Formula sheet for MATH1905 Statistics

- **Calculation formulae**:

  - *For a sample* $x_1, x_2, \ldots, x_n$

Sample mean $\bar{x}$	$\frac{1}{n}\sum_{i=1}^n x_i$
Sample variance s^2	$\frac{1}{n-1}\left[\sum_{i=1}^n x_i^2 - \frac{1}{n}\left(\sum_{i=1}^n x_i\right)^2\right] = \frac{1}{n-1}S_{xx}.$

  - *For paired observations* $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

S_{xy}	$\sum_{i=1}^n x_i y_i - \frac{1}{n}\left(\sum_{i=1}^n x_i\right)\left(\sum_{i=1}^n y_i\right)$
S_{xx}	$\sum_{i=1}^n x_i^2 - \frac{1}{n}\left(\sum_{i=1}^n x_i\right)^2$
S_{yy}	$\sum_{i=1}^n y_i^2 - \frac{1}{n}\left(\sum_{i=1}^n y_i\right)^2$
r	$\frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

For the regression line $y = a + bx$:	
b	$\frac{S_{xy}}{S_{xx}}$
a	$\bar{y} - b\bar{x}$

- **Some probability results:**

For any two events A and B	$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ and $\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B\vert A)$
If A and B are mutually exclusive (m.e.)	$\mathrm{P}(A \cap B) = 0$ and $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B)$
If A and B are independent	$\mathrm{P}(A \cap B) = \mathrm{P}(A)\,\mathrm{P}(B)$

- If $X \sim \mathcal{B}(n, p)$, then :
  $\mathrm{P}(X = i) = \binom{n}{i} p^i (1-p)^{n-i}$, $i = 0, \ldots, n$, $\mathrm{E}(X) = np$ and $\mathrm{Var}(X) = np(1-p)$

- **Some test statistics** and sampling distributions under appropriate assumptions and hypotheses:

$\bar{X} \sim \mathcal{N}\left(\mu, \frac{\sigma^2}{n}\right)$
$\frac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathcal{N}(0,1)$
$\frac{\bar{X}-\mu}{S/\sqrt{n}} \sim t_{n-1}$

$\frac{\bar{X}-\bar{Y}}{S_p\sqrt{\frac{1}{n_x}+\frac{1}{n_y}}} \sim t_{n_x+n_y-2}$, where $S_p^2 = [(n_x-1)S_x^2 + (n_y-1)S_y^2]/(n_x+n_y-2)$
$\sum_i \frac{(O_i-E_i)^2}{E_i} \sim \chi^2_\nu$, for appropriate ν

Table 1: **Some values of the standard normal distribution**: $\Phi(x) = F(z) = \mathrm{P}(Z \leq z)$, where $Z \sim \mathcal{N}(0,1)$. The point tabulated is $1-p$, where $\mathrm{P}(Z \leq z) = 1 - p$.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

Table 2: **Quantiles of the $\mathcal{N}(0,1)$ distribution**: Some percentage points of the standard normal. The point tabulated is z, where $\mathrm{P}(Z > z) = p$, where $Z \sim \mathcal{N}(0,1)$.

				p				
0.25	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001
0.674	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090

Table 3: **Critical values of the t test**: Some percentage points of the t-distribution with ν degrees of freedom. The point tabulated is t, where $\mathrm{P}(t_\nu > t) = p$.

	p								
ν	0.25	0.15	0.10	0.05	0.025	0.01	0.005	0.0025	0.001
1	1.000	1.963	3.078	6.314	12.706	31.821	63.656	127.321	318.309
2	0.817	1.386	1.886	2.920	4.303	6.965	9.925	14.089	22.328
3	0.765	1.250	1.638	2.353	3.182	4.541	5.841	7.453	10.214
4	0.741	1.190	1.533	2.132	2.776	3.747	4.604	5.598	7.173
5	0.727	1.156	1.476	2.015	2.571	3.365	4.032	4.773	5.894
6	0.718	1.134	1.440	1.943	2.447	3.143	3.707	4.317	5.208
7	0.711	1.119	1.415	1.895	2.365	2.998	3.499	4.029	4.785
8	0.706	1.108	1.397	1.860	2.306	2.896	3.355	3.833	4.501
9	0.703	1.100	1.383	1.833	2.262	2.821	3.250	3.690	4.297
10	0.700	1.093	1.372	1.812	2.228	2.764	3.169	3.581	4.144
20	0.687	1.064	1.325	1.725	2.086	2.528	2.845	3.153	3.552
30	0.683	1.055	1.310	1.697	2.042	2.457	2.750	3.030	3.385
50	0.679	1.047	1.299	1.676	2.009	2.403	2.678	2.937	3.261
∞	0.674	1.036	1.282	1.645	1.960	2.326	2.576	2.807	3.090

Table 4: **Quantiles of the χ^2_ν distribution**: Some percentage points of the χ^2-distribution with ν degrees of freedom. The point tabulated is x, where $\mathrm{P}(\chi^2_\nu > x) = p$.

	p					
ν	0.25	0.15	0.10	0.05	0.025	0.01
1	1.323	2.072	2.706	3.841	5.024	6.635
2	2.773	3.794	4.605	5.991	7.378	9.210
3	4.108	5.317	6.251	7.815	9.348	11.345
4	5.385	6.745	7.779	9.488	11.143	13.277
5	6.626	8.115	9.236	11.070	12.833	15.086
6	7.841	9.446	10.645	12.592	14.449	16.812
7	9.037	10.748	12.017	14.067	16.013	18.475
8	10.219	12.027	13.362	15.507	17.535	20.090
9	11.389	13.288	14.684	16.919	19.023	21.666
10	12.549	14.534	15.987	18.307	20.483	23.209