TMA4120

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Høst - 2016 gruppe 3

Øving 2

6.4.5.

$$y'' + y = \delta(t-\pi) - \delta(t-2\pi)$$

$$y(0) = 0, \quad y'(0) = \cancel{3}\,1$$

Transform:

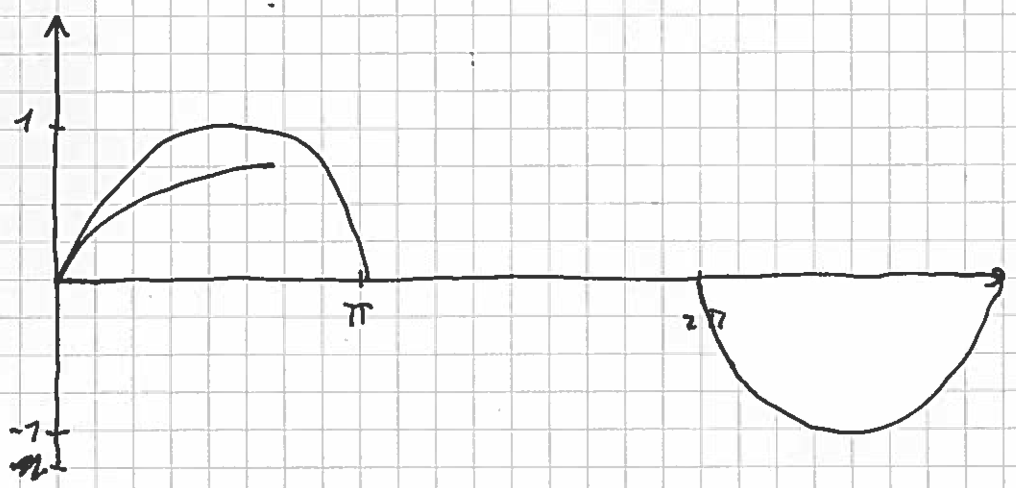
$$s^2 Y(s) - 3 + Y(s) = e^{-s\pi} - e^{-s2\pi}$$

$$Y(s)(s^2+1) = e^{-s\pi} - e^{-s2\pi} + \cancel{3}\,1$$

$$Y(s) = \frac{e^{-s\pi}}{(s^2+1)} - \frac{e^{-s2\pi}}{(s^2+1)} + \frac{\cancel{3}\,1}{s^2+1}$$

$$\mathcal{L}^{-1}(Y(s)) = \sin(t-\pi)\cdot u(t-\pi) - \sin(t)\cdot u(t-2\pi) + \cancel{3}\sin(t)$$

$$= \cancel{3}\sin(t) - \sin(t)\cdot u(t-\pi) - \sin(t)\cdot u(t-2\pi)$$



6.4.14: a)

Theorem: The Laplace transform of a piecewise continuous function $f(t)$ with period p is

$$\mathcal{L}(f(t)) = \frac{1}{1-e^{-ps}} \int_0^p e^{-st} f(t)\, dt$$

Proof: $\mathcal{L}(f(t)) = \int_0^\infty f(t) e^{-st}\, dt$

$$\cancel{= \int_0^p f(t) e^{-st} dt + \int_p^{2p} f(t) e^{-st} dt \ldots \int_{(n-1)p}^{np} f(t) e^{-st} dt}$$

$$\cancel{= \sum_{n=0}^{\infty} \int_{np}^{(n+1)p} f(t) e^{-st} dt}$$

$$\cancel{t = (n-1)p \Rightarrow \sum_{n=1}^{\infty} \int_{p(n-1)}^{np} f((n-1)p) \cdot e^{-s(p(n-1))}}$$

$$F(s) = \int_0^T f(t) e^{-st} dt + \int_T^\infty f(t) e^{-st} dt$$

$$f(t-T) = f(t), \quad t \geq T$$

$$\Rightarrow \int_T^\infty f(t) e^{-st} dt = \int_T^\infty f(t-T) e^{-st} dt = \underline{I}$$

$$\tau = t - T, \quad t = \tau + T, \quad d\tau = dt$$

$$\Rightarrow I = \int_0^\infty f(\tau) e^{-s(\tau+T)} d\tau = e^{-sT} \cdot F(s)$$

