TFY4115 - Høst 2016
Øving 4

Vsevolod Karpov - vsevolok

Oppgave 1

a) $\frac{1}{2}MV^2 = MgL$

$\Rightarrow \underline{\underline{V = \sqrt{2gL}}}$

$S = M\frac{V^2}{L} + Mg = \underline{\underline{3Mg}}$

b) $MV + mv = MV' + mv'$

$v = 0 \Rightarrow MV = MV' + mv'$

$\Rightarrow V' = \frac{M\sqrt{2gL} - mv'}{M}$

$v' = \frac{M\sqrt{2gL} - MV'}{m}$

Elastisk støt $\Rightarrow$

$\frac{1}{2}MV^2 = \frac{1}{2}(MV'^2 + mv'^2)$

$\Rightarrow \sqrt{\frac{MV^2 - mv'^2}{M}} = V'$

$\Rightarrow \underline{\underline{v' = \frac{2M}{m+M} \cdot V}}$

$\underline{\underline{V' = \left(1 - \frac{2m}{m+M}\right)V}}$

c) $S' = M\frac{V'^2}{L} + Mg = M\left(\frac{V^2}{L}\left(\frac{m-M}{m+M}\right)^2 + g\right)$

$s' = m\frac{v'^2}{L} + mg = m\left(\frac{V^2}{L} \cdot \frac{4M^2}{(m+M)^2} + g\right)$

d) $M = 10.0g = 0.01 kg$

$m = 0.02 kg$

$L = 1.00 m$

$g = 9.81 m/s^2$

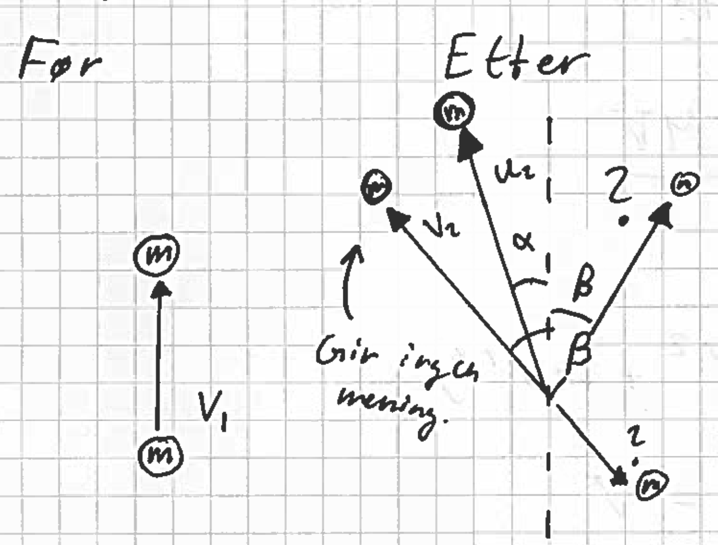
$\Rightarrow \underline{\underline{V = 4.429 m/s}}$, $\underline{\underline{v' = \cancel{0.666}\ 2.952 m/s}}$

$\underline{\underline{V' = -\cancel{0.333}\ -1.476 m/s}}$, $\underline{\underline{S = 0.2943 N}}$

$\underline{\underline{s' = 0.3706 N}}$, $\underline{\underline{S' = 0.1199 N}}$

Oppgave 2

a)



Logisk sett må den ene pucken bevege seg bakover, eller til siden.

Fortegn på vinklene er ikke opgitt!!!

b) $mV_1 = m(V_2\cos(\alpha) + u_2\cos(\beta))$

$0 = V_2 \sin(\alpha) + u_2 \sin(\beta)$

$V_1 = 40 m/s$
$\alpha = 30°$
$\beta = 45°$

$V_2 = -\mu_2 \dfrac{\sin(\beta)}{\sin(\alpha)}$

$\Rightarrow V_1 = -\mu_2 \dfrac{\sin(\beta)}{\tan(\alpha)} + \mu_2 \sin(\beta)$

$= \mu_2 \sin(\beta)\left(1 - \dfrac{1}{\tan(\alpha)}\right)$

$\Rightarrow \mu_2 = \dfrac{V_1}{\sin(\beta) - \dfrac{\sin(\beta)}{\tan(\alpha)}}$

$\Rightarrow \alpha$ må settes til $-30°$
for at svarene skal gi noe mening.

