TTK4240 - Høst 2016
Øving 4
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a) a) $V_o = V_i - V_C$

$$\Rightarrow V_o = V_i - \frac{1}{sC} I(s)$$

$$V_o = I(s) \cdot R \Rightarrow V_i = I(s)\left(R + \frac{1}{sC}\right)$$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{R}{R + \frac{1}{sC}} = \frac{s}{s + \frac{1}{CR}}$$

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{CR}}$$

a) b) $V_o = V_i - V_R$

$$V_o = V_i - I(s) \cdot R$$

$$V_o = \frac{1}{sC} I(s) \Rightarrow V_i = \left(\frac{1}{sC} + R\right) \cdot I(s)$$

$$\Rightarrow H(s) = \frac{V_o}{V_i} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} = \frac{1/RC}{s + \frac{1}{RC}}$$

$$H(j\omega) = \frac{1/RC}{j\omega + 1/RC}$$

a) c) $V_o = V_i - V_L$

$$V_o = V_i - L \cdot s \cdot I(s), \quad V_o = I(s) \cdot R$$

$$\Rightarrow V_i = I(s)(R + L \cdot s)$$

$$\Rightarrow H(s) = \frac{R}{R + L \cdot s} = \frac{R/L}{R/L + s}$$

$$H(j\omega) = \frac{R/L}{R/L + j\omega}$$

(a) d) $V_o = V_i - V_R$

$V_o = V_i - I(s) \cdot R$

$V_o = L \cdot s \cdot I(s) \Rightarrow V_i = I(s)(R + Ls)$

$$\Rightarrow H(s) = \frac{Ls}{R + Ls} = \frac{s}{s + R/L}$$

$$H(j\omega) = \frac{j\omega}{j\omega + R/L}$$

(b) a) $|H(j\omega_c)| = \frac{1}{\sqrt{2}} H_{max}$

$$\Rightarrow |H(j\omega_c)| = \frac{1}{\sqrt{2}}$$

$$H(j\omega) = \frac{j\omega}{j\omega + \frac{1}{CR}} = \frac{\omega^2 + j\frac{\omega}{CR}}{\omega^2 + (\frac{1}{CR})^2}$$

$$|H(j\omega)| = \frac{\sqrt{\omega^4 + \frac{\omega^2}{(CR)^2}}}{\omega^2 + (\frac{1}{CR})^2} = \frac{\omega}{\sqrt{\omega^2 + (\frac{1}{CR})^2}}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{\omega_c}{\sqrt{\omega_c^2 + (\frac{1}{CR})^2}} \Rightarrow \omega_c^2 + \frac{1}{(CR)^2} = 2\omega_c^2$$

$$\Rightarrow \omega_c = \frac{1}{RC}$$

(b) b) $$H(j\omega) = \frac{1/RC}{j\omega + \frac{1}{RC}} = \frac{\frac{1}{(RC)^2} - j\frac{\omega}{RC}}{\omega^2 + (\frac{1}{RC})^2}$$

$$\Rightarrow |H(j\omega)| = \frac{\sqrt{\frac{1}{(RC)^4} + \frac{\omega^2}{(RC)^2}}}{\omega^2 + (\frac{1}{RC})^2} = \frac{1/RC}{\sqrt{\omega^2 + (\frac{1}{RC})^2}}$$

$$\Rightarrow \frac{1/RC}{\sqrt{\omega_c^2 + (1/RC)^2}} = \frac{1}{\sqrt{2}} \Rightarrow 2/(RC)^2 = \omega_c^2 + (\frac{1}{RC})^2$$

$$\Rightarrow \omega_c = \frac{1}{RC}$$

(b) c) $H(jw) = \frac{R/L}{R/L + jw} = \frac{(\frac{R}{L})^2 - j\frac{wR}{L}}{(\frac{R}{L})^2 + w^2}$

$\Rightarrow |H(jw)| = \frac{\sqrt{(\frac{R}{L})^4 + (\frac{wR}{L})^2}}{(\frac{R}{L})^2 + w^2} = \frac{R/L}{\sqrt{(\frac{R}{L})^2 + w^2}}$