$$\Rightarrow F(s) = \int_0^T f(t) e^{-st} dt + e^{-sT} \cdot F(s)$$

$$\Rightarrow F(s) = \frac{1}{1-e^{-sT}} \int_0^T f(t) e^{-st} dt$$

b) $f(t) = \sin(\omega t)(u(t) - u(t - \pi/\omega))$

period $= 2\pi/\omega$

$$\Rightarrow \mathcal{L}(f) = \frac{1}{1-e^{-s2\pi/\omega}} \int_0^{2\pi/\omega} e^{-st} f(t)\,dt$$

$$= \frac{1}{1-e^{-s2\pi/\omega}} \int_0^{\pi/\omega} e^{-st} \sin(\omega t)\,dt$$

$$\cancel{= \frac{1}{1-e^{-s2\pi/\omega}} \Big[t \cdot e^{-st} \sin(\omega t) \Big]_0^{\pi/\omega}}$$

$$= \frac{1}{1-e^{-s2\pi/\omega}} \left[\frac{-e^{-st}(s \cdot \sin(\omega t) + \omega \cdot \cos(\omega t))}{s^2+\omega^2} \right]_0^{\pi/\omega}$$

$$= \frac{e^{-s\pi/\omega} \cdot \omega + \omega}{(1-e^{-s2\pi/\omega})(s^2+\omega^2)} = \underline{\underline{\frac{\omega}{(1-e^{-s\pi/\omega})(s^2+\omega^2)}}}$$

c) $$\mathcal{L}(f) = \frac{1}{1-e^{-s\pi/\omega}} \int_0^{\pi/\omega} e^{-st} \sin(\omega t)\,dt$$

$$= \frac{\omega}{s^2+\omega^2} \cdot \frac{(1+e^{-s\pi/\omega})}{(1-e^{-s\pi/\omega})} \cdot \frac{(e^{s\frac{\pi}{2\omega}})}{(e^{s\frac{\pi}{2\omega}})}$$

$$= \frac{\omega}{s^2+\omega^2} \cdot \frac{e^{s\frac{\pi}{2\omega}} + e^{-s\frac{\pi}{2\omega}}}{e^{s\frac{\pi}{2\omega}} - e^{-s\frac{\pi}{2\omega}}} = \underline{\underline{\frac{\omega}{s^2+\omega^2} \cdot \coth\left(\frac{s\pi}{2\omega}\right)}}$$

6.5:7,

$$h(t) = t * e^t = f(t) * g(t)$$

$$h = \int_0^t f(\tau)\, g(t-\tau)\, d\tau$$

$$\cancel{= \int_0^t t \cdot e^{t-\tau}\, d\tau = t \cdot e^t \int_0^t e^{-\tau}\, d\tau}$$

$$\cancel{= t\, e^t \left[-e^{-\tau}\right]_0^t = t \cdot e^t (1 - e^{-t})}$$

$$\cancel{= t(e^t - 1)}$$

$$e^t \int_0^t \tau\, e^{-\tau}\, d\tau = e^t \cdot \left[e^{-\tau}(-\tau - 1)\right]_0^t$$

$$= e^t\left(e^{-t}(-t-1) + 1\right) = -t - 1 + e^t$$

$$\underline{\underline{= e^t - t - 1}}$$

6.5.13:

$$y(t) + 2e^t \int_0^t y(\tau)\, e^{-\tau}\, d\tau = t \cdot e^t$$

$$y(t) + 2\,(y(t) * e^t) = t \cdot e^t$$

Transform:

$$\cancel{Y(s) + 2\left(Y(s) \cdot \frac{1}{s-1}\right) = \frac{1!}{(s-1)^2}}$$

$$\cancel{Y(s)\left(1 + \frac{2}{s-1}\right) = \frac{1}{(s-1)^2}}$$

$$\cancel{Y(s) = \frac{1}{(s-1)^2} + \frac{s-1}{(s-1)^2} = Y_1(s) + Y_2(s)}$$

$$\cancel{\mathcal{L}^{-1}(Y(s)) = t \cdot e^t + e^t}$$

$$\cancel{Y_2(s) = \frac{A}{(s-1)} + \frac{B}{s-1}}$$

Transform:

$$Y(s) + 2\left(Y(s) \cdot \frac{1}{s-1}\right) = \frac{1}{(s-1)^2}$$

$$Y(s)\left(1 + \frac{2}{s-1}\right) = \frac{1}{(s-1)^2}$$

$$Y(s) = \frac{1}{(s-1)^2\left(1+\frac{2}{s-1}\right)} = \frac{1}{(s-1)^2 + 2s - 2}$$

$$= \frac{1}{s^2 - 2s + 1 + 2s - 2} = \frac{1}{s^2 - 1}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s^2-1}\right) = \underline{\underline{\sinh(t)}}$$

