The directional devivative of fray) in the direction of
- a vector & (unit vector)
eget is bound heroused dellet I I ameed
Dû f = Vf.û
- where $\nabla f = f_{ni} + f_{nj}$
The directional derivated of fix, y, a) is
best for the first the second of the second
$Dxf = \nabla f \cdot \hat{u}$
where of frit fyz +fzk.
On the RHS. is must be a west vector We may
define Dût
to be the directional devivative of fin the director
of a non-curit rector 4 but
The same of the sa
$D_{k}f = D_{k}f, \ \vec{Q} = \frac{y}{ y }$
E.g. 2010 Fram Q2.101
$f: \mathbb{R}^2 \to \mathbb{R}$
(x, y) -> fun-1(x2+3y2)
Pis the point (2,1)
i) The directional derivative of f
at P in the Spection of u=42-7
Duf at (2,1)
$D_{\mathcal{U}}f = D_{\mathcal{U}}f = \nabla f,  \mathcal{U} = \nabla f \xrightarrow{(4,-1)}$
$\nabla f = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} $ $= \langle \frac{27c}{1 + (x^{243}y^2)}, \frac{6y}{(+(x^{243}y^2))} \rangle$
27C 6y
1+(x+3y=), (+(x2+3y=)/

= (= , 3)

Duf = 
$$\nabla f \cdot \hat{\mathcal{U}} = \langle \frac{2}{36}, \frac{3}{36}, \sqrt{4}, -1 \rangle$$

Maximum directional derivative

Suppose of and (xo, xo) are given and it vorces directions

 $Dgf = \nabla f \cdot \hat{\mathcal{U}}$ 

=  $|\nabla f| |\mathcal{U}| \cos \theta$ 

=  $|\nabla f| |\mathcal{U}| \cos \theta$ 

where  $\theta$  is the early between the voctors  $\nabla f$  and  $\hat{\mathcal{U}}$ 
 $|\nabla f| |\mathcal{U}| |\nabla f| |\nabla f|$ 

The notation on RHS is called normal derivatue.

Consider the family of level cure of fire, ys: fix, y) = constant The come through (xo, yo) has equation f(x, y) = f(06, y.) This can solved implicitly for y as a huden of >c near to provided thy to not to. The slope of tangent is the implicit Samuele. In = fx (Xo, Yo) The vector i- 12 7 25 tangent to the level curce. It also fyi-fxj.

(7his allow vertical tangents also) Consider I = fyi - fxq Dzf= 74. Î = (fx2+fy1) (fxi-fxg) \( \sqrt{x^2 \cdot fy^2}. The directional destructure of f is zero in she direction of a tangent so a larel come.

The normal to the level and is VF = fret + fret = 2 ChII, fritfy I fyi - full The direction of maximum directional derivate is sormal to the level auces. The normal despeature is the desivative in the direction of the normal. on = Vf. D = |Vf| Da f=17f1 coso max when  $0=0: \vec{\mathcal{Q}}= \frac{\nabla f}{|\mathcal{A}|}$ min when 0=11: Q=- 2 Zero when  $0=t\frac{2}{2\pi}$ :  $\vec{\mathcal{Q}}=t\hat{\mathcal{T}}$  tangent. Dherel surved of Lucke of 3 variables Suppose  $f: part of R^3 \rightarrow R$   $(x, y, z) \longrightarrow f(x, y, z)$ . The ductional derivatue is Vf. 4 = 17 f / costo

