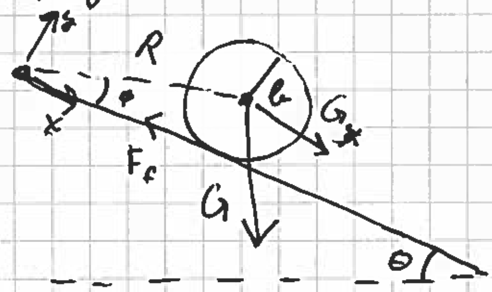
TFY4115 - Høst 2016
Øving 7
Vsevolod Karpov - vsevolok

Oppgave 1a)



$$\sum F = G\sin(\theta) - F_f = ma$$

$$\sum \tau = I_0 \cdot \alpha = \cancel{\vec{r} \times \vec{G}_y} = \vec{R} \times \vec{G}_{xR}$$

$\vec{G}_{xR}$: Komponenten til $\vec{G}_x$ 90° på $\vec{R}$

$$\Rightarrow \sum \tau = \vec{b} \times \vec{G}_x$$

$$\vec{b} = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}, \quad \vec{G}_x = \begin{bmatrix} mg\sin(\theta) \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{vmatrix} i & j & k \\ 0 & b & 0 \\ mg\ldots & 0 & 0 \end{vmatrix} = k \cdot (-bmg\sin(\theta))$$

$$\Rightarrow \sum \tau = -\hat{z}\, bmg\sin(\theta)$$

b) $L = m\vec{R} \times \vec{V} + I_0 \vec{\omega}$

$$\vec{R} = \begin{bmatrix} R\cos(\phi) \\ R\sin(\phi) \\ 0 \end{bmatrix}, \quad \vec{V} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \vec{R} \times \vec{V} = -\hat{z} R\sin(\phi) \cdot V = -\hat{z}\, b V$ (in i papirplanet)

$\vec{\omega} = \frac{\vec{r} \times \vec{V}}{|r|^2} = \frac{\vec{b} \times \vec{V}}{b^2} = \frac{-\hat{z} V b}{b^2} = -\hat{z}\frac{V}{b}$ (ω går med klokka)

$$\Rightarrow L = -\hat{z}\, mVb + \frac{2}{5} m b^2 \cdot \left(-\hat{z}\frac{V}{b}\right)$$

$$= -\hat{z}\,\frac{7}{5} mVb$$

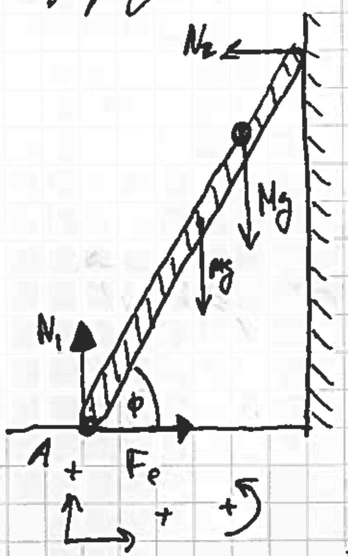
c) $\sum \tau = \sum \frac{dL}{dt}$

$\Rightarrow \sum \tau = \hat{z}\frac{7}{5} m b V \frac{d}{dt} = \hat{z}\frac{7}{5} m b a$

$\sum \tau = -\hat{z}\, b m g \sin(\theta)$

$\Rightarrow a = \frac{5}{7} g \sin(\theta)$

Oppgave 2 a)



Betingelser:

$F_f \geq N_2$

$\sum F_x = F_f - N_2$

$\sum \tau_A = L \cdot N_2 \cdot \sin(\phi) - g\cos(\phi)\left(Mx + m\frac{L}{2}\right)$

$\sum \tau_A = 0$

$\Rightarrow N_2 = \frac{g\left(M\frac{x}{L} + m\frac{1}{2}\right)}{\tan(\phi)}$

$\Rightarrow F_f \geq \frac{g\left(M\frac{x}{L} + m\frac{1}{2}\right)}{\tan(\phi)}$

$\Rightarrow \mu_s g(M+m) \geq \frac{g\left(M\frac{x}{L} + \frac{1}{2}m\right)}{\tan(\phi)}$

