THE UNIVERSITY OF SYDNEY

MATH1901 DIFFERENTIAL CALCULUS (ADVANCED)

Semester 1 **Tutorial Week 7** 2012

1. (*This question is a preparatory question and should be attempted before the tutorial. Answers are provided at the end of the sheet – please check your work.*)

Differentiate the following (don't worry about the domain of the function or its derivative).

(a) $f(x) = e^{x+5}$

(b) $f(x) = (\ln 4)e^x$

(c) $f(x) = xe^x$

(d) $f(x) = \dfrac{x^2 + 5x + 2}{x + 3}$

(e) $f(x) = (x + 1)^{99}$

(f) $f(x) = xe^{-x^2}$

(g) $f(t) = \tan t$

(h) $f(t) = e^{\cos t}$

(i) $f(t) = e^{t \cos 3t}$

(j) $f(t) = \ln\big(\cos(1 - t^2)\big)$

(k) $f(x) = (x + \sin^5 x)^6$

(l) $f(x) = \sin(\sin(\sin x))$

(m) $f(x) = \sin(6 \cos(6 \sin x))$

Questions for the tutorial

2. For each of the following functions f, find $f(f'(x))$ and $f'(f(x))$.

(a) $f(x) = \frac{1}{x}$,

(b) $f(x) = x^2$,

(c) $f(x) = 2$,

(d) $f(x) = 2x$.

3. For the functions given by the following formulas, find the maximum and minimum values on the indicated intervals.

(a) $f(x) = \dfrac{e^x}{x + 1}$ on $[2, 3]$

(b) $f(x) = \dfrac{x}{x^2 + 1}$ on $[-2, 0]$

(c) $f(x) = e^{x^2-1}$ on $[-1, 1]$

4. Consider the function defined by

$$f(x) = \begin{cases} x^2 & \text{for } x \leq 1 \\ e^{ax+b} & \text{for } x > 1. \end{cases}$$

(a) Determine for which values of a and b the function f is continuous at $x = 1$.

(b) Determine for which values of a and b the function f is differentiable at $x = 1$.

5. Use Rolle's Theorem and the IVT to show that the equation $x^2 - x \sin x - \cos x = 0$ has exactly 2 solutions.

6. Define a function f by

$$f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational,} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$$

Show that f is differentiable at 0.

7. Prove that if f is differentiable at a and $f(a) \neq 0$, then $|f|$ is also differentiable at a. Give an example to show why the assumption $f(a) \neq 0$ is necessary.

Extra Questions

8. Define a function f by

$$f(x) = \begin{cases} x^2 \sin \dfrac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

Show that f is differentiable everywhere and that f' is not continuous at 0.

9. Using Rolle's Theorem, prove that a polynomial of degree $n > 0$ has at most n real roots.

Solution to Question 1

(a) $f'(x) = e^{x+5}$
(b) $f'(x) = (\ln 4)e^x$
(c) $f'(x) = e^x + xe^x = (1+x)e^x$
(d) $f'(x) = \dfrac{(x+3)(2x+5) - (x^2+5x+2)}{(x+3)^2} = \dfrac{x^2+6x+13}{(x+3)^2}$
(e) $f'(x) = 99(x+1)^{98}$
(f) $f'(x) = e^{-x^2} - 2x^2e^{-x^2} = (1-2x^2)e^{-x^2}$
(g) $f'(t) = \dfrac{d}{dt}\left(\dfrac{\sin t}{\cos t}\right) = \dfrac{-\sin t \cdot (-\sin t) + \cos t \cdot \cos t}{\cos t \cdot \cos t} = \dfrac{1}{\cos^2 t} = \sec^2 t$
(h) $f'(t) = (-\sin t)e^{\cos t}$
(i) $f'(t) = (\cos 3t - 3t \sin 3t)e^{t \cos 3t}$
(j) $f'(t) = \dfrac{2t \sin(1-t^2)}{\cos(1-t^2)}$
(k) $f'(x) = 6(x + \sin^5 x)^5(1 + 5\sin^4 x \cos x)$
(l) $f'(x) = \cos(\sin(\sin x)) \cos(\sin x) \cos x$
(m) $f'(x) = -36 \cos(6\cos(6\sin x)) \sin(6\sin x) \cos x$