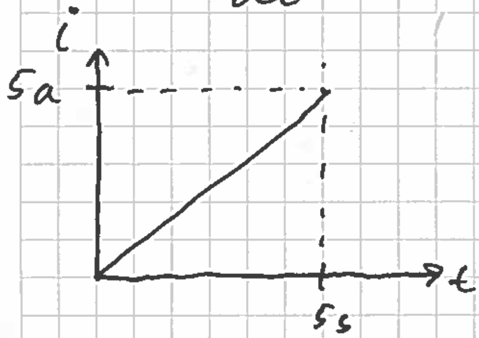
TTK4240 – Høst 2016
Øving 1
Vsevolod Karpov – (vsevolok)

Oppgave 1

a) $V_s = 1V$, $L = 1H$

$$V_s = L \cdot \frac{di}{dt}$$

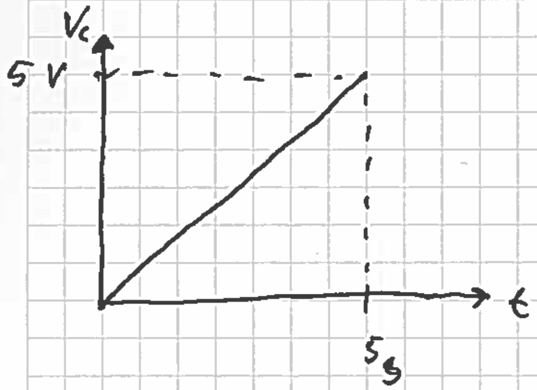
$$1 = \frac{di}{dt} \Rightarrow \underline{\underline{i(t) = t}}$$



b) $I_s = 1A$, $C = 1F$

$$i_c = C \cdot \frac{dV_c}{dt}$$

$$1 = \frac{dV_c}{dt} \Rightarrow \underline{\underline{V_c(t) = t}}$$



c)

$i_c = C \cdot \frac{dv_c}{dt}$ $\cancel{V_c} = V_s = 1V$, $C = 1F$

$V_c = u(t)$ (Heaviside)

$\Rightarrow \cancel{i_c \, dt} =$

$\Rightarrow \underline{\underline{i_c = u'(t) = \delta(t)}}$

Da det er ingen resistans i kretsen, vil kondensatoren lades momentant.

Altså er strømmen delta funksjonen da det går strøm gjennom kondensatoren akkurat når bryteren blir skrudd på og ikke etter det!

d) Det er viktig å notere at under stasjonær oppførsel vil kondensatorer være fulladet og konstant strøm vil gå gjennom spolene mens ingen strøm vil gå gjennom kondensatorene!

ergo:

$\underline{\underline{d)\ i = 1A,\ e)\ i = 0,\ f)\ i = 1A}}$

Oppgave 2 a)

$V_L = L\frac{di}{dt}$, $L = 100\,mH = 0.1\,H$

$V_L = V_s = 80\,V$

$\Rightarrow \underline{\underline{i(t) = 800 \cdot t}}$

b) $i(10ms) = 800 \cdot 0.01 = 8A$

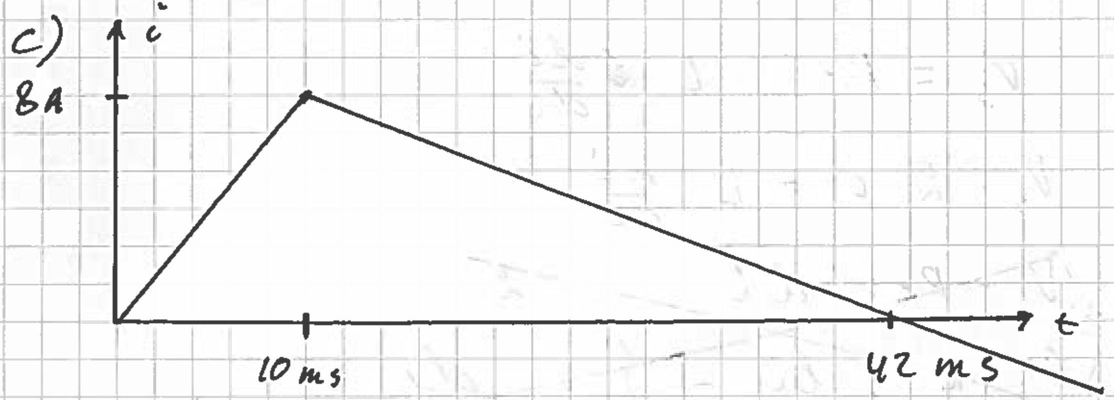
Dette er strømmen gjennom spolen i det V_s endres til $-25\,V$

$\Rightarrow \underline{\underline{i(t) = -250t + 8}}$

$i(t) = 0 \Rightarrow t = \frac{8}{250} = 0.032\,s$

$\Rightarrow T = t + 0.01 = 0.042\,s = \underline{\underline{42ms}}$

c)



Oppgave 3

a) $i_{stasjonær} = \frac{V_s}{R}$

b) $V_s = V_R + V_L = R \cdot i_L + L \cdot \frac{di_L}{dt}$

c) Rett før $t=0$, er det oppgitt at ingen strøm går gjennom spolen.