- where O is the angle between	7
- where $0$ is the angle between $\nabla f = \langle f_{\times}, f_{y}, f_{z} \rangle$ and $\widehat{x}$	
there and the season of the se	
Dût is maximised when 0=0.	
Dûf 25 maximised when $6=0$ .  when $\hat{\mathcal{U}} = \frac{\nabla f}{ \nabla f }$ then	3
$Daflmax =  \nabla f $	K.
with a formed by the the owner.	1
we want to show that this	100
and the second of the second o	
$\frac{\partial f}{\partial n} = \nabla f \cdot \hat{n}$	
- The same and the same of the	
when $\hat{n}$ is normal to the level surface of f.	
Start with	Shar
f(x, y, z) = constant	
Let this define & implicitly in terms of x mdy near a point where fz +0	750
near a point where fz \$0	
sold the second sold the second sold sold sold sold sold sold sold sol	1511
Implicitly differentiate wint x:	
Consider I F By 2 - By	in
英次 + 开资 =0	
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The first the fi	
Similarly	
<del>第二一</del>	1
The first of Ring R	1018
on the surface we have tangent thes in the	
In the surface we have tangent trees on the	
in the kent of the second to ment such and	
2-4k 0200170 = 10-25	

or

fz j-fyk (This allow vertical tangent) The normal to these two vectors is (fz:-fxk)x(fzz-fyk) = fxfz; + fxfzj +fx2k = (felfoy felfy, felfe)  $\nabla f = \langle f_z, f_y, f_z \rangle$ 28 normal to the surfuel. Hence, the maximum directional Lavatre of Duf is the normal desirative. If = Vf. 2 = 1 Vf The normal at (Xo, Yo, 70) can also be seen fun the tangent plane. Consider the case. The equation of the tangent plane at cro, you re = = f(x0, x0) + fr (x0, y0) (x-x0) + fy (x0, x0) (x-x0)

from the equation of a place in Centresian words, (-fx,-fy,1) at (xo, you 7his is the gradient of the scalar z-fox,y) This method works for fox, 2>= c using implicit

Settentiation. 1401- 12-10- H The normal at (Xosta 2) can also be seen The equation of the trapet place let were 3 = + (24 1/2) + fr ca 42 (4 1/2) + fr ca 43 (4 1/2)

2011 eran CLICA) Shotch the set 12-3+2:15 2 (closed disk] 1(6) Factorise P(2) = 24-523+522 +42 +10 gran shot 3-7 75 are of the west 3ti is a wot too x 200 3-7 => also 3+7 is a 200+. 2-(3-i) and 2-(3+i) are factors [2-3+7][2-3-7] = 2-62+10 the his his method I. 02 execut and ETITET NO 22+3+ 1 00 00 00 00 1 + 5+ 5 5 22-62+10 24-5 23+5 22+42+10 24 - 6 = 3 + 10 = 2 23-522-43 file (10,0) = of Z3 -6Z2+10Z 32 -62+10 22-62-10 P(=) = (= 2-62+10)(22+2+1)

```
method 2
    Z4-623+522443+10 = (22-63+10)(22+93+6)
           10=(06 => 6=1
           a-6=5 => a=1
   :. 22+ az+ b = 22+ z+/
    P(2)=(22-62+10)(22+2+1)
1(c) Show that f(x) = \frac{57nx}{x} is decreasing on (0, \pi).

Hints: (05x < 5inx < 1) on (0, \frac{1}{2}\pi)
 Proof:
f'(x) = \frac{\times 606 \times -5 \text{ in}}{\times^2}
                 <0 on (0, $ ₹ ₹)
                by the hint
       on [57,7] both tenes are 50
       i. floodes on Co, 17
       · · fix: is decreasy on (0, Ti)
2(a) f: R2 \ {(0,0)} -> R
     (2,4) -> ln (4x2+y2)
                          2014 E 6-
       P pount (1,2)
     Solution:
                 z = f(x, y)
                   = ln (4224)
```

