TTK4240 - Høst 2016
Øving 5
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1 a)

$$v(t) = V_p \sin(\omega t)$$

$$\Rightarrow V_{rms} = \sqrt{\frac{1}{t}\int_0^t v(\tau)^2 d\tau}$$

$$\int_0^t V_p^2 \sin(\omega\tau)^2 d\tau = V_p^2\left[\frac{\tau}{2} - \frac{\sin(2\omega\tau)}{4\omega}\right]_0^t$$

$$= V_p^2\left(\frac{t}{2} - \frac{\sin(2\omega t)}{4\omega}\right)$$

$$\Rightarrow V_{rms} = V_p\sqrt{\frac{1}{2} - \frac{\sin(2\omega t)}{4\omega t}}, \quad t = 0 \Rightarrow V_{rms} = \frac{V_p}{\sqrt{2}}$$

b) $p(t) = v(t)\,i(t) = \dfrac{(v(t))^2}{R}$

$$= \frac{V_p^2 \sin(\omega t)^2}{R}$$

$$P_{average} = \frac{1}{\frac{2\pi}{\omega}}\int_0^{2\pi/\omega} \frac{V_p^2 \sin(\omega t)^2}{R} dt = \frac{V_p^2}{2R} = V_{rms}^2/R$$

c) $u(t) = V_x \Rightarrow p(t) = V_x \cdot \dfrac{V_x}{R} = \dfrac{(V_x)^2}{R}$

$$\frac{1}{\frac{2\pi}{\omega}}\int_0^{2\pi/\omega} \frac{(V_x)^2}{R} dt = \frac{V_x^2}{R} = \frac{V_p^2}{2R} \Rightarrow V_x = \frac{V_p}{\sqrt{2}}$$

d) $V_x = \dfrac{V_p}{\sqrt{2}} = V_{rms}$

Which means that an AC voltage source can be simplified to a DC source with the same voltage as V_{rms}

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(2) a) $v(t) = V_m \cos(\omega t)$

$V_L = L \cdot \frac{di}{dt}$

$\Rightarrow v(t) = V_L(t) \Rightarrow V_m \cos(\omega t) = L \cdot \frac{di}{dt}$

$\Rightarrow \frac{V_m \cdot s}{s^2+\omega^2} = s \cdot I(s) \cdot L \Rightarrow I(s) = \frac{1}{L} \cdot \frac{V_m}{s^2+\omega^2} = \frac{1}{\omega} \frac{V_m \omega}{s^2+\omega^2} \cdot \frac{1}{L}$

$\Rightarrow \underline{\underline{i(t) = \frac{V_m}{L\omega} \sin(\omega t)}}$

b) $p(t) = V(t) \cdot i(t) = \frac{V_m^2}{L \cdot \omega} \cos(\omega t) \sin(\omega t)$

$= \underline{\underline{\frac{V_m^2}{2L\omega} \sin(2\omega t)}}$

Average power:

$\frac{\omega}{2\pi} \int_0^{2\pi/\omega} \frac{V_m^2}{2L \cdot \omega} \cdot \sin(2\omega t)\, dt = \frac{\omega V_m^2}{4L\pi\omega} \left[-\frac{1}{2\omega} \cos(\omega t) \right]_0^{2\pi/\omega}$

$= \underline{\underline{0}}$ (inductors consume no power over one period)

c) $S = P + jQ = U \cdot \bar{I} = V \cdot \bar{I}$

$v(t) = V_m \cos(\omega t)$

$i(t) = \frac{V_m}{L\omega} \sin(\omega t) = \frac{V_m}{L\omega} \cos(\omega t + \phi)$, $\phi = \frac{3\pi}{2}$

$\Rightarrow \cancel{V(t)}\ V = V_m \cdot e^{j0}$

$i = I = \frac{V_m}{L\omega} e^{j\frac{3\pi}{2}} \Rightarrow \bar{I} = \frac{V_m}{L\omega} e^{j\frac{\pi}{2}}$

$\Rightarrow \underline{\underline{S = V \cdot \bar{I} = \frac{V_m^2}{L\omega} e^{j\frac{\pi}{2}}}}$

$S = \frac{V_m^2}{L\omega} \left(\cos(\frac{\pi}{2}) + j \sin(\frac{\pi}{2}) \right) \Rightarrow \underline{\underline{P = 0,\ Q = \frac{V_m^2}{L\omega}}}$

No active, only reactive power. Makes sense for a circuit that only has an inductor and voltage source.

