PHYS 1901 – Physics 1A (Advanced) Mechanics module

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Rotation of Rigid Bodies

Chapter

9

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Energy in rotation

For a mass at point P

$$KE_i = \frac{1}{2} m_i v_i^2 \Leftarrow v_i = r_i \omega$$

$$= \frac{1}{2} m_i r_i^2 \omega^2$$

$$= \frac{1}{2} (m_i r_i^2) \omega^2$$

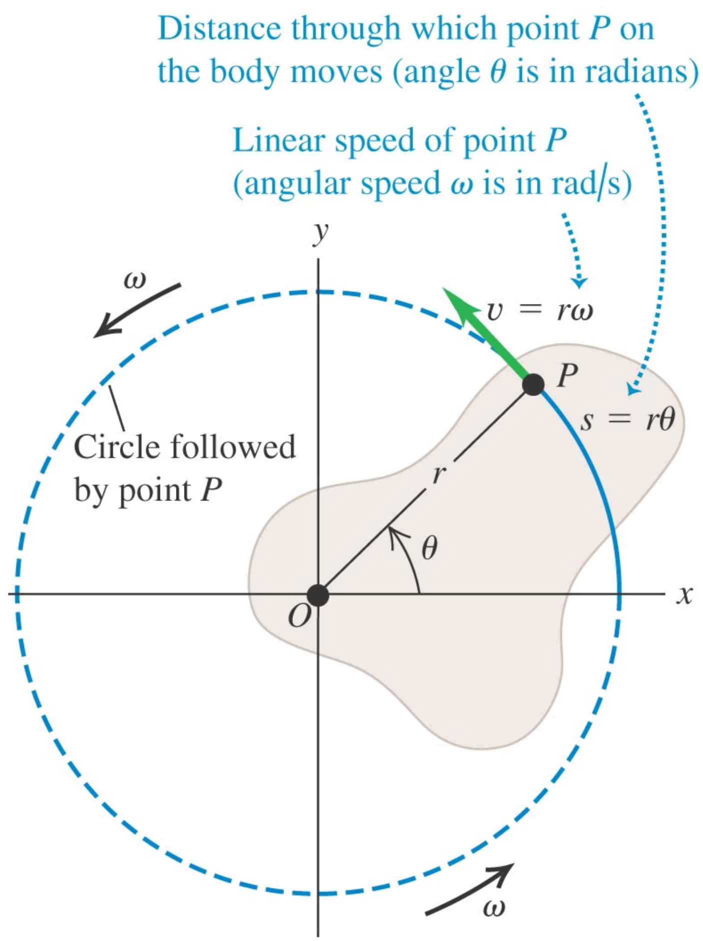
Total kinetic energy

$$KE = \sum_i KE_i$$

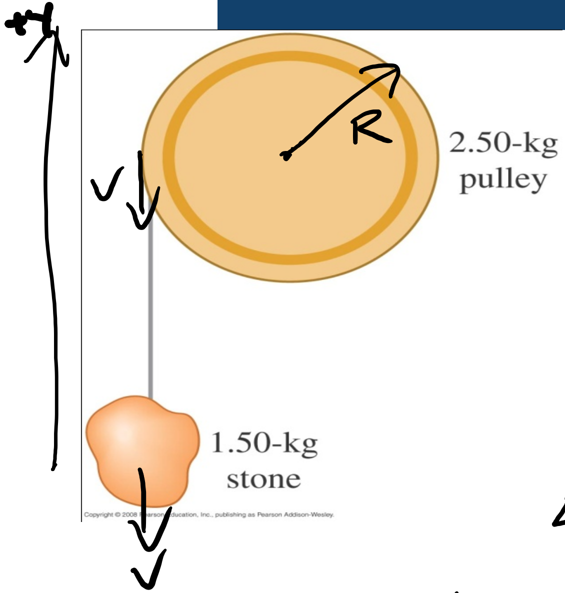
$$= \sum_i \frac{1}{2} (m_i r_i^2) \omega^2$$

$$= \frac{1}{2} \left[\sum_i m_i r_i^2 \right] \omega^2$$

$$\boxed{KE = \frac{1}{2} I \omega^2}$$



Problem 9.49



Pulley is a frictionless solid disk. $I_p = \frac{1}{2} m_p R^2$

Light wire is attached to stone, and wrapped around the disk.

