $\frac{2}{3}$ **(iv)** $\frac{1}{3}$ **(v)** $\frac{5}{6}$
10(a) $\frac{7}{15}$ **(b)** 0 **(c)** $\frac{3}{5}$ **(d)** $\frac{5}{7}$

ISBN: 978-1-107-61604-2 Cambridge University Press

11(a)(i) no **(ii)** $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{20}$, $\frac{3}{5}$ **(b)(i)** no **(ii)** $\frac{1}{2}$, $\frac{3}{10}$, $\frac{3}{20}$, $\frac{13}{20}$ **(c)(i)** yes **(ii)** $\frac{1}{4}$, $\frac{9}{20}$, 0, $\frac{7}{10}$
12 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97
(a) $\frac{1}{4}$ **(b)** $\frac{6}{25}$ **(c)** $\frac{11}{100}$ **(d)** $\frac{19}{50}$
13(a) $\frac{9}{25}$ **(b)** $\frac{7}{50}$ **(c)** $\frac{17}{50}$
14(a) 10 **(b)(i)** $\frac{4}{21}$ **(ii)** $\frac{1}{3}$
15 $\frac{1}{4}$
16(a) $\frac{7}{12}$ **(b)** $\frac{13}{60}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{7}{60}$
17 $\frac{7}{43}$
18(a) $n(A\cup B\cup C)=n(A)+n(B)+n(C)-n(A\cap B)-n(A\cap C)-n(B\cap C)+n(A\cap B\cap C)$
(b) 207 **(c)** $|A\cup B\cup C\cup D|=|A|+|B|+|C|+|D|-|A\cap B|-|A\cap C|-|A\cap D|-|B\cap C|-|B\cap D|-|C\cap D|+|A\cap B\cap C|+|A\cap B\cap D|+|A\cap C\cap D|+|B\cap C\cap D|-|A\cap B\cap C\cap D|$.
It is possible to draw a Venn diagram with four sets, but only if the fourth set is represented not by a circle, but by a complicated loop — the final diagram must have 16 regions.

Exercise **10C** (Page 406)

