THE UNIVERSITY OF SYDNEY
MATH1905 Statistics Advanced

Semester 2	**Tutorial Week 05 – Solutions**	2012

1. To 4 dp: 0.2335,0.1211,0.0282 and $\mathrm{P}(X=3)=0.2668$ so $\mathrm{P}(X \leq 3)=0.2335+0.1211+0.0282+0.2668=0.6496$.

2. Let X denote the count of children with type O blood. $X \sim \mathcal{B}(5,0.25)$ (4dp)

$$\mathrm{P}(X=2)=\binom{5}{2}0.25^2(0.75)^3=0.2637.$$

3.
```
> round(pbinom(5,8,0.4),4)
[a] 0.9502
```

4. $\mathrm{P}(\{6\})=1/6$, let X denotes the number (count) of 6's in seven throws, then $X \sim B(7,1/6)$. To 4dp:

$$\mathrm{P}(X \geq 3)=1-\mathrm{P}(X<3)=1-\mathrm{P}(X \leq 2)=1-0.9042=0.0958$$

5. To 4dp: $\mathrm{P}(X=2)=\dfrac{(0.5)^2}{2}\exp(-0.5)=0.0758$, $\mathrm{P}(X=1)=0.3033$, $\mathrm{P}(X=0)=0.6065$ and so $\mathrm{P}(X \leq 2)=0.0758+0.3033+0.6065=0.9856$. In R with `ppois(2,0.5)`.

6. Let X denote the number of calls during a 1 minute internal: $X \sim \mathcal{P}(0.5)$, the probability of no calls is $\mathrm{P}(X=0)=0.6065$ and the probability of one call is $\mathrm{P}(X=1)=0.3033$.
Let Y denote the number of calls during a 5 minute internal: $Y \sim \mathcal{P}(2.5)$, the probability of no calls during a five minute interval is $\mathrm{P}(Y=0)=\exp(-2.5)=0.082$.

7.
```
> round(1-ppois(4,5),4)
[c] 0.5595
```

8. $\mathrm{E}X=1\times 0.35+2\times 0.3+3\times 0.25+4\times 0.1=2.1$
$\mathrm{E}(1/X)=1\times 0.35+(1/2)\times 0.3+(1/3)\times 0.25+(1/4)\times 0.1=0.6083$
$\mathrm{E}X^2=1\times 0.35+2^2\times 0.3+3^2\times 0.25+4^2\times 0.1=5.4$
$\mathrm{Var}(X)=5.4-(2.1)^2=0.99$

9. (a) The pgf is

$$\begin{aligned}\pi(s) &= \sum_{i=0}^{\infty} pq^i s^i \\ &= \frac{p}{(1-qs)}, \quad |qs| \leq 1.\end{aligned}$$

(b) $\mathrm{E}(Y)=\pi'(1)=\dfrac{q}{p}$, as $\pi'(s)=\dfrac{pq}{(1-qs)^2}$.

(c) We throw the die until a 3 is observed. We denote X the number of throws. The probability that 6 throws are required is (to 4dp)

$$\mathrm{P}(X=6)=(1-1/6)^5\times(1/6)=0.067.$$

The probability that more than 7 throws are required is

$$\mathrm{P}(X>7)=1-\mathrm{P}(X \leq 7)=0.2791.$$

10. (a) $\mathrm{E}Y = 6$, $\mathrm{Var}(Y) = 100$

(b) $\mathrm{E}(Y) = 5$, $\mathrm{Var}(Y) = 25$

(c) $\mathrm{E}(Y) = 0$, $\mathrm{Var}(Y) = 1$

11. $X \sim B(100, 1/6)$, $\mathrm{P}(X < 21) = \mathrm{P}(X \leq 20) = 0.8481$