Rett etter, vil spolen prøve å motvirke strømendringen.

Ergo: $i_L(0^+) = i_L(0^-) = 0$

d) $V_s = R \cdot i + L \cdot \frac{di}{dt}$

$V_s - R \cdot i = L \cdot \frac{di}{dt}$

$\cancel{(V_s - R \cdot i)\,dt = L \cdot di}$

$\cancel{\int_0^t (V_s - Ri)\,dt = \int_0^i L \cdot di}$

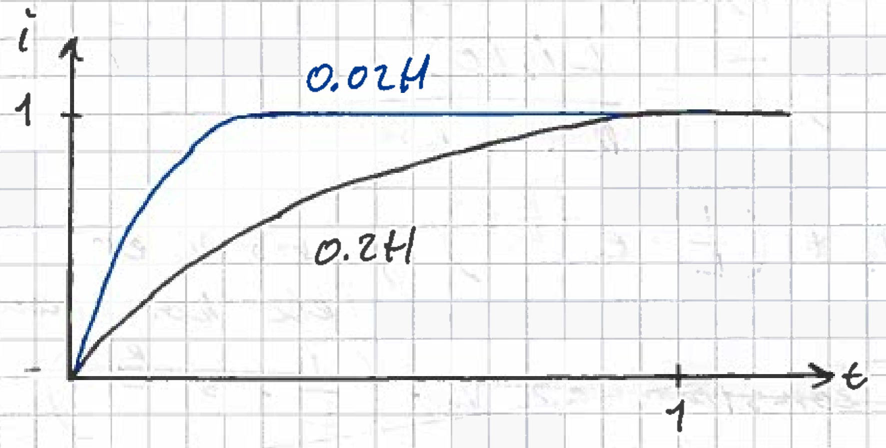
Laplace transform:

$V_s \frac{1}{s} - R \cdot I = L \cdot s \cdot I$

$I(Ls + R) = V_s \frac{1}{s}$

$I = V_s \cdot \frac{1}{s(Ls + R)}$, $\mathcal{L}^{-1}(I) = i(t) = V_s\left(\frac{1}{R} - \frac{e^{-\frac{R}{L}t}}{R}\right)$

$R = 1\,\Omega$, $V_s = 1V$, $L_1 = 0{,}02H$, $L_2 = 0{,}2H$



Stasjonær verdi: $\frac{V_s}{R} = 1A$, som forventet!

Oppgave 4 a)

$$\mathcal{L}(v_L) = \cancel{V_L} = V_L$$

$$\mathcal{L}\left(L \cdot \frac{di}{dt}\right) = L\left(I_L \cdot s - i_L(0)\right)$$

$$\Rightarrow \underline{\underline{V_L = L I_L \cdot s - L \cdot i_L(0)}}$$

b) Med ~~h~~ sprangrespons:

$$v \cancel{V}_s \cdot u(t) = R \cdot i_L + L \cdot \frac{di_L}{dt}$$

Laplace:

$$V_s = R \cdot I_L + L I_L s - L i_L(0)$$

$$\Rightarrow I_L(R + Ls) = V_s + L i_L(0)$$

$$I_L = \frac{V_s}{R + Ls} + \frac{L i_L(0)}{R + Ls}$$

$$\mathcal{L}^{-1}(I_L) = \mathcal{L}^{-1}(I_1) + \mathcal{L}^{-1}(I_2)$$

$$I_1 = \frac{V_s}{R + Ls} \quad , \quad I_2 = \frac{L i_L(0)}{R + Ls}$$

$$\mathcal{L}^{-1}(I_1) = V_s * \left(\frac{1}{L} \cdot e^{-\frac{R}{L}t}\right)$$; hvis V_s er ikke konstant.

