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TFY4115
Øving 1

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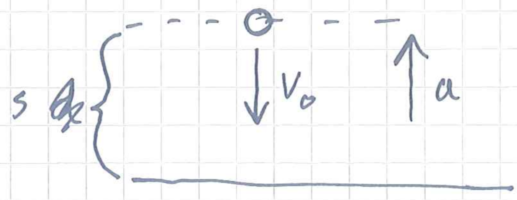
Oppgave 1

$a = -500 \, m/s^2$

$V_0 = 40$



$2as = V^2 - V_0^2 = -V_0^2$

$s = \frac{-V_0^2}{2a} = \frac{40^2}{2 \cdot 500} = \cancel{3.2\,m} = \underline{\underline{1.6\,m}}$

$V = V_0 + at$

$0 = 40 - 500t \Rightarrow t = \frac{40}{500} = \underline{\underline{0.08\,s}}$

Svar: 1.6 m i snøfonnen på 0.08 sekunder.

Oppgave 2

a) $a = -bv^2$

$v'(t) = -b \cdot (v(t))^2$

$\Rightarrow v(t) = \cancel{-b \int_0^t (v(t))^2 \, dt} + V_0 = \underline{\underline{V_0 \cdot e^{-bt}}}$

b) $b = 4.0 \, m^{-1}$, $v_0 = 1.5 \, m/s$

$v(t_1) = v(t_0)/2$

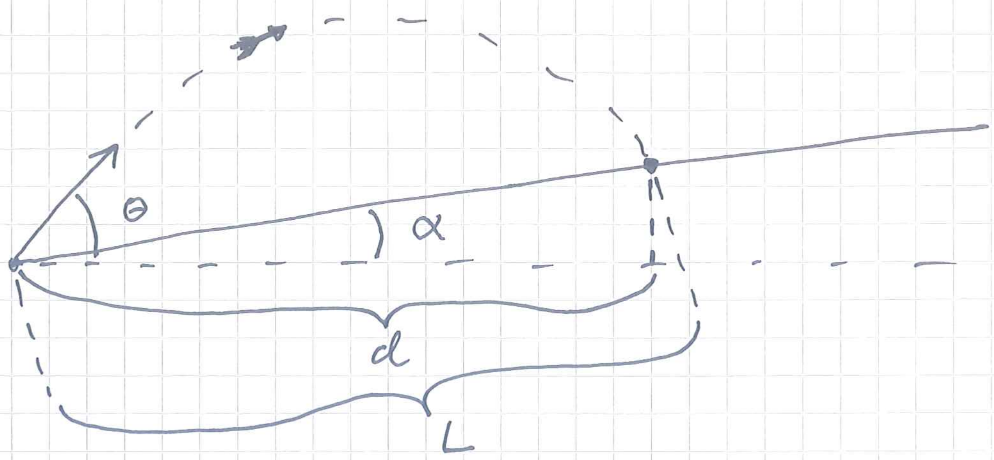
$\Rightarrow e^{-4t_1} = e^{-0}/2 = \frac{1}{2}$

$e^{4t_1} = 2 \Rightarrow 4t_1 = \ln(2)$

$\underline{\underline{t_1 = 0.693/4 = 0.1732}}$

Oppgave 3

a)



b)

$$s(t) = \left(V_{0x} \cdot t \, , \; V_{0y} \cdot t - \tfrac{1}{2} g t^2 \right) \quad / \text{pilens posisjon}$$

$$V_{0x} = V_0 \cdot \cos(\theta) \qquad L_x = d = L \cdot \cos(\alpha)$$

$$V_{0y} = V_0 \cdot \sin(\theta) \qquad L_y = L \cdot \sin(\alpha)$$

Pilen treffer når: $\vec{s}(t) = (L_x, L_y) = s(t_b)$

$$\Rightarrow V_0 \cdot \cos(\theta) \cdot t_b = L \cdot \cos(\alpha) \qquad (I)$$

$$V_0 \cdot \sin(\theta) \cdot t_b - \tfrac{1}{2} g t_b^2 = L \cdot \sin(\alpha) \qquad (II)$$

$$\cancel{t_b = \frac{L \cdot \cos(\alpha)}{V_0 \cdot \cos(\theta)}} \qquad (I)$$

$$\cancel{t_b = \frac{-V_0 \cdot \sin(\theta) \pm \sqrt{V_0^2 \sin^2(\theta) - \frac{L \cdot \sin(\alpha) \cdot g}{2}}}{-g}} \qquad (II)$$