1(a) $\frac{1}{24}$ **(b)** $\frac{1}{28}$ **(c)** $\frac{1}{12}$ **(d)** $\frac{1}{96}$ **(e)** $\frac{1}{42}$ **(f)** $\frac{1}{336}$
2(a) $\frac{1}{12}$ **(b)** $\frac{1}{12}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{3}$
3(a) $\frac{1}{25}$ **(b)** $\frac{2}{25}$ **(c)** $\frac{3}{25}$ **(d)** $\frac{3}{25}$ **(e)** $\frac{4}{25}$ **(f)** $\frac{2}{25}$ **(g)** $\frac{1}{25}$
4(a) $\frac{15}{49}$ **(b)** $\frac{8}{49}$ **(c)** $\frac{6}{49}$
5(a) $\frac{1}{10}$ **(b)** $\frac{3}{10}$ **(c)** $\frac{3}{10}$ **(d)** $\frac{3}{10}$
6(a) $\frac{1}{7}$ **(b)** $\frac{180}{1331}$
7(a)(i) $\frac{13}{204}$ **(ii)** $\frac{1}{17}$ **(iii)** $\frac{4}{663}$ **(iv)** $\frac{1}{2652}$
(b) $\frac{1}{16}$, $\frac{1}{16}$, $\frac{1}{169}$, $\frac{1}{2704}$
8(a)(i) $\frac{2}{3}$ **(ii)** $\frac{1}{3}$ **(b)(i)** $\frac{8}{27}$ **(ii)** $\frac{1}{27}$ **(iii)** $\frac{4}{27}$
9(a) The argument is invalid, because the events ‘liking classical music’ and ‘playing a classical instrument’ are not independent. One would expect that most of those playing a classical instrument would like classical music, whereas a smaller proportion of those not playing a classical instrument would like classical music. The probability that a student does both cannot be discovered from the given data — one would have to go back and do another survey. **(b)** The argument is invalid, because the events ‘being prime’ and ‘being odd’ are not independent — two out of the three odd numbers less than 7 are prime, but only one out of the three such even numbers is prime. The correct argument is that the odd prime numbers amongst the numbers 1, 2, 3, 4, 5 and 6 are 3 and 5, hence the probability that the die shows an odd prime number is $\frac{2}{6}=\frac{1}{3}$. **(c)** The teams in the competition may not be of equal ability, and factors such as home-ground advantage may also affect the outcome of a game, hence assigning a probability of $\frac{1}{2}$ to winning each of the seven games is unjustified. Also, the outcomes of successive games are not independent — the confidence gained after winning a game may improve a team’s chances in the next one, a loss may adversely affect their chances, or a team may receive injuries in one game leading to a depleted team in the next. The argument really can’t be corrected. **(d)** This argument is valid. The coin is normal, not biased, and tossed coins do not remember their previous history, so the next toss is completely unaffected by the previous string of heads.
10 HHH, HHM, HMH, MHH, HMM, MHM, MMH, MMM **(a)** $p\,(\text{HHH})=0{\cdot}9^3=0{\cdot}729$
(b) $0{\cdot}001$ **(c)** $p\,(\text{HMM})=0{\cdot}9\times0{\cdot}1^2=0{\cdot}009$
(d) $p\,(\text{HMM})+p\,(\text{MHM})+p\,(\text{MMH})=3\times0{\cdot}009=0{\cdot}027$ **(e)** $0{\cdot}081$ **(f)** $0{\cdot}243$
11(a) $\frac{1}{36}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{4}$ **(d)** $\frac{1}{36}$ **(e)** $\frac{1}{36}$ **(f)** $\frac{1}{18}$ **(g)** $\frac{1}{12}$
(h) $\frac{1}{12}$ **(i)** $\frac{1}{6}$
12(a) $p\,(\text{CCCCC})=(\frac{1}{5})^5=\frac{1}{3125}$ **(b)** $\frac{1024}{3125}$ **(c)** $\frac{16}{3125}$
(d) $\frac{256}{3125}$ **(e)** $\frac{256}{625}$ **(f)** $\frac{4}{625}$
13(a) $\frac{1}{46\,656}$ **(b)** $\frac{5}{7776}$
14(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{2}$ **(d)** $\frac{1}{3}$
15(a) $\frac{3}{64}$ **(b)** $\frac{17}{64}$ **(c)** $\frac{5}{17}$
16(a)(i) $\frac{3}{4}$ **(ii)** $\frac{31}{32}$ **(iii)** $\frac{1023}{1024}$ **(b)** $1-\dfrac{1}{2^n}=\frac{2^n-1}{2^n}$
(c) 14
17(a) $\left(\frac{5}{6}\right)^n$ **(b)** $a=\frac{5}{6}$, $r=\frac{5}{6}$ **(c)** 13
18(a) $\frac{9}{25}$ **(b)** 11
19(a) $\frac{1}{12\,960\,000}$ **(b)** 233
20(a) $\frac{1}{9}$ **(b)** $\frac{1}{9}$. Retell as ‘Nick begins by picking out two socks for the last morning and setting them aside’. **(c)** $\frac{1}{9}$. Retell as ‘Nick begins by picking out two socks for the third morning and setting them aside’. **(d)** $\frac{1}{63}$ **(e)** $\frac{1}{9\times7\times5\times3}$ **(f)** zero
21 In each part, retell the process of selection as ‘First choose a court for Kia, then choose one of the remaining 11 positions for Abhishek’.
(a) $\frac{3}{11}$ **(b)** $\frac{1}{11}$ **(c)** $\frac{4}{33}$
22 Suppose first that the contestant changes her choice. If her original choice was correct, she loses,

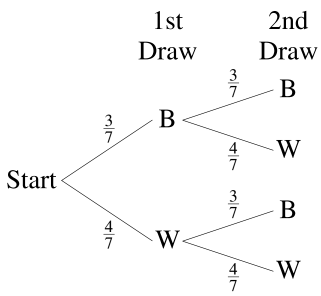
ISBN: 978-1-107-61604-2

otherwise she wins, so her chance of winning is $\frac{2}{3}$. Suppose now that the contestant does not change her choice. If her original choice was correct, she wins, otherwise she loses, so her chance of winning is $\frac{1}{3}$. Thus the strategy of changing will double her chance of winning.

