PHYS 1901 Physics Advanced
Tutorial 6: Waves

A. Qualitative Questions:

1. ***Bungy jumping***

Bungy jumping is an increasingly popular sport, with a growing clientele of “adrenalin junkies” and an increasing number of facilities around the world.

a. Plot a graph with displacement on the vertical axis and time on the horizontal axis for a bungee jump. Zero displacement corresponds to your final equilibrium position.

b. Mark on your graph the region which is approximately simple harmonic motion. Also mark the period of oscillation T for this region.

c. Where are minimum and maximum accelerations reached?

d. Where are minimum and maximum speeds reached?

e. Some people’s retinas become detached when they bungy jump. Why does this happen? and when is it most likely to occur?

2. ***Springs with different lengths***

Imagine you have a spring and you cut it into two pieces, one a third the length of the original spring and one two thirds the length of the original.

a. If you attach equal masses to each new spring, will the extension be the same for each spring? If not, will it be greater for the shorter or longer spring?

b. What can you say about the spring constants of the two new springs?

You have several lengths of steel wire, from which you are making springs.

c. What would happen if you used thicker wire to make the spring?

d. What effect would winding the wire in wider coils have?

e. If you joined two identical springs in parallel, and two in series, what would be the relationship between the spring constants for these arrangements and that for a single spring?

B. Demonstration Questions:

1. ***Oscillations of a spring***

Two identical objects are attached to identical springs. The periods of oscillation of the two objects will also be the same. Answer the following questions assuming that the motion can be modeled as simple harmonic motion.

a. If the mass of one of the objects is increased, how will this affect the period of oscillation?

b. If one of the springs is replaced with one with a larger spring constant, how will this affect the period of oscillation?

c. Does the extension of the spring (i.e. the oscillation amplitude) affect the period of oscillation?

2. ***Total energy of a spring***

Two springs, A and B have the same length when no force is applied to them. When mass m_1 is hung from spring A and a smaller mass m_2 is hung from spring B, the springs are stretched by the same distance.

a. What does this tell you about the spring constants of the two springs?

b. Which system has greater elastic potential energy?

Now if the systems are put into vertical simple harmonic motion with the same amplitude,

c. What happens to the elastic potential energy of the spring-mass systems when they oscillate?

d. How will their periods of oscillation compare?

3. ***Damped oscillations***

Observe the oscillation of the spring when the attached object is immersed in water.

a. Draw displacement-time graphs for oscillations of the object in air and in water.

b. Damping is often achieved by attaching springs and masses in regions prone to vibrations. Identify a few real life situations where such devices may be used.

c. Why are damping devices often used on machinery? Give an example.

4. ***Bouncing balls***

When a tennis ball bounces off the ground, it spends about 5 ms in contact with the ground. When a steel ball bounces off a steel plate, it spends only about 50 μs in contact with the plate. Why is this? [Hint: Think about the spring constant of the two balls.]

<u>C. Quantitative Questions:</u>

1. ***Sumos in a car***

You will need to estimate some masses!

Four sumo wrestlers get into their Daihatsu Charade and it sinks on its suspension by about 80 mm. They take off down the road and hit a speed bump which causes the car to oscillate. You can consider the car to be mounted on four identical springs as far as vertical oscillations are concerned.

a. What frequency does the car oscillate at?

The sumos head off for a drive in the country. They encounter an unsealed country road that is corrugated (has regular bumps).

b. How is it possible to drive such that the car oscillates maximally?

c. If the bumps are 3 m apart, what speed should the sumos drive at if they want to oscillate maximally?

d. How much work is done on the total system (sumos and car) from bottom to top of an oscillation?