Vi får da: $u_2 = 20.7\ m/s$
$V_2 = 29.3\ m/s$

c) $$\frac{\frac{1}{2}m(u_2^2 + V_2^2)}{\frac{1}{2}mV^2} = 0.80$$

Altså har puckene kun 80% av den opprinnelige energien etter kollisjonen.
$\Rightarrow$
Energien tapt = 20 %

Oppgave 3

a) $W = E_1 - E_0 = \frac{1}{2}mV_B^2 - \frac{1}{2}mV_0^2 = \frac{1}{2}mV_B^2$

b) $F_f = mg\mu_k \Rightarrow a = \cancel{m}g\mu_k$

$2ax_k = V_B^2 - V_0^2 \Rightarrow$

$$x_k = \frac{V_B^2}{2\cancel{m}g\mu_k}$$

Evt. $W = F \cdot x_k \Rightarrow \frac{1}{2}mV_B^2 = mg\mu_k x_k$

$$\Rightarrow x_k = \frac{V_B^2}{2g\mu_k}$$

c) $x_k = \frac{1}{2}(v_0 + v_\theta)\cdot t$

$$\Rightarrow t = \frac{2x_k}{v_\theta}, \quad x_k = \frac{v_\theta^2}{2g\mu_k}$$

$$\Rightarrow t = \frac{v_\theta}{g\mu_k}$$

d) $s = v_\theta \cdot t = \frac{v_\theta^2}{g\mu_k}, \quad v_\theta^2 = x_k \cdot 2g\mu_k$

$$\Rightarrow s = \frac{x_k \cdot 2 \cdot g\mu_k}{g\mu_k} = 2x_k$$

e) $W = F \cdot s = mg\mu_k \cdot 2x_k$

$$mg\mu_k \cdot x_k = \frac{1}{2} m v_\theta^2$$

$$\Rightarrow W = m v_\theta^2$$

Oppgave 4

$$\vec{R} = \frac{1}{\sum_{i=1}^{3} m_i} \cdot \sum_{i=1}^{3} m_i \cdot \vec{r_i}$$

$$\sum_{i=1}^{3} m_i = 1u + 1u + 16u = 18u$$

$$\sum_{i=1}^{3} m_i \cdot \vec{r_i} = 1u \cdot \begin{bmatrix} d\cos(52.5^\circ) \\ d\sin(52.5^\circ) \end{bmatrix} + 1u \begin{bmatrix} d\cos(52.5^\circ) \\ -d\sin(52.5^\circ) \end{bmatrix}$$

$$+ 16u \begin{bmatrix} 0 \\ 0 \end{bmatrix} = 2u \begin{bmatrix} d\cos(52.5^\circ) \\ 0 \end{bmatrix}$$

$$\Rightarrow \vec{R} = \frac{2u}{18u}\begin{bmatrix} d\cos(52{,}5°) \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0{,}067d \\ 0 \end{bmatrix}$$

Oppgave 5

a)

$$dv(m - dm) = u_{ex}\,dm$$

b) $a = \frac{dv}{dt} \Rightarrow dv = a \cdot dt$

$$\beta = \frac{dm}{dt} \Rightarrow dm = \beta dt$$

$$\Rightarrow a \cdot dt(m - \beta dt) = u_{ex}\beta dt$$

$$a = \frac{u_{ex}\beta}{m - \beta dt}$$

Siden vi ønsker akselerasjonen i det motoren settes på, er $dt \approx 0$

Dermed:

$$a = \frac{u_{ex}\beta}{m_0} = \frac{3.27 \cdot 10^3 \cdot 480}{2{,}55 \cdot 10^5}$$

$$= 6.155\ m/s^2$$

c) $\beta = \frac{dm}{dt} = dm = \beta dt$

$\Rightarrow m_{tap} = \beta \cdot t$

$dv(m_0 - dm) = u_{ex} dm$

$dv(m_0 - m_{tap}) = u_{ex} \beta dt$

$dv = \frac{u_{ex}\beta}{m_0 - \beta t} dt$

$\Rightarrow \int_0^v dv = \int_0^t \frac{u_{ex}\beta}{m_0 - \beta t} dt$

$\Rightarrow V = u_{ex}\beta \left[\frac{-\ln(m_0 - \beta t)}{\beta} \right]_0^t$

$= u_{ex}\left(-\ln(m_0 - \beta t) + \ln(m_0)\right)$

$\underline{\underline{V(60) = 392 \text{ m/s}}}$

~~Evt~~ Evt: $a = \frac{u_{ex}\beta}{(m_0 - \beta t)}$

massen til rakketten avtar jo med tiden.

$\Rightarrow \frac{dv}{dt} = \frac{u_{ex}\beta}{m_0 - \beta t}$

$dv = \frac{u_{ex}\beta}{m_0 - \beta t} dt$