PHYS 1902 Electromagnetism: 2

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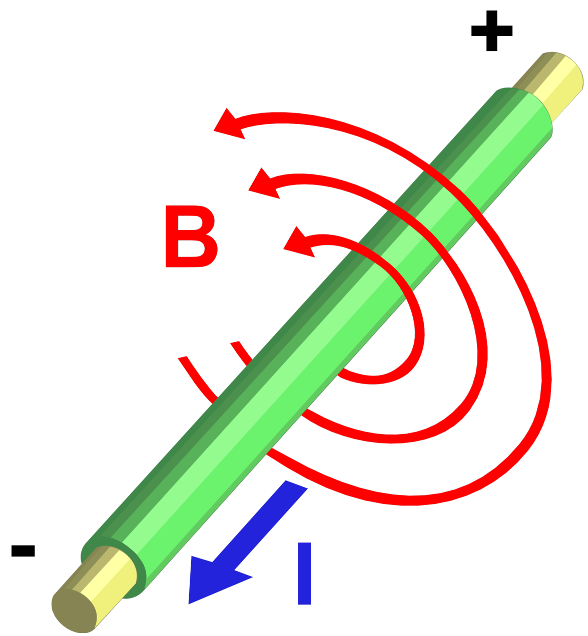
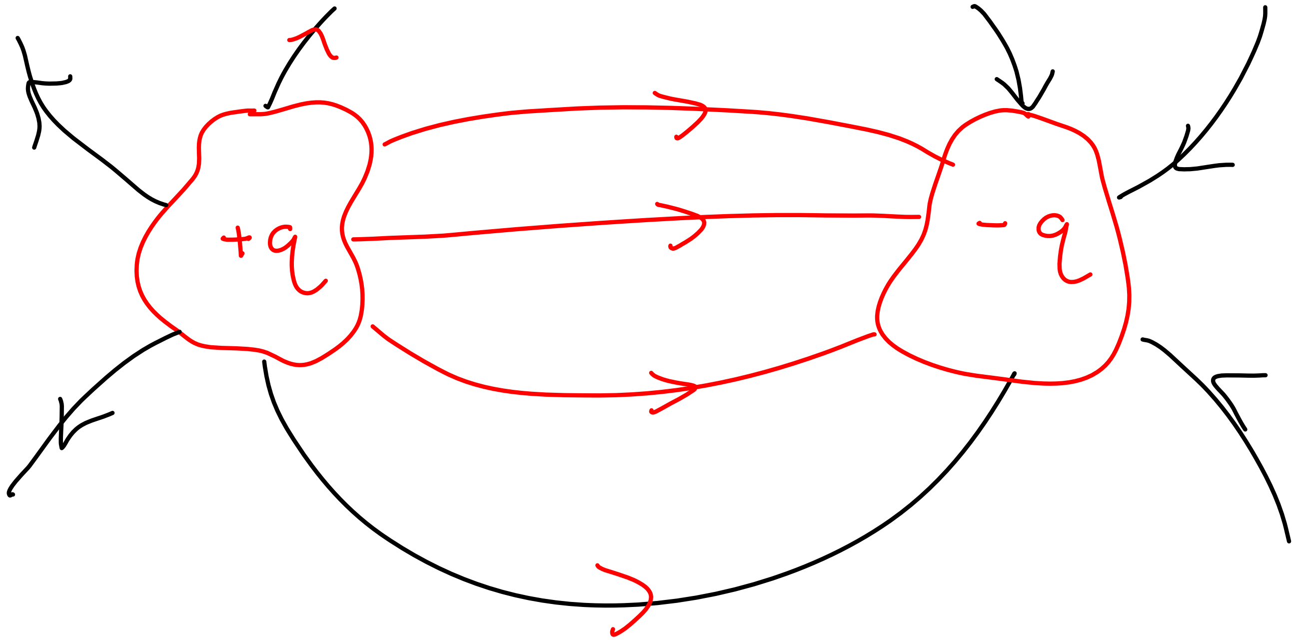


Capacitance

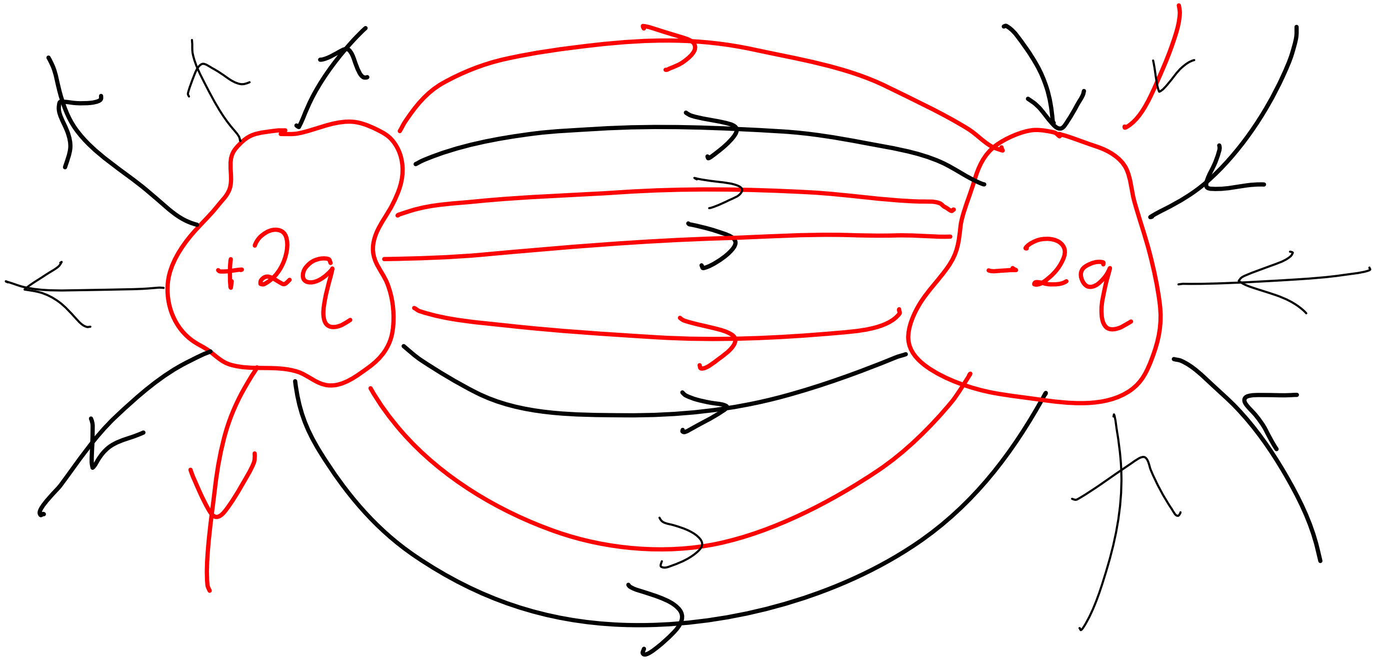


Opposite charges, so attractive electric force

(Something must be holding them apart in a static configuration)

Potential energy is stored in this configuration

Capacitance

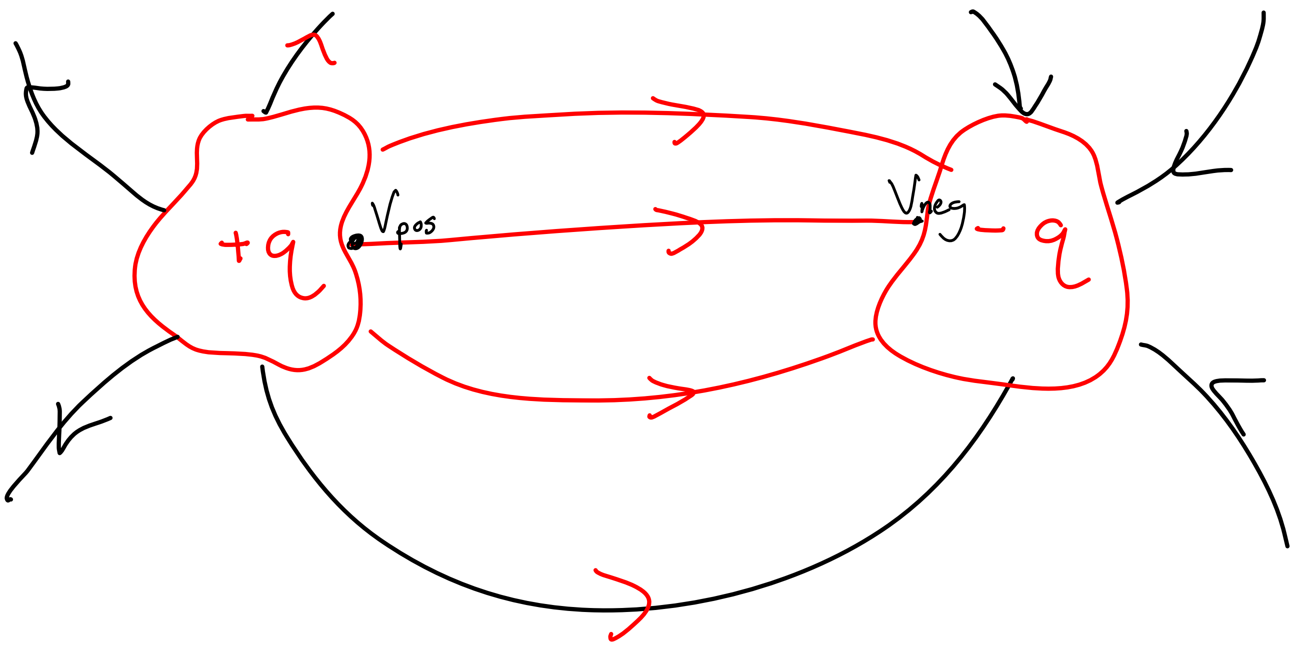


Double the charge (difference)

→ means electric field doubles in strength

→ means voltage doubles

Capacitance



Define capacitance Units = F (Farad)

$$C = \frac{Q}{V} \qquad 1\,F = 1\,\frac{C}{V}$$

Capacitance

Define capacitance $\boxed{C = \frac{Q}{V}}$

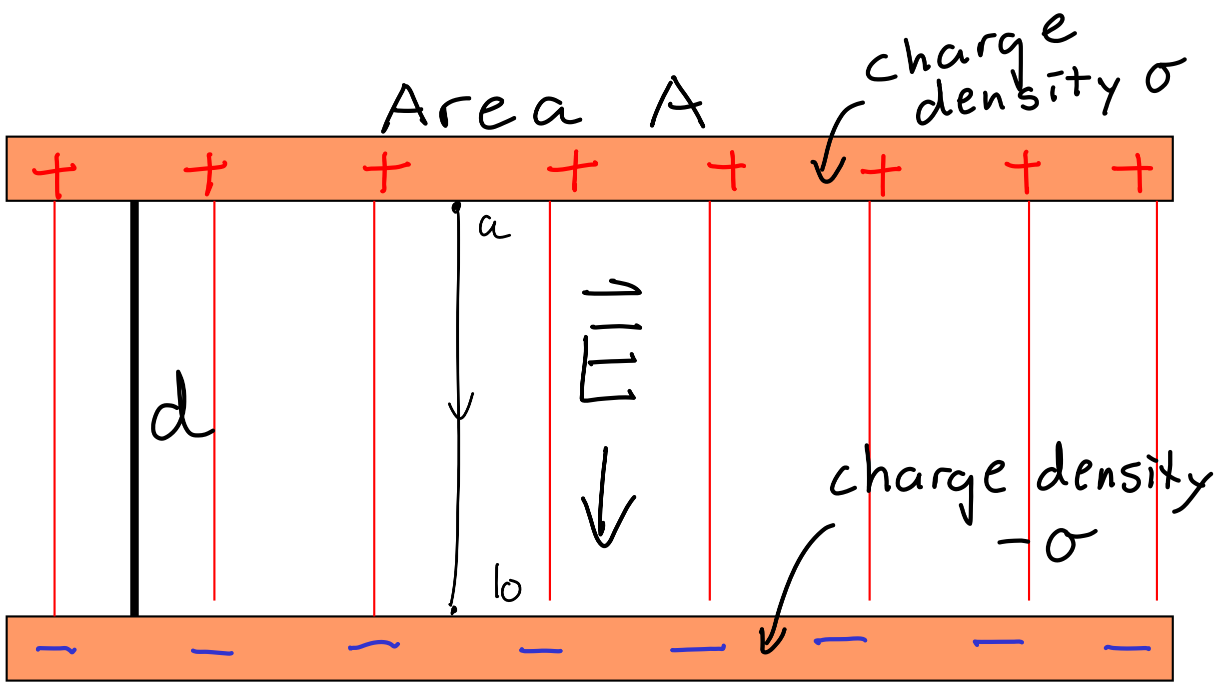
Capacitance: capacity of two conductors for holding electric potential energy

Depends only on the shapes and sizes of conductors (i.e., the configuration)

and on the insulating material between them

NOT on the amount of charge or voltage

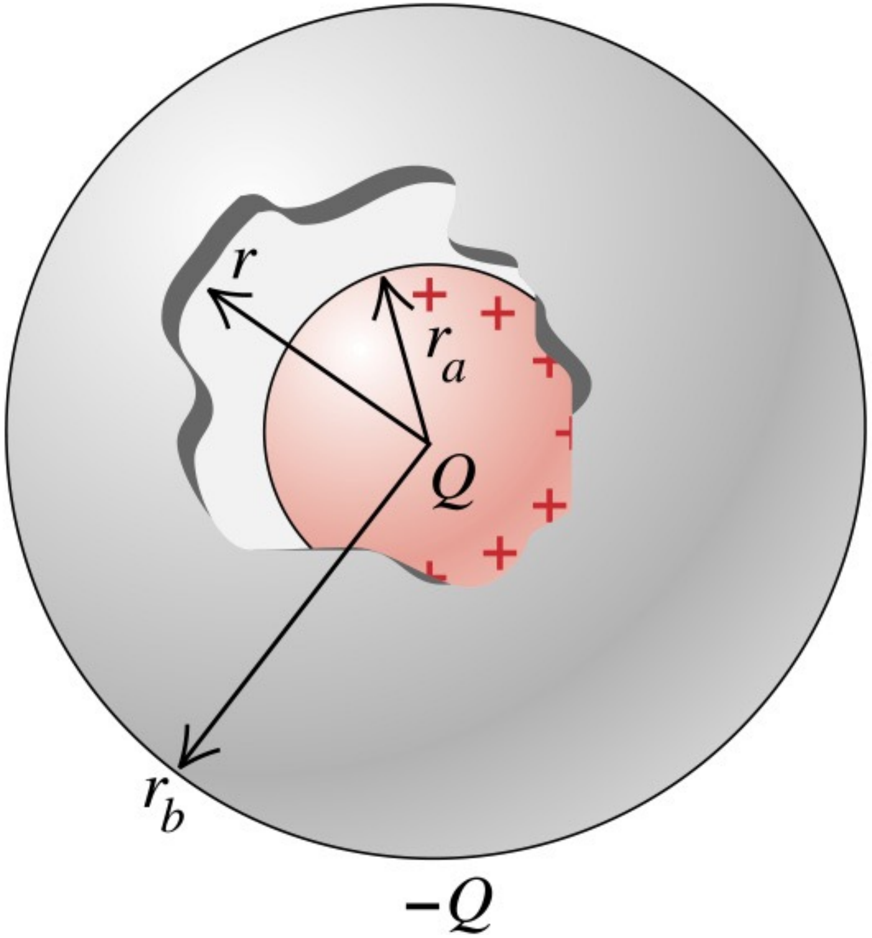
Capacitance of Two Parallel Plates



Plates are close enough that we can use an approx: uniform E field

What is the capacitance?

Spherical Capacitor – 24.3



Concentric spherical conducting shells:

inner shell: radius r_a

outer shell: radius r_b

Both shells are charged, the inner shell with +Q, and the outer with -Q.