(6) directional devivation of z at p in the Irection of  $\mathcal{L}=(3,-2)$ 

Solution: 
$$Out | (1/2) = \nabla f \cdot \hat{y}$$
 $\nabla f(x,y) = \langle fx, fy \rangle$ 
 $= \langle \frac{fx}{qx^2y^2}, \frac{xy}{qx^2y^2} \rangle$ 
 $= \langle \frac{f}{qx^2y^2}, \frac{xy}{qx^2y^2} \rangle$ 
 $= \langle 1, \frac{1}{2} \rangle$ 
 $=$ 

dutin  $\Delta z = fx \ \Delta x + fy \Delta y$   $z - z_0 = fx(x_0 y_0) (x_0 - x_0) + fy(x_0, y_0)(y - y_0)$  z - linf = ((x - 1) + f(y - 2)) z = x + fy + 3 lo 2 - 2

2(b) Any mothed to calculate Taylor poly nomental
2(b) Any method to calculate Taylor poly normal  Ty(x) for fox) = e 2x cos (3x)
THE PARTY OF THE PROPERTY OF CHILD PARTY NAMED AS A PARTY OF THE PARTY
Sub mt total deex
Culm Zotal of WS(3x)
0-6-5 maps ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) ( ) (
pett. Sol.
The same of the sa
ex=1+x+ 5 + 4 + 4
2-)2x: 70
e <sup>22</sup> =1+2x+2x <sup>2</sup> +\frac{4}{3}x^3+\frac{2}{3}x^4+ Hepur A
(os(x)=1- 22 + 29 =
605 (05(3x) = 1- 9x+ ===================================
Multiply, +rungrates at xx=
Multiply, +runecaty at $x^{4}$ : $ \frac{(4(x) = for  e^{2x} \cos 3x)}{e^{2x} (1 + 2x + 2x^{2} + \frac{1}{3}x^{3} + \frac{1}{3}x^{4}) (1 - \frac{1}{3}x^{2} + \frac{1}{3}x^{4})}} $
= (1+2x+2x2+ 1/3x3+2/3x4) (1-2x2+2/2x4)
La Company American Described to Marine Described
= 1+2x + (2-9/2) x2+ (4/3-9) x3+(3/3-9+27/8) x4+
To the second of
$= (+2)x^{-5}/2x^{2} + (-2)/3x^{3} + (-1)/24)x^{4} + \cdots$
くさいするとは、
DEN THE STREET - STREET
LILLER CONTRACTOR LANGE CONTRACTOR CONTRACTO
the second (0.2)
il Equation of the toward who at P.
AND BY PRINT
should be the same of the same
40 4 40 x = 54
The state of the s

8-121-10 DI + 116-3

S-54242+X

Osca) Show that has one and only one zer on [1,10] method I [direction solution not intended sol] lu (2X) - lu (1+ (1+x2)=0 1+ 1-22 = 1 DX= 1+ SI+x2 (2x-1)= 1+x2 4x2-4x+1 = 1+x2. 3x2-4x=0 x(3x-4)=0 X=0 is not admissible. The unique root is It 75 on [1,10] Method I. Intermediate vale Theorem 9(1) = lucrs-lu(1+52) 20 60 = lu 2 - lu (2.414) 9(10) = lu (20) - lu (1+ tui) >0 The sigh change (and continuity of good =) at least one root on C1,10]. To prove that the root is unique, take devinete,  $g'(x) = \frac{1}{x} - \frac{x}{\sqrt{1+x^2}} = \frac{1}{x} - \frac{x}{1+x^2+\sqrt{1+x^2}} > \frac{1}{x} - \frac{x}{1+x^2} > 0$ - · g(x) is in country for x >0

Miz Cancelleg amoun sheders.

In 223+x2-33x+63 (x-3)
25-27x+54 Oscar Shade Share to FIRST ME AND TON · 大村 龙子 he lively expressing to 1929: colored to the second of the second THE CONTRACT OF THE PARTY OF TH I Intermediate Water Traver 9(1)= lang-lal (45) = 20 20 (414) = da (3, 414) Q (10) = La (100) al = (11) A the sign change (and continuity I got = 3) I lavet to me the continuity. To prove that the rook is insigned, take HARDER TO THE CHARLES WE WAY TO ME.