6.5.19:

$$\mathcal{L}(f) = \frac{2\pi s}{(s^2+\pi^2)^2} = 2 \cdot \left(\frac{s}{s^2+\pi^2} \cdot \frac{\pi}{s^2+\pi^2}\right)$$

$$= 2 \cdot \mathcal{L}(\cos(\pi t)) \cdot \mathcal{L}(\sin(\pi t))$$

$$= 2 \cdot (\cos(\pi t) * \sin(\pi t))$$

$$= 2 \int_0^t \cos(\pi \tau) \cdot \sin(\pi(t-\tau))\, d\tau$$

$$= 2\left[\frac{2\pi \tau \sin(\pi t) + \cos(\pi(t - 2\tau))}{4\pi}\right]_0^t$$

$$= \frac{4\pi t \sin(\pi t)}{4\pi} = \underline{\underline{t \cdot \sin(\pi t)}}$$

6.5.16 e)

$$y''(t) + w^2 y = r(t),\ y(0) = K_1,\ y'(0) = K_2$$

Transform:

$$s^2 Y(s) - sK_1 - K_2 + w^2 \cdot Y(s) = R(s)$$

$$Y(s)(s^2+w^2) = R(s) + sK_1 + K_2$$

$$Y(s) = \underbrace{\frac{R(s)}{s^2+w^2}}_{= Y_1} + \underbrace{\frac{K_1 s}{s^2+w^2}}_{= Y_2} + \underbrace{\frac{K_2}{s^2+w^2}}_{= Y_3}$$

$$\mathcal{L}^{-1}(Y_1) = r(t) * \frac{\sin(wt)}{w}$$

$$\mathcal{L}^{-1}(Y_2) = K_1 \cdot \cos(wt)$$

$$\mathcal{L}^{-1}(Y_3) = \frac{K_2}{w} \sin(wt)$$

$$\Rightarrow \underline{\underline{y(t) = r(t) * \frac{\sin(wt)}{w} + K_1 \cdot \cos(wt) + \frac{K_2}{w} \cdot \sin(wt)}}$$

6.6.16:

$$\mathcal{L}(f) = \frac{2s+6}{(s^2+6s+10)^2} = F(s) = G'(s)$$

$$\int^{\infty} F(\bar{s}) \Rightarrow u = s^2+6s+10,\ u' = 2s+6$$

$$\int_s^s F(s) \Rightarrow \int \frac{1}{u^2}\,du = \frac{1}{-1 \cdot u^1} = \frac{1}{-1} \cdot \frac{1}{(s^2+6s+10)}$$

$$\Rightarrow f = \mathcal{L}^{-1}(G'(s)) = -t \cdot g(t)$$

$$g(t) = \mathcal{L}^{-1}(G(s))$$

$$G(s) = \frac{1}{-3}\left(\frac{1}{(s^2+6s+10)}\right) = \frac{1}{-3}\left(\frac{1}{((s+3)^2+1)^1}\right)$$

$$= \frac{1}{-1}\left(\left(\mathcal{L}\left(e^{-3t} \cdot \sin(t)\right)\right)\right)$$

$$\cancel{\Rightarrow g(t) = \frac{1}{-3}\left(e^{-3t}\sin(t) * e^{-3t}\sin(t) * e^{-3t}\sin(t)\right)}$$

$$\cancel{\Rightarrow f(t) = \frac{t}{3}\left(e^{-3t}\sin(t) * e^{-3t}\sin(t) * e^{-3t}\sin(t)\right)}$$

$$\Rightarrow g(t) = -1\left(e^{-3t}\sin(t)\right)$$

$$\Rightarrow f(t) = -t \cdot g(t) = \underline{\underline{t \cdot e^{-3t} \cdot \sin(t)}}$$

