The University of Sydney

MATH1901/06 Differential Calculus (Advanced)

Semester 1 **Short answers to exam questions** **2009**

1. (a) Annulus between two concentric circles, including the circles themselves, radii 1 and 2, centre $-1+i$ in second quadrant, inner circle touching both axes.

(b) Roots: $z = 2 \pm i$ and $z = \pm 2i$.

(c) Surjective because every complex number has a fourth root. Not injective because a nonzero complex number has more than one fourth root.

2. (a) (*i*). Directional derivative: $D_{\mathbf{u}} f(3,1) = \nabla f \cdot \hat{\mathbf{u}} = \dfrac{6\mathbf{i}+8\mathbf{j}}{13} \cdot \dfrac{3\mathbf{i}-2\mathbf{j}}{\sqrt{13}} = \dfrac{2}{13\sqrt{13}}$.

(*ii*). Tangent plane: $z = (6x+8y)/13 + \ln(13) - 2$.

(b) $T_6(x) = 1 - \dfrac{x^2}{3!} + \dfrac{x^4}{5!} - \dfrac{x^6}{7!}$, $f''(0) = -\dfrac{1}{3}$, $f^{(4)}(0) = \dfrac{1}{5}$, $f^{(6)}(0) = -\dfrac{1}{7}$.

(c) Limit is -4. (Use $T_3(x) = x - x^3/6$ for $\sin x$ about $x = 0$.)

3. (a) (*i*). Limit is $a+b$. (Rationalise numerator or use binomial series.)

(*ii*). Limit is $-\infty$. (The logarithm of zero, approached from the right.)

(*iii*). Limit is $2/\pi^2$. (Use l'Hôpital's rule twice.)

(b) Vertical tangent because $\lim\limits_{x\to 0} (x^{3/5} - 0^{3/5})/x = \lim\limits_{x\to 0} x^{-2/5} = +\infty$ (two-sided).
Alternatively, because inverse $y = x^{5/3}$ has a horizontal tangent at $x = 0$.
Also acceptable: $(d/dx)x^{3/5} = (3/5)x^{-2/5} \to +\infty$ as $x \to 0$ (two-sided).

4. (a) Point $(R, 2R)$ occurs at $\theta = \pi/2$. Slope of tangent: $dy/dx = y'(\theta)/x'(\theta) = -1$.
Equation of tangent line: $y = 3R - x$.

(b) (*i*). $f_x(0,y) = 0$, $f_x(0,0) = 0$, $f_y(x,0) = x$, $f_y(0,0) = 0$.

(*ii*). $f_{xy}(0,0) = 0$, $f_{yx}(0,0) = 1$. (Able to be different because f_{xy} and f_{yx} are both discontinuous at $(0,0)$ and because f_y is not differentiable at $(0,0)$.)