23 $\frac{11}{19}$

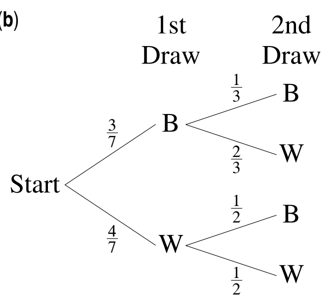
Exercise **10D** (Page 410)

1(a)



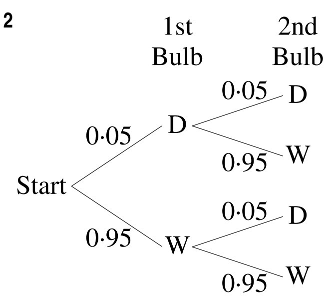
(i) $\frac{16}{49}$ **(ii)** $\frac{24}{49}$

(b)



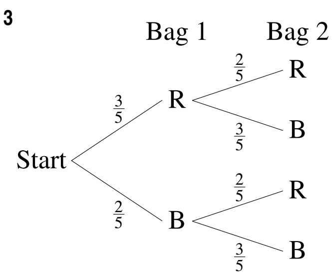
(i) $\frac{3}{7}$ **(ii)** $\frac{4}{7}$

2



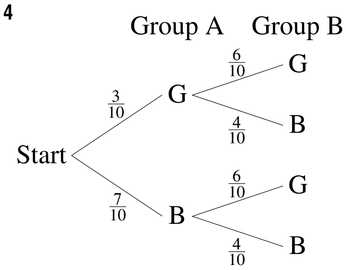
(a) $0{\cdot}25\%$ **(b)** $99{\cdot}75\%$

3



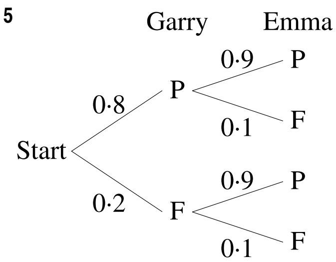
(a) $\frac{12}{25}$ **(b)** $\frac{13}{25}$

4



(a) $\frac{23}{50}$ **(b)** $\frac{27}{50}$

5



(a) 8% **(b)** 18% **(c)** 26% **(d)** 28%

6(a) $\frac{1}{11}$ **(b)** $\frac{14}{33}$ **(c)** $\frac{10}{33}$ **(d)** $\frac{19}{33}$

7(a) $0{\cdot}28$ **(b)** $0{\cdot}50$

8(a) $0{\cdot}01$ **(b)** $0{\cdot}23$

9(a)(i) $\frac{1}{25}$ **(ii)** $\frac{3}{5}$ **(b)** $\frac{1}{20}$, $\frac{3}{5}$

10 $4{\cdot}96\%$

11 $0{\cdot}35$

12(a) $\frac{21}{3980}$ **(b)** $\frac{144}{995}$

13(a) $\frac{9}{25}$ **(b)** $\frac{21}{25}$

14(a) $\frac{3}{10}$ **(b)** $\frac{7}{24}$ **(c)** $\frac{21}{40}$ **(d)** $\frac{11}{60}$

15(a)(i) $\frac{5}{33}$ **(ii)** $\frac{5}{22}$ **(iii)** $\frac{19}{33}$ **(iv)** $\frac{1}{4}$ **(v)** $\frac{19}{66}$ **(vi)** $\frac{47}{66}$

(b) $\frac{25}{144}$, $\frac{5}{24}$, $\frac{5}{9}$, $\frac{1}{4}$, $\frac{25}{72}$, $\frac{47}{72}$

16(a) $\frac{5}{6}$ **(b)** $\frac{5}{12}$ **(c)** $\frac{1}{6}$ **(d)** $\frac{3}{4}$

17 The chance that at least one of them will shoot a basket is $1 - p\,$(they both miss). The boy missing and the girl missing are independent events. The correct answer is $0{\cdot}895$.

18(a) $\frac{4}{9}$ **(b)** $\frac{65}{81}$ **(c)** 4

19 $\frac{4}{11}$

20(a) $\frac{1}{36}$ **(b)** $\frac{1}{46\,656}$ **(c)** $\frac{11}{36}$ **(d)** $\frac{2}{11}$

21(a) $\frac{1}{6}$ **(b)** $\frac{5}{6}$ **(c)** $\frac{1}{5}$

22(a) $\frac{1}{20}$ **(b)(i)** $\frac{57}{8000}$ **(ii)** $\frac{3971}{4000}$

23(a) $\frac{1}{216}$ **(b)** $\frac{5}{72}$ **(c)** $\frac{5}{12}$ **(d)** $\frac{5}{9}$

24(a) $\frac{5}{36}$ **(b)** $\frac{13}{18}$ **(c)** $\frac{5}{36} + \frac{13}{18} \times \frac{5}{36} + \frac{13}{18}^2 \times \frac{5}{36}$ **(d)** $\frac{1}{2}$