6.6.17:

$$F(s) = \ln\left(\frac{s}{s-1}\right), \quad F'(s) = \frac{-1}{s(s-1)} = R(s)$$

$$\int_s^\infty P(\sigma)\,d\sigma = \ln\left(\frac{\infty}{\infty-1}\right) - \ln\left(\frac{s}{s-1}\right) = -\ln\left(\frac{s}{s-1}\right)$$

$$\Rightarrow F(s) = \int_s^\infty G(\sigma)\,d\sigma, \quad G(s) = -F'(s)$$

$$G(s) = \frac{1}{s(s-1)}, \quad \mathcal{L}^{-1}(F(s)) = \frac{g(t)}{t}$$

$$g(t) = \mathcal{L}^{-1}(G(s))$$

$$G(s) = \frac{1}{s} \cdot K(s) \Rightarrow g(t) = \int_0^t K(\tau)\,d\tau$$

$$= \int_0^t e^{\tau}\,d\tau = \underline{e^t - 1 = g(t)}$$

$$\Rightarrow \underline{\underline{\mathcal{L}^{-1}(F(s)) = \frac{e^t-1}{t}}}$$

6.7.13: I: $y_1'' + y_2 = -101 \sin(10t)$

II: $y_2'' + y_1 = 101 \sin(10t)$

$y_1(0) = 0, \; y_1'(0) = 6, \; y_2(0) = 8, \; y_2'(0) = -6$

Transform:

I: $s^2 Y_1 - 6 + Y_2 = -101 \frac{10}{s^2+10^2}$

II: $s^2 Y_2 - 8s + 6 + Y_1 = 101 \frac{10}{s^2+10^2}$

I: $Y_1 = \left(6 - Y_2 - 101 \frac{10}{s^2+10^2}\right)/s^2$

II: $s^2 Y_2 - 8s + 6 + \left(6 - Y_2 - 101 \frac{10}{s^2+10^2}\right)/s^2 = 101 \frac{10}{s^2+10^2}$

II: $s^4 Y_2 - 8s^3 + 6s^2 + 6 - Y_2 - 101 \frac{10}{s^2+10^2} = 101 \frac{10s^2}{s^2+10^2}$

II: $Y_2(s^4 - 1) = 101 \frac{10s^2}{s^2+10^2} + 101 \frac{10}{s^2+10^2} + 8s^3 - 6s^2 - 6$

II: $Y_2 = 101 \left(\frac{10s^2 + 10}{(s^2+10^2)(s^4-1)}\right) + \frac{8s^3 - 6s^2 - 6}{(s^4-1)}$

II: $Y_2 = I_1 + I_2$

$I_1 = 101 \left(\frac{10}{s^2+10^2} \cdot \frac{s^2+1}{s^4-1}\right)$

$= 101 \left(\mathcal{L}(\sin(10t)) \cdot \frac{s^2+1^2}{s^4-1^4}\right)$

$\frac{s^2+1^2}{s^4-1^4} = \frac{s^2+1}{(s^2+1)(s^2-1)} = \frac{1}{s^2-1}$

$\Rightarrow I_1 = 101\left(\mathcal{L}(\sin(10t)) \cdot \mathcal{L}(\sinh(t))\right)$

$\Rightarrow \mathcal{L}^{-1}(I_1) = 101\left(\sin(10t) * \sinh(t)\right)$

$$I_2 = \frac{8s^3 - 6s^2 - 6}{s^4 - 1} = \frac{4s}{s^2+1} + \frac{4s-6}{s^2-1}$$

(Used software to factorize through partial fraction expansion)

$$I_2 = 4\frac{s}{s^2+1} + 4\frac{s}{s^2-1} - 6\frac{1}{s^2-1}$$

$$\underline{\mathcal{L}^{-1}(I_2) = 4\cos(t) + 4\cosh(t) - 6\sinh(t)}$$

$$\mathcal{L}^{-1}(I_1) = 101\left(\sin(10t) * \sinh(t)\right)$$

$$= 101\int_0^t \sin(10\tau)\cdot\frac{e^{(t-\tau)} - e^{(\tau-t)}}{2}\,d\tau$$

$$= 101\left(\frac{e^t}{2}\int_0^t \sin(10\tau)\cdot e^{-\tau} - \frac{e^{-t}}{2}\int_0^t \sin(10\tau)\cdot e^{\tau}\,d\tau\right)$$