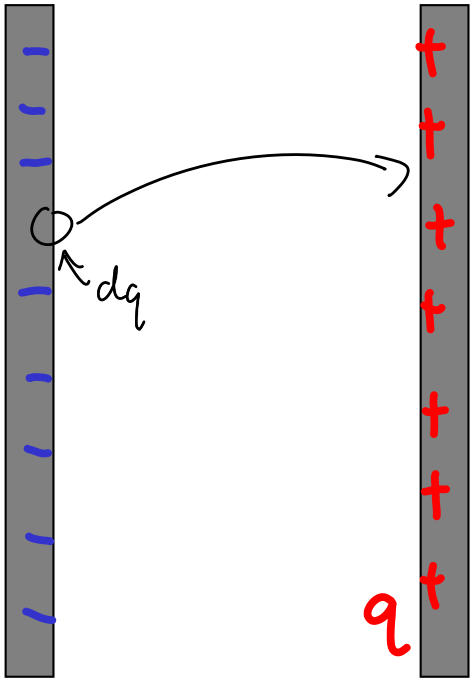
What's the capacitance?

Potential Energy Stored in a Capacitor

Potential energy in a capacitor C with voltage V and charge Q=CV

=

Work needed to "charge" the capacitor, starting from Q=0



Consider point where amount of charge q has been moved

$$C = \frac{q}{V} \Rightarrow V = \frac{q}{C}$$

Move small charge dq

$$dW = dU = V\,dq = \frac{q}{C}\,dq$$

$$W = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\int_0^Q q\,dq = \frac{Q^2}{2C}$$

$$U = \frac{Q^2}{2C}$$

Potential Energy Stored in a Capacitor

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

Puzzle: Alice and Bob argue about the relative "capacity" of two capacitors C and $2C$ to hold potential energy

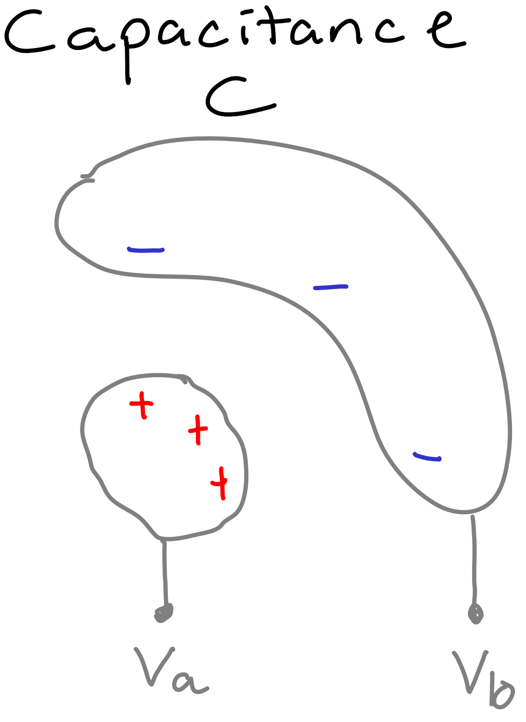
Alice argues that the capacitor $2C$ only holds half the potential energy as C, because $U=Q^2/2C$

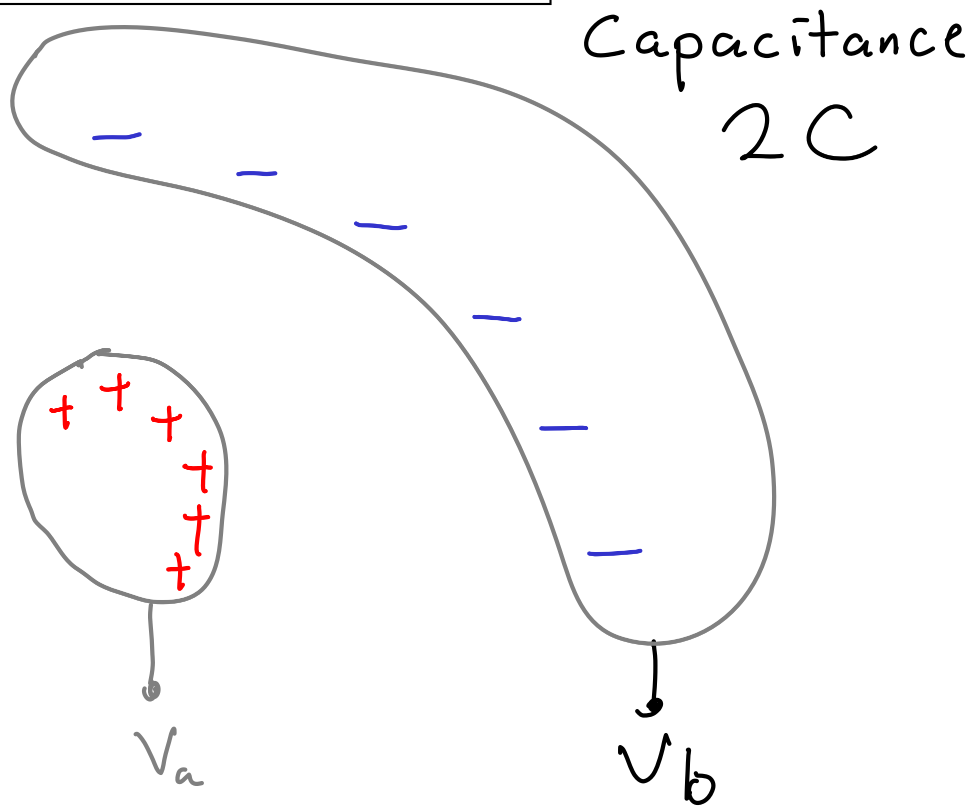
Bob argues the opposite, because $U=CV^2/2$

Which one is correct?

Potential Energy Stored in a Capacitor

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$

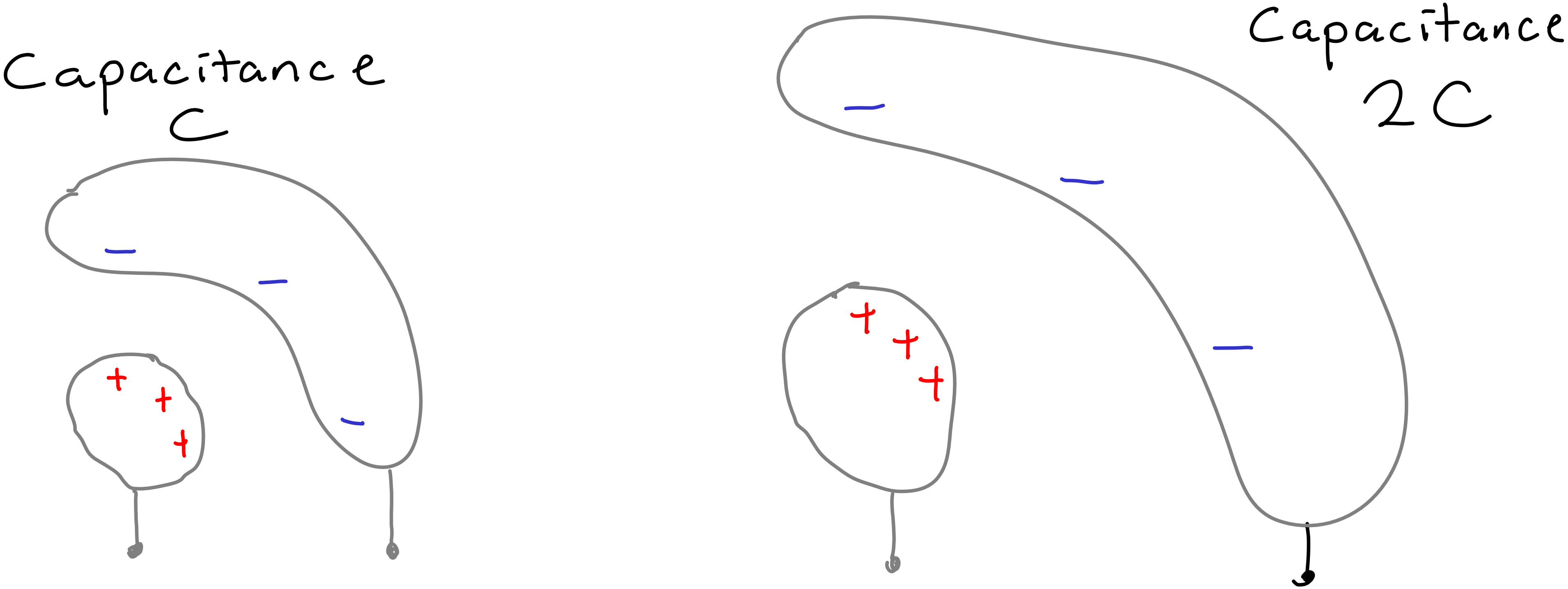




Same voltage

Potential Energy Stored in a Capacitor

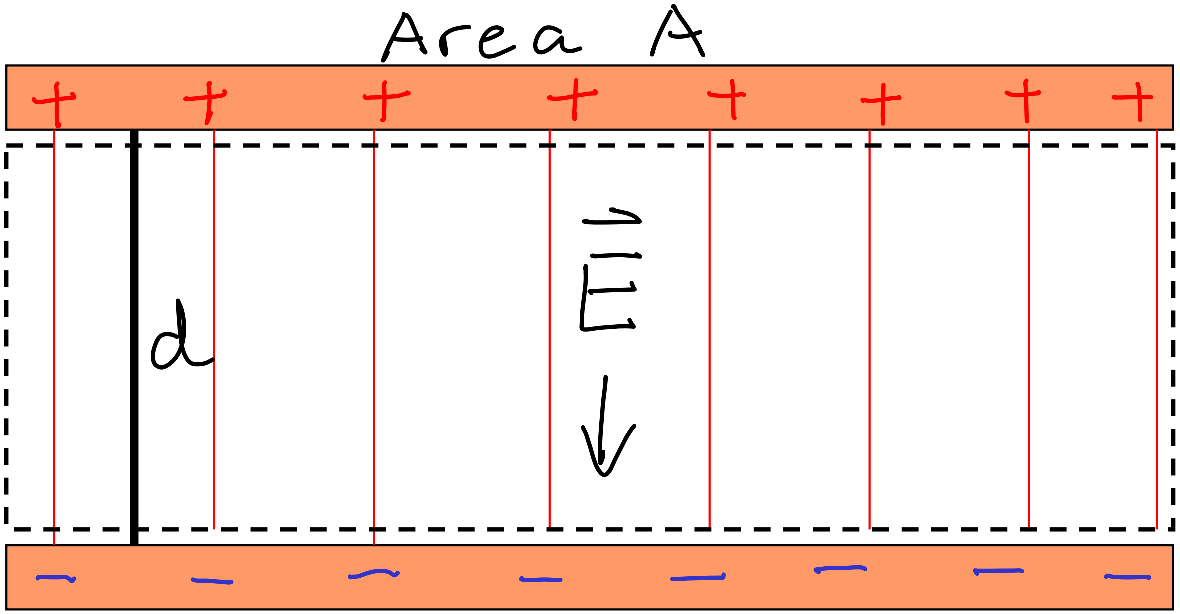
$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV}$$



Same charge Q

Electric Field Energy

Capacitors store potential energy



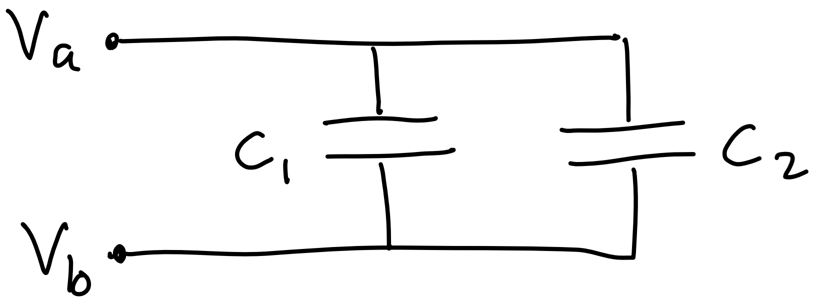
What is the *energy density* between the plates?

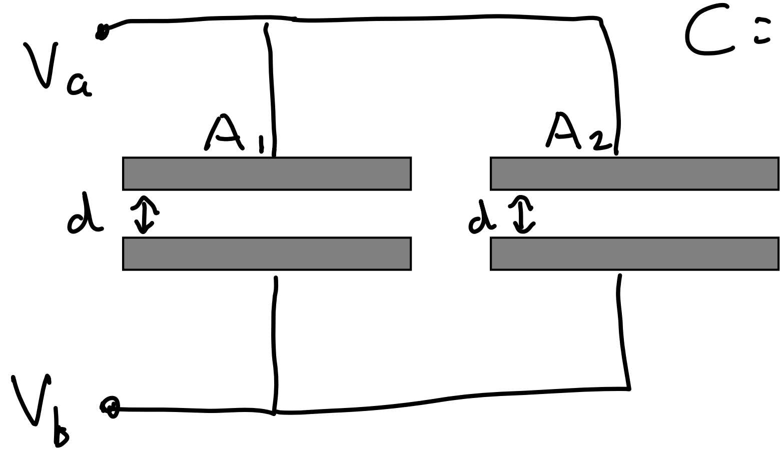
$$u = \frac{\text{potential energy}}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2$$

Capacitors in parallel

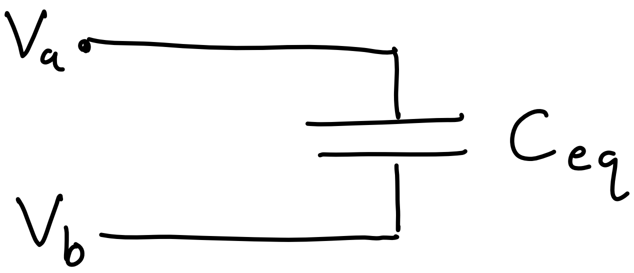
Par plates

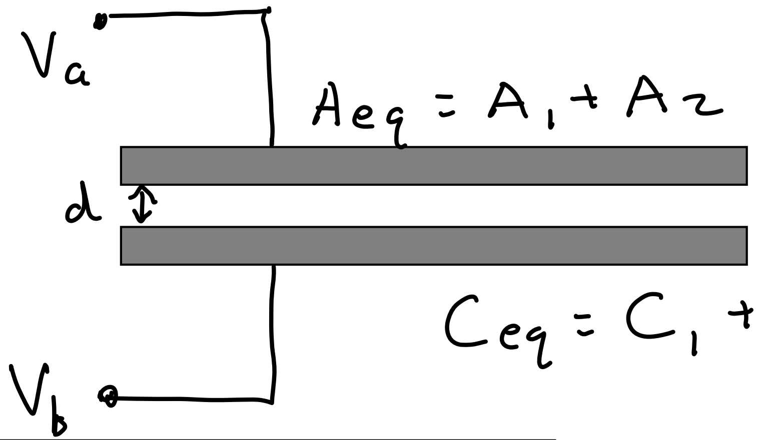
$$C = \epsilon_0 \frac{A}{d}$$





equivalent
to





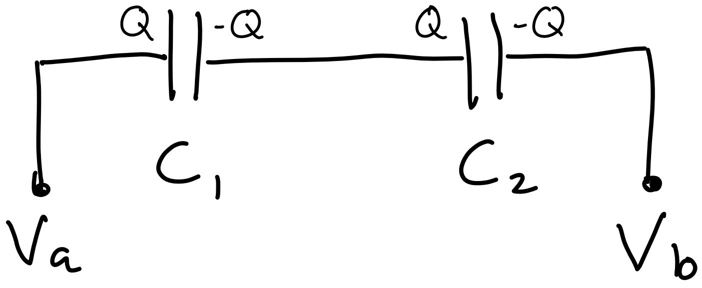
$$A_{eq} = A_1 + A_2$$

$$C_{eq} = C_1 + C_2$$

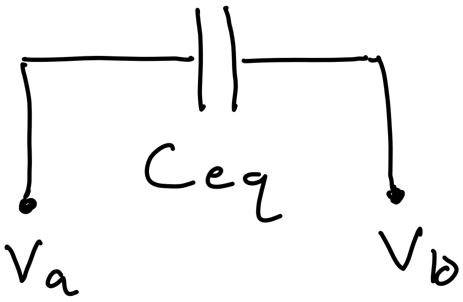
Capacitors in parallel:

