TFY 4115 - Høst 2016
Øving 5

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Oppgave 1 a)

$m = 0.30\ kg\ ,\ R = 0.30 m \quad M = 1.00\ kg$

T. moment til <u>en</u> eik:

$$I_e = \int_0^R r^2 \frac{m}{R} dr = \frac{m}{R} \cdot \left[\frac{1}{3} r^3\right]_0^R = \frac{mR^2}{3} = 0.009$$

T. moment til felgen:

$$\cancel{I_f = \frac{2M}{R^2} \int_0^R r^3 dr = \frac{2M}{R^2} \left[\frac{1}{4} r^4\right]_0^R = \frac{1}{2} MR^2 = 0.045}$$

$$\cancel{I_{tot} = I_f + 8 \cdot I_e = 0.117}$$

$$I_f = 2\pi \rho L \int_R^R r^3 dr\ ,\ \rho = \frac{M}{V} = \frac{M}{\pi(R^2 - R^2)L}$$

$$\Rightarrow I_f = \frac{2M}{R^2 - R^2} \left[\frac{1}{4} r^4\right]_R^R = \frac{1}{2} \cdot \frac{M}{R^2 - R^2} \cdot (R^4 + R^4)$$

$$= \frac{1}{2} \cdot \frac{M}{R^2 - R^2} \cdot (R^2 + R^2)(R^2 - R^2) = MR^2 = 0.09$$

$$\Rightarrow I_{tot} = I_f + 8 \cdot I_e = \underline{\underline{0.162\ kg m^2}} = \underline{\underline{0.16\ kg m^2}}$$

(usikkerhet i opgitte data)

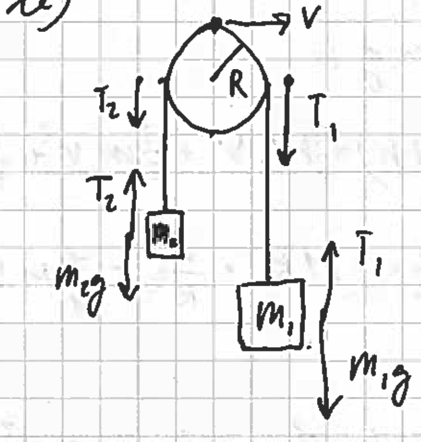
b) $\omega = 2\pi/s$

$$\Rightarrow E_R = \frac{1}{2} I_{tot} \omega^2 = 3.158 J \approx \underline{\underline{3.2 J}}$$

Oppgave ②

a) Rotasjon med klokka, m_1 synker.
Summen av snordragene vil akselerere trinsen $\Rightarrow \{T_1\} + \{T_2\} \neq 0 \Rightarrow \underline{\underline{T_1 \neq T_2}}$

b)



$$a \cdot m_1 = m_1 g - T_1$$
$$a \cdot m_2 = T_2 - m_2 g$$
$$T_1 \cdot R - T_2 \cdot R = I_0 \cdot \dot{\omega} = I_0 \cdot \alpha = I_0 \cdot \frac{a}{R}$$

$$\Rightarrow \underline{\underline{T_1 = m_1(g-a), \quad T_2 = m_2(g+a)}}$$

$$\Rightarrow m_1(g-a)R - m_2(g+a)R = I_0 \cdot \frac{a}{R} = \frac{1}{2} M R a$$

$$\Rightarrow m_1(g-a) - m_2(g+a) = \frac{1}{2} M a$$
$$g(m_1 - m_2) = a\left(\frac{1}{2}M + m_1 + m_2\right)$$

$$\Rightarrow \underline{\underline{a = g \cdot \frac{m_1 - m_2}{m_1 + m_2 + \frac{M}{2}}}}$$

c) For $M \to 0 \Rightarrow T_1 = m_1 g\left(1 - \frac{m_1 - m_2}{m_1 + m_2}\right)$

$$T_2 = m_2 g\left(1 + \frac{m_1 - m_2}{m_1 + m_2}\right)$$

$$\Rightarrow T_1 = \frac{m_1 g(m_1 + m_2) - m_1 g(m_1 - m_2)}{m_1 + m_2} = \frac{m_1^2 g + m_1 m_2 g - m_1^2 g + m_1 m_2 g}{m_1 + m_2}$$