$$= 101\left(\frac{e^t}{2}\left(\frac{1}{2} - \frac{1}{2}e^{-t}(\sin(t)+\cos(t))\right) - \frac{e^{-t}}{2}\left(\frac{1}{2} + \frac{1}{2}e^{t}(\sin(t) - \cos(t))\right)\right)$$

$$= 101\left(\frac{e^t}{4} - \frac{1}{4}\sin(t) - \frac{1}{4}\cos(t) - \frac{e^{-t}}{4} - \frac{1}{4}\sin(t) + \frac{1}{4}\cos(t)\right)$$

$$= 101\left(\frac{1}{2}\sinh(t) - \frac{1}{2}\right.$$

$$= 101\left(\frac{1}{101}\cdot\frac{e^t}{2}\left(10 - e^{-t}(\sin(10t) + 10\cos(10t))\right) - \right.$$

$$\left.\frac{1}{101}\cdot\frac{e^{-t}}{2}\left(10 + e^{t}(\sin(10t) - 10\cos(10t))\right)\right)$$

$$= 10\sinh(t) - \frac{1}{2}\sin(10t) - 5\cos(10t) - \frac{1}{2}\sin(10t) + 5\cos(10t)$$

$$= 10\sinh(t) - \sin(10t)$$

$$\Rightarrow y_2(t) = 4\sinh(t) + 4\cosh(t) + 4\cos(t) - \sin(10t)$$

$$= 2\left(e^t - e^{-t} + e^t + e^{-t}\right) + 4\cos(t) - \sin(10t)$$

$$= 4e^t + 4\cos(t) - \sin(10t)$$

$$y_2(t) = 4e^t + 4\cos(t) - \cancel{\ } \sin(10t)$$

Now, we plug $y_2(t)$ into equation I

$$\text{I: } y_1''(t) + y_2(t) = -101 \sin(10t)$$

$$\Rightarrow y_1''(t) = -100 \sin(10t) - 4e^t - 4\cos(t)$$

Transforming:

$$s^2 Y_1 - 6 = -100 \frac{10}{s^2+10^2} - 4 \frac{1}{s-1} - 4 \frac{s}{s^2+1}$$

$$Y_1 = -100 \frac{10}{s^2(s^2+10^2)} - 4 \frac{1}{s^2(s-1)} - 4 \frac{s}{s^2(s^2+1)} + 6 \cdot \frac{1}{s^2}$$

$$Y_1 = L_1 + L_2 + L_3 + L_4$$

$$\mathcal{L}^{-1}(L_1) = -100 \int_0^t \int_0^\tau \sin(10\sigma)\, d\sigma\, d\tau$$

$$= \cancel{-100 \left(t - \frac{\sin(\sqrt{10}t)}{\sqrt{10}}\right)} = -10t + \sin(10t)$$

$$\mathcal{L}^{-1}(L_2) = -4 \int_0^t \int_0^\tau e^\sigma\, d\sigma\, d\tau = (-t + e^t - 1) \cdot 4 \cdot -1$$

$$\mathcal{L}^{-1}(L_3) = -4 \int_0^t \int_0^\tau \cos(\sigma)\, d\sigma\, d\tau$$

$$= -4 + 4\cos(t)$$

$$\mathcal{L}^{-1}(L_4) = 6t$$

$$\Rightarrow y_1(t) = -10t + \sin(10t) + 4t - 4e^t + 4 - 4 + 4\cos(t) + 6t$$

$$y_1(t) = \sin(10t) + 4\cos(t) - 4e^t$$

$$y_2(t) = -\sin(10t) + 4\cos(t) + 4e^t$$

6. R (review). 39:

$$10y_1'' = -20y_1 + 40y_2 - 40y_1 = 40y_2 - 60y_1$$

$$10y_2'' = -40y_2 + 40y_1 - 20y_2 = 40y_1 - 60y_2$$

$$y_1(0) = y_2(0) = 0, \quad y_1'(0) = 1, \quad y_2'(0) = -1$$

Transform:

$$\text{I}: 10(s^2 Y_1(s) - 1) = 40Y_2 - 60Y_1$$

$$\text{II}: 10(s^2 Y_2 + 1) = 40Y_1 - 60Y_2$$

$$\text{I}: \quad Y_2 = \frac{1}{4}(s^2 Y_1(s) - 1) + \frac{3}{2} Y_1(s)$$

$\Rightarrow$

$$\text{II}: 10\left(s^2\left(\frac{1}{4}(s^2 Y_1(s) - 1) + \frac{3}{2} Y_1(s)\right)\cancel{4} + 1\right)$$