25(a) $\frac{3}{7}$ **(b)** $\frac{16}{49}$ **(c)** $\frac{9399}{16\,807}$ **(d)** $\frac{21}{37}$

26(a) $\frac{1}{3}$ **(b)** $\frac{1}{4}$

27(a) $\frac{1}{25}$ **(b)** $\frac{3}{25}$ **(c)** $\frac{6}{25}$ **(d)** $\frac{19}{25}$

28(a) $\frac{2}{5}$ **(b)** $\frac{9}{25}$ **(c)** $\frac{1622}{3125}$ **(d)** $\frac{10}{19}$

29(a) $\frac{15}{16}$ **(b)** $\frac{2}{3}$

ISBN: 978-1-107-61604-2

30(a) $\dfrac{g}{g+b} \times \dfrac{g-1}{g+b-1} \times \dfrac{g-2}{g+b-2}$ **(b)(iii)** maximum turning point at $g = 5 + 3\sqrt{2}$ and minimum turning point at $g = 5 - 3\sqrt{2}$ **(iv)** 3 green and 2 blue or 4 green and 10 blue marbles

Exercise **10E** (Page 417)

1(a) 30 **(b)** 90 **(c)** 6 **(d)** 6 **(e)** 120 **(f)** 24 **(g)** 15 120 **(h)** 1 814 400

2 There are 6: DOG, DGO, ODG, OGD, GOD, GDO

3 FEG, FGE, FEH, FHE, FEI, FIE, FGH, FHG, FGI, FIG, FHI, FIH

4(a) 360 **(b)** 720

5(a) 120 **(b)** 625

6 60, 36

7 5040

8(a) 40 320 **(b)** 336

9(a) 12 **(b)** 864

10 720

11 48

12(a) 10^7 **(b)** 5×10^6 **(c)** 5^7 **(d)** 32 000

13(a) 10 000 **(b)** 5040 **(c)** 625 **(d)** 1000.

14(a) 3024 **(b)** 336 **(c)** 1344 **(d)** 336 **(e)** 1008

15(a) 6561 **(b)** 729 **(c)** 2916 **(d)** 729 **(e)** 2187

16(a) 6 760 000 **(b)** 3 276 000 **(c)** 26 000 **(d)** 48

17(a) 720 **(b)** 120 **(c)** 24 **(d)** 360 (half of them)

18(a) 144 **(b)** 120 **(c)** 144 **(d)** 2520 (half of the total)

19(a) 720 **(b)** 720 **(c)** 4320

20 622 080

21(a) 24 **(b)** 240

22 1728

23 24

24 2046

25(a) 1152 **(b)** 1152

26(a) 720 **(b)** 120 **(c)** 1680 **(d)** 4200 **(e)** 960 **(f)** 480

27(a) 5040 **(b)** 4320 **(c)** 720 **(d)** 144 **(e)** 720 **(f)** 960 **(g)** 1440

28(a) 7^7 **(b)** 6×7^6 **(c)** 7^6 **(d)** $3^4 \times 4^3 + 4^4 \times 3^3 = 7 \times 12^3$

29(a)(i) 3 628 800 **(ii)** 725 760 **(iii)** 725 760 **(iv)** 2 257 920 **(b)(i)** $2(n-1)!$ **(ii)** $2(n-1)!$ **(iii)** $(n-2)(n-3)(n-2)!$

30 8640

31(a) 40 320 **(b)** 20 160 **(c)** 17 280

32(a) 120 **(b)** 24 **(c)** 95

33(a) 5040 **(b)** 20 160

34(a)(i) 64 **(ii)** 32 **(b)(i)** 340 **(ii)** 170

35(a) 96 **(b)** 36 **(c)** 24

36(a) 5^5 ways **(b)** $5! = 120$ ways **(c)** $5 \times 4^3 = 320$ ways

37(a) 133 **(b)** 104 **(c)** 29 **(d)** 56

38(a) 3 **(b)** 3

39(a) $D(1) = 0$, $D(2) = 1$, $D(3) = 2$, $D(4) = 9$ **(c)** $D(n) = (n-1) \times D(n-1) + (n-1) \times D(n-2)$, $D(5) = 44$, $D(6) = 265$, $D(7) = 1854$, $D(8) = 14\,833$. The successive ratios are approximately 2, 3, 2·667, 2·727, 2·716 981, 2·718 447, 2·718 263. (The convergence to e is proven in question 31 of Exercise 10G.)

