# TFY4115 - Øving 11

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## Oppgave 1 a)

$\Delta U = Q - W$

$1 \to 2$: $\Delta V = 0 \Rightarrow \underline{\underline{W = 0}}$, $p_2 > p_1 \Rightarrow nRT_2 > nRT_1$

$\Rightarrow \underline{\underline{Q > 0}}$ (~~avgir varme~~ tar opp varme)

$2 \to 3$: $\underline{\underline{Q = 0}} \Rightarrow \underline{\underline{W > 0}}$ (gjør arbeid)

$3 \to 4$: $\Delta T = 0 \Rightarrow \Delta U = 0$

$\Rightarrow Q = W$, $W = nRT \ln\left(\frac{V_1}{V_3}\right) < 0$

$\Rightarrow \underline{\underline{Q < 0}}$, $\underline{\underline{W < 0}}$

Arbeid gjøres på systemet og systemet avgir varme.

b) Absorberes når $1 \to 2$. $\Delta V = 0$

$\Rightarrow \Delta U = Q = C_V n \Delta T = \underline{\underline{C_V n (T_2 - T_1)}}$

c) Avgir varme når $3 \to 1$. $\Delta T = 0$

$\Rightarrow Q = W = nRT_1 \ln\left(\frac{V_1}{V_3}\right)$

$$W = \int_{V_3}^{V_1} p\,dV = \int_{V_3}^{V_1} \frac{nRT_1}{V}\,dV = Q = \underline{\underline{nRT_1 \ln\left(\frac{V_1}{V_3}\right)}}$$

d) Adiabatligning: $pV^{\gamma} = \text{const}$

$\Rightarrow TV^{\gamma-1} = \text{konst}$, $T_1 = T_3$, $V_1 = V_2$

$\Rightarrow \frac{V_3}{V_1} = \frac{V_3}{V_2}$, $\frac{T_2}{T_1} = \frac{T_2}{T_3}$

$T_3 V_3^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \frac{V_3}{V_2} = \cancel{\frac{T_2^{1-\gamma}}{T_3^{1-\gamma}}}\ \cancel{\left(\frac{T_2}{T_1}\right)^{\gamma-1}} = \cancel{\frac{V_3}{V_1}}$

$\Rightarrow \cancel{Q_{ut} = nRT_1 \ln\left(\frac{V_1}{V_3}\right) = -nRT_1 \ln\left(\left(\frac{T_2}{T_1}\right)^{\gamma-1}\right)}$

$$\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \Rightarrow \underline{\frac{V_3}{V_2} = \left(\frac{T_2}{T_3}\right)^{\frac{1}{\gamma-1}}}$$

$$\gamma = \frac{C_p}{C_v}, \quad C_p = C_v + R \Rightarrow \frac{1}{\gamma-1} = \frac{C_v}{R}$$

$$\Rightarrow \frac{V_3}{V_2} = \frac{V_3}{V_1} = \left(\frac{T_2}{T_3}\right)^{\frac{C_v}{R}} = \left(\frac{T_2}{T_1}\right)^{\frac{C_v}{R}}$$

$$\Rightarrow Q_{ut} = nRT_1 \ln\left(\frac{V_1}{V_3}\right) = -nRT_1 \ln\left(\frac{V_3}{V_1}\right) = -nRT_1 \ln\left(\left(\frac{T_2}{T_1}\right)^{\frac{C_v}{R}}\right)$$

$$= -nRT_1 \cdot \frac{C_v}{R} \cdot \ln\left(\frac{T_2}{T_1}\right) = \underline{\underline{-nT_1 C_v \ln\left(\frac{T_2}{T_1}\right)}}$$

e) $$\eta = \frac{W}{Q_{inn}} = \frac{W_{ut} + W_{inn}}{Q_{inn}}$$

$$-W_{ut} = \Delta U_{(adiabatisk)} = C_v n (T_1 - T_2)$$

$$\Rightarrow \underline{W_{ut} = C_v n (T_2 - T_1)}$$

$$W_{inn} = \underline{Q_{ut} = -nT_1 C_v \ln\left(\frac{T_2}{T_1}\right)}$$

$$Q_{in} = C_v n (T_2 - T_1)$$

$$\Rightarrow \underline{\underline{\eta = 1 - \frac{T_1}{T_2 - T_1} \ln\left(\frac{T_2}{T_1}\right)}}$$

Oppgave 2 a)

$$ds = \frac{dQ_{rev}}{T} \Rightarrow \cancel{\Delta S = \frac{Q}{T}} \Rightarrow \Delta S_{12} = \int_1^2 \frac{dQ_{rev}}{T}$$

$1 \to 2$: $W = 0 \Rightarrow dU = dQ = C_v n\, dT$

$$\Rightarrow \Delta S = \int_{T_1}^{T_2} \frac{C_v n\, dT}{T} = \underline{\underline{C_v n \ln\left(\frac{T_2}{T_1}\right)}} \quad (\text{for } 1 \to 2)$$

$2 \to 3$: $\Delta Q = 0 \Rightarrow dQ_{rev} = 0 = \Delta \underline{\underline{S = 0}}$.