$$C_{eq} = C_1 + C_2$$

Capacitors in series



equivalent to



$$Q_1 = Q_2 = Q$$

$$C = \frac{Q}{V} \Rightarrow V = \frac{Q}{C}$$

$$V_1 = V_a - V_c = \frac{Q}{C_1}$$

$$V_2 = V_c - V_b = \frac{Q}{C_2}$$

Charge Q

Voltage

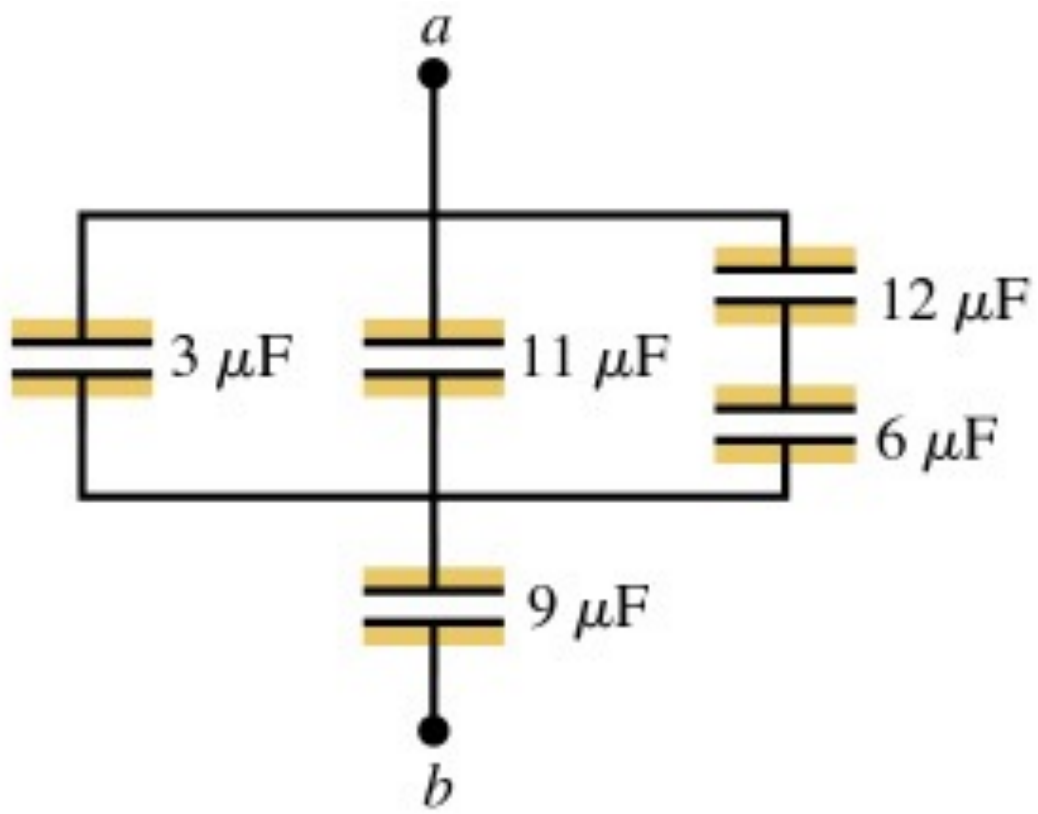
$$V = V_a - V_b$$

$$= V_1 + V_2$$

$$\frac{Q}{C_{eq}} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

Capacitors in serial: $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

Capacitor problem



What is the equivalent capacitance of this combination?

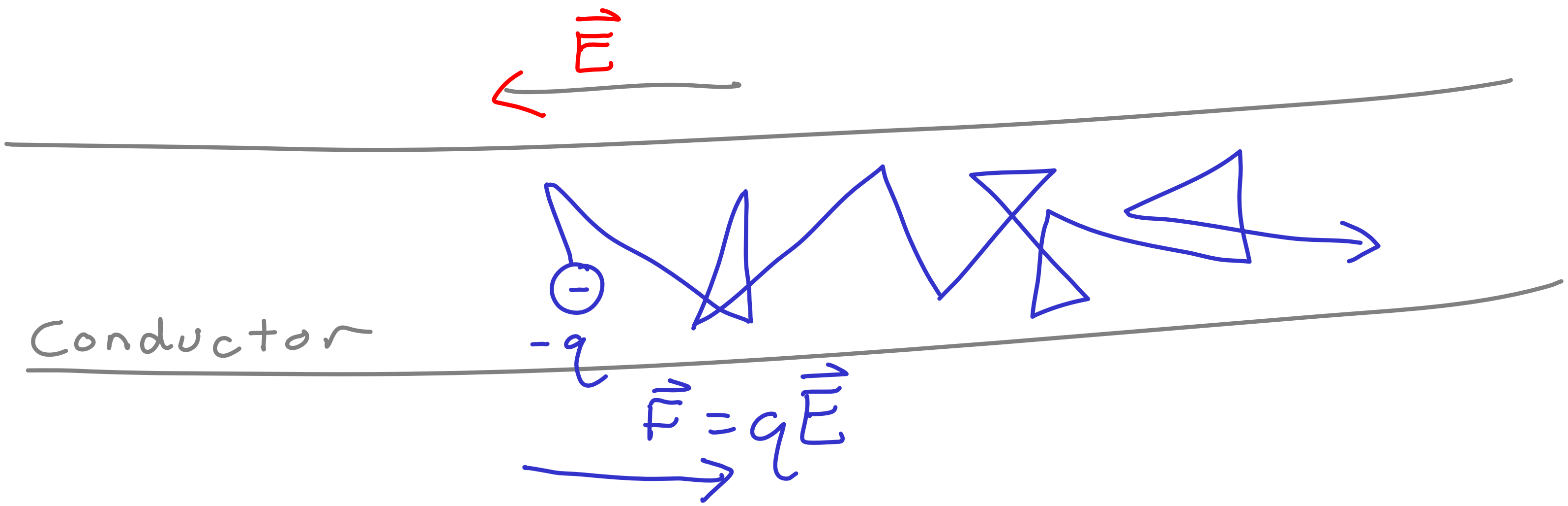
Chapter 25

Current, Resistance, and Electromotive Force

Current

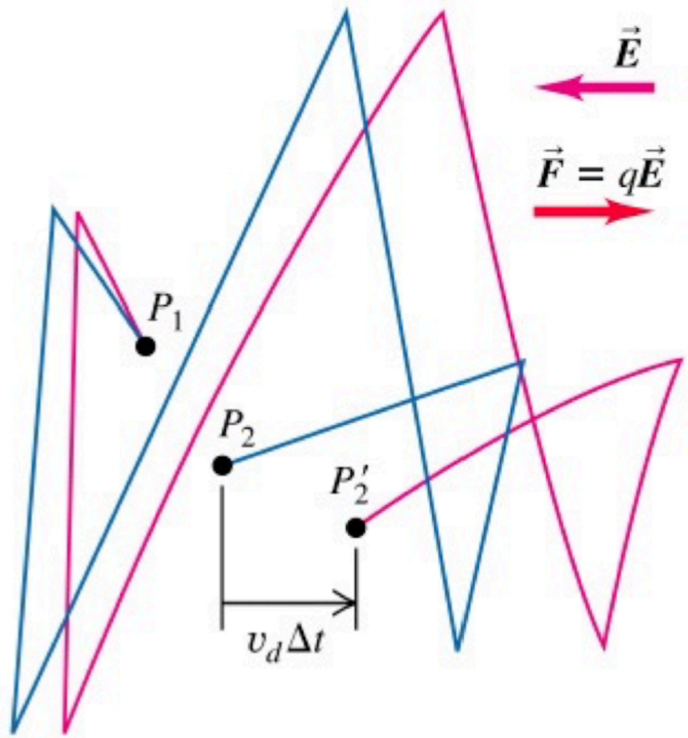
So far, we've studied only *static* situations

Now we start considering charges *in motion*



Drift velocity $\vec{v}_d$

Current



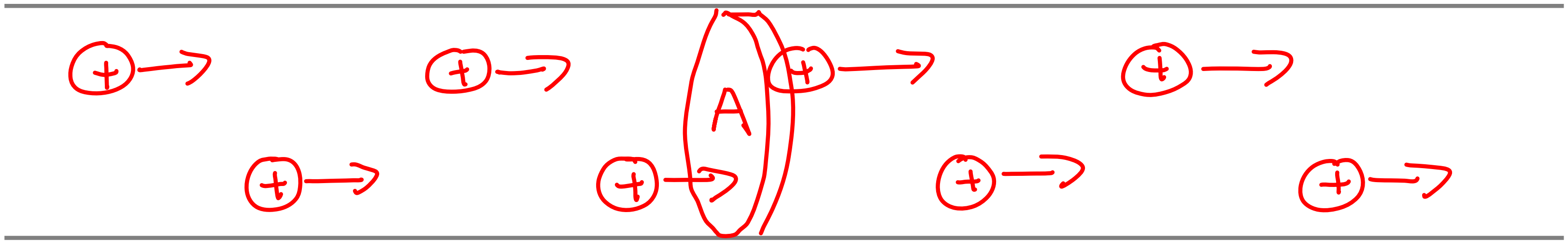
Typical electron trajectory in conductor *without* electric field:

- No net electric force on electrons
- Electrons move randomly within conductor
- No net current

Typical electron trajectory in conductor *with* electric field:

- Electric force $\vec{F} = q\vec{E}$ imposes a small drift on electron's random motion
- There is a net current

Current



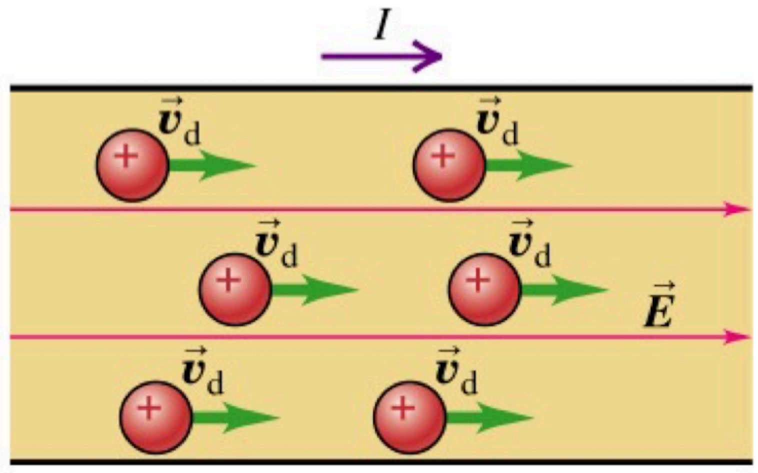
During time dt,

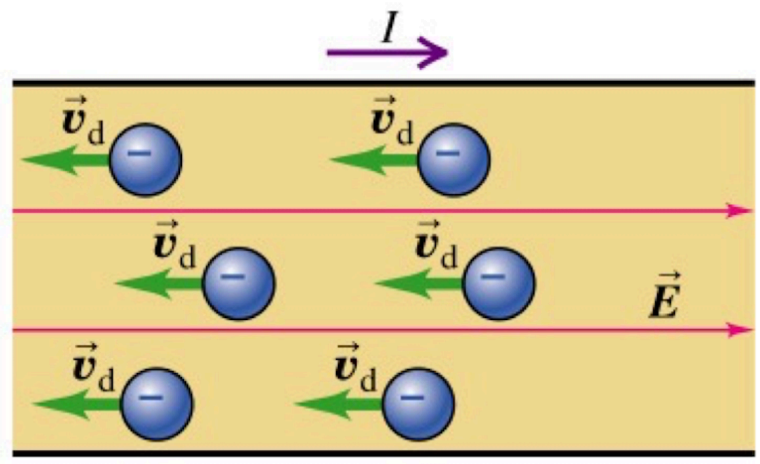
charge passing through A: dQ

Current through A: $I = \frac{dQ}{dt}$

Unit of current: the *Ampere* A = C/s

Current

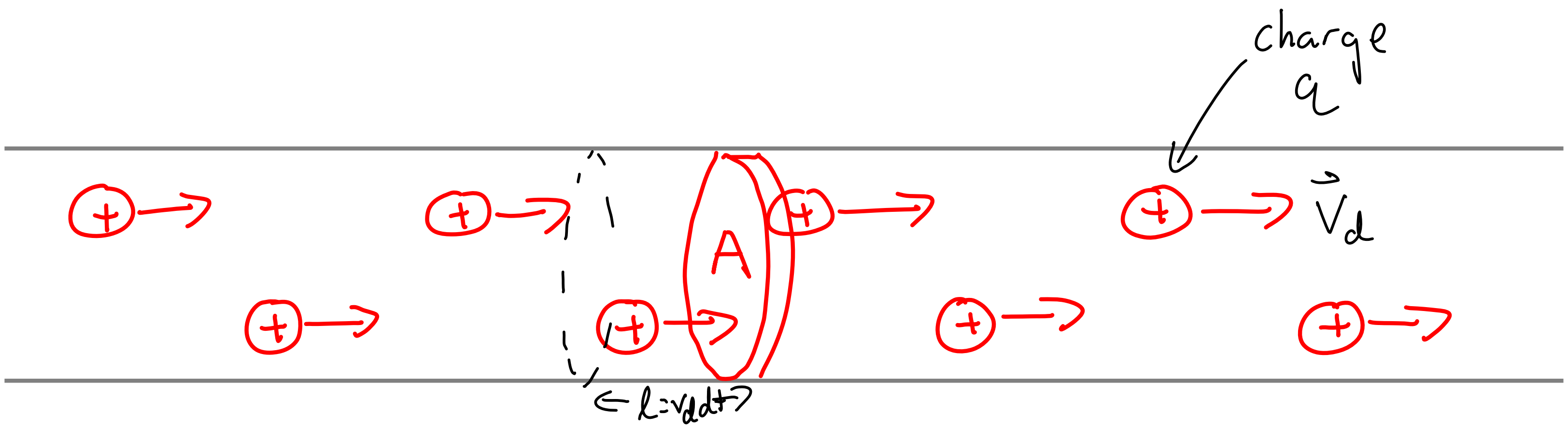




Think of current like a vector.

It "points" in the direction of the flow of positive charge (opposite to the motion of electrons!)

