

Assignment

MATH1903: Integral Calculus and Modelling (Advanced)

Semester 1, 2012

Lecturers: Daniel Daners and James Parkinson

This assignment is worth 10% of your overall assessment for MATH1903

Tuesday 9th of October by 4pm

Please show all working, and present your arguments clearly. After all, mathematics is about communicating your ideas. This is a skill that takes time and effort to master.

Submission Instructions This assignment is due by 4pm on Tuesday 9th October. It should be posted in the locked collection boxes on the verandah of Carlaw Level 3. These boxes are at the end of the verandah closest to Eastern Avenue (not the locked collection boxes near the pyramids on Carlaw Level 3, nor the open pigeonholes). Please do not post your assignment before Tuesday 9th October, since the boxes are also used for the collection of assignments in other units.

Your assignment must be stapled inside a manilla folder, on the front of which you should write the initial of your family name as a LARGE letter. A cover sheet must be signed and attached (see over, or <http://www.maths.usyd.edu.au/u/UG/asscover.pdf>).

See page 25 of the Junior Mathematics Handbook for further information regarding submission of assignments. Assignments without a cover sheet, or in the incorrect format, may not be marked.

1. Let

$$F(x) = \int_0^x t \cosh(t^4) dt$$

Calculate the following. In part (a) your answer will involve $F(1)$.

(a) $\int_0^1 xF(x) dx$

2 Marks

(b) $\lim_{x \rightarrow 0} \frac{F(x)}{x^2}$

1 Mark

2. Calculate the value of the improper integral

3 Marks

$$\int_0^{\frac{\pi}{2}} \left[\operatorname{cosec} x - \frac{1}{x} \right] dx.$$

You may use the standard integral $\int \operatorname{cosec} x dx = -\ln |\operatorname{cosec} x + \cot x| + C$.

3. Does the series

2 Marks

$$\sum_{n=0}^{\infty} \frac{n^4}{4^n}$$

converge? Carefully justify your claim using a suitable test.

4. You are given that the equation

3 Marks

$$y^3 + y^2 + y - 3 = x$$

implicitly defines y as a function of x around $x = 0$, with $y(0) = 1$. Find the second order Taylor polynomial $T_2(x)$ of $y(x)$ about $x = 0$.

5. Consider the non-linear differential equation

$$xy' = y + ax\sqrt{x^2 + y^2}, \quad (1)$$

where $x > 0$, and $a > 0$ is a constant.

- (a) If y solves (1), show that $v := yx^{-1}$ satisfies the differential equation

2 Marks

$$v' = a\sqrt{1 + v^2}$$

- (b) Use the differential equation in part (a) to get the general solution to (1).

3 Marks

- (c) Find the particular solution of (1) with $y(1) = 0$.

1 Mark

6. For $n \in \mathbb{N}$ let

$$f(x) = \frac{1}{x^n(1-x)}$$

- (a) Find the partial fraction decomposition of f .

2 Marks

You may use that $1 - x^n = (1 - x)(x^{n-1} + x^{n-2} + \cdots + x + 1)$

- (b) Find $\int f(x) dx$.

1 Mark

Assignment Cover Sheet

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Semester 1, 2012

This cover sheet must be stapled to the front of your assignment, inside the folder.

This part is to be completed by the student:

Unit name and code MATH1903: Integral Calculus and Modelling

Assignment number Assignment 2

Family Name

Given Names

Student Identification Number

I certify that:

- I have read and understood the *University of Sydney Student Plagiarism: Coursework Policy and Procedure* at <http://www.maths.usyd.edu.au/u/UG/Plagiarism.pdf>;
- this assignment is all my own work, and no part of this assignment has been copied from another person;
- I have not allowed my work to be copied by another person.

Signature: **Date:**

Note:

Legitimate cooperation between students on assignments is encouraged, since it can be a real aid to understanding. It is legitimate for students to discuss assignment questions at a general level, provided everybody involved makes some contribution. However, students must produce their own individual written solutions. Copying someone else's work is plagiarism, and is unacceptable.

The University may impose severe penalties for plagiarism.