(A)

MATH1903 Lecture 1 Thurs 3/8/2017

Welcome to MATH1903

Webpage $\boxed{\text{Google MATH1903}}$

- lecturers { David Easdown Weeks 1–6; Daniel Daners Weeks 7–12 }

- information sheet
  - tutorials start in Week 2

    $\boxed{\text{based on green sheet handed out today}}$

  - Kopystop notes for 2nd half available

    $\boxed{\text{Notes for first half available from webpage}}$

  - good reference books
    - Stewart "Calculus"
    - Spivak "Calculus"

  - assessments
    - First Assignment due 17/10 (5%)
    - First Quiz in Week 6 (10%)

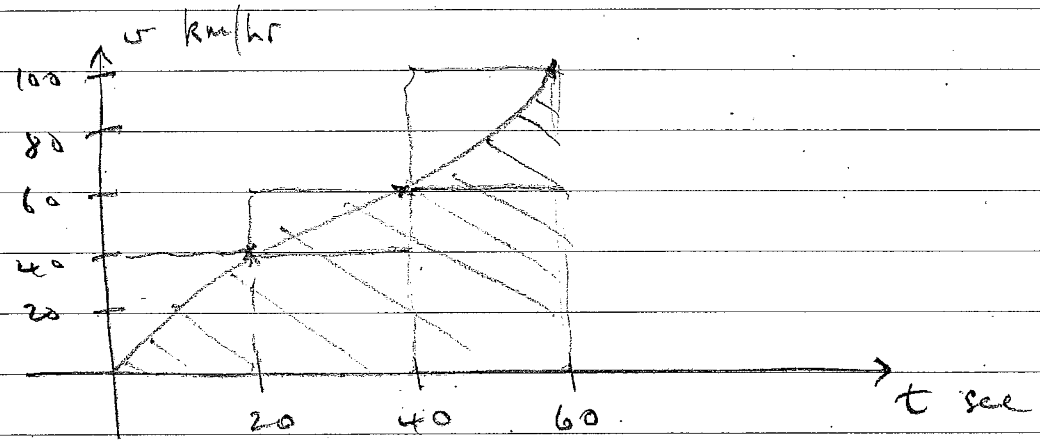
- webpage links to materials
  - exercise sheets Weeks 2–7
    - preambles with summaries of main ideas
    - revision & exploration to try before tutorial
    - starred exercises more difficult
    - short answers at end of sheets (longer worked solutions released later)

Exercises 1, 2, 3 for Week 2 relate to first Assignment, exploring properties of odd & even functions

Notes 1st Instalment (link from webpage)

- table of contents
- Introduction pp 1–18

p1, p2



(area under curve) $\times \frac{1000}{3600}$

gives total distance travelled.

p3 use lower & upper rectangles to bound this

p4, p5 : increasing number of measurements improves accuracy of approximation.

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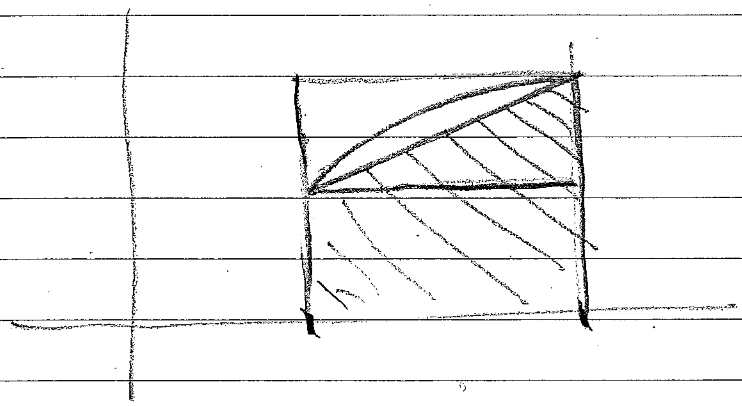
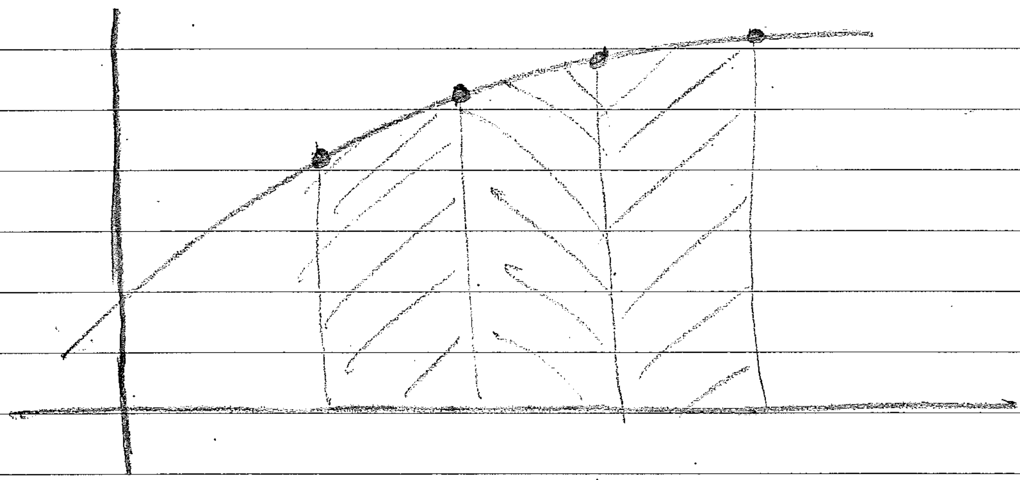
Taking average of lower & upper sums :

