TTK4240 – Høst 2016 1

Øving 3

Vsevolod Karpov – vsevolok.

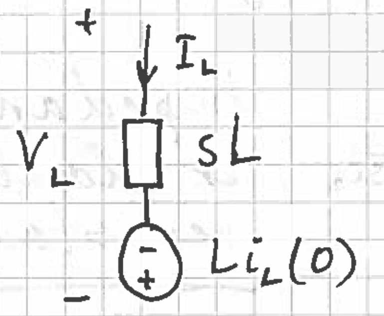
Oppgave 1

a) $V_L = sLI_L - Li_L(0)$

ligner på $R \cdot I$, hvor $R = sL$

konstant. Siden V_L er spenning, må $Li_L(0)$ også være det. Med motsatt fortegn av V_L

Dermed:



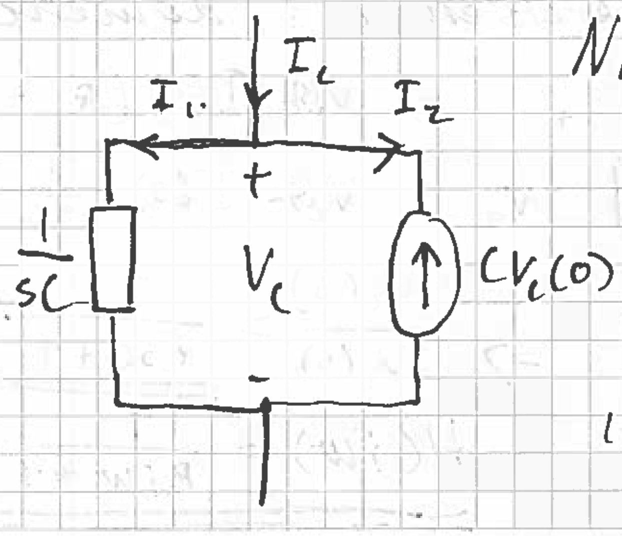
$I_c = sCV_c - CV_c(0) = I_1 + I_2$

=> Summen av strømmene inn i grenen/noden er lik strømmen ut.

$I_1 = \frac{V}{R}$, $V = V_c$, $R = \frac{1}{sC}$

$I_2 = -CV_c(0)$ (En konstant strøm i motsatt retning av I_c, altså en strøm kilde!)

Dermed



NB! Vi vet jo at spenningsfall over en gren (V_c) er lik spenningsfallet over hele parallell koblingen.

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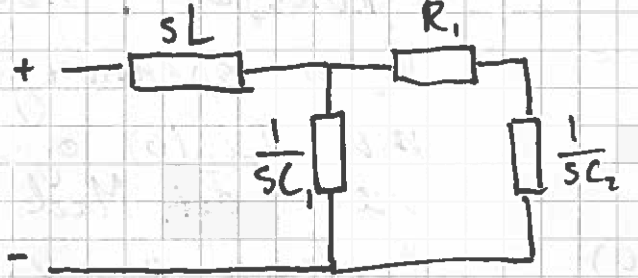
b) $Z_L(s) = \dfrac{V_L(s)}{I_L(s)} = \underline{\underline{sL}}$

$Z_c(s) = \dfrac{V_c(s)}{I_c(s)} = \underline{\underline{\dfrac{1}{sC}}}$

c) Tegner om kretsen til s domenet med alle init. betingelser lik null.

$\Rightarrow$ 

Impedansen er da lik total resistans.

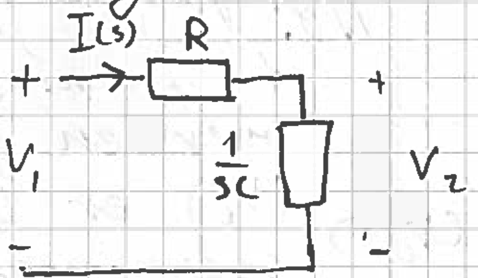
$$\Rightarrow R_{tot} = Z(s) = sL + \underline{\underline{\frac{R_1C_2s + 1}{s^2R_1C_1C_2 + sC_1 + sC_2}}}$$

d) $L = 1H,\ R = 1\,\Omega,\ C_1 = C_2 = 2F$

$$\Rightarrow Z(s) = s + \frac{2s + 1}{s^2 \cdot 4 + 4s}$$

$$s = jw \Rightarrow Z(jw) = \underline{\underline{jw + \frac{2jw + 1}{4jw - 4w^2}}}$$

e) Tegner kretsen i s domenet.



