THE UNIVERSITY OF SYDNEY

PHYS1902 – PHYSICS 1B (ADVANCED)

NOVEMBER 2009

Time allowed: THREE Hours

MARKS FOR QUESTIONS ARE AS INDICATED
TOTAL: 90 marks

INSTRUCTIONS

- **All questions are to be answered.**
- **Use a separate answer book for each section.**
- **All answers should include explanations in terms of physical principles.**

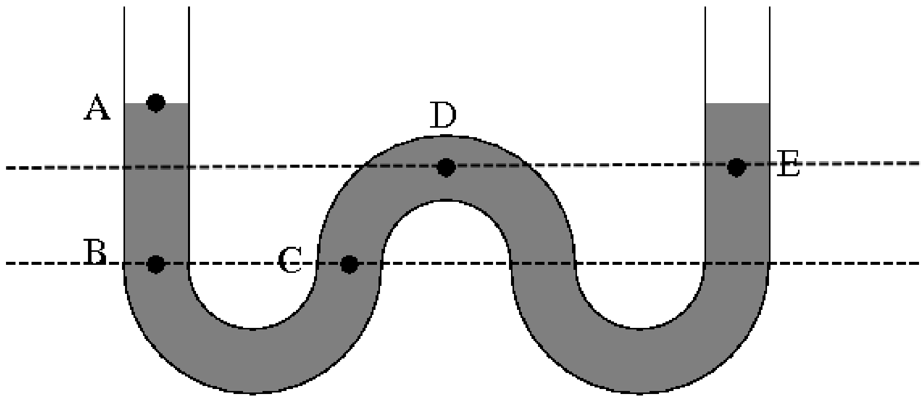
DATA

Density of water	ρ	=	1.00×10^{3} kg.m^{-3}
Density of air	ρ	=	1.20 kg.m^{-3}
Atmospheric pressure	1 atm	=	1.01×10^{5} Pa
Magnitude of local gravitational field	g	=	9.80 m.s^{-2}
Avogadro constant	N_A	=	6.022×10^{23} mol^{-1}
Permittivity of free space	ε_0	=	8.854×10^{-12} F.m^{-1}
Permeability of free space	μ_0	=	$4\pi \times 10^{-7}$ T.m.A^{-1}
Elementary charge	e	=	1.602×10^{-19} C
Speed of light in vacuum	c	=	2.998×10^{8} m.s^{-1}
Planck constant	h	=	6.626×10^{-34} J.s
Rest mass of an electron	m_e	=	9.110×10^{-31} kg
Rest mass of a neutron	m_n	=	1.675×10^{-27} kg
Rest mass of a proton	m_p	=	1.673×10^{-27} kg
Rest mass of a hydrogen atom	m_H	=	1.674×10^{-27} kg
Boltzmann constant	k	=	1.381×10^{-23} J.K^{-1}
Atomic mass unit	u	=	1.661×10^{-27} kg
Rydberg constant	R	=	1.097×10^{7} m^{-1}

SECTION A

Question 1

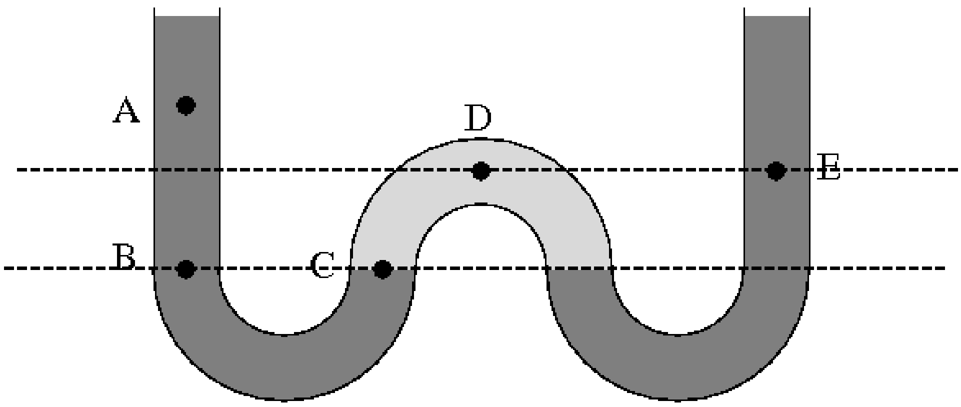
A W-shaped tube is partially filled with water. Both ends of the tube are open and exposed to the atmosphere. Points B and C are at the same height, as are points D and E.



