The University of Sydney

MATH1903 Integral Calculus and Modelling (Advanced)

Semester 2 **Practice Questions for First Quiz** 2017

1. Suppose that $f : \mathbb{R} \to \mathbb{R}$ is continuous and a, b, c and k are constants and $k \neq 0$. Use integration by substitution to verify the following:

$$\int_a^b x f(c - kx^2)\, dx \;=\; \frac{1}{2k} \int_{c-kb^2}^{c-ka^2} f(x)\, dx$$

2. Without doing any antidifferentiation, explain why the following inequalities hold:

$$\frac{\pi}{6\sqrt{3}} \;\leq\; \int_{\pi/6}^{\pi/3} \tan x\, dx \;\leq\; \frac{\pi}{2\sqrt{3}}$$

3. Suppose that f is an odd function and g is an even function such that $\displaystyle\int_0^2 f(x)\, dx = 3$ and $\displaystyle\int_0^2 g(x)\, dx = -3$. Use this information to evaluate the following:

$$\int_{-2}^{2} \big(2 - g(x)\big)\big(f(x) + \sin x + 1\big)\, dx$$

4. Let $\mathcal{C}$ be the curve given by that part of the hyperbola $xy = 1$ in the first quadrant that joins the points $(\frac{1}{100}, 100)$ and $(100, \frac{1}{100})$. Show that the length of $\mathcal{C}$ is given by the following definite integral (but do not try to evaluate it):

$$\int_{1/100}^{100} \frac{\sqrt{x^4 + 1}}{x^2}\, dx$$

5. Use the disc method to calculate exactly the volume of the solid obtained by rotating about the y-axis the region of the plane bounded by $x = 0$, $y = 8$, and $y = x^3$.

6. Use the cylindrical shell method to confirm your answer to the previous question.

7. Evaluate the following:

$$\int_0^{\pi/2} \sin^5\theta \cos^5\theta \, d\theta$$

8. Consider the function $F(x)$ where $x > 0$ and

$$F(x) = \int_{\sin x}^{\sqrt{x}} \cos(t^3)\, dt\,.$$

Find a simplified expression for the derivative $F'(x)$.

9. Let f be a function such that, for $x > 0$,

$$\frac{\sin(\pi x)}{x} \;=\; \int_0^{x^2} f(t)\, dt\,.$$

Evaluate $f(25)$ exactly.

10. Using an appropriate Riemann sum approximation to an appropriate definite integral, explain why the following inequality is true:

$$1 + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{999} \;\geq\; \frac{\ln 1001}{2}$$