$\Rightarrow V_1(s) = I(s)\left(R + \frac{1}{sC}\right)$

$V_2(s) = I(s) \cdot \frac{1}{sC}$

$$\Rightarrow \frac{V_2(s)}{V_1(s)} = \underline{\underline{\frac{1}{Rsc + 1}}} = H(s)$$

$$\underline{\underline{H(jw) = \frac{1}{Rjw + 1}}}$$

Oppgave 2 a)

$R = 10000\,\Omega$, $C = 100 \cdot 10^{-6}\,F$, $V_s = 15V$

fra 1e)

$$V_c(s) = \frac{1}{RsC+1} \cdot V_s(s) \quad , \quad V_s(s) = \mathcal{L}(V_s) = \frac{15}{s}$$

$$\Rightarrow V_c(s) = \frac{15}{s(s+1)} = \frac{1}{s} \cdot 15\,\mathcal{L}(e^{-t})$$

$$\mathcal{L}^{-1}(V_c(s)) = 15\int_0^t e^{-\tau}\,d\tau = 15\left[-e^{-\tau}\right]_0^t = (1-e^{-t}) \cdot 15$$

$$= \underline{\underline{15 - 15e^{-t}}}$$

Tids konstant:

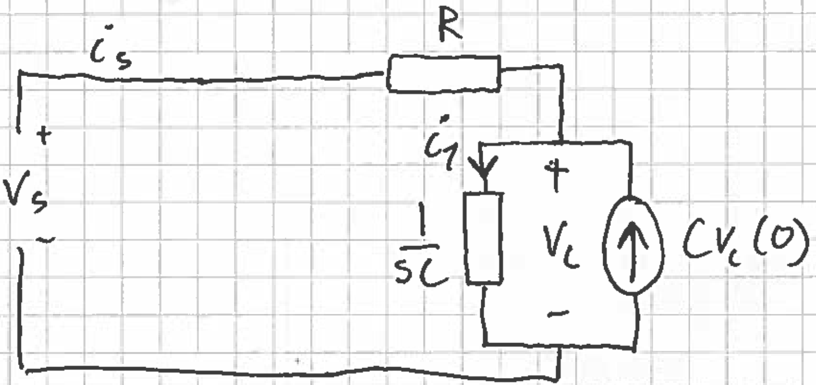
$$\frac{V_c(t) - 0}{15 - 0} = 1 - \frac{1}{e}$$

$$\Rightarrow 1 - e^{-t} = 1 - e^{-1}$$

$$\Rightarrow \underline{\underline{t = 1}}$$

b) $V_c(0) = 5V$

$\Rightarrow$ Kretsen i s domenet



(I) $V_s(s) = I_s(s) \cdot R + V_c(s)$

(II) $i_1 \cdot \frac{1}{sC} = V_c(s)$, $I_s(s) = i_1(s) - CV_c(0)$

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$\Rightarrow$ (II) $\quad i_1(s) = V_c(s) \cdot s \cdot C$

$\Rightarrow I_s(s) = V_c(s) \cdot s \cdot C - C \cdot V_c(0)$

$\Rightarrow$ (II) + (I) $\quad V_s(s) = R \cdot \left(V_c(s) \cdot s \cdot C - C \cdot V_c(0)\right) + V_c(s)$

$$\Rightarrow \frac{15}{s} = V_c(s) \cdot s - 5 + V_c(s)$$

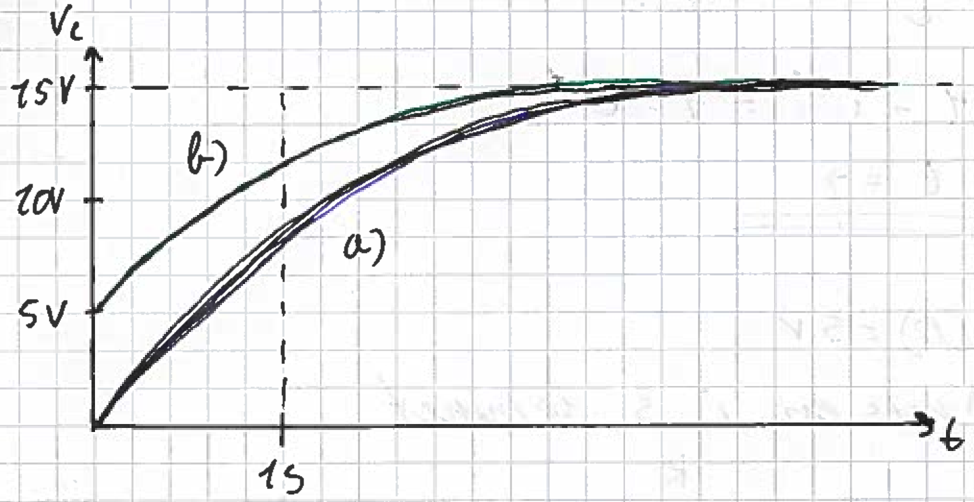
$$\frac{15}{s} + 5 = V_c(s)(s+1)$$

$$\Rightarrow V_c(s) = \frac{15}{s(s+1)} + \frac{5}{s+1}$$

$$\mathcal{L}^{-1}(V_c(s)) = 15 - 15e^{-t} + 5 \cdot e^{-t}$$

$$= \underline{\underline{15 - 10 \cdot e^{-t}}}$$

c)