~~, hvis V_s er konstant $\Rightarrow V_s \cdot \left(\frac{1}{L} \cdot e^{\frac{R}{L}t}\right)$~~

$$\mathcal{L}^{-1}(I_2) = i_L(0) \cdot e^{-\frac{R}{L}t}$$

$$\Rightarrow i(t) = \left(V_s * \frac{e^{-\frac{R}{L}t}}{L} + i_L(0) \cdot e^{-\frac{R}{L}t}\right) \cdot u(t)$$

Hvis V_s er konstant:

Laplace:

$$\frac{V_s}{s} = R \cdot I + L \cdot I \cdot s - L \cdot i(0)$$

$$\Rightarrow I = \frac{V_s}{s(Ls + R)} + \frac{L i(0)}{Ls + R}$$

$$\Rightarrow i(t) = \left(\frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right) + i(0) \cdot e^{-\frac{R}{L}t}\right) \cdot u(t)$$

c) $\quad i_c = C \cdot \frac{dv_c}{dt}$

Laplace:

$$I_c(s) = C\left(V_c(s) \cdot s - v_c(0)\right)$$

d)

$$V_c = V_R$$

$$V_c = i \cdot R$$

$$i = \frac{V_c}{R} = C \cdot \frac{dV_c}{dt}$$

$$\Rightarrow \underline{\underline{V_c'(t) = \frac{V_c}{RC}}}$$

e) Laplace:

$$V(s) \cdot s - V_0 = V(s)/RC$$

$$V(s)\left(s - \frac{1}{RC}\right) = V_0$$

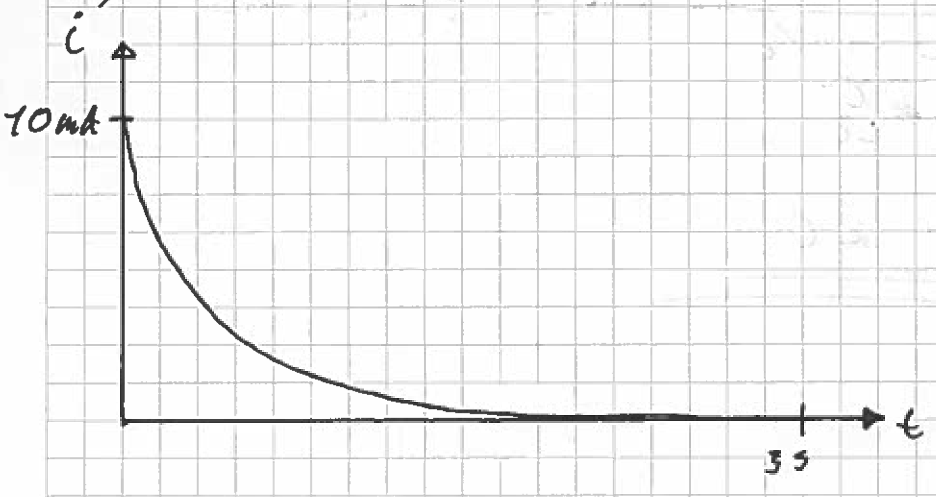
$$V(s) = \frac{V_0}{s - \frac{1}{RC}}$$

$$\Rightarrow \underline{v_c(t) = V_0 \cdot e^{-\frac{1}{RC} \cdot t}}$$

$$i(t) = \frac{V_c(t)}{R} = \underline{\underline{\frac{V_0}{R} \cdot e^{-\frac{t}{RC}}}}$$

$R = 1\,k\Omega$, $C = 1\,mF$, $V_0 = 10\,V$

$\Rightarrow$



Oppgave 5

a) Fra 4b), anta at $i(0) = 0$

$$\Rightarrow i(t) = \frac{V_s}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$V(t) = V_s\left(1 - e^{-\frac{R}{L}t}\right)$$

$$1 - \frac{1}{e} = \frac{V_s\left(1 - e^{-\frac{R}{L}\tau}\right) - V_{start}}{V_{slutt} - V_{start}}$$

$$= \frac{V_s}{V_s}\left(1 - e^{-\frac{R}{L}\tau}\right) = 1 - e^{-\frac{R}{L}\tau}$$

$$\Rightarrow 1 - \frac{1}{e} = 1 - e^{-\frac{R}{L}\tau}$$

$$\Rightarrow \ln(e) = \ln\left(e^{\frac{R}{L}\tau}\right)$$

$$1 = \frac{R}{L}\tau$$

$$\underline{\underline{\tau = \frac{L}{R}}}$$

b) Fra 4e)

$$V(t) = V_0 \cdot e^{-\frac{t}{RC}}$$

$$1 - \frac{1}{e} = \frac{V_0 \cdot e^{-\frac{\tau}{RC}} - V_0}{0 - V_0} = 1 - e^{-\frac{\tau}{RC}}$$

$$\Rightarrow e = e^{\frac{\tau}{RC}}$$

$$\Rightarrow \underline{\underline{\tau = RC}}$$

c)