Exercise **10F** (Page 423)

1(a) 3 **(b)** 12 **(c)** 120 **(d)** 6720 **(e)** 10 080 **(f)** 90 720 **(g)** 4 989 600 **(h)** 45 360 **(i)** 25 740

2 60

3(a) 6 **(b)** 15 **(c)** 20

4(a) 40 320 **(b)** 8 **(c)** 56 **(d)** 560

5(a) 56 **(b)** 20

6(a) 56 **(b)** 5

7(a) 60 **(b)** 24 **(c)** 36 **(d)** 30 (half of them)

8(a)(i) 180 **(ii)** 60 **(iii)** 120 **(iv)** 24 **(b)** 40

9(a) 90 720 **(b)** 720 **(c)** 720 **(d)** 45 360 (half of them)

10 2 721 600

11(a) 1024 **(b)** 256 **(c)** 45 **(d)** 252 **(e)** 56 **(f)** 512 **(g)** 8 **(h)** 70

12(a) 60 **(b)** 60

13(a) 120 **(b)** 60

15(a) 453 600 **(b)** 90 720 **(c)** 5040 **(d)** 10 080 **(e)** 80 640 **(f)** 282 240 **(g)** 15 120

16(a) 3 628 800 **(b)** 4

17(a) 2520 **(b)** 720 **(c)(i)** 600 **(ii)** 480 **(iii)** 360 **(iv)** 240 **(v)** 120 **(d)** 840. Insert the letters U, M, T and R successively into the word EGE. Alternatively, the answer is one third of all arrangements. **(e)** 210 **(f)** 420

18 1 995 840

19 864. The problem can be done by applying the inclusion–exclusion principle from the Extension section of Exercise 10B, or by considering separately the various different patterns.

ISBN: 978-1-107-61604-2 Cambridge University Press

Exercise **10G** (Page 429)

1 There are ${}^5C_2 = 10$ possible combinations: PQ, PR, PS, PT, QR, QS, QT, RS, RT and ST.
2(a) 21 **(b)** 35 **(c)** 15 **(d)** 126
3(a)(i) 45 **(ii)** 45
(b) ${}^{10}C_2 = {}^{10}C_8$, and in general ${}^nC_r = {}^nC_{n-r}$.
4(a) 44 352 **(b)** 34 650
5(a) 70 **(b)** 36 **(c)** 16 **(d)** 1 **(e)** 69
6(a) 126 **(b)** 45 **(c)** 51 **(d)** 75
7(a) 2002 **(b)** 56 **(c)** 6 **(d)** 840 **(e)** 420 **(f)** 1316 **(g)** 715 **(h)** 1287
8(a) 70 **(b)** 5 **(c)** 35
9(a) 792 **(b)** 462 **(c)** 120 **(d)** 210 **(e)** 420
10(a)(i) 252 **(ii)** 126. The number cannot begin with a zero. **(b)** In each part, once the five numbers have been selected, they can only be arranged in one way.
11 13 860
12(a) 1 745 944 200 **(b)** 413 513 100
13(a) 45 **(b)** 120 **(c)** 36 **(d)** 8
14(a) 10 **(b)** 110
15(a) 65 780 **(b)** 1287 **(c)** 48 **(d)** 22 308 **(e)** 288 **(f)** 3744
16(a)(i) ${}^6C_1 + {}^6C_2 = 21$ **(ii)** ${}^5C_2 = 10$ (choose the two people to go in the same group as Laura) **(b)(i)** 4 **(ii)** 3 **(c)(i)** 92 **(ii)** 35
17(a) 2 **(b)** 5 **(c)** 35 **(d)** ${}^nC_2 - n$
18(a) 220 **(b)** 9240 **(c)(i)** 2772 **(ii)** 6468
19(a) 1024 **(b)** 968 **(c)** 466 **(d)** 247
20(a) 16 **(b)** 20 **(c)** 12 **(d)** 8 **(e)** 5
21(a) 252 **(b)** 126
22(a) 315 **(b)** 210
23(a) 12 **(b)** 49 **(c)** 120 **(d)** $(a+1)(b+1)(c+1)$
24(a) 30 **(b)** 24
25(a)(i) 210 **(ii)** 90 **(iii)** 126 **(iv)** 126
27 5151
28 1360
29(a) 3 **(b)** 315 **(c)(i)** 155 925 **(ii)** 10 800