$\tan(\phi) \geq \frac{\frac{1}{2}m + M\frac{x}{L}}{\mu_s(m+M)}$

b) $m = 12$ kg, $M = 80$ kg, $\frac{x}{L} = \frac{8}{10}$

$\mu_{s1} = 0{,}5 \Rightarrow 59.5°$

$\mu_{s2} = 0.4 \Rightarrow 64{,}7°$

$\mu_{s3} = 0.3 \Rightarrow 70{,}5°$

Oppgave 3

a) $x(t) = x_0 \cdot \cos(\omega t + \theta)$

En periode: fra $\omega t = 0$, til $\omega t = 2\pi$

$\omega = \frac{3\pi}{4} \Rightarrow t_1 = \frac{8}{3} \Rightarrow T = \frac{8}{3}$

~~$\frac{8}{3}$ svingninger per sekund~~

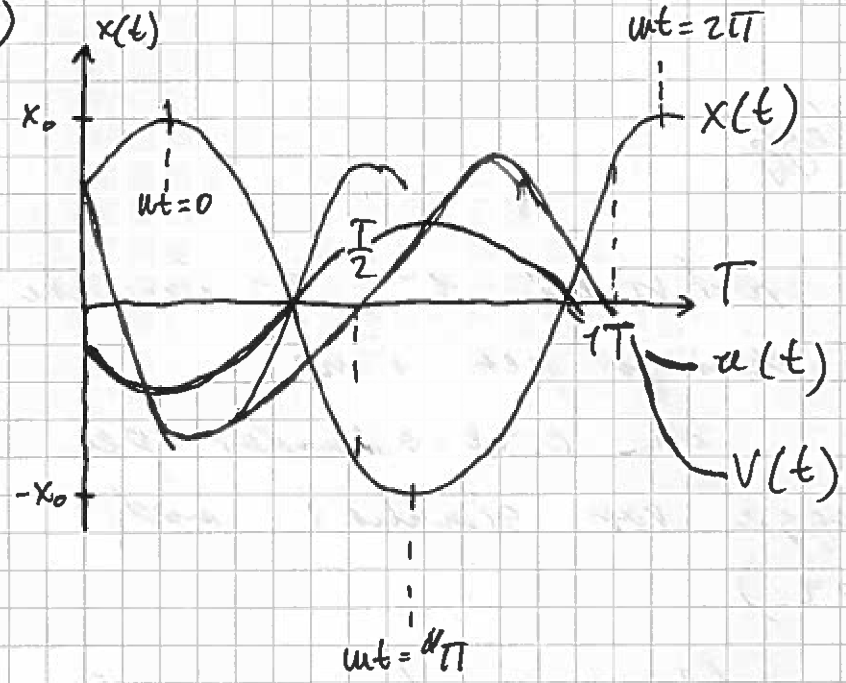
$\frac{8}{3}$ sekunder per svinging

$\Rightarrow f = \frac{3}{8}$ Hz

b) $v(t) = -x_0 \omega \sin(\omega t + \theta)$

$a(t) = -x_0 \omega^2 \cos(\omega t + \theta)$

c)



4

d) v_{max} når $a = 0$

$\cancel{\Rightarrow -x_0 w^2 \cos(wt+\theta) = 0}$

$\cancel{\Rightarrow wt + \theta = \frac{\pi}{2} + k\pi \quad , k \in \mathbb{Z}}$

$\cancel{\Rightarrow t = \frac{\frac{\pi}{2} + k\pi}{w} - \theta}$

$\cancel{\Rightarrow t =}$

$\Rightarrow -x\, w_0^2 \cos(wt + \theta) = 0$

$\Rightarrow wt + \theta = \frac{\pi}{2}$

$\Rightarrow t = \frac{\frac{\pi}{2} - \theta}{w} = \underline{1\,s}$

$v(1) = -x_0 w \sin(w + \theta) = \underline{\underline{1.18\, m/s}}$

Oppgave 4

a) se vedlegg

b) For en startvinkel $1°$, er forsinkelsen ca 0.05 sekunder per time.
For $5°$, ca 0.18 sekunder per time (begge var simulert med $\Delta t = 0.001$)

Dette resulterer i et avvik på c 3.12 sekunder per døgn.

c) Som forventet, gir rekken en mindre periode. Forskjellen vill åpenbart vært mindre hvis vi tok med flere elementer. Resultatet går mot faktisk løsning når antall elementer $\to \infty$