$$= 40Y_1 - 15(s^2 Y_1 - 1) - 90Y_1(s)$$

$$10\frac{s^4}{4} Y_1(s) - \cancel{4}10\frac{s^2}{4} + 15s^2 Y_1(s) + 10$$

$$= -15s^2 Y_1 + 15 - 50Y_1$$

$$\Rightarrow Y_1\left(10\frac{s^4}{4} + 30s^2 + 50\right) = 10\frac{s^2}{4} + 5$$

$$Y_1 = \frac{\frac{10}{4}s^2 + 5}{\left(\frac{10}{4}s^4 + 30s^2 + 50\right)} = \frac{\frac{10}{4}s^2 + 5}{\frac{10}{4}(s^2+10)(s^2+2)}$$

$$= \cancel{\frac{10}{4}}\left(\frac{s^2+2}{(s^2+10)(s^2+2)}\right) = \cancel{\frac{10}{4}}\left(\frac{1}{s^2+10}\right)$$

$$\Rightarrow \mathcal{L}^{-1}(Y_1) = \cancel{\frac{10}{4}} \cdot \frac{1}{\sqrt{10}} \sin(\sqrt{10}\,t) = \underline{\underline{\frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}}}$$

$$y_1(t) = \frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}$$

We plug this into equation II:

$$10\, y_2'' = 40\, y_1(t) - 60\, y_2(t)$$

$$y_2'' = 4 \cdot \frac{\sin(\sqrt{10}\,t)}{\sqrt{10}} - 6\, y_2(t)$$

Transform:

$$s^2 Y_2 + 1 = 4 \cdot \frac{1}{s^2+10} - 6\, Y_2$$

$$Y_2(s^2+6) = 4 \cdot \frac{1}{s^2+10} - 1$$

$$Y_2 = 4 \cdot \frac{1}{(s^2+6)(s^2+10)} - \frac{1}{s^2+6} = I_1 + I_2$$

$$I_1 = 4 \cdot \mathcal{L}\left(\frac{\sin(\sqrt{6}\,t)}{\sqrt{6}}\right) \cdot \mathcal{L}\left(\frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}\right)$$

$$\mathcal{L}^{-1}(I_1) = 4\left(\frac{\sin(\sqrt{6}\,t)}{\sqrt{6}} * \frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}\right)$$

$$= 4 \cdot \int_0^t \sin(\sqrt{6}\,\tau) \cdot \sin(\sqrt{10}\,(t-\tau)) / (\sqrt{6}\sqrt{10})\, d\tau$$

$$= 4/(\sqrt{6} \cdot \sqrt{10}) \cdot \frac{1}{4}\left(\sqrt{10}\sin(\sqrt{6}\,t) - \sqrt{6}\sin(\sqrt{10}\,t)\right)$$

$$= \frac{\sin(\sqrt{6}\,t)}{\sqrt{6}} - \frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}$$

$$\mathcal{L}^{-1}(I_2) = -\frac{\sin(\sqrt{6}\,t)}{\sqrt{6}}$$

$$\Rightarrow y_2(t) = -\frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}$$

$$y_1(t) = \frac{\sin(\sqrt{10}\,t)}{\sqrt{10}}$$

Supplementary B:

$$y'' + 4y' + 4y = 2e^{-2t} + \delta(t-1)\ ,\ t > 0$$

$$y(0) = 0\ ,\ y'(0) = 0$$

Transform:

$$s^2 Y + 4sY + 4Y = 2 \cdot \frac{1}{s+2} + e^{-s}$$

$$Y(s^2 + 4s + 4) = \frac{2}{s+2} + e^{-s}$$

$$Y = \frac{2 + s \cdot e^{-s} + 2 \cdot e^{-s}}{(s+2)(s^2+4s+4)} = \frac{2}{(s+2)^3} + \frac{e^{-s}}{(s+2)^2}$$

$$Y = 2 \cdot \left(\mathcal{L}(e^{-2t})\right)^3 + e^{-s} \cdot \left(\mathcal{L}(e^{-2t})\right)^2$$