Exercise **10H** (Page 435)

1(a) 84 **(b)** $\frac{5}{42}$
2(a) $\frac{1}{210}$ **(b)** $\frac{2}{5}$ **(c)** $\frac{3}{5}$ **(d)** $\frac{4}{15}$
3(a) $\frac{1}{13}$ **(b)** $\frac{46}{455}$ **(c)** $\frac{3}{91}$ **(d)** $\frac{3}{13}$
4(a) $\frac{8}{429}$ **(b)** $\frac{1}{143}$ **(c)** $\frac{140}{429}$ **(d)** $\frac{421}{429}$ **(e)** $\frac{2}{11}$ **(f)** $\frac{1}{3}$
5(a) $\frac{1}{22\,100}$ **(b)** $\frac{1}{5525}$ **(c)** $\frac{11}{850}$ **(d)** $\frac{22}{425}$ **(e)** $\frac{11}{1105}$ **(f)** $\frac{13}{34}$ **(g)** $\frac{16}{5525}$ **(h)** $\frac{6}{5525}$ **(i)** $\frac{741}{1700}$ **(j)** $\frac{64}{425}$
6(a) $\frac{3}{70\,304}$ **(b)** $\frac{1}{2197}$ **(c)** $\frac{1}{64}$ **(d)** $\frac{1}{16}$ **(e)** $\frac{27}{2197}$ **(f)** $\frac{3}{8}$ **(g)** $\frac{6}{2197}$ **(h)** $\frac{3}{2197}$ **(i)** $\frac{27}{64}$ **(j)** $\frac{5}{32}$
7(a) $\frac{1}{10}$ **(b)** $\frac{1}{10}$ **(c)** $\frac{1}{3}$
8(a) $\frac{1}{10}$ **(b)** $\frac{2}{5}$
9(a) $\frac{1}{15}$ **(b)** $\frac{2}{3}$
10(a) $\frac{1}{42}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{2}{7}$ **(d)** $\frac{1}{35}$ **(e)** $\frac{1}{7}$
11(a) $\frac{1}{2}$ **(b)** $\frac{1}{6}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{1}{60}$ **(e)** $\frac{2}{3}$
12(a) $\frac{1}{7}$ **(b)** $\frac{2}{7}$ **(c)** $\frac{1}{7}$ **(d)** $\frac{2}{7}$
13(a) $\frac{1}{3}$ **(b)** $\frac{2}{3}$ **(c)** $\frac{2}{15}$ **(d)** $\frac{1}{5}$
14(a) $\frac{1}{26}$ **(b)** $\frac{5}{13}$ **(c)** $\frac{15}{26}$ **(d)** $\frac{1}{26}$
15(a) 0·403 **(b)** 0·597 **(c)** 0·001 74 **(d)** 0·291
16(a) $\frac{1}{22}$ **(b)** $\frac{125}{1728}$ **(c)** $\frac{5}{144}$
17(a) $\frac{10}{31}$ **(b)** $\frac{15}{31}$ **(c)** $\frac{6}{31}$
18(a) $\frac{1}{60}$ **(b)** $\frac{3}{5}$ **(c)** $\frac{1}{5}$ **(d)** $\frac{2}{5}$ **(e)** $\frac{1}{20}$ **(f)** $\frac{3}{5}$ **(g)** $\frac{3}{10}$ **(h)** $\frac{9}{10}$ **(i)** $\frac{1}{10}$ **(j)** $\frac{3}{5}$
19(a) $\frac{3}{8}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{21}{32}$ **(d)** $\frac{3}{32}$ **(e)** $\frac{17}{64}$
20(a) $\frac{281}{462}$ **(b)** 8
21(a) $\frac{1}{27\,417}$ **(b)** $\frac{28}{703}$
22 In each part, the sample space has ${}^{52}C_5$ members. **(a)** $\frac{352}{833}$. Choose the value of the pair in 13 ways, then choose the cards in the pair in ${}^4C_2 = 6$ ways, then choose the three values of the three remaining cards in ${}^{12}C_3$ ways, then choose the suits of those three cards in 4^3 ways. **(b)** $\frac{198}{4165}$. Choose the values of the two pairs in ${}^{13}C_2$ ways, then choose the suits of the cards in the two pairs in ${}^4C_2 \times {}^4C_2$ ways, then choose the remaining card in 44 ways. **(c)** $\frac{88}{4165}$ **(d)** $\frac{1}{4165}$ **(e)** $\frac{6}{4165}$ **(f)** $\frac{128}{32\,487}$. Choose the lowest card in 10 ways, then choose the suits of the five cards in 4^5 ways. **(g)** $\frac{33}{16\,660}$ **(h)** $\frac{1}{649\,740}$
23(a) $\frac{1}{125}$ **(b)** $\frac{4}{125}$ **(c)** $\frac{16}{125}$ **(d)** $\frac{108}{125}$
24(a) $\frac{48}{125}$ **(b)** $\dfrac{n^2(n-1)^2(n-2)!}{2n^n}$
25(a) 0·0082
(b) $1 - \dfrac{{}^{365}P_n}{365^n}$
(d) 23 **(e)** 45
26(a) $\frac{1}{25}$ **(b)** $\frac{3}{25}$ **(c)** $\frac{19}{25}$
27(b) $\frac{1}{8}$ **(c)** 2^{1-n}