Note! With rms, $S = P = \frac{V_m^2}{2L\omega}$

d) $I \cdot Z = U$

$\Rightarrow \quad Z = \frac{U}{I} = \frac{V_m}{\frac{V_m}{L\omega} e^{j\frac{3\pi}{2}}} = L\omega \cdot \frac{1}{-j} = \underline{\underline{L\omega \cdot j}}$

$H(s) = Z(s) = \frac{U(s)}{I(s)}$

$U(s) = V_m \cdot \frac{s}{\omega^2+s^2}, \quad I(s) = \frac{V_m}{L\cdot\omega}\cdot\frac{\omega}{\omega^2+s^2}$

$\Rightarrow H(s) = L\cdot s \Rightarrow H(j\omega) = \underline{\underline{L\cdot j\omega}}$

Same!

e)

<u>0 kr!!!</u> (Inductor consumes no net power. this of course assumes that all the wires have zero resistance)

3 a) $p_{v(t)} = 20\,ms, \quad p_{i(t)} = 22{,}5 - 2.5 = 20\,ms$

$\Rightarrow p_{v(t)} = p_{i(t)} = p$

First peak for $v(t)$ at $t = 0\,ms$

— " — for $i(t)$ at $t = 2.5\,ms$

$\Rightarrow$ phase angle diff $= \frac{t_{i(t)peak} - t_{v(t)peak}}{p}\cdot 360$

$= \frac{2.5}{20}\cdot 360 = \underline{\underline{45^\circ}}$

b) $v(t) = 5\cdot\cos\left(\frac{360^\circ}{20}t\right)$

$i(t) = 3\cdot\cos\left(\frac{360^\circ}{20}t - 45^\circ\right)$ (in degrees)

$\Rightarrow P(t) = i(t)\cdot v(t) = \underline{\underline{15\cdot\cos\left(\cancel{\frac{360^\circ}{20}t}\ 18^\circ t\right)\cdot\cos\left(18^\circ t - 45^\circ\right)}}$

c) $S = P + jQ = U \cdot \overline{I}$

$U = V = 5 \cdot e^{j0}$

$I = 3 \cdot e^{-45°j} \Rightarrow \overline{I} = 3 \cdot e^{j45°}$

$\Rightarrow S = 3 \cdot 5 \cdot e^{j45°} \cdot e^{j0} = 15 e^{j45°}$

But, with rms values:

$V = \frac{5}{\sqrt{2}} e^{j0}, \quad \overline{I} = \frac{3}{\sqrt{2}} e^{45°j}$

$\Rightarrow \underline{\underline{S = \frac{15}{2} e^{45°j}}}$

$\Rightarrow S = \frac{15}{2} (\cos(45°) + j\sin(45°))$

$\Rightarrow \underline{\underline{P = \sqrt{\frac{1}{8}} \cdot 15, \quad Q = \sqrt{\frac{1}{8}} \cdot 15}}$

Current leads voltage by ~~more than~~ less than $90°$ $\Rightarrow$ capacitive circuit!

d) $Z_{eq} = \frac{U}{I} = \underline{\underline{\frac{5}{3} \cdot e^{45°j}}}$

e) $U = Z \cdot I, \quad S = U \cdot \overline{I}$

$\Rightarrow S = Z \cdot I \cdot \overline{I} = \underline{\underline{Z \cdot |I|^2}}$

$\Rightarrow I = \frac{U}{Z}, \quad \overline{I} = \frac{\overline{U}}{\overline{Z}}$

$\Rightarrow S = U \cdot \frac{\overline{U}}{\overline{Z}} = \underline{\underline{\frac{|U|^2}{\overline{Z}}}} \Rightarrow S = \frac{\frac{25}{2}}{\frac{5}{3}} e^{45°j} = \underline{\underline{\frac{15}{2} e^{45°j}}}$ (match)

$\Rightarrow Z = \frac{S}{|I|^2} = \frac{\frac{15}{2}}{\frac{9}{2}} \cdot e^{45°j} = \underline{\underline{\frac{5}{3} \cdot e^{45°j}}}$ (match)

① a) If we were to connect everything in series, the ~~dish~~ washer would need to be on (closed circuit) in order for the TV to be on and the heater to be on. In general, all three need to be on in order for any to be on! - Bad idea.