$$\cancel{\Rightarrow t_b = \frac{L \cdot \cos(\alpha)}{V_0 \cdot \cos(\theta)}}$$

$$(I) \quad L = \frac{V_0 \cdot \cos(\theta) \cdot t_b}{\cos(\alpha)}$$

$$(I)+(II) \quad V_0 \cdot \sin(\theta) - \tfrac{1}{2} g t_b = V_0 \cdot \cos(\theta) \cdot \tan(\alpha)$$

$$\Rightarrow t_b = 2V_0 \left(-\cos(\theta)\tan(\alpha) + \sin(\theta) \right) / g$$

$$L = \frac{V_0 \cdot \cos(\theta) \cdot t_b}{\cos(\alpha)}$$

$$= \frac{V_0 \cdot \cos(\theta)}{\cos(\alpha)} \cdot \frac{2V_0(\sin(\theta) - \cos(\theta)\tan(\alpha))}{g}$$

$$= \frac{2V_0^2 \cos^2(\theta)}{g \cdot \cos(\alpha)} \cdot \left(\frac{\sin(\theta)}{\cos(\theta)} - \tan(\alpha)\right)$$

$$= \frac{2V_0^2 \cdot \cos^2(\theta)}{g \cdot \cos(\alpha)} (\tan(\theta) - \tan(\alpha))$$

c) Størst L når $L'(\theta) = 0$

$$L'(\theta) = \frac{2V_0^2}{g \cdot \cos(\alpha)} \left(\cos^2(\theta)(\tan(\theta) - \tan(\alpha))\right)'$$

$$\cos^2(\theta)(\tan(\theta) - \tan(\alpha))' = \tan(\alpha) \cdot \sin(2\theta) + \cos(2\theta)$$

$$L'(\theta) = 0 \Rightarrow \tan(\alpha)\sin(2\theta) = -\cos(2\theta)$$

$$\Rightarrow \tan(2\theta) = -\frac{1}{\tan(\alpha)} = \tan(\alpha + \pi/2)$$

$$\theta = \tan^{-1}(\tan(\alpha + \pi/2))/2$$

$$\Rightarrow \theta = \tan^{-1}\left(\tan\left(\alpha + \frac{\pi}{2}\right)\right)/2 = \frac{\alpha}{2} + \frac{\pi}{4}$$

Sjekker for $\alpha = 0$

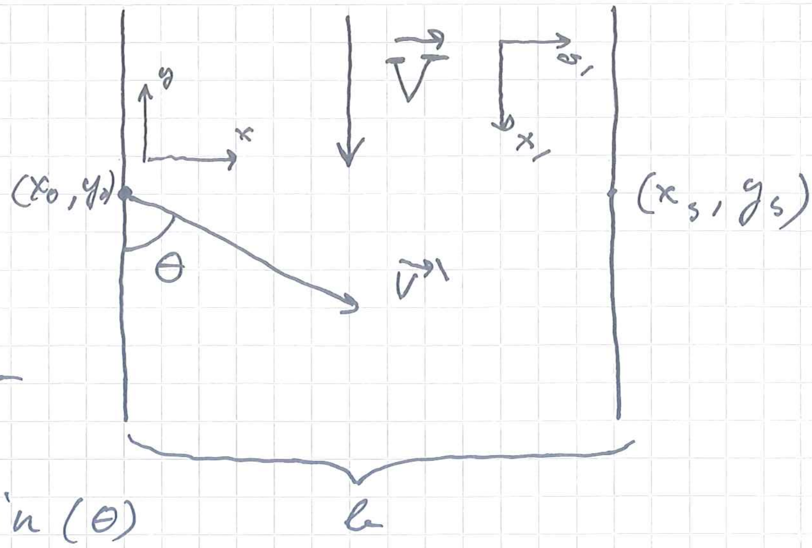
$$\Rightarrow \theta = \tan^{-1}(\infty)/2 = \frac{\pi}{4} = 45^\circ$$

Oppgave 4

a)

$V'_x = V' \cdot \cos(\theta)$

$V'_y = V' \cdot \sin(\theta)$



$\Rightarrow V_x = V'_y = V' \cdot \sin(\theta)$

$V_y = -1 \cdot (V'_x - V) = V - V' \cos(\theta)$

V is the velocity of water, on the figure going in negative y-direction. Therefore, if the boat has positive V'_x, ~~and~~, we know that the boat is moving in negative y-direction relative to land.

