TFY4115 - Høst 2016

Øving 8

Vsevolod Karpov - vsevolok

Ønsker retting!

Oppgave 1.

a)

$$ma = -b\dot{x} - kx$$

$$\Rightarrow \ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\Rightarrow \omega_0 = \sqrt{\frac{k}{m}}, \quad \gamma = \frac{b}{2m}$$

b) Overkritisk: Lang tidsskonstant/responstid

Underkritisk: Oscilerer før stabil tilstand.

Kritisk d.: Ingen oscilasjoner. Kort responstid.

c)

$$x(t) = Ae^{-\gamma t}\cos(\omega_d t + \phi)$$

$$x'(t) = A\left(-\gamma e^{-\gamma t}\cos(\omega_d t + \phi) - \omega_d e^{-\gamma t}\sin(\omega_d t + \phi)\right)$$

$$x''(t) = -\gamma x'(t) - A\omega_d\left(-\gamma e^{-\gamma t}\sin(\omega_d t + \phi) + e^{-\gamma t}\cos(\omega_d t + \phi)\cdot\omega_d\right)$$

$$= -\gamma x'(t) - \omega_d^2 x(t) + A\omega_d\gamma e^{-\gamma t}\sin(\omega_d t + \phi)$$

$$\cancel{\Rightarrow x(t)(-\gamma - \omega_d) + A\omega_d\gamma e^{-\gamma t}\sin(\omega_d t + \phi)}$$

$$\cancel{+ 2\gamma A\left(-\gamma e^{-\gamma t}\cos(\omega_d t + \phi) - \omega_d e^{-\gamma t}\sin(\omega_d t + \phi)\right)}$$

$$\cancel{+ \omega_0^2 \cdot \underbrace{Ae^{-\gamma t}\cos(\omega_d t + \phi)}_{x(t)} = 0}$$

$$\Rightarrow -\gamma x'(t) - \omega_d x(t) + A\omega_d\gamma e^{-\gamma t}\sin(\omega_d t + \phi)$$

$$+ 2\gamma x'(t) + \omega_0^2 x(t) = 0$$

$$x'(t) = -\gamma x(t) - \omega_d e^{-\gamma t}\sin(\omega_d t + \phi)A$$

$\Rightarrow x(t)(\omega_0^2 - \omega_d^2 - \gamma^2) = 0$

Altså en løsning når $\omega_0^2 - \gamma^2 - \omega_d^2 = 0$

$\omega_0^2 > \gamma \Rightarrow \omega_0^2 - \gamma^2 > 0 \Rightarrow \omega_d^2 > 0$

I tråd med logikken om at ω_d^2 må være positiv! ($\omega_d \in \mathbb{R}$)

$$\Rightarrow \underline{\underline{\omega_d = \sqrt{\omega_0^2 - \gamma^2}}}$$

d) Da to foregående verdier behøves må programmet ha kjennskap til de første to foregående verdier, altså $x(t_0)$ og $x(t_1)$!

Se vedlegg:

Det er tydelig at numerisk løsning fungerer fint for alle typer demping så lenge tidsinkrementet er lite nok (helst mindre eller lik 0.01)

e)

i) $A(t) = 6.0 e^{-\gamma t}$

$A(0) = 6.0$

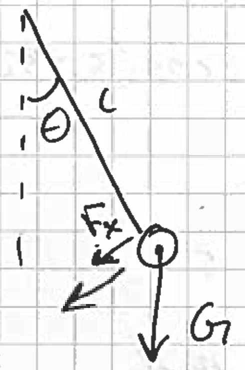
$A(t_1) = 6.0 \cdot e^{-\gamma t_1} = 5.1$

$t_1 = 2 \cdot 60 = 120$

$$\Rightarrow \gamma = -\ln\left(\frac{5.1}{6.0}\right)/120 = \underline{\underline{1.35 \cdot 10^{-3}}}$$

ii) $x(t) = A\cos(\omega_d t + \phi) \cdot e^{-\gamma t}$

Men! Det er jo snakk om en pendel.