2. ***Car engine***

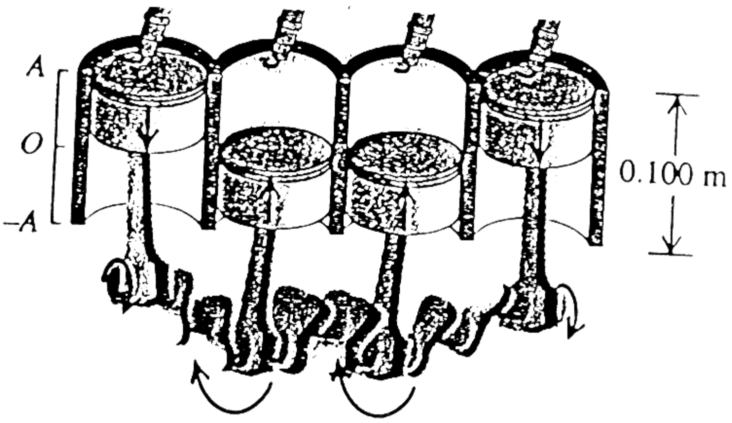
The motion of a piston in a car engine is approximately simple harmonic motion.

a. If the stroke is 0.100 m (see diagram) and the car is revving at 3500 rev/min, what is the acceleration of the piston at the end point of its cycle?

b. If the piston has a mass of 450 g, what net force must be exerted on it at this point?

c. What is the speed and kinetic energy of the piston at the midpoint of its cycle?

d. What average power is required to accelerate the piston from rest to its maximum speed?



Extra Questions:

1. ***Heavy spring***

A mass m hangs down from a spring with mass M and spring constant k. If the mass of the spring can be ignored, the sum of the kinetic and potential energies can be written as:

$$K + U = \tfrac{1}{2} mv^2 + \tfrac{1}{2} kx^2 + mgh$$

a. By taking the time derivative of this expression show that the mass undergoes simple harmonic motion with frequency:

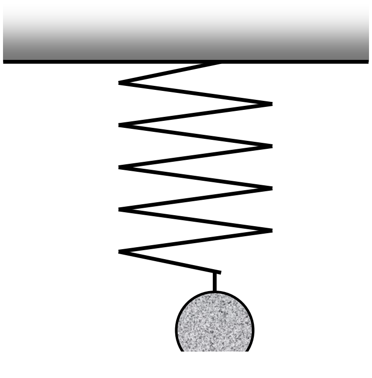
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

b. Now include the mass M of the spring. If the mass m were to travel with velocity v_m, show that the kinetic energy of the spring would be:

$$KE_{spring} = \frac{Mv_m^2}{6}$$

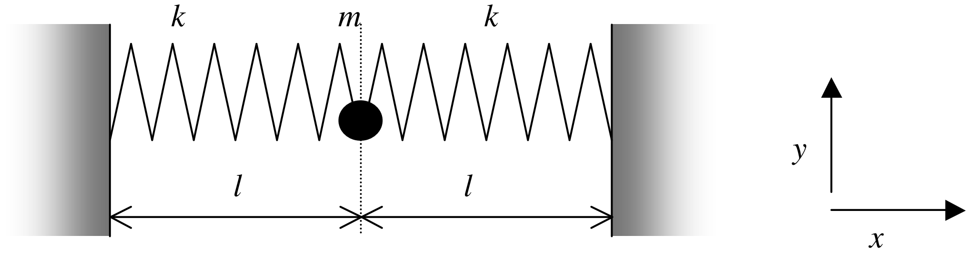
[Hint: Divide the spring into pieces of length dx. Find the mass and kinetic energy of each piece and add them by integration.]

c. Using the result to part **b**, determine the oscillation frequency of the system if the mass M of the spring is included.



2. ***Double spring***

A mass m is attached to two rigid walls by two light springs, as shown below. The springs have spring constant k, and in equilibrium, as below, have length l and are unstretched.



a. Show that the oscillation frequency of the mass in the x direction is

$$f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$

b. Show that if the mass is moved up in the y direction by a distance Δy, there will be a restoring force of the general form:

$$F_y = \alpha\left(\Delta y - \frac{l\Delta y}{\sqrt{\Delta y^2 + l^2}}\right)$$

and find the coefficient α.

c. Assuming that $\Delta y << l$, expand the expression for F_y to show that, to lowest order in Δy, the restoring force has the form:

$$F_y = \beta\,\frac{\Delta y^3}{l^2}$$

and find the coefficient β.

d. Is the oscillation in the y direction simple harmonic motion?