Exercise **10I** (Page 441)

1(a)(i) 120 **(ii)** 24 **(b)(i)** 3 628 800 **(ii)** 362 880
2(a) 10 080 **(b)** 1440
3(a) 24 **(b)** 6 **(c)** 4 **(d)** 12 **(e)** 4
4(a) 5040 **(b)** 144 **(c)** 576 **(d)** 1440 **(e)** 3600 **(f)** 240
5(a) $\frac{3}{10}$ **(b)** $\frac{1}{5}$ **(c)** $\frac{1}{10}$ **(d)** $\frac{9}{10}$

ISBN: 978-1-107-61604-2 Cambridge University Press

6(a) 5040 **(b)** 576 **(c)** 144 **(d)** 2304 **(e)** 1440 **(f)** 3600

7(a) $\frac{1}{12}$ **(b)** $\frac{1}{9}$

8(a) $(n-1)!$ **(b)** $2 \times (n-2)!$ **(c)** $(n-3) \times (n-2)!$ **(d)** $6 \times (n-3)!$

9(a) $39\,916\,800$ **(b)** 165

10 $145\,152$

11(a) 288 **(b)** $\frac{1}{4}$

12 $\dfrac{n!\,(n+1)!}{(2n)!}$

13(a) 60 **(b)** $181\,440$ **(c)** 9

Exercise **10J** (Page 446)

1(a) $\frac{1}{32}$ **(b)** $\frac{5}{16}$ **(c)** $\frac{5}{32}$ **(d)** $\frac{31}{32}$

2 ${}^{6}\mathrm{C}_2\,(\frac{4}{5})^4\,(\frac{1}{5})^2$

3(a) ${}^{7}\mathrm{C}_3\,(\frac{2}{3})^3\,(\frac{1}{3})^4$ **(b)** ${}^{7}\mathrm{C}_5\,(\frac{2}{3})^5\,(\frac{1}{3})^2$ **(c)** $(\frac{1}{3})^7$ **(d)** $1-(\frac{1}{3})^7$

4(a) ${}^{4}\mathrm{C}_3\,(\frac{5}{6})^3\,(\frac{1}{6})$ **(b)** ${}^{4}\mathrm{C}_2\,(\frac{5}{6})^2\,(\frac{1}{6})^2$

5 $(0{\cdot}65)^{12}$

6(a) ${}^{12}\mathrm{C}_3\,(\frac{5}{6})^9\,(\frac{1}{6})^3$ **(b)** ${}^{12}\mathrm{C}_8\,(\frac{5}{6})^4\,(\frac{1}{6})^8$ **(c)** ${}^{12}\mathrm{C}_{10}\,(\frac{5}{6})^2\,(\frac{1}{6})^{10} + {}^{12}\mathrm{C}_{11}\,(\frac{5}{6})^1\,(\frac{1}{6})^{11} + (\frac{1}{6})^{12}$

7(a) $0{\cdot}2009$ **(b)** $0{\cdot}7368$ **(c)** $0{\cdot}2632$

8 $(\frac{5}{6})^{15} + {}^{15}\mathrm{C}_1\,(\frac{5}{6})^{14}\,(\frac{1}{6}) + {}^{15}\mathrm{C}_2\,(\frac{5}{6})^{13}\,(\frac{1}{6})^2$

9(a) $(\frac{9}{10})^{20} + {}^{20}\mathrm{C}_1\,(\frac{9}{10})^{19}\,(\frac{1}{10}) + {}^{20}\mathrm{C}_2\,(\frac{9}{10})^{18}\,(\frac{1}{10})^2$ **(b)** $1-(\frac{9}{10})^{20}$

