THE UNIVERSITY OF SYDNEY
SCHOOL OF MATHEMATICS AND STATISTICS

Assignment 2

MATH1901/1906: Differential Calculus (Advanced) Semester 1, 2017

Web Page: http://sydney.edu.au/science/maths/u/UG/JM/MATH1901/
Lecturer: Daniel Daners

This assignment is due by **23:59 Thursday 18th May 2017**, via Turnitin. A PDF copy of your answers must be uploaded in the Learning Management System (Blackboard) at https://elearning.sydney.edu.au.

Please submit a single PDF document (scan or convert other formats). It should include your name and SID; your tutorial time, day, room and Tutor's name. It is your responsibility to preview each page of your assignment after uploading to ensure each page is included in correct order and is legible (not sideways or upside down) before confirming your submission. After submitting you can go back and view your submission to check it.

The School of Mathematics and Statistics encourages some collaboration between students when working on problems, but students must write up and submit their own version of the solutions.

This assignment is worth 2.5% of your final assessment for this course. Your answers should be well written, neat, thoughtful, mathematically concise, and a pleasure to read. Please cite any resources used and show all working. Present your arguments clearly using words of explanation and diagrams where relevant. After all, mathematics is about communicating your ideas. This is a worthwhile skill which takes time and effort to master. The marker will give you feedback and allocate an overall letter grade and mark to your assignment using the following criteria:

Mark	Grade	Criterion
10	A+	Outstanding and scholarly work, answering all parts of all questions, with clear and accurate explanations and working, appropriate acknowledgement of sources (if appropriate), and at most minor or trivial errors or omissions.
9	A	Very good work, making excellent progress on both questions, but with one or two substantial errors, misunderstandings or omissions throughout the assignment.
7	B	Good work, making excellent progress on one question and good progress on the second, but making more than two distinct substantial errors, misunderstandings or omissions throughout the assignment.
6	C	A reasonable attempt, making good progress on only one of the two questions with some attempt of the remaining question.
4	D	Some attempt, with substantial progress made on only one question.
2	E	No substantial progress made on any of the two questions.
0	F	No credit awarded.

1. (a) Using L'Hôpital's rule, or otherwise, compute

$$\lim_{x\to 1}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right).$$

You will need to rewrite the expression as one fraction.

(b) Compute the Taylor polynomial of order 5 of the function $f(x) := \dfrac{e^{x^2}}{x^2}$ about $x = 1$.

2. (a) Use the Mean Value Theorem to show that $\sqrt{1+x} \le 1 + \dfrac{x}{2}$ whenever $-1 < x < \infty$.

(b) Let $f : \mathbb{R} \to \mathbb{R}$ be a differentiable function. Fix $x_0 \in \mathbb{R}$. Carathéodory's characterisation for differentiability at x_0 asserts that there exists a function $m_{x_0} : \mathbb{R} \to \mathbb{R}$, continuous at x_0, such that

$$f(x) = f(x_0) + m_{x_0}(x)(x - x_0)$$

for all $x \in \mathbb{R}$. In that case $f'(x_0) = m_{x_0}(x_0)$. Assume that f is bijective with inverse $f^{-1} : \mathbb{R} \to \mathbb{R}$ and that $f'(x_0) \neq 0$. You will need to use that the inverse is continuous, but you are not required to show this show this).

Use Carathéodory's characterisation of the derivative to prove that f^{-1} is differentiable at $y_0 := f(x_0)$ and show that

$$(f^{-1})'(y_0) = \frac{1}{f'(f^{-1}(y_0))}$$

(You cannot differentiate $(f \circ f^{-1})(x) = x$ implicitly using the chain rule since this *assumes* that f^{-1} is differentiable!)