(a) Using the diagram above, rank the points A–E according to pressure from largest to smallest. If the pressure at any two points is the same, state so explicitly. Explain your answers.

(b) Suppose that some oil, with a density less than that of water, was injected into the central portion of the tube, as shown in the diagram below. No water is removed, and no air is allowed inside the central portion of the tube. Would the pressure at point E increase, decrease, or stay the same? Explain your answer.

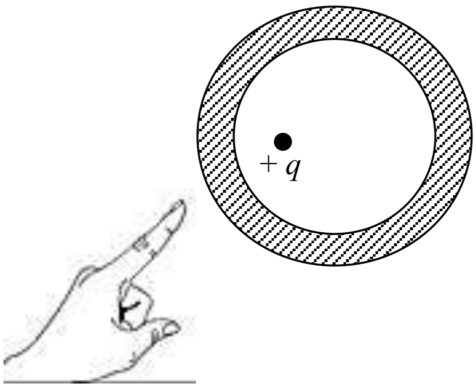
(c) Would the pressure at point D, in the diagram below, be greater than, less than, or equal to the pressure at point E? Explain your answer.



(5 marks)

Question 2

The diagram below shows a cross-sectional view of a charge $+q$ inside a cavity in a metal spherical shell. The shell carries no net charge.



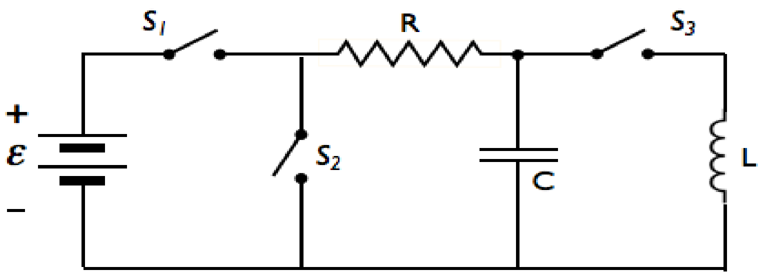
(a) Sketch the distribution of any charges on the sphere.

(b) On the same diagram, sketch some representative electric field lines in the various regions of the diagram, using them to indicate where the field is stronger or weaker (if anywhere).

(c) Someone now touches the outside surface of the spherical shell for a moment and then moves away. Draw a new diagram showing some representative electric field lines. Justify the differences (if any) between your two diagrams.

(5 marks)

Question 3



(a) In the circuit shown above, initially all switches (S_1, S_2, S_3) are open and the capacitor is discharged. Switch S_1 is closed at a time $t = 0$. Sketch the current flowing through resistor R as a function of time indicating the initial value as well as a time scale.

(b) After waiting for a long time with S_1 closed, switch S_1 is opened and switch S_3 is closed (but switch S_2 remains open). Sketch the charge on the capacitor as a function of time, clearly indicating the initial value and a time scale.

(c) With switch S_1 still open and switch S_3 still closed, switch S_2 is now closed. Briefly explain what happens to the energy in the circuit.

(5 marks)

Question 4

A magnet dropped through a hollow copper pipe is observed to fall very slowly. By considering what happens as the magnet moves past a fixed point P on the pipe, carefully explain this observation.

Assume the magnet falls without spinning, with its north pole downwards.

Your explanation should include a diagram and be given in terms of physical principles but without using equations. Your answer should be no more than half a page long, excluding the diagram.

(5 marks)

Question 5

Bremsstrahlung radiation is produced when electrons accelerated through a potential V strike a metal target. The radiation has two components: a continuum component and a line spectrum component.

(a) Briefly explain the origin of the continuum component.

Suppose the radiation is produced by a tungsten target which is then replaced by a nickel target.

(b) Briefly explain why the continuum component changes, or why not, if it does not change.

The continuum component exhibits a maximum frequency given by

$$f_{\max} = \frac{eV}{h}.$$

(c) Explain the origin of this maximum frequency, and hence derive the equation above.

(d) Explain why the existence of a maximum frequency is inconsistent with the classical (wave) picture of light.