Current density



How does current relate to drift velocity?

n: concentration of particles, in m^{-3}

Current density

$$J = \frac{I}{A} = n q v_d$$

$$\vec{J} = n q \vec{v}_d$$

units A/m^2

Resistivity

In *most* metals, there is a simple relationship between the electric field and the current density

Resistivity $\rho = \frac{E}{J}$ Units: $\frac{V\,m}{A}$

Ohmic material

$\Rightarrow \rho$ is ind. of E

Ohm's Law:

Resistivity is independent of the electric field strength

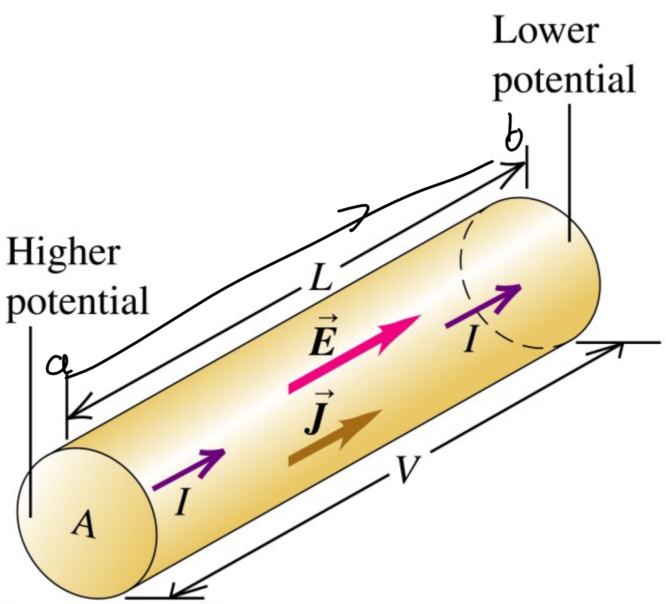
Conductivity: inverse of resistivity

Resistance

For an Ohmic material: $\vec{E} = \rho \vec{J}$

ρ ind. of $\vec{E}$ + $\vec{J}$

Usually more interested in voltage (rather than E) and in current (rather than J)



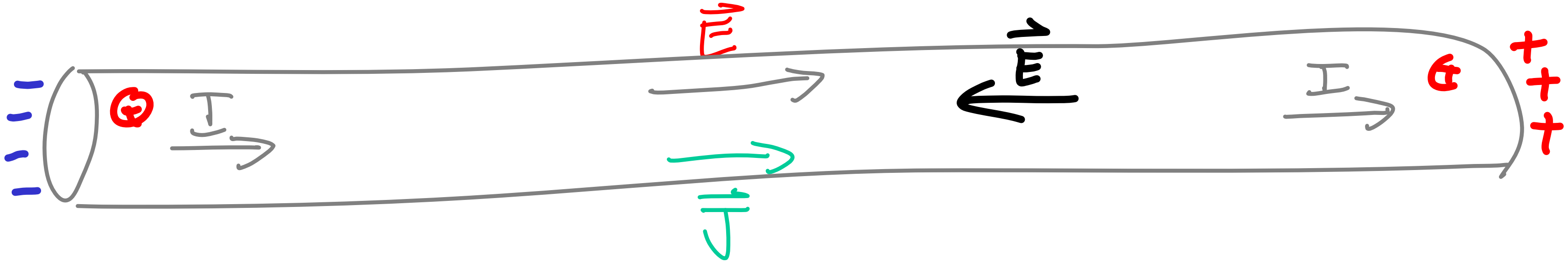
$$V = \left(\frac{\rho L}{A}\right) I \qquad V = IR$$

Resistance: $R = \frac{\rho L}{A}$

Units of *Ohms* $\Omega = \frac{V}{A}$

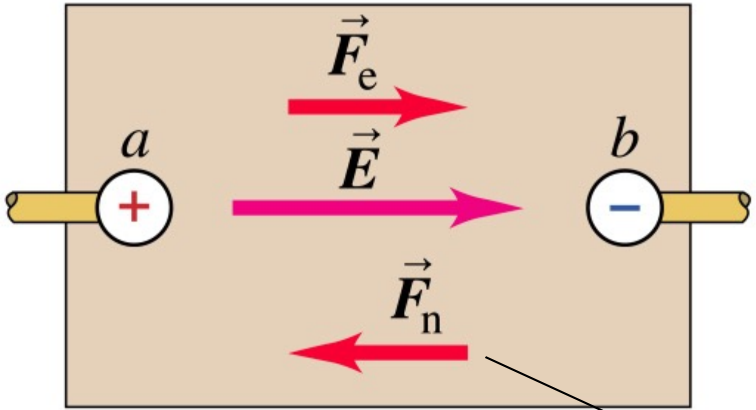
Closed Circuits

To maintain a steady (constant) current, we need to create a *closed path* - a *circuit*



Electromotive Force

Electromotive force (emf) is the pump



Source of emf not connected to a circuit: electric-field force $\vec{F}_e$ has the same magnitude as non-electrostatic force $\vec{F}_n$

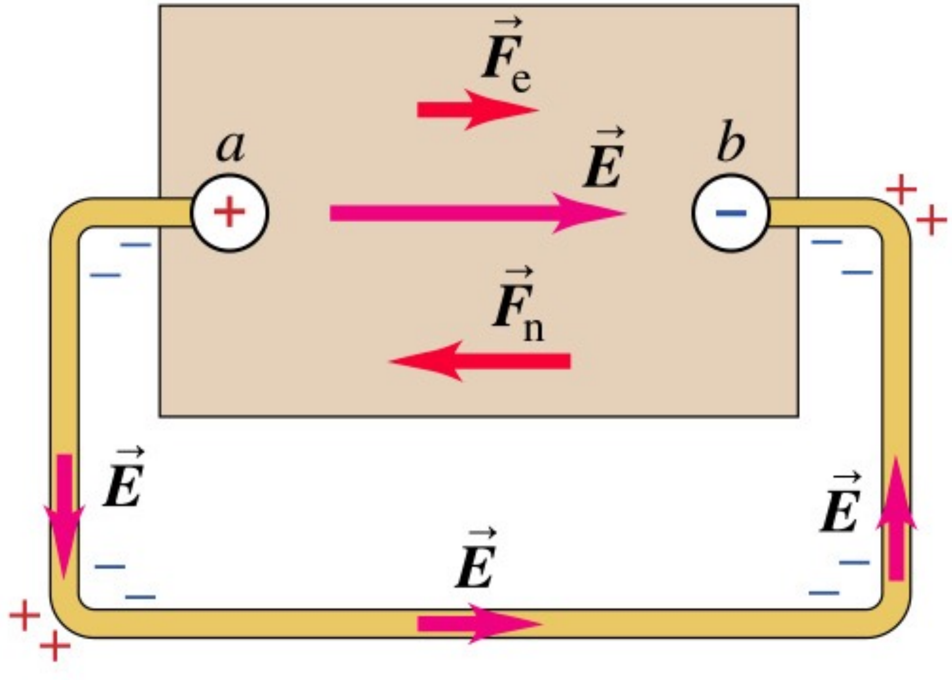
$$\mathcal{E} = V_{ab} = V_a - V_b$$

Units: Volts (V)

Some other force

Electromotive Force

Electromotive force (emf) is the pump



Source of emf connected to a complete circuit: electric-field force $\vec{F}_e$ has a smaller magnitude than non-electrostatic force $\vec{F}_n$

Change in electric potential (voltage) around a loop equals zero

(Think of the fountain)

$$\mathcal{E} - IR = 0$$

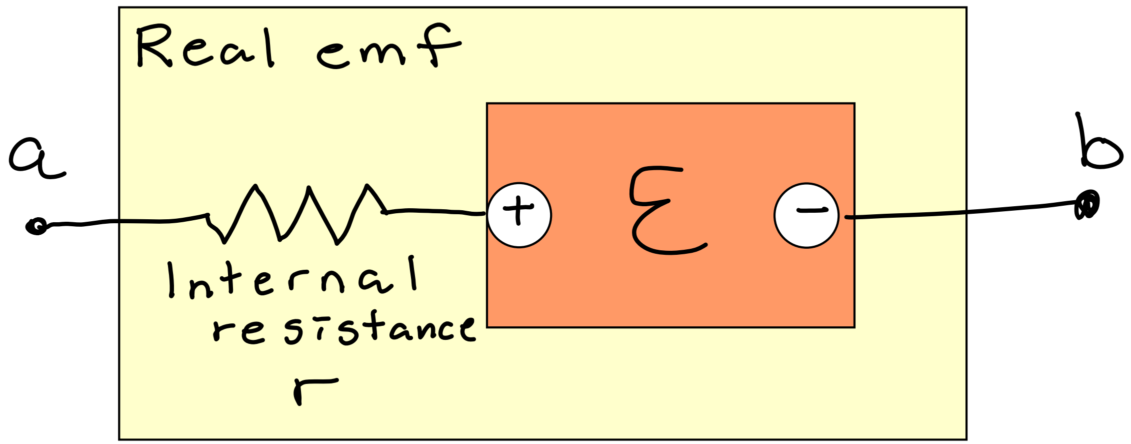
Ideal source of emf:

$$\mathcal{E} = V_{ab}$$
$$= IR$$

Internal Resistance

Electromotive force (emf) in an *idealisation*

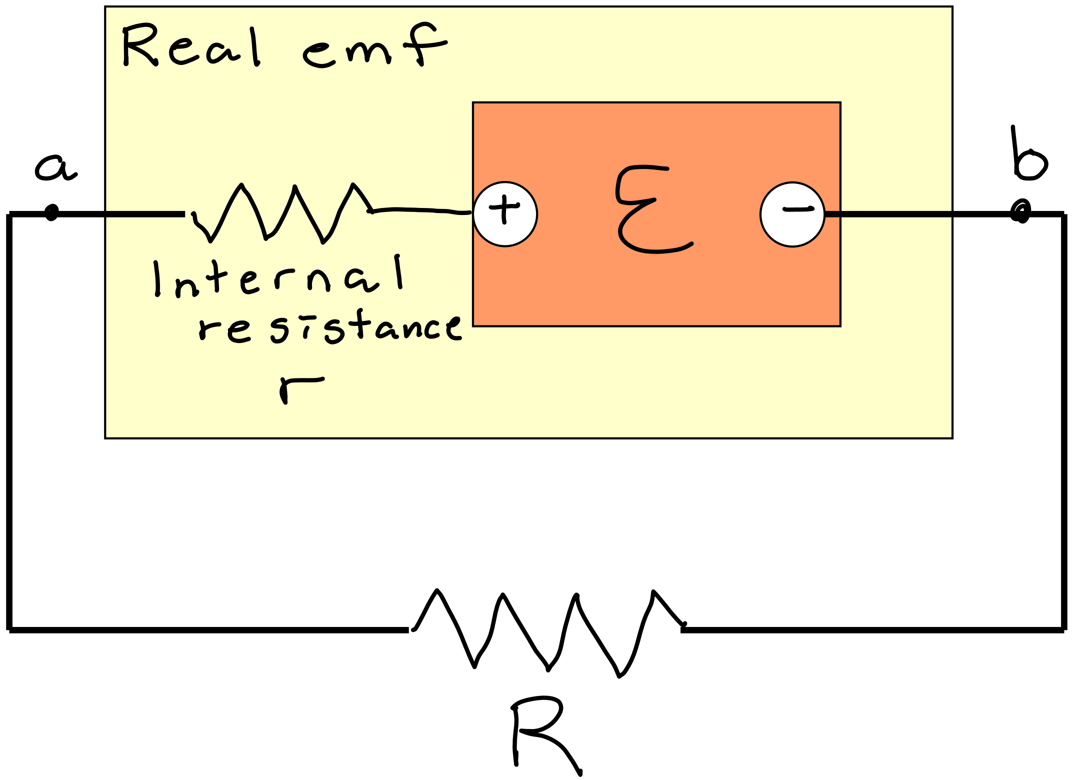
Real devices have internal resistance



$$V_{ab} = \mathcal{E} - I r$$

Terminal voltage, source with internal resistance

Internal Resistance Circuit



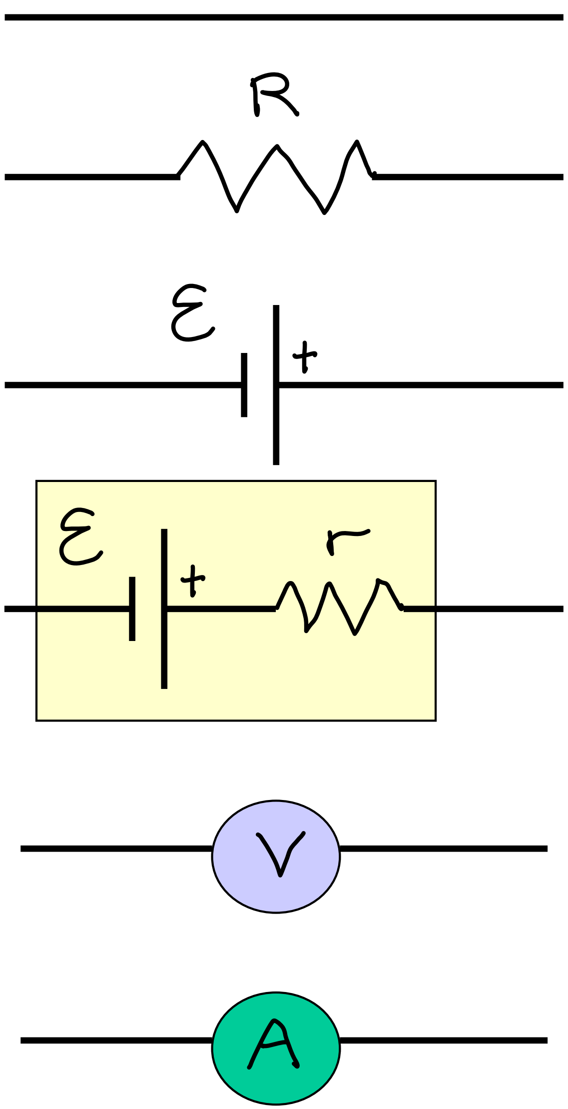
$$V_{ab} = V_{ab}$$

$$\mathcal{E} - Ir = IR$$

$$I = \frac{\mathcal{E}}{R + r}$$

(current, source with internal resistance)

Symbols for Circuits



Conductor with negligible resistance

Resistor

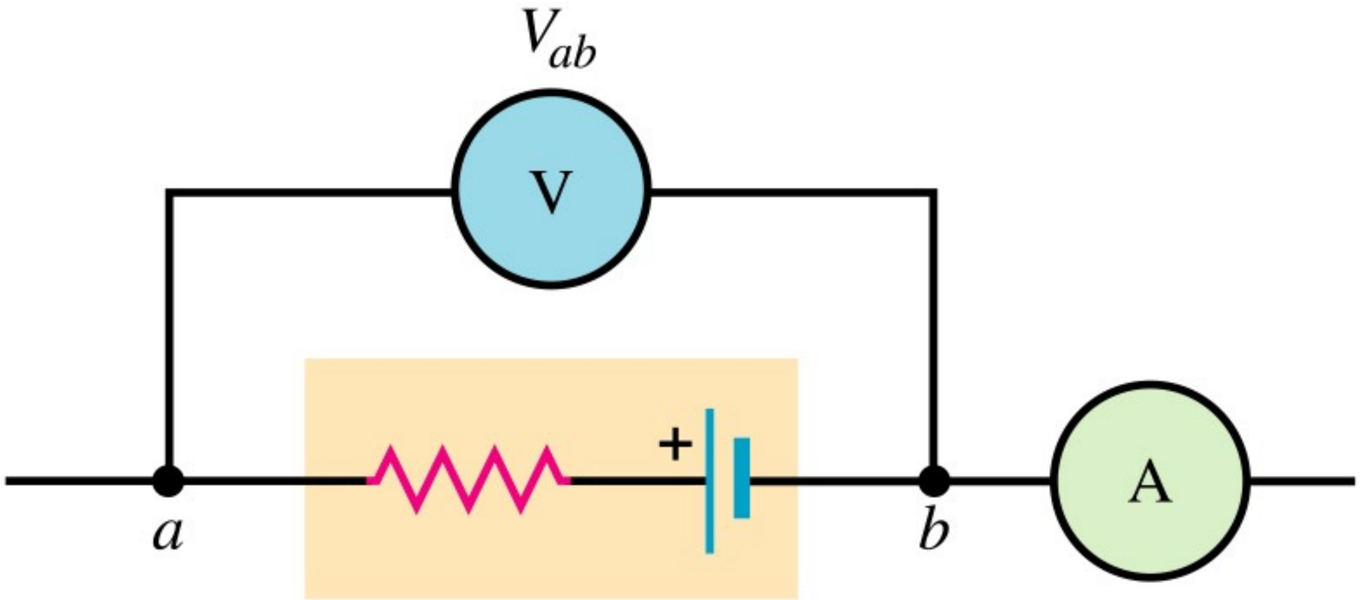
Source of emf (longer line always represents the positive terminal)

Source of emf with internal resistance r (can be on either side)

Voltmeter (measures potential difference)

Ammeter (measures current through it)