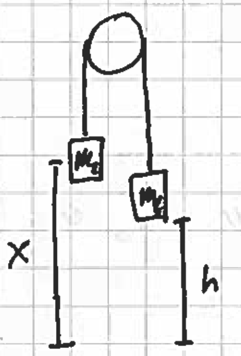
$$= \frac{2 m_1 m_2 g}{m_1 + m_2}, \quad T_2 = \frac{m_2 g(m_1 + m_2) + m_2 g(m_1 - m_2)}{m_1 + m_2}$$

$$= \frac{2 m_1 m_2 g}{m_1 + m_2} \Rightarrow \underline{\underline{T_1 = T_2}}$$

Gir mening da trinsen er masseløs, ingen kraft blir "overført" til trinsen og da må snorkreftene være like!

For $M \to \infty$, $a \to 0$. Altså, hvis trinsen er uendelig massiv, må uendelig stor energi til for å rotere den. Da klossene ikke er uendelig massive, må systemet nærmest stå stille!

d)



$$F_{ør}: E_{tot} = m_2 g x + m_1 g h$$

$$E_{tter}: E_{tot} = m_2 g(x+h) + \frac{1}{2}m_2 V^2 + \frac{1}{2}m_1 V^2 + \frac{1}{2}I_0 \omega^2$$

$$\omega = \frac{V}{R} \Rightarrow \frac{1}{2}I_0\omega^2 = \frac{1}{2}\cdot\frac{1}{2}MR^2\cdot\frac{V^2}{R^2} = \frac{1}{4}MV^2$$

$$E_{nergibevarelse} \Rightarrow m_2 g x + m_1 g h = m_2 g x + m_2 g h + \frac{1}{2}m_2 V^2 + \frac{1}{2}m_1 V^2 + \frac{1}{4}M V^2$$

$$\Rightarrow m_1 g h = m_2 g h + \frac{1}{2}m_2 V^2 + \frac{1}{2}m_1 V^2 + \frac{1}{4}M V^2$$

$$\Rightarrow V^2\left(\frac{1}{2}m_2 + \frac{1}{2}m_1 + \frac{1}{4}M\right) = gh(m_1 - m_2)$$

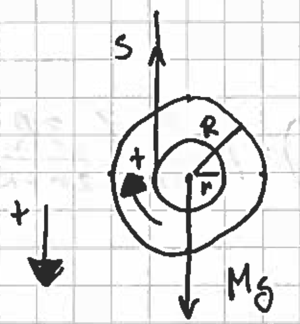
$$\Rightarrow V = \sqrt{gh\frac{(m_1 - m_2)}{\frac{1}{2}(m_2 + m_1 + \frac{1}{2}M)}} = \underline{\underline{\sqrt{2gh\frac{(m_1 - m_2)}{m_2 + m_1 + \frac{M}{2}}}}}$$

Ved formel for akselerasjon fra b):

$$2as = V^2 - V_0^2, \quad V_0 = 0, \quad s = h$$

$$\Rightarrow V = \sqrt{2ha} = \underline{\underline{\sqrt{2gh\frac{m_1 - m_2}{m_2 + m_1 + \frac{M}{2}}}}}$$

Oppgave 3 a)



$$\sum F_y = Ma = Mg - S$$

$$\sum \tau = I_0 \alpha = Sr$$

$$I_0 = \frac{1}{2} M R^2, \quad \alpha = \frac{a}{r}$$

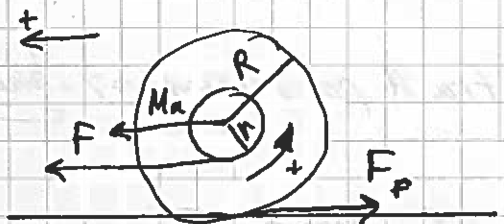
$$\Rightarrow \underline{\underline{S = \frac{1}{2} M \frac{R^2}{r^2} \cdot a}}$$