(5 marks)

Question 6

The magnitude of the magnetic moment associated with a current loop with current I and area A is

$$\mu = IA.$$

(a) In the Bohr model for the Hydrogen atom the electron orbits the nucleus in a plane with radius *r* and speed *v*, and forms a small current loop. The magnitude of the orbital angular momentum *L* of the electron is assumed to have quantised values:

$$L = n\hbar,$$

with $n = 1, 2, ...$ Show that the magnitude of the magnetic moment of the ground state electron in the Bohr model is

$$\mu = \frac{e}{2m}\hbar.$$

(b) The quantum-mechanically correct quantisation of orbital angular momentum for the electron in the Hydrogen atom is that the magnitude of the angular momentum may have possible values

$$L = \sqrt{\ell(\ell+1)}\ \hbar,$$

where $\ell = 0, 1, 2, ..., n-1$ is the orbital quantum number and n is the principal quantum number. Any given component of the angular momentum, say the *z*–component, may have values

$$L_z = m_\ell \hbar,$$

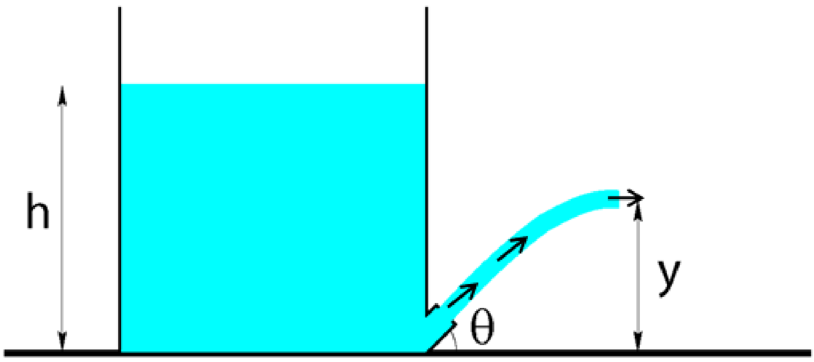
where $m_\ell = 0, \pm 1, \pm 2, ..., \pm \ell$ is the magnetic quantum number.

For a given choice of n and ℓ, it follows that $|L_z| < L$. Briefly explain how this result is a requirement of the Heisenberg uncertainty principle.

(5 marks)

SECTION B
(Please use a separate booklet for this section)

Question 7



A stream of (non-viscous) liquid emerges from a short outlet tube at the base of a large open tank, in which the depth of liquid is h. The tube opening has a cross-sectional area A and is at an angle θ to the ground (as shown in the diagram). You can ignore the height above ground of the end of the outlet tube since it is small compared to h.

(a) Obtain an expression for the speed of the liquid v as it exits the outlet tube.

(b) Show that the maximum height of the stream y can be expressed in terms of θ and h. by $y = h\sin^2\theta$.

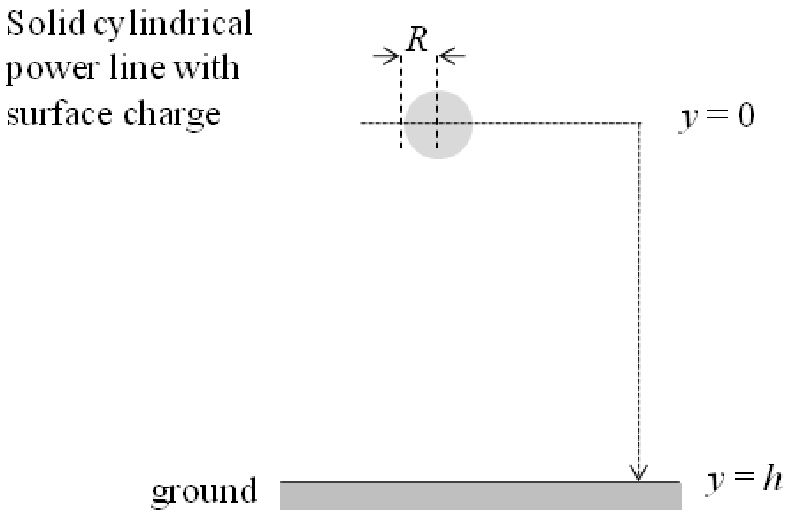
(c) Show that this height y cannot be greater than h

(d) The cross-sectional area of the stream changes after it leaves the outlet tube. Where is this area maximum? Explain why this is so.

(e) Explain qualitatively how the maximum height of the stream would change if the liquid were replaced by a mildly viscous liquid.

(10 marks)

Question 8



A straight, cylindrical, infinitely long power transmission line is at potential V_0 with respect to the ground. The cable has a radius R and a surface charge density σ.