Grid voltage is in RMS $= \frac{V_{peak}}{\sqrt{2}} = \frac{325.3}{\sqrt{2}} \approx \underline{230}$

b) Washer: $\cos(\phi) = 0.95 = \frac{P}{S} = \frac{P}{U \cdot \overline{I}_w}$

$\Rightarrow \overline{I}_w = 0.95 \cdot \frac{U}{P}$, $\cos(\phi) = 0.95 \Rightarrow \phi = 18.19 \approx 18.2°$

$\Rightarrow V = 230 \cdot e^{j0}$

$\underline{I_{wm} = 6 \cdot e^{-18.2 \cdot j}}$ (since inductive, current must lag behind voltage, ergo a minus 18.2°)

TV: $\cos(\phi) = 0.98 \Rightarrow \phi = -11.47$ (since inductive)

$\Rightarrow \underline{I_{TV} = 4 \cdot e^{-11.47 j}}$

Heater: $\cos(\phi) = 1 \Rightarrow \phi = 0$

$\Rightarrow I_h = 7 \cdot e^{j0} = \underline{7}$

$I_{tot} = \underline{6 \cdot e^{-18.2 \cdot j} + 4 \cdot e^{-11.47 j} + 7}$

$= 6\cos(-18.2) + 4\cos(-11.47) + 7 + j(\sin(-18.2) \cdot 6 + \sin(-11.47) \cdot 4)$

$\Rightarrow$

$i_{tot} = 6(\cos(\omega t - 18.2°)) + 4\cos(\omega t - 11.47°) + 7$

inductors in circuit means non constant current and equation above peaks at 16.7 A

c) $Z_{vm} = \frac{U}{I_{vm}} = \frac{230}{6 \cdot e^{-j18.19}} = \underline{38.33 e^{j \cdot 18.19}}$

$Z_{tv} = \frac{U}{I_{tv}} = \frac{230}{4 \cdot e^{-j11.47}} = \underline{57.5 \cdot e^{j11.47}}$

$Z_h = \frac{U}{I_h} = \frac{230}{7} = \underline{32.85}$

$H(j\omega) = Z = R + j\omega L$

$\omega = 2\pi \cdot 50$

$\Rightarrow Z_{vm} = 38.33 \cdot \cos(18.19°) + j \cdot 38.33 \sin(18.19°)$

$\Rightarrow \underline{\underline{R_{vm} = 36{,}41\,\Omega}}, \; L_{vm} = \frac{38.33 \sin(18.19)}{\omega} = 0.03808 \; \underline{\underline{= 0.038\,H}}$

Same approach for other components yields:

$\underline{\underline{R_{tv} = 56.35\,\Omega}}, \; L_{tv} = 0.03639 = \underline{\underline{0.0364\,H}}$

$\underline{\underline{R_h = 32.85\,\Omega, \; L_h = 0}}$

d) $S_{vm} = U \cdot I = 230 \cdot 6 \cdot e^{j18.19} = \underline{\underline{1380 \cdot e^{j18.19}}}$

$\Rightarrow S_{vm} = P_{vm} + jQ_{vm} = 1380 \cdot \cos(18.19°) + j \cdot 1380 \cdot \sin(18.19°)$

$\Rightarrow \underline{\underline{P_{vm} = 1311\,W, \; Q_{vm} = 430.79\,VAr}}$

Same approach for other components yields:

$\underline{\underline{S_{tv} = 920 \cdot e^{j11.47}, \; P_{tv} = 901\,W, \; Q_{tv} = 182.94\,VAr}}$

$\cancel{S_h = 32.85, \; P_h = 32.85W, \; Q_h = 0}$

$\underline{\underline{S_h = 1610, \; P_h = 1610\,W, \; Q_h = 0}}$

e) $I_{tot} = 6 \cdot e^{-j18.19} + 4 \cdot e^{-j11.47} + 7$

$$S_{tot} = V \cdot \overline{I_{tot}} = 230(6e^{j18.19} + 4 \cdot e^{j11.47} + 7)$$

$$\Rightarrow P_{tot} = 230(6 \cdot \cos(18.19) + 4 \cdot \cos(11.47) + 7)$$

$$= \underline{\underline{3822.66\ W}} = P_{vm} + P_{tv} + P_h$$

$$Q_{tot} = 230(6 \cdot \sin(18.19) + 4 \cdot \sin(11.47))$$

$$= \underline{\underline{613.73\ VAr}} = Q_{vm} + Q_{tv} + Q_{pn}$$