Oppgave 3

a)
$$V_s(t) = i(t)\cdot R + V_L(t)$$

Laplace: $V_s(s) = I(s)\cdot R + sL\cdot I(s) - L\cdot i_L(0)$

$$\frac{V\cdot s}{s^2+\omega^2} = I(s)(R+sL) \quad , (i_L(0)=0)$$

$$\Rightarrow I(s) = \frac{V\cdot s}{s^2+\omega^2}\cdot\frac{1}{R+sL}$$

b) 1) sett $I(s) = \dfrac{As+B}{s^2+\omega^2} + \dfrac{C}{R+sL}$

2) $(As+B)\cdot(R+sL) + C\cdot(s^2+\omega^2) = V\cdot s$

$\Rightarrow ARs + BR + As^2L + BsL + Cs^2 + C\omega^2 = V\cdot s$

$\Rightarrow AR + BL = V \quad , (s)$

$BR + C\omega^2 = 0, (1)$

$As^2L + Cs^2 = 0, (s^2)$

Noe som vil løse seg til det opgitt i oppgaven!

c) $\mathcal{L}^{-1}(I(s)) = V\cdot\cos(\omega t) * \frac{1}{L}\cdot e^{-\frac{R}{L}t}$

$$= \int_0^t \frac{V}{L}\cos(\omega\tau)\cdot e^{-\frac{R}{L}(t-\tau)}\,d\tau$$

$$= e^{-\frac{R}{L}t}\cdot\frac{V}{L}\cdot\int_0^t \cos(\omega\tau)\,e^{\frac{R}{L}\cdot\tau}\,d\tau$$

$$= e^{-\frac{R}{L}t}\cdot\frac{V}{L}\cdot\left[\frac{e^{\frac{R}{L}\tau}\left(\frac{R}{L}\cdot\cos(\omega\tau)+\omega\cdot\sin(\omega\tau)\right)}{\frac{R^2}{L^2}+\omega^2}\right]_0^t$$

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$$= e^{-\frac{R}{L}t} \cdot \frac{V}{L} \cdot \left(\frac{e^{\frac{R}{L}t}\left(\frac{R}{L}\cos(\omega t) + \omega \sin(\omega t)\right) - \frac{R}{L}}{\frac{R^2}{L^2} + \omega^2} \right)$$

$$= \frac{\frac{V}{L}\left(\frac{R}{L}\cos(\omega t) + \omega \sin(\omega t)\right) - \frac{R \cdot V \cdot e^{-\frac{R}{L}t}}{L^2}}{\frac{R^2}{L^2} + \omega^2}$$

$V = 10V$, $R = 1\,\Omega$, $L = 5\,mH$, $\omega = 2\pi \cdot 50$ rad/s

$$\Rightarrow \cancel{i(t) = \frac{1000(100\cos(100\pi t) + 100\pi\sin(100\pi t))}{100000 + 10\,000\pi^2} - 100\,000\, e^{-}}$$

$$i(t) = \frac{200(200\cos(100\pi t) + 100\pi \sin(100\pi t))}{40000 + 10\,000\pi^2} - \frac{400\,000 \cdot e^{-200t}}{40000 + 10\,000\pi^2}$$

Simplifiseres til:

$$i(t) = 5.3703 \cdot \cos\left(100\pi t - \frac{57.51}{180}\pi\right) - 7.884 \cdot e^{-200t}$$

d) Stasjonærdel:

$$5.3703 \cdot \cos\left(100\pi t - \frac{57.51}{180}\pi\right)$$

Fordi: $\lim_{t \to \infty} e^{-200t} = 0$

$$\frac{57.51}{180} \cdot \pi = 57.51^{\circ}$$

$$\Rightarrow i_{stasj}(t) = 5.3703\cos(wt - 57.51^{\circ})$$

e) $H(s) = \frac{V(s)}{I(s)}$

$$V(s) = \frac{V \cdot s}{s^2 + w^2}, \quad I(s) = \frac{V \cdot s}{s^2 + w^2} \cdot \frac{1}{R + sL}$$

$$\Rightarrow H(s) = R + sL$$

$s = jw$

$$\Rightarrow H(jw) = R + jwL$$
$$= 1 + jw \cdot 5 \cdot 10^{-3}$$

$$\cancel{H(jw) = 1 + 5 \cdot 10^{-3} \cdot w \cdot j\sin\left(\frac{\pi}{2}\right)}$$

$$H(jw) = w \cdot 5 \cdot 10^{-3}\left(\frac{1}{w \cdot 5 \cdot 10^{-3}} + j\right)$$
$$= w \cdot 5 \cdot 10^{-3}\left(\frac{200}{w} + j\right)$$
$$= w \cdot 5 \cdot 10^{-3}\left(r\cos(\theta) + jr\sin(\theta)\right)$$