$3 \to 1$: Alle prosesser er reversible. For en reversibel kretsprosess er $\Delta S = 0$

$$\Rightarrow \Delta S_{3\to1} = -\Delta S_{1\to2} = \underline{\underline{-C_v n \ln\left(\frac{T_2}{T_1}\right)}}$$

Oppgave 2 b)

$$\Delta S_{12}(T, V) = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

$1 \to 2$: $V_1 = V_2 \Rightarrow \underline{\underline{\Delta S_{12} = nC_V \ln\left(\frac{T_2}{T_1}\right)}}$

$2 \to 3$: $\cancel{T_2 = T_3}$

$$\Delta S = nC_V \ln\left(\frac{T_3}{T_2}\right) + nR\ln\left(\frac{V_3}{V_2}\right)$$
$$= nC_V \ln\left(\frac{T_3}{T_2}\right) - nR\ln\left(\frac{V_2}{V_3}\right)$$

$nRT_1\ln\left(\frac{V_1}{V_3}\right) = -nT_1C_V\ln\left(\frac{T_2}{T_1}\right)$ (se 1c) og 1d))

$V_2 = V_1 \Rightarrow nR\ln\left(\frac{V_2}{V_3}\right) = -nC_V\ln\left(\frac{T_2}{T_3}\right) = nC_V\ln\left(\frac{T_3}{T_2}\right)$

$T_1 = T_3$

$\Rightarrow \Delta_{23}S = nC_V\ln\left(\frac{T_3}{T_2}\right) - nC_V\ln\left(\frac{T_3}{T_2}\right) = \underline{\underline{0}}$

$3 \to 1$:

$$\Delta S = nC_V\ln\left(\frac{T_1}{T_3}\right) + nR\ln\left(\frac{V_1}{V_3}\right)$$

$V_2 = V_1$, $T_1 = T_3 \Rightarrow \Delta S = nR\ln\left(\frac{V_2}{V_3}\right) = nC_V\ln\left(\frac{T_3}{T_2}\right)$

$= nC_V\ln\left(\frac{T_1}{T_2}\right) = -nC_V\ln\left(\frac{T_2}{T_1}\right)$

$\Rightarrow$

$$\underline{\underline{\Delta S_{31} = -\Delta S_{12}}}$$

c) $\Delta S_{tot} = 0$. For kretsprosesser hvor alle prosesser er reversible er $\Delta S = 0$. For universet: $\Delta S = 0$

Oppgave 3 a) $m = 1\ kg$, $V = 1L$

$T_0 = 20°C$, $T_1 = 100°C$

$c'_{vann} = 1\ cal/gK = 4.19\ kJ/kgK$

~~I) Plata endrer ikke temperatur => $\Delta S_p = 0$.~~

~~II)~~

i) $\Delta S = \int_0^1 \frac{dQ}{T}$, temperatur holdes konstant

$\Rightarrow \Delta S = \frac{Q}{T_1}$, $Q = c' \cdot m \cdot \Delta T = 335.2\ kJ$

$\Rightarrow \Delta S = 0.9\ \frac{kJ}{K}$

Men, omgivelsene avgir og mottar ikke entropi. Dermed

$\underline{\underline{\Delta S_p = -0.9\ \frac{kJ}{K}}}$

ii) $\Delta S = \int_0^1 \frac{dQ}{T}$ $\quad c' = \frac{1}{m}\frac{dQ}{dT} \Rightarrow dQ = c' m\, dT$

$\Rightarrow \Delta S_V = c'm \cdot \ln\left(\frac{T_1}{T_0}\right) = 4.19 \cdot \ln\left(\frac{373}{293}\right) = \underline{\underline{1.01\ \frac{kJ}{K}}}$

iii) $\Delta S_{tot} = \Delta S_V + \Delta S_p = 1.01 - 0.9 = \underline{\underline{0.11\ \frac{kJ}{K}}}$

b)

i) $|\Delta S_p| = \int_0^1 \frac{dQ}{T} + \int_1^2 \frac{dQ}{T}$

$= \frac{c' m \cdot \Delta T_{01}}{T_1} + \frac{c' m\, \Delta T_{12}}{T_2}$

$= 4.19\left(\frac{30}{323.15} + \frac{50}{373.15}\right) = 0.95\ kJ/K$

Avgis => negativ fortegn.

$\Rightarrow \underline{\underline{\Delta S_p = -0.95\ \frac{kJ}{K}}}$

ii) $\Delta S_v = c'm \ln\left(\frac{T_1}{T_0}\right) + c'm \ln\left(\frac{T_2}{T_1}\right)$

$= \underline{\underline{1.01 \text{ kJ/K}}}$

iii) $\Delta S_{tot} = \Delta S_v + \Delta S_p$

$= 1.01 - 0.95 = 0.06 \frac{\text{kJ}}{\text{K}} = \underline{\underline{60 \frac{\text{J}}{\text{K}}}}$

c) Dette er en reversibel prosess fordi endringen forigår over lang tid med små inkrementer.