System is released from rest.

(a) How far must the stone fall so that the pulley has 4.50 J of kinetic energy?

(b) What percent of the total kinetic energy does the pulley have?

$$\Delta K = W_g = -\Delta U_g$$

$$K_f - \cancel{K_i} = m_s g h$$

two parts

$$K_{f,s} + K_{f,p} = m_s g h$$

$$K_{f,s} = \frac{1}{2} m_s v_s^2$$

$$K_{f,p} = \frac{1}{2} I_p \omega_p^2$$

$$\boxed{v_s = \omega_p R}$$

$$K_{f,p} = \frac{1}{2}\left(\frac{1}{2} m_p R^2\right)\left(\frac{v_s}{R}\right)^2 = \frac{1}{4} m_p v_s^2$$

$$\frac{1}{4} m_p v_s^2 + \frac{1}{2} m_s v_s^2 = m_s g h$$

Dynamics of Rotational Motion

Chapter

10

Torque

Opening a door requires not only an application of a force, but also how the force is applied;

- It is ‘easier’ pushing a door further away from the hinge.
- Pulling or pushing directly towards/away from the hinge does not work!

From this we get the concept of **torque**.

Torque

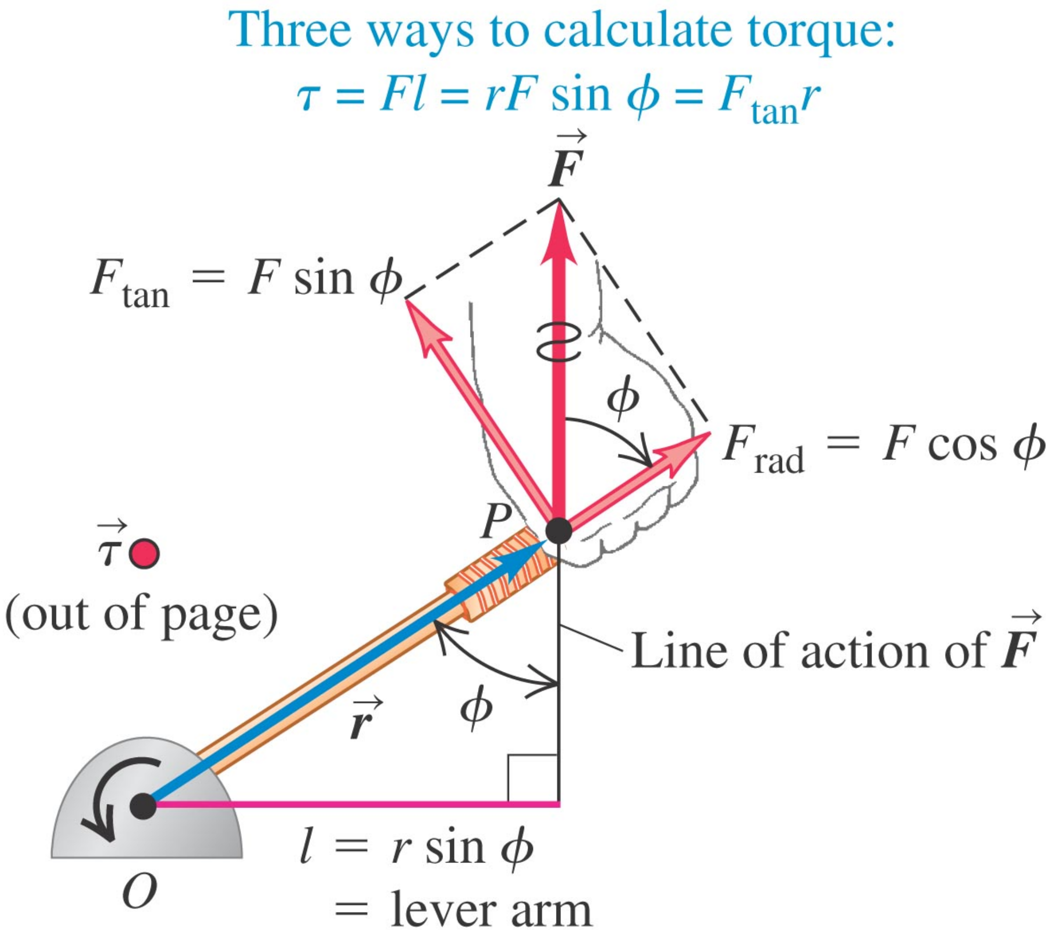
Torque causes angular acceleration

Only the component of force tangential to the direction of motion has an effect

Torque is

$$\tau = rF_{tan} = rF\sin\phi$$