Vi antar: $\theta_0 = 6.0^\circ$

$l = 1.00\,m$

$\Rightarrow F_x = -\sin(\theta)\,mg$

$a = \sin(\theta)\,g$

$\tau = -F_x \cdot l$

$\alpha I = -F_x \cdot l$

$I = m \cdot l^2, \quad \alpha = \frac{a}{l}$

$\Rightarrow \ddot{\theta}\, l = \sin(\theta) \cdot g$

$\frac{d^2\theta}{dt}\, l = \sin(\theta) \cdot g$

$\Rightarrow \cancel{\frac{l}{\sin(\theta)}\, d\theta = g\, dt}$

$\Rightarrow \cancel{-\frac{2\sin(\theta) \cdot l}{\cos(2\theta) - 1} = gt + C}$

$\cancel{t_0 = 0 \Rightarrow \theta_0 = 6.0^\circ = \frac{\pi}{30} \Rightarrow C = 0.21}$

Men: Likningen kan ikke løses for $\theta = 0$

Også er det slik at dette medfører en ikke lineær diff ligning og formelen fra a) kan ikke brukes.

Vi gjør en forenkling (som er vel ok for små vinkler)

$F_x = -\theta\, mg$

$\Rightarrow \theta l = \theta g \Rightarrow \cancel{\theta(t) = C_1 \cdot e^{gt}}$

$\cancel{\theta(0) = \frac{\pi}{30} \Rightarrow C = \frac{\pi}{30}}$

$\cancel{\theta(t_1) = 0 = \frac{\pi}{30} e^{g t_1}}$

$\cancel{\Rightarrow \theta = C_1 e^{3.132t} + C_2 e^{-3.132t}}$

$\cancel{\theta(0) = \frac{\pi}{30}, \quad \theta'(0) = 0}$

$\cancel{\Rightarrow C_1 = C_2 = \frac{\pi}{60}}$

$\Rightarrow \quad \theta(0) = \frac{\pi}{30}, \quad \theta'(0) = 0, \quad \theta(t) = 0.10472\cos(3.43209t)$

$\theta = 0 \Rightarrow \cos(3.15209t) = 0$

$\Rightarrow 3.45209t = \frac{\pi}{2} \Rightarrow t = \frac{\pi}{2\cdot 3.43209} = 0.5$

Altså: 0.5 sekund for å komme ned

$\Rightarrow \omega_0 = \frac{1}{2}$

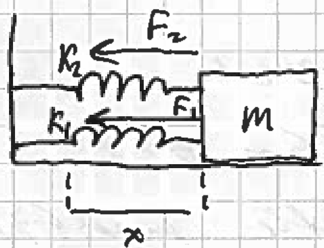
$\Rightarrow \omega_d = \sqrt{\omega_0^2 - \gamma^2} = 0.49999$

$\omega_d = \frac{1}{T_d} \Rightarrow T_d = \frac{1}{\omega_d} = \underline{\underline{2.004\,s}}$

Gir mening. dempingen er jo veldig liten.

Oppgave 2

a)



$\Sigma F = ma = F_1 + F_2 = -(K_1 x + K_2 x)$

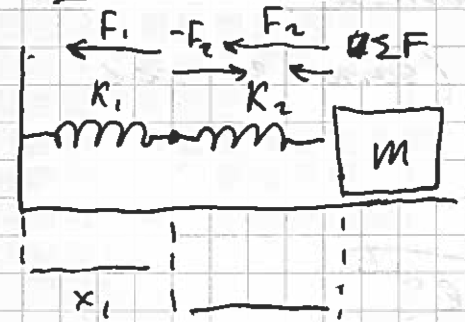
$\Rightarrow \ddot{x} = -x\frac{(K_1 + K_2)}{m}$

$\ddot{x} = -x(\omega_1^2 + \omega_2^2)$

$\ddot{x} = -x\omega^2$

$\Rightarrow \underline{\underline{\omega = \sqrt{\omega_1^2 + \omega_2^2}}}$

b)



$\Sigma F = ma = -K_{eq}(x_1 + x_2)$

$F_1 = -K_1 x_1 = F_2 = -K_2 x_2$

$\Rightarrow x_1 = \frac{K_2}{K_1} x_2$

$\Rightarrow \Sigma F = -K_{eq}\left(x_2\left(\frac{K_2}{K_1} + 1\right)\right)$

$\Sigma F = F_2 \Rightarrow -K_2 x_2 = -K_{eq} x_2\left(\frac{K_2}{K_1} + 1\right)$