(a) Express the electric field (direction and magnitude) at any point outside the cable in terms of the surface charge density σ and the radius R.

(b) Using $V(h) = 0$, where h is the distance from the axis of the cable to the ground, show that σ can be expressed by

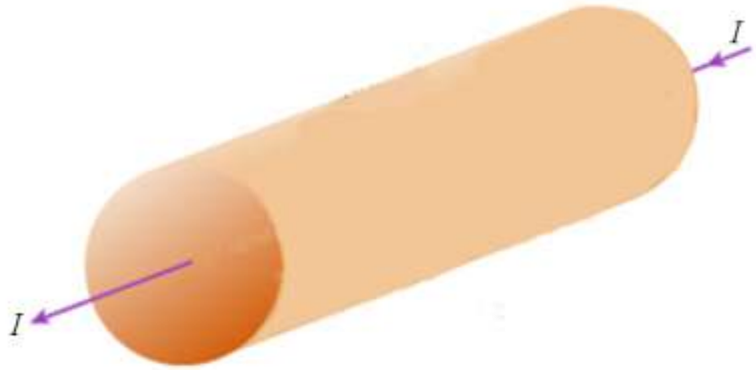
$$\sigma = \frac{\varepsilon_0 V_0}{R \ln(h/R)}.$$

(c) Calculate the electric field just outside the line if $V_0 = 500\,\text{kV}$, the line is 30 m above the ground and is 0.03 m in radius.

(d) The dielectric strength of air is $3.0 \times 10^6\ \text{V.m}^{-1}$. Explain why it would not be a good idea to increase the operating voltage of the line to $750\,\text{kV}$.

(10 marks)

Question 9



Consider a very long *solid* metal cylinder with radius *R*. A current *I* flows along the cylinder to the left with uniform current density.

(a) Using Ampere's Law, derive expressions for the magnitude of the magnetic field $B(r)$ in the range of $r > R$. State all the steps in the argument.

(b) Show that the magnitude of the magnetic field inside the cylinder ($r < R$) is

$$B(r) = \frac{\mu_0 I r}{2\pi R^2}.$$

A long hollow cylindrical shell of radius *2R* is now placed around this same cylinder, such that they share the same axis. A current *I* flows along this outer shell to the *right*, opposite in direction to the current in the inner cylinder.

(c) Calculate the magnetic field of this new configuration at a location outside of this shell ($r > 2R$).

(d) Has the field in the region $R < r < 2R$ changed from that given in part (a)? Justify your answer.

(e) Sketch the magnetic field lines (including their direction) in each region

(i) $r < R$

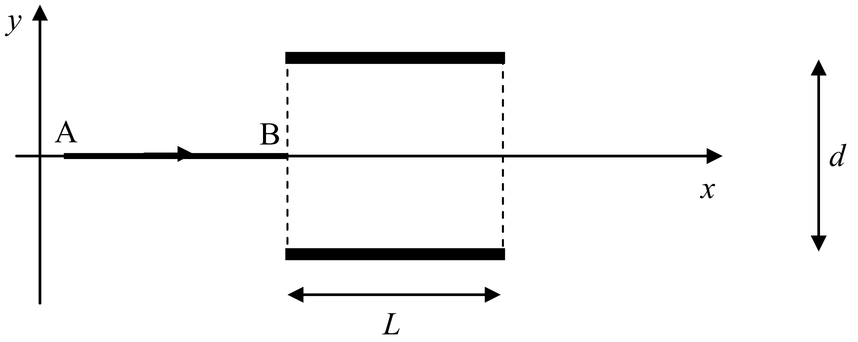
(ii) $R < r < 2R$

(iii) $r > 2R$.

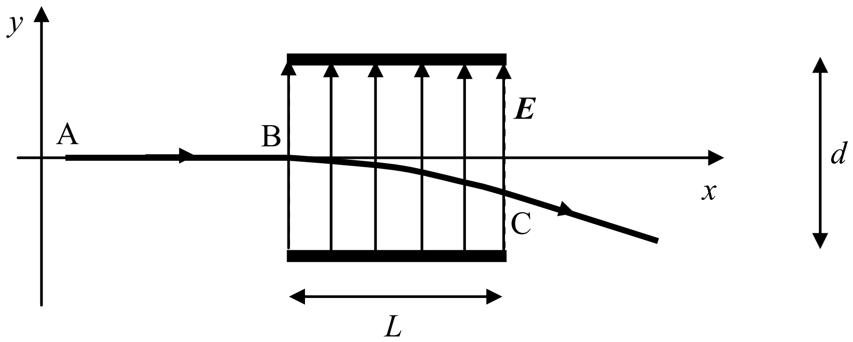
(10 marks)

Question 10

(a) An electron at rest is accelerated from a point A at a potential V_A to a point B at potential V_B. Obtain an expression for the electron's velocity v_B when it reaches B expressed in terms of V_A, V_B, the electron charge e and the electron mass m.