Problem – 25.5



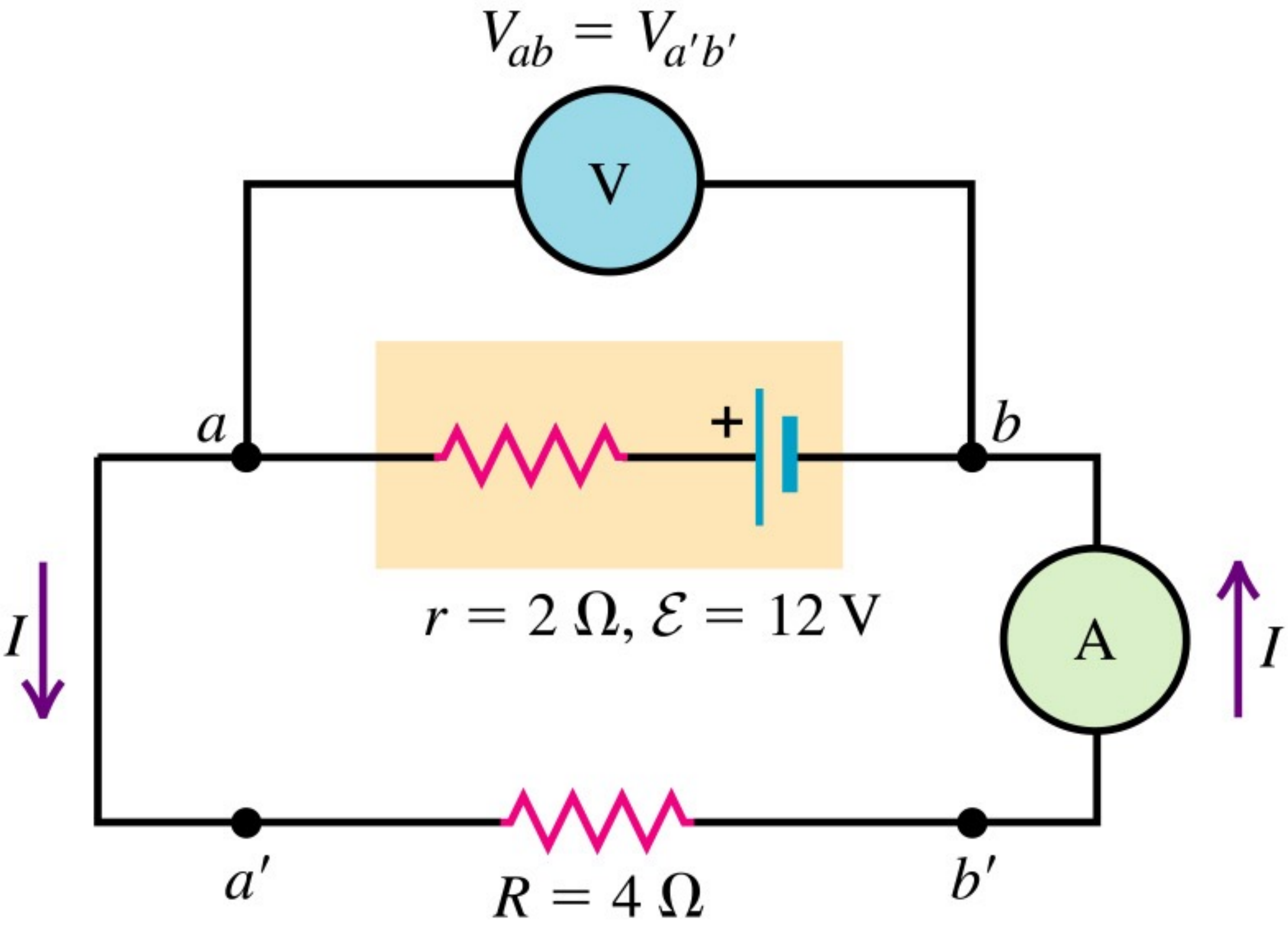
$r = 2\ \Omega,\ \mathcal{E} = 12\ \text{V}$

What is the reading on:

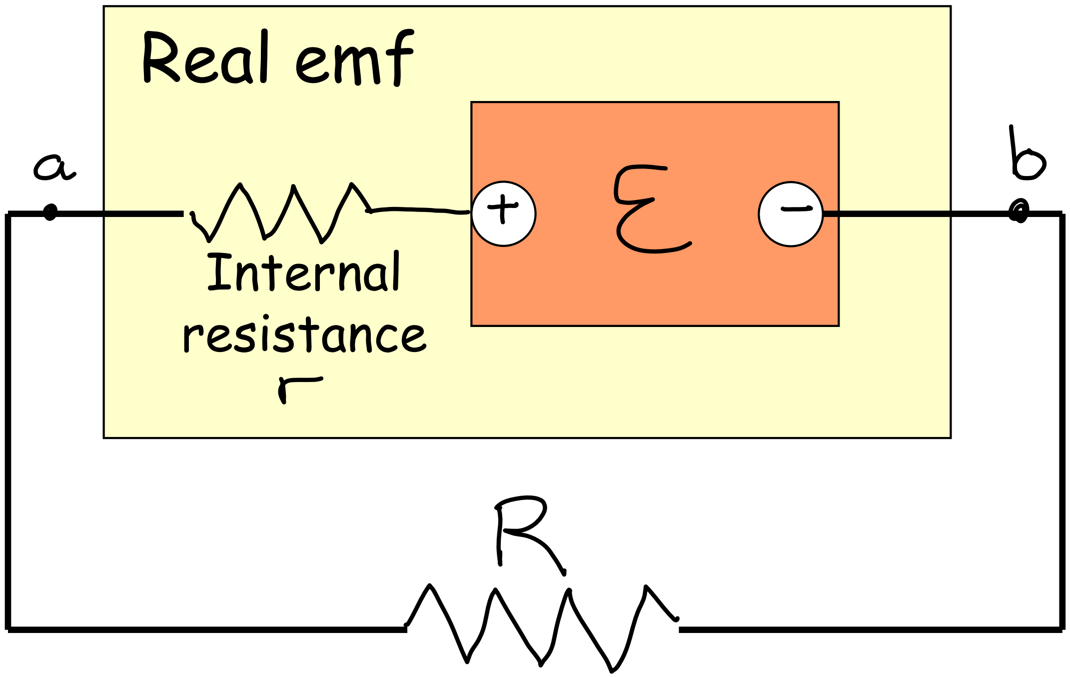
A) the ammeter?

B) the voltmeter?

Problem – 25.5



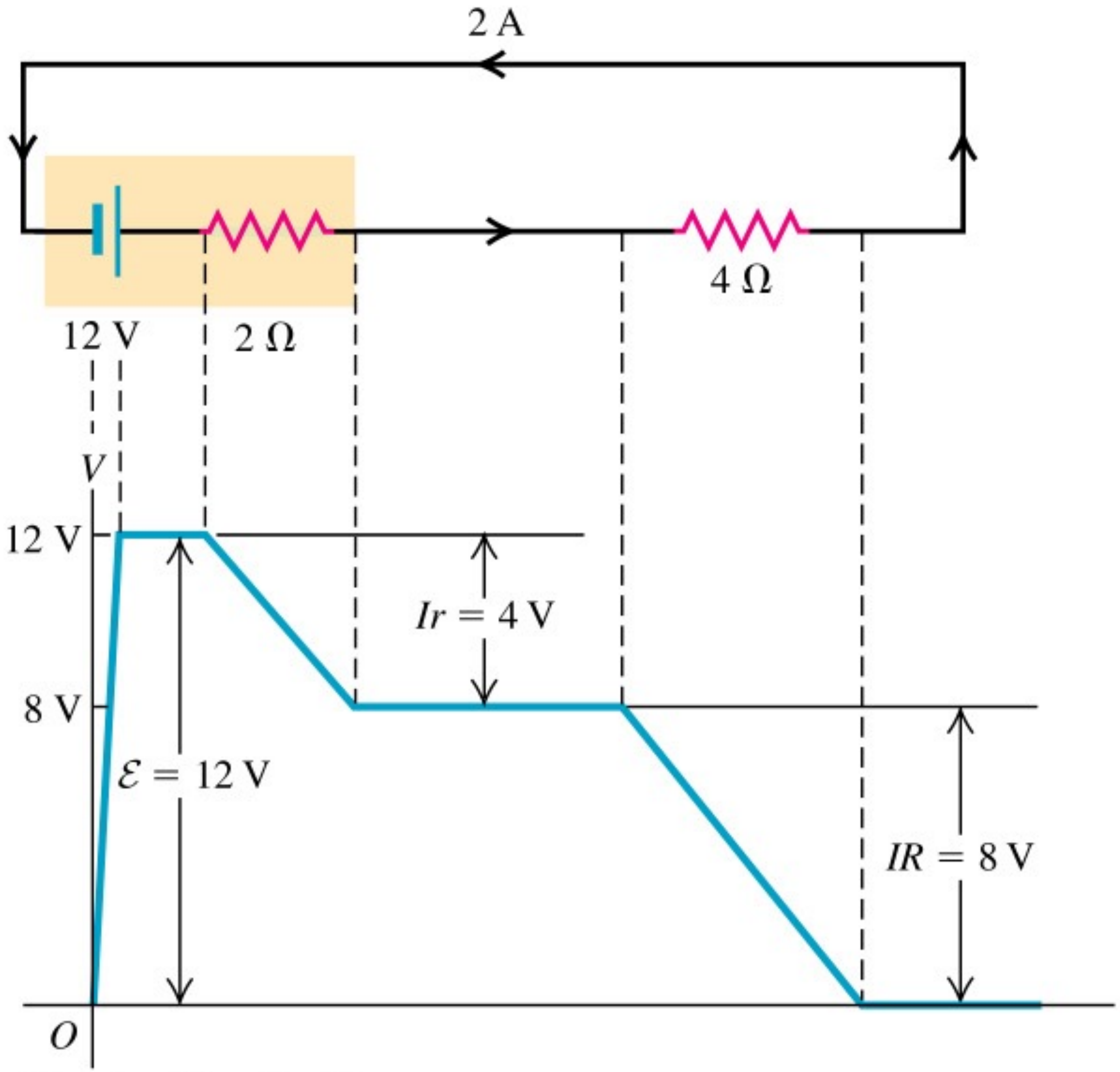
Problem



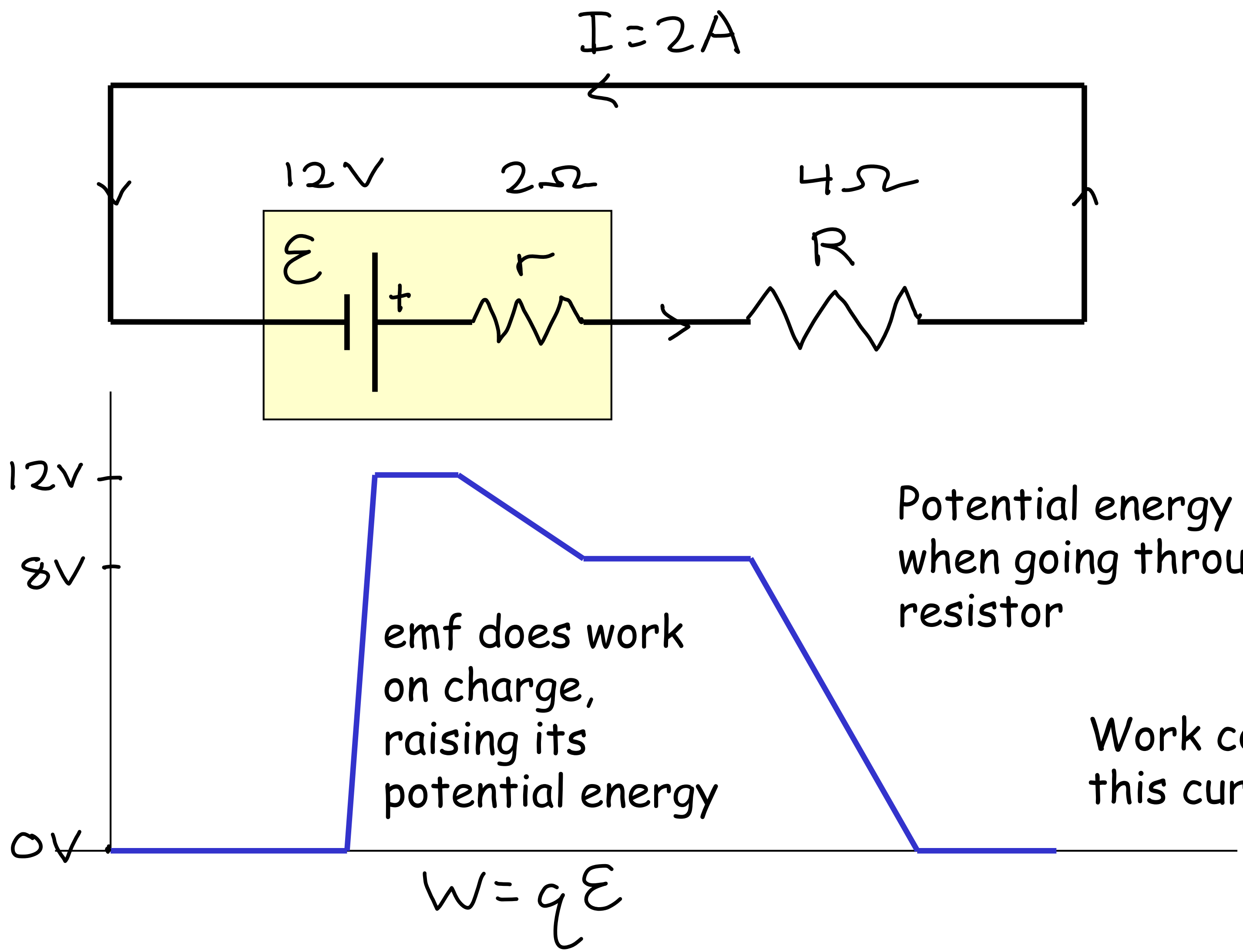
Eight flashlight batteries in series have an emf of about 12 V, similar to that of a car battery.

Could they be used to start a car with a dead battery? Why or why not?

Potential in a Circuit



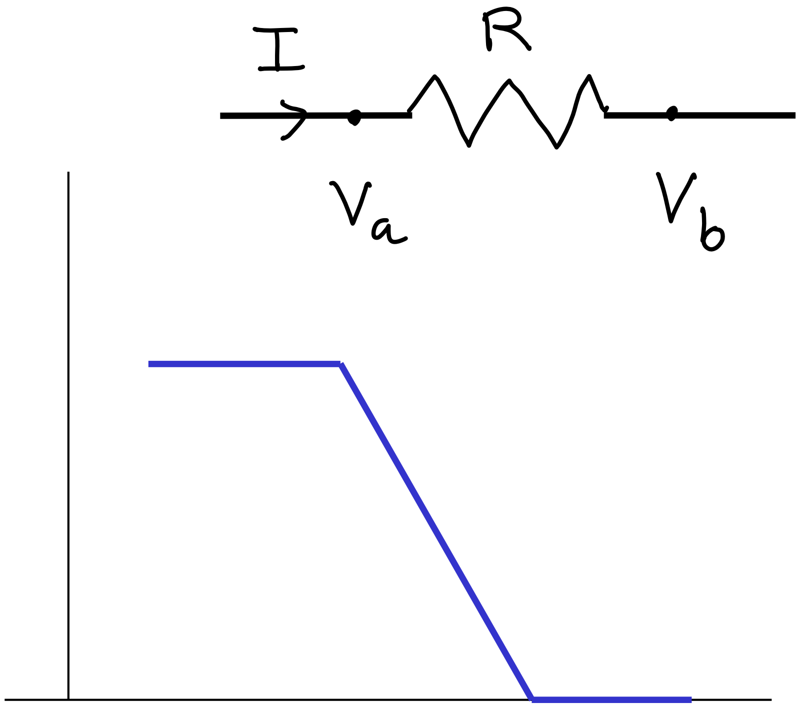
Work in a Circuit



Potential energy is lost when going through a resistor

Work could be done by this current in principle

Power in a Circuit



In time dt:

$$dQ = I\,dt$$

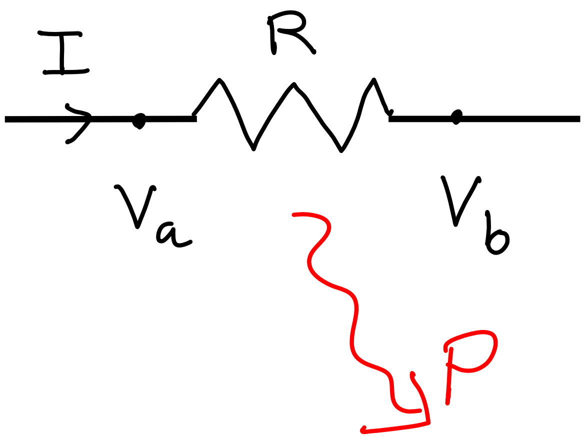
Potential energy decrease in time dt from dQ "falling" through $V_{ab} = V_a - V_b$