$\Rightarrow K_{eq} = \frac{K_2}{\frac{K_2}{K_1} + 1} = \frac{K_2 K_1}{K_2 + K_1}$

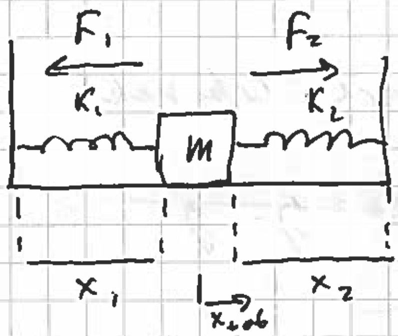
$\Rightarrow \omega = \sqrt{\frac{K_{eq}}{m}}$

$$\omega^2 = \frac{K_2 K_1}{m(K_2+K_1)} \quad , \quad \omega_1^2 = \frac{K_1}{m} \quad , \quad \omega_2^2 = \frac{K_2}{m}$$

$$\Rightarrow \omega^2 = \frac{\omega_1^2 m \omega_2^2}{K_2+K_1} \quad , \quad K_1 = \omega_1^2 m \quad , \quad K_2 = \omega_2^2 m$$

$$\Rightarrow \omega^2 = \frac{\omega_1^2 \omega_2^2}{\omega_1^2+\omega_2^2} \Rightarrow \omega = \sqrt{\frac{\omega_1^2\omega_2^2}{\omega_1^2+\omega_2^2}}$$

c)



$$\Sigma F = ma = F_2 + F_1$$

$$F_2 = -K_2 x_2$$

$$F_1 = -K_1 x_1$$

$$ma = -K_{eq} x_{tot} \quad , \quad x_{tot} = x_1 + x_2$$

$$\Rightarrow ma = -K_2 x - K_1 x = -x(K_1+K_2)$$

$$\Rightarrow K_{eq} = K_1 + K_2$$

$$\Rightarrow \omega = \sqrt{\omega_1^2+\omega_2^2}$$

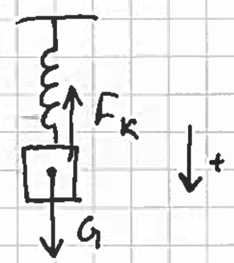
Oppgave 3

a) Likevekt når $G = F_K$

$$\Rightarrow mg = k\Delta x$$

$$\Rightarrow \Delta x = \frac{mg}{k}$$

b) $\Sigma F = ma = G - F_K$



$$\Sigma F = ma = mg - Ky$$

$$\Rightarrow \ddot{y} = g - \frac{K}{m} y$$

$$\Rightarrow y = g + C_1 \sin\left(\sqrt{\tfrac{K}{m}}\,t\right) + C_2 \cos\left(\sqrt{\tfrac{K}{m}}\,t\right)$$

$$y = g + A\cos\left(\sqrt{\tfrac{K}{m}}\,t + \phi\right)$$

$\Rightarrow$ Samme frekvens, kun forskjellig likevekt.

c) $E_{tot} = E_{p0} + E_{k0} + E_{f0}$

$= mgy_0 + \frac{1}{2}mv_0^2 + \frac{1}{2}ky_0^2$

Oppgave 4a)

$\alpha = \frac{60}{12} = 5\,\text{rad}/s^2$

$\sum \tau = r \cdot F = I_0 \cdot \alpha$

$\Rightarrow I_0 = \frac{r \cdot F}{\alpha} = \frac{0.25 \cdot 20}{5} = 1$ B

b) $\sum F = ma = F$

E alle akselererer likt

c) $\sum F = ma$

$\sum F_k = S$, $\sum F_s = S \Rightarrow a_s = a_k$

$\sum \tau_k = 0$, $\sum \tau_s = S \cdot r \Rightarrow \alpha_k = 0$, $\alpha_s \neq 0$

$\Rightarrow$ B sammtidig, men spolen vil rotere når den passerer

d) $\sum F = ma = G_1' - f$

$a_1 = a_2 < a_3$

$\Rightarrow \sum F_1 = \sum F_2 < \sum F_3$

$G_1' = G_2' = G_3' \Rightarrow f_1 = f_2 > f_3$ A