Til Seva:

SAAB
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FALUN
LUKOIL
SAAB
Defence and Security
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FALUN
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LUKOIL

Matte 4k øving 8 Eira Mørch-Thoresen

TMA4120 MTTK

13.4 Cauchy-Riemann Equation

3.) $f(z) = e^{-x}\cos(y) - ie^{-x}\sin(y)$

$f(z) = u(x,y) + iv(x,y)$

C.R. eq:
$u_x = v_y$
$u_y = -v_x$

$u_x = -e^{-x}\cos y$

$v_y = -e^{-x}\cos y$

$u_y = -e^{-x}\sin y = v_x = e^{-x}\sin y$

Analytisk

9.) $f(z) = \dfrac{3\pi^2}{z^3 + 4\pi^2 z}$

$\dfrac{d}{dz} = -\dfrac{3\pi^2(3z^2+4\pi^2)}{(z^3+4\pi^2 z)^2}$ Analytisk når $z \neq 0$

13.) $u = -2xy$ $u_{xx} + u_{yy} = 0 \leftarrow$ harmonic

$u_{xx} = 0$ $u_{yy} = 0$ harmonic

For å finne conjugate harmonic function, må u og v stemme med Cauchy-Riemann eq.

$u = -2xy \Rightarrow u_x = -2y$ $v_y = -2y$

$u_y = -2x$ $v_x = 2x$

v kan da være $x^2 - y^2$

$f(z) = u(x,y) + iv(x,y) = -2xy + i(x^2 - y^2)$

B.5 Exponential function

5.) Find e^z in the form $u+iv$ and $|e^z|$

for $z = 1-3\pi i$

$$\boxed{e^z = e^x(\cos y + i\sin y)}$$

$$e^z = e^1(\cos(-3\pi) - i\sin(-3\pi))$$

$$= e(\cos(3\pi) + i\sin(3\pi)) = -e + 0 = \underline{\underline{-e}}$$

16.) Find Re and Im of $e^{\frac{1}{z}}$

$$e^{\frac{1}{z}} = e^{\frac{1}{x+iy}} = e^{\frac{x-iy}{x^2-y^2}} = e^{\frac{x}{x^2-y^2} - i\frac{y}{x^2-y^2}}$$

$$\mathrm{Re}(e^{\frac{1}{z}}) = e^{\frac{x}{x^2-y^2}}\cos\left(\frac{y}{x^2-y^2}\right)$$

$$\mathrm{Im}(e^{\frac{1}{z}}) = -e^{\frac{x}{x^2-y^2}}\sin\left(\frac{y}{x^2-y^2}\right)$$

13.6 Trigonometric and Hyperbolic Functions

3.) Vis at $\cosh^2(z) - \sinh^2(z) = 1$

$$\cosh^2(z) = \left(\tfrac{1}{2}(e^z + e^{-z})\right)^2 = \tfrac{1}{4}(e^{2z} + 2 + e^{-2z})$$

$$\sinh^2(z) = \left(\tfrac{1}{2}(e^z - e^{-z})\right)^2 = \tfrac{1}{4}(e^{2z} - 2 + e^{-2z})$$

$$\Rightarrow \tfrac{1}{4}(\cancel{e^{2z}} + 2 + \cancel{e^{-2z}}) - \tfrac{1}{4}(\cancel{e^{2z}} - 2 + \cancel{e^{-2z}})$$

$$= \tfrac{1}{4}2 + \tfrac{1}{4}\cdot 2 = \tfrac{1}{2} + \tfrac{1}{2} = \underline{\underline{1}}$$

9.) $\cosh(-2+i)$, $\cos(-1-2i)$ Find in form $x+iy$

$$\cosh(-2+i) = \tfrac{1}{2}\left(e^{(-2+i)} + e^{-(-2+i)}\right) = \tfrac{1}{2}\left(e^{-2+i} + e^{2-i}\right)$$

$$= \tfrac{1}{2}\left(e^{-2}(\cos(1) + i\sin(1)) + e^{2}(\cos(-1) - i\sin(-1))\right)$$

$x+iy$ form:

$$x = \tfrac{1}{2}e^{-2}\cos(1) + \tfrac{1}{2}e^{2}\cos(-1) = 2.03$$

$$y = \tfrac{1}{2}e^{-2}\sin(1) + \tfrac{1}{2}e^{2}\sin(-1) = -3.05$$

$$x+iy = \underline{\underline{2.03 - i\,3.05}}$$

$$\cos(-1-2i) = \tfrac{1}{2}\left(e^{i(-1-2i)} + e^{-i(-1-2i)}\right) = \underbrace{\tfrac{1}{2}\left(e^{2-i} + e^{-2+i}\right)}$$