$$dU = dQ\, V_{ab}$$
$$= V_{ab}\, I\, dt$$

Power (rate at which energy is delivered or extracted from a circuit element):

$$P = \frac{dU}{dt} = V_{ab}\, I$$

Power for a Resistor



Combine Ohm's Law

$$V_{ab} = IR$$

with our new power equation

$$P = V_{ab} I$$

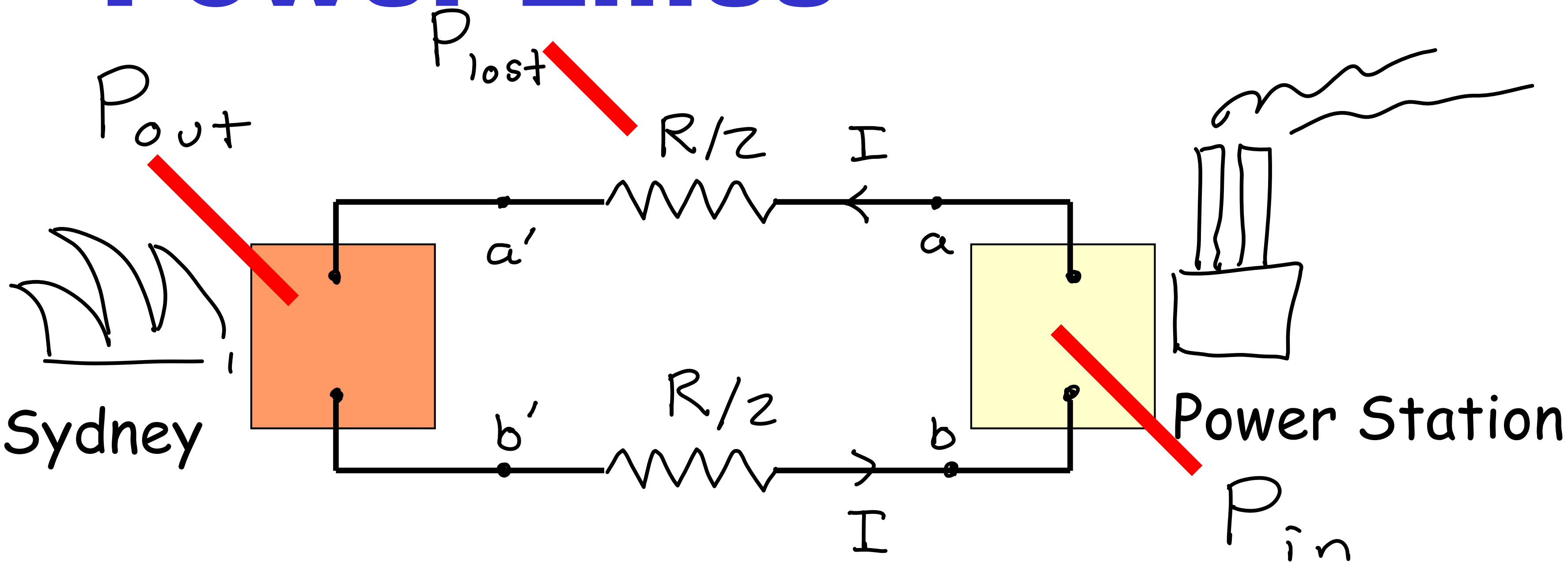
$$\boxed{P = V_{ab} I = I^2 R = V_{ab}^2 / R}$$

(power delivered to a resistor)

In the case of a resistor, energy is lost as heat

Other circuit components (i.e., LEDs) use this energy in other ways

Power Lines



Now $P_{out} = P_{in} - P_{lost}$

$= V_{ab} I - I^2 R$

So minimize I ← fixed

$$P = V_{ab} I = I^2 R = V_{ab}^2 / R$$

(power delivered to a resistor)

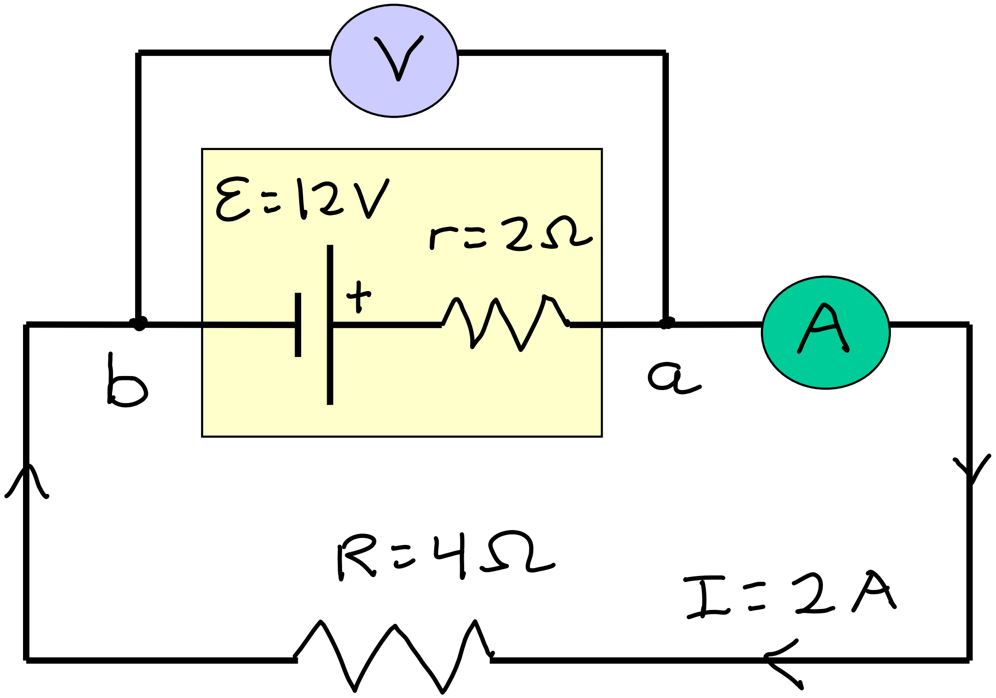
Problems

$$\boxed{P = V_{ab} I = I^2 R = V_{ab}^2 / R}$$

Small aircraft have 24 V electrical systems, even though their requirements are about the same as cars (with 12 V systems).

Aircraft designers say that a 24 V system weighs less, because thinner wires can be used. Why?

Problem – 25.9



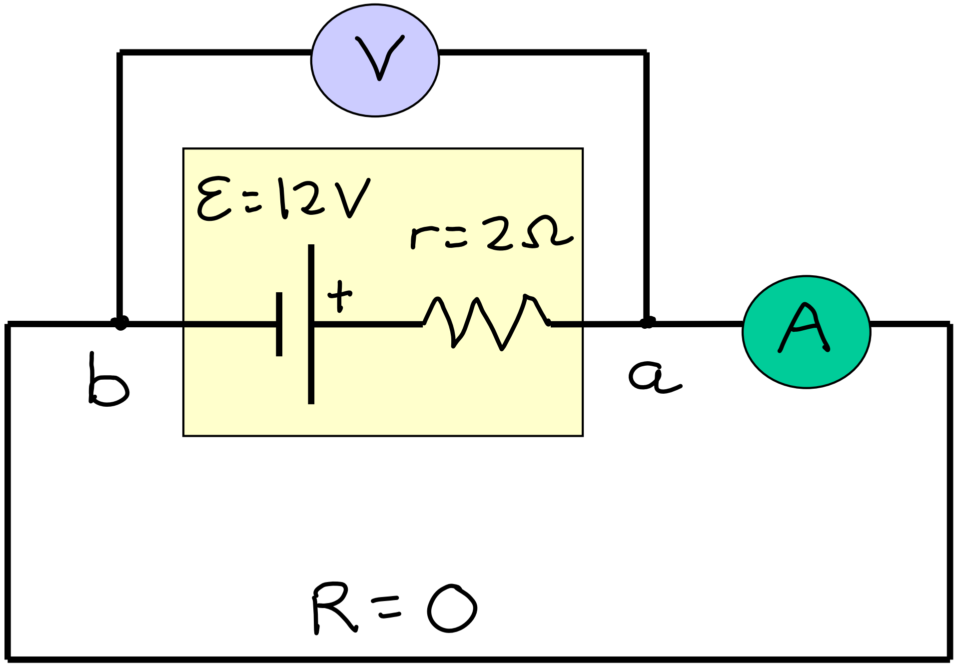
What is rate of:

A) energy conversion (chemical to electrical) by the battery?

B) power dissipation by the battery?

C) power dissipation by the resistor?

Short circuit – 25.11

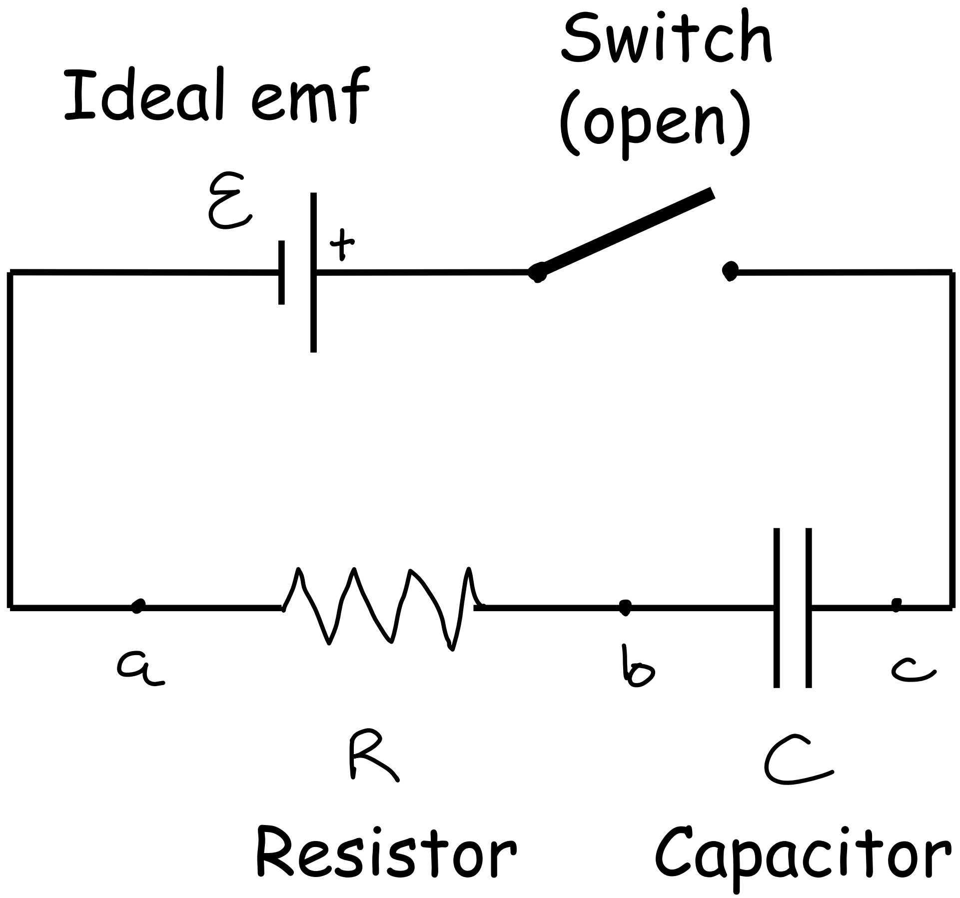


A) What are the meter readings on the voltmeter and ammeter?

B) What is the power dissipation by the battery?

(Sub-)Chapter 26.4
R-C Circuits

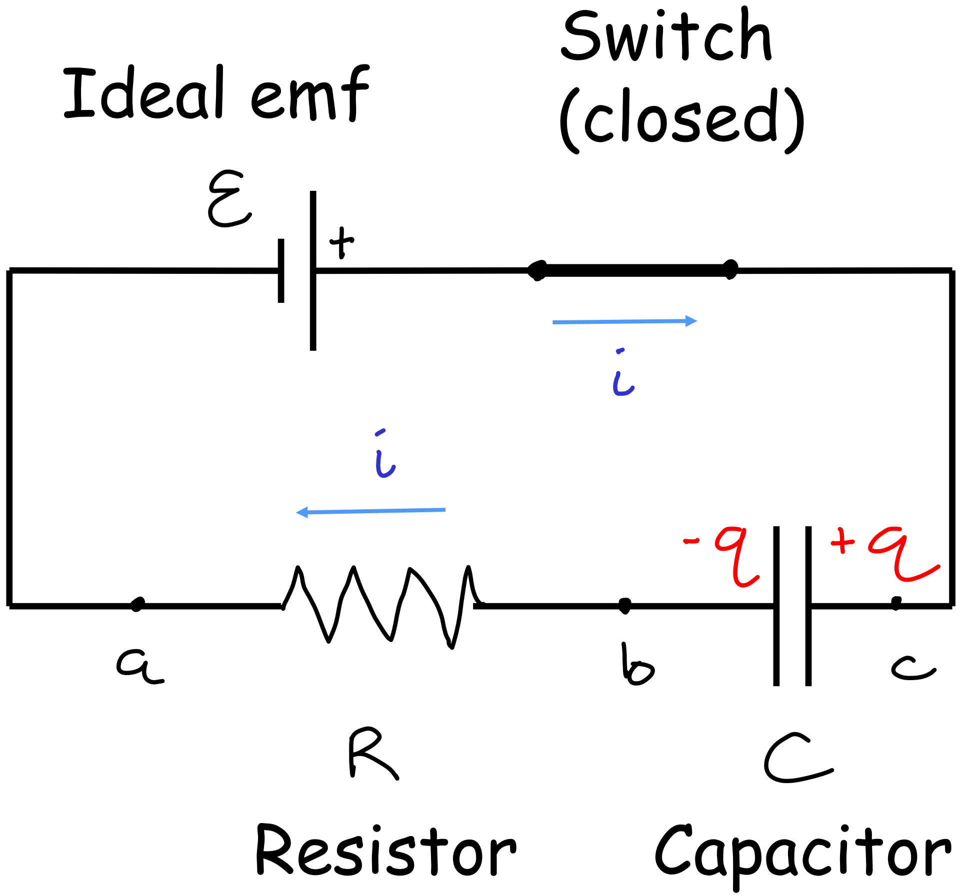
A Capacitor in a Circuit



Closing the switch initiates a non-static (time-dependent) process

What happens?

A Capacitor in a Circuit

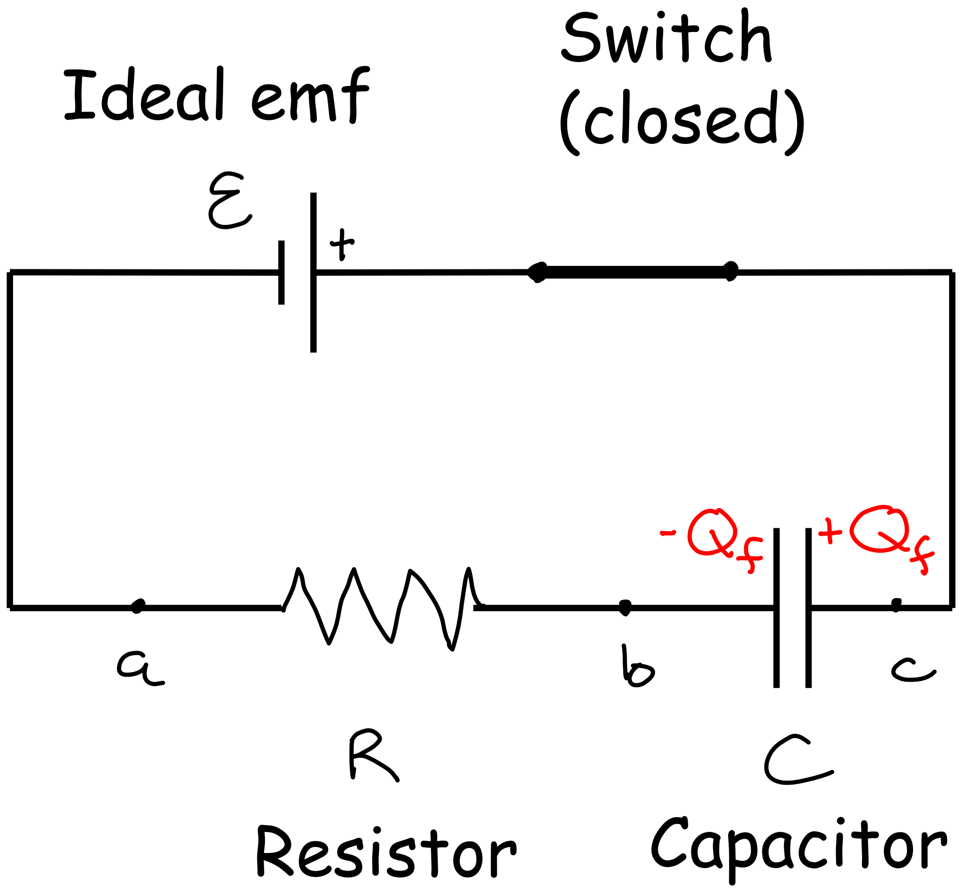


Initially, a current will flow and charge will build up on the capacitor...