$$Ma = Mg - S \Rightarrow a = g - a \cdot \frac{1}{2} \frac{R^2}{r^2}$$

$$\Rightarrow a\left(1 + \frac{1}{2}\frac{R^2}{r^2}\right) = g \Rightarrow a = g \cdot \frac{1}{1 + \frac{1}{2}\frac{R^2}{r^2}} = \underline{\underline{g \frac{2r^2}{2r^2 + R^2}}}$$

$$\Rightarrow S = Mg \cdot \frac{R^2}{2r^2 + R^2} = \underline{\underline{Mg \frac{1}{1 + 2\frac{r^2}{R^2}}}}$$

b)



$$\sum F_x = Ma = F - F_f$$

$$\sum \tau = I_0 \alpha = F_f \cdot R - Fr$$

$$\cancel{\#} \quad \alpha = \frac{a}{R} \Rightarrow \frac{1}{2} M R a = \mu_s M g R - Fr$$

$$\Rightarrow F = MR\left(\mu_s g - \frac{1}{2}a\right)/r$$

$$\Rightarrow a = \frac{F}{M} - \frac{F_f}{M} = \frac{R}{r}\left(\mu_s g - \frac{1}{2}a\right) - \mu_s g$$

$$\Rightarrow a\left(1 + \frac{1}{2}\frac{R}{r}\right) = \mu_s g\left(\frac{R}{r} - 1\right) \Rightarrow a = \mu_s g \left(\frac{\frac{R}{r} - 1}{1 + \frac{R}{r}\frac{1}{2}}\right)$$

$$= \underline{\underline{\mu_s g \left(\frac{2R - 2r}{2r + R}\right)}}, \quad R > r \Rightarrow a > 0 \Rightarrow \underline{\underline{\text{ruller mot venstre}}}$$

$$F_f = F - Ma = MR\left(\mu_s g - \frac{1}{2}a\right)/r - Ma = \frac{R}{r} F_f - \frac{1}{2} a \frac{R}{r} M - Ma$$

$$\Rightarrow F_f\left(1 - \frac{R}{r}\right) = -Ma\left(\frac{1}{2}\frac{R}{r} + 1\right)$$

$$\Rightarrow F_f = -Ma\left(\frac{\frac{R}{2} + r}{r - R}\right), \quad R > r, \ a > 0 \Rightarrow F_f > 0 \Rightarrow$$

$\underline{\underline{\text{Friksjonskrafta virker mot høyre}}}$

Betingelse for rulling:

$$F_f \leq \mu_s Mg \quad , F_f = F - Ma$$

$$\Rightarrow F \leq \mu_s Mg + Ma = \mu_s Mg\left(1 + \frac{2R - 2r}{2r + R}\right) = \mu_s Mg\left(\frac{3R}{2r+R}\right)$$

$$\Rightarrow \underline{\underline{F \leq \mu_s Mg\left(\frac{3R}{2r+R}\right)}}$$

Oppgave 4 a)

$$I_c = \frac{ML^2}{12} \quad , \text{ Steiners sats } \Rightarrow$$

$$I_A = I_c + M\frac{L^2}{4} = \frac{ML^2 + 3ML^2}{12} = \underline{\underline{\frac{ML^2}{3}}}$$

b) $\underline{\underline{p = mv}}$ Ingen ytre krefter $\Rightarrow$ $\underline{\underline{\text{bevart}}}$

c) $\underline{\underline{L = mvl}}$ Torque fra A på staven $\Rightarrow$ $\underline{\underline{\text{ikke bevart}}}$

d) $\cancel{mv + mvL = m\frac{v}{2} + I_A \cdot w}$

$$\cancel{\Rightarrow I_A \cdot w = m\left(\frac{v}{2} + vL\right) = mv\left(\frac{1+2L}{2}\right)}$$