( 3 (b) In ( suh) 2. Hand: replace state with itis taylor pulynomial.
73 (x) about x = 0. Method 2 Taylon.

73 (2) for suche

= x + x = 7200 for man 100 = 11 72 Higher pour do not contistile to the limit so lon (suhx) 1802 lon (1+ 742) 1/22 2 horas (1+ fr) n n=1/22 e = (1+ 1)h = e 16 Method 2 top L'Höpitul L= 200 ( smh 2 ) /x2. Then lu 2 = lu lu (52mhx) (0) = lm du smh x - luxe = ln cuthx - x :. L= e % = ln x (05h x - 52mbx (0) = lu smhx 200 2xcoshxxxinhx

 $(x_{4}) \rightarrow (0,0)$   $\frac{x^{3}+xy+y^{3}}{x^{2}+y^{2}}$ pure Does and exist erse that to pur exist. show fast the unnertating goes to zero. Consider 2 paths
The term 24 > 0 at about the the sure
speed as the denomination. we will prove the limit does not exist. look for 2 puths to (0,0) on which different limit occurs. y-axis not ansidered due to symmetry. x3+x2+X3 = ln 1+2x Different lingts -> the original limit does not rearst.

4 cas Use T3 ox) for sinx and R3 ox) to prove.
Cabont X=0) to prove
<b>1</b>
$=\frac{x^{2}}{6} < \sin x$
at least for OCXCTIX
Sin x = 73(x) + R3(x) where
$S(n) \chi = (3(x) + R3(x))  \text{whel}$ $T_{3(x)} = \chi - \frac{2}{3!}  R_{3(x)} = \frac{f^{(k)}(c)}{4!} \chi^{4}$ $= \chi - \frac{2c}{6}$ $d^{4} = 1$
= x-20 41 5-3 7-8 = 7 - 80 = x 10
$= \frac{d^4}{dx^4} \sin x \Big _{x=c}$
Substitute of the second second
Rz00= 5 m(c) x 4.
· OSECI SILL ME OCC CX EXCENTER OF
Note: If OCXET,
then occutt and
Sinc >0. (a) that they also
:. /23(x):>0, ocx(T.
This implies x-xi c sin x at least for ocxxii
We also pure to show as a show a show a show a
3 XXXXX
oca the we
Lebus - Laborate - Lab
520x20c forx 00x c3
STANKENC JOHN OF WEEK
is some of a so
\$ (x-5mx)=1-105x>0
" X - 87 m X To mousey do all x to de
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t(b). We want to upgrade.
(0.5% k .40)
$x - \frac{x^3}{6} < s_{1n} \times$
to all positive x.  * So faw proved for GCXETT
The face of the same of the sa
Consider x>3 (which one laps OCX ST)
Consider x>3 (which one laps ocx st)
at x=63. 2-2=3-2=-3 -
3 = x x m2 = x = x
5in x >-1
x = x + x = 3
for x73, x-2 25 deanstry beause 1-20 2. x-2 (5inx
2. X-2 (5mx
for all x > 3
together with part (a) 02300
together with point (a)
for all x20 to select the x at 2 3 to x
We also want to show
SINXCX
for all 220
In feeless
sinxexec for x, ocxcz.
for x), ₹, LHS≤1, RHS>15.
· · son x (x for all x >0
d le-8mx9 ≥ 1-cosx ≥0.
Different Kush var de Karal Marie Caral
in x- sinx is many for all xER
O at orgh => >0 for 2>0  i. 2-520 x>0 for all x>0
· x-smx>0 for all >c>0

4(c) explain brittly why to cannot be replaced by The part (a) shows Tolok staxe Tolok at least for ocx c} x- 23 (5/10)C are not to replace To (2) by 2-223 x- 2x3 574x = x- x3 + x5 /20 - d <- f+ x2 2>6-70 when x is small, this faces xzt so he count uple 2-623 with x- 2x and with toxeo