- approximating areas using trapezoids





- leads to the Trapezoidal Rule,

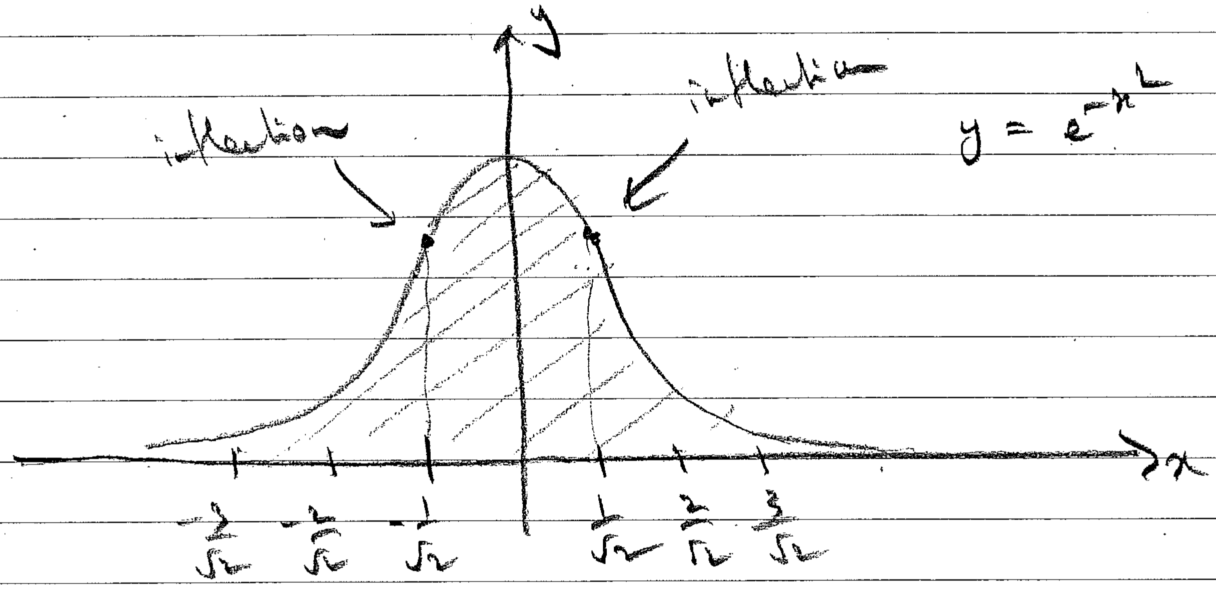
exploiting linear approximations to curves

- Simpson's Rule exploits parabolic approximations

to curves (used by Kepler in 17th century)

- both give elegant formulae for numerical approximations.

<u>Example</u> (Gaussian curve):



$\boxed{\text{Difficult fact (explained later): area under this curve is } \int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi} \approx 1.772}$

Using intervals above, of width $\frac{1}{\sqrt{2}}$, the Trapezoidal Rule gives

$$\text{area} \approx \frac{1}{2\sqrt{2}}\left(e^{-\frac{9}{2}} + 2\left(e^{-\frac{4}{2}} + e^{-\frac{1}{2}} + e^{0} + e^{-\frac{1}{2}} + e^{-\frac{4}{2}}\right) + e^{-\frac{9}{2}}\right)$$

$$\approx 1.764.$$

Using Simpson's Rule we get

$$\text{area} \approx \frac{1}{3\sqrt{2}}\left(e^{-9/2} + 4e^{-4/2} + 2e^{-1/2} + 4e^{0} + 2e^{-1/2} + 4e^{-4/2} + e^{-9/2}\right)$$