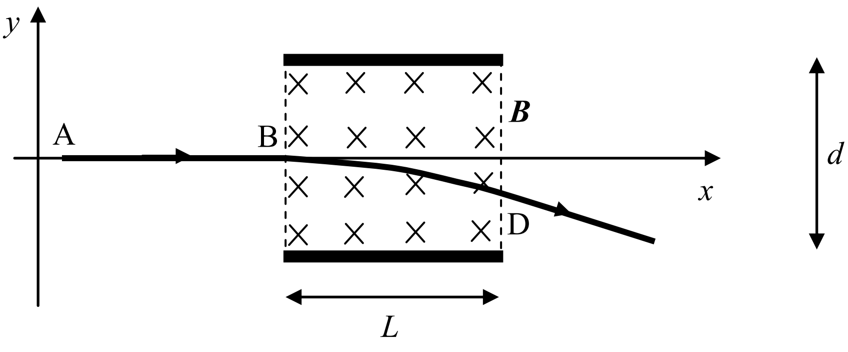
(b) The electron now traverses a uniform electric field $\boldsymbol{E}$ in the space between two charged metal plates. The electric field is zero outside the area of the plates. The plates have a length L and are separated by a distance d with a potential difference V between them.



The electron emerges from the area of the electric field at point C with a velocity v_C. Show that the deflection angle α (the angle between the vectors $\boldsymbol{v_B}$ and $\boldsymbol{v_C}$) is

$$\alpha = \arctan\left(\frac{-eVL}{m\,d\,v_B^2}\right)$$

(c) The electric field is replaced by a magnetic field so that the electron now traverses a uniform *magnetic* field $\boldsymbol{B}$ in exactly the same space between the plates. The magnetic field points into the page in the diagram below. The magnetic field is assumed to be zero outside the area of the plates.



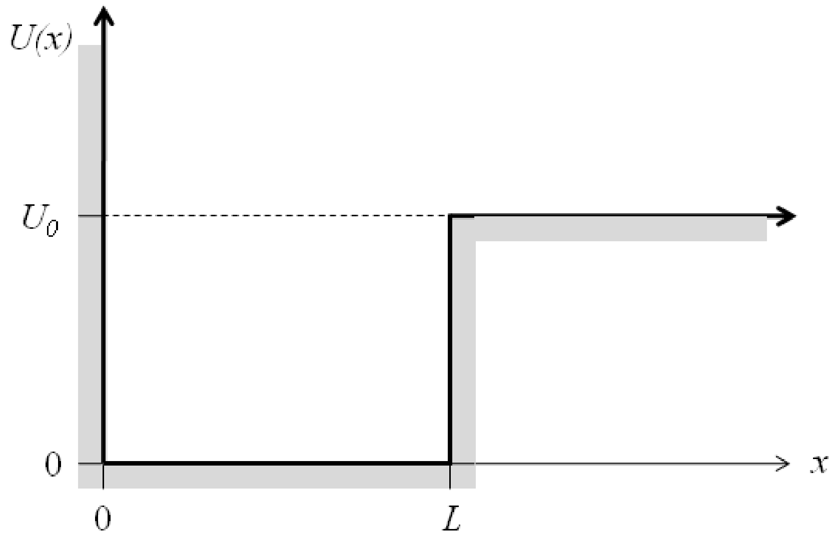
The electron emerges from the area of the magnetic field at point D with a velocity v_D. Show that the deflection angle β (the angle between the vectors $\boldsymbol{v_B}$ and $\boldsymbol{v_D}$) is

$$\beta = \arcsin\left(\frac{-L\,e\,B}{m v_B}\right)$$

(d) Assuming the electrons at B have a small range of velocities about v_B, which of the two ways of deflecting the beam will produce the narrower beam of electrons emerging from the deflection region? Justify your answer.