$r \cdot \cos(\theta) = \frac{200}{w}$

$r = \frac{200}{w\cos(\theta)} \Rightarrow \frac{200}{w} \cdot \tan(\theta) = 1$

$\theta = \tan^{-1}\left(\frac{w}{200}\right), \quad w = 100\pi$

$$\Rightarrow \theta = \tan^{-1}\left(\frac{\pi}{2}\right) = 57.51^{\circ} = 1.00388 \text{ rad}$$

$$\Rightarrow R = r \cdot w \cdot 5 \cdot 10^{-3} = 1.8616 \approx 1.862$$

$$\Rightarrow |H(jw)| = R = 1.862, \quad \angle H(jw) = \theta = 57.51^{\circ}$$

f) $v_s(t) = 10 \cdot \cos(100\pi t)$

$i_{stas,j}(t) = 5.3703 \cdot \cos(100\pi t - 57.51°)$

forhold mellom amplitudene:

$$\frac{10}{5.3703} = \underline{\underline{1.862}}$$

Differansen i fasevinkel:

$$\underline{\underline{57.51°}}$$

Ergo: Amplituden til det komplekse tallet $H(j\omega)$, altså amplituden til transfer funksjonen er lik forholdet mellom amplitudene til spenningssignalet og tilsvarende strøm.

Vinkelen til det komplekse tallet $H(j\omega)$ er lik faseforskyvingen til strømmen i forhold til spenningen.

Oppgave 4

a) $\Theta_m(t) = \int_0^t \Omega_m(t) \Rightarrow \mathcal{L}(\Theta_m(t)) = \frac{\Omega_m(s)}{s}$

$$\Rightarrow \Theta_m(s) = \frac{\Omega_m(s)}{s} \Rightarrow \frac{\Theta_m(s)}{V_m(s)} = \frac{1}{s}\,\frac{\Omega_m(s)}{V_m(s)}$$

$$= \frac{1}{s} \cdot \frac{K}{s\tau + 1} = \underline{\underline{\frac{K}{s(s\tau + 1)}}}$$

b) $$\Omega_m(s) = \frac{V_m(s)K}{\tau s + 1}$$

$$\Theta_m(s) = \frac{V_m(s)K}{s(\tau s + 1)}$$

$$V_m(t) = u(t) \Rightarrow V_m(s) = \frac{1}{s}$$

$$\Rightarrow \Omega_m(s) = \frac{K}{s(\tau s + 1)}, \quad \Theta_m(s) = \frac{K}{s^2(\tau s + 1)}$$

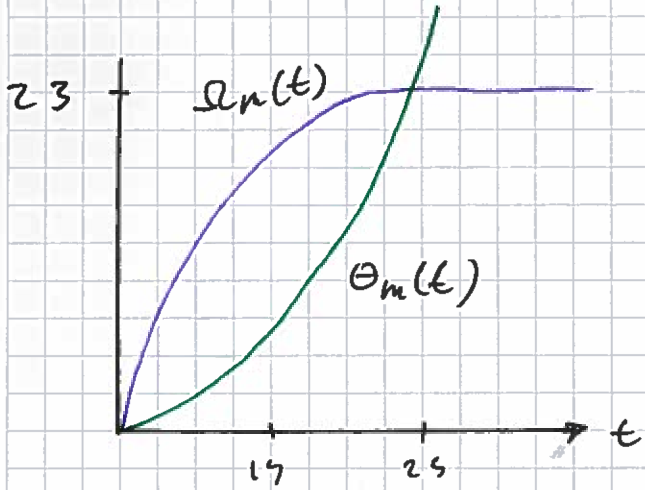
$$\mathcal{L}^{-1}(\Omega_m(s)) = K\left(1 - e^{-\frac{t}{\tau}}\right) = \Omega_m(t)$$

$$\mathcal{L}^{-1}(\Theta_m(s)) = K\left(\tau\left(e^{-\frac{t}{\tau}} - 1\right) + t\right) = \Theta_m(t)$$

$$K = 23, \quad \tau = 0.13$$

$$\Rightarrow \underline{\underline{\Omega_m(t) = 23\left(1 - e^{-\frac{t}{0.13}}\right)}}$$

$$\underline{\underline{\Theta_m(t) = 23\left(0.13\left(e^{-\frac{t}{0.13}} - 1\right) + t\right)}}$$



Dette virker fornuftig da det tar tid før motoren oppnår full hastighet.

Når full hastighet oppnås, ~~øker~~ ~~vinkelen~~ ~~flek~~ er det konstant økning i vinkelen.