$$= I_1 + I_2$$

$$\mathcal{L}^{-1}(I_1) = 2 \cdot \left(e^{-2t} * e^{-2t} * e^{-2t}\right)$$

$$= 2 \cdot \left(\left(\int_0^t e^{-2\tau} \cdot e^{-2(t-\tau)} d\tau\right) * e^{-2t}\right)$$

$$= 2 \cdot \left(e^{-2t} \cdot t * e^{-2t}\right)$$

$$= 2 \cdot \int_0^t \tau \cdot e^{-2\tau} \cdot e^{-2(t-\tau)} d\tau$$

$$= \underline{t^2 \cdot e^{-t}}$$

$$\mathcal{L}^{-1}(I_2) = f(t-1) \cdot u(t-1)\ ,\ f(t) = \mathcal{L}^{-1}(F(s))$$

$$,\ F(s) = \frac{1}{(s+2)^2}$$

$$\mathcal{L}^{-1}\left(\frac{1}{(s+2)^2}\right) = e^{-2t} * e^{-2t} = \underline{t \cdot e^{-t}}$$

$$\Rightarrow y(t) = t^2 \cdot e^{-t} + (t-1) \cdot e^{(-t+1)} \cdot u(t-1)$$

$$= \underline{\underline{e^{-t}\left(t^2 + (t-1) \cdot e \cdot u(t-1)\right)}}$$

Supplementary C:

$$y' + y + \int_0^t y(\tau)e^{t-\tau}\,d\tau = u(t-1)$$

$$y(0) = 1$$

Transform:

$$sY - 1 + Y + Y \cdot \frac{1}{s-1} = e^{-s} \cdot \frac{1}{s}$$

convolution transformed

$$Y\left(s + 1 + \frac{1}{s-1}\right) = e^{-s} \cdot \frac{1}{s} + 1$$

$$Y = \frac{e^{-s}/s + 1}{s + 1 + \frac{1}{s-1}} = \frac{(e^{-s}/s + 1)(s-1)}{s^2 - s + s - 1 + 1}$$

$$= \frac{(e^{-s}/s + 1)(s-1)}{s^2} = \frac{e^{-s} + s}{s^2} \cdot \frac{s-1}{s}$$

$$= \frac{e^{-s}}{s^2} + \frac{1}{s} + 1 - \frac{1}{s} = \frac{e^{-s}}{s^2} + 1$$

$$\mathcal{L}^{-1}\left(\frac{e^{-s}}{s^2}\right) = f(t-1) \cdot u(t-1), \quad f(t) = \mathcal{L}^{-1}(F(s))$$

$$F(s) = \frac{1}{s^2}$$

$$\Rightarrow \mathcal{L}^{-1}\left(\frac{e^{-s}}{s^2}\right) = (t-1) \cdot u(t-1)$$

$$\mathcal{L}^{-1}(1) = \delta(t)$$

$$\Rightarrow y(t) = (t-1) \cdot u(t-1) + \delta(t)$$

$$\frac{e^{-s}+s}{s^2}\cdot\frac{(s-1)}{s}=\frac{se^{-s}+s^2-e^{-s}-s}{s^3}$$

$$=\frac{e^{-s}}{s^2}+\frac{1}{s}-\frac{e^{-s}}{s^3}-\frac{1}{s^2}=Y$$

$$\mathcal{L}^{-1}\left(\frac{e^{-s}}{s^2}\right)=\underline{(t-1)\cdot u(t-1)}$$

$$\mathcal{L}^{-1}\left(\frac{1}{s}\right)=\underline{1}$$

$$\mathcal{L}^{-1}\left(-\frac{e^{-s}}{s^3}\right)=f(t-1)\cdot u(t-1),\ f(t)=\mathcal{L}^{-1}(F(s))$$

$$F(s)=\frac{-1}{s^3}\Rightarrow f(t)=-\frac{t^2}{2}$$

$$\Rightarrow \mathcal{L}^{-1}\left(-\frac{e^{-s}}{s^3}\right)=-\frac{(t-1)^2}{2}\cdot u(t-1)$$

$$\mathcal{L}^{-1}\left(-\frac{1}{s^2}\right)=-t$$

$$\Rightarrow y(t)=\cancel{(t-1)\cdot u(t-1)+1}$$

$$u(t-1)\left(-t^2/2+2t-\frac{3}{2}\right)+1-t$$

$$=\underline{\underline{u(t-1)(t-1)(t-3)+1-t}}$$