... but that capacitor builds up a voltage *drop* given by

$$V_{cb} = \frac{q}{C}$$

A Capacitor in a Circuit



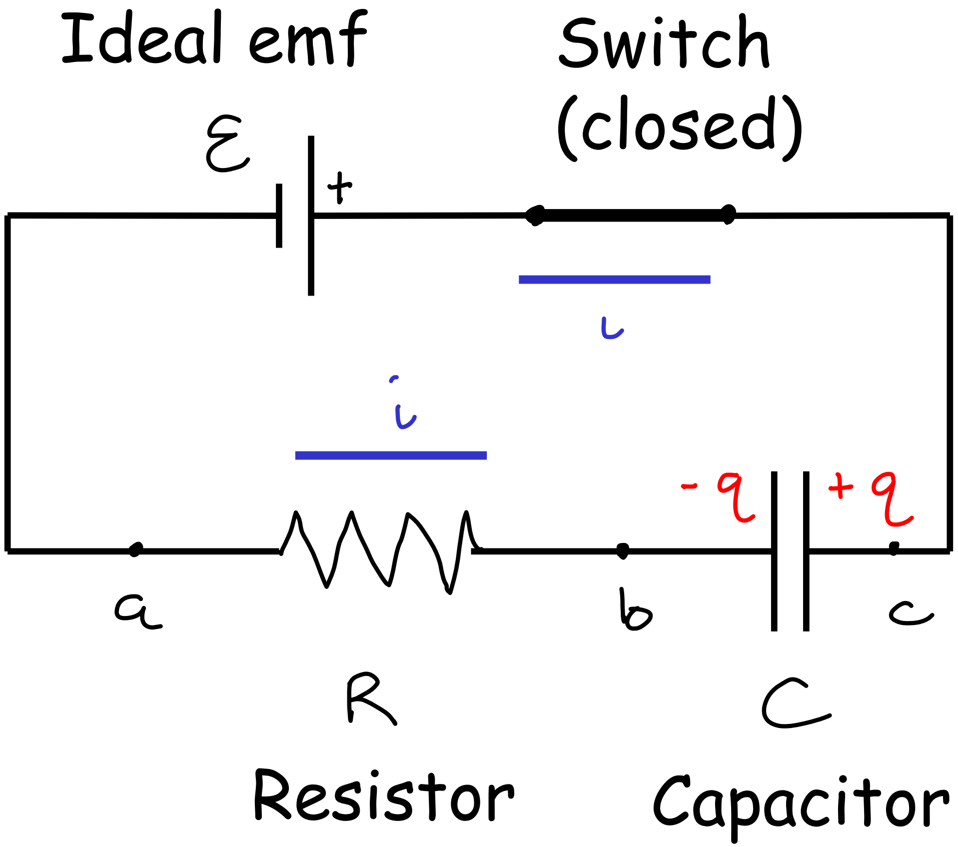
Initially, a current will flow and charge will build up on the capacitor...

... but that capacitor builds up a voltage *drop* that (after a very long time) is

$$V_{cb} = Q_f/C = \mathcal{E}$$

and the current stops

Charging a Capacitor



Use Kirchhoff's loop rule to solve for q and i as functions of time:

$$V_{ca} = V_{ba} + V_{cb}$$

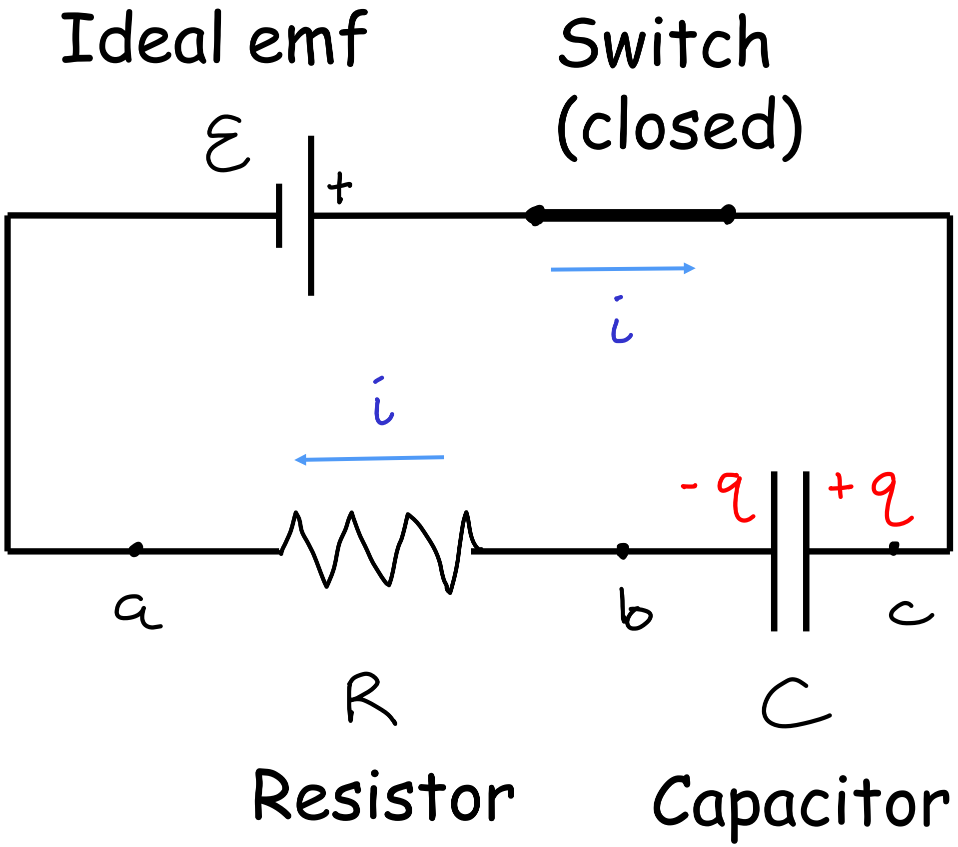
$$\mathcal{E} = iR + q/C$$

At t=0 (switch closed):

$$q(t=0) = 0$$

$$i(t=0) = \mathcal{E}/R$$

Charging a Capacitor



Use Kirchhoff's loop rule to solve for q and i as functions of time:

$$V_{ca} = V_{ba} + V_{cb}$$

$$\varepsilon = iR + q/C$$

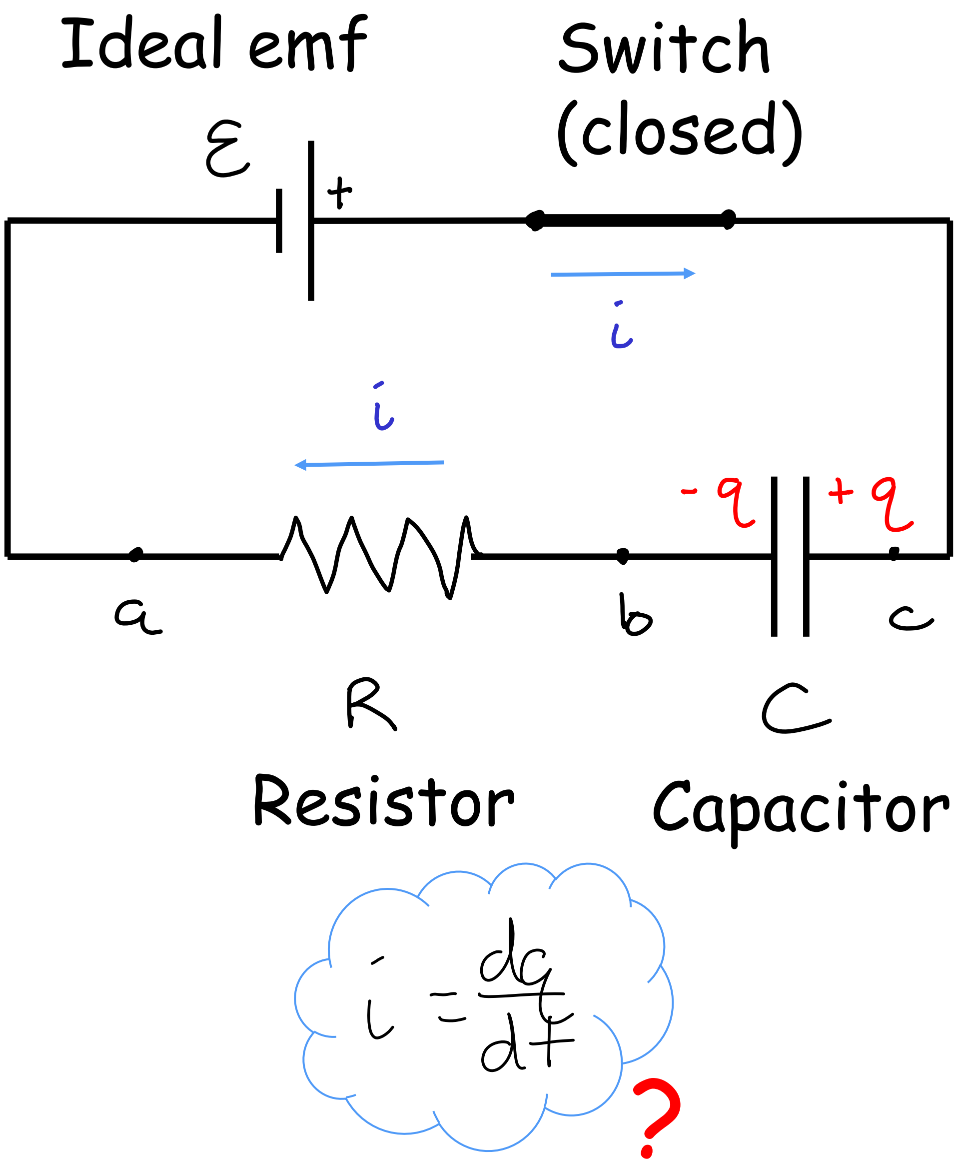
As t -> ∞

$$i(t \to \infty) = 0$$

$$q(t \to \infty) = Q_f = \varepsilon C$$

$$C = \frac{Q_f}{\varepsilon}$$

Charging a Capacitor



Use Kirchhoff's loop rule to solve for q and i as functions of time:

$$V_{ca} = V_{ba} + V_{cb}$$

$$\mathcal{E} = iR + q/C$$

$$\boxed{\mathcal{E} = \frac{dq}{dt}R + q/C}$$

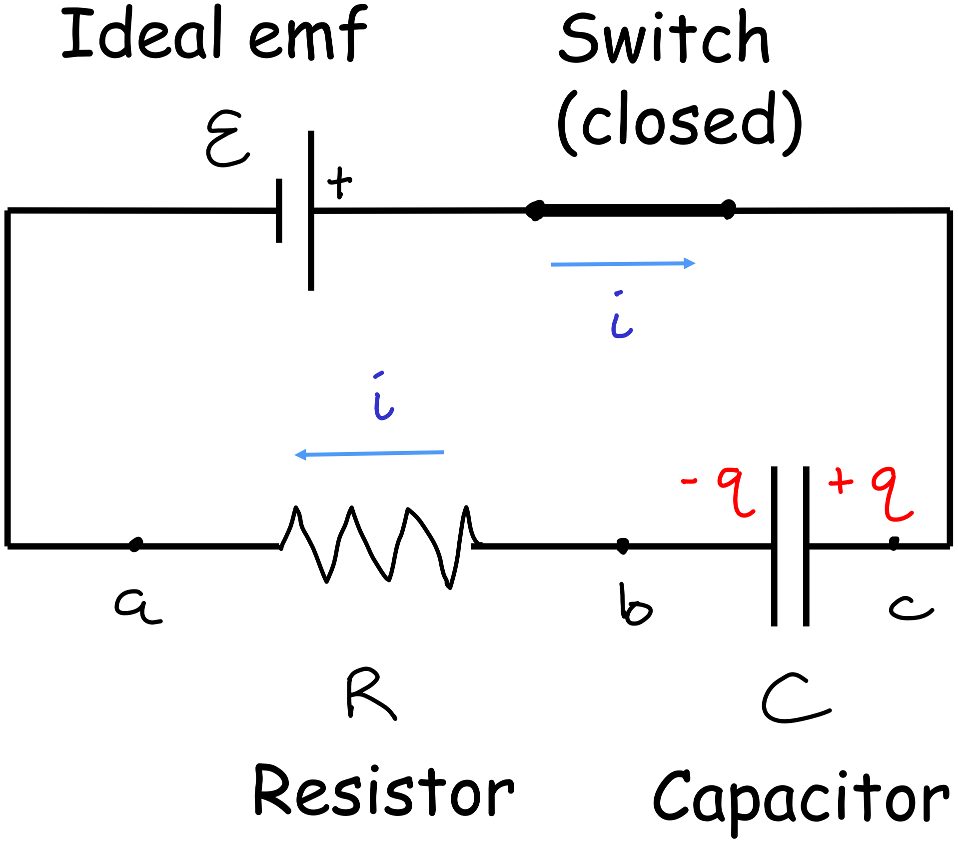
Check:

$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

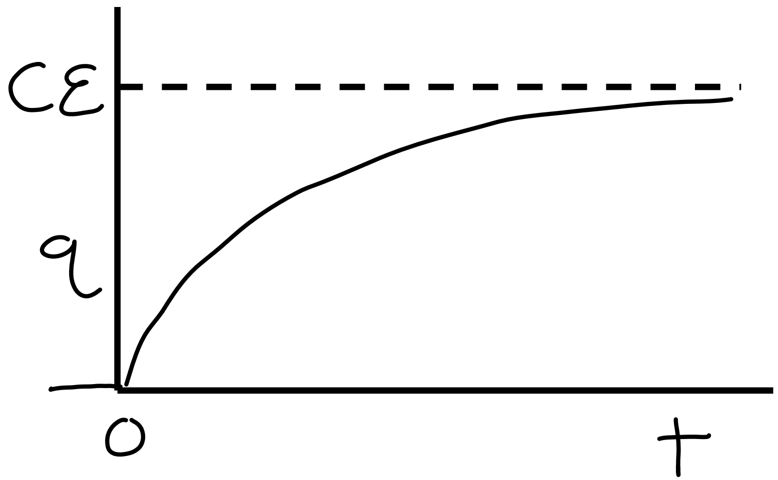
$$q(0) = 0$$

$$q(t \to \infty) = C\mathcal{E}$$

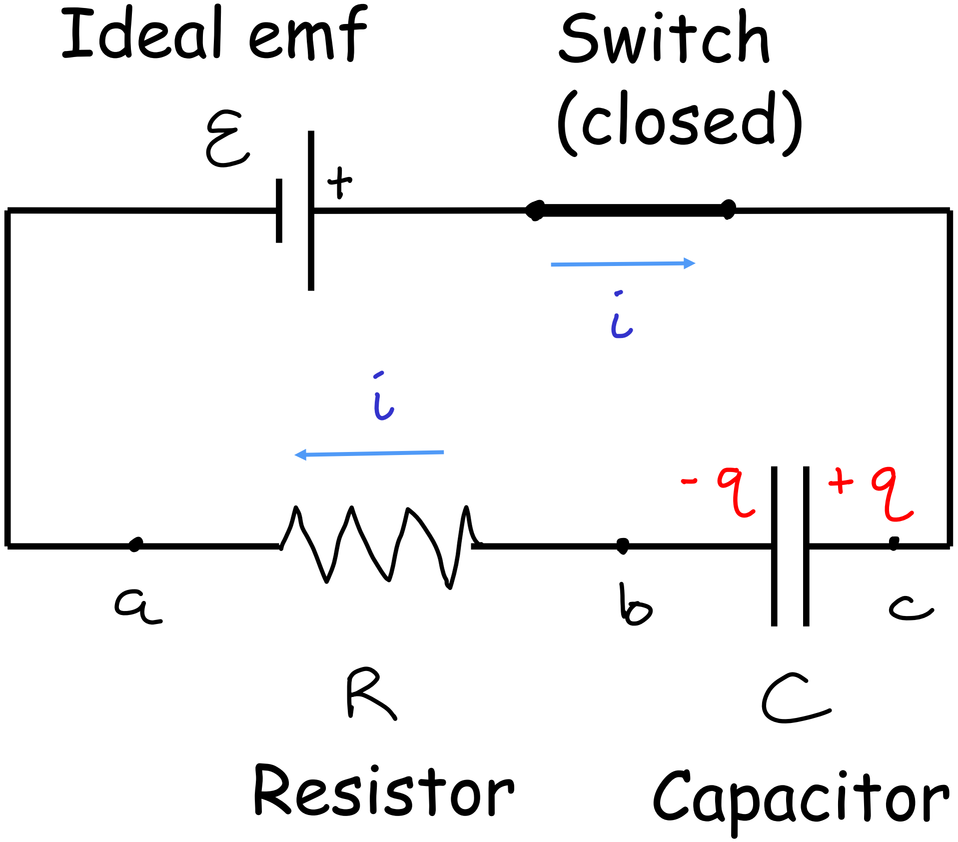
Charging a Capacitor



$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$
$$= q_\infty\left(1 - e^{-t/RC}\right)$$



Charging a Capacitor

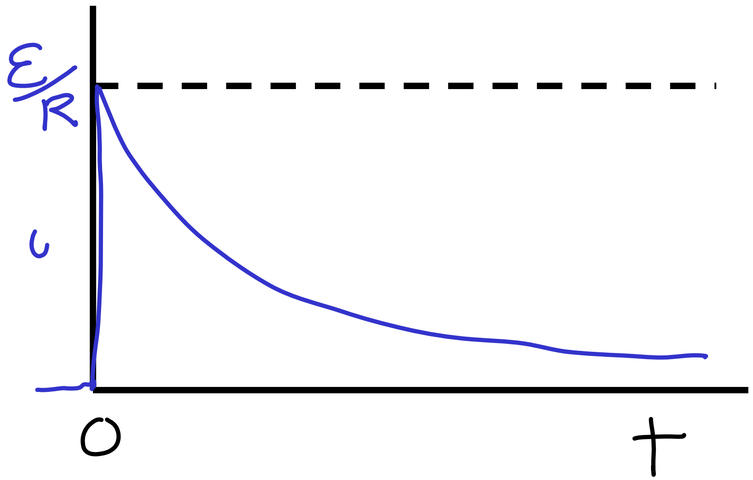


$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right)$$

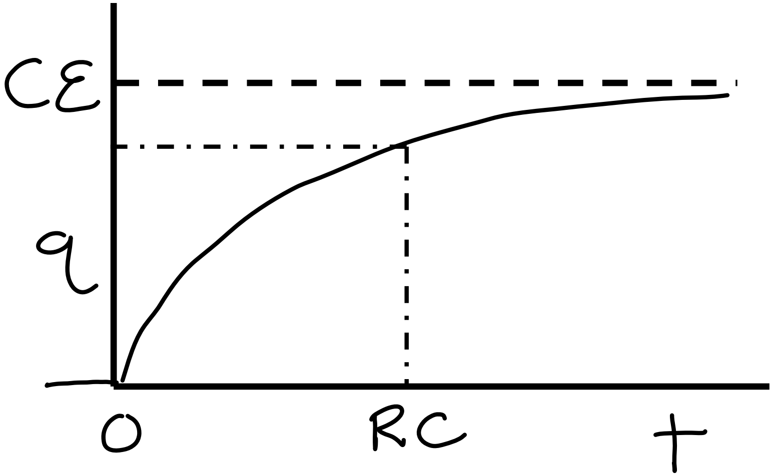
$$i(t) = \frac{dq}{dt}$$

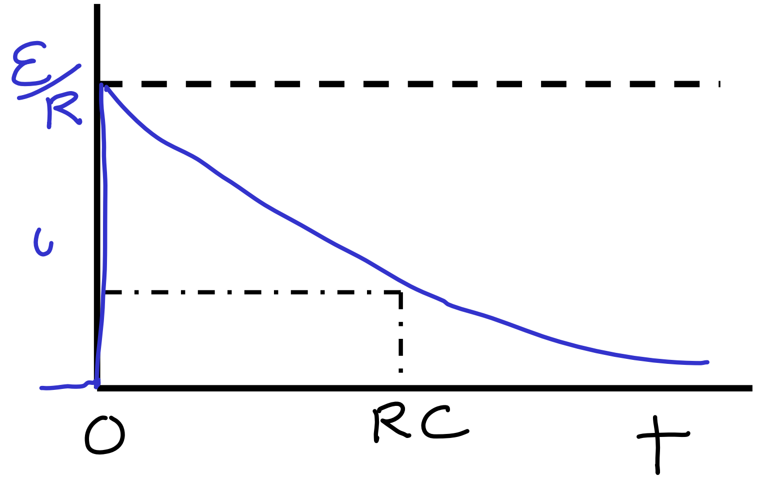
$$= \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$= i_0 e^{-t/RC}$$



Time Constant



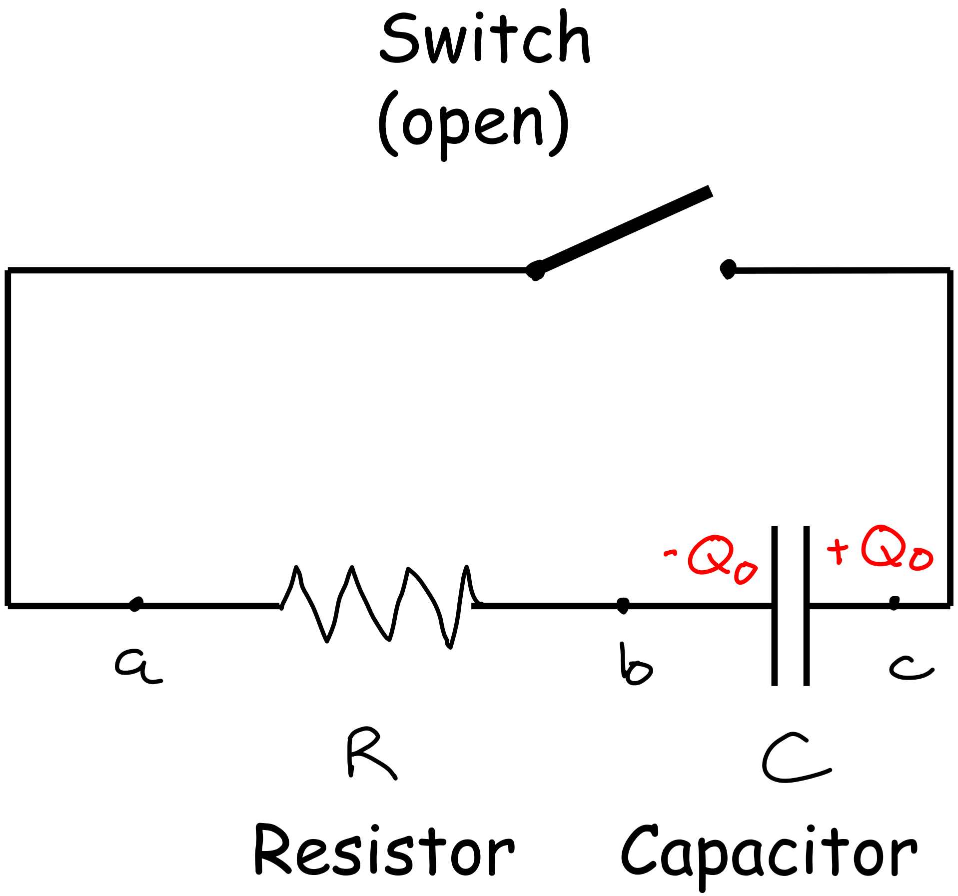


$$q(t=RC) = C\mathcal{E}(1-e^{-1})$$
$$= Q_f(1-e^{-1})$$

$$i(t=RC) = \frac{\mathcal{E}}{R}e^{-RC/RC}$$
$$= \frac{\mathcal{E}}{R}e^{-1} = i(t=0)/e$$

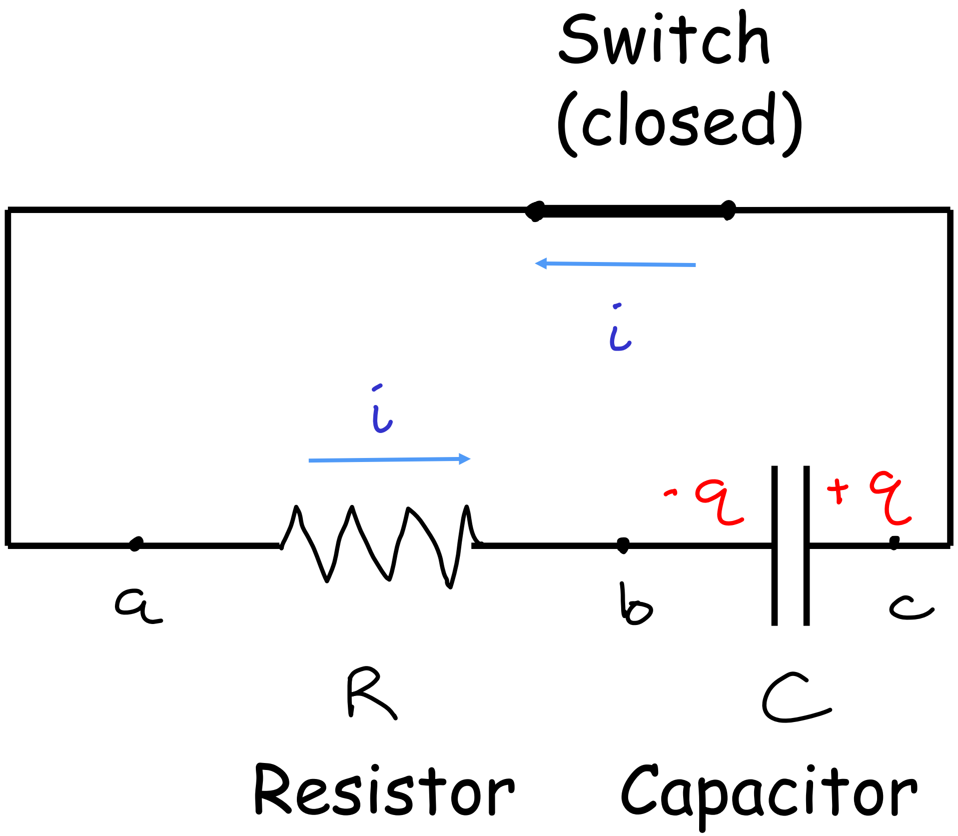
Time constant τ = RC: quantifies *rate* of charging

Discharging a Capacitor



What happens when the switch is closed?

Discharging a Capacitor



$$i = -\frac{dq}{dt}$$?

Capacitor discharges and the current eventually goes to zero

$$V_{ab} + V_{bc} = 0$$

$$-iR + q/C = 0$$

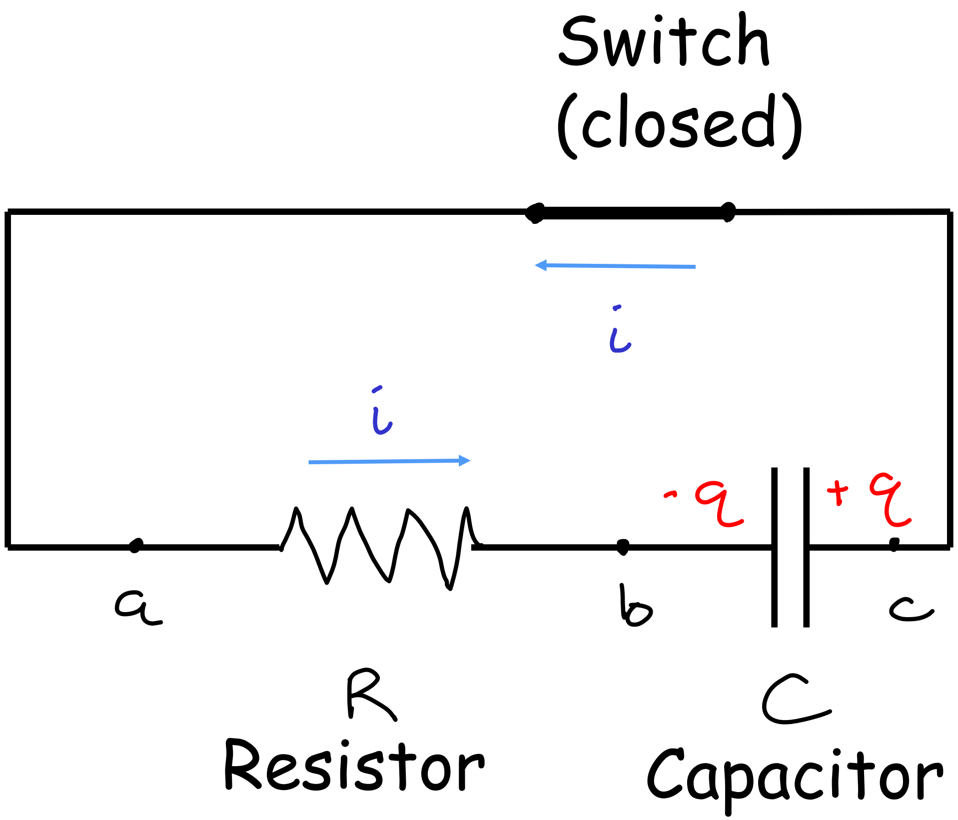
$$iR = q/C$$

$$\frac{dq}{dt} = -\frac{1}{RC}q$$

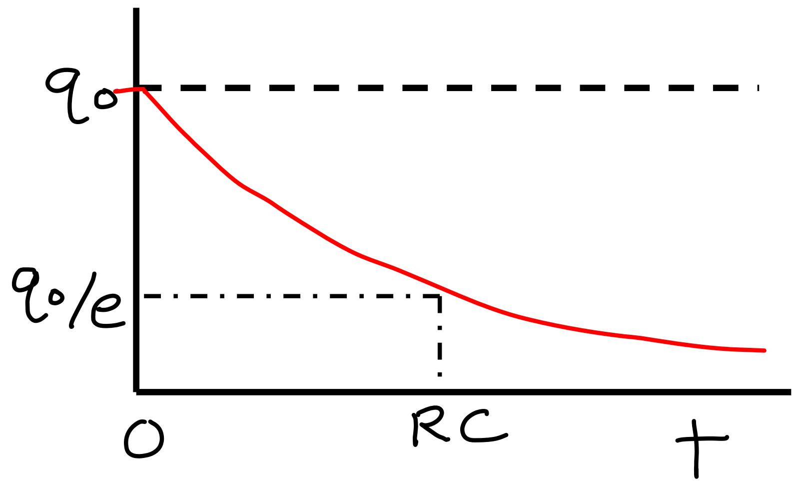
$$q(t) = q_0 e^{-t/RC}$$

$$i(t) = \frac{q_0}{RC} e^{-t/RC}$$

Discharging a Capacitor



$$q(t) = q_0 e^{-t/RC}$$



Discharging a Capacitor

$$i(t) = \frac{dq}{dt} = \frac{q_0}{RC} e^{-t/RC}$$