(10 marks)

Question 11



Consider the 1-D time-independent Schrödinger equation

$$\frac{-\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2}+U(x)\psi(x)=E\psi(x)$$

for an electron in a "half infinite square well", i.e.

$$U(x)=\begin{cases}\infty & \text{for } x\le 0\\ 0 & \text{for } 0<x<L\\ U_0 & \text{for } x\ge L\end{cases}$$

The electron is assumed to have an energy $E < U_0$.

(a) If the electron is considered to be a classical particle, where could it be found for this potential well? Where could it not be found? Briefly explain your answers.

(b) The wave function for the region $0 < x < L$ is

$$\psi_{\mathrm{I}}(x)=Ae^{ikx}+Be^{-ikx}.$$

Using the Schrödinger equation, determine the value of the wave number k.

(c) The wave function in the region $x \ge L$ is

$$\psi_{\mathrm{II}}(x)=Ce^{-\alpha x}.$$

Using the Schrödinger equation, determine the value of α.

(d) The boundary condition on the wave function at $x = 0$ is

$$\psi_I(0)=0,$$

and the boundary conditions at $x = L$ are

$$\psi_I(L)=\psi_{II}(L) \quad \text{and} \quad \psi_I'(L)=\psi_{II}'(L),$$

where a prime denotes the derivative. Applying these boundary conditions, show that the energy E of the electron is defined implicitly by the equation

$$\tan\theta = -\frac{\theta}{\sqrt{\theta_0^2 - \theta^2}},$$

where $\theta = \dfrac{\sqrt{2mE}}{\hbar}L$ and $\theta_0 = \dfrac{\sqrt{2mU_0}}{\hbar}L$.

(e) Sketch the curves

$$y = \tan\theta \quad \text{and} \quad y = -\frac{\theta}{\sqrt{\theta_0^2 - \theta^2}}$$

on the same plot. Hence, or otherwise, present an argument that this potential has no energy states at all if

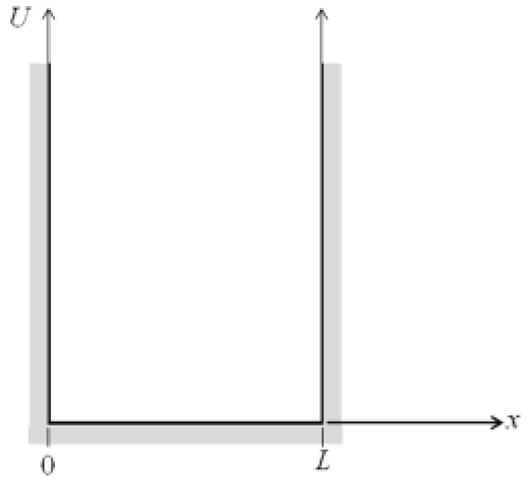
$$\theta_0 < \frac{\pi}{2},$$

i.e., if the potential step satisfies

$$U_0 < \frac{\pi^2\hbar^2}{8mL^2}.$$

(10 marks)

Question 12



Consider an electron in an infinite square well which extends from $x = 0$ to $x = L$ (a "particle in a box"). The time-dependent wave function for the particle in the region of the well may be written

$$\Psi_n(x,t) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \exp\left(-\frac{iE_n t}{\hbar}\right),$$

where the possible energies E_n of the particle are

$$E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}.$$

(a) These wave functions describe stationary (i.e., time-independent) states. Briefly explain what this means, given that the wave functions $\Psi_n(x, t)$ depend on time.

(b) Sketch the ground state ($n = 1$) wave function at time $t = 0$ for the region

$$-\frac{1}{2}L \le x \le \frac{3}{2}L.$$

(c) A reasonable estimate for the uncertainty in the position of the electron is

$$\Delta x = \frac{L}{2}.$$

(i) For the ground state ($n = 1$) write down an estimate for the uncertainty Δp_x in the momentum of the electron. Briefly justify your estimate.

(ii) Show that the estimates of Δx and Δp_x are consistent with the Heisenberg uncertainty principle.

(d) For the ground state ($n = 1$), show that the probability that the electron is located in the region $0 \le x \le \ell$, where $\ell \le L$, is given by

$$p_\ell = \frac{1}{\pi}\left(\frac{\pi\ell}{L} - \frac{1}{2}\sin\frac{2\pi\ell}{L}\right).$$

(10 marks)

This is the end of your questions