ser at dette er det samme som i cos h, og da vet vi allerede svaret

$$\underline{\underline{\cos(-1-2i) = 2.03 - i\,3.05}}$$

18.) Find all solutions to $\cosh(z) = -1$

$$\frac{1}{2}(e^z + e^{-z}) = -1$$

$e^z = s$, $e^{-z} = \frac{1}{s}$

$$e^z + e^{-z} = -2$$

$$s + \frac{1}{s} = -2 \quad / \cdot s$$

$$s^2 + 2s + 1 = 0$$

abc-formel: $s = -1$

$$e^z = -1 \Rightarrow z = \ln(-1) = i\pi$$

$$e^{-z} = \frac{1}{-1} = -1 \Rightarrow -z = \ln(-1) = -i\pi$$

Alle løsninger:

$$\underline{\underline{z = i\pi + 2\pi n i}}$$

$$\underline{\underline{z = -i\pi + 2\pi n i}}$$

13.7 Logarithm. General power. Principal Value.

7.) Find $Ln(z)$ when $z = 8-8i$

$r = \sqrt{8^2+8^2} = 8\sqrt{2}$

$\theta = \arctan(-1) = -\pi/4$

$\boxed{Ln(z) = \ln|z| + i\,Arg(z)}$

$\boxed{\ln(z) = \ln(r) + i\theta}$

$Ln(8-8i) = \ln(8\sqrt{2}) - i\frac{\pi}{4}$

$= \underline{2.426 - 0.7854i}$

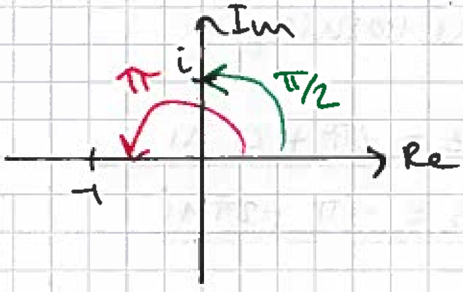
17.) Find all values of $\ln(z)$ and graph some of them

Show that the set of values $\ln(i^2)$ differs from the set of values of $2\ln(i)$

$\ln(i^2) \Rightarrow z_1 = -1$

$r = \sqrt{(-1)^2} = 1$

$\theta = \pi$



$\ln(z_1) = \ln(-1) = \underbrace{\ln(1)}_{0} + i\pi + 2\pi i n = i(\pi + 2\pi n)$

$2\ln(i) \Rightarrow z_2 = i$

$r = \sqrt{1^2} = 1$

$\theta = \pi/2$

$\ln(z_2) = \ln(i) = \ln(1) + i\pi/2 + 2\pi i n = i(\pi/2 + 2\pi n)$

$\Rightarrow 2\ln(z_2) \Rightarrow 2i(\pi/2 + 2\pi n) = i(\pi + 4\pi n)$

$i(\pi + 2\pi n) \neq i(\pi + 4\pi n)$

Flere løsninger enn her

19.) Solve for z

$$\ln(z) = 4-3i$$

$$z = e^{(4-3i)} = e^4(\cos(-3)+i\sin(-3))$$

$$= e^4(\cos(3) - i\sin(3))$$

$$\underline{\underline{= -54.05 - 7.7i}}$$

23.) Find the principal value

$$(1+i)^{1-i} = e^{(1-i)\ln(1+i)} = e^{(1-i)\left[\ln\sqrt{2} + \frac{\pi}{4}i \pm 2\pi n i\right]}$$

$$= e^{\left(\ln\sqrt{2} + \frac{\pi}{4}i - i\ln\sqrt{2} + \frac{\pi}{4}\right)} = \underline{\underline{2.8079 + 1.3179i}}$$

Supplementary R

Find all solutions to $e^{2z} = i$

$\Rightarrow 2z = \ln(i)$

$z = \frac{1}{2}\ln(i)$

$$\boxed{\ln(z) = \ln(r) + i\theta}$$

$r = \sqrt{1^2} = 1$

$\theta = \pi/2$

$\Rightarrow \ln(i) = \ln(1) + \pi/2\, i + 2\pi i n = i\left(\frac{\pi}{2} + 2\pi n\right)$

$\frac{1}{2}\ln(i) = \frac{i}{2}\left(\frac{\pi}{2} + 2\pi n\right) = \underline{\underline{i\left(\frac{\pi}{4} + \pi n\right)}}$

Supplementary S_1

$f(z) = y^3 + Bx^2y + iv(x,y)$

$u_{xx} + u_{yy} = 0$

$u_{xx} = 2By$

$u_{yy} = 6y$

$2By = 6y$

$\Rightarrow B = -3$

$f(z) = y^3 - 3x^2y + iv(x,y)$

Finne v:

Cauchy Riemann:

$$\boxed{\begin{aligned} u_x &= v_y \\ u_y &= -v_x \end{aligned}}$$

$u_x = -6xy$

$v(x,y) = \int_{y_0}^{y} -6xt\,dt = -\frac{6x}{2}y^2 + C(x) = -3xy^2 + C(x)$

Forts. supp. 5

$V_x = -U_y = -3y^2 + 3x^2 = -3y^2 + c'(x)$

$c(x) = x^3 + K$ $\quad \left(\int 3x^2 = x^3 \right)$

$v(x,y) = -3xy^2 + x^3 + K$

$v(0,0) = 0 . \Rightarrow K = 0 .$

$\Rightarrow v(x,y) = -3xy^2 + x^3$