1a) <u>Model</u>: Yields (in g) are values taken by indep. random variables

$X_1, \dots, X_{12} \sim N(\mu_X, \sigma^2)$ and

$Y_1, \dots, Y_{12} \sim N(\mu_Y, \sigma^2)$.

<u>Test</u>: $H_0: \mu_X = \mu_Y$ vs $H_1: \mu_X \neq \mu_Y$ (2-sided).

<u>Statistic</u>: $T = \dfrac{\bar{X} - \bar{Y}}{S_p\sqrt{\frac{1}{12}+\frac{1}{12}}}$, $S_p^2 = \dfrac{S_X^2 + S_Y^2}{2}$ (since sample sizes equal)

$\bar{X} = \frac{1}{12}\sum_{i=1}^{12} X_i$, $\bar{Y} = \frac{1}{12}\sum_{i=1}^{12} Y_i$

$S_X^2 = \frac{1}{11}\sum_{i=1}^{12}(X_i - \bar{X})^2$, $S_Y^2 = \frac{1}{11}\sum_{i=1}^{12}(Y_i - \bar{Y})^2$.

If H_0 true, $T \sim t_{22}$

If T takes the value t_{obs} p-value $= P(|t_{22}| \geq |t_{obs}|)$

$= 2P(t_{22} \geq |t_{obs}|)$

Obs value of S_p^2 is $s_p^2 \approx \dfrac{1.579015 + 0.887272}{2}$

≈ 1.123344

Obs value of $t_{obs} \approx \dfrac{9.491667 - 11.8}{\sqrt{1.233144/6}} \approx -5.0917$

p-value $\approx 2P(t_{22} \geq 5.092)$. From tables

since $P(t_{22} \geq 3.505) \approx 0.001$

our p-value ≤ 0.002 $\left(= 2P(t_{22} \geq 3.505)\right)$

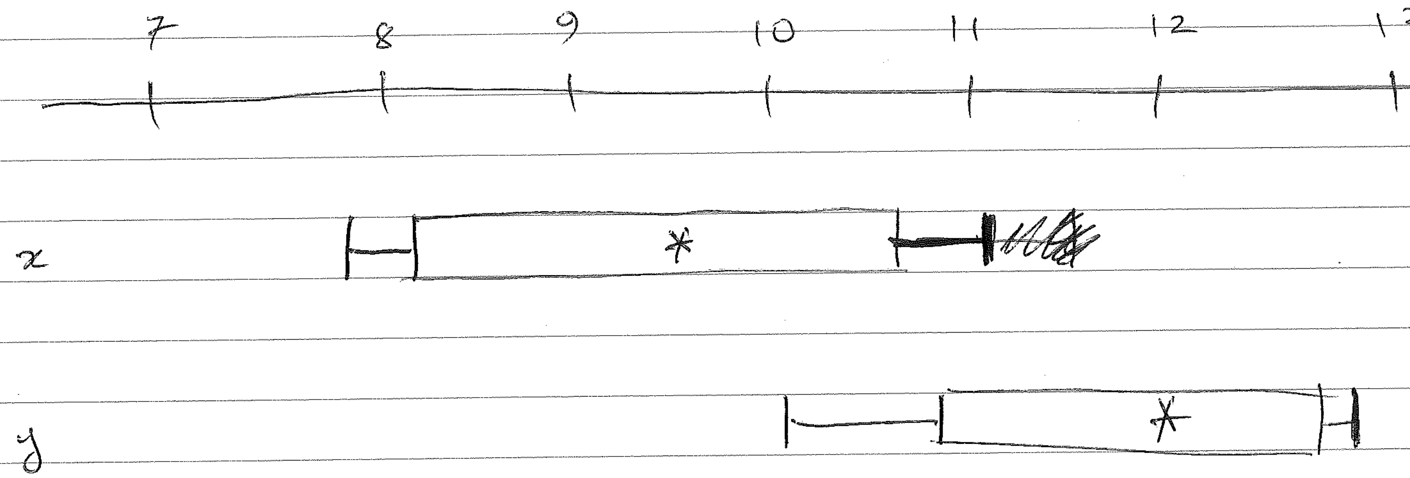
This is significant at the 0.2% level.

This constitutes evidence against H_0.

b) For 12 observations

x_i: $Q_1 = 8.1$, med $= 9.65$, $Q_3 = 10.65$

y_i: $Q_1 = 10.9$, med: 12.05, $Q_3 = 12.65$



No outliers: Both have similar spread and are reasonably symmetric. The assumptions of normality and equal variance are reasonable.

3 a) i) Test of Homogeneity.

ii) Exp. Freq:

81	74	45
81	74	45

Pearson Statistic: $\dfrac{(72-81)^2+(90-81)^2}{81}+\dfrac{(74-78)^2+(74-70)^2}{74}$

$+\dfrac{(45-50)^2+(45-40)^2}{45}$

$=\dfrac{2\times 9^2}{81}+\dfrac{2\times 4^2}{74}+\dfrac{2\times 5^2}{45}$

$=2+\dfrac{32}{74}+\dfrac{50}{45}\approx 3.54344$

(Approx.) p-value $=P\left(\chi^2_{\boxed{2}}\geq 3.54344\right)$ $(r-1)(c-1)$

From tables $0.15 < \text{p-value} < 0.25$.

No evidence to suggest a difference

4 (a) (Tutorial question?)

(b) (i) $\hat{\beta}=S_{xy}/S_{xx}=\dfrac{643}{768.8}\approx 0.8363$

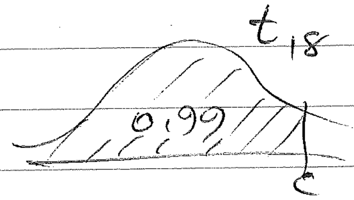
$s.e.=\dfrac{\hat{\sigma}}{\sqrt{S_{xx}}}$, $\hat{\sigma}^2=\dfrac{S_{yy}-\frac{S_{xy}^2}{S_{xx}}}{18}=\dfrac{1537.75-\frac{643^2}{768.8}}{18}$

≈ 55.55

So $\quad se = \sqrt{\dfrac{55.55}{768.8}} \approx 0.26888\ldots$

<u>One-sided</u> confidence interval

$(\hat{\beta} - c.\,se, \infty)$ where



$P(t_{18} \le c) = 0.99$

i.e. $c = 2.552$

So lower confidence limit is

$0.8363 - (2.552 \times 0.268888\ldots)$

≈ 0.150

Further assumptions: * Y_i's independent

* Y_i's normal

* Y_i's have common variance σ^2