Entropi er uavhengig av veien av prosessen. Dermed, entropi for vannet må for reversibel prosess være den samme som for irreversibel prosess fra a) og b)

$\Rightarrow \Delta S_v = 1.01 \text{ kJ/K}$

Entropi er "bevart" for reversible prosesser

$\Rightarrow \Delta S_{tot} = 0$

$\Rightarrow \Delta S_p = -1.01 \text{ kJ/K}$

$\Rightarrow$ i) $-1.01$ kJ/K

ii) $1.01$ kJ/K

iii) $0$ kJ/K

Oppgave 4 a)

$$pV = nRT = N\frac{2}{3}(E_k)$$

$$nRT_2 = nR \cdot 2 \cdot T_1$$

$$\Rightarrow \underline{\underline{E_{k2} = 2 \cdot E_{k1}}}$$

$$\Rightarrow \underline{B}$$

b) $\cancel{pV^{\alpha} = k = 1.41}$

$$\alpha = 1.41$$

$$T^{\alpha} p^{1-\alpha} = K$$

$$\Rightarrow T_1^{\alpha} p_1^{1-\alpha} = T_2^{\alpha} p_2^{1-\alpha} = T_2^{\alpha} \cdot \left(\frac{p_1}{2}\right)^{1-\alpha}$$

$$\Rightarrow T_1^{\alpha} = T_2^{\alpha} \cdot \left(\frac{1}{2}\right)^{1-\alpha}$$

$$\Rightarrow T_2 = T_1 \cdot 2^{\left(\frac{1-\alpha}{\alpha}\right)}$$

$$T_1 V_1^{\alpha-1} = T_2 V_2^{\alpha-1}$$

$$\Rightarrow V_2 = V_1\left(\frac{T_1}{T_2}\right)^{\frac{1}{\alpha-1}} = V_1 \cdot \left(\frac{1}{2^{\left(\frac{1-\alpha}{\alpha}\right)}}\right)^{\frac{1}{\alpha-1}}$$

$$= V_1 \cdot \frac{1}{2^{\left(\frac{1-\alpha}{\alpha^2-\alpha}\right)}} = V_1 \cdot \frac{1}{2^{\left(-\frac{1}{\alpha}\right)}} = V_1 \cdot 2^{\left(\frac{1}{\alpha}\right)}$$

$$= \underline{1.64 \cdot V_1}$$

$$\Rightarrow \underline{\underline{C}}$$

c) $V = \frac{nRT}{p}$

$$\Rightarrow S(T,p) = nC_V \ln\left(\frac{T}{T_0}\right) + nR\ln\left(\frac{\frac{nRT}{p}}{\frac{nRT_0}{p_0}}\right) + S_0$$

$$= nC_V \ln\left(\frac{T}{T_0}\right) + nR\ln\left(\frac{T}{T_0} \cdot \frac{p_0}{p}\right) + S_0$$

$$= nC_v \ln\left(\frac{T}{T_0}\right) + nR\ln\left(\frac{T}{T_0}\right) + nR\ln\left(\frac{p_0}{p}\right) + S_0$$

og $R = C_p - C_v \Rightarrow$

$$\Rightarrow S(T,p) = nC_p \ln\left(\frac{T}{T_0}\right) + nR\ln\left(\frac{p_0}{p}\right) + S_0$$

$$= nC_p \ln\left(\frac{T}{T_0}\right) - nR\ln\left(\frac{p}{p_0}\right) + S_0$$

$\Rightarrow$ (C)

d) $T_0 \to T_1$, $C = C_p = C_v$

$$\Delta S = \int_1^2 \frac{dQ_{rev}}{T} \qquad C = \frac{1}{n}\frac{dQ}{dT} \Rightarrow dQ = Cn\,dT$$

$$\Rightarrow \Delta S = \ln\left(\frac{T_1}{T_0}\right) \cdot C \cdot n$$

$\Rightarrow$ (D)

e) $\Delta S = \int_1^2 \frac{dQ_{rev}}{T}$, her er ~~dT~~ $T$ konstant lik $T_1$

$\Rightarrow dQ_{rev} = Cn\,dT \Rightarrow$

$$|\Delta S| = \frac{Cn(T_1 - T_0)}{T_1}$$, avgir entropi $\Rightarrow$

$$\Delta S = \frac{Cn(T_0 - T_1)}{T_1}$$

$\Rightarrow$ (A)

f) Med all sannsynlighet positiv.

$\Rightarrow$ (A)

g) $P_Q = 2 \cdot 10^3 W$, $T_0 = -10°C$, $T_1 = 30°C$

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$$n_V = \left| \frac{Q_{ut}}{W} \right| = \frac{T_H}{T_H - T_L} = \frac{303}{40}$$

$$\Rightarrow W = \frac{Q_{ut} \cdot 40}{303}$$

$$\Rightarrow P_W = \frac{P_Q \cdot 40}{303} = \underline{\underline{0.26\ kW}}$$

$\Rightarrow$ $\underline{\underline{A}}$

h) $\underline{\underline{D}}$