Ex: $V'_x = 20\,m/s$, $V = -10\,m/s \Rightarrow V_y = -30\,m/s$

$\Rightarrow$ makes sense!

b) ~~For å krysse elva~~: To cross the river:

$V_x \cdot t_1 = b = x_s$

$V' \cdot \sin(\theta) \cdot t_1 = b$ (crosses at t_1)

then:

$V_y \cdot t_1 = V_y \cdot t_2$

On the left side we define the y-coordinate at which the boat will land.

On the right, we say that the same y-distance will be covered over time t_2 with the ~~same~~ land velocity V_y.

$$\Rightarrow \text{(I)} \quad v' \cdot \sin(\theta) \cdot t_1 = b$$

$$\text{(II)} \quad (\bar{V} - v'\cos(\theta)) \cdot t_1 = -v_g \cdot t_2$$

$$t_1 = \frac{b}{v' \cdot \sin(\theta)} \quad \text{(I)}$$

$$\text{(I)}+\text{(II)} \quad (\bar{V} - v'\cos(\theta)) \cdot \frac{b}{-v' \cdot \sin(\theta) \cdot v_g} = t_2$$

$$\Rightarrow T(\theta) = t_1 + t_2$$

$$= \underline{\underline{\frac{b(\bar{V} - v' \cdot \cos(\theta) \mp v_g)}{-v' \cdot \sin(\theta) \cdot v_g}}}$$

c) $T'(\theta) = 0$

$$\Rightarrow R'(\theta) = 0, \quad R = \frac{\bar{V} - v' \cdot \cos(\theta) \mp v_g}{\sin(\theta)}$$

$$R'(\theta) = \frac{v' \cdot \sin^2(\theta) - \overset{\cos}{\cancel{\sin}}(\theta)(\bar{V} - v' \cdot \cos(\theta) \mp v_g)}{\sin^2(\theta)}$$

$$= \frac{v' - \cos(\theta)(\bar{V} \mp v_g)}{\sin^2(\theta)} = 0$$

$$\Rightarrow \cos(\theta)(\bar{V} + v_g) = v'$$

$$\underline{\underline{\theta_{min} = \cos^{-1}\left(\frac{v'}{\bar{V} \pm v_g}\right)}}$$: the angle that gives minimal time.

$b = 150\ m$

$|v'| = 3{,}0\ km/h$

$|\bar{V}| = -2{,}00\ km/h$ (our coordinate system is chosen such that $\bar{V}$ is in ~~oppos~~ negative y-direction)

$v_g = 45{,}0\ km/h$

$\Theta = \cos^{-1}\left(\frac{3}{-7}\right) = 115.37^\circ = 115^\circ$

(we cant be certain about two decimals since the data is not defined with the same presicion)

Ans: 115°

d) $\Theta_{min} = \cos^{-1}\left(\frac{v'}{\bar{V} - v_g}\right)$

If we define $\bar{V} = 0$, $\Rightarrow \Theta_{min} = 126^\circ$, which makes no sense since the logical answer is 90°.

However

$$\cos(\Theta) = \frac{v'}{\bar{V} - v_g}$$ is assuming

that $v_{~~x~~y} = \bar{V} - v' \cdot \cos(\Theta)$!

if $\bar{V} = 0$, $\Rightarrow v_g = -v' \cdot \cos(\Theta)$ and v_g (land speed) $= 0$, since without water stream, no walking will be needed.

$\Rightarrow$ ~~$v_x \cdot$~~ $v_x \cdot t = b$

$$v' \cdot \sin(\Theta) \cdot t = b$$

$$\Rightarrow t(\Theta) = \frac{b}{v' \cdot \sin(\Theta)}$$

$$t'(\Theta) = -\frac{b}{v'} \cdot \frac{1}{\sin^2(\Theta)} \cdot \cos(\Theta) = -\frac{b}{v} \cdot \frac{\cos(\Theta)}{\sin^2(\Theta)}$$

$$t'(\Theta) = 0 \Rightarrow 0 = \frac{\cos(\Theta)}{\sin^2(\Theta)}$$

$$0 = \cos(\Theta) \Rightarrow \underline{\Theta = 90^\circ}$$

So: The formula from c) is not absolutely valid since it leads to division by zero when $\bar{V} = 0 \Rightarrow v_g = 0$.

Simply put, while deriving the formula in c), we assume that $V \neq 0$!

P.S. Beklager for ~~a~~ skifting til engelsk midt i! Gikk på automatgir.