10(a) $0{\cdot}91^{10} + {}^{10}\mathrm{C}_1 \times 0{\cdot}91^9 \times 0{\cdot}09 + {}^{10}\mathrm{C}_2 \times 0{\cdot}91^8 \times 0{\cdot}09^2$

(b) $1 - 0{\cdot}91^{10} - {}^{10}\mathrm{C}_1 \times {\cdot}91^9 \times 0{\cdot}09$

11 $0{\cdot}593$

12 $0{\cdot}000\,786$

13 ${}^{31}\mathrm{C}_3 \times 0{\cdot}95^{28} \times 0{\cdot}05^3$

14(a)(i) $0{\cdot}107\,64$ **(ii)** $0{\cdot}113\,72$ **(b)** $0{\cdot}785\,49$

15(a) 17 **(b)** 7

16(a)(i) $0{\cdot}487$ **(ii)** $0{\cdot}031$ **(b)** We have assumed that boys and girls are equally likely. We have also assumed that in any one family, the events 'having a boy' and 'having a girl' are independent.

17(a) The argument is invalid. Normally, mathematics books are grouped together, so once the shelf is chosen, one would expect all or none of the books to be mathematics books, thus the five stages are not independent events. The result would be true if the books were each chosen at random from the library.

(b) The argument is invalid. People in a particular neighbourhood tend to vote more similarly than the population at large, so the four events are not independent. The result would be true if one chose four streets at random, and then chose a voter randomly from each street.

18(a) $0{\cdot}0124$ **(b)** $0{\cdot}7102$

19(a) $0{\cdot}409\,600$ **(b)** $0{\cdot}001\,126$ **(c)** $0{\cdot}000\,869$

20(a) $0{\cdot}0060$ **(b)** $0{\cdot}0303$

21(a) $\frac{3}{250}$ **(b)(i)** $(\frac{3}{250})^{10}$ **(ii)** ${}^{10}\mathrm{C}_5\,(\frac{3}{250})^5\,(\frac{247}{250})^5$ **(iii)** $(\frac{247}{250})^{10} + 10\,(\frac{247}{250})^9\,(\frac{3}{250})$

22(a) $\frac{7}{16}$ **(b)(i)** ${}^{8}\mathrm{C}_3\,(\frac{9}{16})^5\,(\frac{7}{16})^3$ **(ii)** $1-(\frac{9}{16})^8 - 8(\frac{9}{16})^7\,(\frac{7}{16})^1 - {}^{8}\mathrm{C}_2\,(\frac{9}{16})^6\,(\frac{7}{16})^2$

23(a) 34 **(b)** 22

24(a) 33, ${}^{200}\mathrm{C}_{33}\,(\frac{1}{6})^{33}\,(\frac{5}{6})^{167} \doteqdot 0{\cdot}075\,65$

(b) 20 and 21, ${}^{41}\mathrm{C}_{20}\,(\frac{1}{2})^{41} \doteqdot 0{\cdot}1224$

(c) 2, ${}^{35}\mathrm{C}_2\,(\frac{1}{13})^2\,(\frac{12}{13})^{33} \doteqdot 0{\cdot}2509$ **(d)** 8 and 9, ${}^{35}\mathrm{C}_9\,(\frac{1}{4})^9\,(\frac{3}{4})^{26} = {}^{35}\mathrm{C}_8\,(\frac{1}{4})^8\,(\frac{3}{4})^{27} \doteqdot 0{\cdot}1520$

26(a) $0{\cdot}2048$ **(b)** $0{\cdot}262\,72$

(c)(i) $\dfrac{n(n-1)(n-2)(n-3)}{20 \times 19 \times 18 \times 17}$

27(a) $a^3 + b^3 + c^3 + 3a^2b + 3a^2c + 3ab^2 + 3ac^2 + 3b^2c + 3bc^2 + 6abc$ **(b)(i)** $0{\cdot}102\,96$ **(ii)** $0{\cdot}131\,33$ **(iii)** $0{\cdot}897\,04$

28(c) If a coin is tossed $2n$ times, then the probability P_n of obtaining equal numbers of heads and tails is approximated by $\dfrac{1}{\sqrt{n\pi}}$, in the sense that the percentage error between P_n and this approximation converges to zero as the number of tosses increases.

ISBN: 978-1-107-61604-2 Cambridge University Press