THE UNIVERSITY OF SYDNEY
MATH1905 STATISTICS ADVANCED

Semester 2	**Tutorial Week 11 – Solutions**	2012

1. P = $\mathrm{P}(t_{21} \geq 4.22) = 0.0002$ (4dp). Thus, there is strong evidence that the population mean is greater than 10.

2. Let D_i denote the difference $(A - B)$. Assume $D_i \sim \mathcal{N}(\mu, \sigma^2)$.

(a) Test $H_0 : \mu = 0$ against $H_1 : \mu \neq 0$.

(b) $\overline{d} = 6.2$, $s_d = 15.51$. The observed test-statistic is

$$\tau_{\text{obs}} = \overline{d} \times \sqrt{20}/s_d = 1.78.$$

Since we have a two-sided test,

$$P\text{-value} = 2 \times \mathrm{P}(t_{19} \geq 1.78) = 0.09 \quad (2dp)$$

Thus, $0.05 < P$-value < 0.1 and we conclude that there is not sufficient evidence against the null hypothesis (some marginal evidence). This is a paired experiment where the same cars are used for the two different quotes (self-pairing). The observations are not independent. A two sample test is not appropriate.

(c) The boxplot is roughly symmetric so the normal assumption is not violated.

(d) A test that does not require the normality assumption is the sign-test (here a paired sign-test). Let p_+ denote the probability of a positive difference. If X denotes the number of positive differences then large values of $|X - 10|$ argue in favour of H_1. We test $H_0 : p_+ = 0.5$ against $H_1 : p_+ \neq 0.5$. We observe $x = 14$ positive differences. If H_0 is true we have $X \sim \mathcal{B}(20.0.5)$. With the normal approximation we obtain,

$$P\text{-value} = \mathrm{P}(|X - 10| \geq 4) \simeq \mathrm{P}(|Z| \geq 3.5/\sqrt{5}) = 2\,\mathrm{P}(Z \geq 1.565) = 0.12$$

Exact test: P-value is 0.1153.

3. $s_p^2 = 7.5637$, $\tau_{\text{obs}} = 2.746$, P-value $= 2 \times \mathrm{P}(t_{19} \geq 2.746) = 0.0128$, we have sufficient evidence against H_0.

4. This is a two-sample t test. $H_0 : \mu_x = \mu_y$ against $H_1 : \mu_x \neq \mu_y$. We observe,

$$s_p^2 = 3.4547, \quad s_p = 1.8587$$

$$\tau_{\text{obs}} = 0.3558, \quad P\text{-value} = 2 \times \mathrm{P}(t_{22} \geq 0.3558 = 0.72 \quad (2dp).$$

The data are consistent with H_0.