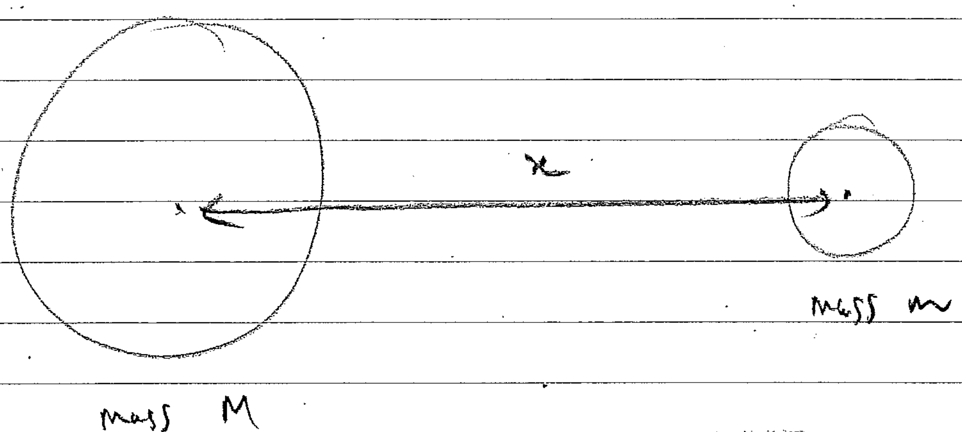
≈ 1.775, agreeing to $\sqrt{\pi}$ to 2 d.p.

(E)

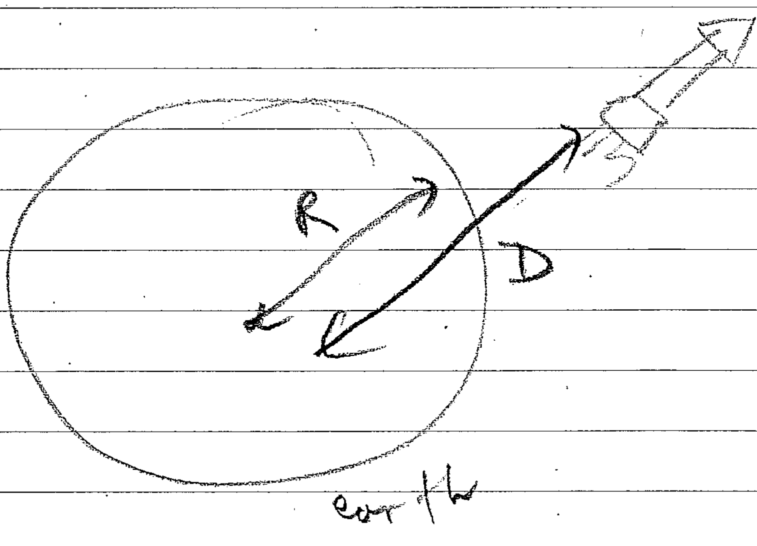
pp 6-10 : birth of calculus

- Newton using pure thought / imagination to derive a formula for the escape velocity of a rocket

in fact approx 11 km/sec



$$\boxed{F = F(x) = G\frac{Mm}{x^2}}$$



(F)

Energy required to get rocket to distance D

$$= \int_R^D \underbrace{F(x)\,dx}_{\text{tiny bit of energy = work expended for a tiny movement } x}$$

Want $\boxed{\underbrace{\lim_{D\to\infty} \int_R^D F(x)\,dx}_{\text{proper integral}} = \underbrace{\int_R^\infty F(x)\,dx}_{\text{called an improper integral}}}$

and put this equal to the kinetic energy associated with a rocket given an escape velocity v_0 :

$$\boxed{\frac{1}{2} m v_0^2 = \int_R^\infty F(x)\,dx}$$

⋮

$$v_0^2 = \frac{2GM}{R}$$

$$\boxed{v_0 = \sqrt{\frac{2GM}{R}} \approx 11 \text{ km/sec}}$$

pp 11-13 : examples of Taylor series.

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots$$

$$e^x + e^{-x} = 2\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)$$

$$\cosh x = \frac{e^x + e^{-x}}{2} = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$$

$$\sinh x = \frac{e^x - e^{-x}}{2} = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\boxed{e^x = \underbrace{\cosh x}_{\text{even}} + \underbrace{\sinh x}_{\text{odd}}}$$

$$\boxed{e^{ix} = \cos x + i \sin x}$$

— famous Euler formula

Put $x = \pi$:

$$\boxed{e^{i\pi} = \cos \pi + i \sin \pi = -1}$$

i.e. $\boxed{e^{i\pi} + 1 = 0}$

Ⓗ

pp 14-18 : exploring exponential & logistic functions:

$\boxed{y = Ae^{kx}}$ solution of $\boxed{y' = ky}$

exponential growth/decay

differential equation

$\boxed{y = \dfrac{L}{1 + Ke^{-kx}}}$ solution of $\boxed{y' = ky - ay^2}$

"growth" factor

"death" factor.

logistic function