Units: N·m



Torque

Like force, torque is a vector quantity (in fact, the other angular quantities are also vectors). The formal definition of torque is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

Magnitude

$$|\vec{\tau}| = |\vec{r}||\vec{F}|\sin\phi$$

where the x is the **vector cross product**.

Note, magnitude is zero if vectors are parallel

In which direction does this vector point?

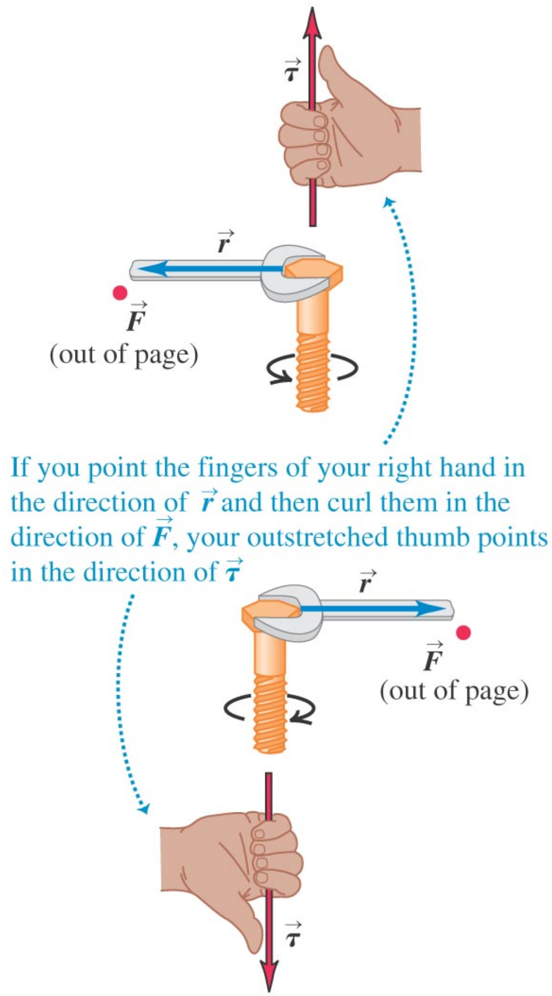
Vector cross product

The magnitude of the resultant vector is

$$|\vec{A} \times \vec{B}| = |\vec{A}|\,|\vec{B}| \sin\phi$$

and is perpendicular to the plane containing vectors A and B.

Right hand rule defines the direction



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Torque and acceleration

At point P, the tangential force gives a tangential acceleration of

Point mass $\quad F_{tan} = m\,a_{tan}$

$\qquad = m(r\alpha)$

Multiply both sides by r

$$r F_{tan} = (m r^2)\alpha$$

Point mass: $\quad \tau = I_{\text{point mass}}\,\alpha$

General formula $\quad \vec{\tau} = I\vec{\alpha}$

Newton's second law for rotational dynamics