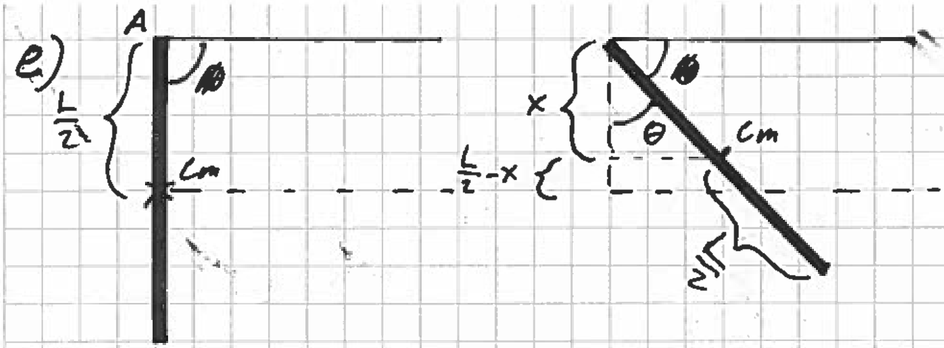
$$\cancel{\Rightarrow w = \frac{mv}{I_A}\left(\frac{1+2L}{2}\right)}$$

$$mvL = m\frac{v}{2}l + I_A \cdot w_0$$

$$\Rightarrow w_0 = m\frac{v}{2}l / I_A = \underline{\underline{\frac{mvl \cdot 3}{2ML^2}}}$$

Notat om b) Da staven blir holdt igjen av A under kollisjonen og beveger seg ikke rett fremover etter kollisjonen, men roterer, regnes dette som ytre kraft, dermed gjelder ikke bevaring av lineær moment.

e)



Hvis vi setter referanse nivå for $E_p = 0$ på $\frac{L}{2}$ ned fra A, har staven ingen potensiel energi når $\theta = 0°$ da stavens massesenter er på potensiel energi nivå null.

Ved et vilkårlig θ, har staven potensiel energi lik Mgh, hvor h er avstanden fra referanse nivået til massesenteret

$$\Rightarrow E_p = Mg\left(\frac{L}{2} - x\right)$$

$$\cos(\theta) = \frac{x}{\frac{L}{2}} = \frac{2x}{L}$$

$$\Rightarrow x = \frac{\cos(\theta)}{L} \cdot 2 \Rightarrow E_p(\theta) = \underline{Mg\frac{L}{2}(1 - \cos(\theta))}$$

$$\Rightarrow E_{tot}(\theta) = \frac{1}{2} I \omega(\theta)^2 + Mg\frac{L}{2}(1 - \cos(\theta))$$

$$E_{tot}(\theta) = \frac{1}{2} I \omega_0^2 \quad \text{(initial energien)}$$

$$\Rightarrow \frac{1}{2} I \omega_0^2 = \frac{1}{2} I \omega(\theta)^2 + Mg\frac{L}{2}(1 - \cos(\theta))$$

$$\Rightarrow \underline{\underline{\omega(\theta)^2 = \omega_0^2 - \frac{3g}{L}(1 - \cos(\theta))}}$$

f) $\theta = \frac{\pi}{2} \Rightarrow \omega(\theta) = 0$

$$\Rightarrow \omega_0^2 = \frac{3g}{L} \Rightarrow \omega_0 = \sqrt{\frac{3g}{L}} = \frac{m}{M} \frac{3vl}{2L^2}$$

$$\Rightarrow \underline{\underline{v = \cancel{\frac{3g}{l}} \frac{M}{m} \cdot \frac{2L^2}{3l} \cdot \sqrt{\frac{3g}{L}}}}$$

g) $\sum F = Mg + M\frac{v^2}{r}$, $v = \omega_0 \cdot \frac{L}{2}$, $r = \frac{L}{2} \Rightarrow \frac{v^2}{r} = \omega_0^2 \cdot \frac{L}{2}$

$$\Rightarrow \sum F = M\left(g + \frac{\omega_0^2 L}{2}\right) = M\left(g + \frac{3g}{2}\right) = \underline{Mg \cdot \frac{5}{2}}$$