$\Rightarrow \frac{1}{\sqrt{2}} = \frac{R/L}{\sqrt{(\frac{R}{L})^2 + w_c^2}} \Rightarrow \underline{\underline{w_c = \frac{R}{L}}}$

(b) d) Same calculation as previously yields $\underline{\underline{w_c = R/L}}$

(c) a) <u>High pass</u>

b) <u>Low pass</u>

c) <u>Low pass</u>

d) <u>High pass</u>

(2)

a) ~~w_c~~ $f_c = 500\ Hz$

$W_c = 500 \cdot 2\pi = \underline{\underline{1000\pi}}$

$\underline{W_c = \frac{1}{RC}}$, ~~$V_o = V_i$~~ $V_{o\,max} = 10\ V$

$V_{w_c} = \frac{1}{\sqrt{2}} \cdot V_{o\,max}$

$P = \frac{V^2}{R} \Rightarrow 2 = \frac{V_{wc}^2}{R} \Rightarrow 2 = \frac{1}{2} \cdot \frac{100}{R} \Rightarrow \underline{\underline{R = 25\ \Omega}}$

$\Rightarrow C = \frac{1}{w_c R} = \underline{\underline{12.73\ \mu F}}$

$\tau = RC = \underline{\underline{0.32 ms}}$, $\underline{\underline{\text{Bandwidth} = [500 Hz, \infty\rangle}}$

3 a) KCL: $\frac{V_o - V_n}{R_2} + i_L = 0$

Ideal op-Amp $\Rightarrow V_n - V_p = 0$, $V_p = 0 \Rightarrow V_n = 0$

$$\Rightarrow \frac{V_o(s)}{R_2} + I(s) = 0$$

$$V_i - V_n = i \cdot R_1 + \frac{1}{C} \cdot \int_{t_0}^{t} i(\tau)\,d\tau$$

$$\Rightarrow V_i = I(s)\left(R_1 + \frac{1}{sC}\right)$$

$$\Rightarrow \frac{V_o(s)}{R_2} + \frac{V_i(s)}{R_1 + \frac{1}{sC}} = 0$$

$$\Rightarrow \frac{V_o(s)}{V_i(s)} = -R_2 \cdot \frac{1}{R_1 + \frac{1}{Cs}} = -R_2 \cdot \frac{s}{R_1\left(s + \frac{1}{CR_1}\right)}$$

$R_1 = R_2 = 10\ k\Omega$, $C = 100\ \mu F$

$$\Rightarrow \underline{\underline{H(s) = -\frac{s}{s+1}}}$$

$H(j\omega) = -\frac{j\omega}{j\omega + 1}$ (This is a H.P. inverting filter)

Since $\frac{R_1}{R_2} = 1$, amplification factor $= 1$ and we can treat it as a passive filter where $\frac{1}{RC} = 1 \Rightarrow$

$W_c = \frac{1}{RC} = 1 \Rightarrow f_c = \frac{1}{2\pi}\ Hz = \underline{\underline{0.16\ Hz}}$

$\underline{\underline{\text{Bandwidth: } [0.16\ Hz, \infty\rangle}}$

4 a) $V_p = 0 \Rightarrow V_n = 0$

KCL: $\frac{V_i - V_n}{R_1} + i_L = 0$

$\Rightarrow \frac{V_i(s)}{R_1} + I_L(s) = 0$

$i_L = \frac{V_o - V_n}{R_2} + i_c = \frac{V_o}{R_2} + C \cdot V_c'(t)$

Since R_2 and C are in parallel: $V_c = V_o$

$\Rightarrow i_L = \frac{V_o}{R_2} + C \cdot V_o'(t)$

$\Rightarrow I_L(s) = \frac{V_o(s)}{R_2} + Cs \cdot V_o(s) = V_o(s)\left(\frac{1}{R_2} + Cs\right)$

$\Rightarrow \frac{V_i(s)}{R_1} + V_o(s)\left(\frac{1}{R_2} + Cs\right) = 0$

$$\Rightarrow \frac{V_o}{V_i} = -\frac{1}{R_1} \cdot \frac{1}{\left(\frac{1}{R_2} + Cs\right)} = -\frac{R_2}{R_1} \cdot \frac{1}{1 + CsR_2}$$

b) $H(s) = -\frac{R_2}{R_1} \cdot \frac{1}{1 + CsR_2} = -\frac{1}{R_1 \cdot C} \cdot \frac{1}{\frac{1}{R_2C} + s}$

