THE UNIVERSITY OF SYDNEY

MATH1905 STATISTICS ADVANCED

Semester 2 **Tutorial Week 6** 2012

1. Let $Z \sim \mathcal{N}(0, 1)$.

(a) Use the standard normal table to find

(i) $\mathrm{P}(Z \leq 1.4)$ (ii) $\mathrm{P}(Z > 0.3)$ (iii) $\mathrm{P}(-0.72 < Z < 0.72)$

(iv) $\mathrm{P}(|Z| > 1.96)$.

Check your answers with `R`.

(b) Use the standard normal table to find z such that

(i) $\mathrm{P}(Z \leq z) = 0.90$ (ii) $\mathrm{P}(Z > z) = 0.95$ (iii) $\mathrm{P}(|Z| < z) = 0.90$.

Check your answers with `R`.

2. If $X \sim \mathcal{N}(10, 16)$, use the standard normal table to find

(a) $\mathrm{P}(X > 12)$ (b) $\mathrm{P}(X < 14)$ (c) $\mathrm{P}(8 < X < 13)$

(d) $\mathrm{P}(X \leq 9.5)$ (e) $\mathrm{P}(|X - 10| < 6)$

Check your answers with `R`.

3. In a photographic process, the developing time of prints may be looked upon as a random variable having a normal distribution with $\mu = 15.4$ seconds and $\sigma = 0.48$ seconds. Find the probability that the time it takes to develop one print is

(a) at least 16 seconds (b) at most 14.2 seconds

(c) between 15.0 and 15.8 seconds.

4. Glaucoma is a disease of the eye that is manifested by high intraocular pressure. The distribution of intraocular pressure in unaffected adults is approximately normal with mean 16 mm Hg and standard deviation 4 mm Hg.

(a) If the normal range for intraocular pressure (in mm Hg) is considered to be 12 – 20, what percentage of unaffected adults would fall within this range?

(b) An adult is considered to have *abnormally high* intraocular pressure if the pressure reading is in the top percentile (1 percent) for unaffected adults. State pressures considered to be abnormally high.

5. The function

$$
\begin{aligned}
f(x) &= Cxe^{-x}, \qquad x > 0, \\
&= 0, \qquad \text{otherwise}
\end{aligned}
$$

is a probability density function.

(a) Find C by solving $\displaystyle\int_0^\infty f(x)dx = 1$.

(b) Calculate $\mathrm{E}(X)$ and $\mathrm{var}(X)$.

(c) Use Chebyshev's inequality to bound $\mathrm{P}(X > 6)$.

(d) Calculate the exact value of $\mathrm{P}(X > 6)$.

MATH1905 QUIZ 1 is in week 7. Review tutorials and problem sheets 1-6.

Assignment 1 for MATH1905 STATISTICS (due on Tuesday, 2nd October, in week 9) will consist of selected questions from the Problem Sheets for weeks 1, 2, 3, 4, 5, 6, 7, 8.

1. Supply true/false answers to the following statements (with reasons or counterexamples).

(a) $\mathrm{E}(3X-5) = 3\,\mathrm{E}(X) - 5$

(b) $\mathrm{E}\left(\dfrac{2}{X}\right) = \dfrac{2}{\mathrm{E}(X)}$

(c) $\mathrm{E}\left(\dfrac{X}{2}\right) = \dfrac{\mathrm{E}(X)}{2}$

(d) $\mathrm{E}(X^2) = [\mathrm{E}(X)]^2$

2. A population is modelled by the random variable X with probability density function

$$\begin{aligned} f(x) &= kx^2(1-x/2), \qquad 0 < x < 2, \\ &= 0, \qquad \text{otherwise.} \end{aligned}$$

(a) Find k.

(b) Calculate $\mathrm{E}(X)$ and $\mathrm{var}(X)$.

(c) Calculate $\mathrm{P}(X \leq 1)$.

(d) Four independent observations are taken from this population. What is the probability that exactly two are less than 1?

3. We throw a fair six-sided die until a 4 is observed.
(1) What is the probability that (stricly) less than 7 throws are required? (4 d.p.)
(2) *Harder.* What is the *expected* number of throws? (Hint: $\sum d(q^k)/dq = d(\sum q^k)/dq$)

4. *Harder.* Let X be a Poisson random variable i.e. $\mathrm{P}(X = k) = \dfrac{\lambda^k}{k!}e^{-\lambda}$, $k = 0, 1, ...$ Where $\lambda > 0$ is a fixed parameter. Show that $\mathrm{E}(X) = \lambda$. (Hint: $e^x = 1 + x + x^2/2! + x^3/3! + ...$, and Q.3(2))