$\Rightarrow H(j\omega) = -\frac{1}{R_1 \cdot C} \cdot \frac{1}{\frac{1}{R_2C} + j\omega}$ (Low pass)

$\Rightarrow H_{max} = \frac{R_2}{R_1}$ $(|H(j\omega_0)|,\ \omega_0 = 0)$

$\Rightarrow \frac{R_2}{R_1} \cdot \frac{1}{\sqrt{2}} = |H(j\omega_c)| = \frac{1}{\sqrt{\left(\frac{1}{R_2C}\right)^2 + \omega_c^2}} \cdot \frac{1}{R_1C}$

$$\Rightarrow \omega_c = \frac{1}{R_2C}$$

c) $C = 100\mu F$, $R_2 = 10 k\Omega$, $R_1 = \{1, 10, 100\}\ k\Omega$

The corners are all at $w = \frac{1}{R_2 C}$, which is consistent with result from b).

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a) $H_{hp}(s)$ is identical ~~with~~ to that found in 3a)

$$\Rightarrow H_{hp}(s) = -\frac{R_1}{R_1} \cdot \frac{s}{s + \frac{1}{C_1 R_1}} = -\frac{s}{s + W_{hp}}$$

where $W_{hp} = \frac{1}{C_1 R_1}$

b) $H_{lp}(s)$ is identical to that found in 4a)

$$\Rightarrow H_{lp}(s) = -\frac{R_2}{R_2} \cdot \frac{1}{1 + s \cdot R_2 C_2} = \frac{1/R_2C_2}{1/R_2C_2 + s}$$

$$= -\frac{W_{lp}}{W_{lp} + s}, \quad \text{where } W_{lp} = \frac{1}{R_2 C_2}$$

c) $$H_{bp}(s) = H_{hp}(s) \cdot H_{lp}(s) = \frac{V_m}{V_i} \cdot \frac{V_o}{V_m} = \frac{V_o}{V_i}$$

$$= -\frac{W_{lp}}{W_{lp} + s} \cdot -\frac{s}{s + W_{hp}} = \frac{W_{lp} \cdot s}{(W_{lp} + s)(W_{hp} + s)}$$

d) $R_1 = R_2 = 1000\,\Omega$

$\omega_{hp} = 500\ rad/s\ , \quad \omega_{lp} = 1000\ rad/s$

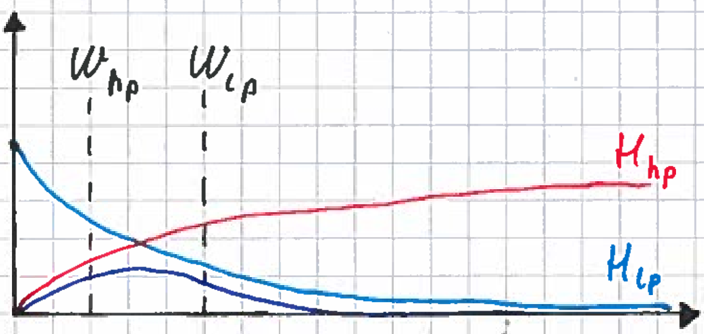
$\cancel{H_{hp\,max} = 1 \Rightarrow \frac{1}{2} =}$

$$\omega_{hp} = \frac{1}{C_1 R_1}$$

$$\Rightarrow C_1 = \frac{1}{\omega_{hp} \cdot R_1} = \underline{\underline{2\,\mu F}}$$

$$C_2 = \frac{1}{\omega_{lp} \cdot R_2} = \underline{\underline{1\,\mu F}}$$

e)



High pass passes frequencies <u>higher</u> than ω_{hp}, low pass passes frequencies <u>lower</u> than ω_{lp}. Combined, only frequencies ~~wit~~ between ω_{hp} & ω_{lp} are passed. So, outside the $[\omega_{hp}, \omega_{lp}]$ band, the power transfer to the output is reduced by more than 50 %.

Ergo: a band pass filter.